1

CANADIAN THESES ON MICROFICHE

I.S.B.N.

THESES CANADIENNES SUR MICROFICHE

National Library of Canada
Collections Development Branch

Canadian Theses on
Microfiche Service

Ottawa, Canada
K1A 0N4

Bibliothèque nationale du Canada
Direction du développement des collections

Service des thèses canadiennes
sur microfiche

NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us a poor photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

THIS DISSERTATION
HAS BEEN MICROFILMED
EXACTLY AS RECEIVED

AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de mauvaise qualité.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30. Veuillez prendre connaissance des formules d'autorisation qui accompagnent cette thèse.

LA THÈSE A ÉTÉ
MICROFILMÉE TELLE QUE
NOUS L'AVONS REÇUE

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Division

Division des thèses canadiennes

Ottawa, Canada
K1A 0N4

60009

128

0-315-12096-7

PERMISSION TO MICROFILM — AUTORISATION DE MICROFILMER

• Please print or type — Écrire en lettres moulées ou dactylographier

Full Name of Author — Nom complet de l'auteur

WILLIAM JOSEPH KELLY

Date of Birth — Date de naissance

12 MARCH 1950

Country of Birth — Lieu de naissance

CANADA

Permanent Address — Résidence fixe

35 SEYMOUR AVE
NEPEAN ONT
K2E 6P7

Title of Thesis — Titre de la thèse

AN INVESTIGATION OF HOT-WIRE ANEMOMETER TECHNIQUES
IN A TURBULENT HIGHLY SHEARED FLOW

University — Université

CARLETON

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

M. ENG.

Year this degree conferred — Année d'obtention de ce grade

1982-83

Name of Supervisor — Nom du directeur de thèse

DR. R. J. KIND

Permission is hereby granted to the NATIONAL LIBRARY OF CANADA to microfilm this thesis and to lend or sell copies of the film

The author reserves other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without the author's written permission

L'autorisation est, par la présente, accordée à la BIBLIOTHÈQUE NATIONALE DU CANADA de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film

L'auteur se réserve les autres droits de publication, ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans l'autorisation écrite de l'auteur

Date

20 Dec 1982

Signature

W. J. Kelly

AN INVESTIGATION OF HOT-WIRE ANEMOMETER TECHNIQUES
IN A TURBULENT HIGHLY SHEARED FLOW

by

William Joseph Kelly, B.Eng.

A Thesis

Presented To The

Faculty Of Graduate Studies And Research

In Partial Fulfillment

Of The Requirements For The Degree

MASTER OF ENGINEERING

Department of Mechanical and Aeronautical Engineering

Faculty of Engineering

Carleton University

Ottawa, Canada

December 1982

The undersigned recommend to the Faculty of Graduate Studies and Research, acceptance of the thesis

"An Investigation of Hot-Wire Anemometer Techniques In
A Turbulent Highly Sheared Flow"

by William Joseph Kelly, B.Eng., in partial fulfillment of the requirements for the degree of Master of Engineering.

Thesis Supervisor

Chairman

Department of Mechanical and Aeronautical Engineering
Faculty of Engineering

December 1982

ABSTRACT

A new method for hot-wire anemometer data reduction is tested experimentally on a wall jet apparatus to determine its suitability for use in investigating a boundary layer control flow. The equations are modified from their original form for analysis of boundary layer flows. Modifications to the boundary layer control apparatus are performed. Experimental results agree well with those of other investigations.

ACKNOWLEDGEMENTS

I wish to express my deepest gratitude to my thesis supervisor, Professor R. J. Kind for his enlightening suggestions, comments and criticisms. His willingness to make himself available for consultation was most appreciated. Financial support during this period has been generously arranged by the Chairman of the Department of Mechanical and Aeronautical Engineering, Professor H. R. Saravanamuttoo, through the Faculty of Graduate Studies and Research.

I would also like to thank Mr. F. M. Yowakim for his advice and for allowing me to use his calibration routine in which he invested a great deal of time and effort.

Thanks also to the staff of the Engineering Technical Centre for their help in simplifying designs and for their expertise in building apparatus.

TABLE OF CONTENTS

CHAPTER 1	INTRODUCTION	1
1.1	Introduction	1
1.2	Outline of Current Study	2
CHAPTER 2	HOT-WIRE ANEMOMETRY	5
2.1	Introduction	5
2.2	Theory of the Hot-Wire Anemometry	7
2.3	Brief Review of Hot-Wire Signal Interpretation Methods	9
2.4	Outline of the Current Study	11
CHAPTER 3	PRESENTATION OF THE NEW METHOD	13
3.1	Introduction	13
3.2	Derivation of the Hot-Wire Equations for Three-Dimensional Flow	13
3.3	Modifications to the New Method	25
3.3.1	3-D Equations Using Bridge Output	25
3.3.2	Re-Worked 3-D Equations	25
3.3.3	2-D Equations	27
CHAPTER 4	DEVELOPMENT OF THE EXPERIMENTAL APPARATUS	31
4.1	Introduction	31
4.2	Existing Equipment	31
4.3	Instrumentation	33
4.4	Design of New Equipment	34
CHAPTER 5	EXPERIMENTAL RESULTS AND DISCUSSION	35
5.1	Introduction	35
5.2	Calibration Procedure	36
5.3	Determination of the Sensitivity Coefficients	38
5.4	Preliminary Experiments	42
5.5	Verification of the Cooling Law and Other Experimental Checks	44
5.5.1	Linearizer Check	44
5.5.2	Verification of the Cooling Law	44
5.5.3	Temperature Effects	46
5.6	Implementation of the Data Acquisition Unit and Associated Results	47
5.7	Summary	53

CHAPTER 6	MODIFICATIONS TO THE BOUNDARY LAYER CONTROL APPARATUS	54
6.1	Introduction	54
6.2	Existing Apparatus and Design of the Forebody	54
6.3	Construction of the Forebody	57
CHAPTER 7	CONCLUSIONS	58
7.1	Conclusions	58
CHAPTER 8	FUTURE WORK	60
8.1	Future Work	60
REFERENCES		61
APPENDIX A	Hot-Wire and Flowchart	96
APPENDIX B	Hot-WireB and Flowchart	99
APPENDIX C	Hot-Wire1 and Flowchart	102
APPENDIX D	Hot-Wire2 and Flowchart	105
APPENDIX E	Hot-Wire3 and Flowchart	108
APPENDIX F	Hot-Wire4 and Flowchart	111
APPENDIX G	Data Acquisition Unit Program 1 and Flowchart	114
APPENDIX H	Data Acquisition Unit Program 2 and Flowchart	117
APPENDIX I	Output of Typical Experimental Run	121
APPENDIX J	Program for Forebody Design and Flowchart	124

LIST OF TABLES

Table 5.1	Comparison of Freestream Velocity Using Cooling Law and Pitot-Static Probe	64
Table 6.1	Forebody Coordinates and Pressure Coefficients	65

LIST OF FIGURES

Figure 1.1	Sketch of Boundary Layer Control Apparatus
Figure 2.1	Cylindrical Hot-Film Sensor and Support Prongs
Figure 2.2	Schematic of Constant Temperature Control Unit
Figure 3.1	Inertial and Wire-Oriented Coordinate System
Figure 3.2	The Hot-Wire Positions Selected
Figure 4.1	Wall-Jet Apparatus
Figure 4.2	Photograph of the Calibration Apparatus
Figure 4.3	Photograph of the Instrumentation
Figure 4.4	Schematic of Instrumentation
Figure 4.5	Photograph of the Flat Plate Apparatus
Figure 5.1	Typical Calibration Program Output
Figure 5.2	Wall Jet Notation
Figure 5.3	Wall Jet Mean Velocity Profile Using Normal Probe
Figure 5.4	Wall Jet Longitudinal Turbulence Profiles in the Outer Flow
Figure 5.5	Linearized Output
Figure 5.6	Geometry for Verification of the Cooling Law
Figure 5.7	Wall Jet Mean Velocity Profiles Using Slanted Probe (With Linearizer)
Figure 5.8	Wall Jet Mean Velocity Profiles Using Slanted Probe (Without Linearizer)
Figure 5.9	Wall Jet Shear Stress Profiles (With Linearizer)
Figure 5.10	Wall Jet Shear Stress Profiles (Without Linearizer)
Figure 5.11	Wall Jet Longitudinal Turbulence Profiles (With Linearizer)
Figure 5.12	Wall Jet Longitudinal Turbulence Profiles (Without Linearizer)
Figure 5.13	Wall Jet Longitudinal Turbulence Profiles
Figure 5.14	Wall Jet Growth
Figure 6.1	Final Forebody Profile Shape
Figure 6.2	Evolution of the Forebody Profile Shape
Figure 6.3	Photograph of Forebody

NOMENCLATURE

A	y intercept of calibration curve
B	slope of calibration curve
C_p	specific heat at constant pressure
d	wire diameter
e_{BR}	AC bridge voltage
e_{lin}	AC linear voltage
E_{BR}	DC bridge voltage
E_{lin}	DC linear voltage
E_o	value of bridge voltage at zero flow
h	pitch sensitivity coefficient of α probe
h_β	pitch sensitivity coefficient of β probe
H_f	total heat transfer rate from the wire
k	yaw sensitivity coefficient of α probe
k_β	yaw sensitivity coefficient of β probe
k_f	thermal conductivity of fluid
k_n	Knudsen number = mean free path/wire diameter
l	length of hot-wire
P_r	Prandtl number = $\mu C_p / k_f$
R_a	resistance of wire at room temperature
R_e	Reynolds number = $\rho U d / \mu$
R_w	resistance of the wire when heated
$\frac{R_w}{R_a}$	overheat ratio of wire = $1 + \alpha_c (T_w - T_a)$

S	slope of calibration curve
T_a	air temperature in the calibration jet
T_f	fluid temperature
T_{JET}	temperature of the wall jet
T_W, T_{WIRE}	wire temperature when heated
u, v, w	fluctuating velocity components
U, V, W	mean flow velocity components
U_B	sensor-based velocity binormal to wire
U_{eff}	effective cooling velocity
U_N	sensor-based velocity normal to wire
U_T	sensor-based velocity tangential to wire
U_∞	freestream velocity
$\overline{uv}, \overline{uw}, \overline{vw}$	Reynolds shear stress components
x, y, z	Cartesian coordinates

Greek Symbols

α	probe angle of slanted probe
α_c	temperature coefficient of resistance of wire
β	probe angle of normal probe
δ	probe pitch angle
ζ	sensor-based coordinate parallel to hot-wire
η	sensor-based coordinate binormal to hot-wire
θ	probe yaw angle
μ	fluid dynamic viscosity
ξ	sensor-based coordinate perpendicular to hot-wire
ψ	probe rotation angle

CHAPTER 1

INTRODUCTION

1.1 Introduction

Tangential injection is a method of controlling boundary layer separation. Fluid injected tangentially imparts additional momentum to the fluid particles in the boundary layer so that the boundary layer remains attached through a larger static pressure rise. Tangential injection can be used, for example, to achieve higher maximum lift coefficients on airfoils and wings.

The flowfield associated with this mixing of a turbulent wall jet (the injected fluid) with a turbulent boundary layer (the upstream flow) on a curved surface is quite complex. This complexity and the variety of possible flow behaviours necessitates that a wide variety of experimental data be considered in the development of prediction methods for flows involving tangential injection.

Previous investigations of the shear layer downstream of the tangential injection slot on a circulation-controlled circular cylinder in a free stream have produced reasonable velocity profiles (Kind et. al. (1979)). This apparatus is shown in Figure 1.1. The purpose of the present work was to investigate a hot-wire anemometry technique proposed by Acrivlellis (1978) to see if it could eventually be applied to measure turbulence parameters (Reynolds stresses) in this boundary layer control flow. The present work involved measurements on a plane

wall jet in still air. By comparing the results with existing data, a decision can be made as to the usefulness of this technique in the more complex flow. Although the method of Acrivlellis has not been tested extensively, the relative simplicity of the equations makes it attractive, and worthy of investigation.

The fact that the flow of interest is rather thin and is characterized by strong velocity gradients raises concern over the problem of spatial resolution. To measure Reynolds shear stresses and the cross-flow components of the normal Reynolds stress it is necessary to orient the hot-wire sensor normal to the wall; thus there is an appreciable variation of mean velocity and other flow parameters over the sensor span. This thesis examines whether the measured flow parameters are affected by the variation mentioned above.

Modifications to the aforementioned boundary layer control apparatus will also be discussed. The modifications involved the use of a computer solution for inviscid flow on arbitrary airfoils, to design a forebody that would thicken the boundary layer at the injection slot.

1.2 Outline of the Current Study

The progression from planning to completion of this thesis involved five major steps, as follows:

- familiarization with the principles of hot-wire anemometry
- derivation of the data-reduction equations
- development of an experimental apparatus
- experimentation and revisions to the equations

- modifications to the boundary layer control apparatus for use in future work.

Chapter 2 of this report discusses pertinent aspects of hot-wire anemometry. An outline of the basic principles of constant temperature hot-wire anemometry is followed by a discussion of some of the methods presently in use for signal interpretation with emphasis on their shortcomings. The chapter closes with a discussion of the method used in this study (proposed by Acrivlellis (1978)), highlighting its advantages.

The method of Acrivlellis (1978) is the subject of Chapter 3. The equations are derived in the original three-dimensional form and then reworked into the various forms required for the present experimental work.

Chapter 4 details the development of the experimental apparatus. This includes discussion of existing equipment, the instrumentation involved, as well as the design of new equipment.

~~Chapter 5, the bulk of the thesis, includes the experimental results and discussion. This chapter outlines the thought process from the initial familiarization with hot-wire anemometry methods, through the testing and revision to the point where acceptable data were obtained.~~

~~The revisions to the boundary layer control apparatus of Figure 1.1 are presented in Chapter 6. These included modifications to a computer program which calculates the pressure distribution around an arbitrary airfoil by replacing the contour with a number of plane uniform two-dimensional vortex sheets whose strengths vary linearly between junction~~

points. By modifying the program with a two-dimensional inviscid flow approximation known as the "method of images", a forebody was designed in hopes of thickening the boundary layer at the injection slot. A thicker boundary layer is desirable to make the flow more representative of most practical applications of boundary layer control by tangential injection. Chapter 6 also describes the construction of the forebody.

CHAPTER 2

HOT-WIRE ANEMOMETRY2.1 Introduction

Hot-wire anemometry is the use of a thin electrically heated wire which when exposed to a flow is cooled by radiation, buoyant convection, conduction along the wire to its end supports and, most importantly, forced convection, for the purpose of measuring a property of the flow, usually mean and fluctuating velocity components. The hot-wire probe consists of a ceramic stem from which two metal prongs protrude. Between the prongs is welded a thin metallic wire, with a typical diameter of $0.5\text{-}5\text{ }\mu\text{m}$ and a typical length of $0.1\text{-}1\text{ mm}$ similar to that shown in Figure 2.1.

Wire materials include platinum, tungsten and platinum-iridium. The usefulness of certain wires for given applications depends on the wire strength, resistance to oxidation, and temperature coefficient of resistance. To date, tungsten hot-wire sensors are the most popular due to their durability and the higher signal to noise ratio obtainable. Details of the properties of various wire materials may be found in Thermo Systems Inc. (1967).

Two main types of control circuits are available for hot-wire operation: constant current and constant temperature. As the terms imply, in one case the electric current through the wire is kept constant and in the other, the wire temperature is kept constant by the control

unit. Modern systems, including that used for the present study, almost invariably use the constant-temperature approach.

A schematic of a constant-temperature anemometry system is shown in Figure 2.2. As the velocity over the sensor wire increases, the sensor tends to cool and its resistance tends to decrease. This decrease in resistance causes a decrease in voltage across the sensor, changing the input to the amplifier. The amplifier in turn responds by increasing the current to the Wheatstone bridge in order to re-establish the equilibrium temperature of the sensor. The gain of the amplifier is sufficiently high that the temperature of the sensor remains virtually constant. The voltage required to drive the current through the sensor to regain the equilibrium temperature is the bridge output and is an indication of the instantaneous velocity of the flow at the sensor. The bridge voltage versus velocity relation is highly non-linear. Often a linearizer is used to linearize the relation, making signal interpretation more convenient.

