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Signal Integrity Analysis of High-Speed Circuits with Embedded Electromagnetic Modules

by

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**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy**

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Abstract

This thesis is concerned with signal integrity analysis of high-speed interconnects. A new incident field coupling algorithm is proposed to account for lossy media in a circuit simulator environment. The algorithm is based on recently developed matrix-pad approximation to the state-transition matrix in MTL analysis. In addition, a physical optics based algorithm is proposed to account for the effects of stratified media on incident fields. For large classes of practical applications, this approach has a comparable accuracy to full wave techniques with little CPU and memory resource requirements. For more complicated interconnect structures which are routed in a 3-D space, a mixed electromagnetic/circuit analysis technique is proposed. The new algorithm is based on the assumption that field behavior of linear circuit systems can be characterized by relatively few terminal parameters. A full-wave finite difference time domain algorithm is employed to extract the terminal characteristics of interconnect systems in the absence of lumped elements. The characteristics are then converted into a linear SPICE-compatible time domain macromodel through a model-reduction technique. The proposed technique overcomes the numerical stability problems of time domain full-wave simulators and speeds up the whole simulation process. A direct access to the vast device libraries of existing circuit simulators is an additional merit of the proposed algorithm.

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Notation

$\mathbf{U}$	Identity Matrix
$\mathbf{0}$	Zero Matrix
$\delta(t)$	Dirac delta function
$\mathbf{I}$	Terminal current vector
$\mathbf{V}$	Terminal voltage vector
$\mathbf{Y}$	Admittance parameter matrix
$\mathbf{C}$	Per-unit-length capacitance matrix
$\mathbf{L}$	Per-unit-length inductance matrix
$\mathbf{G}$	Per-unit-length conductance matrix
$\mathbf{R}$	Per-unit-length resistance matrix
t	Time variable
s	Laplace variable (complex frequency)
ω	Angular frequency
$\mathbf{E}$	Electric field
$\mathbf{k}$	Wave vector
$\mathbf{H}$	Magnetic field
$\mathbf{B}$	Magnetic flux density
$\mathbf{J}$	Current density

ϵ	Permittivity of transmission medium
μ	Permeability of transmission medium
σ	Conductivity of transmission medium
∇	Del operator
Δt	Time increment in FDTD
$\Delta x, \Delta y, \Delta z$	Spatial increment in FDTD in cartesian coordinate system
p	System poles
ϕ	Azimuthal angle in spherical coordinates
θ	Longitudinal angle in spherical coordinates

Abbreviations

ABC	Absorbing Boundary Condition
AWE	Asymptotic waveform evaluation
CFH	Complex frequency hopping
CPU	Central processing Unit
DFT	Discrete fourier transform
EM	Electromagnetic (field)
EMI	Electromagnetic Interference
EMC	Electromagnetic Compatibility
FDTD	Finite Difference Time Domain
FEM	Finite Element Method
IDFT	Inverse Discrete fourier transform
IE	Integral Equation
MCM	Multi-chip module
MMT	Moment matching techniques
MNA	Modified nodal admittance
MTL	Multiconductor Transmission Line
PCB	Printed circuit board
PEEC	Partial Element Equivalent Circuit

PML	Perfectly Matching Layer
RC	Resistor-capacitor (network)
RLC	Resistor-inductor-capacitor (network)
SDA	Spectral Domain Approach
TEM	Transverse Electromagnetic (Field)

Chapter 1

Introduction

The continuous trend in circuit fabrication technology towards dense layouts, low power consumption and fast edge rates has radically changed the design techniques in the past two decades. Deep-submicron technologies for conducting traces has allowed fast edge rate devices and the combination of smaller feature sizes with faster rise times has put the interconnect-dominated design methodologies in front of device-dominated ones. In contemporary high-speed circuit applications, it has become a common practice to observe interconnect problems such as delay, reflection, attenuation, dispersion, crosstalk, ground-bounce. These effects may result in significant distortion in pulses that propagate even a short distance. In addition, the widespread use of MOS technology instead of TTL families has accentuated EMI susceptibility problems. Thus, analysis of both external field effects and parasitic phenomena on signal transmission has become a major design issue at any level of the circuit integration.

The need for precise analysis calls for deliberate consideration of the dispersive nature of the transmission environment and the interaction between conductors in the modeling of traces on printed circuit structures. This can only be achieved with rigorous circuit models which can identify interconnect problems and EMI effects at the design stage and thus save time and effort on fault diagnosis which would otherwise be difficult and costly.

The simplest and the most straightforward model for an interconnect is in the form of RC or RLC networks. For relatively short line segments, this lumped modelling can be accepted as a viable approach. For longer lines distributed modelling becomes essential. Pulse propagation along the interconnects is modeled by telegrapher's equations and multiconductor transmission line (MTL) structures are employed to simulate the parasitic phenomena in the transmission medium [2]-[14].

MTL modeling also lends itself to simulating the susceptibility of the interconnects against external field coupling [14]-[24]. Although some SPICE models have been developed for the analysis of incident field coupling to multiconductor transmission lines [16], they in general suffer two major shortcomings: (1) the models do not account for medium losses, (2) these approaches simplify the problem assuming the incoming field propagates in a homogeneous medium. Such a condition for inhomogeneous media such as a printed circuit structure has no validity at high frequency applications [23], [24].

With the recent advances in semiconductor-processing technology, PCB and microwave designers have run into new challenges [25]. Modern circuit configurations involve vertical integration of complex structures on multilayer substrates and signal transfer is provided by electromagnetic coupling and/or via-hole connections. The third dimension in the design is used to create new revolutionary geometries and to improve circuit performances. Current densities orthogonal to the substrate flow through hole-via connections

and air bridges. High packaging densities produce strong interactions among different circuit elements, not necessarily adjacent to one another. Printed circuit structures with multiple power and ground planes behave like a waveguide which directs the radiated emissions to remotely positioned components. The package interacts with the circuit such that it has to be considered as a part of the circuit itself.

In all the above cases, the complexity of the structures that support electromagnetic field propagation is much higher than conventional design techniques such as MTL can deal with. Such problems can only be solved by three-dimensional electromagnetic tools. These tools are needed especially, to analyze the interaction phenomena between closely spaced elements, to account for external field coupling effects, to characterize packaging effects on circuit performance and to evaluate the behaviors of multichip modules.

In printed circuit applications, interconnect terminals are connected to driver and receiver circuits, which can consist of thousands of lumped linear/nonlinear components. Thus, the incorporation of a conventional circuit simulator with a three-dimensional full-wave simulator is needed for nonlinear analysis. The usual approach in most microwave circuit applications is to extract *s-parameters* of a passive microwave device and then incorporate them with a steady-state circuit simulator [26]. This approach is quite justifiable for the simulation of analog circuits. However, for digital circuit applications a transient analysis is required in the time domain. In the time domain analysis, the effect of transmission medium on pulse propagation can be accounted for by a time domain macromodel generated in the absence of lumped circuit components from the results of full-wave simulator through a model-reduction technique. For model-reduction, the derivatives of the impulse response of the transmission medium are needed. With the derivative information, the model-reduction scheme then reduces a large linear subnetwork into a macromodel that contains a small set of ordinary differential equations. The derivative

calculation, which is the most crucial part of model-reduction algorithms is a challenging task if the system response pertaining to the transmission environment is obtained by measurements or numerically from a full-wave simulator. To our knowledge, there does not exist a robust and generic derivative calculation algorithm for such system responses in the literature.

Another important issue with high speed circuits is the prediction of the radiated emission levels and noise susceptibility of the printed circuit tracks against external fields at the design phase. Today, the most time consuming part of the production cycle is usually the so-called *retro-fit* applications, which involve post-design corrective measurements for EMI compliance. Hence, an accurate prediction of the noise susceptibility and radiated emission levels at the design stage will be a major time saver for the timely mass-marketing of a product.

Considering that current techniques are still far from achieving these goals, the main objective of this thesis project is to fill in the gap by developing fast and accurate analysis techniques that can handle real-life circuits with thousands of components including non-linear driver and receiver circuits along with EMI effects.

1.1 Contributions

The main contributions of this thesis can be listed as follows:

1. A new model is developed to analyse incident field coupling to multiconductor transmission lines in a circuit simulator. The model is based on rational function approximation to exponential state-transition matrix [27].

2. For multilayered printed circuits with interconnects modeled as MTL, the influence of stratified media on the external field coupling problem is investigated. A new approach based on physical optics principle is derived and tested in comparison with more rigorous but computationally more expensive numerical methods [23],[24]
3. For crosstalk analysis of 3D printed circuit structures, a *mixed electromagnetic/circuit analysis* technique is developed. As an extension of mixed electromagnetic/circuit analysis technique, a new algorithm is derived to calculate near-zone radiated fields from 3D printed circuit structures in the presence of nonlinear elements [28], [29]. In the same context, a new algorithm is developed to account for disruption due to incoming fields in 3D printed circuit structures and interconnect discontinuities in a circuit simulator environment [29], [30]
4. A new algorithm based on prony's method is proposed to alleviate the effect of late time transient noise on derivative calculation from discrete time domain data. The calculation of higher-order derivatives of the tabulated data pertaining to terminal characteristics of a circuit structure is a crucial element of the proposed mixed EM/circuit analysis. For this purpose, the validity of the existing derivative calculation techniques is comparatively investigated.

1.2 Organization

This thesis is organized as follows. Chapter 2 is devoted to the problem statement. Prominent analysis techniques are also presented in a literature survey. Chapter 3 presents the preliminaries of the proposed algorithm. A general nonlinear network involving a high-speed transmission medium is formulated as a foundation upon which the proposed algorithm is built in the following chapters. Chapter 4 presents the principles of the model reduction techniques and gives a general overview. In Chapter 5, general MTL equations

are reviewed and a new algorithm is proposed for incident field coupling to lossy multi-conductor transmission lines. Also in this chapter, the effect of the layered substrate is researched in field coupling applications and a new algorithm is proposed to account for the inhomogeneous media in the analysis of field-to-trace coupling. Chapter 7 lays the basics of the proposed mixed EM/circuit analysis. As a preliminary, FDTD method is reviewed concentrating on the circuit applications and some preliminary results are presented. Chapter 8 deals with the derivative calculation for the required model reduction of the structures whose characteristics are obtained in tabulated data format. Various derivative calculation algorithms are compared on several example circuits and a new method is proposed to improve the noise immunity of the time domain integration technique in the derivative evaluation. Conclusions follow in Chapter 9.

Chapter 2

High Speed Interconnect

Problems and Analysis Methods

In the most general sense, an *interconnect* is a signal carrying conductor in a system. Interconnects can be at various levels of design hierarchy, such as packaging structures, multichip modules, printed circuit boards, back-planes and on-chip dies. Depending on the operating frequency, it is possible to observe interconnect related problems such as false switching, signal distortion and logic glitches at almost every system level. It is the purpose of this chapter to pose the main causes of these high-speed interconnect problems and possible analysis techniques.

This chapter begins with the classification of the interconnects. Interconnect problems are presented next. The main analysis techniques to these problems with a literature survey conclude this chapter.

2.1 Classification and Modeling of Interconnects

The classification of interconnects can be based on the number of their electrical dimensions, which in general depend on the physical dimensions, the frequency of excitation and the constitutive parameters of the surrounding medium (Fig. 2.1). An interconnect between two nodes can be modeled in a number of ways with varying accuracy and computational overhead. Generally speaking, an electrical circuit is said to be electrically small if its largest dimension is much smaller than the wavelength. In such systems, circuit components are represented by lumped models and interconnects are simply zero resistance paths connecting two components. The circuit behavior is solely governed by Kirchhoff's voltage and current laws and signal waveforms have no space dependency.

Figure 2.1: Classification of Interconnects

At higher frequencies where interconnect *lengths* are comparable to operational wavelengths, the spacing between them can still be electrically small. The electric field behavior in such circuits is described in terms of voltage and current waves propagating in a one-dimensional space parallel to the line extension direction. There are couple of ways to model such interconnects. The basic application involves discretizing the line into electrically small subsections and model each subsection with an RC tree. RC model is easy to compute, but relatively inaccurate. Accuracy can be improved by adding an inductance in series with the resistor. However, the computational burden due to the excessive number of lumped subsections for long lines may outweigh the computational simplicity. Hence, distributed transmission line models based on telegrapher's equations become more efficient despite the cost of added computational complexity.

Especially in low impedance level microwave circuit applications, two dimensions of an interconnect structure may become comparable to the wavelength while the third dimension is much smaller. 2-D field behavior for such planar structures with basic shapes can be expressed in a relatively simple analytical form. These interconnects fall into the category of planar circuits [26].

In multilayered printed circuit structures, interlayer current transmission takes place in the vertical direction through unconventional interconnects such as vias and holes. Only a 3-D representation of the interconnects can account for the field propagation in such media. Full-wave methods are employed in the analysis of such structures.

2.2 Overview of Interconnect Problems

In electrically small systems where interconnects are trivial short circuits transmission line effects and related EMI problems do not exist. At the other extreme of the frequency spectrum where electrical size is much larger than the excitation wavelength, an electrical structure becomes subject to geometrical optics laws and the circuit definition itself has no meaning.

High-speed interconnect problems with today's circuits are in general associated with *resonant size* electrical systems whose electrical size lies somewhere between these two extremes and is comparable to the physical size. In this frequency range, signals *propagate* through the interconnects undergoing a time delay, attenuation and degradation depending on the physical nature of interconnects. Attenuation which is one of the main causes of signal degradation occurs due to medium losses that become effective at high frequencies. Signal degradation and time delay can create significant functional problems in high-speed digital circuits.

Another serious problem is associated with load mismatches which give rise to reflections. Perhaps, this problem can be best shown with a dramatic example involving a lossless homogeneous line with mismatched terminations [2].

Figure 2.2: A lossless homogeneous line

Consider the trace in Fig. 2.2 with $R_s=0$ and $R_L=\infty$. Assume that the source voltage $V_s(t)$ is a ramp waveform given by

$$V_s(t) = \begin{cases} 0 & , t \leq 0 \\ \frac{1}{\tau_r} V & , t \leq \tau_r \\ 1 V & , t > \tau_r \end{cases}$$

where τ_r is the rise time of the pulse. In a line with length having one-way transit time T , the load voltage for this input pulse is depicted in Fig. 2.3 for various values of τ_r . If R_s and R_L were matched to the line impedance, load voltage waveform would not undergo any distortion. However, since matching cannot always be accomplished, line length should be chosen short enough to satisfy $\tau_r \gg T$ in order to avoid reflection related problems. This solution results in a dense layout with closely spaced interconnects which will be discussed next.

Figure 2.3: Output waveforms for different values of the rise time

In practice, interconnects in circuits are routed in clusters and the electromagnetic fields surrounding the individual traces interact with each other inducing signals in all the adjacent circuits. This undesired mutual interaction between closely spaced interconnects is known as crosstalk and can cause significant functional problems in circuits in addition to attenuation and time delay.

In addition to lumped sources in a circuit, high-speed interconnects can also be driven by signals produced by distant electromagnetic sources. These driving signals which are usually of unintended nature may be in the form of plane waves which may be generated by distant transmitting antennas or may be nonuniform fields generated by nearby radiating structures. The result of this external field coupling may range from temporary malfunctioning to permanent damages depending on the incoming field intensity.

At sufficiently high frequencies, interconnects in principle behave like printed linear antennas. As they may receive undesired distant electromagnetic sources, they also radiate electromagnetic energy onto nearby electronic systems. In most practical cases, radiation from long cables bundles such as backplane ribbon wires is much higher than that from printed traces [3]. However, with the recent printed multilayer technology, stacked power and ground planes can easily guide radiated energy to relatively distant components on the same board, which may give rise to problems difficult to diagnose.

Another problem with high-speed interconnects is caused by a sudden change in the amount of current drawn by a component. This voltage transient can cause a momentary drop in the voltage on the power distribution bus and can be sufficiently large to interfere with the normal operation of other components on the board. This phenomenon is referred to as ground bounce or delta-I noise and is a common problem in high-speed and integrat-

ed circuit designs. In multilayered boards, a PCB or/and IC behave like a resonator with a high quality factor and support damped oscillations due to this voltage surge. The resulting noise can be high enough to jeopardize circuit operation [3].

Interconnect problems are not limited to those discussed so far. In the realm of EMC analysis, for example, conducted emissions are as serious a problem as radiated emissions [3], [4]. In terms of the signal integrity analysis, however, although the causes may differ, the effects can be analyzed with the techniques which will be described in the next section.

2.3 Analysis Techniques-A Literature Survey

It is possible to say that high-speed interconnect problems coexist with digital circuit applications. The problems picked up with the widespread application of digital technology spurred by a demand for faster and smaller devices since the late 70's. Due to the ease of implementation, lumped modeling of interconnects with RC or RLC circuits is still being used in SPICE-like circuit simulators, however at subnanosecond rise-time applications this approach is inefficient and is getting obsolete. Distributed models based on telegrapher's equations are replacing these models in contemporary simulators. Transmission line analysis assumes TEM mode of propagation. Quasi-static approaches are used to determine the propagation constants and characteristic impedances of the lines. Various types of discontinuities are characterized by lumped element models. This section starts with the description of TEM analysis.

2.3.1 TEM Analysis

Depending on the frequency of operation and the structure geometry, the solution of Maxwell's equations can have different complexity levels for interconnect analysis. The simplest of these solution techniques is given by transmission line equations. The fundamental assumption in transmission line representation of an electrical structure is that the field propagation in the transmission medium is dominantly characterized by transverse electromagnetic (TEM) mode. In TEM propagation mode, electric and magnetic fields in the medium are confined to the transverse plane which is perpendicular to conductor axes. The fields in the transverse plane have a static distribution which is described by scalar potential functions which satisfy Laplace's equation. In such transmission media, electromagnetic fields are supported by longitudinal currents flowing in the conductors and are essentially confined to a thin area between signal and return conductors. This modeling reduces the three-dimensional field problem to a one-dimensional one, which greatly simplifies the solution. The medium properties are accounted for in distributed or per-unit-length parameters such as inductance, capacitance and conductance which are calculated from the fields in the transverse plane through a separate electro/magneto-static analysis [2].

At frequencies where the line length is about one tenth of a wavelength or shorter, the distributed line parameters can be approximated as lumped circuit elements and an equivalent T or Π network can be used to model the interconnect with an error about 7% [4] (Fig. 2.4). If the line is electrically longer than the above criterion it can be subdivided into a number of smaller sections whose length satisfy one-tenth-of-a-wavelength condition. However, aside from the error involved, the lumped representation may put an intolerably high burden in terms of the number of components in the simulation of circuits involving large number of interconnects.

