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Suppression of Intermittent Interference using Smart Antennas

**By,
Fayyaz Ahmad Siddiqui,**

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Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of**

Master of Applied Science

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Faculty of Engineering
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Abstract

In this thesis work, we investigate the performance of array processing techniques based on Sample Matrix Inversion (SMI) and the effects of channel estimation (using various training schemes) to combat intermittent interference. This type of interference arises due to an uncontrolled and unpredictable environment such as that of a license-exempted case. In an interference-dominated environment, a smart antenna is the best option to mitigate the interference. Moreover, most current applications of broadband wireless communication systems use short data block lengths. Therefore, antenna weight estimation is usually done only once and these weights are used for the whole packet length. In this work, through simulations we show that in such conditions, weight estimation based on a Pre-amble or Post-amble only scheme fails to track the suddenly appearing interferers, resulting in degradation in the output Signal-to-Interference-plus-Noise Ratio (SINR). Two new, training-based channel estimation techniques are presented which show superior performance over the aforementioned training schemes in this type of environment. The first one uses selection of the better weights calculated either from the Pre-amble or the Post-amble training and the second uses distributed placement of the training symbols throughout the packet. However, between the two, the latter (the Distributed-training scheme) offers the best performance. To deal with frequency selective fading, Orthogonal Frequency Division Multiplexing (OFDM) is already in the IEEE 802.11a and HiperLAN2 Wireless Local Area Network (WLAN) standards. As in OFDM, the output SINR is dominated by the deep nulls in the channel transfer function, adaptive OFDM is used to compensate these nulls. Overall in the thesis, three different simulation environments are simulated. The first one uses an equal average received power scenario from all the interferers and the desired user. Second has all interferers spatially uniformly distributed and the third one, used for OFDM, contains a wideband co-channel interferer. A Stanford University Interim-5 (SUI-5) channel model is used for the multipath environment. For simulating intermittent interference traffic streams, a batch Poisson traffic model is used.

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List of Symbols

β	Block size of interferers
α	Fraction showing interference appearance time
κ	Inner diameter for simulation scenario
λ	Inter burst rate
χ	Log normal random variable
μ	Mean for exponential distribution
ξ	Mean Squared Error (MSE)
ϑ	Outer radius for simulation scenario
Ω	Path loss exponent
τ_d	Inter-element path delay
σ_d^2	Signal variance
σ_{dB}	Lognormal shadowing standard deviation
σ_e^2	Variance of the estimation error
$\xi_{\min}$	Minimum Mean Squared Error (MMSE)
σ_n^2	Noise variance
$\mathbf{h}_1$	Desired signal's steering vector
K	Total number of users
M	Total number of antenna elements
N	Total number of training symbols in a block
$\mathbf{R}$	Correlation matrix
$\mathbf{w}$	Weight vector
$\mathbf{x}(i)$	Array input vector

List of Acronyms

AAA	Adaptive Antenna Array
AOFD	Adaptive Orthogonal Frequency Division Multiplexing
ATM	Asynchronous Transfer Mode
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDF	Cumulative Distribution Function
CGM	Constant Gradient Method
CMA	Constant Modulus Algorithm
DAB	Digital Audio Broadcasting
DVB	Digital Video Broadcasting
FFT	Fast Fourier Transform
ICI	Inter-Carrier Interference
IEEE	International Electrical and Electronics Engineers
IP	Internet Protocol
IPV6	Internet Protocol Version6
ISI	Inter-Symbol Interference
LMS	Least Mean Square
LOS	Line Of Sight
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
OFDM	Orthogonal Frequency Division Multiplexing
PSD	Power Spectral Density
RLS	Recursive Least Squares
Rx	Receiver
SINR	Signal to Interference plus Noise Ratio
SMI	Sample Matrix Inversion
SMI-ZF	Sample Matrix Inversion -Zero Forcing
SMI-ZF-CMA	Sample Matrix Inversion- Zero Forcing- CMA
SNR	Signal to Noise Ratio

SUI-5	Stanford University Interim-5
Tx	Transmitter
U-NII	Unlicensed National Information Infrastructure
WLAN	Wireless Local Area Network

Chapter 1

Introduction

1.1 Objectives

In this thesis, we investigate the performance of array processing technique based on Sample Matrix Inversion (SMI) [1] and the effects of channel estimation (using various training schemes) to combat intermittent interference. In an unlicensed band system, where mobile users using the same channel could be located anywhere (possibly co-located), interference becomes a major source of problem. Since no particular entity has sole ownership of the unlicensed band spectrum, managing the interference requires a global effort, which is usually not feasible. Hence, in order to manage or minimize the interference, highly effective interference cancellation techniques are necessary. Although there exist interference management/minimization techniques such as power control, cell sectorization, etc., such techniques are often inadequate because of poor channel estimations or simply because they are not applicable to the system of interest. In this work, we investigate array signal processing as an interference cancellation technique at the receiver combined with a novel training based block adaptation technique.

Nowadays, most of the current and planned wireless systems are block based and often these blocks are very short as is the case for Asynchronous Transfer Mode (ATM) packets. In a multi-user system with short block transmissions, the interference cannot be predicted and it becomes difficult to have a good estimate of the channel variations. This situation becomes worse when we deal with the uncontrolled and unpredictable interference environment as in license-exempt bands. In this case, the interferers arrive in an asynchronous fashion and will not be present for the whole duration of the desired user's block. Moreover, these interferers appear and reappear at random times. Therefore, we can call them "part time" interferers. For an initial estimation or tracking of such channels, there is an associated problem of interferer's sudden birth and death. Slowly converging algorithms (most of the iterative based techniques) do not work well in these conditions. Therefore, we seek a channel estimation technique that has a fast convergence

rate and can cover the intermittent nature of the interference traffic. Weight estimation based on Pre-amble, Post-amble or Mid-amble training scheme with SMI technique does not have the convergence problem, but how well these training schemes perform in catching the part-time interferers is an important issue to be investigated.

In this work, we show using simulations that in such conditions, weight estimation based on Pre-amble or Post-amble only scheme fails to track the suddenly appearing interferers, resulting in a degradation to the output SINR. Two new semi blind, training-based channel estimation techniques are presented which show superior performance over the aforementioned training schemes in this type of environment. The first one uses the selection of the better weights calculated either from the Pre-amble or the Post-amble training and the second uses distributed placement (Distributed-training) of the training symbols throughout the packet. However, between the two, the latter (the Distributed-training scheme) offers the best performance. The reason behind this is that to null out the “part time” interferers by an array, the training symbols (at least some of them) must lie in the region of the block’s affected portion. The Distributed-training adjusts the weight coefficients so as to suppress interference whose power is proportional to the fraction of the block in which it appears.

For simulating intermittent interference traffic streams, a batch Poisson traffic model is used. Different performance measures like Bit Error Rate (BER), outage probability and effect of different packet sizes are observed. A traffic load in Erlangs is defined in terms of the maximum number of interferers present at a specific time and their block sizes.

In an unlicensed band like Unlicensed National Information Infrastructure (U-NII), we are supposed to have high data rates up to 20 Mb/s. In a multipath environment, inter-symbol interference (ISI) may span up to 50 to 100 symbol intervals. To deal with the multipath environment, OFDM has already been at the foundation of WLAN standard (802.11a and HiperLAN2). But these systems may find themselves operating in close proximity to one another and potentially on the same channel, causing co-channel interference to each other. The properties of this kind of interference will be very similar in nature to the desired signal, which comprises serious multipath environment. In a multipath environment, through simulations, we show that it would be difficult for a conventional OFDM receiver to give acceptable throughput because of the deep nulls in

the channel's transfer function. A combination of adaptive antenna arrays with adaptive OFDM is a better choice to deal with equalization and interference suppression simultaneously. Our simulation uses a three ray (SUI-5) [26] multipath model. Channel estimation is done using time domain interpolation technique. Various training placement schemes are tested for comparison. BER performance of OFDM is compared for a various number of antenna array elements with and without co-channel interference. A comparison between partially and fully adaptive OFDM is made in terms of throughput.

1.2 Thesis Contribution

This thesis proposes techniques to combat uncontrolled and unpredictable interferers in any packet or block based communication system. The following are some of the contributions made;

- In this study, we have dealt with the issues to mitigate the intermittent interferers in unpredictable conditions by using smart antenna processing along with some new adaptation schemes. The system in which we can come across with this kind of interference is the unlicensed band communication system. Both flat and frequency selective fading environments are considered.
- We have developed two novel antenna array coefficients adaptation schemes. One of them uses the training symbols at both ends and is named the Pre-Post training scheme. And the other one has all the training symbols distributed throughout the block. This scheme is named the Distributed-training scheme. Computer simulations are carried out to show that both schemes and especially the second one perform well compared to the conventional Pre-amble only or Post-amble only or any iteration-based technique [see chapter 4].
- A lower bound is also calculated to verify the simulation results. Comparison studies of different adaptation schemes are made with different performance measures like BER, outage probability, transmitter-receiver separation between the desired pair and block length effects [see section 4.2].
- To get the actual intermittent traffic pattern, a simple but comprehensive batch Poisson traffic model is simulated. Traffic load offered by the interferers is

defined in terms of Erlangs. This traffic load in Erlangs actually depicts the average number of actual users at any instant in the system [see section 3.4]

- To deal with multipath environment a comparative study between partially and fully adaptive OFDM is made. SUI-5 [26] model is simulated for the inclusion of the multipath effect in the system. Different channel estimation algorithms and throughput comparisons show the advantages of the fully adaptive OFDM [see chapter 6].

1.3 Thesis Structure

Overview and essential background of adaptive array processing with different combining and block adaptation schemes are given in chapter 2. In this chapter, the problems associated with the intermittent interference situation are presented. We developed and discussed the structure of the problem addressed in the thesis and recent research towards the solution of the problem. Our proposed solution to the problem is also given in this chapter.

Chapter 3 describes the detailed signal and channel model regarding the flat fading environment. Three different simulation scenarios are considered: (1) single interferer, (2) multiple interferers having same average received power as the desired user and (3) multiple interferers spatially uniformly distributed. Interference traffic generation by using a batch Poisson traffic model is also described in this chapter.

Chapter 4 is dedicated to the performance comparison results obtained from the simulations done with flat fading consideration. This chapter is divided in two parts: Simulation results and Lower bound comparison. The first part is sub-divided into two sections: Single interferer scenario and interferences following the batch Poisson traffic model. The second sub-section is further divided into two: the first deals with the simulation scenario of equal average received power from all users (i.e. desired user and the interferers) and the second one accommodates the effect of distances.

Chapter 5 and 6 are devoted to frequency selective fading channels. In chapter 5 we describe the signal and channel model for multipath case, while chapter 6 deals with the simulation scenarios and results with the frequency selective fading environment.

Finally, chapter 7 concludes the thesis and describes some future research work.

Chapter 2

Antenna Arrays, Block adaptation and Problem Formulation

2.1 Adaptive Antenna Array (AAA)

A digital adaptive antenna array consists of an array of elements having an independent processing unit for each element and finally a combiner. This combination of antenna array and signal processing makes a beam former. A beam former is able to separate signals collocated in frequency band but separated in spatial domain. With this device we can separate a desired signal from interfering ones by optimizing the array pattern by adjusting the elemental control weights. To update the controlling weights, sometimes called weight coefficients, several intelligent algorithms are devised in literature. Hence the combination of these adaptation algorithms, antenna arrays and signal processing techniques makes an intelligent system known as smart antenna or adaptive antenna array. A general block diagram of adaptive antenna array or smart antenna is given in Figure 2.1, where w_i is the weight coefficients at each antenna element,

Figure 2.1: Block diagram of a general narrowband linear combiner.

Using smart antennas to combat interference is a very effective technique when the interference is predictable (i.e., constant interference with continuous traffic). According to [3] and references therein, with an M -element, an array processing technique using optimum combining can null out up to $(M-1)$ interferers. Optimum combining requires perfect channel and interference state information. This information, however, may not be readily realizable in a real system. Thus, in a practical system channel estimation based on a technique such as usage of training sequences in the packet, which is known to the receiver, is used. The training is done by reserving a certain number of slots in the data block just for the purpose of channel estimation. We can have three basic training schemes: Pre-amble (training at the beginning of the block), Post-amble (training at the end of the block) and Mid-amble (training at the middle of the block). Based on the information obtained from training, techniques such as SML, or any iterative technique such as Least Mean Square (LMS) or Recursive Least Square (RLS) can be used to calculate the weights [1].

2.2 Block Transmission and Adaptation

Most of the current and planned wireless systems are block based and often these blocks are of very short sizes. The reason for this mode of transmission is the possibilities of coexistence of different services and data rates. In such cases, it is not unrealistic to assume that the length of the block is much shorter than the coherence time of the channel. Therefore, we can assume the fading component to be quasi-stationary in these kinds of environments. Hence, it is reasonable to consider that the weighing coefficients to be used for block adaptation may only be estimated once and should be updated in every block.

Now the question arises about which kind of algorithm should be preferred for this kind of adaptation conditions. Iteration-based blind algorithms have problems such as with their convergence and may be predicted to perform badly in case of short block sizes. The situation with these algorithms would become worse, in case of any sudden change in channel characteristics. Therefore, a preferred estimation mechanism will be the use of a known sequence of training symbols to be used in each and every block. There are

several issues regarding training sequences like their type or structure, the number of symbols per block, the method of their processing and their location in the block.

The processing technique may be iteration-based getting initial information from training and then using iterative methods to track down the data symbols. But they might produce worse performance due to slow convergence properties as described above. SMI method is more appropriate for short block based communication systems as it computes the weighing coefficients once at the end of the training sequence and the weight update is done in every block [1]. The SMI technique was first introduced by Reed and his coworkers in 1974 and was found to adapt rapidly. The convergence of the SMI algorithm is much faster than iterative methods like LMS. But the associated problem with SMI is its complexity and involvement of a matrix inversion. Because of the advancements in Very Large Scale Integration (VLSI) and fast Digital Signal Processing (DSP) techniques, the complexity issue related to the SMI becomes less relevant. If we compare SMI with the RLS algorithm, which is a recursive version of SMI, the SMI uses less computation than RLS, if the weight update rate is once per block [1]. So, we can conclude that SMI is the best-suited algorithm for block adaptation purposes. A good historical note on the development of these algorithms is given in [2].

