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Performance Evaluation and Statistical Analysis of Optical Fiber Communication Systems

by

Habib Besbes

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Ottawa–Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
August, 1991

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acceptance of the thesis,

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submitted by Habib Besbes
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the degree of Master of Engineering

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Abstract

The most meaningful criterion in measuring the performance of an optical fiber communication system is the average bit error rate (BER). The BER is a function of the system parameters as well as the noise appearing in the system. The noise is the sum of thermal and shot noise which follow certain statistical properties. Based on the noise model, the BER can be represented as an integration of a function which is the product of the cumulative distribution function of the thermal noise and the probability density function of the shot noise. However, the latter cannot be expressed explicitly. Thus, it is necessary to introduce an efficient and reliable numerical method to evaluate the error probability.

This dissertation describes the saddlepoint approximation method which is used to evaluate the optical system performance by calculating the average error probability. The method is expanded to include the effect of the impulse response of the receiver and the effect of the intersymbol interference.

To optimize the system performance, a technique is presented for the minimization of the BER. Worst case analysis and statistical tolerancing are highlighted and a method for yield estimation is described.

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List Of Symbols

P_e	average error probability
$P(1 0)$	probability of error in detecting 1 when 0 is being sent
$P(0 1)$	probability of error in detecting 0 when 1 is being sent
p_0, p_1	probabilities of occurrence of 0 and 1, respectively
$h_c(t)$	impulse response of the optical source
$h_F(t)$	impulse response of the fiber
$h_p(t)$	overall impulse response of the source and fiber
$h_R(t)$	impulse response of the receiver
$H_R(f)$	frequency response of the receiver
$x_c(t)$	random pulses generated by the optical source
$x_T(t)$	random variable representing the thermal noise
$v_T(t)$	thermal noise signal at the input of the decision circuit
$v_s(t)$	filtered randomly Poisson process signal at the input of the decision circuit
f_{v_s} and f_{v_T}	probability density function (pdf) of the random variables v_s and v_T
M_{v_T}	moment generating function (mgf) of the thermal noise
M_{v_s}	mgf of the filtered randomly multiplied poisson process v_s
$v(t)$	total signal at the input of the decision circuit
f_0 and f_1	pdf of the random variable $v(t)$ when 0 and 1 are sent respectively

σ_T	root mean square value of the thermal noise
σ_N	root mean square noise value under Gaussian approximation
G	average value of the detector avalanche gain
M_g	moment generating function of the detector avalanche gain
$P(t)$	instantaneous optical power at the output of the fiber
$\lambda(t)$	mean instantaneous rate of primary electrons
s	Laplace operator
s_o	saddlepoint
s_o^-	saddlepoint where 0 is being sent
s_o^+	saddlepoint where 1 is being sent
η	photon counter quantum efficiency
$h\Omega$	energy of a photon
N_o	one sided power spectral density of the thermal Gaussian noise
t_o	decision time
V_d	threshold level
λ_o	dark current rate
k	collision-ionization probability of holes to that of electrons within the diode
R	responsivity of the photodetector
q	electron charge
Bw_R	bandwidth of the receiver
B_r	bit rate

Chapter I

INTRODUCTION

I.1 PROBLEM DESCRIPTION

Communication may be broadly defined as the transfer of information from one point to another [5]. In the case of communication over any distance, it is necessary to use an established system in which the information transfer occurs along an electromagnetic wave which acts as a carrier for the information signal. This simple process has been developed into extremely sophisticated techniques using electromagnetic carrier waves operating at radio frequencies as well as microwave and millimeter wave frequencies. However, communication may also be achieved using an electromagnetic carrier which is selected from the optical range of frequencies [5]. The use of visible optical carrier waves or light for communication has been common for many years [5]. The concept of guided lightwave communication along optical fibers has stimulated a major new technology during the last twenty five years.

Since the lack of suitable light sources and the difficulty of transmitting light through the atmosphere due to disturbances such as rain, snow, fog and dust proved to be serious obstacles to the study of guided light wave communication, research has stimulated considerable interest in this new technology. With the invention of the laser in the early 1960s, a renewed interest in optical communication was stimulated. In 1966, the research of Kao and Hockham and Werts made inroads almost simultaneously in the study of optical communication via dielectric waveguides or optical fibres made from glass. Their studies proposed these options as a solution to the degradation being caused by atmospheric conditions to the optical signal. Nevertheless, despite the problems some modest free space optical communication links have been implemented for applications such as the linking of the television camera to a base vehicle and for data links of a few hundred meters between buildings [5]. There is also some interest in optical communication between satellites in outer space using similar techniques [5].

Theoretically, the greater the carrier frequency, the larger the available transmission bandwidth, which in turn increases the carrying capacity of the communication system [5]. As such, communication which uses optical frequencies increases the potential useable bandwidth by a factor of approximately 10^4 (see Fig 1.1 for the spectrum of fiber communication systems). High carrier frequencies also give a greatly improved system performance by allowing the communication system to concentrate available power within the electromagnetic wave. Initially, these optical fibers demonstrated very high levels of attenuation (up to 1000 dB/km), and could as such not efficiently replace the coaxial cables (5–10 dB/km). Fiber cables could also not be satisfactorily jointed to achieve low loss and to allow

Fig 1.1 Spectrum of optical fiber communication systems

a relatively easy and repeated performance of the process. Fortunately, within the space of 10 years improvements have been made such that, in laboratory testing, fibers have performed with a loss of less than 0.1 dB/km. As a result, low loss fibers (below 0.5 dB/km) such as these have been implemented into optical communication systems worldwide. In parallel with the development of the fiber waveguide, attention was also focused on the other optical components which would constitute the optical fiber communication system [5].

The basic make up of an optical fibre communication system resembles any type of communication system (see Fig 1.2 for a block schematic of a general communication system and optical fiber system). In an electrical communication system, the information signal derives from a message signal and it is then converted by a transmitter into a suitable form for propagation over the transmission medium. In an optical communication system, electrical optical conversion occurs through the means of either a semiconductor laser or a Light Emitting Diode (LED). The research undertaken in this latter system has been greatly increased, especially due to its ability to achieve a reliable information transfer using a light carrier. The transmission medium in an optical fiber communication system consists of an optical fiber cable, and the receiver is an optical detector such as Photodiode (p-n, p-i-n, avalanche) or a phototransistor which is followed by a further electrical stage, thereby providing demodulation of the optical carrier and at present the signal processing usually occurs electrically.

The advantages of an optical fiber communication system are that it allows for enormous potential bandwidth, small size and weight, electrical isolation, etc. But as in any transmission medium, the signal is attenuated and is degraded through contamination by ran-

Fig 1.2 (a) General communication system
(b) Optical fiber communication system

dom signals and noise, as well as distortions by mechanisms within the medium itself. Contrarily, separating the noise generated within the optical system and in the receiver, in addition to any detector response limitations due to the associated circuits, from the information originally contained in the optical fiber communication system plays a vital role in evaluating the system's efficiency. Receiver noise greatly influences the optical fiber communication system, by limiting the receiver sensitivity, thereby being largely responsible for the overall system design. The overall performance of a digital optical fiber system is degraded by shot noise in addition to the usual impairments such as thermal noise and intersymbol interference [5]. With these disadvantages in mind, it is necessary to establish a maximum distance allowed between the transmitter and the receiver. Beyond this distance, it is acknowledged that the system ceases to communicate efficiently and intelligibly. As the length of the application increases, these factors require that repeaters or line amplifiers be installed sporadically, thereby eliminating signal distortion and increasing the signal level before the transmission continues down the length.

Digital light wave system design aims primarily at maximizing the spacing between repeaters, and at the same time maintaining a very specific error performance, as well as providing a power margin for any extenuating circumstances. The most fundamental form of system design is found in a power budget (see Fig 1.3 for power budget). This power budget accounts for the optical power originating at the source, less any power losses occurring from coupling, splices with a fiber loss margin, etc, and the net sum is the amount of power arriving at the receiver. When the power exceeds the sensitivity of the receiver, the error rate performance is met. The power budget also penalizes for any loss in receiver sensitivity due

Fig 1.3 Power loss distribution

to effects such as dispersion, and they are as such counted as actual losses. The maximum system length is calculated as the length at which the received power exactly equals the receiver sensitivity, this latter having been adjusted for all power penalties. The importance of each component within the system is easily recognizable when considering the theoretical application of the power budget. Realistically, system parameters vary significantly, therefore the system will perform with greater efficiency when it is designed to exceed its power budget. This technique is referred to as statistical design [8].

1.2 THESIS OBJECTIVES

The first objective of this thesis is to provide a Computer Aided Design (CAD) tool to measure the system performance of a fiber optic communication system by calculating the error probability in detecting the signal as a function of system model parameters. This CAD tool is based on approximation methods in order to compute a probability distribution by evaluating the inverse Laplace transform of the moment generating function of the shot noise. The saddlepoint approximation method is used throughout this thesis since it is an attractive and efficient method due to its very simple analytic form. Furthermore, it evaluates accurate results with less computational cost compared to other methods previously published.

The second objective of this thesis is to provide a sensitivity and a statistical analysis in order to introduce the concept of system performance optimization. This analysis is based on a simple system model as well as the Gaussian approximation method for the evaluation

of the bit error rate (BER).

Finally, several examples are presented to verify the efficiency of the saddlepoint approximation method, and to demonstrate the usefulness of sensitivity and statistical analysis in combination with optimization.

I.3 THESIS CONTRIBUTIONS

In this thesis, several contributions are presented to address the design of the optical fiber communication system:

1. Generalization of the saddlepoint approximation to include the effect of the receiver impulse response (Chapter III).
2. Generalization of the saddlepoint approximation to include the effect of the intersymbol interference. A new approach, based on the average moment generating function, is used to compute the BER (Chapter IV). Numerical examples are given to demonstrate the efficiency of the proposed saddlepoint method including the effect of the impulse response of the receiver circuit and the intersymbol interference (Chapters III ,IV).
3. Evaluation of the sensitivity of the BER with respect to the fiber optic system parameters (Section V .2 .1).
4. Optimization of the system performance (Section V .3 .1).
5. Statistical and worst case analysis (Section V .3 .3).

I.4 OUTLINE OF THE THESIS

Chapter II **Background and Literature Survey** gives a description of different types of noise and their mathematical characteristics. Different analytic methods for computing the BER are presented and compared.

Chapter III **Generalization of Saddlepoint Approximation to Include the Effect of the Receiver Response** presents the saddlepoint approximation and extends it to include the effect of the receiver impulse response.

Chapter IV **Generalization of Saddlepoint Approximation to Include the Effect of the Intersymbol Interference** extends the saddlepoint approximation method to include the effect of the ISI.

Chapter V **Statistical Analyses and Performance Optimization** presents a practical application of statistical analysis. A minimax algorithm is used to optimize the optical fiber system performance.

Chapter VI **Summary and Conclusions** summarizes the results of the thesis and suggests further research.

Chapter II

BACKGROUND AND LITERATURE SURVEY

II.1 INTRODUCTION

The performance of any communication system, particularly the Optical Fiber System, can be determined through the average error probability (Bit Error Rate) in detecting the signal as a function of certain model parameters such as fiber characteristics, detector avalanche gain, and the receiver circuit. This Bit Error Rate (BER) in the detection process can primarily be attributed to the noise generated in the receiver circuit (thermal noise), as well as the detector noise (shot and dark current noise), the intersymbol interference (ISI), and to the variation in the decision time (Jitter) [1].

Although the statistics of the shot noise follow a Poisson Distribution, other important sources of noise within practical receivers, for example thermal noise, are characterized by a Gaussian probability distribution.

Using the noise model stated above, the BER can be represented as an integration of a function which is the product of the cumulative distribution function of the thermal noise and the probability density function (PDF) of the shot noise statistics and ISI [2]. However, the latter cannot be expressed explicitly, only its moment generating function (MGF) is known and this makes the evaluation of the BER more complex.

Different techniques have been proposed to evaluate the error probability. In most of these techniques some approximations are assumed in order to simplify the mathematical treatment and/or reduce the computational cost. In this chapter, some of the well-known techniques used to evaluate the BER will be presented, which will allow us to compare and contrast them.

A model of an Optical Fiber Communication System is proposed in section II .2 . Based on that model, a description of the different types of noise and their mathematical characteristics is introduced in section II .3 . In section II .4 , the error probability is derived and different analytic methods used to compute the BER are described. Finally, a comparison between these analytic methods is presented in section II .6 .

II .2 MODEL DESCRIPTION OF AN OPTICAL FIBER SYSTEM

A simplified model of an Optical Fiber System is shown in [2] Fig 2. 1 .

A sequence of random pulses $x_c(t)$ is generated by the source and launched into the

Fig 2. 1 (a) Block diagram of a digital optical transmission system
(b) Stochastic Model

fiber optic channel :

$$x_c(t) = \sum_{k=-\infty}^{+\infty} a_k h_c(t - kT) \quad (2.1)$$

where a_k can assume the binary values $\{0, 1\}$ with probabilities p_0 and p_1 , respectively

$h_c(t)$ is the impulse response of the source

It has been widely assumed that optical fibers consist of a linear system which may be described by a frequency response [2], [3]. This response takes into account the attenuation and dispersion of the cable. If $h_F(t)$ is the impulse response of the optical fiber, the light power at the output of the fiber is expressed as follows:

$$P(t) = \sum_{k=-\infty}^{+\infty} a_k h_p(t - kT) \quad (2.2)$$

$$\text{where } h_p(t) = h_c(t) * h_F(t) \quad (2.3)$$

* denotes the convolution product

The optical signal is detected by an avalanche photodetector (APD) or a PIN photodetector and the average current produced by the detector is proportional to the incident optical power. The current signal $x(t)$ at the output of the photodetector circuit results from the sum of a filtered randomly multiplied Poisson process[3], $x_s(t)$, and an independent zero-mean additive thermal noise, $x_T(t)$, assumed to be Gaussian with a one-sided power spectral density N_o [3][4] : $x(t) = x_s(t) + x_T(t)$ (2.4)

The Gaussian noise characterizes the equivalent noise source at the input of the receiver circuit.

The receiving filter with the impulse response $h_R(t)$ equalizes the required signal and reduces noise. The signal at the input of the decision circuit is given by:

$$v(t) = v_s(t) + v_T(t) \quad (2.5)$$

$$\text{where } v_s(t) = x_s(t) * h_R(t) \quad (2.6)$$

$$v_T(t) = x_T(t) * h_R(t) \quad (2.7)$$

The signal $v(t)$ is sampled every T seconds to determine the transmitted signal. At the decision time, t_o , the sampled signal is:

$$v(t_o) = a_0 h_{sys}(t_o) + \left[\sum_{\substack{k=-\infty \\ k \neq 0}}^{+\infty} a_k h_{sys}(t_o - kT) \right] + v_T(t_o) \quad (2.8)$$

$$\text{where } h_{sys}(t) = h_p(t) * h_R(t)$$

The first term of the equation represents the desired signal, while the second and the third terms represent impairments resulting from the intersymbol interference and additive Gaussian noise respectively.

From the sampled values, the decision circuit estimates the peak of the signal $v(t)$ corresponding to a threshold level V_d at the decision time, t_o , for the data a_k being sent.

The overall system performance is dictated by several crucial elements of which the detector is a vital one within the optical fiber communication system. The received optical signal is converted by the detector into an electrical signal, which is in turn amplified before being further processed.

It is essential that the detector performs efficiently with the amplifying and signal pro-

cessing circuit [5]. An important part of this process is to separate the information which is originally transmitted in the optical signal from the noise generated within the rest of the system and within the receiver itself. In addition, any limitations on the detector response imposed by the associated circuits form an integral part of this process.

Since system design can be determined by its performance set by the signal to noise ratio (SNR) at the receiver, we must outline the sources of noise found within optical fiber systems. This is done in the succeeding sections.

II.3 NOISE CONSIDERATIONS

We can define noise as a general term which refers to any extraneous disturbances covering the received signal in a communication system. In optical fiber communication systems, the noise has different origins from that of copper-based systems due to spontaneous fluctuations. Both types of systems have thermal noise generated in the receiver. Although optical fiber systems exhibit little crosstalk, the noise generated within the detector must be considered, as well as the noise properties associated with the electromagnetic carrier [5]. In fiber optic systems, the sources of noise are shown in Fig 2. 2 .

