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Initial Channel Estimation and Frame Synchronization in OFDM Systems for Frequency Selective Channels

by

László Házy, B.Eng.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

December 1996

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**INITIAL CHANNEL ESTIMATION AND FRAME
SYNCHRONIZATION IN OFDM SYSTEMS FOR
FREQUENCY SELECTIVE CHANNELS**

submitted by

László Házy B.Eng

in partial fulfillment of the requirements
for the degree of

Master of Engineering

Professor Mohammed El-Tanany, thesis supervisor

Professor Samy Mahmoud, chair, Department of Systems and Computer Engineering

Carleton University

Date December 20, 1996

Abstract

In the past few years new frequency bands in the 2.5, 28 and 40 GHz region opened up, and thus interest in high speed data communications over wireless channels have largely increased. Given the transmission rates envisioned, from 1 Mb/s to hundreds of Mb/s, and the delay spread of wireless channels being of the order of microseconds, one can infer that frequency selective fading will occur. Adaptive equalization in single carrier communication systems under such channel conditions is complex. For OFDM systems the equalization conditions will be more relaxed and the involved complexity reduced.

In this thesis we address the problem of initial channel estimation and frame synchronization for OFDM systems, topics of crucial importance for the reliable operation of these communication systems. A solution based on the transmission of a pre-frame burst is proposed, and processing algorithms for both channel estimation and frame synchronization have been devised, assuming noncoherent reception, thus avoiding the need for carrier phase information.

An elaborate system simulation is carried out in order to investigate the effect of carrier frequency stability and channel characteristics on the frame timing and channel estimates. Also, error rate simulation is performed in order to analyze the effect of additive noise as well. The conclusion of our work is that the proposed synchronization and channel estimation algorithms are realizable and perform well in the presence of the above mentioned perturbations.

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List of Acronyms

AGC	Automatic Gain Control
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
DAB	Digital Audio Broadcasting
DFT	Discrete Fourier Transform
DSP	Digital Signal Processing
FDM	Frequency Division Multiplex
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
HDTV	High Definition Television
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
IFI	Inter Frame Interference
IIR	Infinite Impulse Response
ISI	Inter Symbol Interference
LMDS	Local Microvawe Distribution System
LS	Least Squares
ML	Maximum Likelihood
MMDS	Multipoint Microvawe Distrib ition System
MMSE	Minimum Mean Square Error
OFDM	Orthogonal Frequency Division Multiplex
PCR	Pulse Compression Ratio
PN	Pseudo Noise
PSK	Phase Shift Keying

List of Acronyms

P/S	Parallel to Serial
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
SAW	Surface Acoustic Wave
SER	Symbol Error Rate
SNR	Signal to Noise Ratio
S/P	Serial to Parallel
XCORR	Cross Correlation

List of Symbols

a_k	transmitted modulated symbols
a'_k	received equalized modulated symbols
$\mathcal{A}(k)$	amplitude coefficient
$\alpha(t, \tau)$	time varying attenuation
$\alpha_n(t)$	time varying attenuation of the n^{th} propagation path
b_n	transmitted M-sequence symbols
$b_n^\dagger$	periodically extended M-sequence
B_c	coherence bandwidth
B_d	Doppler spread
c_n	physical channel
$c(t)$	chirp signal
$C(\omega)$	spectrum of the chirp signal
$C_{n,k}$	n^{th} transmitted symbol on the k^{th} subcarrier
$\hat{C}_{n,k}$	n^{th} estimated symbol on the k^{th} subcarrier
d_k	transmitted data
d'_k	received data
$\delta(n)$	impulse in discrete time
$\delta(t)$	impulse in continuous time
δf	normalized swept bandwidth
$\delta f_c, \delta \omega_c$	normalized carrier frequency offset
Δa_k	symbol estimation error
Δf	swept bandwidth

List of Symbols

$\Delta f_c, \Delta \omega_c$	carrier frequency offset
Δh_n	impulse response estimation error
ΔH_k	frequency response estimation errors
e	error
$f(t)$	time varying frequency
f_0	lowest OFDM subcarrier frequency, and also the lowest frequency swept by the chirp signal
f_c, ω_c	carrier frequency in Hz and in rad/Hz
f_k	frequency of the the k^{th} OFDM subcarrier
f_n	transmit and receive filter impulse response
$F_n(t), F_n(m)$	the n^{th} transmitted OFDM frame
$F'_n(m)$	the n^{th} received OFDM frame
$F_n^\dagger(m)$	cyclically extended (prefixed) OFDM frame
$\mathcal{F}_t\{\bullet\}$	Fourier transform operator with respect to variable t
$\varphi(t)$	phase of chirp signal
$\phi(t)$	low pass equivalent phase of chirp signal
$g_k(t)$	signaling waveform associated with the k^{th} subcarrier
$h(t), h_n$	equivalent channel's impulse response
$\hat{h}(t), \hat{h}_n$	estimated impulse response
$\hat{h}_\Delta(t)$	uncoherently estimated impulse response
$h(t, \tau)$	time variant impulse response
$H(\omega), H_k$	channel's frequency response
$\hat{H}(\omega), \hat{H}_k$	estimated channel's frequency response
$\hat{H}_\Delta(\omega)$	uncoherently estimated frequency response
i, m, n, k	indices
I'_k	unequalized received modulated symbols
K	number of data bits comprised in a modulated symbol

List of Symbols

$\mathcal{K}_0(k), \mathcal{K}_1(k)$	index sets
L	number of paths, and also delay spread in symbol period units
M	number of symbols in the synchronization burst
n_n	filtered gaussian noise
N	number of OFDM subcarriers
N_0	power spectrum density of additive white noise
$\mathcal{N}(k)$	phase coefficient
$P_e(k)$	error probability on the k^{th} subcarrier
$\bar{P}_e$	average error probability over all OFDM subcarriers
P_{e0}	error probability in ideal channel
$P_h(\tau)$	multipath intensity profile
$r(t), r_n$	received signal
R	transmission rate
$\mathcal{R}$	the set of real numbers
$R_h(\bullet)$	autocorrelation function of $h(\bullet)$
$R_\Delta(t), R_\Delta(k)$	uncoherent correlation function
$\mathcal{R}_k\{\bullet\}$	autocorrelation operator with respect to variable t
$s(t), s_n$	transmitted signal
$S(\lambda)$	Doppler power spectrum
$S(\tau, \lambda)$	scattering function
SNR_0	signal to noise ratio in ideal channel
SNR_k	signal to noise ratio on the k^{th} subcarrier
σ_a^2	variance (power) of modulated symbols
σ_s^2	transmitted signal power
$\sigma_z^2, \sigma_n^2, \sigma_\zeta^2$	noise variances
t	time
t_p	time shift of the autocorrelation peak
T	duration of the synchronization burst

List of Symbols

T_c	coherence time
T_d	duration of the OFDM data frame, without prefix
T_e	symbol rate of the synchronization burst
T_F	duration of prefixed OFDM frame
$\hat{T}_F$	estimated frame timing
T_g	duration of guard interval
T_m	multipath spread
T_p	duration of the OFDM cyclic prefix
T_s	symbol period
T_w	rise/fall time
$\tau(t)$	time varying delay
$\tau_n(t)$	time varying attenuation of the n^{th} propagation path
$w(t)$	time domain shaping (window) function
W	bandwidth occupied by a signal
$x(t)$	input signal
$X(\omega)$	frequency response of $x(t)$
$y(t)$	output signal
$Y(\omega)$	frequency response of $y(t)$
z_n	complex white gaussian noise
ζ_k	frequency domain representation of n_n

Introduction

With the advent of broadband applications and the recent availability of adequate frequency bands, wireless high speed communications are becoming very attractive. Typical wireline applications and services such as multimedia and Internet access can now be provided through wireless channels. Examples of wireless broadband applications currently under development and targeting wireless access include high definition television (HDTV), local microwave distribution systems (LMDS), and two-way multipoint microwave distribution systems (MMDS). Given the large bandwidth required by these applications and the physical nature of the wireless channels, frequency selective fading can not be avoided. In such conditions classic solutions involving adaptive equalization become very complex and new approaches to solve the communication problem are needed. Orthogonal frequency division multiplex (OFDM) offers one possible solution by dividing the channel into a number of flat fading subchannels and providing a very good protection against the severe effects of multipath propagation and fading.

1.1 Thesis objectives

The above mentioned desirable properties of OFDM are heavily conditioned upon the availability of an accurate channel and frame timing estimate. There has been much

research work done on tracking the channel and the frame timing however, the problem of initial acquisition was not addressed thoroughly in the open literature.

Our goal is to study this aspect of OFDM signaling and finally to come up with a realizable and economical method to achieve initial frame synchronization and to acquire a reliable initial channel estimate. This is to be done without any prior knowledge on the channel frequency characteristics, assuming only a slow fading so that for purposes of this initial estimation the channel can be considered as time invariant. Further, the behavior of the estimates in the presence of carrier frequency offset and additive white Gaussian noise in frequency selective channels is examined.

1.2 Thesis contributions

In accordance with the objectives of this work we have developed an algorithm for initial frame synchronization and channel estimation in OFDM systems, and we have studied the effect of channel impairments and receiver frequency stability on the proposed estimates. Next, some of the major results are highlighted.

- First, we have identified the requirements on the precision of the two estimates. We determined that frame synchronization should be as accurate as one symbol period, while the requirements on the channel estimate are somewhat relaxed due to the properties of the OFDM signal, as it will be shown later.
- In order to acquire both frame timing and the channel, transmission of a pre-frame burst is proposed. We have chosen two possible implementations, one based on a chirp signal and the other based on an M-sequence.

- Study of the effect of carrier frequency offset and additive noise on the two estimates revealed that achievable performance is similar in a noise only case, while including frequency offset the chirp based synchronization burst proved to outperform the one based on M-sequence.
- Symbol error rate simulations were performed in order to investigate the degradation in system performance due to AWGN, carrier frequency offset, channel frequency profile and the particular synchronization burst. Based on these simulations, we have established that degradation is small for moderate carrier frequency offsets, and more significant for highly frequency selective channels which exhibit deep notches in their frequency characteristics. Moreover, the required overhead, and hence loss in capacity, is well within acceptable limits.

In conclusion, economical and practical initial channel and frame timing estimates have been developed for OFDM systems, estimates which are robust with additive noise and carrier frequency offset, and perform well in frequency selective channel conditions.

1.3 Thesis organization

This document is divided into six main parts, and a brief description of the contents of each is being given below.

Chapter 2 contains some background information on the topics related to the subject of the thesis: an overview of fading channels and models, an introduction to OFDM systems, and general considerations pertaining to initial channel estimation and to frame synchronization. In this chapter we also present a literature survey describing research that has already been done on the subjects of initial frame synchronization and channel estima-

tion in OFDM systems, also giving the main results of these works. The chapter concludes with the problem statement of the thesis.

In chapter 3 a detailed description of the simulation model and of the estimation algorithms are presented. The two choices of pre-frame synchronization bursts, briefly mentioned in section 1.2, are explained in detail and the properties of the two burst signals relevant to the present work are derived.

Experimental results are presented in chapter 4. Based on simulation results we evaluate the accuracy of the proposed estimates, identify conditions on their worst and best performance, and measure the overall degradation in terms of error rate performance.

We conclude our work by drawing final conclusions and by suggesting directions for further research in chapter 5.

Background Information

We provide some background information in the sections herein, covering topics as fading channels, OFDM systems, initial channel estimation, and finally frame synchronization for OFDM systems.

2.1 Fading channels

In this section we provide an overview of fading channels including commonly used mathematical descriptions. Most of the material presented here is taken from [11], [15] and [19].

We start by pointing out that an ideal channel is characterized by a constant (frequency and time invariant) attenuation and delay, and by the existence of only one propagation path. For a typical channel in mobile communications however, usually the opposite is true: there are multiple propagation paths and the parameters are both frequency and time dependent. Multipath is a phenomenon due to the existence of scatterers and reflectors in the propagation medium, and its effect is multiple replicas (echoes) of the transmitted signal being received. Time variation of the parameters is due to the movement of scatterers/reflectors, of the transmitter/receiver, and also to the time varying properties of the physical medium.

Given the above, it is common to talk about resolved and unresolved multipath depending on the time resolution employed in the communication system. It is shown that channels can be modeled as discrete for resolved multipath in which case the low pass equivalent impulse response can be written as:

$$h(t, \tau) = \sum_n \alpha_n(t) e^{-j\omega_c \tau_n(t)} \delta(\tau - \tau_n(t)) \quad (2.1)$$

where $\alpha_n(t)$ is the time varying attenuation while $\tau_n(t)$ is the time varying delay of the n^{th} path. This model is suitable for wide band communications.

