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**Longitudinal and Transverse Diffusion Study of
Argon-Based Gas Mixtures by a TPC/MWPC Detector**

by

Larry Kum-Cheong Wong

A thesis submitted to the Faculty of Graduate Studies and Research in
partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Department of Physics

Carleton University

Ottawa, Ontario

July 1987

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The undersigned recommend to the Faculty of Graduate Studies
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"LONGITUDINAL AND TRANSVERSE DIFFUSION STUDY OF
ARGON-BASED GAS MIXTURES BY A TPC/MWPC DETECTOR"

submitted by Larry Kum-Cheong Wong, B.Sc., M.Sc.

in partial fulfilment of the requirements for
the degree of Doctor of Philosophy

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Abstract

The longitudinal and transverse diffusion coefficients σ_{LT} and the drift velocity v_d of argon-methane mixtures were measured simultaneously by a TPC/MWPC detector. Correlations of these three transport coefficients as a function of drift field E , pressure P and percentage of methane in argon have been investigated. Similar studies on argon-ethane mixtures have also been undertaken. Comparison with other measurements of these transport coefficients and with recent Monte Carlo simulation results are found to be consistent. Theoretical prediction by Parker and Lowke on the condition for $\sigma_L = \sigma_T$ is confirmed by this experiment.

Dedication

This thesis is dedicated to my Lord Jsus Christ.

*"Unless the LORD builds the house,
its builders labor in vain.*

*Unless the LORD watches over the city,
the watchmen stand guard in vain."*

Psalm 127:1-2

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Chapter 1

Introduction

Gaseous particle detectors are some of the most important instruments in high energy physics. Among them, the proportional counter has been used for detecting ionizing particles and radiation [1]. It consists of an isolated wire housed coaxially inside a conducting cylinder. High voltage is applied such that the wire is an anode with respect to the cylinder. Electrons liberated by ionizing radiation or ionizing particles which passed through the cylinder will drift along the radial electric field lines toward the anode wire. Beyond the threshold voltage, the electric field at the surface of the wire is sufficiently high such that an avalanche multiplication process¹ can occur.

For a proportional counter, beyond certain applied anode voltage V ,

¹ Electrons gain energy by acceleration due to the strong electric field near the anode. After acquiring enough energy, these electrons can ionize other molecules themselves. If the rate of ionization is higher than the rate of recombination, avalanche multiplication may start.

the gain-voltage relation is approximately exponential, i.e.,

$$G \propto e^{\alpha V}$$

If increasing the voltage will no longer increase the gain exponentially, the proportional counter is said to be operating in the saturated region or, if the gain does not increase at all, the Geiger-Müller operating region.

The amount of ionization depends on the energy and the type of the particle. Therefore, the pulse height information from the proportional chamber can provide additional information on the energy of the particle (commonly known as the dE/dx measurement [2]).

Single proportional counter has limited usages. A further development of the proportional counter as a particle detector is the multiwire proportional chamber (MWPC) and the drift chamber. For a MWPC, the coordinates of the charged particle tracks can be located by recording which wire "fired" while the total drift time² is being used to calculate the coordinate of a particle track in a drift chamber provided that the drift velocity is known. Reconstruction of the particle trajectories can provide momentum information when the particle travels through a magnetic field.

Tracking accuracy is stretched to its limits in vertex detectors. They are multiwire drift chamber designed specifically for the localization of the

²The signal from the drift chamber provides the 'stop' time t_2 . An external device, the trigger, will provide the start time t_1 to calculate the total drift time t_{drift} :

$$t_{drift} = t_2 - t_1$$

decay vertex for weakly interacting particles, for example, the K_s^0 which has a mean decay length cr of 2.7 cm, the tau lepton τ and the charmed mesons D^{*+0} which have mean decay lengths³ of hundreds of micron. To locate these latter decay vertices, extremely accurate spatial measurement of the order of several tens of μm has to be achieved.

Of all the factors affecting the spatial resolution of a drift chamber (see page 6 for discussions), diffusion is the least known. It depends on the physical properties of the gases being used. Therefore, a better understanding of the physics behind drift and diffusion of electrons in gases is crucial to the success of a vertex detector.

To measure the drift and diffusion of electrons, special gaseous detector has to be constructed. The following section reviews some of these gaseous tracking devices.

1.1 Tracking Detectors

In 1967, Charpak and collaborators [3] successfully developed a multiwire proportional chamber (MWPC). A MWPC consists of a set of proportional wires of equal spacing a sandwiched between two parallel cathode plates of spacing L (Figure 1.1). The electric field between the cathode and the anode plane is very uniform except at the region close to the anode wire [4].

³Due to relativistic effect, the decay length l_{decay} is stretched by an energy dependent Lorentz factor γ , $l_{decay} = \beta\gamma cr$. But particle decay follows an exponential distribution. For a cr of 100 μm , decay length as short as 50 μm or even shorter or as long as 1000 μm or even longer for very high energy machine such as the LEP e^+e^- machine is possible.

Figure 1.1: Cross section of a Multiwire Proportional Chamber.

High energy particles passing through the chamber will ionize gas molecules inside the chamber and these ionized electrons will drift along the electric field lines towards the anodes. As in a proportional tube, the high anode voltage ⁴ will create a strong electric field at the surface sufficient for the avalanche multiplication process to occur. Anode wires which produce a signal in this way will provide the coordinates of the charged particles. Since the anode-to-anode separation a is of the order of mm , the average spatial resolution achieved by a MWPC is $a/2$ or, the r.m.s. spatial resolution is $\sim a/\sqrt{12}$. This is not sufficient for modern tracking devices.

The other common tracking device is the multiwire drift chamber with a basic construction shown in Figure 1.2. The principle of operation of a drift

⁴Or the neighboring electrodes are negative with respect to the anode to create a positive potential difference.

Figure 1.2: Cross section of a multiwire drift chamber.

chamber is similar to a MWPC. The major difference is that the location of ionization is determined by the drift time t_d :

$$z = v_d \times t_d.$$

In general, the drift velocity v_d is a function of the electric field E . If the field is not uniform (which is always true near the anode wire), i.e. $E = E(z)$, then the drift distance z is given by:

$$z = \int_0^{t_d} v_d dt.$$

A further development of the drift chamber is the Time Projection Chamber. A TPC can measure all three spatial coordinates. It consists of a large volume drift space with sense wire/cathode pad readout modules located at the end of the drift space (Figure 1.3). The drift time from the track to the anode wire defines the z -coordinate.

When the drifting electrons hit the sense wire and form avalanches, they induce charges on the cathode pads. From the centroid of this induced charge distribution another coordinate, the lateral x -coordinate, can be calculated. The spatial resolution in the x -direction σ_x can be estimated from the width of this induced charge distribution. The location of the sense wire of the corresponding "hit" will give the third coordinate, the y -coordinate.

1.2 Factors Affecting Spatial Resolution

There are at least six major factors which the spatial resolution σ_x , position resolution along the drift direction, depends on [5] [6], namely:

1. the accuracy of construction;
2. the uniformity of drift field or how well the spatial coordinate-drift time relationship is known;
3. the capability of the detector electronics such as digitizing accuracy⁵, independence of pulse height etc.;
4. ionization fluctuation;
5. track width uncertainty due to δ -rays;
6. the intrinsic diffusion of the electrons.

⁵A time-to-digital converter (TDC) with a resolution of 1 ns cannot be used to achieve an average spatial accuracy of better than 25 μm or a r.m.s. accuracy of $50/\sqrt{12}$ μm for a typical drift velocity of 5 $\text{cm}/\mu\text{s}$.

Figure 1.3: Schematic drawing of the anode-cathode pad readout module of a TPC.

The first three items contribute to the systematic error and usually can be minimized by software calibration methods and better machining while the last three are random in nature. Suppose that the spatial resolution σ_z can be split into four major components;

$$\sigma_z^2 = \sigma_{\text{systematic}}^2 + \sigma_{\text{ionization}}^2 + \sigma_{\text{track}}^2 + \sigma_{\text{diffusion}}^2 \quad (1.1)$$

Let us examine each component separately.

With today's modern numerical controlled machining, a mechanical accuracy of $10 \mu m$ can be achieved. Many drift chambers are designed to achieve several tens of μm so mechanical uncertainties of this magnitude are comparatively tolerable.

The electrostatic field of a complicated detector geometry can be calculated and modified to achieve certain desired accuracy. If one uses a gas mixture such as argon-ethane 50:50 which has saturated⁶ drift velocity a linear spatial coordinate-drift time relationship can be realized and a small local electric field variations can be tolerated.

TDCs with a resolution of $50 ps$ and a range of $100 ns$ are available nowadays. At a drift velocity of $5 cm/\mu s$, the corresponding spatial resolution for a 11-bit device is $2.5 \mu m$ and the maximum drift length is $\sim 5 mm$. If a slower gas such as argon-carbon dioxide mixture or even pure CO_2 is used, same spatial resolution can be attained by a lower resolution TDC.

⁶If drift velocity does not increase nor decrease by further increase of drift field, the drift velocity is said to be saturated.

The ionization contribution to the spatial resolution $\sigma_{\text{ionization}}$ refers to the fluctuation of primary ionization⁷. The distribution of ionized electrons with energy follow the Landau distribution [1]. These electrons are not evenly distributed along the track but are created in clusters at the position of the primary ionization. $\sigma_{\text{ionization}}$ depends on the average distance between two clusters $l_{\text{cluster}} = 1/n$ where n is the average number of primary ion pairs/cm [7]. Higher pressure will reduce $\sigma_{\text{ionization}}$ by increasing ionization n . This ionization contribution is at a maximum for particles passing close to the anode wire. What is also important is the relative direction of the drifting electrons to the particle tracks.

The Landau energy loss distribution mentioned above has a long "tail" in the higher energy region. That means that occasionally there will be some very high energy electrons produced which are known as the δ -rays. For example, according to the calculation of Sauli [1], there are about ten electrons with energy above 15 eV emitted per cm of argon at N.T.P. by a 1 GeV/c proton. It is also estimated that a 1 GeV/c proton will produce about 0.05 electron per cm of argon with energy more than 3 keV and which travels 100 μm . The presence of these δ -rays cause imprecision in the track location and hence the effective track width seen by the sense wire is broader. This imprecision is the σ_{track} contribution to the global spatial resolution σ_z . Although increasing the pressure will reduce the range of the δ -rays, the number of δ -rays will also be increased. The remaining error

⁷ When a high energy particle passing through a gas, a discrete number of electron-ion pairs will be created due to primary ionizing collisions. If the liberated electrons themselves have sufficient energy, they can ionize the gas further producing secondary electron-ion pairs. The sum of the primary and secondary ionization is called total ionization.

contribution is due to the intrinsic diffusion of the drifting electrons.

If the trajectory of an ionized electron, under the influence of an external field, follows a well defined traceable path, the location of the electron after travelling a certain distance can be exactly known. But in reality a drifting electron will undergo many millions of collisions with the molecules and the trajectory cannot be predicted exactly⁸. The best one can predict is the probability of an electron landing at a certain location. Therefore, diffusion will increase the uncertainty of locating a drifting electron and hence decreases the spatial resolution.

The contributions $\sigma_{\text{systematic}}$, $\sigma_{\text{ionization}}$ and σ_{track} can be dealt with by other methods. Thus, this thesis will be concentrating on the diffusion contribution to the spatial resolution.

1.3 Physical Factors Affecting Drift and Diffusion of Ions

There are several known physical factors which affect the drift and diffusion of ions, namely, pressure, temperature, gas mixtures, electric and magnetic fields. In a more fundamental level, these physical factors affect the energy of the electrons, the scattering probability of an electron with the molecules and the inelasticity of the scattering.

The pressure for a fixed volume is a measure of the density of the

⁸Diffusion, which is similar to a three-dimensional random walk problem, is stochastic in nature.

molecules. Higher pressure implies higher probability for a drifting electron colliding with a molecule, therefore, the mean free path will become shorter. The net effect will be a decrease in the drift velocity and diffusion in general.

Temperature is a measure of the kinetic energy of the molecules. At room temperature, the thermal energy of the gas molecules is ~ 0.025 eV. Under certain conditions (cf. Eq. 2.15), this thermal energy of the gas molecules, compared with an energy of eV for the drifting electrons, can be neglected⁹. It is generally assumed that the gas molecules can be regarded as having zero energy, or motionless. For a fixed volume system small temperature fluctuations are taken into account through pressure changes.

Changing the electric field strength will systematically alter the kinetic energy or the "temperature" of the electrons and since electron-molecule cross section is energy dependent, the scattering cross section and the mean free path will be altered. If the energy of the electrons is sufficiently high such that inelastic collisions can occur, the electrons may lose or gain a large fraction of their energy.

Different kinds of molecule have drastically different scattering cross sections. Small changes in the gas mixture also have a drastic effect on drift and diffusion. For example, experimental data from Schmidt [8] showed that at $E/P = 25$ V/cm/bar, 100 ppm of water in argon-methane 90:10 mixture will alter the drift velocity by as much as 10% because cross section

⁹The mean energy of the electrons in a gas like CO₂, which has large vibrational and rotational inelastic cross section, is close to the molecular thermal energy and hence, this molecular thermal energy cannot be neglected.

of water molecule is about 3 orders of magnitude larger than argon or methane.

1.4 Thesis Outline

The purpose of this thesis is to investigate:

1. How drift velocity v_d , the longitudinal diffusion σ_L and the transverse diffusion σ_T vary as a function of electric field E and pressure P .
2. How v_d , $\sigma_{L,T}$ vary by changing the proportion of methane or ethane in argon.
3. How v_d and $\sigma_{L,T}$ are correlated.
4. How does the experimental data on the transport coefficients compare with theoretical prediction.

The transport coefficients v_d , $\sigma_{L,T}$ are measured simultaneously using the same gas sample so as to avoid errors due to different gas mixtures having slightly different compositions and impurities.

Chapter 2 gives an introduction to the theory of drift and diffusion. The Boltzmann Transport Equation will be derived and solved by the so-called "two-term approximation". Formulas for the transport coefficients v_d and $\sigma_{L,T}$ will be expressed explicitly in terms of a distribution function and a scattering cross section (2.20-2.22). Anisotropic nature of diffusion will be discussed and understood by the so-called "bunching mechanism".

A detailed description of the MWPC/TPC detector for the measurement of the drift velocity and the diffusion coefficients will be given in chapter 3. The associated electronic system, the gas monitoring system, the method of triggering and the method of calibration will also be discussed in detail.

