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Generalized Moment-matching Methods for Interconnect Analysis of High-Speed VLSI Systems

by

Eli Chiprout, B. Eng.

This thesis is submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering

Department of Electronics

Faculty of Engineering

Carleton University

Ottawa, Canada, K1S 5B6

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submitted by Eli Chiprout, B.Eng.
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Supervisor
Chair, Department of Electronics

Carleton University
August, 1992

Abstract

As circuit speeds increase, delay and crosstalk due to interconnections have proved to be the dominant limiting factors in high-speed and higher performance systems. Thus efficient and reliable numerical methods to evaluate interconnection delay and crosstalk have become essential tools for modern VLSI systems design.

This thesis describes new approaches to moment-matching used in Asymptotic Waveform Evaluation (AWE). The methods outlined provide increased accuracy and ensure stability in the waveform approximation.

- The entire Padé Table is analyzed and a single most accurate approximation extracted, not only matching low order moments, but giving low error with respect to high order moments.
- Poles generated at circuit nodes other than the output node can be applied to the output node of interest to increase the accuracy. This approach is useful in nonlinear analysis where a macromodel of the linear subnetwork is necessary.
- By using the exponential form of the impulse response, expensive convolution techniques are completely bypassed, providing an efficient approach to nonlinear waveform estimation.

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Firstly, and teachers must come first in any organized society, my grateful appreciation is extended to Prof. Michel Nakhla, for being a catalyst in helping to pull me out of the jaws of industrial intellectual stagnation, to a life of practical research combined with endless mental stimulation. However, the way of a researcher is not always without its frustrations, and my thanks also extend to him for forbearing my strange phone calls at odd times of the day, my barging into his office to debate the merits of this or that new technique, or my complaining when a particular goal seemed unattainable.

Next, my earlier teachers, my parents, are to be thanked, whose patience and complete support has been a gift among many others.

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Elmore delay to include several moments, not just one or two. They have been shown to give an improved delay estimation over an RC-tree model.

2.3 Asymptotic Waveform Evaluation

The Asymptotic Waveform Evaluation (AWE)[4]-[5] is a variable refinement waveform estimator for RLC interconnect models which is not just limited to tree structures. It is also based on the moments of the circuit response, but uses more moments than the RC tree methods. Higher order moments can be used to approximate the response, at an incremental cost in CPU time. Some of the key concepts in the Asymptotic Waveform Evaluation technique are reviewed here.

The AWE can evaluate waveforms with non-monotonic details such as undershoot, overshoot and glitches due to charge sharing, by increasing the order of the approximation as required. Higher orders of approximation are obtained recursively, at an incremental CPU cost. The AWE technique is an efficient method for transient analysis of RLC circuits. Its speed is of about the same order as the RC tree method.

The AWE was proposed using a homogenous and particular solution approach as well as a transfer function approach. Both will be explored below.

2.3.1 Homogenous/Particular Solution Approach

In general, the differential state equations for a lumped, linear, time-invariant network can be expressed as,

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List of Symbols

$\mathcal{R}^N$	Real N-space.
$\hat{z}$	Approximation of a variable z .
G	Conductance matrix of the MNA formulation
C	Inductance matrix of the MNA formulation.
E	Input vector in the MNA formulation.
I	Unitary matrix.
s	Laplace operator.
$\delta(t)$	Dirac Impulse function
M_i	i-th moment vector.
m_i	i-th moment of a particular output.
$V(s)$	Vector of MNA variables in the frequency domain.
$V_i(s)$	Particular output of vector of MMA variables in frequency domain.
$v(t)$	Vector of MNA variables in the time domain.
$v_i(t)$	Particular output of vector of MMA variables in time domain.
N_p	Number poles (eigenvalues) of a linear circuit.
p_i	i-th pole of the linear circuit.
k_i	i-th residue of the linear circuit.
c	Coupling constant.
L	Order of Padé numerator.
M	Order of Padé denominator.
q	Number of approximating poles.
K_i	i-th residue matrix for macromodel.

Chapter 1

Introduction

1.1 Problem Description

Advances in very large scale integrated (VLSI) circuit technologies and their packaging techniques are yielding smaller devices, finer features and tighter integration. As the dimensions of these designs become smaller, the number of interconnections, both on and off-chip, tend to increase dramatically. Simultaneously, high levels of integration allows for larger chip areas, as well as denser packaging of complete devices on Multi-chip Modules (MCMs) and Printed Circuit Boards (PCBs). This trend has tended to yield faster switching times in smaller device sizes and thereby increase the effect of the interconnections. As a

result, the interconnection delay times can often be significantly longer than device switching times. The speed of these circuit becomes primarily dependant on the delay in the interconnections rather than by the device limitations[1].

In addition, as interconnection densities and switching speeds increase, coupling between adjacent interconnects can become prominent. Signals in the active lines can induce transient signals on the neighboring inactive lines via electromagnetic coupling. These transient signals can falsely trigger logic gates and other devices at the terminations of the inactive lines resulting in corrupted data transmission and false machine states. Hence the electromagnetic coupling between neighboring interconnects, or "crosstalk", has become another important factor in determining overall system performance. These effects can appear at the chip, printed circuit board or at the system level, while interconnected blocks could be analog, digital, or mixed signal. Thus accurate transient analysis of coupled interconnect lines is important in the design and simulation of high-speed VLSI systems.

Full simulation of networks containing lumped-element components, distributed interconnects in the form of lossy, coupled transmission lines with frequency dependant parameters, as well as nonlinear elements such as transistors, has proven to be CPU intensive. Recent work has concentrated on calculating an estimated waveform with far less CPU than conventional simulators. It is on these techniques, their strengths and weaknesses, and their extensions that this thesis concentrates.

The commonly used methods for timing estimation of large circuits previously relied upon an RC tree model of the circuit [2]-[3]. However, these techniques have been shown to be inadequate for high-speed applications where crosstalk capacitance, line inductance and the distributed form of the interconnects are significant. Recently, moment-matching methods such as the Asymptotic Waveform Evaluation (AWE) technique [4]-[5] and the Moment Polynomial Nodal Analysis (MPNA) [6] have been proposed for transient waveform estimation of general lumped linear RLC circuits. The MPNA was also shown applicable to distributed RC lines. The AWE method, originally proposed for lumped-element circuits, has been extended to apply to the general case of networks containing lossy, coupled transmission lines [7],[20].

In these moment methods, the fundamental idea is to approximate transient response by matching the first few moments of the Maclaurin series of the exact response to a lower-order response, either in the time or frequency domains. The AWE low-order response consists of a pole/residue model whose first $2q-1$ moments are made equal to the response moments. The MPNA low-order response consists of time-domain exponentials, which is mathematically equivalent to the frequency domain matching of the AWE; both are forms of Padé approximation with its inherent strengths and drawbacks.

The AWE has been shown to give, at times, remarkably accurate results considering the limited CPU time that it consumes. However, there are also situations in which the error of the AWE is large. The method is sensitive both to

network topology and element values as well as to the choice of output at which it is applied. This is the problem of numerical sensitivity [8].

Increasing the order of the approximation sometimes improves the accuracy but does not increase it indefinitely. The matrix to solve the equations quickly becomes ill-conditioned with increasing order (number of poles) eventually yielding right-hand plane poles for known stable systems. Such a transient approximate solutions “blow up” with increasing time and cannot accurately represent the output of a stable system. This is the problem of stability [9].

In order to take advantage of the speed of the AWE in nonlinear waveform estimation, the AWE has been coupled with other techniques such as the Piecewise Decomposition Technique (PDT) [10]. This allows for analysis of interconnect networks with nonlinear terminations. Given an accurate set of approximate poles and residues, the results can be quite accurate. However, the published method requires time-domain convolution which is expensive in CPU usage with increasing number of time points. A linear CPU cost growth with the number of time points is preferred.

1.2 Thesis Objectives

The objective of this thesis is to expand on the use of Padé approximations in the moment-matching techniques. While the connection between moment-matching and Padé approximations is not new, the explicit examination of all the pertinent

details will allow for the exploration of new ways to improve the moment-matching methods.

The properties of Padé approximations as applicable to rational transfer functions will be extensively explored. It will be shown that the AWE is one of many possible approximations, not necessarily the best one. Then, using the properties of the Padé approximation method, the thesis objective will be to develop algorithmic techniques to further enhance the moment-matching method and to reduce or eliminate the problems of numerical sensitivity and stability in the approximation.

The thesis will develop an error criterion which will allow for the choice of the best moment-matching approximation. The ultimate objective will be to extract an accurate set of q poles and residues that best describe the transfer function and therefore yield an accurate transient response.

Further, to precisely model the interconnect delay, the nonlinearities of terminations have to be taken into account. However, such nonlinear terminations pose a problem due to the fact that quasi-transverse electromagnetic propagation modes in lossy transmission lines are defined only in the frequency domain. In contrast to this, nonlinear networks can be in general only be analyzed in the time domain. The second objective of this thesis is to combine the moment-matching methods as applied to the linear subnetwork with nonlinear terminations, to yield a nonlinear waveform estimation method that does not require the expense of convolution, i.e. is linear time-complexity.

1.3 Thesis Contributions

In this thesis, several contributions are made to address the problem of high-speed interconnect analysis:

- An algorithm is introduced which generalizes on the moment matching methods by generating several moment-matching approximations from the Padé Table, (chapter 3).
- Selection criteria are developed in order to select the best moment matching approximations. These criteria are generated from the smallest poles and the unused higher order moments, (chapter 3).
- The developed error criterion is shown useful not only in choosing the best approximation, but in finding accurate system poles based on moments from nodes other than the desired output. This selection of an optimal pole set can lead to better accuracy at the desired output, (chapter 4).
- Optimal pole selection is shown useful in selecting a set of common poles for an $\alpha \times \beta$ reduced-order transfer function description of a linear network, (chapter 5).
- The property of approximate exponentials in the estimated impulse response is used to advantage in allowing for the development of simple non-linear analysis method without the use of convolution. The method of the moment-matching allows transient analysis to bypass expensive numerical inversion techniques from the frequency domain to the time

domain. With the removal of convolution the nonlinear analysis is further speeded up by 30-50 times, (chapter 5).

1.4 Outline Of the thesis

Chapter 2. Background and Literature Survey: some existing moment-matching techniques for the analysis of interconnection delay are reviewed and compared.

Chapter 3. Generalized Moment-matching: introduces the general set of moment-matching approximations possible, as well as two criteria which allow for the selection of the best approximation.

Chapter 4. Optimal Pole Selection: introduces a technique whereby several nodes are selected for a moment-matching approximation but the poles from one node are used for generating residues at another.

Chapter 5. Fast Nonlinear Analysis: introduces a novel analysis method that takes advantage of the approximate poles generated by moment-matching to bypass not only expensive numerical inversion as was done previously but to avoid CPU intensive convolution in non-linear analysis.

Chapter 6. Conclusions: the results of the thesis and suggestions for further research are summarized.

Chapter 2

Background And Literature Survey

2.1 Introduction

As advances in integrated circuit technology continue, delay and crosstalk due to interconnections are becoming dominant limiting factors for higher speed and higher performance systems. For this reason, a considerable amount of work has been done in search of efficient numerical methods and algorithms to predict crosstalk and propagation signal delay caused by interconnects. In this chapter, some important developments in the literature are reviewed and the limitations of their approaches are also presented.

2.2 Elmore Delay and the RC-Tree Model

One of the most widely adopted approaches in timing verification of MOS VLSI circuits is the linear RC-tree model proposed by Rubinstein and Penfield [2], in the early eighties. The RC tree is a network model in which the interconnects are modelled as RC circuits with capacitors from all nodes to ground, no floating capacitors, no resistor loops, and no resistors to ground. Additionally, the nonlinear resistance and capacitance of MOS transistor are approximated by a linear resistor and capacitor, respectively. A typical RC-tree model is shown in Figure 2.1.

