HOW DO YOU TAKE AWAY MORE THAN YOU HAVE? EVALUATING ADULTS' PERFORMANCE ON SIMPLE SUBTRACTION PROBLEMS.

by

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Abstract

Research on the cognitive processes involved in simple subtraction has focused exclusively on problems where the answer is positive (e.g., $4 - 2 = 2$) and has therefore not investigated the additional processes and procedures that may be required for problems yielding negative answers (e.g., $2 - 4 = -2$). The present experiment addressed this issue by using problems that produced positive and negative answers. Problems were presented in both pure (i.e., either problems that yielded all negative or all positive answers) and mixed (i.e., both answer valences) blocks. The results suggest solvers only store one order of subtraction facts. Since problems that produce positive answers are practiced more frequently than those producing negative answers, solvers have developed more effective retrieval and procedural strategies for the former. A model supporting interactive processing of relevant numerical information used in subtraction fact retrieval was created to account for the novel findings reported here.

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INTRODUCTION

The cognitive processes that mediate elementary arithmetic calculation and retrieval have been investigated experimentally for several decades (see Ashcraft, 1995, 1992; Campbell, 2005 for reviews). The purpose behind this research has been to understand the organization and representation of numbers and arithmetic facts within humans' memory networks and to determine what processes and procedures are activated when solvers attempt to produce these facts (Ashcraft, 1992). Although a wealth of knowledge has been collected that provides insight into the cognitive structures that exist for addition and multiplication, few researchers have examined the reverse associations (subtraction and division, respectively). The purpose of this experiment was to investigate the mental processes that are activated during subtraction and to determine if these processes differ depending on the value of the answer (i.e., whether the answer is positive or negative).

Subtraction

Subtraction is an essential part of our every day lives. We use it to compute financial transactions, to determine our available monetary resources after expenses, to understand our financial loans and in solving basic mathematical problems. Understanding the mathematical logic of removing a quantity is taught as early as the first grade; however the processes and procedures adults use to solve simple subtraction problems have been largely ignored in the literature (see Seyler, Kirk, & Ashcraft, 2003). To date, research on subtraction has focused exclusively on problems in which the answer (i.e., the difference (d) between the minuend (c) and the subtrahend (b): $c - b = d$) is a positive numerical value (e.g., $4 - 2 = 2$) and have ignored the additional processes and procedures that may be required when the answer is a negative value (e.g., $2 - 4 = -2$; cf. Lee & Kang, 2002). Producing answers that can be either positive or negative in value depending on the magnitude and order of the minuend and the subtrahend is a unique quality of subtraction and worthy of exploration. This research will address two theoretical questions regarding subtraction. First, are response times and error rates similar for subtraction problems that yield positive or negative answers (e.g., $7 - 3 = 4$ and $3 - 7 = -4$)? Second, do solvers complete a magnitude comparison of the minuend and the subtrahend to determine if the answer is positive (i.e., minuend is larger than the subtrahend) or negative (i.e., minuend is smaller than the subtrahend)? To lay the foundation of my experimental design, I will review research on the strategies people use in subtraction, number comparison tasks using positive and negative numbers, and number comparison in arithmetic.

Strategy Use and Arithmetic Performance

A fundamental assumption in the cognitive sciences is that the response time and error data produced by a cognitive task will provide insight into the mental processes used to complete that task. Accordingly, research has shown that the type of strategy solvers use to answer an arithmetic problem is systematically related to response time and error rates on that problem (Campbell & Gunter, 2002; Campbell & Penner-Wilger, 2006; Campbell & Xue, 2001; LeFevre, DeStefano, Penner-Wilger, & Daley, 2006; Penner-Wilger, Leth-Steensen, & LeFevre, 2002). Arithmetic problems that are solved by retrieving an answer from memory have shorter response times and lower error rates than problems that are solved with a procedural strategy (e.g., counting 3, 4, 5 to solve $2 + 3$).

As such, strategy use is closely linked to arithmetic performance and the major phenomena observed in simple arithmetic performance.

Major Phenomena of Simple Arithmetic Performance

The Problem-Size Effect

The basic effect observed in all arithmetic operations is the problem-size effect; people solve problems with smaller operands (e.g., $3 + 4 = 7$, $5 - 3 = 2$) more quickly and accurately than problems with numerically larger operands (e.g., $9 + 6 = 15$, $13 - 7 = 6$; see Zbrodoff & Logan, 2005 for a review). Two points regarding the problem-size effect in mental arithmetic are important for my research. First, the problem-size effect has been documented in every empirical paper on mathematical cognition (Ashcraft, 1992; Zbrodoff & Logan, 2005). It is the effect that all researchers expect to find and all theories of arithmetic fact retrieval must explain. Second, the slope and size of the problem-size effect has been used extensively as evidence of the mental processes used by solvers to answer arithmetic facts (Ashcraft, 1992; Campbell & Gunter, 2002; LeFevre et al., 2006; LeFevre, Sadesky, & Bisanz, 1996; Seyler et al., 2003). In particular, increases in the problem-size effect indicate solvers shifted from faster retrieval strategies to slower and more error prone calculation strategies (e.g., counting). Thus, any research on mental arithmetic will include problem-size as a central variable and interpret changes in the problem-size effect accordingly.

Strategies in Simple Subtraction

Research has shown that solvers do not choose strategies at random when solving arithmetic problems (e.g., Reder & Ritter, 1992). Instead, a solver's strategy choice depends on the relative efficiency of retrieval strategies (see Siegler & Shrager, 1987;

Siegler & Shipley, 1995) and their familiarity with the physical characteristics of the problem (Reder & Ritter, 1992; Schunn, Reder, Nhouyvanisvong, Richards, & Stroffolino, 1997). For simple subtraction, solvers can choose from a variety of available strategies, including direct retrieval, the corresponding addition reference (e.g., 15 – 9 = 9 + _ = 15), counting by ones (e.g., 5 – 2 = 5, 4, 3) or transformation (e.g., 15 – 9 = 16 – 10 and 6 is the difference; Campbell, 2008, Campbell & Xue, 2001; Geary, Frensch, Wiley, 1993). The strategies that educated adults use to solve simple arithmetic problems depend on the arithmetic task, mathematical skill level, and culture (Campbell & Xue, 2001; LeFevre et al., 2006). Unlike addition and multiplication, which are primarily solved by direct retrieval even on larger problems, subtraction is often solved with a procedural strategy (Campbell, 2008; Campbell & Xue, 2001; LeFevre et al., 2006; Seyler et al., 2003). The percentage of reported strategy use for solving subtraction problems also increases dramatically as the problem-size increases. For Canadian participants, Campbell and Xue (2001) reported 73% retrieval for small problems (minuend < 11) and 42% retrieval for large problems (minuend >= 11). The problem-size effect parallels participants' self reported strategy use as a sharp increase in the problem size effect is observed for minuends greater than ten (LeFevre et al., 2006; Seyler et al., 2003).

To investigate the calculation strategies used by solvers to answer subtraction problems, Campbell (2008) varied the presentation format of addition and subtraction problems. He reasoned that, if solvers typically solve subtraction problems by using the corresponding addition reference (i.e., solving 13 – 7 by asking what could you add to 7 to make 13?), then solvers would be faster and less error prone if the subtraction problem was presented in an addition format (e.g., 7 + _ = 13) rather than subtraction format (e.g., 13 – 7 = _). Consistent with this prediction, people were faster on the addition format problems. However, the format advantage occurred only on large problems (minuend >= 11), not on small problems (minuends < 11). Campbell (2008) concluded that adults use direct retrieval to solve small subtraction problems and addition reference to solve large subtraction problems. In summary, research indicates that solvers rely mainly on direct retrieval for small subtraction problems and procedural strategies that utilize knowledge of addition facts for large subtraction problems.

Semantic Knowledge of Numbers: Evidence from Number Comparison Tasks

Positive Numbers

Although there is still controversy in the literature, performance on number comparison tasks (e.g., which is larger, 4 or 9?) has supported the view that individuals use a mental number line as the cognitive representation of the meaning of numbers (see Fias & Fischer, 2005 for a review). The idea of a linear analogue mental representation of numbers was originally proposed by Moyer and Landauer (1967) to account for the time required by adults to discriminate between two numbers. Using the digits 1 through 9, they discovered that the time to choose the larger (or smaller) digit depended on the relative magnitudes of the two digits. Specifically, the closer the digits were in magnitude (e.g., 8 and 9) the longer the response time to complete the magnitude comparison task. Hence, response time decreased linearly ($r = -0.63$) as the difference between the two numbers increased. This effect later became known as the distance or split effect and has become the hallmark effect in the numerical comparison literature (see Tzelgov & Ganor-Stern, 2005).