Chapter 4 describes in detail the methods of measuring the longitudinal and the transverse diffusion coefficients. The pulse height of the Fe^{55} radioactive source and the cosmic ray signal are used to investigate the stability of the gas gain as a function of time and drift length. This chapter also studies the effect on the longitudinal diffusion coefficient due to triggering and the associated correction factor. The constancy of the diffusion coefficients will be investigated with respect to the gas gain variation, the angular acceptance and the nonlinearity of the electric field.

Chapter 5 summarizes the measurements of the drift velocity, the longitudinal and the transverse diffusion coefficients measurements as a function of electric field and pressure for argon-methane mixtures. Similar measurements for argon-ethane mixtures were also taken to test the validity of the conclusions drawn from the argon-methane measurements. Comparison with other published experimental results and recent Monte Carlo simulation will also be presented. Validity of the theoretical prediction of chapter 2 will be checked. The diffusion coefficients are parametrized as a function of E and P and the results are tabulated in this chapter.

Chapter 6 summarizes the findings of this thesis.

Chapter 2

Theory of Drift and Diffusion

Classical kinetic theory has a long history dating back to the last century. On the theoretical side the transport equation was derived by L. Boltzmann and further developed by J. Maxwell to give the famous Maxwell-Boltzmann distribution function. Einstein wrote down the diffusion equation on the theory of Brownian motion at the beginning of this century. Nernst-Townsend derived formulas relating the drift velocity v_d and the diffusion coefficient D . On the experimental side the first measurement of mobilities of gaseous ions was performed by E. Rutherford and the first measurement of diffusion coefficients of gaseous ions was done by J. Townsend [9].

According to the Boltzmann Transport Equation, if the differential scattering cross section $d\sigma/d\Omega$ of a gas is known then all the transport coefficients can be calculated.

2.1 Einstein Diffusion Equation

Consider the continuity equation relating the flow of particles into or out of a volume to the increase or decrease of particles in the volume. Conservation of particle number implies

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad (2.1)$$

where $\mathbf{J}(\mathbf{r}, t)$ is the particle flux and $n(\mathbf{r}, t)$ is the number of particles per unit volume. The flux $\mathbf{J}(\mathbf{r}, t)$ is caused by two contributions, namely, the flow of particles due to an external force $\mathbf{F}$ and the flow due to diffusion:

$$\mathbf{J} = c\mathbf{F}n - D\nabla n, \quad (2.2)$$

where D is the diffusion coefficient and c , a proportionality constant. The negative sign in (2.2) is due to the fact that particles always diffuse from a higher concentration to a lower concentration, i.e., diffusion flux is antiparallel to the particle density gradient. Substituting (2.2) into (2.1) leads to a Fokker-Planck type of equation:

$$\frac{\partial n}{\partial t} = c\nabla \cdot (\mathbf{F}n) - D\nabla^2 n. \quad (2.3)$$

In the absence of any external fields $\mathbf{F}$, it reduces to the Einstein diffusion equation:

$$\frac{\partial n}{\partial t} = D\nabla^2 n. \quad (2.4)$$

The solution of (2.4) in three dimensions is given by:

$$n(\mathbf{r}, t) = \frac{n_0}{(4\pi Dt)^{3/2}} e^{-r^2/4Dt}. \quad (2.5)$$

In the presence of a uniform electric field (defining $\mathbf{F} = eE\hat{\mathbf{z}}$)¹ the diffusion coefficient D need not necessarily have the same value in all three directions (i.e. isotropic) as assumed in the past. A more general expression for (2.3) is given by:

$$\frac{\partial n}{\partial t} = D_T \left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} \right) + D_L \frac{\partial^2 n}{\partial z^2} + v_d \frac{\partial n}{\partial z} \quad (2.6)$$

L stands for the longitudinal direction and T, the transverse direction with respect to electric field axis. The solution for (2.6) is now

$$n(x, y, z, t) = \frac{n_0}{4\pi D_T t (4\pi D_L t)^{1/2}} \exp \left(-\frac{x^2 + y^2}{4D_T t} \right) \exp \left[-\frac{(z - v_d t)^2}{4D_L t} \right] \quad (2.7)$$

2.2 Boltzmann Transport Equation

The diffusion equation describes only the spatial distribution of particles, it says nothing about their energy distribution. In the absence of any external fields, we expect the energy distribution of ions to be Maxwellian, i.e.,

$$f(\epsilon) \sim \sqrt{\epsilon} e^{-\epsilon/kT}$$

In the presence of external fields, ions will be accelerated by the field and decelerated by ion-molecule collisions. Therefore, the energy distribution $f(\epsilon)$ will no longer be Maxwellian. To calculate this distribution function one has to resort to the Boltzmann Transport Equation.

Consider the distribution function $f(\mathbf{r}, \mathbf{v}, t)$ defined such that¹⁰

$$f(\mathbf{r}, \mathbf{v}, t) d^3x d^3v$$

¹ Purely from dimensional analysis, the factor eF in (2.3) has the dimensions of velocity. In fact, it is the drift velocity v_d with $v_d = ceE$

is the number of ions located within a phase space element $d^3x d^3v$ about $(\mathbf{r}, \mathbf{v})$ at time t . In the presence of an external force $\mathbf{F}$, the centre of the phase space volume $(\mathbf{r}, \mathbf{v})$ will be translated to $(\mathbf{r} + \mathbf{v}\delta t, \mathbf{v} + \frac{\mathbf{F}}{m}\delta t)$ between t and $t + \delta t$. Suppose an ion-molecule collision occurs during $(t, t + \delta)$, the corresponding change in distribution function will be

$$f(\mathbf{r} + \mathbf{v}\delta t, \mathbf{v} + \frac{\mathbf{F}}{m}\delta t) - f(\mathbf{r}, \mathbf{v}) = \left(\frac{\partial f}{\partial t} \right)_{coll} \delta t. \quad (2.8)$$

In fact, (2.8) can be considered as the definition of $\left(\frac{\partial f}{\partial t} \right)_{coll}$. Expanding the first term of (2.8) up to first order of δt will lead to

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} \right) f(\mathbf{r}, \mathbf{v}, t) = \left(\frac{\partial f}{\partial t} \right)_{coll} \quad (2.9)$$

Due to the limited data on the differential cross section of the gas molecule-ion collision, the Boltzmann Transport equation has not been solved analytically. Many authors have tried various techniques to reduce the equation to a solvable form such as the so-called "two-term approximation" by Morse, Allis and Lamar [11], Parker and Lowke [12] [13], Huxley [9], Palladino and Sadoulet [14], Schultz and Gresser [15]; the "three-term approximation" by Ferrari [16]; the path integral method by Skullerud and Kuhn [17]; the moment method by Lin, Robson and Mason [18]; the numerically exact Monte Carlo simulation by Kubo [19], Skullerud [20], [21], Binnie [22], Fraser and Mathieson [23] [24].

The "Two-term approximation" is mathematically the simplest and has been studied some 40 years ago by Morse et. al. By expanding the one-dimensional Maxwell-Boltzmann distribution function $f(z, v, \theta)$ in a series

of Legendre polynomials with $\cos \theta = v_z/v$ and assuming that it is sufficient to keep only the first two terms then $f(z, v, \theta)$ can be reduced to

$$f(z, v, \theta) = f_0(z, v) + f_1(z, v) \cos \theta. \quad (2.10)$$

If we further assume the distribution of electrons is uniform then $f(z, v, \theta) = f(v, \theta)$, or in terms of energy ϵ :

$$F(\epsilon, \theta) = F_0(\epsilon) + F_1(\epsilon) \cos \theta. \quad (2.11)$$

With this approximation, the Boltzmann equation is reduced into two coupled equations² as appeared in Schultz and Gresser's paper 15:

$$eE \frac{\partial}{\partial \epsilon} (\sqrt{\epsilon} F_0) - \frac{eEF_0}{\sqrt{\epsilon}} + \frac{\epsilon F_1}{l_r} = 0 \quad (2.12)$$

$$\frac{eE}{3} \frac{\partial}{\partial \epsilon} (\sqrt{\epsilon} F_1) - \frac{2m}{M} \frac{\partial}{\partial \epsilon} \left(\frac{\epsilon^{\frac{3}{2}} F_0}{l_r} \right) = 0, \quad (2.13)$$

where m, M are the electronic and molecular masses respectively and $l_r(\epsilon)$ is the mean free path between elastic collisions given by

$$l_r(\epsilon) = \frac{1}{N\sigma_r(\epsilon)}$$

where N is the number of molecules per unit volume. With some algebra $F_0(\epsilon)$ is found to be

$$F_0(\epsilon) = C \sqrt{\epsilon} \exp \left(- \int \frac{3\Lambda(\epsilon)\epsilon}{eEl_r(\epsilon)^2} d\epsilon \right). \quad (2.14)$$

²For "three-term approximation" there will be three coupled equations and N-coupled equations for the moment method.

where C is the normalization constant such that

$$\int F_0(\epsilon) d\epsilon = 1.$$

Equation (2.14) is accurate only if the thermal energy of the molecules kT is much smaller than the energy of the drifting electrons. For a more general case with the motion of the molecules is taken into consideration, (2.14) is modified to [25]

$$F_0(\epsilon) = C \sqrt{\epsilon} \exp \left(- \int \frac{3\Lambda(\epsilon)\epsilon}{eEl_e(\epsilon)^2 + 3\Lambda(\epsilon)\epsilon kT} d\epsilon \right). \quad (2.15)$$

Λ , the mean energy gain or loss during collision, equals to $2m/M$ if the scattering cross section is a constant, though in general, it is a function of energy $\Lambda(\epsilon)$:

Although the distribution function is calculable in principle, experimentally, what are being measured are the transport coefficients such as the drift velocity v_d , the longitudinal and transverse diffusion coefficients $D_{L,T}$ as in (2.7) or the r.m.s. dispersion in the longitudinal and transverse direction $\sigma_{L,T}$. Given F_0 and F_1 one should be able to calculate these transport coefficients. Again using the "two-term approximation" and a Fourier transform approach, Parker and Lowke have solved the Boltzmann transport equation and derived formulas for v_d , D_T and a formula relating D_L and D_T all in terms of $F_0(\epsilon)$, $F_1(\epsilon)$ and the momentum transfer scattering cross section $\sigma_m(\epsilon)$.

Parker and Lowke assumed a slightly more general expression for the distribution function given by:

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{r}, \mathbf{v}, t) + \hat{\mathbf{v}} \cdot \mathbf{f}_1(\mathbf{r}, \mathbf{v}). \quad (2.16)$$

where $\hat{v}$ is the unit velocity vector. Equation 2.16 is true only if the distribution function is approximately spherically symmetric, hence, only the first two terms of an expansion in spherical harmonics is sufficient. The corresponding coupled partial differential equations are given by:

$$\sqrt{\frac{2\epsilon}{m}} \frac{\partial f_0}{\partial t} = \frac{4N}{M} \frac{\partial}{\partial \epsilon} \left[\epsilon^2 \sigma_m \left(f_0 + kT \frac{\partial f_0}{\partial \epsilon} \right) \right] - \frac{2\epsilon}{3m} \nabla \cdot \mathbf{f}_1 - \frac{2eE}{3m} \frac{\partial}{\partial \epsilon} (\epsilon \hat{z} \cdot \mathbf{f}_1) \quad (2.17)$$

$$\mathbf{f}_1 = \frac{1}{N\sigma_m} (\nabla f_0 + eE \frac{\partial f_0}{\partial \epsilon} \hat{z}), \quad (2.18)$$

where the electric field $\mathbf{E} = E\hat{z}$. Solving (2.17) and (2.18) gives:

$$f(\epsilon, z, t) = \frac{1}{\sqrt{4\pi D_L t}} \cdot \left(F_0 + F_1 \frac{\partial}{\partial z} + F_2 \frac{\partial^2}{\partial z^2} + \dots \right) \cdot \exp \left[-\frac{(z - v_d t)^2}{4D_L t} \right] \quad (2.19)$$

in which the drift velocity v_d , σ_L and σ_T are calculated from the first two expansion coefficients F_0, F_1 of $f(\epsilon, z, t)$ given by [8]³:

$$v_d = -\frac{1}{3} \sqrt{\frac{2}{m_e}} \frac{eE}{N} \int_0^\infty \frac{\epsilon}{\sigma_m} \frac{\partial F_0}{\partial \epsilon} d\epsilon \quad (2.20)$$

$$D_T = \frac{1}{3} \sqrt{\frac{2}{m_e}} \frac{1}{N} \int_0^\infty \frac{\epsilon}{\sigma_m} F_0(\epsilon) d\epsilon \quad (2.21)$$

$$D_L = D_T + \frac{1}{3} \sqrt{\frac{2}{m_e}} \frac{eE}{N} \int_0^\infty \frac{\epsilon}{\sigma_m} \frac{\partial F_1}{\partial \epsilon} d\epsilon + v_d \int_0^\infty \sqrt{\epsilon} F_1(\epsilon) d\epsilon \quad (2.22)$$

³The expressions in (2.20-2.22) differ from Parker and Lowke by a normalization factor $4\pi/m\sqrt{2/m}$.

The diffusion coefficients $D_{L,T}$ are not measured in general but rather, the r.m.s. dispersion per cm of electron drift $\sigma_{L,T}$ is measured and is related to $D_{L,T}$ by the following formula:

$$\sigma_{L,T}(1cm) = \sqrt{\frac{2D_{L,T}}{v_d}} \quad (2.23)$$

If we define a to be:

$$a(\epsilon) = \int_0^\infty \sqrt{\epsilon} a(\epsilon) d\epsilon,$$

then (2.20-2.22) can be condensed elegantly:

$$\frac{1}{2}(\sigma_L^2 - \sigma_T^2) = \Delta(E, \alpha_i) \quad (2.24)$$

$$F_1 = \frac{g(\epsilon)F_1'(\epsilon)}{g(\epsilon)F_0'(\epsilon)}$$

where

$$g(\epsilon) = \frac{v \epsilon}{\sigma_m(\epsilon)}$$

$$F' = \frac{\partial F}{\partial \epsilon}$$

E : Drift field,

α_i : proportion of the i th gas component.

The diffusion coefficient difference $\Delta(E, \alpha_i)$ can be negative, zero or positive depending upon the values of E_0 , F_1 and their derivatives as well as $\sigma_m(\epsilon)$.

F_0 and F_1 have drift field E as parameter while the scattering cross section $\sigma(\epsilon)$ has α_i as parameter, thus, Δ depends on both E and α_i .

Experimental data by Wagner et. al. [26] and Drumm et. al. [27] and more recent data by Fulda-Quenzer et. al [28] showed that the longitudinal diffusion does deviate from the transverse diffusion significantly at certain region of drift field E . Figure 2.1-2.2 are the plots of D/μ versus normalized electric field where μ is the mobility of electrons in gas given by

$$\mu = \frac{v_d}{E}$$

It is not so obvious from their data that the transverse diffusion coefficient σ_T can be smaller than the longitudinal diffusion coefficient σ_L at low drift field as implied from the theoretical calculation of Parker and Lowke and the Monte Carlo simulation of Fraser and Mathieson (Figure 2.3).

