Geometric Nonlinear Finite Element Analysis of Active Fibre Composite Bimorphs

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Abstract

Active fibre composite-actuated bimorphic actuators were studied in order to measure deflection performance. The deflection of the actuators was a function of the actuating electric potential applied to the active material as well as the magnitude of the axial preload applied to the bimorphic structure. This problem required the use of geometric nonlinear modeling techniques.

Geometric nonlinear finite element analysis was undertaken to determine the deflection performance of Macro Fibre Composite (MFC)- and Hollow Active Fibre (HAFC)-actuated bimorphic structures. A physical prototype MFC-actuated bimorphic structure was manufactured in order to verify the results obtained by the finite element analysis.

Theses analyses determined that the bimorphic actuators were capable of significant deflection. The analyses determined that the axial preload of the bimorphic actuators significantly amplified the deflection performance of the bimorphic actuators. The deflection performance of the bimorphic actuators suggest that they could be candidates to act as actuators for the morphing wing of a micro unmanned air vehicle.

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Nomenclature

AFC	Active Fibre Composite
EAP	Electroactive Polymer
FEA	Finite Element Analysis
FEM	Finite Element Method
HAFC	Hollow Active Fibre Composite
IDE	Interdigitated Electrodes
MAV	Micro Air Vehicle
M+H	MacNeal and Harder
MFC	Macro Fibre Composite
PVDF	Polyvinylidene Difluoride
PZT	Lead Zirconate Titanate
VSSP	Viscous Suspension Spinning Process

Symbols

1, 2, 3	Principal coordinates
A	Area
c	Compliance
D	Electric displacement
d	Piezoelectric Coupling Coefficient (Strain-Charge)
$\bar{E}$	Electric Field
e	Piezoelectric Coupling Coefficient (Stress-Charge)
E	Young's Modulus, Elastic Modulus
F	Deformation Gradient
g	Piezoelectric Coupling Coefficient (Strain-Voltage)
r	Residual Load
G	Shear Modulus
I	Second Moment of Area
K	Stiffness Matrix
L	Length
P	Pressure
q	Piezoelectric Coupling Coefficient (Stress-Voltage)
S	Strain

s	Stiffness
$\hat{s}$	Piola-Kirchoff Stress
T_{mp}	Temperature
T	Stress
t	Thickness
u, v, w	Displacements in x, y, z , respectively
x, y, z	Cartesian coordinates
ϵ	Permittivity
ϵ_0	Permittivity of Vacuum
μ	Poisson's Ratio

Chapter 1: Introduction

Actuators are mechanical devices that translate energy into a prescribed motion. Conventional actuators are typically mechanical actuators, electro-mechanical actuators, pneumatic actuators or hydraulic actuators. However, these actuators can be impractical for applications that require an extremely high degree of precision. Conventional actuators are composed of precisely manufactured components. It is difficult for conventional actuators to deliver precise motion at small scales. Piezoelectric actuators, coupled with sensors generate a feedback that allows for active control. This active control allows piezoelectric actuators to deliver precise motion at small scales. Piezoelectric actuators are therefore suited for small scale applications.

There are some operations that require high actuation power density (actuation power with respect to weight). Some piezoelectric actuators feature much higher power densities as compared with conventional actuators. The deliverable mechanical energy of piezoelectric actuators can be as much as ten times higher per unit mass as compared to comparable conventional actuators [1].

Micro Air Vehicles (MAVs) are one example of operations that require high actuation power density. Piezoelectric actuators are of interest in the field of MAVs as they can replace many conventional actuators.

The behaviour of piezoelectric actuators is examined in this thesis with the assistance of Finite Element Analysis (FEA). The piezoelectric actuators in question undergo large deformations, producing nonlinear deformation effects. As such, the large deformations undergone are described as geometric nonlinear deformation. Therefore, the nonlinear finite element model (FEM) is used in order to take into account these large, geometric nonlinear deformations.

1.1 Piezoelectric Materials

Piezoelectric materials are characterized by the ability to generate electric field when subjected to mechanical stress. This ability is referred to as the piezoelectric effect. Conversely, piezoelectric materials also exhibit the inverse piezoelectric effect (the production of mechanical strain when subjected to electric field).

Piezoelectric materials have been used in order to actuate active structures [2, 3, 4, 5], to pump fluid [6, 7], to sense strains [8, 9], to generate ultrasonic waves in structural health monitoring of components [10, 11, 12] to provide active motion or vibration control to structures [13, 14, 15, 16], in noise suppression [17], to create and detect sound [18, 19] and to generate electricity in energy harvesting applications [20, 21, 22, 23].

Crystalline materials are composed of many repeating atomic cells such as those illustrated in Figure 1-1. The properties of materials are determined largely by the structure of these cells. The atomic cell of piezoelectric materials contains an electric dipole that can be influenced by applied stresses, producing charge displacement.

Likewise, electric fields applied to the material influence the dipole, creating material strain.

Figure 1-1: Atomic structure of PZT-5H (Lead Zirconate Titanate) [25]

(a) Below the Curie Temperature ($T_{mp_{curie}}$): Electric Dipole and Piezoelectric Effect

(b) Above the Curie Temperature: Symmetric, No Dipole

The piezoelectric effect is only exhibited over a limited range of temperatures. The Curie temperature is the upper bound of this temperature range. Below the Curie temperature, the positive and negative centres of charge in the cell have an equilibrium location away from each other, producing magnetic asymmetry, and hence the piezoelectric effect is exhibited. Above this range, the equilibrium locations of charge coincide, and the piezoelectric effect is no longer exhibited [24].

The effect of the Curie temperature is illustrated in Figure 1-1. Below the Curie temperature in Figure 1-1 (a), the cell contains non-coincident centres of charge, which create an electric dipole. This dipole (oriented along the direction of the arrow) can be

influenced by an external electric field and thus exhibits the piezoelectric effect. Above the Curie temperature (illustrated in Figure 1-1 (b)), the centres of charge become coincident. No dipole is created and the piezoelectric effect is not present.

Volumes of material with similarly oriented dipoles are called Weiss domains [25]. A material may contain many Weiss domains. Figure 1-2(a) illustrates a material containing many, randomly oriented Weiss domains (The dipole orientations of the Weiss domains are denoted as arrows). If the Weiss domains within a material form a discernible net orientation (polarization), then the material will exhibit the piezoelectric phenomenon. The piezoelectric phenomenon can be brought about through the poling of the material. Poling consists of reorienting the Weiss with the application of a strong poling electric field. Weiss domains will reorient themselves to align with the poling electric field. Figure 1-2(b) illustrates the reorientation of the Weiss domains within the material in response to poling electric field. Once the poling is complete and the poling electric field is removed, the Weiss domains of the material retain a net dipole orientation and cannot easily return to their original orientations. Figure 1-2(c) illustrates the orientations of the Weiss domains in the newly poled material. The poled material can then be influenced by external electric fields and is said to exhibit the piezoelectric phenomenon. [26] The Weiss domains within the material share a net orientation. The material can be acted upon by external electric fields and is said to exhibit the piezoelectric phenomenon.

Figure 1-2: Material poling and the reorientation of Weiss domains [25]

(a) Randomly oriented Weiss domains, no net pole direction

(b) Poling: Weiss domains align to electric field

(c) Poled material aligned Weiss domains exhibits the piezoelectric phenomenon

The relationship between the intensity of the piezoelectric effect and the degree of poling is observed throughout the poling process. The poling process is illustrated in Figure 1-3. Figure 1-3(a) illustrates a material that is poled in the downwards direction. The Weiss domains of the material are all oriented in a similar direction. The poling electric field (oriented upward) is applied in Figure 1-3(b). A (negative, contracting) strain in the material is observed as a result of the electric field applied in the (upward) direction opposite to that of the Weiss domain orientation (downward). The Weiss domains reorient themselves in order to align with the electric field. The Weiss domains are reoriented (upward) in Figure 1-3(c). It should be noted that not all orientations of

Weiss domains are achievable within polycrystalline materials. In polycrystalline materials, the Weiss domains will reach a net equilibrium orientation nearing that of the poling field. As such, the Weiss domains will not be uniformly oriented. However, a net Weiss domain dipole orientation will be reached that will near the poling field orientation. Figure 1-3(d) illustrates the application of an electric field oriented (upwards) in the same direction of the material poling direction (also upwards). The material will exhibit a (positive) strain in the presence of the electric field. [26]

Figure 1-3: Strain vs. electric field during poling [26]

- (a) Piezoelectric material poled in the downward direction**
- (b) Weiss domains reorient to align with the upward electric field**
- (c) Piezoelectric material poled in the upward direction**
- (d) Actuation of the piezoelectric material in response to the electric field**

The piezoelectric phenomenon is characterized by the relationship between material strain and local charge displacement. The direct piezoelectric effect is the development of

an electric field in response to an applied stress. The magnitude of the charge displacement is proportional to the magnitude of the applied stress and is expressed in [27] as:

$$\mathbf{D} = \mathbf{d}\mathbf{T} \quad (1.1)$$

Where D is the electric displacement per unit volume and T is the stress. d is the piezoelectric coupling coefficient relating stress to electric displacement. Equation 1.1 can be rewritten in matrix form as [27]:

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \quad (1.2)$$

Where, 1, 2, 3 are the Cartesian coordinate system x, y, z. T_1 , T_2 and T_3 represent normal stresses and T_4 , T_5 and T_6 represent shear stresses. It is common to align the 3 direction with the poling direction of the piezoelectric material.

The form of the piezoelectric coupling matrix, d_{ij} , will depend on the point group of the crystal structure of the material in question. For example, Lead Zirconate Titanate (PZT), a popular piezoelectric material with a tetragonal crystal structure with a 4mm point group, has a piezoelectric coefficient matrix of the form [27]:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \quad (1.3)$$

The electric displacement, D , can also result from an externally applied electric field. The magnitude of the electric displacement is proportional to the electric field, E , and the permittivity of the material, ϵ . This relationship is expressed in Equation 1.4 [26].

$$\mathbf{D} = \epsilon \bar{\mathbf{E}} \quad (1.4)$$

The electric displacement also can be a result of both the electric field and applied stresses. Electric displacement can then be re-expressed as [26]:

$$\mathbf{D} = \mathbf{d}\mathbf{T} + \epsilon \bar{\mathbf{E}} \quad (1.5)$$

The converse piezoelectric effect is the phenomenon characterized by the deformation of a material in response to an applied electric field. The strain in the material, S , is proportional to the electric field throughout the material, E , and the piezoelectric coupling of the material, d . Strains in the material can also occur as a result of applied mechanical stress. The magnitude of the strains in the material, S , are proportional to the magnitude of the applied stresses, T , and the compliance of the material, s . Strains of the material resulting from applied electric field and mechanical stress is expressed in Equation (1.6)

$$\mathbf{S} = \mathbf{s}\mathbf{T} + \mathbf{d}\bar{\mathbf{E}} \quad (1.6)$$

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \\ d_{15} & d_{25} & d_{35} \\ d_{16} & d_{26} & d_{36} \end{bmatrix} \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \\ \bar{E}_3 \end{bmatrix}$$

The converse piezoelectric formulation is most useful for describing the motion of piezoelectric actuators. When considering a PZT-5A (a very common piezoelectric material), Equation (1.6) can be reduced to:

$$\mathbf{S} = \mathbf{s}\mathbf{T} + \mathbf{d}\bar{\mathbf{E}} \quad (1.7)$$

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \\ \bar{E}_3 \end{bmatrix}$$

The system of equations relating the electric displacement, D , with the material strains, S , is referred to as the Strain-Charge equation form.

These systems of equations can be presented in more desirable forms that include variables of interest. For example, in Finite Element Analysis, it is frequently more convenient to establish boundary conditions or return results in terms of local voltage or displacement. The method for converting the equation forms in order to include the desired variables is presented in Figure 1-5:

Figure 1-4: Constitutive Equation Forms [28]

Examples of material properties for Lead Zirconate Titanate (PZT-4) are listed in Table 1-1.

Table 1-1: Material and piezoelectric properties of PZT-4 [26]

Material Properties	PZT
Piezoelectric constants	
D_{33} (10^{-12} C/N or m/V)	300
D_{31} (10^{-12} C/N or m/V)	-150
D_{32} (10^{-12} C/N or m/V)	-150
Relative Permittivity, ϵ/ϵ_0 ($\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m)	1800
Young's Modulus (GPa)	
E_1	50
E_2	50
E_3	50
Maximum stress in traction (MPa)	
Direction 1	80
Direction 2	80
Max electric field (10^6 V/m)	1
Density (kg/m^3)	7600

1.2 Piezoelectric Actuators

Piezoelectric actuators are actuators that harness the piezoelectric effect of some materials in order to provide actuation. Piezoelectric actuators consist of piezoelectric material structures furnished with an array of electrodes that supply the actuating electric potential.

