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**The Roles of Phonological
and Spatial Working Memory
in Mental Addition**

by

Patricia Trbovich, B.A.

**A thesis submitted to
The Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of**

Master of Arts

Department of Psychology

Carleton University

Ottawa, Ontario

May 11 2001

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Acceptance of the thesis**

The Roles of Phonological and Spatial Working Memory in Mental Addition

Submitted by

Patricia Trbovich, B.A.

In partial fulfillment of the requirements for

The degree of Master of Arts

Chair

Thesis Supervisor

CARLETON UNIVERSITY
May 11, 2001

Abstract

Participants solved addition problems while performing either a visual-spatial or a phonological memory task at three levels of difficulty. Addition problems were presented at two levels of difficulty; carry (e.g., $23 + 8$) and no-carry (e.g., $23 + 4$). When problems were presented in a horizontal format, increasing the difficulty of a visual-spatial memory load resulted in a decrease in latencies. In contrast, an increase in phonological memory load resulted in an increase in latencies. When problems were presented in a vertical format, latencies increased from a no-load to a low phonological load and decreased from a low to a high phonological load whereas a visual-spatial load did not affect latencies. These findings suggest that both visual-spatial and phonological processing resources are involved in arithmetic performance. The results are discussed with respect to how the solution procedures used in addition may vary with the structural characteristics of the problem.

Acknowledgements

Thanks to Jo-Anne LeFevre, for not only being an amazing supervisor but a great friend. Thank you for going way beyond the call of duty, to help me meet my objective. I hope to return the favour. You have taught me so much. By sharing in your excellence, I am inspired to reach my own.

Thank you Chris Herdman. Thanks to you, I had a story worth telling.

Thanks Mom, your unconditional support has been the source of my strength. I am especially grateful for all the cups of coffee you kept bringing to my desk.

Thanks Dad, I don't know where I'd be without you, but I'd hate to go there and find out.

Thanks Kim and Danny, without your support and encouragement I would have gone mad long ago.

Thank you Mel for your support and help, you gave so selflessly.

And last but not least, thank you Brenda for all your help. The sheer joy of working with you has been wonderful.

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The Roles of Phonological and Spatial Working Memory in Mental Addition

Elementary mental arithmetic is an important aspect of education and is a common everyday activity. Thus, understanding this fundamental skill is clearly of educational and general interest. People presumably use a variety of cognitive processes to solve arithmetic problems. One of the cognitive processes that has been implicated in arithmetic solutions is working memory (Logie & Baddeley, 1987). Working memory is an information storage and processing system that is associated with numerous tasks in a variety of domains that involve the temporary retention and processing of information (Baddeley, 1990). Although relatively few empirical studies have been conducted, there are various ways that working memory might be involved in arithmetic performance (e.g., Hitch, 1978; Hunter, 1979; Logie et al., 1987). For example, working memory may be involved in the storage of problem information (e.g., Baddeley & Logie, 1992), in the execution of calculations (e.g., Logie, Gilhooly, & Wynn, 1994), and in the retention of partial answers (e.g., Noël, Désert, Auburn, & Seron, 2001). The goal of the present research was to explore the role of working memory in mental addition.

Because the working memory model of Baddeley and Hitch (1974) has already proven to be a useful framework for evaluating the role of working memory in mental arithmetic (e.g., Lemaire, Abdi, & Fayol, 1996; Logie et al., 1987; Logie et al., 1994), it was used as a framework for the present research.

Working Memory

Baddeley (1981) proposed that working memory consists of a central executive, a phonological loop and a visual-spatial scratch pad (VSSP). The central executive is in charge of coordinating the actions of the phonological loop and the visual-spatial

scratchpad. In arithmetic tasks, the role of the central executive is to execute the calculations involved in mental addition and to generate approximate answers (De Rammelaere, Stuyven, & Vandierendonck, 1999).

Most research on working memory has focused on the phonological loop that is responsible for mental rehearsal and is involved in temporary storage of verbal information (e.g., Baddeley et al., 1992). The phonological loop consists of a phonological store and an articulatory control process. The phonological store is responsible for holding verbal information in memory for a short time. The articulatory control process is in charge of refreshing the memory trace by means of subvocal rehearsal (Baddeley, 1990). Evidence that the phonological loop plays a role in arithmetical skills comes from a number of studies that used verbal recall tasks (Campbell & Graham, 1985; De Rammelaere et al., 1999; Förs & Hitch, 2000; Lemaire et al., 1996; Logie & Baddeley, 1987; Logie et al., 1994). For example, Logie et al. (1994) found that the phonological loop may function to maintain accuracy in mental arithmetic.

The visual-spatial scratchpad (VSSP) acts as a mental blackboard on which we perform calculations and serves as a temporary storage for visual and spatial material (Logie, 1986). In arithmetic, it may be especially useful for holding partial answers (i.e., for carry information) in memory.

Working memory is conceptualized as a limited resource system. It is therefore possible to assess the role of each working memory component by using a dual-task paradigm. In a dual-task paradigm, participants are asked to perform two tasks at the same time. Usually, the primary task is a memory task and is presented immediately

before, or concurrently with, the secondary task that requires the same resources as the primary task. The dual-task paradigm has helped identify which components of working memory are responsible for carrying out particular cognitive tasks (Logie & Baddeley, 1987). It is used to explore whether two tasks require the same processing resources. If performance on the primary task is affected by performance on the secondary task, the two tasks are assumed to use the same processing resources or processing codes. If there is no decrement in task performance when the tasks are combined, the different tasks are assumed to use different limited processing resources. Therefore, the dual-task paradigm can be used to differentiate the various components of working memory.

A number of researchers have examined the link between mental arithmetic and the various components of working memory using the dual-task methodology (e.g., Adam & Hitch, 1998; Lemaire et al., 1996; Logie & Baddeley, 1987). For example, Logie et al. (1994) required participants to add a series of two-digit numbers that were presented verbally or visually, while performing secondary tasks designed to load the components of working memory. Logie et al. concluded that (a) the phonological loop appears to be responsible for keeping track of running totals and ensuring accuracy in calculation, and that (b) the central executive seems to be responsible for mental arithmetic calculations and the generation of approximate correct answers.

The present experiment was designed to explore the involvement of the phonological loop and the VSSP in arithmetic performance. To do this, two memory load paradigms were used to disrupt phonological and visual-spatial processing, respectively, while participants solved addition problems.

Phonological Loop and Mental Arithmetic

Logie and Baddeley (1987) demonstrated the utility of the working memory approach in their research on mental arithmetic. They provided evidence that the phonological loop is involved in the subvocalization of the running total (Adam & Hitch, 1998; De Rammelaere et al., 1999; Logie et al., 1994). For example, Logie and Baddeley, (1987) found that articulatory suppression disrupted performance on a counting task. Similarly, Noël et al., (2001) found that the phonological loop was involved in the retention of partial results. Participants were asked to add two numbers that were either phonologically or visually similar. Noël et al. observed disruption of arithmetic performance when the numbers were phonologically similar whereas no disruption was found for visually similar numbers. Thus, their findings suggest that the phonological loop, rather than the visual-spatial scratchpad, is involved in temporary retention of addends. In sum, the phonological loop appears to be involved in the retention of problem information and partial results.

