INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600

Carleton University
Ottawa, Canada K1S 5B6

Text complete; leaf 1 omitted in numbering.

Magnetic Propulsion for Spacecraft

by

Walter Gillies, B.Sc.(Hons.)

A thesis submitted to
the faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Engineering

Department of Mechanical and Aerospace Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

August 30, 1996

© Copyright

1996 Walter Gillies

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-22113-X

The undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis

“Magnetic Propulsion for Spacecraft”
submitted by Walter Gillies, B.Sc.(Hons)
in partial fulfillment of the requirements for
the degree of Master of Engineering

Dr. D.L. Russell
Thesis Co-Supervisor

Dr. D. A. Staley
Thesis Co-Supervisor

Dr. R. Bell
Chair, Department of Mechanical and Aerospace Engineering

Carleton University
August 30, 1996

Acknowledgments

All men dream, but not equally. Those who dream by night in the dusty recesses of their minds wake in the day to find that it was vanity, but the dreamers of the day are dangerous men, for they may act their dreams with open eyes, to make it possible.

T.E. Lawrence
The Seven Pillars of Wisdom

Many thanks to my supervisors, Dr. Don Russell and Dr. Doug Staley, whose time and patience were taxed while I dreamt of electric spacecraft.

Walter Gillies

Abstract

Superconducting current loops are proposed as a means of spacecraft propulsion. Magnetic interaction with the geomagnetic field is the propulsive source. The general layout of the propulsion system is developed for both a single loop design and a triple-loop design. Challenges related to the practical design and implementation of superconducting loops as a means of spacecraft propulsion are addressed. Future potential for magnetic propulsion is explored, including interplanetary travel, self-launching capability and interstellar travel. Levitation from earth surface and interstellar flight do not appear practical because present-day materials are not strong enough and superconducting currents are not large enough. However, a single loop used in a polar orbit does appear practical for orbit transfers and interplanetary travel.

Table Of Contents:

<i>Magnetic Propulsion for Spacecraft</i>	ii
<i>Acknowledgments</i>	iv
<i>Abstract</i>	v
<i>Table Of Contents:</i>	vi
<i>Table of Figures:</i>	ix
<i>List Of Variables:</i>	xi
 CHAPTER 1 : INTRODUCTION	1
1.1 <i>Propulsion Requirements</i>	1
1.2 <i>Magnetic Propulsion</i>	1
1.3 <i>The Superconductor Revolution</i>	2
1.4 <i>Objective of Thesis</i>	3
 CHAPTER 2 : LITERATURE REVIEW	4
2.1 <i>Origins</i>	4
2.2 <i>Levitated Superconductors</i>	4
2.3 <i>Early Magnetic Spacecraft</i>	5
2.4 <i>Present Magnetic Spacecraft</i>	6
 CHAPTER 3 : THE GEOMAGNETIC FIELD	8
3.0 <i>General Description</i>	8
3.1 <i>Dipole Approximation</i>	9
3.2 <i>Variations in Ambient Fields and Currents</i>	11
 CHAPTER 4 : LOOP DESIGN	13
4.1 <i>Objective</i>	13
4.2 <i>Magnetic moment of loop</i>	13
4.3 <i>Nonuniform Field Force</i>	15
4.4 <i>Single loop</i>	19
4.4.1 <i>Thrusters</i>	22
4.4.2 <i>Control Moment Gyros</i>	23
4.4.3 <i>Single Loop Applications</i>	23
4.5 <i>Double Loop</i>	25
4.6 <i>Triple Loop</i>	26
4.6.1 <i>Triple Loop Refinement</i>	27
4.6.2 <i>Storing Energy</i>	31
4.6.3 <i>Crew Protection</i>	32
4.7 <i>Magnetic Propulsion Advantages</i>	32
4.8 <i>Design Challenges</i>	33

CHAPTER 5 : LOOP REINFORCEMENT	35
5.1 Objective	35
5.2 Methodology.....	35
5.3 Maximum Current.....	35
5.4 Self-Current Stress.....	36
5.5 Torques between Loops.....	42
5.6 Force from current in tether.....	44
5.7 Tethers to Loop 1	46
5.8 Reinforcement Material	48
5.9 Size of Tethers to Loop 1	49
5.10 Mass of Tethers to Loop 1	51
5.11 Section Modulus	52
5.12 Tethers between loops 2 and 3	54
5.13 Torque between Secondary Loops	56
5.14 Tethers Between Secondary Loops	57
5.15 Secondary Loop Mass	59
5.15.1 Secondary-Secondary Force	60
5.15.2 Secondary-Main Force.....	63
5.15.3 Total Shear and Bending Moment.....	64
5.16 Stress on Curved Beam	66
5.17 Primary Loop Reinforcement	69
 CHAPTER 6 : PERFORMANCE.....	 73
6.1 Overview.....	73
6.2 Drag	73
6.2.1 Variations in Drag	75
6.2.2 Drag in Orbit Program:	76
6.3 Attitude Control	77
6.4 Gravity	77
6.5 Electrostatic Fields.....	78
6.6 Optimum Loop Size:.....	79
6.6.1 Other Authors.....	79
6.6.2 Magnetic Force on Loop	80
6.6.3 Net force equation	84
6.6.4 Single Ring	84
6.6.5 Triple Ring.....	86
6.6.6 Loop Parameter Assumptions	90
6.7 Acceleration Programs	91
6.7.1 Acceleration vs. loop radius	92
6.8 Previous Researchers' Orbit Calculations	93
6.9 Orbit Transfer Programs.....	95
6.9.1 Single Loop.....	95
6.9.2 Triple Loop	103
 CHAPTER 7 : FUTURE DIRECTIONS	 106
7.1 Spaceflight Challenges	106
7.2 Interplanetary Travel.....	106
7.2.1 Solar Wind.....	107
7.2.2 Magsail Orbits	107
7.2.3 Space Sailing.....	110

7.4 <i>Interstellar Travel</i>	111
7.4 <i>Self-Launching Capability</i>	111
7.4.1 Triple Ring	112
7.4.2 Single Loop	114
7.4.3 Radical Assumptions	117
 CHAPTER 8 : PROBLEMS TO ADDRESS	121
8.1 <i>Orbit Control Barriers</i>	121
8.2 <i>Absolute Barriers-Orbit Control</i>	121
8.2.1 Superconductor Materials	122
8.2.2 Power Control	126
8.3 <i>Lesser Barriers-Orbit Control</i>	130
8.3.1 Transportation and Deployment	130
8.3.2 Geomagnetic Inductance	131
8.3.3 Power Required:	132
8.3.4 Orbit type	135
8.3.5 Single Loop Tumbling	136
8.4 <i>Barriers - Advanced Applications</i>	137
 CHAPTER 9 : CONCLUSIONS	138
 LIST OF WORKS CITED:	139
 APPENDIX A	142
A.1 <i>Structure</i>	142
A.2 <i>Single Loop Acceleration</i>	142
A.3 <i>Triple Ring Acceleration</i>	144
A.4 <i>Triple Ring Mass</i>	146
A.6 <i>Single Loop Orbit Program 1</i>	147
A.7 <i>Single Loop Orbit Program 2</i>	150
A.8 <i>Triple Loop Orbit Program</i>	152
A.9 <i>Single Loop Levitation</i>	156
A.9.1 Al Reinforcement	156
A.9.2 UHS Reinforcement	159
A.10 <i>Triple Loop Levitation</i>	161
A.11 <i>Constants File</i>	163
A.12 <i>Air Density File</i>	164
A.13 <i>Drag Force File</i>	165
A.14 <i>Magnetic Force File</i>	166
A.15 <i>Orbit Calculations File</i>	166
A.16 <i>Radial Force File</i>	168
A.17 <i>Single Loop Mass File</i>	168
A.18 <i>Triple Loop Mass File</i>	169
A.19 <i>Flexibility Program</i>	174
A.20 <i>Neutral Axis Calculations</i>	176
 APPENDIX B:	180
B.1 <i>Orbit Coordinates</i>	180

Table of Figures:

FIGURE 3.1 EARTH WITH MAGNETIC FIELD LINES (TASCIONE, 1988).	8
FIGURE 3.2 GEOMAGNETIC FIELD APPROXIMATED AS A DIPOLE.	9
FIGURE 3.3 SPHERICAL COORDINATES FOR EARTH FRAME.	11
FIGURE 4.1 MAGNETIC MOMENT OF CURRENT LOOP (AFTER WANGSNESS (1986), P. 304).....	14
FIGURE 4.2 FIELD OF A MAGNETIC DIPOLE.....	15
FIGURE 4.3 SINGLE LOOP OF CURRENT IN A MAGNETIC FIELD.	19
FIGURE 4.4 SATELLITE ORBIT AROUND THE EARTH SHOWING REGIONS OF MAGNETIC STABILITY.	24
FIGURE 4.5 TWO CURRENT-CARRYING LOOPS AT 90° TO EACH OTHER	25
FIGURE 4.6 TRIPLE LOOP OF SUPERCONDUCTING LOOPS WITH NET MAGNETIC MOMENT , M.....	27
FIGURE 4.7 STABILITY REGIONS FOR MAGNETIC CONTROL SHOWING CIRCULAR ORBIT.	28
FIGURE 4.8 MODIFIED TRIPLE LOOP, VIEW IN DIRECTION OF +Z AXIS (TOWARDS EARTH).....	30
FIGURE 5.1 SUPERCONDUCTING MAGNETIC ENERGY STORAGE SYSTEM (SMES)	36
FIGURE 5.2 OUTWARDS TENSION ON THE MAIN LOOP WHEN CARRYING CURRENT.	37
FIGURE 5.3 VIEW OF CURRENT-CARRYING LOOP IN PLANE OF LOOP.	38
FIGURE 5.4 VECTOR DIAGRAM SHOWING RELATIONSHIP BETWEEN OUTWARD FORCE AND TENSION.	40
FIGURE 5.5 TORQUE ON LOOP 2 DUE TO CURRENT IN LOOP 1, LOOKING ALONG -X AXIS.....	43
FIGURE 5.6 LOOP AREA GIVING MAGNETIC FIELD FOR TETHER STRESS CALCULATION.	45
FIGURE 5.7 VIEW ALONG X AXIS TOWARDS TRIPLE RING, SHOWING TORQUE ON SECONDARY LOOP.....	46
FIGURE 5.8 DOUBLE TETHER ARRANGEMENT FOR CONNECTING MAIN TO SECONDARY LOOPS.....	47
FIGURE 5.9 TENSION ON TETHER AND FORCE ARISING FROM TORQUE ON SECONDARY LOOP.....	49
FIGURE 5.10 STRESS ON A LOADED METAL CYLINDER.	52
FIGURE 5.11 TORQUE ON LOOP 2 DUE TO THE FIELD OF LOOP 1 AND LOOP 3 RESPECTIVELY.	54
FIGURE 5.12 TWO POSSIBLE TETHER CONFIGURATIONS TO CONTAIN THE SECONDARY LOOPS.....	55
FIGURE 5.13 MAGNETIC MOMENT OF LOOP 3 IN THE CENTER FIELD OF LOOP 2.	56
FIGURE 5.14 FORCES DUE TO TETHERS BETWEEN SECONDARY LOOPS.	57
FIGURE 5.15 LOOP CABLE DIMENSIONS.	59
FIGURE 5.16 FIELD LINES OF LOOP 2 CUTTING LOOP 3.....	61
FIGURE 5.17 LOOP 2 IN MAGNETIC FIELD OF LOOP 1.	63
FIGURE 5.18 CURRENT IN LOOP 2 AT ANGLE T (THETA) TO PRIMARY FIELD.	64
FIGURE 5.19 FORCES ON LOOP QUARTER-SECTION.	66
FIGURE 5.20 SECTION OF LOOP WITH NEUTRAL AND CENTROID AXIS.....	67
FIGURE 5.21 TENSION DIRECTIONS ON THE MAIN LOOP TETHERS	69
FIGURE 5.22 TETHER TENSION, COMPRESSION AND BENDING VECTOR DIAGRAM	70
FIGURE 5.23 EXPANSION AND COMPRESSION FORCES ON TETHERED, CURRENT-CARRYING LOOP.....	71
FIGURE 6.1 DIRECTION OF MAGNETIC MOMENT FOR A CURRENT LOOP.	80
FIGURE 6.2 SINGLE-LOOP DESIGN FOR MAGNETICALLY PROPELLED SPACECRAFT.....	85
FIGURE 6.3 FRONTAL VIEW OF SPACECRAFT WITH TRIPLE RING AND TETHERS	87
FIGURE 6.4 LOOP TUBE DIAMETER FOR VARIOUS WALL THICKNESS'	88
FIGURE 6.5 TOTAL MASS VERSUS WALL THICKNESS FOR TRIPLE RING.....	89
FIGURE 6.6: TRIPLE RING WITH 8 TETHER-PAIR ARRANGEMENT.	89
FIGURE 6.7 SATELLITE ORBIT AROUND EARTH SHOWING STABLE THRUST ARC.	94
FIGURE 6.8: TRANSFER FROM CIRCULAR ORBIT RADIUS A TO ELLIPTICAL ORBIT PERIGEE A, APOGEE B.	97
FIGURE 6.9: DEFINITION OF APOGEE, PERIGEE, TRUE ANOMALY AND SEMI-MAJOR AXIS.	98
FIGURE 6.10 SINGLE LOOP CHANGE IN APOGEE WITH ORBIT NUMBER, NO DRAG.	101

FIGURE 6.11 NO DRAG SINGLE LOOP CHANGE IN ΔV PER ORBIT.	102
FIGURE 6.12 SINGLE LOOP CHANGE IN APOGEE WITH TIME, WITH DRAG.....	103
FIGURE 7.1 IMPULSIVE HOHMANN TRANSFER, TYPICALLY USED BY ROCKETS.....	108
FIGURE 7.2 DIRECTION OF MAGSAIL THRUST FROM SOLAR WIND.	109
FIGURE 7.3 MAGSAILS DRAGGING AND SAILING ON THE SOLAR WIND.	110
FIGURE 7.4: SINGLE SUPERCONDUCTING LOOP WITH GYROSCOPIC SHELL.....	115
FIGURE 7.5 MASS AND MASS ABLE TO LIFT VS. CURRENT FOR SINGLE 100 M RADIUS LOOP.	116
FIGURE 8.1 TEMPERATURE, MAGNETIC FIELD AND CURRENT DENSITY LIMITS (WILSON, 1983).....	123
FIGURE 8.2 SIMPLE HOMOPOLAR FLUX PUMP SHOWING PATH OF FLUX SPOT.....	127
FIGURE 8.3 ELECTRONIC HOMOPOLAR FLUX PUMP.....	128
FIGURE 8.4 TRANSFORMER-RECTIFIER TYPE OF FLUX PUMP.....	129
FIGURE B. 1 CLASSICAL ORBIT PARAMETERS FOR AN ELLIPTICAL ORBIT.	180

List Of Variables:

α = angle in radians
 α = flexibility factor comparing curved beam to straight
 θ = co-declination, angle from equatorial plane
 θ = angle from geomagnetic north
 θ = angle of arc subtended
 ϕ = allowable angular deviation from static equilibrium
 ϕ = right ascension, angle from star Aries
 ϕ = area of flux
 ϕ = flux density
 ϕ = azimuth angle
 ρ = density of atmosphere
 σ = electrical conductivity
 E = electromotive force
 π = Pi, 3.141592654
 τ = torque
 v = true anomaly of orbit (see Appendix B)
 ρ_{Al} = density of aluminum
 $\sigma_{allowed}$ = allowed stress
 ρ_l = mass per unit length
 μ_0 = permeability of free space, $4\pi \times 10^{-7}$ Tm/A
 σ_{oc} = stress at outer surface, curved section
 σ_{os} = outer surface stress, straight section
 ΔV = change in velocity
 σ_y = yield stress
 a = acceleration
 A = area, cross section
 a = semi-major axis
 A = total area of loop
 A_θ = area of current loop perpendicular to θ
 a_D = acceleration due to drag
 A_r = area of current loop perpendicular to r
 A_{rad} = radial component of area
 A_{tan} = tangential acting part of loop area
 B = magnetic field
 B_e = magnetic field at earth surface
 B_{ox}, B_{oy}, B_{oz} = magnetic field in x, y and z directions
 B_{rad} = radial part of magnetic field
 B_{tan} = tangential part of magnetic field
 c = distance from neutral axis
 C_D = coefficient of spacecraft drag
 c_o = distance to outer surface from centroid
 d = diameter of tether wire

da = differential element of area
 ds = differential element of curve
 dt = differential torque
 e = eccentricity of orbit
 E = specific energy of orbit
 $\mathbf{F}$ = force
 f = frequency
 F_{12} = force on loop 1 due to secondary loop
 F_{22} = force on loop 2 due to other secondary loop
 F_c = force of compression
 F_{drag} = force of drag
 F_{magnetic} = magnetic force
 F_{net} = net force on satellite
 F_T = force total
 F_{tan} = force tangential to orbit
 Grad = gradient operator
 I = current
 I = moment of inertia
 I_1 = current in loop 1
 I_2 = current in loop 2
 I_x = second moment of inertia of cross section
 L = inductance
 l = length (of wire, tether or loop)
 $\mathbf{M}$ = magnetic moment
 m = mass
 $M(x)$ = bending moment at point x
 M_{max} = maximum bending moment
 m_{super} = mass of superconducting material
 m_{total} = total mass of spacecraft
 N = number of turns in loop
 P = power
 $\mathbf{r}$ = radius from centre of earth, vector from origin
 r = radius of loop
 R_1 = radius of loop 1, main loop
 R_2 = radius of loop 2, secondary loop
 r_A = radius of orbit A
 r_B = radius of orbit B
 R_{body} = radius of satellite body
 r_{earth}, r_e = radius of earth
 R_i = radius to inner edge of loop
 R_o = radius outer edge loop
 r_p = radius of perigee
 r_{super} = radius of superconducting material cross section
 S = section modulus
 T = orbital period

T = tension
 T = tension in tether
 t = thickness of reinforcement wall
 t = time
 T = time period for acceleration
 $T(\theta)$ = thrust function, see equation 6-19
 T_{12Y} = tension from tether linking loop 1 and 2 in y plane
 T_{22Y} = tension between secondary loops in y plane
 t_{23} = torque between loops 2 and 3
 U = magnetic energy
 v = velocity
 V = velocity
 V = voltage
 V_A = Alfven velocity
 x = distance along circumference of loop
 Y = distnace from neutral axis to centroidal axis
 z = distance from plane of loop

Chapter 1 : Introduction

1.1 Propulsion Requirements

Spacecraft require propulsion to attain, change and escape from orbit. Even maintaining an earth orbit below about 600 km requires propulsion to counteract atmospheric drag from the small traces of gas remaining. So far, propulsion forces have been provided exclusively by rockets. Propellant mass must be expended, with the amount depending on the mission. The space station will require occasional refueling by space shuttle for drag make-up thrusting. If a system could replace the chemical thrusting and save even one shuttle flight, one half billion dollars could be saved. For a Mars mission, there may well be a need to shuttle payloads from low to high earth orbit so that the trip can start further from earth's gravity. A system that could repeatedly change orbit without need for fuel mass would be a great advancement.

1.2 Magnetic Propulsion

One possible replacement for rocket propulsion is the use of magnetic forces. In low earth orbit (LEO), by far the largest ambient magnetic field is the geomagnetic field, so it will be considered first. The earth's magnetic field is similar to a fictional bar magnet buried in the core of the earth, tilted at 11.5 degrees from the axis of rotation (Merrill, 1983). It is well known that like poles of two bar magnets repel one another, so one could postulate a very strong magnet lifting itself in the geomagnetic field. It would also be necessary to find some means of controlling and stabilizing

such a magnet, for the situation would be as unstable as trying to balance one magnet over another with like poles opposing. Not only would the levitated magnet tend to flip over, it would also only be in equilibrium at one height, since there is no control over magnetic field strength.

To control field strength, one could use a coil of current-carrying wire. Current flow creates a magnetic field, and if the current flows in a circle, the field resembles that of a bar magnet (Bleaney, 1976). One could change the current in this loop to obtain the required magnetic force for propulsion. The large size of the required currents prevents the concept of superconducting loops from being used for spacecraft propulsion. Development of powerful superconducting materials is addressing this problem.

1.3 The Superconductor Revolution

Superconductivity is the absence of any resistance to current flow. Current can flow in an isolated superconductor indefinitely with no losses. Prior to 1986, superconductivity had only occurred in metals that had been cooled to near absolute zero (Burns, 1992).

In 1986, G. Bednorz and A. Muller found superconductivity occurring in a type of copper oxide at temperatures that, with further research, reached over 80 K. This is warmer than the boiling point of liquid nitrogen (≈ 63 K), which is cheaper to buy than Canadian milk (Burns, 1992). By 1994, temperatures of over 150 K for superconductivity had been claimed (Moon, 1994). These temperatures are much above that of the space environment background temperature of 2.7 K (Zubrin and

Andrews, 1991). The current densities likewise increased by orders of magnitude, to the point where thin films can carry on the order of 10^7 A/cm^2 (Moon, 1994), which means a wire of one square centimeter area could carry a current of 10 million amperes.

1.4 Objective of Thesis

The layout of magnetic propulsion systems utilizing both single and triple rings of superconducting loops interacting with the earth's magnetic field will be developed. Internal and external stress on the loops will be calculated, as will the amount of reinforcement needed. Simulation programs for both the single and triple loop designs will be run for situations with and without drag to estimate times required for a selected orbit transfer. Based on the expected mass of the system for the forces obtained, the practicality of replacing rockets with magnetic propulsion for launching into and travelling through space will be judged. The future potential of superconducting magnetic-loop propulsion as a means of interplanetary and interstellar travel will be outlined, along with calculations showing the barriers that must be overcome to make these developments practical.

Chapter 2 : Literature Review

2.1 Origins

Meissner (1933) is generally credited with the first comprehensive examination of the phenomena of superconductivity, which is the complete absence of electrical resistance in a material. Starting to understand the phenomena of superconductivity allowed study of the possibility of magnetic flight because powerful electromagnets could now be made which required no power input beyond that needed to start the current. However, any system that uses anything but stable attraction forces--a self-levitating system, for example--would need an attitude control system. Careful design and control have resulted in many artificially stabilized, levitated superconductors.

2.2 Levitated Superconductors

In the 1960's, efforts were made to create gyros from levitated, spinning superconductors. The gyros were levitated by inducing a current in the superconductor, which then repelled it from a magnet below. (Culver and Davis, 1957, Bourke, 1964 and Harding, 1965) In the late 1960's and early 1970's, magnetic plasma containment systems were investigated for fusion reactors. The idea was to levitate a superconducting current ring that would generate a plasma containment field without need of physical connections through the plasma (File et al., 1968). These ideas were not practical for production because superconductors could not operate at high enough temperatures at the time.

2.3 Early Magnetic Spacecraft

Until 1986, the limits of current for superconductors compared to the size of the magnetic fields and field gradients in space made the idea of magnetic propulsion unfeasible. Magnetic acceleration for space travel has concentrated on using magnetic guns, either to eject a probe at high velocity (Lemke, 1982) or to eject particles from the spacecraft for thrust (Kom, 1995 and Mauldin, 1992).

Corliss (1960) considered a non-superconducting ring-shaped satellite cutting the earth's magnetic lines of force as a means of generating power and also as a means of propulsion when a current was created in the ring. However, without a superconducting ring, the low thrust to power ratio made the idea unfeasible.

Engelberger (1964) proposed use of the geomagnetic field for propulsion. Engelberger suggested that a superconducting Nb_3Sn wire, tens of kilometers in radius, could be established in a polar orbit and that the current be controlled for propulsion. Since the geomagnetic field changes direction as one orbits from pole to pole, the forces on the loop would alternate with each half orbit. Engelberger's idea came before the development of advanced superconductors.

In 1971, the United States Air Force (USAF) did a review of advanced spacecraft propulsion concepts, including magnetic propulsion, called Project Outgrowth. The USAF team appears to have copied Engelberger's diagrams and equations. The report concluded that there was no significant value to magnetic propulsion with state-of-the-art superconductors in 1971.

Alfven (1972) suggested using pulses of current along a rigid tether to produce forces for orbit transfer. With a magnetic field in space of 5×10^{-9} Tesla and 2 Volt/km of electric field, to get one Newton of force, one would need a spacecraft tens of kilometers long. Even so, Netzer and Kane (1994) wrote about the same idea, with little change except for improved superconductor materials.

2.4 Present Magnetic Spacecraft

The discovery of vastly improved superconductors by Bednoz and Muller in 1986 led to new interest in the use of magnetic forces for spacecraft propulsion. Old ideas were re-examined in the light of new superconductor capabilities.

Ackerman (1989) suggests using an arrangement of two superconducting loops, one rotating at right angles to the other, so as to provide stabilizing control when moving through a magnetic field. Ackerman's concept is by far the most futuristic of any of the investigators, considering currents of hundreds of millions of amperes that allow self-launching of the magnetic craft in the geomagnetic field. However, the mass estimates of Ackerman's proposal do not consider the stresses on the structure created by the magnetic currents.

Zubrin and Andrews (1990) proposed an almost identical design to Engelberger, but with a completely different propulsive effect in mind. They used a Nb_3Sn superconducting wire tens of kilometers in radius, but their propulsion came from deflection of the interplanetary or interstellar plasma winds. For a 10-km radius loop at

earth-distance from the sun, this plasma effect would give an acceleration of 0.008 m/s^2 . Their analysis included the mass for cooling and power generating equipment.

Vulpetti (1990) considered a “field-sail”, with the same design as Zubrin (1990), concentrating on interstellar propulsion using the solar wind. Like Zubrin, Vulpetti did his calculations with a superconducting loop, but also considered a directed beam of particles from an emitter at the starting point for added thrust.

Zubrin (1991) considered the geomagnetic field interaction after a friend (R. Forward) suggested it to him. Geomagnetic field interactions cause forces an order of magnitude greater than the solar forces for identical loops close to the earth. Zubrin noted that if the magnetic sail attitude could be controlled, a force away from the earth could be obtained. Zubrin confined his analysis to the self-stabilized case, which means that the magnetic moment of the loop was allowed to align itself with the earth’s magnetic field. Currents are applied when the loop is favorably positioned, but there is no attitude control over the loop. Zubrin suggests that if superconductor current densities were increased 80 times from their 1991 values, his loop design would be capable of self-driven ground launch to earth orbit, but mass estimates of mechanical reinforcement at the new currents were not included.

Chapter 3 :The Geomagnetic Field

3.0 General Description

The earth has a magnetic field around it. Fig. 3.1 shows the magnetic lines of force.

Figure 3.1 Earth with magnetic field lines (Tascione, 1988).

The source of the earth's magnetic field is not yet known with certainty, but the most popular theory holds that convection movements in the core create a self-charging

dynamo, which means that the movements of charge and mass in the molten iron core of the earth create effects which sustain those same movements (Tascione, 1988).

To simulate the movement of a satellite through the geomagnetic field, it is necessary to calculate magnetic field strength at each orbit position. In practice, the satellite control system will measure the field with on-board magnetometers. Simulation of this situation requires realistic values of field strength. A simple model of the geomagnetic field will also enhance understanding of the control problem.

3.1 Dipole Approximation

In low earth orbit, the geomagnetic field may be approximated as a dipole whose magnetic moment is tilted at 11.5° to the axis of the earth's rotation.

Figure 3.2 Geomagnetic field approximated as a dipole.

The magnetic north pole of the earth lies at about 79° N 69°W (1989) and has an earth surface strength of 3.05×10^{-5} Tesla, though both the position and field strength of the pole vary with time (Corliss, 1960). Field strength decreases as the inverse cube of distance until one leaves the region of the earth's magnetism (the magnetosphere) and enters interplanetary space. The dipole approximation is good to within 20% of the actual field at low earth orbit distances (Merrill, 1983). In polar spherical coordinates, the magnetic field strength, $\mathbf{B}$, is given as:

$$\mathbf{B} = \left(B_e \right) \left(\frac{r_{\text{earth}}}{r} \right)^3 \left((2 \cos \theta) \mathbf{r} + (\sin \theta) \mathbf{q} \right) \quad (3.1)$$

Where:

$\mathbf{B}$ = magnetic field strength

r = distance from center of earth

$\mathbf{r}, \mathbf{q}$ = radius unit vector, tangential unit vector,

r_{earth} = earth's radius

B_e = surface field of earth.

θ = co-declination, angle from equatorial plane

ϕ = right ascension, angle from Aries, a star.

