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Sensitivity Analysis and Optimization of High-Speed VLSI Interconnects using Asymptotic Waveform Evaluation

by

Lup Shun Nelson Liu, B.Eng.

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of**

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
June, 1993**

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**"SENSITIVITY ANALYSIS AND OPTIMIZATION OF
HIGH-SPEED VLSI INTERCONNECTS USING
ASYMPTOTIC WAVEFORM EVALUATION"**

submitted by **Lup Shun Nelson Liu, B.Eng.**
in partial fulfillment of the requirements for
the degree of Master of Engineering

Prof. M. S. Nakhla
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June, 1993

Abstract

An efficient method, based on the *generalized* Asymptotic Waveform Evaluation (GAWE) Technique, is described in this dissertation for time domain adjoint sensitivity analysis of VLSI interconnect networks that contain *distributed* elements. Network sensitivity, which is approximated by a set of poles and residues sensitivities, can be evaluated efficiently with respect to all network components and transmission line parameters. Due to the *recursive* relationship between the time moments of distributed networks, the conventional adjoint approach could not be applied directly. Instead, an alternative approach is presented which solves this problem by introducing the concept of *adjoint moments* for networks with distributed elements. Examples and comparisons with sensitivity determined by perturbation are presented. The proposed technique is useful for identification of critical circuit elements and performance optimization of high speed VLSI designs.

As signal speed increases and circuit designs become more complicated, the need for a design tool for large scale networks becomes increasingly important. A robust method based on GAWE is developed for the optimization of high-speed interconnects. This method combines GAWE and the proposed sensitivity analysis algorithm with a powerful 2-stage minimax optimization package. Networks consisting of an arbitrary number of multiconductor transmission lines joined and terminated by arbitrary linear subnetworks are considered. Physical/geometrical features of transmission line model and elements inside any linear subnetwork can be used as design variables to improve network performance, such as reduced delay time, crosstalk and reflection on printed circuit board and multichip modules.

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List of Symbols

A_k	modified nodal admittance matrix for the linear subnetwork k
a_i	coefficients of the characteristic polynomial of a linear system
B_k	modified nodal admittance matrix for the linear subnetwork k
b	constant vector with entries determined by the independent voltage and current sources
C	symmetric matrix representing the capacitance per unit length of the transmission line
D	length of a transmission line
D_k	selector matrix that maps $i_k(t)$ into the node space
d	distance between two flat conductor transmission lines in parallel
$\delta(t)$	impulse excitation
E	vector of independent sources
$e_{Lj}(\underline{\phi}, t)$	lower error function for an output j with design variables $\underline{\phi}$ at time t
$e_{Uj}(\underline{\phi}, t)$	upper error function for an output j with design variables $\underline{\phi}$ at time t
ϵ_0	dielectric constant in vacuum ($10^{-9}/36\pi$ F/m)
ϵ_r	relative dielectric constant (4.5 for glass-epoxy)
$F_j(\underline{\phi}, t)$	response at the j th output with design variables $\underline{\phi}$ at time t
$f_j(\underline{\phi})$	a vector of error functions associated with the j th output with $\underline{\phi}$ as the design variable
G	symmetric matrix representing the conductance per unit length of the transmission line
Γ	diagonal matrix containing the square-roots of the eigenvalues of the transmission line matrix $Z_p Y_p$
γ_j^2	eigenvalues of the transmission line matrix $Z_p Y_p$
H	constant matrix with entries determined by the linear lumped components

h	height of a printed circuit board (PCB)
$I_k(s)$	vector of currents entering the linear subnetwork k
k_j	complex residues of a linear system
L	symmetric matrix representing the inductance per unit length of the transmission line
λ	parameter which the sensitivity is found with respect to
$\tilde{M}_n$	vector of adjoint moments n for all outputs of a transposed network
M_n	vector of moments n for all outputs
m_n	moment n of an output
N_π	number of linear subnetworks
p_j	complex poles of a linear system
R	symmetric matrix representing the resistance per unit length of the transmission line
S_{v_j}	eigenvectors of the transmission line matrix $Z_p Y_p$
$S_{L_j}(t)$	lower specifications for the j th output at time t
$S_{U_j}(t)$	upper specifications for the j th output at time t
s	Laplace operator
s_j	number of specifications associated with the j th output
T_1	rise/fall time of a trapezoidal waveform
T_2	pulse width of a trapezoidal waveform
U	identity matrix
μ_0	permeability in vacuum ($4\pi \times 10^{-7}$ N/m)
μ_r	relative permeability
$V_k(s)$	vector of node voltages and appended branch currents of the linear subnetwork k
W	constant matrix with entries determined by the linear lumped components
$W_{L_j}(t)$	lower weighting factors for the j th output at time t
$W_{U_j}(t)$	upper weighting factors for the j th output at time t

w	width of the flat conductors in a transmission line
$x(t)$	state vector of a linear system
$Y(s)$	zero initial condition impulse response of a linear system
$Y^*(s)$	approximated transfer function of a linear system
Y_p	complex admittance per unit length of the transmission line
Z_o	characteristic impedance
Z_p	complex impedance per unit length of the transmission line
$z(t)$	vector of node voltage waveforms appended by independent voltage source current waveforms and linear inductor current waveforms

Chapter 1

Introduction

1.1 Problem Description

Recent advances and innovations in large scale integrated circuit manufacturing and chip packaging have yielded larger chips with smaller devices and finer interconnecting lines. High level integration is achieved by the miniaturization of device size such that more and more devices can be put into a single chip. This not only increases the complexity and I/O count of a chip, but also increases the number of interconnections both on chip and inter-chip. As the chip area gets larger, the length of interconnections becomes longer while the switching speed of the circuit becomes faster due to the smaller device sizes. Consequently, the signal rise time becomes comparable to the electrical length of the interconnects and distributed transmission line models have to be used.

Moreover, as longer interconnections are placed closer to each other and as switching speeds become faster, crosstalk due to the electromagnetic coupling between interconnections presents another potential problem in VLSI design where fast pulses in the active lines can induce transient signals on the neighbouring inactive lines. This could trigger logic gates and other devices to produce corrupted data. Improperly terminated interconnections could also introduce signal reflection. Therefore, in designing high-speed systems, both the circuits and the interconnections must be considered or system performance may be impaired. Improperly designed interconnects can exhibit severe signal delay, ringing, inadvertent and false switching [1]-[6]. These phenomena can be observed at the chip, as well as the system level and the interconnected blocks could be analog, digital, or a mix of both.

The development of large-scale integrated circuit is strongly dependent on efficient CAD tools capable of predicting and optimizing circuit performance prior to fabrication, since post-processing tuning is very difficult or practically impossible. Therefore, performance violation due to delay, crosstalk and reflection must be detected and improved before a system is built. Several methods have been proposed for the analysis of networks which contain lossy coupled interconnects [7]-[10]. However, VLSI designs at the chip or system level contain very large number of interconnects. Therefore, a computationally less expensive method is desired especially during the design stage where fast waveform estimation is usually required to point out potential design problems. Recently, the Asymptotic Waveform Evaluation method has been demonstrated as an efficient approach for waveform evaluation of linear lumped networks [11]. Response of a network is approximated by matching the initial boundary conditions and the first $2q-1$

moments of the exact response to a lower order q -pole model. This method provides good compromise between accuracy and CPU requirement for circuit analysis. In [12]-[13], the AWE method has been extended to handle high speed interconnects where network moments are evaluated recursively. The speed is 2 to 3 orders of magnitude faster than conventional simulators with comparable accuracy such as SPICE.

It has been established that the adjoint network and network sensitivity play important roles in automated network design. Such sensitivity is extremely useful for circuit design since it provides the underlying information necessary for design evaluation, critical element identification and performance optimization [14]-[17]. However, despite its usefulness, sensitivity analysis has not been widely used in large scale design automation. The main problem is that circuit simulation and sensitivity analysis are the most time consuming processes in circuit optimization. Recently, an adjoint method based on the AWE technique has been proposed for efficient sensitivity analysis of linear *lumped* circuits [18]. Network sensitivity is approximated by the poles and zeros sensitivities which in turn, can be obtained from the moments sensitivity of the reduced network found by AWE. The evaluation of the moments sensitivity based on the adjoint method has been described for lumped circuits.

1.2 Thesis Objectives

The first objective of this work is to extend the AWE sensitivity method to handle interconnect models which contain *distributed* elements [19]-[20]. Sensitivity can be evaluated with respect to geometrical/electrical parameters of interconnects and lumped components of the linear terminations. Due to the *recursive* relationship between the time

moments of distributed networks, the conventional adjoint approach could not be applied directly. Instead, an alternative approach is presented in this thesis which solves the problem by introducing the idea of *adjoint moments* for networks with distributed elements. As a practical application, the proposed method is applied to the cases where the distributed elements can be modeled as lossless or lossy coupled transmission lines. The proposed method is more efficient than other methods based on FFT or the brute force perturbation technique.

As part of this thesis, a circuit simulator has been implemented for time domain simulation and sensitivity analysis of general transmission line networks. The second objective of this thesis is to combine the simulation and sensitivity results from this simulator with a 2-stage gradient based minimax optimizer for design optimization. The first stage of the optimizer is used to obtain fast convergence to the neighbourhood of the solution and the second stage is used to obtain superlinear final convergence [35], [39]. Transmission line problems commonly encountered in VLSI designs are addressed directly where the design variables can be the geometrical/electrical parameters of transmission lines and/or the linear elements from the terminating subnetworks.

1.3 Thesis Contributions

In this thesis, several contributions are presented to address the design of interconnections in high-speed networks:

1. An efficient method based on the *generalized Asymptotic Waveform Evaluation* technique is developed for sensitivity estimation of distributed networks where the sensitivity can be evaluated with respect to network components and electrical

-
- parameters of transmission lines. (Chapter 4)
2. A complete theory for the adjoint approach to moments sensitivity evaluation is developed and the recursive relationship for moments sensitivity, as well as adjoint moments, have been derived. (Section 4.3-4.4)
 3. A new method is presented to calculate the frequency domain sensitivities of the eigenvalues and eigenvectors and their derivatives with respect to the Laplace operator s . (Section 4.5-4.6)
 4. This method is extended to handle sensitivity with respect to physical/geometrical parameters of transmission line. (Section 5.2)
 5. Several examples comparing the proposed method with the perturbation method demonstrates the accuracy in evaluating network sensitivities. It is found that the proposed method is more efficient when applied to large scale networks. An example is given to show the CPU time savings when this method is applied to networks with different sizes. It is shown in this example that the proposed method can be 2 to 3 orders of magnitude faster than SPICE. (Section 4.8)
 6. The developed tool has been incorporated into a CAD framework [22] for high speed VLSI interconnects where a smooth integration with nonlinear analysis, optimization and statistical simulation can be achieved. Two examples illustrating the usefulness of this package are presented. (Chapter 5)
 7. A generalized interface has been developed to integrate any AWE type simulators into the CAD framework. This interface supports future enhancements to the simulators such as handling non-linear elements and non-uniform transmission lines. (Section 5.5.2)

1.4 Outline of the Thesis

In **Chapter 2 Analysis of Linear Lumped Networks using the AWE Technique**, a brief review of the AWE method for simulation and sensitivity analysis of linear lumped networks is given. The evaluation of network sensitivities from the moments sensitivity is also described in this chapter.

The *generalized* AWE algorithm which allows the simulation of linear lumped and distributed networks is discussed in **Chapter 3 Time Domain Analysis of Transmission Line Networks**.

The concept of *adjoint moments* and the recursive property of the moments sensitivity are introduced in **Chapter 4 Time Domain Sensitivity Analysis of Transmission Line Networks**. A formal proof of the *recursive adjoint* moments sensitivity equation and the evaluation of transmission line stamp sensitivity are given. A summary of the computational steps involved in the proposed method and a number of examples comparing the sensitivities calculated using the proposed method and perturbation are also presented in this chapter.

In **Chapter 5 Time Domain Performance Optimization of Transmission Line Networks**, the sensitivity analysis method from Chapter 4 is extended to handle physical/geometrical design variables. The optimization problem has been formulated into mathematical programming. Problems commonly encountered during transmission line network design are addressed directly. Two examples are used to illustrate the usefulness of this optimization package. The CPU time used in these examples are also given in this chapter.

Chapter 2

Analysis of Linear Lumped Networks using the AWE Technique

2.1 Introduction

The advances in integrated circuit technology are leading to a regime in which delay and crosstalk due to the interconnections will be the dominant limiting factors for higher speed and higher performance systems. For this reason, a considerable amount of work has been done in search of an efficient numerical analysis to predict crosstalk and signal propagation delay in the interconnects. One of the commonly used methods in timing verification of MOS VLSI circuits is the linear RC tree model proposed by Rubinstein, Penfield and Horowitz [23], in the early eighties. The RC tree is an interconnect model in which interconnects are modelled as a RC circuit with capacitors from all nodes to ground, no floating capacitors, no resistor loops and no resistors to ground. A typical RC tree

model from [24] is shown in Figure 2.1.

Figure 2.1: RC tree model

RC tree methods find an approximate waveform that matches the initial conditions and the first moment of the exact response. This is sufficient for monotonic output waveforms. However, for high-speed circuits and printed circuit board level designs, crosstalk capacitance and line inductance can be significant such that the output waveforms become non-monotonic. In this case, the RC tree approach will fail to predict the propagation delay. Besides, there is also a lack of error criteria for testing the validity of the delay time calculated from the RC tree model. Furthermore, the RC tree approach is derived with the assumption that the input excitation is an ideal step waveform but in practice, the propagation signal always exhibits a finite rise-time.

Recently, the Asymptotic Waveform Evaluation technique has been developed by

Pillage *et al.* [11] at Carnegie Mellon University for transient analysis of generalized RLC interconnect models and frequency domain analysis of linear(ized) analog circuits. Unlike the traditional RC tree model, AWE can handle finite input signal rise time and generalized RLC structures. An adjoint sensitivity analysis based on AWE has also been proposed lately for linear lumped networks. This chapter offers a brief review of the key concepts in AWE transient analysis and AWE sensitivity analysis that will be needed in subsequent discussions.

2.2 Time Domain Analysis

AWE is a variable refinement waveform estimator that matches an approximate waveform to the initial conditions and the first $2q-1$ moments of the exact response. Depending upon the details required, this method can be extended to any order by using more moments, which are calculated at an incremental cost in CPU time. This leads to a fundamental extension of the simple Elmore delay method [25] beyond the first moment such that the exact response is approached asymptotically as the number of moments matched is increased. The AWE technique is suitable for the estimation of waveforms with non-monotonic details such as overshoot, undershoot and glitches due to charge sharing.