In the early 1990s, Dehaene and colleagues (Dehaene, Bossini & Giraux, 1993; Dehaene, Dupoux, & Mehler, 1990) documented a second effect in support of a linear analogue representation of numbers. Dehaene et al. (1993) asked participants to indicate the parity (odd or even) of a single number using the two keys, one located on the right side of the participant and one located on the left side. The assignment of the keys to odd or even was counterbalanced across participants. Although accessing a digit's magnitude information was not related to the task of judging parity, they discovered that participants responded to larger numbers faster with their right hand and smaller numbers faster with their left hand. Dehaene and colleagues (1993) named this effect the Spatial Numerical Association of Response Codes (i.e., the SNARC effect). They interpreted the SNARC effect as evidence that numerical magnitude information is spatially coded such that smaller numbers are represented on the left side of space and larger numbers are represented on the right side of space (like a traditional number line). Although there is controversy about the source of the SNARC effect, Dehaene and colleagues have used it to provide additional support for the mental number line hypothesis (Dehaene et al., 1993; Fias & Fischer, 2005; Fischer, 2003a).

Negative Numbers

Research into the mental representation and use of negative numbers is just beginning. There are two hypotheses regarding the mental representation of negative numbers (Fischer, 2003b). The first hypothesis is that individuals may have a magnitude representation of negative numbers that extends from zero, leftwards along the number line (*the holistic hypothesis*). In Western cultures, negative numbers are displayed to the left of positive numbers; therefore, if negative numbers are represented on the number line, they would be associated with the left side of space. The second hypothesis is that negative numbers are not processed on the number line (*componential hypothesis*). Instead, when completing a number comparison task with negative numbers, the numbers are compared as if they were positive numbers and then a rule is applied. For example, when deciding which number is larger, -2 or -4 the absolute magnitude of 2 and 4 is activated. The individual determines that 4 is larger than 2, and then applies the rule that the opposite answer is correct for negative numbers. Thus, if 4 is larger than 2 for positive numbers then, the larger number is -2 for negative numbers. If the componential hypothesis is correct, we would expect small negative numbers to be associated with the right side of space and larger negative numbers would be associated with the left side of space, creating a reverse SNARC effect.

To date the evidence is inconsistent and support has been documented for both hypotheses. Fischer (2003b) had participants select the numerically larger number of a pair of digits ranging from -9 to 9. Participants responded by pressing one of two keys. One key was located on the left side of the participant and the other on their right. He found a regular SNARC effect as participants were faster to respond to negative numbers with their left hand and positive numbers with their right response, which supports the holistic model. Unexpectedly, pairs of negative digits were processed more slowly than mixed pairs (one positive and one negative digit) or positive pairs, suggesting that negative digits require additional processing. Fischer's work suggests adults do have a magnitude representation of negative numbers that extends along the mental number line; however, accessing this information may take longer for negative than for positive digits.

Contrary to Fischer (2003b) who documented a regular SNARC effect using a number comparison task, Nuerk, Iversen, and Willmes (2004) failed to find a SNARC effect for negative numbers using a parity task. In Nuerk et al. (2004), participants pressed one of two buttons to indicate parity of a single presented number ranging from -9 to 9. A reliable SNARC effect was obtained for positive numbers but the effect was not present for negative numbers. The absence of a SNARC effect indicates the participants in Nuerk et al. (2004) did not automatically access a number line representation for the negative numbers but they were able to access the number line for positive numbers. Thus their results support the componential hypothesis.

Fischer and Rotterman (2005) found that the SNARC effect for negative numbers was task specific. When participants made a parity judgment on a single digit (i.e., -9 to 9), a reverse SNARC effect was obtained for negative numbers as smaller negative numbers (e.g., -9) were associated with the right hand response and larger negative numbers (e.g., -3) were associated with a left hand response. However, when a magnitude comparison task is used (participants indicated whether the single presented digit was smaller than or larger than 0) small negative values became associated with the left side of space. Thus, obtaining magnitude information for negative numbers may only occur if the demands of the task require it.

Obtaining a SNARC effect with negative numbers may also depend on the range of numbers used within each block. Shaki and Petrusic (2005) used positive and negative number pairs in a magnitude judgment task. In one condition positive and negative numbers were presented in separate blocks and in the second condition they were intermixed. When the positive and negative values were intermixed a regular SNARC effect was obtained, such that negative numbers were associated with the left side of space and positive numbers were associated with the right side of space. However, when positive and negative numbers were presented in separate blocks, a reverse SNARC effect was obtained. Smaller negative values (e.g., -9) were associated with the right side and larger negative values (e.g., -2) were associated with the left side of space.

In conclusion, existing research on the representation of negative numbers is incomplete. There is evidence, however, that the numerical representation of negative numbers is not identical to that of positive numbers. Unlike the pervasive number line representation for positive numbers, the mental representation of negative numbers varies depending on the experimental task (e.g., magnitude comparison vs. parity judgments) and the experimental design (i.e., blocked vs. mixed trials).

Negative Numbers and Arithmetic

Historically, negative answers in arithmetic (e.g., 2 – 4 = -2) were regarded as impossible by historical mathematical geniuses such as Blaise Pascal, who invented a mechanical calculator for addition and subtraction and researched mathematics in the 1600s (see Dehaene, 1997, p., 87). Even today, with the acceptance of negative entries in monetary transactions (e.g., overdraft protection, lines of credit) negativity is still difficult for the average learner to grasp (Vlassis, 2004). To date, there are no published papers in which negative numbers have been used as operands in arithmetic problems. In an unpublished paper, Das, LeFevre, and Penner-Wilger (2007; see also Das, 2006), showed participants addition and subtraction problems, some of which had negative operands. They concluded that, for problems with negative answers that required addition such as 7 – (-4), solvers used the rule that minus and a negative is a positive, and then simply added the two numbers. Compared to equivalent addition problems such as 7 + 4, the 'negative' addition problems were simply slower. For subtraction problems such as 9 – 5 and 9 + (-5), however, differences between the two formats were small. Das et al. concluded that solvers categorized the problems as either 'addition' or 'subtraction' and then solved them. The presence of a negative operand, per se, did not seem to change solvers' solution strategies.

To date no one has published a paper directly investigating the effects of negative *answers* in subtraction problems. Although Lee and Kang (2002) used both positive and negative answers in a subtraction task, they did not compare performance on problems with negative versus positive answers (e.g., 7 – 2 vs. 2 – 7). Using the digits 1 to 9 in all combinations and a dual task paradigm they investigated verbal and spatial working memory demands on subtraction and multiplication tasks. Multiplication and subtraction tasks were blocked but all orders of the digits were presented in each block such that the subtraction problems consisted of both positive and negative answers within the block. Lee and Kang found verbal interference with a multiplication task and spatial interference with the subtraction task. They concluded that multiplication uses verbal working memory resources whereas subtraction uses spatial working memory resources. The result that subtraction relies on spatial resources is a novel finding in the literature and has proven to be impossible to replicate when positive answers are used (Imbo & LeFevre, in press). Several other researchers have failed to show spatial interference with a subtraction task using only positive answers (see DeStefano & LeFevre, 2004 and Imbo, Rammelaere, & Vandierendonck, 2005 for reviews of working memory in arithmetic).

In summary, researchers have found that negative numbers take longer to process than positive numbers (Fischer, 2003b), they are more difficult to understand in arithmetic operations (Vlassis, 2004), and in mixed blocks (with positive numbers) they behave as if they are represented on a mental number line (Shaki & Petrusic, 2005). It is also possible that problems with negative answers may evoke the use of different solution strategies that rely on different working memory components (cf., Lee & Kang, 2002). Thus, the role of negative numbers (as operands and answers) requires further research.

Number Comparison as a Component in Arithmetic Fact Retrieval

Central to the second research question is addressing the phenomena of number comparison as a central component of arithmetic fact retrieval; when the answers can be either positive or negative, do solvers complete a magnitude comparison of the minuend and the subtrahend to determine if the answer is positive (minuend > subtrahend) or negative (minuend < subtrahend)? Currently, the issue of number comparison in arithmetic fact retrieval has only been addressed for addition (Butterworth, Zorzi, Girelli, & Jonckheere, 2001) and multiplication (Verguts & Fias, 2005; Robert & Campbell, 2008). As the addition reference strategy is often used to solve large subtraction problems (Campbell, 2008), I will review the literature regarding number comparison in addition.