The tangential unit vector $\mathbf{q}$ is in the direction of angle θ . Spherical coordinates r , ϕ and θ are shown in Figure 3.3 below.

Figure 3.3 Spherical coordinates for earth frame.

Note that the magnetic field strength is symmetrical about the magnetic z-axis in figure 3.1. Thus, in equation 3.1, the field is given in two directions only. Note also that the magnetic force on a dipole depends not on the field strength, but rather on the gradient of field strength.

3.2 Variations in Ambient Fields and Currents

Space is filled with all sorts of currents and electric fields (Tascione, 1988). The earth, the sun and some other planetary bodies have magnetic fields and surrounding atmospheres that can carry current. These fields will make a difference to the forces on a loop with large currents flowing through it. Not only that, the fields and currents change with time according to a multitude of factors which are often unpredictable, such as solar activity. Probably the actual implementation of magnetic propulsion would require a merging of modeled and sampled magnetic field data. For orbiting spacecraft, the data

sampled from the previous orbit might be used to predict the field strength expected on the next orbit. The solution will most likely not be as simple as the geomagnetic reference field approximation used in this thesis. However, the dipole approximation used will yield accurate figures to be used in loop size and performance calculations when in the preliminary design phase of a magnetically propelled vehicle.

Chapter 4 : Loop Design

4.1 Objective

It is proposed that a satellite in low earth orbit will use a superconducting loop of current to create forces for propulsion by interaction with the geomagnetic field. A loop of current creates a magnetic dipole that behaves in the same way as a small magnet placed in the field of a large magnet. The forces developed will depend on the magnetic moment, which depends on the area enclosed by any given current. Therefore, a circular loop is chosen as it will enclose the largest area for a given length of superconducting wire. The objective is to find a suitable number of loops and a suitable arrangement of those loops with respect to the spacecraft body.

4.2 Magnetic moment of loop

The definition of magnetic moment for a current distributed about a curve (C) is given as:

$$\mathbf{M} = \frac{I}{2} \int_C \mathbf{r} \times d\mathbf{s} \quad (4.1)$$

where:

$\mathbf{M}$ = magnetic moment of curve C

I = current around curve C

$\mathbf{r}$ = vector from origin to a point on curve C

$d\mathbf{s}$ = differential element of curve C

For a curve that lies in a one plane, the integral part of the expression above is twice the area of the loop, so the expression for magnetic moment is:

$$M = \frac{I}{2} (2 * \text{Area}) = I \pi r^2 \quad (4.2)$$

r = radius of loop

M = magnetic moment of loop

I = current in loop

The magnetic moment vector is in the direction shown in Figure 4.1.

Figure 4.1 Magnetic moment of current loop (after Wangsness (1986), p. 304).

The magnetic field created by the loop is like that of a magnetic dipole, as shown in Figure 4.2 below:

Figure 4.2 Field of a magnetic dipole.

4.3 Nonuniform Field Force

The force ($\mathbf{F}$) on a magnetic dipole moment ($\mathbf{M}$) in a nonuniform field ($\mathbf{B}$) is:

$$\mathbf{F} = (\mathbf{M} \cdot \nabla) \mathbf{B} \quad (4.3)$$

The simple expression above follows from the definition of magnetic induction:

$$\mathbf{F} = \oint_C I d\mathbf{s} \times \mathbf{B}(\mathbf{r}) \quad (4.4)$$

where

$\mathbf{F}$ = force on curve C

I = current

$d\mathbf{s}$ = element of curve C

$\mathbf{B}(\mathbf{r})$ = magnetic induction, or B field

$\mathbf{r}$ = vector from the origin to a point on the curve

Consider a small current loop in an external magnetic induction (B field), which lies in the xy plane, like that of Figure 4.1. To make the integral easy, consider $\mathbf{r}$ in azimuthal coordinates:

$$\mathbf{r} = a \cos \phi \mathbf{x} + a \sin \phi \mathbf{y} \quad (4.5)$$

$$d\mathbf{s} = d\mathbf{r} = a d\phi [-\sin \phi \mathbf{x} + \cos \phi \mathbf{y}] \quad (4.6)$$

$\mathbf{r}$ = vector from origin to current loop

a = radius of current loop

ϕ = angle $\mathbf{r}$ makes with the x axis in the xy plane

$\mathbf{x}, \mathbf{y}$ = unit vectors in the x and y directions

The $d\mathbf{s}$ expression above is used, so that the integral term of 4.4 becomes:

$$d\mathbf{s} \times \mathbf{B}_o = a d\phi \left[x B_{oz} \cos \phi + y B_{oz} - z (B_{ox} \cos \phi + B_{oy} \sin \phi) \right] \quad (4.7)$$

The x, y and z subscripts on the $\mathbf{B}$ term indicate the parts of the magnetic field in the x, y and z axis directions. Since the loop is small, the field does not vary much over it, so the magnetic field can be expanded in a power series about the origin. If the magnetic field is expanded in a power series about the origin and only the first terms retained, the x component of the equation above is found as:

$$(d\mathbf{s} \times \mathbf{B}_o)_x = a d\phi \left\langle B_{oz}(0) \cos \phi + a \left[\left(\frac{\partial B_{oz}}{\partial x} \right) \cos^2 \phi + \left(\frac{\partial B_{oz}}{\partial y} \right) \cos \phi \sin \phi \right] \right\rangle \quad (4.8)$$

The integration about a complete circuit means integrating ϕ from 0 to 2π , which means that the $(\cos \phi)$, $(\sin \phi)$ and $(\cos \phi \sin \phi)$ terms are all zero, while the $(\cos^2 \phi)$ term integrates to π . The x component of the force is found as:

$$F_x = I \oint_C (ds \times \mathbf{B}_o)_x = I \pi a^2 \left(\frac{\delta B_{oz}}{\delta x} \right) = m_z \left(\frac{\delta B_{oz}}{\delta x} \right) \quad (4.9)$$

The external magnetic field is assumed to be generated far away from the loop. Thus, the field is uniform and $\nabla \times \mathbf{B}_o = 0$, which implies $(\delta B_{oz}/\delta x) = (\delta B_{oz}/\delta z)$. Remembering that the magnetic moment only has a component in the z direction, the force in the x direction can be rewritten as:

$$F_x = m_z \left(\frac{\delta B_{ox}}{\delta z} \right) = (\mathbf{m} \cdot \nabla) B_{ox} \quad (4.10)$$

The forces in the y and z directions can be found by exactly the same procedure as:

$$F_y = m_z \left(\frac{\delta B_{oy}}{\delta z} \right) = (\mathbf{m} \cdot \nabla) B_{oy} \quad (4.11)$$

$$F_z = m_z \left(\frac{\delta B_{oz}}{\delta z} \right) = (\mathbf{m} \cdot \nabla) B_{oz} \quad (4.12)$$

Combining the expressions for force in the three directions, one obtains the general expression for force given at the beginning of Section 4.3. For a loop of area A and current I, the equation for magnetic force in an external induction $\mathbf{B}$ may be written as:

$$\mathbf{F} = I(\mathbf{A} \cdot \nabla) \mathbf{B} \quad (4.13)$$

The power series expansion used above for the magnetic field can also be done in spherical coordinates, which are often convenient to use for spacecraft work. See Figure 3.3 for an explanation of spherical coordinates in an earth-centered geomagnetic reference frame. A transformation of equations 4.10 to 4.12 from (x,y,z) coordinates to the spherical coordinates of the earth (r,φ,θ) yields the following new expressions:

$$F_r = IA_r \left(\frac{\delta B_r}{\delta r} \right) + I \frac{A_\theta}{r} \left(\frac{\delta B_r}{\delta \theta} \right) - I \frac{A_\theta}{r} B_\theta \quad (4.14)$$

$$F_\theta = IA_r \left(\frac{\delta B_\theta}{\delta r} \right) + I \frac{A_\theta}{r} \left(\frac{\delta B_\theta}{\delta \theta} \right) + I \frac{A_\theta}{r} B_r \quad (4.15)$$

$$F_\phi = 0 \quad (4.16)$$

I = current through loop

r = radius from centre of geomagnetic field

θ = angle from geomagnetic North pole

ϕ = azimuth angle

A_θ = Area of current loop perpendicular to θ vector

B_θ = Component of magnetic field perpendicular to θ vector

A_r = Area of current loop perpendicular to r vector

B_r = Component of magnetic field perpendicular to r vector

Notice that equation 4.16 gives the azimuthal force F_ϕ as 0. This is a consequence of using the magnetic axis of the earth as a reference. If the reference used was the axis of rotation of the earth, an azimuthal force would appear due to the magnetic field gradient in the ϕ direction. This magnetic force is simply a consequence of the reference frame used. As seen in Figure 4.2, the magnetic field is symmetric about the plane of the magnetic equator, implying that magnetic potential is the same for all angles ϕ about the magnetic axis. In a more intuitive example, one may think of the current loop as a small magnet aligning itself in the field of a much larger magnet. The small magnet would twist to align itself with the field lines of the large magnet and move towards the nearest pole. There would be no force to make it revolve around the magnetic axis of the larger magnet.

This movement towards the nearest pole could be used in orbit by inducing current in the loop during the times when the attraction force is in the desired direction.

However, there will always be a radial downward component to the attractive force (F_r) as well as the desired tangential component (F_θ).

To be able to obtain repulsive forces as well as attractive forces towards the nearest pole, some means of controlling the direction of the magnetic moment vector of the loop must be found. This is the same situation as orientating a small magnet in the field of a large magnet. For instance, for a repelling force, the magnetic moment of the small magnet would have to be opposite the magnetic field lines of the larger magnet, which is the same as saying that like poles repel each other. Similarly, a current loop could be orientated in the geomagnetic field to obtain either repulsive or attractive forces.

4.4 Single loop

With a single loop of current, control over the direction of the magnetic moment vector implies use of an attitude control system (see figure 4.3). To decide on a suitable attitude control system, the disturbance torque on the loop must be known. The torque on a current element in a magnetic field is given in equation 4.17.

Figure 4.3 Single loop of current in a magnetic field.

The torque on the loop of Figure 4.3 is:

$$dt = r \times dF \quad (4.17)$$

dt = differential torque

dF = differential force

r = distance from the loop centre

Using previous equations for force in Section 4.3, one can integrate around the current loop to obtain:

$$d\mathbf{t} = \mathbf{r} \times (I d\mathbf{s} \times \mathbf{B}_o) \quad (4.18)$$

$$\mathbf{t} = \int_{loop} d\mathbf{t} = I \int_{loop} [\mathbf{r} \times (d\mathbf{s} \times \mathbf{B}_o)] \quad (4.19)$$

Using the relation $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$ allows the term in square brackets in the equation above to be written as:

$$d\mathbf{s}(\mathbf{r} \cdot \mathbf{B}_o) - \mathbf{B}_o (\mathbf{r} \cdot d\mathbf{s}) \quad (4.20)$$

Since $|d\mathbf{s}| = |d\mathbf{r}|$ (an infinitesimal change in arc length is the same as an infinitesimal change in radius) the second term above can be changed to :

$$\mathbf{r} \cdot d\mathbf{s} = d\left(\frac{1}{2} \mathbf{r} \cdot \mathbf{r}\right) \quad (4.21)$$

Integrated, the expression above is equal to zero, since the integral of a finite scalar over a closed path is always zero. This leaves only the first term in equation 4.20. Stokes's theorem is used to transform the path integral to a surface integral, so that the combination of equations 4.19 and 4.20 becomes:

$$\mathbf{t} = I \oint (\mathbf{r} \cdot \mathbf{B}_o) d\mathbf{s} = I \int_{Surface} d\mathbf{a} \times \nabla(\mathbf{r} \cdot \mathbf{B}_o) \quad (4.22)$$

$d\mathbf{a}$ = differential area.

ds = differential element of curve

B_o = magnetic field strength

t = torque

r = distance from centre to element being integrated.

I = current in amperes

If B_o is taken as constant, then $\nabla(r \bullet B_o) = B_o$ and

$$t = \left(I \int_{\text{Surface}} da \right) \times B_o \quad (4.23)$$

If the surface area of the loop is A and the current is I , then the magnetic moment, M , equals IA and torque is given as:

$$t = IA \times B_o = M \times B_o \quad (4.24)$$

At 200 km altitude, the geomagnetic field, B_o is given by the equations in Chapter 3 as:

$$B_o = B_e \left(\frac{r_e}{r} \right)^3 (2 \cos \theta \mathbf{r} + \sin \theta \mathbf{q}) \quad (4.25)$$

$$B_o = (3.05 \times 10^{-5} \text{ T}) \left(\frac{6.378 \times 10^6 \text{ m}}{6.578 \times 10^6 \text{ m}} \right)^3 \approx 2.8 \times 10^{-5} \text{ T} \quad (4.26)$$

r = radius from earth centre

B_o = geomagnetic field strength at polar coordinates radius r and angle θ

B_e = geomagnetic field at earth surface (r = earth radius), 3.05×10^{-5} Tesla average.

r_e = radius of the earth, 6378 km.

$\mathbf{r}, \mathbf{q}$ = unit vectors in the r and θ directions (see Figure 3.3)

For a 3 km loop with a current of 50 kA, the maximum torque at 200 km altitude is then:

$$t = M \times B_o = (50 \times 10^3 \text{ A})(\pi)(3 \times 10^3 \text{ m})^2 (2.8 \times 10^{-5} \text{ T}) = 4 \times 10^7 \text{ Nm} \quad (4.27)$$

This torque is for the case when the magnetic moment of the loop is 90° to the geomagnetic field. At 0° or 180° to the field lines, the torque on the loop would be zero because the cross product is proportional to the sine of the angle, and $\sin(0^\circ) = \sin(180^\circ) = 0$. However, at the 180° position, any disturbance would cause the loop to flip over to align itself with the geomagnetic field in the same way that a small bar magnet will flip over when one attempts to balance it over another magnet. To balance a single loop in the unstable 180° position or to damp out oscillations about the stable position, some means of attitude control would have to be used.

4.4.1 Thrusters

As the loop orbits the earth, a continual change in attitude is needed since the geomagnetic field direction is changing relative to the moving satellite. Using thrusters to control the attitude of the loop during the unstable part of the orbit is pointless as the objective of using magnetic forces was to avoid thrust by mass expulsion and its attendant fuel requirement. Large torques could be controlled with large enough rockets and enough fuel, but the mass of the fuel expended would be prohibitive for the type of long-term mission for which a magnetically propelled vehicle would be best suited.

4.4.2 Control Moment Gyros

Control Moment Gyros are spinning wheels mounted in a gimbal. A torque can be created by turning the gimbal axis. In effect, the spacecraft is turned around the gyro, which tends to remain in a fixed orientation in space. Control moment gyros currently used can give torques on the order of 500 N-m for short periods (Mauldin, 1992). Torques on the order of 10^7 N-m, as found in Section 4.4, are much too great for conventional gyro wheels. However, if a big enough gyro wheel were made, or if much higher rotor speeds were possible, the torque could be made large enough. The torque is directly proportional to the mass of the spinning rotor and the square of the rotor radius of gyration. Larger torques imply proportionally larger mass, which is a problem for space applications where mass is critical. However, the use of a gyro for a self-levitating single loop (where attitude control is essential) will be briefly considered.

4.4.3 Single Loop Applications

Even if attitude control of a single loop is difficult, this does not mean that the single loop concept is useless. Far from it: a single loop does not have the complexity of other options, such as double or triple loops and, when the magnetic field is pointed in the same direction as the desired force, the loop can be used for useful thrust without any type of attitude control system.

Figure 4.4 Satellite orbit around the earth showing regions of magnetic stability.

In Figure 4.4, the areas labeled “stable thrust arc region”, are where the magnetic moment of a single loop simply attempts to align itself with the local magnetic field. An attractive force towards the nearest pole is present, which can be used to control polar orbits. No attitude control system is needed for this type of orbit correction, but some means of damping out oscillations about the stable position would have to be found. Calculations of performance of a single loop will be included in Chapter 6.

4.5 Double Loop

Some satellites, such as ISIS, launched in the 1960's, use two circular current loops at 90° to create magnetic dipoles needed for spin-up and planar attitude control of the satellite (Vigneron, 1972). The second loop, at 90° to the first, allows the magnetic moment vector to be placed anywhere within a plane by varying the currents in the two loops. The total magnetic moment vector is the sum of the vectors for the two loops.

Figure 4.5 Two current-carrying loops at 90° to each other

Two loops could place the magnetic moment vector anywhere in a plane, but this is not good enough for complete attitude control because the satellite requires three-dimensional magnetic moment pointing ability as it orbits through the geomagnetic field. With a double loop, attitude control is only possible at the points in orbit when the satellite is favorably oriented with respect to the geomagnetic field. In other words, since the magnetic moment vector can only be controlled in a plane, one would have to wait until that plane is already in the correct attitude with respect to the geomagnetic field. Planar

pointing ability is all that is possible for a double loop, since the magnetic moment vector is only controllable within a plane.

A possible solution is to make the entire spacecraft spin. Then the magnetic moment could be placed where needed by pulsing its current at the times when the orientation is correct. The loop would have to be charged and discharged in response to controller commands. Charging and discharging the loop completely every few seconds would be a challenge and probably a waste of power. Some capacitor system would have to be found to store the charge. There would also be problems keeping the spin constant, since the magnetic torques would affect the spin rate.

For propulsion, the double loop concept does not have the simplicity and freedom from stresses that a single loop does, nor does it have true attitude control. It would be better if the attitude control and propulsion system had no moving parts and would work regardless of satellite attitude. Performance of a double loop will not be investigated.

4.6 Triple Loop

With an arrangement of three loops perpendicular to one another, the attitude of the spacecraft is irrelevant for propulsion or attitude control because currents may be varied to create a total magnetic moment vector in any direction.

Figure 4.6 Triple loop of superconducting loops with net magnetic moment , m .

The attitude and propulsion systems are all one, since the appropriate currents can yield both magnetic forces and torques. Beyond the current switching devices, there are no actuators or other moving parts to the system. Power could be obtained via solar panels or other long-term supplies such as on-board nuclear reactors. Since the loops are superconducting, electrical energy could be converted directly to gravitational or kinetic energy. The only losses would be those associated with controlling the currents. The triple loop will allow force to be applied in any direction, save only those directions that lead to areas of exactly the same magnetic potential.

4.6.1 Triple Loop Refinement

The amount of current required to generate even small forces will be very large compared to the current needed for attitude control. Therefore, during the parts of the orbit where forces are being created, large torques tending to align the spacecraft with the local field will arise unless the magnetic moment vector is at 0° or 180° to the local magnetic field. This torque is the cross product of the magnetic moment and the magnetic field, which is proportional to the sine of the angle between the two terms (which is zero

for 0° or 180°). This torque makes half of the thrust arc stable and half of the thrust arc unstable. The remainder of the orbit is so close to the magnetic pole that only predominantly radial forces could be created.

Figure 4.7 Stability regions for magnetic control showing circular orbit.

The stable part of the thrust arc is the part of the orbit where the loop is being attracted to the nearest magnetic pole. With no attitude control system at all, a single loop would align itself with the local magnetic field, which happens to be roughly tangential to the orbit path. There would be oscillations about this stable position which would have to

be damped. The unstable part of the thrust arc is when the magnetic moment of the loop is being pushed away from the nearest magnetic pole. A single loop without an attitude control system would flip over 180° and try to move towards the nearest pole.

The triple loop design provides a means of stabilizing the unstable thrust arc portion of the orbit via a feedback control system. The thrust arc time can be doubled compared to a single loop with no attitude control. The satellite will also be controllable in attitude at any time, which is useful for pointing payloads toward targets.

Upon reflection, one can see that the arrangement of three equally-sized loops is unnecessary. In the unstable part of the arc, the loops used for attitude control would not need to be nearly as large as the loop being used for tangential thrust *if the loop used for tangential thrust is close to 90° to the local magnetic field*. This is possible because the torque on the main loop depends on the sine of the angle between its magnetic moment and the local magnetic field. When the plane of the loop is 90° to the field, this implies that the magnetic moment of the loop is at either 0° or 180° to the field. Since $\sin(0^\circ) = \sin(180^\circ) = 0$, the torque is also zero for the main loop. On the other hand, the secondary loops, used for attitude control, have magnetic moments at close to 90° to the field, where the torque is a maximum for a given magnetic moment because $\sin(90^\circ) = 1$.

Figure 4.8 Modified triple loop, view in direction of +z axis (towards earth)

As long as the angle the magnetic moment of loop 1 makes with the magnetic field is small, loops 2 and 3 can be much smaller than loop 1, while still counteracting the torque in the unstable part of the orbit. The size ratio of loops 2 and 3 to loop 1 will be set by the angle of deviation, ϕ , at which designers wish the secondary loops to be able to generate equal torque to the main loop. This angle represents the maximum static deviation from unstable equilibrium that can be recovered by the secondary loops without

reducing the current in the main loop. To find the loop size ratio for an allowable static deviation of ϕ , the torques are equated for both loops:

$$\mathbf{t} = \mathbf{M} \times \mathbf{B} = M_1(\sin\phi)B = M_2 \sin(90^\circ - \phi)B \quad (4.28)$$

$\mathbf{M}$ = magnetic moment of loop, $\text{area} \times \text{current}$

$\mathbf{t}$ = torque on loop due to geomagnetic field

$\mathbf{B}$ = geomagnetic field

ϕ = permissible static deviation from unstable equilibrium

Assuming that the geomagnetic field is the same for both loops and that the loops can carry the same current, the loop size ratio can be found as follows:

$$\pi R_1^2 I \sin\phi = \pi R_2^2 I \sin(90^\circ - \phi) \quad (4.29)$$

$$R_2 = R_1 \sqrt{\frac{\sin\phi}{\sin(90^\circ - \phi)}} \quad (4.30)$$

R_1 = Radius of loop 1, the main loop

R_2 = Radius of loop 2, a secondary loop

ϕ = allowable static angle of deviation from unstable equilibrium

Should the main loop be disturbed beyond the point where the control loops can recover it, the result is not catastrophic. One could simply wait for the satellite to fly into the stable part of the thrust arc, which always occurs immediately after the unstable part. Alternatively, the current in the main loop could be reduced so that the smaller loops could re-establish the desired attitude.

4.6.2 Storing Energy

When no control forces or torques are required, current could be stopped by introducing some resistance into the circuit, perhaps by heating the superconductor.

However, dissipating energy is wasteful, especially when the supply may be intermittent, as with solar cells. For the triple loop, one solution is to have current distributed among the loops to align the net magnetic moment vector parallel to the local magnetic field. A control system would have to sense the local field direction, then use the correct combination of the three mutually perpendicular magnetic moments to align the net magnetic moment either directly with or directly against the local field. In this way, energy could be stored without creating torques. However, forces will be created whenever there is a net magnetic moment present. Unwanted forces could be averaged to zero by switching the magnetic moment vector exactly opposite the local magnetic field after a short period. In any event, for the portion of the orbit on the dark side of the earth, or for a single loop design satellite designers will doubtlessly use some other form of batteries, at least as a backup to the energy stored in the superconducting loops.

4.6.3 Crew Protection

For human crews, a benefit to using magnetic forces for propulsion is that charged particles will be deflected by the fields present. Ions that are damaging to spacecraft and crew alike could be deflected with the protection of a magnetic field. The effects of the field itself on biological organisms would have to be tested thoroughly, even though later Chapters will show that the size of the main loop will be on the order of kilometers, which makes the centre field only a few times stronger than ambient magnetic fields on earth.

4.7 Magnetic Propulsion Advantages

Magnetic propulsion via superconducting loops has the following advantages:

- no moving parts to the basic propulsive system
- crew protection from charged particles
- very little fuel used if nuclear powered, none if solar powered
- sustained acceleration possible without large masses of fuel

Attitude control using a triple ring has the following specific advantages:

- works in any spacecraft attitude
- attitude control and propulsion are one system

To realize these advantages, there are numerous design challenges that must first be overcome.

4.8 Design Challenges

To design a magnetically-propelled spacecraft, one must find the stresses on the structure for selected loop sizes and currents, then obtain mass estimates. Once the physical characteristics of mass and volume are known, magnetic force and atmospheric drag may be approximated. Work can then begin on an orbit control system and, for the triple loop arrangement, an attitude control system.

The orbit control system can be open-loop, with the desired impulse of thrust being calculated based on the orbit correction required. The orbit correction will be provided by the main loop. During the unstable part of the thrust arc, the attitude control system on the triple loop will need to maintain the attitude so that the magnetic moment of

loop 1 is at 180° to the local magnetic field. The single loop arrangement will only use the stable part of the thrust arc and allow itself to be aligned with the geomagnetic field.

Once the equations for drag force and thrust on the loop are found, a size for the main loop can be selected based on the expected acceleration using reasonable currents and materials limits. Reinforcement mass is a key concern for the triple loop, since to obtain more magnetic force, one needs larger currents, which imply more reinforcement mass. At each loop size, the internal and external stresses may be calculated so that a mass can be found, based on suitable material strengths (Aluminum alloy is used as an example). Based on the expected performance in terms of the mass of the propulsion system, strengths of materials and superconductor performance required, the feasibility of magnetic propulsion will be assessed.

Chapter 5 : Loop Reinforcement

5.1 Objective

As implied by the title, the objective of this chapter is to find expressions for the thickness and diameter of loop tubing needed to support the stresses imposed on both single and triple loop arrangements. With these expressions, the mass for any main loop radius may be found, along with the acceleration in a typical field. The expressions for loop mass, given main loop radius, will be used for performance calculations in Chapter 6.

5.2 Methodology

A realistic maximum current will be selected, based on examples from superconducting magnetic energy storage (SMES) projects. Fixing the current will fix the estimates used for cooling system and superconducting material mass per meter of loop. The amount of reinforcement material will be found by calculating the stress on the loop and using enough material to support that stress.

5.3 Maximum Current

Peck and Michels (1991) report on a 200 kA Superconducting Magnetic Energy Storage (SMES) system, consisting of a NbTi superconductor stabilized with high purity aluminum. The idea is to store energy so that as demand varies in a town or industrial plant, the power production system does not have to vary its output, but can run at a constant, high efficiency rate. SMES also has applications for emergency power.

Figure 5.1 Superconducting magnetic energy storage system (SMES)

Spadoni (1991) states that the maximum current in a superconducting loop is limited by the voltage used for charging and the heat transfer loss connecting to the room temperature DC power input. Work is continuing on 100 kA to 1000 kA currents, but only 50 kA has been demonstrated by physical cable in useful lengths, and that was for a NbTi low-temperature superconductor. The researchers write confidently about applying high-temperature ceramic superconductors to this application, but to be conservative, 50 kA will be selected as the current available.

5.4 Self-Current Stress

Irrespective of external magnetic fields, there are several forces that must be considered when current exists in the main loop. A radial force that puts tension on the loop exists because the current in the loop creates a magnetic field. Every element of

current in the loop is suspended in the field of all the other current elements, and since the magnetic force is a cross product of current direction and magnetic field, the force is outwards.

Figure 5.2 Outwards tension on the main loop when carrying current.

The force on the wire is thus a tangential tensile force. To obtain the magnetic field along the axis of the loop, one uses the law of Biot and Savart:

Figure 5.3 View of current-carrying loop in plane of loop.

$$dB = \frac{\mu_o I ds \times r}{4\pi r^3} \quad (5.1)$$

Along the axis of the loop, only the part of the field parallel to the z axis ($dB_{||}$) contributes to the field, since there is symmetry about the z axis.

$$B = \int_{\text{loop}} dB_{||} = \int_{\text{loop}} dB \cos\alpha = \int_{\text{loop}} \frac{\mu_o I \sin(90^\circ) \cos\alpha (ds)}{4\pi r^2} \quad (5.2)$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{(R^2 + z^2)^{\frac{1}{2}}} \quad (5.3)$$

Combining and simplifying the above two expressions:

$$dB(z) = \frac{\mu_o I R ds}{4\pi (R^2 + z^2)^{\frac{3}{2}}} \quad (5.4)$$

$$B(z) = \oint_{\text{loop}} dB = \frac{\mu_o I R 2R\pi}{4\pi (R^2 + z^2)^{\frac{3}{2}}} \quad (5.5)$$

$$B(z) = \frac{\mu_o I R^2}{2(R^2 + z^2)^{\frac{3}{2}}} \quad [z \text{ direction}] \quad (5.6)$$

$B(z)$ = magnetic field strength parallel to z axis, at distance z from loop plane

$\mu_o = 4\pi \times 10^{-7} \text{ Tm/A}$, permeability of vacuum

I = current in loop creating field

ds = differential element of loop length

R = radius of loop

r = distance from element of loop length to point P

∞ = see Figure 5.3, angle in radians

Assuming $z = 0$ yields a very simple expression for the magnetic field strength at the centre of a loop of current. This estimate of field strength will be conservative, since the value of z will be greater than zero for most calculations.

$$B = \frac{\mu_o I}{2R} [z \text{ direction}] \quad (5.7)$$

The force on a length of current-carrying conductor in a magnetic field is given by the fundamental electrodynamics equation of

$$\mathbf{F} = I \mathbf{l} \times \mathbf{B} \quad (5.8)$$

l = length of current conductor

I = current in conductor

B = magnetic field

F = force on conductor

Using the field B above and a circular current loop, the force per meter of loop is:

$$\frac{F}{l} = \frac{I \pi 2R (\sin 90^\circ) B}{2\pi R} \quad (5.9)$$

$$\frac{F}{l} = \frac{I \mu_0 I}{2R} = \frac{\mu_0 I^2}{2R} \quad (5.10)$$

R = radius of current loop

μ_0 = permeability of free space, $4\pi \times 10^{-7} \text{ Tm/A}$

F = force on current loop

B = magnetic field strength

l = length of wire, circumference in this case

The expression above is for the radial force on the entire loop due to its own current. To find the tension in the wire, Figure 5.4 is used.

