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TITLE OF THESIS/TITRE DE LA THÈSE A Computer Aided Design of Low Noise Parametric Down Converters
(PDC) and GaAs PET Mixers for Low Cost Satellite Microwave Receivers.

UNIVERSITY/UNIVERSITÉ Carleton

DEGREE FOR WHICH THESIS WAS PRESENTED/
GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE Doctoral

YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ 1977

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**LA THÈSE A ÉTÉ
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A COMPUTER AIDED DESIGN OF LOW NOISE
PARAMETRIC DOWN CONVERTERS (PDC) AND GaAs
FET MIXERS FOR LOW COST SATELLITE MICROWAVE
RECEIVERS

by

PASTEUR LUTA NTAKE B. Eng., M. Eng. (Carleton)

A Thesis Submitted to the Faculty of
Graduate Studies in Partial Fulfilment for the
Degree of
DOCTOR OF PHILOSOPHY (Ph D)

Carleton University
Ottawa, Canada

April, 1977

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The undersigned recommend to the Faculty of Graduate Studies
acceptance of the thesis:

"A Computer Aided Design of Low Noise Parametric Down Converters
(PDC) and GaAs FET Mixers for Low Cost Satellite Microwave Receivers"
submitted by Pasteur Luta Ntake, M. Eng., in partial fulfillment of
the requirements for the degree of Doctor of Philosophy.

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May 1977

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ABSTRACT

This thesis is primarily concerned with the theoretical and the experimental investigation of two types of microwave mixers, the parametric down converter (PDC) and the Field Effect Transistor (FET) mixer for low noise microwave communication receivers.

The possibility of simultaneous low noise figure, high dynamic range and power gain from the PDC offers an alternative to the conventional Tunnel Diode amplifier (TDA)/resistive mixer configuration currently in use in most satellite receivers today. The theoretical analysis of the PDC shows that better noise performance is to be expected if (a) the IF load is cooled, (b) the PDC is operated at high ratios of the IF to the RF (f_o/f_s) frequencies, and (c) low loss (i.e. high Q) diodes such as GaAs diodes are used as the varactors.

Experiments on a PDC using silicon varactor diodes show that good noise figure, large dynamic range, and wide RF bandwidth can be achieved with conversion gain. A conversion gain of 14.5 dB, an output third order intermodulation intercept of 23.6 dBm and a 3 dB bandwidth of 238 MHz have been obtained at a low SSB noise figure of 4.3 dB. The RF and the IF signal frequencies were 6 and 4 GHz, respectively.

The investigation of the GaAs FET mixer operating at 4 GHz with an IF frequency of 70 MHz shows that the reported poor noise performance of the FET mixers were due to improper IF and RF signal terminations and not due to the high $1/f$ noise, as claimed by several authors. An SSB noise figure of 4.8 dB and a conversion gain of 6.1 dB have

been obtained using the NEC V244 FETs when the RF and the IF signals were short-circuited at the drain and the gate, respectively. This low noise figure is believed to be the lowest reported for a FET mixer operating at this IF frequency. The wide RF 3 dB bandwidth of over 600 MHz and the relatively high intercept point of 8.8 dBm of the experimental FET mixer reported in this thesis show that the GaAs FET mixer is a promising candidate for the integrated front-end applications at x-band, and higher frequencies.

The effects of the noise measurement system compression point on low noise figure measurement errors have also been investigated. It is shown that this new type of error correction can be minimized by operating the noise measurement system at noise power levels of at least 30 dB below the third order intermodulation intercept point. Errors of the order of 0.3 dB can be expected if the measurement system is operated at noise power levels of 15 to 20 dB below its intercept point. This error is significant if the noise figure of the device being measured is of the order of two dB.

ACKNOWLEDGMENTS

The author wishes to thank his supervisor, Dr. D. R. Conn of SED Systems Ltd., Saskatoon, for his interest, guidance and useful discussions throughout the course of this work. He would like to thank Dr. A. Kavadas, the President of SED Systems Ltd., Mr. H. Grant, the Manager, Earth Stations Division, and all members of the staff at SED Systems Ltd. for providing the facilities and financial support to carry out this research. Special thanks are also due to Ms. K. Robson for her speedy typing.

Fruitful discussions with Mr. H. Chow and technical help from Mr. M. Noette and Mr. F. Dodd are fully appreciated.

The financial support from Carleton University and the Tanzania Armed Forces is greatly appreciated.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Microwave radio communications require a high transmission/reception performance of the components that make up the system itself. The deployment of the new satellites like the CTS and the ANIK has resulted in an increased activity in the design of low cost, low noise ground station receivers. The major design effort has been in the construction of low noise amplifiers (LNA) using the state of the art Field Effect Transistors (FETs).

The key characteristics of an earth station receiver are its system gain, the overall noise figure, and the receiver dynamic range, which in turn depend directly on the configuration and type of the individual components making up the receiver.

The noise figure and the dynamic range of the receiver are primarily set by the LNA/mixer front end. The present resistive mixers have high noise figures, typically 6-10 dB, and very low dynamic range, due to their low intercept point powers (typically -10 dBm). In order to improve the performance of the receiver front end it is imperative that both the LNA and the mixer subsystems be optimized. It is this motivation that has led the author into investigating the performance characteristics of two new types of mixers for microwave receiver applications, the FET mixer and the parametric down converter (PDC).

The superb performance of the Field Effect Transistor and the parametric devices as low noise figure high dynamic range amplifiers

suggests their possible use as mixers for the microwave receiver front end applications. The parametric down converter (PDC), using varactors, has received no attention at all in the literature because the device's output signal spectrum is inverted and because it presents a negative resistance at both the RF and the IF ports, rendering itself potentially unstable (see Chapter 3). On the other hand, the FET mixer has been reported in the literature [1]-[3]. The reported experimental noise figures (typically 10 dB) have virtually eliminated the hopes of using FETs as mixers for a low noise receiver. The reason given by the authors to explain the FETs' poor noise performance is that the device has a high $1/f$ noise at the IF frequency ($IF = 70$ MHz) [4]. The corner frequency of $1/f$ noise is typically 100 KHz [5]. This means then that other explanations have to be found.

1.2 Thesis Objectives

The primary objectives of this are basically twofold:

(a) To investigate the performance of the PDC (parametric down converter) as a mixer component for use in low noise microwave receivers and repeaters. The possibilities of low noise figure, wide bandwidth, and power gain of the PDC offer future receivers greater dynamic range than the present conventional resistive mixers [6].

(b) The dependence of the FET mixer noise figure on the circuit conditions is investigated. The strong dependence of the FET mixer noise figure on the IF and the RF terminations provides an

explanation of the reported high noise figures.

Other secondary objectives of this research include:

- (a) An investigation into the low noise measurement error caused by the measurement system dynamic range (see Chapter 2, section 6). This error correction has never been reported elsewhere in the literature.
- (b) A development of a computer-aided design (CAD) program that has the capability to analyse and optimize mixer networks using advanced adjoint network techniques to calculate the objective function gradients.

1.3 Thesis Organization

Chapter 2 is a review of the mixer terminology and principles. Section 2.4 discusses the earth station terminal requirements. Its purpose is to point out to the reader the relative importance of the mixer component in a microwave receiver. The noise measurement error correction due to the measurement system dynamic range is discussed in detail in section 2.6. The theoretical development of the PDC is presented in Chapter 3. A simple graphical and/or theoretical design procedure for a low noise figure PDC is detailed in section 3.6. Chapter 4 concerns itself with the FET mixer theory. The effects of the RF and IF terminations are discussed in section 4.6.4. Chapter 5 presents the explanation of the CAD mixer program. The development of the adjoint network techniques and the gradient optimization routines

are treated in section 5.3. The program listing of MIXER is included in Appendix C. Chapter 6 presents the experimental results of the PDC and the FET mixers. The conclusions and recommendations for further extension to this work are included in section 6.4.

CHAPTER 2

MIXERS, EARTH STATIONS, AND NOISE MEASUREMENT ERRORS2.1 Introduction

This chapter discusses the basis principles and terminology used in mixers. The relative importance of a mixer in an earth station microwave receiver is discussed in section 2.5. The results of an investigation on the effects of the measurement-system compression point on noise figure errors is presented in section 2.6. It is shown that noise figure errors of the order of .3 dB are expected if the measurement system is operated at powers of about -15 to -20 dB below its output power compression point. This error is very significant, especially when the noise figure being measured is of the order of two dB.

2.2 Mixer Principles and Terminology

Mixing is defined as the conversion of a low power signal, called the RF signal at a frequency f_s , to another signal, called the IF at a frequency f_o , by combining the RF signal with a higher power signal, called the local oscillator (LO) or pump signal at a frequency f_p , by means of nonlinear devices such as varactors, transistors, vacuum tubes and so on. The nonlinear device output comprises of the following signals:

- (i) the original input signals, the LO and the RF signals,
- (ii) all harmonics of the LO, mf_p and the RF signal nf_s , (m and n

- are integers),
- (iii) a dc level,
- (iv) two primary sidebands, the upper sideband signal at the frequency $f_p + f_s$ and the lower sideband signal at $|f_p - f_s|$
- (v) all higher products at the frequencies $mf_p \pm nf_s$ ($m, n \neq 1$).

The desired output, commonly known as the intermediate frequency (IF), can either be the lower or the upper sideband. In order to extract the IF signal, some form of filtering is necessary to reject all of the harmonics and the unwanted sidebands. The signal power in the unwanted sidebands and harmonics can be recovered and converted to IF by terminating these sidebands appropriately. The sideband of significance is the so-called "image" signal at the frequency $2f_p - f_s$. Some terms and definitions commonly used in mixers and upconverters are described below.

(a) Conversion Gain $-g_c$

The conversion gain (or loss) of a mixer is a measure of the IF power relative to the RF input power. The conversion gain depends on the RF and the IF terminations, the type of the nonlinear device, and the LO drive level. Specific formulae for conversion gain using varactors and FET transistors are covered in Chapters 3 and 4. The conversion loss for resistive mixers is considered in section 2.3.

(b) Noise Figure - F and the Effective Temperature T_e

The noise figure F is defined as the ratio of the noise power available at the output of a network at the output frequency when input termination is at the standard temperature T_0 to the total noise power available from the input source at the input frequency [1]. The noise figure is expressed as:

$$F = \frac{N_o \text{ (available)}}{gN_i \text{ (available)}} \quad 2.2.1$$

where N_i = the available noise power from the signal source,

g = the gain or loss of the network, N_o = the output noise power.

The available noise power from a termination is given by the Nyquist theorem as [1]

$$N_i = KT_0B \quad 2.2.2$$

where K = the Boltzmann constant = 1.38×10^{-23} Joules/ $^\circ K$

B = the measurement bandwidth, and

T_0 = the standard reference temperature = $290^\circ K$.

The effective noise temperature T_e of a network is related to the noise figure F and T_0 by:

$$T_e = T_0(F - 1) \quad 2.2.3$$

The noise figure measurements have to be qualified as to whether

they are single sideband (SSB) or double sideband (DSB). The DSB noise figure refers to the measurements of noise in which the image signal frequency is within the passband of the device under test (DUT). If the image signal frequency is outside the DUT passband, the measured noise figure is referred to as the SSB noise figure. The effect of the image signal response is treated in section 2.6:

(c) The Conversion Compression Point and the 3rd Order Intercept Point - P_I

The concept of the compression point and the intercept point can best be understood by referring to figure 2.1 which is a plot of the signal levels in dBm showing the fundamental and the third order product output levels versus the fundamental input signal level in dBm for an arbitrary device. The compression point is the output power level above which the response in figure 2.1 begins to deviate from linearity. It is customary to define the compression point as the output power level for which the gain (or loss) has deviated from linearity by 1 dB.

The 3rd order intercept point P_I is the output power level at which the input fundamental power equals the third order intermodulation product power. It is expressed as [2]

$$P_I = P_C + 10.6 \text{ dBm} \quad 2.2.4$$

where P_C = the 1dB compression point, and

P_I = the third order intercept power level.

FIG 2.1 INTERCEPT and COMPRESSION DIAGRAM FOR AN ARBITRARY DEVICE

(d) The Dynamic Range - DR

The dynamic range of a mixer is defined as the power range over which the device is useful. The absolute maximum useful range is limited on the lower end by the minimum detectable signal and at the high end by the device saturation level. The "spurious free" dynamic range is related to noise figure F , the gain G , and P_1 by [15]

$$DR = 2/3(P_1 - G - F + 144 - 10 \log B) \text{ dB} \quad 2.2.5$$

where G is the DUT (device under test) gain (dB),

B is the bandwidth in Hz.

(e) Image Focussing

The image signal at the frequency $2f_p - f_s$ has pronounced effects on the gain and the noise figure of the mixer. Image focussing is, therefore, a technique by which the image frequency signal is terminated at a port where it does not interfere with the mixer responses.

(f) Idler Frequencies ($mf_p \pm nf_s$)

This term refers to the harmonic product signal frequencies $mf_p \pm nf_s$. A special name, "the image" frequency, is given to the case when the harmonic product signal frequency is $2f_p - f_s$.

(g) Single and Double Balanced Mixers

A mixer configuration is referred to as single or double balanced, dependent on whether two or four or eight diodes are used in the construction of the mixer, respectively. The manner in which the diodes are connected together depends on the signal isolation between the RF and the

L0, and the type of image focussing required.

2.3 Resistive Mixers

The most used nonlinearity in mixers is the I-V characteristics of the diode in forward bias. The diode current is related to its voltage by:

$$i = I_s \exp(qv/nkT) \quad 2.3.1$$

where v = the diode voltage
 n = the diode ideality factor
 I_s = the saturation current.

The small signal conductance of the diode is given as:

$$g_d = \frac{di}{dv} = \frac{qI_s}{nKT} \exp(qv/nKT) \quad 2.3.2$$

Formulas for conversion gain, minimum noise figure, optimum RF and IF impedances have been derived by assuming that the diode conductance varies with L0 voltage and can be written in a Fourier series as:

$$g_d(t) = \sum_{n=-\infty}^{\infty} g_{dn} \exp(jn\omega_p t) \quad 2.3.3$$

where ω_p = the L0 radian frequency, and
 g_{dn} = the complex Fourier coefficients.

Barber [3] has analysed the above conductance wave-form for sinusoidal and non-sinusoidal L0 voltages and found that the pulse duty ratio (PDR) of the L0 voltage is a fundamental parameter for defining

the conductance mixer operation. K. Johnson [4] extended Barber's work by considering the effects of the image signal termination and the out of band signals on conversion gain and the noise figure. His results are summarized below:

- (a) A lower conversion loss is achievable by reactively terminating the image signal in either an open or a short circuit,
- (b) the open circuited image mixer has less conversion loss, being about .5 dB better than the shorted image mixer,
- (c) the open circuited image mixer is much more strongly affected by the diode series resistance and the out of band signal terminations,
- (d) the constant voltage mixer is superior to the constant current mixer,
- (e) the broadband mixer (image signal termination is equal to the signal termination) has a higher conversion loss and a higher noise figure.

The total conversion loss of a resistive mixer comprises of four individual components:

The first type, the mismatch loss L_m , depends on the degree of the impedance match obtained at both the IF and the RF ports.

The second type is a loss of the RF signal power due to the series resistance R_s . This loss is the ratio of the RF signal power delivered to the average conductance g_{do} . Figure 2.2 is a schematic of a typical

FIG 2.2 A SCHEMATIC DIAGRAM OF A TYPICAL RESISTIVE MIXER DIODE

resistive mixer. It consists of the resistance r_d , a charge storage or diffusion capacitance C_j , the package lead inductance L_p and the capacitance C_p . In terms of the diode parameters R_s , C_j , and $R_o = 1/g_{d0}$, the conversion loss due to R_s is given by [4]

$$L_1 = 10 \log (1 + R_s/R_o + (\omega_s C_j)^2 R_s R_o) \text{ dB} \quad 2.3.4$$

L_1 has a minimum value of

$$L_1(\text{min}) = 10 \log (1 + 2\omega_s R_s C_j) \text{ dB} \quad 2.3.5$$

which occurs at an optimum RF signal impedance given by:

$$R_g(\text{opt}) = 1/\omega_s C_j \text{ ohms} \quad 2.3.6$$

The third type is the loss of the IF signal due to R_s . It is given by:

$$L_2 = 10 \log (1 + R_s/R_{if} + (\omega_o C_j)^2 R_s R_{if}) \text{ dB} \quad 2.3.7$$

The fourth type is the actual loss at the junction. This loss depends on the IV characteristics of the diode, the type of image focussing, and the LO drive level. The conversion loss for various image terminations is included in Appendix A [4]. The total conversion loss L_c for a resistive mixer is given by:

$$L_c = L_m + L_1 + L_2 + L_j \text{ dB} \quad 2.3.8$$

where L_j = the junction conversion loss.

The noise figure of a mixer is given by the well known equation:

$$F = L_c (N_r + F_{if} - 1) \quad 2.3.9$$

where N_r = the diode noise ratio

F_{if} = the IF amplifier noise figure

The noise ratio N_r is the ratio of the total noise output of the diode to the thermal noise output of a resistor having the same value of the resistance as the average diode resistance. Typical values of N_r are between .9 and 1.0. The noise ratio N_r must be modified slightly dependent on the type of image focussing used. Pritchard [5] has expressed the modified noise ratios as:

$$N_m = [N_r(L_c - 1) + 1]/L_c \quad \text{for open and shorted image} \quad 2.3.10$$

and

$$N_m = 2[N_r(L_c/2 - 1) + 1]/L_c \quad \text{for the broadband mixer} \quad 2.3.11$$

Before leaving this section, it is important that a brief mention of the recent advances in the field of resistive mixers be made. The recent developments of very high cut-off frequency GaAs Schottky barrier diodes and the use of double balanced hybrid circuits in controlling the image termination have made significant improvements in the conversion loss and the noise figure. L. Dickens et al of Westinghouse [6] have reported a double balanced hybrid mixer with a conversion loss of 2.2 dB in converting 12 GHz to 70 MHz. Their mixer had a noise figure of 3 dB with an IF 3 dB bandwidth of 40 MHz. Very wide bandwidth

mixers operating in the X-band have been reported by G. Kurpis [7]. His mixers have a 40% IF bandwidth with 10 dB noise figures. G. Stracca et al [8] of the Institute di Electtrotecnica de Electronica in Italy has reported mixers with both low noise figures and low conversion losses. Stracca's mixer performances are summarized below.

RF Frequency	3.6 - 4.2	7.1 - 7.7	12.7 - 13.3
Overall Noise Figure ⁺	3.2 dB	3.7 dB	5.0 dB
IF Noise Figure	1.5 dB	1.5 dB	1.5 dB
IF Frequency	70 mHz	70 mHz	70 mHz
Conversion Loss	2.0 dB	2.6 dB	4.0 dB
Bandwidth	40 mHz	40 mHz	40 mHz

+ Noise figures referred to above are all DSB

Table 2.1 Summary of Stracca's Mixer's Performance [8]

D. Neuf of RHG Electronic Laboratories [9] has reported a patented X-band "Quiet Mixer" that features a 2.5 dB DSB noise figure and a IF bandwidth of 1 GHz. The Quiet Mixer utilizes two double balanced mixers with RF and the LO coupled by a 90° hybrid. Neuf's results were obtained using silicon diodes. RHG Laboratories predict a DSB noise figure of between 1.5 and 2.0 dB at 12 GHz when GaAs chip diodes are used.

2.4 Varactor Down and Up Converters

In the last section mixers that utilize the IV characteristics of a forward biased diode were discussed. Another diode nonlinearity

which can be used in frequency conversion is the variation of the diode reverse bias capacitance with the voltage. Mixers using variable capacitances are commonly known as the parametric up or down converters. Parametric up converters have been treated extensively [10]-[12] in the literature. The parametric down converter (PDC) has not received any treatment in the literature at all. The reasons for its neglect are covered in Chapter 3 where the PDC conversion properties are discussed.

2.5 Earth Stations Requirements

2.5.1 Introduction

There has been quite a considerable interest in the microwave communications industry to develop low cost, low noise receivers for television and radio programming. Satellites like the ANIK have been launched to facilitate communication to remote areas in the Canadian North. Figure 2.3 is a schematic of a typical receiver. The components of the receiver are:

- (i) the receiver antenna,
- (ii) the low noise amplifier (LNA),
- (iii) the down converter, and
- (iv) the baseband signal processing circuitry.

A figure of merit commonly used to classify microwave receivers is the G/T ratio. This ratio is defined as the antenna gain G to the receiver noise temperature T . Typical values for G/T ratio are about 18 to 20 dB/°K. In the following subsection, a sample link calculations for

FIG. 2.3 A TYPICAL DUAL CONVERSION RECEIVER

"Frontier Remote Receiver Earth Station" project [13] is presented. The aim of the subsection is to point out the relative importance of the down converter in an Earth Station receiver.

2.5.2 Satellite Link Calculations

Satellite link calculations are based on the effective isotropic radiated power (EIRP) of the satellite, the station figure of merit, G/T, and the available antenna size and hence the antenna gain. The following specifications pertain to the ANIK satellite and the "Frontier Remote Receiver" project carried out at SED Systems Ltd. [13].

Satellite EIRP (ANIK)	= 35 dBW
Path Loss (PL)	= 196 dB
Antenna gain G (15 foot)	= 42.6 dB
G/T ratio (spec)	> 18.5 @ 4 GHz
Signal to Noise ratio (output)	> 49.0 dB

From the satellite EIRP, the antenna gain G, and the path loss one calculates the received carrier power C dBW as:

$$C = \text{EIRP} - \text{PL} - G = -118.4 \text{ dBW} \quad 2.5.1$$

The path loss, PL dB, is the loss of the signal as it travels from the geosynchronous satellite to the receiver on earth. The path loss is defined as: [17]

$$\text{PL} = 20 \log(4\pi d/\lambda) \text{ dB} \quad 2.5.2$$

where d = distance between the satellite and the earth receiver,
and

λ = free space wavelength (cm)

The antenna gain G is related to its diameter by [17]

$$G = 10 \log (4\pi A / \lambda^2 \epsilon_{ff}) \text{ dB} \quad 2.5.3$$

where A = the antenna area = $\pi D^2 / 4$

D = the antenna diameter

ϵ_{ff} = the antenna efficiency (typically 55%)

Using the specified G/T ratio and the antenna gain G , one calculates the system noise temperature by the equation:

$$\begin{array}{ccccc} (G/T) & = & G & - & 10 \log T \\ \text{specification} & & \text{antenna} & & \text{system} \\ & & \text{gain} & & \text{temperature} \end{array} \quad 2.5.4$$

The calculated system temperature using equation 2.5.4 is $T = 257^\circ\text{K}$.

The system noise temperature is made up of the receiver noise temperature T_R and the antenna temperature T_A (see figure 2.4).

$$T = T_A + T_R + T_m \quad 2.5.4$$

where T_m = the equivalent temperature of the losses between the antenna and the

The antenna noise temperature, T_A for the particular parabolic dish used in the project is given by figure 2.4. For the 30° elevation (worst case) the noise temperature of the antenna $T_A = 21^\circ\text{K}$. Thus, the receiver noise temperature T_R is

$$T_R = 257 - T_A - T_m = 199^\circ\text{K} \quad 2.5.5$$

Worse case conditions
within the 3.7 to 4.2
GHz band
Measurements were
made on a clear dry
day.

FIG 2.4 THE ANTENNA NOISE TEMPERATURE T_A VS THE ELEVATION ANGLE [13]

where $T_m = 28^\circ\text{K}$ (typical value for a loss of 0.4 dB).

The receiver noise figure $F_R(\text{dB})$ required to meet the calculated noise temperature T_R is given by:

$$F_R = 10 \log (1 + T_R/290) = 2.4 \text{ dB} \quad 2.5.6$$

2.5.3 The LNA/Mixer Configuration Requirements

In order to meet the specified G/T ratio of 18.5 dB, a receiver with a maximum noise figure of 2.4 dB is required. The receiver noise figure is made up of the LNA noise figure F_{LNA} and the mixer noise figure F_{mix} contribution according to Friis Formula [14].

$$F_R = F_{\text{LNA}} + (F_{\text{mix}} - 1)/g_{\text{LNA}} \quad 2.5.7$$

(mixer noise contribution)

where g_{LNA} = the low noise amplifier (LNA) power gain.

To minimize the mixer noise contribution, the following specifications (or requirements) must be met by the mixer and/or the LNA:

- (i) The LNA must have very high gain g_{LNA} , and
- (ii) The mixer stage must have a relatively low noise figure.

The LNA gain g_{LNA} cannot be arbitrarily increased without affecting the receiver system dynamic range. The dynamic range DR is related to the system gain by equation 2.2.5. Thus, in order to minimize the mixer noise figure contribution a low noise mixer is essential.

The third order intercept point P_I of a cascaded system is related to the individual component intercept point I_i by [15]

$$\frac{1}{P_I} = \frac{1}{I_1 g_1 g_2 g_3} + \frac{1}{I_2 g_2 g_3} + \frac{1}{I_3} + \dots \quad 2.5.8$$

where I_i and g_i are the intercept point and gain of the i^{th} component.

In the case of the mixer/LNA configuration, the third order intercept point of the configuration is given by:

$$\frac{1}{P_I} = \frac{1}{I_{\text{mix}}} + \frac{1}{I_{\text{LNA}} g_{\text{mix}}} \quad 2.5.9$$

where I_{mix} = the mixer intercept point

g_{mix} = the mixer conversion gain, and

I_{LNA} = the LNA intercept point

Typical values of I_{mix} , g_{mix} , and I_{LNA} for a resistive mixer and the LNA built for the "Frontier Remote Earth Station" project are:

$$g_{\text{mix}} = -6 \text{ dB} = .25$$

$$I_{\text{mix}} = -10 \text{ dBm} = 0.1 \text{ mW}$$

$$I_{\text{LNA}} = +15 \text{ dBm} = 31 \text{ mW}$$

The computed intercept point power $P_I = -10.1 \text{ dBm}$. It is seen from the above simple example that the intercept point of the LNA/mixer

configuration is set primarily by the mixer. Thus, the second requirement for the mixer is that it must have a high dynamic range.

Finally, the mixer must have a wide bandwidth in order to pass the full satellite bandwidth of 500 MHz. In conclusion, it has been shown that the mixer for the earth station receiver must have:

- (a) a relatively low noise figure, to minimize the mixer noise figure contribution,
- (b) a high dynamic range, and
- (c) a wide bandwidth.

The need to investigate the performance of other types of mixers other than the conventional resistive mixer becomes apparent in the light of the low dynamic range, the high noise figures, and the narrow bandwidths of the present resistive mixers.

2.6 Low Noise Figure Measurement Errors

2.6.1 Introduction

In microwave low noise figure measurements using conventional techniques, the results can be distorted and often misleading if factors affecting the accuracy of the results are not properly accounted for. Such factors include:

- (a) the variation of the temperature of the noise source termination,
- (b) the transmission losses between the noise source and the device under test (DUT),

- (c) the differences in VSWR between the "hot" and the "cold" conditions of the noise source,
- (d) the measurement system bandwidth,
- (e) the mismatches between the noise source and the DUT,
- (f) the "second stage contribution",
- (g) the image response, and
- (h) the dynamic range of the noise measurement setup (see fig. 2.5).

The "true" DUT noise figure is obtained after all of the above corrections have been applied. The corrections (a) to (f) have been extensively treated in the literature. The appropriate corrections for the factors (a) to (f) are found in section 2.6.4.

During the course of the low noise amplifier development at SED Systems Ltd., it was found that the noise figure of the LNA, as measured by the measurement system shown in figure 2.5, was dependent on the signal level of the noise source and the image signal response. In the following two subsections, the effects on noise figure of the image signal and the compression points are discussed.

2.6.2 The Effects of the Measurement System Component Compression Point on Noise Figure

Figure 2.5 shows a typical microwave low noise figure measurement setup. It consists of a noise source, a device under test (in this case a LNA 50 dB gain amplifier), an attenuator, a wideband mixer, and the noise meter. The attenuator is used to set up the correct power

DIODE NOISE SOURCE BIAS

FIG. 2.5 LOW NOISE FIGURE MEASUREMENT SETUP (LNA 50 dB GAIN)

levels for the noise meter. The power levels and gain shown on figure 2.5 pertain to the setup at SED Systems Ltd. laboratory.

The low noise figure measurement procedure consists of measuring the system noise figure F_{sys} without the device under test (DUT); then measuring the noise figure F_m of the system with the DUT inserted between the noise source and the high gain amplifier. The noise figure of the DUT, F is calculated from the equation:

$$F_m = F + (F_{sys} - 1)/g_{DUT} \quad 2.6.1$$

where F = the DUT noise figure,

F_{sys} = the measurement system noise figure (without the DUT),
and

F_m = the noise figure of the measurement system including the DUT.

Equation 2.6.1 assumes that:

- (a) the measurement system (the attenuator, the mixer, the noise meter, etc) are operated in a linear range after the DUT is inserted, and
- (b) the image response is outside the DUT passband.

In the case of the measurement setup (fig. 2.5) and the LNA (DUT) built at SED Systems Ltd., neither of the two assumptions are true. Figure 2.6 is a plot of the noise figure and the gain of the FET low

FIG 2.6 THE NOISE FIGURE AND GAIN OF THE FET AMPLIFIER VS THE SIGNAL SOURCE FREQUENCY (SINGLE STAGE FET AMPLIFIER)

noise amplifier at 4 GHz. It is seen that the image response falls within the passband of the DUT violating the second assumption. The high gain of the DUT results in an increased input signal level to the mixer and the first assumption is also not met.

Figure 2.7 is a plot of the output power versus the input power of the measurement setup shown in figure 2.5 (without the DUT). The systems 3rd order intercept point is +5.3 dBm on the output.

The transfer function between the output and the input power can be described by a power series as: [17]

$$e_o = Z\sqrt{P_o} = a_1 e_i + a_2 e_i^2 + a_3 e_i^3 + \dots \quad 2.6.2$$

where e_o = the output voltage,

Z = a constant dependent on the input and the output impedances,

e_i = the input voltage, and

a_i ($i = 1, 2, \dots$) are constants, P_o = the output power

The gain compression/expansion due to the transfer function nonlinearity is given by [17]:

$$g(A) = g_o \left(1 + \frac{3}{4} \frac{a_3}{a_1} A^2 \right)^2 \quad 2.6.3$$

where $g(A)$ = the power gain when an input signal of amplitude A is applied to the system,

g_o = the small signal gain (effects of the nonlinearity being ignored).

FIG 2.7 THE INPUT POWER VS THE OUTPUT POWER OF THE NOISE MEASUREMENT SETUP

The third order harmonic power due to a signal of amplitude A is given by [17]

$$P(3\omega_0) = Z(a_3 A^3/4)^2 \quad 2.6.4$$

The third order intercept point is defined as the point at which the extrapolated output fundamental power response equals the output third order harmonic power response. Using this definition and equation 2.6.4, the power gain can be written as:

$$g(P_o) = g_o (1 \pm 3P_o/P_I)^2 \quad 2.6.5$$

where P_o = the output power, and

P_I = the third order intercept power.

The plus or minus sign refers to the gain expansion or compression, respectively. For most devices, the gain decreases with the increasing power, and therefore, a minus sign will be used throughout this section.

In noise figure measurements using the Y-factor method the lamp (noise source) is alternately turned "on" and "off", corresponding to the high and the low noise output powers, respectively. If the "lamp on" conditions corresponds to the gain g_{off} , then the measured Y factor Y_m can be expressed in terms of g_{on} and g_{off} by: [16]

$$Y_m = \frac{\text{Output noise power with the "lamp on"}}{\text{Output noise power with the "lamp off"}}$$

or

$$Y_m = \frac{T_{on} + (F_m - 1)T_o}{T_o F_m (g_{off}/g_{on})} \quad 2.6.6$$

where T_{on} = the "lamp on" temperature of the noise source

T_o = the "lamp off" temperature ($T_o = 290^\circ \text{K}$)

F_m = the noise figure of the DUT.

The actual noise figure F_{act} can be calculated from equation 2.6.6 if the gains g_{on} and g_{off} are known.

$$F_{act} = (T_{on}/T_o - 1)/(g_{off}/g_{on} Y_m - 1) \quad 2.6.7$$

In practice, the ratio g_{off}/g_{on} is not known. Instead the noise figure is calculated under the assumption that the ratio g_{off}/g_{on} is unity.

Under these circumstances, the apparent noise figure F_{app} is calculated as:

$$F_{app} = (T_{on}/T_o - 1)/(Y_m - 1) \quad 2.6.8$$

The error in dB of the noise figure due to gain compression effects is

$$\Delta F_m = 10 \log\{(Y_m - 1)/(g_{off}/g_{on} Y_m - 1)\} \text{ dB} \quad 2.6.9$$

Figure 2.8 is a plot of the error ΔF_m versus the level of the power P_o in dBm below the measurement system 2nd order intercept point P_I . It is observed from figure 2.10 that if measurements of noise figure are made at noise source powers greater than -20 dB below the intercept point of the measurement system, errors in noise figure larger than 0.25 dB can be expected. The experimental measurements

FIG 2.8 THE ERROR IN NOISE FIGURE DUE TO COMPRESSION EFFECTS VS THE OUTPUT POWER IN dBm BELOW THE INTERCEPT POINT

of the noise error due to the gain compression effects of the FET LNA designed at SED Laboratories confirms the above theory.

2.6.3 The Effects of the Image Signal Response on Noise Measurements

The noise meter in the AILTECH set responds to noise at two frequencies, the wanted signal at $f_0 - 30$ and the image signal at $f_0 + 30$ MHz (see figure 2.6) where f_0 = the local oscillator frequency.

Let the system gain (without the DUT) be g_1 and g_2 at the signal and the image frequencies, respectively. The thermal output noise of the system measured by the AILTECH set is:

$$P_a = KTB (g_1 + g_2) \quad 2.6.9$$

where g_1 = the gain at the signal frequency of the measurement system (without the device under test)

g_2 = the image signal gain; $T^* = T_0 = 290^\circ \text{K}$.

Let the DUT gain at the signal and the image frequencies be g_s and g_m , respectively and let F be the measured noise figure when the DUT is inserted into the measurement setup, the output noise power with the DUT inserted is given by:

$$P_t = FKTB(g_1g_s + g_2g_m) \quad 2.6.10$$

where g_1 and g_2 are the gain of the measurement system (without the DUT) and the gain of the DUT at the signal frequency

g_m and g_s are similarly defined at the image frequency

and the signal frequency, respectively.

F = the measured noise figure (with the DUT inserted)

Now if it is assumed that the DUT noise figure at the signal and the image frequencies are different, then the total output noise power measured by the meter (with the DUT inserted) can be written as:

$$P_t = (F_s g_s g_1 + F_m g_m g_2) KTB + (F_a - 1)(g_1 + g_2) KTB \quad 2.6.11$$

where F_s = the device under test noise figure at the required signal frequency.

F_m = the DUT noise figure at the image frequency,

F_a = the noise figure of the measurement system (without the DUT)

The second term in equation 2.6.11 is the noise power contribution of the LNA to the total output noise of the measurement system. Equating 2.6.11 and 2.6.10 gives:

$$F = \frac{(F_s g_s g_1 + F_m g_m g_2)}{g_1 g_s + g_2 g_m} + \frac{(F_a - 1)(g_1 + g_2)}{g_1 g_s + g_2 g_m} \quad 2.6.12$$

The wanted noise figure F_s can be obtained from equation 2.6.12. Several cases can be considered:

Case A

Assuming that the measurement system (without DUT) has a flat gain g , (i.e. $g_1 = g_2 = g$) then

$$F = \frac{F_s g_s + F_m g_m}{g_s + g_m} + \frac{2(F_a - 1)}{g_s + g_m} \quad 2.6.13$$

Equation 2.6.13 indicates that the measured noise figure F (the system with the DUT) is a sum of weighted contributions of the signal and the image noise figures F_s and F_m , and the noise contribution of the measurement system F_a .