Advantages of the constant-temperature anemometer are as follows:

- low noise level and good frequency response
- convenience in operation
- flexibility of the system
- compatibility with film sensors
- prevention of ~~of~~ sensor burn-out when the cooling velocity past the sensor is suddenly decreased.
- more practical approach for measurements in liquids where large changes in sensor cooling occur when velocity changes

- linearizing is possible
- temperature compensation is possible
- the system gives direct DC output(Thermo Systems Inc. (1967))

2.2 Theory of the Hot-Wire Anemometer

Heat transfer from hot-wires is usually expressed as a dimensionless Nusselt number, equal to the rate of heat transfer to the fluid per unit area, H_f/A , divided by the product of the thermal conductivity of the fluid k_f and a typical temperature gradient $(T_w - T_f)/d$ (Bradshaw (1971)). Thus the Nusselt number is given by

$$\begin{aligned}
 N_u &= \frac{H_f / \pi d l}{k_f (T_w - T_f) / d} \\
 &= \frac{H_f / l}{\pi k_f (T_w - T_f)}
 \end{aligned}
 \tag{2.1}$$

At low speeds the Nusselt number depends primarily on the Reynolds number and the Prandtl number (Bradshaw (1971)). However the Prandtl number is nearly constant for air and thus is not of immediate concern. In most gases, calibration is independent of the Knudsen-number since this depends only on pressure and at low flow speeds the absolute pressure is nearly constant. Thus we come to the relationship of major concern, the variation of Nusselt number with Reynolds number. Here King's relationship,

$$N_u = A + B R_e^{0.5} \tag{Bradshaw (1971)} \tag{2.2}$$

is introduced. More recent work has shown that an exponent of 0.45 gives

a better correlation than an exponent of 0.5. (Bradshaw (1971))

Upon expansion this becomes:

$$\frac{H_f / l}{\pi k_f (T_w - T_f)} = A + B \left(\frac{\rho U d}{\mu} \right)^{.45} \quad (2.3)$$

since $k_f \propto T^{.76}$; $\mu \propto T^{.76}$; and $\rho \propto \frac{1}{T}$ we have (Bradshaw (1971))

$$\frac{H_f}{T^{.76} (T_w - T_f)} \propto A_1 + \frac{B_1 U^{.45}}{T^{.792}} \quad (2.4)$$

or approximately that

$$\frac{H_f}{T_w - T_f} \propto \frac{I^2 R_w}{R_w - R_a} \propto A_1 (T_f) + B_1 U^{.45} \quad (2.5)$$

where A_1 is a function of T_f but B_1 is not. Also since R_w is constant $E \propto I$ and therefore

$$E^2 = A + B U^{.45} \quad (2.6)$$

If E^2 is plotted versus $U^{.45}$ the result is a nearly straight line with A as the intercept corresponding to E_0 , the zero flow voltage, and B is the slope S . In this way anemometer voltages can be related to mean flow velocities. A linearizer may be placed in the circuit to accept the bridge voltage input and give a linearized signal on output. When this is done, equation (2.6) becomes

$$E = S U \quad (2.7)$$

This relationship holds at any instant in time; that is, the system can respond to turbulent velocity fluctuations having frequencies as high as about 100 KHz. This is made possible by the low thermal inertia of the

fine wire sensor and the high frequency-response capability of modern thermal anemometer control systems.

2.3 Brief Review of Hot-Wire Signal Interpretation Methods

Equation (2.7) states the relationship between the linearized voltage and the mean flow velocity. In some cases such as calibration the hot-wire is exposed only to the cooling of a one dimensional flow velocity U . However when probes are operating in turbulent flows it becomes difficult to tell at what velocity the hot-wire cools as a result of the spatial flow. Most existing papers on the calculation of the heat transfer at the hot-wire base calculation on the "cosine law" assumption that heat transfer is determined only by the instantaneous velocity component perpendicular to the wire. Then the component parallel to the hot-wire (tangential velocity component U_T) yields no contributions to the heat transfer. Thus the so-called effective cooling velocity U_{eff} equals $U \cos \alpha$ or U_N , assuming that the velocity vector is parallel to the plane of the prongs of the probe.

According to Champagne et al. (1967) deviations from the cosine law are caused by the tangential velocity component, suggesting that the effective cooling law be calculated from

$$U_{eff}^2 = U_N^2 + k^2 U_T^2 \quad (2.8)$$

where the sensitivity coefficient k makes allowance for the influence of the tangential velocity component. k depends mainly on the wire material, on the hot-wire length-to-diameter ratio, and on the temperature

distribution along the wire, as well as on the flow around the probe tip.

When the velocity vector is not parallel to the plane of the prongs more recent experiments, for example Jorgensen (1971) and Mojola (1974) have suggested that it is necessary to make allowance for the additional hot-wire cooling resulting from the binormal components U_B , by means of the equation:

$$U_{eff}^2 = U_N^2 + k^2 U_T^2 + h^2 U_B^2 \quad (\text{see Figure 3.1}) \quad (2.9)$$

The value of the coefficient h is typically about 1.05.

Usually the velocity normal to the wire provides the majority of the cooling but depending on the flow quantity being determined consideration of the cooling law contributions of U_T and U_B may be important. Further details of the effect of neglecting hot-wire cooling caused by the velocity components parallel and binormal to the wire on hot-wire-signal determination may be found in Acrivellis (1978).

The remaining material in this section is taken from Bradshaw (1971). There are several methods for determining the various mean velocity and turbulence components depending on the type of probe used. Bradshaw (1971) states that there are three common configurations for hot-wire probes. The first and simplest to use is a single wire normal to the flow used for measuring the U and u components. An "X" probe consists of two wires arranged in an "X" at approximately $\pm 45^\circ$ to the flow direction. Ideally, one wire of an "X" probe in the xy plane responds to $u + v$ and $(U + V)$ and the other to $u - v$, and $(U - V)$ according to the simple cosine law, so that the difference between the two wire signals is $2v$ and $2V$.

Disadvantages of cross-wire probes are as follows:

- cross-wire probes require the use of a two channel anemometer
- aerodynamic and thermal interference between sensors and prongs is often encountered
- cross-wire probes are less economical, more fragile, more difficult to repair, and more prone to fouling.

The methods of both Mojola (1974) and Champagne et al. (1967) involve the use of cross-wire probes. The hot-wire response equations are arrived at by complicated expansions of the cooling law in various power series, making the resulting equations difficult to manipulate. As well, in order to simplify the equations it is necessary to neglect some higher order terms making the equations invalid for high turbulence flows.

The final type is the slant-wire probe which looks like half an "X" probe and responds roughly to $u+v$ and $U+V$. If rotated 180° about the probe axis, it will respond to $u-v$ and $U-V$; the difference between the mean squares of the fluctuating signals in these two positions is $4uv$. When used in this way, the slant-wire probe simulates an "X" probe with two perfectly matched wires, but, of course, it cannot be used for instantaneous measurement of the v component. By rotating to additional angles, other flow quantities may be determined.

2.4 Outline of the Current Study

Past difficulties with the interpretation of hot-wire anemometer signals are attributable not to the hot-wire sensor itself but to the

transfer function of the overall system. Only by assuming flows of low turbulence intensity could the various time-averaged flow velocities and Reynolds stresses be found. This assumption allowed experimenters such as Mojola (1974) to neglect the higher-order terms in the binomial series expansion and the nonlinear relationship between the output voltage and the flow parameters to be found. This limited the use of hot-wire anemometers for the study of highly turbulent flows.

Attempts to increase the accuracy of the conventional methods of signal interpretation by taking into account the higher-order terms of the binomial series expansion have met with only limited success.

The new method proposed by Acrivlellis (1978) bases the calculation on squaring the voltage signal before time averaging, thus apparently eliminating the need for the binomial series expansion and consequently also the need to neglect higher-order terms. Acrivlellis (1978) claims several advantages over the conventional method, such as; the new method enables the determination of all 3-D flow quantities without assumptions or loss of accuracy; the probe stem does not have to be aligned in the mean flow direction in order to carry out measurements; and the method uses single wire probes with their advantages of economy and accuracy over multi-wire probes. These claims made the method very attractive, unfortunately as will be shown in this report, the first two of these claims proved to be erroneous.

CHAPTER 3

PRESENTATION OF THE NEW METHOD

3.1 Introduction

This chapter presents the method of Acrivlellis (1978) in its original form along with modifications necessitated by problems encountered with these original equations. As was discovered late in this work, the original method is based on an invalid assumption, but the revised method developed in this chapter can be shown to be valid for the case of low turbulence flows where the probe stem is aligned in the mean flow direction.

Figure 3.1 shows the inertial reference system (x, y, z) which has velocity components $U+u, V+v, W+w$ where the capital letters denote time-averaged velocity components and small letters, fluctuating components. The sensor-based system (ξ, ζ, η) has velocity components U_N, U_T and U_B respectively, where U_N is normal to the hot-wire sensor, U_T is parallel to it and U_B is the binormal velocity vector. The angle between the ξ -axis and the x -axis (probe axis) is the probe angle α , and the angle through which the probe is rotated to obtain the desired equations is ψ . The derivation begins with the transformation of the inertial coordinate system to the sensor-based coordinate system by first rotating the inertial system about the x -axis through an angle ψ and then about the z -axis through the angle α .

3.2 Derivation of the Hot-Wire Equations for Three Dimensional Flow

Transformation from the inertial to the sensor-based system yields the following velocity components:

$$\begin{aligned}
 U_N &= (U+u) \cos \alpha + [(V+v) \cos \psi - (W+w) \sin \psi] \sin \alpha \\
 U_T &= -(U+u) \sin \alpha + [(V+v) \cos \psi - (W+w) \sin \psi] \cos \alpha \\
 U_B &= (V+v) \sin \psi + (W+w) \cos \psi
 \end{aligned} \tag{3.1}$$

Rewriting the cooling law equation (2.9) gives

$$U_{\text{eff}}^2 = U_N^2 + k^2 U_T^2 + h^2 U_B^2$$

If a linearizer is used the following relationship between the linearized output voltage and the effective cooling velocity holds:

$$E = S U_{\text{eff}} \tag{3.2}$$

Substituting equation (3.1) into equation (2.9) and the result into equation (3.2) yields:

$$\begin{aligned}
 \frac{E(\psi, \alpha)}{S} &= \left\{ \{ (U+u) \cos \alpha + [(V+v) \cos \psi - (W+w) \sin \psi] \sin \alpha \}^2 \right. \\
 &\quad + k^2 \{ -(U+u) \sin \alpha + [(V+v) \cos \psi - (W+w) \sin \psi] \cos \alpha \}^2 \\
 &\quad \left. + h^2 [(V+v) \sin \psi + (W+w) \cos \psi]^2 \right\}^{1/2}
 \end{aligned} \tag{3.3}$$

Instead of following the conventional procedure of expanding the right hand side of equation (3.3) into a binomial series and then time averaging, the equation is first squared and then time-averaged giving:

$$\begin{aligned}
\frac{E^2}{S^2} = & (U+u)^2 \cos^2 \alpha + 2(U+u) \cos \alpha [(V+v) \cos \psi \sin \alpha - (W+w) \sin \psi \sin \alpha] + \\
& (V+v)^2 \cos^2 \psi \sin^2 \alpha - 2(V+v) \cos \psi (W+w) \sin \psi \sin^2 \alpha + (W+w)^2 \sin^2 \psi \sin^2 \alpha + \\
& k^2 [(U+u)^2 \sin^2 \alpha - 2(U+u) \sin \alpha [(V+v) \cos \psi \cos \alpha - (W+w) \sin \psi \cos \alpha] + \\
& (V+v)^2 \cos^2 \psi \cos^2 \alpha - 2(V+v) \cos \psi (W+w) \sin \psi \cos^2 \alpha + (W+w)^2 \sin^2 \psi \\
& \cos^2 \alpha] + h^2 [(V+v)^2 \sin^2 \psi + 2(V+v) \sin \psi (W+w) \cos \psi + \\
& (W+w)^2 \cos^2 \psi]
\end{aligned}$$

Time averaging gives:

$$\begin{aligned}
\frac{\overline{E^2}}{S^2} = & (\overline{U^2} + \overline{u^2}) \cos^2 \alpha + 2(\overline{UV} + \overline{uv}) \cos \alpha \cos \psi \sin \alpha - 2(\overline{UW} + \overline{uw}) \\
& \cos \alpha \sin \psi \sin \alpha + (\overline{V^2} + \overline{v^2}) \cos^2 \psi \sin^2 \alpha - 2(\overline{VW} + \overline{vw}) \cos \psi \sin \psi \\
& \sin^2 \alpha + (\overline{W^2} + \overline{w^2}) \sin^2 \psi \sin^2 \alpha + k^2 [(\overline{U^2} + \overline{u^2}) \sin^2 \alpha - 2(\overline{UV} + \overline{uv}) \\
& \sin \alpha \cos \psi \cos \alpha + 2(\overline{UW} + \overline{uw}) \sin \alpha \sin \psi \cos \alpha + (\overline{V^2} + \overline{v^2}) \cos^2 \psi \\
& \cos^2 \alpha - 2(\overline{VW} + \overline{vw}) \cos \psi \sin \psi \cos^2 \alpha + (\overline{W^2} + \overline{w^2}) \sin^2 \psi \cos^2 \alpha] \\
& + h^2 [(\overline{V^2} + \overline{v^2}) \sin^2 \psi + 2(\overline{VW} + \overline{vw}) \sin \psi \cos \psi + (\overline{W^2} + \overline{w^2}) \cos^2 \psi]
\end{aligned}$$

Collecting the terms and noting that $2\cos \alpha \sin \alpha = \sin 2\alpha$ gives:

$$\begin{aligned}
\frac{\overline{E^2}}{\overline{S^2}} &= (\overline{U^2 + u^2}) (\cos^2 \alpha + k^2 \sin^2 \alpha) (\overline{V^2 + v^2}) (\cos^2 \psi \sin^2 \alpha + k^2 \cos^2 \psi \\
&\quad \cos^2 \alpha + h^2 \sin^2 \psi) + (\overline{W^2 + w^2}) (\sin^2 \psi \sin^2 \alpha + k^2 \sin^2 \psi \cos^2 \alpha + h^2 \cos^2 \psi) \\
&\quad + (\overline{UV + uv}) \cos \psi \sin 2\alpha (1 - k^2) - (\overline{UW + uw}) \sin \psi \sin 2\alpha (1 - k^2) - \\
&\quad (\overline{VW + vw}) (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \sin 2\psi \quad (3.4)
\end{aligned}$$

Subtracting the laminar-flow version of eqn.(3.4) from eqn.(3.4), gives:

$$\begin{aligned}
\frac{\overline{e^2}}{\overline{S^2}} &= \overline{u^2} (\cos^2 \alpha + k^2 \sin^2 \alpha) + \overline{v^2} (\cos^2 \psi \sin^2 \alpha + k^2 \cos^2 \psi \cos^2 \alpha \\
&\quad + h^2 \sin^2 \psi) + \overline{w^2} (\sin^2 \psi \sin^2 \alpha + k^2 \sin^2 \psi \cos^2 \alpha + h^2 \cos^2 \psi) + \\
&\quad \overline{uv} (1 - k^2) \cos \psi \sin 2\alpha - \overline{uw} (1 - k^2) \sin \psi \sin 2\alpha - \overline{vw} (\sin^2 \alpha + k^2 \\
&\quad \cos^2 \alpha - h^2) \sin 2\psi \quad (3.5)
\end{aligned}$$

Next, equations (3.4) and (3.5) are applied for each of the six hot-wire positions shown in Figure 3.2. These are $\psi=0^\circ$, 90° , 180° , 270° , 45° and 315° corresponding to positions 1 through 6.

Thus the equations become (for equation (3.4))

Position 1: $\psi=0^\circ$, α° -probe

$$\begin{aligned}
&(\cos^2 \alpha + k^2 \sin^2 \alpha) (\overline{U^2 + u^2}) + (\sin^2 \alpha + k^2 \cos^2 \alpha) (\overline{V^2 + v^2}) \\
&\quad + h^2 (\overline{W^2 + w^2}) + (1 - k^2) \sin 2\alpha (\overline{UV + uv}) = \overline{E_1^2} / \overline{S^2} \quad (3.6)
\end{aligned}$$

Position 2: $\psi=90^\circ$, α° -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) (U^2 + \overline{u^2}) + h^2 (V^2 + \overline{v^2}) \\
 & + (\sin^2 \alpha + k^2 \cos^2 \alpha) (W^2 + \overline{w^2}) \\
 & - (1-k^2) \sin 2\alpha (UW + \overline{uw}) = \overline{E_2^2}/S^2
 \end{aligned} \tag{3.7}$$

Position 3: $\psi=180^\circ$, α° -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) (U^2 + \overline{u^2}) + (\sin^2 \alpha + k^2 \cos^2 \alpha) (V^2 + \overline{v^2}) \\
 & + h^2 (W^2 + \overline{w^2}) - (1-k^2) \sin 2\alpha (UV + \overline{uv}) = \overline{E_3^2}/S^2
 \end{aligned} \tag{3.8}$$

Position 4: $\psi=270^\circ$, α° -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) (U^2 + \overline{u^2}) + h^2 (V^2 + \overline{v^2}) \\
 & + (\sin^2 \alpha + k^2 \cos^2 \alpha) (W^2 + \overline{w^2}) \\
 & + (1-k^2) \sin 2\alpha (UW + \overline{uw}) = \overline{E_4^2}/S^2
 \end{aligned} \tag{3.9}$$

Position 5: $\psi=+45^\circ$, α° -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) (U^2 + \overline{u^2}) + 0.5(\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) \cdot \\
 & (V^2 + \overline{v^2}) + 0.5(\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) (W^2 + \overline{w^2}) \\
 & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha (UV + \overline{uv}) \\
 & - \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha (UW + \overline{uw}) \\
 & - (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) (VW + \overline{vw}) = \overline{E_5^2}/S^2
 \end{aligned} \tag{3.10}$$

Position 6: $\psi = -45^0$, α^0 -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) (U^2 + \overline{u^2}) \\
 & + 0.5(\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) (V^2 + \overline{v^2}) \\
 & + 0.5(\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) (W^2 + \overline{w^2}) \\
 & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha (UV + \overline{uv}) \\
 & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha (UW + \overline{uw}) \\
 & + (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) (VW + \overline{vw}) = \overline{E_6^2} / S^2 \quad (3.11)
 \end{aligned}$$

Position 1: $\psi=0^0$, β^0 -probe

$$\begin{aligned}
 & (\cos^2 \beta + k^2 \sin^2 \beta) (U^2 + \overline{u^2}) + (\sin^2 \beta + k^2 \cos^2 \beta) (V^2 + \overline{v^2}) \\
 & + h^2 (W^2 + \overline{w^2}) + (1-k^2) \sin^2 \beta (UV + \overline{uv}) = \overline{E_1^{*2}} / S^2 \quad (3.12)
 \end{aligned}$$

This last one of these equations was derived from equation (3.4) for an arbitrary probe angle β at $\psi=0$ (position 1). The probe angle β is subject only to the restriction that $\beta \neq \alpha$. Obviously, a different ψ -angle could be chosen for the β -probe instead of position 1 ($\psi=0$). The asterisk at the last equations refers to the β -probe.