Further, Logie et al. (1994) required participants to mentally add two-digit numbers that were auditorily or visually presented while performing either a random letter generation secondary task or an articulatory suppression secondary task. Logie et al. found that the disruption produced by random letter generation was greater than the disruption produced by articulatory suppression. Furthermore, they found that subvocal rehearsal provides a means for maintaining accuracy in mental addition. Lemaire et al. (1996) also found that accuracy in mental arithmetic is dependent upon the phonological loop. Participants were presented with a verification task and with a phonological secondary task, a central executive secondary task, or no secondary task. Lemaire et al.

observed a disruption of performance on true problems by concurrent phonological and central executive tasks and a greater disruption of performance on false problems by the central executive than by the phonological task. Lemaire et al.'s study, however, had a serious limitation. The secondary task used to disrupt phonological processing was also disrupting central executive processing. To explore whether Lemaire et al.'s results were valid, De Rammelaere et al. (1999) replicated Lemaire et al.'s study with a modification: they used a central executive task that did not interfere with the other components of working memory. De Rammelaere et al.'s study provided additional evidence for the role of the central executive in the solution of both true and false sums and for the role of the phonological loop in solving false sums. However, in contrast to Lemaire et al.'s finding that the phonological loop is involved in the solution to true sums, De Rammelaere et al. did not find an effect of phonological load on true sums. Thus, the articulatory suppression task used in Lemaire et al.'s study was probably also interfering with the central executive. Further research is needed to investigate the robustness of this finding.

In sum, there is evidence for the involvement of both the central executive and the phonological loop in the performance of mental arithmetic.

Visual-Spatial Scratchpad

Hitch (1978) demonstrated the utility of the working memory approach in his research on mental arithmetic. He provided evidence that both problem inputs and intermediate results are held in working memory during the conduct of mental arithmetic calculations. Hitch (1978) and Hayes (1973) provided consistent accounts of the role of working memory in the performance of mental arithmetic. For example, Hitch asserted that computations set out upon a sheet of paper in written arithmetic are analogous to

problem information mentally inscribed in working memory. Similarly, Hayes suggested that the visual imagery used in mental arithmetic acts as a substitute for the notations that one would set out on paper during the performance of written arithmetic. Further, Hunter (1979) postulated that in the performance of mental arithmetic, working memory is used to conduct a multiplicity of operations involving the data input, retention, retrieval and tracking of the various stages of calculations. The work of Hitch and Hayes clearly underscores the VSSP's potential importance as the mental worksheet or blackboard of working memory. However, Hunter's work suggests that all three components of working memory must be enlisted to fully explain working memory's multiple functions during the performance of mental arithmetic.

Kulak (1993) conducted an experiment to assess the hypothesis that VSSP resources are involved in the solution of addition problems that require carrying (e.g., $24 + 7$). Participants were asked to solve double + single digit addition problems while retaining either a verbal or a spatial working memory load. Kulak found that participants' performances on carry problems (e.g., $24 + 7$) improved with an increase in VSSP load. Kulak hypothesized that participants' improved performances could be attributed to a shift in the solution procedures used to solve the problems (i.e., switch from a less effective solution procedure such as counting, to a more effective solution procedure such as direct retrieval).

Heathcote (1994) conducted two experiments designed to assess the role of the VSSP in the performance of mental addition. In the first experiment, participants mentally added two three-digit numbers that were auditorily or visually presented (e.g., $468 + 957$) while additional tasks were introduced to load the components of working

memory. He found that the disruption produced by articulatory suppression (i.e., phonological loop) was restricted to the storage of interim results. This is consistent with the views of Logie et al. (1994). In contrast, the disruptive impact of visual-spatial interference (i.e., VSSP) was limited to the retention of carry information (e.g., $17 + 4$ is added as follows: $7 + 4 = 11$, carry the 1; $1 + 1 = 2$; yields a sum of 21). In a second experiment, Heathcote manipulated the spatial arrangement of problem information. His results indicated that problems appearing in a lateral configuration (e.g., $468 + 957$) produced significantly poorer performance than problems presented in a columnar arrangement, thereby suggesting that problem information is held on the VSSP in a columnar display. His findings support the view that mental addition uses a working memory configuration involving both the phonological loop and the VSSP. On this view, the phonological loop seems to provide working memory storage for partial results and assists in the retention of initial problem inputs whereas the VSSP appears to be involved in the retention of both problem information and carry information (Heathcote, 1994). Heathcote concluded that the role of the VSSP is similar to that of a “mental blackboard” upon which material may be visuo-spatially represented while calculations are executed on it. This is consistent with the views of Hitch (1978) and Hayes (1973) as mentioned above.

In summary, studies designed to explore the relation between working memory and arithmetic performance have yielded results that are consistent with the hypothesis that both the phonological loop and the VSSP may mediate arithmetic performance.

Working Memory and Problem Type

Researchers have suggested that more working memory resources are required for

the solution of multi-digit problems (e.g., $23 + 14$) than for the solution of single digit problems (e.g., $5 + 7$) (Ashcraft, Donley, Halas, & Vakali, 1992; Hitch, 1978).

Clarification of how complex mental addition places demands on working memory resources may result in a better understanding of the solution procedures people use when solving arithmetic problems. There are many ways in which we can manipulate problem difficulty. For instance, we can manipulate the size of the digits to be added (e.g., 2 vs. 9) or the number of digits to be added (e.g., $2 + 3$ vs. $2 + 3 + 4$ or $2 + 3$ vs. $2 + 13$).

In the present research, problem type was introduced by means of sums that involved carrying (i.e., carry vs. no-carry). Problem type is an important manipulation to include in experiments designed to evaluate mental arithmetic performance, because it is the most common effect on performance found in mental arithmetic literature. The problem type phenomenon is as follows: when participants solve single-digit addition problems, their performance is slower and more error prone on problems with larger operands (e.g., $9 + 6$) than on problems with smaller operands (e.g., $4 + 3$). Problem type was manipulated in order to verify whether the phonological loop and the VSSP were differentially involved in the solution of carry and no-carry problems.

Other manipulations might help shed some light on whether working memory resources are used in the solution of arithmetic problems. Several researchers have indicated that the way in which problems are presented might affect arithmetic performance (Hayes, 1973; Heathcote, 1994; Smyth et al., 1987). For example, researchers have suggested that vertical displays of digits facilitate addition performance as compared to horizontal displays. Furthermore, LeFevre and Liu (1997) found that Chinese participants were faster to solve arithmetic problems with the smaller operand

first (e.g., $6 + 9$) than problems with the larger operand first (e.g., $9 + 6$). Thus, the solution procedures used by participants may vary with the way the problem is presented.

Working Memory and Problem Presentation

In the present research, presentation of problems was manipulated in two ways. First, participants solved problems presented either in a horizontal or a vertical format. Second, problems were either presented with the single digit first (e.g., $3 + 24$) or with the double digit first (e.g., $24 + 3$). I was particularly interested in whether presentation format (i.e., horizontal vs. vertical) and operand order (i.e., single digit + double digit vs. double digit + single digit) place different demands on working memory resources (i.e., visual-spatial vs. phonological). Several researchers have indicated that vertical displays of digits facilitate addition performance as compared to horizontal displays (Hayes, 1973; Heathcote, 1994; Smyth et al., 1987). Hayes (1973) observed that participants arranged imaged digits into columns to facilitate mental arithmetic calculations. His findings are consistent with the notion that the VSSP provides a blackboard for columnar organization of problem digits in mental addition. Hayes stated that, compared to verbal input, visual images are especially useful because they facilitate the encoding of the spatial properties and relations of digits (e.g., their locations and sizes). Likewise, Smyth et al. (1987) suggested that mental arithmetic might well involve the manipulation of visual images such that digits are arranged into appropriate spatial configurations. Similarly, Heathcote suggested that the VSSP optimizes performance of complex mental arithmetic calculations by permitting columnar organization of initial problem information. These results suggest that spatial alignment of digits may be an important factor to consider when exploring how people mentally solve addition problems. People appear to be faster

at mentally solving addition problems that are presented in a format that facilitates alignment of digits to be added. That is, one of the reasons that a vertical format appears to be better than a horizontal format is that in a vertical format the digits to be added are aligned.

