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INITIAL INVESTIGATIONS OF TRANSONIC TURBINE AERODYNAMICS USING THE CARLETON UNIVERSITY HIGH-SPEED WIND TUNNEL

By

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**B. ENG. (Mechanical Engineering)
M. ENG. (Aerospace Engineering)**

A thesis submitted to
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in partial fulfilment of the requirements
for the degree of

**Doctor of Philosophy
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**OTTAWA-CARLETON INSTITUTE FOR MECHANICAL
AND AEROSPACE ENGINEERING**

**DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING
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**“Initial Investigations of Transonic Turbine Aerodynamics
using the Carleton University High-Speed Wind Tunnel”**

Submitted by Michael Jeffries, B. Eng, M. Eng

in partial fulfilment of the requirements for
the degree of Doctor of Philosophy

Thesis Supervisor S. A. Sjolander

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ABSTRACT

In recent years, Carleton University has been commissioning a high-speed blow down wind tunnel. The wind tunnel was commissioned to support on-going axial turbine research efforts and to provide a facility for transonic turbine cascade research. Much effort has been made at Carleton to investigate the performance of axial turbines at off-design conditions and to develop correlations to be used for predicting turbine off-design performance. The current study was a contribution to the off-design work being done at Carleton.

This thesis describes the first transonic turbine cascade study conducted using the Carleton University High-Speed Wind Tunnel. To document the Carleton wind tunnel capabilities, cascade measurements were taken and compared to four European wind tunnels using the same cascade at equivalent conditions. The development of experimental procedures used at Carleton and estimates of measurement uncertainties are also presented in this text. Measurements at off-design incidences were made using a transonic turbine cascade to increase the body of published data on off-design turbine performance.

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NOMENCLATURE

A	Area
AVDR, Ω	Axial Velocity Density Ratio (see Equation (2.6))
AVR	Axial Velocity Ratio (see Equation (2.7))
α	Blade Metal Angle
β	Flow Angle
c, C	Blade Chord
C_{p_b} , $C_{p_{te}}$	Base Pressure Coefficient (see Equation (2.3))
d, D	Diameter
δ	Uncertainty (see Equation (5.2))
δ^*	Blade Boundary Layer Displacement Thickness
$\Delta\phi_{TET}^2$	Trailing Edge Energy Loss Coefficient (see Equation (2.23))
$\Delta\phi_p^2$	Energy Loss Coefficient (see Equation (2.28))
ζ	Enthalpy Loss Coefficient (see Equation (2.4))
$f_{(Rec)}$	Reynolds Number Correction (see Equation (2.20))
γ	Ratio of Specific Heats (Air = 1.4)
h, H	Blade Span
i	Incidence (see Figure (7.1))
i_s	Stalling Incidence (see Equation (2.26))
K	Correction Factor
M	Mach Number (see Equation (5.8))

m	Pressure Transducer Calibration Slope
μ	Viscosity
P	Static Pressure
P_0	Total Pressure
q, Q	Dynamic Pressure
Re_c	Cascade Reynolds Number Based on Blade Chord (see Equation (5.14))
Re_p	Probe Reynolds Number based on Probe Head Diameter
ρ	Density
S	Blade Spacing or Pitch
t	Blade Trailing Edge Thickness
T	Temperature
T_0	Total Temperature
Tu	Turbulence Intensity
θ	Blade Boundary Layer Momentum Thickness
U	Velocity
χ	Incidence Correlation Parameter (see Equation (2.30))
χ'	Incidence Correlation Parameter (see Equation (2.27))
w	Blade Passage Throat Opening
x	Blade Surface Chord-Wise Coordinate
Y	Total Pressure Loss Coefficient (see Equation (2.2))
Y_t	Total Midspan Loss Coefficient (see Equation (2.25))
Y_p	Profile Loss Coefficient

Y'_{TET}	Trailing Edge Loss Coefficient (see Equation (2.24))
V	Voltage
We	Wedge Angle
z	Pressure Transducer Calibration Zero Pressure Output
Z	Blade Spanwise Coordinate

Subscripts

cr	Critical or Sonic Conditions
des	Design
le	Leading Edge
ms	Mass Averaged
is	Isentropic Conditions
s	Isentropic Conditions or Static Conditions
t	Total Conditions
te	Trailing Edge
th	Theoretical
1	Cascade Inlet Conditions
2	Cascade Exit Conditions

1.0 INTRODUCTION

1.1 Motivation

Modern gas turbine engines are continuously being refined to improve efficiency at both design and off-design conditions. Increased performance and reductions in size of modern gas turbine engines have pushed pressure ratios into the range where the turbine blade rows within the engines are operating at transonic and supersonic conditions. Advances and improvements in axial turbine designs have been achieved from improved correlations obtained through a better understanding of the flow physics within the turbine blade rows. These correlations have been primarily developed using databases comprised of experimental cascade results. Modern computational fluid dynamics (CFD) programs are now also being used in the development of correlations. CFD applications do not remove the need for quality experimental results since most CFD codes must be first validated against experimental measurements.

For many years now, Carleton University has been conducting axial turbine experimental and computational work and developing correlations. One primary area of focus at Carleton has been off-design conditions. Until recently these studies were conducted at low speeds. The need for transonic cascade measurements to extend the correlations to high speeds was recognized and a high-speed transonic cascade wind tunnel was borrowed from the National Research Council of Canada and re-

commissioned as part of the present work. The present study also includes the first transonic turbine cascade study conducted using the Carleton University High-Speed Wind Tunnel.

1.2 Scope of Work

The current study focused on profile losses measured at the midspan of a high pressure axial turbine cascade. There were two primary goals of the current work.

The first goal was to commission the wind tunnel, instrumentation, data acquisition system and data reduction procedures and to assess the quality of experimental results obtained from the Carleton wind tunnel. This was accomplished by performing cascade measurements using a cascade geometry with a published set of cascade results and comparing the results obtained at Carleton. The cascade geometry used in the current study was the same cascade used by Kiock et al. (1986) and Baines et al. (1986) which compared cascade measurements using the same cascade geometry in four European wind tunnels.

The second goal of the current study was to extend the current database of off-design axial turbine measurements at Carleton to include transonic conditions. The current cascade results along with subsequent studies will be used to improve existing

correlations and extend the range of applicability of the correlations to transonic conditions.

The next chapter reviews the published literature pertinent to the current work. In Chapter 3, the wind tunnel and experimental equipment are described. Chapter 4 presents a study which investigated the sampling rates and times required for experimental measurements using the Carleton University High Speed Wind Tunnel. Initial test section commissioning measurements and measurement uncertainty studies are presented in Chapter 5. Chapter 6 presents a comparison of the cascade results obtained at Carleton with equivalent measurements made at the four European wind tunnels. Chapter 7 presents the cascade off-design measurements made in the current study. Finally Chapter 8 presents a comparison of the design point and off-design cascade results to the predicted losses using popular loss prediction methods including Ainley and Mathieson (1951) for the cascade geometry used in this study.

2.0 LITERATURE REVIEW

This chapter reviews the literature relevant to this study. The present work is the initial investigation of cascade measurements using the Carleton University High-Speed Wind Tunnel. The work examines profile losses in a turbine cascade at transonic conditions. The first section of this chapter briefly discusses the present understanding of the flow field within a turbine passage, with particular attention to profile losses under transonic conditions.

The cascade measurements of Kiock et al. (1986) and Baines et al. (1986) are reviewed in Section 2.2. The authors made measurements for the same cascade geometry in four European transonic wind tunnels in order to compare the performance of the four facilities. It was therefore decided to use the same cascade geometry to benchmark the Carleton University High-Speed Wind Tunnel. The European results are examined in detail to provide a basis for the later comparison with the Carleton results. Subsequent studies at Carleton extended the results beyond the range of conditions considered by the Europeans.

Section 2.3 reviews some of the factors, such as Reynolds number, freestream Turbulence Intensity and Axial Velocity Density Ratio (AVDR) which are all known to affect cascade measurements. This review is presented as a basis for interpreting the

cascade measurements made in the present study and for making comparisons to the European wind tunnels.

An additional goal of the present study is to add to the database of cascade results available for developing correlations for use in the design of transonic turbine blades, particularly for predicting losses. The final section of this chapter therefore reviews the Ainley and Mathieson (1951) loss system and its subsequent improvements. These are the most widely used methods for predicting the losses in axial turbines and the present measurements will be compared to them.

2.1 General Description of the Flow in a Turbine Blade Passage

The current study focuses on losses measured at the midspan of an axial turbine blade under transonic conditions. For completeness and to put the midspan losses in context, a brief discussion of overall flow in a turbine blade passage is presented .

The flow in a turbine blade passage is complex and some aspects of the flow are still not completely understood. A general review of the loss mechanisms in turbomachinery is given by Denton (1993). A detailed discussion of midspan losses in turbine cascades at transonic conditions can be found in Mee et al. (1992). The discussion presented in this section is based partly on these references.

Figure 2.1 presents a schematic diagram of the main features of the flow through a turbine rotor passage. There are several distinct flow structures identified in Figure 2.1. The passage vortex is the result of the rolling up of the endwall boundary layer caused by the cross passage pressure gradient. The horse-shoe vortex is caused by the endwall boundary layer separation at the corner of the endwall and blade leading edge. The pressure side leg of the horse-shoe vortex rotates in the same direction as the passage vortex and is believed to merge with the passage vortex to form a single vortex. The suction side leg of the horse-shoe vortex rotates in the opposite direction to the passage vortex. The location of the suction side leg of the horse-shoe vortex has not been fully established and has been reported in different locations by different authors. However, in general it is located close to the blade suction side. Sieverding (1985) suggests that the location of the suction side leg of the horse-shoe vortex is a function of the rotational speed of the passage vortex. The tip-leakage vortex results from the rolling up of the tip-leakage flow as it interacts with the main passage flow.

The losses in a transonic turbine passage are usually divided into several components: profile losses, shock losses, trailing-edge losses, endwall or secondary losses and tip-leakage losses. Profile losses are partly a result of viscous entropy production within the blade boundary layers. The losses which occur at the trailing edge, due to base pressure and the downstream mixing processes, are usually also included in the profile losses. Shock losses are the losses in total pressure resulting from shocks

Figure 2.1: Diagram of the Flow in a Turbine Rotor Passage.

present in the blade passage which occur at transonic conditions. Profile, shock and trailing edge losses are usually considered to occur over the whole blade in the span-wise direction. Endwall losses are usually taken as the losses within the endwall boundary layers as a result of dissipation and mixing losses associated with the secondary flows. Tip clearance losses result from the flow over the blade tip. These losses result from three sources: viscous losses within the tip gap, mixing of the tip clearance flow into the passage flow at the blade suction surface and mixing of the tip-leakage vortex downstream of the tip gap. The endwall and tip-leakage losses are three-dimensional sources of losses. As mentioned, the present work focuses on the losses measured at the midspan of a transonic turbine blade. The remainder of this section will therefore focus on the flow at the midspan, away from the three-dimensional flows at the endwalls.

Based on a simplified control volume analysis, Denton (1993) presents an approximate equation which relates the two-dimensional losses in a cascade blade row under incompressible conditions to the blade boundary layer parameters and the base pressure:

$$Y = \frac{2\theta}{w} + \left(\frac{\delta^* + t}{w} \right)^2 - \frac{Cp_b t}{w} , \quad (2.1)$$

where Y = pressure loss coefficient,

w = passage throat opening,

t = blade trailing edge thickness,

θ = momentum thickness of the blade boundary layers,

δ^* = displacement thickness of the blade boundary layers, and

C_{p_b} = base pressure coefficient.

The total pressure loss coefficient is defined for turbine blades as

$$Y = \frac{P_{01} - P_{02}}{P_{02} - P_2} , \quad (2.2)$$

where P_{01} = cascade inlet total pressure,

P_{02} = mixed-out cascade exit total pressure,

P_2 = mixed-out cascade exit static pressure.

The base pressure coefficient is defined as

$$C_{p_b} = \frac{P_b - P_2}{P_{02} - P_2} , \quad (2.3)$$

where P_b = blade base pressure which is the pressure acting on the blade trailing-edge.

The first term on the right side of Equation 2.1 accounts for the losses due to entropy generation within the blade boundary layers. This term shows that the profile losses are proportional to the momentum thickness of the blade boundary layers. The second term, which contains the blade boundary layer displacement thickness, accounts for the loss due the sudden area expansion at the trailing edge. The blade boundary layers effectively constrict the flow within the passage and thus increases the effective

change in area at the trailing edge. The last term accounts for the low base pressure in the base region immediately behind the trailing edge. The base pressure loss portion is directly proportional to the base pressure coefficient and the thickness of the trailing edge. A detailed discussion of base pressure will be presented later in this section since the base pressure is an important factor influencing the losses at transonic conditions.

As noted, the equation of Denton is applicable to incompressible flows. Under transonic conditions there are additional losses not accounted for in Equation 2.1. Mee et al. (1992) divided the midspan losses in transonic cascades into three components, profile losses, shock losses and mixing losses. The breakdown of losses is shown in Figure 2.2. As already discussed, profile losses are a result of losses due to entropy production in the blade boundary layers and are accounted for in Denton's equation. Shock losses result from shock waves present in the blade passages and result in a decrease in total pressure in the free stream passage flow. The shock losses that occur in the free stream of the blade passage under transonic conditions are not included in Denton's equation. Mixing losses occur due to mixing of the flow downstream of the blade trailing edges. In developing Equation 2.1, Denton assumed uniform flow at the exit of the control volume, therefore mixing losses are also accounted for in Denton's equation. Mee et al. mention that losses from shocks which are isolated to the downstream of the blade trailing edges and not within the blade passages would be included in the mixing losses.

Figure 2.2: Breakdown of Transonic Turbine Midspan Losses (Mee et al. (1992)).

It is worth pointing out that the different losses already discussed are not independent. For example, shocks within the blade passage result in shock wave-boundary layer interaction on the suction surface of a transonic turbine blade. The shock-wave boundary layer interaction would thicken the suction surface boundary layer by applying a strong adverse pressure gradient to the boundary layer locally. If the shock is strong enough, it may cause boundary layer separation, which may reattach to form a separation bubble or remain separated to the blade trailing edge. All these effects may affect the losses through both changes in the momentum thickness and the base pressure.

As noted, the base pressure plays a significant role in the losses at transonic conditions. The base pressure is a result of complex flow structures immediately behind the blade trailing edge. In high pressure transonic turbine blades, the trailing edge thickness of the blades is typically large, of the order of 10 to 20 percent of the throat opening. The thick trailing edge is the result of cooling considerations, such as the need for internal cooling passages near the blade trailing edge. The trailing edge losses can be a large portion of the losses measured at the midspan of a transonic turbine blade. From Figure 2.2, since mixing losses are mainly trailing edge effects, the trailing edge losses for the blades used in the Mee et al. (1982) study were of the order of half the total losses measured at midspan.

The flow at the trailing edge of a transonic turbine blade is very complicated and to date not well understood. Figure 2.3 taken from Denton and Xu (1990) shows the

Figure 2.3: Flow Structure at the Trailing Edge of a Transonic Turbine Blade (Denton and Xu (1990)).

general flow structures at the trailing edge of a turbine blade under transonic or low supersonic conditions. Several features can be seen in Figure 2.3. Close to the trailing edge, the flows on both the suction and pressure surfaces separate from the trailing edge forming two shear layers which bound the triangular shaped region immediately behind the trailing edge. This region which is known as the base region is closed downstream at the point where the suction surface and pressure surface flows meet, called the confluence region. As the flows meet in the confluence region, two compression corners are generated and shocks are formed. The shock formed on the pressure side of the blade will propagate toward the suction surface of the adjacent blade. This is where the shock originates which results in shock wave-boundary layer interactions on the blade suction surface. The shock on the suction surface side will propagate downstream of the blade row and in the case of cascades, may reflect off a free-shear layer or the walls of the test section downstream of the cascade. In the case of an actual engine, this shock may interact with the downstream nozzle row.

Several experimental studies have been reported with regards to base pressure and trailing edge losses. Sieverding et al. (1980) used base pressure measurements from a large database of cascade measurements to develop a correlation for base pressure as a function of back pressure on the cascade, unguided turning downstream of the blade passage throat and the trailing edge wedge angle. It appears that some of the cascade data from the four European wind tunnel study, which is discussed in the next section, was used by Sieverding et al. in developing their correlation. One of the cascade geometries

used by Sieverding et al. appears to be the same geometry used in the four European study and in the current study. The correlation is shown in Figure 2.4. Sieverding et al. shows that suction-side curvature for blades with thick trailing edges increases the base pressure and thus decreases the trailing edge losses. This was also noted by Denton and Xu (1990). By measuring the losses as a function of downstream distance behind the blade trailing edge, Sieverding et al. showed that the majority of losses were generated immediately downstream of the trailing edge. The authors attributed the losses due to the highly viscous and complex flow patterns shown in Figure 2.3.

Xu and Denton (1987) and Xu and Denton (1988) showed that the state of the boundary layers at the trailing edge had a large influence on the base pressure. The authors found that the base pressure tended to decrease with decreasing shape factor of blade boundary layers at the trailing edge. The shape factor is defined as the ratio of the displacement thickness to the momentum thickness and is a measure of the fullness of the boundary layer. Laminar boundary layers or boundary layers close to separation have higher shape factors. For boundary layers with lower shape factors such as a fully turbulent boundary layer, the base pressure was lower as compared to boundary layers close to separation. Xu and Denton suggested that was the result of increased mixing losses in the shear layers shown in Figure 2.3. The higher wall shear stress in the case of a turbulent boundary layer may result in higher shear stresses in the shear layers as the flow separates from the trailing edge to form the base region. Xu and Denton suggested that a higher rate of entrainment of the low velocity fluid in the base region into the shear

Figure 2.4: Sieverding (1980) Base Pressure Correlation as a Function of the Unguided Turning (ϵ) and the Trailing Edge Wedge Angle (δ).

layers could explain the lower base pressures and thus higher losses in the trailing edge region.

2.2 The Four European Wind Tunnels Study

As mentioned at the beginning of this chapter, Kiock et al. (1986) presented a comparison of cascade measurements made using the same turbine cascade geometry in four wind tunnels in Europe. Baines et al. (1986) presented additional information from the same study. The authors examined the systematic differences between the cascade measurements made at the four facilities. The authors termed the differences “Tunnel Effects” and are the result of bias errors in each of the wind tunnels. The study is well recognized and the data have been used to validate Navier-Stokes codes and to provide a bench-mark for testing and commissioning wind tunnels. For example, Killey and Smith (1993) performed the same set of measurements using the cascade geometry to commission the AEROTEK blow-down wind tunnel located in South Africa. The AEROTEK wind tunnel is described in some detail in Hattingh (1991). Killey and Smith (1993) produced comparable cascade results to those of Kiock et al. (1986).

This section presents a detailed review of the results of Kiock et al. and Baines et al. since the same cascade geometry was tested as part of the commissioning of the Carleton University High-Speed Wind Tunnel. The comparisons are presented in Chapter 6.

The four European wind tunnels are located at the Von Karman Institute, Rhode-St.-Genese, Belgium (RG); DFVLR Goettingen, Germany (GO); DFVLR Braunschweig, Germany (BS); and Oxford University, United Kingdom (OX). Each wind tunnel will be referred to by its two letter designation for the remainder of this discussion.

The wind tunnels are of a variety of designs: blow-down, suckdown and closed loop continuous flow. Table 2.1 compares the various wind tunnel parameters for the four European wind tunnels. It is important to note that the uncertainties quoted for the loss coefficients and Mach numbers were based on averaging over multiple blade pitches. Therefore, these uncertainty values are representative of the periodicity or blade-to-blade variations and not the total uncertainty in the actual value of the quantity. The main results taken from Kiock et al. (1986) and Baines et al. (1986) are shown in Figures 2.5 to 2.15.

The measured losses are expressed in terms of the enthalpy loss coefficient which is defined as

$$\zeta = 1 - \left(\frac{V_2}{V_{2,is}} \right)^2, \quad (2.4)$$

where ζ = enthalpy loss coefficient,

V_2 = mixed-out exit velocity and

PARAMETER	WIND TUNNEL			
	RG	GO	BS	OX
Name of Facility	High Speed Cascade Wind Tunnel (C-2) Von Karman Institute, Belgium	Plane Cascade Wind Tunnel (EGG) DFVLR, Goettingen, West Germany	High Speed Cascade Wind Tunnel (HGB) DFVLR, Braunschweig West Germany	O.U.E.L. Blowdown Wind Tunnel Oxford University U.K.
Reference	Kiock et al. (1986)			
Type of Facility	Blowdown	Suckdown	Closed Loop	Blowdown
Test Section (width x height)	50 mm x 180 mm	125 mm x 353 mm	300 mm x 427 mm	300 mm x 420 mm
Length of Upstream Straight End Wall	1000 mm	2550 mm	1550 mm	800 mm
Length of downstream guide flow (mm)	200	650	250 mm	290 mm
Ratio of Inlet Endwall length to width	20	20.4	5.2	2.7
Number of Blades	10	10	7	7
Blade Shape	Figure 3.4			
Difference nominal to real shape (mm)	± 0.031 ± 0.031	± 0.025 ± 0.040	± 0.055 ± 0.083	± 0.055 ± 0.083
Chord, c (mm)	32.6	60	100	100
Aspect Ratio, h/c	1.534	2.083	3	3
Gauge Angle (deg)	67.74 ± 0.10	67.92 ± 0.11	67.96 ± 0.03	67.96 ± 0.03
Pitch (mm)	23.15 ± 0.04	42.58 ± 0.19	70.88 ± 0.10	70.88 ± 0.10
Stagger Angle (deg)	33.14 ± 0.09	33.56 ± 0.12	33.29 ± 0.03	33.29 ± 0.03
Flow Quantities at Cascade Inlet:				
Static Pressure	Side wall taps	Side wall taps	Side wall taps	Side wall taps
Total Pressure	Probe	Probe	Probe	Probe
Flow Angle				
Flow Quantities at Cascade Exit				
Static Pressure				
Total Pressure	Two-finger Probe	Wedge probe	Three-finger probe	Three-finger probe
Flow Angle				
Wake traverse on blade number	5	5	4	4
Axial Distance/Chord	0.64	0.45	0.4	0.4
Probe Motion	Continuous	Continuous	Step-wise	Continuous
Time required for one wake traverse	45 seconds	70 seconds	180 seconds	1 second
Pressure taps on blade number	5 and 6	4 and 5	3 and 5	3 and 5
Pressure Measurement System	Mercury Multi-manometer	Scanivalve	Scanivalve	Individual Transducers
Inlet Flow Angle, β_1	28.95 ± 0.47	29.92 ± 0.13	-	30.04 ± 0.13
Inlet Mach Number, M_1	0.282 ± 0.003	0.282 ± 0.004	0.260 ± 0.000	0.252 ± 0.005
Exit Flow Angle, β_2	67.03 ± 0.15	67.02 ± 0.22	67.33 ± 0.06	67.76 ± 0.1
Exit Mach Number, M_2	0.933 ± 0.006	0.957 ± 0.003	0.828 ± 0.003	0.932 ± 0.004
Loss Coefficient, ζ	0.046 ± 0.007	0.049 ± 0.001	0.039 ± 0.003	0.040 ± 0.003
AVDR	0.929 ± 0.011	0.937 ± 0.008	0.984 ± 0.006	1.012 ± 0.01

Table 2.1: Comparison of Four European Wind Tunnel Parameters and Measurements.

$V_{2,is}$ = exit velocity that would be achieved under isentropic conditions for the same exit static pressure.

For a stationary cascade at adiabatic conditions, the enthalpy loss coefficient can be expressed in terms of the quantities measured in the cascade as

$$\zeta = 1 - \frac{1 - \left(\frac{P_2}{P_{02}} \right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{P_2}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}}}, \quad (2.5)$$

where P_2 = mixed-out cascade exit static pressure,

P_{01} = cascade inlet total pressure,

P_{02} = mixed-out cascade exit total pressure and

γ = ratio of Specific Heats (Air = 1.4).

Figure 2.5 presents the pitch-wise variation of the loss coefficient for the wake traverses in each facility. Also indicated are the mixed-out loss values over each pitch. The figure shows that all facilities had reasonably good periodicity, that is the wakes are similar from blade to blade. The wakes from RG, GO and BS are seen to be very smooth. Considerable scatter in the OX data is clearly evident and was the result of unsteadiness in the operating point of the OX wind tunnel due to its short run times of around 1 second. In addition the transducers at OX were mounted inside the pressure probe so that the frequency response of the whole pneumatic system was much higher

Figure 2.5: Comparison of Wakes Among the Four European Wind Tunnels (Baines et al. (1986).

than at the other facilities. As a result, the OX investigators saw more fluctuation in their data.

The mixing-out of the wakes with downstream distance can also be seen in Figure 2.5. At RG, wake traverses were made at 0.64 blade chord downstream of the blade row as compared to 0.45 at GO. The peak-to-peak values of the loss coefficient are reduced at RG compared to GO indicating more mixing has occurred downstream of the blade row. At BS and OX, the wake traverses were made at 0.4 blade chord behind the cascade. One would have expected that the peak-to-peak loss values would be higher at BS and OX than at GO since the traverses at BS and OX were made closer to the cascade. It appears at BS and OX more mixing of the wakes occurred in a shorter distance downstream from the blade trailing edge. The authors did not suggest an explanation for the differences in the wakes among the wind tunnels.

Kiock et al. and Baines et al. quote the cascade exit conditions and loss coefficients in terms of the mixed out values. The mixing out calculation was developed by Amecke and Safarik (1995). Using a control volume, the mixing calculation uses conservation of mass, momentum and energy to mix the wake flows out to uniform conditions at the exit of the control volume.

Figure 2.6 shows the mixed-out loss coefficients versus exit Mach number. The agreement between the different wind tunnels was quite good for exit Mach numbers

Figure 2.6: Enthalpy Loss Coefficient versus Cascade Exit Mach Number (Kiock et al. (1986)).

from about 0.80 to 0.95, which is the region of primary interest. The agreement in the losses was best for BS, GO, and RG; the loss coefficients agreed to within ± 0.003 . The OX data can be seen to be considerably more scattered due to the unsteadiness of the wind tunnel operating point already noted.

Figure 2.7 shows the mixed-out exit flow angle versus exit Mach number and again good agreement between RG, GO and BS can be seen. With the exception of the OX data, all the data from these tunnels falls within a band of about ± 0.2 degrees. The OX data seems to be shifted to a higher value by approximately 0.5 degrees. The authors suggested that the probe calibration at OX was the likely source of this discrepancy.

Figure 2.8 presents the cascade inlet Mach number as a function of the exit Mach number. The differences were attributed by the authors to the effects of different boundary layer thicknesses at the inlet to the cascade. Sieverding (1993) argues that for a turbine cascade, the acceleration between the inlet and the blade passage throat tends to have a thinning effect on the endwall boundary layer as it passes through the blade passage. The thinning effect produces a divergence of the flow channel in the spanwise direction between the inlet and the throat. The divergence of the flow in the blade passage effectively results in a decrease in the inlet to throat area ratio and therefore increases the inlet Mach numbers. Cascades with thicker inlet endwall boundary layers would have higher inlet Mach numbers as compared to cascades with thin inlet endwall boundary layers. Kiock et al. pointed out that the inlet endwalls were considerably

Figure 2.7: Cascade Exit Flow Angle versus Cascade Exit Mach Number (Kiock et al. (1986)).

Figure 2.8: Cascade Inlet Mach Number versus Cascade Exit Mach Number (Kiock et al. (1986)).

longer compared to the width of the inlet flow channel in the case of RG and GO, than those at BS and OX, as seen in Table 2.1. Figure 2.8 shows that the choking inlet Mach number seems to increase as the length of the inlet walls increases. The authors suggested that the longer inlet endwalls produced a thicker boundary layer at the cascade inlet and thus higher choking cascade inlet Mach numbers. However, the thickness of the endwall boundary layer is not solely a function of the length of the endwalls. It also depends on the roughness of the endwalls. The authors did not measure the thicknesses of the cascade inlet endwall boundary layers at the different wind tunnels.

The Axial Velocity Density Ratio (AVDR) versus exit Mach number is shown in Figure 2.9. The AVDR is an approximate measure of the two-dimensionality of the flow in the cascade at the blade midspan. As will be discussed in Section 2.3.3, an AVDR value equal to unity is necessary to ensure two-dimensional flow at the blade midspan but not sufficient. Kiock et al. (1986) found that the AVDR were close to 1.0 at BS and OX but not at RG and GO. The authors suggested that lower values of AVDR at RG and GO are a result of the higher inlet choking Mach numbers measured at those facilities. The authors noted that an increase in the inlet Mach number of 0.02 decreases the AVDR by approximately 0.08. This is approximately the difference in the AVDR values between the OX and BS wind tunnels and the RG and GO tunnels at an exit Mach number of 0.95. Kiock et al. and Baines et al. concluded that the good agreement in the loss measurements shown in Figure 2.6 suggests that the losses are not strongly influenced by AVDR for axial turbine blades at transonic conditions. However, as will

Figure 2.9: Axial Velocity Density Ratio (AVDR) versus Cascade Exit Mach Number (Kiock et al. (1986)).

be discussed in Section 2.3.3, Rogers et al. (1992) and Whitehouse et. al (1993) found that the losses were quite sensitive to AVDR for turbine blades operating at low speeds.

Figure 2.10 shows the cascade Reynolds numbers, based on cascade exit conditions and blade chord, at each facility as a function of cascade exit Mach number. The RG and GO wind tunnels did not have any form of Reynolds number control and thus the Reynolds number increased with Mach number. Both the BS and OX wind tunnels were able to control the Reynolds number independently of the exit Mach number. For the exit Mach number range of interest, namely 0.8 to 0.95, the Reynolds numbers for all wind tunnels were between 700,000 and 900,000. From the constant Reynolds number facilities, BS and OX, the target Reynolds number for the measurements was apparently about 800,000. Kiock et al. note that variations in Reynolds number within the range of 700,000 to 900,000 had little effect on the cascade aerodynamic characteristics. Figure 2.6 shows the losses increased as the exit Mach number was lowered below 0.8 for the RG wind tunnel. Below an exit Mach number of approximately 0.7 the Reynolds number falls below 700,000 and the authors attributed the rise in losses to the decrease in Reynolds number. Figure 2.11 shows the influence of Reynolds number on the measured cascade parameters including loss coefficient for exit Mach numbers of 0.39 and 0.91. The data was obtained in the BS wind tunnel which allows a large range of Reynolds number variation for a given cascade exit Mach number. It can be seen that as the Reynolds number is lowered the measured losses

Figure 2.10: Cascade Reynolds Number versus Cascade Exit Mach Number (Kiock et al. (1986)).

begin to increase below a Reynolds number of approximately 700,000 for both exit Mach numbers.

Figure 2.11 shows the influence of blade aspect ratio (h/c) on the inlet Mach number, (M_1), exit flow angle (β_2), AVDR (Ω), and loss coefficient (ξ). The aspect ratios investigated were 1.53 and 3.0. These correspond to the range of aspect ratios among the four European wind tunnels. The results seem to be relatively insensitive to the blade aspect ratio at least at the BS wind tunnel. There does appear to be a slight increase in the losses at the low exit Mach number of 0.39 and low aspect ratio. As noted in the discussion of the inlet Mach numbers earlier, the BS wind tunnel inlet endwalls were quite short in length in comparison to some of the other four European wind tunnels. Therefore, the inlet endwall boundary layer was probably very thin. Possibly in the BS wind tunnel, the inlet boundary layer was thin enough that approximately two-dimensional flow would be achievable at the blade midspan for both blade aspect ratios. For facilities with thicker inlet endwall boundary layers, the secondary flows produced by the endwall boundary layers would produce a larger blockage with respect to the flow area within the blade passage. The higher flow blockage could have a larger influence on the flow measured at midspan. Therefore the losses and AVDR might be more sensitive to aspect ratio for facilities with thicker inlet endwall boundary layers and low aspect ratios.