Many types of piezoelectric actuators have been designed using different arrangements of piezoelectric materials and electrodes. The following are examples of piezoelectric actuators to be discussed:

- Piezoelectric plate actuator
- Piezoelectric stack actuator
- Interdigitated electrode monolithic actuator
- Active Fibre Composite (AFC) actuator
- Hollow Active Fibre Composite (HAFC) actuator
- Unimorphic and Bimorphic actuators

Some other piezoelectric actuator configurations are:

- Piezo-hydraulic Pump
- Metal Tube Motor
- Feed-Screw Piezoelectric actuator

1.2.1 Piezoelectric Plate Actuator

The piezoelectric plate actuator consists of a monolithic plate of piezoelectric material. An electric field is applied through the plate. The plate experiences a piezoelectric strain proportional to the intensity of the electric field, E , and the magnitude of piezoelectric coupling coefficient, d . Figure 1-5 illustrates the piezoelectric plate actuator.

The intensity of the electric field is inversely proportional to the thickness of the plate that separates the voltage sources. The strain induced in the plate can then be increased by decreasing the plate thickness.

Figure 1-5: Piezoelectric plate actuator [29]

The actuation strains produced in the piezoelectric plate are outlined in Equation 1.6. When only electric fields and no external stresses are applied, equation 1.6 reduces to equation 1.8 [29]:

$$S_1 = d_{31}\bar{E}_3 + sT = d_{31}\bar{E}_3 + s(0) = d_{31}\bar{E}_3 \quad (1.8)$$

Strains produced in the material are proportional to the orthogonal piezoelectric coupling coefficient, d_{31} . The d_{31} piezoelectric coupling coefficient is not the most beneficial coefficient to use. d_{33} is larger. Different configurations of piezoelectric actuators have been formulated to take advantage of other, larger piezoelectric coupling coefficients. Such configurations include piezoelectric stack actuators and the interdigitated electrode-based actuators.

1.2.2 Piezoelectric Stack Actuator

The piezoelectric stack actuator is designed to take advantage of the d_{33} piezoelectric coupling coefficient.

The piezoelectric stack actuator consists of plates of piezoelectric material separated by sheets of conductive material. A potential difference is created between the alternating conductive sheets creating local electric fields between the conductive sheets. The local electric field direction alternates between adjacent electrodes. Likewise, the poling direction of the piezoelectric material alternates between adjacent electrodes. The electric fields actuate the piezoelectric material located between the conductive sheets. The piezoelectric plate material is poled in the same direction as the local electric field. The

result is a strain through the thickness of the piezoelectric material. The piezoelectric stack actuator is illustrated in Figure 1-6.

The actuation strains, S_3 produced in the piezoelectric stack are a function of the electric field, E_3 and the piezoelectric coupling coefficient. The relationship is outlined in Equation 1.6 [27]. With some rearrangement, Equation 1.6 reduces to:

$$S_3 = nd_{33}\bar{E}_3 \quad (1.9)$$

Since the parallel piezoelectric coupling coefficient d_{33} is typically much larger than the orthogonal piezoelectric coupling coefficient d_{31} (used in the piezoelectric plate actuator), the actuator that takes advantage of this larger coefficient will see a greater actuation performance.

Figure 1-6: Piezoelectric stack actuator [29]

1.2.3 Interdigitated Electrode Monolithic Actuator

The interdigitated electrode monolithic actuator is another configuration designed to take advantage of the d_{33} piezoelectric coupling coefficient.

The interdigitated electrode monolithic actuator consists of a monolithic plate of piezoelectric material sandwiched between two arrays of interdigitated electrodes (IDE) [30]. A potential difference is created between the alternating electrode fingers. This alternating potential difference generates an electric field. An electric field is generated between these alternating electrodes. The electric field leaves the electrode, turns, flows through the piezoelectric material, turns again and arrives at the next adjacent electrode. The piezoelectric material deforms with exposure to the electric field. The interdigitated electrode monolithic actuator is illustrated in Figure 1-7.

The magnitude of the actuator deformation is proportional to the electric field intensity. The electric field intensity is proportional to the potential difference between alternating electrodes and is inversely proportional to the electrode spacing. Since the magnitude of actuator deformation is inversely proportional to the electrode spacing, the reduction of the electrode spacing is desirable. However, there is a limitation to the reduction in electrode spacing due to limits in manufacturability as well as the risk of arcing.

Figure 1-7: Interdigitated electrode monolithic actuator [30]

The actuation strains produced in the monolithic piezoelectric actuator are outlined in Equation 1.10. Equation 1.6 reduces to Equation 1.10 when only electric fields and no external stresses are applied:

$$S_3 = d_{33}\overline{E}_3 \quad (1.10)$$

1.2.4 Active Fibre Composite (AFCs) and Macro Fibre Composites (MFCs)

Monolithic piezoceramic materials are brittle. In piezoceramic composites, the piezoceramic material are manufactured as fibres, which are then embedded into a matrix material. Piezoceramic composites are more resistant to breakage than their monolithic counterparts.

Piezoelectric fibre composites consist of piezoelectric fibres embedded in a material matrix with a pair of interdigitated electrode buses. An electric potential difference is

generated between the interdigitated electrodes. The resulting electric field distribution produces a strain in the piezoelectric fibres. The strain in the fibres leads to the deformation of the entire composite structure. The AFC is illustrated in Figure 1-8.

Figure 1-8: Active Fibre Composite (AFC) actuator [31]

There are two classes of Piezoelectric Fibre Composites: Active Fibre Composites (AFCs) and Macro Fibre Composites (MFCs). These classes of AFCs are segregated based on the geometry of the active fibres embedded in the composite. The term Active Fibre Composites (AFCs) refers to composites containing fibres that have a circular cross-section. Piezoelectric fibre composites containing square or rectangular cross-sectioned fibres are referred to as Macro Fibre Composites (MFCs).

The magnitude of the actuation in AFC and MFC-based actuators is determined by the magnitude of the electric field flux through the piezoelectric material. The magnitude of the electric field flux through the piezoelectric material is determined by the magnitude of the applied electric potential difference between adjacent electrodes and the

spacing between these electrodes, the contact area of the electrode to the piezoelectric material and the thickness of the composite [30]. Figure 1-9 illustrates the electric field flux through the active fibres of circular cross-sectioned AFCs and rectangular cross-sectioned MFCs. The electrodes conform more closely to the shape of the fibre in the MFC than in the AFC. As such, the electric field flux is slightly higher though the rectangular cross-sectioned fibres of the MFC than through the circular cross-sectioned fibres of the AFC. This higher electric field flux through the fibre results in more significant actuation of the MFC.

Figure 1-9: Electric field flow through AFC (left) and MFC (right) (adapted from [30])

The circular cross-sectioned fibres of AFCs can be advantageous. Circular fibres can be manufactured using simple extrusion or spinning methods. Also, the ambiguous rotational orientation of circular fibres leads to eased manufacturing of circular fibre-based composites.

The rectangular cross-sectioned fibres of MFCs also offer some advantages. Rectangular fibres can be manufactured using cutting methods. Also, the electrodes of

MFCs conform more closely to rectangular fibres than to circular fibres resulting in a higher electric field flux, and consequently higher actuation.

Circular cross-sectioned fibres are manufactured using one of two common manufacturing methods: extrusion or Viscous Suspension Spinning Process (VSSP) [32]. Rectangular cross-sectioned fibres are manufactured using one of two manufacturing methods: extrusion or by precision cutting of bulk ceramic.

Both the circular and rectangular fibres can be manufactured via extrusion. Bulk, green material is produced. The material is ground and a feed rod is produced. The feed rod is drawn through a die in the desired cross-section. The fibres of green material are sintered and the fibre is complete. One fibre extrusion process is charted in Figure 1-10.

Figure 1-10: Piezoelectric fibre extrusion process

There is one significant disadvantage to the interdigitated electrode configuration. The interdigitated electrodes create an inhomogeneous electric field path. The inhomogeneous electric field distribution creates an inactive region in the vicinity of the electrodes. Additionally, the inhomogeneous electric field can also cause cracking in the material during poling. [34]. The inhomogeneous electric field path is illustrated in Figure 1-11.

Figure 1-11: Inhomogeneous electric field path [34]

1.2.5 Hollow Active Fibre Composite Actuator

Hollow active fibre-based actuators employ a linear, radial electric field path and avoid the disadvantages associated with nonlinear electric field distributions.

The hollow fibre actuator consists of a hollow fibre of piezoelectric material contained in a conductive matrix material that acts as an electrode. A potential difference is created between the inner and outer walls of the fibre. An electric field is generated in the radial direction, between the inner and outer walls of the fibre. The piezoelectric material is poled in the radial direction. The electric field generated between the inner and outer walls induces strain in the piezoelectric material in the longitudinal direction.

The strain induced in the piezoelectric material is proportional to the intensity of the electric field, $\bar{E}$, and the magnitude of the piezoelectric coupling coefficient, d_{31} . The intensity of the electric field, and therefore, the magnitude of the fibre strain, is inversely proportional to the thickness of the fibre wall. It is desirable to minimize the fibre wall thickness. Limitations on manufacturability are what govern the minimum fibre wall thickness. The hollow fibre actuator is illustrated in Figure 1-12.

Figure 1-12: Hollow Fibre Actuator [4]

Among piezoceramic-base composite actuators, the hollow piezoelectric fibre-based Hollow Active Fibre Composite (HAFC) offers many advantages over the Active Fibre

Composite (AFC), including: increased reliability [8], reduced activation potential [35], a higher maximum actuation potential and a broader options for matrix materials [35]. The HAFC is also hindered by some disadvantages not present in AFCs, including: reduced actuation performance and complexity of fibre manufacturing.

The HAFC offers the advantage of increased reliability over AFCs. Since the electrodes are consistently present throughout the entire length of the hollow fibre, cracks in the fibre will not interrupt the electric field path. As such, hollow fibre composite actuator maintains its actuation performance in the event of fibre cracking better than the Active Fibre Composite (AFC) [8].

The HAFC offers the advantage of increased reliability over AFCs. Since the electrodes are consistently present throughout the entire length of the hollow fibre, cracks in the fibre will not interrupt the electric field path. As such, hollow fibre composite actuator maintains its actuation performance in the event of fibre cracking better than the Active Fibre Composite (AFC) [8].

AFCs are constrained to only having non-conductive matrix materials. Conversely, HAFCs can have any matrix material, independent of conductivity and may in fact be advantageous by having a conductive material matrix.

HAFCs can use a higher actuating electric potential than AFCs. AFCs rely on a series of interdigitated electrodes. In interdigitated electrodes, electric potentials are separated by material matrix. The risk of electrostatic discharge through this matrix material limits the actuating potential of AFCs. In HAFCs, electric potentials are separated by piezoelectric material. The risk of electrostatic discharge is significantly lower. Thus,

HAFCs can use a higher actuating electric potential than AFCs. The actuating electric potential of HAFCs is really only the limit of the material to permit this electric potential.

HAFCs typically have a lower actuation performance than AFCs. AFC actuation is proportional to the parallel piezoelectric coupling coefficient, d_{33} . HAFC actuation is proportional to the orthogonal piezoelectric coupling coefficient, d_{31} . In most materials, the d_{33} coefficient is significantly higher than the d_{31} coefficient. This lower actuation performance of HAFCs can be slightly remedied with the use of higher actuation potentials.

Hollow fibre-based composites are hindered by the added complexity related to the manufacturing of the hollow fibres as compared with the manufacturing of solid cross-section fibres. Solid cross-section fibres can be manufactured by various, simple processes. The manufacturing process of hollow fibres relies on a more complex co-extrusion process. The co-extrusion process is illustrated in Figure 1-13. The piezoceramic material is simultaneously extruded about a polymer / carbon material. Precisely manufactured extrusion dies are required to achieve this simultaneous coextrusion. The polymer / carbon material is subsequently removed during the burnout / sintering process, leaving only the hollow piezoceramic fibre.

Figure 1-13: Hollow Piezoceramic Fibre Coextrusion Process [36]

- (a) Feed rod formation: Carbon / polymer mold for PZT / polymer hollow fibre**
- (b) Feed rod extrusion: Feed rod is (re)extruded until desired size is produced**
- (c) Sintering: Carbon and polymer are removed**

1.2.6 Unimorphic and Bimorphic Actuators

Unimorphic and bimorphic actuators are actuators that undergo bending rather than linear extension, as with the previously discussed actuators. As such, the unimorphic and bimorphic actuators can achieve a larger displacement at the free end than a typical extension-type actuator. Unimorphic and bimorphic structures are illustrated in Figure 1-14.

Unimorphic structures consist of a piezoelectric actuator bonded to a compliant substrate. Extension of the piezoelectric actuator results in a stress distribution about the substrate and bending of the structure. Bimorphs consist of piezoelectric actuators bonded

to both sides of a substrate. Antiparallel actuation of the piezoelectric actuators result in more bending than the comparable unimorphic structure.

These bimorphic actuators are of interest in the field of Micro Air vehicles. MAVs can use the bending properties of the bimorphic actuator to serve as an aileron or flap actuator.

Figure 1-14: Unimorphic (left) and bimorphic (right) structures

1.2.7 Piezohydraulic Pump

The piezohydraulic pump is a pump for driving fluid flow [37]. The piezohydraulic pump employs a stack actuator to drive a diaphragm and subsequently move a fluid. The Piezohydraulic pump would be suitable for hydraulic operation. The piezohydraulic pump is illustrated in Figure 1-15.

