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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RÉCU
THE MEASUREMENT OF UNFROZEN
WATER CONTENT IN FREEZING SOILS BY
TIME DOMAIN REFLECTOMETRY

BY

DANIEL EDWARD PATTERTON

Submitted in Fulfillment of the
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1980
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ABSTRACT

This study has shown that the volumetric unfrozen water content \( \Theta_{uf} \) of freezing soils can be determined from the dielectric property (apparent dielectric constant, \( \kappa_a \)), using Time Domain Reflectometry (TDR) to determine \( \kappa_a \). The technique has been developed previously by others to measure the volumetric water content in unfrozen soils.

This study has shown that \( \Theta_{uf} \) values determined from \( \kappa_a \) measurements compare favorably with published data for soils having similar grain-size characteristics (within 1.5 to 4.0% in \( \Theta_{uf} \)). Simultaneous measurements of \( \Theta_{uf} \) from volume expansion and \( \kappa_a \) measurements on the same soil sample were also made, and the results compared very closely (within 1 to 3%).

The TDR technique appears to be a simple and reliable method for routine laboratory determination of \( \Theta_{uf} \). The method also shows great potential for in situ measurement of \( \Theta_{uf} \).
ACKNOWLEDGEMENTS

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CHAPTER 1

THE UNFROZEN WATER CONTENT OF FREEZING SOILS

1.1 Introduction

The investigation of the properties of freezing soils has an extensive history and many practical applications. For example, soil freezing has been examined in an agricultural context (e.g. winter kill of plant roots), and for its effects on building foundations and highways. Recent plans for pipeline construction and liquid natural gas storage tanks have included geotechnical assessments of the effects of soil freezing and frost heave. In fact, the potential for pipeline rupture, due to differential frost heave, along the proposed MacKenzie Valley pipeline route was a serious design problem (Williams, 1979).

In order to alleviate the problems associated with soil freezing and frost heave, a sound knowledge of the thermal and hydraulic properties of freezing soils is required. In freezing soils these properties are greatly influenced by the unfrozen water content \( \theta_{uf} \), \( \text{cm}^3 \text{cm}^{-3} \), which is a function of the soil's basic physical properties (texture, mineralogy and structure) and temperature. The importance of this unfrozen water has prompted the development of numerous techniques to measure it (e.g. dilatometry, adiabatic calorimetry). However, none of
the techniques are applicable to in situ measurements (with the possible exception of the nuclear magnetic resonance method, NMR); they are laboratory techniques and are generally cumbersome, require elaborate equipment and/or time-consuming experimentation, and generally they cannot be used on undisturbed samples.

This thesis will examine the possibility of using an electromagnetic method (Time Domain Reflectometry, TDR) to measure the volumetric water content of frozen soils, $\Theta_{uf}$.

1.2 The Influence of the Unfrozen Water Content on Thermal and Hydraulic Properties of Freezing Soils

Frozen soil is a multi-phase system comprising solid matter (mineral, organic, ice), air and water. The ability of a soil to store and transport heat and mass is determined by its basic physical properties (structure, texture and mineralogy), temperature and the unfrozen water content. In freezing soils, the common important factor influencing the thermal and hydraulic properties is the liquid (i.e. unfrozen) water content. For a given soil:

1. $\Theta_{uf}$ decreases with decreasing temperature;
2. at a given temperature, $\Theta_{uf}$ is greater the finer the soil texture;
3. the rate of change in $\Theta_{uf}$ for a change in temperature is determined by soil pore size, and the range of pore sizes (i.e. structure and texture);
4. soluble salts in the soil water further depress the freezing point;
5. the $\Theta_{uf}$-T°C relationship (freezing characteristic curve) is hysteretic (i.e. freezing$\Theta_{uf}$ thawing at a given temperature).
If one considers a soil in equilibrium with the atmosphere (no net gain or loss of heat or mass at the soil surface) then the unfrozen water content with depth is essentially determined by the soil's basic physical properties and temperature. A change at the surface boundary (e.g. temperature) will create gradients of potential which give rise to transfers of heat and mass. The ability of the soil to heat and mass is a function of the soil's basic physical properties and the thermal and hydraulic potentials. The soil 'properties' which are used to describe the ability of a soil to hold and transport heat and mass are described in Table 1 for both frozen and unfrozen states.

The thermal properties of freezing soils are largely dependent on temperature since this determines the liquid water content given particular basic physical soil properties. The volumetric heat capacity of freezing soils can be determined from the mass heat capacities of the various soil components and a latent heat term. Since the unfrozen water content can change quite rapidly between 0° and -1°C, the volumetric heat capacity ($C_f$) is usually defined in terms of the heat quantity exchanged over a small temperature (difference (see Williams, 1973). In general, the volumetric heat capacity can be determined from the following:

$$C_f = C_{ms} \cdot \phi_d + \frac{\Theta}{100} \cdot \frac{C_m}{100} + \frac{\Theta_i}{100} \cdot \gamma_d \cdot C_{mi} \cdot \left(\frac{\Delta \Theta}{100 \cdot \Delta T}\right) \cdot \gamma_d \cdot L.$$  \hfill (1)

in J m$^{-3}$; where $T$ is temperature (°C), $\gamma_d$ is bulk density (g m$^{-3}$), $\Theta$ is the gravimetric unfrozen water content, $\Theta_i$ is the ice content (g g$^{-1}$ %), $(\Delta \Theta/100)/T$ corresponds to the slope of the unfrozen water content curve
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<td>Volumetric Heat Capacity ($C_v$)</td>
<td>$C_v$ represents the amount of energy required to raise or lower a unit volume of a substance by 1°C. In frozen soils, it is referred to as the apparent volumetric heat capacity, $C_f$, (in J m$^{-3}$ °C$^{-1}$), which incorporates the latent heat of fusion.</td>
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<td>Thermal Conductivity ($\lambda_u, \lambda_f$)</td>
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<td>Moisture Diffusivity</td>
<td>The moisture diffusivity is the ratio of the hydraulic conductivity to the specific water capacity.</td>
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at temperature T and L is the latent heat term \((334 \text{ J g}^{-1})\). The mass heat capacity of mineral matter \((C_{ms})\) is about \(0.85 \text{ J g}^{-1} \text{ °C}^{-1}\); for water \((C_{mw})\), \(4.19 \text{ J g}^{-1} \text{ °C}^{-1}\) and for ice \((C_{mi})\), \(2.13 \text{ J g}^{-1} \text{ °C}^{-1}\). As suggested by (1), the greater the unfrozen water content the greater is the amount of energy that must be added or removed to change the temperature of the soil since the mass heat capacity of water is greater than that of ice. However, \(C_f\) is generally dominated by the latent heat term.

Determining the thermal conductivities of freezing soils is difficult experimentally especially at temperatures between 0°C and -2.0°C (this relates to the nature of the experiment itself rather than the nature of the soil, see Penner, 1970). The soil’s texture, structure, mineralogy, unfrozen water, ice and air contents are all important factors in determining the thermal conductivity. One can approximate \(\lambda_u\) and \(\lambda_f\) from Johansen’s (1973, 1975) geometric mean equation which attempts to derive an overall thermal conductivity based upon the thermal conductivities of the volume fractions of the various soil components. Quartz has a thermal conductivity of 7.7 W m\(^{-1}\) °C\(^{-1}\), while other common soil minerals (e.g. feldspar and mica) have values of about 2.0 W m\(^{-1}\) °C\(^{-1}\). The thermal conductivities of air, water and ice are 0.024, 0.57 and 2.2 W m\(^{-1}\) °C\(^{-1}\), respectively (Johansen, 1973). For a given soil, the thermal conductivity increases with increases in density and water content. For soils at temperatures below 0°C, \(\lambda_f\) increases as temperature decreases since the unfrozen water content will be decreasing.

In unfrozen soils, the hydraulic conductivity \((k_h)\) is greater the coarser the soil texture since this generally determines the average size of the soil pores. Structure is also important in determining pore
size (i.e. soils in the same textural class can have substantially different hydraulic conductivities if bulk densities are quite different).

In freezing soils, the hydraulic conductivity \( K_f \) is not only controlled by the soil's basic physical properties but temperature must also be considered. At a given temperature, the basic physical properties of the soil determine the amount of unfrozen water that will be present. On this basis only, clays should have a greater \( K_f \) than silts or sands since clays will contain more water capable of movement and less ice blocking soil pores (assuming sample saturation before freezing). An apparent anomaly occurs in the temperature range 0°C to -0.8°C where the \( K_f \) for a silt is greater than that for a clay (Burt, 1974). In this range, a silt will possess substantial amounts of unfrozen water and pore sizes which are more conducive to water movement than those found in clays. At temperatures below about -0.8°C the \( K_f \) for a clay will tend to be higher than that for other soils.

Quantification of the relationships between temperature, \( \Theta_{uf} \) and the various thermal and hydraulic properties of freezing soils has not been fully established to date. In situ measurement of heat and mass transfer (or in soil columns in the laboratory) may provide the necessary data.