Figure 2.4: Lumped interconnect models (a) Π model, (b) T model

In practical PCB or microwave applications, the traces are laid in single or multilayered dielectric substrates. In such configurations, the medium surrounding the conductors is not homogeneous in the cross-sectional dimension. In microstrip structures, for example, top of the strip faces air ($\epsilon = \epsilon_0$) as its downside looks into the dielectric substrate ($\epsilon = \epsilon_r \epsilon_0$). Such inhomogeneous media cannot support pure TEM modes. The phase front which is forced to have two different speeds at any cross-section is then stretched at the interface of two media resulting in a field component in the direction of propagation. This ambiguity in the speed of phase front also give rise to dispersion in the propagating pulse. Dispersion distorts the traveling pulse expanding it in the time scale. The level of dispersion and the magnitude of the field component in the direction of propagation depend on the frequency of operation and the relative permittivity of the substrate material. Pulse propagation in such inhomogeneous media is referred to as of quasi-TEM mode. In quasi-TEM analysis, a pulse is assumed to travel in a homogeneous medium whose relative permittivity is somewhere between those of two interfacing media. The medium inhomogeneity is accounted for in per-unit-length parameters of the conductors, which usually requires numerical analysis.

Following the general principles of TEM mode of propagation, it is possible to model crosstalk effects with coupled or *multiconductor* transmission line equations. The per-unit-length parameters which are scalars in two-conductor cases take the form of a matrix representation with off-diagonal entries accounting for the mutual interaction between the conductors [2]. The voltage and current waves propagating along the lines have vector representations. The solution is usually obtained through a model decomposition technique which transforms the per-unit-length parameter matrices into uncoupled diagonal ones. However, for MTLs with frequency dependent losses, such a decomposition space does not usually exist. Recently developed model-reduction techniques offer efficient alternatives to modal decomposition techniques [33]-[43].

In TEM modeling, the net current at any cross-section of the lines sums up to zero, because the currents on the signal and return conductors are of the same magnitude and oppositely directed. These currents are usually referred to as differential mode currents. In actual case, there exists other currents which are equal in magnitude at a cross-section but are directed in the same direction (Fig. 2.5). These currents, which are not accounted for by TEM modeling are known as common mode or antenna mode currents [2]. Therefore, at points along the lines, there is a combination of both currents only the differential mode component of which is modeled by the transmission line equations. Ordinarily, common mode currents are much smaller in magnitude than the differential mode currents, so they do not substantially affect the result of a crosstalk or external field coupling analysis, provided that transmission line cross-section is electrically small at the frequency of interest. However, common mode currents are the main contributing source in the analysis of radiated emissions, since they add at any cross-section as opposed to differential mode currents which tend to subtract. In this regard, the analysis of radiated emissions by transmission line equations will most likely be inadequate since they do not account for the dominant contributors of the radiation mechanism [44].

Figure 2.5: Decomposition of Currents

The effect of external fields emitted by distant electromagnetic sources was first accounted for in the transmission line equations as distributed forcing functions in [14]. This technique was later extended to MTL equations in [15]. In incident field coupling analysis, the techniques in the literature in general assume a homogeneous medium for the interconnect environment. The inhomogeneity is only accounted for in calculation of the per-unit length parameters of the interconnects as in quasi-TEM analysis. Imposing such conditions for inhomogeneous structures introduces errors in the results at high frequencies [24]. Accounting for the effects of the layered medium requires the calculation of fields in each layer separately. This computation can be done using full wave techniques such as Finite Elements Method (FEM) or Finite Difference Time Domain (FDTD) method. These techniques, however, may require prohibitively high CPU and memory resources. An efficient solution to such cases is proposed in Chapter 5.

As long as the operating wavelengths are much larger than the cross-sectional dimensions conventional transmission line theory provides accurate enough models for the interconnect structures which are represented by telegrapher's equations. However, as higher

frequency ranges are used the separation between the conductors gets electrically large and more accurate models become necessary to account for dispersion and higher order mode effects.

Although consideration of higher order mode effects is an important factor in the analysis it must be also pointed out that the upper cutoff frequency for quasi-TEM mode of propagation extends up to very high frequencies in most printed circuit applications. As an example, take a printed circuit board structure of epoxy-glass material ($\epsilon_r=4.5$) and of 7 mil thickness. These are realistic values commonly used in industrial applications. If the dielectric slab is sandwiched between power and ground planes as in most practical cases, this structure can be considered as a parallel plate waveguide. According to [46] the lowest cutoff frequency for the first higher order mode (TE or TM) is given by

$$f_c = \frac{c}{2d\sqrt{\epsilon_r}} \quad (2.1)$$

where c is the speed of the wavefront in free space and d is the spacing between conducting planes. In this example, up to 416 GHz, quasi-TEM will be the dominant propagation mode. In microstrip type structures which do not have upper power plane substrate modes will accompany to dominant quasi-TEM mode. These modes with imaginary propagation constants decay quickly in the sideward direction and thus will not contaminate the main propagation mode [47].

2.4 Full-Wave Analysis

With the progressive increase in the operational frequencies, 1-D (transmission line) models became inadequate to represent the behavior of microstrip circuits. A second dimension was then included into the mathematical models, leading to the development of

planar circuit concept in early 70's [48]. The planar circuit concept offers an improvement to overcome the limitations inherent to the transmission line representation of microstrip circuits [26]. Dispersion and fringing field effects are taken into account by effective dimension and effective permittivity concepts. This approach provides a rigorous description for discontinuities with a relatively simple analytical treatment. At that time, 3-D full-wave analyses would have required excessive computer resources.

Although some of the basic full-wave techniques were developed more than a century ago, their application to the analysis of printed circuits would wait until the beginning of 70's. All full-wave methods in general, work by transforming the continuum set of integro-differential equations into a set of purely algebraic equations which can be solved on a computer using standard numerical techniques. For electromagnetic modeling there is a fairly natural subdivision of the techniques into integral and differential equation techniques. These methods will now be discussed next.

2.4.1 Integral Equation (IE) Techniques

The characteristic feature of integral equation techniques in interconnect analysis is that they cast the solution for the current density on conducting surfaces in the form of an integral equation, where the unknown current density is a part of the integrand. The solution to the current density can then be obtained by a numerical method such as method of moments which reduces an integral equation into set of algebraic equation by weighted residual techniques. The solution to matrix equation is usually carried out via inversion, elimination or iterative techniques. The crucial part in the formulation of integral equations is to obtain a suitable green's function for the structure to be analyzed. The integral equation then represents the spatial convolution of the green's function and the unknown current distribution in the problem space. The green's function already satisfies the Max-

well's equations and the boundary conditions, within and on the outer boundary of the problem domain, respectively. With this feature, IE techniques are quite suitable for open type structures. However, calculation of the green's function for inhomogeneous transmission media is quite an involved procedure. Furthermore for each different geometry, a different green's function is to be derived. In this respect, especially 3-D IE techniques are in general applicable to simple interconnect geometries.

The above limitations have been mitigated by spectral domain approach (SDA) [26], a special case of IE techniques. SDA emerged as a powerful and popular tool to compute the propagation characteristics of printed circuit configurations in the beginning of 70's [49]. Although SDA can predict 3-D effects such as circuit-package interaction, crosstalk, etc. it is not a 3-D algorithm since it does not account for finite metal thickness and vertical currents.

Another popular IE technique which has close ties with SDA is the mode matching method [50]. This method is particularly useful in the analysis of shielded structures. If the shielding box is rectangular, the fields inside it can be expressed as a weighted sum of homogeneous waveguide modes. Compared to SDA, it has the advantage of accounting for the finite metallization thickness as well as the vertical currents. Compared to differential methods like FDTD, it is less flexible and requires some analytical preprocessing but needs less memory storage and CPU time.

In mid-70's, partial element equivalent circuit (PEEC) modeling took a start as another IE technique [51]. Although PEEC is in principle very similar to the method of moments, it is not a numerical method but a modeling technique. Furthermore, the quasi-static field assumption in the evaluation of the green's function for the transmission medium greatly simplifies the computational efforts. In PEEC, discretized subsections are modeled as

RLC circuits where the individual elements are extracted from the geometry using a quasi-static solution of the Maxwell's equations. As operational frequencies increase PEEC models later would be modified to include retardation effects [52].

2.4.2 Differential Equation Techniques

IE techniques sometimes appear to be not flexible enough to deal with irregularly shaped 3-D structures. Differential equation techniques which are also referred to as *domain discretizing* methods such as Finite Element and Finite Difference Time Domain (FDTD) techniques can easily deal with arbitrarily shaped interconnect configurations and inhomogeneous media. They solve the Maxwell's differential equations in the whole problem domain, i.e the volume in which current and charge can exert influence. They do not require heavy analytical preprocessing unlike IE methods but require usually large memory storage and CPU time.

In the subclassification of the differential equation techniques, the Finite Element Method (FEM) is in general the one with the most analytical rigor [26]. The FEM approximates the problem domain by polygonal subregions (finite elements) and thus casts the Maxwell's partial differential equations into a set of algebraic equations from which the unknown field distribution within each element can be solved by some sparse matrix or iterative techniques. It can be implemented both in the time and the frequency domain.

After its first application to the analysis of printed circuit technology in late 80's [47], FDTD has become very popular in interconnect analysis. The explicit integration scheme FDTD employs gives it a thrust in the competition against other domain discretizing methods by speeding up the computation process. It is also well suited for parallelization of the code. The traditional way of interfacing circuit and field equations in FDTD entails

modeling the lumped circuit component as grid or sub-grid elements in the problem domain [31],[32]. However, the explicit integration scheme, which works to the advantage of FDTD in the field analysis, turns into a handicap in circuit applications from the numerical stability point of view. This is because stiff differential equations [53] representing networks with linear/nonlinear lumped circuit components may require step sizes far below the required numerical stability condition, which usually leads to impractical CPU times.

To overcome the limitations of FDTD in interconnect analysis, *mixed electromagnetic/circuit analysis* technique is proposed in this work. In the proposed algorithm, FDTD is used where it is most efficient, namely *for the analysis of the transmission media in the absence of all lumped elements*. Circuit simulation is achieved through recently developed model-reduction techniques. An overview of the model-reduction algorithms and a detailed discussion of the proposed method are given in Chapters 4 and 7, respectively. The first step in the analysis, however, is the formulation of the circuit equations in the MNA form which is presented in the next chapter.

Chapter 3

Formulation of Circuit Equations

In this chapter, we present a systematic approach to formulate unified network equations including passive EM devices. In the proposed scheme, a general nonlinear network containing EM devices, is initially partitioned into linear and nonlinear subnetworks. The linear subnetwork is then further decomposed into a lumped subnetwork and an EM-based subnetwork.

3.1 General Formulation of Network Equations

Consider a general nonlinear network ϕ which contains nonlinear/linear lumped components and arbitrary linear subnetworks. The linear lumped components may be described by equations in either time-domain or frequency-domain, whereas the nonlinear components may only be described by time-domain equations. The arbitrary linear subnetworks may contain distributed circuits like electrically long interconnects and disconti-

nities such as bends, vias, holes, T-junctions, etc., which are usually described the best in the full-wave domain. Let the linear subnetworks be grouped into a single subnetwork π as shown in Fig. 3.1

Figure 3.1: General Nonlinear Network ϕ Topology

It should be noted that nonlinear subnetwork may also contain linear lumped components. Without loss of generality, the modified nodal admittance (MNA) matrix [54] for the network ϕ can be formulated in the time domain as

$$W_{\phi} \frac{d}{dt} v(t) + G_{\phi} v_{\phi}(t) + L_{\pi} i_{\pi}(t) + F(v_{\phi}(t)) - b_{\phi}(t) = 0, \quad (t \in [0, T]) \quad (3.1)$$

where

- $v_{\phi}(t) \in \mathcal{R}^N$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,
- $W_{\phi} \in \mathcal{R}^{N_{\phi} \times N_{\phi}}$ and $G_{\phi} \in \mathcal{R}^{N_{\phi} \times N_{\phi}}$ are constant matrices describing the lumped memory and memoryless elements of network ϕ , respectively,

- $b_\phi \in \mathcal{R}^{N_\phi}$ is a constant vector with entries determined by the independent voltage and current sources,
- $F(v_\phi)$ is a function describing the nonlinear elements of the circuit,
- $L_\pi = [l_{i,j} \in \{0, 1\}]$, $i \in \{1, \dots, N_\phi\}$, $j \in \{1, \dots, n_\pi\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_\pi(t) \in \mathcal{R}^{n_\pi}$, the vector of currents entering the linear subnetwork π , into the node space $\mathcal{R}^{N_\phi}$ of the network ϕ , N_ϕ is the total number of variables in the MNA matrix.

The linear multi-terminal subnetwork π can be characterized in the frequency-domain by its terminal behavior. The terminal relations for subnetwork π can be represented by frequency-domain equations as

$$I_\pi(s) = Y_\pi(s)V_\pi(s) \quad (3.2)$$

where s is the complex frequency, $Y_\pi(s)$ is the complex frequency-domain admittance representation of the subnetwork π , $V_\pi(s)$ is the vector of terminal voltage nodes that connects subnetwork π to network ϕ .

3.2 Linear Subnetwork Formulation

This section concerns the formulation of the subnetwork π in a matrix form which includes lumped, distributed and EM-based subnetworks, with a systematic extension of MNA.

Figure 3.2: Decomposition of linear subnetwork π

3.2.1 Lumped Components

Considering only the linear lumped components and independent voltage/current sources (3.1) takes the following form

$$W_{\pi} \frac{d}{dt} v_{\pi}(t) + G_{\pi} v_{\pi}(t) - b(t) = 0 \quad (3.3)$$

rearranging (3.3) and writing it in the frequency domain

$$(sW_{\pi}(s) + G_{\pi})V_{\pi}(s) - b(s) = 0 \quad (3.4)$$

where $W_{\pi}, G_{\pi} \in \mathcal{R}^{n_{\pi} \times n_{\pi}}$ are constant matrices describing the lumped memory and memoryless elements of the network, respectively; $\mathcal{R}^{n_{\pi} \times n_{\pi}}$ is the node space of the subnetwork π ; $b \in \mathcal{R}^{n_{\pi}}$ is a constant vector with entries of independent sources of the subnetwork π ; $V(s) \in \mathcal{R}^{n_{\pi}}$ is the vector of s -domain nodal voltages.

3.2.2 Distributed Elements Based on Quasi-TEM Assumption

For linear subnetworks involving distributed elements based on quasi-TEM assumption, the frequency domain equations of a distributed subnetwork consisting of n_d coupled conductors can be written as

$$A_d(s)V_d(s) + B_d(s)I_d(s) = 0 \quad (3.5)$$

where $V_d(s)$ and $I_d(s)$ represent the s -domain terminal voltages and currents of the distributed elements, respectively; $A_d(s)$ and $B_d(s)$ are matrices in terms of line parameters, which are described explicitly in [57]. Combining (3.5) with (3.4)

$$\begin{bmatrix} sW_\pi(s) + G_\pi & L_d \\ A_d(s)L_d^t & B_d(s) \end{bmatrix} \begin{bmatrix} V_\pi(s) \\ I_d(s) \end{bmatrix} = \begin{bmatrix} b(s) \\ 0 \end{bmatrix} \quad (3.6)$$

where $L_d = [l_{i,j} \in \{0, 1\}]$, $i \in \{1, \dots, N_\pi\}$, $j \in \{1, \dots, 2n_d\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_d(t) \in \Re^{2n_d}$, the vector of currents entering the distributed subnetwork, into the node space $\Re^{n_\pi}$ of the subnetwork π .

3.2.3 EM-Based Subnetworks

In this section by EM-based subnetworks, we mean physical structures characterized by their terminal responses. The terminal responses may be obtained by a full-wave simulator as well as by measurement. The frequency domain stencil of such a structure is in the form of (3.2). If we consider an EM-based subnetwork of n_m terminals in a given linear subnetwork π (3.2) can be rewritten as

$$I_m(s) = Y_m(s)V_m(s) \quad (3.7)$$

where $V_m(s)$ and $I_m(s)$ represent the s -domain terminal voltages and currents of the EM-based subnetwork, respectively; $Y_m(s)$ is the short circuit admittance parameters of the EM-based subnetwork of dimension n_m . A detailed derivation of $Y_m(s)$ is given in Chapter 8. Including the EM-based subnetwork, (3.6) can be expanded as

$$\begin{bmatrix} sW_\pi(s) + G_\pi & L_d & L_m \\ A_d(s)L_d^t & B_d(s) & 0 \\ -Y_m(s) & 0 & U \end{bmatrix} \begin{bmatrix} V_\pi(s) \\ I_d(s) \\ I_m(s) \end{bmatrix} = \begin{bmatrix} b(s) \\ 0 \\ 0 \end{bmatrix} \quad (3.8)$$

where $L_m = [l_{i,j} \in \{0, 1\}]$, $i \in \{1, \dots, N_\pi\}$, $j \in \{1, \dots, n_m\}$ with a maximum of one nonzero in each row or column, is a selector matrix that maps $i_m(t) \in \mathbb{R}^{n_m}$, the vector of currents entering the EM-based subnetwork, into the node space $\mathbb{R}^{N_\pi}$ of the linear subnetwork π ; $U \in n_m \times n_m$ stands for the unity matrix.

Transient simulation of (3.8) in the presence of nonlinear elements is associated with two major difficulties. First, large matrix sizes associated with (3.8) render the simulation very slow. Second, due to infinite bandwidth of the distributed/EM-based subnetwork, direct combination of (3.8) with SPICE-like time-domain simulators is not possible. There are mainly two ways of solving (3.8) with nonlinear terminations. The first approach requires the time domain counterpart of (3.7), namely transient admittance parameters. In this case, it is possible to solve (3.1) and (3.7) simultaneously since they are both in the time domain. However, time domain solution requires time consuming convolution operations and it has a little practical value for large networks. Alternatively, model-reduction algorithms can be applied to the linear subnetwork represented by (3.8). The details of this approach will be discussed in the next chapter.

Chapter 4

Review of Model-Reduction

Techniques and Macromodeling

This chapter deals with the solution of the network equations formulated in Chapter 3. The difficulty in combining (3.8) with nonlinear subnetwork as in (3.1) is due to the different nature of the two representations. Equation (3.8) handles the linear subnetwork involving EM-based subnetwork in *the frequency domain*, whereas the general nonlinear circuit is represented in *the time domain*. This problem can be solved by model-reduction algorithms. The objective of model-reduction is to reduce (3.8) into a small set of ordinary differential equations, which can be linked to the time domain equations of nonlinear subnetworks, so as to facilitate the transient simulation.

In this chapter, the principles of model-reduction will be presented first. A review of some prominent model-reduction techniques follows. After a discussion about the calculation of system moments, the generation of a SPICE compatible time domain macromodel concludes this chapter.

4.1 Principles of Model-Reduction Techniques

Network function in general have large number of poles spread over the s -plane. Although the poles with fast attenuation factors normally have negligible effect on the simulation results they cause linear multi-step algorithms to significantly reduce the step size in order to stay within the stability region. In this respect, the system behavior is mainly governed by the poles close to the imaginary axis.

On the other hand, the infinite bandwidth concept associated with distributed/EM-based subnetworks has merely a theoretical significance. In practice, all distributed systems have an upper cut-off frequency determined by the frequency spectrum of their input. Even for a system with infinite number of poles along the imaginary axis, a bandlimited input will result in a bandlimited output. Since one usually has a priori knowledge of the highest operational frequency of the system, it is possible to determine the system behavior with finite number of poles. The finite number of poles which articulate the system response are called the *dominant* poles of the system. In this context, the objective of all model-reduction algorithms is to obtain the dominant poles that govern the system behavior. In the literature, there is a variety of such model-reduction algorithms. In the following part of this chapter, the most prominent ones in circuit applications, namely moment-matching techniques (MMT) will be reviewed.