Case B

Assuming that the image noise figure F_m and the signal noise figure F_s are equal ($F_m = F_s$), then equation 2.6.13 is written as:

$$F = F_s + 2(F_a - 1)/(g_m + g_s) \quad 2.6.14$$

so that F_s can be determined if F , F_a , g_s , and g_m are known. Equation 2.6.14 is the one that is most frequently used in calculating the noise figure of a device under test (DUT). It should be emphasized that equation 2.6.14 holds true under the following assumptions:

- (i) The measurement system (mixers, preamplifiers and the noise measurement set) have a flat gain over the DUT passband.
- (ii) The image noise figure (or temperature) is equal to the required signal noise figure (or temperature).

For the experimental measurements where the Y factor is measured, it is convenient to express equation 2.6.13 in a different form. Let the DUT noise temperatures at the signal and the image frequencies be T_s and T_m , respectively. The noise output with the "lamp on" N_{on} and with the "lamp off" N_{off} can be written as [17]

$$N_{on} = KB(T_s g_s + T_m g_m) + KBT_{on}(g_m + g_s) \quad 2.6.15$$

$$N_{\text{off}} = KB(T_s g_s + T_m g_m) + KBT_{\text{off}}(g_m + g_s) \quad 2.6.16$$

where T_{on} = the effective "lamp on" temperature of the noise source, and

T_{off} = the effective "lamp off" temperature.

The measured (actual) Y factor Y_a is calculated as

$$Y_a = \frac{N_{\text{on}}}{N_{\text{off}}} = \frac{(T_o + T_{\text{on}}) + (T_m + T_{\text{on}})(1 + \hat{g})}{(T_s + T_{\text{off}}) + (T_m + T_{\text{off}})(1 + \hat{g})} \quad 2.6.17$$

where $\hat{g} = g_m/g_s$

Let the image signal temperature T_m be different from that of the signal by a small amount ΔT_s , then

$$T_m = T_s + \Delta T_s$$

and

$$Y_a = \frac{T_s + T_{\text{on}} + T_{\text{err}}}{T_s + T_{\text{off}} + T_{\text{err}}} \quad 2.6.18$$

where $T_{\text{err}} = \frac{\Delta T_s \hat{g}}{1 + \hat{g}}$ = the error temperature

In the limit as ΔT_s or $\hat{g} \rightarrow 0$, $T_{\text{err}} \rightarrow 0$ and the actual Y factor

$Y_a \rightarrow Y_{ao}$ where

$$Y_{ao} = \frac{T_{\text{on}} + T_s}{T_{\text{off}} + T_s} \quad 2.6.19$$

Equation 2.6.19 is the one most commonly used in computing the noise figure of the DUT from Y factor measurements. The assumptions involved in arriving at equation 2.6.19 are:

- (i) The noise temperature of the image signal T_m is equal to the noise temperature of the required signal T_s , and
- (ii) The gain of the image signal is small ($\hat{g} = g_m/g_s = 0$).

In actual experiments neither of the above two assumptions are usually satisfied. The image temperature, T_m is never equal to the signal temperature, T_s and the gain g_m is usually never equal to zero. Under these circumstances, a significant error in the computed noise figure of the DUT can be made if equation 2.6.19 is used blindly. An example of the DUT (FET amplifier built at SED Laboratories) in which the above assumptions are not met is discussed below.

Example

Consider the FET LNA amplifier whose measured noise figure and gain are shown in figure 2.6. The measured noise figure and gain at the signal and image frequencies (see fig. 2.6) are

$$NF_s = 1.74 \text{ dB} \rightarrow F_s = 1.429; T_s = (F_s - 1) T_0 = 0.429 T_0$$

$$NF_m = 2.22 \text{ dB} \rightarrow F_m = 1.667; T_m = 0.667 T_0$$

$$G_s = 6.19 \text{ dB} \rightarrow g_s = 4.159$$

$$G_m = 6.01 \text{ dB} \rightarrow g_m = 3.990$$

$$\hat{g} = g_m/g_s = .959$$

$$\Delta T_s = T_m - T_s = 0.248 T_0$$

The error temperature T_{err} is calculated as

$$T_{err} = \frac{\Delta T_s \hat{g}}{1 + \hat{g}} = 0.116 T_0$$

Assuming a noise source of ENR of 15.5 dB, one calculates $T_{on} = 34.48 T_0$.

The actual Y factor Y_a is calculated from equation 2.6.18 as:

$$Y_a = \frac{T_{on} + T_s + T_{err}}{T_0 + T_s + T_{err}} = 22.669$$

The Y factor Y_{ao} as computed by the most commonly used method is:

$$Y_{ao} = \frac{T_{on} + T_s}{T_0 + T_s} = 24.43$$

The error in the two noise figures is given by:

$$\Delta F = 10 \log(Y_{ao}-1) - 10 \log(Y_a-1) \text{ dB} \quad 2.6.20$$

or

$$\Delta F = .34 \text{ dB (in the above example)}$$

This error is significant especially when the noise being measured is of the order of a dB or two. The only way to correct for this error is to make the ratio of the image gain to the signal gain g_s as small as possible. This can be achieved easily by having a filter at 30 MHz inserted in front of the noise measurement set.

2.6.4 Other Sources of Noise Measurement Errors

The following is a collection of the noise measurement errors and their corrections.

(a) Variation of the Temperature of the Noise Source [16]

If the noise source termination temperature is not $T_0 = 290^\circ \text{K}$, a small correction ΔF_1 dB must be added to the measured noise figure.

$$\Delta F_1 = 10 \log \left[1 - \frac{T_1 - T_0}{T_{\text{on}} - T_0} - \frac{T - T_0}{T_0 F} \right] \text{ dB} \quad 2.6.21$$

where T_1 = the actual temperature of the termination

T_{on} = the "lamp on" temperature

F = the measured noise figure.

(b) The Difference in VSWR Between the "Hot" and the "Cold" Condition of the Noise Source [16]

The error in the measured noise figure due to the differences in the reflection coefficients when the lamp is "on" and when the lamp is "off" is given by:

$$\Delta F_2 = \frac{4.34 \Delta M}{(1 - 1/Y)} \quad 2.6.22$$

and

$$\Delta M = \frac{(1 + |\rho_d| |\rho_{\text{off}}|)^2}{(1 - |\rho_d| |\rho_{\text{on}}|^2)} - 1 \quad 2.6.23$$

where ρ_d = the reflection coefficient of the device under test

ρ_{on} , ρ_{off} are the reflection coefficients of the device with the "lamp on" and the "lamp off", respectively.

Y = the measured Y factor

(c) Transmission Losses

Transmission losses add directly to the noise figure measurements.

(d) Measurement System Bandwidth

If the measurement setup has a bandwidth B (Hz) comparable to the DUT passband, the measured noise figure will NOT be a spot frequency noise figure; instead, the indicated noise figure represents the overall (average) noise figure of the DUT in the passband.

(e) Second Stage Contribution [14]

The overall noise figure of a cascaded system is given by Friis formula (equation 2.5.7). The noise figure of the DUT is obtained from the measured noise figure as:

$$F_D = F_{me} - (F_2 - 1)/g_{DUT}$$

where F_D = the DUT noise figure,

F_{me} = the measured noise with the DUT inserted, and

F_2 = the noise figure of the system without the DUT.

g_{DUT} = the power gain of the DUT

CHAPTER 3: THE PARAMETRIC DOWN CONVERTER THEORY

3.1 Introduction

A parametric device is a periodic time varying nonlinear reactance which may be used to amplify or convert energy from one frequency to another. The literature on parametric devices dates back as early as the Second World War when H. North [1] discovered that his crystal mixer gave a net gain instead of the usual conversion loss. The low noise potential of parametric devices was not realized until late in 1948 when van der Ziel [2] published a paper on the mixing properties of nonlinear reactances. Many authors followed van der Ziel and massive literature on low noise parametric amplifiers was produced. The term parametric amplifier as used in most literature refers to both the amplifiers and the converters. To differentiate parametric amplifiers from the converters, the term Parametric Down Converter (PDC) will be used throughout this thesis.

3.2 Manley and Rowe Relations and The Small Signal Analysis of the PDC

A parametric down converter (PDC) is a mixer in which the output signal frequency f_o is less than the input signal frequency f_s ($f_o < f_s$). The basis for the PDC operation is the Manley and Rowe [3] relations of the power flow into a nonlinear reactance. Manley and Rowe derived a very general set of equations relating power flowing in and out of a nonlinear reactance when two signals of unrelated frequencies are applied. Their results are summarized below.

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{mn}}{(mf_1 + nf_2)} = 0 \quad 3.2.1$$

$$\sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \frac{nP_{mn}}{(mf_1 + nf_2)} = 0 \quad 3.2.2$$

where P_{mn} = power flowing INTO the nonlinear reactance at the frequency $(mf_1 + nf_2)$.

f_1, f_2 = the applied signal frequencies, and

n, m = integers.

In frequency converters (mixers) one is interested in either the lower sideband $|f_1 - f_2|$ or the upper sideband $f_1 + f_2$ signals. The signal power of other sidebands $mf_1 + nf_2$ ($n, m \neq 1$) is usually very small and hence extraction of this power at these sidebands is not usually done. However, the presence of these sidebands has a significant effect on the performance of the converter.

Table 3.1 is a summary of the possible modes of operation of the parametric converter when the desired output signal is either the lower sideband $|f_1 - f_2|$, or the upper sideband $(f_1 + f_2)$. The properties listed in table 3.1 are derived from equations 3.2.1 and 3.2.2 [4]. To be consistent with the mixer terminology the signal frequencies f_1 and f_2 are designated as f_p and f_s . The subscripts s and p refer to the RF signal and the local oscillator (pump) signal frequencies ($f_1 \equiv f_s$, and $f_2 \equiv f_p$).

Mode	Frequency Spectrum ($f_1 = f_p, f_2 = f_s$)	Properties
Upper Sideband Converter		(i) Conversion gain bounded by f_o/f_s (ii) Has positive output and input resistance
Lower Sideband Upconverter		(i) Arbitrary gain (ii) Presents negative impedance to the RF and IF ports
Lower Sideband Downconverter (non-inverting)		(i) Arbitrary gain (ii) Presents negative resistance to the pump circuit
Lower Sideband (inverting) PDC		(i) Arbitrary gain (ii) Present negative resistance to the input signal f and output signal f_o ports.

TABLE 3.1 OPERATION MODES OF THE PARAMETRIC CONVERTER

The parametric down converter (PDC) is characterized by:

- (i) an inverted output signal spectrum relative to the RF spectrum,
- (ii) a negative input resistance at both the RF (f_s) and the IF (f_o) ports, and
- (iii) an arbitrary gain

This mode of operation of the parametric converter has received no attention in the literature because the device is potentially unstable and the output spectrum is inverted. This chapter is devoted to the analysis of the PDC. The parameters defining this mixer such as gain, noise figure, bandwidth, intermodulation products, and so on are derived.

Fig. 3.1 shows a nonlinear elastance $s(t)$ being driven by a large signal amplitude $v_p(t)$ and by a relatively weak signal $v_s(t)$. The nature of the elastance is immaterial for the moment. Let the two signal voltages be

$$v_p = V_b + V_p e^{j\omega_p t} + V_p^* e^{-j\omega_p t}$$

3.2.3

and

$$v_s = V_s e^{j\omega_s t} + V_s^* e^{-j\omega_s t}$$

where V_s, V_p = the complex voltage amplitude containing the phase information of the RF and the pump signals, respectively

Fig 3.1 A PINNED NONLINEAR ELASTANCE

ω_s, ω_p = the radian frequency at the RF and pump signal frequencies, respectively

The time varying elastance is a periodic function of the large pump signal $v_p(t)$ and is expressible by a Fourier series as:

$$S(v_p) = \sum_{n=-\infty}^{\infty} S_n e^{jn\omega_p t} \quad 3.2.4$$

where $S_n = S_{-n}^*$ = the complex Fourier coefficients.

The small signal voltage $v(t)$ across the nonlinear elastance consists of the RF signal at ω_s and the harmonic products of the pump and the RF signals. This voltage can be expressed in a Fourier series as:

$$v(t) = \sum_{m=-\infty}^{\infty} V_m e^{j(m\omega_p + \omega_s)t} \quad 3.2.5$$

where V_m = the complex amplitudes of the voltage across elastance at the frequency $(m\omega_p + \omega_s)$.

The current $i(t)$ through the nonlinear elastance is related to the voltage across it by

$$v = S(t) \int i(t) dt \quad 3.2.6$$

Let us assume that $i(t)$ can be represented in a Fourier series as

$$i(t) = \sum_{k=-\infty}^{\infty} I_k e^{j(k\omega_p + \omega_s)t} \quad 3.2.7$$

where I_k = the complex current amplitudes.

Substituting equations 3.2.4 and 3.2.7 into equation 3.2.6 gives

$$v = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \frac{S_n I_k}{j(k\omega_p + \omega_s)} e^{j[(k+n)\omega_p + \omega_s]t} \quad 3.2.8$$

In order to simplify the analysis, it is assumed that the small signals allowed to exist across the elastance are

- (i) the impressed RF signal at frequency ω_s
- (ii) the sum frequency signal at $\omega_u = \omega_p + \omega_s$, and
- (iii) the IF signal at $\omega_o = \omega_p - \omega_s$.

Using equation 3.2.7 and 3.2.5 three equations for voltage $v(t)$ are obtained, one for each frequency. These equations can be expressed in the following matrix form.

$$\begin{bmatrix} V_s \\ V_o^* \\ V_u \end{bmatrix} = \begin{bmatrix} \frac{S_o}{j\omega_s} & \frac{S_1}{j\omega_o} & \frac{S_1}{j\omega_u} \\ -\frac{S_1}{j\omega_s} & -\frac{S_o}{j\omega_o} & 0 \\ \frac{S_1}{j\omega_s} & 0 & \frac{S_o}{j\omega_u} \end{bmatrix} \begin{bmatrix} I_s \\ I_o^* \\ I_u \end{bmatrix} \quad 3.2.9$$

where $\omega_o = \omega_p - \omega_s$ = output signal radian frequency

$\omega_u = \omega_p + \omega_s$

I_s, I_o, I_u, V_s, V_o , and V_u are the complex quantities

containing the phase information at the RF signal, output and the sum frequency signals.

S_0, S_1 = the Fourier coefficients of the elastance.

3.3 The PDC Conversion Gain - g_c

Fig. 3.2 shows the nonlinear elastance in a three port network. Each port is characterized by a bandpass filter and an impedance seen by the signal flowing in that particular port. In order to calculate the mixer parameters such as gain, noise figure, bandwidth, etc., the three-port network must be reduced to a two-port network. This can be done provided the termination at the unwanted port is known. Let us assume that the impedances seen by the RF, the IF, and the sum frequency signals are Z_g, Z_L , and Z_u , respectively. The sum frequency signal voltage V_u is related to the input signal voltage V_s and current I_s by (matrix equation 3.2.9).

$$V_u = -I_u Z_u = \frac{S_0 I_u}{j\omega_u} + \frac{S_1 I_s}{j\omega_s} \quad 3.3.1$$

Solving for I_u and substituting the result into the equation 3.2.9 gives the following impedance matrix describing the lossless (ideal) elastance conversion properties.

$$\begin{bmatrix} V_g \\ V_0^* \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22}^* \end{bmatrix} \begin{bmatrix} I_s \\ I_0^* \end{bmatrix} \quad 3.3.2$$

FIG 3.2 THE NONLINEAR ELASTANCE IN A THREE PORT NETWORK

$$\text{where } Z_{11} = Z_g + \frac{S_1^2}{\omega_s \omega_u Z_u} + \frac{S_o}{j\omega_s} \quad 3.3.3 (a)$$

$$Z_{12} = -\frac{S_1}{j\omega_o} \quad 3.3.3 (b)$$

$$Z_{21} = \frac{S_1}{j\omega_s} \quad 3.3.3 (c)$$

$$Z_{22} = Z_L + \frac{S_o}{j\omega_o} \quad 3.3.3 (d)$$

Z_g, Z_L, Z_u = the impedances seen by the RF, the IF, and the sum frequency signals, respectively

Fig. 3.3 is a schematic of a reverse biased diode. The nonlinear elastance is the reverse bias capacitance $C_j(v)$. The elements L_p, C_p and R_s represent the package lead inductance, capacitance and series resistance, respectively. For the diode nonlinear capacitance converter, equation 3.3.3 (a) - (d) have to be modified as

$$Z_{11} = R_s + Z_g + \frac{S_o}{j\omega_s} + \frac{S_1^2}{\omega_s \omega_u Z_u} \quad 3.3.4 (a)$$

$$Z_{22} = R_s + Z_L + \frac{S_o}{j\omega_o} \quad 3.3.4 (b)$$

The lead inductance and capacitance are considered to be part of the external circuit loading the elastance converter. The transducer conversion gain g_c (from ω_s to ω_o) is obtained from equations 3.3.3 (b),

FIG 3.3 THE MODEL OF A REVERSE BIASED DIODE [16]

(c) and 3.3.4 (a), (b) as

$$\begin{aligned}
 g_c &= \frac{4R_g R_L |I_o^*|^2}{|V_g|^2} = \frac{\text{Power dissipated in } R_L \text{ at } \omega_o}{\text{Available signal power at } \omega_s} \\
 &= 4R_L R_g |Y_{21}|^2 \quad 3.3.5 \\
 g_c &= \frac{4R_L R_g (S_1/\omega_s)^2}{|Z_{22}^* Z_{11} - Z_{12} Z_{21}|^2}
 \end{aligned}$$

where $R_L = \text{Real}(Z_L)$, $R_g = \text{Real}(Z_g)$, and

Y_{21} = the transfer admittance of the Y matrix formed from the Z matrix in equations 3.3.3 (b), (c) and 3.3.4 (a), (b).

The presence of the upper sideband signal ($\omega_s + \omega_p$) results in an increased input impedance by an amount ΔZ given by (equation 3.3.4 (a))

$$\Delta Z = \frac{S_1^2}{\omega_s \omega_u Z_u} \quad 3.3.6$$

If the upper sideband signal impedance Z_u is not properly controlled, the mixer parameters such as gain, bandwidth, and noise figure will be affected. Furthermore, if Z_u is dispative, (i.e. has a real component) the thermal noise generated by this load will be converted to IF, resulting in an increased noise figure. It is for this reason that the PDC MUST be operated with upper sideband signal open-circuited (i.e. $Z_u \rightarrow \infty$).

At resonance, Z_{22} and Z_{11} can be assumed to be real. In this special case, equation 3.3.5 is written as

$$g_o = \frac{4R_g R_L (S_1/\omega_s)^2}{[(R_g + R_s)(R_L + R_s) - S_1^2/\omega_o \omega_s]^2} \quad 3.3.7$$

The midband gain g_o can further be expressed as:

$$g_o = \frac{4Cr}{(1 - C)^2} \quad , \quad g_o = \text{the midband gain} \quad 3.3.8$$

where $C = \frac{S_1^2}{\omega_o \omega_s (R_g + R_s)(R_L + R_s)}$ 3.3.9

and $r = f_o/f_s = \text{ratio of IF to RF frequencies}$

The constant C can further be expressed in terms of the normalized RF load (R_g/R_s), the IF load R_L/R_s and the diode dynamic quality factor $\tilde{Q}_s$

$$C = \frac{\tilde{Q}_s}{r(1 + R_g/R_s)(1 + R_L/R_s)} \quad 3.3.10$$

$\tilde{Q}_s = S_1/\omega_s R_s = \text{the diode dynamic quality factor at the RF signal frequency } \omega_s.$

$R_s = \text{the series resistance of the diode.}$

3.4 The PDC Noise Figure

Fig. 3.4 is a schematic of the equivalent circuit of the PDC for noise figure calculations. In the PDC, the noise sources are mainly thermal, and shot noise is usually negligible [5]. However, if the pump voltage swings into the forward conduction region a significant forward current may flow resulting in shot noise. In this analysis the diode will be operated in reverse bias, and hence shot noise sources will be neglected.

There are three major sources of noise in a PDC.

(a) Thermal noise from R_s and R_g at $\omega_s - N_1$

The available noise power at the output of the PDC due to the thermal noise generated by R_s and R_g is: (see Appendix F)

$$N_1 = 4KB (T_s R_s + T_g R_g) |Y_{21}|^2 R_L \quad 3.4.1$$

= the conversion gain x the noise power generated by R_s and R_g

where K = Boltzmann constant,

B = measurement bandwidth,

T_s = thermal temperature of R_s

R_g = generator impedance

R_L = IF load resistance,

Y_{21} = the PDC transfer admittance.

T_g = the RF generator resistor thermal temperature

(b) Thermal noise due to R_s at $\omega_0 - N_2$

The available noise power generator by R_s in the output circuit that is delivered to the load is: (see Appendix F)

Fig 3.4, EQUIVALENT CIRCUIT FOR PDC NOISE CALCULATIONS

$$N_2 = 4KBT_S R_S |Y_{22}|^2 R_L \quad 3.4.2$$

(c) Thermal noise generated by R_L at ω_0 - N_3

The noise power generated by the load resistance R_L is amplified by the negative output resistance of the PDC. This reflected signal is then delivered to load. The output reflection coefficient ρ is given by

$$\rho = \frac{Z_L - Z_{out}^*}{Z_L + Z_{out}} \quad 3.4.3$$

where Z_{out} = the output impedance of the PDC, and

Z_L = the load impedance of the PDC.

$$Z_{out} = -(Z_{12}Z_{21}/Z_{11})^*$$

The ~~reflected~~ amplified noise power N_3 is

$$N_3 = KT_L B |\rho|^2 \quad 3.4.4$$

where T_L = the load effective thermal temperature.

The noise figure F is defined as the ratio of the total available noise power at the output of a network to the available power at the output of the same network due to the signal source.

$$F = \frac{N_1 + N_2 + N_3}{KT_g B g_c} \quad 3.4.5$$

where g_c = the PDC conversion gain. Equation 3.4.5 can be simplified to:

$$F = 1 + \frac{T_s}{T_g} \frac{R_s}{R_g} \left[1 + \frac{|Y_{22}|^2}{|Y_{21}|^2} \right] + \frac{|p|^2}{g_c} \frac{T_L}{T_g} \quad 3.4.6$$

Let us assume that the R_g and R_s are at the same ambient thermal temperature $T_0 = 290^\circ \text{K}$. The midband noise figure F_0 can be written as:

$$F_0 = 1 + \frac{R_s}{R_g} \left[1 + \frac{(1+R_g/R_s)^2}{\tilde{Q}_s^2} \right] + \frac{T_L}{T_0} \frac{|p_0|^2}{g_0} \quad 3.4.7$$

where

$$|p_0| = \left| \frac{R_L + R_s + S_1^2 / ((R_g + R_s)(R_L + R_s)\omega_0\omega_s)}{R_L + R_s - S_1^2 / ((R_g + R_s)(R_L + R_s)\omega_0\omega_s)} \right| \quad 3.4.8$$

or

$$|p_0| = \left| \frac{1+C}{1-C} \right| ; g_0 = \text{the midband gain} \quad 3.4.9$$

where C is defined in equation 3.3.10. Equation 3.4.7 can be written in terms of the constant C as:

$$F_0 = 1 + \frac{R_s}{R_g} \left[1 + \frac{(1+R_g/R_s)^2}{\tilde{Q}_s^2} \right] + \left[\frac{T_L}{T_0} \right] \left[\frac{f_s}{f_0} \right] \left[\frac{(1+C)^2}{4C} \right] \quad 3.4.10$$

where $\tilde{Q}_s$ = diode dynamic quality factor,

T_L = load effective temperature,

C = a constant (equation 3.3.10),

R_s = the diode series resistance, and

R_g = the RF signal generator impedance.

Equation 3.4.10 shows clearly the effects of the signal generator impedance R_g , the diode dynamic quality factor Q_s , the effective load temperature T_L , and the ratio of the IF to the RF frequency on the noise figure of the PDC. To minimize the noise figure one must choose a diode with a small series resistance and a high value of the dynamic quality factor. Furthermore, if the load is cooled ($T_L < T_0$), the noise figure will decrease. The last term in equation 3.4.10 will be referred to as the "Load noise contribution" (LNC) throughout this thesis. Once the gain g_0 (equation 3.3.7) and the ratio of the IF to RF frequency are known, the LNC value is fixed. LNC is given by:

$$\text{LNC} = \frac{(1 + C)^2}{4C} \left(\frac{f_s}{f_0} \right) \left(\frac{T_L}{T_0} \right) \quad 3.4.11$$

3.5 The 3 dB Bandwidth

The normalized 3 dB bandwidth P_1 is defined as the ratio of the PDC 3 dB bandwidth $\Delta\omega$ to the input circuit 3 dB bandwidth. For a singly resonated input and output circuit the load and the generator impedances can be written as:

$$Z_g = R_g + R_s + j(X_g - S_0/\omega) \quad 3.5.1$$

$$Z_L = R_L + R_s + j(X_L - S_0/\omega) \quad 3.5.2$$

where S_0 = the average diode elastance,
 R_L = load resistance at the IF frequency,
 R_g = generator resistance at RF frequency,
 R_s = diode series resistance, and
 X_g, X_L = generator and load reactances.

The voltage transfer functions for the input and the output circuit are [6]

$$H_g(\omega) = \frac{1}{1 + jQ_g(\omega/\omega_s - \omega_s/\omega)} \quad 3.5.3$$

$$H_L(\omega) = \frac{1}{1 + jQ_L(\omega/\omega_0 - \omega_0/\omega)} \quad 3.5.4$$

where H_g or H_L is defined to be the ratio of the output current to the input current for the input or the output circuits, respectively.

$$Q_g = \text{input circuit } Q \equiv \omega_s / \Delta\omega_g$$

$$Q_L = \text{the output circuit } Q \equiv \omega_0 / \Delta\omega_L$$

$$\Delta\omega_g = \text{the input circuit 3 dB bandwidth}$$

$$\Delta\omega_L = \text{the output circuit 3 dB bandwidth}$$

$$\omega_0 = \text{the output IF frequency}$$

$$\omega_s = \text{the input RF frequency.}$$

The magnitudes of the transfer functions have the value 1/2 when

$$Q_g^2 (\omega/\omega_s - \omega_s/\omega) = 1, \text{ and} \quad 3.5.5$$

$$Q_L^2 (\omega/\omega_o - \omega_o/\omega) = 1 \quad 3.5.6$$

$$\text{Now, } (\omega/\omega_o - \omega_o/\omega) \approx \frac{2(\omega - \omega_o)}{\omega_o} = \frac{\Delta\omega}{\omega_o} \quad 3.5.7$$

$$\text{and } (\omega/\omega_s - \omega_s/\omega) \approx \frac{2(\omega - \omega_s)}{\omega_s} = \frac{\Delta\omega}{\omega_s} \quad 3.5.8$$

Using equation 3.5.5 to 3.5.8 and the definition of the Q of a circuit, one can express the PDC 3 dB bandwidth as

$$\frac{\Delta\omega}{\Delta\omega_g} = \frac{X_g - S_o/\omega_s}{R_g + R_s} = P_1 \quad 3.5.9$$

$$\text{and } \frac{\Delta\omega}{\Delta\omega_L} = \frac{X_L - S_o/\omega_o}{R_L + R_s} = P_2 \quad 3.5.10$$

where $\Delta\omega$ = the PDC 3 dB bandwidth,

P_1, P_2 = the normalized PDC 3 dB bandwidth to the input and output circuit 3 dB bandwidths, respectively

The conversion gain can be written in terms of P_1 and P_2 as:

$$g_c = \frac{4Cr}{(1 - C)^2 + 2P_1Y(1 - C) + P_1^2(1 - Y^2) + P_1^4Y^2} \quad 3.5.11$$

where $Y = P_2/P_1$ and C is as defined in equation 3.3.10. The ratio $g_c/g_0 = 1/2$ gives the PDC 3 dB bandwidth as the solution to the equation

$$P_1^4Y^2 + P_1^2(Y^2 - 2YC + 1) - (1 - C)^2 = 0 \quad 3.5.12$$

A solution to equation 3.5.12 gives the normalized PDC 3 dB bandwidth $P_1 = \Delta\omega/\Delta\omega_g$ for a given ratio Y of the output to the input circuit 3 dB bandwidth. A plot of the 3 dB bandwidth versus the constant C is shown in figure 3.6.

3.6 The Minimum noise Figure Design

The midband noise figure F_0 (equation 3.4.10) is a function of the load R_L and generator R_g resistances. Differentiating equation 3.4.10 with respect to R_L and R_g and setting results to zero leads to the absurd result of an infinity gain and zero PDC bandwidth. Thus, to achieve some bandwidth at finite gain a sacrifice has to be made in the midband noise figure F_0 .

Now, for a given midband gain g_0 and frequency ratio r , the constant C is obtained from equation 3.3.8 as

$$C = 1 + \frac{2r}{g_0} \pm (4r/g_0(1+r/g_0))^{1/2} \quad 3.6.1$$

Once C is known, the PDC 3 dB bandwidth P_1 is obtained from equation

3.5.12 as

$$P_1 = \left[-(Y^2 - 2YC + 1) \pm \left[(Y^2 - 2YC + 1)^2 + 4Y^2(1 - C)^2 \right]^{1/2} \right]^{1/2} \quad 3.6.2$$

The optimum generator and load resistances are then given by:

$$R_g(\text{opt}) = R_s(\tilde{Q}_s^2 + 1)^{1/2} \quad 3.6.3$$

$$R_L(\text{opt}) = \frac{R_s \tilde{Q}_s^2}{rC(1 + R_g(\text{opt})/R_s)} - R_s \quad 3.6.4$$

Figure 3.5 is a plot of the optimum generator resistance versus the diode dynamic quality factor $\tilde{Q}_s$. The normalized bandwidth P_1 versus the constant C (equation 3.3.8) for various input to the output bandwidth ratios Y is plotted in figure 3.6. The normalized "load noise contribution" (LNC/ r) versus the constant C is shown in figure 3.7.

The design procedure for a PDC is as follows:

- (a) Obtain the dynamic quality factor $\tilde{Q}_s$ of the particular diode that one intends to use (see section 3.8.3 (c)).
- (b) Select the value of the constant C based on the required gain and/or bandwidths (fig. 3.6 or fig. 3.8).
- (c) Calculate the optimum load and generator resistances from equations 3.6.4 and 3.6.3.

The PDC circuit can now be synthesized from the values of $R_L(\text{opt})$ and

Fig 3.5 DYNAMIC QUALITY FACTOR VERSUS THE NORMALIZED
GENERATOR RESISTANCE R_g/R_s

Fig 3.6 THE NORMALIZED BANDWIDTH VS C

$$\text{Normalized load noise Contribution (LNC)} = \frac{(1+C)^2}{4C}$$

$$\text{Noise due to load} = \frac{T_L}{T} \times \frac{T_0}{T_0}$$

$$r = f_0/f_s$$

POTENTIALLY
UNSTABLE REGION

(Unless circulator is used
at input port)

Fig 3.7 THE NORMALIZED LOAD NOISE CONTRIBUTION VS C

Fig 3.8 MIDBAND GAIN (q_0/r) dB VERSUS C

and $R_g(\text{opt})$. The development of the MIC circuit for the PDC is considered in section 3.8.

3.7 Intermodulation Distortion in a PDC

The knowledge of the intermodulation distortion in a mixer is important in communication systems where many channels are to be transmitted. The fidelity of the PDC is described by the linearity of its conversion gain. The purpose of this section is to carry out a large signal analysis assuming that three sinusoids are present at the varactor terminals (i.e. RF, IF, and the LO).

The varactor voltage v is related to the charge $q(t)$ by [9]

$$v(q) = \phi - V_0 \left(\frac{q}{Q_{br} + q_0} \right)^\gamma \quad 3.7.1$$

where $V_0 = V_{br} + \phi$, $\gamma = 1/1-m$

V_{br} = the diode breakdown voltage

ϕ = the diode contact potential

q_0 = the stored charge at the barrier potential ϕ ,

m = the diode index, $m = 1/2$ for abrupt junction diodes
and $m = 1/3$ for the linearly graded junction diodes

q = the charge across the varactor

Q_{br} = the charge on the varactor at the varactor breakdown
voltage V_{br} .

Expanding equation 3.7.1 into a power series gives

$$v(t) = V_b + a_1 q + a_2 q^2 + a_3 q^3 + \dots \quad 3.7.2$$

where V_b = the bias voltage.

a_i ($i = 1, 2, \dots$) are constants.

The desired charge across the varactor terminals are those due to the RF, LO and IF signals. The charge q can be written as:

$$q(t) = Q_b + Q_o \cos \omega_o t + Q_s \cos \omega_s t + Q_p \cos \omega_p t \quad 3.7.3$$

where the subscripts o, s, and p represent the output (IF), the signal (RF), and the pump (LO) signals. Equation 3.7.3 can be expressed in complex notation as:

$$\begin{aligned} q = & Q_b + (q_o \exp(j\omega_o t) + q_o^* \exp(-j\omega_o t)) \\ & + (q_s \exp(j\omega_s t) + q_s^* \exp(-j\omega_s t)) \\ & + (q_p \exp(j\omega_p t) + q_p^* \exp(-j\omega_p t)) \end{aligned} \quad 3.7.4$$

where $q_s = \frac{Q_s}{2}$, $q_o = \frac{Q_o}{2}$, $q_p = \frac{Q_p}{2}$, Q_b = bias charge.

If equation 3.7.3 is substituted into equation 3.7.2 and terms of

corresponding frequencies collected together one gets the following results [7]

Term	Frequency	Amplitude
q q^2	ω_s (RF)	Q_s $Q_s Q_p$
q q^2	ω_p (LG)	Q_p $Q_o Q_s$
q q^2	ω_o (IF)	Q_o $Q_p Q_s$

Table 3.2 Expansion of the power series for three sinusoidal inputs.

The resulting voltages at IF, RF and LO frequencies can be written down as:

$$V_o(\omega_o) = a_1 Q_o + a_2 Q_s Q_p \quad 3.7.5 \text{ (a)}$$

$$V_p(\omega_p) = a_1 Q_p + a_2 Q_s Q_o \quad 3.7.5 \text{ (b)}$$

$$V_s(\omega_s) = a_1 Q_s + a_2 Q_o Q_p \quad 3.7.5 \text{ (c)}$$

The current I_o , I_s , and I_p through the diode are related to the

charges Q_o , Q_s and Q_p by

$$I_o = j\omega_o Q_o \quad 3.7.6 (a)$$

$$I_s = j\omega_s Q_s \quad 3.7.6 (b)$$

$$I_p = j\omega_p Q_p \quad 3.7.6 (c)$$

Substituting equations 3.7.6 (a) and (b) into equations 3.7.5. (a) and (b) gives:

$$V_o = \frac{a_1 I_o}{j\omega_o} - \frac{a_2 I_s I_p}{\omega_s \omega_p} \quad 3.7.7 (a)$$

$$V_p = \frac{a_1 I_p}{j\omega_p} - \frac{a_2 I_o I_p}{\omega_s \omega_o} \quad 3.7.7 (b)$$

$$V_s = \frac{a_1 I_s}{j\omega_s} - \frac{a_2 I_o I_p}{\omega_o \omega_p} \quad 3.7.7 (c)$$

Fig. 3.9 is a schematic showing the PDC model for intermodulation analysis. The diagram depicts a nonlinear reactance (a diode) embedded by three circuits. A bandpass filter is assumed to exist in each circuit that allows only one signal current at a particular frequency to flow through it. Using Kirchhoff's voltage law, three loop equations at each signal frequency can be written as

$$V_o = -I_o Z_L \quad 3.7.8 (a)$$

FIG 3.9 THE PDC CIRCUIT MODEL FOR INTERMODULATION ANALYSIS

$$V_p = V_{gp} - I_p Z_p \quad 3.7.8 (b)$$

$$V_s = V_{gs} - I_s Z_g \quad 3.7.8 (c)$$

where $Z_p = R_p + R_s + jX_p \quad 3.7.9 (a)$

$$Z_L = R_L + R_s + jX_L \quad 3.7.9 (b)$$

$$Z_g = R_g + R_s + jX_g \quad 3.7.9 (c)$$

V_{gp} = the amplitude of the pump voltage source

V_{gs} = the voltage amplitude of the RF signal

Z_p, Z_L, Z_g are the external circuit impedances seen by the pump, IF and RF signal frequencies.