Figure 2.1 Example circuit of an RC Tree. (Adapted from [2].)

2.2.1 Delay Calculation in the RC Tree Model

The estimated signal delay and the error bounds of the delay in the RC tree can be obtained from the following time constants,

$$T_{D_i} = \sum_{k=1}^N R_{ik} C_k \quad (2.1)$$

$$T_{R_i} = \sum_{K=1}^N \frac{R_{ik}^2}{R_{ii}} C_k \quad (2.2)$$

$$T_p = \sum_{K=1}^N R_{kk} C_k \quad (2.3)$$

where,

i and k indicate the output node and internal node, respectively.

R_{ik} is the resistance of the portion of the path between the input and i , that is common with the path between the input and node k .

R_{ii} is the resistance between the input and output i .

R_{kk} is the resistance between the input and node k .

C_k is the capacitance of the lumped capacitor at node k .

To illustrate the resistance terms, consider the RC tree in Figure 2.1. For that network, $R_{ik}=R_1+R_2$, $R_{kk}=R_1+R_2+R_3$, and $R_{ii}=R_1+R_2+R_5$.

T_{D_i} , known as "Elmore time constant" [11], is the estimated elapsed time at which the transient step response reaches 50 percent of its final value for a monotonic waveform; it can also be interpreted as the first moment, or mean, of the impulse response $\dot{v}_i(t)$:

$$T_{D_i} = \int_0^{\infty} t \dot{v}_i(t) dt \quad (2.4)$$

T_{D_i} can merely provide an approximation for the dominant time constant of the step response. The errors in this type single pole model are occasionally large enough to mislead a designer about the delay in a critical path. For this reason, the upper and lower bounds, T_p and T_{R_i} , are the most valuable information for the circuit designers. [12] shows that the ratio $(T_p - T_{R_i})/T_{R_i}$ is not only a good qualitative indicator of the distance between the bounds but also a measure of the worse-case error in the single pole model with monotonic step responses.

2.2.2 Comments on RC Tree Model

The original RC tree model proposed in [2] is a simplified interconnect model; nevertheless, it has been an acceptable and pragmatic approach. Thus, a considerable amount of work [13]-[14] has been done to enhance the linear RC tree model to include resistor loops, initial charges, grounded resistors and even a two-pole-one-zero approximation. As a result of these contributions, the RC tree model for time estimation has been used successfully in many timing simulators and analyzers [15]. The outstanding merit of the RC tree approach is that the single pole approximation can provide a delay estimate for interconnects in MOS circuits

without using those time-consuming numerical integration approaches in circuit simulators, such as SPICE [16]. For low- to mid- frequency MOS integrated circuits the RC model can predict, in some case, the delay to within 10 percent of a SPICE simulation but the computational cost can be thousands of times lower, [17].

However, the RC tree approach still suffers from several severe drawbacks:

- For high-speed circuits and printed circuit board level designs, crosstalk capacitance and line inductance can produce a non-monotonic response where the RC tree approach will fail to predict the propagation delay. An enhance RLC model is therefore needed.
- The RC tree approach is derived by assuming that the input excitation is an ideal step waveform but the propagation signal always exhibits a finite rise-time in practice.
- Interconnects are fundamentally modelled as distributed systems, described by a resistance and capacitance per unit length rather than lumped resistors and capacitors. It would take large number of lumped RC sections to accurately model a distributed line, if at all.

Recently, moment methods have been proposed to increase the accuracy of the RC tree model. Among these is the Asymptotic Waveform Evaluation (AWE) and the Moment Polynomial Nodal Analysis (MPNA). The strength of these techniques lies in the fact that they extend and generalize on the principle of the

Elmore delay to include several moments, not just one or two. They have been shown to give an improved delay estimation over an RC-tree model.

2.3 Asymptotic Waveform Evaluation

The Asymptotic Waveform Evaluation (AWE)[4]-[5] is a variable refinement waveform estimator for RLC interconnect models which is not just limited to tree structures. It is also based on the moments of the circuit response, but uses more moments than the RC tree methods. Higher order moments can be used to approximate the response, at an incremental cost in CPU time. Some of the key concepts in the Asymptotic Waveform Evaluation technique are reviewed here.

The AWE can evaluate waveforms with non-monotonic details such as undershoot, overshoot and glitches due to charge sharing, by increasing the order of the approximation as required. Higher orders of approximation are obtained recursively, at an incremental CPU cost. The AWE technique is an efficient method for transient analysis of RLC circuits. Its speed is of about the same order as the RC tree method.

The AWE was proposed using a homogenous and particular solution approach as well as a transfer function approach. Both will be explored below.

2.3.1 Homogenous/Particular Solution Approach

In general, the differential state equations for a lumped, linear, time-invariant network can be expressed as,

$$\dot{x} = Ax + bu \quad (2.5)$$

where x is the n -dimensional state vector and u is the m -dimensional excitation vector. In order to analyze of the delay and rise time effects of an input waveform, the input is assumed to be in the form,

$$u_p(t) = u_0 + u_1 t \text{ for } t > t_0 \quad (2.6)$$

where u_0 and u_1 , are constant m -dimensional vectors.

For the excitation u_p , the differential-state equation (2.5) has the particular solution

$$x_p(t) = -A^{-1}bu_0 - A^{-2}bu_1 - A^{-1}bu_1 t \quad (2.7)$$

This particular solution is valid provided that the A matrix is not singular. That is not an unreasonable restriction for circuit models of interest because it implies that a circuit has a unique and well-defined DC solution when all of its capacitors are open-circuited and all of its inductors are short-circuited. To complete the solution of (2.5), the homogeneous solution needs to be added, i.e. the solution to,

$$\dot{x}_h = Ax_h \quad (2.8)$$

with the initial condition

$$x_h(0) = x_0 + A^{-1}bu_0 + A^{-2}bu_1 \quad (2.9)$$

where x_0 is the initial state at time zero. The Laplace transform solution of the homogeneous equation is,

$$X_h(s) = (sI - A)^{-1}x_h(0) \quad (2.10)$$

The actual frequency and transient responses at a particular i -th output are given respectively by,

$$X_i(s) = [X_h(s)]_{(i)} = \sum_{j=1}^{N_p} \frac{k_j}{s - p_j} \quad (2.11)$$

$$x_i(t) = [x_h(t)]_{(i)} = \sum_{j=1}^{N_p} k_j e^{p_j t} \quad (2.12)$$

where N_p is the number of actual poles p_j of the circuit, and k_j the corresponding residues of the output. In practice it is complicated and expensive to obtain the actual set of system poles and output residues. In order to approximate this solution, $X_h(s)$ is expanded in a Maclaurin's series,

$$X_h(s) = -A^{-1} (1 + A^{-1}s + A^{-2}s^2 + \dots) x_h(0) = \sum_{n=0}^{\infty} M_n s^n \quad (2.13)$$

In the case of a linear network which contains lumped components, the Modified Nodal Admittance (MNA) matrix equations [18] of the network can be written as,

$$Gv(t) + C \frac{d}{dt} v(t) - E(t) = 0 \quad (2.14)$$

where,

$v(t) \in \mathcal{R}^N$ is the vector of node voltage waveforms appended by current waveforms of linear inductors and independent voltage sources,

$C \in \mathcal{R}^{N \times N}$ and $G \in \mathcal{R}^{N \times N}$ are constant matrices with entries determined by the lumped linear components,

and $E(t) \in \mathcal{R}^N$ is a vector with entries determined by the independent voltage and current sources.

The moments of this network can be shown to be given recursively by,

$$M_{n+1} = (-G^{-1}C) M_n \quad (2.15)$$

with,

$$M_0 = -G^{-1} [v_h(0)] \quad (2.16)$$

The homogenous vector, $v_h(t)$, is related simply to $x_h(t)$. The moments of an output i , $m_n = [M_n]_{(i)}$ ($n = 0, 1, 2, \dots$) form the Maclaurin's series of that output,

$$V_i(s) = \sum_{n=0}^{\infty} m_n s^n \quad (2.17)$$

These are then matched to as many moments as necessary or desirable of a lower-order approximating frequency-domain function of the form,

$$V_i(s) \approx \hat{V}_i(s) = \sum_{j=1}^q \frac{\hat{k}_j}{s - \hat{p}_j} \quad (q \ll N_p) \quad (2.18)$$

The transient approximate homogenous solution is then,

$$v_i(t) \approx \hat{v}_i(t) = \sum_{j=1}^q \hat{k}_j e^{\hat{p}_j t} \quad (2.19)$$

The reciprocal poles of the approximation function $\hat{V}_i(s)$ are found from the roots of the characteristic equation

$$a_0 + a_1 p^{-1} + a_2 p^{-2} + \dots + p^{-q} = 0 \quad (2.20)$$

where the coefficients a_n are found by solving

$$\begin{bmatrix} m_{-1} & m_0 & m_1 & \dots & m_{q-2} \\ m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ \vdots & \vdots & \vdots & \dots & \vdots \\ m_{q-2} & m_{q-1} & m_q & \dots & m_{2q-3} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \vdots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_{q-1} \\ m_q \\ m_{q+1} \\ \vdots \\ m_{2q-2} \end{bmatrix} \quad (2.21)$$

where $m_{-1} = [v_h(0)]_{(i)}$

The corresponding residues are then obtained using the approximate poles from (2.20) and,

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ p_1^{-1} & p_2^{-1} & p_3^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & p_3^{-2} & \dots & p_q^{-2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{-q+1} & p_2^{-q+1} & p_3^{-q+1} & \dots & p_q^{-q+1} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_q \end{bmatrix} = \begin{bmatrix} m_{-1} \\ m_0 \\ m_1 \\ \vdots \\ m_{q-2} \end{bmatrix} \quad (2.22)$$

Once the poles and the residues for the reduced model are found, the approximation for the homogeneous solution can be obtained using equation (2.19). The complete approximate solution for the response at the i -th output is given by.

$$\hat{x}_i(t) = [x_p(t)]_{(i)} + [\hat{x}_h(t)]_{(i)} \quad (2.23)$$

2.3.2 Transfer Function Approach

The AWE can also be explained in terms of the impulse response

$$\dot{x}(t) = Ax(t) + E\delta(t) \quad (2.24)$$

where $\delta(t)$ is the impulse excitation and E is the vector of zeros and ones representing the inputs at which an impulse source is attached.

The Laplace transform solution of (2.24) is

$$X(s) = (sI - A)^{-1}E \quad (2.25)$$

The actual frequency/impulse response at an output i are then respectively given by,

$$V_i(s) = [V(s)]_{(i)} = c + \sum_{j=1}^{N_p} \frac{k_j}{s - p_j} \quad (2.26)$$

$$v_i(t) = [v(t)]_{(i)} = c\delta(t) + \sum_{j=1}^{N_p} k_j e^{p_j t} \quad (2.27)$$

To approximate this solution, $V(s)$ is expanded in a Maclaurin's series

$$V(s) = \sum_{n=0}^{\infty} M_n s^n \quad (2.28)$$

where, in term of the G and C matrices, the moments can be given by,

$$M_{n+1} = (-G^{-1}C)M_n \quad (2.29)$$

and,

$$M_0 = -G^{-1}E \quad (2.30)$$

The moments associated with a particular output i , are then matched to a low order transfer function of the form,

$$V_i(s) \approx \hat{V}_i(s) = \sum_{j=1}^q \frac{\hat{k}_j}{s - \hat{p}_j} \quad (q \ll N_p) \quad (2.31)$$

This transfer function can then be multiplied by the frequency-domain input to achieve the approximate frequency-domain output.