Theoretically the existence of a numerical comparison stage in addition is plausible. When children are developing mental representations for addition facts, they typically learn to add by counting from the largest number rather than counting from the first number (i.e., termed the min strategy because they count on with the minimum addend; Groen & Parkman, 1972). This result suggests that children learn to complete a magnitude comparison of the addends to determine which one is larger and then organize the problem so the smaller numerosity (min) is added to the larger numerosity (max, e.g., 2 + 4 would be reversed to 4 + 2). This strategy would promote the storage of one order of the equations (max + min; Butterworth et al., 2001) and may promote the use of a magnitude comparison stage into adulthood as part of the retrieval process.

The COMP model (Butterworth et al., 2001) implements the theoretical notion that the magnitudes of the addends are a central component of the retrieval process. According to the COMP model, solvers only store one representation of the pairs of addition facts (e.g., 4 + 2 is stored and not 2 + 4). According to Butterworth and colleagues (2001) there are three distinct processes that contribute independently to addition reaction times. The first stage is a numerical comparison stage. During this stage the numerical magnitudes of the addends are compared to determine the numerically larger (max) and smaller (min) addend. The purpose of the comparison stage is to ensure the equation order is max + min so the solver can match the presented equation to the internal memory representation. The second stage, the sum stage, utilizes the max and the min numbers as retrieval cues to obtain the answer from memory or to solve the equation via a procedural strategy such as counting. The third stage is the pronunciation stage which provides the output for the answer. As each stage operates independently to contribute to overall RT, any factor that alters a particular stage will change the equation's overall RT. For example, in the magnitude comparison literature the closer the digits are (e.g., 8 and 9) the longer the RT to complete the magnitude comparison task (Moyer & Landauer, 1967). Therefore the COMP model would predict small split equations (e.g., 5 + 6) would be solved more slowly than large split equations (e.g., 2 + 6) because the magnitude comparison stage would take longer as the split between the numbers decreased.

Using a size congruity manipulation, Robert and Campbell (2008) investigated the presence of a magnitude comparison stage in addition and multiplication using Canadian participants. In the numerical comparison literature, RT increases when numbers are physically congruent with their numerical magnitudes (e.g., 8 | 4) relative to when the trials are incongruent (e.g., 4 | 8; Henik & Tzelgov, 1982). Robert and Campbell reasoned that if solvers used magnitude comparison on arithmetic problems, then 4 + 8 would be solved more slowly than 4 + 8 because of the congruency effect. However, despite adequate statistical power, there was no effect of split or size congruency in the arithmetic trials. Robert and Campbell (2008) concluded that their participants did not use a magnitude comparison to identify the larger and smaller operand when they solved simple addition and multiplication problems.

Although theoretically plausible, the existence of a magnitude comparison stage in addition is not necessary as solvers can obtain the correct answer without knowing the magnitude order of the addends (i.e., both 5 + 8 and 8 + 5 equal 13). However, for subtraction problems the magnitude order is vital in determining the correct answer (i.e., 5 – 8 and 8 – 5 produce different answers; one requires the response "positive three" and the other requires the response "negative three"). Determining if and how the magnitude comparison stage is activated during retrieval and calculation will facilitate the development of theoretical models for subtraction.

The Present Experiment

The first purpose of this experiment was to evaluate adults' performance on a set of simple subtraction problems yielding positive and negative answers. It was hypothesized that subtraction problems yielding negative answers would be solved more slowly and produce a larger problem-size effect than problems producing positive answers. This hypothesis is based on the evidence that negative numbers take longer to process relative to positive numbers (Fischer, 2003b) and they are more difficult to understand in arithmetic operations (Vlassis, 2004). The second purpose of this experiment was to provide evidence of a magnitude comparison stage in subtraction. Research has shown that the processing of negative and positive numbers changes depending on whether the type answer types are blocked or mixed (see Shaki & Petrusic, 2005). In this experiment the pure blocks contained problems that produced either all positive or all negative answers, whereas both answer types were present in the mixed block condition. It is argued that in the pure block condition solvers are less likely to engage in a magnitude comparison because they will quickly learn that the answers are either all positive or all negative. However, in the mixed block condition, the magnitude comparison stage will be highly relevant as solvers need to determine if the equation produces a positive or negative answer. In number comparison tasks, response time increases as the split between the numbers decrease. Thus, if a numerical comparison stage is required when the trials are intermixed then the difference in mean RT between pure and mixed blocks will be larger for small split problems relative to large split problems. Thus, I predicted an interaction between block and split such that small split equations would slow down more in the mixed block condition than large split equations.

METHOD

Participants

Forty-seven volunteers were recruited through the participant pool at Carleton University and received course credit towards their research option in their 1st or 2nd year psychology class. As seven individuals were unable to complete the experiment in the allocated time period the final sample consisted of forty individuals (21 females). Twenty-four participants were educated in Canada, seven were educated in China and nine were educated elsewhere. Participants ranged in age from 18 to 36 years (*Mdn* = 20) and all reported normal or corrected to normal vision. The recruitment information described the purpose of the experiment as investigating the cognitive processes involved in solving simple subtraction problems.

Design

A 2 (Block: pure vs. mixed) x 2 (Answer Valence: positive, negative) x 2 (Problem-size: small, large) x 2 (Split: small, large) repeated measures design was used. The experimental trials consisted of six blocks in total: four pure blocks (i.e., only negative answers or only positive answers) and two mixed blocks (i.e., both answer valences). Participants completed a pure positive block, a pure negative block and a mixed block with the presentation order counter balanced using a Latin square design. Once the first three blocks had been completed, participants were given a short break and then they completed the remaining three blocks in the same presentation order. The pure blocks contained a randomized presentation order of all 81 possible equations whereas the mixed block contained a randomized presentation order of all 162 possible equations. Thus, each participant solved a total of 648 subtraction trials.

Materials

Apparatus. Instructions and stimuli appeared on two high-resolution monitors controlled by an IMac-type personal computer running Mac OS with Superlab Software. The experimenter viewed one monitor and the participant viewed the other. Participants sat approximately 45 cm from the monitor and they responded by speaking into the IMac's microphone. This microphone was controlled by a software clock accurate to ±10 ms. The experimenter recorded the participants' response using the keyboard.

Sign naming trials. The materials used for the sign naming trials consisted of positive and negative signs (i.e., + and -). Each sign was displayed individually in the centre of the monitor at a visual angle of approximately 1.27 ° horizontally and 1.27° vertically for the positive signs and 1.27° horizontally and .64° vertically for the negative signs.

Number naming trials. The number naming trials used positive and negative signed numbers (e.g., -9 and 9) ranging from -9 to 9 excluding 0. Positive digits were displayed at 1.27° vertical and 1.07° horizontal. The negative digits were displayed at a visual angle of 1.89° horizontally and 1.27° vertically. The purpose of the naming trials was to determine the magnitude of the response time difference between pronouncing the words "positive" and "negative". Given that the word 'negative' begins with a voiced consonant while the word 'positive' begins with a voiceless consonant, it is important to isolate articulatory differences that may exist between the two conditions.

Subtraction problems. The subtraction problems (i.e., minuend – subtrahend = difference) were created by arranging integer values ranging from 1 to 18 for the minuend and the subtrahend positions such that the resulting difference ranged from -9 to

9 excluding zero. On negative trials, the larger integer value appeared in the subtrahend position, whereas for positive trials the larger integer value appeared in the minuend position. A complete list of the subtraction problems used in the experiment can be found in Appendix A. There are 81 unique positive problems and 81 unique negative problems. Given that the greatest increase in response time occurs for splits greater than 3 (see Moyer & Landauer, 1967), small split problems were defined as a difference of 3 or less between the minuend and the subtrahend (e.g., $9 - 8 = \_$) and large split problems were those with a difference of 4 or greater (e.g., $9 - 4 = \_$). Subtraction problems were displayed in the center of the screen, with the negative sign at fixation and at a visual angle of approximately 4.28° horizontally and 1.27° vertically.

Measure of Arithmetic Fluency. In addition to the naming and subtraction trials, participants completed an arithmetic fluency test (French, Ekstrom, & Price, 1963). The French kit consists of 120 three-term addition problems, followed by 60 two minus two-digit subtraction (e.g., 27 – 12) interleaved with 60 two-digit by one-digit multiplication problems (e.g., 45 x 7). There are four pages in total, each containing 60 equations. The scores on the French kit were used to describe the general arithmetic fluency of the sample.

