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Spinning Mode Algorithms for the Satellite Attitude Sensor

by

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A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

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Mechanical and Aerospace Engineering
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in partial fulfillment of the requirements for
the degree of Master of Engineering

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Abstract

The Center for Research in Earth and Space Technology (CRESTech) have developed an innovative earth horizon sensor called the Satellite Attitude Sensor (SAS). The SAS utilizes multiple optical paths for a single detector in order to increase its field-of-view while maintaining high resolution. The result is an overlaid image.

This overlaid image has lost important information, and so image features can not be readily associated with a particular optical path. The purpose of this thesis is to develop a spinning mode algorithm that can overcome the ambiguous nature of the overlaid images sufficiently well for attitude determination.

The SAS Overlay Resolution Algorithm (SORA) was shown to resolve SAS's ambiguity problem. SORA operates by examining the apparent mounting phase determined from the image feature, and comparing that to the actual mounting phase for each optical path. A match of the apparent and actual mounting phase determines the optical path taken by the image feature.

Dedication

In memory of Francka Vorobej.

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I would also like to acknowledge Gerry Warner. Gerry adapted the ephemeris algorithms in the Montenbruck text [11] into Visual Basic for a separate project. The ephemeris routines were then translated to C++ by the author and incorporated into the simulation. As well, his comments were insightful, valuable, and greatly appreciated.

My thanks go to Julianna and Godfrey, my parents. Their love and enthusiasm have helped me more than they know.

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Nomenclature

ACS	Attitude Control System
BMS	Body Mounted Sensor
CRESTech	Center for Research in Earth and Space Technology
FOR	Field-of-Regard
FOV	Field-of-View
GEO	Geosynchronous Equatorial Orbit
GTO	Geosynchronous Transfer Orbit
IR	Infrared
ISL	Instrument Services Laboratory
JD	Julian Day
LE	Leading Edge
LEO	Low Earth Orbit
MEO	Mid-geosynchronous Earth Orbit
MJD	Modified Julian Day
RPM	Rotations per Minute
SAS	Satellite Attitude Sensor
SNR	Signal to Noise Ratio
SORA	SAS Overlay Resolution Algorithm
SSTO	Supersynchronous Transfer Orbit
TE	Trailing Edge
UV	Ultraviolet

Chapter 1

Introduction

Horizon sensors are utilized on spacecraft to assist in attitude control. They determine attitude by examining the position of an astronomical body's horizon and use this to infer the position of the body with respect to the spacecraft. This information is then used by the Attitude Control System (ACS) to place the satellite in the correct orientation to perform its mission.

There are two main classes of horizon sensors: spinning and staring. Spinning horizon sensors consist of detectors which are swept through large angular swaths of the sky. The time-varying response of the detectors as they sweep past the Earth can then be used to determine the attitude. Staring sensors consist of many small or a few large detectors arranged to detect multiple locations of the horizon. The locations can be used to determine the attitude of the spacecraft. Figure 1.1 shows how the fields-of-view (FOV) of the staring sensor might be arranged. In both the staring and spinning modes, operation of the sensor is usually restricted to small deviations from a nominal attitude.

Most horizon sensors detect the infrared (IR) band of the electromagnetic spectrum but some detect the visible or ultraviolet (UV) bands. Neither ultraviolet nor infrared sensors suffer from the complications afflicting visible sensors caused by the Earth's terminator. The Earth emits radiation more strongly in the IR band than

Figure 1.1: FOV's for Staring Horizon Sensor

in the UV band, so IR detectors are more commonly used in horizon sensor designs. The main problem with utilizing the IR band is that the Earth's atmosphere causes the horizon to appear fuzzy and indistinct as illustrated in Figure 1.2. The indistinct atmosphere makes locating the real horizon very difficult.

The Centre for Research in Earth and Space Technology (CRESTech) is a Centre of Excellence sponsored by the Government of Ontario. Its mandate is to cooperate with academia, government, and industry in the pursuit of research, innovation, and the development of advanced technology for space and terrestrial science. It attempts to bridge the gap between pure science and its application to real world problems.

In this endeavor, the Instrument Services Laboratory (ISL) group of CRESTech envisioned a means of using an advanced microbolometer design as the detecting

Figure 1.2: The Horizon Viewed From Space

element of an innovative earth horizon sensor. Such a sensor was perceived to have the potential to occupy a significant niche in the attitude sensor market.

In addition to utilizing advanced detectors, CRESTech developed technology to substantially increase the field-of-view (FOV) without sacrificing resolution. The technology entailed the use of multiple mirrors to deposit overlaid images upon the advanced detectors. Each mirror can be thought of as a distinct telescope with each telescope looking in different directions. The overlaid image occurs because the image generated by each telescope is projected onto a single detector array.

Overlays allow a single detector array to have an increased FOV without sacrificing resolution. In addition, it is possible to maintain low aberration levels and compact optical assemblies. In the horizon sensor market, overlays allow the design of a sensor that is as light as the competition, has better accuracy and is more economical.

The innovation of using overlaid images has the consequence of considerably increasing the complexity of the attitude determination algorithms. The complexity

arises due to the inherent degeneracy of the information contained in the overlaid image (i.e. which optical path was followed by a particular image feature is not readily determined, this is referred to as the ambiguity problem). An overlay refers to the image generated by a single telescope.

The Satellite Attitude Sensor (SAS) is the embodiment of these innovative technologies. The SAS utilizes 6 arrays of microbolometers which are significantly larger than those conventionally used. In staring mode the sensor operates by finding the horizon's location in the images generated by the microbolometer arrays. Figure 1.3 shows an image where both horizons are being imaged by a single overlay (i.e. both horizon features are taking the same optical path to the array). In this situation, the chord of the earth seen by the array is smaller than the array. Note that this image is not typical of SAS on a 3-axis stabilized mission. The typical situation would have the Earth centered at the boresight and each overlaid image would have a single earth transition.

The horizon locations are used to determine the relative location of the Earth's center. Due to its large field-of-view and high resolution, the SAS can operate at any altitude below Geosynchronous Equatorial Orbit (GEO), but its design favors Low Earth Orbit (LEO) operation. The ability for a single sensor to operate at many altitudes is a great advantage for satellite design and mission operation.

In addition to its inherent altitude versatility, the SAS design is also capable of operating in a spinning mode for high slew rate missions. Algorithms have already been developed to determine attitude for conventional spinning mode horizon sensors. However, algorithms which are capable of handling the spinning mode overlaid images have not been developed. In staring mode, the SAS employs algorithms which are capable of determining the attitude from an overlaid image; however, they are not suitable at any appreciable slew rate. Algorithms which can be used to determine the spinning mode spacecraft attitude are necessary and form the basis of this thesis.

Inside each optics module of the SAS, there is a detector and a single set of optics. The optics have an entrance mirror that is split to look in four different directions. The images generated from the four different directions are focused onto

Figure 1.3: Staring Mode Image Showing Two Horizon Transitions

the single detector. Each pixel then sees four directions simultaneously. A good analogy is to examine the situation of using overhead slides in a presentation. If two transparencies are placed on the overhead projector, an audience member will not be able to tell by looking at the screen which transparency is on top. With more transparencies the situation is further exacerbated.

The ambiguity problem which arises from the use of overlaid images exists for any signal detected by the sensor. In the spinning mode situation, many possible (and mostly incorrect) solutions will arise due to the overlaid image where each solution will correspond to a different attitude or altitude. Some form of additional information is required in order to choose the one correct solution. The additional information can take the form of *a priori* attitude information, or statistical considerations can be used to eliminate some of the incorrect solutions.

The approaches used to find a suitable solution all involved trying to eliminate

the ambiguity in the solution. It will be shown that the ambiguity is sufficient to prohibit a closed form solution, hence the requirement for an alternative method of attitude determination. The solutions can be classified into four broad classes:

- time based
- space based
- geometry based
- illuminance based

In the chapters to follow, the design for the SAS sensor will be described, the overlay problem will be examined in detail, and several candidate solutions will be presented. The solutions will be presented in order from least feasible to most feasible. Only those solutions deemed sufficiently feasible will be simulated and analyzed. Finally, the incorporation of the most feasible solution into the SAS will be examined.

Chapter 2

The Satellite Attitude Sensor

The Satellite Attitude Sensor can act as a staring horizon sensor by utilizing microbolometer arrays that are significantly larger than those used in conventional horizon sensors. The microbolometer arrays are sensitive to infrared radiation and provide the capability for a wide FOV and excellent resolution.

The engineering model of SAS, on which much of this thesis is based, is designed to operate in a wide range of altitudes centered around an optimal altitude. The models have shown that designing a sensor for all altitudes from LEO to GEO is possible, but it would require greater coverage (i.e. more overlays, wider FOV's, larger arrays or more optics modules) to attain the desired accuracy levels.

2.1 Components of SAS

2.1.1 Detector Arrays

The detector arrays selected for use in the SAS were designed by the National Optics Institute (NOI) of Sainte-Foy, Quebec [9]. The microbolometer is a 50 x 50 μm square multilayer device that is designed to absorb thermal radiation. Figure 2.1 depicts the microbolometer. Each microbolometer stands on pedestals constructed of low thermal conductivity material. The pedestals are angled to increase their length; longer pedestals inhibit thermal conduction from the wafer to the microbolometer. A filter may be added to SAS's optics so that only electromagnetic radiation of the

desired wavelength will be incident on the microbolometers. The earth has a strong output in the thermal radiation band of 14-16 μm .

The microbolometer device acts like a thermistor in that as the temperature of the microbolometer changes, the resistance across the microbolometer also changes. When IR radiation is incident on the microbolometer, the small increase in energy causes a significant increase in temperature of the low mass microbolometer. A constant current is driven through the microbolometer, and a voltage drop indicates that an increase in the amount of IR radiation was received. The microbolometer's small size is important in being able to detect small amount of IR radiation (i.e. a larger change in temperature will occur with a smaller mass given the same amount of IR radiation), and in increasing the thermal response of the device to changing amounts of IR radiation.

Figure 2.1: Detector Arrays

The signals produced by the detectors are subject to a great deal of noise. One of the dominant sources of noise is the microbolometers immediate environmental, because everything not at a temperature of absolute zero emits IR radiation. The reason for the large amount of environmental noise is that the sensor components will be at approximately the same temperature as the Earth. To help overcome this noise, many microbolometers are averaged together to form a single pixel. The detector

array in the engineering model consists of a single column of 256 pixels. Each pixel consists of two rows with 20 microbolometers in each row. Each microbolometer is of the type illustrated in Figure 2.2.