Figure 1-15: Piezohydraulic pump [37]

1.2.8 Metal Tube Motor

The metal tube motor is a piezoelectric actuator that can provide substantial force at very small scales. The metal tube motor is illustrated in Figure 1-16. Synchronized actuation of the X and Y plates produce an elliptical bending motion of the hollow cylinder. This elliptical bending motion can drive a friction-drive rod, producing actuation. The metal tube motor has been used to control the optical zoom function of small cameras. [38]

Figure 1-16: Metal tube motor [38]

1.2.9 Feed-Screw Piezoelectric Actuator

The Feed-screw piezoelectric actuator consists of a piezoelectric stack actuator mounted to a feed-screw carriage. The result is a linear, motion-accumulating piezoelectric actuator that can provide substantial output power. The feed-screw piezoelectric actuator is illustrated in Figure 1-17. The accumulated displacement of the piezoelectric stack would be held in place with an active clamping system. The upper clamp prevents back-driving while the lower clamp allows the stack to advance the load. [39]

Figure 1-17: Piezoelectric feed-screw actuator [39]

1.3 Summary

The goal of this thesis is to determine the actuation capabilities of bimorphic structures actuated by Active Fibre Composites (AFC) and Hollow Active Fibre Composite (HAFC) based actuators.

AFCs are composite materials containing piezoelectric fibres. The piezoelectric fibres undergo strains when exposed to an electric field. The actuating electric fields in AFCs are currently supplied by interdigitated electrodes. These interdigitated electrodes carry some disadvantages, such as nonlinear electric field distributions. These disadvantages can potentially be avoided with the use of HAFCs.

HAFCs are considered more reliable than AFCs. HAFCs do not employ the problematic interdigitated electrodes used in AFCs.

Chapter 2: Nonlinear Finite Element Analysis

2.1 Introduction

Structural analyses can be divided into two categories: linear and nonlinear. Nonlinear analyses consider the nonlinear system response. Nonlinear analyses are more difficult to perform than linear analyses however, they are more widely applicable. Many systems can be satisfactorily analyzed while assuming that the response of the system is linear under small loads and /or applied fields. However, nonlinear analysis techniques are required when analyses employing the linear assumption cannot satisfactorily describe the response of the system to large loads and/or applied fields. There are many sources of nonlinearity. Three common sources of nonlinearity are [40]:

1. Material Nonlinearity
2. Geometric Nonlinearity
3. Boundary Condition Nonlinearity

2.1.1 Material Nonlinearity

The linear material approximation assumes that a material will always respond in a linear manner until it reaches a critical or failure point. These approximations may be

satisfactory for the analysis of a system undergoing small deformation or changes in geometry. Nonlinear material properties must be considered if analyses that make use of this approximation give results with insufficient accuracy.

Some examples of material nonlinearity include material critical failure, hardening, softening, or material response hysteresis. These examples are illustrated in Figure 2-1. In Figure 2-1 (a), the material responds linearly until suffering a sudden material failure. In Figure 2-1 (b) the material undergoes hardening, such as in the case of mesh or fibre-reinforced materials that undergo mesh or fibre reorientation. In Figure 2-1 (c), the material undergoes softening. In Figure 2-1 (d), the material undergoes plastic deformation. The response curve of a plastically deformed material exhibits a hysteresis. The path of the curve depends on the load history of the material.

Figure 2-1: Examples of material response [40]

- a) Linear until failure**
- b) Nonlinear Hardening / Stiffening**
- c) Nonlinear Softening**
- d) Plasticization**

In addition to mechanical stress, other fields can produce strains in materials. For example, the phenomenon of thermal expansion produces material strain caused by a change in temperature. The piezoelectric phenomenon produces strains in materials in the presence of an electric field.

In addition to having a nonlinear stress-strain relationship, materials can exhibit other nonlinear response relationships with other fields. Figure 2-2 illustrates the nonlinear relationship between electric field and strain in piezoelectric materials. The strain produced in a piezoelectric material is not linear with the applied electric field. Nonlinear analyses of systems containing materials that respond nonlinearly to electric fields may be required. However, nonlinear material effects can be insignificant over small ranges of applied electric field such as in the case of PZT-8 in Figure 2-2.

Figure 2-2: Stain vs. electric behaviour for various piezoelectric materials [26]

Moreover, the piezoelectric response varies with the electric field history and strain history of the material. Such material behaviour is illustrated in Figure 2-3. In Figure 2-3 (a), a small amplitude electric field produces a small, approximately linear strain curve. In Figure 2-3 (b, c, d), larger amplitude electric fields produce larger strain and are subject to larger hysteresis. In Figure 2-3 (e), the electric field amplitude exceeds a critical value, the polarization reorients [26]. The scale of strain that occurs in the smaller amplitude loops (a, b) in Figure 2-3 is of the magnitude where the infinitesimal strain assumption could be used. However, the piezoelectric hysteresis effects are significant and cannot simply be ignored. It may be desirable to accommodate material hysteresis in the nonlinear analysis of a system that contains materials with history-dependent properties.

Figure 2-3: Typical strain vs. electric field hysteresis in piezoelectric materials [26]

Material nonlinearity can typically be neglected with the application of the infinitesimal strain assumption. However, it may be necessary to include some nonlinear components of material behaviour in order to obtain sufficiently accurate results.

2.1.2 Geometric Nonlinearity

Geometric nonlinear analyses of structures take into account the effect of changes in the stiffness of the structures as the structures deform. Geometric nonlinearity is a symptom of large deformation / rotation of structures.

One example is a member undergoing tensile stress (Figure 2-4). The tensile stress causes a deformation that reduces the material cross-section due to the Poisson's effect

and consequently changes in the area on which the tensile stress acts. As a result, the stiffness of the structure changes with the applied load.

Figure 2-4: Structure subjected to a stress, resulting in large strain deformation

Rotations of the structure can result in changes in relative direction or magnitude of the load. One example is a lateral load on a slender structure. Figure 2-5 illustrates such changes. In Figure 2-5 (a), a slender, cantilevered beam with a cross-sectional area of A , a length of L , a Young's modulus of E and a second moment of area of I is subjected to a load P at the tip and undergoes a tip displacement u . In the initial configuration, the beam responds to the load with relatively little stiffness. The resistance to tip displacement will be approximately $3EI/L^3$. However, as the beam rotates, it will begin to behave as a beam subjected to a longitudinal load and the stiffness of the structure increases. The resistance to tip displacement will approach EA/L . These changes in structure stiffness are illustrated in Figure 2-5 (b).

Figure 2-5: Slender structure subjected to a tip load

a) Slender structure subjected to a tip load

b) Response curve of slender structure subjected to a tip load

In order to explore this example, a finite element analysis was undertaken. A slender beam with the following specifications was created:

$$A = 4m^2$$

$$L = 100m$$

$$E = 100 \times 10^9 Pa$$

$$I = 1.33m^4$$

$$(\partial P / \partial u)_{initial} \approx \frac{3EI}{L^3} = 4 \times 10^5 \frac{N}{m}$$

$$(\partial P / \partial u)_{final} \approx \frac{EA}{L} = 4 \times 10^9 \frac{N}{m}$$

The finite element model is illustrated in Figure 2-5(a). The beam was assigned a mesh of 20 x 1 x 1 ABAQUS quadrilateral, 20-node C3D20 elements. The beam was fully constrained at one end while a consistently-oriented load was applied at the free end. Tip displacement was computed at the free end.

The beam will have a response curve and corresponding displacement resistance as illustrated in Figure 2-6:

In the initial configuration, the beam responds to the load with relatively little stiffness. The resistance to tip displacement will be $3EI/L^3$ or 4.0×10^5 N/m. However, as the beam rotates, it will begin to behave as a beam subjected to a longitudinal load and the stiffness of the structure increases. The resistance to tip displacement will behave asymptotically and approach EA/L .

Figure 2-6: Displacement resistance of slender beam subjected to tip load

Load interaction can also be a source of nonlinear behaviour. One example is a nonlinear behaviour resulting from load interaction as illustrated in Figure 2-7. A slender, pin-ended beam is subjected to both longitudinal and transverse loads in Figure 2-7 (a). The linear deformation approximation includes the assumption of deformation superposition. When considered individually, each of the loads produces a relatively modest deformation. When subjected to the axial load alone, as in Figure 2-7 (b), the beam undergoes compression, such that the maximum displacement along the longitudinal direction is $u_L = \frac{APL}{E}$. When subjected to the lateral load alone, as in Figure 2-6 (c), the beam undergoes bending, such that the maximum displacement in the

transverse direction is $u_T = \frac{5WL^4}{384EI}$. When combined using the deformations superposition assumption, as in Figure 2-7 (d), there is no contribution into transverse displacement from longitudinal load. However, when the nonlinear analysis is considered, as in Figure 2-7 (e), the transverse displacement depends on both the transverse and longitudinal loads. Thus, the maximum displacement obtained in transverse direction is $u_t = \frac{w}{Pk^2} \left(\sec \frac{kL}{2} - 1 \right) - \frac{wL^2}{8P}$ [42]. The maximum displacement produced by the combined load is far greater than the sum of the displacements caused by the individual loads.

Figure 2-7: Pinned beam subjected to axial point loads and lateral pressure load [42]

- a) Pinned beam subjected to axial point loads and lateral pressure load
- b) Linear analysis of beam subjected to axial loads
- c) Linear analysis of beam subjected to lateral pressure load
- d) Linear analysis of beam subjected to both axial and lateral loads
- e) Nonlinear analysis of beam subjected to both axial and lateral loads

2.1.3 Load Nonlinearity or Boundary Condition Nonlinearity

Finite element analyses must account for boundary condition nonlinearity or load nonlinearity. Boundary condition nonlinearity accounts for changes in boundary conditions with respect to structural deformation. One example of boundary condition nonlinearity is structural interaction. Load nonlinearity accounts for loads that change direction or magnitude with change in structural displacement.

2.2 Finite Element Analysis (FEA)

Solving for structures with nonlinear response schemes can be difficult. Solving difficult nonlinear systems can be accomplished with the use of numerical solving methods, such as finite element analysis.

Structural finite element analysis seeks to determine the response of structures to load cases, usually in the form of displacement or strain. Structures are discretized into geometrically simple elements. The response of these simple elements can be determined. The equations determining the response of an element can be assembled into a system of ordinary or partial differential equations in order to determine the response of the structure.

In the case of piezoelectric structural materials, the interrelated system of partial differential equations consist of [8]:

$$\begin{aligned}\nabla \mathbf{T} + \mathbf{b} &= 0 \\ \mathbf{D} &= 0\end{aligned}\tag{2.1}$$

Where ∇ is the divergence operator, $\mathbf{T}$ is the stress and $\mathbf{b}$ are the body forces and $\mathbf{D}$ is the charge displacement. The interrelationship between stresses and charge displacements is described in depth in chapter 1.

The FEA model is defined by the geometry and relevant features of the subject. The model is meshed by discretizing the structure into a series of geometrically simple elements. Elements are selected on the basis of the problem type and degrees of freedom of interest. The discretization process is typically carried out automatically by the finite element analysis pre-processor.

Elemental stiffness matrices are created based on the problem type and degrees of freedom of interest. The stiffness matrices of individual elements are assembled. The assembled elemental stiffness matrices create the stiffness matrix for the model. The loads and boundary conditions of the problem are assigned to the relevant nodes. The loads and boundary conditions, along with the system stiffness matrix create a system of ordinary or partial differential equations. Solving this system of equations devises the response of the system. The response of a system is:

$$\mathbf{P} = \mathbf{K}\mathbf{u} \quad (2.2)$$

Where displacement, $\mathbf{u}$, or other nodal degree of freedoms are related to load, $\mathbf{P}$, or other force function through the elemental stiffness matrix, $\mathbf{K}$. The stiffness matrix can be made to accommodate nonlinear analyses with the formulation:

$$\mathbf{K} = \mathbf{K}_L + \mathbf{K}_{NL} + \mathbf{K}_\sigma = \bar{\mathbf{K}} + \mathbf{K}_\sigma \quad (2.3)$$

$\mathbf{K}_L$ is the infinitesimal strain stiffness matrix, $\mathbf{K}_{NL}$ is the large deformation stiffness matrix and $\mathbf{K}_g$ is the geometric stiffness matrix.

