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Closed-Form Macromodels for Analysis of High-Speed Interconnects in the presence of Electromagnetic Fields

by

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**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Applied Sciences**

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Abstract

High-speed interconnects are increasingly becoming susceptible to electromagnetic interference. Electromagnetic interference can seriously deteriorate the signal integrity of electrical systems. There is a growing requirement to analyze interconnects in the presence of external electromagnetic fields, in industrial grade circuit simulators, such as SPICE.

This thesis presents efficient algorithms to model and simulate high-speed interconnects exposed to incident fields. The algorithms are based on the recently developed delay extraction based compact and passive transmission line macromodels. The proposed incident field coupling models overcome the mixed frequency/time problem encountered during nonlinear transient simulation. The models are suitable for inclusion in conventional circuit simulators, such as SPICE and provide significant speed-up in terms of CPU time as compared to the existing techniques. Numerical examples are presented to demonstrate the validity and efficiency of the proposed algorithms.

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Glossary of Terms

CAD	Computer Aided Design
CMOS	Complementary Metal Oxide Semiconductor
CPU	Central Processing Unit
DEPACT	Delay Extraction Based Passive and Compact Transmission Line Macromodel
EMC	Electromagnetic Compatibility
EMI	Electromagnetic Interference
IFFT	Inverse Fast Fourier Transform
MLF	Modified Lie Formula
MoC	Method of Characteristics
MRA	Matrix Rational Approximation
MTL	Multiconductor Transmission Line
ODE	Ordinary Differential Equation
PCB	Printed Circuit Board
SPICE	Simulation Program with Integrated Circuits Emphasis
TEM	Transverse Electromagnetic
VLSI	Very Large Scale Integration

CHAPTER 1

Introduction

1.1 Background and Motivation

Advances in VLSI (Very Large Scale Integration) and wireless industry toward higher operating frequencies, miniature designs and increased integration of mixed analog/digital systems is making the signal integrity analysis a challenging task. High-frequency effects (Fig. 1.1) such as signal distortion, dispersion, delay, crosstalk, electromagnetic radiation and interference [1]–[64] are becoming increasingly prominent and present major bottlenecks in the design of high-speed electrical systems. These effects can be detrimental to the functional performance of the system as well as can lead to permanent hardware failures. Hence, it is important to predict the high-frequency effects at early stages of the design cycle. This would significantly reduce any required post-production corrections, resulting in shorter time to market, reduced cost and a more reliable product.

In such a scenario, traditional analysis techniques based solely on circuit theory are no longer sufficient to properly characterize and analyze the high-frequency effects. For instance, a simple wire interconnect can longer be treated as a short-circuit at higher operating frequencies. Rather, interconnects behave as distributed structures at high frequencies and accurate prediction of their behavior requires modeling and analysis based on Maxwell's equations. Subsequently, global circuit analysis [63], [65] requires

incorporation of these distributed interconnect structures along with other circuit components.

Fig. 1.1. High-speed interconnect effects

Interconnections can be from various levels of design hierarchy, such as on-chip, packaging structures, multichip modules, printed circuit boards and backplanes. Depending on the operating frequency and nature of the structure, these interconnects are typically modeled as lumped components (RC or RLC) or distributed elements (frequency independent/dependent RLCG parameters, lossy, coupled) [3]-[15].

With the increasing complexity and operating frequency, the external electromagnetic interference (EMI) also significantly influences the signal integrity of high-speed systems. At higher frequencies, interconnects function as efficient antennas and pick up spurious electromagnetic fields. Coupling of incident electromagnetic fields induces noise disturbances in the system. This can lead to severe deterioration of the signal integrity of the system and can also damage sensitive components. This warrants efficient and accurate EMI analysis of high-speed systems, at various stages of the design cycle.

There are several algorithms available in literature to analyze and simulate high-speed interconnects in the presence of electromagnetic interference [16]–[32]. In the commonly followed approaches, interconnects are modeled as uniform coupled transmission lines with the effect of incident field coupling accounted via the distributed sources along the length of the lines. These techniques are based on the quasi-transverse electromagnetic (TEM) mode of propagation along the lines and are described by appropriate modification to Telegrapher’s equations [6]. The major challenge in incorporating these equations in circuit simulators is of mixed frequency/time [3]. This problem arises, because, the governing interconnect and incident field equations are best described in the frequency-domain, while the nonlinear circuit devices like drivers and receivers are represented only in time-domain. In general, conventional techniques rely on first solving the interconnect and incident field related equations in the frequency-domain and subsequently using convolution to obtain transient responses in the presence of nonlinear terminations. For large systems, these techniques require excessive computational time and memory requirements, and in many cases suffer from numerical instability and ill-conditioning problems.

Also, it is to be noted that, due to high-frequency phenomenon such as skin, edge and proximity effects, the variation of per-unit-length resistance, inductance, capacitance and conductance (R , L , C and G) parameters w.r.t. frequency become significant and cannot be ignored [9], [36], [37]. This poses additional challenges for EMI analysis of high-speed systems.

1.2 Contributions

The focus of this thesis is to develop algorithms for fast and accurate transient analysis of interconnects in the presence of electromagnetic fields. Specifically, the main contributions of this thesis are as follows:

- 1) A semi-analytical formulation for computing equivalent sources due to incident electromagnetic field is proposed [44]-[47]. A recursive convolution based formulation is developed for efficient computation of the sources in time-domain. The new method provides significant computational speed-up as compared to existing techniques.
- 2) A methodology to incorporate frequency-dependent per-unit-length line parameters in the above formulation of equivalent sources is developed [48].
- 3) A fully analytical algorithm to analyze multiconductor transmission lines exposed to EMI is proposed. This algorithm employs the DEFACT macromodel for MTLs [40] and exploits the concept of delay extraction. The algorithm leads to closed-form formulation of equivalent sources in time-domain. The new method avoids any need for numerical computation like inverse Fourier transformation or convolution to compute the equivalent sources in time-domain. A theoretical proof and error analysis for the above new algorithm is also developed [50].
- 4) The scope of the above fully analytical algorithm is extended to incorporate the frequency-dependent per-unit-length interconnect parameters.

1.3 Organization of the Thesis

The rest of the thesis is organized as follows. Chapter 2 discusses the fundamentals of interconnect modeling and the major difficulties encountered in their analysis during circuit simulation. A literature survey of the major available techniques to macromodel interconnects in the presence of EMI is also given in chapter 2. Chapter 3 presents a semi-analytical formulation of sources due to EMI. Numerical examples are provided to

demonstrate the validity and efficiency of the algorithm. In chapter 4, a methodology to incorporate frequency-dependent per-unit-length transmission line parameters in the proposed semi-analytical algorithm is developed. In chapter 5, an algorithm leading to fully analytical formulation of sources due to EMI is proposed. Several numerical examples are given. Chapter 6 provides an extension of the proposed fully analytical algorithm to handle frequency-dependent per-unit-length interconnect parameters. The conclusion and suggestions for further research are presented in Chapter 7.

As discussed in the introduction, interconnects play a major role in determining the signal integrity of high-speed electrical systems. Modeling and simulation of interconnects is one of the critical challenges facing the current circuit simulators. There are two main aspects of interconnect modeling and analysis, in the presence of external electromagnetic interference (EMI). First one is regarding the conventional terminal excitation of the interconnects and the second one is related to the disturbances induced by the incident electromagnetic fields.

This chapter is organized as follows. Sec. 2.1 briefly discusses various interconnect models from the perspective of this thesis. The inhomogeneous Telegrapher's equations describing the quasi-TEM distributed models of interconnects exposed to EMI, are derived from Maxwell's equations in Sec. 2.2. The Sec. 2.3 discusses the major challenges involved in solving the inhomogeneous Telegrapher's equations in time-domain. Several existing modeling/simulation strategies are also examined to address these issues.

2.1 Interconnect Models

Interconnects can exist at various levels of design hierarchy, such as on-chip, packaging structures, multichip modules, printed circuit boards, back-planes and cables as shown in

Fig. 2.1. Depending on the operating frequency, interconnects can cause signal distortion, crosstalk, false switching and logic glitches leading to a failed design. Typical interconnect structures present in packaging and printed circuit boards are shown in Fig. 2.2. Depending on the geometry of the structure and the operating frequency, interconnects can be modeled based on the quasi-transverse electromagnetic (TEM) [3] or full-wave assumptions [51], [60]-[62]. For interconnects that cannot be modeled analytically, modeling techniques based on full-wave analysis or measured data have been developed [53]-[59].

Fig. 2.1. Interconnect hierarchy in high-speed VLSI systems (courtesy [10])

Fig. 2.2. Common interconnect structures in packages and circuit boards

2.1.1 Distributed Models of Interconnects

Interconnects having ideal conductors and surrounded by a homogeneous dielectric medium can support a pure transverse electromagnetic (TEM) mode of propagation [6]. In TEM mode, the electric and magnetic fields are perpendicular to each other and to the direction of propagation. Fig. 2.3 depicts the electric and magnetic field patterns in the cross-section of a stripline structure that supports a pure TEM mode.

Fig. 2.3. Electric and magnetic field contours in pure TEM mode

Interconnects with cross-section comprising of inhomogeneous dielectric mediums cannot support the pure TEM mode. This is because, the field components may no longer obey the TEM characteristics in order to satisfy the boundary conditions at the various medium interfaces. Also, interconnects with imperfect conductors produce longitudinal

components of electric field. However, for most general cases, such violations are not large and the field components in the direction of propagation can be treated as negligibly small. Such propagating modes can still be approximated by TEM modes, which are more precisely referred to as quasi-TEM modes [6]. Typical electric and magnetic field orientation in quasi-TEM modes can be represented as in Fig. 2.4.

Fig. 2.4. Typical quasi-TEM field orientations

In most commonly followed techniques, interconnects are modeled as uniform coupled transmission lines with quasi-TEM assumption. This approach also offers relative ease and low CPU cost as compared to full wave techniques [51]. The advantage of quasi-TEM models is that the signal propagation can be described in terms of usual circuit voltages and currents rather than the electromagnetic field quantities. The governing equations characterizing the quasi-TEM behavior are in the form of partial differential equations, known as Telegrapher's equations [3]. The interconnects are characterized by per-unit-length (p.u.l.) resistance, inductance, capacitance and conductance matrices. The simplest model is the delay-line or the lossless line [8]. For example, industrial circuit simulators such as HSPICE represent such a model, known in their model library as T-element [70].

More complicated models include the conductor and dielectric losses as well as coupling between adjacent lines [52]. At higher operating frequencies, phenomenon like skin, edge and proximity effects start becoming a dominant factor in the interconnect response [36], [37]. These effects lead to non-uniform current distribution in the conductors. The

dielectric permittivity of the medium also tends to vary with the operating frequency. This frequency dependency of the interconnect behavior is generally handled by frequency-dependent per-unit-length (p.u.l.) parameters. The principal difficulty associated with the quasi-TEM distributed models is due to the fact that, they are formulated by differential equations, which are best described in the frequency-domain. Nonlinear circuit elements on the other hand are described in the time-domain as nonlinear ordinary differential equations. In the next section, formulation of interconnect equations in the presence of incident electromagnetic fields is described.

2.2 *Inhomogeneous Telegrapher's Equations*

In order to accurately account for the EMI, the incident field coupling to multiconductor transmission lines is modeled by distributed sources along the lines. This approach was first introduced for single lines [16] and was later extended to handle multiconductor lines [18], [19]. This formulation is based on the quasi-TEM mode of propagation along the lines. The quasi-TEM is a valid assumption, provided the line cross-section is much smaller than the wavelength of interest, and conductor losses and inhomogeneity of the dielectric media are small.

In this section, a derivation of the transmission line equations in the presence of incident fields is shown. The effect of incident field appears as a pair of distributed current and voltage sources added to the Telegrapher's equations.

2.2.1 **Derivation of the first set of Inhomogeneous Telegrapher's Equations**

Consider a $(n+1)$ -conductor transmission line structure as shown in Fig. 2.5.

Fig. 2.5. Geometry of incident field coupling

Fig. 2.6. Transverse plane parameters of the MTL system

Applying Maxwell-Faraday law [66] over the shaded rectangular area ϕ_i enclosed by contour C_i , as shown in Fig. 2.5, we get

$$\oint_{C_i} \vec{E} \cdot d\vec{l} = -s \int_{\phi_i} \vec{B} \cdot d\vec{\phi} \quad (2.1)$$

where $\vec{E}$ and $\vec{B}$ are the phasors of the total electric and magnetic fields, a_n is the unit outward normal vector, s is the complex frequency and the subscript i stands for the i^{th}

conductor. Expanding the integral in LHS of (2.1) and separating the magnetic field $\vec{B}$ into its incident and scattered components $\vec{B}^{inc}$ and $\vec{B}^{sct}$, respectively, we get

$$\begin{aligned} \int_{\rho(x_i, y_i)} E_\rho(\rho, z + \Delta z) d\rho - \int_z^{(z + \Delta z)} E_z(\rho_i, z) dz - \int_{\rho(x_i, y_i)} E_\rho(\rho, z) d\rho + \int_z^{(z + \Delta z)} E_z(\rho_0, z) dz = \\ -s \int_{\phi_i} \vec{B}^{inc} \cdot d\vec{\phi} - s \int_{\phi_i} \vec{B}^{sct} \cdot d\vec{\phi} \end{aligned} \quad (2.2)$$

where, ρ is a parameter in transverse plane as shown in Fig. 2.6 and subscript 0 refers to the reference conductor. Since the structure supports quasi-TEM mode of propagation, the transverse fields are of quasi-static nature. So, the voltage between two points in the transverse plane can be uniquely defined as

$$\begin{aligned} V_i(z) &= - \int_{\rho(x_i, y_i)} E_\rho(\rho, z) d\rho \\ V_i(z + \Delta z) &= - \int_{\rho(x_i, y_i)} E_\rho(\rho, z + \Delta z) d\rho \end{aligned} \quad (2.3)$$

Next, the scattered magnetic field can be related to the corresponding resultant currents through the per-unit-length inductances as

$$\lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \int_{\phi_i} \vec{B}^{sct} \cdot d\vec{\phi} = - \sum_{k=1}^n L_{ik} I_k(z) \quad (2.4)$$

Now, representing the imperfect conductors with per-unit-length resistances, R_i , the total longitudinal electric fields can be related to the currents on the conductors as

$$\begin{aligned}
\int_z^{(z+\Delta z)} E_z(\rho_i, z) dz &= R_i \Delta z I_i(z) \\
-\int_z^{(z+\Delta z)} E_z(\rho_0, z) dz &= R_0 \Delta z \sum_{k=1}^n I_k(z)
\end{aligned} \tag{2.5}$$

It is to be noted that the sum of currents in the n -signal conductors is assumed to be equal and opposite in direction to the current in the reference conductor. In other words, we model only the differential mode currents and do not consider any common mode currents. This is a valid assumption because we are interested only in the terminal response of the transmission line structure, where only the differential mode current flows and the common mode currents cancel out.

Substituting (2.3)–(2.5) in (2.2), we get

$$-V_i(z + \Delta z) + V_i(z) - R_i \Delta z I_i(z) - R_0 \Delta z \sum_{k=1}^n I_k(z) = s \Delta z \sum_{k=1}^n L_{ik} I_k(z) - s \int_{\phi_i} \vec{B}^{\text{inc}} \cdot d\vec{\phi} \tag{2.6}$$

Dividing (2.6) by Δz , taking the limit $\Delta z \rightarrow 0$, and expressing in matrix form, we get

$$\frac{d}{dz} V(z) + (R + sL)I(z) = \begin{bmatrix} \vdots \\ s \int_{\rho(x_P, y_i)} B_n^{\text{inc}} d\rho \\ \vdots \end{bmatrix} \tag{2.7}$$

where, B_n^{inc} is the normal component of the incident magnetic field. The forcing function in the RHS of (2.7) can be more conveniently expressed in terms of only the incident electric fields. To achieve that, we apply Faraday's law around the contour C_i in terms of incident fields alone (given in (2.1)), to get

$$\int_{\rho(x_p, y_i)} E_{\rho}^{inc}(\rho, z + \Delta z) d\rho - \int_z^{(z + \Delta z)} E_z^{inc}(\rho_i, z) dz - \int_{\rho(x_p, y_i)} E_{\rho}^{inc}(\rho, z) d\rho + \int_z^{(z + \Delta z)} E_z^{inc}(\rho_0, z) dz = -s \int_{\phi_i} \vec{B}^{inc} \cdot d\vec{\phi} \quad (2.8)$$

Dividing (2.8) by Δz , taking the limit $\Delta z \rightarrow 0$

$$s \int_{\rho(x_p, y_i)} B_n^{inc} d\rho = -\frac{d}{dz} \int_{\rho(x_p, y_i)} E_{\rho}^{inc}(\rho, z) d\rho + E_z^{inc}(\rho_i, z) - E_z^{inc}(\rho_0, z) \quad (2.9)$$

Substituting (2.9) in (2.7), we obtain the first set of inhomogeneous Telegrapher's equation as

$$\frac{d}{dz} V(z) + (R + sL)I(z) = \begin{bmatrix} \vdots \\ -\frac{d}{dz} \int_{\rho(x_p, y_i)} E_{\rho}^{inc}(\rho, z) d\rho + E_z^{inc}(\rho_i, z) - E_z^{inc}(\rho_0, z) \\ \vdots \end{bmatrix} \quad (2.10)$$

2.2.2 Derivation of the second set of Inhomogeneous Telegrapher's Equations

Fig. 2.7. Elementary volume enclosing a line conductor

Consider a cylindrical surface ϕ_i enclosing elementary volume of i^{th} conductor (Fig. 2.7).

