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**"A STUDY AND MODELING OF TWO DIMENSIONAL
EFFECTS IN BIPOLAR TRANSISTORS"**

by Shawn Searles B.Sc. EE.

**A thesis submitted to the Faculty
of Graduate Studies and
Research in Partial fulfillment
of the requirements for the degree of**

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton Engineering**

July, 1989

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"A Study And Modeling Of Two Dimensional Effects In Bipolar Transistors"

submitted by Shawn Searles in partial fulfillment of the requirements for the degree of Master of Engineering.

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July, 1989.

ABSTRACT

Due to the development of silicon processes that can provide both MOS and bipolar transistors on the same substrate, there has in recent years been increased interest in BJTs. To help optimize the design of integrated circuits, accurate analytic bipolar models that are capable of scaling to any arbitrary device geometry, under all levels of injection, are required. This thesis investigates and generates modeling equations for various fundamental properties of BJTs. The effect of bandgap narrowing on the base transport current is investigated and modelled. The two dimensional phenomenon of emitter debiasing with conductivity modulation of the base is modelled. The effect of the emitter periphery on the base and collector current is studied and modelled. The calculation of the extrinsic base resistance, in two dimensions, is detailed. Finally, a discussion of the unification of the sub-models representing the above effects is performed.

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List Of Symbols And Abbreviations

α	Constant
a	Constant for the high level base resistivity model
A_H	High level emitter debiasing constant
A_L	Low level emitter debiasing constant
$\arg(z)$	Argument or angle of the complex variable z
A_1	Arbitrary constant
A_2	Arbitrary constant
β	Reciprocal of the thermal voltage V_T
b	Emitter width
B	Arbitrary constant for the low level emitter debiasing model
b_B	Ratio of u_n to u_p
B_H	Arbitrary constant for the high level emitter debiasing model
BICMOS	Combined bipolar and CMOS process
BJT	Bipolar junction transistor
BL_0	Initial guess at BL
C_{JC}	Capacitance of the base-collector junction
C_{JE}	Capacitance of the base-emitter junction
D	Effective depth for a circular emitter to inject $2/3 I_{cp}$
d	Base width
Δ_{BL}	Function for I_C from the low level emitter debiasing model
ΔE_g	Change in bandgap energy due to doping
D_n	Electron diffusion coefficient

$\bar{D}_n$	Effective electron diffusion coefficient
D_p	Hole diffusion coefficient
$\bar{D}_p$	Effective hole diffusion coefficient
ϵ	Error bound
E_b	Electric field magnitude in the base
E_C	Conduction band energy
E_g	Bandgap energy
E_n	Electric field for electrons
E_p	Electric field for holes
erf	Error function
E_v	Valence band energy
E_x	Electric field magnitude in the x direction
f	Function for the calculation of q_{bx}
$F(BL)$	Function used to solve for B
g	Function for the calculation of I_C
$G(B_H L)$	Function used to solve for B_H
GP1	Gummel-Poon model number 1
GP2	Gummel-Poon model number 2
GP3	Gummel-Poon model number 3
GP4	Gummel-Poon model number 4
GP5	Gummel-Poon model number 5
i	Square root of negative one
I_B	Base current
I_{bp}	Peripheral base current

I_{bT}	Terminal base current
I_{bTH}	Terminal base current from the high level emitter debiasing model
I_{bTL}	Terminal base current from the low level emitter debiasing model
I_C	Collector current
I_{cp}	Emitter peripheral collector current
I_{cT}	Terminal collector current
I_{cTH}	Terminal collector current from the high level emitter debiasing model
I_{cTL}	Terminal collector current from the low level emitter debiasing model
I_E	Emitter current
I_{sbp}	Saturation current for the peripheral base current
J	Total electron and hole current
J_b	Base current density from the low level emitter debiasing model
J_{bH}	Base current density from the high level emitter debiasing model
J_{N1}	Electron current density near the emitter-base junction
J_{N2}	Electron current density near the collector-base junction
J_n	Electron current density
J_{nH}	Electron current density from the high level emitter debiasing model
J_p	Hole current density
J_s	Saturation current
J_{sb}	Saturation current density for the base
k	Boltzman constant
K_1	Constant
K_2	Constant
K'_2	Constant

K_3	Constant
K_4	Constant
K_{sb}	$2\rho J_{sb}$
L	Emitter length
L_{Film}	Length of the thin film resistor
L_{Test}	Length of the test resistor
m	Effective linear slope of the minority carrier concentration in the base
MOS	Metal Oxide Semiconductor
n	Total electron concentration
n'	Excess electron concentration
n_0	Equilibrium electron concentration
n_i	Intrinsic carrier concentration
n_{ie}	Extrinsic carrier concentration, includes the effect of doping
n_{i0}	Equilibrium intrinsic carrier concentration
n_J	Electron concentration at the emitter-base junction on the base side
N	Total doping concentration
N_0	Peak doping
N_{AB}	Donor concentration in the base
ρ	Base resistivity per unit area
ρ_0	Equilibrium base resistivity per unit area
p	Total hole concentration
p'	Excess hole concentration
p_0	Equilibrium hole concentration
PISCES	Two dimensional device simulator
q	Electron charge

Q	Integrated excess minority charge in the base
Q_{bp}	Equilibrium integrated majority charge in the peripheral base
q_{bt}	Integrated excess minority charge in the base for the periphery
Q_{b0}	Equilibrium integrated majority charge in the base
R_B	Extrinsic base resistance
R_{bc}	Base contact resistance
R_C	Extrinsic collector resistance
R_E	Extrinsic emitter resistance
$R_{\text{Periphery}}$	Resistance of the emitter periphery
R_{Test}	Resistance of the test resistor
R_{Thin}	Resistance of the thin film resistor
$R_{\text{Thin film}}$	Resistance of the thin film
s	Effective film thickness
SRH	Schottky-Read-Hall recombination
$SUPREM$	Semiconductor process simulator
τ	Base transit time underneath the emitter
τ_p	Base transit for the emitter periphery
t	Distance in the complex W plane
T	Temperature
u_c	Diffusion length of a Gaussian distribution
u_n	Electron mobility
$U_{n/p}$	Ratio of the effective electron to hole mobility plus one
u_p	Hole mobility
V	Complex plane

$V(x)$	voltage of the base at position x from the low level emitter debiasing model
V_B	voltage of the base referenced to the emitter
V_{BC}	Base to collector voltage
V_{BE}	Base to emitter voltage
$V_{BE,T}$	Terminal base to emitter voltage
$V_H(x)$	voltage of the base at position x from the high level emitter debiasing model
V_{JC}	Base to collector junction voltage
V_{JE}	Base to emitter junction voltage
V_{PH}	Emitter peripheral voltage from the high level emitter debiasing model
V_{PL}	Emitter peripheral voltage from the low level emitter debiasing model
V_T	Thermal voltage KT
W	Complex plane
x'	Real axis in the complex W plane
$x'_{2/3}$	Depth in the W plane for emission of $2/3 I_{cp}$
x''	Real axis in the complex V plane
y'	Imaginary axis in the complex W plane
y''	Imaginary axis in the complex V plane
$y_{2/3}$	Depth in the Z plane for emission of $2/3 I_{cp}$
y_c	Diffusion length of a Gaussian distribution
Z	Complex plane

1.

INTRODUCTION

In the last few years, there has been increased interest in bipolar junction transistors. The major reason for this increased interest in BJTs can be attributed to the development of silicon processes that provide for both MOS and bipolar transistors on the same substrate. For the effective design and optimization of integrated circuits containing bipolar transistors, accurate models must be developed. These models need to capture both the fundamental mechanisms of BJTs for all levels of injection, along with the distributed effects present in all practical structures. The purpose of this thesis is to investigate and model the fundamental properties of a bipolar transistor that must be accounted for to allow for the accurate simulation of integrated circuit designs.

The current industry standard model for a bipolar transistor, the SPICE adapted version of the Gummel-Poon model [1], is known to be unacceptable for the purpose of accurate integrated circuit simulation. This model for the BJT has weak, one dimensional links to the physics governing the fundamental operation of the device. Due to lack of consideration toward the distributed two dimensional nature of BJTs, the SPICE bipolar model is not capable of scaling. This means that for every different layout of a BJT in an integrated circuit, a different SPICE bipolar model must be developed. Thus, it becomes impossible to know how a circuit will operate if a non-standard bipolar layout is used. This restriction has been tolerated up to now by the circuit designers who use BJTs. However, if a truly optimized circuit is desired, then better models for the bipolar transistor must be developed.

The major problem with the SPICE bipolar model arises from the fact it regards the BJT as a one dimensional device. The unfortunate reality of bipolar transistors is that they are two dimensional in nature. Thus, if a model only regards the one dimensional aspect of the device, then the fundamental distributed effects of BJTs will have been neglected. In an effort to reconcile this problem, the SPICE bipolar model tries to use equivalent one dimensional circuits to cover the truly two dimensional nature of BJTs. The problem with such equivalent circuits is the link to physics is generally lost, making the model applicable to only a single, fixed geometry. This makes it impossible for the model to scale to other transistor layouts, and often results in the complete inability to model some effects seen in the output characteristics of even a single device. Therefore, an understanding of the two dimensional nature of the BJT, leading to a modeling representation of these effects, needs to be developed.

1.1. Modeling Approach

There are two basic ways to model systems that operate in two dimensions. The first is to regard the system as truly two dimensional, and set up all of the governing mathematical equations in two dimensions. This method requires no initial assumptions about the basic operation of the device. Thus, the resulting model will be as accurate as is the understanding and representation of device physics. However, this modeling approach is highly complex in a mathematical sense. Typically, highly complex mathematical systems require correspondingly heavy computation, and often require that numerical techniques be used to solve the equations of the system because analytic closed form relationships can not be obtained. This inevitably results in a model with a long execution time.

The second approach to modeling a two dimensional system, if applicable, is to break the problem down into two coupled one dimensional systems. Basic to this method is the capability to exploit a dominant one dimensional reality in the system being modelled (such as principal current flow in only one direction). Then there is the possibility of creating two separately defined one dimensional models coupled together by appropriate physical relationships. The tenets of this method are three fold: first, conversion from a two dimensional model to one built in terms of coupled one dimensional models greatly reduces the mathematical complexity of the model formulation; second, the model stands a far better chance of being expressed in analytic form; finally, the resulting model should execute far faster than the model obtained from the true two dimensional approach.

The drawback to using coupled one dimensional solutions to model a two dimensional system is loss of accuracy. As was stated in the previous paragraph, to break a two dimensional system down into two coupled one dimensional systems requires that a

dominant one dimensional mechanism exist. Depending on the degree to which this one dimensional mechanism dominates the overall two dimensional nature of the system, the accuracy of the final model can become compromised. Therefore, one is trading off accuracy for mathematical simplicity, resulting in reduced execution time for the overall model.

With integrated circuits becoming increasingly large and complex, the reliance on simulation of the circuits in the design process is increasing. Consequently, there is a need to simulate circuits containing an increasing number of transistors. This poses a problem because for circuit simulation to be most useful it should remain interactive with the designer. To remain interactive, the simulation time must not become excessive. Therefore, it is imperative that the resulting model for the transistor be simple enough to execute quickly in a circuit simulator, and yet be accurate enough to capture the fundamental properties of the device. It is for this reason, execution speed, that the modeling approach of coupled one dimensional systems is most appropriate for the modeling of bipolar transistors, and is used as the basis of the model development in this thesis.

1.2. Direction of Study

To achieve the goal of breaking the bipolar transistor model down into a solution of coupled one dimensional systems, a careful step-by-step study of the bipolar transistor is undertaken. The first step is to take a look at the bipolar transistor as a truly two dimensional entity. Through the use of the numerical, two dimensional simulator PISCES, a typical NPN bipolar transistor structure is studied. By simulating the transistor over various bias ranges, it is possible to use PISCES in such a way as to elucidate the fundamental operation of the device under study. This study of the bipolar transistor leads to the identification of the device mechanisms which are essentially one dimensional, and the device regions where, under appropriate conditions, one dimensional mechanisms are dominant. This study establishes the basis for the modeling in the rest of the thesis.

An important point regarding the modeling performed in this thesis needs to be understood. The focus is on the relationship between emitter and base. Since the emitter-base junction is responsible for virtually all the injected current, this region becomes the fundamental starting point to modeling BJTs. To help reduce the complexity of the problem, effects due to the collector (such as quasi saturation) are not modelled. However, when PISCES is used to study an NPN transistor, effects due to the collector are allowed. This ensures that the results of this thesis are still valid and only require modification when collector effects are finally modelled.

The direction of study then switches to development of one dimensional models. To help guide their development, PISCES is used to test out the validity of the modeling approaches. By using PISCES in this manner, it is possible to quickly sort out the important physical effects that must be taken into account, before the long arduous task of

analytic model development commences.

From this point forward the study is broken down into four parallel modeling assignments.

These areas of concern are:

- 1) The study and modeling of minority current transport through the base. The transport model offered by Gummel and Poon [2] is investigated, commencing with a comparison against data from PISCES. Effects due to bandgap narrowing [3], and a way to include it in the Gummel-Poon model, are also investigated. The possibility of reducing minority current transport through the base to one dimensional flow is at the heart of this study.
- 2) The study and modeling of emitter current crowding or emitter debiasing effects [4] [5]. This is the principal two dimensional effect in a bipolar transistor. It results from the transverse flow of base current through a highly resistive active base region, causing the applied voltage across the emitter to base junction to vary with position.
- 3) The two dimensional study of peripheral injection from the emitter [6] [7]. As emitter areas reduce, the ratio of emitter area to periphery goes down as long as the emitter depth is held relatively constant. Since gains in photolithography cause the lateral dimensions of transistors to shrink faster than the vertical ones, the need to accurately model the parasitic effects of peripheral injection becomes increasingly important as device sizes continue to shrink.
- 4) The study of the extrinsic base resistance [8] [9]. This resistance is due to the fact that the base contact can never be made right at the edge of the active base region. The result is a resistive connection from the base contact to the active base itself. Since a drop of only eighteen millivolts at room temperature will change the output current of a bipolar transistor by a factor of two, it is important that the extrinsic

base resistance be accurately modelled. Furthermore, this region is also two dimensional.

Having studied the above aspects of device modeling individually, the task is to bind them together into one complete device model. This is accomplished by developing coupling equations that link the various one dimensional sub models together. Due to the complexity of the various one dimensional models, the coupling equations will typically require the solution of transcendental equations. As a result, numerical solution techniques are used, requiring that these equations be well posed to insure rapid and guaranteed convergence of the model. It is important to insure the convergence of the coupling equations for all bias ranges which the bipolar model covers. In many semiconductor models, failure of the model occurs due to problems with convergence.

1.3. Thesis Organization

The thesis is broken down into six main chapters plus conclusions. Chapter two studies the true two dimensional nature of the bipolar transistor through the use of PISCES. The second chapter goes on to identify the fundamental effects that must be taken into account to achieve an accurate bipolar model. Chapter three develops and solves a one dimensional model for the minority carrier concentration and current distribution in the active base region. Also, this chapter presents and solves a model for the effect of bandgap narrowing on the flow of minority carriers in the base. Chapter four sets out and solves a model for the effects of emitter debiasing. Included is the solution for both integrated hole and electron currents along the length of the emitter. Chapter five deals with the modeling of peripheral emitter injection. It also discusses modeling techniques used to achieve solutions of the truly two dimensional periphery. Chapter six develops the models for the extrinsic base resistance. Chapter seven gives the coupling equations used to bind the models of the previous chapters into a complete bipolar model. The problem of guaranteed model convergence is also addressed in the seventh chapter. Conclusions for the applicability of the overall model, its limitations, and future work that can be done to improve the model are then presented.

2. A STUDY OF TWO DIMENSIONAL EFFECTS IN BIPOLAR TRANSISTORS

The main objective of this chapter is to study the two dimensional attributes of a bipolar transistor. By gaining an understanding of what is occurring inside of the transistor, it is possible to distinguish such things as the regions of the device where essentially one dimensional conditions exist, overall current flow patterns, and the effects of high level on the current flow. From this study, the fundamental areas requiring detailed modeling can then be identified. This chapter goes on to test the accuracy of the Gummel-Poon model and to investigate improvements to this model. The chapter concludes with a study of the effects of bandgap narrowing in the base.

In order to study the two dimensional attributes of a bipolar transistor, one must be capable of solving the five basic semiconductor relations (Poisson, the two current transport and the two current continuity equations) as true, partial differential equations in two dimensions. In order to do this without invoking any simplifying assumptions, numerical solutions have to be used. The program PISCES was developed for just this purpose: the numerical simulation of semiconductor devices in two dimensions. PISCES is used to simulate a single bipolar NPN transistor with doping profiles obtained from the numerical process simulator SUPREM. By studying solutions obtained for various D.C. bias conditions, it is possible to gain an insight into the operation of bipolar transistors. Thus, PISCES becomes a powerful tool, allowing for the detailed investigation of the internal operation of the device.

2.1. Numerical Study Of Bipolar Transistors

The NPN transistor used as a test structure for this study has the following parameters: Drawn emitter length of 2 μm ; Emitter-base depth of 0.167 μm ; Base-collector depth of 0.483 μm ; Active base width of 0.16 μm at $V_{CB} = 3 \text{ V}$; Peak Emitter doping of $2 \times 10^{20} \text{ cm}^{-3}$; Peak active base doping of $3 \times 10^{17} \text{ cm}^{-3}$. Figure 1 gives a cross section of the NPN transistor together with the doping distributions at various locations. It should be noted that this test structure is a narrow base, vertical mode transistor that is typical of high performance bipolar processes.

To help simplify the problem of understanding the two dimensional effects that occur within a bipolar transistor, the test structure is biased in such a way as to eliminate the effects caused by the collector. By keeping the collector to base potential fixed at three volts, the base-collector junction is sufficiently reverse biased for the effects of quasi saturation to be negligible for most values of base-emitter voltages. This allows the study to concentrate on the interplay between emitter and active base region. In addition, the PISCES simulation included the effects of bandgap narrowing, Shockley-Read-Hall recombination, and concentration and electric field dependent mobility.

Before going on to the study of the PISCES simulations, a discussion of the grid used for the simulations is necessary. PISCES is based on finite elements. As such, the set up of the grid is very important for accurate simulations. A great deal of experimentation on the setup of the grid was carried out before the final simulations were made. These early trials showed that a rectangular grid was important because distorted grids resulted in erroneous results. Furthermore, the grid density should increase as one proceeds from the extrinsic region to the intrinsic region, into the active base and finally to the emitter-base depletion

FIGURE 1. Cross sections of the NPN test transistor

(a) Log base and collector current vs. V_{BE} (b) Beta vs. V_{BE}

(c) Beta vs. log collector current

Simulations for $V_{CB} = 3V$ with and without SRH recombination for V_{BE} ranging from 0.2 to 0.95 volts.

FIGURE 2

region. By scaling the grid in this fashion, accurate simulations result.

2.1.1. A First Look At Bipolar Transistors

The NPN transistor was simulated with $V_{CB} = 3$ V, and V_{BE} swept from 0.2 to 0.95 volts. Plots of the resulting base and collector currents against V_{BE} , and of beta against I_C and V_{BE} are shown in figure 2.

The plots in figure 2 present two sets of simulations; one with SRH recombination and one without. The SRH model in PISCES was set up to reflect modern processing technology [10] (ie. long recombination times). From the plot of I_B vs. V_{BE} , the effect of SRH recombination is clearly evident in the low ranges of V_{BE} . The reason for showing this is to identify the effect of recombination currents, and prove that there is no contribution past a V_{BE} of 0.5 V. This allows for a simplifying assumption, namely that recombination current can be ignored for all operating values with $V_{BE} > 0.5$ V. An interesting point can be seen in the beta vs. V_{BE} plot. As V_{BE} decreases, beta never levels off but continues to rise slightly. This effect can be seen in any BJT that has very low recombination current, as for the NPN device being simulated. The effect is caused by the depletion region of the emitter-base junction expanding as V_{BE} decreases. The result is a smaller active base width, leading to an increase in *collector* current; this causes beta to increase. The phenomenon has the same origin as the Early effect [11].

Further investigation of the plots in figure 2 show that some form of high level phenomenon is occurring at $V_{BE} > 0.7$ V. This is shown in all the plots by the sudden downward motion of the curves. Therefore, when high level phenomena are studied, the

simulations of interest occur for V_{BE} greater than 0.7 V.

Although the ultimate outputs of any device model are the terminal currents for a given bias voltage, it is the internal operation of the device that needs to be understood in order to develop the model itself. Before a study of the PISCES simulations can be made, one must understand what the vector plots are saying. A vector plot is read as follows: 1) The direction of the arrow indicates the direction of the current. 2) The length of the vector indicates the magnitude of the current density normalized to the largest current magnitude for that type of carrier. This results in a greatly magnified scale for the hole current when compared to the electron current. 3) The number or density of arrows indicates how dense the grid is; it has nothing to do with the density of current in that area. 4) The thick broken lines are metallurgical junctions, while the thin broken lines indicate depletion region edges. Figure 3 is an example showing the various features of a vector plot as described above.

The internal study of the bipolar transistor begins with the vector current plots in figures 3 and 4. The entire cross section of the NPN transistor for a V_{BE} of 0.6 and 0.85 V is shown in figure 4, while a blow up of the active base is seen in figure 3. From these plots there are three main observations to be made:

- 1) The electron current entering the device through the collector contact begins by rapidly fanning out until it fills the entire collector sinker. It goes *straight* down into the buried layer where it abruptly turns to a lateral flow. Finally, the electron current flows in the buried layer until the very last moment when it turns sharply upward to enter into the active device region. This occurs at both low and high values of V_{BE} .

- 2) Once the electron current has left the buried layer and entered the active device region, the current flow appears to be nearly vertical (normal to the emitter-base junction), indicating one dimensional flow of electrons. This observation applies to both plots in figure 4 along with the one in figure 3, showing that one dimensional flow of electron current is *independent* of V_{BE} .
- 3) The base current can best be seen in the active base region of figure 3 (remember the hole and electron currents are individually normalized so the hole current scale is greatly magnified). It is clearly seen that the base current is largest near the emitter periphery closest to the base contact. As V_{BE} increases, the base current is seen to segregate even more at the right hand emitter periphery.

From these observations it can be concluded that the electron current flows through the least resistive path in going from the collector contact to the intrinsic collector region. There is no fanning out of the electron current between the intrinsic collector region and the buried layer. This fact is also confirmed for much larger power transistors where the electrons must travel farther from the intrinsic collector to reach the buried layer. There is some fanning out of the electron current, but this is most likely due to diffusion as suggested by Yuan and Eisenstadt [12]. This hypothesis is further borne out by the fact that spreading is seen to increase slightly with increased V_{BE} .

The second observation, relating to one dimensional electron flow in the intrinsic device, is most important. Due to the fact that the one dimensional flow appears to be independent of V_{BE} , it leads to the possibility that for the intrinsic device, electron current density modeling can be reduced to solving a one dimensional problem. The validity of the assumption of one dimensional electron current flow in the intrinsic device is examined

FIGURE 3 Vector plot for the cross section of the emitter-base region.

Hole and electron currents: $V_{BE}=0.600$ and $V_{CE}=3.0$ volts

Hole and electron currents: $V_{BE}=0.850$ and $V_{CE}=3.0$ volts

FIGURE 4 Vector plot of the NPN cross-section for two bias points

further in section 2.1.2 with reference to a more detailed current vector plot.

The third observation, of base current crowding at the emitter periphery nearest the base contact, indicates a two dimensional phenomena. The current crowding can best be explained by a lawn sprinkler fashioned out of a hose by punching holes down its length. At one end of the hose is a cap, while the other end is connected to a faucet. If the water flow density is plotted against position, then a higher density would be seen at the faucet end compared to anywhere else because the water emanating from every hole must come through the hose nearest the faucet. This is exactly what is happening to the base current. The base current is injected along the entire length of the emitter; this injection corresponds to the holes of the hose, while the base current is the analogue of the water flow. Since the base current is constrained to the active base layer (ie. the hose), it will build up at the end nearest the base contact (the faucet), as illustrated in figure 5.

FIGURE 5. Plot of base current crowding

The base current passing through the resistive active base layer causes a voltage drop along the length of the emitter. Furthermore, due to current crowding, the voltage drop is greatest at the emitter periphery nearest the base contact. The result is a position dependent base to emitter voltage, resulting in a position dependent electron current injection; this is a

two dimensional effect and is termed emitter debiasing because the emitter farthest from the base contact will tend to debias. In the "blow-up" of the plots of the intrinsic region to follow, this effect will be seen in much more detail.

The observations so far point to the need for more detailed study of the intrinsic device region to provide further verification and deeper understanding of the property of one dimensional electron flow, and of the fundamental considerations involved in emitter debiasing. The assumption of one dimensional electron flow is the key to reducing the problem of bipolar modeling to one of coupled one dimensional systems. It should be noted that if the electron current is very close to being one dimensional in nature, then the solution obtained from the coupled one dimensional systems is correspondingly close to the one obtained from a true two dimensional model. Therefore, understanding the reason for one dimensional electron flow, in the presence of position varying injection, is the key to accurate bipolar modeling.

2.1.2. A Study Of The Intrinsic Device Region

The study of the NPN transistor now concentrates on the intrinsic device region. To facilitate understanding of what is occurring within the device, three types of plots are generated from PISCES data. The first plot, like the ones shown before, is a vector plot of the hole and electron current densities blown up to show only the intrinsic device region. The second plot shows the total magnitude and the lateral component of the electron current density along a horizontal cross-section at various positions in the base. The third plot shows the dependence of the hole and electron current density magnitudes along a vertical cross-section through the emitter and base. These plots are given for V_{BE} of 0.6, 0.8, 0.85, and 0.94 volts. These plots can be found in figures 6 to 17 and they will be

referenced throughout the thesis.

Starting with the vector plots of electron and hole current densities in figures 6 to 9, we see that the electron current in the base and collector flows essentially vertically, deviating little from vertical flow even at the edges of the emitter. Since the plots have been made from low to very high values of V_{BE} , it is safe to assume that the electron current may be modelled as only one dimensional, even under the condition of non-uniform injection across the emitter (for high values of V_{BE}). This simplifying step is the key to providing a tractable, analytic model for the bipolar transistor.

Further study of the vector plots in figures 6 to 9 show the effects of high level injection. At a V_{BE} of 0.6 V there are no high level effects. As V_{BE} is increased to 0.85 V, high level effects are displayed in the depletion region between base and collector. The strange ellipse of depletion boundary within the depletion region in the collector signifies that the injection of electron carriers is causing conductivity modulation. To understand why the ellipse of depletion region boundary signifies conductivity modulation, the definition of a depletion region boundary in PISCES must be understood. This boundary is taken to be any point in space at which the majority carrier concentration equals half of the doping concentration. Then, the ellipse signifies an enclosed region where the majority carrier concentration is close to or exceeds the doping concentration, implying conductivity modulation. In fact, as V_{BE} is increased to 0.94 V, the entire depletion region in the collector has collapsed, signifying that conductivity modulation has spread throughout the collector.

Carrying on with the study of the vector plots in figures 6 to 9, a clear picture of emitter debiasing and associated Kirk effect [13] can be seen. For the plots of V_{BE} greater than or

equal to 0.85 V, it is observed that the depletion region boundary on the base side of the base-collector junction is no longer planar with respect to the base-collector metallurgical junction. This is evidence of position dependent conductivity modulation caused by emitter debiasing at high injection levels. The Kirk effect is base push-out caused by an increase of the majority carrier concentration in the base induced by minority carrier (electron) injection - ie. by conductivity modulation of the base at high injection levels.