Figure 2.11: Cascade Results for Different Blade Aspect Ratios (Baines et al. (1986)).

The base pressure coefficient versus exit Mach number is shown in Figure 2.12. The base pressure coefficient was defined as Equation 2.3. Negative values of base pressure coefficient indicate the trailing edge pressure is below the downstream static pressure. The influence of base pressure on the losses in a turbine cascade was discussed in Section 2.2. From the Equation 2.1 of Denton (1993), the losses are seen to increase as the base pressure decreases. Once again, the OX data showed discrepancies compared to the other three wind tunnels. This was attributed to an incorrect measurement at OX of the downstream static pressure caused by the flow blockage of the probe stem. Figure 2.12 shows an increase in the base pressure coefficient at cascade exit Mach numbers between 0.85 and 0.95 which corresponds to the range where minimum losses are observed in Figure 2.6.

Figure 2.13 shows the measured blade surface isentropic Mach numbers for an exit Mach number of approximately 0.8, for which the flow was subsonic over the whole blade surface. The blade surface Mach numbers for the transonic case is shown in Figure 2.14. Both figures show relatively good agreement between most of the facilities. The OX data once again shows some differences as compared to the other three wind tunnels particularly on the suction surface after $X/C = 0.5$. As with the base pressure coefficient, the authors attributed the difference in the OX data to an incorrect measurement of the downstream static pressure.

Figure 2.12: Base Pressure Coefficient versus Cascade Exit Mach Number (Kiock et al. (1986)).

Figure 2.13: Isentropic Blade Surface Mach Numbers for the Fully Subsonic Case (Kiock et al. (1986)).

Figure 2.14: Isentropic Blade Surface Mach Numbers for the Transonic Case (Kiock et al. (1986)).

In order to study the effects of Reynolds number of the blade surface Mach numbers, the Reynolds number was varied for constant exit Mach numbers equal to 0.39 and 0.91 as shown in Figure 2.15. The data were taken at the BS wind tunnel. It can be seen from the figure that the Reynolds number has very little influence on the blade loading except close to the trailing edge on the suction surface for X/C greater than 0.8. Baines et al. (1986) found that at an exit Mach number equal to approximately 0.91 and a low Reynolds number of approximately 300,000, a separation bubble appeared at around X/C of 0.6 and extended to roughly 0.65 followed by reattachment as a turbulent boundary layer. At the higher Reynolds numbers, transition occurred at around $X/C = 0.6$ and the suction surface boundary layer remained attached up to the trailing edge.

As mentioned at the beginning of this section, the detailed review of the results of Kiock et al. (1986) and Baines et al. (1986) in this section was presented as a basis for comparison with the data in the present study obtained using the same cascade geometry. The comparison of the Carleton data with the European is presented in Chapter 6. At Carleton, the measurements for the reference cascade were also extended to off-design incidence, which was not considered in the European study. The off-design results are presented in Chapter 7.

Figure 2.15: Effects of Reynolds Number on Isentropic Blade Surface Mach Numbers (Baines et al. (1986)).

2.3 Additional Studies on the Factors Influencing Cascade Loss Measurements

This section reviews some of the additional literature concerning the factors that influence cascade measurements. Factors such as Reynolds number, Turbulence Intensity, Axial Velocity Density Ratio (AVDR) and surface finish all influence the measured losses in cascade blade tests. As discussed in the previous section, the results from the four European wind tunnels results will be used to compare to the results obtained at Carleton using the same cascade geometry. The discussion in this section will be used as further basis for interpreting the results obtained at Carleton and for explaining the differences between the Carleton data and those of the four European wind tunnels.

2.3.1 Reynolds Number

The blade Reynolds number in turbine testing is generally based on cascade exit conditions and the blade chord. The influence of Reynolds number on measured cascade losses was studied over a range of Reynolds numbers between 300,000 and 1,000,000 for exit Mach numbers of 0.39 and 0.91 by Baines et al. (1986) as part of the four European study already discussed in Section 2.2. The authors found that the loss coefficient remained approximately constant for Reynolds numbers greater than 700,000 for the HS2 cascade geometry. Below a Reynolds number of 700,000 the losses were observed to increase as a result of changes in the blade boundary layers.

Ladwig and Fottner (1993) also studied the influence of Reynolds number on cascade turbine blade loss measurements over a range of Reynolds numbers from 50,000 to 1,000,000 at an exit Mach number of approximately 0.59. Ladwig and Fottner used a very similar cascade geometry to that of the four European wind tunnel study. It is also worth noting that the work of Ladwig and Fottner used the Braunschweig (BS) wind tunnel which was also used in the four European wind tunnel study. Ladwig and Fottner also investigated the influence of turbulence intensity which is discussed in the next section.

Figure 2.16 taken from Ladwig and Fottner shows the variation in the total pressure loss coefficient as a function of Reynolds number for two different turbulence intensities. In the range of Reynolds numbers above 500,000 the authors note that the losses are relatively insensitive to the Reynolds number. This was also observed by Baines et al. and Kiock et al. (1986) for Reynolds numbers over 700,000 as already mentioned. The cascade geometries had small differences as far as the inlet metal angle and the thickness of the leading edge between the Ladwig and Fottner study and the work of Baines et al., thus one would expect some differences in their Reynolds number sensitivities. However, both were high pressure turbine blades with close to the same amount of flow turning and thickness-to-chord ratios. For the cascade geometry used in the Ladwig and Fottner study, at a turbulence intensity of about 1.5 percent very small separation bubbles were observed on the aft portion of the blade suction surface. The

Figure 2.16: Pressure Loss Coefficient versus Reynolds Number
(Ladwig and Fottner (1993)).

length of the separation region was nearly constant for Reynolds numbers above 500,000. Therefore the behaviour of the boundary layer on the blade suction surface varied very little for Reynolds numbers above 500,000 and as expected the loss coefficient was essentially constant. As the Reynolds number was reduced below 500,000 the separation region was observed to grow in length. This accounts for the rise in loss coefficient shown in Figure 2.16. At low turbulence and Reynolds numbers below 100,000 laminar separation was observed and the boundary layer never reattached. This explains the very high loss coefficients seen in Figure 2.16 in that range of Reynolds numbers.

2.3.2 Turbulence Intensity

As mentioned in the previous section, Ladwig and Fottner also examined the effects of turbulence intensity on the measured cascade losses at a exit Mach number of 0.59. The authors investigated two turbulence intensity levels, about 1.5 percent and about 6 percent. The turbulence level in the Carleton wind tunnel is approximately 4 percent which is between the values studied by Ladwig and Fottner. Ladwig and Fottner found that at the high turbulence intensity the losses were reduced for all Reynolds numbers compared to the low turbulence case. The higher turbulence forced transition in the higher Reynolds number ranges, above 500,000 which eliminated the separation bubble altogether and thus reduced the losses by a small amount. The higher turbulence had a much larger effect at the very low Reynolds number case of 100,000 where laminar

separation was observed in the low turbulence test. The higher turbulence produced turbulent reattachment of the boundary layer and therefore reduced the losses as compared to the low turbulence level.

2.3.3 Axial Velocity Density Ratio (AVDR)

The Axial Velocity Density Ratio (AVDR) is an approximate measure of the two-dimensionality of the flow in the blade passage. The AVDR is defined as

$$AVDR = \frac{\rho_2 U_2 \cos(\beta_2)}{\rho_1 U_1 \cos(\beta_1)} , \quad (2.6)$$

where ρ_1 = midspan cascade inlet flow density,

U_1 = midspan cascade inlet velocity,

β_1 = midspan cascade inlet flow angle,

ρ_2 = midspan mixed-out cascade exit flow density,

U_2 = midspan mixed-out cascade exit velocity,

β_2 = midspan mixed-out cascade exit flow angle.

In two-dimensional loss measurements which were the goal in the present study, we are interested in the two-dimensionality of the blade surface boundary layers measured at midspan. For two-dimensional flow of the midspan blade boundary layers it is necessary that the AVDR be equal to 1.0 but is not sufficient to ensure two-dimensional flow. It is possible to have an AVDR equal to 1.0 and have non two-

dimensional blade boundary layers at the midspan. The endwall boundary layer at the cascade inlet and within the blade row is responsible for secondary flows within the blade passage. As discussed in Section 2.1, the endwall boundary layer rolls up to form the secondary vortices due to the cross-passage pressure gradient. The cross passage pressure gradient sweeps the endwall boundary layer fluid toward the suction surface. This sweeping action within the blade passage redistributes mass from the endwall boundary layer toward the suction surface from the pressure surface side of the passage. The build up of endwall boundary layer fluid against the blade suction surface forces the blade boundary layer fluid toward the midspan. On the pressure surface the removal of the mass in the blade-endwall region results in a diverging blade boundary layer. It is possible that the AVDR could be sensitive to the blade aspect ratio. For low blade aspect ratios, the relative size of the passage vortices compared to the blade passage cross-sectional area is much larger, thus the effect of the convergence of the blade suction surface boundary layers toward the mid-span would also be larger.

Variations in AVDR are known to affect the measured cascade losses. Rogers et al. (1992) and Whitehouse et al. (1993) investigated the effect of Axial Velocity Ratio (AVR) at incompressible conditions. The AVR is the AVDR at incompressible conditions and is defined as

$$AVR = \frac{U_2 \cos(\beta_2)}{U_1 \cos(\beta_1)} . \quad (2.7)$$

Both Rogers et al. and Whitehouse et al. found that the losses decreased with increasing AVR at incompressible conditions.

Figure 2.17 taken from Saroch (1996) showed that at subsonic conditions, the losses increased with increasing AVDR which contradicts the low speed measurements of Rogers et al. and Whitehouse et al. The work of Saroch was a computational study rather than experimental as that of Rogers et al. and Whitehouse et al. At supersonic conditions the losses decreased with increasing AVDR. In addition, as seen in Figure 2.17, Saroch showed that in the low transonic region, exit Mach number approximately 0.98, that the losses are relatively insensitive to AVDR. Although it is desirable to have an AVDR equal to unity, it may not be critical in the transonic range. This is in agreement with the results of Kiock et al. (1986) as discussed in Section 2.2.

2.3.4 Blade Surface Finish

During the current study, several discontinuities in the original blade profile were removed and additional blade profile coordinates were added mainly near the leading edge for the HS2 cascade. This was done to prevent flat spots on the blade profile which can occur during machining. The computer controlled machining process will put a straight line between the specified blade coordinates. In fact Kiock et al. (1986) mention modifications to the blade profile in order to remove flat spots during the four European wind tunnel tests.

Figure 2.17: Loss Coefficient versus AVDR
(Saroeh (1996)).

The effects of blade surface smoothness was investigated by Radmard et al. (1993). The blades used by Radmard et al. contained flat spots connecting the specified blade profile coordinates which occurred during manufacture. These flat spots resulted in discontinuities in the slope of the blade surface. Radmard et al. found that smoothing out these discontinuities in the slope of the blade surface had very little impact on the blade surface Mach numbers but reduced the losses by an amount which depended on the maximum thickness-to-chord ratio. For a maximum thickness-to-chord ratio of 0.25, which is close to the maximum thickness-to-chord ratio for the HS2 geometry, Radmard et al. measured an 11 percent reduction in the losses for an exit Mach number approximately 0.7 by smoothing and polishing the blades.

2.4 Axial Turbine Loss Prediction Methods

There are several turbine loss prediction methods in the published literature. The most popular of those is Ainley and Mathieson (1951) along with later improvements to their method. Other loss prediction methods have been proposed over the years, e.g. Craig and Cox (1971); however, the Ainley and Mathieson method is still the most widely used. Therefore the present review focuses only on the Ainley and Mathieson method and revisions to that method. In Chapter 8, loss predictions using the loss prediction methods discussed in this section will be compared to the experimental results obtained in the current work. The purpose of the comparison was to evaluate how well the methods predict losses at transonic conditions.

Modifications to the original method of Ainley and Mathieson have been made by Dunham and Came (1970) and Kacker and Okapuu (1982). The Ainley and Mathieson method is a complete loss system which allows all sources of loss in an axial turbine passage to be predicted including profile losses, secondary losses and tip leakage losses at both design and off-design incidence. The present work focuses on losses measured at the midspan in a turbine cascade at transonic conditions. The measurements were obtained at both design and off-design conditions. The present review will concentrate only on the two-dimensional loss portions of the loss prediction methods.

In addition, substantial work at incompressible conditions has been done at Carleton for off-design performance of turbine blades, e.g. Benner et al. (1997). It was of interest to see to what degree the incompressible work could be applied to transonic flows. The work of Benner et al. is reviewed in Section 2.4.2.3.

2.4.1 Loss Predictions at Design Incidence

2.4.1.1 Ainley and Mathieson

The Ainley and Mathieson method was developed using a database of axial turbine experimental data from which the authors formed correlations. This section describes the Ainley and Mathieson method for estimating losses at design incidence.

The total pressure losses in a turbomachinery blade row can be non-dimensionalized in a number of ways. Ainley and Mathieson used the total pressure loss coefficient which is defined as

$$Y = \frac{P_{01} - P_{02}}{q_2} , \quad (2.8)$$

where Y = total pressure loss coefficient,

P_{01} = cascade inlet total pressure,

P_{02} = cascade exit total pressure, and

q_2 = cascade exit dynamic pressure.

Conversion between the total pressure loss coefficient and the enthalpy loss coefficient discussed in Section 2.2 and used in the present work can be made using

$$Y = \frac{\left[1 - \frac{\gamma-1}{2} M_2^2 \left(\frac{1}{1-\zeta} - 1 \right) \right]^{-\frac{\gamma}{\gamma-1}} - 1}{1 - \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{-\frac{\gamma}{\gamma-1}}}, \quad (2.9)$$

where M_2 = blade exit Mach number, and

ζ = enthalpy loss coefficient as defined in Equation 2.5.

The Ainley and Mathieson method nominally applies for a blade Reynolds number of 200,000 based on blade chord. Dunham and Came (1970) which is discussed in the next section offered a correction to the loss predictions of Ainley and Mathieson to account for variations in blade Reynolds number.

At design conditions, the total two-dimensional losses in an axial turbine using the Ainley and Mathieson method are approximated from

$$Y_t = Y_p(i=0) K_{te}, \quad (2.10)$$

where Y_t = total two-dimensional loss coefficient,

$Y_p(i=0)$ = profile loss coefficient at design incidence,

K_{te} = trailing edge loss correction factor.

For two-dimensional profiles losses in axial turbines at design incidence, Ainley and Mathieson assumed that the profile losses were a function of the flow acceleration in the blade passage and the various geometric parameters. For a given outlet flow angle, there are two extremes, a nozzle blade ($\beta_1 = 0$) which produces maximum acceleration and an impulse blade ($\alpha_2 = \beta_1$) which gives zero acceleration. For an arbitrary blade between the two extremes, Ainley and Mathieson used a weighted average to calculate the profile losses,

$$Y_p(i=0) = \left(Y_p(\beta_1=0) + \left(\frac{\beta_1}{\alpha_2} \right)^2 (Y_p(\beta_1=\alpha_2) - Y_p(\beta_1=0)) \right) \left(\frac{t_{\max}/c}{0.2} \right)^{\frac{\beta_1}{\alpha_2}}, \quad (2.11)$$

where $Y_p(i=0)$ = profile loss coefficient at zero incidence or the design inlet flow angle,

$Y_p(\beta_1 = 0)$ = profile loss coefficient for a nozzle blade,

$Y_p(\beta_1 = \alpha_2)$ = profile loss coefficient for an impulse blade,

β_1 = inlet metal angle,

α_2 = exit flow angle,

c = blade chord,

s = blade spacing or pitch

$t_{\max}$ = maximum blade thickness.

Equation 2.11 requires that the exit flow angle be known. Ainley and Mathieson also provide a method of estimating the exit flow angle. The description of the exit flow angle calculation is beyond the scope of the present discussion but is outlined in

Appendix A for completeness. The profile loss coefficient for nozzle blades as a function of the spacing-to-chord and exit flow angle is shown in Figure 2.18. For impulse blades, the profile loss coefficient as a function of blade spacing-to-chord and exit flow angle is shown in Figure 2.19.

The profile loss coefficient calculated using the method of Ainley and Mathieson is based on a trailing edge thickness to blade spacing of 0.02. As mentioned earlier in Section 2.1, high pressure transonic turbines typically have thick trailing edges for cooling reasons. Ainley and Mathieson presents a correction to the loss coefficient for blades with trailing edges of different thicknesses which is shown in Figure 2.20.

2.4.1.2 Dunham and Came

Dunham and Came (1970) made some improvements to the original method of Ainley and Mathieson (1951). Dunham and Came suggested additional corrections to the total loss coefficient to account for Reynolds number and compressibility effects. Dunham and Came suggested the following equation for the total two-dimensional losses

$$Y_t = Y_p K_{Re} K_M K_{te} \quad , \quad (2.12)$$

where Y_p = is the profile loss coefficient calculated from the Ainley and Mathieson,

K_{Re} = Reynolds number correction.

K_M = compressibility correction.

Figure 2.18: Profile Loss Coefficient for Nozzle Blades
(Ainley and Mathieson (1951)).

Figure 2.19: Profile Loss Coefficient for Impulse Blades
(Ainley and Mathieson (1951)).

Figure 2.20: Ainley and Mathieson Trailing Edge Correction (Ainley and Mathieson (1951)).

K_{te} = Ainley and Mathieson trailing edge correction.

As noted in the previous section, the Ainley and Mathieson method was limited to a blade Reynolds number of 200,000. From the discussion presented in Section 2.3, it is clear that as the Reynolds number increases, the losses typically decrease. In fact, from the work of Ladwig and Fottner (1993) for a transonic turbine cascade, the loss at a Reynolds number of 200,000 was twice as high as the loss for the same blades operating at Reynolds numbers of 800,000. Dunham and Came suggested the following correction term for the profile loss coefficient to account for changes in Reynolds number.

$$K_{Re} = \left(\frac{Re_c}{200,000} \right)^{-0.2} \quad (2.13)$$

where Re_c is the actual Reynolds number. As before the Reynolds number is based on the true blade chord and the blade exit Mach number.

The method of Ainley and Mathieson was also limited to subsonic conditions within the turbine blade passages. Dunham and Came suggested a correction for turbine blades operating at high exit Mach numbers. As the exit Mach number increases toward sonic conditions, shock waves begin to form within the blade passage. These introduce another source of loss. Dunham and Came suggested that for exit Mach numbers above 1.0, the Ainley and Mathieson profile loss coefficient be corrected by the multiplier,

$$K_M = [1 + 60 (M_2 - 1)^2] . \quad (2.14)$$

2.4.1.3 Kacker and Okapuu

Further improvements to the original Ainley and Mathieson method were made by Kacker and Okapuu (1982) for design incidence loss predictions only. Using performance data from more modern turbine designs, the authors suggested that as a result of design improvements the profile losses are approximately two-thirds of the profile losses predicted by the Ainley and Mathieson method. The reduction in the profile losses reflected improvements in turbine design. Kacker and Okapuu also suggested improved correlations for the Reynolds number influences on the profile losses, trailing edge losses and shock losses.

To estimate the total mid-span losses, Kacker and Okapuu suggested the equation

$$Y_t = Y_p f_{(Re)} + Y'_{TET} \quad (2.15)$$

where Y_t = total mid-span loss coefficient,

Y_p = profile loss coefficient

$f_{(Re)}$ = Reynolds number correction to the profile loss coefficient, and

Y'_{TET} = trailing edge loss coefficient.

Kacker and Okapuu suggested the following equation for estimating the profile loss coefficient,

$$Y_p = 0.914 \left(\frac{2}{3} Y_p(AM) K_p \right) , \quad (2.16)$$

where $Y_p(AM)$ = profile loss coefficient from Ainley and Mathieson (Equation 2.11),

K_p = compressibility correction.

To account for compressibility effects, the authors suggested that the profile loss as calculated from the Ainley and Mathieson method be corrected by the following term

$$K_p = 1 - K_2(1 - K_1) , \quad (2.17)$$

where K_p is the compressibility correction. The value of K_1 is calculated for $M_2 > 0.2$ using

$$K_1 = 1 - 1.25(M_2 - 0.2) , \quad (2.18)$$

and K_2 is obtained from

$$K_2 = \left(\frac{M_1}{M_2} \right)^2 \quad (2.19)$$

where M_1 is the inlet Mach number and M_2 is the exit Mach number of the blade row.

For changes in Reynolds numbers, Kacker and Okapuu suggested the following corrections. For $Re_c < 200,000$:

$$f_{(Re)} = \left(\frac{Re_c}{200,000} \right)^{-0.4}, \quad (2.20)$$

for $200,000 < Re_c < 1,000,000$:

$$f_{(Re)} = 1.0, \quad (2.21)$$

and for $Re_c > 1,000,000$:

$$f_{(Re)} = \left(\frac{Re_c}{1,000,000} \right)^{-0.2}. \quad (2.22)$$

The method of Kacker and Okapuu differs significantly from the Ainley and Mathieson method in the treatment of the trailing edge losses. Ainley and Mathieson used a correction multiplier on the profile losses, as in Equation 2.10, to account for the increase in losses at the trailing edge. Accounting for the trailing edge losses by using a correction multiplier on the profile losses, Ainley and Mathieson imply that the trailing edge losses are dependent on the same factors and to the same degree as the losses produced in the blade boundary layers. As discussed in Section 2.1, Denton (1993) showed that the losses at the trailing edge are dependent on the blade surface boundary layers but also on the blade base pressure at the trailing edge.

In a similar manner to Ainley and Mathieson for profile losses, Kacker and Okapuu correlated the losses for the trailing edge to the trailing edge losses of nozzle and impulse blades with the same trailing edge geometry. To estimate the trailing edge losses, the authors suggested

$$\Delta\phi_{TET}^2 = \Delta\phi_{TET}^2(\beta_1=0) + \left(\frac{\beta_1}{\alpha_2} \right) (\Delta\phi_{TET}^2(\beta_1=\alpha_2) - \Delta\phi_{TET}^2(\beta_1=0)) \quad , \quad (2.23)$$

where $\Delta\phi_{TET}^2$ = trailing edge energy loss coefficient,

$\Delta\phi_{TET}^2(\beta_1=0)$ = trailing edge loss coefficient for a nozzle blade,

$\Delta\phi_{TET}^2(\beta_1=\alpha_2)$ = trailing edge loss coefficient for an impulse blade.

The trailing edge loss coefficients for nozzle and impulse blades as a function of the trailing edge thickness-to-throat ratio are shown in Figure 2.21.

The energy loss coefficient is converted to a total pressure loss coefficient using

$$Y'_{TET} = \frac{\left(1 - \frac{\gamma-1}{2} M_2^2 \left(\frac{1}{1 - \Delta\phi_{TET}^2} - 1 \right) \right)^{-\frac{\gamma}{\gamma-1}} - 1}{1 - \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{-\frac{\gamma}{\gamma-1}}} \quad . \quad (2.24)$$

Figure 2.21: Trailing Edge Energy Loss Coefficient (Kacker and Okapuu (1982)).

One notable difference in the formula for calculating the total two-dimensional losses used by Kacker and Okapuu as compared to Dunham and Came is that the trailing edge loss is not corrected for Reynolds number. Kacker and Okapuu argued that only the profile losses should be corrected for Reynolds number. The Kacker and Okapuu method suggests that the profile losses and trailing edge losses are relatively independent and that the trailing edge losses do not depend on Reynolds number. As discussed in Section 2.1, this is not entirely the case. The Reynolds number determines the thickness of the blade surface boundary layers at the trailing edge of the blades. The thickness of the suction surface boundary at the trailing edge has an influence on the base pressure, as found by Denton and Xu (1987). Therefore, some dependence on the Reynolds number would be expected since the base pressure plays a significant role in the trailing edge losses downstream of the blade row under transonic conditions.

2.4.2 Loss Predictions at Off-Design Incidence

2.4.2.1 Ainley and Mathieson

In addition to design incidence turbine loss estimation as discussed in Section 2.4.1.1, Ainley and Mathieson also suggest a method for estimating the losses at off-design conditions. Off-design conditions refers to inlet flow angles other than the design inlet flow angle. Incidence is defined as the difference between the inlet flow angle and the design inlet flow angle. The total two-dimensional total pressure loss at off-design conditions can be estimated using

$$Y_t = Y_p(i=0) K_i K_{te} \quad . \quad (2.25)$$

where K_i = correction for off-design incidence.

Figure 2.22 presents the correlation for stalling incidence for blades with a spacing-to-chord ratio of 0.75. Using Figure 2.22 an estimation of the stalling incidence can be made. For variations in spacing-to-chord ratio, Figure 2.23 presents a correction to the stalling incidence. The stalling incidence of the actual blade geometry is then calculated from

$$i_s = i_s(s/c = 0.75) + \Delta i_s \quad , \quad (2.26)$$

where i_s = stalling incidence of the actual blade geometry,

Figure 2.22: Stalling Incidence
(Ainley and Mathieson (1951))

Figure 2.23: Stalling Incidence Correction for Spacing-to-Chord Ratio (Ainley and Mathieson (1951)).

i_s ($s/c=0.75$) = stalling incidence of the blade for a spacing-to-chord ratio of 0.75,

Δi_s = correction term for the spacing-to-chord ratio.

Figure 2.24 presents the correlation for off-design profile loss as a function of the ratio of actual incidence to the stalling incidence. The correction to the profile loss coefficient (K_p), for off-design incidence can be estimated from Figure 2.24.

2.4.2.2 Moustapha et al.

As noted, the loss estimation method of Kacker and Okapuu (1982) is for design incidence only. The authors did not suggest ways of extending the method to off-design conditions. Moustapha et al. (1990) and Benner et al. (1997) presented methods of estimating the additional losses due to off-design incidence for use with the Kacker and Okapuu design-incidence losses.

Moustapha et al. noted the major short-coming of the Ainley and Mathieson method was that the leading edge geometry was not included in the off-design incidence predictions and that the leading edge geometry does affect the incidence characteristics of turbine blades. This had earlier been noted by Mukhtarov and Krichakin (1969) who presented an incidence correlation for off-design profile and secondary losses. The correlation of Mukhtarov and Krichakin was applicable for subsonic and transonic

Figure 2.24: Variation of Profile Loss as a function of Incidence (Ainley and Mathieson (1951)).

conditions. Mukhtarov and Krichakin included in their correlation the leading-edge diameter and the influence of channel acceleration.

Using a large database of cascade results, Moustapha et al. (1990) evaluated the off-design predictions of Ainley and Mathieson and Mukhtarov and Krichakin.

Moustapha et al. found that Ainley and Mathieson tended to over predict the losses but noted that Ainley and Mathieson was based on 1950's blade designs which were less tolerant to off-design conditions as compared to more modern blade designs. Moustapha et al. also found that the Mukhtarov and Krichakin method also largely over-predicted the losses. The authors commented there was some uncertainty on what data the correlation of Mukhtarov and Krichakin was based.

In light of the short-comings of the above mentioned off-design loss predictions, and with the intent of extending the method of Kacker and Okapuu (1982) to off-design conditions, Moustapha et al. developed an improved correlation for the effects of incidence on profile and secondary losses. The authors maintained the blade leading edge diameter and the channel acceleration in their correlation which was originally proposed by Mukhtarov and Krichakin (1969). The authors proposed the following incidence parameter

$$\chi' = \left(\frac{d}{s} \right)^{-1.6} \left(\frac{\cos\beta_1}{\cos\beta_2} \right)^{-2} [\alpha_1 - \alpha_{1(des)}] \quad (2.27)$$

where d = leading edge diameter,

s = blade spacing or pitch,

β_1 = inlet metal angle,

β_2 = exit metal angle,

α_1 = inlet flow angle, and

$\alpha_{1(\text{des})}$ = design inlet flow angle.

Moustapha et al. (1990) then correlated the change in energy loss coefficient with the above incidence parameter,

$$\Delta\phi_p^2 = a_1 \chi' + a_2 \chi'^2 + a_3 \chi'^3 + a_6 \chi'^6, \quad (2.28)$$

where $a_1 = 0.778 \times 10^{-5}$,

$a_2 = 0.56 \times 10^{-7}$,

$a_3 = 0.4 \times 10^{-10}$,

$a_6 = 2.054 \times 10^{-19}$,

for $0 < \chi' < 800$, and

$$\Delta\phi_p^2 = a_1 \chi' + a_2 \chi'^2, \quad (2.29)$$

where $a_1 = -5.1734 \times 10^{-6}$,

$a_2 = 7.6902 \times 10^{-9}$,

for $-800 < \chi' < 0$.

In the above correlation, the $\Delta\phi_p^2$ represents the change in the energy loss coefficient at off-design conditions with respect to the design conditions. Conversion from the energy loss coefficient to a pressure loss coefficient can be made using Equation (2.7)

Moustapha et al. (1990) did not include the effects of compressibility, Reynolds number or turbulence level in their correlation. The authors suggest that using the Kacker and Okapuu (1982) method to first calculate the design loss coefficient would be a suitable approximation. The authors also note that leading edge wedge angle and axial loading distribution are other factors likely to influence the off-design loss characteristics of axial turbine blades.

2.4.2.3 Benner et al.

While the Moustapha et al. (1990) correlation was clearly an improvement over Ainley and Mathieson, it did not predict the off-design performance of all blade rows equally well. Therefore, Benner et al. (1997) suggested a revised version of the correlation of Moustapha et al. which included the leading edge wedge angle in the incidence parameter. The new incidence parameter suggested by Benner et al. was defined as

$$\chi = \left(\frac{d}{s} \right)^{-0.05} We^{-0.2} \left(\frac{\cos\beta_1}{\cos\beta_2} \right)^{-1.4} [\alpha_1 - \alpha_{1(des)}] \quad (2.30)$$

where We = leading edge wedge angle (measured in degrees).

As seen, Benner et al. concluded that the influence of the leading edge diameter was much less than had been suggested by the Moustapha et al. expression. The authors correlated the new incidence parameter to the change in energy loss coefficient as

$$\Delta\phi_p^2 = a_8 \chi^8 + a_7 \chi^7 + a_6 \chi^6 + a_5 \chi^5 + a_4 \chi^4 + a_3 \chi^3 + a_2 \chi^2 + a_1 \chi \quad (2.31)$$

where $a_8 = 3.711 \times 10^{-7}$, $a_7 = -5.318 \times 10^{-6}$,

$a_6 = 1.106 \times 10^{-5}$, $a_5 = 9.017 \times 10^{-5}$, 00

$a_4 = -1.542 \times 10^{-4}$, $a_3 = -2.506 \times 10^{-4}$,

$a_2 = 1.327 \times 10^{-3}$, $a_1 = -6.149 \times 10^{-5}$,

for $\chi \geq 0$, which represents positive incidence and

$$\Delta\phi_p^2 = a_2 \chi^2 + a_1 \chi \quad (2.32)$$

where $a_2 = 1.358 \times 10^{-4}$,

$a_1 = -8.720 \times 10^{-4}$,

for $\chi < 0$ or negative incidence.