The advantage of this technique is that the approximate transfer function is calculated once and then used with any input waveform. In contrast, the particular

solution approach requires a calculation for each different input. Additionally, this method does not require separation of solutions into particular and homogeneous. It is this approach that will be addressed in this thesis.

2.3.3 Transmission Line Moments

In high-speed circuits, the interconnection delay time is often significantly longer than the device switching time. In such case, transmission line effects such as crosstalk and reflections can result from the electromagnetic coupling between interconnects and improper line terminations. In order to accurately predict all those phenomena, interconnects in high-speed digital circuits are best modelled as multiconductor transmission lines. A multiconductor transmission line is assumed to be uniform along its length with an arbitrary cross section which contains n signal conductors and one reference. The interconnection structure of VLSI chips and printed circuit boards is most frequently embedded in a non-homogeneous medium and thus the waves that propagate along the line are not of the transverse electromagnetic (TEM) nature. However, it can be assumed that a quasi-TEM propagation exists in the multiconductor transmission line. This requires that the cross sectional dimensions of the lines are sufficiently smaller than the wavelength of the highest frequency component of the propagation signal. Along a multiconductor transmission line generally n different quasi-TEM modes can propagate. For a lossless line with frequency-independent parameters such modes exist in the time domain, and they can also be defined in the frequency domain. The partial differential equations governing the propagation of voltages and currents in a lossless multiconductor transmission can be written in time domain as

$$\frac{\partial}{\partial x} v(x,t) = -L_{n \times n} \frac{\partial}{\partial t} i(x,t) \quad (2.32)$$

$$\frac{\partial}{\partial x} i(x,t) = -C_{n \times n} \frac{\partial}{\partial t} v(x,t) \quad (2.33)$$

where,

v and i are column vectors of dimension n representing the line voltages and currents.

$L_{n \times n}$ and $C_{n \times n}$ are real, symmetric $n \times n$ matrices representing inductance and capacitance per unit length, respectively, and are related to the physical self and mutual parameters. All the elements of $L_{n \times n}$ are positive whereas $C_{n \times n}$ has positive diagonal elements and negative off-diagonal elements.

However, for a lossy line, in the general case, the quasi-TEM modes can be defined in the frequency domain only. In the frequency domain the multiconductor transmission line can be described by the differential equations

$$V(s) = -Z_p I(s) \quad (2.34)$$

$$I(s) = -Y_p V(s) \quad (2.35)$$

where,

$V(s)$ and $I(s)$ are the vector of complex line voltages and currents, respectively.

Z_p and Y_p are the impedance and admittance per unit length of the line.

$$Z_p = R_{n \times n} + sL_{n \times n} \quad (2.36)$$

$$Y_p = G_{n \times n} + sC_{n \times n} \quad (2.37)$$

$R_{n \times n}$ and $G_{n \times n}$ are the $n \times n$ series resistance and shunt conductance matrices, respectively. Again they are related to the physical self and mutual parameters.

If a network contains k arbitrary distributed linear subnetworks besides the lumped components, the derivation of moments is slightly more complex. Without loss of generality the time-domain modified nodal admittance matrix equations of the network can be written in the form,

$$C \frac{d}{dt} v(t) + G v(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e(t) = 0 \quad (2.38)$$

where,

N is the total number of variables in the MNA formulation, n_k the number of nodes in the linear subnetwork k , N_s the number of linear subnetworks,

$v(t) \in \mathcal{R}^N$, $C \in \mathcal{R}^{N \times N}$ and $G \in \mathcal{R}^{N \times N}$ are as defined above,

$D_k = [d_{i,j}]$, $d_{i,j} \in \{0,1\}$, $i \in \{1,2, \dots, N\}$, $j \in \{1,2, \dots, n_k\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps

$i_k(t) \in \mathcal{R}^{n_k}$, the vector of currents entering the linear subnetwork k , into the node space $\mathcal{R}^N$ of the network π .

$e(t) \in \mathcal{R}^N$ is the vector of independent sources.

Generally, the distributed linear subnetworks can only be described in the frequency domain. These equations of the linear subnetwork k are in the form

$$P_k V_k(s) + Q_k I_k(s) = 0 \quad k = 1, 2, \dots, N_s \quad (2.39)$$

where V_k and I_k represent the Laplace domain terminal voltages and currents of the subnetwork k , respectively.

In the special case where the subnetwork k consists of a lossy multiconductor transmission line system, P_k and Q_k can be described in terms of the line parameters.

With transient input vector $e\delta(t)$, the equations can be put in the form [7],

$$\begin{bmatrix} sC + G & D_1 & D_2 & \dots & D_{N_s} \\ P_1 D_1^t & Q_1 & 0 & \dots & 0 \\ P_2 D_2^t & 0 & Q_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ P_{N_s} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s} \end{bmatrix} \begin{bmatrix} v(s) \\ I_1(s) \\ I_2(s) \\ \vdots \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} e(s) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2.40)$$

or,

$$Y(s) V(s) = E \quad (2.41)$$

In order to expand the response in the form

$$V(s) = \sum_{n=0}^{\infty} M_n s^n \quad (2.42)$$

where,

$$M_n = \frac{\frac{\partial^n}{\partial s^n} [V(s)^{-1} E] \Big|_{s=0}}{n!} \quad (2.43)$$

a recursive equation for the evaluation of the moments can be obtained in the form,

$$[Y_0] M_n = - \sum_{r=1}^n \frac{[V(s)]^{(r)} M_{n-r}}{r!} \quad (2.44)$$

with,

$$[Y_0] M_0 = E \quad (2.45)$$

Next the derivatives of Y are obtained as,

$$[Y]^{(1)} = \begin{bmatrix} C & 0 & 0 & \dots & 0 \\ P_1^{(1)} D_1^t & Q_1^{(1)} & 0 & \dots & 0 \\ P_2^{(1)} D_2^t & 0 & Q_2^{(1)} & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ P_{N_s}^{(1)} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s}^{(1)} \end{bmatrix} \quad (2.46)$$

and,

$$[Y]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ P_1^{(r)} D_1^t & Q_1^{(r)} & 0 & \dots & 0 \\ P_2^{(r)} D_2^t & 0 & Q_2^{(r)} & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ P_{N_s}^{(r)} D_{N_s}^t & 0 & 0 & \dots & Q_{N_s}^{(r)} \end{bmatrix} \quad (r \geq 2) \quad (2.47)$$

The derivatives $P_k^{(r)}$ and $Q_k^{(r)}$ can be determined recursively in terms of the line parameters.

The above technique solves the differential equation and then obtains the derivatives recursively. Another technique to obtain the moments has been proposed which derives a recursive relationship on the differential equations recursively for each moment and then solves a differential equation for each moment [20].

2.3.4 Comments On AWE Technique

The AWE technique is fast compared to full simulation as can be seen from circuit simulation results shown in Table 2.1. It also resolves many of the limitations of the RC tree method and increases its accuracy in a general way.

Table 2.1 CPU comparison between AWE and HSPICE

Circuit	number of transmission lines	node count	lumped element count	CPU seconds	
				AWE	HSPICE
1	35	101	145	2.62	256
2	70	201	290	3.91	5261
3	105	301	435	6.05	7702

However, based on our observations, and published comments [8], the technique has some drawbacks.

- Equation (2.21) quickly becomes ill-conditioned with increasing approximation order. The problem may be postponed temporarily by frequency scaling, i.e. dividing all the moments by the m_{-1} or m_0 . Eventually however, at some order, sometimes low, the matrix yields highly inaccurate poles which are exemplified by poles with positive real parts for known stable systems. This is the problem of stability.

- The method yields different accuracy ranges for different output nodes and for different circuits. It has been noticed that error can be large not only near a source [5], but at other nodes of the circuit [21]. This is the problem of numerical sensitivity.

In his Ph.D. thesis dissertation, Huang [22] has shown that the AWE approximation is equivalent to one form of Padé approximation. In trying to address the general class of Padé approximations that are collectively called the Padé Table, he developed a technique of searching through several approximations and looking for dominant poles that would repeat themselves. Because of the expense of examining each entry of the Padé Table and generating poles at each entry, as the method requires, he restricts himself to diagonal entries of the Table. The results however are not convincing for several reasons.

1. Some accurate system poles may not repeat themselves, or if they do, they may have some error involved without the possibility of determining which one of the poles (if any) was actually the accurate one.
2. Some entries of the Padé Table not near the diagonal may contain accurate poles not found in diagonal Padé entries.

Nothing further has been published using the techniques mentioned in this thesis and it is not clear what are the CPU/accuracy trade-offs.

2.4 Moment Polynomial Nodal Analysis

The Moment Polynomial Nodal Analysis (MPNA) [6] was also developed as a moment-matching method for linear analysis. The differences between this method and the AWE are worth considering.

The MPNA generates moments symbolically, rather than numerically. LU decomposition is performed on polynomials rather than on floating point numbers. The huge expense that this method would require is only partially curtailed by the truncation of the polynomials at certain coefficients. Included in the method are distributed single RC lines.

Since this method was published, the AWE has been shown to be more efficient and extended to more general distributed elements. Additionally, the moment-matching method used suffers from the same problems as that of the AWE. As a result, the MPNA is no longer in vogue as a useful method.

2.5 Nonlinear Analysis: Mixed Frequency/Time Domain

If the nonlinearities of the termination are taken into account, a problem arises due to the fact that quasi-transverse electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain only. In contrast to this, nonlinear network can be in general analyzed in the time domain only. In order to solve the problem, the solutions in the two domains are required. A mixed frequency/time domain approach [37] has been applied to the case of lossy

transmission lines with nonlinear terminations. The method uses the frequency-domain data associated with the linear problem in the absence of nonlinearities. The solution of the nonlinear problem is obtained by time-stepping and convolution. The computational steps involves the evaluation of time domain Green's functions of the transmission lines terminated in quasi-matched loads. To compensate for the change in the termination, complementary networks with negative elements are inserted into the terminal networks. The method is computationally expensive since it requires evaluation of sets of convolution integrals at each time point. In addition, since these methods are targeted towards specific applications, they tend to be restricted, particularly in terms of the topology of the networks to be analyzed.

Another method is presented in [23],[10]-[24]. The method is general and not limited to the analysis of transmission lines. It is based on a piecewise decomposition technique where the nonlinear network is decomposed into linear and nonlinear subnetworks. The linear subnetworks contain all the frequency-dependent components. The terminals of the subnetworks are excited by piecewise linear time-dependent sources. Through solution of the linear subnetworks in the frequency domain, the nonlinear subnetworks in the time domain, and by applying Newton-Raphson iteration technique, the parameters of the piecewise waveforms are adjusted so that the topological and constitutive relations of each subnetwork as well as the topological equations resulting from the interconnection of these subnetworks are satisfied. This method does not impose any restrictions on the topology of the linear or nonlinear subnetworks or on their interconnections. More about this method will be presented in chapter 5.

The methods in [10],[37]-[24] are computationally expensive because they require convolution in order to couple the linear and nonlinear networks for a complete analysis. A method is sought that will eliminate the convolution from the calculations.

2.6 Comparison of Current Research and Other Techniques

The methods in the literature continue to be generalized. The AWE as applied to lossy, coupled transmission lines, is the most general approach given. However, the AWE relies on a form of Padé approximation as has been shown. This type of approximation actually allows an infinite set of possible approximations. The work of this Thesis consists of further generalizing the moment-matching method to enhance the accuracy of the approximation. Several possible approximations are generated from the general Padé Table and one final approximation is selected. The final approximation may not be equal to any in the Padé Table, thereby allowing the examination of many more new approximations. The selection of a single approximation allows all poles to be generated collectively.

In addition, some circuit nodes prove to be difficult to analyze. As a result, several nodes can be examined and using Optimal Pole Selection the best set of poles selected. This method is useful, when one set of poles is needed for a macromodel in nonlinear simulation.