Figure 2.2: Pixel Configuration Used in Engineering Model

Not all of the microbolometers will be part of a pixel; some microbolometers will be shorted in an attempt to calibrate the pixels. Due to imperfections in micromachining, the responsiveness of the microbolometers varies considerably. By switching some microbolometers out of the circuit, a uniform distribution of pixel responses can be achieved.

The aspect ratio of the pixels is such that fine resolution can still be maintained along the length of the array. The resolution in the other direction is not as important in the staring case (where it is assumed that the sensor view is centered on the Earth).

2.1.2 Optics Modules

The optics module consists of the detector array (and the associated integrated circuitry), four entrance mirrors, and a focusing mirror. Figure 2.3 shows the optical path where light enters the module, is reflected off one of the four entrance mirrors, is reflected off the focusing mirror, and is finally incident on the detector array. For example, in the engineering model, the FOV associated with each entrance mirror is nominally $12^\circ \times 1^\circ$. There are 4 mirrors whose FOV's overlap by 1° . Thus the full effective FOV of the overlaid images would be 45° . The FOV is limited in the attempt to achieve 0.05° accuracy in staring mode. At 12° for each overlay, the FOV associated with each pixel will be 0.0475° . After noise and other errors, subpixel transition determination allows the sensor to operate near 0.05° accuracy in staring mode.

Figure 2.3: Optics Module for SAS Engineering Model

The 1° overlap between the overlays in the engineering model means there is no gap in coverage between the FOV's of the overlays. This was included in the design so that the earth to space transition can be tracked from one overlay to the next in staring mode. The overlap has the consequence of reducing the effective FOV of the overlaid image and if a transition occurs in the overlap, then it will be seen in more than one overlay (i.e. it will appear twice in the overlaid image). This has further

consequences in spinning mode which will be discussed later.

The optics modules can be configured in many ways for both staring and spinning modes. A number of characteristics can be adjusted in each module to achieve different sensor configurations. The size of the FOV is rather restricted in order to meet the accuracy requirements in staring mode. The amount of overlap between the overlays is also restricted so that the staring mode algorithm can properly track the horizon from overlay to overlay. The main aspects that can be changed are the number of optics modules in the sensor and the directions in which the modules are pointed. Both for redundancy and increased accuracy, more than 4 modules are desirable in a sensor. More than 3 modules also allows one module to fail without affecting the operation of the sensor.

2.2 SAS in Staring Mode

This section presents how the SAS operates in 3-axis mode. This section serves to explain the appearance the SAS images in staring mode. This explanation will be used as a starting point for explaining how SAS images appear in the spinning mode and how the overlay resolution algorithm works.

If the detector consisted of a single pixel with a single optical path, then the pixel output level would correspond to the image in the FOV. If the Earth is in the FOV, the output from the pixel would correspond to the IR radiance of the earth, Φ_E . If empty space is in the FOV, no noticeable output would be seen from the pixel, other than background noise. If the FOV is partially filled with the image of the Earth and/or it's horizon, then the output level would correspond to the amount of area in the FOV taken up by the Earth, it's atmosphere and empty space. This last idea is useful when explaining what a pixel sees when overlays are being used. If two overlays image the Earth and the others image space, then the output from the overlaid image would correspond to twice the IR radiance of the Earth or $2\Phi_E$. Figure 2.4 shows the detector array's response to the overlaid image.

An examination of the situation when the boresight of the sensor is pointing at nadir can be used in the explanation of an overlaid image. In this attitude, the

Figure 2.4: Response to Overlaid Image

FOV's of the overlays are pointing slightly off nadir. Assuming a LEO situation, the FOV of the first overlay is completely within the horizon of the Earth, as is the second overlay. The third overlay is partially earth and the rest empty space. The fourth overlay consists of empty space. By combining the overlays together, the overlaid image and hence the pixel responses can be determined. The first and second overlays see only the Earth, therefore every pixel in the image will have an output corresponding to at least $2\Phi_E$. Because the fourth sees only empty space, it will not contribute to the output levels of any of the pixels. The third overlay contains the interesting information. It has part of the FOV pointing at the Earth, and so the pixels associated with this part will have an output corresponding to $3\Phi_E$. The other pixels in the array will have an output corresponding to $2\Phi_E$.

Figure 2.5: 3-Axis Attitude Sensing

The information that is useful for attitude determination is that transition between the pixels that have an output corresponding to $2\Phi_E$ and those pixels with an output of $3\Phi_E$. This transition corresponds to the Earth's horizon. Since the SAS sensor is axially symmetric for LEO and GEO configurations, the nadir pointing sensor (the boresight is pointed at the Earth's center) will produce similar images from each of the different detector arrays. Each will show the transition in similar locations as illustrated in Figure 2.5.

When the boresight moves off nadir in some direction, the arrays in that direction will see less of the Earth and the transition will appear to travel toward the boresight. The arrays in the opposite direction will see more of the Earth and the transition will appear to travel away from the boresight. The changing location of the transitions will correspond to the amount of attitude change experienced. If the attitude changes sufficiently so that the third FOV no longer sees any of the Earth,

then the transition will have appeared to move to the end of the array and then jumped to the other end of the array. However, the output levels of the pixel will have also dropped accordingly so that the transition now occurs between pixels with outputs of $2\Phi_E$ and those with outputs of Φ_E .

Chapter 3

State of the Art Spinning Horizon Sensors

Spinning horizon sensors can be used to determine a number of parameters. One can determine the spin-rate, roll error, and spin-axis-to-earth-center angle. The following section will introduce the operation of conventional spinning horizon sensors and the algorithms they employ. The next sections of this chapter will report the state-of-the-art of employing overlaid images, and how more conventional spinning images appear. The last section of this chapter discusses how transition determination is used by spinning horizon sensors.

3.1 Conventional Spinning Horizon Sensors

Spinning horizon sensors incorporate a sensing device that is able to distinguish between the Earth and the background of space. As the sensor's FOV spins, the detector will sweep out a cone in space. When this cone intersects the Earth's horizon circle, the sensor will acquire data that can be used to determine the attitude. This data comes in the form of two transitions. These two transitions occur when the FOV of the detector travels between earth and space. The space-to-earth and earth-to-space transitions are called the leading edge (LE) and trailing edge (TE) horizon crossings respectively.

The simplest of the scanning horizon sensors consists of a single detector. The

information extracted from the single detector is combined with *a priori* attitude information to determine the position of the Earth. The scanning is either accomplished by a fixed sensor being mounted on a spin-stabilized satellite or by the sensor incorporating a spinning mechanism that rotates the field-of-view to sweep out the required cone. An example of a fixed sensor type is illustrated by the Indian Space Research Organization's design [8], and by the Horizon Crossing Indicator produced by EDO Electro-Optics [6]. A good example of a simple scanning horizon sensor is the LightSat Earth Sensor produced by EDO Electro-Optics [5].

Figure 3.1: Conventional Horizon Sensor Response

A typical response from a detector sweeping space is illustrated in Figure 3.1. The figure illustrates the interesting section of the response where the Earth is in the FOV. The response illustrates both the LE (around Modified Julian Day (MJD) 51131.8655) and the TE (around MJD 51131.87275) with a spin-rate of $0.22^\circ/\text{s}$. The period is 27.474 minutes in duration, so the detector experiences a duty cycle of 38.0%. This type of response would be typical of designs like those described in [5], [6], and [8]. By examining the response from a sensor for a sufficient length of time, the spin-rate of the spacecraft can be determined (by looking for repetitions

in the response). Once the spin-rate is determined, it is usually a simple matter to determine the chord lengths. Note that while the spin-rate can be determined by the earth sensor, it can usually be more accurately determined by a sun sensor.

Figure 3.2: Conventional Horizon Sensor Response of Two Detectors

In a similar manner, Figure 3.2 illustrates the responses from two detectors in the same sensor but at different mounting angles. If the detectors are in phase with each other then they will show symmetric responses. The figure illustrates two detectors that are in phase with each other but cut different chords of the Earth. The different chords result in a difference in the duty cycles of the two detectors. The first response in the figure is the same as in Figure 3.1, and the second has a duty cycle of 45.6%. Because the two detectors experience different duty cycles, this must mean that they cut different chords of the earth. Since they cut different chords of the Earth, a full attitude solution can be determined.

State of the art horizon sensors utilize detector arrays of significant size. Anderson [1] has designed staring sensors for both LEO and GEO operation which utilize a number of large detector arrays. Doctor and Burton [4] have designed a sensor that utilizes two large detector arrays but this sensor has a resonant shutter mechanism

that allows it to operate in a pseudo-staring mode. No scanning sensor utilizes a large number of detectors because it is not required. None of the sensors examined were designed to operate in both staring and spinning mode. Horizon sensors were one of the first type of sensors designed and produced. Very little modern research is being done in this mature field. The design of the sensor has not changed much in the last two decades. Advanced electronics and materials have allowed the weight and size of the sensor to be reduced. Accuracy has only improved with improvements in the detecting elements.

3.2 Algorithm for Conventional Spinning Horizon Sensors

The following algorithm is the horizon sensor mathematical model put forth in Wertz [13]. Figure 3.3 illustrates the geometry corresponding to this algorithm. The angle seen by the detector in any time interval is given by:

$$\Omega = (t_o - t_1)\omega \quad (3.1)$$

where ω is the spin rate for the satellite and Ω is the angle swept by the detector.

Let Ω_E be the angle the detector sweeps while seeing the Earth so that:

$$\Omega_E = (t_{LE} - t_{TE})\omega \quad (3.2)$$

Let γ be the angle between the spin-axis and the boresight of the detector and let η be the angle between the spin-axis and the Earth's center. Applying the law of cosines to the appropriate spherical triangle (see Figure 3.4) results in:

$$\cos \rho = \cos \gamma \cos \eta + \sin \gamma \sin \eta \cos \frac{\Omega_E}{2} \quad (3.3)$$

where ρ is the angular radius of the Earth as seen from the sensor's altitude.