Channels with unresolved multipath are being modeled as continuous and their impulse response can be written as:

$$h(t, \tau) = \alpha(t, \tau) e^{-j\omega_c \tau} \quad (2.2)$$

where α and τ are the continuous time counterparts of α_n and τ_n . This model is suitable for narrow band communications.

Both parameters (attenuation and delay) are described by random processes, and considering the usually large value of the carrier frequency $f_c = \omega_c / (2\pi)$ one can infer that small variations in the delay can produce large changes in the $\omega_c \tau$ phase term. Therefore, signal components can add up constructively or destructively in a random manner, phenomenon called fading. In many cases the attenuations and delays corresponding to different paths are uncorrelated and the fading is termed uncorrelated scattering.

A very useful and powerful description of fading channels is possible through correlation functions. We present this development according to the terminology in [15]. The ***multipath intensity profile*** (delay power spectrum) describes the average power output of the channel as function of the delay τ . Mathematically, it is formulated as:

$$P_h(\tau) = R_h(\tau, \Delta t) \big|_{\Delta t=0} = \mathcal{R}_t \{ h(t, \tau) \} \big|_{\Delta t=0} \quad (2.3)$$

where $\mathcal{R}_t\{\cdot\}$ is the autocorrelation operator with respect to t , and $R_h(\cdot)$ is the autocorrelation function of $h(\cdot)$. The outspread of the essentially nonzero region of $P_h(\tau)$ is termed the **multipath spread of the channel** (T_m).

The Fourier transform of the multipath intensity profile gives the **spaced-frequency spaced-time correlation function** of the channel:

$$\begin{aligned} R_H(\Delta f, \Delta t) &= \mathcal{F}_\tau \{ R_h(\tau, \Delta t) \} = \mathcal{R}_t \{ H(\Delta f, t) \} \\ \text{with } H(\Delta f, t) &= \mathcal{F}_\tau \{ h(t, \tau) \} \end{aligned} \quad (2.4)$$

where $\mathcal{F}_\tau\{\cdot\}$ is the Fourier transform operator with respect to the variable τ , and $H(\Delta f, t)$ is the time varying transfer function of the channel. The minimum frequency separation at which the attenuations of two frequency components become decorrelated is termed the **coherence bandwidth** (B_c), and corresponds to the essentially nonzero region of $R_H(\Delta f, \Delta t) \big|_{\Delta t=0}$. It is related to the multipath spread by the relation:

$$B_c \approx \frac{1}{T_m} \quad (2.5)$$

The previous two correlation functions gave a means of characterizing the time invariant multipath channel. The **Doppler power spectrum** characterizes the time variations of the channel and by definition:

$$S(\lambda) = S(\Delta f, \lambda) \big|_{\Delta f=0} = \mathcal{F}_{\Delta t} \{ R_H(\Delta f, \Delta t) \} \big|_{\Delta f=0} \quad (2.6)$$

Time variations in the channel affect the transmitted signal by spectral line broadening. A measure of the spectral broadening is the **Doppler spread** of the channel (B_d),

being the nonzero region of $S(\lambda)$. In the time domain we use the *coherence time* T_c to describe the same effect and the two parameters are related by:

$$T_c \approx \frac{1}{B_d} \quad (2.7)$$

In a manner similar to the definition of the coherence bandwidth, the coherence time is the minimum time separation at which the attenuations of two signals become decorrelated. It also corresponds to the nonzero region of $R_H(\Delta f, \Delta t) \big|_{\Delta f=0}$.

Finally, there is a fourth correlation function, the *scattering function*, often used to describe fading channels:

$$S(\tau, \lambda) = \mathcal{F}_{\Delta t} \{ R_h(\tau, \Delta t) \} \quad (2.8)$$

The different Fourier relationships mentioned here are summarized in Figure 2.1.

Based on the introduced correlation functions, channels can be classified as indicated in Table 2.1, where by W we denote the bandwidth occupied by the signal, and by T_s the symbol period. Flat fading channels are characterized by the same attenuation on all

Table 2.1: Channel classification

Channel type	Condition
Flat fading (frequency flat)	$B_c \gg W$
Frequency selective fading	$B_c \ll W$
Slowly fading (time-flat)	$T_c \gg T_s$
Fast fading (time-selective)	$T_c \ll T_s$

frequencies. In frequency selective fading, different frequency components receive differ-

Figure 2.1: Fourier relations between channel correlation functions

ent attenuations and pulses will broaden in the time domain. This leads to resolvable multipath with a time resolution of $1/W$, and the number of distinguishable paths is given by:

$$L = \left\lfloor \frac{W}{B_c} \right\rfloor + 1 \quad (2.9)$$

Such a channel can be modeled as a tapped delay line with time varying coefficients, as illustrated in Figure 2.2, and the mathematical model is the one in equation (2.1).

As far as the time variations are concerned, if the channel can be considered invariant for the duration of one signaling interval (usually symbol duration) the channel is

Figure 2.2: Tapped delay line model of frequency selective channels

time-flat, otherwise it is time selective. In a mobile terrestrial environment we usually deal with slow fades (time-flat channels).

It is common practice to define the *spreading factor* of the channel as $T_m B_d$, and for values larger than unity the channel is said to be overspread, and considered underspread otherwise. It can be shown that for underspread channels it is possible to choose the transmission rate such that only slow, frequency flat fading will occur.

Correcting the severe degradation in performance due to fading is possible by using diversity techniques. Basically, one should supply to the receiver multiple replicas of the signal based on the assumption that the probability of simultaneous fades of all replicas is low compared to the probability of individual fades. Then, by using an adequate combining technique, significant improvement can be achieved. One particular way to accomplish this goal is to use a wideband signal for which the channel will be frequency selective and hence multipath is resolved. Another solution is to subdivide the wideband

signal into a sufficiently large number of subcarriers so that relative to these the channel behaves as a flat fading channel. This last possibility is being exploited in OFDM systems, which are introduced in the next section.

2.2 Introduction to OFDM

The principles of OFDM have been developed about 25 years ago (see [21]), however, practical interest has increased recently, due in part to advances in signal processing and microelectronics. In the past, as well as in the present, this same modulation scheme is referred to as multi-tone, multicarrier, Fourier transform and orthogonal frequency division multiplex communication. Throughout this document we shall use the latter name and abbreviate it as OFDM.

2.2.1 The OFDM signal

In this section we proceed with a description of the OFDM signal and an OFDM communication system, the description being based on [1], [2], [9] and [16].

The main idea behind OFDM is to split the data stream to be transmitted into N parallel streams of reduced data rate and to transmit each of them on a separate subcarrier. These carriers are made orthogonal by appropriately choosing the frequency spacing between them. Based on this, spectral overlapping among subcarriers is allowed, since the orthogonality will ensure that the receiver can separate the OFDM subcarriers, and a better spectral efficiency can be achieved than by using simple frequency division multiplex. Next we give a mathematical description of the OFDM signal and we present a typical OFDM system.

In its most general form, the OFDM signal can be written as a set of data modulated carriers as follows:

$$s(t) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^{N-1} C_{n,k} g_k(t - nT_S) \right) \text{ with} \quad (2.10)$$

$$g_k(t) = \begin{cases} e^{j2\pi f_k t} & t \in [0, T_S) \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad (2.11)$$

$$f_k = f_0 + \frac{k}{T_S}, \quad k=0 \dots N-1, \text{ where} \quad (2.12)$$

- $C_{n,k}$ is the n^{th} symbol of duration T_S transmitted on the k^{th} carrier
- N is the number of OFDM carriers
- f_k is the k^{th} carrier frequency, with f_0 being the lowest frequency to be used.

We define the n^{th} OFDM frame as the transmitted signal for the n^{th} signaling interval of duration equal to one symbol period T_S , and denote it by $F_n(t)$. Thus:

$$F_n(t) = \sum_{k=0}^{N-1} C_{n,k} g_k(t - nT_S) \quad (2.13)$$

and by substituting $F_n(t)$ in equation (2.10) the relation can be rewritten as

$$s(t) = \sum_{n=-\infty}^{\infty} F_n(t) \quad (2.14)$$

where $F_n(t)$ corresponds to the set of symbols $C_{n,k}$, $k=0 \dots N-1$, transmitted on the corresponding subcarriers f_k .

Demodulation is based on the orthogonality of the carriers $g_k(t)$, namely:

$$\int_{\mathcal{R}} g_k(t) g_l^*(t) dt = T_S \cdot \delta(k-l), \quad (2.15)$$

and therefore the demodulator will implement the relation:

$$C_{n,k} = \frac{1}{T_S} \cdot \int_{nT_S}^{(n+1)T_S} s(t) g_k^*(t) dt \quad (2.16)$$

The block diagram of an OFDM modulator is given in Figure 2.3, while the demodulator is shown in Figure 2.4, where for simplicity we have ignored the filters inherent in all communication systems.

In order to point out the difference between OFDM and FDM (frequency division multiplex) let us consider the power spectral density of the two systems with binary phase shift keying (BPSK) data on all carriers. Further, let the data streams originate from one, rate R , BPSK stream through an appropriate serial to parallel (S/P) conversion. Figure 2.5

Figure 2.3: OFDM modulator

Figure 2.4: OFDM demodulator

illustrates the two spectra indicating the occupied bandwidth W as function of the number of carriers N .

From this figure one can see that the OFDM signal requires less bandwidth as the number of carriers is increased, and in the limit we have $W=R$. This is possible since there is spectral overlapping which is then resolved making use of the orthogonality of the subcarriers as stated in equations (2.15) and (2.16).

In order to make an OFDM system practical a more economical implementation of the modulator and demodulator is required, since according to Figure 2.3 and Figure 2.4 a large number of identical modulator/demodulator blocks are needed. This can be accomplished through discrete time signal processing and by making use of the filtering properties of the discrete Fourier transform (DFT). By sampling the low pass equivalent signal of (2.10), (2.13) and (2.14) at a rate N -times higher than the subcarrier symbol rate $1/T_s$, the OFDM frame can be expressed as:

$$F_n(m) = \sum_{k=0}^{N-1} C_{n,k} g_k(t - nT_s) \Big|_{t = (n + \frac{m}{N})T_s}, m=0 \dots N-1 \quad (2.17)$$

Figure 2.5: OFDM versus FDM power spectrum density

which yields

$$F_n(m) = e^{-j\pi(m+nN)} \cdot \left(\sum_{k=0}^{N-1} C_{n,k} e^{j2\pi k \frac{m}{N}} \right) = \pm (-1)^m \cdot \sum_{k=0}^{N-1} C_{n,k} e^{j2\pi k \frac{m}{N}} \quad (2.18)$$

where the $\pm$ sign depends on the parity of the $n \cdot N$ product and is usually positive since N is taken an even number (out of FFT considerations), while the $(-1)^m$ term can be

associated with a π phase shift of the odd numbered carriers. In the relations above we have considered, based on Figure 2.5, that for N large the lowest frequency in the OFDM spectrum becomes

$$\lim_{N \rightarrow \infty} f_0 = -\frac{R}{2} = -\frac{N}{2T_s} \quad (2.19)$$

where we considered the central frequency of the OFDM spectrum fixed (for example carrier frequency of the corresponding pass band signal).

By performing the sampling as indicated, the OFDM signal is subject to no loss. This is so, since as it can be observed from Figure 2.5 the two sided bandwidth of the OFDM signal (neglecting sidelobes due to the outer subcarriers) is equal to $2|f_0| = \frac{N}{T_s}$, in view of relation (2.19). Then, the sampling rate of N/T_s is exactly the corresponding Nyquist rate, and hence there will be no frequency domain aliasing. For illustrative purposes, Figure 2.6 shows the typical power spectrum of an OFDM signal, where the frequency axis is normalized relative to the inter carrier spacing, $1/T_s$.

In conclusion, up to the before mentioned aspects and to a constant multiplying factor of $1/N$, the sampled OFDM frame can be generated using an inverse DFT (modulation function), and hence the transmitted data can be recovered from the OFDM frame through DFT (demodulation function). A block diagram of the digital OFDM system employing DFT is given at the end of the next section (Figure 2.8), after discussing the need for a cyclic prefix.