The new correlation of Benner et al. (1997) showed much better agreement to the experimental data including the data used by Moustapha et al. (1990) as shown in Figure 2.25. The work of Benner et al. was done at incompressible conditions. The authors

Figure 2.25(a): Moustapha et al. (1990) Correlation.
(b) Improved Correlation of Benner et al. (1997).

noted that the effects of Reynolds number, Mach Number and the blade loading were neglected and required further studies.

As mentioned at the start of Section 2.4, the present study investigated the off-design characteristics of a turbine cascade at transonic conditions. The work is part of an ongoing study at Carleton University into off-design performance of turbine blades. Benner et al. (1997) was also part of that study. It was of interest to compare the predictions of the correlation of Benner et al. to data taken at transonic conditions. Comparisons of the predicted losses from loss prediction methods discussed in this section are presented in Chapter 8.

For computational convenience, the figures of Ainley and Mathieson (1951) presented earlier have been curve-fitted and are presented in Appendix A.

3.0 EXPERIMENTAL APPARATUS AND MEASUREMENTS

The experimental measurements in the current study were performed using the recently commissioned Carleton University High-Speed Wind Tunnel. This chapter briefly describes the wind tunnel, the cascade and the instrumentation used during this work.

3.1 Carleton University High-Speed Wind Tunnel.

The Carleton University High-Speed Wind Tunnel is shown schematically in Figure 3.1. A discussion of the initial commissioning of wind tunnel is presented in Jeffries (1994).

The wind tunnel is located at Carleton University in the Minto Centre for Advanced Studies in Engineering. The wind tunnel is of the blow-down type. Compressed air for the wind tunnel is supplied at a pressure of about 8 atmospheres by a two stage reciprocating compressor located in the basement of the Minto Building. A description of the compressor installation can also be found in Jeffries (1994). The air is stored at about room temperature in four large storage tanks with a combined volume of about 26 cubic metres. Before entering the storage tanks, the air from the compressor is passed through an air drier. The drier lowers the dew point of the air to about -45 degrees Celsius. This prevents condensation in the test section where the air temperature

Figure 3.1: The Carleton University High-Speed Wind Tunnel.

becomes very low due to the acceleration. When the tanks are full, the control valve is opened and the air is discharged through the test section under control of the wind tunnel pressure control system. The pressure control system is a Proportional-Integral control system and is discussed in Jeffries (1994). The test section pressure is maintained approximately constant for as long as there is sufficient air in the tanks. The run is terminated when the pressure in the tanks becomes too low to maintain the test section pressure. This conserves the remaining air in the storage tanks to reduce recharge times. Typical runs are between 30 and 60 seconds in duration and the tanks can be recharged in less than 15 minutes.

The wind tunnel has a cascade test section also shown in Figure 3.1 and in more detail in Figure 3.2. The test section is on long-term loan from the National Research Council of Canada.

The wind tunnel is equipped with an ejector-diffuser system, also shown in Figures 3.1 and 3.2. The ejector-diffuser system is attached downstream of the cascade test section. The ejector-diffuser system operates as a jet-pump. Supersonic air at a Mach number of 1.5 is ejected downstream of the cascade through two nozzles in the ejector system. The ejector nozzles are supplied with high pressure air from the storage tanks. The air from the exit of the cascade and the air from the ejectors are mixed in the mixing tube to provide some pressure recovery. The diffuser on the exit of the mixing tube also provides further pressure recovery. The diffuser rather than the

Figure 3.2: Cascade Test Section and Ejector-Diffuser System.

cascade outlet exhausts to the laboratory at atmospheric conditions. Therefore, the pressure at the exit of the cascade is reduced below atmospheric even without the ejector system in operation.

Used without the ejectors operating, the cascade test section discharges to the laboratory and thus an essentially fixed back pressure. Consequently, any change in the upstream blowing pressure, or cascade inlet total pressure, results in changes in both the Mach numbers and the Reynolds numbers in the cascade. This is a rather inflexible arrangement and limits the usefulness of the test section. The ejector-diffuser system allows control of the cascade back pressure and thus allows the Mach numbers and Reynolds numbers to be varied independently. However, the ejector-diffuser system was not used in the current study.

3.2 HS2 Turbine Cascade.

A single high pressure turbine cascade was used during all of the current work. The cascade geometry is the same as the one used by Kiock et al. (1986) and Baines et al. (1986). The cascade was originally designed and used in a study which compared cascade results from four cascade wind tunnels in Europe for the same cascade geometry. This study was discussed in Chapter 2. By using the same cascade, and comparing the Carleton and the European results, it was possible to assess the quality of the

experimental results obtainable with the Carleton facility. The Carleton data are presented and compared with the European results in Chapter 6.

The blade row geometry is shown to scale in Figure 3.3 along with the nomenclature used to define the geometry. Values of the cascade parameters are listed in Table 3.1. The full cascade as mounted in the Carleton wind tunnel is shown in Figure 3.4. The cascade has been designated HS2 (**H**igh **S**peed **C**ascade **2**) at Carleton. The cascade consists of seven blades and eight complete blade passages. The blades have a chord length of 40 mm and a span of 61 mm. The design exit Mach number of the blades is not specifically quoted in either Kiock et al. (1986), or Baines et al. (1986). Results shown in Chapter 6 suggest that the design exit Mach number is about 0.92.

The cascade is mounted in the test section on a turntable, as shown in Figure 3.5. The turntable allows the incidence of the inlet flow to be varied over a range from +15 degrees to -10 degrees incidence for the HS2 cascade. For blade surface static pressure measurements, blades 3 and 5 (see Figure 3.7(b)) were removed and replaced with blades instrumented with static taps, as shown in Figure 3.4. Blade 3 was instrumented with 18 pressure taps on the blade pressure surface. Blade 5 was instrumented with 23 pressure taps on the suction surface. The last suction surface pressure tap was located on the trailing edge to measure the blade base pressure. The locations of the pressure taps are given in Figure 3.6.

Figure 3.3: Cascade Nomenclature.

HS2 Cascade Parameters	
Chord, C	40.0 mm
Pitch, S	28.4 mm
Throat, O	10.73 mm
Span, H	61.0 mm
Pitch to Chord Ratio, S/C	0.71
Aspect Ratio, H/C	1.525
Throat to Pitch Ratio, O/S	0.378
Stagger Angle, β_s	33.3°
Design Inlet Flow Angle, β_1	30.0°
Gauge Angle, $\cos^{-1}(O/S)$	67.8°
Leading Edge Diameter, D_{le}	3.58 mm
Leading Edge Wedge Angle, We_{le}	75.4°
Trailing Edge Diameter, D_{te}	1.94 mm
Trailing Edge Wedge Angle, We_{te}	11.5°
Unguided Turning, θ_u	20.0°

Table 3.1: HS2 Cascade Geometry.

Figure 3.4: HS2 Cascade Assembly.

Figure 3.5: Cascade Installed in Test Section.

HS2 Static Pressure Tap Locations				
Tap No.	Suction Surface		Pressure Surface	
	X/C	Z/H	X/C	Z/C
1	0.0006	0.705	0.0813	0.705
2	0.0087	0.639	0.1191	0.672
3	0.0264	0.672	0.1572	0.639
4	0.0523	0.705	0.1975	0.705
5	0.0841	0.639	0.2411	0.672
6	0.1206	0.672	0.2886	0.639
7	0.1617	0.705	0.3386	0.705
8	0.2087	0.639	0.3934	0.672
9	0.2613	0.672	0.4524	0.639
10	0.3075	0.705	0.5034	0.705
11	0.3567	0.639	0.5679	0.672
12	0.4083	0.672	0.6186	0.639
13	0.4578	0.705	0.6709	0.705
14	0.5117	0.639	0.7255	0.672
15	0.5600	0.672	0.7827	0.639
16	0.6113	0.705	0.8435	0.705
17	0.6654	0.639	0.9028	0.672
18	0.7135	0.672	0.9539	0.639
19	0.7636	0.705	Note: Z measured from cascade baseplate Diameter of static tap holes = 0.254 mm.	
20	0.8153	0.639		
21	0.8701	0.672		
22	0.9148	0.705		
23	1.0000	0.639		

Figure 3.6: Location of Static Pressure Taps on the HS2 Instrumented Blades.

The instrumented blades were removed and replaced with non-instrumented blades for loss measurements since it is known that the presence of the static taps will affect the blade boundary layers and thus the blade profile losses.

3.3 Test Section Measurements and Data Acquisition

Total and static pressures were measured both upstream and downstream of the cascade. The test section total temperature was also measured in the cascade test section upstream of the cascade. From these measurements, values such as cascade Mach numbers, Reynolds numbers and losses were calculated. This section describes the instrumentation, measurement procedures and data acquisition system used during this work.

3.3.1 Cascade Inlet Measurements

Measurements of total pressure upstream of the cascade were taken using the centre port of a United Sensor Model CT-156 three-hole Cobra-Type probe. The probe was also equipped with a type T thermocouple which allowed the test section total temperature to be measured during the run. The probe stem is 3.96 mm (0.156 inches) in diameter and the height of the test section in the blade span-wise direction is nominally 61 mm. The ratio of the probe diameter to the test section height is thus 6.5 percent. The probe was located about 35 blade chords upstream of the centre blade of the cascade as,

Figure 3.7(a): Location of Cascade Inlet Total Pressure Measurement.

shown in Figure 3.7(a). The probe was also positioned close to the test section side-wall in the pitch-wise direction and along the test-section centre-line in the blade span-wise direction. By placing the probe toward the test section side-wall, any remaining wake caused by the probe would only affect the outer blade passages. The majority of wake measurements downstream of the cascade were made downstream of blades 3, 4 and 5, as shown in Figure 3.7(b).

The cascade inlet static pressure was measured 0.68 blade chord lengths upstream of the blades using a static tap on the top wall of the turntable. The static tap was 0.76 mm (0.030 inches) in diameter. The location of measurements close to the cascade are shown in Figure 3.7(b).

The flow in the blade spanwise direction was measured at three pitch-wise locations to assess the uniformity of the flow at the cascade inlet. Measurements of both total and static pressure were made using a United Sensor Model PAC-T-8-C/C-KL Pitot-Static Probe. The probe was “L”-shaped where the probe head was oriented 90 degrees to the probe stem. The diameter of the probe head and stem was 3.175 mm (0.125 inches). Total pressure port was located 14 stem diameters upstream of the stem. Static pressures were measured from static taps located 4 probe diameters downstream of the total pressure port. The probe was traversed one blade chord upstream of blades 3, 4 and 5, as shown in Figure 3.7(b). The inlet flow was traversed in increments of 1 mm over the

**CASCADE VIEWED
IN HORIZONTAL PLANE**

Figure 3.7(b): Location of Cascade Measurements.

whole inlet passage height. Measurements of the inlet flow uniformity are presented in Chapter 5.

3.3.2 Cascade Exit Measurements

The measurements downstream of the cascade were made downstream of blades 3, 4 and 5 at the blade midspan. The measurements of the flow downstream of the cascade were performed using three probes which are shown in Figure 3.8. These probes were custom designed for use in the Carleton wind tunnel by Islam (1999).

The downstream total pressure measurements were made using a three-hole probe which also allowed flow direction to be measured. The tip of the probe is 1.4 mm (0.054 inches) wide or 4.9 percent of the blade spacing. Some measurements of total pressure were also performed using the Pitot probe shown in Figure 3.8(b). This was done to verify the three-hole probe total pressure measurements. The Pitot probe head is 1.0 mm (0.04 inches) in diameter or 3.5 percent blade spacing.

Static pressure measurements downstream of the cascade were measured using the static probe shown in Figure 3.8(c). The tip outer diameter of the static pressure probe was 1.0 mm (0.04 inches) or 3.5 percent of the blade spacing. Static pressures downstream of the cascade were also measured on the cascade base plate, using a row of 33 static pressure taps located at 40 percent blade chord downstream of the cascade. The

THREE HOLE COBRA PROBE

ENLARGED VIEW OF PROBE TIP

Dimensions are in mm unless otherwise specified

Figure 3.8(a): Three-Hole Probe (Islam (1999)).

Dimensions are in mm unless otherwise specified

Figure 3.8(b): Total Pressure (Pitot) Probe (Islam (1999)).

Dimensions are in mm unless otherwise specified

Figure 3.8(c): Static Pressure Probe (Islam (1999)).

static taps on the cascade base plate were spaced 0.2 blade pitches apart with a diameter of the wall static taps were 0.254 mm (0.010 inches).

The cascade exit flow was traversed at 40 percent of the blade chord downstream of the cascade at the blade midspan. The downstream flow was traversed in 1 mm increments over one complete blade pitch which was found to produce good wake resolution for the HS2 cascade. As will be shown in Chapter 5, good spacial resolution was obtained using uniform step-sizes through cascade exit flow. However, for other cascades such as HS1 (see Jouini(2000)), smaller step-sizes within the blade wakes were used as compared to the step-sizes used in the blade passage freestream flow. This was done since the width of the blade wakes for the HS1 cascade were much narrower than the HS2 cascade.

3.3.3 Experimental Procedure

Total pressure and static pressure traverses of the cascade exit flow were made in separate runs. In addition, due to the short duration of the wind tunnel run, to traverse one complete pitch required 3 wind tunnel runs. The cascade exit flow was traversed using a traverse gear and probe holder which entered the test section flow through the side-wall downstream of the test section. The traverse gear arrangement is shown in Figure 3.9. The traverse gear was designed to allow traversing in both the pitch-wise and span-wise directions. During the current work, measurements downstream of the cascade

Figure 3.9: Probe Traverse Gear Assembly (taken From Jouini (2000)).

were made at the cascade mid-span only, and thus the span-wise motion was not used. The spanwise traverse gear, which holds the probe stem, is mounted on a threaded rod. The threaded rod is driven by a Compu-Motor stepper motor which moves the probe stem in the pitch-wise direction. The stepper motor was operated in full-step mode which corresponded to 200 steps per revolution. The translation of the probe head per motor step was carefully measured. In full-step mode, the probe translates at 0.0098 ± 0.0004 mm per motor step.

3.3.4 Scanivalve and Pressure Transducer

All pressure measurements were all made using a single pressure transducer mounted inside a Scanivalve Corporation model SSS 48C/MK4 48-Port Scanivalve. The transducer was a Kulite Semiconductor miniature pressure transducer with model number XCQ-062-25A and serial number 4568-3B-136. The full scale range of the transducer is 0 to 25 psi absolute. The pressure transducer was re-calibrated daily. The pressure transducer calibration methods will be described in Section 3.4. The transducer was mounted in a custom designed mounting which allowed it to be positioned inside the Scanivalve and very close to the outlet port of the Scanivalve. This minimized the volume between the outlet port of the Scanivalve and the transducer.

The settling time between steps of the Scanivalve was measured by applying different pressures to adjacent ports of the Scanivalve. Figure 3.10 shows the pressure

Figure 3.10: Scanivalve Pneumatic Step Response.

history for a single step on the Scanivalve. The time constant was found to be approximately 0.004 seconds. A delay of 0.02 seconds, which corresponds to five time constants, was implemented into the data acquisition system to allow the pressure to settle before pressure data was taken.

As mentioned earlier, the total and the static pressures downstream of the cascade were taken in separate runs. During a total pressure traverse six individual pressure measurements were recorded at each traverse location: cascade inlet total pressure, cascade inlet static pressure, a downstream reference static pressure measured on the cascade base plate using one of the static ports described earlier, and the three port pressures on the three-hole probe. In a static pressure traverse, four individual pressures were recorded at each traverse location: the static pressure probe measurements were taken in place of the three hole measurements. Homing the Scanivalve was found to significantly delay sequential measurement sets. The valve requires about 2 seconds to home. To increase the number of cascade exit flow traverse points which could be collected in a single wind tunnel run, each of the individual pressures were connected to eight ports on the Scanivalve using six pneumatic manifolds. Each pneumatic manifold had one input port and eight output ports. The output ports of each manifold were arranged and connected to the Scanivalve head as shown in Figures 3.11 (a), (b). This arrangement allowed the six pressure measurements to be obtained eight times in one revolution of the Scanivalve. This eliminated the need to home the Scanivalve after each measurement set and increased the number of cascade exit traverse points that could be

Figure 3.11(b): Data Acquisition Setup for Static Pressure Traverse downstream of the Cascade.

collecting in a single run by a factor of two. All pressures were connected to the pneumatic manifolds using lengths of plastic tubing with an inner diameter of 0.86 mm (0.034 inches). The minimum required lengths of the plastic tubing were used, around 1 m (39.4 inches) in length, to connect the pressure probes and static taps to the manifolds. Each of the eight output ports on the manifolds were connected to the Scanivalve head using short lengths, approximately 152.4 mm (6 inches), of the same plastic tubing.

3.3.5 Data Acquisition System

The data were recorded using a Hewlett-Packard HP3852A Data Acquisition / Control System. The system serial number is 2525A02467. The system is on long-term loan to Carleton from the Ministry of Transportation - Air Accident Investigation Division. The system was equipped with four HP44702B, 13 Bit High Speed Voltmeters. The HP44702B voltmeters are capable of sampling rates up to 100 kHz. Pressure transducer output signals were digitized at a sampling rate of 2000 samples per second using one of the HP44702B voltmeters with the serial number 2707A00735 . The sampling rate of 2 kHz was chosen based on an investigation of the required sampling rates for the Carleton University wind tunnel which is discussed in Chapter 4. The system is also equipped with an HP44701A, 5 ½ Digit Integrating Voltmeter with serial number 2710A02574 and a HP44708A, 20 Channel Relay Multiplexer with Thermocouple Compensation with serial number 2620A00180. Using the multiplexer and HP44701A voltmeter, temperature measurements were made from the thermocouple

on the United Sensor Cobra probe described earlier. The data acquisition was operated using a Pentium 90 Desktop Computer. The data acquisition system was connected to the computer using a Hewlett Packard Interface Bus (HPIB) card installed in the computer. Software to operate the data acquisition system was written in Visual Basic for DOS. The source code and executables can be found on the data acquisition system computer located in the wind tunnel control room.

3.4 Probe and Transducer Calibrations

To perform the measurements needed for this work, the pressure transducer and three-hole probe required calibration. The following sections describe the calibrations of the pressure transducer and three-hole probe.

3.4.1 Pressure Transducer Calibration

The Kulite pressure transducer was calibrated using a pressure chamber which was built on site for this purpose. The pressure chamber was constructed using a length of 8 inch diameter pressure piping. The chamber is instrumented with threaded holes where the pressure transducer and a Mercury manometer were attached to measure the pressure inside the chamber. It was found that day to day calibrations were slightly different, and it is common practice at other facilities to calibrate daily. Therefore, the same procedure was adopted at Carleton.

The calibrations were performed over the range of 0.5 to 2.0 atmospheres, which corresponds to the range of pressures measured in the wind tunnel. For pressures below atmospheric, the pressure chamber was pumped out using a vacuum pump. For pressures above atmospheric, the chamber was pressurized with shop air. The pressure inside the chamber was measured using a Mercury manometer which allowed the pressure to be measured to an accuracy of ± 0.05 inches of mercury. One side of the manometer was connected to the pressure chamber and the other side was open to atmosphere. The daily atmospheric pressure in the lab was measured using a Bourdon tube barometer to an accuracy of ± 0.1 inches of Mercury. The barometer was positioned close to the pressure chamber.

The Kulite transducer calibration is shown in Figure 3.12. Figure 3.12 shows that over a year of use, the slope of the Kulite transducer calibration changed slightly. However, since the transducer was calibrated daily, this change had no impact on the experimental data obtained with the transducer.

3.4.2 Three-Hole Probe Calibration

The three-hole probe was calibrated using a probe calibration rig which was custom designed by Corriveau (1998) to be inserted into the test section with the cascade removed. Thus, the pressure control system for the wind tunnel is used to provide constant flow conditions in the calibration insert. The calibration rig is simply a

Figure 3.12: Kulite Pressure Transducer Calibration.

convergent nozzle which allows the probe to be calibrated in yaw for flow velocities up to sonic. A supersonic nozzle block for the probe calibration rig has also been manufactured but has not been tested and commissioned. The calibration nozzle exhausts into the lab and thus has a fixed back pressure equal to atmospheric. The calibration nozzle Mach number was varied by varying the test section blowing pressure, which is essentially the inlet total pressure to the calibration insert.

The three-hole probe was calibrated in yaw in 1 degree steps over the range of yaw angles from -10 degrees to +10 degrees relative to the probe centre-line. The calibration was performed for Mach numbers from approximately 0.4 to 1.0. The three-hole probe calibration data was expressed in terms of the non-dimensional coefficients suggested by Lewis (1966). The following coefficients are determined as a function of the yaw angle, ϕ :

$$K_1 = \frac{P_1 - P_s}{P_0 - P_s} \quad (3.1)$$

$$K_{12} = \frac{P_1 - P_2}{P_0 - P_s} \quad (3.2)$$

$$K_{13} = \frac{P_1 - P_3}{P_0 - P_s} \quad (3.3)$$

$$K_{23} = \frac{P_2 - P_3}{P_{01} - P_s} \quad (3.4)$$

$$K_{\phi 2} = \frac{P_1 - P_2}{P_1 - P_3} \quad (3.5)$$

$$K_{\phi_3} = \frac{P_1 - P_3}{P_1 - P_2} \quad (3.6)$$

where P_1 , P_2 and P_3 are the three port pressures as defined in Figure 3.13. P_{01} and P_s are the local total and static pressures respectively. The left and right sides of the probe were defined by the probe being oriented with the epoxied side on the right as shown in Figure 3.13. During manufacture, a groove was milled along the length of the probe to allow the internal pneumatic connections to be made within the probe. The groove was then filled with epoxy to protect the internal connections from damage when the probe was placed in the flow. The probe was inserted into the test section with the same orientation used during calibration. The variations of the calibration coefficients with yaw angle and Mach number are shown in Figures 3.14 to 3.19.

The coefficients defined above allow several quantities such as the total pressure, static pressure and flow angle to be extracted from three-hole probe measurements taken in an unknown flow. During the present study, only the coefficients K_1 , K_{ϕ_2} and K_{ϕ_3} were used to calculate the total pressure and flow angle from three-hole probe measurements downstream of the cascade. Static pressures downstream of the cascade were measured directly as described earlier.

The total pressure was calculated from the three-hole probe measurements using the K_1 coefficient. K_1 represents the local dynamic pressure as measured using the three-

Figure 3.13: Three-Hole Probe Numbering System and Orientation.

Figure 3.14: K_1 versus Yaw Angle (ϕ)

Figure 3.15: K_{12} versus Yaw Angle (ϕ).

Figure 3.16: K_{13} versus Yaw Angle (ϕ).

Figure 3.17: K_{23} versus Yaw Angle (ϕ).

Figure 3.18: $K_{\phi 2}$ versus Yaw Angle (ϕ).

Figure 3.19: $K_{\phi 3}$ versus Yaw Angle (ϕ).

hole probe centre port and the local static pressure normalized on the actual local dynamic pressure. From Figure 3.14, it is seen that K_1 is essentially equal to 1 over the range of yaw angles and Mach numbers for which the calibration was performed. Therefore, under subsonic conditions, the total pressure downstream of the cascade was assumed equal to the pressure measured on the three-hole probe centre port for yaw angles within ± 10 degrees relative to the probe centre-line. For supersonic conditions, the pressure measured on the three-hole centre port was corrected for the normal shock at the mouth of the probe port. During the current study the range of flow angles measured were well within the ± 10 degrees limits of the probe calibration.

The flow angle relative to the probe centre-line was calculated using the K_{ϕ_2} and K_{ϕ_3} coefficients from Figures 3.18 and 3.19. Since both coefficients yield an estimate of the flow angle, the value of the flow angle was calculated from each coefficient and the two values were averaged. Between the limits of approximately ± 2 degrees relative to the probe centre-line both coefficients show little variation with Mach number. Since the measured flow angles during the current study were within ± 2 degrees, averaging the two estimates of the flow angle was found to be suitable and no correction for Mach number was required. However, for flow angles larger than ± 2 degrees variations in the probe calibration with Mach number are clearly observed from Figures 3.18 and 3.19. For measured flow angles larger than ± 2 degrees some method of incorporating the change in calibration with Mach number would be required.

Another parameter which affects the probe calibrations is the probe Reynolds number. Dominy and Hodson (1992) investigated the effects of compressibility and Reynolds number and turbulence level on five-hole probe calibrations. The authors found that compressibility did not have a large affect on the probe calibrations. However the probe Reynolds number, based on the tip diameter of the probe, could largely influence the probe calibrations and the uncertainties on the calculated flow angles. Dominy and Hodson examined several different five-hole probe designs and the sensitivity of the calibration to probe Reynolds number, compressibility and turbulence level. One type of probe examined by Dominy and Hodson was the Pyramid type five-hole probe. This type of probe consists of one tube placed in the centre and the other four tubes around the centre tube spaced 90 degrees apart. There are two side ports in the yaw plane of the probe and two side ports in the pitch plane of the probe. To a first approximation, the three hole probe used in the current study is a five-hole Pyramid probe without the two ports in the pitch plane. Therefore it may be reasonable to draw some conclusions regarding the three-hole probe based on the work of Dominy and Hodson. The authors found that between probe Reynolds numbers of 20,000 to 80,000, the yaw errors calculated from the probe calibration were within ± 0.5 degrees relative to the true yaw angle of the flow for pyramid probes. At Reynolds numbers below 20,000 the errors in the yaw angle increased due to the changing nature of the flow around the probe head, the details of which are beyond the scope of the present discussion. Figure 3.20 shows the probe Reynolds number from the calibration data for the three-hole probe in the present work. Figure 3.21 presents an evaluation of the

Figure 3.20: Three-Hole Probe Reynolds Number versus Mach Number.

Figure 3.21: Three-Hole Probe Calibration Extracted Flow Angle versus Actual Probe Yaw Angle.

extracted flow angles from the three-hole probe calibration versus the true flow angle. The evaluation was calculated using the calibration data and feeding it into the data processing routines for extracting the flow angle from three-hole probe measurements. At the two lowest Mach numbers, 0.38 and 0.52, the changes in the probe calibration and the increased error in the extracted flow angles are clearly evident. At these two Mach numbers the probe Reynolds number was below 20,000 as seen in Figure 3.20. Figure 3.21 suggests that the extracted flow angle from the calibration can be resolved to approximately ± 0.5 degrees provided the probe Reynolds number is not below 20,000. In the data presented in later chapters, the cascade Mach numbers were always greater than 0.52, and thus the probe calibration was assumed to be independent of Mach number and probe Reynolds number.

From Figure 2.21, it is also noted that for small changes in flow direction of about ± 2 degrees relative to probe centre-line, the extracted flow angle is accurate to approximately ± 0.2 degrees. In the current study, the probe centre-line was positioned very close to the expected cascade outlet flow direction. The flow angles measured in the current work were within ± 2 degrees of the probe centre-line. Therefore the accuracy of the exit flow angles extracted from the three-hole probe calibration are most likely to be approximately ± 0.2 degrees.

4.0 SAMPLING RATES AND TIMES

This chapter presents the results of the investigation to determine the required sampling rates and times in the Carleton University High-Speed Wind Tunnel.

4.1 Introduction

The pressures in the wind tunnel fluctuate due to the turbulence of the flow. In turbulent flows, fluctuations dictate that the instantaneous pressure and velocity measurements must be time-averaged to obtain accurate mean values of the flow quantities. The CU wind tunnel is a new facility and the present study was the first study conducted using this facility. The required sampling rates and times for pressures and velocities measured in the wind tunnel were not known. Since the Carleton University High-Speed Wind Tunnel is a blow-down wind tunnel the sampling times must be kept to a minimum to maximize the wind tunnel productivity. The goal of the work presented in this chapter was to establish minimum acceptable sampling times and the corresponding sampling rates for the CU wind tunnel.

Sampling rates and times are part of the much larger issue of measurement uncertainty. There is extensive literature regarding estimating measurement uncertainty, some of which will be discussed in the next chapter. Much of the published work on measurement uncertainty focuses on methods of estimating uncertainties on calculated

quantities obtained from a set of individual measurements with known individual uncertainties. For example, in transonic wind tunnel testing, Mach number is an important quantity. The Mach number calculation requires local static and total pressure measurements. Therefore the uncertainty on the Mach number is a function of the uncertainties in the local static and total pressures. The uncertainty in the pressure measurements is a function of the sampling rates and times, among other things. This chapter will focus on the sampling rates and times for pressure measurements in the CU wind tunnel and the resulting uncertainties for the individual pressure measurements. Chapter 5 will discuss uncertainties for calculated quantities such as Mach numbers and loss coefficients.

The subject of sampling times in high speed aeronautical wind tunnels has been discussed by Muhlstein and Coe (1979). The authors present an experimental study on the integration times required to obtain mean static pressure data in transonic flows. The measurements were performed on an aircraft model in a large continuous flow wind tunnel at a free stream Mach number of about 0.8. Three flow conditions were studied, namely attached flow, separated flow and flow with shocks. The data were collected at a sampling rate of 1000 Hz. Muhlstein and Coe used surface mounted pressure transducers in their work. This type of transducer has very fast response times. The authors mention that using probes with slower pneumatic response times would likely increase the required sampling times. The study found that the required integration times ranged from 0.1 seconds for attached flows and up to 2 seconds for the shock flow case. Muhlstein and

Coe suggest that these integration times are typical for large wind tunnels under the examined conditions.

Although Muhlstein and Coe provide some guidance for obtaining mean static pressure, they leave a number of questions unanswered. The effect of sampling rate on the integration times was not addressed. The authors mention that a sampling rate of 1000 Hz was used but did not elaborate on how that value was obtained. It is also not clear to what extent the integration times can be applied in intermittent facilities such as the present one. Intermittent wind tunnels will have variations in the mean pressures due to the effects of the pressure control system. Slow drift or small oscillations in the wind tunnel operating point are to be expected. In addition, one would expect integration times to be a function of the pneumatic response of the pressure measurement system which includes the pneumatic response of the probes and pressure transducers used. In the current study, the pressure probes and static taps were connected to the pressure transducers using lengths of plastic tubing. This arrangement of the instrumentation is the more common one as compared to flush-mounted transducers used in Muhlstein and Coe's study. The pneumatic response of the pressure measurement system used in the current work was thus slower. Finally, the data of Muhlstein and Coe were only for static pressure measurements and whether the same results are applicable to total pressure measurements requires investigation.

Baines et al. (1991) performed an uncertainty analysis specifically related to transonic turbine cascade measurements. The work of Baines et al. was performed using the Oxford University Blow-Down Wind Tunnel and appears to have been done as part of the four European study already discussed in Chapter 2. Baines et al. examined the sensitivity of quantities such as the enthalpy loss coefficient, Mach number and Axial Velocity Density Ratio (AVDR) to the uncertainties on the individual measurements that were used in calculating these quantities. The authors found that the uncertainty in the enthalpy loss coefficient was dominated by the cascade inlet and exit total pressures and much less sensitive to the downstream static pressure. In addition, Baines et al. found that the uncertainties in the AVDR were very sensitive to the cascade inlet Mach number. Therefore, the inlet and exit total pressures and the inlet static pressure were considered as the measurements most likely to dominate the uncertainty in cascade measurements in the current study. Therefore those quantities were used for determining the required sampling rates and times.

4.2 Low Frequency Characteristics of the Pressure Signals

The present section examines the low-frequency characteristics of the pressure measurements made in the Carleton University High-Speed Wind Tunnel.

Figure 4.1 shows the cascade inlet total pressure with time during a run in the CU wind tunnel at a cascade exit Mach number of 0.95 which is close to the design cascade

Figure 4.1: Cascade Inlet Total Pressure as a Function of Wind Tunnel Run Time.

exit Mach number for the HS2 cascade. Overall, the pressure control is quite good. After the end of the start up transient, the drift in the pressure is within a range of around 0.01 bars absolute or less than 1% of the mean value.

