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**Analysis Of Interconnection Delay And Crosstalk
In
High-Speed VLSI Systems**

by

Tak Kuen Tang, B. Eng.

This thesis is submitted to the Faculty of Graduate Studies and research in
partial fulfillment of the requirements for the degree of

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
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Ottawa, Canada
August, 1990**

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The undersigned recommend to the Faculty of Graduate Studies and
Research the acceptance of the thesis:

**“ANALYSIS OF INTERCONNECTION DELAY AND CROSSTALK
IN
HIGH-SPEED VLSI SYSTEMS”**

submitted by Tak Kuen Tang in partial fulfillment of the requirements for
the degree of Master of Engineering.

Prof. M. S. Nakhla
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Abstract

As circuit speed increases, delay and crosstalk due to interconnections are proved to be the dominant limiting factors for higher speed and higher performance systems. Thus an efficient and reliable numerical method to evaluate interconnection delay and crosstalk becomes an essential tool for high-speed VLSI system design.

This dissertation describes a new approach, *generalized* Asymptotic Waveform Evaluation (GAWE), which is based on asymptotic waveform evaluation and piecewise decomposition techniques. GAWE has the following features:

- A unified formulation approach, which is based on the modified nodal admittance matrix, for describing the network equations including multiconductor transmission lines, terminal and interconnecting networks.
- Interconnect models can contain general linear RLC lumped elements, distributed components such as multiconductor transmission lines, and nonlinear terminations.
- GAWE can handle networks with arbitrary number of multiconductor transmission lines and nonlinear terminations with no topological constraints.
- GAWE can be used to evaluate transient response of interconnect circuits with negligible error compared with conventional circuit simulators such as SPICE while two to three orders of magnitude faster.

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For my parents

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List Of Symbols

U	the identity matrix
D_k	a selector matrix
D	length of the multiconductor transmission line
Γ	propagation matrix
γ_m^2	eigenvalues of the transmission line matrix
S_v	a matrix with the eigenvectors in the columns
Z_p	complex impedance per unit length of the transmission line
Y_p	complex admittance per unit length of the transmission line
v	a column vector representing voltages
i	a column vector representing currents
L	$N \times N$ symmetric matrix representing the inductance per unit length of the transmission line
C	$N \times N$ symmetric matrix representing the capacitance per unit length of the transmission line
R	$N \times N$ symmetric matrix representing the series resistance per unit length of the transmission line
G	$N \times N$ symmetric matrix representing the shunt conductance per unit length of the transmission line
s	the Laplace operator
k_n	complex residues
p_n	complex poles
m_n	moments of the output
$\delta(t)$	impulse excitation
$z(t)$	vector of node voltage waveforms appended by independent voltage source current waveforms and linear inductor current waveforms
W, H	constant matrices with entries determined by the lumped linear components
b	a constant vector with entries determined by the independent voltage and current sources
N_s	the number of linear subnetworks

A_k, B_k complex matrices representing the modified nodal admittance stamp of the multiconductor transmission line

$E(x)$ a diagonal matrix whose elements are $\exp(-\gamma_m x)$

C_1, C_2 constant N dimensional vectors

$e(t)$ vector of independent sources

$f(z)$ a function describing the nonlinear elements

N_f number of nonzero entries in $f(z)$

Chapter 1

Introduction

1.1 Problem Description

During recent years, the rapid advances in the development of the large scale integrated circuit technologies and packaging techniques are yielding smaller devices, finer lines and larger chips. As the dimensions of the devices become smaller, more and more devices can be integrated on the same chip such that this not only increases the complexities and I/O counts of the chip but also increases the numbers of interconnections both on chip and inter-chip. At the same time, the higher integrated level results in a larger chip area, causing the length of the interconnections to become longer whereas the circuits are switching faster due to

the smaller device size. As a result, the interconnection delay time is often significantly longer than the device switching time. Thus the speed performance of the circuit is primarily determined by the delay in the interconnections rather than by the device limitations. In addition, as interconnection densities and switching speeds increase, fast pulses in the active lines can induce transient signals on the neighboring inactive lines via electromagnetic coupling; these transient signals can trigger logic gates and other devices at the terminations resulting in corrupted data transmission. Hence the electromagnetic coupling between neighboring lines become another important factor in determining the overall system performance. This can be observed at the chip, printed circuit board as well as system level, and interconnected blocks could be analog, digital, or mixture of both. Thus an accurate delay and crosstalk analysis of coupled interconnection lines is of great importance for the design and simulation of high-speed VLSI systems.

The common methods for interconnect delay estimation in a large circuit rely upon an RC tree model[1]-[2]. However, these methods are not adequate for high-speed application where crosstalk capacitance and line inductance are significant. Recently, an Asymptotic Waveform Evaluation (AWE) technique [3]-[4] has been published for approximating the waveform response of general linear RLC lumped circuits. In the AWE method, the transient portion of the response is approximated by matching the initial conditions and the first $2q-1$ moments of the exact response to a lower q -pole model. AWE is an efficient method for delay estimation in large circuit. However, with higher signal speeds, the electrical length of interconnects becomes a significant fraction of the wavelength. Consequently, the conventional

lumped-impedance interconnection models become inadequate. In order to accurately predict interconnection delay and crosstalk in the high-speed VLSI systems, a distributed transmission line model should be used. Unfortunately, all approaches for transient analysis of the distributed transmission line model in the literature are not suitable for delay and crosstalk estimation in large circuit. Those approaches usually can provide a detailed analysis of delay and crosstalk, but they require more computer time than the circuit designer can normally afford. This creates the need for a computationally less expensive method which can approximate interconnection delay and crosstalk in an efficient way, and still provide enough information to the circuit designer.

1.2 Thesis Objectives

The first objective of this thesis is to generalize the Asymptotic Waveform Evaluation to handle interconnection models which contain distributed elements. As a practical application, the generalized Asymptotic Waveform Evaluation (GAWE) [5]-[7] is applied to the case where the distributed elements can be modelled as lossy coupled transmission lines.

Furthermore, to precisely model the interconnect delay, the nonlinearities of the termination have to be taken into account. In the case where nonlinear terminations are considered, a problem arises due to the fact that quasi-transverse electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain only. In contrast to this, nonlinear networks can be in general

analyzed in the time domain only. The second objective of this thesis is to combine the GAWE with the Piecewise Decomposition Technique (PDT) [8]-[10], which is a powerful method for solving nonlinear network, to efficiently analyze the high-speed interconnects with nonlinear terminations.

1.3 Thesis Contributions

In this thesis, several contributions have been presented to address the high-speed interconnection problems:

- I. A unified formulation for describing the network equations including multiconductor transmission lines, terminal and interconnecting networks. (Section 4.2 and 4.4.1)
- II. A generalized AWE method is developed to handle interconnect models which contain distributed elements. As a practical application, the GAWE is applied to the case where the distributed elements are modelled as lossy multiconductor transmission lines. The GAWE method can handle networks with arbitrary number of multiconductor transmission lines with no topological constraints. (Section 4.3)
- III. A new method is developed for the evaluation of the moments of the multiconductor transmission lines in a recursive closed form. (Section 4.4.2 and 4.5)

- IV. GAWE can be used to evaluate transient response with negligible error compared with HSPICE while being two to three orders of magnitude faster. This fact is substantiated by several numerical examples. (Section 4.6)
- V. A method is presented to provide an efficient numerical solution for the transient analysis of nonlinear networks with frequency dependent parameters. The method is created by combining the GAWE and PDT in a mixed frequency/time domain approach. The method not only handles networks with arbitrary number of multiconductor transmission lines and nonlinear terminations with no topological constraints but also gives a large CPU advantage over HSPICE and other existing approaches. (Section 5.3-5.4).
- VI. Three different approaches are proposed for the estimation of the dominant poles of multiple-input, multiple-output linear networks and comparison between their computational cost is presented. Application to the simulation of distributed elements is described in Section 5.3-5.4.

1.4 Outline Of the thesis

“Chapter 2. Background and Literature Survey” some existing techniques for the analysis of interconnection delay are reviewed and compared.

“Chapter 3. Asymptotic Waveform Evaluation Technique” provides a summary of the key concepts in AWE.

“Chapter 4. Generalized Asymptotic Waveform Evaluation Technique” the derivation of the GAWE in terms of the Modified Nodal Approach (MNA), and the extensions that allow the interconnect model to contain distributed elements are also presented. The accuracy and efficiency of the GAWE are compared with other published techniques using several practical examples.

“Chapter 5. Piecewise Decomposition Technique” examines the key concepts of the piecewise decomposition technique. The concepts of combining the GAWE with the PDT to simulate multiconductor transmission lines with nonlinear terminations are presented and followed by some computational results.

“Chapter 6. Conclusion And Further Research” the results of the thesis and suggestions for further research are summarized.

Chapter 2

Background And Literature Survey

2.1 Introduction

The advances in integrated circuit technology are leading to a regime in which delay and crosstalk due to the interconnections will be the dominant limiting factor for higher speed and higher performance systems. For this reason, a considerable amount of work has been done in search of an efficient numerical analysis to predict crosstalk and propagation signal delay in the interconnects. In this chapter, some important papers in the literature are reviewed and the limitations of their approaches are also presented.

2.2 RC Tree Model

One of the most famous and widely adopted approach in timing verification of MOS VLSI circuits is the linear RC tree model proposed by Rubinstein, Penfield and Horowitz [1], in the early eighties. The RC tree is an interconnect model in which interconnects are modelled as an RC circuit with capacitors from all nodes to ground, no floating capacitors, no resistor loops, and no resistors to ground. Additionally, the nonlinear resistance and capacitance of MOS transistor are approximated by a linear resistor and capacitor, respectively. A typical RC tree model is shown in Figure 2.1.

Figure 2.1 Example circuit for the RC-Tree. (Adapted from [1].)

2.2.1 Delay Calculation in RC Tree

The estimated signal delay and the error bounds of the delay in the RC tree can be obtained from the following time constants,

$$T_{D_i} = \sum_{k=1}^N R_{ik} C_k \quad (2.1)$$

$$T_{R_i} = \sum_{K=1}^N \frac{R_{ik}^2}{R_{ii}} C_k \quad (2.2)$$

$$T_p = \sum_{K=1}^N R_{kk} C_k \quad (2.3)$$

where

i and k indicate the output node and internal node, respectively.

R_{ik} is the resistance of the portion of the path between the input and i , that is common with the path between the input and node k .

R_{ii} is the resistance between the input and output i .

R_{kk} is the resistance between the input and node k .

C_k is the capacitance of the lumped capacitor at node k .

To illustrate the resistance terms, consider the RC tree in Figure 2.1. For that network, $R_{ik}=R_1+R_2$, $R_{kk}=R_1+R_2+R_3$, and $R_{ii}=R_1+R_2+R_5$.

T_{D_i} is known as "Elmore time constant" [11] at which the transient step response would reach 50 percent of its final value for a monotonic waveform; it can also be interpreted as the first moment, or mean, of the impulse response $\dot{v}_i(t)$:

$$T_{D_i} = \int_0^{\infty} t \dot{v}_i(t) dt \quad (2.4)$$

T_{D_i} can merely provide an approximation for the dominant time constant of the step response. The errors in this type single pole model are occasionally large enough to mislead a designer about the delay in a critical path. For this reason, the upper and lower bounds, T_p and T_{R_i} , are the most valuable information for the circuit designers. [28] shows that the ratio $(T_p - T_{R_i})/T_{R_i}$ is not only a good qualitative indicator of the distance between the bounds but also a measure of the worse-case error in the single pole model with monotonic step responses.