Figure 2.6: Typical power spectrum of the OFDM signal

2.2.2 Introduction of a cyclic prefix

When a signal $s(t)$ is passed through a channel with impulse response $h(t)$ the received signal is given by the convolution:

$$r(t) = h(t) * s(t), \quad (2.20)$$

and if the channel is not ideal, there will be inter symbol interference (ISI). It is convenient to view the OFDM signal in terms of data frames, so we can appreciate that the channel will produce ISI within the frame and will also produce inter frame interference (IFI). Considering the discrete time equivalent signal and the channel h_i , $i=0 \dots L$, L being the delay spread of the channel, relation (2.20) becomes

$$r_m = \sum_{i=0}^L h_i \cdot s_{m-i} \quad (2.21)$$

Figure 2.7 shows this convolution sum for the particular case of $L=2$. From this graphical representation it can be seen that introducing a guard interval between two adjacent frames, of length equal to the delay spread of the channel L , will "absorb" the channel delay and hence remove IFI. This may be accomplished by inserting L leading zeros in each frame at the transmitter and removing them at the receiver. However, in order to also eliminate ISI from within the frame, it is better to use a cyclic prefix instead of an all zero guard interval, in which case after dumping the prefix at the receiver one would get the periodic (cyclic) convolution of the transmitted data frame and the channel. The cyclically extended frame can then be written as

$$F_n^\dagger(m) = \begin{cases} F_n(N+m) & , m=L \dots -1 \\ F_n(m) & , m=0 \dots N-1 \end{cases} \quad (2.22)$$

Figure 2.7: Inter Frame Interference in OFDM systems

where

$$F_n(m) = \sum_{k=0}^{N-1} C_{n,k} e^{j2\pi k \frac{m}{N}}, \quad m=0 \dots N-1 \quad (2.23)$$

The received frame after discarding the prefix becomes

$$F'_n(m) = \sum_{i=0}^{N-1} F_n(m-i)_N \cdot h_i \quad (2.24)$$

where $(m-i)_N$ represents the modulo N subtraction, and after DFT the relation gives

$$\hat{C}_{n,k} = \frac{1}{N} \cdot \sum_{m=0}^{N-1} F'_n(m) e^{-j2\pi k \frac{m}{N}} = C_{n,k} \cdot H_k, \quad k=0 \dots N-1 \quad (2.25)$$

where H_k is the channel's transfer function at the subcarrier frequency f_k from relation (2.12). Therefore, by using a cyclic prefix the effect of the channel is transformed into a complex multiplication of the data symbols with the channel coefficients H_k , and all ISI and IFI is removed. In view of these, the complete block diagram of an OFDM system is as shown in Figure 2.8.

2.2.3 Properties of OFDM

After having introduced the OFDM signaling scheme in the previous section, we give here its major advantages and disadvantages.

Advantages of OFDM signaling:

- Makes efficient use of the spectrum by allowing overlap.

Figure 2.8: OFDM communication system

- By dividing the channel into narrowband flat fading subchannels, OFDM is more resistant to frequency selective fading than single carrier systems are.
- Eliminates ISI and ICI through use of a cyclic prefix.
- Using adequate channel coding and interleaving one can recover symbols lost due to the frequency selectivity of the channel.
- Channel equalization becomes simpler than by using adaptive equalization techniques with single carrier transmissions.
- It is possible to use maximum likelihood decoding with reasonable complexity, as discussed in [2].
- OFDM is computationally efficient by using FFT techniques to implement the IDFT and DFT functions. Also, for multiple communication channels, as is the case in digital audio broadcasting (DAB) systems, partial FFT algorithms can be used in order to implement program selection and decimation.

- In conjunction with differential modulation there is no need to implement a channel estimator.
- It is less sensitive to sample timing offsets than single carrier systems are.
- Provides good protection against cochannel interference and impulsive parasitic noise (see [7]).

In terms of drawbacks we mention the following:

- The OFDM signal has a noise like amplitude with a very large dynamic range, therefore it requires RF power amplifiers with a high peak to average power ratio.
- It is more sensitive to carrier frequency offset and drift than single carrier systems due to leakage of the DFT.

2.3 Introduction to channel estimation

In this section we make some general considerations regarding initial channel estimation (acquisition) in digital communications and in particular for OFDM systems. Then, in Chapter 3 the proposed estimation algorithm is described in detail. For the purpose of this section the receiver is assumed to be perfectly synchronized (both data and carrier synchronization).

In order to estimate the channel, transmission of a training signal known to the receiver is required. Based on the nature of this training signal, there are two possibilities we may consider for classifying channel estimation approaches, thus having:

1. Noise like signals of which autocorrelation properties are exploited, and
2. “*Deterministic*” signals with advantageous spectral and/or autocorrelation properties.

Secondly, regarding the position the training signal occupies relative to the data symbols we have:

- Overhead type training sequences transmitted between data organization units such as frames, and
- Sequences embedded into the data transmission so that certain transmitted symbols are reserved for channel estimation purposes instead of data.

In the first case white noise like signals are of interest since their autocorrelation function will approximate an impulse, and a correlator receiver can be used to estimate the channel according to the block diagram of Figure 2.9 and the relations:

$$y(t) = h(t) * x(t) \quad \text{and} \\ \hat{h}(t) = R_{yx}(t) = \int_{\mathcal{R}} y(t+\tau) x(\tau) d\tau = \int_{\mathcal{R}} h(t-\nu) R_x(\nu) d\nu = h(t) * R_x(t) \quad (2.26)$$

Since the autocorrelation $R_x(t) \approx \delta(t)$, the correlator output approximates the desired estimate. For theoretical purposes white noise may be considered, while for practical implementations M-sequences are suitable. A typical periodic (cyclic) autocorrelation function of a length M M-sequence is shown in Figure 2.10, where for large M the side-lobes are negligible and hence $R_x(n) \approx \delta(n)$.

Figure 2.9: Channel estimation using noise like signals

Figure 2.10: M-sequence normalized autocorrelation function

In the second category belong those signals which have a flat amplitude spectrum in the channel's bandwidth. In this case only a phase correction at the receiver is required in order to remove the phase spectrum of the training signal. The block diagram is shown in Figure 2.11 and the mathematical model is given by the relations

$$\begin{aligned} Y(\omega) &= H(\omega) X(\omega) \quad |X(\omega)| = \text{constant} \\ \hat{H}(\omega) &= Y(\omega) \cdot e^{-j \cdot \arg \{X(\omega)\}} = H(\omega) \cdot |X(\omega)| \end{aligned} \quad (2.27)$$

Of course, a correlator receiver can be used as well, and in this case the equation becomes

Figure 2.11: Channel estimation using signals with a flat amplitude spectrum

$$\hat{H}(\omega) = H(\omega) \cdot |X(\omega)|^2, \quad (2.28)$$

which only changes the constant multiplicative factor.

As far as the position of the training signal is concerned, we list some of the pros and cons of the two cases. For overhead type signals we have the following:

- previous data synchronization is not required
- the estimated channel is used only later for equalization, therefore it is not adequate for fast varying channels,

while with training signals embedded into the data frame:

- previous data synchronization is required
- "on-fly" channel estimation is performed, which allows to have equalization done in the same time with channel estimation
- interpolation formulas for the estimate have to be implemented which leads to increased complexity.

In the special case of OFDM the data is transmitted in frames. For an embedded estimator pilot tones can be used with a modulation of the reserved carriers known to the receiver, while with an overhead type estimator an adequate pre-frame burst is transmitted. In both cases the loss in capacity is of concern.

2.4 Frame synchronization for OFDM

The aim of this section is to point out precision requirements on the frame timing estimate, and to investigate on the problem of choosing the nominal frame timing reference.

It should be mentioned from the very beginning that channel estimation and frame synchronization for OFDM systems are closely related, and current algorithms assume knowledge of one of them in order to estimate the other one.

Here we do some qualitative appreciations concerning the frame timing estimate, and will support the results later in Chapter 4 with simulation results. Figure 2.12 presents two possible choices for the nominal frame timing for an OFDM system with frame length of N symbols, and a cyclic prefix of L symbols.

Considering the case when the nominal frame timing reference is chosen in the middle of the prefix (Figure 2.12-a) one will recover a cyclically time shifted frame,

a) Robust nominal frame timing

b) Sensitive nominal frame timing

Figure 2.12: Choosing the nominal OFDM frame timing reference

delimited in the figure with the A marks, and the receiver can compensate for this known time shift. If the channel estimate is already available, relative to the nominal frame timing position, errors larger than one symbol period will result in uncompensated cyclical time shift, which in frequency domain translates in rotation of different OFDM subcarriers by different amounts. This effect will spread each constellation point along a circle centered in the origin, leading thus to irrecoverable loss of the data. On the other hand, when channel estimation is done in conjunction with frame timing estimation by means of a common algorithm, the situation is changed. As far as the frame timing error combined with the length of the channel impulse response does not exceed $L/2$, the estimated frame still contains the correct data. Further, any frame timing error will translate into an appropriate time shift of the channel which is always estimated relative to the actual estimated frame timing, and therefore no performance degradation will occur.

Considering the case when the nominal frame timing reference is chosen at the end of the prefix (Figure 2.12-b) no time shift of the recovered frame occurs. Regardless of whether independent or simultaneous (constrained) channel and frame timing estimation is implemented, for positive frame timing errors in excess of one symbol duration performance degradation will occur due to interference with the next OFDM frame. Symbols from the next frame's cyclic prefix enter the current frame and therefore data will be lost.

In conclusion, a choice of the nominal frame timing reference as in Figure 2.12-a will give more robustness to frame timing estimation errors.

2.5 Literature review

Given that each of the works to be presented considers a different approach to the channel estimation and/or frame synchronization problem, we do our literature survey on a per reference basis.

A low complexity, low overhead, fast synchronization method is presented in [18]. A flat, slowly fading channel is assumed along with the availability of an initial channel estimate, and estimators for carrier frequency offset and for frame and symbol timing are developed. The method relies on the transmission of two consecutive training frames with special data as follows: a pseudo noise (PN) sequence is transmitted in the first frame on even carriers and zeros on the odd ones, and the same PN sequence is transmitted in the second frame on odd frequencies. Also, on the even carriers of the second frame another PN code is sent, and the first frame is arranged to have two identical halves. Frame synchronization is roughly estimated by searching for a particular frame containing zeros on odd frequencies (first training frame), where the symmetry introduced into the frame allows to mitigate the effect of carrier frequency offset. Then, carrier frequency offset is corrected based on correlation with the second frame, and finally frame timing is fine tuned giving the symbol timing. Also, updating the channel estimate is possible: on even frequencies through the first frame and on odd ones through the second frame.

A frame synchronization method for OFDM/FM over mobile radio channels in a pure ALOHA environment is given in [20]. Assuming a known channel which is considered being compensated for, the estimation is based on the use of a certain number of reserved OFDM carriers on which synchronization data is transmitted. A three step algorithm is devised:

- First, transmission is detected by measuring the incoming signal power.

- A coarse synchronization is achieved by correlating the data on the extracted reserved carriers with the locally generated training sequence.
- Fine tuning is performed by finding an optimum estimator for small timing errors.

The described synchronization requires a relatively low overhead (loss of capacity) of approximately 10%, and yields bit error rate (BER) performance within 1.5 dB from perfect estimation.

Channel estimators for tracking mode operation are developed in [3]. A slowly fading channel with known statistics is considered, and minimum mean square error (MMSE) and least squares (LS) estimates are derived based on the known channel autocorrelation function $R_h(\tau)$. In order to reduce the computational complexity of the MMSE a modified algorithm is presented which also improves performance of the LS estimate. In either case a training sequence is being transmitted on the OFDM subcarriers and an already synchronized correlator receiver implementing one of the estimators is assumed. The authors conclude by recommending the LS algorithm for high signal to noise ratio (SNR) conditions, and the modified MMSE and LS for low SNR as a compromise between complexity and performance.

Estimating small frame timing errors is shown to be possible without any additional overhead in [12]. By investigating the joint probability density function (pdf) of the in-phase and quadrature data components on each carrier, it is proved that a gaussian distribution is a good fit for the error range under consideration: $(-T_s, +T_s)$. Using the mean value of this distribution, which is dependent on the actual frame timing error, a ML estimate of the frame timing is computed for QPSK and 16-QAM base modulations.

Due to its simplicity, a very popular channel estimation algorithm is the one proposed in [10], where transmission of pilot tones embedded in the data frame is suggested.

Since obviously frame synchronization is a prerequisite in order to recover the frame, this method would work as a channel tracking algorithm.

Finally, we mention the MMSE channel estimate developed in [5] for frequency selective fading. The idea behind this approach is to update a channel vector $\hat{\mathbf{H}}$ available from a previous step through the computation of an innovation vector $\Delta \hat{\mathbf{H}}$ by means of an adaptive algorithm (Kalman filter). This method is however, only applicable to channel tracking since an initial channel estimate is required by the adaptive algorithm.