2.3 Anisotropic Diffusion

There are several plausible explanations to this anisotropic diffusion problem offered by Parker and Lowke [12]; Huxley and Crompton [9]; Palladino and Sadoulet [14]; and Binnie [22]. The anisotropy can be understood by the so-called "bunching" mechanism⁵.

Due to the fact that distribution of electrons $n(r,t)$ is not spatially uniform, the diffusion current (the second term of Eq. 2.2: $-D\nabla n$) affects

⁵In modern accelerator, anti-particles have to be stochastically cooled to a narrow bunch in phase space. Inside the particle beam some particles move faster and some slower. A sensor will pick up beam signal and then amplifies and filters it. This signal will then be sent ahead in time via a diametrical path to control a "kicker" a ferrite ring or an RF cavity. The overall effect of this kicker is such that those particles which are moving too fast will be decelerated and those which are too slow will be accelerated, i.e., they are bunched. After successive orbiting, phase space of the beam will be squeezed, i.e., particles confined in a small volume with approximately the same momentum

Figure 2.1: D/μ data of Argon at 77°K. E/N is the normalized electric field where N is the number of molecules per unit volume. Solid lines are theoretical calculation by Parker and Lowke [12].

Figure 2.2: Diffusion coefficient D/μ for Ar-methane-isobutane by Drumm et.al. 27

Figure 2.3: D_L, D_T Monte Carlo results for $Ar-CH_4$ 90:10 and 80:20 mixtures with the latter shifted by $1 V cm^{-1} Torr^{-1}$. Solid and dashed curve are simulation results by Fraser and Mathieson [23]. Measurements—filled circles and squares by Siegmund [29] and open circles by Skelding [30].

the energy distribution of the electron swarm. Define $\epsilon_m(z)$ to be the mean energy of the electron distribution which is a function of position. According to the theoretical calculation of Parker and Lowke, ϵ_m is given by

$$\epsilon_m = \epsilon_0 \left[1 - \frac{2\epsilon_0}{3eE \left(1 + 2 \frac{\partial \nu}{\partial \epsilon} \bigg|_{\epsilon_0} \frac{\epsilon_0}{\nu_0} \right)} \frac{1}{n} \frac{\partial n}{\partial z} \right], \quad (2.25)$$

where $\nu(\epsilon)$ is the electron collision frequency; ϵ_0 and ν_0 are the mean energy ϵ_m and collision frequency ν evaluated at the point where density gradient $\partial n / \partial z$ is zero. According to (2.25), electrons at the leading edge where density gradient $\partial n / \partial z < 0$ have mean energy higher than the mean energy of the trailing edge electrons where $\partial n / \partial z = 0$.

Consider the elastic scattering cross section $\sigma_{el}(\epsilon)$ of argon which has a sharp local minimum known as the Ramsauer minimum⁶ (Figure 2.4). Beyond the Ramsauer minimum, the elastic cross section of argon is rising with energy $d\sigma_{el}/d\epsilon > 0$. Hence, electrons at the leading edge will experience more collisions due to their higher mean energy ϵ_m and slow down while electrons at the trailing edge will experience fewer collisions and catch up. The net effect is such that the electron swarm is "bunched" in the longitudinal direction.

The opposite is true if $d\sigma_{el}(\epsilon)/d\epsilon < 0$. In more precise terms according

⁶From the phase shift analysis [31], the total cross section is given by.

$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 \quad k = \frac{1}{a},$$

where δ_0 , a and k are the phase shift, the radius of the atom and the wave vector of the incident electron respectively. If the potential of an atom has the right magnitude, δ_0 can have the value of $n\pi$, i.e., $\sigma = 0$. This is known as the Ramsauer-Townsend effect

Figure 2.4: Electron-argon scattering cross section as a function of energy ϵ .

to Parker and Lowke⁷:

$$\begin{aligned} \text{if } \frac{\partial \nu}{\partial \epsilon} \Big|_{\epsilon_0} < 0, \text{ then } \sigma_L > \sigma_T; \\ \text{if } \frac{\partial \nu}{\partial \epsilon} \Big|_{\epsilon_0} = 0, \text{ then } \sigma_L = \sigma_T; \\ \text{if } \frac{\partial \nu}{\partial \epsilon} \Big|_{\epsilon_0} > 0, \text{ then } \sigma_L < \sigma_T. \end{aligned} \quad (2.26)$$

where collision frequency ν is related to the collision cross section by

$$\nu(v) = Nv\sigma(v),$$

where v is the instantaneous velocity and N , the number of molecules per unit volume. In terms of energy ν becomes:

$$\nu(\epsilon) = N \sqrt{\frac{2\epsilon}{m}} \sigma(\epsilon). \quad (2.27)$$

Transport phenomena can be better understood if this anisotropic nature of diffusion and its dependence on physical parameters such as the drift field, the pressure and the gas compositions, is accurately measured.

⁷Reader should caution that σ is being used to represent the cross section $\sigma(\epsilon)$ and the diffusion coefficients $\sigma_{L,T}$.

Chapter 3

Apparatus

3.1 Detector

The diffusion data shown previously in Figure 2.1 are taken by three different experiments and are therefore subject to three different sets of systematic errors. In fact one can measure all three transport coefficients in a single experiment using a modified MWPC with cathode pad readout. This is particularly important when dealing with gas mixtures as a small amount of impurity or a small change in the mixture can drastically alter the data.

A detector was constructed which consists of a 15 cm long uniform drift field with a multiwire proportional cell at one end (Figure 3.1-3.2). There are 5 anode wires each of 10 μm radius and 20 mm spacing. Underneath the central anode wire are 40 (only 24 implemented) rectangular cathode readout pads which form a Time Projection type of detector (Figure 3.3).

The triangular readout pads underneath the outer anode wires are based

- 20 μm Anode Wire (+HV2)
- 100 μm Guard Wire (Grounded)
- 100 μm Field Shaping Grid Wire (Grounded)

Figure 3.1: Cross sectional view of the MWPC/TPC cell.

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FIGURE 3.2 HAS BEEN FILMED SEPARATELY;
THE CORRESPONDING OVERLAY AND THE FIGURE
HAVE THEN BEEN FILMED TOGETHER.

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LE TRANSPARENT SUPERPOSE ET LA FIGURE FURENT
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C

Figure 3.2: Photograph of the MWPC/TPC cell.

Figure 3.2: Photograph of the MWPC/TPC cell.

oration [32]. These outer triangular pads can be used to form a track in the "y direction" with position calculated from the "charge division" measurement. Since the rectangular cathode readout pads already produced very clean data samples, the triangular pads are not used in this experiment.

The longitudinal diffusion σ_L is determined from a straight line fit to the drift times of the 5 wires so that the fitted position of each wire can be compared with the measured position. The difference between the fitted value and the measured value forms a distribution. From this distribution σ_L can be estimated. The drifting electrons also spread out transversely with respect to the electric field axis (z -axis). The transverse diffusion coefficient σ_T is determined from the width of the induced charge distribution, measured on the cathode readout pads (see Figure 1.3-3.3).

Avalanches from the neighboring wires may also induce charges on the central cathode pads. Theoretical calculations (see Appendix A) indicated that if the pad dimension is $20\text{ mm} \times 2\text{ mm}$ and an anode-to-cathode separation is 2 mm then neighboring charge induction will be negligible and is confirmed by the data. An anode-to-anode separation of 20 mm also reduces cross-talk significantly compared with an earlier prototype of 10 mm spacing.

The drift velocity is determined from the known length of the drift space together with a measurement of the longest drift times recorded. An alternate method for velocity measurement was installed. Three small mirrors separated by a known distance l and oriented at 45° with respect to the anode plane are mounted on top of the detector (Figure 3.4). Ultra-

Figure 3.3: Schematic diagram of the rectangular and triangular (shaded area) cathode readout pads.

violet laser beam with sufficient intensity directed towards these mirrors will provide ionizing tracks which give three well defined drift time distributions. An accurate drift velocity can then be measured. Unfortunately the laser available was not strong enough and the experiment had to rely on the first approach.

Diffusion measurements are sensitive to the presence of impurities, in particular, water and oxygen molecules. These impurities may also change the gas gain significantly. An Fe^{55} radioactive source is planted near the central anode wire for a continuous gas gain monitoring. By monitoring the gas gain continuously we can detect if there is any significant pressure vessel leakage. This Fe^{55} source can also be used as gas gain reference to maintain consistent gas gain from run to run. A proportional tube equipped with another Fe^{55} source is also installed inside the pressure vessel for independent gas gain monitoring.

The pressure vessel has a Datametrix Type 590 pressure transducer of accuracy of 0.05% and a Balzers NV3 vacuum gauge head installed. The accuracy of the gas mixing which was done by partial pressures is estimated to be $\sim \pm 0.5\%$. Before each set of measurements the pressure vessel was evacuated and flushed with fresh gas to prevent possible contamination due to leakage.

Argon gas of 99.998% purity; methane, ethane gases of 99% commercial purity and pre-mixed argon-methane 90:10 and 80:20 ($\pm 1\%$) gas mixtures commonly known as P10 and P20 respectively are being used in this experiment. The mixing error gives rise to a small error in the drift velocity.

Figure 3.A: Cross sectional view of the detector system.

Consider the following example: drift velocity of P10 and P20 are respectively 6.50 ± 0.32 and 4.55 ± 0.23 cm/ μ s at $E = 200$ V/cm. Therefore an error of $\pm 1\%$ in gas mixing will contribute to an error approximately 0.2 cm/ μ s in drift velocity (estimated by linear interpolation) which is well within experimental error.

3.2 Electronics

The anode wires are connected to a positive high voltage supply with each wire isolated from each other by an RC filtering network. Similar RC filtering is also installed for every single guard and grid wires so as to minimize cross-talk. The guard or grid wires can be connected to independent high voltage supply for further anode field optimization. Bertan model 380N and 1792P power supplies of 0.02% stability provide the high voltage for the drift box and the anode respectively.

The anode signals are amplified by a hybrid pre-amplifier RL 723 based on a BNL design [33] with a gain of $0.48 \mu\text{V}/e^-$ (Figure 3.5(a)). A typical anode pre-amp output is shown in Figure 3.6. These pre-amps are installed on the same printed circuit board where the anode wires are soldered. The anode signals are further amplified by a two-stage inverting MWA 110 amplifiers with a total gain of roughly 20 (Figure 3.5(b)). The anode signals are then discriminated by a LeCroy LRS 623 discriminator and then fed to a series of cascaded LeCroy LRS 2229 11-bit TDCs to form the stop signals.

Consider the following example. Using a fast gas with a drift velocity $\sim 60 \mu\text{m}/\text{ns}$, a single TDC of $5 \mu\text{s}$ range will be more than sufficient to cover a drift distance of 15 cm. For a 11-bit TDC the corresponding timing resolution is $2.5 \text{ ns}/\text{TDC count}$, i.e., a spatial resolution of $150 \mu\text{m}/\text{TDC count}$. If this gas has a longitudinal diffusion coefficient $\sim 300 \mu\text{m}/\sqrt{\text{cm}}$, this TDC will be too coarse to resolve tracks close to the wire, for instance, $r = 3 \text{ cm}$.

In order to accommodate gases with a large variation in drift velocity and diffusion, we have chosen to use 5 cascaded TDCs with different resolutions: finer resolution for events close to the wire and coarser resolution for events far away. The LRS 2229 TDCs can be selected from a time range of 100 ns to $5 \mu\text{s}$, hence, the range of the drift time can be up to $25 \mu\text{s}$. That is, this cascaded TDC system can measure a drift velocity ranging from $0.6 \text{ cm}/\mu\text{s}$ up to $30 \text{ cm}/\mu\text{s}$.

The cathode readout pads are connected to the LeCroy's 8-channel hybrid amplifiers HQV 810 with gain up to $500 \text{ mV}/\text{pC}$. The HQV 810's are housed inside the pressure vessel and are connected to the pads via a 20 cm ribbon cable to minimize the noise. These amplified signals are then fed to the LeCroy LRS 2249W ADCs to be digitized. The gate signal for the ADCs is derived from the anode signal of wire #3.

Ten percent of the anode signals is extracted from the MWA 110 main amplifier to provide pulse height information, in particular, wire signal #3

Figure 3.5: Circuit schematics (a) BNL based anode pre-amp RL 723; (b) Two stage MWA 110 main amplifier.

Figure 3.6: Typical shape of an anode signal amplified by an RL 723 pre-amp.

is fed to a LeCroy LRS 3001 qVt multi-channel analyser to collect the Fe^{55} x-ray spectrum.

3.3 Triggering and Calibration

Plastic scintillators provide a 3-fold coincidence cosmic ray trigger with a "transverse" (i.e. the x-direction) angular acceptance of $\sim \pm 7^\circ$ and a "longitudinal" (i.e. the z-direction) angular acceptance of $\sim \pm 27^\circ$. Ideally, the angular acceptance should be as narrow as possible so as to minimize any angular effects (refer to Section 4.5). But tighter angular acceptance drastically reduces statistics. Hence, the above choice is a compromise between statistics and possible angular effects. If required, a looser angular acceptance can always be made tighter by off-line software selection. A

typical set of data requires 24 hours of data taking and approximately 20,000 cosmic ray triggers will be recorded, i.e., a triggering rate of 0.23 per second. After the selections which will be described in chapter 4, only 27 % of the recorded events will survive the selection or ~ 3.7 accepted events per minute.

The coincidence signal from the photomultipliers is used to create the TDC start signal. The wire avalanche signal fed to the RL723 pre-amplifier will provide the TDC stop signal. From this start-stop interval, drift time of the electrons can be accurately calculated. An Ortec 462 time calibrator which provides a sequence of pulses of definite period (± 10 ps accuracy) is being used for TDC time calibration (Figure 3.7). These test pulses are fed to the test pin of the RL723 pre-amplifiers to form the test stop signals.

If the TDCs are not stopped by either the wire avalanche signals or test pulse signals, the TDCs will record an "overflow" i.e., a TDC count of 2048. Each TDC might have a slightly different full-scale time range and might also be slightly different from channel to channel. To make the timing for each channel consistent, the i -th TDC is stopped just before overflow and the $(i-1)$ th TDC is started at the same instant. That is, the TDCs are either stopped by the wire signal or by this common stop signal. The common start/stop signals for the 5 TDCs are created by a combination of cable delays and gate generator delays. Due to the nature of this start/stop arrangement, there is a dead region between two TDCs which can be seen from the drift time spectrum¹.