Equation 3.3 can be used to solve for η . First substitute for $\sin \eta$ and rearrange to get:

$$\cos \rho - \cos \gamma \cos \eta = \pm(1 - \cos^2 \eta)^{\frac{1}{2}} \sin \gamma \cos \frac{\Omega_E}{2} \quad (3.4)$$

Figure 3.3: Geometry of Conventional Scanning Algorithm

Square both sides and collect terms of $\cos \eta$ results in:

$$(\cos^2 \gamma + \sin^2 \gamma \cos^2 \frac{\Omega_E}{2}) \cos^2 \eta - 2 \cos \gamma \cos \rho \cos \eta + \cos^2 \rho - \sin^2 \gamma \cos^2 \frac{\Omega_E}{2} = 0 \quad (3.5)$$

This is a quadratic equation in $\cos \eta$ that has the following solutions:

$$\cos \eta = \frac{\cos \gamma \cos \rho \pm \sin \gamma \cos \frac{\Omega_E}{2} (\cos^2 \gamma + \sin^2 \gamma \cos^2 \frac{\Omega_E}{2} - \cos^2 \rho)^{\frac{1}{2}}}{\cos^2 \gamma + \sin^2 \gamma \cos^2 \frac{\Omega_E}{2}} \quad (3.6)$$

The values for γ and Ω_E are known, so it is only left to find the value of ρ to calculate the value of η . With a known altitude, h , ρ can be easily calculated from the known radius of the Earth, R_E (6300 km) by:

$$\rho = \arcsin \frac{R_E}{R_E + h} \quad (3.7)$$

Figure 3.4: Spherical Triangle Used by Algorithm

Since the spacecraft is spinning, the location of the Earth is described as the angle between the spin-axis and the center of the Earth, but this isn't the only angle that is required. For the purposes of having a robust and flexible attitude sensor, determining the angle between the Sun and the Earth is also desirable. This angle can be determined by the Earth sensor if it is also able to sense the Sun location with sufficient accuracy.

Many simplifications and assumptions were made in the above algorithm. The location of the spin-axis with respect to the spacecraft was assumed known, no nutation or coning was present, and the altitude was assumed known. Many situations may arise where one or more of these simplifications or assumptions may not hold.

Amongst these, the simplest to handle is lack of altitude knowledge. The others require complex estimation procedures, multiple sweeps and/or multiple detectors.

3.3 Overlaid Imaging

A literature search did not reveal any other instruments which utilized overlaid images for attitude determination. In fact, SAS appears to be the first sensor to use overlays to increase its FOV. There have only been two situations where the use of an overlaid image was encountered. These will be discussed in the following sub-sections.

3.3.1 Spectral Imaging

In many spectral imaging applications, there are two telescopes which filter at different wavelengths. Shutters and/or mirrors are employed so that a single detector can be used for both telescopes. In this situation, there is no ambiguity in the information contained by the images because of the use of the resonant shutter. The shutter only allows one overlay to be incident on the detector at a time. The use of overlays in this application is not meant to increase the FOV (usually the two telescopes are pointing at the same object) but instead allows a single detector to see two spectral bands at the same time. Another method that may be used to eliminate the ambiguity in the information is to use a detector that can distinguish between the two (or more) frequencies being observed.

3.3.2 Insect Vision

The only other reference encountered that discussed overlays was a description of the vision of insects. In some insects, the facets of their compound eye generate images on neighboring facets. In this way the visual detector in a facet may see many images. This does not cause the insect a great deal of difficulty because the overlaid images from the neighboring facets are much weaker than the main image and so they can be ignored.

Figure 3.5: Image at $6^\circ/\text{sec}$ (1 rpm)

3.4 The Images in Spinning Mode

The images change dramatically between the staring mode (see Figure 1.3) and the spinning mode. In the spinning mode, the pixel response will lag because of the thermal response of the microbolometers. This lag combined with the dynamic motion will tend to smear the images as illustrated in Figure 3.5 (i.e. the response to an impulsive type of image will be a saw-tooth type pattern). The image in Figure 3.5 is similar to Figure 1.3 in that they are both for the same array at about the same spin-phase; however, Figure 1.3 is the response when the spin is negligible and Figure 3.5 is the response at a spin rate of $6^\circ/\text{s}$ (at one point instant of the spin). Since the array in Figure 3.5 is not aligned with the direction of sweep (i.e. not perpendicular to the spin-axis), then the image produced will not be symmetrical. Because the Earth appears circular, different pixels in a detector array that approaches the Earth at an angle will travel through shorter or longer paths of the Earth's atmosphere. A shorter path will result in a shorter transition. Note that this asymmetry in the image is only seen in the spatial images, such as Figure 3.5. The time based images for individual pixels will show symmetry, such

as Figure 3.8.

Figure 3.6: Image at $24^\circ/\text{sec}$ (4 rpm)

Figure 3.6 more strongly shows the effect of spin on the image. While the horizon profile affects the leading edge and trailing edge horizons, the effect of spin totally hides the horizon profile. Determining where the horizon's location is in this image is very difficult. The Earth subtended an angle of 21° in the simulation which produced this data.

3.5 Earth-Space Transition Determination

In staring mode, the ability to determine which pixel saw the transition is very limited. Not only does the Earth's horizon profile extend over a number of pixels, but oblateness, noise, and the ever-changing characteristics of the Earth's upper atmosphere further complicate the matter. In staring mode the transition can be found at sub-pixel levels by matching an expected horizon profile to the data. This not only pinpoints an acceptable horizon location, but it is more robust in noisy situations. This technique effectively performs a spatial averaging which reduces

Figure 3.7: Image at $180^\circ/\text{sec}$ (30 rpm)

the amount of noise. The details of this technique will not be explored here because it is not applicable to the spinning mode case.

In spinning mode there is extreme difficulty in finding the horizon's location spatially along the array. Figure 3.6 shows an example of an array's image while experiencing a significant spin rate. The spin rate was 4 rpm ($24^\circ/\text{sec}$) and it clearly shows the difficulty in finding the horizon's location. The horizon is around pixel 16. Figure 3.7 shows the situation at an even higher spin rate.

An alternative method for determining the location of the transition is required for spinning mode. Examination of the time based response of a pixel produces a very different scenario. Figures 3.8 to 3.10 show what the time based response of a pixel would be at different spin rates.

Figure 3.8: Full Spin Pixel Response at $6^\circ/\text{sec}$ (1 rpm) Figure 3.9: Full Spin Pixel Response at $24^\circ/\text{sec}$ (4 rpm)

Figure 3.10: Full Spin Pixel Response at $180^\circ/\text{sec}$ (30 rpm)

Chapter 4

The SAS Spin-mode Algorithm Problem

This chapter will examine in detail why overlaid images are a problem to attitude determination algorithms. The following sections will examine the ramifications of the use of overlays in a horizon sensor and the constraints placed on an algorithm to resolve the ambiguity caused by the use of overlaid images.

Figure 4.1: Overlay Principle

4.1 The Overlaid Image Problem

As an attitude sensor, the SAS attempts to measure the transitions in its images which match the expected response from an Earth-to-space or space-to-Earth transition. The SAS will then refer from this transitions the location of a number of points on the Earth's horizon circle. The SAS then uses this information to determine where the horizon circle is located with respect to the sensor. The points located on the horizon are used to infer where the center of the Earth is located. In spinning mode, a chord of the Earth's horizon circle (which can be defined by two points) is used to determine the spin-phase angle for the center of the horizon circle and along with knowledge of the size of the horizon circle, possible solutions for the spin-axis-to-earth-center angle can be determined. Overlays cause problems for using the chords to determine the location of the Earth since the phase information, chord width and mounting angle information is ambiguous, with a number of possibilities for each.

The approach taken to illustrate the overlaid image problem, is to use the equation of a circle with a number of known points to represent the attitude solution. The equation of a circle is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2 \quad (4.1)$$

where (x_c, y_c) is the center of the circle and r is the radius of the circle.

This equation of a circle is an implicit equation (i.e. a point is on the circle if and only if it satisfies the equation). When three points, (x_1, y_1) , (x_2, y_2) , (x_3, y_3) lie on the circle, the following set of equations can be formed:

$$(x_1 - x_c)^2 + (y_1 - y_c)^2 = r^2 \quad (4.2)$$

$$(x_2 - x_c)^2 + (y_2 - y_c)^2 = r^2 \quad (4.3)$$

$$(x_3 - x_c)^2 + (y_3 - y_c)^2 = r^2 \quad (4.4)$$

In this set of three equations, the unknowns are x_c , y_c , and r .

Combining equations 4.2 and 4.3:

$$x_1^2 - x_2^2 - 2x_c(x_1 - x_2) + y_1^2 - y_2^2 - 2y_c(y_1 - y_2) = 0 \quad (4.5)$$

and similarly with equations 4.2 and 4.4:

$$x_1^2 - x_3^2 - 2x_c(x_1 - x_3) + y_1^2 - y_3^2 - 2y_c(y_1 - y_3) = 0 \quad (4.6)$$

Combining equations 4.5 and 4.6 results in a solution for (x_c, y_c) and this result is used to solve for r . This closed form solution for the three unknowns of the circle can be written in the following generalized form:

$$y_c = \frac{1}{2} \frac{(y_1^2 - y_2^2 + x_1^2 - x_2^2)(x_1 - x_3) - (y_1^2 - y_3^2 + x_1^2 - x_3^2)(x_1 - x_2)}{(y_1 - y_2)(x_1 - x_3) - (y_1 - y_3)(x_1 - x_2)} \quad (4.7)$$

$$x_c = \frac{1}{2} \frac{(y_1^2 - y_2^2 + x_1^2 - x_2^2)(y_1 - y_3) - (y_1^2 - y_3^2 + x_1^2 - x_3^2)(y_1 - y_2)}{(x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)} \quad (4.8)$$

$$r = \left[(x_1 - x_c)^2 + (y_1 - y_c)^2 \right]^{\frac{1}{2}} \quad (4.9)$$

As can be seen, with three points that lie on a circle, a closed form solution for the equation of the circle can be formulated. An investigation into trying to exploit other geometrical properties involving the ambiguous points has not resulted in helpful formulations. Every geometrical situation considered resulted in a formula that was equivalent to that demonstrated above (the solution for the circle).

The use of overlays complicates the determination of the horizon points. A simple model to determine where the points are as a function of the overlay number is:

$$x_1 = l_1 N_1 \quad (4.10)$$

$$y_1 = l_1 N_1 \quad (4.11)$$

where l_1 is the measured distance along the array for the pixel that detected the horizon circle and N_1 is the unknown overlay number. Thus, for every point detected on the horizon circle, there is an unknown (the overlay number). Substituting Equations 4.10 and 4.11 into Equations 4.7 to 4.9 results in a set of three equations with six unknowns. The unknowns are the three overlay numbers, N_1, N_2, N_3 , the coordinates for the center of the circle, x_c, y_c and the radius of the circle, r . The use of overlaid images reduces the order of the set of equations so that no closed form solution to the problem will exist.

4.2 Constraints

4.2.1 Physical Constraints

There are a number of constraints that are imposed on the solution to the overlay problem. Many of these are due to the current design of SAS and are grounded in physical reality.

