**The effect of low-energy electrons on the response
of ion chambers to ionizing photon beams**

by

Daniel J. La Russa

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

Department of Physics
Carleton University
Ottawa-Carleton Institute of Physics
Ottawa, Canada
September 23, 2009

Copyright © 2009 Daniel J. La Russa

Library and Archives
Canada

Published Heritage
Branch

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque et
Archives Canada

Direction du
Patrimoine de l'édition

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file *Votre référence*
ISBN: 978-0-494-60110-5
Our file *Notre référence*
ISBN: 978-0-494-60110-5

NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

Canada

Abstract

Cavity ionization chambers are one of the most popular and widely used devices for quantifying ionizing photon beams. This popularity originates from the precision of these devices and the relative ease with which ionization measurements are converted to quantities of interest in therapeutic radiology or radiation protection, collectively referred to as radiation dosimetry. The formalisms used for these conversions, known as cavity theory, make several assumptions about the electron spectrum in the low-energy range resulting from the incident photon beam. These electrons often account for a significant fraction of the ion chamber response. An inadequate treatment of low-energy electrons can therefore significantly effect calculated quantities of interest.

This thesis sets out to investigate the effect of low-energy electrons on 1) the use of Spencer-Attix cavity theory with ^{60}Co beams; and 2) the standard temperature-pressure correction factor, P_{TP} , used to relate the measured ionization to a set of reference temperature and pressure conditions for vented ion chambers. Problems with the P_{TP} correction are shown to arise when used with kilovoltage x rays, where ionization measurements are due primarily to electrons that do not have enough energy to cross the cavity. A combination of measurements and Monte Carlo calculations using the EGSnrc Monte Carlo code demonstrate the breakdown of P_{TP} in these situations when used with non-air-equivalent chambers. The extent of the breakdown is shown to depend on cavity size, energy of the incident photons, and the composition of the chamber. In the worst case, the standard P_{TP} factor overcorrects the response of an aluminum chamber by $\approx 12\%$ at an air density typical of Mexico City. The response of a more common graphite-walled chamber with similar dimensions at the same air density is undercorrected by $\approx 2\%$.

The EGSnrc Monte Carlo code is also used to investigate Spencer-Attix cavity theory as it is used in the formalism to determine the air kerma for a ^{60}Co beam. Following a comparison with measurements in the literature, the air kerma formalism is shown to require a fluence correction factor, K_{fl} , to ensure the accuracy of the formalism regardless of chamber composition and cavity size. The need for such a correction stems from the fact that the cavity clearly distorts the fluence for mismatched cavity and wall materials, and the inability to select the appropriate “cut-off” energy, Δ , in the Spencer-Attix stopping-power ratio. A discussion of this issue is followed by detailed calculations of K_{fl} values for several of the graphite ionization chambers used at national metrology institutes, which range between 0.9999 and 0.9994 with a one standard deviation uncertainty of ± 0.0002 .

Acknowledgments

Whenever I reflect upon my experiences as a graduate student, I quickly lose count of the number of people to whom I owe a great deal of thanks. It has been a pleasure getting to know the people I have worked with over the past few years, and I will never forget how each of them helped me reach this stage. I could not have accomplished this work without their help.

Completing this work would not have been possible without the unending patience and guidance of my supervisor, David Rogers. Not many people were interested in recruiting chemistry students into physics programs, and so I am particularly grateful to him for giving me the opportunity to join his group in the first place. Dave is a supervisor that every graduate student hopes for. He was always available to discuss my research, and to guide me through the problems one inevitably encounters. I doubt there is anyone I could have learned more from.

I would also like to thank the many individuals that I worked closely with during the course of my studies at Carleton, including Elsayed Ali, Randy Taylor, Lesley Buckley, Zdenko Sego, Lilie Wang, Palani Selvam, Bryan Muir, and Rowan Thomson. Our many conversation and debates were memorable, and made my experience at Carleton that much more enjoyable. I wish to thank you all for carefully proof-reading my manuscripts.

Thank you also to Malcolm McEwen, Hong Shen, and several others at the National Research Council of Canada for your assistance with my experiments. Without their help, I would still be working on them. Additional suggestions for my manuscripts and from Malcolm, Carl Ross, and Iwan Kawrakow also proved to be invaluable.

I owe a great deal of thanks to Pat Saull for providing me with detailed data

on air temperature and pressure variations in the Ottawa region, Brian Hooten for providing me with ion chamber schematics, J. E. Burns for drawing our attention to early experimental work on chamber response variations with air pressure, Steve Davis for correcting issues with the reference air density, Vincent Clancy for performing XRF measurements on chamber thimbles, and Doug Cormack for providing us with additional details on his early experiments.

To my friends and family, thank you for your patience while I got through this. I would especially like to thank Bryan Lawrence, Kris Stella, JP Archambault, Matt Hagman, and my father-in law, Bill, for talking me through the tough times, and for just talking to me.

Finally, to my wife and best friend, Natasha, without whom I would never have come this far. Your support over the years went beyond patience and understanding. It required sacrifice and a belief in me that you never questioned. For that I am truly grateful. You are the most honest, caring, and intelligent person I know. Thank you for always being there for me.

This work has been supported in part by the Natural Sciences and Engineering Research Council of Canada, the John Lyndhurst Kingston Memorial Scholarship, and institutional scholarships from the Department of Physics and Carleton University. This research was also enabled through the use of the WestGrid computing resources.

Statement of originality

Discussed herein are the results of the author's research conducted at both Carleton University and at the National Research Council of Canada during the course of the Ph.D. program. Most of these results have been published in the journal *Medical Physics*, and have been presented at both national and international conferences. The details of where the results of these publications may be found in the thesis, along with the author's contributions, are provided in the following list:

- I. D. J. La Russa and D. W. O. Rogers. An EGSnrc investigation of the P_{TP} correction factor for ion chambers in kilovoltage x-rays. *Medical Physics*, **33** (12), 4590 - 4599, 2006.

Winner of the 2007 Farrington Daniels award for the best paper on radiation dosimetry published in *Medical Physics* in 2006

- The results of this paper constitute a large portion of chapter 4 which discusses the breakdown of the P_{TP} correction factor for ion chambers in low-energy x-ray beams.
- The author performed all of the calculations, prepared the manuscript for publication, and made the necessary revisions following the review process. The results from this paper were presented by DJL at the 2006 AAPM annual meeting in Orlando, Florida (2006), and by DWOR at the Absorbed Dose and Air Kerma Dosimetry Workshop in Paris, France (2007).

- II. D. J. La Russa, M. McEwen and D. W. O. Rogers. An experimental and computational investigation of the standard temperature-pressure correction factor for ion chambers in kilovoltage x rays. *Medical Physics*, **34** (12), 4690 - 4699, 2007.

- The results of this paper appear in chapter 5 when discussing experimental verification of the breakdown of the P_{TP} correction factor for ion chambers in low-energy x rays.
- The author assisted in the design of the experiment and recorded and analyzed all experimental measurements. The author performed all the calculations and prepared the manuscript for publication, which included all necessary edits. The results in this paper were presented by DJL at the 2007 CARO-COMP Joint Annual Scientific Meeting in Toronto (2007), and DWOR at the Absorbed Dose and Air Kerma Dosimetry Workshop in Paris, France (2007).

- III. D. J. La Russa and D. W. O. Rogers. Accuracy of EGSnrc calculations at ^{60}Co energies for the response of ion chambers configured with various wall materials and cavity dimensions. *Medical Physics*, **35** (12), 5629 - 5640, 2008.
- Parts of this paper appear in chapter 2 when discussing Fano cavity tests at ^{60}Co energies, and the bulk of the results are presented in chapter 6 when comparing EGSnrc simulations with measurements of chamber response to ^{60}Co beams from the literature.
 - The author performed all the calculations and prepared the manuscript for publication, which included all necessary edits. Results from this investigation were presented by DJL at the Canadian Organization of Medical Physicists (COMP) Annual Scientific meeting in Quebec City (2008).
- IV. D. J. La Russa and D. W. O. Rogers. An EGSnrc investigation of the Spencer-Attix stopping-power ratio and calculations of fluence correction factors for ^{60}Co air kerma standards. *Medical Physics*, **36** (9), 4173 - 4183, 2009.
- The results of this paper are presented in chapter 7 when discussing the limitation of Spencer-Attix cavity theory and the calculation of fluence correction factors for the ^{60}Co air kerma formalism.
 - The author performed all the calculations and prepared the manuscript for publication, which included all necessary edits.
- V. L. L. W. Wang, D. J. La Russa and D. W. O. Rogers. Systematic uncertainties in the Monte Carlo calculation of ion chamber replacement correction factors. *Medical Physics*, **36** (5), 1785 - 1789, 2009.
- The results of this paper are briefly discussed in chapter 7 when discussing the application of techniques for computing the replacement correction factor (P_{repl}) to calculations of fluence correction factors in the air kerma formalism.
 - The author was not the principle contributor to this work, but was rather the first to identify the uncertainty associated with calculating values of P_{repl} using one of the techniques.

Contents

1	Introduction	1
1.1	General introduction	1
1.2	Examples of cavity ionization chambers	3
1.3	The standard temperature-pressure correction factor, P_{TP}	5
1.3.1	Rationale for using P_{TP}	6
1.4	Cavity theory	7
1.4.1	Bragg-Gray cavity theory	9
1.4.2	Spencer-Attix cavity theory	11
1.4.3	Using cavity theory to determine air kerma for ^{60}Co beams	13
1.5	Outline of the thesis	16
2	The EGSnrc Monte Carlo code	17
2.1	Introduction	17
2.2	EGSnrc user-codes	18
2.2.1	Calculations with photon regeneration	21
2.2.2	Calculations of chamber response	22
2.2.3	EGSnrc cross-sections, stopping-powers and transport parameters	22
2.3	Statistical uncertainties on calculated quantities	23
2.4	EGSnrc Fano cavity tests for lead chambers in a ^{60}Co beam	24
2.5	Effect of AE and ECUT in EGSnrc calculations of ion chamber response	26
3	The air kerma formalism for ^{60}Co beams	29
3.1	Formal definition of correction factors	30

3.2	Fluence correction factors for the air kerma formalism	33
3.3	On the direction conversion of chamber response to air kerma	37
3.4	Summary	38
4	An EGSnrc analysis of the P_{TP} correction factor for low-energy x rays	39
4.1	Use of P_{TP} with low-energy x rays	40
4.2	Methods	40
4.2.1	Ion chambers and calculations	40
4.2.2	X-ray spectra	44
4.3	Breakdown of P_{TP}	44
4.4	Contribution of electron ranges to the breakdown of P_{TP}	48
4.5	Effect of the chamber wall material	51
4.6	Effect of photon interactions in the cavity	53
4.7	Summary	53
5	Experimental verification for the breakdown of P_{TP}	55
5.1	Methods	56
5.1.1	Ion chambers	56
5.1.2	X-ray spectra and EGSnrc simulations of the x-ray source . .	56
5.1.3	Experimental measurements	60
5.2	Measured breakdown of P_{TP} and the effect of impurities	64
5.3	Effect of geometry on the calculated response to low-energy x rays . .	70
5.4	Summary	74
6	EGSnrc calculations of chamber response to ^{60}Co beams	76
6.1	Early experimental investigations of ion chamber response and cavity theory	77
6.2	Experimental data	78
6.2.1	Experimental data of Nilsson <i>et al</i>	80
6.2.2	Experimental data of Whyte	81
6.2.3	Experimental data of Attix <i>et al</i>	82

6.2.4	Experimental data of Cormack and Johns	83
6.2.5	Experimental data of Burns <i>et al</i>	84
6.2.6	EGSnrc calculations of chamber response	85
6.2.7	Calculated K_{wall} corrections	86
6.3	EGSnrc comparisons with experiments at ^{60}Co energies	87
6.3.1	Discrepancies between measurements and calculations for the Attix <i>et al</i> and Cormack and Johns chambers	97
6.4	Summary	98
7	Accuracy of the Spencer-Attix stopping-power ratio and fluence cor- rection factors for the ^{60}Co air kerma formalism	100
7.1	Theory	101
7.2	Methods	103
7.2.1	EGSnrc calculations of chamber response	103
7.2.2	Calculation of Spencer-Attix stopping-power ratios	105
7.2.3	Calculation of fluence correction factors (K_{fl})	108
7.3	Limitations of Spencer-Attix cavity theory	108
7.4	Perturbation of charged particle fluence by the cavity	113
7.5	Fraction of chamber response due to charged particles below Δ	115
7.6	Calculations of $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$	117
7.7	Fluence correction factors (K_{fl}) for SA cavity theory and the air kerma formalism	117
7.8	Determining Δ from values of $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$	120
7.9	On calculating K_{fl} directly	124
7.10	K_{fl} values for graphite-walled chambers	124
7.10.1	Uncertainties on K_{fl} for graphite chambers	124
7.11	Summary	131
8	Conclusions	133
	References	138

List of Tables

2.1	List of EGSnrc user-codes used	20
3.1	Correction factors for the air kerma formalism	34
4.1	Mean chord lengths of selected ion chambers	43
4.2	Characteristics of several x ray spectra	49
5.1	Physical characteristics of several thimble ion chambers	58
5.2	Half-value layers and effective energies of several x-ray spectra	59
5.3	Sources of uncertainty on measurements of chamber response per unit response of the monitor chamber	63
7.1	Values of the mean chord length, the low-energy cut-off (Δ), and the wall-to-air Spencer-Attix stopping-power ratio for a list of ion chamber cavity sizes	107
7.2	Fit coefficients for a least-squares fit of a function to stopping-power ratio data	107
7.3	EGSnrc-calculated values of the fluence correction factor for ^{60}Co pri- mary standard ionization chambers used at selected national metrology institutes	125
7.4	Sources of uncertainty on EGSnrc-calculated values of the fluence cor- rection factor	131

List of Figures

1.1	Two types of ion chambers shown in cross section	4
1.2	Example of a thimble ion chamber shown in cross section	5
1.3	The Spencer-Attix stopping-power ratio $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{air}}^{\text{wall}}$ as a function of the low-energy cut-off, Δ	14
2.1	EGSnrc-calculations of chamber response as a function of the cut-off energy of charged particles created and tracked	28
3.1	Schematic of correction factors for the air kerma formalism	35
4.1	Schematic diagrams of ion chambers as modeled with EGSnrc for the investigation of P_{TP}	41
4.2	Schematic diagrams of two mammography chambers as modeled with EGSnrc	42
4.3	EGSnrc calculations of the P_{TP} -corrected response as a function of air density for two thimble ionization chambers	46
	(a) NE 2571 chamber	46
	(b) Exradin A12 chamber	46
4.4	Calculations of the P_{TP} -corrected response for two spherical chambers	47
	(a) graphite walls	47
	(b) C-552 air-equivalent plastic walls	47
4.5	Calculations of P_{TP} -calculated response for two mammography chambers	48
4.6	Spectrum of electrons emerging from slabs of beryllium, graphite and air-equivalent plastic for four incident x-ray spectra	50
4.7	Photon cross-sections per unit mass relative to air for beryllium, graphite, aluminum, and air-equivalent plastic	52

4.8	The calculated fraction of response due photon interactions in the cavity as a function of the average spectrum energy	54
5.1	EGSnrc models of three thimble ion chambers shown in 3D and in cross-section	57
	(a) 3D view	57
	(b) 2D cross-section view	57
5.2	Diagram of the experimental set-up used in the investigation of P_{TP} .	61
5.3	P_{TP} -corrected responses per unit mass as a function of air density for the NE-type thimble ionization chambers	65
	(a) 60 kV	65
	(b) 150 kV	65
5.4	As in figure 5.3 for three modified Exradin A2 chambers	66
	(a) 60 kV	66
	(b) 150 kV	66
5.5	P_{TP} -corrected response of an Exradin A12 chamber for 60 and 100 kV beams	68
5.6	Calculations of N_K as a function of spectrum effective energy for an Exradin A12 chamber	69
5.7	As in figure 5.6 for two NE-type chambers	71
5.8	Calculated responses of the NE 2571 as a function of pressure for two different EGSnrc models	73
6.1	Schematic diagrams of five EGSnrc models of ion chambers used to investigate chamber response to ^{60}Co beams	79
6.2	Relative chamber response to ^{60}Co as a function of cavity size for three different wall materials	88
6.3	Relative chamber response to ^{60}Co as function of the atomic number of the backscatter material for three different build-up materials . . .	90
6.4	Relative chamber response to ^{60}Co as a function of cavity air pressure	91
6.5	Relative response to ^{60}Co as a function of cavity size	93
	(a) with a guard wall (spacer rings)	93
	(b) no guard wall	93

6.6	Relative response to ^{60}Co as a function of cavity size	95
(a)	with a guard wall	95
(b)	no guard wall	95
6.7	Relative response to ^{60}Co as a function of cavity size	96
7.1	Comparison of direct calculations of chamber response with the predictions of Spencer-Attix cavity theory	110
(a)	Whyte chamber	110
(b)	Cormack and Johns chamber	110
7.2	Comparison of direct calculations of chamber response with the predictions of Spencer-Attix cavity theory corrected for non-ideal conditions	111
(a)	Whyte chamber	111
(b)	Cormack and Johns chamber	111
7.3	Comparisons of calculated chamber response and Spencer-Attix cavity theory with correction factors for a copper chamber	112
7.4	Fluence of charged particles in the cavity of an ion chamber due to an unattenuated, unscattered ^{60}Co beam	114
7.5	Fraction of dose to the cavity of an ion chamber due to charged particles with an energy $\leq \Delta$	116
7.6	Calculations of $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ as a function of cavity size	118
7.7	Calculations of K_{fl} as a function of cavity size	119
7.8	The extracted low-energy cut-off, Δ' , as a function of cavity size . . .	122
7.9	Δ' as a function of the atomic number of the wall	123
7.10	Variation in calculated K_{fl} values with density and mean ionization energy of the graphite wall	127
7.11	Calculated K_{fl} values for a graphite chamber as a function of the mean energy of the incident ^{60}Co beam	129

Chapter 1

Introduction

1.1 General introduction

Since the discovery of x rays in 1895, several methods have been established to quantify ionizing radiation fields under a standard set of conditions. In most cases, the goal is to relate the response of a detector to the dose (energy deposited per unit mass) delivered by the radiation field of interest, or some other related quantity. Hence, the task of quantifying ionizing radiation fields has come to be known as dosimetry.

The techniques for quantifying radiation beams include the use of calorimetry, film, luminescent materials, semiconductors, gels and, most prevalently for reference dosimetry, ionization chambers.[?] An ionization chamber, or ion chamber, is a device that consists of a gas filled cavity enclosed between electrically insulated electrodes. When placed in a ionizing photon field, photon interactions in the chamber, which occur predominantly in the chamber wall, set charged particles in motion which then cross the gas-filled cavity. A potential difference applied between the conducting electrodes collects the ionization produced along the path of these charge particles, which

is proportional to the energy deposited by the incident photon field. In the majority of chambers on the market, the cavity is vented to the surrounding atmosphere, and so the density of the air in the cavity is subject to daily fluctuations in temperature and pressure. In order for the reading of vented ion chambers to be independent of these fluctuations, the charge measurement is corrected to yield the reading expected under reference temperature and pressure conditions.

In many cases, the use of ion chambers is preferred over the alternatives listed above due to the precision of these devices, and since the readout of measurements is instantaneous.^{?,1,2} Furthermore, if the characteristics of an ion chamber, such as the cavity volume and necessary correction factors, are sufficiently well known, then it may be used as a primary standard for air kerma. Air kerma will be defined later in section 1.4.3 along with various other dosimetric quantities of interest. Ion chambers made with graphite walls are used as standards for the determination of air kerma for ^{60}Co beams.

The charge or current measured with an ion chamber is related to a quantity of interest, such as dose or air kerma, using cavity theory. More specifically, the premise of cavity theory is to relate the dose to the gas in the cavity of an ion chamber to the dose to the medium surrounding the cavity at the point of measurement (i.e. at the position of the cavity *without* the cavity present). Underlying the use of cavity theory for ion chambers in photon beams are several assumptions about low-energy electrons produced by photons that interact in the chamber. This is discussed later in section 1.4.

Prior to the widespread use and availability of computers, cavity theory was the only means of relating the readout of a dosimeter to a dosimetric quantity of interest. As will be shown, the use of these theories is often limited to specific situations

1.1. GENERAL INTRODUCTION

imposed by various underlying assumptions. Put another way, the assumptions in cavity theory limit the accuracy of its predictions. Inevitably, one is interested in determining dose in situations where cavity theory does not apply, or where a reliable analytical expression cannot be obtained. In these situations, Monte Carlo methods can be used in place of cavity theory to predict chamber response or to calculate any specific conversion factors. The Monte Carlo technique can also be used to test the accuracy and assumptions of cavity theory, particularly as it applies to ion chambers in ionizing photon beams.

1.2 Examples of cavity ionization chambers

Examples of cavity ionization chamber types are shown in figure 1.1 and 1.2. The chamber on the left of figure 1.1 is an example of what is often referred to as a thimble chamber. Thimble chambers consist of an gas filled cavity enclosed within a conducting cylindrical outer wall, called a thimble. Also enclosed in the thimble is a central electrode (collector) that may or may not be made of the same material. The collector is separated from the thimble by a high-voltage (HV) insulator to support the chamber structurally and to prevent leakage currents when a potential is applied to the chamber.⁷ Typical insulating materials include Teflon, polyethylene, or some other form of hydrocarbon-based polymer. Thimble chambers are oriented with their axis of symmetry perpendicular to the incident beam axis.

The chamber on the right in figure 1.1 is referred to as a parallel-plate chamber, or plane-parallel chamber (abbreviated as pp chamber). Typical pp chambers consist of cylindrically symmetric disk-shaped front and back walls with an isolated collecting electrode as part of the back wall. The front wall faces the source with the axis of symmetry of the chamber parallel to the beam axis. For the pp chamber shown in

figure 1.1, the front and back walls are at the same potential and are separated by an isolated central disk-shaped collector. A more common pp chamber configuration without a central collector can be found in a book chapter on ionization chamber instrumentation by DeWerd *et al.*²

Figure 1.1: Schematic diagrams of two different types of ion chambers. The chamber on the left is an example of a thimble ion chamber modeled after the NRC-3C. On the right is an example of a plane-parallel chamber modeled after the BIPM air kerma standard. In each case, the cavity is represented by the enclosed region in white.

Figure 1.2 shows a cross section of a typical thimble ionization chamber configuration used for reference dosimetry, and shows the guard electrode present in the majority of chambers. During use, the guard electrode has the same potential as the collector, and is included to further reduce leakage currents and to electrically shield the collector current through the connecting triaxial cable. The guard also provides

1.2. EXAMPLES OF CAVITY IONIZATION CHAMBERS

electric field uniformity in the sensitive volume of the chamber.^{?,2} The chamber shown in figure 1.2, with the cone shaped top, resembles the design popularized by Baldwin and Farmer³ and, hence, is referred to as a Baldwin-Farmer type chamber.

Figure 1.2: Schematic diagram of a typical cylindrically symmetric thimble ionization chamber. The cavity is represented by the white region enclosed by the thimble.

Thimble chambers are also manufactured with smaller or larger thimbles, hemispherical or flat tops as opposed to cone-shaped tops, and with a variety of chamber wall and electrode thicknesses. Other ion chamber designs include spherically symmetric chambers, often with a cylindrical central collector, and well ion chambers (a.k.a. re-entrant chambers) used to measure brachytherapy sources (seeds of radioactive material). Comprehensive reviews of the different chamber types may be found elsewhere.^{?,1,2}

1.3 The standard temperature-pressure correction factor, P_{TP}

The majority of chambers in use today have cavities that are vented to the surrounding atmosphere. When using an ion chamber with a cavity open to the atmosphere it

1.3. THE STANDARD TEMPERATURE-PRESSURE CORRECTION FACTOR, P_{TP}

is common practice to correct the measured chamber response to the response under reference atmospheric conditions using the standard temperature-pressure correction factor,⁴⁻⁶ P_{TP} , given by

$$P_{TP} = \left(\frac{273.15 + T/^{\circ}C}{273.15 + T_0/^{\circ}C} \right) \left(\frac{P_0}{P} \right), \quad (1.1)$$

where T_0 and P_0 are the reference temperature and pressure, respectively. In North America, $T_0 = 22^{\circ}C$ (295.15 K) and $P_0 = 101.325$ kPa ($\equiv 1$ atm, 760 mmHg), corresponding to a reference density of dry air, $\rho_0 = 1.196$ kg/m³.⁷ Assuming ideal gas law conditions, P_{TP} is formulated such that

$$\rho_0 = \rho P_{TP}, \quad (1.2)$$

where ρ is the air density associated with the temperature and pressure in the cavity of the chamber at the time of measurement.

1.3.1 Rationale for using P_{TP}

The charge or current collected by an ion chamber is the result of ionization produced by electrons as they travel through the cavity. Assuming that the energy deposited per unit charge (of one sign) released, $(W/e)_{\text{air}}$, is independent of beam quality, this ionization is directly proportional to the energy deposited by the electrons, E_{dep} , given as

$$E_{\text{dep}} = \rho t \left(\frac{\overline{dE}}{\rho dt} \right), \quad (1.3)$$

where t is the track-length of an electron in the cavity, and $(\overline{dE}/\rho dt)$ is the mass stopping-power of the air appropriately averaged over the energy of the electron as it slows down traversing t . The mass stopping-power is virtually independent of the

1.3. THE STANDARD TEMPERATURE-PRESSURE CORRECTION FACTOR, P_{TP}

density of the air since the effect of density on collisional energy loss is completely negligible at the energies of interest.⁸ For electrons that completely cross the cavity, the average physical track-length is, to a very good approximation, fixed, and the energy deposited is proportional to the density, ρ . From the relation $\rho_0 = \rho P_{TP}$, it follows that $E_{\text{dep}} P_{TP}$ is also independent of ρ for a given beam energy.

1.4 Cavity theory

As was previously mentioned, the goal in cavity theory is to relate the dose to a detector due to an incident radiation field to the dose to the surrounding medium at the point of measurement. To understand the application of cavity theory to photon beams, consider an arbitrary detector placed in a medium where one is interested in determining the dose. The detector should be sufficiently deep within the medium such that the build-up of charged particles due to the incident photon energy fluence, Ψ , has reached a maximum. Fluence is defined as the number of photons hitting a sphere per unit cross sectional area, or equivalently as photon tracklength per unit volume, with units of cm^{-2} , and energy fluence is defined as the fluence multiplied by the energy of the particle. When the above conditions are satisfied, the medium surrounding the cavity is said to provide “full build-up.”