Applying the equation of continuity, we see

$$\oint_{\phi_i} \vec{J} \cdot d\vec{\phi} = -sQ_{enc} \quad (2.11)$$

Over the end caps i.e. at ϕ_{end} (see Fig. 2.7), we see

$$\int_{\phi_{end}} \vec{J} \cdot d\vec{\phi} = I_i(z + \Delta z) - I_i(z) \quad (2.12)$$

Over the sides of the surface i.e. at ϕ_{side} , we have

$$\int_{\phi_{side}} \vec{J} \cdot d\vec{\phi} = \sigma \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} \quad (2.13)$$

Here, $\vec{E}_\rho^{sct}$ represents the transverse component of the scattered electric field and σ , ϵ are the conductivity and permittivity of the surrounding homogeneous medium, respectively. Recalling the definition of per-unit-length conductances, G_{ik} , between each pair of line conductors, we see

$$\sigma \lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} = \begin{bmatrix} -G_{i1} & \dots & \sum_{k=1}^n G_{ik} & \dots & -G_{in} \end{bmatrix} \begin{bmatrix} V_1^{sct}(z) \\ \vdots \\ V_i^{sct}(z) \\ \vdots \\ V_n^{sct}(z) \end{bmatrix} \quad (2.14)$$

where the superscript sct represents scattered quantities. Also, the charge enclosed by the surface ϕ_i (Fig. 2.7), residing on the surface of conductor is given by Gauss' law [66] as

$$Q_{enc} = \epsilon \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} \quad (2.15)$$

Next, making use of the definition of per-unit-length capacitances, C_{ik} , between each pair of line conductors, we get

$$\epsilon \lim_{\Delta z \rightarrow 0} \frac{1}{\Delta z} \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} = \begin{bmatrix} -C_{i1} & \dots & \sum_{k=1}^n C_{ik} & \dots & -C_{in} \end{bmatrix} \begin{bmatrix} V_1^{sct}(z) \\ \vdots \\ V_i^{sct}(z) \\ \vdots \\ V_n^{sct}(z) \end{bmatrix} \quad (2.16)$$

Note that, $\sum_{k=1}^n C_{ik}$ represents the self-capacitance of the i^{th} conductor.

Substituting (2.12), (2.13), (2.15) into (2.11), and dividing both sides by Δz gives

$$\frac{I_i(z + \Delta z) - I_i(z)}{\Delta z} + \sigma \frac{1}{\Delta z} \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} = -s\epsilon \int_{\phi_{side}} \vec{E}_\rho^{sct} \cdot d\vec{\phi} \quad (2.17)$$

Taking the limit as $\Delta z \rightarrow 0$ and substituting (2.14) and (2.16), we have in matrix notation

$$\frac{d}{dz} I(z) + (G + sC)V^{sct}(z) = 0 \quad (2.18)$$

The above result can be put in terms of total voltages by noting that

$$V(z) = V^{sct}(z) - \int_{\rho(x,y)} E_\rho^{inc}(\rho, z) d\rho \quad (2.19)$$

Substituting (2.19) into (2.18) gives the second set of inhomogeneous Telegrapher's equations in terms of total voltages as

$$\frac{d}{dz}I(z) + (G + sC)V(z) = - (G + sC) \begin{bmatrix} \vdots \\ \int_{\rho(x_p, y_i)} E_{\rho}^{inc}(\rho, z) d\rho \\ \vdots \end{bmatrix} \quad (2.20)$$

2.3 Conventional Techniques for EMI Analysis

In a compact form, the two sets of inhomogeneous Telegrapher's equations as derived in Sec. 2.2, can be expressed as

$$\frac{d}{dz} \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} = Q(s) \begin{bmatrix} V(z, s) \\ I(z, s) \end{bmatrix} + \tilde{F}(z, s) \quad (2.21)$$

where

$$Q(s) = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix} + s \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} \quad (2.22)$$

Here, the forcing function $\tilde{F}(z, s)$ represents the distributed sources along the length of the lines due to incident fields, which can be represented using (2.10) and (2.20) as

$$\tilde{F}(z, s) = \begin{bmatrix} V_d(z, s) \\ I_d(z, s) \end{bmatrix} = \begin{bmatrix} \frac{d}{dz} V_t^{inc}(z, s) + V_z^{inc}(z, s) \\ (G + sC) V_t^{inc}(z, s) \end{bmatrix} \quad (2.23)$$

where

$$\begin{aligned}
 V_t^{inc}(z, s) &= - \begin{bmatrix} \vdots \\ \int_{\rho(x, y)} E_t^{inc}(\rho, z) d\rho \\ \vdots \end{bmatrix} \\
 V_z^{inc}(z, s) &= \begin{bmatrix} \vdots \\ E_z^{inc}(x_i, y_i, z) - E_z^{inc}(x_o, y_o, z) \\ \vdots \end{bmatrix}
 \end{aligned} \tag{2.24}$$

The general solution of (2.21) can be expressed in terms of an exponential matrix function and a convolution term as [6]

$$\begin{bmatrix} V(l, s) \\ I(l, s) \end{bmatrix} = e^{Q(s)l} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} + J(s); \quad J(s) = \int_0^l e^{Q(s)(l-z)} \tilde{F}(z, s) dz \tag{2.25}$$

where V, I represent the Laplace-domain terminal voltage and current vectors of the multiconductor transmission line and l is the length of the line. In the above equation, the matrix exponential $e^{Q(s)l}$ describes the behavior of the interconnect in the absence of EMI ($\tilde{F}(z, s) = 0$). In the presence of incident electromagnetic fields, the additional convolution term $J(s)$ is required to determine the interconnect response. Since the matrix exponential term ($e^{Q(s)l}$) and the convolution term ($J(s)$) do not have a direct time-domain representation, it is difficult to interface them with nonlinear circuit simulators owing to the mixed frequency/time difficulty and the excessive CPU time. In the ensuing of this section, several modeling/simulation strategies are examined to address this problem.

2.3.1 Lumped Segmentation

The simplest solution to the mixed frequency/time problem is to discretize the transmission line using the conventional lumped segmentation model [7]. A typical example of this model is the U-element in the industrial simulator, HSPICE. Extending the idea of lumped segmentation method to incident field analysis, from (2.21), the inhomogeneous Telegrapher's equations can be represented in time-domain as

$$\begin{aligned}\frac{\partial}{\partial z}v(z, t) &= -Ri(z, t) - L\frac{\partial}{\partial t}i(z, t) + v_F(z, t) \\ \frac{\partial}{\partial z}i(z, t) &= -Gv(z, t) - C\frac{\partial}{\partial t}v(z, t) + i_F(z, t)\end{aligned}\tag{2.26}$$

Lumped segmentation method is based on approximating these partial differential equations (2.26) by difference equations

$$\begin{aligned}v_{k+1}(t) - v_k(t) &= -\Delta z Ri_k(t) - \Delta z L\frac{\partial}{\partial t}i_k(t) + \Delta z v_{Fk}(t) \\ i_{k+1}(t) - i_k(t) &= -\Delta z Gv_k(t) - \Delta z C\frac{\partial}{\partial t}v_k(t) + \Delta z i_{Fk}(t)\end{aligned}\tag{2.27}$$

where $k = [1, \dots, N]$, $\Delta z = l/N$, N is the number of sections and l is the length of the interconnect. Equation (2.27) can be implemented by lumped equivalent circuits composed of resistors, inductors, capacitors and independent sources. Fig. 2.8 shows a general lumped component section for a single transmission line.

This lumped segmentation model provides a direct method to discretize field excited interconnects. However, the approximation is valid only if Δz is chosen to be a small fraction of the smallest wavelength of interest. If the operating frequency is high or if the interconnect is electrically long, many lumped sections are required. This leads to large circuit matrices, rendering the method inefficient. A typical rule of thumb [3] for obtaining the number of lumped sections, N is

Fig. 2.8. Lumped section model of a transmission line in the presence of EMI

$$N = \frac{10\tau l}{t_r} \quad (2.28)$$

where, τ is the per-unit-length line delay, l is the length of the line and t_r is the rise-time of the excitation signal. Also, another difficulty with the lumped segmentation approach is that, it cannot easily handle large coupled lines and frequency-dependent p.u.l. parameters.

2.3.2 Method of Characteristics

One of the techniques to model interconnects in the time-domain is the method of characteristics (MoC) [8]. For example, W-element in the HSPICE simulator is based on this technique. Method of characteristics is best suited for long lossless lines. To incorporate lossy and coupled lines, numerical techniques have been proposed for the method of characteristics [33].

The concept of MoC is extended for incident field analysis in the literature [23], [24]. Here, the interconnect is characterized by its transient matched scattering responses, which are numerically computed including frequency-dependent losses. The interconnect

characteristics are then represented in terms of differential operators or rational functions and ideal delays. The resulting model is similar to Branin's method [8] or method of characteristics which was originally proposed to represent lossless lines.

Noting that the matched scattering parameters of a single lossy transmission line in the presence of EMI can be expressed as [43]

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-\gamma(s)l} \\ e^{-\gamma(s)l} & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} \hat{B}_{f1} \\ \hat{B}_{f2} \end{bmatrix} \quad (2.29)$$

$$\gamma(s) = \sqrt{(R + sL)(G + sC)}$$

where

- $\gamma(s)$ is the propagation constant,
- B_1, B_2 represent the scattered current waves at the ports and A_1, A_2 are the corresponding impinging current waves, and
- $\hat{B}_{f1}, \hat{B}_{f2}$ are the equivalent current wave sources accounting for the incident electromagnetic field.

The equivalent sources $\hat{B}_{f1}, \hat{B}_{f2}$ are given by

$$\begin{aligned} \hat{B}_{f1} &= -\frac{1}{2} \int_0^l e^{-\gamma(s)z} (Y_c(s)V_d - I_d) dz \\ \hat{B}_{f2} &= \frac{1}{2} \int_0^l e^{-\gamma(s)(l-z)} (Y_c(s)V_d + I_d) dz \end{aligned} \quad (2.30)$$

where $Y_c(s)$ is the characteristic admittance of the transmission line (given by $Y_c(s) = \sqrt{(G+sC)/(R+sL)}$ and V_d, I_d are the distributed sources due to incident fields (2.23).

Next, the current wave variables can be converted into circuit voltages and currents, by noting that

$$\begin{aligned} A_k &= \frac{1}{2}(Y_c V_k + I_k) \\ B_k &= \frac{1}{2}(Y_c V_k - I_k) \end{aligned} \quad (2.31)$$

for $k=1,2$. Using (2.31), (2.29) can be expressed as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{Y_c(s)}{(1 - e^{-2\gamma(s)l})} \begin{bmatrix} 1 + e^{-2\gamma(s)l} & -2e^{-\gamma(s)l} \\ -2e^{-\gamma(s)l} & 1 + e^{-2\gamma(s)l} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + 2 \begin{bmatrix} -1 & e^{-\gamma(s)l} \\ e^{-\gamma(s)l} & -1 \end{bmatrix} \begin{bmatrix} \hat{B}_{f1}(s) \\ \hat{B}_{f2}(s) \end{bmatrix} \quad (2.32)$$

For circuit simulation purposes, (2.32) needs to be expressed in terms of ordinary differential equations. However, the interconnect characteristics $Y_c(s)$, $e^{\gamma(s)l}$ and the equivalent sources due to EMI $\hat{B}_{f1}, \hat{B}_{f2}$, are transcendental functions of 's' and do not have explicit inverse Laplace transforms. In order to overcome this problem, these functions need either frequency-domain fitting to obtain rational function approximations or time-domain fitting to sums of exponential functions. However, these approaches are based on nonlinear optimization, which can be CPU expensive and can also suffer from issues like non-convergence and non-passivity. It is to be noted that, passivity is a very important property because stable but non-passive models when terminated with other passive models can lead to unstable global transient simulation.

2.3.3 Matrix Rational Approximation (MRA)

To overcome the above discussed passivity related difficulties and to model lossy/coupled transmission lines, matrix rational approximation (MRA) was suggested [34]-[36]. Matrix rational approximation is based on closed form approximation of the interconnect response using pre-determined coefficients, which are computed as a function of per-unit-length interconnect parameters. MRA technique guarantees the passivity of the macromodel by construction. It has been shown that, MRA is more efficient than MoC and lumped segmentation, while providing passive macromodel [38].

The concept of MRA was also extended for incident field analysis [25]. The algorithm here involves two parts. First, the exponential matrix $e^{Q(s)l}$ in (2.25) is expressed as

$$e^{Q(s)l} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \approx [\chi_N(Z)]^{-1} \Omega_N(Z) \quad (2.33)$$

where $\chi_N(s)$, $\Omega_N(s)$ are polynomial matrices, given by

$$\chi_N(Z) = \sum_{i=0}^N \xi_i s^i; \quad \Omega_N(Z) = \sum_{i=0}^N \varsigma_i s^i \quad (2.34)$$

The above approximation is formulated analytically in terms of p.u.l. parameters and constants (i.e. ξ_i and ς_i). These constants are obtained *a priori*, using methods such as Padé approximation or minimax optimization [34], [35]. A major advantage of this technique is that it guarantees the passivity of the macromodel by construction.

In the second step, the matrix rational function approximation of $e^{Q(s)l}$ was used to express the convolution term $J(s)$ in (2.25) as a rational function. Next, converting (2.25) in terms of admittance parameters and using (2.33), we get

$$\begin{aligned}
 \begin{bmatrix} I(0) \\ I(l) \end{bmatrix} &= \begin{bmatrix} -T_{12}^{-1}T_{11} & T_{12}^{-1} \\ T_{21} - T_{22}T_{12}^{-1}T_{11} & T_{22}T_{12}^{-1} \end{bmatrix} \begin{bmatrix} V(0) \\ V(l) \end{bmatrix} + \begin{bmatrix} I_{SC}(0) \\ I_{SC}(l) \end{bmatrix} \\
 \begin{bmatrix} I_{SC}(0) \\ I_{SC}(l) \end{bmatrix} &= \begin{bmatrix} -T_{12}^{-1} & 0 \\ -T_{22}T_{12}^{-1} & U_n \end{bmatrix} J(s)
 \end{aligned} \tag{2.35}$$

Here, $[I_{SC}(0) \ I_{SC}(l)]^T$ represent the equivalent current sources at the interconnect terminals to account for incident field coupling. These equivalent sources can be expressed as a rational function that can then be translated as ordinary differential equations. A disadvantage of this technique is that for long low loss lines, which are more likely to be exposed to EMI, it can result in higher orders of approximation, thus making the transient simulation inefficient. Also, the approach is not fully analytical as it requires intermediate steps requiring the numerical inversion of matrices involving p.u.l. parameters.

In order to overcome the above limitations of mixed frequency/time and excessive CPU expense associated with the existing incident field analysis techniques, new fast/accurate algorithms are proposed in the subsequent chapters.

Proposed Semi-Analytical Formulation of Equivalent Sources

As discussed in chapter 2, interconnects generally tend to serve as the dominant coupling paths for the external electromagnetic radiations. This causes susceptibility to electromagnetic interference (EMI) a major concern for high-frequency system designers. Hence the availability of interconnect models that include the effects of incident fields is becoming an important design requirement.

This chapter presents a new algorithm to macromodel transmission line type interconnects exposed to incident fields. In the proposed methodology [44], the incident field induced effects are formulated to take advantage of the recently developed delay extraction based passive transmission line macromodel (DEPACT) [40]. The algorithm is derived by a suitable application of the modified Lie product formula (MLF) [40] to the inhomogeneous Telegrapher's equations. The algorithm leads to easy implementation in SPICE like industrial-grade circuit simulators. The resulting model consists of simple delay sources and closed-form approximations. The algorithm while guaranteeing the stability of the global transient simulation, provides significant speed-up for EMI analysis of transmission line networks, especially with large delay and low losses.

This chapter is organized as follows. In Sec. 3.1, incident plane waves are analyzed and the distributed sources due to EMI are expressed in a convenient form for use with the proposed algorithm. Sec. 3.2 presents the proposed closed-form formulation of equivalent

sources to model the distributed sources caused by EMI. Sec. 3.3 presents the development of proposed equivalent sources in the time-domain. Several numerical examples are presented in Sec. 3.4.

3.1 Distributed Sources due to Plane Wave EMI Fields

Fig. 3.1. Incident field parameters

For most practical purposes, EMI fields from distant radiating sources can be locally treated as uniform plane waves [6]. In this case, the electric field of the incident wave can be expressed in the usual phasor form as

$$\vec{E}(x, y, z) = E_o(s) (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) e^{-s(\kappa_x x + \kappa_y y + \kappa_z z)} \quad (3.1)$$

where E_o is the complex amplitude and A_x, A_y, A_z are the direction cosines of the electric field vector. Also, from Fig. 3.1 we have

$$\begin{aligned}
A_x &= \sin\theta_E \sin\theta \\
A_y &= -\sin\theta_E \cos\theta \cos\phi - \cos\theta_E \sin\phi \\
A_z &= -\sin\theta_E \cos\theta \sin\phi + \cos\theta_E \cos\phi \\
\kappa_x &= -\frac{\cos\theta}{c} \quad \kappa_y = \frac{\sin\theta \cos\phi}{c} \quad \kappa_z = -\frac{\sin\theta \sin\phi}{c}
\end{aligned} \tag{3.2}$$

with c being the speed of phase front.

A closed-form formulation of the distributed current and voltage sources can then be derived by incorporating (3.1) into (2.23)-(2.24), as shown in the following sections.