2.2.1 Linear Stiffness Matrix, $\mathbf{K}_L$

The linear stiffness matrix relates load on the element and displacement for the case where the element is undergoing infinitesimal strain. The matrix has the form:

$$\mathbf{K}_L = \int_V \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L dV = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L \det[\mathbf{J}] d\xi d\eta d\zeta \quad (2.4)$$

Where $\mathbf{D}$ is the material stiffness matrix that relates stress and strain constitutive Equation:

$$\mathbf{T} = \mathbf{D}\boldsymbol{\epsilon} \quad (2.5)$$

For the simple case of isotropic elastic materials, the $\mathbf{D}$ matrix has the form of the material stiffness matrix. The material stiffness matrix establishes the relationship between stress and strain and has the form:

$$\mathbf{D} = \frac{E}{(1+\nu)(1+2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1+2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1+2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1+2\nu)}{2} \end{bmatrix} \quad (2.6)$$

$\mathbf{B}_L$ is the matrix that maps nodal displacement to strain in the case of infinitesimal strain. Again, for the simple case of an elastic material $\mathbf{B}_L$ is a $6 \times 3n$ matrix, where n is the number of nodes in the element.

$$\mathbf{B}_L = \mathbf{L}\mathbf{N} \quad (2.7)$$

Where N is the interpolation function. For a 20-node quadratic element, some of the shape functions have the following form in the natural coordinate system [45]:

$$\mathbf{N} = [N_1, \dots, N_i, \dots, N_{20}]^T \quad (2.8)$$

$$N_i = \begin{cases} \frac{1}{8}[(1+\xi)(1-\eta)(1-\zeta)(\xi-\eta-\zeta-2)], i=1 \\ \dots \\ \frac{1}{4}[(1-\zeta^2)(1-\xi)(1-\eta)], i=20 \end{cases}$$

Where $\mathbf{L}$ in equation 2.6 is defined in equation 2.8. $\mathbf{L}$ is a gradient function. For a system with three degree of freedom and six strain dimensions, $\mathbf{L}$ has the form:

$$\mathbf{L}^T = \begin{bmatrix} \partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z \\ 0 & \partial/\partial y & 0 & \partial/\partial x & \partial/\partial z & 0 \\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & \partial/\partial x \end{bmatrix} \quad (2.9)$$

$\text{Det } \mathbf{J}$ is the determinant of the Jacobian matrix that maps the deformed element geometry in the Cartesian coordinate system to the ordered matrix geometry in the natural coordinate system, as illustrated in Figure 2-8. This determinant is required to maintain the volume relation between the element in the deformed state to the element in the unreformed state, which is done through a scaling factor known as the determinant of the Jacobian.

Figure 2-8: Jacobian transformation [46]

A three-dimensional Jacobian matrix has the form [46]:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} = \sum_i \begin{bmatrix} \frac{\partial N_i}{\partial \xi} x_i & \frac{\partial N_i}{\partial \eta} x_i & \frac{\partial N_i}{\partial \zeta} x_i \\ \frac{\partial N_i}{\partial \xi} y_i & \frac{\partial N_i}{\partial \eta} y_i & \frac{\partial N_i}{\partial \zeta} y_i \\ \frac{\partial N_i}{\partial \xi} z_i & \frac{\partial N_i}{\partial \eta} z_i & \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix} \quad (2.10)$$

In order to undertake the integration of Equation 2.3, the matrix must be evaluated using a Gaussian integration scheme. The integration is replaced by a summation. The values of the terms of the summation are evaluated at a series of Gauss points and are multiplied by weight factor, $w_{(r, s, t)}$:

$$\begin{aligned} \mathbf{K}_L &= \int_V \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L dV \\ &= \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L \det \mathbf{J} d\xi d\eta d\zeta \\ &= \sum_r \sum_s \sum_t \left(w_{(r, s, t)} \mathbf{B}_L^T \mathbf{D} \mathbf{B}_L | \det \mathbf{J} |_{(r, s, t)} \right) \end{aligned} \quad (2.11)$$

The Gaussian integration scheme for a 20-node quadratic brick element C3D20 is illustrated in Figure 2-9. The location and weight factor of some of the Gaussian integration points are listed in Table 2-1.

Figure 2-9: Gaussian integration scheme [47]

Table 2-1: Examples of Gauss points and weight factors for 20-node quadratic brick element C3D20 [44]

Gauss Point	Location			Weight Factor
	X	Y	Z	
1	-0.77459667	-0.77459667	-0.77459667	0.171467764
2	0.77459667	0	-0.77459667	0.274348422
5	0	0	-0.77459667	0.438957476
14	0	0	0	0.702331962

2.2.2 Large Displacement Stiffness Matrix, K_{NL}

An object undergoing large deformations, displacements and rotations is illustrated in Figure 2-10. This object's initial configuration, x^0 , is transformed to the current configuration, x , through a system of displacements, u and $u+du$. Point P in the current configuration is identified through vector x . Neighboring points, q are identified by the differential vector dx [43].

Figure 2-10: Initial configuration, x^0 and deformed configuration, x [43]

The relationship between the initial configuration differential vector, dx^0 , and the current configuration differential vector, dx , is mapped in the deformation gradient, F [43]:

$$F = \frac{\partial x}{\partial x^0} = \begin{bmatrix} \partial x / \partial x^0 & \partial x / \partial y^0 & \partial x / \partial z^0 \\ \partial y / \partial x^0 & \partial y / \partial y^0 & \partial y / \partial z^0 \\ \partial z / \partial x^0 & \partial z / \partial y^0 & \partial z / \partial z^0 \end{bmatrix} = I + \nabla u \quad (2.12)$$

where ∇u is the displacement gradient:

$$\nabla \mathbf{u} = \begin{bmatrix} \partial u / \partial x & \partial u / \partial y & \partial u / \partial z \\ \partial v / \partial x & \partial v / \partial y & \partial v / \partial z \\ \partial w / \partial x & \partial w / \partial y & \partial w / \partial z \end{bmatrix} \quad (2.13)$$

F reflects the deformation, displacement and rotations of the object and is a function of I and $\nabla \mathbf{u}$. However, strains are only a function of $\nabla \mathbf{u}$. Strains are not a function of displacement or rotations alone.

The deformation gradient can be modified with the gradient function, to give $\bar{\mathbf{F}}$ [43].

$\bar{\mathbf{F}}$ is used to compute $\bar{\mathbf{K}}$.

$$\bar{\mathbf{F}} = \mathbf{F} \mathbf{L} \mathbf{x} = \begin{bmatrix} \partial x / \partial x^0 & \partial x / \partial y^0 & \partial x / \partial z^0 \\ \partial y / \partial x^0 & \partial y / \partial y^0 & \partial y / \partial z^0 \\ \partial z / \partial x^0 & \partial z / \partial y^0 & \partial z / \partial z^0 \end{bmatrix} \begin{bmatrix} \partial / \partial x & 0 & 0 \\ 0 & \partial / \partial y & 0 \\ 0 & 0 & \partial / \partial z \end{bmatrix}^x \begin{bmatrix} \partial / \partial x & 0 & 0 \\ 0 & \partial / \partial y & 0 \\ 0 & 0 & \partial / \partial z \end{bmatrix}^y \begin{bmatrix} \partial / \partial x & 0 & 0 \\ 0 & \partial / \partial y & 0 \\ 0 & 0 & \partial / \partial z \end{bmatrix}^z \quad (2.14)$$

$$\bar{\mathbf{F}} = \begin{bmatrix} \partial x / \partial x^0 & 0 & 0 & \partial y / \partial x^0 & 0 & 0 & \partial z / \partial x^0 & 0 & 0 \\ 0 & \partial x / \partial y^0 & 0 & 0 & \partial y / \partial y^0 & 0 & 0 & \partial z / \partial y^0 & 0 \\ 0 & 0 & \partial x / \partial z^0 & 0 & 0 & \partial y / \partial z^0 & 0 & 0 & \partial z / \partial z^0 \\ \partial x / \partial y^0 & \partial x / \partial x^0 & 0 & \partial y / \partial y^0 & \partial y / \partial x^0 & 0 & \partial z / \partial y^0 & \partial z / \partial x^0 & 0 \\ 0 & \partial x / \partial z^0 & \partial x / \partial y^0 & 0 & \partial y / \partial z^0 & \partial y / \partial y^0 & 0 & \partial z / \partial z^0 & \partial z / \partial y^0 \\ \partial x / \partial z^0 & 0 & \partial x / \partial x^0 & \partial y / \partial z^0 & 0 & \partial y / \partial x^0 & \partial z / \partial z^0 & 0 & \partial z / \partial x^0 \end{bmatrix}$$

The large displacement stiffness matrix, $\bar{\mathbf{K}}$, relates load, $\mathbf{P}$, and displacement, $\mathbf{u}$, for the case where the element is undergoing large displacements or rotations. The matrix has the form:

$$\bar{\mathbf{K}} = \mathbf{K}_L + \mathbf{K}_{NL} = \int_V \mathbf{B} \bar{\mathbf{F}}^T \mathbf{D} \bar{\mathbf{F}} \mathbf{B} dV \quad (2.15)$$

Where $\mathbf{B}$ is the matrix that maps nodal degrees of freedom to the displacement gradient. For a three degree of freedom system with n nodes, $\mathbf{B}$ has the form:

$$\mathbf{B} = \mathbf{L}_{NL} \mathbf{N} = \begin{bmatrix} \frac{\partial}{\partial x^0} & 0 & 0 \\ \frac{\partial}{\partial y^0} & 0 & 0 \\ \frac{\partial}{\partial z^0} & 0 & 0 \\ 0 & \frac{\partial}{\partial x^0} & 0 \\ 0 & \frac{\partial}{\partial y^0} & 0 \\ 0 & \frac{\partial}{\partial z^0} & 0 \\ 0 & 0 & \frac{\partial}{\partial x^0} \\ 0 & 0 & \frac{\partial}{\partial y^0} \\ 0 & 0 & \frac{\partial}{\partial z^0} \end{bmatrix} N_1 \quad \dots \quad \begin{bmatrix} \frac{\partial}{\partial x^0} & 0 & 0 \\ \frac{\partial}{\partial y^0} & 0 & 0 \\ \frac{\partial}{\partial z^0} & 0 & 0 \\ 0 & \frac{\partial}{\partial x^0} & 0 \\ 0 & \frac{\partial}{\partial y^0} & 0 \\ 0 & \frac{\partial}{\partial z^0} & 0 \\ 0 & 0 & \frac{\partial}{\partial x^0} \\ 0 & 0 & \frac{\partial}{\partial y^0} \\ 0 & 0 & \frac{\partial}{\partial z^0} \end{bmatrix} N_n \quad (2.16)$$

2.2.3 Geometric Stiffness Matrix, $\mathbf{K}_\sigma$

The geometric stiffness matrix relates load on the element and displacement for the case where the stiffness of the element is a function of the load acting upon it, such as in the case where the element is experiencing large stresses. The matrix has the form [48]:

$$\mathbf{K}_\sigma = \int_V \mathbf{B}^T \begin{bmatrix} \hat{\mathbf{s}} & 0 & 0 \\ 0 & \hat{\mathbf{s}} & 0 \\ 0 & 0 & \hat{\mathbf{s}} \end{bmatrix} \mathbf{B} dV \quad (2.17)$$

Where $\hat{\mathbf{s}}$ is the second Piola-Kirchoff stress tensor [43]:

$$\hat{\mathbf{s}} = \det \mathbf{F} \mathbf{F}^{-1} \mathbf{T} \mathbf{F}^T = \det \mathbf{F} \mathbf{F}^{-1} \begin{bmatrix} T_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & T_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & T_z \end{bmatrix} \mathbf{F}^T \quad (2.18)$$

The stresses in the element are determined from the strains. The strains in the element are determined from the deformation gradient, $\mathbf{F}$:

$$\mathbf{T} = \mathbf{E}\boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \frac{1}{2}(\nabla \mathbf{u}^T + \nabla \mathbf{u} + \nabla \mathbf{u}^T \nabla \mathbf{u}) = \frac{1}{2}[\mathbf{F}^T \mathbf{F} - \mathbf{I}] \quad (2.19)$$

In order to compute the stress or strain relationship, the simulation must have completed at least one iteration in the solution procedure.

2.3 Solution Methods

There are a number of techniques used to solve nonlinear finite element analyses, including: the incremental (Euler) method, the iterative (Newton-Raphson) method, the corrected incremental method or the modified Newton-Raphson method.

The response curve of the system being analyzed (shown in Figure 2-11) maps the relationship between load, P and displacement, u . Many of the solution schemes rely on finding the tangent to this curve known as the tangent stiffness matrix, k_t . The tangential stiffness matrix represents the local tangent to the response curve [49]:

$$(k_t)_i = \left(\frac{\partial P}{\partial u} \right)_i \quad (2.20)$$

2.3.1 Incremental (Euler) Method

The incremental (Euler) method is illustrated in Figure 2-11. Given a target load, ΔP_B , the system is subjected to a series of incremental loads ΔP_i . For each incremental load applied, the system subsequently undergoes an incremental displacement response, Δu_i .

The magnitude of the incremental displacement is proportional to the incremental load and the inverse of the tangent stiffness matrix, $(k_t)_i$ [49].

$$u_{i+1} = u_i + \Delta u_i = u_i + \left[\left(\frac{\Delta u}{\Delta P} \right)_i \Delta P_{i+1} \right] = u_i + \left((k_t)_i \right)^{-1} \Delta P_{i+1} \quad (2.21)$$

Once the sum of the incremental loads equals the target load for the system, the sum of the incremental displacements undergone by the system should be equal to the displacement of the system when subjected to the target load [49]. Unfortunately, the incremental method solution response will always drift from the system response [49], as shown in Figure 2-11. This drift can be reduced with the use of a corrective term. The corrected incremental method is discussed below.