2.2.2 Comments on RC Tree Model

The original RC tree model proposed in [1] is a simplified interconnect model; nevertheless, it is an acceptable and pragmatic approach. Thus, a considerable amount of work [12]-[13] has been done to enhance the linear RC tree model to include resistor loops, initial charges, grounded resistors and even a two-pole-one-zero approximation. As a result of these contributions, the RC tree model for time estimation has been used successfully in many timing simulators and analyzers [15]. The outstanding merit of the RC tree approach is that the single pole approximation can provide a delay estimate for interconnects in MOS circuit without using those time-consuming numerical integration approaches in circuit simulators, such as SPICE [16]. For low- to mid- frequency MOS integrated circuits the RC model can predict, in some case, the delay to within 10 percent of a SPICE simulation but the computational cost can be thousands of times lower, [14].

However, the RC tree approach still suffers from several severe drawbacks:

1) For high-speed circuit and printed circuit board level designs, crosstalk capacitance and line inductance can be significant such that the response waveforms are non-monotonic, therefore, RC tree approach will fail to predict the propagation delay. Consequently, a more sophisticated RLC model should be used in order to model the interconnects in high-speed circuits.

2) RC tree approach is derived by assuming that the input excitation is an ideal step waveform but the propagation signal always exhibits a finite rise-time in practice.

3) Interconnect is fundamentally a distributed system, described by a resistance and capacitance per unit length rather than lumped resistors and capacitors. It would take large number of lumped RC sections to accurately model a distributed line.

2.3 Asymptotic Waveform Evaluation Technique

The Asymptotic Waveform Evaluation (AWE) technique is proposed by Pillage and Rohrer [3]-[4]. AWE is a variable refinement waveform estimator for linear RLC approximate interconnect models. AWE can evaluate circuits with non-monotone detail, such as undershoot, overshoot and glitches due to charge sharing, by increasing the order of the approximation as required. Higher orders of

approximation are obtained recursively, at an incremental cost in CPU time. More about the AWE technique will be presented in chapter 3.

The AWE technique is an efficient method for transient analysis of RLC circuits. Its speed is about the same order as the RC tree method. However, AWE technique still has two major drawbacks; namely, the interconnect models cannot contain distributed elements or nonlinear components.

2.4 Transmission Line Model

In high-speed circuit, the interconnection delay time is often significantly longer than the device switching time. In such case, transmission line effects such as crosstalk and reflections can result from the electromagnetic coupling between interconnects and improper line terminations. In order to accurately predict all those phenomena, interconnects in high-speed digital circuits are best modelled as multiconductor transmission lines. A multiconductor transmission line is uniform along its length with an arbitrary cross section which contains N signal conductors and one reference. The interconnection structure of VLSI chips and printed circuit boards is most frequently embedded in an inhomogeneous medium and thus the waves that propagate along the line are not of the transverse electromagnetic (TEM) nature. However, it can be assumed that a quasi-TEM propagation exists in the multiconductor transmission line. This requires that the cross sectional dimensions of the lines are sufficiently smaller than the wavelength of the highest frequency component of the propagation signal. Along a multiconductor transmission line

generally N different quasi-TEM modes can propagate. For a lossless line with frequency-independent parameters such modes exist in the time domain, and they can also be defined in the frequency domain. The partial differential equations governing the propagation of voltages and currents in a lossless multiconductor transmission can be written in time domain as

$$\frac{\partial}{\partial x} \mathbf{v}(x,t) = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{i}(x,t) \quad (2.5)$$

$$\frac{\partial}{\partial x} \mathbf{i}(x,t) = -\mathbf{C} \frac{\partial}{\partial t} \mathbf{v}(x,t) \quad (2.6)$$

where

$\mathbf{v}$ and $\mathbf{i}$ are column vectors of dimension N representing the line voltages and currents.

$\mathbf{L}$ and $\mathbf{C}$ are real, symmetric N by N matrices representing inductance and capacitance per unit length, respectively, and are related to the physical self and mutual parameters. All the elements of $\mathbf{L}$ are positive whereas $\mathbf{C}$ has positive diagonal elements and negative off-diagonal elements.

However, for a lossy line, in the general case, the quasi-TEM modes can be defined in the frequency domain only. In the frequency domain the multiconductor transmission line can be described by the differential equations

$$\frac{d}{dx} \mathbf{V}(x) = -\mathbf{Z}_p \mathbf{I}(x) \quad (2.7)$$

$$\frac{d}{dx} \mathbf{I}(x) = -\mathbf{Y}_p \mathbf{V}(x) \quad (2.8)$$

where

$\mathbf{V}(x)$ and $\mathbf{I}(x)$ are the vector of complex line voltages and currents, respectively.

Z and Y are the impedance and admittance per unit length of the line, respectively.

$$Z_p = R + sL \quad (2.9)$$

$$Y_p = G + sC \quad (2.10)$$

R and G the N by N series resistance and shunt conductance, respectively. Again they are related to the physical self and mutual parameters.

Numerous approaches and results have been reported and contributed to the evolution of this transmission line model. These includes 1) normal mode analysis; 2) frequency domain analysis and 3) numerical inversion of Laplace transform approach.

2.5 Normal Mode Analysis

Normal Mode approach [17]-[20] is a time domain analysis where the lossless multiconductor transmission line is replaced by an equivalent circuit consisting of N decoupled transmission lines in conjunction with linear dependent sources at the input and output ports. (2.5) and (2.6) can be combined to yield

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \quad (2.11)$$

$$\frac{\partial^2 i}{\partial x^2} = CL \frac{\partial^2 i}{\partial t^2} \quad (2.12)$$

Generally, the products LC and CL are different, but they have the same eigenvectors because of the symmetry. If λ is the diagonal matrix of the eigenvalues

and M is the modal matrix of the eigenvectors of the product LC , substitute $v=Mu$ and $i = (M^{-1})^t j$ into (2.11) and (2.12)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2}{\partial t^2} \lambda u \quad (2.13)$$

$$\frac{\partial^2 j}{\partial x^2} = \frac{\partial^2}{\partial t^2} \lambda j \quad (2.14)$$

Taking the diagonal matrix λ instead of the product LC , (2.13) and (2.14) represent n uncoupled transmission lines. The linear dependent sources at the input and output ports can be represented by

$$\begin{bmatrix} v \\ j \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & M^t \end{bmatrix} \begin{bmatrix} u \\ i \end{bmatrix} \quad (2.15)$$

2.5.1 Comments On Normal Mode Analysis

The algorithm of the normal mode analysis is easy to implement into a standard circuit simulator including SPICE in that all elements of the equivalent circuit are available. The delay and crosstalk in the interconnects with linear terminations as well as nonlinear terminations can be evaluated. However, the normal mode analysis is only capable of handling lossless transmission lines whereas in practice the lossy property of the interconnects are significant, due to the small line geometries, thus requiring the use of lossy transmission line model. Furthermore, if the transmission line parameters are frequency dependent, the line voltage and current can not be represented in time domain. In VLSI circuit and printed circuit board level design, the computational cost of normal mode approach

would be prohibitive due to the large number of interconnects. In addition, the equivalent circuit of the multiconductor transmission line could create numerical instability problem to the standard circuit simulator owing to the transmission line differential equations.

2.6 Frequency Domain Analysis

Frequency domain analysis [21]-[22] is based on separate formulation for the equations describing the transmission lines and the equations describing the terminal and interconnecting networks. The transmission line equations in the frequency domain are shown in section 2.4. At each frequency, the line eigenmodes are evaluated and they are combined with the equations describing the terminal and interconnecting network to obtain the response of the whole system. Time domain waveforms are obtained using the inverse fast Fourier transform (FFT). The derivation of the line eigenmodes is very similar to the normal mode approach except that the eigenvalues and eigenvectors of the complex matrices ZY are evaluated in the frequency domain. Although frequency domain analysis can evaluate delay and crosstalk in the interconnection where a lossy multiconductor transmission line model is applied, this method has a major difficulty when the analysis has to span a time interval of several line transient times. Even for moderately lossy lines, the duration of the response exceeds many transit times of the transmission line network. This makes the uses of inverse FFT techniques inefficient as a large number of points must be added to the analysis to avoid aliasing problems, thus precluding the use of frequency domain analysis for delay and crosstalk

evaluation in a large network. In addition, the absence of a simple error criterion for the inverse FFT makes it difficult to establish confidence in the results.

2.7 Numerical Inversion Of Laplace Transform (NILT) Approach

The method for numerical inversion of the Laplace transform, and its application to the analysis of linear networks containing lossy multiconductor transmission lines, is proposed by Griffith and Nakhla [6],[23]. The method involves the computation of frequency domain function at preassigned complex points and forming a weighted sum. It exactly inverts a certain number of terms of the Taylor series expansion of the time response and is thus equivalent to the methods used for the integration of differential equations.

2.7.1 The Algorithm Of NILT

Consider the Laplace inversion formula [29]

$$v(t) = \frac{1}{2\pi j} \int_{(c-j\infty)}^{(c+j\infty)} V(s) e^{st} ds \quad (2.16)$$

$$= \frac{1}{2\pi j t} \int_{(c-j\infty)}^{(c+j\infty)} V(z/t) e^z dz \quad (2.17)$$

where $z=st$

Approximate the function e^z in (2.17) by a Pade rational function

$$R_{N,M} = \frac{P_N(z)}{Q_M(z)} = \frac{\sum_{i=0}^N (M+N-i)! \binom{N}{i} z^i}{\sum_{i=0}^M (-1)^i (M+N-1)! \binom{M}{i} z^i} \quad (2.13)$$

which has the first $M+N+1$ terms of its Taylor expansion equal to the Taylor expansion of e^z . Inserting (2.18) into (2.17), the approximation $\hat{v}(t)$ to $v(t)$ becomes

$$\hat{v}(t) = \frac{1}{2\pi j t} \int_{(c'-j\infty)}^{(c'+j\infty)} V(z/t) R_{N,M}(z) dz \quad (2.19)$$

The integral (2.19) can be evaluated by residue calculus by choosing the path of integration along an infinite arc either to the right or to the left. In order that the path along the infinite arc not contributes to the integral, choose M, N such that the function

$$F(z) = V(z/t) R_{N,M}(z) \quad (2.20)$$

has at least two more finite poles than zeros. Then

$$\int_c F(z) dz = \pm 2\pi j (\Sigma \text{ residue at pole inside the closed path}) \quad (2.21)$$

where the sign depends on the direction in which the path is closed. For $N < M$,

$$R_{N,M}(z) = \sum_{i=1}^M \frac{K_i}{z - z_i} \quad (2.22)$$

where z_i are the poles of $R_{N,M}(z)$ and K_i are the corresponding residues. Closing the path of integrating around the poles in the right half plane we get

$$\hat{v}(t) = -(1/t) \sum_{i=1}^M K_i V(z_i/t) \quad (2.23)$$

which is the formula for the calculation of the time response. A collection of tables for the poles z_i and residues K_i can be found in [30] for different values of M and N . It was proved in [31] that the inversion method given by (2.23) is equivalent to the integration of differential equations with the order of integration equal to $M+N$.

The time domain functions can be evaluated using only the poles z_i in the upper half plane. For M even

$$\hat{v}(t) = -(1/t) \sum_{i=1}^{M'} \text{Re} [K'_i V(z_i/t)] \quad (2.24)$$

where $M'=M/2$, $K'_i=2K_i$. When M is odd, $M'=(M+1)/2$ and $K'_i=K_i$ for the residue corresponding to the real pole.

