# Effect of Touchscreen Gestures on Number Line Performance

by

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## Abstract

The goal of this study was to examine whether the type of touchscreen gesture (i.e., drag or tap) affected adults' accuracy in integer and fractional number line tasks. In previous research, the drag gesture was hypothesized to be a more embodied way in which people interact with number lines. Seventy-eight undergraduate students participated in three phases. Phase 1 involved experience placing whole number targets on several number lines (i.e., 0-10, 0-25, 0-50, and 0-75). Phase 2 involved placing fractional amounts (e.g., 5/8) on the same number lines, and Phase 3 involved assessment of the participants' fraction knowledge. The results showed that participants were significantly more accurate using the drag gesture than the tap gesture on fraction number line task (Phase 2). However, using either the tap or drag gesture in Phase 1 (integer number line tasks) did not influence performance in Phase 2 (fraction number line tasks) or Phase 3 (fraction knowledge assessment). In summary, these results did not support the hypotheses that the gesture used in Phase 1 would affect participants' performance in Phases 2 and 3 (fraction number line tasks and fraction knowledge assessment, respectively).

## Acknowledgements

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# Introduction

In recent years, teachers and parents increasingly have used touchscreen devices as educational tools to help children learn mathematics. Shuler (2012) showed that math apps account for the greatest number of subject-specific content on iTunes, for example. Therefore, it is important for researchers to understand how interaction with touchscreen devices can affect children's mathematics understanding. One important issue is whether the type of gesture used when people interact with touchscreens influences their learning and performance. Dubé and McEwan (2015) suggested that dragging a cursor along a number line should support performance in number line tasks, compared to tapping. They also claimed that practicing placing whole numbers on a number line using a drag gesture would support performance on a fractional number line. The goal of the present research was to further explore the use of drag versus touch gestures as adults performed a series of number line tasks.

The theory of embodied cognition supports the view that performance will be influenced by the relation between touchscreen gestures and task requirements. According to this theory, learning is influenced not only by activity in a person's brain but also by the actions of their entire body (Wilson, 2002). The relation between embodied cognition and learning was explored in such areas as science, linguistics, and mathematics (Tran, Smith & Buschkuekhl, 2017). The implementation of embodiment in these domains is evolving, however, because novel interfaces and technologies are constantly being developed. These technological advances allow for direct manipulation and immersive experiences through tools including touchscreen devices, motion sensors, and virtual reality. In this research, I focus on the domain of mathematics and mathematics learning for people using touchscreen devices (Dubé, Alam, Xu, Wen & Kacmaz, 2019; Xie, Peng, Qin, Huang, Tian & Zhou, 2018).

**Background on Embodied Cognition**

The concept of a thought as an internalized action was initially developed by researchers such as Piaget (1964). In Piaget's theory, body movements complement the tendency for learning. Before children can solve a mental problem, they solve real-world problems that involve manipulating objects and moving through space. Piaget distinguished physical and logical-mathematical experiences, both playing an important role in development. Physical experience is acting upon objects and drawing some knowledge about the objects by abstraction from the objects. For example, to discover that this pipe is heavier than this watch, a child would weight them both and find the difference in the objects themselves. Logical-mathematical experience, however, is a type of experience where the knowledge is not drawn from the objects, but it is drawn by the actions effected upon the objects.

Piaget gives an example of a child counting pebbles. First, he put them in a row and counts one by one up to ten. Then he starts to count them in the other direction, and once again he finds ten. After, he put them in a circle and counts them that way, and once again he finds ten. The child did not discover a property of pebbles, he discovered a property of the action of ordering: he discovered that the sum was independent of the order. It is the point of departure of mathematical deduction. The subsequent deduction would consist of interiorizing these actions and then of combining them without any pebbles (Piaget, 1964). This characteristic of a thought as an internalized action is further described in embodied cognition theory.

As specified by the embodied cognition theory, features of our cognition are formed not only by our brains, but also by perception, somatosensory, and motor activities, collectively referred to as "embodiment". Wilson and Foglia (2017) distinguish three roles that the body plays in cognition. First, they describe the role of "**body as a constraint"**, which means that the body functions to constrain the cognitive system, that is some forms of cognition come more naturally because of certain body characteristics, likewise, some kinds of cognition are more difficult because of the body the agent has. Second, they describe the role of **"body as distributor"**, meaning that cognitive tasks are distributed between brain and body. This role draws a connection between embodied cognition and versions of the extended mind idea that appeal to concepts such as realization and scaffolding (Wilson, 2004). Third, they described the role of "**body as regulator"** - in this role, body serves as a real-time feedback-driven regulator of cognitive activity, as in dynamic approaches to cognition (e.g., Port & van Gelder, 1995; Thelen & Smith, 1994). The presence of all three body roles can be found in math cognition (Alibali & Nathan, 2011).

Embodied cognition theory emphasizes that cognition involves acting with a physical body on an environment in which that body is immersed, and that cognition is affected by these action experiences. Action experiences can come in the form of concrete physical experiences such as reaching and grasping or more abstract forms such as gesture (Hostetter & Alabali, 2008; Kontra, Goldin-Meadow & Beilock, 2012). Action is not only a powerful tool for learning during development (Piaget, 1964), but also plays an important role in adulthood (Kontra et al., 2012). For example, research shows that *congruent* actions can facilitate thinking. When people point to objects one by one, they count more proficiently. When people rotate hands in the same direction as mental rotation, they perform better. When gestures are prevented, thinking suffers and as thinking becomes proficient, gesturing diminishes (Chu & Kita, 2008). Thus, bodily movements play an important role in development of cognition as well as in performance during cognitive tasks in adulthood.

**Embodied Cognition and Mathematics**

Lakoff and Núñez (2000) studied mathematics understanding from the perspective of embodied cognition. They developed a view that conceptual knowledge of mathematics is embodied, that is, it is mapped within the sensory-motor system. Whatever people recognize as rational, rule-based, or inferential, is fully embodied in bodily actions. This view of conceptual knowledge as a result of embodied cognition perspective was novel when they proposed it. However, it has since influenced math cognition research and math education. For example, Tran et al. (2017) point out effectiveness of manipulatives, hand gestures, and whole-body movements for mathematics learning. These three domains can benefit math learning by providing an additional representation of math concept, by reducing cognitive load to provide more processing power to deeply think about and problem solve, and by inspiring the use of additional strategies (Tran et al., 2017).

Shapiro and Stolz (2018) also give examples of embodied cognition significance for mathematical education. For example, some work on gesture shows its importance in acquisition of mathematics concepts. Alibali and Nathan (2012) describe pointing gestures that draw attention to objects or locations; iconic gestures created by “drawing” in the air and representing objects or shapes; and metaphoric gestures that convey meaning through metaphoric extensions. They describe how teachers can use these types of gestures to clarify order of operations in an equation or present the relationship between the analogous shapes.

Research on the relationship between embodied cognition and learning mathematics provides evidence that at least some aspects of mathematics are embodied: children actively use their fingers when learning math, and bodily movements help children better understand math concepts (Tran et al., 2017). One example of embodied cognition in mathematics is the use of fingers to count and solve arithmetic problems (Fischer & Brugger, 2011), an activity called embodied numerosity (Moeller et al., 2012). Another example of how bodily activities influence how people process numbers is a sub-base 5 system that seems to develop in Western cultures. When people are asked to show the number 8 with fingers, they typically show five digits on one hand and three digits on the other hand (Domahs, Moeller, Huber, Willmes & Nuerk, 2010). Other researchers have shown that finger representation becomes internalized when children are in school. Thus, finger use provides scaffolding for calculation when children start learning math but benefit of using fingers decreases in later years (Tran et al., 2017).

Another example of embodied cognition in mathematics is related to spatial numerical associations, or SNAs (Cipora et al., 2019). For example, people respond to smaller numbers faster with the left hand, and they respond to large numbers faster with the right hand, which is consistent with an internal mental number line that is organized with smaller numbers on the left and larger numbers on the right. This effect is called spatial-numerical association of response codes, or SNARC-effect, and it is shown to be moderated by the reading direction of a language (Cipora et al., 2019). Overall, there are many examples of how embodiment affects mathematical knowledge.

In addition to the evidence that some aspects of mathematics are embodied, there are studies showing that body movements help in learning mathematics (Chu & Kita, 2008; Ramani & Siegler, 2014; Tran et al., 2017). For instance, Ramani and Siegler (2014) showed that physical interactions with numbers in contexts such as board games help children learn math concepts. Namely, board games with a linear arrangement of numbers improve their numerical magnitude understanding. They looked at such informal activities from a sociocultural perspective, explaining why children who receive the same mathematics instructions demonstrate different levels of understanding. In summary, informal activities that include body movements can promote children's math concept understanding.