There are three main types of noise due to spontaneous fluctuations in optical fiber communication systems: thermal noise, dark current noise and shot noise. The intersymbol interference (ISI) is another type of impairments that causes a degradation in the system performance.

In many problems, it is relatively easy to determine the moment generating function of the

Fig 2. 2 Block schematic of an Optical Fiber System showing the different sources of noise

noise distribution, but quite impossible to carry out in closed form the inverse Laplace transform that converts it to the probability density function [6].

In the following, the different types of noise are described briefly and their moment generating functions are derived in order to be used for the evaluation of the system performance.

II.3.1 Thermal noise

This refers to the spontaneous fluctuation caused by thermal interaction between the free electrons and the vibrating ions in a conducting medium.

The additive thermal noise previously designated as $x_T(t)$ is assumed to be Gaussian with a one-sided power spectral density N_o [3][4]. The spectral density N_o of the equivalent input noise source is evaluated as the sum of the input noise source of each element of the receiver that contributes to the noise:

$$N_o(f) = \sum_{i=1}^n J_i^2 \quad (2.9)$$

where n is the number of circuit elements that contribute to the noise

J_i^2 is the spectral density of the equivalent input noise current source at frequency f in Amp^2/Hz created by element i of the circuit and is given by:

$$J_i^2 = \frac{\delta_i^2}{|H_R(f)|^2} \quad (2.10)$$

δ_i^2 is the spectral density of the output noise voltage at frequency f in volt^2/Hz due

to element i calculated at the output of the receiver

$H_R(f)$, Fourier Transform of $h_R(t)$, is the network transfer function in *ohms*

The corresponding total thermal noise voltage $v_T(t)$ at the output of the receiver is also

Gaussian whose variance is σ_T^2 :

$$\sigma_T^2 = E[v_T^2] = \int_0^{+\infty} N_o(f) |H_R(f)|^2 df \quad (2.11)$$

and the probability density function f_{v_T} :

$$f_{v_T}(x) = \frac{1}{\sigma_T \sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma_T^2}\right] \quad (2.12)$$

The moment generating function of $v_T(t)$ is given by:

$M_{v_T}(s) = E[e^{(sv_T)}]$ which is equivalent to:

$$M_{v_T}(s) = \int_{-\infty}^{+\infty} f_{v_T}(v) e^{(sv)} dv \quad (2.13)$$

Replacing $f_{v_T}(v)$ by its expression, we obtain:

$$M_{v_T}(s) = \exp\left[\frac{s^2 \sigma_T^2}{2}\right] \quad (2.14)$$

II.3.2 Dark current noise

When the detector receives no optical power, some leakage can still occur at the output of the device terminals. This leakage is referred to as the dark current, which clearly contrib-

utes to the total noise and gives random fluctuations about the average particle flow of the photocurrent. It therefore manifests itself as shot noise [5].

II .3 .3 Shot and Avalanche noise

Optical power in the form of photons arrives at the receiver at random time intervals, in a pattern which follows a Poisson distribution. Primary carriers are similarly generated in an avalanche photodetector (APD) receiver, but each primary carrier is then randomly multiplied.

The mean instantaneous rate of primary electrons which is produced by the received optical power and by additional thermal effects is given by $\lambda(t)$ [2][3][7]:

$$\lambda(t) = \frac{\eta}{h\Omega} P(t) + \lambda_o \quad (2 . 15)$$

where η is photon counter quantum efficiency

$h\Omega$ is energy of photon

$\lambda_o > 0$ is dark current rate

Using (2 . 2) , we obtain:

$$\lambda(t) = \frac{\eta}{h\Omega} \sum_{k=-\infty}^{k=+\infty} a_k h_p(t - kT) + \lambda_o \quad (2 . 16)$$

The total number of carriers $E(t)$ generated in one bit time t is [8]

$$E(t) = \sum_{n=1}^{N(t)} g(n) \quad (2 . 17)$$

where $g(n)$ is the number of secondary carriers produced for the n^{th} primary

$N(t)$ is the number of primary carriers generated during $[0, t >$

The filtered randomly multiplied Poisson process is expressed as follows [3]:

$$x_s(t) = \sum_{n=1}^{N(t)} g(n)qh_s(t-t_n) \quad (2.18)$$

where q is the charge of electron

$h_s(t)$ is the impulse response of the detector

t_n is the emission time

The number of primary electrons $N(t)$, the random time points t_n and the positive random number $g(n)$ given in (2.17) and (2.18) are statistically independent random variables [3].

For time varying $P(t)$ the probability density of E is unknown; only its moment generating function $M_E(s)$ is known. Distributions of this type are known as compound distributions.

If $M_N(s)$ and $M_g(s)$ are the moment generating functions of N and g , then we have [8]:

$$M_E(s) = M_N[M_g(s)] \quad (2.19)$$

Since N is Poisson distributed, therefore [24]

$$M_E(s) = \exp[\lambda(t) (M_g(s) - 1)] \quad (2.20)$$

The moment generating function $M_g(s)$ of the detector's avalanche gain g is the solution

of the following equation [9]:

$$s = T(M_g) = \log[M_g] - b \log[1 + a(M_g - 1)] \quad (2.21)$$

where $b = \frac{1}{(1-k)}$

$$a = (1-k) \left(1 - \frac{1}{G} \right)$$

$G = E[g]$ is the average gain of the diode

k is collision-ionization probability of holes to that of electrons within the diode

As in [9], it is possible to use the following inverse transformation

$$M_g = T^{-1}(s) \quad (2.22)$$

in order to obtain a unique solution for (2.22), the complex s plane is cut along the $\text{Re } s$ axis in

$$s^* < \text{Re } s < +\infty \quad (\text{see Fig 2.3}) \quad (2.23)$$

where $s^* = T(M_g^*) \quad (2.24)$

$$M_g^* = \frac{(1-a)}{[a(b-1)]} > 1 \quad (2.25)$$

The signal $v_s(t)$ at the decision circuit is found by deterministically multiplying the detector output current at time τ by the linear channel impulse response and integrating over

Fig 2. 3 's' versus moment generating function of the avalanche gain

all τ , so the corresponding moment generating function is [3]:

$$M_{v_s}(s) = \exp \left[\int_{-\infty}^{+\infty} \lambda(\tau) \left[M_g(sq h_R(t_0 - \tau)) - 1 \right] d\tau \right] \quad (2.26)$$

where $M_g(sq h_R(t_0 - \tau))$ is the solution of the following equation:

$$sq h_R(t_0 - \tau) = \log \left[M_g(sq h_R(t_0 - \tau)) \right] - b \log \left[1 + a \left(M_g(sq h_R(t_0 - \tau)) - 1 \right) \right] \quad (2.27)$$

(2.27) is equivalent to (2.21) where s is replaced by $sq h_R(t_0 - \tau)$.

To include the thermal noise, we use (2.5) which defines the signal $v(t)$ as the sum of two independent random variables $v_s(t)$ and $v_T(t)$. Hence,

$$E \left[\exp(s(v_s + v_T)) \right] = E \left[\exp(s v_s) \right] E \left[\exp(s v_T) \right] \quad \text{or}$$

$$M_v(s) = M_{v_s}(s) M_{v_T}(s) \quad (2.28)$$

Using (2.14) and (2.26), we get

$$M_v(s) = \exp \left[\int_{-\infty}^{+\infty} \lambda(\tau) \left[M_g(sq h_R(t_0 - \tau)) - 1 \right] d\tau + \frac{1}{2} \sigma_T^2 s^2 \right] \quad (2.29)$$

II .3 .4 Intersymbol interference (ISI)

The intersymbol interference (ISI) is due to the dispersion of the transmitted signal on the optical fiber link. In a digital system, this dispersion mechanism causes distortion and a broadening of the transmitted light pulses as they travel along the channel. Fig 2. 4 shows this broadening phenomena where each pulse overlaps with its neighbors and becomes indistinguishable at the receiver [5].

As the ISI becomes more pronounced, an increasing number of errors may be encountered which reduces the sensitivity at the optical receiver. This ISI may be reduced by equalization within the receiver which forces the transmitted binary one to pass through zero at all neighboring decision times [5].

II .4 REVIEW OF ANALYSIS METHODS

II .4 .1 Derivation of error probability

The output electrical signal $v(t)$ of the receiver is sampled every T second. The sampled values at the decision time t_o are compared to the threshold level V_d which in turn permits the original input data to be reconstructed.

For the case of the binary signal, the decision rule is:

1 is obtained if $v(t_o) > V_d$

0 is obtained if $v(t_o) < V_d$

The probability of an erroneous decision is given by the sum of two probabilities:

1. the probability of regenerating 1 when 0 is sent:

$$P(1|0) = P(v(t_o) > V_d \mid a_0 = 0) = \int_{V_d}^{+\infty} f_0(v) dv \quad (2 . 30)$$

2. the probability of regenerating 0 when 1 is sent:

$$P(0|1) = P(v(t_o) < V_d \mid a_0 = 1) = \int_{-\infty}^{V_d} f_1(v) dv \quad (2 . 31)$$

where f_0 and f_1 are the probability density functions of the random variable (RV) $v(t_o)$

when 0 and 1 are sent, respectively

The average error probability is defined by:

$$P_e = p_1 P(0|1) + p_0 P(1|0) \quad (2.32)$$

Let f_{v_r} and f_{v_T} represent the PDF of the RV's v_r and v_T , respectively.

P_e can be written as follows:

$$P_e = p_0 \int_{R_0} f_{v_r}(x|a_0=0) \left\{ \int_{V_d-x}^{+\infty} f_{v_T}(\xi) d\xi \right\} dx + p_1 \int_{R_1} f_{v_r}(x|a_0=1) \left\{ \int_{-\infty}^{V_d-x} f_{v_T}(\xi) d\xi \right\} dx \quad (2.33)$$

where R_0 and R_1 represent the range of definition of x given that $a_0 = 0$ and $a_0 = 1$, respectively

Using the assumption that $v_T(t)$ is a random variable following a Gaussian distribution with a standard deviation σ_T , (2.33) can be written as follows:

$$P_e = \frac{p_0}{2} \int_{R_0} f_{v_r}(x|a_0=0) \left\{ \operatorname{erfc} \left(\frac{V_d-x}{\sqrt{2}\sigma_T} \right) \right\} dx + \frac{p_1}{2} \int_{R_1} f_{v_r}(x|a_0=1) \left\{ \operatorname{erfc} \left(\frac{-V_d+x}{\sqrt{2}\sigma_T} \right) \right\} dx \quad (2.34)$$

The discussion in the succeeding sections will center around various techniques used to evaluate the bit error rate as given by (2.33), and the last section will compare these techniques, demonstrating the advantages and disadvantages of each method.

II.4.2 Gaussian Approximation

In the Gaussian approximation, the total noise, the sum of shot and thermal noise, is

assumed to have a Gaussian distribution with mean square value σ_N^2 given by:

$$\sigma_N^2 = \sigma_s^2 + \sigma_T^2 \quad (2.35)$$

where σ_s and σ_T are the standard deviations (root mean square) of the shot and the thermal noise respectively

When the noise is assumed to have Gaussian distribution with zero mean value, the probability density functions will also be Gaussian and is defined by:

$$p(x) = \frac{1}{\sigma_N \sqrt{2\pi}} \exp \left[-\frac{x^2}{2\sigma_N^2} \right] \quad (2.36)$$

We should mention that σ_s assumes two values, one for the ON pulse, σ_{s1} , and the other for the OFF pulse, σ_{s0} . Therefore, we have two values for σ_N , σ_{N1} and σ_{N0} , and we can write:

$$\sigma_{Ni}^2 = \sigma_{si}^2 + \sigma_T^2 \quad i = 0 \text{ (1) when } 0 \text{ (1) is sent} \quad (2.37)$$

Let V_{M1} and V_{M0} represent the peak of the signal at the input of the decision circuit when 1 and 0 are sent, respectively, and V_d represents the threshold level at the decision circuit.

The average error probability is defined by:

$$\begin{aligned} e &= p_1 P[V_{M1} + \sigma_{N1} < V_d] + p_0 P[V_{M0} + \sigma_{N0} > V_d] \\ &= p_1 P[\sigma_{N1} < V_d - V_{M1}] + p_0 P[\sigma_{N0} > V_d - V_{M0}] \end{aligned} \quad (2.38)$$

Using (2.36), (2.38) can be written as:

$$Pe = \frac{1}{2} \left\{ p_1 \operatorname{erfc} \left[\frac{V_{M1} - V_d}{\sqrt{2} \sigma_{N1}} \right] + p_0 \operatorname{erfc} \left[\frac{V_d - V_{M0}}{\sqrt{2} \sigma_{N0}} \right] \right\} \quad (2.39)$$

where $\text{erfc}(x)$ is the complementary error function defined by:

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-z^2} dz \quad (2.40)$$

σ_{Ni} ($i = 1, 0$) is calculated using (2.37).

The mean square thermal noise voltage at the input of the decision circuit is given by (2.11):

$$\sigma_T^2 = \int_0^{+\infty} N_o(f) |H_R(f)|^2 df \quad (2.41)$$

and the mean square shot noise voltage, assumed to be Gaussian, is given by:

$$\sigma_{si}^2 = \sigma_{vi}^2 - V_{Mi}^2 \quad i = 0 (1) \text{ when } 0 (1) \text{ is sent} \quad (2.42)$$

To simplify the mathematical notations, the subscript "i" will be dropped from the following equations.

In (2.42), σ_{vi}^2 is given by:

$$\sigma_{vi}^2 = \left. \frac{d^2 M_{vi}(s)}{ds^2} \right|_{s=0} \quad (2.43)$$

Using (2.26):

$$M_{vi}(s) = \exp \left[\int_{-\infty}^{+\infty} \lambda(\tau) [M_g(s q h_R(t_o - \tau)) - 1] d\tau \right] \quad (2.44)$$

where $M_{vi}(0) = 1$

the first derivative of $M_{vi}(s)$ with respect to s is:

$$\frac{dM_{vi}}{ds} = M_{vi}(s) \int_{-\infty}^{+\infty} \lambda(\tau) q h_R(t_o - \tau) \frac{dM_g(s q h_R(t_o - \tau))}{ds} d\tau \quad (2.45)$$

$$\text{Let } C(s) = \int_{-\infty}^{+\infty} \lambda(\tau) q h_R(t_o - \tau) \frac{dM_g(s q h_R(t_o - \tau))}{ds} d\tau \quad (2.46)$$

From (2.45) and (2.47):

$$\frac{dM_{v_i}}{ds} = M_{v_i}(s) C(s) \quad (2.48)$$

The second derivative of $M_{v_i}(s)$ with respect to s is:

$$\frac{d^2 M_{v_i}}{ds^2} = M_{v_i}(s) C_1(s) + \frac{dM_{v_i}}{ds} C(s) \quad (2.49)$$

$$\text{where } C_1(s) = \frac{dC(s)}{ds} = \int_{-\infty}^{+\infty} \lambda(\tau) [q h_R(t_o - \tau)]^2 \frac{d^2 M_g(s q h_R(t_o - \tau))}{ds^2} d\tau \quad (2.50)$$

Using (2.43) we obtain:

$$\sigma_{v_i}^2 = \left[[C(s)]^2 + C_1(s) \right] \Big|_{s=0} \quad (2.51)$$

$$\text{where } \left. \frac{dM_g(s)}{ds} \right|_{s=0} = \frac{1}{1-ab} \quad (2.52)$$

$$\left. \frac{d^2 M_g(s)}{ds^2} \right|_{s=0} = \frac{1-a^2b}{(1-ab)^3} \quad (2.53)$$

a and b are defined in (2.21)

To summarize, the error probability is determined by using (2.39) and the values of σ_{N0} and σ_{N1} are calculated using (2.37) where σ_T^2 and $\sigma_{v_i}^2$ are found using (2.41) and (2.42), respectively.

II .4 .3 Series expansion method

This method is described in [4] and [10]. It provides the error probability in terms of the shot noise moments and the Hermite polynomials.

The probability of error given in (2 . 34) is:

$$P_e = p_0 P(1|0) + p_1 P(0|1)$$

$$= \frac{p_0}{2} \int_{R_0} f_{v_i}(x|a_0 = 0) \left\{ \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right) \right\} dx + \frac{p_1}{2} \int_{R_1} f_{v_i}(x|a_0 = 1) \left\{ \operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2} \sigma_T} \right) \right\} dx$$

Let us concentrate on $P(1|0)$ given that the analysis of $P(0|1)$ is similar.