Procedure

The experiment required approximately one hour to complete. Once the consent form had been read and signed, the participants began the naming section of the experiment. Participants were instructed to name a sign (i.e., + or -) as quickly as possible. The alternating positive and negative signs were presented individually on the screen in order (i.e., positive, negative, positive, negative) with each sign presented six times in total. After the sign naming trials participants were instructed to name signed numbers as quickly as possible. For example, participants were instructed to respond "negative nine" to the stimuli '-9' and "positive nine" to '9'. There were six blocks of number naming trials with each block consisting of a random presentation of the 18 integer values between -9 and 9 excluding zero.

Following the naming task participants read the instructions for the subtraction trials on the monitor, which were reiterated aloud by the experimenter. The instructions informed participants that their task was to speak the answer to each subtraction problem as quickly as possible. If the answer was negative, then they were to say "negative" before stating the numerical value. If the answer was positive, then they were to say "positive" before providing the numerical value. Participants were informed that their answer would be incorrect if they failed to designate the sign prior to stating the numerical value.

Each trial started with a fixation dot that flashed twice in the centre of the screen over a 1-second interval. The subtraction problem was displayed on the third flash. Timing began at the onset of the subtraction problem and ended when the voice key triggered the sound activated relay. The problem remained on the screen until the researcher entered the participants' response to the arithmetic problem. To ensure coding accuracy, the next trial was initiated by the researcher. Participants were allowed to take short breaks. Prior to the experimental trials, participants completed 36 practice trials that were randomly selected with the constraint that 18 problems yielded a positive answer and 18 problems yielded a negative answer. The purpose of the practice trials was to ensure that participants understood the task requirements and to adjust the sensitivity of the microphone.

At the end of the experimental trials, participants were shown six individual arithmetic problems; three that produced positive answers (i.e., 5 – 3, 16 – 8, and 15 – 9) and three that produced negative answers (i.e., 3 – 5, 8 – 16 and 9 – 15). The positive problems were always presented first and then its negative equivalent was presented second. Participants were asked to explain how they would solve each problem. After they indicated their strategy for the negative answer of a problem pair, both equations appeared on the screen and participants indicated whether the problems were the same or different in terms of difficulty. Twenty-six participants completed all the questions, 6 completed the first two questions and eight participants were not able to complete any questions in the time allotted for the experiment.

After completing the experimental trials, participants completed the arithmetic fluency test. Participants were given two minutes per page to solve as many equations correctly as they could. Each correctly solved equation was worth one point and scores could range from zero to 240.

RESULTS

In total, participants solved 24,624 trials, of which 1072 (4.4 %) were discarded due to voice key failures or were marked as invalid due to extraneous noise (e.g., sneezing) triggering the microphone. An additional 271 trials (1.1 %) were eliminated as outliers more than three standard deviations from the participants' cell means. For the remaining trials, participants made errors on 1862 trials (7.6 % of all trials). Due to the moderate error rate, both response time and error data were analyzed.

The results section is divided into five sections. First, the number naming trials were analyzed to determine if phonetic differences are present between the words positive and negative spoken as an entirety or with numerical values appended. Second, participants' adjusted response times (i.e., raw response time minus naming time) were analyzed in a repeated measures Analysis of Variance (ANOVA) with factors: block, problem-size, and answer valence. Third, the error data was analyzed. Forth, a 2 (block: pure, mixed) x 2 (split: small, large) repeated measures ANOVA was conducted to determine if a magnitude comparison stage was activated in the mixed block condition. Fifth, participants' performance on the arithmetic fluency test was analyzed. Reported results were significant at $p < .05$, unless otherwise indicated.

Naming Trials

Participants' mean response time to name positive and negative signs was analyzed using a paired sample t-test. For sign naming (i.e., saying 'positive' or 'negative' in response to the sign), there was a trend for participants named negative signs faster than positive signs (606 vs. 642 ms), $t(39) = 1.34$, $p = .09$.

The number naming trials were analyzed in a 2 (answer valence: positive, negative) x 9 (digit: 1, 2, 3, 4, 5, 6, 7, 8, 9) repeated measures ANOVA. Table 1 displays the mean naming time for each numerical value. Participants named positive numbers more slowly than negative numbers, $F(1, 39) = 140.01$, $MSE = 6443$. Naming latencies did not vary across the nine digits, as the interaction between digit and answer valence was not significant, $F < 1$. Thus, I used adjusted response times in the latency analysis to remove the mean naming time to vocalize positive and negative numbers from each answer valence for each participant.

In conclusion, the phonetics of the words "positive" and "negative" were not sufficient to produce significant differences in the sign naming task, however there was a significant difference between naming positive and negative signed numbers. There are two possible explanations for this result. First, it is possible there was simply not enough power in the sign naming task to detect a significant difference between naming positive and negative signs. The sign naming task contained six observations per cell while the number naming task contained 54 observations per cell. To confirm this hypothesis a power analysis was conducted on the sign naming trials using Campbell and Thompson's (2002) methodology. The results confirmed that the sign naming trials had low statistical power (.33).

The second explanation is the task demands were different between naming signs and naming signed numbers. Participants maybe familiar with including the word "negative" before naming a negative number but they maybe unfamiliar with including the word "positive" before a positive number. In mathematics, negative entities are always designated with a value but positive entities are not thus increasing participants' familiarity with indicating negative values in spoken responses. This would result in longer processing times to name positive numbers and may create differences between the two groups.

Table 1.

Mean response time to name all digits separated by answer valence.

Digit	Positive	Negative
1	686	589
2	669	597
3	670	591
4	661	578
5	691	604
6	687	580
7	677	593
8	661	598
9	682	598
Mean	667	590

Response Time Analysis

The purpose of this experiment was to compare adults' performance on subtraction problems that produced a positive or a negative answer when the answer valences are segregated in blocks or intermixed. It was hypothesized that subtraction problems that yield negative answers would be solved more slowly and produce a larger problem-size effect than problems that yielded positive answers. This hypothesis is based on the evidence that negative numbers take longer to process relative to positive numbers (Fischer, 2003b) and they are more difficult to understand in arithmetic operations (Vlassis, 2004). A main effect of block was also predicted as pure block problems would be solved faster than mixed block problems.

The response times entered as the dependent variable in this analysis were adjusted to remove the production time to speak each numerical value. These adjusted response times were created by subtracting the mean production times to utter positive and negative numerical values from the raw response time scores (raw RT – mean production RT) for each individual participant on correct trials only. Adjusted latencies were analyzed in a 2 (problem-size: small, large) x 2 (answer valence: positive, negative) x 2 (block: pure, mixed) repeated measures ANOVA.

As predicted, participants solved problems with positive answers more quickly than those with negative answers (725 vs. 779 ms), $F(1, 39) = 7.38$, $MSE = 30891$. They also solved problems in pure blocks more quickly than in mixed blocks (588 vs. 917 ms), $F(1, 39) = 101.74$, $MSE = 83004$ Thus, problems with negative answers took longer to process than problems that produced positive answers.

Participants solved large problems more slowly than small problems (1059 vs. 445 ms), $F(1, 39) = 38.50$, $MSE = 762312$. This effect is ubiquitous in the literature on simple arithmetic. However, answer valence did not interact with problem-size, $F(1, 39) = 1.62$, $MSE = 10873$, $p = .21$. This lack of interaction between valence and problem-size suggests that the added difficulty of problems with negative answers is due to processing that occurs in a different stage than processing related to problem size.

There were two significant two-way interactions involving block. The interaction between block and answer valence was significant, $F(1, 39) = 6.20$, $MSE = 25546$. In the pure block condition, negative answers were solved more slowly than positive answers (538 vs. 637 ms), but in the mixed block condition there was no difference in participants' performance for positive and negative answers (912 vs. 921 ms). This result is consistent with previous literature using blocked and mixed designs (see Lupker, Brown, & Columbo, 1997). Specifically, when two tasks that produce different response times in pure block trials are intermixed, the response times tend to homogenize resulting in no difference between the two groups. Accordingly, the relation between block and valence may have little to do with arithmetic, and more to do with general task-related factors.