1.3 Methods of Determining the Unfrozen Water Content

Anderson and Morgenstern (1973) provide a comprehensive review of the more commonly used methods of determining the liquid water content of soils at temperatures below 0°C; they include adiabatic calorimetry (e.g. Williams, 1967), dilatometry (e.g. El Khoraihi, 1974) and nuclear magnetic resonance (e.g. Tice, Burrous and Anderson, 1978). \( \Theta_{uf} \) can also
be determined from the theoretical relationship between the soil water characteristic curve and soil freezing (Koopmans and Miller, 1966).

Other approaches have attempted to predict \( \theta_{uf} \) from such soil properties as surface area (Anderson and Tice, 1972) or liquid limit (Tice, Anderson and Banin, 1976).

As with all measurement techniques, there are limitations and assumptions underlying their use. Dilatometry, which measures volume expansion due to freezing, assumes that the sample is completely saturated and that soil water expands to the same extent as bulk water when it freezes. There are little supportive data to suggest that soil water and bulk water expand differently when frozen; however, obtaining a completely saturated sample is difficult and will lead to erroneous results if saturation is not complete. The accuracy in determining unfrozen water contents from adiabatic calorimetry is based on the major assumptions that the heat capacities of the soil components are known and the latent heat of melting ice is known. Although the theoretical basis for these assumptions has not been fully investigated, it is one of the preferred methods—yielding unfrozen water content-temperature data comparable to that obtained by other methods (Anderson and Morgenstern, 1973). This approach, however, requires elaborate equipment and time-consuming experimentation. Determining unfrozen water contents from suction-moisture data is also time consuming. Moreover, this method does not account for the influence of soluble salts which can account for the presence of substantial amounts of unfrozen water at temperatures between 0°C and -0.5°C (El Khorabi, 1975). None of the above techniques are applicable to \( \theta_{uf} \) measurement in situ or in undisturbed soil core samples. Most are time-consuming, require elaborate equipment or calibration to the soil being tested (e.g. nuclear magnetic
resonance), and sensitivities to other variables may exist (e.g. temperature). Moreover, the applicability of data obtained in the laboratory to the field situation can be questioned since sample treatment can alter the freezing characteristic curve (see Hotzel, 1974). Obtaining representative samples from a site may also be a difficult task.

It has been suggested that electromagnetic methods might be used to measure unfrozen water contents (Jumikis and Slusarchuk, 1973). Davis, Topp and Annan (1977) describe such a technique (Time Domain Reflectometry), which they have used to measure the volumetric water content (\( \theta_v \)) of unfrozen soils from the apparent dielectric constant (\( K_a \)). This method also appears to have potential for determining \( \theta_{uf} \) since \( K_a \) is largely unaffected by soil texture, structure, mineralogy, temperature and salt concentration (Davis, 1975; Davis, Topp and Annan, 1977; Topp, Davis and Annan, 1980 in press). The \( K_a \) value for ice is similar to that of dry soil (about 3.0) so it is supposed that ice content would not affect \( K_a \) greatly for a soil at temperatures below 0°C.

A most attractive attribute of this method is that it offers a large degree of flexibility in measurement system design which should permit field use (this method has been used to measure \( \theta_v \) in the field; see Davis and Chudobiak, 1975). The relatively recent application of this method to earth materials necessitates a brief discussion on the electrical properties of soils and how the TDR technique actually is used to detect volumes of water in a soil.
CHAPTER 2

MEASUREMENT OF THE VOLUMETRIC WATER CONTENT OF SOILS FROM THE DIELECTRIC PROPERTY

2.1 Electrical Properties of Soils

If the reader is unfamiliar with electrical terminology, they are referred to the glossary of terms in Appendix I.

There are essentially three electrical properties of soils which can be measured: magnetic permeability, conductivity/resistivity and the dielectric permittivity. The magnetic permeability of soils is similar to that of air, hence, it is generally ignored. Conductivity, \( \sigma \) (in mho m\(^{-1}\)) essentially describes the ease with which an electric current will pass through a substance. A high conductivity denotes easy passage of current. The permittivity of a substance, \( \varepsilon' \), (F m\(^{-1}\)) is usually referenced to air, \( \varepsilon_0 \), \( (8.85 \times 10^{-12} \text{ F m}^{-1}) \) and called the relative permittivity or dielectric constant, \( K' \), \( (K' = \varepsilon'/\varepsilon_0) \) where \( K' \) describes the ability of a substance to store electrical potential energy under the influence of an electric field relative to that of vacuum (vacuum = air). \( K' \) can be defined as the ratio of the capacitance of a capacitor filled with a dielectric to that when it is empty. \( K' \) is dependent upon temperature and frequency, hence, it is not a 'constant' in the true sense of the meaning.

Actually, vacuum; however, air \( \neq \) vacuum.
Selig (1975) states that a soil will behave electrically like a complex series of parallel resistance and capacitance circuits; hence, the dielectric constant is often expressed in complex notation which involves capacitance and conductance. Davis and Annan (1977) define the complex dielectric constant, \( K^* \), as:

\[
K^* = K' + j(K'' + \frac{\sigma_{dc}}{\omega \varepsilon_0})
\]  
(2)

where \( K' \) (\( \varepsilon' / \varepsilon_0 \)) is the 'real' part of \( K^* \); \( K'' \) represents dielectric loss; \( \sigma_{dc} \) is the d.c. conductivity; \( \omega \) is the angular frequency (2\( \pi \)f); \( \varepsilon_0 \) is the free space permittivity and \( j \) equals \( \sqrt{-1} \). An electric loss term, \( \tan \delta \), can also be defined (Davis and Annan, 1977):

\[
\tan \delta = \frac{(K'' + \sigma_{dc} / \omega \varepsilon_0)}{K'}
\]  
(3)

If \( \tan \delta \) is \( \ll 1 \), then \( K^* \) is approximately equal to \( K' \). Other references (e.g. Von Hippel, 1954; Selig, 1975) exclude the \( K'' \) term from (3).

Conductivity \( \sigma \) varies with soil type, structure, water, ice and salt content, temperature and measurement frequency. Its magnitude varies from \( 10^{-10} \) to \( 10 \) mhos cm\(^{-1} \) (Chernyak, 1964). In general clays tend to have a higher conductivity than sands and \( \sigma \) increases with increasing temperature, water content and soluble salts.

The dielectric property of a bulk soil can also be affected by the same sorts of things that influence its conductivity. The dielectric constants of most soil minerals lie in the range of 2 to 5 (Thomas, 1966). \( K' \) for water is much higher, varying slightly with temperature (80.1 at 20°C to 87.7 at 0°C) with minimal frequency dependence from d.c. to about
Figure 1  

K' and Tan δ for Several Soils

DISPERSION CURVES FOR TWO DRY SOILS

source: Von Hippel (1954)

DISPERSION CURVE FOR A WET SOIL

Rubicon sandy loam  
w = 8.5% (g g⁻¹, %)

source: Chudoleisk, Syrett and Hafez (1979)
3 GHz (Von Hippel, 1954). Tanδ is also small in this range (for water
at 25°C, tanδ = 0.04 at 1 MHz and 0.016 at 300 MHz; see Von Hippel, 1954).
Ice has a high d.c. dielectric constant (about 100) but it falls to a
value of about 3.2 at frequencies greater than 3 x 10⁴ Hz (Hoekstra and
Spanogle, 1972); density and temperature have a minimal influence on K'
and Tanδ particularly frequencies in the MHz range (Tanδ decreases from
0.4 at 1 MHz to about 5 x 10⁻⁴ at 1 GHz; Yoshino, 1967).

The dielectric constants of dry soils generally fall within the
range 2.2 and 3.5 at frequencies above about 1 MHz and Tanδ tends to be
small (e.g. Von Hippel, 1954; Thomas, 1966) (see Figure 1). K', though,
is very sensitive to the volume of water present in the soil (Chernyak,
1964; Thomas, 1966; Davis and Chudobiak, 1975; Davis, Topp and Annan, 1977).
When water is present in the soil, the measured dielectric property
can have a substantial frequency variation (see Jumikis and Slusarchuk,
1973). This aspect shall be discussed further in section 2.2.

2.2 Determining Volumetric Water Content from the Dielectric Property

The measurement of soil water content from its dielectric constant is
not a new idea. Most studies have employed capacitance bridge techniques
whereby the dielectric constant is calculated from the ratio of the cap-
acitance of the capacitor filled with the material to that when it is
empty (e.g. Fletcher, 1939; Silva et. al., 1975). Studies such as
Fletcher's (1939) indicated that water content on a mass basis (w, g g⁻¹)
determined from the dielectric constant compared well with those obtained
from the oven drying method (see Figure 2); however, calibration of K'–w
Figure 2

COMPARISON OF GRAVIMETRIC AND CAPACITANCE MEASUREMENTS OF SOIL WATER CONTENT

Source: Flintham, 1938.
is necessary for each soil (if one wished to predict \( w \) from \( K' \)) since soil bulk density is variable. Many authors, as indicated, feel that \( K' \) correlates better with the volumetric water content, \( \Theta_v \) (cm\(^3\) cm\(^{-3}\)) rather than with the conventional gravimetric water content.