3.1.1 Distributed Voltage Sources

Consider the distributed voltage sources as represented by (2.23)-(2.24). It can be noted from the geometry of the transverse plane (Fig. 2.6), that

$$\begin{aligned}
\frac{x}{\rho} &= \frac{x_i}{d} \\
\frac{y}{\rho} &= \frac{y_i}{d} \\
d &= (x_i^2 + y_i^2)^{1/2}
\end{aligned} \tag{3.3}$$

Using the above relations, (2.24) can be simplified as

$$\begin{aligned}
V_i^{inc}(z, s) &= -E_o e^{-s\kappa_z z} \int_0^d \frac{(A_x x_i + A_y y_i)}{d} e^{-s(\kappa_x x_i + \kappa_y y_i) \frac{\rho}{d}} d\rho \\
&= -E_o e^{-s\kappa_z z} \frac{(A_x x_i + A_y y_i)}{s(\kappa_x x_i + \kappa_y y_i)} \left[1 - e^{-s(\kappa_x x_i + \kappa_y y_i)} \right]
\end{aligned} \tag{3.4}$$

Further simplification can be made by exploiting the property of the quasi-TEM mode transmission lines, that their cross-sectional dimensions are much smaller compared to the

smallest wavelength of interest. As per this condition, the exponential term in (3.4) which defines the delay experienced by the incident fields in the transverse plane, can be approximated by its linear Taylor series expansion. This gives

$$\begin{aligned} V_i^{inc}(z, s) &\approx -E_o e^{-s\kappa_z z \frac{(A_x x_i + A_y y_i)}{s(\kappa_x x_i + \kappa_y y_i)}} [s(\kappa_x x_i + \kappa_y y_i)] \\ &= -E_o e^{-s\kappa_z z} (A_x x_i + A_y y_i) \end{aligned} \quad (3.5)$$

Therefore,

$$\frac{d}{dz} V_i^{inc}(z, s) = s\kappa_z E_o e^{-s\kappa_z z} (A_x x_i + A_y y_i) \quad (3.6)$$

Next, V_z^{inc} in (2.24) can be computed for the i^{th} line using (3.1), to yield

$$V_z^{inc}(z, s) = E_o A_z \left(e^{-s(\kappa_x x_i + \kappa_y y_i)} - 1 \right) e^{-s\kappa_z z} \quad (3.7)$$

Approximating the exponential term in (3.7) by its linear Taylor series expansion, we get

$$V_z^{inc}(z, s) \approx -s(\kappa_x x_i + \kappa_y y_i) E_o A_z e^{-s\kappa_z z} \quad (3.8)$$

Hence, from (3.6) and (3.8), the distributed voltage sources for the i^{th} line can be expressed as

$$\begin{aligned} V_d(z, s) &= \frac{d}{dz} V_i^{inc}(z, s) + V_z^{inc}(z, s) \\ &= s\kappa_z E_o e^{-s\kappa_z z} (A_x x_i + A_y y_i) - s(\kappa_x x_i + \kappa_y y_i) E_o A_z e^{-s\kappa_z z} \\ &= sE_o [\kappa_z (A_x x_i + A_y y_i) - A_z (\kappa_x x_i + \kappa_y y_i)] e^{-s\kappa_z z} \end{aligned} \quad (3.9)$$

Let

$$\begin{aligned}\Phi_i^\pm &= A_x x_i \pm A_y y_i \\ \Psi_i^\pm &= \kappa_x x_i \pm \kappa_y y_i\end{aligned}\tag{3.10}$$

Using (3.10), the vector of distributed voltage sources can be expressed in a convenient form as

$$V_d(z, s) = sE_o(s)e^{-s\kappa_z z}\tilde{V}_{F1}\tag{3.11}$$

Here, for a transmission medium without a solid ground plane, we have

$$\tilde{V}_{F1} = \begin{bmatrix} \vdots \\ \kappa_z \Phi_i^+ - A_z \Psi_i^+ \\ \vdots \end{bmatrix}\tag{3.12}$$

In the case of a ground plane backed structure (such as a microstrip), it can be proved on similar lines, using image theory that

$$\tilde{V}_{F1} = \begin{bmatrix} \vdots \\ \kappa_z (\Phi_i^+ + \Phi_i^-) - A_z (\Psi_i^+ - \Psi_i^-) \\ \vdots \end{bmatrix}\tag{3.13}$$

3.1.2 Distributed Current Sources

The distributed current sources as represented in (2.23)-(2.24) can be re-written for uniform plane wave incident fields, using (3.5) as

$$\begin{aligned}
 I_d(z, s) &= (G + sC) \begin{bmatrix} \vdots \\ -E_o e^{-s\kappa_z z} (A_x x_i + A_y y_i) \\ \vdots \end{bmatrix} \\
 &= E_o(s) e^{-s\kappa_z z} (G + sC) \tilde{V}_{F2}
 \end{aligned} \tag{3.14}$$

Next, for a transmission medium without a solid ground plane, we have

$$\tilde{V}_{F2} = \begin{bmatrix} \vdots \\ -\Phi_i^+ \\ \vdots \end{bmatrix} \tag{3.15}$$

Also, for a microstrip structure (which is backed by a ground plane), we have

$$\tilde{V}_{F2} = \begin{bmatrix} \vdots \\ -(\Phi_i^+ + \Phi_i^-) \\ \vdots \end{bmatrix} \tag{3.16}$$

Next, the distributed sources so derived can be put in a more convenient form of

$$\begin{aligned}
 \tilde{F}(z, s) &= \begin{bmatrix} V_d(z, s) \\ I_d(z, s) \end{bmatrix} \\
 &= \tilde{\Gamma}(s) e^{-s\kappa_z z}
 \end{aligned} \tag{3.17}$$

Here, $\tilde{\Gamma}(s)$ describes the interaction of the incident waves with the lines in the transverse plane and from (3.11) and (3.14) we have

$$\tilde{\Gamma}(s) = E_o(s) \begin{bmatrix} sU_n & 0 \\ 0 & (G + sC) \end{bmatrix} \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} \tag{3.18}$$

where U_n is a unity matrix of size n . Note that, n represents the number of signal conductors in the MTL system.

3.2 Proposed Solution of Telegrapher's Equations

Recall from Sec. 2.3 that, the transient analysis of transmission lines with EMI suffers from both the excessive CPU cost and mixed frequency/time difficulty. The reason for this can be explained as follows. As seen in Sec. 2.3, a general solution of the inhomogeneous Telegrapher's equations (2.21) can be expressed in the matrix-exponential form as given by (2.25). However, in this solution the distributed MTL stamp $e^{Q(s)l}$ and the convolution term $J(s)$ due to EMI, do not have explicit time-domain representations. This causes the major difficulty in transient simulation of the transmission lines exposed to EMI.

To enable efficient time-domain representation of the MTL stamp $e^{Q(s)l}$, we employ here, *the recently developed compact and passive delay extraction based transmission line macromodel (DEPACT) [40]*. This macromodel is based on the modified Lie formula [40], where the exponential term is approximated as

$$e^{Q(s)l} = \prod_{k=1}^m \Theta_k + \varepsilon_m; \quad \Theta_k \equiv e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} \quad (3.19)$$

with

$$A = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} \quad (3.20)$$

and the associated error (ε_m) is

$$\|\varepsilon_m\| = \max_{0 \leq s \leq s_{max}} \left\| e^{(A+sB)l} - \prod_{k=1}^m \Theta_k \right\| \cong O\left(\frac{1}{m}\right) \quad (3.21)$$

For the benefit of the reader, a brief description of this macromodel is also given in Appendix-A.

Further, the term $J(s)$ in (2.25) corresponds to the spatial convolution of the distributed sources $\tilde{F}(z, s)$ due to EMI, with the exponential MTL stamp. In effect, this convolution results in translation of the distributed sources along the length of the MTL into equivalent lumped sources at the line terminations. It is to be noted that, since these equivalent sources (represented by $J(s)$) are a function of the exponential MTL matrix $e^{Q(s)l}$, their time-domain formulation is computationally involved and expensive.

In order to avoid this difficulty, in this section, a simplified closed-form formulation of equivalent sources is proposed. The proposed approach avoids the explicit computation of the exponential MTL matrix $e^{Q(s)l}$ in the convolution integral of (2.25) and provides a simplified and semi-analytic closed-form computation for the equivalent sources in time-domain.

Using (3.17), the convolution integral in (2.25) can be simplified as

$$\begin{aligned} J(s) &= \int_0^l e^{Q(s)(l-z)} \tilde{\Gamma}(s) e^{-s\kappa_z z} dz \\ &= e^{Q(s)l} \left(\int_0^l e^{-(Q(s)+s\kappa_z U_{2n})z} dz \right) \tilde{\Gamma}(s) \end{aligned} \quad (3.22)$$

Let

$$H(s) = -Q(s) - s\kappa_z U_{2n} \quad (3.23)$$

Then (3.22) can be simplified as

$$\begin{aligned} J(s) &= e^{Q(s)l} (e^{H(s)l} - U_{2n}) (H(s))^{-1} \tilde{\Gamma}(s) \\ &= -e^{Q(s)l} (H(s))^{-1} \Gamma(s) + e^{-s\kappa_z U_{2n}l} (H(s))^{-1} \tilde{\Gamma}(s) \end{aligned} \quad (3.24)$$

Thus, (2.25) can be re-written as

$$\begin{bmatrix} V(l, s) \\ I(l, s) \end{bmatrix} = e^{Q(s)l} \left\{ \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} + \begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} \right\} + \begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{far} \quad (3.25)$$

where

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = -(H(s))^{-1} \tilde{\Gamma}(s) \quad (3.26)$$

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{far} = e^{-s\kappa_z U_{2n}l} (H(s))^{-1} \tilde{\Gamma}(s) \quad (3.27)$$

represent the equivalent sources at the near and far-ends of the MTL, respectively. Fig. 3.2 shows the pictorial relationship between the equivalent sources due to incident fields as described by (3.25) and the MTL structure.

To enable transient simulation of MTLs exposed to EMI, time-domain representations of the MTL stamp $e^{Q(s)l}$ and the equivalent sources due to EMI are required. *The MTL stamp is efficiently handled using the recently developed delay extraction based DEFACT macromodel (brief details in Appendix-A).* Also, from (3.26)-(3.27), it can be noted that,

Fig. 3.2. Proposed formulation of incident field coupling to MTLs

the proposed formulation of equivalent sources is not dependent on the MTL exponential matrix $e^{Q(s)l}$. This allows efficient computation of equivalent sources in time-domain and in a closed-form manner, which is described in the ensuing section.

3.3 *Semi-Analytical Time-domain Equivalent Sources*

A basic approach to obtain time-domain formulation of equivalent sources due to incident fields is to employ inverse Fourier transformation of the respective sources described in frequency-domain by (3.26)-(3.27). However, this technique becomes computationally expensive and also can suffer from aliasing effects. In order to overcome this problem, a new method is developed, taking advantage of the proposed formulation, where the equivalent sources are not a function of the exponential MTL stamp. The new method allows computation of equivalent sources semi-analytically in terms of rational functions in s and time-delay functions. Details of the proposed method are given below.

In the first step, the equivalent sources due to incident fields (given by (3.26)-(3.27)), are expressed in terms of rational functions in s and time-delay elements. For this purpose, the term $(H(s))^{-1}$ in (3.26) can be rewritten using (3.23) as

$$(H(s))^{-1} = (D + sE)^{-1} \quad (3.28)$$

$$D = \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix}; \quad E = \begin{bmatrix} -\kappa_z U_n & L \\ C & -\kappa_z U_n \end{bmatrix} \quad (3.29)$$

where U_n is the unity matrix of size n . Next, (3.28) can be expressed as

$$(H(s))^{-1} = E^{-1} (DE^{-1} + sU_{2n})^{-1} \quad (3.30)$$

Assuming that the matrix DE^{-1} has $2n$ linearly independent eigenvectors (i.e. diagonalizable), it can be expressed in terms of its eigenvalues and eigenvectors using a similarity transformation as

$$DE^{-1} = \Upsilon \Lambda \Upsilon^{-1} \quad (3.31)$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots)$ is a diagonal matrix containing the eigenvalues of DE^{-1} and Υ contains the eigenvectors of DE^{-1} . Next, substituting (3.31) into (3.30), we can write

$$(H(s))^{-1} = E^{-1} \Upsilon (\Lambda + sU_{2n})^{-1} \Upsilon^{-1} \quad (3.32)$$

For the case of matrix DE^{-1} is not diagonalizable canonical forms can be used. For example, using Schur canonical form [67], DE^{-1} can be written as $DE^{-1} = \Upsilon \Lambda \Upsilon^T$, where the matrix Υ is an orthogonal matrix and Λ is a quasi-upper triangle matrix. Next,

the equivalent sources at the near-end of the MTL as defined by (3.26), can be expressed by using (3.32) as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = -E^{-1}\Upsilon \begin{bmatrix} \frac{1}{s+\lambda_1} & & 0 \\ & \frac{1}{s+\lambda_2} & \\ & & \ddots \\ 0 & & & \frac{1}{s+\lambda_{2n}} \end{bmatrix} \Upsilon^{-1} \tilde{\Gamma}(s) \quad (3.33)$$

and can be re-arranged as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = \begin{bmatrix} \sum_{i=1}^{2n} \frac{\eta_i^{11}}{s-p_i} & \sum_{i=1}^{2n} \frac{\eta_i^{12}}{s-p_i} \\ \sum_{i=1}^{2n} \frac{\eta_i^{21}}{s-p_i} & \sum_{i=1}^{2n} \frac{\eta_i^{22}}{s-p_i} \end{bmatrix} \tilde{\Gamma}(s) \quad (3.34)$$

where η_i are the residue matrices of size n and are defined in terms of entries of E and Υ , and p_i are the poles of $(H(s))^{-1}$ which are the negative of the corresponding eigenvalues λ_i . Note that, the system with n conductors results in rational functions with $2n$ poles (3.34).

Next using (3.18), (3.34) becomes

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = \begin{bmatrix} \sum_{i=1}^{2n} \frac{\eta_i^{11}}{s-p_i} & \sum_{i=1}^{2n} \frac{\eta_i^{12}}{s-p_i} \\ \sum_{i=1}^{2n} \frac{\eta_i^{21}}{s-p_i} & \sum_{i=1}^{2n} \frac{\eta_i^{22}}{s-p_i} \end{bmatrix} \begin{bmatrix} sU_n & 0 \\ 0 & (G+sC) \end{bmatrix} \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} E_o(s) \quad (3.35)$$

which can be further simplified as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = \begin{bmatrix} k^{11} + \sum_{i=1}^{2n} \frac{\mu_i^{11}}{s-p_i} & k^{12} + \sum_{i=1}^{2n} \frac{\mu_i^{12}}{s-p_i} \\ k^{21} + \sum_{i=1}^{2n} \frac{\mu_i^{21}}{s-p_i} & k^{22} + \sum_{i=1}^{2n} \frac{\mu_i^{22}}{s-p_i} \end{bmatrix} \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} E_o(s) \quad (3.36)$$

where

$$\begin{aligned} k^{11} &= \sum_{i=1}^{2n} \eta_i^{11} & k^{12} &= \sum_{i=1}^{2n} \eta_i^{12} C \\ k^{21} &= \sum_{i=1}^{2n} \eta_i^{21} & k^{22} &= \sum_{i=1}^{2n} \eta_i^{22} C \\ \mu_i^{11} &= \eta_i^{11} p_i & \mu_i^{12} &= \eta_i^{12} (G + Cp_i) \\ \mu_i^{21} &= \eta_i^{21} p_i & \mu_i^{22} &= \eta_i^{22} (G + Cp_i) \end{aligned} \quad (3.37)$$

From (3.12)-(3.13) and (3.15)-(3.16), it can be seen that $\begin{bmatrix} \tilde{V}_{F1} & \tilde{V}_{F2} \end{bmatrix}^T$ defines a constant vector based on the incident field angular parameters and line positions in the transverse plane.

In a compact form, the equivalent sources as described by (3.36), can be represented as

$$W(s) = \left(K + \sum_{i=1}^{2n} \frac{\mu_i}{s-p_i} \right) X(s) \quad (3.38)$$

$$W(s) = \begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} ; \quad X(s) = \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} E_o(s) \quad (3.39)$$

$$K = \begin{bmatrix} k^{11} & k^{12} \\ k^{21} & k^{22} \end{bmatrix} ; \quad \mu_i = \begin{bmatrix} \mu_i^{11} & \mu_i^{12} \\ \mu_i^{21} & \mu_i^{22} \end{bmatrix}$$

Thus, the time-domain expressions of the equivalent sources can be obtained from (3.38) in the form of convolution integral as

$$w(t) = Kx(t) + \sum_{i=1}^{2n} \hat{w}_i(t); \quad \hat{w}_i(t) = \mu_i \int_0^t e^{p_i(t-v)} x(v) dv \quad (3.40)$$

Direct solution of the above convolution integral is computationally prohibitive as the response at any time point depends on the entire past history. Thus, direct convolution involving large number of time points can be time consuming. In order to address this difficulty, a recursive solution of the above convolution process is developed in this paper. For the case of trapezoidal rule of numerical integration (details in Appendix-B), (3.40) can be expressed at any time point $t = t_r$ as

$$\hat{w}_i(t_r) = e^{p_i \delta_r} \hat{w}_i(t_{r-1}) + \frac{\delta_r \mu_i}{2} \left(e^{p_i \delta_r} x(t_{r-1}) + x(t_r) \right) \quad (3.41)$$

where, i is the index over number of poles and $\delta_r = t_r - t_{r-1}$.

Also, the equivalent sources at the far-end as given by (3.27) can be written as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{far} = -e^{-s\kappa_z l} \begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} \quad (3.42)$$

From (3.42), it can be seen that, the equivalent sources at the far-end of the MTL can also be expressed in the same form as those at the near-end, but with a time delay based on the time taken by the incident wave to propagate along the length of the line.

The recursive relation given by (3.41) relates the time-domain response at any time point, in terms of the response at the previous time point and the input waveforms at the present and previous time points. This technique allows efficient computation of equivalent sources due to EMI in time-domain. These equivalent lumped sources are then superimposed on the MTL terminations for the time-domain simulation in SPICE like simulators, with the time-domain values being presented in tabulated data or piece-wise linear data form.

Alternatively, the equivalent sources expressed in terms of rational functions in s and time delay elements as given by (3.36) and (3.42), can also be represented in the form of ordinary differential equations (ODEs), using techniques such as Jordan-canonical form of realization [3]. The ODEs can then be directly linked with the non-linear circuit simulator or as an equivalent circuit derived from ODEs [3]. The circuit simulator then obtains the time-domain solution of the equivalent sources while performing the transient simulation of the entire system. The achievable accuracy of the time-domain values of the equivalent sources is then a function of the numerical integration algorithm. One advantage of this approach is that, it treats the source of EMI fields as a part of the overall electrical system being analyzed and can account for any dynamic inter-dependence between the source generating EMI fields and the victim MTL system.