Figure 4.2 shows the time histories during the useful part of a typical run for the three pressure signals investigated here. For the case shown, the cascade exit Mach number was 0.95. Some low frequency fluctuation in the operating point is evident. The frequency spectra of the pressure signals were examined both digitally and with analog instruments. A strong peak was found in the spectra at about 0.5 Hz. The peak was present under all operating conditions of the wind tunnel. It was eventually traced to the pressure control system as shown next.

Figure 4.3 shows the behavior of the test section blowing pressure for two different gain settings on the pressure control system. For the upper curve, a high gain setting was deliberately used to examine the control system behavior. An oscillation is clearly apparent at a frequency of about 0.5 Hz. For the lower curve, the gain was reduced. Figure 4.4 shows the power spectra of the pressure signals for the same two signals. The high-gain data has a strong peak at low frequency. A low-frequency peak still appears to be present for the low-gain data, although its magnitude is reduced and slightly shifted to a higher frequency. The existence of the peaks in both signals was confirmed by the auto-correlations for the data. From additional tests it was concluded that the low-

Figure 4.2: Measured Test Section Pressures as a function of Wind Tunnel Run Time during the Useful Portion of the Wind Tunnel Run.

Figure 4.3: Cascade Inlet Total Pressure Fluctuations for Different Wind Tunnel Pressure Control System Gain Settings.

Figure 4.4: Frequency Spectra for the Two Pressure Signals shown in Figure 4.3.

frequency pressure fluctuations will occur for all gain settings with the present configuration of pressure control valve.

Unless unacceptably long averaging times are used, all pressure measurements made with this facility will have an uncertainty of the same order of magnitude as the peak-to-peak amplitude of the low frequency fluctuations. Therefore it is essential to reduce these fluctuations as much as possible. Unfortunately, while lowering the gains reduces the magnitude of the low-frequency fluctuations it also lengthens the start up transient. Figure 4.1 shows the blowing pressure for a typical run with a low-gain setting. The start-up transient is seen to be around 10 seconds. By comparison, the start up transient was about 6 seconds with the gain setting which gave the upper curve in Figure 4.3. Therefore, choosing a gain setting requires a trade-off between uncertainty level in the data and the useful duration of the run.

4.3 High-Frequency Characteristics of the Pressure Signals

In addition to the low-frequency fluctuations in the pressure signals discussed in Section 4.2, the pressure measurements contain high frequency fluctuations due to the turbulence of the flow. This is evident from Figures 4.3. The magnitude of the uncertainty which these fluctuations introduce into the data is mainly a function of the sampling rates and times used in acquiring the data.

Sampling theory states that data must be sampled at a frequency at least twice the highest frequency found in the data. This minimum sampling frequency is known as the Nyquist sampling frequency. The spectra in Figure 4.4 shows that the inlet total pressure measurements obtained along the centre-line of the test section contain fluctuations in the signal up to about 1000 Hz. This is consistent with Naughton et al. (1993) who commented that by connecting probes and transducers using plastic tubing, the frequency response of the system would likely be limited to 1000 Hz or less.

In order to investigate the sampling times required to account for the high frequency content in the data, the low frequency content should be removed. This is commonly done by filtering. The filtering method used in this work was to average the data over a short time interval and then subtract the interval mean value from the individual data values in that interval. The overall mean for the complete data set was then added to all the data points to return the signals to the original level. An interval averaging-time of about 0.1 seconds was found to remove most of the low frequency content in the data. Figure 4.5 shows the frequency spectra of the high-frequency component of the data after filtering compared to the data with no filtering. It can be seen that the low-frequency data has been effectively removed while the high-frequency data remains unaffected.

Figures 4.6 shows the cumulative mean value as a function of averaging times for the cascade exit total pressure at a Mach number of approximately 0.95 using different

Figure 4.5: Frequency Spectra of the Cascade Inlet Total Pressure before and after filtering to remove Low Frequency Fluctuations.

Figure 4.6: Cumulative Mean Value as a Function of Sampling Time for different Sampling Frequencies.

sampling rates. The cumulative mean value is effectively a running mean value which is calculated using the number of data samples collected as the sampling time increases. The individual mean values were normalized by the long-term mean value for the whole run. The data for each sampling rate was obtained from a single run of over 20 seconds in duration. The data was divided into 1 second intervals and the cumulative means were then calculated for each interval. Five intervals of data are shown in each of the figures. It is seen that the cumulative mean value approaches the long-term mean value, as the averaging time is increased. For sampling rates of 500, 2000 and 10000 Hz, the cumulative mean value appears to have approached a value reasonably close to the long-term mean after approximately 0.1 seconds. At the 100 Hz sampling rate, the cumulative mean continues to vary slightly longer. This suggests that 100 Hz is not a high enough sampling rate.

Figure 4.6 shows that longer sampling times produce mean values closer to the long term mean; however a compromise between uncertainty in the mean pressure measurements and wind tunnel productivity was made. An averaging time of 0.1 seconds was chosen as an acceptable sampling time to account for most of the high-frequency fluctuations for sampling frequencies of 500 Hz or greater. This is mainly due to the fact that the majority of high-frequency content is located between 10 and 100 Hz as shown in Figure 4.5. However a low amplitude peak centred around 500 Hz is observed in Figure 4.5, thus for consistency with the Nyquist frequency, a sampling rate of 2000 Hz was chosen. Clearly longer sampling times would produce more accurate mean values however

longer sampling times were found to reduce wind tunnel productivity to unacceptable levels.

Figure 4.7 shows the cumulative mean values for all the three pressures examined, using the chosen sampling frequency of 2000 Hz and at an exit Mach Number of 0.95. The mean values are reasonably close to the long-term mean value after approximately 0.1 seconds. Figure 4.8 presents the cumulative mean values for the cascade exit total pressure at a sampling frequency of 2000 Hz and for cascade exit Mach numbers spanning the range of Mach numbers examined in the current study. As would be expected, the amplitude of the high-frequency fluctuations increases as the flow velocity increases. However, an averaging time of around 0.1 seconds appears to be sufficient to account for the high-frequency fluctuations in the data for all cases.

4.4 Operating Point Drift and Repeatability

As described in the last section, for an averaging time of 0.1 seconds or longer the uncertainty in the pressure measurements appears to be mainly determined by the low-frequency fluctuations in the wind tunnel operating point. It is not feasible to average the pressure signals long enough to reduce significantly the uncertainty from this source: it would require several seconds of data since the underlying fluctuation has a frequency of about 0.5 Hz. As evident from Figure 4.1, the useful run time for this facility is only a few tens of seconds and therefore such lengthy sampling times are not acceptable.

Figure 4.7: Cumulative Mean Values for a Sampling Frequency of 2000 Hz.

Figure 4.8: Cumulative Mean Values for the Cascade Exit Total Pressure at Different Cascade Exit Mach Numbers.

To incorporate uncertainty in the operating point into the measurement uncertainties, Figure 4.9 presents the mean values over 8 runs under essentially the same conditions for all three pressures examined. The mean values were obtained by time averaging the data in 0.1 second intervals throughout the runs. The mean values were normalized on the long-term mean for the run. It can be seen that the mean values all fall within a range of about ± 0.25 percent of the long-term mean for all runs. From these data, the variation in the mean values was calculated from the long-term mean value using the standard deviation. For all three pressures, the standard deviation of the mean values was roughly ± 0.1 percent of the long-term mean value. It is very common to quote variations in experimental quantities in terms of a 95% confidence level which is twice the standard deviation. Also shown in Figure 4.9 are the 95% confidence limits calculated for each of the three pressures. The largest variation in the mean values for the three pressures examined was for the cascade exit total pressure. For a 95% confidence level, the uncertainty in the measured mean values is approximately $\pm 0.25\%$ of the long-term mean value.

Figure 4.10 shows the mean values for the cascade inlet and exit total pressures after 0.1 seconds of averaging over runs with increasing cascade exit Mach numbers. The mean values for the cascade inlet and exit static pressures are shown in Figure 4.11. The static pressure was measured on one of the static taps along the base plate of the cascade located 0.4 blade chord downstream of the blades. Also shown on the figures is the 95%

Figure 4.9: Mean Values of the Examined Pressures over 8 Runs.

Figure 4.10: Mean Values of the Cascade Inlet and Exit Total Pressures.

Figure 4.11: Mean Values of the Cascade Inlet and Exit Static Pressures.

confidence limits of $\pm 0.25\%$ of the long-term value. For all pressures and exit Mach numbers, the mean values are within the 95% confidence limits at least 95% of the time.

These results indicate that pressure measurements can be obtained to within an accuracy of $\pm 0.25\%$ relative to the long-term mean value under all conditions including different runs by sampling at 2000 Hz and averaging for 0.1 seconds. This uncertainty only accounts for the drift in the operating point of the wind tunnel during a run and the uncertainty due to the high-frequency fluctuations due to turbulence in the flow. The total uncertainty in the absolute value of the measured pressures is also a function of the uncertainty in the pressure transducer calibration. The uncertainty due to the pressure transducer calibrations is discussed in the next chapter.

The study done in this section was performed using probes and static taps attached to individual pressure transducers. During initial cascade testing, it was found that using individual pressure transducers for every measured pressure at times resulted in physically inconsistent measurements. For example: the measurements of the total pressure in the mid-passage at the cascade exit were found at times to be slightly higher than the inlet total pressure. The pressure transducer calibrations appeared to change slightly with temperature. Since different transducers were exposed to different flow temperatures due to the different measurement locations, the changes in the transducer calibrations among different transducers was not uniform. Therefore, it was decided to take all measurements using a single transducer and a Scanivalve system so that any small changes in the

transducer calibrations would be reflected in all pressure measurements equally. The sampling rates and times were re-verified after the change to the single transducer/Scanivalve arrangement and it was found that the new pressure measurement arrangement did not alter the required sampling rates and times. Therefore, the sampling rate of 2000 Hz and a sampling time of 0.1 seconds was kept. It is worth pointing out that other authors have noted the same problem with using multiple transducers. Dominy and Hodson (1993) comment that they used a single transducer and a Scanivalve system while calibrating 5-hole probes to eliminate small errors in the measured pressure differences due to different transducer calibrations.

5.0 CASCADE TEST SECTION COMMISSIONING

Measurements were taken to assess the flow quality in the cascade test section and to estimate the uncertainties in measurements made using the Carleton University High-Speed Wind Tunnel. The results of those studies are presented in this chapter.

5.1 Cascade Test Section Flow Quality

5.1.1 Cascade Inlet Flow Uniformity

Measurements were taken upstream of the cascade to examine the uniformity of the flow at the cascade inlet. Measurements were made using the United Sensor Pitot-Static Probe discussed in Section 3.3.1. Three locations one blade chord upstream of blades 3,4 and 5 were traversed in the spanwise direction, as discussed in Chapter 3.

Figure 5.1 shows the measured Mach numbers as a function of spanwise location across the cascade inlet. The inlet flow is seen to be uniform over about 60 percent of the inlet passage height in the blade spanwise direction. The end-wall boundary layers are approximately 20 percent of the blade span on both top and bottom test section end-walls. The test section has provisions for boundary layer bleed on both end-walls to control the cascade inlet boundary layer thickness. However inlet boundary layer bleed was not used in the current work.

Figure 5.1: Cascade Inlet Mach Number versus Span-wise Height.

The cascade inlet turbulence intensity was also measured by Corriveau (1999) and found to be approximately 4 percent.

5.1.2 Cascade Exit Flow Periodicity

Measurements downstream of the cascade were taken to examine the periodicity of the flow through the blade passages. Periodicity refers to having the same mass flow through each of the blade passages and equal losses on each blade within the cascade. Ideally one would like to have equal mass flow through all blade passages within a cascade. For cascades with limited number of blades, to obtain good quality loss measurements it is required to have at least approximately equal mass flow through some of the blade passages usually within the centre blade passages of a cascade.

Traverses in the pitch-wise direction of total and static pressures were taken at 40 percent blade chord downstream of the cascade. Figure 5.2 shows the measured cascade exit total pressure as a function of pitch-wise location for three different Mach numbers. As can be seen in the figure, the width of the wakes and the peak-to-peak variations in the wakes are similar within the centre blades suggesting reasonably periodic flow within centre of the cascade.

Figure 5.2: Cascade Exit Total Pressure versus Pitch-wise Location

Figure 5.3 presents the isentropic Mach numbers measured downstream of the cascade as a function of pitch-wise location. The isentropic Mach numbers were calculated from the cascade inlet total pressure and the cascade exit static pressure. The cascade exit static pressure was measured at 40 percent blade chord downstream of the blades by traversing the static pressure probe described in Chapter 3 in the blade pitch-wise direction. Within the centre blades of the cascade, the variations in the isentropic Mach numbers in the pitch-wise direction are seen to be similar. Figure 5.3 shows that the isentropic cascade exit Mach number begins to increase in the range of blades 5, 6 and 7. This is believed to be caused by the test section side-walls.

Table 5.1 shows the mixed-out values of cascade exit Mach number (M_2), loss coefficient (ζ), exit flow angle (β_2) and the midspan mass flow per unit area (m/A) for all blade passages in the cascade. The mean values and the standard deviation (σ) were calculated using the values of each of the quantities from the three centre blades namely 3, 4 and 5.

As discussed in Chapter 2, Table 2.1 taken from Kiock et al. (1986) also shows measurements of the variations from blade-to-blade of various quantities including exit Mach number, loss coefficient and cascade exit flow angle. From Table 2.1, the blade-to-blade variations of the cascade exit Mach number were between 0.003 and 0.006 among the four European wind tunnels. The blade-to-blade variations in the Carleton tunnel are seen from Table 5.1 to be between 0.002 to 0.015. The blade-to-blade

Figure 5.3: Cascade Isentropic Exit Mach Number versus Pitch-wise Location

	$M_2 = 0.53$				$M_2 = 0.81$				$M_2 = 0.98$			
Blade	M_2	ζ	β_2	m/A	M_2	ζ	β_2	m/A	M_2	ζ	β_2	m/A
1	0.527	0.067	66.8	202.2	0.803	0.051	66.9	293.6	0.975	0.048	66.9	345.5
2	0.530	0.044	66.9	204.3	0.800	0.042	67.1	294.1	0.981	0.044	67.0	346.0
3	0.528	0.042	67.2	203.8	0.800	0.042	67.3	294.6	0.981	0.054	67.0	343.3
4	0.530	0.048	67.4	203.5	0.804	0.044	67.4	293.7	0.981	0.052	67.4	344.4
5	0.546	0.056	68.1	207.6	0.828	0.041	67.7	297.3	0.984	0.048	67.2	345.0
6	0.581	0.065	70.9	214.9	0.895	0.046	69.3	300.6	0.994	0.051	67.6	343.9
7	0.576	0.068	77.3	213.8	0.972	0.050	76.0	299.7	1.229	0.353	71.3	329.9
Mean	0.534	0.048	67.6	205.0	0.811	0.042	67.5	295.2	0.982	0.051	67.2	329.9
σ	0.010	0.007	0.5	2.3	0.015	0.002	0.2	1.9	0.017	0.003	0.2	1.0

Table 5.1: Variation in the Cascade Quantities from Blade-to-Blade. Mean and Standard Deviation (σ) values calculated from blades 3, 4 and 5.

variations in the loss coefficients among the four European wind tunnels ranged from 0.001 to 0.007. The blade-to-blade variations in the Carleton tunnel shown in Table 5.1 are between 0.002 to 0.007. The blade-to-blade variations in the cascade exit flow angle among the four European wind tunnels was between 0.06 to 0.22 degrees. At Carleton, the blade-to-blade variations were between 0.2 to 0.5 degrees. Therefore it was concluded that the periodicity at the Carleton University wind tunnel was comparable to the periodicity found at the four European wind tunnels.

5.1.3 Test Section Total Temperature

As discussed in Chapter 3, the Carleton wind tunnel is of the blow-down type. During a run, air is discharged from the storage tanks. The polytropic expansion of the air in the storage tanks during a run causes the total temperature of the flow to decrease with time throughout the run. The rate of change in total temperature is a function of the mass flow rate out of the storage tanks. For high mass flow runs, the total temperature will also decrease more rapidly since there is less time for heat transfer into the remaining air in the storage tanks. Other blow-down wind tunnel facilities use various methods such as thermal baffling within the air storage tanks or pebble-bed heaters to reduce the drop in temperature during a run. The Carleton wind tunnel is not at present equipped with a means of total temperature control.

The main problem with the changing total temperature is that the cascade Reynolds number does not remain constant during a run for a constant cascade exit Mach number. The decreasing temperature causes the Reynolds number based on blade chord to increase throughout the run. It was of interest to examine the magnitude of the changes in the flow temperature for different cascade exit Mach numbers. As already mentioned, the rate of decrease in temperature during a run is a function of the mass flow. The mass flow in the test section is determined by the cascade geometry for a given cascade exit Mach number. Therefore the results presented in this section are specific to the HS2 cascade and thus are not general.

Figure 5.4 shows the change in cascade inlet total temperature during a single run of about 30 seconds duration for the HS2 cascade over the range of cascade exit Mach numbers examined in the current work. As can be seen, the temperature difference between the start of the run and the termination of the run increases as the cascade exit Mach number increased. The higher exit Mach number runs required higher mass flow and therefore the decrease in total temperature was greater. For the highest cascade exit Mach number in Figure 5.4, the difference in temperature between the beginning and the end of the run was approximately 18 degrees Celsius.

In the next section, the uncertainty in the Reynolds number will be examined. The variation in Reynolds number due to the temperature change in total temperature will be included in the uncertainty in the quoted cascade Reynolds number.

Figure 5.4: Change in Cascade Inlet Total Temperature versus Cascade Exit Mach Number based on a run time of approximately 30 seconds.

5.2 Estimation of Measurement Uncertainty

In this section, an estimate of the uncertainty in quantities including pressures, Mach numbers, Reynolds numbers, Axial Velocity Density Ratio and loss coefficient will be presented. As discussed in the introduction, the present study was the first transonic turbine cascade research project conducted using the Carleton University High-Speed Wind Tunnel. The uncertainties in the various quantities listed above were not known. The approach taken in the current study was to produce estimates of the uncertainties using uncertainty analysis as a first approximation. It is anticipated that as experience is gained using the Carleton wind tunnel, the estimates of the uncertainties will likely be refined in future work.

The uncertainty analysis was performed using the method suggested by Kline and McClintock (1953). Kline and McClintock appear to have given the first practical guidelines for estimating the uncertainties in calculated results based on the uncertainties in individual measurements. The method of Kline and McClintock is still widely used today.

The uncertainty in a calculated result is a function of the uncertainty in each of the individual measurements used in calculating that result. For example, Mach number is a function of the local static and total pressures and therefore the uncertainty in the

Mach number is a function of the uncertainties in the local static and total pressures. A general outline of the method suggested by Kline and McClintock is given below.

Assuming a result, R , is a function of a number of independent variables, $v_1, v_2, \dots, v_n$, then

$$R = F(v_1, v_2, \dots, v_n) . \quad (5.1)$$

The uncertainty on the result, δR , becomes

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial v_1}\right)^2 \delta_1^2 + \left(\frac{\partial R}{\partial v_2}\right)^2 \delta_2^2 + \dots + \left(\frac{\partial R}{\partial v_n}\right)^2 \delta_n^2} , \quad (5.2)$$

where $\delta_1, \delta_2, \dots, \delta_n$ are the corresponding uncertainties of $v_1, v_2, \dots, v_n$.

Kline and McClintock suggested that results be presented along with an uncertainty interval to a prescribed confidence level. The authors suggest that a 95 percent confidence level be used as a standard. A confidence level of 95 percent can also be represented as a 20 to 1 confidence limit, which means that for 20 measurements of the same quantity, 19 measurements will fall within the prescribed limit. If a set of random data is normally distributed, then a 95 percent confidence level means that the quoted value is estimated to fall within an interval of plus or minus two standard deviations relative to the true value.

One study of particular relevance to the current work was that of Baines et al. (1991). The authors present uncertainty estimates from the Oxford University wind tunnel which appear to have been performed as part of the four European wind tunnel study already discussed in Chapter 2. Results from the Kiock et al. (1986) and Baines et al. (1986) papers are also presented in Baines et al. (1991). Some of the uncertainties quoted by Baines et al. (1991) will be presented later in this section and compared with the uncertainties found for the Carleton wind tunnel.

5.2.1 Uncertainty Estimate for Test Section Pressures

All pressures were measured during the current work using a single absolute pressure transducer as described in Chapter 3. The uncertainty in any pressure measurement is dependent on the uncertainty in the pressure transducer calibration and the fluctuations in the measured pressure as discussed in Chapter 4.

The transducer voltages were converted to pressures using the pressure transducer calibration. The pressure transducer calibration is of the form

$$P = mV + z , \quad (5.3)$$

where P = pressure,

m = slope of the transducer calibration curve,

V = measured transducer voltage, and

z = transducer output voltage for zero absolute applied pressure.

Using the method of Kline and McClintock, the uncertainty in the pressure becomes

$$\delta P = \sqrt{\left(\frac{\partial P}{\partial m} \delta m\right)^2 + \left(\frac{\partial P}{\partial V} \delta V\right)^2 + \left(\frac{\partial P}{\partial z} \delta z\right)^2}, \quad (5.4)$$

where δP is the uncertainty in the pressure,

δm = uncertainty in the slope of pressure transducer calibration.

δV = uncertainty in the voltage, and

δz = uncertainty in the zero pressure voltage of the transducer calibration.

In the case of Equation 5.4, the partial derivatives can be obtained analytically from Equation 5.3, thus

$$\delta P = \sqrt{(V \cdot \delta m)^2 + (m \cdot \delta V)^2 + \delta z^2}. \quad (5.5)$$

The values of the slope and transducer zero pressure output and the uncertainties on those quantities were obtained from the linear regression used to fit a straight line to the pressure transducer calibration data. The linear regression method used is outlined in Press et al. (1986). The least-squares method yields the slope and zero pressure output values by fitting a straight line to the pressure transducer data. From the least-squares method, the standard deviations of the slope and zero pressure output voltage were also obtained. Assuming the pressure transducer calibration data were normally distributed,

for a 95 percent confidence level on pressures obtained from the transducer calibration, the uncertainties on the slope and zero pressure output voltage were taken as twice the standard deviations for the slope and zero pressure outputs respectively.

The uncertainty in the voltage signal was obtained from the results in Chapter 4 for the required sampling rate and times. The uncertainty in the mean voltage after a sampling time of 0.1 seconds at a sampling frequency of 2 kHz was found to be approximately $\pm 0.25\%$ of the absolute value with a 95 percent confidence limit.

Figure 5.5 presents the uncertainty relative to the absolute value in the pressures measured using the transducer over the full range of the pressure transducer. The figure shows that at low pressures, less than 0.5 bars absolute, the relative uncertainty begins to increase sharply. During the current study, the pressures encountered were between 0.7 and 1.8 bars absolute. Therefore, from Figure 5.5 the relative uncertainty in measured pressures is approximately constant and equal to $\pm 0.26\%$ of the absolute pressure.

Figure 5.5: Relative Pressure Uncertainty versus Pressure Level.

5.2.2 Uncertainty Estimates for Flow Angles

The calculation of quantities such as the Axial Velocity Density Ratio (AVDR) require the cascade inlet and exit flow angles. In this section, estimates of the uncertainty for the cascade inlet flow angle and cascade outlet flow angle are given.

The cascade inlet flow angle was obtained from the angle of the turntable. As discussed in Chapter 3, the cascade is mounted on a turntable within the cascade test section which allowed for variation in the incidence of the cascade inlet flow. The test section is equipped with a vernier scale to measure the angle of the turntable relative to the centreline of the test section. The vernier has smallest divisions of 0.1 degrees. The inlet flow angle is a function of the orientation of the inlet side walls upstream of the cascade. It is estimated that the inlet flow angle uncertainty is ± 0.1 degrees.

The cascade exit flow angle was measured using the three-hole probe and extracted from the three-hole probe calibration presented in Chapter 3. The probe calibration yields the flow angle relative to the probe centreline. The uncertainty in the exit flow angle relative to the cascade is a function of the uncertainty in the flow angles extracted from the three-hole probe calibration and the uncertainty in the angle of the probe relative to the trailing edge of the blades. From Chapter 3, it was found that the uncertainty in angles extracted from the three-hole probe calibration was approximately ± 0.2 degrees for angles of misalignment less than ± 2 degrees relative to the probe

centreline. The angle of the probe relative to the trailing edge of the blades was measured using a custom designed probe angle alignment apparatus designed by Jouini (2000). The probe angle alignment apparatus was aligned with the cascade to within ± 0.1 degrees, using the blade trailing edges as a reference line. The probe angle was measured optically using a magnifying glass with divided cross-hairs with divisions of 0.0025 inches. The cross-hairs were aligned at one location along the probe body. The magnifying glass was then moved in the longitudinal direction of the probe to a second location along the probe body. Measuring the displacement in both the longitudinal and lateral directions between the two points on the probe body allowed the angle of the probe to be determined relative to the probe angle measurement apparatus.

To determine the angle of the probe relative to the probe angle measurement apparatus the following equation was used,

$$\phi = \arctan\left(\frac{Y_2 - Y_1}{X_2 - X_1}\right) , \quad (5.6)$$

where ϕ = probe angle relative to the probe angle measurement apparatus,

X_1, X_2 = longitudinal coordinates of the reference points on the probe body and,

Y_1, Y_2 = lateral coordinates of the reference points on the probe body.

Therefore the uncertainty in the probe angle relative to the probe angle measurement apparatus is a function of the uncertainty in the individual measurements of the

longitudinal and lateral coordinates of the reference points on the probe body. Therefore using the method of Kline and McClintock, the uncertainty in the probe angle becomes

$$\partial\phi = \sqrt{\left(\frac{\partial\phi}{\partial Y_2}\delta Y_2\right)^2 + \left(\frac{\partial\phi}{\partial Y_1}\delta Y_1\right)^2 + \left(\frac{\partial\phi}{\partial X_2}\delta X_2\right)^2 + \left(\frac{\partial\phi}{\partial X_1}\delta X_1\right)^2} . \quad (5.7)$$

The uncertainty in the coordinates of the points along the probe body were obtained from the scale divisions for longitudinal and lateral measurements. The longitudinal scale had divisions of 0.01 inches and therefore the accuracy of the longitudinal measurements was ± 0.005 inches. As mentioned above, the magnifying glass used to measure the lateral coordinates had divisions of 0.0025 inches. Therefore the lateral measurements were accurate to ± 0.00125 inches. From Equation 5.7, the uncertainty in the probe alignment angle was estimated as less than ± 0.02 degrees.

From the above uncertainty on the measurement of the probe angle relative to the trailing edge plane of the cascade, the uncertainty in the exit flow angle is dominated by the uncertainty in the angles extracted from the three-hole probe calibration. The uncertainty in the overall exit flow angle is estimated to be approximately ± 0.2 degrees for angles of misalignment less than 2 degrees relative to the probe centreline. As mentioned already in Chapter 3, Dominy and Hodson (1992) investigated pyramid five-hole probes and found that in yaw the flow angles could be extracted from the probe calibration to within ± 0.5 degrees. Therefore, the uncertainty in the cascade exit flow

angles for the Carleton wind tunnel may be slightly higher than ± 0.2 degrees and possibly as high as ± 0.5 degrees.

5.2.3 Uncertainty Estimates for Cascade Mach Numbers

Using the uncertainties in the pressures estimated in Section 5.2.1, the uncertainties in the cascade inlet and exit Mach numbers were estimated. The absolute value of the uncertainty on any pressure measurement depends on the pressure level. To insure the analysis was performed over representative pressures observed during testing, for example the correct range of cascade inlet total pressures, the baseline HS2 data was used in the analysis.

From the isentropic Mach number relationship

$$M = \sqrt{\left(\frac{2}{\gamma-1}\right)\left(\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}, \quad (5.8)$$

where M = Mach number,

P_0 = local total pressure and,

P = local static pressure,

the uncertainty in the Mach number becomes;

$$\delta M = \sqrt{\left(\frac{\partial M}{\partial P_0} \delta P_0\right)^2 + \left(\frac{\partial M}{\partial P} \delta P\right)^2} . \quad (5.9)$$

Figure 5.6 presents the cascade inlet Mach number as a function of the cascade exit Mach number for cascade HS2 and the corresponding uncertainties. The uncertainties on the cascade inlet and exit Mach numbers were obtained from Equation 5.9. The error bars for both cascade inlet and exit Mach numbers are shown in Figure 5.6. The uncertainty on the inlet Mach numbers is approximately ± 0.015 for cascade inlet Mach numbers shown in Figure 5.6. The exit Mach number uncertainty is less than ± 0.004 for all exit Mach numbers greater than 0.8.

Figure 5.6: Typical Uncertainties for Cascade Mach Numbers.

5.2.4 Uncertainty Estimates for Enthalpy Loss Coefficient

During the current study, the enthalpy loss coefficient was used to quote the blade midspan losses. As presented in Chapter 2, the enthalpy loss coefficient is defined as;

$$\zeta = 1 - \frac{1 - \left(\frac{P_2}{P_{0_2}} \right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{P_2}{P_{0_1}} \right)^{\frac{\gamma-1}{\gamma}}}, \quad (5.10)$$

where P_{0_1} = cascade inlet total pressure,

P_{0_2} = mixed-out cascade exit total pressure,

P_2 = mixed-out cascade exit static pressure.

Therefore the uncertainty in the loss coefficient becomes

$$\delta \zeta = \sqrt{\left(\frac{\partial \zeta}{\partial P_{0_1}} \delta P_{0_1} \right)^2 + \left(\frac{\partial \zeta}{\partial P_{0_2}} \delta P_{0_2} \right)^2 + \left(\frac{\partial \zeta}{\partial P_2} \delta P_2 \right)^2} \quad (5.11)$$

The uncertainties on all the pressures were found in Section 5.2.1 to be approximately $\pm$ 0.26 percent of the absolute value.

Figure 5.7 presents the loss coefficient versus the cascade exit Mach number and the corresponding estimated uncertainties in the loss coefficients. The uncertainty in the

Figure 5.7: Enthalpy Loss Coefficient and Uncertainty in the Loss Coefficient.

loss coefficient is very high at low Mach numbers and decreases as the Mach number increases. At an exit Mach number equal to 0.92 which is believed to be close to the design exit Mach number for the HS2 cascade, the uncertainty in the loss coefficient is of the order of ± 0.006 for an absolute value of the loss coefficient equal to 0.035.

The uncertainty analysis shows that the uncertainty in the loss coefficient continues to decrease as the Mach number increases. At high transonic and supersonic cascade exit conditions, one would expect the uncertainty to increase due to the formation of complex shock structures at the cascade exit and possibly moving shocks. Since these effects were not taken into account in the uncertainty analysis, the uncertainty at high transonic and supersonic cascade exit Mach numbers is likely to be higher.

5.2.5 Uncertainty Estimates for Axial Velocity Density Ratio

In Chapter 2, the Axial Velocity Density Ratio (AVDR) was defined and the importance of this parameter discussed. In terms of measured quantities in the cascade, the AVDR can be defined as

$$\Omega = \sqrt{\frac{\left(\frac{P_{02}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \left(\left(\frac{P_{02}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \left(\left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}} \cdot \left(\frac{P_2}{P_1}\right) \cdot \left(\frac{\cos\beta_2}{\cos\beta_1}\right), \quad (5.12)$$

where P_{01} = cascade inlet total pressure,

P_1 = cascade inlet static pressure,

β_1 = cascade inlet flow angle,

P_{02} = mixed-out cascade exit total pressure,

P_2 = mixed-out cascade exit static pressure and,

β_2 = mixed-out cascade exit flow angle.