A way of verifying whether alignment of digits is a critical factor to consider when exploring how people solve addition problems is to manipulate the order in which the operands appear when problems are presented horizontally. In the present research, some problems appeared with the single digit first (e.g., $3 + 24$) and others with the double digit first (e.g., $24 + 3$). If the alignment of digits to be added were an important factor, we would expect performance in the latter case to be faster than in the former, because the two units to be added are adjacent.

Overview of the Present Research

This experiment was designed to evaluate: (a) whether the phonological loop and the visual-spatial scratchpad are involved differently in arithmetic performance; (b) whether the roles of the phonological loop and the VSSP in arithmetic performance vary with problem type (i.e., carry vs. no-carry), and (c) whether the roles of the phonological loop and the VSSP in arithmetic performance vary with the way in which problems are presented (i.e., format; horizontal vs. vertical and operand order; single digit + double digit vs. double digit + single digit).

Overall there is evidence that both visual-spatial and phonological codes could be involved in mental arithmetic performance. Consequently, this experiment was designed to examine the contributions of these two components of working memory.

Phonological Loop

Logie and Baddeley (1987) suggested that the phonological loop is involved in the subvocalization of the intermediate answer (e.g., $17 + 4$ is added as follows: $7 + 4 = 11$, this is part of the answer, carry the 1; $1 + 1 = 2$, this is another part of the answer; yields a sum of 21). Research has also suggested that the phonological loop is involved in the maintenance of accuracy in mental arithmetic (e.g., Logie et al., 1994) as well as in the retention of problem information (e.g., Adam & Hitch, 1998; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Lemaire et al., 1996; Logie et al., 1994; Noël et al., 2001). Therefore, I hypothesized that an increase in phonological memory load would result in an increase in latencies on problem solving.

Problem Type

Problem type is the most common effect found in mental arithmetic literature. Therefore, I expected that carry problems would take longer to solve than no-carry problems.

Researchers have provided evidence that the phonological loop is involved in the subvocalization of the running total (Adam & Hitch, 1998; De Rammelaere et al., 1999; Logie et al., 1994). I expected that carry problems would require longer time to subvocalize because of the extra step of having to perform the carry operation. For example, a no-carry problem such as $17 + 2$ might be added as follows: $7 + 2 = 9$; retain the 9; yields a sum of 19. Whereas adding a carry problem such as $17 + 5$ would involve the following: $7 + 5 = 12$, retain the 2; carry the 1; $1 + 1 = 2$; yields a sum of 22. Therefore, I expected that phonological loads might slow performance on carry problems more than performance on no-carry problems.

Format

Based on Heathcote's 1994 findings, I expected that a vertical format would produce faster responses to addition problems than a horizontal format. I expected that a beneficial presentation format (i.e., vertical vs. horizontal) would improve a person's performance when solving addition problems, since this would eliminate the need for re-arrangement of digits to be added.

Operand Order

I expected that single digit + double digit problems would take longer to solve than double digit + single digit problems. For example, in the problem " $24 + 3$ ", I predicted that it would be easier to determine that the "4" was being added to the "3", than when the problem was presented as " $3 + 24$ ". In the first instance, the units to be added together are adjacent, whereas in the latter case, an incremental step is required to determine which units to add together. Specifically, I only expected operand order to have an effect when problems were presented horizontally. When problems are presented vertically, the digits are aligned in both operand orders whereas, in a horizontal format, the digits are only aligned in the double + single digit order.

Visual-Spatial Scratchpad

There is less evidence that the VSSP is involved in arithmetic performance as opposed to the evidence in favor of the phonological loop. However, the studies that have investigated the involvement of visual-spatial processing in arithmetic performance have suggested that the VSSP plays a role in the retention of carries (e.g., Hayes, 1973; Heathcote, 1994; Kulak, 1993). Therefore, I expected that an increase in VSSP memory load would (a) have an effect on participant's performance and (b) this effect would be

mediated by problem type.

Problem Type

I expected that carry problems would take longer to solve than no-carry problems. Furthermore, Kulak (1993) found that a visual-spatial load did not affect participant's performance on no-carry problems. Conversely, participants were faster at solving carry problems as the visual-spatial load increased. This result was surprising because, if visual-spatial resources were involved in carrying, we would expect an increase in latencies as opposed to a decrease in latencies, as visual-spatial resources were loaded-up. Kulak concluded that participants' improved performances could be attributed to a shift in the types of solution procedures used to solve the addition problems. That is, Kulak hypothesized that participants were shifting from a demanding solution procedure (e.g., counting) to a less-demanding solution procedure (e.g., direct retrieval).

Other researchers believed that VSSP resources had a role in the solution of addition problems and concluded that VSSP acted as "mental blackboard" upon which material may be visuo-spatially represented while calculations are executed on it (Hayes, 1973; Heathcote, 1994; Hitch, 1978). I did not expect the VSSP to play the role of a "mental blackboard" in the present experiment, because the problems remained visible throughout the experiment and therefore, there was no need to visualize the initial problem information. Furthermore, Heathcote (1994) suggested that participants may visualize the "carry" procedure when solving carry problems and therefore the VSSP plays a role in the visualization of the carry operation.

Based on Heathcote's (1994) and Kulak's (1993) results, I predicted that the VSSP would play a bigger role in the solution of carry problems than in the solution of

no-carry problems. More specifically, I expected to find results similar to those of Kulak (1993). That is, I expected that participants' latencies would decrease as a function of VSSP load.

Problem Format

Research suggested that a vertical problem format produces superior performance to a horizontal format (Hayes, 1973; Heathcote, 1994; Smyth et al., 1987). Consequently, the present experiment was designed to see whether a vertical format would produce superior performance to a horizontal format. Additionally, the present research set out to investigate whether the VSSP is differentially involved in arithmetic performance when problems are presented vertically rather than horizontally. Because in vertical presentation, digits are aligned, I predicted that vertical format would produce faster latencies than horizontal format.

Operand Order

I expected that single digit + double digit problems would take longer to solve than double digit + single digit problems, but only when problems were presented horizontally. When problems are presented vertically, the digits are aligned in both operand orders, whereas in a horizontal format the digits are only aligned in the double + single digit order.

Summary of predictions

It was hypothesized that: (a) reaction times would increase with phonological load, especially for carry problems, (b) reaction times to no-carry problems would not be affected by visual-spatial load, whereas reaction time to carry problems would decrease with visual-spatial load, (c) overall, carry problems would take longer to solve than no-

carry problems, (d) reaction time to problems presented in a vertical format would be faster than to problems presented in a horizontal format, (e) reaction times to single digit plus double digit (e.g., $3 + 24$) problems would be longer than reaction times to problems presented with double digit plus single digit (e.g., $42 + 5$), and (f) this effect of operand order would be mediated by format (i.e., only present in horizontal format).