¹In fact, it is better to have an overlapping TDC system. In this way the dead region

Figure 3.7: Block diagram for the detector electronics.

The test pulses from the time calibrator create a periodic sequence of TDC time spectra $\{t_n\}$. Assuming the absolute time to TDC counts is linear, the average time of this time spectrum $\bar{t}_n$ is used to calculate the "slope" m_j^i and the "intercept" c_j^i of the i -th channel of the j -th TDC.

A series of calibration measurements are being taken at intervals of one hour, 24 hours and one week. The stability of the TDC system can be found by comparing the slopes m_j^i and the intercepts c_j^i of the above calibration files over a period of two weeks and the result is summarized below:

$$\begin{aligned} m_j^i &\leq 0.1\% & \forall i, j \\ c_j^i &\leq 1 \text{ ns} & i = 1, \dots, 5; \quad j = 1, \dots, 4 \\ c_j^i &\leq 2 \text{ ns} & i = 1, \dots, 5; \quad j = 5 \end{aligned}$$

between two TDCs can be eliminated. Due to the lack of appropriate electronics at the time when this experiment was running, we have chosen this start/stop configuration

Chapter 4

Data Analysis

The characteristics of the data are described in the following figures. Figure 4.1(a) is the spectrum of the vertical angle of the cosmic ray tracks. The asymmetry of the spectrum is due to the asymmetric positioning of the three scintillators so as to limit the angular acceptance of the cosmic ray tracks to $\sim \pm 27^\circ$. Figure 4.1(b) is a typical drift time spectrum. The spectrum is quite flat but falls off at the far edge. The decrease in events close to the far edge of the chamber can be attributed to the angular acceptance of the 3-fold coincidence cosmic ray trigger. At the far edge, only events which is quite vertical will trigger all 3 scintillators and this reduces statistics. Assuming a nominal chamber length of 15 cm, the maximum drift time t_{max} defines the drift velocity:

$$v_d = \frac{15 \text{ cm}}{t_{max}}$$

The drift time spectrum exhibits sharp dips. They are the dead region between two cascaded TDCs.

Figure 4.2(a) is a typical anode pulse height spectrum. The anode pulse height is proportional to the number of ionizations per 2 cm of sampling length. The anode pulse height spectrum does look Landau-like as expected. Figure 4.2(b) is a scatter plot of anode pulse height versus event number (i.e. time). When plotting the average anode pulse height versus time as in Figure 4.2(c), within experimental error, it is a constant. This is an indication that anode gas gain is very constant during the entire period of data taking.

Figure 4.3(a) is a typical Fe^{55} x-ray spectrum with the escape peak clearly visible. The main peak location, which is a measure of the gas gain, is then fitted with a gaussian curve with the average peak position plotted against event number (Figure 4.3 (b), (c)). The gas gain versus time plot 4.3(c) indicated that the gas gain is stable within 1% over the entire period of 24 hours data taking. This is an indication that the gas has not been contaminated much due to pressure vessel leakage. Even this 1% variation could be attributed to the temperature difference in the afternoon and in the evening.

A scatter plot of anode pulse height versus drift length (Figure 4.4) indicated that pulse height is fairly constant except may be a 6% to 8% dip¹ close to the wire (4.4(b)). This slight dip is due to a weak space charge effect². With a combination of the high gain RL 723 and the MWA 110

¹With the large number of events available, the statistical error associated with the gaussian fitting is approximately 1-2%. Therefore, this 6% to 8% dip is a real effect and not due to statistical fluctuation.

²Space charge effect is due to the high concentration of charges around the avalanche region which distort the local field considerably. As a result, electrons arriving soon after

amplifiers, the cell can be operating with a much lower gas gain such that space charge effect is negligible.

4.1 Longitudinal Diffusion Coefficient Estimation

The diffusion coefficient σ_L is calculated using a track fitting method. The procedures are as follows:

1. Select those events with all 5 drift lengths z_i within the drift range, i.e., $0 < z_i < 15 \text{ cm}$, $i = 1, \dots, 5$.
2. A linear fit is performed on these 5 data points and the slope s of the cosmic ray track from the fit are calculated. Select those events such that $|s| \leq s_{\max}$ ³.
3. Define the residual R_i of wire i to be $R_i = z_i^{\text{data}} - z_i^{\text{fitted}}$, $i = 1, \dots, 5$. Plotting R_i versus drift length z_i to form a scatter plot (Figure 4.5(a)). Divide each scatter plot into 5 slices and fit each slice with a gaussian curve to find its standard deviation w_j , $j = 1, \dots, 5$ (Figure 4.5(b)).
4. Plot w_j^2 against its corresponding average drift length l_j and fit it with a straight line of form $\sigma_z^2 = a^2 + b^2 z$ (Figure 4.5(c)).

the first electron will be screened by this 'space charge' thus effectively reducing the gain of the wire seen by the anode.

³To choose event to be as vertical as possible would require $s_{\max}$ to be as small as possible but this will limit statistics. Section 4.5 will discuss the optimal choice of $s_{\max}$.

(a)

(b)

Figure 4.1: Raw data of a typical data set (a) slope of the cosmic tracks; (b) drift time spectrum.

Figure 4.2: (a) Typical anode pulse height spectrum; (b) Scatter plot of anode pulse height vs. event number; (c) Average anode pulse height vs. event number.

(a)

(b)

(c)

Figure 4.3: (a) A typical Fe^{55} Spectrum; (b) Gaussian fit to the main peak; (c) Main peak versus time.

(a)

(c)

Figure 4.4: (a) Scatter plot of anode pulse height versus drift Length; (b) Average anode pulse height versus drift length.

The number b is the r.m.s. longitudinal dispersion per cm of electron drift $\sigma_L(1cm)$.

The longitudinal coefficients estimated by the 5 wires is tabulated in table 4.1. The reason for $\sigma_{L_1} < \sigma_{L_2} < \sigma_{L_3}$ is purely geometric in nature. The linear fit being used is a least-square fitting algorithm. Suppose z_1 is very much off from the true track, the fitted track will thus be biased towards z_1 . Similarly for z_2, z_3, z_4, z_5 . Consequently, the fitted position z_1^{fitted} is smaller than the true position z_1^{true} . This effect is more pronounced in z_1 than in z_5 .

In order to correct this "fitting bias", a Monte Carlo program was written to simulate the diffusion data and calculate the required correction factor for each wire. These correction factors agreed with the theoretically calculated result (see Appendix B). After applying this correction factor to each wire, their respective $\sigma_L(1cm)$ agreed with each other (Table 4.1).

Wire no.	$\sigma_L(1cm)$ before correction $\mu m/\sqrt{cm}$	Correction factors from Monte Carlo	$\sigma_L(1cm)$ after correction $\mu m/\sqrt{cm}$
1	67.7 ± 0.9	1.6330	110 ± 2
2	85.3 ± 1.2	1.2262	105 ± 2
3	93.0 ± 1.4	1.1593	108 ± 2
4	88.0 ± 1.2	1.2262	108 ± 2
5	66.5 ± 1.0	1.6330	109 ± 2

Table 4.1: $\sigma_L(1cm)$ of each wire channel before and after correction.

Figure 4.5: (a) Residual versus drift length scatter plot; (b) A typical slice of residual distribution with gaussian curve superimposed; (c) Standard deviation of the residual squared versus drift length.

4.2 Transverse Diffusion Coefficient Estimation

Figure 4.6(a) are some typical induced charge distributions of the cathode pad readout and are within 10 cathode pads or 20 mm as designed. Figure 4.6(b) is the total cathode pad charge spectrum which is also Landau-like since total cathode pad charge is proportional to the anode signal. From these measurements the transverse diffusion coefficient σ_T can be estimated. The procedures are given below:

1. Select events with 4 wires having drift length within the acceptable drift range as before.
2. Sum up the induced charge of all the cathode pads to give S but use only events within the total pad charge main peak (Fig. 4.6(b)), i.e., $S_{min} < S < S_{max}$. The choice of the lower and upper cuts S_{min} , S_{max} is such that the bad events can be eliminated as much as possible.
3. A gaussian curve is fitted to the cathode pad induced charge distribution to find its standard deviation σ_c , its centroid $x_{avalanche}$ and its height C .
4. Select only those events such that $x_{avalanche}$ is away from the edges of the readout pads, i.e., $3 \leq x_{avalanche} \leq 21$. This cut will throw away events with truncated distribution which give bad σ_c .

5. Cathode pad charge is measured in terms of ADC counts and LRS 2249W is a 11-bit device which has a maximum of 2048 counts. Select events such that $C \leq 2000$. Events with pad-charges over the dynamic range of the ADC and those double-track events that will fool the fitting algorithm to give incorrect σ_c will be eliminated by this cut.
6. Define $A = C \times \sigma_c$ (A is proportional to the area under the gaussian curve which is also a measure of the total cathode pad charge). Form a scatter plot of A versus anode pulse height of wire # 3 (Figure 4.7). Select only events which fall inside the band because induced charges on the cathode pads should be approximately equal to the anode charge.
7. Plotting σ_c^2 versus drift length x to form a scatter plot. Divide this scatter plot into 4 slices and fit each slice with a gaussian curve to find its average $\langle \sigma_c^2 \rangle_j$, $j = 1, \dots, 4$ as shown in Figure 4.8(a) and (b).
8. Plot $\langle \sigma_c^2 \rangle_j$ against its corresponding average drift length l_j for $j = 1, \dots, 4$ and fit it with a straight line of form $\sigma_c^2 = \alpha^2 + \beta^2 x$ (Figure 4.8(c)). The number β is the transverse diffusion coefficient $\sigma_T(1cm)$ ⁴.

⁴The number $\alpha \sim 1.1 \text{ mm}$ estimated from the data is close to half the width of the rectangular cathode pad of dimension $20\text{mm} \times 2\text{mm}$, i.e., the average spatial resolution estimated by the cathode pads is also $\sim 1.1 \text{ mm}$.

Figure 4.6: (a) Cathode pad induced charge distribution; (b) Total cathode pad charge spectrum.

Figure 4.7: Total pad charge ($\propto C \times \sigma_c$) versus anode pulse height.

(a)

(b)

(c)

Figure 4.8: (a) Scatter plot of σ_c^2 vs. drift length; (b) typical σ_c^2 distribution; (c) $\langle \sigma_c^2 \rangle$ vs. drift length.

4.3 One-Electron Triggering

If there is only one electron to be detected by an anode wire then the limiting accuracy of the position measurement is simply σ_z . For minimum ionizing cosmic track, the number of ionizations per cm is ~ 100 . Suppose there are N drifting electrons available in which their average time of arrival is measured, the limiting accuracy will be $\sigma_z / \sqrt{n}$. For a more general case where k electrons out of the N drifting electrons are required to trigger the electronics, the limiting accuracy σ_k is given by the formula from Sauli [1]:

$$\sigma_k^2 = \frac{\sigma_z^2}{2 \ln N} \sum_{i=k}^N \frac{1}{i^2} \quad (4.1)$$

For the case of $1e^-$ triggering, σ_z is given by:

$$\begin{aligned} \sigma_z &= C(N) \sigma_1 \\ C(N) &= \frac{\sqrt{2 \ln N}}{\sqrt{\sum_{i=1}^N \frac{1}{i^2}}} \\ &\approx \frac{2}{\pi} \sqrt{3 \ln N}. \end{aligned} \quad (4.2)$$

Most of the published data on $\sigma(1cm)$ are quoted under the condition of $1e^-$ triggering. To compare with the published results, the electronics should trigger with only one electron.

In order to be sure one is triggering with one electron, the gain of the anode wire has to be known. An electrostatics computer program originating from CERN was used to calculate the electric field of the chamber in particular the region near the anode wire. An anode voltage V_{anode} ranging

from 1,300 to 1,600 volts is calculated to correspond to an anode field at the surface $E_{surface}$ ranging from 2.2×10^5 to 2.7×10^5 V/cm. According to the measurements of Ritchie [34] for argon-methane 80:20 mixtures (Fig. 4.9), the gas gain G is not less than 10^4 at an electric field at the surface of the anode $E_a = 2.4 \times 10^5$ V/cm. The K in Fig. 4.9 is given by

$$K = E_a r_a,$$

where r_a is the anode radius.

Combining the RL 723 pre-amp and the MWA 110 main amplifier, a single electron will produce a signal not less than 50 mV which is well beyond the minimum threshold level of the LRS 623 discriminator of 30 mV. At this particular gas gain the location of the Fe^{55} main peak P_{55} is used as a reference P_{55}^{ref} . If there are any changes in gas mixtures or changes in drift field E , to maintain 1e⁻ triggering, V_{anode} will be adjusted so that $P_{55} \simeq P_{55}^{ref}$. Although one can use the highest possible gas gain, it may introduce more problems than is desirable. Problems such as space charge effect, anode field nonlinearity and the limited dynamic range of the ADCs will all be exacerbated by too high a gas gain.

Experimental data on energy loss and ion pairs produced per unit length for minimum ionizing particles in several common chamber gases can be found in reference [1]. Based on these data, the number of electrons per 2 cm of sampling length, N_e , is tabulated in Table 4.2 for a selected argon-

⁵To increase the margin of one-electron triggering, another 30% is added to this gain reference P_{55}^{ref} so that for those signals which is 2σ smaller than the average signals will still be able to trigger with one electron.

Figure 4.9: Absolute gas gain measurements by Ritchie [34].

based gas mixtures.

Gas	Number of electrons N_e per 2 cm of sampling length	Correction factor $C(N_e)$ from (4.2)
Ar	188	2.5273
CH ₄	106	2.3880
Ar/CH ₄ 90:10	180	2.5170
Ar/CH ₄ 80:20	171	2.5047
Ar/CH ₄ 65:35	159	2.4873
Ar/CH ₄ 50:50	147	2.4683
Ar/C ₂ H ₆ 65:35	192	2.5323
Ar/C ₂ H ₆ 50:50	195	2.5360

Table 4.2: Number of ionized electrons N_e and their corresponding correction factor $C(N_e)$ for various gas mixtures.

Not all drifting electrons will be collected by the anode wires. When drift field is high, some of the electrons will be collected by the grounded wires or readout pads ⁶. Therefore the number of electrons for triggering N as appeared in (4.2) has to be corrected by this collection efficiency factor η . That same electrostatics computer program is used to estimate η which is a function of anode voltage V_{anode} and drift voltage V_{drift} .

Summarizing, the total number of electrons available for triggering is given by:

$$N = \eta P N_e \quad (4.3)$$

where P is the pressure.

⁶Even the guard or grid wires are connected to negative potential, if drift field is sufficiently high, the resultant charge configuration is such that these "cathodes" will be "anode" like.