The frequency at which \( K' \) for moist soils is determined is also another important consideration. Junikis and Slusarchuk (1973) show that \( K' \) is frequency dependent in the range \( 10^2 \) to \( 10^5 \); a temperature dependence in \( K' \) was also noted. Von Hippel (1954), Tran Lan et al. (1970), Hoekstra and Delaney (1974) and Chudobiak, Syrett and Hafez (1979) present data which suggest that \( K' \) for moist soils is not strongly frequency dependent in the range \( 10^6 \) to \( 10^9 \) Hz (see Figures 1 and 3). Davis, Topp and Annan (1977) discuss the use of Time Domain Reflectometry (TDR) for determining \( \Theta_v \) in this frequency range. In a subsequent paper an empirical relationship is obtained between the apparent dielectric constant, \( K_a \), and \( \Theta_v \) which is only weakly affected by soil texture, structure, mineralogy, salt concentration and temperature (Topp, Davis and Annan, 1980 in press).\(^1\) The relationship is described by (4); the results compare favorably with those obtained by other authors using a variety of techniques (see Topp, Davis and Annan, 1980 in press):

\[
K_a = 3.01 + 10.1 \Theta_v + 143 \Theta_v^2 - 75 \Theta_v^3
\]  

(4)

Topp, Davis and Annan (1980 in press) found that 93\% of the measured data for various mineral soils fell within ±2.5\% in \( \Theta_v \) determined from \( K_a \) using (4).

\(^1\) \( K_a \) is used to denote a measured dielectric constant. \( K_a \neq K' \) is tantissimus.
FIGURE 3

FREQUENCY VARIATION IN $K'$ FOR PYLA SAND

% saturation

$K'$

20

10

67%

75%

100%

265 MHz

Log Frequency (Hz)

10$^6$ 10$^7$ 10$^8$ 10$^9$ 10$^{10}$

Source: Tran Lan et. al., 1971.
2.3 Time Domain Reflectometry

Fellner-Feldegg (1969) was the first to use the technique to investigate the dielectric properties of substances. Suggett (1972), Iskander and Stuchey (1972) and Hasted (1973) have also used the method to determine the dielectric properties of water, alcohols and other biological substances at high frequencies (in the MHz range). The substance being examined is generally placed in a coaxial line, which is essentially a metallic cylinder having a centre rod.

The TDR technique is a form of pulse-reflection measurement which makes use of a special pulse called a step-voltage (Blake, 1969). When a coaxial line (or other form of transmission line) is connected to the output connector of the TDR unit, a broadband pulse travels down the line and is partially reflected back toward the signal generator at every impedance mismatch. Two types of transmission lines used by the author are shown in Figure 4.

If parallel transmission lines are placed in a wet soil sample, the TDR's CRT (cathode ray tube) may display a trace such as is shown in Figure 5 (Appendix II shows the TDR unit used by the author (Figure 20) and it presents information on TDR operation). Point A denotes the location where the parallel lines meet the head connector (see Figure 4). Point B denotes the point where the pulse encounters the end of the transmission line (the open circuited case is shown). Point A can be determined by recording the characteristic trace of the head connector before and after connecting the transmission lines (see Figure 6; Appendix II discusses trace recording
Figure 4

TRANSMISSION LINES

- Coaxial Line: Soil Sample, Centre Connector, Soil Sample
- Parallel Transmission Lines: Line Lengths Used 10 to 35 cm
- Coaxial Line to TDR:
  - Head Connector, Teflon
Figure 5
SAMPLE TDR TRACE FOR A WET SOIL

probe line length = 17.4
travel time (AB) = 3.375 ns
k = 33.68
θ = 40.3%
Figure 6 - Determining the A Point for Travel Time Measurement
procedures and options). Point B is determined by drawing tangents to the trace length where it encounters the open (or short) circuit (as shown in Figure 5). The A to B distance, horizontally, yields the trace length. From this trace length, the one-way travel time of the pulse in the soil can be calculated (see Appendix-H). The velocity of propagation of the pulse in the soil can be determined from the travel time (tt) and the length of the transmission line (L):

$$V_p = \frac{L}{tt}$$  \hspace{1cm} (5)

It should also be noted that, for low loss materials ($\tan\delta \ll 1$), $V_p$ can also be expressed as (Davis and Annan, 1977):

$$V_p = \frac{c}{\sqrt{K_a}}$$  \hspace{1cm} (6)

where $c$ is the free space velocity ($3 \times 10^8$ m s$^{-1}$ = 30 cm ns$^{-1}$). The apparent dielectric constant of a material can be determined if $L$ is known and $tt$ is measured by equating (5) and (6):

$$K_a = \left(\frac{c \times tt}{L}\right)^2$$  \hspace{1cm} (7)

As stated, coaxial and parallel transmission lines have been used by the author. Coaxial lines were used to determine $K_a-\Theta_v$ calibration data for various earth materials (see section 2.5). The parallel transmission line configuration was used for experiments discussed in Chapter 3. The selection of the type of transmission line was dictated by the experiment. Parallel transmission lines have been used in the field (Davis and Chudobiak, 1975) and could be used on samples (e.g. core samples) where it is undesirable to destroy soil structure.
2.4 Precision in Determining $Q_v$ from $K_a$

The precision to which one can determine $K_a$ is essentially a function of the TDR's horizontal scale precision. The horizontal axis of the TDR's CRT is divided into ten divisions, each representing a distance given a particular horizontal scale setting (see Appendix II). The horizontal precision is $\pm 2\%$ of the distance represented by the ten divisions for that particular scale setting. One, therefore, has a certainty in trace length measurement (A to B distance; see Figure 5) of $\pm 0.2$ divisions. Table 2 summarizes the TDR's accuracy in measuring travel time (tt) for two line lengths. The possible variation in $K_a$, and hence, $Q_v$ are also given. The horizontal axis scale setting is converted to a time axis using:

$$\text{time/division} = \frac{\text{metres/division}}{0.3 \text{ m ns}^{-1}} \quad (8)$$

where $0.3 \text{ m ns}^{-1}$ is the free space velocity ($3 \times 10^8 \text{ m s}^{-1}$) (see also Appendix II).

As is shown, precision in $K_a$ measurement is greater when the soil is dry ($\pm 0.2$) than when it is wet ($\pm 1.5$ for $Q_v = 40\%$). The certainty in $Q_v$ based upon the possible variation in $K_a$ (Table 2) is still about $\pm 1.7\%$ when using (4) to determine $Q_v$ from $K_a$. As indicated earlier, a certainty of $\pm 2.5\%$ in $Q_v$ is associated with (4). This apparently lower accuracy is not only attributable to measurement precision but more importantly to variations in the soils used by Topp, Davis and Annan (1980 in press) to obtain (4). One could determine a separate calibration of $K_a$ versus $Q_v$.
## MEASUREMENT ACCURACY IN $K_a$

<table>
<thead>
<tr>
<th>Assumed $K_a$ Water Cont. (%)</th>
<th>Equivalent Line Length (cm)</th>
<th>Horizontal Scale m/div ns/div</th>
<th>#Div.</th>
<th>Travel Time (ns)</th>
<th>$K_a$ Based on tt</th>
<th>$Q_v$ Based (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 0</td>
<td>33.33</td>
<td>33.33</td>
<td>0.10</td>
<td>0.33</td>
<td>1.924 +.066</td>
<td>2.79-3.21</td>
</tr>
<tr>
<td>3 0</td>
<td>15.00</td>
<td>0.10</td>
<td>0.05</td>
<td>0.166</td>
<td>0.866 *-0.033</td>
<td>2.77-3.23</td>
</tr>
<tr>
<td>10.15 20</td>
<td>33.33</td>
<td>0.25</td>
<td>0.833</td>
<td>4.247</td>
<td>3.539 +.166</td>
<td>9.22-11.12</td>
</tr>
<tr>
<td>10.15 20</td>
<td>15.00</td>
<td>0.10</td>
<td>0.333</td>
<td>4.778</td>
<td>1.593 *-.066</td>
<td>9.3 11.00</td>
</tr>
<tr>
<td>25.13 40</td>
<td>33.33</td>
<td>0.25</td>
<td>0.833</td>
<td>6.683</td>
<td>5.569 +.166</td>
<td>23.65-26.65</td>
</tr>
<tr>
<td>25.13 40</td>
<td>15.00</td>
<td>0.10</td>
<td>0.333</td>
<td>7.519</td>
<td>2.506 *-.066</td>
<td>23.82-26.47</td>
</tr>
</tbody>
</table>

1. Assumed $K_a$ values
2. $Q_v$ for assumed $K_a$ determined from (4).
3. Sample line lengths.
4. This is the horizontal scale setting which result in a trace length between 4-10 divisions. It is best to have as long a trace length as possible since horizontal accuracy depends upon the scale setting. (see equation (8)).
5. This is the trace length which should result given the assumed $K_a$ and horizontal scale setting.
6. This is the equivalent travel time, determined from the trace length. The accuracy is +2% of the horizontal scale setting, times 10 divisions.
7. $K_a$ is calculated from the travel time plus or minus the travel time accuracy.
8. $Q_v$, based on the $K_a$ range in 7.
for a particular soil (see section 2.5); however, the certainty in (4) may suffice for most applications. In a qualitative sense, the author has found the reproducibility to be well within the measurement accuracy.

2.5 \( K_a \) versus \( \Theta_v \) for Various Earth Materials

The dielectric properties of various earth materials were examined to familiarize the author with the TDR technique. The materials selected were of a coarser nature (sands, sand-gravels) to complement the data presented in Davis, Topp and Annan (1977) and Topp, Davis and Annan (1980 in press) for soils which were generally finer in texture.