The interaction between block and problem-size was significant, $F(1, 39) = 11.67$, $MSE = 13785$. The problem-size effect (difference between small and large problems) was greater in the mixed block condition than in the pure block condition (659 vs. 568 ms). Because increases in the problem-size effect may indicate that solvers switched from using faster retrieval strategies to slower calculation strategies (Campbell & Xue, 2001; LeFevre et al., 1996; Seyler et al., 2003), the larger problem-size effect in the mixed block condition may reflect a switch such that participants use more procedural strategies in the mixed block condition relative to the pure block condition. There were no further interactions with answer valence, however, suggesting again that processing related to valence and problem size occur in different stages.

Error Data

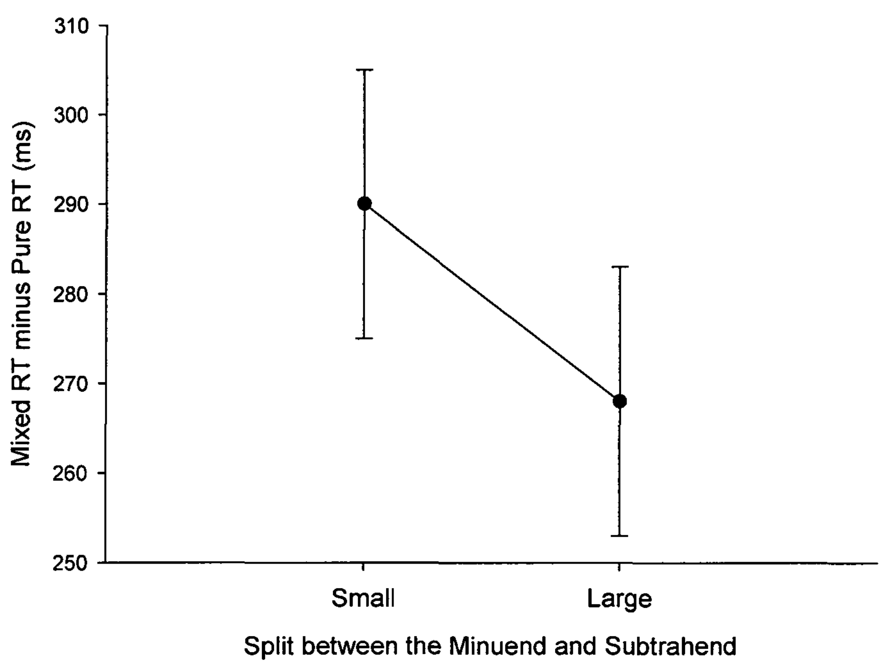
Percentage error was analyzed in a 2 (block: pure, mixed) x 2 (answer valence: positive, negative) x 2 (problem-size: small, large) repeated measures ANOVA. Participants made fewer errors in the pure blocks than in the mixed block condition (6.9 vs. 9.6%) $F(1, 38) = 28.77$, $MSE = .002$. Participants made fewer errors on small problems than on large problems (3.0 vs. 13.5%), $F(1, 39) = 53.33$, $MSE = .017$. They made more errors on problems that produced positive answers relative to problems that produced negative answers (8.7 vs. 7.8%), $F(1, 39) = 5.98$, $MSE = .001$. Further, the interaction between block and answer valence was significant $F(1, 39) = 18.87$, $MSE = .001$. The error rate for problems that produced positive answers increased from the pure to the mixed blocks (6.5 vs. 11.0%) whereas the error rate for problems that produced negative answers did not change from pure to mixed blocks (7.3 vs. 8.2%). This increase in errors on problems with positive answers from the pure to the mixed blocks occurred because participants produced more sign errors (i.e., saying negative instead of positive) in the mixed block condition. Once those errors were removed (156 sign reversal errors) the percentage of error was consistent for problems with positive and negative answers in both the pure (6.3 and 7.2%) and the mixed block conditions (6.9 and 6.8%). Thus, as in the analysis of response times, the results suggest that arithmetic solution procedures and determining answer valence occur at different processing stages.

Evaluating Split for Number Comparison

To determine if a magnitude comparison stage was activated in the mixed block condition the adjusted response times for small problems (subtrahend < 11) were analyzed in a 2 (block: pure, mixed) x 2 (split: small, large) repeated measures ANOVA. Recall that, in number comparison, participants are slower to compare numbers with small splits relative to large split items. Thus, if some of the response time differences from the pure to the mixed block conditions were due to participants performing a magnitude comparison of the operands then we would expect a 2-way interaction between split and block such that smaller split equations (e.g., 3 – 1) should slow down more in the mixed block relative to larger split equations (e.g., 6 – 1). The results, however, do not support the hypothesis that participants perform a number comparison as the interaction between split and block was not significant, $F(1, 39) = 2.09$, $MSE = 2163$, $p = .16$. However, the means are in the predicted direction as smaller split equations slowed down more in the mixed block condition relative (295 vs. 585 ms for pure and mixed blocks) to large split equations (332 vs. 601 ms for pure and mixed blocks). Figure 1 displays the interaction.

Figure 1.

Displays the mean difference in response time between pure and mixed blocks for small and large splits. Higher values indicate the equation slowed down more in the mixed block condition relative to the pure block condition. Error bars are 95% confidence intervals (Masson & Loftus, 2003).



It should be noted that there was no evidence to suggest participants utilized a magnitude comparison stage in the pure or mixed blocks as small split equations were solved significantly faster than large split equations (448 vs. 476 ms), $F(1, 39) = 8.10$, *MSE* = 3916, that is, the opposite result one would expect if a numerical comparison stage was activated in arithmetic. As the split between the numbers is positively correlated with problem-size, this result is likely due to participants using more procedural strategies to solve large split equations than small split equations. Increased use of procedural strategies therefore would produce longer response times for larger split equations.

Analysis of Individual Differences in Arithmetic Skill and Self-reported Strategies

Arithmetic fluency. Overall, participants performed below the expected average of 80 (LeFevre et al., 2003) on the arithmetic fluency test ($M = 70$). Consistent with previous literature, students educated in China ($n = 7$, $M = 105$) scored higher than those educated in Canada ($n = 24$, $M = 65$), $t(28) = 3.58$, or elsewhere ($n = 9$, $M = 58$), $t(14) = 2.93$ (see LeFevre & Liu, 1997). As this sample is relatively low in arithmetic fluency, it is likely that the response times and error rates reported in this experiment will be larger than those reported in other experiments with subtraction.

Open Ended Questions. The arithmetic strategies provided by the open ended questions were coded into eight categories: **1)** knew or retrieved the answer, **2)** counted by ones (e.g., 5 – 3 = 4, 3, 2), **3)** multiplication reference (e.g., 8 x 2 = 16 therefore 16 – 8 = 8), **4)** addition reference (e.g., 5 – 3 = 3 + _ = 5), **5)** computed the absolute value between the two numbers, **6)** flipped the equation and solved the reverse (e.g., 3 – 5 = 5 – 3 = 2), **7)** tens reference (e.g., 15 – 9 = 16 – 10 therefore the answer is 6), or **8)** unsure of how they solved the problem. The percentage use of each strategy for each equation type is presented in Table 2. For negative answers, a common reported strategy was to flip the equation and solve for a positive answer. For example, 9 – 15 would be turned into 15 – 9 and then solved the same way as the positive answer. Many participants spontaneously reported using a visualization strategy that involved flipping the equation to move the larger digit into the minuend position. Thus, it is possible that negative answers may use more visual-spatial resources than positive answers (c.f., Lee & Kang, 2002). Future research is needed in this area.

Table 2.

Percentage of reported strategy use for each equation type.

	Reported Strategy							
	Retrieve	Count	Mult. Reference	Add. Reference	Abs. Value	Flip equation	Tens reference	Unsure
5 - 3	73.0	2.7	0	18.9	5.4	0	0	0
3 - 5	54.1	2.7	0	0	5.4	32.4	0	5.4
16 - 8	25.0	0	55.6	18.5	0	0	0	0
8 - 16	18.5	3.7	37.0	14.8	7.4	18.5	0	0
15 - 9	14.8	11.1	0	11.1	3.7	0	59.3	0
9 – 15	19.2	7.7	0	3.8	3.8	11.5	50.0	3.8

In response to the question of which type of problem was more difficult, 23 % percent of the participants reported that negative answers were equivalent to positive answers in difficulty level for all three problems, 43% always reported negative answers were more difficult than positive answers, and 33% of participants reported that negative answers were more difficult but only for some of the problems. Thus, the majority of participants viewed problems that produced negative answers as more difficult than those with positive answers.