FIGURE 18. Base-push out due to the Kirk effect

The non-planar depletion region boundary is due to emitter debiasing which causes current crowding at the emitter periphery closest to the base contact. The result is preferential injection of electrons at the emitter periphery nearest the base contact, and rapidly decreasing minority carrier injection as one proceeds along the length of the emitter. This position dependent injection causes conductivity modulation and hence base push-out to also be position dependent.

It is important to appreciate that conductivity modulation, or the resistance of the active base layer, as observed and described above, varies with the amount of minority carrier injection. A lowering of the active base resistance due to minority carrier injection will decrease the effect of emitter debiasing, and partially offset the reduction of terminal currents due to emitter debiasing. This has important modeling implications for emitter debiasing. To accurately model emitter debiasing, the effects of conductivity modulation in the base must be accounted for. The plots generated by PISCES clearly show that conductivity modulation in the base is present. Therefore, conductivity modulation can not be ignored for the sake of reducing the complexity of the model. If the effect of conductivity modulation of the base is ignored, then the modeling of emitter debiasing should also be ignored, for the two are inextricably intertwined.

The observations up to this point have led to two major conclusions that will guide the following model development:

- 1) Electron current flows in only one direction, normal to the plane of the emitter-base junction, in the intrinsic device region. This allows the model for the electron current to be fully captured in one dimension.
- 2) Conductivity modulation definitely occurs in the collector and the base regions. The effect of conductivity modulation must therefore be included in the formulation of a model for emitter debiasing.

In view of the latter conclusion, an understanding of the fundamental physical mechanisms involved in emitter debiasing is essential as a basis of modeling this effect. A study of the plots of electron current relationships in figures 10 to 13 is helpful in this regard.

Figures 10 to 13 show the total magnitude and lateral component of the electron current along horizontal cross-sections of the NPN transistor at three different cross-section

depths. The region of interest in these figures is between the two vertical lines indicating the drawn length of the emitter, where peripheral effects can essentially be ignored. From these plots, the most notable feature is the position dependency of the electron current magnitude. As V_{BE} increases, so does the amount of change in the electron current across the length of the emitter. This is the result of emitter debiasing. For a V_{BE} of 0.94 volts, the amount of emitter debiasing is very significant; however, under the emitter, the lateral component is still seen to be a very small fraction of the total electron current. Thus, even in the face of extreme emitter debiasing, causing a definite two dimensional flow of minority carriers, the direction of electron flow remains close to one dimensional. Furthermore, the effect of emitter debiasing is seen to be very significant even for a modest V_{BE} of 0.8 V. This conclusively shows that emitter debiasing, along with conductivity modulation of the base, is a fundamental effect in bipolar transistors that must be accounted for. Any model that neglects these two intertwined effects will have no hope of providing an accurate representation of the behavior of a transistor over a wide range of operating conditions, nor will it be capable of predicting the behavior of varying transistor layouts in a given fabrication technology.

The next salient feature to be noted in figures 10 to 13 is the sudden increase of the lateral component of electron current at the emitter periphery. This indicates a definite two dimensional flow of electron current at the periphery. Thus, while the electron current may be regarded as one dimensional under the emitter, it is certainly two dimensional at the periphery. Therefore, the peripheral injection of electrons can not be simplified, and must be modelled as a true, two dimensional phenomenon.

The effects of the two dimensional peripheral current can be seen in the difference between

the plots at $0.22\text{ }\mu\text{m}$ and $0.38\text{ }\mu\text{m}$ for the total and lateral components of electron current. The flow of the peripheral current is such that it is injected out of a small region at the periphery and then quickly spreads out as it heads toward the collector. Like the base current, the emitter peripheral current also experiences current crowding effects. This is best shown in figure 10 for a V_{BE} of 0.6 V . The plot at $0.22\text{ }\mu\text{m}$ has equal peaks at both ends of the emitter periphery. There is no emitter debiasing or high level effect, so the peaks are due totally to the fact that the peripheral current crowds together at the injection point. As the peripheral current travels toward the collector, it spreads out and the peaks are not observed at the depths of 0.30 and $0.38\text{ }\mu\text{m}$.

For a V_{BE} of 0.6 volts, the peripheral current accounts for 21% of the total electron current. When V_{BE} reaches 0.94 volts, the peripheral current accounts for 31% of the total electron current. For the same doping profile (ie. the same emitter depth), as the emitter area versus periphery decreases, the percentage contribution of the peripheral current to the total current will rise. Since 21% contribution is a substantial amount, the effect of the emitter periphery on the final terminal outputs must be included in any accurate bipolar model. Furthermore, in recent years the advances in photolithography have reduced lateral dimensions at a faster rate than vertical depths. As a result, the effects of peripheral currents will continue to increase in the coming years of bipolar process development.

The final plots in figures 14 to 17 show a vertical cross section in the middle of the intrinsic device region extending from the emitter through the base. Plotted are the magnitudes of electron and hole current density. The single most important feature of these plots is the perfectly flat, constant electron current flow in the intrinsic base. This observation remains true for all bias conditions plotted. Thus, for this transistor, the electron current is free of

recombination effects in the active base. This fact, of constant electron current along the vertical direction in the active base, will prove very important to the Gummel-Poon model and its enhancements that are to be developed.

The reason for the constant electron current in the intrinsic base region was alluded to in the previous paragraph - namely the absence of recombination in the base. Due to the fact that the active base thickness is very small compared to the carrier diffusion length, it is not surprising that recombination is not a factor in the base. For modern bipolar transistors the thickness of the base is such that recombination in the active base is no longer an effect that must be considered while developing a model for bipolar transistors. This does not preclude the fact that recombination and generation will occur in the space charge layers of the collector and emitter.

The observations up to this point have covered most of the significant two dimensional effects in a bipolar transistor. From this study carried out with PISCES, the following fundamental conclusions regarding the operation and modeling of the bipolar transistor can be drawn:

- 1) The electron current flow in the intrinsic base is both one dimensional and constant in the vertical direction, perpendicular to the base-emitter and base-collector metallurgical junctions.
- 2) Conductivity modulation of the base is definitely present for levels of minority carrier injection in the base that correspond to practical operating conditions of the device.
- 3) Emitter debiasing is a significant effect that results in a position varying injection of electrons into the base, and holes into the emitter. Since this effect is due to base resistance, and because conductivity modulation changes the base resistance, the

effects of emitter debiasing and base conductivity modulation must be included simultaneously in the development of the bipolar model.

- 4) The existence and importance of peripheral current from the emitter must be recognised. The flow of the peripheral current is two dimensional with no sign of symmetry. Thus, any model of the peripheral current will have to deal with the effect as a true, two dimensional problem.

These four conclusions dictate the model development that is to follow. They encompass the fundamental attributes of a bipolar transistor that must be included to arrive at an accurate, scalable model.

Hole and Electron currents: $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 6. Current vector plot of the emitter-base, $V_{BE} = 0.60$ V

Hole and Electron currents: $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 7. Current vector plot of the emitter-base, $V_{BE} = 0.80$ V

Hole and Electron currents: $V_{BE}=0.850$ $V_{CE}=3.0$ Volts

FIGURE 8. Current vector plot of the emitter-base, $V_{BE} = 0.85$ V

Hole and Electron currents: $V_{BE}=0.940$ $V_{CE}=3.0$ Volts

FIGURE 9. Current vector plot of the emitter-base, $V_{BE} = 0.94$ V

Electron current at 0.22, 0.30, 0.38 μm : $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 10. Vertical electron current in the base, $V_{BE} = 0.60$ V

Electron current at 0.22, 0.30, 0.38 μm : $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 11. Vertical electron current in the base, $V_{BE} = 0.80$ V

Electron current at 0.22, 0.30, 0.38 μm : $V_{BE}=0.850$ $V_{CE}=3.0$ Volts

FIGURE 12. Vertical electron current in the base, $V_{BE} = 0.85$ V

Electron current at 0.22, 0.30, 0.38 μm : $V_{BE}=0.940$ $V_{CE}=3.0$ Volts

FIGURE 13. Vertical electron current in the base, $V_{BE} = 0.94$ V

Electron and hole current at 8.5 μm : $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 14. Horizontal cross-section of emitter-base, $V_{BE} = 0.60$ V

Electron and hole current at 8.5 μm : $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 15. Horizontal cross-section of emitter-base, $V_{BE} = 0.80$ V

Electron and hole current at 8.5 μm : $V_{BE}=0.850$ $V_{CE}=3.0$ Volts

FIGURE 16. Horizontal cross-section of emitter-base, $V_{BE} = 0.85$ V

Electron and hole current at 8.5 μm : $V_{BE}=0.940$ $V_{CE}=3.0$ Volts

FIGURE 17. Horizontal cross-section of emitter-base, $V_{BE} = 0.94$ V

2.2. Study Of The Gummel-Poon Model

The PISCES simulator is now used in order to investigate the validity of the Gummel-Poon model for base transport. In particular, the influence of two effects not normally represented in the Gummel-Poon model, namely bandgap narrowing and electric field dependent mobility, are studied. To ascertain the effect of these two phenomena, a comparison between the usual formulation of the Gummel-Poon model and the output from PISCES will be made. Furthermore, by re-deriving the Gummel-Poon model, the first step towards a bipolar model that includes the two dimensional effects studied previously will have been made.

This study into the effect of bandgap narrowing and electric field dependent mobility will commence with the derivation of the Gummel-Poon model. As was noted in the previous section, the minority carrier (electron) current flow in the active base can be regarded as one dimensional and constant. These two observations are the cornerstone of the Gummel-Poon model. Since the electron current flow is one dimensional, the current transport equations can be written with total instead of partial derivatives. The device geometry considered is shown in figure 19:

FIGURE 19. Geometric definition of the NPN transistor for theoretical work

The derivation of the Gummel-Poon base transport model begins with the current transport equations.

$$J_n = qn\mu_n E_x + qD_n \frac{dn}{dx} \quad (2.1)$$

$$J_p = qp\mu_p E_x - qD_p \frac{dp}{dx} \quad (2.2)$$

Using eqn (2.2) and solving for E_x gives

$$E_x = \frac{J_p}{qp\mu_p} + \frac{D_p}{\mu_p} \frac{1}{p} \frac{dp}{dx} \quad (2.3)$$

Substituting eqn (2.3) into eqn (2.1) gives

$$\begin{aligned} J_n &= qD_n \frac{dn}{dx} + qn\mu_n \left(\frac{J_p}{qp\mu_p} + \frac{D_p}{\mu_p} \frac{1}{p} \frac{dp}{dx} \right) \\ &= qD_n \left(\frac{dn}{dx} + \frac{n}{p} \frac{dp}{dx} \right) + \frac{n\mu_n}{p\mu_p} J_p \end{aligned} \quad (2.4)$$

Multiply through by (p) to get

$$\begin{aligned} pJ_n &= qD_n \left(p \frac{dn}{dx} + n \frac{dp}{dx} \right) + \frac{n\mu_n}{\mu_p} J_p \\ &= qD_n \left(\frac{d(np)}{dx} \right) + \frac{n\mu_n}{\mu_p} J_p \end{aligned} \quad (2.5)$$

Then by solving eqn (2.5) for the derivative of (pn) gives

$$\frac{d(np)}{dx} = \frac{pJ_n}{qD_n} - \frac{n\mu_n}{qD_n\mu_p} J_p$$

Using

$$\frac{D_i}{\mu_i} = \frac{kT}{q} \Rightarrow D_n\mu_p = D_p\mu_n$$

gives

$$\frac{d(np)}{dx} = \frac{pJ_n}{qD_n} - \frac{nJ_p}{qD_p} \quad (2.6)$$

Integrating eqn (2.6) with respect to (x) gives

$$pn(x_2) - pn(x_1) = \frac{1}{q} \left\{ \int_{x_1}^{x_2} \frac{p(x)J_n(x)}{D_n(x)} dx - \int_{x_1}^{x_2} \frac{n(x)J_p(x)}{D_p(x)} dx \right\} \quad (2.7)$$

Given that $p(x) \gg n(x)$ and $p(x)J_n(x) \gg n(x)J_p(x)$, one has that

$$\int_{x_1}^{x_2} \frac{p(x)J_n(x)}{D_n(x)} dx \gg \int_{x_1}^{x_2} \frac{n(x)J_p(x)}{D_p(x)} dx$$

As was pointed out in figures 14 to 17, recombination in the intrinsic base is negligible. This, causes $J_n(x)$ to be a constant J_n . Using this and substituting the previous assumption into eqn (2.7) gives:

$$J_n = \frac{q[pn(x_2) - pn(x_1)]}{\int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx} \quad (2.8)$$

Eqn (2.8) is the Gummel-Poon model for the transport current J_n . By removing D_n from the integral, and using an effective diffusion coefficient gives with a small degree of approximation:

$$J_n = \frac{q\bar{D}_n[pn(x_2) - pn(x_1)]}{\int_{x_1}^{x_2} p(x) dx}, \text{ where } \bar{D}_n = \frac{KT}{q(x_2 - x_1)} \left[\int_{x_1}^{x_2} \frac{1}{\mu_n} dx \right]^{-1} \quad (2.9)$$

Finally, a low level version of the Gummel-Poon model is achieved by assuming $p(x) \approx N(x)$. This gives:

$$J_n = \frac{q\bar{D}_n n_i^2 \left[e^{\frac{V_{BC}}{V_T}} - e^{\frac{V_{BE}}{V_T}} \right]}{\int_{x_1}^{x_2} N(x) dx} \quad (2.10)$$

The assumptions made to arrive at this model are:

- 1) The electron current flows in only one dimension, normal to the plane of the junctions.

- 2) The electron current is constant throughout the active base region.
- 3) Einstein's equation of $D/\mu = kT/q$ is obeyed.
- 4) The effect of the base current is small. This comes from assuming $pJ_n \gg nJ_p$.
- 5) An effective diffusion coefficient can be used.
- 6) Finally, that $p(x) \approx N(x)$.

The first two assumptions have been shown to be valid in the foregoing PISCES studies. The third assumption of the applicability of the Einstein equation will restrict the application of the Gummel-Poon model to regions where there is no change in mobility due to high electric fields. This requirement restricts the application of this model to the active base region. The fourth assumption will only fail at very high V_{BE} . The final assumption will restrict the simplified Gummel-Poon model, given in eqn (2.10), to low level, for in high level $p(x) > N(x)$. However, the Gummel-Poon model in general has no problems with modeling J_n in the presence of high level injection.

To see if the Gummel-Poon model is accurate, and to see what is the effect of bandgap narrowing and field dependent mobility, a comparison is made between the Gummel-Poon model and the electron current output from PISCES. Three PISCES simulations have been made to see the effect of bandgap narrowing and electric field dependent mobility. The first simulation invokes both the bandgap and the field dependent mobility models that are provided by PISCES. The second simulation removes the field dependent mobility model, while the third simulation removes just the bandgap model. By comparing the three PISCES simulations to the Gummel-Poon model, conclusions can be drawn on the importance of bandgap narrowing and electric field dependent mobility on J_n . From this study, one will know whether these two effects need to be taken into account in the

following bipolar model development.

To gain as much information as possible from the comparison between PISCES and the Gummel-Poon model, the last three assumptions (detailed previously) with regard to the Gummel-Poon model, will also be tested. The model names, model definitions, and the assumptions they are testing are as follows:

$$\text{GP1: } J_n = \frac{q[pn(x_2) - pn(x_1)] + \int_{x_1}^{x_2} \frac{n(x)J_p(x)}{D_p(x)} dx}{\int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx}$$

Remembering J_n is constant, then solving eqn (2.7) for J_n results in this expression. This result removes the assumptions of $pJ_n \gg nJ_p$, $p(x) \approx N(x)$, and the use of an effective diffusion coefficient.

$$\text{GP2: } J_n = \frac{q[pn(x_2) - pn(x_1)]}{\int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx}$$

This result removes the assumptions of $p(x) \approx N(x)$ and the use of an effective diffusion coefficient. By comparing this model to GP1, the effect of $pJ_n \gg nJ_p$ can be determined.

$$\text{GP3: } J_n = \frac{q\bar{D}_n[pn(x_2) - pn(x_1)]}{\int_{x_1}^{x_2} p(x) dx}$$

This result removes the assumptions of $p(x) \approx N(x)$. By comparing this model to GP2, the effect of using an effective diffusion coefficient (2.9) can be seen.

$$\text{GP4: } J_n = \frac{q\bar{D}_n n_i^2 \left[e^{\frac{V_{BC}}{V_T}} - e^{\frac{V_{BE}}{V_T}} \right]}{\int_{x_1}^{x_2} N(x) dx}$$

This is the low level Gummel-Poon model. By comparing this to GP3, the effect of assuming $p(x) \approx N(x)$ will be seen.

Figures 20 to 32 compare the electron transport current density actually given by PISCES and the various forms (GP1 to GP4) of the Gummel-Poon model given above. A fifth model, GP5, was also included consisting of the low-level model GP4 with the addition of the term from GP1 due to the hole current. However, GP5 provides nothing more than

GP1, so it is ignored in the following discussions.

The expressions GP1 to GP5 were evaluated numerically in terms of the position-dependent quantities involved, as obtained from PISCES, eliminating the need to resort to simplifying assumptions. To help see the difference between each of these models and the output from PISCES, all model outputs are normalized to the electron current JN2 at a depth of $0.33\text{ }\mu\text{m}$ (where $0.33\text{ }\mu\text{m}$ is near the collector edge of the base). The PISCES output of electron current density JN1 is taken at a depth of $0.19\text{ }\mu\text{m}$, this being close to the emitter edge of the base. By plotting JN1/JN2, any discrepancy from a value of one indicates that the current flow is two dimensional at that location, and therefore, the models GP1 to GP5 are strictly not valid. In the plots of figures 20 to 32, the various normalized current densities are compared for a V_{BE} of 0.6, 0.8, 0.9, and 0.95 volts, for positions across the base that include the periphery¹.

Starting with figure 20 for $V_{BE} = 0.6\text{ V}$, an error of around 30% is seen for all versions of the Gummel-Poon model compared with the true electron current. This indicates that fundamental effects have not been represented. In figure 21, the electric field dependence on mobility in PISCES is removed, but an improvement of only 2% is then observed. Therefore, the effect of electric field dependent mobility is not a significant factor in the active base, at least not for a V_{BE} of 0.6 V

Further study of figure 21 shows no difference between GP1 and GP2 (the lines are on top of each other). This indicates that the effect of the majority hole current on the electron

¹ For each emitter-base voltage, three plots are shown: one for full PISCES representation, one with the electric field dependent mobility model removed from PISCES, and one with the bandgap narrowing model removed from PISCES.

current is negligible. Thus, it appears that the integral for the hole current can be dropped. Going from GP2 to GP3 shows an increased error of 4%. Therefore, the use of an effective diffusion coefficient appears to cause only a small error (the effective diffusion coefficient is given in 2.9). Finally, going from GP3 to GP4 results in a decreased error of 10%. This indicates that the assumption of $p(x) = N(x)$ is not valid at $V_{BE} = 0.6$ V, though the error introduced is small.

To see the effect of bandgap narrowing, figure 22 shows the effect of removing the bandgap model from PISCES. Immediately the error in GP1 and GP2 has fallen to well under 1%. Therefore, the effect of bandgap narrowing in the base of the bipolar transistor, represents 30% of the electron current. The effect of bandgap narrowing in the base is seen as a fundamental effect that is in need of further study. Furthermore, it appears that any bipolar model that does not include the effect of bandgap narrowing, will be incapable of accurately representing the transport current characteristics, except by some non-physical artifice.

Moving on to a V_{BE} of 0.8 V, figure 23 shows that the difference between GP1 and GP3 has not really changed. Thus the use of an effective diffusion coefficient remains fairly good. However, the difference between GP3 and GP4 has increased dramatically compared with the results for a V_{BE} of 0.6 V. This shows that the use of $p(x) \approx N(x)$ is definitely not valid since the injection level is sufficiently high for $p(x)$ to be significantly larger than $N(x)$. When the mobility model is removed, figure 24 shows the improvement is negligible. Thus, the effect of electric field dependent mobility is still unimportant at a V_{BE} of 0.8 V. However, as before, when the bandgap model is removed in figure 25, a dramatic improvement results, bringing GP1 and GP2 to within an error of less than 1%.

Strong evidence for the need to model bandgap narrowing in the base is present.

At a V_{BE} of 0.9 V, figure 26 shows a tiny difference between GP1 and GP2. However, the difference is so small that the problems of solving the integral for the hole current greatly outweigh the benefits of its inclusion. The difference between GP1 and GP3 is seen to remain relatively constant. This result reinforces the validity of the approximation of an effective diffusion coefficient. There is now a very large difference between GP3 and GP4 due to the large increase of $p(x)$ over $N(x)$ with increasing V_{BE} . This increase in $p(x)$, due to high level, is best appreciated by noting that the difference between GP4 and GP3 increases as one proceeds from left to right (ie. towards the base contact). Once again, when the mobility model is removed from figure 27, the improvement is slight, showing that electric field dependent mobility is not an issue in the active base region. When the bandgap model is removed in figure 28 there is a dramatic improvement in accuracy, also found previously.

The final plots are for a V_{BE} of 0.95 V. Figure 29 shows basically the same things that have been seen before. First of all, the error between GP1 and GP2 is small. The use of an effective diffusion coefficient gives an error of around 4%, as has been the case for all other biases. A huge difference between GP4 and GP1 results when the grossly invalid approximation of $p(x) \approx N(x)$ is made. When the mobility model is removed in figure 30, a rather large improvement results this time. This suggests that for high V_{BE} , the electric fields due to the Webster effect may be affecting the mobility. When the bandgap model is removed in figure 31, the error between GP1 and PISCES has once again fallen under 1%. This shows that the error reduction caused by removing the mobility model was somehow related to the effect of bandgap narrowing; for the error between PISCES and GP1 fell to

under 1% when only the bandgap model, and not the mobility model, was removed from the PISCES simulation. Using only the GP1 model, a comparison between the effects of bandgap narrowing and electric field dependent mobility is shown in figure 32 on page 49.

The following conclusions are drawn from results of the comparison between the various Gummel-Poon models for the electron current density and PISCES:

- 1) Even though the injection from the emitter is in general non-uniform due to emitter debiasing, the electron current density can be represented by a one dimensional Gummel-Poon model. This is established by the accuracy obtained in the case where the bandgap narrowing expression is removed from PISCES. In other words, the model for the electron current density is *independent* of the lateral effects of emitter debiasing.
- 2) The improvement in going from GP2 to GP1 was slight for all ranges of V_{BE} . This means that the complex integral for the hole current in GP1 can be removed.
- 3) The loss of accuracy in going from GP2 to GP3 was constant at around 4% for all ranges of bias. This means that if an improved model for the effective diffusion coefficient is used, the error due to the use of GP3 can be significantly reduced. Furthermore, because the error in GP3 remained nearly constant at 4% for all bias ranges, the effective diffusion coefficient can be accurately extracted for any particular bipolar or BICMOS process.
- 4) The effect of including electric field dependent mobility is totally negligible. Thus, even though the fields in the active base, as a result of doping and the Webster effect, can reach a few kilovolts per centimeter, the mobility remains unaffected.
- 5) The error as a result of assuming $p(x) \approx N(x)$ can exceed 100%. Therefore, the effect of high level, causing $p(x) > N(x)$, must be included in the bipolar model.
- 6) The effect of bandgap narrowing in the base is very significant. Accurate

modeling would appear to demand its representation. It is noted that as long as the device remains out of high level, the error caused by not accounting for bandgap narrowing remains a constant factor. Thus, the use of a constant saturation current will work in low level if this current is extracted from measured data. However, for accurate modeling under practical conditions, which include high level operation, the effect of bandgap narrowing must be properly accounted for. This is essential under all conditions of operation if the electron current is to be predicted for a given device structure, including the observed property that the effect of bandgap narrowing changes under the influence of high level injection (ie. when $p(x) > N(x)$).

The results of this chapter provide a basic qualitative understanding of the two dimensional attributes present in a bipolar transistor. The conclusions drawn from the comparison between the Gummel-Poon model and PISCES will guide the development of the models for electron current density and emitter debiasing. The most important result of this chapter is an understanding of the complex inter-relationships between the various fundamental phenomena involved in the operation of a bipolar transistor. The most surprising result is that bandgap narrowing in the base is a fundamental effect that requires modeling.

Gummel-Poon comparison to PISCES. $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 20. Comparison of G-P trans. eqn. to PISCES, all models, $V_{BE} = 0.60$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 21. Comparison of G-P trans. eqn. to PISCES, const. mob., $V_{BE} = 0.60$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.600$ $V_{CE}=3.0$ Volts

FIGURE 22. Comparison of G-P trans. eqn. to PISCES, const. E_g , $V_{BE} = 0.60$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 23. Comparison of G-P trans. eqn. to PISCES, all models, $V_{BE} = 0.80$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 24. Comparison of G-P trans. eqn. to PISCES, const. mob., $V_{BE} = 0.80$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.800$ $V_{CE}=3.0$ Volts

FIGURE 25. Comparison of G-P trans. eqn. to PISCES, const. E_g , $V_{BE} = 0.80$ V

FIGURE 26. Comparison of G-P trans. eqn. to PISCES, all models, $V_{BE} = 0.90$ V

FIGURE 27. Comparison of G-P trans. eqn. to PISCES, const. mob., $V_{BE} = 0.90$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.900$ $V_{CE}=3.0$ Volts

FIGURE 28. Comparison of G-P trans. eqn. to PISCES, const. E_g , $V_{BE} = 0.90$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.950$ $V_{CE}=3.0$ Volts

FIGURE 29. Comparison of G-P trans. eqn. to PISCES, all models, $V_{BE} = 0.95$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.950$ $V_{CE}=3.0$ Volts

FIGURE 30. Comparison of G-P eqn. to PISCES, const. mob., $V_{BE} = 0.95$ V

Gummel-Poon comparison to PISCES. $V_{BE}=0.950$ $V_{CE}=3.0$ Volts

FIGURE 31. Comparison of G-P trans. eqn. to PISCES, const. E_g , $V_{BE} = 0.95$ V

Using GP1, the effect of mobility and bandgap models is shown.
 $V_{BE}=0.950$, $V_{CE}=3.0$ V

FIGURE 32. Comparison of G-P eqn. to PISCES, various models, $V_{BE} = 0.95$ V

3. A ONE DIMENSIONAL BASE MINORITY CARRIER TRANSPORT MODEL INCLUDING BANDGAP NARROWING

In accordance with the conclusions reached in chapter 2, the objective of this chapter is the development of a one dimensional model for the minority current density in the active base underneath the emitter, including representation for the effect of bandgap narrowing in the base.

As was shown in chapter 2, the Gummel-Poon model is potentially capable of providing a very accurate representation of the minority current density in the active base. In this chapter we shall be concerned with two versions of the Gummel-Poon model:

$$J_n = \frac{q\overline{D}_n n_1^2 \left[e^{\frac{V_{BC}}{V_T}} - e^{\frac{V_{BE}}{V_T}} \right]}{\int_{x_1}^{x_2} N(x) dx} \quad (3.1)$$

$$J_n = \frac{q\overline{D}_n [pn(x_2) - pn(x_1)]}{\int_{x_1}^{x_2} p(x) dx} \quad (3.2)$$

$$\overline{D}_n = \frac{KT}{q(x_2 - x_1)} \left[\int_{x_1}^{x_2} \frac{1}{\mu_n} dx \right]^{-1}$$

Eqn (3.1) applies for low level operation ($V_{BE} < 0.7$ V approximately) for which $p(x) \approx N(x)$. For higher injection levels eqn (3.2) needs to be used, and knowledge of the majority carrier concentration distribution $p(x)$ is required. At both levels of injection the effects of bandgap narrowing in the base need to be included. The objective is to modify eqns (3.1) and (3.2) to include the influence of bandgap narrowing.