If attenuation of the photon fluence in the medium is ignored, then beyond the depth of full build-up, the number of charged particles of a given energy entering a region of interest is exactly balanced by charged particles leaving that region. This condition is known as charged particle equilibrium (CPE).¹ Under CPE conditions, the dose to the medium due to a photon beam is given by

$$D_{\text{med}} \stackrel{\text{CPE}}{=} \int_0^{E_{\text{max}}} \Psi_{\text{med}}(E) \left(\frac{\mu_{\text{en}}(E)}{\rho} \right)_{\text{med}} dE, \quad (1.4)$$

where $\left(\frac{\mu_{\text{en}}(E)}{\rho}\right)_{\text{med}}$ is the mass-energy absorption coefficient for medium “med”, at an energy E , $\Psi_{\text{med}}(E)$ is the photon energy fluence differential with respect to energy, and E_{max} is the maximum incident photon energy.

Now consider the dose to the detector, D_{det} , placed in the medium as described above. If the ranges of the charged particles set in motion by the incident photon beam are small relative to the size of the detector, then the largest contribution to D_{det} comes from photon interactions in the detector material, and

$$D_{\text{det}} \stackrel{\text{CPE}}{=} \int_0^{E_{\text{max}}} \Psi_{\text{det}}(E) \left(\frac{\mu_{\text{en}}(E)}{\rho}\right)_{\text{det}} dE. \quad (1.5)$$

Whenever possible, the material of the detector is chosen such that when it is placed in the medium of interest, the perturbation of the photon fluence in that medium due to the presence of that material is kept at a minimum. If the perturbation to the photon fluence is negligible, then one can assume that $\Psi_{\text{det}}(E) = \Psi_{\text{med}}(E)$ for all E . Hence,

$$\frac{D_{\text{med}}}{D_{\text{det}}} \stackrel{\text{CPE}}{=} \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{det}}^{\text{med}}, \quad (1.6)$$

where

$$\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{det}}^{\text{med}} = \frac{\int_0^{E_{\text{max}}} \Psi_{\text{med}}(E) \left(\frac{\mu_{\text{en}}(E)}{\rho}\right)_{\text{med}} dE}{\int_0^{E_{\text{max}}} \Psi_{\text{med}}(E) \left(\frac{\mu_{\text{en}}(E)}{\rho}\right)_{\text{det}} dE}. \quad (1.7)$$

Methods of calculating values of $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{det}}^{\text{med}}$ are well established. Equations 1.6 and 1.7 are therefore useful for determining D_{med} from measures of D_{det} provided the conditions discussed above are satisfied.

1.4. CAVITY THEORY

1.4.1 Bragg-Gray cavity theory

It should be emphasized that equation 1.6 applies only when the range of charged particles is small relative to the size of the detector. This is not the case with cavity ionization chambers. The cavity radius in thimble ionization chamber for instance ranges between 4 to 6 mm, and so charged particles with energies larger than about 20 keV can completely cross the cavity. This is significant if one considers that for graphite in a ^{60}Co beam, charged particles with energies ≥ 20 keV account for about 98 % of the steady-state electron fluence.

Recognizing that dose is attributed to collisional energy losses of charged particles that were set in motion by incident photons, D_{med} may be written as

$$D_{\text{med}} \stackrel{\delta\text{-CPE}}{=} \int_0^{E_{\text{max}}} \Phi_{\text{med}}^{\text{BG}}(E) \left(\frac{S(E)}{\rho} \right)_{\text{med}} dE, \quad (1.8)$$

where $\Phi_{\text{med}}^{\text{BG}}(E)$ is the differential fluence of primary charged particles with energy E , $\left(\frac{S(E)}{\rho} \right)_{\text{med}}$ is the corresponding *unrestricted* mass collisional stopping-power, and E_{max} in this case is the maximum energy of the charged particle fluence. The superscript “BG” stands for Bragg-Gray and indicates that secondary electrons (a.k.a. knock-on electrons or δ -rays) are not included in the electron fluence spectrum. Omitting secondary electrons from the electron fluence is based on the assumption that all collisional energy losses are deposited locally along the tracks of the primary charged particles. The symbol “ δ -CPE” indicates that there must be charged particle equilibrium of the δ -rays for equation 1.8 to hold since this ensures this assumption is correct. A similar equation may be written for D_{det} for ion chambers filled with air.

In order to relate the dose to the cavity of an ion chamber to D_{med} , Bragg and Gray established the following two conditions:

1.4. CAVITY THEORY

1. The cavity must be small enough such that it negligibly perturbs the fluence of the primary charged particles that exists in the medium in the absence of the cavity.
2. The dose to the cavity must be due entirely to charged particles crossing it. In other words, photon interactions in the cavity must make a negligible contribution to the dose.

The second condition is merely a corollary to the first since both are satisfied if the cavity is small enough, and if the energy of the incident photon beam is sufficiently high such that the majority of electrons depositing dose in the cavity are from the chamber wall. Ma and Nahum⁹ have shown that photon interactions in the cavity of a graphite thimble chamber (6 mm radius) account for less than 1 % of the total for photon energies above 500 keV. Thus, for example, the above two conditions can be reasonably satisfied by using a chamber with a 6 mm cavity radius in a ⁶⁰Co beam (mean photon energy = 1.25 MeV), which was traditionally used in cancer radiotherapy. In this case, one may assume that $\Phi_{\text{air}}(E) = \Phi_{\text{med}}(E)$ for all E , and therefore

$$\frac{D_{\text{med}}}{D_{\text{det}}} = \left(\frac{\bar{S}}{\rho} \right)_{\text{det}}^{\text{med}}, \quad (1.9)$$

where

$$\left(\frac{\bar{S}}{\rho} \right)_{\text{det}}^{\text{med}} = \frac{\int_0^{E_{\text{max}}} \Phi_{\text{med}}(E) \left(\frac{S(E)}{\rho} \right)_{\text{med}} dE}{\int_0^{E_{\text{max}}} \Phi_{\text{med}}(E) \left(\frac{S(E)}{\rho} \right)_{\text{det}} dE}. \quad (1.10)$$

$\left(\frac{\bar{S}}{\rho} \right)_{\text{det}}^{\text{med}}$ in equation 1.10 is the Bragg-Gray ratio of the spectrum-averaged unrestricted mass collisional stopping-powers, loosely referred to as the Bragg-Gray stopping-power ratio.

1.4. CAVITY THEORY

1.4.2 Spencer-Attix cavity theory

It was previously mentioned that the cavity of an ion chamber must be small enough to fulfill the two Bragg-Gray conditions discussed above, and over the years ion chambers have been designed accordingly. However, small cavity sizes make it difficult to ensure CPE of the δ -rays since, by definition, the energy of δ -rays extends to $E_{\max}/2$. In gas-filled cavities, a large number of electron secondaries travel a significant distance from the track of the primary electron and many also have sufficient range to escape the cavity.¹⁰ To resolve this, Spencer and Attix¹¹ took account of “long-range” secondary electrons by explicitly including them in the differential electron fluence, denoted here as $\Phi_{\text{med}}^{\text{SA}}(E)$. Using this formulation, D_{med} is given as

$$D_{\text{med}} = \int_{\Delta}^{E_{\max}} \Phi_{\text{med}}^{\text{SA}}(E) \left(\frac{L_{\Delta}(E)}{\rho} \right)_{\text{med}} dE + \Delta \Phi_{\text{med}}^{\text{SA}}(\Delta) \left(\frac{S(\Delta)}{\rho} \right)_{\text{med}} \quad (1.11)$$

where $\left(\frac{L_{\Delta}(E)}{\rho} \right)_{\text{med}}$ is the *restricted* mass collisional stopping-power at an energy E , and which accounts only for collisional energy losses restricted to values below a cut-off energy, Δ . Thus, values of $\left(\frac{L_{\Delta}(E)}{\rho} \right)_{\text{med}}$ are smaller than values of $\left(\frac{S(E)}{\rho} \right)_{\text{med}}$ for all energies above 2Δ . When E is $\leq 2\Delta$, the restricted and unrestricted stopping-powers are the same. An analogous expression may be written for D_{det} .

The second term in equation 1.11 is called the track-end term which represents the dose deposited by charged particles that have slowed-down to a kinetic energy at or below Δ . Track-end terms were not explicitly included in the original formulation since the energy dissipation of electrons with $E \leq \Delta$ was accounted for via a unique evaluation of the restricted stopping-powers for $\Delta \leq E \leq 2\Delta$.¹² The form of the track-end terms required when conventional restricted stopping-powers are used has been rigorously derived by Nyström and Nahum.¹³

At the time Spencer-Attix cavity theory was formulated, the only available means of calculating $\Phi_{\text{med}}^{\text{SA}}(E)$ was to use the approach of Spencer and Fano,¹⁴ which imposed CPE as an additional constraint and applied only to photon beams. As such, CPE is often erroneously assumed to be required when using Spencer-Attix cavity theory. In fact, as long as $\Phi_{\text{med}}^{\text{SA}}(E)$ can accurately determined, SA cavity theory can be applied. Monte Carlo methods can now be used to calculate $\Phi_{\text{med}}^{\text{SA}}(E)$ to include the effects of attenuation, scatter, and disequilibrium, especially in electron beams.

Using the new expressions for D_{med} and D_{det} given by equation 1.11, equation 1.9 may be replaced with

$$\frac{D_{\text{med}}}{D_{\text{det}}} = \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{det}}^{\text{med}}, \quad (1.12)$$

where

$$\left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{det}}^{\text{med}} = \frac{\int_{\Delta}^{E_{\text{max}}} \Phi_{\text{med}}^{\text{SA}}(E) \left(\frac{L_{\Delta}(E)}{\rho} \right)_{\text{med}} dE + \Phi_{\text{med}}^{\text{SA}}(\Delta) \left(\frac{S(\Delta)}{\rho} \right)_{\text{med}} \Delta}{\int_{\Delta}^{E_{\text{max}}} \Phi_{\text{med}}^{\text{SA}}(E) \left(\frac{L_{\Delta}(E)}{\rho} \right)_{\text{det}} dE + \Phi_{\text{med}}^{\text{SA}}(\Delta) \left(\frac{S(\Delta)}{\rho} \right)_{\text{det}} \Delta}. \quad (1.13)$$

$\left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{det}}^{\text{med}}$ is analogous to $\left(\frac{\bar{S}}{\rho} \right)_{\text{det}}^{\text{med}}$ in equation 1.10, and is therefore called the Spencer-Attix (SA) stopping-power ratio.

The purpose of introducing the cut-off energy, Δ , in equations 1.12 and 1.13 is to explicitly account for secondary electrons with ranges that are large relative to the size of the cavity. As such, Δ is meant to represent the energy of charged particles that have insufficient range to leave the cavity, and one is therefore left with determining what that cut-off energy should be. The most commonly used method of determining Δ for primary standards of air kerma is to equate it to the energy of an electron that has a range in air equal to the mean chord length of the cavity,¹ L , given by $4V/S$,

1.4. CAVITY THEORY

where V is the cavity volume and S is the surface area. However, this is recognized as an approximation and often Δ is assigned a value of 10 keV for a typical ion chamber used for clinical dosimetry. For an air-filled ion chamber in a ^{60}Co beam, the SA graphite-to-air stopping-power ratio, $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{air}}^{\text{gr}}$, needed for graphite-walled chambers is relatively insensitive to the value of Δ above about 10 keV. Figure 1.3 shows values of $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{air}}^{\text{wall}}$ as a function of Δ for graphite, aluminum, copper, and lead wall materials in a ^{60}Co beam (attenuation and scatter removed). The calculation of these values using the EGSnrc Monte Carlo code is discussed in chapters 2 and 7. For graphite, $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{air}}^{\text{gr}}$ increases as Δ is increased since the restricted collisional stopping-powers for graphite increase relative to air as energy decreases. The opposite is the case for the other three wall materials. In general, the mass-restricted collisional stopping-power decreases as the atomic number of the material increases.

1.4.3 Using cavity theory to determine air kerma for ^{60}Co beams

One of the original applications of Bragg-Gray and Spencer-Attix cavity theories was to determine the air kerma (K_{air}) for high-energy photon sources such as ^{60}Co and ^{137}Cs . The word “kerma” is a (pseudo)-acronym for **k**inetic **e**nergy **r**elaxed **p**er unit **m**ass. Thus “air kerma” refers to the kinetic energy released per unit mass in air. Air kerma formed the basis of previous clinical dosimetry protocols,¹⁵ and air kerma standards for ^{60}Co beams are maintained in the various national metrology institutes throughout the world.

Kerma is a dosimetric quantity that applies to photon beams. The kerma in

Figure 1.3: Values of the Spencer-Attix wall-to-air ratio of restricted mass-collision stopping-powers, $\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{air}}^{\text{wall}}$, as a function of the low-energy cutoff, Δ . The points represent values calculated using the SPRRZnrc user-code of EGSnrc, and the solid line represents a least-squares fit to this data of an analytical function (see chapter 7).

any given medium is defined as

$$K_{\text{med}} = \Psi \left(\frac{\bar{\mu}_{\text{tr}}}{\rho} \right)_{\text{med}}, \quad (1.14)$$

where $\left(\frac{\bar{\mu}_{\text{tr}}}{\rho} \right)_{\text{med}}$ is the spectrum averaged mass-energy transfer coefficient, and Ψ is the photon energy fluence as defined in section 1.4. Kerma is distinguished from collision kerma, (K_c) , which is defined for a given medium as

$$(K_c)_{\text{med}} = \Psi \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{med}}. \quad (1.15)$$

As can be seen in equations 1.4 and 1.5, collision kerma is equal to dose under conditions of charged particle equilibrium. Kerma is related to collision kerma by the fraction of energy lost to radiative processes, $\bar{g}_{\text{med}}$:

$$(K_c)_{\text{med}} = K_{\text{med}}(1 - \bar{g}_{\text{med}}) \quad (1.16)$$

Ignoring non-ideal conditions, such as attenuation and scatter of the photon beam, non-parallel incident fluence, and inhomogeneous construction of an ion chamber, the kerma in air (i.e. the “air kerma”) is related to the dose to the air in an ion chamber, D_{air} , by

$$K_{\text{air}} = D_{\text{air}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}^{\text{air}} \frac{1}{(1 - \bar{g}_{\text{air}})}. \quad (1.17)$$

In practice, equation 1.17 must be corrected for various non-ideal conditions using correction factors. These correction factors, combined with equation 1.17, form the air kerma formalism used by the national metrology institutes to determine air kerma for ^{60}Co beams. This formalism is discussed in detail in chapter 3.

1.4. CAVITY THEORY

1.5 Outline of the thesis

The chapters that follow discuss the use of the Monte Carlo methods to investigate the effect of low-energy electrons on both the P_{TP} correction factor and the use of the Spencer-Attix cavity theory for the determination of air kerma. Chapter 2 introduces the EGSnrc Monte Carlo code used throughout this thesis, and describes the methods of calculating stopping-power ratios, mass-energy absorption coefficients, and ion chamber response to ionizing photon beams. In chapter 3, the determination of air kerma using Spencer-Attix cavity theory is described and all necessary correction factors are formally defined. The Monte Carlo techniques used to calculate the various parameters in the air kerma formalism are discussed later in chapter 7.

Chapter 4 begins with a review of the underlying mechanism by which the P_{TP} correction factor breaks down due to low-energy electrons. EGSnrc calculations of response are then used to demonstrate this breakdown for several ion chambers and incident x-ray beam energies. Experimental evidence supporting these findings is presented in chapter 5, as are additional EGSnrc simulations that corroborate the use of this code for calculating the relative response of chambers at low x-ray energies.

In chapter 6, measurements of chamber response to ^{60}Co beams as a function of cavity size and chamber wall material are used to test the accuracy of EGSnrc calculations in this energy range. The comparison with these experiments test both the cross-sections and transport mechanics of the code. Calculations of the same type are then used in chapter 7 to investigate the accuracy of Spencer-Attix cavity theory, and to calculate fluence correction factors for the air kerma formalism. Finally, a summary and concluding remarks for this work are presented in chapter 8.

Chapter 2

The EGSnrc Monte Carlo code

2.1 Introduction

In the vast majority of situations where one is interested in determining dosimetric quantities of interest due to a source of ionizing photons or electrons, the equations describing the transport through a given geometry prohibit the ability to obtain an analytical solution. In such cases, the Monte Carlo technique is the only practical method that can be applied in any energy range to determine quantities of interest for complex geometrical configurations and material inhomogeneities. One of the most popular Monte Carlo codes used in the field radiation dosimetry is the Fortran/Mortran-based EGSnrc Monte Carlo code,¹⁶ aptly named for its use in the simulation of an electron-gamma shower. EGSnrc was adapted from a predecessor code called EGS4,¹⁷ and includes a large number of substantial improvements to the cross-sections and transport algorithms.^{16,18,19} A summary of these improvements may be found in the EGSnrc reference manual.¹⁶

As implied by the name, EGSnrc is restricted to the simulation of photon and

electron transport (including positrons) and excludes the transport of other charged or uncharged particles. Photon interactions simulated within EGSnrc include coherent (Rayleigh) scattering with atoms (or molecules), photo-electric absorption, incoherent (Compton) scattering, and pair/triplet production. These interaction types are discussed in detail by Attix,¹ Khan,²⁰ and Johns and Cunningham.²¹ The production of characteristic x rays resulting from photon and electron interactions that produce atomic excitations are also simulated. For electrons and positrons, transport is simulated using the “condensed history” technique^{22,23} originally described by Berger,²² where the cumulative effect of a large number of interactions associated with the energy loss process are condensed into steps of a size and direction randomly sampled from the appropriate multiple scattering distributions. Radiative energy losses and catastrophic collisional energy loss interactions are accounted for by including bremsstrahlung photons created above a cut-off energy, AP, and secondary “knock-on” electrons (δ -rays) created above a cut-off energy, AE, determined by the set of cross-sections and restricted stopping-powers used in the simulation.

2.2 EGSnrc user-codes

In order to perform a calculation with EGSnrc, a user-code must be written to interface with those parts of the code that handle particle transport. For convenience, EGSnrc is distributed with a set of user-codes,^{24–26} each developed for specific calculations. The CAVRZnrc, CAVSPHnrc and cavity user-codes were developed to calculate the dose in the cavity of an ion chamber. CAVRZnrc and CAVSPHnrc are restricted to geometries that are cylindrically and spherically symmetric, respectively, whereas the cavity user-code, based on the EGSnrc C++ class library, can accommodate more complex geometries. Despite the added flexibility of the cavity user-code,

the process by which geometries are specified by the user is more complex than with the former user-codes. Since a large number of ion chambers are either spherically or cylindrically symmetric, the CAVRZnrc or CAVSPHnrc user-codes are adequate in most cases.

Other user-codes are distributed with EGSnrc to calculate dose distributions in cylindrical (DOSRZnrc) and rectilinear geometries (DOZXYZnrc), Spencer-Attix stopping-power ratios (SPRRZnrc), particle fluence distributions and spectra (FLURZnrc), and values of mass-energy absorption coefficients $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)$ and mass-energy transfer coefficients $\left(\frac{\bar{\mu}_{\text{tr}}}{\rho}\right)$ (“g” user-code). A large user-code capable of accurately and efficiently modeling linear accelerators, ^{60}Co units, and kilovoltage x-ray tubes, called BEAMnrc, is also available. More recent user-codes that are not as of yet distributed with EGSnrc include BrachyDose,^{27,28} which calculates dose distributions and parameters²⁹ specific to brachytherapy, and CSnrc³⁰ which calculates the ratio of responses to ion chambers with highly correlated geometries to a high degree of statistical precision. The user-codes used in this thesis are listed in table 2.1.

The calculation of fluence correction factors discussed later in chapter 7 requires calculations of the Spencer-Attix stopping-power ratio given by equation 1.13. These values are calculated with the SPRRZnrc user-code by simulating the transport of the electron shower in a homogeneous medium “med” to obtain the fluence $\Phi_{\text{med}}(E)$ differential with respect to energy. The contribution to the numerator of equation 1.13 is calculated by summing the energy deposited (E_{dep}) in each condensed history step of a charged particle. The corresponding contribution to the integral term in the denominator of equation 1.13 is calculated by multiplying the E_{dep} by the appropriate ratio of mass stopping-powers. For energies above Δ , the

2.2. EGSNRC USER-CODES

Table 2.1: EGSnrc user-codes used in this thesis.

User-code	Function
CAVRZnrc	Calculates the dose to the cavity of an ion chamber per unit incident fluence. Chamber models are restricted to cylindrical symmetry.
CAVSPHnrc	Same as CAVRZnrc for spherically symmetric geometries.
cavity	A C++ user-code that calculates the dose to the cavity per incident particle. There are very few restrictions on the geometry descriptions and source shape.
SPRRZnrc	Used to calculate Spencer-Attix ratio of mass restricted stopping-power ratios for any given two materials.
FLURZnrc	Calculates the fluence of photons or charged particles (electrons, positrons, or both) in any cylindrically symmetric geometry. It also has the ability to distinguish between primary and secondary particles.
g	Used to calculate values of $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)$ and $\left(\frac{\bar{\mu}_{\text{tr}}}{\rho}\right)$ for a given photon source. Also calculates $\bar{g}$, the fraction of kinetic energy transferred which is lost to radiative processes.
BEAMnrc	Used in this thesis to calculate the photon spectrum from a kilovoltage x-ray tube.

contribution to $\int_{\Delta}^{E_{\max}} \Phi_{\text{med}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{det}}$ is given by

$$E_{\text{dep}} \frac{\left(\frac{L_{\Delta}(E_{\text{mid}})}{\rho} \right)_{\text{det}}}{\left(\frac{L_{\Delta}(E_{\text{mid}})}{\rho} \right)_{\text{med}}}, \quad (2.1)$$

where the restricted mass stopping-powers correspond to the electron energy at the mid-point of the step, E_{mid} . For charged particles that fall below an energy Δ after a step, the contribution to the denominator from the portion of the energy below Δ is calculated as

$$E_{\text{dep}} \frac{\left(\frac{S(\Delta)}{\rho} \right)_{\text{det}}}{\left(\frac{S(\Delta)}{\rho} \right)_{\text{med}}}, \quad (2.2)$$

where $\left(\frac{S(\Delta)}{\rho} \right)_{\text{med}}$ and $\left(\frac{S(\Delta)}{\rho} \right)_{\text{det}}$ are the *unrestricted* mass collisional stopping-powers at $E = \Delta$, which equal the restricted stopping-powers, $\left(\frac{L_{\Delta}(\Delta)}{\rho} \right)_{\text{med}}$ and $\left(\frac{L_{\Delta}(\Delta)}{\rho} \right)_{\text{det}}$, respectively.

2.2.1 Calculations with photon regeneration

The CAVRZnrc and SPRRZnrc user-codes discussed in the previous section along with a modified version of the FLURZnrc code have the added ability to run with a feature known as photon regeneration. When photon regeneration is selected, the energy, position and direction of each incident photon is stored immediately prior to an interaction. After the interaction, the original properties of the primary photon are restored and all scattered photons are discarded, including all photons from relaxation processes, bremsstrahlung x rays, and photons from positron annihilation. This has the effect of removing attenuation and scatter from the simulation and creating regions of true charged particle equilibrium at and beyond the depth of full build-up. Thus, the energy deposited by charged particles under these simulation conditions are solely

from primary photon interactions. The use of these simulation conditions are required for calculating stopping-power ratios for the air kerma formalism, and for calculating the dose to the cavity of an ion chamber in the absence of attenuation and scatter.

2.2.2 Calculations of chamber response

The discussions that follow will often refer to the “response” of ion chambers, regardless of whether it is measured or calculated with EGSnrc. Calculations of response in this work refers to the charge per unit mass of gas in the cavity, per incident fluence. However, rather than being calculated directly, values of chamber response are extracted from calculations with either the CAVRZnrc or cavity user-codes, which calculate dose as discussed in section 2.2. For a cavity filled with air, the dose, D_{air} , is given as

$$D_{\text{air}} = \frac{Q}{m_{\text{air}}} \left(\frac{W}{e} \right)_{\text{air}}, \quad (2.3)$$

where m_{air} is the mass of air and $\left(\frac{W}{e} \right)_{\text{air}}$ is the mean energy lost in dry air per unit charge of one sign released. To avoid confusion, the reader is advised to remember that D_{air} is directly proportional to response, and the two terms may be used interchangeably when referring to relative dose calculations.

2.2.3 EGSnrc cross-sections, stopping-powers and transport parameters

Unless otherwise stated, the transport parameters in all EGSnrc calculations were selected to account for atomic relaxations, spin effects, and the binding energies of electrons in Compton scattering. All calculations made use of XCOM photon cross sections^{31–33} and restricted stopping-powers which include the density effect correc-

tions from ICRU Report 37.⁸ The ICRU density effect corrections are considered to be more accurate than those calculated by Berger and Seltzer.^{34,35} For simulations involving graphite, the correction for the density effect in the stopping-power data was calculated using a mean excitation energy (I value) of 78 eV. Buckley *et al*³⁶ showed that changing the I value from 78 eV to 87 eV increased the calculated response of a graphite chamber by 1.2 %. However it was unclear from that investigation which I value is more appropriate. A more recent report by Wang and Rogers³⁷ suggest that a higher I value for graphite might be appropriate. In this investigation, the ICRU-recommended I value of 78 eV is used in all calculations with graphite unless otherwise stated.

2.3 Statistical uncertainties on calculated quantities

The error bars shown on all the EGSnrc calculations that follow represent the one standard deviation statistical uncertainty. In EGSnrc, the uncertainty, $s_{\bar{X}}$, on any scored quantity $\bar{X}$ is calculated using the history-by-history approach described by Sempau *et al*³⁸ and Walters *et al*,³⁹ where $s_{\bar{X}}$ is given as

$$s_{\bar{X}} = \sqrt{\frac{1}{N-1} \left(\frac{\sum_{i=1}^N X_i^2}{N} - \left(\frac{\sum_{i=1}^N X_i}{N} \right)^2 \right)}. \quad (2.4)$$

In equation 2.4, N is the total number of independent (i.e. primary) histories. During a simulation, $\sum_{i=1}^N X_i^2$ and $\sum_{i=1}^N X_i$ are updated after each initial history is terminated using the following algorithm for scoring a quantity $\bar{X}$:

2.3. STATISTICAL UNCERTAINTIES ON CALCULATED QUANTITIES

```

IF (nhist=X_last) THEN
    X_tmp=X_tmp+delta
ELSE
    X=X+X_tmp
    X2=X2+(X_tmp*X_tmp)
    X_tmp=delta
    X_last=nhist
ENDIF

```

In the above algorithm, `nhist` is an integer that is increased by one each time a new primary history is simulated. It is therefore used to determine whether or not contributions to `X_tmp` (`delta`) are due to the current history or a new history via a comparison with the variable `X_last`. The variable `X_tmp` represents X_i in equation 2.4, while `X` and `X2` represent $\sum_{i=1}^N X_i$ and $\sum_{i=1}^N X_i^2$, respectively. Once the simulation of N independent histories is complete, the values of $\bar{X}$ and $s_{\bar{X}}$ are evaluated. The uncertainty, $s_{\bar{X}}$, decreases as $1/\sqrt{N}$.