4.4 Checks of the Constancy of $\sigma(1cm)$: Gas Gain

The diffusion coefficients measurement may have a slight dependence on gas gain. A series of measurements with identical operation conditions but different V_{anode} (i.e. different gas gain G) are taken to see what this gas gain effect is. Figure 4.10 shows that σ_L is fairly constant for V_{anode} beyond 1400 V and slightly increasing for $V_{anode} < 1350$ V while σ_T has an opposite effect - approximately constant σ_T for V_{anode} beyond 1400 V but decreasing slightly for 1350 V.

The slight gas gain dependence of σ_L can be explained as follows. The chamber electronics will trigger only if the anode signal is beyond a certain discriminator level. For smaller gain, it will take longer for the anode to accumulate sufficient charges to trigger the discriminator. This is the so-called time slewing effect. This uncertainty in triggering will lead to a higher uncertainty in the drift time measurement which implies higher σ_L . This time-slewing effect is clearly seen in Fig. 4.10. σ_L has a higher value when the gas gain is low and is fairly constant when gas gain is sufficiently high. Hence, choosing a higher gain will minimize the dependence of σ_L on gas gain.

The slight dependence of σ_T on gas gain can be understood as follows. The gaussian fitting algorithm will give an accurate standard deviation σ estimation if the distribution is truly gaussian. The dynamic range of the LRS 2249 11-bit ADC is limited, one cannot use too high or too low a gain.

From a simple electrostatic model (Appendix A), the percentage of charges induced on the outer cathode pads is found to be extremely small, e.g., less than 0.3% will be induced on the 6th pad away from the avalanche. If gas gain is low, signals of these outer pads will be below noise level or not being recorded at all. The gaussian fitting algorithm will thus be biased towards a smaller gaussian width σ . If the gain is too high, the top portion of the distribution will be truncated and this fitting algorithm will not work properly.

A Monte Carlo program is written to simulate cathode pad induced charge distribution based on the above electrostatic model. Figure 4.11 is a plot of $\sigma_T^{M.C.} / \sigma_T^{input}$ versus average peak height to noise ratio ⁷ where $\sigma_T^{M.C.}$ is the Monte Carlo prediction of σ_T and σ_T^{input} is the σ_T input to the program. From this Monte Carlo prediction, σ_T increases and decreases with the relative gain. At some optimum peak height to noise ratio, the predicted σ_T will be exactly equalled to the given σ_T .

The reason for σ_T increasing with gain is due to the induced charged distribution being not exactly gaussian but following an $\arctan(x)$ distribution which has a longer "tail". If the whole distribution is recorded, this long "tail" will bias the fitting algorithm to give a wider gaussian width.

From the measurement the average peak height is approximately 900 to

⁷Peak height is defined to be the maximum induced charged on a pad. For a gaussian distribution the "C" of

$$Ce^{-x^2/2\sigma^2}$$

1300 ADC counts ⁸. Without avalanche signals the average noise level is ~ 40 ADC counts. Based on this Monte Carlo simulation, the systematic error of σ_T associated with this gas gain dependence is *estimated* to be $\sim \pm 10\%$ ⁹.

For every set of measurements, V_{anode} will be chosen such that σ_L and σ_T is in the constant region with respect to gain G .

4.5 Checks of the Constancy of $\sigma(1cm)$: Angular Acceptance

If the cosmic tracks are all identically vertical, the initial ionized charge profile can be idealized as a delta function $\delta(z)$ otherwise it has a rectangular profile of width $l/\tan\theta$ where l is the sampling length and θ , the track's inclination with respect to the anode plane (see Figure 4.12).

In general, it is the electrons at the central distribution, which have the shortest track to anode distance (see Figure 4.13), will arrive at the anode first. If the track is sufficiently inclined, the electrons at the leading edge will compete with the central electrons to trigger the electronics. Hence, inclined tracks will increase the triggering timing uncertainty which will lead to an increase in longitudinal diffusion.

Table 4.3 gives σ_L (1cm) of a typical set of data with the corresponding

⁸For gases with a large transverse diffusion, electrons will have a larger lateral spread. The average induced charge on each cathode pad will be correspondingly smaller and hence smaller peak height and vice versa for small σ_T .

⁹This estimated 10% error in σ_T is not included in the graphs shown later. Readers should be aware of this.

Figure 4.10: (a) σ_L ; (b) σ_T as a function of V_{anode} .

Figure 4.11: $\sigma_T^{M.C.} / \sigma_T^{input}$ vs. Peak-Height-to-Noise-Ratio.

Figure 4.12: Ionized electron distribution profile.

Figure 4.13: Electrostatic field map of the MWPC/TPC detector. Solid dots representing the wires and cathode planes. Length measured in cm.

angular cut indicated. There is a systematic decrease in σ_L as predicted by the above simple model. For some data sets, this systematic decrease in σ_L (1cm) is not so obvious. In order to optimize between statistics ¹⁰ and minimum inclination, an angular cut of $|s| < 0.25$ (14°) is chosen which is half-way between maximum angular cut and no cut. The systematic error due to this angular acceptance effect is estimated to be $\sim \pm 5\%$ and will be included in the data presented in Chapter 5.

Angular acceptance	$\sigma_L(1cm)$ in $\mu m/\sqrt{cm}$
$ s \leq 0.50$ (or 27°)	274 ± 17
$ s \leq 0.35$ (or 19°)	271 ± 16
$ s \leq 0.25$ (or 14°)	262 ± 16
$ s \leq 0.20$ (or 11°)	256 ± 16
$ s \leq 0.15$ (or 9°)	245 ± 16
$ s \leq 0.08$ (or 5°)	245 ± 16

Table 4.3: Systematic change in $\sigma_L(1cm)$ with angular acceptance.

Transverse diffusion coefficient estimation does not depend on triggering timing. The width of the scintillators is 15 cm and are 130 cm apart. The "transverse" angular acceptance will be $\sim \pm 7^\circ$. Although this already small angular acceptance may systematically alter the width σ_c of the induced charge distribution by a small amount, this small spread will mainly affect the "intercept" α not the "slope" β where

$$\sigma_T^2(x) = \alpha^2 + \beta^2 x.$$

¹⁰Tighter angular cut will reduce statistics drastically.

It is assumed that angular acceptance of $\pm 7^\circ$ is sufficiently "vertical" so that $\sigma_T(1cm)$ is not expected to be affected by this angular effect.

4.6 Checks of the Constancy of $\sigma(1cm)$: Non-linear Electric Field

The electric field in the drift region is calculated to be sufficiently uniform but highly nonlinear near the anode wire. The drift time will then be slightly altered due to this strong electric field near the proportional wire. The drift velocity of argon-methane 80:20 is parametrized using the data of reference [35]. This parametrization is used to predict the drift velocity in the presence of high field at the proportional wire. At a typical operating anode voltage, the predicted drift velocity at various drift field is tabulated in Table 4.4:

Drift velocity is estimated from the longest drift time recorded and hence is less sensitive to the presence of this high anode field. If the drift field is beyond $150 V/cm$, then

$$|v_d^{\text{parametrized}} - v_d^{\text{predicted}}| \leq 2\%.$$

The uncertainty in drift velocity due to an inability of determining the edges of the drift time spectrum is estimated to be $\sim \pm 4\%$. Therefore, an overall uncertainty in drift velocity is $\sim \pm 5\%$. Since $\sigma_L \propto \sqrt{x} \doteq \sqrt{v_d t}$, a drift velocity uncertainty of 5% implies an uncertainty of $\sim 2.5\%$ in σ_L .

The effect due to this nonlinear anode field on $\sigma_L(1cm)$ is negligible because of the following reasons. From the experimental data, $\sigma_L \propto 1/\sqrt{E}$.

<i>Drift Field</i> <i>V/cm</i>	$v_d^{\text{parametrized}}$ <i>cm/μs</i>	$v_d^{\text{predicted}}$ <i>cm/μs</i>	<i>Difference</i> <i>cm/μs</i>
50	2.50	2.86	+0.36
100	5.00	5.15	+0.15
150	6.30	6.26	-0.04
200	6.60	6.49	-0.11
250	6.44	6.34	-0.10
300	6.28	6.19	-0.09
350	6.12	6.03	-0.08
400	5.96	5.88	-0.08
450	5.80	5.73	-0.07
500	5.64	5.58	-0.06
550	5.48	5.43	-0.05
600	5.32	5.28	-0.04

Table 4.4: Drift velocity prediction in the presence of high anode field.

That is, there is less diffusion at the anode region because of the large electric field. At low drift field, diffusion is dominated in the drift region. At high drift field, diffusion does not vary much both in the drift region and the anode region. Transverse diffusion σ_T , in a first order approximation, can be considered a constant. Hence, large variation in electric field has only marginal effect on σ_T .

Let $E = E(x)$ be the drift field which is a function of x and $\beta = \beta(E)$ be the diffusion coefficient. The r.m.s. dispersion σ is defined as

$$\sigma^2 = \beta^2 x.$$

Divide the path of an electron into N equal segments and each segment contributes a small dispersion σ_i to the overall path. The total dispersion

will then be

$$\begin{aligned}\sigma^2(x) &= \sum_{i=1}^N \sigma_i^2 \\ &= \sum_{i=1}^N \beta^2(E(x_i)) \delta x_i \\ &= \int_0^x \beta^2(E(x)) dx.\end{aligned}$$

To calculate β , the method used in Section 4.1 is by fitting a straight line to $\sigma^2(x)$ at 5 equally spaced points $x_1, \dots, x_5$ where β will be the slope. For simplicity consider a two-point fit:

$$\begin{aligned}\beta &= \frac{\sigma^2(x_2) - \sigma^2(x_1)}{x_2 - x_1} \\ &= \frac{\int_{x_1}^{x_2} \beta^2(E(x)) dx}{x_2 - x_1}.\end{aligned}$$

If the two points are 1 cm away from the anode then $E(x_1) \simeq E(x_2) = E$ which is a constant. Therefore,

$$\sigma^2(1cm) \sim \frac{\beta^2(E) \int_{x_1}^{x_2} dx}{x_2 - x_1} = \beta^2(E).$$

Thus, local field nonlinearity near the anode will not affect the estimation of $\sigma_L(1cm)$ as long as $x_1, \dots, x_5$ are sufficiently away from the anode such that electric field is approximately constant.

Chapter 5.

Experimental Measurements

5.1 Transport Coefficients as a Function of Drift Field for Ar/CH_4 Mixtures

The longitudinal and transverse diffusion coefficients and the drift velocity for Ar/CH_4 90:10, 80:20, 65:35 and 50:50 mixtures at atmospheric pressure are shown in Figures 5.1 to 5.4 respectively. The drift velocities of these mixtures exhibit a local maximum. This drift velocity maximum v_{max} increases when the amount of methane increases. The location of the drift velocity peak, i.e. the corresponding drift field E , also increases with the proportion of methane.

One characteristic exhibited in these graphs is that the transverse diffusion coefficients $\sigma_T(1cm)$ is a constant to first order while the longitudinal diffusion coefficients $\sigma_L^2(1cm)$ is approximately inversely proportional to the drift field E . Another characteristic is that the magnitude of $\sigma_T(1cm)$ at a given E decrease when the proportion of methane in argon α increases.

Parametrizing $\sigma_L^2(1cm)$ according to the following model:

$$\sigma_L^2(1cm) = A_E + \frac{B_E}{E}. \quad (5.1)$$

Parameters A_E and B_E are tabulated in Table 5.1.

Gas mixtures	A_E	B_E
	$10^4 \times \mu m^2 cm^{-1}$	$10^7 \times \mu m^2 V cm^{-2}$
Ar/CH ₄ 90:10	$+3.34 \pm 0.44$	1.31 ± 0.10
Ar/CH ₄ 80:20	$+1.70 \pm 0.29$	1.64 ± 0.09
Ar/CH ₄ 65:35	-0.11 ± 0.59	1.95 ± 0.16
Ar/CH ₄ 50:50	-0.28 ± 0.45	2.11 ± 0.15

Table 5.1: Parametrization of $\sigma_L^2(1cm)$ as a function of drift field for argon-methane mixtures.

Figure 5.5 indicated that the parameter B_E is a monotonically increasing function of α which means that at a fixed E , the term B_E/E is also increasing with α . But A_E is a monotonically decreasing function of α which makes it difficult to see how $\sigma_L(1cm)$ depends on α . There is no theoretical prediction available for the parameters B_E and A_E .

The anisotropy ratio A as quoted in the literature is defined to be D_L/D_T where

$$\sigma_{L,T}(x) = \sqrt{\frac{2x D_{L,T}}{v_d}},$$

i.e., at $x = 1 cm$, D is given by

$$D = \frac{v_d \sigma^2(1cm)}{2}.$$

Therefore,

$$A = \frac{D_L}{D_T} = \frac{\sigma_L^2(1 \text{ cm})}{\sigma_T^2(1 \text{ cm})} \quad (5.2)$$

The ratio A as a function of E/P for the above set of gas mixtures are shown in Figures 5.6 to 5.7. For these four gas mixtures A is approximately inversely proportional to E at 1 atm which is a reflection of the fact that σ_T is approximately a constant and σ_L^2 is inversely proportional to E .

From this set of argon-methane measurements at a given proportion of methane α , σ_L is equalled to σ_T at a certain drift field E_{co} . More precisely,

$$\begin{aligned} \sigma_L^2 - \sigma_T^2 &= 0 & \text{for } E &= E_{co} \\ \sigma_L^2 - \sigma_T^2 &= 0 & \text{for } E &= E_{co} \\ \sigma_L^2 - \sigma_T^2 &= 0 & \text{for } E &= E_{co} \end{aligned}$$

which is exactly predicted by (2.24). If the drift field at this cross-over point E_{co} is plotted against the proportion of methane α , it exhibited an approximately linear relationship as shown in Figure 5.8. Consider the following two models.

$$E_{co} = m_1 \alpha + c_1 \quad \text{Linear Model,} \quad (5.3)$$

$$E_{co} = c_2 \alpha^{m_2} \quad \text{Power Law Model.} \quad (5.4)$$

The fitting parameters m , and c , for these two models ¹ are tabulated in Table 5.2. So far, there is no theoretical prediction on E_{co} as a function.

¹Parametrization was done by a fitting package from CERN called MINUIT

of α . To test how well the linear model predicts E_{co} , the power law model is also considered. The parametrization indicated that the power m_2 of the power law model is 0.819 which is very close to linearity. From their respective χ^2 /degree of freedom, the linear model gives a slightly better fit to the data.

model	m	c	χ^2 /D.O.F.
Linear	$409 \pm 12 \text{ V/cm}$	$23 \pm 4 \text{ V/cm}$	0.506
Power Law	0.819 ± 0.029	$397 \pm 12 \text{ V/cm}$	0.867

Table 5.2: Fitting parameters for the E_{co} - α modelling.