Coaxial lines were used in the experiments. In general, water was added to one end of the coaxial line and permitted to percolate through the sample; hence, water distribution was not always uniform. Figures 7 and 8 summarize the effects of density on \( K_a \) for dry earth materials and the \( K_a \) versus \( \Theta_v \) relationship compared to (4). Accuracy in \( K_a \) measurement is as discussed in section 2.4. The water content of the samples was determined gravimetrically at the end of the \( K_a - \Theta_v \) runs (\( \Theta_v \) calculated from \( w \) since coaxial volume is known); differences in volume of water added and recovered resulted in differences in \( \Theta_v \) of less than 1%.

From Figure 7, it appears that density has a minimal effect upon \( K_a \); the variation could be due to the measurement precision (±0.2 in \( K_a \)), mineralogy or the density differences. The only data point which shows a substantially different \( K_a \) value in the dry state (\( K_a = 4.24 \)) is a crushed stone and sand sample (bulk density = 1.98 g cm\(^{-3}\)). This value
Figure 7  $K_a$ for Dry Earth Materials at Various Bulk Densities

- crushed stone
- sands (various)
- clay loam (Ellwood)
- sand-gravel
- crushed stone and sand

$K_a$

Dry Bulk Density (g cm$^{-3}$)

2.0  1.8  1.6  1.4  1.2  1.0

2  3  4
Figure 8  
$K_a - Q_v$ for Various Earth Materials

- crushed stone: bulk densities from 1.1 to 1.35 g cm$^{-3}$
- various sands: bulk densities from 1.1 to 1.75 g cm$^{-3}$
- sand-gravel: bulk densities from 1.7 to 1.9 g cm$^{-3}$
- crushed stone and sand: bulk density 1.975 g cm$^{-3}$

Solid line is $K_a = 3.01 + 10.1 Q_v + 143 Q_v^2 - 75 Q_v^3$

Dashed lines denote $\pm 2.5$ in $Q_v$.
might be attributed to the dielectric constant of the sample components (i.e. crushed stone may have a high $K'$). In general, $K_a$ falls between 2.6 and 3.0 for a wide range in density.

$K_a$ versus $Q_v$ data were also obtained for most of these samples (see Figure 8). The test results indicate that most of the data fall within $+1.5\%$ of the $K_a-Q_v$ relationship described by (4). The only real exception is the crushed stone and sand sample which had the high dry $K_a$ of 4.24. Other data to be discussed in Chapter 3 suggest (4) may have more general application in determining the volumetric water contents of earth materials.
CHAPTER 3

DETERMINING THE UNFROZEN WATER CONTENT
OF FREEZING SOILS FROM $K_a$

3.1 Introduction

The lack of strong dependence of $K_a$ on such factors as soil type, texture, structure and mineralogy and the similarity of the dielectric constants of ice and dry soil, suggested that the TDR technique might be used to measure the unfrozen water content in freezing soils. The temperature dependence of $K_a$ for a Rideau Clay ($\theta_v = 40\%$) was examined by Davis and Annan (1977) and it proved to be minimal from 0° to 30°C; however, at temperatures below freezing the magnitude of $K_a$ suggests that unfrozen water was detected in the soil (see Figure 9). This author undertook experiments to examine the possibility of determining $\Theta_{uf}$ from $K_a$. The experiments are presented in a sequential order to exemplify the importance of each.

3.2 $K_a$ Measurements on Soils at Temperatures Below 0°C

The relationship between temperature and $K_a$ was examined for various soils which possess distinctly different grain size characteristics. A coarse Ottawa sand was chosen since this sample would not contain unfrozen
THE EFFECT OF TEMPERATURE ON $K_a$

Source: Davis and Annen, 1977

Rideau Clay Sample

40%
water even at temperatures close to 0°C, hence, permitting one to examine the effect of ice content upon $K_a$. A clay (Leda), a clay loam (Ellwood) and a silt loam (Castor) were also selected since they should possess distinctly different freezing characteristic curves.

For the experiments using coarse Ottawa sand, samples were placed in coaxial cylinders; known volumes of water were added and the samples were then frozen at about -18°C for 24 hours in a freezer. One sample was frozen at about -0.5°C in a low temperature chamber. Coaxial lines could not be used for the finer textured materials since temperature uniformity and stability could not be obtained in the low temperature chamber (temperature variations would cause variations in $Q_{uf}$ and possible difficulties in interpreting $K_a$ data). Parallel transmission lines were used instead since the samples (slurried) could be placed in latex covered PVC tubes and suspended in a circulating methanol bath which permits temperature control to better than ±0.01°C for extended periods of time. Figure 10 shows the sample cell. Freezing was initiated by subjecting the sample to a temperature of about -2°C for 30 minutes once it had equilibrated at 0°C for 24 hours. $K_a$ was monitored between temperature steps; when $K_a$ stabilized (i.e. no change in the trace length), a new temperature was imposed on the sample. The finer textured soils were also run through several freezing and warming cycles to examine the effect upon $K_a$. The experiment required about 4 days to complete and up to 24 hours was given to ensure that $K_a$ stabilized (especially at temperatures between 0°C and -1°C since substantial phase change can occur in this range). For all soils examined, at all
Figure 10

PARALLEL TRANSMISSION LINE FREEZING CELL

[Diagram of a parallel transmission line freezing cell with dimensions and notes:]
- Head connector
- Latex collapsed PVC tube
- Transmission lines 2.5 cm
- Cell is immersed in a circulating methanol bath
- Soil sample
- Dimensions: 4.5 cm and 20 cm
Figure 11

$K_a$ for Frozen Ottawa Sand at Different Initial Water Contents

$K_a$ determined at $-18 \, ^\circ C$

$K_a$ determined at $-0.5 \, ^\circ C$

Initial water content, (cm$^3$ cm$^{-3}$ %)
temperatures, no drift in $K_a$ was noted once it had stabilized (in several instances a $K_a$ value was determined when stabilized and measured again 8 hours later; no differences in $K_a$ were noted).

The variation in $K_a$ for temperatures above 0°C is generally quite small. The Leda clay exhibited the largest change, with $K_a$ equal to about 32 at 21°C and 34.8 at 0°C which suggests an apparent increase in water content of about 3% (from (4)). This difference in $K_a$ is expected since the dielectric constant of water increases with decreasing temperature (80.1 at 20°C and 87.7 at 0°C).

Data for $K_a$ versus initial water content for the coarse Ottawa sand are presented in Figure 1. As is shown the variation in $K_a$ is small, yet $K_a$ does appear to increase slightly with increasing ice content, from 2.5 with no ice to 3.5 for a sample that had an initial water content of 35% (essentially saturated). It is apparent that ice content does not account for the $K_a$ values obtained by Davis and Annan (1977) in Figure 9.

The test results for the finer textured samples are shown in Figure 12. They indicate that $K_a$ decreases with decreasing temperature similar to that shown in Figure 9. The Leda clay sample exhibits a large change in $K_a$ from -0.5°C to -1.0°C; the decrease in $K_a$ is much more gradual below -2.0°C. The Ellwood clay loam was similar to the Leda clay; however, $K_a$ changes become more gradual below -1.0°C. The Castor silt loam showed a sharp drop in $K_a$ between 0°C and -0.5°C with more moderate changes below this.

The effect of freezing and warming cycles upon $K_a$ for the Leda clay
Figure 12  \( K_a \) versus Temperature for Several Soils

- Leda clay
- Ellwood clay loam
- Castor silt loam

\( K_a \) at

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>21°C</th>
<th>0°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leda clay</td>
<td>31.99</td>
<td>34.88</td>
</tr>
<tr>
<td>Ellwood clay loam</td>
<td>33.86</td>
<td>33.86</td>
</tr>
<tr>
<td>Castor silt loam</td>
<td>22.60</td>
<td>22.87</td>
</tr>
</tbody>
</table>
Figure 13  $K_a$ versus Temperature for a Leda Clay

*solid lines: first freeze-thaw cycle*
*dashed lines: second freeze-thaw cycle*
is shown in Figure 13. The Leda clay sample showed marked differences in \( K_a \) between the first freezing and warming cycles. The second cycle showed smaller differences which might suggest structural changes in the sample, and hence \( Q_{uf} \) changes, due to the first freeze. The clay loam and silt loam showed similar results; however, they were generally less pronounced.

The test results from Figures 11, 12 and 13 show that:

1. the finer the soil texture, the greater the \( K_a \) value at a given temperature below 0°C;
2. \( K_a \) changes more rapidly with temperature in the range 0°C to -1°C the coarser the texture;
3. the \( K_a - T^\circ C \) relationship is hysteretic (\( K_a \) freezing > \( K_a \) thawing).

These observations are similar to those observed in soil freezing characteristic curves (\( Q_{uf}-T^\circ C \)) (e.g. Williams, 1963).

The experimental results for the coarse Ottawa sand, and the finer textured materials suggest that \( K_a \) measurements might be used to determine the unfrozen water content of freezing soils. The supportive information is summarized as follows:

1. \( K_a \) for ice and dry mineral soils is similar;
2. Ice content (Ottawa sand experiment) does not appreciably affect \( K_a \);
3. the \( K_a \) values for the finer textured soils reflect the characteristics of soil freezing characteristic curves.

