Spectrum Requirements for Interference-Free Wireless Mesh Networks

by

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A thesis submitted to
the Faculty of Graduate and Postdoctoral Affairs
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Electrical and Computer Engineering

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July 2015

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Abstract

In classical channel assignment (CA) in Multi-Radio Multi-Channel (MRMC) Wireless Mesh Networks (WMNs), the number of available frequency channels is assumed to be fixed. Two links that are within the interference range of each other could be assigned the same frequency channel, causing co-channel interference that degrades the network throughput. The objective of this research is to develop a realistic CA method that finds the smallest number of frequency channels required for interference-free communication among the mesh nodes (MNs) in order to achieve the maximum network throughput while maintaining fairness among the multiple network flows in a dynamic MRMC WMN.

As a first step towards achieving this objective, a novel CA method is developed, which ensures interference-free communication among the MNs based on the protocol interference model, and determines a small number of frequency channels required to achieve the maximum network throughput while maintaining fairness among the multiple network flows. Secondly, in order to develop a CA method using a realistic interference model, a novel and computationally simple method of building the conflict graph based on signal-to-interference ratio model with shadowing is developed.

Computationally simple and effective new heuristics are developed to find channel assignments from the conflict graph for the extended coloring problem with cumulative interference constraints. The heuristics are orders of magnitude faster than the exact solution method while consistently returning near-optimum results.

As a final step, the problem of co-channel interference in a dynamic WMN environment is addressed by using beamforming. The novel *Linear Array Beamforming-based Channel Assignment* (LAB-CA) method reduces the number of frequency channels required (NCR) and significantly outperforms the classical omni-directional antenna pattern-based channel assignment (OAP-CA) method in terms of NCR.

The beamforming-based CA framework is extended to incorporate heterogeneous MNs (i.e. nodes having differing numbers of radio interfaces). The LAB-CA method for heterogeneous MNs outperforms OAP-CA for heterogeneous MNs in terms of NCR in both sparse and dense mesh networks. It also provides a further significant reduction in NCR when the number of antennas in the linear antenna arrays of MNs is increased.

To my late father: Chaudhry Suhbat Ali

Acknowledgements

First and foremost, I am thankful to Allah, my creator, who gave me patience, determination, and perseverance to enable me to complete this work. This work would not have been possible without the support of Allah.

I thank my supervisors Professor Roshdy Hafez and Professor John Chinneck for their continual guidance and unrelenting support. Without their tireless, kind, and extremely knowledgeable supervision, this work would not have been possible. Their encouragement always gave me confidence to enable me to continue my research work. I am thankful to the examination committee members including Professor Ekram Hossein, Professor Ian Marsland, Professor Marc St-Hilaire, and Professor Yongyi Mao for their valuable feedback, which led me to improve the quality and presentation of this thesis. I thank my colleagues Dr. Rajab Legnain, Rami Sabouni, and Tamer Beitalmal, who made my stay at Carleton University an enjoyable one.

I am extremely grateful to my parents. The praise that I got from my late father when I did well in education was a source of motivation for me that enabled me to do even better next time. I will always miss that praise. The prayers of my mother have been the source of success for me in my life in general and in this research work in particular. I hope that I will always continue to receive her prayers. I am thankful to my wife Nazli who patiently supported me throughout this long and sometimes stressful journey. I am also grateful to my sons Ehtesham, Absar, and Fasih for their understanding of the demanding requirements of this research work.

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LIST OF ACRONYMS AND ABBREVIATIONS

AF	Array Factor
AG	Array Gain
AND	Average Node-Degree
AP	Access Point
CA	Channel Assignment
CG	Connectivity Graph
CI	Confidence Interval
CRT	Controlled Random Topology
DNT	Direct Neighbor Table
EP	Element Pattern
FNT	Final Neighbor Table
GW	Gateway
ILP	Integer Linear Programming
IR-AND	Interference Range-Average Node-Degree
LAB-CA	Linear Array Beamforming-based Channel Assignment
LAB-CA_HG	Linear Array Beamforming-based Channel Assignment for HomoGeneous mesh nodes
LAB-CA_HT	Linear Array Beamforming-based Channel Assignment for HeTerogeneous mesh nodes
LCR	Links to frequency Channels Ratio
LP	Linear Programming
MaIS	Maximal Independent Set
MAC	Medium Access Control
MILP	Mixed Integer Linear Programming
MIS	Maximum Independent Set
MP	Maximum Power
MPNT	Maximum Power Neighbor Table
MPSP	Minimum Power-based Shortest Path
MR	Mesh Router
MRMC	Multi-Radio Multi-Channel

MRMCMA	Multi-Radio Multi-Channel Multi-Antenna
NCCM	Number of Conflicts in the Conflict Matrix
NCR	Number of frequency Channels Required
NDC	Node-Degree Constraint
NT	Network Throughput
OAP-CA	Omni-directional Antenna Pattern-based Channel Assignment
OAP-CA_HT	Omni-directional Antenna Pattern-based Channel Assignment for HeTerogeneous mesh nodes
OP	Outage Probability
QCBP	Quadratically Constrained Binary Programming
QCP	Quadratically Constrained Program
RTS/CTS	Request to Send/Clear to Send
SINR	Signal-to-Interference-and-Noise Ratio
SIR	Signal-to-Interference Ratio
TCA	Topology Control Algorithm
TDMA	Time Division Multiple Access
TR-AND	Transmission Range-Average Node-Degree
WLAN	Wireless Local Area Network
WMaIS	Weighted Maximal Independent Set
WMN	Wireless Mesh Network

LIST OF SYMBOLS

C	connectivity graph
V	set of vertices of the connectivity graph representing wireless mesh nodes
E	set of edges of the connectivity graph representing wireless links
P	set of commodities, i.e. source-destination pairs
p	a commodity
l_{ij}	wireless link between node i and node j where $(i,j) \in E$
f_{ij}^p	amount of flow of commodity p on link l_{ij}
f_s^p	amount of inflow of commodity p from the source of p
f_d^p	amount of outflow of commodity p from the sink of p
c_{ij}	capacity of link l_{ij}
z_{ij}	its value is 1 when l_{ij} is used for routing; 0 otherwise
dc	constraint on the node-degree of mesh routers
$costl_{ij}$	represents a cost of 0.0001 for each link l_{ij} in multi-path routing
$demand_{sd}$	represents unit flow demands between the source-sink pairs
y	multiplier on the unit flow demand of the commodities
s_p	source of commodity p
d_p	destination or sink of commodity p
$cost2_{ij}$	represents the cost of each link l_{ij} in MPSP routing
$supply_i$	its value is 1 for sources in MPSP routing
$demand_i$	its value is -1 for the sink, i.e gateway, in MPSP routing
F	conflict graph
V_F	set of vertices of the conflict graph representing wireless links
E_F	set of edges of the conflict graph representing conflicts between wireless links
d_{ij}	distance between nodes i and j
R_i	transmission range of node i
R_i'	interference range of node i
m	number of nodes or vertices in the conflict graph
v^*	vertex in the conflict graph which has the highest node-degree

$N(v^*)$	neighborhood of v^* , i.e. nodes that are connected to v^* by an edge
$n1$	set of all the nodes of the conflict graph except v^*
$n2$	set of all the nodes of the conflict graph excluding both v^* and $N(v^*)$
$maxset$	the higher cardinality set among $n1$ and $n2$
L	set of links involved in routing
V_{MaIS}	maximal independent set of vertices
L_{MaIS}	set of links in L corresponding to vertices in MaIS
c_{MaIS}	number of MaISs
V_{MIS}	maximum independent set of vertices
L_{MIS}	set of links in L corresponding to vertices in MIS
c_{MIS}	number of MISs
f	frequency
G_t	gain of transmitter
G_r	gain of receiver
h_t	transmitter antenna height
h_r	receiver antenna height
n	number of mesh nodes in the network
$RxThresh_dBm$	receiver threshold in dBm
$RxThresh_mwatts$	receiver threshold in mW
$SIRThresh_dB$	SIR threshold in dB
$SIRThresh$	SIR threshold in linear scale
$locations_x$	set of x -coordinates of nodes
$locations_y$	set of y -coordinates of nodes
σ	standard deviation for shadowing
OP	outage probability
$Cross_over_dist$	cross-over distance
P_t	transmission power
P_r	received power
$P_{r,xp}$	power received at node x from node p
MT	maximum link throughput in bps

L_{DATA}	length of the payload in bytes
T_{D_DATA}	data transmission delay
T_{D_ACK}	acknowledgement transmission delay
τ	propagation delay and is equal to 1 μ s for 802.11a
T_{SIFS}	short interframe space time and is equal to 16 μ s for 802.11a
T_{DIFS}	distributed coordination function interframe space time and is equal to 34 μ s for 802.11a
$\overline{CW}$	average backoff time
T_P	transmission time of physical layer preamble and is equal to 16 μ s for 802.11a
T_{PHY}	transmission time of the physical layer header and is equal to 4 μ s for 802.11a
T_{SYM}	transmission time for a symbol and is equal to 4 μ s for 802.11a
L_{H_DATA}	MAC overhead in bytes and is equal to 28 bytes
L_{ACK}	acknowledgement size in bytes and is equal to 28 bytes
N_{DBPS}	number of data bits per symbol
CW_{min}	minimum backoff window size and is equal to 15 for 802.11a
T_{slot}	slot time and is equal to 9 μ s for 802.11a
V_{WMaIS}	weighted maximal independent set of vertices
L_{WMaIS}	set of links in L corresponding to vertices in WMaIS
V_{temp}	temporary set of vertices
c_{WMaIS}	number of WMaISs
n_c	maximum number of colors allowed
x_{ih}	its value is 1 when vertex i is assigned the color h ; 0 otherwise
y_h	its value is 1 when color h is used in the solution; 0 otherwise
P_{ik}	represents the maximum power received in milliwatts at vertex i from vertex k and is an element of the conflict matrix at i^{th} row and k^{th} column
$\sum_k P_{ik} x_{kh} x_{ih}$	this term sums up the maximum powers received at vertex i when it is assigned color h from all other vertices which are also assigned color h

ϕ	azimuth angle
θ	elevation angle
N_a	number of antenna elements in the linear antenna array
d_a	distance between the antenna elements in the linear antenna array
β	phase by which the current in each element leads the current of the preceding element in the antenna array
ϕ_o	azimuth angle towards which the beam is steered
$AF(\theta = \pi/2, \phi)$	array factor in the ϕ -plane
$EP(\theta = \pi/2, \phi)$	element pattern in the ϕ -plane
D_o	maximum directivity of a half-wavelength dipole antenna
G_o	maximum gain of an antenna in terms of maximum directivity
e_{cd}	antenna radiation efficiency
$G_e(\theta, \phi)$	gain realized when only one element is excited
$AG(\theta = \pi/2, \phi)$	array gain in the ϕ -plane
AG_{ab}	array gain from node a to node b
ϕ_{ab}	angle between nodes a and b
ND	set of node-degrees of mesh nodes
nd_i	node-degree of a mesh node i
F_J	Jain's fairness index
x_i	throughput of a flow i
N	total number of flows (sources) in the network
q	number of neighbors of a mesh node
N_{CBPS}	number of coded bits per symbol
T_S	symbol duration
B	occupied bandwidth
E_b/N_o	ratio of energy-per-bit and noise
γ	represents the SIR threshold
$NCCM/ L ^2$	ratio of number of conflicts in the conflict matrix and size of the conflict matrix

<i>absmipgap</i>	absolute value of the difference between the best integer solution found by time-out and the lowest bound for any unsolved node remaining in the search tree at time-out
<i>relmipgap</i>	ratio between <i>absmipgap</i> and the best integer solution found by time-out
<i>mipgap</i>	CPLEX parameter that stops the branching process when the current objective function value is within $(100 \times mipgap)\%$ of the optimum
<i>mipemphasis</i>	CPLEX parameter that controls its branching strategy
<i>solutionlim</i>	CPLEX parameter that terminates the search after the i^{th} feasible solution is found
PR_{NCR}	percentage reduction in NCR
PI_{LCR}	percentage improvement in LCR

1. INTRODUCTION

The use of Wireless Local Area Networks (WLANs) has grown tremendously in the past few years due to their ease of deployment and maintenance. However, the access points in these WLANs have to be connected to the backbone network through wired media. Wireless Mesh Networks (WMNs) offer an attractive alternative for providing broadband wireless Internet connectivity by using a wireless backhaul network and eliminating the need for extensive cabling [1].

1.1. Overview

In traditional WLANs, each access point (AP) is connected to the wired network while only a subset of APs is connected to the wired network in WMNs. An AP that is connected to the wired network is called a Gateway (GW); APs without wired connections are called Mesh Routers (MRs), and they connect to the GW through multiple hops. Like routers in a wired network, MRs in a WMN forward each other's traffic to establish and maintain their connectivity. MRs and GWs are similar in design, with the only difference that a GW is directly connected to the wired network, while a MR is not.

Single-radio mesh routers operating on a single frequency channel suffer from hidden and exposed terminal problems which cause collisions and degrade the network throughput. Equipping mesh routers with multiple radios operating on non-overlapping frequency channels can significantly improve the network throughput. In a classical Multi-Radio Multi-Channel (MRMC) WMN architecture, the total number of available non-overlapping frequency channels is assumed to be fixed. Fig. 1.1 shows a sample MRMC WMN in a typical enterprise such as a university where mesh nodes are installed on the rooftop of buildings and the propagation environment is characterized by a strong line-of-sight component.

WMNs aim to provide high-bandwidth broadband connections to a large community, and thus should be able to accommodate a large number of users accessing the Internet. Many companies such as Cisco, Strix Systems, and MeshDynamics are offering products for mesh networking solutions to build small and large-scale broadband

Internet connectivity. The Wireless Mesh Networking Solution [2] offered by Cisco provides city or campus-wide Wi-Fi network that is easy to deploy and manage. The Cisco Aironet 1524SB Outdoor Access Point [3] is a multi-radio node with two 802.11a radios for backhaul connectivity. The third 802.11 b/g radio is dedicated to client access. One of the uses of the Strix Systems' Outdoor Wireless Mesh Solution [4] is to provide 802.11-based building-to-building connectivity across metropolitan areas.

Figure 1.1. A sample enterprise WMN

Figure 1.2. Cisco Aironet 1520 Series Mesh Access Point [3]

WMNs can be used for public safety application such as video surveillance to lower surveillance costs. By providing wireless network connectivity, Firetide mesh networks [5] enable video cameras to be installed without laying cable through walls or between buildings, or drilling holes for cables. WMNs can be set up on-demand to

provide internet access on temporary basis in disaster hit areas, for video surveillance of temporary venues such as sporting events, etc.

WMNs can be used to provide a wireless backhaul in heterogeneous cellular networks. To meet the increasing data rate requirements of the end users, more and more pico-cells are being installed in cellular networks [6]. Currently, each of these pico-cells has to be connected to the wired network to send its traffic to the core network. The infrastructure cost of connecting each pico-cell to the wired network can be saved. Pico-cells, like mesh routers, can connect to the macro-cell through multiple intermediate hops/pico-cells in order to gain access to the wired network. The macro-cell, which is already connected to the wired network, can act as a mesh gateway for the pico-cells to access the core network. In this way, a WMN can be set up between the pico-cells and the macro-cell to provide a wireless backhaul in heterogeneous cellular networks.

1.2. Motivation

The IEEE 802.11a standard [7] provides a fixed number of non-overlapping frequency channels that is 12. These frequency channels can be used simultaneously within the interference range and reused outside the interference range of MRs by assigning them to the radio interfaces of the mesh routers. The classical channel assignment schemes for MRMC WMNs are based on this assumption of a fixed number of available frequency channels. The channel assignment problem for such schemes consists of assigning the available frequency channels to the radios of multi-radio mesh nodes such that co-channel interference is minimized and network throughput is maximized. In such schemes, a link is assigned a conflicting frequency channel if all the available frequency channels have already been assigned within the interference range of that link and a non-conflicting frequency channel is not available. Hence, two links that are within interference range of each other could be assigned the same frequency channel. This causes co-channel interference, which degrades the network throughput. Consider the example shown in Fig. 1.3 where two links that are assigned the same frequency channel by a classical channel assignment scheme [8] suffer from co-channel interference.

As can be seen from Fig. 1.3, links 18-28 and 12-24 are assigned the same frequency channel. The red circular disc in this figure indicates the interference range of

node 18 which is twice its transmission range, i.e. the distance between nodes 18 and 28. As shown in the figure, node 24 of link 12-24 is within the interference range of node 18 of link 18-28. Hence, co-channel interference is encountered by node 24 from node 18 while communicating with node 12 on channel 5. These conflicting frequency channels are unavoidable in the classical channel assignment schemes due to the limitation on the number of available frequency channels where a link is assigned a conflicting frequency channel if a non-conflicting frequency channel is not available. The presence of these conflicting frequency channels degrades the network throughput due to the co-channel interference caused at links using such channels.

Figure 1.3. Co-channel interference problem [8]

This problem can be addressed as follows: instead of working with a fixed number of frequency channels, the number of frequency channels must be determined that provides an interference-free channel assignment that allows the mesh network to achieve the maximum network throughput; interference-free communication among the mesh nodes must be ensured by ensuring that links within interference range of each other are assigned different frequency channels. Hence, a new channel assignment method is required that ensures interference-free communication among the mesh nodes,

and determines the number of frequency channels required in order to achieve the maximum network throughput.

1.3. Contribution

Following is the main contribution of this work:

- A novel interference-free channel assignment method is developed, which ensures interference-free communication among the mesh nodes, achieves the maximum network throughput while maintaining fairness among the multiple network flows, and finds a small number of frequency channels needed for interference-free communication. As part of this contribution,
 - a mixed integer linear programming formulation for multi-path routing is developed;
 - an effective and computationally simple method is developed for constructing the conflict graph and matrix using the SIR (signal-to-interference ratio) model with shadowing;
 - new and effective heuristics are developed for solving the extended coloring problem with cumulative interference constraints;
 - a method to construct the beamforming-based conflict graph and matrix is developed; and
 - an extended beamforming-based channel assignment framework is developed to incorporate heterogeneous mesh nodes (i.e. nodes having differing numbers of radio interfaces).

1.4. Organization

The rest of the thesis is organized as follows. Chapter 2 gives a background on some topics including mixed integer programming, minimum coloring problem and maximum independent set problem so that it is easy for the reader to understand the contents of this thesis.

Chapter 3 gives an overview of the related work. Some well-known classical channel assignment schemes for MRMC WMNs are discussed. Some topology control schemes and the importance of topology control in WMNs are discussed. In addition to channel assignment, some of these schemes also deal with routing in MRMC WMNs.

Some schemes focus on improving the throughput in MRMC WMNs without considering fairness while others try to maximize fairness among the traffic flows as well. The main interference models in WMNs and several channel assignment schemes are discussed with respect to their use of an interference model. Schemes for minimum coloring as well as those using minimum coloring for resource allocation in wireless networks are discussed. The role of directional antennas in MRMC WMNs to improve spatial channel reuse and how such antennas cannot be successful in a dynamic WMN environment is discussed. The role of beamforming for the successful operation of a self-organizing and dynamic MRMC WMN as well as some beamforming-based link scheduling schemes for single-radio single-channel TDMA-based multi-hop wireless networks is discussed. Channel assignment schemes with respect to the heterogeneity of the mesh nodes as well as the size of the mesh network are discussed.

Chapter 4 presents the problem statement and introduces the channel assignment problem for interference-free WMNs. Also, the channel assignment problem is discussed in the context of realistic and dynamic WMNs.

Chapter 5 presents the new channel assignment (CA) method which ensures interference-free communication among the mesh nodes and determines a small number of frequency channels required in order to achieve the maximum network throughput while maintaining fairness among the multiple network flows for interference-free CA in MRMC WMNs using the protocol-based interference model. The model of the network architecture is given and the creation of the connectivity graph using topology control is explained. The problem formulation for routing is presented. The creation of the conflict graph using the protocol model is discussed and the solution of the minimum coloring stage of the channel assignment problem is presented.

In Chapter 6, the impact of three different interference models on CA in MRMC WMNs is studied, namely the protocol model, the SIR model and the SIR model with shadowing. This chapter presents the work on interference modeling for realistic and interference-free channel assignment for MRMC WMNs. It explains the creation of the conflict graph using realistic interference models.

Chapter 7 introduces the extended minimum coloring problem and presents fast and effective methods for wireless channel assignment via new heuristics for solving this

extended coloring problem. Also, an exact quadratically constrained binary programming formulation for this problem is presented.

The fundamentals of beamforming are given in Chapter 8. The novel Linear Array Beamforming-based Channel Assignment method is presented which significantly reduces the number of frequency channels required for interference-free communication among the mesh nodes. Also, the extension of the beamforming-based channel assignment framework to incorporate heterogeneous mesh nodes is presented in this chapter.

Chapter 9 presents the performance evaluation of the interference-free channel assignment method for MRMC WMNs. The impact of topology control algorithm-based connectivity graph vs. maximum power-based connectivity graph; the impact of multi-path routing vs. minimum power-based shortest path routing; the impact of maximal independent set-based vs. maximum independent set-based minimum coloring heuristic; and the impact of three different interference models on channel assignment in MRMC WMNs is investigated and analyzed. Reasonable assumptions are used based on the IEEE 802.11a standard to generate representative results; however, this work can be applied to any scenario of multi-hop MRMC WMNs and is not limited to a specific standard. The performance of linear array beamforming-based channel assignment (LAB-CA) method is evaluated in comparison with omni-directional antenna pattern-based channel assignment (OAP-CA) method in terms of number of frequency channels (NCR). The performance of linear array beamforming-based CA method for heterogeneous mesh nodes (LAB-CA_HT) is compared with linear array beamforming-based CA method for homogeneous mesh nodes (LAB-CA_HG) in terms of NCR. The performance of LAB-CA_HT is evaluated in both dense and sparse wireless mesh networks by comparing its performance with the omni-directional antenna pattern-based channel assignment method for heterogeneous mesh nodes (OAP-CA_HT) in both scenarios. The impact of increasing the number of antennas in the linear antenna arrays of the multi-radio mesh nodes on the number of frequency channels in a dense mesh network is investigated when using LAB-CA_HT.

Chapter 10 presents the conclusions and some directions for future research.

2. BACKGROUND

In this chapter, a brief discussion of some topics which include mixed integer programming, minimum coloring problem and maximum independent set problem is presented. The purpose of providing this information is to make it easy for the reader to understand the contents of this thesis.

2.1. Mixed Integer Programming

Optimization provides a way of allocating the available resources in the best possible manner. Optimization techniques are used on daily basis to optimally solve complex problems in telecommunications, industrial planning, transportation, etc. *Linear programming* is the most widely used optimization technique for constrained programming. In constrained optimization, not only you have to find the best point of the function but you also have to meet certain constraints while doing so.

Linear programming problems, also known as LPs, contain a set of *decision variables*. They represent the unknown quantities that are to be optimized. The function that combines the decision variables to express your goal is called the *objective function*. An LP will either minimize or maximize the value of the objective function. The decisions that must be made are subject to certain restrictions and requirements of a system. These are enforced by including a set of *constraints* in the model. The constraints are represented by mathematical expressions that combine the decision variables to put limits on possible solutions. Also, the decision variables usually have bounds. In an LP, the objective function and the constraints are a linear function of the decision variables. The LP is solved by determining the values of the variables that minimize or maximize the objective function. The most general form of an LP having m constraints and n variables is show below [9].

$$\text{minimize or maximize } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \{ \leq, =, \text{ or } \geq \} b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \{ \leq, =, \text{ or } \geq \} b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \{\leq, =, \text{ or } \geq\} b_m$$

$$l_j \leq x_j \leq u_j \quad \forall j = 1, \dots, n$$

where c_j , $\forall j = 1, \dots, n$, are referred to as objective function coefficients and represent the increase or decrease in Z per unit increase or decrease in x_j ; b_i are the right-hand side values of the constraints and represent the available resources or requirements; a_{ij} are the coefficients of the variables in the constraints and denote how much of resource/requirement i is consumed/satisfied by decision j ; and l_j , u_j represent the lower and upper bounds on the variables.

Integer linear programs (ILPs) are also stated in a similar manner, except that all the decision variables can take on integer values only (or only binary values). A *mixed integer linear program* (MILP) has an added restriction that some of the variables are integer-valued (or binary-valued) and the remaining variables are real-valued. While solving practical problems, it is usually impossible to enumerate all possible solutions to find the best one. Integer linear programming problems as well as mixed integer linear programming problems experience *combinatorial explosion*. Suppose that there are n binary variables. Since there are two choices for each variables, so there are 2^n configurations for these variables and each configuration needs to be evaluated to find the best solution. Using a computer capable of processing 10 trillion operations per second, if $n = 50$, the computer will take 1.5 hours to finish enumerating all possibilities; for $n = 60$, the computer will require 80 days to terminate, and for $n = 70$, the computer will require 262 years [10]. The combinatorial explosion is even worse for integer variables which can take on more values than just the two possibilities for binary variables [9].

Branch and bound is a technique which is commonly employed by popular solvers like CPLEX [11] to solve MILP problems as it can structure the problem to slow down the combinatorial explosion. Such techniques use *implicit enumeration*. They do not *explicitly* examine every possible solution, but instead examine a subset of possible solutions to find the best one. The branch and bound method is based on the fact that the enumeration of integer solutions has a tree structure. The branch and bound avoids growing the whole tree by growing the tree in stages and grows only the most promising nodes at any stage. Although branch and bound slows down the combinatorial explosion; however, it doesn't stop it altogether. In cases where the branch and bound tree is too

large and the time is limited, we can sacrifice the optimal solution in favor of getting a reasonable answer quickly such as the first feasible solution. The branch and bound process can also be stopped when the current solution is within $x\%$ of the optimum [9].

In addition to being popular for solving linear programming problems, CPLEX can also be used for solving mixed integer linear programming problems. One of the algorithmic control parameters of CPLEX is *mipgap*. Depending on the value of this parameter, CPLEX will stop the branching process when the current objective function value is within $(100 \times \text{mipgap})\%$ of the optimum. Increasing *mipgap* allows a solution that is potentially suboptimal to be accepted but the solution time may be significantly shortened as a result. A parameter of CPLEX which deals with the degree of effort used in seeking optimality is *mipemphasis* and controls its branching strategy. The default of 0 for this parameter corresponds to a balance between searching for feasible solutions and proving optimality. A setting of 1 shifts the emphasis strongly toward finding new feasible solutions. A parameter of CPLEX which controls the halting of search is *solutionlim*. It ensures that the search is terminated after the i^{th} feasible solution is found [11].

In binary integer programming that deals with solving binary problems, the variables can have a value of 0 or 1 [9]. Binary quadratic programming belongs to a class of optimization problems that has a quadratic term either in its objective function or in its constraints. Binary programming with quadratic constraints, also known as quadratically constrained binary programming, deals with the problem of maximizing (or minimizing) a linear combination of binary variables, subject to a set of quadratic constraints. The quadratic terms appear in one or more constraints of the problem. Due to its ability to handle quadratically constrained programs [11], CPLEX can be used to solve quadratically constrained binary programming problems.

2.2. Minimum Coloring Problem

A graph G consists of n vertices and m edges. Then, $V(G) = \{v_1, v_2, \dots, v_n\}$ is the set containing vertices of G and $E(G) = \{e_1, e_2, \dots, e_m\}$ is the set containing the edges of G . Each edge in $E(G)$ consists of two distinct vertices in $V(G)$. For an edge $e = \{v_i, v_j\}$ such that $e \in E(G)$, the vertices v_i and v_j are said to be adjacent in G and are referred to as the endpoints of e .

Graph coloring deals with the problem of partitioning a set of objects into different classes according to certain requirements. It is useful in solving various problems and has several applications.

The *vertex coloring* of the graph G is an assignment of colors to vertices such that adjacent vertices receive different colors. The *minimum coloring problem* is the problem of computing a coloring of the vertices in the graph using as few distinct colors as possible [12]. The minimum coloring problem is well-known to be NP-hard for general graphs [13]. An example of minimum coloring is shown in Fig. 2.1 where the given graph is colored using a minimum of two colors.

Figure 2.1. Minimum coloring

The graph shown in Fig. 2.1 can represent the conflict graph of a wireless mesh network where the vertices of the conflict graph represent the wireless links between the wireless mesh nodes and the edges represent conflicts between the wireless links. An edge between a pair of vertices in the conflict graph means that the two wireless links cannot be active simultaneously over the same frequency channel. Also, as shown in figure, two colors, i.e. two frequency channels are required to ensure interference-free communication among the mesh nodes.

A conflict graph similar to the one shown in the Fig. 2.1 can be constructed using the protocol-based interference model where the interference is assumed to be a binary phenomenon. In this model, a conflict is indicated between a pair of wireless links

if any of the two nodes of a link is within the interference range of any of the two nodes of the other link or vice versa. Given a conflict graph which shows conflicts between pairs of wireless links if they are assigned the same frequency channel, assigning frequency channels to wireless links can be cast as a minimum coloring problem.

2.3. Maximum Independent Set Problem

An *independent set* of the graph G is a subset of vertices of G such that none of the vertices in the subset share an edge. In the conflict graph, an independent set of vertices indicates a set of wireless links which can be active simultaneously over the same frequency channel. An independent set is said to be *maximal* if it is not a subset of any larger independent set, or *maximum* if there is no larger independent set in the graph (i.e. it is an independent set with maximum cardinality).

The *maximum independent set* (MIS) problem consists of finding the largest subset of vertices of a graph such that none of these vertices are connected by an edge [12]. In the *weighted maximum independent set problem*, edges have weights. The MIS problem is NP-hard for general graphs [14].

An example of maximum independent sets in a given graph is shown in Fig. 2.2. Figs. 2.2a and 2.2b show that there are two distinct maximum independent sets in the given graph.

Figure 2.2. Maximum independent sets

Fig. 2.3 shows an example of maximal independent sets. Figs. 2.3a, 2.3b, 2.3c, and 2.3d show the four distinct maximal independent sets in the given graph.

Figure 2.3. Maximal independent sets

Note that the two independent sets shown in Fig. 2.4 are not maximal since they are subsets of maximum independent sets shown in Fig. 2.2.

Figure 2.4. Independent sets

Note that vertices in the same maximum or maximal independent set have the same color. In terms of a wireless mesh network, this means that wireless links in the same maximum or maximal independent set are assigned the same frequency channel, and the number of frequency channels required for interference-free communication among the mesh nodes is equal to the number of distinct maximum or maximal independent sets in the given conflict graph.

3. LITERATURE REVIEW

3.1. Classical Channel Assignment for WMNs

There is a great deal of literature [8][15–24] on channel assignment schemes for improving the performance of MRMC WMNs by maximizing the network throughput.

The topology-controlled interference-aware channel-assignment algorithm (TICA) is proposed in [15][16] which improves the network throughput by minimizing the interference within the mesh network using an approach of controlling the network topology based on power control before assigning channels. It assigns a least interfering channel to a link if all available frequency channels have already been assigned within the interference range of that link. It builds an interference level for all frequency channels if all available frequency channels have already been assigned within the interference range of a link. The least interfering channel is a channel that has the minimum interference level which means that assigning this channel to a link would result in minimum interference in the network. A failure recovery mechanism for TICA is proposed in [17] and its performance is evaluated in large-scale networks.

The hidden link problem caused by one-way interference range edge coloring in TICA is identified in [18]. A new algorithm enhanced-TICA (e-TICA) is proposed which uses two-way interference range edge coloring. This algorithm provides a more accurate channel assignment by identifying all least interfering channels leading to an improvement in the fairness among traffic flows without compromising the overall network throughput.

An enhancement of e-TICA, e-TICA version 2 (e-TICA2), is proposed in [8], which employs a minimum spanning tree rooted at the gateway instead of the shortest path tree employed in TICA and e-TICA, to reduce conflicting (least interfering) channels. The presence of relatively long links in some topologies when using TICA and e-TICA leads to a high number of conflicting channels due to their longer interference range. Interference is confined by lowering the transmit power. Since transmit power is directly proportional to the distance between the nodes, a reasonable strategy is to replace

the long links with shorter ones. e-TICA2 minimizes the conflicting channels by using the minimum spanning tree to achieve short link lengths.

Centralized-hyacinth (C-HYA) [19] is a centralized channel assignment algorithm for MRMC WMNs. It assumes that the offered traffic load is known and assigns channels ensuring network connectivity as well as satisfying the bandwidth limitations of each link. First, it estimates the total expected load on each virtual link by summing the load due to each offered traffic flow. Then, it visits each virtual link in decreasing order of expected traffic load and assigns it a channel.

Mesh-based traffic and interference-aware channel assignment (MesTiC) [20] is a fixed, rank-based, greedy algorithm for centralized channel assignment. It visits nodes once in the decreasing order of their rank and assigns channels to radios of mesh nodes based on ranks assigned a priori to the nodes. The rank of each node is computed on the basis of its link-traffic characteristics, topological properties and number of radio interfaces on a node. The rank is directly proportional to the aggregate traffic at a node. This is due to the fact that if a node relays more traffic, assigning it a channel of least interference will increase the network throughput. The rank of a node increases with its traffic. The rank is inversely proportional to the distance of the node and the number of its radio interfaces. The distance of a node is measured as the minimum number of hops from the gateway. Due to the hierarchical nature of a mesh network, the nodes near the gateway are given a higher preference in channel assignment since they are more likely to carry more traffic. Also, the lower the number of radio interfaces of a mesh node, the higher is its priority in channel assignment. Note that C-HYA and MesTiC require the traffic load to be known before assigning channels.