In their review on directed action, gesture, and learning, Nathan and Walkington (2017) concluded that gesture production predicts learning and performance. For example, students are more likely to produce valid proofs for mathematical conjunctures if "dynamic depictive gestures" are present. Depictive gestures are gestures through which speakers directly represent objects or ideas with their bodies, e.g., forming an angle with their two hands (Nathan et al., 2017). They also explained that directed actions can influence mathematical cognition; directed actions from earlier training leave a historical trace expressed through gestures in later performance; and mathematical reasoning and learning are enhanced when actions are coupled with task-relevant speech. Taken together, these findings support a general idea that action plays an important role in acquiring and conveying mathematical concepts.

In summary, research provides evidence that at least some aspects of mathematics are embodied and that bodily movements promote mathematical understanding. Technology makes it possible to learn through different digital devices, so there is growing research on embodiment in human-computer interaction. Combining embodiment in mathematics and the affordances of technology may be a step towards creating effective educational tools that promote mathematical thinking.

**Embodied Cognition in Human-Computer Interaction**

The “third wave” in human-computer interaction, that consider insights from humanities and social sciences, reveals how embodiment matters in post-WIMP (window-icon-mouse-pointer) computing systems where the body is used differently compared to point-and-click interactions (Xambó, Jewitt & Price, 2014). Emerging technology allows learners to have a greater degree of direct interaction with digital environments, to include bodily movement in their interaction, and to be more immersed in those contexts. Touchscreens, in particular, are becoming ubiquitous and provide affordances for direct tapping, sliding, pinching, and rotating gestures. So, it is important to understand how interaction with touchscreen devices affects learning. Researchers cannot directly transfer existing findings on physical interaction with numbers to understanding interaction with numbers through touchscreens because the level of body involvement during digital interactions is different. For example, when people are playing a board game, they are required to move physical objects in three-dimensional space. In contrast, when they interact with a touchscreen, they are required to manipulate digital objects in two-dimensional space. These differences in the relation between the action and the manipulation with physical objects do not let researchers directly transfer findings of studies on embodied cognition to understanding the role of embodied cognition in interaction with digital devices.

To identify the level of embodiment in technology, Johnson-Glenberg, Birchfield, Tolentino, and Koziupa (2014) proposed a taxonomy which can be represented through three components: motoric engagement, gestural congruency, and perception of immersion. They portioned this taxonomy into four categories or degrees ranging from the lowest to the highest level of embodiment in technology (Johnson-Glenberg et al., 2014). Tran et al. (2017), following the example of Johnson-Glenberg et al. (2014), summarizes the four levels of embodiment in technology (see Table 1). Level 1 in Table 1 represents the lowest degree of embodiment which is defined through minimal motoric engagement, no gestural congruency, and a non-immersive experience. In contrast, level 4 represents the highest degree of embodiment through whole body movement, gestural congruency and tangible manipulatives, and a highly immersive experience. There are two moderate degrees of embodiment between these two extreme endpoints.

As we can see from Table 1, touchscreens belong to level 2 of embodiment, which means that they do not allow motoric engagement and are not immersive, but they do allow gestural relevancy, or congruency. Johnson-Glenberg et al. (2014) described a congruent gesture as a gesture linked to the content in a manner that reified the learning construct. They describe that in first- and second-degree lessons, learners are often dealing with “simulations”, that is graphic, visually appealing content that could be interactive. Users move content to construct new entities. They also explain that in the second degree/level of embodiment, the input should have some gestural congruency, for example, drawing a longer vector on screen makes the virtual car go further.

**Table 1**

*Degrees of Embodiment in Technology Proposed by Tran et al. (2017)*

| | Level 1 | Level 2 | Level 3 | Level 4 |
|---|---|---|---|---|
| Motoric engagement | Stationary | Stationary | Partial-body locomotion | Whole-body locomotion |
| Gestural congruency | No congruent gestures<br>No manipulations | Congruent gestures<br>Possible tangible manipulations | Congruent gestures<br>Tangible manipulations | Congruent gestures<br>Tangible manipulations |
| Immersion | Not immersive | Not immersive | Semi-immersive | Immersive |
| Example | Observation on small screen | Interaction with small screen | Motion sensors and large displays | Mixed reality with motions sensors and locomotion |

Continuum on three variables: motoric engagement, gestural congruency, immersion

**Touchscreen Gestures and Mathematics Learning**

Saffer (2009) defines gesture for a gestural interface as any physical movement that a digital system can sense and respond to without the aid of a pointing device, such as a mouse or a stylus. For instance, touchscreens provide affordances for direct tapping, dragging, or rotating gestures. Touchscreen gestures are interesting for math cognition because they allow for an increased degree of gestural congruency such that physical movements correctly simulate cognitive process. However, few researchers have studied the role of touchscreen gestures in people's learning and understanding of mathematics (Dubé & McEwan, 2015; Segal, 2014).

In Segal, Tversky, and Black (2014), 128 seven- and eight-year-old children ($1^{st}$ and $2^{nd}$ grade) performed an arithmetic task and a number line task on a tablet. The arithmetic task involved solving 10 addition problems on a virtual interface that showed blocks arranged in side-by-side piles of two 10-block towers. The number of blocks in each tower was also presented as an Arabic number and was located below each tower. Children were assigned to one of two gesture conditions – congruent or incongruent. Children in the congruent condition tapped with their finger on each individual digital block as they counted to perform the addition. Tapping once on each block was considered a congruent gesture conceptually mapped to the discrete concept of counting. In the incongruent condition, children tapped on the Arabic numbers under the blocks to select a certain number of blocks at once, which was considered incongruent gesture, not conceptually mapped to the discrete concept of counting. Children were more accurate in the congruent than in the incongruent condition, suggesting that the congruent to the math task gesture promotes children's performance.

In the number line task used in Segal et al. (2014), children were required to estimate 23 numbers on a virtual number line (0-100). The computer narrated the questions so children did not need to recognize the symbols. After each answer, the child received an animated feedback with the numbers appearing on the number line from left to right, up to the correct value. In the congruent condition of the 0-100 estimation line task, children slid their finger horizontally (i.e., they used a continuous gesture) on the screen to estimate the number on the number line. In the non-congruent condition of this task, children tapped (i.e., they used a discrete gesture) on the screen to estimate the numbers. The sliding gesture, in this case, is mapped to the concept of continuous magnitude of a number line. It is an action congruent with the mental operation of increasing or decreasing continuously (Segal et al., 2014). Children were more accurate when they used the congruent gesture than the incongruent gesture. In general, Segal et al.'s results suggest that the relation between the gestures and the actions required in the mathematical task is related to performance.

Dubé and McEwen (2015) conducted a number line study similar to that of Segal et al. (2014), except that their participants were skilled adults. Forty adults participated in a study that involved three phases. The *training* phase involving four integer number line tasks, the *near transfer* phase involving four fractional number line tasks, and the *far transfer* phase involved a different continuous quantity task. In the training phase, there were four number line tasks (i.e., lines 0-10, 0-25, 0-50, 0-75) which participants completed on a tablet using either a drag or a tap gesture. In the drag gesture condition, the gesture is assumed to be congruent with the continuous number line. Participants touched the screen in an initial location then moved in a straight line to the target location, while keeping in contact with the screen. In the incongruent (tap) gesture condition, participants touched the screen at the location of the estimate. Dubé and McEwan found that participants who used the drag gesture were more accurate on the integer number lines than those who used the tap gesture.

In the next two phases of the experiment, all participants used the drag gesture to complete two more tasks assessing their understanding of the continuous nature of numbers. The near transfer phase was the same as the training phase, except that participants were placing fractional values instead of integers on the number lines. For example, when participants had to place 3/8 on a 0-10 number line, they were instructed to imagine a 10-foot piece of wood divided into 8 equal pieces and find where 3/8 belongs on this piece of wood. In the far transfer task, participants completed a Continuous Number Manipulation task in which they had to identify fractions of discrete items (i.e., blocks shown on the screen). Participants who used a drag gesture in the training task were more accurate in both the near and far transfer tasks than participants who used the tap gesture. Similar to Segal et al. (2014), Dubé and McEwan concluded that using a gesture that is congruent with the underlying mathematics concept could promote performance during the training phase, that this gesture effect transfers to similar tasks, and, finally, it affects further math concept reasoning.

**Present Study**

The goal of the present study was to explore the role of touchscreen gestures in adults' understanding of math concepts. In particular, I tested whether a congruent continuous drag gesture produces better performance on tasks that require knowledge of the continuous nature of numbers than an incongruent tap gesture. The tap gesture may emphasize a discrete interpretation of number (as in the arithmetic task used by Segal et al., 2014) whereas the drag gesture emphasizes a continuous interpretation of number.

Dubé and McEwen (2015) showed that the drag gesture led to better performance on the number line task than a tap gesture for integer number lines, and that practicing the drag gesture in the training condition led to better performance in the fractional number lines. However, their study had some limitations, as described below. My goal was to replicate their study while also making some changes in study design and data analysis to reduce the limitations of their methodology.

The first limitation of of Dubé and McEwen (2015) was that their analysis did not take into account that the data were nested within participants. Instead, each trial was entered into the analysis as an individual case, which inflates the degrees of freedom and violates the assumption of independence in their analyses. Accordingly, although the effect size for the difference in performance between the tap and drag condition was tiny (i.e., $\eta_p^2 = .003$), it was nevertheless statistically significant because of the inflated degrees of freedom. In the present study, I used an analysis in which the dependence created by using repeated measures was correctly represented in the analysis to test whether the differences between the tap and drag conditions can be replicated in the integer condition and in the fraction condition.