A linear network containing lossy multiconductor transmission lines can be represented in the nodal formulation

$$Y(s) V(s) = I(s) \quad (2.25)$$

where Y represents the node admittance matrix, V the common datum voltages, and I the current source vector. The time domain response can be obtained from

$$\hat{v}(t) = -(1/t) \sum_{i=1}^{M'} \text{Re} [K'_i [Y(z_i/t)]^{-1} [I(z_i/t)]] \quad (2.26)$$

The response at each time point is obtained from the M' solutions to the network equation $[Y(s)]^{-1}[I(s)]$ evaluated at the complex frequencies $s=z_i/t$. An admittance representation of lossy multiconductor transmission line is given by [23]

$$Y = \begin{bmatrix} S_i E_1 S_v^{-1} & S_i E_2 S_v^{-1} \\ S_i E_2 S_v^{-1} & S_i E_1 S_v^{-1} \end{bmatrix} \quad (2.27)$$

where S_i , S_v , E_1 , and E_2 are a function of the eigenvalues and eigenvector whereas these eigenvalues and eigenvectors are a function of the transmission line parameters $[R]$ $[G]$ $[L]$ $[C]$ and the Laplace operator s .

2.7.2 Comments On NILT Approach

NILT is a robust approach for lossy coupled transmission line analysis. It does not suffer from the aliasing problems associated with the inverse FFT-based method. [23] shows that the number of frequency points defined by $M' < 15$ is usually much less than the number of frequency points required by the inverse FFT algorithm for the same accuracy in the output waveforms. This leads to considerable saving in the CPU requirements. The NILT can be used for detailed and accurate analysis. However, very often in the design stage, the circuit designer only requires knowledge of the delay and crosstalk estimation along each path rather than an exact

description of each rising or falling waveform such that the NILT approach become too computationally expensive for timing estimation in the design stage.

2.8 Mixed Frequency/Time Domain Approach

If the nonlinearities of the termination are taken into account, a problem arises due to the fact that quasi-transverse electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain only. In contrast to this, nonlinear network can be in general analyzed in the time domain only. In order to solve the problem, the solutions in the two domains are required. A mixed frequency/time domain approach [22] has been applied to the case of lossy transmission lines with nonlinear terminations. The method uses the frequency-domain data associated with the linear problem in the absence of nonlinearities. The solution of the nonlinear problem is obtained by time-stepping and convolution. The computational steps involves the evaluation of time domain Green's functions of the transmission lines terminated in quasi-matched loads. To compensate for the change in the termination, complementary networks with negative elements are inserted into the terminal networks. The method is computationally expensive since it requires evaluation of set of convolution integrals at each time point. In addition, since these methods are targeted towards specific applications, they tend to be restricted, particularly in terms of the topology of the networks to be analyzed.

Another method is presented in [8]-[10]. The method is general and not limited to the analysis of transmission lines. It is based on a piecewise decomposition

technique where the nonlinear network is decomposed into linear and nonlinear subnetworks. The linear subnetworks contain all the frequency-dependent components. The terminals of the subnetworks are excited by piecewise linear time-dependent sources. Through solution of the linear subnetworks in the frequency domain, the nonlinear subnetworks in the time domain, and by applying Newton-Raphson iteration technique, the parameters of the piecewise waveforms are adjusted so that the topological and constitutive relations of each subnetwork as well as the topological equations resulting from the interconnection of these subnetworks are satisfied. This method does not impose any restrictions on the topology of the linear or nonlinear subnetworks or on their interconnections. More about this method will be presented in chapter 5.

The methods in [22] and [8]-[10] are computationally expensive because they are using FFT and NILT approaches to analyze the linear subnetworks of the nonlinear network. Thus both methods are not efficient for interconnection delay and crosstalk estimation in VLSI systems.

2.9 Comparisons Between The Published Techniques And The Current Research

The methods in the literature are two extreme; the computational cost of RC-tree and AWE methods are very low but they cannot handle coupled transmission lines or nonlinear terminations. On the other hand, the methods which are capable of handling lossy coupled transmission lines and nonlinear terminations require more computer time than the circuit designer can normally afford.

This research has developed a method which not only provides excellent accuracy for transient response analysis of lossy coupled transmission lines with nonlinear terminations but also has the same speed advantage as RC-tree method. Comparisons between the published techniques and the current research are presented in Table 2.1.

Approach Application	RC-Tree [1]-[2]	AWE [3]-[4]	Distributed Transmission Line Model			GAWE (method in this thesis)
			Frequency domain [21]	Time domain [17]-[20]	Mixed Freq/Time domain[22], [8]-[10]	
Inductive	No	Yes	Yes	Yes	Yes	Yes
Distributed elements	No	No	Yes	Yes	Yes	Yes
Topological Constraints	Yes	No	No	No	No	No
Finite Rise time	No	Yes	Yes	Yes	Yes	Yes
Nonlinear	No	No	No	Yes	Yes	Yes
Lossy Trans. Lines	No	No	Yes	No	Yes	Yes
Freq. Dependent Parameters	No	No	Yes	No	Yes	Yes
Accuracy	Low	Good*	High	High	High	Good*
CPU Requirement	Low	Low**	High	High	High	Low**

* Within 5-10% from the exact solution.

** Same order as the RC-tree method.

Table 2.1 Comparisons between the published techniques and the current research

Chapter 3

Asymptotic Waveform Evaluation Technique

3.1 Introduction

Recently, a method, Asymptotic Waveform Evaluation (AWE)[3]-[4], has been developed at Carnegie Mellon University. AWE is a variable refinement waveform estimator for generalized RLC approximate interconnect models and is not just limited to the tree structures. It is based on deriving the moments of the circuit response. Higher order moments can be used to approximate the response, at an incremental cost in CPU time. This makes the method suitable to estimate the circuit responses with non-monotonic detail such as undershoot, overshoot and

glitches. In this chapter, some of the key concepts in Asymptotic Waveform Evaluation technique are reviewed.

3.2 The AWE Algorithm

AWE method can be explained in two different ways: 1) particular solution approach and 2) transfer function approach.

3.2.1 Particular Solution Approach

In general, the differential state equations for a lumped, linear, time-invariant circuit can be expressed as

$$\dot{x} = Ax + bu \quad (3.1)$$

where x is the n -dimensional state vector and u is the m -dimensional excitation vector. In order to investigate of the delay and rise time effects in the interconnects, the input excitations are assumed to be

$$u_p(t) = u_0 + u_1 t \text{ for } t > t_0 \quad (3.2)$$

where u_0 and u_1 , are constant m -dimensional vectors.

For the excitation u_p , the differential-state equation (3.1) has the particular solution

$$x_p(t) = -A^{-1}bu_0 - A^{-2}bu_1 - A^{-1}bu_1 t \quad (3.3)$$

This particular solution is valid provided that the A matrix is not singular. That is not an unreasonable restriction for the interconnect circuit models of interest because that condition actually requires that the circuit have a unique and well defined dc

solution when all of its capacitance are open- circuit and all of its inductances are short-circuit. To complete the solution of (3.2), the homogeneous equation needs to be solved

$$\dot{x}_h = Ax_h \quad (3.4)$$

with the initial condition

$$x_h(0) = x_0 + A^{-1}bu_0 + A^{-2}bu_1 \quad (3.5)$$

where x_0 is the initial state at time zero. The Laplace transform solution of the homogeneous equation is

$$x_h(s) = (sI - A)^{-1}x_h(0) \quad (3.6)$$

To approximate this solution, x_h is expanded in a Maclaurin's series

$$x_h(s) = -A^{-1}(1 + A^{-1}s + A^{-2}s^2 + \dots)x_h(0) \quad (3.7)$$

and then match as many moments as necessary or desirable in terms of lower-order approximating functions. Focusing now on a specific component of $x_h(s)$, say the output j th, its initial conditions and first $2q-1$ moments are characterized as

$$\begin{aligned} [m_{-1}]_j &= [x_h(0)]_j \\ [m_0]_j &= [-A^{-1}x_h(0)]_j \\ [m_1]_j &= [-A^{-2}x_h(0)]_j \\ &\vdots \\ [m_{2q-2}]_j &= [-A^{-2q+1}x_h(0)]_j \end{aligned} \quad (3.8)$$

These moments will then be matched to a lower order frequency-domain function of the form

$$\begin{aligned} X_j^*(s) &= \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \dots + \frac{k_q}{s-p_q} \\ &= \sum_{n=1}^q \frac{k_n}{s-p_n} \end{aligned} \quad (3.9)$$

where p_n and k_n are the complex approximation poles and residues respectively. The time domain representation of $X_j^*(s)$ is of the form

$$x_j^*(t) = \sum_{n=1}^q k_n e^{p_n t} \quad (3.10)$$

The reciprocal poles of the approximation function $x_j^*(t)$ can be found from the roots of the characteristic equation

$$a_0 + a_1 p^{-1} + a_2 p^{-2} + \dots + p^{-q} = 0 \quad (3.11)$$

where the coefficients a_n are found by solving

$$\begin{bmatrix} m_{-1} & m_0 & m_1 & \dots & m_{q-2} \\ m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ \vdots & \vdots & \vdots & \dots & \vdots \\ m_{q-2} & m_{q-1} & m_q & \dots & m_{2q-3} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \vdots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_{q-1} \\ m_q \\ m_{q+1} \\ \vdots \\ m_{2q-2} \end{bmatrix} \quad (3.12)$$

The corresponding residues are then obtained using the approximate poles from (3.12) solving

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ p_1^{-1} & p_2^{-1} & p_3^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & p_3^{-2} & \dots & p_q^{-2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{-q+1} & p_2^{-q+1} & p_3^{-q+1} & \dots & p_q^{-q+1} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_q \end{bmatrix} = \begin{bmatrix} m_{-1} \\ m_0 \\ m_1 \\ \vdots \\ m_{q-2} \end{bmatrix} \quad (3.13)$$

Once the poles and the residues for the reduced model are found, the approximation for the homogeneous solution can be obtained using equation (3.10). The complete approximate solution for the response is equal to

$$x_j(t) = x_{p_j}(t) + x_j^*(t) \quad (3.14)$$

3.2.2 Transfer Function Approach

AWE can also be explained in terms of the impulse response

$$\dot{x} = Ax + b\delta(t) \quad (3.15)$$

where $\delta(t)$ is the impulse excitation.

The Laplace transform solution of (3.15) is

$$X(s) = (sI - A)^{-1}b \quad (3.16)$$

To approximate this solution, $X(s)$ is expanded in a Maclaurin's series

$$X(s) = -A^{-1}(1 + A^{-1}s + A^{-2}s^2 + \dots)b \quad (3.17)$$

(3.17) can be approximated by a lower-order function x_j^* as shown in (3.10)

and the moments of the output j th are characterized as

$$\begin{aligned} [m_0]_j &= [-A^{-1}b]_j \\ [m_1]_j &= [-A^{-2}b]_j \\ &\vdots \\ [m_{2q-1}]_j &= [-A^{-2q}b]_j \end{aligned} \quad (3.18)$$

The coefficients a_n of the characteristic polynomial of the reduced model are related to the moments by

$$\begin{bmatrix} m_0 & m_1 & m_2 & \dots & m_{q-1} \\ m_1 & m_2 & m_3 & \dots & m_q \\ m_2 & m_3 & m_4 & \dots & m_{q+1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ m_{q-1} & m_q & m_{q+1} & \dots & m_{2q-2} \end{bmatrix} \begin{bmatrix} -a_0 \\ -a_1 \\ -a_2 \\ \vdots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_q \\ m_{q+1} \\ m_{q+2} \\ \vdots \\ m_{2q+1} \end{bmatrix} \quad (3.19)$$

the roots of the characteristic polynomial are the reciprocal poles and the residues of $x_j^*(s)$ are related to these poles by

$$\begin{bmatrix} p_1^{-1} & p_2^{-1} & p_3^{-1} & \dots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & p_3^{-2} & \dots & p_q^{-2} \\ p_1^{-3} & p_2^{-3} & p_3^{-3} & \dots & p_q^{-3} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ p_1^{-q} & p_2^{-q} & p_3^{-q} & \dots & p_q^{-q} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_q \end{bmatrix} = \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{q-1} \end{bmatrix} \quad (3.20)$$

$x_j^*(s)$ is the transfer function of the reduced order model such that the frequency response corresponding to an input function $F(s)$ can be easily obtained as $x_j^*(s)F(s)$.