Therefore the uncertainty on the AVDR becomes

$$\delta\Omega^2 = \left(\frac{\partial\Omega}{\partial P_{01}} \delta P_{01} \right)^2 + \left(\frac{\partial\Omega}{\partial P_1} \delta P_1 \right)^2 + \left(\frac{\partial\Omega}{\partial P_{02}} \delta P_{02} \right)^2 + \left(\frac{\partial\Omega}{\partial P_2} \delta P_2 \right)^2 + \left(\frac{\partial\Omega}{\partial \beta_1} \delta \beta_1 \right)^2 + \left(\frac{\partial\Omega}{\partial \beta_2} \delta \beta_2 \right)^2 . \quad (5.13)$$

Figure 5.8 presents the AVDR and the corresponding uncertainties. The data from blades 3, 4 and 5 are presented in the figure. As can be seen, the variation in AVDR appear to be within the limits of the uncertainties on the AVDR values with the exception of exit Mach numbers greater than 1.0. For an exit Mach number of 0.92, the uncertainty in the AVDR is approximately ± 0.035 .

5.2.6 Uncertainty Estimates for Reynolds Number

The cascade Reynolds number used in the current study was based on cascade exit conditions and the cascade blade chord, and therefore

$$Re_c = \frac{\rho_2 U_2 c}{\mu_2} \quad (5.14)$$

where ρ_2 = mixed-out density at the cascade exit,

U_2 = mixed-out cascade exit velocity,

c = blade chord,

μ_2 = viscosity of the flow at the cascade exit.

Figure 5.8: AVDR and Uncertainty on the AVDR.

Using compressible flow relations, the Reynolds number can be written in terms of cascade measured quantities and becomes

$$Re_c = P_2 \cdot \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{1}{RT_{02}}\right)\left(\frac{P_{02}}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\left[\left(\frac{P_{02}}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \left(\frac{c}{\mu}\right) , \quad (5.15)$$

where P_{02} = mixed-out cascade exit total pressure,

P_2 = mixed-out cascade exit static pressure

T_{02} = mixed-out cascade exit total temperature.

The uncertainty in the Reynolds number then becomes

$$\delta Re_c = \sqrt{\left(\frac{\partial Re_c}{\partial P_{02}} \delta P_{02}\right)^2 + \left(\frac{\partial Re_c}{\partial P_2} \delta P_2\right)^2 + \left(\frac{\partial Re_c}{\partial T_{02}} \delta T_{02}\right)^2} . \quad (5.16)$$

The viscosity is a function of temperature and thus not an independent variable and therefore the uncertainty in the viscosity is accounted for in the uncertainty in the total temperature.

There are two issues with regards to the uncertainty in the Reynolds number.

One is the uncertainty in the actual value of Reynolds numbers. The second is the change in the Reynolds number due to the decreasing total temperature throughout the run as discussed in Section 5.1.3.

To traverse one complete pitch, three wind tunnel runs were required as discussed in Chapter 3. The average temperature over the three runs was used to calculate the nominal Reynolds number for the given cascade exit Mach number. Figure 5.9 shows the Reynolds number based on the average total temperature as a function of cascade exit Mach number. Using the average temperature to perform the uncertainty analysis, the uncertainty in the Reynolds numbers shown in Figure 5.9 are ± 4800 and ± 6000 for cascade exit Mach numbers of 0.8 and 1.2 respectively. These uncertainties are much less than the range of Reynolds numbers over which the data were obtained for a given nominal Reynolds number. Figure 5.9 also shows the initial Reynolds number at the start of a single run and the final Reynolds number at the end of the run as a function of cascade exit Mach number. The change in the Reynolds number from the start of a run to the end of a single run will range from between 30,000 and 85,000 for cascade exit Mach numbers between 0.8 and 1.2. Thus the largest source of “uncertainty” in the Reynolds number is the variation in temperature during a wind tunnel run.

For the current work, the Reynolds numbers were greater than 700,000 for all exit Mach numbers greater than 0.8. As discussed earlier in Chapter 2, for Reynolds numbers greater than 700,000 the losses are relatively insensitive to the Reynolds number for the HS2 cascade. Therefore, the changes in Reynolds number experienced in the current work are unlikely to have a large influence on the loss results which are presented in the next chapter.

Figure 5.9: Cascade Reynolds number and Uncertainties including the effects of changes in Total Temperature.

5.2.6 Summary of Uncertainties

Table 5.2 presents a summary of the uncertainties in the quantities above based on a 95% confidence level.

Quantity	Range	Uncertainty Range
Pressure	0.7 - 1.1 (bars)	$\pm 0.26\%$
Inlet Flow Angle	-	$\pm 0.1^\circ$
Exit Flow Angle	$\pm 2^\circ$ relative to probe centre	$\pm 0.2^\circ$
Inlet Mach Number	< 0.285	± 0.01
Exit Mach Number	0.8 to 1.2	$\pm 0.003 - 0.004$
Loss Coefficient	-	± 0.006
AVDR	-	± 0.035
Reynolds Number	$> 700,000$	$\pm 30,000 - 85,000$

Table 5.2 Summary of Uncertainties in Cascade Measured Quantities

As discussed at the start of this section, Baines et al. (1991) performed an uncertainty analysis in conjunction with the four European Wind Tunnel study. The authors quote uncertainty in the cascade exit total pressure as $\pm 0.24\%$ of the absolute value with a 95 percent confidence limit. Baines et al. also quote the uncertainty on the exit Mach number between ± 0.003 and 0.004 for the Oxford University wind tunnel. From Table 5.2, the uncertainties on pressures and the cascade exit Mach numbers at

Carleton are comparable to those of the Oxford tunnel. As noted in Chapter 2, in the four European wind tunnel study, the results from the Oxford wind tunnel showed considerable discrepancies compared to the other three facilities. The discrepancies were attributed by the authors to errors made during testing and not due to the uncertainties in the measurements.

Baines et al. quote the uncertainties on the loss coefficient to be approximately ± 0.004 for an absolute value of the loss coefficient equal to 0.0396 with a 95 percent confidence level. For a loss coefficient of around 0.040 at Carleton, the uncertainty is approximately ± 0.006 . The difference in the uncertainties on the loss coefficients is due to differences in the estimated uncertainties in the cascade inlet total pressures. Baines et al. (1991) quote an uncertainty value which is half that estimated for the Carleton University wind tunnel. The authors mention that the uncertainty in the loss coefficient is dominated by the uncertainty in the cascade inlet total pressure. For the Carleton tunnel, the uncertainty on the cascade inlet total pressure is mostly determined by the fluctuations in the cascade inlet total pressure as delivered by the wind tunnel control system. Therefore, only substantial improvements in control of the test section blowing pressure in the Carleton wind tunnel would significantly reduce the uncertainty in the loss coefficients.

Baines et al. quoted the uncertainty in the AVDR to be ± 0.094 for an AVDR of 1.059. The authors found that the high level of uncertainty was mainly due to the

uncertainty in the exit flow angle. The authors quoted an uncertainty in the exit flow angle of ± 0.5 degrees. The uncertainty on the cascade exit flow angle was found in Section 5.2.2 to be approximately ± 0.2 degrees for the Carleton facility. The lower value of the uncertainty on the AVDR at Carleton as compared to Oxford is consistent with the better estimate of the exit flow angle at Carleton.

The comparison to the Oxford wind tunnel suggests that the uncertainties in measurements made with the Carleton facility are comparable to those of other facilities. In the next Chapter, the Carleton cascade data will be compared to those of the Four European wind tunnels for the same cascade geometry. The agreement of the Carleton data with the European data supports the conclusion that the uncertainty in Carleton measurements are comparable to other well-known wind tunnel facilities.

Finally, the uncertainties presented in Table 5.2 are estimates with a 95 percent confidence limit. Uncertainty estimates are somewhat subjective based on experimental experience with a particular facility and set of equipment. As experience is gained using the Carleton wind tunnel, the uncertainties on the various quantities will likely be reduced or increased accordingly.

6.0 WIND TUNNEL COMMISSIONING: COMPARISON WITH FOUR EUROPEAN WIND TUNNELS

As reviewed in Chapter 2, a series of experiments using the same cascade blade geometry were conducted at four European cascade wind tunnels and a comparison of the results was published in Kiock et al. (1986). The four wind tunnels involved in the study were located at the Von Karman Institute, Belgium (RG), DFVLR Goettingen, West Germany (GO), DFVLR Braunschweig, West Germany (BS) and Oxford University, U.K. (OX). The purpose of the European study was to examine the variations and repeatability of cascade measurements made at different high-quality facilities. Equivalent measurements were made in the Carleton University High-Speed Wind Tunnel (CU), using the same cascade geometry. The cascade geometry was designated RG in the European study; at Carleton, it is designated HS2 (High Speed Cascade 2).

The purposes of the present study were two-fold. The first was to commission the experimental equipment, including the data acquisition and pressure measurement systems, and to develop data processing procedures. As the Carleton wind tunnel is a new facility, the study allowed experience to be gained in transonic cascade research. The second purpose was to document the wind tunnel capabilities by comparing the data obtained at Carleton with that from the four European wind tunnels. This chapter presents the results of the latter study. As will be shown, the Carleton University High-

Speed Wind Tunnel produces results of comparable quality to the four European wind tunnels.

6.1 Summary of Experimental Facilities

Table 6.1 compares geometry of the four European wind tunnels and the Carleton University High-Speed Wind Tunnel. The different tunnels have varying configurations; i.e. suck-down, blow-down and continuous flow. The different tunnels also range in sizes from blade chords of 32.6 mm to 100 mm. The Carleton wind tunnel is very close in size to the RG wind tunnel. Both are also blow-down facilities. Although one would expect to see good agreement in results from all the facilities, one would expect the closest agreement between wind tunnels of similar design and size, as is the case between the Carleton facility and the RG wind tunnel. It is worth noting that both tunnels also have relatively low blade aspect ratios, with Carleton being the lowest among all the tunnels. The aspect ratio of the blades is known to affect the two-dimensionality of the flow in the blade passages. The other tunnels range from 50 to 100 percent higher blade aspect ratios. It should also be noted that the blades manufactured at Carleton (HS2) have much tighter machining tolerances than those used in the other facilities. In addition, the blades were also hand-polished to further improve the surface finish.

Table 6.1 also summarized the instrumentation used at the various facilities, and gives the estimated uncertainties for the results from the reference cascade when it is

PARAMETER	WIND TUNNEL				
	RG	GO	BS	OX	CU
Name of Facility	High Speed Cascade Wind Tunnel (C-2) Von Karman Institute, Belgium	Plane Cascade Wind Tunnel (EGG) DFVLR, Goettingen, West Germany	High Speed Cascade Wind Tunnel (HGB) DFVLR, Braunschweig West Germany	O.U.E.L. Blowdown Wind Tunnel Oxford University U.K.	Carleton University High Speed Wind Tunnel (HSAL) Carleton University, Canada
Reference	Kiock et al. (1986)				Jeffries (1993)
Type of Facility	Blow-down	Suckdown	Closed Loop	Blow-down	Blow-down
Test Section (width x height)	50 mm x 180 mm	125 mm x 353 mm	300 mm x 427 mm	300 mm x 420 mm	60 mm x 200 mm
Length of Upstream Straight End Wall	1000 mm	2550 mm	1550 mm	800 mm	630 mm
Length of downstream guide flow (mm)	200	650	250 mm	290 mm	Long due to Ejector-Diffuser
Number of Blades	10	10	7	7	7
Blade Shape	Figure 3.4				
Difference nominal to real shape (mm)	± 0.031 ± 0.031	± 0.025 ± 0.040	± 0.055 ± 0.083	± 0.055 ± 0.083	± 0.013
Chord, c (mm)	32.6	60.0	100	100	40
Aspect Ratio, h/c	1.534	2.083	3.000	3.000	1.500
Gauge Angle (deg)	67.74 ± 0.10	67.92 ± 0.11	67.96 ± 0.03	67.96 ± 0.03	67.8
Pitch (mm)	23.15 ± 0.04	42.58 ± 0.19	70.88 ± 0.10	70.88 ± 0.10	28.427 ± 0.043
Stagger Angle (deg)	33.14 ± 0.09	33.56 ± 0.12	33.29 ± 0.03	33.29 ± 0.03	33.3
Flow Quantities at Cascade Inlet:					
Static Pressure	Side wall tappings	Side wall tappings	Side wall tappings	Side wall tappings	Side wall tappings
Total Pressure	Probe	Probe	Probe	Probe	Pitot Probe
Flow Angle					Turntable
Flow Quantities at Cascade Exit					
Static Pressure					Static probe
Total Pressure	Two-finger Probe	Wedge probe	Three-finger probe	Three-finger probe	Three-hole probe
Flow Angle					
Wake traverse on blade number	5	5	4	4	4
Axial Distance/Chord	0.64	0.45	0.40	0.40	0.40
Probe Motion	Continuous	Continuous	Step-wise	Continuous	Step-wise
Time required for one wake traverse	45 seconds	70 seconds	180 seconds	1 second	75 seconds
Pressure tappings on blade number	5 and 6	4 and 5	3 and 5	3 and 5	4 and 5
Inlet Flow Angle, β_1	28.95 ± 0.47	29.92 ± 0.13	-	30.04 ± 0.13	30.0 ± 0.05
Inlet Mach Number, M_1	0.282 ± 0.003	0.282 ± 0.004	0.260 ± 0.000	0.252 ± 0.005	0.280 ± 0.001
Exit Flow Angle, β_2	67.03 ± 0.15	67.02 ± 0.22	67.33 ± 0.06	67.76 ± 0.1	67.30 ± 0.11
Exit Mach Number, M_2	0.933 ± 0.006	0.957 ± 0.003	0.828 ± 0.003	0.932 ± 0.004	0.915 ± 0.015
Loss Coefficient, ζ	0.046 ± 0.007	0.049 ± 0.001	0.039 ± 0.003	0.040 ± 0.003	0.038 ± 0.003
AVDR	0.929 ± 0.011	0.937 ± 0.008	0.984 ± 0.006	1.012 ± 0.01	0.936 ± 0.003

Table 6.1: Summary of Wind Tunnel Parameters and Measurements.

operating near its design point. The uncertainties from the Carleton wind tunnel are comparable with and in a number of cases better than, those claimed by the European facilities. It should be noted that the uncertainties in Table 6.1 are based on blade to blade variations in the given quantities and not total measurement uncertainty.

6.2 Cascade Exit Flows

Figure 6.1 shows typical traverse results for local enthalpy loss coefficient as measured from the wake traverse in all five facilities. The results from the four European wind tunnels were taken from Baines et al. (1986) for an exit Mach number approximately 0.9. Also shown are the mixed out flow quantities, averaged over one blade pitch, for each available wake.

The distribution of the local loss coefficients through the wakes are very similar for RG, BS and CU. The results for OX are noticeably poorer than for the other four facilities, for reasons already discussed in Chapter 2. On the other hand, the RG traverses were made somewhat further downstream: at 64 percent blade chord, versus 40 percent blade chord at Carleton. Thus, the RG wake results should show evidence of more mixing. The results suggest that less mixing occurred downstream of the RG cascade as compared to BS and CU. Baines et al. did not comment on the differences in mixing with downstream distance among the different wind tunnels.

Ignoring the OX results, the blade-to-blade variations of the mixed out quantities

Figure 6.1: Local Enthalpy Loss Coefficients versus Cascade Downstream Pitch-wise Location.

seem to be very similar for the remaining four facilities, indicating that all achieve a similar level of periodicity in the flow in the middle part of the cascades.

6.3 Cascade Inlet and Outlet Mach Numbers and Reynolds Numbers

Figure 6.2 presents the cascade inlet Mach numbers as a function of the exit Mach number. The results for CU are shown for three separate blade passages to demonstrate further the periodicity and variation between the passages. The choking Mach number for HS2 is seen from Figure 6.2 to be approximately 0.284. The slight variations in the choking Mach numbers among the different facilities is probably caused by differences in the thickness of the boundary layers on the endwalls of the test section upstream of the cascade. The inlet boundary layers effectively reduce the blade throat-to-inlet area ratio and thus increase the inlet choking Mach number. The choking inlet Mach number is also affected by the two-dimensionality of the flow in the blade passage. In Chapter 5, measurements of the cascade inlet flow showed uniform flow over approximately 60 percent of the flow in the span-wise direction centred around the midspan of the blades. As will be shown in Section 6.4, the Axial Velocity Density Ratio (AVDR) for CU is close to that of RG and is lower than for the other tunnels. The AVDR is an approximate measure of the two-dimensionality of the flow in the blade passages. The other three tunnels, GO, BS, and OX had much higher blade aspect ratios resulting in values of AVDR closer to unity. The trend in the increasing inlet choking Mach numbers among the wind tunnels is consistent with the decreasing values of AVDR which will be shown in Figure 6.4

Figure 6.2: Cascade Inlet and Exit Mach Numbers.

Figure 6.3 presents the Reynolds numbers based on exit conditions and blade chord as a function of the exit Mach number. The BS and OX wind tunnels allow the Reynolds number to be controlled independently of the exit Mach number. Both tunnels were able to maintain the Reynolds number approximately constant for exit Mach numbers greater than 0.7. In the RG and GO wind tunnels, the Reynolds number varied with exit Mach number. Although CU is equipped with an ejector-diffuser system which allows the cascade back pressure to be varied and thus controlling the Reynolds number independently of the exit Mach number, no back pressure control was used during the present measurements. Therefore, the Reynolds number increases with exit Mach number as shown in Figure 6.3. The exit Mach numbers were varied by adjusting the wind tunnel blowing pressure or the inlet total pressure. The Reynolds number and exit Mach number are coupled since varying the inlet total pressure changes the cascade pressure ratio and pressure levels. In the case of the constant Reynolds number facilities, from Figure 6.3, it is clear that the target Reynolds number was around 800,000. In the Carleton tunnel, the Reynolds number of 800,000 is achieved at an exit Mach number of approximately 0.85, which was in the range of Mach numbers of most interest, namely the high subsonic and transonic regions. In fact Kiock et al. (1986) comment that variations in Reynolds numbers between 700,000 and 900,000 did not affect the aerodynamic characteristics of the flow on the blades. Therefore, in that range of Reynolds numbers, changes in Reynolds number are unlikely to have a large influence on the losses.

Figure 6.3: Variation in Cascade Reynolds Number with Cascade Exit Mach Number.

6.4 Cascade Axial Velocity Density Ratio (AVDR)

The Axial Velocity Density Ratio (AVDR) is an approximate measure of the two-dimensionality of the flow in the blade passage as measured at the blade midspan. As discussed in Chapter 2, it is necessary that the AVDR be equal to 1.0 for the blade surface flows at midspan to be two-dimensional, but it is not a sufficient condition.

Figure 6.4 presents the AVDR variation with the exit Mach number. The variation among the different facilities seems quite high. However the value of the AVDR at CU seems to agree reasonably well with both RG and GO in the minimum loss region, which is between exit Mach numbers of 0.9 and 0.95. CU, RG and GO have lower aspect ratio blades than BS and OX. Compared to the Carleton cascade, the blade aspect ratio is twice as high for the BS and OX wind tunnels. The RG and CU are very close to each other at 1.534 and 1.5 respectively. These tunnels showed the lowest values of AVDR which indicated less two-dimensional flow compared to the other wind tunnels. As discussed in Chapter 2, the endwall boundary layers form the passage vortices within the blade passage which are shown in Figure 2.2. The vortices are a result of the cross-passage pressure gradient from which endwall boundary layer fluid is overturned and driven from the pressure surface toward the suction surface. This redistribution of mass creates a converging boundary layer on the blade suction surface and diverging boundary layer on the pressure surface. The convergence results in a thicker boundary layer at midspan and thus higher measured losses. The secondary flows

Figure 6.4: Axial Velocity Density Ratio (AVDR).

or passage vortices are a function of the inlet boundary layer thickness on the endwalls of the test section and not the blade aspect ratio. For blades with low aspect ratios, the secondary flow would occupy a greater portion of the blade passage cross-sectional area and therefore a greater convergence of the suction surface boundary layer than for blades with higher aspect ratios. The trend toward unity in the AVDR among the facilities is consistent with increasing blade aspect ratios. In addition, both the CU and RG tunnels also showed the highest values of the cascade inlet choking Mach number. Neglecting the OX data, the difference in the inlet choking Mach numbers for the other four tunnels is approximately 0.02. Kiock et al. (1986) noted an increase in the inlet Mach number of that amount translates to a decrease in the AVDR of approximately 0.06 which is close to the range of the differences observed in the AVDR as shown in Figure 6.4. Kiock et al. comment that the agreement in the loss measurements and exit flow angles among the four European wind tunnels suggests the AVDR does not affect the losses and flow turning of turbine blades at transonic conditions. As discussed in Section 2.3.3 other authors have found the losses and flow turning to be quite sensitive to AVDR at low speed conditions. The loss results which are presented in the next section for the CU wind tunnel seem to also support the conclusion that the AVDR does not strongly influence the losses at transonic conditions.

6.5 Enthalpy Loss Coefficient

In this section the loss measurements from CU will be compared to those from the four European wind tunnels. Cascade blade loss measurements are influenced by many factors. The influence of Reynolds number, Turbulence Intensity, blade surface finish and Axial Velocity Density Ratio (AVDR) were reviewed in Chapter 2. It is believed that the differences in the losses measured among the different wind tunnels can be mostly explained by the influence of these factors.

Figure 6.5 shows the mixed out losses as a function of the exit Mach number. As can be seen, there is generally good agreement between the CU results and those for the European tunnels for exit Mach numbers around 0.9. For higher exit Mach numbers, the CU data are generally the lowest values measured, while below an exit Mach number of about 0.8 the CU values are highest. These differences are believed to be the result of two different causes.

The lower losses at the higher exit Mach numbers may be mainly the result of higher machining tolerances used, and thus better surface finishes obtained, during manufacturing of the blades at Carleton. Table 6.1 shows the difference between the nominal blade shape and the true blade shape for all facilities. In addition, before manufacturing the Carleton blade set, the profile coordinates were slightly modified to reduce the discontinuities in surface curvature which were present in the original

Figure 6.5: Enthalpy Loss Coefficient.

geometry. Problems with the profile geometry had already been noted by some of the European investigators and Kiock et al. (1986) mention two different geometries being used in the four European wind tunnels. Thus, some of the differences in performance seen in Figure 6.5 and other figures may be due to the slightly different geometry. Of course, the better surface finish should reduce the losses throughout the Mach number range. Figure 6.5 shows that below an exit Mach number of 0.8, the losses are higher than for the other facilities. This suggests the appearance of another factor which has a tendency to increase losses.

From Figure 6.5, the losses were observed to increase as the Reynolds number and exit Mach number decreased. This occurred at exit Mach numbers below 0.8. The same trend was observed in the RG data. As discussed in Chapter 2, measurements made at the BS wind tunnel by Baines et al. (1986), for the present cascade geometry, showed the losses to rise for Reynolds numbers below 700,000. The same trends were observed by Ladwig and Fottner (1993) for another cascade also discussed in Chapter 2. The cascade geometry used in the Ladwig and Fottner study was quite similar to HS2 in the amount of flow turning, thickness-to-chord and inlet and exit Mach numbers. Ladwig and Fottner also investigated two turbulence levels, about 1.5 and 6 percent Turbulence Intensity. In comparison, the Four European Wind Tunnels reported a turbulence level of approximately 1 percent where as at CU the turbulence intensity was around 4 percent. The variations in turbulence levels between the European study and the current study are within the range of turbulence levels studied by Ladwig and Fottner. Thus it is possible

that similar suction surface boundary layer behavior as noted by Ladwig and Fottner was present in the current study. As discussed in Chapter 2, Ladwig and Fottner attributed the increase in losses to the formation of a separation bubble on the suction surface of the blade as the Reynolds number was decreased. The authors also found the losses decreased for higher levels of freestream turbulence.

Comparing the two wind tunnels which showed the increasing losses with decreasing exit Mach number, RG and CU, the losses measured at CU are higher for all exit Mach numbers below 0.8 as compared to RG. From Figure 6.3, the Reynolds numbers for all exit Mach numbers were higher at CU as compared to RG. Therefore between the two facilities, the higher Reynolds number and higher turbulence level at CU should have resulted in lower losses at CU throughout that range of Mach numbers. However as already mentioned the losses at CU were higher than those reported at RG suggesting some other factors such as blade aspect ratio and AVDR were influencing the results for Mach numbers less than 0.8.

For Mach numbers below 0.80, CU has lower value of AVDR than RG, as seen in Figure 6.4 and thus less two-dimensionality of the flow in the blade passages. Rogers et al. (1992) and Whitehouse et al. (1993) found that the losses decreased as the AVR increased at incompressible conditions. As discussed earlier, the suction surface boundary layer converges due to the displacement of endwall boundary layer fluid toward the suction surface as a result of secondary flows in the blade passage. Due to

possible differences in the inlet boundary layers between RG and CU, it is possible that the blade suction surface boundary layer at the midspan is thicker and thus shows somewhat higher losses at CU. The increase in the losses due to suction surface boundary layer convergence should have been observed for all Mach numbers and this is not the case for the transonic and supersonic exit Mach numbers examined. Since there are no data available on the thickness of the suction surface boundary layer or the state of the inlet boundary layers for either of the two facilities, no comparison can be made. Further study of the losses in the subsonic range is required to establish the precise causes of the increased losses at subsonic Mach numbers below around 0.8. For example, a study of the influence of inlet boundary layer thickness on the AVDR and loss measurements in the CU wind tunnel would be very useful. This could be performed by using the boundary layer bleed provisions on the test section to bleed off the cascade inlet boundary layer.

6.6 Exit Flow Angles

Figure 6.6 shows the measured exit flow angles. Again good agreement with the results from most of the four European wind tunnels was obtained, particularly in the transonic region. The CU data also shows the same trends as the other facilities. As the exit Mach number is increased toward sonic, the flow angle increases with respect to the

Figure 6.6: Cascade Exit Flow Angle.

axial direction and toward the gauge angle which is approximately 67.8 degrees for HS2. For subsonic conditions, Islam (1999) argued that the deviation, the difference between exit flow and the blade exit metal angle, would depend on the loading at the aft end of the blade. This loading, and hence the deviation, is a function of several geometric parameters of the blade row.

At choked conditions the exit flow angle becomes a function of the blade geometry downstream of the throat. Figure 6.7 (Sjolander (2000)) shows that at choked conditions, the exit flow angle will always be less than the gauge angle for blades with finite thickness. Figure 6.6 shows that for all facilities with the exception of OX, the exit flow angle was less than the gauge angle at an exit Mach number equal to unity.

To reach supersonic downstream velocities, the streamtube through the cascade must form a convergent-divergent passage. The increase in the area downstream of the throat is achieved by rotating the velocity vector back towards the axial direction. Thus the exit flow angle decreases as the exit flow becomes increasingly supersonic. The data for both CU and GO wind tunnels shows this behavior.

Figure 6.7: (a): Gauge Angle Direction for "Straight-Backed" Blades, (i.e. No Unguided Turning). (b) Gauge Angle Direction for Blades with Unguided Turning.

6.7 Blade Surface Isentropic Mach Numbers

Blade surface isentropic Mach numbers were obtained from pressure measurements on two blades instrumented with static pressure taps. The locations of the instrumented blades within the cascade and the chord-wise locations of the static taps on the blades are described in Chapter 3. Before comparing the data of CU to those of the four European wind tunnels, a brief discussion of the changes to the flow in the blade passage as the exit Mach number is increased from subsonic to supersonic conditions is given.

Figure 6.8 shows the HS2 blade surface Mach numbers for the range of Mach numbers tested. At subsonic exit Mach numbers up to approximately 0.8, the blade loading increases as the cascade exit Mach number increases. Around an exit Mach number of 0.9, the first appearance of supersonic flow on the blade suction surface is observed. This is consistent with Figure 6.2 where the choking exit Mach number is seen to be around 0.9.

For exit Mach numbers greater than 0.99, a shock on the suction surface trailing edge can be seen by the decrease in the Mach numbers which occurs around $X/C = 0.7$. The shock originates at the trailing edge of the adjacent blade on the pressure surface. The complex flow at the trailing edge of a transonic turbine blade was discussed in Section 2.1. The shock is seen to move downstream as the exit Mach number increases.

Figure 6.8: Blade Surface Isentropic Mach Numbers with Increasing Cascade Exit Mach Number.

Baines et al. (1986) presented some Schlieren photographs made at the GO wind tunnel . The shock is visible in the photographs at approximately the same locations on the blade suction surface. At exit Mach numbers of 1.138 and 1.167, the location of the shock on the suction surface moves downstream as the shock becomes more oblique. This was also seen in the Schlieren photographs of Baines et al. The shock wave itself and its interaction with the suction surface boundary layer accounts for part of the increased losses shown in Figure 6.5 at exit Mach numbers greater than roughly 0.95. The majority of losses in this range of exit Mach numbers occur downstream of the blade trailing edges which is further discussed in Section 6.8.

Figures 6.9 and 6.10 show the blade surface Mach numbers for exit Mach numbers of approximately 0.8 and 1.0 respectively. These are the conditions for which blade measurements are available for the four European wind tunnels. On the blade suction surfaces, the CU results fall within the middle of the Four European results. At the higher cascade exit Mach number case shown in Figure 6.10, the peak in the suction surface Mach numbers occurs at around 60 percent along the suction surface which also agrees with the other facilities.

On the blade pressure surface, the surface isentropic Mach numbers seem to be slightly lower as compared to the other wind tunnels for X/C less than 0.5. The inlet flow angle is set using the cascade turntable on which the cascade is mounted. There may be a small difference between the true flow angle relative to the blade and the turntable angle.

Figure 6.9: Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Number of 0.8.

Figure 6.10: Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Number of 1.0.

At the present time the wind tunnel is not equipped to measure the inlet flow angle relative to the blades. However, the inlet side-walls are set accurately relative to the cascade and no large errors in inlet flow angle are expected. Figures 6.11 and 6.12 present the blade surface Mach numbers for variations in the inlet flow angle of 1 degree about the design inlet angle of 30 degrees. The measurements were made at exit Mach numbers of 0.84 and 1.04. As can be seen, the blade loading is not very sensitive to small changes in the inlet flow angle, and therefore the lower suction surface Mach numbers are likely due to another factor.

The lower suction surface Mach numbers are more likely due to differences in the blade geometries. The original blade tested at RG had a flat spot in the blade profile on the pressure surface at approximately 30 percent along the pressure surface. This was identified and the blade coordinates in that area were modified to smooth out the flat spot for the cascades manufactured for GO, BS and OX. The effect of the change in blade profile is clearly seen in the blade surface Mach numbers. In Figures 6.9 and 6.10, the Mach numbers on the forward part of the pressure surface were significantly higher for the RG blade than for the other wind tunnels. Additional smoothing of the coordinates on this part of the blade was done at CU and this probably explains why the Carleton cascade produced the lowest isentropic surface Mach number on this part of the pressure surface for both operating points.

Figure 6.11: Variation in Blade Surface Isentropic Mach Numbers for a 1.0 degree Change in Cascade Inlet Flow Angle at a Cascade Exit Mach Number of 0.84.

Figure 6.12: Variation in Blade Surface Isentropic Mach Numbers for a 1 degree Change in Cascade Inlet Flow Angle for a Cascade Exit Mach Number of 1.04.