2.2.1 MMSE Optimum Combiner

Let $s_1(i)$ denote the desired signal at the array output (assumed known a priori), then the error signal, $e(i)$, can be expressed as:

$$e(i) = s_1(i) - \hat{s}_1(i) = s_1(i) - \mathbf{w}^H \mathbf{x}(i) \quad (2.1)$$

where $\mathbf{x}(i)$ is the array input vector at the i^{th} sample instant, and H denotes a complex conjugate transpose.

$\hat{s}_1(i) = \mathbf{w}^H \mathbf{x}(i)$ is the received signal at the array output and $\mathbf{w}$ is the weight vector to be optimized.

The mean square error is:

$$\begin{aligned} E\{e(i)e(i)^H\} &= E\{[s_1(i) - \hat{s}_1(i)][s_1(i) - \hat{s}_1(i)]^H\} = E\{[s_1(i) - \mathbf{w}^H \mathbf{x}(i)][s_1(i) - \mathbf{w}^H \mathbf{x}(i)]^H\} \\ &= E\{|s_1(i)|^2\} + \mathbf{w}^H E\{\mathbf{x}(i)\mathbf{x}^H(i)\} \mathbf{w} - E\{s_1(i)\mathbf{x}^H(i)\} \mathbf{w} - \mathbf{w}^H E\{\mathbf{x}(i)s_1(i)\} \end{aligned} \quad (2.2)$$

where, E denotes expectation.

Let $\mathbf{h}_1$ denote the cross-correlation vector between the input vector $\mathbf{x}(i)$ of the array and the desired signal $s_1(i)$,

$$\mathbf{h}_1 = E\{\mathbf{x}(i)s_1^*(i)\} = E\{[x_1(i)s_1^*(i) \dots x_M(i)s_1^*(i)]^T\} \quad (2.3)$$

where $*$ denotes complex conjugate.

The array input correlation matrix, $\mathbf{R}$, is given as:

$$\mathbf{R} = E\{\mathbf{x}(i)\mathbf{x}^H(i)\} = E\left\{\begin{bmatrix} x_1(i)x_1^*(i) & \dots & x_1(i)x_M^*(i) \\ \vdots & \ddots & \vdots \\ x_M(i)x_1^*(i) & \dots & x_M(i)x_M^*(i) \end{bmatrix}\right\} \quad (2.4)$$

Substituting $\mathbf{R}$ and $\mathbf{h}_1$ into (2.2), we obtain:

$$\xi = E\{e(i)e(i)^H\} = E\{|s_1(i)|^2\} + \mathbf{w}^H \mathbf{R} \mathbf{w} - \mathbf{h}_1^H \mathbf{w} - \mathbf{w}^H \mathbf{h}_1 \quad (2.5)$$

where, ξ is the mean square error (MSE).

The optimum weights $\mathbf{w}_{\text{opt}}$ is obtained by minimizing ξ with respect to $\mathbf{w}^*$ (MMSE), which is equivalent to taking the partial derivative of ξ with respect to $\mathbf{w}^*$ and setting it to 0:

$$\nabla = \frac{\partial \xi}{\partial \mathbf{w}^*} = \left[\frac{\partial \xi}{\partial w_1^*} \quad \frac{\partial \xi}{\partial w_2^*} \dots \frac{\partial \xi}{\partial w_M^*} \right]^T = \mathbf{R} \mathbf{w} - \mathbf{h}_1 \quad (2.6)$$

$$\text{Setting } \nabla = 0 \Rightarrow \mathbf{R} \mathbf{w} - \mathbf{h}_1 = 0 \Rightarrow \mathbf{w}_{\text{opt}} = \mathbf{R}^{-1} \mathbf{h}_1. \quad (2.7)$$

which is referred to as the Wiener-Hopf equation or the optimum Wiener solution.

Therefore, the minimum mean square error (ξ_{min} , MMSE) can be derived as [4]:

$$\xi_{\text{min}} = E\{|s_1(i)|^2\} - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1 = E\{|s_1(i)|^2\} - \mathbf{h}_1^H \mathbf{w}_{\text{opt}}. \quad (2.8)$$

2.2.2 SMI Weight Estimation

Weights are estimated by using an estimated correlation matrix and an estimated cross correlation vector obtained by averaging over a series of N received samples, based on training symbols. Therefore, the estimated correlation matrix is given as,

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i)\mathbf{x}^H(i) \quad (2.9)$$

The cross correlation vector in fact represents the desired user's signature assuming the training symbols are uncorrelated with unit signal variance. The cross correlation vector should be estimated with the same set of training sequence of length N . The following is the simplest way to estimate the vector by simply correlating the received snapshots with the corresponding training symbols,

$$\hat{\mathbf{h}}_1 = \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) s_1^*(i) \quad (2.10)$$

where, N is the number of training symbols.

The estimated weights, $\hat{\mathbf{w}}$, can then be obtained by using Wiener solution as follows:

$$\hat{\mathbf{w}} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{h}}_1 \quad (2.11)$$

The above SMI weights minimize the sum of squared error magnitudes over the training blocks and work by suppressing the signal components contained in the auto-correlation matrix $\hat{\mathbf{R}}$ that do not match with those contained in the steering vector $\hat{\mathbf{h}}_1$. This can be written as that the weight vector minimizes the following,

$$\frac{1}{N} \sum_{i=1}^N |\hat{\mathbf{w}}^H \mathbf{x}(i) - s_1(i)|^2 = \frac{1}{N} \sum_{i=1}^N |e(i)|^2 \quad (2.12)$$

Considering the SMI algorithm, some relationships between the number of training symbols, number of antenna elements and the output SINR are reviewed analytically in the appendix. The analysis shows that to obtain a performance of output SINR within 3 dB of the optimum one with adaptive antenna arrays, the number of training symbols required should be almost 2 times the number of elements in the array.

2.3 Problems with Intermittent Interference

Most of the current and planned wireless systems are block based and most of the time these blocks are of very short sizes like ATM or Internet Protocol (IP) packets. We will come across the intermittent kind of interference in an environment in which interferers arrive in an asynchronous fashion and will not be present for the whole duration of the desired user block. They may have different bandwidths and symbol rates. Moreover, these interferers get on and off the network at random times. The number of these

interferers may increase by a big amount with the implementation of Internet Protocol Version 6 (IPv6) having 128-bit addresses, allowing thousands of users having different applications. So they all are supposed to create a “Radio without Rules Environment”. The problem with these part time interferers is that they follow a very few rules and as discussed in [6], space processing using smart antennas is one of the promising solutions. An example of such an environment is the license exempt band.

2.3.1 License Exempt Band Environment

The 5 GHz unlicensed UNII bands in North America, is a free spectrum in which everybody is free to transmit at any time with some constraints on transmitted power and power spectral density (PSD). This creates lack of incentive to conserve the shared band. Details (spectrum detail, power and PSD limits etc.) of the unlicensed bands especially UNII can be found in [7]. There are several advantages of this system but when we have to deal with the interference suppression, it becomes a more difficult environment to handle. From upper layer point of view some “etiquettes” and modifications in them are suggested in [7]. Other unlicensed bands (e.g. 2.4 GHz and 900 MHz) have transmission rules, which limit the interference at the expense of spectrum-usage efficiency. The problem associated with UNII bands is that there is no coordination between different users and also no centralized control. Therefore, the effects of interference should be mitigated by receiver signal processing such as adaptive antenna arrays.

2.3.2 Tracking of Part-Time Interferers

Initial estimation and tracking of suddenly appearing (somewhere in between the desired user block) is a key problem in such intermittent interference environments. Existing conventional iterative algorithms may not work well because of their slow convergence. Therefore, we will have to find some channel estimation technique to be used with adaptive antenna array, which converges very quickly and can cover the intermittent traffic of the interferers. Antenna array weights based on Pre-amble, Post-amble or mid-amble training schemes with SMI technique serve well as far as convergence is

concerned. But how these training schemes perform in catching the intermittent interference is an important question to be answered.

In an uncontrollable and unpredictable interference traffic, if there is a desired signal plus $(K-1)$ interferers, the received signal vector at the i^{th} symbol period can be represented as,

$$\mathbf{x}(i) = \sum_{k=1}^K s_k(i) \mathbf{h}_k + \mathbf{n}(i) \quad (2.13)$$

where $s_k(i)$ is the data symbol from the k^{th} source at time i , $\mathbf{h}_k$ is the vector signature at the receiving array from the k^{th} source, and $\mathbf{n}(i)$ is the vector of additive noise samples. All of these vectors are of dimension M , the number of antenna array elements. In what follows, we assume that the data symbols $\{s_k(i)\}$ are independent and mutually uncorrelated, have zero mean, and unit variance. The symbols are assumed synchronized with one another. The components of $\mathbf{n}(i)$ are assumed to be uncorrelated, zero mean, Gaussian random variables, with variance σ_n^2 . This is a standard mathematical model for a system without frequency-selective fading, and where typically the received output signal from each antenna element passes through a matched filter, and is sampled at the symbol rate of the desired signal.

This model can also represent a situation in which some or all of the interferers have different symbol rates. Interferers can have higher or slower symbol rates compared with the desired user. But in either case, our analysis will be unaffected, in general then, $K-1$, the number of interferers, is an effective number of interfering sources, if some of them have higher symbol rates than the desired source.

2.4 Work Done towards the Problem

With continuous interference, different types of training schemes are worked out in the literature including iteration based LMS, RLS, Constant Gradient Method (CGM) etc. and block based adaptation with different combinations of Pre-amble, Post-amble and mid-amble [9]. However, Pre-amble training or Post-amble training is not sufficient to track the intermittent kind of traffic. Several efforts have been made to solve this problem. For example, *Sampei, S.* [10], uses Pre-amble and Post-amble at the both ends of the block and computes the weights based on each of these training sequences alone. Then with the assumption of smooth and slow channel fading variations compared with

the symbol transmission rate, linear interpolation is done between the two weights by using the following equation.

$$w(i) = \frac{i}{T} w_{\text{post-amble}} + \frac{T-i}{T} w_{\text{pre-amble}} \quad (2.13)$$

where, T =total number of the symbols in the block

$w_{\text{pre-amble}}$ = the weight estimated using Pre-amble only

$w_{\text{post-amble}}$ = the weight estimated using Post-amble only

$w(i)$ = the interpolated weight to be used for the i th symbol

For this kind of linear interpolation, the assumption made in the beginning is crucial. This linear interpolation works well for slow varying channel conditions. In an intermittent interference scenario the linear interpolation does not work as the appearance of the interference between the Pre-amble and Post-amble does not contribute to the weight determination as it should.

Actually in an intermittent interference environment, we need a method in which the weight converges very quickly to adapt to the abrupt changes in the channel conditions.

Hara, Y., Fujita, C and Kamio, Y consider in [11], the problem of initial weight computation for rapid convergence with the assumption that we know the spatial signature of the interferers as well. In this method the correlation matrix is estimated by using not only the desired signal's training symbols but also by considering the training symbols of the other users. In ad-hoc kind of networks, where everybody is entering and leaving the system randomly without any centralized control, this kind of assumption is unrealistic.

A very similar problem is addressed in [12] by *Lindskog, E.*, where the appearance of an interferer is outside of the Pre-amble training as shown in Figure 2.2;

Figure 2.2: Lindskog's interference model [12]

In this case, if a co channel interferer is present during the data sequence of a block, then it will remain there and continue until the next block's training sequence arrives. It means that it is assumed that the Pre-amble of the adjacent block will certainly catch the interferer that is not covered by the Pre-amble of the previous block. The problem associated with this assumption is that it assumes the interferer's block is long enough that if it appears after the passage of the Pre-amble then it will remain there for the adjacent training. In actual situations, shorter block lengths of interferers can occur, so this assumption would not be a practical one.

Yonezawa, R., Hirata, K., Kirimoto, T., and Chiba, I in [13] and *Wang, A.K. and Leary, J.* in [14] show a combination of SMI with Constant Modulus Algorithm (CMA) to combat intermittent interference. Three different schemes are compared, named as adjacent frame method (same as [12]), SMI-Zero Forcing (SMI-ZF) (using advantage of the guard interval time to get additional info) and SMI-ZF-CMA. SMI-ZF-CMA used is not totally blind as mentioned in [14] as it takes initial weights from the Pre-amble with SMI technique and then track down using CMA to update the coefficients. This technique performs well compared to the Pre-amble-only training scheme but with the appearance of an interferer it does not converge to an acceptable value even in the steady state condition.

Finally, *Wang and Poor*, in [15], use a sub-space method and compared the results with the conventional SMI algorithm with continuous type of interferences. The sub-space method utilizes the fact that as the correlation matrix does not depend on the transmitted signals we can obtain the correlation matrix from the received symbols of whole of the block. And if somehow we become able to find the steering vector of the desired user based on not only the training (Pre-amble), but also the whole of the block then we will have more information at the receiver. The second part is accomplished by finding the projection of the steering vector obtained from the training onto the whole of the block. The new correlation matrix and steering vector are used to compute the weights to be used. This scheme gives good result in the case of a continuous interference traffic but is not found to be able to track a suddenly appeared interferer within the block.

The methods discussed in the previous section, are not capable of tracking the true sporadic interference environment. To track this kind of channel variation, the

information of the intermittent interferers, irrespective of their location of appearance and duration, should be present in the ingredients of the weights. So to null out the “part-time” interferers by the arrays, the training symbols must lie in the region of the block’s affected portion. This is the basic idea behind the proposed training algorithm.

2.5 Proposed Adaptation Schemes using Distributed-training

The problem associated with the Pre-amble, Post-amble or Mid-amble training symbols is their placement in the block; these symbols are placed consecutively in the data block only at certain location. Such placement is reasonable for tracking a slow time varying channel or interference, but not good enough for tracking suddenly appearing and reappearing interferers anywhere in the block. Hence, we are proposing basically two new schemes:

The first one uses both Pre-amble and Post-amble simultaneously, and final estimation is done by selecting weights giving better output SINR for each symbol, calculated either from the Pre-amble or the Post-amble training (referred as Pre-Post training). In practice, the output SINR is based on the decision directed output after the comparison of the array output and the previous decisions. The second one distributes the known symbols in a periodic fashion throughout the block (Dist training).