Each of these general methods entail more CPU cost than the originally proposed AWE method. However, the accuracy trade-off is a necessary and useful

one. Additionally for nonlinear analysis, a method is proposed that allows the use of the exponential form of the impulse response in order to eliminate convolution and make the nonlinear analysis a linear function of time.

Chapter 3

Generalized Moment-matching

3.1 Introduction

The AWE method has been shown to be fast and often accurate. However, the published method, while being an approximation, can result in a large variation of error in different networks, at different outputs, and with different inputs. By increasing the approximation order of stable networks, right-half plane poles are generated, and the error in the response depends critically on choice of network and output at which the method is applied. Sometimes only two or three left-half plane poles are obtained and all may be inaccurate approximations of the dominant stable network poles. Since the approximate residues of each output are derived

from the approximate poles, and the transient response is derived from both poles and residues, applying the AWE can sometimes give large errors in the approximate system response [5],[8].

It has been noted in [5] that, as with RC tree methods, the error in the approximation of the AWE is large closest to the driven end of the line. Depending on the network, it has been observed that large approximation error can result at additional outputs other than those close to the driven end and, generally, different magnitudes of error result when the AWE is applied to different outputs of a network. This poses a problem since it is not known *a priori* which outputs will yield small error. This is the problem of Sensitivity.

The drawbacks of the moment-matching approximations are these problems of stability and sensitivity. Additionally, it has been noticed for interconnect networks, the output signal can contain severe ringing and signal bounce. As a general rule, these type of signals may require many dominant poles and residues. The moment-matching technique typically yields only a maximum of 8-10 stable poles.

It has been shown in [22] that the AWE is equivalent to performing a near diagonal Padé approximation. This chapter introduces a generalized approach to moment-matching based on the properties of Padé approximations. Rather than using one moment-matching approximation, several are generated from the Padé Table at a minimal additional expense for a large network. Then, using two selection

criteria based on the poles of smallest modulus and the total moment skew, an optimal moment-matching set is selected.

For each entry in the Table, filtering the parasitic and/or right-half plane poles while leaving the remaining poles unaffected, leads to even better accuracy. Several examples are presented which show the stability and visible improvement in the approximate transient response over the published methods at a minimal extra cost for a large circuit.

3.2 Moment-matching Problems

The frequency-domain transfer function of a circuit output i can generally be described by,

$$V_i(s) = c + \sum_{j=1}^{N_p} \frac{k_j}{s - p_j} \quad (3.1)$$

where the p_i are the N_p actual poles of the circuit, the k_i their corresponding residues, and c the coupling constant between the input and the output. The coupling constant is non-zero at certain nodes close to a source and was not considered in the original AWE approach since only the outputs of a circuit were examined. However nonlinear analysis uses, as will be seen, an N-port macromodel of the linear subcircuit be formed, requiring the value of c .

Finding the all the poles (eigenvalues) of large networks is expensive in the case of lumped-element networks, and impossible in the case of distributed

interconnect networks, as these latter contain an infinite number of poles. Moment-matching, however, allows the calculation of a small approximate set of poles. It has been shown in Chapter 2 that moments of an output can be generated inexpensively both for lumped element networks, as well as for linear interconnect networks, modelled as lossy coupled transmission lines.

Given below, for reference, are the generating equations for an approximate characteristic equation used to generate approximate poles in the AWE.

$$\begin{bmatrix} m_{-1} & m_0 & m_1 & \dots & m_{q-2} \\ m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ m_{q-2} & m_{q-1} & m_q & \dots & m_{2q-3} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \cdot \\ \cdot \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_{q-1} \\ m_q \\ m_{q+1} \\ \cdot \\ \cdot \\ m_{2q-2} \end{bmatrix} \quad (3.2)$$

These equations can be concisely described using matrix M and vectors A and N as,

$$MA = N \quad (3.3)$$

There are several problems with these set of equations. The order of the approximation is reflected on the size of the matrix M . The larger the order, the more approximate poles are extracted from the vector A . However the matrix M quickly becomes ill-conditioned with larger dimension, yielding highly inaccurate poles. This is clear in that when the approximation is applied to known stable systems, right

hand plane poles always result as some high order, usually by about order 8-10 or lower. Right hand plane poles mean that the approximation will be unstable with increasing time leading to larger and larger deviations from the output of a true stable system. This is the problem of instability.

This property of the moment-matching approach typically permits only certain low orders of approximation resulting in a small number of approximate poles. Even a well-conditioned matrix can result in a polynomial that yields right-half plane poles for a stable network or approximate poles with relatively large error compared to the real dominant poles. Methods have been attempted to minimize the problem by constraining the poles to the left-hand plane using non-linear optimization [25].

Large approximation errors have been observed when the AWE is applied at outputs near a source. It has also been noted that large error can occur at outputs far from a source. This causes a difficulty since it is not known *a priori* which outputs will give small error magnitudes. Additionally, for the same network topology, the AWE can yield different relative error magnitudes for the same order of approximation when the network impedance parameters are varied. The problem is highlighted when the system is made "stiff" where the closest pole to the origin is much closer than the next nearest poles. This is the problem of sensitivity to network and output choice.

3.3 The Padé Table

In the present method the approximation is based, as in [22], on the general class of Padé approximations where the moments of a Maclaurin expansion of a function are matched to the moments of the rational approximation,

$$V_i(s) \approx \hat{V}_i(s) = \frac{P_L(s)}{Q_M(s)} = \hat{c} + \sum_{j=1}^M \frac{\hat{k}_j}{s - \hat{p}_j} \quad (M \ll N_p) \quad (3.4)$$

where both numerator and denominator polynomials are of order L and M respectively [26] and the $\hat{k}$, $\hat{p}$, $\hat{c}$ are approximations to some of the k , p , c in (3.1). If the denominator polynomial is described by $b_0 + b_1s + b_2s^2 + \dots + b_Ms^M$, the moment-matching equations are given by:

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \dots & m_L \\ m_{L-M+2} & m_{L-M+3} & \dots & m_{L+1} \\ \vdots & \vdots & & \vdots \\ m_L & m_{L+1} & \dots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-1} \\ \vdots \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \vdots \\ m_{L+M} \end{bmatrix} \quad (3.5)$$

in which $m_j = 0$ if $j < 0$. A unique solution for $Q_M(s)$ is generated when $b_0 = 1$. Roots of $Q_M(s)$ yield approximate poles of the output response. The coefficients of the numerator polynomial, $P_L(s)$, follow from the solution of the denominator coefficients but will not be needed in this approach, (approximate residues of each pole will be calculated directly). The resulting approximation is called an $[L/M]$ Padé approximation. The denominator polynomials will be designated as $Q_{[L/M]}$ to indicate the order of the approximation. By inspection, if $m_{-1} = 0$, then (3.2) can be mapped to an $[L = q - 2/M = q]$ Padé approximation with a reversed order of coefficients for the denominator polynomial. (The roots of a polynomial B with

coefficients reversed from polynomial A , and the roots of polynomial A are reciprocals of each other).

$M \backslash L$	0	1	2
0	$\frac{1}{1}$	$\frac{1+z}{1}$	$\frac{2+2z+z^2}{2}$
1	$\frac{1}{1-z}$	$\frac{2+z}{2-z}$	$\frac{6+4z+z^2}{6-2z}$
2	$\frac{2}{2-2z+z^2}$	$\frac{6+2z}{6-4z+z^2}$	$\frac{12+6z+z^2}{12-6z+z^2}$

Table 3.1 Part of the Padé Table for $\exp(z)$

With different possibilities for L and M , it is customary to build a subset of an infinite table where each entry of the table consists of one Padé approximation corresponding to a specific choice of L and M [27]. This array is referred to as a Padé Table. For a large network, the greatest CPU expense is in obtaining the moments, rather than forming a subset of the Padé Table for a small range of L and M . And even here, the greatest cost is in the original LU decomposition, not in the generation of several moments. The approach proposed in this paper requires more moments than the published AWE but is outweighed by the accuracy achieved.

Our goal is to choose one entry, one selection of L and M , of the Padé Table that yields the best set of approximate poles to the dominant output poles. Once these

poles are obtained, the residues may be calculated using (2.22). Therefore, our Table entries need only contain the $Q_{[L/M]}$. Baker and Graves-Morris [27] suggest searching as broad band as possible about the $[L=M/M]$ diagonal entries in the Padé Table in order to search for the poles of the function. However, no clear indication of a search algorithm is given. Chan has remarked [8] on the lack of means of determining the best moment-matching approximation. Huang [22] has developed a search method for poles in the in the Padé Table but, as has been remarked, it is difficult to determine the accuracy of such a method. This thesis develops two criteria for selection of the best entry in the Padé Table.

3.3.1 Smallest Poles

Definition:

Given the network poles $\{p_j, j = 1, 2, \dots, N\}$, the pole(s) of smallest modulus, $\{p_s\}$, are defined as, $\{p_s \in \{p_j\} \mid |p_s| = \min(|p_1|, |p_2|, \dots, |p_N|)\}$.

The number of these poles, n_s , is usually 1 or 2; there being either a smallest pole or a pair of smallest complex conjugate ones. These poles are simply referred to as the smallest pole(s) or $\{p_s\}$. The $\{p_s\}$ and their residues, $\{k_s\}$, may be calculated accurately and independently of any of the other poles. They may therefore be used to gauge the accuracy of an approximation.

To determine the smallest poles, a property of the Padé Table is used. It can be shown [27] that in the Padé Table for a rational function,

$$\lim_{L \rightarrow \infty} |Q_{[L/M]}(s)| \rightarrow \prod_{j=1}^M \left(1 - \frac{s}{p_j}\right) \quad (3.6)$$

where p_j are the actual poles of the network arranged in order of increasing modulus:

$$|p_1| \leq |p_2| \leq \dots \leq |p_M| < \dots \text{ and } |p_M| \neq |p_{M+1}|.$$

In other words, the entries of the Padé Table for a given M will converge to a polynomial with the smallest M poles as its roots, provided those poles are all smaller than any other network pole. The smallest pole(s) may be obtained with a high degree of accuracy by taking L large and solving for the roots of the resulting $Q_{[L/M]}$. For instance, if there is one smallest pole p_s , ($n_s = 1$), (3.6) can be rewritten as,

$$\lim_{L \rightarrow \infty} |Q_{[L/1]}(s)| = 1 - s \left(\lim_{L \rightarrow \infty} \frac{m_{L+1}}{m_L} \right) \rightarrow 1 - \frac{s}{p_s} \quad (3.7)$$

The residues of the smallest poles may be obtained by,

$$\lim_{L \rightarrow \infty} \left(\begin{bmatrix} p_1^{-L} & p_2^{-L} & \dots & p_{n_s}^{-L} \\ p_1^{-1-L} & p_2^{-1-L} & \dots & p_{n_s}^{-1-L} \\ p_1^{-2-L} & p_2^{-2-L} & \dots & p_{n_s}^{-2-L} \\ \vdots & \vdots & \ddots & \vdots \\ p_1^{-n_s+1-L} & p_2^{-n_s+1-L} & \dots & p_{n_s}^{-n_s+1-L} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n_s} \end{bmatrix} = \begin{bmatrix} -m_{L-1} \\ -m_L \\ -m_{L+1} \\ \vdots \\ -m_{L+n_s-2} \end{bmatrix} \right) \quad (3.8)$$

Both (3.6), (3.7) & (3.8) generally demand a more moments than required in the published AWE. The additional moments needed will depend on convergence to the smallest poles.