By expanding the term $\operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right)$ into a power series of x about the point $x = 0$, we

obtain:

$$\operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right) = \sum_n A_n x^n \quad (2 . 54)$$

where A_n represents the Taylor's series coefficient and it is given by:

$$A_n = \frac{1}{n!} \left. \frac{\partial^n}{\partial x^n} \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right) \right|_{x=0} \quad (2 . 55)$$

The n^{th} derivative of $\operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right)$ calculated at $x = 0$ is given by [2]:

$$\left. \frac{\partial^n}{\partial x^n} \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right) \right|_{x=0} = \sqrt{\frac{2}{\pi}} \cdot \frac{(-1)^n}{\sigma_T^n} \cdot \exp \left[-\frac{V_d^2}{2\sigma_T^2} \right] \cdot H_{n-1} \left(\frac{V_d}{\sigma_T} \right) \quad (2 . 56)$$

where $H_n(\xi)$ is a Hermite polynomial generated using the recursive relation [2]:

$$H_{n+1}(\xi) = \xi H_n(\xi) - n H_{n-1}(\xi); \quad H_0(\xi) = 1 \text{ and } H_1(\xi) = \xi \quad (2.57)$$

Using (2.54), $P(1|0)$ can be written as follows:

$$P(1|0) = \frac{1}{2} \sum_n \left\{ A_n \int_{R_0} f_{v_s} x^n dx \right\} \quad (2.58)$$

$$= \frac{1}{2} \sum_n \left\{ A_n M_{v_s}^n \right\} \quad (2.59)$$

where $M_{v_s}^n$ is the n^{th} moment of the the RV v_s ,

For the derivation of the moments $M_{v_s}^n$, we refer to [3] or [7].

II.4.4 Chebyshev approximation

This method is presented in [2]. It is based on deriving a best approximation (in a mini-max sense) for the cumulative distribution function of the thermal noise and uses a sufficient number of higher order moments of the shot noise statistics [2].

II.4.4.1 Description of the algorithm

If $f(x) \in C^n[-a, a]$ then

$$\int_{-a}^{+a} f(x) g(x) dx \cong \tilde{F}^T B^{(m)} A M^{(m)} \quad (2.60)$$

where $\sim$ denotes transpose

$$\tilde{F} = \left[f(0) \quad \frac{f'(0)a}{1!} \quad \dots \quad \frac{f^{(n-1)}(0)a^{n-1}}{(n-1)!} \right]$$

$$\bar{M}^{(m)} = \begin{bmatrix} M_0 & \frac{M_1}{a} & \dots & \frac{M_{m-1}}{a^{m-1}} \end{bmatrix}$$

where M_n is the n^{th} moment of $g(x)$: $M_n = \int_{-a}^a x^n g(x) dx$

$A = \{a_{ij}\}$ is $m \times m$ constant matrix

where $a_{ij} = 0$; $j > i$

$$a_{i,1} = -a_{i-2,1} ; \quad 3 \leq i \leq m$$

$$a_{ij} = 2a_{i-1,j-1} - a_{i-2,j} ; \quad 3 \leq i \leq m \text{ and } 2 \leq j \leq i$$

$$a_{1,1} = 1 ; \quad a_{2,1} = 0 ; \quad a_{2,2} = 1$$

$B^{(m)} = \{b_{ij}^{(m)}\}$ is $n \times m$ constant matrix which is a sub matrix of the $n \times n$ matrix

$$B = \{b_{ij}\}$$

where $b_{ij}^{(m)} = b_{ij} ; \quad 1 \leq i \leq n \text{ and } 1 \leq j \leq m$

$$b_{ij} = 0 ; \quad j > i$$

$$b_{i,1} = \frac{1}{2} b_{i-1,2} ; \quad 2 \leq i \leq n$$

$$b_{i,2} = b_{i-1,1} + \frac{1}{2} b_{i-1,3} ; \quad 2 \leq i \leq n$$

$$b_{i,j} = \frac{1}{2} b_{i-1,j-1} + \frac{1}{2} b_{i-1,j+1} ; \quad 2 \leq i \leq n \text{ and } 2 \leq j \leq n$$

$$b_{1,1} = 1$$

II .4 .4 .2 Application to the evaluation of error probability

The probability of error given in (2 . 34) is:

$$P_e = \frac{p_0}{2} \int_{R_0} f_{v_s}(x|a_0 = 0) \left\{ \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right) \right\} dx + \frac{p_1}{2} \int_{R_1} f_{v_s}(x|a_0 = 1) \left\{ \operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2} \sigma_T} \right) \right\} dx$$

Using (2 . 60), (2 . 34) can be computed as follows [2]:

$$P_e \cong \frac{p_0}{2} \bar{F}_0 B A \Gamma_0 + \frac{p_1}{2} \bar{F}_1 B A \Gamma_1$$

where F_0 and F_1 contain the scaled derivatives of $\operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2} \sigma_T} \right)$ and $\operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2} \sigma_T} \right)$,

respectively (see (2 . 56))

Γ_0 and Γ_1 contain the scaled moments of the RV v_s for $a_0 = 0$ and $a_0 = 1$, respectively

II .4 .5 Gauss Quadrature rule (GQR)

This method is described in [3], [11] and [12].

II .4 .5 .1 Gauss Quadrature Rules Approach

In this approach, we are concerned with an integral of the form [12]:

$$\int_a^b g(x) w(x) dx \quad (2 . 61)$$

where $w(x)$ is a "weight" function

$g(x)$ is an arbitrary function of some wide class

$[a, b]$ is any finite or infinite interval

The integration in (2 . 61) can be approximated using a linear combination of values of the function $g(x)$:

$$\int_a^b g(x) w(x) dx \approx \sum_{i=1}^m w_i g(x_i) \quad (2 . 62)$$

Let $w(x)$ satisfy the two following conditions [12]:

1. $w(x)$ is nonnegative, integrable in $[a, b]$, with $\int_a^b w(x) dx > 0$
2. the products $x^k w(x)$, for any nonnegative integer k , are such that the integrals

$$\int_a^b |x|^k w(x) dx \text{ are definite and finite.}$$

For $w(x)$, it is possible to define a sequence of polynomials $p_0(x)$, $p_1(x)$, ... which are orthonormal with respect to $w(x)$ and in which $p_n(x)$ is of exact degree n so that

$$\begin{aligned} \int_a^b w(x) p_m(x) p_n(x) ds &= 1 \quad \text{when } m = n \\ &= 0 \quad \text{when } m \neq n \end{aligned}$$

The polynomial $p_n = k_n \prod_{i=1}^n (x - x_i)$; $k_n > 0$

has n real roots x_i located in the interval $[a, b]$.

The roots of the orthogonal polynomials play an important role in Gaussian quadrature.

Theorem[11]: Let $g(x) \in C^{2m}[a, b]$, then

$$\int_a^b g(x) w(x) dx = \sum_{i=1}^m w_i g(x_i) + \frac{g^{(2m)}(\xi)}{k_m^2 (2m)!}, \quad (a \leq \xi \leq b)$$

$$\text{where } w_i = -\frac{k_{m+1}}{k_m} \cdot \frac{1}{p_{m+1}(x_i) p'_m(x_i)}, \quad p'_m(x_i) = \left. \frac{dp_m(x)}{dx} \right|_{x=x_i}, \quad i = 1, \dots, m$$

The set of orthogonal polynomials $\{p_j(x)\}_{j=1}^N$ satisfies a three term recurrence relationship:

$$x p_{n-1}(x) = \beta_{n-1} p_{n-2}(x) + \alpha_n p_{n-1}(x) + \beta_n p_n(x); \quad n = 1, 2, \dots, m$$

where $p_{-1}(x) = 0$ and $p_0(x) = 1$

The algorithm consists of two steps [12]:

1. evaluation of the coefficients $\{\alpha_n\}$ and $\{\beta_n\}$ of the three term recurrence relationship by means of the first $2m + 1$ moments of $w(x)$.
2. generation of a symmetric tri-diagonal matrix, whose elements depend on the coefficients $\{\alpha_n\}$ and $\{\beta_n\}$; the weights w_i are found as the first components of the eigenvectors of this matrix whereas the abscissas x_i are the corresponding eigenvalues.

II .4 .5 .2 Application to the evaluation of the error probability

The probability of error given in (2 . 34) is:

$$P_e = \frac{p_0}{2} \int_{R_0} f_{v_s}(x|a_0 = 0) \left\{ \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2}\sigma_T} \right) \right\} dx + \frac{p_1}{2} \int_{R_1} f_{v_s}(x|a_0 = 1) \left\{ \operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2}\sigma_T} \right) \right\} dx$$

The previous description of the Gauss quadrature rule can be applied by using a linear combination of weighted values of the functions $\operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2}\sigma_T} \right)$ and $\operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2}\sigma_T} \right)$:

$$\int_{R_0} f_{v_s}(x|a_0 = 0) \left\{ \operatorname{erfc} \left(\frac{V_d - x}{\sqrt{2}\sigma_T} \right) \right\} dx \cong \sum_{i=1}^m w_i \operatorname{erfc} \left(\frac{V_d - x_i}{\sqrt{2}\sigma_T} \right)$$

$$\int_{R_1} f_{v_s}(x|a_0 = 1) \left\{ \operatorname{erfc} \left(\frac{-V_d + x}{\sqrt{2}\sigma_T} \right) \right\} dx \cong \sum_{i=1}^m w_i \operatorname{erfc} \left(\frac{-V_d + x_i}{\sqrt{2}\sigma_T} \right)$$

II .4 .6 Chernoff Bound method

This method is presented in [13], [14] and [8]. It consists in bounding the probability that the signal v defined in (2 . 5) will lie above or below a threshold level V_d . It is given as follows [14]:

$$P(1|0) = P(v(t_0) > V_d | a_0 = 0) \leq e^{\Psi(t_{ms}) - s\Psi(t_{ms})} \Big|_{\Psi(t_{ms})=V_d} ; \quad s > 0 \quad (2 . 63)$$

$$P(0|1) = P(v(t_0) < V_d | a_0 = 1) \leq e^{\Psi(t_{ms}) - s\Psi(t_{ms})} \Big|_{\Psi(t_{ms})=V_d} ; \quad s < 0 \quad (2 . 64)$$

where $\Psi(t_o, s) = \ln[M_v(s)]$

$$= \int_{-\infty}^{+\infty} \lambda(\tau) [M_g(sqh_R(t_o - \tau)) - 1] d\tau + \frac{1}{2} \sigma^2 s^2$$

($M_v(s)$ is defined in (2 . 29))

$$\Psi'(t_o, s) = \frac{\partial}{\partial s} \Psi(t_o, s)$$

In (2 . 63) and (2 . 64), the equation

$$\Psi'(t_o, s) = V_d \quad (2 . 65)$$

will be solved for s using Newton iteration. This will be described in detail in the succeeding saddlepoint approximation section.

II . 4 . 7 Saddlepoint Approximation

This method is described in [9], [15], [16] and [6]. In these publications, it was assumed that the signal presents no ISI. In addition, the effect of the receiver response was ignored. The saddlepoint approximation is based on the moment generating function of the signal, which is often found to have a much simpler expression than the probability density function.

The approach taken is a special case of the saddlepoint method for approximating integrals of the exponential form [6]:

$$\int_C g(t) \exp[n f(t)] dt \quad (2 . 66)$$

where C is an appropriate contour

n is a parameter that in some sense is "large"

$f(t)$ and $g(t)$ are rational functions of t or functions of like simplicity

The error probability in (2 . 32) is given by:

$$P_e = p_0 P(1|0) + p_1 P(0|1)$$

$$\text{where } P(0|1) = \int_{-\infty}^{v_d} f_1(v) dv \quad (2 . 67)$$

$$P(1|0) = \int_{v_d}^{+\infty} f_0(v) dv \quad (2 . 68)$$

The random variable v can be described most conveniently in terms of the moment generating functions (mgf's) of the signal and noise [13].

$$M_v(s) = E[e^{vs}]$$

$$= \int_{-\infty}^{+\infty} e^{vs} f_j(v) dv ; \quad j = 0, 1 \quad (2 . 69)$$

It follows also that the inverse, given by the Laplace inversion formula, also indicates the probability density function:

$$f_j = L^{-1}[M_v(s)] \quad (2 . 70)$$

$$f_j(v) = \int_{c-i\infty}^{c+i\infty} M_v(s) e^{-vs} \frac{ds}{2\pi i} \quad (s = c + ix) \quad (2 . 71)$$

in which the contour of integration is along a straight line parallel to the $\text{Im}(s)$ axis $(-\infty < \text{Im}(s) < +\infty)$ in the cut plane, defined in (2.23), and lies to the right of all singularities of the integrand [6].

The PDF in (2.67) can be replaced by the inverse Laplace transform of the mgf, resulting in:

$$P(0|1) = \int_{-\infty}^{V_d} \left[\int_{c-i\infty}^{c+i\infty} M_v(s) e^{-vs} \frac{ds}{2\pi i} \right] dv \quad (2.72)$$

By exchanging the variables of integration, we obtain:

$$P(0|1) = \int_{c-i\infty}^{c+i\infty} M_v(s) \left[\int_{-\infty}^{V_d} e^{-vs} dv \right] \frac{ds}{2\pi i} \quad (2.73)$$

We can designate the integration by integrating along the contour to the right of all singularities of the mgf, $M_v(s)$, and by keeping to the left of the origin ($c = \text{Re}(s) < 0$) we obtain:

$$P(0|1) = - \int_{c-i\infty}^{c+i\infty} M_v(s) e^{-sV_d} \frac{1}{s} \frac{ds}{2\pi i} ; \quad c < 0 \quad (2.74)$$

In a similar way:

$$P(1|0) = \int_{V_d}^{+\infty} \left[\int_{c-i\infty}^{c+i\infty} M_v(s) e^{-vs} \frac{ds}{2\pi i} \right] dv \quad (2.75)$$

by exchanging the variables of integration, we obtain:

$$P(1|0) = \int_{c-i\infty}^{c+i\infty} M_v(s) \left\{ \int_{V_d}^{+\infty} e^{-vs} dv \right\} \frac{ds}{2\pi i} \quad (2.76)$$

In bringing the contour to the right of the origin ($0 < c = \text{Re}(s) < s^*$), the integration can be described as:

$$P(1|0) = \int_{c-i\infty}^{c+i\infty} M_v(s) e^{-sV_d} \frac{1}{s} \frac{ds}{2\pi i} ; \quad 0 < c < s^* \quad (2.77)$$

We will focus on $P(1|0)$ since the analysis of $P(0|1)$ is similar.

To prove that the integrand $M_v(s) e^{-sV_d} \frac{1}{s}$ is convex U, we first see that the function

$s(v - V_d) + \log\left(\frac{1}{s}\right)$ where $0 < s < +\infty$ is clearly convex for all v and V_d . Since the

exponential function of a convex function is convex, and the expected value of the convex function is also convex, then the value of the integrand will be convex:

$$E \left[e^{s(v - V_d) + \log\left(\frac{1}{s}\right)} \right] = E \left[e^{(sv)} e^{-sV_d} \frac{1}{s} \right] = M_v(s) e^{-sV_d} \frac{1}{s} \quad (2.78)$$

The integrand in (2.77), being convex over $0 < s < +\infty$, possesses a single minimum at this point, whose location is denoted by s_0 . The saddlepoint method essentially consists of placing the integration contour in (2.77) in order that it passes through s_0 , $c = s_0$. Only the real part of the integrand of (2.77) contributes to $P(1|0)$ (the imaginary part is constant), and because it has a minimum in the real s direction at s_0 , it must have a maximum along

the vertical contour $s = s_0 + i x$ through s_0 , $-\infty < x < +\infty$. The point $s = s_0$ is therefore called a saddlepoint of the integrand.