Other combinations (such as “Dist-Pre”) are also considered for comparison purposes. Dist-Pre works very similar to that of Pre-Post training scheme. In this combination, Pre-amble training symbols are placed along with Distributed-training symbols and the weight vector is selected based on the better SINR.

The idea behind the Distributed-training scheme is the fact that to null out the “part-time” interferers by the arrays, the training symbols must lie in the region of the block’s affected portion. The number of symbols used for training (i.e., the training sequence length) will result in a trade-off between the performance of the weights and the efficiency of the data block (efficiency = $((\text{block length} - \text{training sequence length}) / \text{block length}) * 100\%$). For instance, the more training symbols used the better the performance of the estimated weights, but the lower the efficiency. The training schemes considered are summarized as; (see Figure 2.3).

- Post-amble/Pre-amble training (Post/Pre): All reserved training symbols are placed in the end/beginning of the data block (used for comparison purposes).
- Distributed-training (Dist): All reserved training symbols are distributed throughout the block, whereby each training symbol is placed at a certain periodic interval.
- Dist-Pre training (Dist-Pre): The reserved training symbols are placed in the beginning as in Pre-ample and also distributed throughout the rest of the block and a better set of weights (one that produces a lower mean square error) is selected to be used at each symbol instant.
- Pre-Post-ample training (Pre-Post): Half of the reserved training symbols are placed at the beginning and the other half at the end of the block and a better set of weights (one that produces a lower mean square error) is selected to be used at each symbol instant.

Figure 2.3: Illustration of the different training schemes.

Once the training scheme is determined, the estimated weights can be obtained as discussed in section 2.2.2. Hence, the estimated weight vector, $\hat{\mathbf{w}}$ is given as,

$$\hat{\mathbf{w}} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{h}}_1 \quad (2.14)$$

where, $\hat{\mathbf{R}}$ is the estimated input correlation matrix and $\hat{\mathbf{h}}_1$ is the estimated cross-correlation vector also referred as steering vector. The above SMI weights basically work by suppressing the signal components contained in the auto-correlation matrix $\hat{\mathbf{R}}$ that do not match with those contained in the steering vector $\hat{\mathbf{h}}_1$.

One method to evaluate the performance of these estimated weights is the excess mean square error, which is a measure of how much these weights deviate from the optimum weights. Therefore, the MSE will be equal to the sum of the MMSE found using optimum method and the excess MSE. The excess MSE can be found as:

$$MSE_{\text{excess}}(i) = (\hat{\mathbf{w}}(i) - \mathbf{w}_{\text{opt}}(i))^H \mathbf{R}(i) (\hat{\mathbf{w}}(i) - \mathbf{w}_{\text{opt}}(i)) \quad (2.15)$$

where, $\mathbf{w}_{\text{opt}}(i)$ is the optimum weight for the i^{th} symbol. The actual MSE for a specific symbol is calculated as follows:

$$MSE(i) = MSE_{\text{excess}}(i) + MMSE(i) \quad (2.16)$$

From this MSE, we can find an approximate maximum achievable SINR for the i^{th} symbol as:

$$SINR_{\text{max}} = \frac{E\{|s_1(i)|^2\} - MSE(i)}{MSE(i)} \quad (2.17)$$

From this, the approximate probability of error can be calculated using the following formula for BPSK modulation scheme.

$$Pe \approx \Pr(SINR_{\text{max}}) = Q(\sqrt{2SINR_{\text{max}}}) \quad (2.18)$$

Chapter 3

Signal & Channel Model

Flat Fading Environment

3.1 Introduction

In this chapter we will discuss two radio propagation models used in the simulation. The first one assumes a plane wave model while the second one considers Rayleigh fading model with unknown direction of arrival. A path loss exponent of 4 is used throughout the simulations and also a lognormal shadowing effect with standard deviation of 10 dB is assumed.

Three different simulation scenarios employing different kinds of user distributions are simulated. In the first one, there is only one interferer in the system and its appearance is either at random any time during the desired user block or can be fixed. It is also assumed that the average received power from this interferer is equal to the desired user signal. With this kind of simulation environment, we can form an opinion about the convergence performance of different algorithms with a sudden change in channel conditions. The second simulation model considers all the users residing on the same radius. Hence in this model, we are neglecting the effect of distances and assuming that the average received power from all the users is the same. In the third simulation scenario, we are considering that users are spatially uniformly distributed and the desired user suffers at larger distances because the simulation conditions would become interference dominated. Finally, to simulate bursty kind of interference traffic, a batch Poisson traffic model is described. This simple traffic model is based on a probability distribution for various block sizes and the mean inter-burst time. In all simulations interferers are assumed to arrive in a way that they are always symbol-wise synchronous to the desired user even if they are not block-wise asynchronous.

3.2 Radio Propagation Models

3.2.1 Plane Wave Signal Model

Assume plane waves emitted by K sources impinge on a linear array consisting of M equi-spaced antenna elements. Further, assume that the directions of arrival (DOA) of the K sources, relative to the normal to the array are $\{\theta_1, \theta_2, \dots, \theta_K\}$. Then the output at the l^{th} array element can be expressed as follows (refer to Figure 3.1):

Figure 3.1: K plane wave signal sources impinging on an array with M elements.

$$x_l(i) = \sum_{d=1}^K s_d(i) e^{-jw_c(l-1)\tau_d} + n_l(i), \quad (3.1)$$

where: $s_d(i)$ is the i^{th} message signal emitted by the d^{th} source (for BPSK $s_d(i) = (1 - 2a_d(i))\sqrt{E_b}$, and $l=1,2,\dots,M$)

where for BPSK $a_d = \{0, 1\}$)

w_c is equal to $2\pi f_c$ (f_c is the carrier frequency)

τ_d is the inter-element path delay $= \frac{\Delta}{c}(\sin \theta_d)$ (where $\Delta = \frac{\varpi}{2}$, c is the speed of light, θ_d is

the angle of arrival of source d at the receiver and ϖ is the wavelength)

$n_l(i)$ is the Random Additive White Guassian Noise (AWGN) associated with the l^{th} array element.

The array signal vector can generally be expressed in vector notation as [25]:

$$\mathbf{x}(i) = \mathbf{H}\mathbf{s}(i) + \mathbf{n}(i), \quad (3.2)$$

where: $\mathbf{x}(i) = [x_1(i), x_2(i), \dots, x_M(i)]^T$ and $\mathbf{s}(i) = [s_1(i), s_2(i), \dots, s_K(i)]^T$, where T denotes a vector or matrix transpose.

$\mathbf{H} \sim M \times K$ Vandermonde direction matrix such that:

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K], \text{ and}$$

$$\mathbf{h}_d = [1, e^{-jw_o \tau_d}, \dots, e^{-jw_o (M-1) \tau_d}]^T, \quad (3.3)$$

when no lognormal shadowing effect is incorporated, and:

$$\mathbf{h}_d = \chi_d [1, e^{-jw_o \tau_d}, \dots, e^{-jw_o (M-1) \tau_d}]^T, \quad (3.4)$$

when the effect of lognormal shadowing is incorporated, with χ_d representing

the lognormal random variable, where $10\log(\chi_d)$ is a Gaussian random variable with a standard deviation of 10 dB.

The array correlation matrix can now be expressed as:

$$\mathbf{R} = E[\mathbf{x}(i)\mathbf{x}^H(i)] = E[(\mathbf{H}\mathbf{s}(i) + \mathbf{n}(i))(\mathbf{H}\mathbf{s}(i) + \mathbf{n}(i))^H] = \mathbf{H}\mathbf{S}\mathbf{H}^H + \sigma_n^2 \mathbf{I} \quad (3.5)$$

where, $\mathbf{S} = E[\mathbf{s}(i)\mathbf{s}^H(i)]$ denotes the source correlation matrix, σ_n^2 is the noise variance, and $\mathbf{I}$ is the identity matrix.

3.2.2 Rayleigh Fading Model

In general, in an urban mobile radio environment, plane wave propagation is rarely observed since the radio signal usually encounters large obstacles that lie between the transmitter and receiver. In fact, in a built up environment where there is no Line of Sight (LOS) between the transmitter and receiver, the propagating radio wave could be broken into multiple components either due to scattering or diffraction if it encounters objects that are smaller in dimension compared to its wavelength, or due to reflection if it encounters objects that are larger in dimension compared to its wavelength. In a highly cluttered environment, the envelope of the radio signal usually follows a Rayleigh distribution [16]. Thus, in this model, the signal is assumed to undergo a Rayleigh fading channel whereby it could either encounter scattering, diffraction, or reflection depending on the dimension of the objects encountered compared to the wavelength of the signal.

Figure 3.2 presents the model of K signal sources passing through a flat Rayleigh fading channel. Although Figure 3.2 only shows a single path component from a given source to a given receiving antenna, in reality there could be many multipath components originating from a single source. In this model, we have assumed that the modulated signals of all users are mutually uncorrelated, with zero mean and unit variance and also uncorrelated with the noise. Therefore, the steering vector $\mathbf{h}_l$ as defined in (2.3) is equal to the channel response $\mathbf{h}_l$ used in (3.6). Hence the same notation will be used throughout the thesis. For this model, the output at the l^{th} array element can be expressed as follows (refer to Figure 3.2):

Figure 3.2: K signal sources after undergoing a Rayleigh fading channel.

$$x_l(i) = \mathbf{h}_l^T \mathbf{s}(i) + n_l(i), \quad (3.6)$$

where $\mathbf{h}_l^T$ is the l^{th} row of a channel response matrix (due to Rayleigh fading channel) of all the co-channel links involved including the desired signal source and the interfering signal source(s). When the effects of long-term fading are ignored, $\mathbf{h}_l^T$ is modeled as follows:

$$\mathbf{h}_l^T = [h_{l1} \quad h_{l2} \dots h_{lK}], \quad (3.7)$$

where h_{jl} is a zero-mean complex Gaussian random variable with a unit variance associated with user j and antenna element l . Fading components are independent and uncorrelated.

However, when the long-term effects are considered by way of a propagation path loss exponent Ω and a lognormal shadowing standard deviation σ_{dB} , h_l is modeled as:

$$\mathbf{h}_l^T = [(\frac{1}{d_1})^\Omega \chi_1 h_{1l} \quad (\frac{1}{d_2})^\Omega \chi_2 h_{2l} \dots (\frac{1}{d_K})^\Omega \chi_K h_{Kl}] \quad (3.8)$$

where d_j are the distance of the users from the receiver, and χ_j are the lognormal random variable for $j=1,2,3,\dots,K$.

3.3 Interference Distributions- Simulation Scenarios

For a simulated bit error counting approach, the following block diagram is used to model the whole communication system and the BER is determined by averaging over a certain number of simulated data blocks.

Figure 3.3: Block diagram of the entire communication system.

3.3.1 Single Interferer Scenario

This scenario deals with a single interferer only. This single co-channel interferer is not necessarily present all the time rather it is turned on abruptly during the desired signal block. With this scenario, we can observe the effect of an intermittent interferer on the performance of an adaptation scheme.

3.3.2 Non-Uniformly Distributed Interferers-Equal Average Received Power (Flat Rayleigh Fading)

This is the simplest simulation scenario considered with all the users having equal average received power (See Figure 3.4). Equivalently, in this scenario the angle (θ) of the interferers on the circle is assumed to be uniformly distributed between π and $-\pi$. They are not spatially uniformly distributed and the effect of distance for different users is neglected.

Figure 3.4: Distribution of users in the system (equal average received power model).

3.3.3 Spatially & Uniformly Distributed Interferers Generation (Flat Rayleigh Fading and Shadowing)

In this model, an interferer's location is represented by a random variable R representing the distance of the interferer from the receiver, and an angle θ representing the spatial angle. In order to arrive at a true spatial uniform distribution of locations of interferers, the *p.d.f* of R has to be a function of r (sample value of R), while θ remains uniformly

distributed between $-\pi$ and π . Thus, we obtain the following for a spatially and uniformly distributed location within a ring shape area confined between two radii ϑ and κ for instance.

$$f_R(r) = \frac{2r}{(\vartheta^2 - \kappa^2)} \quad (3.9)$$

Thus, if one were to produce a scatter plot from the above distributions, one would obtain a plot similar to that of Figure 3.5.

Figure 3.5: Spatially uniform distribution of interferers.

Whenever there is a distance attenuation and a lognormal shadowing effect involved, the path loss exponent Ω is assumed equal to 4 and the standard deviation of lognormal shadowing is 10 dB. The simulation area under consideration is a ring-shape area confined between two radii, with the inner radius $\kappa = 100m$ and the outer radius $\vartheta = 1km$. The receiver is placed at the center of this ring and the distance between it and the desired transmitter is a parameter that changes with an incremental step of 100 m along the dotted line, as shown in Figure 3.6. The interfering sources (represented by triangles in Figure 3.5 and Figure 3.6) are randomly placed in the simulation area.

Figure 3.6: Simulation area of a desired Tx-Rx pair and the interfering sources.

3.4 Traffic Model-Interference Traffic Generation

3.4.1 Batch Poisson Traffic Model

The traffic pattern used for each interferer in the following model is that of a Batch Poisson traffic model whereby the inter-arrival rate is modeled by an exponential distribution with a mean parameter μ . The inter-arrival rate λ is actually equal to $1/\mu$. This kind of traffic model is suitable for simulating the bursty traffic [17].

An exponential distribution (with a certain mean, μ , in number of symbols) is used to model the time between the end of one packet and the beginning of the next (i.e., a discrete inter-arrival time). Except for the desired source (the desired source is assumed to generate fixed packets of length 162 symbols), each interfering source generates a variable length data packet (frame/block) according to the probability distribution [18] shown in Table 3.1. In a later section (4.1.2.1.3), we will vary the desired source block length.

Block size (β in symbols)	Probability (ρ).
64	0.6
128	0.06
256	0.04
512	0.02
1024	0.25
1518	0.03

Table-3.1 Probability distribution of various block sizes (bursty traffic).

By changing μ and thus λ , we can simulate a different discrete inter-arrival time from one block to the next. A timing diagram for a small μ , corresponding to a large λ , and for a large μ , corresponding to a small λ , along with the average duty cycle (to be defined later), is given in Figures 3.7 and 3.8, respectively.