2.6 Problem statement

As it results from the previous section, there have been good results achieved in both channel estimation and frame synchronization. However, none of the already known methods is suitable for initial acquisition since they all rely on either an available channel estimate, as is the case in [12], [18], [20], or at least its statistics as in [3], or an available frame timing (see references [5] and [10]).

In this context, we have the problem of initial acquisition of the channel and of the frame timing, with the additional requirement of the estimators having good performance in the presence of carrier frequency offset. This is to be accomplished without any prior knowledge on the channel, only assuming a slow fading so that the channel can be considered as time invariant for initial estimation purposes.

Thus, we would acquire both the channel and the frame timing in noncoherent conditions making frequency offset estimation and correction a later operation, which we do not deal with in this thesis since it has already been studied and solutions are known (see

[7], [8] and [18]). Then, one of the presented algorithms may be used for tracking the channel and the frame synchronization.

The problem just identified in the previous paragraph is of major concern in any communication system, since:

- Any error in the initial estimates will usually propagate unless an adaptive algorithm capable of correcting moderate initial errors is used. Large errors will, however, make the system downright unoperational: training sequences become distorted to a level the receiver cannot recognize them, thus causing all tracking structures to fail.
- As it results from relation (2.25), an imperfectly equalized channel in OFDM systems will lead to significant distortion of the received data through rotation of the signal constellation and through attenuation. Left uncompensated, these will result in serious performance degradation, and in order to overcome this problem we need a reliable channel estimate.
- In OFDM systems it is important to have frame alignment within one symbol interval¹ (as shown later), otherwise severe inter symbol and/or inter frame interference are inherent, leading to irrecoverable loss of data.

In the remainder of this document appropriate estimation algorithms are developed and their performance is analyzed in the presence of additive white gaussian noise and carrier frequency offset.

¹ This consideration only applies when channel estimation and frame synchronization are achieved independently from each other. When an algorithm that estimates the channel and the frame timing simultaneously is used, the timing precision of one symbol interval can be relaxed by proper choice of the nominal frame timing position.

Simulation Model

The purpose of this chapter is to present the functional block diagram of the simulated OFDM system along with the required analytical description, to present in detail the proposed channel and frame timing estimation algorithms, and to derive an expression for the error probability for OFDM given a frequency selective channel. Then, Chapter 4 gives the results of computer simulation.

3.1 Model of the OFDM system

The general block diagram of the considered OFDM system is given in Figure 3.1, where the carrier frequency estimation and tracking functions, as well as the frame timing and channel tracking functions have been neglected. Four blocks can be identified in the figure: OFDM transmitter, OFDM receiver (similar to the ones shown in Figure 2.8), the channel, and an additional channel estimation and frame synchronization circuit. Next we shortly characterize these components.

At the *transmitter* the data source supplied symbols d_k are applied to the baseband modulator which maps K data bits into the modulated symbols a_k , chosen from a QPSK or a 16-QAM constellation. Through serial to parallel conversion (S/P) N symbols are simul-

taneously forwarded to the OFDM modulator, implemented by IDFT as explained in section 2.2. At the output of the OFDM modulator we have:

$$F(n) = \frac{1}{\sqrt{N}} \cdot \sum_{k=0}^{N-1} a_k e^{j2\pi k \frac{n}{N}} \quad n=0 \dots N-1, \quad (3.1)$$

where the symbols $F(n)$, $n=0 \dots N-1$ form an OFDM frame, and N is the number of OFDM carriers, equal to the size of the OFDM frame. The scaling factor in the expression of the IDFT has been modified in order to allow for conservation of power. Adding a cyclic prefix of length L symbols, the OFDM signal (frame) becomes

$$F^\dagger(n) = \begin{cases} F(n+N) & , n=L \dots N-1 \\ F(n) & , n=0 \dots L-1 \end{cases} \quad (3.2)$$

Numerically, in a typical micro cellular environment delay spreads of about $T_m=1 \dots 1.5 \mu s$ can be encountered, while targeted data rates are roughly in the range of $1/T_s=10$ Mbaud/s. This means a required prefix length of at least $L=T_m/T_s=10-15$ symbols and we chose $L=20$ allowing for a safety margin against frame timing errors, and also for more room for the typical exponential delay profiles.

The synchronization burst transmitter of Figure 3.1 will be discussed later, in sections 3.2 and 3.3, in terms of signal structure and properties.

For mathematical purposes we regard the *channel* as incorporating the transmit and receive filters, which are chosen to be identical. Square root raised cosine filters with roll-off of 50% have been employed, and the filters have been implemented as causal FIR filters with coefficients f_n . The physical channel is represented through a FIR filter with coefficients c_n . However, the maximum delay spread should be limited to the value L mentioned above. Complex additive white gaussian noise with variance σ_z^2 is also considered

ered by discrete time implementation z_n . Therefore, the received signal and the equivalent channel is described by the following equations:

$$r_n = h_n * s_n + z_n * f_n = h_n * s_n + n_n \quad (3.3)$$

$$\text{with } h_n = f_n * c_n * f_n \quad (3.4)$$

$$\text{and } n_n = z_n * f_n, \quad (3.5)$$

where h_n is the impulse response of the equivalent overall channel incorporating the physical channel and both transmit and receive filters, while n_n is the equivalent noise seen by the digital receiver. It should also be noted that interpolation is performed through filtering, resulting in a higher data rate at point R in the diagram than at point A.

At the *receiver* the data is down sampled to the symbol rate ($1/T_s$) yielding at point B the same rate as at A. Assuming the frame timing and the channel estimate are available, the processing is done as described below. Based on the frame timing estimate $\hat{T}_F$, the cyclically extended frame is formed followed by removal of the prefix, resulting in the received OFDM frame $F'(n)$. Then OFDM demodulation is performed via DFT (see section 2.2):

$$I'_k = \frac{1}{\sqrt{N}} \cdot \sum_{n=0}^{N-1} F'(n) e^{-j2\pi k \frac{n}{N}} \quad , k=0 \dots N-1. \quad (3.6)$$

This is followed by zero forcing channel equalization (compensation) based on the channel estimate $\hat{H}_k$ corresponding to the k^{th} carrier, obtaining:

$$a'_k = I'_k \cdot \hat{H}_k^{-1} \quad (3.7)$$

Finally, the symbols are demodulated and the estimate d'_k of the transmitted data d_k is obtained.

It can be easily shown that for uncorrelated symbols s_n , the SNR at point B is given by the relation

$$\text{SNR} = \frac{P_s}{P_n} = \frac{\sigma_s^2 \cdot \sum_n |h_n|^2}{\sigma_z^2 \cdot \sum_n |f_n|^2}, \quad (3.8)$$

which after normalizing the transmitter output power ($\sigma_s^2 = 1$), and the filters f_n ($\sum_n |f_n|^2 = 1$) and h_n ($\sum_n |h_n|^2 = 1$), yields

$$\text{SNR} = \frac{1}{\sigma_z^2}. \quad (3.9)$$

3.2 Channel estimation and frame synchronization

In the previous section we mentioned the presence of an auxiliary block in Figure 3.1, the channel estimation and frame timing recovery circuit. Here we present the inner structure of this circuit and describe the proposed estimation algorithms.

In order to accomplish our goal a synchronization burst generator is needed at the transmitter and more details regarding its particular choice will be discussed later in this chapter. We propose the transmission of this signal as an overhead preceding the cyclically extended frame and a generic term of synchronization burst will be used, although it will allow for both channel estimation and frame synchronization. Figure 3.2 presents this structure.

G = guard interval

M = number of symbols in the synchronization burst

Figure 3.2: Overhead with a synchronization burst

A guard interval of duration T_g has been added to both ends of the burst in order to remove the interference, from data frames to the received synchronization signal, interference due to the delay spread of the channel. Therefore, if no other considerations apply, a good choice is given by $T_g = L \cdot T_s$.

At the receiving end the synchronization burst is processed to give the estimates $\{\hat{H}_k\}$ and $\hat{T}_F$. Figure 3.3 shows the block diagram of the channel estimation and frame synchronization circuit, and a description follows. For the proper operation of this block the existence of an AGC circuit which keeps the input power level of y_n at a quasi-constant value is assumed.

The synchronization burst is sampled at rate $1/T_g$ not necessarily the same with the data rate $1/T_s$. The received signal y_n is first down sampled to the appropriate $1/T_g$ rate and then correlation is performed with the locally generated replica of the burst. It is possible to implement this part (shaded on the diagram) in continuous time by using the corresponding matched filter, available for certain type of signals (for example chirp). Since the impulse response has to be estimated for a rate $1/T_s$ data transmission, the output of the correlator is down sampled to this rate (the symbol rate). Considering that the synchroni-

Figure 3.3: Block diagram of channel estimation and frame synchronization

zation burst is chosen to be adequate for channel estimation (as described in section 2.3), the channel estimate up to a yet unknown delay is available after the correlator. This time shift is then estimated by the remaining circuits.

The input buffer implements a sliding time window of length equal to the maximum duration of the channel impulse response. It extracts a possible channel corresponding to the current timing, while the energy computation block and the double threshold detector will decide on the “validity” of it. A typical energy plot versus time at the input of the double threshold detector is given in Figure 3.4. One can notice the flat, high level por-

Figure 3.4: Typical energy plot at the input of the double threshold detector

tion corresponding to a synchronization burst being detected and hence the channel estimated, and the two small valued side regions corresponding to the previous and following OFDM data frames. The flat region and the two side regions are separated by a rising and a falling ramp. The double threshold device will detect the two ramps or equivalently, will detect the presence of the burst. Therefore, the signal labeled ACQ/FAIL in Figure 3.3 may be used as a success/failure indicator validating $\{\hat{H}_k\}$ and $\hat{T}_p$ estimates.

The energy plot does not always have a very flat top (especially true for large delay spreads and/or large noise), therefore a peak detector is used to find the highest value encountered. Each time the instantaneous energy exceeds the previous maximum, the contents of the input buffer is transferred to the current channel buffer, thus having available the most plausible channel up to that point in time. The advantage of this channel estimate

is given by the noise reduction achieved through the front-end correlator. For this reason at high enough SNR values we may consider that only the channel's impulse response is present at the correlator output, since correlation between the noise and the synchronization burst is reduced.

Most synchronization bursts do not have an ideal¹ autocorrelation function, meaning that there might be perturbing sidelobes. If this is the case, false impulses can appear in the estimated impulse response and one would have to eliminate these in order to preserve the estimator's quality. This can be simply achieved by discarding all channel components below a certain small threshold level, considering that inadvertently removed "true" impulses will have little impact given the robustness of the OFDM system to these type of errors, as it is supported with simulation results in Chapter 4. On the other hand, when considering noise also, low level components in the estimated impulse response are more prone to errors and by removing them the negative effects of the noise on the channel estimate can be minimized. As will be seen later, further side lobe degradation occurs due to carrier frequency offset. Therefore, we conclude that removal of the low level components of the estimated channel impulse response is usually required.

The final channel estimate $\{\hat{H}_k\}$ is obtained after an N-point DFT (N equal to the number of subcarriers), thus obtaining the estimated channel transfer function at frequencies corresponding to the OFDM subcarriers.

As far as the frame timing estimate $\hat{T}_p$ is concerned, the output of the peak detector can directly be used to control the receiver's S/P converter. Each time the peak detector's output is set, a new, more plausible channel estimate has been found corresponding to the delay-search done by the energy computing block and the double threshold detector,

¹ In the context of channel estimation an ideal autocorrelation function is an impulse, since by convolving this with the channel impulse response would result in the channel impulse response.

thus the S/P converter's contents should be updated. Since $\hat{T}_F$ is derived based on the energy content of the estimated channel, it will only be dependent on carrier frequency offset induced attenuations, which do occur as shown in the next sections.

In the following we summarize the computational procedure:

- the received signal is down sampled to a rate equal to the corresponding sampling rate of the synchronization burst, $1/T_e$
- correlation with the locally generated replica of the synchronization burst is performed
- the resulting signal is down sampled to the data rate, $1/T_s$, resulting in a time shifted version of the channel's impulse response
- with the help of a sliding time window of duration equal to the maximum channel delay, the energy contained in the "windowed" impulse response is computed
- the frame timing is found where the maximum of this energy is encountered, provided this maximum is above a certain predefined threshold value
- the signal within the time window producing the energy peak is considered as being the estimated impulse response
- low level values in the estimated impulse response are removed since they are unreliable when additive noise and carrier frequency offset are present
- an N-point DFT is performed in order to obtain the estimated frequency response at the OFDM subcarrier frequencies.