3.3 Practical Considerations [4]

The computation of the power of A^{-1} to obtain the moments (3.8) looks to be more complicated than it actually is. In order to illustrate how simply all those required moments can be found using a succession of dc analysis, consider the calculation of the moments for the simple RLC circuit in Figure 3.1. The desirable moments are obtained from the dc circuit in Figure 3.2, where the capacitor and inductor in Figure 3.1 have been replaced by a current source and a voltage source, respectively. The moments that we seek are those for the energy storage elements and all the output variables of interest. For this example we are interested in approximating the impulse response at V_{out} . Thus the set of moments, m_i , comprises the i^{th} moment for V_{out} , the capacitor and the inductor. The initial set of moments m_0 is determined by performing a dc analysis on the circuit in Figure 3.2 with $V_{in}=1$ and open-circuit the capacitor and short-circuit the inductor. The resulting voltage, $[v_c]_0$,

across the current source, in this case equal to V_{out} , and current, $[i_L]_0$, across the voltage source are the initial set of moments m_0 . Finding these moments required first formulating the circuit equations, then LU factorizing these equations to find the voltages and currents. This is equivalent to performing a dc analysis of the circuit. All succeeding moments for this circuit are found using this original dc solution, i.e., the LU factors remain unchanged, with a new set of input stimuli. With V_{in} now set equal to zero and the capacitor current source and inductor voltage source set equal to a value $C[v_c]_0$ and $L[i_L]_0$, respectively, the same dc analysis with this new set of dc inputs yields the set of first moments, m_1 . The next set of moments, m_2 , are found similarly by setting the current source equal to the value $C[v_c]_1$ and the inductor voltage source equal to the value $L[i_L]_1$. Then solve the circuit to obtain the voltage and current for moment, m_2 . This dc analysis is repeated recursively until the desired number of moments are calculated.

Figure 3.1 RLC example circuit.

Figure 3.2 Equivalent circuit for evaluation of moments.

3.4 Numerical Examples

The AWE technique has been implemented as part of this thesis. Several circuits have been analyzed including those are presented in [3]-[4]. To illustrate the accuracy of the AWE technique, consider the RLC circuit shown in Figure 3.3. The input voltage is a pulse with 1 nanosecond rise/fall time and 5 nanoseconds pulse width. The comparison between HSPICE simulation and the AWE approximation is shown in Figure 3.4. At a second order approximation, AWE is not able to completely detect the overshoot, therefore, a higher order approximation is required. At fourth order, AWE response is almost identical to the HSPICE waveform.

To illustrate the efficiency of the AWE, some CPU comparisons with HSPICE are presented in Table 3.1. This table is extracted from [24].

circuit name	node count	element count	CPU seconds	
			AWE	HSPICE
small RC tree	19	35	0.02	0.40
medium sized interconnect	75	121	0.04	0.63
backplane ⁽¹⁾	505	2294	3.54	⁽²⁾
clock distribution line ⁽³⁾	14,000	55,000	300	37271

⁽¹⁾ backplane is modelled by 2000 inductors and some controlled sources, resistors, and capacitor.

⁽²⁾ HSPICE could not simulate of presence of inductor loops.

⁽³⁾ clock distribution line is modelled by 40,000 resistors and 15,000 capacitors.

Table 3.1 CPU comparison between AWE and HSPICE

Figure 3.3 Circuit for example 3.1.

Figure 3.4 Transient response of the circuit shown in Figure 3.3.

3.5 Comments On AWE Technique

I. AWE technique is a robust method to predict the signal delay in the interconnections:

- 1) It can resolve those limitations in RC tree approaches such as floating capacitors, inductors and nonzero input rise times.
- 2) The simple error criterion of considering the difference between two consecutive order of the approximate response waveform can provide the users with good confidence in the results.

II. Based on our observation, some problems associated with AWE are worth to mention:

- 1) When the eigenvalues of A^{-1} are very small, or very large, the powers of A^{-1} , and therefore the moments, change very rapidly. The large variation in moment values may cause the moment matrix in (3.12) and (3.19) to become ill-condition and near singular. For such cases, higher orders of approximation cannot be obtained unless the moment values are scaled. In AWE, frequency scaling approach is used and the scale factor is m_{-1}/m_0 . This frequency scaling can postpone the instability problem in the moment matrix.
- 2) AWE could generate some right half plane poles for a stable, linear circuit especially at higher orders of approximation. Fortunately, all the examples have been analyzed show that a lower order of approximation is enough to accurately predict the signal delay.
- 3) With higher signal speeds, electrical length of interconnects becomes a significant fraction of the wavelength. Consequently, the conventional lumped-impedance interconnection models become inadequate. Instead, a distributed transmission line model should be used. Thus the

AWE technique presented in this chapter becomes impossible to predict crosstalk and signal delay in such interconnects. In next chapter, we will present a *generalized* AWE technique which is capable of predicting crosstalk and signal delay in high-speed circuits where interconnects are modelled as coupled transmission lines.

Chapter 4

Generalized Asymptotic Waveform Evaluation Technique

4.1 Introduction

In order to accurately predict signal delay and crosstalk in high-speed VLSI circuits, interconnects should be modelled as multiconductor transmission lines. However, all approaches for transient analysis of the multiconductor transmission line in the literature require more computer time than the circuit designer can normally afford. In this chapter, a generalized Asymptotic waveform Evaluation (GAWE) technique is proposed. GAWE not only preserves all outstanding merit of the AWE method but also handles networks with arbitrary number of

multiconductor transmission lines with no topological constraints. Furthermore, the computational results of the GAWE show that the evaluation of moments of the network which contains multiconductor transmission lines is very efficient because the moments of the transmission line can be evaluated using a closed form expression.

4.2 Formulation Of The Network Equations

Consider a linear network π which contains linear lumped components and arbitrary linear subnetworks. The arbitrary subnetworks may contain distributed components. Without loss of generality, the Modified Nodal Admittance (MNA) matrix equations [25] of the network π with an impulse input excitation can be written as

$$W \frac{d}{dt} z(t) + H z(t) + \sum_{k=1}^{N_s} D_k i_k(t) - b \delta(t) = 0 \quad (4.1)$$

where

$z(t) \in \mathcal{R}^N$ is the vector of node voltage waveforms appended by independent voltage source current waveforms and linear inductor current waveforms,

$W \in \mathcal{R}^{N \times N}$ and $H \in \mathcal{R}^{N \times N}$ are constant matrices with entries determined by the lumped linear components,

$b \in \mathcal{R}^N$ is a constant vector with entries determined by the independent voltage and current sources,

$D_k = [d_{ij}]$, $d_{ij} \in \{0,1\}$, $i \in \{1,2, \dots, N\}$, $j \in \{1,2, \dots, n_k\}$ with a maximum of one nonzero in each row or column is a selector matrix that maps $i_k(t) \in \mathfrak{R}^{n_k}$ the vector of currents entering the linear subnetworks k , into the node space $\mathfrak{R}^N$ of the network π ,

N_s is number of linear subnetworks,

$\delta(t)$ is the unit impulse function.

Assume the frequency domain equations of the linear subnetwork k to be in the form

$$A_k V_k(s) + B_k I_k(s) = 0 \quad (4.2)$$

where V_k and I_k represent the Laplace domain terminal voltages and currents of the subnetwork k , respectively.

In the special case where the subnetwork k consists of a multiconductor transmission line system, A_k and B_k can be described in terms of the line parameters. The detailed derivation is presented in Section 4.4.

Using (4.2), the Laplace transform of (4.1) is

$$\begin{bmatrix} sW + H & D_1 & D_2 & \dots & D_{N_s} \\ A_1 D_1^t & B_1 & 0 & \dots & 0 \\ A_2 D_2^t & 0 & B_2 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ A_{N_s} D_{N_s}^t & 0 & 0 & \dots & B_{N_s} \end{bmatrix} \begin{bmatrix} Z(s) \\ I_1(s) \\ I_2(s) \\ \cdot \\ \cdot \\ I_{N_s}(s) \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad (4.3)$$

or

$$[Y(s)]X(s) = E \quad (4.4)$$

where

$$X(s) = [Z(s) I_1(s) I_2(s) \dots I_{N_s}(s)]^t,$$

$$E = [b \ 0 \ 0 \ 0 \ \dots \ 0]^t,$$

and t denotes transpose.

Example:

To illustrate the formulation method, consider the network shown in Figure 4.1. The network can be described by (4.3) where:

$$W = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad H = \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G_3 & 0 & G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X(s) = [Z(s) \ I_1(s)]^t$$

$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I_1(s) = [I_a \ I_b \ I_c \ I_d]^t$$

$$Z(s) = [V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ I_5]^t$$

$$b = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^t$$

Figure 4.1 An example circuit contains components with frequency dependent parameters.

A_1 and B_1 are 4 by 4 matrices such that

$$A_1 \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + B_1 \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = 0$$

derivation A_1 and B_1 will be presented in Section 4.4.

4.3 Evaluation Of The Moments

To approximate the transient solution $x(t)$, (4.4) is expanded in Maclaurin's series in the form

$$X(s) = \sum_{n=0}^{\infty} M_n s^n \quad (4.5)$$

where

$$M_n = \frac{\frac{\partial^n [Y^{-1}]}{\partial s^n} \Big|_{s=0}}{n!} E \quad (4.6)$$

The moments $[M_n]_{(i)}$, $n = 0, 1, 2, \dots, 2q-1$ of an output i are then matched to a lower order frequency domain function in the form

$$[Y^*(s)]_{(i)} = \sum_{j=1}^q \frac{k_j}{s - p_j} \quad (4.7)$$

Given the moments $[M_n]_{(i)}$, evaluation of the poles p_j and the residues k_j ; $j=1,2,\dots,q$ is described in details in (3.19) and (3.20).

From (4.6) it can be seen that the derivatives of Y^{-1} are required to evaluate the moments M_n . However it is extremely difficult, in the general case, to explicitly evaluate Y^{-1} as function of s . For this reason, we first write the derivative of Y^{-1} in terms of the derivative of Y . Since

$$[Y] [Y^{-1}] = U \quad (4.8)$$

where U is the identity matrix. Differentiating (4.8) using Leibnitz's rule:

$$\begin{aligned} ([Y] [Y^{-1}])^{(n)} &= [Y]^{(n)} [Y^{-1}] + \sum_{r=1}^n \binom{n}{r} [Y]^{(n-r)} [Y^{-1}]^{(r)} \\ &= 0 \end{aligned} \quad (4.9)$$

or

$$[Y_0] [Y^{-1}]^{(n)} = - \sum_{r=1}^n \binom{n}{r} [Y]^{(r)} [Y^{-1}]^{(n-r)} \quad (4.10)$$

where $^{(n)}$ indicates n th derivative at $s=0$ and $Y_0=[Y]_{s=0}$. Using (4.6) and (4.10), a recursive equation for the evaluation of the moments can be obtained in the form

$$[Y_0] M_n = - \sum_{r=1}^n \frac{[Y]^{(r)} M_{n-r}}{r!} \quad (4.11)$$

with

$$[Y_0] M_0 = E \quad (4.12)$$

Next the derivatives of Y are obtained as

$$[Y]^{(1)} = \begin{bmatrix} W & 0 & 0 & \dots & 0 \\ A_1^{(1)} D_1^t & B_1^{(1)} & 0 & \dots & 0 \\ A_2^{(1)} D_2^t & 0 & B_2^{(1)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ A_{N_s}^{(1)} D_{N_s}^t & 0 & 0 & \dots & B_{N_s}^{(1)} \end{bmatrix} \quad (4.13)$$

and

$$[Y]^{(r)} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ A_1^{(r)} D_1^t & B_1^{(r)} & 0 & \dots & 0 \\ A_2^{(r)} D_2^t & 0 & B_2^{(r)} & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & 0 \\ A_{N_s}^{(r)} D_{N_s}^t & 0 & 0 & \dots & B_{N_s}^{(r)} \end{bmatrix} ; (r \geq 2) \quad (4.14)$$

(4.14) shows that in the *special case* where the network contains lumped components only,

$$[Y]^{(r)} = 0 \quad ; (r \geq 2) \quad (4.15)$$

and (4.11) can be reduced to

$$[Y_0] M_n = -[Y]^{(1)} M_{n-1} \quad (4.16)$$

which is equivalent to the results reported in [3]-[4] for lumped RLC networks.