Additionally, in the model we define a relationship between λ and the offered interference traffic load (measured in Erlangs) as follows:

Let τ_{on} denote the ensemble discrete time average of the on-time duration for a given user given as:

$$\tau_{on} = \sum_{i=1}^6 \beta_i * \rho_i. \quad (3.10)$$

And let τ_{off} denote the discrete time average of the off-time duration for a given user defined as:

$$\tau_{off} = \frac{1}{\lambda}. \quad (3.11)$$

Then, we define an average duty cycle, C_{avg} , as: $C_{avg} = \frac{\tau_{on}}{\tau_{on} + \tau_{off}}$ (3.12)

The interference traffic load, T_L , at a given desired user's frame window is given, in Erlang, as:

$$T_L = C_{avg} * (K - 1) \quad (3.13)$$

where, $K-1$ is the total number of interferers.

In a sense, the parameter λ and the varying block size represent the burstiness of the interference traffic. Moreover, the training sequence length used for channel estimation purposes is 14 symbols out of the desired user's data block/frame.

Figure 3.7: Batch Poisson traffic model (timing diagram, snap shot, avg. duty cycle=67%)

Figure 3.8: Batch Poisson traffic model (timing diagram, snap shot, avg. duty cycle=21%)

Chapter 4

Performance of the Proposed Adaptation Scheme

4.1 Simulation Results

4.1.1 Single Interferer Scenario

Plane wave model described in chapter3 section 3.2.1 is used in this section. Some important parameters are briefly described in the Table 4.1.

Plane Wave Signal Model	
Total number of users	$K = 1$ or 2
Total number of antenna elements	$M = 2$ or 3
Amplitudes of the users	1
Angle of arrival	Random (arbitrarily selected)
Phases	Arbitrarily selected
Log normal shadowing	STD=10dB
AWGN channel	
Appearance of the intermittent interferer	Symbol number 47,81 and 30/90 (with two interferers)
Number of simulation runs	1500 and 5000

Table 4.1 Brief simulation description (Plane wave model)

The simulation result shown in Figure 4.1 compares the adaptation performance of the Distributed-training scheme and the scheme using the sub-space method discussed in [15]. In this specific case, an interferer appears at symbol number 47. A single interferer model with a single block run (snap shot) is used for this simulation. A sharp rise in the squared error can be seen with the sub-space technique while there is no significant change is observed with the Distributed-training scheme. The sub-space method is a good candidate for the system in which the eigen structure of the channel changes very slowly or the required weighing coefficients update is slow [19]. That is why in the current situation appearance of the interference cannot be tracked efficiently by the sub-space method.

Figure 4.1: Effect of sudden appearance of an interferer (Dist vs Sub-space method, $M=2$).

In Figures 4.2, 4.3 & 4.4, three different schemes are compared with the Distributed-training scheme with the same single interferer model. But the difference between these figures and Figure 4.1 is that these results are obtained after averaging over different number of blocks and the interferer is forced to always appear at a specific symbol number (symbol # 81).

The schemes tested are the Pre-amble training scheme, LMS with known reference, LMS in decision directed mode and CMA combined with SMI for initial weight estimation. All of these schemes perform worse than the Distributed-training scheme in terms of MSE after the interferer starts up, although their MSE is lower than that of Distributed-training scheme before the appearance of the interferer. The motivation to compare these results with the Distributed-training scheme is originated from the work done in [14] showing good performance of semi-blind CMA technique. Here in our simulation, CMA is not totally blind as mentioned in [14] as it takes initial weights from the Pre-amble with the SMI technique and then tracks to update the coefficients.

This technique performs well compared to the Pre-amble only training scheme but is not able to converge rapidly, and even after convergence in the steady state it does not attain the mean squared level obtained by the Distributed-training scheme.

Figure 4.5 shows the combination of the results in a single shot. The effect of several intermittent interferers is followed in the next section, while, the effect of two interferers at fixed symbol times (30 and 90) is shown in Figure 4.6. In figures 4.1-4.6 the periodic dips in the MSE are because of the training symbols distributed throughout the block.

Figure 4.2: Effect of sudden appearance of an interferer (Dist training scheme vs Preamble and SMI-CMA, $M=2$).

Figure 4.3: Effect of sudden appearance of an interferer (Dist training scheme vs Preamble and LMS decision directed mode, $M=2$).

Figure 4.4: Effect of sudden appearance of an interferer (Dist training scheme vs Preamble and LMS reference known, $M=2$).

Figure 4.5: Effect of sudden appearance of an interferer (Dist training scheme vs All others on a single shot, $M=2$).

Figure 4.6: Effect of sudden appearance of two interferers (Dist training scheme vs Preamble training scheme, $M=3$).

4.1.2 Interference Following Batch Poisson Traffic Model

Rayleigh fading model described in chapter3 section 3.2.2 is used in this section. Some important parameters are briefly described in the Table 4.2.

Rayleigh Fading Signal Model	
Traffic load (Erlangs)	Varies from 0 to7 Erlangs
Total number of antenna elements (M)	Varies from 2 to10
Inter-arrival rate (λ)	0.033 and 0.0033
Average received SNR	1 and 20 dB
Path loss exponent (Ω)	4 (only used in section 4.1.2.2)
Log normal shadowing	STD=10dB
AWGN channel	
Appearance of the intermittent interferer	Follows traffic model
Number of simulation runs	500 and 1000
Performance measures	BER/Outage/Block length/Histogram/CDF

Table 4.2 Brief simulation description (Rayleigh fading model)

4.1.2.1 Equal Average Received Power Model

4.1.2.1.1 BER Performance

For the following results, the BER performance versus the maximum number of interferers is presented in which the interference follows the traffic pattern described in chapter 3, section 3.4. The weights are obtained using different training schemes described in chapter 2, section 2.5. Figures 4.7-4.12 show the BER versus the total interference traffic load (in Erlangs) in which the desired user and all the interferers are received with equal average powers (i.e., the effect of distance attenuation has been ignored). The training placement schemes are also compared with the optimum performance. Optimum performance curves are obtained by assuming perfect channel knowledge. The value of λ in this case is 0.033 and the average duty cycle is 0.67. The Erlangs traffic load is then proportional to the maximum number of possible interferers.

Each simulation scenario includes Rayleigh fading and lognormal shadowing with a standard deviation of 10dB. Moreover, two average SNRs (signal power to thermal noise power ratio) per antenna element are considered: 1dB, representing a noise-limited system and 20dB, representing an interference-limited system. The average BER is obtained by averaging over 500 simulated blocks.

In this case, the following trends are observed:

1. The Pre-amble (Pre) or Post-amble (Post) offer the worst performance.
2. Pre-Post-amble training is simulated in two ways: The first uses 14+14 training symbols (less efficient) and the second uses 7+7 training symbols (shown in Figure 4.9 for 6 elements and SNR=20 dB and in Figure 4.11 for 10 elements and SNR=20 dB). Curves in both cases offer better BER as compared to the Pre-amble or Post-amble training cases.
3. Focusing on identical efficiency schemes with 14 training symbols the Distributed-training scheme (Dist) offers the best performance compared to Pre-amble, Post-amble or Pre-Post-amble training.
4. One could think that the inferior performance of the Pre-Post-amble training with 7+7 training symbols is due to the insufficient number of training symbols used. In fact results show that even by doubling the number of training symbols, the Pre-Post-amble training is greatly outperformed by the Distributed-training scheme when the number of antenna elements is sufficiently large (greater than 2 times the traffic load).
5. Finally, with the Distributed-training scheme, as long as the number of antenna elements is greater than 2 to 3 times the traffic load in the network, one can achieve a BER performance of 10^{-3} .
6. In Figures 4.8, 4.10 & 4.12, deterioration in BER is due to the noise dominance in the system since SNR is only 1 dB.
7. Finally, Dist-Pre training does not offer significant improvement over Dist training showing that the gains are obtained by the distributed placement of the training symbols within the block and not by the additional Pre-amble training symbols. Since Dist-Pre training is less efficient, Dist is a preferable training scheme.

Figure 4.7: BER comparison between different training schemes ($M=4$, $\text{SNR}=20\text{dB}$).

Figure 4.8: BER comparison between different training schemes ($M=4$, $\text{SNR}=1\text{dB}$)

Figure 4.9: BER comparison between different training schemes (M=6, SNR=20dB).

Figure 4.10: BER comparison between different training schemes (M=6, SNR=1dB).

Figure 4.11: BER comparison between different training schemes (M=10, SNR=20dB).

Figure 4.12: BER comparison between different training schemes (M=10, SNR=1dB).

4.1.2.1.2 Outage Probability

The outage probability (outage rate of error probability) is another performance measure considered in this simulation. The outage probability is defined as “probability that the instantaneous symbol error probability of the system exceeds a specific threshold”. This is equivalent to finding the probability that the output SINR falls below a certain threshold or MMSE exceeds the corresponding MMSE threshold.

Comparison of the outage probability between the Dist and Pre-Post-amble training schemes with the optimum case is made. Simulations are done for two outage criteria.

- 1- MMSE > -10dB (corresponding SINR = 9.5dB)
- 2- MMSE > -14dB (corresponding SINR = 13.8dB)

These simulation curves are obtained with Rayleigh fading and lognormal shadowing. Moreover, the average received SNR per branch is considered to be 20dB. Optimum curves are based on the weights calculated from the exact MMSE technique, calculated from the actual channel and interference parameters. Outage with these three schemes is found with different numbers of antenna elements (4, 8 and 10) and with different traffic loads. The outage probability is obtained by averaging over 1000 blocks. The block length of the desired user is kept fixed (=162 symbols). Results are shown in Figures 4.13-4.18.

Observations:

- As we increase the number of antenna elements, better performance in terms of lower outage is observed.
- The Dist training scheme gives the best results especially with a large number of antenna elements. With 10 maximum possible potential interferers and 10 antenna elements, there is no outage observed before 8 interferers.

Figure 4.13: Outage probability with MMSE>-10dB, M=4

Figure 4.14: Outage probability with MMSE>-14dB, M=4

Figure 4.15: Outage probability with MMSE>-10dB, M=8

Figure 4.16: Outage probability with MMSE>-14dB, M=8

Figure 4.17: Outage probability with MMSE>-10dB, M=10

Figure 4.18: Outage probability with MMSE>-14dB, M=10

4.1.2.1.3 Effect of Block Length

The effect of varying the length of the desired user block is an issue of great interest. In fact there is a trade-off between the desire to reduce the header size and hence the overhead by making the block size large, and the need to reduce the symbol error rates in the presence of intermittent interferences in the channel by using small block lengths, thereby encountering fewer new interferers per block. We will have to determine the optimum length of the desired user block with acceptable output SINR. In future, work can be done to make the length of the block “adaptable”, depending upon throughput, transmission range and power consumption [20].

The effect of the changing block size with Pre-amble, Post-amble and Dist training schemes is simulated. Simulations are done for two criteria:

- 1) By keeping the number of training symbols in the Distributed-training scheme fixed.
- 2) By keeping the period in between the training symbols fixed. In this case, we are allowing the increase of the number of training symbols in the Distributed-training scheme as the length of the block increases. On the other hand, the number of symbols in the Pre-amble and Post-amble is kept fixed (=14 symbols) in both cases.

These simulation curves are obtained with Rayleigh fading channel and lognormal shadowing with standard deviation equal to 10dB. Moreover, the average SNR per branch is considered to be 20dB. The block size is increased with a step size of 20 symbols. The number of antenna elements and maximum number of possible potential interferers are fixed with values of 6 and 4 respectively. The MMSE is obtained by averaging over 1000 blocks. Results are shown in Figures 4.19-4.21.

Observations:

Case1: With a fixed number of training symbols (Pre/Post=14 symbols, Dist=14 symbols), the Distributed-training scheme is giving better results compared to only Pre-amble or Post-amble training. An improvement of about 10dB as compared to Pre-amble or Post-amble training schemes can be seen from the curves.

Case2: With a fixed symbol-period (gap in between two training symbols), in Distributed-training and fixed training symbols in Pre-amble/Post-amble (14 symbols), the performance of the Distributed-training scheme improves compared to the Pre and Post-amble training with increase in the block length. We can see that the Dist training curve crosses the pre and Post-amble training at a certain block length when there are enough training symbols in the Distributed-training scheme. With very short block length, the Distributed-training scheme performs worse due to lack of training symbols. But at this stage Pre and Post-amble training blocks mostly contain training symbols in the block with very few data symbols.

Case3: Dist training scheme is tested with environment similar to case2 but with larger desired user's block sizes (almost two to three times the average inter arrival period). No considerable change in the MSE is observed at higher block sizes as after a specific value (100 in this case), the number of Distributed-training symbols is increasing with the same ratio (as the period is fixed) as the block length. With a fixed traffic load this is an expected result.

Figure 4.19: Effect of desired user block length (14 training symbols).

Figure 4.20: Effect of desired user block length (fixed period of 10 symbols between the Distributed-training symbols)

Figure 4.21: Effect of desired user block length (fixed period of 10 symbols between the Distributed-training symbols with larger block lengths)

4.1.2.1.4 Output SINR Histogram and CDF

Figure 4.22 & 4.23 show the histogram and corresponding CDF of the output SINR with the Distributed-training scheme. Output SINR seems to follow a Log-normal distribution, which is expected to be since the desired signal and the interferer power have lognormal distributions. The mean value is around 30dB.

Figure 4.22: Histogram for the output SINR with Distributed-training scheme, $M=6$, 2.68 Erlangs, Equal avg. received power model.

Figure 4.23: CDF for the output SINR with Distributed-training scheme, $M=4$, 2.68 Erlangs, Equal avg. received power model.

4.1.2.2 Users Spatially Uniformly Distributed Model

4.1.2.2.1 BER

Figures 4.24 & 4.25 show the BER versus the distance separation between the desired transmitter (Tx) and receiver (Rx), whereby losses due to distance attenuation and lognormal shadowing is incorporated, and thus, this represents a case of unequal average interference powers. A path loss exponent of 4 is assumed. For each desired Tx-Rx distance separation, 100 random locations for each interferer are generated, and at each location the number of blocks simulated is 500. Pre-amble and Dist training schemes are compared with 14 training symbols in both cases. In these simulations, the value of λ is 0.033. A similar trend is observed here as in the previous cases, namely the Distributed-training scheme offers the best performance. Furthermore, the following is observed:

1. As the distance between the desired Tx and Rx increases, the BER degrades as expected.
2. The performance difference between the 6 and 10 antenna elements is very small for large distance separations. This may be due to the fact that the system becomes noise-dominant and increasing the number of antenna elements does not help reducing the BER significantly.