3.3.1 Special Case: Lossless MTLs

For the special case of lossless MTL lines, simplified expressions can be obtained for the equivalent sources as given below. For lossless lines, (3.28) becomes

$$(H(s))^{-1} = (sE(s))^{-1} \quad (3.43)$$

since $D = \mathbf{0}$. Using (3.43) and (3.18) and noting that $G = \mathbf{0}$ the equivalent sources at near-end given by (3.26), can be expressed as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{near} = -E^{-1} \begin{bmatrix} U_n & \mathbf{0} \\ \mathbf{0} & C \end{bmatrix} \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} E_o(s) \quad (3.44)$$

Similarly, the far-end equivalent sources given by (3.27) can be expressed as

$$\begin{bmatrix} V_F(s) \\ I_F(s) \end{bmatrix}_{far} = e^{-s\kappa_z U_{2n} l} E^{-1} \begin{bmatrix} U_n & \mathbf{0} \\ \mathbf{0} & C \end{bmatrix} \begin{bmatrix} \tilde{V}_{F1} \\ \tilde{V}_{F2} \end{bmatrix} E_o(s) \quad (3.45)$$

From (3.44)-(3.45), it can be observed that, for the ideal lossless MTLs, the equivalent sources at the line terminations are simply the delayed and amplitude scaled replicas of the incident field waveform. Here, the amplitude scaling factor is a function of both the p.u.l. parameters of the MTL and the angular parameters of the incident field, and the delay factor represents the propagation time for the incident field along the MTL.

3.4 Computational Results

In this section, experimental results are presented to demonstrate the validity and applicability of the proposed algorithm. A long, low-loss three coupled microstrip structure exposed to EMI fields is analyzed. The analysis is carried out for both linear as well as nonlinear terminations. For validation purposes, the obtained results from the

proposed algorithm are compared with the conventional MRA [25] and inverse Fourier transform [6] (for linear terminations) analysis. Performance merits of the proposed method is indicated in terms of the CPU time and the reduction in the order of the required MTL macromodel.

3.4.1 Example 3.1: Gaussian Incident Field

In this experiment, the test structure shown in Fig. 3.3 is considered. The p.u.l. parameters of the microstrip test structure are computed using the HSPICE field-solver [70].

Fig. 3.3. Cross-sectional view of the microstrip test structure (Ex. 3.1)

The structure is exposed to an incident field of Gaussian time-domain waveform,

$$\mathcal{E}(t) = \mathcal{E}_o e^{-((t-t_o)/(\Delta T))^2} \quad \text{with the following parameters:}$$

- Electric field amplitude, $\mathcal{E}_o = 1KV/m$
- $\Delta T = 50ps$
- Initial delay, $t_o = 1ns$

The angular parameters of the incident field are:

- Elevation angle, $\theta = 60^\circ$
- Azimuthal angle, $\phi = -60^\circ$
- Electric field polarization angle, $\theta_E = -90^\circ$

A. Computation of Equivalent Sources (V_F, I_F)

In the first step of the analysis, the equivalent sources due to EMI $\begin{bmatrix} V_F & I_F \end{bmatrix}^T$, given by (3.26)-(3.27), are computed using the proposed recursive convolution based formulation as explained in Sec. 3.3. To validate the proposed formulation, the equivalent sources are also computed using inverse fast Fourier transformation (IFFT). Fig. 3.4 and Fig. 3.5 compare the equivalent sources at the near-end and far-end, respectively using these two approaches. As can be seen, the results match very well.

Fig. 3.4. Near-end equivalent sources due to Gaussian EMI pulse (Ex. 3.1)

Fig. 3.5. Far-end equivalent sources due to Gaussian EMI pulse (Ex. 3.1)

B. Transient Analysis of the Test Structure in a Linear Network

In the next part of the experiment, behaviour of the test structure of Fig. 3.3 is analyzed in a linear network. The above computed equivalent sources due to EMI are connected in the network of Fig. 3.6 as per the proposed formulation outlined in Fig. 3.2. Next, the transient analysis of the network is carried out in SPICE using the above computed sources and the DEPACT MTL macromodel. To validate the results, the induced transient voltages at the MTL ends are also computed by performing the conventional IFFT on the corresponding spectrums. Fig. 3.7 shows a sample of the induced voltages which indicates good match between the IFFT and the proposed approach.

Fig. 3.6. MTL test structure with linear terminations (Ex. 3.1)

Fig. 3.7. Induced voltages due to Gaussian EMI pulse (Ex. 3.1)

3.4.2 Example 3.2: Trapezoidal Incident Field

In this experiment, we consider the same microstrip structure as described in Fig. 3.3 of Example 3.1. Behavior of the test structure subjected to EMI fields having trapezoidal transient waveform is analyzed. Also, effect of conductor and dielectric losses on the equivalent sources is considered. Analysis results for the case of non-linear terminations are also given.

A. Computation of Equivalent Sources (V_F , I_F)

The test structure is exposed to a $1KV/m$ field with trapezoidal amplitude envelope, with $0.1ns$ rise/fall time and $5ns$ pulse duration and same angular parameters as in Example 3.1. Fig. 3.8 and Fig. 3.9 compare the equivalent sources at the near-end and far-end respectively, with and without considering the line losses¹. It can be seen that, for the lossless line assumption, the shape of the equivalent sources is identical to the incident field waveform. However, the line losses disturb the equivalent sources significantly and their effect can not be neglected.

B. Analysis of the Test Structure in a Linear Network

The test structure is connected as shown in Fig. 3.6 of Example 3.1. Transient simulation is carried out in SPICE using the proposed equivalent sources (for the lossy case), as outlined in Sec. 3.3. Also, for accuracy and efficiency comparison purposes, analysis is carried out using both the conventional lumped model (U-element of the HSPICE) and the MRA technique [25]. Fig. 3.10 compares the induced node voltages using all three approaches, and they match reasonably accurately.

1. Please note that these results were also verified using IFFT approach and they matched accurately with the results given by Fig. 3.8 - Fig. 3.9. The IFFT results are not shown here in order to avoid cluttering.

Fig. 3.8. Near-end equivalent sources due to trapezoidal EMI pulse (Ex. 3.2)

Fig. 3.9. Far-end equivalent sources due trapezoidal EMI pulse (Ex. 3.2)

Fig. 3.10. Transient response of terminal voltages due to trapezoidal EMI pulse (Ex. 3.2)

C. Analysis of the Test Structure in a Nonlinear network

In this experiment, the test structure of Fig. 3.3 is connected to CMOS based drivers/receivers as shown in Fig. 3.11. Transient simulation is carried out in SPICE using the proposed approach as outlined in Sec. 3.3. Also, for comparison purposes, analysis is performed using both the conventional lumped model (U-element of the HSPICE) and the MRA technique [25]. Fig. 3.12 compares the induced node voltages using all three approaches, and they match reasonably accurately.

Next, an experiment is conducted to investigate the impact of EMI on signal propagation. Fig. 3.13 shows the transient response at the input of the far-end receiver (node n4), with and without the incident field. It can be observed from Fig. 3.12 and Fig. 3.13 that, the EMI induced voltages can potentially cause bit errors in digital circuits. Also, it can be seen that the induced voltages can drive excessive transient currents into digital devices and can cause permanent damage if protective measures are not taken.

Fig. 3.11. MTL test structure with nonlinear terminations (Ex. 3.2)

Fig. 3.12. Transient response of terminal voltages for nonlinear circuit (Ex. 3.2)

Fig. 3.13. Study of the impact of incident field coupling on the transient response for nonlinear network (Ex. 3.2)

A performance comparison of the proposed algorithm with respect to both the conventional lumped model and the MRA based technique is given below. Table 3.1 indicates the reduction in the macromodel order achieved using the proposed algorithm. Table 3.2 shows the CPU comparison. As seen, the proposed algorithm provides significant speed-up compared to the conventional lumped model (speed-up of 30 for the case of linear and a speed-up of 70 for the case of nonlinear example) as well as MRA model. The speed-up is expected to increase further for interconnects with longer lengths, many coupled conductors and sharper excitations.

Table 3.1: Macromodel order comparison, Example 3.2.

	Model Order		
	Number of MLF cells	MRA order	Number of lumps
Conventional Lumped	-	-	5000
MRA	-	200	-
Proposed	20	-	-

Table 3.2: Performance comparison (SUN Blade CPU), Example 3.2.

	CPU time (secs)			
	Linear circuit	Speed-up	Nonlinear circuit	Speed-up
Conventional Lumped	343.75	30	1917.48	70
MRA	58.11	5	234.67	9
Proposed	12.25		27.6	

In the next chapter, the proposed semi-analytical formulation of incident field coupling is extended to incorporate frequency-dependent p.u.l. MTL parameters.

CHAPTER 4 *Semi-Analytical Algorithm and Frequency-dependent MTL Parameters*

At higher operating frequencies, non-uniform conductor current distribution and frequency-dependent dielectric losses become a dominant factor in the MTL response. Frequency-dependent behavior of p.u.l. impedance ($R(s)+sL(s)$) is largely due to skin, edge and proximity effects. Frequency-dependent variation of p.u.l. admittance ($G(s)+sC(s)$) can be attributed to the fact that the complex dielectric permittivity of the surrounding medium is a function of frequency. In this chapter, a methodology to extend the semi-analytic model for incident field analysis (described in chapter 3) to handle interconnects with frequency-dependent p.u.l. parameters is presented.

4.1 *Proposed Formulation*

In this section, an approach to incorporate frequency-dependent p.u.l. MTL parameters in the proposed formulation of equivalent sources (Sec. 3.2) is developed. The frequency-dependent p.u.l. data can be from electromagnetic simulation, measurement or empirical formula. In this case, positive-real rational functions are used to represent the frequency dependency of p.u.l. MTL parameters as

$$\begin{aligned}
 R(s) + sL(s) &= \frac{N(s)}{f(s)} = \frac{N_a s^a + \dots + N_o}{f_{a-1} s^{a-1} + \dots + 1} \\
 G(s) + sC(s) &= \frac{F(s)}{g(s)} = \frac{F_b s^b + \dots + F_o}{g_{b-1} s^{b-1} + \dots + 1}
 \end{aligned} \tag{4.1}$$

The associated rational functions are obtained by fitting the frequency-dependent data to passive circuit elements [36]. Next, from (3.26), the equivalent sources at the near end of the MTL structure can be represented as

$$W(s) = T(s)X(s) \tag{4.2}$$

Here, $W(s)$ and $X(s)$ are as defined in (3.39), and by using (4.1) $T(s)$ can be expressed as

$$\begin{aligned}
 T(s) &= \begin{bmatrix} s\kappa_z U_n & -(R(s) + sL(s)) \\ -(G(s) + sC(s)) & s\kappa_z U_n \end{bmatrix}^{-1} \begin{bmatrix} sU_n & 0 \\ 0 & (G(s) + sC(s)) \end{bmatrix} \\
 &= \begin{bmatrix} s\kappa_z f(s)g(s)U_n & -g(s)N(s) \\ -f(s)F(s) & s\kappa_z f(s)g(s)U_n \end{bmatrix}^{-1} \begin{bmatrix} sf(s)g(s)U_n & 0 \\ 0 & f(s)F(s) \end{bmatrix} \\
 &= [\Psi]^{-1} \hat{\mathfrak{G}}
 \end{aligned} \tag{4.3}$$

Next, decompose $T(s)$ in terms of its strictly proper and constant parts [68] as

$$T(s) = T_{sp}(s) + T(\infty) \tag{4.4}$$

$$T(\infty) = \begin{bmatrix} \kappa_z f_{a-1} g_{b-1} U_n & -g_{b-1} N_a \\ -f_{a-1} F_b & \kappa_z f_{a-1} g_{b-1} U_n \end{bmatrix}^{-1} \begin{bmatrix} f_{a-1} g_{b-1} U_n & 0 \\ 0 & f_{a-1} F_b \end{bmatrix} \tag{4.5}$$

$$\begin{aligned}
 T_{sp}(s) &= T(s) - T(\infty) \\
 &= [\Psi]^{-1} (\hat{\mathfrak{G}} - \Psi T(\infty)) \\
 &= [\Psi]^{-1} \mathfrak{G}
 \end{aligned} \tag{4.6}$$

Next, define

$$\begin{aligned}\Psi &= \hat{\Psi}_{a+b-1}s^{a+b-1} + \dots + \hat{\Psi}_1s + \hat{\Psi}_0 \\ &= \hat{\Psi}_{a+b-1}(U_{2n}s^{a+b-1} + \dots + \Psi_1s + \Psi_0)\end{aligned}\quad (4.7)$$

and

$$\mathfrak{Y} = \mathfrak{Y}_{a+b-2}s^{a+b-2} + \dots + \mathfrak{Y}_1s + \mathfrak{Y}_0 \quad (4.8)$$

Then, a state-space realization $(\tilde{A}, \tilde{B}, \tilde{C}, T(\infty))$ of matrix transfer function $T(s)$ can be obtained in a canonical form as

$$\begin{aligned}\tilde{A} &= \begin{bmatrix} 0 & 0 & 0 & -\Psi_0 \\ U_{2n} & 0 & 0 & -\Psi_1 \\ & \ddots & & \vdots \\ 0 & 0 & U_{2n} & -\Psi_{a+b-2} \end{bmatrix}; & \tilde{B} &= [\mathfrak{Y}_0 \ \mathfrak{Y}_1 \ \dots \ \mathfrak{Y}_{a+b-2}]^T \\ & & \tilde{C} &= \begin{bmatrix} 0 & 0 & \dots & (\hat{\Psi}_{a+b-1})^{-1} \end{bmatrix}\end{aligned}\quad (4.9)$$

This enables $T(s)$ to be expressed as

$$T(s) = \tilde{C}[sU_h - \tilde{A}]^{-1}\tilde{B} + T(\infty) \quad (4.10)$$

where $h = 2n(a+b-1)$. Equation (4.10) is similar in form to that of (3.32) and using the earlier discussed diagonalization or Schur decomposition [67] techniques (Sec. 3.3) on $\tilde{A}$, a pole-residue formulation of $T(s)$ can be obtained. It is to be noted that, the number of poles in the realization now gets augmented to h (due to rational approximation of p.u.l. parameters) as compared to $2n$ in the case of frequency-independent parameters. For the purpose of illustration, consider a four coupled transmission line structure. Suppose it's p.u.l. parameters are approximated as

$$\begin{aligned}
 R(s) + sL(s) &= \frac{N_3s^3 + N_2s^2 + N_1s + N_o}{f_2s^2 + f_1s + 1} \\
 G(s) + sC(s) &= \frac{F_3s^3 + F_2s^2 + F_1s + F_o}{g_2s^2 + g_1s + 1}
 \end{aligned} \tag{4.11}$$

Here,

- $a = 3$
- $b = 3$
- $n = 4$

Therefore, the number of poles (h) of the matrix transfer function $T(s)$ in (4.10) becomes

$$h = 2n(a + b - 1) = 40 \tag{4.12}$$

Next, having obtained the equivalent sources in pole-residue form, their time-domain formulation can be obtained in a closed-form manner by using the recursive formulation based technique as developed earlier in Sec. 3.3 (3.40)-(3.41).

4.2 Computational Results

In this section, numerical results are presented to demonstrate the validity and efficiency of the proposed algorithm. A two coupled microstrip interconnect with frequency-dependent parameters is analyzed in the presence of EMI. The analysis is carried out for both linear as well as nonlinear terminations. For validation purposes, the obtained results from the proposed algorithm are compared with the conventional MRA [25] and inverse Fourier transform [6] (for linear terminations) analysis.

4.2.1 Example 4.1

A 20cm long two coupled microstrip line in Alumina substrate (Fig. 4.1) is considered. The frequency-dependent complex permittivity of the substrate is modeled by the Debye model as discussed in [41]. The finite conductivity and thickness of the conductors and the ground plane are taken into account. The frequency-dependent p.u.l. parameters of the structure are then extracted using an electromagnetic simulator¹ [69]. Positive-real rational fitting of the frequency-dependent p.u.l. parameters is done using passive circuit elements [36]. The obtained rational function fits are of order 4/3 and 3/2 for p.u.l. impedance ($R+sL$) and p.u.l. admittance ($G+sC$) respectively. Fig. 4.2 and Fig. 4.3 shows sample plots of the p.u.l. MTL parameters and the obtained rational function fittings.

Fig. 4.1. Cross-sectional view of the microstrip test structure (Ex. 4.1)

1. Frequency dependence of the dielectric permittivity was explicitly supplied to the simulator using the Debye model.

A. Computation of Equivalent Sources (V_F, I_F)

The test structure is exposed to a $4KV/m$ incident field with trapezoidal amplitude envelope, having $0.1ns$ rise/fall time, $5ns$ pulse duration. The angular parameters of the incident field are:

- Elevation angle, $\theta = 60^\circ$
- Azimuthal angle, $\phi = -60^\circ$
- Electric field polarization angle, $\theta_E = -90^\circ$

The time-domain equivalent sources obtained by the proposed recursive convolution technique (Sec. 4.1) are shown in Fig. 4.4 and Fig. 4.5, for near and far ends of the MTL, respectively. These figures also compare the computed equivalent sources with those obtained from IFFT analysis [6]. It can be seen that the results match very well.

Fig. 4.2. P.U.L. parameters as a function of frequency (R_{11} and L_{11})

Fig. 4.3. P.U.L. parameters as a function of frequency (G_{11} and C_{11})

Fig. 4.4. Near-end equivalent sources due to trapezoidal EMI pulse (Ex. 4.1)

Fig. 4.5. Far-end equivalent sources due to trapezoidal EMI pulse (Ex. 4.1)

B. Transient Analysis of the Test Structure in a Linear Network

In this experiment, the coupled transmission lines are terminated at both ends by resistive loads ($R=100\text{ Ohms}$) as shown in Fig. 4.6. Transient analysis of the circuit with the above computed equivalent sources is carried out in SPICE as per the proposed algorithm. For accuracy and efficiency comparison purposes, transient simulation is performed using the conventional MRA [25] technique. Fig. 4.7 compares the induced node voltages using these two techniques, which indicates a good match.

Fig. 4.6. MTL test structure with linear terminations (Ex. 4.1)

Fig. 4.7. Transient response of the linear circuit due to EMI (Ex. 4.1)

C. Analysis of the Test Structure in a Nonlinear network

In this experiment, the coupled transmission lines of Fig. 4.1 are connected to non-linear terminations (CMOS based drivers/receivers) as shown in Fig. 4.8. A digital signal of trapezoidal waveform with 0.1ns rise/fall time and 15ns pulse width is applied to the circuit as v_{in} . Fig. 4.9 shows the transient response at the input and output of the receiver at the far end, from both the MRA and the proposed method.