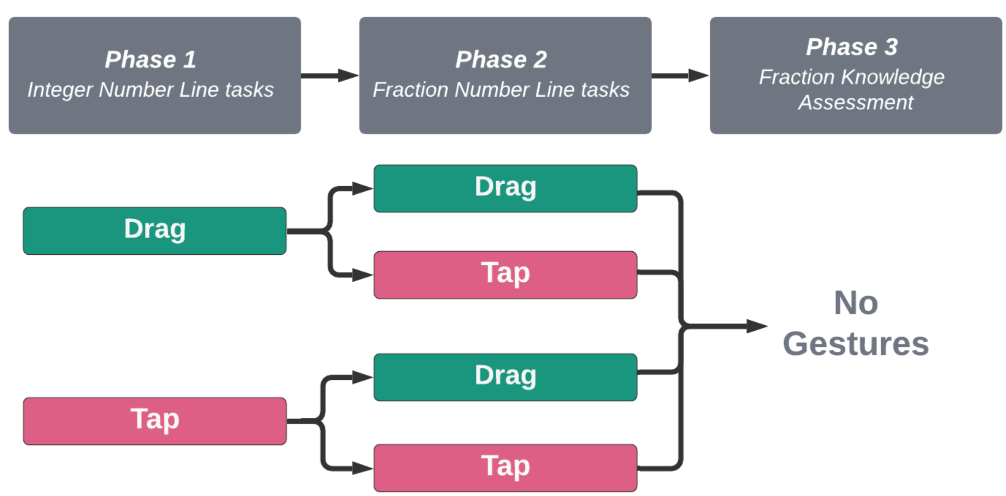
Another limitation was that Dubé and McEwan (2015) concluded that the gesture effect used in the training condition influenced performance in the near transfer condition because it enhanced participants' understanding of the math concept that was practiced in the training phase. However, because all of their participants used the drag gesture in the near transfer task, the better performance for those who also used the drag gesture in the training phase may indicate that the participants trained on using a certain gesture rather than have acquired a math concept. Dubé and McEwan state that the participants should all understand the number line task, and so any training effect could simply be related to familiarity with the particular gesture condition. Moreover, the analyses done in the near transfer condition were also incorrect, with a very small effect size for the tap versus drag comparison. Thus, in the present study, participants were randomly assigned the drag or the tap gesture in the near transfer task. If performance in the near transfer condition was related to training the gesture, then better performance would be expected in the conditions where the gestures were congruent between the training and near transfer tasks (i.e., drag-drag and tap-tap). However, if practicing the number line task in the training condition actually improved the participants' understanding of the continuous nature of numbers, then participants who used drag gesture in the training task would have better performance in the near transfer task, regardless of whether they used the tap or drag motion.

Notably, the use of terms "training" and "transfer" in Dubé and McEwan (2015) study was not appropriate. For two reasons, neither their original study or the present study were training studies. First, the study design does not involve pre- and post-test measures, so it is not possible to compare participants' performance before and after they complete the training task. Second, there was no feedback in the "training" task, so participants were not being explicitly taught knowledge or skills that they could transfer to solving other problems. Thus, it is not correct to talk about near and far transfer in the context of this study. In the present study, I use terms Phase 1, Phase 2, and Phase 3 instead of training, near transfer, and far transfer terms used in Dubé and McEwan (2015) study.

In summary, the goal of the present study was to address the limitations of previous research and to explore how gesture use can affect people's reasoning about math concepts. In Phase 1, I used the same task Dubé and McEwan (2015) did. In this task, participants placed integers on 0-10, 0-25, 0-50, and 0-75 number lines using either a drag or a tap gesture. In Phase 2, participants placed fractions on 0-10, 0-25, 0-50, and 0-75 number lines using either a drag or a tap gesture. For Phase 3, I used a Fraction Knowledge Assessment (FKA; Hallett et al., 2012) that did not involve gesture use. If participants had learned something general about the continuous nature of number magnitudes from their experience in Phase 1 and Phase 2 conditions, then this knowledge should transfer beyond the gesture-based conditions in Phase 3. Design of the present study is summarized in Figure 1.

**Figure 1**

*Study Design*



## Hypotheses

Considering background literature as well as findings and limitations of Dubé and McEwan (2015) study, I have the following hypotheses:

**Hypothesis 1**: Participants who use a drag gesture to place numbers on a number line will be more accurate than those who use a tap gesture in the integer number line tasks (Phase 1) and fraction number line tasks (Phase 2). This hypothesis predicts a main effect of drag versus tap gesture in both of Phases 1 and 2.

**Hypothesis 2**: Participants who use a drag gesture in *both* Phase 1 and Phase 2 of the experiment (i.e., drag-drag) will have the highest accuracy in Phase 2 (i.e., fraction number line tasks) whereas participants who used a tap gesture in both Phase 1 and Phase 2 will have the lowest accuracy (i.e., tap-tap).

**Hypothesis 3**: Participants who use the same touchscreen gesture in Phase 1 and Phase 2 (i.e., drag-drag and tap-tap) will have higher accuracy in the Phase 2 (fraction number line tasks) than participants who used different gestures in Phase 1 and Phase 2 (i.e., drag-tap and tap-drag). This result will suggest that participants perform better when they use a familiar gesture (either drag or tap).

**Hypothesis 4**: Participants who use the drag gesture in Phase 2 (i.e., fractional number line task) will do better in Phase 3 (i.e., fraction knowledge assessment) than those who use the tap gesture.

# Method

## Participants

All the testing took place in the Math Lab at Carleton University. The study was approved by Carleton University Research Ethics Board (i.e., CUREB-B). A pilot study was conducted with two Math Lab members to ensure that all the instructions were clear and that all necessary measurements were collected by the Estimation Line iPad application (Hume & Hume, 2014). Participants signed up for the study using Sona online participant system. All participants were enrolled in PSYC 1001, 1002, 2001, 2002, or CGSC 1001 and received 1% course credit for participation.

The data was collected from a total of 85 participants. Four participants were excluded because of errors made by the experimenter in administering the experimental conditions: Two participants because they re-started the integer number line task, one participant because they started with the fraction number line task, and one participant because they were missing many data points on the 0-75 integer number line. Additionally, three participants were excluded from the analyses due to high error rates on the fraction number line task (i.e., greater than 30%). For adults, error rates greater than 30% suggest that the participant either did not understand the task or was guessing. The final analysis included 78 participants (32 males) aged between 18 and 41 years (*M* = 20.4; *SD* = 3.7).

**Materials and Procedure**

Participants completed the three phases of the experiment in a one-hour session (see *Figure 1*). In Phase 1, participants did four number line tasks with integer targets (0-10, 0-25, 0-50, and 0-75). In the Phase 2, they also did four number line tasks but with fractional targets (as described below). In Phase 3, they completed a fraction knowledge test. The response method (i.e., tap or drag) was counterbalanced across phases 1 and 2 (i.e., the integer and fraction number line tasks) as shown in Figure 1. The number of participants in each of the four experimental conditions (drag-drag, drag-tap, tap-drag, and tap-tap) is shown in Table 2.

**Table 2**

*Distribution of Participants across the Four Experimental Conditions*

| Group | Phase | | Participants | | Age (years) | |
|---|---|---|---|---|---|---|
| | Training | Near transfer | *N* | Male | *M* | *SD* |
| 1 | Drag | Drag | 20 | 11 | 19.9 | 1.8 |
| 2 | Drag | Tap | 19 | 10 | 21.0 | 4.1 |
| 3 | Tap | Drag | 18 | 5 | 20.6 | 5.2 |
| 4 | Tap | Tap | 21 | 6 | 20.2 | 3.1 |

**Phase 1.** In Phase 1 every participant did four integer number line tasks. The 0-10 number line estimation always came first, followed by the 0-25, 0-50, and 0-75 number line tasks. On each number line task, participants were shown a target number and required to indicate its location on a line with the endpoints marked (see Figure 1). The app used to present the number line task on the iPad is called Estimation Line (Hume & Hume, 2014). Before the task started, all participants completed two practice trials where they practiced tapping on a green vertical target line placed on a number line in the tap condition or dragging a red selector to the green target line in the drag condition (see Figure 2). Before they did the practice trials, they saw instructions on the screen. The text of the instruction for the ***drag condition*** was as follows:

"Number-line estimation task

- You will see the numbers at the start and end, but not any in between.

- After you press the green **Go button, you'll see a green target line**.
- **Practice dragging the red selector with your finger.**
- Drag it to the green target line and then let go.
- Then you **press the red Done button**."

The text of the instruction for the ***tap condition*** was as follows:

"Number-line estimation task

- You will see the numbers at the start and end, but not any in between.
- After you press the green **Go button, you'll see a green target line**.
- **Practice touching the red target with your finger.**
- You'll see a red line where you put your finger.
- Then you **press the red Done button**."