Note that this technique does not work as well if $M > n_s$. While (3.6) is still theoretically correct for any $M > n_s$, the matrix in (3.5) to solve it quickly becomes ill-conditioned as L increases. This is a direct result of the fact that the smallest poles numerically dominate the moments m_i for large values of i . Analysis of poles of larger modulus than the $\{p_s\}$ is numerically difficult and must be done using lower orders of L in the Padé Table and the criteria outlined below. The limit to the accuracy of Padé approximations hinges critically on the number of accurate digits in the moments. It is necessary for any implementation to use as many bits as possible on the particular hardware running the approximations.

3.3.2 Selection Criteria

Once the $\{p_s\}$ are calculated, a large band of entries in the Padé Table is determined and the denominator polynomial is calculated for each entry. Two selection criteria allow the successive filtering out of inaccurate Padé entries which will allow extraction of an accurate set of poles and subsequent residues.

Criterion 1:

A table entry is eliminated if its roots do not include the smallest poles, $\{p_s\}$.

A Maclaurin expansion is only valid within its radius of convergence R , which among rational system response functions is equal to the modulus of the smallest pole(s). Padé approximations perform a type of analytic continuation of the function described by the Maclaurin series to give information about the function in the region beyond the radius of convergence. However, as with the Maclaurin series themselves, a Padé approximation decreases in accuracy as the distance is increased

from the origin [27]. If an approximation does not contain the smallest pole(s) as its roots, it is safe to assume that it is not an accurate approximation. Several Table entries may be eliminated in this way without further processing. This is a quick way to zero in on the accurate approximations.

This criterion is easily checked by either extracting the roots of each polynomial in the Table, or with less CPU, by simply substituting the $\{p_s\}$ into the denominator polynomials and verifying that the result is indeed numerically zero. Criterion 1 however, is not a sufficient condition for accuracy.

3.3.2.1 Moment Skew

Expanding (3.4) in a Maclaurin series (with $q \leq M$ poles, to be explained shortly) gives,

$$\hat{V}_i(s) = \left(\hat{c} + \sum_{j=1}^q \frac{-\hat{k}_j}{\hat{p}_j} \right) + \sum_{n=1}^{\infty} \left(\sum_{j=1}^q \frac{-\hat{k}_j}{\hat{p}_j^{n+1}} \right) s^n \quad (3.9)$$

in which the *approximate* moments of the response are,

$$\hat{m}_0 = \hat{c} + \sum_{i=1}^q \frac{-\hat{k}_i}{\hat{p}_i} \quad \hat{m}_j = \sum_{i=1}^q \frac{-\hat{k}_i}{\hat{p}_i^{j+1}} \quad (3.10)$$

Approximate q residues for q approximate poles are calculated using equations that include the coupling constant,

$$\begin{bmatrix} \hat{p}_1^{-1} & \hat{p}_2^{-1} & \dots & \hat{p}_q^{-1} & 1 \\ \hat{p}_1^{-2} & \hat{p}_2^{-2} & \dots & \hat{p}_q^{-2} & 0 \\ \hat{p}_1^{-3} & \hat{p}_2^{-3} & \dots & \hat{p}_q^{-3} & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ \hat{p}_1^{-q-1} & \hat{p}_2^{-q-1} & \dots & \hat{p}_q^{-q-1} & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_1 \\ \hat{k}_2 \\ \vdots \\ \hat{k}_q \\ \hat{c} \end{bmatrix} = \begin{bmatrix} -m_0 \\ -m_1 \\ -m_2 \\ \vdots \\ -m_q \end{bmatrix} \quad (3.11)$$

(3.10) and (3.11) imply that,

$$m_n = \hat{m}_n \quad n = 0, 1, \dots, q. \quad (3.12)$$

A selection criterion for a set of poles and residues is possible by comparing the next moments unused in (3.11) with the corresponding moments of the approximate response. In other words, a comparison is made between m_n and $\hat{m}_n$ for $n > q$. For a given moment m_n and respective approximate moment $\hat{m}_n$, the n -th moment skew, ϵ_n , is defined as the scaled difference between the two values,

$$\epsilon_n = \left| \frac{m_n - \hat{m}_n}{m_n} \right| \quad (3.13)$$

There are an infinite number of these moment skews but the sequence of them converges to zero for any set of poles that passes criterion 1 because the smallest pole(s) dominate both the system moments and the approximate moments. The total moment skew, ϵ , for a particular set of poles and residues is given by,

$$\epsilon = \left(\sum_{i=q+1}^{\infty} \epsilon_n^a \right)^{1/a} \quad a \geq 1 \quad (3.14)$$

A large error in approximation implies a large moment skew. This leads to the second criterion.

Criterion 2:

Of the entries remaining from criterion 1, the set of $q \leq M$ filtered (non-parasitic, non-positive) poles and subsequent residues of each entry that minimizes the total moment skew, ϵ , is the one selected.

A set of M approximate poles is extracted from the each $Q_{[L/M]}$ of the Padé Table entries not rejected by the first criterion. Each set of approximate poles however, is not the final set that will be used for error comparison, because it may contain largely inaccurate poles. Poles known to be inaccurate may be removed from a set without reference to the other $q \leq M$ possibly accurate poles in the set. The nature of Padé approximations is that they may produce both inaccurate and accurate poles in the same approximation. In effect, the known inaccurate poles are "filtered" from each set of approximate poles. This procedure may be carried out because criterion 2 based on the unused moments will determine if a set of poles is collectively accurate.

Inaccurate poles may be in the form of "positive" poles or parasitic ones. Positive poles refer to those poles with positive real parts which are known to be inaccurate when the approximation is of a stable network. These poles often result when actual left-half plane network poles are large relative to their residues and to the smallest pole(s) which causes a large error in their approximation. They often appear large in the approximation but on the positive side of imaginary axis. Rather than rejecting an entire approximation, positive poles are removed from the set of

approximate poles. This implies that all generated approximations will be guaranteed stability.

Parasitic poles are defined to be spurious poles generated by a Padé approximation which are relatively close to the origin and are associated with relatively small residues. These poles are caused by numerical error and do not reflect actual network poles. They can usually be detected. Any poles clearly smaller than the known $\{p_i\}$ are deemed to be spurious. Other parasitic poles must be detected by calculating the approximate residues and the remaining poles. An error margin must be set for determination of small residues which will disqualify remaining parasitic poles. This successive filtering of inaccurate poles allows the examination of many approximations not considered in the published AWE.

Criterion 2 implies that approximate q poles and q residues of the selected Table entry not only match a q -th order approximation, they also minimize the error of higher order approximations. Since the moment-matching is done in the frequency domain, any added accuracy this method provides compared to the AWE can mean a large improvement in the transient response approximation. This is clearly visible in the examples outlined.

3.4 Implementation and CPU Considerations

Difficulties in implementation arise due to numerical error and the setting of margins. The moments will allow for quick converges to the smallest pole(s) if there are no significant poles nearby, otherwise it may take many moments. A tolerance

limit is required typically to recognize convergence or a maximum number of moments. Each additional moment will cost a forward backward substitution from the LU decomposed matrix.

The limit to the accuracy of Padé approximations hinges critically on the number of accurate digits in the moments. It is necessary for any implementation to use as much accuracy as possible on the particular hardware running the approximations.

The Padé Table calculation of denominator polynomials may be done on an entry by entry basis using (3.5). However, [27] lists several published algorithms to calculate the polynomials recursively for each entry without needing matrix solutions. For a large number of Table entries, this approach can prove to be less costly. A polynomial root finding algorithm must be applied only to the few entries left over from criterion 1. Typically, roots of polynomials no greater than degree 10-12 are needed.

The assumptions for this method to be useful is that it is applied to a large, stable circuit. The Padé Table analysis has been timed at 0.2-0.6 CPU seconds on a Sun SPARC II. An LU decomposition for a large transmission line network containing 105 transmission lines and 435 lumped elements was approximately 2.0 CPU seconds, making the extra analysis cost reasonable for higher accuracy achieved.

3.5 Examples

3.5.1 Example 1:

Consider the six-pole circuit shown in Figure 3.1. The output response is sought at node 7. The Padé Table row $M=1$ converges to the smallest pole, $p_s = -1.283258442e8$, as $L \rightarrow 8$ (Table 3.2). Having obtained the smallest pole, the Padé Table is filled for $L=1 \rightarrow 6$ and $M=2 \rightarrow 6$ with the various denominator polynomials. The p_s is then substituted into each polynomial and the error is tabulated (Table 3.3). Following criterion 1, all but the smallest results are eliminated. Several of the approximations generated by the AWE method (bold) are eliminated.

Subsequently criterion 2 is applied, (using $a = 1$ in (24)). The $[0/6]$ approximation yields the lowest moment skew and is the one chosen (Table 3.4).

Figure 3.1 Six pole circuit

The actual network poles and output residues were obtained using eigenvalue methods. The 5th order (and higher) AWE approximation is unstable due to the right-half plane poles and the best stable AWE approximation which was obtained was 4th

order. The transient impulse response of the network is compared to the transient approximation of the 4th order AWE and that of the proposed method in Figure 3.2.

$\begin{smallmatrix} L \\ M \end{smallmatrix}$	0	1	2	...4	...8	...10
1	-1.273825338e8	-1.283625564e8	-1.283262560e8	-1.283258441e8	-1.283258442e8	-1.283258442e8

Table 3.2 $M=1$ part of the Padé Table for node 7: convergence of the smallest pole

$\begin{smallmatrix} L \\ M \end{smallmatrix}$	0	1	2	3	4	5	6
2	3.4296e-4	1.3803e-5	1.1212e-7	2.8857e-9	3.8237e-11	6.8237e-13	1.4055e-14
3	1.4414e-6	4.0866e-9	1.9599e-11	2.0568e-13	1.1090e-15	2.3597e-16	1.9311e-16
4	1.0162e-8	3.2298e-11	3.0458e-13	3.2326e-15	1.8471e-16	1.6156e-16	1.3659e-15
5	3.912e-11	1.4996e-13	1.4596e-15	1.8189e-16	1.6796e-16	4.1998e-16	5.7369e-16
6	1.5252e-16	1.3209e-16	1.2161e-16	1.1287e-16	4.7278e-15	4.5975e-16	8.8702e-16

Table 3.3 Padé Table for node 7: criterion 1 (smallest pole residual error)

$L \backslash M$	0	1	2	3	4	5	6
2							
3						-1.28326e8 (7.25365e8, 7.41344e9) $\epsilon = 2.91e-4$	-1.28326e8 (7.14596e8, 7.406 67e9) $\epsilon = 2.74e-4$
4					-1.28326e8 (-7.23178e8, 7.41245e9) -2.45108e10 $\epsilon = 2.64e-7$	-1.28326e8 -1.65542e9 (-7.28106e8, 7.40707e9) $\epsilon = 2.14e-5$	
5				-1.28326e8 (-7.29453e8, 7.4179e9) $\epsilon = 3.00e-4$	-1.28326e8 (-7.23365e8, 7.41133e9) -1.88244e10 $\epsilon = 7.09e-7$	-1.28326e8 (-7.23170e8, 7.41244e9) -2.44914e10 $\epsilon = 2.65e-7$	-1.28326e8 (-7.2990e8, 7.40825e9) $\epsilon = 2.88e-4$
6	-1.28325e8 (-7.27192e8, 7.41566e9) (2.49390e7, 2.24356e10) -3.18340e10 $\epsilon = 2.98e-14$	-1.28326e8 (-7.27181e8, 7.41563e9) (-2.19596e7, 2.24407e10) -3.18584e10 $\epsilon = 3.09e-14$	-1.28326e8 (-7.27270e8, 7.41559e9) -3.22234e10 $\epsilon = 8.09e-9$	-1.28326e8 -1.59573e9 (-7.30999e8, 7.40652e9) $\epsilon = 2.25e-5$		-1.28326e8 (-7.35593e8, 7.39453e9) $\epsilon = 2.74e-4$	-1.28326e8 (-7.24719e8, 7.41189e9) -3.10044e10 $\epsilon = 7.57e-9$