2.4 EGSnrc Fano cavity tests for lead chambers in a ^{60}Co beam

In 1954,⁴⁰ Fano introduced and formally proved a theorem that allows one to determine the fluence of electrons in a cavity of an ion chamber when the walls and cavity gas are matched in atomic composition. His theorem states that for a uniform field of indirectly ionizing photons (i.e. ionizing photon beams) the field of secondary radiation is also uniform and independent of density variations throughout the medium.^{1,40} This theorem ignores the polarization effect and so is limited to photon energies below

about 1 MeV. Nonetheless, such a finding serves as a useful benchmark to test the transport mechanics of Monte Carlo codes. Under Fano conditions, charged particle equilibrium exists, and so the dose to a cavity filled with low-density wall material will be the same as the collision kerma in the wall when the effects of attenuation and scatter are removed and the cavity gas has the same dosimetric properties as the wall. In other words, the Fano cavity test involves verifying that for a chamber with full build-up walls,

$$\frac{D_{\text{cav}}K_{\text{wall}}}{\overline{E}\left(\frac{\overline{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}} = 1, \quad (2.5)$$

where $D_{\text{cav}}K_{\text{wall}}$ is the dose to the cavity filled with wall gas per unit incident photon fluence, corrected for the effects of attenuation and scatter (using the K_{wall} correction factor), and $\overline{E}\left(\frac{\overline{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}$ is the collision kerma of the wall per unit incident fluence.

Past analysis of the transport mechanics using the Fano cavity test for photons from 10 keV to 1.25 MeV have shown that EGSnrc is accurate to within 0.1 % with respect to its own cross sections and geometry descriptions for chambers made with graphite, aluminum and copper.^{19,41,42} Since part of the investigation that follows relies on the transport mechanics within EGSnrc for calculating the response of lead-walled ion chambers (chapter 6), the Fano test was repeated for lead using a lead-walled plane-parallel chamber with a 1 cm radius and a 2 mm cavity height in a ^{60}Co beam. To satisfy the Fano cavity conditions, the differences in the density effect between the stopping-powers for the lead wall (11.35 g/cm³) and the lead gas in the cavity (0.001205 g/cm³) were removed by using the same density effect corrections for each density (corresponding to that of the bulk density). Thus, the cross sections and stopping-powers are the same per g/cm² in both the wall and cavity. The CAVRZnrc user-code²⁵ was used with photon regeneration (see section 2.2) to calculate $D_{\text{cav}}K_{\text{wall}}$ for a realistic ^{60}Co spectrum⁴³ and the g user-code was used to calculate $\overline{E}\left(\frac{\overline{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}$.

2.4. EGSNRC FANO CAVITY TESTS FOR LEAD CHAMBERS IN A ^{60}Co BEAM

Both calculations used the same cross section data sets and thus this test is not sensitive to the uncertainties in the cross sections, only the internal consistency of the transport algorithms.

EGSnrc calculations of $D_{\text{cav}} K_{\text{wall}} / [\overline{E} \left(\frac{\overline{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}]$ for lead walls yielded a value of 1.0013 ± 0.0004 . The internal consistency of the EGSnrc algorithms have thus been verified to within about 0.13 % for lead wall materials at ^{60}Co energies. This is only slightly worse than for lower-Z walls where a 0.1 % consistency was obtained.^{19,41} In section 7.6, this test is repeated for many different cavity sizes and the root mean squared deviation is 0.06 %.

2.5 Effect of AE and ECUT in EGSnrc calculations of ion chamber response

In any EGSnrc calculation, the user must select the cut-off energy (ECUT) below which charged particles are no longer tracked, and also provide a set of stopping-powers *restricted* to collisional energy losses below a threshold value, AE, where $\text{AE} \leq \text{ECUT}$. Using larger values of ECUT reduces CPU time, and it is therefore desirable to know the highest allowable value of ECUT that one can use without loss of accuracy. It has previously been shown that it is sufficient to use $\text{AE} = \text{ECUT} = 10 \text{ keV}$ (actually $521 \text{ keV} = 10 \text{ keV} + 511 \text{ keV rest mass}$) for graphite chambers and ^{60}Co energies.⁴⁴ Mainegra-Hing *et al.*,⁴⁵ however, showed lower values should be used for chambers with small cavities, and it is unclear which value should be used for chambers made with non-air-equivalent, high-Z materials. Figure 2.1 shows some examples of EGSnrc-calculated values of D_{cav} as a function of ECUT for chambers made with graphite and lead, where $\text{AE} = \text{ECUT}$ in all cases. An ECUT value of 1 keV is

the de facto lower limit in EGSnrc, and is used for the most accurate calculations even though the stopping-powers, which are calculated according to the Bethe-Block formalism, are

known to be inaccurate for high-Z materials (since AE should be $\gg$ K-shell binding energies).⁴⁶ In figure 2.1, it can be seen that an ECUT value of 10 keV is suitable for graphite chambers since it yields the same response, within statistical uncertainties, as calculated with ECUT = 1 keV. Above 10 keV, calculated responses begin to diverge for simulations of the chamber used by Attix *et al*⁴⁷ (0.5 mm cavity height). Simulations of a chamber used by Cormack and Johns⁴⁸ (2.6 mm cavity height) begin to diverge above 20 keV, suggesting that smaller cavity heights are more sensitive to ECUT values. The figure for lead chambers shows that the sensitivity to ECUT is also dependent on the geometry and wall material. The response of the Cormack and Johns chamber with lead walls and a cavity height of 0.7 mm calculated with an ECUT of 10 keV was more than 5 % below the response calculated with an ECUT of 1 keV, even though the cavity height is comparable to the graphite Attix *et al* chamber discussed above. Calculations for a chamber used by Nilsson *et al*⁴⁹ with a similar cavity height show that simply replacing the front wall with polystyrene and adding a guard ring dramatically reduces the sensitivity of the calculated response to ECUT.

To avoid artifacts caused by prematurely terminating particle histories, photons and charged particles were tracked down to a kinetic energy of 1 keV (i.e. ECUT = PCUT = 1 keV) in all calculations. Range rejection of charged particles was used as a time-saving option, where the tracking of charged particles was terminated if their range was too short to reach the cavity. It was confirmed that this approximation had a negligible impact on the results.

2.5. EFFECT OF AE AND ECUT IN EGSNRC CALCULATIONS OF ION CHAMBER RESPONSE

Figure 2.1: EGSnrc calculations of chamber response as a function of the cut-off for the lowest kinetic energy of charged particles created (AE) and tracked (ECUT), where $AE = ECUT$ in all cases, for two of the chambers with graphite walls and three chambers with lead walls (details of the chambers discussed in chapter 6). The Nilsson *et al* chamber had a polystyrene front wall rather than a lead wall (see section 6.2.1). The dimension labels (in mm) represent the cavity height of the chamber. Statistical uncertainties on the data for the lead chamber are smaller than the symbols ($\leq 0.15\%$). (from paper III)

2.5. EFFECT OF AE AND ECUT IN EGSNRC CALCULATIONS OF ION CHAMBER RESPONSE

Chapter 3

The air kerma formalism for ^{60}Co beams

The determination of air kerma (defined in section 1.4.3) for ^{60}Co beams is one of the long-standing objectives of radiation standards laboratories. To this end, the formalism used to relate ion chamber response to air kerma continues to be evaluated and refined. Since this formalism is based on SA cavity theory which, as discussed in section 1.4.2, makes several assumptions about the photon field and the surrounding medium, it is necessary to account for the effects of attenuation and scatter by the wall, beam divergence due to the point-like nature of the source, and the inhomogeneous construction of the chamber. These effects are accounted for using a series of correction factors, which can be defined formally and calculated using Monte Carlo methods.

3.1 Formal definition of correction factors

Correction factors in the air kerma formalism can be defined formally using the approach of Bielajew.⁵⁰ We begin by expressing the dose to the air in the cavity of the ion chamber, D_{air} , as

$$D_{\text{air}} = D_{\text{air}}^0 + D_{\text{air}}^S, \quad (3.1)$$

where D_{air}^0 is the dose deposited in the cavity by charged particles set in motion by primary photon interactions and D_{air}^S is the dose from charged particles set in motion by scattered photons, bremsstrahlung, positron annihilation and fluorescent x rays. In this discussion, it is assumed the cavity is filled with *dry* air so that no correction for the effects of humidity is required. Multiplying and dividing the right side of the above equation by D_{air}^0 gives

$$D_{\text{air}} = \frac{D_{\text{air}}^0 + D_{\text{air}}^S}{D_{\text{air}}^0} D_{\text{air}}^0 = A_{\text{scatt}} D_{\text{air}}^0. \quad (3.2)$$

A_{scatt} is the ratio of the total dose to the dose attributed to interactions by primary photons and therefore acts as a correction for D_{air}^0 for the effect of scattered photons. For N primary photons,

$$D_{\text{air}}^0 = \sum_{i=1}^N \int_0^{E_{\text{max}}} dE \Phi_i(E) \left(\frac{\bar{S}(E)}{\rho} \right), \quad (3.3)$$

where $\int_0^{E_{\text{max}}} dE \Phi_i(E) \left(\frac{\bar{S}(E)}{\rho} \right)$ is the dose deposited by the differential electron fluence $\Phi_i(E)$ resulting from the i^{th} photon, and $\left(\frac{\bar{S}(E)}{\rho} \right)$ is the mass stopping-power of the material.

It was explained in section 1.4 that the dose to the cavity is due primarily to electrons set in motion in the wall of the chamber. The number of electrons entering

the cavity increases as the front wall thickness increases until a maximum is reached at a wall thickness $d_{\max}$. This is known as the thickness of full build-up, and is regarded to be the minimum wall thickness required for ion chambers to measure the response of photon beams. As the thickness of the wall increases beyond $d_{\max}$, the fluence of electrons begins to decrease due to attenuation of the primary photon beam. For a monoenergetic photon beam, the primary photon fluence, Ψ_0 , attenuates exponentially such that the fluence of primary photons at a depth x is $\Psi_0 e^{-\lambda x}$, where λ is an attenuation coefficient characteristic of the material and energy of the incident photons. As a consequence, D_{air}^0 decreases as the chamber wall thickness increases beyond $d_{\max}$. Ideally, for measurements of air kerma, ion chamber response should be independent of wall thickness once full build-up is achieved, and so we define D_{air}^1 , the dose to the air due to primary photons which are not attenuated. For N primary photons, D_{air}^1 is given as

$$D_{\text{air}}^1 = \sum_{i=1}^N e^{\lambda_i x} \int_0^{E_{\max}} dE \Phi_i(E) \left(\frac{\bar{S}(E)}{\rho} \right), \quad (3.4)$$

where λ_i is the appropriate linear attenuation coefficient of the i^{th} incident photon interacting at x , and $\Phi_i(E)$, and $\left(\frac{\bar{S}(E)}{\rho} \right)$ are as defined for equation 3.3. Multiplying and dividing the right side of equation 3.2 by D_{air}^1 gives

$$D_{\text{air}} = A_{\text{scatt}} \frac{D_{\text{air}}^0}{D_{\text{air}}^1} D_{\text{air}}^1 = A_{\text{scatt}} A_{\text{att}} D_{\text{air}}^1, \quad (3.5)$$

where A_{att} is a correction for the effects of attenuation. It is common practice to combine the A_{scatt} and A_{att} into a single correction, A_{wall} , to yield a single correction factor that accounts for the effects of both attenuation and scatter. The “wall” subscript is indicative of the fact that the majority of photon interactions occur in the chamber wall at ^{60}Co energies.

3.1. FORMAL DEFINITION OF CORRECTION FACTORS

Although D_{air}^1 is independent of chamber wall thickness, it is still dependent on the source-to-chamber distance (SSD) due to the diverging nature of the incident photon beam. We therefore introduce the quantity D_{air}^2 which represents the dose to air due to an unattenuated, unscattered, broad parallel photon beam with uniform intensity distribution, where “broad” refers to beams that completely engulf the ion chamber. D_{air} may then be written as

$$D_{\text{air}} = A_{\text{wall}} \frac{D_{\text{air}}^1}{D_{\text{air}}^2} D_{\text{air}}^2 = A_{\text{wall}} A_{\text{an}} D_{\text{air}}^2, \quad (3.6)$$

where A_{an} is a correction for the effects of any axial non-uniformity associated with photon sources being non-parallel.

The above logic can be extended to include corrections for material inhomogeneities (A_{comp}), and for various other corrections for stem and electrode effects (if present), chamber orientation, etc, the product of which can be combined into a single correction, A . Eventually one obtains

$$D_{\text{air}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A \frac{D_{\text{air}}^{\text{Fano}}}{D_{\text{wall}}^{\text{Fano}}} D_{\text{wall}}^{\text{Fano}}, \quad (3.7)$$

where $D_{\text{air}}^{\text{Fano}}$ is the dose to the cavity filled with dry air and $D_{\text{wall}}^{\text{Fano}}$ is the dose to the wall material at the position of the cavity (with the cavity removed). The superscript “Fano” indicates that the respective doses correspond to an unattenuated, unscattered broad parallel photon beam with uniform intensity distribution. In this formalism, the ratio $D_{\text{air}}^{\text{Fano}}/D_{\text{wall}}^{\text{Fano}}$ is approximated by the Spencer-Attix ratio of restricted mass-collision stopping-powers, $(\bar{L}_{\Delta}/\rho)_{\text{wall}}^{\text{air}}$, given by equation 1.13. Due to energy conservation, and since we have charged particle equilibrium, $D_{\text{wall}}^{\text{Fano}}$ is equal to the product of the primary incident photon energy fluence, Ψ_0 , and the appropriately averaged mass-energy absorption coefficient, $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}$. Putting all this together, we

3.1. FORMAL DEFINITION OF CORRECTION FACTORS

have

$$D_{\text{air}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}} \Psi_0 \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}. \quad (3.8)$$

Dividing both sides of equation 3.8 by $K_{\text{air}} = \Psi_0 \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{air}} / (1 - \bar{g}_{\text{air}})$ gives

$$\frac{D_{\text{air}}}{K_{\text{air}}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{air}}^{\text{wall}} (1 - \bar{g}_{\text{air}}). \quad (3.9)$$

In equation 3.9, D_{air} can be replaced by $\frac{Q_{\text{air}}}{m_{\text{air}}} \left(\frac{W}{e} \right)_{\text{air}}$ (defined in section 2.2.2). If we define the reciprocal of the each correction factor A_i in equation 3.9 as $K_i = 1/A_i$, then upon rearranging for K_{air} we obtain the standard expression used to determine air kerma, given as

$$K_{\text{air}} = \frac{Q_{\text{gas}}}{m_{\text{air}}} \left(\frac{W}{e} \right)_{\text{air}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}^{\text{air}} \frac{1}{(1 - \bar{g}_{\text{air}})} K_{\text{h}} K_{\text{wall}} K_{\text{an}} K_{\text{comp}} K, \quad (3.10)$$

where Q_{gas} is the charge collected in the gas in the ion chamber (e.g. humid air), and K_{h} is a correction for the effects of humidity in the air, which was assumed to be dry air while making the above arguments. The use of the K_{h} is described elsewhere.⁵¹ The definitions of the various correction factors in equation 3.10 (except for K_{h}) are summarized in table 3.1 and represented schematically in figure 3.1.

3.2 Fluence correction factors for the air kerma formalism

Since the air kerma formalism makes use of the SA stopping-power ratio, which assumes that the fluence of charged particles in the wall is not perturbed by the cavity, equation 3.10 is, strictly speaking, only an approximation. The effect of the

Table 3.1: Summary of correction factors for the air kerma formalism (equation 3.10).

Correction	Definition	Comment
K_{wall}	$= K_{\text{scatt}} K_{\text{att}} = \frac{D_{\text{air}}^1}{D_{\text{air}}}$	<ul style="list-style-type: none"> • $D_{\text{air}} = \frac{Q_{\text{air}}}{m_{\text{air}}} \left(\frac{W}{e} \right)_{\text{air}}$ • D_{air}^1 = dose due to realistic beam with attenuation and scatter removed
K_{an}	$= \frac{D_{\text{air}}^2}{D_{\text{air}}^1} = \frac{D_{\text{air}}^2}{D_{\text{air}} K_{\text{wall}}}$	<ul style="list-style-type: none"> • D_{air}^2 = dose due to broad-parallel beam with no attenuation and scatter
K_{comp}	$= \frac{D_{\text{air}}^3}{D_{\text{air}}^2} = \frac{D_{\text{air}}^3}{D_{\text{air}} K_{\text{wall}} K_{\text{an}}}$	<ul style="list-style-type: none"> • D_{air}^3 is the same as D_{air}^2 for a chamber of homogeneous construction
K	$= \frac{D_{\text{air}}^4}{D_{\text{air}}^3} = \frac{D_{\text{air}}^4}{D_{\text{air}} K_{\text{wall}} K_{\text{an}} K_{\text{comp}}}$	<ul style="list-style-type: none"> • D_{air}^4 as for D_{air}^3 with other non-ideal characteristics of chamber removed

3.2. FLUENCE CORRECTION FACTORS FOR THE AIR KERMA FORMALISM

Figure 3.1: Schematic showing the correction factors required to apply Spencer-Attix cavity theory in the air kerma formalism. The jagged solid lines in the first two chambers depict charged particle tracks from primary (black lines) and secondary (green line) photons. They are omitted from the other chambers since all charged particles are from primary photon interactions only.

3.2. FLUENCE CORRECTION FACTORS FOR THE AIR KERMA FORMALISM

cavity on the charged particle fluence in the wall is discussed later in chapter 7. To account for any perturbation to $\Phi_{\text{wall}}^{\text{Fano}}$, an additional correction factor can be included in equation 3.10. Such a correction was introduced by Bielajew⁵⁰ by extending the logic of deriving correction factors introduced above. Prior to arriving at equation 3.7, we have

$$D_{\text{air}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A \frac{D_{\text{air}}^{\text{Fano}, \Phi_{\text{air}}}}{D_{\text{air}}^{\text{Fano}, \Phi_{\text{wall}}}} D_{\text{air}}^{\text{Fano}, \Phi_{\text{wall}}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A A_{\text{fl}} D_{\text{air}}^{\text{Fano}, \Phi_{\text{wall}}}. \quad (3.11)$$

$D_{\text{air}}^{\text{Fano}, \Phi_{\text{air}}}$ is the dose to the air in the cavity due to the actual fluence of charged particles in the cavity (Φ_{air}) and $D_{\text{air}}^{\text{Fano}, \Phi_{\text{wall}}}$ is the dose to the air in the cavity due to a charged particle fluence in an infinite wall medium (Φ_{wall}). In both cases, $D_{\text{air}}^{\text{Fano}, \Phi_{\text{air}}}$ and $D_{\text{air}}^{\text{Fano}, \Phi_{\text{wall}}}$ are due to an unattenuated, unscattered broad parallel photon beam. A_{fl} can therefore be defined as a correction for the perturbation to the charged particle fluence due to the presence of the cavity. Proceeding as in section 3.1, we obtain

$$D_{\text{air}} = A_{\text{wall}} A_{\text{an}} A_{\text{comp}} A A_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}} \Psi_0 \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}, \quad (3.12)$$

which is analogous to equation 3.7 with the addition of the fluence correction factor, A_{fl} . An expression for K_{air} can then be obtained in the same way as described in section 3.1. This leads to an equation for $K_{\text{fl}} = 1/A_{\text{fl}}$ given as

$$K_{\text{fl}} = \frac{K_{\text{air}}(1 - \bar{g}_{\text{air}}) \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{air}}^{\text{wall}}}{D_{\text{air}} K_{\text{wall}} K_{\text{an}} K_{\text{comp}} K} \quad (3.13)$$

All parameters on the right-hand-side of equation 3.13 can be calculated with EGSnrc using firmly established methods.⁵² This equation is used in chapter 7 to calculate K_{fl} as a function of cavity height and wall material, and to calculate K_{fl} for a list of graphite-walled chambers used by standards laboratories to determine air kerma for a

3.2. FLUENCE CORRECTION FACTORS FOR THE AIR KERMA FORMALISM

^{60}Co beam. The humidity correction, K_h , is not included here as it is in equation 3.10 since dry air is used in calculations of K_f .

3.3 On the direction conversion of chamber response to air kerma

The use of several distinct correction factors in the air kerma formalism arises from tradition, and the former necessity to account for the various non-ideal conditions separately. Given the proven ability of sophisticated Monte Carlo codes such as EGSnrc to calculate chamber response, this necessity is seemingly eliminated, and it may be argued that a single conversion factor that relates chamber response to K_{air} can be calculated directly. However, as discussed in section 2.2.2, chamber response defined as the charge or ionization per unit mass per unit incident fluence is obtained from Monte Carlo calculations of D_{air} , which requires knowledge of the value of $(\frac{W}{e})_{\text{air}}$. As Burns explains in a recent paper,⁵³ the value of $(\frac{W}{e})_{\text{air}}$ is extracted from experimental measurements of $(\frac{W}{e})_{\text{air}} \left(\frac{\bar{L}_\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ and, as such, is inextricably tied to the values of the stopping-power ratios used in those determinations. In other words, the use of a single conversion factor that relates chamber response to air kerma (which would include the stopping-power ratio) would require accurate knowledge of the value of $(\frac{W}{e})_{\text{air}}$. Since the value of $(\frac{W}{e})_{\text{air}}$ depends on the wall-to-air stopping-power ratio used in its determination, which varies with the density and mean ionization energy used,³⁶ this would introduce a significant amount of uncertainty.

3.4 Summary

The various parameters and correction factors have been rigorously defined for the air kerma formalism using the approach of Bielajew,⁵⁰ which includes a definition for an additional correction factor, K_{fl} , to account for the perturbation of the charged particle fluence due to the presence of the cavity. Calculating each individual correction factor as defined above has the advantage of providing insight into the size of each effect. This is briefly explored in chapter 7.

Chapter 4

An EGSnrc analysis of the P_{TP} correction factor for low-energy x rays

Although the form of the P_{TP} correction factor, given by equation 1.1, is theoretically justified (sections 1.3 and 1.3.1), it has been observed to break down when applied to well ionization chambers used to determine the air kerma strength of brachytherapy seeds. Bohm *et al*⁵⁴ and Griffin *et al*⁵⁵ reported that the relative response to 20 keV photons was overcorrected by as much as 18 % at low air pressures associated with regions of high altitude. They noted the discrepancies were related to the ranges of electrons and the material of the chamber wall. In principle, the same problems may exist for cavity ion chamber measurements used in kilovoltage x-ray beams. If this is the case, use of the standard P_{TP} correction with calibration coefficients in regions high above sea level (asl), such as Calgary (1000 m asl), Denver (1620 m asl), or Mexico City (2240 m asl) may be inappropriate.

4.1 Use of P_{TP} with low-energy x rays

In section 1.3.1, it was shown that the P_{TP} -corrected response is independent of cavity air density (ρ) when all electrons set in motion in the wall have enough energy to cross the cavity. For electrons with insufficient energy to cross the cavity over a range of air densities, the energy they deposit is simply equal to their initial energy and, hence, their contribution to the response is the same regardless of ρ . In this case, the P_{TP} -corrected response is no longer independent of ρ , and the P_{TP} may no longer be appropriate or sufficient to correct ion chamber response. This possibility is investigated extensively in this chapter using EGSnrc simulations of four ion chambers to calculate the response to kilovoltage x-ray beams as a function of cavity air density.

4.2 Methods

4.2.1 Ion chambers and calculations

Several types of geometrically distinct ion chambers were investigated to elucidate any dependence of the P_{TP} correction factor on the dimensions of the cavity and material of the chamber wall/electrode. Cross sections of these chambers as modeled in EGSnrc are shown in figures 4.1 and 4.2. The Farmer-type thimble chambers in figure 4.1 are based on the NE 2571 chamber (figure 4.1a) and the Exradin A12 (figure 4.1b). The former model is taken from previous investigations.^{30,36} Figure 4.1c shows an idealized spherical ion chamber with graphite walls. The radial dimensions of this chamber are the same as the Exradin A4 chamber made with C-552 air-equivalent plastic. Calculations with this chamber geometry were also performed using C-552 plastic as the wall material. The chamber in figure 4.1d is based on the parallel-plate

monitor chamber formerly used with the x-ray standard at the Ionizing Radiation Standards laboratory at the National Research Council of Canada (IRS-NRCC).

Figure 4.1: EGSnrc models of (a) an NE 2571 chamber; (b) an Exradin A12; (c) a large spherical chamber modeled with both graphite and C-552 air-equivalent plastic walls, and (d) a beryllium-walled monitor chamber. (from paper I)

Figure 4.2 shows models of two commonly used mammography chambers, the Capintec PS-033 (figure 4.2a), and the Magna A600 from Standard Imaging (figure 4.2b). The collecting regions of these chambers are defined by the front and back wall and the dashed lines. For each of the chambers in figures 4.1 and 4.2, Table 4.1 lists L , the mean chord length for electrons if they crossed the chamber in straight lines, calculated from the relation $L = 4V/S$, where V is the volume of the cavity and S is the cavity surface area.¹ Also listed are the energies of electrons having a range

4.2. METHODS

Figure 4.2: Geometries of the Capintec PS-033 (a) and the Exradin Magna A600 (b) mammography chambers. Mylar was used as the front wall material of the PS-033 chamber and is 0.5 mg/cm^2 thick in this model. The front wall of the Magna A600 chamber is a 3.6 mg/cm^2 thick kapton window. The collecting regions of these chambers are defined by the dashed lines. (from paper I)

equal to L at the reference air density, ρ_0 , equal to 1.196 kg/m^3 (for $T = 22 \text{ }^\circ\text{C}$ and $P = 101.325 \text{ kPa}$).

Table 4.1: Mean chord lengths (L) for the chambers shown in figure 4.1 and figure 4.2, calculated using the prescription $L = 4V/S$, where V is the volume of the cavity and S is the surface area.¹ For the mammography chambers, L was calculated for the collecting region only, defined by the dashed lines in figure 4.2a and b. The corresponding energies of electrons with a continuous slowing down approximation (CSDA) range in air (R_{CSDA}) equal to L , obtained from interpolating data provided by Attix,¹ are also listed.

Chamber model	L (cm)	e^- energy for $R_{\text{CSDA}} = L$ at ρ_0 (keV)
NE 2571	0.48	14.5
Exradin A12	0.52	15.2
spherical	2.55	38.4
NRC monitor chamber	2.55	38.4
PS-033	0.37	12.5
Magna A600	0.10	5.8

EGSnrc simulations of the spherical and cylindrically symmetric ion chambers were performed with the CAVSPHnrc and CAVRZnrc user-codes,²⁵ respectively. The P_{TP} -corrected response was extracted from calculations of the dose to the air in the cavity per unit incident fluence, D_{cav} . Corrections for changes in air density are inherent in these calculations since D_{cav} is the ratio of E_{dep} over the mass of air in a fixed volume. Thus, to the extent that $(W/e)_{\text{air}}$ is constant with respect to ρ for a given beam quality,

$$D_{\text{cav}} \propto M(\rho) \times P_{TP}, \quad (4.1)$$

where $M(\rho)$ is the chamber response at an air density of ρ . The validity of the P_{TP} correction factor can thus be checked by varying the density of air in the calculation.