The difficulty in assessing the potential of using the TDR technique is that there is no standard method of calibrating a \( K_a-Q_{uf} \) relationship as there is in unfrozen soils where liquid water content is known (see
section 2.5). The next section deals with the measurement of liquid water contents in ice/water mixtures to examine the possibility that (4) may have more general applications in measuring water contents which would lend support to the proposition that \( Q_{uf} \) can be determined from \( k_a \).

3.3 \( k_a \) Measurements in Ice/Water Mixtures

It was felt that the TDR technique should be able to determine liquid water contents in ice/water mixtures since the dielectric constant of ice and dry mineral soils is similar. The procedure entailed adding known volumes of water to a coaxial cylinder containing crushed ice. Due to the nature of the experiment, the water was not uniformly distributed in the sample and the volume of ice was varied depending upon which \( Q_v \) range one was working in. In the range \( Q_v > 20\% \), the sample porosity was about 35-40%. For \( Q_v \) between 10-20\% the porosity was about 20%; for \( Q_v \) less than 10\%, porosity was 10\% or less. The reason why ice content was varied stemmed from a desire to obtain a sample which contained as little air as possible.

For each volume of water added, a TDR reading was taken and \( k_a \) determined. The time required to obtain a reading was approximately 30 seconds. The water was drained after the TDR reading and the volume noted. No substantial differences were found between added and recovered amounts of water suggesting that no substantial phase change had occurred (added water content and recovered water content were within \( \pm 1.5\% \); i.e. if a water volume equal to \( Q_v = 10\% \) was added, the recovered volume may yield a \( Q_v = 8.5 \) or 11.5\%).

The results from the experiment are presented in Figure 14. Also
Figure 14  $K_b$ for Ice/Water and Snow/Water Mixtures

- the author: crushed ice/water
- Sweney and Colbeck (1974) at 6 GHz
  for snow/water

solid line is equation (4) in text
determined for mineral soils
source: Topp, Davis and Annan
(1980 in press)
included are dielectric constant measurements (at 6 GHz) determined by Sweeney and Colbeck (1974) for snow/water mixtures. The $K_a - \Theta_v$ curve, (4), obtained by Topp, Davis and Annan (1980 in press) for unfrozen mineral soils is also shown. The results agree favorably with (4), particularly up to about 20% in $\Theta_v$.

3.4 Assessing the Potential for $\Theta_{uf}$ Measurements from $K_a$

The experimental results in sections 3.2 and 3.3 suggest that (4) might be used to determine $\Theta_{uf}$ from $K_a$ measurements. Accepting this in principle, the $K_a$ values for the Castor silt loam and the Leda clay were converted to $\Theta_{uf}$ using (4) since freezing characteristic curve data for soils with similar grain size characteristics existed in the literature.

The Castor silt loam data were compared to dilatometric data calculated from El Khorabii (1975) for an Onepida silt loam and to suction-moisture data for a Calgary silt loam (Williams and Wood, 1978). El Khorabii's data are sufficient to obtain accurate values for volumetric unfrozen water content. The freezing characteristic data for the Calgary sample were changed to a volumetric basis as outlined in Appendix III. The grain size characteristics are very similar, hence, they should possess similar freezing characteristic curves. The comparison of the $\Theta_{uf} - T^\circ C$ relationship for these soils is presented in Figure 15.

No comparable data could be obtained for clay loams; however, several data sets exist for Leda clays, so the author's Leda sample was compared to them. The grain size characteristics of Leda clays are highly variable (see Figure 16); however, two samples (Penner and GC8) are similar in
Figure 15  
Comparison of Soil Freezing Characteristic
Curves for Several Silt Loams

- Oneida El Khoraihi (1975) from dilatometer tests
- Castor from $K_a$ (4) used to determine $\theta_{uf}$
- Calgary Williams and Wood (1979) from suction-moisture tests

<table>
<thead>
<tr>
<th>Grain Size</th>
<th>$\theta_{uf}$ at 0 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>%sand</td>
<td>%silt</td>
</tr>
<tr>
<td>Oneida</td>
<td>20.5</td>
</tr>
<tr>
<td>Castor</td>
<td>19</td>
</tr>
<tr>
<td>Calgary</td>
<td>23</td>
</tr>
</tbody>
</table>
Figure 16
Grain Size Characteristics for Several Leda Clays

[Graph showing grain size distribution with curves labeled Penner 1970, GCB, TDR, KNB, and 100-7. The x-axis represents log particle size (mm), and the y-axis represents percentage finer than a given size.]
Figure 17

Freezing Characteristic Curves for Several Leda Clays

- Penner 1970 calorimetry
- GC8 Williams 1967 calorimetry
- TDR author from $K_a$ using (4)
- KNB Williams 1967 calorimetry
- 100-7 Williams 1967 calorimetry

see Appendix III also.
texture to the clay used in the $K_a$ experiment. The freezing characteristic curves for the Leda clays taken from published sources were determined calorimetrically. Penner's (1970) sample was the only undisturbed sample. In all cases unfrozen water contents were expressed on a mass basis; these were changed to a volumetric basis as outlined in Appendix III. $K_a$ values for the author's Leda clay sample (Figure 12) were changed to $\Theta_{uf}$ using (5). The comparison with published data is presented in Figure 17.

As is shown in Figure 15, the data for the silt loams are well within 3.0% in $\Theta_{uf}$ at any given temperature. The initial water contents are different; however, initial water content should not affect the $\Theta_{uf}$-T°C relationship (Williams, 1967).

With regard to the Leda clay comparison (Figure 17), it is evident that the TDR results compare well with the finer textured samples (Penner and GC8) over the whole temperature range. Differences below -2.3°C (2 to 4% in $\Theta_{uf}$) might be due to sample differences, experimental error or errors associated with converting gravimetric to volumetric unfrozen water contents. The freezing characteristic curves of the two coarser textured samples (KNB and 100-7) compare favorably over the whole temperature range. The apparent divergence between -0.5°C and -1.5°C could be due to sample differences or experimental in nature. The data from these coarser samples are different than for the finer samples which emphasizes the role that grain size characteristics (and hence pore size and pore size distribution) have upon the $\Theta_{uf}$-T°C relationship.

The favorable comparison of the freezing characteristic curves determined from $K_a$ values to those from published sources, and the information
Figure 18

COMBINED TDR–DILATOMETER

- TDR
- Oil (To Volume Measurement)
- Methanol
- Soil
- Partial Transmission Lines
- Methanol Chamber
- Oil Chamber
- Oil (For Filling)
- 24.0 cm
- 11.5 cm
presented in sections 3.2 and 3.3, suggests that (4) can be used to
determine \( Q_{uf} \) with a certain degree of confidence. A more exacting con-
firmation of this would be to obtain simultaneous measurements of \( Q_{uf} \) from
the TDR technique and some other method on the same soil sample. This
would standardize the factors which affect \( Q_{uf} \) at a given temperature
(i.e. structure, texture, mineralogy, soluble salts etc.). It was felt
that a combined TDR/dilatometer offered the best possible measurement
system.

3.5 Combined TDR/dilatometer Measurements of \( Q_{uf} \)

The combined TDR/dilatometer is essentially a modified dilatometer
which permits both measurement of sample volume expansion due to freezing
and TDR determinations of \( K_a \). The apparatus consists of an inner and outer
cell constructed from steel tubing. The inner cell houses the soil sample
and the silicon oil (used to measure expansion). As the sample freezes,
oil is displaced and the volume can be measured by the change in oil levels
in burettes external to the TDR/dilatometer (see Figure 18). Parallel
transmission lines pass through the plexiglass top plate which covers the
inner cell. The outer cell is essentially a cooling jacket through which
methanol is circulated from a controlled temperature bath (to \( \pm 0.01^\circ C \)).

The sample treatment is similar to that described in El Khoraihi (1975).
Basically, a soil sample is slurried and placed in a latex rubber membrane
and de-aired in a vacuum dessicator connected to a vacuum pump. After
a series of evacuations (over 24 hours), the membrane is placed in the inner
cell so it is suspended from the top. The plexiglass top plate is put in
place and bolted down being careful not to trap any air bubbles. The parallel transmission lines are then installed through holes drilled through the top plate and the TDR probe head is connected. Silicon rubber is used to seal any possible leaks to ensure that any volume expansion does displace oil. After the sealing procedure, the remaining volume of the inner cell is filled with silicon oil from a burette. The volume of oil entering the inner cell is recorded in order to determine the sample volume. The apparatus is then covered with pipe insulation, initial temperature is recorded, and the methanol inflow and outflow tubes and volume measurement burettes are connected. The whole apparatus is then placed in a low temperature incubator (set to 0° ±0.5°C). Oil is allowed to pass through the dilatometer from the oil source so that one of the burettes is full. Methanol at 0°C is circulated through the outer cell and the sample is allowed to equilibrate for at least 24 hours.