Fig. 4.8. MTL test structure with nonlinear terminations (Ex. 4.1)

Fig. 4.9. Transient response of the nonlinear circuit exposed to EMI (Ex. 4.1)

A comparison of the computational efficiency of the proposed algorithm with respect to the conventional MRA technique is given in Table 4.1.

Table 4.1: Performance comparison (SUN Blade CPU), Example 4.1.

	Model Order		CPU time (secs)	
	Num. of MLF cells	MRA Order	Linear circuit	Nonlinear circuit
MRA	-	400	102	354
Proposed	30	1	25	44
Speed-up	13.33		4.08	8.05

As seen from Table 4.1, the proposed algorithm provides significant computational speed-up as compared to the conventional MRA based approach. However, it is to be noted that the proposed algorithm is still not fully analytical. The method involves diagonalization or Schur decomposition of matrices DE^{-1} in (3.31) and $\tilde{A}$ in (4.10). This can lead to additional computational burden especially in the case of large coupled lines. In order to overcome this difficulty, a fully analytical algorithm is proposed in Chapter 5.

CHAPTER 5 ***Proposed Analytical Macromodel of Field Coupling to Interconnects***

In Chapters 3 and 4, a semi-analytical formulation of time-domain equivalent sources due to EMI was presented. However, one of the limitations of the approach is that, it requires computation of eigenvalues of the matrix involving the product of p.u.l. line-parameter matrices (Sec. 3.3). This makes the method not fully analytical, and also can make it CPU expensive for MTL systems with large number of coupled lines and/or with frequency-dependent parameters. In order to overcome this difficulty, a new methodology is presented in this chapter for incident field analysis which leads to fully analytical formulation of equivalent sources in the time-domain [50]. The method also overcomes any need for Fourier transformations or convolution based steps which are associated with the previous approaches. A theoretical proof and error analysis of the proposed algorithm is also presented. It is shown that, the new approach provides significant improvement in accuracy as compared to the semi-analytical techniques described earlier.

5.1 Development of the Proposed Algorithm

In this section, a fully analytical formulation of the equivalent sources in the time-domain, due to incident field coupling is presented. An error bound due to the proposed formulation is also developed. Subsequent discussions and details of the algorithm are presented with respect to the case of uniform plane wave incident fields, with the quasi-

TEM mode of propagation along the MTL.

For the proposed algorithm, it is easier to work with the scattered voltages/currents rather than directly with the circuit voltages/currents. For this purpose, the corresponding required modification to the incident field description (given in chapter 2) is given below.

5.1.1 Incident Field Formulation based on Scattered Voltages

Consider (2.21)-(2.23), and let the scattered voltages be represented by $\hat{V}(z, s)$, which are related to the total voltages $V(z, s)$ [19] as

$$V(z, s) = [\hat{V}(z, s) + V_t^{inc}(z, s)] \quad (5.1)$$

Correspondingly, the Telegrapher's equations described by (2.21), can be modified as

$$\frac{d}{dz} \begin{bmatrix} \hat{V}(z, s) + V_t^{inc}(z, s) \\ I(z, s) \end{bmatrix} = Q(s) \begin{bmatrix} \hat{V}(z, s) + V_t^{inc}(z, s) \\ I(z, s) \end{bmatrix} + \tilde{F}(z, s) \quad (5.2)$$

Above equation can be simplified as

$$\frac{d}{dz} \begin{bmatrix} \hat{V}(z, s) \\ I(z, s) \end{bmatrix} = Q(s) \begin{bmatrix} \hat{V}(z, s) \\ I(z, s) \end{bmatrix} - \begin{bmatrix} \frac{d}{dz} V_t^{inc}(z, s) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -(G + sC)V_t^{inc}(z, s) \end{bmatrix} + \tilde{F}(z, s) \quad (5.3)$$

Further, (5.3) can be expressed as

$$\frac{d}{dz} \begin{bmatrix} \hat{V}(z, s) \\ I(z, s) \end{bmatrix} = Q(s) \begin{bmatrix} \hat{V}(z, s) \\ I(z, s) \end{bmatrix} + F(z, s) \quad (5.4)$$

where the new distributed sources (due to EMI) are obtained using (2.23) as

$$F(z, s) = \begin{bmatrix} V_z^{inc}(z, s) \\ 0 \end{bmatrix} \quad (5.5)$$

Next, the induced distributed sources, given by (5.5) can be evaluated using (2.24), to yield

$$F(z, s) = e^{-s\kappa_z z} \Gamma(s) \quad (5.6)$$

Here, $\Gamma(s)$ describes the interaction of the incident field with the lines in the transverse plane and is given by

$$\Gamma(s) = sE_o(s) \begin{bmatrix} V_{F1} \\ 0 \end{bmatrix} \quad (5.7)$$

The sources relating the total voltages to the scattered voltages, given by (5.1) can be computed using (2.24) as

$$V_t^{inc}(z, s) = e^{-s\kappa_z z} E_o(s) \begin{bmatrix} V_{F2} \\ 0 \end{bmatrix} \quad (5.8)$$

where, for a transmission medium without a solid ground plane, V_{F1} and V_{F2} in (5.7)-(5.8) can be expressed as (similar to the discussion associated with (3.11) and (3.14))

$$V_{F1} = \begin{bmatrix} \vdots \\ -A_z(\kappa_x x_i + \kappa_y y_i) \\ \vdots \end{bmatrix}; \quad V_{F2} = \begin{bmatrix} \vdots \\ -(A_x x_i + A_y y_i) \\ \vdots \end{bmatrix} \quad (5.9)$$

and for a microstrip structure (which is backed by a ground plane), we have

$$V_{F1} = \begin{bmatrix} \vdots \\ -2A_z \kappa_y y_i \\ \vdots \end{bmatrix}; \quad V_{F2} = \begin{bmatrix} \vdots \\ -2A_x x_i \\ \vdots \end{bmatrix} \quad (5.10)$$

It can be noted that, by solving (5.4), results can be expressed back in terms of total voltages (by making use of (5.1)), to yield

$$\begin{bmatrix} V(l, s) \\ I(l, s) \end{bmatrix} = e^{Q(s)l} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} - \begin{bmatrix} V_t^{inc}(0, s) \\ 0 \end{bmatrix} + J_1(s) + \begin{bmatrix} V_t^{inc}(l, s) \\ 0 \end{bmatrix} \quad (5.11)$$

where

$$J_1(s) = \int_0^l e^{Q(s)(l-z)} F(z, s) dz \quad (5.12)$$

A proof that (5.11) is identical to the conventional solution of Telegrapher's equations expressed in total voltage variables (2.25), is given in Appendix-C.

Next, to represent (5.11) in the time-domain, the spatial convolution integral represented by (5.12) presents a major difficulty. To overcome this difficulty, a new algorithm is presented in the following section, facilitating the closed-form computation of the incident field coupling macromodel.

5.1.2 Proposed Analytical Time-domain Macromodel

The new incident field coupling macromodel is developed exploiting the delay extraction and modular features of the recently developed DEFACT macromodel for MTLs [40]. As discussed in Sec. 3.2 and Appendix-A, in this macromodel, the modified Lie formula (MLF) approximates a distributed lossy MTL system with a cascade of MLF cells, with each cell in turn being made of a cascade of lossless MTLs and lossy lumped networks. Exploiting this aspect, in the proposed algorithm, incident fields are first modeled to have coupling with only the lossless MTL sections. Subsequently, the effect of line losses on the incident field coupling is taken care by embedding the lossy sections. This leads to a

fully analytical formulation for distributed sources due to incident field coupling, in the time-domain. Details are given below.

Consider the k^{th} MLF cell (among the cascade of m MLF cells) for a given MTL system of length l . Let the scattered voltages corresponding to this MLF cell be represented by $\hat{V}_1^k, \hat{V}_2^k, \dots, \hat{V}_4^k$ and the total currents be represented by $I_1^k, I_2^k, \dots, I_4^k$. For the purpose of simplicity, dropping the superscript k , these *scattered voltages* and *total currents* can be represented as shown in Fig. 5.1.

Fig. 5.1. k^{th} MLF cell of a cascade of m MLF cells.

Next, the field coupling to this MLF cell can be formulated as follows. For the first lossless section of this cell, the solution of inhomogeneous Telegrapher's equations (5.4) can be expressed ($Q = sB$ in this case) as

$$\begin{bmatrix} \hat{V}_2(s) \\ I_2(s) \end{bmatrix} = e^{\frac{sBl}{2m}} \begin{bmatrix} \hat{V}_1(s) \\ I_1(s) \end{bmatrix} + \int_{z_1}^{z_2} e^{sB(z_2-z)} F(z, s) dz \quad (5.12)$$

$$z_1 = kl/m$$

$$z_2 = (k + 1/2)l/m$$

Note here that $\hat{V}$ represents the scattered voltage. Using (5.6), (5.12) can be simplified to yield

$$\begin{bmatrix} \hat{V}_2(s) \\ I_2(s) \end{bmatrix} = e^{\frac{sBl}{2m}} \left\{ \begin{bmatrix} \hat{V}_1(s) \\ I_1(s) \end{bmatrix} + \begin{bmatrix} V_a^k(s) \\ I_a^k(s) \end{bmatrix} \right\} + \begin{bmatrix} V_b^k(s) \\ I_b^k(s) \end{bmatrix} \quad (5.13)$$

where

$$\begin{aligned} \begin{bmatrix} V_a^k(s) \\ I_a^k(s) \end{bmatrix} &= E_o(s) e^{-s\kappa_z z_1} \begin{bmatrix} \kappa_z U_n & -L \\ -C & \kappa_z U_n \end{bmatrix}^{-1} \begin{bmatrix} V_{F1} \\ 0 \end{bmatrix} \\ &= E_o(s) e^{-s\kappa_z z_1} \Phi \end{aligned} \quad (5.14)$$

and

$$\begin{bmatrix} V_b^k(s) \\ I_b^k(s) \end{bmatrix} = -E_o(s) e^{-s\kappa_z z_2} \Phi \quad (5.15)$$

Here U_n is a unity matrix of size n , and

$$\Phi = \begin{bmatrix} \kappa_z U_n & -L \\ -C & \kappa_z U_n \end{bmatrix}^{-1} \begin{bmatrix} V_{F1} \\ 0 \end{bmatrix} \quad (5.16)$$

It can be observed from (5.13) that $\begin{bmatrix} V_a^k(s) & I_a^k(s) \end{bmatrix}^T$ and $\begin{bmatrix} V_b^k(s) & I_b^k(s) \end{bmatrix}^T$ represent the near-end and far-end equivalent sources, respectively, to account for incident field coupling to the first lossless section of the k^{th} MLF cell, as shown in Fig. 5.2.

Similarly, for the second lossless section of the k^{th} MLF cell (Fig. 5.1), the solution of (5.4) can be expressed as

Fig. 5.2. k^{th} MLF cell incorporating incident field coupling

$$\begin{bmatrix} \hat{V}_4(s) \\ I_4(s) \end{bmatrix} = e^{\frac{sBl}{2m}} \begin{bmatrix} \hat{V}_3(s) \\ I_3(s) \end{bmatrix} + \int_{z_2}^{z_3} e^{sB(z_2-z)} F(z, s) dz \quad (5.17)$$

$$z_3 = (k+1)l/m$$

Using (5.6), (5.17) can be simplified to yield

$$\begin{bmatrix} \hat{V}_4(s) \\ I_4(s) \end{bmatrix} = e^{\frac{sBl}{2m}} \left\{ \begin{bmatrix} \hat{V}_3(s) \\ I_3(s) \end{bmatrix} - \begin{bmatrix} V_b^k(s) \\ I_b^k(s) \end{bmatrix} \right\} + \begin{bmatrix} V_c^k(s) \\ I_c^k(s) \end{bmatrix} \quad (5.18)$$

where

$$\begin{bmatrix} V_c^k(s) \\ I_c^k(s) \end{bmatrix} = -E_o(s) e^{-s\kappa_z z_3} \varphi \quad (5.19)$$

Here, $-\begin{bmatrix} V_b^k(s) & I_b^k(s) \end{bmatrix}^T$ and $\begin{bmatrix} V_c^k(s) & I_c^k(s) \end{bmatrix}^T$ represent the near-end and far-end equivalent sources, respectively for the second lossless section. The full description of the k^{th} MLF cell incorporating the equivalent embedded sources due to incident field coupling is as shown in Fig. 5.2. (Note that V_b and I_b on the far end of the lossy section are connected in reverse directions to accommodate their negative signs).

5.1.3 Frequency-independent P.U.L. Parameters

Further, certain simplifications can be made if the p.u.l. MTL parameters are frequency independent. In this case, the lossy module as represented by $e^{Al/m}$ can be realized exactly, without any approximation as a purely resistive network [44] (see details in Appendix-A). This allows to combine the two EMI induced sources (shown in Fig. 5.2) embedded across every lossy section of a MLF cell into a single source. The resulting k^{th} MLF cell incorporating incident field coupling can be represented as shown in Fig. 5.3.

Fig. 5.3. k^{th} MLF cell for frequency-independent MTL parameters

The new sources appearing in each of the MLF cell can be expressed as (using (5.15))

$$\begin{aligned} \begin{bmatrix} V_d^k(s) \\ I_d^k(s) \end{bmatrix} &= e^{\frac{Al}{m}} \begin{bmatrix} V_b^k(s) \\ I_b^k(s) \end{bmatrix} - \begin{bmatrix} V_b^k(s) \\ I_b^k(s) \end{bmatrix} \\ &= E_o(s) e^{-s\kappa z^2/2} \left[U - e^{\frac{Al}{m}} \right] \Phi \end{aligned} \quad (5.20)$$

Next, the embedded sources between adjacent cells being of opposite polarity would cancel each other. So, the only sources embedded in each of the MLF cells would be

$\begin{bmatrix} V_d^k(s) & I_d^k(s) \end{bmatrix}^T$. Also, it is to be noted that, $\begin{bmatrix} V_a^k(s) & I_a^k(s) \end{bmatrix}^T$ and $\begin{bmatrix} V_c^k(s) & I_c^k(s) \end{bmatrix}^T$ will be

present only for the extreme left and right cells of the entire chain. In other words, the equivalent circuit of a field excited MTL with frequency independent p.u.l. parameters can be represented as shown in Fig. 5.4.

Fig. 5.4. MTL macromodel in terms of MLF cells for frequency-independent p.u.l. parameters

The advantage of the proposed approach is that all the equivalent sources accounting for incident field coupling to a lossy MTL can be formulated analytically in time-domain.

Also, the sources $\left[V_d(s) I_d(s) \right]^T$ need to be computed only once, for the first MLF cell.

The other cells can use these already computed sources simply by adding appropriate delays. It is to be noted that, the method does not require any other numerical methods such as inverse-Fourier transformation or convolution/recursive convolution as required in earlier approaches [24], [25], [44]. Specifically, from (5.14), (5.15), (5.19) and (5.20) we have

$$\begin{aligned}
 \begin{bmatrix} v_a^k(t) \\ i_a^k(t) \end{bmatrix} &= \varepsilon\left(t - \kappa_z k \frac{l}{m}\right) \varphi; & \begin{bmatrix} v_b^k(t) \\ i_b^k(t) \end{bmatrix} &= -\varepsilon\left(t - \kappa_z \left(k + \frac{1}{2}\right) \frac{l}{m}\right) \varphi \\
 \begin{bmatrix} v_c^k(t) \\ i_c^k(t) \end{bmatrix} &= -\varepsilon\left(t - \kappa_z (k+1) \frac{l}{m}\right) \varphi; & \begin{bmatrix} v_d^k(t) \\ i_d^k(t) \end{bmatrix} &= \varepsilon\left(t - \kappa_z \left(k + \frac{1}{2}\right) \frac{l}{m}\right) \left[U - e^{\frac{Al}{m}} \right] \varphi
 \end{aligned} \tag{5.21}$$

where, $\mathcal{E}(t)$ is the incident electric field (inverse laplace transform of $E_o(s)$ in (5.14)) and the index k varies from 0 to $m-1$ (where m is the order of the MTL approximation).

Also, the terminal equivalent sources can be computed by transforming the scattered voltages to total voltages using (5.8) as

$$\begin{aligned} v_t^{inc}(0, t) &= \mathcal{E}(t) \begin{bmatrix} V_{F2} \\ 0 \end{bmatrix} \\ v_t^{inc}(l, t) &= \mathcal{E}(t - \kappa_z l) \begin{bmatrix} V_{F2} \\ 0 \end{bmatrix} \end{aligned} \tag{5.22}$$

It should be noted that, these sources (5.22) are needed at the interface of the MTL with external physical ports (Fig. 5.4).

5.2 Accuracy Considerations for the Proposed Approach

In this section, the accuracy consideration of the proposed technique is presented. In order to quantify the accuracy level achieved by this approach (while providing a fully analytical time-domain macromodel for equivalent sources), a comparison is made with the results obtained from the exact approach (direct solution of the inhomogeneous equations in the Laplace domain). Error bound of the proposed technique in terms of the number of MLF sections employed in the MTL macromodel is also derived. Also a relative error norm between the proposed and the semi-analytical approach presented in chapter 3 is derived.

5.2.1 Derivation of the Error Bound w.r.t. the Exact Approach

The inhomogeneous Telegrapher's equations (5.4) can be solved to obtain an exact

solution in the Laplace domain, as given by (5.11). In this section, first, the exact solution is formulated such that it is suitable for use in deriving the error relation. In the next step, the above formulation is used to derive the error bound.