2.2.1 The AWE Algorithm

To explain AWE in terms of a transfer function approach, consider the state equations for a linear system

$$\dot{x}(t) = Ax(t) + bu(t) \quad (2.1)$$

$$y(t) = c'x(t) + Du(t) \quad (2.2)$$

where $x(t)$ is the state vector, A is the state matrix, b is a selection vector indicating the inputs at which $u(t)$ is applied and D represents the *direct coupling* between the input $u(t)$ and the output $y(t)$. For simplicity, D is assumed to be 0. The transfer function of the system can be expressed as

$$Y(s) = c'(sI - A)^{-1}b \quad (2.3)$$

To approximate this solution, $Y(s)$ is expanded into a Maclaurin's series such that

$$Y(s) = -c'A^{-1}(I + A^{-1}s + A^{-2}s^2 + \dots)b \quad (2.4)$$

or

$$Y(s) = \sum_{n=0}^{\infty} m_n s^n \quad (2.5)$$

where the moments, m_n , are given by

$$m_0 = -c'A^{-1}b \quad (2.6)$$

and

$$m_n = -c'A^{-n-1}b \quad (2.7)$$

The moments m_n for $n=0, 1, \dots, 2q-1$, are matched to a lower order q -pole model in the Laplace domain to form a rational Padé approximation [46] of $Y(s)$. With p_j and k_j as the complex approximating poles and residues, the transfer function is approximated as

$$Y^*(s) = \sum_{n=0}^{2q-1} m_n s^n = \sum_{j=1}^q \frac{k_j}{s - p_j} \quad (2.8)$$

Writing the characteristic polynomial of the reduced model as

$$\sum_{i=0}^q a_i p_j^{q-i} = 0 \quad (2.9)$$

where a_q is set to be 1, the coefficients a_i , for $i=0, 1, 2, \dots, q-1$ are related to the moments by

$$\begin{bmatrix} m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ m_2 & m_3 & m_4 & \dots & m_{q+1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ m_{q-1} & m_q & m_{q+1} & \dots & m_{2q-2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{q-1} \end{bmatrix} = - \begin{bmatrix} m_q \\ m_{q+1} \\ m_{q+2} \\ \vdots \\ m_{2q-1} \end{bmatrix} \quad (2.10)$$

The roots of the characteristic polynomial (i.e. the poles, p_j , of the reduced network) can then be found by using a standard root finding algorithm [26]. Note that (2.10) is only one of many alternative moment matching methods that may be employed to obtain the approximating poles. The corresponding residues, k_j , are related to the poles and moments by

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & p_3^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & p_3^{-2} & \dots & p_q^{-2} \\ p_1^{-3} & p_2^{-3} & p_3^{-3} & \dots & p_q^{-3} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{-q} & p_2^{-q} & p_3^{-q} & \dots & p_q^{-q} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_q \end{bmatrix} = - \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{q-1} \end{bmatrix} \quad (2.11)$$

Once the poles and residues are found by (2.10) and (2.11), the time domain impulse response of the approximating model can be evaluated directly as

$$y^*(t) = \sum_{j=1}^q k_j e^{p_j t} \quad (2.12)$$

2.2.2 Numerical Examples

Example 2.1

To demonstrate the accuracy of the AWE technique, consider the RLC circuit shown in Figure 2.2. The input signal is a pulse with 500ps rise/fall time, 5ns pulse width and 5V amplitude, as shown in Figure 2.3. The transient response at V_{out} by AWE approximation and HSPICE are compared in Figure 2.4. For a second order approximation, AWE is not able to completely detect the overshoot, therefore, a higher order approximation is required. At sixth order, AWE response is almost identical to the HSPICE output.

Example 2.2

To show the efficiency of AWE, some CPU comparisons with HSPICE are presented in Table 2.1. This table is extracted from [27].

Circuit Name	Node Count	Element Count	CPU Seconds	
			AWE	HSPICE
Small RC Tree	19	35	0.02	0.40
Medium Sized Interconnect	75	121	0.04	0.63
Backplane ⁽¹⁾	505	2294	3.54	(2)
Clock Distribution Line ⁽³⁾	14,000	55,000	300	37271

⁽¹⁾ backplane is modelled by 2000 inductors and some controlled sources, resistors and capacitor.

⁽²⁾ HSPICE could not simulate in presence of inductor loops

⁽³⁾ clock distribution is modelled by 40,000 resistors and 15,000 capacitors.

Table 2.1: CPU comparison between AWE and HSPICE

Figure 2.2: Linear lumped circuit for Example 2.1 and Example 2.3.

Figure 2.3: Applied voltage at V_{in} in Example 2.1 and Example 2.3.

Figure 2.4: Transient response at V_{out} in Example 2.1.

2.3 Sensitivity Analysis

The AWE technique has been applied to adjoint sensitivity analysis for linear lumped networks by Lee *et al.* [18] at Carnegie Mellon University. Network sensitivity is approximated by the poles and residues sensitivities of a reduced order q -pole model determined by using a form of Padé approximation. Since the poles and residues completely define the behavior of a system, their sensitivities can serve as a robust mechanism for sensitivity evaluation. This method incurs very little overhead on top of the nominal AWE analysis, which itself outpaces traditional simulators often by at least an order of magnitude. In this section, a brief review of the AWE sensitivity will be given. It is assumed that the order of approximation for sensitivity analysis is the same as in transient analysis.

2.3.1 The AWE Sensitivity Algorithm

To illustrate the AWE sensitivity algorithm in terms of the state equations of a linear circuit, consider the linear system described in Section 2.2.1. Applying the chain rule to (2.7), the moments sensitivity can be computed by using the following identity

$$\frac{\partial A^{-1}}{\partial \lambda} = -A^{-1} \frac{\partial A}{\partial \lambda} A^{-1} \quad (2.13)$$

which yields

$$\frac{\partial m_n}{\partial \lambda} = -\frac{\partial c'}{\partial \lambda} A^{-n-1} b + c' (A^{-1} \frac{\partial A}{\partial \lambda} A^{-n-1} + \dots + A^{-n-1} \frac{\partial A}{\partial \lambda} A^{-1}) b - c' A^{-n-1} \frac{\partial b}{\partial \lambda} \quad (2.14)$$

Define two subsolutions, $\hat{x}_n$ and $\hat{y}_n$

$$\hat{x}_n = A^{-n-1} b \quad (2.15)$$

$$\hat{y}_n = (A^{-n-1})' c \quad (2.16)$$

where $\hat{x}_n$ are the moments of the circuit by a $-c'$ premultiplier and $\hat{y}_n$ are related to the moments of a transposed circuit excited by c . Thus, the sensitivity routines impose an added cost of finding the moments of this adjoint circuit. Since we can use the same L/U factors of the A matrix, this incurs $2q$ extra forward/backward substitutions and no extra matrix factorization.

The moments sensitivity are expressed concisely as

$$\frac{\partial m_n}{\partial \lambda} = -\frac{\partial c'}{\partial \lambda} \hat{x}_n + \sum_{i=-1}^{n-1} \hat{y}_i' \frac{\partial A}{\partial \lambda} \hat{x}_{n-i} - \hat{y}_n' \frac{\partial b}{\partial \lambda} \quad (2.17)$$

The sensitivities of the state equation coefficients $\frac{\partial A}{\partial \lambda}$ and $\frac{\partial b}{\partial \lambda}$ are generated from well-known stencils and, as such, are computed in constant time.

Notice that the $\hat{x}_n$ and $\hat{y}_n$ subsolutions are dependent only on the circuit topology, not on the particular sense element, so their computational cost is shared between multiple sense analyses. Once the poles and residues sensitivities with respect to one element has been found, we can find the sensitivities with respect to other elements by using the same sub-solutions. The time domain sensitivity of the q -pole model with respect to a variable λ can be obtained by

$$\frac{\partial y^*(t)}{\partial \lambda} = \sum_{j=1}^q \left(\frac{\partial k_j}{\partial \lambda} e^{p_j t} + k_j t e^{p_j t} \frac{\partial p_j}{\partial \lambda} \right) \quad (2.18)$$

In order to find the sensitivity of $y^*(t)$, the sensitivities of the poles p_j and residues k_j must be calculated first. With the transfer function in (2.8) written as

$$Y^*(s) = \sum_{n=0}^{2q-1} m_n s^n = \frac{P(s)}{Q(s)} \quad (2.19)$$

where

$$Q(s) = \sum_{i=0}^q a_i s^{q-i} \quad (2.20)$$

is the characteristics polynomial of the model, the j th pole of the network are calculated such that

$$\sum_{i=0}^q a_i p_j^{q-i} = 0 \quad (2.21)$$

Differentiating this equation gives

$$\sum_{i=0}^q \frac{\partial a_i}{\partial \lambda} p_j^{q-i} + \sum_{i=0}^q a_i (q-i) p_j^{q-i-1} \frac{\partial p_j}{\partial \lambda} = 0 \quad (2.22)$$

Therefore, the sensitivity of p_j with respect to a variable λ is given by

$$\frac{\partial p_j}{\partial \lambda} = - \left(\frac{\sum_{i=0}^{q-1} \frac{\partial a_i}{\partial \lambda} p_j^{q-i}}{\sum_{k=1}^q a_{q-k} \cdot k \cdot p_j^{k-1}} \right) \quad (2.23)$$

where $a_q = 1$. (2.23) describes the relationship between the sensitivities of the poles and the sensitivities of the coefficients of $Q(s)$. Since the coefficients are related to the moments by (2.10) which is expressed symbolically here as

$$m_l a = -m_h \quad (2.24)$$

the coefficients sensitivity can be calculated by

$$m_l \frac{\partial a}{\partial \lambda} = - \frac{\partial m_l}{\partial \lambda} a - \frac{\partial m_h}{\partial \lambda} \quad (2.25)$$

The sensitivities of the residues can also be evaluated in the same manner once the sensitivities of the poles and moments are available. Rewriting (2.11) symbolically as

$$Pk = -m \quad (2.26)$$

and taking partial derivative yields

$$P \frac{\partial k}{\partial \lambda} = - \frac{\partial P}{\partial \lambda} k - \frac{\partial m}{\partial \lambda} \quad (2.27)$$

This equation is required for the residues sensitivity calculation. Therefore, once the circuit moments sensitivity are found, (2.18), (2.23), (2.25) and (2.27) could be applied to obtain the network sensitivity.

2.3.2 Numerical Examples

Example 2.3

The AWE sensitivity estimation technique has been implemented as part of this thesis. To illustrate the accuracy of this technique, consider the RLC circuit shown in Figure 2.2. The input impulse is the same as in Example 2.1. The sensitivities of V_{out} with respect to the source impedance, R_S , and the load impedance, R_L , are compared with HSPICE perturbation in Figure 2.5 and Figure 2.6. The sensitivity plots are normalized to represent mV change per 1 percent change in element value.

Example 2.4

Table 2.2, extracted from [18], illustrates the overhead incurred by AWE sensitivity analysis for a range of circuits. In each case, a second-order analysis was performed for the root sensitivities with respect to ten elements. For larger circuits, the overhead is reduced because of the smaller relative cost of a forward and backward substitution with respect to a L/U factorization.

Figure 2.5: Sensitivity of V_{ou} with respect to R_S in Example 2.3.

Figure 2.6: Sensitivity of V_{out} with respect to R_L in Example 2.3.

Circuit Name	Number of Elements	AWE Analysis	AWE Sensitivity	Overhead (%)
Interconnect A	56132	304.020s	3.790s	1.25%
Interconnect B	2294	4.610s	0.190s	4.20%
Op Amp A	308	0.290s	0.036s	12.65%
Comparator	195	0.160s	0.024s	14.74%
Op Amp B	99	0.092s	0.021s	23.38%
Op Amp C	65	0.068s	0.021s	30.04%

Table 2.2: CPU comparison between AWE and AWE Sensitivity.

2.4 Comments on the AWE Technique

The AWE technique is an efficient method for transient analysis and sensitivity analysis of RLC circuits. It can resolve the limitations in RC tree approaches such as floating capacitors, inductors and nonzero input rise times while maintaining the speed to the same order as the RC tree model. By approximating the original network with a lower order q -pole model, the transient response is found by Padé approximation instead of time consuming techniques such as FFT and numerical integration. This provides a good compromise between efficiency and accuracy where detailed waveforms can be obtained by using a higher order approximation.

For the special case of a RC tree driven by an unit step input, the first order AWE approximation reduces to the RC tree models. By using more moments, which are calculated at an incremental CPU cost, this method leads to a fundamental extension of the simple Elmore delay method. It provides a generalized approach to lumped RLC circuit

response approximation, which is more accurate when compared with the RC tree method for delay estimation.

However, based on our experience and published comments [28]-[29], AWE has some drawbacks:

- When the eigenvalues of A^{-1} are very small, or very large, the powers of A^{-1} , and therefore the moments, change rapidly. The large variation in moment values may cause the moment matrix in (2.10) to become ill-condition or nearly singular. In such cases, higher orders of approximation cannot be obtained unless the moment values are scaled. In AWE, frequency scaling approach is used and the scale factor is m_0/m_1 . This frequency scaling can postpone the instability problem in the moment matrix but does not solve the problem completely.
- For a stable linear circuit, AWE could generate some right half plane poles especially at higher orders of approximation.
- There is no obvious error criteria for the AWE technique.

A considerable amount of work has been done to address the above problems [30]-[31], [44]. Although these techniques are not discussed here, any accuracy improvements for the AWE algorithm can be easily incorporated in our proposed sensitivity algorithm.

A simplified approach for the analysis of parasitic coupling between nearby interconnect lines on an integrated circuit can be done by computing a lumped RLC model for the interconnect structure and including this model in simulation. Such an approach gives a certain amount of insight, but does not model the distributed effects accurately enough to make reliable design decision. For high speed signals, the electrical length of

an interconnect can become a significant fraction of the signal wavelength. Therefore, conventional lumped-impedance interconnect model must be replaced by distributed transmission line model. In the next chapter, a generalized AWE technique will be discussed which is capable of predicting crosstalk and signal delay in high-speed circuits where interconnects are modelled as lossless or lossy coupled transmission lines.

Chapter 3

Time Domain Analysis of Transmission Line Networks

3.1 Introduction

The common methods for delay estimation rely upon an RC tree interconnection model [23]. This model is adequate for mid-frequency MOS integrated circuits. However, for high-speed VLSI circuits, multi-chip modules (MCM) and printed circuit boards (PCB), more elaborate interconnect models are required in order to accurately predict the signal delay and crosstalk effects. The models in this case may contain *general RLC lumped and distributed elements* where interconnects are modelled as multiconductor transmission lines. A number of methods [7]-[10] have been proposed for the analysis of lossy coupled transmission lines. However, these methods require more CPU time than a circuit designer can normally afford. To address this issue, the AWE technique has been

extended by Tang *et al.* [12], [24], [32] at Carleton University to handle networks with distributed elements. This results in a *generalized* AWE (GAWE) algorithm that not only preserves all outstanding merit of the AWE method but also handles networks with arbitrary number of multiconductor transmission lines with no topological constraint.

3.2 Formulation of Network Equations

Consider a network π which contains linear lumped components and N_s arbitrary linear subnetworks that may include distributed components. Assume that the Laplace domain equations of the linear subnetwork k to be in the form

$$A_k V_k(s) + B_k I_k(s) = 0 \quad (3.1)$$

where $V_k(s)$ and $I_k(s)$ represent the s domain terminal voltages and currents of this subnetwork, respectively. Without loss of generality, the Modified Nodal Admittance (MNA) matrix [21] equation for this network can be written in the time domain as

$$W \frac{dz(t)}{dt} + H z(t) + \sum_{k=1}^{N_s} D_k i_k(t) - b \delta(t) = 0 \quad (3.2)$$

or in the s domain, when combined with (3.1), as

$$\begin{bmatrix} sW + H & D_1 & D_2 & \dots & D_{N_s} \\ A_1 D_1^t & B_1 & 0 & \dots & 0 \\ A_2 D_2^t & 0 & B_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_{N_s} D_{N_s}^t & 0 & 0 & \dots & B_{N_s} \end{bmatrix} \begin{bmatrix} Z(s) \\ I_1(s) \\ I_2(s) \\ \vdots \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3.3)$$

where b is the source vector,

$Z(s)$ is the vector of node voltages and branch currents for inductors and voltage sources in the s -domain;

$I_k(s)$ are the terminal currents of the transmission line in subnetwork k ;

D_k are the selector matrices;

W and H are constant matrices determined by the linear lumped network and

N_x is the number of linear subnetworks.

From (3.3), the impulse response of the network becomes

$$X(s) = [Y(s)]^{-1} E \quad (3.4)$$

To illustrate the formulation technique, consider the circuit shown in Figure 3.1, which consists of a simple transmission line structure and linear terminations.

Figure 3.1: Example circuit used to illustrate matrices formulation.

This circuit can be described in (3.3) where

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 0 & 1 \\ -G_1 & G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_2 & 0 & 0 & -G_2 & 0 \\ 0 & 0 & 0 & G_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_4 & 0 & 0 \\ 0 & 0 & -G_2 & 0 & 0 & G_2 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Z = [V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6 \ I_E]^t$$

$$I_1 = [I_a \ I_b \ I_c \ I_d]^t$$

$$b = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]^t$$

The derivation of A_1 and B_1 for the transmission line system will be presented in the next chapter.