When the sample reaches equilibrium, the oil levels are recorded again. The noted contraction in the oil levels yields the coefficient of contraction of the oil, the dilatometer, the soil solids and the soil air, per °C. If this contraction is 10% larger than the expected contraction for the oil, then the sample is assumed to be improperly treated (the coefficient of contraction for the oil is known, therefore, an expected contraction can be determined). Numerous attempts were made before a properly treated sample was obtained (in one case, experimental results were obtained for an improperly treated sample, and the volume expansion at -5°C indicated that more water froze than was actually present in the sample). The difficulties with the experiment might be resolved if the
sample container was re-designed (smaller) so that air pressure could be used to test for 'sponginess' due to air present in the sample. A smaller sized cylinder would permit de-airing of the sample in the dilatometer itself, removing the possibility of air re-entering the sample before the top plate can be secured (about 30 minutes elapsed before the top plate could be secured after sample de-airing). Another check of the proper operation of the system is whether the oil levels return to their 0°C levels after a sample is frozen and then thawed. In some cases this, for some unknown reason, was not the case; the differences could not be attributed to particle consolidation.

To initiate sample freezing, the methanol temperature was reduced to about -4°C for 30 to 45 minutes; this is sufficiently long to note a volume expansion in the sample. Soil temperature was controlled by the methanol bath and sample temperature was determined by the bath temperature. Experiments comparing bath temperature and the temperature of the dilatometer showed no appreciable differences (less than 0.05°C). It was observed that about 80-90% of the observed final volume change at temperatures between 0°C and -1°C occurred within 3 to 4 hours. The remaining expansion taking perhaps 2 or more hours. As a precaution the samples were left at each temperature for 24 hours. Preliminary tests (i.e. those in which the samples exhibited unsatisfactory treatment) showed that ice lenses were randomly oriented in the frozen sample. This fact and the low hydraulic conductivities (perhaps 10^-8 to 10^-9 cm s^-1 based on data of Burt and Williams, 1976) would not be conducive to substantial water migration within the sample. Volume expansions are recorded at each temperature and corrected for thermal contraction, as indicated earlier, to yield the true volume expansion. TDR
Figure 19

Combined TDR/Dilatometer

Measurements of $\Theta_{uf}$ for an Ellwood Clay Loam

$\Theta_{uf}$ determined from $K_s$ using (4)

see also Table 3
readings of \( K_a \) are determined at each successive temperature and \( \Theta_{uf} \) calculated from (4).

At the end of the freezing cycle the sample was thawed to about \(+0.2^\circ C\) for 24 hours; the oil levels in the graduated burette should return to their original level; only a 1\% difference was noted (0.15 cm\(^3\) for 15.8 cm\(^3\) of total expansion). The sample was then refrozen and put through another freezing cycle and level check. At the end of both cycles, a gravimetric water content determination was made to determine the volume of water present in the sample. Volume expansion was converted to volume of water that froze and it was subtracted from the total volume of water to yield the unfrozen water. Volumetric unfrozen water content is calculated by dividing the volume of unfrozen water by the initial sample volume plus the volume expansion. The accuracy in \( \Theta_{uf} \) determined by the dilatometer (based upon differences in oil level before and after freeze, differences in expected and actual contraction and the possible variation in sample volume) is probably ±1.5\%. The TDR/dilatometer results are presented in Figure 19 and Table 3.

As is shown, the results determined by the TDR technique compare favorably with those obtained from dilatometer measurements. The largest difference at any one temperature (3\% in \( \Theta_{uf} \)) occurs at \(-0.75^\circ C\) during the first freeze. This difference is an anomaly but not substantial since the certainty in (4) is about 2.5\%. A large difference in \( \Theta_{uf} \) at about \(-0.75^\circ C\) was noted between the first and second freeze cycles. Since both measurement techniques noted this difference (about 8\% in \( \Theta_{uf} \)), it is possible that this reflects changes in sample structure due to the first freeze.
### TABLE 3
COMBINED TDR/DILATOMETER TEST RESULTS

<table>
<thead>
<tr>
<th>Temp. Change (°C)</th>
<th>Volume (cm³)</th>
<th>Sample Volume (cm³)</th>
<th>Unfrozen Water Volume (cm³)</th>
<th>( Q_{uf} ) (cm³ cm⁻³, %)</th>
<th>( K_a ) (cm³ cm⁻³, %)</th>
<th>( Q_{uf} ) (cm³ cm⁻³, %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>414.00</td>
<td>227.06</td>
<td>54.84</td>
<td>36.86</td>
<td>52.50</td>
</tr>
<tr>
<td>-0.72</td>
<td>10.10</td>
<td>424.10</td>
<td>115.46</td>
<td>27.22</td>
<td>13.79</td>
<td>25.75</td>
</tr>
<tr>
<td>-1.48</td>
<td>12.96</td>
<td>426.96</td>
<td>83.86</td>
<td>19.64</td>
<td>10.33</td>
<td>20.50</td>
</tr>
<tr>
<td>-1.98</td>
<td>13.98</td>
<td>427.38</td>
<td>79.22</td>
<td>18.54</td>
<td>9.69</td>
<td>19.25</td>
</tr>
<tr>
<td>-2.81</td>
<td>15.03</td>
<td>429.30</td>
<td>60.98</td>
<td>14.21</td>
<td>7.06</td>
<td>14.00</td>
</tr>
<tr>
<td>-4.34</td>
<td>15.77</td>
<td>429.77</td>
<td>52.76</td>
<td>12.27</td>
<td>6.01</td>
<td>11.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>sample thawed and then refrozen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.75</td>
<td>7.35</td>
<td>421.35</td>
<td>145.84</td>
<td>34.61</td>
<td>18.06</td>
<td>31.50</td>
</tr>
<tr>
<td>-1.20</td>
<td>11.32</td>
<td>425.32</td>
<td>101.98</td>
<td>23.98</td>
<td>11.95</td>
<td>23.00</td>
</tr>
<tr>
<td>-1.86</td>
<td>13.68</td>
<td>427.68</td>
<td>75.90</td>
<td>17.75</td>
<td>8.74</td>
<td>17.50</td>
</tr>
<tr>
<td>-2.84</td>
<td>14.92</td>
<td>428.92</td>
<td>62.20</td>
<td>14.50</td>
<td>7.21</td>
<td>14.25</td>
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<tr>
<td>-3.97</td>
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<td>429.62</td>
<td>54.58</td>
<td>12.70</td>
<td>6.61</td>
<td>13.00</td>
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<tr>
<td>-4.64</td>
<td>15.93</td>
<td>429.93</td>
<td>51.04</td>
<td>11.86</td>
<td>6.25</td>
<td>12.25</td>
</tr>
</tbody>
</table>

Unfrozen water = 227.06 - (volume change from 0°C / 0.0905)

\( Q_{uf} \) from \( K_a \) values is taken to the nearest 0.25%.

\( Q_{uf} \) from the dilatometer results = unfrozen water / sample volume
The results from a previous \( K_a - T^\circ C \) experiment performed on the Ellwood clay loam (Figure 12) were converted to \( Q_{uf} \) using (4) and plotted on Figure 19. This curve also compares closely to the combined TDR/dilatometer results (greatest difference 2% in \( Q_{uf} \)). The difference could be attributed to either measurement precision or sample differences, even though the samples were taken from the same location.

It is felt that the combined TDR/dilatometer results and the data from other sections indicate that the TDR technique is a viable method for measuring unfrozen water contents in freezing soils. The curve relating volumetric water content to \( K_a \), (4), appears to be applicable to mineral soils in either the frozen or unfrozen state.
CHAPTER 4

CONCLUSIONS

The TDR technique shows very great promise for routine measurement of volumetric liquid water content in freezing soils. The method is non-destructive, largely unaffected by soil texture, mineralogy and soluble salts (Davis, Topp and Annan, 1977; Topp, Davis and Annan, 1980 in press). Various experiments have been performed which indicate that:

1. $K_a$ is not strongly sensitive to ice content in soil samples containing coarse sand, ice and air;
2. $K_a$ for finer textured soils suggest that unfrozen water is present in the samples;
3. data for ice/water mixtures compare favorably with the relationship between $K_a$ and $\theta_v$ for unfrozen mineral soils (equation (4));
4. $K_a$ values converted to $\theta_{uf}$ (using (4)) for several soils compare favorably with published data for similar samples;
5. a combined TDR/dilatometer experiment provided simultaneous measurements of $\theta_{uf}$ on the same soil sample; the results compare favorably;
6. there may be a fundamental relationship between $K_a$ and volumetric water content, since equation (4) appears to be applicable to mineral soils in the frozen or unfrozen state.

It is hoped that further work is done to obtain data on $K_a$ as a function of $\theta_v$ and $\theta_{uf}$. Combined TDR/dilatometer experiments need to be performed on a variety of soils, to either calibrate a $K_a$-$\theta_{uf}$ relationship or confirm (4), since measurements are performed on the same soil sample.
thereby removing all extraneous variables which may create differences in the \( Q_{uf} - T^oC \) relationship for 'similar' soils.

The TDR technique shows promise for routine determination of \( Q_{uf} - T^oC \) curves for saturated and unsaturated soils in a relatively short time (2 to 5 days). It should also be possible to obtain freezing characteristic curves for undisturbed core samples provided the parallel probes can be installed in the sample with a minimum of disturbance.