Table 3.4 Padé Table for node 7: criterion 2 (minimum moment skew)

	Actual System	AWE 4th order	AWE 5th order (unstable)	AWE 6th order (unstable)	Proposed method [L=0/M=6]
p_i	-1.2832584e8 (-7.2719171e8 $\pm j 7.4156615e9$) (-2.4947006e7 $\pm j 2.2435630e10$) -3.1834063e10	-1.2832584e8 (-7.0425278e8 $\pm j 7.3789743e9$) -2.5330573e10	-1.2832584e8 (-7.2945278e8 $\pm j 7.4179371e9$) (2.3164665e8 $\pm j 1.7750696e10$)	7.5971377e6 -1.2832584e8 (-7.29419085e8 $\pm j 7.4179911e9$) (2.0744602e8 $\pm j 1.7734305e9$)	-1.2832584e8 (-7.2719184e8 $\pm j 7.4156614e9$) (-2.4938974e7 $\pm j 2.2435625e10$) -3.1834036e10
k_i	9.4334863e7 (-5.0383113e6 $\pm j 1.5112221$) (4.0638252e6 $\pm j 2.4328835e6$) -1.6962861e6	9.4334863e7 (-4.9115303e7 $\pm j 1.5218084e7$) 3.8957442e6	9.4334863e7 (-5.0522910e7 $\pm j 1.5057107e7$) (3.3554783e6 $\pm j 1.4171671e6$)	-9.1072982e-9 9.4334863e7 (-5.0524489e7 $\pm j 1.5059226e7$) (3.3570606e6 $\pm j 1.4157937e6$)	9.4334863e7 (-5.0383114e7 $\pm j 1.5112218e7$) (4.0638283e6 $\pm j 2.4328801e6$) -1.6962902e6

Table 3.5 Actual and approximate poles/residues of circuit in Figure 3.1.

Figure 3.2 Impulse response and approximations (* waveform is almost overlapping the true response)

3.5.2 Example 2:

Figure 3.3 Stable op-amp example

Consider the filter shown in Figure 3.3. The output response is sought at V_{out} . The AWE method yields order 6 as the highest stable approximation. The proposed approach finds a minimum moment skew at $[L=2/M=10]$ which yields 8 filtered poles. A comparison of the approximate transient impulse response of the two methods is given in Figure 3.4. The actual transient impulse response was obtained using eigenvalue analysis.

Figure 3.4 Impulse response and approximations (* waveform is almost overlapping the true response)

3.5.3 Example 3:

Consider the sample interconnect circuit in Figure 3.5. The circuit approximates a transmission line effect. The output is sought at V_{out} . The highest order AWE approximation is 7 (poles), after which, at higher orders, positive poles are encountered. The AWE 7th order nevertheless yields an approximate transient response close to the original response. However, the proposed method increases the accuracy by selecting the lowest moment error at a [5/10] approximation which gives 9 filtered poles. The real system poles are compared to the approximate poles generated by the 7th order AWE as well as those generated by the proposed method in Table 3.6. Residues are compared in Table 3.7. Clearly, most of the poles and all of the residues gain accuracy with the proposed method. Additionally, there are two approximate poles (residues) not found using the highest order stable AWE approximation.

Figure 3.5 RLC interconnect network

Actual System	AWE 7th order		[L=5/M=10]	
Poles	Approx. Poles	% Error	Approx. Poles	% Error
-1.8583557e8 $\pm j$ 1.6096808e8	-1.8583557e8 $\pm j$ 1.6096808e8	0.0	-1.8583557e8 $\pm j$ 1.6096808e8	0.0
-4.5407644e8	-4.5407644e8	9.41e-11	-4.5407644e8	6.94e-12
-5.4105969e9	-5.3837375e9	4.96e-1	-5.4081962e9	4.44e-2
-7.6699449e9	-7.4981994e9	2.24	-7.6524283e9	2.28e-1
-1.7437664e10	-1.9438124e10	11.5	-1.7587814e10	8.61e-1
-3.9734773e10	-3.1921279e10	19.7	-4.9668124e10	24.9
-5.8691526e10 $\pm j$ 1.9263470e10		Inf.	-8.9188622e10	58.4
-8.0519958e11		Inf.	-3.6853250e11	54.3

Table 3.6 Interconnect pole comparison

Actual System	AWE 7th order		Proposed Method [L=5/M=10]	
Residues	Approx. Residues	% Error	Approx. Residues	% Error
-9.7395276e5 $\pm j$ 3.8031794e5	-9.7395276e5 $\pm j$ 3.8031794e5	8.54e-13	-9.7395276e5 $\pm j$ 3.8031794e5	6.02e-13
8.8340210e5	8.8340210e5	1.08e-9	8.8340210e5	1.26e-10
-1.8764976e6	-1.7595713e6	6.23e-2	-1.8657506e6	5.73e-1
-4.9961328e6	-4.7872149e6	4.18	-4.9710776e6	5.01e-1
1.2706096e7	1.7349227e7	3.65e1	1.2907281e7	1.58
-1.4201132e6	-9.7379374e6	5.86e2	-6.8431081e6	3.81e2
-1.6741216e6 $\pm j$ 2.0810412e6		Inf.	1.8478639e6	1.53e2
-6.0600265e2		Inf.	-1.0705571e4	1.67e3

Table 3.7 Interconnect residue comparison

3.5.4 Example 4:

Figure 3.6 Cascaded transmission line network

Consider the cascaded transmission line network in Figure 3.6, proposed by Chang [28]. Two single lossy lines with different resistive line parameters are placed back to back. The parameters of T_1 are $Rl = 25\Omega$, $Gl = \frac{1}{2(\infty)}\Omega^{-1}$, $Ll = 0.1\mu H$, $Cl = 40pF$, where l is the length of the transmission line. The parameters of T_2 are the same except for $Rl = 3.125\Omega$. A 1V pulse of 2.5 ns rise/fall time and 7.5ns duration is placed at the V_a input, and the output waveform is measured at the load resistance. Moment-matching is applied as in the AWE and the highest stable order of approximation obtained is 6th order. By analyzing the entire Padé Table, [2/14] approximation yields 10 filtered poles which gives the best Moment Skew results. The AWE result is compared to the FFT generated waveform in Figure 3.7. The proposed method's result is given in Figure 3.8.

It is to be noted however that as the frequency of the input is increased, (decreased rise time), even the best moment-matching results are not sufficient. There is no more information in the moments about high frequency dominant poles. As a result, it is necessary to resort to other means in order to get an accurate approximation.

Figure 3.7 FFT and AWE response for Figure 3.6

Figure 3.8 FFT and generalized moment-matching response for Figure 3.6

Chapter 4

Optimal Pole Selection

4.1 Introduction

As has been noted the error in the approximation of the AWE is large closest to the driven end of the line. As an example will show, depending on the system, large approximation error can result at different outputs of the system other than those close to the driven end and, generally, different magnitudes of error result when the AWE is applied to different outputs of a system. This poses a problem since it is not known *a priori* which outputs will yield large error. It also creates difficulties when one attempts to define "near to" or "far from" from a

driven end, when there are two or more sources in the circuit, or when the circuit is a complex one with thousands of nodes.

In this thesis, a selection criterion was developed which provides a measure of the relative accuracy of an approximating set of poles and residues generated from the Padé Table. In this chapter, another alternative is proposed to generating several approximating poles and residues. Out of those generated, an optimal set selected based on the derived selection criterion. This method can deal with problems of output sensitivity. The algorithm results in visibly improved accuracy in both transient and frequency analysis as compared to the conventional AWE method. It also allows the selection of a set of approximating poles for the entire circuit in order to apply non-linear analysis.

4.2 Optimal Selection of Poles

The proposed method differs from typical moment-matching methods in that it does not assume that the best set of approximating poles for a particular output are necessarily generated at that same output. It is clear that at each output examined, the set of approximating poles, $\hat{p}_1, \hat{p}_2, \dots, \hat{p}_q$, obtained from moment matching using that output's moments, are approximations of the same system poles, $p_1, p_2, \dots, p_n$. Since these approximate pole sets are generally different from each other, one set of poles may more accurately approximate an output's dominant poles than another set. Given any set of approximating poles, (2.22) may be used to obtain the approximating residues for an output using that output's moments. A greater accuracy of the

approximating dominant poles will imply a greater accuracy in the resulting approximating residues in (2.22) and the approximate frequency/transient response in (2.18) & (2.19). Sets of approximating poles may be derived at any of the other outputs using the available moments and (2.21).

Generally, a set of poles from one output can improve the approximation at another output if both have the same set of dominant poles. Outputs that yield right half plane poles for a given order of approximation may have their poles replaced from an output that yields only left-half plane poles for that order of approximation.

4.2.1 Outline of the Algorithm:

1. The AWE is applied only up to the point of obtaining all the needed output moments of an order q moment matching approximation. The moments for every output in the circuit are a consequence of this procedure and need not be calculated separately.
2. Given the set, $\{v_N\}$, of all outputs in the vector x , a set of outputs, $\{v_s | v_s \in \{v_N\}\}$, is selected including the desired output, $v_o \in \{v_s\}$, and using moment matching, sets of approximate poles, $\{\{p_j\}_s\}$, are derived for each $v \in \{v_s\}$. The set of outputs in $\{v_s\}$ may be selected incrementally as needed until a minimum moment skew or a maximum number of outputs is achieved.

3. Using the acquired sets of poles, $\{\{p_j\}_s\}$, from each selected output $v \in \{v_s\}$, corresponding sets of residues $\{\{k_j\}_{s \rightarrow o}\}$ are obtained using the moments from output v_o .
4. For a given output, one set of poles and corresponding set of residues, $\{p_j\}_i \in \{\{p_j\}_s\}$, $\{k_j\}_{i \rightarrow o} \in \{\{k_j\}_{s \rightarrow o}\}$, is selected as the more accurate approximation based on the preceding error criterion which verifies the skew in one or more additional Maclaurin's series moments not used in the approximation.
5. The selected poles and residues enables, as outlined in the AWE, the approximation of both the transient waveform and the frequency-domain transfer response of the output.
6. It may be desirable to have one set of system poles for a multi-port characterization of a linear network in order to perform nonlinear analysis. In this case a set of poles for each output is extracted, and the set that yields minimum moment skew at all or most of the outputs is selected.

4.3 Examples

4.3.1 Example 1:

Figure 4.1 Six pole circuit

Consider the circuit shown in Figure 4.1. The output response is sought at node 7. The circuit contains 11 possible outputs: the 7 voltage nodes, the 3 inductor currents, and the voltage source current. Two additional outputs, voltages at nodes 5 & 6, are selected for analysis. A set of 5 poles is generated using (2.21) and moments at voltage node 5, (conventional 5th order AWE). Additionally, two sets of poles are generated using these moments at nodes 6 & 7. Using each of the three sets of poles, three sets of residues are generated using (2.22) and the moments at voltage node 7. A moment skew comparison is then made using the next unused moment m_4 at node 7. (Table 4.1).

Output 5 yields the minimum error and is the choice of the three outputs from which to obtain poles. A comparison of the transient impulse response at node 7 and the approximations derived from the published AWE technique and those derived

using Optimal Pole Selection is given in Figure 4.2. The approximate response from the proposed method is almost completely overlapping the true response. A comparison of the actual frequency response to its approximations is given in Figure 4.3 and Figure 4.4. The error in the poles/residues generated by the AWE and Optimal Pole Selection as compared to the actual system poles/residues is given in Table 4.2.