6.8 Trailing Edge Base Pressure

As discussed in Chapter 2, for turbine blades operating at transonic and supersonic exit velocities the losses which occur at the trailing edge are typically the largest contribution to the overall midspan losses. The shock structure at the trailing edge and the subsequent mixing downstream of the blade trailing edge accounts for a large portion of the overall measured midspan losses. The base pressure at the trailing edge plays a large role in determining the losses downstream of the blade as described in Chapter 2. The base pressure is expressed in terms of the base pressure coefficient which is defined as

$$Cp_{te} = \frac{P_b - P_2}{P_{02} - P_2}, \quad (6.1)$$

where P_b = base pressure,

P_2 = mixed-out cascade exit static pressure, and

P_{02} = mixed-out cascade exit total pressure.

Figure 6.13 shows the base pressure coefficient as a function of the exit Mach number. The results from CU show good agreement with the other wind tunnels. The OX data is again out of keeping with the rest of the facilities for the reasons already discussed. The peak in the base pressure coefficient in the CU data seems to occur at a slightly higher exit Mach number than the other tunnels. The positive values in the base pressure coefficient which occur between an exit Mach number of 0.9 and 1.0 are

Figure 6.13: Base Pressure Coefficient.

consistent with the region of minimum loss for the CU data which occurs between the same values of exit Mach number, as observed in Figure 6.5.

For Mach numbers less than 0.8, the RG and CU data agreed very well. As discussed earlier with regards to the losses, the CU data showed somewhat higher losses than that of RG in that range of exit Mach numbers. As concluded in Section 6.5, taking into account improvements to the blade profile at CU, higher turbulence levels and Reynolds numbers at CU, the losses should have been lower at CU than RG at exit Mach numbers below 0.8. The base pressure agreement between these two facilities suggests the same level of trailing edge losses and thus does not explain why CU reported losses higher than RG for subsonic exit Mach numbers. Again further studies are required to investigate the cause of the higher losses in the CU data.

Sieverding et al. (1980) correlated the base pressure against the back pressure on the cascade as a function of unguided turning and the trailing edge wedge angle. Sieverding et al. may have used the RG blade data in the development of their correlation as was presented in the four European wind tunnel study. Although the cascades were not explicitly identified by Sieverding et al., one of the cascades used in the paper appears to have been the same geometry as the RG cascade. Figure 6.14 presents the HS2 data compared to the correlation of Sieverding et al. Good agreement between the HS2 data and the Sieverding et al. correlation further confirms that the Carleton results appear to be in agreement with the results from the four European wind tunnel study.

Figure 6.14: Comparison of HS2 Base Pressure with Sieverding et al. (1980) Correlation.

6.9 Concluding Remarks

The work presented in this chapter has demonstrated that the Carleton facility is capable of producing high quality data which agrees with four well known and well recognized research wind tunnels using the same cascade geometry. Best agreement was obtained for all the wind tunnels at transonic exit Mach numbers. The transonic range is of most practical interest in turbine testing because most modern high pressure turbines operate at transonic conditions. However, some improvements can be made to the Carleton University Wind Tunnel and future cascade designs.

Also discussed in Section 6.5, the losses at subsonic conditions need to be further investigated. By comparing the RG and CU data, and the differences between the two tunnels which are of very similar design and configuration, the losses at subsonic conditions at CU ought to have been lower than those of RG. The reason for this discrepancy is not known at this time. As discussed in Chapter 3, the CU wind tunnel is equipped with an ejector-diffuser system which will allow the blade Reynolds number and Mach numbers to be controlled independently. The ejector-diffuser is currently under commissioning. Studies of the effect of variations in Reynolds numbers for constant exit Mach numbers below 0.8 may reveal the reason for the higher measured losses.

In addition, the loss measurements presented in Section 6.5 show that AVDR does not appear to have a large influence on the losses at transonic conditions. For two-

dimensional loss measurements an AVDR close to unity is always desirable. The AVDR could possibly be improved by reducing the inlet boundary layer thickness using the test section boundary layer bleed provisions. A study such as this may help to further explain the behavior of the losses at exit Mach numbers below 0.8.

In this chapter, the measurements on the HS2 cascade were done under roughly the same conditions as those made at the four European wind tunnels. This was done so that direct comparison between the data of the four European wind tunnels and the Carleton wind tunnel could be made. As discussed in Chapter 2, the data from the four European study has been used for commissioning other wind tunnels and as input test cases for three-dimensional Navier-Stokes codes, e.g. Killey and Smith (1993). In the next chapter, the available data on this cascade geometry will be extended to off-design conditions which to the author's knowledge has yet to have been done.

The data from the four European wind tunnel study are tabulated in Appendix B, and the HS2 data are tabulated in Appendix C.

7.0 EXTENSION OF HS2 CASCADE DATA: OFF-DESIGN CONDITIONS

In the previous chapter, the data from Carleton (CU) using the same cascade geometry (HS2) was compared to that of the four European wind tunnels. Good agreement among most wind tunnels, including CU, was obtained especially at transonic conditions. In this chapter, the extension of the HS2 measurements to off-design conditions is presented. The off-design data are tabulated in Appendix C. Off-design conditions consist of variations in the inlet flow angle relative to the design value. As shown in Figure 7.1(a), incidence (i), is defined as the difference between the inlet flow angle (α_i), and the design inlet flow angle ($\alpha_{i(des)}$). Positive values of incidence indicate the actual inlet flow angle is higher relative to the axial direction than the design inlet flow angle.

During the four European wind tunnel studies, all data were taken at design incidence only. In order to increase the body of published data for this cascade geometry, it was decided to include off-design incidences in the current study. This also represents a continuation and extension of recent cascade turbine research at Carleton which has been focused on off-design conditions, both at low speeds, Benner et al. (1997), and at high speeds, Jouini (2000). The cascade off-design data for HS2 were obtained at four values of incidence: -10, +5, +10 and +15 degrees relative to design.

Figure 7.1: Schematic Diagram of Incidence and Changes to the Inlet Flow with Incidence.

In the next chapter, both the design and off-design data will be compared to some of the available loss prediction methods. These methods were reviewed in Section 2.4.

7.1 Cascade Inlet and Outlet Mach Numbers and Reynolds Numbers

Prior to presenting the cascade inlet Mach number data for the off-design conditions, a brief discussion of what happens to the flow at the cascade inlet as the inlet flow angle changes is required. Considering Figure 7.1(b), as the flow angle increases with respect to the axial direction, the distance between the bounding streamlines (w) for the streamtube entering one blade passage will narrow. This decreases the effective inlet flow area seen by the flow and thus decreases the inlet-to-throat area ratio. Therefore, the cascade inlet Mach number will increase for a given exit Mach number as the inlet flow angle increases.

Figure 7.2 shows the measured cascade inlet Mach numbers as a function of the exit Mach numbers for all incidences examined in the current work, including the design inlet flow angle. As the inlet flow angle increased, the inlet Mach numbers also increased which is consistent with the above discussion.

Also shown in the Figure 7.2 are the ideal inlet choking Mach numbers calculated assuming one-dimensional inviscid flow between the inlet and the passage throat.

Sieverding (1993) argued that the acceleration of the flow between the inlet and the throat

Figure 7.2: Variations in Cascade Inlet Mach Number with Incidence.

in a turbine cascade will result in a thinning of the endwall boundary layers. This results in an effective divergence of the endwalls in the blade passage between the inlet and throat planes. Sieverding indicated that higher inlet choking Mach numbers than ideal values are an indication of such divergence of the blade passage. As seen in Figure 7.2, for incidence values of 0, +5 and +15 degrees, the actual choking Mach numbers are higher than the ideal values, which is consistent with the thinning of the endwall boundary layers. However, at -10 and +10 degrees incidence, the measured inlet choking Mach numbers are seen to be slightly lower than the ideal values which is out of keeping with the trends seen at other values of incidences.

The reason for the inconsistencies in the inlet choking Mach numbers for the -10 and +10 degree incidence cases as compared to the other incidences is not known at this time and requires further study. It is not believed to be a result of an error made during the measurements. The cascade inlet Mach number was calculated from the measured total and static pressures upstream of the cascade, as discussed in Chapter 3. Each pressure was connected to the Scanivalve using lengths of plastic tubing. A leak in either the static or total pressures tubes would not account for the lower than ideal choking inlet Mach numbers without affecting other results such as the loss coefficients. Both the cascade inlet total and static pressures were above atmospheric pressure in all cases and thus a leak in the pressure tubing would reduce the pressure in the tubes. A leak in the inlet total pressure line would reduce the measured cascade inlet Mach number but would also reduce the inferred losses. A problem with the inlet total pressure would also have

shown up as a discrepancy between the upstream total pressure and the total pressure in the blade mid-passage flow downstream of the cascade which should be the same at subsonic conditions. From the wake traverse data, no such discrepancies occurred for -10 and +10 degrees of incidence. A leak in the static pressure tubing would increase the measured Mach numbers at the cascade inlet and therefore would not cause the lower than ideal cascade inlet Mach numbers. In summary, the differences between the ideal and measured cascade inlet Mach numbers at -10 and +10 degrees of incidence do not appear to be the results of experimental problems. They therefore seem to be the result of genuine physical effects which have not yet been identified.

Figure 7.3 shows the cascade Reynolds numbers for all incidence test cases. As seen, the Reynolds numbers were roughly the same for all incidences for a given exit Mach number. Again, as was the case for the design incidence data, the Reynolds numbers were higher than approximately 700,000 for exit Mach numbers greater than 0.8. Therefore, the Reynolds number was not expected to have a large influence on the losses in the transonic region. The reasons for this were discussed in Chapter 6.

Figure 7.3: Cascade Reynolds Number.

7.2 Cascade Axial Velocity Density Ratio (AVDR)

Figure 7.4 presents the measured cascade Axial Velocity Density Ratio (AVDR) with variations in the cascade exit Mach number. For the 0, +5 and +15 degrees of incidence cases, the values of AVDR were roughly the same at approximately 0.92. This was not the case for the -10 and +10 degrees of incidence cases. In the previous section, the inlet choking Mach numbers at incidence values equal to -10 and +10 degrees showed slightly lower than the ideal values and were out of keeping with the trends seen in the data at the other incidence cases. Figure 7.4 shows the AVDR for these two cases to be higher and closer to 1.0. It seems unlikely that the flow at the blade midspan is more two-dimensional at these two values of incidences. This is more likely associated with the inconsistent values of the cascade inlet Mach numbers. Considering Figure 7.2, for values of incidence equal to 0 and +5 degrees, the inlet choking Mach numbers were of the order of 0.02 greater than the ideal value. For incidence values of -10 and +10 degrees, the choking inlet Mach numbers were very close to the ideal value. As discussed, the thickness of inlet endwall boundary is the most likely the cause of the higher than ideal cascade inlet Mach numbers. It is very unlikely that the boundary layers at the cascade inlet were thinner at the -10 and 10 degrees of incidence cases than the other incidence cases.

Figure 7.4: Axial Velocity Density Ratio (AVDR)

7.3 Cascade Enthalpy Loss Coefficients

Figure 7.5 presents the measured enthalpy loss coefficients versus cascade exit Mach number for all incidences examined. The losses are seen to increase for all off-design cases with the exception of the +10 degree incidence case. The +10 degree incidence case was nearly equal to the design incidence loss coefficients for all Mach numbers up to approximately 1.0 which may be a result of changes in the base pressures. The loss coefficients at the extreme cases, -10 and +15 degrees of incidence, showed the largest increase in losses, which is expected. Figure 7.6 shows the measured losses as a function of incidence for several cascade exit Mach numbers. The loss coefficients for the HS2 blade geometry are seen to be insensitive to incidence for all Mach numbers. The losses are observed to be lowest at approximately the design exit Mach number of 0.9.

Moustapha et al. (1990) and Benner et al. (1997) both indicated that the leading edge geometry of a turbine blade had a large influence on its off-design losses. According to Benner et al., the large leading edge wedge angle for the HS2 blade geometry makes this blade relatively insensitive to off-design conditions within the range of incidences tested in the current study. As will be shown in Chapter 8, the correlations of Ainley and Mathieson (1951), Moustapha et al. (1990) and Benner et al. (1997) all predict the HS2 blade geometry to be relatively insensitive to off-design incidence within the range of incidences examined here. The Ainley and Mathieson (1951) loss prediction

Figure 7.5: Enthalpy Loss Coefficient.

Figure 7.6: Changes in Enthalpy Loss Coefficient with Incidence.

method discussed in Chapter 2 predicts the stalling incidence for the HS2 cascade to be roughly +35 degrees. This is over twice as high as the maximum incidence of +15 degrees, examined in the current work. Incidences higher than +15 degrees were not attainable using the CU wind tunnel and the HS2 cascade due to limitations of the cascade test section.

7.4 Exit Flow Angle

Figure 7.7 shows the measured exit flow angles relative to the axial direction as a function of exit Mach number for all incidence values tested. With the exception of the highest incidence case, inlet flow angle equal to +15 degrees incidence, the exit flow angles were seen to be greater than the design case. At +5 degrees of incidence, the highest measured exit flow angles were observed. As the incidence was increased to +10 and +15 degrees, the exit flow angle began to decrease. At +15 degrees of incidence, the exit flow angle is less than the design condition for all Mach numbers. As discussed earlier, Ainley and Mathieson (1951) predict the stalling incidence for the HS2 cascade to be approximately +35 degrees. Therefore it is unlikely that flow separation on the blade suction surface has occurred. As will be shown in the next section, the blade surface isentropic Mach numbers show no signs of separation and the loading on the blade aft end increased as the incidence was increased. Islam (1999) showed that deviation is a function of the blade loading on the aft end of the blade. Therefore the slight reduction in the exit flow angle maybe due to changes in the blade loading on the aft end of the blades.

Figure 7.7: Cascade Exit Flow Angle.

7.5 Blade Surface Isentropic Mach Numbers

Figures 7.8 to 7.13 show the blade surface isentropic Mach numbers as a function of incidence for exit Mach numbers ranging from 0.7 to 1.05. Chapter 6 presented a discussion of the flow behavior on the blade surfaces as the exit Mach number was increased at the design inlet flow angle. The blade surface Mach numbers did not show any signs of separation for all exit Mach numbers and incidences. For all exit Mach numbers, the blade surface Mach numbers distributions seem to be roughly the same for incidences of 0, +5, and +10 degrees. This is consistent with the fact that the losses for the HS2 geometry varied relatively little with incidence. For the outside incidence cases, -10 degrees and +15 degrees, some differences between the blade surface Mach numbers are seen in the figures and are discussed next.

For -10 degrees of incidence, the blade loading on the first 30 percent of the suction surface is close to that of the design case for all exit Mach numbers. After 30 percent of chord, the suction surface Mach numbers are slightly higher for all exit Mach numbers as compared to the design case. As discussed by Denton (1993), the losses in the boundary layer vary with the cube of the boundary layer edge velocity. This may account for some of the increase in losses between the design case and the -10 degree incidence case seen in Figure 7.5. From the blade surface Mach numbers it is also seen that for the first 10 percent of the blade pressure surface the flow is more strongly accelerated compared to the design case. This is probably the result of the stagnation

Figure 7.8: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 0.7.

Figure 7.9: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 0.8.

Figure 7.10: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 0.90.

Figure 7.11: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 0.95.

Figure 7.12: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 1.0.

Figure 7.13: Blade Surface Isentropic Mach Numbers,
Cascade Exit Mach Number = 1.05.

point moving toward suction surface. Higher Mach numbers along the whole blade pressure surface are also observed for all exit Mach numbers. This may also contribute to the higher losses at -10 degrees of incidence.

At the highest incidence, +15 degrees, higher Mach numbers along the suction surface are seen for all exit Mach numbers. As with the -10 degree case, the blade surface Mach numbers on the suction surface are somewhat higher compared to the design case and these may account for the increase in the losses also seen at +15 degrees incidence.

As noted, the blade surface Mach numbers did not show any signs of boundary layer separation. Baines et al. (1986) found that for the current blade geometry, at design incidence, an exit Mach number of 0.91, and a Reynolds number of 800,000, the suction surface boundary layer underwent transition to a turbulent boundary layer at around X/C of 0.6. The turbulent boundary layer remained attached on the blade suction surface until the trailing edge. In figure 7.10, the off-design blade surface Mach numbers at approximately the same exit Mach number and Reynolds number are shown which also did not show signs of separation. However, the last blade surface static pressure taps were located at $X/C = 0.92$ on the suction surface and $X/C = 0.95$ on the pressure surface. Thus it is not possible to identify suction surface separation if it is occurring very close to the trailing edge.

7.6 Cascade Base Pressure

The base pressure is shown in Figure 7.14 as a function of exit Mach number for all incidences examined. The base pressure at transonic conditions has a large effect on the losses. For all off-design incidences with the exception of the +10 degree incidence case, the base pressure was found to be somewhat lower than the design case.

In Chapter 2, Denton (1993)'s equation which relates the boundary layer parameters and the base pressure coefficient to total pressure loss coefficient was given as

$$Y = \frac{2\theta}{w} + \left(\frac{\delta^* + t}{w} \right)^2 - \frac{C_{p_b} t}{w} , \quad (7.1)$$

The first two terms on the right hand side of Equation 7.1 are functions of the blade surface boundary layers. Since the blade loading measurements in the previous section showed little change in the blade surface Mach numbers with incidence, these terms would vary relatively little with incidence. The last term on the right-hand side of Equation 7.1 contains the base pressure coefficient as shown in Figure 7.14 as a function of incidence and cascade exit Mach number.

For the off-design incidences of -10, +5 and +15 degrees the base pressure is lower than at the design case. From Figure 7.5, the loss coefficients at the -10, +5 and +15 degrees incidence were higher than at design conditions for all cascade exit Mach

Figure 7.14: Base Pressure Coefficient.

numbers. At the +10 degrees incidence case, the base pressure is nearly equal to that of the design incidence for all exit Mach numbers. Correspondingly the losses appeared to be approximately equal to the losses at design conditions also seen in Figure 7.5.

7.7 Concluding Remarks.

For the HS2 cascade, the losses were found to be relatively insensitive to incidence. As will be shown in the next chapter, for the HS2 cascade this relative insensitivity of the losses to incidence is consistent with and provides support for the recently developed correlation of Benner et al. (1997) for profile losses at off-design incidences. In the next chapter, the results from this chapter and the previous chapter will be compared to the loss prediction methods described in Section 2.4.

There were some unexpected results particularly at the +10 degrees of incidence case. It was not possible to identify any experimental or physical explanation for the discrepancies between the data at +10 degrees incidence and the other incidences examined.

8.0 COMPARISON OF HS2 DATA WITH LOSS PREDICTION METHODS

8.1 Introduction

The most widely used methods for predicting axial turbine losses were discussed in Chapter 2. In this chapter, the results of cascade measurements for the HS2 cascade will be compared to the loss prediction methods discussed in Chapter 2. Comparisons for both design incidence and off-design incidences will be made. The purpose of the comparisons was to evaluate the loss prediction methods for turbines operating at transonic conditions. Of particular interest was the loss predictions at off-design incidence. As already mentioned, the current study of the losses at off-design conditions for the HS2 cascade was made as part of a larger study being conducted at Carleton into off-design turbine losses.

8.2 Loss Predictions for Design Incidence

In this section, the losses measured at the design incidence for the HS2 cascade will be compared to the losses predicted using the method of Ainley and Mathieson (1951) and subsequent improvements to that method due to Dunham and Came (1970) and Kacker and Okapuu (1982). The losses are expressed in terms of the total pressure loss coefficient which is defined as

$$Y = \frac{P_{0_1} - P_{0_2}}{P_{0_2} - P_2} \quad (8.1)$$

where P_{0_1} = cascade inlet total pressure,

P_{0_2} = mixed-out cascade exit total pressure, and

P_2 = mixed-out cascade exit static pressure.

8.2.1 Ainley and Mathieson

Figure 8.1 compares the losses measured from the HS2 cascade with the predictions of the original Ainley and Mathieson (1951) method. Shown in the figure is the profile losses (Y_p) and the trailing edge losses (Y_{TE}) which combined equal the total midspan losses (Y_T). It is important to note that the additional losses which occur in the blade passage at transonic conditions due to shocks are not accounted for in the Ainley and Mathieson method since the method is limited to subsonic conditions. For the HS2 cascade, shocks begin to appear in the trailing edge region for exit Mach numbers greater than about 0.92 and, as seen, shock losses increase the total midspan losses. Compared to the HS2 data, Ainley and Mathieson (AM) tends to over-predict the losses for exit Mach numbers below 1.0. The AM method likely over-predicts the losses for a couple of reasons. The AM method was based on 1940's and 1950's blade designs and on blade

Figure 8.1: Comparison of HS2 Loss Coefficients to Ainley and Mathieson (1951) Loss Prediction.

loss data at a blade Reynolds number of 200,000. The HS2 data was taken at much higher Reynolds numbers, over twice that of the AM data for all exit Mach numbers.

The trailing edge losses predicted by the AM method seen in Figure 8.1 are about one quarter of the total midspan losses. For transonic blades, the AM method seems to under-predict the trailing edge losses. Mee et al. (1992) found that the trailing edge losses comprised about half of the total midspan losses for transonic turbine blades. The geometry of the cascade used by Mee et al. was similar to the HS2 cascade but had much thinner trailing edge. The trailing edge thickness-to-blade spacing of the cascade used by Mee et al. was 0.023. The trailing edge thickness-to-blade spacing for the HS2 cascade is 0.068. Based on the Denton (1993) equation discussed in Chapter 2, one would expect that the trailing edge losses would be a larger portion of the total midspan losses for the HS2 cascade as compared to the cascade used by Mee et al. Further discussion of the influence of the trailing edge thickness is presented in Section 8.2.3.

8.2.2 Dunham and Came

Dunham and Came (1970) suggested modifications to the Ainley and Mathieson method to account for variations in Reynolds number and the increase in losses at supersonic conditions. Figure 8.2 shows the measured HS2 losses compared to the predictions using the AM method with the modifications made by Dunham and Came which is designated here as AMDC. There is much better agreement in the total losses at

Figure 8.2: Comparison of HS2 Loss Coefficients to Dunham and Came (1970) Loss Prediction.

the lower Mach numbers due to the Reynolds number correction. As well, AMDC also predicts the rise in losses at supersonic conditions, but the rise is predicted to start at an exit Mach number of 1.0. From the HS2 data, the rise in the losses due to supersonic conditions within the blade passages are seen to start at approximately 0.92. Having the shock losses appear at an exit Mach number equal to 1.0 is probably not reasonable since there are generally already patches of supersonic flow inside the blade passage, and therefore shocks, before the average exit Mach number reaches 1.0. Thus it might be reasonable for the correlation to include shock losses starting at an exit Mach number somewhat less than 1.0.

AMDC predicts the trailing edge losses decreased slightly as compared to the AM trailing edge losses shown in Figure 8.1. This was a result of the Reynolds number correction being applied to the trailing edge losses in the AMDC method. Kacker and Okapuu (1982) commented that the trailing edge losses are likely to be somewhat independent of the blade Reynolds number and suggested that only the profile losses be corrected for Reynolds number.

8.2.3 Kacker and Okapuu

Kacker and Okapuu (KO) revised the method of AMDC for design incidence only. The authors suggested that for modern blade designs the profile losses are approximately two-thirds of the profile losses calculated from the AM method. KO also

suggested that the trailing edge losses are for the most part independent of Reynolds number, as already mentioned. Figure 8.3 shows the profile, trailing edge and total midspan losses predicted by KO for the HS2 cascade. The overall midspan losses are over-predicted for the HS2. The most notable difference between the KO and the AMDC methods is that KO predicts much higher trailing edge losses. Kacker and Okapuu stated that modern high pressure turbine blades typically have thicker trailing edges for cooling reasons. Therefore, the trailing edge losses would be a much larger portion of the overall losses. As seen in Figure 8.3, at an exit Mach number of around 0.92, KO predicts that the trailing edge losses are about two-thirds of the overall midspan losses. The KO trailing edge losses are approximately equal to total midspan losses measured experimentally for the HS2 cascade. Therefore in the case of the HS2 cascade geometry, the KO method apparently over-predicts the total losses because it over-predicts the trailing edge losses. Further support for this conclusion can be obtained from Denton's (1993) analysis for the trailing edge losses.

The equation of Denton (1993) was already discussed in Chapter 2. From Denton's equation, the losses at the trailing edge can be approximated as

$$Y_{TE} = \left(\frac{\delta^* + t}{w} \right)^2 - \frac{C_{pb} t}{w} , \quad (8.1)$$

where Y_{TE} = trailing edge losses,

t = trailing edge thickness,

Figure 8.3: Comparison of HS2 Loss Coefficients to Kacker and Okapuu (1982) Loss Prediction.

w = blade passage throat opening,

δ^* = blade surface boundary layer displacement thickness, and

C_{pb} = base pressure at the blade trailing edge.

The first term on the right hand side of Equation 8.1 approximates the mixing loss due to the sudden expansion in the flow area at the trailing edge. The second term represents the contribution from the base pressure. To approximate the magnitude of the trailing edge losses for the HS2 cascade, the two terms in Equation 8.1 were estimated from the HS2 geometry and experimental data with a simplifying assumption. During the present work, no boundary layer traverses were made on the blade surfaces. Therefore the displacement thickness of the blade surface boundary layers at the trailing edge were not known.

However as seen in Equation 8.1, the displacement thickness only increases the magnitude of the sudden expansion losses above those resulting from the thickness of the trailing edge relative to the blade throat opening. Therefore, setting the displacement thickness equal to zero results in the minimum possible value of the sudden expansion loss. Using the base pressure data at design incidence which were presented in Chapters 6, the second term on the right hand side of Equation 8.1 was estimated. Figure 8.4 shows the relative magnitudes of the sudden expansion losses compared to those due to the base pressure. For the HS2 geometry, the dominant term for the trailing edge losses is clearly the sudden expansion losses. The overall trailing edge losses shown in Figure 8.4 are about twice those predicted by AMDC but are lower than those predicted by Kacker

Figure 8.4: Trailing Edge Losses Based on Denton (1993).

and Okapuu. Thus, while KO correctly predict a strong influence of trailing edge thickness, they appear to over-predict it for the case of HS2.

For completeness Figure 8.5 presents the predicted exit flow angle from the Ainley and Mathieson method and the measured HS2 exit flow angle with changes in exit Mach number. The Dunham and Came (1970) and Kacker and Okapuu (1982) revisions did not alter the original correlations for exit flow angle. Therefore, Figure 8.5 is applicable to those methods as well. The method of calculating the AM exit flow angle is presented in Appendix A. Both the predicted and the measured flow angles were less than the gauge angle, which is expected for finite thickness blades. The predicted flow angles showed the flow angle increases with exit Mach number up to an exit Mach number of 1.0 whereas the measured flow angle peaked at a slightly lower value of exit Mach number. The agreement between the predicted and the measured exit flow angles is seen to be better closer to the HS2 design exit Mach number of about 0.92.

Figure 8.5: Comparison of HS2 Exit Flow Angle to Ainley and Mathieson (1951) Prediction.

8.3 Loss Predictions for Off-Design Incidence

8.3.1 Ainley and Mathieson

The Ainley and Mathieson (1951) loss system also allows for loss predictions to be made at off-design incidence. Since the Ainley and Mathieson (AM) method is limited to subsonic conditions, a comparison with only the subsonic HS2 data will be made.

Figure 8.6 compares the midspan losses predicted using Ainley and Mathieson method together with the Dunham and Came (1970) correction for Reynolds number (AMDC) for an exit Mach number of 0.80. For this case, the AMDC method predicts the losses quite well. However, Moustapha et al. (1990) and Benner et al. (1997) found that AM tended to over-predict the increase in losses with incidence for many other modern turbine cascades. Likewise the agreement between off-design measurements of Jouini (2000) and the AMDC method was not nearly as good as for the HS2 cascade. This suggests that the good agreement between the HS2 results and the AMDC maybe somewhat fortuitous. In Chapter 7, the leading edge wedge angle of the HS2 cascade was suggested as the likely reason that the HS2 cascade was found to be not very sensitive to off-design incidence. The AMDC correlation does not include any influence of leading edge geometry at all.

Figure 8.6: Comparison of HS2 Loss Coefficient to Ainley and Mathieson (1951) Loss Prediction at Off-Design Incidence.

8.3.2 Moustapha et al. and Benner et al.

Moustapha et al. (1990) and Benner et al. (1997) presented new correlations for the changes in midspan turbine losses with incidence. The Moustapha et al. correlation included the leading edge diameter as the correlating parameter and Benner et al. added the leading edge wedge angle.

Figure 8.7 presents a comparison of the Moustapha et al. and Benner et al. predictions with the HS2 data. The figure shows the change in the total pressure loss coefficient relative to the design incidence value as a function of incidence. Both the Moustapha et al. and Benner et al. correlations were limited to subsonic conditions and therefore the comparison is made for the HS2 data at an exit Mach number of 0.8 only. Figure 8.7 shows that for the HS2 geometry Benner et al. predicts the changes in loss with incidence quite well. Both correlations correctly predict the losses for HS2 are relatively insensitive to incidence but the Benner et al. predictions are closer to the measured values.

In other studies in the Carleton University High-Speed Wind Tunnel, Jouini (2000) performed off-design measurements for a transonic cascade with a very different cascade geometry to that of HS2. Jouini notes some shortcomings with the Benner et al. correlation but his results also suggest that it maybe possible to improve the Benner et al. correlation further with some fairly simple modifications.

Figure 8.7: Comparison of HS2 Change in Loss Coefficient to Moustapha et al. (1990) and Benner et al. (1997).

One of the major contributions of the present study was to extend the database of cascade results to include transonic axial turbine loss data. Also noted earlier, the correlation of Benner et al. was limited to subsonic conditions. Improvements are planned to the Benner et al. correlation to extend its range of applicability into the transonic range based on the contributions of the current study and subsequent studies done at Carleton.

9.0 CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

The Carleton University High-Speed Wind Tunnel is a fully operational transonic cascade research facility. By comparing the results of transonic cascade measurements with the results of the Four European wind tunnel study using the same cascade and at the equivalent operating conditions, it was concluded that the Carleton wind tunnel is capable of producing results for transonic cascades which are comparable in quality with those from other recognized facilities.

It was found that to achieve good quality measurements, several experimental procedures needed to be followed. Pressure transducer calibrations should be performed daily and that day's transducer calibration should be used in processing that day's cascade measurements. It was found that the pressure transducer calibration would drift from day-to-day and common practice at other facilities was to calibrate daily. The same practice was adopted at Carleton. As discussed in Chapter 3, originally different transducers were used for each measured test section pressure. It was found that the pressure transducer calibrations were somewhat sensitive to temperature and the transducer calibrations changed in varying degrees depending on the temperature of the pressure being measured. Therefore a single pressure transducer was used for all pressures so that any change in the transducer calibration would be reflected in all

measured pressures. In Chapter 4, it was found that using a sampling rate of 2000 Hz and a sampling time of 0.1 seconds provided an acceptable balance between the accuracy of the measured pressure and the productivity of the wind tunnel. From Chapter 5, it was concluded that the uncertainty levels at Carleton are equivalent to the uncertainty levels at other facilities.

From the transonic off-design measurements made on the HS2 cascade, presented in Chapter 7, it was found that the HS2 cascade was relatively insensitive to off-design incidence within a range of -10 to +15 degrees of incidence.

9.2 Recommendations

As discussed in previous chapters, further studies are required to investigate some unusual results. The cause of the high losses at low Reynolds number and cascade exit Mach numbers as described in Chapter 6 requires further investigation.