3.3 Moments Evaluation

To approximate the transient solution $x(t)$, (3.4) is expanded in MacLaurin's series in the form

$$X(t) = \sum_{n=0}^{\infty} M_n t^n \quad (3.5)$$

where

$$M_n = \frac{\partial^n [Y^{-1}]_{s=0}}{n!} E \quad (3.6)$$

The moment vectors of this network can be evaluated recursively with

$$[Y]_0 M_0 = E \quad (3.7)$$

$$[Y]_0 M_n = - \sum_{r=1}^n \frac{[Y]_0^{(r)} M_{n-r}}{r!} \quad \text{for } n > 0 \quad (3.8)$$

where $[Y]_0 = Y(s)$ at $s = 0$. The derivatives of $[Y]_0$ are obtained as

$$[Y]_0^{(1)} = \begin{bmatrix} W & 0 & 0 & \dots & 0 \\ A_1^{(1)} D_1' & B_1^{(1)} & 0 & \dots & 0 \\ A_2^{(1)} D_2' & 0 & B_2^{(1)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ A_{N_n}^{(1)} D_{N_n}' & 0 & 0 & \dots & B_{N_n}^{(1)} \end{bmatrix} \quad (3.9)$$

and

$$[Y]_0^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ A_1^{(r)} D_1' & B_1^{(r)} & 0 & \dots & 0 \\ A_2^{(r)} D_2' & 0 & B_2^{(r)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ A_{N_n}^{(r)} D_{N_n}' & 0 & 0 & \dots & B_{N_n}^{(r)} \end{bmatrix} \quad \text{for } r > 1 \quad (3.10)$$

The derivatives $A_k^{(r)}$ and $B_k^{(r)}$ can be determined recursively in terms of the line parameters. In the special case where a network contains only linear lumped components

$$[Y]_0^{(r)} = 0 \quad \text{for } r > 1 \quad (3.11)$$

and (3.8) becomes equivalent to (2.7). Once the moments are found by (3.7) and (3.8), a reduced order pole-residue model can be constructed by using the moment matching technique presented in the previous chapter.

3.4 Numerical Examples

Example 3.1

An example circuit consists of three lossy coupled transmission lines is shown in Figure 3.2. The length of transmission line #1, #2 and #3 are 0.5mm, 0.4mm and 0.3mm respectively. The electrical parameters are identical for these lines.

$$R = \begin{bmatrix} 7.5 & 1.5 \\ 1.5 & 7.5 \end{bmatrix} \Omega/mm \quad L = \begin{bmatrix} 49.46 & 6.33 \\ 6.33 & 49.46 \end{bmatrix} nH/mm$$

$$G = \begin{bmatrix} 0.01 & -0.001 \\ -0.001 & 0.01 \end{bmatrix} S/mm \quad C = \begin{bmatrix} 6.28 & -0.49 \\ -0.49 & 6.28 \end{bmatrix} pF/mm$$

The input waveform, as shown in Figure 3.3, is a pulse with 10ps rise/fall time, 4ns pulse width and 1V amplitude. The output waveforms at V_{out} and V_{cross} are shown in Figure 3.4 and Figure 3.5 respectively. Note that this circuit cannot be analyzed by HSPICE since the lossy transmission lines are characterized in the frequency domain.

Figure 3.2: Interconnect model with lossy coupled transmission lines for Example 3.1.

Figure 3.3: Input signal at V_{in} in Example 3.1.

Figure 3.4: Transient response at V_{out} in Example 3.1.

Figure 3.5: Transient response at V_{cross} in Example 3.1.

Example 3.2

Table 3.1 compares the run time of the GAWE algorithm and HSPICE. It shows that the additional computational cost due to the existence of transmission lines is negligible for GAWE. This table is extracted from [24].

Number of Transmission Lines	Node Count	Lumped Element Count	CPU Time (seconds)	
			GAWE	HSPICE
35	101	145	2.62s	256s
70	201	290	3.91s	5261s
105	301	435	6.05s	7702s

Table 3.1: CPU comparison between GAWE and HSPICE.

3.5 Summary

In this chapter, an approach for the time domain analysis of transmission line networks based on the AWE technique has been discussed where the time domain response is evaluated in terms of the dominant poles and residues. This method provides significant speed improvement when compared with previous literature. The computational results of GAWE show that the evaluation of moments of a network which contains multiconductor transmission lines is very efficient because the moments of the transmission line can be evaluated by using a closed form expression.

Chapter 4

Time Domain Sensitivity Analysis of Transmission Line Networks

4.1 Introduction

Performance optimization of lossless/lossy interconnects has been recently addressed [16]- [17]. However, VLSI designs at the chip or system level contain very large number of interconnects. Application of powerful gradient based optimizers on such large scale problem is usually limited by the CPU cost involved during the sensitivity evaluation. The direct perturbation approach can be prone to error and is very inefficient when the number of design variables becomes large. Therefore, a computationally less expensive method is desired for the sensitivity analysis of high speed VLSI interconnects. This tool, by itself, can also be used to identify critical elements that affect the system performance such as rise/fall time, overshoot/undershoot, crosstalk, reflection and time delay.

In this chapter, we will present an efficient sensitivity analysis method based on GAWE where network sensitivity is approximated by the poles and residues sensitivities of a reduced model. As part of this thesis, a circuit simulator has been implemented for the simulation and sensitivity analysis of general linear lumped and distributed networks. This simulator, dGAWE -- *generalized AWE with sensitivity*, has been incorporated into a CAD framework for high speed VLSI interconnects where a smooth integration with nonlinear analysis, optimization and statistical simulation can be achieved.

4.2 Poles and Residues Sensitivities

From Section 2.3, it seems plausible that if we can find the moments sensitivity of a distributed network, the time domain sensitivity of this network can be obtained readily by a simple moment matching technique. Indeed, as we will show in this chapter, once the moments sensitivity are available, the poles and residues sensitivities of the GAWE model can be found by using equations (2.23), (2.25) and (2.27) from Section 2.3.

$$\frac{\partial p_j}{\partial \lambda} = - \left(\frac{\sum_{i=0}^{q-1} \frac{\partial a_i}{\partial \lambda} p_j^{q-i}}{\sum_{k=1}^q a_{q-k} \cdot k \cdot p_j^{k-1}} \right) \quad (4.1)$$

$$m_i \frac{\partial a}{\partial \lambda} = - \frac{\partial m_i}{\partial \lambda} a - \frac{\partial m_h}{\partial \lambda} \quad (4.2)$$

$$P \frac{\partial k}{\partial \lambda} = - \frac{\partial P}{\partial \lambda} k - \frac{\partial m}{\partial \lambda} \quad (4.3)$$

Finally, the approximating network's sensitivity can be found directly as

$$\frac{\partial y^*(t)}{\partial \lambda} = \sum_{j=1}^q \left(\frac{\partial k_j}{\partial \lambda} e^{p_j t} + k_j t e^{p_j t} \frac{\partial p_j}{\partial \lambda} \right) \quad (4.4)$$

In the next section, an efficient method will be presented for recursive moments sensitivity calculation by using the adjoint method.

4.3 Moments Sensitivity using the Adjoint Method

There are two possible scenarios for sensitivity analysis. The first case is when sensitivity of *all* outputs with respect to *one* parameter is required. The second case is when sensitivity of *one* output with respect to *many* parameters is needed. For the first case, the *direct method*, a partial differentiation of (3.8) gives

$$[Y]_0 \frac{\partial M_n}{\partial \lambda} = - \sum_{r=1}^n \frac{1}{r!} \left(\frac{\partial [Y]_0^{(r)}}{\partial \lambda} M_{n-r} + [Y]_0^{(r)} \frac{\partial M_{n-r}}{\partial \lambda} \right) - \frac{\partial [Y]_0}{\partial \lambda} M_n \quad (4.5)$$

This equation describes the relationship required for the sensitivity evaluation of the moments of all outputs, M_n , with respect to one network parameter λ . For each additional λ , the right hand side of (4.5) has to be re-formulated and an extra forward/backward substitution is required. For the second case, the *adjoint method*, the sensitivity of the moment of one output, m_n , with respect to many parameters can be found by using only one forward/backward substitution. This method is more efficient when compared with the direct method especially if the number of design variables is large. Therefore, in practice, the adjoint method is more desirable and is applicable to problems such as performance optimization where a few signals at vital ports are optimized with respect to a large set of design variables.

As shown in the previous chapter, the moments sensitivity for a linear lumped network can be computed by applying the adjoint method directly. However, for networks with distributed elements, the higher derivatives (≥ 2) of the MNA matrix are no longer zero. This leads to a general recursive function for moments evaluation

$$[Y]_0 M_n = - \sum_{r=1}^n \frac{1}{r!} [Y]_0^{(r)} M_{n-r}, \quad (4.6)$$

where

$$[Y]_0 M_0 = E \quad (4.7)$$

Suppose the output of interest is a linear combination of the output vector $x^*(t)$. i.e.

$$v_{out} = d^T x^*(t) \quad (4.8)$$

$$m_n = d^T M_n \quad (4.9)$$

where m_n is the n th moment of v_{out} and d is a selector vector. In order to find the sensitivity of v_{out} , the sensitivities of m_n , for $n=0, 1, \dots, 2q-1$ have to be calculated first. If the adjoint method is applied to (4.7), the moment zero sensitivity can be easily obtained by using the identity

$$\frac{\partial [Y]_0^{-1}}{\partial \lambda} = -[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} [Y]_0^{-1} \quad (4.10)$$

as

$$m_0 = d^T M_0 \quad (4.11)$$

$$= d^T [Y]_0^{-1} E \quad (4.12)$$

$$\frac{\partial m_0}{\partial \lambda} = d^T \frac{\partial [Y]_0^{-1}}{\partial \lambda} E \quad (4.13)$$

$$= -d'[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} [Y]_0^{-1} E \quad (4.14)$$

$$= -d'[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} M_0 \quad (4.15)$$

However, when we try to find the sensitivity of moment one, m_1 , from (4.6)

$$m_1 = d' M_1 \quad (4.16)$$

$$= -d'[Y]_0^{-1} [Y]_0^{(1)} M_0 \quad (4.17)$$

$$\frac{\partial m_1}{\partial \lambda} = d'[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} [Y]_0^{-1} [Y]_0^{(1)} M_0 - d'[Y]_0^{-1} \frac{\partial [Y]_0^{(1)}}{\partial \lambda} M_0 - d'[Y]_0^{-1} [Y]_0^{(1)} \frac{\partial M_0}{\partial \lambda} \quad (4.18)$$

$$= -d'[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} M_1 - d'[Y]_0^{-1} \frac{\partial [Y]_0^{(1)}}{\partial \lambda} M_0 - d'[Y]_0^{-1} [Y]_0^{(1)} \frac{\partial M_0}{\partial \lambda} \quad (4.19)$$

It becomes clear that (4.19) is undefined since it requires the moment zero sensitivities of all outputs $\frac{\partial M_0}{\partial \lambda}$ but only $\frac{\partial m_0}{\partial \lambda}$ is available from the last step. Therefore, this approach is infeasible.

In this section, a new approach will be presented for moments sensitivity evaluation of linear lumped and distributed networks. The proposed technique can solve the above problem while preserving the efficiency of the conventional adjoint method. This new approach would be best explained by first defining what we called, the *Adjoint Moments*.

4.3.1 Definition of the Adjoint Moments

By defining the adjoint moment zero, $\tilde{M}_0$, as

$$[Y']_0 \tilde{M}_0 = d \quad (4.20)$$

(4.15) can be rewritten as

$$\frac{\partial m_0}{\partial \lambda} = -\tilde{M}_0' \frac{\partial [Y]_0}{\partial \lambda} M_0 \quad (4.21)$$

Since $\tilde{M}_0$ does not depend on λ , it can be reused when the sensitivity of m_0 with respect to another variable is required. Hence, the cost of a single forward/backward substitution in calculating $\tilde{M}_0$ is amortized over all of the variables. For higher adjoint moments, $\tilde{M}_n$ is defined as

$$[Y']_0 \tilde{M}_n = -\sum_{r=1}^n \frac{1}{r!} [Y']_0^{(r)} \tilde{M}_{n-r} \quad (4.22)$$

By comparing (4.20) and (4.22) with (4.6) and (4.7), it can be seen that $\tilde{M}_n$ are actually the moment vectors of a transposed network, $[Y']$, with d as the input excitation. Since $[Y]$ does not change, the same L/U factors from (4.7) can be used again in (4.20) and (4.22). Therefore, only one L/U decomposition, which has been done during the transient analysis, is required for all adjoint moments calculation.

4.3.2 Evaluation of the Moments Sensitivity

Combining (4.22) with (4.5) leads to a recursive equation for successive moments sensitivity evaluation (to be proven in the next section)

$$\frac{\partial m_n}{\partial \lambda} = -\sum_{k=0}^n \tilde{M}_k' \sum_{r=0}^{n-k} \frac{1}{r!} \frac{\partial [Y]_0^{(r)}}{\partial \lambda} M_{n-k-r} \quad (4.23)$$

This equation suggests that regardless of the number of design variables, only $2q$ forward/backward substitutions (for adjoint moments calculation) are required for the evaluation of the $2q$ moments sensitivity where q is the order of approximation. The

computational cost can be further reduced by defining a subsolution

$$\xi_k = \sum_{r=0}^k \frac{1}{r!} \frac{\partial [y]_0^{(r)}}{\partial \lambda} M_{k-r} \quad (4.24)$$

which is to be evaluated for each λ . Since (4.24) does not change with different outputs (i.e. different d vectors), it can be reused in (4.23) when the sensitivity at a different output is required. Equation (4.23) can now be rewritten as

$$\frac{\partial m_n}{\partial \lambda} = - \sum_{k=0}^n \tilde{M}'_k \xi_{n-k} \quad (4.25)$$

which involves only *vector* multiplication. Therefore, for the sensitivity of i outputs and j design variables, (4.22) has to be evaluated i times, which requires $2q \times i$ forward/backward substitutions, and (4.24) has to be calculated j times. After that, (4.25) can be used to find the sensitivity of all i outputs with respect to all j design variables.

With the formulation technique described in Section 3.2, the sensitivities of $[y]$ and its derivatives are obtained as

$$\frac{\partial [y]_0}{\partial \lambda} = \begin{bmatrix} \frac{\partial H}{\partial \lambda} & 0 & 0 & \dots & 0 \\ \frac{\partial A_1}{\partial \lambda} D'_1 & \frac{\partial B_1}{\partial \lambda} & 0 & \dots & 0 \\ \frac{\partial A_2}{\partial \lambda} D'_2 & 0 & \frac{\partial B_2}{\partial \lambda} & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ \frac{\partial A_{N_s}}{\partial \lambda} D'_{N_s} & 0 & 0 & \dots & \frac{\partial B_{N_s}}{\partial \lambda} \end{bmatrix}_{s=0} \quad (4.26)$$

$$\frac{\partial [Y]_0^{(1)}}{\partial \lambda} = \begin{bmatrix} \frac{\partial W}{\partial \lambda} & 0 & 0 & \dots & 0 \\ \frac{\partial A_1^{(1)}}{\partial \lambda} D'_1 & \frac{\partial B_1^{(1)}}{\partial \lambda} & 0 & \dots & 0 \\ \frac{\partial A_2^{(1)}}{\partial \lambda} D'_2 & 0 & \frac{\partial B_2^{(1)}}{\partial \lambda} & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \vdots \\ \frac{\partial A_{N_s}^{(1)}}{\partial \lambda} D'_{N_s} & 0 & 0 & \dots & \frac{\partial B_{N_s}^{(1)}}{\partial \lambda} \end{bmatrix}_{s=0} \quad (4.27)$$

$$\frac{\partial [Y]_0^{(n)}}{\partial \lambda} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ \frac{\partial A_1^{(n)}}{\partial \lambda} D'_1 & \frac{\partial B_1^{(n)}}{\partial \lambda} & 0 & \dots & 0 \\ \frac{\partial A_2^{(n)}}{\partial \lambda} D'_2 & 0 & \frac{\partial B_2^{(n)}}{\partial \lambda} & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \vdots \\ \frac{\partial A_{N_s}^{(n)}}{\partial \lambda} D'_{N_s} & 0 & 0 & \dots & \frac{\partial B_{N_s}^{(n)}}{\partial \lambda} \end{bmatrix}_{s=0} \quad \text{for } n \geq 2 \quad (4.28)$$

In the case when the subnetwork consists of a multiconductor transmission line system, the sensitivities of $A_k^{(n)}$ and $B_k^{(n)}$ are defined in terms of the line parameters. The detailed derivation of $A_k^{(n)}$, $B_k^{(n)}$ and their corresponding sensitivities will be described in Section 4.5.