R_s = the diode series resistance.

The diode parasitics are assumed to be part of the external circuits.

Equating equations 3.7.8 to 3.7.7 results in the following relationships:

$$\frac{I_o}{I_s} = \frac{-a_2 I_p}{\omega_s \omega_p (Z_L - ja_1/\omega_o)} \quad 3.7.10$$

$$V_{gp} = (Z_p - ja_1/\omega_p) I_p - a_2 I_s I_o / \omega_s \omega_p \quad 3.7.11$$

Equation 3.7.10 and 3.7.11 can be solved simultaneously to eliminate I_p . The result is:

$$\frac{I_o}{I_p} = \frac{-a_2 V_{gp} / \{\omega_s \omega_p (Z_p - ja_1/\omega_p)(Z_L - ja_1/\omega_o)\}}{1 + a_2^2 I_s^2 / \{\omega_s \omega_p (Z_p - ja_1/\omega_p)(Z_L - ja_1/\omega_o)\}} \quad 3.7.12$$

or

$$\frac{I_o}{I_s} = \frac{h_o}{1 + (h_1 I_s)^2} \quad 3.7.13$$

where h_o and h_1 are defined as:

$$h_o = \frac{-a_2 V_{gp}}{\omega_s \omega_p (Z_p - ja_1/\omega_p)(Z_L - ja_1/\omega_o)} \quad 3.7.14$$

$$h_1 = \frac{a_2^2}{\omega_s \omega_p (Z_p - ja_1/\omega_p)(Z_L - ja_1/\omega_o)} \quad 3.7.15$$

The PDC conversion gain g_c is defined as:

$$g_c = 4 \cdot \left| \frac{I_o}{I_s} \right|^2 \frac{R_L}{R_g} = \frac{\text{Power delivered to } R_L \text{ at } \omega_o}{\text{The available power at } \omega_s} \quad 3.7.16$$

Substituting equation 3.7.13 into 3.7.16 gives

$$g_c = \frac{4R_L}{R_g} \frac{h_o^2}{[1 + (h_1 I_s)^2]^2} \quad 3.7.17$$

To evaluate the intermodulation characteristics of the PDC, the current gain (equation 3.7.13) is expanded in a series for $|h_1 I_s| \ll 1$ as follows

$$I_o = h_o I_s \sum_{n=0}^{\infty} (-1)^{n+2} (h_1 I_s)^{2n} \quad 3.7.18$$

Assuming an RF signal current of amplitude A_s is applied to the input of the PDC, equation 3.7.18 may be written as:

$$I_o = h_o A_s [1 - (h_1 A_s)^2 i_s^3 + (A_s h_1)^4 i_s^5 + \dots] \quad 3.7.19$$

where $I_s = A_s \cos \omega_s t$, and

$$i_s = \cos \omega_s t$$

ω_s = the RF radian frequency.

Equation 3.7.19 has been put in a series form for easy identification of the intermodulation distortion terms. The second term in the bracket causes gain compression or expansion [17]. Following an analysis similar to that in chapter 2 section 5; the current gain compression can be expressed as:

$$\frac{I_o}{I_s} = h_o (1 - 3A_s^2 h_1^2 / 4)^2 \quad 3.7.20$$

Substitution of equation 3.7.20 into the conversion gain equation 3.7.16 gives

$$g_c(A_s) = \frac{4R_L h_o^2}{R_g} (1 - 3A_s^2 h_1^2 / 4)^2 \quad 3.7.21$$

Equation 3.7.21 is in a form similar to equation 2.6.3 which is repeated here for convenience

$$g_c(A_s) = g_{co} (1 - 3a_3 A_s^2 / 4a_1)^2 \quad 3.7.22$$

where a_1 and a_3 are the series coefficients of the expansion of the gain when a signal of amplitude A_s is applied (see equation 2.6.3)

g_{co} = the gain when the compression effects are ignored.

In equation 3.7.21, g_{co} is defined as

$$g_{co} = 4R_L h_o^2 / R_g \quad 3.7.23$$

Following the same arguments as in chapter 2 section 5, the gain compression (equation 3.7.21) can be written as (see equation 2.6.5)

$$g_c = g_{co} (1 - 3P_o / P_I)^2 \quad 3.7.24$$

where P_o = the output power of the PDC, and

P_I = the third order intercept point

A comparison of equation 3.7.21, 3.7.22 and 3.7.24 results in the following relations

$$\frac{P_o}{P_I} = \frac{A_s^2 h_1^2}{4}, \text{ and} \quad 3.7.25$$

$$h_1^2 = a_3/a_1 \quad 3.7.26$$

The output power of the device is related to the input current amplitude A_s by:

$$P_o = g_{co} (A_s^2 / 2R_L) \quad 3.7.27$$

where g_{co} = the conversion gain when compression effects are ignored,

A_s = the input RF signal current amplitude, and

R_L = the IF load

Substituting equation 3.7.27 into 3.7.25 gives

$$P_I = 2g_{co}/R_L h_1^2 \quad 3.7.28$$

where g_{co} and h_1 are defined in equations 3.7.23 and 3.7.15. At resonance, the gain g_c attains its maximum value g_o and the intercept point can then be written as

$$P_I = 2g_o/R_L h_{10}^2 \quad 3.7.29$$

where h_{10} = the midband value of h_1 (equation 3.7.15)

$$h_{10} = \frac{a_2^2}{\omega_s \omega_p (R_p + R_s)(R_L + R_s)} \quad 3.7.30$$

g_o = the midband conversion gain

R_p = the pump circuit resistance, R_L = the IF load

R_s = the diode series resistance

The constant a_2 can be evaluated by expanding equation 3.7.2 in a Taylor series about the bias Q_b .

$$a_2 = \left. \frac{2\partial^2 V}{\partial q^2} \right|_{q=Q_b(\text{bias charge})} \quad 3.7.31$$

The reverse biased diode charge q is related to the voltage V by [9]

$$V = \phi - V_0 (q/Q_b)^\gamma \quad 3.7.32$$

where ϕ = the contact potential, $V_0 = V_{br} + \phi$,

Q_b = the breakdown voltage charge, V_{br} = the breakdown voltage

$\gamma = 1/(1-m)$

m = the diode index, $m = 1/2$ for abrupt junction diodes
and $1/3$ for linearly graded junction diodes.

Performing the differentiation results in the value

$$a_2 = -2\gamma(\gamma-1)(V_{br}+\phi) q^{\gamma-2}/Q_b^\gamma \Big|_{q=Q_b} \quad 3.7.33$$

or

$$a_2 = \frac{-2\gamma(\gamma-1)(V_{br}+\phi)}{Q_b^2} \left[\frac{Q_b}{Q_b} \right]^{\gamma-2} \quad 3.7.34$$

The breakdown voltage charge Q_b is related to the maximum elastance at breakdown S_{max} by [9]

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$$Q_B = (V_{br} + \phi) / S_{max} \quad 3.7.35$$

Substituting 3.7.35 into 3.7.34 gives

$$a_2 = \frac{-2\gamma(\gamma-1)S_{max}^2 (V_{br} + \phi)^{(\gamma-2)/\gamma}}{(V_{br} + \phi)^{2(\gamma-1)/\gamma}} \quad 3.7.36$$

Consider the case of the abrupt junction diode $\gamma=2$, the constant a_2 becomes

$$a_2 = -4(V_{br} + \phi) \quad 3.7.37$$

For the linearly graded junction diode $\gamma=3/2$, a_2 is given by

$$a_2 = \frac{-1.5}{(V_{br} + \phi)^{1/3} (V_{br} + \phi)^{1/3}} \quad 3.7.38$$

Equations 3.7.38 and 3.7.37 show that the intercept point P_1 of the linearly graded junction diode depends on the bias voltage whereas the abrupt junction diode intercept point is not directly dependent of the bias voltage $-V_b$. To increase the intercept point of the PDC, one must choose a diode with a large breakdown voltage $-V_{br}$. However, the required pump power to achieve the same conversion gain has to be increased (see equations 3.7.14 and 3.7.17).

3.8 Varactor Diode Selection

The parameters of the diode chosen for a parametric down converter must permit operation at the LO, the RF, the IF, and the upper side-band frequencies. The following procedure is recommended for the selection of the diode.

3.8.1 The diode cut-off frequency - f_c

The selected diode must have a high cut-off frequency - f_c . The cut-off frequency is related to the series resistance R_s and the zero bias capacitance C_0 by [4].

$$f_c = \frac{1}{2\pi R_s C_0} \quad 3.8.1$$

A high cut-off frequency means that the diode series resistance is small and hence the corresponding losses are negligible (see chapter 2). Dickens et al [8] have shown that GaAs diodes have higher cut-off frequency than silicon diodes. Thus, GaAs diodes are the natural choice for low loss mixers.

3.8.2 The diode reactance - X_d

The diode must present "reasonable" impedances at the RF, IF, LO, and upper sideband frequencies. Since most microwave integrated circuits are operated in a 50 ohm system, the diode reactance should lie in the range 10 to 200 ohms. The diode reactance at the lower IF frequency is given by:

$$X_d = \frac{1}{2\pi f_0 C_0} \quad 3.8.2$$

The diode zero bias capacitance must lie in the range

$$1/f_0 < C_0 < 15/f_0 \quad \text{pFs} \quad 3.8.3$$

where f_o = the IF output frequency (in GHz)

3.8.3 The power handling capability

In a parametric down converter, power is supplied to the diode by the pump (LO) and RF signal generators. The pump generator must provide sufficient power to drive the diode hard enough in order to achieve frequency conversion. The pump power is related to V_{br} , R_s , f_p and f_c by [9]:

$$P_p = 8P_n(f_p/f_c)^2 m_p^2 \quad 3.8.4$$

where $P_n = (V_{br+\phi})^2 / R_s$, m_p = the pump signal modulation index $\leq .25$

Equation 3.8.4 indicates that for systems where the pump power is the constraint, one must operate at low pump frequencies and choose diodes with lower breakdown voltages and higher cutoff frequencies.

3.8.4 The Diode Parasitics

The diode terminal impedance at various frequencies is greatly modified by the package parasitics L_p and C_p (figure 3.3). The diode packaging and the associated parasitics must be chosen to minimize the ratio C_p/C_o . If the capacitance C_p is too large, the variation of $C_j(v)$ with the pump voltage is masked by the small shunting reactance due to C_p .

3.9 The Design of the Microwave Integrated Circuits (MIC)

3.9.1 Introduction

The need to control the impedance levels at the various frequencies was pointed out in section 3.3. An MIC circuit for the PDC must perform the following functions:

- (a) present the correct impedance levels at the four frequency of interest (LO, RF, IF and upper sideband frequencies).
- (b) provide the necessary isolation between the output IF, the RF, and the LO circuits.

Figure 3.10 is a schematic diagram illustrating the impedance terminology used in this thesis. The diode parasitics L_p and C_p are considered as part of the external circuit. The networks N_o , N_s , and N_p are designed to provide the isolation and the impedance transformation. The circuit is constructed on teflon substrate ($\epsilon_r = 2.35$, $h = 10$ mils).

3.9.2 The Design of Auxiliary Circuits

The bandpass and impedance transforming filters designed on an insertion-loss basis were found to be very unsatisfactory because of their uncontrolled out of band frequency response. Thus, circuits whose response can be controlled at the four frequencies of interest (IF, RF, LO, and upper sideband) are required. Figure 3.11 shows the response of a bandpass/band reject filter using coupled transmission lines. This filter has the following desired characteristics.

FIG. 3.10 ILLUSTRATION OF THE IMPEDANCE TERMINOLOGY

(a) Filter Structure

FIG 3.11 THE BANDPASS/BAND REJECT FILTER & ITS RESPONSE

- (a) At the centre frequency (i.e. the frequency at which the stubs are a quarter of a wavelength) the circuit behaves as a band reject filter. The response of this structure at other frequencies is virtually unaffected by the presence of the coupled line.
- (b) The filter presents an open circuit impedance at port A and looks like a short circuit when viewed from terminal B. Thus an infinity number of reactive terminations can be achieved.
- (c) The filter is narrowband. The measured 3 dB bandwidth is 100 mHz.

Another filter configuration commonly used in mixer designs is the travelling wave ring directional filter. Basically, this filter utilizes a full wavelength resonator coupled to four ports as shown in figure 3.12. The filter has the following characteristics.

- (a) a bandpass response between port 1 and port 3,
- (b) a band reject response between port 1 and port 2, and
- (c) signals whose frequencies are not a multiple of the resonator wavelength are transmitted unattenuated between port 1 and port 2.

Figure 3.13 shows the measured response of the travelling wave ring filter. The insertion loss for the "off-resonance" signals is typically less than .2 dB. The physical parameters describing the filter are indicated in figure 3.13. The even and odd mode impedances

Dimensions of the filter

$S/H = .5$

$W_1/H = 3.1$

$W_2/H = 1.25$

$H = 10 \text{ mils}$

$Z_{00} = 68.7 \Omega$

$Z_{0e} = 97.6 \Omega$

FIG 3.12 THE TRAVELLING WAVE DIRECTIONAL FILTER

FIG 3.13 TRAVELLING WAVE DIRECTIONAL FILTER RESPONSE

Z_{oo} and Z_{oe} are calculated by T. Bryant and J. Weiss [10] computer program.

The two types of filters described above are used in the PDC design. The directional filter is used to couple the LO power to the varactor, whereas the coupled transmission line filter (fig. 3.11(a)) is used to control the upper sideband signal impedance.

3.9.3 The Development of the PDC Circuit

To test the foregoing theory, a parametric down converter circuit was designed on teflon substrate ($\epsilon_r = 2.35$, $h = 10$ mil). In this section a detailed design procedure for a PDC circuit is presented.

(a) Choice of Frequencies

The first step in the PDC circuit design is to choose the RF and the LO frequencies. The LO frequency f_p is always greater than the RF frequency f_s . The frequencies chosen for this thesis are:

$$\begin{aligned} f_p &= 10 \text{ GHz} \\ f_s &= 6 \text{ GHz} \\ f_o &= f_p - f_s = 4 \text{ GHz} \\ f_u &= f_p + f_s = 16 \text{ GHz} \end{aligned}$$

(b) Diode Selection

The diode is chosen following the guidelines in section 3.8. For the PDC circuit in this thesis a Varian silicon diode was chosen. Its characteristics are shown in figure 3.16.

(c) Estimation of the Dynamic Quality Factor $\bar{Q}_s$

To calculate the dynamic quality factor $\bar{Q}_s$, one needs to know the value of the fundamental elastance coefficient S_1 . The diode elastance under reverse bias can be written as [9]:

$$S(v) = S(V_b)[1 + R_r \cos \omega_p t]^m \quad 3.9.1$$

where $R_r = V_p / (V_b + \phi)$

$$S(V_b) = S_{\max}[(\phi + V_b)/(V_{br} + \phi)]^m \quad 3.9.2$$

V_b = the bias voltage

V_{br} = the diode breakdown voltage

ϕ = the contact potential

m = the diode index

V_p = the pump signal voltage

$S_{\max}$ = maximum elastance at V_{br}

A Fourier analysis of the time waveform in equation 3.9.1 results in the elastance ratios $S_1/S(V_b)$ and $S_0/S(V_b)$. The variation of these coefficients with pump voltage amplitude (i.e. R_r) for the abrupt and the linearly graded junction diodes are shown in figure 3.14 and figure 3.15.

FIG 3.14 RATIO OF $S_1/S(V_b)$ AS FUNCTION OF PUMP VOLTAGE (R_r)

FIG 3.15 THE VARIATION OF $S_0/S(V_b)$ WITH PUMP VOLTAGE (R_r)

Assuming that the diode is fully driven (i.e. $R_r = 1$), one obtains the elastance ratio S/S_0 from figures 3.14 and 3.15 as:

$$a = S_1/S_0 = [S_1/S(V_b)]/[S_0/S(V_b)] \quad 3.9.3$$

where S_0 and S_1 are the average and fundamental Fourier coefficients of the elastance $S(t)$.

The dynamic quality factor Q_s is obtained from Q_m by the formula: [13]

$$\hat{Q}_s = aQ_m(f_m/f_s)((\phi+V_b)/(\phi+V_m))^m \quad 3.9.4$$

where m = the diode index

f_s = the RF signal frequency,

V_b = the desired bias voltage, Q_m = the diode quality factor at the frequency f_m

V_m = the voltage at which the diode quality factor Q_m was measured.

For the Varian diodes VAT-111-16 used in this thesis

Q_m = 4500 at the bias voltage V_m = -4 volts

f_m = 50 mHz

m = 0.5

f_s = the RF signal frequency (6 GHz)

ϕ = 0.7 volts

The elastance ratio $a = S_1/S_0$ is read from figures 3.14 and 3.15 to be $a = .333$. The computed dynamic quality factor $\hat{Q}_S$ is $\hat{Q}_S = 13.4$ at a bias voltage of 4.7 volts.

(d) The Optimum Source and Load Resistances

In order to obtain the optimum design of the IF load and the RF source resistance for minimum noise figure design, either the bandwidth and/or the midband gain must be specified. For a specified gain $g_0 = 15$ dB, the constant $C = .745$ (see figure 3.8). The corresponding normalized 3 dB bandwidth $\Delta\omega/\Delta\omega_s = .325$ (fig. 3.6). The optimum load and generator resistances are obtained from either equations 3.6.3 and 3.6.4 or figure 3.5. The calculated optimum load and generator resistances for the diode in question are:

$$R_g(\text{opt}) = 13.4 R_S = 12.5\Omega, \text{ and}$$

$$R_L(\text{opt}) = 23.9 R_S = 22.3\Omega. \quad R_S = .9\Omega \text{ for VAT-111-16 diodes.}$$

(e) MIC Circuit Synthesis/Computer Optimization

An MIC microstrip circuit configuration capable of providing the above impedance loading can now be designed. Figure 3.16 is a schematic of the microwave circuit used in this thesis. The RF to IF signal isolation is provided by $\lambda/4$ stubs at IF placed in the RF path and vice versa. The impedance transformations required to achieve the above optimum resistances are accomplished by varying the positions of the $\lambda/4$ stubs and the line lengths and impedances of the connecting transmission lines.

FIG 3.16 THE PDC CIRCUIT CONFIGURATION

The realization of $R_L(\text{opt})$ and $R_g(\text{opt})$ cannot be easily done by a simple Smith Chart design. The impedance matching problem involves two frequencies f_s and f_o and hence optimum circuit conditions at one of the frequencies does not automatically result in optimum conditions at the other frequency. It was, therefore, decided to develop a computer program to analyse and optimize the PDC circuit. The computer program is described in Chapter 5. Table 3.3 is a summary of the line lengths and characteristic impedances after the computer optimization (see fig. 3.18) of the circuit.

Element Number i	Impedance Z_i (ohms)	Line Length θ_i (degrees)
1	29.3	40.00
2	24.7	70.28
3	62.5	92.30
4	64.7	42.96
5	24.7	21.93
6	23.8	15.18
7	35.4	24.21

NB: All stubs are 50 ohms and $\lambda/4$ at either f_s or f_o .
The line lengths in table 3.3 are at the centre frequency $f_s = 6\text{GHz}$.

Table 3.3 Line Impedances/Lengths of the MIC Circuit
(fig. 3.16)

The theoretical responses of the circuit in figure 3.16 are tabulated below in tables 3.4 and 3.5.

Frequency	Required Impedance	Theoretical Impedance seen by the diode
6.2	$23 + j0$	19.1 $-j2.8$
6.1		22.4 $-j1.7$
6.0		24.0 $-j1.1$
5.9		23.8 $+j.6$
5.8		22.4 $+j4.0$

Table 3.4 The PDC Circuit Response at RF

Frequency	Required Impedance	Theoretical Impedance seen by the diode
3.8	$13 + j0$	14.1 $+j1.4$
3.9		14.0 $+j0.6$
4.0		14.5 $+j0.$
4.1		14.0 $-j0.6$
4.2		13.6 $-j1.8$

Table 3.5 The PDC Circuit Response at IF

3.10 Summary and Conclusions

The theoretical noise figure and conversion gain of the circuit discussed in the previous section is shown in Chapter 6 figure 6.2 and figure 6.4, respectively. The actual line width to achieve the impedances shown in table 3.2 were obtained from a computer program [11] specifically developed for this purpose. Its listing is included in Appendix B. The corrections due to microstrip line discontinuities were applied by using another program developed by H. Sankar [12].

In conclusion, this chapter has presented a small signal analysis of the parametric down converter (PDC). The analyses show that the PDC can simultaneously provide conversion gain and relatively low noise figure. A large signal analysis of the PDC has been carried out and the third order intercept point has been related to measurable quantities like the breakdown voltage of the diode, the pump impedance, and the IF load. Also a simple procedure for predicting the optimum RF and IF loads has been developed.

CHAPTER 4: THE FIELD EFFECT TRANSISTOR MIXER

4.1 Introduction

It has been widely recognized that GaAs Field Effect Transistors (FET) have very low noise figures of about 3-4 dB at X-band. [1]-[3]. The Schottky barrier contact, together with the high mobility of the GaAs materials have made it possible to attain high cut off frequencies, as high as 50-80 GHz; that is, several times higher than that of silicon bipolar transistors.

Most of the work today on GaAs have been concentrated on the application of FETs as small signal amplifiers [4], oscillators [5], and as power amplifiers [6]. The application of GaAs FET as mixers has been lightly treated in the literature. R. Purcel and J. Stich [7] have indicated briefly the possibility of using GaAs FET as front-end mixers for communication receivers. Their results, which are mostly experimental, show that the FET mixers have very poor noise figures. The experimental work of R. Purcel and D. Masse [9] show that their FET mixer has a conversion gain of 6 dB and a poor SSB noise figure of 10.6 dB at an RF and IF frequency of 8 GHz and 30 MHz, respectively.

The intention of this chapter is to investigate the causes of the reported high noise figures of FET mixers. Expression relating noise figure, conversion gain, and bandwidth to circuit terminations are derived. It is shown that the RF and the IF terminations have a very significant effect on the FET mixer noise figure.

4.2 The Device Physics

The FET (field effect transistor), as the name implies, is a device whose current through it can be varied by an electric field applied to a terminal. Basically, there are two types of FETs, depending on whether the controlling field is applied to a semiconductor reverse biased junction or at an insulating layer. In this thesis, only the GaAs Metal to Semiconductor field effect transistor (GaAs MESFET) will be analysed.

Figure 4.1 is a schematic of a GaAs MESFET. The device consists of a semiconductor substrate, usually GaAs, two contacts at both ends, called the drain and the source. The substrate material is mostly n-type because of the large electron mobility. The current flow from the drain to the source is by the majority carriers and can be controlled by a voltage applied to a third terminal called the gate. The applied gate voltage causes the metal to semiconductor space charge layer to either widen or shorten. The widening and shortening of this layer has a modulating effect on the current through the device.

The drain current through the FET is given by: [19]

$$I_d = qWh(x)v(E,x)n(x) \quad 4.2.1$$

where

- q = the electronic charge
- W = the width of the gate
- h = the conducting layer thickness
- v = the electron drift velocity
- n = the density of conduction electrons.
- x = the distance from source to drain

L_g = gate length E_p = critical field

FIG 4.1 SCHEMATIC OF A MESFET

In GaAs material, the electron drift velocity v depends on the applied electric field as shown in fig. 4.2 [8]. As the applied field increases, the electrons reach a peak drift velocity v_p . Further increases in the applied field cause the velocity to decrease and limit to a velocity v_s of about 1.5×10^7 cm/sec resulting in the saturation of the drain current. Another effect that causes drain current saturation is the reduction in size of the conducting channel near the drain end. The conducting channel height $h(x)$ decreases from the source to the drain end due to the applied gate to drain potential. If the drain to gate potential is further increased, the drain current gets constrained to a small channel height h_0 at the drain end. Further increases in this potential result in a saturated drain current. The gate to drain potential at which the drain current starts to saturate is known as the "pinch off" voltage - V_p . The position of the minimum channel height (h_0) shifts towards the source end as the gate to drain voltage is further increased. The movement in position of h_0 results in a widened drain end and more current is injected into the channel. The net result is a drain to source resistance R_{ds} when the device is operated in the drain current saturation mode.

When the junction FET theory of Shockley [11] is applied to the GaAs MESFET with a small gate length to height ratio L_g/a , significant deviations are encountered between the theory and the observed experimental results. The discrepancies are due to the dependence of the drift velocity on the applied electric field. An analysis by Zuleeg [10] of the pinch off effect of Silicon FETs with a small L_g/a ratio show that the current saturation is due to velocity limiting

FIG 4.2 ELECTRON DRIFT VELOCITY VS ELECTRIC FIELD FOR GaAs [8]

effects. K. Lehovec et al [12] have analysed the voltage to current relationships of GaAs MESFET. Their results are summarized below:

$$I_d = I_p \frac{[3(U_m^2 - t^2) - 2(U_m^3 - t^3)]}{1 + Z(U_m^2 - t^2)} \quad 4.2.2$$

where I_p = the saturation current in the Shockley case where saturation effects are ignored. ($Z=0$, $U_m = 1$).

$$Z = \mu V_p / L_g v_s$$

μ = the low field mobility

V_p = the pinch off voltage

v_s = the saturation velocity

$$t^2 = -(v_g + \phi) / (V_p + \phi)$$

v_g = applied gate voltage

ϕ = the junction contact potential, and U_m is the normalized reduced drain potential given by:

$$U_m^3 + 3U_m(1/Z - t^2) + 2t^3 - 3/Z = 0 \quad 4.2.3$$

The device transconductance g_m and the drain resistance R_{ds} are given by: [12]

$$g_m = \frac{\partial I_d}{\partial v_g} = g_{m0} (U_m - t) / (1 + Z(U_m^2 - t^2)) \quad 4.2.4$$

$$R_{ds} = E_c L_g (1 + Z(U_m^2 - t^2)) / I_d \quad 4.2.5$$

where $E_c = 13v_s / \mu$

This model of junction FET is appropriate for devices such as NEC V244 or V388 where the gate length L_g is of the order of a micron and the field can exceed 3 KV/cm even at low (~ 3 v) drain voltages.

4.3 The Small Signal Model of a FET

The small signal behaviour of a $GaAs$ FET can be described by the equivalent circuit shown in figure 4.3 [13]. In the diagram, the input gate circuit is represented by the gate metallization resistance R_m , a gate to source capacitance C_{gs} and the channel resistance R_i . The capacitance C_{dg} represents the gate to drain capacitance, and R_{ds} represents the effects of the channel resistance. The transconductance g_m represents the small signal relationship between the drain current and the gate voltage. R_d and R_s are the metallization resistances of the drain and source, respectively. Typical element values for NEC V244 FET are shown on figure 4.3. Not shown on figure 4.3 is the package parasitics which affect the performance of the device at high frequencies. Figure 4.4 shows the packaged FET parasitics. The most important parasitic element is the gate lead inductance. At high frequencies L_g , C_{gs} , and L_s form a resonant circuit. Above the resonance frequency the gate inductance dominates the input impedance and the device stops to behave as a voltage controlled device, rather it behaves as a current controlled device similar to bipolar transistors.

4.4 The FET Mixer Conversion Gain - g_c

Figure 4.5 is a schematic of a FET when used as a mixer [9]. In

$R_{ds} = 450\Omega$ @ $I_d = 10\text{mA}$ $g_m = 30 \text{ mmhos}$

FIG 4.3 THE INTRINSIC MESFET MODEL

FIG 4.4 THE PACKAGED MESFET

FIG 4.5 THE FET MIXER MODEL

the following analysis the drain to gate capacitance is ignored. Its effect on gain and noise figure will be considered later. The output drain current source has been transformed into an equivalent voltage source $U(t) = R_{ds} g_m v_g(t)$. The drain resistance is a function of the gate voltage (equation 4.2.5). However, the conversion properties of R_{ds} will be ignored in this thesis; instead R_{ds} will be assumed to be equal to the average drain resistance. The package parasitics are considered as part of the external circuits loading the FET.

The analysis of the FET mixer is carried out in the same manner as that of the PDC (Chapter 3). The nonlinear element in this case is the transconductance $g_m(v_g)$. Let us assume that a large LO signal is applied to the gate of the FET. The voltage $U(t)$ becomes time-dependent on the LO voltage and can be expressed as:

$$U(t) = R_{ds} g_m(t) v_g(t) \quad 4.4.1$$

The time varying transconductance g_m can be expressed in a Fourier series as

$$g_m(t) = \sum_{k=-\infty}^{\infty} g_k \exp(jk\omega_p t) \quad 4.4.2$$

where $\omega_p = 2\pi f_p$ = the radian frequency of the pump signal

g_k = the k^{th} harmonic Fourier coefficient, given by

$$g_k = \frac{1}{2\pi} \int_{t_0}^{t_0 + 2\pi/\omega_p} g_m \exp(-jk\omega_p t) d(\omega_p t) \quad 4.4.3$$

where t_0 = an arbitrary time origin. The time origin can always

be chosen such that

$$g_k = g_{-k}^* \quad 4.4.4$$

The input current I_{in} and the input voltage V_{in} are related by the following (see fig. 4.5) equation:

$$V_{in} = I_{in}(R_i + R_s + R_m - j/\omega_{in} C_{gs}) + I_{out} R_s \quad 4.4.5$$

Similarly, the output voltage is related to the output current by:

$$V_{out} = I_{out}(R_{ds} + R_d + R_s) + I_{in} R_s - U(t) \quad 4.4.6$$

where

I_{in} , I_{out} , V_{in} , V_{out} , ω_{in} , and ω_{out} represent any of the possible currents and voltages at the frequency $n\omega_p + \omega_s$ ($n = -\infty$ to $+\infty$).

To simplify the analysis, only three signals will be allowed to exist at both the input and the output terminals. These signals are:

- (i) RF signal at the frequency f_s ,
- (ii) IF signal at the output frequency $f_o = f_s - f_p$, and
- (iii) LO signal at the frequency f_p .

The effects of the image signal on the mixer performance is considered in section 4.9. Following a procedure similar to that in Chapter 3, a set of equations relating the input RF currents and voltages to the output IF voltages and currents can be written down as:

$$V_{is} = (R_{dt} - j/\omega_s C_{gs}) I_{is} + R_s I_{os} \quad 4.4.7$$

$$V_{io}^* = (R_{gt} - j/\omega_o C_{gs}) I_{io}^* + R_s I_{od}^*$$

$$V_{os} = (R_s + jR_{ds}g_o/\omega_s C_{gs})I_{is} + R_{dt}I_{os} + jR_{ds}g_l I_{id}^*/\omega_o C_{gs} \quad 4.4.9$$

$$V_{od}^* = (R_s + jR_{ds}g_o/\omega_o C_{gs})I_{od}^* + jR_{ds}g_l I_{id}^*/\omega_s C_{gs} + R_{dt}I_{od}^* \quad 4.4.10$$

where g_o, g_l = the average and the fundamental Fourier coefficients of g_m .

$$R_{gt} = R_m + R_i + R_s$$

$$R_{dt} = R_s + R_d + R_{ds}$$

$$I_{is} = \text{the input current at the RF } -(\omega_s)$$

$$I_{os} = \text{the output current at the RF frequency}$$

$$I_{id} = \text{the input current at the IF } (\omega_o)$$

$$I_{od} = \text{the output current at the IF}$$

$$V_{is}, V_{os}, I_{id}, \text{ and } I_{od} \text{ are similarly defined}$$

$$\omega_o = 2\pi(f_s - f_p)$$

Let the impedances seen by the signals at the two ports be designated by Z_{id} , Z_{od} , Z_{is} , and Z_{os} .

where

$$\begin{aligned} Z_{id} &= \text{the IF signal impedance at the input port (gate)} \\ Z_{is} &= \text{the RF signal impedance at the input port (gate)} \\ Z_{os} &= \text{the RF signal impedance at the output port (drain)} \\ Z_{od} &= \text{the IF signal impedance at the output port (drain)} \end{aligned}$$

The impedance matrix Z describing the relationship between the RF and

and the IF currents is obtained from equation 4.4.7 to 4.4.10 as:

$$\begin{bmatrix} V_{is} \\ V_{od}^* \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22}^* \end{bmatrix} \begin{bmatrix} I_{is} \\ I_{od}^* \end{bmatrix} \quad 4.4.11$$

$$\text{where } Z_{11} = R_m + R_i + R_s - jX_s - \frac{R_s(R_s + jg_o X_s R_{ds})}{(Z_{os} + R_{dt})} \quad 4.4.12$$

$$Z_{12} = \frac{-jX_s g_1 R_s}{(Z_{os} + R_d + R_{ds} + R_s)(R_{gt} + Z_{id} - jX_d)^*} \quad 4.4.13$$

$$Z_{21} = jX_s g_1 \quad 4.4.14$$

$$Z_{22} = R_{dt} - \frac{R_s(R_s + jR_{ds} g_o X_s)}{(R_{gt} + Z_{id} - jX_d)} \quad 4.4.15$$

$$X_s = 1/\omega_s C_{gs} \quad 4.4.16$$

$$X_d = 1/\omega_o C_{gs} \quad 4.4.17$$

The conversion gain g_c can be written as:

$$g_c = \frac{4R_g}{R_L} \frac{|V_{od}|^2}{|V_{gs}|^2} = \frac{\text{The IF power delivered to load } R_L}{\text{The available RF power}} \quad 4.4.18$$

where V_{gs} = the RF signal source voltage. Assuming that the external circuit presents impedances Z_g and Z_L to the RF and the IF signals, respectively, then equation 4.4.8 can be solved directly to yield the conversion gain g_c as:

$$g_c = \frac{4R_g R_L |Z_{21}|^2}{|(Z_{22} + Z_L)(Z_{11} + Z_g) - Z_{12}Z_{21}|^2} \quad 4.4.19$$

where $R_g = \text{Real}(Z_g) =$ the real part of the impedance seen by
by the RF in the gate external circuit

$R_L = \text{Re}(Z_L) =$ the real part of the impedance seen by the
IF signal in the drain external circuit.

The midband conversion gain can be calculated after the terminations
of the IF and RF signals at the gate and the drain are known.

4.5 The Effect of the Feedback Capacitance - C_{dg}

The effect of the feedback capacitance, C_{dg} , can be easily analyzed by considering the admittance matrix of the circuit in figure 4.5. R_m and R_s will be ignored for the moment. A simple circuit analysis of figure 4.5 gives:

$$I_{in} = Y_{11}V_{in} + Y_{12}V_{out} \quad 4.5.1$$

$$I_{out} = Y_{21}V_{in} + Y_{22}V_{out} \quad 4.5.2$$

$$\text{where } Y_{11} = Y_c \{ (f_B + 1/(R_s g_m + 1))(1 + R_s/R_{ds}) \} \quad 4.5.3$$

$$Y_{12} = -Y_c (f_B + R_s/R_{ds})(1/(R_s g_m + 1)) \quad 4.5.4$$

$$Y_{21} = -Y_c (f_B + R_s/R_{ds})(1/(R_s g_m + 1)) + g_m \quad 4.5.5$$

$$Y_{22} = Y_C \{ f_\beta + (R_S + R_i) / (R_{ds} (R_S g_m + 1)) \} + 1 / (R_{ds} (R_S g_m + 1)) \quad 4.5.6$$

$Y_C = j\omega C_{gs}$ = the gate capacitance admittance at the radian frequency ω , and $f_\beta = C_{dg}/C_{gs}$.