Figure 5.4 Vector diagram showing relationship between outward force and tension.

The length of arc enclosed by an angle θ is $R\theta$. From the diagram, it can be deduced that the outward force is reacted by the downward (in the diagram) part of the tension vectors.

$$(F)l = 2T \sin\left(\frac{\theta}{2}\right) \quad (5.11)$$

$$T = \frac{Fl}{2 \sin\left(\frac{\theta}{2}\right)} = \left(\frac{\mu_o I^2}{2R}\right) \frac{(R\theta)}{2 \sin\left(\frac{\theta}{2}\right)} \quad (5.12)$$

F = magnetic force per meter

l = length of cable considered

μ_o = permeability of free space

R = radius of loop

I = current in loop

θ = angle of arc subtended by segment length l .

Simplifying and taking the limit of this expression as $\theta \rightarrow 0$, one obtains:

$$T = \lim_{\theta \rightarrow 0} \frac{\mu_o I^2 \theta}{4 \sin\left(\frac{\theta}{2}\right)} = \frac{\mu_o I^2}{2} \quad \left(\text{from } \sin \frac{\theta}{2} \approx \frac{\theta}{2} \text{ as } \theta \rightarrow 0\right) \quad (5.13)$$

The nominal value for current is 50 kA.

$$T = \frac{(4\pi \times 10^{-7} \frac{Tm}{A})(50 \times 10^3 A)^2}{2} = 1570 \text{ N} \quad (5.14)$$

This is a conservative estimate of tension, since the field experienced by the loop will not be as strong as the centre field under maximum current conditions. 137 MPa is used as a typical allowable tensile stress for aluminum (See Section 5.8).

$$\text{Area} = \frac{\text{Tension}}{\sigma_{Al}} \quad (5.15)$$

$$\text{Area} = \frac{1570 \text{ N}}{137 \times 10^6 \text{ Pa}} = 1.1 \times 10^{-6} \text{ m}^2 \quad (5.16)$$

where:

σ_{Al} = allowed stress for aluminum

This area, equivalent to a solid wire with a radius of less than 2 mm, is the area used when deciding on reinforcement for a single loop. However, the triple loop will be shown to require a larger amount of reinforcement due to other loads.

5.5 Torques between Loops

The net torque of all loops due to each other's fields is zero, as with any closed system. However, there will be large torques between the loops, since their magnetic moments are perpendicular to each other. To find the reinforcement needed to resist the bending moments of one loop due to the field of another, the worst case scenario of maximum currents in both loops will be used.

Figure 5.5 Torque on loop 2 due to current in loop 1, looking along -x axis.

The torque between loops may be understood as arising because the magnetic moment vectors of the loops try to align with each other, just as a magnet tries to align with a magnetic field. This is the major cause of stress on the loops and tethers when the triple-ring arrangement is used. As an example, consider a 500 meter radius main loop and a 210 meter radius secondary loop.

$$\text{Moment} = \mathbf{M}_{\text{mag}} \times \mathbf{B} \quad (5.17)$$

$$\text{Magnetic moment} = M_{\text{mag}} = \pi R_2^2 I_2 \quad (5.18)$$

$$M_{\text{mag}} = \pi (210 \text{ m})^2 (50 \times 10^3 \text{ A}) = 6.93 \times 10^9 \text{ m}^2 \text{ A} \quad (5.19)$$

$$B_1 = \frac{\mu_0 I_1}{2R_1} = \frac{\left(4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}\right) (50 \times 10^3 \text{ A})}{2 (500 \text{ m})} = 6.28 \times 10^{-5} \frac{\text{N}}{\text{Am}} \quad (5.20)$$

$$\text{Moment} = M_{\text{mag}} (\sin 90^\circ) B_1 = (6.93 \times 10^9 \text{ m}^2 \text{ A}) \left(6.28 \times 10^{-5} \frac{\text{N}}{\text{Am}} \right) \quad (5.21)$$

$$\text{Moment} = 4.35 \times 10^5 \text{ Nm} \quad (5.22)$$

This is the moment which acts on the entire secondary loop, which will be supported with 2 tethers for each loop.

5.6 Force from current in tether

The tethers between loops may carry current if they are used to transfer current between loops, or if the power source happens to be located at the spacecraft centre. The tethers would then experience bending forces, as they would be carrying current through the magnetic field of the loops. The question is: how strong must the attachment points be for the tethers and the loop joints because of currents that may be carried in the tether?

If flux pumps are used to control the current in the loops, then the current in the tethers to the flux pumps will much less than the loop currents¹, probably 0.2%. To be extra conservative, let us say that the magnetic field on the tether is equal to that which would be produced by a loop one quarter the size of the main loop, rather than the centre field of the main loop, which would be weaker.

¹ see chapter 7 section on flux pumps.

Figure 5.6 Loop area giving magnetic field for tether stress calculation.

The magnetic field, and the force on the tether due to the field are found as follows:

$$B = \frac{\mu_o I}{2 (\text{Radius})} = \frac{\mu_o I}{2 \left(\frac{R_1}{2} \right)} = \frac{\mu_o I}{R_1} \quad (5.23)$$

$$F = I \mathbf{l} \times \mathbf{B} = I_{\text{tether}} (R_1) (\sin 90^\circ) \left(\frac{\mu_o I_{\text{loop}}}{R_1} \right) = \mu_o I_{\text{tether}} I_{\text{loop}} \quad (5.24)$$

$$F = \left(4\pi \times 10^{-7} \frac{N}{A^2} \right) (100 A) (50 \times 10^3 A) = 6.3 N \quad (5.25)$$

B = magnetic field strength

I = current in Amperes

R₁ = radius of loop 1

μ_0 = permeability of free space
 l = length of current carrier

Calculations in the next few sections will show that this 6 N force is completely insignificant compared to the forces that will arise due to the loops' interactions with each others magnetic fields in the triple loop arrangement. That is, the tension in the tethers from restraining twisting between loops will be the determining factor for tether size in the triple ring arrangement.

5.7 Tethers to Loop 1

The torques created between the loops will cause large stresses in the tethers.

Figure 5.7 View along x axis towards triple ring, showing torque on secondary loop.

If single tethers were used at A, B, C and D, then the torque, which acts as shown for current in the main loop and in the secondary loop from D to A, will cause bending and torsion. There would be torsion along tethers A and D and bending for tethers B and C. For minimum mass, torsion is better than bending to handle a load, and tension is a better way than both. To minimize the amount of material needed, good engineering design calls for as much of the load as possible to be held in tension. For this reason, single tether designs can not compete with multi-tether designs because the mass of the single tethers would be excessive. Below is a tether design that will allow most of the load between the secondary and main loops to be held in tension:

Figure 5.8 Double tether arrangement for connecting main to secondary loops.

To visualize the loads on the tethers, imagine the main and secondary loops twisting to align their magnetic moments with each other. The tethers are assumed to be

flexible, so that no load is held in compression or bending. There are a total of 8 tethers from the main loop to the secondary loops. There will also be 8 smaller tethers from the secondary loops to the spacecraft centre.

5.8 Reinforcement Material

For example purposes, consider the reinforcement material used in the tether and loops to be aluminum alloy. In practice, the reinforcement may well be some other material, but aluminum is a realistic material to use for aerospace applications. The sample material used will be aluminum alloy 2014-T6 with a safety factor of 3. From Table 5.1 below, one can see that there are both stronger and weaker alloys of aluminum.

aluminum type	σ_y , yield stress (deformation)	σ_u , ultimate stress (failure)	density, kg/m ³
Pure Al	20 MPa	70 MPa	2710
Alloy 6061-T6	270	310	2700
Alloy 2014-T6	410	480	2800
Alloy 7075-T6	480	550	2800

Table 5.1 Yield and ultimate stress for types of aluminum.

The safety factor of 3 is included by dividing the yield stress by 3.

$$\sigma_{\text{allowed}} = \frac{\sigma_y}{3} = \frac{410 \text{ MPa}}{3} = 137 \text{ MPa} \quad (5.26)$$

137 MPa is the allowable stress (σ_{allowed}) that will be used in subsequent calculations of the mass of reinforcement needed.

5.9 Size of Tethers to Loop 1

The size, and thus the mass, of the tethers from the main loop to the secondary loops will be determined by the allowable stress on the tether material and the tension to which they are subjected. As an example, consider the 500 m radius main loop and 210 meter radius secondary loops. The tension in each tether can be deduced from Figure 5.9.

Figure 5.9 Tension on tether and force arising from torque on secondary loop.

From Figure 5.9,

$$\theta = \tan^{-1}\left(\frac{R_2}{R_1}\right) = \tan^{-1}\left(\frac{210\text{ m}}{500\text{ m}}\right) = 22.8^\circ \quad (5.27)$$

$$\text{Tension} = \frac{F}{\cos\theta} \quad (5.28)$$

$$F = \frac{\text{moment}}{R_2} \quad (5.29)$$

$$\text{Tension} = \frac{\text{Moment}}{(2 \text{ tethers})R_2 \cos\theta} \quad (5.30)$$

θ = angle at which tether pulls on secondary loop

F = force at 90° to plane of secondary loop, caused by moment

moment = moment in Nm for secondary loop in field of loop 1

R_1, R_2 = radii of loops 1 and 2

Notice that the moment is divided over the two tethers holding the loop. The moment from the interaction of loops 1 and 2 at maximum current was found previously. The diameter of wire needed to resist the tension is found below assuming aluminum 2014-T6 is used with 137 MPa allowable stress, as calculated in Section 5.8.

$$\text{Tension} = \frac{4.35 \times 10^5 \text{ Nm}}{2(210 \text{ m})(\cos 22.8^\circ)} = 1124 \text{ N} \quad (5.31)$$

$$\sigma_{\text{allowed}} = \frac{\text{Tension}}{\text{Area}} = \frac{\text{Tension}}{\pi \left(\frac{d^2}{4} \right)} \quad (5.32)$$

$$d^2 = \frac{\text{Tension}(4)}{\pi \sigma_{\text{allowed}}} \quad (5.33)$$

$$d = \left(\frac{4(1124 \text{ N})}{(137 \times 10^6 \text{ Pa}) \pi} \right)^{\frac{1}{2}} = 0.003 \text{ m} \quad (5.34)$$

d = diameter of tether

Tension = tension on the tether due to secondary loop in field of main loop

σ_{allowed} = allowed stress on aluminum, taken as 137 MPa .

An aluminum wire of 3 mm diameter should thus be able to react the static forces on the secondary loop with a safety factor of 3. Given the size of the structure, however, one would expect various vibrations that may well cause the forces to be larger than those experienced in the static situation. A detailed design calculation, including possible vibrations, would increase the safety factor of 3 upwards.

5.10 Mass of Tethers to Loop 1

The mass of wire per meter, given the diameter, is the volume of wire per meter multiplied by the density of aluminum alloy. The sample wire for a 500 meter main loop is used as an example.

$$\rho_l = \rho_{Al} \left(\frac{d^2}{4} \right) \pi \quad (5.35)$$

$$\rho_l = \left(2800 \frac{\text{kg}}{\text{m}^3} \right) \left(\frac{(0.003\text{m})^2}{4} \right) \pi = 0.0063 \frac{\text{kg}}{\text{m}} \quad (5.36)$$

ρ_l = mass per meter of wire

ρ_{Al} = density of aluminum 2014-T6 alloy, 2800 kg/m³

d = diameter of wire used for tether

There are 8 tethers from the main loop to the secondary loops, and 8 tethers from the secondary loops to the spacecraft centre. Assuming the size of tethers is the same as in Section 5.9, the total mass is found below.

$$\text{mass} = 8\rho_l \sqrt{R_1^2 + R_2^2} + 8\rho_l \sqrt{R_2^2 + R_{\text{Body}}^2} \quad (5.37)$$

$$\text{mass} = 8(0.006 \frac{\text{kg}}{\text{m}}) \left(\sqrt{500^2 + 210^2} + \sqrt{210^2 + 10^2} \right) = 36 \text{ kg} \quad (5.38)$$

This mass is for the tethers from the main loop to the secondary loops, and also from the secondary loops to the spacecraft centre. However, this mass does not include the tethers between secondary loops: they are a different size because the forces between

secondary loops are different from the forces between the main loop and a secondary loop.

Tethers between secondary loops are included in Section 5.12.

5.11 Section Modulus

The concept of section modulus is used to determine the amount of reinforcement that is needed for a given bending load on the loops.

$$S = \frac{M_{\max}}{\sigma_{\text{allowed}}} \quad (5.39)$$

S = section modulus

$M_{\max}$ = maximum bending moment

σ_{allowed} = allowable stress on material

The section modulus may be calculated from the moment of inertia and the distance from the neutral axis. The neutral axis is the line along which there is no stress.

Consider a solid, circular cylinder which is being loaded by a bending moment:

Figure 5.10 Stress on a loaded metal cylinder.

It can be seen that the maximum stresses are furthest from the neutral axis. Let c be the distance from neutral axis to the outer edge. Then the section modulus is given by:

$$S = \frac{I}{c} \quad (5.40)$$

S = section modulus

I = moment of inertia about axis moment is acting

c = distance from the neutral axis

Section moduli vary greatly from one cross section to another, and the type of cross section chosen will have a great effect on the mass of structure required to support a given load. Section modulus gives a measure of the ability of a particular cross-section to resist bending loads. Some examples of section moduli are given below:

Type of cross section area	section modulus	
solid circle	$S = \frac{\pi d^3}{32}$	d = diameter
solid square	$S = \frac{\pi \sqrt{\pi} d^3}{48}$	d = side length
pipe, or hollow circle	$S = \frac{\pi d^2 t}{4}$	d = diameter t = thickness of shell

Table 5.1 Examples of section moduli for various cross-sections.

By calculating the area of each cross section for the same section modulus, one can see that a square requires less material than a circle to resist the same bending moment. Likewise, a hollow cylinder requires less than both as a consequence of having nearly all its material far from the neutral axis, so that all material is used to support stresses. A hollow cylinder will be used for the triple loop arrangement since it is an efficient way to

support bending stress. The single loop arrangement can be made from a simple cable with little regard for cross section resistance to bending because the largest stress for the single-loop case will be tension.

5.12 Tethers between loops 2 and 3

The force created by secondary loops is always directly against a tether when they carry current in the field of loop 1. However, when current is present in both secondary loops, their magnetic moments will tend to align with each other, creating torsion between the loops that can not be supported by the tethers from the main loop to the secondary loops.

Figure 5.11 Torque on loop 2 due to the field of loop 1 and loop 3 respectively.

For example, if loop 2 has a moment clockwise as viewed above, tethers 1 and 3 would be tight and 4 and 2 would be slack. If tethers 1 and 3 both behave the same way under load, loop 3 will feel no load at all from loop 2, but will simply rotate clockwise by whatever amount tether stretching will allow. The connection between the secondary

rings at the “top” and “bottom” of loop 2 (viewed above) does not have to support forces on loops 2 or 3 due to the field of loop 1: the tethers take that load.

If loops 2 and 3 have current in them simultaneously, there will be a torque that tends to line the loops up in the same plane with magnetic moments in the same direction. This is the moment about the x-axis in the previous Figure. This moment could be resisted either by making the loop connections and the loops themselves strong enough to resist the load in bending, or by using another set of tethers connected to the main loop in the XY plane, or by using a set of tethers between the secondary loops. The best option is the one that will use the least mass, so the “strong and rigid” option is a non-starter since it is much easier to carry loads in tension with a tether rather than in bending or torsion. As for connections to the main loop or connections between secondary loops, it can be seen that 8 tethers would be required for the main loop-to-secondary connection, while only 4 tethers could be used between the secondary loops.

Figure 5.12 Two possible tether configurations to contain the secondary loops.

Since the second configuration of 4 tethers requires much less tether length, it will be selected for initial calculations of tether mass.

5.13 Torque between Secondary Loops

In the worst case, both secondary loops would be operating at full current, 50 kA. For example purposes, the standard 210 m secondary loops, used previously, will be used here. The field strength acting on the dipole moment of one loop is the centre field of the other loop. From Figure 5.13, it can be seen that the magnetic moment of loop 2 acts at the centre field point of loop 3.

Figure 5.13 Magnetic moment of loop 3 in the centre field of loop 2.

The torque between the loops in this situation would be:

$$\mathbf{t}_{23} = \mathbf{M}_{\text{mag}} \times \mathbf{B} \quad (5.41)$$

$$\mathbf{M}_{\text{mag}} = \pi R_2^2 \mathbf{I}, \quad \mathbf{B} = \frac{\mu_o \mathbf{I}}{2R_2} \quad (5.42)$$

$$\tau = \pi R_2^2 I (\sin 90^\circ) \frac{\mu_o I}{2R_2} = \frac{\pi \mu_o I^2 R_2}{2} \quad (5.43)$$

$$\tau = \frac{\pi}{2} \left(4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2} \right) (50 \times 10^3 \text{ A})^2 (210 \text{ m}) = 1.04 \times 10^6 \text{ N m} \quad (5.44)$$

This torque between secondary loops is larger than the torque experienced by a secondary loop due to the field of loop 1, since the current in all loops has the same maximum and loop 1 is further away from the secondary loops than the secondary loops are from each other. When estimating the size of the secondary loop reinforcement and tethers between the secondary loops, this torque is used.

5.14 Tethers Between Secondary Loops

Tension between secondary loops is directed at a 45° angle to the force at the connection due to torque on the loop. The tension is found from Figure 5.14.

Figure 5.14 Forces due to tethers between secondary loops.

The worst-case torque between secondary loops is as found by equation 5.45, remembering that two tethers absorb this torque.

$$\text{Force} = \frac{M_{\max}}{2R_2} \quad (5.45)$$

$$\text{Tension} = \frac{\text{Force}}{\cos\theta} = \frac{M_{\max}}{2R_2 \cos\theta} \quad (5.46)$$

$$\text{Tension} = \frac{1.04 \times 10^6 \text{ Nm}}{2(210 \text{ m}) \cos 45^\circ} = 3502 \text{ N} \quad (5.47)$$

$M_{\max}$ = maximum torque between secondary loops

R_2 = radius of secondary loops

θ = angle tension acts to force

The area of the wire may be found from Section 5.9, using the new tension.

$$d = \left(\frac{4(\text{Tension})}{\sigma_{\text{allowable}} \pi} \right)^{\frac{1}{2}} \quad (5.48)$$

$$d = \left(\frac{4(3502 \text{ N})}{137 \times 10^6 \text{ Pa} \pi} \right)^{\frac{1}{2}} = 0.0057 \text{ m} \quad (5.49)$$

d = wire diameter

$\sigma_{\text{allowable}}$ = allowable stress on aluminum alloy

A 6 mm wire diameter should be able to support the static stress. The length of each tether is found by the Pythagorean theorem as the secondary loop radius multiplied by $\sqrt{2}$. An estimate of mass is found for a 210 m secondary loop.

$$\text{total_mass}_{23} = 4\sqrt{2} R_2 \rho_{\text{Al}} \pi \left(\frac{d}{2} \right)^2 = \sqrt{2} R_2 \rho_{\text{Al}} \pi d^2 \quad (5.50)$$

$$\text{total_mass}_{23} = \sqrt{2} (210 \text{ m}) (2800 \text{ kg}) \pi (0.006 \text{ m})^2 = 94 \text{ kg} \quad (5.51)$$

R_2 = radius of secondary loop

ρ_{Al} = density of aluminum alloy used

d = diameter of tether

The mass calculation programs of Chapter 6 will change the thickness and length of the tethers according to the calculations used above.

5.15 Secondary Loop Mass

To find the mass needed for secondary loop reinforcement, the force at every point along the loop is found. Integrating the force by the distance along the loop yields the bending moment. From the moment, the amount of reinforcement material needed can be found. The total magnetic force is the sum of the magnetic forces from the other secondary loop, the magnetic force from the primary loop, the self-induced tension and the geomagnetic field interactions. The thickness of the cable (see Figure 5.15) will be found first.

Figure 5.15 Loop cable dimensions.

To find an approximate mass for reinforcement needed for a given stress, the hollow circle section modulus from Section 5.11 is used. The thickness is set at a fixed value, then the diameter needed at each point on the loop calculated. For simplicity, the loop size is found by using the largest bending moment present, that between the magnetic moments of different loops. This approximation ignores the self-induced loop tension and any torsion that may develop. Following Bowes, Russell and Suter (1984), one obtains:

$$S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{\pi d^2 t}{4} \quad (5.52)$$

rearranging,

$$d = \left(\frac{4M_{\max}}{\sigma_{\text{allow}} \pi t} \right)^{\frac{1}{2}} \quad (5.53)$$

S = section modulus

$M_{\max}$ = maximum bending moment on loop, found previously

σ_{allow} = allowable stress on material

d = diameter of loop tube reinforcement

t = thickness of loop reinforcement

The volume of reinforcement can be found by a summation of all the different cross-sections multiplied by the length of each cross-section. The calculations can be repeated for a range of different thickness values to arrive at an optimum thickness in terms of reinforcement material mass. For the mass calculations, a quarter section of secondary loop will be used. Each quarter section experiences the same loads and is anchored by tethers at each end, so multiplying by four will yield the total mass.

5.15.1 Secondary-Secondary Force

The force on a current carrying wire in a magnetic field is

$$dF(x) = Idl \times B(x) \quad (5.54)$$

$dF(x)$ = magnetic force at point x

x = distance along arc of loop

I = current

dl = differential element of length

$B(x)$ = magnetic field at point x

Figure 5.16 Field lines of loop 2 cutting loop 3.

The magnetic field strength is approximated to be constant for any plane in z , with $z = 0$ taken as the plane of loop 2. The magnetic field direction is not fixed, but due to the symmetry of the 2 secondary loops, it is always at 90° to the current. The law of Biot-Savart gives the field strength at any z -plane.

$$B(z) = \frac{\mu_0 I R_2^2}{2(R_2^2 + z^2)^{\frac{3}{2}}} \quad (5.55)$$

$B(z)$ = magnetic field strength
 μ_0 = permeability of free space
 I = current in secondary loop
 R_2 = radius secondary loop
 z = distance from plane of loop

The starting and ending values of field strength are used to create a straight-line relationship between field strength and arc length over a quarter-loop. This approximation is used to obtain a simple expression of field strength at any arc length x . This linear approximation of field strength will have no error at each end of the quarter-section.

$$\frac{1}{4} \text{ loop circumference} = \frac{\pi 2 R_2}{4} \quad (5.56)$$

$$\text{field start} = \frac{\mu_o I R_2^2}{2(R_2^2 + 0)^{\frac{3}{2}}} = \frac{\mu_o I}{2 R_2} \quad (5.57)$$

$$\text{field end} = \frac{\mu_o I R_2^2}{2(R_2^2 + R_2^2)^{\frac{3}{2}}} = \frac{\mu_o I}{R_2 2^{\frac{5}{2}}} \quad (5.58)$$

$$\text{slope} = \frac{\frac{\mu_o I}{2 R_2} \left(1 - \frac{1}{2^{\frac{3}{2}}}\right)}{\frac{\pi R_2}{2}} = \frac{\mu_o I \left(1 - 2^{-\frac{3}{2}}\right)}{\pi R_2^2} \quad (5.59)$$

$$\text{field}(x) = B(x) = \frac{\mu_o I}{2 R_2} \left(1 - \frac{2 \left(1 - 2^{-\frac{3}{2}}\right) x}{\pi R_2}\right) \quad (5.60)$$

x = arc length, or length along loop from crossing point

This field equation is used with the differential force expression below.

$$dF_{22} = I B(z) \sin(90) dx \quad (5.61)$$

$$dF_{22} = \frac{\mu_o I^2}{2 R_2} \left(1 + \frac{2 \left(1 - 2^{-\frac{3}{2}}\right) x}{\pi R_2}\right) dx \quad (5.62)$$

F_{22} = force on loop 2 due to other secondary loop

This is the differential force on an element of loop arc length dx .

5.15.2 Secondary-Main Force

The main loop has a significantly larger radius than the secondary loops (see Figure 5.17). The main loop field is thus approximated as being constant in direction with respect to the secondary loops. In other words, the curvature near the centre of the field is assumed to be negligible.

Figure 5.17 Loop 2 in magnetic field of loop 1.

The magnetic field is then taken from the law of Biot-Savart.

$$B(z) = \frac{\mu_0 I R_1^2}{2(R_1^2 + z^2)^{\frac{3}{2}}} \quad (5.63)$$

$$\text{field start} = \frac{\mu_0 I}{2R_1} \quad (5.64)$$

$$\text{field end} = \frac{\mu_0 I R_1^2}{2(R_1^2 + R_2^2)^{\frac{3}{2}}} \quad (5.65)$$

$$\text{field}(x) = B(x) = \mu_0 I \left[\frac{1}{2R_1} - \left(\frac{\frac{1}{R_1} - \frac{R_1^2}{(R_1^2 + R_2^2)^{\frac{3}{2}}}}{\pi R_2} \right) x \right] \quad (5.66)$$

$$dF_{12}(x) = I dx \sin(\theta) B(x) \quad (5.67)$$

F_{12} = force on loop 1 due to current in loop 2

I = current

$B(x)$ = magnetic field at arc length distance x
 R_1, R_2 = radius of loops 1 and 2

Figure 5.18 Current in loop 2 at angle t (theta) to primary field.

define:

$$\theta = \frac{x}{R_2} \quad (\text{radians}) \quad (5.68)$$

$$\alpha = \frac{\frac{1}{R_1} - \frac{R_1^2}{(R_1^2 + R_2^2)^{\frac{3}{2}}}}{\pi R_2} \quad (\text{for simplicity of } B(x)) \quad (5.69)$$

Notice that x is the distance in arc length along the quarter-section of loop.

5.15.3 Total Shear and Bending Moment

The total force on a secondary loop due to the other secondary loop (F_{22}) is:

$$F_{22} = \frac{\mu_o I^2}{2R_2} \int_0^{\frac{\pi}{2}} \left(1 + \frac{2(1 - 2^{\frac{-3}{2}})x}{\pi R_2} \right) dx \quad (5.70)$$

$$F_{22} = \frac{\mu_o I^2}{2R_2} \left[x + \frac{\left(1 - 2^{\frac{-3}{2}}\right) x^2}{\pi R_2} \right] \quad (5.71)$$

Force on loop 1 due to the field of loop 2 (F_{12}):

$$F_{12}(x) = I \int_0^{\frac{\pi}{2}} \sin\left(\frac{x}{R_2}\right) \mu_o I \left[\frac{1}{2R_1} - \alpha x \right] dx \quad (5.72)$$

integrate by parts

$$\begin{aligned} u &= \frac{1}{2R_1} - \alpha x & dv &= \sin\left(\frac{x}{R_2}\right) dx \\ du &= -\alpha dx & v &= -R_2 \cos\left(\frac{x}{R_2}\right) \end{aligned} \quad (5.73)$$

$$F_{12}(x) = \mu_o I^2 \left[\left(\frac{1}{2R_1} - \alpha x \right) - R_2 \cos\left(\frac{x}{R_2}\right) - \int_0^{\frac{\pi}{2}} -R_2 \cos\left(\frac{x}{R_2}\right) (-\alpha) dx \right] \quad (5.74)$$

$$F_{12}(x) = \mu_o I^2 \left[\left(\frac{1}{2R_1} - \alpha x \right) - R_2 \cos\left(\frac{x}{R_2}\right) - R_2^2 \alpha \sin\left(\frac{x}{R_2}\right) - \left(\frac{1}{2R_2} \right) (-R) \right] \quad (5.75)$$

α , θ and other variables are the same as in Section 5.15.2

Total shear force is the sum of self-induced tension, primary-secondary force and secondary-secondary force. For static loads, the total moment is the integral of the shear force by the distance along the loop circumference.

$$M(x) = \int_0^{\frac{\pi}{2}} [x(F_{12}(x) + F_{22}(x))] dx + T_{12y}(0) + T_{22y}(R_2) \quad (5.76)$$

$M(x)$ = total moment at point x along loop circumference

T_{12y} = tension from tether linking loop 1 and 2 in y plane

T_{22y} = tension from tether linking secondary loops in y plane

Stress on the beam will be calculated using the moment found above. The free-body diagram of the loop quarter-section below shows the worst case scenario for stress.