4.2 Asymptotic Waveform Evaluation (AWE)

Obtaining a low order approximation of a network transfer function by using a single rational approximant is commonly referred to as Asymptotic Waveform Evaluation (AWE) in the literature [33], [34]. In the following sections, the nature of the rational approximant will be briefly explained.

4.2.1 Padé Approximation

Padé approximation is a special case of rational function approximation in which a truncated power series expansion of a function is fit to a quotient of two polynomials. This type of approximation has proven to be very suitable for network transfer functions because network functions have an analytic continuation with poles in the complex s -plane. While such poles can ruin the convergence of an infinite power series in s , a rational approximation, by contrast, will stay sound as long as its denominator order is high enough to account for any nearby poles. In the usual process, the coefficients of the power series are obtained from Taylor series expansion of a network function around a complex frequency s and are referred to as system moments. The process of obtaining rational polynomial coefficients from system moments is called moment matching. For example, a transfer function with moments m_i can be expressed as

$$h(s) = m_0 + m_1s + m_2s^2 + \dots \quad (4.1)$$

From (4.1), a set of equations could be derived to obtain a rational approximation of order L/M for $h(s)$

$$h(s) = \frac{a_0 + a_1s + a_2s^2 + \dots + a_Ls^L}{1 + b_1s + \dots + b_Ms^M} \quad (4.2)$$

where the coefficients a_i and b_i are obtained by

$$\begin{bmatrix} m_{L-M+1} & m_{L-M+2} & \dots & m_L \\ m_{L-M+2} & \dots & \dots & m_{L+1} \\ \dots & \dots & \dots & \dots \\ m_L & m_{L+1} & \dots & m_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-2} \\ \dots \\ b_1 \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ \dots \\ m_{L+M} \end{bmatrix} \quad (4.3)$$

$$\begin{aligned}
a_0 &= m_0 \\
a_1 &= m_1 + b_1 m_0 \\
a_2 &= m_2 + b_1 m_1 + b_2 m_0 \\
&\dots \\
&\dots \\
a_L &= m_L + \sum_{i=1}^{\min(L, M)} b_i m_{L-i}
\end{aligned} \tag{4.4}$$

4.2.2 Practical Aspects of AWE

In practice, the AWE expansion is normally done at the origin of the complex plane. Fig. 4.1 outlines the general steps of an AWE simulation. First the moments of the MNA matrix are obtained. This procedure is normally simple for circuits composed of lumped elements only, but gets more complicated when distributed elements are involved. In the case of EM-based subnetworks whose characteristics are usually not in the closed form, moment calculation itself is a problem which has to be addressed separately (Chapter 8.) All the same, the CPU cost of moment calculation step is negligible compared to L/U decomposition of the large MNA matrix. After obtaining the system moments, the elements corresponding to the desired output are selected from the moment vector. Padé approximation is then applied using (4.3) and (4.4) to obtain a rational transfer function. The poles of the system transfer function are the roots of the denominator polynomial. The residues can be calculated by the method detailed in [58] or as follows

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & \dots & p_L^{-1} & -1 \\ p_1^{-2} & p_2^{-2} & \dots & p_L^{-2} & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ p_1^{-L-1} & p_2^{-L-1} & \dots & p_L^{-L-1} & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_L \\ c \end{bmatrix} = - \begin{bmatrix} m_0 \\ m_1 \\ \vdots \\ m_{L+M} \\ m_L \end{bmatrix} \tag{4.5}$$

Where p , k , c stand for the system pole, residue and constant, respectively. Next the approximate system response in the time domain is stated as

$$h(t) = c\delta(t) + \sum_{i=1}^L k_i e^{p_i t} \quad (4.6)$$

where $\delta(t)$ is the dirac delta function.

Figure 4.1: General steps of Moment Matching.

4.2.3 Limitations of AWE

Although the algorithm for Padé approximation is relatively simple, determining the optimum order of the approximation is however less obvious an issue. This problem is discussed in detail in [36] and [59]. This type of order reduction is generally accurate for RC networks, but could fail for some RLC networks and networks with distributed elements if the denominator order of the approximant is not high enough to capture all the dominant poles. This has led to the development of multi-point expansion techniques as will be seen in the following sections. The following is a list of the general properties which have the most impact on MMTs.

1. As one goes high in the order of the rational approximant, the entries of the Toeplitz matrix in (4.3) tend to get linearly dependent which renders it ill-conditioned. This implies that one could rarely expect to detect more than 8-9 accurate poles from a single expansion.
2. Padé approximation often produces unstable poles on the right hand side of the complex plane.
3. Padé approximation provides no estimates for error bounds.

Here, one should also mention Cauchy method which is used to generate a transfer function using sampled data points and higher order derivatives [55]. As an interpolation technique, Cauchy method is computationally more efficient than AWE approach. The latest combination of Cauchy method with singular value decomposition enables one to determine the order of approximation in advance [56]. However, from stability point of view it doesn't offer any advantages over AWE. Like Padé approximation, unstable poles are produced in the process of approximation. Despite its superiority over AWE in interpola-

tion, it is not suitable for model-reduction and macromodeling since it cannot guarantee a passive model unlike Complex Frequency Hopping and Krylov subspace based methods which will follow.

4.3 Complex Frequency Hopping

For systems with many dominant poles such as transmission line networks, a single Padé expansion will not be able to capture all the dominant poles. For such networks, *Complex Frequency Hopping* (CFH) has been developed [35], [36], [59]. CFH extends the process of moment matching to multiple expansion points in the complex plane near or on the imaginary axis using a binary search algorithm. With a minimized number of frequency point expansions, enough information is obtained to enable the generation of an approximate transfer function that matches the original function up to a predefined highest frequency of interest. By using the information from all the expansion points, CFH ensures that all dominant poles are captured as illustrated in Fig. 4.2. In addition, CFH provides clear bounds on the region of accuracy and with a post-processing technique, it restores the passivity of the generated macromodel [58].

Figure 4.2: Comparison of CFH method against AWE

Complex Frequency Hopping relies on a binary search algorithm to determine the expansion points and to minimize the number of expansions. Binary search algorithm can be based on either transfer function approach or evaluation of the system poles and zeros. In the following sections both approach will be succinctly presented.

4.3.1 CFH with Transfer Function Approach

This is a variation of CFH that uses the transfer functions obtained at various expansion points to evaluate the system frequency response. IFFT is then used for transient analysis. In this regard, this approach can be considered as an interpolation scheme that speeds up the calculation of a frequency domain impulse response. The output of CFH

consists of a set of transfer functions corresponding to the various expansion points as illustrated in Fig. 4.3. Each transfer function is checked for consistency with adjacent transfer functions, which guarantees accuracy within the region of validity.

Figure 4.3: Illustration of Transfer Function Approach

The binary search algorithm employed in CFH transfer function approach can be summarized as follows,

1. *Calculate two transfer functions at the origin ($s=0$) and at the highest frequency of operation ($s=j\omega_{max}$).*
2. *Check the transfer functions at the midpoint $s=j\omega_{max}/2$ between the two extremes. If the mismatch between two is below a prescribed error value, stop the algorithm. Else, calculate another transfer function at $s=j\omega_{max}/2$.*
3. *Continue to search in a similar fashion between $s=0$ and $s=j\omega_{max}/2$ and between $s=j\omega_{max}/2$ and $s=j\omega_{max}$ halving the each frequency interval until the mismatch between two successive transfer function is below the prescribed error value.*

4.3.2 CFH with Pole-Match Approach

In this approach, poles of the transfer function are evaluated at each expansion point. A matching pole which would be obtained at two successive expansion points is used to set the region of accuracy for the those frequency points locally. For this purpose, relative location of a matching pole lying farthest from the expansion point determines the radius of region of accuracy (R_{acc}) for that expansion. Once the R_{acc} is set stable poles sitting in the circular region are accepted as true system poles. Unstable and spurious poles (those outside the R_{acc}) are simply rejected (Fig. 4.4).

Figure 4.4: Illustration of Pole-Matching Algorithm

In this case, the binary search algorithm employed for CFH transfer function approach can be modified as follows,

1. Calculate the system poles at the origin ($s=0$) and at the highest frequency of operation ($s=j\omega_{max}$).

2. *If at least one common pole exists between $s=0$ and $s=j\omega_{max}$ set the R_{acc} for each expansion and collect the poles lying within each R_{acc} as true system poles and stop the algorithm. Else, expand at the midpoint $s=j\omega_{max}/2$ between the two extremes.*
3. *Continue to search in a similar fashion between $s=0$ and $s=j\omega_{max}/2$ and between $s=j\omega_{max}/2$ and $s=j\omega_{max}$ halving the each frequency interval until a common pole is found.*

4.4 Krylov Subspace-Based Methods

Krylov-subspace based methods such as Lanczos and Arnoldi algorithms have emerged as promising options in model-reduction applications. The power of these algorithms stems from the fact that they tend to converge to the dominant eigenvalues of a system by some finite matrix-vector multiplications. The details of these algorithms can be found in [37]-[39]. The direct application of Krylov-subspace based methods to network functions requires the network equations in state-space form [37]. Until lately, Krylov-space techniques concerned only with RLC networks. This is because, lumped-distributed hybrid systems could not be accommodated in state-space forms. The recent advances in the area made it possible to handle distributed interconnect elements and measured subnetworks along with lumped circuit components by a second level of reduction with passivity compensation ([42], [43]) The rest of this section briefly reviews the technique on including distributed elements/EM devices in a Krylov based formulation.

In the discussion below, distributed interconnects and EM-based subnetwork are both assumed to be represented by their terminal characteristics and for the sake of clarity, no distinction has been made between them [29]. From (3.1), it is possible to extract the lin-

ear multiport subnetwork π with n_π ports and N distributed elements/passive EM-devices. Let the number of ports associated with the m th distributed element/EM-device be denoted by n_m . The MNA equations for the entire linear subnetwork can be written as

$$W_\pi \dot{v}(t) + G_\pi v(t) + \sum_{m=0}^{N_m} L i_m(t) = P b_\pi(t); \quad i_\pi(t) = P^T v(t) \quad (4.7)$$

where $P = [p_{i,j} \in \{0, 1\}]$ is a selector matrix, with $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n_\pi\}$ that maps the port voltages into the node space $\mathcal{R}^n$ of the network and $L = l_{i,j} \in \{0, 1\}$ is a selector matrix with $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n_m\}$ with a maximum of one non-zero in each row or column, that maps $i_m(t) \in \mathcal{R}^{n_m}$, the vector of terminal currents entering the distributed/EM-based subnetwork m , into the node space $\mathcal{R}^n$ of the network. The terminal voltages/currents of the distributed/EM-based subnetwork m are related by y-parameters, given by (3.7). Assuming zero initial conditions, the Laplace transform of (4.7) can be written as

$$Y_\pi(s)X(s) = P B_\pi(s); \quad I_\pi = P^T X(s) \quad (4.8)$$

where $B_\pi(s) = \mathcal{L}\{b_\pi(t)\}$ and

$$Y_\pi(s) = \begin{bmatrix} sW_\pi(s) + G_\pi & L \\ Y_m(s)L^t & U \end{bmatrix} \quad (4.9)$$

Once the derivatives of $Y_\pi(s)$ matrix is computed, the moments of $X(s)$ can be obtained and are used in the construction of a Krylov subspace (denoted by K). Performing an orthonormal decomposition on K one has, $K = QR$, where, $Q^T Q = U_q$, $U_q \in \mathbb{R}^{q \times q}$ is an identity matrix and q is the order of reduction. The Q thus obtained is used in the congruence transformation of the variable ($x \rightarrow \hat{x}$),

$$X(s) = Q_{nxq} \hat{X}(s) \quad (4.10)$$

to reduce the original system of n variables to a system of q variables. The reduced order system now becomes

$$\begin{aligned} \hat{Y}_\pi(s) \hat{X}(s) &= \hat{P} B_\pi; & I_\pi &= \hat{P}^T \hat{X}(s) \\ \hat{Y}_\pi(s) &= Q^T Y_\pi(s) Q; & \hat{P} &= Q^T P \end{aligned} \quad (4.11)$$

Due to the distributed nature of the reduced-order model, a direct time-domain realization may not possible. Hence, a second level of reduction is performed on this macromodel using Block CFH and passivity compensation.

4.5 Calculation of The System Moments

All model reduction techniques discussed so far require the calculation of the system moments. Starting from the MNA equations

$$Y_\pi(s) X(s) = B_\pi(s) \quad (4.12)$$

one could obtain

$$\sum_n Y_{\pi,n}(s-s_0)^n \sum_n M_n(s-s_0)^n = \sum_n B_{\pi,n}(s-s_0)^n \quad (4.13)$$

where the coefficients $Y_{\pi,n}$ and $B_{\pi,n}$ are the moments of the MNA matrix and the input vector respectively. For an impulse excitation at port i , $B_{\pi,0}(i)=1$ with the remaining entries equal to zero and $B_{\pi,n}=0$, $n>0$. The moments M_n of the output vector $X(s)$ are the unknowns to be calculated. Recursive formulae for M_n can be derived by expanding (4.13) and equating equal powers of $(s-s_0)$

$$Y_{\pi,0}M_0 = B_{\pi,0} \quad (4.14)$$

$$Y_{\pi,0}M_n = B_{\pi,n} - \sum_{r=1}^n Y_{\pi,r}M_{n-r} \quad (4.15)$$

Note that $Y_{\pi,0}$ does not change on the right hand side, and CPU cost of determining all the moments is only one L/U decomposition. The moments of the transfer function at the desired output correspond to one row of the moment vectors M_n . A recursive relation for the evaluation of moments can be obtained in the form

$$[Y_{\pi}(\alpha)]M_n^i = - \sum_{r=1}^n \frac{\left. \frac{d^r}{ds^r} Y_{\pi}(s) \right|_{s=\alpha}}{r!} M_{n-r}^i; \quad [Y_{\pi}(\alpha)]M_0^i = b_i \quad (4.16)$$

To evaluate the moments M_n^i using (4.16), the derivatives of $Y_{\pi}(s)$ (denoted by $Y_{\pi}^{(r)}$) are needed. They can be obtained using (4.9) as

$$\frac{d}{ds}[Y_{\pi}] = \begin{bmatrix} W & 0 \\ \frac{d}{ds}Y_m L^t & U \end{bmatrix}; \quad \frac{d^r}{ds^r}[Y_{\pi}] = \begin{bmatrix} 0 & 0 \\ \frac{d^r}{ds^r}Y_m L^t & U \end{bmatrix}; \quad (r \geq 2) \quad (4.17)$$

For distributed transmission line interconnects, Y_m has a closed form expression and calculation of its derivatives has been extensively studied in [57] and [59]. The derivatives, $Y_m^{(r)}$ corresponding to the EM-based subnetwork can be computed according to the methods described in Chapter 8.

4.6 Time Domain Macromodel Generation

After obtaining a pole-residue representation for the system parameters $Y_\pi(s)$ can now be approximated as [29]

$$Y_\pi(s) = \begin{bmatrix} c^{1,1} + \sum_{i=1}^q \frac{k_i^{1,1}}{s-p_i} & c^{1,2} + \sum_{i=1}^q \frac{k_i^{1,2}}{s-p_i} & \dots & c^{1,n_\pi} + \sum_{i=1}^q \frac{k_i^{1,n_\pi}}{s-p_i} \\ \vdots & \vdots & \ddots & \vdots \\ c^{n_\pi,1} + \sum_{i=1}^q \frac{k_i^{n_\pi,1}}{s-p_i} & c^{n_\pi,2} + \sum_{i=1}^q \frac{k_i^{n_\pi,2}}{s-p_i} & \dots & c^{n_\pi,n_\pi} + \sum_{i=1}^q \frac{k_i^{n_\pi,n_\pi}}{s-p_i} \end{bmatrix} \quad (4.18)$$

where p_i is the i th pole in the common pole-set, k_i^{lj} are the corresponding residues for Y_π^{ij} , respectively. The poles and residues thus obtained are used to form first-order differential equations as

$$\begin{aligned} \frac{d}{dt}[\mathbf{x}_\pi(t)] - [\mathbf{A}_\pi][\mathbf{x}_\pi(t)] - [\mathbf{B}_\pi][\mathbf{i}_\pi(t)] &= 0 \\ [\mathbf{v}_\pi(t)] - [\mathbf{C}_\pi][\mathbf{x}_\pi(t)] + [\mathbf{D}_\pi][\mathbf{i}_\pi(t)] &= 0 \end{aligned} \quad (4.19)$$

where $\mathbf{i}_\pi$ and $\mathbf{v}_\pi$ are the vectors of terminal currents and voltages of the linear subnetwork π , $\mathbf{x}_\pi$ is the state vector, $\mathbf{A}_\pi$ is the state matrix, $\mathbf{B}_\pi$ is a matrix which relates inputs to states, $\mathbf{C}_\pi$ is a matrix relating states to the outputs and $\mathbf{D}_\pi$ is a matrix relating inputs directly to

output. Equation (4.19) can be easily coupled with the differential equations representing nonlinear components by using the relation $v_\pi = L_\pi^t v_\phi$ as

$$\begin{aligned} \frac{d}{dt}[x_\pi(t)] - [A_\pi][x_\pi(t)] - [B_\pi][i_\pi(t)] &= 0 \\ [L_\pi]^t[v_\phi(t)] - [C_\pi][x_\pi(t)] + [D_\pi][i_\pi(t)] &= 0 \\ W_\phi \frac{d}{dt}v(t) + G_\phi v_\phi(t) + L_\pi i_\pi(t) + F(v_\phi(t)) - b_\phi(t) &= 0 \end{aligned} \quad (4.20)$$

Using standard nonlinear equation solvers or any of the general-purpose circuit simulators, the unified set of differential equations represented by (4.20) can be solved to yield unified transient solutions for the entire nonlinear circuit consisting of high frequency interconnect subnetworks. For those simulators such as HSPICE which do not directly accept the differential equations as input, the time domain macromodel can be converted into an equivalent subcircuit. The details of this conversion can be found in the Appendix.

Chapter 5

TEM Approach to Field Coupling

Analysis in High-Speed Circuits

MTL modeling of interconnects is quite an established technique in the analysis of high-speed circuits. A great deal of research has also been done for field coupling to transmission lines [14]-[24]. However, the interest in SPICE compatible models is fairly recent for incident field coupling to MTLs [16], [22]-[24]. The incident field coupling model developed in [16] is based on method of characteristics [5] and cannot account for *medium losses* in a circuit simulation environment.

In this chapter, an efficient time-domain macromodel for incident field coupling to lossy multiconductor transmission lines is presented. The development of the model involves two parts. First, the exponential matrix describing the Telegrapher's equations is translated to ordinary differential equations through a closed-form matrix-rational approximation [45]. Next, the forcing functions are also converted to rational-functions in a semi-analytic approach. Once a stamp for the forcing functions is generated (which depend on the incident field and the geometry of the structure), it can be superimposed on the source vector in field coupling analysis [27].

This chapter is organized as follows. In Section I, MTL equations with external field effects are derived from Maxwell's equations. Section II presents the details of the MTL stamp involving incident fields and reviews the concept of closed-form matrix-rational approximation of the exponential matrix describing Telegrapher's equations. Section III deals with the formulation of forcing functions with the matrix-rational approximation of the exponential matrix. In Section IV, generation of a time domain macromodel for circuit simulation is explained. Section V presents the numerical results.