Hybrid multi-channel protocol (HMCP) [21] is a hybrid channel assignment scheme that applies a fixed assignment for some radios and a dynamic assignment for other radios. Out of the available interfaces at each node, X interfaces are assigned statically to X channels, and these interfaces are designated as “fixed interfaces.” The fixed interfaces stay on the specified channels for long durations of time. The remaining interfaces can frequently switch between any of the remaining channels, based on the data traffic, and are designated as “switchable interfaces.” The following example illustrates the protocol operation. Suppose that node A has packets to send to node C via

node B. Nodes A, B, and C have their fixed interfaces on channels 1, 2, and 3, and switchable interfaces on channels 3, 1, and 2 respectively. Node A switches its switchable interface from channel 3 to channel 2 and transmits the packet since the fixed interface of node B is using channel 2. Node B switches its switchable interface from channel 1 to channel 3 and forwards the packet which is received by node C using its fixed interface. A co-ordination mechanism is required for enabling neighbors of a node X to know about the channels used by fixed interfaces of node X to ensure that its neighbors can communicate with it on-demand. Also, time synchronization is required to decide when to switch an interface from one channel to another.

Breadth first search-channel assignment (BFS-CA) [22] is a centralized channel assignment scheme aimed at improving the capacity of the WMN backbone. This scheme computes interference and bandwidth estimates based on the number of interfering radios, where an interfering radio is a simultaneously operating radio that is visible to a mesh router but is external to its network. Each mesh router then derives two separate channel rankings. The first ranking depends on the increasing number of interfering radios. The second depends on the increasing channel utilization. A mesh router then merges the two rankings by taking the average of the individual ranks. The resulting ranking is used for channel assignment. As indicated by its name, this scheme uses a breadth first search to assign channels. The search begins with links emanating from the gateway while links fanning outwards towards the edge of the network are given lower priority. If all the available frequency channels have already been assigned within the interference range of a link and a non-conflicting channel is not available, BFS-CA randomly selects a channel and assigns it to that link.

Maxflow-based channel assignment and routing (MCAR) algorithm [23] is a centralized channel assignment and routing algorithm that splits the channel assignment into two stages. In the first stage, links are sorted into groups based on the flows they carry, while in the second stage, a channel is selected for each group and is assigned to all links of this group. If it is possible to do so, different channels are assigned to groups containing interfering links. This algorithm uses a flow rate computation method which is independent of the traffic demands and aims to maximize the network throughput.

A centralized tabu search-based channel assignment algorithm is proposed in [24] to address the problem of assigning channels to communication links in a MRMC WMN with the objective of minimizing the total network interference. In the first phase, the algorithm starts with a random initial solution. Then a sequence of solutions is created in an attempt to reach a solution with minimum network interference. In the second phase, violations of interface constraint are removed to get a feasible channel assignment solution. This algorithm uses interference and traffic models as input.

In all of these channel assignment schemes, the number of available frequency channels is assumed to be fixed. Therefore links within the interference range of each other could be assigned the same frequency channel, causing co-channel interference that degrades the network throughput.

3.2. Topology Control in WMNs

Topology control using transmit power control is a useful technique for reducing the co-channel interference in a WMN in order to increase the network capacity. This is done by adjusting the transmission power of a mesh node. The main goal of a topology control algorithm (TCA) is to minimize the co-channel interference, enhance spatial channel reuse, and maintain network connectivity through the selection of minimum transmission power for each radio interface of each mesh node. It controls the network topology by selecting the nearest nodes as communication neighbors for each node in the network and builds a connectivity graph with a small node-degree to mitigate co-channel interference and enhance spatial channel reuse as well as preserve network connectivity with the use of minimal power, as less transmit power translates to less interference. Mesh nodes transmit at the minimum power required to maintain connectivity with their immediate neighbors. This leads to multi-hop communication instead of long direct links and results in lower interference in the network [8].

The following requirements can serve as essential guidelines towards the design of an effective topology control algorithm [25]:

- The network connectivity should be preserved with the use of minimum possible transmission power. This is the most important objective of a topology control algorithm as less transmit power translates to less interference.

- It is desirable that all links be bi-directional. Bi-directional links are required to facilitate link level acknowledgments and ensure the existence of reverse paths for routing protocols. Changing power levels can create uni-directional links, which can happen when a node i 's power level is high enough for a node j to hear it, but not vice versa.
- The node-degree in the topology derived under the algorithm should be small. A small node-degree helps to mitigate co-channel interference and enhance spatial channel reuse. As node-degree increases, more nodes which are farther away from a node are selected as its communication neighbors. This results in longer links in the connectivity graph with higher transmission powers causing more interference.

Two types of topology control schemes for wireless networks have been proposed in the literature [26]. In homogeneous topology control, all nodes are assumed to use the same transmission power and the topology control problem consists of finding the minimum value of the transmission power that ensures network connectivity. In non-homogeneous topology control, nodes are allowed to choose different transmission powers provided they do not exceed the maximum transmission power.

Non-homogeneous topology control is further divided into three categories. In location-based schemes, exact node positions are known. In direction-based schemes, it is assumed that nodes do not know their position, but they can estimate the relative direction of each of their neighbors. In neighbor-based schemes, nodes are assumed to know only the ID of the neighbors, and are able to order them according to some criterion such as link quality.

In the local minimum spanning tree (LMST) algorithm [25], each node builds its local minimum spanning tree independently and only keeps as its neighbors those on-tree nodes that are one hop away in the final topology. It is a location-based scheme as it assumes that each node is equipped with the ability to gather its location information. The topology constructed under LMST preserves the network connectivity. The degree of any node in the resulting topology is bounded by 6. The feature of small node-degree is desirable as it reduces interference. LMST is composed of the following three phases: information exchange; topology construction; and determination of transmission power.

The objective of the enhanced local minimum shortest-path tree (ELMST) algorithm [27] is to find a topology that preserves certain network requirements, namely, decrease in average node-degree, an averagely low power consumption which results in a longer network lifetime, and a maintenance of connectivity in the resultant network topology. It is a location-based topology control scheme and consists of the following two main phases: construction of local minimum shortest path tree; and removal of unidirectional links. Each node gathers its neighbor information to construct its local minimum shortest path tree.

A drawback of multi-hop communication in WMNs is that it consumes multiple radio resources to transmit a frame due to multiple intermediate hops between a mesh router and the gateway. As a wireless channel is shared by the ongoing transmissions at various hops, spatial reuse requires simultaneous transmissions at low data rates by using a robust modulation and coding scheme in order to avoid co-channel interference. The objective of the maximum minimum power (MMP) algorithm [28] is to maintain the best possible modulation and coding scheme for each link while decreasing the transmission power as much as possible.

In the local mean algorithm (LMA) [29], all nodes start with same initial transmission power. Every node periodically broadcasts a *Life* message. Upon the reception of a *Life* message, each node responds with a *Live-Acknowledgement* message. The number of responses is counted and if it is greater than a pre-set threshold *NodeMaxThres*, the node decreases transmission power and repeats the algorithm. However, if the number of responses is less than or equal to *NodeMaxThres*, the node does not change the transmission power and repeats the algorithm after some delay. LMA is a typical power control algorithm, putting emphasis on power conservation while preserving network connectivity. To ensure connectivity of the wireless network, *NodeMaxThres* is set to either 4 or 5 for all nodes.

The goal of *Select x for less than x* topology control algorithm [8][15] is to minimize the co-channel interference among the mesh nodes in a WMN. To achieve this goal, it controls the network topology by selecting the nearest nodes as communication neighbors for each node in the network. Since transmit power is proportional to the

distance between the nodes, the shorter the distance, the lower the transmit power. Less transmit power translates to less interference which leads to better spatial channel reuse.

3.3. Routing in WMNs

In addition to channel assignment, the schemes in [8][15][19–24] also deal with routing in MRMC WMNs.

In previous work [8][15], minimum power-based minimum spanning tree and minimum power-based shortest path tree were employed for degree constrained routing. In TICA and e-TICA, a shortest path tree with a maximum node-degree of four is constructed from the connectivity graph using Dijkstra's algorithm. The metric for path selection is minimum transmission power. The number of neighbors of a mesh router is bounded by the number of its radio interfaces and each node has four data radios. If any node in the shortest path tree has more than four links, those four links are selected for that node which have the minimum weight.

The channel assignment schemes in [30][31] use minimum spanning tree for constructing network topology. The motivation for using minimum spanning tree in these channel assignment schemes is that shorter links resulting from minimum spanning tree imply more capacity in WMNs by reducing co-channel interference with nearby links using the same frequency channel. e-TICA2 employs a minimum spanning tree rooted at the gateway to minimize conflicting channels.

C-HYA does not require any specific routing mechanism. For evaluation purposes, traffic is routed using either shortest path routing or randomized multi-path routing. The shortest path routing is based on standard Bellman-Ford algorithm with minimum hop-count metric. The multi-path routing algorithm tries to achieve load balancing by distributing the traffic among multiple available paths. The exact set of paths between a source-destination pair is chosen randomly out of the set of available paths with sufficient bandwidth.

For the performance evaluation of MesTiC, the experiments were performed using the dynamic source routing (DSR) protocol. The authors claim that their simulation platform can be extended to incorporate different routing schemes for mesh networks. A multi-channel routing metric is proposed in [21] which incorporates the impact of interface switching cost in HMCP. The routing metric selects channel diverse routes

while accounting for interface switching cost. For the evaluation of HMCP, the routing metric is incorporated into a source-initiated on-demand routing protocol similar to DSR.

The optimized link state routing (OLSR) protocol is used for route selection while evaluating BFS-CA through simulations. The MCAR scheme uses a flow rate computation method for routing, which aims to maximize the network throughput and is not dependent on any particular traffic profile. For the performance evaluation of the tabu search-based channel assignment scheme [24], the routes are computed statically using the smallest number of hops as the metric and do not change for the lifetime of the simulation.

3.4. Fairness in WMNs

The channel assignment schemes in [15][19–24] focus on improving the throughput in MRMC WMNs without considering fairness.

In previous work [8][18], channel assignment algorithms are proposed to improve throughput as well as fairness in MRMC WMNs. e-TICA uses two-way interference range edge coloring which implies that links formed by nodes that are within the interference range of each other will not be allocated the same frequency channel, provided that there is a channel available for allocation. It identifies all conflicting channels providing a more accurate channel assignment strategy which improves fairness among flows in the network without compromising the overall network throughput.

Unfairness in medium access arises in MRMC WMNs due to some mesh nodes operating on a conflicting channel and suffering from co-channel interference. e-TICA2 improves medium access fairness among the mesh nodes by reducing the conflicting channels by using minimum spanning tree that is rooted at the gateway. The motivation behind using minimum spanning tree is to achieve short link lengths which result in the medium being shared efficiently by reducing conflicting channels.

The authors have proposed an algorithm in [32] to improve the fairness by differentiating the traffic among the connections in a wireless multi-hop network. It tries to assign a higher priority to those traffic connections that experience delay greatly larger than the average, so that their delay is reduced, and the fairness of the system is improved.

In [33], it is shown that the CSMA/CA protocol suffers from starvation when the receiving and sensing ranges are equal. However, this unfairness of medium access among links in a multi-hop wireless network is reduced when these two ranges are sufficiently different. A dynamic contention window algorithm is proposed in [34] for improving fairness among flows in a WMN that is based on employing multiple queues per node, different back-off parameters and extended interframe space values.

In [35], a fair binary exponential back-off algorithm is proposed for providing medium access fairness among different nodes in a WMN. Mesh nodes that have successfully transmitted a data packet are not permitted to eagerly transmit more data packets. By delaying the transmission of successive packets by adapting the contention window, the algorithm is able to reduce the degree of starvation and improve the channel usage of flows from nodes that are multiple hops from the gateway.

The problem of finding a rate allocation along with a flow allocation and a transmission schedule for a set of end-to-end communication sessions such that the network throughput of a multi-channel WMN is maximized is studied in [36]. Three different schemes are compared in terms of network throughput and fairness of rate allocation. The maximum throughput rate allocation scheme achieves the highest throughput but results in a severe unfairness on rate allocation. The max-min guaranteed maximum throughput rate allocation scheme performs best in terms of fairness since rates allocated to all sessions are almost the same but offers very low network throughput. The proportional rate fair allocation scheme achieves a good tradeoff between throughput and fairness even though it does not achieve an absolutely even distribution.

The authors have shown in [37] that the network throughput as well as fairness in a multi-channel WMN increases as the number of available radio interfaces per mesh router or the number of available orthogonal frequency channels increases. The approach in [38] deals with congestion control and channel assignment in MRMC WMNs. It achieves significant gains in terms of network utilization and establishing source rate fairness. This is a decomposition approach which divides the problem into a congestion control sub-problem and a channel assignment sub-problem. The two sub-problems are sequentially and iteratively solved until termination.

In [39], the authors deal with the problem of joint channel assignment, link scheduling, and routing for throughput optimization in MRMC wireless networks. It is shown that the fairness and throughput achieved by the proposed method is within a constant factor of the optimum value. An algorithm is proposed in [40] for joint channel, capacity, and flow assignment in MRMC WMNs. Maximizing the overall network throughput without including any fairness constraints may lead to starvation of nodes which are far from the gateway. On the other hand, just maximizing the fairness (i.e., maximizing the minimum throughput that is achieved by any node) may lead to under-utilization of network resources. So the algorithm first tries to maximize the fairness and then uses the remaining unused network resources to maximize the overall network throughput.

The divide-and-conquer approach in [41] splits the joint routing and channel assignment problem into a number of local optimization sub-problems that are executed sequentially. The number of sub-problems is equal to the maximum number of hops to the gateway. Each sub-problem is solved using a branch-and-cut method. This approach improves the network performance in terms of fairness among the traffic flows.

3.5. Interference Modeling in WMNs

Several channel assignment schemes have been proposed [8][15][18–24][30][31] which use the protocol model [42] for modeling interference in WMNs due to its simplicity and ease of implementation. This model considers the interference to be a binary phenomenon. In this model, a conflict is indicated if the transmitter or receiver of a link is within the interference range of the transmitter or receiver of the other link or vice versa.

The study in [43] concludes that cumulative interference must be taken into account to obtain accurate performance results for multi-hop wireless networks and recommends the use of the cumulative interference model, i.e. SIR model [42]. This model considers the cumulative effect of interference at the receiving node where the incoming desired signal is received correctly if its SIR is above the required SIR threshold. The study also states that although the physical model is necessary for accuracy, it is not suitable for large-scale multi-hop wireless networks due to its high

computational complexity. A few schemes discussed earlier [40][41] have used this model for modeling interference.

3.6. Minimum Coloring for Resource Allocation in Wireless Networks

Exact methods guarantee an optimum solution for minimum coloring but can require an excessive amount of computation time, whereas heuristics can find reasonable solutions much more quickly. Several greedy heuristics for the classical minimum coloring problem have been proposed in the literature. Some well-known examples include *SEQ* [44], *DSATUR* [45], *RLF* [46], and *IGA* [47]. *SEQ* is a very simple and commonly used greedy heuristic coloring algorithm. It takes each vertex of the graph sequentially and assigns to it the smallest indexed color that does not appear in one of its neighbors. *DSATUR* arranges the vertices in the decreasing order of degrees. A vertex with highest degree is colored with color 1. A vertex with highest saturation degree is chosen (ties are broken randomly) where the saturation degree of a vertex is defined as the number of different colors to which it is adjacent. The chosen vertex is colored with the lowest numbered color with which it does not conflict. The algorithm stops when all the vertices are colored.

RLF (Recursive Largest First) colors the vertices one class at a time using the following greedy process. Let C be the color class to be constructed which is initially empty. Let V' be the set of uncolored vertices that can be placed in C ; V' initially contains all the currently uncolored vertices. Let U be the set of uncolored vertices that cannot be placed in C ; U is initially empty. Choose the first vertex x_0 from V' that has the maximum number of adjacent vertices in V' . Place x_0 in C and move all the vertices in V' that are adjacent to x_0 from V' to U . While V' remains nonempty: choose the first vertex x in V' that has the maximum number of adjacent vertices in U ; add x to C and move all the vertices in V' that are adjacent to x from V' to U . Considering only vertices in $V \setminus C$, it constructs the next independent set as the second color class and so on.

IGA (Iterated Greedy Algorithm) iteratively colors the graph using the *SEQ* algorithm. At each iteration, the order of the color classes is changed according to some criteria which may lead to a reduction in the number of colors used. The various reordering heuristics include reversing the order of the color classes, placing the color classes in a random order, placing the classes with largest cardinality first or largest fit,

placing the smallest cardinality classes first or smallest fit, placing the classes in increasing order by degree sum of the group, and placing the classes in decreasing order by degree sum of the group. The authors used a probabilistic choice of the reordering move to be performed. A ratio of 50:50:30 (largest first : reverse : random) was shown to be efficient.

The algorithm in [48] selects the maximum independent set from the set of uncolored vertices. The vertices of the selected set are assigned to a new color. This process is repeated until the whole graph is colored. A greedy randomized adaptive search procedure [49][50] (*GRASP*) was used in [51] for coloring sparse graphs. It consists of two phases: construction phase and improvement phase. The construction phase uses a randomized version of *RLF* to generate initial colorings. The improvement phase applies a local search procedure to the initial solution in the hope of finding an improvement. *GRASP* is an iterative technique in which each iteration provides a solution. The incumbent solution over all *GRASP* iterations is the final result.

Schemes using minimum coloring for resource allocation in wireless networks have been proposed. The authors investigate the problem of determining the number of frequency bands required for multi-radio multi-hop wireless networks in [52] such that all co-located transmitters and receivers can be simultaneously activated. Instead of using the link interference graph, the authors develop a method using minimum coloring to find the number of frequency bands directly from the network connectivity graph. They use a simplistic interference model based on duplexing constraints in which a node is restricted from simultaneously transmitting and receiving on the same frequency band.

In [53], the authors use minimum coloring for TDMA link scheduling in a multi-hop wireless network to maximize its throughput. They construct the conflict graph using the protocol interference model or the RTS/CTS interference model [54]. By using minimum coloring for the link interference graph, the authors in [55] investigate the problem of obtaining tight delay guarantees for throughput-optimal link scheduling in wireless ad-hoc networks.

In [56], the authors propose a scheduling algorithm based on minimum coloring to determine a minimum length schedule for maximizing the network throughput in TDMA based wireless mesh networks. The interference model used considers that

transmissions may interfere in two ways which are typically referred to as primary and secondary interference [57]. The authors assume that there are enough orthogonal channels available to eliminate all the secondary interference, i.e. the interference that arises between two links when the receiver of one link is within the interference range of the transmitter of the other link. They consider the primary interference only while constructing the link conflict graph in which the links conflict with each other if they share a common sender or receiver.

3.7. Independent Sets for Resource Allocation in Wireless Networks

Algorithms for solving the maximum independent set (MIS) problem have been widely used for resource allocation in multi-hop wireless networks.

The scheduling scheme in [58] uses independent sets for feasible link scheduling in TDMA-based WMNs. The method in [36] uses maximal independent sets for link scheduling in multi-channel WMNs. In [59], the authors proposed a polynomial time approximation scheme for computing an independent set from the link interference graph, as large as $(1 - \epsilon)$ times the cardinality of the MIS. Given the number of channels among other inputs, the method is then used to develop a channel assignment for MRMC WMNs such that the number of links in the communications graph that can be active simultaneously is maximized.

A polynomial computing method in [60] searches for the critical maximal independent set (i.e. a small set of critical maximal independent sets) that needs to be scheduled for optimal resource allocation. In [61], the authors use solutions for the maximum weighted independent set problem to develop approximation algorithms for link scheduling and compute a maximum (concurrent) multi-flow in multi-radio multi-channel multi-hop wireless networks. Weighted maximal independent sets have also been used for link scheduling in multi-hop wireless networks [62] and in ultra wideband wireless ad-hoc networks [63].

3.8. Beamforming in WMNs

Numerous channel assignment schemes [8][15–24][30][31][41][54][59][64][65] have been proposed in the literature for MRMC WMNs which typically assume a wireless mesh node to be equipped with omni-directional antennas in a classical WMN

architecture. During an active communication of a mesh node with its neighbor, the circular radiation pattern of its omni-directional antenna prevents mesh nodes within its circular range from transmission using the same frequency channel.

To improve the spatial channel reuse by reducing the co-channel interference, DMesh [66] proposes the use of directional antennas for mesh nodes in a MRMC WMN. It combines spatial separation from directional antennas with frequency separation from orthogonal frequency channels to improve the throughput. It assumes that each mesh router can have multiple radio interfaces and each radio interface has its own directional antenna. DMesh consists of three main stages: tree formation to provide connectivity to the gateway using directional antennas; routing to correctly deliver packets along the tree; and channel assignment to separate spatially contending transmissions whenever possible to increase the throughput. It extends the optimized link state routing protocol for tree formation, to set up routing, and to perform channel assignment.

Directional antennas have also been used in [67] to propose a directional transmission and reception TDMA-based medium access control algorithm for wireless ad-hoc networks. A study of joint routing and scheduling optimization problem in TDMA-based WMNs is conducted in [68] where nodes are assumed to be equipped with directional antennas. It is assumed that the WMN's topology is fixed, and each node knows its location as well as the locations of its neighbors. Also, each node is able to point its directional antenna's main lobe in the direction of the neighbor it wants to communicate with.

Topology control for MRMC WMNs that use directional antennas is studied in [69]. This consists of the following problem: how to construct a network that connects all mesh routers to the gateway, set proper orientation for the directional antennas of the mesh nodes, and assign channels to radios, such that the end-to-end throughput of the network is maximized. The interdependence of network construction, antenna orientation setting and channel assignment on each other complicates this problem. The proposed solution breaks this interdependent cycle. First, it constructs a network topology such that the traffic load among links is balanced. Then, antenna orientations and channel assignment are computed to minimize the network interference in order to increase the network throughput.

In [70], the authors study the problem of building topologies for nomadic WMNs where the degree of any node is less than its number of available directional mesh radios. However, such antennas are non-steerable and always point in the same direction in which they were manually directed at the time of installation. They cannot be successful in a dynamic WMN environment undergoing regular topological changes.

For the successful operation of a self-organizing and dynamic MRMC WMN, the use of beamforming is required. Unlike the non-steerable beam of a directional antenna, the main beam of an antenna array can be pointed in any desired direction.

Beamforming-based link scheduling schemes have been proposed for single-radio single-channel TDMA-based multi-hop wireless networks [71][72]. In [71], the authors proposed spatial reuse only protocol which uses multiple antennas of a wireless node for beamforming to prevent a transmitter's signal from reaching nearby undesired receivers. A multi-antenna beamforming technique is proposed in [72] to suppress the interference and to improve the spatial reuse. However, both [71] and [72] have used the simplistic protocol model for modeling interference.

3.9. Heterogeneous Mesh Nodes

A MRMC WMN consists of mesh nodes that are equipped with multiple radio interfaces. In a MRMC WMN architecture consisting of homogeneous mesh nodes, the node-degree of all mesh nodes in the network is the same, i.e. all mesh nodes have same number of radio interfaces for data communication with neighbors. In a heterogeneous WMN, different mesh nodes have different number of radio interfaces.

The channel assignment schemes in [15][19][20][21][24][41][54][59][64][65] assume that the node-degrees of all mesh nodes in the network are the same. TICA uses four radio interfaces per each mesh node. The simulation experiments were performed using two radios per node for channel assignment in MesTiC. In all simulations for evaluating the performance of HMCP, nodes in the network were equipped with two radio interfaces.

To generate experimental results, the number of radio interfaces per node are varied from one to five in C-HYA. In [24], the simulation experiments are conducted using one to twelve radios per node. To analyze the behavior of G-PaMeLa [41], each node in the WMN is equipped with 3 or 4 interfaces depending on the scenario. For

conducting experiments, the number of radios per node are varied from 1 to 4 in [54], from 2 to 8 in [59], and from 2 to 12 in [65]. In the distributed hyacinth (D-HYA) scheme [64], the number of radio interfaces are varied for different experiments and is either 1 or 2 or 4.

BFS-CA requires a certain number of mesh routers with a certain number of radio interfaces to be placed at certain hops from the gateway. While evaluating the performance of MCAR, the authors consider two classes of topologies with 25 and 50 mesh nodes. In the 25-node network, 60% of the mesh routers are assigned 2 radios while the remaining ones are assigned 3 radios. In the 50-node network, 20% of nodes have 2 radios, 40% have 3 radios and 40% have 4 radios. For each class, 20 different topologies are generated having different placements of nodes.

3.10. Dense WMNs

For their performance evaluation, different channel assignment schemes have used different sizes of wireless mesh networks.

To evaluate the performance of TICA, a 36 node mesh network was used. C-HYA used a mesh network consisting of 100 nodes. The simulation experiments were performed using a 25 node mesh network in MesTiC. In HMCP, ten topologies were generated and each topology had 50 mesh nodes.

A topology consisting of 30 mesh nodes was used for the performance evaluation of BFS-CA scheme. To evaluate the performance of MCAR, the authors considered two classes of topologies with 25 and 50 mesh nodes respectively. In G-PaMeLa, two different topologies consisting of 36 nodes and 37 nodes were considered for its performance analysis. A total of 60 mesh nodes is used for simulation experiments in [54]. For all experiments in [59], 30 mesh routers are used.

D-HYA used a 60 node mesh network. To study the performance of their channel assignment algorithm [65], the authors used two different deployments consisting of 36 and 50 mesh nodes respectively. Using extensive simulations to study the performance of their channel assignment method, the authors consider two sets of networks in [24]. Their dense network is generated by placing 50 mesh nodes in a 500×500 square meter area whereas their sparse network is generated by placing the same number of nodes in an 800×800 square meter area.

4. THE CHANNEL ASSIGNMENT PROBLEM

Problem Statement

The objective of this research is to develop a realistic channel assignment method for dynamic MRMC WMNs that finds the smallest number of frequency channels required for interference-free communication among the mesh nodes in order to achieve the maximum network throughput while maintaining fairness among the multiple network flows, given the location of the mesh nodes and the number of their half-duplex radio interfaces.

This problem involves:

- building the connectivity graph for the given mesh network;
- routing of flows over the connectivity graph from the source mesh nodes to the gateway which is sink for all flows such that the maximum network throughput is achieved under flow conservation, node-degree, half-duplex, fairness, and link capacity constraints;
- interference modeling via building the conflict graph for the set of links involved in routing where the vertices of the conflict graph represent the wireless links between mesh nodes while the edges represent the conflicts between the wireless links; and
- computing a coloring of the vertices of the conflict graph using as few distinct colors as possible which is the same as finding the minimum number of frequency channels to use such that there is no interference. The goal is to assign the smallest number of frequency channels to the links involved in routing such that all of these links can be active simultaneously without any co-channel interference.

To find out the true optimum solution in terms of the smallest number of frequency channels required for interference-free communication among the mesh nodes, this problem should be solved as a single integrated problem. However, this makes the problem too complex; especially the minimum coloring part of this problem is well-known to be NP-hard. That's why this problem is broken down into stages, namely, connectivity graph, routing, interference modeling, and minimum coloring. Except for the first stage (i.e. the connectivity graph stage), each stage is solved by using the solution of the previous stage among other inputs.

4.1. Channel Assignment for Interference-Free Communication in WMNs

The classical channel assignment schemes for MRMC WMNs [8][15–24] are based on the assumption of a fixed number of available frequency channels. The channel assignment problem for such schemes consists of assigning the available frequency channels to the radios of multi-radio mesh nodes such that co-channel interference is minimized and network throughput is maximized. In such schemes, a link is assigned a conflicting frequency channel if all the available frequency channels have already been assigned within the interference range of that link and a non-conflicting frequency channel is not available. Hence, two links that are within interference range of each other could be assigned the same frequency channel. The presence of these conflicting frequency channels causes co-channel interference at links using such channels, which degrades the network throughput. Such conflicting frequency channels are unavoidable in the classical channel assignment schemes due to their limitation on the number of available frequency channels.

To address this problem, this work aims to study the channel assignment problem from a different perspective. Instead of working with a fixed number of available frequency channels, it searches for the smallest number of frequency channels that provides an interference-free channel assignment that allows the mesh network to achieve the maximum network throughput. It ensures interference-free communication among the mesh nodes by ensuring that links within interference range of each other are assigned different frequency channels.

Most of these channel assignment schemes [15–24] focus on improving the network throughput while some of them try to improve the fairness [8][41]. Throughput and fairness do not normally go hand in hand and increasing one generally decreases the other. The trade-off relationship means that supporting fairness generally sacrifices the overall network throughput. An aim of this work is to develop a new channel assignment method that achieves maximum network throughput and ensures fairness among the multiple network flows at the same time.

4.2. Channel Assignment for WMNs using Realistic Interference Models

The protocol model is widely used to model interference for channel assignment in wireless mesh networks [8][15–24][30][31][54][59][64][65] due to its simplicity and

ease of implementation. This simple model assumes that interference is a binary phenomenon that occurs within the interference range of the nodes of any active link. The SINR (signal-to-interference-and-noise ratio) model, also known as the physical model, is more accurate. It considers the cumulative effect of interference at the receiving node where the incoming desired signal is received correctly if its SINR is above a certain threshold, commonly known as the SINR threshold.

Neither the protocol model nor the SINR model reflect the reality of signal propagation in wireless links. In a real wireless channel, the RF signal is reflected from and around nearby objects, which causes the signal strength to fluctuate in space. This effect is commonly known as *shadowing* and is usually accounted for by adding a random component to the received signal strength. A model that accounts for shadowing will be referred to as *SINR model with shadowing*. SIR instead of SINR can be used since co-channel interference is generally much stronger than noise. An aim of this work is to determine the number of frequency channels required for interference-free communication among the mesh nodes based on a realistic interference model.

Schemes using minimum coloring, maximal independent sets and weighted maximal independent sets for resource allocation in wireless networks have been proposed [52–63]. Given the conflict graph in which vertices represent the wireless links between pairs of mesh nodes and edges represent conflicts between the wireless links, the channel assignment problem requires the assignment of a frequency channel to each vertex in the conflict graph so that the total number of distinct frequency channels is minimized. This is a standard minimum coloring problem in which colors represent frequency channels and this problem is well-known to be NP-hard for general graphs.

However the cumulative effect of interference when using the SIR model with shadowing complicates the coloring problem. When there are multiple links using the same frequency channel, the cumulative interference may be enough to cause an unacceptable level of interference on some link using that frequency channel, even though the interference on each pair of links is below the tolerance. An aim of this work is to address this extended coloring problem by developing fast heuristics that incorporate the cumulative interference constraints.

4.3. Improving Channel Assignment using Beamforming in Dynamic WMNs

The channel assignment schemes [8][15–24][30][31][41][54][59][64][65] for MRMC WMNs typically assume a multi-radio mesh node to be equipped with omni-directional antennas. When a mesh node is communicating with a neighbor using a certain frequency channel, other mesh nodes within its circular range must refrain from transmission using the same frequency channel due to the circular radiation pattern of its omni-directional antennas.

Directional antennas reduce co-channel interference and enhance frequency reuse [66–70]. However, these antennas are non-steerable and always point in the same direction in which they were oriented at the time of installation. Hence they are less useful in a dynamic WMN environment that undergoes regular topological changes as new mesh nodes join the network or existing mesh nodes leave it. The term “dynamic” here does not imply mobility or fast changing topology, but it rather means that the network experiences slow topology changes due to adding or removing mesh nodes.

Beamforming can be used to reduce co-channel interference and enhance spatial channel reuse [73]. Link scheduling schemes for single-radio single-channel TDMA-based multi-hop wireless networks [71][72] use multiple antennas of a wireless node for beamforming to suppress the co-channel interference and to improve the spatial reuse.

Unlike the non-steerable beam of a directional antenna such as a parabolic antenna, the main beam of an antenna array can be pointed in any desired direction by controlling the progressive phase difference between the antenna elements of the array. The steerable beam pattern provided by the linear antenna array of a mesh node can be used in the operation of a self-organizing and dynamic WMN. An aim of this work is to address the problem of co-channel interference in a dynamic MRMC WMN by using beamforming to minimize the number of frequency channels by improving spatial reuse. Note that advanced interference cancellation techniques, such as null steering, side-lobe cancellation, etc. are outside the scope of this thesis.

The channel assignment schemes in [15][19][20][21][24][41][54][59][64][65] assume that all mesh nodes have same number of radio interfaces for data communication with neighbors. In a heterogeneous WMN architecture, different number of radio interfaces is assigned to different mesh nodes. Also, different channel assignment

schemes have used different sizes of wireless mesh networks for their performance evaluation. To model a more realistic WMN, an aim of this work is to develop a beamforming-based channel assignment framework that incorporates heterogeneous mesh nodes and to evaluate its performance in sparse as well as dense mesh networks.

A wireless mesh network architecture where the number of antennas in the linear antenna array of a multi-radio mesh node is greater than its number of radio interfaces can be referred to as a *multi-radio multi-channel multi-antenna* (MRMCMA) WMN. An aim of this work is to study the impact of increasing the number of antennas in the linear antenna arrays of the multi-radio mesh nodes on the number of frequency channels in a dense MRMCMA WMN.

5. INTERFERENCE-FREE CHANNEL ASSIGNMENT METHOD

In this chapter, a novel channel assignment method is developed [74][75] that ensures an interference-free channel assignment for MRMC WMNs. It finds a small number of frequency channels required to achieve interference-free communication among the mesh nodes under the protocol-based interference model to enable the mesh network to achieve the maximum network throughput with fairness among the flows at the mesh nodes, given their location and the number of their half-duplex radio interfaces.

5.1. Introduction

Unlike the classical channel assignment schemes for MRMC WMNs, the proposed channel assignment method tackles the channel assignment problem in MRMC WMNs from a different perspective. It eliminates the assumption of a fixed number of available frequency channels and searches for the smallest number of frequency channels that provides interference-free communication among the mesh nodes that allows the mesh network to achieve the maximum network throughput. It provides an interference free channel assignment by ensuring that links within interference range of each other are assigned different frequency channels; this ensures interference-free communication among the mesh nodes. This work is the first to propose an interference-free channel assignment method for MRMC WMNs that enables the mesh network to achieve maximum network throughput while maintaining fairness.