Figure 2-11: Incremental (Euler) method [48]

2.3.2 Iterative Method (Newton-Raphson) Method

The Newton-Raphson Method is illustrated in Figure 2-12. At a given starting point, A (Figure 2-12 (A)) and a starting load, P_A , the resulting response displacement of the system due to P_A is measured to be u_A and the tangential stiffness matrix at point A , $(k_t)_A$ is determined. The target load is P_B . The displacement u_B that would result from load P_B is sought. A truncated Taylor series of about A is then [48]:

$$P_B = f(u_B) = f(u_A + \Delta u) = P_A + \left(\frac{\partial P}{\partial u} \right)_A \Delta u = P_A + (k_t)_A^{-1} \Delta u \quad (2.22)$$

The residual or out-of-balance load term, g , is a measure of the difference between the target load, P_B , and the internal load that would produce the iterated displacement, $P_i = f(u_i)$ [48]:

$$r(u_i) = P_B - P_i(u_i) \quad (2.23)$$

The next displacement iteration, u_{i+1} , occurs at the intercept of P_B and the tangent from current load iteration $P_i(u_i)$ [48]:

$$u_{i+1} = u_i + K_i^{-1} r = u_i + K_i^{-1} (P_B - P_i(u_i)) \quad (2.24)$$

The iteration is repeated until the residual, r , is zero or is sufficiently small.

Figure 2-12: Newton-Raphson method [48]

The Newton-Raphson Method seeks to eliminate the out-of-balance force term, g . The seeking process may take many iterations. The number of iterations required may be reduced by accepting an out-of-balance load that is “sufficiently small”. Accepting such imperfections may introduce a significant cumulative error into complex systems that rely on many iterative processes.

2.3.3 Corrected Incremental Method

The response solution given by the Incremental Method will lead to drift from the system response. This drift can be reduced or eliminated with the use of a corrective term. The

corrected incremental displacement incorporates the residual force term from Newton-Raphson method [48]:

$$r_i(u_i) = P_{External} - P_{Internal}(u_i) \quad (2.25)$$

g_i is the same term introduced in the Newton-Raphson method. r_i is the residual load term (i.e. difference between the load P_E and the load that would produce displacement u_i , $P_i(u_i)$.)

For a given state load, P_i , and displacement, u_i , the stiffness matrix, $(k_i)_i$ and residual load, r_i are determined. The increase in displacement due to an applied load is proportional to the stiffness matrix and the difference between the residual [48]:

$$u_{i+1} = u_i + (k_i)_i^{-1} (P_{i+1} - P_{iR}) = u_i + (k_i)_i^{-1} [P_{i+1} - (P_i - r_i)] \quad (2.26)$$

The response scheme given by the corrected incremental method is illustrated in Figure 2-14.

Figure 2-13: Incremental method with load correction [48]

The corrected incremental method requires less incrimination and delivers a response that more closely follows the system response than the original, uncorrected incremental method.

The difference between the Newton-Raphson Method and the Corrected Incremental Method is thus: The Newton Raphson Method iterated displacement until the load that would produce such displacement reached the target load. The Corrected Incremental Method increments load until the target load is reached and the resulting displacement is measured.

Chapter 3: Validation

3.1 Introduction

A validation of the capabilities of the ABAQUS finite element analysis software and of the capabilities of the finite element analyst was undertaken. This validation took the form of a series of linear elastic tests, geometric nonlinear elastic tests and piezoelectric tests. Linear elastic tests were furnished by MacNeal and Harder [50]. Geometric nonlinear elastic test were furnished by Sze [51] and by Benham and Crawford [42]. Piezoelectric tests were furnished by Martinez [44] and by the reproduction of experiments conducted in [4].

ABAQUS, the finite element analysis software package, was used for all tests. Testing was undertaken using the ABAQUS 20-node elastic element (C3D20), 20-node elastic reduced-order-integration element (C3D20R) and the 20-node piezoelectric element (C3D20E).

3.2 Linear Elastic Problems: MacNeal and Harder Tests

MacNeal and Harder established a set of tests to determine the accuracy of finite element analysis tools in various applications [50]. The tests consisted of simple structures with geometries undergoing simple load cases that are easily solved analytically for which reliable computational results are available. These simple problems test the capability of

the software and of the analyst to implement simple geometries, simple boundary conditions and simple load cases. Furthermore, the tests evaluate the capability of the software to correctly implement element discretization, assembly, processing and post-processing.

MacNeal and Harder provided theoretical results for each test presented in [50]. Agreement with these theoretical results is the metric used to gauge the success of the ABAQUS analysis. In some cases, these theoretical results are unachievable due to sparse meshing. MacNeal and Harder also provided the most accurate results achievable with sparse meshing. Achieving results that are comparable or better than those obtained by MacNeal and Harder (M+H) is the secondary metric used to gauge the success of the ABAQUS analysis.

The MacNeal and Harder problem sets have been nondimensionalized. While not expressed, a self-consistent system of units are used. The magnitude and order of the values presented are consistent.

The MacNeal and Harder tests were undertaken using the ABAQUS 20-node elastic element (C3D20), the 20-node reduced-order-integration element (C3D20R) and the 20-node piezoelectric element (C3D20E). The C3D20E element returned results that were identical to those returned by the C3D20 element and are not presented.

The tests undertaken include:

- Patch test
- Straight beam test (with rectangular, parallelogram and trapezoidal elements)
- Curved beam test

- Twisted beam test
- Rectangular plate test
- Scordelis-Lo roof test

3.2.1 Patch Test

The Patch Test is designed to be simple, yet test all features of Finite Element Analysis: application of boundary conditions, meshing, element interaction, solution and post-processing.

The patch test consisted of a unit cube. The cube was assigned a Young's modulus 1.0×10^6 and a Poisson's ratio of 0.25. The mesh was assigned a geometry defined by the cube geometry and a series of internal nodes. The elements of the patch test as well as the coordinates of the internal nodes are included in Figure 3-1.

Figure 3-1: Patch test geometry and nodal coordinates [50]

A displacement field was assigned to the exterior nodes of the structure as follows:

$$u = \frac{(2x + y + z)}{2} \cdot 10^{-3}$$

$$v = \frac{(x + 2y + z)}{2} \cdot 10^{-3}$$

$$w = \frac{(x + y + 2z)}{2} \cdot 10^{-3}$$

The theoretical resultant stress and strain fields are as follows:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 10^{-3}$$

$$\sigma_x = \sigma_y = \sigma_z = 2000$$

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 400$$

The results of the patch test are tabulated in Table 3-1.

Table 3-1: Results for patch test

Element C3D20				
Load	ABAQUS Result	Theoretical Result	% Diff	Agree?
Stress	2000	2000	0.00	Yes
Shear	400	400	0.00	Yes
Strains	1.00E-03	1.00E-03	0.00	Yes

Element C3D20R				
	ABAQUS Result	Theoretical Result	% Diff	Agree?
Stress	2000	2000	0.00	Yes
Shear	400	400	0.00	Yes
Strains	1.00E-03	1.00E-03	0.00	Yes

The finite element analysis of the patch test provided results that were consistent with the predicted results. ABAQUS passed the patch test.

3.2.2 Straight Beam Test

The straight beam test consisted of a straight beam constructed from rectangular, parallelogram and trapezoidal elements.

The beam had dimensions of $6.0 \times 0.2 \times 0.1$. The beam was assigned a mesh that was subdivided into six elements along its length. The beam geometry is depicted in Figure 3-2. The beam was assigned a Young's modulus of 1.0×10^7 and a Poisson's ratio of 0.3. The beam was fully constrained at one end and was subjected to a unit load in either the axial, lateral out-of-plane or lateral in-plane directions. The tip displacement due to this load was measured. The tip deflections are tabulated Table 3-2.

Figure 3-2: Straight beam, with

- a) Rectangular elements
- b) Parallelogram elements
- c) Trapezoidal elements

Table 3-2: Results for the straight beam test

Rectangular Elements							
Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	2.983E-05	3.00E-05	0.57	Yes	2.98E-05	0.10	Yes
In-Plane	1.049E-01	0.1081	2.96	Yes	1.05E-01	0.10	Yes
Out-of-Plane	4.151E-01	0.4321	3.93	Yes	4.15E-01	0.02	Yes

Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	2.997E-05	3.00E-05	0.10	Yes	3.00E-05	0.10	Yes
In-Plane	1.049E-01	0.1081	2.96	Yes	1.05E-01	0.10	Yes
Out-of-Plane	4.151E-01	0.4321	3.93	Yes	4.15E-01	0.02	Yes

Parallelogram Elements							
Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	2.87E-05	3.00E-05	4.33	Yes	2.83E-05	1.41	Yes
In-Plane	1.05E-01	0.1081	2.87	Yes	1.04E-01	0.96	Yes
Out-of-Plane	4.13E-01	0.4321	4.42	Yes	3.76E-01	9.84	No

Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	3.00E-05	3.00E-05	0.00	Yes	3.00E-05	0.00	Yes
In-Plane	1.07E-01	0.1081	1.02	Yes	1.05E-01	1.90	Yes
Out-of-Plane	4.15E-01	0.4321	3.96	Yes	4.13E-01	0.48	Yes

Trapezoidal Elements							
Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	2.983E-05	3.00E-05	0.57	Yes	2.98E-05	0.10	Yes
In-Plane	1.043E-01	0.1081	3.52	Yes	1.04E-01	0.29	Yes
Out-of-Plane	3.765E-01	0.4321	12.87	No	3.77E-01	0.13	Yes

Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Extension	2.998E-05	3.00E-05	0.07	Yes	3.00E-05	0.07	Yes
In-Plane	1.043E-01	0.1081	3.52	Yes	1.04E-01	0.29	Yes
Out-of-Plane	3.765E-01	0.4321	12.87	No	3.77E-01	0.13	Yes

The models returned results that were consistent with the results set out in [50] or with those set out by M+H. ABAQUS passed the straight beam test.

3.2.3 Curved Beam Test

The curved beam test consisted of applying load to the free end of a curved beam. The beam had a cross-section of 0.2 x 0.1 and an arc-length of 90°, as illustrated in Figure 3-3. The beam was assigned a Young's modulus of 1.0×10^7 and a Poisson's ratio of 0.25. The beam was assigned a mesh that was sectioned into six elements along its length. The beam was subjected to a unit load at the free end. The tip displacements resulting from this load are tabulated in Table 3-3.

Figure 3-3: Curved beam [50]

Table 3-3: Results for the curved beam test

Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
In-Plane	0.076423	0.08734	12.50	No	0.076423	0.00	Yes
Out-of-Plane	0.473575	0.5022	5.70	No	0.475081	0.32	Yes

Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
In-Plane	0.087864	0.08734	0.60	Yes	0.088213	0.40	Yes
Out-of-Plane	0.475081	0.5022	5.40	No	0.480103	1.05	Yes

The model returned results that were consistent with the analytical results set out in [50]. ABAQUS passed the curved beam test.

3.2.4 Twisted Beam Test

The curved beam test consisted of applying load to the free end of a twisted beam. The twisted beam was a 12.0 x 1.1 x 0.32 beam subject to a 90° twist along its length. The beam was assigned a mesh with a 12 x 2 x 1 distribution. The beam geometry and element distribution is illustrated in Figure 3-4. The beam was constrained at one end and subjected to in-plane and out-of-plane unit loads at the free end. The beam was assigned a Young's modulus of 29.0×10^6 and a Poisson's ratio of 0.22. The tip deflection resulting from the end loads are tabulated in Table 3-4.

Figure 3-4: Twisted beam [50]

Table 3-4: Results for twisted beam geometry

Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
In-Plane	0.005375	0.005424	0.90	Yes	0.005375	0.00	Yes
Out-of-Plane	0.001747	0.001754	0.40	Yes	0.001745	0.11	Yes

Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
In-Plane	0.005386	0.005424	0.70	Yes	0.005386	0.00	Yes
Out-of-Plane	0.001752	0.001754	0.11	Yes	0.001752	0.00	Yes

The 20-node elastic element passed the twisted beam test. The model returned results that were consistent with the results set out in [50]. The model returned results that were fairly consistent with the theoretical results. ABAQUS passed the test.

3.2.5 Rectangular Plate Test

MacNeal and Harder proposed two geometries and two load cases for the rectangular plate test. The two plate geometries are illustrated in Figure 3-5. The first plate had dimensions of 2.0 x 2.0 x 0.01 and an aspect ratio of 1.0. The second plate had dimensions of 10.0 x 2.0 x 0.01 and an aspect ratio of 5.0. For both geometries, the plates were assigned a mesh with 8 x 8 x 1 element distribution.

The first load case prescribed fully constrained plate edges and a point load of 4.0×10^{-4} applied at the centre of the plate. The second load case prescribed simply supported plate edges and a pressure of 1.0×10^{-4} applied to the entire surface of the plate.