When participants were done with two experimental trials, they saw instructions for the actual experimental trials. The text of the instruction for the ***drag condition*** is as follows:

Number line estimation tasks

- You will see the numbers at the start and end, but not once in between.
- After you press the green **Go button, you'll see a number**.
- **Drug the red selector to where you think the number should go on the number line**
- Then you **press the red Done button**"

The text of the instruction for the ***tap condition*** was as follows:

"Number line estimation tasks

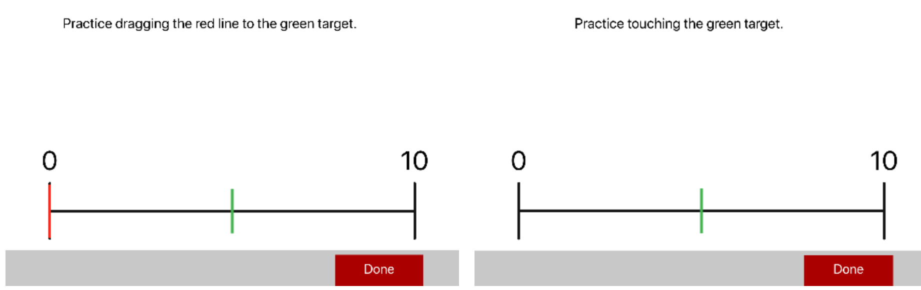
- You will see the numbers at the start and end, but not once in between.

- After you press the green **Go button, you'll see a number**.
- **Put your finger where you think the number should go on the number line**
- You'll see a red line where you put your finger.
- Then you **press the red Done button**"

Before the actual trials started, participants had seen the notification: *"Now you are going to place numbers on a 0---10 number line"*. Participants were verbally instructed to be as accurate as possible, but to not spend too much time on each trial.

**Figure 2**

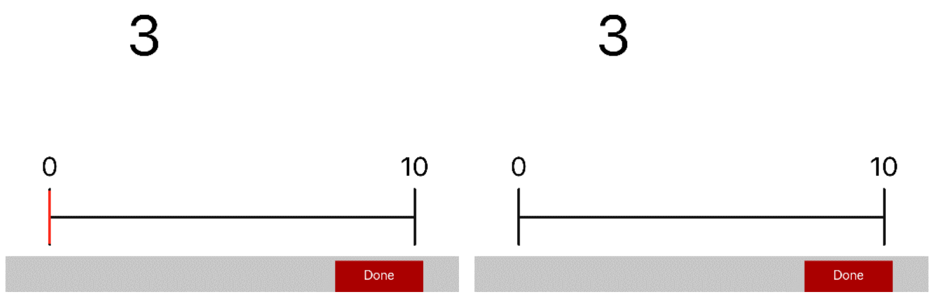
*Practice Trials for Integer Number Line (Training Phase) in the Drag Condition (left) and Tap Condition (right)*




Next, they completed 10 trials for 0-10 number line, and 20 trials for each of 0-25, 0-50, and 0-75 number lines. For a list of all of the stimuli in the Phase 1 tasks, see Appendix A. When the number line changed, participants saw the notification: *"Now you are going to place number on a 0---25 number line"*. Participants were randomly assigned to one of the two conditions for Phase 1. Participants in the ***drag condition*** used a drag gesture during the number line tasks, whereas participants in the ***tap condition*** used a tap gesture. An example of a trial for the integer number line task is shown in Figure 3.

**Figure 3**

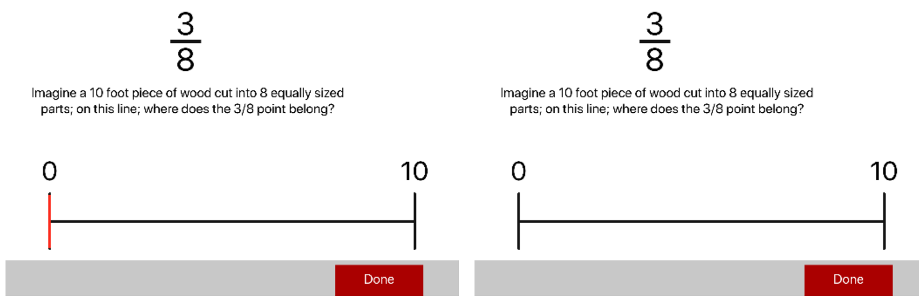
*Examples of Trials for the Integer Number Line Task in the Drag Condition (Left) and the Tap Condition (Right)*




**Phase 2.** The second phase involved indicating fractional values on a number line. This phase included practice and four sets of trials with number lines ranging from 0 to 10, 25, 50, or 75. For example, for the 0-25 number line, participants were asked to imagine a 25-foot piece of wood cut into 12 equally sized parts; and to think of where the value of 5/12 point belonged on this line. In order to make sure the instruction was clearly understood, the first two trials for each number line included the instruction text. These two trials were not randomized and were the same for all participants (see *Figure 4*).

**Figure 4**

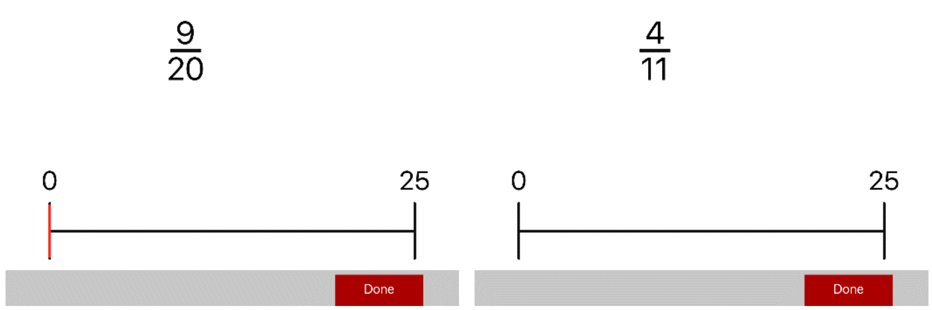
*Practice Trials on the 0-10 Fraction Number Line in the Drag and Tap Conditions*




In the experimental trials, the participants were presented with fractions not accompanied with any text (see Figure 5). There were 10 trials for the 0-10 number line and 20 trials for each of the 0-25, 0-50, and 0-75 number lines. All the trials of this task are listed in Appendix B. Before the number line changed, participants had seen the notification, for example: *"Now you are going to place numbers on a 0---25 number line".*

**Figure 5**

*Example Trials in the Near Transfer Task for the Drag (left) and Tap (right) Conditions*



For Phase 2 of the experiment, participants were assigned a gesture (drag or tap) to form the following four groups:

***Drag - drag*** - participants who used the drag gesture in both Phase 1 and Phase 2.

***Drag - tap*** - participants who used the drag gesture in Phase 1 and the tap gesture in Phase 2.

***Tap - drag*** - participants who used the tap gesture in Phase 1 and the drag gesture in Phase 2.

***Tap - tap*** - participants who used the tap gesture in both Phase 1 and Phase 2.

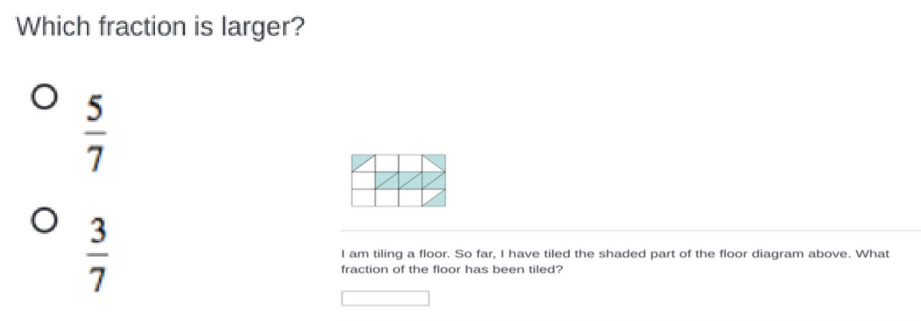
The goal of creating these four groups was to make sure the participants got trained on a math concept and not simply on using a certain touchscreen gesture during the tasks. If the participants gained a better understanding of the continuous nature of numbers during the task sessions, they would demonstrate the best results in Phase 1 in ***drag-drag*** (congruent continuous gesture) condition, and the worst results in the ***tap-tap*** (incongruent discrete gesture) condition. However, if the participants simply got trained to use a certain gesture during the tasks, they would demonstrate better results in both ***drag-drag*** and ***tap-tap*** conditions, and worse results in the ***tap-drag*** and ***drag-tap*** conditions because they had to learn a new touchscreen gesture in the Phase 2 in the two latter conditions.