3.1. A Model For The Majority Carrier Concentration In The Base

In order to capture the effects of high level injection in the Gummel-Poon model an accurate representation of the majority carrier concentration in the base ($p(x)$ for an NPN transistor) needs to be found. In the study of emitter debiasing, the two dimensional nature of this phenomena is modelled by taking the one dimensional vertical flow of minority carriers and coupling it to a one dimensional lateral debiasing phenomenon caused by a resistive effect. The essence of the solution to emitter debiasing is that an analytic closed form model results. To achieve this goal of a closed form model, it is very important that the one dimensional model for the electron current density remain simple. Therefore, in the development of the model for the majority carrier concentration, simplicity must prevail.

The reason for placing such strict limitations on the model development for the flow of minority carriers in the base is because it is only an intermediate model. The effect of emitter debiasing was shown in chapter 2 as a phenomenon that simply can not be ignored. Any bipolar model that begins by assuming the effect of emitter debiasing (also termed base current crowding) is small enough to be ignored is simplistic and is limited to low level modeling. It is sometimes possible to forcibly manipulate models, like the SPICE bipolar model, to fit characteristics that include the effects of emitter debiasing. However, the result is a model that is incapable of scaling to other device layout dimensions. Thus, true representation of the effect of emitter debiasing is the key to an accurate bipolar model, and any intermediate model, like the one dimensional minority current model, must not compromise the emitter debiasing model that is developed in chapter 4.

3.1.1. Using Differential Equations For The Solution Of $p(x)$

For the general (or high level) form of the Gummel-Poon model (eqn (3.2)), the distribution $p(x)$ is needed. The model development for the majority carrier concentration starts by assuming quasi neutrality. This assumption is justified as long as the boundaries x_1 and x_2 do not enter the depletion regions since, as illustrated in figure 33, the space charge over most of the active base is small with respect to the minority carrier concentration.

Thus, assuming quasi neutrality:

$$n'(x) = p'(x)$$

where

$$n'(x) = n(x) - n_0(x)$$

and

$$p'(x) = p(x) - N_{AB}(x)$$

$$n'(x) = p(x) - N_{AB}(x)$$

or

$$p(x) = n'(x) + N_{AB}(x) \quad (3.3)$$

Since for even low ranges of V_{BE} , $n(x) \gg n_0(x)$, the above equation becomes

$$n'(x) \approx n(x)$$

Thus, if a solution for the excess minority carrier concentration $n'(x)$ is possible, eqn (3.3) will give the solution for $p(x)$. Using ambipolar theory as shown by Warner and Grung[14], the current transport equations for holes and electrons (eqns (2.2) and (2.1)) give the electric field as:

$$E_x = \frac{(J - J_n) + qD_p \frac{dp}{dx}}{q\mu_p}$$

where $J = J_n + J_p$. Or using eqn (3.3) gives:

$$E_x = \frac{(J - J_n) + qD_p \frac{dn'}{dx} + qD_p \frac{dN_{AB}}{dx}}{q(n' + N_{AB})\mu_p}$$

Substituting this back into the electron transport equation (eqn (2.1)) gives:

$$J_n = qD_n \frac{dn}{dx} + qn'b_B\mu_p \frac{(J - J_n) + qD_p \frac{dn'}{dx} + qD_p \frac{dN_{AB}}{dx}}{q(n' + N_{AB})\mu_p}$$

$$\text{where } b_B = \frac{\mu_n}{\mu_p} = \frac{D_n}{D_p}$$

Furthermore, for almost all bias ranges $\frac{dn}{dx} \approx \frac{dn'}{dx}$, and $(J - J_n) \rightarrow 0$, giving

$$J_n = qD_n \frac{dn'}{dx} + n'b_B \frac{qD_p \frac{dn'}{dx} + qD_p \frac{dN_{AB}}{dx}}{(n' + N_{AB})}$$

or, simplifying,

$$J_n = qD_n \frac{2n' + N_{AB}}{n' + N_{AB}} \frac{dn'}{dx} + \frac{qD_n n'}{n' + N_{AB}} \frac{dN_{AB}}{dx} \quad (3.4)$$

Eqn (3.4) represents a non-linear ordinary differential equation for the solution of the excess minority carrier concentration in the base of an NPN transistor. The inherent assumption in eqn (3.4) is that $n(x) \gg n_0(x)$; however, this is perfectly valid for virtually all but the smallest V_{BE} . Before a solution of eqn (3.4) is attempted, it is worthwhile to appreciate the physical significance of its two terms.

The first term of the right hand side of eqn (3.4), containing the derivative of n' , represents the diffusion and high level drift of the minority carriers in the base. The second term represents the effect of non-linear doping in the base on the flow of the minority carriers. It

is interesting to see the effect of these two terms on J_n . Let x increase from emitter to collector. Then the derivative of n' must be monotonically decreasing, so that the first term is always negative. Now, for a typical doping in the base (such as for the test NPN transistor of chapter 2), the derivative of N_{AB} is negative, and the second term is also always negative, representing the effect of the aiding field due to the non-linear doping in the base. Therefore, both terms in eqn (3.4) add together to form J_n .

To gain a deeper understanding of eqn (3.4), we take the limiting cases of low and extreme high level operation. In low level operation $n' \ll N_{AB}$ giving the Moll-Ross model:

$$J_n(\text{low level}) \approx qD_n \frac{dn'}{dx} + \frac{qD_n n'}{N_{AB}} \frac{dN_{AB}}{dx} \quad (3.5)$$

But the built in electric field due to the doping in the base is:

$$E_b = \frac{KT}{q} \frac{1}{N_{AB}} \frac{dN_{AB}}{dx}$$

When E_b is substituted into eqn (3.5) the following low level equation for n' results:

$$J_n(\text{low level}) \approx qD_n \frac{dn'}{dx} + qD_n \frac{q}{KT} n' E_b = qD_n \frac{dn'}{dx} + q\mu_n n' E_b \quad (3.6)$$

This result confirms the previous analysis of eqn (3.4).

In the case of extreme high level where $n' \gg N_{AB}$, eqn (3.4) becomes:

$$J_n(\text{high level}) \approx q2D_n \frac{dn'}{dx} \quad (3.7)$$

Therefore, when compared to eqn (3.6), the result of high level injection is the flow of minority carriers as if by diffusion with double the diffusion coefficient (this is termed the Webster effect [15]).

It is very interesting to note that the solutions of eqns (3.6) and (3.7) will, in the limit, produce linear relationships for $n'(x)$. Taking eqn (3.6) and letting n' go to zero, but remain large enough so that $n' \gg n_0$, then, due to the restrictions in device fabrication, the drift term under extreme low level vanishes compared to the diffusion term giving:

$$n'(\text{extreme low level}) = \frac{J_n}{qD_n} x + \text{Constant}$$

Moving to the other extreme limit, the solution of eqn (3.7) gives:

$$n'(\text{extreme high level}) = \frac{J_n}{q2D_n} x + \text{Constant}$$

Therefore, in the limit as J_n becomes very small or very large, $n'(x)$ becomes linear with respect to x (this is only true as long as the base doping is low enough so that D_n is a constant). Thus, the general shape of $n'(x)$ is known in the two extreme cases. But how $n'(x)$ evolves from one extreme to the other remains locked within the non-linear equation given as eqn (3.4).

The eqns (3.4), (3.6) and (3.7) together represent the complete, low level, and extreme high level ordinary differential equations for the solution of $n'(x)$ respectively. Due to the non-linear nature of eqn (3.4), it appears that no closed form analytic solution is possible. In fact, even if a constant base doping N_{AB} is used, only an implicit solution for eqn (3.4) will result (see appendix A for this solution and a series solution of eqn (3.4)). From the proceeding analysis of eqn (3.4), it is only possible to understand qualitatively the forces driving the flow of minority carriers. Therefore, other methods for the solution of the majority carrier concentration, needed for the Gummel-Poon model, have to be pursued.

FIGURE 33. Horizontal cross section of the base showing the space charge, $V_{BE}=0.9\text{V}$

3.1.2. Using Charge Control Theory For The Solution Of $p(x)$

The previous section has provided some rough qualitative insight into the relation between current density and carrier concentrations in the active base region, but solution of the differential equation (3.4) for $n'(x)$, and hence $p(x)$, is intractable. In the interests of achieving a reasonably simple model for $p(x)$ or, more to the point, of the integral of $p(x)$ over the active base as required by the Gummel-Poon model, the charge control approach is now adopted [16].

Define:

$$Q = q \int_{x_1}^{x_2} n'(x) dx = J_n \tau$$

where τ is the effective base transit time. Then by eqn (3.3)

$$q \int_{x_1}^{x_2} p(x) dx = q \int_{x_1}^{x_2} N(x) dx + \tau J_n \quad (3.8)$$

The integral for $N(x)$ will be independent of J_n until Kirk effect or "base push out" results. Since phenomena associated with the collector are not being treated in this thesis, the integral for $N(x)$ is *independent* of J_n . This allows eqn (3.8) to capture the effect of high level conductivity modulation on the majority carrier concentration in the base. It should be noted that in a strict sense the quantity τ has a dependency on J_n . The problem with defining τ is that in low level it takes on one value, and in high level another. Since τ is used to model the effects of high level, it makes sense to extract and optimize τ for high level modeling. Furthermore, in low level, the term $J_n \tau$ is by definition negligible. Therefore, τ is to be viewed as a high level parameter for D.C. modeling that has very

loose ties to the A.C. models from which it is defined.

The reason for resorting to charge control theory for the solution of the integrated charge density Q is to maintain a model that includes the basic effects of high level injection, and yet does not introduce complexity to the point where further model developments will become intractable. Furthermore, models that rely on charge control theory, such as the SPICE bipolar model [1], can be made to represent the device characteristics with a high degree of accuracy. The problem is that it is typically very difficult to predict with any certainty the parameter τ before the bipolar process is created. However, τ will be layout independent because it is the vertical and not the lateral geometry that determines its value and the vertical geometry is fixed by the process being used. Therefore, the use of a charge control model at this stage of the bipolar model development will permit subsequent model scaling to cover all layout geometries.

3.2. A One Dimensional Model For The Minority Current Density In the Active Base

Using charge control theory with τ a constant process parameter, a simple yet accurate model for J_n is achieved. Substituting eqn (3.8) into eqn (3.2), the following model for J_n results:

$$J_n = \frac{q^2 \bar{D}_n [pn(x_2) - pn(x_1)]}{Q_{b0} - \tau J_n} \quad \text{where} \quad Q_{b0} = q \int_{x_1}^{x_2} N(x) dx$$

$$\Rightarrow J_n = \frac{Q_{b0} - \sqrt{4q^2 \bar{D}_n \tau (pn(x_1) - pn(x_2)) + Q_{b0}^2}}{2\tau} \quad (3.9)$$

Eqn (3.9) represents the one dimensional model for the electron current traversing the base, and is close to the equation given by Gummel and Poon. The terms $pn(x_1)$ and $pn(x_2)$ represent the pn products at the emitter and the collector edges of the active base region respectively. This equation will give the classic high and low level relation to V_{BE} . It is also possible to bring in the Early effect by allowing Q_{b0} to become a function of V_{BE} and V_{CE} as discussed further in chapter 4 on emitter debiasing.

Expressing the pn products in eqn (3.9) in terms of V_{BE} and V_{BC} :

$$J_n = \frac{Q_{b0} - \sqrt{4q^2 n_i^2 \bar{D}_n \tau (e^{\beta V_{BE}} - e^{\beta V_{BC}}) + Q_{b0}^2}}{2\tau} \quad \text{where} \quad \beta = \frac{1}{V_T} \quad (3.10)$$

A numerical problem with eqns (3.10) and (3.9) can occur when the term Q_{b0}^2 dominates over the rest of the radical. Depending on the floating point accuracy of the computer used to execute the model given by eqns (3.9) and (3.10), round off error can result in intolerable errors. To remedy this problem, and to see that eqns (3.9) and (3.10) degrade into the low level asymptote given by the simple Gummel-Poon model (eqn (3.1)), eqn

(3.9) is expanded in Maclaurin series in terms of $(pn(x_1) - pn(x_2))$. For low level operation this gives:

$$J_n(\text{low level asymptote}) = \frac{-q^2 \overline{D}_n (pn(x_1) - pn(x_2))}{Q_{b0}} \quad (3.11)$$

- the low level expression of eqn (3.2). To eliminate the effects of round off error we use the following:

$$J_n = \left\{ \begin{array}{ll} \frac{Q_{b0} - \sqrt{4q^2 \overline{D}_n \tau (pn(x_1) - pn(x_2)) + Q_{b0}^2}}{2\tau}, & \text{for } 4q^2 \overline{D}_n \tau (pn(x_1) - pn(x_2)) \geq \epsilon Q_{b0}^2 \\ \frac{-q^2 \overline{D}_n (pn(x_1) - pn(x_2))}{Q_{b0}}, & \text{for } 4q^2 \overline{D}_n \tau (pn(x_1) - pn(x_2)) < \epsilon Q_{b0}^2 \end{array} \right\} \quad (3.12)$$

Where ϵ is adjusted, for the specific computer being used, to minimize roundoff error.

Equations (3.9), (3.10), (3.11), and (3.12) are strictly only applicable when the emitter is forward biased and the collector is reverse biased. Generally speaking, however, the restrictions imposed on these equations can be relaxed to include moderate forward bias on the collector. As long as the electron concentration at the collector is far lower than at the emitter, then the use of a constant τ will suffice. In more concrete terms, as long as $V_{BC} < V_{BE} - 5V_T$, equations (3.9) to (3.12) may be used.

Eqn (3.12) is in fact quite close to the SPICE bipolar model. This model is complex enough to cover the major high level effects. Yet, eqn (3.12) remains simple enough to allow for the development of a tractable closed form model that includes the complex two dimensional effects observed in chapter 2.

3.3. A Model For The Effect Of Bandgap Narrowing

It has been seen in chapter 2 that bandgap narrowing in the base has a major effect on the base transit current J_n . In this section a modified Gummel-Poon model is developed as a representation of the bandgap narrowing effect.

We begin by defining the energy gap as:

$$E_C - E_V = E_g + \Delta E_g \quad (3.13)$$

where E_C and E_V are the conduction and valence band energies respectively, and ΔE_g represents bandgap narrowing due to impurity doping with respect to the intrinsic or lightly doped case. The effective intrinsic carrier concentration is given by:

$$n_{ie}^2 = n_{i0}^2 e^{\frac{-\Delta E_g}{kT}}$$

where n_{i0} is the intrinsic carrier concentration in absence of bandgap narrowing. Then

$$\Delta E_g = -2kT \ln\left(\frac{n_{ie}}{n_{i0}}\right) \quad (3.14)$$

Now, a spatially varying bandgap causes the electric fields for holes E_p and electrons E_n to differ: ie.

$$E_p = \frac{1}{q} \nabla E_V$$

and

$$E_n = \frac{1}{q} \nabla E_C$$

using eqn (3.13)

$$E_n = \frac{1}{q} (\nabla(\Delta E_g) + \nabla E_V) \quad (3.15)$$

Substituting eqn (3.14) into eqn (3.15) one obtains:

$$E_n = -\frac{2kT}{q} \nabla(\ln(n_{ie})) + E_p \quad (3.16)$$

Eqn (3.16) gives the relationship between the two electric fields and the effective intrinsic

carrier concentration n_{ie} . In view of the different electric fields for holes and electrons, the current transport equations become:

$$\left. \begin{aligned} J_n &= qn\mu_n E_n + qD_n \nabla n \\ J_p &= qp\mu_p E_p - qD_p \nabla p \end{aligned} \right\} \quad (3.17)$$

Converting to one dimensional analysis, the last equation gives:

$$E_p = \frac{J_p}{qp\mu_p} + \frac{D_p}{\mu_p} \frac{1}{p} \frac{dp}{dx}$$

Using this with eqn (3.16), J_n is obtained from eqn (3.17) as:

$$\begin{aligned} J_n &= qn\mu_n \left(\frac{J_p}{qp\mu_p} + \frac{D_p}{\mu_p} \frac{1}{p} \frac{dp}{dx} - \frac{2kT}{q} \frac{d(\ln(n_{ie}))}{dx} \right) + qD_n \frac{dn}{dx} \\ &= \frac{n\mu_n}{p\mu_p} J_p - 2qnD_n \frac{d(\ln(n_{ie}))}{dx} + qD_n \left(\frac{dn}{dx} + \frac{n}{p} \frac{dp}{dx} \right) \end{aligned}$$

Multiplying through by p/qD_n and reducing, the exact differential gives:

$$\frac{pJ_n}{qD_n} = \frac{nJ_p}{qD_p} - 2pn \frac{d(\ln(n_{ie}))}{dx} + \frac{d(pn)}{dx}$$

Integrating and remembering that J_n can be considered a constant in the active base gives

$$J_n = \frac{pn(x_2) - pn(x_1) + \int_{x_1}^{x_2} \frac{nJ_p}{qD_p} dx - 2 \int_{x_1}^{x_2} pn \frac{d(\ln(n_{ie}))}{dx} dx}{\int_{x_1}^{x_2} \frac{p(x)}{qD_n(x)} dx} \quad (3.18)$$

When eqn (3.18) is compared to the GP1 model in chapter 2, the first three terms in the numerator and the term in the denominator are common. The fourth term in the numerator containing n_{ie} is the difference, and is due to the effects of bandgap narrowing. From the results obtained in chapter 2, the third term in the numerator of (3.18) may be dropped while $D_n(x)$ can be removed from the integrand and replaced with an effective diffusion coefficient $\overline{D}_n$. This results in a modified Gummel-Poon model:

$$J_n = q\bar{D}_n \frac{pn(x_2) - pn(x_1) - 2 \int_{x_1}^{x_2} pn \frac{d(\ln(n_{ie}))}{dx} dx}{\int_{x_1}^{x_2} p(x) dx} \quad (3.19)$$

Furthermore, it is readily seen that if bandgap narrowing is neglected, then eqn (3.19) reduces to the Gummel-Poon analysis of the bipolar transistor.

Before continuing with the solution of eqn (3.19), one should pause to see whether the effect of bandgap narrowing in the base is detrimental to J_n . Given that x increases from emitter to collector, and that $x=x_1$ is at the emitter edge of the active base while $x=x_2$ is at the collector edge of the active base, J_n must be negative. In the additional term in the numerator of eqn (3.19), the derivative of n_{ie} is monotonically decreasing, due to the reduction of the doping with x for typical base dopings found in an NPN transistor. Thus, the effect of this term is to reduce J_n and hence the collector current.

In order to make use of eqn (3.19) for modeling purposes, the numerator term for the effect of bandgap narrowing in the base needs to be evaluated. For this purpose, the pn product and the value of n_{ie} are required as a function of position over the range $x_1 \leq x \leq x_2$ in the base. It will be shown, however, that the latter can be avoided.

Starting with the problem of finding a model for the pn product along the entire width of the base, eqns (3.4) and (3.3) are useful. In section 3.1.1., even though eqn (3.4) did not allow for the solution of $n'(x)$, it does give a solution for the pn product. Using quasi neutrality, eqn (3.3) gives with negligible approximation

$$pn = n^2 + n'N_{AB}$$

now eqn (3.4) is

$$J_n = \frac{qD_n(2n' + N_{AB})\frac{dn'}{dx} + qD_n n' \frac{dN_{AB}}{dx}}{n' + N_{AB}}$$

or

$$\begin{aligned} \frac{J_n}{qD_n} &= \frac{\frac{d(n'^2 + n'N_{AB})}{dx}}{n' + N_{AB}} \\ &= \frac{(n' + N_{AB})\frac{dn'}{dx} + n' \frac{d(n' + N_{AB})}{dx}}{n' + N_{AB}} \end{aligned} \quad (3.20)$$

Thus, dividing by n'

$$\frac{1}{n'} \frac{dn'}{dx} + \frac{1}{n' + N_{AB}} \frac{d(n' + N_{AB})}{dx} = \frac{J_n}{qn'D_n}$$

Integrating with respect to x ,

$$\ln(n') + \ln(n' + N_{AB}) = A_1 + \frac{J_n}{qn'D_n} \int \frac{dx}{n'}, \text{ where } A_1 \text{ is an arbitrary constant}$$

Thus

$$pn = n'^2 + n'N_{AB} = A_2 e^{\frac{J_n}{qn'D_n} \int \frac{dx}{n'}} \quad (3.21)$$

If a linear relation between n' and x is assumed (as is found to be a reasonable approximation for all operating levels in PISCES simulations - see, for example, figure 33), then a solution for the pn product is possible. Let $n'(x)$ be given by $n' = mx + b$. Then eqn (3.21) gives the pn product as:

$$\begin{aligned} pn = pn(x) &= A_2 e^{\frac{J_n}{qn'D_n} \ln(mx + b)} \\ &= A_2 (mx + b)^{\frac{J_n}{qn'D_n}} \end{aligned} \quad (3.22)$$

Assuming idealized reverse bias boundary conditions at the collector boundary $x=x_2$ of the

quasi-neutral base, the two boundary conditions on p_n needed to solve eqn (3.22) are:

$$p_n(x=x_2) = 0 \quad \text{and} \quad p_n(x=x_1) = p_{n1}$$

Applying the boundary condition at the collector,

$$p_n = A_2 m (x - x_2)^K$$

where

$$K = \frac{J_n}{qm\bar{D}_n}$$

Applying the boundary conditions at the emitter, the solution for $p_n(x)$ is obtained as

$$p_n(x) = p_{n1} \left(\frac{x_2 - x}{x_2 - x_1} \right)^K \quad (3.23)$$

With reference to the distributions of n' at low and extreme high level, as presented at the end of section 3.1.1, it is evident that the value of K ranges between 1 and 2 between these respective conditions.

Presently, the relationship between K and J_n is not known. To use eqn (3.23) in the solution of eqn (3.19), it is imperative that K be solved for in terms of J_n . The solution for K begins with eqn (3.4). Assuming that the effect of the electric field due to non-uniform base doping is small, as is borne out by PISCES simulations, the second RHS term of eqn (3.4) can be neglected, giving

$$J_n = qD_n \frac{2n' + N_{AB}}{n' + N_{AB}} \frac{dn'}{dx} \quad (3.24)$$

With the derivative of n' with respect to x given by m (constant), integrating eqn (3.24) with respect to x , using an effective diffusion coefficient, and invoking charge control theory gives:

$$K = \frac{2J_n\tau - Q_{b0}}{J_n\tau - Q_{b0}} \quad (3.25)$$

$$\text{Where } \lim_{J_n \rightarrow 0} (K) = 1 \quad \text{and} \quad \lim_{J_n \rightarrow \infty} (K) = 2$$

The details of the derivation for K are given in appendix B.

The result for K given in eqn (3.25) is, in retrospect, not surprising. K signifies the effect of the integrated majority charge on J_n . With eqn (3.8) for the integrated majority charge given, the relation between J_n and the integral of $p(x)$ is known. It is then expected that eqn (3.8) should manifest itself in the solution of K , providing confidence in the overall methods used to solve for K .

At this point the first part of the task of solving the term in eqn (3.19) for the effect of bandgap narrowing has been achieved: the pn product along the entire length of the active base is known and given by eqn (3.23), with K defined in eqn (3.25).

We now turn to the consideration of the position dependent intrinsic carrier density n_{ie} . This quantity is a function of the bandgap, but the relationship between the bandgap and the base doping is in general not known. Returning to eqn (3.19), and reiterating, the problem that requires solution is the evaluation of:

$$2 \int_{x_1}^{x_2} pn \frac{d(\ln(n_{ie}))}{dx} dx \quad (3.26)$$

Now, because the variation of $\ln(n_{ie})$ between x_1 and x_2 is small compared to that of the pn product, it is possible to express eqn (3.26), using the mean-value theorem, as:

$$\begin{aligned} 2 \int_{x_1}^{x_2} pn \frac{d(\ln(n_{ie}))}{dx} dx &\approx \frac{2}{x_2 - x_1} \int_{x_1}^{x_2} pn dx \int_{x_1}^{x_2} \frac{d(\ln(n_{ie}))}{dx} dx \\ &= \ln\left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)}\right) \frac{2}{x_2 - x_1} \int_{x_1}^{x_2} pn dx \end{aligned} \quad (3.27)$$

Eqn (3.27) allows for the effect of bandgap narrowing without having to specify how the

bandgap varies with doping. The only requirement of eqn (3.27) is the evaluation of the natural log term, which can be determined by experimental extraction. Furthermore, if the doping at the edges of the depletion regions are known, then eqn (3.27) can be used to find the relation between doping and the bandgap. See appendix C for an experimental procedure to determine the relation between doping and the bandgap.

Using eqns (3.23) and (3.25) gives:

$$\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} pn \, dx = \frac{pn_1}{K + 1} = pn_1 \left(\frac{J_n \tau - Q_{b0}}{3J_n \tau - 2Q_{b0}} \right) \quad (3.28)$$

When eqn (3.28) is substituted into eqn (3.27) the result for the effect of bandgap narrowing is

$$2 \int_{x_1}^{x_2} pn \frac{d(\ln(n_{ie}))}{dx} \, dx \approx 2pn_1 \ln \left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)} \right) \left(\frac{J_n \tau - Q_{b0}}{3J_n \tau - 2Q_{b0}} \right) \quad (3.29)$$

As a check on the result given by eqn (3.29), a numerical study beginning with eqn (3.26) and ending with eqn (3.29) is performed. Beginning with the original equation, eqn (3.26) is compared to eqn (3.27). For the purpose of this comparison a model for n_{ie} , and the base doping together are required. The model for n_{ie} was taken from PISCES because comparisons to the PISCES data will be made later on. The base doping distribution was obtained by fitting to the base doping in the test transistor studied in chapter 2, giving:

$$N_{AB}(x) = 4.7 \times 10^{17} e^{-(3.86x)^2} \text{ cm}^{-3}, \text{ where } x \text{ is in } \mu\text{m}$$

Under low level conditions the pn product is linear. Eqn (3.26) then gives a value of $0.2819pn_1$ where $pn_1 = pn(x=x_1)$. Eqn (3.27) on the other hand gives a result of $0.2792pn_1$. Therefore, the error in the approximation in going from eqn (3.26) to eqn

(3.27), under low level, is -0.96% . Under high level conditions the pn product is quadratic. Eqn (3.26) then gives a value of $0.1857pn_1$ while eqn (3.27) gives a value of $0.1861pn_1$. The error under high level conditions has fallen to 0.22% . Thus, the approximation in going from eqn (3.26) to eqn (3.27) is negligible.

The accuracy of equation 3.29 is now tested. Figure 20 for the low level case of $V_{BE} = 0.6$ V shows that the error caused by not including the effect of bandgap narrowing is $0.22pn_1$. Then, eqn (3.29), under low level conditions, gives a value of $0.28pn_1$ for the bandgap narrowing term. Therefore, the error in using eqn (3.29) for the evaluation of eqn (3.26) is 27% . The reason for this large error is in low level the pn product is not linear, but a linear function multiplied by the doping. When the non-linear doping is included into eqn (3.29) the result is a value of $0.21pn_1$ for the bandgap narrowing term. This compares to the PISCES value, from above, of $0.22pn_1$ - an error of only 4.5% . This error is mainly due to the fact that the exact doping was not used but a fitted analytic result was used instead. These comparisons show that eqn (3.29) is indeed a good solution for the bandgap narrowing term (eqn (3.26)).