5th order conventional AWE at node 7		Poles derived from node 6 and subsequent residues		Poles derived from node 5 and subsequent residues	
Poles	Residues	Poles	Residues	Poles	Residues
-3.4511e6	1.0523e6	-3.4512e6	1.0523e6	-3.4512e6	1.0523e6
-4.7398e7	-1.3281e6	-4.7395e7	-1.3391e6	-4.7395e7	-1.3386e6
-2.0015e8	9.9068e4	-2.0015e8	2.3522e5	-2.0015e8	2.2731e5
-4.6420e6 + j 3.3176e8	8.8327e4 - j 4.9057e4	-4.5571e6 + j 3.3179e8	2.5843e4 + j 2.3874e4	-4.5560e6 + j 3.3179e8	2.9471e4 + j 1.9642e4
-4.6420e6 - j 3.3176e8	8.8327e4 + j 4.9057e4	-4.5571e6 - j 3.3179e8	2.5843e4 - j 2.3874e4	-4.5560e6 - j 3.3179e8	2.9471e4 - j 1.9642e4
$\epsilon_4=2.6864e-6$		$\epsilon_4=1.2443e-7$		$\epsilon_4=3.8156e-8$	

Table 4.1 Selection of Poles and Residues for output 7

Figure 4.2 Transient impulse response and approximations at node 7 (The proposed method waveform is almost overlapping the true response).

Figure 4.3 Frequency response and approximations at node 7

Figure 4.4 Phase and approximations at node 7

Actual system		Pole Error %		Residue Error %	
Poles	Residues at output 7	Conventional AWE	Proposed Method	Conventional AWE	Proposed Method
-3.4512e6	1.0523e6	2.95e-4	4.19e-6	1.21e-3	1.57e-5
-4.7395e7	-1.3387e6	6.80e-3	9.60e-5	8.00e-1	1.13e-2
-2.0016e8	2.2916e5	4.49e-3	4.96e-4	56.8	8.07e-1
-4.5543e6 + j 3.3178e8	2.8623e4 + j 2.0632e4	2.74e-2	7.16e-4	260	3.70
-4.5543e6 - j 3.3178e8	2.8623e4 - j 2.0632e4	2.74e-2	7.16e-4	260	3.70
-9.9900e9	-1.0306e-1				

Table 4.2 Percent error in approximating poles and residues at node 7 .

4.3.2 Example 2:

In Figure 4.5, an RC tree is shown, as given in [4]. The desired output is the voltage at node 11. An additional output, the voltage at node 10, is compared. Poles derived from each output and residues derived from node 11 moments are compared in Table 4.3 and the percent error from the actual system poles and residues at node 11 is compared in Table 4.4.

Figure 4.5 RC tree network

6th order conventional AWE at node 11		Poles derived from 10 and subsequent residues	
Poles	Residues	Poles	Residues
-1.7309e9	7.5412e7	-1.7309e9	7.5417e7
-1.3208e10	1.3471e9	-1.3203e10	1.3347e9
-2.2811e10	1.5307e8	-2.2828e10	2.1474e8
-6.0643e10	5.4911e10	-6.0621e10	5.4668e10
-1.1297e12	-6.6688e10	-1.1238e12	-6.3690e10
-7.9129e12	1.0201e10	-9.1269e12	7.3969e9
$\epsilon_5=9.2009e-6$		$\epsilon_5=4.1634e-7$	

Table 4.3 Selection of poles and residues for output 11

A selection of the poles from node 10 is made. Clearly, the poles generated at node 10 are more accurate approximation of the dominant poles at node 11, than poles generated from node 11 itself.

Actual system		Pole Error %		Residue Error %	
Poles	Residues at output 11	Conventional AWE	Proposed Method	Conventional AWE	Proposed Method
-1.7309e9	7.5417e7	1.17e-3	5.30e-7	6.14e-3	2.78e-4
-1.3203e10	1.3352e9	3.68e-2	1.21e-5	8.93e-1	3.78e-2
-2.2827e10	2.1217e8	7.08e-2	2.62e-5	27.86	1.21
-6.0623e10	5.4679e10	3.42e-2	1.95e-5	4.24e-1	196e-1
-3.0266e11	-2.4628e2				
-6.6920e11	-5.5799e6				
-2.0598e12	-4.2607e5				
-1.1236e12	-6.3775e10	5.412e-1	1.40e-4	4.57	1.33e-1
-9.1359e12	7.4789e9	13.39	9.78e-4	36.4	1.096
-1.6417e13	1.1807e-5				

Table 4.4 Percent error in approximating poles and residues at node 11

Chapter 5

Fast Nonlinear Analysis

5.1 Introduction

Transient analysis of large networks containing both lumped and distributed elements including lossy, coupled transmission lines with arbitrary nonlinear terminations has proven to be CPU intensive. Several methods have been proposed for the analysis of networks containing coupled and/or lossy transmission lines [7],[29]-[35]. Some of these methods have been modified to apply to networks with nonlinear terminations [10],[24],[36],[37]. Additionally, other new approaches have been proposed [38]-[39] for the analysis of nonlinear distributed networks. Of these methods, all except lumped-element model reductions, require

time-domain convolution and/or numerical inversion from the frequency domain to the time domain.

Any convolution method used implies that for each time point calculation all previous time points must be taken into account. The CPU expense can quickly become large with increasing number of time points. This is especially true for mismatched transmission lines. Djordjevic et al. [33] have noted that convolution is the most time-consuming portion of the calculations and attempt to minimize it by quasi-matching the transmission lines. Schutt-Aine and Mittra [36] also make note of the expense of convolution which they are able to avoid, with considerable speed-up, only for the lossless case.

Additionally, most methods require some form of numerical inversion algorithms such as FFT [33] or Numerical Inversion of Laplace Transform (NILT) [35], sometimes applied in numerous iterations [39], to transform frequency points into time-domain descriptions. This is an additionally large expense for large system and with many frequency points. In [10] a new approach was proposed for nonlinear waveform estimation which does not require numerical inversion methods. However, the method still requires convolution.

In this chapter, an extension of waveform estimation which requires neither convolution nor numerical inversion methods is outlined [40]. It is ideally suited for very large systems with many components. The method relies directly on the approximate poles and residues obtained from the Asymptotic Waveform Evaluation (AWE) and its generalization (GAWE) which have previously been successfully

applied to linear waveform estimation. The AWE and GAWE have been shown to give an estimated transient response with 2-3 orders of magnitude speed-up over SPICE. With elimination of convolution, the nonlinear solution speed-up is increased further by 25 to 50 times. In addition, the proposed method can handle the general case where the network contains lossy, multiconductor transmission lines, whereas conventional circuit simulators such as SPICE cannot. While being fast, the method is ideal for implementation in terms of parallel computation which would increase its efficiency.

5.2 Response of the Linear Subnetwork

Consider a nonlinear network π which contains an arbitrary linear subnetwork. The linear subnetwork may contain distributed components. The actual frequency response of the linear subnetwork due to an impulse applied at a single input and measured at single output can be given by,

$$H(s) = c + \sum_{n=1}^{N_p} \frac{k_n}{s - p_n} \quad (5.1)$$

where p_n is the n -th pole of the response and k_n its corresponding residue. Distributed systems imply an infinite number of transcendental poles, ($N_p = \infty$). However, by applying the Generalized Asymptotic Waveform Evaluation (GAWE) technique to the modified nodal admittance matrix equations of the linear subnetwork networks, it is possible to extract an approximate q -pole transfer function of this network.

Let the approximate response due to a single input applied only at x_i and measured at output v_j be described by,

$$H^{[i,j]}(s) = \hat{c}^{[i,j]} + \sum_{n=1}^q \frac{\hat{k}_n^{[i,j]}}{s - \hat{p}_n^{[i,j]}} \quad (q \ll N_p) \quad (5.2)$$

where $\hat{p}_n^{[i,j]}$ is the n -th approximate dominant pole of the response and $\hat{k}_n^{[i,j]}$ its corresponding n -th approximate residue.

The poles of a system are its fundamental modes. Obtaining their approximation is not equivalent to simply forming a rational function approximation. It is to be noted that several improvements to the method of obtaining these fundamental modes is outlined in this thesis.

We characterize the entire linear subnetwork by a minimal $\alpha \times \beta$ input-output approximate transfer function description. The α inputs x_i to be described will be either voltage nodes with attached independent voltage sources or attached augmenting voltage sources in the form of nonlinear terminations where the voltage is a function of some variable(s), or branch currents with attached independent current sources or attached augmenting nonlinear terminations where the current is a function of some variable(s).

The β output ports $v_j \in v(t)$ are the desired voltage nodes and branch current outputs, and all voltage nodes and branch currents which are the arguments of the nonlinear functions yielding the inputs mentioned. (See example in Figure 5.1).

Figure 5.1 Example nonlinear network.

The general $\alpha \times \beta$ hybrid characterization is given by,

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_\alpha \end{bmatrix} = H_{\alpha \times \beta}(s) \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_\beta \end{bmatrix} \quad (5.3)$$

or,

$$y = H_{\alpha \times \beta}(s) x \quad (5.4)$$

where, using (5.2), $H_{\alpha \times \beta}(s)$ is given by,

$$\begin{bmatrix} \hat{c}_n^{[1,1]} & \hat{c}_n^{[1,2]} & \dots & \hat{c}_n^{[1,\alpha]} \\ \hat{c}_n^{[2,1]} & \hat{c}_n^{[2,2]} & \dots & \hat{c}_n^{[2,\alpha]} \\ \vdots & \vdots & & \vdots \\ \hat{c}_n^{[\beta,1]} & \hat{c}_n^{[\beta,2]} & \dots & \hat{c}_n^{[\beta,\alpha]} \end{bmatrix} + \sum_{n=1}^q \begin{bmatrix} \frac{\hat{k}_n^{[1,1]}}{s - \hat{p}_n^{[1,1]}} & \frac{\hat{k}_n^{[1,2]}}{s - \hat{p}_n^{[1,2]}} & \dots & \frac{\hat{k}_n^{[1,\alpha]}}{s - \hat{p}_n^{[1,\alpha]}} \\ \frac{\hat{k}_n^{[2,1]}}{s - \hat{p}_n^{[2,1]}} & \frac{\hat{k}_n^{[2,2]}}{s - \hat{p}_n^{[2,2]}} & \dots & \frac{\hat{k}_n^{[2,\alpha]}}{s - \hat{p}_n^{[2,\alpha]}} \\ \vdots & \vdots & & \vdots \\ \frac{\hat{k}_n^{[\beta,1]}}{s - \hat{p}_n^{[\beta,1]}} & \frac{\hat{k}_n^{[\beta,2]}}{s - \hat{p}_n^{[\beta,1]}} & \dots & \frac{\hat{k}_n^{[\beta,\alpha]}}{s - \hat{p}_n^{[\beta,\alpha]}} \end{bmatrix} \quad (5.5)$$

To make the mathematical analysis simpler, it is assumed that the same q approximate dominant poles characterize each of the input-output transfer functions; i.e. $p_n^{[i,j]} = p_n, \forall [x_i, v_j]$, giving,

$$H_{\alpha \times \beta}(s) \approx \hat{C} + \sum_{n=1}^q (s - \hat{p}_n)^{-1} \begin{bmatrix} \hat{k}_n^{[1,1]} & \hat{k}_n^{[1,2]} & \dots & \hat{k}_n^{[1,\alpha]} \\ \hat{k}_n^{[2,1]} & \hat{k}_n^{[2,2]} & \dots & \hat{k}_n^{[2,\alpha]} \\ \vdots & \vdots & & \vdots \\ \hat{k}_n^{[\beta,1]} & \hat{k}_n^{[\beta,1]} & \dots & \hat{k}_n^{[\beta,\alpha]} \end{bmatrix} \quad (5.6)$$

or in compact form,

$$H_{\alpha \times \beta}(s) \approx \hat{C} + \sum_{n=1}^q \hat{K}_n (s - \hat{p}_n)^{-1} \quad (5.7)$$

While each of the entries can have different approximate poles, this increases the number of variables that need to be simulated in the time domain as will be seen. Choosing a similar set of poles is much more efficient and useful. This can be done

by generating poles at all of the entries and then by checking which set of poles will give the least Moment Skew (chapter 3) at most or all of the entries.