3.3 Synchronization burst analysis

In this section we derive some of the relevant properties of the two synchronization signals considered for channel and frame timing estimation: chirp signal and M-sequence. To be specific, only autocorrelation and spectral properties are of concern.

3.3.1 The chirp signal

The choice of a chirp as a synchronization burst is based on its well established reputation in radar technology and on its known useful properties. It is also of interest the availability of solid state matched filters at low cost (SAW devices) which would allow for such an implementation of the front-end correlator from Figure 3.3, rather than by using a digital signal processor (DSP).

The chirp signal is characterized by a frequency ramp modulation as shown in Figure 3.5. Therefore, the instantaneous frequency and phase can be written as

$$\frac{1}{2\pi} \cdot \frac{d}{dt} \varphi(t) = f(t) = f_0 + \frac{\Delta f}{T} \cdot t \quad , t \in [0, T] \quad (3.10)$$

$$\text{and} \quad \varphi(t) = 2\pi f_0 t + \pi \frac{\Delta f}{T} \cdot t^2 + \varphi_0, \quad (3.11)$$

where φ_0 is an arbitrary initial phase, Δf is the swept bandwidth, and the product $\Delta f \cdot T$ is known as the pulse compression ratio (PCR) in radar. The bandpass chirp signal may therefore be expressed as

Figure 3.5: Frequency ramp modulation of the chirp signal

$$s(t) = \cos[\varphi(t)], \quad (3.12)$$

and the low pass equivalent for a carrier frequency of f_c becomes:

$$c(t) = e^{j[\varphi(t) - 2\pi f_c t]}. \quad (3.13)$$

By choosing f_c in the middle of the swept bandwidth, $f_c = \frac{f_0 + f_1}{2}$, the low pass equivalent of relation (3.11) becomes (with $\varphi_0 = 0$)

$$\varphi(t) = -\pi \cdot \Delta f \cdot t + \pi \cdot \frac{\Delta f}{T} \cdot t^2, \quad t \in [0, T], \quad (3.14)$$

and by sampling with rate $1/T_e$ the discrete time equation is

$$\varphi(n) = \varphi(nT_e) = -\pi \cdot \delta f \cdot n + \pi \cdot \frac{\delta f}{M} \cdot n^2, \quad n \in [0, M-1], \quad (3.15)$$

where $\delta f = \Delta f \cdot T_e$ is the normalized swept bandwidth and M the number of samples. In this case the pulse compression ratio is given by $PCR = M \cdot \delta f$. Figure 3.6 gives the typical spectrum of the chirp signal also indicating the margins of the swept bandwidth. The frequency axis is normalized to the sampling rate of the synchronization burst.

In order to use the chirp for channel estimation purposes we have to consider its power spectrum, defined as the Fourier transform of the autocorrelation function. The mathematical expression is given later, when we consider the effect of carrier frequency offset on the autocorrelation function and on the power spectrum. A typical power spectrum plot is shown in Figure 3.7.

One can see that the spectrum is not flat, there being significant ringing which will perturb the channel estimation. A possible counter measure is to stretch the spectrum to a grade where it becomes sufficiently flat in the channel's bandwidth. Nevertheless, besides

Figure 3.6: Spectrum of the chirp signal

Figure 3.7: Chirp signal power spectrum

of being impractical, a significant amount of power would be lost, since only a very narrow central part of the spectrum would be useful for channel estimation purposes. A much better solution arises by observing that there is a maximum delay spread L we are dealing with. As a result, only the first $L \cdot \frac{T_s}{T_c}$ samples around both sides of the origin of the chirp signal's autocorrelation are involved in channel estimation, and thus it is reasonable to investigate the power spectrum given by these samples only. The result is shown in Figure 3.8. There is a noticeably flat region useful for channel estimation however, a still too large extra bandwidth and significant ringing is present, the latter being of concern in presence of carrier frequency offset. In order to strongly reduce the spectral ringing and therefore extend the useful frequency range as well, a time window has been applied to the chirp signal. In its natural form in relation (3.13) an implicit rectangular window (envelope) is assumed. By trial and error a satisfactory window has been found the shape of which is given by the relation

Figure 3.8: Power spectrum after time domain truncation

$$w(t) = \begin{cases} \sin\left(\frac{\pi}{2} \cdot \frac{t}{T_w}\right) & , t \in [0, T_w) \\ 1 & , t \in [T_w, T - T_w] \\ \sin\left(\frac{\pi}{2} \cdot \frac{T-t}{T_w}\right) & , t \in (T - T_w, T] \end{cases} \quad (3.16)$$

and represents a square root Hanning weighting of the discontinuities in the original rectangular window. Figure 3.9 presents the two windows, and Figure 3.10 shows the resulting power spectrum. Also drawn is the channel's frequency range, represented by the inner pair of lines. We have obtained this power spectrum by taking into account the central part of the autocorrelation function only, since as previously explained only these samples are involved in channel estimation. It should be noted, however, that the same power spectrum is obtained if we consider the full length autocorrelation function instead of the "truncated" one, and therefore the weighting proves very useful by eliminating the ringing almost completely. In any case, a maximum ringing of 0.1 dB in the channel's bandwidth is achieved by using the chirp signal parameters as specified at the end of this section. This is important because the described procedure becomes applicable even if the channel is characterized by a large delay spread.

Figure 3.9: Rectangular and square root Hanning weighting

Figure 3.10: Power spectrum after time domain shaping and truncation

It is of interest to study the correlation function of the chirp signal $c(t)$ with its frequency shifted version in order to evaluate the effect of carrier frequency offset

$\Delta f_c = \frac{\Delta \omega_c}{2\pi}$ on the channel estimate. Thus, we need to compute

$$R_A(t) = [c(t) \cdot e^{j\Delta\omega_c t}] * c^*(-t) \quad (3.17)$$

In Appendix A this relation is found to be approximated for small frequency offsets $\Delta f_c \ll \Delta f$ by

$$R_A(t) \approx T \left[1 - \frac{|\Delta f_c|}{\Delta f} \right] \cdot e^{j \frac{\Delta \omega_c T}{2}} \cdot e^{j \frac{\Delta \omega_c t}{2}} \cdot R(t - t_p), \quad (3.18)$$

where $R(t)$ is the chirp signal autocorrelation function without frequency offset, and t_p is the position of the peak given by

$$t_p = -\frac{\Delta f_c}{\Delta f} \cdot T. \quad (3.19)$$

In view of these two relations, the carrier frequency offset has the following effects on the correlation function:

- attenuation dependent on Δf_c , which is negligible given the small frequency offset condition
- constant phase rotation of $\pi \cdot \Delta f_c T$
- time shift of the peak location with an amount given by t_p
- frequency offset equal to half of the initial value

In order to point out the effect of the above results on the channel estimate, we write the received signal in presence of receiver carrier frequency offset as

$$y(t) = [c(t) * h(t)] \cdot e^{j\Delta\omega_c t} \quad (3.20)$$

which after correlation becomes

$$\hat{h}_\Delta(t) = y(t) * c^*(-t) = \left[[c(t) * h(t)] \cdot e^{j\Delta\omega_c t} \right] * c^*(-t) \quad (3.21)$$

where we use the index Δ to point out that channel estimation is performed in presence of carrier frequency offset.

Rewriting in frequency domain we have

$$\hat{H}_A(\omega) = C(\omega - \Delta\omega_c) H(\omega - \Delta\omega_c) C^*(\omega) = \mathcal{F}\{R_A(t)\} \cdot H(\omega - \Delta\omega_c) \quad (3.22)$$

and by substituting relation (3.18) in we get

$$\hat{H}_A(\omega) = T \cdot e^{j\frac{\Delta\omega_c T}{2}} e^{-j\omega t_p} \cdot e^{j\frac{\Delta\omega_c t_p}{2}} \cdot \left| C(\omega - \frac{\Delta\omega_c}{2}) \right|^2 \cdot H(\omega - \Delta\omega_c) \quad (3.23)$$

At this point it should be noted that for transmitter carrier frequency offset a similar result would have been obtained, except for the frequency shift in the channel estimate.

Considering again $\Delta f_c \ll \Delta f$ (which is reasonable considering that Δf is at least equal to the channel bandwidth), we may neglect the $\frac{\Delta\omega_c t_p}{2}$ phase term. Then, the $\frac{\Delta\omega_c T}{2}$ constant phase term can be easily compensated for after estimating the frequency offset and hence we may disregard this term as well. Further, the linear phase term due to time shift of the position of the correlation peak will modify the estimated phase with a maximum value at the edges of the spectrum equal to

$$-t_p \omega_{\max} = \frac{\Delta f_c}{\Delta f} T \cdot \frac{2\pi}{2T_s} = \frac{\Delta f_c}{\Delta f} \cdot \pi M \cdot \frac{T_c}{T_s} \quad (3.24)$$

and usually is large enough to have to be taken into account. This phase shift is related to the time shift in the peak position and as such may produce timing errors for large frequency offset values, while for small frequency offsets will spread each constellation point along an arc of size given by the relation above.

The frequency shift of $\Delta\omega_c$ in the estimated transfer function will have an effect depending on the actual channel, being more important for channels with sudden phase and/or amplitude variations, and we shall consider it in Chapter 4. Considering the

$\left| C\left(\omega - \frac{\Delta\omega_c}{2}\right) \right|^2$ term, we have to design the power spectrum of the chirp signal to be flat over the channel's bandwidth extended with $\frac{(\Delta\omega_c)_{max}}{2}$ on both sides. Since $(\Delta\omega_c)_{max}$ is usually a small value compared to the channel bandwidth, this condition is easily satisfied.

For larger frequency offsets however, some of the above considerations do not hold. Specifically, there will be an attenuation factor of $\left[1 - \frac{|\Delta f_c|}{\Delta f}\right]$, and the correlation function will no longer be symmetric around its peak, additional phase terms being thus generated. These effects will be more visible in the simulation results of Chapter 4.

In designing the parameters of the chirp signal the following numerical values have been chosen: the channel bandwidth is limited by the transmit and receive filters to the value $1.5 \cdot \frac{1}{T_s}$, and using Figure 3.10 we find a required swept bandwidth of about $\Delta f = 2.2 \cdot \frac{1}{T_s}$. Since the sampling frequency of the chirp should be larger than Δf to avoid serious frequency domain aliasing, a minimum value of $3 \cdot \frac{1}{T_s}$ is needed however, from software implementation considerations we have chosen $\frac{1}{T_c} = 4 \cdot \frac{1}{T_s}$. For good performance (high pulse compression ratio) a duration of $T = 400 \cdot T_c$ (PCR=220 and M=400), while for little loss due to windowing $T_w = 44 \cdot T_c = 0.11 \cdot T$ have been selected. It is found that this value of T_w corresponds to an actual transmitter power loss of 0.5 dB. Finally, as mentioned in section 3.2 we have to guard the synchronization burst with zeros on both sides to avoid interference from adjacent frames.

A subtle advantage of using a chirp as a synchronization burst is due to the signal's constant envelope. Since the OFDM signal is very spiky in nature, the RF power amplifier should work at a reduced average to peak power ratio. In turn, when the chirp is transmitted this power limitation can be removed and hence higher transmit power is available for the synchronization burst, gains of about 4 dB being quite common as shown in [4]. Of course, the same considerations hold for the weighted chirp signal as well.

3.3.2 M-sequences

This is the other choice we have considered for a synchronization burst, the white noise like autocorrelation properties of the M-sequence being of interest.

A very detailed analysis of M-sequences and shift register generators is found in [22], while a good overview of the subject is given in [14]. In Figure 3.11 the general shift register structure for M-sequence generation is presented.

The periodic autocorrelation function for M-sequences of length M is given by:

$$R_b(k) = \begin{cases} 1 & ,k \text{ multiple of } M \\ -\frac{1}{M} & ,\text{otherwise} \end{cases} \quad (3.25)$$

and in Figure 2.10 this function is plotted. For M large (long sequence) the expression above becomes an impulse and hence an ideal channel estimate could be achieved. However, this can be severely limited by carrier frequency offset as explained later.

In order to evaluate the effect of the correlation sidelobes we consider the error introduced in the estimation of each component of the channel impulse response:

$$e = -\frac{1}{M} \cdot \sum_{n=0}^L h_n \quad (3.26)$$

L being the maximum delay spread. The relation above is constrained through filter normalization (see relation (3.8) and (3.9)) by

$$\sum_{n=0}^L |h_n|^2 = 1 \quad (3.27)$$

From equations (3.26) and (3.27) the following upper bound on the error can be found

$$|e| \leq \frac{\sqrt{L}}{M} \quad (3.28)$$

where equality holds when all h_n are in phase and of equal magnitude.