Analogously with the above equation (3.5) for the squared AC voltage for the same hot-wire positions 1-6 yields the following additional system of equations:

Position 1: $\psi=0^\circ$, α° -probe

$$\begin{aligned} & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{v^2} \\ & + h^2 \overline{w^2} + (1-k^2) \sin 2\alpha \overline{uv} = \overline{e_1^2}/S^2 \end{aligned} \quad (3.13)$$

Position 2: $\psi=90^\circ$, α° -probe

$$\begin{aligned} & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + h^2 \overline{v^2} \\ & + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{w^2} \\ & - (1-k^2) \sin 2\alpha \overline{uw} = \overline{e_2^2}/S^2 \end{aligned} \quad (3.14)$$

Position 3: $\psi=180^\circ$, α° -probe

$$\begin{aligned} & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{v^2} + h^2 \overline{w^2} \\ & - (1-k^2) \sin 2\alpha \overline{uv} = \overline{e_3^2}/S^2 \end{aligned} \quad (3.15)$$

Position 4: $\psi=270^\circ$ (-90°), α° -probe

$$\begin{aligned} & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + h^2 \overline{v^2} + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{w^2} \\ & + (1-k^2) \sin 2\alpha \overline{uw} = \overline{e_4^2}/S^2 \end{aligned} \quad (3.16)$$

Position 5: $\psi=+45^\circ$, α° -probe

$$\begin{aligned} & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + 0.5 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) \overline{v^2} \\ & + 0.5 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) \overline{w^2} \\ & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha \overline{uv} \\ & - \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha \overline{uw} \\ & - (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \overline{vw} = \overline{e_5^2}/S^2 \end{aligned} \quad (3.17)$$

Position 6: $\psi = -45^\circ$, α° -probe

$$\begin{aligned}
 & (\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + 0.5 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) \overline{v^2} \\
 & + 0.5 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) \overline{w^2} \\
 & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha \overline{uv} \\
 & + \frac{1}{\sqrt{2}} (1-k^2) \sin 2\alpha \overline{uw} \\
 & + (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \overline{vw} = \overline{e_6^2} / S^2
 \end{aligned} \tag{3.18}$$

Position 1: $\psi = 0^\circ$, β° -probe

$$\begin{aligned}
 & (\cos^2 \beta + k^2 \sin^2 \beta) \overline{u^2} + (\sin^2 \beta + k^2 \cos^2 \beta) \overline{v^2} + h^2 \overline{w^2} \\
 & + (1-k^2) \sin 2\beta \overline{uv} = \overline{e_1^2} / S^2
 \end{aligned} \tag{3.19}$$

Taking into account

$$E = \overline{E} + e \quad \text{or} \quad \overline{E^2} = \overline{\overline{E}^2} + \overline{e^2} \tag{3.20}$$

Subtracting equation (3.8) from equation (3.6) yields:

$$\frac{\overline{E_1^2} - \overline{E_3^2}}{S^2} = 2(1-k^2) \sin 2\alpha (\overline{uv} + \overline{uw}) \tag{3.21}$$

Likewise equation (3.15) - equation (3.13) gives:

$$2(1-k^2) \sin 2\alpha \overline{uv} = \frac{\overline{e_1^2} - \overline{e_3^2}}{S^2}$$

Therefore

$$\overline{uv} = \frac{\overline{e_1^2} - \overline{e_3^2}}{2S^2 (1-k^2) \sin 2\alpha} \tag{3.22}$$

since

$$\overline{E}^2 = \overline{E}^2 + \overline{e}^2 \quad \text{equation (3.21) becomes}$$

$$\frac{\overline{E}_1^2 + \overline{e}_1^2 - \overline{E}_3^2 - \overline{e}_3^2}{S^2} = 2(1-k^2) \sin 2\alpha (UV + \overline{uv})$$

Substituting equation (3.22) above yields:

$$UV + \frac{\overline{e}_1^2 - \overline{e}_3^2}{2S^2(1-k^2) \sin 2\alpha} = \frac{\overline{E}_1^2 + \overline{e}_1^2 - \overline{E}_3^2 - \overline{e}_3^2}{2S^2(1-k^2) \sin 2\alpha}$$

$$UV = \frac{\overline{E}_1^2 - \overline{E}_3^2}{2S^2(1-k^2) \sin 2\alpha} \quad (3.23)$$

Similarly equation (3.9) - equation (3.7) gives

$$2(1-k^2) \sin 2\alpha (UW + \overline{uw}) = \frac{\overline{E}_4^2 - \overline{E}_2^2}{S^2}$$

and equation (3.17) - equation (3.15) yields

$$2(1-k^2) \sin 2\alpha \overline{uw} = \frac{\overline{e}_4^2 - \overline{e}_2^2}{S^2}$$

thus

$$\frac{\overline{e}_4^2 - \overline{e}_2^2}{2S^2(1-k^2) \sin 2\alpha} = \frac{\overline{E}_4^2 - \overline{e}_4^2 - \overline{E}_2^2 - \overline{e}_2^2}{2S^2(1-k^2) \sin 2\alpha}$$

$$UW = \frac{\overline{E}_4^2 - \overline{E}_2^2}{2S^2(1-k^2) \sin 2\alpha} \quad (3.24)$$

equation (3.11) - equation (3.10) gives:

$$\frac{\overline{E}_6^2 - \overline{E}_5^2}{S^2} = \frac{2}{\sqrt{2}} (1-k^2) \sin 2\alpha (UW + \overline{uw}) + 2(\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)(VW + \overline{vw})$$

equation (3.7) - equation (3.9) yields:

$$-2 (1-k^2) \sin 2\alpha (UW + \overline{UW}) = \frac{\overline{E_2^2} - \overline{E_4^2}}{S^2} \quad (3.26)$$

multiplying equation (3.25) by $\sqrt{2}$ and substituting for $2(1-k^2) \sin 2\alpha (UW + \overline{UW})$ from equation (3.26) gives:

$$\sqrt{2} \frac{(\overline{E_6^2} - \overline{E_5^2})}{S^2} = \frac{-\overline{E_2^2} + \overline{E_4^2}}{S^2} + 2\sqrt{2} (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) (VW + \overline{VW}) \quad (3.27)$$

equation (3.18) - equation (3.17) gives:

$$\frac{2}{\sqrt{2}} (1-k^2) \sin 2\alpha \overline{UW} + 2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \overline{VW} = \frac{\overline{e_6^2} - \overline{e_5^2}}{S^2}$$

Also

$$\frac{\overline{e_4^2} - \overline{e_2^2}}{S^2} + 2\sqrt{2} (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) \overline{VW} = \sqrt{2} \frac{(\overline{e_6^2} - \overline{e_5^2})}{S^2}$$

Simplifying equation (3.27) and substituting for the AC voltages yields:

$$\begin{aligned} \frac{\sqrt{2} (\overline{E_6^2} + \overline{e_6^2} - \overline{E_5^2} - \overline{e_5^2}) + \overline{E_2^2} + \overline{e_2^2} - \overline{E_4^2} - \overline{e_4^2}}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} &= VW + \overline{VW} \\ VW &= \frac{\sqrt{2} (\overline{E_6^2} + \overline{e_6^2} - \overline{E_5^2} - \overline{e_5^2}) + \overline{E_2^2} + \overline{e_2^2} - \overline{E_4^2} - \overline{e_4^2}}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \\ &= \frac{\sqrt{2} (\overline{e_6^2} - \overline{e_5^2}) - \overline{e_4^2} + \overline{e_2^2}}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \\ VW &= \frac{\sqrt{2} (\overline{E_6^2} - \overline{E_5^2}) + \overline{E_2^2} - \overline{E_4^2}}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \quad (3.28) \end{aligned}$$

Also

$$\overline{UV} = \frac{\overline{e_1^2} - \overline{e_3^2}}{2S^2 (1-k^2) \sin 2\alpha} \quad (3.29)$$

$$\overline{UW} = \frac{\overline{e_4^2} - \overline{e_2^2}}{2S^2 (1-k^2) \sin 2\alpha} \quad (3.30)$$

$$\overline{VW} = \frac{(\overline{e_6^2} - \overline{e_5^2}) \sqrt{2} - \overline{e_4^2} + \overline{e_2^2}}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \quad (3.31)$$

$$U^2 = \frac{UV \cdot UW}{VW}$$

$$U^2 = \frac{\overline{e_4^2} - \overline{e_2^2}}{2 S^2 (1-k^2) \sin 2\alpha} \cdot \frac{\overline{e_1^2} - \overline{e_3^2}}{2 S^2 (1-k^2) \sin 2\alpha} \cdot \frac{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)}{(\overline{e_6^2} - \overline{e_5^2}) \sqrt{2} + (\overline{e_2^2} - \overline{e_4^2})}$$

$$U^2 = \frac{(\overline{e_4^2} - \overline{e_2^2}) (\overline{e_1^2} - \overline{e_3^2}) (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)}{\sqrt{2} S^2 (1-k^2)^2 \sin^2 2\alpha [(\overline{e_6^2} - \overline{e_5^2}) \sqrt{2} + (\overline{e_2^2} - \overline{e_4^2})]} \quad (3.32)$$

$$\text{Similarly } V^2 = \frac{UV \cdot VW}{UW}$$

$$V^2 = \frac{\overline{e_1^2} - \overline{e_3^2}}{2 S^2 (1-k^2) \sin 2\alpha} \cdot \frac{[\sqrt{2} (\overline{e_6^2} - \overline{e_5^2}) + \overline{e_2^2} - \overline{e_4^2}]}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \cdot \frac{2 S^2 \sin 2\alpha (1-k^2)}{\overline{e_4^2} - \overline{e_2^2}}$$

$$V^2 = \frac{(\overline{e_1^2} - \overline{e_3^2}) [(\overline{e_6^2} - \overline{e_5^2}) \sqrt{2} + \overline{e_2^2} - \overline{e_4^2}]}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) (\overline{e_4^2} - \overline{e_2^2})} \quad (3.33)$$

$$\text{Also } W^2 = \frac{UW \cdot VW}{UV}$$

$$W^2 = \frac{E_4^2 - E_2^2}{2 S^2 (1-k^2) \sin 2\alpha} \cdot \frac{\sqrt{2} (E_6^2 - E_5^2) + E_2^2 - E_4^2}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \cdot \frac{2 S^2 (1-k^2) \sin 2\alpha}{E_1^2 - E_3^2}$$

$$W^2 = \frac{(E_4^2 - E_2^2) [(E_6^2 - E_5^2) \sqrt{2} + E_2^2 - E_4^2]}{2\sqrt{2} S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) (E_1^2 - E_3^2)} \quad (3.34)$$

For the fluctuating velocity components adding equation (3.13) and equation (3.15) gives:

$$2(\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + 2(\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{v^2} + 2h^2 \overline{w^2} = \frac{\overline{e_1^2} + \overline{e_3^2}}{S^2}$$

$$(\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{v^2} + h^2 \overline{w^2} = \frac{\overline{e_1^2} + \overline{e_3^2}}{2S^2} \quad (3.35)$$

Adding equation (3.14) and equation (3.16) gives

$$(\cos^2 \alpha + k^2 \sin^2 \alpha) \overline{u^2} + h^2 \overline{v^2} + (\sin^2 \alpha + k^2 \cos^2 \alpha) \overline{w^2} = \frac{\overline{e_2^2} + \overline{e_4^2}}{2S^2} \quad (3.36)$$

Substituting equation (3.29) in equation (3.19) yields

$$(\cos^2 \beta + k_\beta^2 \sin^2 \beta) \overline{u^2} + (\sin^2 \beta + k_\beta^2 \cos^2 \beta) \overline{v^2} + h_\beta^2 \overline{w^2} +$$

$$(1-k_\beta^2) \sin 2\beta \frac{\overline{e_3^2} - \overline{e_1^2}}{-2 S^2 (1-k^2) \sin 2\alpha} = \frac{\overline{e_1^2}}{S_\beta^2}$$

$$(\cos^2 \beta + k_\beta^2 \sin^2 \beta) \overline{u^2} + (\sin^2 \beta + k_\beta^2 \cos^2 \beta) \overline{v^2} + h_\beta^2 \overline{w^2} =$$

$$\frac{\overline{e_1^2}}{S_\beta^2} + \frac{\overline{e_3^2} - \overline{e_1^2}}{2S^2} \frac{\sin 2\beta}{\sin 2\alpha} \frac{(1-k_\beta^2)}{1-k^2} \quad (3.37)$$

3.3 Modifications to the New Method

3.3.1 3-D Equations Using Bridge Output

The remaining sections in this chapter will present the modifications to the equations, that were necessitated in light of experimental results to be presented later.

The first attempt to improve the accuracy of the experiment involved removing the electronic linearizer from the circuit and linearizing the bridge output mathematically. At first the method was tested only for mean velocity components; the original results were only slightly modified by the revised interpretation method. The method involved first subtracting the value of the bridge output for zero flow from the reading. The result is then multiplied by the appropriate coefficients given by a curve fitting routine incorporated in the calibration program to be discussed later. Multiplying by the slope of the calibration curve gave the mathematically linearized output. Appendix A includes the data-reduction program when the linearizer was used while Appendix B shows the program for calculating mean velocities when the bridge voltage was mathematically linearized. As will be discussed later, this revision did not solve the problems. The equations were therefore reworked.

3.3.2 Reworked 3-D Equations

Examination of the equations showed that in some cases they were very sensitive and problems would be encountered if $\frac{2}{E_1} \approx \frac{2}{E_3}$ or $\frac{2}{E_2} \approx \frac{2}{E_4}$ which was usually the case with this apparatus. This sensitivity was a result of Equations (3.6) through (3.11) and (3.13) through (3.18) having been differenced rather than summed to solve for the mean velocity

components. Thus the equations were reworked by summing wherever possible to obtain expressions for the mean velocity components and Reynolds stresses.

Adding equations (3.6) and (3.8) gives:

$$2(U^2 + \overline{u^2}) (\cos^2 \alpha + k^2 \sin^2 \alpha) + 2(V^2 + \overline{v^2}) (\sin^2 \alpha + k^2 \cos^2 \alpha) + 2h^2 (\overline{w^2} + \overline{w'^2}) = \frac{\overline{E_1^2} + \overline{E_3^2}}{S^2} \quad (3.38)$$

For the AC voltage one obtains

$$2 \overline{u^2} (\cos^2 \alpha + k^2 \sin^2 \alpha) + 2 \overline{v^2} (\sin^2 \alpha + k^2 \cos^2 \alpha) + 2 h^2 \overline{w^2} = \frac{\overline{e_1^2} + \overline{e_3^2}}{S^2} \quad (3.39)$$

Expanding equation (3.38) using $\overline{E^2} = \overline{E^2} + \overline{e^2}$ and substituting from equation (3.39) yields

$$U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + V^2 (\sin^2 \alpha + k^2 \cos^2 \alpha) + h^2 \overline{w^2} = \frac{\overline{E_1^2} + \overline{E_3^2}}{2S^2} \quad (3.40)$$

Similarly for equations (3.7) and (3.9) one obtains

$$U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + V^2 h^2 + \overline{w^2} (\sin^2 \alpha + k^2 \cos^2 \alpha) = \frac{\overline{E_2^2} + \overline{E_4^2}}{2S^2} \quad (3.41)$$

Adding equations (3.10) and (3.11) and eliminating the AC voltages gives

$$2U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) V^2 + (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) W^2 + \sqrt{2} (1-k^2) \sin 2\alpha UV = \frac{\frac{2}{E_5} + \frac{2}{E_6}}{S^2}$$

Substituting for UV from equation (3.23) yields the third equation

$$2U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + V^2 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) + W^2 (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) = \frac{\frac{2}{E_5} + \frac{2}{E_6}}{S^2} - \frac{1}{\sqrt{2}} \frac{\frac{2}{E_1} - \frac{2}{E_3}}{S^2} \quad (3.42)$$

The FORTRAN program HOT-WIRE1 shown in Appendix C using the IMSL subroutine LEQTF1F was written to solve this system of equations, however problems were encountered (as will be discussed later) and further work on the equations was necessary.