In case the subnetwork k consists of a multiconductor transmission line system, we need to solve two sets of Telegrapher's equations which govern the propagation of voltages and currents in the system, to obtain the A_k and B_k matrices. The detailed derivation of A_k and B_k and the corresponding derivatives are described in the next section.

4.4 The Multiconductor Transmission Line System

4.4.1 The Multiconductor Transmission Line Stamp

Consider a multiconductor transmission line formed by $(N+1)$ conductors, one of which is chosen as the reference ground. This multiconductor transmission line can be treated as an $2N$ port network with a Modified Nodal Admittance representation of the form

$$AV + BI = 0 \quad (4.17)$$

where

V is the $2N \times 1$ vector for the terminal voltage of the line with respect to the reference conductor.

I is the $2N \times 1$ vector for the terminal current of the line with respect to the reference conductor.

A and B are the $2N \times 2N$ complex matrices and can be described in terms of the line parameters.

The reference directions for the voltage and current signals on a multiconductor transmission line are shown in Figure 4.2. Let x represent the length along the multiconductor transmission line, with $x=0$ corresponding to the near end, and $x=D$ corresponding to the far end. In the frequency domain the multiconductor transmission line can be described by the following differential equations as shown in Chapter 2

$$\frac{d}{dx}V(x) = -Z_p I(x) \quad (4.18)$$

$$\frac{d}{dx}I(x) = -Y_p V(x) \quad (4.19)$$

The two differential equations can be combined into a set of wave equation

$$\frac{d^2}{dx^2}V(x) = Z_p Y_p V(x) \quad (4.20)$$

$$\frac{d^2}{dx^2}I(x) = Y_p Z_p I(x) \quad (4.21)$$

which will have solution of the form

$$V_m(x) = V_m(0) \exp(\pm \gamma_m x) \quad (4.22)$$

$$I_m(x) = I_m(0) \exp(\pm \gamma_m x) \quad (4.23)$$

where γ_m is the complex propagation mode constant. $V_m(0)$ and $I_m(0)$ are vectors of complex constants. The plus and minus sign in (4.22) and (4.23) represent the reflected wave and the incident wave, respectively.

Substituting the solution form into the wave equation (4.20) yields

Figure 4.2 A multiconductor transmission line.

$$\left\{ \gamma_m^2 U - Z_p Y_p \right\} V_m(0) = 0 \quad (4.24)$$

where U is an identity matrix. $V_m(0)$ will have nontrivial solutions if γ_m^2 satisfies the eigenvalue problem

$$\det \left\{ \gamma_m^2 U - Z_p Y_p \right\} = 0 \quad (4.25)$$

In general there exist n distinct eigenvalue γ_m^2 , $m=1, N$. Each eigenvalue has its corresponding eigenvector S_m . Let Γ be a diagonal matrix whose elements are the complex propagation constants $\{\gamma_1, \gamma_2, \dots, \gamma_N\}$, and S_v be a matrix with the eigenvectors $\{S_1, S_2, \dots, S_N\}$ in the columns. Solution for the voltages can now be written as

$$V(x) = S_v [E(x)] C_1 + S_v [E(x)]^{-1} C_2 \quad (4.26)$$

where

$E(x)$ is a diagonal matrix whose elements are $\exp(-\gamma_m x)$, $m=1, \dots, N$.

C_1 and C_2 are constant N dimensional vector coefficients.

From (4.26) and (4.18), we get

$$I(x) = S_i [E(x)] C_1 - S_i [E(x)]^{-1} C_2 \quad (4.27)$$

where

$$S_i = Z_p^{-1} S_v \Gamma \quad (4.28)$$

A relationship between the near and far end voltage can be obtained from (4.26) as

$$\begin{bmatrix} V(0) \\ V(D) \end{bmatrix} = \begin{bmatrix} S_v & S_v \\ S_v E(D) & S_v [E(D)]^{-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (4.29)$$

Similarly, a relationship between the near and far end current from (4.27) can be written as

$$\begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} S_i & -S_i \\ S_i E(D) & -S_i [E(D)]^{-1} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad (4.30)$$

Using (4.30) and (4.29), we get

$$\begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} S_i & -S_i \\ S_i E(D) & -S_i [E(D)]^{-1} \end{bmatrix} \begin{bmatrix} S_v & S_v \\ S_v E(D) & S_v [E(D)]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} V(0) \\ V(D) \end{bmatrix} \quad (4.31)$$

To get the correct sense as required for the MNA representation as Figure 4.2, we have to multiply $I(D)$ by -1. After some algebraic manipulation, (4.31) can be written as

$$AV + BI = 0 \quad (4.32)$$

where

$$E_1 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) + \exp(\gamma_m D)}{2} \right\}, m = 1, \dots, N \quad (4.33)$$

$$E_2 = \text{diagonal} \left\{ \frac{\exp(-\gamma_m D) - \exp(\gamma_m D)}{2} \right\}, m = 1, \dots, N \quad (4.34)$$

$$A = \begin{bmatrix} S_v E_1 S_v^{-1} & -U \\ S_i E_2 S_v^{-1} & 0 \end{bmatrix} \quad (4.35)$$

$$B = \begin{bmatrix} S_v E_2 S_i^{-1} & 0 \\ S_i E_1 S_i^{-1} & -U \end{bmatrix} \quad (4.36)$$

$$V = \begin{bmatrix} V(0) \\ V(D) \end{bmatrix} = \begin{bmatrix} V_{near} \\ V_{far} \end{bmatrix}$$

$$I = \begin{bmatrix} I(0) \\ I(D) \end{bmatrix} = \begin{bmatrix} I_{near} \\ I_{far} \end{bmatrix}$$

A and B are function of the eigenvalues γ_m^2 and the eigenvectors S_m of (4.24). These eigenvalues and eigenvectors are function of the transmission line parameters R, L, G, C and the Laplace operator s.

4.4.2 The Transmission Line Moments

As mentioned in previous sections, in order to find the moments of the network, we need to find the derivative of the A and B matrices. From (4.35) and (4.36), the derivative of A and B can be written as

$$A^{(r)} = \begin{bmatrix} [S_v E_1 S_v^{-1}]^{(r)} & 0 \\ [S_i E_2 S_v^{-1}]^{(r)} & 0 \end{bmatrix} \quad (4.37)$$

$$B^{(r)} = \begin{bmatrix} [S_v E_2 S_i^{-1}]^{(r)} & 0 \\ [S_i E_1 S_i^{-1}]^{(r)} & 0 \end{bmatrix} \quad (4.38)$$

Let

$$S_v E_1 S_v^{-1} = \phi_a \quad (4.39)$$

or

$$S_v E_1 = \phi_a S_v \quad (4.40)$$

Differentiating (4.40), we get

$$\Phi_a^{(n)} S_v = - \sum_{r=1}^n \binom{n}{r} \phi_a^{(n-r)} S_v^{(r)} + \sum_{r=1}^n \binom{n}{r} S_v^{(n-r)} E_1^{(r)} + S_v^{(n)} E_1 \quad (4.41)$$

Similarly,

$$\Psi_a^{(n)} S_v = - \sum_{r=1}^n \binom{n}{r} \Psi_a^{(n-r)} S_v^{(r)} + \sum_{r=1}^n \binom{n}{r} S_i^{(n-r)} E_2^{(r)} + S_i^{(n)} E_2 \quad (4.42)$$

$$\Phi_b^{(n)} S_i = - \sum_{r=1}^n \binom{n}{r} \Phi_b^{(n-r)} S_i^{(r)} + \sum_{r=1}^n \binom{n}{r} S_v^{(n-r)} E_2^{(r)} + S_v^{(n)} E_2 \quad (4.43)$$

$$\psi_b^{(n)} S_i = - \sum_{r=1}^n \binom{n}{r} \psi_b^{(n-r)} S_i^{(r)} + \sum_{r=1}^n \binom{n}{r} S_i^{(n-r)} E_1^{(r)} + S_i^{(n)} E_1 \quad (4.44)$$

where

$$\psi_a = S_i E_2 S_v^{-1}$$

$$\phi_b = S_v E_2 S_i^{-1}$$

$$\psi_b = S_i E_1 S_i^{-1}$$

Next, the derivatives of E_1 , E_2 , S_i and S_v are evaluated recursively in terms of the derivatives of the eigenvalue γ_m^2 and the eigenvector S_m as follows

$$E_1^{(n+1)} = \text{diagonal} \left[\begin{aligned} & \frac{-D}{2} \left([e^{-\gamma_m D} - e^{\gamma_m D}]^{(n)} \gamma_m^{(1)} \right) - \\ & \frac{D}{2} \sum_{r=1}^n [e^{-\gamma_m D} - e^{\gamma_m D}]^{(n-r)} \gamma_m^{(r+1)} \end{aligned} \right] ; m = 1, \dots, N \quad (4.45)$$

$$E_2^{(n+1)} = \text{diagonal} \left[\begin{aligned} & \frac{-D}{2} \left([e^{-\gamma_m D} + e^{\gamma_m D}]^{(n)} \gamma_m^{(1)} \right) - \\ & \frac{D}{2} \sum_{r=1}^n [e^{-\gamma_m D} + e^{\gamma_m D}]^{(n-r)} \gamma_m^{(r+1)} \end{aligned} \right] ; m = 1, \dots, N \quad (4.46)$$

$$S_v^{(n)} = [S_m^{(n)}] \quad ; \quad m = 1, \dots, N \quad (4.47)$$

$$S_i^{(n)} = (Z_p^{-1})^{(n)} S_v \Gamma + \sum_{r=1}^n \binom{n}{r} (Z_p^{-1})^{(n-r)} (S_v \Gamma)^{(r)} \quad (4.48)$$

where

$$\begin{aligned}
(Z_p^{-1})^{(n)} Z_p &= \sum_{r=1}^n \binom{n}{r} (Z_p^{-1})^{(n-r)} Z_p^{(r)} \\
(S_v \Gamma)^{(n)} &= S_v^{(n)} \Gamma + \sum_{r=1}^n \binom{n}{r} (S_v^{(n-r)} \Gamma^{(r)}) \\
\Gamma^{(n)} &= \text{diagonal}[\gamma_m^{(n)}] \quad ; \quad m = 1, \dots, N
\end{aligned}$$

To evaluate the derivatives of γ_m and S_m , we use (4.44)

$$\left[\gamma_m^2 U - Z_p Y_p \right] S_m = 0 \tag{4.49}$$

The first derivative of (4.49) can be obtained as

$$\left[2\gamma_m \frac{\partial}{\partial s} \gamma_m U - \frac{\partial}{\partial s} Z_p Y_p \right] S_m + \left[\gamma_m^2 U - Z_p Y_p \right] \frac{\partial S_m}{\partial s} = 0 \tag{4.50}$$