5.2.1.1 Exact Approach Formulation

It can be seen from (5.11) that, the effective equivalent sources due to EMI, when translated to the far-end of the MTL terminals can be expressed as

$$F_{T1} = \begin{bmatrix} V_t^{inc}(l, s) \\ 0 \end{bmatrix} - e^{Q(s)l} \begin{bmatrix} V_t^{inc}(0, s) \\ 0 \end{bmatrix} + J_1(s) \quad (5.23)$$

Next, using (5.12) and (5.6), the spatial convolution in $J_1(s)$ can be expressed as

$$J_1(s) = e^{Q(s)l} \int_0^l e^{-\hat{Q}(s)z} dz \Gamma(s) = e^{-s\kappa_z l} \left[e^{\hat{Q}l} - U \right] (\hat{Q})^{-1} \Gamma(s) \quad (5.24)$$

where

$$\hat{Q} = Q + s\kappa_z U \quad (5.25)$$

or

$$\hat{Q} = A + s\hat{B}; \quad \hat{B} = B + \kappa_z U \quad (5.26)$$

For the purposes of comparison with the proposed technique, the result obtained in (5.24) is re-written as

$$J_1(s) = e^{-s\kappa_z l} \left[\prod_{k=1}^m e^{\hat{Q}l/m} - U \right] (\hat{Q})^{-1} \Gamma(s) \quad (5.27)$$

Using the formula of a sum of geometric series, (5.27) can be restated as

$$J_1(s) = \sum_{k=0}^{m-1} \left(\prod_{j=1}^{m-k} e^{Ql/m} \right) P_k \quad (5.28)$$

where the term P_k is given by

$$P_k = e^{-s\kappa_z kl/m} \left[U - e^{-\hat{Q}l/m} \right] (\hat{Q})^{-1} \Gamma(s) \quad (5.29)$$

5.2.1.2 Derivation of the Error Bound (w.r.t. the Exact Approach)

Consider the proposed time-domain macromodel expressed in terms of MLF cells and embedded sources, as given in Fig. 5.4. To derive the error bound, in the first step, the equivalent sources embedded inside each of the MLF cells (as seen in Fig. 5.3) are translated to the near end of the corresponding MLF cell, as shown in Fig. 5.5. In the second step, the contributions due to all of these equivalent sources are summed together at the extreme far-end of the MTL macromodel, which is then compared with the expression corresponding to the exact approach.

Fig. 5.5. Proposed analysis using MLF cells with embedded sources

For the first lossless section, the equivalent sources at the near end of the k^{th} MLF cell can be expressed as

$$M_k^1 = e^{\frac{sB(s)l}{2m}} \int_{kl/m}^{(2k+1)l/2m} e^{sB(s)\left((2k+1)\frac{l}{2m}-z\right)} F(z, s) dz \quad (5.30)$$

Introducing a change of variables $\tilde{z} = z - kl/m$ and using (5.6), (5.30) can be simplified to yield

$$\begin{aligned} M_k^1 &= e^{-s\kappa_z kl/m} \int_0^{l/2m} e^{-s\hat{B}(s)\tilde{z}} d\tilde{z} \Gamma(s) \\ &= e^{-s\kappa_z kl/m} \left[U - e^{-s\hat{B}(s)l/2m} \right] (s\hat{B})^{-1} \Gamma(s) \end{aligned} \quad (5.31)$$

For the second lossless section, the sources at the near end of the k^{th} MLF cell can be expressed as

$$M_k^2 = e^{\frac{sB(s)l}{2m}} e^{\frac{Al}{m}} e^{\frac{sB(s)l}{2m}} \int_{(2k+1)l/2m}^{(k+1)l/m} e^{sB(s)\left((k+1)\frac{l}{m}-z\right)} F(z, s) dz \quad (5.32)$$

With a change of variables $\tilde{z} = z - (2k+1)l/2m$ (5.32) can be simplified as

$$\begin{aligned} M_k^2 &= e^{-s\kappa_z kl/m} e^{\frac{s\hat{B}(s)l}{2m}} e^{\frac{Al}{m}} \int_0^{l/2m} e^{-s\hat{B}(s)\tilde{z}} d\tilde{z} \Gamma(s) \\ &= e^{-s\kappa_z kl/m} e^{\frac{s\hat{B}(s)l}{2m}} e^{\frac{Al}{m}} \left[U - e^{-\frac{s\hat{B}(s)l}{2m}} \right] (s\hat{B})^{-1} \Gamma(s) \end{aligned} \quad (5.33)$$

Therefore, the net equivalent sources at the near-end of the k^{th} MLF cell can be expressed as

$$\begin{aligned}
M_k &= M_k^1 + M_k^2 \\
&= e^{-s\kappa_z kl/m} \left[U + e^{\frac{s\hat{B}(s)l}{2m}} e^{\frac{Al}{m}} \right] \left[U - e^{\frac{s\hat{B}(s)l}{2m}} \right] (s\hat{B}(s))^{-1} \Gamma(s)
\end{aligned} \tag{5.34}$$

Next, the effective equivalent sources at the MTL far-end can be obtained by summing the contributions of the sources associated with each of the MLF cells to give

$$J_2(s) = \sum_{k=0}^{m-1} \left(\prod_{j=1}^{m-k} e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} \right) M_k \tag{5.35}$$

Then, from (5.11), the net equivalent sources at the far-end of the MTL line can be written as

$$F_{T2} = \begin{bmatrix} V_t^{inc}(l, s) \\ 0 \end{bmatrix} - \left(\prod_1^m e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} \right) \begin{bmatrix} V_t^{inc}(0, s) \\ 0 \end{bmatrix} + J_2 \tag{5.36}$$

where J_2 is given by (5.35).

Next, the approximation error involved in the proposed approach can be expressed as

$$\varepsilon_m = F_{T1} - F_{T2} \tag{5.37}$$

Using (5.23), (5.36) and letting $\alpha = e^{\frac{Ql}{m}}$, $\beta = e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}}$ and $L = \begin{bmatrix} V_t^{inc}(0, s) \\ 0 \end{bmatrix}^T$,

(5.37) can be expressed as

$$\varepsilon_m = -(\alpha^m - \beta^m)L + \sum_{k=0}^{m-1} (\alpha^{m-k} P_k - \beta^{m-k} M_k) \tag{5.38}$$

Simplification of the above relation yields

$$\|\varepsilon_m\| \leq \|\alpha^m - \beta^m\| \|L\| + \sum_{k=0}^{m-1} \|\alpha^{m-k} P_k - \beta^{m-k} M_k\| \quad (5.39)$$

Using (5.29), P_k can be expanded in power series to give

$$\begin{aligned} P_k &= e^{-s\kappa_z kl/m} \left[U - \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \left(\frac{\hat{Q}l}{m} \right)^j \right] (\hat{Q})^{-1} \Gamma(s) \\ &= \frac{l}{m} e^{-s\kappa_z kl/m} \left[U - \frac{l}{2m} \hat{Q} + \frac{1}{6} \left(\frac{l}{m} \right)^2 (\hat{Q})^2 + \dots \right] \Gamma(s) \end{aligned} \quad (5.40)$$

From (5.34), M_k can also be expanded in power series to give

$$\begin{aligned} M_k &= e^{-s\kappa_z kl/m} \left[U + \left(\sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \left(\frac{s\hat{B}l}{2m} \right)^j \right) \left(\sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \left(\frac{Al}{m} \right)^j \right) \right] \\ &\quad \cdot \left[U - \left(\sum_{j=0}^{\infty} \frac{(-1)^j}{j!} \left(\frac{s\hat{B}l}{2m} \right)^j \right) \right] (s\hat{B}(s))^{-1} \Gamma(s) \\ &= \frac{l}{m} e^{-s\kappa_z kl/m} \left[U - \frac{l}{2m} \hat{Q} + \frac{1}{6} \left(\frac{l}{m} \right)^2 \left(\frac{3}{2} A^2 + (s\hat{B})^2 + \frac{9}{4} sA\hat{B} \right) + \dots \right] \Gamma(s) \end{aligned} \quad (5.41)$$

Substituting the above results in (5.39) we get

$$\begin{aligned} \|\varepsilon_m\| &\leq \|\alpha^m - \beta^m\| \|L\| + \\ &\quad + \sum_{k=0}^{m-1} \left(\frac{l}{m} \|\alpha^{m-k} - \beta^{m-k}\| + \frac{1}{2} \left(\frac{l}{m} \right)^2 \|\alpha^{m-k} - \beta^{m-k}\| \|\hat{Q}\| + \dots \right) \|\Gamma(s)\| \end{aligned} \quad (5.42)$$

Further, it is to be noted that

$$\|\alpha^r - \beta^r\| \leq r \mathfrak{I}^{r-1} \|\alpha - \beta\| \quad (5.43)$$

where $\mathfrak{I}$ is $\max(\|\alpha\|, \|\beta\|)$. Proof of (5.43) is given in Appendix-D. Using (5.43), (5.42)

can be simplified as

$$\begin{aligned}\|\varepsilon_m\| &\leq m\mathfrak{T}^{m-1}\|L\|\|\alpha - \beta\| + \frac{l}{m}\|\Gamma(s)\| \sum_{k=0}^{m-1} ((m-k)\mathfrak{T}^{m-k-1})\|\alpha - \beta\| + \dots \\ &\leq m(\|L\| + l\|\Gamma(s)\|)\mathfrak{T}^m\|\alpha - \beta\|\end{aligned}\quad (5.44)$$

Now expanding $\alpha - \beta$ in power series gives

$$\begin{aligned}\alpha - \beta &= \sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{QL}{m}\right)^j - \left(\sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{sBL}{2m}\right)^j \right) \left(\sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{AL}{m}\right)^j \right) \left(\sum_{j=0}^{\infty} \frac{1}{j!} \left(\frac{sBL}{2m}\right)^j \right) \\ &= \left(\frac{l}{m}\right)^3 \zeta(s) + \frac{1}{m^4}[\dots] + \dots\end{aligned}\quad (5.45)$$

where $\zeta(s)$ is given by

$$\zeta(s) = \frac{s}{24}[(A^2B + BA^2 - 2ABA) - 2s(AB^2 + B^2A - 2BAB)] \quad (5.46)$$

Using the above relations, (5.44) can be further simplified to get

$$\|\varepsilon_m\| \leq \frac{l^3}{m^2} e^{(\|A\| + s\|B\|)l} \|\zeta(s)\| (\|L\| + l\|\Gamma(s)\|) \quad (5.47)$$

It can be seen that as $m \rightarrow \infty$, the above expression goes to zero. This implies

$$\lim_{m \rightarrow \infty} F_{T2} = F_{T1} \quad (5.48)$$

The associated approximation error (ε_m) is then given by

$$\|\varepsilon_m\| = \max_{0 \leq s \leq s_{max}} \|F_{T1} - F_{T2}\| \cong O\left(\frac{1}{m}\right) \quad (5.49)$$

5.2.2 Derivation of the Error Bound w.r.t. Semi-Analytical Approach

In this section, the approximation error associated with the proposed technique compared to the semi-analytical technique (Chapter 3) is derived. It will be evident at the end of this section, that the proposed technique achieves significant improvement in the accuracy level compared to the semi-analytical approach described in chapter 3.

From (3.25)-(3.27), it can be seen that the effective far end equivalent sources as per the semi-analytical technique, can be expressed as

$$F_{T3} = \left(\prod_1^m e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} - e^{-s\kappa_z Ul} \right) Q^{-1} \tilde{\Gamma}(s) \quad (5.50)$$

where $\tilde{\Gamma}(s)$ is as given by (3.18). Comparing with the exact expression of the far-end sources (3.25)-(3.27), the approximation error associated with this technique can be expressed as

$$\varepsilon_{m2} = \left(\prod_1^m e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} - e^{-s\kappa_z Ul} \right) Q^{-1} \tilde{\Gamma}(s) - \left(e^{Ql} - e^{-s\kappa_z Ul} \right) Q^{-1} \tilde{\Gamma}(s) \quad (5.51)$$

Above can be simplified as

$$\varepsilon_{m2} = \left(\prod_1^m e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} - e^{Ql} \right) Q^{-1} \tilde{\Gamma}(s) \quad (5.52)$$

Therefore, we get

$$\|\varepsilon_{m2}\| \leq \left\| \prod_1^m e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} - e^{Ql} \right\| \|Q^{-1}\| \|\tilde{\Gamma}(s)\| \quad (5.53)$$

From (5.43) and (5.45), the above relation can be simplified as

$$\|\varepsilon_{m2}\| \leq \frac{l^3}{m^2} e^{(\|A\| + s\|B\|)l} \|\zeta(s)\| \|\tilde{\Gamma}(s)\| \|Q^{-1}\| \quad (5.54)$$

Fig. 5.6 presents a comparison of the approximation error associated with the proposed analytical technique (given by (5.47)) and the semi-analytical technique (given by (5.54)) for a typical set of line parameters, with respect to the number of MLF cells (m). Fig. 5.7 shows the error rate versus length of the line for a fixed number of MLF cells.

As can be seen, the proposed fully analytical technique leads to considerable increase in accuracy levels as compared to the earlier technique, although both the approaches rely on the MLF based MTL macromodel [40]. This can be interpreted on the basis of the fact that, the semi-analytical technique simply lumps the field induced equivalent sources at the line terminations while the proposed fully analytical approach in this chapter more accurately capture the distributed nature of the sources due to incident fields by exploiting the structure and features of the MLF macromodel of the transmission line.

Fig. 5.6. Comparison of the error norm using the analytical and semi-analytical approaches, w.r.t the number of MLF cells (m)

Fig. 5.7. Comparison of the error norm using the analytical and semi-analytical approaches, w.r.t line length (for fixed m)

5.3 Computational Results

This section presents numerical examples to demonstrate the computational aspects of the proposed algorithm. Transient analysis is carried out in the presence of biexponential transient waveform EMI fields. Both linear and nonlinear circuit terminations are considered. The obtained results are also compared with the conventional inverse Fourier transform [6] (for linear terminations) and the semi-analytical technique (chapter 3).

5.3.1 Example 5.1

In this example, the proposed algorithm is applied to determine the transient response of a 30cm long low loss three conductor microstrip structure (Fig. 5.8), exposed to EMI fields. HSPICE field solver [70] is used to obtain the per-unit-length parameters of the line. The

waveform of the incident electric field $\mathcal{E}(t)$ is described by a double exponential function, given by

$$\mathcal{E}(t) = \mathcal{E}_o(\exp(-\hat{\alpha}t) - \exp(-\hat{\beta}t)) \quad (5.55)$$

where

- $\hat{\alpha} = 4 \times 10^8$
- $\hat{\beta} = 10^9$

and with the following angular parameters

- Elevation angle, $\theta = 40^\circ$
- Azimuthal angle, $\phi = -50^\circ$
- Horizontal polarization, $\theta_E = -90^\circ$

Fig. 5.8. Cross-section of the test structure (Ex. 5.1)

A. Transient Analysis of the Test Structure in a Linear Network

The line is terminated in matched resistive loads of 120Ω as shown in Fig. 5.9 and the

analysis is carried out using the proposed fully analytical algorithm. For validation purpose, obtained results are compared with the inverse Fourier transformation based technique [6]. Simulation is also performed using the semi-analytical technique (chapter 3). Fig. 5.10 depicts the transient response of the voltages induced at the line ends. As seen, results from all three approaches match reasonably accurately.

Fig. 5.9. MTL test structure with linear terminations (Ex. 5.1)

Fig. 5.10. Transient responses of the linear circuit due to EMI (Ex. 5.1)

B. Analysis of the Test Structure in a Nonlinear network

In this experiment, the test structure of Fig. 5.8 is terminated with nonlinear loads as shown in Fig. 5.12. Effect of the incident field described by (5.55) on the digital transmission through the coupled transmission line terminated with CMOS drivers/receivers is examined. A digital signal of trapezoidal waveform with 0.1ns rise/fall time and 5ns pulse width is applied to the circuit as v_{in} . Fig. 5.12 shows the transient response at the input and output of the receiver at the far-end.

Fig. 5.11. MTL test structure with nonlinear terminations (Ex. 5.1)

Fig. 5.12. Transient response of the nonlinear circuit exposed to EMI (Ex. 5.1)

Comparison of the computational efficiency of the proposed algorithm with respect to the semi-analytical (chapter 3) technique and the conventional lumped model based approach is given in Table 5.1 and Table 5.2, respectively.

Table 5.1: Performance comparison (SUN Blade CPU) w.r.t. semi-analytical model, Example 5.1

(all CPU time in secs)	Semi-Analytical Model (Chapter 3)			Proposed Model (m=10)	Speed-up
	Recursive Convolution	SPICE Simulation	Total CPU Time		
Linear	16.37	12.13	28.5	4.10	6
Nonlinear	16.37	70.2	86.57	16.4	5

Table 5.2: Performance comparison (SUN Blade CPU) w.r.t. lumped model, Example 5.1

(all CPU time in secs)	Linear circuit	Speed-up	Nonlinear circuit	Speed-up
Conventional Lumped	604.61	150	2704.23	165
Proposed	4.10		16.4	

In the next chapter, the proposed analytical incident field coupling macromodel is extended to MTLs with frequency-dependent p.u.l. parameters.

In Chapter 5, a fully analytical algorithm was proposed to macromodel multiconductor transmission lines in the presence of electromagnetic interference. However, as discussed in chapter 4, phenomenon like skin, edge and proximity effects cannot be neglected at higher operating frequencies. The consequent non-uniform conductor current distribution and dielectric properties result in frequency-dependent variation in the p.u.l. MTL parameters. In this chapter, the scope of the analytical incident field coupling macromodel is extended to interconnects with frequency-dependent p.u.l. MTL parameters.

6.1 Proposed Formulation

To incorporate the frequency-dependent p.u.l. parameters the modified Lie formula (3.19) needs to be appropriately reformulated. For this purpose, a maximum possible delay is extracted without affecting transmission line passivity conditions. Details are given in the Appendix-A (see eq. (A.5)-(A.7)). The resulting realization of a single MLF cell is as shown in Fig. A.1.

Next, the incident field coupling to the k^{th} MLF cell is formulated as given by (5.13) and (5.18). In this case, the k^{th} MLF cell incorporating the equivalent lumped sources can be represented as shown in Fig. 6.1, with the embedded sources given by (5.21).

Fig. 6.1. k^{th} MLF cell for frequency-dependent MTL parameters

Further, it is to be noted that, the equivalent sources between adjacent MLF cells being of opposite polarity would cancel each other. So, the resulting model will have sources

$\begin{bmatrix} V_b^k(s) & I_b^k(s) \end{bmatrix}^T$ across the lossy section for each of the interior MLF cells. The left

outermost MLF cell will have the source $\begin{bmatrix} V_a^k(s) & I_a^k(s) \end{bmatrix}^T$ connected to its input side and

the right outermost MLF cell will have the source $\begin{bmatrix} V_c^k(s) & I_c^k(s) \end{bmatrix}^T$ connected to its output

side. The complete form of the equivalent circuit model of a field excited MTL with frequency-dependent p.u.l. parameters is shown in Fig. 6.2.