The proposed channel assignment method consists of four stages, namely, connectivity graph, routing, interference modeling and minimum coloring. To minimize the number of frequency channels required, the existing *Select x for less than x* topology control algorithm [8][15] is used to build the connectivity graph (CG). It builds a CG with a small node-degree to mitigate the co-channel interference and enhance spatial channel reuse as well as preserve network connectivity with the use of minimal power, as less transmit power translates to less interference. The results prove the effectiveness of using topology control for building the connectivity graph and show that the TCA-based CG approach requires fewer frequency channels than the classical approach of building the CG based on maximum power.

A single mesh gateway is assumed. All mesh nodes, except the GW, are sources of flow. The GW is the sink for all flows. The routing problem in MRMC WMNs is formulated as a mixed integer linear program (MILP) whose objective is to maximize the network throughput while maintaining fairness among the multiple flows under flow conservation, fairness, half-duplex, node-degree, and link capacity constraints. By *network throughput*, it means the total amount of flow that reaches the GW from all sources. The multi-path routing approach is used to achieve the maximum network throughput. It provides better network throughput than the classical approach of minimum power-based shortest path (MPSP) routing.

The protocol model is used to model the interference in a MRMC WMN and to build the conflict graph for the set of links involved in routing. The vertices of the conflict graph represent the wireless links between the mesh nodes while the edges represent the conflicts between the wireless links.

For interference-free communication among mesh nodes, the problem of assigning frequency channels to the links involved in routing is similar to the minimum coloring problem for the conflict graph, which is known to be NP-hard for general graphs [13]. The goal is to assign the smallest number of frequency channels to the links involved in routing such that all of these links can be active simultaneously. This amounts to finding the smallest number of maximal independent sets (MaISs) from the conflict graph. The number of frequency channels required is equal to the number of MaISs. Two effective new minimum coloring heuristics are developed to determine a small number of frequency channels required for interference-free channel assignment.

5.2. Network Architecture

It is assumed that each mesh node is equipped with multiple radio interfaces. One of these radios is used for control traffic (*control radio*), while the other radios are used for data traffic (*data radios*). The *node-degree* of a mesh node is defined as the number of neighbors it can communicate with simultaneously for data communication. For example, for a node-degree of two, it is considered that each mesh node is equipped with two data radios, and can communicate with at most two of its neighbors at the same time. The radio interfaces are assumed to be half-duplex; hence a mesh node cannot send and receive at the same time using the same radio interface. However, a mesh node can

transmit or receive at a radio interface while transmitting or receiving at its other radio interfaces at the same time, which is similar to out of band full duplex communication.

Each radio interface of a multi-radio mesh node is equipped with an omni-directional antenna. Roofnet [76] is an experimental WMN built by Massachusetts Institute of Technology. Similar to Roofnet, it is assumed that each mesh node has omni-directional antennas installed on the roof of a building and the propagation environment is characterized by a strong line-of-sight component. It is assumed that the radio interfaces of nodes can be tuned to different non-overlapping frequency channels and their control radios are tuned to a common frequency channel for communication of the control traffic.

A single mesh gateway is assumed that is always available and that cannot fail like a mesh router. All mesh nodes, except the GW, are sources of flow and the GW is the sink for all flows. The node-degree of all mesh nodes in the network is the same.

5.3. Connectivity Graph

Two different approaches are used for building the CG $C(V,E)$, where vertices V correspond to the wireless nodes and the edges E correspond to the wireless links between the nodes. The first approach is the classical way based on maximum transmission power while the second approach uses previously proposed *Select x for less than x TCA*.

The *Select x for less than x TCA* builds a CG using topology control to mitigate the co-channel interference and enhance spatial channel reuse as well as ensures network connectivity. It controls the network topology by selecting the nearest nodes as communication neighbors for each node in the network. Since transmit power is proportional to the distance between the nodes, the shorter the distance, the lower the transmit power. Less transmit power translates to less interference which leads to better spatial channel reuse.

Each MR broadcasts a *Hello* message at maximum transmission power containing its node ID and position over the control channel using the control radio. From the information in the received *Hello* messages, each MR arranges its neighboring nodes in ascending order of their distance. The result is the *maximum power neighbor table* (MPNT). Then each MR sends its MPNT along with its position and node ID to the GW

over the control channel. For each MR in the network, the GW builds a *direct neighbor table* (DNT) by selecting at least x nearest nodes for that MR. If required, the GW then converts some uni-directional links in the DNT of a mesh node into bi-directional links, which results in the *final neighbor table* (FNT). Bi-directional links are required for link level acknowledgments and to ensure the existence of reverse paths.

The CG for the maximum power (MP)-based approach is built using the MPNT, and the CG for the TCA-based approach is built using the FNT. Node locations are assumed to be known. In order to achieve a strongly connected topology, a maximum transmission range of 164 m is assumed for all mesh nodes.

5.4. Routing

5.4.1. Problem Formulation for Multi-Path Routing

The multi-path routing problem in multi-radio WMNs is formulated as an MILP. The index $p \in P$ is called a commodity. Let:

- P be the commodities, i.e. source-destination pairs $(s_1, t_1), \dots, (s_P, t_P)$;
- f_{ij}^p be a variable denoting the amount of flow of commodity p on link l_{ij} where $(i,j) \in E$;
- f_s^p be a variable denoting the amount of inflow of commodity p from the source of p ;
- f_d^p be a variable denoting the amount of outflow of commodity p from the sink of p ;
- c_{ij} be an input parameter denoting the capacity of link l_{ij} ;
- z_{ij} be a binary variable such that $z_{ij} \in \{0, 1\}$ is 1 when the link l_{ij} is used for routing and 0 otherwise;
- dc be an input parameter denoting the constraint on the node-degree of the mesh routers, such that $dc \in \{2,3,4,5,6\}$;
- $cost_{l_{ij}}$ be an input parameter containing a cost of 0.0001 for each link l_{ij} ;
- $demand_{sd}$ be an input parameter representing flow demands between the source-sink pairs and is equal to 1 for all commodities; and
- y be a variable denoting the multiplier on the unit flow demand of the commodities.

Objective

Given the connectivity graph, the link capacity, the node-degree constraint and the unit flow demand between the source-sink pairs, the objective is to maximize y ,

which is the multiplier on the unit flow demand of the commodities, so as to achieve the maximum total flow in the network using multiple paths between a source-sink pair. In this work, the *maximum network throughput* means the maximum possible total network flow, i.e. the maximum possible flow that can reach the GW from all sources.

$$\max \left(y - \sum_{(i,j) \in E} (z_{ij} \times \text{cost} l_{ij}) \right) \quad (5.1)$$

The small value of $\text{cost} l_{ij}$ in (5.1) prevents redundant flow-loops and does not affect the result.

Constraints

The following are the flow balance constraints for the source nodes, sink (GW) node and the intermediate nodes, respectively. For the source nodes (or the sink node), the sum of the outgoing flow of a commodity is equal to the sum of incoming flow for that commodity and the amount of inflow (or outflow) of that commodity from the source (or sink) of that commodity. For the intermediate nodes, the sum of the incoming flow of a commodity is equal to the sum of the outgoing flow for that commodity:

$$\sum_j f_{sj}^p - \sum_i f_{is}^p - f_s^p = 0 \quad \text{for all } s \text{ and for all } p \quad (5.2)$$

where $s, i, j \in V$,

$$f_d^p + \sum_i f_{id}^p - \sum_j f_{dj}^p = 0 \quad \text{for all } d \text{ and for all } p \quad (5.3)$$

where $d, i, j \in V$,

$$\sum_j f_{ji}^p - \sum_k f_{ik}^p = 0 \quad \text{for all } i \text{ and for all } p \quad (5.4)$$

where $i \in V \setminus \{s_p, d_p\}, j, k \in V$ and $p \in P$.

The following represent the constraints on the node-degree of the mesh routers in the WMN. At any mesh node in the network, there is a limit on the number of radio-interfaces, and hence on the number of incoming plus outgoing links. If the limit is 4, i.e. if $dc = 4$, then at each mesh node n_i :

$$\sum_k z_{ki} + \sum_j z_{ij} \leq 4 \quad \text{for all } i \quad (5.5)$$

where $i, j, k \in V$.

Varying x in the *Select x for less than x* TCA for different node-degree constraints ensures that the amount of total flow in the network increases equally for the TCA-based and MP-based CG approaches with an increase in the node-degree constraint. It creates a certain amount of connectivity in the CG for a certain node-degree constraint and hence a certain number of links for the GW. For example, for node-degree constraints of 2 and 3 the *Select 3 for less than 3* TCA is used to ensure at least 3 links for the GW in the CG; for the node-degree constraint of 4 the *Select 4 for less than 4* TCA is used to ensure at least 4 links for the GW in the CG, and so on. Since the *Select 2 for less than 2* TCA mostly leads to a disconnected network in the case of random and controlled random topologies, the *Select 3 for less than 3* TCA is used for the node-degree constraint of 2. The amount of total flow in the network depends on the number of links for the GW (sink) node for a certain node-degree constraint. For example, if the capacity of each link is 24 Mbps then the maximum possible total network flow, i.e. the maximum network throughput is 48 Mbps for a node-constraint of 2.

The following constraints introduce fairness among flows of the multiple commodities. A change in y which is the multiplier on the unit flow demand of the commodities ensures that the inflows for all commodities are changed equally:

$$f_s^p \geq y \times demand_{sd} \quad \text{for all } s \text{ and } d \text{ and for all } p \quad (5.6)$$

where $s \in \{sources\}$, $d \in \{sinks\}$, and $p \in P$.

The following constraints represent the half-duplex nature of the radio-interfaces of the mesh routers, which means that a link can be used for either transmission or reception but not for both:

$$z_{ij} + z_{ji} \leq 1 \quad \text{for all } i \text{ and } j \quad (5.7)$$

where $i, j \in V$.

The following are the link capacity constraints that ensure that the sum of flows of commodities on a link does not exceed the link capacity:

$$\sum_p f_{ij}^p \leq c_{ij} \times z_{ij} \quad \text{for all } i \text{ and } j \quad (5.8)$$

where $i, j \in V$ and $p \in P$.

A capacity of 24 Mbps is assumed for each link based on the fact that the maximum throughput, also known as goodput, of an IEEE 802.11a link operating at a maximum data rate of 54 Mbps is approximately 24 Mbps [77]. All link capacities are non-negative integers:

$$c_{ij} \geq 0 \text{ for all } i \text{ and } j \quad (5.9)$$

where $i, j \in V$.

The flow of a commodity p on a link is non-negative but is not required to be an integer:

$$f_{ij}^p \geq 0 \text{ for all } i \text{ and } j \text{ and for all } p \quad (5.10)$$

where $i, j \in V$ and $p \in P$.

The AMPL language [78] is used to model the multi-path routing problem and IBM CPLEX 12.2 [11] is used to solve the resulting problem.

5.4.2. Problem Formulation for Minimum Power-based Shortest Path Routing

The MPSP routing problem is formulated in two stages. In the first stage, shortest paths are determined between the source-sink pairs. The metric for path selection is minimum transmission power. For the two CG approaches, the gateway calculates the minimum transmission power required to reach each of the nodes in the FNT or the MPNT of a node by using the appropriate propagation model. The Free Space model is used for short distances and the Two-ray ground reflection model is used for longer distances, depending on the value of the Euclidean distance in relation to the cross-over distance [79]. If the distance between two nodes u and v is less than or equal to the cross-over distance, i.e. $d(u,v) \leq \text{Cross_over_dist}$, the Free Space model is used whereas if $d(u,v) > \text{Cross_over_dist}$, the Two-ray model is used.

In the second stage, the flows of individual commodities are maximized equally so as to achieve the maximum total flow in the network, using the shortest paths determined in the first stage.

First Stage

This stage is formulated as the following MILP. Let:

- $cost2_{ij}$ be an input parameter containing the cost of each link l_{ij} , which is minimum transmission power,
- $supply_i$ be an input parameter which is 1 for sources
- $demand_i$ be an input parameter which is -1 for the sink (gateway)

Objective

Shortest paths are determined between the source-sink pairs using the following objective:

$$\min \sum_{p \in P} \left(\sum_{(i,j) \in E} (f_{ij}^p \times cost2_{ij}) \right) \quad (5.11)$$

Constraints

The following are the flow balance constraints for this stage:

$$\sum_j f_{ij}^p = supply_i + \sum_h f_{hi}^p \text{ for all } i \text{ and for all } p \quad (5.12)$$

where $i, j, h \in V$ and i is the source of p ,

$$\sum_j f_{ij}^p = demand_i + \sum_h f_{hi}^p \text{ for all } i \text{ and for all } p \quad (5.13)$$

where $i, j, h \in V$ and i is the sink of p ,

$$\sum_j f_{ij}^p = \sum_h f_{hi}^p \text{ for all } i \text{ and for all } p \quad (5.14)$$

where $i \in V \setminus \{s_p, d_p\}$ and $j, h \in V$.

The node-degree, half-duplex and link capacity constraints are exactly the same as in (5.5), (5.7) and (5.8), respectively. Note that because of the integer demand and supply flows and the unimodularity property of the network matrix, the continuous flows f_{ij}^p will have binary values in the solution.

Second Stage

This stage is formulated as the following linear program:

Objective

The multiplier on the unit flow demand of the individual commodities is maximized so as to achieve the maximum total flow in the network, using the shortest paths determined in the first stage:

$$\max y \quad (5.15)$$

The additional term in (5.1) is not required in (5.15) since there is no possibility of flow-loops due to pre-established shortest paths.

Constraints

The constraints for this stage are exactly the same as in (5.2), (5.3), (5.4), and (5.6). In addition, this stage requires the following link capacity constraints:

$$\sum_p f_{ij}^p \leq c_{ij} \text{ for all } i \text{ and } j \text{ and for all } p \quad (5.16)$$

where $i, j \in V$ and $p \in P$.

As in the case of multi-path routing, the AMPL language is used to model the MPSP routing problem, and IBM CPLEX 12.2 is used to solve the resulting problem.

5.5. Conflict Graph

The *interference-free* channel assignment problem involves the assignment of a minimum number of frequency channels to the radios of mesh nodes in such a way that two nodes that are sufficiently close to each other are assigned different frequency channels, while those that are sufficiently far apart can share the same frequency channel. This involves forming a *conflict graph*, which is also sometimes known as the *link interference graph*. The vertices of the conflict graph represent the wireless links between the mesh nodes while the edges in the conflict graph represent the conflicts between the wireless links. An edge between two vertices in the conflict graph indicates that the two wireless links cannot be active simultaneously over the same frequency channel.

The protocol-based interference model is used to build the conflict graph. Its input consists of the *links involved in routing*, i.e. the output of the routing problem, and the

node locations. A conflict is indicated between a pair of links if any of the two nodes of a link is within the interference range of any of the two nodes of the other link or vice versa.

Let d_{ij} denote the distance between nodes i and j , R_i be the transmission range of node i , and R_i' be the interference range of node i . In the conflict graph F , the vertices correspond to the links in the connectivity graph C . An edge between the vertices l_{ij} and l_{pq} in F indicates that the links l_{ij} and l_{pq} in C cannot be active simultaneously. Note that links l_{ij} and l_{pq} , which are involved in routing, are bidirectional links at the MAC and physical layer levels. So, for a link l_{ij} which is involved in routing, it is checked eight times with every other link that is involved in routing while building the conflict graph. Therefore, an edge is drawn between the vertices l_{ij} and l_{pq} if any of the following is true:

- $d_{ip} \leq R_i'$ or $d_{iq} \leq R_i'$ or $d_{jp} \leq R_j'$ or $d_{jq} \leq R_j'$ or
- $d_{pi} \leq R_p'$ or $d_{pj} \leq R_p'$ or $d_{qi} \leq R_q'$ or $d_{qj} \leq R_q'$.

This means that a conflict is indicated between links l_{ij} and l_{pq} by drawing an edge between these two vertices in the conflict graph if either node p or node q of link l_{pq} is within the interference range of node i or node j of link l_{ij} or node i or node j of link l_{ij} is within the interference range of node p or node q of link l_{pq} . While building the conflict graph, the interference range is assumed to be twice the transmission range.

5.6. Minimum Coloring

The *minimum coloring problem* is the problem of computing a coloring of the vertices in the conflict graph F using as few distinct colors as possible; this is the same as the problem of finding the minimum number of frequency channels to use such that there is no interference. The minimum coloring problem is well-known to be NP-hard for general graphs. A greedy heuristic for its solution consists of these steps: (a) find a maximal independent set of vertices and assign the members of this set to the same frequency channel, (b) remove these vertices from the conflict graph, and (c) repeat until all vertices are colored (assigned a frequency channel). The heuristic solution amounts to finding a small number of maximal independent sets in the conflict graph. The number of frequency channels (colors) required for interference-free communication among the mesh nodes is equal to the number of MaISs.

Since finding a maximum independent set is itself NP-hard [14], heuristic algorithms are used for this step of the solution to the interference-free channel assignment problem.

5.6.1. MaIS-based Heuristics for Minimum Coloring

Three greedy heuristic algorithms are used to find MaISs. The first algorithm, *Maximum Node-Degree Start*, selects a vertex from the conflict graph with the maximum node-degree (i.e. maximum numbers of conflicts) and introduces that vertex into the maximal independent set under construction. The algorithm then checks the other vertices of the conflict graph and puts them in the set if they do not have an edge with the vertices already in the set. To find an MaIS, the worst case computational complexity of this algorithm, summarized in Algorithm 5.1, is $O(m^2)$ where m is the number of nodes in the conflict graph. The second algorithm, *Minimum Node-Degree Start*, starts by selecting a vertex from the conflict graph with the minimum node-degree. Note that if the conflict graph has multiple vertices with maximum node-degree (Maximum Node-Degree Start) or minimum node-degree (Minimum Node-Degree Start), then the starting vertex is selected randomly from among them. After removing a MaIS from the conflict graph, ties for the starting vertex in the revised conflict graph are again broken randomly if required. The third algorithm, *Random Start*, starts by selecting a vertex at random from the conflict graph.

These randomized heuristics find MaISs very quickly but may return different results each time, so they are run multiple times. For a given topology of MRs, each of the three algorithm variants is run 25 times on the conflict graph and the best solution is taken over all 75 runs.

5.6.2. MIS-based Heuristic for Minimum Coloring

The greedy minimum coloring heuristic presumably works best if the *maximum* independent set can be found at each step instead of just a *maximal* independent set. Wilf's algorithm [80] is used for finding the maximum independent set of a given graph G . It starts by choosing a vertex v^* from the graph which has the highest node-degree. If v^* has at least two neighbors, the computational complexity of Wilf's Algorithm to find an MIS in the given graph is $O(1.47^m)$, which reduces to $O(1.39^m)$ if v^* has at least three

neighbors, where m is the number of nodes in the graph. After selecting v^* , the algorithm builds two sets. The set $n1$ contains all the nodes of the graph except v^* and the set $n2$ excludes both v^* and $N(v^*)$, the neighborhood of v^* , i.e. the nodes that are connected to v^* by an edge. *maxset* is the set which has the higher cardinality of the two sets. The vertices in *maxset* (and their incident edges) are deleted from G and the procedure is repeated until G has no edges. The MIS consists of the vertices in the disconnected G . The pseudo-code of the procedure using Wilf's algorithm is shown in Algorithm 5.2.

Note that the procedures related to the connectivity graph, routing, conflict graph and minimum coloring are carried out at the GW, which then sends each MR its channel assignment and routing information over the control channel using the control radio. Based on the frequency channel assigned to a MR to communicate with a neighbor and its distance to that neighbor, each MR applies power control and adjusts its transmission power accordingly.

5.7. Control Mechanism to Support a Dynamic WMN

The proposed channel assignment method supports a dynamic WMN environment that undergoes regular topological changes as new mesh nodes join the network or existing mesh nodes leave it (e.g. due to node failure). To achieve this, all mesh nodes exchange periodic *keep-alive* messages using maximum transmission power on the control frequency channel using their control radios. The *keep-alive* message from a node which contains its node ID and its position tells its neighbors within its maximum transmission range that the node is active. If a mesh node fails then its maximum-transmission-range neighbors will not receive its periodic *keep-alive* message. After receiving *keep-alive* messages from its neighbors, a mesh node builds a new MPNT using the information received from these messages and sends its MPNT along with its position and node ID to the GW over the control frequency channel. Based on the received MPNTs, the GW performs a new channel assignment and sends this information to all mesh nodes in the network on the control frequency channel. Each mesh router receives the new channel assignment information on the control frequency channel over its control radio and switches its radios to new frequency channels in case the received channel assignment is different from its existing channel assignment.

If a new mesh node joins the network, it sends a *Hello* message using maximum transmission power to inform mesh nodes within its maximum transmission range of its presence. Note that a *Hello* message is the same as a *keep-alive* message but has a different name as it is used to announce the presence of a new node. Whenever there is a topology change as an existing mesh router leaves/fails or a new mesh router joins the network, the exchange of *keep-alive* or *Hello* messages over the common control frequency channel will lead to a quick reassignment of frequency channels. In this way, fast reconfiguration of the network can be achieved in dynamic mesh environments in future dynamical models.

If a data radio is used for control traffic, it will have to switch to the control frequency channel every time it has to send control information. The delay caused by channel switching and transmission of control information will disrupt the data traffic on the link [81]. For this reason, the use of a dedicated control radio is proposed on every mesh node for control traffic. The cost of multiple radios is no longer a prohibitive factor due to the decreasing cost of wireless interface cards [82].

Because of the ad hoc nature of the mesh network, communication between mesh nodes is not synchronized, so a dedicated time slot cannot be used as the control channel. Instead a frequency channel is dedicated as the common control channel for the exchange of control traffic. This allows the proposed channel assignment method to achieve interference-free communication among the mesh nodes, but at the cost of occupying a frequency channel. A data frequency channel should not be used for control traffic as this will cause severe interference for those links using the same frequency channel for data traffic since the *keep-alive* as well as *Hello* messages are sent using maximum transmission power.

Inputs:

- L : set of links involved in routing
- $F(V_F, E_F)$: conflict graph

Outputs:

- V_{MaIS} : MaIS of vertices
- L_{MaIS} : Set of links in L corresponding to vertices in MaIS
- c_{MaIS} : number of MaISs

BEGIN

```

1.  $c_{MaIS} \leftarrow 0$ 
2.  $m \leftarrow |L| = |V_F|$ 
3. While  $m \geq 1$ :
4.   If  $F$  is fully connected or  $m = 1$ :
5.     For  $d = 1$  to  $m$ :
6.       Initialize an empty  $V_{MaIS}$  and  $L_{MaIS}$ 
7.       Add vertex  $d$  to  $V_{MaIS}$  and the corresponding link to  $L_{MaIS}$ 
8.       Output  $V_{MaIS}$  and  $L_{MaIS}$ 
9.        $c_{MaIS} \leftarrow c_{MaIS} + 1$ 
10.    end For
11.    Output  $c_{MaIS}$  and exit
12.  end If
13.  Initialize an empty  $V_{MaIS}$  and  $L_{MaIS}$ 
14.  Add vertex of  $F$  with the maximum node-degree to  $V_{MaIS}$  and the corresponding
    link to  $L_{MaIS}$ 
15.  For  $i = 1$  to  $m$ :
16.    If vertex  $i$  does not have an edge with vertices in  $V_{MaIS}$ :
17.      Add vertex  $i$  to  $V_{MaIS}$  and the corresponding link to  $L_{MaIS}$ 
18.    end If
19.  end For
20.  Output  $V_{MaIS}$  and  $L_{MaIS}$ 
21.   $c_{MaIS} \leftarrow c_{MaIS} + 1$ 
22.   $V_F \leftarrow V_F \setminus V_{MaIS}$ 
23.   $L \leftarrow L \setminus L_{MaIS}$ 
24.   $m \leftarrow |L|$ 
25. end While
26. Output  $c_{MaIS}$ 
END

```

Algorithm 5.1. Maximum Node-Degree Start

Inputs:

- L : set of links involved in routing
- $F(V_F, E_F)$: conflict graph

Outputs:

- V_{MIS} : MIS of vertices
- L_{MIS} : Set of links in L corresponding to vertices in MIS
- c_{MIS} : number of MISs

BEGIN

```

1.  $c_{MIS} \leftarrow 0$ 
2.  $m \leftarrow |L| = |V_F|$ 
3. While  $m \geq 1$ :
4.   If  $F$  is fully connected or  $m = 1$ :
5.     For  $d = 1$  to  $m$ :
6.       Initialize an empty  $V_{MIS}$  and  $L_{MIS}$ 
7.       Add vertex  $d$  to  $V_{MIS}$  and the corresponding link to  $L_{MIS}$ 
8.       Output  $V_{MIS}$  and  $L_{MIS}$ 
9.        $c_{MIS} \leftarrow c_{MIS} + 1$ 
10.    end For
11.    Output  $c_{MIS}$  and exit
12.  end If
13.   $T(V_T, E_T) \leftarrow F(V_F, E_F)$ 
14.  While  $T$  has any edges:
15.     $v^* \leftarrow$  vertex of  $T$  with the maximum node-degree
16.     $n1 \leftarrow T \setminus v^*$ 
17.     $N(v^*) \leftarrow$  set of vertices of  $T$  connected to  $v^*$ 
18.     $n2 \leftarrow T \setminus \{v^* \cup N(v^*)\}$ 
19.     $maxset \leftarrow$  the higher cardinality set among  $n1$  and  $n2$ 
20.     $V_T \leftarrow V_T \setminus maxset$ 
21.  end While
22.  Initialize an empty  $V_{MIS}$  and  $L_{MIS}$ 
23.  For  $g = 1$  to  $|V_T|$ :
24.    Add vertex  $g$  to  $V_{MIS}$  and the corresponding link to  $L_{MIS}$ 
25.  end For
26.  Output  $V_{MIS}$  and  $L_{MIS}$ 
27.   $c_{MIS} \leftarrow c_{MIS} + 1$ 
28.   $V_F \leftarrow V_F \setminus V_{MIS}$ 
29.   $L \leftarrow L \setminus L_{MIS}$ 
30.   $m \leftarrow |L|$ 
31. end While
32. Output  $c_{MIS}$ 
END

```

Algorithm 5.2. Procedure of finding MISs using Wilf's Algorithm

6. REALISTIC INTERFERENCE MODELING

This chapter deals with the problem of building the conflict graph based on a realistic interference model. The impact of different interference models on channel assignment in MRMC WMNs is studied, namely the protocol model, the SIR model and the SIR model with shadowing. The goal is to determine the number of frequency channels required to achieve interference-free communication among the mesh nodes based on a realistic interference model, i.e. SIR model with shadowing. A novel channel assignment method is developed for finding a small number of frequency channels needed for interference-free communication in MRMC WMNs under realistic conditions [83].

6.1. Introduction

The protocol model is widely used to model interference for channel assignment in wireless mesh networks. This simple model assumes that interference is a binary phenomenon that occurs within the interference range of the nodes of any active link. The SINR model is more accurate. It considers the cumulative effect of interference at the receiving node where the desired signal is received correctly if its SINR is above the SINR threshold.

Neither of these models reflects the reality of signal propagation in wireless links where the RF signal is reflected from and around nearby objects, which causes the signal strength to fluctuate in space. This effect is commonly known as *shadowing* and a model that accounts for shadowing will be referred to as *SINR model with shadowing*. The goal is to use this model to devise a realistic and interference-free channel assignment method for MRMC WMNs. SIR is used instead of SINR since co-channel interference is generally much stronger than noise.

To achieve this objective, a novel and computationally simple yet effective method of building the conflict graph based on the SIR model is developed, which is then extended for the SIR model with shadowing to construct a conflict graph that incorporates shadowing [83]. Note that in the SIR models, the interference is controlled but it still exists. By *interference-free* communication under these models, it means when

the desired incoming signal's SIR is above the required SIR threshold to be correctly received at the receiver. A new channel assignment method is then developed that uses the conflict graph constructed using the SIR model with shadowing and finds a small number of frequency channels needed for interference-free communication in MRMC WMNs under realistic conditions. Unlike any previous channel assignment schemes for MRMC WMNs, this work is the first to propose a channel assignment method that is based on a realistic interference model, i.e. the SIR model with shadowing.

The results show that the channel assignment using the realistic SIR model with shadowing requires more frequency channels for network throughputs at different node-degree constraints as compared to using simpler interference models.

6.2. Interference Models

Once the links involved in routing are known, the conflict graph F can be built using the three different interference models, namely the protocol model, the SIR model and the SIR model with shadowing. Note that the creation of the conflict graph based on the protocol model was presented in Section 5.5.

6.3. Conflict Graph based on SIR Model

The method to construct a conflict graph and matrix based on the SIR model is shown in Algorithm 6.1. An essential part of the method for channel assignment is the construction of the *conflict matrix* that represents conflicts between the links involved in routing. The set of links involved in routing is L , so the conflict matrix is of size $|L| \times |L|$. Unlike the conflict matrix based on the protocol model which consists of ones and zeros indicating conflicts and no conflicts, the conflict matrix for the SIR model consists of elements that are either one (indicating a conflict between a pair of links; this appears as an edge in the conflict graph) or the maximum power received at a link from any other link. $RxThresh_mwatts$ is the power threshold required at the receiver in mW to correctly receive the desired incoming signal. If the ratio of the $RxThresh_mwatts$ to the maximum of the four received powers at link l_{xy} from link l_{pq} is less than the SIR threshold, a conflict is indicated between links l_{xy} and l_{pq} by placing a one in the conflict matrix. If this ratio is above the SIR threshold, there could be no direct conflict between the two links and they could be active simultaneously over the same frequency channel. In this

case, the maximum of the four received powers at link l_{xy} from link l_{pq} is placed in the conflict matrix for use later while building *weighted maximal independent sets* (WMAISs) during the minimum coloring stage. The necessary nomenclature follows below:

- f (frequency);
- G_t (gain of transmitter);
- G_r (gain of receiver);
- h_t (transmitter antenna height);
- h_r (receiver antenna height);
- n (number of mesh nodes);
- $RxThresh_dBm$ (receiver threshold in dBm);
- $SIRThresh_dB$ (SIR threshold in dB);
- L (set of links involved in routing);
- $locations_x$ (set of x -coordinates of nodes); and
- $locations_y$ (set of y -coordinates of nodes).

Instead of working with four received powers at link l_{xy} from a link l_{pq} , the maximum of these received powers is used to determine the conflict. This significantly reduces the overall computational complexity; specifically, the complexity of the related step of building a WMAIS during the minimum coloring stage is reduced from $O(m^4)$ to $O(m)$, where m is the number of routing links. If the ratio of $RxThresh_mwatts$ and the maximum received power at link l_{xy} from link l_{pq} is above the SIR threshold as well as the ratio of $RxThresh_mwatts$ and the maximum received power at link l_{pq} from link l_{xy} is above the SIR threshold, this indicates that the two links can be active simultaneously over the same frequency channel and could potentially become members of the same WMAIS.

The distance between the nodes is calculated using the Euclidean distance formula. The link transmission power or the received interference power is calculated using the Free Space propagation model for short distances and the Two-Ray ground reflection propagation model for longer distances, depending on the Euclidean distance between the nodes $d(x,y)$ in relation to the cross-over distance $Cross_over_dist$. If $d(x,y) \leq Cross_over_dist$, the Free Space propagation model is used, whereas if $d(x,y) >$

Cross_over_dist, the Two-Ray propagation model is used. Using the Free Space model, transmission power is calculated as

$$P_t = \frac{RxThresh(4\pi d)^2}{G_t G_r \lambda^2}, \quad (6.1)$$

and received power is calculated as

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}. \quad (6.2)$$

Using the Two-Ray model, transmission power is calculated as

$$P_t = \frac{RxThresh(d)^4}{G_t G_r h_t^2 h_r^2}, \quad (6.3)$$

and received power is calculated as

$$P_r = \frac{P_t G_t G_r h_t^2 h_r^2}{(d)^4}, \quad (6.4)$$

where G_t and G_r are transmitter and receiver antenna gain respectively, h_t and h_r are transmitter and receiver antenna height respectively and $RxThresh$ is the power threshold required by the radio interface of the receiving node to correctly understand the desired incoming signal [79].

The concept of conflict table and conflict matrix used in Algorithm 6.1 is explained via the following example. Consider the set of links involved in routing L (shown in Table 6.1) for topology #1 when the node-degree constraint is 3 and the link data rate is 12 Mbps. The corresponding routing graph is shown in Fig. 6.1. $L(i,1)$ represents the 1st node in the i^{th} link in L , and $L(i,2)$ represents the 2nd node in the i^{th} link in L , for example $L(3,1)$ indicates node 3 and $L(3,2)$ indicates node 1 (i.e. the two nodes in link #3). In Algorithm 6.1, when i is equal to 3 in the *for* loop on Step #9, the distance between node 3 and node 1 is stored in $L(3,3)$ and the transmission power for the link $l_{3,1}$ (calculated using (6.1) or (6.3)) is stored in $L(3,4)$. $|L| = m = 35$.

In Algorithm 6.1, when i is equal to 3 and j is equal to 14, the entries in the conflict table are as follows and the goal is to find the ratio of $RxThresh_mwatts$ and maximum power received at link $l_{3,1}$ (3rd link in L) from link $l_{14,15}$ (14th link in L). Note that the value of $RxThresh_dBm$ is -79 dBm and that of $RxThresh_mwatts$ is 1.2589×10^{-8}

mW. Also for the link data rate of 12 Mbps, the value of *SIRThresh_{dB}* is 5.78 dB and that of *SIRThresh* is 3.7844.