Bringing all the results of this chapter together, eqn (3.19) becomes:

$$J_n = -q^2 \overline{D_n} (pn(x_1) - pn(x_2)) \frac{1 + \Delta \text{Bandgap}}{Q_{b0} - \tau J_n} \quad (3.30)$$

where

$$Q_{b0} = q \int_{x_1}^{x_2} N(x) dx$$

and

$$\Delta \text{Bandgap} = 2 \ln \left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)} \right) \left(\frac{J_n \tau - Q_{b0}}{3J_n \tau - 2Q_{b0}} \right) \quad (3.31)$$

Eqn (3.30) models the effect of bandgap narrowing in a manner consistent with the formulation of the Gummel-Poon one-dimensional base transport model, and requires only the specification of the natural log factor. Furthermore, eqn (3.30) is valid only when the pn product at the emitter $[pn(x_1)]$ is much larger than the pn product at the collector. It is noted that as the current level increases the effect of the term $\Delta\text{Bandgap}$ drops. However, under low level conditions (ie. when $J_n\tau \ll Q_{b0}$), the effect of bandgap narrowing can be represented by a suitable constant "multiplier" in the Gummel-Poon model (eg. by changing $\bar{D}_n$).

4. A MODEL FOR THE EFFECT OF EMITTER DEBIASING WITH CONDUCTIVITY MODULATION IN THE BASE

The phenomenon known as emitter debiasing or base current crowding is becoming increasingly important as the active base width of BJTs decreases. Due to the effects of emitter debiasing the efficiency of the emitter, in terms of current injection, rapidly falls causing a corresponding reduction of collector current. Furthermore, because of the non-constant injection across the length of the emitter, a two dimensional current distribution in the active base region results as has been seen in the PISCES studies in chapter 2. Since the severity of the effect depends on the emitter length, simple scaling rules are not applicable for practical vertical mode BJT geometries. Therefore, a model for emitter debiasing is necessary to achieve an accurate scalable model for bipolar transistors.

Stated simply, emitter debiasing is due to the lateral flow of base current (holes for the case of an NPN transistor) through the highly resistive active base region. The result is a lateral voltage drop across the length of the emitter, causing a spatially varying injection of minority carriers into the base. Up to this point in time only the simple model of Hauser [4], [5] has been available for emitter debiasing. This model assumes that the conductivity of the base remains constant over all V_{BE} . However, it has been shown in chapter 2 that conductivity modulation exists in the base even for moderate emitter-base voltages of 0.8 V, and the importance of this phenomenon in the present context is clear from the work of Fletcher [17] and Misawa [18]. To quickly show the effect of conductivity modulation on emitter debiasing, for large emitter-base voltages $I_b \propto e^{\frac{3V_{BE}}{4V_T}}$ with conductivity modulation (as is proven later in this chapter), and $I_b \propto e^{\frac{V_{BE}}{2V_T}}$ without [4]. It is concluded that conductivity modulation in the base must be taken into account in the development of a realistic emitter

debiasing model.

4.1. A Physical Model For The Effects Of Emitter Debiasing

As was shown in figure 5 in chapter 2, the fact that all the base current injected into the emitter must flow out through one edge¹ of the active base causes the largest position dependent ohmic voltage drop to occur at the emitter edge closest to the base contact. Therefore, the rate of change of debiasing with respect to position along the emitter is not constant. The result is a coupling between the vertically injected current and the lateral voltage drop due to emitter debiasing - a two dimensional phenomenon. However, it has been seen in chapter 2 that the injected current **only** depends on the pn product at the point of injection and flows essentially normal to the base-emitter junction (ie. vertically). That is the flow of injected current can be treated as one dimensional in the vertical direction. Furthermore, because the active base width is generally quite small compared to the emitter length, the lateral voltage drop due to emitter debiasing can be treated as one dimensional in the lateral (horizontal) direction. This allows the model for emitter debiasing to be cast in terms of two, coupled one dimensional models instead of one, complex two dimensional model, and be expressed in closed form.

From the above properties it follows that the lateral voltage drop due to emitter debiasing is mainly due to the **lateral** flow of base current in the active base layer. Furthermore, in modern BJTs there is no noticeable recombination in the active base region. This removes the direct dependency between electron and hole current densities, the base current being only due to back injection into the emitter. Also, because the emitter is much more heavily

¹ In the event the BJT has two symmetric base contacts, the results presented in this chapter for one base contact may still be used by dividing the emitter in two, calculating the currents and then doubling all of them to arrive at the final terminal currents. This method was investigated by Hauser.

doped than the base, high level effects on the base current can be ignored. Finally, because the current densities are being modelled with the pn products (Misawa boundary conditions [18]) and not the junction potentials, the only voltage drop that needs to be considered is the ohmic drop due to transverse base current flow. Accordingly, for an NPN transistor, the model for emitter debiasing is depicted in figure 34.

FIGURE 34. Schematic representation of the emitter debiasing model

In this n -section model, ρ_i is the base resistance in section i , and is conductivity modulated. I_j is the base current injected into the emitter at node j . $V_{j-1,j}$ is the voltage between nodes $j-1$ and j . V_j is the voltage at node j referenced to ground. $V_B = V_{BE}$, where the emitter is treated as a uniformly constant potential reference, here shown as the ground reference.

The voltage at node i is:

$$\begin{aligned} V_i &= V_B - \sum_{j=1}^i V_{j-1,j} \quad \text{where} \quad V_{j-1,j} = \rho_j \sum_{k=j}^n I_k \\ &= V_B - \sum_{j=1}^i \rho_j \sum_{k=j}^n I_k \end{aligned} \quad (4.1)$$

The order of summation is now changed as illustrated in figure 35, giving

$$V_i = V_B - \left(\sum_{k=1}^i \rho_k \right) \left(\sum_{k=i+1}^n I_k \right) - \sum_{k=i}^i I_k \sum_{j=1}^k \rho_j \quad (4.2)$$

FIGURE 35. Change of order of summation

Converting eqn (4.1) into integral form and noting that L is the emitter length corresponding to $i = n$, then the voltage at position x , with respect to the emitter is

$$V(x) = V_B - \int_0^x \rho(u) \left[\int_u^L J_b(z) dz \right] du \quad (4.3)$$

where $\rho(u)$ is conductivity modulated. Converting eqn (4.2) into integral form gives:

$$V(x) = V_B - \left(\int_0^x \rho(u) du \right) \left(\int_x^L J_b(u) du \right) - \int_0^x J_b(u) \left[\int_0^u \rho(z) dz \right] du \quad (4.4)$$

Eqn (4.3) is most useful for further model development, while eqn (4.4) gives better insight into the operation of emitter debiasing. The second term of eqn (4.4), consisting of the product of two integrals, represents the debiasing due to the cumulative base current to the right of position x , injected between positions x and L , flowing through the base resistance from the edge of the emitter (closest to the base contact) to position x . The third term represents the debiasing due to base current injected between positions 0 (edge of the emitter) and x . Thus, there are two currents that cause debiasing: the first is a cumulative current flowing past the point of interest going toward the emitter edge farthest from the base contact; the second current is a distributed current that causes an ever increasing rate of emitter debiasing as the emitter edge closest to the base contact is approached.

Continuing with the model development, the derivative of eqn (4.3) is taken with respect to x . This gives:

$$\frac{dV}{dx} = -\rho(x) \int_x^L J_b(z) dz \quad (4.5)$$

Differentiating again:

$$\frac{d^2V}{dx^2} = \rho(x)J_b(x) - \frac{d\rho(x)}{dx} \int_x^L J_b(z) dz$$

The term containing the integral can then be replaced by solving eqn (4.5) and substituting back. This gives:

$$\begin{aligned} \frac{d^2V}{dx^2} &= \rho(x)J_b(x) + \frac{1}{\rho(x)} \frac{d\rho(x)}{dx} \frac{dV}{dx} \\ &= \rho(x)J_b(x) + \frac{d \ln(\rho(x))}{dx} \frac{dV}{dx} \end{aligned} \quad (4.6)$$

Eqn (4.6) is a second order, non-linear ordinary differential equation for the solution of emitter debiasing with conductivity modulation of the base included. The first step in solving this equation is to reduce the order by one. This is done by noting that:

$$\frac{d^2V}{dx^2} = \frac{1}{2} \frac{d}{dV} \left(\left(\frac{dV}{dx} \right)^2 \right)$$

Substituting this into (4.6) and changing to the derivative of $\ln(\rho)$ with respect to V gives

$$\frac{1}{2} \frac{d}{dV} \left(\left(\frac{dV}{dx} \right)^2 \right) = \rho(V)J_b(V) + \frac{d \ln(\rho(V))}{dV} \frac{dV}{dx} \frac{dV}{dx}$$

Thus

$$\frac{dE}{dV} = 2\rho(V)J_b(V) + 2 \frac{d \ln(\rho(V))}{dV} E \quad (4.7)$$

where

$$E = \left(\frac{dV}{dx} \right)^2$$

Eqn (4.7) is a first order, linear, non-homogeneous differential equation, the solution of which is:

$$E e^{\int -2 \frac{d \ln(\rho(V))}{dV} dV} = \int 2\rho(V)J_b(V) e^{\int -2 \frac{d \ln(\rho(V))}{dV} dV} dV$$

Reducing the integrals and the exponentials gives:

$$\frac{E}{\rho^2(V)} = 2 \int \frac{J_b(V)}{\rho(V)} dV$$

or

$$E = \left(\frac{dV}{dx}\right)^2 = 2\rho^2(V) \int \frac{J_b(V)}{\rho(V)} dV + A_1 \quad (4.8)$$

where A_1 is an arbitrary constant.

The geometry of the base and emitter regions are given in figure 36.

FIGURE 36. Geometry of the emitter-base for the emitter debiasing model

Finally, using eqn (4.8) and taking square roots, rearranging, and integrating:

$$\int dx = x = - \int \frac{dV}{\sqrt{2\rho^2(V) \int \frac{J_b(V)}{\rho(V)} dV + A_1}} + A_2 \quad (4.9)$$

Eqn (4.9) is the general emitter-debiasing model, the solution of which is the focus of the remaining portion of the chapter.

4.2. Emitter Debiasing Model With Constant Base Conductivity

Before proceeding further with the general model development of section 4.1, we consider the form of model that results if there is no conductivity modulation in the base. As pointed out by Hauser, in many bipolar processes typical BJTs will experience emitter debiasing at a sufficiently low V_{BE} value for conductivity modulation to be neglected. While the assumption of no conductivity modulation in the base may in such cases lead to a prediction of the onset of emitter debiasing, it results in a greatly overestimated amount of emitter debiasing at higher levels of operation, making the BJT appear to inject less current than it does in practice. This could cause a design for a driver cell in an integrated circuit to be made much larger than required and thereby waste valuable silicon chip area. The constant conductivity model of Hauser is, however, useful for reference purposes.

Hauser's model for emitter debiasing was based upon the divergence of current in an infinitesimal box that covers the width of the active base layer. Furthermore, the base current was assumed to be generated from the recombination of minority carriers in the base, an inappropriate assumption for modern BJTs, where recombination in the base is insignificant. A direct comparison with Hauser's model is made by assuming constant base resistivity ρ : eqn (4.9) then becomes:

$$x = - \int \frac{dV}{\sqrt{\int 2\rho J_b(V) dV + A_1}} + A_2 \quad (4.10)$$

where, as previously noted, the base current density J_b is unaffected by high level effects.

Even in the case of extreme emitter debiasing, the emitter edge farthest from the base contact will be sufficiently forward biased for $J_b(V)$ to be expressed as

$$J_b(V) = J_{sb} e^{V(x)/V_T} \quad (4.11)$$

where J_{sb} is the base saturation current. This applies regardless of whether or not constant resistivity is assumed.

Substituting this into eqn (4.10) and integrating gives:

$$x = - \int \frac{dV}{\sqrt{V_T K_{sb} e^{V(x)/V_T} + A_1}} + A_2$$

where

$$K_{sb} = 2\rho J_{sb}$$

or

$$x = - \frac{2V_T}{\sqrt{-A_1}} \tan^{-1} \left(\sqrt{\frac{V_T K_{sb} e^{V(x)/V_T} + A_1}{-A_1}} \right) + A_2 \quad (4.12)$$

where

$$A_1 < 0$$

Thus, as $V(x)$ increases the radical in eqn (4.12) must also increase so that x decreases as required physically.

Solving eqn (4.12) for $V(x)$ we obtain

$$V(x) = 2V_T \ln \left[\sqrt{\frac{-A_1}{K_{sb} V_T}} \sec \left(A_2 - \frac{x \sqrt{-A_1}}{2V_T} \right) \right] \quad (4.13)$$

and from eqn (4.11),

$$J_b(x) = \frac{-A_1}{2\rho V_T} \sec^2 \left(A_2 - \frac{x \sqrt{-A_1}}{2V_T} \right) \quad (4.14)$$

The constraints on these two equations are that $A_1 < 0$, and $0 < A_2 < \pi/2$.

2

The solution for the arbitrary constants A_1 and A_2 begins by going back to eqn (4.5). Solving for the integral in eqn (4.5) by using eqn (4.14), and remembering that the base has a constant resistivity gives:

$$\frac{dV}{dx} = \sqrt{-A_1} \left[\tan \left(A_2 - \frac{L\sqrt{-A_1}}{2V_T} \right) - \tan \left(A_2 - \frac{x\sqrt{-A_1}}{2V_T} \right) \right]$$

But differentiating eqn (4.13) gives

$$\frac{dV}{dx} = -\sqrt{-A_1} \tan \left(A_2 - \frac{x\sqrt{-A_1}}{2V_T} \right)$$

Equating these two equations it is required that:

$$\tan \left(A_2 - \frac{L\sqrt{-A_1}}{2V_T} \right) = 0$$

Setting the argument of the tangent equal to zero (instead of an arbitrary integer multiplied by π) to satisfy the constraints presented above for A_1 and A_2 , we obtain

$$A_2 = \frac{L\sqrt{-A_1}}{2V_T} \quad (4.15)$$

Letting $B = \frac{A_2}{L} = \frac{\sqrt{-A_1}}{2V_T}$ and substituting into eqns (4.13) and (4.14) gives:

$$V(x) = 2V_T \ln \left[2B \sqrt{\frac{V_T}{K_{sb}}} \sec(B(L-x)) \right] \quad (4.16)$$

$$J_b(x) = \frac{2B^2 V_T}{\rho} \sec^2(B(L-x)) \quad (4.17)$$

where the constant B has yet to be specified.

These two equations are identical to those presented by Hauser. This shows that the model presented in this chapter does in fact degrade into the low level solution when conductivity modulation in the base is removed.

The value of B follows from the boundary conditions for $V(x)$ at $x = 0$. If the voltage drop due to the base current travelling through the extrinsic base region from the emitter edge to

the base contact is assumed negligible, then the terminal voltage V_{BE} will appear at the emitter edge where $x=0$, so that $V(0) = V_{BE}$. Eqn (4.16) then gives

$$B = \frac{1}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}} \cos(BL), \text{ where } 0 < B < \frac{\pi}{2L} \quad (4.18)$$

This transcendental equation for B must be solved numerically. A more detailed discussion regarding the solution of this equation and the removal of the restriction on zero extrinsic base resistance will be given in chapter 7.

Now, integrating eqn (4.17) over the entire length of the emitter gives the terminal base current as:

$$I_{bT} = \int_0^L J_b(x) dx = \frac{2BV_T}{\rho} \tan(BL) \quad (4.19)$$

It is now worthwhile to consider the cases of extreme low level and extreme high level operating conditions.

Extreme low level

$$B \text{ (low level)} = \frac{1}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}} \quad (4.20)$$

This shows as I_{bT} goes to zero, B approaches zero also. Substituting for BL from eqn (4.20) with $\tan(BL) \approx BL$, eqn (4.19) gives

$$\begin{aligned} I_{bT}(\text{low level}) &= \frac{2BLV_T}{L\rho} BL = \frac{2V_T}{L\rho} \left(\frac{L}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}} \right)^2 \\ &= LJ_{sb} e^{\frac{V_{BE}}{V_T}} \end{aligned} \quad (4.21)$$

This shows that eqns (4.16) to (4.19) do degenerate into the known low level solution for bipolar transistors, corresponding to no emitter debiasing.

Extreme high level

Although eqn (4.19) does not apply for cases where V_{BE} is large because of conductivity modulation in the base, it is very interesting to find out the form of I_{bT} when V_{BE} is allowed to become infinitely large. The result of this analysis will be useful later, while it also reaffirms Hauser's analysis of emitter debiasing.

When V_{BE} becomes very large, BL approaches $\pi/2$. Taking the first step in a Newton-Raphson solution for eqn (4.18) and using $\pi/2$ as a starting point gives:

$$BL \text{ (high level)} = \frac{\pi}{2} \frac{A}{A + 1}$$

where

$$A = \frac{L}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}}$$

Eqn (4.19) for I_{bT} then gives

$$I_{bT}(\text{high level}) = \frac{2BV_T}{\rho} \tan\left(\frac{\pi}{2} \frac{A}{A + 1}\right)$$

Since BL is close to $\pi/2$, $\tan(BL) \approx (\pi/2 - BL)^{-1}$, giving

$$I_{bT}(\text{high level}) = \frac{2V_TA}{L\rho} = \sqrt{\frac{2V_TJ_{sb}}{\rho}} e^{\frac{V_{BE}}{2V_T}} \quad (4.22)$$

This shows that the base current bears a square root relationship to the low level case. The important point to note, however, is that the multiplier in front of the exponential has no dependence on the length of the emitter. Thus, at some large V_{BE} , all bipolar transistors will emit the same amount of current (as long as the depth of the emitters are the same), regardless of the length of the emitter. This property, even though it is obtained from consideration of the constant base conductivity case which is not physically valid for high level, is found to apply also when conductivity modulation of the base is represented in the model, though of course the current value is different (increased).

The final equation that is needed for the constant base conductivity model is an expression for the collector terminal current I_{cT} . Eqn (3.10) relates the electron current density J_n to the emitter-base voltage assuming no bandgap narrowing effect². Eqn (3.10) allows for the effects of conductivity modulation in the base on J_n , and while it is strictly inconsistent to model the effect of conductivity modulation on one hand while disregarding it on the other, the results obtained will become useful when conductivity modulation is included in the emitter debiasing model in section 4.3.

Using eqn (3.10), I_{cT} is given by:

$$\begin{aligned} I_{cT} &= \int_0^L J_n(x) dx \\ &= \frac{1}{2K_3} \int_0^L \left(K_1 - \sqrt{K_2' K_3 (e^{V(x)/V_T} - e^{V_{bc}/V_T}) + K_1^2} \right) dx \\ &\quad \text{where } K_1 = Q_b, \quad K_3 = \tau, \text{ and } K_2' = 4q^2 n_i^2 D_n \end{aligned}$$

Using eqn (4.16) for $V(x)$, the above equation for I_{cT} becomes:

$$\begin{aligned} I_{cT} &= \frac{1}{2K_3} \int_0^L \left(K_1 - \sqrt{K_2 \sec^2(B(L-x)) + K_4} \right) dx \\ \text{where } K_2 &= \frac{K_2' K_3 4B^2 V_T}{K_{sb}}, \quad K_4 = K_1^2 - K_2' K_3 e^{V_{bc}/V_T}, \text{ and } K_{sb} = 2\rho J_{sb} \end{aligned}$$

This integral has a closed-form solution (see appendix D), giving

$$I_{cT} = \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{\Delta_{BL} + K' \sin(BL)}{\Delta_{BL} - K' \sin(BL)} \right) + \alpha \sin^{-1}(\alpha K' \sin(BL)) \right\} \quad (4.23)$$

² Since the effect of emitter debiasing on J_n is much more pronounced than the effect of bandgap narrowing, the low level result for bandgap narrowing, where the diffusion coefficient is changed by a constant multiplier, is used instead of the full bandgap narrowing model.

$$\text{where } \Delta_{BL} = \sqrt{1 - (\alpha K')^2 \sin^2(BL)}, \quad \alpha^2 = \frac{K_4}{K_2}, \quad \text{and } K' = \frac{1}{\sqrt{1 + \alpha^2}}$$

Combining eqn (4.23) with eqns (4.16) to (4.19), a complete model for emitter debiasing without conductivity modulation results.

As a final check of the validity of the low level model for emitter debiasing, eqn (4.23) should become eqn (4.19) multiplied by the beta of the transistor as V_{BE} becomes small enough for high level effects to be ignored. Appendix E shows this to be true. Therefore, emitter debiasing does not cause the intrinsic beta of the transistor to change. The only way beta can change is if components of base or collector current are present that do not follow the relationships developed here (such as hole current injection into the surface oxide around the emitter).

4.3. Emitter Debiasing Model With Base Conductivity Modulation

A model for emitter debiasing that includes the high level effect of conductivity modulation in the base is now presented. The development begins by returning to the general relationship of eqn (4.9) for the base voltage $V(x)$. Two further sub-models are required to solve eqn (4.9): the first is a model for the base current density, given by eqn (4.11) which is still valid for the case of conductivity modulation in the base; the second, derived below, is a model for the base resistivity.

4.3.1. Base Resistivity Model

At a point x , ρ is defined in terms of the base voltage $V = V(x)$ as follows:

$$\rho = \rho(V) = \left[q \int_{y_E}^{y_C} (n(V,y)\mu_n + p(V,y)\mu_p) dy \right]^{-1}$$

Where y_C is at the base side of the collector-base depletion region, and y_E is at the base side of the emitter-base depletion region. From the charge control model for the excess minority charge and quasi-neutrality one has:

$$q \int_{y_E}^{y_C} n dy = -\tau J_n$$

and

$$q \int_{y_E}^{y_C} p dy = Q_b - \tau J_n$$

where

$$Q_b = q \int_{y_b}^{y_c} N_{AB} dy$$

Then, assuming an effective mobility as was done with the diffusion coefficient

$$\begin{aligned} \rho(V) &= [Q_b \bar{\mu}_p - \tau J_n (\bar{\mu}_n + \bar{\mu}_p)]^{-1} \\ &= \frac{1}{\bar{\mu}_p (Q_b - \tau J_n U_{n/p})} \end{aligned} \quad (4.24)$$

where

$$U_{n/p} = \frac{\bar{\mu}_n}{\bar{\mu}_p} + 1$$

In the zero bias condition, $V_{BC} = V_{BE} = 0$, we define:

$$\rho_0 = \frac{1}{\bar{\mu}_p Q_{b0}}$$

where

$$Q_{b0} = q \int_{y_{b0}}^{y_{c0}} N_{AB} dy$$

so that

$$\rho(V) = \frac{\rho_0 Q_{b0}}{(Q_b - \tau J_n U_{n/p})} \quad (4.25)$$

ρ_0 is a process constant. The reason $Q_{b0} \neq Q_b$ is because the depletion region boundaries are not fixed in space, but move with changing bias. To model this motion of the depletion region boundaries, the method employed by Boothroyd [20] is used. Since junction capacitances can be extracted very accurately, it is possible to use these capacitances to calculate the change in Q_b due to bias. Then:

$$\frac{Q_b}{q} = \int_{y_b}^{y_c} N_{AB} dy = \frac{Q_{b0}}{q} - \int_0^{V_{JC}} C_{JC} dV - \int_0^{V_{JE}} C_{JE} dV \quad (4.26)$$

Where V_{JC} and V_{JE} are the junction voltages, while C_{JC} and C_{JE} are the junction

capacitances given by the simple power rule for the junction.

There is some difficulty in determining the junction voltages. Depending on how the final bipolar model is cast, the junction potentials can be derived in various ways, as detailed in Warner and Grung [21].

The focus of this chapter is to model the distributed effects due to emitter debiasing. Since the injection varies with position x , the junction voltages will also vary with position x . This will cause the model for $\rho(V)$ to become very complicated. However, it is possible to invoke two simplifying assumptions to reduce the complexity of $\rho(V)$.

The first assumption relates to the junction potential at the emitter, V_{JE} . Under normal, forward mode operation, the change in Q_b due to change of V_{BE} (i.e. due to change in position of the emitter boundary of the neutral base) is very small. Therefore, for the purpose of defining Q_b , V_{JE} can be assumed constant over the entire length of the active base, and its value can be taken from the known boundary condition at $x=0$. The second simplifying assumption is for V_{JC} . Accurate modeling of V_{JC} is essential because it controls the Early effect. Typically, the change in voltage from one side of the active base to the other is never more than 200 mV, while, V_{BC} is usually more than ten times larger than this voltage drop. Therefore, V_{JC} can also be regarded as constant over x , its value being taken from the known boundary condition at $x=0$. Further justification for this assumption comes from the fact that it is most important to model V_{JC} accurately near the point of greatest injection. By modeling V_{JC} as described above, the junction potential is defined at the point of greatest injection, while the accuracy of V_{JC} then decreases with the amount of injection.

On the above basis, by combining eqns (4.25), (4.26) with the Gummel-Poon expression for J_n (given by eqn (3.10)), the final model for the base resistivity becomes:

$$\rho(V) = \frac{\rho_0 Q_{b0}}{\left(Q_b - U_{np} \frac{Q_b - \sqrt{4q^2 n^2 D_n (e^{V(x)/V_T} - e^{V_{bc}/V_T}) + Q_b^2}}{2} \right)} \quad (4.27)$$

Where ρ_0 is the zero bias resistivity per unit depth into the cross section of the device, while Q_b and Q_{b0} are defined by eqns (4.26) and (4.25) respectively.

4.3.2. Base Potential Distribution $V(x)$

Eqns (4.27) and (4.11) define the two functions required by eqn (4.9), which needs to be solved in order to model emitter debiasing with conductivity modulation in the base. However, because of the square root term in eqn (4.27), when $\rho(V)$ is integrated in eqn (4.9) for the first time, a term raised to the three halves results. The next integration then requires the solution to a square root of a third order polynomial. As is stated by Gradshteyn and Ryzhik [22], integrals of this form result in a solution of simple transcendental and elliptic equations. Since this solution must also be inverted, the chance of a closed form analytic solution is very remote.

As it stands, eqn (4.9) embodies the entire essence of emitter debiasing. By carefully examining the ordinary differential equation (4.6) that created eqn (4.9) it is possible to reduce the problem somewhat. The first term $\rho(x)J_b(x)$ is the principal term of the RHS of eqn (4.6) and embodies the major effect of emitter debiasing. From the V_{BE} where emitter debiasing begins to occur, up to extreme amounts of emitter debiasing, the first term in eqn (4.6) dominates the O.D.E. Thus, eqn (4.6) can be reduced to:

$$\frac{d^2V}{dx^2} \approx \rho(x)J_b(x) \quad (4.28)$$

Since $\rho(x)$ and $J_b(x)$ are known as functions of V , eqn (4.28) can alternatively be written as

$$\frac{d^2V}{dx^2} = \frac{1}{2} \frac{d}{dV} \left(\left[\frac{dV}{dx} \right]^2 \right) \approx \rho(V)J_b(V)$$

Thus

$$\left[\frac{dV}{dx} \right]^2 = \int 2\rho(V)J_b(V) dV + A_1$$

or

$$x = - \int \frac{dV}{\sqrt{\int 2\rho(V)J_b(V) dV + A_1}} \quad (4.29)$$

Eqn (4.29) is much simpler than eqn (4.9); however, when eqns (4.11) and (4.27) are substituted in, the solution still remains intractable. Currently, unless the complexity of eqn (4.27) can be further reduced, it appears to be necessary to resort to series approximations or numerical methods for the solution of eqn (4.29). If a Laurent series in terms of $e^{V(x)/V_T}$ is taken of eqn (4.27), and $\rho(V)$ equated to the first term, then the essence of conductivity modulation is still retained. The result is

$$\rho(V) \text{ (High level)} = \frac{\rho_0 Q_{b0}}{q U_{np} n_i \sqrt{D_n \tau} e^{V(x)/2V_T}} \quad (4.30)$$

Where U_{np} is defined in eqn (4.24), and Q_{b0} is defined in the equation before eqn (4.25).