Equations (4.20)-(4.25) represent the adjoint method for moments sensitivity evaluation of linear lumped and distributed networks. For the special case when only linear lumped components exist in the network π , (4.28) becomes 0 for $n \geq 2$ and (4.23) becomes equivalent to the approach presented in Section 2.3. A formal proof of the recursive sensitivity equation, (4.23), by mathematical induction is given in the next

section.

4.4 Proof of the Recursive Moments Sensitivity Equation

From the *Second Principle of Induction* [19], if an open sentence $P(n)$ is a true statement for $P(0)$ and with the hypothesis that $P(y)$ is also true when $y \leq p$, then $P(n)$ is a true statement for $n \geq 0$ if $P(y)$ can be proven to be true for $y \leq p + 1$. In this section, the recursive adjoint sensitivity equation is to be proven by mathematical induction. From Chapter 3, the moments of a distributed network has the following recursive relationship

$$[Y]_0 M_0 = E \quad (4.29)$$

$$[Y]_0 M_n = - \sum_{k=1}^n \frac{1}{k!} [Y]_0^{(k)} M_{n-k} \quad (4.30)$$

which shows that the current moment depends on all previously calculated moments. The adjoint moments defined in Section 4.3.1 also have a similar recursive relationship. By replacing the selection vector d with an identity matrix U , the adjoint moments equations can be modified as

$$[Y^T]_0 \tilde{M}_0 = U \quad (4.31)$$

$$[Y^T]_0 \tilde{M}_n = - \sum_{k=1}^n \frac{1}{k!} [Y^T]_0^{(k)} \tilde{M}_{n-k} \quad (4.32)$$

Note that in this case, $\tilde{M}_n$ is no longer the adjoint moment vector for a specified output. Instead, it is now a matrix consisting of the adjoint moment vectors for all outputs. This effectively changes the original vector equation (4.22) into a matrix equation. (4.23) should also be rewritten in the vector form as

$$\frac{\partial \mathbf{M}_n}{\partial \lambda} = - \sum_{k=0}^n \tilde{\mathbf{M}}_k' \sum_{r=0}^{n-k} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} \mathbf{M}_{n-k-r} \quad (4.33)$$

where $\frac{\partial \mathbf{M}_n}{\partial \lambda}$ is still a vector of moment n sensitivity for all outputs with respect to λ . Once this vector equation is proven to be true, the scalar version as shown in (4.23) can be obtained by premultiplying (4.33) by $\mathbf{d}'$.

Directly differentiating (4.30) with respect to a network parameter λ gives

$$[\mathbf{Y}]_0 \frac{\partial \mathbf{M}_n}{\partial \lambda} = - \sum_{k=1}^n \frac{1}{k!} \left(\frac{\partial [\mathbf{Y}]_0^{(k)}}{\partial \lambda} \mathbf{M}_{n-k} + [\mathbf{Y}]_0^{(k)} \frac{\partial \mathbf{M}_{n-k}}{\partial \lambda} \right) - \frac{\partial [\mathbf{Y}]_0}{\partial \lambda} \mathbf{M}_n \quad (4.34)$$

$$= - \sum_{k=0}^n \frac{1}{k!} \frac{\partial [\mathbf{Y}]_0^{(k)}}{\partial \lambda} \mathbf{M}_{n-k} - \sum_{k=1}^n \frac{1}{k!} [\mathbf{Y}]_0^{(k)} \frac{\partial \mathbf{M}_{n-k}}{\partial \lambda} \quad (4.35)$$

$$\frac{\partial \mathbf{M}_n}{\partial \lambda} = - [\mathbf{Y}]_0^{-1} \sum_{k=0}^n \frac{1}{k!} \frac{\partial [\mathbf{Y}]_0^{(k)}}{\partial \lambda} \mathbf{M}_{n-k} - [\mathbf{Y}]_0^{-1} \sum_{k=1}^n \frac{1}{k!} [\mathbf{Y}]_0^{(k)} \frac{\partial \mathbf{M}_{n-k}}{\partial \lambda} \quad (4.36)$$

The sensitivity equation in (4.33) can be proven to be essentially the same as (4.36). i.e.

$$- [\mathbf{Y}]_0^{-1} \sum_{k=0}^n \frac{1}{k!} \frac{\partial [\mathbf{Y}]_0^{(k)}}{\partial \lambda} \mathbf{M}_{n-k} - [\mathbf{Y}]_0^{-1} \sum_{k=1}^n \frac{1}{k!} [\mathbf{Y}]_0^{(k)} \frac{\partial \mathbf{M}_{n-k}}{\partial \lambda} = - \sum_{k=0}^n \tilde{\mathbf{M}}_k' \sum_{r=0}^{n-k} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} \mathbf{M}_{n-k-r} \quad (4.37)$$

4.4.1 Proof for the Moments Sensitivity Equation (4.33)

The left and the right hand sides of (4.37) are represented by *L.H.S.* and *R.H.S.* respectively.

Step 1: For $n = 0$

$$R.H.S. = -\tilde{M}'_0 \frac{\partial [Y]_0}{\partial \lambda} M_0 \quad (4.38)$$

$$L.H.S. = -[Y]_0^{-1} \frac{\partial [Y]_0}{\partial \lambda} M_0 \quad (4.39)$$

Substituting (4.31) into (4.39) for $[Y]_0^{-1}$ in terms of $\tilde{M}'_0$

$$L.H.S. = -\tilde{M}'_0 \frac{\partial [Y]_0}{\partial \lambda} M_0 = R.H.S. \quad (4.40)$$

Therefore, (4.33) is true for $n = 0$.

Step 2: Assume that (4.33) is true for $n \leq p$ where p is any integer greater than or equal to zero.

Step 3: When $n = p + 1$

$$R.H.S. = -\sum_{k=0}^{p+1} \tilde{M}'_k \sum_{r=0}^{p+1-k} \frac{1}{r!} \frac{\partial [Y]_0^{(r)}}{\partial \lambda} M_{p+1-k-r} \quad (4.41)$$

$$= -\tilde{M}'_0 \sum_{r=0}^{p+1} \frac{1}{r!} \frac{\partial [Y]_0^{(r)}}{\partial \lambda} M_{p+1-r} - \sum_{k=1}^{p+1} \tilde{M}'_k \sum_{r=0}^{p+1-k} \frac{1}{r!} \frac{\partial [Y]_0^{(r)}}{\partial \lambda} M_{p+1-k-r} \quad (4.42)$$

$$L.H.S. = -[Y]_0^{-1} \sum_{k=0}^{p+1} \frac{1}{k!} \frac{\partial [Y]_0^{(k)}}{\partial \lambda} M_{p+1-k} - [Y]_0^{-1} \sum_{k=1}^{p+1} \frac{1}{k!} [Y]_0^{(k)} \frac{\partial M_{p+1-k}}{\partial \lambda} \quad (4.43)$$

$$= -\tilde{M}'_0 \sum_{k=0}^{p+1} \frac{1}{k!} \frac{\partial [Y]_0^{(k)}}{\partial \lambda} M_{p+1-k} - [Y]_0^{-1} \sum_{k=1}^{p+1} \frac{1}{k!} [Y]_0^{(k)} \frac{\partial M_{p+1-k}}{\partial \lambda} \quad (4.44)$$

By inspection, the first term in (4.42) and (4.44) are identical. Therefore, what is left to be proven becomes

$$-[\mathbf{Y}]_0^{-1} \sum_{k=1}^{p+1} \frac{1}{k!} [\mathbf{Y}]_0^{(k)} \frac{\partial M_{p+1-k}}{\partial \lambda} = - \sum_{k=1}^{p+1} \tilde{\mathbf{M}}_k' \sum_{r=0}^{p+1-k} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} M_{p+1-k-r} \quad (4.45)$$

Under the assumption in Step 2, $\frac{\partial M_{p+1-k}}{\partial \lambda}$ in (4.45) can be replaced by (4.33) for $k=1, 2, \dots, p+1$. Therefore, the left hand side of (4.45) becomes

$$[\mathbf{Y}]_0^{-1} \sum_{k=1}^{p+1} \frac{1}{k!} [\mathbf{Y}]_0^{(k)} \sum_{i=0}^{p+1-k} \tilde{\mathbf{M}}_i' \sum_{r=0}^{p+1-k-i} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} M_{p+1-k-i-r} \quad (4.46)$$

$$= \sum_{k=1}^{p+1} \sum_{i=0}^{p+1-k} \frac{1}{k!} [\mathbf{Y}]_0^{-1} [\mathbf{Y}]_0^{(k)} \tilde{\mathbf{M}}_i' \sum_{r=0}^{p+1-k-i} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} M_{p+1-k-i-r} \quad (4.47)$$

The double summation in (4.47) can be visualized graphically as shown in Figure 4.1. The terms in the $k-i$ plane are summed in the direction indicated by the arrows as k increases from 1 to $p+1$. The same result can be achieved by summing the terms in this $k-i$ plane in a different fashion as shown in Figure 4.2. The terms are summed along the arrows as i moves from 0 to p . With the transformation

$$k = \tau \quad (4.48)$$

$$i = \sigma - \tau \quad (4.49)$$

the terms in the $k-i$ plane can be mapped into a new $\sigma-\tau$ plane as shown in Figure 4.3. Several terms are marked on Figure 4.2 and Figure 4.3 to show the actual transformation. By grouping the terms in the $\sigma-\tau$ plane along the σ -axis as σ increases from 1 to $p+1$, the same result as in (4.47) can be obtained. Therefore (4.47) becomes

$$\sum_{\tau=1}^{p+1} \left(\sum_{\tau=1}^{\sigma} \frac{1}{\tau!} [\mathbf{Y}]_0^{-1} [\mathbf{Y}]_0^{(\tau)} \tilde{\mathbf{M}}_{\sigma-\tau}' \right) \sum_{r=0}^{p+1-\sigma} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} \mathbf{M}_{p+1-\sigma-r} \quad (4.50)$$

With the following intermediate equation (to be proven later)

$$\tilde{\mathbf{M}}_n' = - \sum_{k=1}^n \frac{1}{k!} \tilde{\mathbf{M}}_{n-k}' [\mathbf{Y}]_0^{(k)} [\mathbf{Y}]_0^{-1} = - \sum_{k=1}^n \frac{1}{k!} [\mathbf{Y}]_0^{-1} [\mathbf{Y}]_0^{(k)} \tilde{\mathbf{M}}_{n-k}' \quad (4.51)$$

(4.50) becomes

$$- \sum_{\sigma=1}^{p+1} \tilde{\mathbf{M}}_{\sigma}' \sum_{r=0}^{p+1-\sigma} \frac{1}{r!} \frac{\partial [\mathbf{Y}]_0^{(r)}}{\partial \lambda} \mathbf{M}_{p+1-\sigma-r} = R.H.S. \text{ of (4.45)} \quad (4.52)$$

Therefore, the adjoint equation is valid if (4.51) can be proven to be true.

Figure 4.1: Graphical interpretation of the double summation in (4.47).

Figure 4.2: A different way of adding the terms in the k - i plane.

Figure 4.3: The terms represented in the new σ - τ plane.

4.4.2 Proof for the Intermediate Equation (4.51)

In the following equations, *L.H.S.* and *R.H.S.* are used to represent the left and the right hand sides of (4.51).

Step 1: For $n = 1$

$$L.H.S. = -\tilde{M}'_0 [Y]_0^{(1)} [Y]_0^{-1} \quad (4.53)$$

Substituting (4.31) into (4.53), it can be rewritten as

$$L.H.S. = -[Y]_0^{-1} [Y]_0^{(1)} \tilde{M}'_0 = R.H.S. \quad (4.54)$$

Therefore, (4.51) is true for $n = 1$.

Step 2: Assume that (4.51) is true for $n \leq p$ where p is any integer greater than or equal to one.

Step 3: For $n = p + 1$

$$L.H.S. = -\sum_{k=1}^{p+1} \frac{1}{k!} \tilde{M}'_{p+1-k} [Y]_0^{(k)} [Y]_0^{-1} \quad (4.55)$$

$$= -\frac{1}{(p+1)!} \tilde{M}'_0 [Y]_0^{(p+1)} [Y]_0^{-1} - \sum_{k=1}^p \frac{1}{k!} \tilde{M}'_{p+1-k} [Y]_0^{(k)} [Y]_0^{-1} \quad (4.56)$$

Since $\tilde{M}'_n$ is assumed to be $-\sum_{k=1}^n \frac{1}{k!} [Y]_0^{-1} [Y]_0^{(k)} \tilde{M}'_{n-k}$ for $n \leq p$ under the assumption made in Step 2, (4.56) becomes

$$\begin{aligned} & -\frac{1}{(p+1)!} \tilde{M}'_0 [Y]_0^{(p+1)} [Y]_0^{-1} + \\ & \sum_{k=1}^p \frac{1}{k!} \sum_{r=1}^{p+1-k} \frac{1}{r!} [Y]_0^{-1} [Y]_0^{(r)} \tilde{M}'_{p+1-k-r} [Y]_0^{(k)} [Y]_0^{-1} \end{aligned} \quad (4.57)$$

Figure 4.4 represents the original sequence of summation for the second term in

(4.57). Each term is summed together along the arrows as k increases from 1 to p

By summing these terms in a different way as represented by Figure 4.5, (4.57)

can be rearranged as

$$\begin{aligned}
 & -\frac{1}{(p+1)!} \tilde{M}'_0 [Y]_0^{(p+1)} [Y]_0^{-1} + \\
 & \sum_{r=1}^p \frac{1}{r!} [Y]_0^{-1} [Y]_0^{(r)} \sum_{k=1}^{p+1-r} \frac{1}{k!} \tilde{M}'_{p+1-k-r} [Y]_0^{(k)} [Y]_0^{-1}
 \end{aligned} \tag{4.58}$$

$$= -\frac{1}{(p+1)!} [Y]_0^{-1} [Y]_0^{(p+1)} \tilde{M}'_0 - \sum_{r=1}^p \frac{1}{r!} [Y]_0^{-1} [Y]_0^{(r)} \tilde{M}'_{p+1-r} \tag{4.59}$$

$$= -\sum_{r=1}^{p+1} \frac{1}{r!} [Y]_0^{-1} [Y]_0^{(r)} \tilde{M}'_{p+1-r} = R.H.S. \tag{4.60}$$

Therefore, by the *Second Principle of Induction*, (4.37), (4.33) and thus (4.23) are all true for any integer n greater than or equal to zero.

In the following sections, the actual calculation of $[Y]^{(n)}$ and its sensitivity as required in (4.23) for transmission line networks will be considered.

Figure 4.4: Original sequence of summation for the second term in (4.57).

Figure 4.5: New sequence of summation used in the second term of (4.58).

4.5 The Multiconductor Transmission Line System

Consider a multiconductor transmission line system with N signal conductors and a ground wire as shown in Figure 4.6. This multiconductor transmission line can be treated as a $2N$ port network with a MNA representation of the form

$$AV + BI = 0 \quad (4.61)$$

where A and B are $2N \times 2N$ complex matrices that can be described in terms of the line parameters;

V is a $2N \times 1$ vector of the terminal voltages;

I is a $2N \times 1$ vector of the terminal currents.

It is assumed that the transmission line is uniform along its length, D , and the wave propagation along the transmission line is of a quasi-TEM nature. Hence, the transmission line behavior can be completely described in terms of its $N \times N$ parameter matrices: the resistance per unit length, R , due to conductor losses; the inductance per unit length, L , due to the electromagnetic induction; the conductance per unit length, G , due to dielectric losses; and the capacitance coefficients per unit length, C , due to the electrostatic effects. Note that these matrices may contain frequency dependent elements (e.g., due to *skin effect* or *dielectric dispersion*) although they are often taken to be constants.

