Characteristic Mode Approach to Understanding a Capacitively Fed Crossed Bow-tie Antenna

by

Austin B. Balcaen, B.Eng

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Carleton University
Ottawa, Ontario

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Austin B. Balcaen

Abstract

The Theory of Characteristic Modes and subsequently characteristic mode analysis is a unique way to analyze antennas. It allows the response of the antenna to be decomposed into the sum of its parts. Each part (mode) has a basic response and is more intuitive to understand. In this thesis, two structures are analyzed using characteristic mode analysis; a simple bow-tie antenna and a complex capacitively fed crossed bow-tie antenna. The bow-tie antenna was modified through visual inspection of the eigencurrent for two excited modes. Perpendicular stubs placed at an eigencurrent null for the third order mode and close to an eigencurrent maximum for the first order mode reduced the modal resonance of the third order mode by 1.8 GHz while reducing the modal resonance of the first order mode by 25 MHz. S-parameter measurements of the fabricated bow-tie structure with and without stubs closely correlated to simulated results.

Three characteristic modes were found to make up the response of the capacitively fed crossed bow-tie antenna. The modes exhibited coupling effects in their eigenvalues, eigencurrents, and modal admittances due to eigenvalue crossing avoidances and were decoupled into auxiliary modes to remove these effects. The auxiliary modes were described as a ground plane mode, a dipole dominant mode, and a bow-tie dominant mode. The ground plane mode was shown to have similar characteristic attributes to the lowest frequency mode of a circular sheet. The dipole mode and bow-tie mode synchronously coupled with each other which shifted their uncoupled resonant frequencies by 0.3 GHz from 1.8 GHz to 1.5 GHz and 2.1 GHz respectively. The dipole mode exhibited an in-phase dipole/bow-tie eigencurrent relationship. The bow-tie mode exhibited an anti-phase dipole/bow-tie eigencurrent relationship. The eigencurrent on each side of the bow-tie and on the dipole were shown to have a half wavelength distribution. The wide bandwidth for the antenna was attributed to the dual resonance created due to the synchronously coupled modes.

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List of Acronyms

TCM	Theory of Characteristic Modes
CMT	Coupled Mode Theory
CMA	Characteristic Mode Analysis
MIMO	Multiple-input Multiple-output
CM	Characteristic Mode
CA	Characteristic Attribute
EFIE	Electric Field Integral Equation
MoM	Method of Moments
PEC	Perfect Electric Conductor
CCE	Capacitive Coupling Element
ICE	Inductive Coupling Element
GNSS	Global Navigation Satellite System
AR	Axial Ratio
PCV	Phase Center Variation

Nomenclature

ω	angular frequency, rad/s
f	frequency, Hz
λ	wavelength, m
H	magnetic field intensity, A/m
E	electric field intensity, V/m
J	surface current density, A/m ²
J_n	eigencurrent or characteristic current, A/m ²
λ_n	eigenvalue
δ_{mn}	Kronecker delta
W_e	electric energy, J
W_m	magnetic energy, J
MS_n	modal significance
α_n	characteristic angle, °
a_n	modal weighting coefficient
V_n^i	modal excitation coefficient
Y_{in}	input admittance, S
Y_n	modal admittance, S
G_n	modal conductance, S
B_n	modal susceptance, S
K, G	coupling coefficient
Γ	reflection coefficient

Chapter 1

Introduction

1.1 Motivation

Antennas are present everywhere around us. At times they are noticeable, such as those radio towers, on top of buildings, and on cars. Most of the time, they are not noticeable. Hidden inside your cellphone, streamlined to follow the shape of aircraft, even inside electrical outlets and light bulbs. The growing demand to have everything wireless and antennas invisible introduces many design limitations such as small size, low weight, and aesthetically pleasing structures. All of which can not compromise the increasingly demanding requirements for antenna performance. The use of electromagnetic modeling techniques and associated software packages are vital in antenna design. As antennas become more involved to fit into difficult design limitations, its shape moves farther away from well known structures such as dipole, loop, and patch antennas. Without being reliant on the designer's knowledge and experience, it is difficult to realize the mechanisms that are major contributors to the performance of complex antenna structures. Eventually, the process becomes trial and error using educated or blind guesses through the use of simulators and bench testing to meet design constraints. The Theory of Characteristic Modes (TCM) allows the decomposition of these complicated structures into basic responses like that of a dipole.