We write the integrand as $e^{\Psi(s)}$:

$$M_v(s) e^{-sV_d} \frac{1}{s} = e^{\Psi(s)} \quad \text{or}$$

$$\Psi(s) = \log[M_v(s)] - sV_d + \log\left(\frac{1}{s}\right) \quad (2.79)$$

$$(\text{In the case of } P(011), \Psi(s) \text{ is given by } \Psi(s) = \log[M_v(s)] - sV_d + \log\left(\frac{1}{s}\right)) \quad (2.80)$$

Taylor's series expansion of $\Psi(s)$ around the point $s = s_0$ gives:

$$\Psi(s) = \Psi(s_0) + \frac{1}{2} \Psi''(s_0)(s-s_0)^2 + \frac{1}{6} \Psi'''(s_0)(s-s_0)^3 + \dots + (k!)^{-1} \Psi^{(k)}(s_0)(s-s_0)^k + \dots \quad (2.81)$$

$$\text{where } \Psi^{(k)}(s_0) = \left. \frac{d^k \Psi(s)}{ds^k} \right|_{s=s_0} \quad (2.82)$$

Since s_0 is an extremum of $\Psi(s)$, the first derivative disappears from the series.

By expanding the exponential function in Taylor's series, we obtain:

$$\exp \left[\sum_{k=3}^{\infty} (k!)^{-1} \Psi^{(k)}(s_0) (s-s_0)^k \right] = 1 + \frac{1}{6} \Psi'''(s_0)(s-s_0)^3 + \dots + b'_k (s-s_0)^k + \dots \quad (2.83)$$

With $s = s_0 + ix$ on the contour, the integral in (2.77) becomes

$$P(1|0) = \exp[\Psi(s_o)] *$$

$$\int_{-\infty}^{+\infty} \exp\left[-\frac{1}{2}\Psi''(s_o)x^2\right] \left[1 + \frac{1}{6}\Psi'''(s_o)(ix)^3 + \frac{1}{24}\Psi^{(4)}(s_o)x^4 + \dots + b'_k(ix)^k + \dots\right] \frac{dx}{2\pi}$$

$$P(1|0) = \frac{1}{\sqrt{2\pi \Psi''(s_o)}} \exp[\Psi(s_o)] (1 + R) \quad (2.84)$$

$$\text{where } R = \frac{\Psi^{(4)}(s_o)}{8[\Psi''(s_o)]^2} + \dots + (-1)^k \left[\frac{(2k)!}{2^k k!} \right] \frac{b'_{2k}}{[\Psi''(s_o)]^k} + \dots \quad (2.85)$$

The terms of odd order vanish.

In the zero order approximation to the right-hand tail probability $P(1|0)$, the residuum R is neglected and we obtain [9][13][6]:

$$P(1|0) \approx \frac{1}{\sqrt{[2\pi \Psi''(s_o)]}} \exp[\Psi(s_o)] ; \quad s_o > 0 \quad (2.86)$$

The left-hand tail distribution is similarly given by

$$P(0|1) = \frac{1}{\sqrt{2\pi \Psi''(s_o)}} \exp[\Psi(s_o)] (1 + R) \quad (2.87)$$

where R has the same form as in (2.84), but now s_o is the saddlepoint of the integrand of (2.74) lying on the real s axis immediately to the left of the origin ($s_o < 0$).

II.5 COMPUTATION OF THE ERROR PROBABILITY IN THE PRESENCE OF INTERSYMBOL INTERFERENCE

In the previously described methods, it was assumed that there is no ISI in the signal. However, in a practical system, the ISI has to be taken into account.

As mentioned previously in section II.3.4, the presence of ISI increases the number of errors in detecting the signal. Different approaches can be used for the computation of the error probability in the presence of ISI. These approaches are listed below:

1. Using the Exhaustive technique which consists essentially in computing the conditional error probability for each of the possible sequences of data and in averaging over all the sequences.
2. Averaging the MGF of the shot noise over all possible sequences (see [2] for the derivation of the average of the MGF).

More details about computing the error probability in the presence of ISI using the above stated approaches will be presented in chapter IV.

II.6 COMPARISON BETWEEN DIFFERENT ANALYSIS METHODS AND CONCLUSIONS

We have described in the preceding section some of the well-known analytic methods for the evaluation of fiber optic performance. The principle methods which have been chosen are the Exhaustive techniques, the Gaussian approximation, the series expansion

method, the Gauss quadrature rules (GQR), the Chebyshev approximation, Chernoff bound and the Saddlepoint approximation. These are the most common methods used in evaluating the BER, and we will next compare the effectiveness of each.

Firstly, for the Exhaustive technique, the computational cost is usually quite high and this limits the number of interfering samples which can be accounted for. The computation involved grows exponentially with the number N of neighbor pulses. It is nevertheless useful in order to verify the validity of the other methods [2][4][12].

For the Gaussian approximation, the shot noise is modelled as additive Gaussian process with its variance equal to the second moment of the Poisson process. However, it has been reported that the Gaussian approximation can lead to underestimates of the optimal avalanche photodetector gain, and overestimates of received error probability [2].

The series expansion method is a simple one. The amount of computation involved only increases linearly with N [2][4][12]. However, its convergence is slow and it provides oscillating results when channel distortion, the signal-to-noise ratio, or the number of levels increases.

The GQR is a more accurate method than the series expansion method. The generation of the integration rule is, however, difficult and it is sensitive to round off errors. The numerical procedure becomes increasingly ill conditioned as the number of moments of the random variable representing the ISI or the shot noise is increased. Analyses based on the GQR approximation are particularly useful for linear lightwave systems where thermal noise and ISI are the dominant degradations [2][8].

Another approach is to bound the error probability. However, the derived bounds are

often too loose, which reduces their usefulness [2][4][12].

Finally, the Chebyshev approximation derived in [2] has been reported to give very accurate results. However, the computational effort is relatively high.

The saddlepoint approximation seems to be a very attractive method since its analytic form is very simple and it does not require the derivation of higher moments. It is mainly based on calculating a saddlepoint using simple numerical routines like Newton iterations and then replacing the saddlepoint's value in a simple formula. However, the saddlepoint approximation method, as reported in previous publications, suffers from two main disadvantages which make it useless for most practical applications:

1. The detector shot noise was modeled as Poisson process. In reality, it is filtered Poisson process. In other words, the effect of the receiver impulse response has to be taken into account. This, as will be shown in the next chapters, changes some of the fundamental steps in the computational algorithm.
2. The effect of intersymbol interference (ISI) is neglected. Again, for practical systems, this has to be taken into account.

In the following two chapters, we will expand and generalize the saddlepoint approximation by taking into account the impulse response of the receiver and the ISI. In chapter V, an algorithm is described for the computation of the sensitivity of the error probability based on the saddlepoint approximation.

Chapter III

GENERALIZATION OF THE SADDLEPOINT APPROXIMATION TO INCLUDE THE EFFECT OF THE RECEIVER RESPONSE

III.1 INTRODUCTION

The saddlepoint approximation method has been described for computing the probability distribution of the sum of the Gaussian noise and the avalanche photodetector output using the Personick–Mc Intyre model of the random electron multiplication in the diode [9][15].

In their publications (see [9], [15], [13] and [6]), C.W.Helstrom, X.Schumacher and J.J.O'Reilly have presented this method and have applied it to evaluate the error probability in optical fiber communication systems. The saddlepoint approximation was used to evaluate the error in counting the number of electrons generated by the avalanche photodetector where the detector shot noise was modeled as Poisson process. However, in a practical fiber communication system, the output of the avalanche photodetector is filtered by the receiver circuit. Computation of the bit error rate that involves the impulse response of the receiver was not taken into consideration in the previously mentioned publications.

In this chapter we will expand the saddlepoint approximation to include the effects of the receiver impulse response. This leads to fundamental changes in the steps of the computational algorithm. It should be mentioned that the intersymbol interference, not considered in the previous publications dealing with the saddlepoint approximation, is also part of a practical communication system. However to simplify the mathematical analysis, we will first ignore the effect of the ISI (i.e, in this chapter we consider transmission of a single isolated pulse) and the discussion of its effect will be presented in the next chapter.

The saddlepoint approximation method appears to be particularly attractive owing to its simple analytic form and the requirement of much less computation effort than the other methods presented in chapter II. It is based on calculating a saddlepoint using basic numerical routines such as Newton iteration, and once the saddlepoint's value has been determined, the approximation formula can be simply evaluated.

The saddlepoint approximation method is described in section III .2 . Its application to the evaluation of the BER is given in sections III .3 . Several examples are presented in section III .6 . Finally, section III .7 presents a summary and conclusion.

III .2 SADDLEPOINT APPROXIMATION ALGORITHM

Using the saddlepoint approximation discussed in section II .4 .7 , the probability of error is given by:

$$P_e = p_0 P(1|0) + p_1 P(0|1) \quad (3 . 1)$$

where for the zero order approximation the residuum R given in (2 . 85) is neglected and we have:

$$P(1|0) \approx \frac{1}{\sqrt{2\pi} \Psi''(s_o^+)} \exp[\Psi(s_o^+)] ; \quad s_o^+ > 0 \quad (3 . 2)$$

$$P(0|1) \approx \frac{1}{\sqrt{2\pi} \Psi''(s_o^-)} \exp[\Psi(s_o^-)] ; \quad s_o^- < 0 \quad (3 . 3)$$

s_o^+ and s_o^- are the saddlepoints where the function $\Psi(s)$ defined in (2 . 79) or (2 . 80) is minimum.

$\Psi(s)$ can be expressed as follows:

$$\Psi(s) = \log[M_v(s)] - sV_d - \log(\pm s) ; \quad \begin{cases} \pm s = -s & \text{when 1 is sent} \\ \pm s = s & \text{when 0 is sent} \end{cases} \quad (3 . 4)$$

The saddlepoint s_o^+ is determined by solving the equation

$$\Psi'(s) = \frac{d\Psi(s)}{ds} = 0 \quad (3 . 5)$$

Because $\Psi(s)$ possesses a single minimum in $0 < s < s^*$ ($\Psi(s)$ is a convex U function of s as was shown previously), (3 . 5) is efficiently solved by Newton's method. Starting with the initial value $s_1^+ > 0$, a new value s_2^+ is determined by

$$s_2^+ = s_1^+ - \frac{\Psi'(s_1^+)}{\Psi''(s_1^+)} \quad (3 . 6)$$

and the process is repeated until the change in s is less than a fraction ϵ . The latest value of $\Psi''(s)$ is immediately available for substitution into (3 . 2).

s_0^- is also found by solving (3.5) iteratively with the initial value selected to the left of the origin ($s_1^- < 0$).

III.3 EVALUATION OF THE BIT ERROR RATE

In this section, we will apply the saddlepoint approximation to evaluate the bit error rate in optical fiber communication systems.

In the "conventional" saddlepoint method presented in the previously mentioned publications (see [9], [13] and [16]), the effect of the receiver impulse response was not taken into consideration.

Firstly, we will present this "conventional" method, then we will introduce the receiver impulse response and show the difference in the computational steps. This difference is basically due to the fact that the detector shot noise was modeled as Poisson process, where in a practical system, it is a filtered Poisson process due to the presence of the receiver circuit. Consequently, this difference appears in the expression of the exponent $\Psi(s)$ given in (3.4).

In the "conventional" saddlepoint approximation, the exponent $\Psi(s)$ is expressed as follows [9]:

$$\Psi(s) = \log[M_E(s)] + \frac{1}{2}\sigma_T^2 s^2 - sV - \log(\pm s) ; \quad \begin{cases} \pm s = -s & \text{when 1 is sent} \\ \pm s = s & \text{when 0 is sent} \end{cases} \quad (3.7)$$

where $M_E(s)$, given in (2 . 20), is the MGF of the RV $E(t)$

$E(t)$, given in (2 . 17), is the total number of carriers generated at the output of the photodetector in one bit time t

V is the decision level

In (2 . 20), $M_E(s)$ is expressed as follows:

$$M_E(s) = \exp[\lambda(t) (M_g(s) - 1)] \quad (3 . 8)$$

where $M_g(s)$ is the solution of

$$s = T(M_g) = \log[M_g] - b \log[1 + a (M_g - 1)] \quad (3 . 9)$$

(see (2 . 21) for the definition of all the parameters in (3 . 9))

As we mentioned previously, the ISI is ignored in this chapter, which gives the following expression of λ :

$$\lambda(t) = \frac{\eta}{h\Omega} a_0 h_p(t) \quad (3 . 10)$$

Replacing $M_E(s)$ by its expression given in (3 . 8), (3 . 7) will be:

$$\Psi(s) = \lambda(t) [M_g(s) - 1] + \frac{1}{2} \sigma_I^2 s^2 - s V - \log(\pm s); \begin{cases} \pm s = -s & \text{when 1 is sent} \\ \pm s = s & \text{when 0 is sent} \end{cases} \quad (3 . 11)$$

The saddlepoints s_o^+ and s_o^- are the roots of

$$\Psi'(s) = \lambda(t) \frac{dM_g}{ds} + \sigma_I^2 s - V - \frac{1}{s} = 0 \quad (3 . 12)$$

(3 . 12) can be expressed as a function of M_g through the transformation (3 . 9) and can be

solved using Newton's iteration for values M_{go}^+ , M_{go}^- of M_g corresponding to s_o^+ , s_o^- through:

$$s_o^\pm = T(M_{go}^\pm) \quad -\infty < s_o^- < 0, \quad 0 < M_{go}^- < 1; \quad 0 < s_o^+ < s^*, \quad 1 < M_{go}^+ < M_g^*$$

(see (2.24) for the definition of s^* and M_g^*)

Starting with an initial value M_g^i , a new value M_g^{i+1} is determined by

$$M_g^{i+1} = M_g^i - \frac{\Psi'(s^i)}{\Psi''(s^i)} \frac{dM_g}{ds} \bigg|_{M_g=M_g^i}, \quad s^i = T(M_g^i) \quad (3.13)$$

The second derivative is given by

$$\Psi''(s) = \lambda(t) \frac{d^2 M_g}{ds^2} + \sigma_T^2 - V + \frac{1}{s^2} \quad (3.14)$$

$$\text{where } \frac{dM_g}{ds} = \frac{M_g l_1}{l_2} \quad (3.15)$$

$$\frac{d^2 M_g}{ds^2} = \frac{dM_g}{ds} \frac{l_1^2 - a^2 b M_g^2}{l_2^2} \quad (3.16)$$

$$l_1 = 1 + a(M_g - 1) \quad (3.17)$$

$$l_2 = l_1 - abM_g \quad (3.18)$$

The process is repeated until the change in M_g is less than a small value ε . Finally, (3.2)

and (3.3) are computed using the values of s_o^+ and s_o^- found through the transformation (3.9).

We recapitulate that the saddlepoints are found by solving (3.12) for M_g using the transfor-

mation T expressed in (3.9) which makes the problem easy to solve. In the following, we will see that this transformation T cannot be performed when the receiver impulse response is taken into consideration.

By introducing the receiver in the system, the signal v at the input of the decision circuit is found as the sum of the filtered randomly multiplied Poisson process and the additive thermal noise assumed to be Gaussian and both are convoluted with the impulse response of the receiver. The MGF of the RV v is given in (2.29) by:

$$M_v(s) = \exp \left[\int_{-\infty}^{+\infty} \lambda(\tau) [M_g(sq h_R(t_0 - \tau)) - 1] d\tau + \frac{1}{2} \sigma_T^2 s^2 \right]$$

Replacing $M_v(s)$ by its expression in (3.4), we obtain:

$$\Psi(s) = \int_{-\infty}^{+\infty} \lambda(\tau) [M_g(sq h_R(t_0 - \tau)) - 1] d\tau + \frac{1}{2} \sigma_T^2 s^2 - sV_d - \log(\pm s) \quad (3.19)$$

$$\begin{cases} \pm s = -s & \text{when 1 is sent} \\ \pm s = s & \text{when 0 is sent} \end{cases}$$

For simplicity, we will use M_g in the following discussion to denote $M_g(sq h_R(t_0 - \tau))$.

M_g is the solution of:

$$qh_R(t_0 - \tau)s = \log[M_g] - b \log[1 + a(M_g - 1)] \quad (3.20)$$

(3.20) is equivalent to (3.9) where s is replaced by $s_t = sq h_R(t_0 - \tau)$.