3.3.3 2-D Equations

Due to the failure of the equations of the previous section, the analysis was reduced to two dimensions assuming $W=0$. Thus equation (3.40) becomes:

$$U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + V^2 (\sin^2 \alpha + k^2 \cos^2 \alpha) = \frac{\frac{2}{E_1} + \frac{2}{E_3}}{2S^2} \quad (3.43)$$

Similarly equation (3.41) becomes

$$U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + V^2 h^2 = \frac{\frac{2}{E_2} + \frac{2}{E_4}}{2S^2} \quad (3.44)$$

equation (3.43) - equation (3.44) yields

$$V^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2) = \frac{\frac{2}{E_1} + \frac{2}{E_3} - \frac{2}{E_2} - \frac{2}{E_4}}{2S^2}$$

$$V^2 = \frac{\frac{2}{E_1} + \frac{2}{E_3} - \frac{2}{E_2} - \frac{2}{E_4}}{2S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \quad (3.45)$$

Substituting equation (3.45) into equation (3.44) gives

$$U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) + \frac{(\overline{E_1^2} + \overline{E_3^2} - \overline{E_2^2} - \overline{E_4^2}) h^2}{2S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} = \frac{\overline{E_2^2} + \overline{E_4^2}}{2S^2}$$

$$U^2 = \left\{ \frac{\overline{E_2^2} + \overline{E_4^2}}{2S^2} + h^2 \frac{(\overline{E_4^2} + \overline{E_2^2} - \overline{E_1^2} - \overline{E_3^2})}{2S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \right\} / (\cos^2 \alpha + k^2 \sin^2 \alpha)$$

(3.46)

Similarly, for the fluctuating quantities one can obtain:

$$\overline{u^2} = \left\{ \frac{\overline{e_2^2} + \overline{e_4^2}}{2S^2} + \frac{h^2 (\overline{e_4^2} + \overline{e_2^2} - \overline{e_1^2} - \overline{e_3^2})}{2S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)} \right\} / (\cos^2 \alpha + k^2 \sin^2 \alpha)$$

(3.47)

$$\overline{v^2} = \frac{\overline{e_1^2} + \overline{e_3^2} - \overline{e_2^2} - \overline{e_4^2}}{2S^2 (\sin^2 \alpha + k^2 \cos^2 \alpha - h^2)}$$

(3.48)

Since the analysis would now be carried out using 4 positions and only one probe, determination of $\overline{w^2}$ would not be possible. Equations (3.23), (3.24) (3.29) and (3.30) are also satisfactory. The program HOT-WIRE2 in Appendix D was written to perform the data reduction using the above equations.

The program HOT-WIRE3 in Appendix E was written to mathematically linearize the bridge output. The DC voltages were converted as outlined earlier, but the AC voltages were converted as follows:

$$E_{lin} = (C01 + C02 (\overline{E_{BR}} - E_0) + C03 (\overline{E_{BR}} - E_0)^2 + C04 (\overline{E_{BR}} - E_0)^3 + C05 (\overline{E_{BR}} - E_0)^4) S$$

(3.49)

where C01 to C05 are the coefficients of the polynomial linearizing function. Since $\Delta E_{1in} = e_{1in}$

$$\begin{aligned} \frac{\Delta E_{1in}}{\Delta E_{BR}} &= \frac{\delta E_{1in}}{\delta E_{BR}} = \frac{e_{1in}}{e_{BR}} = C02 + 2C03 (E_{BR} - E_0) + 3C04 (E_{BR} - E_0)^2 \\ &\quad + 4C05 (E_{BR} - E_0)^3 S \end{aligned} \quad (3.50)$$

Therefore

$$\begin{aligned} e_{1in} &= (C02 + 2C03 (E_{BR} - E_0) + 3C04 (E_{BR} - E_0)^2 + 4C05 (E_{BR} - E_0)^3 S) e_{BR} \\ &\quad (3.51) \end{aligned}$$

HOT-WIRE4 in Appendix F was written to solve the system of equations for the mean velocities in the two-dimensions by iteration, as outlined below.

Equations (3.45) and (3.44) were added to give

$$\begin{aligned} 2(\cos^2 \alpha + k^2 \sin^2 \alpha) U^2 + (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) V^2 \\ = \frac{\overline{E_1}^2 + \overline{E_2}^2 + \overline{E_3}^2 + \overline{E_4}^2}{2S^2} \end{aligned} \quad (3.52)$$

The program first assumes $V^2=0$ and solves for U^2 using

$$\begin{aligned} U^2 &= \left(\frac{\overline{E_1}^2 + \overline{E_2}^2 + \overline{E_3}^2 + \overline{E_4}^2}{2S^2} - (\sin^2 \alpha + k^2 \cos^2 \alpha + h^2) V^2 \right) \\ &\quad / 2(\cos^2 \alpha + k^2 \sin^2 \alpha) \end{aligned} \quad (3.53)$$

Then using this value of U^2 , V^2 is recalculated using:

$$V^2 = \left(\frac{\frac{2}{E_2} + \frac{2}{E_4}}{2S^2} - U^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) \right) / h^2 \quad (3.54)$$

the process is repeated until the values of U^2 and V^2 converge.

As was mentioned at the outset of this chapter the method used by Acrivlellis (1978) to separate the mean and fluctuating components of velocity is not strictly correct, for the case of a three-dimensional flow of any turbulence intensity (Bartenwerfer (1979)). Fortunately, it can be shown for flows such as those investigated in this thesis, where the mean velocity (U) vector is aligned with the probe stem and the turbulence levels are low, that is where $V=0$, and $W=0$ and u, v and $w \ll U$, that the mean and fluctuating quantities may be separated into separate equations, and U, u and $\overline{uv}$ may be determined as outlined above.

CHAPTER 4

DEVELOPMENT OF THE EXPERIMENTAL APPARATUS4.1 Introduction

Very little experimental data obtained by use of the hot-wire technique proposed by Acrivlellis (1978) has been published to date. Before the method could be used to test the boundary layer control flow it would first have to be verified on a well documented flow. As was mentioned earlier, the flow chosen was a wall jet in quiescent air over a flat plate. The results of these tests could easily be compared with published data and thus a decision as to the expected usefulness of the method in the boundary layer control application could be made.

Although a portion of the flat-plate apparatus was already in existence, considerable effort was put into design and construction of auxiliary equipment, selection of instrumentation and assembly of the overall apparatus.

4.2 Existing Equipment

Figure 4.1 shows a cross-section of the apparatus used for this study. For this application only one slot was used; the plate over the second plenum chamber was removed and the resulting opening covered with cardboard. This allowed the probe to be positioned near the floor of the apparatus and moved further downstream of the slot. The plate is 381 mm long and 304.8 mm wide. 584.2 x 203.2 mm side plates are present to

assist in maintaining two-dimensional flow. The slot has a nominal thickness t of 1.587 mm uniform to within $\pm 0.8\%$. The flow from the slot discharges tangentially to the flat polished steel plate. A static pressure tap in the plenum chamber allows the calculation of the jet velocity at slot exit. The chamber was supplied with air from both sides by a variable-speed centrifugal blower.

The hot-wire probe was traversed using a DISA type 55E40 traversing mechanism mounted above the working section. A 10:1 gear ratio was selected, giving the mechanism a maximum traversing length of 100 mm with a resolution of 0.02 mm.

The calibration apparatus was developed by F. M. Yowakim. The flow was produced by a variable-speed centrifugal blower that supplied air to a settling chamber with screens and discharged through an axisymmetric nozzle of exit diameter 3.81 cm. The probe holder was supported on a vice arrangement, so that the probe could be both rolled and yawed relative to the flow. When calibrating, a pitot-static tube connected to a pressure transducer was placed in the flow. Both the hot-wire probe and the pitot-static probe were positioned about 6.5 cm downstream of the nozzle exit plane, in the potential core of the jet from the nozzle. The pressure transducer was calibrated using an inclined water manometer, in the range from zero to 50 m/s (0-1200 mv). The calibration apparatus is shown in Figure 4.2.

4.3 Instrumentation

The traversing mechanism was controlled by means of a DISA Type 52C01 External Stepper motor and a DISA Type 52B01 Sweep drive unit. The motor performed 48 steps per revolution; 1 step corresponded to a probe movement of 0.021 mm. The sweep drive unit was accurate to 0.01 mm.

A TSI model 1053A constant temperature anemometer control unit was used with DISA 55P11 and 55P12 normal and inclined hot-wire probes respectively. On some runs the anemometer signal was linearized using a TSI model 1052 polynomial signal linearizer. The sensor resistance and thus the overheat ratio were controlled with a TSI model 1056 variable decade. Power was supplied by a TSI model 1051-1 monitor and power supply. The stability of operation of the anemometer system was monitored using a Tektronix 2213 60 MHz oscilloscope. RMS voltages were measured with a Bruel and Kjaer type 2425 voltmeter while DC signals were measured with a DISA type 55D31 digital voltmeter. This instrumentation is shown in Figures 4.3 and 4.4. Eventually, a Hewlett Packard 3054A automatic data acquisition/control system was used. This consisted of a HP 85 computer, a 3437A system voltmeter, a 3456A digital voltmeter, and a 3497A data acquisition/control unit.

The calibration instrumentation consisted of a Hewlett Packard 6217A power supply to drive a Rosemount 831A3 pressure transducer. When calibrated the pressure transducer had a near linear relationship between cm of water and millivolts in the range from zero to 50 m/s with the slope varying only in the fourth decimal place. The transducer output was monitored by a Sabtronics model 2000 multi-meter. During calibration

the DC anemometer output was monitored on an HP 3440A digital voltmeter. This meter was also used to set the polynomial coefficients on the linearizer, as it could be read to 0.001 volts and had a variable sweep time.

4.4 Design of the New Equipment

Once the traversing mechanism was attached to the stepper motor, it was necessary to support the assembly above the working section. This was accomplished with an aluminium bracket that could be clamped between the walls of the flat plate apparatus. With the motor and traversing mechanism mounted on this bracket, the guide tube moved perpendicular to the flow surface of the apparatus.

A drilled aluminium block allowed the probe to be supported on the guide tube near the flow surface and aligned with the flow as well as rolled to any desired angle. These angles were checked with a Precision Tool & Instrument Co., Ltd., microscope type 2150. Figure 4.5 shows the completed apparatus.

CHAPTER 5

EXPERIMENTAL RESULTS AND DISCUSSION5.1 Introduction

This chapter will follow the progression from the initial familiarization with the hot-wire to the point where acceptable results were obtained.

The familiarization with the hot-wire involved the use of a normal probe to obtain a calibration relationship between the bridge output voltage and the velocity in the potential core of the wall jet. The flow was then traversed at several downstream locations. Using the calibration curve established earlier, plots of U/U_m vs. $y/y_{1/2m}$ and $\frac{(u^2)^{1/2}}{U_m}$ vs. y/y_m were prepared and found to compare well with the results of Kind and Suthanthiran (1973). These results provided the confidence needed to move onto a slanted probe, and the more involved data reduction techniques. Using these techniques required the choice of a calibration procedure, the determination of the pitch and yaw sensitivity coefficients h and k (see Equations (5.8) and (5.9)) and several other modifications and checks to be discussed in detail in this chapter.

As the experimentation progressed, the effects of refinements such as mathematically linearizing the bridge output were studied. Eventually, the data acquisition/control unit was used which required the rewriting of existing FORTRAN programs in BASIC. Once this unit was on line and the equations had been reduced to a workable form, results with and

without the linearizer were obtained and compared with existing data.

5.2 Calibration Procedure

The calibration program was written by F. M. Yowakim as part of his Ph.D. thesis which has not yet been submitted. Thus a listing of the program will not appear in this thesis; however, a discussion of the program will.

According to Bradshaw (1971), universal calibrations like King's law or the .45 law give a useful guide to the best way of plotting results, but uncertainties about the properties of the probe and the fluid are such that individual calibration of probes is essential if good accuracy is to be obtained. The calibration method used in this thesis did just that. The data acquisition/control unit was used to take up to 21 sets of readings of the bridge output voltage of the anemometer and the output of the pressure transducer connected to the pitot tube. These readings were taken by the HP 3456A digital voltmeter using the system subroutine AVDCV which programs the voltmeter to average DC voltages readings over 10 power line cycles (0.17 secs.). The readings are then linearized using a fourth degree polynomial curve fitting routine for the TSI model 1052 linearizer. On output the program gives the coefficients to be set on the polynomial linearizer and S , the slope of the linearized calibration curve. Finally, the program plots the bridge output voltage vs. the air velocity and the linear output against the air velocity.

A typical output is shown in Figure 5.1. The first column is the reading being taken while the second and third are the transducer output in millivolts and the bridge output in volts respectively. VTRAN is the transducer voltage in millivolts with the zero reading subtracted off. VELOC is the result of converting the transducer voltage to a velocity using:

$$VELOC = \left(\frac{2 \rho H_2 O g R_a}{P_a} \right)^{1/2} (h \cdot T_2)^{1/2} \quad (5.1)$$

$$VELOC = .7459 \cdot (0.0134 \cdot VTRAN \cdot T_2)^{1/2}$$

where .0134 is the value of the pressure transducer calibration factor and T_2 is the air temperature in °K. BOUTPUT is the bridge output. NFLOW is the normalized flow or the value of the voltage that falls on a perfect straight line between 0 and 10 volts. COUTPUT (calculated output) is the value of the linearizer output that should be obtained if the given linearizer coefficients are set. PERROR is the percentage error between NFLOW and COUTPUT.

Before calibrating it was necessary to set the decade resistance, the reference set, and the stability and trim settings on the anemometer control unit. The procedures followed were as outlined in the owner's manual.

With these operations completed, the probe was positioned in the calibration vice such that the wire was normal to the nozzle flow and the probe had no pitch. In this orientation $U_B = U_T = 0$ and equation (3.2) reduces to:

$$U_{eff}^2 = U_N \quad (5.2)$$

This eliminates the need to know k and h before calibrating. A pitot-static probe was placed in the potential core behind and below the anemometer probe so as to avoid any thermal or aerodynamic interference. At this location (approximately $1\frac{1}{2}$ nozzle diameters from the nozzle) Yowakim found the potential core to be at least 13 mm wide thus the wire and the pitot tube would see the same velocity. Thus U_N becomes equal to the velocity in the potential core of the nozzle and S can be determined from $E = S U_{eff}$ where E is the linearized voltage output of the anemometer and U_{eff} is the potential core velocity sensed by the pitot-static tube.

Once the calibration is completed the linearizer coefficients are set, along with the zero and span. The zero and span are set by first feeding the value of the bridge output for zero flow to the input jack with the HP power supply and adjusting the output to read zero and then feeding the value of the bridge output at maximum flow and adjusting the span to read 10 volts.

5.3 Determination of the Sensitivity Coefficients

Previous studies such as Champagne (1967) and Mojola (1974) have shown the importance of k , the yaw sensitivity coefficients in accounting for the tangential cooling velocity encountered with inclined wire probes. Jørgensen (1971) however showed that because of the sensitivity to movements of the velocity vector out of the sensor prong plane - such as may occur in turbulent flows - the "overall" directional characteristic of the probe must be composed of a pitch correction (h) in addition to the yaw correction (k).

Thus the effective cooling velocity acting on a sensor is:

$$U_{\text{eff}}^2 = U_N^2 + k^2 U_T^2 + h^2 U_B^2$$

(see Figure 3.1) where

$$U_{\text{eff}}^2 = U_N^2 + k^2 U_T^2 \quad \text{for } \theta = 0$$

$$U_{\text{eff}}^2(\alpha) = U(\theta=0)^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) \quad \text{for } \theta = 0 \quad (5.3)$$

similarly

$$U_{\text{eff}}^2(\theta) = U(\alpha=0)^2 (\cos^2 \theta + h^2 \sin^2 \theta) \quad \text{for } \alpha = 0 \quad (5.4)$$

In order to evaluate the yaw and pitch factors it is first necessary to choose a calibration expression. The best results were obtained when King's law was used.

$$E^2 = A + B U^{1/2} \quad (5.5)$$

Substituting for U in equation (5.3) gives

$$\left(\frac{E(\alpha)^2 - A}{B} \right)^4 = \left(\frac{E(0)^2 - A}{B} \right)^4 (\cos^2 \alpha + k^2 \sin^2 \alpha) \quad (5.6)$$

$$\frac{\left(\frac{E(\alpha)^2 - A}{B} \right)^4}{E(0)^2 - A} = \cos^2 \alpha + k^2 \sin^2 \alpha \quad (5.7)$$

$$k = \frac{1}{\sin \alpha} \left\{ \frac{\left(\frac{E(\alpha)^2 - A}{B} \right)^4}{E(0)^2 - A} - \cos^2 \alpha \right\}^{1/2} \quad \text{for } \theta=0 \quad (5.8)$$

similarly

$$h = \frac{1}{\sin \theta} \left\{ \frac{\left(\frac{E(\theta)^2 - A}{B} \right)^4}{E(0)^2 - A} - \cos^2 \theta \right\}^{1/2} \quad \text{for } \alpha=0 \quad (5.9)$$

where $E(\alpha)$, $E(\theta)$, and $E(0)$ are for the same flow speed in the calibration jet.

Jørgensen (1971) noted that the yaw factor varied as a function of α , however since α had a constant value of 45° in this study, the value of k was determined for $\alpha=45^\circ$. Jørgensen also stated that using the value of h selected for $\theta=90^\circ$ would reduce errors on $U(0)$ thus this angle was chosen for the determination of h . The mean values of the sensitivity coefficients for the slanted 55P12 sensor obtained were:

$$k = 0.33$$

$$h = 1.19$$

When the linearizer was used the equations became somewhat more simplified since the calibration relationship became

$$E = S U_{\text{eff}} \quad (5.10)$$

Therefore equation (5.3) becomes

$$\frac{E(\alpha)^2}{S^2} = \frac{E(0)^2}{S^2} (\cos^2 \alpha + k^2 \sin^2 \alpha) \quad (5.11)$$

$$k = \frac{1}{\sin \alpha} \left\{ \frac{E(\alpha)^2}{E(0)^2} - \cos^2 \alpha \right\}^{1/2} \quad (5.12)$$

similarly

$$h = \frac{1}{\sin \theta} \left\{ \frac{E(\theta)^2}{E(0)^2} - \cos^2 \theta \right\}^{1/2} \quad (5.13)$$

Using the linearizer, the following values of the sensitivity coefficients were obtained:

$$k = .222$$

$$h = 1.183$$

Differences between the values obtained with and without the linearizer are attributable in part to errors introduced by the polynomial linearizer. Rodi (1975) notes that the yaw factor k is very sensitive to small measurements errors and discrepancies point to the difficulty in measuring k .