DISCUSSION

The present experiment compared subtraction performance when the answer valences were positive or negative, segregated in separate blocks or in mixed blocks. In general, subtraction problems that produced a positive answer were solved faster than subtraction problems that produced a negative answer and pure blocks were solved faster than mixed blocks. Answer valence did not interact with problem-size, indicating solvers used the same solution strategy for both types of answer valences. Thus, processing of the problem was not affected by answer valence. Furthermore, the predicted interaction between split and block was not significant.

Processing Stages in Mental Arithmetic

The fundamental question is: why were subtraction problems that yielded a negative answer answered more slowly than problems that produced a positive answer? To correctly solve an equation like 5 – 2 or 2 – 5 an individual needs to encode the problem then convert the problem into the appropriate internal code(s) which are used to access the answer from long-term memory. Once the answer is retrieved, the individual then produces the answer using the appropriate response modality (Blankenberger, 2001; Campbell, 1994, 1992; Campbell & Clark, 1989; 1992; Campbell & Epp, 2005; Dehaene, 1992; Dehaene & Cohen, 1995; McCloskey, 1992). Thus, the differences in solving the equations could be due to the encoding, processing or production stage. Due to the high degree of visual similarity between the subtraction problems that yield a positive answer and subtraction problems that yield a negative answer (i.e., 5 – 2 and 2 – 5 share all the same visual components, just in different orders), it is unlikely encoding would differ depending on the answer valence of the problem. Also, it is unlikely that the differences occurred in the production stage, as the response time analysis removed each participant's production times. Thus, the differences in responding must be due to the processing stage. However, because answer valence did not interact with problem-size, dealing with answer valence does not appear to affect retrieval or execution of a procedural strategy. Thus, the results suggest that subtraction problems that yield a negative answer require an *extra* processing stage that is not present for the problems that yield a positive answer.

The hypothesis that determining answer valence involves a separate processing stage than calculation is consistent with other research. LeFevre and Liu (1997) found response time advantages for Chinese participants based upon the order of the operands in a multiplication problem. In particular, Chinese participants were faster to solve multiplication problems when the equation was presented with the smaller operand on the left (e.g., 3 x 5). This position advantage was attributed to greater practice of the min-left equations. Chinese participants report completing multiplication tables with the smaller operand located on left. They never report practicing multiplication problems with the larger operand located on the left. This frequent practice would promote more efficient retrieval for the highly practiced order (see Campbell & Epp, 2005; Reder & Ritter, 1992; Rickard, 2005; Schunn, et al., 1997). LeFevre and Liu concluded that Chinese participants transformed multiplication problems that had the larger number on the left (e.g., 5 x 3) into the complimentary equation (e.g., 3 x 5) so they could efficiently retrieve the answer. The re-ordering of the equation required a measurable amount of time to complete and contributed significantly to multiplication response times.

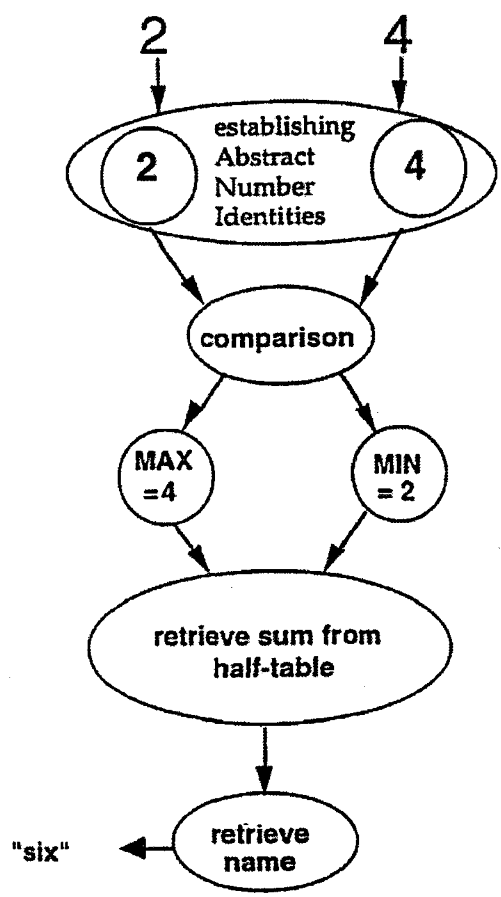
The learning process for subtraction problems in Canada is very similar to the learning process of multiplication fact in China. All participants in this experiment reported practicing problems that produced a positive answer more frequently than problems that produced a negative answer. Furthermore, 21 % of the participants in this experiment spontaneously reported using a strategy where they reversed the equation when the larger number was located on the right. For example, when faced with the equation 2 – 5, a common strategy was to report reversing the equation into 5 – 2 and then solve the complementary problem. Because the equation was reversed, the participants reported the answer must be negative. This reordering process would take a measurable amount of time, and because the same calculation procedures were used for each answer valence, problem-size and answer valence would not interact.

Number Comparison in Arithmetic

The second goal of this experiment was to find evidence for number comparison as a component in arithmetic fact retrieval using the split between the minuend and the subtrahend. In the numerical comparison literature, it takes participants longer to decide which number is larger (or smaller) for small splits relative to large splits (Moyer & Landauer, 1967; Robert & Campbell, 2008). It was argued that a numerical comparison stage would be most relevant in the mixed block condition relative to the pure block condition. Thus, the difference between small splits from the pure to mixed blocks would be greater than the difference between large splits. This predicted interaction was not significant, indicating participants were not engaging in a magnitude comparison stage in the mixed block condition.

Although the results are inconsistent with the hypothesis stated at the onset of this paper, they are consistent with previous literature that has failed to find evidence of a number comparison stage in arithmetic (Robert & Campbell, 2008; Robert, Jensen, & LeFevre, 2009). The COMP model (Butterworth et al., 2001) assumes that arithmetic fact retrieval is a serial process as each process contributes independently to response times (figure 2 displays the COMP).

Figure 2. The COMP model from Butterworth et al. (2001 p. 1011).

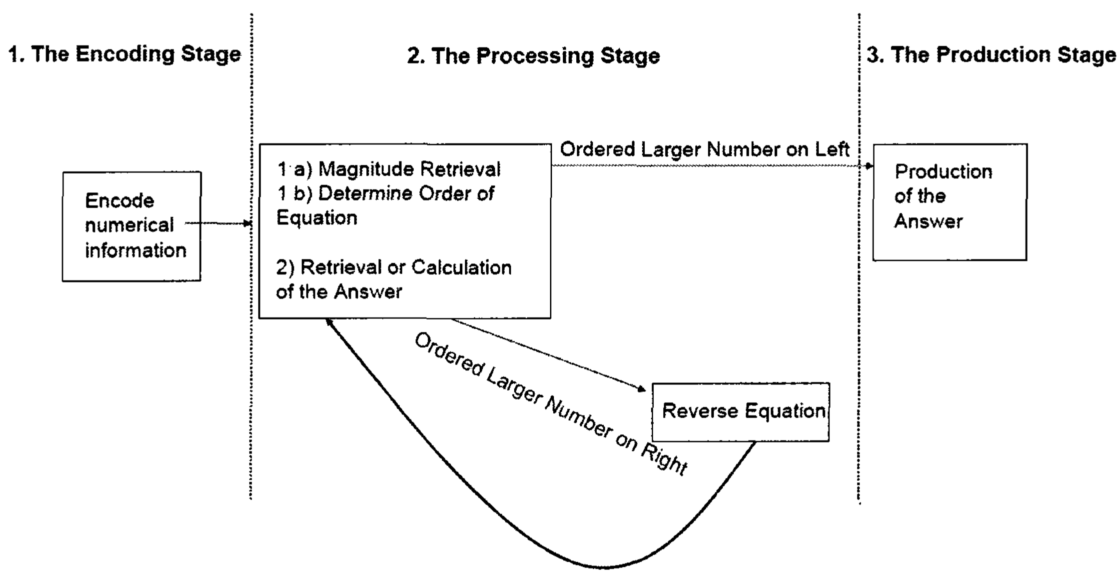


As shown in the figure, each stage must be completed before the next stage begins.

Thus, the numerical comparison stage must be completed prior to retrieving (or calculating) the arithmetic fact and producing the answer. This assumption implies that the numerical comparison stage is directly measurable by factors that affect number comparison. However, a numerical comparison of the minuend and the subtrahend seemingly was required in the present experiment and yet response times did not increase significantly as the split between the minuend and the subtrahend decreased. This result suggests that the numerical comparison stage happens in parallel with the retrieval/calculation stage.