(4.50) can be rewritten in a matrix form

$$\begin{bmatrix} \gamma_m^2 U - Z_p Y_p & -2\gamma_m S_m \end{bmatrix} \begin{bmatrix} \frac{\partial S_m}{\partial s} \\ \frac{\partial \gamma_m}{\partial s} \end{bmatrix} = \left[\left(\frac{\partial}{\partial s} Z_p Y_p \right) S_m \right] \tag{4.51}$$

(4.51) represents n equations with $(n+1)$ unknowns. In order to solve it, we need to introduce one more equation. Let the eigenvector S_m be normalized such that

$$S_m^t S_m = 1 \quad (4.52)$$

or

$$S_m^t \frac{\partial S_m}{\partial s} = 0 \quad (4.53)$$

Adding (4.53) into (4.51),

$$\begin{bmatrix} \gamma_m^2 U - Z_p Y_p & -2\gamma_m S_m \\ S_m^t & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial S_m}{\partial s} \\ \frac{\partial \gamma_m}{\partial s} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial}{\partial s} Z_p Y_p \right) S_m \\ 0 \end{bmatrix} \quad (4.54)$$

Differentiating (4.54), we get

$$T \begin{bmatrix} S_m^{(n+1)} \\ \gamma_m^{(n+1)} \end{bmatrix} + \sum_{r=1}^n \binom{n}{r} T^{(k)} \begin{bmatrix} S_m^{(n)} \\ \gamma_m^{(n)} \end{bmatrix} = \sum_{r=0}^n \binom{n}{r} [Z_p Y_p]^{(n-r+1)} S_m^{(r)} \quad (4.55)$$

where

$$T = \begin{bmatrix} \gamma_m^2 U - Z_p Y_p & -2\gamma_m S_m \\ S_m^t & 0 \end{bmatrix} \quad (4.56)$$

From (4.37)-(4.56), it can be seen that the additional computational cost due to the existence of multiconductor transmission line is negligible, since the corresponding moments are evaluated in a closed-form. This fact is substantiated by the CPU measurements reported in Section 4.6.

4.5 Lossless Multiconductor Transmission Line

In some applications, the losses in the interconnection lines do not significantly affect the signal propagation. For example, GaAs technology [26], the separation between the interconnection lines is about a few micrometers and the distance between the lines and the ground plane is about several hundred micrometers; therefore, the electromagnetic coupling are very important. Moreover, because of the short interconnection length, the losses in the interconnection line do not significantly affect the signal slew rate and also play a negligible part on the induced signal to the inactivated lines. Thus it is possible in the first approximation to consider the lines as lossless multiconductor transmission line. Equation (4.24) becomes

$$\left[\gamma_m^2 U - s^2 LC \right] S_m = 0 \quad (4.57)$$

It is impossible to find the eigenvalues of the $s^2 LC$ matrix at $s=0$. Instead, the eigenvalues of the matrix LC are found. From the basis of eigenvalues, if λ is an eigenvalue of matrix A , $k\lambda$ is also an eigenvalue of kA . In this case, the Laplace operator s is taken as the constant k and also from the basis of eigenvectors, we know

that the eigenvectors S_m are not effected in either case. Equation (4.35) and (4.36) become

$$A = \begin{bmatrix} S_v E_1 S_v^{-1} & -U \\ L^{-1} S_v \Gamma E_2 S_v^{-1} & 0 \end{bmatrix} \quad (4.58)$$

$$B = \begin{bmatrix} S_v E_2 \Gamma S_v^{-1} L & 0 \\ L^{-1} S_v E_1 S_v^{-1} L & -U \end{bmatrix} \quad (4.59)$$

where

γ_m^* are the eigenvalues of the matrix LC

S_v is a constant (independent of Laplace operator s)

$\Gamma = \text{diagonal} \{ \gamma_m^* \}, m = 1, N$

$$E_1 = \text{diagonal} \left\{ \frac{\exp(-s\gamma_m^* D) + \exp(s\gamma_m^* D)}{2} \right\}, m = 1, \dots, N$$

$$E_2 = \text{diagonal} \left\{ \frac{\exp(-s\gamma_m^* D) - \exp(s\gamma_m^* D)}{2} \right\}, m = 1, \dots, N$$

The derivatives of A and B can be found as follows,

$$A^{(r)} = \begin{bmatrix} S_v E_1^{(r)} S_v^{-1} & 0 \\ L^{-1} S_v \hat{\Gamma} E_2^{(r)} S_v^{-1} & 0 \end{bmatrix} \quad (4.60)$$

$$\mathbf{B}^{(r)} = \begin{bmatrix} \mathbf{S}_v \mathbf{E}_2^{(r)} \hat{\Gamma} \mathbf{S}_v^{-1} \mathbf{L} & 0 \\ \mathbf{L}^{-1} \mathbf{S}_v \mathbf{E}_1^{(r)} \mathbf{S}_v^{-1} \mathbf{L} & 0 \end{bmatrix} \quad (4.6i)$$

where

$$\mathbf{E}_1^{(r)} = \text{diagonal} \left\{ \frac{(-\gamma_m^* D)^{(r)} e^{-s \gamma_m^* D} + (\gamma_m^* D)^{(r)} e^{s \gamma_m^* D}}{2} \right\}, m = 1, \dots, N$$

$$\mathbf{E}_2^{(r)} = \text{diagonal} \left\{ \frac{(-\gamma_m^* D)^{(r)} e^{-s \gamma_m^* D} - (\gamma_m^* D)^{(r)} e^{s \gamma_m^* D}}{2} \right\}, m = 1, \dots, N$$

Since only matrix $\mathbf{E}_1$ and $\mathbf{E}_2$ are a function of the Laplace operator s , finding the moments of lossless multiconductor transmission line is much simpler than that of lossy multiconductor transmission line.

4.6 Computational Results And Comparisons

Example1

An interconnect circuit which contains lossless transmission lines is shown in Figure 4.3. The **generalized** Asymptotic Waveform Evaluation is compared with the HSPICE response in Figure 4.4. The applied voltage is a pulse with 1 nanosecond rise/fall time and 10 nanoseconds pulse width. Using sixth-order approximation, the GAWE and HSPICE waveforms are indistinguishable.

Figure 4.3 Circuit for example 1, interconnect model with transmission lines.

Figure 4.4 Transient response of the circuit shown in Figure 4.3 with lossless transmission lines.

Example 2

The transmission lines in Example 1 are replaced by lossy lines with $R=75\Omega/\text{m}$ and $G=0.01\text{S/m}$. The GAWE is compared with the numerical inversion of Laplace transform (NILT) in Figure 4.5. The sixth-order GAWE waveform completely matches the NILT waveform. Note that this circuit can not be analyzed using HSPICE since the lossy transmission lines are characterized in the frequency domain.

Figure 4.5 Transient response of the circuit shown in Figure 4.3 with lossy transmission lines.

Example 3

Figure 4.6 shows a circuit consists of two lossy coupled transmission lines. Both lines are of the same length, 0.1m. Line 1 has the following parameters:

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix} \text{ nH/m} \quad \mathbf{C} = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix} \text{ pF/m}$$

$$\mathbf{R} = \begin{bmatrix} 75 & 15 \\ 15 & 75 \end{bmatrix} \Omega/\text{m} \quad \mathbf{G} = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix} \text{ S/m}$$

and the parameters for line 2 are:

$$\mathbf{L} = \begin{bmatrix} 494.6 & 63.3 & 7.8 & 0.0 \\ 63.3 & 494.6 & 63.3 & 7.8 \\ 7.8 & 63.3 & 494.6 & 63.3 \\ 0.0 & 7.8 & 63.3 & 494.6 \end{bmatrix} \text{ nH/m} \quad \mathbf{C} = \begin{bmatrix} 62.8 & -4.9 & -0.3 & 0.0 \\ -4.9 & 62.8 & -4.9 & -0.3 \\ -0.3 & -4.9 & 62.8 & -4.9 \\ 0.0 & -0.3 & -4.9 & 62.8 \end{bmatrix} \text{ pF/m}$$

$$\mathbf{R} = \begin{bmatrix} 50 & 10 & 1 & 0.0 \\ 10 & 50 & 10 & 1 \\ 1 & 10 & 50 & 10 \\ 0.0 & 1 & 10 & 50 \end{bmatrix} \Omega/\text{m} \quad \mathbf{G} = \begin{bmatrix} 0.1 & -0.01 & -0.001 & 0.0 \\ -0.01 & 0.1 & -0.01 & -0.001 \\ -0.001 & -0.01 & 0.1 & -0.01 \\ 0.0 & -0.001 & -0.01 & 0.1 \end{bmatrix} \text{ S/m}$$

The applied voltage is a pulse with 1 nanosecond rise/fall time and 3 nanoseconds pulse width. The comparisons between GAWE and NILT are show in Figure 4.7 - 4.10.

Figure 4.6 Circuit for example 3, interconnect model with lossy multiconductor transmission lines.

Figure 4.7 Transient response at node 3 of the circuit shown in Figure 4.6.

Figure 4.8 Transient response at node 4 of the circuit shown in Figure 4.6.

Figure 4.9 Transient response at node 7 of the circuit shown in Figure 4.6.

Figure 4.10 Transient response at node 15 of the circuit shown in Figure 4.6.

Example 4

A circuit which contains 35 transmission lines is shown in Figure 4.11. The circuit is obtained by cascading the interconnect model shown in Figure 4.2. The second order and fourth order GAWE responses are compared with HSPICE shown in Figure 4.12.

Figure 4.11 Circuit for example 4, interconnect model with 35 transmission line.

Figure 4.12 Transient response of the circuit shown in Figure 4.11.

Example 5 CPU Comparisons

To show the efficiency of the GAWE, we compare its run time with that of HSPICE in Table 4.1. All comparisons are made on a SUN 3/60 workstation. The GAWE and HSPICE output waveforms for the circuits #1, 2 and 3 are shown in Figures 4.12, 4.13 and 4.14, respectively. The run time distribution of circuit #3 is shown in Figure 4.15. As expected, most of the CPU time goes into I/O processing and the L/U factorization of the circuit equations. The computational cost of the moments is minimal. Again, this agrees with the results reported in [4] for lumped components and substantiate our claim in Section 4.4.2 that the additional computational cost due to existence of transmission lines is negligible. It should be mentioned that no special effort has been made to optimize the efficiency of the GAWE program. It is estimated that an additional 25 percent reduction in the CPU time is possible.

It is to be noted that higher orders of approximation provide an improved waveform estimate at an incremental cost to the first-order approximation as illustrated in Figure 4.16. This conclusion is in complete agreement with the results reported in [4] for linear lumped networks.

circuit #	number of transmission line	node count	lumped element count	CPU seconds	
				GAWE	HSPICE
1	35	101	145	2.62	256
2	70	201	290	3.91	5260.5
3	105	301	435	6.05	7702

Table 4.1 CPU comparison between GAWE and HSPICE

Figure 4.13 Transient response of the circuit # 2.

Figure 4.14 Transient response of the circuit # 3.

Figure 4.15 CPU time distribution of the circuit #3.

Figure 4.16 Incremental CPU time for circuit #3.

4.7 Summary

I. In this chapter, several contributions have been presented to address the high-speed interconnection problems:

1) A unified approach for describing the network equations including multiconductor transmission lines, terminal and interconnecting networks.