Figure 5.19 Forces on loop quarter-section.

The mass obtained from these force equations is an approximation only. Including dynamic loads and torsion could well increase the total reinforcement mass required. Also, Figure 5.19 shows a straight beam, while the loop is actually curved. The straight beam approximation is justified in the next section.

5.16 Stress on Curved Beam

The maximum stress on a straight beam under a bending moment is on the outside edge of the beam and may be simply found (Bowes et al., 1984).

$$\sigma_{os} = \frac{Mc_o}{I_x} \quad (5.77)$$

σ_{os} = stress in Pa. for straight beam outer surface

M = bending moment, force times distance
 c_o = distance to outer edge from centroid axis
 I_x = second moment of inertia of cross section

Since the loop quarter-section is curved rather than being a straight beam, a correction factor is applied when calculating the stress. From Bowes et al. (1984):

$$\sigma_{oc} = \frac{M(c_o + Y)}{YAR_o} \quad (5.78)$$

$$Y = R_c - R_n \quad (5.79)$$

$$R_n = \frac{A}{\int_{R_i}^{R_o} \frac{dA}{R}} \quad (5.80)$$

R_c = radius from centre loop to centroid of cross section
 R_n = radius from centre loop to neutral axis
 Y = distance from neutral axis to centroidal axis
 A = area of loop cross section
 R_o = outer radius of loop
 R_i = inner radius of loop
 c_o = distance from centroid to outer wall

Figure 5.20 section of loop with neutral and centroid axis.

The integral in the neutral axis equation above was solved numerically (see Appendix A, program “flex.cpp” and function “na.cpp”). Beams which are slim compared to their radius of curvature closely approximate straight beams with regard to stress under bending load. A measure of how accurate this approximation is may be obtained by taking the ratio of the straight-beam stress (equation 5.78) to the curved beam stress (5.79) at either the inner or outer surface. If this ratio is close to 1, then the approximation is good. This ratio is found by equation 5.81 in program “flex.cpp”.

$$\frac{\sigma_{oc}}{\sigma_{os}} = \frac{\frac{M(c_o + Y)}{YAR_o}}{\frac{Mc_o}{I}} = \left(1 + \frac{Y}{c_o}\right) \frac{I}{YAR_o} \quad (5.81)$$

σ_{oc} = stress at outer surface, curved section

σ_{os} = stress at outer surface, straight section

I = second moment of inertia of cross section

(see Figure 5.20 and equations above for other variables)

Another measure of the accuracy of the straight-beam approximation is a “flexibility factor”, explained in Bowes, Russell and Suter (1984), equation 2-41. Flexibility is a measure of resistance to bending load. If the ratio of curved beam to straight beam flexibility is close to 1, then the approximation of using straight beam bending equations is good.

$$\bar{\alpha} = \frac{\text{flexibility (curved)}}{\text{flexibility (straight)}} = \frac{\frac{MI}{EYAR_c}}{\frac{MI}{EI}} \quad (5.82)$$

$$\bar{\alpha} = \frac{I}{YAR_c} \quad (5.83)$$

α = flexibility of curved beam compared to straight
 l = length along centroidal axis of beam
 M = moment on beam
 (see Figure 5.20 and equations above for other variables)

The program “flex.cpp” (Appendix A) found the ratio of curved beam to straight beam stress (the “stress factor”) to be 0.9769 for the inner surface and the flexibility factor to be 0.9761 for a 500 m radius main loop with a 0.80 m radius loop cross section of wall thickness 1 cm. Thus, the straight-beam equations used should yield an answer that only differs from the curved beam equations by about 4%. Given that the bending equations are a rough approximation to begin with, the straight beam equations are acceptable.

5.17 Primary Loop Reinforcement

There is both bending and compression on the main loop due to the tether tension.

Figure 5.21 Tension directions on the main loop tethers

Current in the main loop will cause an expansion force, which will resist the compressive effect of the tether. The compressive force will be compared to the current-induced expansive force. The size of the radial inward and bending forces can be found Figure 5.22 and the tension from chapter 5.

Figure 5.22 Tether tension, compression and bending vector diagram

$$F_c = T \sin \theta = 2686 \text{ N} \sin(22.8^\circ) = 1041 \text{ N} \quad (5.84)$$

F_c = compression force

T = tension

θ = found in Section 5.9, for example 500 m radius main loop

The expansion force will be calculated using the centre field of the loop, which is assumed constant in the plane of the loop. The part of the expansion force per meter that directly opposes tether compression will then be integrated over a half loop. A half loop is selected since if all three loops had full current in them, there could be four tethers under tension and a half loop would be connected to all four. The same calculations as for the secondary loop are used.

$$F = I(\text{length}) \times B = I\pi(2R_1) \sin 90^\circ \left(\frac{\mu_o I}{2R_1} \right) = \pi \mu_o I^2 \quad (5.85)$$

$$\frac{\text{Force}}{\text{meter}} = \frac{F}{\pi 2 R_1} = \frac{I^2 \pi \mu_o}{\pi 2 R_1} = \frac{\mu_o I^2}{2 R_1} \quad (5.86)$$

F = force

I = current

R₁ = radius of loop 1

μ_o = permeability of vacuum

The force opposing the tether tension is the integral of the force in the y direction in Figure 5.23.

Figure 5.23 Expansion and compression forces on tethered, current-carrying loop.

$$\text{Total force} = \int_0^{\pi} \frac{\mu_o I^2}{2 R_1} R_1 \sin(\theta) d\theta \quad (5.87)$$

$$F_T = \frac{\mu_o I^2}{2} (-\cos\theta) \Big|_0^{\pi} = \frac{\mu_o I^2}{2} (1 + 1) = \mu_o I^2 \quad (5.88)$$

$$F_T = 3142 \text{ N} \quad (5.89)$$

F_T = total expansion force in y direction for loop quarter-section

Tension forces causing compression are far enough below (1041 N vs. 3142 N) expansion forces that bending rather than buckling will be used for reinforcement mass

estimates of the main loop. For the main loop, the mass per quarter section will be found by calculating the mass needed to resist bending, as for the secondary loops. The computer code using the above expressions for mass is found in Appendix A. The code was run with a 500 m radius main loop and 210 m radius secondary loops and the total mass was found.

thickness of tube wall (m)	tube cross-section radius, (m)	mass triple ring reinforcement (kg)
0.001	0.135	22351.2
0.002	0.095	31609.4
0.003	0.078	38713.4
0.004	0.067	44702.4
0.005	0.060	49978.8
0.006	0.055	54749.0
0.007	0.051	59135.7
0.008	0.048	63218.7
0.009	0.045	67053.6
0.010	0.043	70680.7
0.011	0.041	74130.5
0.012	0.039	77426.8
0.013	0.037	80588.4
0.014	0.036	83630.5
0.015	0.035	86565.8
0.016	0.034	89404.8
0.017	0.033	92156.3
0.018	0.032	94828.1
0.019	0.031	97426.6

Table 5.2 Mass of triple ring, radius $l = 500.0$ m, with varying tube wall thickness.

Table 5.2 clearly shows that the best mass is at the minimum practical thickness of 0.001 m. In theory, a very thin, large diameter tube would be best.

Chapter 6 : Performance

6.1 Overview

With the mass of the spacecraft and the magnetic force on it, one can calculate acceleration. With acceleration and a knowledge of how long thrust can be obtained each orbit, one can calculate orbit transfer times and times to escape from low earth orbit. In this chapter, the performance at various loop radii for both triple and single loops will be found. By trying many different loop radii with the equations developed for loop reinforcement, drag and magnetic force, one can see how the acceleration is related to loop size and thickness of reinforcement. Decisions can then be made about which way to drive the design: either towards a very large loop with a small current, or towards a compact loop with a large current.

6.2 Drag

It is a complex problem to accurately predict the amount of drag to be expected for a given satellite shape and density of atmosphere. Wertz (1991) gives a general expression for the acceleration due to atmospheric drag on a satellite:

$$a_D = -\frac{1}{2} \rho \frac{C_D A V^2}{m} \quad (6-1)$$

ρ = atmospheric density

C_D = coefficient of drag ≈ 2.2

A = cross sectional area

m = mass of satellite

V = velocity with respect to the atmosphere

Echo I, a large balloon satellite orbiting at 1600 km, experienced a drag force exceeding expected aerodynamic drag by a factor of 50. Drell et al. (1965) explained the drag as an electromagnetic coupling between the vehicle and plasma (charged particles) of the earth's magnetosphere. Momentum of the vehicle was transferred to the plasma. Drell (1965) gives this drag as:

$$F_D = ma = \rho V V_a A \quad (6-2)$$

$$V_a = \sqrt{\frac{B^2}{4\pi \rho}} \quad (6-3)$$

F_D = force due to drag
 m = mass of satellite
 a = acceleration of satellite due to drag
 ρ = mass density of plasma
 V = vehicle velocity relative to plasma
 A = frontal area of vehicle
 V_a = Alfven velocity, after D. Alfven
 B = geomagnetic field transverse to flight path.

In the magnetosphere, $V_a \cong 100$ km/s for most satellite orbits, which have flight paths nearly transverse to the geomagnetic field. The relative magnitudes of the atmospheric drag and the electromagnetic drag at 800 km will be compared below. The plasma density is found from Tascione (1988) for 800 km altitude.

$$\rho_{\text{plasma}} = 10^4 \text{ particles/cm}^{-3} = 4.4 \times 10^{-16} \text{ kg/m}^3$$

$$A = 100 \text{ m}^2$$

$$V = 8 \text{ km/s}$$

$$V_{\text{Alfven}} = 100 \text{ km/s for typical orbit}$$

$$\rho_{\text{air}} = \text{air density} = 10^{-14} \text{ kg/m}^3 \text{ at 800 km.}$$

$$\text{Alfven drag} = (4.4 \times 10^{-16} \text{ kg / m}^3)(8 \times 10^3 \text{ m / s})(100 \times 10^3 \text{ m / s})(100 \text{ m}^2) \quad (6-4)$$

$$\text{Alfven drag} = 3.5 \times 10^{-5} \text{ N}$$

$$\text{Atmospheric Drag} = \frac{1}{2}(10^{-14} \text{ kg / m}^3)(2.2)(100 \text{ m}^2)(8 \times 10^3 \text{ m / s})^2 \quad (6-5)$$

$$\text{Atmospheric Drag} = 7.0 \times 10^{-5} \text{ N}$$

Electromagnetic drag is thus half of atmospheric drag at 800 km for a flight path transverse to the geomagnetic field. Further calculations show that as altitude increases, electromagnetic drag becomes progressively larger than atmospheric drag. However, thought must be given to the type of orbit before one includes Alfven drag. The magnetic field used in calculating Alfven velocity is the field transverse to the flight path. Since a magnetically propelled satellite would likely be in a polar orbit in order to travel along the magnetic field lines, the Alfven velocity would be very small, since almost none of the field is transverse to the flight path. Alfven drag will thus not be included in the calculations of drag in the orbit simulation program in this thesis. Detailed calculations of performance, prior to actually building a magnetically propelled spacecraft would certainly need to consider Alfven drag, especially for orbits that do not follow geomagnetic field lines.

6.2.1 Variations in Drag

Any heating of the earth's atmosphere will produce density changes. Heating implies expansion of the atmosphere, which means that the amount of drag a satellite experiences in low earth orbit increases. Geomagnetic storms, associated with solar activity, dump large numbers of particles into the high-latitude atmosphere, causing one type of heating. Most heating will be near the auroral zone (northern lights), so polar

orbiting satellites experience the greatest effect. Auroral substorms can also produce a horizontal circulation pattern across the polar cap, with wind speeds of 1 to 2 km/s. For a low-altitude polar orbiter moving at 6 or 7 km/s, the drag of the circulation can be significant (Tascione, 1988). As satellite altitude changes, as for any elliptical orbit, atmospheric density and drag will change as well.

6.2.2 Drag in Orbit Program:

Drag in the orbit program will be calculated using the atmospheric drag formula above. Atmospheric density will be taken as the maximum values at each altitude, as given by Larson and Wertz (1991), which means that the 11-year solar cycle, associated with drag increases, will be considered as active. The data table below will be included, in expanded form, as a look-up table (called density.cpp) in the program (see Appendix A).

Altitude, km above earth surface	Atmospheric Density, kg/m ³	Delta V to maintain Altitude, mean m/s per year
100	5.25×10^{-7}	5.09×10^6
200	2.41×10^{-10}	2.30×10^3
300	1.87×10^{-11}	1.76×10^2
400	2.62×10^{-12}	2.43×10^1
500	4.76×10^{-13}	4.35×10^0
600	9.89×10^{-14}	8.91×10^{-1}
700	2.36×10^{-14}	2.09×10^{-1}

Table 6.1 Atmospheric density and delta V needed to maintain altitude.

Notice the third column above, which gives the delta-V, or change in velocity, needed to maintain altitude for a spacecraft with a ballistic coefficient (a measure of mass to surface area) of 100 kg/m^2 . Above about 600 km, the delta-V needed is so small that orbits usually last greater than 10 years, which is more than most satellite lifespans. In other words, atmospheric drag is not significant above about 600 km unless the mission is more than 10 years long or has very stringent orbit requirements.

6.3 Attitude Control

The attitude controller for the triple loop design controls the currents in loops 2 and 3 (the secondary loops) during the unstable parts of the thrust arc. Near the magnetic poles, little tangential thrust can be produced, so the main loop would be available for attitude control. The attitude controller of the triple loop arrangement is assumed to keep the main loop in equilibrium during the unstable part of the thrusting arc in the orbit transfer programs of this chapter.

6.4 Gravity

Gravity causes the satellite to accelerate towards the centre of the earth. In Low Earth Orbit (LEO), the gravity of the moon, the sun and the other planets is so small compared to that of the earth that it is typically treated as a perturbation to the orbit. Furthermore, most LEOs are nearly circular: otherwise they encounter too much

atmosphere (or, worse yet, the earth surface) at perigee. For a circular orbit close to the earth, the force balance for gravity is:

$$F_{\text{gravity}} = G \frac{m_i m_{\text{earth}}}{r^2} = F_{\text{acceleration}} = m \frac{v^2}{r} \quad (6-6)$$

G universal gravitational constant
 m_i mass of satellite
 m_{earth} mass of earth
 v velocity of satellite
 F force
 r radius from earth centre to satellite

Assuming a spherical earth¹, no part of the gravity force is tangential to a circular orbit, so gravity is not explicitly included in calculations to find an optimum loop size for tangential acceleration. However, the effect of gravity is implicitly included since if atmospheric drag decreases the velocity, then the gravity force will cause an undesirable radial movement towards the centre of the earth. This effect is the primary orbit perturbation which the magnetic loop will correct and is the basic explanation of how the Keplerian equations used in the orbit transfer programs of this chapter work. Equation 6.6 will be used to find a velocity for circular orbits in the orbit transfer programs used in this chapter. Velocity is needed in the calculations of atmospheric drag.

6.5 Electrostatic Fields

Sometimes spacecraft become charged, building up a different electrostatic potential from the surrounding plasma in the same way that rubber shoes on a polyester

¹ Ignores triaxiality, or oblate spheroid shape of earth.

rug can build up a static charge on a person. On a satellite, a large charge could create electrostatic forces. However, the charge-buildup phenomena has only been observed on geosynchronous satellites. For low earth orbit vehicles, charging is rarely observed because low earth orbits are within the ionosphere (Tascione, 1988).

The ionosphere is a region where the atmosphere is partially ionized due to photodissociation. Photodissociation is the breaking apart of molecules by high-energy solar radiation to form ions (hence, the ionosphere). An arbitrary upper limit for the ionosphere was set at 2000 km, which means a low earth orbit satellite will be within it continually (Tascione, 1988). In the presence of ions, an electrostatic field will cause current to flow, neutralizing a charge imbalance (Corlis, 1960). Charge buildup and electrostatic forces will not be included for this thesis.

6.6 Optimum Loop Size:

6.6.1 Other Authors

Zubrin (1991), Vulpetti (1990) and Engelberger (1964) all propose using superconducting loops with radii on the order of tens of kilometers for spacecraft propulsion. The reason for the large radii is that the maximum current that can be carried by a superconducting wire is limited. If superconducting current allowed is small, the loop size must be large for an appreciable magnetic moment. Thick wires presently carry much less current density and lose their superconducting state with much weaker magnetic disturbances than thin wires or thin films. Present-day technology thus inclines the design

towards either a thin wire or a superconducting film spread over a large surface area, as with a large-diameter hollow tube.

6.6.2 Magnetic Force on Loop

The magnetic force on a current loop is equal to the dot product of the magnetic moment and the gradient of the local magnetic field. (Chapter 4 develops this expression for magnetic force) The magnetic moment vector is perpendicular to the plane of the loop and equal in size to the current multiplied by the area of the loop.

$$\mathbf{F}_{\text{mag}} = \mathbf{M} \cdot \nabla \mathbf{B} \quad (6-7)$$

$$\mathbf{M} = I (\text{Area}) \quad (6-8)$$

$\mathbf{M}$ = magnetic moment vector

$\mathbf{B}$ = geomagnetic field

I = current in loop

Area = area of loop

Figure 6.1 Direction of magnetic moment for a current loop.

During the thrusting part of each orbit, the magnetic moment vector will be aligned with the local geomagnetic field, which means only part of the area of the main loop can

be used to increase satellite velocity unless the field happens to be parallel to the orbit. The other part of the loop area will be creating a radial downwards force because the geomagnetic field makes the loop tilt with respect to its flight path. This also means that the ratio of “radial area” to “tangential area” is equal to the ratio of the radial and tangential magnetic field components.

$$\frac{A_{rad}}{A_{tan}} = \frac{B_{rad}}{B_{tan}} \quad (6-9)$$

A_{rad} = radial component of area

A_{tan} = tangential component of area

B_{rad} = radial component of magnetic field

B_{tan} = tangential component of magnetic field

The radial and tangential areas squared and added equal the total area of the loop squared. Combined with the magnetic field expressions given in Chapter 3, the radial and tangential areas may be found as a function of the angle from magnetic north.

$$A_{rad} = \frac{B_{rad}(A_{tan})}{B_{tan}} = \frac{B_{rad}(A^2 - A_{rad}^2)^{\frac{1}{2}}}{B_{tan}} \quad (6-10)$$

$$(A_{rad})^2 \left(1 + \left(\frac{B_{rad}}{B_{tan}} \right)^2 \right) = \left(\frac{B_{rad}}{B_{tan}} \right)^2 A^2 \quad (6-11)$$

$$\frac{B_{rad}}{B_{tan}} = \frac{B_{earth} \left(\frac{r_{earth}}{r} \right)^3 2 \cos \theta}{B_{earth} \left(\frac{r_{earth}}{r} \right)^3 \sin \theta} = \frac{2 \cos \theta}{\sin \theta} \quad (6-12)$$

$$A_{rad} = \frac{\left(\frac{2 \cos \theta}{\sin \theta} \right) A}{\left(1 + \left(\frac{2 \cos \theta}{\sin \theta} \right)^2 \right)^{\frac{1}{2}}} = \frac{2 \cos \theta A}{\left(\sin^2 \theta + \frac{4 \sin^2 \theta \cos^2 \theta}{\sin^2 \theta} \right)^{\frac{1}{2}}} \quad (6-13)$$

Use the identity $\cos^2\theta + \sin^2\theta = 1$, then use the sum of squares to replace $A_{\tan}$.

$$A_{\text{rad}} = \frac{2 \cos\theta A}{(1 - \cos^2\theta + 4 \cos^2\theta)^{\frac{1}{2}}} = \frac{2 \cos\theta A}{(1 + 3 \cos^2\theta)^{\frac{1}{2}}} \quad (6-14)$$

$$A_{\text{un}}^2 = A^2 - A_{\text{rad}}^2 \quad (6-15)$$

$$A_{\text{un}}^2 = A^2 \left(1 - \frac{4 \cos^2\theta}{(1 + 3 \cos^2\theta)} \right) = A^2 \left(\frac{1 + 3 \cos^2\theta - 4 \cos^2\theta}{1 + 3 \cos^2\theta} \right) \quad (6-16)$$

$$A_{\text{un}} = A \left(\frac{\sin\theta}{(1 + 3 \cos^2\theta)^{\frac{1}{2}}} \right) \quad (6-17)$$

A = total area enclosed by loop

θ = angle from magnetic north

The tangential force is then the tangential area multiplied by the tangential part of the gradient in magnetic field, found in Chapter 3. The radial force will not be considered in the orbit control program, but rather the perigee will be kept constant. The justification for this is that the radial force is small compared to the tangential force because the thrust arc selected is mainly away from the magnetic poles, where the field is vertical. Also, the effect of the radial inwards force during the thrust arc is offset by the non-impulsive nature of the thrust, which would tend to make the perigee increase over time in the absence of the radial inwards force. The maximum magnetic force the loop can create in the θ , or tangential, direction to directly counter the drag is expressed following Zubrin (1991). Force variation is described with a “thrust function”, $T(\theta)$, which scales the magnetic force according to position from magnetic north.

$$F_{tan} = (A_{tan})(I)(\text{Grad } B_{tan}) = \frac{-3(\sin 2\theta)B_{earth}(IA)r_{earth}^3}{2(3\cos^2 \theta + 1)^{\frac{1}{2}}(r)^4} \quad (6-18)$$

$$\text{define: } T(\theta) = \frac{\sin 2\theta}{2(3\cos^2 \theta + 1)^{\frac{1}{2}}} \quad (6-19)$$

B_{earth} = magnetic field at earth surface, 3.05×10^{-5} Tesla

r_{earth} = radius of earth, 6.378×10^6 meters

r = position from earth centre

$T(\theta)$ = thrust function, defined here.

θ = angle from magnetic axis of earth

I = current carried in loop

A_{tan} = area for tangential thrust, as found above.

A = area of loop

Substituting for $T(\theta)$ and the constants and multiplying through by r_{earth} , one obtains:

$$F_{tan} = (-1.46 \times 10^{-11})IA \left(\frac{r_{earth}}{r} \right)^4 T(\theta) \quad (6-20)$$

This is the expression for the force tangential to the orbit which will be used in the orbit simulation program. The thrust function, $T(\theta)$, highlights a shortcoming of magnetic propulsion: over the magnetic pole ($\theta = 0^\circ$) or over the magnetic equator ($\theta = 90^\circ$), no force can be created in the tangential direction if the earth's magnetic field is assumed to be a dipole. Of course, the field is not a perfect dipole. There are always some gradients at any point in the field, but as the dipole approximation suggests, certain locations in orbit will need very large currents to generate the desired forces. Directly over a magnetic pole, for instance, radial forces will be a maximum, but tangential forces will be very small. The situation is like holding a small magnet directly over the pole of a larger magnet. The force on the small magnet is straight down towards the pole. There

is no tangential force because the small magnet is already over the pole of the large magnet.

The loop size question can be resolved with a specification of the acceleration desired at a specific location. Toward $\theta = 0^\circ$, the radius of the loop will tend toward infinity as $\sin(2\theta) \rightarrow 0$. As $\theta \rightarrow 45^\circ$, the radius necessary for a given magnetic force reaches a minimum. Once the altitude changes, the density of the atmosphere and the magnetic field will change as well and the loop size required for a given acceleration will change.

6.6.3 Net force equation

To find the best loop radius, one can find the best tangential acceleration as loop radius varies with constant current and constant drag per m^2 of area. The net force on the satellite is the magnetic force less the drag:

$$F_{\text{net}} = F_{\text{magnetic}} - F_{\text{drag}} \quad (6-21)$$

Rearranging equation 6-1, and using $F = ma$, the expression for atmospheric drag is:

$$F_{\text{drag}} = \frac{1}{2} \rho C_D A V^2 \quad (6-22)$$

ρ = atmospheric density
 C_D = coefficient of drag
 A = spacecraft frontal area
 V = velocity of spacecraft

In the orbit program, the density of the atmosphere and velocity of the spacecraft will change with position. Larger loop radii do not necessarily imply greater accelerations for all loop designs, since the increased thrust from a larger-radius current loop will be offset by greater mass and larger loop area.

6.6.4 Single Ring

When shuttling payloads from low to high orbits, attitude control is not necessary if only the stable part of the thrust arc is used. Most of the single-loop reinforcement is needed for self-induced tension rather than bending. Most of the tether force will be opposed by the self-induced radial outwards force created by the current in the loop.

Figure 6.2 Single-loop design for magnetically propelled spacecraft.

For a single loop design, the frontal area consists of the magnetic loop, the satellite body and the tethers between the body and the loop. The area and mass calculations for a single-loop design are straightforward when the loop is of uniform thickness. The tension in the loop is found as described in Chapter 5, then the area of reinforcement found for that tension. For the tethers, the same equations as for the triple loop are used, except that the tension term is much smaller because there are no bending forces to restrain.

$$\text{Tension} = \frac{\mu_o I^2}{2} \quad (6-23)$$

$$\text{Area}_{\text{section}} = \frac{\text{Tension}}{\sigma_{\text{allow}}} \quad (6-24)$$

$$\text{diameter} = \left(\frac{4(\text{Tension})}{\sigma_{\text{allow}} \pi} \right)^{\frac{1}{2}} \quad (6-25)$$

Area_{section} = area of loop reinforcement cross section

Tension = loop tension caused by own current

σ_{allow} = allowable stress on reinforcement material

diameter = diameter of loop cable

Since bending forces are not a significant consideration for the single loop, the superconductor cable does not have to be a hollow tube, but can be a simple cable. This means that the diameter equation can be used for either the loop cable or the tether cables, so long as the appropriate tension is used. For both single and triple rings, the total mass is:

$$\text{mass}_{\text{total}} = \text{mass}_{\text{reinf}} + \text{mass}_{\text{tethers}} + \text{mass}_{\text{super}} + \text{mass}_{\text{system}} + \text{mass}_{\text{body}} \quad (6-26)$$

mass_{total} = total mass of the spacecraft

mass_{reinf} = mass of reinforcement material

mass_{tethers} = mass of the tethers

mass_{super} = mass of superconductor used

mass_{system} = mass of cooling, solar paneling and systems dependent on loop size

mass_{body} = mass of centre spacecraft body, including payload

A size for the single loop will be selected based on self-acceleration. The selected loop will be used in the orbit transfer program described in Section 6.7.

6.6.5 Triple Ring

The frontal area of the satellite is composed of the three loops, the tethers and the main body area.

Figure 6.3 Frontal view of spacecraft with triple ring and tethers

Remembering that there are two main loop tethers for each edge of the secondary loops, the area of the spacecraft that meets atmospheric drag is approximated by using all the tethers. Tethers hidden behind the tethers shown in the Figure above are too far apart to be considered as blocked from drag. Loops are symmetrical about their quarter-section, so loop 1 uses 4 times its quarter-sections area. However, loops 2 and 3 are edge-on, so their area is approximated as 3 times quarter-section area, implying some blocking of the trailing edge of the loop, but not complete blocking. There are 2 secondary loops, so the multiplier of the quarter-section area summation is 6.

Area =

$$4 \sum_{n=1}^{\text{sections}} d_n \frac{R_1}{\text{sections}} + A_{\text{body}} + 6 \sum_{n=1}^{\text{sections}} d_n \frac{R_2}{\text{sections}} + 8 (d_{\text{tether1}}) R_1 + 4 (d_{\text{tether2}}) \sqrt{2} R_2 \quad (6-27)$$

Area = area of spacecraft for drag calculations

sections = number of sections used for numerical integration of area

d_n = diameter of section n

$d_{\text{tether1}}, d_{\text{tether2}}$ = diameter of tethers 1 and 2

R_1, R_2 = radius of loops 1 and 2, on the order of hundreds of meters

A_{body} = frontal area of spacecraft body

The programs in the thesis use the subroutine “triple.cpp” (see Appendix A) to calculate the diameter of loop section for different thicknesses of reinforcement. As an example, the tube diameter for various wall thicknesses is shown below for a 500 m main loop radius with a 50 kA current in both main and secondary loops. The methods of Chapter 5 were used to arrive at the wall thickness values.