4.5.1 The Multiconductor Transmission Line Stamp

Define the line impedances matrix, Z_p , and the line admittances matrix, Y_p , as

$$Z_p = R + sL \quad (4.62)$$

$$Y_p = G + sC \quad (4.63)$$

Figure 4.6: A multiconductor transmission line.

The voltages $V(x)$ and currents $I(x)$ along a transmission line for TEM mode propagation are related by the well-known Telegrapher equation

$$\frac{d[V(x)]}{dx} = -[Z_p][I(x)] \quad (4.64)$$

$$\frac{d[I(x)]}{dx} = -[Y_p][V(x)] \quad (4.65)$$

These two differential equations can be combined into a set of wave equations

$$\frac{d^2[V(x)]}{dx^2} = [Z_p][Y_p][V(x)] \quad (4.66)$$

$$\frac{d^2[I(x)]}{dx^2} = [Y_p][Z_p][I(x)] \quad (4.67)$$

that have solution of the form

$$V_j(x) = V_j(0) e^{\pm \gamma_j x} \quad \text{for } j=1, 2, \dots, N \quad (4.68)$$

$$I_j(x) = I_j(0) e^{\pm \gamma_j x} \quad \text{for } j=1, 2, \dots, N \quad (4.69)$$

where γ_j is the complex propagation mode constant. The positive and negative signs of (4.68) and (4.69) represent the backward wave and the forward wave respectively.

Substituting (4.68) back into the wave equation (4.66) gives

$$\{\gamma_j^2 U - Z_p Y_p\} V_j(0) = 0 \quad (4.70)$$

which will have non-trivial solutions if γ_j^2 satisfies the eigenvalue problem

$$\det \{\gamma_j^2 U - Z_p Y_p\} = 0 \quad (4.71)$$

In general, there exists N distinct eigenvalues γ_j^2 , for $j=1, 2, \dots, N$. Each eigenvalue has its corresponding eigenvector S_{γ_j} . For

$$S_v = \{S_{v_1}, \dots, S_{v_N}\} \quad (4.72)$$

the solution for the voltages can be written as

$$V(x) = S_v[E(x)]C_1 + S_v[E(x)]^{-1}C_2 \quad (4.73)$$

where $E(x)$ is a diagonal matrix whose elements are $e^{-\gamma_j x}$, for $j=1, 2, \dots, N$;

C_1 and C_2 are constant N dimensional vector coefficients.

From (4.73) and (4.64), we get

$$I(x) = S_i[E(x)]C_1 - S_i[E(x)]^{-1}C_2 \quad (4.74)$$

where

$$S_i = Z_p^{-1}S_v\Gamma \quad (4.75)$$

$$\Gamma = \text{diagonal} \{\gamma_j\} \quad \text{for } j=1, 2, \dots, N. \quad (4.76)$$

A relationship between the near and far end voltages can be obtained from (4.73) as

$$\begin{bmatrix} V(0) \\ V(D) \end{bmatrix} = \begin{bmatrix} S_v & S_v \\ S_v E(D) & S_v[E(D)]^{-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (4.77)$$

Similarly, a relationship between the near and far end currents from (4.74) can be written as

$$\begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} S_i & -S_i \\ S_i E(D) & -S_i[E(D)]^{-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (4.78)$$

Combining (4.77) and (4.78) gives

$$\begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} S_i & -S_i \\ S_i E(D) & -S_i[E(D)]^{-1} \end{bmatrix} \begin{bmatrix} S_v & S_v \\ S_v E(D) & S_v[E(D)]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} V(0) \\ V(D) \end{bmatrix} \quad (4.79)$$

To get the correct sense as required for the MNA representation as shown in Figure 4.6, $I(D)$ has to be multiplied by -1. After some algebraic manipulation, (4.79) can be written in the form of

$$AV + BI = 0 \quad (4.80)$$

where

$$A = \begin{bmatrix} S_v E_1 S_v^{-1} & -U \\ S_i E_2 S_v^{-1} & 0 \end{bmatrix} \quad (4.81)$$

$$B = \begin{bmatrix} S_v E_2 S_i^{-1} & 0 \\ S_i E_1 S_i^{-1} & -U \end{bmatrix} \quad (4.82)$$

$$E_1 = \text{diagonal} \left\{ \frac{e^{-\gamma_j D} + e^{\gamma_j D}}{2} \right\} \quad \text{for } j=1, 2, \dots, N \quad (4.83)$$

$$E_2 = \text{diagonal} \left\{ \frac{e^{-\gamma_j D} - e^{\gamma_j D}}{2} \right\} \quad \text{for } j=1, 2, \dots, N \quad (4.84)$$

$$V = \begin{bmatrix} V(0) \\ V(D) \end{bmatrix} = \begin{bmatrix} V_{near} \\ V_{far} \end{bmatrix} \quad (4.85)$$

$$I = \begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} I_{near} \\ I_{far} \end{bmatrix} \quad (4.86)$$

4.5.2 The Transmission Line Moments and Moments Sensitivity

As discussed previously, in order to find the moments and moments sensitivity of a transmission line network, we need the transmission line stamp derivatives and their corresponding sensitivities. From (4.81) and (4.82), the transmission line MNA stamp

sensitivities are

$$\frac{\partial A}{\partial \lambda} = \begin{bmatrix} \frac{\partial}{\partial \lambda}(S_v E_1 S_v^{-1}) & 0 \\ \frac{\partial}{\partial \lambda}(S_i E_2 S_i^{-1}) & 0 \end{bmatrix} \quad (4.87)$$

$$\frac{\partial B}{\partial \lambda} = \begin{bmatrix} \frac{\partial}{\partial \lambda}(S_v E_2 S_i^{-1}) & 0 \\ \frac{\partial}{\partial \lambda}(S_i E_1 S_i^{-1}) & 0 \end{bmatrix} \quad (4.88)$$

To evaluate the MNA stamp sensitivities for a transmission line system, each term inside (4.87) and (4.88) is defined separately as

$$S_v E_1 S_v^{-1} = \alpha_A \quad (4.89)$$

$$S_i E_2 S_i^{-1} = \beta_A \quad (4.90)$$

$$S_v E_2 S_i^{-1} = \alpha_B \quad (4.91)$$

$$S_i E_1 S_i^{-1} = \beta_B \quad (4.92)$$

The sensitivities of (4.89)-(4.92) are found by

$$\frac{\partial \alpha_A}{\partial \lambda} S_v = \frac{\partial S_v}{\partial \lambda} E_1 + S_v \frac{\partial E_1}{\partial \lambda} - \alpha_A \frac{\partial S_v}{\partial \lambda} \quad (4.93)$$

$$\frac{\partial \beta_A}{\partial \lambda} S_v = \frac{\partial S_i}{\partial \lambda} E_2 + S_i \frac{\partial E_2}{\partial \lambda} - \beta_A \frac{\partial S_v}{\partial \lambda} \quad (4.94)$$

$$\frac{\partial \alpha_B}{\partial \lambda} S_i = \frac{\partial S_v}{\partial \lambda} E_2 + S_v \frac{\partial E_2}{\partial \lambda} - \alpha_B \frac{\partial S_i}{\partial \lambda} \quad (4.95)$$

$$\frac{\partial \beta_B}{\partial \lambda} S_i = \frac{\partial S_i}{\partial \lambda} E_1 + S_i \frac{\partial E_1}{\partial \lambda} - \beta_B \frac{\partial S_i}{\partial \lambda} \quad (4.96)$$

which require the sensitivities of S_v , S_i , E_1 and E_2 .

Recall that $S_v = \{S_{v_j}\}$, for $j=1, 2, \dots, N$, is a matrix consisting of all voltage eigenvectors of a transmission line system such that it satisfies the following equation

$$(\gamma_j^2 U - Z_p Y_p) S_{v_j} = 0 \quad (4.97)$$

the corresponding sensitivity has the relationship

$$(\gamma_j^2 U - Z_p Y_p) \frac{\partial S_{v_j}}{\partial \lambda} + (2\gamma_j \frac{\partial \gamma_j}{\partial \lambda} - Z_p \frac{\partial Y_p}{\partial \lambda} - \frac{\partial Z_p}{\partial \lambda} Y_p) S_{v_j} = 0 \quad (4.98)$$

With the sensitivities of γ_j and S_{v_j} unknown, $N+1$ equations are required to obtain the solution. (4.98) represents the first N equations. The last equation used is

$$S'_{v_j} S_{v_j} = 1 \quad (4.99)$$

which normalizes the magnitude of the eigenvectors. Differentiating (4.99)

$$\frac{\partial S'_{v_j}}{\partial \lambda} S_{v_j} + S'_{v_j} \frac{\partial S_{v_j}}{\partial \lambda} = 0 \quad (4.100)$$

$$S'_{v_j} \frac{\partial S_{v_j}}{\partial \lambda} = 0 \quad (4.101)$$

Combining (4.98) and (4.101) leads to a matrix equation

$$\begin{bmatrix} \gamma_j^2 U - Z_p Y_p & 2\gamma_j S_{v_j} \\ S'_{v_j} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial S_{v_j}}{\partial \lambda} \\ \frac{\partial \gamma_j}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} (Z_p \frac{\partial Y_p}{\partial \lambda} + \frac{\partial Z_p}{\partial \lambda} Y_p) S_{v_j} \\ 0 \end{bmatrix} \quad (4.102)$$

which can be used to evaluate the sensitivities of the eigenvalues and eigenvectors one by one. For the current eigenvectors $S_i = Z_p^{-1} S_v \Gamma$, their sensitivities can be evaluated from

$$Z_p \frac{\partial S_i}{\partial \lambda} = \frac{\partial S_v}{\partial \lambda} \Gamma + S_v \frac{\partial \Gamma}{\partial \lambda} - \frac{\partial Z_p}{\partial \lambda} S_i \quad (4.103)$$

For the sensitivities of E_1 and E_2 , partial differentiation gives

$$\frac{\partial E_1}{\partial \lambda} = \text{diagonal} \left\{ \frac{-\frac{\partial \gamma_j}{\partial \lambda} D e^{-\gamma_j \rho} + \frac{\partial \gamma_j}{\partial \lambda} D e^{\gamma_j \rho}}{2} \right\} \quad (4.104)$$

$$\frac{\partial E_1}{\partial \lambda} = -D \frac{\partial \Gamma}{\partial \lambda} E_2 \quad (4.105)$$

and

$$\frac{\partial E_2}{\partial \lambda} = \text{diagonal} \left\{ \frac{-\frac{\partial \gamma_j}{\partial \lambda} D e^{-\gamma_j \rho} - \frac{\partial \gamma_j}{\partial \lambda} D e^{\gamma_j \rho}}{2} \right\} \quad (4.106)$$

$$\frac{\partial E_2}{\partial \lambda} = -D \frac{\partial \Gamma}{\partial \lambda} E_1 \quad (4.107)$$

With the sensitivities of S_v , S_i , E_1 and E_2 obtained from (4.102), (4.103), (4.105) and (4.107), the sensitivity of $[y]_0$, and thus the sensitivity of m_0 , can be found by using (4.26) and (4.21).

To complete the sensitivity analysis, successive moments sensitivity have to be evaluated. This requires the sensitivities of $[y]_0^{(n)}$ where $n = 1, 2, \dots, 2q-1$. From (4.81) and (4.82), the derivative of the transmission line MNA stamp can be written as

$$A^{(n)} = \begin{bmatrix} (S_v E_1 S_v^{-1})^{(n)} & 0 \\ (S_i E_2 S_v^{-1})^{(n)} & 0 \end{bmatrix} \quad \text{for } n > 0 \quad (4.108)$$

$$B^{(n)} = \begin{bmatrix} (S_v E_2 S_i^{-1})^{(n)} & 0 \\ (S_i E_1 S_i^{-1})^{(n)} & 0 \end{bmatrix} \quad \text{for } n > 0 \quad (4.109)$$

Once again, writing each term inside (4.108) and (4.109) separately as

$$S_v E_1 S_v^{-1} = \alpha_A \quad (4.110)$$

$$S_i E_2 S_v^{-1} = \beta_A \quad (4.111)$$

$$S_v E_2 S_i^{-1} = \alpha_B \quad (4.112)$$

$$S_i E_1 S_i^{-1} = \beta_B \quad (4.113)$$

and applying *Leibnitz's Rule* to (4.110)-(4.113) yields

$$\alpha_A^{(n)} S_v = \sum_{r=0}^n \binom{n}{r} S_v^{(n-r)} E_1^{(r)} + \sum_{r=1}^n \binom{n}{r} \alpha_A^{(n-r)} S_v^{(r)} \quad (4.114)$$

$$\beta_A^{(n)} S_v = \sum_{r=0}^n \binom{n}{r} S_i^{(n-r)} E_2^{(r)} + \sum_{r=1}^n \binom{n}{r} \beta_A^{(n-r)} S_v^{(r)} \quad (4.115)$$

$$\alpha_B^{(n)} S_i = \sum_{r=0}^n \binom{n}{r} S_v^{(n-r)} E_2^{(r)} + \sum_{r=1}^n \binom{n}{r} \alpha_B^{(n-r)} S_i^{(r)} \quad (4.116)$$

$$\beta_B^{(n)} S_i = \sum_{r=0}^n \binom{n}{r} S_i^{(n-r)} E_1^{(r)} + \sum_{r=1}^n \binom{n}{r} \beta_B^{(n-r)} S_i^{(r)} \quad (4.117)$$

Differentiating (4.114)-(4.117) with respect to λ

$$\begin{aligned} \frac{\partial \alpha_A^{(n)}}{\partial \lambda} S_v &= \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial S_v^{(n-r)}}{\partial \lambda} E_1^{(r)} + S_v^{(n-r)} \frac{\partial E_1^{(r)}}{\partial \lambda} \right] - \\ &\quad \sum_{r=1}^n \binom{n}{r} \left[\frac{\partial \alpha_A^{(n-r)}}{\partial \lambda} S_v^{(r)} + \alpha_A^{(n-r)} \frac{\partial S_v^{(r)}}{\partial \lambda} \right] - \alpha_A^{(n)} \frac{\partial S_v}{\partial \lambda} \end{aligned} \quad (4.118)$$

$$\begin{aligned} \frac{\partial \beta_A^{(n)}}{\partial \lambda} S_v &= \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial S_i^{(n-r)}}{\partial \lambda} E_2^{(r)} + S_i^{(n-r)} \frac{\partial E_2^{(r)}}{\partial \lambda} \right] - \\ &\quad \sum_{r=1}^n \binom{n}{r} \left[\frac{\partial \beta_A^{(n-r)}}{\partial \lambda} S_v^{(r)} + \beta_A^{(n-r)} \frac{\partial S_v^{(r)}}{\partial \lambda} \right] - \beta_A^{(n)} \frac{\partial S_v}{\partial \lambda} \end{aligned} \quad (4.119)$$

$$\begin{aligned} \frac{\partial \alpha_B^{(n)}}{\partial \lambda} S_i &= \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial S_v^{(n-r)}}{\partial \lambda} E_2^{(r)} + S_v^{(n-r)} \frac{\partial E_2^{(r)}}{\partial \lambda} \right] - \\ &\quad \sum_{r=1}^n \binom{n}{r} \left[\frac{\partial \alpha_B^{(n-r)}}{\partial \lambda} S_i^{(r)} + \alpha_B^{(n-r)} \frac{\partial S_i^{(r)}}{\partial \lambda} \right] - \alpha_B^{(n)} \frac{\partial S_i}{\partial \lambda} \end{aligned} \quad (4.120)$$

$$\begin{aligned} \frac{\partial \beta_B^{(n)}}{\partial \lambda} S_i &= \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial S_i^{(n-r)}}{\partial \lambda} E_1^{(r)} + S_i^{(n-r)} \frac{\partial E_1^{(r)}}{\partial \lambda} \right] - \\ &\quad \sum_{r=1}^n \binom{n}{r} \left[\frac{\partial \beta_B^{(n-r)}}{\partial \lambda} S_i^{(r)} + \beta_B^{(n-r)} \frac{\partial S_i^{(r)}}{\partial \lambda} \right] - \beta_B^{(n)} \frac{\partial S_i}{\partial \lambda} \end{aligned} \quad (4.121)$$

which require the evaluation of $\frac{\partial S_v^{(n)}}{\partial \lambda}$, $\frac{\partial S_i^{(n)}}{\partial \lambda}$, $\frac{\partial E_1^{(n)}}{\partial \lambda}$ and $\frac{\partial E_2^{(n)}}{\partial \lambda}$.