5.1 Derivation of Transmission Line Equations with External Field Effects

As mentioned in Chapter 2, the fundamental assumption in transmission line formulation is that the electric field intensity vector and the magnetic field intensity vector lie in a plane transverse to the line axis. Applying Maxwell-Faraday law,

$$\vec{\nabla} \times \vec{E} = -s\vec{B} \quad (5.1)$$

where $\vec{E}$ and $\vec{B}$ are the phasors of the total electric and magnetic fields and s is the complex frequency, respectively. Integrating (5.1) over the rectangular area enclosed by the path shown in the Fig. 5.1 yields

$$\oint (\nabla \times \vec{E}) d\vec{A} = -s \int \vec{B} \hat{a}_n d\vec{A} \quad (5.2)$$

Figure 5.1: Interconnects excited by an arbitrary incident field

where a_n is the unit vector in the outward normal direction. By Stoke's theorem

$$\oint_{c_m} \vec{E} d\vec{l} = -s \int_z^{(z+\Delta z)} \int_{\rho(x_m, y_m)} \vec{B} \cdot \vec{a}_n d\rho dz \quad (5.3)$$

where the subindex m stands for the m th conductor. Expanding the integral in LHS of (5.3) and noting that the integration is confined to the $\rho(x,y)$ - z plane

$$\begin{aligned} \oint_{c_m} \vec{E} d\vec{l} = & \int_{\rho(x_m, y_m)} E_\rho(\rho, z + \Delta z) d\rho - \int_z^{(z+\Delta z)} E_z(\rho_m, z) dz \\ & - \int_{\rho(x_m, y_m)} E_\rho(\rho, z) d\rho + \int_z^{(z+\Delta z)} E_z(\rho_0, z) dz \end{aligned} \quad (5.4)$$

where the subindex 0 stands for the reference conductor. Considering that Δz is an incremental distance

$$\begin{aligned} \oint_{c_m} \vec{E} d\vec{l} = & \int_{\rho(x_m, y_m)} E_\rho(\rho, z + \Delta z) d\rho - E_z(\rho_m, z) \Delta z - \int_{\rho(x_m, y_m)} E_\rho(\rho, z) d\rho + E_z(\rho_0, z) \Delta z \end{aligned} \quad (5.5)$$

from which (5.3) can be rewritten as

$$\int_{\rho(x_m, y_m)} E_\rho(\rho, z + \Delta z) d\rho - E_z(\rho_m, z) \Delta z - \int_{\rho(x_m, y_m)} E_\rho(\rho, z) d\rho + E_z(\rho_0, z) \Delta z = -s \int_z^{(z + \Delta z)} \int_{\rho(x_m, y_m)} \vec{B} \hat{a}_n d\rho dz \quad (5.6)$$

Dividing (5.6) by Δz and taking the limit as $\Delta z \rightarrow 0$

$$\frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_\rho(\rho, z) d\rho - [E_z(\rho_m, z) - E_z(\rho_0, z)] = -s \int_{\rho(x_m, y_m)} \vec{B} \hat{a}_n d\rho \quad (5.7)$$

The fields above are the total fields propagating along the conductors. In scattered field formalism they are comprised of incident and scattered fields. Decomposition in the cartesian coordinate system gives the following relations

$$\begin{aligned} E_x &= E_x^{inc} + E_x^{scat} \\ E_y &= E_y^{inc} + E_y^{scat} \\ E_z &= E_z^{inc} + E_z^{scat} \end{aligned} \quad (5.8)$$

Eq. (5.7) can be written with this new convention as

$$\begin{aligned} \frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_\rho^{scat}(\rho, z) d\rho + \frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_\rho^{inc}(\rho, z) d\rho \\ - [E_z(\rho_m, z) - E_z(\rho_0, z)] = -s \int_{\rho(x_m, y_m)} B_n^{inc} d\rho - s \int_{\rho(x_m, y_m)} B_n^{scat} d\rho \end{aligned} \quad (5.9)$$

Separating incident and scattered fields in (5.9) yields

$$\begin{aligned} \frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_{\rho}^{scat}(\rho, z) d\rho + s \int_{\rho(x_m, y_m)} B_n^{scat} d\rho - [E_z(\rho_m, z) - E_z(\rho_0, z)] = \\ \frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_{\rho}^{inc}(\rho, z) d\rho - s \int_{\rho(x_m, y_m)} B_n^{inc} d\rho \end{aligned} \quad (5.10)$$

Since the structure supports TEM mode of propagation the transverse fields are of static nature. Hence, the electric field can be written as the gradient of a scalar potential

$$V^{scat}(z) = - \int_{\rho(x_m, y_m)} E_{\rho}^{scat}(\rho, z) d\rho \quad (5.11)$$

At this stage, it is useful to define per-unit-length self and mutual inductance L_{mm} and L_{ml} , respectively

$$\begin{aligned} L_{m,m} &= \frac{- \int_{\rho(x_m, y_m)} \vec{B} \hat{a}_n d\rho}{\int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(\rho_m, z) \hat{a}_z d\rho} \\ L_{m,l} &= \frac{- \int_{\rho(x_m, y_m)} \vec{B} \hat{a}_n d\rho}{\int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(\rho_l, z) \hat{a}_z d\rho} \end{aligned} \quad (5.12)$$

where the current density at the ends of the trace can be expressed as

$$\int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(\rho_m, z) \hat{a}_z d\rho = I(z) \quad (5.13)$$

Combining (5.12) and (5.13) gives

$$\int_{\rho(x_m, y_m)} \vec{B} \hat{a}_n d\rho = -L_{mm} I(z) \quad (5.14)$$

Similar relation holds for the mutual inductance, too. Total fields can be related to surface current density along the line as follows

$$Z_{mm} = \frac{E_z(\rho_m, z) - E_z(\rho_0, z)}{\int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(\rho_m, z) \hat{a}_z d\rho} \quad (5.15)$$

Neglecting the skin effect losses and using (5.13) total fields can be expressed as follows

$$E_z(\rho_m, z) - E_z(\rho_0, z) = R_{mm} I(z) \quad (5.16)$$

By using (5.11), (5.14) and (5.16); (5.10) can be expressed in a matrix form

$$\frac{\partial}{\partial z} V^{scat}(z) + sLI(z) + RI(z) = \frac{\partial}{\partial z} \int_{\rho(x_m, y_m)} E_{\rho}^{inc}(\rho, z) d\rho + s \int_{\rho(x_m, y_m)} B_n^{inc} d\rho \quad (5.17)$$

In circuit theory, it is more convenient to deal with voltage and current values corresponding to total fields. Since the total voltage at any cross section of the line can be expressed as

$$\begin{aligned} V(z) &= V^{scat}(z) + V^{inc}(z) \\ V(z) &= V^{scat}(z) - \int_{\rho(x_m, y_m)} E_{\rho}^{inc}(\rho, z) d\rho \end{aligned} \quad (5.18)$$

from which (5.17) can be expressed as

$$\frac{\partial}{\partial z} V(z) + sLI(z) + RI(z) = s \int_{\rho(x_m, y_m)} B_n^{inc} d\rho \quad (5.19)$$

From (5.7) it follows that

$$\begin{aligned} \frac{\partial}{\partial z} V(z) + sLI(z) + RI(z) = \\ -\frac{\partial}{\partial z} \int_{\rho(x,y)} E_{\rho}^{inc}(\rho, z) d\rho + E_z^{inc}(\rho, z) - E_z^{inc}(\rho_0, z) \end{aligned} \quad (5.20)$$

Now, consider the Maxwell-Ampere's law

$$\nabla \times \vec{H} = \vec{J} + s\epsilon \vec{E} \quad (5.21)$$

Integrate (5.21) over the surface enclosing elementary volume of the trace (Fig. 5.2)

$$\oint \nabla \times \vec{H} \cdot d\vec{A} = \oint \vec{J}(z) \cdot d\vec{A} + s \oint \epsilon \vec{E}^{scat} \cdot d\vec{A} \quad (5.22)$$

Figure 5.2: Elementary volume enclosing incremental trace

For the above geometry, current density at trace ends can be expressed as

$$\begin{aligned} \int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(y, z) \hat{a}_z dy &= I(z) \\ \int_{-\frac{w}{2}}^{\frac{w}{2}} J_s(y, z + \Delta z) \hat{a}_z dy &= I(z + \Delta z) \end{aligned} \quad (5.23)$$

Combining the above expressions, one obtains

$$\int_{ENDS} \vec{J}(z) d\vec{A} = I(z + \Delta z) - I(z) \quad (5.24)$$

On the cylindrical surface

$$\int_{CYL} \vec{J}(z) d\vec{A} = \Delta z \left(2 \int_0^{\Delta x} \sigma \vec{E}_y^{scat} dx + 2 \int_{-\frac{w}{2}}^{\frac{w}{2}} \sigma \vec{E}_x^{scat} dy \right) \quad (5.25)$$

where σ is the conductivity of the medium surrounding the trace. Hence on the surface enclosing the elementary volume

$$\oint \vec{J}(z) \cdot d\vec{A} = I(z + \Delta z) - I(z) + \Delta z \left(2 \int_0^{\Delta x} \sigma \vec{E}_y^{scat} dx + 2 \int_{-\frac{w}{2}}^{\frac{w}{2}} \sigma \vec{E}_x^{scat} dy \right) \quad (5.26)$$

Dividing (5.26) by Δz and taking the limit as $\Delta z \rightarrow 0$

$$\oint \vec{J}(z) \cdot d\vec{A} = \frac{\partial}{\partial z} I(z) + 2 \int_0^{\Delta x} \sigma \vec{E}_y^{scat} dx + 2 \int_{-\frac{w}{2}}^{\frac{w}{2}} \sigma \vec{E}_x^{scat} dy \quad (5.27)$$

Defining per-unit-length self and mutual capacitance C_{mm} and C_{ml} , respectively

$$C_{mm} = \frac{2 \int_{-w/2}^{w/2} \epsilon \vec{E}^{scat} \hat{a}_x dy + 2 \int_0^{\Delta x} \epsilon \vec{E}^{scat} \hat{a}_y dx}{\int_{\rho(x_m, y_m)} E_{\rho}^{scat}(\rho, z) d\rho} \quad (5.28)$$

$$C_{ml} = \frac{2 \int_{-w/2}^{w/2} \epsilon \vec{E}^{scat} \hat{a}_x dy + 2 \int_0^{\Delta x} \epsilon \vec{E}^{scat} \hat{a}_y dx}{\int_{\rho(x_l, y_l)} E_{\rho}^{scat}(\rho, z) d\rho}$$

since

$$\oint \epsilon \vec{E}^{scat} \cdot d\vec{A} = 2 \int_{-w/2}^{w/2} \epsilon \vec{E}^{scat} \hat{a}_x dy + 2 \int_0^{\Delta x} \epsilon \vec{E}^{scat} \hat{a}_y dx \quad (5.29)$$

Using (5.11) the second term on the RHS of (5.22) can be expressed as follows

$$\oint \epsilon \vec{E}^{scat} \cdot d\vec{A} = C_{mm} V^{scat} \quad (5.30)$$

A similar relation holds for the mutual capacitance. Assume that conductivity of the surrounding medium is uniform then,

$$2 \int_0^{\Delta x} \sigma \vec{E}_y^{scat} dx + 2 \int_{-\frac{w}{2}}^{\frac{w}{2}} \sigma \vec{E}_x^{scat} dy = \sigma \int_{cyl} \vec{E}^{scat} d\vec{A}' \quad (5.31)$$

where A' is the surface enclosing the cylindrical portion of the elementary volume. The RHS of the (5.31) can be expressed as

$$\sigma \int_{cyl} \vec{E}^{scat} d\vec{A}' = \frac{\sigma}{\epsilon} C_{mm} V^{scat} \quad (5.32)$$

Defining

$$G = \frac{\sigma}{\epsilon} C \quad (5.33)$$

Eq.(5.22) becomes

$$\oint \nabla \times \vec{H} \cdot d\vec{A} = \frac{\partial}{\partial z} I(z) + G V^{scat}(z) + s C V^{scat}(z) \quad (5.34)$$

Since LHS of (5.34) is simply zero over the closed surface, the LHS is the second transmission line equation in terms of scattered fields. It can be expressed in terms of total voltage by using (5.18)

$$\frac{\partial}{\partial z}I(z) + \mathbf{G}V(z) + s\mathbf{C}V(z) = -\mathbf{G} \int_{\rho(x,y)} \mathbf{E}_{\rho}^{inc}(\rho, z)d\rho - s\mathbf{C} \int_{\rho(x,y)} \mathbf{E}_{\rho}^{inc}(\rho, z)d\rho \quad (5.35)$$

which is the second transmission line equation with external field effects.

5.2 Calculation of an MTL Stamp with Incident Field Effects

Equations (5.20) and (5.35) can be combined in a compact form

$$\frac{d}{dz} \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} = -(\mathbf{D} + s\mathbf{E}) \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} + \mathbf{F}(z, s) \quad (5.36)$$

where

$$\mathbf{D} = \begin{bmatrix} 0 & \mathbf{R} \\ \mathbf{G} & 0 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 & \mathbf{L} \\ \mathbf{C} & 0 \end{bmatrix} \quad (5.37)$$

and

$$\mathbf{F}(s, z) = \begin{bmatrix} -\frac{\partial}{\partial z} \int_{\rho(x,y)} \mathbf{E}_{\rho}^{inc}(\rho, z)d\rho + \mathbf{E}_z^{inc}(\rho, z) - \mathbf{E}_z^{inc}(\rho_0, z) \\ -\mathbf{G} \int_{\rho(x,y)} \mathbf{E}_{\rho}^{inc}(\rho, z)d\rho - s\mathbf{C} \int_{\rho(x,y)} \mathbf{E}_{\rho}^{inc}(\rho, z)d\rho \end{bmatrix} \quad (5.38)$$

For circuit simulation, the multiconductor transmission line (MTL) stamp is derived from the solution of (5.36), which can be put into a state space representation form,

$$\frac{d}{dz}\mathbf{x}(s, z) = -\boldsymbol{\varphi}(s)\mathbf{x}(s, z) + \mathbf{F}(z, s) \quad (5.39)$$

where

$$\mathbf{x}(s, z) = \begin{bmatrix} \mathbf{V}(z, s) \\ \mathbf{I}(z, s) \end{bmatrix} \quad (5.40)$$

and

$$\varphi(s) = \mathbf{D} + s\mathbf{E} \quad (5.41)$$

Due to the addition of the sources $\mathbf{F}(z, s)$, equation (5.39) is an inhomogeneous first order differential equation in z . Its solution can be found from the sum of the homogeneous solution and the particular solution. The homogeneous equation is

$$\frac{\partial}{\partial z} \mathbf{x}(z, s) + \varphi(s) \mathbf{x}(z, s) = 0 \quad (5.42)$$

whose solution with a prescribed initial state z_0 can be obtained as

$$\mathbf{x}(z, s) = e^{-(z-z_0)\varphi(s)} \mathbf{x}(z_0, s) \quad (5.43)$$

The homogeneous solution in (5.43) is often referred to as the *state transmission matrix* in linear systems literature. The particular solution of (5.39) can then be found by convolution integral

$$\mathbf{x}(z, s) = \int_{z_0}^z e^{(z-\gamma)\varphi(s)} \mathbf{F}(s, \gamma) d\gamma \quad (5.44)$$

The final solution for the inhomogeneous Telegrapher's equations is the sum of the homogeneous solution and the particular solution

$$\mathbf{x}(z, s) = e^{-(z-z_0)\varphi(s)} \mathbf{x}(z_0, s) + \int_{z_0}^z e^{(z-\gamma)\varphi(s)} \mathbf{F}(s, \gamma) d\gamma \quad (5.45)$$

If the near end of the interconnect is taken to be at the origin of the axis ($z_0 = 0$) and transmission lines of length l ($z = l$) are considered, near and far end voltages and currents can be related by *state transition matrix* $T(s)$

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = T(s) \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + b(s) \quad (5.46)$$

where the *state transition matrix* $T(s)$ and *input vector* $b(s)$ are explicitly written as,

$$T(s) = e^{-\Phi l} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (5.47)$$

$$b(s) = e^{-\Phi l} \int_0^l e^{\Phi z} F(z, s) dz \quad (5.48)$$

The transmission line stamp with forcing functions therefore becomes

$$A \begin{bmatrix} V(0) \\ V(l) \end{bmatrix} + B \begin{bmatrix} I(0) \\ I(l) \end{bmatrix} = b(s) \quad (5.49)$$

where

$$A = \begin{bmatrix} -T_{11} & U_m \\ T_{21} & 0 \end{bmatrix} \quad B = \begin{bmatrix} T_{12} & 0 \\ -T_{22} & U_m \end{bmatrix} \quad (5.50)$$

where U_m is the m -by- m identity matrix where m is the total number of the conductors.

Eq. (5.49) can be inserted in the *Modified Nodal Analysis* (MNA) matrix equations as

$$\begin{bmatrix} (H_\phi + sW_\phi) & D_1 & \dots & D_{N_t} \\ A_1 D_1^t & B_1 & 0 & 0 \\ \dots & 0 & \dots & 0 \\ A_{N_t} D_{N_t}^t & 0 & 0 & B_{N_t} \end{bmatrix} \begin{bmatrix} V(s) \\ I_1(s) \\ \dots \\ I_{N_t}(s) \end{bmatrix} = \begin{bmatrix} e_\phi(s) \\ b_1(s) \\ \dots \\ b_{N_t}(s) \end{bmatrix} \quad (5.51)$$

Solution to (5.51) accounts for crosstalk effects as well as the external field excitation on the conducting traces represented as MTL.

The MTL stamp in (5.49) is expressed in the frequency domain whereas a transient solution to the general nonlinear circuit equation of (3.1) can be obtained in the time domain. Two equations can be incorporated through a model order reduction algorithm as explained in Chapter 4.

Alternatively, a time domain macromodel for MTL equations can be obtained through a matrix-Pad approximation to the state transition matrix T . The exponential matrix $e^{\Phi l}$ of (5.47) can then be written as

$$e^{\Phi l} \approx [P_{N,M}(\Phi l)]^{-1} Q_{N,M}(\Phi l) \quad (5.52)$$

where $P_{N,M}(\Phi l)$ and $Q_{N,M}(\Phi l)$ are *polynomial matrices* that can be expressed in terms of a closed-form Pad rational function as

$$\begin{aligned} P_{N,M}(\Phi l) &= \sum_{j=0}^N \frac{(N+M-j)!N!}{(N+M)!j!(N-j)!} (-\Phi l)^j \\ Q_{N,M}(\Phi l) &= \sum_{j=0}^M \frac{(N+M-j)!M!}{(N+M)!j!(M-j)!} (\Phi l)^j \end{aligned} \quad (5.53)$$

The Pad rational function of (5.53) can be expressed in terms of products of lower order sections obtained by the poles and zeros of the Pad approximation. For $M=N=n$, the passivity of the proposed macromodel is guaranteed, as well as an error criteria for selecting the order of the approximation exists. Once the exponential matrix is converted into a rational form, an equivalent representation in terms of ordinary differential equations can be obtained [45]. The differential equations can be integrated with conventional circuit

simulators such as SPICE in the form of a time-domain macromodel. For incident field effects, a time domain modeling of $b(s)$ is also needed, which is the subject of the next section.