In this thesis, the use of characteristic mode analysis (CMA) in decomposing antenna structures is explored. Initially, a simple bow-tie shape is modified using characteristic modes and associated eigencurrents to find the exact mechanism to manipulate in order to get the desired response. The response of a complex antenna structure is deconstructed into the responses of basic shapes. A capacitively fed crossed bow-tie antenna was able to be expressed as the sum of a circular sheet and the coupling between a dipole and bow-tie like structure.

1.2 Introduction to Characteristic Modes

In the 1960's and 1970's, the Theory of Characteristic Modes (TCM) was developed into what is used today. The initial idea of the theory came from Garbacz where he stated PEC shapes had characteristic modes that were dependent only on the size and shape of the PEC [1]. The theory was eventually reformulated by R. Harrington and J. Mautz to make use of the electric field integral equation [2]. Thus, allowing for the eigencurrents and characteristic fields to be conveniently obtained [3]. However, TCM has not been studied extensively until recently as roughly 70% of the publications relating to TCM are from 2006 or later [3]. Most publications will have cited the work done in [2].

Much of the work done using TCM is related to MIMO and cellphone antenna design such as in [4]–[6]. This is because CMA allows for flexible placement of ports and the excitation of multiple modes to create a multiband response. The work done in [7]–[9] expands on ways to physically excite each characteristic mode. In [10], a method is developed to synthesize the shape of an antenna to operate at a frequency or multiple frequencies before the feed is placed. In [11], a couple methods are shown to determine feed locations. One method finds the feed location where certain characteristic modes are excited over others. Another finds the optimal feed location over frequency for a given field distribution. In his dissertation, Fabrs analyzes the first few characteristic modes of many basic shapes [12]. All antennas can be broken down into a combination of the modes for these shapes. A loop antenna and a dipole, having similar radiation patterns, are combined in [13] to create a wideband antenna with a stable radiation pattern. In [14]–[16], Coupled Mode Theory (CMT) is shown to be relevant to antenna analysis through the use of TCM. The works stated in this paragraph provides a brief glimpse into what has or can be done using TCM. Select papers are mentioned again in subsequent chapters of this thesis.

1.3 Thesis Overview

The structure of the thesis is as follows. In chapter 1, the history of TCM is briefly discussed and a handful of interesting works related to TCM are mentioned. TCM is expanded upon in chapter 2. The derivation of TCM into the state it is used today is shown. Characteristic attributes obtained from performing CMA are explained and how modes are excited is discussed. CMT is also discussed along with examples of how it relates to CMA from eigenvalue crossing avoidance and synchronous coupling.

In chapter 3, CMA is performed on a bow-tie antenna. The eigencurrents of the first two odd modes are analyzed to determine where to place perpendicular stubs. Increasing the length of the stubs is shown to reduce the modal resonance of the third order mode without affecting the first order mode. A wideband impedance bandwidth

is created from a dual resonance by shifting the resonance of the third order mode close to the first order mode. The bow-tie antennas with and without stubs were fabricated and the measured results correlate well with the simulated results.

In chapter 4, a complex capacitively fed crossed bow-tie antenna is analyzed using characteristic modes. Characteristic attributes such as the eigenvalue, eigencurrent, and modal admittance are discussed and shown to be affected by coupling due to eigenvalue crossing avoidances. The characteristic modes are decomposed into auxiliary modes to remove the effects of crossing avoidance coupling. The same characteristic attributes are discussed in relation to the auxiliary modes and three modes are found; a ground plane mode, a dipole dominant mode, and a bow-tie dominant mode. The dipole mode and the bow-tie mode are shown to synchronously couple together.