Fig. 6.2. MTL macromodel in terms of MLF cells for frequency-dependent p.u.l. parameters

For the case of frequency-dependent p.u.l. parameters, similar remarks can be made about the proposed macromodel as observed in Sec. 5.1.2. All the embedded sources accounting for incident field coupling to a given lossy MTL system can be formulated analytically in time-domain, as given by (5.21). Also, the sources $\left[V_b(s) \ I_b(s) \right]^T$ need to be computed only once, for the first MLF cell. The other cells can use these sources simply as their delayed versions. This leads to additional computational savings and ease of implementation. Also, the proposed macromodel does not require any numerical method such as inverse-fourier transformation or convolution/recursive convolution as required in existing approaches [24], [25], [44].

Next, for the purpose of error analysis, modified Lie relation given by (A.6) is used. This MLF can be easily incorporated in the expression of effective far-end equivalent sources (F_{72}), given by (5.36). Following a similar approach for error analysis, it can be shown that the approximation error in this case is also of $O\left(\frac{1}{m^2}\right)$ as given by (5.49).

6.2 Computational Results

This section presents numerical examples to demonstrate the computational aspects of the proposed algorithm. Transient analysis is carried out in the presence of trapezoidal transient waveform EMI fields. Both linear and nonlinear circuit terminations are considered. The obtained results are also compared with the conventional inverse Fourier transform [6] (for linear terminations) and the semi-analytical technique (chapter 4).

6.2.1 Example 6.1

In this example, transient field coupling analysis is carried out for a 40cm long low loss coupled microstrip line (Fig. 6.3) with frequency-dependent p.u.l. parameters. The non-uniform current distribution in the line conductors at higher frequencies is taken into account, that leads to frequency-dependent p.u.l. resistance and inductance. An Alumina substrate is considered with the dielectric constant of 8.9 at d.c. and 8.87 with a loss tangent of 0.0077 at 50 GHz. Frequency dependent behaviour of the substrate's complex permittivity is modeled using the Debye-model [41]. Subsequently, the fitted model for the complex permittivity is used to extract the p.u.l. line parameters using a field solver¹ [69]. The obtained p.u.l. parameters are then fitted to positive real rational functions using pre-determined passive circuit elements, as described in [36].

A $4KV/m$ horizontally polarized incident field with trapezoidal amplitude envelope, having $0.2ns$ rise/fall time and $5ns$ pulse duration is considered to illuminate the test structure. The direction of incidence is given by the azimuthal angle of -50° and elevation angle of 40° .

Fig. 6.3. Cross-sectional view of the test structure (Ex. 6.1)

1. Frequency dependence of the dielectric permittivity was explicitly supplied to the simulator using the Debye model.

A. Transient Analysis of the Test Structure in a Linear Network

The coupled transmission lines are terminated at both ends by matched resistive loads ($R = 100\Omega$) as shown in Fig. 6.4.

Fig. 6.4. MTL test structure with linear terminations (Ex. 6.1)

Induced voltages at the line terminations are computed in SPICE using the proposed analytical model (Sec. 6.1). Fig. 6.5 compares the transient response with that obtained from the semi-analytical technique (Chapter 4) and the IFFT approach [6]. As seen, all the three approaches match reasonably well.

Fig. 6.5. Transient response of the linear circuit due to EMI (Ex. 6.1)

B. Analysis of the Test Structure in a Nonlinear network

In this experiment, the test structure of Fig. 6.3 is terminated with CMOS based nonlinear drivers and receivers (Fig. 6.6). A digital signal of trapezoidal waveform with 0.2ns rise/fall time and 8ns pulse width is applied to the circuit as v_{in} . Fig. 6.7 shows the transient response at the input and output of the receiver at the far end.

Fig. 6.6. MTL test structure with nonlinear terminations (Ex. 6.1)

A comparison of the computational efficiency of the proposed model with respect to the semi-analytical technique (chapter 4) is given in Table 6.1.

Table 1: Performance comparison (SUN Blade CPU), Example 6.1

Example 2	Semi-Analytical Model (Chapter 4)			Proposed Model (m=28)	Speed-up
	Recursive Convolution	SPICE Simulation	Total CPU Time		
Linear	56.85	61.24	118.09	21.02	6
Nonlinear	56.85	102.72	159.57	32.13	5

Fig. 6.7. Transient response of the nonlinear circuit exposed to EMI (Ex. 6.1)

This chapter contains a summary of the work that was presented in this thesis. In addition, the directions for future work are also discussed.

7.1 Summary

In this thesis, efficient techniques were presented for transient simulation of multiconductor transmission lines exposed to electromagnetic interference. The proposed macromodels are suitable for inclusion in nonlinear circuit simulators such as SPICE. The macromodels take advantage of the delay extraction based compact and passive transmission line model (DEPACT). The first algorithm (chapters 3 and 4) models the disturbances induced by incident electromagnetic fields as equivalent sources, in the time-domain, and are computed semi-analytically. The second proposed method (chapters 5 and 6) provides further improvements by computing the equivalent sources fully analytically. Also, a theoretical error analysis is presented to quantify the accuracy levels achieved by these methods. The major advantages of the proposed algorithms are as follows.

- 1) The proposed methods result in significantly lower order macromodels for long as well as short lossy-coupled lines, while preserving the macromodel passivity. This is because, the algorithms provide for delay extraction from the transmission line

response and approximates the rest of the response via passive matrix rational approximation.

- 2) The proposed model (chapters 5 and 6) provide further computational savings as they formulate the EMI induced transient effects analytically. Also, they do not require any numerical processing like inverse Fourier/Laplace transformation as required in existing techniques to compute the EMI induced effects.
- 3) Techniques to incorporate frequency-dependent per-unit-length transmission line parameters has also been developed (chapters 4 and 6) to handle the issue of non-uniform current distributions at higher frequencies.
- 4) The approximation error associated with the proposed macromodels is also theoretically derived (chapter 5). This can be used for the selection of an appropriate order for the modified Lie formula approximation over the frequency bandwidth of interest.

7.2 *Future Research*

Some suggestions for further research based on the results of this thesis are listed below:

- 1) **Large coupled interconnects:** It is observed that, the computational cost of simulating multiconductor structures increases rapidly with the number of coupled lines. In order to address this issue, a technique based on waveform relaxation and transverse partitioning of the MTL has been recently suggested [42]. It would be of research interest to explore the coupling of the proposed incident field macromodel
-

with the waveform relaxation method. This can provide further CPU savings during incident field analysis of large number of coupled lines.

- 2) **Nonuniform incident fields:** The proposed work assumes uniform plane wave incident fields, which is a good approximation for distant radiating sources. However, for the case of interference from nearby sources, the fields can no longer be treated as plane waves. Additional work is needed to handle the nonuniform incident fields. One approach could be to use the plane wave spectrum method to represent the nonuniform incident fields as a superposition of plane waves. The obtained spectrum of plane waves could then be handled by the algorithms presented in this thesis.
 - 3) **Nonuniform interconnects:** In many practical scenarios, interconnects need to be treated as nonuniform coupled transmission lines. An algorithm based on integrated congruence transformation [64] has been recently suggested to analyze such cases. Extension of this method to incorporate the proposed incident field macromodel would be of significant research interest.
 - 4) **Inhomogeneous medium effects:** In the presented work, the perturbation of the incident field by the surrounding medium has been ignored. The effect of the dielectric medium has only been accounted for in the generation of per-unit-length line parameters. However, for accurate EMI analysis, it has been shown [31] that the effect of inhomogeneous medium is quite significant. A physical optics based approach [32] has been suggested to address this issue. Further work is required to incorporate such approaches with the methods presented here.
-

- 5) **Sensitivity Analysis:** The presented techniques in this thesis, provide a closed-form characterization of the influence of external electromagnetic fields. Therefore, they can be used for fast sensitivity analysis of the MTL networks to EMI.

(Delay Extraction based Passive and Compact MTL Macromodel)

The distributed MTL stamp $e^{Q(s)l}$ does not have an explicit time-domain representation. This presents a major difficulty in transient simulation of transmission lines in industrial circuit simulators, such as SPICE.

In order to overcome the above problem, the DEPACT algorithm [39], [40] provides an elegant solution. It employs delay extraction from the exponential MTL stamp $e^{Q(s)l}$ and approximates the rest of the MTL response via passive matrix rational approximation (MRA) [35]. This avoids higher-order MRA approximation that may be required for long low loss lines. It results in significantly lower order macromodels for long as well as short lossy-coupled lines, while preserving the macromodel passivity. Details of the algorithm are as follows.

From (2.25), the exponential MTL stamp can be written as

$$e^{Q(s)l} = e^{(A+sB)l} \quad (\text{A.1})$$

where, the matrices A and B are as given by (3.20), repeated here for convenience

$$A = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix} \quad (\text{A.2})$$

From (A.1), it can be noted that, the delay part in the MTL response is essentially contributed by the matrix B . For the purpose of accurate delay extraction from the

exponential MTL stamp, modified Lie product formula was developed [40]. According to this formula, the product

$$\prod_{k=1}^m \Theta_k + \epsilon_m; \quad \Theta_k \equiv e^{\frac{sBl}{2m}} e^{\frac{Al}{m}} e^{\frac{sBl}{2m}} \quad (\text{A.3})$$

converges asymptotically to $e^{(A+sB)l}$ as $m \rightarrow \infty$. The associated error (ϵ_m) in this case is given by [40]

$$\|\epsilon_m\| = \max_{0 \leq s \leq s_{\max}} \left\| e^{(A+sB)l} - \prod_{k=1}^m \Theta_k \right\| \cong O\left(\frac{1}{m^2}\right) \quad (\text{A.4})$$

In the context of this thesis, each of the product term in (A.3) is referred to as a modified Lie formula (MLF) cell. So, the products represented by (A.3) can be viewed as a cascade of m MLF cells. It is to be noted that, each of the k^{th} MLF cell consists of a cascade of lossy and lossless transmission line subnetworks, as shown in Fig. A.1.

Fig. A.1. Macromodel realization of the k^{th} MLF cell in (A.3)

Realization of the Lossless and Lossy Modules of the MLF Cells

The lossless modules are macromodeled by the MoC approach [8] or pure delay elements in circuit simulators. The lossy subnetworks are macromodeled using the passive MRA [35]. The resulting macromodels are of lower orders, since a significant delay portion is already extracted from these terms. It is to be noted that, the passivity of the entire

macromodel is now guaranteed, as the passivity of each subnetwork (represented by delay lines and passive MRA) is preserved.

In general, realization of the lossy modules by closed form MRA is achieved by first extracting the maximum possible delay without violating the passivity of the transmission line. From (A.3), the exponential matrix of a lossy module can be expressed as

$$e^{\frac{Q_a(s)l}{m}} = e^{\frac{(A(s) + sB_a(s))l}{m}}; \quad B_a(s) = B(s) - B(\infty) = \begin{bmatrix} 0_n & -L_a(s) \\ -C_a(s) & 0_n \end{bmatrix} \quad (\text{A.5})$$

such that $L_a(s)$, $C_a(s)$ are positive definite matrices. The exponential matrix in (A.5) can be further expressed using a N^{th} order passive matrix-rational approximation as [36]

$$\chi_N(Z) e^{\frac{Q_a(s)}{m}} \approx \Omega_N(Z) \quad (\text{A.6})$$

where $\chi_N(s)$, $\Omega_N(s)$ are polynomial matrices, given by

$$\chi_N(Z) = \sum_{i=0}^N \xi_i s^i; \quad \Omega_N(Z) = \sum_{i=0}^N \zeta_i s^i \quad (\text{A.7})$$

The above approximation is formulated analytically in terms of p.u.l. parameters and constants (i.e. ξ_i and ζ_i). These constants are obtained *a priori*, using methods such as Padé approximation or minimax optimization, while ensuring macromodel passivity. Significant reduction in the approximation order is achieved as the major portion of the delay has already been extracted.

Frequency-independent p.u.l. MTL parameters

In the case of frequency-independent p.u.l. parameters, the lossy modules can be simply realized by passive, purely resistive network (i.e., without further approximation, such as

MRA [35] etc.). In this case (A.5) can be written as

$$e^{\frac{Q(s)l}{m}} \equiv e^{\frac{Ql}{m}} = e^{\frac{Al}{m}} \quad (\text{A.8})$$

Thus, it can be noted that, the resulting lossy section is a purely resistive network. For instance, for single conductor transmission lines, the MLF cell shown in Fig. A.1 (which constitutes the approximation (A.3) of the MTL exponential stamp), can be accurately represented (by replacing the lossy module with its equivalent resistive form) as shown in Fig. A.2.

Fig. A.2. Exact realization of the k^{th} MLF cell (A.3) for a single conductor transmission line with frequency-independent parameters

Further, it can be seen that $\gamma = \sqrt{RG}$ is always a positive real value and thus the resistor values shown in Fig. A.2, are also positive. Since the resistor values are all positive, passivity of the resistive network is preserved. Similar considerations hold for the case of coupled lines. Thus, the passivity of the entire macromodel is now guaranteed as the passivity of each subnetwork represented by delay lines and passive resistive networks is preserved.

***Recursive Convolution Formulation
using the Trapezoidal Rule of
Numerical Integration***

In this section, details of derivation of (3.41) is given. From (3.40), we see

$$\hat{w}_i(t) = \mu_i \int_0^t e^{p_i(t-v)} x(v) dv \quad (\text{B.1})$$

At any time point $t = t_r$, we see

$$\hat{w}_i(t_r) = \mu_i \int_0^{t_r} e^{p_i(t_r-v)} x(v) dv \quad (\text{B.2})$$

Dividing the integration interval over two parts, from 0 to t_{r-1} and t_{r-1} to t_r , we get

$$\hat{w}_i(t_r) = \mu_i \left[\int_0^{t_{r-1}} e^{p_i(t_{r-1} + \delta_r - v)} x(v) dv + \int_{t_{r-1}}^{t_r} e^{p_i(t_{r-1} + \delta_r - v)} x(v) dv \right] \quad (\text{B.3})$$

which can be further simplified as

$$\hat{w}_i(t_r) = \mu_i \left[e^{p_i \delta_r} \int_0^{t_{r-1}} e^{p_i(t_{r-1} - v)} x(v) dv + \int_0^{\delta_r} e^{p_i(\delta_r - v)} x(v + t_{r-1}) dv \right] \quad (\text{B.4})$$

Using (B.2), (B.4) becomes

$$\hat{w}_i(t_r) = e^{p_i \delta_r} \hat{w}_i(t_{r-1}) + \mu_i e^{p_i \delta_r} \int_0^{\delta_r} e^{-p_i v} x(v + t_{r-1}) dv \quad (\text{B.5})$$

Now using trapezoidal rule of integration to solve the integral in (B.5) over the small interval 0 to δ_r , we get

$$\hat{w}_i(t_r) = e^{p_i \delta_r} \hat{w}_i(t_{r-1}) + \frac{\delta_r \mu_i}{2} [e^{p_i \delta_r} x(t_{r-1}) + x(t_r)] \quad (\text{B.6})$$

Equivalence of Scattered Field and Total Field Representations

This section gives a proof that, the solution obtained from the inhomogeneous Telegrapher's equations expressed in terms of scattered field variables (5.11) is identical to the solution obtained from the total field representation (2.25).

Note that, (2.25) can be re-written as

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = e^{Ql} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + \int_0^l e^{Q(l-z)} \tilde{F}(z) dz \quad (C.1)$$

Substituting from (2.23), the above equation becomes

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = e^{Ql} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + \int_0^l e^{Q(l-z)} \begin{bmatrix} \frac{d}{dz} V_t^{inc}(z) + V_z^{inc}(z) \\ (G + sC) V_t^{inc}(z) \end{bmatrix} dz \quad (C.2)$$

(C.2) can be further expressed as

$$\begin{aligned} \begin{bmatrix} V(l) \\ I(l) \end{bmatrix} &= e^{Ql} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + \int_0^l e^{Q(l-z)} \begin{bmatrix} \frac{d}{dz} V_t^{inc}(z) \\ 0 \end{bmatrix} dz + \int_0^l e^{Q(l-z)} \begin{bmatrix} V_z^{inc}(z) \\ 0 \end{bmatrix} dz + \\ &\quad + \int_0^l e^{Q(l-z)} \begin{bmatrix} 0 \\ (G + sC) V_t^{inc}(z) \end{bmatrix} dz \end{aligned} \quad (C.3)$$

Next, define the second term in the above summation as

$$\psi = \int_0^l e^{Q(l-z)} \begin{bmatrix} \frac{d}{dz} V_t^{inc}(z) \\ 0 \end{bmatrix} dz \quad (C.4)$$

Making further manipulations on (C.4), we have

$$\begin{aligned} \psi &= e^{Ql} \int_0^l e^{-Qz} \frac{d}{dz} \begin{bmatrix} V_t^{inc}(z) \\ 0 \end{bmatrix} dz \\ &= e^{Ql} \left\{ \left(e^{-Qz} \begin{bmatrix} V_t^{inc}(z) \\ 0 \end{bmatrix} \right) \Big|_0^l - \int_0^l e^{-Qz} (-Q) \begin{bmatrix} V_t^{inc}(z) \\ 0 \end{bmatrix} dz \right\} \\ &= e^{Ql} \left\{ e^{-Ql} \begin{bmatrix} V_t^{inc}(l) \\ 0 \end{bmatrix} - \begin{bmatrix} V_t^{inc}(0) \\ 0 \end{bmatrix} + \int_0^l e^{-Qz} (Q) \begin{bmatrix} V_t^{inc}(z) \\ 0 \end{bmatrix} dz \right\} \\ &= \begin{bmatrix} V_t^{inc}(l) \\ 0 \end{bmatrix} - e^{Ql} \begin{bmatrix} V_t^{inc}(0) \\ 0 \end{bmatrix} + \int_0^l e^{Q(l-z)} (Q) \begin{bmatrix} V_t^{inc}(z) \\ 0 \end{bmatrix} dz \end{aligned} \quad (C.5)$$

Next, using (2.22), the above equation can be further simplified as

$$\psi = \begin{bmatrix} V_t^{inc}(l) \\ 0 \end{bmatrix} - e^{Ql} \begin{bmatrix} V_t^{inc}(0) \\ 0 \end{bmatrix} - \int_0^l e^{Q(l-z)} \begin{bmatrix} 0 \\ (G + sC) V_t^{inc}(z) \end{bmatrix} dz \quad (C.6)$$

Substituting the simplified form of the term ψ , given by (C.6) in (C.3), yields

$$\begin{bmatrix} V(l) \\ I(l) \end{bmatrix} = e^{Ql} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + \int_0^l e^{Q(l-z)} \begin{bmatrix} V_z^{inc}(z) \\ 0 \end{bmatrix} dz + \begin{bmatrix} V_t^{inc}(l) \\ 0 \end{bmatrix} - e^{Ql} \begin{bmatrix} V_t^{inc}(0) \\ 0 \end{bmatrix} \quad (C.7)$$

It can be seen that, (C.7) is identical to (5.11), the solution of the Telegrapher's equations obtained from the scattered field representation.