The drawback to eqn (4.30) stems from the fact that it is an asymptotic expression where the accuracy decreases as $V(x)$ approaches zero. Therefore, spatial regions in which there is no conductivity modulation will have their resistivities increased past what they really should be. This results in an over estimation of the amount of emitter debiasing in the regions in which conductivity modulation does not occur (see figure 37).

FIGURE 37. Base resistivity along the emitter for various models

One strong factor is noted that works in the favor of this approximation for $\rho(V)$: the current density $J_b(V)$ decreases much more rapidly than the resistivity increases as V is reduced. Thus, the effect of increasing $\rho(V)$ is offset by the rapidly diminishing base current. This property applies, however, only if emitter debiasing becomes significant simultaneously with the occurrence of conductivity modulation. Fortunately (for our purposes), good bipolar processes are so designed that this is the case.

The approximation of eqn (4.30) is considered to be a reasonable first order representation of conductivity modulation under **high level** conditions of operation. The objective in developing the final form of emitter debiasing model is to blend a high level model using the approximation of $\rho(V)$ in eqn (4.30) with the model of section 4.2 for operation with negligible conductivity modulation.

Eqn (4.29) is now solved using eqn (4.30) for $\rho(V)$, and eqn (4.11) for J_b . The simplified equation becomes

$$x = - \int \frac{dV}{\sqrt{\frac{K_{sb}}{a} \int e^{V(x)/2V_T} dV + A_1}}$$

where

$$a = \frac{qU_{np}n_i\sqrt{D_n\tau}}{Q_{b0}} \text{ and } K_{sb} = 2\rho_0J_{sb}$$

giving

$$x = - \int \frac{dV}{\sqrt{2\frac{K_{sb}V_T}{a}(e^{V(x)/2V_T} + A_1)}}$$

Letting $w = e^{V(x)/2V_T}$ gives $2V_Tdw = w dV$. Substituting this into the above gives:

$$\begin{aligned} x &= -\sqrt{\frac{2aV_T}{K_{sb}}} \int \frac{dw}{w\sqrt{(w + A_1)}} \\ &= \sqrt{\frac{8aV_T}{-K_{sb}A_1}} \sec^{-1}\left(\sqrt{\frac{w}{-A_1}}\right) + A_2 \end{aligned} \quad (4.31)$$

Replacing the arbitrary constant A_1 with a new variable B_H given by

$$B_H = \sqrt{\frac{-K_{sb}A_1}{8aV_T}} ; -A_1 = \frac{B_H^2 8aV_T}{K_{sb}}$$

and solving for w in terms of x :

$$w^2 = e^{V(x)/V_T} = \frac{64a^2B_H^4V_T^2}{K_{sb}^2} \sec^4(B_H(x - A_2)) \quad (4.32)$$

From boundary conditions imposed by the original model (eqn (4.3)) $A_2 = L$. Then the high level model for emitter debiasing with conductivity modulation in the base becomes:

$$V_H(x) = 4V_T \ln \left\{ 2B_H \sqrt{\frac{2aV_T}{K_{sb}}} \sec(B_H(x - L)) \right\} \quad (4.33)$$

while from eqn (4.11)

$$J_{bH}(x) = J_{sb} \frac{64a^2B_H^4V_T^2}{K_{sb}^2} \sec^4(B_H(x - L)) \quad (4.34)$$

Assuming the extrinsic base resistance from the edge of the active base to the base contact small enough to be neglected, then $V(0) = V_H(0) = V_{BE}$ and eqn (4.33) gives

$$B_H L \text{ (High level)} = \frac{L}{2} \sqrt{\frac{K_{sb}}{2aV_T}} e^{\frac{V_{BE}}{2V_T}} \cos(B_H L) \quad (4.35)$$

where

$$0 < B_H L < \frac{\pi}{2}$$

When eqns (4.33) to (4.35) are compared to eqns (4.16) to (4.18), similar relationships are seen. In fact, the only differences are in the values of the constants and the exponential factor in the definition of B and B_H . The latter difference is very significant. In eqn (4.18) the exponential is raised to $V_{BE}/2V_T$, where in eqn (4.35) the exponential is raised to $V_{BE}/4V_T$. Therefore, the low level solution for the arbitrary constant B will saturate at a value of $\pi/(2L)$ at a much lower V_{BE} than will the corresponding high level solution of B_H . This fact has important ramifications on the current leaving the terminals of the BJT.

4.3.3. Base And Collector Currents

To complete the high level model for emitter debiasing the terminal currents I_{bT} and I_{cT} need to be found. The base current is obtained from eqn (4.34) as

$$I_{bT} \text{ (High Level)} = \int_0^L J_{bH}(x) dx = \frac{64a^2 B_H^3 J_{sb} V_T^2 \tan(B_H L) (\tan^2(B_H L) + 3)}{3K_{sb}^2} \quad (4.36)$$

The collector current is

$$I_{cT} \text{ (High Level)} = \int_0^L J_{nH}(x) dx \quad (4.37)$$

where the Gummel-Poon expression of eqn (3.10), together with eqn (4.34) for J_{bH} gives

$$J_{nH}(x) = \frac{Q_b - \sqrt{4q^2 n_i^2 D_n \tau \left(\frac{J_{bH}(x)}{J_{sb}} - e^{V_{BC}/V_T} \right) + Q_b^2}}{2\tau}$$

with Q_b given by eqn (4.26).

The integral of eqn (4.37) for the terminal collector current appears to have no closed form analytic solution. Therefore, numerical integration must be used to solve for I_{cT} .

It is now possible to show that I_{bT} is not proportional to $\frac{V_{BE}}{e^{2V_T}}$, as suggested by Hauser, but instead exhibits a less pronounced reduction due to emitter debiasing. As in the low level model for emitter debiasing, when V_{BE} becomes very large, $B_H L$ approaches $\pi/2$. Taking the first step in a Newton-Raphson solution for eqn (4.35) and using $\pi/2$ as a starting point gives:

$$B_H L \text{ (Extreme high level)} = \frac{\pi}{2} \frac{A}{A + 1} \quad (4.38)$$

where

$$A = \frac{L}{2} \sqrt{\frac{K_{sb}}{2aV_T}} e^{\frac{V_{BE}}{4V_T}}$$

Since $B_H L$ under extreme high level approaches $\pi/2$, then:

$$\tan^3(B_H L) \approx (\pi/2 - B_H L)^{-3} \gg \tan(B_H L)$$

Substituting this, along with eqn (4.38) into eqn (4.36), gives the terminal base current as

$$\begin{aligned} I_{bT} \text{ (Extreme high level)} &= \frac{64a^2(B_H L)^3 J_{sb} V_T^2 \tan^3(B_H L)}{3K_{sb}^2 L^3} \\ &= \frac{J_{sb}}{3} \sqrt{\frac{8aV_T}{K_{sb}}} e^{\frac{3V_{BE}}{4V_T}} \end{aligned}$$

Substituting in the definitions of a and K_{sb} into the above equation gives:

$$I_{bT} \text{ (Extreme high level)} = \frac{1}{3} \sqrt{\frac{4V_T q U_{n/p} n_i J_{sb} \sqrt{D_n \tau}}{\rho_0 Q_{b0}}} e^{\frac{3V_{BE}}{4V_T}} \quad (4.39)$$

Performing the same substitutions into eqn (4.37) for I_{cT} and noting that $J_{bH} \gg Q_b^2$

$$I_{cT} \text{ (Extreme high level)} = -qn_i \sqrt{\frac{4V_T D_n q U_{n/p} n_i \sqrt{D_n \tau}}{\tau \rho_0 J_{sb} Q_{b0}}} e^{\frac{V_{BE}}{4V_T}} \quad (4.40)$$

It is seen that no dependence on emitter length is present in eqns (4.39) and (4.40) - the same result as obtained in the low level model for emitter debiasing. Since there is no

dependence on emitter length, once emitter debiasing reaches the point where eqns (4.39) and (4.40) apply, all bipolar transistors with the same emitter depth will emit the **same** terminal currents. Thus, the important circuit design technique of using emitter area to ratio currents in an integrated circuit over a range of V_{BE} is not available under such high level conditions.

The second point to note in eqns (4.39) and (4.40) is the injection indices. In high level theory without the inclusion of emitter debiasing, the injection indices for I_{cT} and I_{bT} are 2 and 1 respectively. In Hauser's model for emitter debiasing (even though the assumption of a constant conductivity is invalid) the high level injection indices for I_{cT} and I_{bT} are 4 and 2 respectively. When the high level model for emitter debiasing, given by eqns (4.33) to (4.40), is used then the injection indices for I_{cT} and I_{bT} are 4 and $4/3$ respectively. This shows a dramatic difference to the classical and even Hauser's model for emitter debiasing.

4.4. A Complete Model For Emitter Debiasing Including The Effect Of Conductivity Modulation In The Base

In section 4.1 the theoretical basis of a model for emitter debiasing with conductivity modulation was presented but did not lead to a closed form model because of the untractable form of the integral in eqn (4.9). To circumvent this problem, two asymptotic cases were modelled. In section 4.2, eqns (4.16) to (4.19) and eqn (4.23) detail the low level model for emitter debiasing when there is no conductivity modulation in the base. Section 4.3 then gives a high level model for emitter debiasing when conductivity modulation is present throughout the entire base region. The high level model for emitter debiasing is contained within eqns (4.33) to (4.37). A complete emitter debiasing model is now defined by combining the two asymptotic models.

Since the ultimate requirement of any transistor model is to provide a relationship between the terminal voltages and the terminal currents, the emphasis, when connecting the two asymptotic models for emitter debiasing, will be placed on the equations governing the terminal currents. Thus, eqns (4.19) and (4.23) for the low level model, are to be joined in some way to eqns (4.36) and (4.37) for the high level model.

The process of bringing the two asymptotic models together begins with an analysis of the current density equations versus position. If for suitable values of the various physical parameters in the emitter debiasing models, the respective base current density distributions (eqn (4.17) for low level and eqn (4.34) for high level) are plotted for various bias levels, a very interesting result occurs (see figure 38).

Plot a) in figure 38 shows that the low level model dominates over the high level model

over the entire length of the emitter, and thus is representative of the true current distribution in the device. Moving to plot c) in figure 38, the high level model dominates over the low level model, and it now becomes representative of the true current distribution of the device. Therefore, from just plots a) and c) in figure 38, the transition from one asymptotic model to the other is very rapid indeed. Plot b) in figure 38 shows that when the transition between the models is occurring, they are so close together that the error in picking one or the other to represent the device is small. Thus, the transition from the low level model to the high level model, in which one or the other represents the true behavior of the device, occurs over a change in V_{BE} of no more than 10 mV at room temperature.

Under low level conditions where the low level model applies, the high level model overestimates the base resistivity and causes the effect of emitter debiasing to be more pronounced than it should be - reducing the terminal currents when compared to the low level model. Now, as V_{BE} approaches and passes the value at which high level effects cause conductivity modulation in the base, the high level model for emitter debiasing applies. When conductivity modulation occurs in the base, the low level model overestimates the base resistivity causing the effect of emitter debiasing to be more significant than it is, thereby reducing the terminal currents when compared to the high level model. Thus, when the V_{BE} where high level effects occur is passed, the underestimation of the terminal currents passes from the high level model to the low level model. Therefore, the high and low level models for emitter debiasing must cross over each other at some V_{BE} .

Since the range of V_{BE} in which the transition between the two models for emitter debiasing is very small, it is possible to construct very simple bridging equations for the overall

emitter debiasing model, as follows:

$$I_{cT} = \text{MAX}(I_{cT}(\text{low level, eqn (4.23)}), I_{cT}(\text{high level, eqn (4.37)})) \quad (4.41)$$

$$I_{bT} = \text{MAX}(I_{bT}(\text{low level, eqn (4.19)}), I_{bT}(\text{high level, eqn (4.36)})) \quad (4.42)$$

The problem that could perhaps occur with eqns (4.41) and (4.42) is a large discontinuity in the first or second derivatives of I_{cT} or I_{bT} ; however, this does not occur. Figure 39 gives a plot of $\log I_{cT}$ and I_{bT} against V_{BE} for the same physical parameters used in figure 38. As can be seen in figure 39, there is no discernable discontinuity in the plots. Even when a linear plot is made of figure 39 (see figure 40) in the region where the discontinuity occurs, it is not visible. The reason for such a continuous result is the rapid transition from the low level model to the high level model, which is based on the difference between the exponentials in eqns (4.18) and (4.35) for the solution of B and B_H respectively. Therefore, eqns (4.41) and (4.42) along with the two asymptotic models in sections 4.2 and 4.3 give an essentially continuous model for emitter debiasing with conductivity modulation in the base.

FIGURE 38 Comparison of the high and low level models at the cross over point

The solid line is the low level model. The broken line is the high level model.

FIGURE 39 Plot of I_c and I_b **FIGURE 40** Blow Up of I_c and I_b

5. A MODEL FOR THE PERIPHERAL CURRENT

In recent years advances in photolithography have caused the lateral dimensions of devices to reduce faster than the vertical dimensions. As a result, the ratio of the current emanating from the planar portion of the emitter to the periphery of the emitter has decreased. Therefore, the emitter peripheral current, although not dominant, now accounts for larger portions of the total current in bipolar transistors. Thus, a better understanding of the origins and physics governing the emitter peripheral current, along with accurate modeling of this phenomenon is required.

From chapter 2, the NPN test transistor with an emitter length of 2 μm and emitter width of 0.16 μm , the emitter peripheral current accounted for 21% of the total electron current at a V_{BE} of 0.6 V. Therefore, neglecting the emitter peripheral current would result in a large error. Furthermore, as emitter debiasing occurs and the collector current becomes preferentially injected from the periphery, the emitter peripheral current will account for an even larger percentage of the total current emanating from the BJT. These facts clearly point to the importance of the emitter periphery in modern bipolar transistors.

This chapter studies the origins and salient properties of emitter peripheral current, and develops a model for its representation.

5.1. A Study Into The Origin And Nature Of Peripheral Current

For all bipolar devices, including ones with walled emitters, there must be an essentially two dimensional emitter peripheral region. This stems from the fact that for conventional bipolar processes that implant/diffuse the emitter into the base, or for highly advanced poly emitter structures that simply sit on top of the base, the emitter must present a lateral boundary to the base. Therefore, the emitter periphery is a fact of BJTs, and at best its effects can only be reduced but not eliminated; thus, understanding and modeling these effects is essential if the device is to be accurately characterized.

FIGURE 41. Cross-sections showing the location of the emitter periphery.

In advanced bipolar processes, where the emitter is made of poly-silicon and sits on top of the base, the emitter peripheral current is simply due to the edge of the emitter itself. Just as a capacitor has a fringing field at the edge, a poly emitter, like the one in figure 41, has a fringing current distribution at the periphery. As a result, the current flowing in the emitter

periphery will be essentially two dimensional and behave in a different way to the current emanating from the bottom portion of the emitter. This necessitates a different model or at least different parameters for the emitter peripheral current as compared to the current from the bottom of the emitter.

Bipolar processes typically have implanted/diffused emitters. Even processes that use poly emitters usually are such that dopant from the poly emitter diffuses out into the base to form the actual emitter. Thus, even though there are BJTs with poly emitters as described in the previous paragraph, virtually all the bipolar processes around today use diffused emitters. For this reason, the focus of this chapter is on peripheral currents emanating from diffused emitter sources.

The emitter periphery, in terms of injection, is similar to the planar portion of the emitter. The emitter periphery is capable of injecting current, governed by the same general physical rules, as the rest of the emitter. The difference between the emitter periphery and the planar portion of the emitter is the metallurgical junctions of emitter-base and collector-base junctions are not parallel. This causes a truly two dimensional current distribution as was seen in chapter 2. A second major difference between the emitter periphery and the planar portion of the emitter is in regard to doping. In the planar portion of the emitter the doping concentration has contours that are parallel to the emitter-base junction. In the periphery, because of the nature of diffusing impurities into the emitter and base, the doping concentration has contours that are not parallel to the emitter-base junction. As a result, the doping varies as the emitter periphery is traversed. Together, these two differences between the emitter periphery and the planar portion of the emitter result in changes to the peripheral current injection relationship when compared to the current injected from the rest of the emitter.

Beginning with the problem of non-parallel emitter-base and collector-base junctions, an understanding into the origin of the two dimensional current distribution of the emitter periphery is possible. Using the basic concepts of the Gummel-Poon model, current injection from the emitter into the base is governed by two things: the pn product at the emitter-base and collector-base junctions, and the majority charge integrated along the path that the injected current takes through the base. Under low level conditions the pn products along the emitter-base and collector-base junctions can be considered a constant throughout the entire transistor. Therefore, there is no difference in the pn product between the emitter periphery and the planar portion of the emitter under low level conditions. However, the current density differs, even under low level conditions, between the emitter periphery and the planar portion of the emitter. In fact, the current density changes throughout the entire emitter periphery itself.

The reason for the two dimensional current distribution along the emitter periphery is the different paths the current takes through the base. As one proceeds from the bottom of the emitter periphery toward the surface of the transistor, each injected electron must traverse a longer path through the base to reach the collector. In doing so, even with a homogeneous base doping, the integrated majority charge (Q_b) along the current's path increases as the emitter periphery is traversed from the bottom to the surface, and the current density decreases correspondingly. In fact, once the non-homogeneous base doping is accounted for, the current density decreases at an even faster rate than the homogeneous base doping case.

The above effect is increased due to the influence of non-homogenous base doping. Once high level conditions are present in the base, the amount of injected current depends on the

doping. High level effects reduce the current injected from the emitter as opposed to what the low level model predicts. By increasing the doping, however, one also increases the V_{BE} that must be attained to cause high level effects in the injected current. Thus, due to the increase of base doping, high level effects tend to occur at increasing V_{BE} values as the periphery is traversed from the bottom of the emitter periphery towards the surface. The result is two fold: firstly, the planar portion of the emitter will enter into high level before the entire emitter periphery; secondly, there is a range of V_{BE} for which part of the emitter periphery is under the influence of high level effects (that portion closest the bottom of the emitter), while the rest of the periphery is under low level conditions (that portion of the emitter periphery closest the surface). This causes the injection index, defined by eqn (5.0), to differ between the emitter peripheral current and the current from the bottom of the emitter.

$$J_n = J_{ns} e^{\frac{V_{BE}}{nV_T}} \quad (5.0)$$

where J_{ns} is the saturation current and n is the injection index.

The above discussion of the origins of the emitter peripheral current conclusively shows that the details of the emitter periphery are quite different than the planar portion of the emitter. At best, one can use the same model for the emitter periphery as is used for the planar portion of the emitter by simply changing the pertinent model parameters. In the worst case, and to be realistic in physical terms, an entirely different model for the emitter periphery is needed.

5.2. The Physics Of Peripheral Current

At this point, the qualitative understanding of the emitter periphery must be augmented by a study into the physics of this phenomenon. From such a study it is possible to develop a model for the emitter periphery.

The approach used is via conformal mapping of the emitter and base regions. By conformally mapping this region into a suitable plane, the two dimensional attributes of the emitter periphery can be reduced to a one dimensional analysis. The drawback is that the conformal mapping approach can be applied to solve Laplace's equation but not the general Poisson equation. This restriction requires that the base be uniformly doped, that high level effects which introduce space charge be avoided, and that source/sinks caused by generation/recombination not be present. However, as was shown in figure 33 in chapter 3, even under high level conditions, the effects of space charge did not manifest themselves until $V_{BE} > 0.90$ V. Furthermore, in the base, generation/recombination is not an issue for modern BJTs. Thus, the only significant restriction imposed by conformal mapping is the requirement of uniform doping.

Hurkx [6] and Jos [7] have both looked at the problem of conformally mapping the emitter-base region to achieve a solution for the current densities and the terminal currents. Their results are for uniform doping under low level operation, but with some judicious approximations, it is possible to modify their work to cover non-uniform doping and high level. The approach of Hurkx and Jos is the basis of the present model development. Jos

begins by creating an ideal representation of the emitter-base region¹, with uniform doping in each region, in the $z = x + iy$ plane (as shown in figure 42). Then, through the use of the Schwarz-Christoffel [23] transformation, the base region in the z plane is mapped into the upper half of the $w = x' + iy'$ plane, where the boundary conditions for the entire base region in the z plane are placed onto the real axis in the w plane. Figure 42 illustrates this transformation, with corresponding points numbered in the two planes. In particular, the part of the real axis in the w plane from '3' to '1' at $+\infty$, corresponds to the emitter-base junction. The z plane extends to infinity in both directions making points 1' and 1'' exist at $-\infty$, while points 2' and 2'' exist at $+\infty$. Furthermore, when a number has a prime attached that means in one plane the point is split into two locations while in the other plane it is just one point. The transformation is expressed by:

$$\frac{dz}{dw} = -\frac{d}{\pi} \frac{1}{w+1} \sqrt{\frac{w-t}{w+1}} \quad (5.1)$$

where

$$t = 2\left(\frac{d+b}{d}\right)^2 - 1$$

giving explicitly

$$z = -\frac{d}{\pi\alpha} \ln\left(\frac{\alpha s - 1}{\alpha s + 1}\right) + \frac{d}{\pi} \ln\left(\frac{s-1}{s+1}\right) + id\left(\frac{1}{\alpha} - 1\right) \quad (5.2)$$

where

$$s = \sqrt{\frac{w-t}{w-1}} \quad \text{and} \quad \alpha = \sqrt{\frac{2}{t+1}}$$

The problem with the transform in the w plane is that it involves a mixture of boundary conditions on electron density n and the derivative of n (termed Dirichlet and Neumann conditions respectively) along the x' axis. Due to the boundary condition placed on the

¹ To help maintain a tractable model, Jos used rectangular instead of the realistic circular structures to represent a BJT. Section 5.3 addresses how the results obtained by Jos can be extended to cover circular structures.

derivative of n , there is no straight forward method for the solution for n . To solve for the electron density n , the w plane is transformed into the upper half of the v plane where symmetry leads simply to the solution. The transform is:

$$w = \frac{v^2 + 1}{2v} \quad (5.3a)$$

where $v = x'' + iy''$.

FIGURE 42. Schwarz-Christoffel transformation of the emitter-base region.

The transform in eqn (5.3a) is such that points 2 and 3 remain in the same location in both the v and the w planes. These two points are also branch points for the transform. When going along the real axis in the w plane for $x' \geq 1$, the reverse transform is:

$$v = w + \sqrt{w^2 - 1} \quad (5.3b)$$

However, due to the branch cut from -1 to 1 in the w plane, when going along the real axis for $x' \leq 1$, the reverse transform is:

$$v = w - \sqrt{w^2 - 1} \quad (5.3c)$$

Furthermore, for w real and between -1 and 1, the transform given by eqn (5.3a) maps onto the upper semi-circle in the v plane. Figure 43 shows the v plane along with the current flow and the boundary conditions. The emitter-base junction is along the positive

real axis starting at $x'' = 1$ and going to infinity. The collector-base junction is along the negative real axis starting at $x'' = -1$ and going to negative infinity. The emitter periphery is between points 3 and 4.

FIGURE 43. Second transformation of the emitter-base region.

Thus, by transforming the problem into the v plane, the Neumann boundary has been moved from the real axis of the w plane to the upper semi-circle in the v plane. Furthermore, we are at liberty to choose the boundary conditions for the portion of the real axis that is not already specified. By setting $n = 0$ (corresponding to a reverse biased collector-base junction) for $x'' \leq 0$ and $n = n_j$ for $x'' > 0$, then all the specified boundary conditions are met. Furthermore, due to the symmetry introduced by this choice of boundary conditions along the x'' axis, the flux crossing the unit circle is zero - satisfying the Neumann boundary condition. The solution for these boundary conditions along the x'' axis in the v plane is given by inspection as:

$$n = n_j \left(1 - \frac{\arg(v)}{\pi} \right) \quad (5.4a)$$

Since eqn (5.4a) is analytic, it satisfies the Cauchy-Reimann equations (ie. is harmonic), and thus solves the Laplace equation. Therefore, due to the uniqueness theorem for

conformal transforms, eqn (5.4a) is the solution of the electron concentration n over the entire base region. Eqn (5.4a) is then converted into a complex concentration N , which proves to be useful in further calculations, by satisfying the Cauchy-Reimann equations and letting $n = \text{Re}\{N\}$. The complex concentration N is given by

$$N = \frac{in_J}{\pi} \ln(v) + n_J \quad (5.4b)$$

Since the emitter peripheral current is being sought, eqn (5.3b) is substituted into eqn (5.4b), giving in the w plane:

$$N = \frac{in_J}{\pi} \ln(w + \sqrt{w^2 - 1}) + n_J = \frac{in_J}{\pi} \cosh^{-1}(w) + n_J$$

As long as the device is operating in low level, then the complex current J_n is proportional to the derivative of N in all of the transform-related planes, giving:

$$J_n = qD_n \frac{dN}{dw}$$

Its is now possible to solve for the emitter peripheral current by integrating the above equation for J_n along the x' axis, in the range $1 \leq x' \leq t$, corresponding to the emitter periphery. Since J_n is harmonic, the direction in which w goes to zero in the derivative is unimportant. Therefore, J_n is equally defined by the derivative of N with respect to x' .

The emitter peripheral current I_{cp} is:

$$I_{cp} = \text{Im} \left\{ \int_1^t J_n(x') dx' \right\} = \text{Im} \left\{ \int_1^t qD_n \frac{dN}{dx'} dx' \right\} = \frac{qD_n n_J}{\pi} \cosh^{-1}(t) \text{ A/cm} \quad (5.5)$$

Where t is defined in eqn (5.1). The imaginary component is taken because in the complex concentration N , the imaginary part corresponds to current flow. Eqn (5.5) is the result given by Hurkx.

As is shown by the work done by Jos, the emitter peripheral current has virtually no effect

on the emitter current emanating from the bottom of the emitter when the emitter width b is much smaller than the emitter length L . Therefore, the modeling of the emitter can be broken down into two separate models: one model for the conventional bottom or planar emitter current (as was done in the previous chapters), and a second model for the periphery. From this point on, effort concentrates on modeling the periphery, using the result of eqn (5.5) as the starting point. It is emphasized that this result is for a uniformly doped base operating in low level.

Eqn (5.5) suggests that under low level conditions I_{cp} is proportional to the electron current emanating from the planar portion or bottom of the emitter (termed I_{cb}). However, as argued early in this chapter, this is not so in general. The plots of I_{cb} ratioed to I_{cp} from PISCES simulations (same device structure as in chapter 2) indicates a strong bias dependence of this ratio down to V_{BE} of 0.7 V (see figure 44). The plot bot/sideN is the ratio of the integrated current along the entire bottom of the emitter to I_{cp} . Emitter debiasing, which affects I_{cb} , exerts a strong influence on this plot, enhancing the effects of the bias dependance on I_{cp} . A clearer indication of the properties of I_{cp} is obtained if only a small portion (5 %) of the current emanating from the bottom of emitter, at the periphery nearest the base contact, is used to represent I_{cb} . This becomes the plot of part bot/sideN.