Figure 3.11: Shift register M-sequence generator

It is of interest to study the periodic correlation function of the M-sequence b_n with its frequency shifted version in order to evaluate the effect of carrier frequency offset

$\delta f_c = \frac{\delta \omega_c}{2\pi}$ (where $\delta f_c = \Delta f_c T_e$ is the normalized offset) on the channel estimate. Thus we need to compute

$$R_{\Delta}(k) = \frac{1}{M} \cdot \sum_{n=0}^{M-1} b_n e^{j\delta \omega_c n} b_{(n+k)_M} \quad (3.29)$$

In Appendix B the following general relation has been derived:

$$R_{\Delta}(k) = \begin{cases} \frac{\sin(\pi M \delta f_c)}{M \cdot \sin(\pi \delta f_c)} \cdot e^{j \frac{(M-1)\delta \omega_c}{2}} & , k \text{ multiple of } M \\ \frac{1}{M} \cdot \left[\sum_{n \in \mathcal{K}_0(k)} e^{jn\delta \omega_c} - \sum_{n \in \mathcal{K}_1(k)} e^{jn\delta \omega_c} \right] & , \text{otherwise} \end{cases} \quad (3.30)$$

where $\mathcal{K}_0(k)$ and $\mathcal{K}_1(k)$ are index sets of which meaning is given in the appendix, being unimportant in the foregoing discussion.

The useful term in this relation is the main pulse obtained for $k = 0$, the other terms representing the sidelobes. By comparing the effects of frequency offset from this equation to the case of a chirp signal the following remarks can be made:

- There is a constant phase term similar to the one for chirp, and having the same mathematical expression.
- An attenuation term is also present, but it has a different behavior, being well approximated by $\text{sinc}(\pi M \delta f_c)$. First, it exhibits a larger flat top where little attenuation results, and then a rapid roll off where frame synchronization

becomes more difficult since the double threshold detector of Figure 3.3 may fail to lock on. While for chirp one needs $\Delta f_c \ll \Delta f \approx \frac{1.5}{T_s}$ in order to neglect attenuation, for M-sequences the condition is more restrictive, being roughly approximated by $\pi M \delta f_c \leq \pi/4$ (for attenuation less than 10%), or equivalently $\Delta f_c \leq \frac{1}{4MT_s}$, where we have considered $T_c = T_s$.

- The estimated channel is also frequency shifted by $\delta\omega_c$ since relations similar to (3.20), (3.21) and (3.22) hold here as well.

As far as the correlation sidelobes are concerned, both sets $\mathcal{K}_0(k)$ and $\mathcal{K}_1(k)$ are finite and therefore there is an upper bound on the side lobe level, which however does not necessarily vanish as M gets larger but may tend towards a certain constant. Usually, a trade-off between side lobe levels and maximum main lobe attenuation factor has to be made, and in these conditions the presence of the threshold device on the output of the channel estimation branch of Figure 3.3 becomes mandatory.

Yet another problem arises since the received data is given by $y_n = b_n * h_n$ and after correlation $\hat{h}_n = b_n * h_n * b_{-n}$ instead of having periodic convolution. This can be corrected by periodically extending the transmitted M-sequence b_n at both ends with L symbols, obtaining

$$b_n^\dagger = \begin{cases} b_{n+M} & , n=-L \dots -1 \\ b_n & , n=0 \dots M-1 \\ b_{n-M} & , n=M \dots M+L-1 \end{cases} \quad (3.31)$$

Figure 3.12 shows the correlation $b_n^\dagger * b_{-n}$ for $M=63$ and $L=20$, which contains a

Figure 3.12: Correlation with a cyclically extended M-sequence

term due to periodic correlation around the main pulse and linear correlation terms further away. For sufficiently long M-sequences the useful term becomes an impulse, as mentioned at relation (3.25), guarded with approximately zeros at both sides. Therefore, the periodic extension not only transforms a linear correlation into a periodic one, but also provides the guard intervals required according to Figure 3.2. Finally, the desired channel estimate is given by the convolution of the useful term of this correlation with the channel impulse response, and we have $\hat{h}_n \equiv h_n$.

Numerically, we have chosen an M-sequence with length $M=255$ symbols for small sidelobes in the periodic correlation (considering both zero and non zero frequency

offset values) and a transmission rate equal to the data rate, $\frac{1}{T_e} = \frac{1}{T_s}$, since this will give sufficient resolution in estimating the OFDM frame timing. With these values and a maximum delay spread of $L=20$ symbols, the error from equation (3.28) is $|e| \leq 1.7\%$ and therefore channel components below a level of approximately 2% have to be discarded. In the presence of carrier frequency offset however, this threshold has to be significantly increased. Considering Figure B.1-b which is reprinted here as Figure 3.13, a relation similar to (3.28) can be evaluated and we find that $|e| \leq 28.6\%$. This holds in the pessimistic case where all sidelobes are equal to the maximum value of Figure 3.13, they are all in phase, and the particular channel impulse response has a uniform delay profile with a maximum delay spread of $L=20$ symbols.

Figure 3.13: M-sequence noncoherent correlation function

3.4 A symbol error rate expression

Consider the OFDM receiver from Figure 3.1 and recall equation (3.3) and (3.5):

$$r_n = h_n * s_n + n_n \quad (3.32)$$

$$n_n = z_n * f_n \quad (3.33)$$

where n_n are uncorrelated gaussian noise samples, due to the square root raised cosine shape of f_n . Due to the cyclic prefix the linear convolution transforms into a periodic one, and after removing the prefix and performing the DFT we obtain:

$$I'_k = H_k \cdot a_k + \zeta_k \quad (3.34)$$

where $\zeta_k = \text{DFT}(n_n)$ are identically distributed gaussian random variables with variance

$$\sigma_\zeta^2 = \sigma_n^2 = \sigma_z^2 = N_0 \cdot \frac{1}{T_s} \quad (3.35)$$

The SNR for the data on the k^{th} carrier is therefore given by

$$\text{SNR}_k = \frac{\sigma_a^2 \cdot |H_k|^2}{\sigma_\zeta^2} = \frac{\sigma_a^2}{\sigma_n^2} \cdot |H_k|^2 = \text{SNR}_0 \cdot |H_k|^2 \quad (3.36)$$

and the corresponding error probability is

$$P_e(k) = P_{e,0} [\text{SNR}_0 \cdot |H_k|^2] \quad (3.37)$$

In equation (3.37) we have assumed the phases of H_k are correctly estimated and compensated for, that is the signal constellation is not rotated. In this case the overall average symbol error rate is the mathematical average of the per carrier error rates, i.e.

$$\bar{P}_e = \frac{1}{N} \cdot \sum_{k=0}^{N-1} P_e(k) \quad (3.38)$$

where N is the number of OFDM carriers and the expression of $P_e(k)$ depends on the particular data modulation involved (QPSK, QAM).

It is important to note that for frequency selective channels with deep nulls certain OFDM carriers will yield very high error rates which will globally induce an error rate floor. To avoid this kind of performance degradation, channel coding and interleaving has to be used in any practical system, thus allowing for the recovery of data symbols erased by the channel. In references [1], [2], [9] and [16] the performance of such systems are discussed in greater detail.

Simulation Results

Based on the system model and parameter choice described in Chapter 3 we shall present the results of computer simulations and discuss the results. Throughout this chapter 16-QAM modulation is mainly used since PSK type modulations are mostly sensitive to the OFDM carrier phases but not to amplitudes.

4.1 Effect of channel estimation errors

In a general form, the estimated channel impulse response can be written as the sum of the real impulse response and an estimation error Δh_n :

$$\hat{h}_n = h_n - \Delta h_n \quad (4.1)$$

After OFDM demodulation the received signal is given by

$$I'_k = a_k \cdot (\hat{H}_k + \Delta H_k) \quad (4.2)$$

which after channel equalization yields the estimated symbols a'_k given by

$$a'_k = a_k \cdot \left(1 + \frac{\Delta H_k}{\hat{H}_k} \right) = a_k + a_k \cdot \frac{\Delta H_k}{\hat{H}_k} \quad (4.3)$$

The last term in the equation will produce a misplacement of the signal in the constellation. The effect of this misplacement is more significant for symbols corresponding to the outer constellation points than is for inner constellation points (this can also be observed experimentally later, in the plots of section 4.4). Based on this observation, transmission schemes involving amplitude modulation with large constellations are more sensitive to channel estimation errors.

To investigate the influence of estimation errors, consider a simple case when the channel consist of two pulses $h_n = \alpha \cdot \delta(n - p) + \beta \cdot \delta(n - q)$ and the estimated channel is $\hat{h}_n = \alpha \cdot \delta(n - p)$. Then, equation (4.3) becomes

$$a'_k = a_k + a_k \cdot \frac{\beta \cdot w_N^{kq}}{\alpha \cdot w_N^{kp}} = a_k + a_k \cdot \frac{\beta}{\alpha} \cdot e^{j \frac{2\pi}{N} k(p-q)} = a_k + \Delta a_k \quad (4.4)$$

$$\text{where } w_N = e^{j \frac{2\pi}{N}}.$$

Since the transmitted symbols a_k are random, after a sufficiently long time the same symbol a_k will have been transmitted on all OFDM subcarriers at least once. Therefore, we can conclude that channel estimation errors will produce spreading of each constellation point around the nominal (transmitted) position. More precisely, instead of receiving the correct point in the constellation we will receive N points scattered around the nominal position. For the particular case chosen above, the spreading will be along a circle centered at a_k , while for more realistic situations a more complex curve is obtained.

Typical constellation spreads for two different grades of channel estimation errors in an OFDM-QPSK system are shown in Figure 4.1. Based on these qualitative representations we conclude that uncompensated ISI, or equivalently channel estimation errors,

Figure 4.1: Constellation spread due to channel estimation errors

produces a rather well definable distortion, giving the OFDM system high robustness as compared to non-OFDM systems where due to ISI symbols would start mixing together. Since plots like the ones in Figure 4.1 characterize the amount of uncompensated ISI present in the received signal (whatever its source would be), we can consider these figures as equivalents to an eye diagram for OFDM systems.

4.2 Effect of frame synchronization errors

In this section we point out the need for and the precision of frame synchronization by a simple analytical reasoning sustained with simulation results. In the context of the discussion concerning the optimum choice of the nominal frame timing reference from

Chapter 2 (section 2.4), here we analyze the worst case when timing reference is chosen at the end of the cyclic prefix.

Consider two cases: when frame timing is within one symbol duration T_s , and the other case when it is larger. In the first case the received symbols are expressed by

$$r_{n+\Delta} = \sum_i s_i h[(n-i)T_s + \Delta T_s] \quad (4.5)$$

meaning the receiver sees a different (time shifted) channel but the symbols transmitted in an OFDM frame are being kept within the same frame. The modified channel is estimated as it is and compensated for, so there will be no degradation.

In the second case, although the channel is still correctly estimated, the structural integrity of the data frame is lost by foreign symbols entering the frame and perturbing all constellation points. Also, the correct part of the reconstructed frame will be time shifted with respect to the transmitted one, and this will give a different phase rotation for each OFDM carrier. For small frame synchronization errors the second effect is dominant (Figure 4.2-a), while for large errors both effects are noticeable (Figure 4.2-b).

4.3 Simulated channel profiles

As stated at the beginning of this document, we are concerned with frequency selective channels. As a general rule, we assume an exponential delay profile which will, however, not limit the generality of our conclusions. Further, randomly generated channel impulse responses have also been investigated and the results presented through the remainder of the thesis hold for them as well. For the purpose of detailed analysis we have selected the following channel profiles shown in Figure 4.3:

Figure 4.2: Constellation spread due to frame synchronization errors

- ideal channel, which is used as a good reference case for which results are readily available (channel #0)
- exponential¹ power delay profile with unresolved multipath and smooth frequency response² (channel #1)
- exponential power delay profile with resolved multipath and abrupt frequency response³ (channel #2)
- uniform power delay profile with small delay spread and with zeros in the frequency response, which is a worst case scenario (channel #3)

¹ Channels with exponential power delay profiles are commonly assumed in digital communication systems, representing a fairly good model, and also it is mathematically more easy to deal with (see [19]).

² This channel is expected to be *less sensitive* to frequency shifts in the estimated frequency response, and to false pulses which are not removed from the estimated impulse response.