Figure 6.4 Loop tube diameter for various wall thicknesses.

The mass of the triple ring will vary with the thickness of tube reinforcement needed. Figure 6.5 shows that the total spacecraft mass is least for the minimum thickness, but this is not automatically the best option, because minimum thickness means maximum loop diameter and thus increased drag, as shown by Figure 6.4. See “triple.cpp” in Appendix A for mass calculations.

Figure 6.5 Total mass versus wall thickness for triple ring.

One possible way to reduce the amount of reinforcement needed for the triple loop is to use more tethers to resist the bending moments between the loops. For example, if one wished to double the number of tethers, the arrangement below might be used.

Figure 6.6: Triple ring with 8 tether-pair arrangement.

The new arrangement with 8 tether-pairs could be used in the acceleration program with the appropriate new loads over each loop section. Consequently, a slight reduction in mass may be expected at the expense of complexity. However, further calculations will show that the triple ring is so massive that even reducing its mass by an order of magnitude would not make it competitive with a single loop for either orbit control or any of the other applications suggested.

6.6.6 Loop Parameter Assumptions

The minimum practical thickness of the hollow loop wall will be set at 1 mm. 1 mm is selected because other parts of the spacecraft will need to be connected to the loops at various points, creating stress concentration factors much above those considered for the bending, tension and torsion. With very thin walls, tears will develop at attachment points. Also, the diameter of the hollow loop will become very large as the thickness becomes very small, creating transport problems.

The area of the superconducting cable will be taken from the superconducting magnetic energy storage prototype cable mentioned previously, which had a radius of about 0.013 m for the complete 50 kA cable. For conservatism, the area of a round 0.02 m radius cable will be assumed. 5000 kg/m^3 , the density of copper oxide (CuO), will be used since virtually all high temperature superconductors are copper oxides. The mass of solar panel, reflecting layers and other systems dependent on the size of the loop will be set at 0.05 kg per meter of loop.

6.7 Acceleration Programs

There are two acceleration programs, one for the single-loop model and one for the three-loop model. Both programs are listed in Appendix A, the main difference between them being the manner in which reinforcement mass is calculated (see Chapter 5). A function calculates the mass of superconductor material, then adds the mass of solar panel and reflecting (or cooling) material needed for any given loop radius. Total mass for a loop is the reinforcement mass, the superconductor mass and the systems mass (solar and cooling).

$$m_{\text{super}} = (2\pi R_1 + 2\pi (2R_2))\pi (r_{\text{super}})^2 (\text{den}_{\text{super}}) \quad (6.28)$$

$$m_{\text{system}} = (2\pi R_1 + 4\pi R_2) \left(1 \frac{\text{kg}}{\text{m}} \right) \quad (6.29)$$

$$m_{\text{loop}} = m_{\text{reinforc}} + m_{\text{super}} + m_{\text{system}} \quad (6.30)$$

m_{super} = mass of superconducting material

r_{super} = radius of superconducting cable

$\text{den}_{\text{super}}$ = density of superconducting material (CuO, 5000 kg/m³)

m_{system} = solar and reflecting systems mass, taken as 0.05 kg/m of loop

“ m_{system} ”, the solar and reflecting system mass used, is defined per meter of loop, so as loop size increases, the systems mass will increase proportionately. From equation 6.21, the tangential acceleration of the loop, considering atmospheric drag and magnetic forces, is:

$$a = \frac{F_{\text{net}}}{m_{\text{total}}} = \frac{\pi R_1^2 I_1 \bullet \nabla B - \frac{1}{2} \rho C_D V^2 [\text{Area}]}{m_{\text{total}}} \quad (6.31)$$

a = acceleration (tangential to orbit)

F_{net} = magnetic force less drag

ρ = density of atmosphere

C_D = coefficient of drag

V = velocity of spacecraft

I = current in loop

R_1 = radius of loop 1

6.7.1 Acceleration vs. loop radius

To find the best radius for the main loop, a number of different radii were tried and the acceleration found at each. Table 6.2 shows the variation of spacecraft mass, acceleration and area with increasing loop radius for a single loop spacecraft. The single loop acceleration program was run with drag calculated at an altitude of 300.0 km (the same perigee as Zubrin and Andrews) and a current of 50 kA.

Loop Radius, meters	Acceleration, m/s^2	Mass, kg, total	Area meeting drag, m^2	Magnetic force Newtons
600	0.0002	1255	66	0.203
1200	0.0005	1510	82	0.814
1800	0.0010	1765	98	1.831
3000	0.0022	2275	131	5.086
6000	0.0057	3550	211	20.343
9000	0.0095	4825	292	45.772
12000	0.0133	6100	372	81.372
15000	0.0172	7375	453	127.143
18000	0.0212	8650	533	183.086
21000	0.0251	9925	614	249.201
27000	0.0330	12475	775	411.944
30000	0.0370	13750	855	508.573

Table 6.2 Single loop acceleration change with radius, including drag

As the above table shows, single loop acceleration at a fixed current is best for the largest radius possible. The increase in mass and drag with loop radius is more than

compensated for by the increased magnetic force on a single loop. The next table shows spacecraft mass, acceleration and area for various main loop radii of a triple-loop spacecraft, again with a superconducting current of 50 kA and drag calculation for density and velocity conditions at a 300 km circular orbit.

Radius loop, m	main	Acceleration, ms ⁻²	Mass, kg	Reinforcement thickness, m	Reinforcement diameter, m
50		0.00000005	4043	0.0100	0.0790
100		0.00000030	7446	0.0090	0.1178
150		0.00000055	9472	0.0050	0.1935
200		0.00000080	10933	0.0030	0.2885
300		0.00000130	15400	0.0020	0.4327
500		0.00000232	22434	0.0010	0.7901
1000		0.00000459	59348	0.0010	1.1174
2000		0.00000776	163307	0.0010	1.5802
3000		0.00001017	297609	0.0010	1.9353
4000		0.00001220	456448	0.0010	2.2347
4950		0.00001390	627005	0.0010	2.4860

Table 6.3 Acceleration of triple ring for various radii, including drag.

From even a casual look at the tables, it is obvious that the triple loop design will require massive reinforcement compared to the single loop. The large mass of the reinforcement means that putting such a spacecraft into LEO would be extremely expensive. Acceleration would be extremely small because of the drag of the structure and large mass. For satellite orbit control, the mass of a the triple ring design is prohibitive.

6.8 Previous Researchers' Orbit Calculations

Magnetic propulsion might be used to escape from low earth orbit (LEO), as well as to change from one orbit to another. Zubrin and Andrews (1991) calculate

performance for a 31 km radius loop with a 50 kA current and a 14 ton payload. 5 tonnes (5000 kg) are assumed for the superconducting loop (“magsail”) and 1 tonne for power and systems. Since their sample magsail consists of a single superconducting loop with no attitude control, it is only switched on for $\theta = 20^\circ$ to $\theta = 80^\circ$, where θ is the polar angle from the magnetic pole. This range of angles represents the stable portion of the thrust arc.

Figure 6.7 Satellite orbit around earth showing stable thrust arc.

Zubrin and Andrews consider a transfer from a circular 300 km orbit to a geosynchronous polar elliptical orbit with a apogee (furthest point) of 71090 km and perigee (closest point) of 636 km. The self-acceleration of Zubrin’s sample magsail is

0.0206 m/s², which by computer integration (not shown) requires 160 orbits in 34 days to meet the required ΔV (change in velocity) for the transfer. Zubrin and Andrews give the following flight times for their loop:

Orbit #	Time	Perigee	Apogee	Velocity
0	0 days	290 km	310 km	7.7 km/s
60	4.9	383	6076	8.75
120	15	451	21723	9.7
160	34.3	521	67265	10.3

Table 6.4: Orbit transfer flight times, Zubrin and Andrews (1991).

The same orbital transfer will be used in the single loop orbit program of this thesis and the results compared to Zubrin and Andrew's table above.

6.9 Orbit Transfer Programs

6.9.1 Single Loop

The orbit transfer program uses the acceleration values found in the single loop acceleration program (see Appendix B) and basic orbital mechanics to find the time and number of orbits necessary to achieve a given transfer. Thrust is only used for the stable part of the orbit, which is a quarter-section or less of the total orbit. Perigee (closest point) is kept small and apogee (furthest point) steadily increases. The perigee is kept close because F_θ , the tangential magnetic thrust, falls off as (radius)⁴, as noted in the magnetic force section. The impulse per orbit is maximized by keeping perigee small and

using as much of the thrust arc as possible per orbit before the radial component of the magnetic field becomes too large.

A factor of 0.9 is included because the magnetic poles are rotating with the earth, while the satellite orbit is not. The average loss in force was found as 0.9 by Zubrin and Andrews (1991). The delta-V required for any particular orbit transfer is calculated using Keplerian equations and assuming a Hohmann transfer (after Larson and Wertz, 1992). The time under acceleration for a given delta-V is found from elementary physics.

$$\Delta V = \sqrt{Gm_e} \left[\left(\frac{2}{r_A} - \frac{1}{\text{avg}} \right)^{\frac{1}{2}} - \left(\frac{1}{r_A} \right)^{\frac{1}{2}} \right] \quad (6.32)$$

$$\text{avg} = \frac{r_A + r_B}{2} \quad (6.33)$$

$$V = V_o + at \quad (6.34)$$

$$t = \frac{V - V_o}{a} = \frac{\Delta V}{a} \quad (6.35)$$

ΔV = velocity needed for transfer

Gm_e = gravitational constant and mass of earth, $3.986 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$

r_A, r_B = radius of orbits A and B

avg = average of orbit radius A and B

V = final velocity

V_o = initial velocity

a = acceleration

t = time

The ΔV equation above is for a transfer from a circular orbit of radius A to an elliptical orbit with perigee equal to radius A and apogee equal to radius B. The magsail thrust arc is not impulsive, but neither is it constant thrust, since the thrust portion is

always less than a quarter orbit for a single loop and less than a half-orbit for a triple loop. The impulsive Hohmann transfer equation for ΔV will be used as an approximation. However, ΔV in the orbit program will assume gravity losses of 5% from the ideal ΔV for the maneuver because of the semi-constant thrust transfer instead of the impulsive, single orbit Hohmann transfer. 5% is used because Zubrin and Andrews (1991) calculated a 5% loss for their magnetic propulsion transfer.

Figure 6.8: Transfer from circular orbit radius A to elliptical orbit perigee A, apogee B.

Since both air density and magnetic field change with altitude, it was necessary to make the drag and magnetic force subroutines functions of altitude. For an elliptical orbit, altitude above the earth is changing constantly during the thrust arc. To obtain an accurate delta-V expected from a given thrust arc, the thrust arc was numerically integrated. The velocity and total energy of the orbit were updated after each integration

step. At the end of a thrust arc, the orbit energy was used to update the shape and size of the next orbit. Altitude for any angle from perigee for a given elliptical orbit was found by using a combination of Newton's second law and his law of gravitation, which is the position equation:

$$\frac{\partial^2 r}{\partial t^2} + \frac{GM_e}{r^3} r = 0 \quad (6.36)$$

r = radius from earth center, see diagram below.

G = gravitational constant

M_e = mass of earth

t = time, used for second derivative

Figure 6.9: Definition of apogee, perigee, true anomaly and semi-major axis.

Notice that the earth is one focus of the ellipse, while the other focus is twice the semi-major axis away. For such an elliptical orbit, standard geometric equations were

used to find position for any angle from perigee. Altitude was found by subtracting the radius of the earth from the position.

$$r = \frac{a(1 - e^2)}{1 + e \cos v} \quad (6.37)$$

$$e = 1 - \frac{r_p}{a} \quad (6.38)$$

a = semi-major axis of orbit, half distance between two foci of the ellipse

e = eccentricity of orbit, measure of circularity

v = true anomaly, angle from perigee, radians, as in Figure 6.9

r_p = radius at perigee, closest point to earth in orbit

The acceleration time for each segment of the thrust arc is found by using the change in a quantity called the mean anomaly (M in equation 6.40) for a given change in true anomaly (angle from perigee, see Figure 6.9) over the arc segment. The orbit control program assumed an orbit with the perigee mid-way through the thrust arc and subdivided the thrust arc into segments of equal change in true anomaly. For each segment of thrust arc, the time of flight was found by use of other canonical Keplerian orbit elements. See Appendix B for an explanation of Keplerian elements.

$$t - t_o = \frac{(M - M_o)}{\eta} \quad (6.39)$$

$$M = E - e \sin E \quad (6.40)$$

$$E = \cos^{-1} \left[\frac{e + \cos v}{1 + e \cos v} \right] \quad (6.41)$$

$$\eta = \left(\frac{(G)M_e}{a^3} \right)^{\frac{1}{2}} \quad (6.42)$$

$t - t_o$ = time of flight

M = mean anomaly in radians

E = eccentric anomaly

e = eccentricity

v = true anomaly

η = mean motion

$G M_e$ = gravitational constant and mass of earth.

The magnetic force and drag force for a given loop radius were found using the altitude as found above. The acceleration time for a segment is the time-of-flight. The delta-V for a segment is the acceleration (total force minus drag divided by mass) multiplied by the acceleration time. This delta-V was used to update the energy of the orbit after the calculation for each segment.

$$\epsilon = \frac{V^2}{2} + \frac{GM_e}{r} \quad (6.43)$$

ϵ = energy of orbit per unit mass of spacecraft

V = velocity, found by adding delta-V to previous velocity

r = position from earth center

At the end of a thrust arc, the energy of the orbit was used to find a new semi-major axis for the next thrust arc. A new semi-major axis automatically changed the eccentricity, mean anomaly, eccentric anomaly and mean motion, but the perigee was assumed to be fixed. Fixing the perigee is equivalent to assuming that the thrust arc is an impulsive delta-V input. The size of the semi-major axis is a direct measure of the energy of the orbit, so the new semi-major axis is found from Larson and Wertz (1992) as:

$$a = \frac{-GM_e}{2\epsilon} \quad (6.44)$$

The orbit program was run twice with the same size of loop as Zubrin and Andrews (1991), first by neglecting drag, as they did, then including drag. The graphs below show that, without drag, the orbits and time to achieve a transfer from a 300 km circular orbit to an elliptical orbit with apogee 71090 km are almost identical to the table

of Zubrin and Andrews given previously: 155 orbits for the thesis program vs. 161 for Zubrin and Andrews. Note that Zubrin assumes a 14 ton payload and 6 tons for the rest of the spacecraft. The mass estimates in the thesis show that a loop of the same size as Zubrin would allow a 7 ton payload, and 13 tons for the rest of the spacecraft. All the same, the total mass is made to be 20 tons, so that the results are comparable.

Figure 6.10 Single loop change in apogee with orbit number, no drag.

As apogee increases, the speed of the spacecraft during the thrusting arc will also increase. As a result, the delta-V possible during each thrusting arc will decrease even if perigee is constant. This effect is shown in Figure 6.11.

31.6 km Single Loop, No Drag

Figure 6.11 No drag single loop change in delta V per orbit.

Under identical conditions, the same program was run with drag. Not only is there drag during the thrusting arc, reducing the delta-V available, but there is also drag for the rest of the orbit, which is significant until the apogee is greater than about 600 km. Figures 6.11 and 6.12 (below) show that the acceleration of the single ring is strong enough that drag does not significantly affect its performance for the given transfer.

Figure 6.12 Single loop change in apogee with time, with drag.

The total time for the orbit transfer is 36 days with drag, 34.9 days without drag, in excellent agreement with Zubrin and Andrew's estimate of 34.3 days without drag.

6.9.2 Triple Loop

A program similar to the single loop program with drag was run for the triple loop. Naturally, the area and mass calculations were different, but the thrust arc was also doubled, since the triple loop may use the unstable part of the thrust arc as well as the stable part. Since a 50 kA current and a 31 km radius main loop were not sufficient to counteract drag (see Table 6.5), the current and radius were changed in an effort to see if any combination of current and loop size could make the triple ring work. However, all orbits decay from the start perigee of 300 km and go below 150 km in less than 2 days, no matter what current or radius is used.

orbit	days	delta V (m/s)	apogee, km
	1	0.06	-1.15
	3	0.19	-2.93
	7	0.44	-2.70
	10	0.63	-2.69
	15	0.94	-0.82
	20	1.25	-0.81
	25	1.56	-0.80
	30	1.87	-0.79

Table 6.5 Triple ring, 31 km radius, mass 9,746,818 kg and current 50 kA.

orbit	days	del V(m/s)	apogee(km)
	1	0.06	-1.15
	3	0.19	-2.94
	7	0.44	-2.71
	10	0.63	-2.70
	15	0.94	-0.82
	20	1.25	-0.81
	25	1.56	-0.80

Table 6.6: Triple ring, radius 31 km, mass 97,147,734 kg and current 500 kA.

orbit	days	del v	apogee
1	0.06	-1.19	296
3	0.19	-2.97	275
7	0.44	-2.74	235
10	0.63	-2.74	207
15	0.94	-0.86	186
20	1.25	-0.85	172
25	1.56	-0.83	158

Table 6.7: Triple ring, radius 1 km, mass 565,724 kg, and current 500 kA.

orbit	days	del v	apogee
1	0.06	-1.13	296
3	0.19	-2.84	276
7	0.44	-2.62	238
10	0.63	-2.62	212
15	0.94	-0.82	185
20	1.25	-0.81	172
25	1.56	-0.80	158

Table 6.8: Triple ring, radius 1 km, mass 59348 kg and current 50 kA.

Tables 6.6 to 6.8 show that larger currents do not yield better performance because the mass and area of reinforcement increase to counteract the larger torque between loops that larger currents bring. To see if the mass could be reduced, a smaller radius loop (1 km) was used for the last two tables, but the result was still a decay in orbit below 150 km in less than two days.

Chapter 7 : Future Directions

7.1 Spaceflight Challenges

Traveling through space cosmic distances will take a long time using rockets. Mars would be a two year round trip using rockets, and the size of the ship would require many shuttle flights to assemble it in earth orbit (Kumar, 1994). Fuel mass required on a rocket-driven spacecraft increases exponentially with distance of the mission and mass of the payload. For the Apollo moon missions, 2700 tonnes of mass on the ground were needed to lift 40 tonnes of mass to the moon . For a 1 kg probe to be accelerated to 100 km/s, one would need a 70 million tonne rocket. This does not include any fuel for deceleration at destination, and certainly not any return trip. Worse yet, 100 km/s is slow when one considers that a one-way trip to the nearest star with planets (10 light years) would take 30 000 years (Mauldin, 1992). Clearly, alternatives to rockets such as magnetic loops must be investigated in order to travel cosmic distances.

7.2 Interplanetary Travel

Zubrin and Andrews (1991) describe the use of superconducting current loops, which they call magsails, for interplanetary travel. The mechanism for propulsion is deflection of interplanetary plasma winds. Accelerations on the order of 0.01 ms^{-2} are calculated for a single loop of radius 30 km. The great advantages of a magsail are that multiple trips can be made per mission and that trips are not subject to a launch window, as are Hohmann transfers.

Tangential force (or “lift”) can be generated by turning the loop at an angle to the solar wind, hence the name “magnetic sail” or “magsail”.

7.2.1 Solar Wind

A flux of a few million charged particles per cubic meter flows by the earth from the sun. The earth’s magnetosphere deflects this plasma wind, protecting life on earth from the various ill effects of high-energy particles. The solar wind is visible when charged particles manage to penetrate the magnetosphere in the vicinity of the magnetic poles, creating the Northern Lights (Aurora Borealis) and Southern Lights (Aurora Australis). Solar wind velocity is 400-600 km/s, which sets an upper limit to the velocity a magsail can attain by drag, or repulsion of the particle wind.

Since the solar wind consists of charged particles, the number of particles affected by the magnetic field of the loop is much greater than those that pass through the loop area. In fact, the area of effective total reflection of particles is calculated by Zubrin and Andrews as 75 times the area of the loop for a 50 kA current. The larger the current for a given loop size, the larger the area of effective total reflection.

7.2.2 Magsail Orbits

A trip from earth-orbit distance from the sun to the orbit of Mars is calculated by Zubrin and Andrews (1991) to take 273 days for a 5-tonne magsail with 3 tonnes of insulation, shielding and power equipment and 37.5 tonnes of payload. This is only slightly longer than the Hohmann transfer time (see Figure 7.1), but it should be remembered that the

estimates of this thesis suggest that the payload mass estimates of Zubrin and Andrews may be optimistic, given the loop reinforcement required.

Figure 7.1 Impulsive Hohmann transfer, typically used by rockets.

Once at the orbit radius of Mars, the magsail could circularize its orbit and, while remaining under power, stay in the orbit of Mars while traveling at a different velocity from Mars. The magsail force is canceling some of the sun's attractive force, so the magsail can be traveling about 4.6 km/s slower than Mars while at the same distance from the sun. This means that a magsail can launch at any time: there is no launch window as with an impulsive Hohmann transfer because the magsail can wait for the destination planet to catch up with it once at the same orbit distance from the sun.

Figure 7.2 Direction of magsail thrust from solar wind.

In the reference frame of a nonrotating sun, the magsail orbit transfer would look similar to a Hohmann transfer, except that at the destination orbit, the tangential speed would be the same as at the initial orbit. If the initial orbit were at earth distance from the sun, and one watched the magsail from the earth, it would appear to move radial outwards from the sun to the new orbit.

The advantage of the magsail becomes even greater after the cargo is delivered: now the spacecraft can return for another trip. The magnetic field of the destination planet could be used for an initial impulse out of orbit. If the next destination is in the direction of the solar

wind, then the magsail could proceed as previously. However, if the mission is to return to earth for another load, then the magsail could either use the gravity of the sun alone to return, or do some sailing to speed up the process.

7.2.3 Space Sailing

All of the preceding discussion has assumed that the magsail does not change its angular momentum about the sun. No lift is generated; the magsail simply reflects the charged particles of the solar wind, like a sailboat traveling straight downwind. To generate lift, or “sail”, one would need to deflect the solar wind at an angle.

Figure 7.3 Magsails dragging and sailing on the solar wind.

If lift were to be used, the single loop proposed by Zubrin and Andrews would need some form of attitude control. They suggest changing the spacecraft centre of mass by shifting the payload on a set of tethers, thus offsetting the centre of pressure from the centre of mass and creating a torque. Using lift allows trip times to be reduced and flexibility in

missions to be increased. For example, using lift means that it is not necessary for elliptical orbits to be used to meet up with a certain planet. The Mars mission times in Zubrin and Andrew's 1991 paper did not consider using planetary magnetic fields, but it is obvious that the travel times would be reduced using planetary magnetic fields as well as the solar wind. Flexibility, large payload capability and multi-trip missions make magnetic propulsion extremely attractive for interplanetary missions.

7.4 Interstellar Travel

The solar wind is about 500 km/s, so the maximum theoretical speed a magsail could ever hope to attain from the solar wind using drag alone is 500 km/s. Of course, as the spacecraft travels further from the sun, the density of the solar wind decreases, so that the efficiency of solar wind propulsion must also decrease. It would be a fantastic feat to obtain 300 km/s from the solar wind. Is 300 km/s fast or slow? In the case of interstellar travel, it is slow. Desirable, "close" interstellar destinations are on the order of 30 light-years away. Light travels at 300 000 km/s, so that a 300 km/s solar wind-powered spacecraft would take 30, 000 years to reach the destination. Acceleration and deceleration times would add a few thousand years, but given average human lifespans of ≈ 72 years, the point is moot.

7.4 Self-Launching Capability

Zubrin and Andrews (1991) claim that 80 times the best current densities available today would be needed to obtain self-launching capability from earth surface. Ackerman (1989) also suggests magnetic self-levitation. With extremely large currents available,

Ackerman assumes a loop size of less than 100 meters and a total mass on the order of 1000 kg. The estimates in this thesis show that very large currents will not yield self-levitation because the reinforcement mass grows faster with current than the magnetic force.

7.4.1 Triple Ring

For hovering flight, a single ring with no attitude control is not adequate. To find if self-levitation is possible to achieve using a triple ring arrangement, an iterative procedure was used in the program “l3.cpp” (see Appendix A). A starting radius was selected, then the mass that could be lifted at a starting current calculated and compared to the mass of the spacecraft, including reinforcement. The current was then incremented, the mass of the spacecraft and the mass that could be lifted recalculated, and the difference between the mass that could be lifted and the mass of the structure recorded. The current at which there was the smallest difference between the mass of the spacecraft and the mass that could be lifted was selected as the best current for that radius. Then the radius was increased and the procedure repeated. The mass that could be lifted was found as follows.

$$(\text{Area})(\text{current})(\text{grad_B}) = (\text{mass})(\text{gravity}) \quad (7.1)$$

Area = area of main loop

current = current in main loop

grad_B = gradient in the magnetic field

mass = mass that can be levitated

gravity = 9.71 m/s² at earth surface

The gradient in the magnetic field is needed in the radial direction, not the tangential, as with the orbit programs. The derivative of the magnetic field as given in Chapter 3 is:

$$\frac{\delta B}{\delta r} = B_{\text{earth}} (r_{\text{earth}})^3 \left(\frac{-3}{r^4} \right) (2 \cos \theta \mathbf{r} + \sin \theta \mathbf{q}) \quad (7.2)$$

B_{earth} = magnetic field at earth surface

r_{earth} = radius of earth

r = distance from centre of earth

θ = angle from magnetic north

$\mathbf{r}, \mathbf{q}$ = radial and tangential unit vectors.

Notice that anywhere other than a magnetic pole, where $\theta = 0$ or π , there will be a tangential force as well as a radial one. This means that the loop would have to tilt to fly upwards at any point other than directly over a magnetic pole. See program “mag_for_rad.cpp” in Appendix A, where the conditions are assumed to be at earth surface, at a magnetic pole, for the details of the radial magnetic force calculation. The results for the triple ring are tabulated in Table 7.1 for currents from 4500 A to 1 000 000 A.

radius l (m)	mass able to lift, total (kg)	best mass (kg)	current (A)
40	0.0001	1171	4500
50	0.0001	1233	4500
60	0.0001	1300	4500
70	0.0002	1372	4500
80	0.0003	1449	4500
90	0.0003	1530	4500
100	0.0004	1615	4500
110	0.0005	1703	4500
120	0.0006	1796	4500
130	0.0007	1891	4500
140	0.0008	1991	4500
150	0.0009	2093	4500
160	0.0011	2198	4500
170	0.0012	2307	4500
180	0.0013	2418	4500
190	0.0015	2532	4500

Table 7.1 Mass that can be lifted by magnetic force and spacecraft mass for triple ring.

It has been shown in Chapter 6 that triple ring acceleration decreases when larger loop radii are used. Smaller loop radii were used here to find if reinforcement mass at smaller sizes would grow less quickly than magnetic lift. With stress as approximated in the “triple.cpp” routine (Appendix A), the mass of the triple ring grows much faster than the lift force as current is increased. The program selects the best current by finding the smallest difference between the total mass and the total lift. In all cases, it is the smallest or starting current which is selected.

The problem is that reinforcement mass increases more quickly with current than lift, so that the triple ring, as proposed, will always be too massive to fly, regardless of the size of superconducting currents available. It would then appear that, using aluminum reinforcement in the configuration of this thesis, the self-levitation postulated by Ackerman (1989) would not be possible. Some other configuration, or a reinforcement material that is much lighter and stronger than aluminum, would have to be used.

7.4.2 Single Loop

Since the triple loop seems too massive for levitation, let us assume a single loop with an attitude stabilization system such as a controllable-speed gyroscopic shell. Before considering how such an attitude control system may work, consider the practicality of achieving levitation. As in the triple-loop lift program, the lifting force and the total mass are compared for many different loop radii and current sizes. The attitude control system is included by assuming gyro mass equal a circular shell of aluminum one mm thick and of the

same area as the main loop (optimistic, but still excessively massive, as shall be seen). Figure 7.4 shows the proposed design of a single loop gyro-stabilized superconducting magnetic spacecraft.

Figure 7.4: Single superconducting loop with gyroscopic shell.

The single-loop lifting program (Appendix A, "11.cpp") is the same as the triple-loop program except that the mass calculations are for the single, not the triple loop and the starting current is larger. As with the triple loop, the best current is selected as the current where the difference between the total mass and the mass that can be lifted is smallest. Also as with the triple loop, the best current is the starting current since single loop reinforcement mass grows at a faster rate than magnetic force for earth-surface conditions and aluminum reinforcement.