Noise One of the major problems to overcome, when implementing a solution for SAS, is the high level of noise in the imagery. The expected signal-to-noise ratio (SNR) of a single microbolometer is on the order of 0.2 when the update rate is around 10 Hz (i.e. integration time ≈ 0.1 seconds). Because each pixel consists of a 2×20 array of microbolometers, the SNR for the pixel is increased significantly. Not all of the microbolometers would be used for imaging at one time, and one design iteration had 36 of the 40 available microbolometers being used for the pixel. This would raise the SNR mentioned above for a single microbolometer to about 1.2 for the pixel.

Orientation of the Arrays and Overlays The orientation of the overlays with respect to boresight of the sensor and the spin-axis of the spacecraft is important to how the algorithm can operate. When the orientation of an overlay is perpendicular to the spin-axis, then all of the pixels in the array will be detecting the same point on the earth (i.e. they all have the same spin-axis to pixel angle). While this scenario greatly increases the redundancy in detecting this point, the detection of multiple points has shown to be of greater use in spinning sensors.

Update Rate The update rate is tied to the amount of noise that will be experienced by the sensor and it is limited by the capabilities of the sensor's electronic components. It was conservatively assumed that the minimum update rate available would be about 10 Hz; however, operation at 100 Hz will also be explored. With shorter integration times, the amount of blurring caused by the spacecraft spinning at a given rate can be reduced. The reduced amount of blurring will result in the

achievement of higher spin rates while maintaining the same accuracy (assuming of course that the noise characteristics do not change). Note that this is not the system update to the spacecraft ACS.

Size and Shape of Pixels At first glance, the size but not the shape of the pixel will place constraints on the spin-mode algorithm. In fact, quite the opposite is true. Most conventional horizon sensors have relatively large FOV's with low noise characteristics. The low noise and the use of an electronic percentage-of-peak indicator can accurately determine when the pixel sees the horizon. The ideal shape for horizon detection is a circle because in the case of a circle, every 'edge' is the same distance from the circle's center...so the direction at which the horizon meets the circular FOV is not relevant. In the case of high aspect ratio rectangular pixels, the direction at which the horizon meets the rectangular FOV can greatly affect where the location of the horizon is perceived.

Low Processing Capability Space computing is fraught with many hazards. The higher levels of background radiation induce Single Event Upsets (SEU's), bit flips and other failures. Because of this, the electronic components tend to utilize older and slower computing technology. The algorithm design must be able to operate within the expected computing capability of SAS. At most SAS will utilize 6 or 7 8086 class processors (which will be integrated with the imaging array electronics), but it is more likely that only 1 or 2 processors would be available.

Small Memory Footprint For many of the same reasons that low processing capabilities is a constraint, so too is the amount of memory that can be placed on the spacecraft. For this reason, the algorithm cannot be complex or highly involved. It must be simple and straightforward and be easily converted to the machine language utilized by the processor.

Lack of Calibrated Signal Levels A lack of a calibrated signal means that when more than one overlay sees the Earth, the signal level cannot be used to determine

how many of the overlays see the Earth. The SAS design was conceived without the need for radiometric accuracy in the signals. The incident Earth radiation is a small portion of the total radiation incident on the array, so calibration would be difficult unless some form of thermal control (active cooling) could be used to reduce the level of the background radiation. For this reason it is not possible to determine what the response of a microbolometer will be in the future.

4.2.2 Mission Constraints

Other constraints are imposed by the expected operational concept for the satellite's mission. It is envisioned that small satellites (which would benefit greatly from a small versatile horizon sensor), and large communication satellites (which must carry both staring and spinning sensors for mission and transfer orbit operations respectively) would be most likely to employ SAS in spinning mode. Other missions will be more likely to use SAS only in staring mode.

Slow ADC Computer The spacecraft's Attitude, Determination and Control computer is affected in the same way that SAS's controller is by the space environment. For this reason, the ADC computer will not be very fast (compared to land-based computer systems) and will not be able to take on some of the processing that may be required by the SAS algorithms.

Increased Redundancy Whenever possible, satellite designers build redundancy into the satellite's systems to increase the probability that the satellite will successfully complete its mission. If SAS can sense both the Sun and the Earth, then a failure in the Sun sensor can be compensated by SAS. When successfully sensing the Sun, SAS would provide attitude information in the form of the spin-axis-to-Sun angle, the Sun-to-Earth-center phase angle, and the Sun-to-Earth-center elevation angle.

Nadir Angle Output As an Earth sensor, the most useful form of attitude information that the SAS can provide is the satellite's spin-axis-to-Earth-center angle.

This angle is referred to as the nadir angle in this thesis.

Limited Ephemeris Availability The spacecraft control computer may not provide ephemeris data to SAS. It will be assumed in this thesis that only the altitude will be known or provided. The altitude will be used to determine the angle the Earth subtends rather than trying to determine this from the images.

Transition Determination Conventional attitude determination algorithms make use of the ability to distinguish between LE and TE transitions. In fact, identifying the type of transition is almost as important as identifying the presence of a transition. The overlay resolution algorithm will assume that the SAS will provide adequate transition determination.

4.3 Applicability of Conventional Algorithms

For simplicity, the following considers a situation without complex dynamics (i.e. the satellite is cylindrical in shape and has a uniform mass distribution). It has been assumed that the spacecraft neither nutates nor has a coning action. The sensor considered is a body-mounted sensor (BMS) consisting of an infrared detector fixed onto a spinning spacecraft.

The FOV of the detector will sweep out a cone in space. The situation of interest occurs when the FOV encounters the Earth during one of its sweeps. Such a situation will generate a response from the detector with two events of major interest, the LE and the TE transitions. The major piece of information provided by these transitions is the time when they occurred. This can then be used to infer the chord length of the Earth at the location where the detector crossed the Earth's body. With knowledge of how the boresight is situated in relation to the spin-axis and knowing what part of the Earth that the sensor cut, it is possible to determine where the Earth is located with respect to the spacecraft's coordinate frame.

A pixel with 4 overlays can possibly see 4 TE and 4 LE transitions. Since there is no way of knowing which TE and LE transitions match, there are 16 possible

combinations which can be used in Equation 3.2. This can be rewritten and the index j used to indicate the possible solutions and the index i used to indicate the possible overlay:

$$\Omega_{E_j} = (t_{LE_i} - t_{TE_i})\omega \quad (4.12)$$

Each pixel (or detector) has its own specific angle between the spin-axis and its pointing direction, however; this mounting angle is dependent on the overlay. There are four possible mounting angles, γ_i , which need to be combined with Equation 4.12. Equation 3.6 can now be written as:

$$\cos \eta = \frac{\cos \gamma_i \cos \rho \pm \sin \gamma_i \cos \frac{\Omega_{E_j}}{2} (\cos^2 \gamma_i + \sin^2 \gamma_i \cos^2 \frac{\Omega_{E_j}}{2} - \cos^2 \rho)^{\frac{1}{2}}}{\cos^2 \gamma_i + \sin^2 \gamma_i \cos^2 \frac{\Omega_{E_j}}{2}} \quad (4.13)$$

Equation 4.13 illustrates the problem in finding an attitude solution when overlays are involved. There are two groups of solutions for η , each one based on 16 possible values for Ω_{E_j} and four possible values for γ_i . All together, there are 128 possible solutions for η .

4.4 Desirability of an Overlaid Spin-mode Algorithm

Missions that utilize a spin-stabilized mode usually carry a set of attitude sensors dedicated to operation in this mode. SAS has been designed for operation in three-axis stabilized mode; however, it has the flexibility and meets the performance requirements for operation in a spin-stabilized mode as well. The ability to operate in both modes would greatly increase the marketability of the sensor to satellite manufacturers, since a reduced set of sensors would be sufficient for attitude determination in both modes.

To meet the goal of operation in both stabilization modes, it is necessary to develop two complimentary algorithms. The algorithm for three-axis stabilized operation has been developed and successfully tested in the lab. An algorithm needs to be developed to achieve the goal of operation in spin stabilized mode. This thesis

addresses the desirability of operating the SAS in spin-stabilized mode by developing and simulating an algorithm that permits the operation of SAS on a spinning spacecraft.

Since SAS was designed as a three-axis stabilized sensor, the spin-mode algorithm is not as straightforward as its three-axis counterpart. The spin-mode algorithm will differ from those commonly used on other spin-mode type earth sensors because SAS uses overlays. The overlays cause the attitude information to be ambiguous and for this reason there will be a significant departure between the conventional algorithms and the ones that would be successful for SAS.

Chapter 5

The SAS Overlay Resolution Algorithm

A number of factors were considered in judging whether a particular overlay resolution algorithm was feasible. Beyond the primary consideration that the algorithm must be able to overcome the overlay problem, the algorithm should also have little or no impact on the current design of SAS.

The SAS Overlay Resolution Algorithm (SORA) needs to operate under a number of conditions. Algorithms already exist which overcome the overlay problem on 3-axis stabilized spacecraft. The algorithm being pursued in this thesis needs to work on a spinning spacecraft. The algorithm should have adequate performance for all altitudes below GEO. LEO and GEO altitudes can be considered especially important because the majority of missions utilize these two orbit configurations.

Many spinners spin at low frequencies for mission operations, but in Geosynchronous Transfer Orbit (GTO) the spin rate can be moderate (10-30 rpm) to high (60-100 rpm). The accuracy achieved in the spin configuration will of course be dependent on the spin rate. At low rates it should approach the accuracy of the three-axis case. At moderate spin rates it should be on the order of 0.1° . Operation at high spin rates is not critical because most spacecraft missions will not be Earth observing at these spin rates and an alternative sensor, such as a sun sensor, will be utilized. In addition, the market for sensors which operate at high spin rates has decreased as communication spacecraft utilize liquid apogee engines (LAE) instead of the solid variety. The lower thrust from the LAE means that moderate spin rates of 2 to 5 rpm are adequate.

Coding of the algorithm must take place within the limits of the SAS design. It must not utilize a great deal of memory or CPU usage. The memory footprint should be around 10 kB and it should be able to run sufficiently quickly on 1 to 6 CPU's of the 8086 class. Space computing capabilities are always limited, especially for such a small/low-cost device.

The SORA must make use of the SAS configuration. Aspects of the configuration which will affect the algorithm are:

- Size of the arrays
- Relative position of the overlays
- Number of overlays
- Size and shape of the pixels (each pixel is 2x20 array of 50 μm square bolometers with a 52 μm pitch)
- Signal-to-Noise-Ratio
- Number of pixels in the array
- Update (sampling) rate

This thesis examines and evaluates the various solutions. Recommendations for improvement to the sensor for spinning operation will be given and the best solution will be outlined.