Based on the results in this thesis, it appears that the TDR technique could be used for in situ measurement of \( Q_{uf} \) using parallel transmission lines installed horizontally with depth in an excavated pit or by using the surface installed probe configuration described by Davis and Chudobiak (1975). (Resolution with depth is obtained by using probes which have physical-electrical discontinuities along their length). The use of the TDR technique, in situ or in the lab, could be used to provide the necessary data for testing numerical models of heat and mass transfer in soils, models which could be applied to assessing (and minimizing) the effects of soil freezing and frost heave in roadways and around buildings and pipelines.
APPENDIX I

THE MKSA SYSTEM OF UNITS AND
DEFINITIONS OF ELECTRICAL UNITS

This system is based on four base units which represent the quantities length, mass, time and electric current intensity. This system is essentially a subset of the Système International (SI). The definitions were derived from Semioli and Shubert (1974) and may differ slightly from definitions given in other sources.

<table>
<thead>
<tr>
<th>BASE UNIT</th>
<th>COMMON NOTATION</th>
<th>QUANTITY</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>l</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>m</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>time</td>
<td>t</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>electric current</td>
<td>I</td>
<td>ampere</td>
<td>A</td>
</tr>
</tbody>
</table>

Other quantities have been given special names and symbols.

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>UNIT</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>force</td>
<td>Newton (kg m s(^{-2}))</td>
<td>N</td>
</tr>
<tr>
<td>work</td>
<td>joule (kg m(^2) s(^{-2}))</td>
<td>J</td>
</tr>
<tr>
<td>power</td>
<td>watt (J s(^{-1}))</td>
<td>W</td>
</tr>
<tr>
<td>charge</td>
<td>coulomb (A s)</td>
<td>C</td>
</tr>
<tr>
<td>electrical potential</td>
<td>volt (W A(^{-1}) J c(^{-1}))</td>
<td>V</td>
</tr>
<tr>
<td>capacitance</td>
<td>farad (c V(^{-1}))</td>
<td>F</td>
</tr>
<tr>
<td>resistance</td>
<td>ohm (V A(^{-1}))</td>
<td>(\Omega)</td>
</tr>
</tbody>
</table>

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DEFINITIONS OF THE UNITS

Newton
(unit of force) One newton is the force which is required to give an acceleration of one m s$^{-2}$ to a mass of one kilogram.

Joule
(unit of work, energy or quantity of heat) One joule represents the work done when a force of one newton is displaced through a distance of one metre in the direction of the force.

Watt
(unit of power) One watt is one joule per second; essentially, the rate at which work is done.

Coulomb
(unit of charge) One coulomb can be defined as the quantity of charge which repels an equal and similar charge in vacuum when placed one metre apart with a force of $9 \times 10^9$ N.

Ampere
(unit of electric current intensity) The current is one ampere when a charge is transferred from one point in a conductor to another at a rate of one coulomb per second.

Volt
(unit of electrical potential) One volt is the potential created when one joule of work is required to transfer one coulomb of charge.

Ohm
(unit of resistance) One ohm is the resistance through which a potential difference of one volt will produce a current of one ampere.

OTHER DEFINITIONS

Alternating Current (a.c.) A current in which the charge flow periodically reverses. It implies a sinusoidal variation of current and voltage.

Angular frequency Simply defined as $2\pi$ times frequency.
Capacitance A conducting body has a capacitance of one farad if a charge of one coulomb is required to raise its potential by one volt.

Conductance The reciprocal of resistance, in mho's where 1 mho = 1/ohm.

Conductivity The reciprocal of resistivity, 1/Ωm.

Current Current describes the movement of electrons through an electrical conducting medium.

Dielectric Essentially any substance which has electrical insulating properties. A charge at one part of a dielectric is not readily communicated to any other part. There are no true dielectrics but only materials which exhibit dielectric qualities.

Dielectric Constant or Relative permittivity The dimensionless ratio of the permittivity of a substance to the permittivity of free space (see section 2.1).

Frequency The number of cycles completed in unit time. Direct current (d.c.) has zero frequency since it is non-sinusoidal; a.c. can be described in terms of frequency.

Impedance Impedance is the apparent opposition to the flow of alternating current. It is analogous to the actual resistance to a direct current.

Period The time required to complete one cycle. If one cycle is completed in one second the frequency is one cycle per second which equals one hertz (1 Hz).

Power Power describes the time rate at which work is done.

Resistance Resistance is the property of a conductor which determines the current produced by a given difference of potential (i.e. the opposition offered by a body to the passage of a steady current).

Resistivity Resistivity is the resistance that a cubic centimetre of a substance offers to the passage of electricity when the current is perpendicular to two parallel faces.
APPENDIX II

USING THE TEKTRONIX 1502 TDR

A.11.1 Introduction

Figure 20 and Table 4, in this appendix have been included to familiarize the reader with the Tektronix 1502 TDR control panel.

The Tektronix 1502 TDR is used by the author since it is reasonably priced and field portable. This TDR provides the CABLE connector with a 200 mV step-voltage which has an incident rise time of less than 0.1 ns. This rise time contains a wide bandwidth of frequencies up to a possible maximum of 3.5 GHz. The TDR's CRT is calibrated in units of mρ/DIV vertically and M/DIV horizontally. The vertical scale represents the ratio of the reflected to incident voltage (or voltage reflection coefficient, ρ).

If a cable is open circuited (i.e. infinite impedance: \( R_L = \infty \)), \( ρ = 1 \); for a short circuited cable (zero impedance; \( R_L = 0 \)), \( ρ = -1 \). When a cable is terminated with its characteristic impedance there is no reflection and \( ρ = 0 \). One can convert the reflected pulse amplitude to impedance since \( ρ \) is dependent upon the characteristic impedance, \( Z_0 \), of the line under test and the load on the cable, \( R_L \) (or the impedance of a discontinuity within a cable):

\[
ρ = \frac{R_L - Z_0}{R_L + Z_0}
\]

therefore

\[
R_L = \frac{Z_0}{1 + \frac{1}{1 - ρ}}
\]

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<table>
<thead>
<tr>
<th>No.</th>
<th>Control</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CABLE</td>
<td>BNC connector, delivers 110 ps rise time pulse and receives reflected pulse</td>
</tr>
<tr>
<td>2</td>
<td>FOCUS</td>
<td>visual controls for CRT</td>
</tr>
<tr>
<td>3</td>
<td>INTENSITY</td>
<td>visual controls for CRT</td>
</tr>
<tr>
<td>4</td>
<td>POSITION/FINE</td>
<td>vertical position control of the CRT</td>
</tr>
<tr>
<td>5</td>
<td>V/DIV</td>
<td>vertical scaling control</td>
</tr>
<tr>
<td>6</td>
<td>POWER</td>
<td>ON-OFF</td>
</tr>
<tr>
<td>7</td>
<td>GAIN</td>
<td>for adjusting gain of the vertical amplifier</td>
</tr>
<tr>
<td>8</td>
<td>NOISE FILTER</td>
<td>reduces noise and improves trace appearance</td>
</tr>
<tr>
<td>9</td>
<td>BATTERY</td>
<td>battery level indicator</td>
</tr>
<tr>
<td>10</td>
<td>ZERO REF CHECK</td>
<td>returns trace to reference setting</td>
</tr>
<tr>
<td>11</td>
<td>ZERO REF SET</td>
<td>horizontal position control</td>
</tr>
<tr>
<td>12</td>
<td>MULTIPLIER</td>
<td>horizontal scale setting factor: 0.1 or 1.0</td>
</tr>
<tr>
<td>13</td>
<td>DISTANCE</td>
<td>moves CRT display to any location in cable</td>
</tr>
<tr>
<td>14</td>
<td>METRES/DIV</td>
<td>horizontal scale selector</td>
</tr>
<tr>
<td>15</td>
<td>CABLE DIELECTRIC</td>
<td>permits selection of $v_p$</td>
</tr>
<tr>
<td>16</td>
<td>RECORD</td>
<td>activates x-y or y-t recorder</td>
</tr>
<tr>
<td>17</td>
<td>AC LINE FUSES</td>
<td></td>
</tr>
</tbody>
</table>
The horizontal axis is readily converted to a time axis from (8) when CABLE DIELECTRIC is set for air (all buttons out).

There are several other pieces of information which can be determined from the display on the TDR's CRT. First, the impedance of a transmission line can be determined (or an impedance mismatch within a line) from $\rho$, where $Z_0$ is the impedance of a 'known' cable (or impedance mismatch in a line). If the vertical scale setting is known, the impedance at any point along the trace displayed on the CRT can be determined. Also, the length of a cable can be determined directly off the CRT display if the CABLE DIELECTRIC is set for the type of cable being tested. The dielectric constant of an 'unknown' cable can also be determined if the cable length is known, from (9).

A.II.2 TDR Operational Procedure

The first step in the procedure is to calibrate the device which records the information presented on the TDR's CRT. Photographing the CRT can be used in the field unless a Y-t module is used (the 1502 TDR has this option).

Operational Procedure

1. Set the TDR for air dielectric (VARIABLE control fully clockwise or all the CABLE/DIELECTRIC buttons in the default position)

2. Pull the power switch on.

If remote X-Y recorder is used:

3. Connect a two-wire cable to the X-axis jacks on the TDR and the X-Y recorder.

4. Set the X and Y axis scales on the X-Y recorder to accommodate 1 V full scale.
5. Push up the RECORD switch on the TDR; this initiates the recording of the trace displayed on the crt. The length of the trace will equal 10 divisions on the TDR's crt. Set the Y-axis on the X-Y recorder to the same setting as used for the X-axis. Go to step 9

If a Y-t recorder is used:

6. Since the horizontal axis is time, connect the X cables from the TDR to the Y connectors on the Y-t recorder. Set the chart speed of the Y-t recorder somewhere between 20 to 60 cm min⁻¹.