Figure 4.24: Effect of desired user Tx-Rx separation distance with Pre-amble training scheme

Figure 4.25: Effect of desired user Tx-Rx separation distance with Distributed-training scheme

Figure 4.26: Comparison of the performance of Distributed-training with Pre-amble.

Figure 4.26 is overlapping the two results in Figures 4.24 and 4.25, to compare the performance of Distributed-training with the Pre-amble training.

4.1.2.2.2 Outage Comparison

Figure 4.27 shows the outage probability versus the desired user transmitter-receiver separation. The total interference traffic load in this case is 2.68 Erlangs. Only the Pre-amble training scheme is compared with the Distributed-training scheme. Pre-amble and Dist training schemes are compared with 14 training symbols in both cases. In these simulations, the value of λ is 0.033. The curves for the Pre-amble training and Distributed-training schemes with a threshold of $\text{SINR} < 9.5\text{dB}$ are shown. As long as the number of elements is less than or equal to the number of interferers ($M=3$), the Distributed and Pre-amble training schemes have the same performance. With more elements say with 10 elements and at a distance of 550m, the difference in outage between the two schemes is an order of magnitude.

Figure 4.27: Comparison of Distributed-training and Pre-amble training scheme in terms of outage vs desired user Tx-Rx separation

Figure 4.28 shows comparison of Dist and Pre-amble training schemes in terms of outage with different traffic loads in Erlangs. Better performance of the Distributed-training scheme can be seen from the figure. With a traffic load of 3 Erlangs and at 0.1km Tx-Rx separation, there is no outage at all with Distributed-training while with Pre-amble, the outage probability is much higher; i.e. 10^{-1} .

Figure 4.28: Comparison of Distributed-training and Pre-amble training scheme in terms of outage vs total offered traffic load

4.1.2.2.3 Output SINR Histogram

Figures 4.29 & 4.30 show the histograms of the output SINR (Distributed-training scheme) at different distances from the desired transmitter. Separations of 0.3 and 0.6 km between the desired user and transmitter are considered. We can see the decline in the mean values of the SINRs with increase in the separation distance.

Figure 4.29: Histogram for the output SINR with Distributed-training scheme, $M=10$, Max. number of interferers=4 (2.68 Erlangs), (0.3km)

Figure 4.30: Histogram for the output SINR with Distributed-training scheme, $M=10$, Max. number of interferers=4 (2.68 Erlangs), (0.6km)

4.2 Lower bound Comparison

Configuration of the estimator

A comparison is made between the simulated results of the proposed training scheme with a lower bound. In the following, a lower bound is found analytically and then by using Monte-Carlo simulations, the equation obtained by this lower bound expression is plotted and compared with the simulated one.

Figure 4.31: Illustration of the suddenly appearing interferer after a fraction of the desired user block.

Figure 4.31 shows the sudden appearance of an interferer in a system already having $K-1$ interferers. The fraction (region-I) before the appearance is termed α and the affected portion (region-II) by the additional interferer (K th) is termed $(1-\alpha)$. The lines in the desired user block are showing the training symbols distributed throughout the block.

Assuming the data symbols of all users are mutually uncorrelated, with zero mean and unit variance, the ensemble-average correlation matrix for the first region as indicated in Figure 4.31 would be,

$$\mathbf{R}_1 = E(\mathbf{x}\mathbf{x}^H) = \sum_{k=1}^{K-1} \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I}$$

where, $\mathbf{h}_k$ represents the steering vector of the users and σ_n^2 is the noise variance.

The optimum antenna weights for region-I are

$$\mathbf{w}_1 = \mathbf{R}_1^{-1} \mathbf{h}_1$$

and the MMSE for the region-I will be

$$MMSE_1 = 1 - \mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_1$$

Similarly, for the region-II, the correlation matrix would be,

$$\mathbf{R}_2 = E(\mathbf{x}\mathbf{x}^H) = \sum_{k=1}^K \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I} = \mathbf{R}_1 + \mathbf{h}_K \mathbf{h}_K^H \quad (4.1)$$

The resulting antenna weights are

$$\mathbf{w}_2 = \mathbf{R}_2^{-1} \mathbf{h}_1 \quad (4.2)$$

the MMSE for the second region would be

$$MMSE_2 = 1 - \mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_1$$

Now with the proposed Distributed-training scheme, the correlation matrix can be formulated by averaging with the use of affected fraction, such as (assuming $\mathbf{R}_1$ and $\mathbf{R}_2$ are known therefore corresponding to a lower bound on MSE)

$$\mathbf{R} = \alpha \mathbf{R}_1 + (1 - \alpha) \mathbf{R}_2 = \mathbf{R}_1 + (1 - \alpha) \mathbf{h}_K \mathbf{h}_K^H = \mathbf{R}_2 - \alpha \mathbf{h}_K \mathbf{h}_K^H$$

the corresponding antenna weights are,

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{h}_1 \quad (4.3)$$

the MSE in region-II is the sum of MMSE and the excess MSE in region-II

$$MSE_i = MMSE_i + ExcessMSE_i$$

$$MSE_2 = MMSE_2 + (\mathbf{w} - \mathbf{w}_2)^H \mathbf{R}_2 (\mathbf{w} - \mathbf{w}_2) \quad (4.4)$$

to find excess MMSE in region-II, from (4.3),

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{h}_1 = [\mathbf{R}_2 - \alpha \mathbf{h}_K \mathbf{h}_K^H]^{-1} \mathbf{h}_1$$

By using matrix inversion lemma [4],

$$\mathbf{w} = \mathbf{R}_2^{-1} \mathbf{h}_1 + \frac{\alpha \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

from (4.2) we can have,

$$\mathbf{w} - \mathbf{w}_2 = \frac{\alpha \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

from (4.4) the excess MMSE in region-II is,

$$ExcessMSE_2 = \frac{\alpha^2 \mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{|1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2}$$

or,

$$ExcessMSE_2 = \frac{\alpha^2 |\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2 \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}{|1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2} \quad (4.5)$$

Similarly, the excess MSE in region-I is,

$$= (\mathbf{w} - \mathbf{w}_1)^H \mathbf{R}_1 (\mathbf{w} - \mathbf{w}_1)$$

and as

$$\mathbf{w} = \mathbf{R}^{-1} \mathbf{h}_1 = [\mathbf{R}_1 + (1 - \alpha) \mathbf{h}_K \mathbf{h}_K^H]^{-1} \mathbf{h}_1$$

again using matrix inversion lemma,

$$\mathbf{w} = \mathbf{R}_1^{-1} \mathbf{h}_1 + \frac{(1 - \alpha) \mathbf{R}_1^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_1}{1 + (1 - \alpha) \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K}$$

in the same way as above the excess MSE in region-I is,

$$ExcessMSE_1 = \frac{(1 - \alpha)^2 \mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_1}{|1 + (1 - \alpha) \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K|^2} \text{ or,}$$

$$ExcessMSE_1 = \frac{(1 - \alpha)^2 |\mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_K|^2 \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K}{|1 + (1 - \alpha) \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K|^2} \quad (4.6)$$

finding excess MSE₁ in terms of $\mathbf{R}_2$,

$$\text{As, } \mathbf{R}_1 = \mathbf{R}_2 - \mathbf{h}_K \mathbf{h}_K^H$$

using matrix inversion lemma,

$$\mathbf{R}_1^{-1} = \mathbf{R}_2^{-1} + \frac{\mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1}}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

Therefore, $\mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_K$ can be written as,

$$\mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_K = \mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K + \frac{(\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K)(\mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K)}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

or,

$$\mathbf{h}_1^H \mathbf{R}_1^{-1} \mathbf{h}_K = \mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K \left(\frac{1}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K} \right)$$

Similarly, the other component $\mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K$ of (4.6) can be found as,

$$\mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K = \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K + \frac{|\mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

or,

$$\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K = \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K \left(\frac{1}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K} \right)$$

by putting these values in (4.6) the excess MSE in region-I is,

$$\begin{aligned} \text{ExcessMSE}_1 &= \frac{(1-\alpha)^2 |\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2 \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}{\left(1 + (1-\alpha) \frac{\mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K} \right)^2 (1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K)^3} \text{ or,} \\ \text{ExcessMSE}_1 &= \frac{(1-\alpha)^2 |\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2 \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}{(1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K)^2 (1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K)} \end{aligned} \quad (4.7)$$

To reduce MMSE in either of the regions, we have to reduce the excess MSE. From (4.5) and (4.7), we can see common terms in the excess MSE of both regions. In order to minimize the excess MSE, the term $\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K$ should be small. The term $\mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K$ is actually the (1-MMSE) for detecting the interferer's steering vector. Therefore, to reduce the excess MSE this term should be close to one. The third term in the expression is $1 - \alpha \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K$, which can be written as $1 - \alpha + \alpha(1 - \mathbf{h}_K^H \mathbf{R}_1^{-1} \mathbf{h}_K)$. The term in the parentheses is the MMSE in detecting the steering vector of the interferer and hence to minimize the excess MSE α should be close to unity.

Figure 4.32 shows the comparison of the lower bound and the simulated curves with a block length of 162. The simulated results are close to the lower bound.

Figure 4.32: Comparison of the lower bound and the simulated curves (block length=162, number of interferers=1)

If we increase the block length, we should expect this difference to be reduced. This can be seen in Figure 4.33 where everything is same as in Figure 4.32 but the block length is made one thousand. We can see the difference in the two curves is much closer than the previous case.

Figure 4.33: Comparison of the lower bound and the simulated curves (block length=1000, number of interferers=1)

A similar effect is observed if we make the assumption of larger block size and considers appearance of two interferers during the desired block length. The effect on MSE with their appearances can be viewed in Figure 4.34.

Figure 4.34: Comparison of the lower bound and the simulated curves (block length=1000, number of interferers=2)

Now the excess MSE resulted from Pre-amble training is to be found so that a comparison can be made.

Figure 4.35: Illustration of the suddenly appearing interferer after a fraction of the desired user block (Pre-amble training).

In region-I,

$$\mathbf{R}_1 = E(\mathbf{x}\mathbf{x}^H) = \sum_{k=1}^{K-1} \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I}$$

$$\mathbf{w}_1 = \mathbf{R}_1^{-1} \mathbf{h}_1$$

the estimated correlation matrix will be,

$$\mathbf{R} = \mathbf{R}_1$$

and

$$\mathbf{w} = \mathbf{R}_1^{-1} \mathbf{h}_1$$

In region-II,

$$\mathbf{R}_2 = E(\mathbf{x}\mathbf{x}^H) = \sum_{k=1}^K \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I} = \mathbf{R}_1 + \mathbf{h}_K \mathbf{h}_K^H$$

$$\mathbf{w}_2 = \mathbf{R}_2^{-1} \mathbf{h}_1$$

the estimated correlation matrix would not change,

$$\mathbf{w} = \mathbf{R}_1^{-1} \mathbf{h}_1 = [\mathbf{R}_2 - \mathbf{h}_K \mathbf{h}_K^H]^{-1} \mathbf{h}_1$$

using matrix inversion lemma,

$$\mathbf{w} = \mathbf{R}_2^{-1} \mathbf{h}_1 + \frac{\mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

from (4.2) we can have,

$$\mathbf{w} - \mathbf{w}_2 = \frac{\mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}$$

the excess MMSE in region-II is,

$$ExcessMSE_2 = \frac{\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_1}{|1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2}$$

or,

$$ExcessMSE_2 = \frac{|\mathbf{h}_1^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2 \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K}{|1 - \mathbf{h}_K^H \mathbf{R}_2^{-1} \mathbf{h}_K|^2} \quad (4.8)$$

With Pre-amble training, all the training symbols are in the beginning of the frame and most of the time we may say that in the first region, the performance of the Pre-amble would be very close to the optimum one. And it may be possible to have performance better than the Distributed-training scheme because of the higher number of training symbols in region I. But in region-II, where the additional interferer is present, the excess MSE with Pre-amble has the similar expression as in the case of the Distributed-training scheme. The difference

is the α^2 term in the numerator and multiplication of α with the second term in the denominator. This concludes that the performance with Pre-amble training will be worse than the Distributed-training by a factor of α^2 . As α approaches 1 the performance of Pre-amble training will approach that of the Distributed one. But in the presence of an intermittent interferer α will always be less than one, making the Pre-amble performance worse than the Distributed-training scheme.

Figure 4.36 shows the worst performance of Pre-amble in the second region. In the second region, Pre-amble is most of the time not able to catch the additional interferer (sometimes, Pre-amble would be able to catch the interferer when it appears in the very beginning of the desired user block).

Figure 4.36: Worst performance of Pre-amble in the second region (block length=1000, number of interferers=1)

Chapter 5

Frequency Selective Fading Environment

With high data rates, such as 20Mb/s or higher, there exists the inherent problem of multipath interference, which may span up to 50 to 100 symbol intervals. To combat ISI caused by the multipath interference, OFDM has been the foundation of the IEEE 802.11a wireless LAN standard. But due to the free nature of the unlicensed band, there may be many systems operating in close proximity to each other and they could pose (if operating on the same frequency channels) severe co-channel interference to one another. In this case, it would be difficult for a conventional OFDM receiver to survive. Hence, a combination of adaptive antenna arrays with OFDM is one method of dealing with both the problems of multipath and co-channel interference.

In this section we will describe some OFDM fundamentals including the system model of OFDM with adaptive antenna arrays and different training schemes for estimation. First uncoded OFDM systems will be analyzed and then a simulation of adaptive OFDM based on training is presented.

5.1 OFDM Fundamentals

In OFDM systems, the input high-rate data stream is divided into several low-rate streams that are transmitted in parallel, thereby increasing the symbol duration. As the symbol duration increases for lower rate parallel sub-carriers, the relative time dispersion due to multipath delay spread is decreased. In other words, since the duration of each sub-carrier is much larger than the delay spread of a multipath fading channel, the channel can be regarded as frequency non-selective from the view point of each sub-carrier. Residual ISI can be eliminated by inserting a guard time in each OFDM symbol that has duration longer than the maximum channel delay spread. To maintain orthogonality between the sub-carriers, in this guard time, the OFDM symbol is cyclically extended (with a cyclic prefix) [21]. In this way inter-carrier interference (ICI)

can be avoided. Some of the important features of OFDM are explained briefly in the following sections.