Main loop radius (m)	Total mass (kg)	Mass able to lift, Total (kg)	Current (A)
50	2020	0.01149	500000
100	3039	0.04594	500000
150	4059	0.10337	500000
200	5079	0.18377	500000
250	6099	0.28714	500000
300	7118	0.41349	500000
350	8138	0.56280	500000
400	9158	0.73508	500000
450	10178	0.93034	500000
500	11197	1.14857	500000

Table 7.2 Single loop self-levitation current selected for smallest lift-mass difference.

To observe how the single loop mass changes with increased current at a radius fixed at 100 meters, see Figure 7.5.

Figure 7.5 Mass and mass able to lift vs. current for single 100 m radius loop.

The reinforcement mass increases tremendously with current, but the mass that it is possible to lift increases so little that the line appears to lie along the X -axis in Figure 7.5. As with the triple loop, the mass of reinforcement grows much more quickly than the mass that can be lifted. With aluminum reinforcement in the configuration selected, the single loop will not self-levitate for any current value.

7.4.3 Radical Assumptions

Even if technology evolves to allow very large superconducting currents, a fundamental problem appears to be the mass of reinforcement needed to contain the forces that those same currents induce. Aluminum is simply too massive for the craft to fly in the configuration suggested. However, there are prospects for much stronger materials to be used, as the data taken from Friedrich et al (1984) shows in Table 7.6.

	Yield or Fracture Stress, M Pa	density, g/cm ³
Al Alloy, HS	550 (yield)	2.7
Steel, UHS	3000 (yield)	7.8
Ti Alloy, HS	1500 (yield)	4.5
Maraging Steel, Ni-Co-Mo, bulk	3500 (fracture)	7.8
Maraging Steel, wires	4000 (fracture)	7.9
Si - whisker	7000 (fracture)	2.3
B - Fiber	3500 (fracture)	2.6
C - Fiber	4378 (fracture)	1.7
S - Glass	4585 (fracture)	2.5
Kevlar 49	3600 (fracture)	1.45
Al ₂ O ₃ whisker	43 000 (fracture)	3.96

Table 7.3 Yield and fracture stresses of high strength (HS) materials.

Some explanation of the high strength (HS) and Ultra-High Strength (UHS) yield and fracture strengths listed above is in order. Yield stress is the stress at which a sample of material under stress begins to permanently deform; it happens gradually. Fracture stress is the stress at which a sample of the material would break. There is no gradual elongation of such a sample in tension; the material is fine until the fracture stress, then it suddenly breaks completely. This fracturing is a feature of most high strength materials, and the stronger the version of a particular alloy or fiber, the more brittle it becomes. The last six materials on the table are whiskers or fibers, which means that they can not be used directly for structure

materials, but are a volume fraction in composite materials. The composite will typically have less than half the strength to density ratio of the whisker or fiber, often much less than half. It is a great challenge to make practical structural materials with the strengths in the table above.

The single-ring lifting program was modified to run with an Al_2O_3 composite material of fracture strength 9000 M Pa and density 10 g/cm^3 . The reduced strength and greater density allow for the composite reinforcement of the whiskers. With a safety factor of 3, the allowable stress was set at 3000 M Pa. See Appendix A, program "l2.cpp" for the program.

radius, main loop (m)	Total mass (kg).	Total mass able to lift, (kg)	current (A)
50	1026	0.01149	500000
100	1053	0.04594	500000
150	1079	0.10337	500000
200	1106	0.18377	500000
250	1132	0.28714	500000
300	1159	0.41349	500000
350	1185	0.56280	500000
400	1212	0.73508	500000
450	1238	0.93034	500000
500	1265	1.14857	500000

Table 7.4 Ultra-high strength reinforcement single-loop.

current , (A)	total mass (kg)	mass able to lift (kg)
500000	1053	0.0459
1000000	1152	0.0919
1500000	1316	0.1378
2000000	1546	0.1838
2500000	1843	0.2297
3000000	2204	0.2757
3500000	2632	0.3216
4000000	3126	0.3675
4500000	3685	0.4135
5000000	4310	0.4594

Table 7.5 Single loop of radius 100 m with UHS reinforcement

Tables 7.4 and 7.5 show clearly that even with ultra-high strength materials for reinforcement, the single loop is still too massive to fly. Increasing either current or loop radius results in a greater mass increase than the gain in mass that can be lifted. Fantastically strong materials or a design that somehow contains the self-induced stress would be needed before magnetic levitation from earth surface could be considered as practical, even after superconductors evolve to carry mega-amp currents.

Chapter 8 : Problems to Address

8.1 Orbit Control Barriers

Although superconductors exist that carry large enough currents to make magnetic propulsion for orbit control appear possible with a single loop, there remain numerous practical barriers that must be overcome. These barriers can be divided into absolute barriers and lesser barriers. Absolute barriers are fundamental problems that must be solved before development of magnetic propulsion can proceed. These include superconductor limits, material fabrication and power system problems. Lesser barriers are problems that are challenging, but unlike the absolute barriers, they are almost certain to be solved. Lesser barriers include transportation and deployment of the loops, reinforcement of the tethers and control of vibrations.

8.2 Absolute Barriers-Orbit Control

By far the greatest impediments to practical use of magnetic propulsion are the limitations imposed by present-day superconductor technology. Until the critical current densities, critical temperatures and critical magnetic fields of superconductors improve, the superconducting loops will require insulation and cooling systems to function in the near-earth space environment. The mass of such cooling systems would almost certainly make rocket propulsion more practical for nearly all missions less than decades long. Closely linked with superconductor limits is the problem of fabricating suitable materials to support high-current superconductivity in space while remaining flexible and resistant to

damage. Finally, a practical means of controlling the currents in the superconducting loops will require a concentrated development effort.

8.2.1 Superconductor Materials

A superconductor is a perfect conductor. There is no internal resistance to current flow at all, which means that once started, a current will persist indefinitely if there are no further external interactions with it. Superconductors must be used because normal conductors simply take too much power to make magnetic propulsion practical. For example, most metallic conductors such as copper have conductivity in the range of $10^7 \text{ m}\Omega^{-1}$, so the power for a typical current loop can be found as follows:

$$P = \frac{I^2 l}{\sigma A} \quad (8.1)$$

I = current, 50 kA

l = length of wire, 3000 m

σ = conductivity, $10^7 \text{ m}\Omega^{-1}$

A = area of wire, $3 \text{ mm}^2 = 3 \times 10^{-6} \text{ m}^2$

$$P = \frac{(50 \times 10^3 \text{ A})^2 (3 \times 10^3 \text{ m})}{10^7 \text{ m}\Omega^{-1} (3 \times 10^{-6} \text{ m}^2)} = 2.5 \times 10^{11} \text{ W} \quad (8.2)$$

A solar panel would have to use 100% of the solar radiation passing through a 182 km^2 area to supply this power (assuming 1376 W/m^2 solar energy at earth orbit). Such a large power requirement makes magnetic propulsion feasible only with superconductors.

Prior to the 1980's, superconductivity only occurred at temperatures close to absolute zero and could only support very small currents in very weak magnetic fields. In the 1980's, a new class of ceramic superconductors were developed which allowed higher

temperatures, larger current densities and stronger magnetic field strengths before returning to the normal conducting state. The graph below shows the three limits on superconductivity: current density, magnetic field strength and temperature. If any one of these is too high, the superconducting effect is destroyed and the material “goes normal”.

Figure 8.1 Temperature, magnetic field and current density limits (Wilson, 1983).

Starting from absolute zero temperature with zero magnetic field and no current density, one can destroy the superconducting effect either by heating, increasing the magnetic field, or increasing the current beyond the surface defined on the axis above. The required current densities and resistance to external magnetic fields necessary for magnetic propulsion have been shown in laboratory conditions on small samples. Unfortunately, the superconducting properties of a material change as one tries to scale up the size of the wire. This is one of the primary reasons why superconducting power lines, superconducting electric motors, or magnetic propulsion systems have not been produced in quantity yet. To make the concept of magnetic propulsion commercially viable, a

breakthrough in the size of superconducting current that it is possible to carry in a wire is necessary.

So far, superconducting wires are composites of a superconductor and normal conducting material. The superconducting state is stabilized by using many superconducting filaments in a low resistivity conducting matrix. Stabilized means the superconductor is protected against quenching. Quenching is a transition to normal conducting state, caused by some disturbance, either magnetic, thermal or mechanical. Even if the wire is operating well below the critical temperature, magnetic field and current density, a temporary disturbance will cause a flux jump, which means a temporary disruption of the superconducting state. Once resistance exists, there is further heating, which further destroys the superconducting effect, leading to more resistance and heating until the superconducting current is destroyed or “quenched”.

One can stabilize the superconductor by fine subdivision of the superconductor volume so that the heat capacity of the material limits the heating or by damping the flux change rate by use of a highly conducting normal conductor (such as Cu or Al) to reduce heat generation and allow heat removal. A problem is that the highly conducting material between the superconducting filaments can introduce new instabilities. Also, the induced currents in the support matrix change with the wire length, which means that long lengths of wire will have different performance than short lengths. However, Barone et. al. (1991) report that over a certain critical length, superconducting properties are nearly constant as the induced currents behave as if the wire is infinite.

Ceramic High-Temperature Super Conductors (HTSC) are greatly superior to previous Nb-Ti alloys in terms of critical current density, temperature and magnetic field. But most HTSC compounds have failed in application because of low critical current density in bulk form, poor workability into multifilamentary wires and high cost. Only NbTi and Nb₃Sn alloys, which usually operate at about 4.2 K, have been used in true large-scale industrial applications. Moreover, HTSC critical parameters are not exactly defined, especially in bulk forms like wires. Critical current and critical temperature tend to depend on the application rather than being an absolute property of the material.

Initially, one may believe that a superconducting loop in space must have little problem with temperature because background radiation temperature in space is a mere 2.7 K. However, this temperature is for deep space, away from sunshine or planetary atmospheres. Since the thesis presumes that the magnetic forces are used to counter atmospheric drag, the satellite is in contact with the atmosphere of the earth. All is not lost, though, because the atmosphere is so thin at these altitudes that air temperature has no appreciable effect on a satellite. In fact, the temperature of the satellite is controlled by the radiant heat of the sun and the reflectance of the satellite surface (King, 1987). If the surface of the loop was a blackbody (a perfect absorber), it would heat up fast and superconducting would be unattainable. On the other hand, a perfect reflector would enable the loop to absorb none of the solar radiation. Perfect blackbodies and perfect reflectors do not exist, but it is obvious that the loop will need a coating or covering of the closest thing to a perfect reflector that can be developed. At the same time, this coating should allow heat to escape from the ring so that the ring may cool down after heating up

due to resistance heating, should the superconducting effect be lost after too large a current was inadvertently supplied for the conditions. The requirements for good radiation of heat and good rejection of incoming radiation are contradictory, so design of the insulation will be an interesting challenge in itself.

8.2.2 Power Control

Once power is produced by solar cells or other sources, it must be transferred to current in the superconducting loops as required. One can not simply connect positive and negative leads to the superconducting loop, since those leads would break the superconducting path, as well as potentially creating heat problems. Instead, it is necessary to use a device such as a flux pump.

A flux pump is a device where power can be introduced either mechanically or electrically with small heat input to a low temperature region. Once in the low temperature region of the superconductor, the power is converted to a low-volt, high-current supply. There are two general types of flux pump: the homopolar generator and the transformer- rectifier system (Barone et al, 1991).

Figure 8.2 Simple homopolar flux pump showing path of flux spot

The homopolar generator consists in its simplest form of a magnet which can rotate over a superconducting plate. The magnetic field generated is greater than the critical field of the plate material, but is less than the critical field of the superconducting wire. The flux spot created by the magnet quenches superconductivity locally on the plate, but there is always a superconducting path through the plate. Rotation moves the flux spot over the plate to the coil circuit. Since the coil circuit has a high critical field, the wire is not quenched and the flux is left trapped in the circuit. Work is done moving the magnet away from the coil circuit. There is also an electronic version of the homopolar

generator. In this case, a three phase current supplied to coils can make a moving flux spot. Diodes ensure that the flux over the plate is of only one polarity.

Figure 8.3 Electronic homopolar flux pump.

The voltage supplied by a flux pump is equal to the area of the flux spot swept by the magnet multiplied by the frequency of the sweep (Barone et al., 1991).

$$V = f\phi \quad (8.3)$$

V = voltage

f = frequency of sweep in Hz

ϕ = flux area swept by magnet

A problem with obtaining high voltages from a homopolar flux pump is that frequency is limited by materials to about 50 Hz. The time to energize a coil is given as:

$$T = \frac{LI}{V} \quad (8.4)$$

T = time in seconds

L = inductance of the coil

I = current in the coil

V = voltage supplied

LI is in Volt-s and, for a given frequency, one may find the area of the flux spot needed to energize a coil in a given time. For 1000 Vs, one needs 17 minutes with 1 V if the maximum flux is 1 T and the area swept is 200 cm². For larger power outputs, the area and mass of the homopolar generator become undesirably high. The transformer-rectifier system, an alternative to the homopolar generator, is pictured below.

Figure 8.4 Transformer-rectifier type of flux pump.

The transformer core must be of material with low eddy current and hysteresis losses, which are problems with superconductors which carry alternating current (AC). The rectifiers are devices known as cryotrons, which means that they can be switched from the superconducting to the normal state by applying a magnetic field. The resistance

in the normal state limits the reverse current to a small fraction of the forward current in the superconducting state, and the current is thus rectified. AC is introduced into the primary, low current circuit, which induces AC of opposite phase in the high-current windings. When the voltage is, for example, negative in one secondary circuit, that circuit's cryotron rectifier is made resistive. The other cryotron is made superconducting and forward current flows. When the current alternates, the rectifiers are switched, and current flows in only one direction in the high-current loop.

Using this arrangement, large power outputs are possible without need for large areas to pump flux over. However, using this type of flux pump will depend on cryotron development. Both zero field and zero voltage over the cryotron rectifier are needed to make it superconducting, and some form of liquid cooling ($N_{2(l)}$ or $He_{(l)}$) is usually used. This arrangement would be difficult to make robust and reliable enough for a space application.

8.3 Lesser Barriers-Orbit Control

8.3.1 Transportation and Deployment

At the present time, there is no launch capability for loops with diameters on the order of 1 km. This implies that the superconducting loops must be assembled in orbit. Flexible superconducting tape is now available. The tape can carry 1.3×10^6 A per cm^2 at liquid nitrogen temperatures and is flexible enough to be bent around a pencil with only a 9% current loss. Los Alamos National Labs are now making $5\text{ cm} \times 1\text{ cm}$ strips that hold their superconducting ability in 2 Tesla magnetic fields. Evidently, these strips are

representative of longer lengths because the strips are not single crystals. Development of thallium based superconductors is continuing. Thallium would allow superconductivity at 125 K, which is well above the temperature of space, so there is great hope that using simple reflective insulation would allow superconductivity with only space as a cryocooler (Scott, 1995). However, according to the Los Alamos scientists, the technology will need several years of development before even terrestrial commercial applications are practical.

8.3.2 Geomagnetic Inductance

Inductance is a measure of how much magnetic flux is changing with time within a conductor. Magnetic flux can change because the conductor moves through the magnetic field, or because the magnetic field changes over time. Geomagnetic inductance is inductance caused by movement through or change in the geomagnetic field. Since the superconducting loops are continuously traveling through the geomagnetic field, there is a voltage induced in them.

$$\mathcal{E} = -\frac{d\phi}{dt} \quad (8.5)$$

$$\mathcal{E} = \frac{d\phi}{dt} = \int_{\text{Surface}} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a} + \oint_{\text{Curve}} (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{s} \quad (8.6)$$

$\mathcal{E}$ = electromotive force, volts

ϕ = magnetic flux density, Weber/m²

$\mathbf{v}$ = velocity of loop

$\mathbf{B}$ = geomagnetic field strength

$d\mathbf{s}$ = element of loop curve length

$d\mathbf{a}$ = element of surface area

$\mathbf{v}$ = velocity

The geomagnetic field is changing in a quasi-predictable fashion over thousands of years and also in an unpredictable fashion on timescales as short as a minute (Tascione, 1988). Both these types of changes would affect the $\partial B/\partial t$ term above, with the short term changes, caused by solar activity, dominating. The ambient magnetic field would need to be continuously monitored in application, so that loop currents could be adjusted as the geomagnetic field strength changes. The current controller would also have to consider the induced emf caused by satellite movement through the geomagnetic field.

8.3.3 Power Required:

When current in a loop is made to change, the flux through the loop will also change. A self-induced, or back-emf, will be present when the current is changing (Wangsness, 1986). The flux change creates an emf that opposes the current change. The change in flux through the loop for a given change in current is related to the properties of the loop via the self-inductance, L , which follows directly from Faraday's Law.

$$\mathcal{E}_{self} = -L \frac{dI}{dt} \quad (8.7)$$

$\mathcal{E}_{self}$ = self-induced electromotive force (emf)

L = self-inductance for loop

I = current

A combination of the Biot-Savart Law and Maxwell's equations give self-inductance as (Wangsness, 1986):

$$L = \frac{\mu_0}{4\pi} \oint_{Curve} \frac{ds_i \cdot ds_j}{R_{ij}} \quad (8.8)$$

μ_0 = permeability of free space

R_{ij} = distance from opposing elements of current

ds = elements of curve length

L = self-inductance

For a circle, $ds=Rd\theta$ and $R_{ij}=2r$, R =radius.

$$L = \frac{\mu_o R}{4\pi 2} \int_0^{2\pi} \int_0^{2\pi} \frac{R^2 d\theta d\theta}{2R} \quad (8.9)$$

$$L = \frac{\mu_o R}{4\pi 2} \oint_{0 \rightarrow 2\pi} (2\pi) d\theta \quad (8.10)$$

$$L = \frac{\mu_o R 2\pi (2\pi)}{8\pi} \quad (8.11)$$

$$L = \frac{\mu_o R \pi}{2} \quad (8.12)$$

The self-inductance of each ring of current can be calculated as above, or by considering the definition of inductance with the assumption that the magnetic field in the plane of a current-carrying loop is constant.

$$L = \frac{N \Phi_B}{I} = \frac{NBA}{I} \quad (8.13)$$

$$BA = \left(\frac{\mu_o I}{2R} \right) (\pi R^2) = \frac{\mu_o I \pi R}{2} \quad (8.14)$$

$$L = \frac{(1)\mu_o I \pi R}{2I} = \frac{\mu_o \pi R}{2} \quad (8.15)$$

L = inductance

N = number of turns in loop, 1 for case considered

B = magnetic field inside the loop, as calculated in chapter 5.

A = area of loop

I = current in loop

It is gratifying that the results by two different methods agree. Whenever the current in the loop is made to change, this self-inductance will determine how much

voltage is required for a given change in current. When the current is constant in a superconducting loop, there is no voltage needed to maintain the current, so this self-inductance would only be considered during power-up and power-down of the loop.

The current in the superconducting loop can not be turned on permanently. If it was, the loop would not gain any net velocity over an orbit because half the attractive part of the orbit would be against the flight path and half in the same direction. For example, if the orbit carried the loop from the South magnetic pole to the North magnetic pole, the magnetic force from the South pole to the magnetic equator would slow the satellite down. There is approximately a 20° part of the arc between the unstable and stable parts of the thrust arc mentioned in chapter 4 where the current would have to reverse direction in the main loop for the triple ring or be turned on for the single ring. A realistic nominal case to find the time available for a complete current change is 500 km altitude.

$$T = \frac{2\pi R}{V} \quad (8.16)$$

$$\frac{GM_e}{R^2} = \frac{V^2}{R} \quad \text{where } GM_e = 3.986005 \times 10^{14} \frac{\text{m}^3}{\text{s}^2} \quad (8.17)$$

$$V = \sqrt{\frac{GM_e}{R}} \quad \text{where } R = R_e + 500 \text{ km} \quad (8.18)$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM_e}{R}}} = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{GM_e}} \quad (8.19)$$

T = orbital period

V = orbital velocity, circular orbit

R = radius of orbit from center of earth

R_e = radius of earth, 6378140 meters

GM_e = universal gravitational constant and mass of earth

Time for current change is 20°/360° of the orbital period.

$$\text{time to change} = \frac{2\pi(6378140 \text{ m} + 500 \times 10^3 \text{ m})^{\frac{3}{2}}}{\sqrt{3.986005 \times 10^{14} \frac{\text{m}^3}{\text{s}^2}}} \times \frac{20^\circ}{360^\circ} = 315 \text{ s} \quad (8.20)$$

Thus, for a 94 minute orbit period (500 km circular), the loop current must be changed in about 5.3 minutes if both the stable and unstable parts of the thrust arc are being used. If a single-loop option is being used, there will be more time, but any current in the loop before crossing the magnetic equator into the stable portion of the thrust arc is working against the velocity of the spacecraft. The power required to charge the loop may be deduced from the potential energy stored in the loop and the time, calculated above, in which the loop must be fully charged.

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \left(\frac{\mu_o \pi R}{2} \right) I^2 = \frac{\mu_o \pi R I^2}{4} \quad (8.21)$$

$$\text{Power} = \frac{\Delta U}{\Delta t} = \frac{\left(4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}} \right) \pi (500 \text{ m}) (50 \times 10^3 \text{ A})^2}{4(5.3 \text{ min})(60 \text{ s/min})} \quad (8.22)$$

$$\text{Power} = 3880 \text{ W}$$

Electrical power on the order of 4 kW is thus required for a 500 m radius loop.

8.3.4 Orbit type

Not all orbits can use magnetic propulsion effectively. Also, at certain points in any orbit, no force can be created in the desired direction. This is because directly over the magnetic poles, there can be no tangential force on the loops. All force would be radial, either directly up or directly down. If it so happens that a tangential impulse is

needed near the poles, it will not be possible. Certainly, there are always some tiny gradients in the field, but the currents required to use them are astronomically high. A more practical solution is simply to wait an orbit or two until the ground track of the satellite has moved with respect to the magnetic pole. Nearly all polar orbits have a ground track that shifts substantially with time, so a favorable field orientation is bound to occur sometime, but it is essential that it be considered when planning a mission.

8.3.5 Single Loop Tumbling

When the single loop has current in it, the magnetic moment aligns with the geomagnetic field with libration (oscillations). Since the geomagnetic field changes direction over the thrust arc, the loop will be turning. With nothing to prevent it, the loop will tumble for the remainder of the orbit until the current is turned on again for the beginning of the next thrust arc, where the moment of the loop will rapidly realign with the geomagnetic field. This realignment from a tumbling state will introduce new forces on the structure that have not been accounted for in the mass estimates of reinforcement. Some strategy to limit tumbling and vibration of the single loop will have to be used to stop it from vibrating or twisting apart as current is turned on and off. A possible solution is to introduce a small current during parts of the arc where no thrust is required. This small current would cause enough torque to align with the local field, but any forces created would be orders of magnitude smaller. In this way, the single loop could turn in a slow and controlled manner throughout its orbit.

8.4 Barriers - Advanced Applications

To be used for interplanetary travel, interstellar travel or self-launching flights, magnetic propulsion requires that all of the above problems for orbit control be solved, but on a larger scale. Since best loop performance is for very large loop radii, interplanetary or interstellar missions would require extremely large loop radii. All the problems for orbit control are an order of magnitude harder for interplanetary efforts, and two orders of magnitude harder for interstellar, since interstellar missions would require a means of accelerating faster than the 500 km/s solar wind. As for self-launching capability, one would need not only fantastically large superconducting currents that do not quench in their own fields, but also extremely strong reinforcement material that is also much lighter than Aluminum. Interstellar magnetic propulsion and self-launching capability require stunning breakthroughs in superconductor current limits and materials strengths before even being considered for development.

Chapter 9 : Conclusions

Magnetic propulsion appears practical to develop for a single loop used for satellite orbit control. The triple ring design appears too massive to use for orbit control with aluminum as the structural material. Interplanetary travel using magnetic propulsion should be practical at roughly the same time as orbit transfer using magnetic propulsion. Interstellar travel and self-levitation from the earth's surface using magnetic propulsion do not appear possible with the design proposed and present day materials limits, even if superconducting currents increase by orders of magnitude.

The most efficient loop design for acceleration is a large-radius loop with a relatively small current. A transfer from a 300 km circular orbit to a 70, 000 km apogee elliptical orbit would take about 34 days with a 50 kA current in a 30 km radius loop of the design proposed. Adding drag does not increase the transfer time by more than about 1 day.

For short duration missions, or missions that require large ΔV in a short time, rockets would be more effective than magnetic sails. The flexibility of using a rocket in any orbit and in any direction is a great advantage. Magnetic propulsion, if it can be developed in the single-loop form proposed, appears better than using rockets for missions that last many years, or missions that require continual changes in velocity while in polar orbit. For example, a low-to-high polar orbit shuttle in preparation for a Mars mission may well use magnetic propulsion, since the loops could continue making trips until the solar cells degrade. Reusability means that magnetic propulsion has a future.

List of Works Cited:

- Ackerman, J., (1989). *To Catch a Flying Star*, Univelt, San Diego, California.
- Alfven, H., (1972). "Spacecraft Propulsion: New methods", *Science*, Vol 176, p. 167-168.
- Arkadiev, V., (1945). "Hovering of a Magnet of a Superconductor", *Journal of Physics* (Moscow), Vol. 9, No. 2, p. 148.
- Barone, A., Morini, A., and Frunzio, L. (1991). "School on European Training on Technologies and Industrial Applications of Superconductivity", 1st, Sept., 1991, Naples, Italy.
- Bleaney, (1976). *Electricity and Magnetism*, 3rd edition, Oxford University Press.
- Bond, A., and Martin, A., (1986). "Project Daedalus Reviewed", *Journal of the British Interplanetary Society*, Vol. 39, p. 385-390.
- Bourke, R., ((1964). Oct.) "A Theoretical and Experimental Study of a Superconducting Magnetically-Supported Spinning body", NASA contractor report CR-108, Stanford University, Stanford, California.
- Bowes, W. H., Russell, L.T., and Suter, G.T., (1984). *"Mechanics of Engineering Materials"*, John Wiley & Sons, New York.
- Burns, G., (1992). *"High Temperature Superconductivity: An introduction"*, Academic Press, New York.
- Corliss, W.R., (1960). *Propulsion Systems for Space Flight*, McGraw-Hill, Toronto, p. 12-158.
- Cowan, E.W., (1968). *Basic Electromagnetism*, Academic Press, New York, p. 177-179.
- Culver, W., and Davis, M., (1957, Jan.). "An Application of Superconductivity to Inertial Navigation", Rand Corporation Report RM-1852.
- Drell, S.D., Foley, H. M. and Ruderman, M.A., (1965). "Alfven Drag", *Physical Review Letters*, Vol. 14, p. 171-175.
- Engelberger, J.F., (1964). "Space propulsion by magnetic field interaction", *Journal of Spacecraft and Rockets*, Vol. 1, p. 347-349.
- File, J., Martin, G., Mills, R., Upham J., (1968). "Stabilized, Levitated Superconducting Rings", *Journal of Applied Physics*, Vol 39, No. 2, p. 2623.

Friedrich, K, Hornbogen, E. and Bochum, A., (1984). "Ultra-High Strength Materials", Institut für Werkstoffe Ruhr-Universität Bochum, Reihe 5, Nr. 82.

Furth, H.P. (1990). "Magnetic confinement fusion", *Science*, 28 Sept. p. 178.

Gere and Timoshenko (1993). "Mechanics of Materials", 3rd SI edition, Chapman & Hall, London.

Harding, J., (1965). "Force on a Superconducting Sphere in a Magnetic Field: the General Case", NASA, Jet Propulsion Laboratory Technical Release No. 32-806.

Haymes, R.C., (1971). "*Introduction to Space Science*", 1st ed., Wiley, New York, p. 222-284.

King-Hele, D.G. (1987). "*Satellite Orbits in an Atmosphere: Theory and Applications*", Blackie and Son, Glasgow, Britain.

Kom, C.H., and Brunet, Y., (1995). "Magnetic field modelisation of an annular MHD thruster", *Journal de Physique III*, Vol. 5, No. 1, p. 91-102.

Lemke, E.H., (1982). "Magnetic acceleration of interstellar probes", *Journal of the British Interplanetary Society*, Vol. 35, p. 498-503.

Mauldin, J.H., (1992). "*Prospects for interstellar travel*", Science and Technology Series Vol. 80, Univelt, San Diego, CA.

Mead, F. B.(ed.) (1971, June). "Project Outgrowth: Advanced Propulsion Concepts", Air Force Rocket Propulsion Lab Report AFRPL-TR-72-31, Edwards AFB, California.