$$\frac{\partial S_v^{(n)}}{\partial \lambda} = \left[\frac{\partial S_{v_j}^{(n)}}{\partial \lambda} \right] \quad \text{for } j=1, 2, \dots, N \quad (4.122)$$

Taking the first derivative of (4.97) and (4.99) with respect to s and following the approach used in (4.97)-(4.102), the following matrix equation can be derived

$$\begin{bmatrix} \gamma_j^2 U - Z_p Y_p & 2\gamma_j S_{v_j} \\ S_{v_j}' & 0 \end{bmatrix} \begin{bmatrix} S_{v_j}^{(1)} \\ \gamma_j^{(1)} \end{bmatrix} = \begin{bmatrix} (Z_p Y_p)^{(1)} S_{v_j} \\ 0 \end{bmatrix} \quad (4.123)$$

which is expressed symbolically as

$$T\Phi^{(1)} = \begin{bmatrix} (Z_p Y_p)^{(1)} S_{v_j} \\ 0 \end{bmatrix} \quad (4.124)$$

Differentiating this equation with respect to s for n times

$$T\Phi^{(n+1)} = \begin{bmatrix} \sum_{r=0}^n \binom{n}{r} [Z_p Y_p]^{(n-r+1)} S_v^{(r)} \\ 0 \end{bmatrix} - \sum_{r=0}^{n-1} \binom{n}{r} T^{(n-r)} \Phi^{(r+1)} \quad (4.125)$$

Taking partial derivative of the above equation with respect to λ

$$T \frac{\partial \Phi^{(n+1)}}{\partial \lambda} = -\frac{\partial T}{\partial \lambda} \Phi^{(n+1)} - \sum_{r=0}^{n-1} \binom{n}{r} \left[\frac{\partial T^{(n-r)}}{\partial \lambda} \Phi^{(r+1)} + T^{(n-r)} \frac{\partial \Phi^{(r+1)}}{\partial \lambda} \right] +$$

$$\begin{bmatrix} \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial [Z_p Y_p]^{(n-r+1)}}{\partial \lambda} S_v^{(r)} + [Z_p Y_p]^{(n-r+1)} \frac{\partial S_v^{(r)}}{\partial \lambda} \right] \\ 0 \end{bmatrix} \quad (4.126)$$

With $\frac{\partial S_v}{\partial \lambda}$ and $\frac{\partial \Gamma}{\partial \lambda}$ from (4.102) as a starting point, $\frac{\partial S_v^{(n)}}{\partial \lambda}$ and $\frac{\partial \Gamma^{(n)}}{\partial \lambda}$ may be evaluated recursively from (4.126).

For the sensitivity of $S_i^{(n)}$, write

$$S_i = Z_p^{-1} S_v \Gamma \quad (4.127)$$

$$Z_p S_i = S_v \Gamma \quad (4.128)$$

and apply *Leibnitz's Rule*

$$\sum_{r=0}^n \binom{n}{r} Z_p^{(n-r)} S_i^{(r)} = \sum_{r=0}^n \binom{n}{r} S_v^{(n-r)} \Gamma^{(r)} \quad (4.129)$$

$$Z_p S_i^{(n)} = \sum_{r=0}^n \binom{n}{r} S_v^{(n-r)} \Gamma^{(r)} - \sum_{r=0}^{n-1} \binom{n}{r} Z_p^{(n-r)} S_i^{(r)} \quad (4.130)$$

Differentiating (4.130) with respect to λ yields

$$\begin{aligned}
 Z_p \frac{\partial S_i^{(n)}}{\partial \lambda} = & -\frac{\partial Z_p}{\partial \lambda} S_i^{(n)} + \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial S_v^{(n-r)}}{\partial \lambda} \Gamma^{(r)} + S_v^{(n-r)} \frac{\partial \Gamma^{(r)}}{\partial \lambda} \right] - \\
 & \sum_{r=0}^{n-1} \binom{n}{r} \left[\frac{\partial Z_p^{(n-r)}}{\partial \lambda} S_i^{(r)} + Z_p^{(n-r)} \frac{\partial S_i^{(r)}}{\partial \lambda} \right]
 \end{aligned} \quad (4.131)$$

Now, only the sensitivities of $E_1^{(n)}$ and $E_2^{(n)}$ are left to be found.

$$E_1 = \text{diagonal} \left\{ \frac{e^{-\gamma_j D} + e^{\gamma_j D}}{2} \right\} \quad (4.132)$$

$$E_1^{(1)} = \text{diagonal} \left\{ \frac{-\frac{\partial \gamma_j}{\partial s} D e^{-\gamma_j D} + \frac{\partial \gamma_j}{\partial s} D e^{\gamma_j D}}{2} \right\} \quad (4.133)$$

$$= -\text{diagonal} \left\{ \frac{\partial \gamma_j}{\partial s} D \right\} \text{diagonal} \left\{ \frac{e^{-\gamma_j D} - e^{\gamma_j D}}{2} \right\} \quad (4.134)$$

$$= -D \frac{\partial \Gamma}{\partial s} E_2 \quad (4.135)$$

which gives the following relationship when *Leibnitz's Rule* is applied

$$E_1^{(n+1)} = -[D \Gamma^{(1)} E_2]^{(n)} \quad (4.136)$$

$$= -D \sum_{r=0}^n \binom{n}{r} \Gamma^{(r-r+1)} E_2^{(r)} \quad (4.137)$$

Differentiating the above equation with respect to λ

$$\frac{\partial E_1^{(n+1)}}{\partial \lambda} = -D \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial \Gamma^{(n-r+1)}}{\partial \lambda} E_2^{(r)} + \Gamma^{(n-r+1)} \frac{\partial E_2^{(r)}}{\partial \lambda} \right] \quad (4.138)$$

Similarly, the sensitivity of $E_2^{(n)}$ can be found as

$$E_2^{(1)} = -D \frac{\partial \Gamma}{\partial s} E_1 \quad (4.139)$$

$$E_2^{(n+1)} = -D \sum_{r=0}^n \binom{n}{r} \Gamma^{(n-r+1)} E_1^{(r)} \quad (4.140)$$

$$\frac{\partial E_2^{(n+1)}}{\partial \lambda} = -D \sum_{r=0}^n \binom{n}{r} \left[\frac{\partial \Gamma^{(n-r+1)}}{\partial \lambda} E_1^{(r)} + \Gamma^{(n-r+1)} \frac{\partial E_1^{(r)}}{\partial \lambda} \right] \quad (4.141)$$

Although the above equations may look tedious to derive and evaluate, only minimal CPU cost is required in the actual calculation. Since the left hand side square matrices have all been L/U factored during the transient analysis, only forward/backward substitution is required to get the sensitivities of $[\bar{y}]_0^{(n)}$. For larger circuits, this becomes advantageous because of the smaller relative CPU cost of forward/backward substitution with respect to a L/U factorization.

Note that the MNA stamp of a transmission line can serve as a macro model of a transmission line system. In this case, the eigensystem would have to be solved only once for each class of transmission line system since any two transmission lines belonging to the same class would have identical R , G , L and C matrices. For some special cases, the above equations can be simplified. These cases will be investigated in the next section.

4.6 Special Cases

4.6.1 Lossless Multiconductor Transmission Lines

In certain applications, the losses in interconnection lines do not significantly affect the signal propagation. For example, in GaAs technology [33], the separation between interconnects is about a few micrometers and the distance between the lines and the

ground plane is about several hundred micrometers; therefore, the electromagnetic coupling between adjacent lines becomes very important. Moreover, because of the short interconnection length, the losses in the interconnects do not significantly affect the signal slew rate and also play a negligible part on the induced signal to the inactivated lines. Thus it is possible in the first approximation to consider the lines as lossless multiconductor transmission lines.

With $R = 0$ and $G = 0$, (4.97) and (4.102) become

$$(\gamma_j^2 U - s^2 LC) S_{v_j} = 0 \quad (4.142)$$

$$\begin{bmatrix} \gamma_j^2 U - s^2 LC & 2\gamma_j S_{v_j} \\ S_{v_j}' & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial S_{v_j}}{\partial \lambda} \\ \frac{\partial \gamma_j}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} s^2 (L \frac{\partial C}{\partial \lambda} + \frac{\partial L}{\partial \lambda} C) S_{v_j} \\ 0 \end{bmatrix} \quad (4.143)$$

It is impossible to find the eigenvalues of the $s^2 LC$ matrix at $s=0$. Instead, the eigenvalues of the matrix LC are found first. From the basis of eigenvalues, if ϕ is an eigenvalue of matrix A , $k\phi$ is also an eigenvalue of matrix kA . In this case, the Laplace operator s is taken as the constant k . Again, from the basis of eigenvectors, we know that the eigenvectors S_{v_j} are not effected in either case.

4.6.2 Sensitivity w.r.t. Linear Lumped Components

In the case when we are only interested in the sensitivities with respect to linear lumped components, the moments sensitivity equation can be simplified since the sensitivities of the transmission line MNA stamps become zero.

A. When the design variable is a resistor or a conductor

$$\frac{\partial [Y]_0}{\partial \lambda} = \begin{bmatrix} \frac{\partial H}{\partial \lambda} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}_{s=0} \quad (4.144)$$

$$\frac{\partial [Y]_0^{(n)}}{\partial \lambda} = 0 \quad \text{for } n \geq 1 \quad (4.145)$$

Hence, the moments sensitivity equations can be modified as

$$\frac{\partial m_0}{\partial \lambda} = -\tilde{M}'_0 \frac{\partial [Y]_0}{\partial \lambda} M_0 \quad (4.146)$$

$$\frac{\partial m_n}{\partial \lambda} = -\sum_{k=0}^n \tilde{M}'_k \frac{\partial [Y]_0}{\partial \lambda} M_{n-k} \quad \text{for } n \geq 1 \quad (4.147)$$

B. When the design variable is an inductor or a capacitor

$$\frac{\partial [Y]_0^{(1)}}{\partial \lambda} = \begin{bmatrix} \frac{\partial W}{\partial \lambda} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}_{s=0} \quad (4.148)$$

$$\frac{\partial [Y]_0^{(n)}}{\partial \lambda} = 0 \quad \text{for } n \neq 1 \quad (4.149)$$

Again, the moments sensitivity equations can be modified as

$$\frac{\partial m_0}{\partial \lambda} = 0 \quad (4.150)$$

$$\frac{\partial m_n}{\partial \lambda} = - \sum_{k=0}^{n-1} \tilde{M}'_k \frac{\partial [Y]_0^{(1)}}{\partial \lambda} M_{n-k-1} \quad \text{for } n \geq 1 \quad (4.151)$$

4.6.3 Sensitivity w.r.t. Transmission Line Length

When $\lambda=D$, the length of a transmission line system, the transmission line MNA stamp sensitivity can be simplified since the impedances matrix Z_p and the admittances matrix Y_p do not depend on D . Hence, the sensitivities of the eigenvalues and eigenvectors with respect to D become zero. Thus, the transmission line MNA stamp sensitivity can be rewritten as

$$\frac{\partial A^{(n)}}{\partial D} = \begin{bmatrix} S_v \frac{\partial E_1^{(n)}}{\partial D} S_v^{-1} & 0 \\ S_i \frac{\partial E_2^{(n)}}{\partial D} S_v^{-1} & 0 \end{bmatrix} \quad (4.152)$$

$$\frac{\partial B^{(n)}}{\partial D} = \begin{bmatrix} S_v \frac{\partial E_2^{(n)}}{\partial D} S_i^{-1} & 0 \\ S_i \frac{\partial E_1^{(n)}}{\partial D} S_i^{-1} & 0 \end{bmatrix} \quad (4.153)$$

4.7 Summary of Computational Steps

The procedures for time domain simulation and sensitivity evaluation of distributed coupled transmission line networks based on GAWE can be summarized as follows:

- /* MNA matrix preparation */
- Formulate the MNA matrix Y where $YX = E$, as shown in (3.4);

- L/U decompose $[Y]_0$;
- /* Evaluation of M_n , $\tilde{M}_n$ and $\frac{\partial m_n}{\partial \lambda}$ for $n=0, 1, \dots, 2q-1$. */
- For $n=0$ to $2q-1$
 - If $n=0$ then
 - /* Calculation of moment zero and adjoint moment zero */
 - Solve (3.7) for moment zero, M_0 ;
 - For $V_Index=1$ to Num_of_Vout
 - Define d and solve (4.20) for adjoint moment zero, $\tilde{M}_0$;
 - Next V_Index ;
 - Else
 - Compute $[Y]_0^{(n)}$ by (3.9) and (3.10);
 - /* Calculation of successive moments and adjoint moments */
 - Solve (3.8) by forward/backward substitution to obtain M_n ;
 - For $V_Index=1$ to Num_of_Vout
 - Solve (4.22) by forward/backward substitution to get $\tilde{M}_n$;
 - Next V_Index ;
 - End If;
 - /* Moment n sensitivity for all outputs w.r.t. all variables */
 - For $Var_Index=1$ to $Num_of_Variables$
 - Compute $\frac{[Y]_0^{(n)}}{\partial \lambda}$ using (4.26)-(4.28);
 - Evaluate ξ_n from (4.24);

- For $V_Index=1$ to Num_of_Vout
- Solve (4.21) and (4.23) for $\frac{\partial m_n}{\partial \lambda}$ of each output;
- Next V_Index ;
- Next Var_Index ;
- Next n ;
- For $V_Index=1$ to Num_of_Vout
- /* Evaluation of the transient response */
- With m_n , find a_i and p_j by (2.10) and a root finding subroutine;
- Solve for k_j by using (2.11);
- Compute $[x^*(t)]$ for output V_Index by (2.12);
- /* Evaluation of the time domain sensitivity */
- For $Var_Index=1$ to $Num_of_Variables$
- With $\frac{\partial m_n}{\partial \lambda}$, find $\frac{\partial a_i}{\partial \lambda}$ and $\frac{\partial p_j}{\partial \lambda}$ by (4.2) and (4.1);
- Find $\frac{\partial k_j}{\partial \lambda}$ from (4.3);
- Finally, $\frac{\partial [x^*(t)]}{\partial \lambda}$ for output V_Index can be found by (4.4);
- Next Var_Index ;
- Next V_Index ;

4.8 Results and Comparisons

In this section, the results obtained from dGAWE is compared with the sensitivities obtained by perturbing the design variable λ . To highlight the transmission line effects, we have kept the input signal rise time in these examples to be much smaller than the time delay introduced by the transmission lines. The sensitivity plots are normalized to represent mV change per 1 percent change in λ . It should be mentioned that the perturbation technique is computationally expensive since it requires an extra time domain analysis for each design variable. Note that for the examples analyzed in this section, a simple moment matching technique is used for poles and residues extraction. However, various techniques such as [30] and [31] can also be used.

Example 4.1

An interconnect circuit consists of terminal networks and seven single lossless transmission lines is shown in Figure 4.7. The applied voltage is a pulse with 10ps rise/fall time, 5ns pulse width and 5V amplitude as shown in Figure 4.8. The transient response at V_{out} from GAWE is compared with HSPICE response in Figure 4.9. The sensitivities with respect to various parameters are shown in Figure 4.10 - Figure 4.13 which demonstrate excellent agreement with the results obtained by perturbation using HSPICE.

Figure 4.7: Circuit for Example 4.1. Transmission lines with terminal networks.

Figure 4.8: Input waveform used in Example 4.1.

Figure 4.9: Transient response at V_{out} of Example 4.1.

Figure 4.10: Sensitivity of voltage V_{ou} w.r.t. R_s in Example 4.1.