5.1 Infeasible Algorithms

A non-invertible function is a mathematical function whose inputs can not be determined from its output. Unfortunately, non-invertible functions occur frequently in engineering. The loss of information caused by the use of overlays makes it difficult for SAS to determine the location and size of the horizon circle. The problem is easily handled in one direction; with a known attitude and altitude, it is easy to

calculate which overlays and pixels the horizon circle will fall in. The reverse, which is the solution to the problem addressed in this thesis, is very difficult.

Another aspect that is peculiar to many problems is the trade-off between memory and processing speed. In some cases, much of the processing can be performed ahead of time and the result can be stored for subsequent analysis. A characteristic of computers for spacecraft is that trading off memory for processing power does not benefit the design since memory is limited, expensive, heavy (for shielding) and unreliable. In the SAS design, a great deal of processing may be required to run the algorithm. If this is the case, then preprocessing may be necessary at the expense of additional memory.

Note that every attempt to exploit geometrical properties involving the overlaid images did not result in helpful formulations. Every geometrical situation considered resulted in a formula that was equivalent to the solution for the circle.

5.1.1 'Equation of a Circle' Based Algorithms

When attempting to solve for the equation of the circle, there will always be 3 fewer equations than unknowns when 4 overlays are involved. However, the unknowns are not completely unknown. There is some relationship between them because the points are constrained to lie on a circle. From the set of 4 LE and TE transition points, three sets of 3 points can be formed. Each set of 3 points can generate 64 possible solutions, only one of which is correct. In general, each set will result in solutions which differ from the solutions of the other sets. The true solution exists for any set of points chosen, so the solutions generated by a set of 3 points must contain the true solution. All solutions that are not common to each set can be considered to not be the true solution. Figure 5.1 illustrates some circle solutions given 3 points for which the overlay is unknown. Note that only some of the solutions associated with these 3 points are shown.

With enough sets of 3 points, it is possible to reduce in size the set of 64 possible solutions. Reducing it down to a single solution, which would be the required solution, would be the goal of the algorithm. Note that this algorithm operates

Figure 5.1: True and Incorrect Circle Solutions

from a single image that contains more than 3 points (in this context the image is considered to be the output of the 6 arrays taken at the same instant of time). Two images taken shortly after one another could be analyzed to give an approximate attitude to the spinning case. Additional images would be used to refine the attitude estimate. This algorithm should also work well in 3-axis mode.

A slight alternative to the above method, would be to examine the situation at two different sequential instances. In this situation only the components of the center of the circle in the equations are changing with time, and the subsequent images can be used to eliminate solutions.

On the surface the solution outlined above appears to be a feasible solution for both the spinning and staring modes. However, there is too much similarity between

the possible solutions generated to allow elimination to be useful. Other problems include:

- the blurring of the images along the direction of spin, turning the horizon circles into ellipses.
- it is only rarely and at specific nadir angles that more than 3 points will be observed simultaneously by the sensor

This algorithm may still be feasible for a LEO 3-axis mode situation if the use of ephemeris data (specifically altitude) is used to eliminate as many solutions as possible before comparisons between solution sets is made.

5.1.2 'Insect Vision' Based Algorithms

This proposed solution looks at a scenario where the detectors will signal when they see an Earth-to-space transition or a space-to-Earth transition. Upon examining the signals from a single detector, a determination of the direction that the Earth crossed the detector cannot be made. With two detectors, and one detector fired before the other, then a coarse estimate as to which direction the Earth crossed the detectors can be made. With more detectors, the estimate of the direction can be made with more accuracy. When a sufficient number of detectors are used, the problem of overlays can be ignored. This method is not unlike how an insect detects motion with a compound eye. The rate of the spinning spacecraft can be estimated from the time between firings of the detectors. Figure 5.2 shows the geometry of this situation. As can be seen from the figure, the determination of the path of object is more exact as more detectors are used.

This algorithm is basically an elimination procedure operating on all possible attitude solutions. With one detector, only whether or not the detector has swept across the Earth can be determined. If two detectors are used, then the sequence that they encounter horizon events can be used to eliminate possible solutions (approximately 1/2 of the possible solutions). As the number of detectors increases, the number of eliminations increases and the sub-set of possible solutions get smaller.

Figure 5.2: Geometry of Insect Vision

Given n detectors, only a finite number of attitudes can be determined. The number of possible attitudes and the arrangement of the sensors will determine the accuracy and resolution of the solution. The optimal number of possible attitudes (with no considerations for poor topologies) with n detectors is given by:

$$\sum_{i=0}^n \frac{n!}{i!}$$

For four detectors, arranged at the points of a square, there are 65 possibilities. For 6 detectors, arranged at the points of a hexagon, there are 1957 possibilities. Now, multiply the number of locations by a factor of 4 because of the use of overlays, and there will be 24 locations which would correspond to 1 686 553 615 927 922 350 000 000 possibilities. As can be seen, as the number of detectors and overlays increase, the number of unique attitudes also increases and the subset of possible solutions associated with any sequence gets smaller. Note that the number of possibilities just mentioned is for the detection of a point source. The

Earth is an extended object which has the effect of greatly reducing the number of possibilities (which depends on the size of the Earth) and greatly increases the detected sequences.

The procedure of determining the coarse attitude estimate would then involve comparing the real sequence with the unknown attitude parameters to a simulated/measured sequence with *a priori* known attitude parameters. This could be a horrendous task computationally because of the infinite number of attitude parameters.

In order to fix the position of the spin-axis in inertial space to within 5° in both coordinates, approximately 2600 possible sequences would need to be resolved. This number can be reduced by limiting the solution set to expected mission attitudes, but greater accuracy will quickly increase the number. In addition, because the Earth subtends a significant angle even at GEO, the angular size of the Earth cannot be eliminated from the sequences. This can greatly increase the size of the sequence, especially in LEO. It was experimentally found that approximately 3500 possible sequences were needed to achieve 5° accuracy in both the spin-axis location and the angular size of the Earth. It was also determined that at least 12 detectors would be required to achieve this level of accuracy with 4 overlays. The number of detectors available on the SAS can easily achieve much better accuracy.

The form of simulated solutions will depend on the capabilities of SAS. If SAS had a supercomputer then the solutions might be simulated in real time. Alternatively, the solutions can be calculated beforehand and stored in ROM within SAS. Approximately 100 kilobytes would be required to store approximately 3500 solutions when 12 detectors with 4 overlays each are examined. Using more detectors will allow better attitude estimates but more memory will be required to store the solutions (not only do the number of solutions increase with a decrease in step size, but the length of the sequences increases with an increase in the number of detectors and overlays).

The amount of processing and memory required to implement this algorithm makes it infeasible to implement, even at unacceptable levels of accuracy.

5.1.3 'Signal Level' Based Algorithms

The use of signal levels to get the overlay number can reduce the number of unknowns to 3 in the 3 equations. The set of equations can then be solved in a normal manner. While this solution has feasibility in overcoming the overlay problem, the signals available on SAS can not be calibrated sufficiently to make this a viable option.

Figure 5.3: A Signal Level Solution

In the spinning situation, this algorithm is not as straightforward as initially

thought. In LEO where the size of the Earth is very large, the algorithm is relatively straight forward; however, in GEO where the size of the Earth is small, there are multiple possibilities to the signal level solution, as illustrated in Figure 5.3. While this solution is valid for dealing with the overlay problem in general it is not applicable for SAS and wasn't analyzed in this thesis work.

5.1.4 'Macro Pixel' Based Algorithms

Averaging (or adding) all of the pixels in an array can be performed to get the equivalent of 6 low noise but large macropixels. These macropixels would behave much like the high-resolution photocells used in other earth sensors. There is still a problem with overlays but in general a signal level for each photocell can be found that can be calibrated to the amount of Earth that the array views.

Knowledge of how much of each array is seeing the Earth can be used to determine how far the array is pointing from the center of the Earth. Once it is known how far each array is pointing from the center of the Earth it will be possible to calculate a good attitude estimate. The problem with this algorithm is that it is not expected to achieve the required accuracy. The macro-pixels and high level of noise in the sensor can be compared to similar sensors which don't utilize overlays. These sensors have lower noise and in practice don't attain accuracies better than 0.1° .

Note that this algorithm is similar to the signal level algorithm in that the signal levels will be related to the macropixel values. For example, if 65% has a signal level of 2 and the rest has a signal level of 3, then the macropixel value can easily be determined as $0.65 \times 2 + 0.35 \times 3 = 2.35$.

5.1.5 'A Priori' Based Algorithms

If the coarse attitude is known, then it is possible to quickly determine the overlays that will see the horizon circle. Once the overlays are known then the solution to the problem is straight forward. This solution may involve a Kalman filter that provides the attitude estimate which then is used to estimate the overlays which is

then used to find an improved attitude estimate which is then used to update the Kalman filter. The problem with this solution is how to initialize the filter (or obtain the initial attitude acquisition). Another problem with this solution is the amount of processing that is required to simulate the attitude. This extra processing may be performed by the spacecraft's attitude computer, but then this is putting the solution outside of the SAS device. Depending on the performance, this solution may be a mode that is used once one of the other solutions is used to determine the initial attitude.

5.2 Feasible Algorithm

After analyzing several possible candidates, only one method was found to be feasible. Not only is it able to determine the attitude in the face of overlay ambiguity, it is implementable into the current design of SAS. This algorithm is based on the conventional horizon sensor algorithm presented in Section 3.2 and developed further with Equations 4.14 and 4.15. The set of possible solutions need to be reduced to the one correct solution. The following algorithm explains how the overlay information can be retrieved from the signals produced by a single pixel. For the remainder of this thesis, SORA refers to the algorithm about to be described.

This algorithm is based on utilizing a pair of transitions. The pair will consist of one LE transition and one TE transition and so the SAS must be able to distinguish between a LE and TE transition. A line joining the locations of the pair on the earth horizon circle will form a chord across the earth image. The chord can be thought of as the path traced out by the detector's FOV as it passes across the face of the Earth. Because of the overlay problem, a single detector will see multiple LE and TE events. These multiple events correspond to multiple chords, one for each overlay. However, the ambiguity in the attitude information arises because we cannot determine which LE corresponds to a particular TE (and vice versa). Essentially, the overlay problem boils down to trying to determine which LE-TE pairs are correct by eliminating all of the erroneous pairs. Both the correct and erroneous pairs are chosen from a set generated by taking in combination all of the

LE and TE events. The set must contain all possible pair combinations in order for the algorithm to be successful.