4.2. METHODS

To relate the calculated results to physical models, additional Monte Carlo calculations of the spectra of electrons emerging from walls of various materials were done using the EGSnrc user-code FLURZnrc.²⁵

In order to simulate a change in air pressure within the cavity, a series of interaction cross sections was created for dry-air densities ranging from 0.602 kg/m³ to 1.566 kg/m³, corresponding to pressures ranging from 0.5 atm to 1.3 atm at 22 °C. Setting up the calculations in this way is convenient and assumes that air density is directly related to pressure at a constant temperature. This assumption is valid to within 0.1 % over the range of pressures used in this study according to van der Waals approximations for deviations from ideal conditions.

4.2.2 X-ray spectra

Representative narrow (NS) and high-rate (HR) x-ray spectra used as sources in the calculations were obtained from a catalogue produced by Ankerhold.⁵⁶ For the mammography chambers, a 26 kV mammography spectrum was obtained from IPEM Report 78,⁵⁷ and is similar to the spectrum published by Boone *et al.*⁵⁸ A ⁶⁰Co spectrum published by Mora *et al.*⁴³ was used for calculations with the NE 2571 and spherical chambers in ⁶⁰Co beams. All calculations used a point source with a 100 cm source-to-surface distance (SSD) and a beam radius large enough to cover the entire chamber.

4.3 Breakdown of P_{TP}

The relative EGSnrc-calculated responses as a function of air density for the thimble ionization chambers are shown in figure 4.3. If the variation in the calculated response

as a function of pressure was properly accounted for by the P_{TP} correction factor, the P_{TP} -corrected response would be unity as a function of air density. However, for the NE 2571 chamber (figure 4.3(a)), the P_{TP} -corrected response is below unity by as much as 1.5 % for the 20 and 60 kV spectra, and by over 2 % for the 40 kV spectrum at an air density typical of Mexico City ($\rho/\rho_0 = 0.76$). The relative deviation for the 120 kV spectrum is much less, and the response is constant within 0.25 % for the 250 kV and ^{60}Co beams. The insert shows the calculated P_{TP} -corrected response for the 20, 40, and 60 kV spectra for air densities ranging 5 % above and below the reference density, ρ_0 . Here, the deviation of the normalized response is as much as 0.6 % for the 40 kV spectrum at $\rho/\rho_0 = 0.95$, however, the pressure rarely deviates to that extent for regions near sea level.⁵⁹

Calculations for the Exradin A12 chamber are shown in figure 4.3(b) using the same scale as in figure 4.3(a). Although the cavity sizes of these two chambers are comparable, the P_{TP} -corrected response of the latter is within 1 % of unity over the full range of air densities tested for the 20, 40 and 60 kV beams. The normalized response when irradiated by the remaining x-ray spectra is nearly constant over the whole range of air densities tested.⁵⁹ The most significant differences between the NE 2571 and the Exradin A12 are the materials of the walls and collectors.

Calculations of the P_{TP} -corrected response of the spherical chambers are shown in figure 4.4. As with the thimble chambers, large deviations (e.g. ≈ 17 % for the 40 kV spectrum at $\rho/\rho_0=0.5$) are observed for the graphite-walled chamber, while relatively negligible variation is observed for the chamber with air-equivalent plastic walls.

Figure 4.5 shows the P_{TP} -corrected response of two mammography chambers (figure 4.2) in a 26 kV mammography beam, and shows that the breakdown of the

4.3. BREAKDOWN OF P_{TP}

(a)

(b)

Figure 4.3: EGSnrc calculations of the P_{TP} -corrected response of the two thimble ionization chambers shown in figure 4.1. The results for the NE 2571 chamber are shown in (a) and results for the Exradin A12 are shown in (b). (from paper I)

4.3. BREAKDOWN OF P_{TP}

(a)

(b)

Figure 4.4: As in figure 4.3 for a large spherical chamber with graphite walls (a) and C-552 air-equivalent plastic walls(b).
(from paper I)

P_{TP} correction factor also applies to these types of chambers. The breakdown is larger for the A600 chamber despite having a smaller cavity size than the PS-033.

Figure 4.5: P_{TP} -corrected responses for the Capintec PS-033 and Exradin Magna A600 as a function of air density for a 26 kV mammography beam. (from paper I)

4.4 Contribution of electron ranges to the breakdown of P_{TP}

Since ion chamber response is the result of ionization produced by electrons as they cross the cavity, one might expect the failure of the P_{TP} correction factor to be directly related to the range of electrons set in motion in the wall by the incident beam. To explore the above notion in detail the FLURZnrc user-code was used to calculate

4.4. CONTRIBUTION OF ELECTRON RANGES TO THE BREAKDOWN OF P_{TP}

the spectra of electrons emerging from slabs of beryllium, graphite, and C-552 plastic shown in figure 4.6 for the 20, 40, 60 and 120 kV beams. The average electron energies are listed in table 4.2 along with the corresponding ranges in dry air at the reference density (ρ_0). The average energies of electrons emerging from each of the three wall materials does not increase in proportion to the incident x-ray beam energy. This makes it difficult to account for the extent of the breakdown in the P_{TP} correction factor in terms of electron ranges alone.

As an example, contrary to what is observed (figure 4.3(a)) it would be expected that the relative response of the 20 kV beam incident on the NE 2571 chamber be different from that due to the 40 kV spectrum since the respective average electron energies differ by about 50 %, and the corresponding distributions of electron energies differ even more significantly. Similar arguments may be made for the spherical graphite chamber with the 40 and 60 kV beams, and for the mammography chambers.

Table 4.2: Average photon energies for the narrow (NS) and high-rate (HR) x-ray spectra from the PTB⁵⁶ used in this study, along with the average electron energies emerging from a slab of graphite (0.073 g/cm²), a slab of beryllium (0.092 g/cm²), a slab of C-552 plastic (0.076 g/cm²), and the corresponding CSDA ranges in air for these mean energies. The average electron energies were calculated using FLURZnrc²⁵ and the CSDA ranges were interpolated from range data provided by Attix.¹

PTB spectra	avg. photon energy (keV)	avg. e ⁻ energy			R_{CSDA} (air) at ρ_0	
		graphite	C-552	Be	graphite	Be
		(keV)			(cm)	
20 kV (HR)	13.1	8.6	8.7	8.5	0.20	0.19
40 kV (NS)	33.3	22.2	22.6	19.3	0.99	0.78
60 kV (NS)	47.9	27.1	30.4	17.0	1.40	0.63
120 kV (NS)	100.4	23.2	31.2	17.7	1.07	0.67
250 kV (NS)	207.1	54.1	57.0	52.3	4.56	4.33

4.4. CONTRIBUTION OF ELECTRON RANGES TO THE BREAKDOWN OF P_{TP}

Figure 4.6: EGSnrc-calculated spectra of electrons emerging from slabs of beryllium (dashed and dotted line, 0.09 g/cm^2), graphite (solid line, 0.06 g/cm^2) and C-552 air-equivalent plastic (dashed line, 0.09 g/cm^2) upon irradiation by broad-beam PTB spectra listed in table 4.2. The low-energy peaks (absent in 20 kV case) are Compton secondaries, which are roughly the same intensity in each material, and become relatively more important as the photon energy increases due to the increase in Compton cross section. The higher-energy electrons are from the photoelectric interactions, which are strongly material dependent and relatively less important at higher photon energies. The distribution of the high-energy peaks produced by the 40, 60 and 120 kV beams are similar for each material but differ significantly in intensity.

(from paper I)

4.4. CONTRIBUTION OF ELECTRON RANGES TO THE BREAKDOWN OF P_{TP}

4.5 Effect of the chamber wall material

The effect of the chamber wall material can be seen by comparing the response of the chambers with graphite and air-equivalent C-552 plastic walls along with the spectra of electrons emerging from the respective materials. Despite the similarities in cavity size, almost no breakdown of the P_{TP} correction factor is observed for the chambers with C-552 plastic walls (figure 4.3(b) and 4.4(b)), even when the range of electrons is short in comparison to the mean chord length of the cavity. This may be explained using the theory of Fano discussed in section 2.4, which states that for a homogeneous medium the fluence of electrons is uniform and independent of density variations in the medium.^{1,40} Thus, ignoring the effects of attenuation, chambers with air-equivalent walls will have charged-particle equilibrium between the chamber wall and the air in the cavity regardless of air density, and therefore variation in the P_{TP} -corrected response as a function of ρ is not expected.

If charged particle equilibrium is required for the P_{TP} correction to work, then the breakdown of P_{TP} may depend on the degree of charged particle disequilibrium, which is related to the extent that the photon cross-sections between the material in the wall and the air in the cavity are different. Figure 4.7 shows the ratio of photon cross-sections *per g/cm²* of several materials with air. At the photon energies considered in this study, the mismatch in the photon cross sections between graphite and air is substantial,⁴⁴ differing by as much as a factor of two at 10 keV. In most cases,⁵⁹ the magnitude of breakdown of the P_{TP} correction factor for a given energy correlates with the extent that the photon cross sections between the wall and the air are different. There are exceptions to this. For the 20 kV beam incident on the graphite chambers, the overall variation is the same (figure 4.3(a)) or less (figure 4.4(b)) than the variation exhibited by the 40 and 60 kV beams despite there being a larger mis-

match in wall/air cross sections at lower energies. For the NE 2571 chamber, this can be partially explained by the aluminum collector which has a compensating effect, but this does not explain the variation in response observed with the spherical chambers. Rather, in the latter case, the variation is influenced by the large cavity size. This is explained in the following section.

Figure 4.7: Ratios of the total photon interaction cross sections in g/cm^2 as a function of photon energy for aluminum, beryllium, graphite, and C-552 air-equivalent plastic with respect to air.^{31,32} Similar data for mylar and kapton ratios to air are less than unity, with the ratio with mylar closer to unity.

(from paper I)

4.5. EFFECT OF THE CHAMBER WALL MATERIAL

4.6 Effect of photon interactions in the cavity

The overall magnitude of the breakdown is also influenced by the fraction of the response due to photon interactions in the air of the cavity, denoted here as F_{air} . Ma and Nahum,⁹ as well as Borg et. al.⁴⁴ have shown F_{air} is non linear as a function of incident photon energy for thimble ion chambers. Figure 4.8 shows values of F_{air} for the three chamber geometries as a function of the average photon energy for each of the x-ray spectra listed in table 4.2. In each case F_{air} is highest for the 20 kV beam, approaching unity for the spherical graphite chamber and NRC monitor chamber with no electrode. A large F_{air} reduces the impact that non-air-equivalent walls have on the breakdown of the P_{TP} correction factor since a relatively small fraction of the response is due to electrons from the chamber wall. This explains why the breakdown of P_{TP} is less for the 20 kV beam.

4.7 Summary

The calculations discussed above demonstrate the breakdown of the standard temperature-pressure correction factor for several geometrically distinct ion chambers irradiated by low-energy x-ray spectra. The breakdown of P_{TP} is due to low-energy electrons that do not have enough energy to cross the cavity at air densities of interest. Although this is the primary cause, the extent of the breakdown is also influenced by the mismatch of photon cross sections between the chamber wall and air, and the fraction of the response due to photon interactions in the cavity.

Figure 4.8: Fraction of response due to photon interactions in the cavity (F_{air}) as a function of the average photon energy for the NE 2571 chamber (a), spherical graphite chamber (b), and NRC monitor chamber (c) with (dashed line) and without (dashed and dotted line) the aluminum electrode. The average photon energies refer to those listed in table 4.2 for the five PTB spectra used in this study. The corresponding beam qualities are shown in (b). F_{air} is estimated as the ratio of chamber response with high and low values of ECUT in the chamber walls and electrode, and is calculated at an air density of $\rho/\rho_0 = 1$. Computing F_{air} in this way ignores the contribution from backscattered electrons which is assumed to be negligible. Previous studies^{9,44} suggest a peak in F_{air} around a mean energy of 80 keV. However, a beam quality with an average energy of 80 keV was not considered here. (from paper I)

Chapter 5

Experimental verification for the breakdown of P_{TP}

Having identified the breakdown of the P_{TP} correction factor at low x-ray energies with Monte Carlo simulations, it is prudent to fully scope the effect experimentally. Experimental evidence has been reported previously in a publication written in German by Will and Rakow,⁶⁰ and later in internal reports at the National Physical Laboratory (NPL) by J. E. Burns and D. H. Pritchard⁶¹ for an NE 2561 chamber (now NE 2611), but those results were only considered in the context of chamber air-equivalence. This chapter presents a more comprehensive set of experimental data confirming the breakdown of the P_{TP} correction factor at low-energies for several ion chambers made with non-air-equivalent materials. The results of these experiments are reproduced by Monte Carlo simulations of the experimental set-up as a means of corroborating the previous calculations, and to demonstrate the efficacy of the EGSnrc code at low photon energies.

5.1 Methods

5.1.1 Ion chambers

The ion chambers used in this investigation are shown in figure 5.1 as they were modeled with the “cavity” user-code of EGSnrc. The geometry descriptions of these chambers were obtained from manufacturer documentation and from previous reports.^{36,62,63} Ignoring differences in guard design, the chambers studied may be grouped according to the geometrical configuration of the cavity: those having cavity dimensions similar to the NE 2571, and those similar to an Exradin A2 but with a thin electrode. An Exradin A12 was also investigated due to its calibration history and familiarity in the lab. The chambers listed, although similar in dimension, are composed of various wall and electrode materials, thereby permitting an investigation into the respective influences of the materials on chamber response as a function of air density. Table 5.1 lists these materials for each chamber, along with the nominal collecting volume and mean chord length of the cavity, L , calculated using $L = 4V/S$.¹ To gauge the effect of impurities, the composition and impurity level of the dural (an aluminum alloy) and graphite NE 2505/3 thimbles was measured at NRC using x-ray fluorescence spectroscopy with an uncertainty of $\pm 10\%$ on the impurities. These results are reported in paper II.⁶⁴

5.1.2 X-ray spectra and EGSnrc simulations of the x-ray source

In all experimental measurements the Comet MXR-320 x-ray system at NRC was used as a source of x rays. Table 5.2 lists the beam qualities used when measuring the response as a function of air density ($M(\rho)$) along with those used to obtain air

(a)

(b)

Figure 5.1: EGSnrc models of the chambers used in the experimental investigation of P_{TP} , shown in 3D (a) and in cross-section (b). From left to right, the models represent the Exradin A2, the Exradin A12, and the NE 2571. The Exradin chambers were modeled as a homogeneous material since information about the construction of the stems was not available from the manufacturer.

Table 5.1: Physical characteristics of the ionization chambers shown in figure 5.1. The diameter of the electrodes are all 1.0 mm, including the Exradin A2 chambers which normally come with a 4.6 mm diameter electrode. In all cases, the wall thickness was sufficient to provide full build-up for each beam quality used in this investigation. The mean chord length, L , represents the average straight line distance an electron must travel to cross the cavity, given by $L = 4V/S$ where V is the volume of the cavity and S is the surface area.

Chamber	Wall		Electrode material	Nominal collecting volume (cm ³)	$L = 4V/S$ (cm)
	Material	Thickness (mm)			
NE 2571	graphite	0.36	aluminum	0.6	0.48
NE 2505/3 (modified)	graphite	0.36	aluminum	0.6	0.48
	dural	0.09	aluminum	0.7	0.52
Exradin A19	C-552	0.5	C-552	0.6	0.46
Exradin A12	C-552	0.5	C-552	0.6	0.54
Exradin A2 (modified)	C-552	1.0	C-552	0.7	1.24
	C-552	1.0	aluminum	0.7	
	aluminum	1.0	C-552	0.7	
	aluminum	1.0	aluminum	0.7	

Table 5.2: X-ray beams used in the investigation. The half-value layer (HVL) is defined as the thickness of material (Al or Cu) required to reduce the measured air kerma to one-half of the original value at 1 m distance from the source. The calculated HVLs for the simulated spectra were determined using equation 12 from a paper by Mainegra-Hing and Kawrakow.⁶⁵ The effective energies (E_{eff}) in this case are defined as the energy of a monoenergetic photon beam having the same HVL as the corresponding spectrum.

<u>For N_K curves</u>						
Tube kV	Filtration (mm)		HVL (mm)		E_{eff} (keV)	
	Al	Cu	expt	cal'n	expt	cal'n
50	1.032	-	1.102 (Al)	1.048	22.74	22.63
60*	1.032	-	1.209 (Al)	1.219	23.49	23.90
100	3.63	-	4.022 (Al)	4.038	37.09	38.16
135	1.00	0.25	0.488 (Cu)	0.461	59.96	58.86
180	1.25	0.50	0.991 (Cu)	0.940	79.55	77.66
250	1.00	1.70	2.533 (Cu)	2.414	122.52	122.11

<u>For $M(\rho)$ vs. ρ</u>			
Tube kV	Filtration (mm)		Avg. calc. photon E (keV)
	Al	Cu	
60	-	-	22.9
100 [†]	3.63	-	50.9
150 [‡]	1.00	0.246	70.0
150	1.00	0.495	76.0

*Only used for the calibration of the NE 2571.

[†]Only used with the Exradin A12.

[‡]Only used with the NE 2571.

kerma calibration coefficients (N_K) for three of the chambers. Spectral distributions of these beams were calculated using a BEAMnrc⁶⁶ simulation of the x-ray source by combining the methods of Mainegra-Hing and Kawrakow,⁶⁵ and Ali and Rogers.⁶⁷ The calculated spectra were found to be visually similar to the corresponding spectra in IPEM Report 78,⁵⁷ calculated using the method of Birch and Marshall.⁶⁸ Half-value layers (HVL) of the calculated spectra used to obtain calibration coefficients were determined using equation 12 of reference,⁶⁵ and were found to be within 5.5 % of experimental values. This agreement is within the range reported in previous studies,^{69–71} and confirms that the x-ray source was well modeled. Values of air kerma per unit incident photon fluence (K_{air}) for each of the x-ray spectra used were calculated using the EGSnrc “g” user-code.

5.1.3 Experimental measurements

The ability of the P_{TP} correction factor to correct the response of each of the chambers listed in table 5.1 was tested by measuring as a function of pressure the response of the chamber placed within an air-tight vessel made of polymethylmethacrylate (PMMA). A diagram of the experimental set-up is shown in figure 5.2. The air pressure within the vessel, and presumably in the cavity of the chambers, was reduced using a vacuum pump, and monitored using a high-precision electronic barometer calibrated against a secondary standard. The temperature within the vessel was monitored using a platinum sensor connected to an external DMM via a vacuum-sealed feed-through. An additional feed-through connected the ion chamber to an electrometer which controlled the bias and measured the ionization. The ion chambers were positioned with their cavities centered in the field of view and supported in the vessel by an aluminum stand outside of the field.

Figure 5.2: Schematic of the experimental setup. The ion chambers were supported by an aluminum stand within a cylindrical PMMA vessel (0.65 cm thick walls) and laser-aligned in the center of the field, approximately 123 ± 1 cm from the source. The field size was collimated to a diameter of 9 cm at an SSD of 100 cm. Note that the diagram is not drawn to scale, and the x-ray tube and monitor chamber are only shown for perspective. (from paper II)

Before each data set was acquired, the ion chamber in the vessel was pre-irradiated until a stable reading was reached relative to the monitor chamber.⁶³ The air pressure in the vessel was varied at random to values as low as 0.5 atm (i.e. measurements were not obtained in order of decreasing or increasing air pressure). At each pressure, the chamber was allowed to stabilize which also provided enough time for the temperature to re-equilibrate. The final measured response was then taken as the average of the last seven to ten stable charge measurements by the chamber per unit charge measured by the monitor chamber. The standard deviation of the mean for each measurement was 0.03 % or less. Measurements at atmospheric pressure were repeated after the pressure in the vessel had been ramped in order to confirm that hysteresis effects were negligible. The total uncertainties on these measurements were estimated to be 0.14 % ($k = 1$) using the ISO Guide to the Expression of Uncertainty in Measurement,^{72,73} and summarized in table 5.3. Air kerma calibration coefficients (N_K) for three of the chambers were also obtained for x-ray beams listed in the top part of table 5.2 following the standard calibration procedures at NRC.

All chamber response measurements were corrected by P_{TP} (equation 1.1) using measurements of temperature and pressure within the vessel. Data for each chamber were normalized by the response under reference temperature and pressure conditions extracted from a least squares of the measured data sets to a second-order polynomial. Corresponding EGSnrc calculations were normalized using a χ^2 minimization between the measured and calculated results. Normalizing and comparing results in this way avoids emphasis on any one particular measured or calculated value.

Table 5.3: Estimated uncertainties associated with the experimentally measured P_{TP} -corrected chamber response per unit response of the monitor chamber.

Component	Std. unc. (%)	Comment
Type A:		
Repeatability (short term)	0.03	Standard deviation of the mean of 7 - 10 readings once stable reading was reached
Pressure correction (zero offset)	0.04	Based on calibrations with reference pressure gauge
Pressure correction (linearity)	0.08	Based on calibrations with reference pressure gauge
Type B:		
Temperature stability	0.01	Analysis assumed constant T Uncertainty estimated from experimental data as temperature stabilized after pressure change
Reference pressure gauge	0.02	From uncertainty in calibration
Humidity	0.05	Not monitored inside vessel
Hysteresis of pressure change	0.05	Estimated from repeated measurements after changing pressure
Drift of system	0.08	Could be chamber, x-ray tube, or monitor chamber (pre-irradiation effects considered)
Overall:	0.14	Consistent with long term repeatability (between independent measurements)

5.2 Measured breakdown of P_{TP} and the effect of impurities

P_{TP} -corrected responses measured and calculated as a function of cavity air density for the NE-type chambers are shown in figure 5.3. The variation observed with chambers with graphite and dural walls confirms the shortcomings of the P_{TP} correction factor discussed in chapter 4. At an air density of $\rho/\rho_0=0.76$, typical of Mexico City, the response of the graphite-walled chamber is undercorrected by 2 % for the 60 kV beam, while the response of the chamber with dural walls is overcorrected by 13 %. For the 150 kV beam, the variation is significantly less. The P_{TP} -corrected response of the air-equivalent chamber was constant for both incident spectra. All of these results are consistent with the effect of the wall material discussed in section 4.5.

Monte Carlo simulations with the cavity user-code demonstrates that EGSnrc is able to calculate the variation in the P_{TP} -corrected response at these energies to within 1.5 % for the chamber with dural walls, and within 0.5 % for the other chambers. The larger discrepancy in the former case is due to the significant ambiguity of the measured composition of the dural wall.⁶⁴ The insert of figure 5.3(a) shows that EGSnrc is sensitive to the presence of impurities. The composition of graphite used in the simulation of the NE 2505/3 was taken from x-ray fluorescence spectroscopy measurements (section 5.1.1) while the simulations of the NE 2571 used pure graphite.⁶⁴ The difference between the measured and calculated responses of these two chambers for the 150 kV spectrum is negligible and so is not included here.

Additional measurements and calculations for the set of Exradin A2 chambers are shown in figure 5.4 for the same two incident x-ray beams. A graphite thimble was not available for this chamber, but results were obtained for the C-552 air-equivalent

(a)

(b)

Figure 5.3: Comparisons of calculated (dotted lines) and measured (solid lines) P_{TP} -corrected response as a function of cavity air density for the NE-type chambers with graphite, dural, and C-552 air-equivalent walls. Results are shown for the 60 (a) and 150 kV beam (b) spectra listed in table 5.2. (from paper II)

5.2. MEASURED BREAKDOWN OF P_{TP} AND THE EFFECT OF IMPURITIES

Figure 5.4: As in figure 5.3 for the modified Exradin A2 chambers.
(from paper II)

5.2. MEASURED BREAKDOWN OF P_{TP} AND THE EFFECT OF IMPURITIES

chamber with an aluminum electrode. In general, the overall variation of the P_{TP} -corrected response is less than that observed with the NE-type chambers, which is attributable to the smaller cavity size of this chamber (see table 5.1) and the small difference in composition between aluminum and dural.⁶⁴ As with the NE-type chambers, EGSnrc calculations are within 0.5 % of measurements for this set of chambers.

Figure 5.5 shows the P_{TP} -corrected response of an Exradin A12 chamber measured and calculated for 60 and 100 kV beams (table 5.2). Unlike the other air-equivalent chambers, an unexpected 1.6 % variation in the measured, P_{TP} -corrected response was observed with the 60 kV beam. Corresponding EGSnrc calculations at this energy did not reflect this variation when the chamber was modeled with C-552 plastic walls. Note that these 100 kV calculations were normalized by the average value since there is no discernible variation with air density. Calculations for the 100 kV beam were within 0.5 % of measurements as in the previous cases.

Calculations for the 60 kV beam were repeated with impurities included in the wall of the chamber model. Due to the calibration history of this chamber at NRC, a chemical analysis of the wall was not obtained to avoid the possibility of damage to the chamber. Since the objective is to merely gauge the effect of a realistic set of impurities, the composition with impurities in the EGSnrc model was taken from an analysis by Seuntjens *et al*⁴¹ (≈ 463 ppm) for another sample of C-552. Figure 5.5 shows the effect of these impurities on the calculated response as function of air density is negligible for both the 60 and 100 kV beams.

In the investigation by Seuntjens *et al*,⁴¹ the effect of impurities on chamber response was determined by calculating the air kerma calibration coefficient (N_K) as a function of the effective energy (E_{eff}), or energy response curve, using EGSnrc since

5.2. MEASURED BREAKDOWN OF P_{TP} AND THE EFFECT OF IMPURITIES

Figure 5.5: Measurements and EGSnrc calculations of an Exradin A12 chamber for 60 and 100 kV beams. The dotted line represents calculations performed with 463 ppm impurities included in the chamber wall material. Refer to the text for details.
(from paper II)

5.2. MEASURED BREAKDOWN OF P_{TP} AND THE EFFECT OF IMPURITIES

N_K is particularly sensitive to compositional changes.^{41,64} N_K in this case is defined as

$$N_K \propto \frac{K_{\text{air}}}{D_{\text{cav}}}, \quad (5.1)$$

where K_{air} and D_{cav} are the air kerma per unit fluence and dose per unit fluence to the cavity for a particular incident spectrum, respectively. Figure 5.6 shows measured and EGSnrc-calculated values of N_K as a function of E_{eff} for the Exradin A12 chamber. The x-ray spectra corresponding to each value of E_{eff} are listed in table 5.2. EGSnrc calculations were normalized to the value measured for the 250 kV beam ($E_{\text{eff}} \approx 120$ keV) since the effect of impurities on the calculated response at that energy is negligible.