5.3 Formulation of The Forcing Function $b(s)$

In the case of uniform plane wave excitation of traces, it is possible to express the incoming field analytically (see Fig. 5.3)

$$\vec{E}(x, y, z) = E_0(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)e^{-s\kappa_x x}e^{-s\kappa_y y}e^{-s\kappa_z z} \quad (5.54)$$

where E_0 is the amplitude, A_x, A_y, A_z are the components of the incoming wave along x, y, z axes of the rectangular coordinate system describing the line [2]

$$\begin{aligned} A_x &= \sin\theta_p \sin\theta \\ A_y &= -\cos\theta \cos\phi \sin\theta_p - \cos\theta_p \sin\phi \\ A_z &= -\cos\theta \sin\phi \sin\theta_p - \cos\theta_p \cos\phi \end{aligned} \quad (5.55)$$

with

$$A_x + A_y + A_z = 1 \quad (5.56)$$

and

$$\kappa_z = -\frac{\sin\theta \sin\phi}{c}; \quad \kappa_x = -\frac{\cos\theta}{c}; \quad \kappa_y = -\frac{\sin\theta \cos\phi}{c} \quad (5.57)$$

with c being the speed of phase front.

Figure 5.3: The Geometry of Incident Field Coupling

Assuming that the reference conductor is positioned at the origin, (5.48) can be written as [27]

$$b(s) = e^{Zl} \int_0^l e^{-Zz} \Gamma(s) e^{-s\kappa_z z} dz \quad (5.58)$$

The term $\Gamma(s)$ in (5.58) describes the interaction of the incoming fields with the lines in the transverse plane as

$$\Gamma(s) = \begin{bmatrix} \tilde{V}_F(s) \\ \tilde{I}_F(s) \end{bmatrix} \quad (5.59)$$

For a transmission medium with no solid ground plane, the terms on the RHS of (5.59) can be expressed as

$$\tilde{V}_F(s) = \begin{bmatrix} \vdots \\ \left(-\frac{\kappa_z \Phi_i^+}{\psi_i^+} + A_z\right)(e^{-s\psi_i^+} - 1) \\ \vdots \end{bmatrix} \quad (5.60)$$

$$\tilde{I}_F(s) = \left(\frac{G}{s} + C\right) \begin{bmatrix} \vdots \\ \frac{\Phi_i^+}{\psi_i^+}(e^{-s\psi_i^+} - 1) \\ \vdots \end{bmatrix} \quad (5.61)$$

and for a microstrip structure, which is backed by a ground plane we have

$$\tilde{V}_F(s) = \begin{bmatrix} \vdots \\ -\kappa_z \left(\Phi_i^+ \frac{e^{-s\psi_i^+} - 1}{\psi_i^+} + \Phi_i^- \frac{e^{-s\psi_i^-} - 1}{\psi_i^-} \right) + A_z (e^{-s\psi_i^+} - e^{-s\psi_i^-}) \\ \vdots \end{bmatrix} \quad (5.62)$$

and

$$\tilde{I}_F(s) = \left(\frac{G}{s} + C\right) \begin{bmatrix} \vdots \\ \Phi_i^+ \frac{e^{-s\psi_i^+} - 1}{\psi_i^+} + \Phi_i^- \frac{e^{-s\psi_i^-} - 1}{\psi_i^-} \\ \vdots \end{bmatrix} \quad (5.63)$$

In (5.60)-(5.63), the subscript i refers to the i th conductor and Φ_i , ψ_i are defined as [2]

$$\Phi_i^\pm = A_x x_i \pm A_y y_i; \quad \psi_i^\pm = \kappa_x x_i \pm \kappa_y y_i \quad (5.64)$$

Next, define

$$H = -Z - s\kappa_z U_{2m} \quad (5.65)$$

where U_{2m} is the identity matrix of size $2m$ -by- $2m$. The solution to (5.48) results in

$$b(s) = e^{Zl} (e^{Hl} - U_{2m}) H^{-1} \Gamma(s) \quad (5.66)$$

which can be further simplified as follows

$$b(s) = (e^{-s\kappa_z U_{2m}} - e^{Zl}) H^{-1} \Gamma(s) \quad (5.67)$$

5.4 Time Domain Macromodel for SPICE

Eq. (5.46) can be rewritten as

$$\begin{bmatrix} V(s, l) \\ I(s, l) \end{bmatrix} = \begin{bmatrix} T_{11}(s, l) & T_{12}(s, l) \\ T_{21}(s, l) & T_{22}(s, l) \end{bmatrix} \begin{bmatrix} V(s, 0) \\ I(s, 0) \end{bmatrix} + b(s, l) \quad (5.68)$$

An equivalent representation in terms of the admittance parameters can be written as (the term s is omitted for simplicity)

$$\begin{aligned} \begin{bmatrix} I(0) \\ I(l) \end{bmatrix} &= \begin{bmatrix} -T_{12}(l)^{-1} T_{11}(l) & T_{12}(l)^{-1} \\ T_{21}(l) - T_{22}(l) T_{12}(l)^{-1} T_{11}(l) & T_{22}(l) T_{12}(l)^{-1} \end{bmatrix} \begin{bmatrix} V(0) \\ V(l) \end{bmatrix} \\ &+ \begin{bmatrix} -T_{12}(l)^{-1} & 0 \\ -T_{22}(l) T_{12}(l)^{-1} & U_m \end{bmatrix} \begin{bmatrix} V_{FT}(l) \\ I_{FT}(l) \end{bmatrix} \end{aligned} \quad (5.69)$$

The second term on R.H.S. of (5.69) represents the short circuit current sources. These sources can be superimposed at the terminals of the line during a transient simulation (Fig. 5.4).

Figure 5.4: Equivalent Circuit Representation of Field Coupling to Transmission Lines

In order to obtain a SPICE model for the short circuit current sources, $I_{sc}(0)$ and $I_{sc}(l)$ are expressed in terms of rational functions in s and delay elements, as follows:

Using (5.67) and (5.69), the short circuit current sources can be written as

$$\begin{bmatrix} I_{sc}(0) \\ I_{sc}(l) \end{bmatrix} = \begin{bmatrix} -T_{12}(l)^{-1} & 0 \\ -T_{22}(l)T_{12}(l)^{-1} & U_m \end{bmatrix} \left(\left(e^{-s\kappa_z U_{2m}} - \begin{bmatrix} T_{11}(l) & T_{12}(l) \\ T_{21}(l) & T_{22}(l) \end{bmatrix} \right) \mathbf{H}^{-1} \Gamma(s) \right) \quad (5.70)$$

Eq (5.70), can be simplified to

$$\begin{bmatrix} I_{sc}(0) \\ I_{sc}(l) \end{bmatrix} = \mathbf{W} \mathbf{H}^{-1} \Gamma(s) \quad (5.71)$$

where

$$\mathbf{W} = \begin{bmatrix} W_{11}(l) & W_{12}(l) \\ W_{21}(l) & W_{22}(l) \end{bmatrix} \quad (5.72)$$

$$\begin{aligned}
W_{11}(l) &= -T_{12}(l)^{-1}(e^{-s\kappa_z U_m} - T_{11}(l)) \\
W_{12}(l) &= U_m \\
W_{21}(l) &= -T_{22}(l)T_{12}(l)^{-1}(e^{-s\kappa_z U_m} - T_{11}(l)) - T_{21}(l) \\
W_{22}(l) &= e^{-s\kappa_z U_m}
\end{aligned} \tag{5.73}$$

The term $\Gamma(s)$ as described by (5.59)-(5.64) contributes to current sources with a single pole at the origin along with time delay functions composed of incident field angular parameters and line positions in the transverse plane.

Next, the term H^{-1} is converted to a rational-function form (in terms of poles and residues) as follows. From (5.65), we note that the term H^{-1} can be written as

$$H^{-1} = (D + s\tilde{E})^{-1} \tag{5.74}$$

where

$$\tilde{E} = \begin{bmatrix} -\kappa_z U_m & L \\ C & -\kappa_z U_m \end{bmatrix} \tag{5.75}$$

Eq. (5.74) can be rewritten as

$$H^{-1} = (U_{2m} + sD^{-1}\tilde{E})^{-1}D^{-1} \tag{5.76}$$

Assuming that the matrix $D^{-1}\tilde{E}$ has $2m$ distinct eigenvalues (i.e. diagonalizable), it can be expressed in terms of its eigenvalues and eigenvectors using a similarity transformation as

$$D^{-1}\tilde{E} = F\Lambda F^{-1} \tag{5.77}$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots)$ is a diagonal matrix containing the eigenvalues of $D^{-1}\tilde{E}$ and F contains the eigenvectors of $D^{-1}\tilde{E}$.

Next, substituting (5.77) into (5.76), we can write

$$H^{-1} = F(U_{2m} + s\Lambda)^{-1}F^{-1}D^{-1} \quad (5.78)$$

The poles of H^{-1} are the negative reciprocal of the eigenvalues λ_i and the corresponding residues of the poles are obtained in terms of the entries of D and F .

In case the matrix $D^{-1}\tilde{E}$ is not diagonalizable, canonical forms can be used. For example, using Schur canonical form [34], $D^{-1}\tilde{E}$ can be written as $D^{-1}\tilde{E} = F\Lambda F^t$, where the matrix F in this case is an orthogonal matrix and Λ is a quasi-upper triangle matrix. Alternatively, Jordan canonical form can be used. However, computation of the Schur form is usually numerically more stable.

Similarly, the matrix W of (5.71) can be expressed in a rational form using the closed-form Padé approximation of (5.52) to represent the state-transition matrix e^{Zl} . For example, replacing T in (5.73) by its equivalent Padé approximation, $W_{11}(l)$ can be expressed as

$$\begin{aligned} W_{11}(l) &= T_{12}(l)^{-1}T_{11}(l) - T_{12}(l)^{-1}e^{-s\kappa_z} \\ &= \left(K_1 b + \sum_{i=0} \frac{r_{1i}}{s - p_i} \right) + \left(K_2 b + \sum_{i=0} \frac{r_{2i}}{s - p_i} \right) e^{-s\kappa_z} \end{aligned} \quad (5.79)$$

where p_i represent the poles of the rational function, r_{1i} , r_{2i} are the corresponding residue matrices of dimensions $m \times m$, and K_1, K_2 are predetermined constants based on closed-form Padé approximation.

Using (5.78) and (5.79), the short-circuit-current sources in (5.71) can be described as rational-functions in s and time-delay elements, which can be easily converted to equivalent time-domain sources suitable for SPICE simulation.

5.5 Numerical Example

In this section, a lossy microstrip structure with two conductors is used to validate the proposed algorithm for incident field coupling analysis. In the first part of this example, the structure is analyzed in the presence of incident fields with matched loads. In the second part, the structure is terminated by nonlinear elements and transient analysis in the presence of field coupling is studied.

Figure 5.5: Coupled Lossy Microstrip Lines as a Test Circuit

The incident field is a TM-polarized plane wave illuminating the lines at $\theta=45^\circ$ and $\phi=0^\circ$ angles. The per-unit-length parameter matrix of the structure is calculated by a static field solver, and is as follows,

$$L = \begin{bmatrix} 623.3 & 132.9 \\ 132.9 & 623.3 \end{bmatrix} nH/m \quad C = \begin{bmatrix} 50.18 & -7.359 \\ -7.359 & 50.18 \end{bmatrix} pF/m$$

$$R = \begin{bmatrix} 42.73 & 2.401 \\ 2.401 & 42.73 \end{bmatrix} \Omega/m \quad G = \begin{bmatrix} 2.514 & -0.2865 \\ -0.2865 & 2.514 \end{bmatrix} m \quad \Omega/m$$

Following the steps from (5.58) through (5.69), the near and far-end short-circuit current sources due to incoming fields are obtained in closed form. In Fig. 5.6, the sources obtained with the proposed method are compared to those obtained with conventional frequency domain analysis [2]. As seen the results are indistinguishable.

Figure 5.6: Near End Equivalent Current Sources in The Frequency Domain

Here, it should be noted that there is no phase delay between two ends of the line due to the azimuthal incidence angle ($\phi=0^\circ$), hence both sources are identical. Next, a SPICE model is generated from this pole-residue set as explained above. All the calculations so far are based on impulse excitation of the line. For a transient simulation in SPICE, a time domain envelope waveform is needed. A gaussian pulse waveform with an amplitude of 1KV/m and a bandwidth of 10 GHz is used for this purpose. The pulse is defined in a piecewise manner in SPICE. For validating the results, coupling to the test structure is also obtained in the frequency domain through conventional analysis [2]. The result is multiplied with the spectrum of the gaussian pulse and the time domain response is obtained by discrete (inverse) Fourier transform (IDFT). Fig. 5.7 shows the comparison of results from both the approaches, and they match accurately.

Figure 5.7: Coupled voltage waveform for the linear circuit

The microstrip structure of Fig. 5.5 is connected to a CMOS inverter circuit for nonlinear simulation (Fig. 5.8). In addition to external fields, the circuit is driven by a voltage source having a rectangular waveform with an amplitude of 5 volts , switching transition time of 0.1 ns and duty cycle of 2 ns . The objective of this example is to predict the effect of incident fields on the circuit operation. For this purpose, (a) the circuit is initially analyzed with no external field disruption, (b) with the effect of external fields, using the proposed SPICE model. Both results are plotted in Fig. 5.9. It is to be noted that, it is difficult to handle the incident field analysis of such lossy multiconductor structures with nonlinear terminations using conventional frequency-domain approach.

As a final remark, it is to be noted that incident field in free space will be modified due to the dielectric slab surrounding the traces. In the examples of this section, to simplify the

problem, incident field is assumed to be that with the conductors and the slab *removed*. However, it can be accurately computed as described in the next chapter without compromising on the closed form expression of the equivalent current sources.

Figure 5.8: (a) Nonlinear Test Circuit (b) CMOS inverter schematics

Figure 5.9: Output Waveform of The Nonlinear Test Circuit

Chapter 6

Analysis of Medium Effects on Incident Field Coupling

In quasi-TEM approach to incident field coupling analysis, inhomogeneity of the medium is accounted for only in per-unit-length parameters of the interconnects. The effect of the dielectric medium on the incoming field is usually neglected for the sake of simplicity. In the previous chapter, incident field has been assumed to propagate in a homogeneous and obstacle-free medium. However, it is obvious that the incident field in free space will be modified due to the dielectric slab surrounding the traces. The forcing functions expressed in (5.20) and (5.35) account for any modification incident fields can undergo due to medium effects.

Exploiting this property, this chapter is concerned with the calculation of the fields in a planarly layered medium with the conductors removed. Next, the effect of dielectric medium on incoming fields is analyzed and is shown that the usual assumption of fields propagating in a homogeneous medium may result in intolerable errors at high frequencies. Finally, an efficient method is proposed for the accurate simulation incident field coupling

to traces immersed in a planarly layered inhomogeneous medium. The fields in the layered medium are calculated using physical optics technique. The Chapter is concluded with numerical examples.

6.1 Field Coupling to Traces in Planarly Layered Media

In layered media, E_p^{inc} and E_z^{inc} will take on different values from those in free space. The modified fields in the layered medium can be determined by the physical optics method ([60], [61]). According to physical optics, a plane wave incident on a planarly layered medium undergoes multiple reflections and refractions, which results in two sorts of waves in each layer, (1) transmitted and (2) reflected waves. Assuming the reference conductor is placed at the origin of the coordinate system, those waves can be written as follows

$$\begin{aligned}\vec{E}^i(x, y, z) &= (A_{x,i}\hat{a}_x + A_{y,i}\hat{a}_y + A_{z,i}\hat{a}_z)e^{-s\kappa_x x}e^{-s\kappa_y y}e^{-s\kappa_z z} \\ \vec{E}^r(x, y, z) &= (B_{x,i}\hat{a}_x - B_{y,i}\hat{a}_y - B_{z,i}\hat{a}_z)e^{s\kappa_x x}e^{-s\kappa_y y}e^{-s\kappa_z z}\end{aligned}\quad (6.1)$$

where A_i and B_i are the amplitudes of the waves travelling in the same direction as the incident and reflected waves, respectively, with the subscript i referring to the layer number. Calculation of the coefficients A_i and B_i in (6.1) is the objective of the next section. The total field within the i th layer is the sum of the transmitted and reflected fields. In the transverse plane, the total electric field $E_{p,i}$ can be written as follows

$$\begin{aligned}E_{p,i}(x, y, z) &= [(A_{x,i}e^{-s\kappa_x x} + B_{x,i}e^{s\kappa_x x})\hat{a}_x \\ &\quad + (A_{y,i}e^{-s\kappa_x x} - B_{y,i}e^{s\kappa_x x})\hat{a}_y]e^{-js\kappa_y y}e^{-s\kappa_z z}\end{aligned}\quad (6.2)$$

The line integral of the transverse field between the ground trace and the m th trace positioned in the i th layer is written as follows

$$V(x_m, y_m, z) = \int_{\rho(x_m, y_m)} \vec{E}_{\rho, i}(\rho, z) d\vec{\rho} \quad (6.3)$$

If the reference and the signal traces don't lie in the same layer the integration in the transverse plane must include the effect of each layer in-between. Solution of (6.3) yields transverse component of the forcing function in the i th layer and at the m th conductor position as

$$V_i(x_m, y_m, z) = - \left[\frac{e^{-s(\kappa_{x, i} x_m + \kappa_{y, i} y_m)} - 1}{\kappa_{x, i} x_m + \kappa_{y, i} y_m} (A_{x, i} x_m + A_{y, i} y_m) - \frac{e^{j(\kappa_{x, i} x_m - \kappa_{y, i} y_m)} - 1}{\kappa_{x, i} x_m - \kappa_{y, i} y_m} (B_{x, i} x_m - B_{y, i} y_m) \right] \frac{e^{-js\kappa_z z}}{s} \quad (6.4)$$

The longitudinal component of the forcing function E_L can be written in a similar fashion

$$\vec{E}_L(x_m, y_m, z) = (E_{z, i}(x_m, y_m, z) - E_z(x_0, y_0, z)) \hat{a}_z \quad (6.5)$$

where

$$E_{z, i}(x_m, y_m, z) = (A_{z, i} e^{-s\kappa_{x, i} x_m} - B_{z, i} e^{s\kappa_{x, i} x_m}) e^{-s\kappa_{y, i} y_m} e^{-s\kappa_z z} \quad (6.6)$$

Setting the reference conductor at the origin of the coordinate system ($x_0=y_0=0$), one obtains

$$\begin{aligned} \vec{E}_L(x_m, y_m, z) = & [A_{z,i}(e^{-s(\kappa_x x_m + \kappa_y y_m)} - 1) \\ & - B_{z,i}(e^{s(\kappa_x x_m - \kappa_y y_m)} - 1)] e^{-s\kappa_z z} \hat{a}_z \end{aligned} \quad (6.7)$$

The expressions (6.4) and (6.7) are of the same form as those obtained in [2] for the homogeneous case, except that the field amplitudes A and B in this case depend on frequency. Incorporating these equations to (5.20) and (5.35) one obtains

$$\frac{\partial}{\partial z} V(s, z) + sLI(s, z) + RI(s, z) = V_F(s, z)$$

$$\frac{\partial}{\partial z} I(s, z) + GV(s, z) + sCV(s, z) = I_F(s, z)$$

where the entries of RHS in the above expressions are

$$V_{F,m}(s, z) = -\frac{\partial}{\partial z} V_t(x_m, y_m, z) + E_L(x_m, y_m, z) \quad (6.8)$$

$$I_{F,m}(s, z) = -sCV_t(x_m, y_m, z) - GV_t(x_m, y_m, z) \quad (6.9)$$

Having obtained the expressions for transverse and longitudinal fields, the next step is to find the field coefficients, which will be investigated in the upcoming section.