The first derivative of Ψ with respect to s is:

$$\Psi'(s) = \int_{-\infty}^{+\infty} \lambda(\tau) q h_R(t_o - \tau) \frac{dM_g}{ds_t} d\tau + \sigma_T^2 s - V_d - \frac{1}{s} \quad (3.21)$$

$$\text{where } \frac{dM_g}{ds_t} = \frac{M_g l_1}{l_2} \quad (3.22)$$

$$l_1 = 1 + a(M_g - 1)$$

$$l_2 = l_1 - abM_g$$

The saddlepoints are found by solving the equation

$$\Psi'(s) = 0 \quad (3.23)$$

In the conventional saddlepoint approximation, the above equation is solved for M_g using the transformation T given in (3.9). However, this transformation cannot be applied when the impulse response of the receiver is included since in (3.20) the variable of integration τ appears in $h_R(t_o - \tau)$ in the left term of the equality which makes M_g not only a function of s but also of τ . Consequently, (3.23) has to be solved for s using Newton's iteration.

At iteration number $i + 1$, a new value of s is found by using (3.6)

$$s_{i+1} = s_i - \frac{\Psi'(s_i)}{\Psi''(s_i)}$$

The second derivative is given by:

$$\Psi''(s) = \int_{-\infty}^{+\infty} \lambda(\tau) [q h_R(t_o - \tau)]^2 \frac{d^2 M_g}{ds_t^2} d\tau + \sigma_T^2 + \frac{1}{s^2} \quad (3.24)$$

where
$$\frac{d^2 M_g}{ds_i^2} = \frac{dM_g}{ds_i} \frac{l_1^2 - a^2 b M_g^2}{l_2^2} \quad (3.25)$$

The integrations in (3.21) and (3.24) are computed numerically using Simpson's rule. For each value of τ fixed by the numerical integration subroutine, (3.20) is solved for $M_g(s_i q h_R(t_o - \tau))$ and the value found is then replaced in (3.22) and (3.25).

Newton's iteration method is used to determine $M_g(s_i q h_R(t_o - \tau))$ and this is described in the following.

Using (3.20), define $\xi(M_g)$ as follows:

$$\xi(M_g) = \log[M_g] - b \log[1 + a(M_g - 1)] - q h_R(t_o - \tau) s_i \quad (3.26)$$

Its derivative with respect to M_g is given by

$$\xi'(M_g) = \frac{d\xi}{dM_g} = \frac{1}{M_g} - \frac{ba}{1 + a(M_g - 1)} \quad (3.27)$$

$M_g(s_i q h_R(t_o - \tau))$ is found by solving

$$\xi(M_g) = 0 \quad (3.28)$$

A trial value M_g^i at any stage is replaced by

$$M_g^{i+1} = M_g^i - \frac{\xi(M_g^i)}{\xi'(M_g^i)} \quad (3.29)$$

One stops the iteration when the change in M_g^i is less than a precision ϵ .

The previous steps can be summarized in the following chart (see Fig 3.1).

Fig 3.1 Computational steps of the bit error rate when introducing the receiver impulse response

III.4 OPTIMIZATION CONSIDERATION USING THE SADDLEPOINT APPROXIMATION

The decision level V_d is chosen to minimize the probability of error. It has been found that the optimum threshold level V_{opt} is the solution of [9]

$$p_0 f_0(V) - p_1 f_1(V) = 0 \quad (3.30)$$

where f_0 and f_1 are defined in (2.30) and (2.31), respectively

p_0 and p_1 are the probabilities of occurrence of 0 and 1, respectively

To prove (3.30), the following property is stated first.

$$\text{Property: If } g \text{ is a function of } \lambda \text{ defined as: } g(\lambda) = \int_{\Phi(\lambda)}^{\Psi(\lambda)} f(x) dx \quad (3.31)$$

then its first derivative with respect to λ is given by:

$$\frac{\partial g(\lambda)}{\partial \lambda} = \int_{\Phi(\lambda)}^{\Psi(\lambda)} \frac{\partial f(x)}{\partial \lambda} dx + \frac{\partial \Psi(\lambda)}{\partial \lambda} f(\Psi(\lambda)) - \frac{\partial \Phi(\lambda)}{\partial \lambda} f(\Phi(\lambda)) \quad (3.32)$$

The error probability is given by:

$$P_e = \frac{1}{2} \left[\int_V^{+\infty} f_0(x) dx + \int_{-\infty}^V f_1(x) dx \right] \quad (3.33)$$

Using (3.32), the derivative of P_e with respect to V is

$$\frac{\partial P_e}{\partial V} = p_0 [-f_0(V)] + p_1 [f_1(V)] \quad (3.34)$$

V_{opt} is the solution of $\frac{\partial P_e}{\partial V} = 0$, that leads to the expression given in (3.30)

The same saddlepoint approximation can be applied to (2.71) where f_0 and f_1 are given

$$\text{by } f(v) = \int_{c-i\infty}^{c+i\infty} M_v(s) e^{-vs} \frac{ds}{2\pi i} \quad (3.35)$$

We write the integrand in (3.35) as $e^{\Phi(s)}$:

$$M_v(s) e^{-sv} = e^{\Phi(s)} \quad \text{or}$$

$$\Phi(s) = \log[M_v(s)] - sv \quad (3.36)$$

Replacing $M_v(s)$ given in (2.29), the following expression is obtained:

$$\Phi(s) = \int_{-\infty}^{+\infty} \lambda(\tau) [M_g(sq h_R(t_0 - \tau)) - 1] d\tau + \frac{1}{2} \sigma_T^2 s^2 - sv \quad (3.37)$$

The first derivative of Φ with respect to s is

$$\Phi'(s) = \int_{-\infty}^{+\infty} \lambda(\tau) q h_R(t_0 - \tau) \frac{dM_g}{ds_t} d\tau + \sigma_T^2 s - v \quad (3.38)$$

where $s_t = sq h_R(t_0 - \tau)$

The saddlepoint is the solution of

$$\Phi'(s) = 0 \quad (3.39)$$

At the saddlepoint $\bar{s}$, we have

$$f(v) = \frac{1}{\sqrt{2\pi \Phi''(\bar{s})}} \exp\left[\Phi(\bar{s})\right] \quad (3.40)$$

where the second derivative is given by

$$\Phi''(s) = \int_{-\infty}^{+\infty} \lambda(\tau) [qh_R(t_0 - \tau)]^2 \frac{d^2 M_g}{ds_t^2} d\tau + \sigma_T^2 \quad (3.41)$$

Using (3.40), (3.30) can be written as follows:

$$\left[2\pi \Phi''(\bar{s}_0)\right]^{\frac{1}{2}} \exp\left[\Phi(\bar{s}_0)\right] - \left[2\pi \Phi''(\bar{s}_1)\right]^{\frac{1}{2}} \exp\left[\Phi(\bar{s}_1)\right] = 0 \quad (3.42)$$

We define a function $X(v)$ by

$$X(v) = \left[2\pi \Phi''(\bar{s}_0)\right]^{\frac{1}{2}} \exp\left[\Phi(\bar{s}_0)\right] - \left[2\pi \Phi''(\bar{s}_1)\right]^{\frac{1}{2}} \exp\left[\Phi(\bar{s}_1)\right] \quad (3.43)$$

V_{opt} is the solution of

$$X(v) = 0 \quad (3.44)$$

(3.44) can be solved using Newton's iterations.

A trial value v^i at any stage is replaced by

$$v^{i+1} = v^i - \frac{X(v^i)}{X'(v^i)} \quad (3.45)$$

$$\text{where } X'(v) = \bar{s}_1 - \bar{s}_0 \quad (3.46)$$

III .5 COMPUTATIONAL CONSIDERATIONS

From the previous studies, the saddlepoint approximation method appears to be analytically simple. However, one problem does pose itself and is encountered in the case of computing the BER when 0 is being sent. This is due to the fact that for certain system parameters, the saddlepoint is very close to the point s^* defined in (2 . 24). Therefore, we can clearly see in this case that the BER is highly sensitive to the system parameter and to the roundoff error caused by the numerical steps used to find the saddlepoint.

However, when "0" is sent and assuming that the dark current is negligible, the shot noise is almost absent and the dominant noise is the thermal noise which is Gaussian. Hence, the Gaussian approximation is a good method to apply in the case of "0" is being sent.

In the following examples, the saddlepoint approximation is used to compute the BER when 1 is sent, while for the case of 0 being sent the Gaussian approximation is used.

III .6 EXAMPLES

Several examples are considered in this section to show the effectiveness of the method described in the previous sections as a simulation tool for estimation of fiber optic system performance.

III .6 .1 Example 1 [2]

In this example, a 45 – Mbit/s binary system is considered. The optical source gen-

erates statistically independent equally likely symbols. The optical pulse at the photodetector's input is assumed to be Gaussian in the form

$$h_p(t) = A e^{-(t/t_f)^2}$$

where $t_f = FWHM$ (Full Width Half Maximum of the Gaussian pulse)

The APD's responsivity is assumed to be equal to 0.5 and its ionization ratio $k = 0.1$. The overall impulse response of the receiver is given by

$$h_R(t) = A e^{-(t/t_R)^2}$$

where t_R is related to the bandwidth of the receiver Bw_R by [5]

$$Bw_R = \frac{0.133\sqrt{2}}{t_R} Hz$$

The thermal noise is assumed to be Gaussian with $\frac{\sigma_T}{V_o} = 0.075$, where V_o is the average output at the decision time.

The effect of the threshold level variations on the Bit Error Rate (BER) is shown in Fig 3. 2 for $t_R = 3ns$, $t_f = 1ns$, avalanche gain = 50 and received optical power = $-47 dBm$. In Fig 3. 3 and Fig 3. 4, the BER as a function of detector avalanche gain using Saddlepoint and Gaussian approximation is shown for threshold level 50% and 40% respectively. Fig 3. 5 shows the variation of the BER with respect to the detector avalanche gain for fixed threshold level $V_d = 50\%$ while in Fig 3. 6 the threshold level is chosen for its optimum (see Fig 3. 7). Fig 3. 8 shows the degradation of the receiver sensitivity due to tolerances $\pm 10\%$ in the threshold level.

The effect of the fiber dispersion is shown in Fig 3. 9 . The receiver sensitivity is computed for avalanche gain = 50 and threshold level = 40% .

Finally, Fig 3. 10 shows the BER as function of the receiver bandwidth.

Fig 3. 2 Error probability versus threshold level

for received optical power = -47 dBm and detector avalanche gain = 50

Fig 3. 3 Error probability versus detector avalanche gain

for received optical Power = -47 dBm and threshold level = 50%

Fig 3. 4 Error probability versus detector avalanche gain

for received optical power = -47 dBm and threshold level = 40%

Fig 3.5 Error probability versus detector avalanche gain

for threshold level = 50%

Fig 3. 6 Error probability versus detector avalanche gain

for optimum threshold level and received optical power = -47 dBm

Fig 3. 7 Optimum threshold level versus detector avalanche gain

for received optical power = -47 dBm

Fig 3. 8 Effect of the threshold level variation on the receiver sensitivity

a- Optimum threshold

b- Threshold is +10% from optimum

c- Threshold is -10% from optimum

Fig 3. 9 Effect of fiber dispersion on the receiver sensitivity

for detector avalanche gain = 50 and threshold level =40%

a- FWHM = 1ns

b- FWHM = 2ns

c- FWHM = 4ns

Fig 3. 10 Effect of receiver bandwidth on the bit error rate

for avalanche gain =50, threshold level =40% and received optical power = -47 dBm

III.6.2 Example 2: Application of the saddlepoint approximation to a fiber system with a practical receiver

It has been assumed previously that the receiver has a Gaussian impulse response, but this is not the case in a real communication system.

In this example, a real receiver which consists of a preamplifier circuit (see Fig 3. 11) followed by a filter/equaliser is considered. The required Filter/Equaliser response was realized by a 5th order lowpass filter (Bessel Filter) (see Fig 3. 12). For a small-signal equivalent circuit, the avalanche photodiode is replaced by a current source parallel with the junction capacitor (the junction resistor is neglected). The small-signal equivalent circuit for the transistor is shown in Fig 3. 13

An AC and transient analysis were performed using HSPICE and the result of the frequency response and the impulse response of the receiver are shown in Fig 3. 14 and Fig 3. 15 respectively.

A binary system is considered where the bit rate is $Br = 12.6 \text{ Mbit/s}$. The APD's prime responsivity, R , is assumed to be equal to 0.55 and its ionization ratio $k = 0.02$.

The pulses generated by the source are assumed to be in the form:

$$h_c(t) = \begin{cases} \frac{1}{t_s} & |t| \leq \frac{t_s}{2} \\ 0 & |t| > \frac{t_s}{2} \end{cases} \quad \text{where } t_s \text{ is chosen equal to } \frac{1}{Br}$$

The fiber impulse response is approximated by the Gaussian shape:

$$h_F(t) = \frac{1}{\sqrt{2\pi} t_f} \exp\left[-\left(\frac{t}{\sqrt{2} t_f}\right)^2\right] \quad \text{where } t_f \text{ is chosen such that the fiber bandwidth is equal}$$

to 100 MHz .

The optical pulse at the photodetector input is given by the convolution product of the receiver impulse response by the fiber impulse response and this leads us to the following expression of $h_p(t)$:

$$h_p(t) = \frac{1}{2t_s} \left[\operatorname{erfc} \left(\frac{t - t_s/2}{\sqrt{2} t_f} \right) - \operatorname{erfc} \left(\frac{t + t_s/2}{\sqrt{2} t_f} \right) \right]$$

The photodetector current at the input of the receiver has the following expression:

$$I_p(t) = q G \lambda(t) = R G h_p(t)$$

for an "ON" pulse, we have:

$$I_p(t) = q \frac{\eta}{h\Omega} \frac{1}{2t_s} \left[\operatorname{erfc} \left(\frac{t - t_s/2}{\sqrt{2} t_f} \right) - \operatorname{erfc} \left(\frac{t + t_s/2}{\sqrt{2} t_f} \right) \right]$$

The voltage at the output of the receiver is given by:

$$v(t) = I_p(t) * h_R(t)$$

The current pulse at the input and the voltage pulse at the output of the receiver are shown in Fig 3. 16 and Fig 3. 17 respectively.

The impulse response of the receiver can be approximated by a Gaussian function as shown in Fig 3. 18 . Variation of the bit error rate with respect to the threshold level, detector avalanche gain and received optical power are shown in Fig 3. 19 through to Fig 3. 21 .

Fig 3. 12 Filter/Equalizer

Fig 3. 13 Small-signal transistor equivalent circuit

Fig 3. 14 Frequency response of the receiver

Fig 3. 15 Impulse response of the receiver

Fig 3. 16 Current at the input of the receiver

Fig 3. 17 Voltage at the output of the receiver

Fig 3. 18 Impulse response of the:

a. practical receiver

b. approximated Gaussian impulse response of the receiver

Fig 3. 19 Bit error rate versus threshold level

for received optical power = -58 dBm and detector avalanche gain =70

a. using the impulse response of the receiver

b. using the approximated Gaussian impulse response of the receiver

Fig 3. 20 Error probability versus detector avalanche gain

for threshold level = 50%, received optical power=-58 dBm

a. using the impulse response of the receiver

b. using the approximated Gaussian impulse response of the receiver

Fig 3. 21 Effect of the threshold level variation on the receiver sensitivity

a– optimum threshold

b– threshold is +10% from optimum

c– threshold is -10% from optimum

III.7 SUMMARY AND CONCLUSIONS

The saddlepoint approximation method as presented in previous publications was first reviewed. This method consists of evaluating the error in counting the number of electrons generated by the avalanche photodetector where the detector shot noise was modeled as Poisson process. However, in a practical fiber communication system, the output of the avalanche photodetector is filtered by the receiver circuit and the shot noise can no longer be modeled as Poisson process.

The saddlepoint approximation method has been generalized in this chapter to include the effect of the receiver impulse response in order to compute the error probability for practical fiber optic communication systems.

Expansion of the saddlepoint approximation to include the effect of the receiver impulse response alters some of the basic computational steps of the original algorithm. These changes are explained in detail in this chapter. To show the flexibility of the developed tool, several practical examples were considered where the dependency of the bit error rate on the system parameters such as the impulse response of the receiver, threshold level, detector avalanche gain, received optical power, thermal noise, bit rate ...etc were demonstrated.

Chapter IV

GENERALIZATION OF THE SADDLEPOINT APPROXIMATION TO INCLUDE THE EFFECT OF THE INTERSYMBOL INTERFERENCE

IV .1 INTRODUCTION

The intersymbol interference (ISI) can have a dominant effect in detecting and reconstructing the data being initially sent. The ISI occurs as a result the dispersion of the transmitted signal on the fiber link and the limited bandwidth of the receiver which causes a pulse broadening manifested in an overlaps of the pulses with their neighbors. As the ISI becomes more pronounced, an increasing number of errors may be encountered, which reduce the sensitivity at the optical receiver.