The values suggested by Jørgensen are

$$k = .32$$

$$h = 1.08$$

It should be noted that these values are for the DISA 55P11 normal probe. Differences in the experimentally determined values of h and that given by Jørgensen may result from the variation in prong geometry of the slanted probe as opposed to the normal probe, since h is in essence a measure of prong interference.

For the experiments in this study, values of k and h were taken to be 0.33 and 1.19 respectively.

To determine the effect of the variation of k on the experimental results, one set of data was reduced using a worst case value of $k=0$. At high speeds (above 10 m/s) the difference in the mean velocity was approximately five percent. The error increased from five to fifteen percent between one and ten m/s and is very high (125 percent) for velocities under one m/s. This would account partially for discrepancy of results in the outer portion of the wall jet flow, where the mean velocity approaches zero.

Errors in $\overline{uv}$ are constant at about 10%, while errors in $(\overline{u^2})^{1/2}$ range from approximately 5 to 10 percent.

5.4 Preliminary Experiments

As mentioned earlier, initial experiments were carried out with a normal probe. For this case $U_T = U_B = 0$ and signal interpretation is straightforward. The resulting plots of U/U_m versus, $y/y_{1/2m}$ and $(\overline{u^2})^{1/2}/U_m$ vs y/y_m appear as Figures 5.3 and 5.4. Figure 5.2 illustrates the quantities mentioned above. These results are in fair agreement with those of Kind and Suthanthiran (1973); the discrepancy in these preliminary results can be attributed to the simplified calibration procedure used and the need to interpolate local slopes off the calibration curve to calculate the longitudinal turbulence velocity.

Initial trials using a slanted probe and the method suggested by Acrivelllis (1978) to determine all mean and fluctuating velocity components as well as Reynolds shear stresses in a three-dimensional flow were not promising. One major difficulty that presented itself immediately, was in taking the data manually, since the reading usually changed before it could be written down making it difficult to determine the appropriate value to be recorded. Increasing the voltmeter time constants did little to alleviate the problem since signal fluctuations were still occurring, making accurate reading difficult.

It was hoped that by eliminating the electronics of the linearizer, less fluctuation in the output voltage would be noticed. This would

also eliminate the problem of drift in the linearizer coefficients. Thus trials were run with the bridge output being mathematically linearized. Once again the results showed little improvement and as it turned out the drift in the linearizer coefficients could be virtually eliminated by leaving the anemometer power supply on continuously.

At this point it was realized that the major problem lay in the inherent sensitivity of the original equations (3.23) to (3.25) and (3.29) to (3.37). These equations involved taking the square root of difference of pairs of voltages that were typically within 0.01 to 0.15 volts of each other, thus frequently leading to imaginary solutions. As well, for small values of V where, $E_1 \approx E_3$ and of W where $E_2 \approx E_4$ the denominators of equations (3.33) and (3.34) approach zero, giving a singularity at about the kind of flow conditions under study.

The equations were therefore re-worked by summing rather than differencing wherever possible, as outlined in Chapter 3. The resulting set of equations turned out to be algorithmically singular. A solution could only be found if a second probe (a normal probe) was introduced, giving an additional, independent, equation.

Since the flow had been checked for two-dimensionality by Kind and Suthanthiran (1973), who found the maximum velocity, thickness and shape of the velocity profiles to be invariant with spanwise position (except very near the side walls), the equations were reduced to two dimensions. The resulting equations are then (3.23), (3.24), (3.29), (3.30), (3.45) and (3.46).

5.5 Verification of the Cooling Law and Other Experimental Checks

5.5.1 Linearizer Check

This check consisted of first calibrating the probe as discussed in section 5.2. Once the calibration was complete and the proper coefficients had been set on the linearizer, readings of the linear output and the transducer voltage were taken at several flow velocities. The transducer voltages were converted to velocities using equation (5.1) and then a plot was made of the linear output vs. the velocity read by the pitot-static probe. As can be seen from Figure 5.5 the output was approximately linear and the slope of the line is within 4% of that calculated by the program.

The linearizer may be quickly checked by using the HP power supply to input a certain bridge voltage read from the calibration output and monitoring the linear output to see that it corresponds with the appropriate COUTPUT from the program.

5.5.2 Verification of the Cooling Law

The procedure to verify the cooling law was to first calibrate the probe and set the linearizer coefficients, and then to determine k and h as outlined previously. The probe would then be placed in the flow at a known yaw, roll and pitch angle, and the linear voltage output and the transducer voltage were read. The velocity read by the pitot-static tube (and transducer) can be calculated using equation (5.1). Also

$$E_{\text{LINEAR}}^2 = S^2 U_{\text{eff}}^2$$

$$U_{\text{eff}}^2 = \frac{E_{\text{LIN}}^2}{S^2}$$

$$U_{\infty}^2 = U_{\text{eff}}^2 \times \frac{1}{\left(\frac{U_{\text{eff}}^2}{U_{\infty}^2}\right)} \quad (5.14)$$

Thus U_{eff} can be calculated from the linear output voltage and the slope of the calibration curve. If a relationship for $\left(\frac{U_{\text{eff}}}{U_{\infty}}\right)^2$ can be found then the velocity U_{∞}^2 can be calculated using the cooling law and compared with U_{∞}^2 as given by the pitot-static tube.

From Figure 5.6:

$$U = U_{\infty} \cos\theta \cos\delta$$

$$V = U_{\infty} \cos\theta \sin\delta$$

$$W = U_{\infty} \sin\theta \quad (5.15)$$

As shown earlier

$$U_N = U \cos\alpha + V \cos\psi \sin\alpha - W \sin\psi \sin\alpha$$

$$U_T = -U \sin\alpha + V \cos\psi \cos\alpha - W \sin\psi \cos\alpha$$

$$U_B = V \sin\psi + W \cos\psi$$

Substituting for U, V and W gives

$$U_N = U_{\infty} \cos\theta \cos\delta \cos\alpha + U_{\infty} \cos\theta \sin\delta \cos\psi \sin\alpha - U_{\infty} \sin\theta \sin\psi \sin\alpha \quad (5.16)$$

$$U_T = -U_{\infty} \cos\theta \cos\delta \sin\alpha + U_{\infty} \cos\theta \sin\delta \cos\psi \cos\alpha - U_{\infty} \sin\theta \sin\psi \cos\alpha \quad (5.17)$$

$$U_B = U_\infty \cos\theta \sin\delta \sin\psi + U_\infty \sin\theta \cos\psi \quad (5.18)$$

Also

$$U_{\text{eff}}^2 = U_N^2 + k^2 U_T^2 + h^2 U_B^2$$

Thus

$$\begin{aligned} \frac{U_{\text{eff}}^2}{U_\infty^2} &= [\cos\theta \cos\delta \cos\alpha + \cos\theta \sin\delta \sin\alpha \cos\psi - \sin\theta \sin\psi \sin\alpha]^2 \\ &+ k^2 [-\cos\theta \cos\delta \sin\alpha + \cos\theta \sin\delta \cos\psi \cos\alpha - \sin\theta \sin\psi \cos\alpha]^2 \\ &+ h^2 [\cos\theta \sin\delta \sin\psi + \sin\theta \cos\psi]^2 \end{aligned} \quad (5.19)$$

Trials were carried out at three different roll angles, ψ , with the difference between the value of U_∞ calculated using the cooling law and that found with the pitot-static tube varying from 0.04% to 6.4% (see Table 5.1).

5.5.3 Temperature Effects

One of the major difficulties in using hot-wires to measure velocities, is the effect of fluid temperature changes on the heat transfer from the sensor. Since the hot-wire responds both to velocity fluctuations and temperature fluctuations, one must somehow compensate for variation in temperature.

In this study, the problem was partially remedied by running the test rig, before taking any measurements, until an equilibrium temperature in the air leaving the slot was reached. The effect on the velocity

fluctuation measurements of the warm wall jet mixing with the cooler ambient air was minimized by running the wire at as high a temperature as possible. An overheat ratio of 1.8 was felt to be the maximum safe operating value for the probe used. Bradshaw (1971) confirms that this is a valid means of minimizing temperature effects. A rough estimate of the maximum percentage error introduced by temperature variation is 7% based on the following relationship:

$$\% \text{ error} = \frac{2 (T_{\text{JET}} - T_a)}{(T_{\text{WIRE}} - T_a)} \times 100 \quad (5.20)$$

Thermo-Systems, Inc. (1968)

5.6 Implementation of the Data Acquisition Unit and Associated Results

Early trials with the data acquisition unit involved manually triggering the unit to take seven average DC voltages readings at each position. Each reading was taken over ten line power cycles, thus providing the average. From these readings the high and low values were eliminated and the remaining values averaged to obtain what was hoped would be a good representative value of the average DC voltage. Although values of the longitudinal mean velocity U were easily obtained, the V component of velocity provided difficulty. Examination of equation (3.45) showed that for V to be real, $(\overline{E_1^2} + \overline{E_3^2}) > (\overline{E_2^2} + \overline{E_4^2})$ was required. From the data in these early trials, it was found that although the average values may have given imaginary solutions, values could be picked from the seven taken at each position, which would give a solution. In other words, by choosing different values from within the range of the seven readings taken

for E_1 and/or E_3 , the product $\overline{E_1}^2 + \overline{E_3}^2$ could be made to be less than, equal to, or greater than $\overline{E_2}^2 + \overline{E_4}^2$ and thus the solution for V could be made to be imaginary, zero or real. Thus it was impossible to determine V since slight variation in the signal (at any one location) between one reading and the next often meant the difference between a solution or no solution for V . The sensitivity of the equations makes them unsuitable for determining V and W .

Having determined that V could not be found using the aforementioned data reduction technique, the author decided to concentrate on obtaining U , $\overline{u^2}$ and $\overline{uv}$, using equations (3.29), (3.46) and (3.47). $\overline{v^2}$ was also calculated using equation (3.48) but once again due to the sensitivity of this equation the results were of little use. In fact, as came to light late in the course of this work, Acrivlellis' (1978) method is only valid when V , W , $(\overline{u^2})^{1/2}$, $(\overline{v^2})^{1/2}$ and $(\overline{w^2})^{1/2}$ are all small relative to U . Fortunately, these conditions are approximately satisfied in the present flow at positions not too close to the outer edge of the wall jet, and equations (3.29), (3.46) and (3.47) are suitable for data reduction.

To speed up the process of acquiring data, a program was written for the data acquisition unit. When triggered the unit would take 100 readings of the voltage and 100 values of the AC voltage and average them. The sampling rate was approximately 2.5 1/sec. The procedure was carried out at twenty locations in the flow for the four sensor positions, taking approximately two hours for an entire profile. This was a significant reduction from the six hours required to do a run manually. Although drift in the calibration parameters was not a problem in these experiments,

(with the typical difference in S from one day to the next being in the order of half a percent) by reducing the time of the run one reduces the risk of calibration drift. The unit was also programmed to perform the data reduction calculations (using equations (3.23), (3.24), (3.29), (3.30), (3.45) and (3.46)) immediately following each experimental run thus eliminating the time consuming procedure of entering the 160 data points for each trial on the University's central computer as had been done up to this point. The program shown in Appendix G accepts linearizer voltage inputs. The program pauses after each average reading is printed to allow the user to move the probe. After every twenty readings the program pauses to allow the user to roll the probe and return it to the starting location. The only input required is S the slope of the calibration curve, which must be set before each run. By changing three lines (40, 120, 300) the program can be made to accommodate any number of probe locations per traverse.

The program shown in Appendix H was designed to accept bridge voltages and then mathematically linearize them. This program is similar to the previous one except that in addition to the slope the user must input $B1$ to $B5$, the five linearizer coefficients obtained from the calibration program output. Appendix I shows a typical output of this program during an experimental run.

Figure 5.7 shows the mean velocity profiles obtained when the linearizer was incorporated in the anemometer circuit. The results agree with those of Kind and Suthanthiran (1973), and Irwin (1973). Scatter in the outer region may be attributed to higher percentage errors

encountered with the linearizer at low flow velocities. This is evident in Figure 5.1 where the percentage error is highest at low flow velocities. This is further substantiated by the fact that the curve obtained by mathematically linearizing the bridge output (Figure 5.8) does not show this scatter.

As is evident from these profiles, the method is not suited to near wall measurements. Since measurements are taken at four roll angles at each position in the traverse, the probe must be positioned far enough from the wall to allow it to be rolled through 360 degrees, making near wall measurements impossible.

The turbulent shear stress profiles presented in Figure 5.9 were obtained with the linearizer in the circuit. Once again these results show more scatter in the outer region than those obtained without the linearizer. (Figure 5.10).

It is not surprising that some scatter exists in these profiles since among available sets of data presently in existence, there is considerable discrepancy. The maximum value of these profiles compares well with that of Irwin (1973) but is about 10 percent higher than the values suggested by Guitton (1970) and Wilson and Goldstein (1976).

The profile drops in the correct manner near the outer edge of the wall jet, unlike the results of Giles et al (1966). The value of $y/y_{1/2m}$ at which the maxima occur compares well with those of Guitton (1970) and Wilson and Goldstein (1976).

It is interesting to note that the point of zero shear stress occurs before the velocity maximum. The mean value of $y/y_{1/2m}$ was found to be 0.167 while the value of $y/y_{1/2m}$ for zero shear stress can be extrapolated from Figure 5.10 to be about 0.1. Thus the shear stress falls to zero at about 60% of the height to the velocity maximum, a result confirmed by Launder and Rodi (1981). The physical explanation of these results is as follows: in a wall jet in still air the excess velocity is large compared with the stagnant external flow, and the turbulence of the boundary-layer region near the wall is less vigorous than that in the outer layer and hence diffusion of outer-layer properties into the boundary-layer takes place.

Figure 5.11 presents the longitudinal turbulence profiles obtained at various stations with the linearizer in the circuit. Results without the linearizer are shown in Figure 5.12 and once again show little difference from those with the linearizer. The curves peak at the correct distance from the wall but have a maximum value of $(\overline{u^2})^{1/2}/U_m$ about 25 percent higher than the results presented in Rodi and Launder (1981). However, Figure 5.13 shows that the results of this study do agree with those of Kind and Suthanthiran (1973) obtained for the same apparatus. Thus the differences appear to be caused by characteristics of the flow in this particular apparatus and not by the data reduction technique.

The difference mentioned above may be related to the fact that the wall-jet growth rate for this apparatus is higher than the conventionally accepted value, thus the flow is mixing and decelerating faster and one would expect a somewhat higher level of turbulence. Figure 5.14 shows

the growth rate $dy_{m/2}/dx$ which was found to be .088 in this study. A value of 0.081 was found by Kind and Suthanthiran (1973) and although both results are higher than the value of .07 found by Bradshaw and Gee (1960), Patel (1962), and Garthshore and Hawaleshka (1964) they are close to the value of 0.085 found by Schwarz and Cosart (1961) and Wilson and Goldstein (1976).

The reasons for the higher value of growth rate on this apparatus are still unclear. Kind and Suthanthiran (1973) were unable to change the value by varying the turbulence level of the flow leaving the blowing slots from 0.3% to 2% or by varying the temperature difference between the blowing air and the ambient air from 2.8 to 16.4°C.

In general, the good collapse of profiles from different streamwise positions in the flow shows that the results are self consistent. The fact that the experiments gave the same non-dimensional answers near the slot and further downstream indicates that spatial resolution is not a problem whether one is dealing with the strong gradients of near slot flow or the reduced gradients of the flow far downstream of the slot. Except for the results of $(\overline{u^2})^{1/2} / U_m$, experimental results agree well with those of other experimenters as can be seen from Figures 5.7, 5.8, 5.9 and 5.10. Although the results of $(\overline{u^2})^{1/2} / U_m$ are self consistent they do not agree totally with those of most other experimenters as discussed above.

The present results therefore indicate that the final method used in this thesis work can be used in the boundary layer control apparatus to determine U , $\overline{uv}$ and $\overline{u^2}$, assuming sufficiently low turbulence levels exist.

Turbulence levels in the boundary layer control flow are expected to be about the same as those in the plane still-air wall jet flow: on the one hand, the convex flow curvature of the boundary layer control flow de-stabilizes the flow in the outer regions of the wall jet, tending to increase turbulence intensity, but on the other hand, the mean flow velocity at the outer edge of the wall jet is non-zero and this decreases the local turbulence intensity.

5.7 Summary

This chapter has presented the experimental results obtained with both a normal and a slanted hot-wire. Results obtained with and without the linearizer differ little except for some scatter in the outer flow regions of the curves obtained when the linearizer was used.

Checks of the linearizer and the cooling law in three dimensions proved successful as did the determination of the pitch and yaw sensitivity coefficients h and k .

Once the equations had been reduced to a less sensitive form and the data acquisition/control unit was implemented, the present results for U , $(\overline{u^2})^{1/2}$ and $\overline{uv}$ were found to be self consistent and to compare well with previous studies. This suggests that the same techniques can be used in the boundary layer control apparatus of Figure 1.1 without excessive error due to spatial resolution.