**Phase 3.** After they finished the number line tasks, participants completed the Fraction Knowledge Assessment (FKA; Hallett et al., 2012) via the Qualtrics survey tool (see Figure 6). For all the questions in this task, see Appendix C. Participants were instructed to be as accurate as possible, but to not spend too much time on each item. Before participants started the task, they had seen the following instruction on the screen:

*"Please answer the following questions to the best of your ability. You may use pencil and paper in front of you to do your work. These questions will be timed, so try to complete them as quickly as possible. Once you click next, the timer will start! Any questions?"*

**Figure 6**

*Examples of Fraction Knowledge Assessment (FKA) Questions on Qualtrics*




All participants completed the same version of this task. I intentionally did not ask participants to use touchscreen gestures in the far transfer task to make sure gestures did not interfere with the task results.

## Results

Most of the analyses were done using SPSS statistics Version 25, except for Bayesian analysis that was conducted using JASP Version 0.11.1.0. The significance level was set at .05. When homogeneity of variance was violated, Greenhouse-Geisser corrected degrees of freedom are reported. Performance on all of the number line tasks were calculated as the mean percent of absolute error (PAE) for each number line. The absolute value of the difference between the target and the indicated location was divided by the length of the number line and then multiplied by 100. Post-hoc tests used the Bonferroni correction.

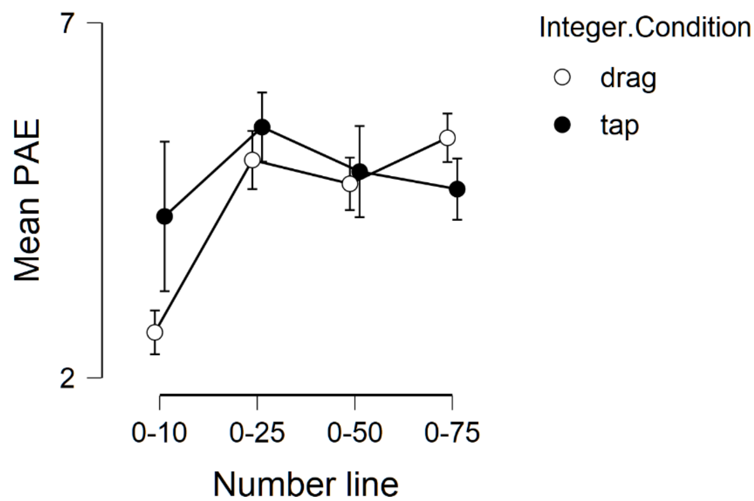
**Phase 1**

To evaluate the impact of gesture on number line performance (Hypothesis 1), percentage of absolute error (PAE) in Phase 1 (integer number lines) were analyzed using a 4 (number line: 0-10, 0-25, 0-50, 0-75) x 2 (gesture: tap, drag) mixed ANOVA. Hypothesis 1 was that participants in the drag condition would be more accurate than those in the tap condition. However, although mean PAE was lower in the drag condition ($M$ = 4.46, $SE$ = .311) than in tap condition ($M$ = 4.84, $SE$ = .311), the difference was not significant, $F(1, 76) = .77$, $MSE = 11.65$, $p = .383$, $\eta_p^2 = .01$. Thus, I did not replicate the gesture advantage for the drag condition reported by Dubé and McEwan (2015). [Note that the effect size was larger in the present research, but with the appropriate analysis (I calculated mean percent absolute error for each participant instead of entering each of 70 task trials as a separate case in the analysis), it was not statistically significant].

There was a main effect of number line length, $F(1.84, 139.59) = 4.36$, $MSE = 85.31$, $p = .017$, $\eta_p^2 = .54$. Participants had the lowest PAE in the 0-10 condition ($M$ = 3.5%), and similar PAEs in the other three conditions ($Ms$ = 5.3%, 4.8%, 5.0% respectively). Subsequent post-hoc test for pairwise comparisons showed that PAE on the 0-10 number line was significantly different than that for the 0-25 number lines: $t(139.59) = 3.04$, $p = .019$, $d = .5$. No other differences were significant. The interaction between number line length and gesture is shown in Figure 7 and Table 3. The interaction was not significant, $F(1.84, 139.59) = 1.54$, $MSE = 30.1$, $p = .22$, $\eta_p^2 = .02$. Note that I also did not replicate the finding reported by Dubé and McEwan of increasing error across the four number lines (see their Figure 3). Participants were only more accurate on the 0-10 number line.

**Figure 7**

*Percent absolute error for each number line across drag and tap integer number line task conditions.*




*Note.* Error bars are standard errors of the mean

**Table 3**

*Means and Standard Deviations of PAE For Integer Number Lines in Phase 1 for Each Number Line*

| Number line | Drag condition | | Tap condition | |
|---|---|---|---|---|
| | *M* | *SD* | *M* | *SD* |
| 0-10 | 2.64 | 1.58 | 4.27 | 7.42 |
| 0-25 | 5.07 | 2.37 | 5.53 | 2.24 |
| 0-50 | 4.73 | 2.52 | 4.91 | 4.02 |
| 0-75 | 5.38 | 2.65 | 4.66 | 2.07 |

In addition to frequentist analyses, I evaluated the impact of gesture on the integer number line tasks (Phase 1) accuracy by conducting a 4 (number line: 0-10, 0-25, 0-50, 0-75) by 2 (condition: drag, tap) Bayesian mixed ANOVA. The factor of the number line length was included in the model. Models involving the main effect of gesture and the model including Gesture x Number line length interaction were both compared to models not involving these factors. As a result, Bayes factors for the inclusion of these two effects were derived, which quantify the extent to which the data support the inclusion of the main effect or interaction effect of gesture. These were $BF_{10} = 0.272$ for the main effect models and $BF_{10} = 0.061$ for the interaction model. So, an estimated Bayes factor (null/alternative) suggested that the data were 3.7 times more likely to occur under a model that does not include an effect of gesture. The data were 16.4 times more likely to occur under a model that does not include interaction between gesture and number line length. Thus, it is more likely that gestures did not affect accuracy in the integer number line tasks (Phase 1).

In summary, the results of the analysis did not support Hypothesis 1. There was no difference in performance for participants in the tap and drag conditions, and thus no evidence for an effect of gesture-concept match producing better performance. The Bayesian analyses support the null hypothesis.

**Phase 2**

***Drag versus Tap Gesture***

To see if gesture use affected participants' accuracy placing fractional values on a number line (Hypothesis 1), PAEs in Phase 2 (fraction number lines) were analyzed in a 4 (number line: 0-10, 0-25, 0-50, 0-75) by 2 (gesture: drag, tap) mixed ANOVA. There was a main effect of gesture, $F(2.45, 186.29) = 4.12$, $MSE = 407.8$, $p = .046$, $\eta_p^2 = .051$. Participants in the drag condition were more accurate than those in the tap condition (6.2% vs. 8.5%). This result provides evidence for Hypothesis 1, supporting the view that the congruent drag gesture which mimics the continuity of fractional quantities supports participants' performance in this task.

There was a main effect of number line length, $F(2.45, 186.29) = 6.14$, $MSE = 58.67$, $p = .001$, $\eta_p^2 = .075$. Descriptive statistics are shown in Table 4. Post-hoc tests for pairwise comparisons, conducted using Bonferroni procedure, showed that there was significant difference in mean PAE between 0-10 and 0-75 number lines: $t(186.29) = 3.87$, $p = .001$, $d = .6$. Thus, participants' performance improved as they had more practice in the fractional number line task.

**Table 4**

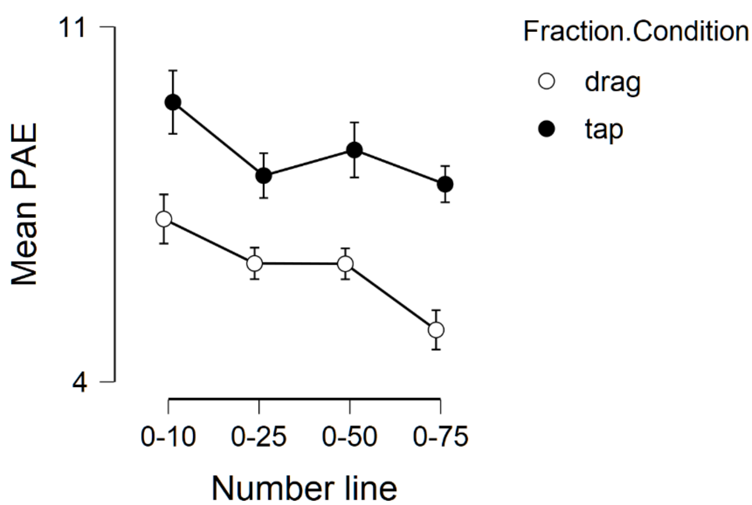
*Means and Standard Deviations of PAE for Fraction Targets in Phase 2 for Each Number Line*

| | | | 95% CI | |
|---|---|---|---|---|
| Number line | *M* | *SE* | *Lower Bound* | *Upper Bound* |
| 0-10 | 8.36 | .79 | 6.78 | 9.94 |
| 0-25 | 7.20 | .55 | 6.11 | 8.29 |
| 0-50 | 7.45 | .56 | 6.3 | 8.6 |
| 0-75 | 6.46 | .56 | 5.35 | 7.57 |

The interaction between number line length and gesture is presented in Figure 7. This interaction was not significant, $F(2.45, 186.29)$, $MSE = 5.2$, $p = .617$, $\eta_p^2 = .007$. As shown in Figure 7, error declines over conditions, suggesting that practice with the fractional values was resulted in improved performance in both drag and tap conditions. Note that the pattern across number lines was the opposite to that found in Dubé and McEwan, where error increased across the number lines.