Four tests were conducted. These tests consisted of a:

- Plate with an aspect ratio of 1.0 with clamped edges and a point load at the centre of the plate
- Plate with an aspect ratio of 1.0 with simply supported edges, subjected to a pressure load

- Plate with an aspect ratio of 5.0 with clamped edges and a point load at the centre of the plate
- Plate with an aspect ratio of 5.0 with simply supported edges, subjected to a pressure load

Figure 3-5: Plate geometry: (left) aspect ratio 1.0 and (right) aspect ratio 5.0 [50]

Table 3-5: Results for rectangular plate test

Aspect Ratio 1.0, Element C3D20							
	ABAQUS	Theoretical			M+H		
	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Clamped edges, Point load	4.03E-06	4.062E-06	0.79	Yes	4.03E-06	0.00	Yes
Simply supported edges, pressure load	4.60E-06	5.60E-06	17.86	No	4.60E-06	0.00	Yes

Aspect Ratio 1.0, Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Clamped edges, Point load	4.09E-06	4.062E-06	0.69	Yes	4.09E-06	0.00	Yes
Simply supported edges, pressure load	5.28E-06	5.60E-06	5.71	No	5.29E-06	0.19	Yes

Aspect Ratio 5.0, Element C3D20							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Clamped edges, Point load	1.33E-05	1.297E-05	2.54	Yes	1.33E-05	0.00	Yes
Simply supported edges, pressure load	2.35E-06	7.23E-06	67.50	No	2.70E-06	12.96	No

Aspect Ratio 5.0, Element C3D20R							
	ABAQUS	Theoretical			M+H		
Load	Tip Displacement	Tip Displacement	% Diff	Agree?	Tip Displacement	% Diff	Agree?
Clamped edges, Point load	1.30E-05	1.297E-05	0.23	Yes	1.31E-05	0.76	Yes
Simply supported edges, pressure load	6.27E-06	7.23E-06	13.28	No	6.27E-06	0.00	Yes

In most cases, ABAQUS provided results that were consistent with the theoretical results set out in [50]. In the cases where ABAQUS returned results that were significantly different from theoretical results, the ABAQUS results were consistent with results obtained by MacNeal and Harder [50]. The results obtained by MacNeal and Harder were the most accurate obtainable by a finite element model with such sparse meshing. ABAQUS passed the flat plate test.

In the case of the test involving the pressure loaded, simply supported, aspect ratio 5.0 plate with C3D20 elements, the finite element analysis returned results that were inconsistent with those set out in [50]. In this case, both MacNeal and Harder [50] and the finite element analysis returned results that were consistently inconsistent with the theoretical results.

3.2.6 Scordelis-Lo Roof test

The Scordelis-Lo roof consisted of a curved plane. The curved plane had a radius of curvature of 25.0, a length of 50.0, a thickness of 0.25 and an arc length of 80 degrees. The plane was assigned a Young's modulus of $E = 4.32 \times 10^8$ and a Poisson's ratio of $\mu = 0.0$. The Scordelis-Lo roof is illustrated in Figure 3-6.

The plane was loaded to 90 per unit area in the negative Z direction. The curved edges of the plane were constrained in place. The Z-component of displacement of the plane edges is tabulated and compared to expected values in Table 3-6.

Figure 3-6: Scordelis-Lo roof [50]

Table 3-6: Results for Scordelis-Lo roof test

	ABAQUS	Theoretical			M+H		
	Displacement	Displacement	% Diff	Agree?	Displacement	% Diff	Agree?
C3D20	2.519E-01	3.024E-01	16.70	No	2.455E-01	2.59	Yes
C3D20R	3.086E-01	3.024E-01	2.05	Yes	3.021E-01	2.15	Yes

ABAQUS produced results that were inconsistent with the theoretical results. However, ABAQUS did produce results that were consistent with those obtained by MacNeal and Harder (M+H). ABAQUS passed the Scordelis-Lo roof test.

3.3 Geometric Nonlinear Tests

Validation of the modeler's ability to implement geometric nonlinear problems was verified by undertaking geometric nonlinear benchmark tests. The first test consisted of a

heavily loaded flat beam. The second test consisted of a bar loaded both axially and transversely.

In each benchmark test, geometry and loading condition is supplied [42, 51]. The theoretical solution is also supplied [42, 51]. The ABAQUS model is considered to have succeeded if it returned results that agreed with the theoretical solutions.

3.3.1 Heavily Loaded Flat Beam Test

The test consisted of a flat beam subjected to a consistently oriented shear load applied to the end face of the beam.

The plate had dimensions of 10.0 x 1.0 x 0.1 m. The beam was assigned a mesh that was segmented into 8 and 16 elements along its length. The beam was assigned a Young's modulus of $E = 1.2 \times 10^6 \text{ N/m}^2$ and a Poisson's ratio of 0.0.

One end of the beam was fully constrained. The other end of the beam was subjected to a constantly oriented out-of-plane shear stress. The stress ranged in magnitude from 0 N/m^2 to 40 N/m^2 .

The vertical and horizontal components of tip displacement were measured and compared to the solution presented in [51]. The finite element model of the beam (with 16 elements) is presented in Figure 3-7. The results are presented in Figure 3-8.

Figure 3-7: Flat beam with 16 elements (left) and displacement resulting from load (right)

Figure 3-8: Results for heavily loaded flat plate: vertical (y) and horizontal (x) components of tip displacement

ABAQUS returned results that were consistent with those provided in [51]. The finite flat beam with 16 elements along its length provided more accurate results than the model with only eight elements along its length.

3.3.2 Dually-Loaded Beam Test

This benchmark test consisted of a beam loaded in both the axial and transverse direction. The beam had dimensions of 50.0 x 1.0 x 1.0 m. The beam was assigned a mesh with a distribution of 200 (50 x 2 x 2) elements.

One end of the beam was fixed in space but was allowed to rotate in the direction of the applied transverse load. The beam was loaded with an axial point load and a transverse pressure load. The axial load was set to 1000 N. The transverse pressure load varied logarithmically (10^3 , 10^4 , 10^5 N/m²). The theoretical solution giving beam tip displacement was given in [42]. The exercise was undertaken once while considering the geometric nonlinear effects (NLGeom on) and again while ignoring geometric nonlinear effects (NLGeom off). The beam is presented in Figure 3-9. The results are presented in Table 3-7.

Figure 3-9: Dually-loaded beam

Table 3-7: Results for dually-loaded beam test

Axial load (N)	Lateral load (N/m ²)	Theoretical deflection (m)	NLGeom Off		NLGeom ON	
			Deflection (m)	% Diff	Deflection (m)	% Diff
1000	1.00E+02	4.90E-02	4.90E-02	0.10%	4.89E-02	0.22%
1000	2.50E+02	1.23E-01	1.22E-01	0.49%	1.22E-01	0.65%
1000	5.00E+02	2.45E-01	2.48E-01	1.10%	2.45E-01	0.20%
1000	7.50E+02	3.68E-01	3.67E-01	0.30%	3.67E-01	0.35%
1000	1.00E+03	4.89E-01	4.89E-01	0.02%	4.89E-01	0.02%
1000	2.50E+03	1.23E+00	1.22E+00	0.90%	1.22E+00	0.33%
1000	5.00E+03	2.45E+00	2.38E+00	2.81%	2.45E+00	0.33%
1000	7.50E+03	3.68E+00	3.48E+00	5.52%	3.67E+00	0.33%
1000	1.00E+04	4.89E+00	4.47E+00	8.53%	4.89E+00	0.02%
1000	2.50E+04	1.23E+01	8.78E+00	28.38%	1.22E+01	0.33%
1000	5.00E+04	2.45E+01	Failed*	N/A	2.45E+01	0.31%

The finite element model of the beam that considered the geometric nonlinearity effect successfully returned results that were consistent with the theoretically determined solution. The finite element model that did not consider the geometric nonlinearity effect returned results that diverged greatly from the mathematically determined solution when larger loads were applied.

3.4 Piezoelectric Tests

Validation of the modeler's ability to implement piezoelectric problems was verified by undertaking piezoelectric tests. The first piezoelectric tests consisted of simple, mathematically determinable problems furnished in [44]. Other piezoelectric tests consisted of reproducing experimental results published in [4]

The piezoelectric problems undertaken were:

- Piezoelectrically-Extended Straight Beam Test
- Piezoelectrically Sheared Straight Beam Test
- Piezoelectrically-Actuated Bimorphic Beam Test
- Piezoelectric hollow active fibre
- Piezoelectric Hollow Active Fibre Composite (HAFC)

The experimental problems undertaken in [4] were the actuation of a hollow active fibre and the actuation of a hollow active fibre composite.

3.4.1 Piezoelectrically-Extended Straight Beam Test

The Piezoelectrically-Extended Straight Beam test consisted of the simulation of extension of a beam due to the piezoelectric effect. The beam geometry for these tests was the same as those in defined section 3.1.2. The beam was assigned a mesh that was subdivided into six elements with (a) rectangular, (b) parallelogram and (c) trapezoidal geometry. The piezoelectric material used (PZT-5A) was assigned Young's moduli of $E_{11} = E_{22} = 60.6 \times 10^9 \text{ N/m}^2$ and $E_{33} = 48.3 \times 10^9 \text{ N/m}^2$, Poisson's ratios of $\mu_{12} = 0.2896$ and $\mu_{13} = \mu_{23} = 0.408$, shear moduli of $G_{12} = 23.6 \times 10^9 \text{ N/m}^2$ and $G_{13} = G_{23} = 2.29 \times 10^9 \text{ N/m}^2$, permittivities of $k_{11} = k_{22} = 2.77 \times 10^{-8} \text{ F/m}$ and $k_{33} = 3.01 \times 10^{-8} \text{ F/m}$ and piezoelectric coefficients of $d_{15} = d_{24} = 741 \times 10^{-12} \text{ C/N}$, $d_{31} = d_{32} = -274 \times 10^{-12} \text{ C/N}$ and $d_{33} = 593 \times 10^{-12} \text{ C/N}$. The beam was constrained at one end and a potential difference of 1 V (or an electric field intensity of 0.01 V/mm) was established through the thickness of the beam.

The displacement of the free end was measured. A schematic of the setup is illustrated in Figure 3-10. The results are tabulated in Table 3-8.

Figure 3-10: Straight beam undergoing piezoelectric extension [44]

Table 3-8: results for piezoelectrically-extended straight beam test

	Displacement, u (m)		
	Theoretical	ABAQUS	% Diff
Rectangular Elements (Type a)	1.644E-08	1.628E-08	0.973
Parallelogram Elements (Type b)	1.644E-08	1.630E-08	0.859
Trapezoidal Elements (Type c)	1.644E-08	1.635E-08	0.576

The finite element analysis returned tip displacement results that were consistent with the theoretical tip displacement. The Piezoelectrically-extended straight beam test was passed successfully.

3.4.2 Piezoelectrically-Sheared Straight Beam Test

The piezoelectrically sheared straight beam test consists of the simulation of the shearing of a beam due to the piezoelectric effect. The beam was assigned geometry used for the

straight beam test in section 3.1.2. The beam was subdivided into six elements with (a) rectangular, (b) parallelogram and (c) trapezoidal geometry. The Piezoelectric material used (PZT-5A) was assigned material properties described in section 3.3.1. The beam was constrained at one end and a potential difference of 1 V (or an electric field intensity of 0.01 V/mm) was established through the thickness of the beam. The displacement of the free end was measured. A schematic of the setup is illustrated in Figure 3-11. The results are tabulated in Table 3-9.

Figure 3-11: Straight beam undergoing piezoelectric shear [44]

Table 3-9: Results for piezoelectric sheared straight beam test

	Displacement, w (m)		
	Theoretical	ABAQUS	% Diff
Rectangular Elements (Type a)	4.446E-08	4.446E-08	0.000
Parallelogram Elements (Type b)	4.446E-08	4.446E-08	0.000
Trapezoidal Elements (Type c)	4.446E-08	4.446E-08	0.000

As seen in table 3-9, the finite element analysis returned tip displacement results that were consistent with the theoretical tip displacement. The Piezoelectrically sheared straight beam test was passed successfully.

3.4.3 Piezoelectrically-Actuated Bimorphic Beam Test

The piezoelectrically-actuated bimorphic beam test consists of the simulation of the deformation of a bimorphic beam due to the piezoelectric effect. The beam used had the geometry used for the straight beam test in section 3.2.2. The beam was subdivided into six elements with (a) rectangular, (b) parallelogram and (c) trapezoidal geometry. The elements were then subdivided again through the thickness of the plate. The piezoelectric material used (PZT-5A) was assigned material properties recorded in section 3.4.1. The beam was constrained at one end and a potential difference of 1 V (or an electric field intensity of 0.01 V/mm) was established through the thickness of the beam. The displacement of the free end was measured. A schematic of the setup is illustrated in Figure 3-12. The results are tabulated in Table 3-10.

Figure 3-12: bimorphic beam undergoing piezoelectric deformation [44]

Table 3-10: Results for piezoelectrically-actuated bimorphic beam test

	Displacement, w (m)		
	Theoretical	ABAQUS	% Diff
Rectangular Elements (Type a)	1.4796E-08	1.465E-08	0.947
Parallelogram Elements (Type b)	1.4796E-08	1.465E-08	0.947
Trapezoidal Elements (Type c)	1.4796E-08	1.465E-08	0.947

As seen, the finite element analysis returned tip displacement results that were consistent with the theoretical tip displacement. The piezoelectrically-actuated bimorphic beam test was passed successfully.

3.4.4 Piezoelectric Hollow Active Fibre test

Experiments have been conducted in order to determine the actuation performance of hollow piezoelectric fibres embedded in an epoxy matrix. The results of these experiments were reported in [4].

Finite element analyses were conducted in order to validate the ability of the analyst to produce successful analyses using the piezoelectric element C3D20E. Finite element analyses were created, reproducing the geometry and conditions of the hollow active fibres and the hollow active fibre composites (HAFCs) used in [4].