5.1.1 Spectral Efficiency

OFDM is a multi-carrier system with many low-rate sub-channels (P). With a proper selection of spacing between the sub-carriers, they can be made orthogonal to each other. Because of orthogonality between the sub-carriers, their spectral overlapping becomes possible. And if this orthogonality could be maintained in the transmission, then these overlapped sub-carriers can easily be separated at the receiver.

5.1.2 Cyclic Prefix

In order to remove residual ISI, a guard time is inserted into each OFDM symbol. The guard time selected should be greater than the expected maximum delay spread. The guard time could consist of no signal at all but in this case, ICI would arise, resulting in loss of orthogonality between the sub-carriers. This will cause ICI, as within the Fast Fourier Transform (FFT) interval at the receiver, the sub-carriers would not remain orthogonal to each other. For orthogonality at the receiver, there should be integer number of cycles difference between sub-carriers within an FFT interval. This is depicted in Figure 5.1 [21]:

Figure 5.1: No (zero) signal in guard time (ICI)

To remove ICI, the last part of the OFDM symbols is copied in the guard time. This portion of the OFDM symbol is called Cyclic Prefix. The idea is shown in Figure 5.2;

Figure 5.2: Cyclic prefix in guard time (no ICI)

When a cyclic prefix is added in the OFDM symbol, it always has an integer number of cycles within the FFT interval, as long as the maximum delay spread is smaller than the cyclic prefix. Hence, multipath signals with delays smaller than the cyclic prefix cannot cause ICI. In [21], simulation results are shown with different levels of Inter-symbol interference (ISI) due to two-ray multipath delay exceeding the guard time by a fraction of the FFT interval. With delay spread exceeding about 10% of the FFT interval, the constellation is really distorted.

5.1.3 Realization Complexity

OFDM system has the ability to deal with large delay spreads with a reasonable implementation complexity. In single-carrier systems, time domain equalization leads to complexity, which rises linearly or quadratically, with delay spread. While in OFDM, complexity increases only logarithmically with delay spread due to FFT processing, which is used to demodulate the sub-carriers. In [21], a comparison is made with respect to the processing complexity of an OFDM modem and single-carrier modem with time domain equalization, for the case in which both systems are able to tolerate the same amount of delay spread. The system description and the required processing complexity for both systems are provided in Table 5.1 [21].

	Single-carrier system (time domain equalization)	802.11a OFDM standard
Equalizer used	DFE	-
Modulation used	GMSK	
Feed-Forward Taps	20	-
Feed-Back Taps	20	-
Delay spread	250nsec	250nsec
Data rate	24Mbps	24Mbps
FFT points	-	64-point
COMPLEXITY		
Multiplication per second	960×10^6	96×10^6

Table 5.1: Processing complexity comparison between single-carrier and OFDM

From Table 5.1, it can be observed that if complexity is measured in terms of the number of multiplication operations per second, the time-domain equalizer of the single-carrier system is 10 times more complex as compared to the OFDM system. Moreover, this complexity grows with different ratios in both of the systems. In the single-carrier system, the equalizer complexity grows quadratically with the bandwidth-delay spread product, while in OFDM the hardware complexity is roughly proportional to the logarithm of the maximum delay spread. Therefore, OFDM offers better performance and complexity tradeoff than conventional time-domain equalization with single carrier modulation. Single carrier modulation competes successfully with OFDM in complexity only if it uses frequency-domain processing for equalization [23].

5.1.4 Salient Properties of OFDM

In the following, we list the advantages and disadvantages associated with an OFDM system.

Advantages:

- 1- OFDM is more resistant defiant to the effects of frequency selective fading than a single-carrier system. This is due to division of the channel into narrow band flat fading sub-channels.
- 2- ISI and ICI can be eliminated using cyclic prefix.
- 3- Equalizer design in OFDM is much simpler than time-domain adaptive processing techniques with single-carrier transmissions.
- 4- OFDM system can be combined with spatial processing to provide diversity and suppression of co-channel interference.
- 5- Using adequate channel coding and interleaving one can recover symbols lost due to the frequency selectivity of the channel.
- 6- Implementation complexity of OFDM is significantly lower than that of a single-carrier system with a time-domain equalizer.

Disadvantages:

- 1- Power Efficiency: As the OFDM signal contains many tones, it has a high peak-to-average power ratio. As a result, it requires Radio Frequency (RF) power amplifiers with a high Peak to Average Power Ratio. These kinds of amplifiers are expensive.
- 2- OFDM is basically designed for wired LAN, and to implement it in wireless environment, powerful error coding and interleaving is required.
- 3- In OFDM, because of the close spacing in frequency in its sub-carriers, there is always a possibility of carrier frequency offsets and drift compared to the single carrier system.

Furthermore, it is shown in [23] that a single-carrier conventional modulation combined with frequency domain equalization can produce the same performance as an OFDM system, and with almost the same computational complexity. According to [23], we can obtain all advantages of OFDM systems with single carrier systems with frequency domain adaptive equalization, and can avoid almost all disadvantages of OFDM. Recently both modes of operations are provided in 802.16a broadband wireless access standard.

5.2 OFDM Signal Model

The OFDM signal can be written as [22],

$$s(t) = \sum_{n=-\infty}^{\infty} \left[\sum_{k=0}^{P-1} C[n,k] g_k(t - nT_s) \right]$$

$$g_k(t) = \begin{cases} e^{j2\pi f_k t} & \dots\dots t \in (0, T_s) \\ 0 & \dots\dots \text{otherwise} \end{cases} \quad (5.1)$$

$$f_k = f_0 + \frac{k}{T_s} \dots\dots, k = 0, 1, 2, \dots, P-1$$

where:

$C[n,k]$ is the n^{th} symbol transmitted on the k^{th} carrier.

P is the total number of sub-carriers.

f_k is the k^{th} carrier frequency with f_0 is the lowest frequency to be used.

T_s is the duration of each data symbol on each sub-carrier.

The n^{th} OFDM symbol can be written as:

$$F_n(t) = \sum_{k=0}^{P-1} C[n,k] g_k(t - nT_s) \quad (5.2)$$

where, $F_n(t)$ corresponds to the set of symbols $C[n,k]$, $k=0, 1, \dots, P-1$, transmitted on the corresponding sub-carriers f_k . Therefore, by using (5.2), the OFDM signal can be written as:

$$s(t) = \sum_{n=-\infty}^{\infty} F_n(t) \quad (5.3)$$

A simplified block diagram of complete OFDM system and the receiver structure for the adaptive array are shown in Figure 5.3 and 5.4 respectively.

Figure 5.3: Block diagram of an OFDM system.

Figure 5.4: Block diagram of OFDM receiver with MMSE antenna diversity combiner.

5.3 Multipath Channel Model

Slow time varying and frequency selective channel is assumed. SUI-5 channel model is taken as reference whose Power-Delay profile is depicted in Figure 5.5 [26].

Figure 5.5: Average power-delay profile for SUI-5 model.

Each multipath finger undergoes independent Rayleigh fading. Clark's model is used to generate the Rayleigh fading samples. In this model the concept of in-phase and quadrature-phase modulation paths is used. They are passed through a spectral filter defined by the following equation [16].

$$S_{Ez}(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{f - f_c}{f_m} \right)^2}} \quad (5.4)$$

where f_m is the maximum Doppler frequency and f_c is the carrier frequency. Processing is done in frequency domain and to get the time domain correlated fading samples an

Inverse Fast Fourier Transform (IFFT) is used. A complete block diagram and procedure to implement this fading model is described in [16].

Figure 5.6 illustrates the simulated normalized Rayleigh fading envelope. Figure 5.7 shows the Doppler power spectrum, and in figure 5.8, the histogram of the resulted fading samples is shown. Finally, in figure 5.9, the Cumulative Distribution Functions (CDF) obtained from simulated data and the theoretical one are compared.

Figure 5.6: Simulated Rayleigh fading envelope (using Clarke's model)

Figure 5.7: Doppler power spectrum

Figure 5.8: Histogram of the Rayleigh fading samples

Figure 5.9: CDF comparison for Rayleigh envelope between simulated & theoretical

5.4 Received Signal Structure

For an M -branch receiver antenna arrays system with $K-1$ co-channel interferers, the received signal after FFT processing from the m^{th} antenna element ($m=1,2,3,\dots,M$), at the k^{th} sub-carrier ($k=1,2,3,\dots,P$) of the n^{th} OFDM symbol when the desired signal and interferers are completely synchronized, can be written as:

$$X_m[n,k] = H_m^{(1)}[n,k]C_1[n,k] + U_m[n,k] \quad (5.5)$$

where,

$H_m^{(1)}[n,k]$ = frequency response for the desired user (user 1) from m^{th} antenna element.

$$U_m[n, k] = \sum_{l=2}^K H_m^{(l)}[n, k] C_l[n, k] + N_m[n, k] \quad (5.6)$$

$N_m[n, k]$ is AWGN.

5.5 Channel Equalization

If the number of sub-channels is very large in an OFDM system, the channel transfer function can be regarded as nonselective (flat) within the bandwidth of individual sub-channels. But the problem associated with the OFDM system is that the useful signal at the receiver is recovered from the entire bandwidth and not only from some individual components. Therefore, as each sub-channel in OFDM undergoes different attenuation and phase rotation, equalization is still required.

Either zero forcing or MMSE criteria can be used to equalize the channel for each sub-carrier. As zero forcing has a noise enhancement problem it is better to use the efficient MMSE solution. Following are the two equations for the two equalization solutions;

The zero forcing complex coefficient for k^{th} sub-carrier is given as

$$W_m[n, k] = \frac{1}{H_m[n, k]} \quad (5.7)$$

while, the MMSE complex coefficient for k^{th} sub-carrier is given as [27],

$$W_m[n, k] = \frac{H_m^*[n, k]}{|H_m[n, k]|^2 + \frac{\sigma_n^2}{\sigma_d^2}} \quad (5.8)$$

where, σ_n^2 is the variance of the noise and σ_d^2 is the variance of the data symbol.

In our simulations, we will be using MMSE channel equalization as it is more efficient.

5.6 Beam Forming using Adaptive Antenna Array

SMI algorithm would be used to calculate the parameters required for the optimum combining [24]. Let $q_{ij}[n, k]$ be the instantaneous correlation of the received

signals from the i^{th} and j^{th} antenna elements corresponding to the same block and sub-carrier, which can be expressed as:

$$q_{ij}[n,k] = E\{X_i[n,k]X_j^*[n,k]\} = \sum_{l=1}^K H_i^{(l)}[n,k]H_j^{(l)*}[n,k] + \sigma_n^2 \delta(i-j) \quad (5.9)$$

By using SMI the MMSE equalization coefficients can be calculated as:

$$\mathbf{W}[n,k] = \mathbf{R}[n,k]^{-1} \mathbf{H}^{(1)*}[n,k] \quad (5.10)$$

where,

$\mathbf{I} = M \times M$ identity matrix

and $\mathbf{R}[n,k]$ is defined below as:

$$\mathbf{R}[n,k] = \begin{pmatrix} q_{11}[n,k] & q_{12}[n,k] & \dots & q_{1M}[n,k] \\ q_{21}[n,k] & q_{22}[n,k] & \dots & q_{2M}[n,k] \\ \dots & \dots & \dots & \dots \\ q_{M1}[n,k] & q_{M2}[n,k] & \dots & q_{MM}[n,k] \end{pmatrix} \quad (5.11)$$

And:

$\mathbf{H}^{(1)}[n,k]$ is a $M \times 1$ steering vector and defined as

$$\mathbf{H}^{(1)}[n,k] = (\mathbf{H}_1^{(1)}[n,k] \ \mathbf{H}_2^{(1)}[n,k] \ \dots \ \mathbf{H}_M^{(1)}[n,k])^T \quad (5.12)$$

where the $\mathbf{H}[n,k]$ can be written as,

$$\mathbf{H}[n,k] = [\mathbf{H}^{(1)}[n,k] \ \mathbf{H}^{(2)}[n,k] \ \mathbf{H}^{(3)}[n,k] \ \mathbf{H}^{(4)}[n,k] \ \dots \ \mathbf{H}^{(K)}[n,k]]$$

The number of rows of $\mathbf{H}$ is equal to the number of antenna elements and the number of columns is equal to the total number of co-channel users present in the system.

With the weighing factors, the desired signal at the output can be found as:

$$\mathbf{Y}[n,k] = \mathbf{W}^H[n,k] \mathbf{X}[n,k] \quad (5.13)$$

where, $\mathbf{X}[n,k]$ is the received signal vector at the k^{th} sub-carrier and for the n^{th} OFDM symbol, given as:

$$\mathbf{X}[n,k] = (X_1[n,k] \ X_2[n,k] \ \dots \ X_M[n,k])^T$$

Chapter 6

Frequency Selective Fading Simulation Scenario and Results

6.1 Channel Estimation using Different Training Schemes

In OFDM system, the channel's transfer function or frequency response has to be estimated to be used for equalization purposes as well as for the determination of antenna array coefficients. To estimate the channel several OFDM based standards, like Digital Video Broadcast Terrestrial (DVB-T) and Integrated Services Digital Broadcasting Terrestrial (ISDB-T), use known training pilots inserted in the data sub-carriers [28]. As the receiver already knows the data modulated on these training sub-carriers, the channel's transfer function on these sub-carriers can be easily estimated. Then depending on the pilot locations, interpolation is used to estimate the channel response for the rest of the sub-carriers. The number of pilot carriers depends on the training overhead a system can afford.

In our simulation scenario, all the sub-carriers constitute one OFDM symbol and a group of five OFDM symbols makes an OFDM block. With the consideration of OFDM signal as an OFDM block, channel estimation and pilot position can be viewed as one of the following two scenarios [21].

6.1.1 One-Dimensional Channel Estimation

Considering a slow changing channel, it is possible to do only an estimation in the frequency domain for all symbols containing pilots either by using Pre-amble (all symbols in the first OFDM symbol are known) or with estimation through some interpolation technique.