**Figure 8**

*Percent Absolute Error For Each Number Line Across Drag And Tap Fraction Number Line Tasks Conditions.*




*Note.* Error bars are standard errors of the mean.

Additionally, I conducted a 4 (number line: 0-10, 0-25, 0-50, 0-75) by 2 (gesture: drag, tap) Bayesian mixed ANOVA. The factor of number line length was included in the model. Models involving the main effect of gesture and the model including Gesture x Number line length interaction were both compared to models not involving these factors. There was $BF_{10}$ = 1.625 for the main effect model and $BF_{10}$ = 0.099 for the interaction model. So, an estimated Bayes factor (null/alternative) suggested that the data were 1.625 times more likely to occur under a model that includes an effect of gesture.

This level of support is weak (Jarosz & Wiley, 2014). The data were also 10.1 times more likely to occur under a model that does not include an interaction between gesture and number line length. Thus, it is likely that gesture had an effect on participants' accuracy in the fraction number line task (Phase 2) but that gesture was equally important across all number line lengths.

In summary, we conducted the same analysis for the integer and fraction number line tasks to test Hypothesis 1 and see whether use of congruent drag gesture would increase participants' accuracy in the number line estimation tasks. There was no evidence for an effect of gesture effect in the integer number line task. However, participants who used the drag gesture in the fraction number line task were more accurate than those who used the tap gesture, which does support Hypothesis 1, although according to the Bayesian analysis the evidence was only weak. Previous research showed that embodiment of math fades as children get older and gain more math knowledge, which suggests that embodiment plays role in more difficult tasks, possibly in those tasks where some scaffolding is required (Chu & Kita, 2011; Tran et al., 2017). I suggest that the integer number line task was too easy for adult participants, whereas the fraction number line task is more difficult and thus some adults might have engaged their embodied cognition. In particular, embodied cognition could have affected performance of those participants who were not proficient in fractions. This suggestion, however, needs to be further studied. It could be helpful, for example, to see how touchscreen gestures affect performance of people with different levels of fraction knowledge.

Another possible explanation for the higher accuracy in the drag condition of the fraction number line task could be that the drag gesture provided participants with the visual cue while they were making their estimate and, thus, allowed them revise their estimates before choosing a final location. Although there was no “retry” option which means participants could not change their estimate after they let their finger go off the screen, they were able to move the bar back and force before lifting their finger. I observed that some participants spent more time on trials in the drag than in the tap condition trying to place the bar accurately. Because there was no “retry’ option in the tap condition either, participants were not able to change their estimate after they touched the line. Thus, even though there was a weak evidence for the hypothesis that the drag gesture promoted performance on the fraction number line, I cannot argue that the gesture effect was observed because the drag gesture was congruent with the concept of the number line continuity; other factors could have influenced the effect of gesture.

***Effect of gesture Used in Phase 1 on Accuracy in Phase 2***

To determine whether gesture used in the integer number line tasks affected participants’ accuracy in the fraction number line tasks (Phases 1 and 2; Hypothesis 2 and 3), mean PAE on the fractional number lines was analyzed in a 4 (number line: 0-10, 0-25, 0-50, 0-75) by 4 (gesture group: drag-drag, drag-tap, tap-drag, tap-tap) ANOVA. Descriptive statistics for the four gesture groups are shown in Table 6. The main effect of gesture group was not significant, $F(3, 74) = 1.78$, $MSE = 178.5$, $p = .158$ ; $\eta_p^2 = .08$. Post-hoc tests for individual comparisons, conducted with the Bonferroni procedure to account for differences in sample sizes, showed that there were no significant differences in PAE between any of four groups. For the fraction number line tasks (Phase 2), even descriptive statistics were not in favour of the specific practice hypothesis – participants in tap-tap group had higher mean PAE than participants in the drag-tap group.

**Table 5**

*Estimated Marginal Means and Standard Deviations of PAE in Phase 2 Across Gesture Conditions*

| Gesture Condition | *M* | *SE* | 95% CI *Lower Bound* | 95% CI *Upper Bound* |
|---|---|---|---|---|
| Drag-drag | 5.38 | 1.12 | 3.15 | 7.61 |
| Drag-tap | 8.74 | 1.15 | 6.45 | 11.03 |
| Tap-drag | 7.16 | 1.18 | 4.81 | 9.51 |
| Tap-tap | 8.31 | 1.09 | 6.13 | 10.48 |

The effect of number line length was significant, $F(2.45, 180.98) = 6.1$, $MSE = 48.24$, $\eta_p^2 = .08$. As we can see from the Table 6, the highest PAE was on 0-10 number line, and the lowest PAE was on the 0-75 number line. Post-hoc pairwise comparisons, conducted with the Bonferroni procedure, showed that there was a significant difference in PAE between 0-10 and 0-75 number lines, $t(180.98) = 3.84$, $p = .002$, $d = .6$. The interaction between line length and condition is shown in Figure 8. It was not statistically significant.

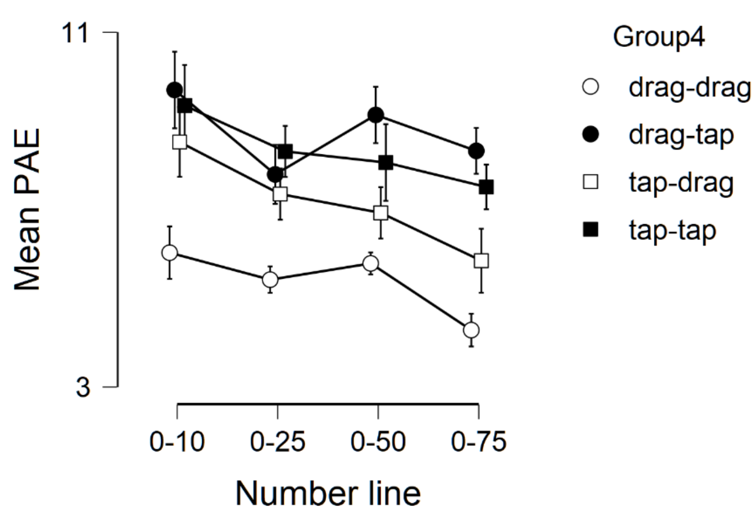
**Table 6**

*Means and Standard Deviations of PAE for Fraction Targets in the Phase 2 for Each Number Line*

| Number line | *M* | *SE* | 95% CI | |
|---|---|---|---|---|
| | | | *Lower Bound* | *Upper Bound* |
| 0-10 | 8.40 | .80 | 6.81 | 9.89 |
| 0-25 | 7.22 | .55 | 6.12 | 8.32 |
| 0-50 | 7.48 | .58 | 6.32 | 8.63 |
| 0-75 | 6.49 | .56 | 5.37 | 7.61 |

**Figure 9**

*Mean PAEs for Each Number Line with Fraction Targets (Phase 2).*




*Note.* Error bars represent standard error of the mean.

Additionally, we evaluated the impact of gesture on the fraction number line tasks (Phase 2) accuracy conducting a 4 (number line: 0-10, 0-25, 0-50, 0-75) by 4 (group: drag-drag, drag-tap, tap-drag, tap-tap) Bayesian mixed ANOVA. The factor of the number line length was included in the null model. Models involving the main effect of gesture and the model including Gesture x Number line length interaction were both compared to models not involving these factors. As a result, Bayes factors for the inclusion of these two effects were derived, which quantify the extent to which the data support the inclusion of the main effect or interaction effect of gesture. These were $BF_{10} = 0.717$ for the main effect models and $BF_{10} = 0.011$ for the interaction model. So, an estimated Bayes factor (null/alternative) suggested that the data were 1.4 times more likely to occur under a model that does not include an effect of gesture. The data were also 91 times more likely to occur under a model that does not include an interaction between gesture and number line length. Thus, it is more likely that gesture group did not affect fraction number line task accuracy. Hypotheses 2 and 3 were not supported.

**Phase 3**

In order to see if gesture used in Phase 2 affected participants' conceptual fraction knowledge (Hypothesis 4), percentage of incorrect responses in Phase 3 (i.e., fraction knowledge assessment) were analyzed using a 2 (Phase 2 gestures: drag, tap) one-way ANOVA. Although descriptive statistics suggested that participants who used a drag gesture in Phase 2 made fewer errors in the fraction knowledge assessment than those who used a tap gesture ($M$ = 27.1% and 30.3%, respectively), the effect of gesture on fraction knowledge assessment performance was not significant, $F(1, 77) = .62$, $MSE = 203.06$, $p = .435$, $\eta^2 = .008$.

I also conducted a Bayesian ANOVA for Phase 3. The percentage of errors in the fraction knowledge assessment was entered as a dependent variable and gesture (drag, tap) was entered as a fixed factor. There was $BF_{10} = 0.238$ indicating that the data were 4.2 times more likely to occur under the model that does not include gesture type. Thus, Bayesian analysis was in favour of the null hypothesis.