The present research design and methodology is an adaptation of Kulak's (1993) research, where participants completed: (a) a phonological memory task that involved remembering a list of pronounceable non-words, or (b) a spatial memory task that involved remembering the positions of asterisks in a random array. The tasks were presented at two levels of difficulty; low (2 non-words and 4 asterisks respectively), and high (4 non-words and 8 asterisks respectively). Following initial presentation of the memory task, participants solved an addition problem. The addition problems were double plus single-digit problems; half of the problems involved carrying (e.g., $27 + 4$) and half had no carry operation (e.g., $27 + 2$). Kulak found that participants' latencies did not change on no-carry problems as a function of load. Furthermore, latencies were not affected by change in phonological load. However, participants responded more quickly to carry problems in the high, relative to the medium, load condition. Kulak argued that the more rapid latencies to carry problems in the high load condition might have resulted from overtaxing of participants' visual-spatial resources. The combination of a carry problem with high spatial processing may have forced them to adopt an alternate, more efficient, solution procedure such as fact retrieval.

Interpretation of Kulak's results was compromised because a memory load control (i.e., no-load condition) was not included in the experimental design. This

limitation in Kulak's approach eliminated the possibility of determining what pattern of results would have occurred if no visual-spatial interference was present. The present research design is an adaptation of Kulak's (1993) design with two important changes. First, a memory load control task was added. The same no-load control item (1 asterisk) was used for the phonological task as for the visual-memory task to facilitate comparisons between these two memory task conditions. The inclusion of a no-load control in the present experimental design permitted an assessment of results when no memory interference is present. The no-load condition established a baseline against which results from load tasks could be compared. Second, the phonological memory loads were modified in the following ways: (a) the low phonological load was changed from two non-words to one non-word, and the high phonological load was changed from four non-words to three non-words, so that participants would have sufficient amount of time to rehearse the items, (b) the probe non-word in the present experiment was only different by a single sound from the presented nonword (e.g., GUB vs. GUP) as opposed to being a totally different consonant-vowel-consonant (cvc) combinations, so that participants would be encouraged to rehearse the items as opposed to using a visual strategy to remember the items, and (c) the non-words in the present experiment were made easier to rehearse by using nonwords that are easily pronounced (i.e., NOF) as opposed to including hard to pronounce nonwords (i.e., BOJ).

Method

Participants

Ninety-six participants took part in this experiment. Participants received either course credit (as partial fulfillment of a course requirement) or ten dollars for their

participation.

Apparatus

An IBM 80286 computer was used to present stimuli on a video monitor and to record responses. Response latencies were detected using a headset containing a built-in microphone. Participants were seated approximately 50 cm in front of the video monitor. Trials were self-paced with participants pushing the center key on a three-button response panel to start each trial. Memory-load task responses were made using the remaining two keys.

Materials

As shown in Appendix A, there were 36 arithmetic problems. Each problem was presented twice, one with the single digit first (e.g., $5 + 43$) and once with the double digit presented first (e.g., $43 + 5$). Eighteen of the problems involved carrying (e.g., $6 + 37$) and eighteen did not involve a carry operation (e.g., $3 + 84$). All sums were under 100 (range of 41 to 93 for carry and 35 through 89 for no-carry). An equal number of problems in each the carry and no-carry sets had sums in the decade 30 through 90 (three from each set). Thus phonological onset was controlled across problem size. The “single” digit components of the problems in each set consisted of the numbers from 3 to 8 and each were used an equal number of times per set. Stimulus lists were constructed such that the easy load, the intermediate load and the hard load were presented an equal number of times and with the equivalent set of arithmetic problems (i.e., equal number of problems with sums in the decade 30 through 90) in each condition. The stimulus sets were constructed such that each level of addition problem (no-carry vs. carry) was presented an equal number of times under the three load conditions. Six randomly

ordered lists of the same 72 addition problems were generated.

Procedure

Participants were instructed that they would be doing two different tasks at the same time: half of the participants (i.e., 48 participants) completed a phonological memory task and an arithmetic task, whereas the other half of the participants (i.e., 48 participants) completed a visual-spatial memory task and an arithmetic task. Participants were assigned randomly to the different memory task conditions. Every participant first completed 20 practice trials. Trial-by trial feedback on both the arithmetic and the memory tasks was given in the practice block.

Procedure for the Phonological Memory Task

Each trial began with the word "READY" shown in the center of the screen. The ready prompt flashed three times and then the memory load appeared. Participants saw three kinds of memory loads: a single asterisk in the center of the screen, one non-word in the center of the screen or three non-words in the center of the screen. They were required to rehearse the verbal memory items (i.e., nonwords) subvocally because they were asked to compare it to another one. After 500 ms, an arithmetic problem was presented in the center of the screen and participants were required to answer the problem as quickly and rapidly as possible by speaking into the microphone. As soon as they responded, a second memory load appeared in the center of the screen. If the first memory load had one asterisk, the second also had one asterisk and was always in the same position in the middle of the screen. If the first memory set had one or three non-words, the second had a single non-word probe. Participants were asked to decide whether the probe item had been in the initial list. The experimenter recorded answers to

the arithmetic problems. Participants were asked to subvocally rehearse the non-words while attempting to solve the addition problem and to place the same emphasis on both tasks.

The verbal memory items were pronounceable non-words (e.g., GUB, NOF, SUL). These items were consonant-vowel-consonant (cvc) combinations. The cvcs were constructed so that they did not spell a word, proper name, or common abbreviation. Each cvc combination was only used once and cvc items were selected randomly on each trial. Thus, participants saw different combinations of problems and cvc items (i.e., specific problem was not always matched with a specific cvc). The probe cvc only differed by a single letter from the memory item cvc (see Appendix B for an example of the experimental procedure). Half of the probe cvs matched the memory item cvc in each problem category.

Procedure for Visual-Spatial Memory Load

Each trial began with the word “READY” shown in the center of the screen. The ready prompt flashed three times and then the pattern appeared. Participants saw three kinds of memory patterns, a single asterisk in the center of the screen, a pattern of four asterisks in the center of the screen or a pattern of eight asterisks in the center of the screen. They were required to remember the memory pattern because they were asked to compare it to another one. After 500 ms, an arithmetic problem was presented in the center of the screen and participants were required to answer the problem as quickly and as accurately as possible by speaking in the microphone. As soon as they responded, a second pattern of asterisks appeared in the middle of the screen. If the first pattern had one asterisk, the second also had one asterisk and was always in the same position in the

middle of the screen. If the first pattern had 4 or 8 asterisks the second set had 4 or 8 asterisks respectively, but one asterisk may have moved. Participants had to determine whether the second pattern of asterisks was the same or different from the first pattern. The experimenter recorded answers to the arithmetic problems. Participants were instructed to keep the asterisk pattern in memory while attempting to solve the addition problem and to place the same emphasis on both tasks (see Appendix C for an example of the experimental procedure).

Arithmetic Fluency Test

Participants were also asked to complete a posttest package that included a paper-pencil test of arithmetic that contained addition, subtraction and multiplication problems (French, Ekstrom, & Price, 1963) and two surveys which asked questions concerning their educational background and experiences.

Results and Discussion

Because this experiment had four between-subjects groups, participants' mean arithmetic fluency scores were used to verify that the groups were equivalent in overall computational skill. Fluency score was the total number of problems solved correctly across the 4 pages of this test (max 240). The mean, standard deviation and range of scores on the fluency test are presented in Table 1 for each group of participants. Scores on the arithmetic fluency test were analyzed in a 2 (memory task; visual-spatial vs. phonological) x 2 (format; vertical vs. horizontal) between-subjects ANOVA. No main effects or interactions were significant, suggesting that the four groups were reasonably similar in arithmetic fluency. To verify the overall skill level, a t-test was performed to see whether the overall mean for the four groups differed significantly from the cut-off

point used to determine if a score represents high skill (i.e., cut-off was 80). The result of this t-test showed that the overall group mean was significantly smaller than the cut-off score (71 vs. 80), $t(95) = 30.59$, $p < .001$. Therefore, participants in this study were mostly low skill.