6.2 Calculation of The Fields within The Layers

In order to solve (6.8) and (6.9), fields in the layered medium must be calculated with conducting traces removed and the dielectric medium in place. For a plane wave whose wavefront normal to the xz -plane and which is incident on a layered medium at an angle θ with respect to the x -axis (Fig. 6.1), equation (6.2) takes on the following form

Figure 6.1: Plane wave incidence on a slab

$$\vec{E}_{p,i}(x, y, z) = (A_{x,i}e^{-jk_ix\cos\theta_i} + B_{x,i}e^{jk_ix\cos\theta_i})e^{-jk_iz\sin\theta_i} \quad (6.10)$$

where

$$k_i = \frac{\omega}{c_i} \quad (6.11)$$

with ω being angular frequency. The boundary conditions at $x=d_i$ are

$$\begin{aligned} E_{i,z}(d_i, y, z) &= E_{i+1,z}(d_i, y, z) \\ E_{i,y}(d_i, y, z) &= E_{i+1,y}(d_i, y, z) \\ H_{i,z}(d_i, y, z) &= H_{i+1,z}(d_i, y, z) \\ H_{i,y}(d_i, y, z) &= H_{i+1,y}(d_i, y, z) \end{aligned} \quad (6.12)$$

From Maxwell-Faraday equation, one obtains the following relations

$$\begin{aligned}
H_x &= \frac{k_z}{\omega\mu} (A_y e^{-jk_x x} + B_y e^{jk_x x}) e^{-jk_z z} \\
H_z &= \frac{k_x}{\omega\mu} (-A_y e^{-jk_x x} + B_y e^{jk_x x}) e^{-jk_z z} \\
H_y &= \frac{k}{\omega\mu \cos\theta} (A_z e^{-jk_x x} - B_z e^{jk_x x}) e^{-jk_z z}
\end{aligned} \tag{6.13}$$

where k_x , k_y and k_z are the x , y and z components of the wave vector respectively. At the boundaries tangential components of the electric field are continuous, whereas tangential components of the magnetic field are discontinuous by the amount of surface current density. Assuming a lossless medium, the following basic equations are obtained at each boundary

$$K_i (A_{y,i} e^{-jk_{x,i} h_i} - B_{y,i} e^{jk_{x,i} h_i}) = A_{y,i+1} e^{-jk_{x,i+1} h_i} - B_{y,i+1} e^{jk_{x,i+1} h_i} \tag{6.14}$$

$$A_{y,i} e^{-jk_{x,i} h_i} + B_{y,i} e^{jk_{x,i} h_i} = A_{y,i+1} e^{-jk_{x,i+1} h_i} + B_{y,i+1} e^{jk_{x,i+1} h_i} \tag{6.15}$$

where

$$K_i = \frac{k_{x,i}}{k_{x,i+1}} \tag{6.16}$$

for TE polarized waves, and

$$K_i = \frac{k_i \cos\theta_{i+1}}{k_{i+1} \cos\theta_i} \tag{6.17}$$

for TM polarized waves. Along with the Snell's law of refraction,

$$|k_i| \sin\theta_i = |k_{i+1}| \sin\theta_{i+1} \tag{6.18}$$

the field amplitudes A and B in (6.14) and (6.15) can be solved for and substituted in (6.4) and (6.7) to evaluate (6.8) and (6.9), respectively.

6.3 Validation of The Physical Optics-Based Model

To validate the results of the physical optics based method, the finite difference time domain method (FDTD) was applied in two separate steps: in the first step, fields in the absence of the traces and with the dielectric medium in place are calculated using FDTD field solver. In the second step, partial derivatives in (5.20) and (5.35) are discretized in space and time and approximated with difference equations. The field values are then substituted in the forcing functions and the following expressions are obtained

$$\begin{aligned}
 \frac{1}{\delta z}(V_{k+1}^n - V_k^n) + \frac{L}{\delta t} \left(I_k^{n+\frac{1}{2}} - I_k^{n-\frac{1}{2}} \right) + \frac{R}{2} \left(I_k^{n+\frac{1}{2}} + I_k^{n-\frac{1}{2}} \right) = \\
 -\frac{1}{\delta z} \left(\int_{\rho(x,y)} E_{T,k+1}^n d\rho - \int_{\rho(x,y)} E_{T,k}^n d\rho \right) \\
 + \frac{1}{2} (E_{L,k+1}^n(x_m, y_m, z) + E_{L,k}^n(x_m, y_m, z)) \\
 - \frac{1}{2} (E_{L,k+1}^n(x_0, y_0, z) + E_{L,k}^n(x_0, y_0, z))
 \end{aligned} \tag{6.19}$$

$$\begin{aligned}
 \frac{1}{\delta z} \left(I_k^{n+\frac{1}{2}} - I_{k-1}^{n+\frac{1}{2}} \right) + \frac{C}{\delta t} (V_k^{n+1} - V_k^n) + \frac{G}{2} (V_k^{n+1} + V_k^n) = \\
 -\frac{C}{\delta t} \left(\int_{\rho(x,y)} E_{T,k}^{n+1} d\rho - \int_{\rho(x,y)} E_{T,k}^n d\rho \right) \\
 -\frac{G}{2} \left(\int_{\rho(x,y)} E_{T,k}^{n+1} d\rho + \int_{\rho(x,y)} E_{T,k}^n d\rho \right)
 \end{aligned} \tag{6.20}$$

where δz and δt are cell size and time step value, respectively. Equations (6.19) and (6.20) are solved in a bootstrapping fashion with appropriate terminal conditions. Considering that this approach was validated in [21] with experimental results, it is especially preferred for the validation of the proposed method in this paper. Summary of the numerical results is presented in the next section.

6.4 Numerical Examples

In this section, the proposed approach is compared with previous techniques where incident field coupling is calculated neglecting the inhomogeneity of the medium. In the discussions and figures below, the latter approach and physical optics based model are referred to as Approach-I and Approach-II, respectively. The test structure whose physical parameters are given in Fig. 6.2 was taken as a dielectric board sandwiched between two traces.

Figure 6.2: Test structure and incoming field parameters

The electrical parameters of the test structure were computed by method of moments. In all examples, azimuthal incidence angle ϕ and longitudinal incidence angle θ was set to -90° and 30° , respectively and terminations were matched to the line characteristic impedance. The comparison was made in the time domain for an incident field in the form of a gaussian pulse envelope waveform whose amplitude falls 120 dB below the peak value at 15 Ghz. As can be seen from Figs. 6.3 and 6.4, a fairly good agreement is obtained between the proposed physical optics based model (Approach-II) and the FDTD full wave analysis.

Figure 6.3: Coupled voltage waveform for a slab with $\epsilon_r=4.5$

At normal incidence of the incoming field, Approach-I doesn't cause a substantial error. However, as the angle of incidence is declined from the normal to $\theta=30^\circ$, a large error is observed especially at the far end of the line. Fig. 6.3 illustrates this case for an epoxy-

glass substrate ($\epsilon_r=4.5$). The error is relatively small for a duroid substrate ($\epsilon_r=2.2$) whose relative permittivity is comparably small (Fig. 6.4a). For a silicon substrate ($\epsilon_r=11.7$), however, whose relative permittivity is high, a larger error is observed (Fig. 6.4b).

Figure 6.4: Coupled voltage waveform for two slabs with different permittivities

The numerical examples clearly point out the shortcomings of the Approach-I in circuit applications. The reason of this deficiency can be explained as follows. With the first illumination at the near end of the trace, a voltage waveform is excited. Consequently, there are two kinds of waves travelling towards the far end; (1) incident electromagnetic wave in the medium, (2) voltage and current waves on the traces. According to Snell's law of refraction, the axial component of the wave vector is the same in all layers. However, the component in the direction of inhomogeneity depends on the relative permittivity of the medium. If it is not accounted for properly as in Approach-II, the results will be erroneous. The field magnitude, on the other hand, is hardly affected unless the edge effects

and surface waves begin to dominate at grazing incidence angles. Hence, when an arbitrarily polarized incident field is decomposed to its TE and TM polarized parts the effect of the inhomogeneous medium on the TE polarized part is not substantial.

Figure 6.5: far end coupled voltage waveform for a slab with $\epsilon_r=4.5$

As the incidence angle is increased to higher values ($\theta=85^\circ$) some deviation between FDTD and physical optics results are observed (Fig. 6.5a). This stems from the fact that physical optics method is based on the assumption that layered medium planarly extends to infinity. Hence, edge effects cannot be accounted for and simulation of an endfire excitation based on the proposed method is not possible. Finally, Fig. 6.5b depicts the case where mismatched near and far end terminations were taken as 566Ω for an epoxy-glass substrate.

To sum up, in a small angular zone close to the horizontal, accuracy of the physical optics results suffer. The width of this angular zone mainly depends on the frequency contents of the incident field and the relative permittivity of the layered medium. For more complex structures, a mixed EM/circuit analysis algorithm is proposed in the next chapter.

Chapter 7

Mixed EM/Circuit Analysis of High-Speed Circuits

This chapter presents a hybrid method based on recently developed model-reduction techniques to analyze complex 3D circuit structures with full-wave FDTD algorithm. In the proposed algorithm, *FDTD is used where it is most efficient, namely for the analysis of the EM-based subnetwork only*. Circuit simulation is achieved through a model-reduction based scheme, which reduces the large linear subnetwork including passive electromagnetic (EM) devices into a macromodel having small set of ordinary differential equations. Transient analysis is performed in a SPICE-like circuit simulator by linking the macromodel with the time-domain equations which represent the nonlinear elements.

The organization of this chapter is as follows. Section II reviews the basics of the FDTD method concerning circuit structures. A new algorithm for the analysis of crosstalk, radiated emissions and incident field coupling is given in Section III. In Section IV, numerical examples are presented.

7.1 Review of The FDTD Method

The finite difference time domain method is a computational tool that directly solves Maxwell's curl equations or their equivalent integral equations in the discrete time and space domains [62]. In the analysis, continuous distribution of electromagnetic fields in a finite volume of space is initially sampled at discrete points, then the solution is obtained through an explicit leapfrog algorithm which entails marching in the time and space. The capabilities of FDTD in the analysis of printed circuit structures can be listed as

1. *Calculation of broadband response with a single excitation.*
2. *Analysis of arbitrary 3-D model geometries.*
3. *Flexibility in handling complex material properties*
4. *Modeling of frequency dependent materials*

In the forthcoming sections, basics of the FDTD method will be reviewed with a special emphasis on microstrip and PCB applications.

7.1.1 General Formulation and Stability Considerations

Starting with Maxwell's equations in the form

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \left(\sigma + \epsilon \frac{\partial}{\partial t} \right) \mathbf{E}\end{aligned}\tag{7.1}$$

FDTD discretizes the problem domain both in time and space replacing the differential equations by second order central difference approximations

$$\begin{aligned}
& \frac{E(i+1, j, k)_z^n - E(i, j, k)_z^n}{\Delta x} - \frac{E(i, j, k+1)_x^n - E(i, j, k)_x^n}{\Delta z} \\
& = \mu \frac{H(i, j, k)_y^{n+\frac{1}{2}} - H(i, j, k)_y^{n-\frac{1}{2}}}{\Delta t} \\
& \frac{H(i, j, k)_z^{n-\frac{1}{2}} - H(i, j-1, k)_z^{n-\frac{1}{2}}}{\Delta y} - \frac{H(i, j, k)_y^{n-\frac{1}{2}} - H(i, j, k-1)_y^{n-\frac{1}{2}}}{\Delta z} \\
& = \sigma \frac{E(i, j, k)_x^n + E(i, j, k)_x^{n-1}}{2} + \epsilon \frac{E(i, j, k)_x^n - E(i, j, k)_x^{n-1}}{\Delta t}
\end{aligned} \tag{7.2}$$

where i, j, k represent spacial indices for a three dimensional Yee-cell [62], n is the time index; $\Delta x, \Delta y, \Delta z$ are spacial discretization in cartesian coefficients, Δt is the time step. Similar relations can be written for the remaining field components. The electric and magnetic field points are interlaced half a cell size in space and half a time step in time to simulate the physical phenomenon. To obtain a satisfactory resolution, cell size must be at least one tenth of the shortest wavelength of interest [31]. Considering that (7.2) accounts for material interaction of each field component separately, modeling anisotropic materials with diagonal permittivity or permeability tensors is straightforward in FDTD.

The numerical stability of FDTD depends on the temporal integration interval. For a three dimensional rectangular grid, Courant stability condition given as

$$v_{max} \Delta t \leq \frac{1}{\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}} \tag{7.3}$$

with v_{max} being the maximum speed of the wavefront, determines the upper limit for the time step Δt for a stable solution. It should be noted that once the grid size is set, this stability condition strictly depends on the material properties enclosed in the problem domain. This subject will be reconsidered in the discussion of lumped element modeling in FDTD.

7.1.2 Absorbing Boundary Conditions

In open region problems, the spatial domain of the fields is unbounded in one or more coordinate directions. In this case, the solution region must be large enough to enclose the structure of interest in the computation domain while the outer perimeter must be terminated by a suitable boundary condition that emulates the extension of the domain to infinity. Outer boundary conditions of this type are called absorbing boundary conditions (ABCs). In the literature, there is a variety of such ABCs whose computational complexities are balanced by their capability of suppressing reflected waves from the outer boundaries. Due to their easy implementation and relatively high suppression level, Mur's first and second order ABCs [63] have become quite popular in most FDTD applications. For this reason, Mur's ABCs are used in the numerical examples of this chapter. For an absorbing wall in the zy -plane, the first order ABC reads as

$$E_{i,j,k}^{n+1} = E_{i-1,j,k}^n + \frac{c\Delta t - \Delta x}{c\Delta t + \Delta x}(E_{i-1,j,k}^{n+1} - E_{i,j,k}^n) \quad (7.4)$$

which is tantamount to say that the future electric field value at location i at the $n+1$ time step is estimated from the field at the previous location and at previous time step.

The second-order Mur estimate can be written as

$$\begin{aligned}
E_{i,j,k}^{n+1} = & -E_{i-1,j,k}^{n-1} + \frac{c\Delta t - \Delta x}{c\Delta t + \Delta x}(E_{i-1,j,k}^{n+1} - E_{i,j,k}^n) \\
& + \frac{2\Delta x}{c\Delta t + \Delta x}(E_{i-1,j,k}^n - E_{i,j,k}^n) \\
& + \frac{\Delta x(c\Delta t)^2}{2(\Delta y)^2(c\Delta t + \Delta x)}(E_{i,j+1,k}^n - 2E_{i,j,k}^n \\
& + E_{i,j-1,k}^n + E_{i-1,j+1,k}^n - 2E_{i-1,j,k}^n + E_{i-1,j-1,k}^n) \\
& + \frac{\Delta x(c\Delta t)^2}{2(\Delta z)^2(c\Delta t + \Delta x)}(E_{i,j,k+1}^n - 2E_{i,j,k}^n \\
& + E_{i,j,k-1}^n + E_{i-1,j,k+1}^n - 2E_{i-1,j,k}^n + E_{i-1,j,k-1}^n)
\end{aligned} \tag{7.5}$$

The above expression uses values from the preceding two time steps. At the intersection of the planes first-order Mur estimate has to be used. To suppress the reflection from outer boundaries efficiently, the test structure must be located at a sufficient distance from the outer boundary. A practical criterion for Mur's ABCs is 15-20 cells [31].

Recently, perfectly matching layer (PML) technique has been proposed [64], which lowers the reflection from the outer boundary by several orders of magnitude over the existing methods. The basic idea behind PML is to decompose the electric fields into TE and TM components at the boundaries creating four coupled equations rather than the usual three. This extra degree of freedom allows one to derive a non-physical anisotropic absorbing medium with a wave impedance independent of the incidence angle and frequency of the outgoing waves. A spacing of 4-5 cells from the test structure is considered as a sufficient distance to insert PML to absorb outgoing waves.

ABCs are in general designed for absorbing propagating modes only. Especially in dealing with PCB or microstrip type structures, evanescent (nonpropagating) modes will be generated in addition to the propagating modes due to dielectric interfaces. Though much lower in magnitude compared to the propagating modes, evanescent waves eventu-

ally reach the outer periphery and are reflected back to the problem domain. This reflection usually causes instability in the FDTD analyses which span long transient runs. Due to this *late time instability* inherent in FDTD method, it is not possible to run a simulation indefinitely long. One way of circumventing this *late time instability* phenomenon is to cut the analysis prematurely and apply an extrapolation technique to estimate the late time transient waveform.

7.1.3 Compensation of Curved Surfaces

The geometry of the microstrip or printed circuit structures are in general well suited to finite difference gridding scheme. The interconnect discontinuities, however, may not conform to grid edges in most cases. There are several ways to model such curved edges/surfaces. The most straightforward way would be to apply a staircase approximation to them. In highly resonant structures, this approximation may give rise to shifts to the center frequency of the resonances or even introduce spurious resonances. To mitigate this negative effect, a very high grid resolution can be resorted to, which may in turn increase the computational cost severely. To ease up the computational cost in terms of space quantization, fine gridding can be used locally around the curved geometries although this does not help the decrease in the time step. In the literature rigorous solutions have been proposed. The most prominent techniques include:

1. The use hybrid numerical methods which aid the FDTD computation at selected regions by utilizing either finite-element time domain or the integral equation time domain methods [65].
2. Nonorthogonal space gridding which uses usual mesh generation schemes to construct nonrectangular grids which are continuously and globally stretched to conform to the curved surface [66].

3. Locally distorted grid models are quick and relatively simple alternative to computationally involved nonorthogonal mesh scheme. This technique deforms only space grids adjacent to material surface by a contour path integration method to conform to the curved surface [67]. This approach, however, lacks a numerical stability criterion unlike nonorthogonal mesh modeling.