CHAPTER 6

MODIFICATIONS TO THE BOUNDARY LAYER CONTROL APPARATUS6.1 Introduction

In the boundary layer control (BLC) flow shown in Figure 1.1 the boundary layer approaching the blowing slot is very thin; this is not typical of many applications where BLC is used. Therefore, a forebody was designed to extend upstream of the circular cylinder in order to produce a thicker boundary layer. This chapter discusses the existing BLC apparatus, the purpose of the forebody, and the design and construction of the forebody.

6.2 Existing Apparatus and Design of the Forebody

The existing apparatus consists of a 12.7 cm diameter circular cylinder mounted in the 50.8 x 76 cm working section of a low speed wind tunnel. The cylinder has a tangential injection slot with a mean thickness of 0.91 mm. At either end of the cylinder are porous end walls for suction to suppress end effects and a slot is located on the working section wall near the cylinder for tangential injection in the side wall as well. The existing apparatus is shown in Figure 1.1.

The forebody was designed to thicken the boundary layer at the slot for reasons mentioned in the introduction. The forebody was to, if possible, keep a favourable or only moderately adverse pressure gradient near the nose and to have a zero or slightly adverse pressure gradient

near the slot. The fact that the forebody boundary layer might be slightly three-dimensional was not of great concern since as long as uniformity was reasonably well maintained at and downstream of the slot, a little crossflow would not have a significant effect on the flow of interest, namely the flow downstream of the slot. The overall length of the forebody was to be approximately 63 cm with a leading edge radius of 2.54 cm.

The forebody was designed using a computer solution for incompressible flow over airfoils of arbitrary section. The program solves the flow over an arbitrary section shape by replacing the contour by a number of plane uniform two-dimensional Vortex sheets whose strengths vary linearly between junction points. It was originally written as a course-work requirement but was modified for this special case. The modification was to incorporate the method of images in the program to access the effects of the wind-tunnel wall on the forebody.

Since the original program is only suitable for single closed contours, the forebody and its image must be joined by an additional panel. Thus the forebody was modelled using twenty panels, then a twenty first panel joined the forebody to its image at their trailing edges (the trailing edge being the furthest downstream location on the cylinder), and the image was modelled by an additional twenty panels. This arrangement is shown in Figure 6.1. Since the panel between the trailing edge junction points was not really desired, the boundary condition equation for the control point on this panel was deleted and the strengths of the vortices at either end of it were set equal to zero. Thus this so-called "false panel" accomplished the purpose of completing the closed

contour but then is overwritten to ensure that it induces zero velocity everywhere. Fortunately, one is also able to set the strengths of the vortices on the trailing edges equal to zero to satisfy the Kutta condition. A copy of the modified program and flowchart can be found in Appendix J.

Four quite different profiles were tried to obtain the final design. The first shape was a symmetric body 63.5 cm in length with a leading edge radius of 2.54 cm and a thickness to chord ratio of 0.27. Although this produced a favourable pressure gradient at the leading edge, the pressure gradient at the slot was not as desired.

The second shape with a thickness to chord ratio of 0.2 and leading edge radius of 1.25 cm was still symmetrical and did little to improve on the first.

The third shape was designed with a bulge near the leading edge (on the wall side) in hopes of producing a more adverse pressure gradient over a greater portion of the body. Using an analysis similar to that done for separation calculations in a conical diffuser, a rough estimate of the minimum allowable distance between the forebody and the wall to avoid separation was obtained. This distance of approximately 2.5 cm was used for the third profile but the resulting coefficients of pressure were too high.

For the fourth shape, the forebody was only permitted to come within 3.81 cm of the wall. This then lowered the maximum coefficient of pressure to the desired value of approximately four while maintaining the desired pressure gradients at the leading edge and the slot. Figure 6.1

shows the forebody and its image while Table 6.1 shows the coordinates of the control points and the coefficients of pressure. Figure 6.2 shows the evolution of the profile shape.

6.3 Construction of the Forebody

Once the profile had been decided upon, strips of styrofoam were cut to the desired shape using electrically heated wire and a plywood template. The shaped styrofoam was then laminated and coated with a plaster compound. The resulting forebody shape was made uniform by repeated coats of a joint filler compound followed by sanding. The resulting finish was left slightly rough to aid in boundary layer development. The forebody is shown in Figure 6.3.

Although measurements were not carried out on the modified apparatus it is felt that the boundary layer at the slot will be significantly thickened by the forebody.

CHAPTER 7

, CONCLUSIONS7.1 Conclusions

The experimental results presented in Chapter 5 highlight several features of this flow and the methods used to examine it. First, the procedure for determining the pitch and yaw sensitivity coefficients h and k gives acceptable results if one is very careful to determine angles as accurately as possible and to have point rotation of the probe. Despite disagreement amongst various authors on the value of k it is interesting to note that variation in k has little effect on mean velocities over 10 m/s and causes errors ranging from 5 to 10 percent in $\overline{uv}$ and $(\overline{u^2})^{1/2}$ at these mean velocities.

The cooling law check discussed in Chapter 5 gave very good results which would suggest that it is advantageous to include h , the pitch sensitivity coefficient, in the universal cooling law.

Temperature effects can be minimized by using as high an overheat ratio as possible, in this case 1.8.

The problem of the high growth rate encountered with the present apparatus is still unresolved.

Except for slightly less scatter in the outer flow region, there is little difference between results obtained with and without the linearizer.

The method of Acrivlellis (1978) is not suited to highly three-dimensional turbulent flows. If one assumes low turbulence levels and $V=0$ and $W=0$ the equations can be reworked into a form suitable for determining U , $\overline{u^2}$, and $\overline{uv}$. Thus the method would be suitable for the analysis of these flow parameters in the boundary layer control apparatus.

Most importantly, no problem with spatial resolution was encountered. Despite the fact that the wire was oriented normal to the wall (suggesting that there was a significant variation in mean velocity across the span of the sensor) the profiles collapsed well whether in the severe gradients of the upstream flow or the more moderate gradients of the downstream flow.

CHAPTER 8

FUTURE WORK8.1 Future Work

- If one wishes to determine the shear stress profile in the flow near the injection slot of the boundary-layer control apparatus, then the present method is suitable; however, if determination of V , $\overline{v^2}$ and $\overline{w^2}$ is necessary, then the present method is unsuitable.

With the forebody already mounted in the wind tunnel work section, it is a simple matter to complete the apparatus for experimental measurements of the shear stress profile.

REFERENCES

- Acrivlellis, M. An Improved Method for Determining the Flow Field of Multidimensional Flows of Any Turbulence Intensity, DISA Information, No.23, 1978, 11-15.
- Acrivlellis, M. Flow Field Dependence on Hot-Wire Probe Cooling Law and Probe Adjustments, DISA Information, No.23, 1978, 17-23.
- Bartenwerter, W., and Acrivlellis, M. Letters to the Editor - Comments On: The Method for Analyzing Hot-Wire Signals, DISA Information, No.23, 1979, 39-42.
- Bertin, J.J. and Smith, M.L. Aerodynamics for Engineers, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1979, 388-389.
- Bradshaw, P. An Introduction to Turbulence and Its Measurement, Pergamon Press, Oxford, 1971, 106-132.
- Bradshaw, P., and Gee, M.T. Turbulent Wall Jets Without an External Stream, Aero. Res. Council, R&M, 3252, A60.
- Champagne, F.H., and Sleicher, C.A. Turbulence measurements with inclined hot-wires, Part II, Hot-Wire Response Equations, J. Fluid Mech., vol.28, part 1, 1967, 177-182.
- Champagne, F.H., Sleicher, C.A., and Wehrmann, D.H. Turbulence measurements with inclined hot-wires, Part I, Heat transfer experiments with inclined hot-wires, J. Fluid Mech., vol.28, part 1, 1967, 153-175.
- Comte-Bellot, G. Hot-Wire Anemometry, Annual Rev. of Fluid Mech., Vol.8, 1976, 209-231.
- Del Toro, V. Principles of Electrical Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1972, 28.

- Gartshore, I., and Hawaleshka, O. The design of a two-dimensional blowing slot and its application to a turbulent wall jet in still air, Tech. Note 64-5, Department of Mech. Eng., McGill University, Montreal, 1964.
- Giles, J.A., Hays, AP., and Sawyer, R.A. Turbulent wall jets on logarithmic spiral surfaces, Aero. Quart., Vol.17, 1966, 201.
- Guitton, D.E. Some contributions to the study of equilibrium and non-equilibrium wall jets over curved surfaces, Ph.D. thesis, Mech. Eng. Dept., McGill University, 1970.
- Irwin, H.P.A. Measurements in a self-preserving plane wall-jet in a positive pressure gradient, J. Fluid Mech., Vol.61, part 1, 1973, 33-63.
- Jørgensen, F.E. Directional Sensitivity of Wire and Fiber Film Probes, DISA Information, No.11, 1971, 31-37.
- Kind, R.J., Gooden, K., and Dvorak, F.A. Measurements of Flows with Tangential Injection and Comparison with Prediction Methods, AIAA Journal, Vol.17, No.7, 1979, 730-735.
- Kind, R.J. and Suthanthiran, K. The interaction of two opposing plane turbulent wall jets, J. Fluid Mech., Vol.58, part 2, 1973, 389-402.
- Launder, B.E., and Rodi, W. The Turbulent Wall Jet, Prog. Aerospace Sci., Vol.19, 1981, 81-128.
- Mojola, O.O. A Hot-Wire Method for 3-D Shear Forces, DISA Information No.16, 1974, 11-14.
- Patel, R.P. Self-preserving, two-dimensional turbulent jets and wall jets in a moving stream, M.Eng. Thesis, Dept. of Mech. Eng., McGill University, Montreal, 1962.
- Rodi, W. A New Method of Analyzing Hot-Wire Signals in Highly Turbulent Flow, and its Evolution in a Round Jet, DISA Information, No.17, 1975, 9-18.
- Schwarz, W.H., and Cosart, W.P. The two-dimensional wall-jet, J. Fluid Mech., Vol.10, 1961, 481.

Thermo-Systems, Inc.

Hot Film and Hot Wire Anemometry,
Theory and Application Bulletin TB5,
Thermo-Systems Inc., Minnesota.

Thermo-Systems, Inc.

Operating Manual, 1054 A Hot-Wire
Anemometer, Thermo-Systems, Inc.,
Minnesota, 1968.

Wilson, D.J., and
Goldstein, R.J.

Turbulent Wall Jets With Cylindrical
Streamwise Surface Curvature,
Transactions of the ASME, Vol.98,
1976, 550-557.

$$\theta=20^\circ \quad \delta=24^\circ \quad \psi=0^\circ \quad \alpha=45^\circ$$

U_∞ from cooling law	U_∞ from pitot- static probe	% Difference
14.697 m/s	14.366	2.30
22.876	21.990	4.03
28.918	27.747	4.22
33.255	33.267	- .036
37.666	38.050	-1.02
41.371	42.586	-2.8

$$\theta=-30^\circ \quad \delta=22^\circ \quad \psi=270^\circ \quad \alpha=45^\circ$$

U_∞ from cooling law	U_∞ from pitot- static probe	% Difference
14.948	14.49	3.16
23.298	22.1546	5.16
29.916	28.255	5.88
35.698	33.651	6.08
40.782	38.737	5.28
44.093	43.140	2.21

$$\theta=30^\circ \quad \delta=23.5^\circ \quad \psi=90^\circ \quad \alpha=45^\circ$$

U_∞ from cooling law	U_∞ from pitot- static probe	% Difference
13.550	14.480	-6.4
21.290	21.730	-2.02
28.470	28.252	0.77
34.540	33.960	1.70
39.220	38.757	1.19
42.290	43.162	-2.02

Table 5.1 Comparison of Freestream Velocity Using
Cooling Law and Pitot-Static Probe

AIRFOIL COORDINATES AND PRESSURE COEFFICIENTS

X	Y	CP
.0952	5.4784	.915
.4613	6.3623	.215
1.1378	7.0388	-.841
2.0217	7.4049	-1.061
4.7500	7.4750	-.537
9.2500	7.3500	-.263
16.0000	6.7500	-.190
22.0000	5.8750	-.063
25.0000	4.7500	.387
27.0000	3.5000	.967
27.0000	2.5000	-1.421
25.0000	1.8750	-2.429
22.0000	1.6250	-4.024
16.0000	1.7500	-3.595
9.2500	2.1250	-1.994
4.7500	2.3750	-1.578
2.0217	2.5952	-1.777
1.1378	2.9613	-1.393
.4613	3.6378	-.087
.0952	4.5217	.863
.0000	.0000	1.000
.0952	-4.5217	1.000
.4613	-3.6378	.757
1.1378	-2.9613	-.590
2.0217	-2.5952	-1.261
4.7500	-2.3750	-1.354
9.2500	-2.1250	-1.868
16.0000	-1.7500	-3.431
22.0000	-1.6250	-3.833
25.0000	-1.8750	-2.935
27.0000	-2.5000	-.276
27.0000	-3.5000	.866
25.0000	-4.7500	-.146
22.0000	-5.8750	-.391
16.0000	-6.7500	-.541
9.2500	-7.3500	-.621
4.7500	-7.4750	-1.113
2.0217	-7.4049	-2.126
1.1378	-7.0388	-2.109
.4613	-6.3623	-.728
.0952	-5.4784	.740

Table 6.1 Forebody Coordinates and Pressure Coefficients

FIG. I.1 SKETCH OF APPARATUS AND FLOW FIELD

Figure 2.1 Cylindrical Hot-Wire Sensor and Support Prongs

Figure 2.2 Schematic of Constant Temperature Control Unit

Figure 3.1 Inertial and Wire-Oriented Coordinate System

Figure 3.2 The Hot-Wire Positions Selected

Figure 4.1 Wall-Jet Apparatus

Figure 4.2 Photograph of the Calibration Apparatus

Figure 4.3 Photograph of the Instrumentation

Figure 4.4 Schematic of Instrumentation

Figure 4.5 Photograph of the Flat Plate Apparatus

Figure 5.2 Wall Jet Notation

Figure 5.3 Wall Jet Mean Velocity Profiles Using Normal Probe.

∇ $x/t = 93.23$, $+$ $x/t = 68.66$, $\blacksquare$ $x/t = 77.48$,
 — Kind, 1973

Figure 5.4 Wall Jet Longitudinal Turbulence Profiles for Normal Probe in the Outer Flow, ∇ $x/t=93.23$, $+$ $x/t=68.66$, $\blacksquare$ $x/t=77.48$, — Kind (1973).

Figure 5.5 Linearized Output

Figure 5.6 Probe Angles for Cooling Law Check

Figure 5.7 Wall Jet Mean Velocity Profiles Using Slanted Probe (with linearizer). ∇ $x/t=137.32$, $\blacksquare$ $x/t=135.43$, $+$ $x/t=140.16$, $\blacklozenge$ $x/t=178.58$, --- Kind (1973), — Irwin (1973).

Figure 5.8 Wall Jet Mean Velocity Profiles Using Slanted Probes
 (without linearizer), ∇ $x/t=162.84$, $\blacksquare$ $x/t=172.59$,
 $+$ $x/t=109.72$, $\blacklozenge$ $x/t=119.69$,
 --- Kind (1973), — Irwin (1973).

Figure 5.9 Wall Jet Shear Stress Profiles (with linearizer)
 ∇ $x/t=137.32$, $\blacksquare$ $x/t=135.43$, $+$ $x/t=140.16$, $\blacklozenge$ $x/t=178.58$,
 — Irwin (1973), - - Guitton (1970),
 — — Giles et al (1966), ···· Wilson and Goldstein (1976).

Figure 5.10 Wall Jet Shear Stress Profiles (without linearizer)

▼ $x/t=162.84$, □ $x/t=172.59$, + $x/t=107.72$, ♦ $x/t=119.69$.

— Irwin (1973), --- Guitton (1970),

- · - Wilson and Goldstein (1976), — — — Giles et al. (1966).

Figure 5.11 Wall Jet Longitudinal Turbulence Profiles (with linear).
 ∇ $x/t=137.32$, $\blacksquare$ $x/t=135.43$, $+$ $x/t=140.16$, $\blacklozenge$ $x/t=178.58$
 --- Guitton (1970).

22
OF / DE

Figure 5.12 Wall Jet Longitudinal Turbulence Profiles (without linearizer)
 ♦ $x/t=119.69$, ■ $x/t=177.95$, + $x/t=107.72$,
 — Guitton (1970).

Figure 5.13 Wall Jet Longitudinal Turbulence Profiles.
 ∇ $x/t=137.32$, $\blacksquare$ $x/t=119.69$, $+$ $x/t=107.72$,
 --- Kind (1973).