7. Set the Y-axis scale on the Y-t recorder to accept 1 V.

8. Push up the RECORD switch on the TDR. The slow ramp which will appear on the Y-t recorder represents the 100 mV/DIV that the TDR produces. The distance from the start to the end of the ramp equals 10 divisions on the TDR's crt.

Once the scaling factor (i.e. trace length on the recorder used divided by 10 divisions on the TDR), the starting point of the transmission line used (i.e. coaxial or parallel line) must be known. This is the A point discussed in Chapter II.

9. Connect a 50Ω cable to the transmission line (for parallel lines, the cable is an integral part of the line)

10. Drain static charge from the line by connecting a 50Ω terminator and cable adapter (supplied with the TDR) to the end of the cable.

11. Connect the cable to the CABLE connector of the TDR.

Repeat steps 12 to 14 for the transmission line system with and without the actual lines of transmission connected to the probe head for various
horizontal scale settings.

12. Adjust the DISTANCE dial and the POSITION control until the open circuit appears on the crt.

13. Set the mΩ/DIV scale so that the trace appears completely on the screen vertically.

14. Record the crt display.
APPENDIX III

CALCULATING $Q_{uf}$ FROM $w$

This section includes freezing characteristic curve data and the method of converting $w$ to $Q_{uf}$ for comparing to $Q_{uf}$ determined from $K_{s}$ using the TDR technique. With the exception of El Khorabi's (1975) data the following was assumed:

- mass of the solids ($M_{s}$) = 1 g
- water content at 0°C ($w$, g g$^{-1}$) = saturation (i.e. no air)
- particle density ($\rho_{d}$) = 2.65 g cm$^{-3}$

Therefore,

- volume of the solids ($V_{s}$) = 0.377 cm$^{3}$ ($V_{s} = M_{s}/\rho_{d}$)
- volume of water ($V_{w}$) = $M_{w}$ (by definition)
- total volume ($V_{t}$) at 0°C = $V_{s} + V_{w}$

As the soil freezes, the water that turns to ice expands by 9.05%; therefore, $V_{t}$ must be corrected:

$$V_{t}(at \ T°C) = V_{t}(at \ 0°C) \times (V_{w}(0°C) - V_{w}(T°C)) \times 0.0905$$

The unfrozen water content expressed volumetrically is:

$$Q_{uf} = V_{w}(T°C)/V_{t}(T°C)$$

El Khorabi (1975) provided sufficient data to calculate $Q_{uf}$, so his data and method of conversion will be presented separately.
### Oneida Silt Loam (El Khoraiwi, (1975) from dilatometric methods)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>Observed Vol. (cm³)</th>
<th>Thermal Vol. Contr. Change (cm³)</th>
<th>Volume Change (cm³)</th>
<th>Vₜ</th>
<th>Mₚ</th>
<th>Mᵢ</th>
<th>Qᵤᵤ (cm³ cm⁻³ %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>144.02</td>
<td>73.02</td>
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<tr>
<td>-0.328</td>
<td>4.361</td>
<td>0.0</td>
<td>4.361</td>
<td>148.38</td>
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<td>48.18</td>
<td>16.73</td>
</tr>
<tr>
<td>-0.370</td>
<td>4.434</td>
<td>0.001</td>
<td>4.435</td>
<td>148.45</td>
<td>24.02</td>
<td>49.00</td>
<td>16.13</td>
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<tr>
<td>-0.586</td>
<td>4.703</td>
<td>0.005</td>
<td>4.708</td>
<td>148.73</td>
<td>21.00</td>
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<td>14.12</td>
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<tr>
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<td>0.017</td>
<td>4.958</td>
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<td>18.24</td>
<td>54.78</td>
<td>12.24</td>
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<tr>
<td>-1.943</td>
<td>5.109</td>
<td>0.035</td>
<td>5.144</td>
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<td>16.18</td>
<td>56.84</td>
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<td>-3.009</td>
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<td>-3.994</td>
<td>5.251</td>
<td>0.079</td>
<td>5.330</td>
<td>149.35</td>
<td>14.12</td>
<td>58.90</td>
<td>9.45</td>
</tr>
<tr>
<td>-5.199</td>
<td>5.273</td>
<td>0.103</td>
<td>5.376</td>
<td>149.39</td>
<td>13.62</td>
<td>59.40</td>
<td>9.12</td>
</tr>
</tbody>
</table>

Thermal contraction = 0.0215 cm³ °C⁻¹

\[ Mᵢ = \text{volume change} / 0.0905 \]

\[ Mₚ = Mₚ(0°C) - Mᵢ \]

\[ Vₜ = Vₜ(0°C) + \text{volume change} \]

### Calgary Silt Loam (Williams and Wood, (1978) from suction moisture data; the points represent those taken from a best fit curve)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>W (g g⁻¹)</th>
<th>Qᵤᵤ (cm³ cm⁻³ %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.4165</td>
<td>0.524</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.22</td>
<td>0.277</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.185</td>
<td>0.233</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.125</td>
<td>0.157</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.0825</td>
<td>0.104</td>
</tr>
<tr>
<td>-1.5</td>
<td>0.075</td>
<td>0.098</td>
</tr>
<tr>
<td>-2.0</td>
<td>0.075</td>
<td>0.074</td>
</tr>
<tr>
<td>-3.0</td>
<td>0.0725</td>
<td>0.091</td>
</tr>
</tbody>
</table>
### Leda Clay (Penner) (Penner, 1970 from calorimetric methods for an undisturbed sample; the points represent those taken from a best fit curve)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>( w ) (g g(^{-1} ))</th>
<th>( \Theta_{uf} ) (cm(^3) cm(^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.80</td>
<td>0.68</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.50</td>
<td>0.415</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.325</td>
<td>0.267</td>
</tr>
<tr>
<td>-2.0</td>
<td>0.22</td>
<td>0.179</td>
</tr>
<tr>
<td>-3.0</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>-4.0</td>
<td>0.14</td>
<td>0.113</td>
</tr>
<tr>
<td>-5.0</td>
<td>0.125</td>
<td>0.101</td>
</tr>
</tbody>
</table>

### Leda Clay (GC8) (Williams, 1967 from calorimetric methods for a disturbed sample; the points represent those taken from a best fit curve)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>( w ) (g g(^{-1} ))</th>
<th>( \Theta_{uf} ) (cm(^3) cm(^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.57</td>
<td>0.602</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.355</td>
<td>0.367</td>
</tr>
<tr>
<td>-1.5</td>
<td>0.23</td>
<td>0.235</td>
</tr>
<tr>
<td>-2.0</td>
<td>0.175</td>
<td>0.178</td>
</tr>
<tr>
<td>-3.0</td>
<td>0.125</td>
<td>0.127</td>
</tr>
<tr>
<td>-4.0</td>
<td>0.085</td>
<td>0.086</td>
</tr>
<tr>
<td>-5.0</td>
<td>0.075</td>
<td>0.076</td>
</tr>
</tbody>
</table>

### Leda Clay (100-7) (Williams, 1967 from calorimetric methods for a disturbed sample; the points represent those taken from a best fit curve)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>( w ) (g g(^{-1} ))</th>
<th>( \Theta_{uf} ) (cm(^3) cm(^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.31</td>
<td>0.451</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.20</td>
<td>0.287</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.13</td>
<td>0.184</td>
</tr>
<tr>
<td>-2.0</td>
<td>0.07</td>
<td>0.099</td>
</tr>
<tr>
<td>-3.0</td>
<td>0.047</td>
<td>0.066</td>
</tr>
<tr>
<td>-4.0</td>
<td>0.037</td>
<td>0.052</td>
</tr>
<tr>
<td>-5.0</td>
<td>0.03</td>
<td>0.042</td>
</tr>
</tbody>
</table>
Leda Clay (KNB) (Williams, 1967 from calorimetric methods for a disturbed
sample; the points represent those taken from a best fit
curve, these data also appears in Burt, 1974)

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>w (g cm⁻¹)</th>
<th>ρuf (cm³ cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.35</td>
<td>0.48</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.26</td>
<td>0.354</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.239</td>
<td>0.324</td>
</tr>
<tr>
<td>-0.85</td>
<td>0.203</td>
<td>0.274</td>
</tr>
<tr>
<td>-1.10</td>
<td>0.189</td>
<td>0.255</td>
</tr>
<tr>
<td>-1.30</td>
<td>0.168</td>
<td>0.226</td>
</tr>
<tr>
<td>-1.50</td>
<td>0.149</td>
<td>0.20</td>
</tr>
<tr>
<td>-1.70</td>
<td>0.123</td>
<td>0.165</td>
</tr>
<tr>
<td>-1.90</td>
<td>0.098</td>
<td>0.130</td>
</tr>
<tr>
<td>-2.55</td>
<td>0.063</td>
<td>0.084</td>
</tr>
<tr>
<td>-4.3</td>
<td>0.025</td>
<td>0.033</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


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