Merrill, R., McElhinny, M., (1983). "*The Earth's Magnetic Field*", International Geophysics Series Vol. 32, Academic Press, New York, p 248-345.

Olsen, E., (1966). "*Applied Magnetism: A study in Quantities*", Philips Technical Library, Springer-Verlag, New York.

Peck, S.D. and Michels, P.H. (1991). "Test results from the 200 kA SMES/ETM conductor", *IEEE Transactions on Magnetics*, Vol. 27, p.1724.

Priest, E.R., (1985). "*Solar System Magnetic Fields*", 1st ed., Reidel Publishing, Hingham, p. 226-285.

Scott, W.B. (1995). "Flexible Superconductors Show Commercial Promise", *Aviation Week & Space Technology*, Oct. 2., p. 40-42.

Tascione, T.F., (1988). *Introduction to the Space Environment*, 1st ed., Orbit Book Company, Malabar, Florida, pp. 10-386.

Vulpetti, G. , (1990). “*Achievement of Rectilinear Trajectories in the Solar System by Non-Rocket Propulsion*”, 41st Congress of the International Astronautical Federation, Dresden, GDR, IAF-90-303, Oct 6-12.

Waite, J., Burch, J., and Moore, R., (1989). “*Solar System Plasma Physics: Geophysical Monograph 54*”, 1st ed., American Geophysical Union, Washington, p. 11-159.

Wangsness, R. K., (1986). *Electromagnetic Fields*, 2nd ed., Wiley, New York, p. 280-350.

Wertz, J.R. (Ed.), (1978). *Spacecraft Attitude and Determination Control*, Kluwer Academic Publishers, Dordrecht, The Netherlands, p. 58-235.

Wertz, J.R. and Larson, W.J. (Eds.), (1991). *Space Mission Analysis and Design*, Kluwer Academic Publishers, Boston, USA, p. 300-368.

Wilson, M. N., (1983). “Superconducting Magnets”, *Oxford Science Publications Monographs on Cryogenics*, Clarendon Press, Oxford.

Zaslavsky, A. (1980). “On the limitations of the shearing stress formula”, *International Journal of Mechanical Engineering Education*, Vol. 8, No. 1, p.13-18.

Zubrin, R., (1991). “The use of magnetic sails to escape from low earth orbit”, *AIAA 27th Joint Propulsion Conference*, June 24-26, Sacramento, California.

Zubrin, R.M., and Andrews, D.G., (1991). “Magnetic Sails and Interplanetary Travel”, *Journal of Spacecraft and Rockets*, Vol. 28, No. 2, p. 197-203.

Appendix A

A.1 Structure

Below are found C++ programs used to find mass, acceleration and orbit transfer times for both single and triple loops. After the programs themselves are various subroutines that are used as “include files” in all programs.

A.2 Single Loop Acceleration

```
/* single loop acceleration program */
/* calculates acceleration of a single superconducting loop for a range of
radii */

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp>
#include <c:\wg\density.cpp>
#include <c:\wg\mag_for.cpp>
#include <c:\wg\single.cpp>

/*****
double loop_area(double radius1)
/* returns area for drag of superconducting loop, given radius */
{
return(Pi*2.*radius1*diameter_wire);
}
*****/
double tether_area(double radius1)
/* returns area of tethers from loop to main body*/
{
return(4.*radius1*diameter_tether);
}
*****/
main(void)
{
double radius1; /* radius of superconducting loop */
```

```

double mass_total; /* total mass of spacecraft */
double area_total; /* total mass of spacecraft meeting drag */
double accel; /* acceleration of spacecraft */
double current; /* current in loop, amperes */
double altitude; /* m above earth surface */
double velocity; /* velocity through atm for drag calc. */
double true_anom; /* true anomaly, assume perigee magnetic north */
double position; /* distance from center of earth */
double body_mass; /* mass spacecraft body */

short n;

FILE *fp;

fp=fopen("a1.out","w");

current = 50E3;
radius1=0.; /* start radius of main loop */
altitude = 300E3; /* altitude where thrust takes place */
position = altitude + radius_earth;
true_anom = 0. ; /* at 30 deg from mag north=perigee */
body_mass = 1000.;

fprintf(fp,"Single loop with drag, altitude %5.1lf km, current %5.1lf\n",
        altitude/1000., current);

fprintf(fp,"radius  accel    mass  area  mag_force\n");

for (n=1;n<52;n++)
{
radius1=radius1+600.;

mass_total=loop_mass(radius1,current)+body_mass+tether_mass(radius1);
area_total = loop_area(radius1)+tether_area(radius1)+body_area;

accel = (mag_force(radius1,current,position,true_anom)-
        .5*atm_density(altitude)*Cd*pow(velocity,2.)*(area_total))
        /(mass_total);

fprintf(fp,"%5.0lf,  %5.4lf,  %6.0lf,  %6.0lf,  ",
        radius1,accel,mass_total,area_total);

fprintf(fp,"%7.3lf\n",mag_force(radius1,current,
position,true_anom));
};

```



```

    return(0);
}

```

A.3 Triple Ring Acceleration

```

/* Program a3.cpp. Acceleratin of 3 ring arrangement
   finds acceleration at various loop sizes for triple ring.
   estimates drag and mass increase with size
*/

```

```

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp> /*common constants between programs */
#include <c:\wg\density.cpp> /*file of atmospheric density */
#include <c:\wg\orb_calc.cpp> /* basic orbit calculations */
#include <c:\wg\mag_for.cpp> /*magnetic force routine */
#include <c:\wg\triple.cpp> /* mass and area calculations triple loop */
/*****
main(void)

{
double thickness,increment,radius1,radius2;
short n,m,loop; /* loop is a flag for loop 1 or 2 */
double accel; /* acceleration of the loop */
double best_accel; /* maximum acceleration at given radius */
double area_total; /* total area meeting atmospheric drag */
double mass_total; /* total mass of spacecraft */
double best_thick; /* thickness selected for best mass */
double best_mass; /* mass at best acceleration */
double altitude; /* m above earth surface */
double position; /*distance from earth center to satellite position */
double true_anom; /* true anomaly of satellite position */
double current; /* current in the loops */
double semi_major; /*semi-major axis of orbit */
double mass_body; /* mass of spacecraft body */

FILE *fp;

fp=fopen("a3.out","w");

increment = 0.001; /* increment thickness by meters */

```

```

radius1 = 0.0; /* start radius 1, m */
altitude = 800E3; /* start at 800 km */
position = radius_earth+altitude;
true_anom = 0.; /*take measurement at perigee */
semi_major = position; /*circular orbit */
mass_body = 2000. ; /* 2000 kg of payload, container */
current = 50E3;

fprintf(fp,"Triple ring with drag \n");
fprintf(fp,"radius1 best_accel best_mass thickness \n");

for (m=1;m<30;m++) /* different tries for radius 1 */
{
radius1=radius1+200.;
thickness = 0.001; /* reset thickness for loop wall */
best_accel = 0.0; /* start accelertion */

for(n=1;n<=10;n++) /* different tries for thickness */
{
area_total= body_area
+area_loops(radius1,0.2)
+area_tethers(radius1,current);

mass_total= mass_reinf(radius1,thickness,current)+mass_super(radius1)+
mass_system(radius1)+mass_body;

accel = (mag_force(radius1,current,position,true_anom)-
.5*atm_density(altitude)*Cd*
pow(velocity(semi_major,position),2.)*(area_total))
/(mass_total);

if (n == 1) /* start best at first value */
{
best_accel=accel;
best_thick = thickness;
best_mass = mass_total;
};

if (best_accel < accel)
{
best_accel=accel; /* finding best acceleration*/
best_thick = thickness;
best_mass = mass_total;
};

```

```

thickness = thickness + increment;

}; /* end thickness iteration */

fprintf(fp," %6.0lf, %9.8lf, ", radius1,best_accel);

fprintf(fp,"%10.0lf, %5.4lf\n",
best_mass, best_thick);
};
fclose(fp);
return 0;
}

```

A.4 Triple Ring Mass

```

/*
  Program m3.cpp
  mass of 3 ring arrangement.
  gives mass of primary and secondary loops for various thicknesses
  uses a standard loop size of 500 m main and 210 m secondary
  uses Aluminum alloy reinforcement
*/
#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <io.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <iostream.h>
#include <string.h>
#include <c:\wg\const.cpp> // basic constants used in programs
#include <c:\wg\triple.cpp> // triple loop mass calculations
/*****
main(void)

{
double thickness; /* wall of loop reinforcement */
double mass_body; /* mass of spacecraft body in kg */
double increment; /*amount thickness increased each iteration */
double radius1; /*radius of superconducting loop */
double current; /* max current in all loops */
double mass_reinforce; // mass of reinforcement
double radius_tube; // radius of loop circular cross section

short n,loop; /* loop is a flag for loop 1 or 2 */

```

```

FILE *fp;

fp=fopen("m3.out","w");

thickness = 0.001; /* start thickness for loop wall */
mass_body = 2000.; /* kg */
increment = 0.001; /* increment thickness by meters */
current = 50E3; /*current in loops */
radius1 = 500.; /* 500 meter main loop */

fprintf(fp,"radius1 %5.1lf \n",radius1);
fprintf(fp,"thickess tube_radius mass_reinf    mass_total \n");

for(n=1;n<20;n++) /* different tries for thickness */
{
    mass_reinforce = mass_reinf(radius1,thickness,current);

    radius_tube = mass_reinforce/
        (Pi*2.*thickness*Pi*radius1*2.* density_Al*3.);

    fprintf(fp,"%5.3lf,  %5.3lf,  %5.1lf,  %5.1lf,\n",
        thickness,radius_tube,mass_reinforce,
        mass_reinforce+mass_super(radius1)+
        mass_system(radius1)+mass_body+mass_tether(radius1,current));

    thickness = thickness + increment;
}

fclose(fp);
return 0;
}

```

A.6 Single Loop Orbit Program 1

```

/* single loop orbit transfer program, no drag included */
/* acceleration taken from accel1.cpp */

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp> /*constants definition file */
#include <c:\wg\mag_for.cpp> /* magnetic force file */

```

```

#include <c:\wg\orb_calc.cpp> /* basic orbit calculations file */
/*****
main(void)
{
    double radius1; /* radius of superconducting loop */
    double final_apogee; /* apogee new orbit */
    double perigee; /* perigee of orbit in meters */
    double apogee; /* apogee of orbit in meters */
    double current; /* current in loop, amperes */
    double theta1, theta2; /* start and end angles of thrust arc */
    double segments; /* how many segments thrust arc divided into */
    double position; /* vector from earth center to position */
    double true_anom; /* true anomaly, radians */
    double delta_true_anom; /* delta true anomaly, segment of thrust arc */
    double del_V; /* delta-velocity of satellite */
    double semi_major; /* semi-major axis of orbit */
    double accel_time; /* time for acceleration */
    double accel; /* acceleration of spacecraft */
    double mass_total; /* total mass of spacecraft */
    double time; /* time to complete transfer */

    short orbit; /* number of orbit, multiple orbits for transfer */
    short n; /* incremental counter */

    FILE *fp;

    fp=fopen("o1.out","w");

    current = 50E3;
    radius1=31.6E3; /* radius of main loop */
    theta1 = 20.*Pi/180.; /* start thrust arc */
    theta2 = 80.*Pi/180.; /* end thrust arc */
    mass_total = 20E3; /* total mass, kg, of satellite */
    del_V = 0.0; /* start delta V */
    orbit = 0;
    perigee= 300E3+radius_earth; /* circular starting orbit */
    apogee = perigee; /* start simulation */
    final_apogee = 71090E3+radius_earth; /* ending apogee elliptical orbit */
    segments=25.; /*divide thrust arc into segments*/
    position = perigee+radius_earth; /* start orbit */
    delta_true_anom = (theta2-theta1)/segments;
    semi_major = (perigee+apogee)/2.;
    time =0.;

```

```

fprintf(fp,"Perigee %4.0lf km loop radius %6.0lf m\n\n",
(perigee-radius_earth)/1000., radius1);
fprintf (fp,"orbit days  del_V (m/s)  apogee(km)\n");

while (apogee<final_apogee && orbit<1000)
/* if orbit gets too large, something is wrong */
{
true_anom = -(30.*Pi/180.); /* start thrust @ 80, perigee theta=50 */

for (n=1; n<segments; n++)
{
position = semi_major*(1.-pow(eccent(perigee,semi_major),2.))/
(1.+eccent(perigee,semi_major) *cos(true_anom));

if (n==1) /* same accel time through 1 thrust arc */
accel_time=(mean_anom(eccent(perigee,semi_major),true_anom)
-mean_anom(eccent(perigee,semi_major),true_anom+delta_true_anom))
/mean_motion(semi_major);
accel = (mag_force(radius1,current,position,true_anom))
/mass_total;
del_V = del_V+accel*accel_time;

true_anom =true_anom+delta_true_anom;

}

orbit = orbit + 1;
time = time+period(semi_major);

semi_major= -GMe/(pow((velocity(semi_major,position)+del_V)
,2.))-2.*GMe/position);

apogee= 2.*semi_major-perigee;

fprintf (fp," %d,    %5.2lf,    %4.2lf,    %8.0lf,\n",
orbit,time,del_V,(apogee-radius_earth)/1000.);

del_V = 0. ; /* reset to zero for next orbit */

}; /* end while */

return(0);
}

```

A.7 Single Loop Orbit Program 2

```

/* single loop orbit transfer program, same as orbl except with drag */
/* acceleration taken from accel1.cpp */

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp> /*constants definition file */
#include <c:\wg\density.cpp> /* atmospheric density file */
#include <c:\wg\drag.cpp> /* drag calculations */
#include <c:\wg\mag_for.cpp> /* magnetic force calculations */
#include <c:\wg\orb_calc.cpp> /* basic orbit calculations */

/*****
main(void)
{
    double radius1; /* radius of superconducting loop */
    double final_apogee; /* apogee new orbit */
    double perigee; /* perigee of orbit in meters */
    double apogee; /* apogee of orbit in meters */
    double current; /* current in loop, amperes */
    double theta1, theta2; /* start and end angles of thrust arc */
    double thrust_seg; /* how many segments thrust arc divided into */
    double drag_seg; /* how many segments drag arc divided into */
    double position; /*vector from earth center to position */
    double true_anom; /* true anomaly, radians */
    double delta_true_anom; /* delta true anomaly, segment of thrust arc */
    double del_V; /* delta-velocity of satellite */
    double semi_major; /* semi-major axis of orbit */
    double accel_time; /* time for acceleration */
    double accel; /* acceleration of spacecraft */
    double area_total; /* area for drag on spacecraft */
    double mass_total; /* total mass of spacecraft */
    double time; /* time to complete transfer */

    short orbit; /* number of orbit, multiple orbits for transfer */
    short n; /* incremental counter */

    FILE *fp;

    fp=fopen("o2.out","w");

```

```

current = 50E3;
radius1=31.6E3; /* radius of main loop */
theta1 = 20.*Pi/180.; /* start thrust arc */
theta2 = 80.*Pi/180.; /* end thrust arc */
area_total = 885.; /*total area, m2, of satellite */
mass_total = 20E3; /* total mass, kg, of satellite */
del_V = 0.0; /* start delta V */
orbit = 0;
perigee= 300E3+radius_earth; /* circular starting orbit */
apogee = perigee; /* start simulation */
final_apogee = 71090E3+radius_earth; /* ending apogee elliptical orbit */
thrust_seg=25.; /*divide thrust arc into segments*/
position = perigee+radius_earth; /* start orbit */
delta_true_anom = (theta2-theta1)/thrust_seg;
semi_major = (perigee+apogee)/2.;
time =0.;

fprintf(fp,"Perigee %4.0lf km loop radius %6.0lf m\n\n",
(perigee-radius_earth)/1000., radius1);
fprintf(fp,"orbit days del_V(m/s) apogee(km)\n");

while (apogee<final_apogee && orbit<1000)
/* if orbit gets too large, something is wrong */
{
true_anom = -(30.*Pi/180.); /* start thrust @ 80, perigee theta=50 */

for (n=1; n<thrust_seg; n++)
{
position = semi_major*(1.-pow(eccent(perigee,semi_major),2.))/
(1.+eccent(perigee,semi_major) *cos(true_anom));

if (n==1) /* same accel time through 1 thrust arc */
accel_time=(mean_anom(eccent(perigee,semi_major),true_anom)
-mean_anom(eccent(perigee,semi_major),true_anom+delta_true_anom))
/mean_motion(semi_major);

accel = (mag_force(radius1,current,position,true_anom)-
drag_force(position, area_total, velocity(semi_major,position)))
/mass_total;

del_V = del_V+accel*accel_time;

true_anom =true_anom+delta_true_anom;

```



```

}
/* now the drag part of the orbit */
drag_seg = (2.*Pi-(theta2-theta1))/delta_true_anom;

for (n=1; n<drag_seg; n++)
{
position = semi_major*(1.-pow(eccent(perigee,semi_major),2.))/
(1.+eccent(perigee,semi_major)*cos(true_anom));

/* same accel time through 1 thrust arc */
accel_time=(mean_anom(eccent(perigee,semi_major),true_anom)
-mean_anom(eccent(perigee,semi_major),true_anom+delta_true_anom))
/mean_motion(semi_major);
accel =drag_force(position, area_total, velocity(semi_major,position))
/mass_total;

del_V = del_V+accel*accel_time;

true_anom =true_anom+delta_true_anom;

}

orbit = orbit + 1;
time = time+period(semi_major);

semi_major=-GMe/(pow((velocity(semi_major,position)+del_V)
.2.))-2.*GMe/position);
apogee= 2.*semi_major-perigee;
fprintf(fp," %d,  %5.2lf,  %4.2lf,  %8.0lf,\n",
orbit,time,del_V,(apogee-radius_earth)/1000.);

del_V = 0. ; /* reset to zero for next orbit */

}; /* end while */

return(0);
}

```

A.8 Triple Loop Orbit Program

```

/* orb3.cpp
orbit calculations for a triple-ring arrangement
includes drag
set the condition number from 1 to 4 for different current and

```

```

    main loop radius conditions
*/

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp> /*common constants between programs */
#include <c:\wg\density.cpp> /*file of atmospheric density */
#include <c:\wg\orb_calc.cpp> /* basic orbit calculations */
#include <c:\wg\mag_for.cpp> /*magnetic force routine */
#include <c:\wg\drag.cpp> /*drag force calculations */
#include <c:\wg\triple.cpp> /* mass and area calculations triple loop */
/*****/
main(void)

{
    double radius1; /* radius of superconducting loop */
    double final_apogee; /* apogee new orbit */
    double perigee; /* perigee of orbit in meters */
    double apogee; /* apogee of orbit in meters */
    double current; /* current in loop, amperes */
    double theta1, theta2; /* start and end angles of thrust arc */
    double thrust_seg; /* how many segments thrust arc divided into */
    double drag_seg; /* how many segments drag arc divided into */
    double position; /*vector from earth center to position */
    double true_anom; /* true anomaly, radians */
    double delta_true_anom; /* delta true anomaly, segment of thrust arc */
    double del_V; /* delta-velocity of satellite */
    double semi_major; /* semi-major axis of orbit */
    double accel_time; /* time for acceleration */
    double accel; /* accleration of spacecraft */
    double area_total; /* area for drag on spacecraft */
    double mass_total; /* total mass of spacecraft */
    double time; /* time to complete transfer */
    double thickness; /*thickness of reinforcement wall */
    double mass_body; /* mass of spacecraft body */
    double termination_orbit; /*orbit to stop loop at */

    short orbit; /* number of orbit, multiple orbits for transfer */
    short n,m,loop; /* loop is a flag for loop 1 or 2 */
    short condition; /* condition of loop currents and sizes */

```

```

FILE *fp;
mass_body = 2000.; /* 2000 kg of payload, container */
thickness = .001; /* practical lower limit reinforcement thickness */
theta1 = 20.*Pi/180.; /* start thrust arc */
theta2 = 80.*Pi/180.; /* end thrust arc */
del_V = 0.0; /* start delta V */
orbit = 0;
perigee = 300E3+radius_earth; /* circular starting orbit */
apogee = perigee; /* start simulation */
final_apogee = 71090E3+radius_earth; /* ending apogee elliptical orbit */
thrust_seg = 25.; /* divide thrust arc into segments */
position = perigee+radius_earth; /* start orbit */
delta_true_anom = (theta2-theta1)/thrust_seg;
semi_major = (perigee+apogee)/2.;
termination_orbit = 1000; /* terminates after this orbit if no success */
time = 0.;

condition = 1; /* set the condition of loop radius and current */

if (condition==1)
{
current = 50E3;
radius1 = 1000.; /* radius of main loop */
fp = fopen("o3t1.out", "w");
}
else if (condition==2)
{
current = 500E3;
radius1 = 1000.; /* radius of main loop */
fp = fopen("o3t2.out", "w");
};
if (condition==3)
{
current = 50E3;
radius1 = 31000.; /* radius of main loop */
fp = fopen("o3t3.out", "w");
};
if (condition==4)
{
current = 500E3;
radius1 = 31000.; /* radius of main loop */
fp = fopen("o3t4.out", "w");
};

area_total = body_area

```

```

+area_loops(radius1,thickness,current)
+area_tethers(radius1,current);

mass_total= mass_reinf(radius1,thickness,current)+mass_super(radius1)+
    mass_system(radius1)+mass_body;

fprintf(fp,"Perigee %4.0lf km radius1 %5.0lf mass %8.0lf area %6.0lf\n",
    (perigee-radius_earth)/1000., radius1, mass_total,area_total);

fprintf(fp," current %6.0lf\n\n", current);

fprintf (fp,"orbit  days  del_V(m/s)  apogee(km)\n");

while (apogee<final_apogee && orbit<termination_orbit)

{
true_anom = -(30.*Pi/180.); /* start thrust @ 80, perigee theta=50 */

for (n=1; n<thrust_seg; n++)
{
position = semi_major*(1.-pow(eccent(perigee,semi_major),2.))/
    (1.+eccent(perigee,semi_major) *cos(true_anom));

if (n==1) /* same accel time through 1 thrust arc */
accel_time=(mean_anom(eccent(perigee,semi_major),true_anom)
    -mean_anom(eccent(perigee,semi_major),true_anom+delta_true_anom))
    /mean_motion(semi_major);

accel = (mag_force(radius1,current,position,true_anom)-
    .5*atm_density(position-radius_earth)*Cd*
    pow(velocity(semi_major,position),2.)*(area_total))
    /(mass_total);

del_V = del_V+accel*accel_time;

true_anom =true_anom+delta_true_anom;
}

/* now the drag part of the orbit */
drag_seg = (2.*Pi-(theta2-theta1))/delta_true_anom;

for (n=1; n<drag_seg; n++)
{
position = semi_major*(1.-pow(eccent(perigee,semi_major),2.))/
    (1.+eccent(perigee,semi_major) *cos(true_anom));

```

```

/* same accel time through 1 thrust arc */
accel_time=(mean_anom(eccent(perigee,semi_major),true_anom)
-mean_anom(eccent(perigee,semi_major),true_anom+delta_true_anom))
/mean_motion(semi_major);

accel =drag_force(position, area_total, velocity(semi_major,position))
/mass_total;

del_V = del_V+accel*accel_time;

true_anom =true_anom+delta_true_anom;
}

orbit = orbit + 1;
time = time+period(semi_major);

semi_major=-GMe/(pow((velocity(semi_major,position)+del_V)
,2.))-2.*GMe/position);

apogee= 2.*semi_major-perigee;

if (orbit%5 ==0 || orbit < 25)
{
fprintf(fp," %d, %5.2lf, %4.2lf, %8.0lf,\n",
orbit,time,del_V,(apogee-radius_earth)/1000.);
}

if (apogee < radius_earth+150E3)
orbit=termination_orbit; /* orbit decayed into earth */

del_V = 0. ; /* reset to zero for next orbit */

}; /* end while */

return(0);
}

```

A.9 Single Loop Levitation

A.9.1 AI Reinforcement

/*

11.cpp: single loop self-levitation program

calculates mass and lifting ability of single superconducting loop
for a range of radii. Shows mass change with current for 100 m loop
assumes Aluminum alloy for reinforcement material.

2 files output: ll.out and m1.out

```

*/
#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp>
#include <c:\wg\rad_for.cpp>
#include <c:\wg\single.cpp>
/*****/
double system_mass (double radius1)
/* finds mass of attitude control system for single loop stabilization
by gyroscopic shell. Assumes a shell of the size of the loop made
from Aluminum */
{
return (Pi*pow(radius1,2)*.001*density_Al);
}
/*****/
main(void)
{
double radius1; /* radius of superconducting loop */
double mass_total; /* total mass of spacecraft */
double current; /* current in loop, amperes */
double true_anom; /* true anomaly, assume perigee magnetic north */
double position; /* distance from center of earth */
double mass_lift; /* mass that can be lifted by magnetic force */
double altitude; /* altitude above earth surface, m */
double theta; /* angle from magnetic north, radians */
double body_mass; /* mass spacecraft body */
double best_mass_lift; /* point with least difference between mass_lift */
double best_mass_total; /* and mass_total has "best" values */
double best_current; /* current least difference lift, total mass */
double min_diff; /* minimum difference between mass_lift and mass_total */

FILE *fp, *fp2;

fp=fopen("ll.out","w");
fp2 = fopen("m1.out","w");

```

```

altitude = 0.; /*altitude where thrust takes place */
position = altitude + radius_earth;
theta = 0.; /* at a magnetic pole */
body_mass = 1000.;

fprintf(fp,"Single loop self-levitation current \n");

fprintf(fp,"radius   mass_total   mass_lift   current \n");

fprintf(fp2," Single loop radius 100 m \n");
fprintf(fp2," current   mass_total   mass_lift\n");

for (radius1=0.;radius1< 500.;)
{
radius1=radius1+50.;
current = 0.;
mass_lift=0.; /* start the loop making mass_lift less than mass_total*/
mass_total=1.;
min_diff = 1E9; /* start at high value so 1st value below replaces */

while (mass_lift < mass_total)
{
current = current +5E5;

mass_lift =fabsl(mag_for_rad(radius1,current,theta,position)/9.81);

mass_total=loop_mass(radius1,current,stress_allow_A1,density_A1)
+body_mass+tether_mass(radius1,density_A1);
+ system_mass(radius1);

if (min_diff > (mass_total-mass_lift))
{
min_diff = mass_total-mass_lift;
best_mass_lift = mass_lift;
best_mass_total = mass_total;
best_current = current;
};

if (current>1E8) break;

if (radius1 == 100. )
fprintf(fp2," %7.0lf %7.0lf %7.4lf\n", current, mass_total,mass_lift);
}; /* end while loop */

```

```

fprintf(fp, "%5.0lf, %6.0lf, %8.5lf, %9.0lf, \n",
radius1, best_mass_total, best_mass_lift, best_current);

}; /* end for loop */
fclose (fp);
return(0);
}

```

A.9.2 UHS Reinforcement

/* l2.cpp: single loop self-levitation program

The same as l1.cpp, except for Ultra-high strength composite reinforcement instead of Aluminum

calculates mass and lifting ability of single superconducting loop for a range of radii. Shows mass change with current for 100 m loop
2 files output: l2.out and m2.out

```

*/
#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp>
#include <c:\wg\rad_for.cpp>
#include <c:\wg\single.cpp>
/*****
double system_mass (double radius1)
/* finds mass of attitude control system for single loop stabilization
by gyroscopic shell. Assumes a shell of the size of the loop made
from Aluminum */
{
return (Pi*pow(radius1,2)*.001*density_fiber);
}
*****/
main(void)
{
double radius1; /* radius of superconducting loop */
double mass_total; /* total mass of spacecraft */
double current; /* current in loop, amperes */
double true_anom; /* true anomaly, assume perigee magnetic north */
double position; /* distance from center of earth */
double mass_lift; /* mass that can be lifted by magnetic force */

```



```

double altitude; /* altitude above earth surface, m */
double theta; /*angle from magnetic north, radians */
double body_mass; /*mass spacecraft body */
double best_mass_lift; /*point with least difference between mass_lift*/
double best_mass_total; /*and mass_total has "best" values */
double best_current; /* current least difference lift, total mass */
double min_diff; /* minimum difference between mass_lift and mass_total*/

```

```

FILE *fp, *fp2;

```

```

fp=fopen("l2.out","w");
fp2 = fopen("m2.out","w");

```

```

altitude = 0.; /*altitude where thrust takes place */
position = altitude + radius_earth;
theta = 0.; /* at a magnetic pole */
body_mass = 1000.;