Figure 5.14 Wall Jet Growth

Case #4

Figure 6.1 Final Forebody Profile Shape

Figure 6.2 Evolution of the Forebody Profile Shape

--- Trial #1 -.-.- Trial #2 -.-.- Trial #3
—— Final Decision

Figure 6.3 Photograph of Forebody

COMPUTER PROGRAM VARIABLE NAMES

HOT-WIRE

E	DC voltage
EL	RMS voltage
LEQTIF	Matrix solving subroutine
A	Input matrix of dimension Z by Z containing the coefficient matrix of the equation $AX=B$
B	Input matrix of dimension Z x M containing right-hand sides of the equation $AX=B$
WKAREA	Work area of dimension greater than or equal to Z
IA	Row dimension of A and B exactly as specified in the dimension statement of the calling program
IDGT	If IDGT is greater than 0 the elements of A and B are assumed to be correct to IDGT decimal digits and the routine performs an accuracy test
IER	Error parameter IER=129 A is algorithmically singular IER=34 Accuracy test failed
Z	Order of A, number of rows in B
M	Number of right-hand sides
FV	Fluctuating velocities u, v, w
K	Yaw sensitivity coefficient k of α probe
KB	Yaw sensitivity coefficient k of β probe
H	Pitch sensitivity coefficient h of α probe
HB	Pitch sensitivity coefficient h of β probe
S	Slope of α probe calibration curve
SB	Slope of β probe calibration curve
ALPH	α
BETA	β

ELS	RMS voltage of β probe
UV,UW,VW	Reynold's stresses $\overline{uv}$, $\overline{uw}$, $\overline{vw}$
N	Number of hot-wire positions chosen
L	Number of locations in hot-wire traverse
HOT-WIRE B	
EZ	DC voltage at zero flow
D1 to D5	Coefficients of the polynomial linearizer obtained from calibration program
HOT-WIRE 1	
MV	Mean velocities, U, V, W
HOT-WIRE 2	
UVL, UWL	Reynold's shear stresses $\overline{uv}$, $\overline{uw}$
UL, VL	Fluctuating velocity components $(\overline{u^2})^{1/2}$, $(\overline{v^2})^{1/2}$
HOT-WIRE 3	
EZL	RMS voltage at zero flow
C01 to C05	Coefficients of polynomial linearizer obtained from calibration program
HOT-WIRE 4	
VSQ	u^2
VSW	v^2

PROGRAMS FOR DATA ACQUISITION UNIT

DATA1

C0	Channel 0 on control unit
C1	Channel 1 on control unit
P	Digital voltmeter assignment
E	DC voltage
F	RMS voltage

R	Probe angle alpha
U	Mean velocity cmpt U
U1	UV
U2	UW
U3	$\overline{UV}$
U4	$\overline{UW}$
U5	$(\overline{u^2})^{1/2}$
U6	$(\overline{v^2})^{1/2}$
I	Number of locations in hot-wire traverse
J	Number of probe positions required
K	Number of voltage readings taken of each channel at each location

DATAB

Z	DC voltage at zero flow
Z1	RMS voltage at zero flow
B1 to B5	Coefficients of the polynomial linearizer
Init	Subroutine to initialize the HP interbus
DCV	Subroutine to take DC voltage readings

FOREBODY DESIGN PROGRAM

EX(I)	X coordinate of junction pt
X(I)	Y coordinate of junction pt
XX	ΔX
YY	ΔY
EXX(I)	X coordinate of control pt
WHY(I)	Y coordinate of control pt
A(I,M)	Matrix of influence coefficients

X(M), B(M)	Matrix of vortex strengths on input and output respectively
B(M)	Right-hand side of equation $AX=B$ on input
AA,BB,CC,DD	Coefficients in expressions for velocities parallel and normal to vortex sheet
THETA	θ angle between vortex sheet and line to control pt at which velocities are being calculated
PHI	ϕ angle between horizontal and vortex sheet
L(M)	l length of panel
R	Distance from junction pt on one panel to control pt on another
ALPHA	α angle of attack of airfoil
N	n number of panels
CP(M)	Coefficient of static pressure

APPENDIX A

COMPUTER PROGRAM HOT-WIRE

```

1 - 1.000 DIMENSION E(6),EL(6),A(3,3),B(3),FV(3),WKAREA(9)
2 - 2.000 REAL K,ALPH,H,S,E,EL,A,B
3 - 3.000 INTEGER N,M,IA,IDGT,IER,Z
4 - 4.000 READ*,K,KB,H,HB,S,SB,N,ALPH,BETA,Z,M,IA,IDGT
5 - 5.000 L=1
6 - 6.000 10 READ*,(E(I),I=1,N),(EL(I),I=1,N),ELS
7 - 7.000 D=1.414213562
8 - 8.000 C1=E(1)**2-E(3)**2
9 - 9.000 C2=E(4)**2-E(2)**2
10 - 10.000 C3=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)-H**2
11 - 11.000 C4=D*(S**2)*((1+K**2)**2)*((SIN(2*ALPH))**2)
12 - 12.000 C5=(E(6)**2-E(5)**2)*D+E(2)**2-E(4)**2
13 - 13.000 C6=2*(S**2)*(1-(K**2))*(SIN(2*ALPH))
14 - 14.000 U=SQRT((C1*C2*C3)/(C4*C5))
15 - 15.000 V=SQRT((C1*C5)/(2*D*(S**2)*C3*C2))
16 - 16.000 W=SQRT((C2*C5)/(2*D*(S**2)*C3*C1))
17 - 17.000 UV=(EL(1)**2-EL(3)**2)/(C6)
18 - 18.000 UW=(EL(4)**2-EL(2)**2)/(C6)
19 - 19.000 VW=((EL(6)**2-EL(5)**2)*D+EL(2)**2-EL(4)**2)/(2*D*(S**
20 - 20.000 *2)*C3)
21 - 21.000 98 FORMAT(2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4
22 - 22.000 *,//)
23 - 23.000 99 FORMAT(6X,"U",9X,"V",9X,"W",9X,"UV",8X,"UW",8X,"VW")
24 - 24.000 WRITE(6,99)
25 - 25.000 WRITE(6,98)U,V,W,UV,UW,VW
26 - 26.000 C
27 - 27.000 C
28 - 28.000 C
29 - 29.000 C
30 - 30.000 C
31 - 31.000 C
32 - 32.000 C
33 - 33.000
34 - 34.000
35 - 35.000
36 - 36.000
37 - 37.000
38 - 38.000
39 - 39.000
40 - 40.000
41 - 41.000
42 - 42.000
43 - 43.000
44 - 44.000
45 - 45.000
46 - 46.000
47 - 47.000
48 - 48.000 100
49 - 49.000
50 - 50.000 101
51 - 51.000
52 - 52.000
53 - 53.000
54 - 54.000 2
55 - 55.000
56 - 56.000
57 - 57.000

CALCULATION OF THE FLUCTUATION VELOCITIES

A(1,1)=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
A(1,2)=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)
A(1,3)=H**2
A(2,1)=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
A(2,2)=H**2
A(2,3)=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)
A(3,1)=(COS(BETA))**2+(KB**2)*((SIN(BETA))**2)
A(3,2)=(SIN(BETA))**2+(KB**2)*((COS(BETA))**2)
A(3,3)=HB**2
B(1)=(EL(1)**2+EL(3)**2)/(2*(S**2))
B(2)=(EL(2)**2+EL(4)**2)/(2*(S**2))
B(3)=(ELS**2)/(SB**2)+((EL(3)**2-EL(1)**2)/(2*(S**2))
*)*((SIN(2*BETA))/(SIN(2*ALPH)))*((1-(KB**2))/(1-(K**2)
*))
CALL LEQTLF (A,M,Z,IA,B,IDGT,WKAREA,IER)
FORMAT(2X,"FLUCTUATION VELOCITIES, u,v,AND w")
WRITE(6,100)
FORMAT(2X,F8.4,/)
DO 2 J=1,3
FV(J)=SQRT(B(J))
WRITE(6,101)FV(J)
CONTINUE
L=L+1
IF(L.LE.20)GO TO 10
END

```



```

1 - 1.000 DIMENSION A(42,42),AA(42),BB(42),CC(42),DD(42),THETA
2 - 2.000 *(43),R(43),X(42),B(42),PHI(42),L(42),EX(42),
3 - 3.000 *Y(43),CP(42),WHY(42),EXX(42),WKAREA(42)
4 - 4.000 INTEGER M,N,IA,IDGT,IER,Z,W
5 - 5.000 REAL L,A,B,WKAREA
6 - 6.000 READ*,ALPHA,N,(EX(M),M=1,N+2),(Y(M),M=1,N+2),Z,
7 - 7.000 *IA,IDGT,W
8 - 8.000 M=1
9 - 9.000 PHI(21)=1.57079
10 - 10.000 DO 1 I=1,N+1
11 - 11.000 WHY(I)=Y(M)+(Y(M+1)-Y(M))/2
12 - 12.000 EXX(I)=EX(M)+(EX(M+1)-EX(M))/2
13 - 13.000 YY=Y(M+1)-Y(M)
14 - 14.000 XX=EX(M+1)-EX(M)
15 - 15.000 IF(XX.EQ.0.)GO TO 2
16 - 16.000 PHI(I)=ATAN2(YY,XX)
17 - 17.000 2 L(M)=(((EX(M)-EX(M+1))**2)+((Y(M)-Y(M+1))**2))**.5
18 - 18.000 B(M)=-SIN(ALPHA-PHI(I))
19 - 19.000 M=M+1
20 - 20.000 1 CONTINUE
21 - 21.000 BB(0)=0
22 - 22.000 DD(0)=0
23 - 23.000 AA(21)=0
24 - 24.000 CC(21)=0
25 - 25.000 BB(22)=0
26 - 26.000 DD(22)=0
27 - 27.000 AA(42)=0
28 - 28.000 CC(42)=0
29 - 29.000 PIE=3.141592654
30 - 30.000 PHI(0)=0
31 - 31.000 PHI(22)=0
32 - 32.000 PHI(42)=0
33 - 33.000 B(1)=0
34 - 34.000 B(21)=0
35 - 35.000 B(22)=0
36 - 36.000 B(42)=0
37 - 37.000 I=1
38 - 38.000 5 DO 4 M=1,N+2
39 - 39.000 IF(M.EQ.N+2)GO TO 10
40 - 40.000 XXA=EXX(I)-EX(M)
41 - 41.000 YYA=WHY(I)-Y(M)
42 - 42.000 R(M)=SQRT(XXA*XXA+YYA*YYA)
43 - 43.000 XXB=EXX(I)-EX(M+1)
44 - 44.000 YYB=WHY(I)-Y(M+1)
45 - 45.000 R(M+1)=SQRT(XXB*XXB+YYB*YYB)
46 - 46.000 THETA(M)=ATAN2(YYA,XXA)-PHI(M)
47 - 47.000 THETA(M+1)=ATAN2(YYB,XXB)-PHI(M)
48 - 48.000 IF(ABS(ABS(THETA(M)-THETA(M+1))-PIE).LE.0.00001)
49 - 49.000 *THETA(M)=0.0000;THETA(M+1)=PIE
50 - 50.000 IF(THETA(M).LT.0.0.AND.THETA(M+1).GT.0.0)THETA(M+1)
51 - 51.000 *-THETA(M+1)-2*PIE
52 - 52.000 IF(THETA(M).GT.0.0.AND.THETA(M+1).LT.0.0)THETA(M)=
53 - 53.000 *THETA(M)-2*PIE
54 - 54.000 A1=R(M)*SIN(THETA(M))/(2.*PIE*L(M))
55 - 55.000 IF(ABS(SIN(THETA(M))).LE.0.001.OR.ABS(SIN(THETA(M+1))))
56 - 56.000 *.LE.0.001)A5=0.;GO TO 15
57 - 57.000 TH=ABS(SIN(THETA(M+1))/SIN(THETA(M)))
58 - 58.000 A5=A1*ALOG(TH)
59 - 59.000 15 A4=A1*(THETA(M+1)-THETA(M))
60 - 60.000 A6=R(M)*COS(THETA(M))/L(M)
61 - 61.000 A2=1.-A6
62 - 62.000 A3=A6/(2.*PIE)
63 - 63.000 A7=ALOG(R(M)/R(M+1))
64 - 64.000 AA(M)=A2*(THETA(M)-THETA(M+1))/(2.*PIE)-A5
65 - 65.000 BB(M)=A3*(THETA(M)-THETA(M+1))+A5

```

```

1 - 1.000 DIMENSION E(6)
2 - 2.000 REAL K,ALPH,H,S,E
3 - 3.000 INTEGER N
4 - 4.000 READ*,K,H,S,N,ALPH,EZ,D1,D2,D3,D4,D5
5 - 5.000 L=1
6 - 6.000 10 READ*,(E(I),I=1,N)
7 - 7.000 D=1.414213562
8 - 8.000 DO 3 I=1,6
9 - 9.000 E(I)=E(I)-EZ
10 - 10.000 E(I)=D1+D2*E(I)+D3*(E(I)**2)+D4*(E(I)**3)+D5*(E(I)**4)
11 - 11.000 E(I)=S*E(I)
12 - 12.000 3 CONTINUE
13 - 13.000 C1=E(1)**2-E(3)**2
14 - 14.000 C2=E(4)**2-E(2)**2
15 - 15.000 C3=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)-H**2
16 - 16.000 C4=D*(S**2)*((1+K**2)**2)*((SIN(2*ALPH))**2)
17 - 17.000 C5=(E(6)**2-E(5)**2)*D+E(2)**2-E(4)**2
18 - 18.000 C6=2*(S**2)*(1-(K**2))*(SIN(2*ALPH))
19 - 19.000 U=SQRT((C1*C2*C3)/(C4*C5))
20 - 20.000 V=SQRT((C1*C5)/(2*D*(S**2)*C3*C2))
21 - 21.000 W=SQRT((C2*C5)/(2*D*(S**2)*C3*C1))
22 - 22.000 UV=(E(1)**2-E(3)**2)/(C6)
23 - 23.000 UW=(E(4)**2-E(2)**2)/(C6)
24 - 24.000 VW=((E(6)**2-E(5)**2)*D+E(2)**2-E(4)**2)/(2*D*(S**
25 - 25.000 2)*C3)
26 - 26.000 98 FORMAT(2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4,2X,F8.4
27 - 27.000 *,//)
28 - 28.000 99 FORMAT(6X,"U",9X,"V",9X,"W",9X,"UV",8X,"UW",8X,"VW")
29 - 29.000 WRITE(6,99)
30 - 30.000 WRITE(6,98)U,V,W,UV,UW,VW
31 - 31.000 L=L+1
32 - 32.000 IF(L.LE.20)GO TO 10
33 - 33.000 END

```

APPENDIX C

COMPUTER PROGRAM HOT-WIRE1

```

1 - 1.000 DIMENSION E(6),A(3,3),B(3),WKAREA(9),MV(3)
2 - 2.000 REAL K,ALPH,H,S,E,A,B
3 - 3.000 INTEGER N,M,IA,IDGT,IER,Z
4 - 4.000 READ*,K,H,S,N,ALPH,Z,M,IA,IDGT
5 - 5.000 L=1
6 - 6.000 10 READ*,(E(I),I=1,N)
7 - 7.000 A(1,1)=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
8 - 8.000 A(1,2)=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)
9 - 9.000 A(1,3)=H**2
10 - 10.000 A(2,1)=A(1,1)
11 - 11.000 A(2,2)=A(1,3)
12 - 12.000 A(2,3)=A(1,2)
13 - 13.000 A(3,1)=((COS(ALPH))**2+(K**2)*((SIN(ALPH))**2))
14 - 14.000 A(3,2)=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)+H**2
15 - 15.000 A(3,3)=A(3,2)
16 - 16.000 B(1)=(E(1)**2+E(3)**2)/(2*(S**2))
17 - 17.000 B(2)=(E(2)**2+E(4)**2)/(2*(S**2))
18 - 18.000 B(3)=(E(5)**2+E(6)**2)/(S**2)-.707107*(E(1)**2-E(3)**2)
19 - 19.000 *)/(S**2)
20 - 20.000 CALL LEQTF (A,M,Z,IA,B,IDGT,WKAREA,IER)
21 - 21.000 100 FORMAT(2X,"MEAN VELOCITIES, U,V,AND W")
22 - 22.000 WRITE(6,100)
23 - 23.000 101 FORMAT(2X,F8.4,/)
24 - 24.000 DO 2 J=1,3
25 - 25.000 MV(J)=SQRT(B(J))
26 - 26.000 WRITE(6,101)MV(J)
27 - 27.000 2 CONTINUE
28 - 28.000 L=L+1
29 - 29.000 IF(L.LE.20)GO TO 10
30 - 30.000 END

```

APPENDIX D

COMPUTER PROGRAM HOT-WIRE2

```

1 - 1.000 DIMENSION E(4),EL(4)
2 - 2.000 REAL K,ALPH,H,S,E,EL
3 - 3.000 INTEGER N
4 - 4.000 READ*,K,H,S,N,ALPH
5 - 5.000 L=1
6 - 6.000 10 READ*,(E(I),I=1,N),(EL(I),I=1,N)
7 - 7.000 C1=2*(S**2)
8 - 8.000 C2=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)-H**2
9 - 9.000 C3=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
10 - 10.000 C4=C1*(1-(K**2))*(SIN(2*ALPH))
11 - 11.000 U=SQRT(((E(2)**2+E(4)**2)/C1+((H**2)*(E(4)**2+E(2)**2
12 - 12.000 *-E(1)**2-E(3)**2)/(C1*C2))/C3)
13 - 13.000 UV=(E(1)**2-E(3)**2)/C4
14 - 14.000 UW=(E(4)**2-E(2)**2)/C4
15 - 15.000 UVL=(EL(1)**2-EL(3)**2)/C4
16 - 16.000 UWL=(EL(4)**2-EL(2)**2)/C4
17 - 17.000 UL=SQRT(((EL(2)**2+EL(4)**2)/C1+((H**2)*(EL(4)**2+
18 - 18.000 *EL(2)**2-EL(1)**2-EL(3)**2)/(C1*C2))/C3)
19 - 19.000 VL=SQRT(((EL(1)**2+EL(3)**2-EL(2)**2-EL(4)**2)/(C1*C2))
20 - 20.000 98 FORMAT(1X,F8.4,1X,F8.4,1X,F8.4,1X,F8.4,1X,F8.4
21 - 21.000 *,1X,F8.4,1X,F8.4,/)
22 - 22.000 99 FORMAT(5X,"U",7X,"UV",7X,"UW",7X,"UVL",7X,"UWL"
23 - 23.000 *,7X,"UL",7X,"VL")
24 - 24.000 WRITE(6,99)
25 - 25.000 WRITE(6,98)U,UV,UW,UVL,UWL,UL,VL
26 - 26.000 L=L+1
27 - 27.000 IF(L.LE.20)GO TO 10
28 - 28.000 END