I am proposing a modification to Butterworth et al.'s (2001) model that incorporates parallel processing of numerical information (Figure 3). Like Butterworth's model, the first stage is the encoding stage. In this stage the numerical quantities are identified. Once the numerical information has been encoded, the solver moves to the processing stage where the magnitude comparison stage happens in parallel with the retrieval or calculation of the answer. The processing stage assumes solvers immediately begin to activate all relevant information related to the digits within the arithmetic problem. So not only is the digits' magnitude information becoming activated but the solvers' knowledge of arithmetic facts and procedures is also activated simultaneously. The idea of simultaneous activation of magnitude and arithmetic information is consistent with previous literature. For Arabic digits, studies have shown that magnitude information is automatically activated (see Tzelgov & Ganor-Stern, 2005) and also arithmetic fact knowledge is also automatically activated (Lemaire, Barrett, Fayol, & Abdi, 1994). The retrieval or calculation processes utilize the magnitude information to determine if the equation is ordered with the larger number located on the left. If the larger number is located on the left, then the retrieval or calculation procedures proceeds normally, the individual obtains the answer and proceeds to the production stage. If the equation is ordered with the larger number located on the right, the individual mentally reverses the equation and then utilizes the same retrieval or calculation strategies that were activated upon the presentation of the problem. Once the problem is ordered appropriately the individual obtains the answer and moves to the production stage. As numerical comparisons take approximately half the time of arithmetic operations (see Robert & Campbell, 2008) the limiting factor in the processing stage is the retrieval or calculation of the arithmetic fact. Thus, any disruption to the numerical comparison stage would have to be quite substantial before changes in response times would be observed.

Figure 3. Proposed model of subtraction



Limitations

The main limitation of this experiment was using a vocal response to measure response time. In mathematical cognition experiments, participants typically indicate the numerical value but not the polarity of the answer. Thus, requiring them to state 'negative' or 'positive' before the numerical value adds additional processing. Further, most experimental manipulations do not have different vocal responses between the conditions. Although every attempt was made to reduce this bias by subtracting each participant's mean pronunciation time for positive and negative numbers, the results suggest that the response modality increased the working memory load, especially for subtraction problems that produced positive answers in the mixed block condition. Forcing participants to initiate a response with the words "positive" and "negative" increased the error rate in the mixed block condition (156 sign reversal errors) as participants had to activate and produce 'positive' and inhibit 'negative' on answers where they would usually respond with neither label. Although differences between subtraction problems with different answer valences were still observed, due to the larger working memory demands placed on positive answers this experiment may not provide an accurate picture of any differences that may exist between positive and negative answer valences. Furthermore, subtracting naming times from raw response times requires the assumption that the vocal responding and articulatory process are consistent for the arithmetic operations and naming tasks. This assumption, although validated by some researchers (see Butterworth et al., 2001) is not validated by others (LeFevre & Liu, 1997). The results should be interpreted with caution until replicated using a different response modality.

Conclusions

The results of this experiment suggest solvers only store one order of subtraction facts. Because equations that produce a positive answer are practiced more frequently than problems that produce a negative answer, solvers have developed more effective retrieval and procedural strategies for equations that produce a positive answer. To account for the results, a new model of subtraction fact retrieval was created. The model supports simultaneous interactive processing of all relevant numerical information. After the problem is encoded, solvers begin to access numerical information which includes numerical magnitude and arithmetic fact knowledge. The magnitude information is used to determine the order of the equation (i.e., max – min). This information is passed to the retrieval and calculation procedures. As the retrieval or calculation procedures are already activated, the solver can effectively utilize this knowledge to obtain the answer. As the numerical magnitude information is extracted faster than arithmetic operations, disruptions to numerical comparison stage in arithmetic will only be observable if they are quite substantial.

REFERENCES

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44,* 75 – 106.

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition, 1,* 3 – 34.

Blankenberger, S. (2001). The arithmetic tie-effect is mainly encoding based. *Cognition,82,* B15 – B24.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *Quarterly Journal of Experimental Psychology, 54A,* 1005 – 1029.

Campbell, J. I. D. (2008). Subtraction by addition. *Memory & Cognition, 36,* 1094 – 1102.

Campbell, J. I. D. (2005). *Handbook of Mathematical Cognition.* New York: Psychology Press.

Campbell, J. I. D. (1994). Architectures for numerical cognition. *Cognition, 53,* 1 – 44.

Campbell, J. I. D. (1992). In defence of the encoding-complex approach: Reply to McCloskey, Macaruso, & Whetstone. In J. I. D. Campbell (Ed.), *The Nature and Origin of Mathematical Skills* (pp. 539 – 556). Amsterdam: Elsevier Science.

Campbell, J. I. D. (1991). Conditions of error priming in number fact retrieval. *Memory & Cognition, 19,* 197 – 209.

Campbell, J. I. D. & Clark, J. M. (1992). Numerical cognition: An encoding-complex perspective. In J. I. D. Campbell (Ed.), *The Nature and Origin of Mathematical Skills* (pp. 457 – 491). Amsterdam: Elsevier Science.

Campbell, J. I. D., & Clark, J. M. (1989). Time course of error priming in number fact retrieval: Evidence for excitatory and inhibitory mechanisms. *Journal of Experimental Psychology: Learning, Memory and Cognition, 15,* 920 – 929.

Campbell, J. I. D. & Epp, L. J. (2005). Architectures for arithmetic. In. J. I. D. Campbell (Ed.). *Handbook of Mathematical Cognition* (pp. 347 – 360). New York: Psychology Press.

Campbell, J. I. D. & Gunter, R. (2002). Calculation, culture, and the repeated operand effect. *Cognition, 86,* 71 – 96.

Campbell, J. I. D. & Penner-Wilger, M. (2006). Calculation latency: The μ of memory and the τ of transformation. *Memory & Cognition, 34,* 217 – 226.

Campbell, J. I. D. & Thompson, V. A. (2002). More power to you: Simple power calculations for treatment effects with one degree of freedom. *Behaviour Research Methods, Instruments, and Computers, 34,* 332 – 337.

Campbell, J. I. D. & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General, 130,* 299 – 315.

Das, R. (2006). Mental arithmetic and negative numbers: Assessing the unknown. Undergraduate Honours Thesis, Department of Psychology, Carleton University.

Das, R., LeFevre, J.-A., & Penner-Wilger, M. (unpublished manuscript). Negative numbers in simple arithmetic.

Dehaene, S. (1997). *The Number Sense: How the Mind Creates Mathematics.* New York: Oxford University Press.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44,* 1 – 42.

Dehaene, S., Bossini, S., &Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122,* 371 – 396.

Dehaene, S., & Cohen, L. (1995). Toward an anatomical and functional model of number processing. *Mathematical Cognition, 1,* 83 – 120.

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is number comparison digital? Analogical and symbolic effects in two digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16,* 626 – 641.

DeStefano, D. & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16,* 353 – 386.

Geary, D. C., Frensch, P. A., Wiley, J. G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology and Aging, 8,* 242 – 256.

Fias, W. & Fischer, M. H. (2005). Spatial representation of numbers. In. J. I. D. Campbell (Ed.). *Handbook of Mathematical Cognition* (pp. 43 – 54). New York: Psychology Press.

Fischer, M. H. (2003a). Spatial representations in number processing – Evidence from a pointing task. *Visual Cognition, 10,* 493 – 508.

Fischer, M. H. (2003b). Cognitive representation of negative numbers. *Psychological Science, 14,* 278 – 282.

Fischer, M. H. & Rottmann, J. (2005). Do negative numbers have a place on the mental number line? *Psychological Science, 47,* 22 – 32.

French, J. W., Ekstrom, R. B., & Price, L. A. (1963). *Kit of Reference Test for Cognitive Factors.* Princeton, N. J.: Educational Testing Services.

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79,* 329 – 343.

Henik, A., Tzelgov, J. (1982). Is three greater than five: The relation between physical and semantic size in comparison tasks. *Memory & Cognition, 10,* 389 – 395.

Imbo, I., Rammelaere, S., & Vandierendonck, A. (2005). New insights in the role of working memory in carry and borrow operations. *Psychologica Belgica, 45,* 101 - 121.

Imbo, I., & LeFevre, J.-A. (in press). Cultural differences in complex addition: Efficient Chinese versus adaptive Belgians and Canadians. *Journal of Experimental Psychology: Learning, Memory, and Cognition.*

Lee, K.-M. & Kang, S.-Y. (2002) Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition, 83,* B63 – B68.