Only trials in which responses to both the arithmetic problem and the secondary task were correct were included in the analyses of latencies. Percentages of errors on the arithmetic task and on the secondary memory tasks were also computed. The mean latencies and mean percentages of error were analyzed in separate sets of ANOVAs with two between-subject factors; 2 (memory task; visual-spatial vs. phonological) x 2 (format; vertical vs. horizontal) and three repeated measure factors; 2 (operand order; single digit + double digit vs. double digit+ single digit) x 2 (problem type; no-carry vs. carry) x 3 (load; no-load vs. low vs. high load). The results of this analysis of variance are summarized in Table 2 (alpha of .05).

Main Effects

As expected, participants solved carry problems more slowly than no-carry problems (2004 ms vs. 1421 ms). Likewise, participants made more errors on carry problems than on no-carry problems (9% vs. 3%). Carry problems were more difficult than no-carry problems. No other error effects were found¹. Participants responded more rapidly when problems were presented in a vertical format than when problems were presented horizontally (1571 ms vs. 1853 ms). As predicted, vertical presentation of addition problems facilitated problem solution.

¹ Proportion of errors on the arithmetic task were also analyzed using an arc sin transformation but the results were the same.

Two-Way Interactions

Participants solved single + double digit problems more slowly than double + single digit problems (1762 ms vs. 1662 ms). As shown in Figure 1 (note: all figures display the “95% confidence interval”), the effect of operand order interacted with format. In horizontal format, participants solved problems presented with the single digit first more slowly than problems presented with the double digit first. Conversely, no operand order effect was found in the vertical format. This result supports the idea that the way in which problems are presented can make a difference in how easily the problems can be solved. This result is consistent with the hypothesis that presenting problems vertically or with the double digit first, enhances people’s performance on addition tasks, because the digits to be added are already aligned. Thus, vertical format and double + single digit format appeared to allow participants to easily determine which digits should be added. In sum, the vertical format appeared to be more beneficial than the horizontal format for both single + double digit and double + single digit problems. However, the format effect was more pronounced for problems presented with the single digit first than for problems presented with the double digit first. In summary, these results support two main hypotheses. First, the vertical format is more beneficial than the horizontal format because the digits to be added are aligned and because a vertical presentation is especially good for encoding the spatial properties and relations of digits (e.g., their locations and sizes). Second, the operand order has an effect on arithmetic performance because the alignment of digits to be added is better when the double digit is presented first as opposed to when the single digit is presented first.

There was a significant interaction of operand order and problem type. Latencies

to carry problems were not affected by operand order, whereas latencies to no-carry problems were slower when problems were presented with the single digit first than when problems were presented with the double digit first (see Figure 2). This result suggests that people might be solving carry and no-carry problems differently. That is, the solution procedure that people adopted when confronted with a no-carry problem was more heavily influenced by the order in which the operands appeared than the solution procedures used for solving carry problems. For no-carry problems, participants may have found it easier to solve the problems when the units to be added were aligned (i.e., side by side or units over units) because it was easier to find the units that needed to be added together, and therefore retrieval of the answer was faster. For a problem like $24 + 3$, for instance, total reaction time may be composed of the time required to look at 24 and say “twen...ty”, and time for retrieving $4 + 3$. This solution procedure would necessitate more time if the “single” digit appeared first (i.e., $3 + 24$) because an incremental step would be required for switching the digits around (i.e., from $3 + 24$ to $24 + 3$). Furthermore, this solution procedure could not be used when solving carry problems. For the problem $24 + 8$, for instance, it would not be possible to say “twen...ty” and then retrieve the answer to $4 + 8$. An incremental step would be needed to perform the carry operation. Thus, this is an example of a solution procedure that would only work when solving no-carry problems. This result supports the hypothesis that different solution procedures may be used when solving carry as opposed to no-carry problems.

Three Way Interaction: Memory Task x Format x Load

Horizontal Format. There was a significant interaction of memory task, format

and load (as shown in Figure 3). In the horizontal format, latencies increased as a function of phonological load, whereas latencies decreased as a function of visual-spatial load. This result is consistent with the result of Kulak (1993) for the visual-spatial load condition. Kulak had hypothesized that the decrease in latencies as a function of load occurred because participants changed their solution procedures when visual-spatial load increased. Specifically, Kulak hypothesized that participants were adopting a more efficient solution procedure when their VSSP was loaded, such as a phonological solution procedure or direct retrieval. Kulak's results, however, did not show an effect of phonological load. Thus, the improvement over Kulak's design, in the present experiment, appears to have yielded a different pattern of results for the phonological condition. In sum, Kulak's results suggested that participants' latencies decreased as a function of visual-spatial load, whereas an increase in phonological load had no effect. The present experiment, however, found that latencies decreased as a function of visual-spatial load and increased as a function of phonological load.

Furthermore, in the present experiment, pairwise comparisons revealed that in the highest load (i.e., load 2), the visual-spatial and phonological loads produced results that were significantly different. Thus, participants in the VSSP condition responded faster when their VSSP resources were overtaxed than when they had no secondary load, or a low secondary load. In contrast, participants' latencies, in the phonological condition, increased as a function of load. It is difficult to make a statement about whether this pattern supports Kulak's hypothesis that participants changed their solution procedure when the visual-spatial load increased, because different people participated in the VSSP and phonological conditions in the present experiment. A better way of understanding

what might be happening is to look at the pairwise comparisons across the different loads for each memory task individually. These latter analyses revealed that as visual-spatial load increased, there was no significant decrease in reaction time. Therefore, the results of the present experiment, for the visual-spatial load, appeared to be going in the same direction as Kulak's (1993) results, but were not statistically significant. Furthermore, the pairwise comparisons revealed that as phonological loads increased, participants' latencies were significantly increasing. Thus, the results of the present experiment, for phonological load, were different than Kulak's (1993) results that showed no effect of phonological load. The difference in results between the present study and Kulak's (1993) study could be due to several factors: (a) the present study was not designed to replicate Kulak's (1993) study, therefore these two studies did not have exactly the same stimuli, (b) Kulak did not have a no-load condition, therefore it was not possible to differentiate between how people solved arithmetic problems when they did and did not have a load task, and (c) the present study used an improved phonological task, therefore the phonological load was presumably more effective in the present study.

In sum, the results of the present study, for the horizontal format, showed a decrease in latencies as visual-spatial load increased, and an increase in latencies as phonological load increased. The pattern, however, was caused by a significant increase in latencies as phonological load increased. The latter result is consistent with the idea that the phonological loop may be involved in mental arithmetic (Campbell & Graham, 1985; De Rammelaere et al., 1999; Fürst & Hitch, 2000; Lemaire et al., 1996; Logie & Baddeley, 1987; Logie et al., 1994). More specifically, research has found that the phonological loop may be involved in the retention of problem information and partial

results, as well as in the maintenance of accuracy. The present result supports the idea that the phonological loop may be involved in the retention of partial results. The present experiment, however, could not verify whether the phonological loop was involved in the retention of initial problem information because problems remained visible throughout the experiment.