2) A *generalized* AWE method is described to handle interconnect models which contain distributed elements. As a practical application, the GAWE is applied to the case where the distributed elements are modelled as lossy multiconductor transmission lines. The GAWE method is useful for both delay and crosstalk estimation.

3) A new method is developed for the evaluation of the moments of the multiconductor transmission line in a recursive closed form.

II. The CPU measurements show that GAWE method not only provides an excellent accuracy for transient response analysis but also gives a large CPU advantage over HSPICE.

III. In general, a relatively low order GAWE approximation is adequate. However, in some cases, a higher order approximation may be required to show the detail of the transient response. Fortunately, the additional cost of higher approximation was shown to be very small.

Chapter 5

Analysis Of High-Speed Interconnects With Nonlinear Terminations

5.1 Introduction

In chapter 4, the termination networks were modelled using linear elements. Although linear elements are easy to analyze, they can never represent nonlinear effects precisely. However, if the nonlinearities of the termination are taken into account, a problem arises due to the fact that quasi-transverse electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain only. In contrast to this, nonlinear network can be in general analyzed in the time domain only. Several methods have been proposed for the analysis of lossy

multiconductor transmission lines with nonlinear terminations. In most cases, the lossy transmission line has been analyzed either by using a series of lumped models in the time domain [27], or by using the frequency domain transformation [22], but both approaches suffer those drawbacks mentioned in Chapter 2. In addition, none of these methods are capable of providing an efficient numerical solution for transient analysis of nonlinear networks with frequency dependent parameters.

Recently, an alternative method has been proposed by Nakhla [8]-[10]. The method is general and not limited to the analysis of transmission lines. It is based on a piecewise decomposition technique where the nonlinear network is decomposed into linear and nonlinear subnetworks. The linear subnetworks contain all the frequency-dependent components. The terminals of the subnetworks are excited by piecewise linear time-dependent sources. Through solution of the linear subnetworks in the frequency domain, the nonlinear subnetworks in the time domain, and by applying Newton-Raphson iteration technique, the parameters of the piecewise waveforms are adjusted so that the topological and constitutive relations of each subnetwork as well as the topological equations resulting from the interconnection of these subnetworks are satisfied. This method does not impose any restrictions on the topology of the linear or nonlinear subnetworks or on their interconnections. In addition, it can be easily implemented using general circuit formulation technique such as modified nodal admittance approach. In this chapter the key concepts of the piecewise decomposition technique are reviewed and then the GAWE method is combined with the piecewise decomposition technique to efficiently analyze the interconnection problems.

5.2 The Piecewise Decomposition Technique (PDT)

5.2.1 Basis of the Method

Consider a network shown in Figure 5.1 which consists of linear lumped components, linear components with frequency-dependent parameters, and nonlinear components. Decompose the network into linear subnetwork L and nonlinear subnetworks $T_1, T_2, \dots, T_n$ such that all components with frequency-dependent parameters are contained within the linear subnetwork. The nonlinear subnetworks contain the nonlinear elements. Augment the subnetworks with M independent voltage sources $x_1(t), x_2(t), \dots, x_m(t)$, shown in Figure 5.2. Let $Q(t)$ and $P(t)$ be the vectors containing the resulting currents at the terminals of the linear and nonlinear subnetworks, respectively. Define an error function $\Phi(t) = \|Q(t) - P(t)\|$. The problem can be posed as finding the set of waveforms $x_m(t); m=1, \dots, M$ which can reduce the error function to zero for all t .

Figure 5.1 An arbitrary network

Figure 5.2. Decomposition of the arbitrary network in Figure 5.1 into linear and nonlinear subnetworks.

5.2.2 Formulation Of The Network Equations

Equation (4.1) is the modified nodal admittance matrix equation for a linear network. If the network contains nonlinear elements, (4.1) becomes

$$W \frac{d}{dt} z(t) + H z(t) + \sum_{k=1}^{N_s} D_k i_k(z) - e(t) - f(z(t)) = 0 \quad (5.1)$$

where

$z(t) \in \mathcal{R}^N$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,

$e(t) \in \mathcal{R}^N$ is the vector of independent sources.

$f(z): \mathcal{R}^N \rightarrow \mathcal{R}^N$ is a function describing the nonlinear elements of the circuit.

Example:

To illustrate the formulation method, consider the network shown in Figure 5.3. The network can be described by (5.1) where:

$$W = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & C & 1 \\ 0 & 0 & 0 & 0 & 1 & C \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & -G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -G_3 & 0 & G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$D_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad e = [0 \ 0 \ 0 \ 0 \ 0 \ v_i]^t$$

$$z(t) = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ I_5]^t$$

$$f = [f_1 \ f_2 \ 0 \ 0 \ 0 \ 0]^t$$

$$i_1 = [I_a \ I_b \ I_c \ I_d]^t$$

Figure 5.3 An example circuit contains nonlinear terminations.

5.2.3 Application Of The PDT

Applying the concepts of the PDT, the network is decomposed into linear and nonlinear subnetworks. The linear subnetworks are augmented by a set of independent sources. This means the nonzero entries in $f(z(t))$ are replaced by a set of independent waveforms $x(t)$

Let

$$f(z(t)) = D_f x(t) \quad (5.2)$$

where

$$x(t) \in \mathbb{R}^{N_f}$$

$D_f = [d_{ij}]$, $d_{ij} \in \{0,1\}$, $i \in \{1,2, \dots, N\}$, $j \in \{1,2, \dots, N_f\}$ is a selector matrix, and N_f is the number of nonzero entries in $f(z(t))$.

Using (5.2), (5.1) is reduced to a set of linear differential equations in the form

$$W \frac{d}{dt} z(t) + H z(t) + \sum_{k=1}^{N_s} D_k i_k(t) - e(t) - D_f x(t) = 0 \quad (5.3)$$

Let $x(t) = [x_m(t)]$; $m \in \{1,2, \dots, N_f\}$ be a piecewise linear function described by

$$x(t) = \sum_{k=1}^L (B_k + A_k) [u(t - t_{k-1}) - u(t - t_k)] \quad (5.4)$$

where

L is the number of linear segments

$$A_k = [a_{1k} \ a_{2k} \ a_{3k} \ \dots \ a_{N_f k}]^t$$

$$B_k = [b_{1k} \ b_{2k} \ b_{3k} \ \dots \ b_{N_f k}]^t$$

$u(t)$ is the unit step function

a_{mk} and b_{mk} are parameters defining the segment k of waveform $x_m(t)$ in the interval

$t_{k-1} \leq t \leq t_k$ such that

$$a_{mk} = \frac{x_{mk} - x_{m(k-1)}}{t_k - t_{k-1}}$$

$$b_{mk} = \frac{x_{m(k-1)} t_k - x_{mk} t_{k-1}}{t_k - t_{k-1}}$$

Let the interval of analysis $t \in [0, T]$ be divided into I equal length intervals such that $t_k - t_{k-1} = \Delta$ and $L\Delta = T$. (5.4) can be rewritten in the form

$$x(t) = \sum_{k=1}^L u_h(t_k - t_{k-1}) (B_k + A_{k-1}) + \sum_{k=1}^L u_g(t_k - t_{k-1}) A_k \quad (5.5)$$

where

$$u_h = \text{diagonal} \{ u(t) - u(t - \Delta) \}$$

$$u_g = \text{diagonal} \{ t(u(t) - u(t - \Delta)) \}$$

Substitute (5.5) the piecewise linear representation of $x(t)$ into (5.3) yielding

$$z(t) = \sum_{k=1}^L H(t_k - t_{k-1}) (B_k + A_k t_{k-1}) + \sum_{k=1}^L G(t_k - t_{k-1}) A_k + F(t) \quad (5.6)$$

where

The vector $\mathbf{F}(t)$ represents the output $\mathbf{z}(t)$ corresponding to the independent sources $\mathbf{e}(t)$ when the augmenting sources are set to zero,

$\mathbf{H}(t)$ and $\mathbf{G}(t)$ are time domain response matrices defined as follows:

$\mathbf{H}(t)=[h_{ij}(t)]; i=1,\dots,N$ and $j=1,\dots,N_f$, $h_{ij}(t)$ is the response i corresponding to the input waveform $x_j = u(t) - u(t - \Delta)$ at port j with all other sources are set to zero,

$\mathbf{G}(t)=[g_{ij}(t)]; i=1,\dots,N$ and $j=1,\dots,N_f$, $g_{ij}(t)$ is the response i corresponding to the input sources $x_j = t(u(t) - u(t - \Delta))$ at port j with all other sources are set to zero.

The linear subnetwork may contain components with frequency dependent parameters such that it needs to be analyzed in frequency domain. The techniques for evaluation of the linear subnetwork response are presented section 5.3.

Example:

To illustrate the application of PDT, we consider the circuit in Figure 5.3 again. Applying (5.2) to the circuit in Figure 5.3 is equivalent to replacing all the nonlinear resistors by independent sources $x(t)$ as shown in Figure 5.4. The elements of $\mathbf{F}(t)$ vector are obtained from the circuit shown in Figure 5.5, where all independent sources $x(t)$ are open-circuit. The elements of $\mathbf{H}(t)$ and $\mathbf{G}(t)$ matrices can be evaluated as shown in Figure 5.6 and 5.7, respectively.

Figure 5.4 The linear subnetwork of the circuit shown in Figure 5.3.

Figure 5.5 Evaluation of the $F(t)$ vector elements of the circuit shown in Figure 5.4.

Figure 5.6.a Evaluation of the elements in the first column of $H(t)$ matrix of the circuit shown in Figure 5.4.

Figure 5.6.b Evaluation of the elements in the second column of $H(t)$ matrix of the circuit shown in Figure 5.4.

Figure 5.7.a Evaluation of the elements in the first column of $G(t)$ matrix of the circuit shown in Figure 5.4.

Figure 5.7.b Evaluation of the elements in the second column of $G(t)$ matrix of the circuit shown in Figure 5.4.

5.2.4 Solution Of The Nonlinear Equations

Discretization of (5.2) at L time points, we obtain L nonlinear equations in the form

$$f(z(t_r^*)) - D_f x(t_r^*) = 0 \quad (5.7)$$

where $t_{r-1} \leq t_r^* \leq t_r$ and $r \in \{1, 2, \dots, L\}$.

Let $x_r = x(t_r^*)$ and $z_r = z(t_r^*)$. At each time point (5.7) represents a set of N_f nonlinear equations in the N_f unknowns $x_r = \{x_{1r}, x_{2r}, \dots, x_{N_f r}\}$. Using (5.6) z_r can be represented as an explicit function of x_r :

$$z_r = R_r x_r + S_r \quad (5.8)$$

where

$$R_r = \frac{G(t_r^* - t_{r-1})}{\Delta}$$

$$S_r = \sum_{k=1}^r \left[H(t_r^* - t_{k-1}) - \frac{G(t_r^* - t_{k-1})}{\Delta} \right] x_{k-1} + \sum_{k=1}^{r-1} \left[\frac{G(t_r^* - t_{k-1})}{\Delta} \right] x_k + F(t_r^*)$$

Solution of (5.7) and (5.8) are obtained using Newton Raphson iterations in the form

$$J^{(i)} [x_r^{(i+1)} - x_r^{(i)}] = -[f(z_r^{(i)}) - D_f x_r^{(i)}]$$

and the Jacobian matrix J is evaluated as

$$J = \left[\frac{df}{dz} \right] R_r - \frac{t_r^* - t_{r-1}}{\Delta} D_f$$

The iterations are repeated until $\|x_r^{(i+1)} - x_r^{(i)}\|$ is less than a predefined tolerance value. The entries time domain solution is calculated by carrying out Newtons iterations until convergence for each time point $r \in \{1, 2, \dots, N\}$.