It is a general rule that if the scattering cross section is high then drift velocity will be low and vice versa because high cross section means more collisions which will slow down the drift. It is conjectured that the drift velocity curve is correlated ² to the cross section in such a way that the local maximum of the drift velocity curve corresponds to the Ramsauer local minimum of the cross section; the *positive* slope of the drift velocity curve corresponds to the *negative* slope of the cross section and vice versa.

Define v_{co} to be the drift velocity at drift field E_{co} and v_{max} to be the maximum drift velocity. If the ratio $R_{co} \equiv v_{co}/v_{max}$ is plotted against α (see Figure 5.9 and Table 5.3), R_{co} is approximately a constant.

If the above conjecture about drift velocity is correct and since v_{co} is

²Correlation in the sense that at certain drift field E it defines the average electron energy ϵ of the electron swarm. From this average electron energy we can calculate the corresponding cross section $\sigma(\epsilon)$.

Gas mixtures	E_{co} V/cm	v_{co} $cm/\mu s$	v_{max} $cm/\mu s$	R_{co}
Ar/CH ₄ 90:10	62 ± 5	3.84 ± 0.19	5.10 ± 0.26	0.753 ± 0.053
Ar/CH ₄ 80:20	108 ± 5	5.25 ± 0.26	6.67 ± 0.33	0.787 ± 0.056
Ar/CH ₄ 65:35	163 ± 5	6.00 ± 0.30	7.75 ± 0.39	0.774 ± 0.055
Ar/CH ₄ 50:50	228 ± 5	6.66 ± 0.33	8.50 ± 0.43	0.784 ± 0.055

Table 5.3: Drift velocity ratio R_{co} for Ar-CH₄ mixtures.

always on the positive slope of the drift velocity curve, it implies that this cross-over point where $\sigma_L = \sigma_T$ corresponds to a point on the negative slope of the cross section rather than corresponding to the Ramsauer minimum. How then can it be reconciled with (2.26) where it is theoretically predicted by Parker and Lowke that the cross-over point corresponds the local minimum of the collision frequency curve $\nu(\epsilon)$, i.e., $\partial\nu/\partial\epsilon = 0$? According to (2.27), the condition $\partial\nu/\partial\epsilon = 0$ implies

$$\frac{\partial}{\partial\epsilon} \sqrt{\sigma(\epsilon)} = 0$$

or

$$\frac{\partial\sigma(\epsilon)}{\partial\epsilon} = -\frac{\sigma(\epsilon)}{2\epsilon} \quad (5.5)$$

Since ϵ and $\sigma(\epsilon)$ are both positive numbers, the slope of the cross section $\partial\sigma(\epsilon)/\partial\epsilon$ is always negative which is exactly as predicted. Therefore, the theoretical prediction by Parker and Lowke and the above drift velocity conjecture and the experimental measurements are all consistent with one another. The new set of conditions replacing (2.26) is now given by:

$$\begin{aligned}
 \frac{\partial \sigma(\epsilon)}{\partial \epsilon} &= \frac{\sigma(\epsilon)}{2\epsilon} \text{ then } \sigma_L = \sigma_T; \\
 \frac{\partial \sigma(\epsilon)}{\partial \epsilon} &= \frac{\sigma(\epsilon)}{2\epsilon} \text{ then } \sigma_L = \sigma_T; \\
 \frac{\partial \sigma(\epsilon)}{\partial \epsilon} &= \frac{\sigma(\epsilon)}{2\epsilon} \text{ then } \sigma_L = \sigma_T.
 \end{aligned}
 \tag{5.6}$$

Figure 5.1: (a) Diffusion coefficients; (b) drift velocity of Ar/CH₄ 90:10 at 1 atm.

2 of/de 2

Figure 5.2: (a) Diffusion coefficients; (b) drift velocity of Ar/CH_4 80:20 at 1 atm.

Figure 5.3: (a) Diffusion coefficients; (b) drift velocity of Ar/CH_4 65:35 at 1 atm.

Figure 5.4: (a) Diffusion coefficients; (b) drift velocity of Ar/CH₄ 50:50 at 1 atm.

Figure 5.5: Plotting the fitting parameter B_E vs. the % of CH_4 in Ar.

Figure 5.6: D_L/D_T versus E/P for Ar/CH_4 90:10 (a) and 80:20 (b).

Figure 5.7: D_L/D_T versus E/P for Ar/CH_4 65:35 (a) and 50:50 (b).

5.2 Transport Coefficients as a Function of Pressure for Ar/CH₄ Mixtures

It is known experimentally that diffusion coefficients $\sigma_{L,T}$ decrease with an increase in pressure. At a fixed drift field, a series of measurements were taken at different pressures. Figures 5.10 and 5.11 are the diffusion coefficients and drift velocity measurements as a function of pressure P for Ar/CH₄ 90:10 mixture at a drift field of 200 V/cm and 300 V/cm respectively. Both set of measurements for $\sigma_L(1cm)$ and $\sigma_T(1cm)$ exhibit $1/\sqrt{P}$ characteristics. Figure 5.12 is their corresponding anisotropy ratio plots. It is relatively flat at this region of E/P .

Parametrizing $\sigma_{L,T}^2(1cm)$ according to the following model:

$$\sigma_{L,T}^2(1cm) = A_P^{L,T} + \frac{B_P^{L,T}}{P}. \quad (5.7)$$

Parameters $A_P^{L,T}$ and $B_P^{L,T}$ are tabulated in Table 5.4.

Drift Field V/cm	A_P^L $10^4 \times \mu m^2 cm^{-1}$	B_P^L $10^4 \times \mu m^2 cm^{-1} Atm$
200	$+1.82 \pm 0.73$	7.71 ± 0.89
300	-0.81 ± 0.75	8.91 ± 1.02
	A_P^T	B_P^T
200	$+2.7 \pm 1.1$	28.8 ± 1.1
300	$+4.4 \pm 1.0$	25.9 ± 1.1

Table 5.4: Parametrization of $\sigma_{L,T}^2(1cm)$ as a function of pressure for Ar/CH₄ 90:10 mixture

In Section 5.1 it has been established that the longitudinal diffusion σ_L^2

Figure 5.8: Plotting the cross-over drift field E_{co} vs. the % of CH_4 in Ar.

Figure 5.9: Drift velocity ratio v_{co}/v_{max} vs. % of CH_4 in Argon.

is inversely proportional to the drift field E . From the study of pressure dependence, σ_L^2 is also inversely proportional to the pressure P . One possible parametrization linking the dependence of σ_L^2 on both E and P is given by the following model:

$$\sigma_L^2 = C_0 + \frac{C_1}{EP} + \frac{C_2}{E} + \frac{C_3}{P} \quad (5.8)$$

It is an experimental fact that E/P is an appropriate scaling variable for the gas gain mechanism and drift velocity but it is not so obvious that it is true for other transport phenomena. Diffusion coefficients and drift velocity measurements for Ar/CH_4 90:10 taken at different drift fields and pressures are tabulated as a function of E/P in Table 5.5. From these measurements, drift velocity does obey this E/P scaling while $\sigma_{L,T}$ clearly violate it.

In fact, from the pressure and the drift field study, there is an indication that EP is an appropriate scaling variable for the longitudinal diffusion, i.e.,

$$\sigma_L^2 = C_0 + \frac{C_1}{EP} \quad (5.9)$$

To test this scaling behaviour, another series of measurements are taken for Ar/CH_4 90:10 mixture at 1.25 atm. as a function of drift field (Figure 5.13 and 5.14). Fitting all the Ar/CH_4 90:10 measurements of various drift fields and pressures, which is tabulated in Table 5.6, according to (5.9) gives

$$C_0 = (2.03 \pm 0.23) \times 10^4 \quad \mu m^2 cm^{-1},$$

$$C_1 = (15.66 \pm 0.64) \times 10^6 \quad \mu m^2 V cm^{-2} Atm,$$

E/P V/cm/torr	E V/cm	P Atm.	$\sigma_L(1\text{cm})$ $\mu\text{m}/\sqrt{\text{cm}}$	$\sigma_T(1\text{cm})$ $\mu\text{m}/\sqrt{\text{cm}}$	v_d cm/ μs
0.053	50	1.25	631 $\pm$ 46	492 $\pm$ 12	2.87 $\pm$.14
0.066	50	1.00	633 $\pm$ 51	513 $\pm$ 13	3.50 $\pm$.17
0.079	75	1.25	486 $\pm$ 27	514 $\pm$ 12	3.90 $\pm$.20
0.088	67	1.00	549 $\pm$ 45	535 $\pm$ 19	4.00 $\pm$.20
0.099	75	1.00	462 $\pm$ 30	550 $\pm$ 12	4.39 $\pm$.22
0.105	100	1.25	392 $\pm$ 24	535 $\pm$ 11	4.40 $\pm$.22
0.132	100	1.00	404 $\pm$ 29	554 $\pm$ 10	4.90 $\pm$.25
0.132	200	2.00	252 $\pm$ 16	395 $\pm$ 22	4.87 $\pm$.24
0.150	200	1.75	251 $\pm$ 20	440 $\pm$ 20	5.00 $\pm$.25
0.158	150	1.25	297 $\pm$ 21	525 $\pm$ 12	4.96 $\pm$.25
0.175	133	1.00	346 $\pm$ 21	532 $\pm$ 19	5.20 $\pm$.26
0.175	200	1.50	258 $\pm$ 18	458 $\pm$ 17	5.04 $\pm$.25
0.197	150	1.00	335 $\pm$ 21	559 $\pm$ 11	5.00 $\pm$.25
0.197	300	2.00	199 $\pm$ 16	373 $\pm$ 23	5.01 $\pm$.25
0.211	200	1.25	276 $\pm$ 15	517 $\pm$ 12	4.90 $\pm$.25
0.266	300	1.75	212 $\pm$ 16	411 $\pm$ 16	4.95 $\pm$.25
0.263	200	1.00	295 $\pm$ 20	559 $\pm$ 11	4.94 $\pm$.25
0.263	250	1.25	255 $\pm$ 14	534 $\pm$ 12	4.57 $\pm$.23
0.263	300	1.50	222 $\pm$ 17	457 $\pm$ 13	4.72 $\pm$.24
0.316	300	1.25	244 $\pm$ 14	533 $\pm$ 10	4.44 $\pm$.22
0.351	200	0.75	354 $\pm$ 22	663 $\pm$ 12	4.30 $\pm$.22
0.351	267	1.00	283 $\pm$ 16	574 $\pm$ 15	4.55 $\pm$.23
0.368	350	1.25	240 $\pm$ 19	544 $\pm$ 12	4.17 $\pm$.21
0.395	300	1.00	278 $\pm$ 18	553 $\pm$ 11	4.17 $\pm$.21
0.421	400	1.25	229 $\pm$ 19	549 $\pm$ 11	4.02 $\pm$.20
0.438	333	1.00	257 $\pm$ 20	577 $\pm$ 16	4.10 $\pm$.20
0.474	450	1.25	231 $\pm$ 14	537 $\pm$ 13	3.80 $\pm$.19
0.526	200	0.50	430 $\pm$ 26	754 $\pm$ 16	3.60 $\pm$.18
0.526	300	0.75	326 $\pm$ 28	640 $\pm$ 12	3.70 $\pm$.19
0.526	400	1.00	259 $\pm$ 18	569 $\pm$ 11	3.70 $\pm$.19
0.526	500	1.25	232 $\pm$ 15	544 $\pm$ 10	3.61 $\pm$.18
0.592	450	1.00	265 $\pm$ 17	574 $\pm$ 10	3.47 $\pm$.17
0.658	500	1.00	262 $\pm$ 16	577 $\pm$ 12	3.35 $\pm$.17
0.789	300	0.50	471 $\pm$ 41	704 $\pm$ 18	3.10 $\pm$.16

Table 5.5: $\sigma_{L,T}$, v_d tabulated as a function of E/P for Ar/CH₄ 90:10.

with a χ^2 per degree of freedom of 1.133. With the above model, the fitted

E [V/cm]	P [Atm.]						
	0.50	0.75	1.00	1.25	1.50	1.75	2.00
50			633 $\pm$ 51	631 $\pm$ 46			
75			462 $\pm$ 30	486 $\pm$ 27			
100			404 $\pm$ 29	392 $\pm$ 24			
150			335 $\pm$ 21	297 $\pm$ 21			
200	430 $\pm$ 26	354 $\pm$ 22	294 $\pm$ 20	276 $\pm$ 15	258 $\pm$ 18	251 $\pm$ 20	252 $\pm$ 16
250				255 $\pm$ 14			
267			283 $\pm$ 16				
300	451 $\pm$ 40	326 $\pm$ 28	278 $\pm$ 18	244 $\pm$ 14	222 $\pm$ 17	212 $\pm$ 16	199 $\pm$ 16
333			257 $\pm$ 20				
350				240 $\pm$ 19			
400			259 $\pm$ 18	230 $\pm$ 19			
450			265 $\pm$ 17	231 $\pm$ 14			
500			262 $\pm$ 16	232 $\pm$ 15			

Table 3.6: Tabulated $\sigma_L(1cm)$ for Ar/CH₄ 90:10 as a function of E and P .

$\sigma_L(1cm)$ on the average differs with the measured $\sigma_L(1cm)$ by less than 7% at most of the 'EP' region except when the drift field is $E \leq 50$ V/cm or when the pressure is $P \leq 0.5$ Atm.

$$\frac{|\sigma_L^{meas} - \sigma_L^{fitted}|}{\sigma_L^{fitted}} \leq 7\%.$$

If experimental uncertainties are taken into consideration, experimental measurements agreed with the calculated $\sigma_L(1cm)$ according to (5.9) in all our experimentally accessible 'EP' region. A three dimensional plot of $\sigma_L(1cm)$ versus E and P is given in Figure 5.15.

σ_L^{fitted} , the square-root of $\sigma_L^{2\,fitted}$, is used here for the purpose of comparing with the experimental measurements σ_L^{meas} .

If the relative magnitudes of σ_L and σ_T are indeed correlated to the slope of the collision frequency $d\nu/d\epsilon$ as in (2.26) such that $\sigma_L < \sigma_T$ when $d\nu/d\epsilon > 0$ and vice versa, then the $1/\sqrt{E}$ dependence for σ_L is not expected to obey at high drift field. When the drift field increase, average electron energy $\bar{\epsilon}$ will also increase. The slope of the elastic cross section of argon (see Figure 2.4) will turn from positive to negative at about 10 eV. Similarly for the slope of $\nu(\epsilon)$, it is also expected to turn from positive to negative, i.e., from bunching to anti-bunching. Thus, σ_L will not keep on decreasing with E but will catch up with σ_T at very high drift field. This effect has been demonstrated in Binnie's Monte Carlo simulation [22] for Ar/CO₂ mixtures (Fig. 5.16).