Carrying out the mixer analysis similar to the one in section 4.4, the following matrix relating the input RF current and voltage to the output IF current and voltage results:

$$\begin{bmatrix} I_{is} \\ I_{od}^* \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_{is} \\ V_{od}^* \end{bmatrix} \quad 4.5.7$$

$$\text{where } Y_{11} = Y_{CS} K_1 - \frac{(Y_{CS} K_2 + g_o) K_2}{(Y_{OS} + K_3 + Y_{CS} f_\beta)} \quad 4.5.8$$

$$Y_{12} = \left[\frac{K_2 Y_{cd} g_1}{K_1 Y_{cd} + Y_{id}} \right] * \left[\frac{Y_{CS} K_2}{(Y_{OS} + K_3 + Y_{CS} f_\beta)} \right] \quad 4.5.9$$

$$Y_{21} = g_1$$

$$Y_{22} = K_3 + Y_{cd} f_\beta - \frac{Y_{cd} K_2 (g_o + K_2 Y_{cd})}{Y_{cd} K_1 + Y_{id}} \quad 4.5.10$$

$$K_1 = f_\beta + 1 / (R_S g_o + 1) \quad 4.5.11$$

$$K_2 = -f_\beta - R_S / R_{ds} (R_S g_o + 1) \quad 4.5.12$$

$$K_3 = 1/R_{ds}(1+R_sg_o) \quad 4.5.13$$

$$Y_{cd} = j(\omega_p - \omega_s)C_{gs} \quad 4.5.14$$

$$f_B = f_\beta + (R_i + R_s)/R_{ds}(R_sg_o + 1) \quad 4.5.15$$

Y_{id} = the admittance seen by the IF signal at the input gate circuit

Y_{os} = the admittance seen by the RF at the output drain circuit

$$Y_{cs} = j\omega_s C_{gs}$$

The feedback capacitance C_{dg} increases the input admittance and the reverse transfer admittance, Y_{12} . The result is a reduced conversion gain. The effect of C_{dg} can be clearly seen by evaluating the term K_2 (equation 4.5.12). For the NEC V244 FET transistors, $f_\beta = C_{dg}/C_{sg} = .25$, whereas the R_s feedback contribution to K_2 is of the order 10^{-2} . K_2 is a multiplicative factor in the admittance Y_{12} . The effect of the capacitance C_{dg} on noise figure will be considered in the following section.

4.6 The Noise Figure of the FET Mixer

4.6.1 Introduction

The basic theoretical work on noise in FETs has been done by van der Ziel [14]. His analysis was restricted to the operation of the FET below the pinch off. Beachtold [15] has extended van der Ziel's analysis to FETs with very short gate lengths biased beyond pinch off. Beachtold's analysis shows that the effective noise temperature of GaAs FETs is increased due to the intervalley scattering of the electrons from the low mobility to the high mobility valleys. However, his analysis ignores the effects of carrier velocity saturation. H. E. Statz [16] has derived the small signal noise behaviour of the FETs taking into account the saturation velocity behaviour. Statz' analysis shows that for small gate-length FETs, the contribution of the saturation velocity region cannot be neglected.

In this section the noise behaviour of the FET as a mixer will be developed. The effects of the RF and IF terminations will be examined.

4.6.2 Noise Sources

There are many causes of noise in solid state devices; nevertheless, these can all be classified under one or more of the following categories:

- (a) thermal noise
- (b) shot noise, and
- (c) $1/f$ noise.

Thermal noise is caused by the random motion of the carriers in a conducting medium. The noise voltage or current generated by a conducting medium with a resistance R_g at a temperature T and in a small bandwidth B is given by the Nyquist theorem as: [20]

$$\overline{v^2} = 4KTBR_g$$

4.6.1

$$\overline{i^2} = 4KTBR_g/R_g$$

where K = Boltzmann's constant

$\overline{v^2}$ and $\overline{i^2}$ are the mean square noise voltage and current, respectively.

Shot noise is caused by the discreteness of the charge carriers and the random fluctuations in the number available for conduction at any given time. The shot noise current resulting from a dc current I flowing is given by

$$\overline{i^2} = 2qIB$$

4.6.2

where q = the electronic charge

The thermal noise and the shot noise spectral densities are independent of frequency and hence the name "white" noise.

The cause of 1/f noise is the least understood. 1/f noise is mostly pronounced at low frequencies (of the order of 100KHz and below). The contributors to the FET 1/f noise include: [17]

(a) carrier trapping at crystal dislocations,

- (b) formation of an additional depletion region on the channel surfaces resulting from surface contamination, and
- (c) the generation and recombination of the carriers in the space charge region.

1/f noise can be minimized by passivation of the FET surface and by having a buffer layer between the active epitaxial layer and the semiconductor substrate.

Figure 4.6 shows the equivalent circuit noise model of a GaAs MESFET for the frequency range beyond 1/f noise (100KHz) [16]. The average noise current $\overline{i_{nd}}$ represents the thermal channel noise. Based on the transmission line properties, Klaasen [17] expressed this drain noise current as:

$$\overline{i_{nd}^2} = 4KT B g_m [P(V_{gs}, V_{gd}) + S(\omega C_{in})^2 / g_m] \quad 4.6.3$$

where P is a factor dependent on the gate to source, V_{gs} and gate to drain, V_{gd} voltages [17].

S is a constant also dependent on V_{gs} and V_{gd} .

C_{in} = the input gate circuit capacitance, and

g_m = the saturation conductance.

For mixer applications, the FET is usually operated in the saturation region. The limiting values for P and S are given by [17].

$$P(V_{gs}, V_{gd}) = 2/3, \text{ and}$$

FIG 4.6 NOISE FIGURE MODEL OF FET

$$S(V_{gs}, V_{gd}) = .15, V_{gd} = V_p = \text{the pinch off voltage.}$$

The transconductance g_m in equation 4.6.3 is replaced by the average conductance g_o (equation 4.4.3). The mean squared drain noise current $\overline{i_{nd}^2}$ at the IF frequency is then expressed as:

$$\overline{i_{nd}^2}(\omega_o) = 4KTBg_o[2/3 + (\omega_o C_{in})^2 S/g_o] \quad 4.6.4$$

The noise voltage fluctuations in the channel result in an induced gate capacitance charge fluctuations. Its effect is represented by the noise current $\overline{i_{ng}^2}$ at the gate terminals. The noise currents i_{ng} and i_{nd} are partially correlated because they arise from the same process (channel thermal noise). According to Statz et al [16], the correlation coefficient is purely imaginary since there is a capacitive 90 phase shift between the gate and the drain currents. The correlation coefficient C between the drain current i_{nd} and i_{ng} is defined as [20]

$$jC = \frac{\overline{i_{nd} i_{ng}^*}}{\sqrt{\overline{i_{nd}^2} \overline{i_{ng}^2}}} \quad 4.6.5$$

where $j = \sqrt{-1}$

Klaasen [17] found that the correlation coefficient C between the induced gate and drain current has a real and imaginary parts given by:

$$C \approx 0.13\omega C_{in}/g_m + j0.35 \quad 4.6.6$$

This expression differs from that of van der Ziel [14] and Statz et

al [16] who found C to be given by

$$C \approx j0.4 \quad 4.6.7$$

The differences arise from the assumptions involved in Klaasen's analysis. Klaasen's analysis is based on the assumption that the transistor is a distributed transmission line. For the frequencies (4GHz) and the devices used in this thesis ($C_{in} = C_{gs} = .2\text{fps}$, $g_m \approx 20\text{mmhos}$), equation 4.6.7 is used. The gate mean square induced noise $\overline{i_{ng}^2}$ is given by [15].

$$\overline{i_{ng}^2} = 4KTB(\omega C_{in})^2 / g_m R$$

Where C_{in} = the input gate capacitance

g_m = the transconductance, and

ω = the radian frequency of operation

R = factor dependent on biasing conditions. ($R = .3$)

The significant signals flowing in the gate circuit are the RF and the LO. The LO signal noise contribution will be ignored in this analysis. For this simplified case, equation 4.6.8 can be written as:

$$\overline{i_{ng}^2}(\omega_s) = 4KTB(\omega_s C_{in})^2 / g_o R \quad 4.6.9$$

where ω_s = the RF signal frequency.

A rigorous treatment of the FET mixer noise should have the gate induced noise current sources at the FR, the IF, the LO and at the image

$(2\omega_p - \omega_s)$ frequencies. The input capacitance C_{in} is calculated as:

$$C_{in} = I_m(Y_{in})/\omega_s \quad 4.6.10$$

where $Y_{in} = Y_{11} - Y_{12}Y_{21}/(Y_{os} + Y_{22}) \quad 4.6.11$

Y_{11} , Y_{12} , Y_{21} , and Y_{22} are given by equations 4.5.8 to 4.5.10, and

Y_{os} = the RF signal termination at the drain circuit.

The effect of the RF termination on noise figure is through the input capacitance C_{in} . For a shorted RF at the output C_{in} equals the gate capacitance C_{gs} , otherwise C_{in} takes on the value dependent on the RF, the IF terminations, and the feedback factor $f_B = C_{dg}/C_{gs}$.

4.6.3 The FET Mixer Noise Figure

To calculate the FET mixer noise figure, the total output short-circuit noise current i_{out}^2 must be evaluated. Fig. 4.7 is a schematic for calculating the total output short-circuit noise current. Let the mixer be represented by the Y matrix given by equations 4.5.8 to 4.5.10. The total output short-circuit current is given by:

$$i_{out} = (V_g + V_{ng})Y_{21} + (V_{nd} + V_1)Y_{22} \quad 4.6.12(a)$$

where V_1 = the thermal noise voltage generated by R_d and R_s at the IF frequency,

FIG 4.7 MODEL FOR CALCULATING THE FET MIXER NOISE FIGURE

V_4 = the thermal noise voltage generated by R_m , R_s , and R_g at the RF frequency ω_s ,

V_{ng} and V_{nd} are the equivalent induced gate noise voltage and the channel noise voltage, respectively.

Equation 4.6.12(a) is divided into two parts as follows

$$i_{out} = \underbrace{(Y_{21}V_4 + Y_{22}V_1)}_{\text{part 1}} + \underbrace{(Y_{21}V_{ng} + Y_{22}V_{nd})}_{\text{part 2}} \quad 4.6.12(b)$$

The first part of equation 4.6.12(b) consists of the terms of the noise current that are not correlated, whereas the second part contains the noise currents that are correlated. The mean square noise current $\overline{i_{out}^2}$ is given by

$$\left| \overline{i_{out}^2} \right| = \left| \frac{[(Y_{21}V_4 + Y_{22}V_1) + (Y_{21}V_{ng} + Y_{22}V_{nd})]^2}{2} \right| \quad 4.6.13$$

Upon performing the squaring operation and taking the average of the result, the following mean square current is obtained

$$\left| \overline{i_{out}^2} \right| = |Y_{21}|^2 \overline{V_4^2} + |Y_{22}|^2 \overline{V_1^2} + |(Y_{21}V_{ng} + Y_{22}V_{nd})|^2 \quad 4.6.14$$

Observe that equation 4.6.14 is obtained after neglecting all the terms that are uncorrelated. The noise voltages V_{nd} , V_{ng} , V_1 and V_4 are defined as

$$\overline{V_4^2} = 4KTB(R_s + R_m + R_g) \quad 4.6.15$$

$$\overline{V_1^2} = 4KT B(R_s + R_d), \quad 4.6.16$$

$$V_{nd} = R_{ds} i_{nd}; \quad V_{ng} = (R_i - j/\omega_s C_{gs}) i_{ng} \quad 4.6.17$$

i_{nd} and i_{ng} are defined in equations 4.6.3 and 4.6.9, respectively.

Let i_{ngo} and i_{ndo} be defined as

$$i_{ngo} = Y_{21} V_{ng} = Y_{21} (R_i - j/\omega_s C_{gs}) i_{ng} \quad 4.6.18$$

$$\text{and} \quad i_{ndo} = Y_{22} R_{ds} i_{nd} \quad 4.6.19$$

Also let i_{ngo} be divided into two components as:

$$i_{ngo} = i_c + i_n \quad 4.6.20$$

where i_c = the part of i_{ngo} that is correlated to i_{ndo} , and

i_n = the part of i_{ngo} that is not correlated to i_{ndo} .

The last part of equation 4.6.14 can be expressed as:

$$\begin{aligned} \overline{|Y_{21} V_{ng} + Y_{22} V_{nd}|^2} &= \overline{|i_{ngo} + i_{ndo}|^2} \\ &= \overline{|i_n|^2} + \overline{|i_{ndo}|^2} \left| 1 + \frac{i_c}{i_{ndo}} \right|^2 \end{aligned} \quad 4.6.21$$

The term i_c/i_{ndo} is the complex constant expressing the correlation between i_{ngo} and i_{ndo} .

$$\overline{\begin{bmatrix} i_c \\ i_{ndo} \end{bmatrix}} = \overline{\begin{bmatrix} i_c & i_{ndo}^* \\ i_{ndo} & i_{ndo}^* \end{bmatrix}} = \overline{\begin{bmatrix} (i_{ngo} - i_n) i_{ndo}^* \\ i_{ndo}^2 \end{bmatrix}} = \overline{\begin{bmatrix} i_{ngo} i_{ndo}^* \\ i_{ndo}^2 \end{bmatrix}} \quad 4.6.22$$

Since $i_c i_{ndo}^* = 0$. (uncorrelated).

Let the correlation coefficient between i_{ngo} and i_{ndo} be defined as [20]

$$jC_{nd} = \frac{\overline{i_{ngo}^2 i_{ndo}^*}}{\sqrt{\overline{i_{ndo}^2} \overline{i_{ngo}^2}}}; \quad C_{nd} = \text{the correlation coefficient between } i_{ngo} \text{ and } i_{ndo}. \quad 4.6.23$$

Using equation 4.6.23, the complex constant i_c/i_{ndo} (equation 4.6.22) can be rewritten as

$$\overline{\begin{bmatrix} i_c \\ i_{ndo} \end{bmatrix}} = jC_{nd} \frac{\sqrt{\overline{i_{ngo}^2} \overline{i_{ndo}^2}}}{\overline{i_{ndo}^2}} \quad 4.6.24$$

The last term of equation 4.6.14 is expressable as

$$|\overline{Y_{21}V_{ng} + Y_{22}V_{nd}}|^2 = \overline{i_n^2} + \overline{i_{ndo}^2} + \left| \frac{jC_{nd} \sqrt{\overline{i_{ngo}^2} \overline{i_{ndo}^2}}}{\overline{i_{ndo}^2}} \right|^2 \quad 4.6.25$$

The uncorrelated noise current i_n^2 is related to i_{ngo} by: [20]

$$\overline{i_n^2} = \overline{i_{ngo}^2} - \overline{i_c^2} = \overline{i_{ngo}^2} (1 - C_{nd}^2) \quad 4.6.25$$

The available mean square short-circuit current noise at the mixer output due to the RF source termination is

$$\overline{i_{R_g}^2} = |Y_{21}|^2 \overline{V_{R_g}^2} \quad 4.6.26$$

where

$\overline{V_{R_g}^2}$ = the mean square RF source termination thermal noise voltage. $\overline{V_{R_g}^2}$ is given by

$$\overline{V_{R_g}^2} = 4KTBR_g \quad 4.6.27$$

where

- T = the RF termination temperature
- B = the measurement bandwidth (Hz)
- R_g = the termination (ohms)
- K = Boltzmann constant

The mixer noise figure is defined as [20]

$$F = \frac{|\overline{i_{out}^2}|}{|\overline{i_{R_g}^2}|} \quad 4.6.28$$

After some lengthy simple algebraic manipulations, equation 4.6.28 is expressible as: (See Appendix H)

$$F = 1 + \frac{R_m + R_s}{R_g} + \frac{|R_i - jX_s|^2 (1 - |C_{nd}|^2) g_{gn}}{R_g}$$

$$+ \frac{R_s + R_d}{R_g} \left| \frac{Z_{11} + Z_g}{Z_{21}} \right|^2 +$$

$$\frac{g_{dn} R_{ds}^2}{R_g |Z_{21}|^2} \left| \frac{Z_g + Z_{11} - \frac{jC(R_i - jX_s)Z_{21}(g_{gn}/g_{dn})^{1/2}}{R_{ds}}}{R_{ds}} \right|^2 \quad 4.6.29$$

where $g_{gn} = (\omega_s C_{in})^2 / g_o R \quad 4.6.30$

$$g_{dn} = 2/3 g_o + (\omega_o C_{in})^2 S \quad 4.6.31$$

g_o = the fundamental Fourier coefficient of transconductance g_m

C_{in} = the input capacitance as defined by equation 4.6.10

R , and S are defined in equations 4.6.3 and 4.6.8, respectively

Z_g = the RF signal impedance in the gate circuit

Z_{11} , Z_{21} , Z_{22} , and Z_{12} are the impedances formed by inverting the matrix in equation 4.5.7

C = the correlation coefficient between i_{ng} and i_{nd} (equation 4.6.5)

$$X_s = 1/\omega_s C_{gs} \quad 4.6.32$$

ω_s , ω_o = the RF and the IF radian frequencies

C_{gs} = the gate to source capacitance

The noise figure F (equation 4.6.29) can be put in the following form:

$$F = 1 + \frac{R_n}{R_g} + \frac{G_n}{R_g} |Z_g + Z_{cor}|^2 \quad 4.6.33$$

where $R_n = R_s + R_m + |(R_i - jX_s)|^2 (1 - |C_{nd}|^2) g_{gn} \quad 4.6.34$

$$G_n = \frac{g_d R_{ds}^2}{|Z_{21}|^2} \quad 4.6.35$$

Z_{cor} = the FET mixer correlation impedance

$$Z_{cor} = \frac{Z_{11} - jC(R_i - jX_s)Z_{21}(g_{gn}/g_{dn})^{1/2}}{R_{ds}} \quad 4.6.36$$

Considering the noise figure as a function of Z_g , then from simple calculus one obtains the minimum noise figure and the optimum source impedance $[R_g(\text{opt}) + jX_g(\text{opt})]$ as:

$$R_g(\text{opt}) = [\text{Real}(Z_{cor}) + R_n/G_n]^{1/2} \quad 4.6.37$$

$$F_{\min} = 1 + 2G_n [\text{Real}(Z_{cor}) + R_g(\text{opt})] \quad 4.6.38$$

$$X_g(\text{opt}) = \text{Imaginary}(Z_{cor}) \quad 4.6.39$$

4.6.4 The Effects of IF and RF Terminations on Noise Figure

To study the effects of the IF and the RF terminations on the FET mixer noise figure, equations 4.6.37 to 4.6.39 were analysed on a computer using the data of the NEC V244 FET. The transistor analysed has the following element values (see Figure 4.5):

$$C_{dg} = .05 \text{ pfs}$$

$$g_m = 30 \text{ mmhos @ } I_d = 10 \text{ mA (small signal value)}$$

$$R_s = 5.0 \text{ ohms}$$

$$R_d = 5.0 \text{ ohms}$$

$$R_{ds} = 450 \text{ (small signal value)}$$

$$C_{gs} = .2 \text{ pfs, } R_m = .8 \text{ ohms, } R_i = 3.1 \text{ ohms}$$

The above data refers to a particular NEC V244 FET device at SED Systems, Ltd. Other NEC V244 FETs have different parameters.

The values of g_o , g_1 and R_{ds} to be used in equations 4.6.29 to 4.6.39 are calculated in section 4.7. The results of the computer analysis are summarized below:

- (a) the minimum noise figure F_{min} increases rapidly with the feedback capacitance ration $f_B = C_{dg}/C_{gs}$ (see Fig.

4.8). This observation suggests a figure of merit to be used when choosing FETs for mixer applications. Based on this observation alone, the NEC V244 FET ($f_{\beta} = .25$) should have a better mixer noise performance than the NEC V388 ($f_{\beta} = .75$) though the V388 has better noise figure than the V244 FET when used as an amplifier.

- (b) For a given transistor ($f_{\beta} = \text{constant}$), the mixer circuit in which both the RF and the IF signals are shorted at the drain and the gate circuits, respectively, shows a much improved noise performance. Thus, for the minimum mixer noise figure design, a shorted RF and IF signals at the output and the input of the transistor, respectively, is necessary.
- (c) The circuit with both the RF and the IF signal shorted shows the least change in noise figure with feedback factor $f_{\beta} = C_{dg}/C_{gs}$. This behaviour is to be expected since the RF voltage or current at the output port is not fed back to the input circuit where it could be converted to IF, amplified and delivered to the load.
- (d) Biasing the FET gate circuit very close to pinch off voltage results in an increased noise figure. This change in the noise performance is due to the increased noise conductance G_n (equation 4.6.29). The conductance G_n depends directly on R_{ds} , which in turn

FIG 4-8 VARIATION OF NOISE FIGURE WITH FEEDBACK CAPACITANCE

increases with the gate bias voltage (see figure 4.10). Most reported experiments on FET mixers in the literature have had the transistors biased at or near pinch off voltage. This partially explains the poor noise figure performance of the FET mixers reported in the literature.

4.7 Calculation of g_o , g_1 , and R_{ds}

In order to predict the noise performance of the FET mixer, the conversion conductances g_1 and the resistance R_{ds} must be evaluated accurately. To calculate g_o , g_1 or R_{ds} , a Fourier analysis of the transconductance g_m and the drain conductance g_{ds} must be carried out. The variation of the conductance g_m and g_{ds} with the gate voltage was discussed in section 4.2, and is repeated here for convenience.

$$g_m = \begin{cases} \frac{g_{mo}(U_m - t)}{1 + Z(U_m^2 - t^2)} & , \quad t < 1 \\ 0 & , \quad t \geq 1 \end{cases} \quad 4.7.1$$

$$g_{ds} = \begin{cases} \frac{I_d}{E_c L_g (1 + Z(U_m^2 - t^2))} & ; \quad t < 1 \\ 0 & ; \quad t \geq 1 \end{cases} \quad 4.7.2$$

where $t^2 = -(v_g + \phi)/(V_p + \phi)$, V_p = the gate pinch off voltage

U_m is given by equation 4.2.3

L_g = the gate length (manufacturer's data sheets)

Z , E_c and I_d are defined in section 4.2.

The gate voltage waveform is assumed to be a sinusoid at the pump frequency f_p . It is also assumed that the bias and the pump voltage amplitudes are adjusted in such a manner as to avoid driving the FET into forward conduction (see figure 4.9).

The average drain resistance R_{ds} is equal to the reciprocal of the average drain conductance. The variation of R_{ds} with the gate bias V_b for the NEC V244 FET is shown in figure 4.10.

Observe that the average drain resistance R_{ds} differs significantly from the small signal value. For example, the small signal value is 400 ohms, whereas the value of R_{ds} when the FET is used as a mixer is about 940 ohms at the gate bias of 2.5 volts. As a rule of thumb, the drain resistance is about twice the value of the small signal value of R_{ds} .

Figure 4.11 shows the computed device conductance g_0 and g_1 for the same transistor. It is seen that a broad maximum of g_1 is obtained when the gate bias exceeds 2 volts.

4.8 Bandwidth of the FET Mixer

The normalized 3 dB bandwidth P_1 of the FET mixer is defined as the ratio of the mixer 3 dB bandwidth to the input circuit 3 dB bandwidth. The mixer conversion gain g_c is given by equation 4.4.19 as:

FIG 4.9 THE LO VOLTAGE WAVEFORM

FIG 4.10 THE VARIATION OF THE DRAIN RESISTANCE WITH THE GATE VOLTAGE - V_b

FIG 4.11 THE VARIATION g_0 & g_1 WITH GATE BIAS FOR NEC V244 FET

$$g_c = \frac{4R_g R_L |Z_{21}|^2}{|(Z_{22} + Z_L)(Z_{11} + Z_g) - Z_{12}Z_{21}|^2} \quad 4.8.1$$

Assuming that the IF and the RF are short circuited at the input and the output, respectively, equation 4.8.1 can be rewritten as:

$$g_c = \frac{4R_g R_L |Z_{21}|^2}{(R_{gt} + R_g)^2 (R_L + R_{dt})^2 |1 + jP_1)(1 + jP_2)|^2} \quad 4.8.2$$

$$\text{where } P_1 = \frac{X_g - 1/\omega_s (C_{dg} + C_{gs})}{R_g + R_m + R_i + R_L} \quad 4.8.3$$

$$P_2 = \frac{X_L}{R_{ds} + R_d + R_s + R_L} \quad 4.8.4$$

$$R_{gt} = R_m + R_i + R_s$$

$$R_{dt} = R_d + R_{ds} + R_s$$

P_1, P_2 = the normalized 3 dB bandwidths of the input and the output circuits,

X_g and X_L are the generator and load reactances...

Observe that the transfer impedance $Z_{12} = 0$. (equation 4.5.9) since $Y_{os} = Y_{id} = \infty$ (i.e. shorted RF and IF at the drain and the gate, respectively).

Let the ratio of the normalized input and the output 3 dB bandwidth be defined as:

$$Y = P_2/P_1, \text{ then} \quad 4.8.5$$

the conversion gain can be written as:

$$g_c = \frac{4R_g R_L |Z_{21}|^2}{(R_{gt} + R_g)^2 (R_{dt} + R_L)^2 |(1 + jP_1)(1 + jYP_1)|^2} \quad 4.8.6$$

The ratio of the conversion gain g_c to the midband gain is equal to 1/2 at the 3 dB bandwidth points. The resulting 3 dB bandwidth equation is:

$$Y^2 P_1^4 + P_1^2 (Y^2 + 1) - 1 = 0 \quad 4.8.7$$

A solution to the above equation gives the FET mixer normalized 3 dB bandwidth $P_1 = \Delta\omega/\Delta\omega_s$ for a given ratio of the output to the input circuit 3 dB bandwidths Y . If it is assumed that the input to the output bandwidth ratio Y is unity, then equation 4.8.7 can be solved directly to give:

$$P_1 = \Delta\omega/\Delta\omega_s = 0.64, \text{ and} \quad 4.8.8$$

$$Y = \Delta\omega_s/\Delta\omega_o = 1. \quad 4.8.9$$

where $\Delta\omega_s$ = the 3 dB bandwidth of the input circuit,

• $\Delta\omega_0$ = the 3 dB bandwidth of the output circuit, and

$\Delta\omega$ = the 3 dB bandwidth of the mixer

Solving equation 4.8.9 and 4.8.8 gives $\Delta\omega$ as:

$$\Delta\omega = .64\Delta\omega_0 \quad 4.8.10$$

Observe that the FET mixer bandwidth does not depend on the gain as was the case with the PDC (Chapter 3); instead it depends on the RF and the IF circuit bandwidth $\Delta\omega_s$ and $\Delta\omega_0$.

4.9 The Effect of the Image Signal (at $2\omega_p - \omega_s$)

The image signal at the frequency $2\omega_p - \omega_s$ generated by the non-linear transconductance g_m will flow through the drain external circuits. In order to recover this power and convert it to the required IF signal, some means of feeding back the image signal to the gate capacitance must be devised. This feedback process is necessary because the output drain current depends only on the gate voltage.

In the FET model discussed earlier, there are feedback mechanisms already in existence; the source contact resistance R_s (providing a voltage feedback), and the drain gate capacitance C_{dg} (providing a current feedback). Both of these mechanisms have severe effects on the gain and noise figure of the mixer. In fact, it is shown in figure 4.8 that the feedback capacitance must be minimized in order to obtain minimum FET mixer noise figure.

The conversion process of the image signal to the IF involves the second harmonic Fourier coefficient g_2 of the nonlinear transconductance g_m . This harmonic Fourier coefficient is very close to a zero value. In fact, if it is assumed that the transconductance g_m varies between a fixed value over half the LO cycle and zero over the remaining half, the value of g_2 is exactly equal to zero.

It is for the above reasons that the effect of the image signal on the mixer performance was considered to be negligible.

4.10 The Development of the FET Mixer Circuit

4.10.1 Introduction

To test the foregoing theory on FET mixers, a microstrip circuit fabricated on a teflon substrate ($\epsilon_r = 2.35$, $h = 10$ mils) was constructed. The RF and IF frequencies chosen are:

$$f_s = 4 \text{ GHz} - \text{RF}$$

$$f_o = 70 \text{ MHz} - \text{IF}$$

$$f_p = 3.93 \text{ GHz}$$

In the following section, a description of the auxiliary circuits needed for proper FET mixer operation are described.

4.10.2 The Design of Auxiliary Circuits

There are two auxiliary circuits required for the FET mixer. The power combiner is used to couple the RF and the LO powers

to the FET gate input. The power combiner consists of a pair of quarter wave sections having a characteristic impedance of 70.7 ohms which are series terminated by a 100 ohm resistor. The combiner circuit response is shown in figure 4.12. The circuit has a maximum insertion loss of 3.7 dB. The theoretical value is 3.0 dB. There is a minimum isolation of over 25 dB between ports 2 and 3 over the 500 MHz bandwidth.

The FET mixer IF impedance is very high, about 700-1200 ohms at 70 MHz. It is necessary to transform this high impedance to 50 ohms with very little signal attenuation. Figure 4.13 is a schematic of the transforming network used to step down the impedance from a nominal value of 800 ohms to 50 ohms. The network is a Pi network widely used in vacuum tube transmitters. The element values are chosen from the computer tables prepared by F. Davis [18]. From the return loss measurements the transforming circuit has 3 dB bandwidth of about 30 MHz.

4.10.3 The MIC FET Mixer Microstrip Circuit

The design of the FET mixer follows the theory developed earlier in this chapter. Figure 4.14 shows the microstrip circuit configuration. It consists of a power combiner, and three transmission lines that are designed to provide the correct impedance loading for minimum noise figure. The IF short circuit at the gate is provided by a quarter wave transmission line that is shorted by a large ($C=820$ pfs) bypass capacitor. Two taps at the drain output provide the LO and RF signal short-circuit. The IF power is coupled to the load through a lumped impedance transformer discussed in the last section.

FIG 4.12 EXPERIMENTAL RESULTS OF THE POWER COMBINER

FIG 4.13 THE IF IMPEDANCE TRANSFORMING FILTER

FIG 4.14 THE FET MIXER CIRCUIT

This circuit was analysed and optimized for minimum noise figure. The characteristic impedances and line lengths are tabulated below (see fig. 4.14)

Element Number i	Impedance Z_i (ohms)	Line* Length (degrees) θ_i
1	80	70.5
2	22	90.0
3	50	90.0
4	50	90.0

* All line lengths are referenced to the RF centre frequency
 $f_s = 4\text{GHz}$.

Table 4.1 The Impedance and Line Lengths of the FET Mixer Circuit

The predicted noise figure and conversion gain of this circuit are shown in figures 6.7 and 6.10.

4.11 Summary and Conclusions

This chapter has dealt with the theoretical analysis of the FET mixer. The predicted noise figure and conversion gain for the circuit discussed in the previous section are shown in figures 6.7 and 6.10, respectively of Chapter 6. Also the correlation between the theoretical and the experimental results is discussed in Chapter 6.

The theoretical analysis of the FET mixer shows that the noise

figure is strongly affected by the RF and the IF terminations. The FET mixer circuit with the RF and the IF signals short-circuited at the gate and the drain, respectively, has the best noise performance. This observation explains the poor noise figures of FET mixers reported in the literature today. Also the investigation reveals that the drain to gate capacitance C_{dg} has a significant effect on the noise figure of the mixer. The analysis shows that a FET with a smaller gate to drain capacitance will perform better as a mixer.

Other results of this chapter include the derivation of the optimum noise impedance of the FET mixer.

CHAPTER 5

COMPUTER AIDED DESIGN OF MIXERS5.1 Introduction

Considerable effort has been made in the last decade in the field of computer aided design (CAD) of microwave circuits. Presently, there are a number of specialized programs [1]-[3] that can analyse and optimize microwave networks. Programs like COMPACT [1] have the capabilities to handle complex topologies. However, many of these programs suffer from the following deficiencies:

- (a) They cannot perform analysis or optimization of microwave circuits in which more than one frequency is allowed to be present. This means that mixer circuits cannot be handled by these programs.
- (b) Except in very few cases, these CAD programs cannot analyse circuits with many ports.

This chapter discusses a CAD program called MIXER that can analyse and optimize mixer circuits.

5.2 Circuit Analysis5.2.1 Introduction

Figure 5.1 is a schematic showing the general microwave circuit configuration for mixer applications. It consists of several input and output ports with signals flowing in and out at different frequencies. Each of the networks N_i ($i = 1, 2, \dots$) is made up of individual circuit

FIG 5.1 THE GENERAL NETWORK FOR MIXER ANALYSIS/OPTIMIZATION

elements which may be interconnected in any desired manner. The network N_0 represents the inevitable mixer parasitics such as capacitances and lead inductances. The microwave circuit is described by the Floating Nodal Admittance matrix [4]. In the following section a brief description of the formation of the Nodal matrix is presented.

5.2.2 The Floating Nodal Matrix -Y

The indefinite admittance matrix is the basis for the formulation of the network in MIXER. The construction of the floating nodal matrix follows the rules of basic network circuit theory [4]. The first step in the analysis and construction of the Y matrix describing the circuit is to construct a topological graph and label the circuit nodes (see fig. 5.6 for an example). Several rules are to be observed when numbering the nodes:

- (a) For lumped elements, such as inductors, capacitors, and resistors, both nodes are to be numbered.
- (b) The ground node is labelled zero.
- (c) Transmission lines are in fact three terminal networks. The third terminal, the ground node, need not be numbered as long as all the transmission lines have the same common ground.
- (d) Stubs (open or short) are transmission lines with a known termination. Thus, in MIXER the stubs are considered as lumped elements whose second node is taken to be the ground node. Only the "connected" node should be numbered.

- (e) Coupled transmission line elements are five terminal networks. The element has different admittance description dependent on which of the nodes are taken as the input and the output nodes. The allowable coupled transmission line configuration is shown in table 5.1.
- (f) The nodes of the circuit are numbered consecutively from 1 to N where N denotes the total number of nodes in the circuit. The "ideal" mixer nodes must be labelled first; otherwise no attention should be paid to the position of the element in the circuit when numbering the nodes. The ground node is numbered zero.

The allowable element types and their admittance description are shown in table 5.1. Also shown on the same table is the element code number and the parameters that must be provided with each element. Using basic circuit theory, an admittance matrix is formed as illustrated below.