One of the Hollow Active Fibres presented in [4] had an aspect ratio (ratio of wall thickness to outer radius) of 0.42, an outer wall radius of 450 μm and wall thickness of 190 μm . The piezoelectric material used (PZT-5A) was assigned material properties recorded in section 3.4.1. Application of an electric potential difference of 57 V between the inner and outer walls of the fibre generated an electric field intensity of 300 V/mm. The fibre in [4] produced a longitudinal strain of 80 microstrain.

A finite element model was created, reproducing the hollow fibre in [4]. The quarter-fibre finite element model had a length of 77 mm. When actuated, the fibre extended 6.15 μm , resulting in an average strain of 79.2 microstrain. The actuated fibre model is illustrated in Figure 3-13. The finite element analysis returned results that were consistent with those published in [4]. The finite element analysis succeeded in reproducing the piezoelectric fibre experiment.

Figure 3-13: Actuated quarter-fibre

3.4.5 Piezoelectric Hollow Active Fibre Composite (HAFC) Test

Experiments were also conducted on Active Fibre Composites (AFCs) in [4]. These AFCs consisted of the hollow piezoelectric fibre embedded in an epoxy matrix material. The fibre was the same 0.42 aspect ratio used previously. The matrix was applied such that the composite had a fibre volume fraction of 4.6%. The same actuating potential of 57 V (300 V/mm) was applied. The composite produced an average strain of approximately 45 microstrain.

The matrix material was Insulcast 501 / Insulcure 24. The matrix material had a Young's Modulus of $E = 1.29 \times 10^9$ Pa and a Poisson's ratio of $\mu = 0.3$.

The hollow active fibre composite consisted of the same hollow piezoelectric fibre embedded in an epoxy matrix. A model of a quarter fibre was created. The hollow fibre had a wall thickness of 190 μm and an external radius of 450 μm . The fibre was embedded in an epoxy matrix such that the fibre-to-matrix volume fraction was 4.6%.

This corresponded to having a quarter fibre with an external diameter of $450\text{ }\mu\text{m}$ embedded in an epoxy matrix measuring $1685\text{ }\mu\text{m}$ by $1400\text{ }\mu\text{m}$. The entire HAFC was 77 mm long. A quarter cell of the HAFC is illustrated in Figure 3-14.

Figure 3-14: One quarter of a unit cell of the hollow active fibre composite

The finite element model of the HAFC experienced a longitudinal strain of $3.69\text{ }\mu\text{m}$, resulting in an average strain of 48 microstrain, which was consistent with [4]. The finite element model successfully reproduced the experimental HAFC.

3.5 Conclusions

Tests were created to gauge the capabilities of the finite element analysis software as well as the capabilities of the finite element analyst. These tests included linear elastic tests, geometric nonlinear tests and piezoelectric tests. The modeler created finite element models that successfully returned the expected solutions to each of the test cases.

Chapter 4: Analysis and Results

4.1 Introduction

A piezoceramic bimorph actuator was studied. The reliability of Hollow Active Fibre Composites to damage was also considered as a part of this study. The piezoceramic bimorph actuator was designed to create a morphing wing for a fixed-wing aircraft. The morphing capability was used to replace conventional ailerons. A schematic of the wing is presented in Figure 4-1. The morphing wing was developed for a micro, unmanned air vehicle (MAV) that is capable of performing the prop-hanging maneuver. The prop-hanging maneuver is a high angle of attack flight configuration where the aircraft is essentially hovering. Hover is a desirable capability for MAVs on missions such as reconnaissance or inspection. The lifting force holding the aircraft aloft is provided exclusively by the propulsion system (i.e. thrust is greater than weight.)

The prop-hanging maneuver requires powerful roll control. In order to provide such roll control, the MAV wing must be capable of large aileron deflection. A piezoceramic bimorph actuator-based morphing wing would be capable of large deflection of the part of the morphing wing which is replacing the aileron. The deflection ability of the piezoceramic bimorph actuator was the subject of study.

The prop-hanging maneuver also requires an experienced pilot. The piezoceramic bimorph actuator relies on an active feedback control system for operation. A control system could operate the piezoceramic bimorph wing while reducing the need for pilot experience.

Furthermore, the MAV would have very tight design envelopes for size, structural weight and power consumption. The piezoceramic bimorph actuator has excellent specific power and low energy consumption.

The piezoelectric material-based morphing wing would consist of a series of Macro Fibre Composite (MFC)-actuated or Hollow Active Fibre Composite (HAFC)-actuated bimorphic structures. These bimorphic structures would deliver the desired aileron deflection required to perform the prop-hanging maneuver. Piezoelectric material-based morphing wing is illustrated in Figure 4-1.

Figure 4-1: Macro Fibre Composite (MFC)-based morphing wing [54]

It is hypothesized that the aileron deflection performance of the bimorphic structures could be further amplified with the application of an axial precompression force [2]. Such an axial precompression force could be delivered by Electro-Active Polymer (EAP) skin attached to the bimorphic structures, delivering a controlled precompressive force.

The goal of this thesis is to determine and compare the aileron tip displacement performance of an axially-precompressed, Macro Fibre Composite (MFC)-actuated and Hollow Active Fibre Composite (HAFC)-actuated bimorphic structures. A physical MFC-actuated bimorphic structure was manufactured. A finite element model of this MFC-actuated bimorphic structure was also created. The finite element model of the MFC-actuated bimorphic structure was used to examine the effects of load conditions not achievable by the physical actuator. A finite element model of the HAFC-actuated bimorphic structure was created. HAFC actuators were selected in part due to their increased reliability during fibre cracking as compared with MFC actuators. An examination of the reliability of HAFCs was also undertaken.

4.2 Macro Fibre Composite-Actuated Bimorphic Structure

A Macro Fibre Composite-actuated bimorphic structure was created consisting of two Macro Fibre Composite (MFC) actuators (Figure 4-2(a)) bonded to an aluminum substrate (Figure 4-2(b)). Actuation was brought about with the application of an actuating electric potential of 1500 V. Measurements were taken of the out-of-plane tip displacement.

The macro fibre composite actuators consisted of lead zirconate titanate (PZT-5A) fibres in an epoxy matrix, sealed in Kapton. The macro fibre composite actuators were M-8528-P1 actuators provided by Smart-Materials, Inc.

The M-8528-P1 macro fibre composite actuator is illustrated in Figure 4-2(a). The M-8528-P1 actuator has dimensions of 110 mm x 45 mm. The active region (the region containing the active fibres and the interdigitated electrodes) had dimensions of 85 mm x 28 mm. The active fibres had cross-section dimensions of $350\text{ }\mu\text{m} \times 175\text{ }\mu\text{m}$. The active fibres were spaced $235\text{ }\mu\text{m}$ apart. The space between the active fibres is occupied by an epoxy matrix. This epoxy matrix was composed of epoxy DOW DER 330. The interdigitated electrodes of the MFC were spaced $500\text{ }\mu\text{m}$ apart.

Figure 4-2: Macro fibre composite

(a) M-8528-P1 macro fibre composite actuator

(b) Cross section of bimorphic structure containing an M-8528-P1 actuator

Thin aluminum sheet 88.9 μm (0.035") thick was used as the substrate of the bimorphic actuator.

Epoxy DOW DER 330 was used as an adhesive between the aluminum substrate and the M-8528-P1 Macro Fibre Composite Actuators. The adhesive layer had an approximate thickness of 38.1 μm (0.0015in).

The entire bimorphic structure had a measured thickness of 765 μm . The measured thicknesses of the aluminum substrate (88.9 μm), the adhesive layers (38.1 μm each) and the active fibres (175 μm each) could be accounted for. The remaining 250 μm was attributed to the thickness of the epoxy matrix and the surrounding Kapton layers. Hence, the epoxy matrix and Kapton layers were attributed a thickness of 31.25 μm each.

The Macro Fibre Composite-actuated bimorphic structure was clamped in a fixture. An actuating electric potential difference of 1500 V was established between the interdigitated electrodes using a voltage amplifier (not shown). An axial load of 15 N, 30 N and 40 N was measured and applied to the structure using a series of pre-stressed rubber bands. The tip displacement of the structure was measured using a Keyence LK-086 laser displacement sensor. The Keyence LK-086 had a measurement range of 30 mm and a resolution of 3 μm . Displacement was measured at the edge of the active region of bimorphic actuator. The test setup is illustrated in Figure 4-3. The resulting tip displacements are presented in Figure 4-8.

Figure 4-3: Experimental setup [54]

An analytical model predicting the tip deflections of the MFC-actuated bimorphic structure was created by Dr. Eric Chen of the Institute of Aerospace Research at the National Research Council Canada. The analytical model was created using equations set out in [52] and [53]. A composite beam containing piezoelectric materials is illustrated in Figure 4-4.

Figure 4-4: Composite beam undergoing tip deflection due to applied load and piezoelectric stresses [53]

The displacement distribution of a composite structure is given as:

$$\frac{d^2 z}{dx^2} = \frac{T_L(L-x) + M_L - \sum_i E_i Z_i A_i d_{33} V_i - T_a(z_L - z(x))}{EI} \quad (4.1)$$

Where,

- T_L is the lateral load
- M_L is the moment
- E_i is the Young's modulus of layer i
- A_i is the cross-sectional area of layer i
- d_{33} is the piezoelectric coupling coefficient
- V_i is the electric field at layer i
- T_a is the axial load
- E is the composite young's modulus
- I is the second moment of area

The parameters used in the model are tabulated in Table 4-1. The cross-sectional area of a layer, A_i , is determined by the thickness of the layer and by the width of the beam. The bimorph was assigned a width of 35 mm and a length, L , of 95 mm.

Table 4-1: Material parameters of the bimorphic structure

Material	Young's Modulus, E (Pa)	Thickness, t (m)	Piezoelectric Coupling, d_{31} (V/m)
PZT	61.52×10^9	191×10^{-6}	171×10^{-12}
Aluminum	70×10^9	76×10^{-6}	0
Adhesive	3.378×10^9	31×10^{-6}	0

The mathematically-predicted tip deflections are also presented in Figure 4-8.

A finite element model of the MFC-actuated bimorphic structure was also created.

4.3 Finite Element Model of Macro Fibre Composite-Actuated Bimorphic Structure

A finite element analysis of the bimorphic structure was produced. The goal of the finite element analysis was to determine the tip displacement performance of the actuated bimorphic structure. A representative finite element model was created. The finite element analysis was simplified and reduced to the width of a single active fibre (plus a proportional quantity of accompanying, surrounding, inactive materials). The reduced finite element model of the bimorphic structure was composed of 15,288 three dimensional, 20-node, piezoelectric (C3D20E) elements and had dimensions of 98 x 0.585 x 0.765 mm. The electrodes were 85 μm in width and were separated by a 415 μm space.

The physical bimorphic actuator was clamped at one end. The finite element model only included the segment of the actuator that was not clamped. Hence, the finite element model was 98 mm long while the physical actuator was 114 mm long.

The finite element model used a mesh of 15,288 C3D20E elements (13 x 3 x 4 elements per mm, 98 mm long). The representative finite element model is show in Figure 4-5.

Figure 4-5: Finite element model of the MFC-actuated bimorphic structure

A cross-section of the finite element model of the bimorphic structure, with dimensions, is illustrated in Figure 4-6.

Figure 4-6: Cross-section of the finite element model

In order to remain consistent with the physical bimorph, 13 mm of the model, starting from the unclamped end, was made inactive. In the inactive length of the model, the piezoelectric fibre material was removed, replaced with epoxy matrix material.

The material properties used in the finite element model of the bimorphic structure are listed in Table 4-2.

Table 4-2: Material properties of active fibre composite bimorph structure

	Aluminum	Adhesive	Kapton	Matrix Epoxy	Piezoelectric Fibre
		DOW DER 330	DuPont 100-HN	DOW DER 330	PZT-5A
Young's modulus (GPa)	70.00	3.38	2.05	3.38	60.90
Poisson's ratio	0.33	0.33	0.36	0.33	0.35
Permittivity					
K11 (F/m)	6.20E-05	3.52E-06	3.01E-11	3.52E-05	1.53E-08
K22 (F/m)	6.20E-05	3.52E-06	3.01E-11	3.52E-05	1.53E-08
K33 (F/m)	6.20E-05	3.52E-06	3.01E-11	3.52E-05	1.51E-08
Piezoelectric Coefficients					
d15, d24 (pC/N)	0.00	0.00	0.00	0.00	585.00
d31, d32 (pC/N)	0.00	0.00	0.00	0.00	-171.00
d33 (pC/N)	0.00	0.00	0.00	0.00	374.00

The actuation of the Active Fibre Composite bimorph structure was achieved by applying an electric potential of 1500 V to alternating electrodes. Other electrodes were grounded. On one side of the aluminum substrate, the piezoelectric fibre was poled in the same direction as the applied electric field, resulting in fibre extension. On the other side of the aluminum substrate, the piezoelectric fibre was poled in the direction opposite to that of the applied electric field, resulting in fibre contraction. This coupling of fibre extension and contraction produced the out-of-plane bending of the bimorphic structure. The actuation of the bimorphic structure was further amplified by the application of an axial compressive load to the structure. These load conditions are illustrated in Figure 4-7.

Figure 4-7: Load conditions applied to active fibre bimorph structure

The finite element analysis was conducted using the manufacturer-suggested actuation potential of 1500 V. The theoretical maximum actuating potential that could be applied to the material is 2000 V. The finite element analysis was also conducted using this theoretical maximum actuation potential of 2000 V.