Figure 5.6: Calculated air kerma calibration coefficients (N_K) as a function of spectrum effective energy (E_{eff}) for the Exradin A12 chamber. (from paper II)

5.2. MEASURED BREAKDOWN OF P_{TP} AND THE EFFECT OF IMPURITIES

The EGSnrc calculations of N_K shown in figure 5.6 indicate that the effect of impurities on N_K values is significant. Increasing the amount of impurities taken from the Seuntjens *et al* study⁴¹ by 50 % (to 695 ppm) brought calculated values of N_K to within 0.6 % of measurements for $E_{\text{eff}} \geq 35$ keV. The agreement between measured and calculated values of N_K for the 60 kV beam ($E_{\text{eff}} = 23.5$ keV, table 5.2) also improved as impurities were added to the chamber model, but a 2 % discrepancy remains unaccounted for. Incidentally, the effective energy of this spectrum is comparable to the 60 kV beam used in figure 5.5, suggesting that ability to accurately calculate the relative P_{TP} -corrected response is contingent on the ability to calculate the energy response curve. This is supported by the calculations of N_K for the NE-type chambers shown in figure 5.7. In this case, calculated values of N_K are within 1 % of experiment or less over the whole range of effective energies. Attempts to resolve the discrepancies in figures 5.5 and 5.6 are discussed in detail in paper II.⁶⁴

5.3 Effect of geometry on the calculated response to low-energy x rays

Potential errors in the geometry descriptions of the Exradin A12 were also investigated as a possible explanation for the remaining discrepancy in the calculated energy response curve discussed above. For instance, changing the cavity size may influence the response if the average range of electrons in air is on the order of the dimensions of the cavity. In this case, the average energy of an electron entering the cavity is about 25 keV, which has a CSDA range in air of ≈ 1.2 cm, which is slightly more than twice the mean chord length, L , of the cavity (≈ 0.52 cm, table 4.1). Additionally, the chamber wall thickness may significantly influence the absolute response at this

Figure 5.7: As in figure 5.6 for the two NE-type chambers.
(from paper II)

5.3. EFFECT OF GEOMETRY ON THE CALCULATED RESPONSE TO LOW-ENERGY X RAYS

energy due to attenuation and beam hardening. Reducing the wall thickness by 15 % (0.0075 cm) in the model of the A12 with impurities while keeping the outer dimensions the same reduced beam hardening by the wall and increased the cavity size. This modification is consistent with the tolerances of construction for this chamber, and brought the value of N_K calculated at 60 kV to within 1.2 % of the measured value relative to the value at 250 kV. However, the reduced wall thickness had a negligible effect on the calculated response to the 60 kV beam as a function of air density. Thus, as with impurities, changes in chamber dimensions have more of an effect on values of N_K than on the response as a function of air density.

The effect of chamber dimensions was also investigated for the NE 2571. Figure 5.8 shows the P_{TP} -corrected response calculated for two different geometrical configurations of the NE 2571 chamber free in air (i.e. without the PMMA vessel) with the 40 kV PTB spectrum used in chapter 4 (table 4.2). The CAVRZnrc configuration refers to the geometry which is also shown in figure 4.1a of chapter 4, and the data in the solid line shows the associated CAVRZnrc calculations taken from figure 4.3(a). Calculations using the exact same geometry in the “cavity” user-code are given by the dotted line and serve as a check that two user-codes give the same result. The dashed line represents calculations that included the more accurate model of the chamber used throughout the rest of this study. In this case, the P_{TP} -corrected response deviates an additional 1 % below the response corresponding to reference temperature and pressure conditions. This suggests that accurately modeling the chamber dimension is essential in order to predict the breakdown of the P_{TP} correction factor to at least within 1 % at low x-ray energies and hence all comparisons are for realistic ion chamber models. It should be noted, however, that no difference was observed in a similar comparison made between realistic and approximated geometries of the air-equivalent Exradin A12 (without impurities). Such a result is not

5.3. EFFECT OF GEOMETRY ON THE CALCULATED RESPONSE TO LOW-ENERGY X RAYS

surprising if the chamber is air-equivalent.

Figure 5.8: Calculations of the P_{TP} -corrected response of the NE 2571 chamber *free in air* (no PMMA vessel) as a function of air density for two different geometrical configurations: the one used in this study, and the one shown in figure 4.1. A 40 kV PTB spectrum was used as the incident beam (see table 4.2). The solid line shows data from figure 4.3(a) using the CAVRZnrc user-code. The dotted line shows the same calculations using the cavity user-code and the identical chamber geometry as that defined by CAVRZnrc. The dashed line shows the calculations of the cavity user-code using a more realistic model of the chamber shown in figure 5.1. (from paper II)

The geometry-sensitivity of these calculations for the NE 2571 has potential implications at higher energies. However, both models of the NE 2571 chamber with a ^{60}Co beam showed no variation in the P_{TP} -corrected response as a function of air density (under the normal situation for a chamber with a build-up cap). This result is consistent with previous reports for different graphite-walled chambers,^{47,48,74} and

5.3. EFFECT OF GEOMETRY ON THE CALCULATED RESPONSE TO LOW-ENERGY X RAYS

implies that geometrical specifications in Monte Carlo models are much less important for these types of calculations with higher energy photons. As a check, measured values of the response to a ^{60}Co beam for the NE 2571 and Exradin A12 chambers (with build-up caps) were obtained using the Co unit at NRC and showed no variation in response over the range of pressures investigated here (data not shown).

5.4 Summary

Measurements of the response of ion chambers in a variable-pressure environment have been used to experimentally confirm the breakdown of the P_{TP} correction factor for several non-air-equivalent ion chambers in low-energy x rays. Consistent with previous reports,^{54,55,59,61} the extent of the breakdown is larger for lower energy x-ray spectra, and is also related to the difference between the photon cross sections of air and the material of the chamber wall and/or collector. Overall, the experimental data were well matched by the EGSnrc Monte Carlo code, which confirms its ability to calculate ion chamber response accurately at low x-ray energies, and validates the calculations with this code presented in chapter 4 provided contributions from impurities are ignored, along with approximations to the chamber geometries.

It was also found that the reliability of EGSnrc in predicting the breakdown of the P_{TP} correction factor for a given chamber can be tested by comparing calculated and measured values of N_K in the energy range of interest. Values of N_K are highly sensitive to chamber dimensions and composition, and can therefore be used as a stringent test of a Monte Carlo chamber model. Thus, for those users of non-air-equivalent ion chambers in regions requiring large P_{TP} corrections, it appears the breakdown of the P_{TP} correction factor may be accurately predicted using Monte Carlo simulations provided the Monte Carlo model can accurately calculate N_K val-

ues over the energy range of interest. However, until the connection between accurate calculations of N_K and chamber response versus air density can be further improved, these predictions are best confirmed with direct experimental measurements of chamber response over a range of cavity air densities. The difficulties associated with modeling the Exradin A12 chamber (figure 5.5 and 5.6) prove that Monte Carlo calculations at these energies should be used with caution. Issues associated with the breakdown of the P_{TP} correction factor may be avoided altogether by using chambers made with air-equivalent materials known to be free of impurities.

Chapter 6

EGSnrc calculations of chamber response to ^{60}Co beams

The contribution of low-energy electrons to the response of ion chambers in high-energy photon beams, and in ^{60}Co beams in particular, was briefly discussed in chapter 1.4. Rather than affecting the P_{TP} correction factor as in the case with x rays, the low-energy portion of the electron fluence due to a ^{60}Co beam instead affects the evaluation of the Spencer-Attix stopping-power ratio used to convert ion chamber response into dosimetric quantities of interest. Understanding the role of low-energy electrons on the response of chambers to high-energy photons (i.e. ^{60}Co beams) is the goal of the following two chapters. In the present chapter, measurements from the literature are used to determine the accuracy with which EGSnrc can calculate ion chamber response as a function of cavity dimension and chamber wall material at ^{60}Co energies. In chapter 7, EGSnrc calculations of chamber response are used to test the accuracy of the Spencer-Attix stopping-power ratio and to calculate fluence correction factors for the ^{60}Co air kerma formalism.

6.1 Early experimental investigations of ion chamber response and cavity theory

In the mid-to-late 1950's and early 1960's, several experiments^{47, 74-79} were performed with the aim of investigating the effects of cavity size and wall material on the response of chambers free-in-air. Experiments focused on ^{60}Co beams, but other photon sources were also tested, including ^{137}Cs , ^{198}Au , betatron radiation, and orthovoltage x rays. Contrary to the predictions of the Bragg-Gray (BG) cavity theory (section 1.4.1), it was observed that chamber response varies with changes in cavity height (distance between front and back wall of a plane-parallel chamber) or cavity air pressure.⁷⁶⁻⁷⁹ At the time, interest in these experiments was driven, in part, by the cavity theory introduced by Spencer and Attix (SA) in 1955¹¹ (section 1.4.2), which has the potential to predict the variation mentioned above since it was formulated to explicitly account for the production of δ -rays, and to take into account the cavity size dependence of chamber response.

Although it was found that SA cavity theory could predict some of the variation in response with changes in cavity dimension, which is regarded as a significant improvement over BG cavity theory, some discrepancies with measurements were still observed that are large by present standards (e.g. discrepancies as much as 10 % in ^{60}Co beams^{47, 76} and 20 % for a lead chamber in ^{198}Au ⁴⁷). However, ignoring those few large discrepancies, the response predicted by SA cavity theory was often within a few percent of measured values for a range of wall materials. Given the level of precision of these comparisons, uncertainties on the measured values were rarely discussed. Yet despite the challenges involved with experiments of this type,⁵³ it is reasonable to assume that the measurements were, on average, reproducible to within a few percent or less. This is comparable to the level of uncertainty on the cross sections^{80, 81} and

stopping-powers⁸ used in modern Monte Carlo codes such as EGSnrc,^{16,18} which are estimated to be 1-2 % at ^{60}Co energies. Thus, in addition to early tests of cavity theory, these experiments are also well suited to test the cross sections in EGSnrc via calculations of the change in response associated with changes in wall material, at least at the level of experimental precision. Previously, the EGSnrc Monte Carlo code has been evaluated for its ability to calculate the ratio of responses of chambers with different walls,^{36,82} but only a few comparisons are made with experiment. The primary aim in this chapter is to test the EGSnrc code more thoroughly by comparing with a broad range of measurements in ^{60}Co beams.

Simulations of these experiments also present the opportunity to test the transport mechanics of EGSnrc through calculations of the response as a function of cavity height or cavity air pressure. These comparisons with experiment will, at best, only test the transport mechanics at the level of experimental precision, assumed to be ≈ 1 % in most cases. However, recent high-precision measurements (standard uncertainty of ≈ 0.015 %) by Burns *et al*⁵³ of the response to ^{60}Co as a function of cavity volume were also simulated for the opportunity to better confirm the accuracy of the transport algorithms when graphite-walled ion chambers are simulated.

6.2 Experimental data

The majority of experiments of the type discussed above were concisely summarized in a paper by Burlin,⁷⁶ who later contributed additional experiments and discussion in a series of papers on the topic.⁷⁶⁻⁷⁹ Unfortunately, the discussion of his own experiments did not include some important information about the chamber wall thicknesses required for Monte Carlo calculations. However, papers by Attix *et al*,⁴⁷ Whyte,⁷⁴ and Cormack and Johns⁴⁸ contain most of the information needed for calculations.

Additional measurements of the same type by Nilsson *et al.*,⁴⁹ intended for an investigation into perturbation effects, were also simulated. Combined, these experiments provide a comprehensive set of ^{60}Co data covering various chamber geometries and wall materials. In a more recent paper, Burns *et al.*⁵³ reported high-precision measurements but only for a graphite chamber.⁵³ Based on information in the original papers, schematic diagrams of the chambers used in the studies mentioned above are shown in figure 6.1 as they were modeled in this investigation using EGSnrc. Additional details of each experiment are briefly described in the following sections.

Figure 6.1: Schematic diagrams of the five ion chambers investigated in this study as modeled with EGSnrc using the CAVRZnrc user-code.²⁵ An outer layer of lucite (0.19 g/cm^2) was used to support the front wall of the Attix *et al.* chamber when lead walls were used (not shown). The inner lining of the walls in the Nilsson *et al.* chamber were coated with 0.88 mg/cm^2 mylar foils (not shown), one of which was aluminized (back wall/collector) and the other coated with graphite. Sensitive regions of the Attix *et al.* and Nilsson *et al.* chambers are indicated. All the chambers are shown as they were oriented in experiments to detect a fluence of photons incident from the left. Diagrams are not drawn to scale.
(from paper III)

6.2.1 Experimental data of Nilsson *et al*

As part of an investigation into perturbation effects, Nilsson *et al*⁴⁹ used a custom-made, cylindrically symmetric plane-parallel ionization chamber to record, among other things, the response as a function of cavity height in a ^{60}Co beam. The results of interest to this investigation are presented in figure 5 of their paper.⁴⁹ Measurements were obtained for the chamber configured with either polystyrene (C_8H_8), aluminum or lead as a backscatter material (back wall, about 0.5 g/cm^2 thick) and a slab of polystyrene used as a build-up material (front wall, about 0.5 g/cm^2 thick to provide full build-up). The cavity height was taken to be equal to the thickness of ring-shaped spacers placed between the front and back wall which, according to the diagram provided by the authors, had an inner diameter larger than the collimated beam. By determining the cavity height this way, the authors reported an uncertainty in the cavity height of about 0.02 mm, which results in a 2 % uncertainty in chamber response per unit mass at a cavity height of 1 mm. They also reported an uncertainty in the cavity height associated with the bending of the mylar foils (which line the inside of both walls) induced by the applied potential. The potential was adjusted to maintain a constant electric field strength at all cavity heights. Therefore, the walls/liner were likely distorted to the same extent at each cavity height, and reduced the cavity volume relatively more at smaller cavity heights. The effect of this bending was not accounted for in the measurements since, as the authors explain, doubling the potential resulted in only a 1 % decrease in the relative response at a cavity height of 1 mm (which could result from a 0.01 mm distortion). However, these small distortions have a much larger effect at smaller cavity heights, and the implications of not accounting for the effect on the measured response are discussed in section 6.3.

In addition to the measurements discussed above, the response as a function

of the atomic number of backscatter material was also reported for build-up materials of polystyrene, aluminum and lead, and these results were compared to Monte Carlo calculations using the EGS4 Monte Carlo code¹⁷ (from figure 7 of their paper). It is assumed that these measurements were performed with a cavity height of 1 mm.

Since the aim of these experiments was to investigate perturbation effects, and since their interest was to make comparisons with Monte Carlo calculations in some cases, no corrections for attenuation and scatter were reported to have been applied to any of their measurements.

6.2.2 Experimental data of Whyte

Experiments by Whyte,⁷⁴ published in 1957, varied the cavity air pressure rather than the chamber cavity height. However, both methods have the effect of changing the average path length in air (in g/cm²) that an electron must travel to cross the cavity, and so in SA cavity theory varying air pressure is equivalent to changing the cavity dimension. In this experiment, a large air-tight cylindrical ionization chamber made of aluminum was used. The chamber could be fitted with an inner liner and central electrode (≈ 1 cm diameter) made of any desired material. The air pressure in the cavity was controlled by a pump connected directly to the cavity, and the chamber was oriented with its electrode pointed toward a 1.1 TBq ⁶⁰Co source placed 30 cm away (see caption of figure 6.1). The relative measurements of response were corrected for the effects of attenuation and scatter by the wall and aluminum liner by adding sheaths of wall material of known thicknesses to the *outside* of the chamber (i.e. outside the aluminum liner) and measuring the resulting reduction in ionization current. The correction, denoted here by $K_{\text{wall}}^{\text{expt}}$, was taken as the ratio of the response corresponding to zero wall thickness (determined from a linear extrapolation) with

the response corresponding to the nominal thickness, which was not reported but inferred from the schematic diagram provided in the paper. It was assumed that no additional corrections for the mean center of electron production (K_{cep}) were applied to the measurements since it was not mentioned in his paper. Section 6.2.7 discusses how these results were “reverse-corrected” in order to be compared with the EGSnrc calculations since the corrections Whyte used were not reported. The original results of this experiment are from figure 4 of his paper⁷⁴ for wall and electrode materials of beryllium, graphite, aluminum and copper.

6.2.3 Experimental data of Attix *et al*

In 1958, Attix *et al*⁴⁷ measured the response of a large cylindrically symmetric plane-parallel ion chamber to a filtered ^{60}Co beam as a function of cavity height. The 37 GBq ^{60}Co source was filtered by 12 mm of lead, 2.4 mm of tin, 0.5 mm of copper and 0.8 mm of aluminum, presumably in that order, to attenuate the low-energy portion of the spectrum as much as possible. The authors report a 1 m source-to-chamber distance (SCD) for measurements with x-rays, but the low activity of the ^{60}Co source coupled with heavy filtration likely required a shorter SCD (e.g. between 10 and 30 cm) in order to produce the 10^{-12} A currents they wished to measure. Wall materials of graphite, aluminum, copper, tin, and lead were used and the thicknesses of each were reported. Each back wall was supported on a 0.17 g/cm^2 thick layer of polyethylene, and the front wall of the lead chamber was supported on a 0.19 g/cm^2 thick layer of lucite. Measurements were obtained with and without ring-shaped spacers separating the chamber walls. These spacers, which were made of the same material as the walls, had a diameter small enough to sit within the irradiated field, but had an inner diameter large enough to leave a 2 cm thick guard ring of

air around the 5 cm diameter sensitive region. Cavity heights were determined by measuring the capacitance of the walls for the given applied potential, which had the advantage, particularly for thinner high-Z wall materials, of accounting for any distortions of the walls due to the applied potential, and subsequently reducing the uncertainty in the measured cavity volume. Corrections for the effects of attenuation and scatter were applied to the final measurements using measured values of $K_{\text{wall}}^{\text{expt}}$ determined using the same procedure employed by Whyte⁷⁴ discussed above. As such, the original measurements, from figure 10 of their paper⁴⁷ for a ^{60}Co beam, had to be “reverse-corrected” (refer to section 6.2.7) in order to make comparisons with EGSnrc calculations. It was assumed that no K_{cep} corrections were applied to the measured values.

6.2.4 Experimental data of Cormack and Johns

The earliest experimental data investigated in this study were published by Cormack and Johns⁴⁸ in 1954. Similar to Attix *et al*⁴⁷ and Nilsson *et al*,⁴⁹ their experiment made use of a cylindrically symmetric plane-parallel ionization chamber with interchangeable front and back walls. Measurements of the relative ionization current as a function of cavity height resulting from a 111 TBq ^{60}Co source (80 cm SCD, D. Cormack, personal communication, 2006) were recorded for wall materials of graphite, aluminum, copper, silver and lead. The thicknesses required to provide full build-up were determined from the maximum response obtained from measurements of ionization as a function of wall thickness (using a different chamber similar in design and a $4\times 4\text{ cm}^2$ field size). As in the Nilsson *et al* experiment, the cavity height was inferred from the thickness of ring-shaped spacers made of the same material as the wall, and which were within the $10\times 10\text{ cm}^2$ field. Measurements were repeated for the

graphite and lead chambers without the use of spacers, but how the cavity height was determined in this experiment was not discussed. No wall corrections ($K_{\text{wall}}^{\text{exp}}$) were mentioned and so it is assumed they were not used. The results used for comparison with EGSnrc are from figure 5 of their paper.⁴⁸

6.2.5 Experimental data of Burns *et al*

The most recent set of experiments investigated here were reported by Burns *et al*⁵³ in 2007. The goal of their experiment was to derive the air-kerma rate in a ^{60}Co beam from a differential measure of the ionization current with respect to cavity volume using a graphite plane-parallel chamber. Obtaining the air-kerma rate this way is, in principle, more accurate since the measures of cavity volume are considered differentially, and the need to rely on a single measure of cavity volume is avoided. When configured with the smallest cavity height, the chamber used is geometrically similar to the BIPM primary standard. Information on the dimensions and material densities are reported in their paper. For their experiments, cavity heights were varied from 5.15 to 10.13 mm and the cavity volume in each configuration was precisely determined before and after each measurement using a three-dimensional co-ordinate measuring machine. Independent measurements of the ionization were repeated at least three times for five cavity heights (i.e. the chamber was disassembled and re-assembled between measurements), and the standard uncertainty on the repeated measurements was typically 1.5 parts in 10^4 . Experimentally derived corrections for the effects of ion recombination and diffusion, stem scatter, the presence of inhomogeneous materials, and chamber orientation were applied in all cases. Measurements were also corrected for the effects of beam axial non-uniformity (K_{an}) and attenuation and scatter by the walls (K_{wall}) using correction factors calculated with the

PENELOPE Monte Carlo code. Details about the calculated correction factors are discussed in their paper as well as in an earlier paper by Burns.⁸³ For comparisons with EGSnrc calculations, the final measured results, from figure 3 of their paper,⁵³ were divided by their reported K_{wall} and K_{an} values, and normalized by the response of the BIPM standard (also with K_{wall} and K_{an} corrections removed).

6.2.6 EGSnrc calculations of chamber response

All of the ion chambers used in the experiments discussed above are cylindrically symmetric, and so the geometries of each were modeled as shown in figure 6.1 using the CAVRZnrc user-code.²⁵ The only notable approximation made to the geometry descriptions was for the Whyte chamber,⁷⁴ where the electrode was modeled as a perfect cylinder rather than with the hemispherical end as shown in his paper. The effect of this approximation was confirmed to be negligible via calculations with a more accurate geometry (for copper walls at 1 atm) modeled with the EGSnrc C++ class library in the cavity user-code.

Since descriptions of the ^{60}Co sources and enclosures were not given in enough detail to model the sources in the Monte Carlo simulations, a ^{60}Co spectrum calculated by Mora *et al*⁴³ was used as an input in the form of a collimated, isotropic point source in all cases except for the Attix *et al* experiment.⁴⁷ Input spectra for the latter simulation were calculated using FLURZnrc,²⁵ which scored the photon fluence in a 0.1 cm thick vacuum layer (5 cm in diameter) due to a Mora *et al* ^{60}Co point source filtered as described in section 6.2.3. The scattered photons from the filter are included in the spectra computed in this way. This process was repeated for SCDs of 8, 10, 15, 30, 50, 80 and 100 cm since it was not reported which SCD was used for the ^{60}Co measurements. Simulations of the Attix *et al* chamber were also performed

using a phase space from a BEAMnrc⁸⁴ simulation as an input to investigate the effect of electron contamination, and to explain some discrepancies between EGSnrc calculations and measurements discussed in the next section.

In all cases, the calculated results were normalized to the experimental data via multiplication by a factor a such that $\sum (e_i - ac_i)^2$ is minimized, where e_i and c_i are the i^{th} experimental and corresponding EGSnrc-calculated values, respectively. It can be shown that $a = \sum e_i c_i / \sum c_i^2$. The root mean squared deviation (RMSD) of a data set is given by $\sqrt{\sum (e_i - ac_i)^2 / n}$.

6.2.7 Calculated K_{wall} corrections

Since the response of an ion chamber is proportional to the product of the stopping-power ratio and $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}^{\text{air}}$ after correction for effects such as wall attenuation and scatter, Attix *et al*⁴⁷ and Whyte⁷⁴ corrected measurements of chamber response for the effects of attenuation and scatter using measured K_{wall} correction factors, denoted here as $K_{\text{wall}}^{\text{expt}}$. The goal in this case was to compare the chamber responses without these corrections but, unfortunately, the respective $K_{\text{wall}}^{\text{expt}}$ corrections used were not reported in either of these studies. It is assumed here that the $K_{\text{wall}}^{\text{expt}}$ corrections are equivalent to the response of the chamber divided by the response measured when the thickness of the walls was doubled. The values of $K_{\text{wall}}^{\text{expt}}$ were thus determined using calculated responses. Determining the corrections in this way, although the common practice at the time, has since been shown to be inaccurate.^{85–87} The experimental results of Attix *et al*⁴⁷ and Whyte⁷⁴ without K_{wall} corrections are referred to as reverse-corrected responses.

6.3 EGSnrc comparisons with experiments at ^{60}Co energies

Measurements and EGSnrc calculations of the response as a function of cavity height for the plane-parallel chamber used by Nilsson *et al*⁴⁹ are shown in figure 6.2. Included in figure 6.2 are the experimental data as they were originally presented along with the same data corrected for a possible systematic error in the cavity volume associated with the inward bending of the wall due to the applied potential (see section 6.2.1). As with the original measured data, the corrected experimental results were normalized relative to the response of the polystyrene (C_8H_8) chamber at a cavity height of 1 mm. The cavity heights used for the corrected experimental data were reduced by 0.01 mm for the polystyrene back wall, 0.02 mm for the aluminum back wall and 0.03 mm for the lead back wall, chosen arbitrarily to give the best fit with calculated results. It can be seen that a 0.03 mm reduction in the cavity height increased the measured response by 6 % at a cavity height of 0.5 mm for the lead chamber. Normalized collectively to the corrected experimental results, the root-mean-squared deviation (RMSD) of EGSnrc calculations from experiment, expressed as a fraction of the experimental value for each wall material and cavity height is 1.4 %. The average RMSD for each wall material normalized independently is also 1.4 %.

Although Nilsson *et al* experimentally confirmed the bending of the mylar foils lining the inside of the chamber walls, the use of cavity height corrections with their experimental data to account for the associated decrease in cavity volume deserves further justification. In their paper, the decrease in response observed with increasing cavity height was attributed to a decrease in the relative contribution to ionization from backscattered electrons. The broad angular distribution of these low-energy electrons results in a large fraction of them directed outside the collecting volume

Figure 6.2: Measured and EGSnrc-calculated response as a function of cavity height for the plane-parallel ion chamber used by Nilsson *et al.*⁴⁹ The front wall is made of polystyrene (C_8H_8) in all cases. The dashed lines represent experimental data corrected for the possibility of small reduction in the cavity height due to the applied potential between the front and back wall (details discussed in the text). Statistical uncertainties on the EGSnrc results are 0.2 % or less, and the experimental measurements were reported to be reproducible to within 0.8 % (1σ). (paper III)

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ^{60}CO ENERGIES

without an equal number directed back in (recall the outer walls are outside the beam), where the fraction escaping increases with cavity height. According to this reasoning, however, the response at a cavity height of 0.5 mm should be larger than at 1 mm, which is not the case in the uncorrected measurements for the chamber configured with aluminum and lead backscatter materials (figure 6.2). One can also expect a larger response at smaller cavity heights from the predictions of SA cavity theory since the SA ratio of mass-restricted stopping-powers, $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{med}}^{\text{air}}$, increases with decreasing cavity size when the atomic number of the wall is larger than the atomic number of the cavity gas, as demonstrated in the other experiments.^{47,78} The fact that the measured data without cavity height corrections does not reflect these expectations or the results of previous experiments suggests that cavity height corrections cannot be ignored in this case. The corrections to the cavity heights used here were chosen to show that a small correction can have a big effect on the comparison with EGSnrc calculations, and does not imply that those corrections are the most appropriate.

Figure 6.3 shows comparisons between their measurements and EGS4 calculations of chamber response as a function of backscatter material with calculations using EGSnrc (using a 1 mm cavity height and assuming no bending of the walls). Three build-up materials (polystyrene, aluminum and lead) were investigated. For the extreme case of a lead backscatter material, calculations using EGS4 underestimated the experimental response by as much as 8 % for a lead build-up material. The largest discrepancy using EGSnrc is 3 %, and the RMSD is 1.4 %.