Vertical Format. The results found for the vertical condition (see Figure 3) showed that an increase in visual-spatial load did not affect participants' performances on the arithmetic task. Conversely, going from a no-load to a low phonological load, increased participants' latencies, whereas going from a low to a high phonological load decreased participants' latencies. These results suggest that (a) participants may be relying on their phonological resources when there is no secondary load, (b) when a low phonological load is present, they continue to rely on their phonological resources and therefore their latencies increase because the two tasks (i.e., arithmetic and subvocal rehearsal) are competing for the same processing resources, and (c) when their phonological resources are overtaxed (i.e., load 2), participants use another solution procedure that requires less or no phonological resources, resulting in improved performance because there is less competition for the phonological resources.

This is one possible explanation for the above results. However, there are other factors that need to be considered before a firm conclusion can be made about what is occurring. For instance, an assessment must be made as to whether participants are making significantly more errors in the higher load condition, because the decrease in latency from a low load to a high load condition might be the result of a speed-accuracy trade-off. Analyses of the percentages of errors on the arithmetic task show that there was

no speed-accuracy trade-off (see Table 3). Furthermore, performance on the secondary task must be evaluated. Were participants really putting an emphasis on both tasks? Analyses on the percentages of errors on the secondary task showed that participants seemed to be putting an emphasis on both tasks. As expected, participants made more errors on the secondary task as load increased (see Table 4). However, there did not appear to be a trade-off between performing the arithmetic task and the secondary task. As shown in Table 4, participants made 18.4% errors on the secondary task for vertical format in the high phonological load condition. In order to verify whether this is a high error rate, we can compare this error rate (i.e., 18.4%) to the percent error made on the same secondary task (i.e. phonological) in the horizontal format. As shown in Table 4, the error rate on the secondary phonological task in the high load (i.e., load 2), for horizontal format, is 26.3%. That is, the error rate on the secondary task is higher on the condition where participants were slowest on the arithmetic task (see Figure 3, phonological-horizontal-load 2) than the error rate on the condition where participants were fast on the arithmetic task (see Figure 3, phonological-vertical-load 2). If participants were trading-off the secondary task for the arithmetic task, we would expect a higher error rate on the secondary task when the latency on the arithmetic task was fastest. This was not the case, and therefore there did not appear to be a trade-off between the two tasks (i.e., arithmetic task and subvocalization task). Thus, the hypothesis that when problems are presented vertically, participants are faster at solving problems in the high phonological load, because they are switching solution procedures seems valid. Further research, however, is needed to verify this hypothesis. A productive avenue for research would be to replicate the present experiment, but also ask participants to

describe the solution procedures they are using.

As mentioned earlier, participants' latencies were not affected by visual-spatial load, in the vertical format. Non-significant results are usually not very interesting. In dual-task studies, however, it might be very important to explore why the two tasks are not interacting. For instance, in the present experiment, it is important to verify whether there is a trade-off between the two tasks. Are participants giving up the secondary visual-spatial task, and only concentrating on the primary arithmetic task? As shown in Table 4, participants are making the most errors (30.3%) on the secondary task in the vertical-spatial-high load condition. Thus, we might suspect that participants were giving up the secondary visual-spatial task in order to perform the arithmetic task. It is possible that a vertical presentation of problems forced participants to use visual-spatial resources to solve the arithmetic problem. Therefore, participants might have given up on the visual-spatial memory task, and consequently made more errors on this secondary task. What would we expect if participants were giving up the visual-spatial secondary task, because they needed the visual-spatial resources to solve the arithmetic problems? We would expect that in the vertical-phonological condition participants would solve the problems more quickly, because they would be free to use their visual-spatial resources. Such a result would be consistent with Heathcote's view that vertical problem format accelerates performance by eliminating the need to re-arrange horizontal data into vertical displays, thereby freeing up VSSP resources that are needed to perform addition. However, the present research shows that, in the vertical – phonological condition, participants were not faster at solving the arithmetic problems – indeed, they were slower. Furthermore, when problems were presented vertically, participants' latencies

were slower as the phonological memory load (i.e., subvocalization) increased from no-load to low load. Therefore, these results support the earlier hypothesis that participants rely on phonological resources to solve the problems, even when problems are presented vertically. Accordingly, the present research does not support Heathcote's hypothesis.

Researchers have suggested that a vertical format is more beneficial than a horizontal format when solving addition problems (Hayes, 1973; Heathcote, 1994; Smyth et al., 1987). The present experiment supports this hypothesis. When problems are presented vertically, participants were faster at responding to the arithmetic problems than when problems were presented horizontally. As mentioned above, Heathcote suggested that a vertical format was beneficial because it alleviated the need to re-arrange problems into a columnar format, thereby freeing up the VSSP resources for performance of addition. Heathcote's hypothesis is not supported by the present findings. Participants do not appear to be faster at solving problems in a vertical format because their VSSP resources were freed up. Moreover, participants did not seem to be using VSSP resources to solve the problems, but rather they appeared to be using phonological resources. There is no evidence of format re-arrangement. The present findings support my hypothesis that vertical format is more beneficial than the horizontal format, simply because the digits to be added are aligned. Moreover, vertical format appears to be beneficial for phonological solution procedures as well as for visual-spatial solution procedures. That is, the format effect does not appear to be mediated by the memory task. The arithmetic performance benefit that a vertical format provides appears to be due to a better organization of information.

In summary, vertical presentation appears to be more beneficial than horizontal presentation even when people are using phonological solution procedures. Furthermore, the hypothesis that vertical presentation is better than horizontal presentation because the digits to be added are aligned, is supported by the present findings.

Pairwise comparisons revealed that for vertical format, participants were significantly slower at solving problems in the low load (i.e., load 1) condition than in the no load condition. Thus, having participants subvocalize one nonword while performing the arithmetic task appeared to have slowed their performance. Pairwise comparisons also revealed that for vertical format, in the low load condition (i.e., load 1), there was a significant difference between the two memory tasks conditions (i.e., visual-spatial vs. phonological). Therefore, the phonological memory task appeared to have disrupted arithmetic performance more than the visual-spatial arithmetic task. No other significant differences were found in the pairwise comparisons.

In sum, the low load subvocalization task (i.e., one nonword) appeared to have increased participants' latencies on the arithmetic task. This result supports the hypothesis that the phonological loop is responsible for maintaining partial results in mental arithmetic (e.g., Adam & Hitch, 1998; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Lemaire et al., 1996; Logie et al., 1994; Noël et al., 2001). The decrease in latencies from a low to a high phonological load could be due to a switch in solution procedures when phonological resources are overtaxed. Further research is needed, however, to explore this possibility. The visual-spatial secondary task did not appear to impede performance on the arithmetic task.

Three Way Interaction: Problem Type by Format by Load

As shown in Figure 4, there was an interaction of problem Type, format and load. Latencies to no-carry problems were not affected by load, whereas latencies to carry problems were affected differently by load depending on the format in which they were presented. It is hard to interpret these findings because they do not specify whether we are looking at visual-spatial or phonological loads. However, from the previous results (Figure 3), I was able to conclude that the phonological load was showing this effect. That is, latencies were increasing as a function of load in the horizontal format, whereas latencies were increasing from a no-load to a low load and decreasing from a low load to a high load in the vertical format. These same results were found for phonological loads in the previous interaction of memory task, load, and format (see Figure 3).