Y =

Nodes	1	2	3		N	N+1
1	Y_{11}	Y_{12}	Y_{13}		Y_{1N}	1
2	Y_{21}	Y_{22}	...			0
.						
.						
.						
N	Y_{N1}	Y_{N2}	...		Y_{NN}	0
N+1	1	0	...		0	0

5.2.1

Ground node

N = total number of nodes in circuit

Element Name	Element Symbol	Element Code	Parameters	Admittance Matrix
			$B(K), K=1, 3$	$Y_{11} \quad Y_{12}$ $Y_{21} \quad Y_{22}$
Resistor		4	$R, 0.0, 0.0,$	$\frac{1}{R} \quad -\frac{1}{R}$ $-\frac{1}{R} \quad \frac{1}{R}$
Capacitor		5	$C(\text{pfs}), 0.0, 0.0,$	$j\omega C \quad -j\omega C$ $-j\omega C \quad j\omega C$
Inductor		6	$L(\text{nH}), 0.0, 0.0,$	$j/\omega L \quad -j/\omega L$ $-j/\omega L \quad j/\omega L$
Shorted Stub		3	$Z(\text{ohm}), \theta(\text{degrees}), 0.0$	$-j\cot(\theta)/Z \quad j\cot\theta/Z$ $j\cot(\theta)/Z \quad -j\cot(\theta)/Z$

Table 5.1 The Admittance Description of the Elements in MIXER

Element Name	Element Symbol	Element Code	Parameters	Admittance
Open Circuit Stub		2	$Z(\Omega), \theta(\text{deg}), 0.0$	$\frac{j}{Z} \begin{bmatrix} \tan(\theta) & -\tan(\theta) \\ -\tan(\theta) & \tan(\theta) \end{bmatrix}$
Series Line		1	$Z(\Omega), \theta(\text{deg}), 0.0,$	$-\frac{j}{Z} \begin{bmatrix} \cot(\theta) & -\csc(\theta) \\ -\csc(\theta) & \cot(\theta) \end{bmatrix}$
Parallel Coupled		7	Z_{oe}, Z_{oo}, θ	$-j \begin{bmatrix} Y_1 \cot \theta & -Y_2 \csc(\theta) \\ -Y_2 \csc(\theta) & Y_1 \cot(\theta) \end{bmatrix}$ $Y_1 = (Y_e + Y_o)/2$ $Y_2 = (Y_e - Y_o)/2$ $Y_e = 1/Z_{oe}, Y_o = 1/Z_{oo}$

Table 5.1 The Admittance Description of the Elements in MIXER (continued)

Y_{ij} = the sum of the admittances between node i and j
as obtained from table 5.1

The circuit analysis is carried out by assuming that a signal source of unity voltage amplitude at a frequency ω is applied to the circuit in place of the "ideal" mixer (see figure 5.1). The matrix equation $\bar{I} = Y\bar{V}$ is solved for the node voltages $\bar{V} = (1, v_2, v_3, v_4, \dots, v_{N+1})^T$. From the node voltage solutions the input impedance is calculated as:

$$Z_{in}(\omega) = -V_{in}(\omega)/I_{in} \quad 5.2.2$$

where I_{in} is the input current to the network, and

$V_{in} = 1$ volt, ω = the radian frequency

To evaluate the circuit input impedance at any other frequency, the input voltage frequency is changed to the desired frequency value and the analysis process is repeated. The mixer conversion gain noise figure, and bandwidth are evaluated using the impedances calculated above (see Chapters 3 and 4).

5.3 Circuit Optimization

5.3.1 Introduction

Circuit optimization by a computer becomes very necessary when either the synthesis procedures of the circuit from the classical theory is inapplicable, for example, if the circuit structure to be designed is too complicated to permit a simple formal synthesis procedure.

or when the number of variables is more than four (say). This situation is particularly true in the case of mixer circuits where several circuit conditions have to be met simultaneously. In the following subsections an optimization routine is described.

5.3.2 The Optimization Objective Function - E

The optimization process involves the calculation of the minimum of a function E, where

$$E = E(\bar{U}) \quad 5.3.1$$

and

$$\bar{U} = (u_1, u_2, u_3 \dots u_k)^T \quad 5.3.2$$

E is called the objective function, and the column vector $\bar{U}$ has k real independent variables. The variables $\bar{U}$ can take on a fixed range of values. The constraint comes from the practical consideration of realizing the circuit from the computer solution. To incorporate these constraints on variables, a transformation of variables $\bar{U}$ into a new set $\bar{X}$ is necessary. The new vector $\bar{X}$ can take on values from minus infinity to infinity. In MIXER the following transformation is used:

$$u_i = a_i + b_i \tan^{-1}(x_i) \quad 5.3.3$$

where u_i = the i^{th} "practical" variable (e.g. Z, θ , L or C),

x_i = the i^{th} unconstrained variable that takes on the values $-\infty$ to $+\infty$.

$$a_i = (u_i(\text{max}) + u_i(\text{min}))/2 \quad 5.3.4$$

$$b_i = (u_i(\max) - u_i(\min))/2\pi \quad 5.3.5$$

$u_i(\max)$ = the maximum allowable value of u_i

$u_i(\min)$ = the minimum allowable value of u_i

Other transformation may be used [5]. The choice depends on the complexity of the constraints and the ease with which equation 5.3.3 can be evaluated.

The objective function in MIXER is defined as:

$$E = \sum_i^{N_f} C_i |S_i - S_R|^2 \quad 5.3.6$$

where C_i = a constant

S_i = the circuit response at the i^{th} frequency point

S_R = the required circuit response

N_f = the number of frequency points over which the circuit is to be analysed or optimized.

In the above formulation S_i can be either one or a weighted sum of the following responses:

- (a) the mixer conversion gain g_c
- (b) the input reflection coefficient ρ_{in}
- (c) the output reflection coefficient ρ_{out}
- (d) the mixer noise figure F , and/or
- (e) the mixer bandwidth.

The responses are directly related to the input impedance of the mixer circuit and can be evaluated using the equation developed in Chapters 3 and 4.

5.3.3 The Choice of Optimization Variables - $\bar{U}$

The choice of the optimization variable is as important as the optimization process itself. The particular vector $\bar{U}$ chosen depends on the circuit in question and the objective function to be minimized. For example, if the noise figure of a FET mixer is to be minimized, only the circuit elements that affect the input gate impedance will be allowed to vary, since the noise figure in FETs depends strongly on the gate input impedance (see Chapter 4).

MIXER allows the user to select any of the element parameters as variables in the optimization. The element parameter to be optimized is indicated to the program by putting a minus sign in front of the parameter in the input data. The input data for any of the elements is as follows:

$M, N1, N2, -B(1), B(2), -B(3)$

where M = the element type

$N1$ = the first node of the element

$N2$ = the second node of the element

$B(K)$ = the parameters describing the element (e.g. $Z, \theta, L, C, Z_{oe}, Z_{oo}$, etc.)

The minus sign behind $B(1)$ and $B(3)$ indicates that $B(1)$ and $B(3)$ are selected for optimization.

5.3.4 The Optimization Method

There are various optimization routines available in the literature today [7]. Most of these fall into two categories:

- (a) the direct search routines, and
- (b) the routines that use the gradient information to minimize the function.

A recent survey [7] has shown that the gradient techniques are more efficient than the direct search methods, especially when combined with the adjoint network method of computing the objective function gradients. MIXER combines both the gradient techniques and the linear search methods to minimize the error function [8].

Briefly, the optimization program performs a "steepest descent" method to minimize E . The initial step size is taken to be equal to the gradient of the objective function as

$$x_i^{k+1} = x_i^k - \alpha E / \alpha x_i^k \quad 5.3.7$$

where x_i^k = the i^{th} variable at step k .

The step size is doubled until the function ceases to decrease. At this point the objective function minimum is bounded between two sets of the vector $\bar{X}$ (say $\bar{X}^{m+1}$ and $\bar{X}^m$). A quadratic interpolation between $\bar{X}^{m+1}$ and $\bar{X}^m$ is carried out to further bind the minimum. From this step onwards the linear search is carried out as:

$$x_i^{k+1} = x_i^k + \alpha^k S_i^k \quad 5.3.8$$

$$S_i^k = -(\partial E / \partial x_i^k) / |\nabla E|^k \quad 5.3.9$$

where $|\nabla E|$ = the norm of the gradient vector,

α^k is a constant that is chosen to minimize the error function in that particular search direction.

In MIXER, α^k is chosen as:

$$\alpha^k = |\nabla E|^k / |\nabla E|^{k+1} \quad 5.3.10$$

where $|\nabla E|^k$ = the norm of the gradient vector at step k .

5.3.5 Calculation of the Gradients

The gradient vector required by the optimization routine is computed using the adjoint network techniques. Fig. 5.2(a) and (b) is a schematic of a two-port network and its adjoint network. An adjoint network is an auxiliary network that has the same topology as the original network but its admittance matrix is the transpose of the original network. The gradient vector with respect to an adjustable parameter $\bar{X}$ is obtained from equation 5.3.6 as:

$$\nabla E = 2 \sum_{f=1}^{N_f} \text{Re} (C_f S_f^* \nabla S_f) \quad 5.3.11$$

where $\nabla S_f = \partial S_f / \partial \bar{X}$, N_f = the number of frequency points 5.3.12

Let S_i be the reflection coefficient $|\rho|^2$ defined as:

$$S_i = |\rho_i|^2 = \left| \frac{Z_{in} - Z_g^*}{Z_{in} + Z_g} \right|^2 \quad 5.3.13$$

(a) The Original Network

(b) The Adjoint Network

FIG 5.2 THE ORIGINAL AND THE ADJOINT NETWORK

where Z_{in} = the input impedance of the mixer circuit

Z_g = the optimum generator impedance (say)

The gradient vector ∇E is given by:

$$\nabla E = 2 \sum_i^{N_f} \text{Real} (\rho_i^* \partial \rho_i / \partial \bar{X}) \quad 5.3.14$$

$$\partial \rho_i / \partial \bar{X} = -2Z_g^* (\partial Z_{in} / \partial \bar{X}) / (Z_{in} + Z_g)^2 \quad 5.3.15$$

The gradient vector, ∇E for other mixer parameters, such as noise figure, gain, bandwidth can be evaluated in the same manner. In general the gradient vector can be written as: (see appendix D)

$$\nabla E = \sum_i^{N_f} \text{Re}(W \partial Z_{in} / \partial \bar{X}) \quad 5.3.16$$

where $W = -2Z_g^* / (Z_{in} + Z_g)^2$. W depends on the response function S_i being considered.

To evaluate ∇E one must evaluate $\partial Z_{in} / \partial \bar{X}$. The gradient vector $Z_{in} / \partial \bar{X}$ is evaluated using the adjoint network method [9]. The adjoint network method is based on Tellegen's [9] theorem. Tellegen's theorem asserts that:

$$\sum_{\text{all branches}} (V_b \hat{I}_b - \hat{V}_b I_b) = \sum_{\text{all ports}} (V_p \hat{I}_p - \hat{V}_p I_p) = 0 \quad 5.3.17$$

where V_b, I_b are the branch voltages and currents, respectively of the original network

V_p, I_p are the port voltages and currents, respectively
of the original network

$\hat{V}_b, \hat{I}_b, \hat{V}_p,$ and $\hat{I}_p$ are similarly defined but refer to the
adjoint network.

Perturbing the elements in the original network and noting that
Tellegens theorem still holds for increment voltages $\Delta V_b, \Delta V_p$ and
currents ΔI_p and ΔI_b results in a sensitivity equation [9]

$$G\Delta\bar{X} = \Delta V_i \hat{I}_i + \Delta V_o \hat{I}_o - \Delta I_i \hat{V}_i - \Delta I_o \hat{V}_o \quad 5.3.18$$

where G is called the sensitivity vector

$\Delta\bar{X}$ = the incremental change in the variable element
parameters

ΔV_i = the input port incremental voltage resulting from
perturbing the elements $\bar{X}$

ΔI_i = the input incremental current

ΔV_o and ΔI_o are similarly defined but refer to the original
network output port

$\hat{I}_o, \hat{I}_i, \hat{V}_o,$ and $\hat{V}_i$ refer to the adjoint network.

The derivative $\partial Z_{in}/\partial\bar{X}$ can be calculated using equation 5.2.2 repeated
here for convenience:

$$Z_{in} = -V_i/I_i \quad 5.3.19$$

$$\partial Z_{in}/\partial\bar{X} = V_i/I_i^2 (\partial I_i/\partial\bar{X}) \quad 5.3.20$$

since $V_1 = \text{constant} = 1 \text{ volt}$.

Now, let us excite the original network and the adjoint network as:

$$\begin{aligned} V_1 &= 1 \text{ volt} \\ V_0 &= 0 \end{aligned} \quad \text{for the original network, and}$$

$$\begin{aligned} \hat{V}_1 &= 1 \text{ volt} \\ \hat{V}_0 &= 0 \end{aligned} \quad \text{for the adjoint network}$$

$$\Delta V_1 = V_0 = 0$$

Substituting the above into equation 5.3.18 gives

$$G \Delta \bar{X} = -\Delta I_1 \hat{V}_1 \quad 5.3.21$$

or

$$\frac{\Delta I_1}{\Delta \bar{X}} = \frac{\partial I_1}{\partial \bar{X}} = \frac{-G}{\hat{V}_1}$$

The derivative $\partial Z_{in}/\partial \bar{X}$ (equation 5.3.19) can now be evaluated once the sensitivity vector G and $\hat{V}_1$ are known. The gradient vector ∇E is computed from the knowledge of $\partial Z_{in}/\partial \bar{X}$.

The sensitivity vector G for most elements is readily available in the literature [9]. G depends on the original and the adjoint network branch voltages. For example, the sensitivity vector G of a resistive element in a network is given as:

$$G = \hat{V}V/R^2 \quad 5.3.22$$

where V , and $\hat{V}$ are the original and the adjoint network voltages across the resistor, respectively.

A tabulation of the G vector for most elements is found in Appendix E.

5.3.6 The Program Termination Criteria

Theoretically, the minimum value of a function $E(\bar{X})$ is obtained when the gradient vector $\nabla E = 0$ and the Hessian vector $\nabla^2 E$ is positive definite. In practice this criteria cannot be met because of round-off errors and the excessive computer time required. For MIC circuits, certain practical constraints lead to practical termination criteria

- (a) The circuit fabrication tolerances and achievable impedance levels provide one of the termination criteria. If the difference between two consecutive optimization vectors is "small", the program is terminated.

$$\max_{i=1,2,\dots,N} \left\{ 2|x_i^{k+1} - x_i^k| / (x_i^{k+1} + x_i^k) \right\} \leq \epsilon_1 \quad 5.3.23$$

where ϵ_1 is a small number $\cdot 10^{-4}$ (say)

x_i^k = the i^{th} adjustable parameter at step K

- (b) The objective function itself E gives another termination criteria. The optimization routine is terminated when the difference in the error function at two consecutive steps is less than a small number ϵ_2 .

$$|E^{m+1} - E^m| \leq \epsilon_2 \quad 5.3.24$$

(c) If the norm of the gradient vector $|\nabla E|$ is less than a small value ϵ_3 the program is terminated

$$|\nabla E| \leq \epsilon_3$$

5.3.25

(d) Another, but less elegant, termination criteria is to perform an analysis based on parameters generated by the computer program after a certain number of iterations. If the solution meets the specification then the solution vector is taken to be an optimal solution.

To ensure that a global minimum is obtained, another starting point is tried. If the resultant solution is the "same" as the previous solution vector, then an optimum solution has been obtained. The master check to all computer solutions is to build the circuit from the computer solution and check the theoretical results against the experimental result. All of the above criteria were used in the mixer circuit development.

5.4 The Computer Program Description

5.4.1 Introduction

A simplified flow chart of MIXER is shown in figure 5.3. It depicts the program's general organization without any detailed description of the subroutines.

The main program is mainly concerned with the input and output of data. It inputs the network configuration, the reference data (such as the optimization constants C_1 , the reference frequency for transmission

FIG 5.3 MIXER MAIN PROGRAM FLOWCHART

lines, etc.). After the data is read in, it is printed out.

The next step is to evaluate the circuit response and the objective function at the base point. The circuit analysis is carried out by subroutine FUNC. FUNC calls the subroutines MAT and SOLV, which set up the nodal matrix and solves for the node voltages, respectively. The gradient vector needed by subroutine OPT to perform objective function minimization is computed by FUNC. Subroutine OPT uses another subroutine called UPDATE that updates the solution vector at every optimization step. In the following subsection, the individual subroutines will be described.

5.4.2 Subroutine FUNC

Figure 5.4 is a macro flow chart of subroutine FUNC. FUNC is divided into 3 major subsections. The first section concerns itself with the setting up and solution of the admittance nodal matrix. The second section deals with the computation of the gradient vector $\partial Z_{in}/\partial X$ using the nodal voltages of the original and the adjoint networks. The last subsection calculates the mixer performances such as the conversion gain, the noise figure, the RF and the IF impedances, etc., over a range of the RF input signal frequencies. The objective function E and the gradient vector ∇E are calculated using the computed mixer performances and the input impedance gradient vector $\partial Z_{in}/\partial X$, respectively. The circuit responses (gain, noise figure, and the RF and the impedances are printed out if no optimization (NOPT = 0) is required.

The main input and output variables in FUNC are:

FIG. 5.4 THE FLOW GRAPH OF SUBROUTINE FUNC (continued)

FIG. 5.4 THE FLOW GRAPH OF SUBROUTINE FUNC (continued)

Variable Name	Explanations
N	Number of variables
PV	Array of the "actual" variables (e.g. Z,L,C) that are to be optimized
P	Array of the "unconstrained" variables
NOPT	Integer NOPT = 1 optimization required. NOPT = 0 no optimization
$\bar{V}$	An array of nodal voltages of the admittance matrix of the original network
VH	Array of nodal voltages of the adjoint network
G	The gradient vector ∇E
DZ	The vector $\partial Z_{in} / \partial \bar{X}$
ZIN(KF)	A vector containing the input impedance of the mixer circuit at various frequencies - KF
GAMIF	The reflection coefficient at the IF frequency
GAMRF	The reflection coefficient at the RF frequency
GAIN	The conversion gain of the circuit
OF	The objective function
Y	The nodal admittance matrix

Table 5.2 The Input and the Output Variables in FUNC

It should be noted that the original network nodal voltage $\bar{V}$ and the adjoint network voltages $\bar{VH}$ are equal, since the elements in table

5.1 are reciprocal. FUNC can be modified slightly by the user if some of the elements are not reciprocal. The modification includes the setting up of another adjoint network (i.e. the admittance matrix $\hat{Y} = Y^T$) and performing a **second nodal analysis** using subroutine SOLV.

5.4.3 Subroutine MAT

This subroutine sets up the admittance matrix Y from the node connection data. The main variables are:

Variable Name	Variable Explanation
F	The frequency at which the nodal matrix is being set up
Y	The nodal matrix (complex)
Y ₁₁ , Y ₁₂ , Y ₂₁ , Y ₂₂	The admittance components of an element connected to nodes N1(I) and N2(I)
N1(I), N2(I)	The arrays contain the node information to which element I is connected
CIN	Array containing the current/voltage excitation vector

Table 5.3 The Input and Output Variables in Subroutine MAT

5.4.4 Subroutine OPT

This subroutine performs the minimization of the objective function E as explained in section 5.3.4. The flow graph of OPT is shown in figure 5.5. The main variables of OPT are:

Variable Name	Explanations
P	Array containing the "unconstrained" variables
N	Number of variables
AMBDA	Step size α
S	Array containing the gradient vector $S = -\nabla E / \nabla E $
E	Objective function
G	The gradient vector ∇E
LIMIT	Maximum number of allowable optimization steps

Table 5.4. The Variables in Subroutine OPT

5.4.5 Subroutine SOLV

Subroutine SOLV uses the Gauss Jordan elimination with the maximum pivotal strategy [10]. The flow chart and variable names are not included in this report as they are readily available in the cited reference.

FIG. 5.5 THE FLOWCHART OF SUBROUTINE OPT

FIG. 5.5 THE FLOW CHART OF SUBROUTINE OPE (continued)

5.5 The Mixer Network Design Example

The most general network that MIXER can analyse and/or optimize is shown in fig. 5.1 of section 5.2. To illustrate the use of MIXER in network design, the PDC circuit discussed in Chapter 3 will be used as an example.

Figures 5.6 and 5.7 show the PDC circuit, repeated here for convenience, and the topological graph of the same network. In the example, there are 14 nodes.

The input data to MIXER is divided into two groups. These groups are:

- (a) The reference data
- (b) Network configuration

The reference data is made up of the following variables:

Variable Names	Explanation
F0	Reference frequency for transmission lines (GHz)
FMIN, FMAX	The minimum and the maximum frequency for which analysis is required.
DF	Small increment in frequency (GHz)
NODE	The number of nodes
ZGOPT	Optimum generator impedance (complex number)
ZLOPT	Optimum load impedance (complex)
C ₁ , C ₂ , C ₃	Weighting constants for the conversion gain,

Table 5.5(a) The Reference Data Variables

3

4

OF/DE

OPY RESOLUTION TEST CHART
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continued)

Variable Names	Explanation
S_1	generator reflection coefficient, and the load reflection coefficient, respectively
GR	The diode elastance (see Chapter 3) (darafs)
NOPT	The required gain in dB
	NOPT = 1 if optimization is required = 0 for analysis only

Table 5.5(a) The Reference Data Variables (continued)

The network configuration data comprises of the following:

Variable Name	Explanation
M	Element type
N1(NS), N2(NS)	The first and second nodes of an element
B(1), B(2), B(3)	The element's parameter values
	Lumped element B(1) = L, or C, or R B(2) = B(3) = 0.
	Transmission line: B(1) = Z ohms B(2) = θ degrees B(3) = 0.
	Coupled transmission line: B(1) = Z_{oo} ohms B(2) = Z_{oe} ohms B(3) = θ degrees

Table 5.5.(b) The Network Configuration Data

FIG 5.6 THE PDC CIRCUIT DESIGN EXAMPLE

FIG 5.7 THE TOPOLOGICAL GRAPH OF THE PDC CIRCUIT

The computer input data is as follows:

Data	Explanation
4.0, 3.75, 4.25, .05, 15, 15	FO, FMIN, FMAX, DF, NODE, NOUT
1.0, 0.0, 1.0, 1.0, .13, 15.	NOPT, C1, C2, C3, S1, GR
23.0, 0.0, 14.0, 0.0	ZLOPT, ZGOPT
5, 1., 2., 2.5, 0.0, 0.0	M, N1, N2, B(1), B(2), B(3)
4, 2, 3, .9, 0.0, 0.0	The element parameters for
6, 3, 4, .4, 0.0, 0.0	the mixer parasitics
6, 4, 5, 1000., 0.0, 0.0	M, N1, N2, B(1), B(2), B(3)
4, 5, 0, 470., 0.0, 0.0	
1, 4, 6, -50, -30, 0.0	
1, 6, 14, -50, -30., 0.0	
1, 14, 7, -50, -30., 0.0	
1, 7, 8, -50, -30., 0.0	These data refer to the
1, 6, 11, -50, -30., 0.0	transmission lines.
1, 11, 12, -50, -30., 0.0	Minus signs mean the
1, 12, 13, -50, -30., 0.0	variables are to be optimized
2, 14, 0, 50, 60, 0.0	
2, 7, 0, 50, 60, 0.0	These data are for stubs
2, 11, 0, 50, 90, 0.0	
2, 12, 0, 50, 90, 0.0	

Table 5.6 Input Data to MIXER Program

5, 8, 9, 100, 0.0, 0.0	The element data refers to the capacitors in the circuit
5, 13, 10, 100, 0.0, 0.0	
5, 4, 0, 15, 0.0, 0.0	

4, 9, 0, 50.0, 0.0,	These data are for the resistors in the circuit
4, 10, 0, 50.0, 0.0	

Table 5.6 Input Data to MIXER Program (continued)

From the input data table there are 14 variables that are to be varied to match the circuit impedance at the RF and the IF frequencies to the optimum RF and IF impedance (the number of parameters with -ve sign in front of it). Observe that the element's position in the circuit does not influence the order in which the data is read in.

The results of the optimization run are contained in table 5.7(a), (b), (c), and (d). Table 5.7(a) and (b) is the print out of the reference data and network configuration as discussed in tables 5.5(a) and (b). The output serves as a check on the correctness of the input data to the computer. Table 5.5(c) shows the intermediate steps during the optimization run. The objective function, which in this case is the sum of the reflection coefficients ($C_2 = C_3 = 1$; $C_1 = C_4 = 0$, see table 5.7(a) and table 5.5(a) for explanations) starts at a value of 6.945. The variable parameters are printed at 4 per line. The variables at each optimization step are printed in such a way that there is a one to one correspondence between each successive column and row of each

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REFERENCE DATA IS AS FOLLOWS

TRANSMISSION LINE REF FREQ.	=	4.000	GHZ
MINIMUM FREQ.	=	5.700	GHZ
MAXIMUM FREQ.	=	6.300	GHZ
INCREMENTS IN FREQUENCY	=	.100	GHZ
PUMP FREQUENCY	=	10.000	GHZ
REQUIRED CONVERSION GAIN	=	15.000	DB
OPTIMUM CONSTANT C1	=	0	
OPTIMIZ CONSTANT C2	=	1	
OPTIMIZ CONSTANT C3	=	1	
OPTIMIZ CONSTANT C4	=	0	
OPTIMIZ. CONTROL VARIABLE	=	0	
NUMBER OF NODES IN CIRCUIT.	=	14	
IF OPTIMUM IMPEDANCE	=	14.000	.000
RF OPTIMUM IMPEDANCE	=	23.000	.000

The reference data (see table 5.5(a))

The optimization control variable

NOPT = 0 and hence only the circuit analysis is carried out.

Table 5.7(a) Reference Data for the Mixer Circuit Design Example

NETWORK CONFIGURATION DATA

ELE. TYPE NODE(1) NODE(2) ELEM. PARAM. B(1), B(2), B(3)

5	1	2	2.500	.0000	.0000
4	2	3	.9000	.0000	.0000
6	3	4	.4000	.0000	.0000
6	4	5	.1000E+06	.0000	.0000
4	5	0	470.0	.0000	.0000
1	4	6	-50.00	-30.00	.0000
1	6	14	-50.00	-30.00	.0000
1	14	7	-50.00	-30.00	.0000
1	7	8	-50.00	-30.00	.0000
1	6	11	-50.00	-30.00	.0000
1	11	12	-50.00	-30.00	.0000
1	12	13	-50.00	-30.00	.0000
2	14	0	50.00	60.00	.0000
2	7	0	50.00	60.00	.0000
2	11	0	50.00	90.00	.0000
2	12	0	50.00	90.00	.0000
5	8	9	820.0	.0000	.0000
5	13	10	820.0	.0000	.0000
5	4	0	.1500	.0000	.0000
4	9	0	50.00	.0000	.0000
4	10	0	50.00	.0000	.0000

Network configuration

print out

Table 5.7(b) The Network Configuration Data

optimization step. For example, the variable whose value is 50 in the initial step (circled with a 1 superscript) corresponds to the variable with the values 34.92 and 25.13 (circled with a 1 superscript) in first and second optimization steps. Observe that only three optimization steps and the initial steps are shown on table 5.7(c). Intermediate steps are not included. The last set of data on table 5.7(c) shows the variables and objective function at the 30th step.

The optimization procedure was forced to stop by a counter LIMIT, set at a maximum of 30 iterations. The resultant variable parameter values were taken as the optimum solution because the difference in values between the variables in the 29th step (not shown) and the last optimization step was negligible. The difference in values was observable in the 2nd decimal place only. For line impedances (ohms) and line lengths (degrees), this order of accuracy is sufficient. To ensure that the solution is not a local minimum, the initial values of variables were changed and the optimization process repeated. The resulting optimization gave values that were the "same" as the previous optimization run.

Table 5.7(d) is an output of the mixer circuit response before and after optimization. As expected, the noise figure and gain of the mixer circuit with the initial parameters is bad. The second data on table 5.7(d) is the result of the optimization. The theory predicts a PDC maximum gain of 15.1 dB and a low noise figure of 4.1 dB. Observe that the IF and the RF impedances of 23 and 14 ohms have been realized with the optimum variables parameters generated by the MIXER optimization routines.

OPTIMIZATION IS REQUIRED

NUMBER OF VARIABLES= 14

OBJ. FUNC.	VARIABLES P(I), I=1,N														Initial objective function variable and variables
6.945	50.00	30.00	50.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
	50.00	30.00	50.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
	50.00	30.00	50.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
	50.00	30.00	50.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
3.992	43.93	38.82	30.11	54.16	30.12	61.98	38.11	19.34	25.12	59.30	24.11	64.09	43.23	18.18	61.98
	58.12	61.23	54.16	30.12	61.98	38.11	19.34	25.12	59.30	24.11	64.09	43.23	18.18	61.98	38.11
	34.92	28.34	30.12	61.98	38.11	19.34	25.12	59.30	24.11	64.09	43.23	18.18	61.98	38.11	19.34
	39.12	28.47	30.12	61.98	38.11	19.34	25.12	59.30	24.11	64.09	43.23	18.18	61.98	38.11	19.34
2.876	43.71	36.56	25.12	59.30	24.11	64.09	43.23	18.18	27.43	60.22	21.12	69.87	44.12	17.24	64.09
	65.10	85.43	25.12	59.30	24.11	64.09	43.23	18.18	27.43	60.22	21.12	69.87	44.12	17.24	43.23
	25.13	25.55	24.11	64.09	43.23	18.18	61.98	38.11	27.43	60.22	21.12	69.87	44.12	17.24	25.12
	41.44	27.19	24.11	64.09	43.23	18.18	61.98	38.11	27.43	60.22	21.12	69.87	44.12	17.24	27.19
1.092	28.41	40.03	27.43	60.22	21.12	69.87	44.12	17.24	40.00	92.34	21.93	24.72	70.28	42.96	40.03
	63.45	108.54	21.12	69.87	44.12	17.24	61.98	38.11	40.00	92.34	21.93	24.72	70.28	42.96	108.54
	23.19	22.21	21.12	69.87	44.12	17.24	61.98	38.11	40.00	92.34	21.93	24.72	70.28	42.96	22.21
	37.38	25.19	21.12	69.87	44.12	17.24	61.98	38.11	40.00	92.34	21.93	24.72	70.28	42.96	25.19
0.014	29.31	40.00	24.72	70.28	42.96	75.18	61.98	38.11	24.72	70.28	42.96	75.18	61.98	38.11	40.00
	62.50	92.34	21.12	69.87	44.12	17.24	61.98	38.11	24.72	70.28	42.96	75.18	61.98	38.11	92.34
	24.70	21.93	21.12	69.87	44.12	17.24	61.98	38.11	24.72	70.28	42.96	75.18	61.98	38.11	21.93
	35.41	24.21	21.12	69.87	44.12	17.24	61.98	38.11	24.72	70.28	42.96	75.18	61.98	38.11	24.21

Table 5.7(c) The Optimization Variables

THE CIRCUIT RESPONSE

FS(GHZ)	GAIN(DB)	ZRF(OHMS)	ZIF(OHMS)	NOISE F(DB)	
5.70	-33.0	59.9	59.9	17.9	33.0
5.80	-31.6	55.6	55.6	16.0	31.6
5.90	-30.2	52.2	52.2	13.8	30.2
6.00	-28.8	49.7	49.7	11.3	28.9
6.10	-27.5	47.9	47.9	8.74	27.6
6.20	-26.3	46.7	46.7	6.27	26.3
6.30	-25.1	46.1	46.1	3.95	25.2
STOP					

The initial circuit gain, RF impedance, IF impedance and noise figure before optimization

THE CIRCUIT RESPONSE

FS(GHZ)	GAIN(DB)	ZRF(OHMS)	ZIF(OHMS)	NOISE F(DB)	
5.70	13.4	22.1	13.3	-2.1	4.3
5.80	14.2	22.4	13.6	-1.8	4.3
5.90	14.9	23.8	14.5	-.6	4.2
6.00	14.7	24.0	14.5	.0	4.2
6.10	15.1	22.4	14.0	.6	4.2
6.20	14.3	19.1	14.1	1.4	4.3
6.30	13.3	18.7	14.3	2.0	4.4
STOP					

After optimization the circuit response is printed out.

Table 5.7(d) Results of a Mixer Circuit Analysis before and After Optimization

CHAPTER 6 EXPERIMENTAL RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

6.1 Introduction

This chapter presents the experimental results of the 6 to 4 GHz parametric down converter (PDC) and discusses the results of an experimental GaAs FET mixer. The design of the auxiliary circuits and their experimental evaluation were discussed in Chapters 3 and 4. The mixing properties and performances of the PDC, the FET mixer and the resistive mixers are compared in section 6.4.3. Recommendations for further studies are presented in the last subsection.

6.2 The Experimental Results of the PDC [1]

6.2.1 The Conversion Gain g_c

The PDC MIC circuit discussed in Chapter 3 was fabricated on a Teflon substrate ($\epsilon_r = 2.35$, $h = 10$ mil). The Varian silicon diode, whose parameters are shown in Table 6.1, was used as the variable elastance. To measure the conversion gain, the experimental setup shown in figure 6.1 was used. The RF signal input consists of the signal generator at 6GHz, an input circulator, a dual directional coupler and the bias network. The input circulator is used to isolate the signal source from the high level RF signal power that is reflected from the PDC. Bias to the PDC is supplied through the bias tee at the input of the PDC. The incident and the reflected RF power are monitored with the aid of the dual directional coupler. The IF output consists of the coupler and a bias tee network. The bias tee is used for DC blocking

FIG 6.1 THE PDC CONVERSION GAIN SETUP

purposes.

The conversion gain is calculated from the measured RF input power (excluding losses due to the couplers and the circulator) and the IF output power.

Parameter	Value (Data Sheet)
R_s	0.9 ohms
Breakdown voltage V_{br}	24 volts
Q_m @ 50 mHz at $V_m = 4$ volts	4500
C_j ($V = 0$)	2.5 pfs
Package cap. C_p	0.15 pfs
Diode index m	-0.5
Package inductance L_p	0.4 nH

Table 6.1 The VAT 111-16 Diode Parameters.

Figure 6.2 shows the variation with frequency of the experimental conversion gain of the PDC circuit discussed in Chapter 3. Shown on the same figure is the theoretical conversion gain of the same circuit. The theoretical conversion gain shown on figure 6.2 does not include the effect of the upper sideband coupled transmission line filter. The measured conversion gain is 14.5 dB at 6 GHz. The 3 dB bandwidth is 238 mHz. The theoretical 3 dB bandwidth is much larger than the experimental bandwidth because of the narrowband upper sideband filter. The two peaks in the theoretical response is a result of the two resonators used in matching the output and the input circuits to the optimum RF

FIG 6.2 THE CONVERSION GAIN OF THE PDC VS THE RF FREQUENCY

and IF impedances. It should be noted that the theoretical conversion gain response in Figure 6.2 does not include the upper sideband filter response.

6.2.2 Measurement of the PDC Noise Figure

Figure 6.3 is a schematic showing the experimental setup used in measuring the noise figure. It consists of a white noise source of known excess noise ratio (ENR = 15.53 dB) at 6 GHz, a second mixer which is used to convert the PDC IF output (4 GHz) to a 70 MHz IF signal, a high gain 70 MHz amplifier (~50 dB), and the noise receiver set (AILTECH noise set type 7330).

To measure the noise figure of the PDC, the following procedure is followed.

(a) The conversion gain of the PDC (device under test - DUT) is measured accurately over the frequency band of interest (5.8 to 6.2 GHz)

(b) An automatic or manual noise figure measurement is made of the measurement system without the PDC. In the case of the manual measurements (i.e. the Y factor method) the DSB noise figure is calculated by the equation:

$$F_{Sy}(\text{DSB}) = \text{ENR} - 10 \log (Y_m - 1) \text{ dB} \quad 6.2.1$$

where Y_m = the IF attenuation, or the ratio of the noise power with the "lamp on" to the noise power with the "lamp off"

F_{sy} = the measurement system noise figure

The Y factor method consists of adjusting the IF attenuator to give the same meter reading with the "lamp on" and "off" conditions.

FIG 6.3 THE NOISE FIGURE MEASUREMENT SET

The single sideband noise figure (SSB) is computed from the double sideband measurement as: [5]

$$F_{Sy} (SSB) = F_{Sy} (DSB) + 10 \log (1+g_2/g_1) \text{ dB} \quad 6.2.2$$

where g_2 = the image signal gain of the measurement system
 g_1 = the signal gain of the measurement system

The measurement system gain g_1 and g_2 are assumed equal in the following calculations.

(c) The next step is to insert the PDC in the measurement system and measure the total DSB noise figure $F_T(DSB)$ of the PDC and the measurement system. The SSB noise figure $F_T(SSB)$ of the system and the PDC is calculated from $F_T(DSB)$ using the equation similar to equation 6.2.2.