The effect of the ISI in the calculation of the bit error rate using the saddlepoint approximation was not treated previously.

In section IV .2 , the signal at the input of the decision circuit will be described as

2

of/de

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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a function of the thermal noise, shot noise and ISI. In section IV .3 , different techniques are described to compute the error probability in the presence of intersymbol interference using the saddlepoint approximation. Examples are presented in section IV .4 and results are compared to those obtained by previously published methods.

IV .2 SIGNAL AT THE INPUT OF THE DECISION CIRCUIT

The current signal $x(t)$ at the output of the photodetector circuit is the sum of a filtered randomly multiplied Poisson process, $x_s(t)$, and an independent zero-mean additive thermal noise, $x_T(t)$, assumed to be Gaussian.

$$x(t) = x_s(t) + x_T(t)$$

where $x_s(\cdot)$ is given in (2 . 18) by

$$x_s(t) = \sum_{n=1}^{N(t)} g(n) q h_s(t - t_n)$$

$x(t)$ is filtered by a receiver circuit resulting in a signal $v(t)$ at the input of the decision circuit

$$v(t) = v_s(t) + v_T(t)$$

$$= \sum_{n=1}^{N(t)} g(n) q h(t - t_n) + \int_{-\infty}^{+\infty} x_T(\tau) h_R(t - \tau) d\tau$$

where $h(t) = h_s(t) * h_R(t)$

The signal $v(t)$ can be separated into the average of the signal and a zero-mean random process [3],[7]

$$v(t) = s(t) + \left\{ v_s(t) - s(t) \right\} + v_T(t) \quad (4.1)$$

where $s(t) = \left\{ E[v(t)] \right\}_{\lambda_s=0}$ represents the data signal

$$E[v(t)] = q E[g] \int_{-\infty}^{+\infty} \lambda(\tau) h(t-\tau) d\tau \quad (4.2)$$

Using the expression of $\lambda(t)$ given in (2.16), (4.2) can be expressed as follows:

$$E[v(t)] = \frac{\mu q}{h\Omega} E[g] \sum_{k=-\infty}^{+\infty} a_k \int_{-\infty}^{+\infty} h_p(\tau - kT) h(t-\tau) d\tau + \lambda_{oq} E[g] \int_{-\infty}^{+\infty} h(t-\tau) d\tau \quad (4.3)$$

Combining (4.1) with (4.3), we obtain [7]

$$v(t) = \sum_{k=-\infty}^{k=+\infty} a_k r(t - kT) + n_s(t) + v_T(t) \quad (4.4)$$

where $r(t)$ is the overall baseband pulse response given by: $r(t) = \frac{\mu q}{h\Omega} E[g] h_p(t) * h(t)$

$n_s(t)$ is the shot noise given by: $n_s(t) = \sum_{n=1}^{N(t)} g(n) q h(t - t_n) - \left\{ E[v_s(t)] \right\}_{\lambda_s=0}$

$v_T(t)$ is the thermal noise given by: $v_T(t) = \int_{-\infty}^{+\infty} x_T(\tau) h_R(t-\tau) d\tau$

At the decision time t_o , we have

$$v(t_o) = a_0 r_0 + I(t_o) + n_s(t_o) + v_T(t_o) \quad (4.5)$$

where $r_0 = \frac{\mu q}{h\Omega} E[g] \int_{-\infty}^{+\infty} h_p(\tau) h(t_o - \tau) d\tau$ is the desired signal

$$I(t_o) = \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} a_k v(t_o - kT) \text{ represents the intersymbol interference}$$

$n_s(t_o)$ and $v_T(t_o)$ represent the shot and thermal noise, respectively

IV .3 EVALUATION OF THE BER IN THE PRESENCE OF ISI

In the previous section, the term $I(t_o)$ given in (4 . 5) representing the ISI was ignored. In this section, the effect of the ISI will be taken into consideration and different techniques for the evaluation of error probability will be presented. The first method uses an exhaustive technique. The second one, based on evaluating the average moment of the shot noise is more efficient and relatively easy to implement.

IV .3 .1 Evaluation of the BER using exhaustive technique

Rewrite (3 . 19), we have:

$$\Psi(s) = \int_{-\infty}^{+\infty} \lambda(\tau) [M_g(sq h_R(t_o - \tau)) - 1] d\tau + \frac{1}{2} \sigma_T^2 s^2 - sV_d - \log(\pm s)$$

$$\begin{cases} \pm s = -s & \text{when 1 is sent} \\ \pm s = s & \text{when 0 is sent} \end{cases} \quad (4.6)$$

Replacing $\lambda(\tau)$ by its expression given in equation (2.16), we obtain:

$$\Psi(s) = \int_{-\infty}^{+\infty} \left[\lambda_o + D(\tau) + \frac{\eta}{h\Omega} \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} a_k h_p(\tau - kT) \right] [M_g(sq h_R(t_o - \tau)) - 1] d\tau +$$

$$+ \frac{1}{2} \sigma_T^2 s^2 - sV_d - \log(\pm s) \quad (4.7)$$

$$\text{where } D(\tau) = \begin{cases} \frac{\eta}{h\Omega} h_p(\tau) & \text{if 1 is sent} \\ 0 & \text{if 0 is sent} \end{cases} \quad (4.8)$$

$$\frac{\eta}{h\Omega} \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} a_k h_p(\tau - kT) \text{ represents the ISI}$$

In the following, M_g denotes $M_g(sq h_R(t_o - \tau))$ and s_t denotes $sq h_R(t_o - \tau)$.

The first derivative of $\Psi(s)$ with respect to s gives:

$$\Psi'(s) = \int_{-\infty}^{+\infty} \left[\lambda_o + D(\tau) + \frac{\eta}{h\Omega} \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} a_k h_p(\tau - kT) \right] q h_R(t_o - \tau) \frac{dM_g}{ds} d\tau +$$

$$+ \sigma_T^2 s - V_d - \frac{1}{s} \quad (4.9)$$

and the second derivative is:

$$\Psi''(s) = \int_{-\infty}^{+\infty} \left[\lambda_o + D(\tau) + \frac{\eta}{h\Omega} \sum_{\substack{k=-\infty \\ k \neq 0}}^{+\infty} a_k h_p(\tau - kT) \right] [qh_R(t_o - \tau)]^2 \frac{d^2 M_g}{ds^2} d\tau + \sigma_I^2 + \frac{1}{s^2} \quad (4.10)$$

The same technique described in chapter-III for the evaluation of the BER using the saddlepoint approximation method is applied where the functions $\Psi(s)$, $\Psi'(s)$ and $\Psi''(s)$ are replaced by their new expressions which includes the effect of the ISI.

In the exhaustive technique, a truncated N-pulse-train approximation of the true channel is considered and the error probability is calculated by evaluating the conditional error probability of each of 2^N possible data sequences and averaging over all 2^N sequences.

The computational cost of this method is usually quite expensive and this limits the number of interfering samples which can be accounted for. The computation involved grows exponentially with the number N of neighbor pulses. It is nevertheless useful in order to verify the validity of the other methods [2][16][12]. In addition, the exhaustive technique is essential in cases where the nonlinear behavior of the system components has to be taken into account.

IV.3.2 Evaluation of the BER using the average moment of the shot noise

In this section, the effect of intersymbol interference on the system performance is evaluated using the average moment of the shot noise.

IV .3 .2 .1 Derivation of the average moment of the shot noise

The moment generating function of the shot noise given in equation (2 . 26) can be written as follows:

$$M_{v_i}(s) = \exp \left[\int_{-\infty}^{+\infty} \lambda(\tau) \Phi(s, \tau) d\tau \right] \quad (4 . 11)$$

$$\text{where } \Phi(s, \tau) = M_g(sq h_R(t_o - \tau)) - 1 \quad (4 . 12)$$

$\lambda(\tau)$ can be expressed by:

$$\lambda(\tau) = \lambda_o + D(\tau) + \frac{\eta}{h\Omega} \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} a_k h_p(t_o - kT) \quad (4 . 13)$$

$$\text{where } D(\tau) = \begin{cases} \frac{\eta}{h\Omega} h_p(\tau) & \text{if 1 is sent} \\ 0 & \text{if 0 is sent} \end{cases} \quad (4 . 14)$$

Replacing the expression of $\lambda(\tau)$ in $M_{v_i}(s)$, we obtain:

$$M_{v_i}(s) = \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) [\lambda_o + D(\tau)] d\tau \right] * \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \frac{a_k \eta}{h\Omega} h_p(t_o - kT) d\tau \right] \quad (4 . 15)$$

Removing the sum from the second exponential we get:

$$M_{v_i}(s) = \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) [\lambda_o + D(\tau)] d\tau \right] * \prod_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \frac{a_k \eta}{h\Omega} h_p(t_o - kT) d\tau \right] \quad (4 . 16)$$

$$\text{Let } M_k(s)|_{a_k} = \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \frac{a_k \eta}{h\Omega} h_p(\tau - kT) d\tau \right] \quad (4.17)$$

Knowing that a_k takes the binary values $\{0, 1\}$ with probabilities p_0 and p_1 respectively, $M_k(s)|_{a_k}$ assumes the values $M_k(s)|_{a_k=0}$ and $M_k(s)|_{a_k=1}$ with the same probabilities, respectively.

Averaging $M_k(s)|_{a_k}$, we obtain:

$$\begin{aligned} \langle M_k(s)|_{a_k} \rangle &= p_0 M_k(s)|_{a_k=0} + p_1 M_k(s)|_{a_k=1} \text{ equivalent to} \\ \langle M_k(s)|_{a_k} \rangle &= p_0 + p_1 \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \frac{\eta}{h\Omega} h_p(\tau - kT) d\tau \right] \end{aligned} \quad (4.18)$$

Replacing $M_k(s)|_{a_k}$ by its average value in $M_v(s)$, we obtain the average moment of the shot noise [2]:

$$\begin{aligned} \langle M_v(s) \rangle &= \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) [\lambda_o + D(\tau)] d\tau \right] * \\ &\quad \prod_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \left[p_0 + p_1 \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \frac{\eta}{h\Omega} h_p(\tau - kT) d\tau \right] \right] \end{aligned} \quad (4.19)$$

If $U(s, k) = p_1 \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) \frac{\eta}{h\Omega} h_p(\tau - kT) d\tau \right]$, we have:

$$\langle M_{\nu_i}(s) \rangle = \exp \left[\int_{-\infty}^{+\infty} \Phi(s, \tau) [\lambda_o + D(\tau)] d\tau \right] * \prod_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} [p_0 + U(s, k)] \quad (4.20)$$

IV.3.2.2 Evaluation of the BER

Instead of using the expression of $M_{\nu_i}(s)$ given in (4.26), we use its average expression, that leads to the following expression of $\Psi(s)$

$$\Psi(s) = \int_{-\infty}^{+\infty} \Phi(s, \tau) [\lambda_o + D(\tau)] d\tau + \left[\sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \log(U(s, k) + p_0) \right] + \frac{1}{2} \sigma_T^2 s^2 - sV_d - \log(\pm s) \quad (4.21)$$

The first derivative of $\Psi(s)$ with respect to s is:

$$\begin{aligned} \Psi'(s) = \int_{-\infty}^{+\infty} [\lambda_o + D(\tau)] q h_R(t_o - \tau) \frac{dM_g}{ds}(sq h_R(t_o - \tau)) d\tau + \\ + \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \left[\frac{U'(s, k)}{U(s, k) + p_0} \right] + \sigma_T^2 s - V_d - \frac{1}{s} \end{aligned} \quad (4.22)$$

$$\text{where } U'(s, k) = U(s, k) \int_{-\infty}^{+\infty} \frac{\eta}{h\Omega} h_p(\tau - kT) q h_R(t_o - \tau) \frac{dM_g}{ds_i} d\tau \quad (4.23)$$

$$s_i = sq h_R(t_o - \tau)$$

and the second derivative is given by:

$$\begin{aligned} \Psi''(s) = & \int_{-\infty}^{+\infty} [\lambda_o + D(\tau)] [qh_R(t_o - \tau)]^2 \frac{d^2 M_g}{ds_i^2} d\tau + \\ & + \sum_{\substack{k=-\infty \\ k \neq 0}}^{k=+\infty} \left[\frac{U'(s, \tau) [U(s, \tau) + p_0] - U'^2(s, \tau)}{[U(s, \tau) + p_0]^2} \right] + \sigma_T^2 + \frac{1}{s^2} \end{aligned} \quad (4.24)$$

$$\begin{aligned} \text{where } U'(s, k) = U'(s, k) & \int_{-\infty}^{+\infty} \frac{\eta}{h\Omega} h_p(\tau - kT) qh_R(t_o - \tau) \frac{dM_g}{ds_i} d\tau + \\ & + U(s, \tau) \int_{-\infty}^{+\infty} \frac{\eta}{h\Omega} h_p(\tau - kT) [qh_R(t_o - \tau)]^2 \frac{d^2 M_g}{ds_i^2} d\tau \end{aligned} \quad (4.25)$$

The saddlepoints can be determined and the performance is evaluated in the presence of ISI using the same process as in chapter II .

IV .4 **EXAMPLE**

IV .4 .1 **Example-1**

Consider the system described previously in section III .6 .1 To include the ISI effect, a truncated 11-pulse train is considered. To show the accuracy of the saddlepoint approximation using the average moments, comparison of the BER as a function of the threshold level with the results obtained using the Chebyshev approximation[2] is shown in

Fig 4. 1 where an excellent agreement is obtained. As a second test for its accuracy, Fig 4. 2 shows the results obtained using the average moments to those obtained using the exhaustive technique. Again, an excellent agreement is observed.

Fig 4. 1 Comparison between three approximation methods:

Gaussian, Chebyshev and Saddlepoint approximation

**Fig 4. 2 Comparison between the exhaustive technique
and the saddlepoint approximation using the average moment of the shot noise**

IV .4 .2 Example-2

A 50 Mbit/s binary system is considered in this example. The optical pulses at the input of the photodetector are assumed to be Gaussian:

$h_p(t) = A e^{-(t/t_f)^2}$ where $t_f = FWHM = 8.18 \cdot 10^{-8}$ corresponding to a 230 Mhz bandwidth. The ADP's responsivity is assumed to be equal to 0.5 and the ionization ratio $k=0.1$.

An RC receiver circuit is considered. Its impulse response is given by:

$h_R(t) = A e^{-(t/RC)}$ where $\frac{1}{RC} = 1.469 \cdot 10^{18}$ equivalent to a 64 Mhz bandwidth.

For a truncated 3-pulses train, the variation of the BER with respect to the threshold level is shown in Fig 4. 3 for received optical power = -52.1 dBm and detector avalanche gain =110. In Fig 4. 4 , the effect of the avalanche gain on the BER for a fixed threshold level is shown while in Fig 4. 5 the threshold level is chosen at its optimum value (see Fig 4. 6). Fig 4. 7 shows the degradation of the receiver sensitivity due to tolerances $\pm 10\%$ in the threshold level. Finally, the effect of the bit rate of the system on the BER is shown in Fig 4. 8 .

**Fig 4. 3 Bit error rate versus threshold level
for a system with an RC-receiver circuit**

Fig 4. 4 Error probability versus detector avalanche gain
for received optical power = -52.1 dBm and threshold level = 50%

Fig 4. 5 Error probability versus detector avalanche gain

for received optical Power = -52.1 dBm and optimum threshold level

Fig 4. 6 Optimum threshold level versus detector avalanche gain

for received optical power = -52.1 dBm

Fig 4. 7 Effect of the threshold level variation on the receiver sensitivity

for detector avalanche gain =110

a- optimum threshold

b- threshold is +10% from optimum

c- threshold is -10% from optimum

Fig 4. 8 Bit error rate versus bit rate of the system
for threshold level = 50%, detector avalanche gain =110
and received optical power =-52.1 dBm

IV .4 .3 Example-3

Consider the system described in section-III .6 .2 . The effect of the ISI is added by considering a truncated 4-pulse train as shown in Fig 4. 9 . It is clear that the ISI is very pronounced and a relatively high BER is expected. Using the saddlepoint approximation, the BER was found to be equal to 0.3. Trying to improve the BER, the bit rate is reduced to 8-Mbit/s and the results are shown in Fig 4. 10 . The BER as function of the threshold level is shown in Fig 4. 11 . This example shows clearly that the equalizer design for this system is not optimum. It also demonstrates the need for an optimization tool which can be used to minimize the BER, in the presence of thermal noise, shot noise, and ISI, as function of the system parameters including the receiver components. This optimization tools will be further described in the next chapter.