³ This channel is expected to be *more sensitive* to frequency shifts in the estimated frequency response, and to false pulses which are not removed from the estimated impulse response.

a) Flat channel (#0)

b) Channel with smooth frequency profile (#1)

c) Channel with abrupt frequency profile (#2)

d) Channel with uniform power delay profile (#3)

Figure 4.3: Characteristics of the simulated channels

In the following sections we shall evaluate the performance of the proposed channel and frame timing estimators over each of the selected channels.

4.4 Performance of estimators in the presence of carrier frequency offset

It is shown in [13] that carrier frequency offset in OFDM systems produces a common rotation of all constellation points and an additive noise like spreading. The first effect is referred to as the common component while the second one as the foreign component. Since both are disturbing, frequency offset estimation and compensation is essential.

In this thesis it is assumed that those functions do exist and hence the above mentioned effects can be disregarded. However, initial channel estimation and frame synchronization are performed in the presence of carrier frequency offset. According to the discussions in 3.3.1 and 3.3.2, we can account for relatively small frequency offsets through a constant phase rotation plus an attenuation factor, a frequency shift of the estimated channel, and through an additional linear phase term for chirp based estimation.

In order to appreciate the quality of the channel estimator we have first performed an analysis in a noiseless case. The evaluation of the sensitivity to both additive noise and frequency offset is the subject of the next section. We proceed with the evaluation from two points of view:

- As it results from 4.1, we can evaluate the channel estimator by inspecting the signal constellation after equalization by considering the amplitude and phase spread at each signal location as a measure of performance.

- By visually comparing the estimated channel amplitude and phase characteristics to the known real ones, we can identify such effects as frequency shifted channel estimates, attenuations, imperfectly compensated constant phase rotations and additional linear phase components.

First, we show the results obtained when the two synchronization bursts (chirp and M-sequence based) are used for channel estimation without compensating for the non idealities pointed out in sections 3.3.1 and 3.3.2, only the common phase rotation being compensated for. In Figure 4.4 the non ideal nature of the chirp based estimator is illustrated for a frequency offset of 4000 ppm (parts per million with respect to the bandwidth occupied by the OFDM signal, that is $\frac{1}{T_s}$) by considering an ideal (flat) channel (#0). Inspecting the impulse response one can notice the timing error due to shift in the position of the correlation peak. For the same reason the constellation is spread along a curve with angular measure given by relation (3.24). However, there is an additional phase component which by measurements is found to be half of the value given by relation (3.24). In order to improve performance we should derotate the constellation accordingly, based on the availability of a carrier frequency offset estimate. Further, as read from the amplitude spectrum, there is no gain loss, as it has been predicted in section 3.3.1. However, we can notice distortions at the band edges, which are mainly due to the estimated channel being frequency shifted, as it results from equation (3.23), and so, when folded about the Nyquist frequency, the estimated channel does not have the proper phase and amplitude.

In Figure 4.5 the non ideal character of the M-sequence based channel estimator is illustrated for a frequency offset of 1000 ppm, for the same flat channel (#0). In the estimated impulse response we notice the presence of false pulses due to the very high side-lobe level of the incoherent correlation function. The side lobes are also responsible for the rippled amplitude spectrum and for the larger phase errors, all of which lead to a rather

a) estimated impulse response
(absolute values)

b) estimated spectrum

c) signal constellation

Figure 4.4: Non ideal character of the chirp based channel estimator

a) estimated impulse response
(absolute values)

b) estimated spectrum

c) signal constellation

d) signal constellation after
discarding low level pulses

**Figure 4.5: Non ideal character of the M-sequence
based channel estimator**

random spreading of the data symbols in the signal constellation. To improve performance we have to discard low valued pulses in the estimated impulse response, as it has been predicted in section 3.3.2. Further, in the impulse response one can notice the existence of gain loss (approx. 0.9 dB) which is due to the presence of carrier frequency offset, as detailed in the same section. In order to point out this effect more precisely, the signal constellation obtained after low level pulse removal is also presented. It follows that in case of large frequency offsets we need to compensate for the attenuation factor of equation (3.30), compensation to be based on a separate carrier frequency offset estimator. This compensation is necessary for large frequency errors, since without it the frame synchronization circuit may not lock on to the received synchronization burst if the magnitudes are seriously degraded.

In conclusion, for the chirp based estimate we have to compensate for an additional phase rotation of which value has been experimentally found as being half of the one given by relation (3.24). However, when there is noise in the system, low level pulses in the estimated impulse response have to be discarded since they are unreliable. Moreover, by experimenting we have found that the impact of this removal is equivalent with derotating the constellation, and therefore only the correction by low level pulse removal is being implemented for a threshold level of 3%, which was found to be satisfactory. For the M-sequence based estimate we have to remove low level impulses from the estimated impulse response, and optionally (depending on the magnitude of the frequency offset) to compensate for the attenuation factor of equation (3.30). A threshold level of 8% has been found as necessary, while attenuation compensation has been neglected since, as demonstrated by the following results, no more than 1 dB gain loss is registered for frequency offsets that would otherwise still allow for acceptable channel estimates.

The results in Figure 4.6 through Figure 4.9 allow us to evaluate the performance of the chirp based channel estimate with the carrier frequency offset, which is expressed in

a) $\delta f_c = 500$ ppm b) $\delta f_c = 2000$ ppm c) $\delta f_c = 4000$ ppm

Figure 4.6: Chirp based estimation: flat channel (#0)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 2000$ ppm c) $\delta f_c = 4000$ ppm

Figure 4.7: Chirp based estimation: channel with smooth frequency profile (#1)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 2000$ ppm c) $\delta f_c = 4000$ ppm

Figure 4.8: Chirp based estimation: channel with abrupt frequency profile (#2)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 2000$ ppm c) $\delta f_c = 4000$ ppm

Figure 4.9: Chirp based estimation: channel with uniform power delay profile(#3)

ppm as a parameter. We notice that for offsets lower than 500 ppm the received signal constellations are very close to ideal and therefore the channel estimate is considered accurate. For 2000 ppm there is some spreading for the flat channel (#0) and for the channel with a smooth frequency profile (#1), spreading mainly due to asymmetry of the sidelobes of the chirp correlation function which in turn results from the time shift of the peak combined with the time dependent magnitude weighting from equation (A.7). At 4000 ppm the estimated frequency responses are also given to show that large frequency offsets not only produce an additional linear phase term (shift of peak location) but also amplitude errors due to sidelobe distortion, and due to frequency shift of the estimated channel. On the same figures the amplitude and phase estimation error graphs are also plotted. The channel with abrupt frequency profile (#2) turns out to be more sensitive to frequency offsets due to its pronounced frequency selective character. As far as the channel with uniform power delay profile (channel #3) is concerned, the frequency characteristic has very deep notches, virtually zeros. Symbols transmitted on such highly attenuated OFDM subcarriers are received under very low SNR conditions and therefore channel equalization is not performed for the corresponding carriers, instead the data is declared erased by the channel. In the signal constellation, this effect manifests itself as received symbols being grouped around the origin, as can be seen in Figure 4.8. In Figure 4.8-c, along with the estimated frequency response, the equalization threshold at -20 dB is also drawn. One should notice that large amplitude and phase estimation errors occur close to the notches in the amplitude characteristic, and therefore one should balance between a lower equalization threshold value and larger estimation errors.

Figure 4.10 through Figure 4.13 present similar results for M-sequence based estimation. In general, the same remarks hold with one exception: the estimation of the flat channel is perfect (up to the already mentioned gain loss). This is due to the noncoherent correlation function of M-sequences. If the perturbing sidelobes can be safely removed,

a) $\delta f_c = 500$ ppm b) $\delta f_c = 1000$ ppm

Figure 4.10: M-sequence based estimation: flat channel (#0)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 1000$ ppm

Figure 4.11: M-sequence based estimation: channel with smooth frequency profile (#1)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 1000$ ppm

Figure 4.12: M-sequence based estimation: channel with abrupt frequency profile (#2)

a) $\delta f_c = 500$ ppm b) $\delta f_c = 1000$ ppm

Figure 4.13: M-sequence based estimation: channel with uniform power delay profile (#3)

which is possible for flat channels only, all is left is a scaled impulse which is an ideal estimator. In all other cases removal of the low level components improves the performance, however the harm done by the random character of the sidelobes is usually irreversible and this shows in more spread constellations.

By comparing the figures corresponding to the same channel but different synchronization bursts the superiority of the chirp signal for larger frequency offsets can be established.

4.5 Performance of estimators in the presence of additive noise

In order to evaluate our estimates in more realistic system conditions we have to include the effect of both carrier frequency offset and additive noise. Since considerations as the ones in the previous section may be meaningless when noise is present, we have performed error rate simulations for the channels considered in section 4.3. The resulting symbol error rate curves are plotted against the theoretical bound developed in section 3.4.

During the simulation the following have been considered:

- In order to reduce the simulation time, OFDM frames with a large number of subcarriers have been chosen, numerically $N=8192$.
- The physical channel is assumed to be fading slowly enough to be considered time invariant for the duration of one OFDM frame. For simulation purposes however, time variations of the channel (fading) have not been considered, since as far as the initial channel and frame timing estimation is concerned the channel is assumed stationary.

- For each simulated SNR value the channel is estimated once, before the transmission of the first frame, and the estimated impulse response is used to equalize all subsequent frames.
- For each simulated SNR value a sufficient number of frames have been transmitted so as to detect at the receiver between 50 and 100 erroneous symbols, more at higher error probability values and less at low error probability values. This will ensure a sufficiently reliable error rate reading.

In Figure 4.14 through Figure 4.17 the simulated symbol error rate plots are presented for frequency offsets of 500, 1000 and 2000 ppm for chirp based estimator, and for offsets of 500 and 1000 ppm for the M-sequence based estimator. As predicted from the noiseless analysis, the chirp signal gives higher robustness to the channel estimate. For frequency offsets of 500 ppm there is no degradation in performance, for 1000 ppm about 1.0-1.5 dB, and for 2000 ppm 2-3 dB loss is encountered. The performance degradation is mainly due to carrier frequency offset induced channel estimation errors which will spread the received constellation, as illustrated in section 4.1. The channel with uniform power delay profile exhibits an error floor due to the equalization threshold mentioned in the previous section. This error floor is due to the highly attenuated (20 dB and more) OFDM subcarriers which are not equalized. By changing the value of the equalization threshold the error floor level will also change however, the performance is already much degraded (error rate bound of 2×10^{-2} at 30 dB SNR) and there is no practical reason in trying to lower the equalization threshold. In turn, channel coding and interleaving has to be used in order to combat the effects of the pronounced frequency selectivity of the channel.

When we use an M-sequence as synchronization burst, for 1000 ppm frequency offset an error rate floor is noticeable for all simulated channels, except the flat channel. The presence of error floors can be explained by considering the large constellation spread

a) Chirp based estimation

b) M-sequence based estimation

Figure 4.14: Error rate curves: flat channel (#0)

2 OF 2

PM-1 3 1/2 x 4 PHOTOGRAPHIC MICROCOPY TARGET
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a) Chirp based estimation

b) M-sequence based estimation

Figure 4.15: Error rate curves: channel with smooth frequency profile (#1)

a) Chirp based estimation

b) M-sequence based estimation

Figure 4.16: Error rate curves: channel with abrupt frequency profile (#2)

a) Chirp based estimation

b) M-sequence based estimation

Figure 4.17: Error rate curves: channel with uniform power delay profile (#3)

under noiseless conditions, as it is seen from the results of section 4.4. The exception in the case of the flat channel (#0) is to be considered in the view of the discussion provided at the end of the previous section, and therefore the performance degradation at frequency offset of 1000 ppm is due to the uncompensated attenuation factor only. From Figure 4.17 we can see that the error rate floor in the case of the channel with uniform power delay profile is frequency offset dependent, as opposed to the chirp based estimator case when it was not. This observation leads us to the conclusion that for the chirp based channel estimator the error floor is due to the equalization threshold only, while for the M-sequence based channel estimator there is an additional contribution due to the frequency error. By recalling Figure 4.9 and Figure 4.13 and comparing them, it is easy to predict the above mentioned behavior.

The error rate simulations have been complemented with confidence interval evaluations. For each channel and synchronization burst pair the confidence interval associated with the obtained error rate was determined at one selected SNR value. This has been accomplished by simulation, where the following were considered:

- The particular SNR value is chosen such that the corresponding symbol error rate is between 10^{-4} and 10^{-3} in order to assure a realizable simulation time.
- With the above selected SNR value a total of 25 runs have been performed.