Figure 5.10: (a) Diffusion coefficients; (b) drift velocity of Ar/CH₄ 90:10 as a function of pressure at $E = 200$ V/cm.

Figure 5.11: (a) Diffusion coefficients; (b) drift velocity of $\overline{Ar}/CH_4$ 90:10 as a function of pressure at $E = 300$ V/cm.

Figure 5.12: D_L/D_T versus E/P for Ar/CH_4 90:10 mixture at different pressure: (a) $E = 200 V/cm$; (b) $E = 300 V/cm$.

Figure 5.13: (a) Diffusion coefficients; (b) drift velocity of Ar/CH_4 90:10 at 1.25 atm.

Figure 5.14: D_L/D_T versus E/P of Ar/CH_4 90:10 mixture at 1.25 atm.

Figure 5.15: 3-D plot of $\sigma_L(1cm)$ versus E and P .

5.3 Transport Coefficients as a Function of Drift Field for Ar/C_2H_6 Mixtures

The measurements presented in Section 5.1 indicated that the ratio v_{co}/v_{max} is approximately a constant independent of the proportion of methane in argon. It is of interest to test whether this phenomenon is true for other gas mixtures. Argon-ethane mixtures are being chosen to test this phenomenon due to its different drift velocity characteristics, i.e., the cross section of ethane could be very different from that of methane.

Figures 5.17 and 5.18 are the diffusion coefficients and drift velocity

Figure 5.16: Binnie's Monte Carlo simulation for Ar/CO₂ mixtures. (a) $\sigma_L(1\text{cm})$; (b) σ_L/σ_T as a function of E at 1 atm.

measurements for Ar/C_2H_6 65:35 and 50:50 mixtures respectively while Figure 5.19 is their corresponding anisotropy ratio plots. The characteristics of these graphs are very similar to that of the results of argon-methane mixtures. Transverse diffusion $\sigma_T(1cm)$ has a slightly negative slope while longitudinal diffusion coefficient $\sigma_L(1cm)$ also has the same $1/\sqrt{E}$ dependence. The drift velocity curves do not have a local maximum, instead, it saturates approximately at $E \geq 500 V/cm$. The anisotropy ratio plots also exhibits a $1/E$ dependence as in argon-methane mixtures, a consequence of the $1/E$ dependence of σ_L^2 .

The ratio v_{co}/v_{maz} tabulated in Table 5.7 is consistent with being a constant. The parametrization of $\sigma_L^2(1cm)$ as appeared in Section 5.1 for argon-ethane mixtures is tabulated in Table 5.8.

Gas mixtures	E_{co} V/cm	v_{co} $cm/\mu s$	v_{maz} $cm/\mu s$	R_{co}
Ar/C_2H_6 65:35	100 ± 5	2.80 ± 0.14	4.84 ± 0.24	0.58 ± 0.04
Ar/C_2H_6 50:50	120 ± 5	2.64 ± 0.14	4.85 ± 0.24	0.54 ± 0.04

Table 5.7: Drift velocity ratio R_{co} for Ar/C_2H_6 mixtures.

Figure 5.17: (a) Diffusion coefficients; (b) drift velocity of Ar/C_2H_6 65:35 at 1 atm.

Figure 5.18: (a) Diffusion coefficients; (b) drift velocity of Ar/C_2H_6 50:50 at 1 atm.

Figure 5.19: D_L/D_T versus E/P for Ar/C_2H_6 65:35 (a) and 50:50 (b).

Gas mixtures	A_E	B_E
	$10^4 \times \mu m^2 cm^{-1}$	$10^6 \times \mu m^2 V cm^{-2}$
Ar/C ₂ H ₆ 65:35	1.61 ± 0.24	9.21 ± 0.69
Ar/C ₂ H ₆ 50:50	1.17 ± 0.21	8.63 ± 0.62

Table 5.8: Parametrization of $\sigma_L^2(1cm)$ as a function of drift field for Ar/C₂H₆ mixtures

5.4 Comparison with Published Experimental Results

There are few published experimental data on transverse diffusion coefficients and much less on longitudinal diffusion coefficients. A compilation of drift and diffusion of electrons in gases can be found in the report by Peisert and Sauli 36. Some of the more recent measurements on diffusion coefficients are the one by Piuz 37 and the one by Fulda-Quenzer et. al. 28. These two experiments are commonly known as the "time-of-flight" experiments. The drift velocity and the longitudinal diffusion coefficient are calculated from the average drift time of a known drift length and the width of the drift time spectrum respectively. The detector of Piuz has a total drift length of 3 cm while the drift lengths used in the Fulda-Quenzer's experiment have a range from 5 cm to 80 cm.

The major difference between this experiment and that of Piuz or Fulda-Quenzer's is that they use a laser beam as the source of ionization while cosmic rays are being used in this experiment. The intensity of their laser beam is claimed to be so low that the probability of producing two electrons

is negligible, and hence they satisfy the "one-electron triggering criterion". Although the detector of Fulda-Quenzer et. al. can measure all three transport coefficients v_d and σ_T , they are not measured simultaneously. In order to measure σ_T , they have to replace the grid which separates the drift space from the sense wire by a rectangular metal slit. In doing so, two gas samples with slightly different contents and impurities might have been used.

Figures 5.20 and 5.21 are respectively the measurements of Piuz for Ar/CH_4 80:20 and Ar/C_2H_6 50:50 with the measurements taken in this experiment superimposed. The estimated systematic errors of the Piuz's measurements have a range from 12% to 30%. Figure 5.22 and 5.23 are respectively the longitudinal and transverse diffusion coefficients by Fulda-Quenzer et. al.. Although the source of ionization and the method of estimation of transport coefficients are completely different, within experimental uncertainty, the v_d and the $\sigma_L(1cm)$ measurements agreed with their results but not so well with Fulda-Quenzer's $\sigma_T(1cm)$ measurements.

The $\sigma_T(1cm)$ discrepancy can possibly be explained by the following argument. In the experiment by Fulda-Quenzer et. al., a $100\ \mu m$ wide rectangular slit is placed perpendicularly to the sense wire (Fig. 5.24). The slit can be translated parallel to the wire and the electron impact density along the direction of the sense wire can be measured. The transverse diffusion coefficient is determined from the width of this gaussian shaped electron impact density. Ideally, the slit at a position x should only collect those electrons drifted around the region at $x \pm 50\mu m$. Due to the presence of

Figure 5.20: Measurements for Ar/CH_4 80:20 by Piuz [37], (a) diffusion coefficient $\sigma_L(1\text{cm})$; (b) drift velocity.

Figure 5.21: Measurements for $\text{Ar}/\text{C}_2\text{H}_6$ 50:50 by Piuz 37. (a) diffusion coefficient $\sigma_L(1\text{cm})$; (b) drift velocity.

Figure 5.22: Longitudinal diffusion coefficients by Fulda-Quenzer [28] et. al.

Figure 5.23: Transverse diffusion coefficients by Fulda-Quenzer [28] et.al.

Figure 5.24: Slit/sense wire geometry of the detector of Fulda-Quenzer, et. al.

this metal slit the electric field near the slit is highly distorted in such a way that electrons drifted around the region $x = (50 \cdot \delta x) \mu m$ will also be collected. The net effect is such that there are more electrons collected at the gaussian tail which will increase the width of the distribution systematically. Hence, their transverse diffusion coefficient measurements are systematically higher.

5.5 Comparison with Monte Carlo Simulation Results

Monte Carlo simulation offers a completely different approach towards electron transport phenomena. Instead of solving the Boltzmann Transport Equation analytically, one can trace the path of an electron ⁴ or an ensemble of electrons.

Given the total cross section of the gas (either from experimental data or assuming a certain model), mean time between collisions can be generated stochastically. Due to lack of experimental data on differential and inelastic cross section, most Monte Carlo studies assumed isotropic scattering with scattering angle generated randomly.

For the ensemble approach, electrons are allowed to drift in three dimensions for a fixed period of time T . The drift velocity is simply the average displacement of the electron distribution divided by T . The diffusion coefficients can be deduced from the scatter of a large number of displacements in the x , y and z directions.

For the path tracing approach (or the time average approach), the location of a single electron is sampled periodically. The diffusion coefficients

⁴ According to the ergodic theorem, time averaging is equivalent to ensemble averaging. Therefore, it is more efficient computationally to trace an electron than tracing an ensemble of electrons. Recent Monte Carlo studies do take advantage of this method pioneered by Kubo [19] and Skullerud [21].

are given by the following correlation function [23] [21]:

$$\mathbf{D} = \lim_{t \rightarrow \infty} \left\langle |\mathbf{v}(t) - \mathbf{v}_d| \int_0^t |\mathbf{v}(t') - \mathbf{v}_d| dt' \right\rangle \quad (5.10)$$

where $\mathbf{v}(t)$ is the instantaneous velocity and $\mathbf{v}_d$, the drift velocity. Assuming no magnetic field is present, the diffusion tensor $\mathbf{D}$ is diagonal. The transverse diffusion coefficient will then be the diagonal element D_{xx} or D_{yy} while the longitudinal diffusion coefficient is the component D_{zz} .

Monte Carlo simulation results by Fraser and Mathieson [23] on the anisotropy ratio D_L/D_T are shown in Figure 5.25. The solid and dashed curves are their simulation results. The filled circles and squares, the open circles, the filled triangles are the measurements by Siegmund et. al. [29], Skelding [30] and by this experiment respectively. Despite the assumption that scattering is isotropic, the simulation results by Fraser and Mathieson are within 1 to 2 standard deviation from our measurements except in a small region between 0.12 to 0.25 V/cm/torr for Ar/CH₄ 90:10. In this region our measurements agree with the measurements by Skelding and both disagree with the Monte Carlo model.

If the argon-methane data is extrapolated to pure methane, the cross-over point E_{co} in which $\sigma_L = \sigma_T$ can be predicted by (5.3) or (5.4). According to the linear model, for pure methane,

$$E_{co} = 432 \pm 14 \text{ V/cm} \quad \text{or} \quad E_{co}/P = 0.568 \pm 0.019 \text{ V cm}^{-1} \text{ Torr}^{-1}$$

while

$$E_{co} = 397 \pm 15 \text{ V/cm} \quad \text{or} \quad E_{co}/P = 0.522 \pm 0.020 \text{ V cm}^{-1} \text{ Torr}^{-1}$$

for the Power-Law model. This extrapolation agrees very well with Fraser and Mathieson's simulation result shown in Figure 5.26 in particular the simulation curve based on the elastic scattering cross sections of Duncan and Walker [38] where

$$E_{co}/P = 0.53 \text{ V cm}^{-1} \text{ Torr}^{-1} \text{ or } E_{co} = 403 \text{ V/cm.}$$

The Monte Carlo prediction by Fraser and Mathieson on the absolute values of σ_L are also quite reasonable. Using the parametrization in Section 5.1 to predict the value of σ_L at a drift field of 760 V/cm or $E/P = 1 \text{ V cm}^{-1} \text{ Torr}^{-1}$, within experimental error, the simulation results of Fraser and Mathieson agree with this experiment (see Table 5.9).

Gas mixtures	σ_L μm	$\sqrt{\text{cm}}$
	Fraser and Mathieson's Monte Carlo	Extrapolation from experimental data
Ar/CH ₄ 90:10	213	220 ± 11
Ar/CH ₄ 80:20	175	198 ± 10
Ar/CH ₄ 50:50	172	158 ± 16

Table 5.9: Comparing Monte Carlo prediction of $\sigma_L(1\text{cm})$ at $E/P = 1 \text{ V cm}^{-1} \text{ Torr}^{-1}$ with experimental data.

Figure 5.25: D_L/D_T Monte Carlo results for Ar-CH₄ 90:10 and 80:20 mixtures with the latter shifted by $1 V cm^{-1} Torr^{-1}$. Solid and dashed curve are simulation results by Fraser and Mathieson. Measurements: filled circles and squares by Siegmund; open circles by Skelding; filled triangles by this experiment.

Figure 5.26: D_L/D_T Monte Carlo results for pure methane based on (a) elastic cross sections of Duncan and Walker [38]; (b) elastic cross sections of Kleban and Davis [39]; (c) hybrid elastic cross sections of (a) and (b). Measurements—open squares by Fulda-Quenzer et. al. [28]; filled circles by this experiment.

Chapter 6

Conclusions

The MWPC/TPC chamber is the only ¹ detector successfully measuring the three transport coefficients *simultaneously* with a reasonable accuracy, namely, the drift velocity, the transverse and the longitudinal diffusion coefficients. Since they are measured under the same conditions, systematic errors are greatly reduced.

The characteristics of the drift and diffusion measurements for argon-methane and argon-ethane mixtures are summarized below:

1. $\sigma_L(1cm) \sim 1/\sqrt{E}$.
2. $\sigma_T(1cm) \sim a + b \cdot E$ in first order. In the case for argon-methane mixtures, b is small and positive while for argon-ethane mixtures, b is small and negative.
3. $\sigma_{L,T}(1cm) \sim 1/\sqrt{P}$.

¹Although the experiment by Fulda-Quenzer et al measured both longitudinal and transverse diffusion coefficients by a single detector, they are not being measured simultaneously but are separately measured using a different electrode configuration

4. $\sigma_T(1cm)$ decreases in magnitude when the proportion of methane or ethane in argon increases (Fig. 6.1). There is only a marginal decrease in $\sigma_L(1cm)$ with α for argon-ethane mixtures. In fact, for argon-methane mixtures, $\sigma_L(1cm)$ increases with α when $E < 300 V/cm$ and decreases marginally with α when $E \geq 300 V/cm$ (Fig. 6.2).
5. Maximum drift velocity v_{max} increases with the proportion of methane α in argon. The drift field E at v_{max} also increases with α . No simple rule for argon-ethane mixtures is found.
6. E/P is a good scaling variable for the drift velocity but obviously not a good scaling variable for the diffusion coefficients.

The anisotropic nature of diffusion under a uniform electric field for argon-methane and ethane mixtures is clearly observed and accurately measured. Preliminary measurements for Ar/CO_2 90:10 mixture, which is not presented in this thesis, also exhibited this anisotropy. Although one cannot infer from these measurements that the anisotropic nature of diffusion is true for all gases, the measurements from this experiment indicate that it is a general behaviour for argon-based mixtures.