Fig 4. 9 Effect of the ISI

Train pulse (1 1 1 0 1) at the input of the decision circuit;

$B_r = 12.6$ Mbit/s, $t_s = 1/B_r$

Fig 4. 10 Effect of the ISI

Train pulse (1 1 1 1 0 1) at the input of the decision circuit;

$B_r=8$ Mbit/s, $t_s=0.5/B_r$, received optical power = -59.7 dBm

and detector avalanche gain = 60

Fig 4. 11 Bit error rate versus threshold level

for $t^s = T/2$, $B_r = 8$ Mbit/s, received optical power = -59.7 dBm and

detector avalanche gain=60

IV .5 DISCUSSION AND CONCLUSION

The effect of the ISI on the system performance was added in this chapter. For the evaluation of the bit error rate, two methods were implemented, namely the exhaustive technique and the saddlepoint approximation using the average moment of the shot noise. The former consists of calculating the error probability of each 2^N possible sequences and averaging over all 2^N sequences, hence it increases the computational cost of the method. Nevertheless, the exhaustive technique is a good way to check the accuracy of other approximation techniques.

The average moment of the shot noise when the ISI is present was derived. Based on its expression, the saddlepoint approximation was applied. Several examples were considered which demonstrated the accuracy of this technique compared to the exhaustive approximation and other previously published methods. In the last example where a practical receiver is considered, the bit error rate was found to be high even when the bit rate was reduced. Consequently, an improvement in the design of the receiver circuit is needed and this involves an optimization technique which will be the subject of the next chapter.

Chapter V

STATISTICAL ANALYSIS AND PERFORMANCE **OPTIMIZATION**

V.1 INTRODUCTION

The performance of Optical Fiber Communication Systems in the presence of inter-symbol interference is studied in the previous chapters, including the effect of the receiver response. The average error probability depends on the system model parameters such as fiber characteristics, detector avalanche gain, received optical power, threshold level, the receiver circuit...etc. This Bit Error Rate in the detection process is affected by noises such as thermal noise, shot noise, intersymbol-interference (ISI), and by the variation in the decision time (Jitter). However, an improvement of the system performance can be done by using optimization techniques to design the model parameters.

To demonstrate the ideas in this chapter, we will present this concept of optimization

based on a simple system model as well as on the Gaussian approximation method for the evaluation of the BER.

The system model chosen is the one stated in example III.6.1. Based on this model and using the Gaussian approximation for the evaluation of the BER, the gradient will be derived in section V.2. This latter is necessary to evaluate the sensitivity of the BER vis-à-vis certain model parameters of the system. In section V.3.1, we discuss the optimization of the BER and in section V.3.3, a worst case analysis of the Optical Fiber System is presented. This is based on estimating the worst system performance corresponding to specific probability distributions of system parameters.

In section V.3.4, the yield is calculated by simulation. Estimation of the yield using worst case analysis is also considered.

Each section first presents a general case scenario, which is then followed by some examples related to the Fiber Optic System.

V.2 SENSITIVITY ANALYSIS

V.2.1 Derivation of the BER Gradient using Gaussian Approximation

Based on the Gaussian approximation method, the error probability derived in II.4.2 is given by:

$$P_e = \frac{1}{2} \left\{ p_1 \operatorname{erfc}\left[\frac{V_{M1} - V_d}{\sqrt{2}\sigma_{N1}}\right] + p_0 \operatorname{erfc}\left[\frac{V_d - V_{M0}}{\sqrt{2}\sigma_{N0}}\right] \right\} \quad (5.1)$$

$$\sigma_N^2 = \sigma_s^2 + \sigma_T^2 \quad (5.2)$$

where σ_s and σ_T are the standard deviations (root mean square) of the shot and the thermal noise respectively

(see II.4.2 for the definition of the parameters in (5.1))

The use of exact gradient information $\frac{\partial P_e}{\partial x}$ significantly improves the effectiveness of an optimization algorithm [17]. The sensitivities of the Bit Error Rate with respect to different parameters of the system are derived in this section to be used in the optimization of the performance and in the worst case analysis.

The following parameters of the system are chosen to evaluate the sensitivity:

V_d is the threshold level at the decision circuit

G is the Avalanche gain of the photodetector

Bw_R is the bandwidth of the receiver

P_o is the optical power at the input of the receiver (at the far end of the fiber)

k is the ionization factor of the detector

N_o is the spectral power density of the thermal noise at the input of the receiver

Using equations (5.1) and (5.2), the sensitivity is derived as follows:

$$\frac{\partial P_e}{\partial V_d} = \frac{1}{4} \sqrt{\frac{2}{\pi}} \left[\frac{1}{\sigma_{N1}} \exp - \left(\frac{V_{M1} - V_d}{\sqrt{2}\sigma_{N1}} \right)^2 + \frac{1}{\sigma_{N0}} \exp - \left(\frac{V_d - V_{M0}}{\sqrt{2}\sigma_{N0}} \right)^2 \right] \quad (5.3)$$

$$\frac{\partial P_e}{\partial x} = \frac{\partial P_e}{\partial \sigma_{N1}} \frac{\sigma_{N1}}{\partial x} + \frac{\partial P_e}{\partial \sigma_{N0}} \frac{\partial \sigma_{N0}}{\partial x}; \quad (5.4)$$

where $x \in \{G, Bw_R, P_o, k, N_o\}$.

$$\frac{\partial P_e}{\partial \sigma_{N1}} = \frac{1}{2\sqrt{\pi}} \left(\frac{V_{M1} - V_d}{\sqrt{2}\sigma_{N1}^2} \right) \exp \left[- \left(\frac{V_{M1} - V_d}{\sqrt{2}\sigma_{N1}} \right)^2 \right] \quad (5.5)$$

$$\frac{\partial P_e}{\partial \sigma_{N0}} = \frac{1}{2\sqrt{\pi}} \left(\frac{V_d - V_{M0}}{\sqrt{2}\sigma_{N0}^2} \right) \exp \left[- \left(\frac{V_d - V_{M0}}{\sqrt{2}\sigma_{N0}} \right)^2 \right] \quad (5.6)$$

$$\frac{\partial \sigma_N}{\partial x} = \left[\sigma_S \frac{\partial \sigma_S}{\partial x} + \sigma_T \frac{\partial \sigma_T}{\partial x} \right] \left(\frac{1}{\sigma_N} \right) \quad (5.7)$$

V.3 STATISTICAL ANALYSIS

V.3.1 Optimization of the Bit Error Rate

Optimization of the Bit Error Rate has been presented and performed in several papers using simple methods such as Newton iteration, as well as using a limited number of optimization variables (e.g: V_d , G). However, the optimization problem will be difficult to implement if a large number of variables are introduced. In order to overcome this obstacle, some special optimization techniques are used to tackle the problem.

The formulation of the Minimax optimization problem can be stated as follows [17]:

- Given:
- *Set of optimization variables $\underline{\Phi} = [\Phi_1 \ \Phi_2 \dots \dots \Phi_n]^T$
 - *Independent parameter ψ
 - *Non linear continuously differentiable function $F(\underline{\Phi}, \psi)$: "performance"
 - *Upper specification $S_u(\psi)$
 - *Lower specification $S_l(\psi)$
 - *Weighting factors (positive) $w_u(\psi)$, $w_l(\psi)$

Let: $\underline{f}(\underline{\Phi}) = [f_1(\underline{\Phi}) \ f_2(\underline{\Phi}) \ \dots \ f_n(\underline{\Phi})]^T$ be a set of error functions

where $f_j(\underline{\Phi}) = w_u(\psi_j) [F(\underline{\Phi}, \psi_j) - S_u(\psi_j)]$ if an upper specification is imposed

or $f_j(\underline{\Phi}) = w_l(\psi_j) [S_l(\psi_j) - F(\underline{\Phi}, \psi_j)]$ if a lower specification is imposed

$$\text{Find: } \min_{\underline{\Phi}} \left[U(\underline{\Phi}) \equiv \max_{j=1, m} \{ f_j(\underline{\Phi}) \} \right] \quad (5.8)$$

$$\text{subject to constraints: } \underline{g}(\underline{\Phi}) \geq 0 \quad \underline{g} = [g_1 \ g_2 \ \dots \dots \ g_{n_g}]^T \quad (5.9)$$

$$\underline{h}(\underline{\Phi}) = 0 \quad \underline{h} = [h_1 \ h_2 \ \dots \dots \ h_{n_h}]^T \quad (5.10)$$

The optimization of the Bit Error Rate is performed by using the available optimizer, namely the "Minimax with exact gradient". It consists of two stages which combine the robustness of a first-order method of the Gauss-Newton type with the speed of the quasi-Newton method [18][17]. The optimizer requires user-supplying first order derivatives of the

error functions with respect to the optimization variables. Such information is obtained by the sensitivity analysis described in section-V.2.1. The optimizer automatically generates approximate second order derivatives by a BFGS (Broyden-Fletcher-Goldfarb-Shanno) update to provide fast convergence.

V.3.2 Examples

V.3.2.1 Example 1

In this example, the optimization variables are $\underline{\Phi} = [V_d \ G \ Bw_R \ P_o \ k \ N_o]^T$, the weighting factors are equal to 1 and no specifications are imposed.

In this case $F(\underline{\Phi}, L)$ is equal to $\log[P_e(\underline{\Phi})]$ which is measured for a specific length of fiber, L .

The objective function is: $U(\underline{\Phi}) \equiv \log[P_e(\underline{\Phi})]$ (5.11)

The constraints are: $0.1 < V_d < 1$.

$$10. < G < 250.$$

$$1.Mhz < Bw_R < 500.Mhz$$

$$-47.1dBm < P_o < -47.dBm$$

$$0.1 < k < 0.9$$

$$4.02211 \cdot 10^{-22} < N_o < 4.03 \cdot 10^{-22}$$

No specifications are imposed.

Results are shown in Table 5. 1 and Table 5. 2 where two different starting points are used.

Table 5. 1 : Performance optimization at starting point-1

Design variables	At the starting point	optimum solution
V_d	0.3	0.232837
G	120.	108.96004
Bw_R	90.	129.08731
P_o	-47.05	-47
k	0.5	0.1
N_o	4.023E-22	4.02211E-22
Objective function: Log(P_e)	-8.1083424185568	-30.8588101
P_e	3.01017E-03	3.9645057E-14

Table 5. 2 : Performance optimization at starting point-2

Design variables	At the starting point	optimum solution
V_d	0.5	0.232916
G	70.	108.95681
Bw_R	150.	128.85942
P_o	-47.05	-47
k	0.6	0.1
N_o	4.023E-22	4.02211E-22
Objective function: Log(P_e)	-6.77957	-30.8588146
P_e	1.136765E-03	3.9644879E-14

Since P_e increases when k and/or N_o increases, and P_e decreases when P_o increases, P_o should converge to the upper limit and N_o and k to the lower limit of their constraint boundaries at the optimum solution (see Table 5. 1 and Table 5. 2 and constraints listed above in example-1). Consequently, the set of optimization variables $\underline{\Phi} = [V_d \ G \ Bw_R \ P_o \ k \ N_o]^T$ can be reduced to $\underline{\Phi} = [V_d \ G \ Bw_R]^T$.

V.3.2.2 Example-2

In this example, the set of optimization variables is $\underline{\Phi} = [V_d \ G \ Bw_R]^T$, the weighting factors are equal to 1, the independent parameter is the length of the fiber L and no lower specifications are imposed.

The objective function is: $U(\underline{\Phi}) \cong \max_{j=1,m} \{ f_j(\underline{\Phi}) \}$

where: $f_j(\underline{\Phi}) = w_u(\psi_j) [F(\underline{\Phi}, \psi_j) - S_u(\psi_j)]$

The specifications are:

$$S_u(L) = \text{Log}(2.00 \cdot 10^{-12}) \approx -26.9379 \quad 0 \leq L \leq 1 \text{ Km}$$

$$S_u(L) = \text{Log}(1.05 \cdot 10^{-11}) \approx -25.2796 \quad 1. \leq L \leq 2. \text{Km}$$

$$S_u(L) = \text{Log}(1.00 \cdot 10^{-09}) \approx -20.7233 \quad 2. \leq L \leq 3. \text{Km}$$

The constraints are: $0.1 < V_d < 1$

$$10 < G < 250$$

$$1 \text{ Mhz} < Bw_R < 500 \text{ Mhz}$$

Knowing that $\text{Log}(P_e)$ is a monotonically increasing function of L , by using the optimization of the performance for the three lengths ($L_1 = 1$, $L_2 = 2$, $L_3 = 3$) we obtain the same result as we would by evaluating the performance at any additional points on the curve. The number of error functions, f_j , is then equal to 3 or $m = 3$. The optimized variables from the solution are substituted back into (5.1) to evaluate the optimum performance for other lengths ranging between 1 and 3 km (see Fig-5.1).

Table 5.3 shows the results of the optimization.

Table 5.3 : Performance optimization with specifications		
Design variables	At the starting point	optimum solution
V_d	0.3	0.24584924
G	120.	134.90823
Bw_R	90.	65.0936
f_1 $\text{Log}(P_e)$ P_e	1.4739666198516 -25.463907315517 8.7330D-12	-4.9075091835324D-02 -26.986949027204 1.9388D-12
f_2 $\text{Log}(P_e)$ P_e	2.8350238349512 -2.444622023814 1.7882D-10	-4.9075091834865D-02 -25.328720950600 9.9973D-12
f_3 $\text{Log}(P_e)$ P_e	1.9972067044513 -18.726059132495 7.3684D-09	-4.9075091836666D-02 -20.772340928783 9.5211D-10

At the optimum point, the three functions, f_j , are active, and the minimax objective is equal to $f_2 = -4.9075091834865D-02$.

Fig-5. 1 Performance optimization

a. Before optimization ...+....+.....+

b. After optimization (without specification) _____

c. After optimization (with specification) _____

V .3 .3 Worst Case analysis

The system's performance can be affected by any inconsistencies in the manufacturing of the products. The performance must be evaluated based on any such inconsistencies in order to adjust design parameters, hence one can perform the yield prediction at each step of the manufacturing procedure. In order to avoid the high cost of this procedure, the performances can be checked under a set of worst case conditions in order to determine their acceptability.

This section is divided into two parts. In the first part, the tolerances assigned to the design variables are uniformly distributed, and in the second, the variables are assumed to follow a Gaussian probability distribution.

V .3 .3 .1 Worst case analysis with uniformly distributed tolerances

The worst case analysis problem can be formulated as follows:

Given: *Set of variables $\Phi = [\Phi_1 \ \Phi_2 \dots \dots \Phi_n]^T$

*Set of error functions $f(\Phi) = [f_1(\Phi) \ f_2(\Phi) \ \dots \dots \ f_m(\Phi)]^T$

where $f_j(\Phi) = w_u(\psi_j) [F(\Phi, \psi_j) - S_u(\psi_j)]$ if an upper specification is impose

$f_j(\Phi) = w_l(\psi_j) [S_l(\psi_j) - F(\Phi, \psi_j)]$ if a lower specification is imposed

$w_u(\psi)$, $w_l(\psi)$ are weighting factors (positive)

ψ is an independent parameter

$S_u(\psi)$, $S_l(\psi)$ are upper and lower specifications

*Nominal point $\underline{\Phi}^0 = [\Phi_1^0 \ \Phi_2^0 \dots \Phi_n^0]^T$ and the corresponding set of

tolerances $\underline{\varepsilon} = [\varepsilon_1 \ \varepsilon_2 \dots \varepsilon_n]^T$

Let's define: *The constraint region R_c is the region of points $\underline{\Phi}$ such that all performance specifications and constraints are satisfied by the circuit.