The measured (reverse-corrected, see section 6.2.7) and EGSnrc-calculated response for the chamber used by Whyte⁷⁴ as a function of cavity air pressure are shown in figure 6.4. The RMSD of the EGSnrc calculations, normalized collectively to all the experimental results, is 0.5 %. However, the RMSD is within 0.3 % if the calculated and measured data sets are normalized separately for each wall material.

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ⁶⁰CO ENERGIES

Figure 6.3: Measured and EGS4/EGSnrc-calculated responses as a function of the atomic number of the back wall material for the plane-parallel chamber investigated by Nilsson *et al.*⁴⁹ Results are shown for polystyrene (C_8H_8), aluminum and lead used as build-up materials. A 1 mm cavity height was assumed for the calculations. Measurements and EGS4 calculations appear as they were originally presented.⁴⁹ Statistical uncertainties on the EGSnrc calculations are 0.2 % or less, and the experimental measurements were reported to be reproducible to within 0.8 % (1σ). Statistical uncertainties on the EGS4 calculations were better than 1 %.

(from paper III)

Figure 6.4: Measured (reverse-corrected, section 6.2.7) and EGSnrc-calculated response as a function of cavity air pressure for the cylindrical ion chamber used by Whyte.⁷⁴ The experimental results are normalized to the response of the graphite chamber at 1 atm. Statistical uncertainties on the calculated responses are 0.2 % or less. Experimental uncertainties were not reported.

(from paper III)

The measured (reverse-corrected, see section 6.2.7) and EGSnrc-calculated response as a function of cavity height for the Attix *et al* plane-parallel chamber with spacer rings are shown in figure 6.5(a). The EGSnrc results shown are for an SCD of 10 cm since this yielded the best results, and are normalized with respect to the measured data for aluminum, copper and tin. The normalized results calculated for these three materials are generally within 2 % of measurements (RMSD of 1.3 %), with a maximum discrepancy of 3 % (tin chamber, 10.3 mm cavity height). For the graphite and lead chambers, the relative calculated responses are as much as 9 % below and 17 % above measurements, respectively. Figure 6.5(b) shows the same comparison for the chambers without spacers. As in figure 6.5(a), the calculations were normalized with respect to the measured data for aluminum, copper and tin, and the RMSD for these three materials is approximately 1 %. Discrepancies for the graphite and lead data are as large as for the chambers with spacers. If the absolute discrepancies observed with these chambers in figure 6.5(a) and 6.5(b) are ignored, then the relative agreement as a function of cavity height for a given wall material is within 2 % (RMSD of approximately 1 %).

The possibility that electron contamination from the source could account for the observed discrepancies was also investigated since the wall of the graphite chamber is not quite thick enough for full build-up.^{47,88} The BEAMnrc user-code⁸⁴ was used to score a phase space at the position of the front face of the ion chamber due to a ⁶⁰Co point source (Mora spectrum⁴³) filtered as described in section 6.2.3, which was then used as an input to the CAVRZnrc calculations. Relative to the response of the aluminum chamber, the calculated response of the graphite chamber was within 1 % of measurements for the chamber with side walls when electron contamination is accounted for (results shown in figure 6.5(a)). This is a significant improvement over the results discussed above for the graphite chamber. However, using this same phase

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ⁶⁰CO ENERGIES

(a)

(b)

Figure 6.5: Measured (reverse-corrected, section 6.2.7) and EGSnrc-calculated response as a function of cavity height for the plane-parallel chamber used by Attix *et al*⁴⁷ with (a) and without (b) ring-shaped spacers. Measured data are normalized to the response for the graphite chamber at the smallest cavity height. EGSnrc results for aluminum, copper, and tin wall materials are normalized collectively to the respective experimental results as discussed in section 6.2.6 Statistical uncertainties on the calculated responses are 0.2 % or less. Experimental uncertainties were not reported. (from paper III)

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ^{60}Co ENERGIES

space source as an input to the calculations with the lead chamber did not resolve the discrepancies with those measurements.

Measurements and EGSnrc-calculated ionization currents for the Cormack and Johns chamber⁴⁸ with spacers are compared in figure 6.6(a). Results were collectively normalized to experimental values using all the results except those for lead. The RMSD is 1.5 % for atomic number ranging from 6 (graphite) to 47 (silver), with a maximum discrepancy of 2.5 % (silver wall, 8.6 mm cavity height). Discrepancies with the lead chamber exceed 35 % and increase with increasing cavity height. The results for the chamber without spacers are shown in figure 6.6(b) for graphite and lead wall materials (measurements for the other wall materials were not provided). In this case, a much better agreement between calculations and measurements was obtained with the lead chamber relative to graphite (RMSD of 1.5 %). Normalized independently, the average RMSD was ≈ 1.4 % for each wall material excluding the lead chamber with spacers.

The results of measurements and EGSnrc calculations of the response as a function of cavity height for the graphite chamber used by Burns *et al*⁵³ is shown in figure 6.7. The fractional RMSD of the EGSnrc calculations from the measured values is 0.03 %, although there is a clear, statistically significant difference in the respective trends (maximum discrepancy of 0.04 %). The overall variation of the measured data is 0.7 % while only a 0.6 % variation was calculated by EGSnrc. By comparison, a 0.8 % variation was calculated by Burns *et al*⁵³ using the PENELOPE Monte Carlo code (see figure 2 of their paper).

The experiments by Burns *et al* were simulated to stringently test the transport mechanics in EGSnrc, and the 0.03 % agreement with this experiment confirms previous checks on the transport mechanics based on the Fano cavity test. However,

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ⁶⁰CO ENERGIES

(a)

(b)

Figure 6.6: Comparison of EGSnrc-calculated ionization currents as a function of cavity height with those measured by Cormack and Johns⁴⁸ using a plane-parallel chamber with (a) and without (b) spacer rings. The measured values appear as originally presented. Statistical uncertainties on the calculated results are 0.2 % or less. Uncertainties on the experimental results were not reported.

(from paper III)

6.3. EGSNRC COMPARISONS WITH EXPERIMENTS AT ^{60}Co ENERGIES

Figure 6.7: Measured and EGSnrc-calculated response of the variable-volume chamber used by Burns *et al.*⁵³ The reported K_{wall} and K_{an} corrections were removed from the experimental data for this comparison. The measured results are normalized to the response of the BIPM primary standard (also with K_{wall} and K_{an} corrections removed). The uncertainty bars on the measured data, where visible, represent the statistical standard deviation of the mean of the measurements for each cavity height. The EGSnrc calculations have statistical uncertainties of 0.01 %. Experimental uncertainties represent the standard deviation of repeated measurements and are within 0.015 %.

(from paper III)

in their paper, Burns *et al*⁵³ discuss several experimental challenges associated with determining the response at this level of accuracy. In particular, the observed trend in the experimental data may be misrepresenting the true variation due to the effect of a physical phenomenon which is similar to a systematic offset in the cavity volume determination. One possibility is that the “effective” cavity volume is overestimated by the mechanical measurements from which it is derived due to a constant volume region in the cavity where charge is not collected.⁵³ The volume reduction required to bring consistency in the results is estimated to be about 10 mm³ (0.15 % of cavity volume at smallest cavity height), and taking this into account increases the variation of the experimental data shown in figure 6.7 by 10 % (from 0.7 % to 0.77 %). As a consequence of taking this into account, the RMSD of the EGSnrc calculations would double from 0.03 % to 0.06 % (but with a maximum discrepancy of only 0.11 %). Although this level of agreement is still comparable to the results of Fano cavity tests, it is clear that the comparison is very sensitive to the active volume of the cavity.

6.3.1 Discrepancies between measurements and calculations for the Attix *et al* and Cormack and Johns chambers

Several possibilities were investigated to try to resolve the discrepancies seen in figures 6.5(a)-(b) and 6.6(a) for the lead chambers used by Attix *et al* and Cormack and Johns chambers. The details are discussed elsewhere.⁸⁸ For the Attix *et al* chamber, the discrepancy could not be accounted for by the details of the incident source, the source-to-chamber distance, or impurities in the wall. Nor could this discrepancy be explained by calculated values of $K_{\text{wall}}^{\text{expt}}$ used to reverse-correct the data. If one ignores corrections for attenuation and scatter, axial non-uniformity, and possible fluence corrections, then the relative response of the chamber for any given wall material

is proportional to $\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{en}}{\rho}\right)_{\text{air}}^{\text{wall}}$. Under these ideal conditions, the response of the lead chamber should be approximately three times larger than the response of the graphite chamber. The EGSnrc calculations in figures 6.5(a) and (b) reflect this difference moreso than the measurements, and so it is possible that the measured data were improperly normalized.

For the Cormack and Johns chamber with lead walls, the discrepancies between calculations and measurements only applies to the lead chamber with spacers (figure 6.6(a)). Notice, however, that the ionization *measured* with the lead chamber, relative to that measured with graphite chamber, increases when the spacers are removed. This is opposite to what is observed with the measurements by Attix *et al* in figures 6.5(a)-(b), and contrary to what is expected given that the placing a high-Z wall material on each side of the cavity region should increase the fluence of electrons in the cavity, thereby increasing the relative response. Based on this, one can only speculate that the values published by Cormack and Johns for the lead chamber with spacers represent data for another combination of wall materials.

6.4 Summary

In this investigation, the EGSnrc Monte Carlo code was evaluated for its ability to calculate ion chamber response to ^{60}Co using experimental data as benchmarks. With the exception of the Attix *et al* results for two wall materials and one set of results for Cormack and Johns, the RMSD of EGSnrc calculations of chamber response in a ^{60}Co beam as a function of cavity height or cavity air pressure are generally within 1.5 % when compared collectively for a variety of chamber wall materials and cavity dimensions/air pressures which show variations of up to 300 %. The

level of agreement one can expect in these comparisons depends on the accuracy of both the underlying cross sections used and on the uncertainty on the experimental measurements, which was assumed to be $\approx 1\%$ for the older experiments. These comparisons are therefore consistent with expectations. When the results for each wall material are considered independently, the comparisons are much less dependent on the uncertainties in the cross sections and the RMSD of EGSnrc calculations and measurements range between 0.03-1.4 %. This is comparable to the estimated uncertainty on the experiments except those of Burns *et al.*⁵³ Simulations of the latter high-precision experiments were within 0.03-0.06 % depending on the cavity volume assumed in the measurements, which experimentally confirms the accuracy of the transport mechanics used by EGSnrc at these energies established in previous theoretical investigations.

Chapter 7

Accuracy of the Spencer-Attix stopping-power ratio and fluence correction factors for the ^{60}Co air kerma formalism

In the previous chapter, it was briefly mentioned that the experiments simulated with EGSnrc were originally intended to evaluate Spencer-Attix (SA) cavity theory. Given the proven ability of EGSnrc to calculate the response of ion chambers to ^{60}Co beams, particularly as a function of cavity size, one can use the calculated results presented in chapter 6 to test SA cavity theory as it applies to the air kerma formalism. Comparing the theory to Monte Carlo-calculated results ensures that any observed discrepancies are not attributed to uncertainties in cross-section data since a consistent set of cross-sections is used in all calculations. As will be shown, SA cavity theory is, in general, unable to predict chamber response at the level of accuracy required for primary

and secondary standards laboratories although for graphite-walled ion chambers it is remarkably accurate. Following this comparison, a more in-depth analysis of SA cavity theory will be performed using the calculated response of an idealized plane-parallel chamber in order to determine the exact source of its limitations.

7.1 Theory

Applied to ion chambers, the defining equation for SA cavity theory (equation 1.12) can be used to relate the dose to the air in the cavity of an ion chamber, D_{air} , to the dose to the wall medium, D_{wall} , given by

$$\frac{D_{\text{air}}}{D_{\text{wall}}} = \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}}, \quad (7.1)$$

where $\left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}}$ is given by *the inverse of* equation 1.13 with “det” and “med” replaced with “air” and “wall”, respectively. In the absence of attenuation and scatter, which is required in order for the above equation to hold, $D_{\text{wall}} = \Psi \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}$, where $\left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}}$ is the mass-energy absorption coefficient appropriately averaged over the incident fluence, Ψ . Inserting this into equation 7.1, the dose to air *per incident fluence* (D_{air}/Ψ) becomes

$$\frac{D_{\text{air}}}{\Psi} = \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho} \right)_{\text{wall}} \quad (7.2)$$

It should be emphasized that equation 7.2 only applies under the conditions described in section 1.4. In practice, values of D_{air} , whether calculated or measured, must be corrected for the effects of attenuation and scatter (K_{wall}), axial non-uniformity of the beam (K_{an}), inhomogeneous construction of the chamber (K_{comp}), and other non-ideal conditions (K). Applying these correction factors to equation 7.2 yields an expression

for D_{air}/Ψ given by

$$\frac{D_{\text{air}}}{\Psi} = \frac{\left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}}{K_{\text{wall}}K_{\text{an}}K_{\text{comp}}K}, \quad (7.3)$$

where K_{wall} , K_{an} , K_{comp} , and K are as defined in chapter 3. Equation 7.3 provides a way of predicting the response per unit incident fluence of an ion chamber using SA cavity theory. The ability of SA cavity theory to account for the cavity-size dependence of chamber response presented in chapter 6 (figures 6.2, 6.4-6.7) is attributed to the low-energy cut-off, Δ , in the SA stopping-power ratio.

In Monte Carlo investigations of ion chamber response, the geometry and physics of the simulation can be defined such that the various correction factors discussed in equation 7.3 (except for K_{wall}) are, by definition, unity.^{52,83} Under these simulation conditions, the accuracy of SA cavity theory is limited by the accuracy of the SA stopping-power ratio. Recognizing this, Borg *et al*⁴⁴ introduced an additional correction factor, K_{SA} , to account for the discrepancy between the response of ion chambers and the predictions of SA cavity theory. They calculated K_{SA} as a function of energy (from 100 keV to a few MeV) for the graphite chambers used for ^{60}Co air kerma standards at NRC, BIPM, and NIST, and showed it ranged between 0.9970 and 1.0005 for a ^{60}Co beam depending on the chamber and the value of Δ used in the calculation of stopping-power ratio. Buckley *et al*³⁶ later showed that K_{SA} is unity for one particular graphite thimble chamber at ^{60}Co energies, but is a 0.5 % correction (1.0050 ± 0.0003) for the same chamber configured with an aluminum thimble. Neither study, however, determined whether the K_{SA} correction was due to fluence perturbations by the cavity, or due to the wrong choice of Δ .

Unlike the previous investigations discussed above, this chapter focuses on the accuracy of SA cavity theory as it applies to the air kerma formalism for ^{60}Co beams (discussed in chapter 3). Since the discussion that follows excludes low-energy photon

beams where several assumptions of SA cavity theory potentially break down, the K_{SA} correction discussed above is replaced by a fluence correction (K_{fl}) to account for the perturbation of the charged particle fluence in the wall, Φ , by the cavity. Under simulation conditions where $K_{an} = K_{comp} = K = 1.000$, K_{fl} is given as

$$K_{fl} = \frac{\left(\frac{\bar{L}_\Delta}{\rho}\right)_{wall}^{air} \Psi\left(\frac{\bar{\mu}_{en}}{\rho}\right)_{wall}}{D_{air} K_{wall}} = \frac{\left(\frac{\bar{L}_\Delta}{\rho}\right)_{wall}^{air} (K_c)_{wall}}{D_{air} K_{wall}}, \quad (7.4)$$

where $(K_c)_{wall}$ is the collision kerma in the wall. The accuracy of SA cavity theory as it is applied in the air kerma formalism can be revealed through calculated values of K_{fl} given by equation 7.4 as a function of cavity height and wall material. Values of K_{fl} that are unity within statistical uncertainties are an indication that the underlying assumptions of the theory are applicable.

7.2 Methods

7.2.1 EGSnrc calculations of chamber response

Following the approach of the original experiments in the 1950s and 60s, SA cavity theory is extensively tested via comparisons with EGSnrc-calculated values of chamber response. Comparisons are made with the calculated response, rather than the measured response, in order to ensure that any observed discrepancies are not attributed to uncertainties in cross-section data since a consistent set of cross-sections are used in all calculations.^{44,52,89} Several sets of ion chamber response calculations at ^{60}Co energies are used as a benchmark for testing the prediction of SA cavity theory, including simulations of experiments by Whyte⁷⁴ and Cormack and Johns⁴⁸ presented in chapter 6, and calculations of response for an “idealized” plane-parallel

(pp) chamber configured with a variety of wall materials and cavity heights. The latter pp chamber was simulated without an insulator or collector (i.e., homogeneous walls), and no guard ring was included. To provide full build-up, the walls were set slightly thicker than the CSDA range of the maximum-energy electrons in the charged particle fluence spectrum. The radius of the cavity was 1.2 cm and was filled with dry air in the simulations.

In order to investigate the importance of correction factors, the predictions of SA cavity theory are compared with chamber response calculations of the Whyte and Cormack chambers using both equations 7.2 and 7.3. Calculations of response for the former comparison are taken from chapter 6. For comparisons using equations 7.3, calculations of response were performed using the photon regeneration technique discussed in section 2.2 for a broad parallel beam incident on the chambers modeled with a single wall material. The calculated dose to air per unit incident fluence under these simulation conditions is denoted as $D_{\text{air}}^{\text{Fano}}$, where the superscript “Fano” is used to represent the fact that $D_{\text{air}}^{\text{Fano}}$ is calculated for a parallel beam in the absence of attenuation and scatter. $D_{\text{air}}^{\text{Fano}}$ can be equated to D_{air} by

$$D_{\text{air}}^{\text{Fano}} = D_{\text{air}} K_{\text{wall}} K_{\text{an}} K_{\text{comp}} K, \quad (7.5)$$

In other words, $D_{\text{air}}^{\text{Fano}}$ is directly proportional to the fully corrected chamber response. Determining $D_{\text{air}}^{\text{Fano}}$ in this way is equivalent to calculating D_{air} and each of the individual correction factors as defined in chapter 3.

Calculations of $D_{\text{air}}^{\text{Fano}}$ were also performed for the idealized pp chamber. The purpose in this case is to evaluate the source of any discrepancy between the fully corrected chamber response and SA cavity theory. As briefly mentioned previously, discrepancies are potentially due to the perturbation of the electron fluence by the

cavity, or by the wrong choice of Δ in the SA stopping-power ratio. As such, these two parameters are isolated from each other so that they can be calculated using firmly established methods. Using equation 7.4, the product of K_{fl} and $\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{air}}^{\text{wall}}$ is given by

$$K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}} = \frac{(K_{\text{c}})_{\text{wall}}}{D_{\text{air}}^{\text{Fano}}}. \quad (7.6)$$

From the Fano theorem^{1,40} (section 2.4), $(K_{\text{c}})_{\text{wall}} = D_{\text{wall gas}}^{\text{Fano}}$, where “wall gas” refers to the dose to the cavity filled with low-density wall material with the same dosimetric properties as the wall. Therefore,

$$K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}} = \frac{D_{\text{wall gas}}^{\text{Fano}}}{D_{\text{air}}^{\text{Fano}}}. \quad (7.7)$$

As with the Fano cavity tests discussed in section 2.4 the wall gas has the same density as air (1.205 kg/m³). Comparing calculations of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{air}}^{\text{wall}}$ from equations 7.6 and 7.7 is equivalent to performing the Fano cavity test for a range of wall materials and cavity dimensions (i.e., comparing $D_{\text{wall gas}}^{\text{Fano}}$ to $(K_{\text{c}})_{\text{wall}}$), which serves as a useful check of the transport mechanics of the code in addition to Fano tests performed previously. All calculations of chamber response in this chapter were performed with the CAVRZnrc user-code. Calculations of collision kerma were calculated with the g user-code.

7.2.2 Calculation of Spencer-Attix stopping-power ratios

The SA stopping-power ratios required for the calculation of K_{fl} are calculated with SPRRZnrc user-code following the approach described by Borg *et al*⁴⁴ and discussed in section 2.2. We are interested in stopping-powers for an unattenuated, unscattered photon beam as the theory requires. Therefore, as for the CAVRZnrc calculations,

these calculations were also performed with the photon regeneration option selected. The stopping-power ratio was scored in the central region of a uniform cylinder of wall material surrounded with enough material to provide full build-up. In principle, the results are independent of geometry.^{25,44} As in section 1.4.2, the values of Δ for these calculations, which determine which set of restricted stopping-powers are used in the simulations, were set to the kinetic energy of an electron with a CSDA range in air equal to L , given by $4V/S$. Since L depends on cavity size, separate PEGS4 data sets of restricted stopping-powers were created for each cavity height, where the low-energy cut-off (AE) is set equal to the corresponding value of Δ (actually set to $\Delta + 511$ keV rest mass). This is unlike the calculation of chamber response where AE was set to 1 keV, regardless of cavity size. Table 7.1 lists the values of L and Δ for each cavity height along with the wall-to-air stopping-power ratios for graphite, aluminum, copper, and lead wall materials.

In section 1.4.2, calculations of SA wall-to-air stopping-power ratios were calculated for values of Δ from 1 keV to 60 keV, in 1 keV intervals. These values have been fit to a functional form for $\left(\frac{\bar{L}_\Delta}{\rho}\right)_{\text{air}}^{\text{wall}}$, given as

$$\left(\frac{\bar{L}_\Delta}{\rho}\right)_{\text{air}}^{\text{wall}} = a + b \ln(\Delta) + \frac{c}{\sqrt{\Delta}} + \frac{d}{\Delta^2}, \quad (7.8)$$

using the method of least squares, where a , b , c and d represent the best fit coefficients (see table 7.2) but which do not possess physical meaning. Equation 7.8 reflects the monotonic trend of the stopping-power ratios as a function of Δ . In all cases, the fit was within 0.1 % of the calculated values, and within 0.03 % for the graphite-to-air stopping-power ratio.

7.2. METHODS

Table 7.1: Values of the mean chord length and Δ at each cavity height for the plane-parallel chamber used in this study. The mean chord length, L , was estimated as $4V/S$, while values of Δ were taken as the kinetic energy of an electron with a CSDA range in dry air = L . Also listed are the Spencer-Attix wall-to-air stopping-power ratios for graphite, aluminum, copper and lead calculated using SPRRZnrc. Stopping-powers are from ICRU Report 37.⁸

cavity height (cm)	L (cm)	Δ (keV)	$\left(\frac{L\Delta}{\rho}\right)_{\text{air}}^{\text{wall}}$			
			graphite	Al	Cu	Pb
0.05	0.096	5.7	1.00260(8)	0.85476(3)	0.7037(1)	0.4817(2)
0.075	0.141	7.1	1.00232(8)	0.85741(2)	0.7091(1)	0.4915(2)
0.1	0.185	8.3	1.00217(8)	0.85908(2)	0.71268(7)	0.4975(2)
0.15	0.267	10.3	1.00180(8)	0.86138(2)	0.71724(5)	0.5056(2)
0.2	0.343	12.0	1.00151(8)	0.86289(2)	0.72024(5)	0.5107(2)
0.3	0.480	14.5	1.00123(8)	0.86457(2)	0.72364(4)	0.5166(2)
0.4	0.600	16.6	1.00112(8)	0.86581(2)	0.72604(4)	0.5205(2)
0.5	0.706	18.2	1.00099(8)	0.86659(2)	0.71747(4)	0.5227(2)
0.6	0.800	19.6	1.00098(8)	0.86721(2)	0.72869(4)	0.5249(2)
0.8	0.960	21.8	1.00073(8)	0.86800(2)	0.73040(4)	0.5276(2)
1.0	1.091	23.4	1.00066(8)	0.86859(2)	0.73148(3)	0.5293(2)
1.2	1.200	24.8	1.00067(8)	0.86901(2)	0.73232(3)	0.5306(2)

Table 7.2: Values of fit coefficients for the least-squares fit of equation 7.8 to values of the wall-to-air SA stopping-power ratio as a function of Δ for ^{60}Co beams ($1 \text{ keV} \leq \Delta \leq 60 \text{ keV}$) using stopping-powers from ICRU Report 37.⁸

material	a	b	c	d
beryllium	0.9176(4)	-0.00168(7)	0.0140(6)	0.0009(3)
graphite	1.0023(4)	-0.00077(8)	0.0040(7)	0.0008(3)
aluminum	0.8674(1)	0.00322(2)	-0.0433(2)	-0.0036(1)
copper	0.7317(2)	0.00583(5)	-0.0897(4)	-0.0156(3)
silver	0.6535(4)	0.00697(9)	-0.1176(8)	-0.0310(4)
tin	0.6294(6)	0.0074(1)	-0.115(1)	-0.033(5)
lead	0.529(1)	0.0098(2)	-0.144(2)	-0.100(1)

7.2.3 Calculation of fluence correction factors (K_{fl})

Values of K_{fl} were obtained by dividing the calculated value of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ (equations 7.6 or 7.7) at each cavity height by the corresponding value of $\left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ listed in table 7.1 (i.e. assuming the $L = 4V/S$ prescription is correct). For these calculations, the average of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ calculated from equations 7.6 and 7.7 was used since, as will be shown, the two methods give nearly the same numeric result and should be equivalent. Using the average value of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ also reduces the statistical uncertainty on K_{fl} .

Values of K_{fl} were also calculated for several of the graphite-walled ion chambers used as ^{60}Co air kerma standards at national metrology institutes (NMIs). The graphite densities and dimensions of the various chambers were obtained from the references listed in the BIPM report by Allisy-Roberts *et al.*,⁹⁰ but were modeled without insulating materials as the theory requires. As with all calculations with graphite in this investigation, the graphite stopping-powers for these calculations correspond to those with an ICRU-recommended⁸ mean ionization energy (I value) of 78 eV. While there is a real issue about what the proper I value is for graphite, using a different I value should have a negligible effect if the same I value is used for all calculations of K_{fl} for a given wall material. The effect of the graphite density and I value on calculations of K_{fl} is explored by calculating K_{fl} as a function of both. The effect of the incident spectrum and transport parameters on K_{fl} is also investigated.

7.3 Limitations of Spencer-Attix cavity theory

Comparisons between calculations of chamber response without correction factors and the predictions of the basic SA cavity theory (equation 7.2) are shown in figure 7.1

for the chambers used by Whyte and Cormack. Results are shown for wall materials ranging from beryllium to silver. Relative to the response of the graphite chamber at atmospheric pressure (Whyte chamber) or with a 0.95 cm cavity height (Cormack and Johns chamber), the response predicted by SA cavity theory differs in magnitude from CAVRZnrc calculations, and hence from measurements, by as much as 8.5 %. Equation 7.2 also fails to predict the response with changes in cavity height or air pressure in some cases. However, the predicted variation in response of the graphite chamber, to the extent that there is variation, is within 0.3 % and 1.5 % for the Whyte⁷⁴ and Cormack and Johns⁴⁸ chambers, respectively.