(d) The PDC noise figure is calculated by: [5]

$$NF = NF_T + (NF_{sy} - 1)/g_s \quad 6.2.3$$

where NF = the noise figure factor for the PDC,

NF_T = the SSB noise figure factor for the system with the PDC,

NF_{sy} = the noise factor of the system without the PDC,
 and

g_s = the signal gain of the PDC.

Figure 6.4 shows a plot of the experiment SSB noise figure variation with frequency. A minimum SSB noise figure of 4.3 dB was obtained at the IF signal frequency of 4 GHz. The theoretical SSB noise figure is

FIG 6.4 THE NOISE FIGURE CHARACTERISTICS OF THE PDC

4.1 dB. The theoretical noise figure in Fig. 6.4 does not include the upper sideband filter response. The 4.3 dB noise figure does not include the 0.3 dB insertion loss of the bias network and connectors.

6.2.3 The Variation of the Conversion Gain with the LO Power, the Bias, and the RF Power

The measurements of the variations of the conversion gain with the LO power, the bias, and the RF power were made at a fixed RF frequency of 6 GHz. The 1 dB compression point is obtained from figure 6.5 as 13 dBm. The third order intercept point is calculated using equation 2.2.4 as $P_I = 23.6$ dBm at the output of the PDC. From figure 6.5 it is seen that the gain changes rapidly with the increasing LO power. A change of one dBm in the LO power results in a 3 dB change in the conversion gain. This means that the PDC LO signal source must be power stabilized for proper operation.

For a fixed pump power ($P_p = 26$ dBm), a decrease in the bias voltage from the optimum bias value results in a large change in the conversion gain. However, an increase in the bias voltage from the optimum value does not cause a great change in the conversion gain. This behaviour is to be expected since the diode dynamic quality factor $\tilde{Q}_s$ does not change significantly with the bias voltage for a forward-driven varactor diode [4]. However, driving the diode into forward bias results in an increased noise figure. The increase is due to the shot-noise resulting from the forward bias current flowing through the diode.

FIG 6.5 THE VARIATION OF GAIN WITH LO POWER, RF POWER, and BIAS

6.2.4 A Summary of the PDC Performance

Table 6.2 is a summary of the PDC performance.

RF frequency	6 GHz
LO (pump) frequency	10 GHz
IF frequency	4 GHz
Bandwidth	238 mHz
Conversion gain	14.5 dB
1 dB Compression	13 dBm
Intercept point	23.6 dBm
Pump power	26 dBm
Noise figure	4.3 dB (SSB)

Table 6.2 A Summary of the PDC Performance

In order to compare the PDC with other existing types of mixers, the four GHz IF is converted to 70 mHz using a resistive mixer with a noise figure of 6.0 dB. Using Friis' Formula for the cascaded system, one calculates the midband noise figure as [5]

$$NF = NF(PDC) + (NF(res.mix.) - 1) / g_c(PDC) \quad 6.2.4$$

where g_c = the PDC gain (14.5 dB) = 28.18

$NF(PDC)$ = the noise factor of the PDC (4.3 dB) = 2.69

$NF(res.mix.)$ = the noise factors of the resistive mixer
(6 dB) = 3.98

The calculated noise figure of the two cascaded mixers is:

$$F = 10 \log (NF) = 4.5 \text{ dB}$$

6.2.5

The variation with RF frequency of the cascaded PDC mixer and the resistive mixer is shown in figure 6.4.

6.3 The FET Mixer Experimental Results

6.3.1 Introduction

The circuit discussed in Chapter 4 was fabricated on teflon substrate ($\epsilon_r = 2.35$, $h = 10$ mils). The FET device used in the experiments was an NEC V244 chip mounted in a package designed at CRC (Communications Research Centre), Ottawa. The device characteristics are tabulated below.

Parameter (manufacturers specif.)	Value
Pinch off voltage	2.95 volts
Drain Saturation current I_{DSS}	60 mA
Maximum gate to source voltage V_{GS}	-10 v
Maximum power dispatation	27 dBm
Maximum RF input power	20 dBm
Maximum available gain @ 4 GHz	17 dB
Minimum noise figure	1.8 dB

Table 6.3 NEC V244 FET Ratings

6.3.2 Conversion Gain Measurements

Figure 6.6 is a schematic of the conversion gain measurement setup. It consists of an RF signal source at 4 GHz, a power combiner, and a bias network at the input to the FET mixer. The LO and the RF powers are coupled to the FET gate by the power combiner. The IF power is measured by the power meter at the output of the FET mixer. The conversion gain measurement procedure consists of carefully measuring the RF input power and the IF output power. The spectrum analyser is used to ensure that the LO and the RF powers at the FET mixer output are much less than the IF power being measured. In all measurements the LO power at the FET mixer output was at least 30 dB below the IF power level. The losses of the coupling elements are shown on figure 6.6. These losses are taken into account when computing the conversion gain.

Figure 6.7 shows the variation with RF frequency of the conversion gain of the FET mixer circuit discussed in Chapter 4. Shown on the same diagram is the theoretical conversion gain of the same circuit. The measured conversion gain is 5.1 dB at 4.1 GHz. The theoretical gain is 7.2 dB. The differences between the theoretical and the experimental conversion gain is due to the IF matching circuit which was tuned for minimum noise figure. Figure 6.8 shows the instantaneous 3 dB bandwidth of the FET mixer. The mixer has an IF instantaneous bandwidth of 30 MHz which is 42 per cent of the IF 70 MHz centre frequency.

6.3.3 The FET Mixer Noise Figure Measurements

The experimental setup for the FET noise figure is shown in figure 6.9. The 20 dB coupler is used in place of the power combiner to minimize

FIG 6.6 THE FET MIXER CONVERSION GAIN EXPERIMENT SETUP

FIG 6.7 THE THEORETICAL AND EXPERIMENTAL CONVERSION GAIN OF THE FET MIXER

FIG 6.8 THE CONVERSION GAIN VS THE IF SIGNAL FREQUENCY

FIG 6.9 THE EXPERIMENTAL SETUP FOR FET MIXER NOISE MEASUREMENTS

the RF losses. The waveguide at the input filters out the 70 MHz noise that might otherwise be amplified by FET and be delivered to the load as excess noise.

To measure the noise figure of the FET mixer a procedure similar to the one described in section 6.2.2 is followed. The measured RF and the IF signal insertion losses of 1.9 dB and 1.5 dB, respectively were used in calculating the FET mixer noise figure. The measured DSB noise figure is converted to the SSB noise figure by adding the factor $10 \log (1 + g_m/g_s)$ where g_m and g_s are the image and the signal gains. The gain g_m and g_s are obtained from figure 6.7.

Figure 6.10 shows the measured SSB noise figure variation with the RF signal frequency. Shown on the same graph is the theoretical noise figure response of the FET mixer. Also shown on the same figure is the measured noise figure of the FET mixer whose IF signal shorting stub (figure 4.14) was obtained with the RF and the IF signals shorted at the drain and the gate ports, respectively. The predicted minimum noise figure is 4.6 dB. The minimum noise figure of the FET whose IF is open circuited at the gate port is 8.5 dB. The differences in noise figures supports the author's claim that the reported high noise figures of the FET mixers is solely due to the uncontrolled terminations of the IF and the RF signals and not due to the high corner frequency of the $1/f$ noise as explained by several authors [2]-[3].

Figure 6.11 shows the variation of the FET mixer noise figure with the gate bias voltage. The noise figure has an optimum value of 4.8 dB at a bias voltage of -2.35 volts. Biasing the FET at the pinch off

FIG 6.10 THE THEORETICAL AND EXPERIMENTAL SSB FET MIXER NOISE FIGURE

FIG 6.11 THE VARIATION OF THE NOISE FIGURE OF THE FET MIXER WITH GATE BIAS VOLTAGE

voltage ($V_p = 2.95$ v) resulted in an increased noise figure. This behaviour is expected because the noise conductance G_n (see equation 4.6.29) is a function of the drain resistance R_{ds} which in turn increases with the decreased gate bias voltage (see figure 4.10).

6.3.4 The Variation of Gain with the LO Power, the RF Power, and the Gate Bias Voltage

Figure 6.12 shows the variation of the IF output power with the RF input power. The measured output power third order intercept point is about 8.9 dBm. The LO power was kept at a constant value of -0.2 dBm. This LO power level corresponds to the minimum noise figure value used in section 6.3.3.

The variation of the conversion gain with LO power, and the gate bias voltage is shown in figure 6.13. Observe that the FET mixer conversion gain remains relatively constant with increased gate bias. However, the conversion gain decreases rapidly as the gate bias is decreased. This phenomenon can be explained by noticing that the conversion transconductance g_1 (see figure 4.11) has a broad maximum from a gate bias of 2.0 to about 3.2 volts. The conversion gain increases with the LO power to an optimum point at about zero dBm, and then decreases with further LO power increase. Observe that the optimum LO power and bias for conversion gain differs somewhat from that for minimum noise figure. The difference is small but is noticeable during measurements.

FIG 6.12 THE INTERMODULATION CHARACTERISTICS OF THE FET MIXER

FIG 6.13 THE VARIATION OF CONVERSION GAIN WITH THE GATE BIAS VOLTAGE AND THE LO POWER

6.3.5 A Summary of the FET Mixer Performance

Table 6.4 summarizes the performance of the FET mixer discussed in the previous subsections.

Parameter	Value
RF frequency	4.0 GHz
IF frequency	70 mHz
Conversion gain	6.1 dB @ 4.1 GHz
Noise figure (SSB)	4.8 dB @ 4.1 GHz
Third order intercept (at output)	8.9 dBm
Bandwidth (3 dB BW) - RF	600 mHz
Bandwidth (3 dB BW) - IF	30 mHz
Pump power	-0.0 dBm

Table 6.4 A Summary of the FET Mixer Performance

6.4 Conclusion and Recommendations

6.4.1 Conclusions

The primary purpose of this research is to investigate the performance of a new type of mixer, namely the parametric down converter (PDC) and to investigate the causes of the reported poor noise performance of the FET mixer. The following remarks are directed towards this aim:

1. The parametric down converter has been analysed and the results of this investigation are:

(a) The PDC has a high dynamic range. The third order

intercept point is +13 dBm on the output.

(b) A very low noise figure of 4.3 dB, and a wide 3 dB bandwidth of 238 MHz were obtained using ordinary silicon varactor diodes. Lower noise figures are expected if one uses the low loss GaAs diodes.

(c) The PDC noise figure is a strong function of the load effective temperature. An improved noise performance can be expected if the load is thermally cooled or if the IF amplifier has a relatively low noise figure.

(d) The conversion gain and noise figure depend on the ratio f_o/f_s where f_o and f_s are the output IF and the input RF signal frequencies, respectively. Best performance of the PDC is obtained by operating the PDC at high ratios of f_o/f_s .

2. The theoretical and the experimental evaluation of the FET mixer show that:

(a) The noise figure of the FET mixer depends strongly on the IF and the RF signal terminations. The circuit whose RF and IF signal are shorted at the drain and the gate, respectively has a better noise performance since the IF noise voltage at the gate is shorted and hence cannot be amplified by the FET mixer which is biased as an amplifier.

(b) The ratio of the drain-gate capacitance $-C_{dg}$ to the gate-source capacitance $-C_{gs}$ has a significant effect on the noise

figure of the FET mixer. The noise figure increases rapidly with the ratio C_{dg}/C_{sg} , especially for the mixer circuit for which the IF signal is open-circuited at the gate. The ratio C_{dg}/C_{sg} can be used as a figure of merit when choosing a FET for mixer applications.

(c) The wide RF bandwidth of the FET mixer (~600 MHz) suggests its possible use as the tunable mixer in the front-end of a dual conversion microwave receiver (see figure 2.3).

(d) The common practice of biasing the FET near or at the pinch off voltage in order to achieve higher conversion gain has been shown to result in poorer noise figures. The theoretical analysis show that the increase in noise figure is due to the increased noise conductance G_n (see equation 4.6.35).

3. Other results of this research include an investigation into the effects of the measurement system compression point on low noise figure measurement errors. This error correction has never been reported elsewhere in the literature. It is shown both experimentally and analytically that if noise figure measurements are made at noise power levels of -15 to -20 dBm below the system intercept point, errors of the order of 0.3 dB can be expected. This error is quite significant if the noise figure of the device under test is of the order of a dB or two.

A computer program with the capability to analyse and optimize mixer circuits was developed. The program uses the adjoint network

method to calculate the objective function gradient vector needed for optimization purposes.

Table 7.1 is a comparison of the performance of the PDC, the diode resistive mixer, and the FET mixer. The parametric down converter (PDC) reported in this thesis has an IF signal frequency of 4 GHz. In order to compare its performance with that of the FET and the diode mixers, a resistive mixer with a 6 dB SSB noise figure is used to convert the 4 GHz IF to the 70 MHz signal. It is seen from table 6.1 that the PDC has very definite advantages in having a high gain, good power handling capability and a comparable noise figure. These characteristics alone make the PDC an attractive candidate to replace the Tunnel diode/resistive mixer configuration currently in use in most satellite receivers. However, the need for a high power stabilized pump source makes it less attractive for satellite transponder applications where pump power is at a premium. The pump power requirement can be reduced by using diodes with low breakdown voltages. A possible application of the PDC is in the 12 GHz satellite ground station receivers.

The FET mixer, on the other hand, has comparable noise figure and a wide RF bandwidth. These properties of the FET mixer, in conjunction with its low LO power requirements and conversion gain, make it suitable for satellite transponder and ground station applications. The relative cost of the FET itself is still a hindrance to the wide application of the device.

The resistive mixer has a very wide RF bandwidth. However, to achieve low noise figures, a sacrifice has to be made in its bandwidth.

Mixer type	IF and RF frequency	Conversion gain (dB)	SSB Noise figure (dB)	3 dB RF Bandwidth (mHz)	Required LO power	1 dB output compression point	Gain variation for ± 1 dB change in LO power	Circuit Complexity
PDC (this thesis)	RF=6 GHz IF=70 mHz	14.5 dB	4.5 dB	238 mHz	23 dBm	13 dBm	± 3 dB	Circuit is relatively complex to make because of the need to control the impedance of the upper sideband signal.
GaAs FET Mixer (this thesis)	RF=4.0 GHz IF=70 mHz	6.1 dB	4.8 dB	600 mHz	0.0 dBm	-1.8 dBm	± 1 dB	The FET mixer circuit is simple to construct because all of the unwanted harmonic and sidebands have to be shorted for optimum operation.
An optimized GaAs Diode Resistive Mixer ⁺⁺	RF=4.0 GHz IF=70 mHz	-2.0 dB	3.2 dB	40 mHz	0.0 dBm	-6 dBm	± 1 dB	The circuit is very complicated. To achieve these results the image sideband signal and the harmonics of the LO were properly terminated resulting in complex circuit.
A Typical ⁺⁺⁺ Balanced Resistive Mixer	RF=4 GHz IF=70 mHz	-7.0 dB	8.5 dB	1 GHz	+7 dBm	0 dBm	± 1 dB	The circuit is not complex since a simple hybrid circuit is required to couple the RF and the LO to the two diodes and also to provide RF/IF isolation.

Table 6.5 A Comparison of the Performance of the PDC, the FET and the Diode Resistive Mixers

+ The noise figure of the PDC is adjusted to account for the 6 dB noise figure of the resistive mixer used to convert the 4 GHz IF to 70 MHz.

++ G. B. Stracca et al., IEEE Trans. on Microwave Theory and Tech., August 1967.

+++ The performance reported in the fourth row are typical of most balanced resistive mixers available on the market today.

Table 6.5 A comparison of the performance of the PDC, the FET and the Diode Resistive Mixers. (continued)

The resistive diode's poor power handling capability, poor noise figure, and conversion loss are some of the reasons why the author believes that the FET mixer or the PDC will eventually replace the resistive mixer in future receivers. However, it should be pointed out that the resistive diode mixer is still less costly to manufacture when compared to the two types of mixers discussed in this thesis.

In conclusion, a new type of mixer, the parametric down converter (PDC) has been analysed both experimentally and theoretically. The PDC has been shown to have a good noise figure, conversion gain and good power handling capability, making it a suitable candidate for satellite ground station applications. The causes of the reported poor noise performance of the FET mixer have been related to the IF and the RF signal terminations and NOT due to the high corner frequency of the $1/f$ noise as suggested by several authors [2]-[3].

6.4.2 Recommendations for Future Work

The parametric down converter (PDC) has demonstrated the capability to provide conversion gain, wide RF bandwidth, and good noise figure. However, the PDC noise figure depends on the IF load temperature T_L and on the RF to IF frequency ratio. To achieve better noise performance the PDC must be operated with a high f_o/f_s ratio and/or with a cooled load. The load noise contribution (LNC) is given by equation 3.4.11 as:

$$\text{LNC} = \frac{(1+C)^2}{4C} \left[\frac{T_L}{T_o} \right] \left[\frac{f_s}{f_o} \right] \quad 6.4.1$$

where C is a constant that determines the PDC conversion gain,

T_L = the load temperature

T_O = the ambient temperature

f_s and f_o are the RF and the IF frequencies, respectively.

Equation 6.4.1 shows that for widely separated RF (f_s) and IF (f_o) frequencies, the IF load must be cooled in order to maintain the same load noise contribution (LNC). As an example, consider the case of an uncooled PDC ($T_L = T_O$) with a midband gain of 15 dB ($C = .745$, see Chapter 3) operating at an RF frequency f_s of 12 GHz and an IF frequency f_o of 4 GHz. Its minimum noise figure is computed from equation 3.4.10, as:

$$\begin{aligned} F &\cong 10 \log (1 + \text{LNC}) \\ &= 6.09 \text{ dB} \end{aligned} \quad 6.4.2$$

Assume that the same PDC IF load is cooled to 145° K, then the PDC noise figure is calculated to be

$$\begin{aligned} F &\cong 10 \log [1 + (T_L/T_O)(f_s/f_o)(1 + C)^2/4C] \\ &= 4.0 \text{ dB} \end{aligned} \quad 6.4.3$$

The two dB reduction in noise figure is due to the cooling of the IF load from 290° K to 145° K. The noise figure of an IF amplifier with an effective temperature of 145° K is (see equation 2.2.3).

$$F = 10 \log (1 + T_L/T_O) = 1.8 \text{ dB} \quad 6.4.4$$

An IF amplifier with a noise figure of 1.8 dB at 4 GHz is probably easier to design than a 4 dB LNA/resistive mixer configuration at 12 GHz.

The above simple example serves to illustrate the trade off between the ratio f_s/f_o and the effective IF load temperature. It is the belief of the author that a thorough investigation between the ratio f_s/f_o and the load temperature T_L be carried out with the aim of producing an optimum PDC/IF LNA configuration.

In order to improve the RF bandwidth of the PDC, work should be done in designing wideband bandpass/band reject filter structures. For example, coupled transmission line filters could be studied with the aim of generating a filter structure providing an open circuit condition to the upper sideband signal. Furthermore, GaAs diodes should be experimented with in order to reduce the noise figure of the PDC. The large variation of the PDC conversion gain with the LO power needs an investigation into the design of simple power stabilized local oscillator.

The theoretical analysis and experiments on FET mixers presented in this thesis refer to the FET mixer in which the LO and the RF signals are fed into the gate. The nonlinear element responsible for conversion is the FET transconductance g_m . However, if the RF and the LO signal are fed into the gate and the drain, respectively, frequency conversion is possible through the nonlinear characteristics of the drain resistance R_{ds} . This type of a FET mixer has been reported in the literature [5]. However, the analysis as presented by the author does not indicate what are the optimum RF and IF impedances, nor does it specify the factors that affect the minimum noise figure. Therefore an attempt should be made to extend the FET mixer analysis in this thesis

to the FET mixer with the drain LO injection.

The FET mixer performance presented in this thesis and in the literature today have been obtained by using FETs that were originally designed as amplifiers. A study into the effects on noise figure of the FET physical parameters such as the gate length, the doping profiles and so on should be made with the aim of reducing the FET mixer noise figure to that of the corresponding FET amplifier operating at the same RF frequency.

The problem of coupling and isolation of the RF and the LO signals can be simplified greatly by the use of the dual gate FET. Therefore, a study of the dual gate FET mixer is necessary if simple mixer circuits are to be fabricated.

The computer program MIXER presented in Chapter 5 can be extended to include other elements not shown in table 5.1. To save storage and computer time, sparse matrix techniques should be applied in solving the nodal admittance matrix.

APPENDIX A

THE JUNCTION CONVERSION LOSS L_j OF RESISTIVE MIXERS

The diode conductance g_d is assumed to have the following variation due to the LO voltage

$$g_d = g_{d0} + \sum_{n=-\infty}^{\infty} g_{dn} \exp(jn\omega_p t) \quad A.1$$

The conversion loss can be written in terms of the Fourier coefficients g_{dn} , for various conditions of image terminations as:

(a) The Broadband mixer (image impedance is equal to the signal impedance).

$$L_j = \frac{2(1 + \sqrt{1 - \epsilon_1})}{(1 - \sqrt{1 - \epsilon_1})} \quad A.2$$

where $\epsilon_1 = 2\gamma_1^2 / (1 + \gamma_2)$ A.3

$$\gamma_n = g_{dn} / g_{d0} \quad (n=1, 2, \dots, \infty) \quad A.4$$

(b) The shorted Image mixer

$$L_j = \frac{1 + \sqrt{1 - \epsilon_2}}{1 - \sqrt{1 - \epsilon_2}} \quad A.5$$

$$\epsilon_2 = \gamma_1^2 \quad A.6$$

(c) The Open Circuited Image Mixer

$$L_j = \frac{1 + \sqrt{1 - \epsilon_3}}{1 - \sqrt{1 - \epsilon_3}} \quad A.7$$

$$\epsilon_3 = \gamma_1^2(1-\gamma_2)/(1+\gamma_2)(1-\gamma_1^2)$$

A.8

APPENDIX BTHE MICROSTRIP CIRCUIT DESIGN PROGRAM PACKAGEB.1 Introduction

This appendix contains a listing of three mostly used programs for microstrip circuit design.

- (a) The first program, MSWH, calculates W/H given the desired characteristic impedance Z_0 and the operating frequency.

The input data is as follows:

ZOG = the desired impedance, (OHMS)

FREQ = the operating frequency (GHz)

ER = dielectric constant

T = The thickness of the conductor (microns)

The output data consists of the variables:

FREQ = the operating frequency

Z0 = the characteristic impedance

EFF = the effective dielectric constant

WG = the guide wavelength (λ_g cm)

ALPHA = the attenuation in dB/ λ_g (α)

- (b) The second program MSLINE calculates the characteristic impedance Z_0 given the ratio W/H. The input data is as follows:

WOVERH = the ratio W/H

ER = the dielectric constant

FREQ = the operating frequency (GHz)

H_1 and T = the thickness of substrate (mm) and
copper conductor (microns), respectively.

B.2 Method of Solution

The characteristic impedance Z_{OC} is computed by Wheelers synthesis formula [B.1]

$$Z_{OC} = \frac{120\pi/(\epsilon_r)^{1/2}}{W/H + 0.882 + [(\epsilon_r + 1)/\pi\epsilon_r][\text{Log}_c(W/H + 1.88) + 0.758] + [(\epsilon_r - 1)/\epsilon_r^2(.164)]} \quad \text{B.1}$$

The effective dielectric constant is computed by the equation [B.1]

$$\epsilon_{eff} = (Z_c/Z_0)^2 \quad \text{B.2}$$

where Z_0 = the characteristic impedance as computed by
equation B.1

Z_c = the characteristic impedance computed from equation
B.1 with $\epsilon_r = 1$.

The effect of effective dielectric constant dispersion on the dielectric constant is accounted for by O.P. Jain [B.2] equation as:

$$\epsilon_{eff} = 3 \times 10^{-6} (\epsilon_r + 1)(\epsilon_r - 1)H(Z_0/3)^{1/2} (W^1/H)(f - f_0) + \epsilon_{eff} \quad \text{B.3}$$

where f_0 = frequency in GHz below which dispersion is
negligible [B.2]

$$\text{or } f_0 = \frac{6.0}{(\epsilon_r - 1)^{1/4}} \cdot \left(\frac{Z_0}{H}\right)^{1/2} \quad \text{B.4}$$

$$W^1 = W + T/\pi[1 + \log_e(2H/T)]$$

B.5

T = the metallization thickness in mils

W^1 = the effective line width for $W/H \geq \pi/2$

The characteristic impedance is then calculated as:

$$Z_0 = Z_{oc} \sqrt{(\epsilon_{eff0}/\epsilon_{eff})}$$

B.6

The computation of W/H for a given Z_0 is by an iterative method using the above equations. The computed characteristic impedance is compared to the given impedance Z_0 for a particular estimate of W/H . W/H is either incremented or decreased dependent on whether the computed Z_0 is smaller or larger than the given impedance, respectively.

B.3 The Program Listing/Example

The program listing including an example is included in the following pages. The computer prompts the user as to which input data is required. The input data is entered by a free format of the form FORMAT (G.0).

!FLAG MSLINE
WITH>

INPUT THE RATIO W/H
?2.976
READ SUBSTR. THICKNES(MILS)
?10.
READ THE COND. THICKNESS(MICRONS)
?33.
READ DIEL CONSTANT ER
?2.35
READ FREQ(GHZ)
?4.

FREQUENCY =	4.00 GHZ
IMPEDANCE ZO(OHMS)=	49.994 OHMS
THE RATIO W/H=	2.976
EFFECTIVE DIELECTRIC CONST.=	1.989
WAVE LENGTH=	5.3142 CMS
ATTEN. CONSTANT=	.116 DB/WAVE LEN
STOP	

FLAG MSWH
WITH>

READ IN THE REQUIRED IMPEDANCE(OHMS)

?50.

READ IN THE DIEL. CONSTANT }

?2.35

INPUT SUBSTRATE HEIGHT(MILS)

?10.

INPUT METALLIC CONDUCTOR THICKNESS(MICRONS)

?33.

INPUT THE FRQ. OF OPERATION (GHZ)

?4.

FREQUENCY =

4.00 GHZ

IMPEDANCE ZO(OHMS)=

50.000 OHMS

THE RATIO W/H=

2.976

EFFECTIVE DIELECTRIC CONST.=

1.99

WAVE LENGTH=

5.3142 CMS

ATTEN. CONSTANT=

.116 DB/WAVE LEN

STOP

```

.100 C
.500 C
1.000 C
2.000 C
3.000 C
4.000 C
5.000 C
6.000 C
7.000 C
8.000 C
9.000 C
10.000 C
11.000 C
12.000 C
13.000 C
14.000
15.000
16.000
17.000 66
18.000
19.000
20.000
21.000
22.000
23.000
24.000
25.000
26.000
27.000
28.000
29.000
30.000
31.000 105
32.000
33.000
34.000
35.000
36.000
37.000
38.000 C

PROGRAM MSWH
*****
THIS PROGRAM COMPUTES THE RATIO W/H FOR A
GIVEN ZO(OHMS)

INPUT DATA
*****

ZOG-----REQUIRED IMPEDANCE
ER-----DIELECTRIC CONSTANT
H-----THE SUBSTATE THICKNESS(MILS)
T-----THE COPPER THICKNESS(MICRONS)
FREQ-----FREQ OF OPERATION

COMMON /OND/ EEF,EEFG,EEFO,ZOF,T
COMMON /OUT/ YEND,FREQ,ZOG,WOVERH,ALPHA,WLOSS
PRINT 66
FORMAT(////)
OUTPUT(108)"READ IN THE REQUIRED IMPEDANCE(OHMS)"
READ 105,ZOG
OUTPUT(108)"READ IN THE DIEL. CONSTANT"
READ 105,ER
OUTPUT(108)"INPUT SUBSTRATE HEIGHT(MILS)"
READ 105,H
OUTPUT(108)"INPUT METALLIC CONDUCTOR THICKNESS(MICRONS)"
READ 105,T
T=T+.025
OUTPUT(108)"INPUT THE FRQ. OF OPERATION (GHZ)"
READ 105,FREQ
T=T*1.E-4
H=2.54E-3*H
FORMAT(G.0)
CALL MSWH(ZOG,FREQ,WOVERH,H,ER)
CALL MSLOSS(WOVERH,H,T,ZOG,FREQ,ALPHA)
WLOSS=ALPHA*YEND
CALL OUTPUT
CALL EXIT
END

```

```

39.000 C
40.000 C
41.000 C
42.000 C
43.000 C
44.000 C
45.000 C
46.000 C
47.000 C
48.000 C
49.000 C
50.000 C
51.000 C
52.000 C
53.000 C
54.000 C
55.000 C
56.000 C
57.000 C
58.000 C
59.000 10
60.000 20
61.000 C
62.000 C
63.000 C
64.000 C
65.000 C
65.100 C
66.000 C
67.000 C
68.000 C
69.000 C
70.000 30
71.000 C
72.000 C
73.000 C
74.000 C
75.000 C

SUBROUTINE MSWH

THIS ROUTINE USES AN ITERATIVE METHOD TO CALCULATE THE
VALUE OF W/H FOR A GIVEN IMPEDANCE AT THE FREQ. OF INTE
REST.

OUTPUT:-
WOVERH=====W/H

SUBROUTINE MSWH(ZOG,FREQ,WOVERH,H,ER)
DEPENDENT ON THE VALUE OF THE IMPED. A FIRST EST. IS
MADE

IF(ZOG,LT.50.) GO TO 10
Y=-.015*ZOG+1.63
IF(Y,LT.0.) Y=.05
GO TO 20
Y=-.105*ZOG+6.12
CALL MSZOG(Y,H,ER,FREQ,Z1)

DEPENDENT ON WETHER Z1>YORZOG<Y;Y IS INCREASED OR DECRE
ASED BY AN AMOUNT PROPORTIONAL TO THE DIFFERENCE BETWEEN
Z1 AND ZOG;THE NEW VALUE OF Y IS USED TO CALCULATE Z2.
IF Z1 AND Z2 DO NOT BRACKET ZOG,Z2 BECOMES Z1 AND A NEW
VALUE
OF Y IS CALCULATED ,GIVING A NEW VALUE QF Z2. THE PROCESS
IS REAPEATED UNTIL ZOG IS BRACKETED BETWEEN Z1 AN Z2

R=ABS(Z1-ZOG)
YOLD=Y
TST=Z1-ZOG
RR=ABS(TST)
IF(RR,LE.1.E-3) GO TO 100
IF(TST,LT.0.) GO TO 40
Y=Y+R

```

```

76.000      GO TO 50
77.000      IF(Y,GT,R) GO TO 45
78.000      Y=Y-Y/2.
79.000      GO TO 50
80.000      Y=Y-R
81.000      50  CALL MSZOG(Y,H,ER,FREQ,Z2)
82.000      IF(Z2,GT,ZOG,AND,Y,LT,YOLD) GO TO 60
83.000      IF(Z2,LT,ZOG,AND,Y,GT,YOLD) GO TO 70
84.000      Z1=Z2
85.000      GO TO 30
86.000      C
87.000      C  AN ITERATIVE PROCEDURE IS USED TO CONVERGE TO THE VALUE
88.000      C  OF W/H THAT GIVES Z0 TO WITHIN .001 OHMS
89.000      C
90.000      60  YTMP=Y
91.000      Y=YOLD
92.000      YOLD=YTMP
93.000      ZTMP=Z1
94.000      Z1=Z2
95.000      Z2=ZTMP
96.000      70  DELTA=(Y-YOLD)*(Z2-ZOG)/(Z1-Z2)
97.000      80  YTMP=Y+DELTA
98.000      CALL MSZOG(YTMP,H,ER,FREQ,ZTMP)
99.000      IF(ZTMP,LE,ZOG) GO TO 90
100.000     DELTA=DELTA/2.
101.000     GO TO 80
102.000     90  Y=YTMP
103.000     IF(ABS(ZTMP-ZOG),LE,1.E-3) GO TO 100.
104.000     Z2=ZTMP
105.000     GO TO 70
106.000     100 WOVERH=Y
107.000     RETURN
108.000     END
109.000     C

```

*

```

1.000 C
2.000 C
3.000 C
4.000 C
5.000 C
6.000 C
7.000 C
8.000 C
9.000 C
10.000 C
11.000 C
12.000 C
13.000 C
14.000 C
15.000
16.000
17.000
18.000
19.000
20.000
21.000
22.000
23.000
24.000
25.000
26.000
27.000
28.000
29.000
30.000
31.000
32.000
33.000
34.000
35.000
36.000
37.000
38.000
39.000
40.000 C

PROGRAM MSLINE
*****
THIS IS THE MAIN PROGRAM FOR CALCULATING THE CHARACTER
ISTIC IMPED. OF A LINE GIVEN THE FOLLOWING

INPUT DATA:-
WOVERH-----W/H
ER-----DIELECTRIC CONSTANT
H-----THE SUBSTATE THICKNESS(MILS)
T-----THE COPPER THICKNESS(MICRONS)
FREQ-----FREQ OF OPERATION

COMMON /ONO/ EEF,EEFG,EEFO,ZOF,T
COMMON /OUT/ YEMD,FREQ,ZOG,WOVERH,ALPHA,WLOSS
PRINT 66
FORMAT(////)
OUTPUT(108)"INPUT THE RATIO W/H"
READ 105,WOVERH
OUTPUT (108)"READ SUBSTR. THICKNES(MILS)"
READ 105,H
OUTPUT(108)"READ THE COND. THICKNESS(MICRONS)"
READ 105,T
T=T+.05
OUTPUT(108)"READ DIEL CONSTANT ER"
READ 105,ER
OUTPUT(108)"READ FREQ(GHZ)"
READ 105,FREQ
T=T*1.E-4
H=H*2.54E-3
FORMAT(5G15.4)
CALL MSZOG(WOVERH,H,ER,FREQ,ZOG)
CALL MSLOSS(WOVERH,H,T,ZOG,FREQ,ALPHA)
WLOSS=ALPHA*YEMD
CALL OUTPUT
CONTINUE
CALL EXIT
END

```

```

41.000 C
42.000 C
43.000 C
44.000 C
45.000
46.000
47.000
48.000
49.000
50.000
51.000
52.000
53.000
54.000
55.000
56.000
57.000
58.000
59.000
60.000
61.000
62.000
63.000
64.000
65.000
66.000
67.000
68.000
69.000
70.000 C

```

THE ATTENUATION CONSTANT IS COMPUTED IN THIS SUBROUTINE

```

SUBROUTINE MSLOSS(WOVERH,H,T,ZOG,FREQ,ALPHA)
DATA PI,EN/3.1415926,2.7182818/
REAL INV2PI
INV2PI=.1591549
W=WOVERH*H
CLOG=ALOG(4.*PI*W/T)
IF(WOVERH.GE.INV2PI) GO TO 10
DELT=(T/PI)*(CLOG+1.)
GO TO 20
DELT=(T/PI)*(ALOG(2.*H/T)+1.)
WNEW=W+DELT
RESURF=8.68E-3*SQR(T/FREQ)
RATIO=WNEW/H
C1=8.68*INV2PI*(1.-(RATIO/4.))**2
IF(WOVERH.GT.INV2PI) GO TO 30
C2=(1.+1/RATIO+(1./(PI*RATIO))*(CLOG+T/W))
GO TO 40
C2=(1.+1/RATIO+(1./(PI*RATIO))*(ALOG(2.*H/T)-T/H))
IF(WOVERH.LE.2.) GO TO 40
C1=8.68/(RATIO+(2./PI)*ALOG(2.*PI*EN*(.5*RATIO+.94))**2)
C2=C2*(RATIO+(2.*INV2PI*RATIO)/(.5*RATIO+.94))
COEFF=C1*C2
ALPHA=(COEFF*RESURF)/(ZOG*H)
RETURN
END

```

*


```

71.000 SUBROUTINE MSZOG COMPUTES THE EFFECTIVE DIELECTRIC CONS
72.000 TANT AND ZOG FOR A GIVEN W/H--WOVERH
73.000
74.000
75.000 SUBROUTINE MSZOG(WOVERH,H,ER,FREQ,ZOG)
76.000 COMMON /ONO/ EEF,EEFG,EEFO,ZOF,T
77.000 COMMON /OUT/YEMD,FRQG,ZOGG,W0,ALPHA,WLOSS
78.000 PI=ATAN(1.)*4.
79.000 X1=3.E-6
80.000 XO=WOVERH+.882+(ER+1.)/(PI*ER)*(ALOG(WOVERH+1.88)+.758)
81.000 XC=WQVERH+.882+2/PI*(ALOG(WOVERH+1.88)+.758)
82.000 ZO=120.*PI/(SQRT(ER)*XO)
83.000 ZC=120.*PI/XC
84.000 EEFO=(ZC/ZO)**2
85.000 HP=H/2.54*1000.
86.000 WP=WOVERH*H/2.54*1000.
87.000 TP=T/2.54*1000.
88.000 FO=6./(ER-1.):**.25*SQRT(ZO/HP)
89.000 EEFG=EEFO
90.000 IF(FREQ.LE.FO) GO TO 10
91.000 WP=WP+TP/PI*(1.+ALOG(2.*HP/TP))
92.000 IF(WOVERH.GT.4.) GO TO 13
93.000 EEFG=X1*(ER-1.)*(ER+1.)*HP*SQRT(WP*ZO/HP)*(FREQ-FO)+EEFO
94.000 GO TO 10
95.000 13 EEFG=X1*(ER-1.)*(ER+1.)*HP*SQRT(ZO/3)*WP/HP*(FREQ-FO)+EEFO
96.000 10 ZOG=ZO*SQRT(EEFO/EEFG)
97.000 YEMD=29.98/(FREQ*SQRT(EEFG))
98.000 RETURN
99.000 END

```

*

```

99.100 C      OUTPUT IS THE SUBROUTINE FOR PRINTING THE DATA
100.000 C
101.000
102.000      SUBROUTINE OUTPUT
103.000      COMMON /ONO/ EEF,EEFG,EEFO,ZOF,T
104.000      COMMON /OUT/ YEMD,FREQ,ZOG,WOVERH,ALPHA,WLOSS
105.000      PRINT 103,FREQ,ZOG,WOVERH,EEFG,YEMD,WLOSS
106.000      FORMAT(///,5X,"FREQUENCY =",21X,F10.2," GHZ",
107.000      1//,5X,"IMPEDANCE ZO(OHMS)=",15X,F10.3," OHMS",
108.000      1//,5X,"THE RATIO W/H=",19X,F10.3,
109.000      1//,5X,"EFFECTIVE DIELECTRIC CONST.=",4X,F10.3,
110.000      1//,5X,"WAVE LENGTH=",22X,F10.4," CMS",
111.000      1//,5X,"ATTEN. CONSTANT=",16X,F10.3," DB/WAVE LEN")
112.000      RETURN
          END

```

*

APPENDIX C
MIXER PROGRAM LISTING

C.1 Introduction

This appendix contains the listing of MIXER. The program flowcharts and an example are discussed in Chapter 5.