The SORA is based on the timing of the signals generated by the edges of the earth image. Because the overlays are not in phase with each other, the multiple signals seen by a pixel will be slightly out-of-phase with each other. This must be true since the corresponding spin-phase angle calculated from the pixel response must correspond to that pixel's mounting phase angle. The phase angles of the signals can be checked by comparing the center of the response with the known pixel mounting phase angle of the overlays. The pixel's mounting phase angle is similar to the mounting phase of the boresight, but instead it relates the position of the pixel to the boresight about the spin-axis.

All of the phases can be referenced to when SAS's boresight crosses the plane containing the spin axis and the center of the Earth. The time this occurs at will be referred to as the boresight time. The boresight time can be easily found by averaging all of the LE and TE times for all of the pixels of a single spin. For this discussion, a single spin is defined as going from a spin-phase of -180° to $+180^\circ$, where 'zero phase' is defined to occur at the boresight time.

Figure 5.4 illustrates some situations when the boresight time algorithm will produce correct and incorrect results. Note that 'zero phase' will not necessarily occur at the greatest average of all pixel responses, but the response will always have symmetry (since the sensor has a symmetric design) and the 'zero phase' will correspond to the point of symmetry. The illustrations associated with incorrect results show that some information, either LE or TE times, is not available thus leading to incorrect boresight time results.

The following development illustrates the steps taken for the SORA. It starts by examining the LE and TE transition times of a pixel during a single spin. In the following example, a pixel has three associated overlays and each overlay encountered a LE and TE event. The signal from the pixel resulted in 6 transitions being determined (the following example times are in MJD):

- LE: 50894.0008391211, 50894.0008518526, 50894.0008611119

- TE: 50894.0010428250, 50894.0011469917, 50894.0012094918

These times represent the transitions from three different overlays, since there are three each of LE and TE. These times can be grouped into 9 possible pairs, of which only three are correct. The three correct pairs will correspond to the three overlays which imaged the Earth onto the detector. The times will be designated as:

$$t_{LE,i} = \{50894.0008391211, 50894.0008518526, 50894.0008611119\}, i = \{1, 2, 3\} \quad (5.1)$$

$$t_{TE,j} = \{50894.0010428250, 50894.0011469917, 50894.0012094918\}, j = \{1, 2, 3\} \quad (5.2)$$

The boresight time needs to be determined first. Since it is simply the average of all of the transition times from all of the pixels in all of the arrays of the sensor (assuming that the sensor configuration is axially symmetric), then:

$$\bar{t}_B = \sum_{k=1}^n \left[\frac{\sum_{i=1}^{m_k} t_{LE,i} + \sum_{j=1}^{m_k} t_{TE,j}}{\sum_{k=1}^n 2m_k} \right] \quad (5.3)$$

where n is the total number of pixels in the sensor and m_k is the number of overlays experiencing a LE or TE transition for pixel k . It is important that all of the times for the full spin are included in the boresight time calculation. Figure 5.4 shows situations where the boresight time calculation will generate incorrect results. In this example the boresight time was found to be $BT = 50894.0010482138$. Note that the boresight time was found from all of the LE and TE transitions detected by the sensor, not just those times noted for the single pixel in Equations 5.1 and 5.2.

The average for each correct pair is related to when the overlay saw the center of the Earth (i.e. crossed the plane defined by the spin-axis and the center of the Earth). The average is simply determined by:

$$\bar{t}_{i,j} = \frac{t_{LE,i} + t_{TE,j}}{2} \quad (5.4)$$

The averages for the 9 possible pairs are listed in Table 5.1.

The relative phase for each pair can be calculated based on the difference between the boresight time and the average times for the pairs multiplied by the spin rate.

$$\phi_{i,j} = \omega(\bar{t}_B - \bar{t}_{i,j}) \quad (5.5)$$

Figure 5.4: Boresight Time Situations

The SORA assumes that the spin rate is known (by the use of a sun sensor) and if it is not known there are a number of simple algorithms that can be used to determine the spin rate (i.e. from successive boresight calculations by the SAS). The sample data being used was acquired from a simulation set up for a spin rate of $2^\circ/\text{s}$. The apparent relative phases are tabulated in Table 5.2 .

The actual relative phases of the overlays for the pixel being examined are (for overlay 1,2,3, and 4): 0.052503819, 0.161052785, 0.281034902, and 0.423021915. This information would be available from calibration procedures performed on SAS prior to operation. It depends on: the relative geometry of the pixel with respect to

LE-TE Pair ($i - j$)	Average ($\bar{t}_{i,j}$)
1-1	50894.0009409731
2-1	50894.0009473388
3-1	50894.0009519685
1-2	50894.0009930564
2-2	50894.0009994221
3-2	50894.0010040518
1-3	50894.0010243064
2-3	50894.0010306722
3-3	50894.0010353018

Table 5.1: Averages for LE-TE Pairs

the boresight, the orientation of the SAS around its boresight, and the orientation of the SAS with respect to the spin-axis.

The apparent relative phases can now be compared to the actual relative phases. The best three matches of pairs that do not reuse TE or LE transition times will indicate which overlays actually saw the Earth. LE-TE pair 2-3 has the best match of all the possible combinations...to overlay 1. Now there are four pairs left. The next best match is with LE-TE pair 1-2 with overlay 2. This leaves only one pair left which can be matched with overlay 3. The simulation used to generate the LE-TE transitions confirms that these are the correct overlays and times for the pixel being examined. The matches were (from worst to best, in radians): 0.009234232, 0.005298029, and 0.000400418. The worst match corresponded to an error of 0.53° . It may be possible to reduce this error by changing the transition determination algorithm to trigger earlier (a lower threshold) or later (a higher threshold), but it is limited by the update rate and spin rate.

The above method is slightly simplistic but is adequate to illustrate the solution. It is not always the case that the single smallest difference between apparent and actual relative phases correctly predicts the LE-TE pair for an overlay. If the first LE-TE pair is matched to the incorrect overlay, then it is likely that all subsequent

LE-TE Pair ($i - j$)	Apparent Relative Phase (radians)
1-1	0.3234304724
2-1	0.3042318231
3-1	0.2902691332
1-2	0.1663508140
2-2	0.1471521647
3-2	0.1331894747
1-3	0.0721028873
2-3	0.0529042380
3-3	0.0389415480

Table 5.2: Apparent Relative Phases for LE-TE Pairs

matches will also be incorrect. A much more robust method, and the one that was used in the simulation, considers all of the possible sets of matches for a pixel and chooses the set which has the lowest sum of absolute differences. An enhancement to a method would be to not consider any matches where the difference between the apparent and actual relative phases was larger than a predetermined limit. The limit would be set such that it encompasses all expected errors and eliminates obviously incorrect matches. This algorithm is useful regardless of how many overlays see the Earth. For example, if only one of a possible four overlays see the Earth, then there will be only a single LE and TE pair to determine. Which overlay the pair belongs to can be easily found by finding the best apparent relative phase.

The SORA can quickly determine which overlays saw the Earth and hence overcome the ambiguity caused by the use of overlaid images. If this algorithm determines the correct overlays, then the problem is solved. Any error in the spin-axis/nadir angle would be a result of the determination of the time of the LE or TE events, errors in the image synthesis or other system errors typical of horizon sensors.

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5.2.1 A SORA-like Algorithm for the Sun

The solution must also consider determination of the position of the Sun with either the spin-axis to sun angle (the Sun angle) along with the spin-phase between the Sun and the Earth's center or the Earth-center to sun angle. Detection of the Sun is much easier than detection of the Earth. The Sun is small and bright and will only be seen by a few pixels at a time. The Sun can be distinguished from the Earth whenever the pixel signal goes above what is expected from the Earth, plus a margin for noise. The noise margin must be reasonably large so that false detections of the Sun would occur infrequently.

The algorithm which resolves the ambiguity problem for the Sun operates by examining the possible sun angles derived from a given image. Based on the configuration of SAS used for the spinning mode, the Sun should be seen by at least two different arrays every spin. From the two images that see the Sun, the earth-sun phase and the sun angle can be compared for the different overlays. Because the phase and angle are nearly orthogonal, the likelihood of matching more than two incorrect solutions is slim. The pixel which sees the Sun has associated with it a mounting angle and phase.

The phase of the Sun is found in the same way as outlined above in determining the phase of the Earth. There are four possible angle and phase combinations for each image. The sun angle and phase combination which is consistent with all of the images over an entire spin is the desired solution.

Chapter 6

Implementing the Solution

The SORA algorithm, described in Chapter 5, resolves the overlay problem so that an attitude determination algorithm can take the LE and TE times and determine a spin-axis/nadir angle. In this chapter, the key components of the simulator are explained.

The simulator serves two purposes. It primarily serves as a testing tool to validate the proposed algorithm; however, in the course of acting as a test tool it also demonstrates the ability of an algorithm to utilize a state to predict which overlays the transitions will fall upon. However, while this *a priori* solution resolves the overlay problem, it is not feasible based on the amount of processing required.

Once the simulator has determined the images and where the LE and TE transitions occur, then the overlay resolution algorithm is used to solve the overlay problem. Once the overlay problem is solved, an attitude determination algorithm would be used to determine the attitude.

6.1 Earth, Moon, and Sun Positions

The SAS sensor is primarily designed to sense the Earth; however, sensing other objects such as the Sun and Moon can not be avoided. Unfortunately, sensing the Moon and Sun can interfere with SAS's performance. If the Moon and the Sun could be ignored, then we could simply consider the Earth to be the center of the reference system. In addition, there is the possibility that the Sun can provide useful

information in the form the sun/nadir angle.

In order to properly model the effects of the Moon and Sun, relative positions of all three objects need to be simulated. The relative positions can be used to determine the directions to the respective object, as illustrated in Figure 6.1. The direction is important when determining what is in the sensor's FOV. The positions are generated by a simple ephemeris predictor taken from [11].

Figure 6.1: Earth, Moon and Sun Directions From Satellite

6.2 Satellite Position and Attitude

The position of the satellite relative to the object dictates how the object will appear to the SAS. Of primary interest is the size of the Earth which is dictated by the distance from the Earth. For example, in LEO the Earth can subtend an angle of 160° , taking up almost half of the sky. In GEO the Earth can subtend an angle of

17°. This large amount of variation in the size of the Earth needs to be simulated to ensure that the algorithm can handle these extremes.

The satellite's position also determines the directions to the Sun, Moon and Earth. This will be useful in order to determine which objects come in the SAS FOV while the satellite is spinning. Just as important, when the object comes into the FOV and how long it stays there needs to be determined. This is done in conjunction with the image synthesis algorithms to be discussed later.