## Discussion

The goal of this study was to explore the role of touchscreen gestures in adults' performance on number line tasks. In particular, I studied whether the type of gesture affected participants' accuracy on a number line task, and whether a congruent continuous drag gesture is more effective for understanding the continuous nature of numbers than an incongruent discrete tap gesture. The results show that participants' accuracy was not affected by the type of gesture type (i.e., drag or tap) they used to place integers on a number line. This result was supported by Bayesian analysis. The results also showed that participants were more accurate using a drag gesture when they were placing fractions on a number line. However, Bayesian analysis showed that there was only weak support for the hypothesis that this gesture effect actually exists.

There was no evidence that the combination of gestures that participants used in Phases 1 and 2 affected their accuracy in Phase 2 and thus no support for the hypothesis that gesture type affects knowledge acquisition in this task. Furthermore, there was no evidence that the combination of gestures participants used in Phases 1 and 2 affected participants' accuracy in Phase 3, and thus no support for the view that the gestures used in the first two phases of the experiment affected math understanding. These nonsignificant results were supported by Bayesian analysis.

The results also show that participants who used the same gesture in Phases 1 and 2 (i.e., drag-drag and tap-tap experimental groups) did not perform better than those who used different touchscreen gestures in Phases 1 and 2 (i.e.., drag-tap and tap-drag experimental groups). In other words, there was no evidence for gesture practice effect. Participants who used the drag gesture in both Phases 1 and 2 had the best performance; however, participants who used the tap gesture in both phases had worse performance than participants who used different gestures in these phases.

Why did the touchscreen gesture only affect participants' accuracy in the fraction number line task in this experiment? Previous research showed that embodied cognition plays the biggest role in the early stages of math concept acquisition, and that the effect of embodied cognition fades as students acquire understanding of a certain math concept. For example, children's finger use provides scaffolding for calculation when they start learning math, but those finger representations get internalized in the later stages of math learning (Domahs et al., 2010; Tran et al., 2017). These findings could help to explain why gesture affected participants' accuracy on the fraction number line, but not on the integer number line in the present study: The fraction number line task was more difficult than integer number line for some participants, and as a result their embodied cognition was engaged. Segal et al. (2014) showed that 1$^{st}$ and 2$^{nd}$ grade children's accuracy on the integer number line was affected by touchscreen gestures, in support of the view that embodiment plays a stronger role in the early stages of math learning.

Besides, Schneider et al. (2018) found that the correlation between math competence (measured by counting tasks, mental and written arithmetic tasks, school grades, and standardized tests of math achievement) and number line estimation increases with age, possibly due to the more frequent use of fractions by older than younger children. This correlation with whole number line estimation is the strongest during the elementary school. Together, the high correlation between whole-number estimation and math competence in elementary school (Schneider et al., 2018) and the important role of embodiment on early stages of math concept acquisition (Chu & Kita, 2011; Tran et al., 2017) may explain why Segal et al. (2014) found a touchscreen gesture effect on whole number line accuracy in elementary school children whereas there was no evidence for an effect in adult participants. In order to see an effect of embodiment through the touchscreen gesture, there has to be a correlation between the task performance and math competence because the effect of embodiment depends on math competence.

Overall, fraction estimation is more closely related to math competence than whole-number estimation is (Schneider et al., 2018). Those results, and that finding that embodiment plays a role in early stages of math concept acquisition (Chu & Kita, 2011, Tran et al., 2017), together might also explain why the touchscreen gesture effect was found only for the fractional number line in the present study. Adult participants might have had different levels of fraction knowledge and only those who struggled with the task engaged embodied cognition.

The fact that fraction estimation is more closely related to math competence than to whole-number estimation can be explained by the greater complexity of fractions and fraction estimation strategies as compared to whole numbers and whole number strategies (Schneider et al., 2018). Rinne and Jordan (2017; as cited in Schneider et al., 2018) state that the greater complexity might allow for wider ranges of solution behavior on the task, which allows for a finer differentiation between children with different math aptitudes

(Schneider et al., 2018). The relations touchscreen gesture and different strategies on number line tasks should be further studied to explore this question.

Moreover, since the drag gesture is believed to promote performance in number line tasks because it is congruent with the continuous nature of numbers (Segal, 2014), people might think about fractions as continuous more than they do so about integers, thus, a drag gesture effect is present in the fraction number line task, and not in the integer one.

It is worth mentioning that number line tasks confound people's ability to correctly estimate the location of a number on the line with their fine motor and visual-spatial abilities that allow them to actually place the number. Previous research showed that spatial ability predicts growth in number line knowledge, separately from children's knowledge of the number system (LeFevre et al., 2013). Performance on the number line requires special abilities because children need to determine an approximate location of a number along a continuum and indicate its spatial location (LeFevre et al., 2013). Moreover, Di Lonardo et al. (2020) showed that people have a leftward bias when they place numbers on traditional direction (left-to-right) number lines. These findings should be a caution for researchers when they design a study, especially if the study includes touchscreen gestures.

**Comparison to Dubé and McEwan (2015)**

As I mentioned earlier, I tried to replicate Dubé and McEwan (2015). The present study does not replicate their results. Notably, however, the effect sizes were similar to those reported by Dubé and McEwan and, in the present study, more appropriate statistical techniques were used to evaluate those effects. In their study, the inflated degrees of freedom led to Type I errors, specifically, incorrectly rejecting the null hypothesis. In the present study I used Bayesian analyses to show that there was support for the null hypothesis.

It is probably not very surprising that there was no effect of which gesture was used in the integer number line (training phase). Dubé and McEwan (2015) stated that they chose the number line task because it drew parallel between their study and previous research (Siegler & Ramani, 2009) where physical interactions with real world were shown to affect understanding of a specific math concept. So, the first phase was more accurately characterized as one of practice using the touchscreen gesture. This specific practice with the gesture did not carry over to the second phase, however. In Phase 2, participants who used the gesture that is more consistent with the fraction task (i.e., dragging the cursor to the location on the number line) did show somewhat better performance than people who used the tap gesture. Thus, there was some support for the more general idea that the consistency between the task and the touchscreen gesture positively affected behavior (Segal, 2014; Dubé & McEwan, 2015).

**Limitations of the Present Research**

One of the limitations of the present study could be that there were differences in participants' experience with the touchscreen gesture that were confounded with the nature of the gesture. Participants in the tap condition had a very short interaction with the touchscreen device on each trial because they only touched the digital number line once, whereas participants in the drag condition spent more time interacting with the touchscreen dragging the bar back and forth before they placed it in the desired spot on the line. This gesture design might be an analog of having a "retry" option: participants in the drag gesture condition were able to place a cursor on a few different spots on the line before they were satisfied with its location and lifted their finger off the screen. Participants in the tap condition could only place the cursor on the line once. In other words, a cursor in the drag condition was a visual cue that provided participants with an affordance to adjust its location. In future work, researchers could allow participants to drag the cursor in only one direction, or allow them to have a few attempts to tap, which could help to erase difference in “retry” possibilities (Segal et al., 2014).

Another limitation of the present study is that the tasks were rather simple for adult participants. As it was mentioned above in this section, embodied cognition plays role on the early stages of math concept acquisition, but most adult participants must already have a good understanding of the concept of number continuity. It could be helpful to do similar research with children whose number continuity understanding is not yet fully formed.

**Implications for Future Research**

More research has to be done on embodiment of mathematics in adults. In future work, it is important to use appropriate tasks for adult participants. If embodied cognition plays a bigger role in the early stages of math concept understanding, math concepts that are more difficult and less familiar math concepts to adults should be chosen for further research. I think at this point it makes sense to research whether touchscreen gestures affect adults’ accuracy/performance in math tasks, and only in case such effect is found, it would be appropriate to see whether touchscreen gestures affect transfers to further math reasoning.

Research on how touchscreen gestures affect children's math understanding may be more interesting. The present study shows that type of gesture does not influence adults' performance on number line or fraction knowledge tests. The reason why the gesture effect does not transfer to adults' math concept reasoning could be that adults have already acquired a high level of that math concept understanding. However, research on children shows that the effect of embodiment in children' performance in math tasks exists, so their further math concept understanding could be affected by the gesture type.