Chapter 5 summarizes the work done and results obtained in chapters 3 and 4. Future work that can be performed to continue what was done in these chapters is also discussed.

Chapter 2

Characteristic Modes and Coupled Mode Theory

2.1 Formulation of Characteristic Modes

The most conventional way to derive TCM is presented by R. Harrington and J. Mautz [2]. This thesis will not explain the method by R. Harrington and J. Mautz in detail as it is sufficiently explained in other works such as [3], [12], [11]. In concise terms, this method uses the Electric Field Integral Equation (EFIE) to create the relationship between electric fields and surface currents on a PEC surface. From the EFIE, an impedance operator, $Z(\cdot)$, can be defined.

Using method of moments (MoM) to discretize the $Z(\cdot)$ operator, an $N \times N$ impedance matrix, $\mathbf{Z} = \mathbf{R} + j\mathbf{X}$, is obtained. Where $\mathbf{R}$ and $\mathbf{X}$ are the real and imaginary Hermitian parts of the impedance matrix [3]. From this, one can consider a weighted eigenvalue equation eventually obtaining the generalized eigenvalue equation for characteristic modes.

$$\mathbf{X}\mathbf{J}_n = \lambda_n \mathbf{R}\mathbf{J}_n \quad (2.1)$$

$\mathbf{J}_n$ is the eigencurrent or characteristic current, λ_n is the eigenvalue, and the subscripts m, n represent different modes. In (2.1), there is no excitation and as such the eigencurrent only depends on the shape and size of the PEC structure and are independent of any excitation source [12]. The modes are orthogonal and generally normalized to radiate unit power according to:

$$\langle \mathbf{J}_m^*, \mathbf{R}\mathbf{J}_n \rangle = \delta_{mn} \quad (2.2)$$

where δ_{mn} is the Kronecker delta and the operation in (2.2) is the inner product for the complex Hilbert space as defined in (2.3) [2].

$$\langle B, C \rangle = \oint\!\!\!\oint_S \mathbf{B} \cdot \mathbf{C} \, dS \quad (2.3)$$

When the eigencurrents are normalized according to (2.2) then the orthogonality relationships become:

$$\begin{aligned} \langle \mathbf{J}_m, \mathbf{R}\mathbf{J}_n \rangle &= \langle \mathbf{J}_m^*, \mathbf{R}\mathbf{J}_n \rangle = \delta_{mn} \\ \langle \mathbf{J}_m, \mathbf{X}\mathbf{J}_n \rangle &= \langle \mathbf{J}_m^*, \mathbf{X}\mathbf{J}_n \rangle = \lambda_n \delta_{mn} \\ \langle \mathbf{J}_m, \mathbf{Z}\mathbf{J}_n \rangle &= \langle \mathbf{J}_m^*, \mathbf{Z}\mathbf{J}_n \rangle = (1 + j\lambda_n) \delta_{mn} \end{aligned} \quad (2.4)$$

The orthogonality relationships show that an eigencurrent's polarization and magnitude are orthogonal with other eigencurrents. If one mode has strong currents at a specific point on the PEC surface then all the orthogonal modes have weak currents at the same point [3]. This is also the case for the characteristic electric and magnetic fields.

2.2 Characteristic Attributes

Solving the eigenvalue and eigencurrent from the generalized eigenvalue equation stated in (2.1) is the basis of CMA. The eigenvalue, λ_n , can range from $-\infty$ to $+\infty$ and directly pertains to an object's reactive power or the difference in stored magnetic and electric energy as shown in (2.5) [11].

$$\lambda_n = 2\omega(W_m - W_e) \quad (2.5)$$

where ω is the angular frequency, W_m is the modal magnetic energy and W_e is the modal electric energy. In (2.5), each mode has been normalized according to (2.2).