7.1.4 Consideration of Medium Losses

With increasing frequencies, consideration of conductor losses is becoming more significant in circuit analysis. The addition of frequency dependent conductor losses in a time domain solution is not a straightforward task. Almost in all proposed techniques [31], [68], a first order impedance boundary condition which relates tangential field components is considered in the frequency domain

$$\mathbf{E} - \mathbf{n} \cdot (\mathbf{n} \cdot \mathbf{E}) = \bar{\mathbf{Z}} \cdot (\mathbf{n} \times \mathbf{H}) \quad (7.6)$$

where $\mathbf{n}$ is the unit normal vector pointing outward direction from the conducting surface and $\bar{\mathbf{Z}}$ is the surface impedance tensor. In the frequency domain, each element of the surface impedance tensor reads as

$$Z_{mn}(\omega) = (1 + j) \sqrt{\frac{\omega \mu}{2\sigma}} \quad (7.7)$$

for good conductors. In the above expression, μ and σ are permeability and conductivity parameters, respectively. To Incorporate with the FDTD equations, define

$$Z'_{mn}(\omega) = \frac{Z_{mn}(\omega)}{j\omega} \text{ and consider a wave propagation in the } x\text{-direction}$$

$$E_x(\omega) = Z'_{xy}(\omega) j\omega H_y(\omega) \quad (7.8)$$

with the time domain counterpart,

$$E_x(t) = Z'_{xy}(t) * \frac{\partial}{\partial t} H_y(t) \quad (7.9)$$

A direct convolution operation in the time as in (7.9) would be impractical. To get around this difficulty, the time domain surface impedance can be represented in a finite pole-residue summation form

$$Z'_{xy}(t) = \sum_{i=1}^N k_i e^{-p_i t} \quad (7.10)$$

The above representation of time domain surface impedance allows a fast recursive convolution operation with less storage requirements. The details of this approach can be found in [31]. However, with the inclusion of the surface impedance concept in FDTD, the Courant stability condition in (7.3) is no longer valid. Hence caution has to be exercised to avoid numerical instability in the calculations.

7.1.5 Lumped Element Modeling in FDTD

Circuit equations are mathematical abstractions of field equations. In this context, it is possible to accommodate lumped circuit components in the FDTD problem domain. Considering that conductivity and relative permittivity are directly related to resistance and capacitance, respectively, a resistor or a capacitor element can be straightforwardly modeled by modifying the constitutive parameters of the grid location to which the lumped circuit element is to be connected [31]. However, especially in capacitance modeling, changing the medium permittivity results in a change in the velocity of the phase front which affects the time step according to (7.3). Another way of modeling lumped elements in FDTD is to modify the Maxwell-Ampere equation to account for the lumped current

$$\nabla \times \mathbf{H} = \mathbf{J}_d + \mathbf{J}_c + \mathbf{J}_l$$

$$\mathbf{J}_d = \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (7.11)$$

$$\mathbf{J}_c = \sigma \mathbf{E}$$

and $\mathbf{J}_l$ is the current density through the lumped element which is assumed to be connected *in parallel* with cell. For example, in *z-direction*, local current density is related to the total element current I_l as

$$\mathbf{J}_l = \frac{I_l}{\Delta x \Delta y} \quad (7.12)$$

Next, the lumped element can be modeled by using its terminal voltage-current relation. For example, a resistor is modeled as follows

$$\begin{aligned} I_l &= \frac{V}{R} \\ V &= -E \Delta z \\ I_l &= \frac{-E \Delta z}{R} \end{aligned} \quad (7.13)$$

In (7.13), once the lumped current is expressed in terms of field quantity, it is straightforward to insert the above relation into FDTD updating equations as

$$\begin{aligned} & \frac{H(i, j, k)_z^{n-\frac{1}{2}} - H(i, j-1, k)_z^{n-\frac{1}{2}}}{\Delta y} - \frac{H(i, j, k)_y^{n-\frac{1}{2}} - H(i, j, k-1)_y^{n-\frac{1}{2}}}{\Delta z} \\ &= \sigma \frac{E(i, j, k)_x^n + E(i, j, k)_x^{n-1}}{2} + \epsilon \frac{E(i, j, k)_x^n - E(i, j, k)_x^{n-1}}{\Delta t} \\ & \quad - \Delta z \frac{E(i, j, k)_x^n + E(i, j, k)_x^{n-1}}{2R\Delta x\Delta y} \end{aligned} \quad (7.14)$$

The details of this method with its extension to other lumped elements can be found in [32] and [68].

One of the major problems in extending FDTD analysis to include lumped components is due to the stiff nature of the lumped circuit equations. Condition (7.3) is not sufficient to ensure the numerical stability of the FDTD solution. In the presence of stiff nonlinear sub-circuits, Δt has to be reduced to impractically small values that impede the whole simulation process. In this regard, the techniques discussed in this section are in general applicable to very simple circuit configurations with an academic level of interest. In this thesis, lumped resistor modeling is used only as a bench test for the proposed technique. The impracticality of lumped element modeling in FDTD for large circuits has become one of the key motivations of the proposed mixed EM/circuit analysis which will be introduced in following sections.

7.1.6 Source Considerations

At the beginning of the FDTD calculation a certain field distribution must be defined as a forcing function in the problem domain. For circuit structure analyses, excitation is given in the form of a finite duration pulse. With this sort of excitation, it is possible to analyze arbitrary n -ports in a broadband frequency spectrum, which is determined by the pulse form and its time length. To avoid sudden changes and discontinuities in the time domain evolution of the excitation pulse, the time domain envelope waveform has to be smooth in shape. Usually a gaussian or smooth cosine functions are preferred for this purpose. Another consideration in the excitation is the specification of the field on the excitation plane. The excitation source is polarized in the vertical plane perpendicular to the extension of the conductor. Assuming a conductor extending in y -direction with a thickness in z -direction, the FDTD updating equation at the source location is given as

$$\bar{\epsilon}_z|_{i,j,k}^n = E_z|_{i,j,k}^{n-1} + \frac{\Delta t}{\epsilon_0} \left(\frac{H_y|_{i+\frac{1}{2},j,k}^{n-\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k}^{n-\frac{1}{2}}}{\Delta x} - \frac{H_x|_{i+\frac{1}{2},j,k}^{n-\frac{1}{2}} - H_x|_{i-\frac{1}{2},j,k}^{n-\frac{1}{2}}}{\Delta y} \right) + \frac{v^n}{\Delta z} \quad (7.15)$$

Although the mode distribution is not known accurately in the time domain, in practice, it is observed that vertical polarization of the excitation in the source plane dominantly results in the TEM mode of propagation [47].

7.2 Terminal Characterization from FDTD Results

As discussed in Chapter 3, the proposed scheme depends on the circuit decomposition which mainly involves lumped and EM-based subcircuits. Lumped circuit equations are governed by Kirchhoff's laws and are therefore formulated in terms of nodal/port voltages and branch currents. Electromagnetic field behaviour pertaining to EM-based subnetwork, on the other hand, obeys Maxwell's equations. The simulation of the overall nonlinear circuit is performed by properly connecting the lumped circuit nodes to the linear n -ports of the EM-based subnetwork expressed as a terminal characteristic matrix. This discussion is based on the assumption that the EM-based subnetwork dominantly supports TEM mode of propagation, i.e.

$$\nabla_t \times E = 0 \quad (7.16)$$

where ∇_t is the transverse component of the *del* operator. In this case, voltage and current variables are *uniquely* related to electromagnetic fields in the *transverse plane*

$$\begin{aligned}
 V &= -\oint_c \mathbf{E}_t \cdot d\mathbf{l} \\
 I &= \oint_c \mathbf{H}_t \cdot d\mathbf{l}
 \end{aligned}
 \tag{7.17}$$

The voltage and current obtained from (7.17) are independent of the path chosen, which is consistent with the theory of lumped elements.

When higher order modes begin to dominate at high frequencies or due to interconnect discontinuities (7.17) no longer holds and voltage and current have to be determined from *surface integrations*. However, the dependence of voltage and current on the integration path in the transverse plane contradicts the theory of lumped elements. This is because the only aspect of the electromagnetic field to which the lumped element is exposed is the port voltage as determined from (7.17). Hence, to be consistent with lumped circuit theory, (7.17) must be satisfied at the interface where the EM-based subnetwork is incorporated with lumped elements, i.e. at interconnect terminals. In this case, the natural representation of terminal characteristics for mixed EM/circuit algorithm is in the form of admittance parameters. This single mode assumption is very suitable for time domain methods like FDTD which are not subject to mode considerations, as far as the determination of the electromagnetic field is concerned [69].

Following the above argument, FDTD technique can be directly used to determine the terminal voltage and current waveforms of the multiport EM-based subnetwork ϕ . The terminal characteristic equations for the EM based subnetwork can be written in the form

$$i_\phi(t) = y_\phi(t) * v_\phi(t) \tag{7.18}$$

where $i_\phi(t)$, $y_\phi(t)$ represent the time-domain terminal current and voltage vectors, $y_\phi(t)$ represents the time-domain admittance matrix of the EM based subnetwork ϕ .

'*' represents the convolution operation. For example, in the case of two port network, (7.18) becomes,

$$\begin{aligned} i_1(t) &= y_{11}(t)*v_1(t) + y_{12}(t)*v_2(t) \\ i_2(t) &= y_{21}(t)*v_1(t) + y_{22}(t)*v_2(t) \end{aligned} \quad (7.19)$$

where $y_{11}(t)$, $y_{12}(t)$, $y_{21}(t)$, $y_{22}(t)$, the inverse Fourier transform of $Y_{11}(s)$, $Y_{12}(s)$, $Y_{21}(s)$, $Y_{22}(s)$ are the transient admittance parameters and are defined as follows.

$$y_{11}(t) = \left. \frac{\{i_1(t)\}}{\{v_1(t)\}} \right|_{v_2(t) = 0} \quad (7.20)$$

where $i_1(t)$ and $v_1(t)$ are obtained from the FDTD analysis and $\frac{\{ \}}{\{ \}}$ denotes the deconvolution operation. Similar equations can be obtained for the remaining transient admittance parameters.

In coupled interconnects, short-circuiting the terminals in FDTD analysis results in long transients in the case of a low loss medium. Furthermore, the spacial and temporal offsets between voltages and currents introduce numerical errors to the calculation of admittance parameters [31]. To mitigate these problems, instead of shorting-circuiting, the EM-based subnetwork could be terminated with reasonably matching loads. For linear terminations, terminal currents can be obtained from (Fig. 7.1)

$$\begin{aligned} i_1(t) &= \frac{v_s(t) - v_1(t)}{R_1} \\ i_2(t) &= \frac{-v_2(t)}{R_2} \end{aligned} \quad (7.21)$$

Figure 7.1: Terminal Characteristics

This configuration forces the transients to die out quickly as it avoids any numerical error due to spatial and temporal offset, since only terminal voltages are needed in the calculation of the admittance parameters.

Once the admittance parameters are obtained, they are inserted in MNA matrix as in (3.8) and model-reduction/macromodeling phase can be done as described in Chapter 4 [70]. The details of moment calculation for the time domain macromodel generation will be explained in the next chapter. The time domain macromodel obtained this way contains all the field domain information concerning interconnect discontinuities for a rigorous crosstalk analysis in the circuit domain. In the following sections, extension of mixed EM/circuit analysis to computation of radiated emissions and incident field coupling will be investigated. The analysis techniques proposed in the following two sections are based on the assumption that incoming field and fields scattered/radiated by EM-based subnetwork do not couple to the rest of the circuit external to the subnetwork through any other path than at terminals via port voltages and currents. This assumption is justified by the consideration that the circuit external to EM-based subnetwork is composed of lumped elements only.

7.3 Radiated Emission Analysis

The radiated emission analysis of 3D circuit structures with nonlinear lumped components is a two-stage algorithm based on substitution theorem. In the proposed algorithm, the general nonlinear network involving the macromodel of the EM-based subnetwork is initially analyzed with a circuit simulator to calculate the waveforms at macromodel ports. The next stage encompasses a full-wave FDTD analysis in which port waveforms are substituted as lumped forcing functions at the ports of the physical EM-based subnetwork. This two-stage approach concentrates the effect of non linearity in the port waveforms in FDTD analysis and saves the designer from the burden of handling nonlinear elements in the meshed problem domain with slow-paced electromagnetic simulator. In the second stage, computation of radiated emissions in the near zone of the transmission medium is straightforward [28].

Figure 7.2: Proposed Radiated Emission Algorithm

7.3.1 Near-Field Analysis

In the case of near-zone analysis, for cells in which lumped sources are connected, the FDTD updating equation for a *z-directed source* is given by (7.15). FDTD analysis is then used to compute the radiated fields in the time domain.

By the circuit simulation, it is possible to obtain voltage/current only at macromodel ports. In practice, however, interconnect terminals and the linear subnetwork macromodel ports may not coincide if interconnects are linearly loaded (Fig. 7.3).

Figure 7.3: Circuit with interconnect ports different from those of linear subnetwork

In this case, assuming the linear subnetwork contains linear lumped elements and EM-based subnetwork only, (3.8) can be rewritten as

$$\begin{bmatrix} sW_{\pi}(s) + G_{\pi} & L_m \\ -Y_m(s) & U \end{bmatrix} \begin{bmatrix} V_{\pi}(s) \\ I_m(s) \end{bmatrix} = \begin{bmatrix} b(s) \\ I_{port}(s) \end{bmatrix} \quad (7.22)$$

From the above relation, it is straightforward to calculate the voltages at the interconnect terminals. It should be noted that moments of the equivalent sources $I_{port}(s)$ are also needed in model-reduction analysis for (7.22).

7.3.2 Far-Field Analysis

There are mainly two options for the calculation of far zone radiation pattern. The first approach evokes Schelkunoff's surface equivalence theorem [71] according to which current densities $\mathbf{J}_s$ and $\mathbf{M}_s$ are calculated from the tangential components of electric and magnetic fields on a surface enclosing the radiating structure

$$\begin{aligned}\mathbf{J}_s &= \hat{n} \times \vec{H} \\ \vec{M}_s &= -\hat{n} \times \vec{E}\end{aligned}\tag{7.23}$$

By constructing auxiliary electric and magnetic vector potentials, far zone fields can be calculated in a straightforward manner [72]. Despite its conceptual simplicity, with domain discretizing methods such as FDTD, this approach may require huge storage capabilities especially for open type structures.

Alternatively, integral equation techniques can be employed for far-field radiation analysis,

$$\mathbf{E}(\mathbf{r}, t) = \int \int_{S_{-\infty}}^t \vec{G}(\mathbf{r}, t; \mathbf{r}', t') \mathbf{J}(\mathbf{r}', t') dt' ds \tag{7.24}$$

where $\vec{G}(\mathbf{r}, t; \mathbf{r}', t')$ is the appropriate dyadic Green's function for the EM-based subnetwork. In this approach, surface current distribution $\mathbf{J}(\mathbf{r}', t')$ on the conductor surfaces which is already available from FDTD analysis is used. Although this approach reduces the storage requirements its advantage is usually eroded by the efforts to evaluate the Green's function for inhomogeneous structures. For microstrip-like single layer structures, a simplified Green's function is proposed in [73].

7.4 Incident Field Coupling Analysis

Analysis of incident field coupling to 3D circuit structures with nonlinear lumped components is based on superposition theorem. Being a two-stage algorithm, it can be considered as the reciprocal of radiated emission analysis. In the 3D transmission medium, equivalent current sources due to external fields are initially calculated by full-wave FDTD simulation short-circuiting the interconnect ports. These equivalent current sources are superimposed at macromodel ports in a following circuit simulation in the second stage of the algorithm [30]. In this context, (7.18) can be modified as follows

$$i_{\phi}(t) = y_{\phi}(t) * v_{\phi}(t) + i_{sc}(t) \quad (7.25)$$

where $i_{sc}(t)$ represent the equivalent current sources at *the interconnect terminals* due to external fields calculated by full-wave FDTD method. The above equation gives the equivalent sources at the interconnect terminals. However, as in radiation analysis, if interconnect terminals and the linear subnetwork macromodel ports are not the same (7.22) can be modified as

$$\begin{bmatrix} sW_{\pi}(s) + G_{\pi} & L_m \\ -Y_m(s) & U \end{bmatrix} \begin{bmatrix} V_{\pi}(s) \\ I_m(s) \end{bmatrix} = \begin{bmatrix} b(s) \\ I_{sc}(s) \end{bmatrix} \quad (7.26)$$

From (7.26), equivalent sources at macromodel ports can be directly calculated. As in radiation analysis case, moments of the equivalent sources $I_{sc}(s)$ are needed in model-reduction analysis for (7.26).

In the generation of time domain macromodel, superposition of the short circuit terminal currents modify (4.19) as follows.

$$\begin{aligned} \frac{d}{dt}[x(t)] - [A][x(t)] - [B][i_{\pi}(t) + i'_{sc}(t)] &= 0 \\ [v_{\pi}(t)] - [C][x(t)] + [D][i_{\pi}(t) + i'_{sc}(t)] &= 0 \end{aligned} \quad (7.27)$$

where $i'_{sc}(t)$ stands for the short circuit equivalent sources at the *macromodel* ports.

Equation (7.27) can be coupled with the differential equations representing nonlinear components and simulated using a conventional circuit simulator (SPICE) with the effect of incident fields. Graphical illustration of the steps involved in the new algorithm are given in Fig. 7.4

Figure 7.4: Proposed Field Coupling Algorithm

7.5 Numerical Examples

In this section, computational examples pertaining to radiation and field coupling analysis will be given. The numerical examples presented here demonstrate the validity and efficiency of the proposed method in the calculation of near zone radiated emissions and field coupling problems.

7.5.1 Example 1: Radiation from Coupled Bent Strips

This example shows the validity of the proposed method and gives a comparison with direct FDTD analysis. A set of three coupled bent strips, shown in Fig. 7.5 is taken as the test structure. The substrate is of duroid material ($\epsilon_r=2.2$) backed by a ground plane. All the conductors are assumed to be perfect ($\sigma=\infty$) and of zero thickness and are terminated resistively with $R = 95\Omega$.

Figure 7.5: Test structure for Example 1

The circuit is excited at the terminal of the innermost conductor by a trapezoidal pulse with rise/fall time of 120 ps and duty cycle of 200 ps. For excitation in FDTD analysis, the model for a voltage source in series with a resistor [32] is used. The internal resistance of the source is taken to be equal to the load resistance. The resistors are modeled during FDTD according to [32]. The SPICE equivalent model for the coupled interconnect structure was generated with 11 poles. The observation point is taken half an inch above the corner of the innermost strip (since highest level of radiation is expected at the discontinuity). As seen in Fig. 7.6, a good agreement for radiated field components has been obtained between direct application of FDTD full wave analysis and the proposed algorithm.

Figure 7.6: Radiated field at observation point P_1

7.5.2 Example 2: Radiation from CMOS Amplifier

In this example we perform radiation analysis in the presence of nonlinear elements. Consider the CMOS amplifier circuit shown in Fig. 7.7, which is excited by a pulse having an amplitude of 8V, duration of 0.6ns and rise/fall times of 2.8ns. The network is analyzed

at two different observation points. The first observation point is taken half an inch above the corner of the innermost strip and the corresponding radiated fields are given in Fig. 7.8.

Figure 7.7: CMOS amplifier circuit

The second observation is made within the substrate corner and the field component parallel to the excitation direction (z-component) is given in Fig. 7.8. As expected the response has a lower intensity, since the observation point is far from the discontinuity (the other two field components are practically zero in this case).

Figure 7.8: Radiated field at observation points $P1$ and $P2$

This example highlights the full advantage of the proposed method in radiation problems. First, it establishes the re-usability of macromodels: since the macromodel for the EM based subnetwork is already available from Example 1, it is recycled here at no CPU cost. Second, significant CPU speed-up is achieved during nonlinear simulation. Applying the proposed method, it was observed that the addition of nonlinear components increased the CPU time by a factor of less than 1% compared to the linear case. Contrary to this, a brute-force full-wave FDTD analysis with three MOSFETs would be either numerically unstable or prohibitively CPU expensive.