Without the need for numerical inversion, the approximate $\alpha \times \beta$ transient impulse response is immediately given by the closed form,

$$h_{\alpha \times \beta}(t) \approx \hat{C}\delta(t) + \sum_{n=1}^q \hat{K}_n \exp(\hat{p}_n t) \quad (5.8)$$

In the following equations it is assumed that $\hat{C}$ is a zero matrix, thereby simplifying the analysis. This is no great loss and can be easily rectified as the $\hat{C}$ simply indicates direct coupling between the input and the output, so all that needs to be added to the output waveform is $\hat{C}$ times the input waveform.

5.3 Response of the Nonlinear Network

The required response vector $y(t)$ and its approximation $\hat{y}(t)$ consists of the subset of α entries of $v(t)$, is due to a general $x(t)$ input of β non-zero entries consisting of the subset of entries. $\hat{y}(t)$ can be given by the convolution of the approximate transient $\alpha \times \beta$ transfer response and the input waveforms $x(t)$ as,

$$\hat{y}(t) = \sum_{n=1}^q \int_0^t \hat{K}_n \exp(\hat{p}_n(t-\tau)) x(\tau) d\tau = \sum_{n=1}^q \hat{y}^{(n)}(t) \quad (5.9)$$

Using the trapezoidal rule for integration from $t = 0$ to $t = \Delta t$,

$$\hat{y}^{(n)}(\Delta t) = \frac{\Delta t}{2} (\hat{K}_n [x(0) + \exp(\hat{p}_n \Delta t) x(\Delta t)]) \quad (5.10)$$

Substituting $t = t + \Delta t$ in (5.9) gives,

$$\hat{y}^{(n)}(t + \Delta t) = \int_0^{t + \Delta t} \hat{K}_n \exp(\hat{p}_n(t + \Delta t - \tau)) \mathbf{x}(\tau) d\tau \quad (5.11)$$

Combining (5.9) and (5.11) results in,

$$\begin{aligned} \hat{y}^{(n)}(t + \Delta t) = & \exp(\hat{p}_n \Delta t) [\hat{y}^{(n)}(t)] + \\ & \frac{\Delta t}{2} (\hat{K}_n [\mathbf{x}(t + \Delta t) + \exp(\hat{p}_n \Delta t) \mathbf{x}(t)]) \end{aligned} \quad (5.12)$$

The last equation is a recursive one, which says that the transient output at a time point depends only on the output at the previous time point and the present and previous input waveform time points. The convolution equations are eliminated.

In the formulation of (5.3) the nonlinear elements were treated as augmenting sources. If $\mathbf{x}(t)$ is further expanded in terms of its linear and nonlinear components, $\mathbf{g}(t)$ and $f(\mathbf{y}(t))$, and $\hat{\mathbf{y}}(t) \approx \mathbf{y}(t)$ is used, it gives,

$$\hat{\mathbf{y}}(t + \Delta t) = C_1 + C_2 f(\hat{\mathbf{y}}(t + \Delta t)) \quad (5.13)$$

where,

$$\begin{aligned} C_1 = & \sum_{n=1}^q \exp(\hat{p}_n \Delta t) [\hat{y}^{(n)}(t)] + \\ & \frac{\Delta t}{2} \sum_{n=1}^q \hat{K}_n \{ \mathbf{g}(t + \Delta t) + \exp(\hat{p}_n \Delta t) [f(\hat{\mathbf{y}}(t)) + \mathbf{g}(t + \Delta t)] \} \end{aligned} \quad (5.14)$$

is a constant which depends on the previous time point and the present and previous input waveform timepoints, and

$$C_2 = \frac{\Delta t}{2} \sum_{n=1}^q \hat{K}_n \quad (5.15)$$

Every time point may be calculated as a solution to nonlinear equation in (5.13) whose constants depend only on the previous time point solution.

The steady state solution and initial condition can be given by letting the input sources remain constant and allowing all time point solutions be equal, i.e.,

$$\hat{y}(t) = \hat{y}(0) \quad \underline{e}(t) = \underline{e}(0) \quad (5.16)$$

This gives,

$$\hat{y}_j(0) = \frac{\Delta t}{2} K_j \frac{(1 + \exp(p_j \Delta t))}{(1 - \exp(p_j \Delta t))} [f(\hat{y}(0)) + \underline{e}(0)] \quad (5.17)$$

Taking $\lim_{\Delta t \rightarrow 0}$ gives,

$$\hat{y}_j(0) = \frac{-K_j}{p_j} [f(\hat{y}(0)) + \underline{e}(0)] \quad (5.18)$$

or,

$$\hat{y}(0) = \left(\sum_{j=1}^q \frac{-K_j}{p_j} \right) [f(\hat{y}(0)) + \underline{e}(0)] \quad (5.19)$$

This is a non-linear equation which can be solved by standard methods.

It is to be noted that all transfer functions, whether frequency responses (Green's functions or other forms of impulse responses, may be manipulated in this way by first converting the responses as

$$\sum_i \frac{k_i}{s - p_i}$$

or in the time domain as

$$\sum_i k_i \exp(p_i t)$$

and then applying the above equations.

Other nonlinear approaches have also been proposed. One approach applies the macromodel approach to form a set of differential equations which can be combined with the other circuit equations [41]. Another approach uses the macromodel approach with a piecewise linear waveform input [42].

5.4 CPU Considerations

The linear waveform estimation of the GAWF was shown to be 2-3 orders of magnitude faster in comparison to HSPICE. When convolution is avoided for the nonlinear segments, the speed-up is maintained. It is difficult to compare the speed-up to all other nonlinear methods, however, the speed-up of this method over the same method requiring convolution can be theoretically estimated.

Let the number of time points needed be N_t , usually $100 \leq N_t \leq 500$, and let the number of multiplications needed per time point due the number of nonlinear elements and sources be $N_{\alpha \times \beta}$ which reflects the size of the matrix K_j , and let the order of approximations or the number of poles be N_q , usually $4 \leq N_q \leq 10$. Then,

assuming that the convolution is calculated only at the interface of the sources and the nonlinear elements, and assuming that N_{iter} iterations are needed per nonlinear solution of a time point for both convolution and non-convolution methods, the number of multiplications needed to perform an entire convolution will be generally given by,

$$\sum_{i=1}^{N_t} \sum_{j=1}^i N_{iter} N_{\alpha \times \beta} = \sum_{i=1}^{N_t} i N_{iter} N_{\alpha \times \beta} = \frac{N_t(N_t+1)N_{iter}N_{\alpha \times \beta}}{2} \quad (5.20)$$

Without convolution and using q poles, the number of multiplications is reduced to,

$$\sum_{i=1}^{N_t} N_{iter} N_{\alpha \times \beta} N_q = N_t N_{iter} N_{\alpha \times \beta} N_q \quad (5.21)$$

The ratio of (5.20) to (5.21) is,

$$\frac{N_t+1}{2N_q} \quad (5.22)$$

If, as is usually the case for interconnects, the poles are complex conjugates, the speedup is further increased by a factor of 2 as only one of the poles needs to be simulated. Given typical values of $N_t = 200$ and $N_q = 6$, results in a speed-up of 33. The range of speed-up will generally vary between 25 and 50. This is a conservative estimate since it does not include the speed-up due to avoiding numerical inversion techniques compared to other methods. The following examples demonstrate the relative accuracy of the method.

5.5 Examples

5.5.1 Example 1:

Consider the circuit in Figure 5.2 which illustrates a single lossy transmission line terminated by a nonlinear diode whose equation is given by,

$$i_d = \sum_{n=1}^{10} a_n v_d^n$$

The values of the transmission line parameters is given by, $R=0.1 \Omega/\text{m}$, $G=0.1 \text{ S/m}$, $C=200 \text{ pF/m}$, $L=25 \text{ nH/m}$, $d=1.0 \text{ m}$. The circuit was first simulated without transmission line losses and compared to SPICE and subsequently with a lossy line. Both results were simulated using the source switched on at $t=0$ with rise time of 0.1 ns . The results are given in Figure 5.3

Figure 5.2 RLCG transmission line with non-linear diode

Figure 5.3 Transient response of Figure 5.2

5.5.2 Example 2:

A lossy, coupled transmission line circuit is given in Figure 5.4. The nonlinear current sources are functions of the output voltage. The source is a 5V pulse of 1ns rise/fall time and 3ns duration. The output voltage is given, first in the lossless, uncoupled case as compared to HSPICE, and then in the lossy, coupled case.

Figure 5.4 RLCG nonlinear coupled transmission line network

Figure 5.5 Transient response of Figure 5.4

Chapter 6

Conclusions

6.1 Summary

This thesis has focused on the waveform estimation methods associated with high-speed networks that are modelled containing lossy, coupled transmission lines and lumped nonlinear terminations. It was shown that the AWE techniques for waveform estimation of interconnect circuits can be extremely fast. However, the accuracy and stability issues needed to be addressed more clearly.

An enhanced moment-matching algorithm was introduced which increases the accuracy of the AWE while ensuring the stability of the approximation. This technique requires more CPU time than previous methods but the extra time is

limited to about 0.2-0.6 seconds on a SPARC II and is independent of circuit size. Large circuits therefore can benefit from the added accuracy at a minimal extra cost.

This thesis introduced the concept of selection criteria to choose the best moment-matching approximation. The second criterion, Moment Skew, has been shown useful for increasing the accuracy even further by selecting poles generated at nodes other than the output node (Optimal Pole Selection), and by allowing the selection of one set of common poles for all input-output pairs in nonlinear analysis.

Nonlinear waveform estimation using moment-matching was speeded up with the elimination of convolution equations required in all complete simulation methods. This allowed speedup of 25-50 times over the same methods using convolution increasing the speed of the moment-matching techniques as applied to nonlinear networks.

These techniques allow for the complete integration of a linear/nonlinear waveform estimation package that is fast and efficient.

6.2 Suggestions For Further Research

- I. While moment matching in general is very quick and accurate, it has been empirically observed that as the rise time of the input is decreased, (increased frequency of operation), the error of the response approximation can become large. This is due to high frequency poles becoming more dominant. Padé approximation can only obtain poles close to the origin.

However in high-frequency circuits dominant poles can be far from the origin but close to the imaginary axis. Getting these high frequency dominant poles requires other moment-matching expansions. [22] uses a technique of frequency shifting to obtain a clearer view of stiff system poles. This technique would be useful to address some of the issues of high-frequency poles by expanding near those poles rather than $s = 0$. This approach requires many problems to be solved including where to expand and how to include the information from two expansions together. I have already begun researching the problem and obtained, by preliminary results, higher accuracy at increased CPU cost. Using more than a single expansion point means an extra LU decomposition per expansion point. However the accuracy increases with each expansion point.

- II. Nonlinear waveform estimation for large circuits may be used to address some difficult problems of periodic nonlinear systems. This can be done by assuming a period and checking for a steady state solution in that time frame. This would be a much faster method than current methods.
- III. Nonuniform lines, where the line parameters are functions of the distance along the line, are an important research extension. Moments could be generated for non-uniform lines and coupled to the rest of the network. Generalized moment-matching would still be applicable in this case.
- IV. Generalized moment-matching is also useful for optimization and sensitivity analysis. Already, the AWE has been analyzed in terms of

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optimization. Further research needs to be done on the extension of these techniques using the new methods presented in this thesis.

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