Table 6.1. Set of Links Involved in Routing for Topology #1

No.	Link	No.	Link	No.	Link	No.	Link	No.	Link
1	1 8	8	8 13	15	16 15	22	23 22	29	30 35
2	2 1	9	9 2	16	17 16	23	24 23	30	31 32
3	3 1	10	10 17	17	18 24	24	25 19	31	32 26
4	4 3	11	11 17	18	19 13	25	26 21	32	33 27
5	5 11	12	12 24	19	20 19	26	27 21	33	34 27
6	6 5	13	13 14	20	21 15	27	28 22	34	35 34
7	7 14	14	14 15	21	22 16	28	29 28	35	36 35

Figure 6.1. Routing Graph for Topology #1

$conflict_table(14,1) = 3$ (node 3 in the 3rd link in L , i.e. $l_{3,1}$)
 $conflict_table(14,2) = 1$ (node 1 in the 3rd link in L , i.e. $l_{3,1}$)
 $conflict_table(14,3) = 14$ (node 14 in the 14th link in L , i.e. $l_{14,15}$)
 $conflict_table(14,4) = 15$ (node 15 in the 14th link in L , i.e. $l_{14,15}$)
 $conflict_table(14,5) = 1.5196 \times 10^{-9}$ mW (power received at node 3 from node 14)
 $conflict_table(14,6) = 1.8248 \times 10^{-9}$ mW (power received at node 3 from node 15)
 $conflict_table(14,7) = 1.8905 \times 10^{-9}$ mW (power received at node 1 from node 14)
 $conflict_table(14,8) = 1.5020 \times 10^{-9}$ mW (power received at node 1 from node 15)
 $conflict_table(14,9) = 1.8905 \times 10^{-9}$ mW (maximum power received at $l_{3,1}$ from $l_{14,15}$)
 $conflict_table(14,10) = 6.6591$ (ratio of $RxThresh_mwatts$ and maximum power received at $l_{3,1}$ from $l_{14,15}$)

Note that the transmission power for $l_{14,15}$ (calculated earlier and stored in $L(14,4)$) is used while calculating the received powers at $l_{3,1}$ from $l_{14,15}$ in the conflict table. The relevant entry in the conflict matrix when $i2$ is equal to 14 is

$conflict_matrix(3,14) = 1.8905 \times 10^{-9}$ mW,

since the ratio of the $RxThresh_mwatts$ and maximum power received at $l_{3,1}$ from $l_{14,15}$ is 6.6591, which is greater than the required SIR threshold of 3.7844. This maximum received power at $l_{3,1}$ from $l_{14,15}$ is used later while building weighted maximal independent sets during the minimum coloring stage.

Similarly, when the algorithm checks the maximum power received at $l_{17,16}$ (16th link in L) from $l_{23,22}$ (22nd link in L), it finds it to be 9.9868×10^{-9} mW which results in $conflict_matrix(16,22) = 1$,

since the ratio of the $RxThresh_mwatts$ and maximum power received at $l_{17,16}$ from $l_{23,22}$ is 1.2606 which is less than the required SIR threshold of 3.7844. This value of 1 indicates a conflict between links $l_{17,16}$ and $l_{23,22}$ which means that these two links cannot be active simultaneously over the same frequency channel. Note that for two links l_{xy} (link number s in L) and l_{pq} (link number t in L), a one in either or both $conflict_matrix(s,t)$ and $conflict_matrix(t,s)$ indicates a conflict between these links. There is no direct conflict between these two links if there is a non-one value in both $conflict_matrix(s,t)$ and $conflict_matrix(t,s)$.

Inputs:

- $f, G_t, G_r, h_t, h_r, n, RxThresh_dBm, SIRThresh_dB, L, locations_x, locations_y$.

Output:

- *conflict_matrix*: Conflict graph

BEGIN

```

1.  $RxThresh\_mwatts \leftarrow 10^{(RxThresh\_dBm/10)}$ 
2.  $SIRThresh \leftarrow 10^{(SIRThresh\_dB/10)}$ 
3.  $m \leftarrow |L|$ 
4. For  $i = 1$  to  $n$ :
5.   For  $j = 1$  to  $n$ :
6.      $dist\_all(i,j) \leftarrow$  distance between node  $i$  and node  $j$ 
7.   end For
8. end For
9. For  $i = 1$  to  $m$ :
10.   $L(i,3) \leftarrow$  distance between the nodes of link  $i$ 
11.   $L(i,4) \leftarrow$  transmission power for link  $i$  using (6.1) or (6.3)
12. end For
13. Initialize an  $m \times m$  conflict_matrix of all ones
14. For  $i1 = 1$  to  $m$ :
15.   Initialize an empty conflict table
16.   For  $j = 1$  to  $m$ :
17.      $conflict\_table(j,1) \leftarrow$  node  $x$  of link  $i1$ 
18.      $conflict\_table(j,2) \leftarrow$  node  $y$  of link  $i1$ 
19.      $conflict\_table(j,3) \leftarrow$  node  $p$  of link  $j$ 
20.      $conflict\_table(j,4) \leftarrow$  node  $q$  of link  $j$ 
21.      $P_{r,xp} \leftarrow$  power received at node  $x$  from node  $p$  using (6.2) or (6.4)
22.      $P_{r,xq} \leftarrow$  power received at node  $x$  from node  $q$  using (6.2) or (6.4)
23.      $P_{r,yp} \leftarrow$  power received at node  $y$  from node  $p$  using (6.2) or (6.4)
24.      $P_{r,yq} \leftarrow$  power received at node  $y$  from node  $q$  using (6.2) or (6.4)
25.      $conflict\_table(j,5) \leftarrow P_{r,xp}$ 
26.      $conflict\_table(j,6) \leftarrow P_{r,xq}$ 
27.      $conflict\_table(j,7) \leftarrow P_{r,yp}$ 
28.      $conflict\_table(j,8) \leftarrow P_{r,yq}$ 
29.      $conflict\_table(j,9) \leftarrow \max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
30.      $conflict\_table(j,10) \leftarrow RxThresh\_mwatts / \max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
31.   end For
32.   For  $i2 = 1$  to  $m$ :
33.     If  $conflict\_table(i2,10) > SIRThresh$ :
34.        $conflict\_matrix(i1,i2) \leftarrow conflict\_table(i2,9)$ 
35.     end If
36.   end For
37. end For
38. Output conflict_matrix
END

```

Algorithm 6.1. Conflict Graph and Matrix based on SIR Model

6.4. Conflict Graph based on SIR Model with Shadowing

The conflict graph for the SIR model is extended to include shadowing. The transmission power using the Free Space model with shadowing is calculated as

$$P_t = \frac{RxThresh(4\pi d)^2}{G_t G_r \lambda^2} \times 10^{-x/10}, \quad (6.5)$$

where x is the lognormal random variable. The transmission power using the Two-Ray model with shadowing is calculated as

$$P_t = \frac{RxThresh(d)^4}{G_t G_r h_t^2 h_r^2} \times 10^{-x/10}. \quad (6.6)$$

While creating the conflict graph and matrix for the SIR model with shadowing, the link transmission power is calculated using either (6.5) or (6.6), depending on the cross-over distance. The inputs for the conflict graph for this model, in addition to those in the pseudo-code in Fig. 6.1, include the standard deviation for shadowing σ and the outage probability OP . The worst case computational complexity of the method to construct the conflict graph for this model is $O(n^2 + m + m(m + m)) = O(n^2 + m + 2m^2) \approx O(n^2 + m^2)$, where n is the number of mesh nodes in the network.

If p_c is the probability that the received power P_r is greater than or equal to the receiver threshold $RxThresh$, then the outage probability OP is $(1 - p_c)$. Let us consider the following example where the transmission power P_t for a 100 m long link between two nodes is calculated using (6.5) such that the probability that $P_r \geq RxThresh$ is 90%. In other words, the outage probability OP is 10%. In this example, it is considered that $f = 5.805$ GHz, $G_t = 1$, $G_r = 1$, $RxThresh = -65$ dBm, $d = 100$ m, and $\sigma = 3$ dB. Using (6.5), one can proceed as follows:

$$RxThresh + 20\log(4\pi d) = 10\log P_t + 10\log G_t + 10\log G_r + 20\log \lambda + \frac{x}{10}(10\log 10)$$

$$-65 + 61.9842 = 10\log P_t + 0 + 0 - 25.7336 + x$$

$$x = 22.7178 - 10\log P_t$$

Now $\mathbb{P}(P_r \geq RxThresh) \geq 0.9$ can be written in terms of Q -function as

$$Q\left(\frac{z - u}{\sigma}\right) \geq 0.9$$

$$Q\left(\frac{22.7178 - 10\log P_t}{3}\right) \geq 0.9$$

Since $1 - Q(-z) = Q(z)$, the above equation can be written as

$$1 - Q\left(\frac{10\log P_t - 22.7178}{3}\right) \geq 0.9$$

$$Q\left(\frac{10\log P_t - 22.7178}{3}\right) \leq 0.1$$

For $Q(z) \leq 0.1$, one can find that $z \geq 1.2816$ from the Q -function table. The above equation can now be written as

$$\frac{10\log P_t - 22.7178}{3} \geq 1.2816$$

$$10\log P_t \geq 26.5626$$

$$P_t \geq 453.1688$$

Hence, using the Free Space model with shadowing, a transmission power of 453.1688 mW is required for a 100 m link to achieve an OP of 10% at the receiver when the operating frequency is 5.805 GHz and the standard deviation for shadowing is 3 dB. Decreasing the outage probability will result in an increase in the transmission power required. For example, decreasing the outage probability to 5% in the above example will increase the transmission power to 582.422 mW. Also, an increase in σ will lead to an increase in the transmission power required. For example, increasing σ to 5 dB in the above example will increase the transmission power to 817.6312 mW.

6.5. Channel Assignment using Realistic Interference Modeling

A novel channel assignment method is developed that uses the conflict graph that is built using a realistic interference model, i.e. the SIR model with shadowing. The proposed channel assignment method finds a small number of frequency channels needed for interference-free communication in MRMC WMNs under realistic conditions. It is compared to the channel assignment methods that use simpler interference models, namely, channel assignment method using protocol model (presented in Chapter 5) and channel assignment method using SIR model. The results show that it requires the highest number of frequency channels among the three channel assignment methods due to its

use of the SIR model with shadowing which is the most realistic of the three interference models.

Like the interference-free channel assignment method presented in Chapter 5 that uses the protocol model, this method as well as the channel assignment method using the SIR model consists of the following four stages, namely, connectivity graph, routing, interference modeling and minimum coloring. The network architecture used for these methods is the same as the one presented in Section 5.2.

6.5.1. Connectivity Graph

To minimize the number of frequency channels, the connectivity graph is created using the *Select x for less than x* TCA given in Section 5.3. The *Select x for less than x* TCA enhances spatial channel reuse by selecting the nearest neighbors for each node in the network.

6.5.2. Routing

As presented in Section 5.4.1, multi-path routing is used to maximize the network throughput. Given the connectivity graph, the objective is to maximize the network throughput while maintaining fairness among the multiple network flows under flow conservation, node-degree, half-duplex, fairness, and link capacity constraints. The AMPL language is used to model the multi-path routing problem, and IBM CPLEX 12.2 is used to solve the resulting problem.

A value for the link capacity is assumed based on the fact that maximum goodputs (maximum link throughputs) of an IEEE 802.11a link operating at data rates of 12, 24, 36 and 54 Mbps are 9.18, 15.52, 20.03, and 24.73 Mbps, respectively [77]. The link capacity (maximum link throughput) for a particular IEEE 802.11a link data rate is calculated using [77]

$$MT = \frac{8 \times L_{DATA}}{T_{D_DATA} + T_{D_ACK} + 2\tau + T_{DIFS} + T_{SIFS} + \overline{CW}} \quad (6.7)$$

$$T_{D_DATA} = T_P + T_{PHY} + T_{SYM} \times \left\lceil \frac{16 + 6 + (8 \times L_{H_DATA}) + (8 \times L_{DATA})}{N_{DBPS}} \right\rceil \quad (6.8)$$

$$T_{D_ACK} = T_P + T_{PHY} + T_{SYM} \times \left\lceil \frac{16 + 6 + (8 \times L_{ACK})}{N_{DBPS}} \right\rceil \quad (6.9)$$

$$\overline{CW} = \frac{CW_{min} \times T_{slot}}{2} \quad (6.10)$$

where

MT is the maximum link throughput in bps;

L_{DATA} is the length of the payload in bytes;

T_{D_DATA} is the data transmission delay;

T_{D_ACK} is the ACK (acknowledgement) transmission delay;

τ is the propagation delay and is equal to 1 μ s for 802.11a;

T_{SIFS} is the short interframe space time and is equal to 16 μ s for 802.11a;

T_{DIFS} is the distributed coordination function interframe space time and is equal to 34 μ s for 802.11a;

$\overline{CW}$ is the average backoff time;

T_P is transmission time of physical layer preamble and is equal to 16 μ s for 802.11a;

T_{PHY} is the transmission time of the physical layer header and is equal to 4 μ s for 802.11a;

T_{SYM} is the transmission time for a symbol and is equal to 4 μ s for 802.11a;

L_{H_DATA} is the MAC overhead in bytes and is equal to 28 bytes;

L_{ACK} is the ACK size in bytes and is equal to 28 bytes;

N_{DBPS} is the number of data bits per symbol;

CW_{min} is the minimum backoff window size and is equal to 15 for 802.11a; and

T_{slot} is the slot time and is equal to 9 μ s for 802.11a.

Substituting (6.8), (6.9) and (6.10) in (6.7) and simplifying, one gets

$$MT = \frac{8 \times L_{DATA}}{159.5 + 4 \times \left\lceil \frac{246}{N_{DBPS}} \right\rceil + 4 \times \left\lceil \frac{246 + (8 \times L_{DATA})}{N_{DBPS}} \right\rceil} \times 10^6 \quad (6.11)$$

Using (6.11) and assuming a payload size of 1000 bytes, the maximum link throughput for the link data rate of 54 Mbps ($N_{DBPS} = 216$) is calculated to be 24.73 Mbps.

6.5.3. Interference Modeling

The conflict graph for the channel assignment method that is presented in Chapter 5 and that uses the protocol model is created using the procedure given in Section 5.5.

The conflict graph and matrix for the channel assignment methods using the SIR model and the SIR model with shadowing are constructed using the procedures presented earlier in Sections 6.3 and 6.4 respectively.

6.5.4. Minimum Coloring

For the channel assignment method presented in Chapter 5 that uses the conflict graph that is generated based on the protocol model, the three heuristic algorithms based on the classic greedy approach presented earlier in Section 5.6.1 are used to find MaISs from the conflict graph.

For the channel assignment methods that use the conflict graph that is generated based on the SIR model or the SIR model with shadowing, the MaIS-based heuristic algorithms are enhanced to find weighted maximal independent sets from the conflict graph. These new algorithms, which can be referred to as the WMaIS-based heuristic algorithms for minimum coloring, incorporate the cumulative interference constraint. They are presented in the next chapter, i.e. Chapter 7.

7. HEURISTIC AND EXACT SOLUTION FOR MINIMUM COLORING WITH CUMULATIVE INTERFERENCE CONSTRAINTS

Given a conflict graph F , which shows conflicts between wireless links, assigning frequency channels to wireless links can be cast as a standard minimum coloring problem. However the coloring problem is complicated by the fact that acceptably small levels of interference between pairs of links using the same frequency channel can accumulate to cause an unacceptable level of total interference at a given link. In this chapter, new and effective heuristics are developed for solving this extended coloring problem [83][84] and compared to an exact quadratically constrained binary programming (QCBP) formulation [84].

7.1. Introduction

Given the conflict graph and the fact that a frequency channel is the most important and scarce resource in a wireless mesh network, the requirement is to assign a frequency channel to each node in the conflict graph so that the total number of distinct frequency channels is minimized but no arc connects two nodes having the same frequency channel. This is a standard minimum coloring problem in which colors represent frequency channels. The minimum coloring problem is well known to be NP-hard for general graphs.

However there is a further complication. “No interference” means that the amount of interference between two wireless links using the same frequency channel is below a certain threshold, but it is usually not zero. Hence when there are multiple links using the same frequency channel, the cumulative interference may be enough to cause an unacceptable level of interference on some link using that frequency channel, even though the interference on each pair of links is below the tolerance. This complicates the coloring problem.

This work addresses this extended coloring problem by developing fast heuristics for solving this problem. Randomized heuristic algorithms are developed which find a sequence of *weighted maximal independent sets* in the conflict graph and assign the same frequency channel to all of the nodes in each WMaIS, while considering the cumulative

interference constraint. This is the first such solution algorithm for the minimum coloring stage of the channel assignment problem in MRMC wireless mesh networks that considers the effect of cumulative interference. Note that the use of the SIR model with shadowing while building the conflict graph makes it possible to incorporate the cumulative interference constraint into these heuristics and find WMaISs from the conflict graph.

The heuristics are compared to an exact quadratically constrained binary programming formulation of the extended coloring problem. The proposed heuristics return very good results (requiring at most two frequency channels more than the true optimum solution) while being several orders of magnitude faster than the exact QCBP formulation. The short solution times are desirable since a quick solution is required due to the dynamic nature of wireless mesh networks.

7.2. Conflict Matrix and Cumulative Interference

If all of the co-channel interference in a wireless mesh network could be accurately represented as conflicts between pairs of wireless links, then the problem of assigning frequency channels to links could be directly cast as a minimum coloring problem. However this is not the case: acceptably small amounts of interference between pairs of links using the same frequency channel may accumulate to an unacceptable total amount of interference at a given link. Thus there can be a *group conflict* that cannot be represented by a simple conflict graph. For this reason a conflict matrix was introduced in Section 6.3 to represent all of the interference data needed to solve this extended coloring problem.

The conflict graph and matrix represent conflicts between the links involved in routing. They are constructed using the methods presented in Chapter 6. Let L be the set of links involved in routing; the size of the conflict matrix is $|L| \times |L|$ and all elements are non-zero. Vertices in the conflict graph F correspond to links in L . The elements of the conflict matrix are of one of two types: either (i) the value 1 representing a conflict between a pair of links (this appears as an edge in the conflict graph), or (ii) the maximum power received at a link from another link (these values are significantly different from 1.0 so there is no confusion). This knowledge of maximum received

powers provided by the conflict matrix is used while building WMaISs based on the cumulative interference constraint.

7.3. Randomized Greedy Heuristics with Cumulative Interference Constraints

For the conflict graph and matrix generated based on the SIR model or the SIR model with shadowing, the MaIS-based heuristic algorithms presented in Section 5.4.3 are enhanced to handle the cumulative interference constraint as described below. The main idea is to check this new constraint whenever a wireless link is considered for assignment to a particular color.

The proposed randomized greedy heuristics for the solution of the extended minimum coloring problem have these steps: (1) find a WMaIS of vertices that satisfies the cumulative interference constraint and assign the members of this set to the same frequency channel, (2) remove these vertices from the conflict graph, and (3) repeat until all vertices are colored. The number of frequency channels needed for interference-free communication among the mesh nodes is equal to the number of WMaISs. Since finding a maximum independent set is itself NP-hard, heuristics are also used for this step.

These algorithms incorporate the cumulative interference constraint, i.e. the cumulative SIR at every vertex in a WMaIS must be greater than the SIR threshold for interference-free communication on links using the same frequency channel, where the *cumulative SIR* is the ratio of *RxThresh_mwatts* to the sum of the maximum powers received at that vertex from all other vertices in the WMaIS. This accounts for the cumulative interference at that vertex in the WMaIS from all other vertices in this set.

Given the conflict graph $F = (V_F, E_F)$ where V_F is the set of vertices and E_F is the set of arcs in the conflict graph, the first heuristic algorithm for the extended minimum coloring problem, namely, *Enhanced Maximum Node-Degree Start* starts by selecting a vertex from the conflict graph with the maximum number of incident arcs and creates a new set of vertices V_{WMaIS} having this single entry. Another vertex i in V_F is added to V_{WMaIS} provided it satisfies two conditions:

- it does not directly conflict with vertices already in V_{WMaIS} (Step #18), and
- all vertices in V_{temp} , which is equal to $V_{WMaIS} \cup i$, satisfy the cumulative interference constraint considering the power received from all other vertices in V_{temp} (Step #21).

The pseudo-code for the Enhanced Maximum Node-Degree Start heuristic is shown in Algorithm 7.1. The worst case computational complexity of this algorithm to find a WMaIS is $O(m^2)$.

The second heuristic, *Enhanced Minimum Node-Degree Start*, and the third heuristic, *Enhanced Random Start*, differ only in how the initial node is selected. Enhanced Minimum Node-Degree Start heuristic starts by selecting a vertex from the conflict graph with the minimum number of incident arcs. Enhanced Random Start heuristic starts by selecting a vertex at random from the conflict graph. It is usually true that there are multiple candidates for the initial node in the first two algorithms, in which case ties are broken randomly. This work also investigated a variant of the algorithms in which the order of the links is randomized before Step #17; it was discarded because it worsened the results.

These heuristic algorithms are very quick, so each variant is run 25 times on the conflict graph and the best solution is taken over all 75 runs.

The proposed heuristics may appear similar to *RLF*, but there are important differences. Both Algorithm 7.1 and *RLF* start by selecting a vertex from the conflict graph with the maximum number of incident arcs. Besides checking the cumulative interference constraint in Step #21, the major difference is in selecting vertices to add to the independent set under construction. *RLF* selects a vertex with the maximum number of incident arcs from among the remaining vertices, while these heuristics simply check each vertex in the conflict graph sequentially and put it in the weighted maximal independent set under construction if it does not conflict with any vertices already in the set, while also satisfying the cumulative interference constraint in Step #21.

The procedure of building a WMaIS using Algorithm 7.1 is explained with the help of the following example. The goal is to assign frequency channels to links in L (the set of links involved in routing) such that all of these links can be active simultaneously, which means finding WMaISs in the conflict graph for these links. Consider the L shown in Table 6.1. From the conflict graph based on the conflict matrix for L , the vertex with the maximum node-degree, i.e. the vertex in the conflict graph having the most conflicts with other vertices in the conflict graph, is selected and added to an empty WMaIS. This is vertex number 20 in the conflict graph, i.e. the twentieth link in L which is $l_{21,15}$; so the

current WMaIS is $V_{WMaIS} = \{20\}$. Vertex 20 conflicts with 28 other vertices (including itself) and has no conflict with the following 7 vertices in the conflict graph: vertex 6 (link $l_{6,5}$ in L), vertex 10 (link $l_{10,17}$ in L), vertex 11 (link $l_{11,17}$ in L), vertex 12 (link $l_{12,24}$ in L), vertex 17 (link $l_{18,24}$ in L), vertex 29 (link $l_{30,35}$ in L), vertex 35 (link $l_{36,35}$ in L).

In the next step, vertex 20 ($l_{21,15}$) and vertex 6 ($l_{6,5}$) are placed in a temporary set. Since the ratio of $RxThresh_mwatts$ (1.2589×10^{-8} mW) and the maximum power received at $l_{21,15}$ from $l_{6,5}$ (1.9297×10^{-9} mW) is 6.5241 which is above the required SIR threshold of 3.7844, and since the ratio of $RxThresh_mwatts$ and the maximum power received at $l_{6,5}$ from $l_{21,15}$ (7.6919×10^{-10} mW) is 16.3669 which is also above the required SIR threshold, vertex 6 is added to the WMaIS. Now the current WMaIS is $V_{WMaIS} = \{6, 20\}$. Note that the maximum received powers are available from the conflict matrix.

Vertices 10, 11, 12 and 17 cannot become members of the current WMaIS because they conflict with vertex 6 in the conflict graph, which is a member of the current WMaIS.

Next vertex 6 ($l_{6,5}$), vertex 20 ($l_{21,15}$), and vertex 29 ($l_{30,35}$) are placed in a temporary set and the cumulative SIR at every vertex is checked. First, the algorithm checks for cumulative SIR at $l_{6,5}$. The maximum power received at $l_{6,5}$ from $l_{21,15}$ is 7.6919×10^{-10} mW and from $l_{30,35}$ is 1.1963×10^{-9} mW, totaling 1.9655×10^{-9} mW. The ratio of $RxThresh_mwatts$ and the cumulative maximum received power is 6.4052 which is above the required SIR threshold of 3.7844. Second, it checks for cumulative SIR at $l_{21,15}$. The maximum power received at $l_{21,15}$ from $l_{6,5}$ is 1.9297×10^{-9} mW and from $l_{30,35}$ is 2.4653×10^{-9} mW, totaling 4.3949×10^{-9} mW. The ratio of $RxThresh_mwatts$ and the cumulative maximum received power is 2.8645. This ratio is below the required SIR threshold, so vertex 29 ($l_{30,35}$) cannot become a member of the current WMaIS. Note that the algorithm would have checked for the cumulative SIR at vertex 29 ($l_{30,35}$) if the cumulative SIR at vertex 20 ($l_{21,15}$) had been above the required SIR threshold.

Lastly the algorithm places vertex 6 ($l_{6,5}$), vertex 20 ($l_{21,15}$), and vertex 35 ($l_{36,35}$) in a temporary set and checks the cumulative SIR at every vertex. First, it checks for cumulative SIR at $l_{6,5}$. The maximum power received at $l_{6,5}$ from $l_{21,15}$ is 7.6919×10^{-10} mW and from $l_{36,35}$ is 1.0131×10^{-9} mW, totaling 1.7823×10^{-9} mW. The ratio of $RxThresh_mwatts$ and the cumulative maximum received power is 7.0636 which is above

the required SIR threshold of 3.7844. Second, it checks for cumulative SIR at $l_{21,15}$. The maximum power received at $l_{21,15}$ from $l_{6,5}$ is 1.9297×10^{-9} mW and from $l_{36,35}$ is 2.8498×10^{-9} mW, totaling 4.7795×10^{-9} mW. The ratio of $RxThresh_mwatts$ and the cumulative maximum received power is 2.6340. Since this ratio is below the required SIR threshold, vertex 35 ($l_{36,35}$) cannot become a member of the current WMaIS. In this way, the algorithm constructs the first WMaIS as $V_{WMaIS} = \{6, 20\}$. This means that the links $l_{6,5}$ and $l_{21,15}$ in L can be active simultaneously and are assigned the same frequency channel.

7.4. Exact QCBP Formulation

The true minimum number of frequency channels can be found by formulating the extended coloring problem as a quadratically constrained binary program. The standard binary linear programming formulation for the basic coloring problem is well-studied [85][86][87][88]. The QCBP formulation for the extended coloring problem follows below.

Given the conflict graph F , this work defines the following parameters and variables:

- n_c is an input parameter which represents the maximum number of colors allowed;
- x_{ih} is a binary variable such that $x_{ih} = 1$ when vertex i is assigned the color h and 0 otherwise;
- y_h is a binary variable such that $y_h = 1$ when color h is used in the solution and 0 otherwise;
- $RxThresh_mwatts$ is an input parameter which represents the power threshold required at the receiver in milliwatts to correctly receive the desired incoming signal;
- $SIRThresh$ is an input parameter which represents the SIR threshold (i.e. the SIR requirement for a particular link data rate) in linear scale; and
- P_{ik} is an input parameter which represents the maximum power received in milliwatts at vertex i from vertex k and is an element of the conflict matrix at i^{th} row and k^{th} column.

Inputs:

- L
- $F(V_F, E_F)$: conflict graph
- $SIRThresh_dB$
- $RxThresh_dBm$

Outputs:

- V_{WMaIS} : WMaIS of vertices
- L_{WMaIS} : Set of links in L corresponding to vertices in WMaIS
- c_{WMaIS} : number of WMaISs

BEGIN

```

1.  $c_{WMaIS} \leftarrow 0$ 
2.  $m \leftarrow |L| = |V_F|$ 
3.  $SIRThresh \leftarrow 10^{(SIRThresh\_dB/10)}$ 
4.  $RxThresh\_mwatts \leftarrow 10^{(RxThresh\_dBm/10)}$ 
5. While  $m \geq 1$ :
6.   If  $F$  is fully connected or  $m = 1$ :
7.     For  $d = 1$  to  $m$ :
8.       Initialize an empty  $V_{WMaIS}$  and  $L_{WMaIS}$ 
9.       Add vertex  $d$  to  $V_{WMaIS}$  and the corresponding link to  $L_{WMaIS}$ 
10.      Output  $V_{WMaIS}$  and  $L_{WMaIS}$ 
11.       $c_{WMaIS} \leftarrow c_{WMaIS} + 1$ 
12.    end For
13.    Output  $c_{WMaIS}$  and exit.
14.  end If
15.  Initialize an empty  $V_{WMaIS}$  and  $L_{WMaIS}$ 
16.  Add vertex of  $F$  with the maximum node-degree to  $V_{WMaIS}$  and the corresponding link
    to  $L_{WMaIS}$ 
17.  For  $i = 1$  to  $m$ :
18.    If vertex  $i$  does not have an edge with vertices in  $V_{WMaIS}$ :
19.       $V_{temp} \leftarrow V_{WMaIS} \cup i$ 
20.    end If
21.    If the cumulative SIR at every vertex in  $V_{temp}$  is greater than  $SIRThresh$ :
22.      Add vertex  $i$  to  $V_{WMaIS}$  and the corresponding link to  $L_{WMaIS}$ 
23.    end If
24.  end For
25.  Output  $V_{WMaIS}$  and  $L_{WMaIS}$ 
26.   $c_{WMaIS} \leftarrow c_{WMaIS} + 1$ 
27.   $V_F \leftarrow V_F \setminus V_{WMaIS}$ 
28.   $L \leftarrow L \setminus L_{WMaIS}$ 
29.   $m \leftarrow |L|$ 
30. end While
31. Output  $c_{WMaIS}$ 
END

```

Algorithm 7.1. Enhanced Maximum Node-Degree Start

Objective

Given a limit of n_c colors, the objective is to minimize the number of colors used:

$$\min \sum_{h=1}^{n_c} y_h \quad (7.1)$$

Constraints

Each vertex is assigned exactly one color:

$$\sum_{h=1}^{n_c} x_{ih} = 1 \quad \text{for all } i \in V. \quad (7.2)$$

For an arc E_{ij} connecting vertices i and j , V_i and V_j cannot be assigned the same color:

$$x_{ih} + x_{jh} \leq y_h \quad \text{for all } h \quad (7.3)$$

where $E_{ij} \in E$, $h = 1 \dots n_c$.

It is a binary decision as to whether to assign color h to vertex i :

$$x_{ih} \in \{0,1\} \quad (7.4)$$

where $i \in V$, $h = 1 \dots n_c$.

It is a binary decision as to whether color h is used:

$$y_h \in \{0,1\} \quad (7.5)$$

where $h = 1 \dots n_c$.

This formulation allows many equivalent solutions because colors are interchangeable. If a solution uses k out of the n_c available colors, then there are $\binom{n_c}{k}$ possible combinations of k colors. Once the k colors are chosen, they can be permuted in $k!$ ways. For a solution consisting of k colors, there are $\binom{n_c}{k} k!$ equivalent solutions [86].

The following constraints are used in [87][88] to eliminate some of the equivalent solutions in a standard minimum coloring problem, thereby reducing the binary program's search space by ensuring that color $h + 1$ is used only if color h has already been assigned. This eliminates those symmetrical k -colorings for any feasible k -coloring that use colors with labels higher than k .

$$y_h \leq \sum_i x_{ih} \quad \text{for all } h \quad (7.6)$$

where $i \in V, h = 1 \dots n_c$.

$$y_h \geq y_{h+1} \quad \text{for all } h = 1 \dots n_c - 1. \quad (7.7)$$

This work extends model (7.1)–(7.7) to include the *cumulative interference constraints*. The term $\sum_k P_{ik} x_{kh} x_{ih}$ sums up the maximum powers received at vertex i when it is assigned the color h from all other vertices which are also assigned the color h . The following constraints ensure that the cumulative SIR at every vertex sharing the same frequency channel (color) is greater than the SIR threshold where the cumulative SIR is the ratio of the receiver threshold to the sum of the maximum powers received at that vertex from all other vertices using the same frequency, denoted here by the color h .

$$\left(\frac{RxThresh_mwatts}{\sum_k P_{ik} x_{kh} x_{ih}} \right) \geq SIRThresh \quad \text{for all } i \text{ and for all } h \quad (7.8)$$

where $k \neq i$, and $i, k \in V$.

Constraints (7.8) can be re-written as:

$$\left(\frac{\sum_k P_{ik} x_{kh} x_{ih}}{RxThresh_mwatts} \right) \leq \frac{1}{SIRThresh} \quad \text{for all } i \text{ and for all } h \quad (7.9)$$

Note that the introduction of these constraints to the model (7.1)–(7.7) transforms it into a QCBP. This extended coloring problem is much harder to solve as compared to the standard minimum coloring problem due to these quadratic constraints. IBM's commercial solver CPLEX is used to solve the QCBP formulation of the extended coloring problem due to its ability to handle quadratically constrained programs (QCPs).

8. BEAMFORMING-BASED CHANNEL ASSIGNMENT

In a classical multi-radio multi-channel wireless mesh network architecture, mesh nodes are equipped with omni-directional antennas. Due to the circular radiation pattern of such antennas, when a mesh node communicates with its neighbor on a certain frequency channel, other mesh nodes within its circular range must remain silent. Directional antennas have been proposed as a way to improve spatial reuse. Since these antennas are non-steerable, they are not suitable for a dynamic WMN. In this chapter, the problem of co-channel interference in a dynamic WMN environment is addressed by proposing a novel *Linear Array Beamforming-based Channel Assignment* (LAB-CA) method [89][90] that uses beamforming to reduce the number of frequency channels required for interference-free communication among the mesh nodes. Then, an extended beamforming-based channel assignment framework is developed to incorporate heterogeneous mesh nodes [91].

8.1. Introduction

Channel assignment schemes for MRMC WMNs typically assume a multi-radio mesh node to be equipped with multiple omni-directional antennas. Using these, a mesh node can communicate with several neighbors simultaneously on different frequency channels. When a mesh node is communicating with a neighbor using a certain frequency channel, other mesh nodes within its circular range must not transmit on the same frequency channel due to the circular radiation pattern of its omni-directional antennas. This eliminates co-channel interference and enables successful communication between the two nodes.

Directional antennas reduce co-channel interference and enhance spatial channel reuse, but these are non-steerable and always point in the same direction. Hence they are less useful in a dynamic WMN environment that undergoes regular topological changes as new mesh nodes join the network or existing mesh nodes leave it (e.g. due to node failure). This work addresses the problem of co-channel interference in a dynamic MRMC WMN while minimizing the number of frequency channels required for interference-free communication among the mesh nodes. It improves the spatial channel

reuse via beamforming by using the multiple omni-directional antennas of a multi-radio mesh node in the form of a linear antenna array.