It is clear from figure 7.1 that correction factors for non-ideal conditions can not be ignored when using SA cavity theory to predict chamber response. Figure 7.2 shows the same comparison discussed above with the inclusion of K_{wall} , K_{an} , and K_{comp} correction factors (i.e. using equation 7.3). Although some discrepancies larger than 1 % still remain for chambers with high- Z walls, the overall agreement is significantly improved. For the Whyte chamber, the improvement is mostly due to the effect of the K_{wall} and K_{an} corrections on the relative magnitude of the responses rather than the response as a function of pressure. For the Cormack and Johns chamber, the use of correction factors improved agreement both in terms of the relative magnitude of the predicted response and the variation with cavity height.

The effect of each individual correction for the Whyte chamber with copper walls is shown in figure 7.3. In this example, the effect on response of the K_{wall} correction *relative to* the K_{wall} correction for graphite is shown to be much larger than the relative effect of the K_{an} and K_{comp} corrections. A 1.5-2.0 % discrepancy remains between the calculated response and the fully-corrected SA prediction in this case, and similar discrepancies are observed with the Cormack and Johns chamber. These remaining discrepancies are not related to uncertainties in cross-section data since

7.3. LIMITATIONS OF SPENCER-ATTIX CAVITY THEORY

(a)

(b)

Figure 7.1: EGSnrc-calculated values of response as a function of cavity air pressure for the Whyte chamber⁷⁴ (a), and as a function of cavity height for the Cormack and Johns chamber⁴⁸ (b). Also shown is the product of $\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{wall}}^{\text{air}}$ and $\left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}$ for each wall material. (from paper IV)

7.3. LIMITATIONS OF SPENCER-ATTIX CAVITY THEORY

(a)

(b)

Figure 7.2: As in figure 7.1 with values of $\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{wall}}^{\text{air}} \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{wall}}$ corrected by calculated values of $K_{\text{wall}} K_{\text{an}} K_{\text{comp}}$.
(from paper IV)

7.3. LIMITATIONS OF SPENCER-ATTIX CAVITY THEORY

the cross-sections used were common to both sets of calculations in this comparison. Rather, this remaining discrepancy is attributed to the perturbation of the charged particle fluence by the cavity as discussed in chapter 3, or an incorrect selection of Δ . It is clear from this result that an additional fluence correction (K_{fl}) is required to ensure the accuracy of SA cavity theory in general, or an improved method to select Δ .

Figure 7.3: As in figures 7.1 and 7.2 for the Whyte chamber with copper walls. The relative effect of each correction factor on the comparison is also shown by dividing the values of $\left(\frac{\bar{L}\Delta}{\rho}\right)_{Cu}^{air} \left(\frac{\mu_{en}}{\rho}\right)_{Cu}$ by one correction at a time. The effect of the K_{comp} correction is small ($< 0.1\%$) relative to the effect of K_{wall} and K_{an} and is therefore not included in this plot. (from paper IV)

7.4 Perturbation of charged particle fluence by the cavity

The need for a fluence correction can be seen by examining the effect of a cavity on the charged particle fluence in the wall medium. Figure 7.4 shows the EGSnrc-calculated spectrum of charged particles in the cavity region of the idealized plane-parallel ion chamber described in section 7.2.1 for two cavity sizes and wall materials. Photon regeneration was used in these calculations since we are interested in the case where the effects of attenuation and scatter have been removed. Otherwise, this calculation is the same as one performed in a previous investigation.⁹¹

Fluence spectra were calculated for cavities filled with either air or wall material. As shown, the air cavity appears to have only a small effect on the fluence in the graphite wall regardless of cavity size. For the extreme case of a lead chamber, the air cavity reduces the charged particle fluence significantly at low energies (i.e. < 0.1 MeV).

Given the results shown in figure 7.4, one could argue that fluence corrections should be included in the air kerma formalism regardless of whether or not they could be made unity via an appropriate selection of Δ in the SA stopping-power ratio. However, the fluence perturbations occur primarily at low energies, and may therefore not affect the evaluation of the SA stopping-power ratio since it only considers the fluence above Δ (recall that the track-end term does not include charged particles *created* below Δ , only those that “slow down” to that energy). For example, the fluence perturbation for the graphite chamber is only visible below about 10 keV, and about 50-60 keV for the lead chamber (1.0 cm cavity height). Calculations for aluminum and copper chambers reveal that fluence perturbations occur primarily

Figure 7.4: Fluence of charged particles in the cavity region of an idealized plane-parallel ion chamber in a ^{60}Co beam. The effects of attenuation and scatter have been removed by using photon regeneration. Results are shown for 0.1 and 1.0 cm cavity heights, and for wall materials of graphite and lead. (from paper IV)

below 30 and 40 keV, respectively. Thus the need for fluence corrections will depend on whether or not there exists a value of Δ that can exclude them, and whether or not this value can be determined *a priori*.

7.5 Fraction of chamber response due to charged particles below Δ

Figure 7.4 shows one example where the low-energy electron fluence in the wall medium can be significantly perturbed by the cavity. Using SA cavity theory in these situations may therefore be inappropriate if one considers the contribution of low-energy electrons to chamber response. Figure 7.5 shows the fraction of the total response due to electrons below a cut-off energy, Δ , $[D_{\text{air}}^{\text{Fano}}(E_{e^-} < \Delta)]$ as a function of cavity size as calculated with EGSnrc. The value of Δ used with each cavity height is given in table 7.1, and is the same value that would be used with the SA stopping-power ratio.

As for the results in figure 7.4, the calculations were performed with photon regeneration to remove the effects of attenuation and scatter. Regardless of wall material, the contribution of low-energy electrons to chamber response, expressed as a fraction of the total response, increases monotonically with cavity size, and ranges between ≈ 20 and 30 %. This contribution also increases as the atomic number of the wall increases. Notice that $D_{\text{air}}^{\text{Fano}}(E_{e^-} < \Delta)$ is larger than the track-end term in the SA stopping-power ratio, $\Phi_{\text{wall}}(\Delta) \left(\frac{S(\Delta)}{\rho} \right)_{\text{air}} \Delta$ (equation 1.13), since it *includes* the electrons created below Δ in addition to those that slow down to Δ from higher energies.

Figure 7.5: Fraction of dose due to charged particles with an energy below Δ (track-ends) as a function of cavity height for the idealized plane-parallel ion chamber in a ^{60}Co beam. The value of Δ varies with cavity height as in table 7.1, but AE and ECUT were set to 512 keV in all cases. Results for four wall materials are shown. As in figure 7.4, the effects of attenuation and scatter have been removed by using photon regeneration.

7.6 Calculations of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$

Values of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ obtained from CAVRZnrc calculations as a function of cavity height using the two methods described in section 7.2.1 are shown in figure 7.6 for graphite, aluminum, copper and lead wall materials. The differences between the results of the two methods should be negligible due to the Fano theorem. The agreement between the two sets of calculations serves to further validate the transport mechanics of the EGSnrc code, which were discussed earlier in chapter 2 (section 2.4). Of particular significance is the agreement observed with copper and lead, where the root mean squared deviation of the ratio from unity is 0.09 and 0.06 %, respectively. Statistical uncertainties on the calculated values were 0.1 % or less. Similar agreement was also obtained for wall materials of beryllium and tin.

7.7 Fluence correction factors (K_{fl}) for SA cavity theory and the air kerma formalism

In this section the size of K_{fl} is investigated on the assumption that the standard $L = 4V/S$ method for assigning Δ is correct. Figure 7.7 shows values of K_{fl} as a function of cavity height for the same four wall materials as in figure 7.6. The approach to calculating K_{fl} is described in section 7.2.3. For the graphite chamber, K_{fl} values are very close to unity, although on average they are below unity by 0.03 %. Values of K_{fl} close to unity are expected for graphite chambers since graphite is nearly air-equivalent at ^{60}Co energies. By the Fano theorem, the perturbation of the charged particle fluence by the cavity is negligible, regardless of cavity size and air density.

As the atomic number (Z) of the wall increases relative to the effective Z of

$$7.6. \text{ CALCULATIONS OF } K_{\text{FL}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{AIR}}^{\text{WALL}}$$

Figure 7.6: Values of $K_f \left(\frac{\bar{L}_\Delta}{\rho} \right)_{air}^{wall}$ calculated for graphite, aluminum, copper, and lead wall materials.
(from paper IV)

7.7. FLUENCE CORRECTION FACTORS (K_{FL}) FOR SA CAVITY THEORY AND THE AIR KERMA FORMALISM

Figure 7.7: Values of K_{fl} as a function of cavity height calculated on the assumption that the standard prescription for Δ is correct and using the average value of $K_{fl} \left(\frac{\bar{L}\Delta}{\rho} \right)_{air}^{wall}$ from figure 7.6 divided by the corresponding value of $\left(\frac{\bar{L}\Delta}{\rho} \right)_{air}^{wall}$ from table 7.1. (from paper IV)

the air, values of K_{fl} increase above unity. As with the graphite chamber, there does not appear to be a trend in K_{fl} with cavity height for the aluminum chamber, but the corrections are larger (average value of 1.0018(1)). A slight trend is observed for the copper chamber, and values of K_{fl} approach 1 % in some cases. For the lead chamber, the value of K_{fl} is strongly dependent on the cavity size, ranging from 1.0385(6) at a cavity height of 0.05 cm to 1.0703(6) at a cavity height of 1.2 cm. This is qualitatively consistent with figure 7.4 where the electron fluence spectrum was shown to differ more for the large cavity. Similar results were obtained for the chamber with tin walls, only with less variation. The K_{fl} corrections for the chamber modeled with beryllium walls, the Z of which is less than the effective Z of air, are less than unity (average value of 0.9991(1)) and also slightly less on average than the K_{fl} values for the graphite chamber. In general, based on these results, K_{fl} diverges from unity as the difference between the (effective) Z of air and wall material increases.

7.8 Determining Δ from values of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$

The calculation of K_{fl} corrections discussed in the previous section assumes *a priori* that the choice of Δ in the calculation of the SA wall-to-air stopping-power ratio is appropriate. Janssens⁹² proposed a different definition of Δ which takes into account the fact that many low-energy electrons do not escape because they backscatter from the walls even if they have the energy to cross the cavity. Although this proposed change may be appropriate, it makes Δ more difficult to determine.

Insight into the value of Δ that should be used can be obtained by analyzing the calculated values of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ assuming $K_{\text{fl}} \equiv 1.000$; i.e., one may equate values of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ to $\left(\frac{\bar{L}_{\Delta'}}{\rho} \right)_{\text{air}}^{\text{wall}}$ to determine the corresponding value of Δ' . If the extracted values of Δ' are independent of wall material, but not necessarily the same

7.8. DETERMINING Δ FROM VALUES OF $K_{\text{FL}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{AIR}}^{\text{WALL}}$

as the value of Δ corresponding to L , then this would indicate that the standard prescription for choosing Δ is wrong, and no K_{fl} correction would be needed. Using the average value of $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ at each cavity height in figure 7.6, equation 7.8 along with the fit coefficients in table 7.2 were used to systematically find the corresponding values of Δ' as a function of cavity height for each wall material. The results are shown in figure 7.8 with the data for graphite and beryllium wall materials separated from the data for the other materials for clarity of presentation. Uncertainties on Δ' were estimated from the standard deviation (σ) on the $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ values (in figure 7.6) from which it was derived, where the upper limit on Δ' is taken from $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}} + \sigma$ and the lower limit from $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}} - \sigma$. As a result, the uncertainties on Δ' for the graphite chamber are larger since the SA graphite-to-air stopping-power ratio is relatively insensitive to this parameter. Also shown in figure 7.8 for comparison is the variation in Δ expected from the $L = 4V/S$ prescription. With the exception of the graphite and beryllium chambers at cavity heights ≤ 0.2 cm, Δ' is larger than the kinetic energy of an electron with a CSDA range in air equal to L . The difference between the extracted and expected values increases with cavity height. For the tin and lead chambers, values of Δ' were not obtained above a cavity height of 0.8 cm and 0.4 cm, respectively, since $\left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ data were not calculated for $\Delta > 60$ keV. However, it was confirmed through calculations of $\left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ with larger Δ values that corresponding values of Δ' can be obtained for all the cavity heights for these two chambers.

If one accepts the definition for Δ (or Δ') proposed by Janssens,⁹² then to account for charged particles that cross but backscatter into the cavity the value needed to avoid K_{fl} corrections should be larger than the kinetic energy of an electron needed to travel a distance L in air. Based on this reasoning, it should be expected that Δ' increases as the atomic number of the chamber wall increases since the backscatter

7.8. DETERMINING Δ FROM VALUES OF $K_{\text{FL}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{AIR}}^{\text{WALL}}$

Figure 7.8: Values of Δ' for $\left(\frac{\bar{L}\Delta'}{\rho}\right)_{\text{air}}^{\text{wall}}$ required to avoid a non-unity value of K_{fl} as a function of cavity height. Also shown are the values of Δ' equal to the kinetic energy of an electron with a CSDA range in air given by $L = 4V/S$.
 (from paper IV)

7.8. DETERMINING Δ FROM VALUES OF $K_{\text{FL}} \left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{AIR}}^{\text{WALL}}$

coefficient for electrons also increases with Z for a given energy.⁴⁶ For the chamber investigated here, figure 7.9 shows that Δ' does in fact increase as Z of the wall increases for a cavity height of 0.1 cm, although the uncertainty on the value for the graphite wall ($Z = 6$) is relatively large. The same cannot be said, however, for the trend in Δ' with atomic number at larger cavity heights. At cavity heights of 0.8 and 1.2 cm, Δ' clearly decreases from $Z = 4$ (beryllium) to $Z = 13$ (aluminum) and then begins to increase again, contrary to the expectation mentioned above. In any case, the value of Δ' needed to avoid K_{fl} corrections clearly depends on Z in addition to the cavity dimensions.

Figure 7.9: Values of Δ' for $\left(\frac{\bar{L}\Delta'}{\rho}\right)_{\text{air}}^{\text{wall}}$ as a function of the atomic number of the chamber wall material for cavity heights of 0.1, 0.4, 0.8 and 1.2 cm. (from paper IV)

7.8. DETERMINING Δ FROM VALUES OF $K_{\text{FL}} \left(\frac{\bar{L}\Delta}{\rho}\right)_{\text{AIR}}^{\text{WALL}}$

7.9 On calculating K_{fl} directly

The calculation of the K_{fl} corrections shown in figure 7.7 were based on assumptions about the value of Δ for a given cavity size. Ideally, however, one should be able to calculate K_{fl} independently of these assumptions, perhaps by using the methods to calculate P_{repl} for chambers in-phantom. In a recent paper,⁹¹ methods were presented for directly calculating the replacement correction factor, P_{repl} . This corresponds to the K_{fl} correction for in-air calculations. However, in endeavoring to use these methods here for high- Z materials it became clear that these methods are not applicable. This is discussed further elsewhere.⁹³

7.10 K_{fl} values for graphite-walled chambers

The EGSnrc-calculated values of K_{fl} for the graphite-walled ion chambers used at NMI laboratories are shown in table 7.3. All values are calculated to a statistical precision of 1 part in 10^4 . Values ranged between 0.9996 and 0.9999 and there does not appear to be a trend with cavity size or chamber type. Also listed in table 7.3 are the SA stopping-power ratios used to calculate K_{fl} and the associated values of Δ .

7.10.1 Uncertainties on K_{fl} for graphite chambers

The uncertainties on K_{fl} quoted in table 7.3 for the graphite chambers used at the NMIs represent one standard deviation statistical uncertainties for a given set of transport parameters and cross-section data sets. The following sections discuss the sensitivity of calculated K_{fl} values for graphite chambers to the density and I value

Table 7.3: EGSnrc-calculated values of K_{fl} for the graphite-walled chambers used as ^{60}Co air kerma standards at various national metrology institutes. Details about the various chambers may be found in a report by Allisy-Roberts *et al*⁹⁰ and references therein. The uncertainties on $\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{air}}^{\text{wall}}$ and K_{fl} represent the statistical uncertainties for one standard deviation.

NMI	chamber type	ρ_{graphite} (g/cm ³)	$L = 4V/S$ (cm)	Δ (keV)	$\left(\frac{\bar{L}_{\Delta}}{\rho}\right)_{\text{air}}^{\text{wall}}$ ± 0.00002	K_{fl} ± 0.0001
BEV	cylindrical	1.72	0.65	17.3	1.00105	0.9999
		1.80	0.65	17.3	1.00105	0.9999
NMi	spherical	1.8045	1.40	27.1	1.00056	0.9999
NIST	spherical 30cc	1.74	2.57	38.6	1.00018	0.9999
	50cc	1.74	3.07	42.8	1.00002	0.9997
ARPANSA	BIPM-type	1.73	0.41	13.3	1.00138	0.9997
VNIIM	cylindrical 1cc	1.634	0.66	17.6	1.00110	0.9997
	30cc	1.634	2.26	35.8	1.00024	0.9999
NRC	cylindrical	1.66	0.76	19.0	1.00101	0.9998
PTB	cylindrical HRK1	1.775	0.45	14.0	1.00136	0.9998
	cylindrical HRK2	1.775	0.67	17.9	1.00107	0.9996
	spherical HRK3	1.775	0.40	13.0	1.00140	0.9997
SMU	cylindrical	1.71	0.64	17.2	1.00104	0.9999
ZMDM	cylindrical	1.75	0.64	17.2	1.00104	0.9999
NMIJ	cylindrical 6cc	1.85	1.25	25.3	1.00068	0.9997
	63cc	1.85	2.76	40.3	1.00011	0.9997
NCM	cylindrical	1.75	0.64	17.2	1.00104	0.9999
LNMRI	cylindrical	1.71	0.64	17.2	1.00104	0.9999
LNE-LNHB	cylindro-spherical	1.81	1.60	29.3	1.00047	0.9999
ENEA	cylindrical	1.75	0.64	17.2	1.00104	0.9999
MKEH	cylindrical	1.75	0.65	17.3	1.00105	0.9998
BIPM	parallel-plate	1.84	0.41	13.2	1.00145	0.9996

7.10. K_{FL} VALUES FOR GRAPHITE-WALLED CHAMBERS

of graphite, incident spectrum, photon cross-sections, and electron transport parameters.

Effect of density and mean ionization energy on K_{R} calculations

In previous investigations, it was shown that, combined, the density effect and I value of graphite can have a large effect (i.e. 1.5 % or more) on the calculated response of graphite chambers³⁶ and on calculations of the SA graphite-to-air stopping-power ratio.⁵² Although these large effects likely cancel when computing values of K_{R} , the effects of both the density effect and I value on K_{R} were examined at the level of statistical uncertainty considered here.

Figure 7.10 shows calculations of K_{R} for the plane-parallel graphite chamber as a function of the density of the graphite wall. Calculations were repeated for cavity heights of 0.15 and 0.6 cm and I values of 78 and 87 eV. For each calculation, the density effect for graphite was calculated using the ESTAR program from NIST.⁹⁴ Values of the calculated response varied by nearly 2 % from the lowest value ($I = 78$ eV, $\rho = 1.7$ g/cm³) to the highest value ($I = 87$ eV, $\rho = 2.26$ g/cm³). Despite this variation, calculations of K_{R} are independent of both the density and I value. The average value of K_{R} is 0.99989(4) for the 0.15 cm cavity height and 0.99973(3) for the 0.6 cm cavity height. In each case, the root mean squared deviation is ≈ 0.008 %, which is comparable to the statistical uncertainty on each value. Since only four of the twelve values deviate from the average by more than one standard deviation, there is no indication of any statistically significant variation. As such, the systematic uncertainty associated with the restricted stopping-powers used is estimated to be 1 part in 10^4 .

7.10. K_{FL} VALUES FOR GRAPHITE-WALLED CHAMBERS

Figure 7.10: Calculations of K_{FL} for plane-parallel graphite chambers of two cavity sizes as a function of graphite density and I values used to calculate the graphite stopping-powers.
(from paper IV)

Effect of the incident spectrum

The effect of the incident spectrum on calculated values of K_{fl} was investigated using five different photon sources at ^{60}Co energies. These include a 1.25 MeV monoenergetic photon source, a spectrum representing a bare ^{60}Co source with two equiprobable photon lines at 1.175 and 1.334 MeV, a ^{60}Co spectrum published in 1988,⁹⁵ and two ^{60}Co spectra calculated by Mora *et al* for 10×10 and 30×30 field sizes.⁴³ The 10×10 spectrum is that used throughout the rest of this investigation.

Figure 7.11 shows that the variation of calculated K_{fl} values for the realistic spectra is approximately 0.01 % (RMSD = 0.004 %), which is comparable to the statistical uncertainties on those calculations. A slightly larger 0.02-0.03 % increase in K_{fl} is observed as one changes from a realistic spectrum to a monoenergetic or bare ^{60}Co source. All of this implies that calculations of K_{fl} are insensitive to the details of the incident spectrum, which contributes no more than 0.01 % to the overall uncertainty on K_{fl} .

The effect of the photon cross-sections was also investigated by comparing values of K_{fl} calculated with cross-sections compiled by Storm and Israel⁹⁶ with those calculated using the XCOM cross-sections from NIST.^{31,32} Values of K_{fl} for 0.6 cm cavity height were the within 0.003 % in this comparison. The differences between these cross-sections for graphite are less than 0.003 % at ^{60}Co energies (1.25 MeV), but increases to over 1 % at lower energies. Increasing the photon cross sections more dramatically by 1 and 5 % also had about a 0.01 % effect on the value of K_{fl} .

7.10. K_{FL} VALUES FOR GRAPHITE-WALLED CHAMBERS

Figure 7.11: Values of K_{FL} calculated for various incident photon spectra at ^{60}Co energies.
(from paper IV)

Effect of electron transport parameters

In this investigation, an ECUT of 512 keV was used for all ion chamber calculations but there may be a contribution to the overall uncertainty on K_{fl} at the level of precision considered here due to terminating charged particle histories at this energy. Using the parallel plate chamber with 0.05 and 0.6 cm cavity heights, values of K_{fl} were compared for cut-off energies of 512, 513, 516 and 521 keV (where AE=ECUT in all cases). For the 0.6 cm cavity height, all calculated values of K_{fl} (with a statistical precision of 0.004 %) are within 0.005 % of values calculated with ECUT = 512 keV. Calculations are also within 0.005 % for the chamber with a 0.05 cm cavity height (the smallest cavity size considered in this paper) when ECUT = 513 keV, but differ by 0.05 % and 0.1 % for ECUT values of 516 and 521 keV, respectively. In any case, using a 1 keV kinetic energy cut-off should contribute no more than 1 part in 10^4 to the uncertainty in K_{fl} .

EGSnrc parameters related to charged particle transport were also investigated concerning the effect on calculated K_{fl} values. Excluding spin effects from the multiple scattering algorithm in EGSnrc changed calculated K_{fl} values by less than 0.005 % (0.6 cm cavity height). Similar results were obtained for electron impact ionization on and off.

Summary of systematic uncertainties

Table 7.4 summarizes the various contributions to the systematic uncertainty on calculated K_{fl} values for the graphite chambers discussed above. The overall uncertainty is approximately 0.02 %. Although this is comparable to the average size of the correction, there were no cases where the calculated correction was unity or above. One can safely say that K_{fl} is within 0.04 % of unity for all graphite-walled ion chambers

7.10. K_{FL} VALUES FOR GRAPHITE-WALLED CHAMBERS

studied, with an average value of 0.9998.

Table 7.4: Summary of one standard deviation uncertainties on the calculation of K_{fl} for graphite chambers.

Component	Std. unc. (%)	Comment
stopping-powers	0.01	Combines the effect of density and I value
incident spectrum	0.01	Applies to realistic ^{60}Co spectra
photon cross-sections	0.01	Estimated from comparison of K_{fl} values calculated with two different photon cross-sections
transport parameters	0.01	
overall	0.02	Including statistical uncertainties

7.11 Summary

In this chapter, EGSnrc calculations of chamber response were used to evaluate the accuracy of Spencer-Attix cavity theory as it applies to the air kerma formalism. Using simulations of two classic experiments as an example, it was shown that SA cavity theory generally fails to predict the response of ion chambers to ^{60}Co beams when no fluence correction factors are used. Following this, precise values of $K_{\text{fl}} \left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}^{\text{wall}}$ were indirectly obtained from calculations of chamber response as a function of cavity size and wall material. Contrary to the assumptions of SA cavity theory, unit values of K_{fl} are not obtained in general when the low-energy cut-off Δ in the Spencer-Attix stopping-power ratio, $\left(\frac{\bar{L}\Delta}{\rho} \right)_{\text{air}}$, is set to the kinetic energy of an electron with a CSDA range in air equal to $4V/S$. However, when nearly air-equivalent walls such

as graphite are used, K_{fl} values are within 0.04 % of unity since Fano conditions are approximated. This applies to the graphite ion chambers used for ^{60}Co air kerma standards at national metrology institutes. For other wall materials, the size of the K_{fl} correction increases as the atomic number of the wall increases. As an alternative to using K_{fl} corrections, one can select a low-energy cut-off Δ' in the SA stopping-power ratio such that K_{fl} corrections can be avoided. It is not obvious how to calculate Δ' from first principles but it can be obtained, as was shown, using Monte Carlo methods. It was hoped that a simple relationship existed between Δ' and cavity size, independent of the composition of the wall, but this is clearly not the case. Thus, given the complexity of calculating Δ' , it may be preferable to incorporate K_{fl} corrections in the air kerma formalism and always calculate the product of $K_{\text{fl}} \left(\frac{\bar{L}_{\Delta}}{\rho} \right)_{\text{air}}^{\text{wall}}$ rather than the individual factors.

Chapter 8

Conclusions

This thesis focused on the effects of low-energy electrons on ion chamber response to photon beams as it applies to 1) the standard temperature-pressure correction factor (P_{TP}) used with low-energy x rays; and 2) Spencer-Attix (SA) cavity theory as used for the determination of air kerma in ^{60}Co beams. When using vented ion chambers, the P_{TP} correction factor is always used to yield the response corresponding to reference temperature and pressure conditions, and so it is important to understand when it can be accurately applied, the mechanism by which it breaks down, and how this breakdown can be accounted for. Even with high incident photon energies where the P_{TP} correction works as expected, low-energy electrons are still an important consideration due to their influence on the formalisms used to convert chamber response to dosimetric quantities of interest. The complexity of this influence consequently leads to the introduction of fluence correction factors (K_{fl}) in the air kerma formalism based on SA cavity theory. In practice, these corrections are small for graphite-walled chambers but are nonetheless required in order to maintain an optimal level of accuracy in general.