Figure 6.2: Orbital Parameters of a Keplerian Orbit

The orbit is modeled as a simple two body Keplerian orbit. The orbit of the spacecraft is modeled using 6 classical orbital parameters. These parameters are: true anomaly (θ), inclination (i), eccentricity (e), semi-major axis, argument of perigee (ω), and right ascension of the ascending node (Ω). Figure 6.2 illustrates some of these parameters. In this model, the true anomaly is time dependent and so a position can be calculated for any desired time.

Because a device mounted to the spacecraft will move as the spacecraft moves, the attitude of the spacecraft determines where a mounted device (such as the SAS) will be pointed. Hence, the attitude of the spacecraft and its relative position,

determines if the SAS will sense a particular object.

The overlay problem to be solved by this thesis only involves spinning spacecraft. An algorithm has already been developed for SAS when used on 3-axis stabilized spacecraft. Modeling the true dynamics of a spinning spacecraft are not necessary for this thesis, since nutation will not have an effect on the overlay resolution algorithm. In conventional spinning horizon sensors, the nutation is not taken into account by the attitude algorithm; rather, it is determined from the determined attitude errors as a systematic and predictable bias. Simple pure rotation is a sufficient model of a spinning spacecraft for this thesis. The model used in this thesis simply applies a small rotation about a predetermined axis. The size of the rotation depends on the time since the last applied rotation. This model was taken from [13].

In this thesis the attitude can refer to one of two things: the orientation of the spin axis with respect to inertial space, or the instantaneous orientation of the spacecraft. In most cases, references to the spacecraft's attitude will refer to the latter.

Figure 6.3: Sensor Position on Spacecraft

6.3 Sensor Position and Configuration

The sensor position about the spin-axis of the spacecraft is not important since the only reference on the spacecraft is the location of the SAS. It is assumed that location of the sensor defines the x-z plane (body coordinates) of the spacecraft where the z-axis is the spin axis. However, to facilitate testing a mounting phase can be designated. The mounting phase is treated as a bias and essentially moves the sensor in a circular manner around the spin axis. For example, a mounting phase of 90° will put the SAS in the y-z plane instead of the default x-z plane. The mounting phase, Ω_M is illustrated in Figure 6.3.

Figure 6.4: The SAS FOV Swept Through a Revolution

The relative direction of the sensor to the spin-axis determines where the bore-sight will point during the rotation of the spacecraft. This is defined by a spin-axis-to-boresight angle, which is γ_B in Figure 6.3. As the sensor sweeps through the sky, the cone which represents the FOV, will form a solid of revolution which appears as a hollow cone (see Figure 6.4). The figure shows a situation where the spin-axis/boresight angle is not near 90° . When near 90° , the FOV will sweep through a

region that will appear as the region between two cones, facing opposite directions, and joined at their vertex.

Figure 6.5: Rotated Sensor FOV's

The sensor can be rotated about the boresight by an angle called the boresight phase. This has the effect of changing the orientation of the imaging arrays with respect to the spin-axis. It is designated as ϕ_B in Figure 6.3. The effect of rotating the sensor about the boresight is shown in Figure 6.5. Figure 6.5 a) shows a situation where every pixel of array 1 will see the exact same chord. The arrows in the figure

denote the apparent direction of the FOV with respect to the Earth. In Figure 6.5 b), many more chords of the Earth will be sampled by the sensor. Figure 6.5 c) shows a situation where fewer chords will be detected in the vicinity of the boresight when compared to b). The boresight phase depicted in b) is the most favorable; however, the ideal phase will not be studied in this thesis. All of the tests performed with the simulator will use a boresight phase of 10° .

The array size is another configurable parameter. It is the angular amount the array subtends. The array size in conjunction with the number of pixels determines the FOV for each pixel. For example, if the array size is 12° and there are 16 pixels in the array, then each pixel will have a FOV of 0.75° . The array size, θ_{array} is illustrated in Figure 6.6.

Figure 6.6: Example SAS Configuration

The number of overlays determines how many optical paths are imaging onto a single array. Each overlay has the same FOV as the array size, indeed overlay size may be a better term to use. The number of arrays determines how many arrays are spread around the boresight of the sensor. In this simulation, the arrays are evenly spaced and they are oriented so that the long axis of the array intersects with the boresight of the sensor.

The inner angle, θ_{inner} , is the angle between the boresight of the sensor and the first pixel of the first overlay (i.e. the closest pixel to the boresight). The overlay gap, θ_{gap} is the angle between the overlay FOV's. Unlike the illustration

in Figure 6.6, the gap does not need to be positive. A negative gap would indicate that there is overlap between the overlays. Some degree of overlap may be the final design choice as it facilitates tracking of the Earth's horizon from overlay to overlay when in 3-axis mode.

While all of these parameters can be varied in the simulation, a standard configuration will determine the majority of the parameters. The standard configuration is outlined in Table 6.3. This configuration was chosen to optimally test the overlay resolution algorithm. Because each pixel is essentially tested individually, many aspects of the configuration do not need to be varied. Aspects which will be varied for the testing are: the orbit, the SNR, the spin rate, and the update rate.

Parameter	Standard Value
Number of Arrays	3
Number of Overlays	4
Number of Pixels	32
Array Size	11°
Inner Angle	6°
Overlay Gap	-1°
Boresight Phase	10°
Spin-axis Boresight Angle	90°

Table 6.1: Standard Sensor Configuration

6.4 Transition Determination

Transitions are determined by examining how the signal from a pixel changes with time. A sudden jump up or down by a predetermined amount indicates that a transition occurred. There are two transition determination algorithms used by the simulator. The first operates on image data with no overlay problem (i.e. before the images are combined...see the following section). In this situation there is no

noise to confound the transition determination and a relatively low level is used. The second transition algorithm operates on the overlay image data.

A great deal of problems are inherent in the transition algorithm. When the update rate increases, the transition appears to ramp up more slowly between two successive updates. If the algorithm triggers on two successive updates at one update rate, then a lower trigger level is required for higher update rates. The spin rate also affects the transition determination since the ramp will appear more steep as the spin rate increases.

6.5 Image Simulation

Once the attitude of the sensor and direction to an object is determined, it is possible to determine how the image of the Earth's horizon will fall on the imaging array. How the image falls on the array will determine each pixel's response. Examination of the response over time will yield the earth transitions and when they occur.

To simulate the image, each overlay is treated as if it had its own array. The orientation of each mirror is known...or more precisely can be calculated from the configuration parameters. For an actual SAS sensor, this information would be gleaned from calibration operations. The orientation of each mirror is used to determine how the cone, representing the projection of the horizon circle of the Earth, intersects with the plane of each overlay's array.

The intersection of the cone and the plane is a conic section. In this case, the nature of the conic section is not limited and may take the form of a circle, ellipse, parabola or hyperbola (see Figure 6.7). However, this complexity was avoided in the simulation by always using a circle. Two constraints are placed on the circle: the center for the circle corresponds to the center of the Earth (i.e. the focus of the conic section) and the point nearest to the center of the array is correctly located. These two constraints combine to determine the size of the circle. Note that this means that the circle is smaller than the conic section. Figure 6.8 illustrates the situation and approximates the GEO case with a good array size of 10-12°. If the size of the image of the Earth's horizon is always larger than the size of the array,

Figure 6.7: Projection of Earth Image on Array

the circle is a very good approximation.

Figure 6.8: Circle Approximation of Earth Image

The use of circles instead of parabolas and hyperbolas will introduce errors into the attitude determination aspect of the simulation, but it will not affect the transition determination and overlay resolution of the simulator. The use of circles greatly decreases the complexity and increases the speed of the simulator. Note that the use of conic sections may not give the accuracy required for a true model of the

detected Earth's image.

The correct method of determining the image of the Earth on the array plane would be to use ray-tracing through the optical system. It is believed that only this method would accurately model the image sufficiently to test the attitude determination aspects of the simulation. This is beyond the scope of this thesis for two reasons: the first reason is the amount of effort required to write the ray-tracing code, and the second reason is that attitude determination algorithms are readily available in the literature of horizon sensors.

The signal generated by each overlay is then added together to generate an overlaid image. The final component of the image to be considered is the noise in the signal. Noise is simulated by utilizing the signal-to-noise-ratio supplied by the user. The simulator assumes that the noise is gaussian. The noise is not added to the image immediately, instead it is stored separately and added before processing. Originally this was so that both images (with and without noise) could be plotted and compared.

Note that the thermal aspects of the SAS are not modeled by the simulator because it is not needed to test the overlay resolution algorithm. However, a full system analysis of SAS in spinning mode would need to model the thermal aspects in order to account for the response lag of the microbolometers and its effect on the overlaid images.

Chapter 7

The Simulation Results

The simulator is designed to test aspects of the SAS Overlay Resolution Algorithm (SORA). Aspects of the algorithm investigated by this thesis are: sensitivity to noise, response to higher spin-rates, behaviour in different orbit configurations, and response to higher update rates. The first section of this chapter will discuss each aspect in detail and the following section will present the test cases. The test results as well as a discussion of the test results can be found in the last section of this chapter.

7.1 The Simulations

The performance and operation of SORA depends on a number of aspects discussed below. The SORA depends on calculation of the apparent pixel phase and so is directly dependent on any errors in the transition times. All of these aspects can affect the transition determination algorithms. While these aspects do not directly affect the SORA, they serve to characterize how the SORA is affected by the reduced performance of the transition determination algorithm. Unfortunately, it is difficult, if not impossible, to independently test the performance of the SORA and the performance of the transition determination algorithm.

Noise The sensitivity of SORA to noise will be tested at two SNR levels: no noise, and at a SNR of 3.2. The SNR of 3.2 was chosen as it corresponds to the

expected level of noise when combining the signals from 256 microbolometers (i.e. 8 pixels of the sort discussed in Chapter 2...see Figure 2.2). The SNR of a single microbolometer was assumed to be 0.2. The test results from these two levels will be compared to see how the SORA responds to noise.

Spin Rate The spin rate is the rate at which the sensor will slew across the sky. A number of things happen when the spin rate is increased. The most critical of these is that the error in the determined time of the LE or TE transitions will increase because of the thermal response of the microbolometers and the resolution of the sensing strategy. With this increased error comes a higher rate of failure of the overlay resolution algorithm since the error will propagate to the apparent pixel phase calculation. The amount of error can be compounded further by low update rates.

Orbit Configuration The orbit, especially the altitude of the spacecraft, determines how large the Earth will appear to the sensor. In LEO, most of the overlays will see the Earth and so the overlay resolution algorithm will need to deal with 8 LE and TE transitions for each pixel. In GEO, many pixels will see fewer than 8 transitions and so will place an additional burden on the SORA. The burden arises because only 1, 2, or 3 of the possible 4 overlays used in the test will be correct. There is then the possibility that errors will make the unused overlay a better, but incorrect, match.