Unlike the non-steerable beam of a directional antenna, the main lobe or main beam of a linear antenna array can be pointed in any desired direction by controlling the progressive phase difference between the antenna elements of the array. The steerable beam pattern provided by the linear antenna array of a mesh node can be used in the operation of a self-organizing and dynamic WMN. Through beamforming, the novel linear array beamforming-based channel assignment method [89][90] reduces the number of frequency channels required for interference-free communication among the mesh nodes in a dynamic MRMC WMN. In fact, the results show that LAB-CA significantly outperforms classical omni-directional antenna pattern-based channel assignment OAP-CA in terms of the number of frequency channels for all node-degrees at all link data rates.

A MRMC WMN consists of mesh nodes that are equipped with multiple radio interfaces. The node-degree of a mesh node is the number of radio interfaces that it possesses for data communication with its neighbors. In a MRMC WMN architecture consisting of *homogeneous mesh nodes*, the node-degree of all mesh nodes in the network is the same. This work also explores a MRMC WMN consisting of *heterogeneous mesh nodes*, i.e. mesh nodes having different node-degrees, in order to model a more realistic WMN architecture. It extends the beamforming-based channel assignment framework to incorporate heterogeneous mesh nodes. The performance of linear array beamforming-based CA method for heterogeneous mesh nodes LAB-CA_HT is compared with linear array beamforming-based CA method for homogeneous mesh nodes LAB-CA_HG. It is found that LAB-CA_HT requires more frequency channels as compared to LAB-CA_HG [91].

The performance of LAB-CA_HT is evaluated in both sparse and dense wireless mesh networks [91]. This is done by comparing its performance with the omni-directional antenna pattern-based channel assignment method for heterogeneous mesh nodes OAP-CA_HT in both scenarios. The results show that LAB-CA_HT outperforms OAP-CA_HT in terms of number of frequency channels in sparse as well as dense mesh

networks. It performs even better in dense mesh networks and provides a reduction of up to 69% in the number of frequency channels.

This work also explores the impact of increasing the number of antennas in the linear antenna arrays of the multi-radio mesh nodes on the number of frequency channels in a dense MRMCA WMN [91]. It is found that LAB-CA_HT provides a further reduction of up to 54% in the number of frequency channels when the number of antennas in the linear antenna arrays of mesh nodes exceeds their node-degree by six.

8.2. Network Architecture

For LAB-CA, it is assumed that the omni-directional antennas of the data radios of a mesh node can be arranged in the form a linear antenna array for beamforming. Using multiple beams equal to the number of its multiple omni-directional antennas in its array, a mesh node can communicate with multiple neighbors simultaneously. For example, a mesh node with a node-degree of four has four data radios and four antennas in its array. Using all four antennas, the mesh node can form up to four beams to communicate with up to four neighbors simultaneously.

In a homogeneous WMN, the node-degrees of all mesh nodes in the network are the same. In a heterogeneous WMN, mesh nodes have different node-degrees chosen using a uniform random distribution between two and some upper limit. The GW is the sink of all flows and hence the maximum possible network throughput depends on its node-degree. For a fair comparison of LAB-CA_HG and LAB-CA_HT, this work assumes that the GW has the highest node-degree in the heterogeneous architecture. For example, for a heterogeneous mesh network where the node-degree of mesh nodes is randomly selected between two and four, the node-degree of the GW is set at four.

In LAB-CA_HG, when the node-degree of all mesh nodes is six and a mesh node needs to communicate with two neighbors simultaneously, it uses all antennas of its six-antenna linear array to form two beams directed towards the two neighbors. On the other hand, in LAB-CA_HT, when the node-degree of a mesh node is randomly selected between two and six, and a mesh node is assigned a node-degree of three and that mesh node needs to communicate with two neighbors simultaneously, then it uses all antennas of its three-antenna linear array to form two beams directed towards its two neighbors.

The beam formed by a six-antenna linear array is more focused than that formed by a three-antenna linear array. This concept is illustrated using Fig. 8.3 in Section 8.6.

8.3. Fundamentals of Linear Antenna Array Design

Each mesh node is equipped with multiple radio interfaces and each radio interface is equipped with an omni-directional antenna. Note that a vertically polarized half-wavelength dipole antenna is an omni-directional antenna in the xy -plane or the ϕ -plane, i.e. at elevation angle $\theta = 90^\circ$. The multiple antennas of a mesh node constitute a linear antenna array which is based on the following assumptions:

- The array consists of N_a elements equally spaced at distance d_a apart.
- The antenna elements are positioned along the x -axis, as shown in Fig. 8.1.

Figure 8.1. Linear antenna array along x -axis

- The distance d_a between the elements in the array is $\lambda/2$.
- All elements have identical amplitudes of the excitation current.
- The phase β by which the current in each element leads the current of the preceding element is the same.
- The elements in the array are half-wavelength dipoles.

- The first element of the array is located at the origin.
- Far-field conditions are assumed such that $r \gg d_a$.
- The antenna elements of the array are assumed to be lossless.
- There is no mutual coupling between antenna elements of the array.
- The height of the linear antenna array of a mesh node from the ground is the same for all nodes.
- The antenna elements are vertically polarized, which means that the antenna elements of the array of a mesh node are oriented parallel to the z -axis.

The array factor AF of the linear antenna array described above and shown in Fig. 8.1 is given by [92]

$$AF(\theta, \phi) = 1 + e^{j(kd_a \sin\theta \cos\phi + \beta)} + e^{j2(kd_a \sin\theta \cos\phi + \beta)} + \dots + e^{j(N_a - 1)(kd_a \sin\theta \cos\phi + \beta)} \quad (8.1)$$

where $k = 2\pi/\lambda$. As stated earlier, the height of the linear array of a mesh node is the same for all nodes. So, a mesh node needs to steer the multiple beams of its linear array in the xy -plane (ϕ -plane) only to communicate with its multiple neighbors. Hence it is required to find the array factor only in the ϕ -plane. Therefore, AF reduces to

$$AF(\theta = \pi/2, \phi) = 1 + e^{j(kd_a \cos\phi + \beta)} + e^{j2(kd_a \cos\phi + \beta)} + \dots + e^{j(N_a - 1)(kd_a \cos\phi + \beta)} \quad (8.2)$$

For phased array [92],

$$\beta = -kd_a \cos\phi_o \quad (8.3)$$

From (8.2) and (8.3), AF can be re-written as

$$AF(\theta = \pi/2, \phi) = 1 + e^{jkd_a(\cos\phi - \cos\phi_o)} + e^{j2kd_a(\cos\phi - \cos\phi_o)} + \dots + e^{j(N_a - 1)kd_a(\cos\phi - \cos\phi_o)} \quad (8.4)$$

By controlling the progressive phase difference between the elements by controlling ϕ_o , the maximum radiation of the beam or the main-lobe can be pointed in any desired direction.

The element radiation pattern, also known as element pattern EP , of this linear antenna array which consists of half-wavelength dipoles is given by [92]

$$EP(\theta, \phi) = \cos((\pi/2) \times \sin \theta) / \cos \theta \quad (8.5)$$

At elevation angle $\theta = 90^\circ$, EP reduces to unity in the xy -plane or the ϕ -plane.

$$EP(\theta = \pi/2, \phi) = \cos((\pi/2) \times \sin(\pi/2)) / \cos(\pi/2) = 1 \quad (8.6)$$

The maximum directivity of a half-wavelength dipole antenna is given by [92]

$$D_o = 4\pi(U_{max}/P_{rad}) \cong 1.643 \quad (8.7)$$

The maximum gain of an antenna in terms of maximum directivity is given by [92]

$$G_o = e_{cd} D_o \quad (8.8)$$

where e_{cd} is the antenna radiation efficiency. Since the antenna elements of the array are assumed to be lossless, therefore,

$$G_o = D_o = 1.643 \quad (8.9)$$

which is the maximum gain of a half-wavelength dipole antenna over an isotropic source.

Provided there is absolutely no mutual coupling between the antenna elements of a linear array, all the elements have identical patterns and are excited by equal amplitudes of the excitation current, the array gain AG is given by [93]

$$AG(\theta, \phi) = N_a G_e(\theta, \phi) \quad (8.10)$$

where $G_e(\theta, \phi)$ is the gain realized when only one element is excited. At elevation angle $\theta = 90^\circ$, the maximum AG is

$$AG(\theta = \pi/2, \phi)_{max} = N_a G_e(\theta = \pi/2, \phi)_{max} = N_a G_o \quad (8.11)$$

where G_o is the maximum gain of a half-wavelength dipole.

The normalized array gain of a linear antenna array is given by [94]

$$AG(\theta, \phi)_{norm} = |EP(\theta, \phi)|^2 |AF(\theta, \phi)|^2 \quad (8.12)$$

Note that

$$|AF(\theta = \pi/2, \phi)|_{max}^2 = N_a^2 \quad (8.13)$$

From (8.11), (8.12) and (8.13), the generalized form of array gain of this linear antenna array in the ϕ -plane can be written as

$$AG(\theta = \pi/2, \phi) = \frac{1}{N_a} G_0 |EP(\theta = \pi/2, \phi)|^2 |AF(\theta = \pi/2, \phi)|^2 \quad (8.14)$$

Since EP reduces to unity in the ϕ -plane, so AG reduces to

$$AG(\theta = \pi/2, \phi) = \frac{1}{N_a} G_0 |AF(\theta = \pi/2, \phi)|^2 \quad (8.15)$$

Note that the maximum value of AG in (8.15) reduces to (8.11).

Fig. 8.2 shows a plot of $|AF(\theta = \pi/2, \phi)|^2$ and $|AG(\theta = \pi/2, \phi)|$ for different values of ϕ_o when $N_a = 2$. Note that the maximum value of $|AF(\theta = \pi/2, \phi)|^2$ is equal to N_a^2 . It should also be noted that the maximum value of $|AG(\theta = \pi/2, \phi)|$ remains constant for different values of ϕ_o , is equal to N_a times G_o and is independent of the element pattern.

Figure 8.2. Plot of $|AF(\theta = \pi/2, \phi)|^2$ and $|AG(\theta = \pi/2, \phi)|$ for different values of ϕ_o ($N_a = 2$)

8.4. Conflict Graph and Matrix for LAB-CA

To construct the conflict graph and matrix for LAB-CA using the SIR model with shadowing, the method in Algorithm 6.1 is extended to include beamforming. The pseudo-code of this beamforming version of the conflict graph and matrix for LAB-CA is given in Algorithm 8.1. Its inputs include the number of antenna elements in the linear antenna array N_a , and the distance between the elements of the array d_a , in addition to those in the pseudo-code in Algorithm 6.1.

To determine the array gain AG_{ab} from node a to b using (8.15), the angle ϕ_{ab} between mesh nodes a and b is calculated, and ϕ and ϕ_o are set equal to ϕ_{ab} . Due to reciprocity, AG_{ab} is the same as AG_{ba} .

The algorithm considers two links l_{xy} and l_{pq} and checks if l_{pq} conflicts with l_{xy} as follows. It calculates $P_{r,xp}$ (power received at node x from node p), $P_{r,xq}$ (power received at node x from node q), $P_{r,yp}$ (power received at node y from node p), and $P_{r,yq}$ (power received at node y from node q) using (6.2) or (6.4). To calculate $P_{r,xp}$, it calculates $P_{t,pq}$ using (6.5) or (6.6) which is the transmission power required when node p is transmitting to q ; array gain AG_{px} from node p to x ; and array gain AG_{xp} from node x to p . To calculate AG_{xp} using (8.15), it calculates ϕ_{xp} , the angle between node x and p ; ϕ_{xy} , the angle between node x and y ; and substitutes them for ϕ and ϕ_o , respectively.

Consider the following example which explains the procedure of building the conflict table and matrix using Algorithm 8.1 for LAB-CA. The procedure of building the conflict table and matrix based on the SIR model with shadowing for OAP-CA is also given for comparison.

For this example, the same set of links involved in routing L is considered that is shown in Table 6.1. The corresponding routing graph for this topology, which is topology #1, is shown in Fig. 6.1 when the node-degree constraint is 3 and the link data rate is 12 Mbps. For link data rate of 12 Mbps, the value of $RxThresh_dBm$ is -79 dBm and that of $RxThresh_mwatts$ is 1.2589×10^{-8} mW; also, the value of $SIRThresh_dB$ is 5.78 dB and that of $SIRThresh$ is 3.7844. The outage probability OP is 0.1 and standard deviation for shadowing σ is 3 dB. Since the node-degree constraint is 3, so the number of antenna elements in the linear antenna array of mesh nodes N_a is 3 for LAB-CA.

Inputs:

- $f, G_t, G_r, h_t, h_r, n, RxThresh_dBm, SIRThresh_dB, L, locations_x, locations_y, \sigma, OP, N_a, d_a$.

Output:

- *conflict_matrix*: Conflict graph

BEGIN

```

1.  $RxThresh\_mwatts \leftarrow 10^{(RxThresh\_dBm/10)}, m \leftarrow |L|, SIRThresh \leftarrow 10^{(SIRThresh\_dB/10)}$ 
2. For  $i = 1$  to  $n$ :
3.   For  $j = 1$  to  $n$ :
4.      $dist\_all(i,j) \leftarrow$  distance between node  $i$  and node  $j$ 
5.   end For
6. end For
7. For  $i = 1$  to  $m$ :
8.    $L(i,3) \leftarrow$  distance between the nodes of link  $i$ 
9.    $L(i,4) \leftarrow AG_{ab} \leftarrow$  array gain of  $beam_{ab}$  for nodes  $a$  and  $b$  of link  $i$  using (8.15)
10.   $L(i,5) \leftarrow P_{t,ab}$  for link  $i$  with  $G_t$  and  $G_r$  equal to  $AG_{ab}$  using (6.5) or (6.6)
11. end For
12. Initialize an  $m \times m$  conflict_matrix of all ones
13. For  $il = 1$  to  $m$ :
14.   Initialize an empty conflict table
15.   For  $j = 1$  to  $m$ :
16.      $conflict\_table(j,1) \leftarrow$  node  $x$  of link  $il$ 
17.      $conflict\_table(j,2) \leftarrow$  node  $y$  of link  $il$ 
18.      $conflict\_table(j,3) \leftarrow$  node  $p$  of link  $j$ 
19.      $conflict\_table(j,4) \leftarrow$  node  $q$  of link  $j$ 
20.      $AG_{xp} \leftarrow$  array gain of  $beam_{xy}$  with  $\phi = \phi_{xp}$  and  $\phi_o = \phi_{xy}$  using (8.15)
21.      $AG_{px} \leftarrow$  array gain of  $beam_{pq}$  with  $\phi = \phi_{px}$  and  $\phi_o = \phi_{pq}$  using (8.15)
22.      $P_{r,xp} \leftarrow$  power received at  $x$  from  $p$  with  $G_t = AG_{px}, G_r = AG_{xp}$  and  $P_{t,pq} = L(j,5)$  using (6.2) or (6.4)
23.      $conflict\_table(j,5) \leftarrow P_{r,xp}$ 
24.      $AG_{xq} \leftarrow$  array gain of  $beam_{xy}$  with  $\phi = \phi_{xq}$  and  $\phi_o = \phi_{xy}$  using (8.15)
25.      $AG_{qx} \leftarrow$  array gain of  $beam_{qp}$  with  $\phi = \phi_{qx}$  and  $\phi_o = \phi_{qp}$  using (8.15)
26.      $P_{r,xq} \leftarrow$  power received at  $x$  from  $q$  with  $G_t = AG_{qx}, G_r = AG_{xq}$  and  $P_{t,qp} = L(j,5)$  using (6.2) or (6.4)
27.      $conflict\_table(j,6) \leftarrow P_{r,xq}$ 
28.      $AG_{yp} \leftarrow$  array gain of  $beam_{yx}$  with  $\phi = \phi_{yp}$  and  $\phi_o = \phi_{yx}$  using (8.15)
29.      $AG_{py} \leftarrow$  array gain of  $beam_{pq}$  with  $\phi = \phi_{py}$  and  $\phi_o = \phi_{pq}$  using (8.15)
30.      $P_{r,yp} \leftarrow$  power received at  $y$  from  $p$  with  $G_t = AG_{py}, G_r = AG_{yp}$  and  $P_{t,pq} = L(j,5)$  using (6.2) or (6.4)
31.      $conflict\_table(j,7) \leftarrow P_{r,yp}$ 
32.      $AG_{yq} \leftarrow$  array gain of  $beam_{yx}$  with  $\phi = \phi_{yq}$  and  $\phi_o = \phi_{yx}$  using (8.15)
33.      $AG_{qy} \leftarrow$  array gain of  $beam_{qp}$  with  $\phi = \phi_{qy}$  and  $\phi_o = \phi_{qp}$  using (8.15)
34.      $P_{r,yq} \leftarrow$  power received at  $y$  from  $q$  with  $G_t = AG_{qy}, G_r = AG_{yq}$  and  $P_{t,qp} = L(j,5)$  using (6.2) or (6.4)
35.      $conflict\_table(j,8) \leftarrow P_{r,yq}$ 
36.      $conflict\_table(j,9) \leftarrow \max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
37.      $conflict\_table(j,10) \leftarrow RxThresh\_mwatts / \max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
38.   end For

```

```

39.   For  $i2 = 1$  to  $m$ :
40.       If  $\text{conflict\_table}(i2,10) > \text{SIRThresh}$ :
41.            $\text{conflict\_matrix}(i1,i2) \leftarrow \text{conflict\_table}(i2,9)$ 
42.       end If
43.   end For
44. end For
45. Output  $\text{conflict\_matrix}$ 
END

```

Algorithm 8.1. Conflict Graph and Matrix for LAB-CA

When $i1$ is equal to 3 and j is equal to 14, the entries in the conflict table for OAP-CA are as follows:

$\text{conflict_table}(14,1) = 3$
 $\text{conflict_table}(14,2) = 1$
 $\text{conflict_table}(14,3) = 14$
 $\text{conflict_table}(14,4) = 15$
 $\text{conflict_table}(14,5) = 3.6829 \times 10^{-9} \text{ mW}$
 $\text{conflict_table}(14,6) = 4.4225 \times 10^{-9} \text{ mW}$
 $\text{conflict_table}(14,7) = 4.5819 \times 10^{-9} \text{ mW}$
 $\text{conflict_table}(14,8) = 3.6404 \times 10^{-9} \text{ mW}$
 $\text{conflict_table}(14,9) = 4.5819 \times 10^{-9} \text{ mW}$
 $\text{conflict_table}(14,10) = 2.7476$

The relevant entry in the conflict matrix when $i2$ is equal to 14 is

$\text{conflict_matrix}(3,14) = 1$,

since the ratio of the $RxThresh_mwatts$ and maximum power received at $l_{3,1}$ from $l_{14,15}$ is 2.7476, which is less than the required SIR threshold of 3.7844.

The corresponding entries in the conflict table for LAB-CA are as follows when $i1$ is equal to 3 and j is equal to 14:

$\text{conflict_table}(14,1) = 3$
 $\text{conflict_table}(14,2) = 1$
 $\text{conflict_table}(14,3) = 14$
 $\text{conflict_table}(14,4) = 15$

$$\phi = \phi_{3,14} = 341.76^\circ = 5.9648 \text{ radians}$$

$\phi_o = \phi_{3,1} = 278.61^\circ = 4.8628 \text{ radians}$ (Note that the knowledge of node locations is available at the gateway which is used while calculating ϕ and ϕ_o . One can recall from the topology control stage that each node is assumed to know its location. After building its MPNT, each node sends its MPNT along with its position, and node ID to the GW over the control frequency channel using its control radio.)

$$AG_{3,14} = 0.2091$$

$$\phi = \phi_{14,3} = 161.76^\circ = 2.8233 \text{ radians}$$

$$\phi_o = \phi_{14,15} = 96.24^\circ = 1.6797 \text{ radians}$$

$$AG_{14,3} = 0.3128$$

$conflict_table(14,5) = P_{r,3,14} = 9.9130 \times 10^{-12} \text{ mW}$ (which is the power received at node 3 from node 14 when: node 3 is steering its main beam in the direction of node 1, and node 14 is steering its main beam in the direction of node 15)

$$\phi = \phi_{3,15} = 2.03^\circ = 0.0355 \text{ radians}$$

$$\phi_o = \phi_{3,1} = 278.61^\circ = 4.8628 \text{ radians}$$

$$AG_{3,15} = 0.3339$$

$$\phi = \phi_{15,3} = 182.03^\circ = 3.1771 \text{ radians}$$

$$\phi_o = \phi_{15,14} = 276.24^\circ = 4.8213 \text{ radians}$$

$$AG_{15,3} = 0.4297$$

$conflict_table(14,6) = P_{r,3,15} = 2.6114 \times 10^{-11} \text{ mW}$ (which is the power received at node 3 from node 15 when: node 3 is steering its main beam in the direction of node 1, and node 15 is steering its main beam in the direction of node 14)

$$\phi = \phi_{1,14} = 14.28^\circ = 0.2494 \text{ radians}$$

$$\phi_o = \phi_{1,3} = 98.61^\circ = 1.7212 \text{ radians}$$

$$AG_{1,14} = 0.4071$$

$$\phi = \phi_{14,1} = 194.28^\circ = 3.3910 \text{ radians}$$

$$\phi_o = \phi_{14,15} = 96.24^\circ = 1.6797 \text{ radians}$$

$$AG_{14,1} = 0.3598$$

$conflict_table(14,7) = P_{r,l,14} = 2.7625 \times 10^{-11}$ mW (which is the power received at node 1 from node 14 when: node 1 is steering its main beam in the direction of node 3, and node 14 is steering its main beam in the direction of node 15)

$$\phi = \phi_{l,15} = 34.28^\circ = 0.5984 \text{ radians}$$

$$\phi_o = \phi_{l,3} = 98.61^\circ = 1.7212 \text{ radians}$$

$$AG_{l,15} = 0.5415$$

$$\phi = \phi_{l5,l} = 214.28^\circ = 3.7400 \text{ radians}$$

$$\phi_o = \phi_{l5,14} = 276.24^\circ = 4.8213 \text{ radians}$$

$$AG_{l5,l} = 0.5030$$

$conflict_table(14,8) = P_{r,l,15} = 4.0814 \times 10^{-11}$ mW (which is the power received at node 1 from node 15 when: node 1 is steering its main beam in the direction of node 3, and node 15 is steering its main beam in the direction of node 14)

$conflict_table(14,9) = 4.0814 \times 10^{-11}$ mW (which is the maximum power received at $l_{3,l}$ from $l_{14,15}$)

$conflict_table(14,10) = 308.4549$ (which is the ratio of the $RxThresh_mwatts$ and maximum power received at $l_{3,l}$ from $l_{14,15}$)

The relevant entry in the conflict matrix when $i2$ is equal to 14 is

$$conflict_matrix(3,14) = 4.0814 \times 10^{-11} \text{ mW},$$

since the ratio of the $RxThresh_mwatts$ and maximum power received at $l_{3,l}$ from $l_{14,15}$ is 308.4549, which is much greater than the required SIR threshold of 3.7844.

Note that the ratio of $RxThresh_mwatts$ and maximum power received at $l_{3,l}$ from $l_{14,15}$ was less than the required SIR threshold in case of OAP-CA. This led to an entry of one in the (3,14) element of the conflict matrix indicating a conflict which means that these two links could not be simultaneously active over the same frequency channel when using OAP-CA.

On the other hand, when using Algorithm 8.1 in case of LAB-CA, the ratio of $RxThresh_mwatts$ and maximum power received at $l_{3,l}$ from $l_{14,15}$ is found out to be much greater than the required SIR threshold. This indicates that these two links could be active simultaneously over the same frequency channel and could potentially become members of the same WMaIS. So the maximum of the four received powers at $l_{3,l}$ from $l_{14,15}$ is

placed in the (3,14) element of the conflict matrix. This information is used later while building weighted maximal independent sets during the minimum coloring stage.

The use of beamforming in Algorithm 8.1 while building the conflict table and matrix leads to a highly improved spatial reuse in case of LAB-CA as compared to that for OAP-CA. This leads to a reduced number of conflicts as well as lower values of maximum received powers in the conflict matrix, which result in significantly fewer WMaISs during the minimum coloring stage and a significant reduction in the number of frequency channels when using LAB-CA.

8.5. Linear Array Beamforming-based Channel Assignment

A novel channel assignment method, LAB-CA, is developed that uses the beamforming-based conflict graph and matrix presented in Section 8.4. LAB-CA uses beamforming by utilizing the multiple omni-directional antennas of a multi-radio mesh node in the form of a linear antenna array and reduces the number of frequency channels required for interference-free communication among the mesh nodes in a dynamic MRMC WMN. The steerable beam pattern provided by the linear antenna array in LAB-CA can be used in the operation of a self-organizing and dynamic WMN. It is compared to the classical omni-directional antenna pattern-based channel assignment method. It significantly outperforms OAP-CA in terms of number of frequency channels.

Like the channel assignment methods presented earlier in this work, LAB-CA and OAP-CA have four stages, namely, connectivity graph, routing, interference modeling and minimum coloring. The network architecture used for OAP-CA is the same as presented in Section 5.2 while the one used for LAB-CA is presented in Section 8.2.

The *Select x for less than x* TCA is used to build the connectivity graph as it helps to minimize the number of frequency channels by mitigating co-channel interference and enhancing spatial reuse through the use of topology control. It controls the network topology by selecting the nearest neighbors for each node in the network. Given the topology-controlled connectivity graph, multi-path routing is used to maximize the network throughput while maintaining fairness among the multiple network flows.

For OAP-CA, Algorithm 6.1 is used to construct the conflict graph and matrix based on the SIR model with shadowing for realistic interference-free channel assignment in MRMC WMNs. For LAB-CA, Algorithm 8.1 is used to construct the

beamforming-based conflict graph and matrix. The randomized heuristic algorithms presented in Chapter 7 are used to find WMaISs from the conflict graphs and matrices returned by OAP-CA and LAB-CA. Using these heuristics to find WMaISs, the overall greedy minimum coloring heuristic is very quick, so it is run multiple times. Each of the three algorithm variants, namely, Enhanced Maximum Node-Degree Start, Enhanced Minimum Node-Degree Start, and Enhanced Random Start, is run 25 times on the conflict graph and the best solution (i.e. requiring the fewest frequency channels) is taken over all 75 runs.

8.6. Channel Assignment for Beamformed Heterogeneous WMNs

To model a more realistic WMN architecture, the beamforming-based channel assignment framework presented in Section 8.5 is extended to incorporate heterogeneous mesh nodes as follows.

First, the *Select x for less than x* TCA is extended so that it is able to accommodate heterogeneous mesh nodes. In the extended algorithm, each mesh node sends its node-degree information to the GW. Given a node-degree of x for a mesh node, the GW selects at least x nearest nodes as neighbors for that mesh node. The multi-path routing formulation presented in Section 5.4.1 is also extended to accommodate heterogeneous mesh nodes. All other constraints in the existing formulation remain the same except for the node-degree constraint. Finally, the method for constructing a beamforming-based conflict graph and matrix, i.e. Algorithm 8.1, is extended to accommodate heterogeneous mesh nodes. In addition to the inputs for Algorithm 8.1, the inputs for the extended beamforming-based conflict graph and matrix for heterogeneous mesh nodes include the set of node-degrees assigned to the mesh nodes.

The new channel assignment method using the extended channel assignment framework for heterogeneous mesh nodes is referred to as linear array beamforming-based CA method for heterogeneous mesh nodes LAB-CA_HT. The performance of LAB-CA_HT is evaluated in comparison with LAB-CA_HG (earlier referred to as LAB-CA in this work). It is found that LAB-CA_HT requires more frequency channels as compared to LAB-CA_HG.

8.6.1. Extended *Select x for less than x* TCA

The *Select x for less than x* TCA is extended to build the connectivity graph for LAB-CA_HT.

In the extended algorithm, each mesh node broadcasts a *Hello* message at maximum transmission power containing its node ID and position over the control channel using the control radio. From the information in the received *Hello* messages, each mesh node arranges its neighboring nodes in ascending order of their distance which results in its maximum power neighbor table. Each mesh node then sends its MPNT along with its position, its node-degree x and its node ID to the GW over the control channel using its control radio. For each node in the network, the GW runs the *extended Select x for less than x* TCA. Based on the node-degree x of a mesh node, the GW builds its direct neighbor table by selecting at least x nearest nodes as communication neighbors for that node. If required, the GW then converts some uni-directional links in the DNTs of mesh nodes into bi-directional links, which results in their final neighbor table.

For a mesh node that is assigned a node-degree of 2 or 3, the GW ensures at least 3 nearest nodes as its neighbors by using the *extended Select 3 for less than 3* TCA; for a mesh node that is assigned a node-degree of 4, the GW ensures at least 4 nearest nodes as its neighbors by using the *extended Select 4 for less than 4* TCA; and so on. Like *Select 2 for less than 2* TCA, the *extended Select 2 for less than 2* TCA mostly leads to a disconnected network in the case of random and controlled random topologies; so the *extended Select 3 for less than 3* TCA is used for a mesh node that has been assigned a node-degree of 2.

8.6.2. Extended Multi-Path Routing Formulation

To accommodate heterogeneous mesh nodes in LAB-CA_HT, all other constraints in the formulation of multi-path routing (presented in Section 5.4.1) remain the same except for the node-degree constraint. Let ND be the set of node-degrees of mesh nodes and nd_i be an input parameter denoting the node-degree of a mesh node i . Then, the following represent the constraints on the node-degree of the mesh nodes in the WMN.

$$\sum_k z_{ki} + \sum_j z_{ij} \leq nd_i \quad \text{for all } i \quad (8.16)$$

where $i, j, k \in V$, and $z_{ij} \in \{0, 1\}$ is 1 when the link l_{ij} is used for routing and 0 otherwise.

8.6.3. Extended Beamforming-based Conflict Graph and Matrix

Algorithm 8.1 is extended for LAB-CA_HT to accommodate heterogeneous mesh nodes as shown in Algorithm 8.2. Added inputs in Algorithm 8.2 include ND , i.e. the set of node-degrees of the mesh nodes. Note that while calculating the link transmission power in Step #11, reciprocity cannot be applied as AG_{ab} is no longer the same as AG_{ba} since the node-degrees of mesh nodes a and b could be different. The node-degrees of mesh routers are also needed while calculating the received power. For example, to calculate the power received at node x from node p in Step #23, the node-degree of node x is needed to know the number of antennas while calculating AG_{xp} and the node-degree of node p is needed to know the number of antennas while calculating AG_{px} since the node-degrees of mesh nodes x and p could be different in a heterogeneous mesh network.

Fig. 8.3 shows a plot of $|AG(\theta = \pi/2, \phi)|$ for different values of N_a when $\phi_o = 60^\circ$ showing that the beam formed by a six-antenna array is more focused than that formed by a three-antenna array.

8.7. Channel Assignment for Beamformed Heterogeneous Dense WMNs

The performance of LAB-CA_HT is evaluated in comparison with OAP-CA_HT in both sparse and dense mesh networks. To evaluate the performance of LAB-CA_HT in a sparse mesh network, 36 mesh nodes are distributed in a $500 \text{ m} \times 500 \text{ m}$ area whereas to evaluate its performance in a dense mesh network, 100 mesh nodes are distributed in the same area.

The results show that LAB-CA_HT outperforms OAP-CA_HT in terms of number of frequency channels in sparse as well as dense mesh networks. It performs even better in dense mesh networks and provides a greater reduction in the number of frequency channels. Note that OAP-CA_HT is the same as OAP-CA except for its connectivity graph and routing stages which are described in Sections 8.6.1 and 8.6.2 respectively.

8.8. Channel Assignment for Beamformed Heterogeneous Dense MRMCMA WMNs

A wireless mesh network architecture where the number of antennas in the linear antenna array of a multi-radio mesh node is greater than its number of radio interfaces is referred to as a *multi-radio multi-channel multi-antenna* WMN. In the MRMC WMN architecture, the node-degree of mesh nodes is equal to the number of antennas in their linear antenna arrays. For example, a node-degree of 2 for a mesh node indicates two data radio interfaces and two antennas for that node. In the MRMCMA WMN architecture, a node-degree of 2 for a mesh node also indicates two data radio interfaces but the number of antennas in its antenna array is more than 2, i.e. 3 or more. Like in a MRMC WMN, a mesh node in a MRMCMA WMN with a node-degree of 2 can communicate with at most two of its neighbors at the same time over its two data radio interfaces but using better (i.e. more focused) beams due to the 3 or more antennas in its antenna array.

The impact of increasing the number of antennas in the linear antenna arrays of mesh nodes on the number of frequency channels in a dense MRMCMA WMN is also investigated in this work. The results show that LAB-CA_HT provides a further significant reduction in the number of frequency channels in MRMCMA WMNs when the number of antennas in the linear antenna arrays of mesh nodes is greater than their node-degree.

Figure 8.3. Plot of $|AG(\theta = \pi/2, \phi)|$ for different values of N_a ($\phi_o = 60^\circ$)

Inputs:

- $f, G_t, G_r, h_t, h_r, n, RxThresh_dBm, SIRThresh_dB, L, locations_x, locations_y, \sigma, OP, d_a, ND$.