```

```

fprintf(fp,"Single loop self-levitation current \n");

```

```

fprintf(fp,"radius    mass_total  mass_lift  current \n");

```

```

fprintf(fp2," Single loop radius 100 m \n");
fprintf(fp2," current    mass_total  mass_lift\n");

```

```

for (radius1=0.;radius1< 500.;)
{
radius1=radius1+50.;
current = 0.;
mass_lift=0.; /* start the loop making mass_lift less than mass_total*/
mass_total=1.;
min_diff = 1E9; /* start at high value so 1st value below replaces */

```

```

while (mass_lift < mass_total)
{
current = current +5E5;

```

```

mass_lift=fabsl(mag_for_rad(radius1,current,theta,position)/9.81);

```

```

mass_total=loop_mass(radius1,current,stress_allow_fiber,density_fiber)+
body_mass+tether_mass(radius1,density_fiber);
+ system_mass(radius1);

```

```

if (min_diff > (mass_total-mass_lift))
{

```

```

    min_diff = mass_total-mass_lift;
    best_mass_lift = mass_lift;
    best_mass_total = mass_total;
    best_current = current;
};

if (current>1E8) break;

if (radius1 == 100. )
    fprintf(fp2," %7.0lf %7.0lf %7.4lf\n", current, mass_total,mass_lift);
}; /* end while loop */

fprintf(fp, "%5.0lf, %6.0lf, %8.5lf, %9.0lf,\n",
radius1,best_mass_total,best_mass_lift,best_current);

}; /* end for loop */
fclose (fp);
return(0);
}

```

A.10 Triple Loop Levitation

```

/* lift.cpp, triple ring.  Fix radius.  Fix altitude.
   calculates mass various currents can lift against gravity
   calculates mass of spacecraft for various currents (reinforce changes)
   objective: current for self-levitation, mass @ that current
*/
#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <fcntl.h>
#include <sys\stat.h>
#include <string.h>
#include <c:\wg\const.cpp> /*common constants between programs */
#include <c:\wg\orb_calc.cpp> /* basic orbit calculations */
#include <c:\wg\rad_for.cpp> /* radial magnetic force routine */
#include <c:\wg\triple.cpp> /* mass and area calculations triple loop */
/*****
main(void)

{
double thickness,radius1; /*thickness of loop radius1 */
double mass_total; /* total mass of spacecraft */
double best_thick; /* thickness selected for best mass */

```

```

double best_mass; /* mass at best acceleration */
double altitude; /* m above earth surface */
double position; /* distance from earth center to satellite position */
double current; /* current in the loops */
double mass_body; /* mass of spacecraft body */
double mass_lift; /* mass that can be lifted */
double theta; /* angle from magnetic north, radians */
double smallest_diff; /* smallest difference between lift and mass */
double best_current; /* current at smallest difference */
double best_lift; /* mass that can be lifted at best current */
double increment; /* amount to increment thickness by */

short n; /* counter for loop */

FILE *fp;

fp=fopen("l3.out","w");

increment = 0.001; /* increment thickness by meters */
mass_body = 1000.; /* kg of payload, container */
altitude = 0.; /* start km */
theta = 0.; /* start angle from magnetic pole */
position = radius_earth+altitude;
best_mass = mass_body;
mass_lift = 0.;

fprintf(fp,"Triple ring Lift \n");
fprintf(fp,
"radius1 best_lift best_mass best_current mass_lift mass \n" );

for (radius1=0.;radius1<200.;)
{
current = 4E3; /* start current */
radius1=radius1+10.;
smallest_diff = 777777.;

while (mass_lift<best_mass)
{
thickness = 0.001; /* reset thickness for loop wall */
current = current+500.;

for(n=1;n<=10;n++) /* different tries for thickness */
{
mass_total= mass_reinf(radius1,thickness,current)+mass_super(radius1)+
mass_system(radius1)+mass_body;

```

```

thickness = thickness + increment;

if (n==1)
    best_mass = mass_total;

if (best_mass > mass_total)
    best_mass=mass_total;

}; /* end thickness iteration */

mass_lift=-mag_for_rad(radius1,current,theta,position)/9.81;

if ((mass_total-mass_lift)< smallest_diff)
{
    best_lift=mass_lift;
    smallest_diff = mass_total-mass_lift;
    best_current = current;
};

if (current>500000.) break;
}; /* end of current while loop */

fprintf(fp,"%7.0lf, %7.4lf, %7.0lf, %9.0lf, %7.4lf, %7.0lf,\n",
    radius1,best_lift,best_lift+smallest_diff,best_current,
    mass_lift,best_mass );

}; /* end of radius iteration */

fclose(fp);
return 0;
}

```

A.11 Constants File

```

/* include file const.cpp
   contains constants common to all other programs of orbit control
   and mass/acceleration calculations
*/

#define Pi 3.141592654
#define radius_earth 6378E3 /* radius of earth, m */
#define density_Al 2800.0 /* density of aluminum alloy 2014-T6 */
#define perm_free 4.0 * Pi * (pow10(-7)) /* permeability of vacuum */

```

```

#define stress_allow_Al 137E6 /* Pa of stress allowed on Al */
#define GMe 3.986E14 /* universal gravitational constant for earth */
#define B_surface 3.05E-5 /*magnetic field at earth surface Tesla*/
#define Cd 2.2 /* coefficient of drag */
#define diameter_wire 0.003 /* diameter of loop wire */
#define diameter_tether 0.002 /* diameter of tether wire */
#define radius_super 0.00126 /*radius solid superconducting wire */
#define density_super 5000. /*kg/m3 of CuO, basic HTSC material */
#define body_area 50. /* m2 of spacecraft center body area */
#define deviation 10*Pi/180 /* deviation from unstable equilibrium */
#define density_fiber 1000. /* density of UHS composite */
#define stress_allow_fiber 3000E6 /* allowable stress on UHS composite */
#define r_outer_start 0.05 // start value hollow circle radius
#define delta_r_outer 0.05 // increment radius outer

```

A.12 Air Density File

```

/*****/
double atm_density(double altitude)
/* density of atmosphere at any altitude given in m */
/* atmospheric densities given are maximums, for peak solar cycle */
{
double density;
altitude = altitude/1000.;

if (altitude>0. && altitude < 100.)
density = (10.);
else if (altitude>=100. && altitude < 150.)
density = (6E-7);
else if (altitude>=150. && altitude < 200.)
density = (2E-9);
else if (altitude>=200. && altitude < 250.)
density = (3E-10);
else if (altitude>=250. && altitude < 300.)
density = (1E-10);
else if (altitude>=300. && altitude < 350.)
density = (4E-11);
else if (altitude>=350. && altitude < 400.)
density = (2E-11);
else if (altitude>=400. && altitude < 450.)
density = (1E-11);
else if (altitude>=450. && altitude < 500.)
density = (5E-12);
else if (altitude>=500. && altitude < 550.)

```

```

density = (3E-12);
else if (altitude>=550. && altitude < 600.)
density = (1.5E-12);
else if (altitude>=600. && altitude < 650.)
density = (8E-13);
else if (altitude>=650. && altitude < 700.)
density = (4E-13);
else if (altitude>=700. && altitude < 750.)
density = (2.7E-13);
else if (altitude>=750. && altitude < 800.)
density = (1.6E-13);
else if (altitude>=800. && altitude < 850.)
density = (9.4E-14);
else if (altitude>=850. && altitude < 900.)
density = (5.7E-14);
else if (altitude>=900. && altitude < 950.)
density = (3.5E-14);
else if (altitude>=950. && altitude < 1000.)
density = (2.2E-14);
else if (altitude>=1000. && altitude < 1250.)
density = (1.4E-14);
else if (altitude>=1250. && altitude < 1500.)
density = (2.8E-15);
else if (altitude>=1500. && altitude < 2000.)
density = (1.1E-15);
else if (altitude>2000.)
density = (0.0);

return(density);
}
/*****/

```

A.13 Drag Force File

```

/*****/
double drag_force (double position,double area_total, double velocity)

/* calculates drag force on satellite of area_total at atmospheric density
found at altitude.*/

{
double altitude; /* altitude above earth surface in km */

altitude = (position - radius_earth);

```

```

return(0.5*Cd*atm_density(altitude)*area_total*pow(velocity,2.));

}
/*****/

```

A.14 Magnetic Force File

```

/*****/
double mag_force(double radius1, double current, double position,
double true_anom)
/* find the magnetic force on loop of radius1 with current in it at altitude
above the surface of the earth and angle theta from magnetic north
*/
{
double moment; /* magnetic moment of loop 1 */
double thrust; /* thrust function due position in geomagnetic field*/
double grad_B; /* gradient in geomagnetic field */
double theta; /* angle from magnetic north */

theta = 50.*Pi/180.-true_anom; /* assume perigee center thrust arc */

thrust =0.5*sin(2*theta)/sqrt(3*pow(cos(theta),2.)+1);

grad_B= pow((radius_earth/position),4.)*thrust*1.46E-11;

moment = current*Pi*pow(radius1,2.);

/* assume .9 loss for earth rotation, poles moving */
return(0.9*moment*grad_B);
}

```

A.15 Orbit Calculations File

```

/* basic orbit calculation routines used by orb1, orb2, orb3.cpp */

/*****/
double velocity (double semi_major, double position)
/* returns velocity at position in elliptical orbit of semi major axis */
{
return (sqrt(2*GMe*(1./position-1./(2.*semi_major))));
}
/*****/
double delV_total(double orbit_radiusA, double orbit_radiusB)

```

```

/* returns the required delta V for a transfer from an initial orbit
   with orbit_radiusA to final elliptical orbit with perigee A and apogee B
*/
{
double semi_major; /* semi-major axis of orbit */
double delV ; /* delta V for transfer to elliptical orbit */

semi_major = (orbit_radiusA+orbit_radiusB)/2.;
delV = sqrt(GMe)*abs(sqrt(2/orbit_radiusA-1/semi_major)-
  sqrt(1/orbit_radiusA));
return(delV);
}
/*****/
double thrust_arc(double theta1,double theta2,double perigee)
/* finds length of thrust arc, approximated as a circle segment
   with a radius of perigee
*/
{
return (perigee*abs(theta2-theta1));
}
/*****/
double vel_perigee(double perigee,double apogee)
/* finds velocity at perigee for an elliptical orbit */
{
double semi_major; /* semi-major axis of orbit */

semi_major = (apogee+perigee)/2.;
return(sqrt(2.*GMe*(1./perigee-1./(2.*semi_major))));
}
/*****/
double eccent (double perigee, double semi_major)
/* returns eccentricity given perigee and semi major axis of orbit */
{
return (1.-perigee/semi_major);
}
/*****/
double mean_motion (double semi_major)
/* returns mean motion of earth satellite given semi major axis of orbit*/
{
return (sqrt(GMe/pow(semi_major,3)));
}
/*****/
double mean_anom (double eccent, double true_anom)
/* returns mean anomaly given orbit eccentricity and true anomaly at
   point in question */

```



```

{
double eccent_anom; /* eccentric anomaly for orbit */

eccent_anom = (acos((eccent+cos(true_anom))/
(1.+eccent*cos(true_anom))));

return (eccent_anom-eccent*sin(eccent_anom));
}
/*****/
double period (double semi_major)
/*period of orbit in days given semi major axis*/
{
double period; /* period of orbit */
/* period = sqrt(2.*Pi*pow(semi_major,3.)/GMe); */
period = pow((semi_major/331.2E3),1.5)/(60.*24.);
return(period);
}
/*****/

```

A.16 Radial Force File

```

/*****/
double mag_for_rad(double radius1, double current,
double theta, double position)
/* returns the magnetic force in the radial direction */
{
double radial; /*radial force function of theta */

radial= pow((3.*pow(cos(theta),2.)+1.),.5);
return(-3.*radial*B_surface*Pi*pow(radius1,2.)*current*
pow(radius_earth,3.)/pow(position,4.));
}
/*****/

```

A.17 Single Loop Mass File

```

/*
include file single.cpp: calculations used for mass of single loop
used by a1.cpp and l1.cpp
*/

/*****/
double area_reinf(double current, double stress_allow)

```

```

/* area cross section of reinforcement needed in main loop to
counter tension from current in the loop */

{
double tension; /* tension in loop due to own current */

tension = (perm_free * pow(current,2.))/2.;
return (tension/stress_allow);
}
/*****/
double loop_mass(double radius1, double current, double stress_allow,
double density_reinf)
{
double vol_super; /* volume of superconducting material*/
double vol_reinf; /* volume of reinforcement material */
double system_mass; /* mass of cooling, reflecting, power systems */

vol_reinf=2.*Pi*radius1*area_reinf(current,stress_allow);
vol_super=2.*Pi*radius1*Pi*pow(radius_super,2.);
system_mass = 2.*Pi*radius1*0.005; /* systems */

return(vol_super*density_super+vol_reinf*density_reinf+system_mass);

}
/*****/
double tether_mass(double radius1, double density_reinf)
/* finds mass of tethers from main loop to center body */
{
return(radius1*Pi*pow(diameter_tether,2.)*density_reinf);
/* 4 tethers cancel with factor of 4 denominator */
}
/*****/

```

A.18 Triple Loop Mass File

```

/*
include file: triple.cpp
calculations associated with triple loop mass, area and loads
*/
/*****/
double radius2(double radius1)
/* calculates radius loops 2 and 3 for given deviation and radius of 1 */
{
return (radius1*sqrt(sin(Pi / 2 -deviation)/sin(deviation)));
}

```

```

/*****/
double tension12 (double radius1, double current)
/* tension between loop1 and secondary loops */
{
double angle; /*angle in radius that tether makes with loop1 plane */
double moment; /* in N-m that tether supports */

angle = atan(radius2(radius1)/radius1);
moment = Pi*pow(radius2(radius1),2.0)*pow(current,2.0)*perm_free/
(2.*radius1);

return (moment/(2.*radius2(radius1)*cos(angle)));
}
/*****/
double area_tethers(double radius1, double current)
/* returns frontal area of tethers between the loops, used for drag*/
{
double angle; /* angle between tether and loop plane */
double moment; /* torque between loops due to current in loops */
double tension22; /* tension on tethers between secondary loops */
double diameter; /* of tether */
double tether1,tether2; /*frontal area of tethers 1 and 2 */

/* first for tethers from loop1 to loops 2 and 3 */
diameter=sqrt(4*tension12(radius1,current)/(Pi*stress_allow_A1));
tether1 = radius1*diameter; /*frontal area of tethers loop 1 to loops 2,3*/

/* now tethers between secondary loops */
angle = 45*Pi/180; /* secondaries are the same size */
moment = Pi*radius2(radius1)*pow(current,2.)*perm_free/(2);
tension22=moment/(2*radius2(radius1)*cos(angle));
diameter=sqrt(4*tension22/(Pi*stress_allow_A1));
tether2 = sqrt(2)*radius2(radius1)*diameter; /*frontal area tethers 2,3*/

return(8*tether1+4*tether2);
}
/*****/

double deltaX (double radius, double sections)
/* gives length of an element of straight beam used to represent quarter
loop */
{
return (Pi*radius/(2.*sections));
}

```

```

}
/*****/
double mass_super(double radius1)
/* mass of superconducting material */
{

return((2*Pi*radius1+4*Pi*radius2(radius1))*Pi*pow(radius_super,2.)*
density_super);

}
/*****/
double mass_system(double radius1)
/* mass of the solar, cooling and power distribution systems */
/* mass of all systems assumed to be 0.05 kg/m */
{
return((Pi*2*radius1+4*Pi*radius2(radius1))*0.05);
}
/*****/
double area_loops(double radius1, double r_outer)
// returns the area of loops meeting atmospheric drag
// given radius main loop and outer radius of loop cross section
{
double area1, area2; /* areas of primary & secondary loops */

area1 = 2.*Pi*radius1*2.*r_outer;
area2 = 2.*Pi*radius2(radius1)*2.*r_outer;

/* add main loop area to two secondary loops */

return (area1+2.*area2);
}
/*****/
double force22(double x,double radius1,double deltaX, double current)
/* force on current element deltaX in secondary loop due to current
in other secondary loop */
{
return(pow(current,2.)*perm_free*deltaX*(1.+2.*(1.-pow(2.,-1.5))*x/(Pi*
radius2(radius1)))/(2.*radius2(radius1)));
}
/*****/
double force21(double x,double radius1,double deltaX, double current)
/* force on current element deltaX in secondary loop due to current
in loop 1 */
{
return(pow(current,2.)*perm_free*deltaX*sin(x/radius2(radius1))*(1./

```

```

(2.*radius1)-((1./radius1-pow(radius1,2.)/(pow((pow(radius1,2.)+pow(
radius2(radius1,2.)),1.5)))/(Pi*radius2(radius1)*x)));
}
/*****
double radius_outer(double thickness, double moment)
// returns outer radius of loop cross section
// uses bending beam straight-beam equations
// thicknes of hollow circle cross section
// moment on cross section
// for a straight beam:
// radius_outer = sqrt(moment1/(stress_allow_A1*thickness*Pi))
{
    return (sqrt(fabs(moment/(stress_allow_A1*thickness*Pi))));
}
*****/
double mass_primary(double radius1, double thickness,double current)

// finds mass of the primary loop, given the radius and thickness
// worst-case tension is assumed, with entire loop made with same
// cross-section. Worst-case bending stess assumed over entire loop.

{
    double mass; /* of a quarter section of main loop */
    double moment1; /* moment up to point x */

    moment1 = (tension12(radius1,current)*cos(Pi/6.)-100.*perm_free*
    pow(current,2.)/(2.*radius1))/(Pi*2.*radius1/8.);
    // max. moment loop 1 due current in loop 2, less expansion
    //      force from primary loop
    mass = Pi*2.*radius_outer(thickness,moment1)*thickness
    *Pi*radius1*2. * density_A1;

    /* mass is for a straightened quarter-section of primary loop */
    return(mass);
}
*****/
double mass_second(double radius1, double thickness,double current)

/* finds mass of one secondary loop, given the radius of loop 1,
the thickness of loop 2 reinforcement, the current in all loops (assumed
to be max. and height of square cross-section of reinforcement */
{

    double delta_m; /* differential mass element */

```

```

double mass2; /* total mass element */
double x ; /* distance along arc of secondary loop quarter section */
double sections; /*number sections divided into for integration*/
double moment2; /* moment at point x along beam */
double shear_force; /*shear force at any point x along arc */
short n;

sections = 1000.0; /* number of parts quarter loop divided into */
mass2 = 0.0 ; /* start mass2 */
moment2 = 0.; /* moment at end of beam */
x = 0.; /* start value for distance along beam */

for (n=1; n<sections; n++)
{
    shear_force =force21(x,radius1,deltaX(radius2(radius1)
        ,sections), current)
    +force22(x,radius1,deltaX(radius2(radius1),sections),current);

    moment2 = moment2+deltaX(radius2(radius1),sections)*shear_force;

    delta_m = Pi*2.*radius_outer(thickness,moment2)
    *thickness*deltaX(radius2(radius1),sections) * density_Al;

    mass2 = mass2 + delta_m;

    x = x + deltaX(radius1,sections);

}; /* end for*/
/* mass is for a straightened quarter-section of secondary loop */
return (4.*mass2);
}
/*****/
mass_tether(double radius1,double current)
/* finds mass of tethers from main to secondaries and between
secondary loops */
{
    double angle; /* angle between tether and loop plane */
    double moment; /* torque between loops due to current in loops */
    double tension; /* tension on tether */
    double diameter; /* of tether */
    double mass_tether1,mass_tether2; /*mass of tethers 1 and 2 */

    /* first for tethers from loop1 to loops 2 and 3 */

```

```

angle = atan(radius2(radius1)/radius1);
moment = Pi*pow(radius2(radius1),2.0)*pow(current,2.0)*perm_free/(2*radius1);
tension=moment/(2*radius2(radius1)*cos(angle));
diameter=sqrt(4*tension/(Pi*stress_allow_A1));
mass_tether1 = (sqrt(pow(radius1,2.)+pow(radius2(radius1),2.))
+sqrt(pow(radius2(radius1),2.)+100.))
*density_A1* pow(diameter,2.)*Pi/4.;
/* includes the body to secondary tethers, makes same diameter */

/* now tethers between secondary loops */
angle = 45*Pi/180; /* secondaries are the same size */
moment = Pi*radius2(radius1)*pow(current,2.)*perm_free/(2);
tension=moment/(2*radius2(radius1)*cos(angle));
diameter=sqrt(4*tension/(Pi*stress_allow_A1));
mass_tether2 = sqrt(2.)*radius2(radius1)*density_A1*Pi*diameter/4.;

return(8*mass_tether1+4*mass_tether2);

}
/*****/
double mass_reinf (double radius1,double thickness,double current)
/* total reinforcement mass is the sum of primary and secondary mass */
{

return (mass_primary(radius1,thickness,current)+2.*mass_second(radius1,
thickness,current));

}
/*****/

```

A.19 Flexibility Program

```

// flex.cpp finds flexibility of given curved beam compared to straight
// calculates the neutral axis position
// assumes hollow circle cross section used for reinforcement

#include <math.h>
#include <stdlib.h>
#include <stdio.h>
#include <io.h>
#include <fcntl.h>
#include <sys\stat.h>

```

```

#include <iostream.h>
#include <string.h>
#include <c:\wg\const.cpp> // basic constants used in program
#include <c:\wg\na.cpp> // neutral axis calculations

//*****
main(void)
{
double flexibility; // comparism to straight beam flexibility
double area; // area of loop cross section
double second_moment; // of area about loop center
double r_outer; // outer radius of cross section
double thickness; // thickness of loop reinforcement
double radius1; // radius of primary loop

FILE *fp;

fp = fopen("flex.out","w");

r_outer = .8;
thickness = 0.01;
radius1 = 500.;

area=area_tube(r_outer,thickness);

second_moment =(pow(r_outer,4.)-pow((r_outer-thickness),4.))*Pi/4.;

flexibility = second_moment/(neutral_axis(r_outer,radius1,thickness)
*area*(radius1+r_outer));

fprintf(fp,"stress factor, inner %5.4lf\n",
(1.-neutral_axis(r_outer,radius1,thickness)/(r_outer-thickness))*
second_moment/(neutral_axis(r_outer,radius1,thickness)*area*radius1));

fprintf(fp,"stress factor, outer %5.4lf\n",
(1.+neutral_axis(r_outer,radius1,thickness)/r_outer)*
second_moment/(neutral_axis(r_outer,radius1,thickness)*area*(radius1
+2.*r_outer)));

fprintf(fp,"calculated area*10^6 %7.5lf\n", Pi*(pow(r_outer,2.)
-pow((r_outer-thickness),2.))*1E6);
fprintf(fp,"integrated area*10^6 %7.5lf\n",area*1E6);
fprintf(fp,"second moment*10^6 %7.5lf\n ",second_moment*1E6);
fprintf(fp,"neutral axis %7.5lf\n\n",
neutral_axis(r_outer,radius1,thickness));

```



```
fprintf(fp," flex factor %8.7lf\n ", flexibility);

return(0);
}
```

A.20 Neutral Axis Calculations

```
// na.cpp neutral_axis
// include file to calculate the neutral axis position from the centroid
// of the hollow loop used for reinforcement

/*****/
double neutral_axis(double r_outer, double radius1, double thickness)

// returns neutral axis position from centroid of cross section
// performs integral for second term of equation for neutral axis position
// as found in curved beam equations from Bowes, Russell and Suter, p. 129

// r_outer is radius of outer surface of circular cross section
// radius1 is radius primary loop
// thickness is thickness of cross sectional reinforcement

{
double area; // cross sectional area
double delta_r; // increment in radius for integration
double delta_area; // increment of cross section area
double r; // radius from center of primary loop
double integral; // integral of area over radius for cross section
double theta_outer; // angle from vertical to point on outer surface
double theta_inner; // angle to point on inner surface of cross section
double length_outer; // horizontal length from center cross section
double length_inner; // horizontal length to point inner surface

short sections; // number of sections divided into for integration
short n; // counter

sections = 5000.;
area = 0.;
delta_area=0.;
integral = 0.;

// for radius1 < r < radius1 + thickness
delta_r = thickness/sections;
r = radius1;
```

```

for (n=1; n<sections; n++)
{
integral = integral + delta_area/r;
theta_outer = acos((r-r_outer-radius1)/r_outer);
delta_area = fabs(2.*r_outer*sin(theta_outer)*delta_r);
area = area + delta_area;
r = r + delta_r;
}
// for radius1+thickness < r < radius1 + 2*r_outer - 2*thickness

delta_r = (2.*r_outer-2.*thickness)/sections;
r = radius1+thickness;

for (n=1; n<sections; n++)
{
integral = integral + delta_area/r;
if (fabs(-r+radius1+r_outer)/(r_outer-thickness) >= 1.)
{
theta_inner = 0.;
theta_outer = 0.;
}
else
{
theta_outer = acos((-r+radius1+r_outer)/r_outer);
theta_inner = acos((-r+radius1+r_outer)/(r_outer-thickness));
};
length_outer = fabs(r_outer*sin(theta_outer));
length_inner = fabs((r_outer-thickness)*sin(theta_inner));
delta_area = (length_outer-length_inner)*2.*delta_r;
area= area+ delta_area;
r = r + delta_r;
}
// for radius1+2*r_outer-thickness < r < radius1+2*r_outer
delta_r = thickness/sections;

// r = radius1+2.*r_outer-thickness;

for (n=1; n<sections; n++)
{
integral = integral + delta_area/r;
theta_outer = acos((r-r_outer-radius1)/r_outer);
delta_area = fabs(2.*r_outer*sin(theta_outer)*delta_r);
area = area + delta_area;
r = r + delta_r;
}

```

```

    }
    return(fabs(radius1+r_outer-area/integral));
}
/*****
double area_tube (double r_outer, double thickness)
// returns cross sectional area of hollow circle of outer radius
// r_outer and thickness
{
    double area; // area of cross section
    double delta_r; // increment in radius for integration
    double delta_area; // increment of cross section area
    double r; // radius from center of primary loop
    double theta_outer; // angle from vertical to point on outer surface
    double theta_inner; // angle to point on inner surface of cross section
    double length_outer; // horizontal length from center cross section
    double length_inner; // horizontal length to point inner surface

    short sections; // number of sections divided into for integration
    short n; // counter

    sections = 5000.;
    area = 0.;
    delta_area=0.;

    delta_r = thickness/sections;
    r = 0.;

    for (n=1; n<sections; n++)
    {
        theta_outer = acos((r-r_outer)/r_outer);
        delta_area = fabs(2.*r_outer*sin(theta_outer)*delta_r);
        area = area + delta_area;
        r = r + delta_r;
    }

    delta_r = (2.*r_outer-2.*thickness)/sections;

    for (n=1; n<sections; n++)
    {
        if (fabs(-r+r_outer)/(r_outer-thickness) >= 1.)
        {
            theta_inner = 0.;
            theta_outer = 0.;
        }
        else

```

```

{
theta_outer = acos((-r+r_outer)/r_outer);
theta_inner = acos((-r+r_outer)/(r_outer-thickness));
};
length_outer = fabs(r_outer*sin(theta_outer));
length_inner = fabs((r_outer-thickness)*sin(theta_inner));
delta_area = (length_outer-length_inner)*2.*delta_r;
area= area+ delta_area;
r = r + delta_r;
}

delta_r = thickness/sections;

for (n=1; n<sections; n++)
{
theta_outer = acos((r-r_outer)/r_outer);
delta_area = fabs(2.*r_outer*sin(theta_outer)*delta_r);
area = area + delta_area;
r = r + delta_r;
}
return(area);
}

```

Appendix B:

B.1 Orbit Coordinates

The position and orbit of a satellite is defined by six parameters, a , e , i , Ω , ω , and a time variable, usually the time of perigee passage. These parameters are known as the Keplerian orbit elements

Figure B. 1 Classical orbit parameters for an elliptical orbit.

a = semimajor axis

Ω = Right ascension of ascending node: angle from Aries, which is towards the sun at vernal equinox, to the ascending node vector from the earth center eastward.

i = Inclination angle between equatorial and orbit planes, about the ascending node vector.

ω = Argument of perigee: angle between the vector from earth center to the perigee and the ascending node vector, in the orbit plane.

v = True anomaly

e = Eccentricity = c/a

c = Distance from orbit center to apogee (furthest point).

The ascending node is the point where the orbit cuts the equatorial plane from South to North, and the ascending node vector is the line from earth center to the ascending node. The semimajor axis and eccentricity define the shape of the orbit. For a circular orbit, $e = 0$ and ω is not defined.