It is seen that for high level operation ($V_{BE} > 0.8$ V), there is a definite declining trend in both plots, the relative contribution of I_{cp} to the total injected current increasing with injection level. The true effect of I_{cp} is represented by the part bot/sideN plot, which does not include any influence due to emitter debiasing. The results of figure 44 for high level operation show that the high level parameters for the periphery are indeed different than those for the bottom of the emitter. The figure also shows that high level affects I_{cb} before

I_{cp} - as was predicted earlier.

FIGURE 44. Normalized plot of I_{cp} ratioed to I_{cb} vs. V_{BE} .

If eqn (5.5) for i_{cp} were used as it stands, even for low level operation, the emitter peripheral current would be greatly over estimated. The effect of the actual non-uniform base doping is to reduce I_{cp} compared to the uniform base doping case. This is due to the increase in Q_b with increase of base doping as the peripheral sidewall is traversed from the planar emitter-base junction to the base surface. It is therefore necessary to modify eqn (5.5) in some way to represent this effect.

A true two dimensional analysis of the emitter periphery with non-uniform doping, including high level effects, is clearly untractable. Instead of attempting such an approach, one is guided by remembering the goal of a good transistor model for use in a circuit simulation environment - namely the accurate modeling of a transistor for a **known**

manufacturing process. It is not necessary to go beyond predicting the operation of any transistor formed in a given process, without attempting to predict the effects of process changes. Therefore, it is appropriate in modeling the peripheral current to sacrifice the ability to model process changes in order to make possible the use of a simple model that relies on parameter extraction from the **fixed** process.

With the above considerations in mind, it is noted that for a given process every periphery in every type of device will be exactly the same. Therefore, even though the emitter periphery is a very complex region requiring two dimensional analysis to predict its operation, once the operation is known it becomes a constant, independent of transistor layout, for a given bipolar process. This allows a simple model, like eqn (3.10) for J_n , to be used to solve for I_{cp} by extracting the pertinent device parameters for low and high level operation from the process. A drawback is that the model is not capable of predicting the current from the emitter periphery before the process is created. However, this restriction is acceptable in the context of a model designed to simulate transistor level circuits.

5.3. Model Development For The Peripheral Current

The model development for the emitter peripheral current begins with eqn (5.5) and the conformal transforms used to achieve it. Assuming that a simple classic one dimensional model, like eqn (3.10), can be used to model the periphery under low and high level operation (unlike eqn (5.5) which covers only low level), then the emitter peripheral current becomes:

$$I_{cp} = \frac{Q_{bp} - \sqrt{4q^2 n_i^2 D_n \tau_p (e^{\beta V_{be}} - e^{\beta V_{bc}}) + Q_{bp}^2}}{2\tau_p} \text{ A/cm where } \beta = \frac{1}{V_T} \quad (5.6)$$

where the parameters Q_{bp} and τ_p need to be determined by an appropriate experimental procedure for a given process.

To help guide and verify the experimental extraction of Q_{bp} and τ_p , an approximate analytic starting point will be developed. It must be stressed that these analytic results are a rough approximation, and if accurate values are required, then experimental extraction must be used.

Analytic approximations for Q_{bp} and τ_p are now developed following the approach of Jos [7] who, for a uniform doped device, modelled the total collector current emanating from the emitter as:

$$I_c = J_s(L + 2bg) \quad (5.7)$$

Where L is the emitter length and b the emitter width. J_s is the saturation current density from classic one dimensional analysis of the emitter bottom in absence of emitter peripheral current, while g is actually a function of emitter width b and the base width d . The function g transforms the length b into an effective length that has constant current

density J_s along it. The factor 2 represents the two sides of the emitter periphery. It should be noted that all results presented for the emitter periphery are per unit depth into the cross-section of the device.

Jos then modelled the total minority base charge q_{bt} as:

$$q_{bt} = \tau J_s (L + 2bf) \quad (5.8)$$

where τ is the classic one dimensional transit time for the bottom of the emitter. The function f transforms the length b into an effective distance with associated constant minority charge of τJ_s .

The functions f and g used in eqns (5.8) and (5.7), may be derived numerically using the conformal transformations of section 5.2. To arrive at a simple tractable model for these two functions, Jos calculated these two functions in this manner for a variety of d/b values and then empirically fit curves to the data, giving:

$$f \approx \left(\frac{b}{d}\right)^{0.85} + 0.37\left(\frac{d}{b}\right) + 2.55 \quad (5.9)$$

$$g \approx 1 + 0.6\frac{d}{b} \quad (5.10)$$

Where eqns (5.9) and (5.10) are valid for $1/5 \leq d/b \leq 5$. Eqns (5.7) and (5.8) then give the effective transit time of the periphery as:

$$\tau_p = \frac{q_{bt} - \tau J_s L}{I_c - L J_s} = \frac{\tau f}{g} \quad (5.11)$$

Eqn (5.11) gives an upper bound for the parameter τ_p because non-uniform doping has not been included. With non-uniform doping τ_p is reduced due to the effects of the built in aiding field and the reduction of the the current emanating from the emitter periphery nearest the surface (since this current travels the farthest, reducing it causes a significant reduction to τ_p). Furthermore, eqn (5.7) for I_c becomes an upper bound when the base

doping is non-uniform.

The equations for τ_p and I_c are for a uniformly doped base. To allow for the effects of non-uniform base doping begin by finding the point $y = y_{2/3}$, from the bottom of the emitter periphery, where two thirds of the total, integrated peripheral current is injected. The reason for choosing two thirds of the peripheral current is that two thirds represents a standard deviation of a gaussian distribution - a reasonable measure with reference to a gaussian. With reference to figure 45, $y_{2/3}$ is defined in the z plane. The corresponding value in the w plane is $x'_{2/3}$ which lies in the range $1 \leq x' \leq t$.

FIGURE 45. Definition of $y_{2/3}$ in the z plane.

Using eqn (5.5) for I_{cp} , the point in the w plane that corresponds to $y = y_{2/3}$ (termed $x'_{2/3}$) may be found:

$$\frac{\int_{w=x'_{2/3}}^{w=t} J_n(w) dw}{I_{cp}} = \frac{\cosh^{-1}(t) - \cosh^{-1}(x'_{2/3})}{\cosh^{-1}(t)} = \frac{2}{3}$$

giving

$$x'_{2/3} = \cosh\left(\frac{\cosh^{-1}(t)}{3}\right) \quad (5.12)$$

where t is defined in eqn (5.1). To transform $x'_{2/3}$ into $y_{2/3}$ eqn (5.2) is used. Since $x'_{2/3}$ is between points 4 and 3, the variable s must be purely imaginary. This allows eqn (5.2) to be written as

where

$$z = -\frac{d}{\pi\alpha} \ln\left(\frac{-1 + i\alpha s'}{1 + i\alpha s'}\right) + \frac{d}{\pi} \ln\left(\frac{-1 + is'}{1 + is'}\right) + id\left(\frac{1}{\alpha} - 1\right)$$

$$s' = \sqrt{\frac{t-w}{w-1}}, \quad \alpha = \sqrt{\frac{2}{t+1}}$$

Thus

$$\begin{aligned} z &= -\frac{d}{\pi\alpha} \left(\frac{1}{2} \ln\left(\frac{1 + \alpha^2 s'^2}{1 + \alpha^2 s'^2}\right) + i \arg(-\alpha s') - i \arg(\alpha s') \right) + \\ &\quad + \frac{d}{\pi} \left(\ln\left(\frac{1 + s'^2}{1 + s'^2}\right) + i \arg(-s') - i \arg(s') \right) + id\left(\frac{1}{\alpha} - 1\right) \\ &= -\frac{id}{\pi\alpha} (\arg(-\alpha s') - \arg(\alpha s')) + \frac{id}{\pi} (\arg(-s') - \arg(s')) + id\left(\frac{1}{\alpha} - 1\right) \end{aligned}$$

Simplifying gives

$$z = \frac{i2d}{\pi} \left(\frac{\tan^{-1}(\alpha s')}{\alpha} - \tan^{-1}(s') \right) \quad (5.13)$$

Up to this point all the equations produced from the conformal transformation of the emitter-base region have been based on a rectangular emitter. This shape for the emitter is only an approximation to make the conformal transform tractable. In reality, the emitter has rounded peripheries as depicted in figure 41. To represent the true geometry, $y_{2/3}/(ib)$ is taken to mean the fractional arc length of a circle with radius b to emit two thirds of the peripheral current (see figure 46). Then the effective depth D at which a circular emitter injects two thirds of the peripheral current is:

$$D = b \cos\left(\frac{y_{2/3}}{ib} \frac{\pi}{2}\right) \quad (5.14)$$

FIGURE 46. Definition for the area of a circular emitter required to inject $2/3$ of I_{cp} .

Using eqn (5.14), a definition for the percentage of the depth from the bottom of the emitter required to emit two thirds of the peripheral current becomes:

$$\% \text{ depth for circular emitter} = \frac{b - D}{b} \quad (5.15)$$

Figure 47 plots the results of eqn (5.15) and $y_{2/3}$ against the ratio b/d of emitter width to neutral base width. Figure 47 shows that the cross-section of the emitter required to inject two thirds of the peripheral current decreases with increasing b/d . Also, circular emitters require smaller cross-sections than rectangular structures to emit the same peripheral currents.

FIGURE 47. Plot of the cross-sections required to emit $2/3$ of I_{cp} vs. emitter width.

As can be seen for a wide range of ratios b/d , which would include all possible devices, for a circular structure the depth at which two thirds of the emitter peripheral current is emitted varies from around 40% to 20% of the junction depth, and decreases as b/d increases. Since this result is for a uniformly doped base, it represents an upper bound.

The decrease in the effective cross section of the periphery as b/d increases is explained by the following argument. Let d , the neutral base width, remain fixed. As b increases, the current emanating from the surface of the device must travel a longer path through the base to reach the collector. This longer path will increase Q_{bp} and thus reduce the magnitude of the current. As a result, less current is injected from the periphery near the device surface, reducing the fraction of the cross section required to emit two thirds of the peripheral current.

With the above results, it is now possible to develop a better approximation, under low level conditions, for the actual Q_{bp} and τ_p values when non-uniform base doping is accounted for. Starting with Q_{bp} , its value is proportional to the effective emitter length bg of eqn (5.7) from the analysis by Jos. However, this length needs to be adjusted to allow for the non-uniform doping of the base. Thus, an effective multiplier is desired to transform the length bg into one which includes the non-uniform doping and the circular emitter structure. Assuming that Q_{bp} is proportional to the vertical integration of the base doping from D down to the width of the emitter b , then:

$$Q_{bp} \approx Q_{b \text{ Jos}} \frac{\int_{u=D}^{u=b} N_{AB}(u) du}{(b-D) N_{AB}(b)} \quad (5.16)$$

where a change of variables $u = b - y$ has been made to move the origin to the device surface (this is the normal location of the origin for N_{AB}). D is given by eqn (5.14), $N_{AB}(u)$ is the base doping with $u=0$ at the device surface, and $Q_{b \text{ Jos}} = Q_b bg$ where Q_b is defined in eqn (4.26). This definition takes the uniform base doping case (given by $N_{AB}(b)$), and ratios it to the increased doping along the effective emitter peripheral cross-

section, $b - D$.

Assuming $N_{AB}(u)$ to be gaussian with a surface concentration N_0 and a diffusion length u_c , then:

$$N_{AB}(u) = N_0 e^{-u^2/u_c^2}$$

and eqn (5.16) becomes:

$$Q_{bp} = Q_{b Jos} \frac{u_c \sqrt{\pi} [\operatorname{erf}(b/u_c) - \operatorname{erf}(D/u_c)]}{2(b - D) e^{-(b/u_c)^2}} \quad (5.17)$$

In this equation, $Q_{b Jos} = bgQ_b$ (classic Gummel-Poon), b is the emitter width, d is the base width, and g is given by eqn (5.10). From eqns (5.12), (5.13) and (5.14)

$$D = b \cos \left(\frac{d}{b} \left(\frac{\tan^{-1}(\alpha s'_{2/3})}{\alpha} - \tan^{-1}(s'_{2/3}) \right) \right)$$

where

$$s'_{2/3} = \sqrt{\frac{t - x'_{2/3}}{x'_{2/3} - 1}}, \quad x'_{2/3} = \cosh \left(\frac{\cosh^{-1}(t)}{3} \right), \quad t = 2 \left(\frac{d+b}{d} \right)^2 - 1, \quad \alpha = \sqrt{\frac{2}{t+1}}$$

Eqn (5.17) will, for any given bipolar process, need to be calculated only once. Therefore, Q_{bp} can be calculated outside of the bipolar model and fed in as a process parameter. It is stressed that this result is approximate, relying on many assumptions and is for low level operation.

To verify the accuracy of eqn (5.17) and the analysis that created it, a check against the predictions of a true two dimensional simulator, PISCES, is given. The parameters from the PISCES simulation are:

$d = 0.22 \mu\text{m}$, $b = 0.17 \mu\text{m}$, and $u_c = 0.238 \mu\text{m}$. Then using the above equations gives:

$$t = 5.28512, \quad \alpha = 0.56410, \quad x'_{2/3} = 1.32252 \quad \text{and} \quad D = 0.1114 \mu\text{m}$$

Low level operation is required since the previous work is not valid for high level. Now, to find the current flowing out of the emitter periphery we begin with the work by Jos. Eqn (5.7) gives

$$I_{cp} \text{ (low level)} = J_s b g = J_s (0.17) (1 + 0.6d/b) = (0.302) J_s$$

This value for I_{cp} is based on a uniformly doped base. To include the effects of a non-uniformly doped base eqn (5.17) is used. This gives:

$$\frac{u_c \sqrt{\pi} [\operatorname{erf}(b/u_c) - \operatorname{erf}(D/u_c)]}{2(b-D)e^{-(b/u_c)^2}} = \frac{0.4218(0.6877 - 0.4920)}{2(0.0586)0.6004} = 1.173$$

The above factor of 1.173 gives the proportional increase in the majority charge in the emitter peripheral region over the case for a uniformly doped base. Thus, this factor of 1.173 is the proportional **decrease** in the emitter peripheral current compared to the case of a uniformly doped base, and the ratio of the emitter current emanating from the bottom of the emitter to that from the periphery of the emitter is:

$$\frac{J_s L}{J_s b g / (0.173)} = \frac{2 \mu\text{m}}{0.302 \mu\text{m} / (0.173)} = 7.77 \quad (5.18)$$

Now, at a V_{BE} of 0.6 V no emitter debiasing or high level effects are present. Thus, an accurate comparison between the value calculated in eqn (5.18) and the corresponding result from PISCES simulation can be made. At the midpoint of the emitter, PISCES gives the electron current density as 3.22 A/cm². The emitter drawn length L is 2 μm , giving a total current from the bottom of the emitter of 6.44×10^{-8} A/ μm . Now, the total electron current is 8.142×10^{-8} A/ μm , placing the emitter peripheral current at $0.5(8.142 \times 10^{-8} - 6.44 \times 10^{-8}) = 0.851 \times 10^{-8}$ A/ μm . This gives the ratio of the emitter current emanating from the bottom of the emitter to that from the periphery of the emitter as:

$$6.44/0.851 = 7.568$$

This number compares to the theoretical calculation in eqn (5.18) to within an error of +2.67%. The theory of Jos for a uniformly doped device gives the current ratio as 6.622, an error of -12.5%. These results do not prove the approximations used to generate eqn (5.17), but they do show that the theoretical calculations provide a good starting point for the experimental extraction of Q_{bp} and τ_p .

Up to this point, only eqn (5.17) for the calculation of Q_{bp} has been provided. Without the inclusion of τ_p , eqn (5.6) can not model high level effects on I_{cp} . Under low level conditions with uniform doping, eqn (5.11) gives the value of τ_p . However, from the arguments throughout this chapter, non-uniform doping contributes an aiding field, along with a reduction of emitter peripheral current from the surface; both tending to decrease τ_p from the uniform doped case. Furthermore, as high level effects occur, the value of τ_p changes from its low level value. This makes the accurate theoretical calculation of τ_p virtually impossible.

From experience with the SPICE bipolar model (which is based on eqn (5.6)), it has been found that eqn (5.6) can accurately model a fixed device by experimentally extracting and optimizing its parameters. Since the emitter periphery is fixed within a given process, eqn (5.6) can be used to model I_{cp} without compromising the scalable feature of the rest of the bipolar model. Therefore, τ_p should be found only through experimental extraction, with eqn (5.11) being used to place an upper bound on the final extracted value. By determining τ_p in this fashion, eqn (5.6) can accurately model I_{cp} with high level effects included.

It is concluded that by using the classic one dimensional model for the current I_{cp} , given by eqn (5.6), the emitter peripheral current can be reasonably modelled. The parameter Q_{bp} can then be roughly calculated by using eqn (5.17), and an upper bound placed on τ_p by eqn (5.11). Finally, to ensure that eqn (5.6) for the calculation of I_{cp} is as accurate as possible, Q_{bp} and τ_p are found through experimental extraction and optimization.

5.4. Peripheral Contribution To The Base Current

The analysis up to this point has been concerned with the effect of the emitter periphery on the collector current. Peripheral injection also influences the base current and this needs to be included in a realistic bipolar model.

The base current is essentially due to the back injection of holes into the emitter, recombination of minority carriers in the base being negligible. Emitter back injection is governed by the shape of the emitter periphery and doping distribution, together with recombination in the emitter volume and at the emitter contact. The situation is complex and intractable analytically, particularly since the conformal transformation approach is not applicable (the presence of recombination current sinks violates Laplace's equation). Therefore, the analysis of the proceeding section can not be used. Thus, there is no simple way to theoretically calculate the effect of the emitter periphery on the base current.

For the base current, however, there is a very important physical simplification, namely the emitter doping is typically high enough to preclude any high level effects. Furthermore, the periphery is a constant for any given bipolar process. Therefore, a simple low level model can be used to include the effects of the emitter periphery on the base current. The model for the peripheral base current is:

$$I_{bp} = I_{sbp} e^{V/V_T} \quad (5.19)$$

where I_{sbp} is the base peripheral saturation current. The parameter I_{sbp} is then left to experimental extraction.

The model for I_{bp} is very simple, and as long as the emitter doping is high enough to

ensure high level effects can not exist in the base, as should be the case in any practical device, the model will be accurate. Since I_{sbp} can not be treated analytically, the effect of the emitter periphery on the base current can only be determined after the process is created. However, as with the collector current, for a given bipolar process the emitter periphery is a constant, so the peripheral effect on the base current can be characterized using only one parameter, I_{sbp} .

6. A MODEL FOR THE EXTRINSIC BASE RESISTANCE

A very important modeling consideration for bipolar transistors is the calculation of the extrinsic base resistance associated with the flow of the base current from the periphery of the emitter to the base contact. The reason for the importance of accurately calculating the extrinsic base resistance is that any voltage drop in this resistance reduces the forward voltage drop applied to the emitter-base junction and has an exponential effect on the terminal currents.

The extrinsic base resistor is broken down into three distinct regions as illustrated in figure 48. These regions are:

- 1) The resistance due to the base current bending to enter the base contact.
- 2) The thin film resistance from the base contact up to the emitter periphery.
- 3) The resistance due to the base current crowding around the emitter periphery at the junction of the extrinsic and intrinsic base regions.

FIGURE 48. Cross-section showing the three regions that comprise the extrinsic base.

6.1. The Resistance Of The Base Contact

The calculation of the extrinsic base resistance begins with the base contact, region 1. Hébert and Roulston [8], and Hall [9], state that for all useful situations the effect of the thin film resistance in region 2 can be considered independent of the resistance of the base contact. This allows the resistance of the base contact to be calculated independently of the resistance of the thin film.

Hall solved for the resistance of the base contact by conformally mapping the region, assuming uniform doping of the base material (see figure 49). The result obtained by Hall for the base contact resistance was:

$$R_{bc} = \rho \left(a - \frac{2}{\pi} \ln \left(\sinh \left(\frac{a\pi}{2} \right) \right) \right) \quad (6.1)$$

FIGURE 49. This is the layout of the base contact that Hall used for his model. The depth of the film is s , the contact length is as , and the thin film length is Ns .

where ρ is the resistivity of the film in Ohms per square of cross-section times the unit depth into the page, s is the film thickness, N is the number of squares for the thin film region, and a is the number of squares for the base contact region. Eqn (6.1) is valid only for uniform doping, and is not strictly applicable for an actual bipolar transistor where the

doping is not uniform. However, by selecting an effective resistivity ρ along with a modified film thickness s , eqn (6.1) can be made to accurately predict the resistance of the base contact.

Before determining the effective ρ and s for eqn (6.1) to be valid, a plot of eqn (6.1) against $\log(a)$ is presented in order to gain a qualitative understanding for the base contact resistance. It is convenient to plot the normalized resistance of R_{bc}/ρ , the units of which are 'squares'.

This plot clearly shows that the base contact resistance increases rapidly as the base contact length becomes smaller than the film thickness s , and saturates for contact lengths greater than twice the film thickness. This saturation of the base contact resistance is very important because it points to the possibility that the contact resistance can become independent of the geometry of the contact. In fact, for virtually all conceivable bipolar processes one has that $a \geq 2$. Therefore, the resistance of the base contact can be considered independent to the geometry of the contact itself.

The saturation value of the base contact resistance is found by taking the limit of eqn (6.1)

as a becomes very large. This gives:

$$\lim_{a \rightarrow \infty} R_{bc}/\rho = \lim_{a \rightarrow \infty} \left\{ a - \frac{2}{\pi} \ln \left(\sinh \left(\frac{a\pi}{2} \right) \right) \right\} = \lim_{a \rightarrow \infty} \left\{ a - \frac{2}{\pi} \ln \left(\frac{e^a}{2} \right) \right\} = \frac{2}{\pi} \ln(2) = 0.441 \quad (6.2)$$

By assuming the normalized base contact is a constant 0.441 squares, then for all $a \geq 2$ the error is less than one quarter of one percent.

The preceding result, however, can only be applied to a "real" BJT with non-uniform doping if an effective P and s can be found to modify Hall's result given by eqn (6.1). Attention is now turned toward the determination of these effective values.

The term "effective film thickness" is taken to refer to the depth of an equivalent thin film, in terms of resistance, of uniform doping. If such a representation can be shown to be valid for a given non-uniform base doping profile, then eqn (6.1) can be applied for modeling this part of the extrinsic base resistance.

To see how Hall's result (eqn (6.1)) relates to the base contact resistance with non-uniform doping, PISCES was used to simulate the structure. The doping was chosen to be gaussian given by:

$$N_{AB}(y) = N_0 e^{-(y/y_c)^2}$$

where $y = 0$ is at the surface of the device. The PISCES simulations were based on the different structures shown in figure 50. The first structure is a simple thin film resistor used to calibrate the resistivity of the film. The second and third structures represent base contact geometries for comparison to Hall's result.

FIGURE 50. Cross-sections of the resistors used in the PISCES simulations.

In the PISCES simulations three different values for y_c were used; the corresponding resistance values given by PISCES for the above three test structures are given in table 1.

$y_c(\mu\text{m})$	Test (Ω)	Res #1(Ω)	Res #2(Ω)
0.33	128.07	88.00	45.41
0.66	66.17	46.88	24.91
1.20	46.26	33.56	18.28

TABLE 1. Resistance values from PISCES simulations of the structures in figure 50.

To prove that the base contact does indeed have a non-zero contribution to the extrinsic base resistance, a simple test on the above data can be made. If the base contact had zero resistance, then resistor #1 would be given by a resistance of $4/6 \times (\text{test resistance})$. For $y_c = 0.33 \mu\text{m}$, then $4/6(128.07) = 85.38$, a difference of 3.07 % when compared to the actual value for resistor #1. For $y_c = 1.20 \mu\text{m}$, then $4/6(46.26) = 30.84$, a difference of 8.82 %. It can be seen that the base contact does indeed have a resistance associated with it, and as the extrinsic base becomes smaller, the effect of the base contact on the extrinsic base resistance becomes larger.

The 'equivalent uniform doping' model is now defined by choosing the effective thickness s to be equal to y_c if y_c is less than the actual base region depth. Otherwise, s is made equal to the base depth. Assuming the former situation to apply, with $s = y_c$, the value of the effective ρ is found from PISCES simulations of the test resistor structure. Converting the total resistance of the test structure into Ohms per square by dividing by the effective film thickness s gives:

$$\rho = \frac{R_{\text{test}}}{L_{\text{test}}/s} \quad (6.3)$$

We may now evaluate the resistance of resistor #1 by breaking the structure into two parts. Part one is the thin film resistance and is given by:

$$R_{\text{Thin}} = \rho \frac{L_{\text{film}}}{s} \quad (6.4)$$

where L_{Test} is the length of the test resistor ($6 \mu\text{m}$ in this case), s is the effective film thickness, L_{film} is the length of just the thin film region (ie. length of the resistor minus the length of the base contact), and a is the base contact length divided by s .

The second part of the resistance is the base contact resistance, R_{bc} and is calculated by eqn

(6.1). Then the resistance of resistor #1 is $R_{\text{Thin}} + R_{\text{bc}}$. The resistance of resistor #2 is evaluated by the same procedure as for resistor #1, replacing the variables L_{test} and L_{film} with the appropriate values. It is stressed that when the base depth is less than y_c , then s is set equal to the base depth.

Table 2 compares the results obtained from the theoretical procedure above with the resistance values from PISCES simulations.

		% error between theoretical and simulated results for resistor #1.	% error between theoretical and simulated results for resistor #2.
y_c (μm)	∞	1.01	1.30
	0.33	0.55	0.86
	0.66	0.94	1.58
	1.20	2.00	4.00

TABLE 2. Comparison of theoretical and PISCES resistance values.

The case where $y_c = \infty$ is used to calibrate the error in the numerical simulation. Since letting y_c go to infinity creates a uniform doping, the results presented from theory should exactly equal the numerical; any error must then be attributed to numerical problems. Thus, the results for resistor #1 are good to ± 1 %, while the results for resistor #2 are good to ± 1.3 %. The results of table 2 show that with the effective film thickness set to one diffusion length, y_c , of the doping distribution, the theoretical results are very good.

The only point at which the theoretical calculations differ from the numerical calculations is

when y_c is somewhat larger than the actual base depth. However, when y_c is much greater than the actual base depth there is no error. Luckily, for most bipolar processes y_c is less than the actual base depth. Therefore, as demonstrated above, Hall's results, modified for non-uniform doping, may be used to accurately calculate the base contact resistance.

It is worthwhile to see if using an effective film thickness of $s = y_c$ can be improved upon. The following will also provide a method for the experimental extraction of the effective film thickness s .

For any one test structure it is possible to determine an effective depth at which the theoretical and simulated results exactly match. Using eqn (6.1), with the definition of eqn (6.3) for P , then

$$R(\text{simulated}) = \left[\frac{R_{\text{Test}}}{L_{\text{Test}}} s \right] \left[\frac{L_{\text{film}}}{s} + a - \frac{2}{\pi} \ln \left(\sinh \left(\frac{a\pi}{2} \right) \right) \right] \quad (6.5)$$

Eqn (6.5) is in general a transcendental equation that requires numerical methods to solve. However, if $a \geq 1.58$ then the following equation, accurate to within 1 %, can be used instead.

$$R(\text{simulated}) \approx \left[\frac{R_{\text{Test}}}{L_{\text{Test}}} s \right] \left[\frac{L_{\text{film}}}{s} + \frac{2}{\pi} \ln(2) \right]$$

Rearranging gives:

$$s = \frac{\pi(R(\text{simulated})L_{\text{Test}} - R_{\text{Test}}L_{\text{film}})}{2R_{\text{Test}}\ln(2)} \quad (6.6)$$

Table 3 shows the results obtained from eqn (6.6) for the various y_c values used in PISCES simulations.