Output:

- *conflict_matrix*: Conflict graph

BEGIN

```

1.  $RxThresh\_mwatts \leftarrow 10^{(RxThresh\_dBm/10)}, m \leftarrow |L|, SIRThresh \leftarrow 10^{(SIRThresh\_dB/10)}$ 
2. For  $i = 1$  to  $n$ :
3.   For  $j = 1$  to  $n$ :
4.      $dist\_all(i,j) \leftarrow$  the distance between node  $i$  and node  $j$ 
5.   end For
6. end For
7. For  $i = 1$  to  $m$ :
8.    $L(i,3) \leftarrow$  distance between the nodes of link  $i$ 
9.    $L(i,4) \leftarrow AG_{ab} \leftarrow$  array gain of  $beam_{ab}$  with  $\phi = \phi_o = \phi_{ab}$  and  $N_a$  equal to  $ND(L(i,1))$  using (8.15)
10.   $L(i,5) \leftarrow AG_{ba} \leftarrow$  array gain of  $beam_{ba}$  with  $\phi = \phi_o = \phi_{ba}$  and  $N_a$  equal to  $ND(L(i,2))$  using (8.15)
11.   $L(i,6) \leftarrow P_t$  for link  $i$  with  $G_t$  and  $G_r$  equal to  $AG_{ab}$  and  $AG_{ba}$  using (6.5) or (6.6)
12. end For
13. Initialize an  $m \times m$  conflict_matrix of all ones
14. For  $il = 1$  to  $m$ :
15.   Initialize an empty conflict table
16.   For  $j = 1$  to  $m$ :
17.      $conflict\_table(j,1) \leftarrow$  node  $x$  of link  $il$ 
18.      $conflict\_table(j,2) \leftarrow$  node  $y$  of link  $il$ 
19.      $conflict\_table(j,3) \leftarrow$  node  $p$  of link  $j$ 
20.      $conflict\_table(j,4) \leftarrow$  node  $q$  of link  $j$ 
21.      $AG_{xp} \leftarrow$  array gain of  $beam_{xy}$  with  $\phi = \phi_{xp}, \phi_o = \phi_{xy}, N_a = ND(L(il,1))$  using (8.15)
22.      $AG_{px} \leftarrow$  array gain of  $beam_{pq}$  with  $\phi = \phi_{px}, \phi_o = \phi_{pq}, N_a = ND(L(j,1))$  using (8.15)
23.      $P_{r,xp} \leftarrow$  power received at  $x$  from  $p$  with  $G_t = AG_{px}, G_r = AG_{xp}$  and  $P_{t,pq} = L(j,6)$  using (6.2) or (6.4)
24.      $conflict\_table(j,5) \leftarrow P_{r,xp}$ 
25.      $AG_{xq} \leftarrow$  array gain of  $beam_{xy}$  with  $\phi = \phi_{xq}, \phi_o = \phi_{xy}, N_a = ND(L(il,1))$  using (8.15)
26.      $AG_{qx} \leftarrow$  array gain of  $beam_{qp}$  with  $\phi = \phi_{qx}, \phi_o = \phi_{qp}, N_a = ND(L(j,2))$  using (8.15)
27.      $P_{r,xq} \leftarrow$  power received at  $x$  from  $q$  with  $G_t = AG_{qx}, G_r = AG_{xq}$  and  $P_{t,qp} = L(j,6)$  using (6.2) or (6.4)
28.      $conflict\_table(j,6) \leftarrow P_{r,xq}$ 
29.      $AG_{yp} \leftarrow$  array gain of  $beam_{yx}$  with  $\phi = \phi_{yp}, \phi_o = \phi_{yx}, N_a = ND(L(il,2))$  using (8.15)
30.      $AG_{py} \leftarrow$  array gain of  $beam_{pq}$  with  $\phi = \phi_{py}, \phi_o = \phi_{pq}, N_a = ND(L(j,1))$  using (8.15)
31.      $P_{r,yp} \leftarrow$  power received at  $y$  from  $p$  with  $G_t = AG_{py}, G_r = AG_{yp}$  and  $P_{t,pq} = L(j,6)$  using (6.2) or (6.4)
32.      $conflict\_table(j,7) \leftarrow P_{r,yp}$ 
33.      $AG_{yq} \leftarrow$  array gain of  $beam_{yx}$  with  $\phi = \phi_{yq}, \phi_o = \phi_{yx}, N_a = ND(L(il,2))$  using (8.15)
34.      $AG_{qy} \leftarrow$  array gain of  $beam_{qp}$  with  $\phi = \phi_{qy}, \phi_o = \phi_{qp}, N_a = ND(L(j,2))$  using (8.15)
35.      $P_{r,yq} \leftarrow$  power received at  $y$  from  $q$  with  $G_t = AG_{qy}, G_r = AG_{yq}$  and  $P_{t,qp} = L(j,6)$  using (6.2) or (6.4)
36.      $conflict\_table(j,8) \leftarrow P_{r,yq}$ 