In this section, proof of (5.43) is given. For any two matrices α and β , it can be noted that

$$\begin{aligned}
 \alpha^m - \beta^m &= \alpha^m + \sum_{k=0}^{m-2} \alpha^{k+1} \beta^{m-1-k} - \sum_{k=0}^{m-2} \alpha^{k+1} \beta^{m-1-k} - \beta^m \\
 &= \sum_{k=0}^{m-1} \alpha^{k+1} \beta^{m-1-k} - \sum_{k=1}^{m-1} \alpha^k \beta^{m-k} - \beta^m \\
 &= \sum_{k=0}^{m-1} \alpha^{k+1} \beta^{m-1-k} - \sum_{k=0}^{m-1} \alpha^k \beta^{m-k} \\
 &= \sum_{k=0}^{m-1} \alpha^k (\alpha - \beta) \beta^{m-k-1}
 \end{aligned} \tag{D.1}$$

Then, from (D.1) we see

$$\begin{aligned}
 \|\alpha^m - \beta^m\| &\leq m \|\alpha\|^k \|\beta\|^{m-1-k} \|\alpha - \beta\| \\
 &\leq m \mathfrak{S}^{m-1} \|\alpha - \beta\|
 \end{aligned} \tag{D.2}$$

where $\mathfrak{S} = \max(\|\alpha\|, \|\beta\|)$.

References

- [1] H. B. Bakoglu, *Circuits, Interconnections and packaging for VLSI*, Addison-Wesley, Reading MA, 1990.
- [2] W. W. M. Dai (Guest Editor), "Special issue on simulation, modeling, and electrical design of high-speed and high-density interconnects," *IEEE Transactions on Circuits and Systems*, vol. 39, no. 11, pp. 857-982, Nov. 1992.
- [3] R. Achar and M. Nakhla, "Simulation of high-speed interconnects," *Proc. of the IEEE*, vol. 89, pp. 693-728, May 2001.
- [4] M. Nakhla and R. Achar, *Multimedia Book Series on Signal Integrity*, OMNIZ Global Knowledge Corporation (www.omniz.com), ON: 2002.
- [5] A. Deustsch, "Electrical characteristics of interconnections for high-performance systems," *Proceedings of the IEEE*, vol. 86, no. 2, pp. 315-355, Feb. 1998.
- [6] C. Paul, *Analysis of Multiconductor Transmission Lines*, New York: John Wiley and Sons Inc., 1994.
- [7] T. Dhane and D. Zutter, "Selection of lumped element models for coupled lossy transmission lines", *IEEE Transactions on Computer-Aided Design*, vol. 11, pp. 957-967, July 1992.
- [8] F. H. Branin, Jr., "Transient Analysis of Lossless Transmission Lines," *Proc. of the IEEE*, vol 55, pp. 2012-2013, 1967.
- [9] R. Khazaka, J. Poltz, M. Nakhla, and Q. J. Zhang, "A fast method for the simulation of lossy interconnects with frequency dependent parameters," *Proceedings IEEE Multi-Chip Module Conference*, (Santa Cruz, CA), pp. 95-98, February 1996.
- [10] R. Achar, "Model-reduction techniques for high-speed interconnect analysis", Ph.D. thesis, Carleton University, Ottawa, 1998.
- [11] E. Chiprout, and M. Nakhla, "Analysis of interconnect networks using complex frequency hopping (CFH)," *IEEE Transactions on Computer-Aided Design*, pp. 186-200, Feb 1995.

-
- [12] R. Achar and M. Nakhla, Minimum Realization of Reduced-Order Models of High-Speed Interconnect Macromodels, Chapter III: *Signal Propagation on Interconnects*, pp. 23-45, Kluwer Publishers, 1998.
 - [13] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Transactions on Computer-Aided Design*, vol. 9, pp. 352-366, Apr. 1990.
 - [14] E. Chiprout and M. Nakhla, *Asymptotic Waveform Evaluation and Moment Matching for Interconnect Analysis*, Boston: Kluwer Academic Publishers, 1993.
 - [15] A. R. Djordjević and T. K. Sarkar, "Analysis of lossy transmission lines with arbitrary nonlinear terminal networks", *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-34, pp. 660-666, June 1986.
 - [16] C. D. Taylor, R. S. Satterwhite, and C. W. Harrison, "The response of a terminated two-wire transmission line excited by a nonuniform electromagnetic field," *IEEE Transactions on Antennas Propagation*, vol. 13, pp.987-989, Nov. 1965.
 - [17] A. A. Smith, "A more convenient form of the equations for the response of a transmission line excited by nonuniform fields," *IEEE Transactions on Electromagnetic Compatibility*, 151-152, Aug. 1973.
 - [18] C. R. Paul, "Frequency response of multiconductor transmission lines illuminated by an incident electromagnetic field," *IEEE Transactions on Microwave Theory and Techniques*, vol. 22, pp. 454-457, Apr. 1976.
 - [19] A. K. Agrawal, H. J. Price and S. H. Gurbaxani, "Transient response of multiconductor transmission lines excited by a nonuniform electromagnetic field," *IEEE Transactions on Electromagnetic Compatibility*, vol. 22, pp. 119-129, May 1980.
 - [20] Y. Kami and R. Sato, "Circuit-concept approach to externally excited transmission lines," *IEEE Transactions on Electromagnetic Compatibility*, vol 27, pp. 177-183, Nov. 1985.
 - [21] N. Ari and W. Blumer, "Analytic formulation of the response of a two-wire transmission line excited by a plane wave," *IEEE Transactions on Electromagnetic Compatibility*, vol. 30, pp. 437-448, Nov. 1988.
 - [22] C. R. Paul, "A SPICE model for multiconductor transmission lines excited by an in-
-

-
- cident electromagnetic field," *IEEE Transactions on Electromagnetic Compatibility*, vol. 36, No. 4, pp. 342-354, Nov. 1994.
- [23] I. Maio and F. Canavero, "Field Coupling and crosstalk in distributed networks with nonlinear loads", *IEEE Transactions on Electromagnetic Compatibility*, vol. 37, pp. 599-606, Nov. 1995.
- [24] I. Maio and F. Canavero, "Analysis of crosstalk and field coupling to lossy MTLs in a SPICE environment", *IEEE Transactions on Electromagnetic Compatibility*, Vol. 38, No. 3, Aug. 1996.
- [25] I. Erdin, A. Dounavis, R. Achar and M. Nakhla, "A SPICE model for Incident field coupling to multiconductor transmission lines," *IEEE Transactions on Electromagnetic Compatibility*, pp. 449-461, Nov. 2001.
- [26] R. K. Das and W. T. Smith, "Incident field coupling analysis of multiconductor transmission lines using asymptotic waveform evaluation," *Proc. IEEE Intl. Symp. Electromagnetic Compatibility*, pp. 265-270, Aug. 1996.
- [27] I. Wuyts and D. Zutter, "Circuit model for plane-wave incidence on multiconductor transmission lines," *IEEE Trans Electromagnetic Compatibility*, vol 36, no. 3, pp. 206-212, Aug. 1994.
- [28] P. Bernardi, R. Cicchetti and C. Pirone, "Transient response of a microstrip line circuit excited by an external electromagnetic source," *IEEE Transactions on Electromagnetic Compatibility*, vol. 34, No. 2, pp. 100-108, May 1992.
- [29] R. Khazaka and M. Nakhla, "Analysis of high-speed interconnects in the presence of electromagnetic interference," *IEEE Transactions on Microwave Theory and Techniques*, vol. 46, no. 7, pp. 940-947, Jul. 1998.
- [30] I. Erdin and M. Nakhla, "Mixed Circuit/EM Analysis of Field Coupling to High Speed Interconnects in Inhomogeneous Medium," *IEEE Int. Symposium on Electromagnetic Compatibility*, pp. 1811-1814, August 1999.
- [31] M. Leone and H. L. Singer, "On the coupling of an external electromagnetic field to a printed circuit board trace", *IEEE Transactions on Electromagnetic Compatibility*, Vol. 41, pp. 418-424, Nov. 1999.
-

-
- [32] I. Erdin, R. Khazaka and M. Nakhla, "Simulation of high-speed interconnects in a multilayered medium in the presence of incident field" *IEEE Transactions on Microwave Theory and Techniques*, Vol. 46, pp. 2251-2257, Dec. 1998.
 - [33] N. Orhanovic, P. Wang, and V. K. Tripathi, "Generalized method of characteristics for time-domain simulation of multiconductor lossy transmission lines," *Proc. IEEE Symp. Circuits syst.*, May 1990.
 - [34] A. Dounavis, X. Li, M. Nakhla and R. Achar, "Passive Closed-Form Transmission-Line Model for General Purpose Circuit Simulators," *IEEE Transactions on Microwave Theory and Techniques*, vol. 47, no. 12, pp. 2450-2459, Dec. 1999.
 - [35] A. Dounavis, R. Achar and M. Nakhla, "A general class of passive macromodels for lossy multiconductor transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, pp. 1686-1696, Oct. 2001.
 - [36] A. Dounavis, X. Li, M. Nakhla and R. Achar, "Efficient passive circuit models for distributed networks with frequency-dependent parameters", *IEEE Transactions on Advanced Packaging*, Vol. 23, No. 3, pp. 382-392, Aug. 2000.
 - [37] A. Deutch et. al., "When are transmission-line effects important for on-chip interconnections", *IEEE Transactions Microwave Theory Tech.*, Oct. 1997.
 - [38] I. M. Elfadel, H. M. Huang, A. E. Ruehli, A. Dounavis and M. Nakhla "A Comparative Study of Two Transient Analysis Algorithms for Lossy Transmission Lines with Frequency-Dependent Data," *IEEE Transactions on Advanced Packaging*, vol. 25, no. 2, pp 143-153, May 2002.
 - [39] A. Dounavis, N. Nakhla, R. Achar and M. Nakhla, "Delay Extraction and Passive Macromodeling of Lossy Coupled Transmission Lines", *Proc. IEEE 12th EPEP-2003*, pp. 295-298, Oct. 2003.
 - [40] N. Nakhla, A. Dounavis, R. Achar and M. Nakhla, "DEPACT: Delay Extraction-Based Passive Compact Transmission-Line Macromodeling Algorithm", *IEEE Transactions on Advanced Packaging*, Vol. 28, pp. 13-23, Feb. 2005.
 - [41] K. M. Coperich Branch, J. Morsey, A. C. Cangellaris and A. E. Ruehli, "Physically consistent transmission line models for high-speed interconnects in lossy dielec-
-

-
- trics", *IEEE Transactions on Advanced Packaging*, Vol. 25, pp. 129-135, May 2002.
- [42] N. Nakhla, A. Ruehli, M. Nakhla, and R. Achar, "Simulation of Coupled Interconnects using waveform relaxation and transverse partitioning", *IEEE 13th Topical Meeting on Electrical Performance of Electronic Packaging*, pp. 25-28, Oct. 2004.
- [43] J. E. Schutt-Aine and R. Mittra, "Scattering parameter transient analysis of transmission lines loaded with nonlinear terminations," *IEEE Transactions on Microwave Theory and Techniques*, vol. 36, pp. 529-536, 1988.
- [44] G. Shinh, N. Nakhla, R. Achar, M. Nakhla, A. Dounavis and I. Erdin, "Efficient SPICE macromodel for EMI analysis of electronic packages and high speed interconnects", *Proceedings IEEE Electrical Performance of Electronic Packaging (EPEP)*, pp. 277-280, Oct. 2004.
- [45] G. Shinh, N. Nakhla, R. Achar, M. Nakhla, A. Dounavis and I. Erdin, "Fast transient analysis of incident field coupling to multiconductor transmission lines", *Submitted to IEEE Transactions on Electromagnetic Compatibility (currently in the second review stage)*.
- [46] G. Shinh, N. Nakhla, R. Achar, M. Nakhla, A. Dounavis and I. Erdin, "Fast transient analysis of incident field coupling to multiconductor transmission lines", *Proceedings 10th Int. Symposium on Antenna Technology and Applied Electromagnetics (ANTEM)*, pp. 281-284, July 2004.
- [47] G. Shinh, N. Nakhla, R. Achar, M. Nakhla and I. Erdin, "An accurate and fast computer-aided design tool for electromagnetic compatibility analysis of microsystems", *Symposium on Microelectronics Research & Develeopment in Canada (TEXPO)*, Sep. 2004.
- [48] G. Shinh, N. Nakhla, R. Achar, M. Nakhla and I. Erdin, "Analysis of multiconductor transmission lines with frequency-dependent parameters and incident electromagnetic fields", *Proceedings IEEE MTT-S International Microwave Symposium*, pp. wepc3.1-wepc3.4, June 2005.
- [49] G. Shinh, R. Achar, N. Nakhla, M. Nakhla and I. Erdin, "Efficient and accurate EMC analysis of high-frequency VLSI subnetworks", *Submitted to 19th International Symposium on VLSI design*, Hyderabad, India, Jan 2006.
-

-
- [50] G. Shinh, N. Nakhla, R. Achar, M. Nakhla and I. Erdin, "ATMIC: Analytical Time-Domain Macromodel for Incident Field Coupling to MTLs with frequency-dependent parameters", *In Preperation to submit to IEEE Transactions Advanced Packaging*.
- [51] R. Achar, M. S. Nakhla and Q. J. Zhang, "Addressing high frequency issues in VLSI interconnects with full wave model and Complex Frequency Hopping (CFH)", *Proceedings IEEE Int. Conf. on Computer-Aided Design*, pp.53-56, Nov. 1995.
- [52] T. R. Arabi, A. T. Murphy, T. K. Sarkar, R. F. Harrington and A. R. Djordjevic, "On the modeling of conductor and substrate losses in multiconductor, multidielectric transmission lines systems", *IEEE Transactions on Microwave Theory and Techniques*, vol. 39, July 1991.
- [53] R. Achar and M. Nakhla, "Efficient transient simulation of embedded subnetworks characterized by S-parameters in the presence of nonlinear elements", *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-46, pp. 2356-2363, Dec. 1998.
- [54] W. T. Beyene and J. E. Schutt-Aine, "Efficient transient simulation of high-speed interconnects characterized by sampled data," *IEEE Transactions on Components, Packaging, and Manufacturing Technology*, Part B, vol. 21, no. 1, pp. 105-114, Feb. 1998.
- [55] L. M. Silveria, I. M. Elfadel, J. K. White, M. Chilukuri and K. S. Kundert, "An efficient approach to transmission line simulation using measured or tabulated S-parameter data," *in Proc. ACM/IEEE Design Automation conf.*, pp. 634-639, June 1994.
- [56] R. Achar, P. Gunupudi, M. Nakhla and E. Chiprout, "Passive interconnect reduction algorithm for distributed/measured networks", *TCAS-II*, pp. 287-301, April 2000.
- [57] D. Saraswat, R. Achar and M. Nakhla, "Passive macromodels of microwave subnetworks characterised by measured/simulated data", *IEEE International Microwave Symposium*, pp. 999-1002, June 2003.
- [58] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting", *IEEE Transactions on Power Delivery*, vol. 14, no. 3, pp. 1052-1061, July 1999.
-

-
- [59] J. Morsey and A. C. Cangellaris, "PRIME: Passive Realization of Interconnect Models from Measured Data", *IEEE 10th Topical Meeting on Electrical Performance of Electronic Packaging*, Cambridge, Massachusetts, pp. 47-50, Oct.01.
- [60] T. Mangold and P. Russer, "Full-wave modeling and automatic equivalent-circuit generation of millimeter-wave planar and multilayer structures", *IEEE Transactions on Microwave Theory and Techniques*, vol. 47, no. 6, pp. 851-858, June 1999.
- [61] R. Achar, M. A. Kolbehdari and M. Nakhla, "A unified approach for mixed EM/circuit simulation using model-reduction techniques", *IEEE International Microwave Symposium Digest*, June 1997, pp. 1017-1021, Denver, USA.
- [62] M. A. Kolbehdari, M. Nakhla, R. Achar and M. Srinivasan, "Solution of EM problems using reduced-order models by complex frequency hopping", *13th IEEE Annual Review of Progress in Applied Computational Electromagnetics*, CA, Mar. 1997.
- [63] C. W. Ho, A. E. Ruehli and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Transactions on Circuits and Systems*, vol. CAS-22, pp. 504-509, June 1975.
- [64] E. Gad and M. Nakhla, "Simulation and sensitivity analysis of nonuniform transmission lines using integrated congruence transform", *IEEE Transactions on Advanced Packaging*, vol. 28, no. 1, pp. 32-44, Feb. 2005.
- [65] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*, New York: Van Nostrand Reinhold, 1983.
- [66] C. Balanis, *Advanced Engineering Electromagnetics*, John Wiley & Sons, 1989.
- [67] J. W. Demmel, *Applied Numerical Linear Algebra*, Philadelphia: SIAM, 1997.
- [68] C. T. Chen, *Linear system theory and design*. New York: Holt, Rinehart and Winston, 1984
- [69] Maxwell 2D Extractor, Ansoft Corp. Ver. 4.5.
- [70] Synopsys-HSPICE Manual, Release 2003.1
-