6.1.2 Two-Dimensional Channel Estimation

As the mobile channels are fading in both time and frequency, an estimation technique should be able to estimate the time varying amplitudes and phases of all sub-

carriers. In a two-dimensional estimator, known pilots are used and the channel is estimated by performing two-dimensional interpolation. This is achieved by using estimates in both time and frequency using pilots shown in case 3 of Figure 6.1. The pilot spacing should fulfill the Nyquist sampling rate.

In OFDM, we will have to take care about minimum sub-carrier spacing and symbol spacing between the pilots. By reducing the spacing, better channel estimation with lesser complexity can be achieved. But on the other hand, increasing the number of pilots will degrade the capacity and SNR available for the data symbols. Therefore, the pilot density is a compromise between SNR loss and estimator's performance.

6.1.3 Use of Training Symbols

Two-dimensional channel estimators are well suited for the channels that vary slowly in both time and frequency. Applications with continuous transmission such as Digital Audio Broadcasting (DAB) or Digital Video Broadcasting (DVB) are typical examples. 2-D estimators are not good for the applications with packet data transmission like wireless LAN. There are two basic reasons [21].

1. The packet length is short enough that the channel could be assumed as constant during the length of the packet. Therefore, there is no need for the estimator to take care about fading in time.
2. Pilots are scattered over several OFDM data symbols, therefore there is a considerable delay for the first channel estimate calculations. In TCP packet transmission, any delay in the reception of a packet will also delay the acknowledgment resulting in degradation in overall system's throughput.

Use of one or more known OFDM training symbols can solve these problems in packet transmission systems. With the use of a training sequence, the packet starts with an OFDM symbols for which all data values are known. These training symbols can be used for both channel estimates as well as a frequency offset estimate. A few pilots can be followed by these training symbols for tracking purposes after the initial training. This

kind of proposal is given in IEEE 802.11a OFDM standard, with two training symbols at the beginning for channel estimation with two pilot sub-carriers used for the frequency synchronization.

6.1.4 Choice of Pilot Locations

From the above discussion of various channel estimation methods, one can conclude that the choice of the channel estimation technique depends on the channel environment, the efficiency and the performance required from the system. The following diagram summarizes the possible placement of the pilot symbols in an OFDM block.

Figure 6.1: Pilots (Black) scenarios embedded into info sub-carriers (case1-3)

The first scheme shown uses scattered pilot carriers with 1-D interpolation in only frequency axis. 1-D interpolation will be used for estimation of the channel response. This technique is good for fast fading conditions but its interpolated frequency response may result in degradation of performance. The second one uses known pilot symbols on all the sub-carriers in the first OFDM symbol. With this scheme, estimation based on Preamble is done and in the rest of the OFDM block, estimates for each sub-carrier are based on the initial estimate. This scheme is already proposed in 802.11a WLAN standard. This scheme is good for slow time varying systems but having very low frequency correlation between the adjacent sub-channels. Third scheme uses 2-D interpolation in both time and frequency axes. This scheme is recently proposed in 802.16 standard along with boosting the power levels at the pilot locations by 3dB relative to the power at data carriers [30].

Case 3 will give a much more efficient interpolation in frequency axis than case 1, since $H_m^{(l)}[n,k]$ varies much more slowly with time n than with frequency k .

In our simulations, we have used the first two techniques to estimate the channel frequency response.

6.1.5 Channel Estimation using Time Domain Interpolation Technique

First, we will consider the estimation of the channel for the case when the pilot sub-carriers are inserted uniformly in between the info data sub-carriers as shown in case 1 of Figure 6.1. There are several interpolation techniques that can be used to interpolate the channel transfer function. An efficient interpolation technique is used in order to estimate the channel at data sub-carriers by using the channel information at the pilot sub-carriers. Let N_p denote the number of pilot sub-carriers, which are inserted among P total sub-carriers. We define $\hat{H}_m^{(1)}[n, p_i]$ as the frequency response for the i^{th} pilot placed at sub-carrier p_i of user 1, such that:

$$\hat{H}_m^{(1)}[n, p_i] = \left(\frac{X[n, p_i]}{C_1[n, p_i]} \right) \quad (6.1)$$

After obtaining the estimated channel by using the transmitted pilot tones, we convert it into the time domain by using IFFT. Then the resulting sequence is padded with zeros to length P . The estimate of the channel at all frequencies is obtained by taking the FFT. This interpolated estimated frequency response could be used to find the equalizer's coefficients. This kind of interpolation is a form of time-domain interpolation.

One example of interpolated channel frequency response with only four tones out of 64 sub-carriers being used as pilots with two antenna elements is shown in Figure 6.2 (using case 1 of Figure 6.1).

Figure 6.2: Interpolated channel frequency response (using case1)

We can say that for highly time dispersive channels, estimation using only few scattered pilots will not be enough. Therefore, a training scheme shown as case 2 of Figure 6.1 is used which contains a Pre-amble, having known data on all sub-carriers in the first symbol. Moreover, results show that without adapting the modulation schemes at the transmitter according to the channel condition, one cannot get acceptable performance in terms of bit errors due to deep fades in the channel frequency response.

6.2 Simulation Scenario

With Pre-amble pilot sub-carrier scheme (case 2), an OFDM system is simulated to observe the BER and throughput performance with both partially and fully adaptive modulation techniques.

A total of 64 sub-carriers per OFDM symbol with BPSK modulation scheme are used. An OFDM block is considered to have 5 OFDM symbols. The average received SNR ranges from 1 dB to 45 dB. Lognormal shadowing with standard deviation of 10dB and a maximum Doppler frequency of 100 Hz are assumed. In case of co-channel interferer equal average received power is assumed. As discussed in chapter 5 section 5.3, SUI-5 channel model is used to simulate the channel. Each SUI-5 multipath component undergoes independent Rayleigh fading.

The performance of the uncoded OFDM is dominated by the lowest SNR value in the signal bandwidth. As in OFDM system the total bandwidth is divided into several narrowband sub-channels, it allows the system designer to obtain some flexibility in selecting different constellation schemes for each and every sub-carrier. Therefore, we can modulate each sub-carrier according to some criterion like received SNR which depends on the channel response at each sub-carrier. This leads us to use an adaptive modulation scheme in OFDM systems. Each modulation scheme provides a trade off between spectral efficiency and the bit error rate. With a certain acceptable BER (P_e target) higher order modulation can be used to increase the spectral efficiency.

For throughput comparison two schemes named “partially adaptive” and “fully adaptive” are simulated. In partially adaptive scheme, either BPSK modulation or “no transmission” is used. In fully adaptive modulation scheme, different higher-level modulation schemes are used whose selection depends on observed SINR on each sub-channel. Throughput performance with partially adaptive and fully adaptive modulated OFDM system is simulated using a specific target BER. Fully adaptive OFDM simulation is based on output SINR grid that is obtained by using uncoded AWGN waterfall curves [29], assuming that the interference has approximately the statistical properties of Gaussian noise.

In our adaptive modulation scheme, we will try to improve the throughput of the system while keeping a specific BER threshold. The implementation of such scheme depends on the channel environment and the feedback information. Ideal feedback connection (no delay or error) is assumed. An SINR grid is formulated according to the AWGN waterfall curves [29] for BER performance with a threshold of 10^{-5} . Grid used is given in Table 6.1.

SINR level (dB)	Modulation scheme
$\text{SINR} \leq 10$	No transmission
$13 > \text{SINR} > 10$	BPSK
$13 > \text{SINR} > 10$	QPSK
$17 > \text{SINR} > 13$	8QAM
$21 > \text{SINR} > 17$	16QAM
$24 > \text{SINR} > 21$	32QAM
$\text{SINR} > 24$	64QAM

Table 6.1 SINR grid (for target $\text{BER} = 10^{-5}$) [29]

No clustering or grouping of the sub-channels is done, therefore, the SINR on each and every sub-carrier is observed individually, and hence the signal constellation on each sub-channel is decided on the basis of the observed SINR in that sub-channel. As shown in Table 6.1, in case of deep nulls in the channel's frequency response (very low output SINR), 'no transmission' choice is selected while in case of good conditions a higher-level modulation scheme is chosen. In this way, bit rate is optimized on each sub-carrier link and improvement in the resultant throughput is expected.

All simulation is carried out with different antenna elements (2, 4 and 6). BER is obtained by averaging over 500 OFDM blocks.

6.3 BER Performance

6.3.1 No Co-channel Interferer

In Figure 6.3, BER with no co-channel interferer with fixed BPSK modulation is simulated. In the absence of any co-channel interferer the antenna elements are only providing diversity gain in the system. For higher SNRs, more than 6-7dB improvement can be observed in the system with 4 antenna elements compared with 2 elements case.

Figure 6.3: BER comparison with OFDM system (no co-channel Interferer)

6.3.2 With Co-channel Interferer

A wideband co-channel interferer with similar OFDM structure as that of the desired user is introduced in the system. This co-channel interferer may appear in the system in three ways, either exactly synchronous to the desired user, block asynchronous or fully asynchronous. These three cases are depicted in Figure 6.4.

Figure 6.4: Synchronous, block asynchronous and fully asynchronous interferer.

Only synchronous and block asynchronous interferers will be considered in this thesis. Degradation in performance, with true asynchronous case is expected, because with true asynchronous co-channel interferer, there would be a loss of orthogonality between the sub-carriers causing severe ICI. Moreover, the receiver structure presented in chapter 5, section 5.6 is only valid for the perfectly synchronized co-channel interferers.

Figures 6.5 to 6.7 compare the BER of OFDM with and without co-channel interference. With synchronous case a very small degradation in BER is observed but with block asynchronous, a huge degradation can be observed, which is an expected result. Antenna diversity is mitigating the fading effects and with higher number of antenna elements, fading may completely be removed. Moreover, better BER performance at higher SNRs may be caused because of shadowing effects on the interferers.

Figure 6.5: BER comparison with OFDM system (co-channel Interferer, M=2)

Figure 6.6: BER comparison with OFDM system (co-channel Interferer, M=4)

Figure 6.7: BER comparison with OFDM system (co-channel Interferer, M=6)

6.4 Throughput Comparison of Partially Adaptive and Fully Adaptive OFDM

6.4.1 No Co-channel Interferer

Figure 6.8 shows the results for the throughput of partially adaptive OFDM system. In this scenario a target BER of 10^{-5} is set and according to the water fall AWGN curve, if the SNR level is less than 10 dB, no transmission is done while any level of SNR greater than 10 dB results in BPSK transmission. There is no co-channel interferer in the system. At higher SNRs like case of 30 dB and with 6 elements we can achieve maximum throughput from the system. Similarly, with fewer antenna elements the saturation points come at higher values of SNRs, which is an expected result.

Figure 6.9 shows the throughput performance with fully adaptive modulation scheme and no co-channel interferer. In the fully adaptive OFDM scheme, we have used the SNR grid shown in Table 6.1. We can observe that with fully adaptive OFDM, we can achieve very

good throughput with moderate SNR compared to the partially adaptive OFDM. Here we can see the effect of utilization of good channel conditions.

Figure 6.8: Partially adaptive OFDM throughput comparison (no co-channel Interferer)

Figure 6.9: Fully adaptive OFDM throughput comparison (no co-channel Interferer)

6.4.2 With Co-channel Interferer

Figures 6.10-6.15 show the effect of co-channel interferer along with the outcome of antenna diversity. The effect of a co-channel interferer, on the partially and fully adaptive OFDM is observed in this section. With a block asynchronous interferer worse performance could be expected from the BER curves as antenna arrays could not do much against block asynchronous interferer.

Figure 6.10: Partially adaptive OFDM throughput comparison (M=2)

Figure 6.11: Fully adaptive OFDM throughput comparison (M=2)

Figure 6.12: Partially adaptive OFDM throughput comparison (M=4)

Figure 6.13: Fully adaptive OFDM throughput comparison (M=4)

Figure 6.14: Partially adaptive OFDM throughput comparison (M=6)

Figure 6.15: Fully adaptive OFDM throughput comparison (M=6)

Chapter 7

Conclusion and Future Work

7.1 Conclusion

A new antenna adaptation scheme named “Distributed-training scheme” is introduced. Different iterative based and Pre-amble or Post-amble training schemes are compared with the proposed scheme. These comparisons are made under the intermittent interference traffic environment, which is expected in an uncontrolled environment like Unlicensed Band. Combination of partially adaptive OFDM and fully adaptive OFDM with adaptive antenna array processing is simulated. Performance improvement in terms of diversity gain and co-channel interference suppression is observed. Based on our simulation results, the following is observed and concluded:

- 1) For the same number of training symbols used, the Pre- or Post-amble alone training scheme shows inferior performance to other training schemes. Meanwhile, apart from the optimum combining and especially with lower interference traffic Erlangs, the Distributed-training scheme offers far superior performance to other training schemes. This superiority is due to the fact that the Distributed-training scheme is able to constantly track the interference, while the other training schemes are less able to track the intermittent interference. The reason behind this is that to null out the part time interferers by the arrays, the training symbols (at least some of them) must lie in the region of the block’s affected portion.
- 2) With the Distributed-training scheme, as long as the number of antenna elements is 2-3 times the number of Erlangs of interference traffic, a BER of 10^{-3} can be obtained.
- 3) The advantage gained from having a larger number of antenna elements is quite noticeable at small distance separations between the desired user and the receiver. But as this distance separation increases, this advantage disappears as the BER curves for the different number of antenna elements approach one another. This may be due to the fact that the system is becoming interference dominant.

- 4) Acceptable performance can be achieved with larger block sizes, under intermittent interference traffic, with Distributed-training scheme; while it is not acceptable with conventional Pre-amble or Post-amble training schemes.
- 5) Under Distributed-training scheme, when the number of Erlangs of interference traffic is well below the number of antenna elements (1.3 Erlangs for 4 elements and 3.8 Erlangs for 10 elements), no outage is observed for the given MMSE threshold value; meanwhile, above these set values the outage probability starts to increase sharply as the interference traffic Erlangs increases.
- 6) A lower bound is calculated and validity of the simulated curves is observed from the figures showing comparison between the simulated and lower bound curves.
- 7) Different pilot based training schemes are used in an OFDM system for estimation purposes. As the bit errors in an OFDM system are generally concentrated in some severely faded sub-carriers adaptive OFDM is considered. Fully adaptive OFDM shows acceptable throughput performance.
- 8) With OFDM system antenna diversity gives the expected improvements.