C.2 Program Listing

The program listing of MIXER is contained in the following pages.

```

1.000 C
2.000 C
3.000 C
4.000 C
5.000 C
6.000 C
7.000 C
8.000 C
9.000 C
10.000 C
11.000 C
12.000 C
13.000 C
14.000 C
15.000 C
16.000 C
17.000 C
18.000 C
19.000 C
20.000 C
21.000 C
22.000 C
23.000 C
24.000 C
25.000 C
26.000 C
27.000 C
28.000 C
29.000 C
30.000 C
31.000 C
32.000 C
33.000 C
34.000 C
35.000 C
36.000 C
37.000 C
38.000 C
39.000 C
40.000 C

MIXER ANALYSES AND OPTIMIZES MICROWAVE CCTS USING
ADJOINT NETWORK TECHNIQUES TO COMPUTE THE GRADIENTS
REQUIRED BY THE OPT. SUBROUTINE.
=====
INPUT VARIABLES ARE:--
FO=REFERENCE FREQ. FOR TRANSMISSION LINES
FMIN=THE MINIMUM FREQ.(GHZ) FOR ANALYSIS
FMAX=THE MAXIMUM FREQ(GHZ) FOR ANALYSIS
DF =SMALL ICREMENT IN FREQ.(GHZ)
NODE =THE NUMBER OF NODES IN A CCT.
ZGOPT=OPTIMUM RF IMPEDANCE IN (OHMS)==COPMLEX NUMBER
ZLOPT= OPT. IF IMPEDANCE IN OHMS
C1,C2,C3= THE WEIGHTS FOR GAIN,REFL. COEFF,AND NOISE
FIGURE
S1=THE DIODE ELASTANCE(DARAFS)
GR= GAIN REQUIRED IN DB
NOPT=1 IF OPT. IS REQ,D
      =0 IF NO OPT. IS REQ,D
=====

THE NETWORK CONFIGRATION DATA IS:--

M=ELEMNT TYPES
N1(NS) THE FIRST NODE OF ELEMENT NS
N2(NS)= THE SECOND NODE
B(1),B(2),B(3)= THE ELEMENT PARAMETERS
SEE TABLE 5.5 (b) FOR EXPLANATIONS
=====

THE ELEMENT TYPES ARE:--
1-----TRANSMISSION LINE
2-----OPEN STUB
3-----SHORTED STUB
4=RESISTOR
5=CAPACITOR
6=INDUCTOR
7-----PARARELL. COUPLED LINE

```

```

41.000 C *****
*2 THE MAIN PROGRAM
42.000 C IMPLICIT REAL*8(A-H,O-Z)
43.000 C DOUBLE COMPLEX ZGOPT,ZLOPT
44.000 C DIMENSION B(3),G(40),S(80),F(40),FV(40)
45.000 C COMMON /ONE/ E(100),PR(300),FO,FMIN,FMAX,DF,GR,N1(200),
46.000 C 1N2(200)
47.000 C COMMON /TWO/ NE(200),NOP(200),LIS(200),NS,NODE,NP
48.000 C COMMON /THREE/ ZGOPT,ZLOPT,C1,C2,C3,C4,S1,FP,RS,NOPT
49.000 C COMMON /PRINT/ IPRINT
50.000 C
51.000 C READ IN THE REF. DATA
52.000 C
53.000 C OUTPUT(108)
54.000 C OUTPUT(108) REFERENCE DATA IS AS FOLLOWS
55.000 C OUTPUT(108) *****
56.000 C OUTPUT(108)
57.000 C READ 2,FO,FMIN,FMAX,DF,GR,FP,RS
58.000 C READ 2,NOPT,C1,C2,C3,C4,S1,NODE
59.000 C READ 2,ZGOPT,ZLOPT
60.000 C PRINT 199,FO,FMIN,FMAX,DF,GR,FP,C1,C2,C3,C4,NOPT,NODE
61.000 C 1,ZLOPT,ZGOPT
62.000 C 199 FORMAT(5X,"TRANSMISSION LINE REF FREQ.=",F10.3," GHZ",/
63.000 C 1,5X,"MINIMUM FREQ.",14X,"=",F10.3," GHZ",//
64.000 C 1,5X,"MAXIMUM FREQ.",14X,"=",F10.3," GHZ",//
65.000 C 1,5X,"INCREMENTS IN FREQUENCY",5X,"=",F10.3," GHZ",//
66.000 C 1,5X,"PUMP FREQUENCY",14X,"=",F10.3," GHZ",//
67.000 C 1,5X,"REQUIRED CONVERSION GAIN",4X,"=",F10.3," DB",//
68.000 C 1,5X,"OPTIMUM CONSTANT C1",9X,"=",I10,//
69.000 C 1,5X,"OPTIMIZ CONSTANT C2",9X,"=",I10,//
70.000 C 1,5X,"OPTIMIZ CONSTANT C3",9X,"=",I10,//
71.000 C 1,5X,"OPTIMIZ CONSTANT C4",9X,"=",I10,//
72.000 C 1,5X,"OPTIMIZ. CONTROL VARIABLE =",I10,//
73.000 C 1,5X,"NUMBER OF NODES IN CIRCUIT =",I10,//
74.000 C
75.000 C
76.000 C

```

```

77.000 1,5X,"IF OPTIMUM IMPEDANCE",9X,"=",2F10.3,///
78.000 1,5X,"RF OPTIMUM IMPEDANCE",9X,"=",2F10.3,////)
79.000 PRINT 197
80.000 OUTPUT(108)"NETWORK CONFIGURATION DATA"
81.000 OUTPUT(108)"*****"
82.000 2 FORMAT(7G15.5)
83.000 C NV=0
84.000 NS=0
85.000 NP=0
86.000 C
87.000 C READ IN THE CONFIG. DATA
88.000 C
89.000 C
90.000 197 FORMAT(////)
91.000 PRINT 196
92.000 OUTPUT(108)" "
93.000 196 FORMAT(2X,"ELE.TYPE",2X,"NODE(1)",2X,"NODE(2)",2X,
94.000 1"ELEM. PARAM. B(1),B(2),B(3)" )
95.000 30 NS=NS+1
96.000 DO 77 KK=1,3
97.000 77 B(KK)=0.0
98.000 READ 2, M,N1(NS),N2(NS),(B(K),K=1,3)
99.000 IF(M.EQ.99) NS=NS+1; GO TO 999
100.000 PRINT 22,M,N1(NS),N2(NS),B(1),B(2),B(3)
101.000 22 FORMAT( 3I8,9X,3G10.4)
102.000 IF(N1(NS).EQ.0) N1(NS)=NODE+1
103.000 IF(N2(NS).EQ.0) N2(NS)=NODE+1
104.000 E(NS)=M
105.000 GO TO (1,1,1,4,4,4,6) ,M
106.000 C
107.000 C THE ELEMENT IS EITHER A STUB OR TRANS LINE
108.000 C DETERMINE IF THERE ANY VARIABLE PARAMETERS
109.000 1 DO 3 I=1,2
110.000 NP=NP+1
111.000 PR(NP)= DABS(B(I))
112.000 IF(B(I).GE.0.) GO TO 3

```

*

```

113.000 NV=NV+1
114.000 LIS(NV)=NP
115.000 NE(NV)=NS
116.000 NOP(NV)=I
117.000 3 CONTINUE
118.000 GO TO 30
119.000 C
120.000 C THIS IS ALUMPED ELEMENT
121.000 C DETERMINE IF THERE IS A VARIABLE PARAMTER
122.000 C
123.000 4 NP=NP+1
124.000 PR(NP)=DABS(B(1))
125.000 IF(B(1).GE.O.) GO TO 30
126.000 NV=NV+1
127.000 LIS(NV)=NP
128.000 NE(NV)=NS
129.000 NOP(NV)=1
130.000 GO TO 30
131.000 C
132.000 C THE ELEM. IS A COUPLED LINE
133.000 C ONLY THE COUPLED LINE LENGTH IS ALLOWED TO VARY
134.000 C
135.000 6 CONTINUE
136.000 DO 24 I=1,3
137.000 NP=NP+1
138.000 PR(NP)=DABS(B(I))
139.000 IF(B(I).GE.O.DO) GO TO 24
140.000 NV=NV+1
141.000 LIS(NV)=NP
142.000 NE(NV)=NS
143.000 NOP(NV)=3
144.000 24 CONTINUE
145.000 GO TO 30
146.000 999 CONTINUE
147.000 C
148.000 IPRINT=1

```

*

```
149.000 C      CALL FUNC TO EVAL CCT AT THE BASE POINT
150.000 C
151.000 C      CALL FUNC(NV,P,PV,OF,G)
152.000 C      IF (NOPT.EQ.0) GO TO 987
153.000 C
154.000 C      CALL SUBR. OPT TO PERFORM CIRCUIT OPTIMIZATION
155.000 C
156.000 C      CALL OPT(NV,P,G,S,OF,LIMIT)
157.000 C      IPRINT=1
158.000 C      987 NORI=0
159.000 C      CALL FUNC(NV,P,PV,OF,G)
160.000 C      CALL EXIT
161.000 C      END
```

*


```

162.000 C
163.000 C
164.000 C
*
165.000 C
166.000 C
167.000 C
168.000 C
169.000 C
170.000 C
171.000 C
172.000 C
173.000 C
174.000 C
175.000 C
176.000 C
177.000 C
178.000 C
179.000 C
180.000 C
181.000 C
182.000 C
183.000 C
184.000 C
185.000 C
186.000 C
187.000 C
188.000 C
189.000 C
190.000 C
191.000 C
192.000 C
193.000 C
194.000 C
195.000 C
196.000 C
197.000 C
*

*****
SUBROUTINE FUNC
SUBROUTINE FUNC EVALUATES THE CCT, RESPONSES AT RF AND IF
FREQUENCIES BY CALL SUBROUTINE MAT AND SOLVE TO
SET UP THE NODAL MATRIX AND THE SOLVE IT, RESPECTIVELY
THE MAJOR VARIABLES ARE:-
N=NO. OF OPTIMIZATION VARIABLES
FV=ARRAY OF ACTUAL VARIABLE PARAMETERS
P=ARRAY OF UNCONSTAINED VARIABLE PARAMETERS
V=ORIGINAL NETWORK NODAL VOLTAGES
VH=ADJOINT NETWORK NODAL VOLTAGES
G=ARRAY OF THE OBJ. FUNT. GRADIENTS
ZIN(KF)=THE MIXER CCT. INPUT IMPED. AT FREQ. -KF
OF=OBJ. FUNCTION
GAMAIF,GAMARF=THE IF AND THE RF REFL. COEFF.
Y=NODAL MATRIX
GAIN= CONVERSION GAIN(DB)
FN= NOISE FIGURE(DB)
SUBROUTINE FUNC(N,P,PV,OP,G)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE COMPLEX ZGOPT,ZLOFT,ZIN(4),TEMP,Y(50,50),CIN(50),
1CJ,V(50)
DOUBLE COMPLEX VH(50),Y1,Y2,Y3,CG,DZ(3,50),D,DD
DOUBLE COMPLEX GAML,GAMG,GL,Z12,Z21,ZOUT,DZOUT,DD1
COMMON /ONE/ E(100), PR(300),FO,FMIN,FMAX,DF,GR,N1(200),
1N2(200)
COMMON /TWO/ NE(200),NOP(200),LIS(200),NS,NODE,NP
COMMON /THREE/ ZGOPT,ZLOFT,C1,C2,C3,C4,S1,FP,RS,NOPT
EQUIVALENCE(V(1),CIN(1))
DIMENSION FV(40),G(40),F(40),JFIV(50)

```

```

198.000 COMMON /PRINT/IPRINT
199.000 PI=DATAN(1.DO)*4.DO
200.000 C
201.000 C CALL SBR. UPDATE TO UPDATE THE VARIABLE PARAMETERS
202.000 C
203.000 CALL UPDATE(P,PV,N)
204.000 DO 120 I=1,N
205.000 G(I)=0.DO
206.000 CONTINUE
207.000 CJ=(0.DO,1.DO)
208.000 CF=0.DO
209.000 C
210.000 C .BEGIN CIRCUIT ANALYSIS
211.000 C
212.000 FS=FMIN
213.000 777 DO 911 KF=1,2
214.000 IF(KF.EQ.1) F=FS
215.000 IF(KF.EQ.2) F=FF-FS
216.000 TEMPO=F/FO/180.*PI
217.000 CALL MAT(F,CIN,Y,TEMPO)
218.000 ITEMP=0
219.000 NO=NODE+1
220.000 C
221.000 C CAL SUBR. SOLV TO COMPUTE THE ORIGINAL NETWORK
222.000 C VOLTAGES V(50)
223.000 C
224.000 CALL SOLV(Y,CIN,NO,NO,JPIV,ITEMP)
225.000 C
226.000 C THE CIRCUIT IS RECIPROCAL--HENCE SET THE ADJOINT
227.000 C VOLTAGES VH(50) EQUAL TO THE ORIGINAL VOLTAGES V(50)
228.000 C
229.000 C DO 20 I=1,NO
230.000 VH(I)=V(I)
231.000 CONTINUE
232.000 CG=CIN(NO)
233.000 V(NO)=0.DO

```

*

```

234.000      VH(ND)=0.
235.000      TEMP=1./CG**2
236.000 C    CALCULATE THE CIRCUIT INPUT IMPEDANCE ZIN(F)
237.000 C
238.000 C    ZIN(KF)=-1.D0/CG
239.000 C
240.000 C    IS OPT. REQ,D?
241.000 C
242.000 C    IF(NOPT. EQ. 0) GO TO 911
243.000 C
244.000 C    DETERMINE THE VARIABLE ELEMENTS AND THE VARIABLE PARAM.
245.000 C
246.000 C    DO 40 I=1,N
247.000 C    333
248.000      L=NE(I)
249.000      K=LIS(I)
250.000      M=E(L)
251.000      MK=NOF(I)
252.000      NOD1=N1(L)
253.000      NOD2=N2(L)
254.000      GO TO (1,2,3,4,5,6,7),M
255.000 C
256.000 C    THE ELEM. IS A TRANS. LINE
257.000 C
258.000 C    GO TO (11,12),MK
259.000 C
260.000 C    THE VARIABLEPARAM. IS THE IMPED. OF THE TRAN.
261.000 C    LINE
262.000 C
263.000 C    11
264.000      ZO=PR(K)
265.000      RAD=TEMPO*PR(K+1)
266.000      Y1=-CJ/(ZO**2*DTAN(RAD))
267.000      Y2=CJ/(ZO**2*DSIN(RAD))
268.000      Y3=Y1*(V(NOD1)*VH(NOD1)+V(NOD2)*VH(NOD2))
269.000      Y3=Y3+Y2*(V(NOD1)*VH(NOD2)+VH(NOD1)*V(NOD2))
      DZ(KF,I)=TEMP*Y3

```

*

```

270.000 GO TO 40
271.000 C
272.000 C THE LINE LENGTH IS VARIABLE
273.000 C
274.000 12 CONTINUE
275.000 ZO=PR(K-1)
276.000 RAD=TEMPO*PR(K)
277.000 Y1=-CJ*TEMPO*(1.+DTAN(RAD)**2)/(ZO*DTAN(RAD)**2)
278.000 Y2=CJ*TEMPO*DCOS(RAD)/(ZO*DSIN(RAD)**2)
279.000 DZ(KF,I)=Y3*TEMP
280.000 GO TO 40
281.000 C
282.000 C THE ELEMENT IS ANY OPEN STUB
283.000 C
284.000 2 CONTINUE
285.000 GO TO (13,14),MK
286.000 C
287.000 C THE STUB IMPED. IS VARIABLE
288.000 C
289.000 13 ZO=PR(K)
290.000 RAD=TEMPO*PR(K+1)
291.000 Y3=CJ*DTAN(RAD)/ZO**2*VH(NOD1)*V(NOD1)
292.000 DZ(KF,I)=Y3*TEMP
293.000 GO TO 40
294.000 C
295.000 C THE STUB LENGTH IS VARIABLE
296.000 C
297.000 14 RAD=TEMPO*PR(K)
298.000 ZO=PR(K-1)
299.000 Y3=-CJ*TEMPO*(1.DO+DTAN(RAD)**2)/ZO*V(NOD1)*VH(NOD1)
300.000 DZ(KF,I)=Y3*TEMP
301.000 GO TO 40
302.000 C
303.000 C THE ELEM. IS A SHORTED STUB
304.000 C
305.000 3 CONTINUE

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306.000      GO TO (15,16),MK
307.000 C
308.000 C      THE SHORTED STUB IMPED. IS VARIABLE
309.000 C
310.000 C      15      ZO=PR(K)
311.000      RAD=TEMPO*PR(K+1)
312.000      Y3=-CJ*VH(NOD1)*V(NOD1)/ZO**2/DTAN(RAD)
313.000      DZ(KF,I)=TEMP*Y3
314.000      GO TO 40
315.000 C
316.000 C      THE STUB LENGTH IS VARIABLE
317.000 C
318.000 C      16      CONTINUE
319.000      RAD=TEMPO*PR(K)
320.000      ZO=PR(K-1)
321.000      Y3=-CJ*TEMPO*VH(NOD1)*V(NOD1)/(ZO*DSIN(RAD)**2)
322.000      DZ(KF,I)=TEMP*Y3
323.000      GO TO 40
324.000 C
325.000 C      THE VARIABLE ELEM. IS A RESISTOR
326.000 C
327.000 C      CONTINUE
328.000      DZ(KF,I)=TEMP*(V(NOD1)-V(NOD2))*(VH(NOD1)-VH(NOD2))/PR(K)
329.000      1**2
330.000      GO TO 40
331.000 C
332.000 C      THE ELEM. IS A CAPACITOR
333.000 C
334.000 C      5      DZ(KF,I)=-2.*PI*FCJ*TEMP*(V(NOD1)-V(NOD2))*(VH(NOD1)
335.000      1-VH(NOD2))/1000.
336.000      GO TO 40
337.000 C
338.000 C      THE ELEMENT IS AN INDUCTOR
339.000 C
340.000 C      6      DZ(KF,I)=TEMP*(VH(NOD1)-VH(NOD2))*(V(NOD1)-V(NOD2))/
341.000      1(2.*PI*FCJ*PR(K)**2)

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342.000 C
343.000 C
344.000 C
345.000 C
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373.000
374.000 C
375.000 C
376.000 C
377.000

GO TO 40
THE ELEM. IS ACOUPLED TRANS. LINE
CONTINUE
THE VARIABLE ELEMENT IS THE COUPLED LINE LENGTH
RAD=TEMPO*PR(K+2)
YS=1./PR(K)-1./PR(K+1)
YD=1./PR(K)+1./PR(K+1)
CU1=-(YS*V(NOD1)/DTAN(RAD)-YD*V(NOD2)/DSIN(RAD))
CU2=YD*V(NOD1)/DSIN(RAD)+YS*V(NOD2)/DTAN(RAD)
Y3=CJ*(CU1*V(NOD2)+CU2*V(NOD1))
DZ(K,I)=TEMP*Y3
CQNTINUE
CONTINUE
40
911

THIS SECTION COMPUTES THE MIXER PERFORMANCE VALUES
NOISE FIGURE, GAIN, AND REFLECTION COEFF.
CALCULATE THE GAIN
WS=2.*PI*FS*1.D9
WD=2.*PI*(FP-FS)*1.D9
RG=DREAL(ZIN(2))-RS
RL=DREAL(ZIN(1))-RS
D=ZIN(2)*DCONJG(ZIN(1))-S1**2/(WD*WS)
GC=4.*RL*RG*(S1/WS)**2/CDABS(D)**2
GAIN=10.*DLOG10(GC)
GRR=GR/10.
GRR=10.**GRR
CALCULATE REFLECTION COEFF. AT RF AND IF
GAML=(ZLOPT-DCONJG(ZIN(1)))/(ZLOPT+ZIN(2))

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378.000      GAMG=(ZGOPT-DCONJG(ZIN(1)))/(ZGOPT+ZIN(2))
379.000 C
380.000 C      COMPUTE THE NOISE FIGURE
381.000 C
382.000      Z12=-S1/CJ/WO
383.000      Z21=S1/(CJ*WS)
384.000      ZOUT=-DCONJG(Z12*Z21/ZIN(2))
385.000      GL=(ZOUT-DCONJG(ZIN(1)))/(ZOUT+ZIN(1))
386.000      FN=1.+RS/RG*(1.+CDABS(ZIN(2)/Z21)**2)+CDABS(GL)**2/GC
387.000      FN=10.*DLOG10(FN)
388.000 C
389.000 C      COMPUTE THE GRADIENTS IF OPT. IS REQ'D
390.000 C
391.000      IF(NOPT.EQ.0) GO TO 999
392.000      DO 404 K=1,N
393.000 C
394.000 C      COMPUTE THE OBJ. FUNC. GRAD W.R.T THE REFLEC. COEFF.
395.000 C
396.000      DGAML=DREAL(2*DCONJG(GL)*(-(ZLOPT+ZIN(1))*DCONJG(DZ(1,K))
397.000      1-(ZLOPT-DCONJG(ZIN(1))*DZ(1,K))/(ZLOPT+ZIN(1))**2)
398.000      DGAMG=DREAL(2.*DCONJG(GL)*(-(ZGOPT+ZIN(2))*DCONJG(DZ(2,K))-
399.000      1-(ZGOPT-DCONJG(ZIN(2))*DZ(2,K))/(ZGOPT+ZIN(2))**2)
400.000 C
401.000 C      COMPUTE THE GRAD W.R.T. THE GAIN --GC
402.000 C
403.000      DD=ZIN(2)*DCONJG(DZ(1,K))+DCONJG(ZIN(1))*DZ(2,K)
404.000      DGC1=GC*DREAL(DZ(1,K)/RL+DZ(2,K)/RG-2.*DCONJG(D)*DD/CDABS
405.000      1(D)**2)
406.000      DGC=2.*(GC-GRR)*DGC1
407.000 C
408.000 C      COMPUTE THE GRADIENT VECTOR W.R.T. NOISE FIGURE.
409.000 C
410.000      DZOUT=DCONJG(Z21*Z12)/QCONJG(ZIN(2))*2*DCONJG(DZ(2,K))
411.000      DRG=-RS/RG**2*(1.+CDABS(ZIN(2)/Z21)**2)*DREAL(DZ(2,K))
412.000      DZG=RS/RG*DREAL(2.*DCONJG(ZIN(2))*DZ(2,K))/CDABS(Z21)**2
413.000      DD1=(ZOUT+ZIN(1))*(DZOUT-DCONJG(DZ(1,K)))

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414.000 DD1=DD1-(ZOUT-DCONJG(ZIN(1)))*(DZOUT+DZ(1,K))
415.000 DPD=2.*DREAL(DCONJG(GL)/GC*DD1)
416.000 DFGC=-CDABS(GL)**2/GC**2*DDGC1
417.000 DFN=DFGC+DRG+DZG+DPD
418.000 C
419.000 G(K)=G(K)+C1*DDGC+C2*DGAMG+C3*DGAML+C4*DFN
420.000 404 CONTINUE
421.000 999 CONTINUE
422.000 C
423.000 C IF NO OPT. IS REQ'D THEN PRINT RESPONSE AND STOP
424.000 C
425.000 C IF(NOPT.EQ.1) GO TO 888
426.000 C IF(IPRINT.EQ.0) GO TO 603
427.000 PRINT 555
428.000 555 FORMAT('//////////')
429.000 OUTPUT(108)
430.000 OUTPUT(108) 'THE CIRCUIT RESPONSE'
431.000 OUTPUT(108) '*****'
432.000 PRINT 556
433.000 556 FORMAT('////')
434.000 PRINT 604
435.000 604 FORMAT('FS(GHZ)',4X,'GAIN(DB)',5X,'ZRF(OHMS)',15X,'ZIF(OH
MS)',
436.000 1,4X,'NOISE F(DB)',//)
437.000 603 PRINT 111,FS,GAIN,ZIN(2),ZIN(2),FN
438.000 IPRINT=0
439.000 111 FORMAT(G10.3,G11.3,2G10.3,2X,2G10.3,2X,G11.3)
440.000 888 CONTINUE
441.000 FS=FS+DF
442.000 OF=OF+(GAIN-GR)**2*C1+C2*CDABS(GAMG)**2+C3*CDABS(GAML)+C4
*FN
443.000 IF(FS.GT. FMAX) GO TO 666
444.000 GO TO 777
445.000 666 IF(NOPT.EQ.0) STOP
446.000 RETURN
447.000 END

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448.000 C
449.000 C
450.000 C
451.000 C
452.000 C
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454.000 C
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481.000 C
482.000 C
483.000 C

SUBROUTINE MAT IS USED TO SETUP THE NODAL MATRIX Y
SUBROUTINE MAT(F,CIN,Y,D)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE COMPLEX Y(50,50),CIN(50),CJ,Y11,Y12,Y21,Y22
COMMON /ONE/ E(100),PR(300),FO,FMIN,FMAX,DF,GR,N1(200),
IN2(200)
COMMON /TWO/ NE(200),NOP(200),LIS(200),NS,NODE,NP
PI=DATAN(1.D0)*4.D0
CJ=(0.D0,1.D0)

SETUP THE CURRENT VECTOR CIN

DO 10 K=1,50
CIN(K)=0.D0
DO 20 J=1,50
Y(K,J)=(0.D0,0.D0)
CONTINUE
CONTINUE

DETERMINE THE ELEMENT TYPE AND THE NODES TO WHICH IT IS
CONNECTED. COMPUTE THE Y MATRIX CORRESPONDING TO THAT
ELEMENT.

L=0.
DO 30 I=1,NS
M=E(I)
L1=L+1
L2=L+2
L3=L+3
GO TO (1,2,3,4,5,6,7)M

ELEMENT IS A TRANS. LINE

CONTINUE
Y11=-CJ/(DTAN(0*PR(L2))*PR(L1))

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*

```

484.000 Y12=CJ/(DSIN(O*PR(L2))*PR(L1))
485.000 Y21=Y12
486.000 L=L+2
487.000 Y22=Y11
488.000 GO TO 170
489.000 C
490.000 C
491.000 C
492.000 2
493.000 Y11=CJ*DTAN(O*PR(L2))/PR(L1)
494.000 Y12=Y11
495.000 Y21=Y12
496.000 Y22=Y11
497.000 L=L+2
498.000 GO TO 170
499.000 C
500.000 C
501.000 C
502.000 3
503.000 Y11=-CJ/(DTAN(O*PR(L2))*PR(L1))
504.000 Y12=-Y11
505.000 Y21=Y12
506.000 Y22=Y11
507.000 L=L+2
508.000 GO TO 170
509.000 C
510.000 C
511.000 C
512.000 4
513.000 Y11=1./PR(L1)
514.000 Y12=-Y11
515.000 Y21=Y12
516.000 Y22=Y11
517.000 L=L+1
518.000 GO TO 170
519.000 C

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520.000 C
521.000 C
522.000 5
523.000
524.000
525.000
526.000
527.000
528.000 C
529.000 C
530.000 C
531.000 6
532.000
533.000
534.000
535.000
536.000
537.000
538.000 C
539.000 C
540.000 C
541.000 7
542.000
543.000
544.000
545.000
546.000
547.000
548.000
549.000 C
550.000 170
551.000
552.000
553.000 91
554.000
555.000

ELEMENT IS A CAPACITOR
Y11=CJ*2.*PI*F*PR(L1)/1000.
Y12=-Y11
Y21=Y12
Y22=Y11
L=L+1
GO TO 170

ELEMENT IS AN INDUCTOR
CONTINUE
Y11=-CJ/(2.*PI*F*PR(L1))
Y12=-Y11
Y21=Y12
Y22=Y11
L=L+1
GO TO 170

ELEMENT IS COUPLED LINE
CONTINUE
YS=1./PR(L1)-1./PR(L2)
YD=1./PR(L1)+1./PR(L2)
Y11=-CJ*YS/DTAN(0*PR(L3))
Y12=CJ*YD/DSIN(0*PR(L3))
Y21=Y12
Y22=Y11
L=L+3
CONTINUE
NODE1=N1(I)
NODE2=N2(I)
Y(NODE1,NODE1)=Y(NODE1,NODE1)+Y11
Y(NODE2,NODE2)=Y(NODE2,NODE2)+Y22
Y(NODE1,NODE2)=Y(NODE1,NODE2)+Y12

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```

556.000      Y(NODE2, NODE1)=Y(NODE2,NODE1)+Y21
557.000      CONTINUE
558.000      NN=NODE+1
559.000      DO 567 KK=1,NN
560.000      Y(NN,KK)=0.
561.000      Y(KK,NN)=0.
562.000      CONTINUE
563.000      Y(NN,1)=1.00
564.000      Y(1,NN)=(1.00,0.00)
565.000      CIN(NN)=(1.00,0.00)
566.000      RETURN
567.000      END
568.000 C
569.000 C
570.000 C
-- X TO ABORT.

```

*X

*

```

556.000      Y(NODE2, NODE1)=Y(NODE2,NODE1)+Y21
557.000      CONTINUE
558.000      NN=NODE+1
559.000      DO 567 KK=1,NN
560.000      Y(NN, KK)=0.
561.000      Y(KK, NN)=0.
562.000      CONTINUE
563.000      Y(NN, 1)=1.00
564.000      Y(1, NN)=(1.00, 0.00)
565.000      CIN(NN)=(1.00, 0.00)
566.000      RETURN
567.000      END

```

*

```

568.000 C
569.000 C
570.000 C
571.000 C
572.000 C
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576.000 C
577.000 C
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603.000

SUBROUTINE SOLV
SUBROUTINE SOLVE IS USED TO OBTAIN A SOLUTION TO
TO A SET SIMULTANEOUS EQUATIONS
AX=B
METHOD OF SOLUTION GAUSSIAN ELLIMIN. USING THE
LARGEST PIVOT
SUBROUTINE SOLV(A,B,N,M,JPIV,II)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE COMPLEX A(50,50),B(50),PIV,PIVA,SUM
DIMENSION JPIV(50)
IF(II.EQ.2) GO TO 999
DO 4 I=1,N
JPIV(I)=I
II=II+1
IF(CDABS(A(I,I)).LE.1,D-30) GO TO 1
GO TO 15
CONTINUE
IF(I.EQ.N) GO TO 20
DO 14 J=II,N
IF(CDABS(A(J,I)).LE.1,D-30) GO TO 14
JPIV(I)=J
GO TO 16
CONTINUE
GO TO 20
DO 2 K=1,N
IPIV=JPIV(I)
PIV=A(IPIV,K)
A(IPIV,K)=A(I,K)
A(I,K)=PIV
IF(I.EQ.N) GO TO 3
DO 8 JI=II,N

```

*

```

604.000      A(I,JI)=A(I,JI)/A(I,I)
605.000      CONTINUE
606.000      DO 44 J=I1,N
607.000      DO 46 K=I1,N
608.000      A(J,K)=A(J,K)-A(J,I)*A(I,K)
609.000      CONTINUE
610.000      CONTINUE
611.000      CONTINUE
612.000      C
613.000      IFB=1
614.000      DO 61 I=1,N
615.000      IPIV=JPIV(I)
616.000      IF(IPIV.LE.I) GO TO 61
617.000      J=I
618.000      PIVA=B(I)
619.000      PIV=B(IPIV)
620.000      B(IPIV)=PIVA
621.000      JPIV(J)=-IPIV
622.000      J=IPIV
623.000      IPIV=JPIV(J)
624.000      PIVA=PIV
625.000      IF(IPIV.GT.0) GO TO 62
626.000      CONTINUE
627.000      DO 63 I=1,N
628.000      JPIV(I)=IABS(JPIV(I))
629.000      GO TO (65,165),IFB
630.000      DO 31 K=1,N
631.000      SUM=(0,DO,0,DO)
632.000      IF(K.EQ.1) GO TO 41
633.000      MM=K-1
634.000      DO 51 J=1,MM
635.000      SUM=SUM+A(K,J)*B(J)
636.000      CONTINUE
637.000      B(K)=(B(K)-SUM)/A(K,K)
638.000      CONTINUE
639.000      DO 91 LL=1,N

```

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```

640.000      K=N-LL+1
641.000      SUM=0.00
642.000      IF(K.EQ.N) GO TO 81
643.000      KK=K+1
644.000      DO 71 J=KK,N
645.000          SUM=SUM+A(K,J)*B(J)
646.000          B(K)=B(K)-SUM
647.000      CONTINUE
648.000      GO TO 30
649.000      OUTPUT (108)'MATRIX IS SINGULAR'
650.000      STOP
651.000      CONTINUE
652.000      RETURN
653.000      IFB=2
654.000      GO TO 64
655.000      165 DO 131 K=2,N
656.000          L=K-1
657.000          SUM=0.00
658.000          DO 151 J=1,L
659.000              151 SUM=SUM+A(J,K)*B(J)
660.000              B(K)=B(K)-SUM
661.000          131 CONTINUE
662.000      C
663.000      DO 191 LL=1,N
664.000          K=N-LL+1
665.000          SUM=0.
666.000          IF(K.EQ.N) GO TO 181
667.000          KK=K+1
668.000          DO 171 J=KK,N
669.000              171 SUM=SUM+A(J,K)*B(J)
670.000              181 B(K)=(B(K)-SUM)/A(K,K)
671.000          191 CONTINUE
672.000          RETURN
673.000      C
674.000      C
675.000