7.5.3 Example 3: Incident Field Coupling to CMOS Inverter

In this example, incident field coupling to a three-layer board in Fig. 7.9 is analyzed in the presence of both linear and nonlinear terminations. This is a quasi-resonant structure with top and bottom metallization modeled as perfect electric conductors. The layers adjacent to metallization are glue layers; between them lies the signal layer. The bottom metal plate also serves as the reference plane. The relative permittivity of the board is set to 4.5.

Figure 7.9: Test structure for incident field coupling examples

The network with linear terminations is given in Fig. 7.10. For the validation of proposed algorithm, comparison with the direct full-wave FDTD method was done in the time-domain. Incident field considered here is a plane wave with TM polarization, having the form of a gaussian pulse with an amplitude of 1KV/m and bandwidth of 28GHz. It illuminates the board as an endfire excitation for the upper face traces (i.e. $\theta=90^\circ$, $\phi=-90^\circ$). As seen from Fig. 7.11, a good agreement is obtained between the two techniques.

Figure 7.10: Linear circuit for incident field coupling example

Next, the incident field analysis is performed in the presence of nonlinear elements. Linear terminations of Fig. 7.10 are replaced by several nonlinear components (a CMOS inverter circuit), as shown in Fig. 7.12. The network is excited by an input pulse having an amplitude of 5 volts, 2.5 ns duration and 0.5 ns rise/fall times. In addition to the input pulse, the network is subjected to an external field whose amplitude in this case is increased by 100 times compared to the linear case (100KV/m, a value on the order of a lightning strike). Macromodel obtained for the linear case analysis is re-used here and the transient response is obtained using HSPICE. A comparison of responses with and without external field disturbances is given in Fig. 7.13.

The CPU cost of the proposed method for nonlinear analysis has two components: computation of the macromodel and the simulation using HSPICE. It is to be noted that the cost involved in the macromodel generation is comparable to that of full-wave FDTD analysis in the presence of linear terminations. The additional CPU cost for nonlinear sim-

ulation using the macromodel and HSPICE is very marginal over the linear case (about 5% for this example). Contrary to this, a brute-force full-wave FDTD analysis in the presence of nonlinear terminations would be either numerically unstable or prohibitively CPU expensive.

Figure 7.11: Coupled voltage waveforms at ports (a) *A* and (b) *B*

Figure 7.12: CMOS inverter circuit

Figure 7.13: Output voltage waveform of the CMOS inverter circuit

Chapter 8

Moment Calculation Techniques for Discrete Data

In transient analysis, incorporation of EM-based subnetwork into circuit equations is in general realized through model-reduction techniques. The common ground of model-reduction algorithms is that they all require moments (scaled derivatives) of the impulse response of the system to be reduced. For interconnects modeled as multiconductor transmission lines, system response is available in a closed form expression and the relevant moment calculation was presented in Chapter 5. However, in the case of time-domain measurements, evaluation of higher-order derivatives of discrete data is not an easy task because a high precision is required on the derivatives for the accuracy of time domain macromodel. Hence, numerical differentiation is out of question in the solution. There remain mainly two ways of solution to this problem; (1) analytical substitution, (2) numerical integration in the time domain. In this chapter, these methods will be investigated in depth.

This chapter opens with a brief review of the existing frequency domain methods. Evaluation of the moments in the time domain through numerical integration follows. Then, moment calculation based on prony's method is described. Finally, numerical examples conclude this chapter.

8.1 Analytical Substitution

In this method, tabulated data is initially approximated by a closed form expression then derivatives of the approximant are obtained next. This technique was proposed in [74], [75] for moment calculation from tabulated data. It is based on fitting the discrete parameter data in the *s-domain* to a rational transfer function in the least-squares sense. Despite its relative simplicity, due to the ill-conditioned nature of the least-squares polynomial fitting problem, this is not a reliable technique. This approach is applied to a simple example to find the moments.

8.1.1 Example: Single Microstrip Line

Take a simple microstrip line as shown in Fig. 8.1. The strip transmission line length is 1 cm long and per-unit-length parameters of the line are found to be $L=3.03 \times 10^{-7}$ H/m, $C=12.5 \times 10^{-11}$ F/m. We applied an FDTD analysis to this structure and find the y-parameters through a discrete fourier transform (DFT) as in Fig. 8.2. The parameters are calculated as $Y_{11}=-4.489425181976351 \times 10^9$ and $Y_{12}=4.489425181976351 \times 10^9$ at $s=0$.

Figure 8.1: Single Microstrip Line

As proposed in [74], we approximated the y-parameters with a rational transfer function up to 8 GHz by a least squares curve fitting algorithm. Due to ill-conditioned nature of the curve fitting, it is not possible to span a larger frequency band for this example. The MNA matrix for this circuit simply reads as in Fig. 8.1.

Figure 8.2: Admittance parameters of the microstrip

With the original y-parameters inserted in the MNA matrix and with matched terminations, we calculated the dc voltage ($s=0$) at node 3 as 0.5V. Then, we recalculated it with the approximated y-parameters. Table-I shows the results comparatively. Although the normalized approximation error is reasonably low for 8 Ghz bandwidth, the voltage value calculated with approximated y-parameters wildly differs from the correct result. Only with a reduced approximation bandwidth of 2.5 GHz, we were able to obtain the correct result. However, this held only for the zeroth derivative, i.e function itself. Because of the mismatches on higher order derivatives, the model reduction algorithm did not converge to a solution.

TABLE-I

$Y_{11}(0)$	$Y_{12}(0)$	Y_{11} Norm. error	Y_{12} Norm. error	$V_3(0)$, volts	BW Ghz
$-4.48942481775176 \times 10^9$	$4.489425155657584 \times 10^9$	4.094×10^{-10}	4.458×10^{-11}	2.959×10^{-5}	8
$-4.489425181976346 \times 10^9$	$4.489425181976351 \times 10^9$	1.2685×10^{-20}	2.7635×10^{-21}	0.499	2.5

As seen from this example, direct approximation of the y-parameters in the frequency domain is not practical. In the following section, we will investigate an alternative technique.

8.2 Integration in The Time Domain

Instead of direct calculation of the derivatives of y-parameters, the terminal waveforms can be calculated first and derivatives of the y-parameters can be obtained indirectly as in [29]. The terminal voltage-current relationship in the s domain can be expressed in terms of Y-parameters,

$$I(s) = Y(s)V(s) \quad (8.1)$$

with the following definitions

$$\begin{aligned} I(s) &= \begin{bmatrix} I_1^{(1)}(s) & I_1^{(2)} & \dots & I_1^{(m)} \end{bmatrix} \\ V(s) &= \begin{bmatrix} V_1^{(1)}(s) & V_1^{(2)} & \dots & V_1^{(m)} \end{bmatrix} \end{aligned} \quad (8.2)$$

where the superscript indices refer to measurement numbers. For model-reduction, derivatives of admittance parameters with respect to complex frequency s are needed, which also requires the derivatives of current and voltage waveforms. Zero order derivatives of admittance parameters $Y(s)$ can be calculated as follows

$$\begin{aligned} I^t(s) &= V^t(s) Y^t(s) \\ Y^t(s) &= (V^t(s))^{-1} I^t(s) \end{aligned} \quad (8.3)$$

where superscript t denotes transpose operation. The higher order derivatives of $Y(s)$ can be calculated by Leibnitz rule in an iterative manner.

$$Y^t(s)^{(n)} = (V^t(s))^{-1} (I^t(s)^{(n)} - V^t(s)^{(n)} Y^t(s)) + \sum_{i=1}^{n-1} \binom{n}{i} V^t(s)^{(n-i)} Y^t(s)^{(i)} \quad (8.4)$$

with n being the derivative order. According to (8.4) only derivatives of the terminal voltage waveforms will be needed in the process.

From basic properties of Laplace transform the following relationship holds

$$\frac{d^n}{ds^n} V(s) = \int_0^\infty (-1)^n t^n v(t) e^{-st} dt \quad (8.5)$$

For terminal waveforms obtained in the time domain, (8.5) can be directly used to evaluate the higher order derivatives of the impulse response. Now, we will scrutinize the validity of above approach with another example.

8.2.1 Example: Microstrip Line

The previous example is tried with the time domain integration technique this time. As seen in Fig. 8.3, the frequency domain response using the macromodel shows good agreement to CPU intensive DFT approach. In the generation of macromodel, the network function needed to be evaluated only at 8 points.

Figure 8.3: Voltage waveform at node 3

8.2.2 Example: A large circuit

In this example, time domain integration technique is tested against a relatively large circuit of Fig. 8.4. The circuit contains 13 EM-based subnetworks with 24 lines in coupled bunches of 3 conductors and 15 coupled discontinuities. The overall size of the MNA matrix is 134-by-134. The circuit is excited with the rectangular pulse of previous examples and the response at the output is observed. As in previous examples, moment matching algorithm is compared to direct DFT analysis. As seen from Fig. 8.5, a good match is ob-

served between two approaches. With model reduction, 52 frequency point evaluation of the network function is required. The same response is obtained by DFT with 500 frequency point evaluation.

Figure 8.4: Circuit of Example-8.2.2

Figure 8.5: (a) Frequency (b) time domain response of the circuit at node A

8.2.3 Effect of Late Time Transient Noise

It is to be noted that the accuracy of the moments in numerical integration technique depends on the behaviour of the late-time transient data. If the impulse response dies out quickly the above approach gives good results as seen from examples 4 and 5. However, for long transient impulse responses, such as of high-Q structures, time domain data has to be truncated at a certain time point, or else noise at the tail of the pulse will likely be amplified by the t^n factor of the integrand and system parameters will not be accurate. The premature truncation of the data, on the other hand, may result in insufficient information which yields inaccurate system moments, as well. A possible solution to such cases is to use extrapolation techniques to estimate the late time transient response. In this case, a small portion of the waveform with a decaying tendency is characterized in the time domain and the late time transient response is deduced without full-wave simulation. For this purpose extrapolation techniques such as Prony's method [76] or matrix pencil method [77] can be used. A more recent technique depends on approximation of the time frequen-

cy data by hermite polynomials, which seems to be promising to reduce the whole EM simulation time [78]. In this work, we employed Prony's technique, an established method to estimate the late time transient response. The method is briefly reviewed in the following section.

8.2.4 Review of Prony's Method

If the measured terminal waveform data is sampled at equidistant time points impulse response $v[n]$ is related to $V(z)$ by the *z-transform*

$$V(z) = \sum_{k=0}^{\infty} v[n]z^{-k} \quad (8.6)$$

where $z=e^{sT}$ with T being the sampling period. $V(z)$ can be modeled as an IIR filter for which the transfer function is given as

$$V(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1z^{-1} + \dots + b_Mz^{-M}}{1 + a_1z^{-1} + \dots + a_Nz^{-N}} \quad (8.7)$$

Eq. (8.7) can be written in the time domain as a convolution operation

$$b(t)=v(t)*a(t) \quad (8.8)$$

Using the first $K+1$ terms of impulse response, (8.7) can be written as a matrix multiplication

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_M \\ 0 \\ \vdots \end{bmatrix} = \begin{bmatrix} v_0 & 0 & \dots & 0 \\ v_1 & v_0 & & \\ v_2 & v_1 & v_0 & 0 \\ \vdots & & & \\ v_M & & & \vdots \\ \vdots & & & \\ v_K & \dots & v_{K-N} \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} \quad (8.9)$$

To uncouple the calculations of a_n and b_n , matrices can be partitioned as follows

$$\begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ \Upsilon_1 \vdots V_2 \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{\mathbf{a}} \end{bmatrix} \quad (8.10)$$

where $\mathbf{b}$ is the vector of the $M+1$ numerator coefficients and $\tilde{\mathbf{a}}$ is the vector of the N denominator coefficients ($a_0=1$). Υ_1 is the vector of the last $K-M$ terms of the impulse response, V_1 is the $(M+1)$ -by- $(N+1)$ partition of and V_2 is the $(K-M)$ -by- N remaining part. $\tilde{\mathbf{a}}$ can be solved from the lower $K-M$ equations

$$\Upsilon_1 = -V_2 \tilde{\mathbf{a}} \quad (8.11)$$

If V_2 is not a square matrix $\tilde{\mathbf{a}}$ has to be solved in a least-squares sense

$$\tilde{\mathbf{a}} = -(V_2' V_2)^{-1} V_2' \Upsilon_1 \quad (8.12)$$

and finally the upper $M+1$ equations yield $\mathbf{b}$

$$\mathbf{b} = V_1 \mathbf{a} \quad (8.13)$$

8.2.5 Example: Band-pass filter

In this example, we consider the band-pass filter in Fig. 8.6. The terminal waveforms of the circuit below was calculated with FDTD analysis and the simulation was terminated at 5000th time step.

Figure 8.6: Band-pass filter

As seen from Fig. 8.7, both source and load end waveforms have significant magnitude at 5000th time point. To estimate the late time transient responses, data between 4000th and 5000th time steps were decimated by 20 and extrapolated to higher time points. The extended data was then used to extract the admittance parameters of this structure by DFT and time domain integration techniques separately. Since it is impossible to tell the results of two approaches to naked eye in a broad band only the 4th resonance of Y_{11} is plotted in Fig. 8.8a after a calculation involving 20,000 time points. Similar agreement is observed at other resonant frequencies. The overall normalized error between the results of DFT and time domain integration technique comes out to be $2.707990129433e-04$ for 20,000 time points. As the number of data points is increased the deviation between two methods

grows (Fig. 8.8b). Although this example illustrates an extreme case with a long transient data, the time domain integration technique successfully extracts the parameters. In other examples, maximum number of data points were about 2000 time steps and fewer.

Figure 8.7: (a) Source end, (b) load end terminal response of band-pass filter

Figure 8.8: (a) Y_{11} parameter of band-pass filter, (b) normalized error

The superiority of moment generation algorithms from time domain data gives an edge to time domain measurement techniques and electromagnetic simulation methods in model-reduction and macromodel generation process. In this context, mixed-EM/circuit simulation methods based on FDTD or TLM algorithms are more efficient compared to others employing frequency domain differential or integro-differential numerical electromagnetic field solvers.

Chapter 9

Conclusions

In this thesis, high-speed interconnect problems have been explored. Depending on the complexity of the problem, either of quasi-TEM approach and mixed circuit/EM method has been employed in the analysis. For interconnect structures which can be modeled as MTLs, a new field coupling algorithm has been proposed to account for the medium losses in a circuit simulator environment. The method is based on matrix-Padé approximation to the state-transition matrix of MTL equations. In addition, effects of stratified media have been analyzed in field coupling problems. The assumption of incident field propagation in a homogeneous environment was shown to generate inaccurate results in high-speed circuit applications. A physical optics based method have been proposed to account for the layered medium effect on the incoming fields. For large classes of practical applications, this approach was shown to provide similar accuracy to full wave techniques, while requiring a fraction of the CPU and memory resources.

For more complicated transmission media in which a field propagation in all three dimensions has to be considered, a new simulation algorithm, namely *mixed EM/circuit analysis technique* has been proposed. The algorithm is based on combining model-reduction techniques with time-domain full-wave electromagnetic simulators and generates a SPICE-compatible time domain macromodel for 3-D transmission media for the analysis of EMI effects, such as crosstalk, radiation and incident field coupling associated with high-speed interconnects embedded in nonlinear circuits. The results have shown that this hybrid approach overcomes the numerical stability problems of time domain full-wave simulators and speeds up the whole simulation process while preserving the accuracy of the full-wave solution. The method can account for inhomogeneity as well as anisotropy of the media in EMI analysis. In addition, the proposed technique has the extra advantage of being able to use the vast device libraries of existing circuit simulators.

Different derivative calculation algorithms were compared for the proposed *mixed EM/circuit analysis technique* and it was shown that time domain integration technique is the most suitable one for this purpose. The noise susceptibility of the time domain integration technique against slowly dying transients was improved with prony's method.

Despite its high precision and accuracy, some difficulties and areas of further research still remain with the mixed EM/circuit analysis technique. First of all, for an impulse response which is measured or is computed numerically, a method that will preserve the passivity in the generation of a time domain macromodel is still a concern. The uncertainty of full-wave simulation or measurement results due to noise may affect the accuracy of the time domain macromodels negatively.

Although the accuracy of full-wave electromagnetic simulators are far above any other simulation technique, this advantage is usually balanced or sometimes is outweighed by the temporal computational cost, which may extend from hours to days in some large problems. For irregular type conducting traces which do not conform to conventional MTL modeling, nonuniform transmission line models can be applied as a relatively quick but reasonably accurate solution technique. The usual and simplest way of modeling nonuniform transmission lines is to treat them as cascaded uniform transmission lines. Although this approach gives good results for lines whose characteristic impedance undergoes little changes at transitions, they may not model sudden discontinuities such as a slot in the ground adequately. Hence, more research has to be invested in the analysis of nonuniform transmission lines.

Perhaps, one of the best uses of mixed EM/circuit analysis technique would be in the broad-band modeling of printed circuit discontinuities such as bends and vias in high-speed digital circuit applications. Considering the difficulties of working with tabulated data generated by electromagnetic simulators in time domain macromodeling, other electromagnetic techniques of semi-analytic nature can be used in the characterization of such discontinuities. Planar circuit concept is one of these techniques. Although it is efficient in the modeling of bends and stubs in two dimensions, it cannot be used in via modeling. Another technique for discontinuity analysis is the mode-matching method. It is an efficient technique in particular for shielded printed circuit structures. For open geometries, however, it is difficult to use. The macromodel tool developer must be vigilant to the developments in electromagnetic modeling area to improve the quality of the mixed EM/circuit analysis technique.

Appendix

Conversion of Differential Equations into Equivalent Subnetworks

Consider a simple case of two port network with two states represented in the form

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\ \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \end{aligned} \quad (\text{A.1})$$

The above expression can be explicitly rewritten as

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + b_{11}v_1 + b_{12}v_2 & (a) \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + b_{21}v_1 + b_{22}v_2 & (b) \\ i_1 &= c_{11}x_1 + c_{12}x_2 + d_{11}v_1 + d_{12}v_2 & (c) \\ i_2 &= c_{21}x_1 + c_{22}x_2 + d_{21}v_1 + d_{22}v_2 & (d) \end{aligned} \quad (\text{A.2})$$

In the above equations, the port currents and voltages are represented by i_1 , i_2 and v_1 , v_2 , respectively. An equivalent network representing (A.2) can be constructed as in Fig. A-1. Each state in the macromodel requires a separate node in the equivalent circuit and are represented by nodes n_1 , n_2 . The state variables x_1 , x_2 can be represented by the capacitor voltages. These capacitors are denoted by C_{n1} , C_{n2} and the voltages across them by v_{n1} , v_{n2} . State matrix entries, a can be represented by voltage controlled voltage sources. With this assignment, (A.2) is fully realized through equivalent circuits shown in Fig. A-1. Generalization of the above discussion to more states or ports is straightforward.

Fig. A-1:Equivalent subcircuit realization for (A.2)

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