The closer a mode's eigenvalue is to zero, the more efficiently the mode radiates as the modal energies cancel each other. When the energies completely cancel each other out, the mode is resonant and the eigenvalue is equal to zero. Similarly, if $\lambda_n < 0$ then the stored electric energy is dominant and the mode is capacitive while if $\lambda_n > 0$ then the stored magnetic energy is dominant and the mode is inductive.

The Q factor of a mode, which can be used to understand how sharp a modes resonance is, can be approximated from the slope of the eigenvalue over frequency

and is shown in (2.6) [17]. However, this equation is only valid when the mode is resonant. A large Q factor represents a mode with a sharp resonance, while a small Q factor represents a mode with a large bandwidth.

$$Q_n \approx \omega \frac{\partial \lambda_n}{\partial \omega} \quad (2.6)$$

Since the eigenvalue can range from $-\infty$ to $+\infty$ it is sometimes more intuitive to express it in terms of modal significance instead.

$$MS_n = \left| \frac{1}{1 + j\lambda_n} \right| \quad (2.7)$$

The modal significance, MS_n , ranges from 0 to 1 with the modal significance being equal to 1 representing the mode at resonance.

Another term used is the characteristic angle, α_n , which represents the phase lag between the characteristic electric field, $\mathbf{E}_n$, and the characteristic current, $\mathbf{J}_n$, on the PEC surface [3]. Similarly, the difference between the characteristic angle of two modes is the phase difference between the two characteristic currents [11].

$$\alpha_n = 180^\circ - \tan^{-1} \lambda_n \quad (2.8)$$

If the characteristic electric field and the characteristic current of a mode are out of phase by 180° , i.e. $\alpha_n = 0$, then the mode is resonant. If $90^\circ < \alpha_n < 180^\circ$ then the mode is capacitive and if $180^\circ < \alpha_n < 270^\circ$ then the mode is inductive just as with an eigenvalue being positive or negative.

In Mier's dissertation [11], the characteristic phase is used to obtain circular polarization in a crossed dipole antenna without the use of external phase shifters. In this example, the lengths of the dipoles are changed such that the difference in characteristic phase between the fundamental modes of the dipoles is 90° . This corresponds to a phase shift of 90° between the two dipoles and circular polarization is achieved.

The eigencurrent or characteristic current, $\mathbf{J}_n$, provides useful insight into the field response of each modes and the modes overall contribution to the response of the antenna structure. The surface current on the structure is the weighted sum of all the characteristic currents and since the electric and magnetic fields are linearly related to the currents [2], the same relationship can be stated:

$$\begin{aligned}
\mathbf{J} &= \sum_n a_n \mathbf{J}_n \\
\mathbf{E} &= \sum_n a_n \mathbf{E}_n \\
\mathbf{H} &= \sum_n a_n \mathbf{H}_n
\end{aligned} \tag{2.9}$$

where a_n is the modal weighting coefficient (MWC) defined by (2.10).

The MWC is important as it combines the eigenvalue, an intrinsic property of each mode and independent of an excitation source, with an excitation source. Therefore, as stated in (2.9), it expresses the contribution of each mode's characteristic current or fields the overall response of the antenna.

$$a_n = \frac{V_n^i}{1 + j\lambda_n} \tag{2.10}$$

The modal excitation coefficient, V_n^i , represents the coupling between an external excitation source, $\mathbf{E}^i$, and the characteristic current, $\mathbf{J}_n$. The coupling is dependent on the position, magnitude, phase, and polarization of the excitation source and the characteristic current [3].

$$V_n^i = \langle \mathbf{J}_n, \mathbf{E}^i \rangle = \oiint_S \mathbf{J}_n \cdot \mathbf{E}^i dS \tag{2.11}$$