The resulting confidence intervals are also shown in the previous error rate curves, with the exception of the set corresponding to the channel with uniform power delay profile (#3). For this channel the error rate floor is very high and the obtained confidence intervals were found too small to produce a significant plot.

Conclusions and Recommendations

In this work we have addressed the problem of initial channel estimation and frame synchronization for OFDM systems in presence of carrier frequency offset and additive noise, considering channels with small delay spread (microcellular environment).

5.1 Thesis conclusions

Based on the system model of section 3.1 and on the proposed estimation model in 3.2 we have built a complex simulation tool enabling us to investigate the effect of the following on the channel and frame timing estimates:

- type of the synchronization burst
- synchronization burst parameters
- channel characteristics
- carrier frequency offset
- SNR in additive noise
- modulation scheme (although we have mainly looked at 16QAM base modulation since it is more sensitive to the type of estimation errors we have countered)

Based on the simulation results of Chapter 4 and on some theoretical considerations made in Chapter 3 we conclude the following:

-
- A combined initial channel and frame timing estimation algorithm was developed, and its performance with carrier frequency offset and additive noise has been studied.
 - The channel estimation algorithm performs well even in the presence of carrier frequency offsets. In low frequency offset conditions (up to 500 ppm) both synchronization bursts yield about the same performance, with no significant performance degradation. For larger offset values (1000 to 2000 ppm) however, the chirp based estimator outperforms the M-sequence based estimator due to the different frequency offset dependence of the two noncoherent correlation functions.
 - Sidelobes of the correlation functions can be significantly distorted by frequency offset, reason for which components of the estimated channel impulse response below a certain threshold level have to be discarded as being unreliable.
 - The frame timing estimate is very robust. In the M-sequence based estimator case it is mainly affected by frequency offset induced gain loss. In the chirp based estimator case it is mainly affected by frequency offset induced time shift of the correlation peak.
 - Both synchronization signals represent approximately the same overhead, small compared to the usual OFDM frame lengths of 2048 to 8192 symbols.

5.2 Recommendations for further study

Considering the large interest in OFDM signaling, the applicability of the proposed channel and frame timing estimation algorithms to channels with large delay spread should also be considered. As pointed out at the end of section 3.3.1, the time domain weighted chirp signal should not give further performance degradation, however for the M-sequence case the effect is yet unknown.

This leads to our second suggestion, for further study of the noncoherent correlation function of M-sequences. Here, the sidelobe structure should be analyzed and even an approximate analytical result regarding the sequence length and frequency offset dependence would be useful. Moreover, such an analysis would allow for considering channels with large delay spread for which significant sidelobe induced distortions are expected.

Noncoherent Correlation of Chirp Signals

Here we discuss the properties of the correlation between a chirp signal and its frequency shifted version. Consider the low pass equivalent $c(t)$ of a chirp which according to relations (3.13) and (3.14) can be written as

$$c(t) = e^{j\phi(t)} \text{ with} \quad (\text{A.1})$$

$$\phi(t) = -\pi \cdot \Delta f \cdot t + \pi \cdot \frac{\Delta f}{T} \cdot t^2, \quad t \in [0, T]. \quad (\text{A.2})$$

The required correlation function for a frequency offset of $\Delta f_c = \frac{\Delta \omega_c}{2\pi}$ is given by

$$R_\Delta(t) = [c(t) \cdot e^{j\Delta \omega_c t}] * c^*(-t) = \int_I e^{j[\phi(t-\tau) - \phi(-\tau) + \Delta \omega_c(t-\tau)]} d\tau \quad (\text{A.3})$$

where interval I is

$$\begin{aligned} I &= (t-T, 0] & \text{for } t \in (0, T] \\ I &= (-T, 0] & \text{for } t = 0 \\ I &= (-T, t] & \text{for } t \in [-T, 0) \end{aligned} \quad (\text{A.4})$$

By considering the expressions of $\phi(t-\tau)$ and of $\phi(\tau)$ equation (A.3) can be written in a simpler form as

$$R_{\Delta}(t) = e^{j[\phi(t) + \Delta\omega_c t]} \int_1 e^{-j\left[\left(\frac{\Delta\omega_c}{T} + \Delta\omega_c\right)\tau\right]} d\tau. \quad (\text{A.5})$$

By evaluating this relation for each of the three cases above we end up with

$$R_{\Delta}(t) = \frac{2 \cdot \sin\left[\left(\frac{\Delta\omega_c}{T} + \Delta\omega_c\right) \left(\frac{T - |t|}{2}\right)\right]}{\left(\frac{\Delta\omega_c}{T} + \Delta\omega_c\right)} \cdot e^{j\frac{\Delta\omega_c}{2}(T+t)} \quad t \in (-T, T) \quad (\text{A.6})$$

or in an equivalent form

$$R_{\Delta}(t) = T \left(1 - \frac{|t|}{T}\right) \cdot \text{sinc}\left[\left(\frac{\Delta\omega_c}{T} + \Delta\omega_c\right) \left(\frac{T - |t|}{2}\right)\right] \cdot e^{j\frac{\Delta\omega_c}{2}(T+t)} \quad t \in (-T, T) \quad (\text{A.7})$$

This is an attenuated $\text{sinc}(\bullet)$ function with modified zero crossings given the second order argument. Further, there is a constant phase rotation of $\frac{\Delta\omega_c T}{2}$ and a remaining frequency offset equal to half of the initial one. From the argument of the $\text{sinc}(\bullet)$ function the following zero crossings are found for a given order k :

$$\begin{aligned} t_k &= \frac{T}{2} \cdot \left[\left(1 - \frac{\Delta f_c}{\Delta f}\right) + \sqrt{\left(1 + \frac{\Delta f_c}{\Delta f}\right)^2 - \frac{4k}{\Delta f \cdot T}} \right] \quad t_k > 0 \\ t_k &= \frac{T}{2} \cdot \left[\left(1 + \frac{\Delta f_c}{\Delta f}\right) - \sqrt{\left(1 - \frac{\Delta f_c}{\Delta f}\right)^2 + \frac{4k}{\Delta f \cdot T}} \right] \quad t_k < 0 \end{aligned} \quad (\text{A.8})$$

The peak of the correlation is located where the argument of the $\text{sinc}(\bullet)$ function is zero, that is at

$$\frac{\Delta\omega_c \cdot t_p}{T} + \Delta\omega_c = 0 \Rightarrow t_p = -\frac{\Delta f_c}{\Delta f} \cdot T \quad (\text{A.9})$$

meaning that the peak is shifted from its original $t_p = 0$ position with an amount dependent on Δf_c . Then, the peak value is being attenuated due to the amplitude term and therefore

$$|R_{\Delta}(t_p)| = T \left(1 - \frac{|\Delta f_c|}{\Delta f}\right). \quad (\text{A.10})$$

For all practical purposes we must have $|t_p| \ll T$ which is equivalent to $\Delta f_c \ll \Delta f$, and then relation (A.7) can be written as

$$R_{\Delta}(t) = T \left(1 - \frac{|\Delta f_c|}{\Delta f}\right) \cdot e^{j\frac{\Delta\omega_c T}{2}} \cdot e^{j\frac{\Delta\omega_c t}{2}} \cdot R(t - t_p) \quad , t \in (-T, T) \quad (\text{A.11})$$

where $R(t)$ is the autocorrelation function without carrier frequency offset.

In this small frequency offset case for orders k of the zero crossings that satisfy $4|k| \ll \Delta f \cdot T = \text{PCR}$ (pulse compression ratio), the zeros are approximately located at

$$t_k \approx \frac{k}{\Delta f}, \quad (\text{A.12})$$

thus being equally spaced, and if PCR is large enough all significant zeros will satisfy this property.

However, it should be noted that in reality the correlation function is no longer symmetric around its peak due to time shift of the peak, and to the time dependent attenuation factor. Hence, there is an error term in addition to equation (A.11), error dependent on the magnitude of t_p , and usually the complete equation in (A.7) has to be considered. Nevertheless, relation (A.11) gives us an insight as what to expect.

Noncoherent Correlation of M-sequences

Here we discuss the properties of the periodic correlation between an M-sequence and its frequency shifted version. Consider an M-sequence $\{b_n\}$ of length M symbols, and a normalized frequency shift of $\delta f_c = \frac{\delta\omega}{2\pi}$. The needed correlation function is given by

$$R_\Delta(k) = \frac{1}{M} \cdot \sum_{n=0}^{M-1} b_n e^{j\delta\omega_c n} b_{(n+k)_M} \quad (\text{B.1})$$

and according to Figure 3.11 $b_n = (-1)^{a_n}$, therefore

$$R_\Delta(k) = \frac{1}{M} \cdot \sum_{n=0}^{M-1} e^{j\delta\omega_c n} (-1)^{a_n \oplus a_{n+k}} \quad (\text{B.2})$$

However, the modulo 2 sum $a_n \oplus a_{n+k}$ can be written as

$$a_n \oplus a_{n+k} = \begin{cases} 0 & , k \text{ multiple of } M \\ a'_n & , \text{otherwise} \end{cases} \quad (\text{B.3})$$

where $b'_n = (-1)^{a'_n}$ is just another M-sequence of length M. It is known that in any M-sequence there are one more ones than zeros, and by denoting with $\mathcal{K}_0(k)$ the set of

indices n for which a'_n is zero and by $\mathcal{K}_1(k)$ the set of indices for which it is one, relation (B.2) can be expressed as

$$R_{\Delta}(k) = \begin{cases} \frac{\sin(\pi M \delta f_c)}{M \cdot \sin(\pi \delta f_c)} \cdot e^{j \frac{(M-1) \delta \omega_c}{2}} & , k \text{ multiple of } M \\ \frac{1}{M} \cdot \left[\sum_{n \in \mathcal{K}_0(k)} e^{jn \delta \omega_c} - \sum_{n \in \mathcal{K}_1(k)} e^{jn \delta \omega_c} \right] & , \text{otherwise} \end{cases} \quad (\text{B.4})$$

If we consider very small frequency offsets only,

$$n \cdot \delta \omega_c \ll 1 \quad , n=0 \dots M-1, \quad (\text{B.5})$$

the following simplified form can be obtained:

$$R_{\Delta}(k) = \begin{cases} e^{j \frac{(M-1) \delta \omega_c}{2}} & , k \text{ multiple of } M \\ \frac{1}{M} \cdot \left[-1 + j \delta \omega_c \left(\sum_{n \in \mathcal{K}_0(k)} n - \sum_{n \in \mathcal{K}_1(k)} n \right) \right] & , \text{otherwise} \end{cases} \quad (\text{B.6})$$

where $\sin(x) \equiv x$ and $e^{jx} \equiv 1 + j \cdot x$ has been considered. Further, this can be written in the more compact form

$$R_{\Delta}(k) = \begin{cases} e^{j \frac{(M-1) \delta \omega_c}{2}} & , k \text{ multiple of } M \\ -\frac{1}{M} \cdot \mathcal{A}(k) \cdot e^{j \frac{\mathcal{N}(k) \delta \omega_c}{2}} & , \text{otherwise} \end{cases} \quad (\text{B.7})$$

where

$$\mathcal{N}(k) = 2 \left(\sum_{n=\mathcal{N}(k)}^n n - \sum_{n=\mathcal{N}(k)}^n n \right) \quad (\text{B } 8)$$

and

$$\mathcal{A}(k) = \sqrt{1 + \left(\frac{\mathcal{N}(k) \delta \omega_c}{2} \right)^2} = 1 \quad (\text{B } 9)$$

$\mathcal{N}(k)$ is a phase coefficient dependent on the particular M sequence and on the index k of the correlation, while $\mathcal{A}(k)$ is an amplitude weighting.

In Figure B.1 the noncoherent correlation function for a length $M=255$ sequence is presented for zero and for 1000 ppm (parts per million relative to the symbol rate) frequency offsets. The attenuation of the main lobe given by the $\text{sinc}(\pi M \delta f_c)$ term is clearly noticeable (absolute values of the correlation are represented for $\delta f_c=1000$), while the side lobe structure is random looking with a maximum value of 6.4% of the main lobe

a) $\delta f_c = 0$ ppm

b) $\delta f_c = 1000$ ppm

Figure B.1: M-sequence noncoherent correlation function

As it results from the relations developed, the sidelobe structure depends on the particular choice of M-sequence for a given length M (different feed back connections in the shift register generator), and also depends on M . Since deriving a closed form and useful expression for the sidelobes is difficult, we used computer simulation to study this problem, and we have observed an approximately linear dependence of the maximum sidelobe value on the frequency offset for values up to about 1000 ppm.

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