```

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37.          $conflict\_table(j,9) \leftarrow \max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
38.          $conflict\_table(j,10) \leftarrow RxThresh\_mwatts/\max(P_{r,xp}, P_{r,xq}, P_{r,yp}, P_{r,yq})$ 
39.     end For
40.     For  $i2 = 1$  to  $m$ :
41.         If  $conflict\_table(i2,10) > SIRThresh$ :
42.              $conflict\_matrix(i1,i2) \leftarrow conflict\_table(i2,9)$ 
43.         end If
44.     end For
45. end For
46. Output  $conflict\_matrix$ 
END

```

Algorithm 8.2. Extended Conflict Graph and Matrix for LAB-CA_HT

9. PERFORMANCE EVALUATION

In this chapter, the following are evaluated:

1. The performance of interference-free channel assignment method using TCA-based CG is evaluated in comparison with the one using MP-based CG [74].
2. The performance of interference-free channel assignment method using multi-path routing is evaluated in comparison with the one using minimum power-based shortest path routing [75].
3. The performance of interference-free channel assignment method using MaIS-based heuristic for minimum coloring is evaluated in comparison with the one using MIS-based heuristic for minimum coloring [75].

This performance comparison is carried out in terms of network throughput, fairness, solution time of the routing stage in clock seconds, number of (frequency) channels required (NCR), and links to (frequency) channels ratio (LCR) at different node-degree constraints (NDCs). LCR is the ratio of the number of links involved in routing to the number of frequency channels required. Note that an average LCR of 2 for a topology means that on the average, 2 links are assigned the same frequency channel for that topology. So, a higher LCR indicates a better performance. The worst-case LCR is 1, meaning that each link involved in routing requires a different frequency channel, so there is no spatial channel reuse.

4. The performance of channel assignment method using the SIR model with shadowing is evaluated in comparison with the ones using the protocol model and the SIR model based on NCR and network throughput (NT) at different NDCs [83].
5. The performance of the heuristic solution for the minimum coloring problem with cumulative interference constraints is evaluated in comparison with the exact QCBP solution for this extended minimum coloring problem based on NCR, solution time in seconds, and difference (or maximum possible difference) from the true minimum number of frequency channels required [84].
6. The performance of the linear array beamforming-based channel assignment method LAB-CA is evaluated in comparison with the omni-directional antenna pattern-based

channel assignment method OAP-CA in terms of NCR at different NDCs and at different links data rates [89][90].

7. The performance of the linear array beamforming-based channel assignment method for heterogeneous mesh nodes LAB-CA_HT is evaluated in comparison with the linear array beamforming-based channel assignment method for homogeneous mesh nodes LAB-CA_HG in terms of NT, the solution time of the routing stage, and NCR at different NDCs and at different link data rates [91].
8. The performance of LAB-CA_HT is also evaluated in sparse as well as dense mesh networks in comparison with the omni-directional antenna pattern-based channel assignment method for heterogeneous mesh nodes OAP-CA_HT. This performance comparison is carried out in terms of NCR, LCR, NT, and solution time of the routing stage at different NDCs at a link data rate of 54 Mbps [91].
9. The performance of LAB-CA_HT is evaluated in dense MRMCMC WMNs in comparison with dense MRMC WMNs in terms of NCR for different numbers of antennas at a link data rate of 54 Mbps [91].

9.1. Experimental Setup

Network Topology

A controlled random topology (CRT) is used for the evaluation, in which the physical terrain, consisting of a 500 m $\times$ 500 m area, is divided into cells and a mesh node is placed randomly within each cell using a uniform random distribution. Twenty-five different CRTs consisting of 36-node networks are considered. Node 15 is the selected as the gateway for all CRTs.

As stated earlier, all mesh nodes except the GW are set as sources of flow (i.e. data traffic); GW is the sink for all flows. In other words, there are 35 source-sink pairs and 35 flows in the network.

The mean values and statistics on the 95% confidence intervals of all measures are collected for the twenty-five different CRTs. The confidence intervals are reported in tables, and the mean values of most measures are graphed so that trends are immediately apparent.

Software Used

MATLAB version 7.8 [95] has been used to implement the following algorithms: *Select x for less than x* TCA, conflict graph for the protocol model, MaIS-based heuristics, and MIS-based heuristic in Chapter 5; conflict graph and matrix for the SIR model, and conflict graph and matrix for the SIR model with shadowing in Chapter 6; heuristics for the extended coloring problem in Chapter 7; and beamforming-based conflict graph and matrix for LAB-CA, *extended Select x for less than x* TCA, and beamforming-based conflict graph and matrix for LAB-CA_HT in Chapter 8.

AMPL language [78] is used to model the MILP formulation of the multi-path routing problem and the QCBP formulation of the extended coloring problem. IBM's CPLEX solver version 12.2 [11] has been used to solve the resulting problems.

Hardware Used

The software programs including MATLAB, AMPL and CPLEX were run on a desktop PC having Intel Core 2 Quad Q8200 processor running at 2.33 GHz, and 8 GB of memory. The operating system was Windows Vista Business with Service Pack 2.

9.2. Channel Assignment using TCA-based vs. MP-based Connectivity Graph

9.2.1. Simulation Setup

To study the impact of utilizing topology control on the interference-free channel assignment method presented in Chapter 5, the performance of channel assignment using TCA-based CG is compared with the one using MP-based CG in terms of (i) fairness, (ii) network throughput, (iii) solution time of routing stage, (iv) NCR, and (v) LCR, for different node-degrees. For both connectivity graph approaches, multi-path routing is used in combination with the MaIS-based heuristic for minimum coloring. All CPLEX solver parameters are at their default settings, except $mipgap=0.01$. This speeds the MILP solution at the possible expense of a small degradation in the objective value, though the optimum was found in all cases. For the channel assignment using TCA-based CG, if the solver finds a sub-optimum solution for the multi-path routing problem for node-degree constraints of 2 or 3, the gateway moves to a higher TCA to build the CG, i.e. *Select 4 for less than 4 TCA* or *Select 5 for less than 5 TCA*, until an optimum solution is found. Note that for a node-degree constraint of 2, the maximum value of total network flow (maximum network throughput) is 48 Mbps; for a node-degree constraint of 3, this value is 72 Mbps; and so on.

9.2.2. Results

a) Fairness

For maximum network throughput while maintaining fairness, the objective is to maximize the value of y in (5.1), where y represents the multiplier on the unit flow demand between any source and the GW. The amount of flow from each source reaching the GW (sink) is equal to $y \times demand_{sd}$, as per (5.6). However, due to a unit flow demand between each source-sink pair, the value of y in Table 9.1 represents the amount of flow from each source reaching the GW. This indicates that flows from all of the 35 sources are maximized equally. *Jain's fairness index* [96] F_J , which is a measure of fairness among the multiple network flows, is given by

$$F_J = (\sum_{i=1}^N x_i)^2 / (N \times \sum_{i=1}^N x_i^2) \quad (9.1)$$

where x_i is the throughput of a flow i and N is the total number of flows (sources) in the network. For both CG approaches, the flows from all sources are maximized equally. So, absolute fairness is achieved and the value of F_J is 1 in all CRTs for all node-degree constraints.

b) Network Throughput

The results in Table 9.1 indicate that the total flow in the network (the flow reaching the gateway (sink)), increases equally for the MP-based and TCA-based CG approaches with an increase in the node-degree constraint. For a node-degree constraint of 2 in Table 9.1, y is 1.3714. The total flow in the network (or network throughput) is equal to the number of sources times y , i.e. $35 \times 1.3714 = 48$ Mbps, for both CG approaches, which is equal to the maximum total network flow (maximum network throughput) for the node-degree constraint of 2. In fact, the network throughput achieved, for all node-degree constraints, is maximum for both CG approaches, as shown in Table 9.1.

The centralized approach leads to an optimum solution in terms of network throughput while also ensuring fairness among the network flows since the required global network information is available at the GW.

Table 9.1. Comparison of Network Throughput (Mbps) – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	y	Network Throughput (Mbps)
2	MP-based	1.3714	48
	TCA-based		
3	MP-based	2.0571	72
	TCA-based		
4	MP-based	2.7428	96
	TCA-based		
5	MP-based	3.4285	120
	TCA-based		
6	MP-based	4.1142	144
	TCA-based		

c) Solution Time of Routing Stage

Table 9.2 compares the two CG approaches in terms of the solution time in clock seconds of the multi-path routing stage. It shows the average values of the solution times for the 25 CRTs along with the 95% confidence interval (CI) at different node-degree constraints. The TCA-based CG approach is clearly faster than the MP-based CG approach.

Table 9.2. Comparison of Solution Time – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	Solution Time (secs)	95% CI for Solution Time (secs)
2	MP-based	656.33	296.43–1016.22
	TCA-based	138.24	102.00–174.49
3	MP-based	11.31	10.32–12.30
	TCA-based	3.04	2.62–3.45
4	MP-based	10.89	9.83–11.96
	TCA-based	3.70	3.13–4.28
5	MP-based	9.74	8.32–11.17
	TCA-based	3.61	2.80–4.43
6	MP-based	8.82	7.26–10.38
	TCA-based	3.99	3.10–4.89

d) Number of Channels Required

Fig. 9.1 and Table 9.3 show that the TCA-based CG approach requires fewer frequency channels than the MP-based CG approach for all node-degree constraints. Using multi-path routing in combination with MaIS-based minimum coloring heuristic at a node-degree constraint of 2, the TCA-based CG approach uses one third fewer frequency channels compared to the MP-based CG approach.

Table 9.3. Comparison of NCR – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	NCR	95% CI for NCR
2	MP-based	17.88	17.31–18.45
	TCA-based	13.44	12.78–14.10
3	MP-based	19.72	19.20–20.24
	TCA-based	13.80	13.19–14.41
4	MP-based	22.04	21.22–22.86
	TCA-based	17.80	17.15–18.45
5	MP-based	23.48	22.54–24.42
	TCA-based	20.88	19.70–22.06
6	MP-based	27.60	26.67–28.53
	TCA-based	25.56	24.58–26.54

Figure 9.1. Comparison of NCR – TCA-based vs. MP-based CG Approach

e) Links to Channels Ratio

Fig. 9.2 and Table 9.4 show that LCR for the TCA-based CG approach is higher than for the MP-based CG approach for all node-degree constraints. Using multi-path routing in combination with the MaIS-based minimum coloring heuristic at a node-degree constraint of 2, the LCR for the TCA-based CG approach is one third higher than that for the MP-based CG approach.

Table 9.4. Comparison of LCR – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	LCR	95% CI for LCR
2	MP-based	2.03	1.96–2.09
	TCA-based	2.72	2.59–2.85
3	MP-based	1.89	1.83–1.94
	TCA-based	2.71	2.61–2.82
4	MP-based	1.74	1.68–1.80
	TCA-based	2.16	2.08–2.24
5	MP-based	1.51	1.45–1.57
	TCA-based	1.79	1.70–1.87
6	MP-based	1.46	1.41–1.51
	TCA-based	1.59	1.53–1.65

Figure 9.2. Comparison of LCR – TCA-based vs. MP-based CG Approach

As the node-degree constraint increases, so does the number of links emanating from a mesh node. Since the links emanating from a node must be assigned different frequency channels, this increases the NCR as well as decreases the LCR for both CG approaches, as shown in Figs. 9.1 and 9.2. However, the TCA-based CG approach still outperforms the MP-based CG approach by controlling the network connectivity using topology control. To illustrate the reason for this, see Figs. 9.3 and 9.4 where connectivity and interference are highlighted when using the TCA-based approach.

The average node-degree (AND) for the entire network can be calculated as:

$$AND = \frac{1}{n} \sum_{j=1}^n q_j \quad (9.2)$$

where q is the number of neighbors of a mesh node, and n is the total number of mesh nodes in the network. The *transmission range-average node-degree* (TR-AND) and the *interference range-average node-degree* (IR-AND) are defined as the average number of transmission range neighbors and the average number of interference range neighbors of nodes, respectively. Figs. 9.3 and 9.4 along with Tables 9.5 and 9.6 compare TR-AND

(connectivity) and IR-AND (interference) for the two CG approaches, respectively. With the TCA-based CG approach, the connectivity and interference can be controlled, as can be seen from these figures. However as the network connectivity increases by increasing x in the *Select x for less than x* TCA, so does the interference in the network. This increases the NCR as well as decreases the LCR at higher node-degree constraints with the TCA-based approach.

Since the solver finds a sub-optimum solution of the multi-path routing problem for the node-degree constraint of 2 with the TCA-based CG approach for many of the CRTs, the gateway moves from *Select 3 for less than 3* TCA to a higher TCA to build the CG until an optimum solution is found. This is the reason for the higher connectivity and interference in Figs. 9.3 and 9.4 for the node-degree constraint of 2 with the TCA-based CG approach.

For the connectivity graph of CRT 4 based on *Select 3 for less than 3* TCA which is shown in Fig. 9.5, the solver is not able to find a solution for node-degree constraint of 2. As can be seen from this figure, there are four possible ways for nodes 5, 6 and 12 to reach the GW (sink) which is node 15. Nodes 5 and 6 can go through node 12 in which case it will require a node-degree of three. In other words, node 12 will require two data radios for nodes 5 and 6 and one to connect to node 11. Nodes 5 and 12 can go through node 6 in which case it will require a node-degree of three. Nodes 6 and 12 can go through node 5 in which case it will require a node-degree of three. Nodes 5, 6 and 12 can go through node 11 in which case it will require a node-degree of four. Since it is not possible to satisfy a node-degree constraint of 2 for this connectivity graph, the solver is unable to find a solution. So, the gateway moves from *Select 3 for less than 3* TCA to *Select 4 for less than 4* TCA to build the connectivity graph which is shown in Fig. 9.6. For the connectivity graph shown in this figure, the solver is able to find the optimum solution using more connectivity provided by *Select 4 for less than 4* TCA.

For the connectivity graph of CRT 1 based on *Select 3 for less than 3* TCA which is shown in Fig. 9.7, the solver finds a sub-optimal solution for node-degree constraint of 2. Using the connectivity provided by *Select 3 for less than 3* TCA, it finds the following two paths for the flows from the 35 different sources to reach the GW (sink):

Path 1:

4→3→2→1→8→14→7→13→19→20→31→25→26→32→33→34→27→21→15

Path 2:

36→35→30→29→28→22→23→24→18→6→12→5→11→9→10→17→16→15

Note that for the node-degree constraint of 2, the GW (sink) has two links and the capacity of each link is 24 Mbps. Due to the fairness constraint, the amount of flow from each source reaching the GW is the same and is equal to 1.33 Mbps. Using path 1, flows from 18 sources reaches the GW. So, the total amount of flow reaching the GW through path 1 is 23.94 Mbps. Path 2, on the other hand, is used by the flows of 17 sources to reach the GW and the total amount of flow reaching the GW through this path is 22.61 Mbps. The two paths do not carry the maximum amount of flow and the solution is sub-optimal.

To achieve the optimal solution, the gateway moves from *Select 3 for less than 3* TCA to *Select 4 for less than 4* TCA to build the connectivity graph which is shown in Fig. 9.8. Using the additional connectivity provided by the connectivity graph shown in this figure, optimal solution is obtained by the solver. More connectivity results in the following two paths for the flows from the 35 different sources to reach the gateway (sink):

Path 1:

4→9→2→3→1→7→8→13→19→25→31→32→33→34→27→26→20→14→15

Path 2:

4→11→5→10→17→16→22→29→36→35→30→24→12→6→18→23→28→21→15

In this case, the amount of flow from each source reaching the GW is 1.3714 Mbps. A flow of 0.6857 Mbps from node 4 uses path 1 while a similar amount of flow from node 4 uses path 2 to reach the GW. Both paths are used by flows of same number of sources and an equal amount of flow, which is 23.99 Mbps, reaches the GW from the two paths. So, the solver is able to find the optimum solution using more connectivity provided by the *Select 4 for less than 4* TCA.

Table 9.5. Comparison of TR-AND – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	TR-AND	95% CI for TR-AND
2	MP-based	7.95	7.79–8.11
	TCA-based	4.33	4.04–4.62
3	MP-based	7.95	7.79–8.11
	TCA-based	3.74	3.58–3.89
4	MP-based	7.95	7.79–8.11
	TCA-based	4.64	4.58–4.69
5	MP-based	7.95	7.79–8.11
	TCA-based	5.61	5.55–5.67
6	MP-based	7.95	7.79–8.11
	TCA-based	6.36	6.29–6.43

Figure 9.3. Comparison of TR-AND – TCA-based vs. MP-based CG Approach

Table 9.6. Comparison of IR-AND – TCA-based vs. MP-based CG Approach

Node-Degree Constraint	CG Approach	IR-AND	95% CI for IR-AND
2	MP-based	15.92	15.75–16.09
	TCA-based	10.00	9.51–10.50
3	MP-based	15.92	15.75–16.09
	TCA-based	9.05	8.67–9.44
4	MP-based	15.92	15.75–16.09
	TCA-based	10.60	10.30–10.90
5	MP-based	15.92	15.75–16.09
	TCA-based	12.03	11.80–12.26
6	MP-based	15.92	15.75–16.09
	TCA-based	12.90	12.69–13.12

Figure 9.4. Comparison of IR-AND – TCA-based vs. MP-based CG Approach

Figure 9.5. Connectivity Graph of CRT 4 using *Select 3 for less than 3 TCA*

Figure 9.6. Connectivity Graph of CRT 4 using *Select 4 for less than 4 TCA*

Figure 9.7. Connectivity Graph of CRT 1 using *Select 3 for less than 3 TCA*

Figure 9.8. Connectivity Graph of CRT 1 using *Select 4 for less than 4 TCA*

9.3. Channel Assignment using Multi-Path vs. MPSP Routing

9.3.1. Simulation Setup

To study the impact of employing multi-path routing on the interference-free channel assignment method presented in Chapter 5, the performance of channel assignment using multi-path routing is compared with the one using MPSP routing in terms of (i) fairness, (ii) network throughput, (iii) solution time of routing stage, (iv) NCR, and (v) LCR, for different node-degree constraints. For each routing approach, TCA-based CG as well as the MP-based CG is used in combination with the MaIS-based minimum coloring heuristic. For both routing approaches, CPLEX is asked to search for the first feasible solution, which significantly reduces the solution time at the cost of a small degradation in the network throughput. Also, *mipemphasis=1* is used in CPLEX for both routing approaches. For the TCA-based CG approach, *Select 3 for less than 3* TCA is used for the node-degree constraint of 2 and 3; *Select 4 for less than 4* TCA is used for the node-degree constraint of 4, and so on. Since the first feasible solution of the two routing approaches is used for this comparison, so unlike the previous comparison in Section 9.2, the gateway does not move to a higher TCA in search of the optimum, unless the solver cannot find a solution as explained earlier using Figs. 9.5 and 9.6.

The MP-based CG performs poorly as compared to the TCA-based CG for both routing approaches in terms of solution time of the routing stage, NCR and LCR and is slightly better than TCA-based CG in terms of network throughput. Therefore, for the sake of brevity, the results for each routing approach are presented with TCA-based CG only in combination with the MaIS-based minimum coloring heuristic.

9.3.2. Results

a) Fairness

As indicated by the results in Table 9.7, the flows (y -values) are maximized equally in both routing approaches. Absolute fairness is achieved for both routing approaches in all CRTs for all node-degree constraints.

b) Network Throughput

The results in Table 9.7 and Fig. 9.9 indicate that the total amount of flow in the network (or network throughput) increases for both routing methods with an increase in the node-degree constraint. However, the increase in network throughput is significantly higher with multi-path routing, especially at higher node-degree constraints. For example, using the TCA-based CG, the network throughput is 87.3% of the maximum network throughput of 144 Mbps at a node-degree constraint of 6 with multi-path routing as compared to a network throughput of 41.9% of the maximum network throughput with MPSP routing for the same node-degree constraint. Note that the small degradation in the network throughput with multi-path routing is due to finding the first feasible solution vs. finding the optimum solution. The main reason for the lower network throughput of MPSP routing is that while determining the shortest paths between the sources and the GW, maximizing the total network flow is not an objective.

Table 9.7. Comparison of Network Throughput (Mbps) – Multi-Path vs. MPSP Routing Approach

Node-Degree Constraint	Routing Approach	y	Network Throughput (Mbps)	95% CI for Network Throughput (Mbps)
2	Multi-Path	1.160	40.59	37.95–43.23
	MPSP	1.199	41.96	40.40–43.53
3	Multi-Path	1.902	66.56	62.15–70.97
	MPSP	1.314	46.00	42.19–49.82
4	Multi-Path	2.382	83.37	78.19–88.55
	MPSP	1.487	52.03	47.55–56.52
5	Multi-Path	2.981	104.33	97.01–111.66
	MPSP	1.603	56.11	51.33–60.89
6	Multi-Path	3.593	125.74	116.67–134.81
	MPSP	1.724	60.35	54.99–65.71

c) Solution Time of Routing Stage

Table 9.8 compares the two routing approaches in terms of the solution time in clock seconds. For multi-path routing, the solution times are reduced as compared to those in Table 9.2. However this reduction occurs as a trade-off in the form of a decrease in the network throughput. As indicated in Table 9.8, MPSP routing is faster as compared to multi-path routing in terms of the solution time. Since the objective of the multi-path routing is to maximize the total network flow, it takes the solver more time to find the first feasible solution for it as compared to MPSP routing.

d) Number of Channels Required

The results in Fig. 9.10 and Table 9.9 indicate that fewer frequency channels are required by the MPSP routing approach than are required by the multi-path routing approach.

e) Links to Channels Ratio

Fig. 9.11 and Table 9.10 show that LCR with MPSP routing is higher than that for multi-path routing.

Since the objective of multi-path routing is to maximize the total flow in the network by using multiple paths between the mesh nodes and the GW, more nodes have high node-degrees in its output, which means more conflicts in the conflict graph. This leads to a higher NCR and a lower LCR as compared to MPSP routing. Although MPSP routing has smaller solution times, lower NCR and higher LCR, it performs poorly vs. multi-path routing in terms of network throughput at higher node-degree constraints. There is a trade-off between the network throughput and the NCR for the two routing approaches. For example, using the TCA-based CG and a node-degree constraint of 6, multi-path routing provides 2.08 times more network throughput than MPSP routing but is 1.67 times costlier than MPSP routing in terms of NCR.

Figure 9.9. Comparison of Network Throughput (Mbps) – Multi-Path vs. MPSP Routing Approach

Table 9.8. Comparison of Solution Time – Multi-Path vs. MPSP Routing Approach

Node-Degree Constraint	Routing Approach	Solution Time (secs)	95% CI for Solution Time (secs)
2	Multi-Path	13.79	8.60–18.98
	MPSP	1.64	1.47–1.82
3	Multi-Path	1.68	1.48–1.88
	MPSP	0.25	0.23–0.27
4	Multi-Path	1.75	1.43–2.08
	MPSP	0.24	0.24–0.25
5	Multi-Path	1.89	1.37–2.42
	MPSP	0.26	0.26–0.27
6	Multi-Path	2.37	1.69–3.05
	MPSP	0.28	0.28–0.29

Table 9.9. Comparison of NCR – Multi-Path vs. MPSP Routing Approach

Node-Degree Constraint	Routing Approach	NCR	95% CI for NCR
2	Multi-Path	11.84	11.47–12.21
	MPSP	11.16	10.81–11.51
3	Multi-Path	13.12	12.77–13.47
	MPSP	11.96	11.46–12.46
4	Multi-Path	17.36	16.82–17.90
	MPSP	13.60	13.07–14.13
5	Multi-Path	21.88	20.71–23.05
	MPSP	14.44	13.84–15.04
6	Multi-Path	24.88	23.51–26.25
	MPSP	14.88	14.40–15.36

Figure 9.10. Comparison of NCR – Multi-Path vs. MPSP Routing Approach

Table 9.10. Comparison of LCR – Multi-Path vs. MPSP Routing Approach

Node-Degree Constraint	Routing Approach	LCR	95% CI for LCR
2	Multi-Path	2.97	2.88–3.07
	MPSP	3.15	3.06–3.25
3	Multi-Path	2.81	2.74–2.88
	MPSP	2.97	2.84–3.10
4	Multi-Path	2.10	2.03–2.17
	MPSP	2.60	2.50–2.70
5	Multi-Path	1.70	1.61–1.78
	MPSP	2.46	2.35–2.56
6	Multi-Path	1.55	1.47–1.62
	MPSP	2.37	2.29–2.45

Figure 9.11. Comparison of LCR – Multi-Path vs. MPSP Routing Approach

9.4. Channel Assignment using MaIS-based vs. MIS-based Minimum Coloring Heuristic

9.4.1. Simulation Setup

To study the impact of using MaIS-based minimum coloring heuristic on the interference-free channel assignment method presented in Chapter 5, the performance of channel assignment using MaIS-based minimum coloring heuristic is compared with the one using MIS-based minimum coloring heuristic in terms of (i) NCR, and (ii) LCR for different node-degree constraints. For each heuristic approach, the two routing approaches are used in combination with the two CG approaches. During routing, the CPLEX solver is asked to search for the first feasible solution and *mipemphasis=1* is used in CPLEX to reduce the solution time.

Since MP-based CG mostly performs poorly as compared to TCA-based CG for both minimum coloring heuristics, the results are presented for each heuristic with TCA-based CG only in combination with the two routing approaches.

9.4.2. Results

a) Number of Channels Required

The results in Fig. 9.12 and Table 9.11 show that fewer frequency channels are required by the MaIS-based heuristic than the MIS-based heuristic approach. Using the TCA-based CG in combination with multi-path routing at a node-degree constraint of 2, the MaIS-based heuristic approach uses 25.6% fewer frequency channels than the MIS-based heuristic approach.

b) Links to Channels Ratio

The results in Fig. 9.13 and Table 9.12 show that the LCR for the MaIS-based heuristic approach is higher than that for the MIS-based heuristic approach. Using the TCA-based CG in combination with multi-path routing at a node-degree constraint of 2, MaIS-based heuristic approach results in 24.7% higher LCR as compared to MIS-based heuristic approach.

The MaIS-based heuristic approach not only outperforms the MIS-based heuristic approach in terms of NCR and LCR but is also much more efficient in terms of

computational complexity. It is observed that the MaIS-based heuristic returns fewer frequency channels than the MIS-based heuristic because the cardinalities of the MaISs are more even in size whereas the MIS-based heuristic first finds a few large cardinality MISs, leaving a larger number of small MISs, for an overall larger number of frequency channels.

Table 9.11. Comparison of NCR – MaIS-based vs. MIS-based Heuristic Approach

Node-Degree Constraint	Minimum Coloring Heuristic	Multi-Path Routing		MPSP Routing	
		NCR	95% CI for NCR	NCR	95% CI for NCR
2	MaIS-based	11.84	11.47–12.21	11.16	10.81–11.51
	MIS-based	14.88	14.26–15.50	13.64	13.05–14.23
3	MaIS-based	13.12	12.77–13.47	11.96	11.46–12.46
	MIS-based	16.28	15.65–16.91	14.60	13.99–15.21
4	MaIS-based	17.36	16.82–17.90	13.60	13.07–14.13
	MIS-based	20.52	19.93–21.11	16.08	15.49–16.67
5	MaIS-based	21.88	20.71–23.05	14.44	13.84–15.04
	MIS-based	24.48	23.36–25.60	17.00	16.37–17.63
6	MaIS-based	24.88	23.51–26.25	14.88	14.40–15.36
	MIS-based	27.40	26.20–28.60	17.40	16.87–17.93

Figure 9.12. Comparison of NCR – MaIS-based vs. MIS-based Heuristic Approach

Table 9.12. Comparison of LCR – MaIS-based vs. MIS-based Heuristic Approach

Node-Degree Constraint	Minimum Coloring Heuristic	Multi-Path Routing		MPSP Routing	
		LCR	95% CI for LCR	LCR	95% CI for LCR
2	MaIS-based	2.97	2.88–3.07	3.15	3.06–3.25
	MIS-based	2.38	2.28–2.48	2.59	2.49–2.70
3	MaIS-based	2.81	2.74–2.88	2.97	2.84–3.10
	MIS-based	2.28	2.19–2.37	2.43	2.33–2.53
4	MaIS-based	2.10	2.03–2.17	2.60	2.50–2.70
	MIS-based	1.78	1.72–1.83	2.20	2.12–2.28
5	MaIS-based	1.70	1.61–1.78	2.46	2.35–2.56
	MIS-based	1.51	1.45–1.56	2.08	2.00–2.16
6	MaIS-based	1.55	1.47–1.62	2.37	2.29–2.45
	MIS-based	1.40	1.35–1.44	2.03	1.96–2.09

Figure 9.13. Comparison of LCR – MaIS-based vs. MIS-based Heuristic Approach

9.5. Comparison of Channel Assignment using Different Interference Models

9.5.1. Simulation Setup

To study the impact of using the SIR model with shadowing on the realistic interference-free CA method presented in Chapter 6, the performance of channel assignment using the SIR model with shadowing is compared to channel assignments that use simpler interference models, namely, channel assignment using the protocol model and channel assignment using the SIR model, in terms of (i) network throughput, and (ii) NCR, for different node-degree constraints at different link data rates.

For constructing the conflict graph based on the SIR model and the SIR model with shadowing, the frequency is assumed to be 5.805 GHz, G_t and G_r to be 1, and h_t and h_r to be 3 m. The receiver thresholds are taken as -79 , -74 , -70 and -65 dBm for link data rates of 12, 24, 36, and 54 Mbps, respectively as per IEEE 802.11a. The link capacity is set equal to the maximum goodput at a particular link data rate. The SIR requirements for different data rates are shown in Table 9.13 and are calculated using

$$SIR_{(dB)} = (\text{coded } E_b/N_o)_{(dB)} + 10 \log_{10}(N_{CBPS}/(T_s \times B)). \quad (9.3)$$

Note that N_{CBPS} represents the number of coded bits per symbol in (9.3). As per the IEEE 802.11a standard, a symbol duration (T_s) of 4 μ sec and an occupied bandwidth (B) of 16.6 MHz is used while calculating the required SIR. Note that the values of coded E_b/N_o [97] shown in the table are taken at a bit-error-rate of 10^{-5} which is specified as the required bit-error-rate in 802.11a. These reasonable assumptions are used to generate representative results, but this work can be applied to any scenario of multi-hop MRMC WMNs and is not limited to a specific standard.

Table 9.13. SIR Requirement for Different Data Rates

Link Data Rate (Mbps)	Modulation	Code Rate	Coded bits per symbol	Coded E_b/N_o (dB) [97]	Required SIR (dB)
12	QPSK	1/2	96	4.18	5.78
24	16-QAM	1/2	192	6.32	10.93
36	16-QAM	3/4	192	8.59	13.20
54	64-QAM	3/4	288	12.04	18.41

As stated in Section 6.5, TCA-based CG is used in combination with multi-path routing. Also, *mipemphasis=1* is used in CPLEX and it is asked to search for the first feasible solution of the routing stage. This significantly reduces the solution time at the cost of a small degradation in the network throughput. If the solver cannot find a solution of the multi-path routing problem for node-degree constraints of 2 or 3, the gateway moves to a higher TCA to build the CG, i.e. *Select 4 for less than 4 TCA* or *Select 5 for less than 5 TCA*, until the first feasible solution is found.

9.5.2. Results

a) Network Throughput

The amount of flow from each source reaching the GW (sink) is equal to $y \times demand_{sd}$ as per (5.6). However, due to a unit flow demand between each source-sink pair, the value of y in Table 9.14 represents the amount of flow from each source reaching the GW. This indicates that all of the 35 flows in the network are maximized equally. For a NDC of 2 in Table 9.14 at a link capacity of 24.73 Mbps for a link data rate of 54 Mbps, y is 1.1270. The network throughput (NT), i.e. the total amount of flow in the network, is equal to the number of sources times y , i.e. $35 \times 1.1270 = 39.45$ Mbps.

Note that using (6.11) and assuming a payload size of 1000 bytes, the maximum link throughput for the link data rate of 54 Mbps ($N_{DBPS} = 216$) is calculated to be 24.73 Mbps.

The total amount of flow in the network depends on the number of links for the GW node. The number of links for the GW increases with an increase in the NDC. For example, at a link capacity of 24.73 Mbps for a link data rate of 54 Mbps, the maximum network throughput is 49.46 Mbps for a NDC of 2, 74.19 Mbps for a NDC of 3, and so on. The results in Table 9.14 indicate that the network throughput increases with an increase in the NDC. However, there is a small degradation in the achieved network throughput as compared to its maximum value which is due to finding the first feasible solution vs. finding the optimum solution.

b) Number of Channels Required

The mean values and statistics on the 95% confidence intervals of the NCR are collected for the 25 different CRTs used. The confidence intervals for different node-

degree constraints are reported in a table. The mean values are graphed so that trends are immediately apparent.

In Figs. 9.14, 9.15, 9.16, 9.17 and 9.18 as well as in Tables 9.15, 9.16, 9.17, 9.18 and 9.19, γ represents the SIR threshold, σ represents the standard deviation for shadowing and OP represents the outage probability. Note that the red line in Figs. 9.14, 9.17 and 9.18 represents the maximum value of NCR. This is the worst case scenario in which each link involved in routing requires a different frequency channel. For example, if the number of links involved in routing for a CRT is 35, then the number of frequency channels required is also 35 in the worst case scenario.

Table 9.14. Network Throughput (Mbps)

Link Data Rate (Mbps)	NDC	y	NT (Mbps)	95% CI for NT (Mbps)	Maximum NT (Mbps)
12	2	0.4497	15.74	15.00–16.48	18.36
	3	0.7079	24.78	22.93–26.63	27.54
	4	0.9029	31.60	29.80–33.40	36.72
	5	1.2466	43.63	41.86–45.40	45.90
	6	1.5063	52.72	49.99–55.45	55.08
24	2	0.7259	25.40	23.77–27.04	31.04
	3	1.2406	43.42	40.55–46.29	46.56
	4	1.5634	54.72	50.96–58.47	62.08
	5	2.1075	73.76	70.86–76.67	77.60
	6	2.5350	88.72	85.37–92.08	93.12
36	2	0.9265	32.43	30.47–34.39	40.06
	3	1.6281	56.98	54.76–59.20	60.09
	4	1.9580	68.53	63.93–73.13	80.12
	5	2.7212	95.24	91.16–99.32	100.15
	6	3.2263	112.92	107.89–117.95	120.18
54	2	1.1270	39.45	37.10–41.80	49.46
	3	2.0302	71.06	68.63–73.48	74.19
	4	2.5764	90.17	84.76–95.59	98.92
	5	3.3913	118.70	113.61–123.79	123.65
	6	3.8762	135.67	127.25–144.08	148.38

i) Comparison of the Three Interference Models

Fig. 9.14 provides a comparison of the three interference models based on NCR for a link data rate of 24 Mbps. γ is 10.93 dB, σ is 3 dB and OP is 10%.

The simplistic protocol model has the lowest NCR among the three models. The SIR model with shadowing is the most realistic thereof the models, and has the highest NCR as expected. With the increase in the network throughput with NDC, the NCR increases for all three models. The network throughput depends on the number of links for the GW and increases with an increase in the NDC. Also, the number of links emanating from a mesh node increases with an increase in the NDC. Since the links emanating from a node must be assigned different frequency channels, this increases the NCR.

ii) SIR Model

Fig. 9.15 shows the NCR with SIR model for different data rates. NCR increases with the increase in γ as the data rate increases. An increase in γ means less tolerance for interference in the network. This results in more conflicts in the conflict graph which leads to a higher NCR.

iii) SIR Model with Shadowing

Figs. 9.16, 9.17 and 9.18 show the NCR for the SIR model with shadowing for different γ (data rate), σ and OP . The NCR for the SIR model with shadowing increases with an increase in γ or σ or with a decrease in OP . An increase in σ or a decrease in OP translates to an increase in the transmission power of mesh nodes. Higher transmission power causes more interference in the network which results in more conflicts in the conflict graph, thereby leading to a higher NCR.

**Table 9.15. NCR – Comparison of the three interference models
at a link data rate of 24 Mbps
($\gamma = 10.93$ dB, $\sigma = 3$ dB, $OP = 10\%$)**

Node-Degree Constraint	Interference Model	NCR	95% CI for NCR	Maximum value of NCR
2	Protocol Model	11.80	11.38–12.22	35.00
	SIR Model	22.84	22.22–23.46	
	SIR Model with Shadowing	31.56	31.02–32.10	
3	Protocol Model	13.56	12.91–14.21	36.56
	SIR Model	25.16	24.21–26.11	
	SIR Model with Shadowing	33.76	33.06–34.46	
4	Protocol Model	17.80	16.98–18.62	36.80
	SIR Model	30.36	29.59–31.13	
	SIR Model with Shadowing	35.76	35.09–36.43	
5	Protocol Model	21.04	20.10–21.98	36.20
	SIR Model	32.48	31.73–33.23	
	SIR Model with Shadowing	35.64	34.93–36.35	
6	Protocol Model	24.92	23.72–26.12	39.00
	SIR Model	35.72	34.76–36.68	
	SIR Model with Shadowing	38.36	37.52–39.20	

Table 9.16. NCR for SIR model – Comparison for different link data rates

Link Data Rate (Mbps)	NDC	NCR	95% CI for NCR
12	2	14.16	13.89–14.43
	3	15.76	15.36–16.16
	4	18.00	17.53–18.47
	5	21.40	20.40–22.40
	6	24.72	23.59–25.85
24	2	22.84	22.22–23.46
	3	25.16	24.21–26.11
	4	30.36	29.59–31.13
	5	32.48	31.73–33.23
	6	35.72	34.76–36.68
36	2	28.76	28.24–29.28
	3	30.72	30.05–31.39
	4	33.48	32.83–34.13
	5	35.36	34.66–36.06
	6	37.92	37.00–38.84
54	2	34.72	34.48–34.96
	3	36.68	36.33–37.03
	4	36.68	36.14–37.22
	5	36.76	36.23–37.29
	6	38.88	38.00–39.76

Figure 9.14. NCR – Comparison of the three interference models at a link data rate of 24 Mbps

Figure 9.15. NCR for SIR model – Comparison for different link data rates

Table 9.17. NCR for SIR model with shadowing – Comparison for different link data rates
($\sigma = 3$ dB, $OP = 10\%$)

Link Data Rate (Mbps)	NDC	NCR	95% CI for NCR
12	2	19.56	19.15–19.97
	3	22.24	21.72–22.76
	4	26.48	25.88–27.08
	5	30.32	29.36–31.28
	6	34.56	33.52–35.60
24	2	31.56	31.02–32.10
	3	33.76	33.06–34.46
	4	35.76	35.09–36.43
	5	35.64	34.93–36.35
	6	38.36	37.52–39.20
36	2	33.96	33.54–34.38
	3	35.56	35.02–36.10
	4	35.84	35.36–36.32
	5	36.64	36.14–37.14
	6	38.76	37.79–39.73
54	2	34.96	34.88–35.04
	3	36.96	36.61–37.31
	4	36.76	36.20–37.32
	5	36.80	36.27–37.33
	6	38.92	38.03–39.81

Figure 9.16. NCR for SIR model with shadowing – Comparison for different link data rates

Figure 9.17. NCR for SIR model with shadowing – Comparison for different σ s at a link data rate of 12 Mbps

Table 9.18. NCR for SIR model with shadowing – Comparison for different σ s at a link data rate of 12 Mbps ($\gamma = 5.78$ dB, $OP = 10\%$)

σ (dB)	NDC	NCR	95% CI for NCR
3	2	19.56	19.15–19.97
	3	22.24	21.72–22.76
	4	26.48	25.88–27.08
	5	30.32	29.36–31.28
	6	34.56	33.52–35.60
4	2	22.60	22.05–23.15
	3	25.32	24.56–26.08
	4	29.68	28.96–30.40
	5	32.96	32.05–33.87
	6	36.64	35.61–37.67
6	2	29.52	28.93–30.11
	3	32.08	31.37–32.79
	4	33.84	33.24–34.44
	5	35.72	34.91–36.53
	6	38.68	37.83–39.53
8	2	33.28	32.86–33.70
	3	35.32	34.70–35.94
	4	35.72	35.06–36.38
	5	36.92	36.25–37.59
	6	39.28	38.47–40.09

**Table 9.19. NCR for SIR model with shadowing – Comparison for different OP s at a link data rate of 12 Mbps
($\gamma = 5.78$ dB, $\sigma = 4$ dB)**

OP (%)	NDC	NCR	95% CI for NCR
10	2	22.60	22.05–23.15
	3	25.32	24.56–26.08
	4	29.68	28.96–30.40
	5	32.96	32.05–33.87
	6	36.64	35.61–37.67
8	2	23.92	23.47–24.37
	3	26.68	26.02–27.34
	4	30.44	29.77–31.11
	5	33.68	32.84–34.52
	6	37.28	36.30–38.26
5	2	26.60	26.13–27.07
	3	29.36	28.55–30.17
	4	32.24	31.55–32.93
	5	34.80	33.89–35.71
	6	38.08	37.21–38.95
2	2	30.28	29.71–30.85
	3	32.88	32.31–33.45
	4	34.40	33.75–35.05
	5	36.40	35.67–37.13
	6	38.88	38.02–39.74

Figure 9.18. NCR for SIR model with shadowing – Comparison for different OP s at a link data rate of 12 Mbps

9.6. Comparison of OAP-CA and LAB-CA

9.6.1. Simulation Setup

The performance of linear array beamforming-based channel assignment method LAB-CA presented in Chapter 8 is evaluated in comparison with omni-directional antenna pattern-based channel assignment method OAP-CA in terms of NCR for different NDCs at different link data rates.

For constructing the conflict matrices of the two channel assignment methods, the frequency is assumed to be 5.805 GHz, G_t and G_r to be 1 for OAP-CA, and h_t and h_r to be 3 m. The receiver thresholds are taken as -79 , -74 , -70 and -65 dBm for link data rates of 12, 24, 36, and 54 Mbps, respectively as per IEEE 802.11a. The SIR requirements are calculated as 5.78, 10.93, 13.2, and 18.42 dB for link data rates of 12, 24, 36, and 54 Mbps, respectively as per Table 9.13. The standard deviation for shadowing σ is set to 3 dB and the outage probability OP is set to 10%.

As stated in Section 8.5, TCA-based CG is used in combination with multi-path routing. The first feasible solution of the network throughput for multi-path routing is obtained using the CPLEX solver. This significantly reduces the solution time of the routing stage at the cost of a small degradation in the network throughput.

9.6.2. Results

The mean values and statistics on the 95% confidence intervals of the NCR are collected for the 25 different CRTs used for both channel assignment approaches. For brevity, mean values of NCR for different NDCs at different link data rates are graphed in Figs. 9.19, 9.20, 9.21, and 9.22 so that trends are immediately apparent. Note that γ represents the required SIR threshold in these figures.

The red line shows the worst case maximum value of NCR in which each link involved in routing requires a different frequency channel. LAB-CA requires significantly fewer frequency channels than OAP-CA for all node-degrees at all link data rates.

For a certain node-degree constraint, NCR for OAP-CA as well as NCR for LAB-CA increases with γ as the data rate increases. Higher γ means less tolerance for

interference in the network. This results in more conflicts in the conflict graph, leading to a higher NCR.

NCR for OAP-CA increases with NDC. As NDC increases, more nodes which are farther away from a node are selected as its data communication neighbors. This results in longer links in the connectivity graph. A longer link requires a higher transmission power, which causes more interference in the network. This results in more conflicts in the conflict graph, thereby leading to a higher NCR. NCR for LAB-CA, on the other hand, decreases with NDC because the beam formed by the array in LAB-CA becomes narrower as the number of antennas increases with NDC. The higher the NDC, the larger the number of antennas in the linear antenna array and the narrower the beam formed by the array; hence there is better spatial reuse, fewer conflicts in the conflict graph, and lower NCR when using LAB-CA.

Figure 9.19. NCR at a link data rate of 12 Mbps – OAP-CA vs. LAB-CA

Figure 9.20. NCR at a link data rate of 24 Mbps – OAP-CA vs. LAB-CA

Figure 9.21. NCR at a link data rate of 36 Mbps – OAP-CA vs. LAB-CA

Figure 9.22. NCR at a link data rate of 54 Mbps – OAP-CA vs. LAB-CA

9.7. Channel Assignment in Homogeneous vs. Heterogeneous Beamformed WMNs

9.7.1. Simulation Setup

The performance of linear array beamforming-based channel assignment method for heterogeneous mesh nodes LAB-CA_HT presented in Section 8.6 is evaluated in comparison with linear array beamforming-based channel assignment method for homogeneous mesh nodes LAB-CA_HG (or LAB-CA) presented in Section 8.5 in terms of (i) network throughput, (ii) solution time of routing stage, and (iii) NCR, for different NDCs at different link data rates.

In a homogeneous beamformed WMN, i.e. for LAB-CA_HG, the node-degrees of all mesh nodes in the network are the same. In a heterogeneous beamformed WMN, i.e. for LAB-CA_HT, mesh nodes are assigned different node-degrees. The node-degrees of mesh nodes are chosen using a uniform random distribution between two and some upper limit.

For a fair comparison of LAB-CA_HG and LAB-CA_HT, the GW is assigned the highest node-degree in the heterogeneous architecture. For example, for a heterogeneous mesh network where the node-degree of mesh nodes is randomly selected between two and four, the node-degree of the GW is set at four.

To construct the conflict matrices for the two channel assignment methods, the frequency is assumed to be 5.805 GHz, and h_t and h_r to be 3 m. The receiver thresholds are taken as -79 , -74 , -70 and -65 dBm as per IEEE 802.11a, and the SIR requirements are calculated as 5.78, 10.93, 13.2, and 18.42 dB as per Table 9.13 for link data rates of 12, 24, 36, and 54 Mbps, respectively. σ is set to 3 dB and OP is set to 10%. To reduce the solution time of the multi-path routing stage at the cost of a small degradation in the network throughput, the first feasible solution for network throughput is obtained using the CPLEX solver.

9.7.2. Results

The mean values and 95% confidence intervals of all measures are collected for the 25 different CRTs used for both channel assignment approaches. The solution times of the routing stage are presented in tabular form. For brevity, the mean values of network throughput and NCR for different NDCs at different link data rates are graphed

so that trends are apparent. The value "4, 2 to 4" for the NDC in a figure or a table means that all mesh nodes have a node-degree of 4 in LAB-CA_HG whereas in LAB-CA_HT, mesh nodes are randomly assigned a node-degree between two and four.

a) Solution Time of Routing Stage

Table 9.20 compares the two channel assignment methods at different link data rates in terms of solution time of the routing stage.

LAB-CA_HT has slightly higher solution times than LAB-CA_HG. Note that a node-degree of 2 is a hard constraint to satisfy during the routing of flows and due to the presence of such nodes in the heterogeneous beamformed WMN, it takes the solver more time to find the solution for network throughput.

Table 9.20. Solution Time of Routing Stage – LAB-CA_HG vs. LAB-CA_HT

Link Data Rate (Mbps)	NDC	Solution Time of Routing Stage (secs)			
		LAB-CA_HG		LAB-CA_HT	
		Mean	95% CI	Mean	95% CI
12	3, 2 to 3	1.52	1.27–1.78	2.07	1.79–2.36
	4, 2 to 4	1.77	1.48–2.07	2.36	2.05–2.67
	5, 2 to 5	1.44	1.23–1.65	2.83	2.50–3.16
	6, 2 to 6	1.79	1.59–1.99	2.80	2.28–3.31
24	3, 2 to 3	1.57	1.33–1.81	2.10	1.83–2.37
	4, 2 to 4	1.77	1.45–2.09	2.29	2.04–2.54
	5, 2 to 5	1.91	1.40–2.42	2.94	2.52–3.36
	6, 2 to 6	1.92	1.68–2.16	2.63	2.07–3.18
36	3, 2 to 3	1.48	1.27–1.70	1.94	1.69–2.19
	4, 2 to 4	1.86	1.48–2.24	2.25	2.05–2.44
	5, 2 to 5	1.49	1.19–1.79	2.72	2.32–3.11
	6, 2 to 6	1.94	1.61–2.27	3.10	2.37–3.82
54	3, 2 to 3	1.61	1.39–1.83	2.09	1.84–2.33