From the previous results, I was able to conclude that visual-spatial loads did not interact with format. Therefore, it seemed logical to assume that the interaction found in the present result (problem type x format x load Figure 4) was due to an increase in phonological load as opposed to a visual-spatial load or a combination of visual-spatial and phonological loads. However, to verify this assumption, I decided to look at the graph of the interaction of problem type, memory task, load and format (see Figure 5). This latter figure confirmed the assumption that the interaction found in Figure 4 was due to an increase in phonological load as opposed to a visual-spatial load. The problem type by format by load interaction suggested that carry problems were affected by an increase in phonological load whereas no-carry problems were not. Furthermore, these results suggested that when problems were presented horizontally, solution latencies to carry problems increased as phonological load increased. When problems were presented

vertically, however, latencies to carry problems increased from a no-load to a low phonological load and decreased from a low to a high phonological load. In sum, these results suggest that (a) phonological resources are more involved in the solution of carry problems than no-carry problems, and (b) when problems are presented horizontally, participants continue to rely on phonological resources to solve carry problems, as phonological load increases. In contrast, when carry problems are presented vertically, participants rely on phonological solution procedures when confronted with a low phonological load but appear to switch to more efficient solution procedures when confronted with a high phonological load. In order to accomplish this switch in solution procedures when their phonological resources were overtaxed, participants would have had to reduce the demands of the phonological resource. This could be accomplished by a speed-accuracy trade-off (i.e., reallocating resources between the two tasks), or by switching solution procedures (i.e., going from a phonological solution procedure to a visual-spatial solution procedure or direct retrieval). Analyses of errors on the arithmetic problem task (see Table 5) and secondary phonological task (see Table 6) do not show evidence of a speed-accuracy trade-off on either task. However, the possibility of the solution procedure shift still remains. Thus, participants may have switched either from a phonological solution procedure to direct retrieval, or from a demanding to a less demanding solution procedure. Further research is needed to substantiate which of these hypotheses is correct.

In summary, it appears that (a) when carry problems are presented horizontally, latencies decrease with visual-spatial load, whereas latencies increase with phonological loads and, (b) when carry problems are presented vertically, latencies are unaffected by a

visual-spatial load whereas, latencies increase from a no-load condition to a low phonological load condition and decrease from a low phonological load to a high phonological load condition.

Analysis of errors on the secondary tasks

Percent errors to the memory probe items were analyzed in an ANOVA with two between-subject factors; 2 (memory task; visual-spatial vs. phonological) x 2 (format; vertical vs. horizontal) and three repeated measure factors; 2 (operand order; single digit + double digit vs. double digit + single digit) x 2 (problem type; no-carry vs. carry) x 2 (low vs. high load).² The results for the no-load condition were not included in this analysis. There was no variance in the no-load condition because participants did not have to make a decision in this condition; they always knew that the probe item would match the memory item. Problems that required a carry operation disrupted participants' performance on the secondary tasks more than problems that did not require a carry operation, (19.8 % vs. 15.9 %), $F(1, 92) = 14.38$, $p < .001$, $Mse = 204$. Participants made more errors in the high load conditions than in the low load conditions (22.8% vs. 12.9%), $F(1, 92) = 75.07$, $p < .001$, $Mse = 249$. As predicted, determining whether the probe items were the same or different as the memory items was harder when there was a large number of items in the memory set than when there was a small number of items in the memory set.

As shown in Figure 6, there was an interaction between format and memory task, $F(1, 92) = 45.00$, $p < .001$, $Mse = 674$. In the phonological memory task, participants

² Proportion of errors on the secondary tasks were also analyzed using an arc sin transformation but results were the same.

made more errors when problems were presented in a horizontal format than when problems were presented in a vertical format. This result supports the idea that participants were using phonological resources when solving problems in a horizontal format. Conversely, in the visual-spatial memory task, participants made more errors when problems were presented in a vertical format. This result supports the idea that when performing the arithmetic task, participants may have switched from a phonological solution procedure to a visual-spatial solution procedure when problems were presented vertically.

General Discussion

This research set out to investigate the cognitive processing involved in solving arithmetic problems. Solution procedures for addition generally require retrieval of basic facts in combination with storage of partial results and possibly carry procedures. The present experiment was designed to test the role of the visual-spatial scratchpad and the phonological loop in the performance of mental addition.

The multicomponent model of working memory (Baddeley & Hitch, 1974) has been a useful framework for studying the resources involved in arithmetic performance. There is very little evidence that the visual-spatial scratchpad is involved in arithmetic performance (e.g., Hayes, 1973; Heathcote, 1994). However, the results of studies concerned with both the visual-spatial scratchpad and the phonological loop are consistent with the hypothesis that the component of working memory most involved in arithmetic performance is the phonological loop (e.g., Adam & Hitch, 1998; Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Lemaire et al., 1996; Logie et al., 1994; Noël

et al., 2001).

The results of the present experiment suggest that the phonological loop plays a more substantial role than the VSSP in the performance of mental arithmetic. Specifically, this research illustrates that phonological interference disrupted participants' abilities to perform mental arithmetic problems that involved carrying. In this study, the secondary phonological memory task was a subvocal rehearsal task. This supports other research which shows that the phonological component of working memory is involved in mental arithmetic, because people mentally rehearse or subvocalize partial answers (Adam & Hitch, 1998; De Rammelaere et al., 1999; Logie et al., 1994). This is particularly important in the data retention component of carry problems.

As mentioned above, the present research did not demonstrate that the VSSP played a significant role in the performance of the mental arithmetic exercises undertaken in this study. Although other research points to VSSP involvement in mental arithmetic (Hayes, 1973; Heathcote, 1994; Hitch, 1978), arithmetic problems used in these studies involved more complex problems (e.g., triple digit addition) as compared to single + double digits problems involved in the present research. The greater complexity of problems used in the former studies may partially explain this variance in our findings. Similarly, these same studies provided participants with a brief vocal or visual problem input stimulus, whereas the present research provided participants with an ongoing visual problem presentation. This latter approach eliminated the need for participants to mentally maintain original problem information, a function which these other researchers (Hayes, 1973; Heathcote, 1994; Hitch, 1978) ascribe to the VSSP. Taken together, the foregoing two factors probably account for differences in the findings of these authors

and the present research.

A further possible explanation for these differences in findings may be that the VSSP load task used in the present experiment was not sufficiently rigorous. To evaluate this possibility, an analysis was undertaken of the percent error on the secondary memory load tasks. This evaluation revealed that for both the VSSP and phonological secondary load tasks, the percent error increased as the load increased (see Table 4). This demonstrates that participants were performing both primary and secondary tasks simultaneously. Furthermore, the VSSP secondary load task reflected the highest percent error rate as a function of load, clearly eliminating any question of the rigor of this task. Moreover, this analysis also demonstrates that the VSSP is more strongly influenced by problem format than by degree of problem difficulty. Specifically, the highest percentage of errors on the secondary memory tasks occurred when the VSSP was loaded in a vertical format. This finding is consistent with Heathcote's (1994) observation that the columnar presentation of problems uses VSSP resources. However, arithmetic performance does not coincidentally deteriorate with this mounting VSSP load, thereby pointing to a phonological rather than a VSSP solution strategy. This latter finding is clearly demonstrated by a dramatic increase in arithmetic performance latencies as the phonological load task increases from a no-load to a low-load condition (see Figure 3). In sum, the foregoing findings strongly illustrate that phonological resources are involved in the solution of mental arithmetic.

The present research clearly shows that a vertical rather than a horizontal problem format facilitates mental arithmetic performance. As noted above, the highest percentage of errors on the secondary memory tasks occurred when the VSSP was loaded in a

vertical format. This finding is consistent with Heathcote's (1994) observation that the columnar presentation of problems uses VSSP resources. However, arithmetic performance did not coincidentally deteriorate with this mounting VSSP load. Therefore, unlike Heathcote, I do not tie arithmetic performance to use of the VSSP. My findings show that arithmetic performance did not deteriorate with increasing secondary VSSP load, neither in the horizontal nor vertical format. My findings suggest that the phonological loop facilitates arithmetic performance by permitting subvocal rehearsal to achieve retention of partial answers in carry problems. Specifically, in the present study, arithmetic performance deteriorated when phonological load was increased, for both horizontal and vertical problem display formats.