The effect of the Axial Velocity Density Ratio (AVDR) on transonic turbine measurements also requires further study. In the present study, the AVDR values were around 0.92 at the blade midspan which suggested that the flow at the blade midspan was not two-dimensional. Jouini (2000) using the Carleton wind tunnel reported AVDR values greater than 1.0 using a different cascade geometry also indicating non two-dimensional flow at the blade midspan. It is felt the cascade inlet boundary layer is likely

the cause of the AVDR values not being equal to 1.0. As shown in Chapter 5, the inlet boundary layer is approximately 40 percent of the cascade inlet passage height. It is not known whether the AVDR values of around 0.92 for the HS2 cascade had a large impact on the cascade results in the current study. Based on the comparison of the data from the Four European wind tunnels, Kiock et al. (1986) concluding that the agreement between the different facilities, even though the values of AVDR ranged from between 0.92 and 1.0 among the different wind tunnels, suggested that the AVDR did not strongly influence the measurements at transonic conditions. However, Jouini (2000) who performed a subsequent study to the current work using the Carleton wind tunnel and a different cascade found that AVDR not equal to 1.0 may have had an impact on the data. The influence of the AVDR on cascade results at the Carleton wind tunnel could be investigated by bleeding off the cascade inlet boundary layer to reduce the boundary layer thickness at the cascade inlet. The test section is already equipped with boundary layer bleed provisions which were not used in the current work.

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APPENDIX A

Ainley and Mathieson

A.1: Ainley and Mathieson Exit Flow Angle Calculation

Ainley and Mathieson (1951) presented a method for estimating the exit flow angle. The authors suggested the following equation for the exit flow angle,

$$\alpha_2 = \alpha_2^* - 4(s/e) . \quad (\text{A.1})$$

This equation is for blades with curved surfaces downstream of the blade passage throat on the suction surface of the blade. Equation A.1 is applicable for exit Mach numbers less than 0.5 and for a radius of curvature (e), downstream of the blade passage throat. The value of α_2^* is estimated as a function of the gauge angle (β_g) of the blades from Figure A.1. The gauge angle is defined as

$$\beta_g = \cos^{-1} \left(\frac{o}{s} \right) \quad (\text{A.2})$$

where o = blade throat, and

s = blade spacing or pitch.

At an exit Mach number of 1.0, Ainley and Mathieson held that for blades with a straight-backed suction surface from the throat to the trailing edge on the suction

Figure A.1: Exit Flow Angle as a Function of Gauge Angle for Straight-Backed Blades (Ainley and Mathieson (1951)).

surface, the exit flow angle would be equal to the gauge angle. In actuality, for straight-backed blades with a finite thickness, the exit flow angle will be less than the gauge angle which was discussed in Chapter 6. For blades with curved suction surfaces downstream of the passage throat, the exit flow angle will also be less than the gauge angle. Ainley and Mathieson (1955) suggested the following equation for the exit flow angle for curved-backed blades at an exit Mach number equal to 1.0:

$$\alpha_2 (M_2 = 1.0) = \cos^{-1} \left(\frac{o}{s} \right) - f(s/e) \sin^{-1} \left(\frac{o}{s} \right) . \quad (A.3)$$

The value of the correction term, $f(s/e)$, for curved-backed blades is found from Figure A.2. For exit Mach numbers between 0.5 and 1.0, Ainley and Mathieson (1951) suggest that a linear variation in flow angle from the flow angle at an exit Mach number equal to 0.5 to the flow angle at an exit Mach number equal to 1.0.

As mentioned above, Ainley and Mathieson stated that the exit flow angle will be equal to the gauge angle for straight-backed blades for an exit Mach number of unity. The Ainley and Mathieson correlation was based on older turbine designs which typically had very thin trailing edges. The approximation of the exit flow angle being equal to the gauge angle may have been based on infinitely thin blades such as a thin cambered plate.

Ainley and Mathieson (1951) was limited to exit Mach numbers up to sonic. For supersonic exit Mach numbers, the exit flow angle greatly simplifies and becomes a

Figure A.2: Correction for Exit Flow Angle for Curved-Backed Blades (Ainley and Mathieson (1955)).

function of the change in area between the throat and exit plane of the blade passage and the exit Mach number. To reach supersonic downstream velocities, the streamtube through the cascade must form a convergent-divergent passage. The increase in the area downstream of the throat is achieved by rotating the velocity vector back towards the axial direction. Thus the exit flow angle decreases as the exit velocity becomes increasingly supersonic. From Horlock (1973), the exit flow angle for a turbine at supersonic exit conditions becomes

$$\cos \alpha_2 = \cos \alpha_2^* \left(\frac{A}{A^*} \right), \quad (\text{A.4})$$

where α_2 is the exit flow angle,

α_2^* is the flow angle at the blade passage throat,

A is the cross-sectional flow area at the blade passage exit plane, and

A^* is the cross-sectional flow area at the blade passage throat.

The ratio of areas can then be calculated from one-dimensional compressible flow using

$$\frac{A}{A^*} = \frac{1}{M_2} \left(\frac{1 + \left(\frac{\gamma - 1}{2} \right) M_2^2}{\left(\frac{\gamma + 1}{2} \right)} \right)^{\frac{1}{2} \left(\frac{\gamma + 1}{\gamma - 1} \right)}, \quad (\text{A.5})$$

where M_2 is the exit Mach number at the blade passage exit plane. From the above two equations, the exit flow angle under supersonic conditions can be estimated for any supersonic exit Mach number.

The off-design calculations are done for a spacing-to-chord ratio, s/c , equal to 0.75. Figure A.3 presents the correction (K_{α_2}), for the exit flow angle for variations in spacing-to-chord ratio. This correction is required to calculate the exit flow angle if the actual blade had a spacing-to-chord ratio equal to 0.75. For blades with spacing-to-chord ratios other than 0.75, the exit flow angle for the actual blade with a spacing-to-chord ratio of 0.75 can be estimated using

$$\alpha_2(s/c=0.75) = \frac{\alpha_2}{K_{\alpha_2}} \quad , \quad (A.6)$$

where $\alpha_2 (s/c=0.75)$ is the exit flow angle for a spacing-to-chord ratio of the 0.75,

α_2 is the actual exit flow angle for the blade, and

K_{α_2} is the correction multiplier from Figure A.3.

Figure A.3: Correction for Exit Flow Angle with Pitch-to-Chord Ratio (Ainley and Mathieson (1951)).

A.2: Ainley and Mathieson Curve Fits

Figure 2.18: Profile Loss Coefficient for Nozzle Blades.

Polynomial Surface Fit, (Verhiel and Sjolander (1994)).

$$Y_p (\beta_1 = 0^\circ) = \sum A_{I,J} \alpha_2^I \left(\frac{S}{C} \right)^J, \quad (\text{A.7})$$

where $Y_p (\beta_1 = 0^\circ)$ = Loss Coefficient for a Nozzle Blade,

α_2 = exit flow angle,

S = blade pitch,

C = blade chord, and

$A_{I,J}$	J			
	0	1	2	3
0	0.358716	-1.43508	1.57161	-0.496917
1	-0.0112815	0.0548594	-0.0555387	0.014165
2	0.000175083	-0.000824937	0.000652287	-7.30141E-05
3	-8.61323E-07	3.95998E-06	-1.89698E-06	-4.9954E-07

Table A.1: Curve Fit Coefficients for Profile Loss Coefficient for Nozzle Blades.

Figure 2.19: Profile Loss Coefficient for Impulse Blades

Polynomial Surface Fit, (Verhiel and Sjolander (1994)).

$$Y_p (\beta_1 = \alpha_2) = \sum A_{I,J} \alpha_2^I \left(\frac{S}{C} \right)^J, \quad (\text{A.8})$$

where

A _{I,J}	J		
I	0	1	2
0	0.0995503	0.182837	0.01603
1	0.00621508	-0.0283658	0.011249
2	-7.10628E-05	0.000327648	-0.000122645

Table A.2: Curve Fit Coefficients for Profile Loss Coefficient for Impulse Blades.

Figure 2.20: Ainley and Mathieson Trailing Edge Correction.

$$K_{TE} = \sum A_I \left(\frac{t_{TE}}{S} - 0.02 \right)^I, \quad (\text{A.9})$$

where t_{TE} = trailing edge thickness, and

I	A_I
0	1.0
1	5.035529
2	25.968185
3	-79.884556

Table A.3: Curve Fit Coefficients for Ainley and Mathieson Trailing Edge Correction.

Figure 2:21: Trailing Edge Energy Loss Coefficient.

Polynomial Curve Fits, (Verhiel and Sjolander (1994))

$$\Delta\phi^2_{TET} = \sum A_I \left(\frac{t_{TE}}{o} \right)^I, \quad (A.10)$$

where o = blade throat opening, and

	$\Delta\phi^2_{TET}$	
A_I	Nozzle Blades	Impulse Blades
1	0.0232143	0.0387381
2	0.575714	1.13857
3	-0.45	-0.916667

Table A.4: Curve Fit Coefficients for Trailing Edge Energy Loss Coefficient.

Figure 2.22: Stalling Incidence

Polynomial Surface Fit,

$$i_s (S/C = 0.75) = \sum A_{I,J} \left(\frac{S}{C} \right)^I \alpha_2^J, \quad (\text{A.11})$$

where

A _{I,J}						
I	J					
	0	1	2	3	4	5
0	121.436716	-14.367594	0.698921	-0.016328	1.863006E-04	-8.144731E-07
1	-27.465405	2.126647	-0.050904	6.131222E-04	-3.057781E-06	0
2	-13.972526	0.745408	-2.21048E-03	-2.047726E-04	0	0
3	-2.657717	-6.918847E-03	-3.240528E-04	0	0	0
4	-16.513662	0.357324	0	0	0	0
5	1.297642	0	0	0	0	0

Table A.5: Curve Fit Coefficients for Stalling Incidence.

Figure 2.23: Stalling Incidence Correction For Spacing-To-Chord Ratio

Polynomial Surface Fit,

$$\Delta i_s = \sum A_{I,J} \left(\frac{S}{C} \right)^I \alpha_2^J, \quad (\text{A.12})$$

where

A_{IJ}						
I	J					
	0	1	2	3	4	5
0	3.284005E-05	6.857181E-04	9.030831E-03	9.151048E-05	-4.866106E-06	3.80422E-08
1	-0.057781	-0.762316	-0.055593	9.08152E-04	-6.04925E-06	0
2	11.149079	3.319741	0.025508	1.704351E-04	0	0
3	19.797311	-4.736992	-0.043097	0	0	0
4	2.848971	4.301418	0	0	0	0
5	-59.080857	0	0	0	0	0

Table A.6: Curve Fit Coefficients for Stalling Incidence Correction for Spacing-to-Chord Ratio.

Figure 2.24: Variation in Profile Loss as a Function of Incidence

Polynomial curve fit,

$$K_i = \sum A_I \left(\frac{i}{i_{stall}} \right)^I \quad (A.13)$$

where i = incidence,

i_{stall} = stalling incidence, and

I	A _I
0	1.0
1	-0.13724288
2	0.6338455
3	0.33368595
4	0.16933259
5	0.043550643
6	0.004171815

Table A.7: Curve Fit Coefficients for Variation in Profile Loss as a Function of Incidence.

Figure A.1: Exit Flow Angle as a Function of Gauge Angle for Straight-Backed Blades

Polynomial curve fit,

$$\alpha_2^* = \sum A_I \left(\cos^{-1} \left(\frac{O}{S} \right) \right)^I, \quad (\text{A.14})$$

where α_2^* = exit flow angle for straight-backed blades,

O = throat opening,

S = blade spacing, and

I	A _I
0	-11.091418
1	1.1494518

Table A.8: Curve Fit Coefficients for Exit Flow Angle as a Function of Gauge Angle for Straight-Backed Blades.

Figure A.2: Correction for Exit Flow Angle for Curved-Backed Blades

Polynomial curve fit,

$$f(S/e) = \sum A_I \left(\frac{S}{e} \right)^I, \quad (\text{A.15})$$

where $f(S/e)$ = correction term for curved-backed blades

S = blade spacing,

e = radius of curvature of blade downstream of the throat, and

I	A_I
0	0.0
1	0.054486737
2	0.098092767
3	0.10155696

Table A.9: Curve Fit Coefficients for Correction for Exit Flow Angle for Curved-Backed Blades.

Figure A.3: Correction for Exit Flow Angle with Pitch-to-Chord Ratio

Polynomial curve fit,

$$K_{\alpha_2} = \sum A_I \left(\frac{S}{C} - 0.75 \right)^I, \quad (\text{A.16})$$

where $f(S/e)$ = correction term for curved-backed blades

S = blade spacing,

C = blade chord, and

I	A_i
0	1.0
1	-0.35750048
2	-0.16895657
3	-0.05317116

Table A.10: Curve Fit Coefficients for Correction for Exit Flow Angle with Pitch-to-Chord Ratio.

APPENDIX B

Four European Wind Tunnel Data Taken from Kiock et al. (1986) and Baines et al. (1986)

B.1: Von Karman Institute, Belgium (RG)

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.420	0.191	3.37	66.12	0.050	0.937
0.521	0.222	4.23	66.33	0.048	0.945
0.652	0.256	5.36	66.62	0.044	0.936
0.772	0.275	6.66	66.90	0.044	0.925
0.861	0.281	7.65	67.08	0.039	0.929
0.935	0.280	8.47	67.19	0.046	0.928

Table B.1: Cascade Results from RG.

M_2	$C_{p,TE}$
0.605	-0.049
0.744	-0.033
0.785	-0.024
0.836	-0.020
0.866	0.020
0.913	0.041
0.969	-0.008

Table B.2: Base Pressure Coefficients for RG.

$M_2 = 0.80$				$M_2 = 1.0$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0248	0.000	0.0693	0.235	0.0248	0.000	0.0693	0.238
0.0050	0.273	0.1287	0.234	0.0050	0.288	0.1287	0.238
0.0099	0.433	0.2030	0.233	0.0099	0.455	0.2030	0.233
0.0297	0.546	0.2822	0.232	0.0297	0.551	0.2822	0.233
0.1040	0.619	0.3762	0.232	0.1040	0.627	0.3762	0.233
0.2030	0.711	0.4703	0.253	0.2030	0.728	0.4703	0.253
0.3020	0.753	0.5644	0.304	0.3020	0.778	0.5644	0.313
0.4059	0.851	0.6535	0.351	0.4059	0.869	0.6535	0.364
0.4901	0.928	0.7426	0.443	0.4901	1.021	0.7426	0.455
0.5693	0.954	0.8119	0.536	0.5693	1.183	0.8119	0.546
0.6188	0.949	0.8663	0.603	0.6188	1.264	0.8663	0.637
0.6832	0.912	0.9208	0.706	0.6832	1.112	0.9208	0.763
0.7327	0.917			0.7327	1.112		
0.7822	0.897			0.7822	1.072		
0.8347	0.902			0.8347	1.061		
0.8812	0.892			0.8812	1.087		
0.9307	0.902			0.9307	1.152		
0.9851	0.892			0.9851	1.173		
1.0000	0.820			1.0000	1.007		

Table B.3: Blade Surface Isentropic Mach Numbers from RG.

B.2: DFVLR, Goettingen, West Germany (GO)

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR	$C_{p,TE}$ B4	$C_{p,TE}$ B5
0.292	0.131	3.78	66.34	0.042	0.977	0.000	-0.065
0.393	0.172	4.90	66.40	0.038	0.975	0.002	-0.045
0.488	0.209	5.92	66.40	0.038	0.976	0.000	-0.029
0.587	0.234	6.73	66.55	0.038	0.696	0.007	-0.028
0.684	0.253	7.70	67.76	0.040	0.964	0.004	-0.024
0.782	0.269	8.27	66.91	0.041	0.958	0.008	-0.016
0.874	0.275	8.67	67.33	0.042	0.945	0.029	-0.004
0.914	0.278	8.34	67.41	0.043	0.939	0.049	0.037
0.959	0.281	8.93	67.12	0.049	0.937	0.031	-0.049
1.055	0.281	8.98	66.69	0.058	0.933	-0.020	-0.053
1.131	0.283	8.80	66.12	0.070	0.924	-0.073	-0.139
1.168	0.279	8.06	62.95	0.127	0.965	-0.200	-0.259
1.185	0.279	8.57	65.12	0.075	0.941	-	-
1.225	0.279	7.96	62.16	0.128	0.954	-0.245	-0.327

Table B.4: Cascade Results from GO.

$M_2 = 0.80$				$M_2 = 1.0$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0248	0.021	0.0644	0.227	0.0248	0.010	0.0644	0.233
0.0099	0.309	0.0990	0.222	0.0099	0.303	0.0990	0.222
0.0149	0.402	0.1337	0.217	0.0149	0.414	0.1337	0.217
0.0248	0.423	0.1683	0.206	0.0248	0.485	0.1683	0.212
0.0594	0.521	0.2574	0.201	0.0594	0.536	0.2574	0.202
0.0990	0.577	0.3069	0.191	0.0990	0.607	0.3069	0.197
0.1535	0.644	0.3614	0.206	0.1535	0.677	0.3614	0.212
0.2208	0.701	0.4059	0.227	0.2208	0.718	0.4059	0.233
0.2891	0.727	0.4554	0.247	0.2891	0.768	0.4554	0.253
0.3564	0.763	0.5050	0.268	0.3564	0.809	0.5050	0.273
0.4158	0.845	0.5594	0.278	0.4158	0.919	0.5594	0.303
0.5495	0.897	0.6139	0.325	0.5495	1.137	0.6139	0.334
0.6089	0.907	0.6584	0.361	0.6089	1.132	0.6584	0.374
0.6535	0.969	0.7030	0.402	0.6535	1.081	0.7030	0.425
0.7030	0.892	0.7624	0.443	0.7030	1.072	0.7624	0.465
0.7475	0.887	0.7970	0.500	0.7475	1.072	0.7970	0.525
0.7822	0.876	0.8515	0.546	0.7822	1.102	0.8515	0.586
0.8218	0.871	0.8861	0.603	0.8218	1.127	0.8861	0.647
0.8641	0.856	0.9257	0.660	0.8641	1.152	0.9257	0.718
0.9010	0.845	0.9604	0.732	0.9010	1.163	0.9604	0.910
0.9455	0.856	1.0000	0.794	0.9455	1.183	1.0000	0.970
0.9802	0.861			0.9802	1.092		
1.0000	0.806			1.0000	1.016		

Table B.5: Blade Surface Isentropic Mach Numbers from GO.

B.3 DFVLR, Braunschweig, West Germany (BS)

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR	$C_{p,TE}$
0.392	0.166	4.90	66.83	0.038	0.989	-0.094
0.587	0.225	6.73	66.91	0.038	0.987	-0.069
0.784	0.256	7.86	67.05	0.038	0.993	-0.065
0.830	0.259	7.98	67.34	0.040	0.980	-0.069
0.875	0.263	7.85	67.27	0.039	0.987	-0.045
0.918	0.266	7.76	67.37	0.039	0.986	-0.049
0.967	0.264	7.71	67.27	0.058	0.977	-0.155

Table B.6: Cascade Results from BS.

$M_2 = 0.80$				$M_2 = 1.0$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0243	0.000	0.0495	0.144	0.0243	0.040	0.0495	0.182
0.0049	0.289	0.0792	0.227	0.0049	0.283	0.0792	0.233
0.0000	0.340	0.0990	0.217	0.0000	0.374	0.0990	0.233
0.0194	0.449	0.1287	0.186	0.0194	0.425	0.1287	0.217
0.0243	0.474	0.1584	0.196	0.0243	0.455	0.1584	0.207
0.0437	0.495	0.1980	0.175	0.0437	0.516	0.1980	0.202
0.0777	0.546	0.2475	0.134	0.0777	0.556	0.2475	0.192
0.0971	0.572	0.2871	0.165	0.0971	0.586	0.2871	0.212
0.1359	0.608	0.3564	0.191	0.1359	0.637	0.3564	0.202
0.1748	0.650	0.4455	0.237	0.1748	0.667	0.4455	0.243
0.2136	0.691	0.5248	0.268	0.2136	0.718	0.5248	0.283
0.2621	0.711	0.6040	0.309	0.2621	0.738	0.6040	0.334
0.3107	0.732	0.6931	0.381	0.3107	0.768	0.6931	0.394
0.3495	0.778	0.7624	0.453	0.3495	0.819	0.7624	0.495
0.3981	0.825	0.8317	0.536	0.3981	0.879	0.8317	0.566
0.4369	0.876	0.9010	0.639	0.4369	0.950	0.9010	0.728
0.4854	0.897			0.4854	1.021		
0.5243	0.907			0.5243	1.102		
0.5680	0.918			0.5680	1.152		
0.6117	0.907			0.6117	1.112		
0.6505	0.918			0.6505	1.082		
0.6893	0.918			0.6893	1.072		
0.7282	0.918			0.7282	1.072		
0.7670	0.907			0.7670	1.092		
0.8058	0.897			0.8058	1.112		
0.8447	0.887			0.8447	1.142		
0.8738	0.871			0.8738	1.142		
0.9126	0.856			0.9126	1.132		
0.9417	0.845			0.9417	1.112		
1.0000	0.825			1.0000	1.082		

Table B.7: Blade Surface Isentropic Mach Numbers from BS.

B.4: Oxford University, U.K. (OX)

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.667	0.238	8.11	67.77	0.040	0.990
0.701	0.244	8.06	67.84	0.040	0.998
0.709	0.244	8.16	67.91	0.040	1.005
0.717	0.244	8.06	67.98	0.043	0.996
0.756	0.247	8.21	68.27	0.049	1.002
0.880	0.251	8.04	67.91	0.036	1.007
0.898	0.251	8.27	68.13	0.037	0.997
0.918	0.251	8.04	68.06	0.040	0.998
0.928	0.251	8.05	67.77	0.038	1.019
0.933	0.251	8.06	67.84	0.040	1.013
0.955	0.251	8.04	67.91	0.041	1.011
0.978	0.253	8.11	67.77	0.060	1.029

Table B.8: Cascade Results from OX.

M_2	$C_{p,TE}$
0.491	-0.339
0.699	-0.188
0.785	-0.127
0.850	-0.073
0.913	-0.106
0.940	-0.118
0.985	-0.163

Table B.9: Base Pressure Coefficients for OX.

$M_2 = 0.80$				$M_2 = 1.0$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0243	0.000	0.0534	0.155	0.0243	0.000	0.0534	0.152
0.0097	0.289	0.0777	0.217	0.0097	0.273	0.0777	0.212
0.0000	0.361	0.1068	0.201	0.0000	0.364	0.1068	0.202
0.0194	0.434	0.1311	0.196	0.0194	0.425	0.1311	0.202
0.0243	0.489	0.1650	0.196	0.0243	0.465	0.1650	0.192
0.0437	0.516	0.2039	0.175	0.0437	0.516	0.2039	0.172
0.0583	0.536	0.2427	0.159	0.0583	0.546	0.2427	0.162
0.0777	0.567	0.2913	0.170	0.0777	0.571	0.2913	0.182
0.1019	0.598	0.3641	0.186	0.1019	0.607	0.3641	0.192
0.1359	0.639	0.4369	0.217	0.1359	0.647	0.4369	0.253
0.1748	0.670	0.5243	0.258	0.1748	0.677	0.5243	0.263
0.2233	0.722	0.6019	0.309	0.2233	0.738	0.6019	0.344
0.2621	0.732	0.6796	0.381	0.2621	0.748	0.6796	0.384
0.3107	0.753	0.7573	0.454	0.3107	0.768	0.7573	0.455
0.3495	0.804	0.8252	0.536	0.3495	0.809	0.8252	0.545
0.3981	0.845	0.9029	0.644	0.3981	0.869	0.9029	0.677
0.4369	0.907			0.4369	0.950		
0.4854	0.938			0.4854	1.021		
0.5340	0.964			0.5340	1.092		
0.5728	0.959			0.5728	1.173		
0.6117	0.959			0.6117	1.213		
0.6505	0.964			0.6505	1.213		
0.6893	0.969			0.6893	1.183		
0.7282	0.959			0.7282	1.142		
0.7670	0.959			0.7670	1.112		
0.8058	0.954			0.8058	1.132		
0.8447	0.943			0.8447	1.132		
0.8736	0.928			0.8736	1.142		
0.9126	0.923			0.9126	1.163		
0.9515	0.897			0.9515	1.173		
1.0000	0.814			1.0000	1.082		

Table B.6: Blade Surface Isentropic Mach Numbers from OX.

APPENDIX C.

HS2 Cascade Data and Results.

C.1: HS2 Cascade Results at Design Incidence

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.519	0.231	4.79	66.82	0.056	0.886
0.683	0.264	6.36	66.94	0.046	0.915
0.811	0.280	7.56	67.04	0.040	0.924
0.897	0.281	8.60	67.19	0.037	0.936
0.936	0.280	8.96	67.02	0.044	0.946
0.975	0.284	9.39	66.82	0.046	0.940
1.035	0.278	10.03	66.63	0.055	0.956
1.088	0.275	10.69	65.49	0.068	0.958

Table C.1: HS2 Results for Blade Number 3 at Design Incidence.

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.530	0.226	4.83	67.07	0.051	0.910
0.700	0.265	6.41	67.23	0.043	0.915
0.827	0.280	7.59	67.24	0.040	0.926
0.922	0.278	8.55	67.40	0.035	0.939
0.958	0.284	8.99	67.44	0.040	0.921
1.030	0.287	9.38	67.29	0.048	0.911
1.065	0.284	10.17	67.01	0.051	0.922
1.135	0.280	10.85	66.53	0.054	0.936
1.156	0.282	11.14	65.76	0.065	0.937

Table C.2: HS2 Results for Blade Number 4 at Design Incidence.

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.534	0.225	4.85	67.42	0.058	0.911
0.706	0.263	6.47	67.48	0.049	0.914
0.838	0.276	7.69	67.48	0.043	0.923
0.924	0.280	8.57	67.43	0.040	0.934
0.958	0.282	8.93	67.45	0.046	0.924
0.998	0.286	9.39	67.40	0.045	0.912
1.067	0.282	10.12	67.10	0.052	0.926

Table C.3: HS2 Results for Blade Number 5 at Design Incidence.

M_2	$C_{p,TE}$
0.674	-0.037
0.799	-0.035
0.909	0.020
0.948	0.049
0.991	-0.002
1.052	-0.053
1.138	-0.034
1.209	-0.083

Table C.4: HS2 Base Pressure Coefficients at Design Incidence.

$M_2 = 0.80$				$M_2 = 1.0$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.317	0.0813	0.216	0.0006	0.322	0.0813	0.225
0.0087	0.363	0.1191	0.186	0.0087	0.382	0.1191	0.195
0.0264	0.426	0.1572	0.167	0.0264	0.447	0.1572	0.177
0.0523	0.478	0.1975	0.150	0.0523	0.497	0.1975	0.156
0.0841	0.554	0.2411	0.138	0.0841	0.570	0.2411	0.149
0.1206	0.592	0.2886	0.137	0.1206	0.606	0.2886	0.134
0.1617	0.648	0.3386	0.152	0.1617	0.669	0.3386	0.152
0.2087	0.699	0.3934	0.169	0.2087	0.722	0.3934	0.181
0.2613	0.718	0.4524	0.200	0.2613	0.748	0.4524	0.214
0.3075	0.729	0.5034	0.218	0.3075	0.765	0.5034	0.239
0.3567	0.775	0.5679	0.261	0.3567	0.826	0.5679	0.270
0.4083	0.870	0.6186	0.309	0.4083	0.937	0.6186	0.319
0.4578	0.899	0.6709	0.383	0.4578	0.993	0.6709	0.395
0.5117	0.930	0.7255	0.411	0.5117	1.096	0.7255	0.422
0.5600	0.923	0.7827	0.469	0.5600	1.178	0.7827	0.493
0.6113	0.920	0.8435	0.544	0.6113	1.175	0.8435	0.576
0.6654	0.940	0.9028	0.629	0.6654	1.129	0.9028	0.677
0.7135	0.928	0.9539	0.780	0.7135	1.108	0.9539	0.880
0.7636	0.910			0.7636	1.099		
0.8153	0.919			0.8153	1.153		
0.8701	0.887			0.8701	1.167		
0.9148	0.862			0.9148	1.177		
1.0000	0.834			1.0000	1.021		

Table C.5: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.8 and 1.0.

$M_2 = 0.493$				$M_2 = 0.674$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.246	0.0813	0.176	0.0006	0.296	0.0813	0.210
0.0087	0.285	0.1191	0.155	0.0087	0.340	0.1191	0.180
0.0264	0.329	0.1572	0.151	0.0264	0.397	0.1572	0.162
0.0523	0.368	0.1975	0.138	0.0523	0.451	0.1975	0.143
0.0841	0.409	0.2411	0.122	0.0841	0.516	0.2411	0.138
0.1206	0.438	0.2886	0.122	0.1206	0.544	0.2886	0.135
0.1617	0.472	0.3386	0.125	0.1617	0.599	0.3386	0.138
0.2087	0.498	0.3934	0.147	0.2087	0.636	0.3934	0.165
0.2613	0.508	0.4524	0.155	0.2613	0.655	0.4524	0.194
0.3075	0.517	0.5034	0.169	0.3075	0.665	0.5034	0.218
0.3567	0.521	0.5679	0.194	0.3567	0.695	0.5679	0.244
0.4083	0.567	0.6186	0.238	0.4083	0.763	0.6186	0.285
0.4578	0.567	0.6709	0.284	0.4578	0.773	0.6709	0.352
0.5117	0.568	0.7255	0.304	0.5117	0.784	0.7255	0.373
0.5600	0.560	0.7827	0.341	0.5600	0.776	0.7827	0.435
0.6113	0.560	0.8435	0.375	0.6113	0.773	0.8435	0.491
0.6654	0.560	0.9028	0.418	0.6654	0.783	0.9028	0.557
0.7135	0.552	0.9539	0.488	0.7135	0.764	0.9539	0.665
0.7636	0.545			0.7636	0.752		
0.8153	0.542			0.8153	0.754		
0.8701	0.521			0.8701	0.730		
0.9148	0.519			0.9148	0.718		
1.0000	0.488			1.0000	0.705		

Table C.6: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.493 and 0.674.

$M_2 = 0.799$				$M_2 = 0.909$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.317	0.0813	0.216	0.0006	0.329	0.0813	0.232
0.0087	0.363	0.1191	0.186	0.0087	0.380	0.1191	0.206
0.0264	0.426	0.1572	0.167	0.0264	0.442	0.1572	0.191
0.0523	0.478	0.1975	0.150	0.0523	0.493	0.1975	0.172
0.0841	0.554	0.2411	0.138	0.0841	0.571	0.2411	0.153
0.1206	0.592	0.2886	0.137	0.1206	0.610	0.2886	0.151
0.1617	0.648	0.3386	0.152	0.1617	0.669	0.3386	0.164
0.2087	0.699	0.3934	0.169	0.2087	0.720	0.3934	0.193
0.2613	0.718	0.4524	0.200	0.2613	0.743	0.4524	0.228
0.3075	0.729	0.5034	0.218	0.3075	0.761	0.5034	0.248
0.3567	0.775	0.5679	0.261	0.3567	0.812	0.5679	0.273
0.4083	0.870	0.6186	0.309	0.4083	0.926	0.6186	0.316
0.4578	0.899	0.6709	0.383	0.4578	0.970	0.6709	0.395
0.5117	0.930	0.7255	0.411	0.5117	1.036	0.7255	0.430
0.5600	0.923	0.7827	0.469	0.5600	1.034	0.7827	0.494
0.6113	0.920	0.8435	0.544	0.6113	1.021	0.8435	0.571
0.6654	0.940	0.9028	0.629	0.6654	1.061	0.9028	0.672
0.7135	0.928	0.9539	0.780	0.7135	1.065	0.9539	0.854
0.7636	0.910			0.7636	1.055		
0.8153	0.919			0.8153	1.084		
0.8701	0.887			0.8701	1.032		
0.9148	0.862			0.9148	1.003		
1.0000	0.834			1.0000	0.918		

Table C.7: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.799 and 0.909.