5.3 Evaluation Of The Linear Subnetwork

In order to obtain $H(t)$, $G(t)$ and $F(t)$, the linear subnetwork is first analyzed in the frequency domain and then the results are converted to the time domain. To obtain the results, we can use three different approaches; 1) inverse fast Fourier transform (FFT), 2) numerical inversion of Laplace transform (NILT), 3) the *generalized Asymptotic Waveform Evaluation* (GAWE) method.

The FFT has a major difficulty when the analysis has to span a time interval of several line transient times. Even for moderately lossy lines, the duration of the response exceeds many transit times of the transmission line network. This makes the uses of inverse FFT techniques inefficient as a large number of points must be added to the analysis to avoid aliasing problems, thus precluding the use of FFT to analyze the linear subnetwork which contains components with frequency-dependent parameters. In addition, the absence of a simple error criterion for the inverse FFT makes it difficult to establish confidence in the results obtained in this case.

The use of NILT in [23] shows reliable results. However, it is still computationally expensive since it requires the solution of the linear subnetwork at several frequency points.

GAWE method is the most efficient approach for finding $\mathbf{H}(t)$, $\mathbf{G}(t)$ and $\mathbf{F}(t)$ in terms of the computational cost and memory storage requirement. To illustrate this, we consider the linear subnetwork as a multiple-input, multiple-output system. While evaluating the elements of $\mathbf{H}(t)$, $\mathbf{G}(t)$ and $\mathbf{F}(t)$, we consider only one input at a time. This means that all but one input is set to zero and the output response is evaluated at all other ports. The process is repeated until all the elements of $\mathbf{H}(t)$, $\mathbf{G}(t)$ and $\mathbf{F}(t)$ are known. Although for every input, we need to evaluate the moments for each output port, fortunately, only *one* LU decomposition is required for evaluating all the elements of $\mathbf{H}(t)$, $\mathbf{G}(t)$ and $\mathbf{F}(t)$. In order to clearly demonstrate the advantage of using GAWE, we compare the required numbers of LU decomposition and forward/backward substitution in the three different approaches because, in general, LU decomposition and forward/backward substitution operations contribute the major computational cost for evaluating the $\mathbf{H}(t)$, $\mathbf{G}(t)$ and $\mathbf{F}(t)$. The comparison is presented in table 5.1.

The comparison shows that even for a typical medium size interconnect network GAWE could be at least thousand times faster than FFT approach and hundred times faster than NILT method. The speed advantage of using GAWE with PDT to solve nonlinear termination problem will be further illustrated using the numerical examples described in Section 5.4.

method \ operation	FFT	NILT	GAWE
LU	N_L	$M \cdot N_t$	1
F/B	$N_f \cdot N_L$	$M \cdot N_t \cdot N_f$	$N_f \cdot 2 \cdot P$

N_L is the number of frequency points and usually in the range of $2000 \leq N_f \leq 20000$,

N_t is the number of time points and usually in the range of $50 \leq N_t \leq 200$,

M is the poles in the Pade approximation and usually in the range of $5 \leq M \leq 11$,

N_f is the number of nonlinear elements,

P is the order of approximation.

Table 5.1 Major computational cost comparison

The second advantage of using GAWE method is that it requires a very small amount of memory storage because only the approximate poles and residues are needed to represent the linear subnetwork. Hence, the time domain outputs $F(t)$, $H(t)$ and $G(t)$ required for the solution of the nonlinear equation (5.7) can be evaluated in a closed form. This approach provides a compromise between memory storage and computational cost. Fortunately, the order of the GAWE approximation is usually low, therefore, the extra computational cost should be very small. On the other hand, FFT and NILT approaches have to store the complete waveform of each element of $F(t)$, $H(t)$ and $G(t)$, thus requiring a very large amount of memory storage. Particularly, if the network is badly mismatched, the reflected signals will exist on the lines for a relatively long period of time.

For a multiple-input, multiple-output linear network, the characteristic polynomial is unique such that all the elements of $H(t)$, $G(t)$ and $F(t)$ have the same poles in the frequency domain. However, GAWE method can only find the approximate poles of the network. This means that for accurate approximation we need to evaluate the poles at each output of the linear subnetwork. Therefore, the pole-finding process has to be repeated N_p times where N_p equal to the number of elements in $F(t)$ and $H(t)$. Although the order of the characteristic polynomial is usually low (3rd-7th), the CPU time required to find the poles N_p could contribute a significant portion to the total computational cost for solving the linear subnetwork. Therefore, three approaches have been used in the implementation stage to reduce N_p :

- 1) Using one set of poles for all the elements of $F(t)$, $H(t)$ and $G(t)$ such that only **one** pole-finding process is required. ($N_p=1$)
- 2) Using different poles for different inputs, $N_p=N_f + 1$, where N_f is the number of nonlinear elements.
- 3) Every output has its own set of poles, therefore, $N_p=N_p * N + N$, where N is size of the nonlinear network. This approach has been used in all the examples in Section 5.4

The comparison of the accuracy among those three different approaches is illustrated via an example in Section 5.4.

5.4 Computational Results And Comparisons

Example1

An interconnect circuit which contains linear and nonlinear lumped elements is shown in Figure 5.8. The applied voltage is a pulse with 1 nanosecond rise/fall time and 4 nanoseconds pulse width. The GAWE transient response is compared with the HSPICE response in Figure 5.9.

Figure 5.8. Circuit for example 1, interconnect models contain linear and nonlinear lumped elements.

Figure 5.9 Transient response of the circuit shown in Figure 5.8.

Example 2

An interconnect circuit which contains lossless transmission lines, linear and nonlinear lumped elements is shown in Figure 5.10. The GAWE response is compared with the HSPICE response in Figures 5.11-5.13. The nonlinear functions are defined as: $f=50i+21.5i^{1/3}$ and $g=50i+i^{1/3}$. It should be noted that output waveforms from the GAWE method are indistinguishable from the HSPICE waveforms.

Figure 5.10 Circuit for example 2, interconnect models contain lossless transmission lines and nonlinear terminations.

Figure 5.11 Transient response of the circuit shown in Figure 5.10.

Figure 5.12 Transient response of the circuit shown in Figure 5.10.

Figure 5.13 Transient response of the circuit shown in Figure 5.10.

Example 3

Figure 5.14 shows a circuit consists of lossy coupled transmission lines and nonlinear elements. The two coupled transmission lines have the same parameters and length as those in example 3 of Chapter 4. Note that this circuit cannot be analyzed using HSPICE since the lossy transmission lines are characterized in the frequency domain. The comparisons between GAWE and NILT are shown in Figures 5.15-5.16.

Figure 5.14 Circuit for example 3, interconnect models contain lossy coupled transmission lines and nonlinear terminations.

Figure 5.15 Transient response at node 7 of the circuit shown in Figure 5.14.

Figure 5.16 Transient response at node 4 of the circuit shown in Figure 5.14.

Example 4

In Section 5.4, three different approaches were proposed to reduce N_p (number of times we need to factorize the approximating characteristic polynomial). To compare the accuracy of the three approaches, the circuit shown in Figure 5.10 was analyzed using the GAWE technique. Comparison between the GAWE response and HSPICE is shown in Figures 5.17-5.18. The results show that the response from approach # 3 is the most accurate as expected. Approach # 1 and 2 have about 20% error in the peak value but for signal delay measurement they are in good agreement with HSPICE. It should be mentioned that for this example, the average CPU time required for one factorization of the approximating characteristic polynomial was measured to be 0.08 seconds while the total CPU time for approach # 3 is about 3.2 seconds. All CPU measurements are made on a SUN3/60 workstation.

Figure 5.17 Comparisons between the three different approaches for finding the dominant poles and HSPICE.

Figure 5.18 Comparisons between the three different approaches for finding the dominant poles and HSPICE.

Example 5 CPU Comparisons

To show the efficiency of the GAWE, a circuit which contains 35 transmission lines and nonlinear terminations is consider. The GAWE output waveforms are compared with HSPICE response in Figure 5.19-5.21 while the CPU comparison is given in Table 5.1. All CPU measurements are made on a SUN3/60 workstation.

number of transmission line	node count	CPU seconds	
		GAWE	HSPICE
35	101	5.61	260

Table 5.2 CPU comparison between GAWE and HSPICE

Figure 5.19 Transient response of the circuit in example 5.

Figure 5.20 Transient response of the circuit in example 5.

Figure 5.21 Transient response of the circuit in example 5.

5.5 Summary

I. In this chapter, the following contributions have been presented to address the high-speed interconnection with nonlinear termination problems:

- 1) The Asymptotic Waveform Evaluation Technique (chapter 3) has been extended to handle nonlinear elements by combining it with the Piecewise Decomposition Technique (PDT).
- 2) A method was presented to provide an efficient numerical solution for the transient analysis of nonlinear networks with frequency dependent parameters. The method combines the GAWE (chapter 4) and PDT in a mixed frequency/time domain approach.
- 3) Three different strategies were described for estimating the poles of the required characteristic polynomial and comparison between their relative computational cost was given.

II.. The GAWE method not only gives a large CPU time advantage over HSPICE but also provides excellent accuracy for transient response analysis of nonlinear network with distributed components.

Chapter 6

Conclusions And Further Research

6.1 Summary

This research was motivated by the problem of providing an efficient numerical analysis to predict propagation signal delay and crosstalk in the interconnects of a high-speed VLSI system where interconnects are modelled as distributed transmission lines. A *generalized* Asymptotic Waveform Evaluation (GAWE) method based on the asymptotic waveform evaluation technique and the piecewise decomposition technique was presented to address the high-speed interconnection problems. GAWE is an efficient approach for transient response estimation for interconnect circuits which contain general RLC lumped elements,

distributed components such as multiconductor transmission lines, and nonlinear terminations. GAWE provides a unified formulation approach, which is based on the modified nodal admittance matrix, for describing the network equations including multiconductor transmission lines, terminal and interconnecting networks. Furthermore, GAWE method can handle networks with arbitrary number of multiconductor transmission lines and nonlinear terminations with no topological constraints. The computational cost due to the existence of multiconductor transmission line is negligible since the corresponding moments are evaluated in a recursive closed form. The numerical result shows that GAWE not only provides an excellent accuracy for transient response analysis but also gives a large CPU advantage, 2-3 order of magnitude, over conventional circuit simulators such as SPICE. Because of its generality and efficiency GAWE should be applicable to the analysis and design of interconnections in high-speed VLSI systems and printed circuit boards.

6.2 Suggestions For Further Research

- I. In GAWE method, the approximated transient response is evaluated by expanding the exact response in Taylor series at zero frequency ($t = \infty$) such that the approximate transient response has its largest numerical error around time equal to zero. One of the possible way to improve the accuracy of the approximate response around time equal to zero is to expand the exact response in Taylor series at frequency equal to infinity ($t = 0$). Thus the

approximate transient response can be determined using a combination of moments evaluated at frequency equal to zero and infinity.

II. The GAWE technique can be considered as expanding the transfer function of the linear subnetwork using Pade approximation. An alternative approach which could lead to more efficient and accurate solution is to use chebyshev rational approximation [32].

III. The selection of the step size in PDT is important. It is because in GAWE method the transient response of the linear subnetwork is evaluated by expanding the transfer function in Taylor series at zero frequency ($t = \infty$) hence, the estimated transient response has its largest errors around time equal to zero. If the time step is chosen too small such that it is within the "large" error region, the accuracy of the nonlinear solution could be unacceptable. On the other hand, if the time step is chosen to be too large, the resolution in the output waveform could also be unacceptable. An algorithm for choosing the optimal step size need to be explored.

IV. The possibility of applying the GAWE to other applications such as analysis of networks with non-uniform transmission lines should also be explored.

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