The resulting deflections of the finite element model bimorph are compared to the results for the physical bimorph deflection in Figure 4-8. There is a strong agreement between results obtained by the physical model, the finite element model and the analytical model. This strong agreement indicates that the tip deflection resulting from an actuation potential of 1500 V is correct. Given that the finite element model produced correct results for an actuating potential of 1500 V, it can be assumed that the finite element model will also produce correct results for an actuating potential of 2000 V, assuming no hysteresis.

Figure 4-8: Tip deflection of an MFC-actuated bimorph subjected to an axial preload and an actuating potential of either 1500 V or 2000 V

As shown in Figure 4-8, the MFC-actuated bimorph achieved an out-of-plane tip displacement of 11.1 mm due to the manufacturer-suggested actuation potential of 1500 V and without axial preload. This out-of-plane tip displacement could be amplified to 16 mm using an axial preload of 40 N.

The maximum actuating electric potential theoretically sustainable by the PZT material is 2000 V. The MFC-actuated bimorph achieved an out-of-plane tip displacement of 14.77 mm due to this actuation potential of 2000 V and without axial

preload. This out-of-plane tip displacement could be amplified to 23.2 mm using an axial preload of 40 N.

4.4 Finite Element Model of Hollow Active Fibre Composite (HAFC) Actuator

A finite element analysis was conducted to determine the out-of-plane bending performance of a Hollow Active Fibre Composite (HAFC)-actuated bimorphic structure.

HAFC actuators were selected in part due to their increased reliability during fibre cracking as compared with MFC actuators [8]. The reliability of HAFC during fibre cracking was examined and presented in [8].

4.4.1 HAFC Actuator Reliability During Fibre Cracking

A finite element model was created to study the effect of transverse fibre cracking on HAFC. Transverse fibre cracking is a common problem challenging the reliability of MFCs. MFCs tend to develop transverse fibre cracks near the interdigitated electrodes due to inhomogeneous electric fields during poling. Transverse fibre cracking is not the only composite failure mechanism that can occur in MFC, however, it is the most common. Other mechanism of composite failure in MFC include fiber delamination

The model consisted of a lamina of hollow piezoelectric fibres embedded in an epoxy matrix. The lamina is illustrated in Figure 4-9. The lamina was assigned dimensions of 77 mm long 2.8 mm tall 16.85 mm wide. The hollow fibres embedded in the lamina had a diameter of 900 μm and wall thickness of 190 μm . The fibres were assigned the material properties of PZT-5H and had a Young's Modulus of $E_{11} = E_{22} = 60.6 \times 10^9 \text{ Pa}$ and $E_{33} =$

48.3×10^9 Pa, Poisson's ratios of $\mu_{12} = 0.2896$ and $\mu_{13} = \mu_{23} = 0.408$, shear moduli of $G_{12} = 23.6 \times 10^9$ Pa and $G_{13} = G_{23} = 2.29 \times 10^9$ Pa, permittivities of $k_{11} = k_{22} = 2.77 \times 10^{-8}$ F/m and $k_{33} = 3.01 \times 10^{-8}$ F/m and piezoelectric coefficients of $d_{15} = d_{24} = 741 \times 10^{-12}$ C/N, $d_{31} = d_{32} = -274 \times 10^{-12}$ C/N and $d_{33} = 593 \times 10^{-12}$ C/N. Polarization direction about the fibre circumference was taken into account when assigning these material properties. The matrix material was assigned the material properties of Insulcast 501 / Insulcure 24 and had a young's modulus of $E = 1.29 \times 10^9$ Pa and a Poisson's ratio of $\mu = 0.3$.

Figure 4-9: Finite element model of HAFC lamina [8]

An electrode bus, located at both ends of the lamina could establish potential difference of 57 V (300 V/mm) between the inner and outer walls of the hollow active fibres. The fibres were polled in the radial direction. This electric potential difference and poling are illustrated in Figure 4-10. The actuation potential caused an extension of the lamina.

Figure 4-10: Hollow active fibre with electrode placement and poling

Cracks were simulated in the fibres of the lamina. The cracks regions were modeled so that the piezoelectric material was removed and substituted for an epoxy material with $1/1000^{\text{th}}$ of the stiffness of the matrix epoxy material. The crack material had a Young's Modulus of $E = 1.29 \times 10^9$ Pa and a Poisson's ratio of $\mu = 0.3$. The crack isolated a segment of the piezoelectric material from the actuating electric potential. The electric potential placement and the effect of the crack is illustrated in Figure 4-11. The degradation of the elongation performance of the lamina was studied as cracks were introduced. The results are presented in Figure 4-12.

Figure 4-11: Broken fibre effects [8]

(a) HAFC lamina with electric potential and

(b) Cracked HAFC with electric potential removed from inactive regions

Figure 4-12: Lamina extension performance with introduction of fibre cracks [8]

The lamina underwent 100.0% extension when no fibre cracks were present. When five cracks were present, the lamina extension performance degraded to 99.3% of that of the undamaged lamina. The lamina undergoes little elongation performance degradation.

The lamina suffers very little extension performance degradation as cracks are introduced. With the dual electrode bus system, the active fibres are not separated from the actuating electric potential and therefore do not suffer much loss of actuation performance.

The reliability and performance of HAFC lamina during sensing was also examined.

4.4.2 HAFC Sensor Reliability During Fibre Cracking

The finite element of the model used in section 4.3.1 was reused in order to examine the reliability of HAFC lamina during sensing. The lamina was fully constrained at one end, while the other end was subjected to a fixed extension of 35 mm (45.4 microstrain). The potential difference established between the inner and outer walls of the fibre was monitored as cracks were introduced to the fibres. The results are illustrated in Figure 4-13. The lamina, undergoing sensing is illustrated in Figure 4-14.

Figure 4-13: Lamina sensing performance with introduction of fibre cracks [8]

The lamina underwent 100.0% extension when no fibre cracks were present. When five cracks were present, the lamina extension performance degraded to 96.5% of that of the undamaged lamina. The lamina undergoes little sensing performance degradation as cracks are introduced.

Figure 4-14: Voltage distribution on damaged sensor lamina [8]

HAFC actuators have been proven to be very reliable during actuation. The use of HAFCs to actuate a bimorphic structure has been explored.

The examination of the reliability of HAFC presented here was also presented in [8].

4.4.3 Finite Element Analysis of Hollow Active Fibre Composite-Actuated Bimorphic Structures

The HAFC-actuated bimorph was tailored in order to be similar to the Macro Fibre Composite (MFC) –actuated bimorph analyzed in the in section 4.1. The HAFC-actuated bimorph was similar to the MFC-actuated bimorph such that the HAFC-actuated bimorph was designed to have the same fibre volume fraction as the MFC-actuated bimorph. Also,

the actuation potential that was applied to the HAFC-actuated bimorph was selected in order to yield electric field intensity similar to that used in the MFC-actuated bimorph. The cross-section of the HAFC-actuated bimorphic structure, with dimensions, is illustrated in Figure 4-15. The dimensions of the MFC-actuated and HAFC-actuated bimorphic structures are tabulated in Table 4-2.

Figure 4-15: Cross-section of the HAFC-actuated bimorphic structure

Table 4-3: Characteristics and dimensions of the MFC- and HAFC-actuated bimorphs

	Macro fibre composite-actuated bimorph	Hollow active fibre composite-actuated bimorph
Aluminum thickness (mm)	88.9	88.9
Adhesive thickness (mm)	31.25	31.25
Fibre length (mm)	85	85
Fibre height (μm)	175	n/a
Fibre width (μm)	350	n/a
Fibre inner radius (μm)	n/a	50
Fibre outer radius (μm)	n/a	200
Matrix width (μm)	585	441.7
Matrix height (μm)	237.5	441.7
Fibre volume fraction	0.629	0.629
Automatic electric potential (V)	1500	723
Electric field intensity (V/mm)	4820	4820

The HAFC-actuated bimorph was fully constrained at one end. The actuating electric potential difference of 723 V was established between the inner and outer walls of the hollow fibre. The electric potential distribution is illustrated in Figure 4-16.

Figure 4-16: Electric potential distribution of HAFC-actuated bimorph

One of the fibres was poled such that the poling direction was parallel to the direction of the actuating electric potential. The result was the axial extension of the hollow fibre. The fibre on the opposite face of the bimorph was poled in the direction opposite to that of the actuating electric field. The result was the axial contraction of the hollow fibre. The anti-parallel actuation of the hollow fibres on the opposite faces of the bimorph resulted in the bending actuation of the bimorphic structure. The displaced structure created in the finite element analysis is illustrated in Figure 4-17. The finite element analysis produced an out-of-plane tip displacement of 7.41 mm.

Figure 4-17: Out-of-plane tip deflection of HAFC-actuated bimorph

4.5 Summary

Finite element analyses were created of a Macro Fibre Composite (MFC)-actuated bimorphic structure as well as of a similar Hollow Active Fibre Composite (HAFC)-actuated bimorphic structure. The MFC-actuated structure generated an out-of-plane tip displacement of 18 mm when exposed to an actuating electric field intensity of 4820 V/mm. An HAFC-actuated bimorphic structure was designed to have similar properties and a similar fibre volume fraction. This similar HAFC-actuated bimorphic structure generated an out-of-plane tip displacement of 7.41 mm when exposed to the same actuating electric field intensity of 4820 V/mm.

The bending performance of the HAFC-actuated bimorph was noticeably smaller than that of the similar MFC-actuated bimorph. This result is likely due to the piezoelectric characteristics of the fibres used in the MFCs and HAFCs. MFC actuation is proportional to the parallel piezoelectric coefficient, d_{33} . HAFC actuation is proportional to the orthogonal piezoelectric coefficient, d_{31} . Since the d_{31} for the selected material was nearly half of that of the d_{33} (see table 4-2), a reduction in actuation performance was expected.

However, HAFC-based actuators offer some advantages over MFC-based actuators such as increased reliability and decreased actuation electric potential.

Chapter 5: Conclusions

The deflection performance of a Macro Fibre Composite (MFC)-actuated and Hollow Active Fibre Composite (HAFC)-actuated bimorphic actuators were studied. The study also measured the amplification effects of axial preload on deflection performance. In order to conduct this study, a physical MFC-actuated structure was manufactured. A Finite Element Model was also created and compared to an analytical model.

The physical model consisted of two M-8528-P1 MFCs bonded to both sides of an aluminum substrate. The 98 mm long bimorphic structure achieved approximately 11 mm tip deflection (6.4°) when an actuating electric potential of 1500 V (4820 V/mm actuating electric field intensity) was applied to the MFCs. The tip deflection of the above bimorphic structure would increase to approximately 17 mm (9.8°) when an axial preload of 40 N was applied in addition to the 1500 V actuating electric potential. The finite element and analytical models produced similar results. The finite element model further predicted that a tip deflection of 23 mm (13.2°) could be achieved with the application of an axial compressive force of 80 N and an actuating electric potential of 1500 V. The finite element model also predicted that a tip deflection of 15 mm (13.2°) could be achieved with the application of an actuating electric potential of 2000 V.

The deflection performance of a Hollow Active Fibre Composite (HAFC) –actuated bimorphic structure was also studied. A finite element model of an HAFC was created and was assigned material properties, fibre volume fraction and actuating electric field intensity similar to that of the M-8528-P1 MFC-actuated bimorphic structure. The HAFC-actuated bimorphic structure was exposed to a 4820 V/mm actuating electric field intensity. The 98 mm bimorph achieved a tip deflection of 7.41 mm (4.32°). The tip deflection performance of the HAFC-actuated bimorphic structure was approximately half of that of the MFC-actuated bimorphic structure. The finite element model confirmed this result.

The actuation performance of an HAFC is proportional to the orthogonal piezoelectric coefficient d_{31} . Meanwhile, the actuation performance of an MFC is proportional to the parallel piezoelectric coefficient d_{33} . Since the value of d_{31} of the PZT studied was approximately half of the value of the d_{33} , a decrease in tip deflection performance was anticipated.

HAFCs provide many advantages over MFCs, such as increased reliability, resistance to fibre breakage and lower required actuation electric potential. In some cases, these advantages of HAFCs could outweigh any decreases in actuation performance when compared with MFCs.

It has been shown that MFC and HAFC-based bimorphic structures deliver large actuation performance. It has also been shown that the actuation performance of the bimorphic structures could be amplified with the use of a compressive axial preload. The

actuation performance of these bimorphic structures suggest that they could be potential candidates to be morphing wing actuators.

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Wickramasinghe, V.K., Chen, Y., Martinez, M., Wong, F., Kernaghan, R., "Design and Verification of a Smart Wing for an Extremely Agile Micro-Air-Vehicle", Accepted for publication to Journal of Smart Material Structures, Aug. 2011.

Wickramasinghe, V.K., Chen, Y., Martinez, M., Kernaghan, R., Wong, F., "Design and Verification of a Smart Wing for an Extremely Agile Micro-Air-Vehicle", Proceedings of: 50th AIAA/ASME/ASCE/AHS/ASC Structures, structural Materials and Dynamics Conference, art. No. 2009-2132, 2009.

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