Figure 4.11: Sensitivity of V_{out} w.r.t. L_1 in Example 4.1.

Figure 4.12: Sensitivity of V_{ou} w.r.t. C of transmission line #2 in Example 4.1.

Figure 4.13: Sensitivity of V_{out} w.r.t. the length of transmission line #1 in Example 4.1.

Example 4.2

To demonstrate the efficiency of the proposed method, the circuit in Figure 4.7 is cascaded repeatedly as shown in Figure 4.14 to form networks with 35, 70 and 105 single lossless transmission lines. The CPU requirement of dGAWE and HSPICE perturbation are tabulated in Table 4.1 along with the speedup ratio. For each case, a seventh order approximation is performed in dGAWE. The comparison was made on a Sun SPARC 2 workstation.

The input waveform for the 105 single lossless transmission lines circuit is shown in Figure 4.15. It is a pulse with 10ps rise/fall time, 20ns pulse width and 5V amplitude. The output waveform at V_{out} and its sensitivity with respect to R_s are compared with HSPICE in Figure 4.16 and Figure 4.17, respectively.

Number of Interconnects	Number of Parameters	CPU Seconds		Speedup Ratio
		HSPICE	7th order dGAWE	
7	10	35.20	0.37	95.14
35	10	293.15	0.78	375.83
70	10	678.70	1.41	481.35
105	10	3576.71	2.65	1349.70

Table 4.1: CPU comparison between sensitivity calculation using HSPICE and proposed method.

Figure 4.14: Circuit for Example 4.2, obtained by cascading the circuit in Figure 4.7.

Figure 4.15: V_{in} for the 105 lossless transmission lines circuit in Example 4.2.

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Figure 4.16: Response at V_{out} of the 105 lossless transmission lines circuit in Example 4.2.

Figure 4.17: Sensitivity of V_{out} w.r.t. R_s in the 105 lossless transmission lines circuit.

Example 4.3

The circuit in Figure 4.18 consists of 3 lossy coupled transmission lines. The length of the transmission lines #1, #2 and #3 are 0.5mm, 0.4mm and 0.3mm respectively. The electrical parameter matrices for these lines are identical:

$$R = \begin{bmatrix} 7.5 & 1.5 \\ 1.5 & 7.5 \end{bmatrix} \Omega/mm \quad L = \begin{bmatrix} 49.46 & 6.33 \\ 6.33 & 49.46 \end{bmatrix} nH/mm$$

$$G = \begin{bmatrix} 0.01 & -0.001 \\ -0.001 & 0.01 \end{bmatrix} S/mm \quad C = \begin{bmatrix} 6.28 & -0.49 \\ -0.49 & 6.28 \end{bmatrix} pF/mm$$

The applied voltage, as shown in Figure 4.19, is a pulse with 10ps rise/fall time, 4ns pulse width and 1V amplitude. The output waveform at V_{out} is shown in Figure 4.20. The sensitivities with respect to various parameters are shown in Figure 4.21 - Figure 4.25. It should be noted that this circuit cannot be analyzed using HSPICE since the lossy transmission lines are characterized in the frequency domain. Therefore, GAWE perturbation is used for comparison.

Figure 4.18: Interconnect model with lossy coupled transmission lines for Example 4.3.

Figure 4.19: Input signal at V_{in} in Example 4.3.

Figure 4.20: Transient response at V_{out} in Example 4.3.

Figure 4.21: Sensitivity of V_{out} w.r.t. R_I in Example 4.3.

Figure 4.22: Sensitivity of V_{out} w.r.t. C_I in Example 4.3.

Figure 4.23: Sensitivity of V_{out} w.r.t. D of transmission line #1 in Example 4.3.

Figure 4.24: Sensitivity of V_{ow} w.r.t. $R_{1,1}$ of transmission line #1 in Example 4.3.

Figure 4.25: Sensitivity of V_{ow} w.r.t. $L_{1,1}$ of transmission line #1 in Example 4.3.

4.9 Summary

In this chapter, an efficient method has been presented for sensitivity analysis of networks consisting of arbitrary number of lossless or lossy multiconductor transmission lines with no topological constraint. An adjoint approach has also been presented for the sensitivity evaluation of general recursive functions. The moment level adjoint sensitivity method can be used to evaluate the time domain sensitivity of transmission line circuits with negligible error when compared with conventional circuit simulators such as SPICE while being two to three orders of magnitude faster.

A major advantage of this method is that it allows the time domain sensitivity of general distributed elements to be calculated without using computationally expensive lumped models or numerical inverse transform techniques. It is observed that the proposed technique is particularly useful for VLSI designs since:

- It can handle lossless/lossy, single/coupled interconnects;
- As shown in Example 4.2, the speedup ratio increases with the size of the circuit being analyzed. This is because for larger circuits (and thus larger MNA matrix), the relative cost for forward/backward substitution is smaller when compared with the L/U decomposition.

This technique is also useful for performance optimization especially when the number of design variables and number of specifications are large since *only one L/U decomposition is required for both transient and sensitivity analysis for any number of time points.*

Chapter 5

Time Domain Performance Optimization of Transmission Line Networks

5.1 Introduction

The development of large-scale network simulation and optimization techniques have been motivated by the requirements of the VLSI era [34]-[35]. As signal speed increases, the transmission line effects of interconnects become the dominant factor limiting the overall performance of VLSI systems. Improperly designed interconnects can result in increased signal delay, ringing, inadvertent and false switching. Therefore, on-chip and off-chip interconnections must be treated as an integral part of a design or system performance would be impaired. This problem can be found at chip level, printed circuit board (PCB) level, multichip modules (MCM) and chip packaging where the number of interconnects can be very large. Consequently, efficient and reliable CAD tools capable of

predicting and optimizing interconnect and circuit performance prior to fabrication are of great importance since post-processing tuning is very difficult or practically impossible in VLSI systems. Unfortunately, accurate simulation and sensitivity analysis of large scale networks are highly time-consuming. Hence, due to time limitation, optimization for networks of such scale is usually impossible during the design stage; especially when the transmission line effects have to be taken into consideration. Besides, circuit layouts change frequently during the design stage. Thus, a fast optimization method for large scale network with arbitrary number of transmission lines becomes important for optimal "tailoring" of circuit elements to satisfy or even exceed the design specifications.

In this chapter, the optimization of transmission line networks is formulated into mathematical programming. Elements inside any linear subnetwork or physical/geometrical features of transmission line model may be used as design variables. The adjoint sensitivity algorithm from Chapter 4 is extended to handle physical/geometrical parameters. The dGAWE simulator is then integrated with a powerful two-stage minimax optimization package inside SysCAD [22] for simultaneous minimization of delay, crosstalk and reflections at all vital connection ports. The proposed technique has an advantage of being one to two orders of magnitude faster than existing optimization methods [16]-[17] without sacrificing much of the accuracy.

5.2 Sensitivity w.r.t. Physical Parameters of T.L.s

Physical/geometrical parameters of a multiconductor transmission line, as depicted in Figure 5.1, include the conductor's width, distance between two conductors, length of transmission line and height of PCB. The R , L , G and C parameter matrices can be

computed from the physical/geometrical parameters using either quasi-static analysis [36], [37] or empirical formulas [38]. Suppose λ is a physical/geometrical parameter of a transmission line with N signal conductors. A simple approach for finding the sensitivity of v_{out} with respect to λ is

$$\frac{\partial v_{out}}{\partial \lambda} = \sum_{i=1}^N \sum_{j=1}^N \left[\frac{\partial v_{out}}{\partial R_{i,j}} \frac{\partial R_{i,j}}{\partial \lambda} + \frac{\partial v_{out}}{\partial L_{i,j}} \frac{\partial L_{i,j}}{\partial \lambda} + \frac{\partial v_{out}}{\partial G_{i,j}} \frac{\partial G_{i,j}}{\partial \lambda} + \frac{\partial v_{out}}{\partial C_{i,j}} \frac{\partial C_{i,j}}{\partial \lambda} \right] \quad (5.1)$$

This involves the sensitivity calculation of v_{out} with respect to all $(4 \times N^2)$ elements inside the parameter matrices. For example, a 4-conductor transmission line has 65 electrical parameters (R , L , G , C matrices and length D) but only 10 physical/geometrical parameters (widths of 4 conductors, distance between adjacent conductors, length of conductors, height of printed circuit board and dielectric constant). Since the number of physical/geometrical parameters is much smaller than the number of electrical parameters, efficient optimization should bypass (5.1) and use the physical parameters directly on the variables for differentiation of the right hand sides of (4.102) and (4.126).

5.2.1 Evaluation of Transmission Line Parameter Matrices

The equations used to calculate the L and C parameter matrices of the transmission line structure in Figure 5.1 are provided in [38]. This transmission line consists of two flat conductors near a ground plane. Assuming the transmission line is lossless, we have

$$C_{1,1} = C_{2,2} \approx \epsilon_r \epsilon_o K_{C1} \left(\frac{w}{h} \right) + \frac{\epsilon_r \epsilon_o}{4\pi} K_{L1} K_{C1} \left(\frac{w}{h} \right)^2 \ln \left[1 + \left(\frac{2h}{d} \right)^2 \right] \text{F/m} \quad (5.1)$$

$$C_{1,2} = C_{2,1} \approx -\frac{\epsilon_r \epsilon_o}{4\pi} K_{L1} K_{C1} \left(\frac{w}{h} \right)^2 \ln \left[1 + \left(\frac{2h}{d} \right)^2 \right] \text{F/m} \quad (5.2)$$

$$L_{1,1} = L_{2,2} \approx \frac{\mu_r \mu_o}{K_{L1}} \left(\frac{h}{w} \right) - \frac{\mu_r \mu_o}{4\pi} \ln \left[1 + \left(\frac{2h}{d} \right)^2 \right] \text{ H/m} \quad (5.3)$$

$$L_{1,2} = L_{2,1} \approx \frac{\mu_r \mu_o}{4\pi} \ln \left[1 + \left(\frac{2h}{d} \right)^2 \right] \text{ H/m} \quad (5.4)$$

$$K_{L1} = \frac{120\pi}{Z_{o(\epsilon_r=1)}} \left(\frac{h}{w} \right) \quad (5.5)$$

$$K_{C1} = \left[\frac{120\pi}{Z_{o(\epsilon_r=1)}} \sqrt{\frac{\epsilon_{r(\text{eff})}}{K_{L1} \epsilon_r}} \left(\frac{h}{w} \right) \right]^2 \quad (5.6)$$

$$Z_{o(\epsilon_r=1)} \approx 60 \ln \left(\frac{8h}{w} + \frac{w}{4h} \right) \Omega \quad \text{for } \frac{w}{h} \leq 1 \quad (5.7)$$

$$Z_{o(\epsilon_r=1)} \approx \frac{120\pi}{\frac{w}{h} + 2.42 - 0.44 \left(\frac{h}{w} \right) + \left(1 - \frac{h}{w} \right)^6} \Omega \quad \text{for } \frac{w}{h} \geq 1 \quad (5.8)$$

$$\epsilon_{r(\text{eff})} = \frac{\epsilon_r + 1}{2} + \left(\frac{\epsilon_r - 1}{2} \right) \left(\frac{1}{\sqrt{1 + \frac{10h}{w}}} \right) \quad (5.9)$$

where Z_o is the characteristic impedance;

ϵ_o is the dielectric constant in vacuum ($10^{-9}/36\pi$ F/m);

ϵ_r is the relative dielectric constant (4.5 for glass-epoxy);

μ_o is the permeability in vacuum ($4\pi \times 10^{-7}$ H/m);

μ_r is the relative permeability;

w is the conductor's width;

d is the distance between adjacent conductors and

h is the height of the PCB.

Figure 5.1: Two flat conductor transmission lines in parallel near a ground plane.

5.3 Formulation of Error Function

Consider the case when a network π with arbitrary number of multiconductor transmission lines is excited by a trapezoidal signal of rise/fall time T_1 and pulse width T_2 . Let ϕ be a vector of design variables and $F_j(\phi, t)$ be the j^{th} circuit response (e.g. nodal voltage) at time t . Furthermore, assume that the upper and lower specs for the j^{th} response be $S_{Uj}(t)$ and $S_{Lj}(t)$. Then the error functions are defined as:

$$e_{Uj}(\phi, t) = W_{Uj}(t) [F_j(\phi, t) - S_{Uj}(t)] \quad (5.10)$$

$$e_{Lj}(\phi, t) = W_{Lj}(t) [F_j(\phi, t) - S_{Lj}(t)] \quad (5.11)$$

where $W_{Uj}(t)$ and $W_{Lj}(t)$ are positive weighting factors for the upper and lower specifications for output j at time t . To directly address the problems that are commonly encountered during interconnect design, the following subsections will discuss the

formulation of error functions in order to minimize various transmission line effects.

5.3.1 Time Delay

If the maximum allowable delay time is T_D for a signal to reach a threshold value V_T , then the weighted error function for propagation delay can be written as

$$e_{L_j}(\underline{\phi}, t) = W_D [F_j(\underline{\phi}, t) - V_T] \quad \text{for } t = T_D \quad (5.12)$$

5.3.2 Signal Rise Time

Due to transmission line effects and terminating subnetworks, the rise time of an output signal may violate the design specification. Assuming that the required rise time is T_{RT} for a signal to rise from below V_L to V_H , the weighted error function can be formulated as

$$e_{U_j}(\underline{\phi}, t) = W_{RT} [F_j(\underline{\phi}, t) - V_H] \quad \text{for } t = T_D - \alpha T_{RT} \quad (5.13)$$

$$e_{L_j}(\underline{\phi}, t) = W_{RT} [F_j(\underline{\phi}, t) - V_L] \quad \text{for } t = T_D + \beta T_{RT} \quad (5.14)$$

where $\alpha > 0$, $\beta > 0$ and $\alpha + \beta = 1$.

5.3.3 Crosstalk

Due to electromagnetic coupling, an inactive conductor may pick up signals (noises) from fast adjacent active conductors. These signals will trigger logic gates if their magnitude is sufficiently large. To reduce these signals, we can specify the error functions as

$$e_{Uj}(\underline{\phi}, t) = W_C [F_j(\underline{\phi}, t) - V_C] \quad \text{for } t > 0 \quad (5.15)$$

$$e_{Lj}(\underline{\phi}, t) = W_C [F_j(\underline{\phi}, t) - V_C] \quad \text{for } t > 0 \quad (5.16)$$

such that their magnitude are kept under V_C .

5.3.4 Reflection

Ideally, for a delay time of T_D , the output signal should decay to zero after $2T_1 + T_2 + T_D$. However, due to reflections, the signal may continue to exist for an extended period of time. To suppress the magnitude of reflection to a level of V_{RF} , the weighted error functions can be formulated as

$$e_{Uj}(\underline{\phi}, t) = W_{RF} [F_j(\underline{\phi}, t) - V_{RF}] \quad \text{for } t > 2T_1 + T_2 + T_D \quad (5.17)$$

$$e_{Lj}(\underline{\phi}, t) = W_{RF} [F_j(\underline{\phi}, t) - V_{RF}] \quad \text{for } t > 2T_1 + T_2 + T_D \quad (5.18)$$

5.4 Formulation of Minimax Optimization

Let I_{Uj} (or I_{Lj}) contain indices i if an upper (or lower) specification is imposed on the j^{th} output $F_j(\underline{\phi}, t_i)$ and s_j be the total number of specification points associated with the j^{th} output such that

$$I_{Uj} = \{i_{1,j}, i_{2,j}, \dots, i_{k,j}\} \quad (5.19)$$

$$I_{Lj} = \{i_{k+1,j}, i_{k+2,j}, \dots, i_{s,j}\} \quad (5.20)$$

Note that I_{Uj} and I_{Lj} are not necessarily disjoint (i.e. we may have simultaneous specifications). Let n be the number of outputs to be optimized and

$$f_j(\underline{\phi}) = [f_{j,1}(\underline{\phi}) \dots f_{j,k}(\underline{\phi}) \dots f_{j,s_j}(\underline{\phi})]^t \quad (5.21)$$

be the error functions associated with output j where

$$f_{j,i}(\underline{\phi}) = e_{U_j}(\underline{\phi}, t_i) \quad \text{for } i \in I_{U_j} \quad (5.22)$$

$$f_{j,i}(\underline{\phi}) = -e_{L_j}(\underline{\phi}, t_i) \quad \text{for } i \in I_{L_j} \quad (5.23)$$

Then the problem statement is to find $\underline{\phi}$ such that the objective function

$$U(\underline{\phi}) = \max\{f_{1,1}(\underline{\phi}), f_{1,2}(\underline{\phi}), \dots, f_{n,s_n}(\underline{\phi})\} \quad (5.24)$$

is to be minimized subject to the constraints

$$g(\underline{\phi}) \geq 0 \quad (5.25)$$

and

$$h(\underline{\phi}) = 0 \quad (5.26)$$

Therefore, a positive (negative) $U(\underline{\phi})$ means that the specifications are violated (exceeded) and $U(\underline{\phi}) = 0$ means that the specification is just met.