7.2 Future Work

For future work, some further issues involving flat fading environment with Distributed-training scheme, and frequency selective fading with OFDM could be investigated. Some of them can be described as follows;

1. With flat fading, combination of Distributed-training scheme and adaptive array processing gives very promising results. But in case of severe co-channel interference or in the case of different parties of perhaps different architectures, or different traffic patterns co-existing while using this same unlicensed band carrier frequency, it is necessary to devise some higher order protocols to control the situation.
2. The traffic model used in this work is a simple one and may not be able to cover some of the actual traffic situations in a real unlicensed environment. Therefore, some simulations should be done with some different traffic models like 4IPP traffic model.

3. In OFDM case, channel estimation with 2-D interpolation could be one of the major future step in continuation to this work to cover the time variation of the channel as well.
4. Different other bit loading algorithms must be tried to obtain better throughput results. For that purpose, the channel response could be observed and grouping or clustering of adjacent sub-carrier techniques could be used.
5. OFDM system with interleaving and coding should be simulated with the same channel conditions to have a throughput comparison with the adaptive modulation scheme.
6. A fully asynchronous co-channel interferer in OFDM could be simulated. Worst performance compared to the synchronous and block asynchronous cases would be expected.
7. More co-channel interferers could be simulated with the OFDM system. And on the top, they could follow same batch Poisson or any other kind of traffic pattern.

APPENDIX

A.1 Analysis of SMI Algorithm

In this appendix, the following two relationships are reviewed analytically based on the material in [3] and [4];

- Relation between the number of training symbols and misadjustment (weight error vector)
- Relation between the output SINR and the number of elements/number of training symbols.

A.1.1 Error Caused by Estimation

The received desired symbol can be written as [3] and [4],

$$s_1(i) = \mathbf{w}_{\text{opt}}^H \mathbf{x}(i) + e(i) \quad (\text{A1})$$

where, $e(i)$ is the error caused by the estimation. This error is assumed to be white with zero mean and variance $\sigma_e^2 = 1 - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1$, where $\mathbf{h}_1$ denotes the desired user's steering vector.

As the estimated correlation matrix $\hat{\mathbf{R}}$ is independent of the training symbols and cross-correlation vector $\hat{\mathbf{h}}_1$ only depends on the transmitted training symbols see (2.10), (A1) can be written as,

$$\begin{aligned} \hat{\mathbf{h}}_1 &= \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) [\mathbf{w}_{\text{opt}}^H \mathbf{x}(i) + e(i)]^* \\ &= \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) \mathbf{x}^H(i) \mathbf{w}_{\text{opt}} + \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) e^*(i) \\ &= \hat{\mathbf{R}} \mathbf{w}_{\text{opt}} + \frac{1}{N} \sum_{i=1}^N \mathbf{x}(i) e^*(i) \end{aligned} \quad (\text{A2})$$

the estimated weight vector is,

$$\hat{\mathbf{w}} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{h}}_1 = \hat{\mathbf{R}}^{-1} \hat{\mathbf{R}} \mathbf{w}_{\text{opt}} + \frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) e^*(i)$$

$$\hat{\mathbf{w}} = \mathbf{w}_{\text{opt}} + \frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) e^*(i) \quad (\text{A3})$$

The weight error vector ϕ can be written as,

$$\phi = \hat{\mathbf{w}} - \mathbf{w}_{\text{opt}} = \frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) e^*(i) \quad (\text{A4})$$

We want to find relation between the weight error vector ϕ (causing excess MSE) with the number of training symbols (N). For this we define correlation matrix of the weight error vector as,

$$\mathbf{Z} = E[\phi \phi^H]$$

Using the expectation property of random variables,

$$E[x] = E[E\{x | y\}]$$

$$E[\phi] = E\left[\frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) e^*(i)\right] = E\left[E\left\{\frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) e^*(i) \mid \mathbf{x}(i), i = 1, 2, 3, \dots, N\right\}\right] = 0$$

since $e(i)$ is assumed to be zero mean.

The expectation of the sample correlation matrix is

$$\mathbf{Z} = E\left[\frac{1}{N^2} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \sum_{j=1}^N \mathbf{x}(i) E\{e^*(i) e(j)\} \mathbf{x}^H(j) \hat{\mathbf{R}}^{-1}\right]$$

As $e(i)$ is assumed to be a white process

$$E[e^*(i) e(j)] = \begin{cases} 1 - \mathbf{h}_1^H \hat{\mathbf{R}}^{-1} \mathbf{h}_1, & i = j \\ 0, & i \neq j \end{cases}, \text{ and therefore}$$

$$\mathbf{Z} = E\left[\frac{1}{N^2} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) \mathbf{x}^H(i) \hat{\mathbf{R}}^{-1}\right] (1 - \mathbf{h}_1^H \hat{\mathbf{R}}^{-1} \mathbf{h}_1)$$

or,

$$\mathbf{Z} = \frac{(1 - \mathbf{h}_1^H \hat{\mathbf{R}}^{-1} \mathbf{h}_1)}{N} E\left[\frac{1}{N} \hat{\mathbf{R}}^{-1} \sum_{i=1}^N \mathbf{x}(i) \mathbf{x}^H(i) \hat{\mathbf{R}}^{-1}\right]$$

and from (2.9)

$$\mathbf{Z} = \frac{(1 - \mathbf{h}_1^H \hat{\mathbf{R}}^{-1} \mathbf{h}_1)}{N} E[\hat{\mathbf{R}}^{-1}] \quad (\text{A5})$$

Using the independence assumption as in [4],

1. $\mathbf{x}(i)$ is iid
2. $\mathbf{e}(i)$ is statistically independent of $\mathbf{x}(i)$.

and using the Appendix J in [4], the $\hat{\mathbf{R}}$ can be described by a complex Wishart distribution.

The expectation of $\hat{\mathbf{R}}^{-1}$ can be written as

$$E[\hat{\mathbf{R}}^{-1}] = \frac{N}{N-M-1} \mathbf{R}^{-1}$$

$$\therefore \mathbf{Z} = \frac{(1 - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1)}{N-M-1} \mathbf{R}^{-1} \quad (\text{A6})$$

With a higher number of training symbols, $\mathbf{Z}$ would decrease linearly and hence the excess mean square error due to the error weight vector will reduce resulting in a reduction of the overall MSE, as we know that

$$\text{MSE} = \text{MMSE (optimum)} + \text{Excess MSE (due to estimation error)}$$

A.1.2 Output SINR Vs - Number of Training Symbols & Antenna Elements

Expressing the correlation matrix in a split form with $\mathbf{R}$ showing the correlation matrix of all the users while $\mathbf{R}_{\text{int}}$ represents the correlation matrix for the interferers and noise only;

$$\mathbf{R} = \sum_{k=1}^K \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I} \quad (\text{A7})$$

$$\mathbf{R}_{\text{int}} = \sum_{k=2}^K \mathbf{h}_k \mathbf{h}_k^H + \sigma_n^2 \mathbf{I} \quad (\text{A8})$$

where K is the total number of co-channel users in the system, it can be shown [4],

$$\mathbf{R}^{-1} = \psi \mathbf{R}_{\text{int}}^{-1} \text{ where, } \psi = \frac{1}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)}$$

The desired user's signal power at the array output with estimated weights is

$$S = E[|\hat{\mathbf{w}}^H \mathbf{h}_1|^2]$$

The error weight vector is given as,

$$\boldsymbol{\varphi} = \hat{\mathbf{w}} - \mathbf{w}_{\text{opt}}$$

$$\hat{\mathbf{w}} = \boldsymbol{\phi} + \mathbf{w}_{\text{opt}}$$

$$\therefore S = E[|(\mathbf{w}_{\text{opt}} + \boldsymbol{\phi})^H \mathbf{h}_1|^2] = \mathbf{w}_{\text{opt}}^H \mathbf{h}_1 \mathbf{h}_1^H \mathbf{w}_{\text{opt}} + E[\boldsymbol{\phi}^H \mathbf{h}_1 \mathbf{h}_1^H \boldsymbol{\phi}] + 2\text{Re}\{\mathbf{w}_{\text{opt}}^H \mathbf{h}_1 \mathbf{h}_1^H E[\boldsymbol{\phi}]\}$$

The received signal output power can be split into two,

$$S = S_{\text{opt}} + S_{\text{err}}$$

where, S_{opt} is the received power in optimum conditions and the S_{err} is the excess power because of excess error,

$$S_{\text{opt}} = \mathbf{w}_{\text{opt}}^H \mathbf{h}_1 \mathbf{h}_1^H \mathbf{w}_{\text{opt}} = (\mathbf{w}_{\text{opt}}^H \mathbf{h}_1)^2 = (\mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1)^2 = (\mathbf{h}_1^H \psi \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)^2 = \psi^2 (\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)^2$$

$$S_{\text{opt}} = \frac{(\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)^2}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)^2}$$

$$S_{\text{err}} = E[\boldsymbol{\phi}^H \mathbf{h}_1 \mathbf{h}_1^H \boldsymbol{\phi}] = E[\text{tr}\{\boldsymbol{\phi}^H \mathbf{h}_1 \mathbf{h}_1^H \boldsymbol{\phi}\}]$$

The total variance of a square matrix is equal to the sum of all diagonal elements.

$$S_{\text{err}} = \text{tr}\{\mathbf{h}_1^H E[\boldsymbol{\phi} \boldsymbol{\phi}^H] \mathbf{h}_1\} = \text{tr}\{\mathbf{h}_1^H \mathbf{Z} \mathbf{h}_1\}$$

$$= \frac{(1 - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1)}{(N - M - 1)} \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1$$

$$\text{As } \psi = \frac{1}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)}$$

$$\mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1 = \frac{(\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)}$$

and

$$1 - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1 = 1 - \frac{(\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)} = \frac{1}{1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1}$$

putting above,

$$S_{\text{err}} = \frac{1}{(N - M - 1)} \frac{\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1}{(1 + \mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)^2} \quad (\text{A9})$$

The ratio between them would be given as,

$$\frac{S_{\text{err}}}{S_{\text{opt}}} = \frac{1}{(N - M - 1) (\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1)} \quad (\text{A10})$$

Similarly the noise plus interference power at the output of the combiner would be,

$$I = E[(\mathbf{w}_{opt}^H \mathbf{R}_{int} \mathbf{w}_{opt})] = E[(\mathbf{w}_{opt} + \boldsymbol{\varphi})^H \mathbf{R}_{int} (\mathbf{w}_{opt} + \boldsymbol{\varphi})]$$

The noise plus interference output power can also be split into two,

$$I_{opt} = \mathbf{w}_{opt}^H \mathbf{R}_{int} \mathbf{w}_{opt} = (\boldsymbol{\psi} \mathbf{R}_{int}^{-1} \mathbf{h}_1)^H \mathbf{R}_{int} (\boldsymbol{\psi} \mathbf{R}_{int}^{-1} \mathbf{h}_1) = \boldsymbol{\psi}^2 \mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1 = \frac{\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1}{(1 + \mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)^2}$$

$$\begin{aligned} I_{err} &= E[\boldsymbol{\varphi}^H \mathbf{R}_{int} \boldsymbol{\varphi}] = E[\text{tr}\{\boldsymbol{\varphi}^H \mathbf{R}_{int} \boldsymbol{\varphi}\}] = \text{tr}\{E[\boldsymbol{\varphi} \boldsymbol{\varphi}^H] \mathbf{R}_{int}\} = \text{tr}\{E[\boldsymbol{\varphi} \boldsymbol{\varphi}^H] (\mathbf{R} - \mathbf{h}_1 \mathbf{h}_1^H)\} \\ &= \frac{(1 - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1)}{N - M - 1} (M - \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1) \\ &= \frac{M + (M-1)(\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)}{(N - M - 1)(1 + \mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)^2} \end{aligned} \quad (\text{A11})$$

The ratio would be,

$$\frac{I_{err}}{I_{opt}} = \frac{M + (M-1)(\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)}{(N - M - 1)(\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)} \quad (\text{A12})$$

Now the Signal to Noise plus Interference ratio at the output of the array combiner,

$$\text{Output SINR} = \frac{S_{opt} + S_{err}}{I_{opt} + I_{err}} = \frac{S_{opt}}{I_{opt}} \left(\frac{1 + S_{err}/S_{opt}}{1 + I_{err}/I_{opt}} \right)$$

$$\text{Optimum SINR} = \frac{S_{opt}}{I_{opt}} = \mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1$$

Now we define the Normalized SINR as,

$$\text{Normalized SINR} = \left(\frac{1 + S_{err}/S_{opt}}{1 + I_{err}/I_{opt}} \right) = 1 - \frac{(M-1)(1 + \mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1)}{M + (N-2)\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1} \quad (\text{A13})$$

the 10log of normalized SINR is actually the difference between the actual output SINR and the optimum SINR in dB units, therefore to achieve Δ dB difference in both SINRs, we can formulate an equation as [3],

$$\Delta dB = 10 \log \left(\frac{M + (N-2)\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1}{M + (N-2)\mathbf{h}_1^H \mathbf{R}_{int}^{-1} \mathbf{h}_1} \right)$$

hence the required number of training symbols with specific number of antenna elements can be,

$$N = \frac{(M-1)}{(1-\Delta)} \frac{\Delta(M-1)}{(1-\Delta)\mathbf{h}_1^H \mathbf{R}_{\text{int}}^{-1} \mathbf{h}_1} + 2$$

By putting different values, we get that to achieve an output SINR within say 3 dB of the optimum performance, the N should be at least two times the number of elements M. (approximating the second term in the product factor by 1 due to the assumption that the optimum SINR is much higher than unity).

$$N = \frac{(M-1)}{(1-\Delta)} + 2$$

$$-10 \log \Delta = 3dB$$

$$N = \frac{(M-1)}{(1-0.5)} + 2 = 2(M-1) + 2 = 2M$$

Therefore, $N \geq 2M$ is the condition to achieve the 3dB performance. This result indicates to use more and more training symbols with the increase of number of antenna elements to achieve a typical output performance. There is actually a tradeoff between efficiency and the performance you require.

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