In chapters 4 and 5, the effect of low-energy electrons is discussed in the context of the P_{TP} correction factor for ion chambers in kilovoltage x rays. In chapter 4, EGSnrc Monte Carlo calculations were used to show that the P_{TP} -corrected response depends on cavity air density for ion chambers with non-air-equivalent walls. This is contrary to what is expected based on the assumption that electrons from the wall completely cross the cavity. In x-ray beams, a large fraction of the electrons from the wall of the chamber stop in the cavity, and this results in the breakdown of the normal P_{TP} correction. Although the problem is related to low-energy electrons, the magnitude of the breakdown was shown to depend on the material of the chamber wall. For instance, if the mass cross-sections of the chamber wall are larger than that of air and electrons from the wall dominate the response, then the response will be over-corrected by P_{TP} as the air density in the cavity is decreased. The reverse is true when the mass cross-sections of the wall are smaller than that of air, and the magnitude of problem roughly corresponds to the difference between the cross-sections of the wall and air. Countering the effect of the wall is the fraction of photon interactions in the cavity contributing to the response (F_{air}). As F_{air} increases, the over/under-correction by P_{TP} is reduced. The situation is further complicated by the fact that both the effect of the wall and F_{air} depend on the energy of the incident photon spectrum, and the presence of any components that differ in composition from the wall (e.g. collector or insulator). Regardless, it is clear that low-energy electrons are not the only factor to consider in the breakdown of P_{TP} .

In chapter 5 the breakdown of the P_{TP} correction factor predicted by EGSnrc calculations was corroborated through measurements of chamber response as a function of cavity air pressure. These experiments also served as a benchmark of the EGSnrc Monte Carlo code for chamber response calculations with low-energy x rays. With the exception of one chamber, EGSnrc calculations were typically within 0.5 %

of measurements. In the case of one air-equivalent Exradin A12 chamber, the discrepancies between measurements and calculations were akin to the presence of impurities in the chamber wall that were not included in the EGSnrc model. The shortcoming of the model was better revealed through failed attempts to calculate the energy response curve for this chamber, where chamber dimensions and composition have a larger effect. Although incorporating impurities in the model of the A12 did resolve the discrepancies observed with experiment, the relative P_{TP} -corrected response of the other air-equivalent chambers was generally constant within 1 % over the range of photon energies and air densities evaluated. Based on this result, it is advised that the use of chambers with air-equivalent walls be given priority at low x-ray energies, especially in high-altitude regions where large P_{TP} corrections are required. If an air equivalent chamber is not available, the breakdown of the P_{TP} correction can be reliably predicted using the EGSnrc Monte Carlo code provided the chamber composition and dimensions are well known.

The second objective of this thesis was to investigate the effect of low-energy electrons on the response of ion chambers to ^{60}Co beams. The use of EGSnrc for this purpose was justified in chapter 6 via simulations of experiments that measured the response of chambers to ^{60}Co as a function of both cavity size and wall material. The calculated results were compared collectively for all wall materials and, in most cases, the agreement with experiments was consistent with the presumed uncertainties on the measurements and the underlying cross-sections used in the calculations. Large discrepancies were observed in a few instances, but can be attributed to lack of full build-up in the chamber wall or a possible error in the normalization of experimental data. Although the general ability of EGSnrc to simulate the influence of the wall serves as an important verification of the cross-sections, the results of comparisons for each wall material considered separately are more relevant to a sub-

sequent investigation of cavity theory. EGSnrc simulations of one particular set of high-precision measurements with a variable-volume graphite chamber were within 0.03 %, which is comparable to the systematic uncertainties on the measurements. Such a result validates the use of EGSnrc-calculated responses as a benchmark for testing the predictions of SA cavity theory.

In chapter 7, EGSnrc calculations of ion chamber response to ^{60}Co beams were used to test the predictions of SA cavity theory with the aim of resolving any discrepancies. By using calculated responses as a benchmark, the comparisons are independent of cross-sections as long as they are consistent in both calculations. Low-energy electrons in this case influence the predictions of SA cavity theory since one is required to determine the portion of the low-energy fluence spectrum to include in the evaluation of the SA stopping-power ratio. This is achieved through the selection of a cut-off energy, Δ . It was thought that Δ was only dependent on cavity size, but it was shown to also depend on wall material, and a way to determine it from first principles was not found. The complex dependence on Δ is attributed to the perturbation of the electron fluence in the wall by the cavity, particularly at low-energies. This was confirmed by the need for fluence corrections (K_{fl}) in the air kerma formalism to account for discrepancies with chamber response when conventional methods were used to calculate Δ . The size of the K_{fl} correction increased as the cross-sections and stopping-powers of the wall and air diverged. For graphite chambers, K_{fl} corrections ranged between 0.01-0.04 % below unity, exhibited no dependence on cavity size, and were relatively insensitive to electron transport parameters, cross-sections, and details of the incident spectrum. The average value calculated for the graphite chambers used at various national metrology institutes is 0.9998. Larger corrections calculated for other chamber wall materials imply that fluence corrections are required for SA cavity theory in general.

The investigation of ion chamber response to ionizing photon beams in this work has revealed the effects of low-energy electrons. For vented ion chambers in low-energy x rays, the potential breakdown of the P_{TP} correction factor caused by low-energy electrons can be predicted using EGSnrc simulations provided the geometry and composition of the chamber, as well as details of the incident spectrum, are well known. The ability to accurately calculate chamber response at low energies provides a means of overcoming the breakdown of P_{TP} without having to replace the chamber or repeat measurements. Additionally, EGSnrc can also be used to resolve problems related to low-energy electrons for chambers in ^{60}Co beams, where SA cavity theory is used to determine air kerma. Although protocols for clinical dosimetry are no longer based on the air kerma, the formalism provides a convenient way to analyze SA cavity which still forms the basis for all of clinical dosimetry. The K_{fl} corrections calculated for the ion chambers used at the various national metrology institutes, however small in some cases, are nonetheless important to further refine the accuracy of Spencer-Attix cavity theory and the air kerma formalism for graphite chambers in ^{60}Co beams. This level of precision may exceed that of experiments at present, but until the formalism is refined to this extent, there will little incentive to overcome current experimental limitations.

References

- [1] F. H. Attix, *Introduction to Radiological Physics and Radiation Dosimetry*, Wiley, New York, 1986.
- [2] L. A. DeWerd, S. D. Davis, L. J. Bartol, and F. Grenzow, Ionization chamber instrumentation, in *Clinical dosimetry measurements in radiotherapy*, edited by D. W. O. Rogers and J. E. Cygler, Medical Physics Publishing, 2009.
- [3] E. G. A. Aird and F. T. Farmer, The design of a thimble chamber for the Farmer dosimeter, *Phys. Med. Biol.* **17**, 169 – 174 (1972).
- [4] P. R. Almond, P. J. Biggs, B. M. Coursey, W. F. Hanson, M. S. Huq, R. Nath, and D. W. O. Rogers, AAPM’s TG–51 Protocol for Clinical Reference Dosimetry of High-Energy Photon and Electron Beams, *Med. Phys.* **26**, 1847 – 1870 (1999).
- [5] C.-M. Ma, C. W. Coffey, L. A. DeWerd, R. Nath, C. Liu, S. M. Seltzer, and J. Seuntjens, AAPM protocol for 40 – 300 kV x-ray beam dosimetry in radiotherapy and radiobiology, *Med. Phys.* **28**, 868 – 893 (2001).
- [6] IAEA, *Absorbed Dose Determination in Photon and Electron Beams; An International Code of Practice*, volume 277 of *Technical Report Series*, IAEA, Vienna, 1987.

- [7] D. R. Lide, *CRC Handbook of Chemistry and Physics: 80th Edition*, CRC Press LLC, Boca Raton, FL, 2000.
- [8] ICRU, Stopping powers for electrons and positrons, ICRU Report 37, ICRU, Bethesda, MD., 1984.
- [9] C.-M. Ma and A. E. Nahum, Bragg-Gray theory and ion chamber dosimetry for photon beams, *Phys. Med. Biol.* **36**, 413 – 428 (1991).
- [10] A. E. Nahum, Cavity theory, stopping power ratios, correction factors, in *Clinical Dosimetry Measurements in Radiotherapy*, edited by D. W. O. Rogers and J. E. Cygler, Medical Physics Publishing, Madison, Wisconsin, 2009.
- [11] L. V. Spencer and F. H. Attix, A theory of cavity ionization, *Radiat. Res.* **3**, 239 – 254 (1955).
- [12] A. E. Nahum, Water/Air Stopping-Power Ratios for Megavoltage Photon and Electron Beams, *Phys. Med. Biol.* **23**, 24 – 38 (1978).
- [13] U. H. Nyström and A. E. Nahum, A re-examination of the Spencer-Attix cavity integral, *Phys. Med. Biol.* **37**, 2143 – 2146 (1992).
- [14] L. V. Spencer and U. Fano, Energy Spectrum Resulting from Electron Slowing Down, *Phys. Rev.* **93**, 1172 – 1181 (1954).
- [15] AAPM TG-21, A protocol for the determination of absorbed dose from high-energy photon and electron beams, *Med. Phys.* **10**, 741 – 771 (1983).
- [16] I. Kawrakow and D. W. O. Rogers, The EGSnrc Code System: Monte Carlo simulation of electron and photon transport, NRC Technical Report PIRS-701 v4-2-2-5, National Research Council of Canada, Ottawa, Canada. <http://www.irs.inms.nrc.ca/inms/irs/EGSnrc/EGSnrc.html>, 2007.

- [17] W. R. Nelson, H. Hirayama, and D. W. O. Rogers, The EGS4 code system, Report SLAC-265, Stanford Linear Accelerator Center, Stanford, California, 1985.
- [18] I. Kawrakow, Accurate condensed history Monte Carlo simulation of electron transport. I. EGSnrc, the new EGS4 version, *Med. Phys.* **27**, 485 – 498 (2000).
- [19] I. Kawrakow, Accurate condensed history Monte Carlo simulation of electron transport. II. Application to ion chamber response simulations, *Med. Phys.* **27**, 499 – 513 (2000).
- [20] F. M. Khan, *The Physics of Radiation Therapy*, Lipincott Williams and Wilkins, Baltimore, Maryland, 3rd edition, 2003.
- [21] H. E. Johns and J. R. Cunningham, *The physics of radiology*, 4th ed., Charles C. Thomas, Springfield, Illinois, 1983.
- [22] M. J. Berger, Monte Carlo Calculation of the penetration and diffusion of fast charged particles, in *Methods in Comput. Phys.*, edited by B. Alder, S. Fernbach, and M. Rotenberg, volume 1, pages 135 – 215, Academic, New York, 1963.
- [23] I. Kawrakow and A. F. Bielajew, On the condensed history technique for electron transport, *Nuclear Instruments and Methods* **142B**, 253 – 280 (1998).
- [24] D. W. O. Rogers, I. Kawrakow, J. P. Seuntjens, B. R. B. Walters, and E. Mainegra-Hing, NRC User Codes for EGSnrc, Technical Report PIRS-702(RevB), National Research Council of Canada, Ottawa, Canada, 2003.
- [25] D. W. O. Rogers, I. Kawrakow, J. P. Seuntjens, and B. R. B. Walters, NRC User Codes for EGSnrc, NRC Technical Report PIRS-702, National Research Council of Canada, Ottawa, Canada. <http://www.irs.inms.nrc.ca/inms/irs/EGSnrc/EGSnrc.html>, 2000.

- [26] I. Kawrakow, egsp: the EGSnrc C++ class library, Technical Report PIRS-899, National Research Council of Canada, Ottawa, Canada, 2005.
- [27] G. Yegin, R. E. P. Taylor, and D. W. O. Rogers, BrachyDose: A New Fast Monte Carlo Code for Brachytherapy Calculations, *Med. Phys.* **33**, 2075 – 2075 (abs) (2006).
- [28] R. E. P. Taylor, G. Yegin, and D. W. O. Rogers, Benchmarking BrachyDose: voxel-based EGSnrc Monte Carlo calculations of TG-43 dosimetry parameters, *Med. Phys.* **34**, 445 – 457 (2007).
- [29] R. E. P. Taylor and D. W. O. Rogers, Generating a consistent TG-43 Dosimetry Parameter Database using the EGSnrc user-code BrachyDose, Abstract book of Third McGill International Workshop on Monte Carlo Techniques in Radiotherapy Delivery and Verification , p44(abstract) (2007).
- [30] L. A. Buckley, I. Kawrakow, and D. W. O. Rogers, CSnrc: correlated sampling Monte Carlo calculations using EGSnrc, *Med. Phys.* **31**, 3425 – 3435 (2004).
- [31] M. J. Berger and J. H. Hubbell, XCOM: Photon cross sections on a personal computer, Report NBSIR87-3597, National Institute of Standards Technology (NIST), Gaithersburg, MD 20899, U.S.A., 1987.
- [32] M. J. Berger, J. H. Hubbell, S. M. Seltzer, J. S. Coursey, and D. S. Zucker, XCOM: Photon cross section database (version 1.2), Report, NIST, Gaithersburg, MD, 1999, see <http://physics.nist.gov/xcom>.
- [33] F. Hobeila and J. P. Seuntjens, Effect of XCOM photoelectric cross-sections on dosimetric quantities calculated with EGSnrc, IAEA-CN-96/17P in Vol 1, Proc. of Int'l Symp. on Standards and Codes of Practice in Medical Radiation Dosimetry, (IAEA, Vienna) , 177 – 186 (2003).

- [34] M. J. Berger and S. M. Seltzer, Stopping power and ranges of electrons and positrons, NBS Report NBSIR 82-2550-A (second edition), NBS, Gaithersburg, MD 20899, U.S.A., 1983.
- [35] S. Duane, A. F. Bielajew, and D. W. O. Rogers, Use of ICRU-37/NBS collision stopping powers in the EGS4 system, NRCC Report PIRS-0173, Ottawa, March (1989).
- [36] L. A. Buckley, I. Kawrakow, and D. W. O. Rogers, An EGSnrc investigation of cavity theory for ion chambers measuring air kerma, Med. Phys. **30**, 1211 – 1218 (2003).
- [37] L. L. W. Wang and D. W. O. Rogers, The replacement correction factor for the BIPM flat cavity ion chamber and the value of W/e , Med. Phys. **35**, 4410 – 4416 (2008).
- [38] J. Sempau, A. Sánchez-Reyes, F. Salvat, H. O. ben Tahar, S. B. Jiang, and J. M. Fernández-Varea, Monte Carlo simulation of electron beams from an accelerator head using PENELOPE, Phys. Med. Biol. **46**, 1163 – 1186 (2001).
- [39] B. R. B. Walters, I. Kawrakow, and D. W. O. Rogers, History by history statistical estimators in the BEAM code system, Med. Phys. **29**, 2745 – 2752 (2002).
- [40] U. Fano, Note on the Bragg-Gray cavity principle for measuring energy dissipation, Radiat. Res. **1**, 237 – 240 (1954).
- [41] J. P. Seuntjens, I. Kawrakow, J. Borg, F. Hobeila, and D. W. O. Rogers, Calculated and measured air-kerma response of ionization chambers in low and medium energy photon beams, in *Recent developments in accurate radiation dosimetry*,

- Proc. of an Int'l Workshop*, edited by J. P. Seuntjens and P. Mobit, pages 69 – 84, Medical Physics Publishing, Madison WI, 2002.
- [42] D. W. O. Rogers, Fifty years of Monte Carlo simulations for medical physics, *Phys. Med. Biol.* **51**, R287 – R301 (2006).
 - [43] G. Mora, A. Maio, and D. W. O. Rogers, Monte Carlo simulation of a typical ^{60}Co therapy source, *Med. Phys.* **26**, 2494 – 2502 (1999).
 - [44] J. Borg, I. Kawrakow, D. W. O. Rogers, and J. P. Seuntjens, Monte Carlo study of correction factors for Spencer-Attix cavity theory at photon energies at or above 100 keV, *Med. Phys.* **27**, 1804 – 1813 (2000).
 - [45] E. Mainegra-Hing, I. Kawrakow, and D. W. O. Rogers, Calculations for plane-parallel ion chambers in ^{60}Co beams using the EGSnrc Monte Carlo code, *Med. Phys.* **30**, 179 – 189 (2003).
 - [46] E. S. M. Ali and D. W. O. Rogers, Benchmarking EGSnrc in the kilovoltage energy range against experimental measurements of charged particle backscatter coefficients, *Phys. Med. Biol.* **53**, 1527 – 1543 (2008).
 - [47] F. H. Attix, L DeLa Vergne, and V. H. Ritz, Cavity ionization as a function of wall material, *J. Res. of the NBS* **60**, 235 – 243 (1958).
 - [48] D. V. Cormack and H. E. Johns, The measurement of high-energy radiation intensity, *Radiation Res.* **1**, 133 – 157 (1954).
 - [49] B. Nilsson, A. Montelius, and P. Andreo, A study of interface effects in ^{60}Co beams using a thin-walled parallel plate ionization chamber, *Med. Phys.* **19**, 1413 – 1421 (1992).

- [50] A. F. Bielajew, Ionization cavity theory: a formal derivation of perturbation factors for thick-walled ion chambers in photon beams, *Phys. Med. Biol.* **31**, 161 – 170 (1986).
- [51] D. W. O. Rogers and C. K. Ross, The Role of Humidity and Other Correction Factors in the AAPM TG-21 Dosimetry Protocol, *Med. Phys.* **15**, 40 – 48 (1988).
- [52] D. W. O. Rogers and I. Kawrakow, Monte Carlo calculated correction factors for primary standards of air-kerma, *Med. Phys.* **30**, 521 – 543 (2003).
- [53] D. T. Burns, C. Kessler, and P. Roger, Air-kerma determination using a variable-volume cavity ionization chamber standard, *Phys. Med. Biol.* **52**, 7125 – 7135 (2007).
- [54] T. D. Bohm, S. L. Griffin, J. Paul M. DeLuca, and L. A. DeWerd, The effect of ambient pressure on well chamber response: Monte Carlo calculated results for the HDR 1000 Plus, *Med. Phys.* **32**, 1103 – 1114 (2005).
- [55] S. L. Griffin, L. A. DeWerd, J. A. Micka, and T. D. Bohm, The effect of ambient pressure on well chamber response: Experimental results with empirical correction factors, *Med. Phys.* **32**, 700–709 (2005).
- [56] U. Ankerhold, Catalogue of X-ray spectra and their characteristic data -ISO and DIN radiation qualities, therapy and diagnostic radiation qualities, unfiltered X-ray spectra-, Technical Report PTB-Dos-34, Physikalisch Technische Bundesanstalt, Braunschweig, Germany, 2000.
- [57] K. Cranley, B. J. Gilmore, G. W. Fogarty, and L. Desponds, Catalogue of diagnostic x-ray spectra and other data - CD edition (electronic version prepared by D Sutton), Technical Report Report 78, The Institute of Physics and Engineering in Medicine (IPEM), 1997.

- [58] J. M. Boone, T. Yu, and A. Seibert, Mammography spectrum measurement using x-ray diffraction device, *Phys. Med. Biol.* **43**, 2569 – 2582 (1998).
- [59] D. J. La Russa and D. W. O. Rogers, An EGSnrc investigation of the P_{TP} correction factor for ion chambers in kilovoltage x-rays, *Med. Phys.* **33**, 4590 – 4599 (2006).
- [60] W. Will and A. Rakow, Measuring of the standard ionic dose in compact and liquid media during x-ray and gamma irradiation by energies between 50 keV and 3 MeV, *Strahlentherapie. English translation in Rad. Sci. Trans.* 8, 1973 **141**, 166 – 175 (1971).
- [61] J. E. Burns and D. H. Pritchard, Effect of non air-equivalence of NPL secondary-standard therapy level ionization chamber, Technical Report Rad. Sci. 40, National Physical Laboratory, Teddington, U. K., 1977.
- [62] A. E. Nahum, W. H. Henry, and C. K. Ross, Response of carbon and aluminium walled thimble chamber in ^{60}Co and 20 MeV electron beams, *Proc. of VII Int. Conf. on Med. Phys, in Med. and Biol. Engin. and Comp.* **23**, 612 – 613 (1985).
- [63] J. P. McCaffrey, B. Downton, H. Shen, D. Niven, and M. McEwen, Pre-irradiation effects on ionization chambers used in radiation therapy, *Phys. Med. Biol.* **50**, N121 – N133 (2005).
- [64] D. J. La Russa, M. McEwen, and D. W. O. Rogers, An experimental and computational investigation of the standard temperature-pressure correction factor for ion chambers in kilovoltage x rays, *Med. Phys.* **34**, 4690 – 4699 (2007).
- [65] E. Mainegra-Hing and I. Kawrakow, Efficient x-ray tube simulations, *Med. Phys.* **33**, 2683 – 2690 (2006).

- [66] D. W. O. Rogers, B. A. Faddegon, G. X. Ding, C.-M. Ma, J. Wei, and T. R. Mackie, BEAM: A Monte Carlo code to simulate radiotherapy treatment units, *Med. Phys.* **22**, 503 – 524 (1995).
- [67] E. S. M. Ali and D. W. O. Rogers, Efficiency improvements of x-ray simulations in EGSnrc user-codes using Bremsstrahlung Cross Section Enhancement (BCSE), *Med. Phys.* **34**, 2143 – 2154 (2007).
- [68] R. Birch and M. Marshall, Computation of Bremsstrahlung X-ray spectra and comparison with spectra measured with a Ge(Li) detector, *Phys. Med. Biol.* **24**, 505 – 517 (1979).
- [69] F. Verhaegen, A. E. Nahum, S. Van de Putte, and Y. Namito, Monte Carlo modelling of radiotherapy kV x-ray units, *Phys. Med. Biol.* **44**, 1767 – 1789 (1999).
- [70] M. R. Ay, M. Shahriari, S. Sarkar, M. Adib, and H. Zaidi, Monte Carlo simulation of x-ray spectra in diagnostic radiology and mammography using MCNP4C, *Phys. Med. Biol.* **49**, 4897 – 4917 (2004).
- [71] G. Ullman, M. Sandborg, D. R. Dance, M. Yaffe, and G. A. Carlsson, A search for optimal x-ray spectra in iodine contrast media mammography, *Phys. Med. Biol.* **50**, 3143 – 3152 (2005).
- [72] ISO, *Guide to the Expression of Uncertainties in Measurement*, International Organisation for Standardization, Geneva, Switzerland, 1993.
- [73] R. E. Bentley, *Uncertainty in Measurement: The ISO Guide*, 6th ed, National Measurement Laboratory CSIRO, Australia, 2003.
- [74] G. N. Whyte, Measurement of the Bragg-Gray stopping power correction, *Radiation Res.* **6**, 371 – 379 (1957).

- [75] J. R. Greening, An experimental examination of theories of cavity ionization, Brit. J. Radiol. **353**, 254–262 (1957).
- [76] T. E. Burlin, An experimental examination of theories relating the absorption of γ -ray energy in a medium to the ionization produced in a cavity, Phys. Med. Biol. **6**, 33–53 (1961).
- [77] T. E. Burlin, The limits of validity of cavity ionization theory, Brit. J. Radiol. **35**, 343–348 (1962).
- [78] T. E. Burlin, A general theory of cavity ionization, Br. J. Radiol. **39**, 727 – 734 (1966).
- [79] T. E. Burlin, Further examination of theories relating the absorption of gamma-ray energy in a medium to the ionization produced in a cavity, Phys. Med. Biol. **11**, 255–266 (1966).
- [80] J. H. Hubbell, Photon Mass Attenuation and Energy-absorption Coefficients from 1 keV to 20 MeV, Int. J. Appl. Radiat. Isot. **33**, 1269 – 1290 (1982).
- [81] J. H. Hubbell, Review of photon interaction cross section data in the medical and biological context, Phys. Med. Biol. **44**, R1 – R122 (1999).
- [82] I. Kawrakow and D. W. O. Rogers, The EGSnrc System, a status report., in *Advanced Monte Carlo for Radiation Physics, Particle Transport Simulation and Applications: Proc. of the Monte Carlo 2000 Meeting Lisbon*, edited by A. Kling, F. Barao, M. Nakagawa, L. Távora, and P. Vaz, pages 135 – 140, Springer, Berlin, 2001.
- [83] D. T. Burns, A new approach to the determination of air kerma using primary-standard cavity ionization chambers, Phys. Med. Biol. **51**, 929 – 942 (2006).

- [84] D. W. O. Rogers, B. Walters, and I. Kawrakow, BEAMnrc Users Manual, NRC Technical Report PIRS-509(A) rev K, National Research Council of Canada, Ottawa, Canada. <http://www.irs.inms.nrc.ca/BEAM/beamhome.html>, 2007.
- [85] D. W. O. Rogers and A. F. Bielajew, Wall attenuation and scatter corrections for ion chambers: measurements versus calculations, *Phys. Med. Biol.* **35**, 1065 – 1078 (1990).
- [86] L. Büermann, H. M. Kramer, and I. Csete, Results supporting calculated wall correction factors for cavity chambers, *Phys. Med. Biol.* **48**, 3581 – 3594 (2003).
- [87] J. P. McCaffrey, E. Mainegra-Hing, I. Kawrakow, K. R. Shortt, and D. W. O. Rogers, Evidence for using Monte Carlo calculated wall attenuation and scatter correction factors for three styles of graphite-walled ion chambers, *Phys. Med. Biol.* **49**, 2491 – 2501 (2004).
- [88] D. J. La Russa and D. W. O. Rogers, Accuracy of EGSnrc calculations of the response of various ion chambers to ^{60}Co beams, *Med. Phys.* **35**, 5629 – 5640 (2008).
- [89] D. J. La Russa and D. W. O. Rogers, Accuracy of Spencer-Attix cavity theory and calculations of fluence correction factors for the air kerma formalism, *Med. Phys.* **36**, 4173 – 4183 (2009).
- [90] P. J. Allisy-Roberts, D. T. Burns, and C. Kessler, Summary of the BIPM.RI(I)-K1 comparison for air kerma in ^{60}Co gamma radiation, *Metrologia* **44**, 06006 (2007).
- [91] L. L. W. Wang and D. W. O. Rogers, Calculation of the replacement correction factors for ion chambers in megavoltage beams by Monte Carlo simulation, *Med. Phys.* **35**, 1747 – 1755 (2008).

- [92] A. C. A. Janssens, Modified energy-deposition model, for the computation of the stopping-power ratio for small cavity sizes, *Phys. Rev. A* **23**, 1164 – 1176 (1981).
- [93] L. L. W. Wang, D. J. La Russa, and D. W. O. Rogers, Systematic uncertainties in the Monte Carlo calculation of ion chamber replacement correction factors, *Med. Phys.* **36**, 1785 – 1789 (2009).
- [94] M. J. Berger, ESTAR, PSTAR and ASTAR: Computer Programs for Calculating Stopping-Power and Ranges for Electrons, Protons, and Helium Ions, NIST Report NISTIR-4999 (Washington DC), available on-line at <http://physics.nist.gov/Star> (1992).
- [95] D. W. O. Rogers, G. M. Ewart, A. F. Bielajew, and G. van Dyk, Calculation of Electron Contamination in a ^{60}Co Therapy Beam, in “Proceedings of the IAEA International Symposium on Dosimetry in Radiotherapy” (IAEA, Vienna), Vol 1 , 303 – 312 (1988).
- [96] E. Storm and H. I. Israel, Photon cross sections from 1 keV to 100 MeV for elements $Z=1$ to $Z=100$, *Atomic Data and Nuclear Data Tables* **7**, 565 – 681 (1970).