$y_c(\mu\text{m})$	s/y_c	
	Res #1	Res #2
0.20	0.7708	0.8158
0.33	0.8438	0.8734
0.66	0.8623	0.8880

TABLE 3. Table of the effective depth s for various y_c .

This table shows that using an effective film thickness of $s = y_c$ is quite a reasonable approximation. To help reduce the data of the above table, the mean and standard deviation of all the data are 0.84 and 5.1% of the mean respectively. Therefore, using an effective film thickness s of $(0.84)y_c$ will give an error in the model of the base contact resistance, for cases where y_c is less than the base depth, of under 0.6 %.

To summarize the results for the model of the base contact, the resistance of the contact is given by:

$$R_{bc} = R(\text{base contact}) = \rho \left(a - \frac{2}{\pi} \ln \left(\sinh \left(\frac{a\pi}{2} \right) \right) \right)$$

$$\rho = \frac{R_{\text{test}}}{L_{\text{test}}} (0.84)y_c$$

where

$$a = \frac{\text{base contact length}}{(0.84)y_c}$$

and y_c is the gaussian diffusion length.

This resistance is for a unit depth into the cross section of the device and must be divided by the device depth to give the actual base contact resistance.

6.2. The Resistance Of The Thin Film

The second part of the extrinsic base resistance is simply the thin film resistance, given by:

$$R_{\text{Thin Film}} = \frac{\rho}{s} (\text{Number of squares in the plan view}) \quad (6.7)$$

The reason for dividing ρ by the effective film thickness s is because all of the results of the previous section were based on resistances per unit depth. Historically, when the thin film resistance has been calculated, the thin film thickness was incorporated into the resistivity term because this thickness was a process constant and did not need to be known. However, in the previous section, determining this thickness was very important. Therefore, in order to preserve the historical method of calculating the thin film resistance, eqn (6.7) is used.

6.3. The Resistance Of The Emitter Periphery

The third component of the extrinsic base resistance is due to the emitter periphery. The emitter periphery forces the base current down from the surface of the device into the intrinsic base region. Due to the non-uniform doping of the base, when the base current is pushed down from the surface, it must travel through an ever increasing resistive layer until it reaches the active base. As a result, the emitter periphery can contribute a significant portion of the total extrinsic base resistance.

Hall has developed an analytic model for the resistance of the emitter periphery for a uniformly doped base through the use of conformal mapping. However, it is the non-uniformity of the base doping that has a major influence on the resistance of the emitter periphery. Therefore, the non-uniform doping can not be neglected, making Hall's model in-applicable.

In treating the resistance of the base contact region, an effective film thickness was used to allow for the non-uniform doping in the base. This technique, however, does not lend itself to the emitter periphery because, depending on the base doping and the depth of the emitter, a different rule for an effective film thickness is required. However, as in the modeling of the emitter peripheral current in chapter 5, the fact that for a given bipolar process every possible device will have an identical emitter periphery can be used to an advantage, the emitter periphery being characterized by a process constant. The only assumption made in assuming a constant peripheral resistance is that conductivity modulation effects will be ignored.

To help understand the resistance of the emitter periphery, numerical simulations with

PISCES are used. The objective of the simulation is to study the effect of the emitter depth on the resistance of the emitter periphery. To help simplify the study, a rectangular emitter is used instead of a curved emitter periphery as illustrated in figure 51. Using a rectangular emitter places a lower bound on the resistance of the emitter periphery, for any circular emitter structure will have a higher resistance when compared to the rectangular case

FIGURE 51

The doping profile used for this test structure is gaussian, as defined for the base contact resistance model in section 6.1. The diffusion length of the doping profile is again y_c . The resistance of the emitter periphery is found by first simulating the above structure as a thin film test resistor (as in figure 50a) by making the emitter depth zero. Then the test structure is re-simulated with the desired emitter depth. By taking the resistance of the test structure with the desired emitter depth and subtracting off the resistance of the test structure with zero emitter depth, the emitter peripheral resistance is found. A plot of the emitter peripheral resistance for various values of y_c is found in figure 52. It is stressed that all these results are per unit depth of the structure as viewed from a cross-section.

The effective resistivity per square¹, based on the measurements of the test structure with zero emitter depth, for y_c equal to 0.2, 0.33, and 0.66 μm was found to be 5.92, 5.97, and

¹ This is based on the cross-section of the device and not the plan view as is normally done. Thus, all these results are per unit depth into the cross-section.

6.11 Ohms per square respectively. It can then be seen from figure 52 that for an emitter depth of $(1.25)y_c$, approximately 3.5 squares of resistance is associated with the periphery. Furthermore, the increase of the peripheral resistance is exponential with respect to the emitter depth normalized to y_c . When the emitter depth is forced below $2y_c$, the peripheral resistance increases to about 57 squares. Therefore, a very strict limit is placed on the depth of the emitter relative to the base doping profile. The rule of thumb is that the emitter depth, including depletion layer thickness, should be less than $(1.5)y_c$.

FIGURE 52. Results of the PISCES simulation of the emitter peripheral resistance

As was stated previously, the emitter peripheral resistance should be extracted experimentally from the bipolar process. A good empirical fit to the data in figure 52 gives for this specific case:

$$R_{\text{periphery}} \approx (0.21)40.3^{\text{emitter depth}/y_c} \quad (6.8)$$

showing the exponential rise in the peripheral resistance with respect to emitter depth over y_c .

The final recommendation for the calculation of the peripheral resistance is to rely on experimental extraction for the following three reasons:

- 1) The accurate calculation of the emitter peripheral resistance is important because it can be a large portion of the extrinsic base resistance.
- 2) Analytic and empirical equations are quite limited in accuracy and/or range of applicability.
- 3) The peripheral resistance is a constant for a given bipolar process. Thus, once the resistance is determined for any one process, it is a constant over that process.

Each section of this chapter has dealt with a region of the extrinsic base resistance. Together, these three regions form the complete extrinsic base resistance. Since each region is in series, the total extrinsic base resistance is given by the sum of the base contact, thin film, and emitter peripheral resistance.

7. ON THE FORMULATION OF THE OVERALL BIPOLAR MODEL

Up to this point in the thesis, various aspects of the internal physical mechanisms of a bipolar transistor, that need to be characterized in a complete unified model, have been studied. Many issues have been addressed, and appropriate models developed, however, the task of unifying these sub models remains. The sub models that have been developed up to this point are:

- 1) An extension of the basic Gummel-Poon base transport model to include the effects of bandgap narrowing.
- 2) A coupled two dimensional model for the effect of emitter debiasing with conductivity modulation in the base. Included are the equations for the base and collector terminal currents with high level effects.
- 3) A model for the emitter peripheral current.
- 4) A model for the calculation of the extrinsic base resistance.

Sub models 2), 3), and 4) are to be coupled with the one dimensional base transport model 1) to form a unified bipolar model¹.

The backbone of the bipolar model lies in the intrinsic region. Therefore, the emitter debiasing model, which applies to the entire intrinsic region, becomes the foundation of the overall bipolar model. The requirements of the emitter debiasing model need to be understood so that the other two sub models may then be integrated together to form the overall model.

¹ This is in the sense of modeling effects in the base and emitter. Modeling of the collector is beyond the scope of this thesis.

The emitter debiasing model, as it has been currently developed, requires the base-emitter voltage to be defined at the emitter periphery nearest to the base contact. Furthermore, it has been assumed in the development of the emitter debiasing model that the transverse base current from the emitter periphery farthest from the base contact is negligible². This assumption was made for two reasons:

- 1) The peripheral base current from the side farthest from the base contact is typically small compared to the total base current. Therefore, the effect of the peripheral current is also small. Furthermore, as emitter debiasing occurs, the contribution of the emitter periphery farthest from the base contact rapidly diminishes.
- 2) If a non-zero current at the emitter periphery farthest from the base contact is allowed in the emitter debiasing model, then two non-linear coupled equations need to be solved, greatly increasing the modeling complexity.

With this justifiable assumption, the emitter debiasing model requires that only the V_{BE} at the emitter periphery nearest the base contact be given as a boundary condition. No other information is needed.

Turning to the third sub model, which represents the emitter peripheral effects, since there are in general two emitter peripheries, as shown in figure 41 of chapter 5, the emitter-base voltage at both ends of the active base region are required in order to specify the peripheral currents. However, once the emitter debiasing model has been solved for, the voltage at both ends of the base, and hence the two emitter peripheral currents, will be known. Thus, in absence of extrinsic resistances, the total base, collector, and emitter currents would at

² In the case of a symmetric double base contact where the transistor is broken down along the line of symmetry, there is no transverse base current from the periphery farthest from the base contact.

this point be known for a specific V_{BE} .

These two sub-models complete the model for the intrinsic device. In the absence of extrinsic resistances, such as the extrinsic base resistance of the fourth sub-model, the device currents are defined, for given applied voltages V_{BE} , V_{BC} , in terms of the one dimensional Gummel-Poon base transport equation in association with the emitter debiasing and peripheral current sub-models. An implementation of the bipolar transistor model for this case is now proposed before considering the more complex general situation with extrinsic resistances in series with the base, collector, and emitter terminals.

7.1. A Bipolar Model Without Terminal Resistances

As was stated at the start of this chapter, the integration of the sub models into a unified bipolar model begins with the emitter debiasing model, which serves as the backbone for the overall bipolar model. Since there are no terminal resistances, the voltages at the internal device terminals become the applied V_{BE} and V_{BC} . This allows the emitter debiasing model to be solved for immediately, after which the peripheral currents can be calculated. Thus, the main task in this section is to implement the emitter debiasing model.

(a) Emitter debiasing model

The solution of the emitter debiasing model is arrived at through the determination of the arbitrary constants B and B_H (derived in chapter 4). These constants characterize the emitter debiasing model at low and high levels of operation. Eqns (4.18) and (4.35) are used for the solution of B and B_H respectively. Repeating these two equations:

$$BL \text{ (Low level model)} = \frac{L_e}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}} \cos(BL) \quad (7.1)$$

$$B_H L \text{ (High level model)} = \frac{L_e}{2} \sqrt{\frac{K_{sb}}{2aV_T}} e^{\frac{V_{BE}}{4V_T}} \cos(B_H L) \quad (7.2)$$

where

$$0 < B_H L, BL < \frac{\pi}{2}$$

Eqns (7.1) and (7.2) are both transcendental, requiring numerical solution techniques as presented shortly.

Once B and B_H have been determined, the base and collector terminal currents emanating from the intrinsic portion of the transistor can be determined. The low level equations for the solution of these currents are given by eqn (4.19) and (4.23). They are:

$$I_{bTL} = \frac{2BV_T}{\rho} \tan(BL) \quad (7.3)$$

$$I_{cTL} = \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{\Delta_{BL} + K' \sin(BL)}{\Delta_{BL} - K' \sin(BL)} \right) + \alpha \sin^{-1}(\alpha K' \sin(BL)) \right\} \quad (7.4)$$

where

$$K_1 = Q_b, \quad K_3 = \tau, \quad K_2' = 4q^2 n_i^2 D_n, \quad K_2 = \frac{K_2' K_3 4B^2 V_T}{K}$$

$$K_4 = K_1^2 - K_2' K_3 e^{V_{bc}/V_T}, \quad K = 2\rho J_{sb}, \quad \Delta_{BL} = \sqrt{1 - (\alpha K')^2 \sin^2(BL)}$$

$$\alpha^2 = \frac{K_4}{K_2}, \quad \text{and} \quad K' = \frac{1}{\sqrt{1 + \alpha^2}}$$

The high level equations for the solution of the base and collector currents are given by eqns (4.36) and (4.37) as follows:

$$I_{bTH} = \frac{64a^2 B_H^3 J_{sb} V_T^2 \tan(B_H L) (\tan^2(B_H L) + 3)}{3K_{sb}^2} \quad (7.5)$$

$$I_{cTH} = \int_0^L J_{nH}(x) dx \quad (7.6)$$

where

$$a = \frac{q U_{n/p} n_i \sqrt{D_n \tau}}{Q_{b0}}$$

and

$$J_{nH}(x) = \frac{Q_b - \sqrt{4q^2 n_i^2 D_n \tau \left(\frac{J_{bH}(x)}{J_{sb}} - e^{V_{bc}/V_T} \right) + Q_b^2}}{2\tau}$$

$J_{bH}(x)$ is given by eqn (4.34).

The solution procedure is thus as follows, beginning with a given V_{BE} . B and B_H are first found using eqns (7.1) and (7.2), and then substituted into eqns (7.3) through (7.6), arriving at values for the intrinsic region terminal currents according to the low and high level emitter debiasing model relationships. The base and collector currents are then given

by:

$$I_{bT} = \text{MAX}(I_{bTL}, I_{bTH}) \quad (7.7)$$

$$I_{cT} = \text{MAX}(I_{cTL}, I_{cTH}) \quad (7.8)$$

It is possible to reduce the amount of computation needed for eqn (7.8). Since I_{bTL} and I_{bTH} are analytic equations, the time required to calculate them is relatively small. I_{cTH} on the other hand requires numerical integration. Therefore, if it is possible to circumvent the calculation of I_{cTH} a great deal of time is saved. Now, for virtually every case, if $I_{bTL} > I_{bTH}$ then $I_{cTL} > I_{cTH}$, so that $I_{cT} = I_{cTL}$. Furthermore, if $I_{bTL} < I_{bTH}$ then $I_{cTL} < I_{cTH}$, giving $I_{cT} = I_{cTH}$. Therefore, the information gained by calculating I_{bT} indicates whether the low or high level form of the emitter debiasing model is to be used.

(b) Emitter peripheral currents

At this point the contribution of the intrinsic region to the total base and collector currents is known. The current contribution from the emitter periphery is now to be added. The model for the emitter peripheral current is given by eqns (5.6) and (5.19). They are repeated here:

$$I_{cp} = \frac{Q_{bp} - \sqrt{4q^2 n_i^2 D_n \tau_p (e^{\beta V} - e^{\beta V_{bc}}) + Q_{bp}^2}}{2\tau_p} \text{ A/cm} \quad (7.9)$$

where

$$\beta = \frac{1}{V_T}$$

$$I_{bp} = J_{sbp} e^{V/V_T} \quad (7.10)$$

and V refers to the emitter-base voltage at either end of the emitter periphery.

Eqns (7.9) and (7.10) for the emitter peripheral current must be evaluated twice - once for

the periphery nearest the base contact, and once for the periphery farthest from the base contact. Since extrinsic resistances are here taken to be zero, the former is V_{BE} . The voltage at the other end of the periphery is obtained from the emitter debiasing model, using either the low or high level form as employed for the calculation of I_{bT} above.

Repeating eqns (4.16) and (4.33), the low and high level emitter debiasing models give the voltage V as a function of position as:

$$V(x) \text{ (low level)} = 2V_T \ln \left[2B \sqrt{\frac{V_T}{K_{sb}}} \sec(B(L-x)) \right] \quad (7.11)$$

$$V_H(x) \text{ (high level)} = 4V_T \ln \left\{ 2B_H \sqrt{\frac{2aV_T}{K_{sb}}} \sec(B_H(x-L)) \right\} \quad (7.12)$$

The voltage at the periphery furthest the base contact is then found by setting $x = L$ in the above two equations, giving:

$$V_{PL} \text{ (low level)} = V_T \ln \left(4B^2 \frac{V_T}{K_{sb}} \right) \quad (7.13)$$

$$V_{PH} \text{ (high level)} = V_T \ln \left\{ 16B_H^4 \left(\frac{2aV_T}{K_{sb}} \right)^2 \right\} \quad (7.14)$$

Therefore, if the low level model was chosen for I_{bT} then V_{PL} is used. Otherwise V_{PH} is used. The emitter peripheral current is then calculated by setting V in eqn (7.9) and (7.10) first to V_{BE} and then to either V_{PL} or V_{PH} . These two peripheral currents are then added together to give the total peripheral current. This completes the evaluation of the bipolar model for the case of zero extrinsic resistances. The sequence of calculation for the terminal currents is shown in figure 53.

(c) Numerical procedure for calculation of B and B_H .

The constants B and B_H are calculated by standard Newton-Raphson techniques using eqns (7.1) and (7.2) respectively. Rewriting eqns (7.1) and (7.2) suitably for this purpose:

$$F(BL) = A_L \cos(BL) - BL \quad (7.15)$$

$$G(B_H L) = A_H \cos(B_H L) - B_H L \quad (7.16)$$

where

$$A_L = \frac{L}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE}}{2V_T}}, \quad A_H = \frac{L}{2} \sqrt{\frac{K_{sb}}{2aV_T}} e^{\frac{V_{BE}}{2V_T}}$$

and

$$0 < B_H L, \quad BL < \frac{\pi}{2}$$

Since the function F for the solution of BL is of the same form as the function G used to solve $B_H L$, the procedure used to solve for BL can also be used to solve for $B_H L$; A_i being set to either A_L or A_H as appropriate.

FIGURE 53. Flow diagram for the intrinsic bipolar model.

Starting with an initial guess of BL_0 , the Newton-Raphson iterative procedure is expressed by

$$BL_{n+1} = BL_n - \frac{F(BL)}{F'(BL)} \Big|_{BL=BL_n} \quad (7.17)$$

Differentiating the function F with respect to BL gives:

$$F'(BL) = -A_i \sin(BL) - 1$$

where

$$A_i = A_L \text{ or } A_H$$

Substituting this into eqn (7.17)

$$BL_{n+1} = BL_n + \frac{A_i \cos(BL) - BL}{A_i \sin(BL) + 1} \Big|_{BL=BL_n} \quad (7.18)$$

Eqn (7.18) is then iterated until $F(BL)$ is arbitrarily close to zero, or the change from BL_n to BL_{n+1} is less than some value. The last BL_{n+1} is then set equal to BL .

Whenever the Newton-Raphson technique is used, a problem with convergence may occur. If the function being solved for (in this case $F(BL)$) has points of inflection, then it is quite possible that the Newton-Raphson technique could oscillate and never converge. To determine if oscillation is a possibility in $F(BL)$, a plot where A_i is set to three different values is shown in figure 54.

FIGURE 54. Plot of $F(BL)$ used to solve for BL without R_B .

It is clear that the function $F(BL)$ is very well behaved - its first derivative is always negative and there are no points of inflection. Furthermore, if one makes the first guess of $BL_0 = \pi/2$, then a nearly straight line from BL_0 to the solution where $F(BL) = 0$ is observed. Therefore, the Newton-Raphson technique will converge rapidly and unconditionally with the starting point of $BL_0 = \pi/2$.

It has been found that eqn (7.18) with $BL_0 = \pi/2$ converges within a maximum of three iterations for all bias levels to a precision of six decimal places. Furthermore, as the device enters high level operation the number of iterations can fall to only one iteration. To speed up the process, the first iteration is calculated analytically. Setting BL_n to $\pi/2$ in eqn (7.18) gives as the starting point:

$$BL_0 = \frac{\pi}{2} - \frac{\frac{\pi}{2}}{A_i + 1} = \frac{\pi}{2} \frac{A_i}{A_i + 1}$$

This is much simpler to calculate than a single iteration through eqn (7.18), and therefore represents a significant savings in the solution of B and B_H .

(d) Operational considerations

To save additional simulation time, the emitter debiasing model can be circumvented if the transistor is operating in either the very low or very high level region. In the very low level region there is no emitter debiasing. Therefore, the voltage along the entire length of the emitter is constant, and there is no need to solve the emitter debiasing model. When the transistor is operating in the very high level region, then emitter debiasing has occurred to such a degree that all the current basically emanates from the emitter periphery nearest the base contact. When this occurs, the base and the collector currents both follow an

exponential behavior, given by eqns (4.39) and (4.40), that depend only on the emitter-base voltage at the emitter periphery nearest the base contact. Therefore, the peripheral current from the side farthest from the base contact can be ignored under very high level operation. This again makes the calculation of the emitter debiasing model unnecessary.

To take advantage of these two time saving measures, a check for very low and very high level operation is required. Then, if it is found that the transistor is indeed in one of these regions of operation, the emitter debiasing model can be skipped, greatly reducing the execution time of the overall bipolar model. The task becomes one of defining rules for the determination of very low and very high level operation.

We begin by defining a rule for the onset of very low level operation. There are basically two different ways to approach this definition. The first is to find the V_{BE} at which the voltage difference from one side of the emitter to the other is some small fraction of V_T . Alternatively, the V_{BE} at which the low level asymptote for the base current is within some arbitrary value of the actual base current may be found. Using the first method, which gives an indication of the extent to which the emitter has debiased

$$V(0) - V(L) = QV_T \quad (7.19)$$

where Q is required to be sufficiently small.

Using the low level model for emitter debiasing, the voltage as a function of position given by eqn (4.16) results in:

$$\left. \begin{aligned} V(0) &= 2V_T \ln \left[2B \sqrt{\frac{V_T}{K_{sb}}} \sec(BL) \right] \\ V(L) &= 2V_T \ln \left[2B \sqrt{\frac{V_T}{K_{sb}}} \right] \end{aligned} \right\} \quad (7.20)$$

so that eqn (7.19) gives

$$Q = 2 \ln(\sec(BL))$$

Then the V_{BE} at which extreme low level operation occurs is found by substituting the previous result into eqn (7.20) for $V(0)$. This gives:

$$V_{BE} \text{ (very low level)} = V(0) = 2V_T \left\{ \ln \left[\cos^{-1} \left(e^{\frac{Q}{2}} \right) \frac{2}{L} \sqrt{\frac{V_T}{K_{sb}}} \right] + \frac{Q}{2} \right\} \quad (7.21)$$

where a suitable value of Q is 0.01.

Thus, whenever V_{BE} is equal to or lower than the V_{BE} given by eqn (7.21), the voltage along the entire emitter length can be considered a constant, giving

$$\left. \begin{aligned} I_{bT} &= L J_{sb} e^{V_{BE}/V_T} \\ I_{cT} &= L \frac{Q_b - \sqrt{4q^2 n_i^2 D_n \tau (e^{V_{BE}/V_T} - e^{V_{BC}/V_T}) + Q_b^2}}{2\tau} \end{aligned} \right\} \quad (7.22)$$

Next, the definition for the onset of very high level operation is required. For this purpose, a value of V_{BE} is defined at which the high level asymptote for the base current is within some arbitrary value of the actual base current. Since we are dealing with very high level operation, the high level model for emitter debiasing is used: eqn (4.39) gives:

$$I_{bT} \text{ (very high level)} = \frac{J_{sb}}{3} \sqrt{\frac{8aV_T}{K_{sb}}} e^{\frac{3V_{BE}}{4V_T}} \quad (7.23)$$

Eqn (4.36) gives I_{bT} for all bias conditions. Knowing that in very high level $B_H L \approx \pi/2$, then $\tan^3(B_H L) \gg \tan(B_H L)$. Inserting this condition in eqn (4.36) gives:

$$I_{bT} \approx \frac{64a^2 B_H^3 J_{sb} V_T^2 \tan^3(B_H L)}{3K_{sb}^2} \quad (7.24)$$

where this value of I_{bT} is set to some fraction P of the high level asymptote value of eqn (7.23). Then

$$P \sqrt{\frac{8aV_T}{K_{sb}}} e^{\frac{3V_{BE}}{4V_T}} = \frac{64a^2B_H^3V_T^2 \tan^3(P_H L)}{K_{sb}^2} \quad (7.25)$$

The closer P is to unity, the better will be the approximation of using the extreme high level asymptote. Using eqn (7.2) for the definition of $B_H L$ gives:

$$e^{\frac{3V_{BE}}{4V_T}} = 8B_H^3 \sec^3(B_H L) \sqrt{\frac{2aV_T}{K_{sb}}} \left(\frac{2aV_T}{K_{sb}} \right)$$

Substituting this into eqn (7.25) gives:

$$P = \sin^3(B_H L) \quad (7.26)$$

Using this relationship and eqn (4.33), the definition for the V_{BE} at the onset of extreme high level operation becomes:

$$V_{BE} \text{ (very high level)} = V_H(0) = 4V_T \ln \left\{ \frac{2 \sin^{-1}(P^{1/3})}{L \sqrt{1 - P^{2/3}}} \sqrt{\frac{2aV_T}{K_{sb}}} \right\} \quad (7.27)$$

where a suitable value for P is 0.99.

Whenever V_{BE} is greater than the V_{BE} given by eqn (7.27), then the base and collector currents are given by eqns (4.39) and (4.40). Furthermore, the peripheral current farthest from the base contact can be neglected from the total current of the transistor.

It is instructive to evaluate the V_{BE} values of eqns (7.21) and (7.27) for typical values of the various parameters involved (taken to match PISCES simulations in chapter 2). The parameters are:

$$\rho = 7400 \, \Omega, \quad J_{sb} = 3.96 \times 10^{-12} \, \text{A/cm}^2, \quad L = 2 \times 10^{-4} \, \text{cm}, \quad Q_{b0} = 4 \times 10^{-7} \, \text{C}, \quad U_{n/p} = 2.9, \\ D_n = 20.8 \, \text{cm}^2/\text{s}, \quad \tau = 9.048 \times 10^{-12} \, \text{s}.$$

This makes $a = 2.308 \times 10^{-7}$ and $K_{sb} = 5.861 \times 10^{-8}$.

Then, with $Q = 0.01$ (1% error), the V_{BE} for very low level operation is:

$$V_{BE} \text{ (very low level)} = 0.6947 \text{ V}$$

And, with $P = 0.99$ (1% error), the V_{BE} for very high level operation is:

$$V_{BE} \text{ (very high level)} = 1.17 \text{ V}$$

Thus, when V_{BE} is between 0.6947 and 1.17 V the complete emitter debiasing model must be used. For all other $V_{BE} < 0.6947$ or > 1.17 V, the simple exponential model for the BJT will suffice. It is noted that for V_{BE} greater than 0.7 V the effect of emitter debiasing is important. Therefore, over most of the useful operating range, the two dimensional nature of the BJT can not be ignored.

7.2. A Bipolar Model With Terminal Resistances

The device currents I_C , I_B , and I_E are defined in terms of the internal voltages V_{BE} , and V_{BC} applied to the intrinsic model of section 7.1. For the actual device, however, these voltages are not known explicitly and need to be determined from the applied terminal voltages by allowing for the voltage drops in the extrinsic regions, which in turn depend on the device currents. The simplest representation of the extrinsic regions is shown in figure 55, involving three series resistances: the extrinsic base resistance of the fourth sub-model, an emitter contact resistance and a simple collector resistance.

It is possible to take many approaches for the inclusion of the extrinsic resistances in the model. The first method simply brings all these resistances into the model. This requires that a reasonably sophisticated numerical problem solver be included in the overall bipolar model. The second approach is to adopt the representation of figure 55 with two nodes and one 'terminal resistance' associated with each device terminal. Then, let the circuit simulator, which is an efficient numerical problem solver, determine the voltage drop across the terminal resistances. Other approaches will bring some of the extrinsic resistance representation into the bipolar model while letting the rest remain outside.

The argument for including the terminal resistances into the overall bipolar model is that the solution algorithm could be optimized. The circuit simulator must be robust enough to handle any circuit, but the specific case of the terminal resistances certainly does not require the capacity of a general circuit simulator. Another reason for including the terminal resistances is that the arbitrary constant for the emitter debiasing model already requires numerical simulation. Therefore, including the terminal resistances may possibly prove to

be only a small increase in execution time.