Whether a visual or a phonological secondary load task is involved, arithmetic performance improved when problems were presented in a vertical rather than horizontal format. This is due to the fact that problem inputs were aligned (i.e., units over units, tens over tens, etc.). I do not support Heathcote's argument that people use the VSSP to rearrange horizontal problems into vertical formats. My findings demonstrate that participants are using phonological resources to solve problems, whether they are presented in a horizontal or a vertical format. Vertical displays facilitate arithmetic performance, due to alignment of problem inputs. In sum, based on my findings, it is my contention that individuals use the phonological loop to subvocalize and retain partial solutions, regardless of format.

The present results demonstrated the importance of working memory, especially the phonological loop, for combining temporary information retention with ongoing processing operations in mental arithmetic. The results of the present study are of interest

because they help understand the processing codes involved in mental addition. If we are better able to understand the nature of the processing requirements involved in mental addition, we could improve the teaching of arithmetic as well as pinpoint the locus of individual difficulties.

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Table 1 -

All Groups: Scores on the Arithmetic Fluency Test

	N	Minimum	Maximum	Mean	SD
Horizontal-Visual-Spatial	24	38	114	69.4	19.4
Horizontal-Phonological	24	41	108	67.5	19.7
Vertical-Visual-Spatial	24	40	134	77.0	28.0
Vertical-Phonological	24	30	123	69.6	22.9

Table 2
Analyses of Variance for Latencies and Percent Errors

Source	df	Latencies	F %Errors
Memory Task	1	2.60	1.70
Format	1	9.53**	.10
Memory Task X Format	1	0.07	.01
MSE	92	2384695.9	389.98
Operand Order	1	23.86**	.52
Operand Order X Memory Task	1	0.04	1.20
Operand Order X Format	1	15.02**	1.75
Operand Order X Format X Memory Task	1	0.08	.94
MSE	92	119624.46	75.60
Problem Type	1	186.05**	61.58**
Problem Type X Memory Task	1	0.59	1.00
Problem Type X Format	1	2.27	.15
Problem Type X Format X Memory Task	1	0.07	.52
MSE	92	525373.59	183.70
Load	2	0.77	.39
Load X Memory Task	2	2.91	1.05
Load X Format	2	2.24	.33
Load X Memory Task X Format	2	3.50*	.09
MSE	184	148512.61	102.43

(Table 2 Continued...)

Source	df	<u>F</u>	
		Latencies	% Errors
Operand Order X Problem Type	1	4.82*	.20
Operand Order X Problem Type X Memory Task	1	0.11	2.49
Operand Order X Problem Type X Format	1	0.13	.41
Operand Order X Problem Type X Memory Task X Format	1	0.35	.11
MSE	92	120789.49	84.70
Operand Order X Load	2	0.84	1.76
Operand Order X Load X Memory Task	2	0.28	3.00
Operand Order X Load X Format	2	0.18	.59
Operand Order X Load X Format X Memory Task	2	0.63	.09
MSE	184	138896.60	85.35
Problem Type X Load	2	0.03	1.48
Problem Type X Load X Memory Task	2	0.86	.23
Problem Type X Load X Format	2	3.10*	.79
Problem Type X Load X Memory Task X Format	2	0.02	.20
MSE	184	146255.73	112.64
Operand Order X Problem Type X Load	2	0.12	.07
Operand Order X Problem Type X Load X Memory Task	2	0.05	1.64

(Table 2 Continued...)

Source	df	Latencies	<u>F</u> % Errors
Operand Order X Problem Type X Load X Format	2	0.38	.16
Operand Order X Problem Type X Load X Memory Task X Format	2	0.46	.06
MSE	184	132702.12	89.51

* $p < .05$ ** $p < .01$

Table 3

Percent Errors on the Arithmetic Problems as a Function of Memory Task, Format and Load

	No-Load	Load 1	Load 2
Memory Task = Phonological			
Format			
Horizontal	6.5	6.7	7.4
Vertical	5.6	6.7	7.0
Memory Task = Visual-Spatial			
Format			
Horizontal	5.4	5.5	5.0
Vertical	4.9	6.0	4.0

Table 4

Percent Errors on the Secondary Tasks as a Function of Memory Task, Format and Load

	No-Load	Load 1	Load 2
Memory Task = Phonological			
Format			
Horizontal	0	18.9	26.3
Vertical	0.2	5.7	18.4
Memory Task = Visual-Spatial			
Format			
Horizontal	0.7	6.1	16.1
Vertical	0.2	21.0	30.4

Table 5

Percent Errors on the Arithmetic Problems as a Function of Problem Type, Format and Load

	No-Load	Load 1	Load 2
Problem Type = No-Carry			
Format	4.0	3.0	2.2
Horizontal	2.3	2.6	2.3
Vertical			
Problem Type = Carry			
Format	7.8	9.2	10.2
Horizontal	8.1	10.2	8.7
Vertical			

Table 6

Percent Errors on the Secondary Tasks as a Function of Problem Type, Format, and Load

	No-Load	Load 1	Load 2
Problem Type = No-Carry			
Format	0.3	14.5	23.8
Horizontal	0.0	14.1	26.9
Vertical			
Problem Type = Carry			
Format	0.3	10.5	18.6
Horizontal	0.3	12.7	21.9
Vertical			

Figure 1

Mean Latencies (ms) as a Function of Operand Order and Format

Figure 2

Mean Latencies (ms) as a Function of Operand Order and Problem Type

Figure 3

Mean Latencies (ms) as a Function of Memory Task, Format, and Load

Figure 4

Mean Latencies (ms) as a function of Problem Type, Format, and Load

Figure 5

Mean Latencies as a Function of Problem Type, Memory Task, Format, and Load

Figure 6

Percent Errors on the Secondary Tasks as a Function of Memory Task and

Format

Appendix A

List of Problems

No-Carry	Sum	Carry	Sum
$3 + 84$	87	$3 + 89$	92
$3 + 45$	48	$3 + 58$	61
$3 + 52$	55	$3 + 49$	52
$4 + 53$	57	$4 + 57$	61
$4 + 31$	35	$4 + 79$	83
$4 + 35$	39	$4 + 68$	72
$5 + 43$	48	$5 + 36$	41
$5 + 62$	67	$5 + 47$	52
$5 + 63$	68	$5 + 78$	83
$6 + 32$	38	$6 + 48$	54
$6 + 83$	89	$6 + 37$	43
$6 + 73$	79	$6 + 85$	91
$7 + 72$	79	$7 + 65$	72
$7 + 52$	59	$7 + 86$	93
$7 + 81$	88	$7 + 54$	61
$8 + 61$	69	$8 + 75$	83
$8 + 71$	79	$8 + 64$	72
$8 + 41$	49	$8 + 36$	44

Appendix B

Experimental Procedure: Phonological Task

1. Initial Memory Items

GUB NOF SUL

2. Solve arithmetic problem

7 + 81

3. Decide whether probe item was included in the initial memory items

GUP

Appendix C

Experimental Procedure: Spatial Memory Task

1. Initial spatial pattern

* *

*

*

2. Solve arithmetic problem

$$7 + 81$$

3. Decide whether second pattern is the same or different as the first pattern

* *

*

*