If the excitation source is a gap voltage source of 1 V then the input admittance, Y_{in} , of an antenna can be calculated from the surface current at a point, P . In (2.9) the surface current is the sum of the modal weighting coefficient and characteristic current, therefore, the input admittance can be expressed in terms of the sum of the modal admittances, Y_n , as shown in (2.12) [12]. Notice (2.12) also implies that the equivalent circuit for a structure is equal to the parallel combination of its characteristic mode's equivalent circuits [14]. The modal admittance can be broken down into its modal conductance, G_n , and modal susceptance, B_n .

$$Y_{in} = \frac{\mathbf{J}(P)}{1V} = \sum_n a_n \mathbf{J}_n(P) = \sum_n Y_n = \sum_n G_n + jB_n \tag{2.12}$$

2.3 Excitation of Characteristic Modes

The generalized eigenvalue equation, (2.1), expressed that CMA can be performed in absence of an excitation source. Therefore, one can obtain insight on where to place an excitation and the type of excitation to use by analyzing the characteristic fields. The stronger the coupling between the excitation and a mode's eigencurrent or characteristic fields (as they are linearly related), the more significant the mode will be in the overall response of the antenna. Field overlays on a structure are visual, as such, it is sufficient to determine good locations for an excitation from visual analysis.

Figure 2.1: Eigencurrent distribution on a long thin PEC sheet for the lowest resonant characteristic mode a) without an ICE and b) with an ICE. c) Characteristic attributes for the thin PEC sheet and the reflection coefficient of a dipole with the same dimensions.

For example, if CMA is performed on a long thin PEC sheet as shown in Fig. 2.1(a), the lowest resonant mode's eigencurrent displays a half wavelength distribution across its structure. From (2.11), the largest possible V_n^i , would occur if $\mathbf{E}^i$ is equal to $\mathbf{J}_n$ where $\mathbf{J}_n$ is at its maximum. Therefore, the ideal way to couple to the lowest resonant mode is to place an inductive coupling element (ICE) in the middle of the sheet. The thin sheet then becomes a half-wavelength dipole where the majority of fields on the antenna are due to the lowest resonant mode of the PEC sheet. In Fig. 2.1(c), it can

be seen that the MWC and MS are at their maximum around the same frequency the dipole is at resonance. Similarly to placing an ICE at a characteristic current (or magnetic field) maximum to excite a mode, a capacitive coupling element (CCE) can be placed at a characteristic voltage (or electric field) maximum [7].

ICEs and CCEs are usually physically realized as being much smaller than a wavelength such that they contribute minimal radiation. In the long thin sheet example, implementation of the ICE would be simple as the sheet can be cut in the middle and then the inner and outer conductors of a coaxial cable can be attached to separate halves of the sheet. For other structures it may not be as simple and certain methods may be more effective than others at coupling the excitation to the characteristic mode. Examples of other methods are to incorporate a non resonant wire loop into a PEC sheet as an ICE [8], and a parallel plate [7] or a L-shaped probe [18] as a CCE.

2.4 Coupled Mode Theory and Eigenvalue Crossing Avoidance

Coupled Mode Theory (CMT) is a topic not normally discussed when regarding the analysis of antennas. Instead, its relevance in electromagnetics is focused on devices related to electromagnetic oscillation and wave propagation such as waveguides, fiber optics, and oscillators [19]. The emergence of the theory of characteristic modes in the recent decade has allowed for CMT to become relevant in antenna analysis as the characteristic modes on a structure can be seen to couple with one another. Instances of which will be discussed later in this section.

CMT is described in [19] and [20] but to briefly explain, consider the two weakly coupled resonators, r_1 and r_2 , in (2.13).

$$\frac{dr_1}{dt} = j\omega_1 r_1 + jK_{12}r_2 \quad (2.13a)$$

$$\frac{dr_2}{dt} = j\omega_2 r_2 + jK_{21}r_1 \quad (2.13b)$$

where ω_1 and ω_2 are the uncoupled resonant frequencies of r_1 and r_2 respectively. K_{12} and K_{21} are the coupling coefficients which for weak coupling are $|K_{12}| \ll \omega_1$ and $|K_{21}| \ll \omega_2$. Therefore, the energy stored due to coupling is negligible and the total energy in the two resonators should stay constant allowing (2.14) to be stated [20].