LeFevre, J.-A., DeStefano, D., Penner-Wilger, M., & Daley, K. E. (2006). Selection procedures in mental subtraction. *Canadian Journal of Experimental Psychology, 60,* 209 – 220.

LeFevre, J.-A., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition, 3,* 31 – 62.

LeFevre, J.-A., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem-size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22,* 216 – 230.

Lemaire, P., Barrett, S. E., Fayol, M., & Abdi, H. (1994). Automatic activation of addition and multiplication facts in elementary school children. *Journal of Experimental Child Psychology, 57,* 224 – 258.

Lumpker, S. J., Brown, P., & Colombo, L. (1997). Strategic control in a naming task: Changing routes or changing deadlines? *Journal of Experimental Psychology: Learning, Memory, and Cognition, 23,* 570 – 590.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44,* 107 – 157.

Moyer, R. S. & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature, 215,* 1519 – 520.

Nuerk, H. C., Iversen, W., & Willmes, K. (2004). Notational moderation of the SNARC and the MARC (Linguistic markedness of response codes) effect. *Quarterly Journal of Experimental Psychology, 57A,* 835 – 863.

Penner-Wilger, M., Leth-Steensen, C., & LeFevre, J.-A. (2002). Decomposing the problem-size effect: A comparison of response time distributions across cultures. *Memory & Cognition, 30,* 1160 – 1167.

Reder, L. M. & Ritter, F. E. (1992). What determines initial feelings of knowing? Familiarity with question terms, not with the answer. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 18,* 435 – 451.

Rickard, T. C. (2005). A revised identical elements model of arithmetic fact representation. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 31,* 250 – 257.

Robert, N. D. & Campbell, J. I. D. (2008). Simple addition and multiplication: No comparison. *European Journal of Cognitive Psychology, 20,* 123 – 138.

Robert, N. D., Jensen, C., & LeFevre, J.-A. (2009, 11). *Eyeing the difference: Chinese performance on magnitude comparisons and simple arithmetic.* Poster presented at the meeting of Psychonomics, Boston.

Schunn, C. D., Reder, L. M., Nhouyvanisvong, A., Richards, D. R., & Stroffolino, P. J. (1997). To calculate or not to calculate: A source confusion model of problem familiarity's role in strategy selection. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 23,* 3 – 29.

Seyler, D., J., Kirk, E. P., & Ashcraft, M. H. (2003). Elementary Subtraction. *Journal of Experimental Psychology: Learning Memory, and Cognition, 29,* 1339 – 1352.

Shaki, S. & Petrusic, W. M. (2005). On the mental representation of negative numbers: Context-dependent SNARC effects with comparative judgments. *Psychonomic Bulletin & Review, 12,* 931 – 937.

Siegler, R. S. & Shipley, C. (1995). Variation, selection, and cognitive change. In. T. Simon and G. Halford (Eds.) *Developing Cognitive Competence: New Approaches to Process Modeling* (pp. 31 – 76). Hillsdale, NJ: Erlbaum.

Siegler, R. S. & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In. C. Sophian (Ed.) *Origins of Cognitive Skills* (pp. 229 – 260). Hillsdale, NJ: Lawrence Erlbaum Associates.

Tzelgov, J. & Ganor-Stern, D. (2005). Automaticity in processing ordinal information. In. J. I. D. Campbell (Ed.). *Handbook of Mathematical Cognition* (pp. 55 – 66). New York: Psychology Press.

Verguts, T. & Fias, W. (2005a). Interacting neighbors: A connectionist model of retrieval in single-digit multiplication. *Memory & Cognition, 33,* 1-16.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and Instruction, 14,* 469 – 484.

Zbrodoof, N. J., & Logan, G. D. (2005). What everyone finds: The problem-size effect. In J. I. D. Campbell (Ed.) *Handbook of Mathematical Cognition* (pp. 331 – 346). New York: Psychology Press.

APPENDIX A: Subtraction Stimuli

Small Problems				Large Problems			
Positive Answer		Negative Answer		Positive Answer		Negative Answer	
Sm. Split	Lg. Split	Sm. Split	Lg. Split	Sm. Split	Lg. Split	Sm. Split	Lg. Split
2 – 1 =	5 – 1 =	1 – 2 =	1 – 5 =	11 – 8 =	11 – 2 =	9 – 11 =	2 – 11 =
3 – 2 =	6 – 2 =	2 – 3 =	6 – 2 =	11 – 9 =	11 – 3 =	8 – 11 =	3 – 11 =
4 – 3 =	7 – 3 =	3 – 4 =	3 – 7 =		11 – 4 =		4 – 11 =
5 – 4 =	8 – 4 =	4 – 5 =	4 – 8 =		11 – 5 =		5 – 11 =
6 – 5 =	9 – 5 =	5 – 6 =	5 – 9 =		11 – 6 =		6 – 11 =
7 – 6 =	10 – 6 =	6 – 7 =	6 – 10 =		11 – 7 =		7 – 11 =
8 – 7 =	6 – 1 =	7 – 8 =	1 – 6 =		12 – 3 =		3 – 12 =
9 – 8 =	7 – 2 =	8 – 9 =	2 – 7 =		12 – 4 =		4 – 12 =
10 – 9 =	8 – 3 =	9 – 10 =	3 – 8 =		12 – 5 =		5 – 12 =
3 – 1 =	9 – 4 =	1 – 3 =	4 – 9 =		12 – 6 =		6 – 12 =
4 – 2 =	10 – 5 =	4 – 2 =	5 – 10 =		12 – 7 =		7 – 12 =
5 – 3 =	7 – 1 =	5 – 3 =	1 – 7 =		12 – 8 =		8 – 12 =
6 – 4 =	8 – 2 =	4 – 6 =	2 – 8 =		12 – 9 =		9 – 12 =
7 – 5 =	9 – 3 =	5 – 7 =	3 – 9 =		13 – 4 =		4 – 13 =
8 – 6 =	10 – 4 =	6 – 8 =	4 – 10 =		13 – 5 =		5 – 13 =
9 – 7 =	8 – 1 =	9 – 7 =	1 – 8 =		13 – 6 =		6 – 13 =
10 – 8 =	9 – 2 =	8 – 10 =	2 – 9 =		13 – 7 =		7 – 13 =
4 – 1 =	10 – 3 =	1 – 4 =	3 – 10 =		13 – 8 =		8 – 13 =
5 – 2 =	9 – 1 =	2 – 5 =	1 – 9 =		13 – 9 =		9 – 13 =
6 – 3 =	10 – 2 =	3 – 6 =	2 – 10 =		14 – 5 =		5 – 14 =
7 – 4 =		4 – 7 =			14 – 6 =		6 – 14 =
8 – 5 =		5 – 8 =			14 – 7 =		7 – 14 =
9 – 6 =		6 – 9 =			14 – 8 =		8 – 14 =

10 – 7 =		7 – 10 =			14 – 9 =		9 – 14 =
					15 – 6 =		6 – 15 =
					15 – 7 =		7 – 15 =
					15 – 8 =		8 – 15 =
					15 – 9 =		9 – 15 =
					16 – 7 =		7 – 16 =
					16 – 8 =		8 – 16 =
					16 – 9 =		9 – 16 =
					17 – 8 =		8 – 17 =
					17 – 9 =		9 – 17 =
					18 – 9 =		9 – 18 =

Appendix B: Ethics

Approval

Department of Psycology

Ethics Committee for Psychology Research with Human Participants

Certificate of Ethics Approval

Principal Investigator	Department	StudyNumber
Nicole Robert	**Psychology**	**08-107**

Institution(s) where research will be conducted:

Carleton University

Co-Investigators and other researchers:

Researcher	Study Role	Position
Jo-Anne LeFevre	Faculty Sponsor	Faculty

Study Title: **Using Eye Movements to understand Simple Multiplication, Addition and Subtraction**

Approval Date: **01/26/2009** Validity Term: **9 Month** Approval Type:

Submitted Date	Study Component	Approval Date
01/26/2009	Addendum	01/26/2009

Comments:

Certification

The protocol describing the above-named project has been reviewed by the Committee for Ethics in psychology Research and the research procedures were found to be acceptable on ethical grounds for research involving human participants.

No Signature

Chair, Ethics Committee for Psychology Research

This Certificate of Approval is valid for the above term provided there is no change in the research procedures.

Close

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