Recalling the calculation by Parker and Lowke (2.22):

$$D_L = D_T + \frac{1}{3} \sqrt{\frac{2}{m_e}} \frac{eE}{N} \int_0^\infty \frac{\epsilon}{\sigma_m} \frac{\partial F_1}{\partial \epsilon} d\epsilon + v_d \int_0^\infty \sqrt{\epsilon} F_1(\epsilon) d\epsilon.$$

Unless the last two terms of (2.22) is zero or negligible, which may not be the case in general, diffusion under the influence of a uniform electric field is almost always anisotropic.

The cross-over point E_{co} exhibits a linear relationship with the proportion of methane in argon α . If this linear relationship is extrapolated to $\alpha = 1$, i.e., 100% methane, the cross-over point E_{co} from this extrapolation is consistent with the value predicted by the Monte Carlo simulation by Fraser and Mathieson. One can now simply read out E_{co} directly from this linear graph for any argon-methane mixture.

If this linear characteristics is also true for some other gases, for instance, gases A and B, then knowing the E_{co} of pure A and B would be sufficient to predict the behaviour of any A_xB_{1-x} mixtures. The converse is also useful. Suppose one want to know E_{co} for gas B but this cannot be measured due to some other limitation ², E_{co} of pure B can still be predicted by extrapolating the data of A_xB_{1-x} .

The ratio v_{co}/v_{max} , where v_{co} is the drift velocity at E_{co} , is independent of α and v_{co} is on the positive slope of the drift velocity curve for the gas used here. This confirmed the theoretical predictions of Parker and Lowke on the bunching of electrons ³ given by (2.26) and (5.6):

$$\begin{array}{llll} \text{if } \frac{\partial \nu}{\partial \epsilon} < 0 & \text{or} & \frac{\partial \sigma(\epsilon)}{\partial \epsilon} < \frac{\sigma(\epsilon)}{2\epsilon} & \text{then } \sigma_L < \sigma_T; \\ \text{if } \frac{\partial \nu}{\partial \epsilon} = 0 & \text{or} & \frac{\partial \sigma(\epsilon)}{\partial \epsilon} = \frac{\sigma(\epsilon)}{2\epsilon} & \text{then } \sigma_L = \sigma_T; \\ \text{if } \frac{\partial \nu}{\partial \epsilon} > 0 & \text{or} & \frac{\partial \sigma(\epsilon)}{\partial \epsilon} > \frac{\sigma(\epsilon)}{2\epsilon} & \text{then } \sigma_L > \sigma_T. \end{array}$$

²In our case, to get a reasonable gas gain for pure methane the anode wire may not be able to withstand such a high voltage. Therefore, no pure methane measurement was taken.

³When the bunching mechanism is operating, the diffusion parallel to the field is smaller than the diffusion orthogonal to it, i.e., $\sigma_L < \sigma_T$. Anti-bunching is when $\sigma_L > \sigma_T$.

where $\nu(\epsilon)$ and $\sigma(\epsilon)$ are the collision frequency and the scattering cross section respectively. The ratio v_{co}/v_{max} for Ar/ C_2H_6 mixtures is also a constant and v_{co} is also on the positive slope of the drift velocity curve reinforce this theoretical prediction.

A parallel prediction associated with the above finding suggested that v_{max} is correlated to the local minimum of the scattering cross section $\sigma(\epsilon)$ (also known as the Ramsauer minimum) in the sense that the average electron energy ϵ at v_{max} is exactly the same energy region of the Ramsauer minimum. Also, the average electron energy ϵ corresponding to the positive slope of the drift velocity curve $dv/dE > 0$ is at the energy region of the negative slope of the cross section $\partial\sigma(\epsilon)/\partial\epsilon < 0$ and vice versa⁴.

If the above predictions are indeed correct, one can apply it to find the optimal electric field such that the longitudinal diffusion of a given gas mixture is a minimum. This information is valuable in the designing of high resolution drift chambers such as the vertex detectors.

Anisotropy ratio D_L/D_T for Ar/ CH_4 90:10 and Ar/ CH_4 80:10 agree very well with the Monte Carlo simulation by Fraser and Mathieson despite their unphysical assumption that scattering is isotropic. Not only the simulated anisotropy ratio is reasonably accurate, their Monte Carlo simulation also predicted the magnitude of the σ_L with a very reasonable accuracy. If the scattering cross section, and hence the collision frequency, is known either from experimental data or from theory then one can use this Monte

⁴This prediction works very well for the argon-methane mixtures where their drift velocity curves exhibit a local maximum. How this prediction works at the region where the drift velocity saturates, i.e., $dv_d/dE = 0$, is not known.

Carlo approach to estimate the above-mentioned cross-over point E_{co} and the optimal electric field.

It is experimentally known that E/P is a good scaling variable for transport phenomena such as the drift velocity or the gas gain mechanism but there is no strong experimental evidence to support E/P is a good scaling variable for the diffusion coefficients⁵. The measurements from this experiment that $\sigma_L \propto 1/\sqrt{E}$ and also $\propto 1/\sqrt{P}$, or more precisely, $\sigma_L \propto 1/\sqrt{EP}$ suggested that EP might be a more plausible scaling variable for σ_L ⁶. Simple scaling rule for σ_T cannot be deduced from this experiment.

⁵ Although E/P is used as a variable in many published experimental results to present diffusion measurements, there is no theoretical or experimental basis to prove that it is a good scaling variable for diffusion other than the fact that it is being used widely in many other transport phenomena.

⁶ At least valid at the 'EP' region studied by this experiment

Figure 6.1: $\sigma_T(1cm)$ as a function of the proportion of methane and ethane in argon. σ_T data for Ar/C_2H_6 are shifted by $-100 \mu m/\sqrt{cm}$ for clarity. Error bars are suppressed.

Figure 6.2: $\sigma_L(1cm)$ as a function of the proportion of methane and ethane in argon. σ_L of Ar/C_2H_6 data are shifted by $-125 \mu m/\sqrt{cm}$ for clarity. Error bars are suppressed.

Appendix A.

Electrostatic Model Calculation

If a point charge Q_0 is located at (x_0, y_0, L) , the induced charge distribution $\sigma(x, y)$ on the cathode plane is given by:

$$\sigma(x, y) = -\frac{Q_0 L}{2\pi r^3}$$
$$r^2 = L^2 + (x - x_0)^2 + (y - y_0)^2.$$

Suppose the cathode plane is divided into rectangular pads of dimension (l, w) as in Figure A.1, the charge induced on pad (m, n) is given by:

$$Q(m, n) = -\frac{Q_0}{2\pi} \sum_{i=1}^2 \sum_{j=1}^2 (-1)^{i+j} \arctan \left[\frac{x_{im} y_{jn}}{L \sqrt{L^2 + x_{im}^2 + y_{jn}^2}} \right], \quad (\text{A.1})$$

where

$$x_{\{2,1\}m} = (m \pm \frac{1}{2})l - x_0,$$

$$y_{\{2,1\}n} = (n \pm \frac{1}{2})w - y_0.$$

The induced charge distribution of the central pads calculated according to (A.1) are shown in Figure A.2. If we consider the MWPC/TPC geometry with only the central cathode pad column (i.e. $m = 0$) and the anode wire at $x_0 = 0$, then $Q(n)$ is given by:

$$\begin{aligned} Q(n) &= -\frac{Q_0}{\pi} \sum_{j=1}^2 (-1)^j \arctan \left[\frac{l}{2L \sqrt{L^2 + l^2 + y_{jn}^2}} \right] \\ &= -\frac{Q_0}{\pi} \arctan \left[\frac{y}{2L \sqrt{L^2 + l^2 + y^2}} \right] \Big|_{y_{1n}}^{y_{2n}} \end{aligned} \quad (\text{A.2})$$

Compared with the result of Endo et al [40] when $l \rightarrow \infty$, the induced charge $Q(n)$ on the n -th strip placed at $a < y < a + w$ is given by:

$$\begin{aligned} Q(n) &= -\frac{Q_0}{4L} \int_a^{a+w} \text{sech}\left(\frac{\pi y}{2L}\right) dy \\ &= -\frac{Q_0}{\pi} \arctan(\exp^{\frac{\pi y}{2L}}) \Big|_a^{a+w} \quad \text{for } a > 0. \end{aligned} \quad (\text{A.3})$$

Transverse diffusion simulation are calculated according to the above electrostatic model using (A.1) and (A.2).

(a) Elevation View

$Q(x_0, y_0, L)$

(b) Plan View

Figure A.1: Geometry of the electrostatic model

(a)

(b)

Figure A.2: Calculated cathode pad induced charge distribution:

(a) Point charge located at the center of a pad;

(b) Point charge located between two pads.

Appendix B

Fitting Correction Calculation

The following calculation is originated from Dr. Michael Losty [41]. Consider a linear fit $x = az + b$ to the data points x_i , $i = 1, \dots, 5$ positioned at $z = 2, 1, 0, 1, 2$ units and assuming all have the same resolution σ . The residual of the i -th wire is given by

$$R_i = x_i - (az_i + b),$$

where

$$a = \frac{N \sum x_i z_i - (\sum x_i)(\sum z_i)}{N \sum z_i^2 - (\sum z_i)^2},$$

and

$$b = \frac{(\sum x_i)(\sum z_i^2) - (\sum x_i z_i)(\sum z_i)}{N \sum z_i^2 - (\sum z_i)^2}.$$

The variance of the i -th residual is

$$\delta^2 R_i = \sum_k \left(\frac{\partial R_i}{\partial x_k} \right)^2 \sigma^2.$$

After some algebra $\delta^2 R_i$ is found to be

$$\delta^2 R_i = A_i^2 \sigma^2 = \left[1 - \frac{1}{N} - \frac{(z_i - \langle z \rangle)^2}{\sum_k (z_k - \langle z \rangle)^2} \right] \sigma^2,$$

where $\langle z \rangle = \sum z_i = 0$ in our case. Thus, true resolution δR , will be reduced by the above factor A . To correct the measured resolution, the 5 wires have to be corrected by $1/A$, which in our case are $\sqrt{10/4}$, $\sqrt{10/7}$, $\sqrt{10/8}$, $\sqrt{10/7}$, $\sqrt{10/4}$ or 1.5811, 1.1952, 1.1180, 1.1952, 1.5811 respectively. The 3% difference from the Monte Carlo result could be due to the angular acceptance built into the simulation model.

Bibliography

- 1 F. Sauli, *Principles of Operation of Multiwire Proportional and Drift Chambers*, CERN Internal Report 77-09.
- 2 Ludlam, et. al., *I.E.E.E. Trans. Nucl. Sci.* **NS-28** (1981) 439.
- 3 G. Charpak, et. al., *Nucl. Inst. Meth.* **62** (1968) 235.
- 4 G. A. Erskine, *Nucl. Inst. Meth.* **105** (1972) 565.
- 5 OPAL Detector Technical Proposal, CERN/LEPC/83-4.
- 6 J. Va'vra, *SLAC-PUB-9727*.
- 7 A. Walenta et. al., *Proc. of the International Conference on Instrumentation for Colliding Beam Physics*, SLAC-Report 250, June 1982.
- 8 B. Schmidt, *The IV International Wire Chamber Conference*, Vienna 1986.
- 9 L. Huxley and R. Crompton, *The Diffusion and Drift of Electrons in Gases*, John Wiley and Sons, New York.
- 10 K. Huang, *Statistical Mechanics*, John Wiley and Sons, New York.

- 11 P. M. Morse, W. P. Allis and E. S. Lamar, *Phys. Rev.* **48** (1935) 412.
- 12 J. H. Parker and J. J. Lowke, *Phys. Rev.* **181** (1969) 290.
- 13 J. J. Lowke and J. H. Parker, *Phys. Rev.* **181** (1969) 302.
- 14 V. Palladino and B. Sadoulet, *Nucl. Inst. Meth.* **128** (1975) 323.
- 15 G. Schultz and J. Gresser, *Nucl. Inst. Meth.* **151** (1978) 413.
- 16 L. Ferrari, *Physica* **81A** (1975) 276.
- 17 H. R. Sküllerud & S. Kuhn, *J. Phys. D: Appl. Phys.* **16** (1983) 1225.
- 18 S. L. Lin, R. E. Robson & E. A. Mason, *J. Chem. Phys.* **71** (1979) 3483.
- 19 R. Kubo, *Rep. Prog. Phys.* **29** (1966) 255.
- 20 H. R. Sküllerud, *J. Phys. B: Atom. Molec. Phys.* **2** (1969) 85.
- 21 H. R. Sküllerud, *J. Phys. B: Atom. Molec. Phys.* **6** (1973) 728.
- 22 D. M. Binnie, *Nucl. Inst. Meth.* **A234** (1985) 54.
- 23 G.W. Fraser and E. Mathieson, *Nucl. Inst. Meth.* **A247** (1986) 544.
- 24 G.W. Fraser and E. Mathieson, *Nucl. Inst. Meth.* **A247** (1986) 566.
- 25 L. S. Frost and A.V. Phelps, *Phys. Rev.* **127** (1962) 1621.
- 26 E. Wagner, F. Davis and G. Hurst, *J. Chem. Phys.* **47** (1967) 3138.
- 27 H. Drumm, et. al., *DESY 80/38* (1980).

- 28 F. Fulda-Quenzer, et. al., *Nucl. Inst. Meth.* **A235** (1985) 517.
- 29 O. H. W. Siegmund, *IEEE Trans. Nucl. Sci.* **NS-30** (1983) 950.
- 30 R. Skelding, *Ph.D. Thesis, U. of Leicester* (1984).
- 31 H. A. Bethe and R. Jackiw, *Intermediate Quantum Mechanics*, Benjamin Cummings, 1986.
- 32 J. Allison et.al. *Nucl. Inst. Meth.* **A236** (1985) 284.
- 33 R. A. Boie, A. T. Hrisoho and P. Rehak, *I.E.E.E. Trans. Nucl. Sci.* **NS-28** (1981) 603.
- 34 E. Ritchie, *M.Sc. Thesis, Carleton University*, 1985.
- 35 B. Jean-Marie, V. Lepeltier and D. L'Hote, *Nucl. Inst. Meth.* **159** (1979) 213.
- 36 A. Peisert and F. Sauli, *CERN Report* 84-08.
- 37 F. Piuz, *Nucl. Inst. Meth.* **205** (1983) 425.
- 38 C. W. Duncan and I. C. Walker, *Trans. Faraday Soc.* **68** (1972) 1514.
- 39 P. Kleban and H. T. Davis, *J. Chem. Phys.* **68** (1978) 2999.
- 40 I. Endo, et. al., *Nucl. Inst. Meth.* **188** (1981) 51.
- 41 M. Losty, *private communication*, 1986.

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