Update Rate A higher update rate can compensate for higher spin rates. However, it is assumed that the SAS imaging arrays will not be able to support update rates higher than 20 Hz. The tests will be performed at two update rates: 10 Hz and 20 Hz. The error in the transition times is on the order of $\pm \frac{1}{2} \text{Update Period}$. This will correspond to an error of $\pm \frac{\text{Spin Period}}{\frac{1}{2} \text{Update Period}}$ in the pixel phase. This means that as the spin rate changes, so must the update rate in order to maintain acceptable errors for the SORA.

Case	Orbit	SNR	Spin Rate ($^{\circ}/s$)	Update Rate (Hz)
1	LEO	0.0	6	10
2	GEO	0.0	6	10
3	LEO	3.2	6	10
4	GEO	3.2	6	10
5	LEO	0.0	24	10
6	GEO	0.0	24	10
7	LEO	3.2	24	10
8	GEO	3.2	24	10
9	LEO	0.0	60	10
10	GEO	0.0	60	10
11	LEO	3.2	60	10
12	GEO	3.2	60	10

Table 7.1: Test Cases with a 10 Hz Update Rate

7.2 Test Cases

Twenty-four tests were performed to characterize the performance of the SORA. The results consist of the number of transitions which pass and the number which fail. The test case parameters are summarized in Table 7.1 and Table 7.2.

Noise Test cases were designed to test noise by running every case with a noise level of 0.0 and 3.2. The results from the corresponding pairs of test cases (i.e. test cases 1 and 3) were then examined to see how the SORA performed with only noise varying. It was expected that GEO cases at low update rates and high spin rates would be most affected by noise (i.e. test cases 10 and 12).

Spin Rate Test cases were designed to test the spin rate by running sets of test cases where the only changing parameter was the spin rate. The results from the corresponding set of test cases (i.e. test cases 1, 5, and 9) were then examined. It

was expected that as the spin rate increases the performance of the SORA would decrease.

Orbit Configuration Test cases were designed to test the orbit configuration by running a corresponding GEO test case for every LEO test case. The results of all of the LEO test cases (i.e. odd numbered test cases) were then compared to the GEO test cases (i.e. even numbered test cases). It was expected that the SORA performance would decrease in GEO where fewer transitions would be encountered and some pixels would not see the Earth with every overlay.

Update Rate Test cases were designed to test the update rate by running every test at two different update rates (see Table 7.2. The results from the corresponding pairs of test cases (i.e. test cases 1 and 13) were then examined. It was especially interesting to see how the spin rate and update rate combine to affect the SORA. It was expected that as the update rate increases, the performance of the SORA would increase so that the increased update rate would be able to compensate for the increased spin rate (i.e. test cases 1 and 17 should be comparable).

7.2.1 Pass/Fail Criterion

The pass/fail criterion was found by running a transition determination algorithm on data for which the overlay information was known in order to generate the expected transition times. The image data without overlay information was then run through the overlay resolution algorithm and the transition times were compared to the expected transition times. Note that the transition times in both cases were stored for each pixel of each overlay of each array.

This pass/fail criterion is useful when examining the tests of the SORA since the transition times for the overlays will differ. If they do not differ then the SORA will always produce the correct answer since the times could be interchanged amongst the overlays and they would still be correct. The pass/fail criterion greatly simplifies the quantification of the SORA performance.

Case	Orbit	SNR	Spin Rate ($^{\circ}/s$)	Update Rate (Hz)
13	LEO	0.0	6	20
14	GEO	0.0	6	20
15	LEO	3.2	6	20
16	GEO	3.2	6	20
17	LEO	0.0	24	20
18	GEO	0.0	24	20
19	LEO	3.2	24	20
20	GEO	3.2	24	20
21	LEO	0.0	60	20
22	GEO	0.0	60	20
23	LEO	3.2	60	20
24	GEO	3.2	60	20

Table 7.2: Test Cases with a 20 Hz Update Rate

7.3 The Results

Table 7.3, at the end of this section, summarizes the results for the test cases. The first 12 cases deal with the SAS operating at a 10 Hz update rate. At this update rate and low spin rates, the SORA operates perfectly. A detailed discussion of the results appears in the following subsections. Conclusions on the performance of the SORA can be found in the last chapter of this thesis.

Note that, independent of the spin rate or update rate, the SAS will experience the same number of transitions if everything else is kept constant. All of the LEO cases show 1536 transitions; however, the GEO cases show a range on the number of transitions. This points to problems in the transition algorithms.

7.3.1 Noise

One peculiarity shown by the results is that noise occasionally increased the success rate of the SORA. This result must be treated with caution. Essentially, the success

rate is a comparison of two different transition determination algorithms, with the SORA being the main difference between them. The noise will tend to jiggle the LE and TE times, and so there is the possibility that the noise will help obscure some bias between the transition determination algorithms. This being said, noise does not appear to have a significant effect on the success of the SORA algorithm.

The test case pair, (1, 3), showed that noise has a small effect on the SORA algorithm at low spin rates. Test case pairs (5, 7), (9, 11), (10, 12), (14, 16), and (22, 24) show situations where the performance of the SORA increases as noise increases. However, other pairs, (6, 8), (13, 15), (17, 19), (18, 20), and (21, 23) show that the performance of the SORA decreases as the noise increases. Regardless, other parameters show a much greater effect on the performance of the SORA.

7.3.2 Spin Rate

At high spin rates, SORA does not operate well at either the 10 Hz or 20 Hz update rates. Test cases 1 to 4 and 13 to 16 show that the SORA operates near perfectly at low spin rates. Test cases 17 to 20 show that a higher update rate can compensate for moderate spin rates. The higher update rate appears to help the GEO test cases more than the LEO test cases at the moderate spin rates, but at high spin rates the LEO test cases are helped more than the GEO test cases. Figure 7.1 shows a graph which is proposed to illustrate this situation.

A number of factors contribute to the failure of the SORA at higher spin rates. Foremost among them is the error in the LE and TE times. The LEO and GEO cases behave differently because in the GEO case, the chord is shorter so any error in the times correspond to greater error in the chord when compared to the LEO case.

No check is made on how close apparent mounting phase comes to the expected mounting phase. It is thought that such a check could be used in some type of filter which could sift out the good LE-TE pairs produced by the SORA. Such a filter might allow operation at high spin rates if the update rate cannot be increased higher than 20 Hz.

7.3.3 Orbit Configuration

It was expected that the GEO cases would be harder on the SORA since there would be fewer transitions per pixel. The results show that this is not always true. When the GEO tests are compared to the corresponding LEO test, the LEO tests have better results at low and moderate spin rates when the update rate was 10 Hz, and at low and high spin rates when the update rate was 20 Hz. Figure 7.1 shows why this might be happening. In general, GEO cases are easier for SORA until a certain limiting spin rate is reached. This limit depends on the update rate. The LEO cases are not as easy for SORA, but at high update rates the effect is not as pronounced when the limit is reached.

Figure 7.1: Success Rate vs Spin Rate

7.3.4 Update Rate

While the SORA operated well at low spin rates regardless of the update rate, an update rate of 20 Hz was barely adequate for moderate spin rates. Test cases 9 to 12, and 21 to 24 show that SORA does not operate well at high spin rates if the update rate is 10 or 20 Hz. It is unclear at this time if the current design of SAS can achieve update rates higher than 20 Hz.

Case	Pass	Fail	Success Percentage
1	1536	0	100
2	604	0	100
3	1536	0	100
4	604	0	100
5	1205	331	78.5
6	208	272	43.3
7	1231	305	80.1
8	178	306	36.8
9	387	1149	25.2
10	114	208	35.4
11	394	1142	25.7
12	127	219	36.7
13	1532	4	99.7
14	608	3	99.5
15	1525	11	99.3
16	610	1	99.8
17	1393	143	90.7
18	610	4	99.3
19	1385	151	90.2
20	604	11	98.2
21	1081	455	70.4
22	162	222	42.2
23	1068	468	69.5
24	184	232	44.2

Table 7.3: Results for the Test Cases

Chapter 8

Summary and Conclusions

8.1 Conclusions

An overlay resolution algorithm is required for SAS to operate in spinning mode. The overlay resolution algorithm (SORA) presented in this thesis is such an algorithm.

The SORA has been demonstrated to work for a simulated model of the SAS. It was found that at low spin rates the SORA algorithm worked very well. A significantly higher update rate is required for the SORA to operate at high spin rates. The SORA was also found to be robust with respect to noise.

The nadir angle algorithm presented in Chapter 5 is based on the principles used by the SORA. Since the SORA has been demonstrated to work, the nadir angle algorithm would operate by taking the output of the SORA algorithm and passing it through a conventional spinning attitude determination algorithm to generate the nadir angle.

Since the SORA allows SAS to operate without *a priori* attitude information, it would not require the assistance of a host of other sensors (star sensors, sun sensor, or magnetometers) to assist it in determining the attitude. The SAS could then play an increased role as one of the main attitude sensors as opposed to a sensor that is used only for fine tuning the attitude determination.

The test cases indicate that higher spin rates cause the greatest difficulty for the SORA. This is somewhat offset by higher update rates and it is suggested that SAS operate at the highest update rate possible. If there is a single design change to be

made to SAS to facilitate operation in spinning mode, it would be to increase the update rate of the device.

8.2 Recommendations for Future Research

Many chords of the Earth are detected by the SAS instrument. There is a distinct relationship between the chords and their angle from the spin-axis. This information can be used to form a filter which can reduce or eliminate incorrect SORA results. This geometrically based filter would utilize the same equations as the attitude determination algorithms.

The dependence on the transition determination algorithm could be reduced by constructing an algorithm which will work for both required situations. The two situations in the simulator are: no overlay problem and no noise, and an overlay problem with noise. Even if no single algorithm could be found, running both algorithms on the same set of data will show how the transition determination algorithms are going to affect the test results independently of the SORA.

Testing of the sun algorithm should be performed. The sun angle algorithm is intimately tied to the attitude determination algorithm. Since the attitude determination algorithm had problems in this simulator, the sun angle algorithm could not be properly tested.

Many sensors employ resonant shutters (especially spectral imagers). Employing some type of shutter mechanics to resolve the ambiguity problem should be examined in future work.

It may be possible that the SORA algorithm can operate without knowledge of a transition being LE or TE. This should be examined in detail since the SAS may have problems determining the transition type.

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