$M_2 = 0.948$				$M_2 = 0.991$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.327	0.0813	0.223	0.0006	0.322	0.0813	0.225
0.0087	0.382	0.1191	0.193	0.0087	0.382	0.1191	0.195
0.0264	0.441	0.1572	0.180	0.0264	0.447	0.1572	0.177
0.0523	0.493	0.1975	0.155	0.0523	0.497	0.1975	0.156
0.0841	0.571	0.2411	0.150	0.0841	0.570	0.2411	0.149
0.1206	0.603	0.2886	0.159	0.1206	0.606	0.2886	0.134
0.1617	0.668	0.3386	0.165	0.1617	0.669	0.3386	0.152
0.2087	0.720	0.3934	0.183	0.2087	0.722	0.3934	0.181
0.2613	0.744	0.4524	0.208	0.2613	0.748	0.4524	0.214
0.3075	0.757	0.5034	0.238	0.3075	0.765	0.5034	0.239
0.3567	0.817	0.5679	0.278	0.3567	0.826	0.5679	0.270
0.4083	0.931	0.6186	0.323	0.4083	0.937	0.6186	0.319
0.4578	0.982	0.6709	0.398	0.4578	0.993	0.6709	0.395
0.5117	1.069	0.7255	0.425	0.5117	1.096	0.7255	0.422
0.5600	1.082	0.7827	0.493	0.5600	1.178	0.7827	0.493
0.6113	1.053	0.8435	0.572	0.6113	1.175	0.8435	0.576
0.6654	1.080	0.9028	0.674	0.6654	1.129	0.9028	0.677
0.7135	1.099	0.9539	0.872	0.7135	1.108	0.9539	0.880
0.7636	1.091			0.7636	1.099		
0.8153	1.140			0.8153	1.153		
0.8701	1.137			0.8701	1.167		
0.9148	1.089			0.9148	1.177		
1.0000	0.944			1.0000	1.021		

Table C.8: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.948 and 0.991.

$M_2 = 1.052$				$M_2 = 1.138$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.315	0.0813	0.204	0.0006	0.324	0.0813	0.217
0.0087	0.369	0.1191	0.179	0.0087	0.373	0.1191	0.188
0.0264	0.431	0.1572	0.153	0.0264	0.432	0.1572	0.175
0.0523	0.487	0.1975	0.141	0.0523	0.488	0.1975	0.156
0.0841	0.564	0.2411	0.131	0.0841	0.567	0.2411	0.143
0.1206	0.599	0.2886	0.135	0.1206	0.603	0.2886	0.148
0.1617	0.667	0.3386	0.139	0.1617	0.668	0.3386	0.157
0.2087	0.717	0.3934	0.167	0.2087	0.719	0.3934	0.179
0.2613	0.741	0.4524	0.193	0.2613	0.742	0.4524	0.210
0.3075	0.756	0.5034	0.224	0.3075	0.758	0.5034	0.232
0.3567	0.832	0.5679	0.259	0.3567	0.840	0.5679	0.266
0.4083	0.938	0.6186	0.306	0.4083	0.936	0.6186	0.320
0.4578	0.991	0.6709	0.389	0.4578	0.992	0.6709	0.402
0.5117	1.098	0.7255	0.417	0.5117	1.100	0.7255	0.422
0.5600	1.196	0.7827	0.490	0.5600	1.199	0.7827	0.489
0.6113	1.328	0.8435	0.569	0.6113	1.336	0.8435	0.575
0.6654	1.395	0.9028	0.673	0.6654	1.552	0.9028	0.674
0.7135	1.297	0.9539	0.882	0.7135	1.552	0.9539	0.888
0.7636	1.197			0.7636	1.407		
0.8153	1.221			0.8153	1.338		
0.8701	1.213			0.8701	1.345		
0.9148	1.208			0.9148	1.360		
1.0000	1.119			1.0000	1.204		

Table C.9: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 1.052 and 1.138.

M₂ = 1.167			
Suction Surface		Pressure Surface	
X/C	M	X/C	M
0.0006	0.327	0.0813	0.235
0.0087	0.375	0.1191	0.195
0.0264	0.443	0.1572	0.186
0.0523	0.494	0.1975	0.172
0.0841	0.570	0.2411	0.165
0.1206	0.607	0.2886	0.160
0.1617	0.668	0.3386	0.174
0.2087	0.718	0.3934	0.188
0.2613	0.745	0.4524	0.223
0.3075	0.761	0.5034	0.245
0.3567	0.841	0.5679	0.278
0.4083	0.938	0.6186	0.323
0.4578	0.996	0.6709	0.401
0.5117	1.102	0.7255	0.430
0.5600	1.195	0.7827	0.500
0.6113	1.336	0.8435	0.582
0.6654	1.550	0.9028	0.681
0.7135	1.689	0.9539	0.887
0.7636	1.560		
0.8153	1.392		
0.8701	1.366		
0.9148	1.377		
1.0000	1.305		

Table C.10: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Number of 1.167.

C.2: HS2 Cascade Results at Off-Design Incidence

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.707	0.225	6.42	67.48	0.046	0.980
0.835	0.237	7.43	67.49	0.043	-
0.922	0.237	8.47	67.75	0.040	0.988
0.956	0.236	8.82	67.67	0.045	0.988
0.990	0.235	9.12	67.64	0.049	1.001
1.053	0.238	9.95	67.47	0.053	0.978

Table C.11: HS2 Results for -10 Degrees Incidence.

M_2	$C_{p,TE}$
0.807	-0.056
0.912	-0.008
0.950	0.003
0.986	-0.035
1.053	-0.081

Table C.12: HS2 Base Pressure Coefficients at -10 Degrees Incidence.

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.379	0.191	3.59	67.31	0.065	0.872
0.605	0.254	5.78	67.39	0.048	0.926
0.755	0.288	7.20	67.68	0.043	0.904
0.859	0.301	8.39	67.58	0.042	0.907
0.906	0.297	8.83	67.80	0.038	0.921
0.946	0.297	9.21	67.86	0.042	0.919
1.017	0.296	10.07	67.10	0.047	0.941
1.099	0.302	10.82	67.78	0.043	0.934

Table C.13: HS2 Results for +5 Degrees Incidence.

M_2	$C_{p,TE}$
0.651	-0.089
0.776	-0.078
0.884	-0.020
0.927	0.004
0.969	-0.038
1.031	-0.095

Table C.14: HS2 Base Pressure Coefficients at +5 Degrees Incidence.

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.678	0.275	6.28	67.26	0.044	0.980
0.813	0.289	7.48	67.41	0.040	1.000
0.907	0.295	8.61	67.53	0.037	0.997
0.949	0.297	9.03	67.55	0.038	0.992
0.989	0.301	9.26	67.34	0.045	0.985
1.055	0.295	9.99	67.15	0.051	0.999

Table C.15: HS2 Results for +10 Degrees Incidence.

M_2	$C_{p,TE}$
0.787	-0.039
0.891	0.000
0.938	0.036
0.980	0.016
1.046	-0.052

Table C.16: HS2 Base Pressure Coefficients at +10 Degrees Incidence.

M_2	M_1	$Re_c \times 10^{-5}$	β_2	ζ	AVDR
0.409	0.233	3.87	66.87	0.061	0.909
0.619	0.311	5.90	67.03	0.050	0.921
0.759	0.349	7.32	67.03	0.045	0.911
0.865	0.362	8.41	66.93	0.043	0.920
0.913	0.364	8.84	67.03	0.038	0.923
0.955	0.366	9.24	67.13	0.042	0.914
1.031	0.368	10.02	66.77	0.051	0.912
1.109	0.367	10.75	66.38	0.050	0.914

Table C.17: HS2 Results for +15 Degrees Incidence.

M_2	$C_{p,TE}$
0.699	-0.029
0.811	-0.032
0.920	-0.005
0.963	-0.005
1.003	-0.049
1.053	-0.075

Table C.18: HS2 Base Pressure Coefficients at +15 Degrees Incidence.

$M_2 = 0.69$				$M_2 = 0.81$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.258	0.0813	0.307	0.0006	0.268	0.0813	0.314
0.0087	0.320	0.1191	0.250	0.0087	0.331	0.1191	0.245
0.0264	0.383	0.1572	0.226	0.0264	0.400	0.1572	0.227
0.0523	0.438	0.1975	0.215	0.0523	0.462	0.1975	0.222
0.0841	0.513	0.2411	0.209	0.0841	0.542	0.2411	0.217
0.1206	0.550	0.2886	0.201	0.1206	0.592	0.2886	0.216
0.1617	0.607	0.3386	0.205	0.1617	0.654	0.3386	0.222
0.2087	0.651	0.3934	0.228	0.2087	0.706	0.3934	0.230
0.2613	0.672	0.4524	0.248	0.2613	0.727	0.4524	0.249
0.3075	0.683	0.5034	0.266	0.3075	0.745	0.5034	0.264
0.3567	0.714	0.5679	0.285	0.3567	0.793	0.5679	0.297
0.4083	0.779	0.6186	0.321	0.4083	0.884	0.6186	0.339
0.4578	0.790	0.6709	0.374	0.4578	0.915	0.6709	0.402
0.5117	0.800	0.7255	0.401	0.5117	0.950	0.7255	0.435
0.5600	0.796	0.7827	0.455	0.5600	0.943	0.7827	0.491
0.6113	0.793	0.8435	0.514	0.6113	0.938	0.8435	0.565
0.6654	0.800	0.9028	0.580	0.6654	0.955	0.9028	0.647
0.7135	0.788	0.9539	0.686	0.7135	0.945	0.9539	0.795
0.7636	0.775			0.7636	0.927		
0.8153	0.776			0.8153	0.931		
0.8701	0.748			0.8701	0.897		
0.9148	0.736			0.9148	0.876		
1.0000	0.725			1.0000	0.853		

Table C.19: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.69 and 0.81 at -10 Degrees Incidence.

$M_2 = 0.91$				$M_2 = 0.95$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.286	0.0813	0.332	0.0006	0.289	0.0813	0.339
0.0087	0.350	0.1191	0.265	0.0087	0.349	0.1191	0.269
0.0264	0.421	0.1572	0.250	0.0264	0.420	0.1572	0.258
0.0523	0.482	0.1975	0.243	0.0523	0.484	0.1975	0.242
0.0841	0.565	0.2411	0.233	0.0841	0.565	0.2411	0.239
0.1206	0.606	0.2886	0.230	0.1206	0.611	0.2886	0.233
0.1617	0.671	0.3386	0.237	0.1617	0.678	0.3386	0.238
0.2087	0.728	0.3934	0.249	0.2087	0.738	0.3934	0.258
0.2613	0.758	0.4524	0.269	0.2613	0.762	0.4524	0.276
0.3075	0.776	0.5034	0.293	0.3075	0.780	0.5034	0.291
0.3567	0.831	0.5679	0.319	0.3567	0.833	0.5679	0.321
0.4083	0.940	0.6186	0.361	0.4083	0.948	0.6186	0.364
0.4578	0.984	0.6709	0.424	0.4578	0.996	0.6709	0.428
0.5117	1.051	0.7255	0.456	0.5117	1.088	0.7255	0.462
0.5600	1.050	0.7827	0.515	0.5600	1.106	0.7827	0.525
0.6113	1.036	0.8435	0.594	0.6113	1.072	0.8435	0.601
0.6654	1.077	0.9028	0.692	0.6654	1.097	0.9028	0.699
0.7135	1.080	0.9539	0.871	0.7135	1.115	0.9539	0.891
0.7636	1.071			0.7636	1.108		
0.8153	1.093			0.8153	1.158		
0.8701	1.050			0.8701	1.162		
0.9148	1.020			0.9148	1.116		
1.0000	0.939			1.0000	0.974		

Table C.20: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.91 and 0.95 at -10 Degrees Incidence.

$M_2 = 0.99$				$M_2 = 1.05$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.285	0.0813	0.332	0.0006	0.285	0.0813	0.340
0.0087	0.346	0.1191	0.263	0.0087	0.353	0.1191	0.275
0.0264	0.417	0.1572	0.246	0.0264	0.424	0.1572	0.268
0.0523	0.480	0.1975	0.237	0.0523	0.483	0.1975	0.247
0.0841	0.566	0.2411	0.230	0.0841	0.570	0.2411	0.245
0.1206	0.611	0.2886	0.219	0.1206	0.613	0.2886	0.244
0.1617	0.679	0.3386	0.226	0.1617	0.683	0.3386	0.250
0.2087	0.734	0.3934	0.246	0.2087	0.740	0.3934	0.263
0.2613	0.760	0.4524	0.274	0.2613	0.764	0.4524	0.277
0.3075	0.781	0.5034	0.288	0.3075	0.780	0.5034	0.301
0.3567	0.844	0.5679	0.320	0.3567	0.855	0.5679	0.326
0.4083	0.953	0.6186	0.358	0.4083	0.963	0.6186	0.370
0.4578	1.002	0.6709	0.425	0.4578	1.013	0.6709	0.438
0.5117	1.110	0.7255	0.454	0.5117	1.120	0.7255	0.463
0.5600	1.193	0.7827	0.521	0.5600	1.218	0.7827	0.528
0.6113	1.195	0.8435	0.598	0.6113	1.350	0.8435	0.603
0.6654	1.142	0.9028	0.697	0.6654	1.449	0.9028	0.705
0.7135	1.130	0.9539	0.900	0.7135	1.334	0.9539	0.908
0.7636	1.114			0.7636	1.229		
0.8153	1.163			0.8153	1.252		
0.8701	1.178			0.8701	1.248		
0.9148	1.190			0.9148	1.240		
1.0000	1.036			1.0000	1.141		

Table C.21: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.99 and 1.05 at -10 Degrees Incidence.

$M_2 = 0.67$				$M_2 = 0.78$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.346	0.0813	0.192	0.0006	0.375	0.0813	0.186
0.0087	0.381	0.1191	0.168	0.0087	0.415	0.1191	0.174
0.0264	0.428	0.1572	0.160	0.0264	0.464	0.1572	0.173
0.0523	0.469	0.1975	0.147	0.0523	0.504	0.1975	0.169
0.0841	0.529	0.2411	0.140	0.0841	0.576	0.2411	0.164
0.1206	0.556	0.2886	0.139	0.1206	0.600	0.2886	0.166
0.1617	0.603	0.3386	0.152	0.1617	0.658	0.3386	0.169
0.2087	0.638	0.3934	0.179	0.2087	0.701	0.3934	0.177
0.2613	0.655	0.4524	0.201	0.2613	0.721	0.4524	0.202
0.3075	0.665	0.5034	0.224	0.3075	0.732	0.5034	0.226
0.3567	0.689	0.5679	0.248	0.3567	0.774	0.5679	0.267
0.4083	0.753	0.6186	0.287	0.4083	0.868	0.6186	0.312
0.4578	0.764	0.6709	0.354	0.4578	0.894	0.6709	0.386
0.5117	0.774	0.7255	0.371	0.5117	0.924	0.7255	0.411
0.5600	0.764	0.7827	0.430	0.5600	0.921	0.7827	0.468
0.6113	0.764	0.8435	0.491	0.6113	0.915	0.8435	0.545
0.6654	0.773	0.9028	0.553	0.6654	0.933	0.9028	0.630
0.7135	0.761	0.9539	0.660	0.7135	0.921	0.9539	0.779
0.7636	0.744			0.7636	0.902		
0.8153	0.746			0.8153	0.908		
0.8701	0.722			0.8701	0.876		
0.9148	0.710			0.9148	0.859		
1.0000	0.669			1.0000	0.829		

Table C.22: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.67 and 0.78 at +5 Degrees Incidence.

$M_2 = 0.88$				$M_2 = 0.93$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.386	0.0813	0.210	0.0006	0.394	0.0813	0.207
0.0087	0.430	0.1191	0.189	0.0087	0.430	0.1191	0.200
0.0264	0.478	0.1572	0.177	0.0264	0.479	0.1572	0.184
0.0523	0.522	0.1975	0.164	0.0523	0.524	0.1975	0.166
0.0841	0.591	0.2411	0.162	0.0841	0.594	0.2411	0.161
0.1206	0.620	0.2886	0.166	0.1206	0.619	0.2886	0.158
0.1617	0.679	0.3386	0.177	0.1617	0.683	0.3386	0.180
0.2087	0.725	0.3934	0.193	0.2087	0.734	0.3934	0.205
0.2613	0.747	0.4524	0.216	0.2613	0.752	0.4524	0.226
0.3075	0.762	0.5034	0.245	0.3075	0.764	0.5034	0.244
0.3567	0.816	0.5679	0.280	0.3567	0.822	0.5679	0.278
0.4083	0.925	0.6186	0.330	0.4083	0.934	0.6186	0.328
0.4578	0.968	0.6709	0.398	0.4578	0.983	0.6709	0.407
0.5117	1.035	0.7255	0.432	0.5117	1.069	0.7255	0.433
0.5600	1.029	0.7827	0.492	0.5600	1.086	0.7827	0.499
0.6113	1.018	0.8435	0.572	0.6113	1.055	0.8435	0.579
0.6654	1.056	0.9028	0.669	0.6654	1.084	0.9028	0.679
0.7135	1.058	0.9539	0.854	0.7135	1.101	0.9539	0.871
0.7636	1.050			0.7636	1.094		
0.8153	1.065			0.8153	1.145		
0.8701	1.016			0.8701	1.135		
0.9148	0.991			0.9148	1.083		
1.0000	0.915			1.0000	0.947		

Table C.23: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.88 and 0.93 at +5 Degrees Incidence.

$M_2 = 0.97$				$M_2 = 1.03$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.398	0.0813	0.204	0.0006	0.389	0.0813	0.205
0.0087	0.437	0.1191	0.196	0.0087	0.427	0.1191	0.179
0.0264	0.488	0.1572	0.182	0.0264	0.480	0.1572	0.172
0.0523	0.527	0.1975	0.163	0.0523	0.523	0.1975	0.164
0.0841	0.596	0.2411	0.157	0.0841	0.592	0.2411	0.155
0.1206	0.624	0.2886	0.162	0.1206	0.625	0.2886	0.161
0.1617	0.685	0.3386	0.180	0.1617	0.684	0.3386	0.170
0.2087	0.736	0.3934	0.200	0.2087	0.730	0.3934	0.191
0.2613	0.757	0.4524	0.222	0.2613	0.750	0.4524	0.220
0.3075	0.771	0.5034	0.252	0.3075	0.766	0.5034	0.247
0.3567	0.831	0.5679	0.285	0.3567	0.839	0.5679	0.278
0.4083	0.941	0.6186	0.330	0.4083	0.946	0.6186	0.327
0.4578	0.995	0.6709	0.405	0.4578	0.999	0.6709	0.404
0.5117	1.097	0.7255	0.428	0.5117	1.105	0.7255	0.433
0.5600	1.177	0.7827	0.499	0.5600	1.203	0.7827	0.500
0.6113	1.152	0.8435	0.584	0.6113	1.333	0.8435	0.580
0.6654	1.116	0.9028	0.684	0.6654	1.384	0.9028	0.683
0.7135	1.111	0.9539	0.885	0.7135	1.127	0.9539	0.888
0.7636	1.102			0.7636	1.186		
0.8153	1.160			0.8153	1.218		
0.8701	1.172			0.8701	1.208		
0.9148	1.178			0.9148	1.214		
1.0000	1.017			1.0000	1.119		

Table C.24: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.97 and 1.03 at +5 Degrees Incidence.

$M_2 = 0.67$				$M_2 = 0.79$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.400	0.0813	0.143	0.0006	0.432	0.0813	0.160
0.0087	0.428	0.1191	0.142	0.0087	0.457	0.1191	0.159
0.0264	0.466	0.1572	0.143	0.0264	0.499	0.1572	0.158
0.0523	0.494	0.1975	0.145	0.0523	0.535	0.1975	0.158
0.0841	0.543	0.2411	0.144	0.0841	0.599	0.2411	0.157
0.1206	0.566	0.2886	0.147	0.1206	0.623	0.2886	0.155
0.1617	0.611	0.3386	0.153	0.1617	0.667	0.3386	0.163
0.2087	0.643	0.3934	0.173	0.2087	0.704	0.3934	0.183
0.2613	0.656	0.4524	0.201	0.2613	0.724	0.4524	0.208
0.3075	0.664	0.5034	0.220	0.3075	0.735	0.5034	0.231
0.3567	0.689	0.5679	0.247	0.3567	0.775	0.5679	0.269
0.4083	0.754	0.6186	0.288	0.4083	0.865	0.6186	0.306
0.4578	0.761	0.6709	0.354	0.4578	0.893	0.6709	0.381
0.5117	0.766	0.7255	0.372	0.5117	0.923	0.7255	0.408
0.5600	0.764	0.7827	0.429	0.5600	0.915	0.7827	0.469
0.6113	0.759	0.8435	0.487	0.6113	0.914	0.8435	0.547
0.6654	0.765	0.9028	0.547	0.6654	0.929	0.9028	0.631
0.7135	0.754	0.9539	0.656	0.7135	0.916	0.9539	0.774
0.7636	0.739			0.7636	0.901		
0.8153	0.742			0.8153	0.905		
0.8701	0.720			0.8701	0.871		
0.9148	0.701			0.9148	0.853		
1.0000	0.691			1.0000	0.822		

Table C.25: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.67 and 0.79 at +10 Degrees Incidence.

$M_2 = 0.89$				$M_2 = 0.94$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.445	0.0813	0.160	0.0006	0.449	0.0813	0.182
0.0087	0.471	0.1191	0.158	0.0087	0.478	0.1191	0.170
0.0264	0.511	0.1572	0.157	0.0264	0.519	0.1572	0.164
0.0523	0.550	0.1975	0.157	0.0523	0.551	0.1975	0.161
0.0841	0.611	0.2411	0.157	0.0841	0.613	0.2411	0.166
0.1206	0.637	0.2886	0.154	0.1206	0.637	0.2886	0.164
0.1617	0.688	0.3386	0.165	0.1617	0.694	0.3386	0.169
0.2087	0.728	0.3934	0.187	0.2087	0.743	0.3934	0.196
0.2613	0.750	0.4524	0.212	0.2613	0.753	0.4524	0.226
0.3075	0.762	0.5034	0.238	0.3075	0.762	0.5034	0.250
0.3567	0.816	0.5679	0.270	0.3567	0.820	0.5679	0.281
0.4083	0.923	0.6186	0.322	0.4083	0.933	0.6186	0.322
0.4578	0.967	0.6709	0.397	0.4578	0.980	0.6709	0.402
0.5117	1.026	0.7255	0.426	0.5117	1.060	0.7255	0.434
0.5600	1.026	0.7827	0.489	0.5600	1.071	0.7827	0.500
0.6113	1.016	0.8435	0.569	0.6113	1.045	0.8435	0.579
0.6654	1.055	0.9028	0.669	0.6654	1.084	0.9028	0.676
0.7135	1.054	0.9539	0.849	0.7135	1.095	0.9539	0.869
0.7636	1.039			0.7636	1.091		
0.8153	1.056			0.8153	1.134		
0.8701	1.006			0.8701	1.121		
0.9148	0.979			0.9148	1.055		
1.0000	0.910			1.0000	0.940		

Table C.26: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.89 and 0.94 at +10 Degrees Incidence.

$M_2 = 0.98$				$M_2 = 1.05$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.452	0.0813	0.158	0.0006	0.457	0.0813	0.172
0.0087	0.480	0.1191	0.165	0.0087	0.483	0.1191	0.170
0.0264	0.519	0.1572	0.171	0.0264	0.521	0.1572	0.172
0.0523	0.556	0.1975	0.161	0.0523	0.557	0.1975	0.173
0.0841	0.620	0.2411	0.158	0.0841	0.620	0.2411	0.173
0.1206	0.640	0.2886	0.155	0.1206	0.643	0.2886	0.174
0.1617	0.693	0.3386	0.171	0.1617	0.693	0.3386	0.178
0.2087	0.743	0.3934	0.195	0.2087	0.738	0.3934	0.204
0.2613	0.762	0.4524	0.226	0.2613	0.758	0.4524	0.225
0.3075	0.773	0.5034	0.253	0.3075	0.774	0.5034	0.249
0.3567	0.831	0.5679	0.285	0.3567	0.840	0.5679	0.282
0.4083	0.940	0.6186	0.330	0.4083	0.947	0.6186	0.332
0.4578	0.992	0.6709	0.404	0.4578	1.002	0.6709	0.408
0.5117	1.092	0.7255	0.430	0.5117	1.108	0.7255	0.434
0.5600	1.161	0.7827	0.502	0.5600	1.203	0.7827	0.503
0.6113	1.132	0.8435	0.582	0.6113	1.323	0.8435	0.583
0.6654	1.109	0.9028	0.682	0.6654	1.331	0.9028	0.685
0.7135	1.104	0.9539	0.884	0.7135	1.214	0.9539	0.890
0.7636	1.102			0.7636	1.145		
0.8153	1.161			0.8153	1.192		
0.8701	1.172			0.8701	1.190		
0.9148	1.171			0.9148	1.207		
1.0000	0.998			1.0000	1.112		

Table C.27: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.98 and 1.05 at +10 Degrees Incidence.

$M_2 = 0.70$				$M_2 = 0.81$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.441	0.0813	0.200	0.0006	0.469	0.0813	0.214
0.0087	0.465	0.1191	0.192	0.0087	0.486	0.1191	0.212
0.0264	0.501	0.1572	0.197	0.0264	0.526	0.1572	0.212
0.0523	0.527	0.1975	0.190	0.0523	0.559	0.1975	0.210
0.0841	0.582	0.2411	0.182	0.0841	0.619	0.2411	0.205
0.1206	0.601	0.2886	0.189	0.1206	0.642	0.2886	0.205
0.1617	0.642	0.3386	0.199	0.1617	0.692	0.3386	0.218
0.2087	0.681	0.3934	0.218	0.2087	0.732	0.3934	0.237
0.2613	0.688	0.4524	0.245	0.2613	0.747	0.4524	0.253
0.3075	0.695	0.5034	0.265	0.3075	0.760	0.5034	0.273
0.3567	0.736	0.5679	0.286	0.3567	0.811	0.5679	0.298
0.4083	0.770	0.6186	0.343	0.4083	0.872	0.6186	0.368
0.4578	0.800	0.6709	0.361	0.4578	0.920	0.6709	0.382
0.5117	0.811	0.7255	0.403	0.5117	0.951	0.7255	0.432
0.5600	0.804	0.7827	0.465	0.5600	0.946	0.7827	0.493
0.6113	0.800	0.8435	0.517	0.6113	0.943	0.8435	0.569
0.6654	0.812	0.9028	0.586	0.6654	0.958	0.9028	0.653
0.7135	0.800	0.9539	0.691	0.7135	0.948	0.9539	0.793
0.7636	0.782			0.7636	0.927		
0.8153	0.780			0.8153	0.982		
0.8701	0.758			0.8701	0.898		
0.9148	0.736			0.9148	0.878		
1.0000	0.729			1.0000	0.846		

Table C.28: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.70 and 0.81 at +15 Degrees Incidence.

$M_2 = 0.70$				$M_2 = 0.81$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.485	0.0813	0.228	0.0006	0.485	0.0813	0.224
0.0087	0.504	0.1191	0.223	0.0087	0.512	0.1191	0.221
0.0264	0.545	0.1572	0.221	0.0264	0.544	0.1572	0.219
0.0523	0.575	0.1975	0.217	0.0523	0.576	0.1975	0.213
0.0841	0.632	0.2411	0.209	0.0841	0.635	0.2411	0.206
0.1206	0.661	0.2886	0.211	0.1206	0.660	0.2886	0.212
0.1617	0.716	0.3386	0.220	0.1617	0.718	0.3386	0.218
0.2087	0.760	0.3934	0.246	0.2087	0.762	0.3934	0.242
0.2613	0.776	0.4524	0.264	0.2613	0.779	0.4524	0.265
0.3075	0.789	0.5034	0.285	0.3075	0.789	0.5034	0.288
0.3567	0.851	0.5679	0.310	0.3567	0.850	0.5679	0.317
0.4083	0.926	0.6186	0.379	0.4083	0.938	0.6186	0.376
0.4578	0.998	0.6709	0.402	0.4578	1.008	0.6709	0.397
0.5117	1.068	0.7255	0.450	0.5117	1.091	0.7255	0.448
0.5600	1.077	0.7827	0.516	0.5600	1.137	0.7827	0.521
0.6113	1.050	0.8435	0.594	0.6113	1.097	0.8435	0.595
0.6654	1.088	0.9028	0.697	0.6654	1.105	0.9028	0.699
0.7135	1.097	0.9539	0.871	0.7135	1.127	0.9539	0.891
0.7636	1.080			0.7636	1.116		
0.8153	1.112			0.8153	1.169		
0.8701	1.068			0.8701	1.177		
0.9148	1.019			0.9148	1.147		
1.0000	0.944			1.0000	0.991		

Table C.29: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 0.92 and 0.96 at +15 Degrees Incidence.

$M_2 = 1.00$				$M_2 = 1.05$			
Suction Surface		Pressure Surface		Suction Surface		Pressure Surface	
X/C	M	X/C	M	X/C	M	X/C	M
0.0006	0.493	0.0813	0.220	0.0006	0.492	0.0813	0.228
0.0087	0.515	0.1191	0.220	0.0087	0.510	0.1191	0.222
0.0264	0.551	0.1572	0.220	0.0264	0.543	0.1572	0.223
0.0523	0.581	0.1975	0.218	0.0523	0.578	0.1975	0.217
0.0841	0.643	0.2411	0.218	0.0841	0.638	0.2411	0.217
0.1206	0.661	0.2886	0.219	0.1206	0.662	0.2886	0.218
0.1617	0.717	0.3386	0.221	0.1617	0.718	0.3386	0.224
0.2087	0.766	0.3934	0.243	0.2087	0.766	0.3934	0.243
0.2613	0.780	0.4524	0.264	0.2613	0.779	0.4524	0.264
0.3075	0.797	0.5034	0.288	0.3075	0.791	0.5034	0.287
0.3567	0.859	0.5679	0.315	0.3567	0.855	0.5679	0.316
0.4083	0.943	0.6186	0.382	0.4083	0.952	0.6186	0.384
0.4578	1.014	0.6709	0.404	0.4578	1.018	0.6709	0.403
0.5117	1.109	0.7255	0.455	0.5117	1.117	0.7255	0.450
0.5600	1.197	0.7827	0.526	0.5600	1.219	0.7827	0.525
0.6113	1.210	0.8435	0.600	0.6113	1.344	0.8435	0.597
0.6654	1.153	0.9028	0.703	0.6654	1.451	0.9028	0.702
0.7135	1.140	0.9539	0.903	0.7135	1.331	0.9539	0.904
0.7636	1.121			0.7636	1.230		
0.8153	1.174			0.8153	1.246		
0.8701	1.184			0.8701	1.221		
0.9148	1.203			0.9148	1.229		
1.0000	1.062			1.0000	1.135		

Table C.30: HS2 Blade Surface Isentropic Mach Numbers for Cascade Exit Mach Numbers of 1.00 and 1.05 at +15 Degrees Incidence.