```

END

*


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676.000 C
677.000 C
678.000 C
679.000 C
680.000 C
681.000
682.000
683.000
684.000
685.000
686.000 1
687.000
688.000

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SUBROUTINE PRMCHG IS USED TO ICREMEMENT(ADJUST) THE
 UNCONSTRAINED VARIABLES P.

```

    SUBROUTINE PRMCHG(P,AMBDA,SIN)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION S(80),P(40)
      DO 1 J=1,N
        P(J)=P(J)+S(J)*AMBDA
      CONTINUE
      RETURN
      END

```

*

```

689.000 C
690.000 C
691.000 C
692.000 C
693.000 C
694.000 C
695.000 C
696.000 C
697.000 C
698.000
699.000
700.000
701.000 155
702.000
703.000
704.000
705.000
706.000
707.000 222
708.000
709.000 600
710.000
711.000
712.000
713.000 601
714.000
715.000
716.000 602
717.000
718.000 88
719.000 100
720.000
721.000
722.000
723.000
724.000 1

SUBROUTINE OPT IS USED TO MINIMIZE A FUNC
USING CONJUGATE GRADIENT METHOD.
P-----THE UNCONSTAINED VARIABLES
PV-----THE ACTUAL PARAMETERS BEING VARIED
OF-----THE OBJECTIVE FUNCTION
G-----THE GRADIENT VECTOR
N-----THE NUMBER OF VARIABLES
SUBROUTINE OPT(N,P,G,S,OF,LIMIT)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION P(40),G(40),S(80),PV(40)
PRINT 155
FORMAT(////////////////////)
ICOUNT=0
EST=0.
LIMIT=30.
N1=N+1
PRINT 222
FORMAT(///)
PRINT 600
FORMAT("OPTIMIZATION IS REQUIRED")
OUTPUT(108) "*****"
PRINT 222
PRINT 601,N
FORMAT("NUMBER OF VARIABLES=",I10,/)
CALL FUNC(N,P,PV,OF,G)
PRINT 602
FORMAT("OBJ. FUNC.",20X,"VARIABLES P(I),I=1,N")
WRITE(6,88) OF, (PV(I),I=1,N)
FORMAT(G8,3,4X,G10,2,3(/,12X,G10,2))
DO 99 I=1,N1
ICOUNT=ICOUNT+1
OLDF=OF
GG=0.
DO 1 J=1,N
GG=GG+G(J)*G(J)

```

```

725.000 IF(GG.EQ.0) GO TO 50
726.000 IF(I.GT.1) GO TO 3
727.000 DO 2 J=1,N
728.000 S(J)=-G(J)
729.000 GO TO 4
730.000 BETA=GG/OLDGG
731.000 DO 5 J=1,N
732.000 S(J)=-G(J)+BETA*S(J)
733.000 YB=OF
734.000 VB=0.
735.000 SS=0.
736.000 DO 6 J=1,N
737.000 JJ=JAN
738.000 S(JJ)=PV(J)
739.000 VB=VB+G(J)*S(J)
740.000 SS=SS+S(J)*S(J)
741.000 CONTINUE
742.000 IF(VB.GT.0.) GO TO 20
743.000 ALFA=2.*(EST-OF)/VB
744.000 AMBDA=1./DSQRT(SS)
745.000 IF(ALFA.LE.0.) GO TO 7
746.000 IF(ALFA**2*SS.GE.1.DO) GO TO 7
747.000 AMBDA=ALFA
748.000 ALFA=0.
749.000 YA=YB
750.000 VA=VB
751.000 CALL PRMCHG(P,AMBDA,S,N)
752.000 CALL FUNC(N,P,PV,OF,G)
753.000 PRINT 888,OF,(PV(KK),KK=1,N)
754.000 YB=OF
755.000 VB=0.
756.000 DO 10 J=1,N
757.000 VB=VB+G(J)*S(J)
758.000 IF(VB) 11,12,21
759.000 IF(YB.GT.YA) GO TO 21
760.000 AMBDA=AMBDA*ALFA

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*

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761.000 ALFA=AMBDA
762.000 IF(SS*AMBDA,LT,1.D10) GO TO 15
763.000 23 CONTINUE
764.000 RETURN
765.000 21 T=0.
766.000 13 IF(AMBDA.EQ,0) GO TO 12
767.000 Z=3.*(YA-YB)/AMBDA+VA+VB
768.000 W=DSQRT(Z**2-VA*VB)
769.000 1111 CONTINUE
770.000 ALFA=AMBDA*(VB+W-Z)/(VB+2.*W-VA)
771.000 CALL PRMCHG(P,T-ALFA,S,N)
772.000 CALL FUNC(N,P,PV,OF,G)
773.000 WRITE(6,888),OF, (PV(KK),KK=1,N)
774.000 888 FORMAT(88.3,4X,4G10.2,3(/,12X,4G10.2))
775.000 OUTPUT YA,YB,OF
776.000 IF(OF.LE.YA.AND.OF.LE.YB) GO TO 12
777.000 IF(OF.GT.YA.AND.OF.GT.YB) GO TO 50
778.000 VC=0.
779.000 DO 19 J=1,N
780.000 19 VC=VC+G(J)*S(J)
781.000 IF(VC.LE.0.D0) GO TO 18
782.000 YA=OF
783.000 VA=VC
784.000 AMBDA=ALFA
785.000 T=AMBDA
786.000 GO TO 13
787.000 18 YB=OF
788.000 VB=VC
789.000 AMBDA=AMBDA-ALFA
790.000 GO TO 21
791.000 12 IF(ICOUNT,LT,N1) GO TO 20
792.000 DO 27 J=1,N
793.000 K=J+N
794.000 IF(DABS(2.D0*(S(K)-FV(J))/(S(K)+PV(J))),GT,1.0D-2) GO TO
22
795.000 KKK=J
796.000 27 CONTINUE

```

*

```
797.000 IF(KKK.EQ.N) GO TO 50
798.000 22 IF(DABS((OF-OLDF)/(OF+OLDF)),LT,1.D-4) GO TO 50
799.000 20 OLDGG=GG
800.000 IF(ICOUNT.GT.LIMIT) GO TO 50
801.000 99 CONTINUE
802.000 GO TO 100
803.000 50 CONTINUE
804.000 RETURN
805.000 END
```

```

806.000 C
807.000 C
808.000
809.000
810.000
811.000
812.000
813.000
814.000
815.000
816.000
817.000
818.000
819.000
820.000
821.000
822.000
823.000
824.000 C
825.000 C
826.000 7
827.000
828.000
829.000 C
830.000 C
831.000 C
832.000 8
833.000
834.000
835.000 C
836.000 C
837.000 C
838.000 4
839.000
840.000 15
841.000

SUBROUTINE UPDATE(P,PV,N)
IMPLICIT REAL*8(A-H,O-Z)
COMMON /ONE/ E(100),PR(300),FO,FMIN,FMAX,DF,GR,N1(200),
1N2(200)
COMMON /TWO/ NE(200),NOP(200),LIS(200),NS,NODE,NP
DIMENSION P(40),PV(40)
DATA IF/1/
PI=DATAN(1.D0)*4.D0
DO 10 I=1,N
K=LIS(I)
L=NE(I)
MM=NOP(I)
M=E(L)
GO TO (1,1,4,4,8),M
GO TO (7,8),MM
10

DEFINE THE MIN.--A;AND THE MAX.--B OF LINE IMPED.

A=10.
B=80.
GO TO 15

DEFINE THE MIN. AND MAX. OF LINE LENGTH.

A=0.
B=360.
GO TO 15

DEFINE MIN. MAX. OF LUMPED ELMENTS

A=0.
B=1000.
CONTINUE
GO TO (16,17),IF

```

841.000 GO TO (16,17),IF
842.000 CONTINUE
843.000 P(I)=DTAN((PR(K)-(A+B)/2.)*PI/(B-A)),
844.000 PR(K)=(A+B)/2.+(B-A)/PI*DTAN(P(I))
845.000 PV(I)=PR(K)
846.000 CONTINUE
847.000 IF=2
848.000 RETURN
849.000 END

APPENDIX D

THE GRADIENT VECTOR OF THE OBJECTIVE FUNCTION

The objective function E is defined in Chapter 5 as:

$$E = \sum_{i=1}^{N_f} |S_i - S_R|^2 \quad D.1$$

where S_i = the circuit response

S_R = the desired response, and

N_f = the number of frequency points.

In this appendix, the gradient vector of the three circuit responses, reflection coefficients, noise figure, and gain will be derived.

(a) The Circuit Reflection Coefficient

In mixer design one usually knows the optimum RF and IF impedances and would like to realize these impedances from a particular circuit. The appropriate objective function is:

$$E = \sum_{j=1}^{N_f} |\rho|^2 \quad D.2$$

$$\text{where } \rho = \frac{Z_{op} - Z_L^*}{Z_{op} + Z_L} = 1 - \frac{2R_L}{Z_g} \quad D.3$$

$Z_{op} = R_{op} + jX_{op}$ = the optimum impedance

$Z_L = R_L + jX_L$ = the circuit impedance

The gradient vector can be written down as:

$$\nabla E = \sum_{i=1}^{N_f} 2 \operatorname{Real} (\rho^* \partial \rho / \partial \bar{X}) \quad \text{D.4}$$

where $\bar{X} = (x_1, x_2, x_3 \dots x_k)^T =$ the vector of variable elements.

Equation D.4 can be written in terms of the circuit impedance and its derivative as:

$$\nabla E = 2 \sum_{i=1}^{N_f} \operatorname{Re} \left\{ \left[\frac{Z_{op} - Z_L^*}{Z_{op} + Z_L} \right]^* \left[\frac{\partial R_L / \partial \bar{X}}{Z_{op} + Z_L} - \frac{R_L Z_L / \partial \bar{X}}{(Z_{op} + Z_L)^2} \right] \right\} \quad \text{D.5}$$

where $\partial R_L / \partial \bar{X} = \operatorname{Real} \partial Z_L / \partial \bar{X}$

(b) The Circuit Conversion Gain g_c

Let the required gain be g_r , then the objective function E is:

$$E = \sum_{i=1}^{N_f} |g_c - g_r|^2 \quad \text{D.6}$$

$$\text{where } g_c = \frac{4R_L R_g S_1^2 / \omega_s^2}{|(Z_{11} + Z_g)(Z_{22} + Z_L)^* - S_1^2 / \omega_s \omega_0|^2} \quad \text{D.7}$$

R_L, R_g = the real parts of Z_g and Z_L

Z_L, Z_g = the circuit IF and RF impedances, respectively

Z_{11} and Z_{22} are defined in Chapter 3.

After some tedious manipulation, the gradient vector is given by

$$\nabla E = \sum_{i=1}^{N_f} 2g_c(g_c - g_r) \text{Real} \left[\frac{\partial R_L / \partial \bar{X}}{R_L} + \frac{\partial R_g / \partial \bar{X}}{R_g} - \frac{2 D^* \partial D / \partial \bar{X}}{|D|^2} \right] \quad D.8$$

$$\text{where } D = (Z_{11} + Z_g)(Z_{22} + Z_L)^* - S_1^2 / \omega_s \omega_o \quad D.9$$

$$\partial R_L / \partial \bar{X} = \text{Real} (\partial Z_L / \partial \bar{X}) \quad D.10$$

$$\partial R_g / \partial \bar{X} = \text{Real} (\partial Z_g / \partial \bar{X}) \quad D.11$$

$$\partial D / \partial \bar{X} = (Z_{11} + Z_g)(\partial Z_L / \partial \bar{X})^* + (Z_{22} + Z_L)^*(\partial Z_g / \partial \bar{X}) \quad D.12$$

(c) The Gradient of the Noise Figure - F

In the particular case of the PDC, the noise figure is given

as:

$$E = \sum_{i=1}^{N_f} |F|^2 \quad D.13$$

$$F = 1 + R_s / R_g (1 + |Z_{11} + Z_g|^2 / |Z_{21}|^2) + |P_L|^2 / g_c \quad D.14$$

In this case the noise figure is a function Z_g , P_L , and g_c .

$$\partial F / \partial \bar{X} = \sum_{i=1}^{N_f} (\partial F / \partial R_g + \partial F / \partial Z_g + \partial F / \partial P_L + \partial F / \partial g_c) \quad D.15$$

$$\text{where } \frac{\partial F}{\partial R_g} = - \frac{R_s}{R_g^2} \left[1 + |Z_{11} + Z_g|^2 / |Z_{21}|^2 \right] \quad D.16$$

$$\frac{\partial F}{\partial Z_g} = \frac{2R_s}{R_g} \operatorname{Re} \left[\frac{(Z_{11} + Z_g)^*}{|Z_{21}|^2} \frac{\partial Z_g}{\partial \bar{X}} \right] \quad D.17$$

$$\frac{\partial F}{\partial p_L} = \frac{2}{g_c} \operatorname{Re} \left(\rho_L^* \frac{\partial}{\partial \bar{X}} \right) \quad D.18$$

where $\rho_L = \frac{Z_{out} - Z_L^*}{Z_{out} + Z_L}$, and D.19

$$Z_{out} = \frac{S_1^2 / \omega_0 \omega_s}{(Z_g + Z_{11})^*} \quad D.20$$

$$\begin{aligned} \frac{\partial \rho_L}{\partial \bar{X}} = & [(Z_{out} + Z_L) (\frac{\partial Z_{out}}{\partial \bar{X}} - \frac{\partial Z_L^*}{\partial \bar{X}}) \\ & - (Z_{out} - Z_L^*) (\frac{\partial Z_{out}}{\partial \bar{X}} + \frac{\partial Z_L}{\partial \bar{X}})] / (Z_{out} + Z_L)^*{}^2 \end{aligned} \quad D.21$$

$$\frac{\partial Z_{out}}{\partial \bar{X}} = \frac{-S_1^2 / \omega_s \omega_0}{(Z_g + Z_{11})^*{}^2} \frac{\partial Z_g}{\partial \bar{X}} \quad D.22$$

$$\frac{\partial F}{\partial g_c} = \frac{-|\rho_L|^2}{g_c^2} \frac{\partial g_c}{\partial \bar{X}} \quad D.23$$

$$= -2|\rho_L|^2 \operatorname{Re} \left[\frac{\partial \rho_L / \partial \bar{X}}{R_L} + \frac{\partial R_g / \partial \bar{X}}{R_g} - \frac{2D^* \partial D / \partial \bar{X}}{|D|^2} \right] \quad D.24$$

APPENDIX ESENSITIVITY EXPRESSIONS OF SOME LUMPED AND
UNIFORMLY DISTRIBUTED ELEMENTSE.1 Introduction

This appendix contains some of the sensitivity function G of the mostly used elements in microwave circuit design work. The tables are photocopies from Bandler et al's paper [E.1].

E.2 Tables of the Sensitivity Functions G

The tables for G are tabulated in the following pages.

TABLE II
SENSITIVITY EXPRESSIONS FOR SOME LUMPED AND UNIFORMLY DISTRIBUTED ELEMENTS

Element	Equation	Sensitivity (component of G)	Increment (component of $\Delta\phi$)
Resistor	$V = RI$ $I = GV$	II $-V\hat{V}$	ΔR ΔG
Inductor	$V = j\omega LI$ $I = \frac{1}{j\omega} \Gamma V$	$j\omega II$ $-\frac{1}{j\omega} V\hat{V}$	ΔL $\Delta \Gamma$
Capacitor	$V = \frac{1}{j\omega} SI$ $I = j\omega CV$	$\frac{1}{j\omega} II$ $-j\omega V\hat{V}$	ΔS ΔC
Transformer	$\begin{bmatrix} V_p \\ I_s \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} I_p \\ V_s \end{bmatrix}$	$V_s I_p + I_s \hat{V}_s$	Δn
Gyrator	$V = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix} I$	$I_p I_s - I_s \hat{V}_s$	$\Delta \alpha$
Voltage controlled voltage source	$\begin{bmatrix} I_p \\ V_s \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V_p \\ I_s \end{bmatrix}$	$V_p I_s$	$\Delta \mu$
Voltage controlled current source	$I = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} V$	$-V_p \hat{V}_s$	Δg_m
Current controlled voltage source	$V = \begin{bmatrix} 0 & 0 \\ r_m & 0 \end{bmatrix} I$	$I_p I_s$	Δr_m
Current controlled current source	$\begin{bmatrix} V_p \\ I_s \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} I_p \\ V_s \end{bmatrix}$	$-I_p \hat{V}_s$	$\Delta \beta$
Short-circuited uniformly distributed line	$V = Z \tanh \theta I$ $I = Y \coth \theta V$	$\tanh \theta II$ $Z \operatorname{sech}^2 \theta II$ $-\coth \theta V\hat{V}$	ΔZ $\Delta \theta$ ΔY
Open-circuited uniformly distributed line	$V = Z \coth \theta I$ $I = Y \tanh \theta V$	$Y \operatorname{cosech}^2 \theta V\hat{V}$ $\coth \theta II$ $-Z \operatorname{cosech}^2 \theta II$ $-\tanh \theta V\hat{V}$ $-Y \operatorname{sech}^2 \theta V\hat{V}$	$\Delta \theta$ ΔZ $\Delta \theta$ ΔY $\Delta \theta$
Uniformly distributed line	$V = Z \begin{bmatrix} \coth \theta & \operatorname{cosech} \theta \\ \operatorname{cosech} \theta & \coth \theta \end{bmatrix} I$ $I = Y \begin{bmatrix} \coth \theta & -\operatorname{cosech} \theta \\ -\operatorname{cosech} \theta & \coth \theta \end{bmatrix} V$	$\frac{1}{Z} V\hat{V}$ $-\frac{1}{\sinh \theta} V^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} I$ $-\frac{1}{Y} I^T \hat{V}$ $-\frac{1}{\sinh \theta} I^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \hat{V}$	ΔZ $\Delta \theta$ ΔY $\Delta \theta$
Short-circuited lossless transmission line	$V = jZ \tan \beta l I$ $I = -jY \cot \beta l V$	$j \tan \beta l II$ $jZ \sec^2 \beta l II$ $j \cot \beta l V\hat{V}$ $-jY \beta \operatorname{cosec}^2 \beta l V\hat{V}$	ΔZ Δl ΔY Δl
Open-circuited lossless transmission line	$V = -jZ \cot \beta l I$ $I = jY \tan \beta l V$	$-j \cot \beta l II$ $jZ \operatorname{cosec}^2 \beta l II$ $-j \tan \beta l V\hat{V}$ $-jY \beta \sec^2 \beta l V\hat{V}$	ΔZ Δl ΔY Δl
Lossless transmission line	$V = -jZ \begin{bmatrix} \cot \beta l & \operatorname{cosec} \beta l \\ \operatorname{cosec} \beta l & \cot \beta l \end{bmatrix} I$	$\frac{1}{Z} V\hat{V}$	ΔZ

TABLE II (Cont.)

Element	Equation	Sensitivity (component of G)	Increment (component of $\Delta\phi$)
Lossless transmission line	$I = -jY \begin{bmatrix} \cot \beta l & -\csc \beta l \\ -\csc \beta l & \cot \beta l \end{bmatrix} V$	$-\frac{\beta}{\sin \beta l} V^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} f$ $-\frac{1}{Y} I^T \hat{V}$ $-\frac{\beta}{\sin \beta l} I^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \hat{V}$	Δl ΔY Δl
Rectangular waveguide operating in H_{10} mode	as for lossless transmission line with $Z = b\lambda_0$, β replaced by $\beta_0 = 2\pi/\lambda_0$, where $\lambda_0 = \lambda/\sqrt{1 - (\lambda/2a)^2}$	$-\frac{\lambda_0^2}{4a^2} V^T \begin{bmatrix} 1 & \frac{\beta_0 l}{\sin \beta_0 l} \\ \frac{\beta_0 l}{\sin \beta_0 l} & 1 \end{bmatrix} f$ $\frac{1}{b} V^T f$ $-\frac{\beta_0}{\sin \beta_0 l} V^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} f$	Δa Δb Δl
Uniform-RC line	as for uniformly distributed line with $Z = \sqrt{\frac{R}{sC}}$ and $\theta = \sqrt{sRC}$	$\frac{1}{2R} V^T \begin{bmatrix} 1 & -\frac{\theta}{\sinh \theta} \\ -\frac{\theta}{\sinh \theta} & 1 \end{bmatrix} f$ $-\frac{1}{2C} V^T \begin{bmatrix} 1 & \frac{\theta}{\sinh \theta} \\ \frac{\theta}{\sinh \theta} & 1 \end{bmatrix} f$	ΔR ΔC
Uniform coaxial line	as for lossless transmission line with $Z = \frac{1}{2\pi} \frac{Z_0}{\sqrt{\epsilon_r}} \ln \frac{d_o}{d_i}$ and $\beta = \beta_0 \sqrt{\epsilon_r}$	$\frac{1}{d_o \ln \frac{d_o}{d_i}} V^T f$ $-\frac{1}{d_i \ln \frac{d_o}{d_i}} V^T f$ $-\frac{\beta_0 \sqrt{\epsilon_r}}{\sin \beta_0 \sqrt{\epsilon_r} l} V^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} f$ $-\frac{1}{2\epsilon_r} \left(V^T f + \frac{\beta_0 \sqrt{\epsilon_r} l}{\sin \beta_0 \sqrt{\epsilon_r} l} V^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} f \right)$	Δd_o Δd_i Δl $\Delta \epsilon_r$
Coupled lines (1) capacitance matrix description	$I = c \begin{bmatrix} C \coth \theta & -C \csc \theta \\ -C \csc \theta & C \coth \theta \end{bmatrix} V$ where $C \triangleq \begin{bmatrix} C_{01} + C_{12} & -C_{12} \\ -C_{12} & C_{02} + C_{12} \end{bmatrix}$ (see text and Fig. 11)	$-c[V_{12} V_{12}] \begin{bmatrix} \coth \theta & -\csc \theta \\ -\csc \theta & \coth \theta \end{bmatrix} \begin{bmatrix} \hat{V}_{12} \\ \hat{V}_{12} \end{bmatrix}$ $-c[V_{22} V_{22}] \begin{bmatrix} \coth \theta & -\csc \theta \\ -\csc \theta & \coth \theta \end{bmatrix} \begin{bmatrix} \hat{V}_{22} \\ \hat{V}_{22} \end{bmatrix}$ $-cV^T \begin{bmatrix} 1' \coth \theta & -1' \csc \theta \\ -1' \csc \theta & 1' \coth \theta \end{bmatrix} \hat{V}$ $-\frac{1}{\sinh \theta} I^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \hat{V}$ (see text for definitions of $1'$, 1 , and 0)	ΔC_{01} ΔC_{02} ΔC_{12} $\Delta \theta$
Coupled lines (2) even- and odd-mode description for symmetrical arrangement	$I = -j \begin{bmatrix} Y_M \cot \beta l & -Y_M \csc \beta l \\ -Y_M \csc \beta l & Y_M \cot \beta l \end{bmatrix} V$ where $Y_M \triangleq \frac{1}{2} \begin{bmatrix} Y_e + Y_o & Y_e - Y_o \\ Y_e - Y_o & Y_e + Y_o \end{bmatrix}$ and where Y_e and Y_o are the even- and odd-mode admittances	$-\frac{1}{2Y_o} V^T \begin{bmatrix} I_{1e} + I_{1o} \\ I_{1e} + I_{1o} \\ I_{1e} + I_{1o} \\ I_{1e} + I_{1o} \end{bmatrix}$ $-\frac{1}{2Y_o} V^T \begin{bmatrix} I_{1e} - I_{1o} \\ I_{1e} - I_{1o} \\ I_{1e} - I_{1o} \\ I_{1e} - I_{1o} \end{bmatrix}$ $-\frac{\beta}{\sin \beta l} I^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \hat{V}$	ΔY_o ΔY_e Δl

APPENDIX FTHE AVAILABLE NOISE POWER P_{av} DELIVERED TO THELOAD - R_L

F.1 Introduction

Consider a circuit shown in figure F.1 whose input voltage and current are related to the output voltage and current by the admittance matrix as:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{F.1.1}$$

Let V_1 and V_2 be the thermal noise voltages sources generated by a termination R_1 and R_2 (see figure F.1). The output current I_2 is expressable in terms of V_1 and V_2 as:

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \text{F.1.2}$$

The power delivered to the load R_L is given by

$$P_L = |I_2|^2 R_L \quad \text{F.1.3}$$

or
$$P_L = |y_{21}V_1 + y_{22}V_2|^2 R_L \quad \text{F.1.4}$$

$$P_L = |y_{21}|^2 R_L |V_1|^2 + |y_{22}|^2 R_L |V_2|^2 + 2 \operatorname{Real}(y_{21}y_{22}^* V_1 V_2^*) \quad \text{F.1.5}$$

Now the average power delivered to the load is

$$P_{av} = |y_{21}|^2 |V_1|^2 R_L + |y_{22}|^2 R_L |V_2|^2 \quad \text{F.1.6}$$

if V_1 and V_2 are uncorrelated.

The noise voltage V_1 and V_2 are related to the terminations R_1 and R_2 by:

$$|V_1|^2 = 4KT_1BR_1, \text{ and} \quad \text{F.1.7}$$

$$|V_2|^2 = 4KT_2BR_2 \quad \text{F.1.8}$$

where T_1 and T_2 are the thermal temperature of R_1 and R_2

K = the Boltzmann constant

B = the measurement bandwidth (Hz).

Substituting F.1.7 and F.1.8 into F.1.6 gives

$$P_{av} = 4[|y_{21}|^2 T_1 R_1 + |y_{22}|^2 T_2 R_2] KTB \quad \text{F.1.9}$$

$$\overline{V_1^2} = 4KT_1 R_1$$

$$\overline{V_2^2} = 4KT_2 R_2$$

FIG F.1 THE NOISE-FREE NETWORK WITH TWO NOISE SOURCES AT THE INPUT AND THE OUTPUT

APPENDIX H

DETAILED DERIVATION OF THE FET NOISE FIGURE

EXPRESSION

H.1 Introduction

This appendix develops in some detail equation 4.6.29 of Chapter 4. The noise figure expression is repeated here for convenience. All symbols and definitions in this appendix are the same as those of Chapter 4 section

$$F = 1 + \frac{R_m + R_s}{R_g} + \frac{|R_1 - jX_s|^2 (1 - |C_{nd}|^2) g_{gn}}{R_g} + \frac{R_s + R_d}{R_g} \left| \frac{Z_{11} + Z_g}{Z_{21}} \right|^2 + \frac{g_{dn} R_{ds}^2}{R_g |Z_{21}|^2} \left| \frac{Z_{11} + Z_g - \frac{jC(R_1 - jX_s)Z_{21}(g_{gn}/g_{dn})^{1/2}}{R_{ds}}}{Z_{21}} \right|^2 \quad \text{H.1}$$

H.2 Development of Equation H.1

The noise figure is defined by equation 4.6.28 as

H.2

$$F = \frac{\overline{i_{out}^2}}{\overline{i_{R_g}^2}} = \frac{\text{the output short-circuit current due to all noise sources}}{\text{the output short-circuit current due to the generator}}$$

The mean square current $\overline{i_{out}^2}$ is given by equation 4.6.14 as:

$$|\overline{i_{out}^2}| = \underbrace{|(V_{R_m} + V_{R_s} + V_{R_g})Y_{21}|^2}_{\text{Term 1}} + \underbrace{|(Y_{22}V_{nd} + Y_{21}V_{ng})|^2}_{\text{Term 2}} + \underbrace{|Y_{22}(V_{R_s} + V_{R_d})|^2}_{\text{Term 3}} \quad \text{H.3}$$

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The noise currents in term one are due to the thermal noise voltages of R_m , R_s , R_g in the input circuit (see fig. 4.6). The noise currents in term two of equation H.3 are due to the channel thermal noise (V_{nd}) and the induced gate noise (V_{ng}). The last term consists of the thermal noise due to R_s and R_d at the output circuit of the FET.

Upon squaring equation H.3 and noting that term 1 and term 3 are not correlated to term 2 nor to each other, the following equation results:

$$\overline{|i_{out}^2|} = (\overline{V_{R_{ms}}^2} + \overline{V_{R_s}^2} + \overline{V_{R_g}^2}) |Y_{21}|^2 + (\overline{V_{R_s}^2} + \overline{V_{R_d}^2}) |Y_{22}|^2 + \frac{|(Y_{22}V_{nd} + Y_{21}V_{ng})^2|}{|Y_{22}V_{nd} + Y_{21}V_{ng}|^2} \quad \text{H.4}$$

The last term in equation H.4 can be expressed as

$$\frac{|(Y_{22}V_{nd} + Y_{21}V_{ng})^2|}{|Y_{22}V_{nd} + Y_{21}V_{ng}|^2} = \frac{|(Y_{22}V_{nd} + Y_{21}V'_{ng} + Y_{21}V''_{ng})^2|}{|Y_{22}V_{nd} + Y_{21}V'_{ng} + Y_{21}V''_{ng}|^2} \quad \text{H.5}$$

where V'_{ng} = part of the gate noise voltage that is correlated to V_{nd} ,

V''_{ng} = the part of the gate noise voltage that is not correlated to V_{nd} .

Upon performing squaring and averaging of equation H.5, the following result is obtained

$$\overline{|Y_{22}V_{nd} + Y_{21}V_{ng}|^2} = |Y_{21}|^2 \overline{|V_{ng}''|^2} + \overline{|V_{nd}^2|} \left| \overline{Y_{22} + Y_{21} \frac{V_{ng}'}{V_{nd}}} \right|^2 \quad \text{H.6}$$

where V_{ng}'/V_{nd} = the complex correlation coefficient relating V_{nd} to V_{ng} . The term V_{ng}'/V_{nd} can be written as

$$\begin{aligned} \frac{V_{ng}'}{V_{ng}} &= \frac{\overline{V_{ng}' V_{nd}^*}}{\overline{V_{nd} V_{nd}^*}} = \frac{\overline{(V_{ng} - V_{ng}'') V_{nd}^*}}{\overline{V_{nd}^2}} \\ &= \frac{\overline{V_{ng} V_{nd}^*}}{\overline{V_{nd}^2}}; \text{ since } \overline{V_{ng}'' V_{nd}^*} = 0 \text{ (not correlated)} \end{aligned} \quad \text{H.7}$$

Let the complex correlation coefficient between V_{ng} and V_{nd} be defined as

$$C_{nd} = \frac{\overline{V_{ng} V_{nd}^*}}{\overline{V_{nd}^2}} \quad \text{H.8}$$

Equation H.4 can be expressed as:

$$\overline{i_{out}^2} = (\overline{V_{R_m}^2} + \overline{V_{R_s}^2} + \overline{V_{R_g}^2}) |Y_{21}|^2 + (\overline{V_{R_s}^2} + \overline{V_{R_d}^2}) |Y_{22}|^2 +$$

$$|Y_{21}|^2 |V_{ng}|^2 + |V_{nd}|^2 |Y_{22} + Y_{21} C_{nd}|^2 \quad H.9$$

The noise voltages V_{nd} and V_{ng} are related to the drain channel noise current and the gate induced current by equation 4.6.17 as

$$V_{nd} = R_{ds} i_{nd} = 4KT_o B g_{dn} R_{ds} \quad H.10$$

$$V_{ng} = (R_i - jX_s) i_{ng} = 4KT_o B g_{gn} (R_i - jX_s) \quad H.11$$

where $g_{dn} = g_o P$

$$g_{gn} = (\omega_s C_{gs})^2 R / g_o$$

$$X_s = 1 / \omega_s C_{gs}$$

C_{gs} = the gate to source capacitance

g_o = the fundamental Fourier coefficient

R and P are factors dependent on bias

ω_s = the RF radian frequency. The rest of the symbols are the same as in Chapter 4 section

Substituting H.10 and H.11 into H.8 gives

$$C_{nd} = \frac{(R_i - jX_s)}{R_{ds}} \left[\frac{i_{nd}^* i_{ng}}{i_{nd}^2} \right] \quad \text{H.12}$$

The correlation coefficient between i_{ng} and i_{nd} is defined in equation 4.6.5 as

$$jC = \frac{i_{nd}^* i_{ng}}{(i_{nd}^2 i_{ng}^2)^{1/2}} \quad \text{H.13}$$

Substitution of equation H.13 into H.12 gives

$$C_{nd} = \frac{jC(R_i - jX_s)}{R_{ds}} \left(\frac{G_{gn}}{g_{dn}} \right)^{1/2} \quad \text{H.14}$$

The uncorrelated gate noise voltage V_{ng}'' is related to V_{ng} by:

$$V_{ng} = V_{ng}' + V_{ng}'' \quad \text{H.15}$$

Squaring equation H.15 and taking the average results in

$$\overline{V_{ng}''^2} = \overline{V_{ng}^2} - \overline{V_{ng}'^2} = \overline{V_{ng}^2} (1 - |C_{nd}|^2) \quad \text{H.16}$$

Upon substituting equations H.16 and H.14 into H.9 the following result is obtained

$$\begin{aligned} \overline{i_{out}^2} = & (\overline{V_{R_m}^2} + \overline{V_{R_g}^2} + \overline{V_{R_s}^2}) |Y_{21}|^2 + (\overline{V_{R_s}^2} + \overline{V_{R_d}^2}) |Y_{22}|^2 \\ & + |Y_{21}|^2 (1 - C^2) \overline{V_{ng}^2} + \overline{V_{nd}^2} \left| \frac{(Y_{22} + jY_{21}C(R_i - jX_s)) \left(\frac{g_{gn}}{g_{dn}} \right)^{1/2}}{R_{ds}} \right|^2 \end{aligned} \quad H.17$$

Now, the output short circuit current due the generator termination can be written as

$$\overline{i_{R_g}^2} = \overline{|Y_{21}V_R|^2} = |Y_{21}|^2 \overline{V_{R_g}^2} \quad H.18$$

Substituting H.18 and H.17 into H.2 gives

$$\begin{aligned} F = & \frac{\overline{V_{R_g}^2} + \overline{V_{R_m}^2} + \overline{V_{R_s}^2}}{\overline{V_{R_g}^2}} + \frac{(\overline{V_{R_s}^2} + \overline{V_{R_d}^2}) |Y_{22}|^2}{\overline{V_{R_g}^2} |Y_{21}|^2} + \frac{\overline{V_{ng}^2} (1 - |C_{nd}|^2)}{\overline{V_{R_g}^2}} \\ & + \frac{\overline{V_{nd}^2}}{\overline{V_{R_g}^2} |Y_{21}|^2} \left| Y_{22} + Y_{21}C_{nd} \right|^2 \end{aligned} \quad H.19$$

The thermal noise voltages V_{R_m} , V_{R_g} , V_{R_s} , V_{R_d} , etc. are defined by the Nyquist theorem as:

$$\overline{V_{R_m}^2} = 4KTBR_m \quad H.20$$

$$\overline{V_{R_g}^2} = 4KTBR_g$$

H.21

$$\overline{V_{R_s}^2} = 4KTBR_s$$

H.22

$$\overline{V_{R_d}^2} = 4KTBR_d$$

H.23

Substituting equation H.21 to H.23 into H.19 results in equation H.1.

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