However, the problem with including the terminal resistances within a 3-node model stems from the highly coupled nature of the problem. In order to include these resistances an iterative technique must be used. This forces the bipolar model to include a numerical problem solver. Furthermore, if resistances in the circuit are placed in series with the terminals of the BJT, the circuit simulator can not reduce the complexity of the problem because the terminal resistances would be inside the model, and beyond the control of the simulator.

FIGURE 55. Identification of the intrinsic and extrinsic regions of a BJT.

It is concluded that to include the extrinsic base, emitter, and collector resistances within the bipolar model is not realistic. This conclusion is evident from the flow diagram in figure 56 for an iterative scheme required to solve for a bipolar model with terminal resistances.

The flow diagram in figure 56 requires numerous iterations around the intrinsic model without terminal resistances. Furthermore, this type of iterative solution technique is provided by the highly efficient circuit simulator. Therefore, inclusion of the terminal

resistances is unnecessary, since the circuit simulator itself is best suited to solve this problem.

FIGURE 56. Flow diagram for the intrinsic and extrinsic bipolar model.

It is possible, however, to reach a compromise where some of the terminal resistances are brought into the model and the rest remain outside for the simulator to deal with. As an example, the extrinsic base resistance R_B is brought into the model. To help simplify the problem, the peripheral current is assumed small enough so that it may be ignored for the purpose of calculating the voltage drop across the extrinsic base resistance. Then, eqns (7.1) and (7.2) for the solution of B and B_H respectively are modified to include the extrinsic base resistance. Concentrating on eqn (7.1), the voltage V_{BE} is modified as follows:

$$V_{BE} \rightarrow V_{BE,T} - I_{bTL} R_B \quad (7.28)$$

where $V_{BE,T}$ is the voltage between the external device terminals, as defined in figure 55, with zero emitter resistance.

Then substituting eqn (7.3) for I_{bTL} into eqn (7.28) gives:

$$V_{BE} \rightarrow V_{BE,T} - \frac{2R_B BL V_T}{\rho L} \tan(BL) R_B$$

Substituting this into eqn (7.1) gives:

$$BL = \frac{L}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE,T}}{2V_T}} e^{-\frac{R_B BL}{\rho L} \tan(BL)} \cos(BL)$$

Putting this into a form suitable for the Newton-Raphson technique gives:

$$F(BL) = A e^{-C BL \tan(BL)} \cos(BL) - BL \quad (7.29)$$

where

$$A = \frac{L}{2} \sqrt{\frac{K_{sb}}{V_T}} e^{\frac{V_{BE,T}}{2V_T}}$$

and

$$C = \frac{R_B}{\rho L}$$

Solution of eqn (7.29) gives the value of BL while including the effects of the extrinsic base resistance R_B . The problem with eqn (7.29) is that there is a point of inflection in this relationship. This makes eqn (7.29) unstable and difficult to solve. To see this, a plot of eqn (7.29) with $A=3$ and $C=10, 1, 0.1$, and 0.01 is shown in figure 57.

As can be seen in figure 57, starting at either $Bl = \pi/2$ or 0 , will result in oscillations that never converge. In fact, only a very limited number of "good" initial conditions will provide convergence. Further, the speed of convergence for the solution of eqn (7.29) is low. Due to the presence of the factor $\text{EXP}[-BL \tan(BL)]$, eqn (7.29) is highly non-linear.

Application of the linear Newton-Raphson technique results in a large number of iterations to reach the solution. In the case where $A=3$ and $C=0.1$ (this corresponds to a V_{BE} of around 0.9 V and a typical extrinsic base resistance), ten iterations are required. This is a large number of iterations compared to the case of no extrinsic base resistance where only two iterations are required. Furthermore, due to the complexity of eqn (7.29) compared to eqn (7.1), even a single iteration through eqn (7.29) takes longer than one pass through eqn (7.1). Therefore, inclusion of the extrinsic base resistance causes a substantial increase in the number of iterations to solve for BL .

FIGURE 57. Plot of $F(BL)$ used to solve for BL with R_B .

Up to this point, only eqn (7.1) for the solution of B has been addressed. When eqn (7.2) for the solution of B_H is modified to accept the extrinsic base resistance, an even more difficult equation, when compared to that with eqn (7.29), results. And use of a second order Newton-Raphson technique provides little help in convergence, because the second order technique is not capable of following the exponential behavior inherent when the extrinsic base resistance is added. Thus, inclusion of just one of the terminal resistances, namely the extrinsic base resistance, proves to be difficult and time consuming.

Furthermore, it was assumed that the peripheral currents could be neglected; their inclusion adds to the problem. It is concluded therefore, that inclusion of all the terminal resistances within the overall device model, as represented by the flow diagram in figure 56, is impractical.

This section has discussed the basic requirements needed to include the terminal resistances into the bipolar model, and has indicated the numerical difficulties that would be incurred if such an approach were adopted. Thus, the task of calculating the effect of the terminal resistances is best left up to the circuit simulator, the bipolar model being partitioned into intrinsic and extrinsic sections, as shown in figure 55. The model algorithm involved is then that of the intrinsic model without terminal resistances as portrayed by the flow diagram in figure 53.

8.

CONCLUSIONS

This thesis has investigated the fundamental D.C. mechanisms of bipolar junction transistors under all levels of injection, with a view to representing these mechanisms analytically in an accurate device model. The main thrust of this thesis is to study and uncover the underlying physics of bipolar transistors so that regardless of operating conditions, a model that is capable of scaling to cover any arbitrary device geometry results. Following the study of each major effect, pertinent modeling equations are developed and justified, with their limitations duly noted. The major study and model development in this thesis surrounds the effects of bandgap narrowing, emitter peripheral currents, emitter debiasing in two dimensions, and extrinsic base resistance. The final accomplishment of this thesis is a discussion regarding the unification of the various sub models into a bipolar model structure.

The thesis begins with a two dimensional numerical study of an NPN transistor using PISCES. This study establishes the following attributes of bipolar operation:

- 1) The minority current density in the base, even when influenced by the distributed effect of emitter debiasing, flows in a one dimensional manner normal to the emitter-base and collector-base junctions.
- 2) The effect of emitter debiasing is definitely present in all practical structures, and needs to be modelled except for operation at very low levels.
- 3) The emitter periphery, although not dominant, does contribute a substantial amount to the total current emanating from a BJT. Furthermore, the emitter periphery must be considered in two dimensions.
- 4) Bandgap narrowing, caused by non-uniform doping in the base, has an appreciable

effect on the base transport current (30% for the transistor under study).

- 5) Electric field dependent mobility is not an issue in the base region, since the electric field strength is never high enough to have a significant effect.
- 6) Under virtually all bias conditions the assumption of quasi-neutrality is justified in the base region.
- 7) Finally, in the original model formulation for the base transport current given by Gummel and Poon, the accuracy can be better than 1% under all levels of injection regardless of the doping profiles and the distributed effect of emitter debiasing, as long as bandgap narrowing is not a factor.

From these findings the model development for the rest of the thesis was determined.

The main work of the thesis has been on four aspects of bipolar transistor modeling that are vital to the development of an accurate overall model of the device: base transport, emitter debiasing, peripheral emitter currents and extrinsic base resistance. Original contributions have been made in each of these areas.

Detailed studies established the accuracy of the Gummel-Poon equation for the base transport current as conventionally formalized for a base of uniform bandgap. Since bandgap narrowing in the base of a BJT has a significant effect on the base transport current, the possibility of analytic modifications to the Gummel-Poon model to include these effects was investigated. The resultant model, termed the modified Gummel-Poon model, has all the capabilities of the original Gummel-Poon model, with the added ability to model the effect of bandgap narrowing in the base. A further use of the modified Gummel-Poon model is the ability to extract, from experimental data, the exact relationship between the doping concentration and the bandgap energy. Comparisons of the modified Gummel-Poon model to PISCES simulations show that even in the presence of bandgap narrowing

the accuracy of the modified Gummel-Poon model can be better than 1%.

Up to the time of the present work, only a simple low level model for emitter debiasing, as presented by Hauser, has been available. The distributed effect of emitter debiasing invalidates the simple scaling rules used by the SPICE bipolar model. Emitter debiasing is caused by the transverse flow of base current through the highly resistive active base layer. The result is a position-varying injection of current into the emitter and the base. Since emitter debiasing is closely coupled to the length of the emitter, neglecting this effect makes the resultant model incapable of scaling. To ameliorate this problem, a complete emitter debiasing model was developed that included the effects of conductivity modulation in the base. This model is analytic, requiring only simple numerical calculation of a single boundary condition. All the pertinent equations relating the terminal voltages to the terminal currents are also provided. Calculations for a typical NPN transistor have shown that for $V_{BE} > 0.7$ V emitter debiasing is an important effect, meaning that for most of the useful operating range of BJTs, emitter debiasing must be accounted for.

With the advances of photolithography, the lateral dimensions of devices have been decreasing at a faster rate than the vertical. As a result, the contribution of the emitter periphery to the total current emanating from a BJT has increased. This places a need on the accurate modeling of the emitter peripheral current in order to achieve a scalable model. Through conformal transformation, the emitter peripheral current was studied and a model developed. The model was also extended to cover non-uniform doping, circular emitter peripheral structures, and high level operation. By including the effects of the emitter periphery, the resultant bipolar model becomes much more accurate. Furthermore, in future devices, the importance of the emitter periphery in terms of terminal currents will increase, maintaining the need for an accurate emitter periphery model.

The extrinsic region of the device, although its effect on current injection is indirect, nevertheless has an appreciable effect on BJT operation. The base resistance associated with the extrinsic region of the bipolar transistor was investigated. Accurate modeling of the extrinsic base resistance is paramount to the final accuracy of the model because any error is magnified exponentially. The current flow in the extrinsic base resistance was broken down into three distinct regions:

- 1) The base contact.
- 2) The thin film resistance connecting the base contact to the intrinsic base region.
- 3) The resistance of the emitter periphery. Since the base current travels through the extrinsic base near the surface of the device, when it enters into the intrinsic base region it must pass around the emitter periphery, contributing to the total extrinsic base resistance.

Each of the three regions of the extrinsic base resistance was studied and appropriate models developed. In the case of the base contact region, previous analysis by conformal transformation was modified to allow for non-uniform doping. It was concluded that the peripheral resistance was not suitable for analytic modeling, and that since this resistance is a constant for a given process, it should be determined through experimental extraction.

The final work of this thesis discusses the unification of the various sub-models into a bipolar model. Two approaches are presented: one that does not include extrinsic resistances into the model, termed an intrinsic model, and one that does include the extrinsic resistances into the model. The problems incurred with the numerical solution of the constants in the emitter debiasing model were also addressed. This included the guarantee of rapid and unconditional convergence of the overall model. Measures were also introduced to reduce the execution time of the overall bipolar model by detecting when

effects contained within sub-models become negligible under certain operating conditions. It is then concluded that the inclusion of the extrinsic resistances into the bipolar model should not be attempted, making the intrinsic bipolar model the overall bipolar model.

8.1. Recommendations For Further Research

The first recommendation is to implement the model of chapter 7. The implementation of the model should then be compared to either fabricated devices or PISCES simulated devices. In both cases, the device must either be made with an N^+ collector, or biased in such a manner as to preclude collector effects. The reason for doing this is the model equations developed throughout this thesis assume the collector effects to be negligible.

To form a truly complete bipolar model, the collector region must be adequately modelled. To include collector effects both the one dimensional base transport model and the one dimensional base conductivity model must be further developed to include:

- 1) The Kirk effect or base push out.
- 2) Quasi-saturation.
- 3) High level boundary conditions at the collector boundary of the neutral base.

Then, the base transport and conductivity models need to be reestablished in the emitter debiasing model to form a complete quasi-2D representation of BJTs.

Other recommendations for future research include the solution of the emitter debiasing model without invoking the assumption of low or high level operation. By doing this it will be possible to extend the bipolar model to devices in which the effect of emitter debiasing occurs at a V_{BE} far above the one for high level effects. However, this will most likely require numerical solution techniques.

The final area that requires better models is the emitter peripheral region. The study of high level effects on the emitter periphery has been mostly qualitative. Therefore, improved

models that properly cover high level effects on the emitter periphery are needed.

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APPENDIX A

SOLUTION OF THE DIFFERENTIAL EQUATION FOR J_n

Following are two solutions of eqn (3.4) for the excess carrier concentration $n'(x)$ in the base. Rearranging this equation:

$$J_n = qD_n \frac{2n' + N_{AB}}{n' + N_{AB}} \frac{dn'}{dx} + \frac{qD_n n'}{n' + N_{AB}} \frac{dN_{AB}}{dx} \quad (A.1)$$

A general solution for the non-linear differential equation defined by eqn (A.1) appears impossible. An implicit solution, however, does exist if the doping in the base is constant. Letting the doping N_{AB} be constant with respect to x , eqn (A.1) becomes:

$$J_n = qD_n \frac{2n' + N_{AB}}{n' + N_{AB}} \frac{dn'}{dx} \quad (A.2)$$

Solving eqn (A.2) for $n'(x)$ gives:

$$x = \frac{qD_n}{J_n} (2n' - N_{AB} \ln(N_{AB} + n')) + \text{constant} \quad (A.3)$$

Therefore, it is possible to obtain a solution for $n'(x)$, but even for the case of constant doping the solution remains implicit.

A second method for solving eqn (A.1) is through the use of series approximations. To solve eqn (A.1) with a Taylor series, all of the derivatives of $n'(x)$ need to be calculated and the solution becomes:

$$n(x) = \sum_{i=0}^{\infty} \frac{n'(x_1)}{i!} (x - x_1)^i \quad (A.4)$$

By manipulating eqn (A.1) into the form of:

$$\frac{dn'}{dx} = f(x, n')$$

then by recursive means it is possible to solve for all the derivatives. This gives:

$$\frac{d^j n'}{dx^j} = \frac{d^{j-1} f(x, n')}{dx^{j-1}}$$

Using eqn (A.4), and expanding about the emitter edge given by $x=x_1$, the first derivative of $n'(x)$ is:

$$\frac{dn'}{dx} = \frac{K_n(n'(x_1) + N_{AB}) - n'(x_1) \frac{dN_{AB}}{dx}}{2n'(x_1) + N_{AB}} \quad (A.5)$$

where

$$K_n = \frac{J_n}{qD_n}$$

The second derivative of $n'(x)$ is then found by differentiating eqn (A.5) to give:

$$\frac{d^2 n'}{dx^2} = \frac{-N_{AB} \frac{dn'}{dx} \left(\frac{dN_{AB}}{dx} + K_n \right) + K_n n' \frac{dN_{AB}}{dx} - n' \frac{d^2 N_{AB}}{dx^2} (2n' + N_{AB}) + n' \left(\frac{dN_{AB}}{dx} \right)^2}{(2n' + N_{AB})^2}$$

This procedure can be repeated as many times as desired, however, the complexity is rapidly increasing. Furthermore, the error at the collector edge can be quite large if only a few terms of the Taylor series are used. Finally, the Taylor series depends on J_n , requiring that J_n and the integral of $p(x)$ be solved for simultaneously by using the Gummel-Poon model and the Taylor series solution of eqn (A.1).

APPENDIX B

CALCULATION OF K FOR THE BANDGAP NARROWING MODEL

This chapter sets out to detail the derivation of the function K from the bandgap narrowing model given in eqn (3.23). The derivation of K begins with the differential equation given in eqn (3.24). Repeating eqn (3.24):

$$J_n = qD_n \frac{2n' + N_{AB}}{n' + N_{AB}} \frac{dn'}{dx} \quad (B.1)$$

Remembering the assumption that the derivative of n' with respect to x is a constant given by m , then rearranging eqn (B.1), integrating with respect to x , and using an effective diffusion coefficient gives:

$$\int_{x_1}^{x_2} (n' + N_{AB}) dx \frac{J_n}{q\bar{D}_n m} = \int_{x_1}^{x_2} (2n' + N_{AB}) dx \quad (B.2)$$

Then, by the results of charge control theory given by eqn (3.8), eqn (B.2) becomes:

$$(J_n \tau + Q_{b0}) \frac{J_n}{q\bar{D}_n m} = 2J_n \tau + Q_{b0}, \text{ where } Q_{b0} = \int_{x_1}^{x_2} N_{AB} dx \quad (B.3)$$

Thus, as defined in chapter 3

$$K = \frac{J_n}{qm\bar{D}_n} \\ = \frac{2J_n \tau - Q_{b0}}{J_n \tau - Q_{b0}}$$

where

$$J_n < 0$$

APPENDIX C

USE OF THE BANDGAP NARROWING MODEL IN EXPERIMENTAL EXTRACTION OF n_{ie}

From the results for the effect of bandgap narrowing in the base on J_n , it is possible to setup an experimental procedure for the measurement of the relationship between n_{ie} and doping. The model for the inclusion of bandgap effects on J_n is:

$$J_n = -q^2 \overline{D}_n (pn(x_1) - pn(x_2)) \frac{1 + \Delta \text{Bandgap}}{Q_{b0} - \tau J_n}$$

$$\text{where } Q_{b0} = q \int_{x_1}^{x_2} N(x) dx \quad \text{and} \quad \Delta \text{Bandgap} = 2 \ln \left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)} \right) \left(\frac{J_n \tau - Q_{b0}}{3J_n \tau - 2Q_{b0}} \right)$$

Now, if Q_{b0} can be determined then it is possible to determine the $\Delta \text{Bandgap}$ term. For low level conditions where the collector is reverse biased, J_n is given by:

$$J_n = -\frac{q^2 \overline{D}_n n_{ie}^2(x_1)}{Q_b} \left(1 + \ln \left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)} \right) \right) e^{\frac{V_{BE}}{V_T}} \quad (\text{C.1})$$

With Q_{b0} known and $\overline{D}_n$ estimated, the term in front of the exponential can be determined by plotting J_n against log of V_{BE} and finding the saturation current J_{ns} . Then:

$$J_{ns} = \frac{q^2 \overline{D}_n n_{ie}^2(x_1)}{Q_b} \left(1 + \ln \left(\frac{n_{ie}(x_2)}{n_{ie}(x_1)} \right) \right) \quad (\text{C.2})$$

If the doping is known at $x=x_1$ and $x=x_2$, it is possible to find $n_{ie}(x_1)$ and $n_{ie}(x_2)$ by performing experiments on many different structures with various dopings at the emitter and collector edges of the active base region. The key to performing these experiments is to know exactly what the doping is at x_1 and x_2 . To guarantee the knowledge of the doping at these two points, two EPI layers of different doping concentrations are used to form the base. Thus, even as the depletion boundary layer moves due to bias, the doping at each boundary will be a constant and is therefore known.

Following is a cross section of the test transistor, where the two different EPI layers for the base have concentrations P_1 and P_2 .

FIGURE 58. Cross-section of the device for experimental extraction of n_{ie} .

APPENDIX D

SOLUTION OF I_{cT} WITH HIGH LEVEL EFFECTS

The purpose of this appendix is to detail the derivation of the terminal collector current I_{cT} from the low level model for emitter debiasing. The derivation begins with the definition of I_{cT} . Using eqn (3.10) for J_n the following result is obtained:

$$I_{cT} = \int_0^L J_n(x) dx$$

$$= \frac{1}{2K_3} \int_0^L (K_1 - \sqrt{K_2' K_3 (e^{V(x)/V_T} - e^{V_{bc}/V_T}) + K_f^2}) dx$$

where $K_1 = Q_b$, $K_3 = \tau$, and $K_2' = 4q^2 n_i^2 D_n$

Using eqn (4.16) to solve for $V(x)$, the above equation becomes:

$$I_{cT} = \frac{1}{2K_3} \int_0^L (K_1 - \sqrt{K_2 \sec^2(B(L-x)) + K_4}) dx$$

where $K_2 = \frac{K_2' K_3 4B^2 V_T}{K_{sb}}$, $K_4 = K_f^2 - K_2' K_3 e^{V_{bc}/V_T}$, and $K_{sb} = 2\rho J_{sb}$

Changing variables so that $u = B(L-x)$ gives $du = -Bdx$. The above integral becomes:

$$I_{cT} = \frac{-1}{2BK_3} \int_{BL}^0 (K_1 - \sqrt{K_2 \sec^2(u) + K_4}) du \tag{D.1}$$

$$= \left[\frac{-K_1 u}{2BK_3} \right]_{u=BL}^{u=0} + \frac{\sqrt{K_2}}{2BK_3} \int_{BL}^0 \sqrt{\sec^2(u) + K_4/K_2} du$$

Concentrating on the integral in eqn (D.1) gives:

$$\int_{BL}^0 \sqrt{\sec^2(u) + K_4/K_2} du = \int_{BL}^0 \frac{\sqrt{1 + \alpha^2 \cos^2(u)}}{\cos(u)} du$$

$$= \sqrt{1 + \alpha^2} \int_{BL}^0 \frac{\sqrt{1 - \frac{\alpha^2}{1 + \alpha^2} \sin^2(u)}}{\cos(u)} du, \text{ where } \alpha^2 = K_4/K_2$$

From integral number 33 in section 2.583 in Gradshteyn and Ryzhik [19], the solution to the above integral is:

$$\int_{BL}^0 \sqrt{\sec^2(u) + K_4/K_2} du = \left[\frac{1}{2} \ln \left(\frac{\Delta + K' \sin(u)}{\Delta - K' \sin(u)} \right) + \alpha \sin^{-1}(\alpha K' \sin(u)) \right]_{u=BL}^{u=0}$$

where $\Delta = \sqrt{1 - (\alpha K')^2 \sin^2(u)}$, and $K' = \frac{1}{\sqrt{1 + \alpha^2}}$

Substituting this result into eqn (D.1) gives:

$$I_{cT} = \frac{K_1 L}{2K_3} + \frac{\sqrt{K_2}}{2BK_3} \left[\frac{1}{2} \ln \left(\frac{\Delta + K' \sin(u)}{\Delta - K' \sin(u)} \right) + \alpha \sin^{-1}(\alpha K' \sin(u)) \right]_{u=BL}^{u=0}$$

$$\Rightarrow I_{cT} = \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{\Delta_{BL} + K' \sin(BL)}{\Delta_{BL} - K' \sin(BL)} \right) + \alpha \sin^{-1}(\alpha K' \sin(BL)) \right\}$$

Where $\Delta_{BL} = \sqrt{1 - (\alpha K')^2 \sin^2(BL)}$

APPENDIX E

VERIFICATION OF THE DERIVATION FOR THE EMITTER DEBIASING MODEL

This appendix shows that eqn (4.23) for I_{cT} (here eqn (E.2)) degrades into eqn (4.20) for I_{bT} (here eqn (E.1)) multiplied by the beta of the transistor as V_{BE} becomes very small, so proving that emitter debiasing does not effect the beta of the intrinsic device region, and verifying the derivation of the emitter debiasing model.

$$I_{bT} = \int_0^L J_b(x) dx = \frac{2BV_T}{\rho} \tan(BL) \quad (E.1)$$

$$I_{cT} = \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{\Delta_{BL} + K' \sin(BL)}{\Delta_{BL} - K' \sin(BL)} \right) + \alpha \sin^{-1}(\alpha K' \sin(BL)) \right\} \quad (E.2)$$

$$\text{where } K_1 = Q_b, K_3 = \tau, K_2' = 4q^2 n_i^2 D_n$$

$$K_2 = \frac{K_2' K_3 4B^2 V_T}{K_{sb}}, K_4 = K_1^2 - K_2' K_3 e^{V_{bc}/V_T}, K_{sb} = 2\rho J_{sb}$$

$$\Delta_{BL} = \sqrt{1 - (\alpha K')^2 \sin^2(BL)}, \alpha^2 = \frac{K_4}{K_2}, K' = \frac{1}{\sqrt{1 + \alpha^2}}$$

In low level $B \rightarrow 0$ giving $K_2 \rightarrow 0$ while $\alpha \rightarrow \infty$ and $\alpha K' \rightarrow 1$

$$\Delta_{BL} \approx \sqrt{1 - \sin^2(BL)} = \cos(BL)$$

Substituting this into eqn (E.2) gives:

$$I_{cT} = \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{1 + K' \tan(BL)}{1 - K' \tan(BL)} \right) + \alpha \sin^{-1}(\alpha K' \sin(BL)) \right\} \quad (E.3)$$

Since $\alpha K' \rightarrow 1$, a Taylor expansion of $\sin^{-1}(ax)$ about $x = 1$ gives:

$$\sin^{-1}(ax) \approx \sin^{-1}(a) + (x - 1) \frac{a}{\sqrt{1 - a^2}} \Rightarrow \sin^{-1}(\alpha K' \sin(BL)) \approx BL + (\alpha K' - 1) \frac{\sin(BL)}{\cos(BL)}$$

Substituting this into eqn (E.3) gives:

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$$I_{cT} \approx \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2BK_3} \alpha BL - \frac{\sqrt{K_2}}{2BK_3} \left\{ \frac{1}{2} \ln \left(\frac{1 + K' \tan(BL)}{1 - K' \tan(BL)} \right) + \frac{\alpha^2 - \alpha \sqrt{1 + \alpha^2}}{\sqrt{1 + \alpha^2}} \tan(BL) \right\} \quad (E.4)$$

Since $BL \approx 0$ in low level, the Maclaurin series for the natural log term is:

$$\ln \left(\frac{1+x}{1-x} \right) \approx 2x$$

Then

$$\frac{1}{2} \ln \left(\frac{1 + K' \tan(BL)}{1 - K' \tan(BL)} \right) \approx K' \tan(BL)$$

assuming that $K_1^2 \gg K_2' K_3 e^{V_{bc}/V_T}$ then $K_4 = K_1^2$ and $\alpha^2 = \frac{K_1^2}{K_2}$

Substituting the above two results into eqn (E.4) gives:

$$I_{cT} \approx \frac{K_1 L}{2K_3} - \frac{\sqrt{K_2}}{2K_3} \frac{K_1 L}{\sqrt{K_2}} - \frac{\sqrt{K_2} K'}{2BK_3} \tan(BL) (1 + \alpha^2 - \alpha \sqrt{1 + \alpha^2}) \quad (E.5)$$

because $\alpha^2 \gg 1$ then $\sqrt{\alpha^2 + 1} \approx \alpha + \frac{1}{2\alpha}$. Substituting into eqn (E.5) gives:

$$-I_{cT} \approx \frac{\sqrt{K_2} K'}{4BK_3} \tan(BL) \quad (E.6)$$

Now $K_2 = \frac{K_2' K_3 4B^2 V_T}{2\rho J_{sb}}$, while $J_{sb} = \frac{K_2'}{\beta 4K_1}$ where β is the beta of the transistor

Therefore

$$K_2 = \frac{\beta K_1 K_3 8B^2 V_T}{\rho}$$

Substituting this, K' and α^2 into eqn (E.6) gives:

$$-I_{cT} \approx \beta \frac{2BV_T K_1}{\rho \sqrt{K_2 + K_1^2}} \tan(BL)$$

But in low level $K_2 \ll K_1$ making I_{cT} become:

$$-I_{cT} \approx \beta \frac{2BV_T}{\rho} \tan(BL)$$

This is just eqn (E.1) multiplied by beta. Therefore, when high level effects are not

present, $-I_{cT}/I_{bT}$ is just beta. This shows that emitter debiasing does not change the beta of the intrinsic device. Since this result is expected, it helps to show the validity of the emitter debiasing model presented by eqns (4.16), (4.17), (4.18), (4.19), and (4.23).

END

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