```

APPENDIX E

COMPUTER PROGRAM HOT-WIRE3

```

1 - 1.000 DIMENSION E(4),EL(4)
2 - 2.000 REAL K,ALPH,H,S,E,EL
3 - 3.000 INTEGER N
4 - 4.000 READ*,K,H,S,N,ALPH,EZ,EZL,CO1,CO2,CO3,CO4,CO5
5 - 5.000 L=1
6 - 6.000 10 READ*,(E(I),I=1,N),(EL(I),I=1,N)
7 - 7.000 DO 3 I=1,4
8 - 8.000 E(I)=E(I)-EZ
9 - 9.000 EL(I)=EL(I)-EZL
10 - 10.000 EL(I)=(CO2+2*CO3*E(I)+3*CO4*E(I)**2+4*CO5*E(I)**3)*S*EL
(I)
11 - 11.000 E(I)=CO1+CO2*E(I)+CO3*(E(I)**2)+CO4*(E(I)**3)+CO5*(E(I)
**4)
12 - 12.000 E(I)=S*E(I)
13 - 13.000 3 CONTINUE
14 - 14.000 C1=2*(S**2)
15 - 15.000 C2=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)-H**2
16 - 16.000 C3=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
17 - 17.000 C4=C1*(1-(K**2))*(SIN(2*ALPH))
18 - 18.000 U=SQRT(((E(2)**2+E(4)**2)/C1+((H**2)*(E(4)**2+E(2)**2
19 - 19.000 *-E(1)**2-E(3)**2)/(C1*C2))/C3)
20 - 20.000 UV=(E(1)**2-E(3)**2)/C4
21 - 21.000 UW=(E(4)**2-E(2)**2)/C4
22 - 22.000 UVL=(EL(1)**2-EL(3)**2)/C4
23 - 23.000 UWL=(EL(4)**2-EL(2)**2)/C4
24 - 24.000 UL=SQRT(((EL(2)**2+EL(4)**2)/C1+((H**2)*(EL(4)**2+
25 - 25.000 *EL(2)**2-EL(1)**2-EL(3)**2)/(C1*C2))/C3)
26 - 26.000 VL=SQRT((EL(1)**2+EL(3)**2-EL(2)**2-EL(4)**2)/(C1*C2))
27 - 27.000 98 FORMAT(1X,F8.4,1X,F8.4,1X,F8.4,1X,F8.4,1X,F8.4,1X,F8.4
28 - 28.000 *,1X,F8.4,/)
29 - 29.000 99 FORMAT(5X,"U",7X,"UV",7X,"UW",7X,"UVL",7X,"UWL"
30 - 30.000 *,7X,"UL",7X,"VL")
31 - 31.000 WRITE(6,99)
32 - 32.000 WRITE(6,98)U,UV,UW,UVL,UWL,UL,VL
33 - 33.000 L=L+1
34 - 34.000 IF(L.LE.20)GO TO 10
35 - 35.000 END

```

APPENDIX F

COMPUTER PROGRAM HOT-WIRE4

```

1 - 1.000 DIMENSION E(4)
2 - 2.000 REAL K,ALPH,H,S,E,USQ,VSQ
3 - 3.000 INTEGER N,F
4 - 4.000 READ*,K,H,S,N,ALPH
5 - 5.000 L=1
6 - 6.000 F=0
7 - 7.000 10 READ*,(E(I),I=1,N)
8 - 8.000 C1=(E(1)**2+E(3)**2)/(2*(S**2))
9 - 9.000 C2=(E(2)**2+E(4)**2)/(2*(S**2))
10 - 10.000 C3=(SIN(ALPH))**2+(K**2)*((COS(ALPH))**2)
11 - 11.000 C4=(COS(ALPH))**2+(K**2)*((SIN(ALPH))**2)
12 - 12.000 VSQ=0
13 - 13.000 20 USQ=(C1+C2-((C3+(H**2))*VSQ))/(2*C4)
14 - 14.000 VSQ=(C2-(USQ*C4))/(H**2)
15 - 15.000 F=F+1
16 - 16.000 IF(F.LE.20)GO TO 20
17 - 17.000 WRITE (6,100)
18 - 18.000 100 FORMAT(2X,"U SQUARED",2X,"V SQUARED")
19 - 19.000 WRITE (6,101)USQ,VSQ
20 - 20.000 101 FORMAT(2X,F8.4,3X,F8.4)
21 - 21.000 L=L+1
22 - 22.000 IF(L.LE.6)GO TO 10
23 - 23.000 END

```

APPENDIX G

DATA ACQUISITION PROGRAM WITH LINEARIZER

```

10  REM *****PROGRAM*****
20  DIM C(100),I(100),F(100)
30  DIM E(100),L(100),U(100),V(100)
40  DIM C(100),I(100),F(100)
50  C(0)=0: I(0)=0: F(0)=0
60  P1=3456
70  GOSUB 530: CALL Init
80  FOR I=1 TO 100
90  GOSUB 540
100 NEXT I
110 FOR J=1 TO 4
120 FOR I=1 TO 20
130 E(I,J)=C(I*100)/100
140 F(I,J)=C(I*100)/100
150 PRINT "E= ",E(I,J), "EL= ",F(I,J)
160 PAUSE
170 FOR K=1 TO 100
180 GOSUB 540
190 NEXT K
200 NEXT I
210 PAUSE
220 NEXT J
230 K1= 33: H=1: 19: 0: 5= 312798
240 R= 785
250 D1=2*5^2
260 D2=SIN(R)^2+K1^2*COS(R)^2-4^2
270 D3=COS(P)^2+K1^2*SIN(P)^2
280 D4=D1*(1-K1^2)*SIN^2(P)
290 D5=H^2
300 FOR I=1 TO 20
310 D6=E(I,2)^2+E(I,4)^2
320 D7=E(I,1)^2+E(I,3)^2
330 U=SQR(D5-D1-D5*(D6-D7)/D1*
340 D2)/D3
350 U1=(E(I,1)^2-E(I,3)^2)/D4
360 U2=(E(I,4)^2-E(I,2)^2)/D4
370 U3=(F(I,1)^2-F(I,3)^2)/D4
380 U4=(F(I,4)^2-F(I,2)^2)/D4
390 D8=F(I,2)^2+F(I,4)^2
400 D9=F(I,1)^2+F(I,3)^2
410 U5=SQR(D8/D1-D5*(D8-D9)/D1*
420 D2)/D3
430 U6=SQR(ABS(D9-D8)/D1*D2)
440 PRINT "U= ",U: "UV= ",U1
450 PRINT "UW= ",U2: "UWL= ",U3
460 PRINT "UWL= ",U4: "UL= ",U5
470 PRINT "UL= ",U6
480 NEXT I
490 P1=0
500 GOSUB 1440: CALL Dec
510 C(K)=P
520 C(K)=C(K-1)+C(K)
530 P1=1
540 GOSUB 1440: CALL Dec
550 C1(K)=F
560 C1(K)=C1(K-1)+C1(K)
570 RETURN
580 END

```

APPENDIX H

DATA ACQUISITION PROGRAM WITHOUT LINEARIZER

```

1000 REM *****PROGRAM#10DATA#1
1010 DIM C(100),C1(100),F(100)
1020 E=20.4:F=10.4
1030 X=0 @ C1(0)=0
1040 Y=2452
1050 GOSUB 530 : CALL Init
1060 X=1 TO 100
1070 GOSUB 540
1080 NEXT X
1090 FOR J=1 TO 4
1100 FOR I=1 TO 20
1110 U=C1(100)/100
1120 V=C0(100)/100
1130 PRINT "E= ",E(I,J),"EL= " F
1140 GOSUB 550
1150 FOR K=1 TO 100
1160 GOSUB 540
1170 NEXT K
1180 NEXT I
1190 GOSUB 550
1200 NEXT J
1210 U=C1(100)/100
1220 V=C0(100)/100
1230 PRINT "EZ= ",Z,"EYL= ",Z1
1240 K1=33 @ H=1 19 @ S=215022
1250 U=024532
1260 V=186428
1270 W=72195
1280 X=-1 90386
1290 Y=0 805527
1300 Z=785
1310 D1=2*S^2
1320 D2=SIN(R)^2+K1^2*COS(R)^2-H^2
1330 D3=COS(R)^2+K1^2*SIN(R)^2
1340 D4=D1*(1-K1^2)*SIN(2*F)
1350 H=H^2
1360 FOR J=1 TO 4
1370 FOR I=1 TO 20
1380 E(I,J)=E(I,J)-Z
1390 E(I,J)=(B2+2*B3*E(I,J)+3*B4*
1400 E(I,J)^2+4*B5*E(I,J)^3)*S*F
1410 E(I,J)=Z1)
1420 E(I,J)=B1+B2*E(I,J)+B3*E(I,J)
1430 +B4*E(I,J)^2+B5*E(I,J)^3
1440 E(I,J)=S*E(I,J)
1450 NEXT I
1460 NEXT J
1470 FOR I=1 TO 20
1480 F(I,2)^2+E(I,4)^2
1490 F(I,1)^2+E(I,3)^2
1500 D=D6/D1-D5*(D6-D7)/D1*
1510 D3)
1520
1530
1540 F(I,1)^2-E(I,3)^2) D4
1550 F(I,4)^2-E(I,2)^2) D4
1560 F(I,1)^2-F(I,3)^2) D4
1570 F(I,4)^2-F(I,2)^2) D4
1580 F(I,2)^2+F(I,4)^2

```

```

400 D2=F(I,2)^2-F(I,4)^2
410 D5=GOSUB D3 D1-D5*(D6-D7)
420 D6=GOSUB D3 D1-D6*(D6-D7)
430 PRINT "U= ",U,"V= ",V
440 PRINT "UW= ",U2,"VL= ",V2
450 PRINT "UWL= ",U4,"VL= ",V4
460 PRINT "UL= ",U6
470 NEXT I
480 P1=0
490 GOSUB 1440 : CALL Dcm
500 C0(I)=P
510 C0(K)=C0(K-1)+C0(K)
520 P1=1
530 GOSUB 1440 : CALL Dcm
540 C1(K)=F
550 C1(K)=C1(K-1)+C1(K)
560 RETURN
570 END

```

POOR PRINT
Epreuve illisible

APPENDIX I

OUTPUT OF TYPICAL EXPERIMENTAL RUN

APPENDIX J

FOREBODY DESIGN PROGRAM

```

1 - 1.000 DIMENSION A(42,42),AA(42),BB(42),CC(42),DD(42),THETA
2 - 2.000 *(43),R(43),X(42),B(42),PHI(42),L(42),EX(42),
3 - 3.000 *Y(43),CP(42),WHY(42),EXX(42),WKAREA(42)
4 - 4.000 INTEGER M,N,IA,IDGT,IER,Z,W
5 - 5.000 REAL L,A,B,WKAREA
6 - 6.000 READ*,ALPHA,N,(EX(M),M=1,N+2),(Y(M),M=1,N+2),Z,
7 - 7.000 *IA,IDGT,W
8 - 8.000 M=1
9 - 9.000 PHI(21)=1.57079
10 - 10.000 DO 1 I=1,N+1
11 - 11.000 WHY(I)=Y(M)+(Y(M+1)-Y(M))/2
12 - 12.000 EXX(I)=EX(M)+(EX(M+1)-EX(M))/2
13 - 13.000 YY=Y(M+1)-Y(M)
14 - 14.000 XX=EX(M+1)-EX(M)
15 - 15.000 IF(XX.EQ.0.)GO TO 2
16 - 16.000 PHI(I)=ATAN2(YY,XX)
17 - 17.000 2 L(M)=(((EX(M)-EX(M+1))**2)+((Y(M)-Y(M+1))**2))**.5
18 - 18.000 B(M)=-SIN(ALPHA-PHI(I))
19 - 19.000 M=M+1
20 - 20.000 1 CONTINUE
21 - 21.000 BB(0)=0
22 - 22.000 DD(0)=0
23 - 23.000 AA(21)=0
24 - 24.000 CC(21)=0
25 - 25.000 BB(22)=0
26 - 26.000 DD(22)=0
27 - 27.000 AA(42)=0
28 - 28.000 CC(42)=0
29 - 29.000 PIE=3.141592654
30 - 30.000 PHI(0)=0
31 - 31.000 PHI(22)=0
32 - 32.000 PHI(42)=0
33 - 33.000 B(1)=0
34 - 34.000 B(21)=0
35 - 35.000 B(22)=0
36 - 36.000 B(42)=0
37 - 37.000 I=1
38 - 38.000 5 DO 4 M=1,N+2
39 - 39.000 IF(M.EQ.N+2)GO TO 10
40 - 40.000 XXA=EXX(I)-EX(M)
41 - 41.000 YYA=WHY(I)-Y(M)
42 - 42.000 R(M)=SQRT(XXA*XXA+YYA*YYA)
43 - 43.000 XXB=EXX(I)-EX(M+1)
44 - 44.000 YYB=WHY(I)-Y(M+1)
45 - 45.000 R(M+1)=SQRT(XXB*XXB+YYB*YYB)
46 - 46.000 THETA(M)=ATAN2(YYA,XXA)-PHI(M)
47 - 47.000 THETA(M+1)=ATAN2(YYB,XXB)-PHI(M)
48 - 48.000 IF(ABS(ABS(THETA(M)-THETA(M+1))-PIE).LE.0.00001)
49 - 49.000 *THETA(M)=0.0000;THETA(M+1)=PIE
50 - 50.000 IF(THETA(M).LT.0.0.AND.THETA(M+1).GT.0.0)THETA(M+1)
51 - 51.000 *-THETA(M+1)-2*PIE
52 - 52.000 IF(THETA(M).GT.0.0.AND.THETA(M+1).LT.0.0)THETA(M)=
53 - 53.000 *THETA(M)-2*PIE
54 - 54.000 A1=R(M)*SIN(THETA(M))/(2.*PIE*L(M))
55 - 55.000 IF(ABS(SIN(THETA(M))).LE.0.001.OR.ABS(SIN(THETA(M+1))))
56 - 56.000 *.LE.0.001)A5=0.;GO TO 15
57 - 57.000 TH=ABS(SIN(THETA(M+1))/SIN(THETA(M)))
58 - 58.000 A5=A1*ALOG(TH)
59 - 59.000 15 A4=A1*(THETA(M+1)-THETA(M))
60 - 60.000 A6=R(M)*COS(THETA(M))/L(M)
61 - 61.000 A2=1.-A6
62 - 62.000 A3=A6/(2.*PIE)
63 - 63.000 A7=ALOG(R(M)/R(M+1))
64 - 64.000 AA(M)=A2*(THETA(M)-THETA(M+1))/(2.*PIE)-A5
65 - 65.000 BB(M)=A3*(THETA(M)-THETA(M+1))+A5

```

```

66 - 66.000 CC(M)=A7*A2/(2.*PIE)-A4+1./(2.*PIE)
67 - 67.000 DD(M)=A3*A7+A4-1./(2.*PIE)
68 - 68.000 10 A(I,M)=BB(M-1)*SIN(PHI(M-1)-PHI(I))+AA(M)*SIN(PHI(M)
69 - 69.000 *-PHI(I))+DD(M-1)*COS(PHI(M-1)-PHI(I))+CC(M)*COS(
70 - 70.000 *PHI(M)-PHI(I))
71 - 71.000 4 CONTINUE
72 - 72.000 I=I+1
73 - 73.000 IF(I.EQ.N+2)GO TO 7
74 - 74.000 GO TO 5
75 - 75.000 7 DO 11 M=1,N+1
76 - 76.000 A(1,M)=0
77 - 77.000 A(21,M)=0
78 - 78.000 A(22,M)=0
79 - 79.000 A(42,M)=0
80 - 80.000 11 CONTINUE
81 - 81.000 A(1,1)=1
82 - 82.000 A(21,21)=1
83 - 83.000 A(22,22)=1
84 - 84.000 A(42,42)=1
85 - 85.000 CALL LEQTLF (A,Z,W,IA,B,IDGT,WKAREA,IER)
86 - 86.000 WRITE(6,120)
87 - 87.000 WRITE(6,121)
88 - 88.000 120 FORMAT(10X,'AIRFOIL COORDINATES AND PRESSURE'
89 - 89.000 *, 'COEFFICIENTS',/)
90 - 90.000 100 FORMAT(2X,F8.4,4X,F8.4,6X,F10.3,/)
91 - 91.000 121 FORMAT(7X,'X',11X,'Y',13X,'CP',/)
92 - 92.000 DO 6 M=1,N+1
93 - 93.000 C=(B(M)+B(M+1))/2
94 - 94.000 CP(M)=1.-C**2
95 - 95.000 WRITE(6,100) EXX(M),WHY(M),CP(M)
96 - 96.000 6 CONTINUE
97 - 97.000 END

```

END

0	6	H	0	9	H	8	4
---	---	---	---	---	---	---	---

FIN