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## Appendices

### Appendix A

List of numbers for the integer number line tasks

| Trial set | | | |
|---|---|---|---|
| 0-10 | 0-25 | 0-50 | 0-75 |
| 1 | 2 | 3 | 2 |
| 2 | 3 | 4 | 6 |
| 3 | 4 | 7 | 9 |
| 4 | 5 | 8 | 12 |
| 5 | 6 | 11 | 15 |
| 6 | 7 | 14 | 21 |
| 7 | 8 | 18 | 24 |
| 8 | 9 | 19 | 29 |
| 9 | 10 | 22 | 33 |
| 10 | 11 | 23 | 36 |
| | 14 | 27 | 39 |
| | 15 | 28 | 41 |
| | 16 | 31 | 44 |
| | 17 | 32 | 48 |
| | 18 | 35 | 53 |
| | 19 | 38 | 57 |
| | 20 | 42 | 62 |
| | 21 | 43 | 66 |
| | 22 | 46 | 70 |
| | 23 | 47 | 72 |

**Appendix B**

List of numbers for the fraction number line tasks

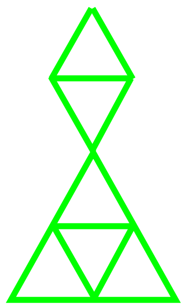
| Number line set | | | | |
|---|---|---|---|---|
| | 0-10 | 0-25 | 0-50 | 0-75 |
| Denominator less than 25 | 3/8 | 2/13 | 14/29 | 2/19 |
| | 3/7 | 2/7 | 15/26 | 13/14 |
| | 4/9 | 4/11 | 13/20 | 2/19 |
| | 3/5 | 5/12 | 22/25 | 3/17 |
| | 2/3 | 3/7 | 4/15 | 4/18 |
| | 7/9 | 9/20 | 7/19 | 4/21 |
| | 5/6 | 8/17 | 8/9 | 5/16 |
| | 6/7 | 9/19 | 10/23 | 6/7 |
| | 7/8 | 11/16 | 11/14 | 8/20 |
| | 4/6 | 13/18 | 11/16 | 9/24 |
| Denominator more than 25 | 3/8 | 17/50 | 32/77 | 34/70 |
| | 3/7 | 24/49 | 39/91 | 27/48 |
| | 4/9 | 30/55 | 16/31 | 17/30 |
| | 3/5 | 19/32 | 13/25 | 60/96 |
| | 2/3 | 18/30 | 25/40 | 53/75 |
| | 7/9 | 44/62 | 18/28 | 26/35 |
| | 5/6 | 20/28 | 29/45 | 71/87 |
| | 6/7 | 28/37 | 65/84 | 82/98 |
| | 7/8 | 35/45 | 72/90 | 49/51 |
| | 4/6 | 8/25 | 7/50 | 2/23 |

## Appendix C

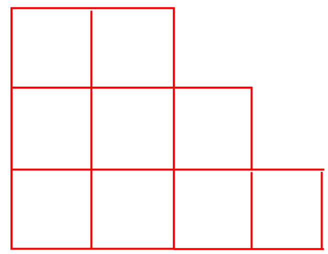
Question from Fraction Knowledge Assessment

1. Shade in two-thirds of each of these shapes:

(a)



(b)



(c)

2. Please solve this problem:

$$\frac{4}{7} + \frac{9}{14} =$$

3. Choose the larger fraction in each pair.

(a) $\frac{3}{7}$ $\frac{5}{7}$ (b) $\frac{3}{5}$ $\frac{3}{4}$ (c) $\frac{2}{5}$ $\frac{3}{10}$ (d) $\frac{4}{5}$ $\frac{5}{6}$

4. Please solve this problem:

$$\frac{4}{5} - \frac{5}{8}$$

5. Please solve this problem:

$$4\frac{2}{5} + 3\frac{3}{4} =$$

6. Mary and John both have pocket money. Mary spends $\frac{1}{4}$ of hers, while John spends $\frac{1}{2}$ of his.

(a) Is it possible for Mary to have spent more than John?

____________________

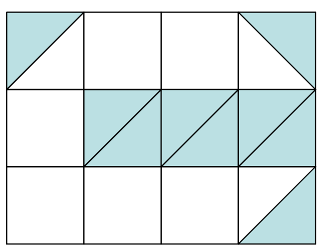
(b) Why do you think so?

____________________________________________

7. Please solve this problem:

$$\frac{3}{5} \times \frac{5}{12}$$

8. I am tiling a floor. So far, I have tiled the shaded part of the floor diagram below.



What fraction of the floor has been tiled? ________

9. Please solve this problem:

$$\frac{2}{3} \div \frac{2}{6}$$

10. Estimate the sum of $\frac{12}{13} + \frac{7}{8}$. Which of the following numbers is closest to this sum?

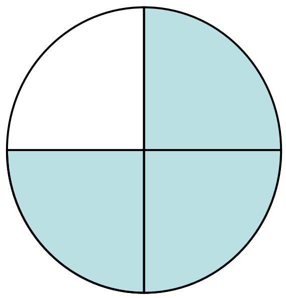
(a) 1 (b) 2 (c) 19 (d) 21

| |
|---|
| 11. Which number should go in place of the "?" in order to make these two fractions equal?<br><br>$\frac{6}{9} = \frac{?}{3}$ |
| 12. Choose the larger fraction in each pair.<br><br>(a) $\frac{4}{9}$ $\frac{2}{3}$ (b) $\frac{6}{5}$ $\frac{5}{6}$ (c) $\frac{7}{6}$ $\frac{83}{90}$ (d) $\frac{14}{13}$ $\frac{13}{12}$ |
| 13. Please solve this problem:<br><br>$1 - \frac{5}{12}$ |
| 14. Put these fractions in order of size, from the smallest on the left to the largest on the right.<br><br>$\frac{3}{4}$ $\frac{2}{5}$ $\frac{5}{4}$ $\frac{2}{3}$ |
| 15. Which number should go in place of the "?" in order to make these two fractions equal?<br><br>$\frac{2}{?} = \frac{8}{28}$ |
| 16. How many possible fractions are between $\frac{1}{4}$ and $\frac{1}{2}$? |

17. Please solve this problem and show your workings:

$$2 \times \frac{1}{8}$$

18. Shade in $\frac{1}{6}$ of the checkered part of the disc.



What fraction of the whole disc have you shaded?

20. Which number should go in place of the "?" in order to make these two fractions equal?

$$\frac{4}{5} = \frac{?}{15}$$

21. Circle EACH and EVERY fraction below that is equal to $\frac{2}{3}$.

(a) $\frac{9}{12}$ (b) $\frac{5}{6}$ (c) $\frac{8}{12}$ (d) $\frac{12}{15}$ (e) $\frac{12}{16}$ (f) $\frac{6}{9}$

22. Please solve this problem:

$$1\frac{1}{5} - \frac{3}{5}$$

| |
| --- |
| 23. A relay race is run in stages of $\frac{1}{8}$ km each. Each runner runs one stage. How many runners would be required to run a total distance of $\frac{3}{4}$ km? |
| 24. Please solve this problem: $$\frac{3}{4} \div \frac{3}{8}$$ |

**Appendix D**

Effect of Gestures used in Phases 1 and 2 on Fraction Knowledge Assessment

In order to see if the combination of gestures used in Phases 1 and 2 affected participants' conceptual fraction reasoning, percentage of incorrect responses in Phase 3 (i.e., fraction knowledge assessment) were analyzed using a 4 (groups: drag-drag, drag-tap, tap-drag, tap-tap) x 1 (fraction knowledge assessment results) one-way ANOVA. Table 7 summarizes descriptive statistics, which show that on average, percentage of errors was the lowest in the drag-drag group and the highest in the drag-tap group. The effect of group on fraction knowledge assessment performance was not significant, $F(3, 74) = 1.24$, $MSE = 403.38$, $\eta^2 = .048$. Post-hoc test for pairwise comparisons did not show significant difference in the percentage of errors between any of the four gesture groups.

**Table 7**

*Means and Standard Deviations for Percentage of Incorrect Responses In Fraction Knowledge Assessment (Phase 3)*

| | | | 95% CI | |
|---|---|---|---|---|
| Group | *M* | *SD* | *Lower Bound* | *Upper Bound* |
| Drag-drag | 23.33 | 13.33 | 17.09 | 29.57 |
| Drag-tap | 33.68 | 22.30 | 22.93 | 44.43 |
| Tap-drag | 31.30 | 15.68 | 23.50 | 39.09 |
| Tap-tap | 27.30 | 19.37 | 18.49 | 36.12 |

We also conducted a Bayesian ANOVA for Phase 3. The percentage of errors in the fraction knowledge assessment was entered as a dependent variable and gesture group (drag-drag, drag-tap, tap-drag, tap-tap) was entered as a fixed factor. There was $BF_{10}$ = 0.258 indicating that the data were 3.86 times more likely to occur under the model that does not include gesture group.

In summary, although the descriptive statistics suggested that participants in the drag-drag group did better in both Phase 2 and Phase 3 than those in the other groups, the effect of gesture was not statistically significant for either dependent variable. Bayesian analyses are in favour of the null hypothesis suggesting that gesture did not have an effect on participants' reasoning about the concept of number continuity. In summary, there is no evidence that practicing the drag gesture in Phase 1 promotes better understanding of number continuity than practicing with a tap gesture. We did not find evidence for a specific practice effect for the gestures. The previous analysis showed that participants in the drag-drag and tap-tap groups made fewer errors on the far transfer task than participants in tap-drag and drag-tap groups (M = 23.33% and 27.3% vs. 33.68% and 31.3%. However, the comparisons among all conditions in the present analyses were in favour of the null hypothesis suggesting that gesture condition was not related to participants' fraction knowledge assessment.