	4, 2 to 4	2.19	1.76–2.63	2.55	2.28–2.82
	5, 2 to 5	1.85	1.43–2.28	3.16	2.67–3.65
	6, 2 to 6	2.49	2.00–2.97	3.53	2.90–4.15

b) Network Throughput

Fig. 9.23 compares the two channel assignment methods at different link data rates in terms of network throughput. These results show that the two methods have similar performance in terms of NT.

Figure 9.23. Network Throughput (Mbps) at different link data rates – LAB-CA_HG vs. LAB-CA_HT

Table 9.21 compares the two CA methods at different link data rates in terms of the number of CRTs where the first feasible solution for NT is the optimum solution. Note that the CA method that has more CRTs in which the first feasible solution is the optimum solution achieves a slightly better NT. For example, for NDC of "4, 2 to 4" at a link data rate of 54 Mbps, the number of CRTs in which the first feasible solution is the optimum solution is 13 for LAB-CA_HG and 16 for LAB-CA_HT in Table 9.21, and the corresponding NT is 90.17 Mbps for LAB-CA_HG and 97.79 Mbps for LAB-CA_HT.

Table 9.21. Number of CRTs with Optimum Solution of NT – LAB-CA_HG vs. LAB-CA_HT

Link Data Rate (Mbps)	NDC	Number of CRTs with Optimum Solution of NT	
		LAB-CA_HG	LAB-CA_HT
12	3, 2 to 3	16	11
	4, 2 to 4	8	17
	5, 2 to 5	19	22
	6, 2 to 6	18	12
24	3, 2 to 3	17	10
	4, 2 to 4	11	15
	5, 2 to 5	19	20
	6, 2 to 6	16	9
36	3, 2 to 3	14	8
	4, 2 to 4	6	17
	5, 2 to 5	18	20
	6, 2 to 6	17	12
54	3, 2 to 3	19	8
	4, 2 to 4	13	16
	5, 2 to 5	21	19
	6, 2 to 6	16	13

c) Number of Frequency Channels Required

Fig. 9.24 compares the two channel assignment methods at different link data rates in terms of NCR. LAB-CA_HT requires more frequency channels than LAB-CA_HG.

In LAB-CA_HG at a NDC of six, a mesh node uses all six antennas in its linear antenna array to form a beam. On the other hand, in LAB-CA_HT when the NDC is between two and six and a mesh node is randomly assigned a node-degree of two, it uses a two-antenna linear array to form a beam. The beams formed by mesh nodes with a

node-degree of two, three, four or five in LAB-CA_HT are less focused than those formed by mesh nodes with a six-antenna linear array. The more focused beams formed by the six-antenna linear array of all mesh nodes in LAB-CA_HG lead to better spatial reuse, fewer conflicts and lower NCR.

Figure 9.24. NCR at different link data rates – LAB-CA_HG vs. LAB-CA_HT

Also, statistics were collected on average link transmission power and $NCCM/|L|^2$ for the two CA approaches. They are presented in Table 9.22, where NCCM is the number of conflicts in the conflict matrix and $|L|$ is the number of links involved in routing. $NCCM/|L|^2$ indicates the fraction of possible conflicts. A lower $NCCM/|L|^2$ is better; its maximum value is 1 when the conflict graph is fully connected, meaning that all the elements in the conflict matrix are equal to 1.

The average link transmission power and $NCCM/|L|^2$ for both CA approaches decrease as NDC increases. The beam formed by the linear antenna array becomes narrower as the number of antennas increases with the increase in NDC, resulting in a

decrease in $NCCM/|L|^2$. Also, the array gain of the linear antenna array increases as the number of antennas increases with the increase in NDC, which results in a decrease in the average link transmission power. The decrease in average link transmission power as well as $NCCM/|L|^2$ leads to a lower NCR.

Table 9.22. Average Link Transmission Power and $NCCM/|L|^2$ – LAB-CA_HG vs. LAB-CA_HT

Link Data Rate (Mbps)	NDC	LAB-CA_HG		LAB-CA_HT	
		Average Link Transmission Power (mW)	$NCCM/ L ^2$	Average Link Transmission Power (mW)	$NCCM/ L ^2$
12	3, 2 to 3	0.60	0.2356	0.87	0.2711
	4, 2 to 4	0.41	0.2345	0.67	0.2553
	5, 2 to 5	0.31	0.2297	0.54	0.2477
	6, 2 to 6	0.24	0.2300	0.47	0.2413
24	3, 2 to 3	1.87	0.4305	2.80	0.4980
	4, 2 to 4	1.32	0.3885	2.19	0.4593
	5, 2 to 5	0.98	0.3632	1.69	0.4160
	6, 2 to 6	0.76	0.3256	1.49	0.3882
36	3, 2 to 3	4.62	0.5190	7.01	0.6070
	4, 2 to 4	3.33	0.4605	5.30	0.5426
	5, 2 to 5	2.47	0.4274	4.24	0.4979
	6, 2 to 6	1.91	0.3896	3.69	0.4674
54	3, 2 to 3	14.62	0.7159	21.40	0.7974
	4, 2 to 4	10.52	0.6477	16.90	0.7266
	5, 2 to 5	7.69	0.6018	13.62	0.6918
	6, 2 to 6	6.07	0.5408	11.88	0.6545

$NCCM/|L|^2$ for LAB-CA_HG is lower than that for LAB-CA_HT due to the more focused beams formed by the homogeneous antenna arrays of mesh nodes in LAB-

CA_HG and results in a lower NCR for LAB-CA_HG. Also, for a given node-degree constraint, an increase in the receiver threshold (i.e. $RxThresh_dBm$ in Algorithms 8.1 and 8.2) leads to a an increase in the average link transmission power as the link data rate increases. A higher link transmission power causes more interference and more conflicts in the conflict graph which results in an increase in NCR for both CA approaches with the increase in link data rate for a given NDC.

9.8. Channel Assignment using Heuristic vs. Exact Solution for Minimum Coloring with Cumulative Interference Constraints

9.8.1. Simulation Setup

The performance of the heuristic solution presented in Chapter 7 is evaluated for the minimum coloring stage of the beamforming-based channel assignment methods, LAB-CA_HG and LAB-CA_HT, on conflict graphs generated from 25 different controlled random topologies. For some CRTs, the node-degree of all mesh nodes is fixed at a homogeneous value of six (LAB-CA_HG), and for others the node-degree of each mesh node is assigned a heterogeneous random value between two and six using a uniform random distribution (LAB-CA_HT). Different link data rates are used.

The main metrics for evaluation of the heuristic solution are (i) the number of frequency channels used, (ii) solution time in seconds, and (iii) difference (or maximum possible difference) from the true minimum number of frequency channels required. The true minimum value is provided by an exact QCBP formulation written in the AMPL modeling language and solved using IBM's CPLEX 12.2 solver.

Binary programming solutions can take a very long time, especially in problems such as this one that have a great deal of symmetry. To speed the solution the search space is reduced by setting the maximum number of colors that can be used equal to the number of frequency channels required returned by the heuristic solution, which will be a close upper bound on the true minimum number of channels required. A formulation is also used that helps to limit the number of equivalent symmetric solutions in the search space [87][88]. All CPLEX parameters are set at their default values, and the solution time is limited to 3600 seconds. Where the QCP solver terminates successfully the *exact difference* is calculated between the NCR for the heuristic and the optimum solution. When the QCP solver times out the best bounding function value at time-out is used to calculate the *maximum possible difference* between the NCR for the heuristic and the optimum solution. Note that the true difference may be much lower than the maximum possible difference, and in fact may be zero, however the QCP solver is not able to determine that within the time limit.

9.8.2. Direct Conflicts

The conflict graph reflects only the direct conflicts between pairs of wireless links (direct conflicts as well as smaller interference quantities are captured in the conflict matrix). Still, the number of direct conflicts in the conflict graph (or conflict matrix) is a measure of how difficult the channel assignment problem is. Table 9.23 shows the *number of (direct) conflicts in the conflict matrix* (NCCM) for LAB-CA_HT and LAB-CA_HG. The average number of conflicts in the conflict matrix for the twenty-five CRTs is 378, 538 and 876 for LAB-CA_HG at link data rates of 12 Mbps, 24 Mbps and 54 Mbps respectively; and 390, 627 and 1058 for LAB-CA_HT at link data rates of 12 Mbps, 24 Mbps and 54 Mbps respectively. The number of conflicts in the conflict matrix increases as the link data rate increases for both LAB-CA_HT and LAB-CA_HG.

Many coloring instances are available in the literature, but cannot be used here since they lack the interference data needed to construct the conflict matrix. The conflict matrix is essential to solving the extended coloring problem because it allows to detect a group conflict which cannot be represented by a simple conflict graph.

9.8.3. Results

The evaluation of the heuristic in terms of NCR is presented in Tables 9.24, 9.25 and 9.26 for LAB-CA_HT at link data rates of 54 Mbps, 24 Mbps and 12 Mbps respectively; and in Tables 9.27, 9.28 and 9.29 for LAB-CA_HG at link data rates of 54 Mbps, 24 Mbps and 12 Mbps respectively.

In Table 9.24 for CRT #2, the optimum solution NCR is 18, the heuristic solution NCR is 19, and the exact difference between the heuristic and optimum solutions is 1 channel. Non-zero values of *absmipgap* and *relmipgap* are returned by the CPLEX QCP solver when it exceeds the time limit. *Absmipgap* is the absolute value of the difference between the best integer solution found by time-out and the lowest bound for any unsolved node remaining in the search tree at time-out. *Relmipgap* is the ratio between *absmipgap* and the best integer solution found by time-out. For example, in Table 9.24 for CRT #4, the best integer solution found by time-out is 14 (as shown in the QCBP NCR column), *absmipgap* is 2 and *relmipgap* $\times$ 100 is 14.29. This means that the NCR of 14 is within 2 channels (14.29%) of the optimum solution, i.e. the true optimum NCR

could be 12 or 13 or 14. Since the heuristic NCR is 16, the maximum possible difference is $16 - 12 = 4$.

As shown in Tables 9.24–9.29, there are no cases with a maximum possible difference of 4 or greater except for CRT #4 and CRT #8 in Table 9.24. The *absmipgap* was 2 in both cases. These cases were re-run without any time limit and with n_c equal to 12 for CRT #4 and 11 for CRT #8. CPLEX found 12 to be infeasible for CRT #4 after 980.84 secs and 11 to be infeasible for CRT #8 after 181072.26 secs. This means that the optimum solution for CRT #4 cannot be 12 but could be 13 or 14. The heuristic solution for this CRT is 16, so the maximum possible error of the heuristic is only 3 channels. Similarly, the optimum solution for CRT #8 cannot be 11 but could be 12 or 13. The heuristic solution for this CRT is 15, so the maximum possible error of the heuristic solution is 3 channels.

The evaluation of the heuristic in terms of exact difference and maximum possible difference from the optimum solution is presented in Figs. 9.25 and 9.26 for LAB-CA_HT at different link data rates, and in Figs. 9.27 and 9.28 for LAB-CA_HG at different link data rates. As shown in these figures and Tables 9.24–9.29, the heuristics provide reasonable solutions for NCR in comparison with the true optimum. They find the true optimum solution for 11 CRTs for LAB-CA_HT and for 18 CRTs for LAB-CA_HG.

Where the optimum solution is known (i.e. CPLEX does not time out), the error of the heuristic is at most one channel more than the optimum solution for 33 out of 45 CRTs for LAB-CA_HT (Fig. 9.25), and for 45 out of 53 CRTs for LAB-CA_HG (Fig. 9.27). The maximum error is at most two channels for all CRTs (Figs. 9.25 and 9.27).

When the true optimum is not known, the maximum possible error is found by comparing to the bound returned by the QCP solver at time-out. The maximum possible heuristic error in this case is at most two channels for 63 out of 75 CRTs for LAB-CA_HT (Fig. 9.26), and for 66 out of 75 CRTs for LAB-CA_HG (Fig. 9.28). The maximum possible error for all CRTs is at most three channels (Figs. 9.26 and 9.28).

The heuristic is significantly faster than the exact QCBP method, by about three orders of magnitude on average. The average time difference between the QCBP solution

and the heuristic solution is around 1418 seconds over the test set. There is only a single instance in which the QCBP solution is faster (by 0.125 seconds).

The exact QCBP solution time increases with the increase in the number of conflicts in the conflict matrix as this creates a larger problem that is more difficult to solve. This is seen as the link data rate increases for both LAB-CA_HT (Tables 9.24–9.26) and LAB-CA_HG (Tables 9.27–9.29).

Table 9.23. Number of Conflicts in the Conflict Matrix at different link data rates for LAB-CA_HT and LAB-CA_HG

CRT	LAB-CA_HT			LAB-CA_HG		
	NCCM (54 Mbps)	NCCM (24 Mbps)	NCCM (12 Mbps)	NCCM (54 Mbps)	NCCM (24 Mbps)	NCCM (12 Mbps)
1	1045	607	395	914	499	376
2	1068	635	407	849	597	363
3	979	609	363	842	553	342
4	960	637	325	891	585	380
5	1150	704	423	809	524	409
6	1147	640	406	939	494	371
7	1092	735	459	823	537	376
8	875	528	326	766	488	322
9	968	534	373	820	536	376
10	1162	754	420	967	587	389
11	1099	632	424	894	479	352
12	1057	667	419	1003	607	394
13	1081	527	341	807	459	335
14	1014	612	403	808	547	390
15	1019	631	426	862	615	443
16	1032	612	431	878	537	362
17	1056	566	381	874	466	395
18	955	574	348	861	479	361
19	1039	713	422	824	574	367
20	1087	533	386	907	576	386
21	1135	712	367	869	527	335
22	1037	509	369	951	551	440
23	1219	753	423	914	499	376
24	1091	673	384	940	654	406
25	1090	570	338	881	479	393

**Table 9.24. NCR using LAB-CA_HT: Heuristic vs. QCBP solution
(link data rate: 54 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	18	1.681	16	0	0	132.610	2	2
2	19	1.556	18	0	0	31.497	1	1
3	16	1.563	15	0	0	1293.870	1	1
4	16	1.628	14	2	14.29	3640.510	-	4
5	19	1.569	18	1	5.56	3600.990	-	2
6	20	1.603	20	0	0	100.603	0	0
7	21	1.571	21	0	0	24.674	0	0
8	15	1.599	13	2	15.38	3641.650	-	4
9	17	1.602	15	1	6.67	3629.010	-	3
10	20	1.686	19	0	0	155.627	1	1
11	18	1.538	18	1	5.56	3617.850	-	1
12	18	1.669	17	0	0	1911.290	1	1
13	17	1.702	16	1	6.25	3606.300	-	2
14	18	1.643	16	1	6.25	3602.140	-	3
15	18	1.581	17	0	0	211.110	1	1
16	18	1.552	17	2	11.76	3660.250	-	3
17	17	1.597	16	1	6.25	3640.240	-	2
18	16	1.575	15	1	6.67	3617.060	-	2
19	19	1.625	19	0	0	24.507	0	0
20	19	1.630	18	2	11.11	3909.370	-	3
21	20	1.694	19	0	0	100.793	1	1
22	20	1.542	19	0	0	55.849	1	1
23	20	1.601	19	0	0	178.201	1	1
24	20	1.593	20	0	0	30.592	0	0
25	18	1.565	16	0	0	253.082	2	2

**Table 9.25. NCR using LAB-CA_HT: Heuristic vs. QCBP solution
(link data rate: 24 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	12	1.736	11	1	9.09	3600.560	-	2
2	13	1.650	11	0	0	71.090	2	2
3	12	1.621	12	0	0	8.877	0	0
4	13	1.681	12	0	0	86.362	1	1
5	14	1.698	12	1	8.33	3611.990	-	3
6	14	1.658	12	0	0	58.251	2	2
7	14	1.733	13	0	0	107.391	1	1
8	10	1.566	9	0	0	47.830	1	1
9	11	1.613	10	1	10.00	3600.440	-	2
10	14	1.604	14	0	0	37.005	0	0
11	12	1.612	11	0	0	64.357	1	1
12	13	1.682	12	0	0	65.590	1	1
13	11	1.685	10	1	10.00	3600.500	-	2
14	13	1.753	13	0	0	17.096	0	0
15	13	1.630	12	1	8.33	3600.390	-	2
16	13	1.687	11	1	9.09	3606.210	-	3
17	11	1.603	10	0	0	59.186	1	1
18	12	1.622	11	2	18.18	3656.220	-	3
19	13	1.603	13	0	0	17.270	0	0
20	11	1.598	9	0	0	96.093	2	2
21	14	1.741	13	1	7.69	3618.120	-	2
22	10	1.675	9	1	11.11	3606.890	-	2
23	14	1.838	12	1	8.33	3609.890	-	3
24	14	1.647	13	0	0	29.515	1	1
25	11	1.645	10	0	0	318.788	1	1

**Table 9.26. NCR using LAB-CA_HT: Heuristic vs. QCBP solution
(link data rate: 12 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	9	1.634	8	0	0	9.282	1	1
2	10	1.691	8	0	0	48.501	2	2
3	9	1.700	8	1	12.50	3600.290	-	2
4	9	1.709	7	0	0	60.559	2	2
5	10	1.722	9	1	11.11	3600.330	-	2
6	10	1.880	8	0	0	90.340	2	2
7	11	1.721	11	0	0	414.307	0	0
8	7	1.637	7	0	0	2780.240	0	0
9	9	1.666	7	0	0	17.490	2	2
10	10	1.777	9	0	0	13.574	1	1
11	10	1.719	9	1	11.11	3600.620	-	2
12	11	1.786	9	1	11.11	3608.050	-	3
13	8	1.770	7	0	0	3549.510	1	1
14	10	1.733	8	0	0	55.474	2	2
15	10	1.653	9	0	0	384.777	1	1
16	11	1.718	9	0	0	68.921	2	2
17	9	1.637	8	0	0	218.229	1	1
18	9	1.722	8	1	12.50	3600.440	-	2
19	11	1.709	10	0	0	30.218	1	1
20	10	1.742	9	2	22.22	3629.250	-	3
21	9	1.850	8	1	12.50	3600.490	-	2
22	9	1.693	8	1	12.50	3604.500	-	2
23	11	1.752	9	0	0	45.365	2	2
24	10	1.803	10	0	0	7.940	0	0
25	8	1.688	7	1	14.29	3600.350	-	2

**Table 9.27. NCR using LAB-CA_HG: Heuristic vs. QCBP solution
(link data rate: 54 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	15	1.649	13	1	7.69	3618.600	-	3
2	14	1.551	13	0	0	2881.960	1	1
3	14	1.584	13	1	7.69	3609.410	-	2
4	15	2.221	13	1	7.69	3616.540	-	3
5	14	1.636	12	1	8.33	3645.270	-	3
6	15	1.782	13	1	7.69	3617.310	-	3
7	16	1.560	14	0	0	85.167	2	2
8	13	1.676	11	0	0	156.612	2	2
9	16	1.543	14	1	7.14	3625.000	-	3
10	18	1.613	18	0	0	46.156	0	0
11	15	1.692	13	0	0	120.686	2	2
12	17	1.615	17	1	5.88	3640.110	-	1
13	14	1.724	12	0	0	133.550	2	2
14	14	1.590	13	0	0	81.199	1	1
15	16	1.530	16	1	6.25	3600.540	-	1
16	16	1.580	15	0	0	19.048	1	1
17	14	1.573	13	1	7.69	3613.520	-	2
18	14	1.497	13	1	7.69	3600.460	-	2
19	14	1.530	13	0	0	58.407	1	1
20	16	1.582	15	0	0	33.619	1	1
21	16	1.544	16	0	0	15.366	0	0
22	17	1.547	16	0	0	16.349	1	1
23	15	1.629	13	1	7.69	3620.210	-	3
24	17	1.538	17	1	5.88	3618.400	-	1
25	15	1.637	13	1	7.69	3601.180	-	3

**Table 9.28. NCR using LAB-CA_HG: Heuristic vs. QCBP solution
(link data rate: 24 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	11	1.733	10	0	0	4.978	1	1
2	12	1.596	10	0	0	44.336	2	2
3	11	1.667	10	1	10.00	3600.550	-	2
4	11	1.698	10	0	0	22.246	1	1
5	10	1.586	9	2	20.55	3621.610	-	3
6	10	1.646	8	0	0	79.732	2	2
7	12	1.634	11	0	0	26.427	1	1
8	9	1.585	8	1	12.50	3600.360	-	2
9	11	1.612	10	0	0	14.150	1	1
10	12	3.223	12	0	0	12.527	0	0
11	9	1.615	8	0	0	347.181	1	1
12	11	1.632	11	0	0	12.137	0	0
13	9	1.733	8	0	0	571.806	1	1
14	12	1.613	10	0	0	36.879	2	2
15	11	1.645	11	0	0	10.686	0	0
16	11	1.582	11	0	0	4.836	0	0
17	10	1.635	9	0	0	22.838	1	1
18	10	1.648	10	3	28.52	3655.120	-	3
19	12	1.586	12	0	0	3.995	0	0
20	12	1.623	11	0	0	31.226	1	1
21	12	1.519	12	0	0	5.243	0	0
22	11	1.657	10	1	10.00	3600.570	-	2
23	11	1.685	10	0	0	4.930	1	1
24	12	1.647	11	1	9.09	3600.430	-	2
25	9	1.597	8	0	0	23.494	1	1

**Table 9.29. NCR using LAB-CA_HG: Heuristic vs. QCBP solution
(link data rate: 12 Mbps)**

C R T	Heuristic Solution		QCBP Solution				Exact Difference	Maximum Possible Difference
	NCR	Time (secs)	NCR	absmipgap	relmipgap $\times 100$	Time (secs)		
1	9	1.654	9	0	0	2.402	0	0
2	8	1.625	7	0	0	10.826	1	1
3	8	1.576	8	0	0	1.451	0	0
4	8	1.689	8	0	0	7.066	0	0
5	9	1.659	8	0	0	13.790	1	1
6	9	1.697	8	0	0	3361.870	1	1
7	10	1.766	9	0	0	9.594	1	1
8	8	1.571	7	0	0	1448.080	1	1
9	8	1.677	8	0	0	1323.150	0	0
10	10	1.646	9	0	0	8.143	1	1
11	7	1.609	6	0	0	14.758	1	1
12	9	1.662	8	1	12.50	3600.320	-	2
13	8	1.710	7	0	0	8.518	1	1
14	9	1.759	8	0	0	19.251	1	1
15	10	1.578	10	0	0	2.184	0	0
16	9	1.608	8	0	0	7.254	1	1
17	9	1.640	8	1	12.50	3600.350	-	2
18	9	1.639	8	0	0	9.126	1	1
19	8	1.607	8	0	0	3.136	0	0
20	10	1.633	10	0	0	4.212	0	0
21	8	1.680	8	0	0	2.964	0	0
22	9	1.725	9	0	0	6.240	0	0
23	9	1.664	9	0	0	2.356	0	0
24	9	1.661	8	1	12.50	3600.330	-	2
25	9	1.710	7	0	0	96.954	2	2

Figure 9.25. Exact difference between Heuristic Solution and QCBP Solution using LAB-CA_HT at different link data rates

Figure 9.26. Maximum possible difference between Heuristic Solution and QCBP Solution using LAB-CA_HT at different link data rates

Figure 9.27. Exact difference between Heuristic Solution and QCBP Solution using LAB-CA_HG at different link data rates

Figure 9.28. Maximum possible difference between Heuristic Solution and QCBP Solution using LAB-CA_HG at different link data rates

9.9. Channel Assignment in Sparse vs. Dense Beamformed Heterogeneous WMNs

9.9.1. Simulation Setup

The performance of LAB-CA_HT presented in Chapter 8 is evaluated in sparse as well as dense mesh networks. This is done by comparing its performance with the omnidirectional antenna pattern-based channel assignment method for heterogeneous mesh nodes OAP-CA_HT in terms of (i) NCR, (ii) LCR, (iii) NT, and (iv) solution time of the routing stage for different node-degree constraints.

For sparse mesh networks, 25 different controlled random topologies are generated by distributing 36 mesh nodes in a $500 \text{ m} \times 500 \text{ m}$ area whereas for dense mesh networks, 25 different controlled random topologies are generated by distributing 100 mesh nodes in the same area.

To construct the conflict matrices for OAP-CA_HT and LAB-CA_HT, the frequency is assumed to be 5.805 GHz, G_t and G_r to be 1 for OAP-CA_HT, and h_t and h_r to be 3 m. For this comparison, σ is 3 dB, OP is 10% and link data rate is set at 54 Mbps. So, the receiver threshold is taken as -65 dBm , and the required SIR threshold is 18.42 dB as per Table 9.13. To reduce the solution time of the multi-path routing stage at the cost of a small degradation in the network throughput, the first feasible solution for network throughput is obtained using the CPLEX solver.

9.9.2. Results

The mean values and 95% confidence intervals for all measures are collected for the 25 different CRTs for both CA approaches in dense as well as sparse scenarios; these are reported in tables.

a) Number of Frequency Channels Required

Tables 9.30 and 9.31 compare NCR between LAB-CA_HT and OAP-CA_HT in sparse and dense mesh networks, respectively. PR_{NCR} in these tables represents the percentage reduction in NCR when using LAB-CA_HT and is given by

$$PR_{NCR} = \left(\frac{NCR_{OAP-CA_HT} - NCR_{LAB-CA_HT}}{NCR_{OAP-CA_HT}} \right) \times 100. \quad (9.4)$$

Table 9.30. NCR in Sparse Mesh Networks – OAP-CA_HT vs. LAB-CA_HT

NDC	OAP-CA_HT		LAB-CA_HT		PR_{NCR}
	NCR	95% CI for NCR	NCR	95% CI for NCR	
2 to 3	36.00	35.66–36.34	21.28	20.59–21.97	40.9
2 to 4	37.56	37.12–38.00	19.52	18.88–20.16	48.0
2 to 5	36.40	35.83–36.97	17.80	17.28–18.32	51.1
2 to 6	38.20	37.30–39.10	17.44	16.83–18.05	54.3

Table 9.31. NCR in Dense Mesh Networks – OAP-CA_HT vs. LAB-CA_HT

NDC	OAP-CA_HT		LAB-CA_HT		PR_{NCR}
	NCR	95% CI for NCR	NCR	95% CI for NCR	
2 to 3	94.76	94.05–95.47	39.32	38.79–39.85	58.5
2 to 4	97.32	96.58–98.06	35.64	34.94–36.34	63.4
2 to 5	100.28	99.73–100.83	32.80	32.17–33.43	67.3
2 to 6	100.20	99.73–100.67	30.28	29.52–31.04	69.8

These results clearly show that LAB-CA_HT requires significantly fewer frequency channels as compared to OAP-CA_HT for all NDCs in sparse as well as dense mesh networks. In fact, it performs even better in dense mesh networks and provides a reduction of up to 69% in NCR.

Tables 9.32 and 9.33 compare the two CA approaches in terms of average link transmission power and $NCCM/|L|^2$ for sparse and dense scenarios respectively.

The average link transmission power and $NCCM/|L|^2$ for OAP-CA_HT increase with the increase in NDC. As NDC increases, more nodes which are farther away are selected as data communication neighbors of a node. This results in longer links in the

connectivity graph which increases the average link transmission power for the network as well as $NCCM/|L|^2$, leading to a higher NCR.

Table 9.32. Average Link Transmission Power and $NCCM/|L|^2$ in Sparse Mesh Networks – OAP-CA_HT vs. LAB-CA_HT

NDC	L	OAP-CA_HT		LAB-CA_HT	
		Average Link Transmission Power (mW)	$NCCM/ L ^2$	Average Link Transmission Power (mW)	$NCCM/ L ^2$
2 to 3	36.04	351.10	0.9937	21.40	0.7974
2 to 4	37.60	398.68	0.9940	16.90	0.7266
2 to 5	36.44	445.89	0.9966	13.62	0.6918
2 to 6	38.24	476.66	0.9966	11.88	0.6545

Table 9.33. Average Link Transmission Power and $NCCM/|L|^2$ in Dense Mesh Networks – OAP-CA_HT vs. LAB-CA_HT

NDC	L	OAP-CA_HT		LAB-CA_HT	
		Average Link Transmission Power (mW)	$NCCM/ L ^2$	Average Link Transmission Power (mW)	$NCCM/ L ^2$
2 to 3	99.96	115.80	0.9610	7.05	0.5910
2 to 4	101.7	129.71	0.9679	5.45	0.5219
2 to 5	103.1	146.58	0.9780	4.62	0.4846
2 to 6	102.5	162.49	0.9806	3.95	0.4437

On the other hand, the average link transmission power and $NCCM/|L|^2$ for LAB-CA_HT decrease as NDC increases. In LAB-CA_HT, the beam formed by the linear antenna array becomes narrower as the number of antennas increases with the increase in NDC, which results in a decrease in $NCCM/|L|^2$. Also, the array gain of the linear antenna array increases as the number of antennas increases with the increase in NDC,

which results in a decrease in the average link transmission power. The decrease in average link transmission power as well as $NCCM/|L|^2$ leads to a lower NCR.

b) Links to Channels Ratio

Table 9.34 compares LCR between sparse and dense mesh networks when using LAB-CA_HT. PI_{LCR} in this table represents the percentage improvement in LCR when using LAB-CA in dense mesh networks and is given by

$$PI_{LCR} = \left(\frac{LCR_{\text{dense}} - LCR_{\text{sparse}}}{LCR_{\text{sparse}}} \right) \times 100. \quad (9.5)$$

LAB-CA_HT has a higher LCR in dense mesh networks and provides an improvement of up to 53% in dense mesh networks as compared to sparse mesh networks.

Table 9.34. LCR using LAB-CA_HT – Sparse vs. Dense

NDC	Sparse Mesh Networks		Dense Mesh Networks		PI_{LCR}
	LCR	95% CI for LCR	LCR	95% CI for LCR	
2 to 3	1.70	1.65–1.76	2.54	2.51–2.58	49.4
2 to 4	1.94	1.87–2.01	2.86	2.80–2.92	47.4
2 to 5	2.06	2.00–2.12	3.15	3.09–3.21	52.9
2 to 6	2.21	2.13–2.28	3.40	3.32–3.48	53.8

As can be seen from Tables 9.32 and 9.33, the average link transmission powers for LAB-CA_HT in dense mesh networks are less than those in sparse mesh networks. Nodes in a dense mesh network are closely spaced. The links between the nodes in the connectivity graph are shorter, which leads to lower average link transmission powers, which in turn translates to less interference, as reflected by a lower $NCCM/|L|^2$ in dense mesh networks as compared to sparse mesh networks. Less interference improves the spatial channel reuse leading to a greater reduction in the number of frequency channels and a higher LCR in dense vs. sparse networks.

c) Network Throughput

Tables 9.35 and 9.36 compare network throughput and throughput per node (TPN) respectively between sparse and dense mesh networks. NT in dense mesh networks is similar to that in sparse mesh networks. Note that the total amount of flow in the network depends on the number of links for the GW node, which increases with an increase in NDC. This results in an increase in NT with an increase in NDC. At a NDC of 2 to 3, there are three links for the GW and the maximum NT is equal to *capacity per link* $\times$ *no. of links* = 24.73 Mbps $\times$ 3 = 74.19 Mbps; at NDC of 2 to 4, there are four links for the GW and the maximum NT is 98.92 Mbps and so on. Note that there is a small degradation in the achieved network throughput as compared to its maximum value, due to finding the first feasible solution as opposed to finding the optimum solution.

Table 9.35. Network Throughput (Mbps) – Sparse vs. Dense

NDC	Sparse Mesh Networks		Dense Mesh Networks		Max. NT (Mbps)
	NT (Mbps)	95% CI for NT (Mbps)	NT (Mbps)	95% CI for NT (Mbps)	
2 to 3	66.40	62.90–69.90	62.49	57.56–67.41	74.19
2 to 4	97.79	97.14–98.45	93.30	88.56–98.05	98.92
2 to 5	118.06	113.63–122.50	118.91	115.11–122.71	123.65
2 to 6	135.08	128.04–142.12	147.42	145.87–148.98	148.38

Table 9.36 shows that TPN in dense mesh networks is less than that in sparse mesh networks. Note that all mesh nodes, except the GW, are sources of flow and the GW is the sink of all flows. In Table 9.35, the maximum NT at a NDC of 2 to 3 is 74.19 Mbps. In a sparse mesh network, this maximum NT is shared by 35 sources which results in a maximum TPN of 2.1197 Mbps. On the other hand, in a dense mesh network, this maximum NT is shared by 99 sources which results in a lower maximum TPN of 0.7493 Mbps.

Table 9.36. Throughput per Node (Mbps) – Sparse vs. Dense

NDC	Sparse Mesh Networks			Dense Mesh Networks		
	TPN (Mbps)	95% CI for TPN (Mbps)	Max. TPN (Mbps)	TPN (Mbps)	95% CI for TPN (Mbps)	Max. TPN (Mbps)
2 to 3	1.8972	1.7972–1.9972	2.1197	0.6312	0.5815–0.6809	0.7493
2 to 4	2.7941	2.7754–2.8128	2.8262	0.9424	0.8945–0.9904	0.9991
2 to 5	3.3732	3.2465–3.4999	3.5328	1.2011	1.1627–1.2395	1.2489
2 to 6	3.8595	3.6584–4.0605	4.2394	1.4891	1.4734–1.5048	1.4987

d) Solution Time of Routing Stage

The solution times for dense mesh networks are higher than those for sparse mesh networks, at about 169–350 seconds for dense mesh networks and 2.05–3.55 seconds for sparse mesh networks. The number of links in the connectivity graph is much higher for dense mesh networks (e.g. 526 links in the connectivity graph of CRT #1 at NDC of 2 to 6) than that for sparse mesh networks (e.g. 194 links in the connectivity graph of CRT #1 at NDC of 2 to 6). This results in a much bigger routing problem for dense mesh networks, which causes the solver to take more time to find the solution for network throughput.

9.10. Channel Assignment in MRMC vs. MRMCMCMA Beamformed Heterogeneous Dense WMNs

9.10.1. Simulation Setup

The impact of increasing the number of antennas in the linear antenna arrays of the multi-radio mesh nodes on the performance of LAB-CA_HT is investigated. Its performance is evaluated in dense MRMCMCMA vs. dense MRMC WMNs in terms of NCR for different numbers of antennas.

For this comparison, σ is 3 dB, OP is 0.1 and link data rate is set at 54 Mbps. To generate a dense mesh network, 100 mesh nodes are distributed in a $500 \text{ m} \times 500 \text{ m}$ area.

In the MRMC WMN, the node-degree of mesh nodes is equal to the number of antennas in their linear antenna arrays. For example, a node-degree of 2 for a mesh node indicates two data radio interfaces and two antennas for that node. In the MRMCMCMA WMN, a node-degree of 2 for a mesh node also indicates two data radios but the number of antennas in its antenna array is more than 2, i.e. 3 or more.

9.10.2. Results

Table 9.37 shows the average link transmission power, $NCCM/|L|^2$ and NCR when using LAB-CA_HT in a MRMCMCMA vs. a MRMC dense WMN.

Note that N_a in Table 9.37 indicates the number of antennas in the linear antenna array of a multi-radio mesh node. A N_a of "2 to 6" in a heterogeneous MRMC WMN means that a mesh node is randomly assigned a node-degree between 2 and 6 and the number of antennas in its linear antenna array is equal to its node-degree. A mesh node that is randomly assigned a node-degree of six (or six antennas) in the heterogeneous MRMC WMN has seven antennas when N_a is "3 to 7" in the heterogeneous MRMCMCMA WMN in Table 9.37; eight antennas when N_a is "4 to 8"; and so on.

PR_{NCR} (i.e. the percentage reduction in NCR) in Table 9.37 is calculated using

$$PR_{NCR} = \left(\frac{NCR_{\text{LAB-CA_HT}_{\text{MRMC}}} - NCR_{\text{LAB-CA_HT}_{\text{MRMCMCMA}}}}{NCR_{\text{LAB-CA_HT}_{\text{MRMC}}}} \right) \times 100. \quad (9.6)$$

LAB-CA_HT performs better in dense MRMCMCMA vs. dense MRMC WMNs and provides a reduction of up to 54% in NCR when N_a is increased from "2 to 6" to "8 to

12". As N_a increases, the array gain increases and the average link transmission power decreases. Also, the array beam becomes more focused with the increase in N_a which reduces $NCCM/|L|^2$. The decrease in average link transmission power as well as $NCCM/|L|^2$ leads to a lower NCR when using LAB-CA_HT in dense MRMCMAs.

Table 9.37. Average Link Transmission Power, $NCCM/|L|^2$ and NCR using LAB-CA_HT – MRMCM vs. MRMCM Dense Mesh Networks

LAB-CA_HT _{MRMC}				LAB-CA_HT _{MRMCM}				PR _{NCR}
N_a	Average Link Transmission Power (mW)	$NCCM/ L ^2$	NCR	N_a	Average Link Transmission Power (mW)	$NCCM/ L ^2$	NCR	
2 to 6	3.95	0.4437	30.28	3 to 7	2.38	0.3382	24.12	20.34
				4 to 8	1.61	0.2697	20.24	33.16
				5 to 9	1.17	0.2246	18.00	40.55
				6 to 10	0.90	0.1918	16.08	46.90
				7 to 11	0.71	0.1689	14.72	51.39
				8 to 12	0.57	0.1516	13.72	54.69

10. CONCLUSION

10.1. Conclusions

TCA-based vs. MP-based Connectivity Graph

The channel assignment using *TCA-based CG approach* outperforms the one using the classical MP-based CG approach in terms of number of frequency channels. Using this approach *reduces the number of frequency channels* by controlling the network connectivity using topology control. The TCA-based CG controls the network topology by selecting the nearest neighbors for each node in the network. Since transmission power is proportional to the distance between the nodes, the shorter the distance, the lower the transmission power. Less transmission power translates to less interference which leads to better spatial channel reuse and lower NCR.

The topology control approach of the TCA-based CG to reduce the interference by controlling the network connectivity leads to a trade-off between connectivity and optimality of the solution for the multi-path routing. Although, the lower connectivity with the TCA-based CG results in achieving a lower NCR but at the same time it occasionally leads to a sub-optimum solution of the network throughput especially at lower node-degree constraints.

The higher connectivity with the MP-based CG results in more links in the CG which means more options for the solver to explore during the solution of the multi-path routing problem. Hence when using MP-based CG, it takes the solver a longer time to find the optimal solution for the network throughput as compared to when using TCA-based CG.

Multi-Path vs. Minimum Power-based Shortest Path Routing

The channel assignment using *multi-path routing* outperforms the one using MPSP routing in terms of network throughput. The multi-path routing is formulated as a MILP. Using this approach *achieves maximum network throughput* for MRMC WMNs while ensuring fairness among the multiple network flows at the same time. The MILP formulation makes it possible to trade-off the optimality of the solution for the network

throughput with the solution time. Consequently, the solution times of the first feasible solution for the multi-path routing are found to be smaller than those of the optimal solution. This reduction occurs as a trade-off in the form of a decrease in the network throughput.

A trade-off is observed between network throughput and NCR for multi-path routing and MPSP routing. With an increasing node-degree constraint, multi-path routing provides more network throughput than MPSP routing but becomes costlier than MPSP routing in terms of NCR at the same time. The main reason for the lower network throughput of MPSP routing is that while determining the shortest paths between the sources and the GW, maximizing the total network flow is not an objective. Since the objective of multi-path routing is to maximize the total flow in the network by using multiple paths between the mesh nodes and the GW, more nodes have high node-degrees in its output, which means more conflicts in the conflict graph. This leads to a higher NCR as compared to MPSP routing.

MaIS-based vs. MIS-based Minimum Coloring Heuristic

The channel assignment using *MaIS-based minimum coloring heuristic* outperforms the one using MIS-based minimum coloring heuristic in terms of the number of frequency channels. Using this approach *returns fewer frequency channels* than the MIS-based heuristic because the cardinalities of the MaISs are more even in size whereas the MIS-based heuristic first finds a few large cardinality MISs, leaving a larger number of small MISs, for an overall larger number of frequency channels.

The MaIS-based heuristic is computationally simple and therefore more efficient than the MIS-based heuristic.

Realistic Interference Modeling

The channel assignment using the *SIR model with shadowing*, which is the *most realistic* of the models, has the *highest NCR*, which clearly demonstrates that the simplistic protocol model gives overly optimistic results. This has important implications for future research in this area using simplistic interference models.

With the increase in the network throughput with node-degree constraint, NCR increases for all three channel assignments, namely, channel assignment using SIR model

with shadowing, channel assignment using SIR model, and channel assignment using protocol model. The network throughput depends on the number of links for the GW and increases with an increase in the node-degree constraint. Also, the number of links emanating from a mesh node increases with an increase in the node-degree constraint. Since the links emanating from a mesh node must be assigned different channels, this increases the NCR.

NCR for the channel assignment using SIR model increases with the increase in the required SIR threshold γ as the data rate increases. An increase in γ means less tolerance for interference in the network. This results in more conflicts in the conflict graph which leads to a higher NCR.

NCR for the channel assignment using SIR model with shadowing increases with an increase in γ or standard deviation for shadowing σ or with a decrease in outage probability OP . An increase in σ or a decrease in OP translates to an increase in the transmission power of mesh nodes. This causes more interference in the network which results in more conflicts in the conflict graph, thereby leading to a higher NCR.

LAB-CA vs. OAP-CA

LAB-CA requires significantly fewer frequency channels than OAP-CA. NCR for OAP-CA increases with NDC. As NDC increases, more nodes which are farther away from a node are selected as its data communication neighbors. This results in longer links in the connectivity graph. A longer link requires a higher transmission power, which causes more interference in the network. This results in more conflicts in the conflict graph, thereby leading to a higher NCR. NCR for LAB-CA, on the other hand, decreases with NDC because the beam formed by the array in LAB-CA becomes narrower as the number of antennas increases with NDC. The higher the NDC, the larger the number of antennas in the linear antenna array and the narrower the beam formed by the array; hence there is better spatial reuse, fewer conflicts in the conflict graph, and lower NCR when using LAB-CA.

LAB-CA_HT vs. LAB-CA_HG

LAB-CA_HT requires more frequency channels as compared to LAB-CA_HG. The beams formed by mesh nodes in LAB-CA_HT are less focused than those formed by

mesh nodes in LAB-CA_HG. The narrower beams in LAB-CA_HG lead to better spatial reuse, lesser conflicts (reflected by a lesser $NCCM/|L|^2$) and lower NCR.

LAB-CA_HT has slightly higher solution times of the routing stage than LAB-CA_HG. A node-degree of 2 is a hard constraint to satisfy during the routing of flows and due to the presence of such nodes in the heterogeneous network, it takes the solver more time to find the solution for network throughput.

LAB-CA_HT and LAB-CA_HG have similar performance in terms of network throughput. The channel assignment method that has more CRTs in which the first feasible solution is the optimum solution achieves a slightly better network throughput.

Heuristic vs. Exact Method for Extended Minimum Coloring

The *heuristic solution for the extended minimum coloring problem* with cumulative interference constraints consistently *returns near-optimum results*. Where the optimum solution is known, the exact difference between heuristic solution and optimum solution is at most two frequency channels. When the true optimum is not known, the maximum possible difference between heuristic solution and optimum solution is at most three frequency channels.

The heuristic solution is also *significantly faster* than the exact QCBP solution, by about three orders of magnitude on average.

LAB-CA_HT vs. OAP-CA_HT in Sparse vs. Dense Mesh Networks

LAB-CA_HT requires significantly fewer frequency channels as compared to OAP-CA_HT for all node-degree constraints in sparse as well as dense mesh networks. The average link transmission power and $NCCM/|L|^2$ for OAP-CA_HT increase with the increase in node-degree constraint. As node-degree constraint increases, more nodes which are farther away are selected as data communication neighbors of a node. This results in longer links in the connectivity graph and the average link transmission power for the network as well as $NCCM/|L|^2$ increases, leading to a higher NCR.

The average link transmission power and $NCCM/|L|^2$ for LAB-CA_HT on the other hand decrease as NDC increases. In LAB-CA_HT, the beam formed by the linear antenna array becomes narrower as the number of antennas increases with the increase in NDC, which results in a decrease in $NCCM/|L|^2$. Also, the array gain of the linear

antenna array increases as the number of antennas increases with the increase in NDC, which results in a decrease in the average link transmission power. The decrease in average link transmission power as well as $NCCM/|L|^2$ leads to a lower NCR.

LAB-CA_HT has a higher LCR in dense mesh networks and provides an improvement of up to 53% in dense mesh networks as compared to sparse mesh networks. The average link transmission powers for LAB-CA_HT in dense mesh networks are less than those in sparse mesh networks. Nodes in a dense mesh network are closely spaced. This results in shorter links between the nodes in the connectivity graph which leads to lower average link transmission powers. The low transmission power translates to less interference which is reflected by a lower $NCCM/|L|^2$ in dense mesh networks as compared to sparse mesh networks. Less interference means better spatial reuse which leads to a greater reduction in the number of frequency channels and a higher LCR in dense vs. sparse networks.

Throughput-per-node in dense mesh networks is less than that in sparse mesh networks. In a sparse mesh network, the network throughput is shared by 35 sources. On the other hand, in a dense mesh network, the network throughput is shared by 99 sources which results in a lower throughput-per-node.

The *solution times of the routing stage* for dense mesh networks are higher than those for sparse mesh networks. The number of links in the connectivity graph is much higher for dense mesh networks than that for sparse mesh networks. This results in a much bigger routing problem for dense mesh networks, which causes the solver to take more time to find the solution for network throughput.

LAB-CA_HT in MRMCMMA vs. MRMC Dense WMNs

LAB-CA_HT performs better in dense MRMCMMA vs. dense MRMC WMNs. It provides a reduction of up to 54% in NCR when the number of antennas in the linear antenna arrays of mesh nodes N_a is increased from "2 to 6" to "8 to 12". As N_a increases, the array gain increases and the average link transmission power decreases. Also, the array beam becomes narrower with the increase in N_a which reduces $NCCM/|L|^2$. The decrease in average link transmission power as well as $NCCM/|L|^2$ leads to a lower NCR when using LAB-CA_HT in dense MRMCMMA WMNs.

10.2. Future Work

Following are some aspects of this work which can be extended to enhance the channel assignment framework presented in this thesis:

- As can be seen from Fig. 8.2, the narrowness of the beam formed by the linear antenna array depends on the direction towards which the beam is steered. While selecting the nearest neighbors during topology control to build the connectivity graph, the direction of the mesh nodes in the maximum power neighbor table of a mesh node can be considered. Those nodes can be selected as neighbors of a mesh node that will result in narrower/better beams which may lead to a reduction in the number of frequency channels.
- As discussed in Section 10.1, the selection of the routing mechanism affects the network throughput as well as the number of frequency channels. Also, the solution times for multi-path routing greatly increase in dense mesh networks. A new routing heuristic can be developed for the routing stage of the channel assignment framework that can provide a better solution for network throughput as compared to MPSP routing and requires fewer frequency channels as compared to multi-path routing but is also highly efficient in terms of the solution time.
- Compared to sparse mesh networks, the throughput per node is lower and the solution times of the routing stage are higher in dense mesh networks. The throttling of the throughput per node in dense mesh networks can be alleviated by adding more gateways. Also, when using multiple gateways, mesh nodes in a dense mesh network can be divided into groups, one group per gateway. This will divide the computational complexity among the multiple gateways, which will reduce the solution times.
- Instead of using the same highest available data rate for all links, link data rates for the conflict graph and minimum coloring stages can be selected based on the amount of flow being carried over a link that is determined during the multi-path routing stage. This will lead to a reduction in link data rates for those links that carry less flow. Lower data rates will mean lower SIR thresholds during the conflict graph and

minimum coloring stages which may lead to a reduction in the number of frequency channels.

- Using null steering, the array weights can be selected such that a null is placed in the direction of interference while the main lobe or main beam is steered in the direction of interest [92]. An N_a -element array can provide up to $N_a - 1$ nulls. However, the array maximum or the main beam cannot be close to a null. Also, too many nulls cannot be placed close together without incurring severe pattern distortion [98].

Using side-lobe cancellation techniques, the array weights can be set so as to create an array factor with no side-lobes. However, suppressing the side-lobes comes at the cost of broadening the main beamwidth [99].

Note that in beamforming that is used in Algorithms 8.1 and 8.2, the antenna elements are assumed to have equal amplitudes of the excitation current and the array weights (or amplitude coefficients) are set to unity while calculating the array factor using (8.4). The method to construct the beamforming-based conflict graph and matrix for heterogeneous mesh nodes in Algorithm 8.2 can be enhanced to incorporate null steering or side-lobe cancellation. The use of such techniques by a mesh node while steering its main beam in the desired direction can nullify its interference at some of its nearby neighbors which may lead to a reduction in the number of frequency channels.

- Other greedy minimum coloring heuristic algorithms can be modified for solving the extended coloring problem at the minimum coloring stage of the channel assignment problem in wireless mesh networks. For example, the *RLF* algorithm can be modified by adding the cumulative interference constraint and its performance can be compared with Algorithm 7.1 in terms of number of frequency channels and solution time.
- Instead of finding the smallest number of frequency channels, the extended coloring problem can be set up to find the minimum bandwidth required to achieve the maximum network throughput. Unlike binary variables in the current formulation which cause combinatorial explosion, this new formulation will result in continuous variables leading to an extended coloring problem which may be easier to solve.

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