5.5 System Architecture of SysCAD

5.5.1 Overview of the CAD Framework

A powerful two-stage optimization package is used to solve the minimax problem in (5.24). Stage 1 is used to obtain fast convergence to the neighbourhood of the solution and stage 2 is used to obtain superlinear final convergence [35], [39]. This combines the robustness of a first-order method of the Gauss-Newton type with the speed of the quasi-Newton method [34]. At the beginning of the optimization process, approximated gradient information would be calculated inside the optimizer for sensitivity verification.

Please note that the weighting factors mentioned here after would be a combination of W in (5.10), (5.11) and the sign in (5.22), (5.23).

Figure 5.2 gives an overview of the SysCAD architecture. Simulation and sensitivity analysis results from the dGAWE simulator is fed to the optimization module. This module, according to the sensitivity information, would make “smart” adjustments to the circuit components, which includes linear lumped and distributed parameters. These design variables would then be passed back to dGAWE for another iteration of optimization.

Figure 5.2: Overview of the CAD framework.

5.5.2 Special Consideration for AWE Simulators

In dGAWE (and all other AWE simulators), once the dominant poles, residues and their sensitivities are extracted from the network, time domain response and sensitivities can be generated for all time points. During optimization, the error functions (and thus the circuit responses) at many time points would be required. Therefore, the ϕ vector passed from the optimizer would stay constant most of the time except that the responses at a different time point is required. In this case, we can speed up the process by storing the poles, residues and their sensitivities every time the circuit's configuration has been changed. When the outputs and sensitivities at another time point is needed, (2.8) and (4.4) can be applied directly to obtain the results. By doing it this way, we can save the time for MNA matrix formulation, L/U decomposition and forward/backward substitutions when the design variables are not changed.

Therefore, dGAWE (and all other AWE simulators) would be advantageous when the number of error functions is large (and it usually is very large) since the poles, residues and their sensitivities would have to be calculated only once for each ϕ vector. The circuit responses and their sensitivities at another time point can be calculated by (2.8) and (4.4) with incremental CPU cost. The following is an outline of the interface between SysCAD and dGAWE:

```
Subroutine dfunction(n,m,xarray,yarray,dyarray,k,ss,iflag);
```

```
{ xarray is an 'n' dimension double precision array containing the  
  current values of all design parameters while the first element in  
  this array is reserved for Time }
```

```
{ yarray is an 'm' dimension double precision array containing the
  outputs at the specified time point }

{ dyarray is an 'm' by 'n' double precision array containing the
  sensitivity of all outputs in yarray with respect to all
  design variables in xarray at the specified time point }
```

```
if firstrun then begin
```

```
  read input file and store information in circon and cirpara
end;
```

```
if (firstrun) or (parameter_changed) then begin
```

```
  firstrun = false;
```

```
  update cirpara with xarray
```

```
{ if this is the first run and the element values inside the input
  file do not match the values passed from the optimizer, the
  later one would be used. }
```

```
  initialize variables;
```

```
  formulate MNA matrix;
```

```
  find circuit moments and their sensitivities for all outputs;
```

```
  find and store the poles, residues and their sensitivities for
  all outputs;
```

```
end;
```

```
calculate all outputs and sensitivities at the specified time
  point by (2.8) and (4.4);
```

```
{ Note that if no element has been changed in  $\phi$ , the outputs
  and their sensitivities at another time point can be
  evaluated based on the stored poles, residues and their
  sensitivities. }
```

5.6 Results

Example 5.1

Figure 5.3 shows an interconnect model with seven single transmission lines. The input waveform is shown in Figure 5.4 with 1ns rise/fall time and 5ns pulse width. The design specification requires that a threshold voltage of 0.65V is to be reached at V_{out} for $t=1.5ns$ and the magnitude of V_{sub} is to be kept under 0.1V. i.e.

$$V_{out} \geq 0.65V \quad \text{for } t=1.5ns$$

$$V_{sub} \leq 0.1V \quad \text{for } t \geq 0ns$$

The total number of error functions is 12.

The design variables include R_1, R_2, R_3, R_4 and the length of transmission line #1 to transmission line #7. The total length for the path between the input to V_{sub} is fixed at $0.06m \pm 0.01m$ and the path between the input to V_{out} would be kept constant at 0.10m. i.e.

$$L_1 + L_2 = 0.06m \pm 0.01m$$

$$L_4 + L_5 + L_7 = 0.10m$$

The weighting factors are chosen such that the error functions from V_{out} and V_{sub} will have similar values when the optimization process starts. The objective function before optimization is 116.723. After 30 iterations, the objective function has been reduced to -0.014 which means that all design specifications have been satisfied while all design variables are kept within their allowable range. Table 5.1 summarizes the maximum and minimum allowable range, the original and the optimal values for all design variables.

Figure 5.5 - Figure 5.6 show the waveforms at V_{out} and V_{sub} before and after optimization. With the optimal values, V_{out} increases from 0.5333V to 0.6657V at 1.5ns

and the maximum magnitude of V_{sub} decreases from 0.1028V to 0.0952V. The total CPU time used in this example including I/O, simulation, sensitivity analysis and optimization is 25.29 seconds. This timing is made on a Sun SPARC 2 workstation.

Design variables	Minimum	Original values	Maximum	Optimal values
R_1	15 Ω	25 Ω	50 Ω	25.186 Ω
R_2	15 Ω	25 Ω	50 Ω	25.109 Ω
R_3	15 Ω	25 Ω	50 Ω	25.025 Ω
R_4	15 Ω	25 Ω	50 Ω	24.978 Ω
D_1	0.5cm	3.0cm	8.0cm	6.500cm
D_2	0.5cm	3.0cm	8.0cm	0.500cm
D_3	0.5cm	3.0cm	8.0cm	0.500cm
D_4	0.5cm	5.0cm	8.0cm	0.809cm
D_5	0.5cm	3.0cm	8.0cm	8.000cm
D_6	0.5cm	4.0cm	8.0cm	1.718cm
D_7	0.5cm	2.0cm	8.0cm	1.191cm

Table 5.1: List of design variables for Example 5.1 before and after optimization.

Figure 5.3: Circuit for Example 5.1. Transmission lines with terminal networks.

Figure 5.4: Input signal for Example 5.1.

Figure 5.5: Response at V_{out} in Example 5.1 before and after optimization.

Figure 5.6: Response at V_{sub} in Example 5.1 before and after optimization.

Example 5.2

The circuit used in this example is shown in Figure 5.7 which contains lossless multiconductor transmission lines. The L and C parameter matrices are calculated from the physical/geometrical properties of the transmission lines according to Section 5.2.1. The length of transmission line #1, #2 and #3 are 0.05m, 0.04m and 0.03m respectively. The height of the PCB, width and distance of the microstrip lines are the same for the three transmission lines:

$$\text{width} = 0.58\text{mm}, \text{distance} = 2.49\text{mm} \text{ and height} = 1.17\text{mm}$$

The output ports of interest are nodes V_{out1} , V_{out2} , V_{cross1} and V_{cross2} . Simultaneous optimization is applied to these nodes such that the delay time of V_{out1} and V_{out2} will be reduced and the crosstalk of V_{cross1} and V_{cross2} will be minimized. Only the 12 physical/geometrical parameters of the transmission lines are used as design variables in this example. The microstrip structure is assumed to be fabricated on glass epoxy (i.e. $\epsilon_r = 4.5$). The input signal of this circuit is a 1 volt amplitude, 1.5ns rise/fall time, 4.5ns pulse width trapezoidal waveform as shown in Figure 5.8.

The design specifications require that V_{out1} and V_{out2} to reach a threshold voltage of 0.6V at time $t=1.5\text{ns}$ and the magnitude of crosstalk at V_{cross1} and V_{cross2} to be less than 5mV. A specification of 1.1V is also placed on V_{out1} and V_{out2} between the time range of 2ns to 6ns to reduce the overshoot in the waveforms. The total number of error functions in this example is 128. The weighting factors are chosen so that the initial error functions from crosstalk and delay time specs are close when compared with each other.

A constraint is placed on the total length of transmissions lines such that

$$D_1 + D_2 + D_3 = 0.12\text{m}$$

The conductors of a microstrip line should not touch each other. i.e.

$$d_1 - w_1 > 0$$

$$d_2 - w_2 > 0$$

$$d_3 - w_3 > 0$$

The transmission lines would be fabricated on a single PCB; therefore, the height between the conductors to ground plane should be consistent for all microstrip lines.

$$h_1 = h_2 = h_3$$

The initial objective function is 0.1504. After 50 iterations of optimization, this value is reduced to -0.004093. The negative value means that all design specifications are satisfied. Table 5.2 summaries the maximum and minimum allowable range, the original and the optimal values for all design variables. Note that D_1 has the minimum value (2.0cm min.) and the rest of the transmission line length is shared between D_2 and D_3 . Besides, the distance between the two flat conductors in transmission lines #2 and #3 are far apart (4.0mm max.) to reduce crosstalk.

The sensitivities of V_{out1} , V_{out2} , V_{cross1} and V_{cross2} with respect to various physical/geometrical parameters of the transmission lines are shown in Figure 5.9 - Figure 5.12. These plots are normalized to represent mV change per 1 percent change in λ . These results demonstrate excellent agreement with sensitivities obtained by the perturbation technique. The waveforms for V_{out1} , V_{out2} , V_{cross1} and V_{cross2} before and after optimization are presented in Figure 5.13 - Figure 5.16. Significant reduction in delay time and crosstalk level can be observed. The total CPU time used in this example including I/O, simulation, sensitivity analysis and optimization is 90.01 seconds. This timing is made on a Sun SPARC 2 workstation.

Design variables	Minimum	Original values	Maximum	Optimal values
D_1	2.00cm	5.00cm	15.00cm	2.0000cm
w_1	0.20mm	0.58mm	0.80mm	0.7168mm
d_1	1.00mm	2.49mm	4.00mm	1.0000mm
h_1	0.50mm	1.17mm	3.00mm	1.6154mm
D_2	2.00cm	4.00cm	15.00cm	4.4142cm
w_2	0.20mm	0.58mm	0.80mm	0.2000mm
d_2	1.00mm	2.49mm	4.00mm	4.0000mm
h_2	0.50mm	1.17mm	3.00mm	1.6154mm
D_3	2.00cm	4.00cm	15.00cm	5.5858cm
w_3	0.20mm	0.58mm	0.80mm	0.2000mm
d_3	1.00mm	2.49mm	4.00mm	4.0000mm
h_3	0.50mm	1.17mm	3.00mm	1.4233mm

Table 5.2: List of design variables for Example 5.2 before and after optimization.

Figure 5.7: Interconnect model with lossless coupled transmission lines for Example 5.2.

Figure 5.8: Input signal for Example 5.2.

Figure 5.9: Sensitivity of V_{owl} with respect to D_1 in Example 5.2.

Figure 5.10: Sensitivity of V_{crossI} with respect to h_2 in Example 5.2.

Figure 5.11: Sensitivity of V_{out2} with respect to d_3 in Example 5.2.

Figure 5.12: Sensitivity of V_{cross2} with respect to w_3 in Example 5.2.

Figure 5.13: Response at V_{out1} in Example 5.2 before and after optimization.

Figure 5.14: Response at V_{cross1} in Example 5.2 before and after optimization.

Figure 5.15: Response at V_{out2} in Example 5.2 before and after optimization.

Figure 5.16: Response at V_{cross2} in Example 5.2 before and after optimization.

5.7 Summary

The sensitivity estimation method from Chapter 4 is extended in Section 5.2 to handle physical/geometrical parameters of transmission lines. This method is more efficient than the chain-rule method as shown in (5.1) since physical parameters are directly used to calculate the eigenvalues and eigenvectors sensitivities.

A time domain design optimization method for multiconductor transmission line circuits has been proposed in this chapter. Two circuits with single and coupled lossless transmission lines are used as examples. In each case, the specifications are met and the resulting figures are plotted. As demonstrated by the CPU timing in Example 5.1 and Example 5.2, this method is very efficient. Optimization can usually be done in minutes instead of hours and this method allows physical/geometrical parameters of transmission model to be used as design variables.

This chapter also addresses the problems commonly found on transmission line networks, such as reflections, rise-time slowdown, increased delay and crosstalk, and suggests methods for controlling these effects in order to maintain distortion-free propagation of high-speed signals.

Chapter 6

Conclusions and Further Research

6.1 Summary

This research was motivated by the need of an efficient automated design tool to solve the interconnect problems in high-speed VLSI systems where interconnects are modelled as distributed transmission lines. A *generalized* AWE adjoint sensitivity analysis method (dGAWE) based on the asymptotic waveform evaluation technique was presented in this thesis to determine the sensitivities of high-speed circuits. The simulation and sensitivity results are then fed to an optimization package to minimize transmission line effects such as delay, crosstalk and reflection.

dGAWE is an efficient approach for time domain sensitivity estimation of interconnect circuits which contain general RLC lumped elements and distributed

components such as lossless/lossy multiconductor transmission lines. In addition, the sensitivity can be determined with respect to the electrical and physical/geometrical parameters of the transmission lines. A unified formulation based on the modified nodal admittance matrix is used to describe the network equations. A complete theory for the adjoint approach to moments sensitivity evaluation is developed and the recursive relationship for moments sensitivity, as well as adjoint moments, has been derived. This method can handle networks with arbitrary number of multiconductor transmission lines with no topological constraint. The computational cost due to the existence of multiconductor transmission line is negligible since the corresponding moments and moments sensitivity are evaluated in a recursive closed form. The numerical examples in this thesis demonstrate excellent agreement with sensitivities produced using the perturbation technique while at the same time providing significant CPU savings.

When dGAWE is combined with an optimization package, the result is an efficient automated design tool which allows a circuit designer to solve interconnection problems efficiently. Minimization of transmission line effects has been formulated into a minimax optimization problem where the error function formulation for delay time, signal rise time, crosstalk and reflection are presented. Optimizations with design variables that include linear lumped elements and/or physical/geometrical features of transmission lines have been presented. In each case, the design specifications have been met and significant improvement is observed in terms of signal integrity. Because of its generality and efficiency, dGAWE should be applicable to the analysis and design of interconnections in high-speed VLSI systems, multichip modules and printed circuit boards.

6.2 Suggestions for Further Research

1. dGAWE approximates the time domain sensitivity by expanding the exact sensitivity in Taylor series at zero frequency (i.e. $t=0$) such that the approximated sensitivity has the largest numerical error around time equal to zero. New methods for Asymptotic Waveform Evaluation such as [30]-[31] and [44] can be used to improve the stability and accuracy of GAWE.
2. dGAWE should be extended to handle non-linear components. This can be done by combining the existing technique with the Piecewise Decomposition Technique in [24]. Once this is done, the existing SysCAD architecture will support circuit optimization with non-linear elements as design variables.
3. The dGAWE method should also be extended to handle non-uniform transmission lines. Once again, the existing dGAWE/SysCAD interface will support circuit optimization with non-uniform transmission lines as design variables.
4. Since the proposed adjoint equations can be applied to general recursive functions, the possibility of combining these equations with numerical integration techniques for time domain adjoint sensitivity analysis should be explored.

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