*The tolerance region R_t :

$$R_t = \{ \underline{\Phi} / \Phi_i^0 - t_i \leq \Phi_i \leq \Phi_i^0 + t_i, \ i = 1, 2, 3, \dots, n \}$$

$$\text{The vertices of } R_t \text{ are: } \underline{\Phi}^k = \underline{\Phi}^0 + \begin{bmatrix} \varepsilon_1 & 0 & 0 & 0 \\ 0 & \varepsilon_2 & 0 & 0 \\ 0 & 0 & \varepsilon_3 & \vdots \\ \vdots & \vdots & 0 & \vdots \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & \varepsilon_n \end{bmatrix} * \begin{bmatrix} \mu_1^k \\ \mu_2^k \\ \vdots \\ \vdots \\ \vdots \\ \mu_n^k \end{bmatrix}, \text{ where } \mu_i^k = \pm 1$$

$$\underline{\Phi}^k = \underline{\Phi}^0 + \underline{E} \underline{\mu}^k \quad (5.12)$$

assumption: * R_c is not empty

* R_c is bounded and simply connected

* R_c is at least one-dimensionally convex

* f_j is monotonic in R_t , $j = 1, \dots, m$

Find: *for each f_j , the worst case vertex $\underline{\Phi}^k = \underline{\Phi}^0 + \underline{E} \underline{\mu}^k$ (5.13)

$$\text{where } \mu_i^k = \text{sign} \left[\frac{\partial f_j}{\partial \Phi_i} \right] \quad (5.14)$$

$$\text{*worst case performance } f^{wc} = \max_{j=1, \dots, m} [f_j^{wc}] \quad (5.15)$$

where f_j^{wc} is evaluated at the worst case vertex of f_j

V.3.3.1.1 Example

In this example, the set of variables Φ and the error function $f(\Phi)$ are the same as in V.3.2.2 (example-2), the nominal point Φ^0 is the optimum point found in V.3.2.2 and the set of tolerance $\underline{\varepsilon}$ is defined such that $\varepsilon_i = 5\%$ of Φ_i^0 ,

We define the vertex number as indicated in Table 5.4 and Fig-5.2.

Table 5.4 : Vertex identification								
μ_1	+	+	+	+	-	-	-	-
μ_2	+	+	-	-	+	+	-	-
μ_3	+	-	+	-	+	-	+	-
	vertex#1	vertex#2	vertex#3	vertex#4	vertex#5	vertex#6	vertex#7	vertex#8

The predicted worst case result is found by using the sensitivity information as defined in (5.14) (see Table 5.5) and verified by the calculation of the worst case at all vertices (see Table 5.6).

Fig-5. 2 Vertices representation in the three dimensional space

In the following table, for each f_j $j = 1, 2, 3$; the worst case vertex and the corresponding value of f_j are evaluated.

Table 5. 5 : Worst case performance using sensitivity information	
Error function: f_j^{wc}	At vertex #
$f_1=1.5204895019153$	2
$f_2=1.3986418223666$	1
$f_3=3.1900106646531$	7

The predicted worst case vertex is $\underline{\Phi}^7 = \begin{bmatrix} V_d^7 \\ G^7 \\ Bw_R^7 \end{bmatrix} = \begin{bmatrix} 0.23355677718945 \\ 128.16281998804 \\ 68.348296946356 \end{bmatrix}$ corre-

sponding to $f^{wc} = f_3^{wc} = 3.1900106646531$, and the worst case performance is

$P_e=2.4288686472128D-08$

In the following table, for each vertex Φ^k $k = 1, 2, \dots, 8$, $f_j^{wc} = \max_{j=1,3} \{f_j^{wc}\}$ is evaluated

Table 5. 6 : Worst case performance at each vertex	
Vertex #	f^{wc}
1	$f_1^{wc}=1.4449694470662$
2	$f_1^{wc}=1.5204895019153$
3	$f_3^{wc}=0.14075176281749$
4	$f_1^{wc}=1.3357838020720D-02$
5	$f_3^{wc}=0.10755542707058$
6	$f_1^{wc}=1.4438963718764D-02$
7	$f_3^{wc}=3.1900106646531$
8	$f_3^{wc}=3.1356325522016$

The predicted worst case corresponds to the worst case found in Table 5. 6 .

V.3.3.2 Worst-case analysis with statistically independent variable disturbances

The problem formulation can be formulated as follows [19]:

given: *set of design parameters $\underline{\Phi} = [\Phi_1 \ \Phi_2 \dots \Phi_n]^T$ with associated probability

density functions (pdf) p_{Φ_i} $i = 1, \dots, n$

*set of performances of a design $\underline{R} = [R_1 \ R_2 \ R_3 \dots R_m]^T$

*variable τ , such that $\tau = +1$ when an upper specification is imposed and -1 when a lower specification is imposed to the performance

evaluate: the pdf of R_j , p_{R_j} , $j = 1, \dots, m$

and find: the worst case value of R_j , R_j^{wc} , such that the probability of occurrence up to this worst case value is equal to a preassigned probability ($prob$). Or in other words:

$$prob = \int_{-\infty}^{R_j^{wc}} p_{R_j}(r) dr, \quad \text{if } \tau = +1 \quad (5.16)$$

$$prob = 1 - \int_{-\infty}^{R_j^{wc}} p_{R_j}(r) dr, \quad \text{if } \tau = -1 \quad (5.17)$$

For each component of $\underline{\Phi}$, it is possible to define a worst case direction with respect

to the specification on performance R_j as $\mu_i = \text{sign} \left[\tau \frac{\partial R_j}{\partial \Phi_i} \right] \quad i = 1, 2, \dots, n$

If we assume that the functional relationship between Φ_i and R_j is monotonic and the components of $\underline{\Phi}$ are assumed to be statistically independent, the worst case analysis problem can be reformulated as:

given: set of design parameters $\underline{\Phi} = [\Phi_1 \ \Phi_2 \dots \dots \Phi_n]^T$ with associated probability

density functions (pdf) $p_{\Phi_i} \quad i = 1, \dots, m$, and a probability ($prob$)

find: for each performance R_j , the worst case values of $\underline{\Phi}$, $\underline{\Phi}^{wc}$, such that

$$prob = \prod_{i=1}^n prob_{\Phi_i}^{\mu_i} \quad (5.18)$$

$$\text{where} \quad \text{prob}_{\Phi}^i = \int_{-\infty}^{\Phi_i^{\text{rc}}} p_{\Phi_i}(\psi) d\psi, \quad \text{if } \mu_i \text{ is positive} \quad (5.19)$$

$$\text{prob}_{\Phi}^i = 1 - \int_{-\infty}^{\Phi_i^{\text{rc}}} p_{\Phi_i}(\psi) d\psi, \quad \text{if } \mu_i \text{ is negative} \quad (5.20)$$

Furthermore, if we assume that the component of $\underline{\Phi}$ can be estimated by normal distributions, the worst case values of $\underline{\Phi}$ can be calculated from

$$\Phi_i^{\text{wc}} = m_i + \mu_i N^{-1}(\text{prob}_{\Phi}^i) \sigma_i \quad (5.21)$$

where: m_i and σ_i are the estimated mean and variance of the pdf of Φ_i

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x \exp\left[-\frac{x^2}{2}\right] dx \quad (5.22)$$

From the worst case design parameter $\underline{\Phi}^{\text{wc}}$, it is possible to calculate the worst case performance, R_j^{wc} .

V.3.3.2.1 Example

Since the inverse of the function N^{-1} cannot be expressed explicitly, equation (5.21) cannot be applied directly, thus the Newton iteration method is needed in order to find the worst case $\underline{\Phi}^{\text{wc}}$ by solving (5.19) or (5.20).

Let $\underline{\Phi} = [V_d \quad G \quad Bw_R]^T$ be a set of design variables. Each element of $\underline{\Phi}$, Φ_i , follows a Gaussian distribution where the mean m_i is equal to the optimum solution found in

example-2 $\underline{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.24584923914679 \\ 134.90823156636 \\ 65.093616139387 \end{bmatrix}$ and σ_i is equal to a certain percentage of the

optimum solution Φ_i (see Table-7,8,9,10,11)

It is assumed that $prob_{\Phi}^1 = prob_{\Phi}^2 = prob_{\Phi}^3 = \sqrt[3]{prob}$

In the following tables, for each function f_j we find the worst case values of Φ , Φ^{wc} , and the corresponding value of f_j^{wc} .

In Table 5. 7 through to Table 5. 10 below, we fix $prob$ at 100% and we try to find an approximate σ_i $i = 1, 2, 3$; this gives the worst case performance close to the one found in V.3.2.2 (example-2). This is found for $7\sigma_i = 5\%$ of Φ_i^0 (see Table 5. 9).

Table 5. 7 : Worst case performance-1 $prob=100\%; \quad 3 \sigma_i = 5\% \text{ of } \Phi_i^0 \iff \sigma_i = 0.016667 \Phi_i^0$			
f_j^{wc}	V^{wc}	G^{wc}	Bw_R^{wc}
f1=4.0439169478899	0.27974869344195	153.51029490257	56.118041413148
f2=3.8914940464799	0.27974869344195	153.51029490257	74.069197451132
f3=8.0445320411448	0.21194986815717	116.30616292846	74.069197451132

In Table 5. 7 , the worst case is given by f_3 corresponding to $P_e = 3.1167062720539D-06$.

Table 5. 8 : Worst case performance-2 $prob=100\%; \quad 5 \sigma_i = 5\% \text{ of } \Phi_i^0 \iff \sigma_i = 0.01 \Phi_i^0$			
f_j^{wc}	V^{wc}	G^{wc}	Bw_R^{wc}
f1=2.4944214284592	0.26618885074503	146.06949903896	59.708285805512
f2=2.3517038446239	0.26618885074503	146.06949903896	70.478964321856
f3=5.1537638660112	0.22550955552284	123.74696410745	70.478964321856

In Table 5. 8 , the worst case is given by f_3 corresponding to $P_e = 1.7308172226786D-07$.

Table 5. 9 : Worst case performance-3 <i>prob=100%; 7 $\sigma_i = 5\%$ of $\Phi_i^0 \iff \sigma_i = 0.007 \Phi_i^0$</i>			
f_j^{wc}	V^{wc}	G^{wc}	Bw_R^{wc}
f1=1.7597438181205	0.26008697464707	142.72111745690	61.323873512166
f2=1.6317493095166	0.26008697464707	142.72111745690	68.863358853240
f3=3.6819861336279	0.23161150484373	127.09534567582	68.863358853240

In Table 5. 9 , the worst case is given by f_3 corresponding to $P_e = 3.9725215357957D-08$.

Table 5. 10 : Worst case performance-4 <i>prob=100%; 10 $\sigma_i = 5\%$ of $\Phi_i^0 \iff \sigma_i = 0.005 \Phi_i^0$</i>			
f_j^{wc}	V^{wc}	G^{wc}	Bw_R^{wc}
f1=1.2566080100566	0.25601908212360	140.49886547377	62.400943678345
f2=1.1425669671465	0.25601908212360	140.48826547377	67.786288600388
f3=2.6411131360945	0.23567943157101	129.32759765894	67.786288600388

In Table 5. 10 , the worst case is given by f_3 corresponding to $P_e = 1.4028810895375D-08$.

Table 5. 11 : Worst case performance-5 <i>prob=70%; 3 $\sigma_i = 5\%$ of $\Phi_i^0 \iff \sigma_i = 0.0166667 \Phi_i^0$</i>			
f_j^{wc}	V^{wc}	G^{wc}	Bw_R^{wc}
f1=0.5989379376420	0.25082955710336	137.64114988041	63.774975063081
f2=0.51199609714391	0.25082955710336	137.64114988041	66.412257215693
f3=1.2567874553898	0.24086892119021	132.17531325231	66.412257215693

In Table 5. 11 , the worst case is given by f_3 corresponding to $P_e = 3.5141140860579D-09$ with a probability of 70%.

V .3 .4 Yield estimation

During a manufacturing process, outcomes are spread over a region which can be described by a nominal design, $\underline{\Phi}^0$, along with a statistical distribution of parameters [20]. For N statistical outcomes generated, the production yield can be estimated by

$$\text{Yield} \approx \frac{\text{number of acceptable outcomes}}{N} \quad (5 . 23)$$

V .3 .4 .1 Example 1:

Using the same data as in V .3 .3 .2 .1, we try to estimate the Yield by simulation. A specified mean value m_i and standard deviation σ_i of the design variables are given to a random generator that follows a Gaussian distribution. At each outcome, the performance is evaluated and compared to the specified constraints (see Fig–5. 1). The yield is then found by using (5 . 23).

Fig–5. 3 shows the estimated Yield as a function of the number of simulation runs. It is found that the Yield stabilizes at 21%.

Fig-5. 3 Yield estimation

V.3.4.2 Example 2:

The following formulation can be used to find a relation between the worst case performance and the yield.

given: *set of design parameters $\underline{\Phi} = [\Phi_1 \ \Phi_2 \dots \dots \Phi_n]^T$ with associated probability

density functions (pdf) p_{Φ_i} , $i = 1, \dots, m$

find: *a probability ($prob$), and the worst case values of $\underline{\Phi}$, $\underline{\Phi}^{wc}$, such that all performance specifications and constraints at $\underline{\Phi}^{wc}$ are satisfied. Or equivalently

$$U = 0 \quad (5.24)$$

$$\text{where } U \equiv \max_{j=1}^m (f_j) \quad (5.25)$$

$$prob = \prod_{i=1}^n prob_{\Phi_i}^i \quad (prob \text{ is defined in V.3.3.2}) \quad (5.26)$$

U is plotted as function of $prob$ (see Fig-5.4) and (5.24) is solved using the secant method.

The solution for $prob$ is 14% when $U = 0$. This represents the minimum value of the Yield which can be explained by using Fig-5.5:

$\underline{\Phi}^{wc}$ is at the boundary of the acceptable region. The shaded area T represents the area of total outcomes and the acceptable region is given by $R_A = A + B + C$.

$prob$ is equal to $\frac{A}{T}$, where the yield is equal to $\frac{A+B+C}{T}$, hence $prob \leq yield$

Fig-5. 4 Yield estimation using worst case analysis

Fig-5. 5 Two dimensional space representation of the worst case point
corresponding to the minimum value of the yield

V.4 CONCLUSIONS

A brief discussion was given on the derivation of the BER's gradient which constitutes an essential element to build the more detailed analysis of the optimization of the BER. We are able to optimize the performance of a linear Optical Fiber Communication System, initially omitting any constraining specifications and then enhancing the analysis by adding limiting specifications.

The worst case performance analysis, with the use of fluctuating parameters, established a minimum level of optimization possible with the BER. Finally, using the probability distribution of the design parameters we are able to estimate the yield by simulation. The minimum value of the yield is evaluated by using worst case analysis. Further improvements can be achieved by considering a real receiver model where a more accurate system performance can be estimated. It is possible to improve the yield through special optimization techniques.

Chapter–VI

SUMMARY AND CONCLUSIONS

VI.1 SUMMARY

This research was motivated by the growing attention being focused on the optical fiber communication system and the advantages that it presents in the transfer of information. Despite its advantages of potential bandwidth, small signal and weight, electrical isolation, its immunity to interference ... etc, the successful performance of the system is impeded by thermal noise and shot noise as well as the additional obstacle of the intersymbol interference.

In this thesis, a Computer Aided Design (CAD) was developed in order to measure the fiber optic communication system performance. The proposed method, based on the saddlepoint approximation, is used to evaluate the average error probability, and it is expanded to take into account the effect of the receiver impulse response and the ISI.

A brief discussion of various approximation methods was included in order to provide a base for comparison of the techniques used in evaluating the BER. The saddlepoint

approximation proved to be an effective method of evaluation, and several practical examples were given to demonstrate its efficiency.

Through a sensitivity and statistical analysis based on a simple system model, optimization techniques, worst case analysis and yield evaluation were introduced. The effectiveness of the optimization techniques is then demonstrated again through the use of examples.

VI.2 SUGGESTIONS FOR FURTHER RESEARCH

1. Evaluation of the sensitivity of the BER with respect to the receiver components can be achieved through merging the developed tool in this thesis with a circuit simulator such as SPICE. The combined program can be used in conjunction with an optimization package to adjust the receiver parameters in order to minimize the BER.
2. Expand the sensitivity and statistical analysis by using the saddlepoint approximation method.
3. Detailed thermal noise computation can be achieved by combining the developed tool with a circuit simulator.
4. Take into account the effect of nonlinearity of the laser as well as the nonlinear behavior of certain components of the receiver circuit.
5. The system could be expanded to take into effect coding schemes.

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