$$K_{12} = K_{21}^* = K \quad (2.14)$$

To find the eigensolutions, the solution of the coupled resonators is assumed to be

$$r_1(t) = A_1 e^{j\omega t} \quad r_2(t) = A_2 e^{j\omega t} \quad (2.15)$$

then (2.14) and (2.15) is substituted into (2.13) to obtain the coupled resonant frequencies, ω_+ and ω_- , shown in (2.16).

$$\omega_{\pm} = \frac{\omega_1 + \omega_2}{2} \pm \sqrt{\left(\frac{\omega_1 - \omega_2}{2}\right)^2 + |K|^2} \quad (2.16)$$

Now, the two resonators can be expressed as the summation of two modes (or eigensolutions) each with their own coupled resonant frequency. In (2.17), this is shown for the case of synchronous coupling where the uncoupled resonant frequencies, ω_1 and ω_2 , are equal.

$$\begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = c_1 \begin{bmatrix} K \\ |K| \end{bmatrix} e^{j\omega_+ t} + c_2 \begin{bmatrix} K \\ -|K| \end{bmatrix} e^{j\omega_- t} \quad (2.17)$$

where c_1 and c_2 are constants determined from initial conditions. From (2.17) it can be seen that each resonator is made up of two modes where one mode will have the resonators in-phase and the other will have the resonators anti-phase. If K is assumed to be less than zero then the higher frequency mode, ω_+ , would be the anti-phase mode and the lower frequency mode, ω_- would be the in-phase mode [20].

Figure 2.2: Normalized modal charge distribution on U-slot antenna for a) the in-phase mode, CM_1 , and b) the anti-phase mode, CM_3 . The images are obtained from [14].

In [14], the two excited characteristic modes of a U-slot antenna are shown to be coupling. The charge distributions on the structure for the modes, shown in Fig. 2.2, exhibit the in-phase and anti-phase characteristic discussed in the previous paragraph and the difference in the resonant frequencies of the excited modes follows

the relationship stated in (2.16). The same is shown in [15] for the excited modes of a stacked patch antenna.

A different approach is taken in [16], where characteristic modes are shown to couple when their eigenvalues exhibit a phenomenon called eigenvalue crossing avoidance. At the crossing avoidance, the eigenvectors of the coupled modes are a blend of each mode and eventually exchange characteristics after the crossing avoidance. Using CMT as the framework, a method is developed to decouple the modes into auxiliary modes. For the auxiliary modes, a crossing avoidance no longer occurs and the eigenvectors do not exchange characteristics. An example of an eigenvalue crossing avoidance between two modes and the decoupled auxiliary eigenvalues are shown in Fig. 2.3. The eigenvalues, λ_1 and λ_3 , exhibit a crossing avoidance where the decoupled eigenvalues, λ_a and λ_b do not.

Figure 2.3: Eigenvalues and the decoupled auxiliary eigenvalues for CM₁ and CM₃ of a thin dipole. Plot is obtained from [16].

In [16], the auxiliary eigenvalues are obtained from inverting (2.18). Notice the similarities to (2.16), where G is the coupling coefficient synonymous to K . Subscripts ₁ and ₂ represent the two modes that are coupling with one another.

$$\lambda_{1,2} = \frac{\lambda_a + \lambda_b}{2} \pm \sqrt{\left(\frac{\lambda_a - \lambda_b}{2}\right)^2 + G^2} \quad (2.18)$$

The auxiliary eigenvectors, $\mathbf{J}_a$ and $\mathbf{J}_b$, are obtained from the characteristic mode eigenvectors in (2.19),

$$\begin{aligned}\mathbf{J}_a &= \frac{\gamma}{\gamma^2 + 1}(\mathbf{J}_1 - \mathbf{J}_2) \\ \mathbf{J}_b &= \frac{1}{1 + \gamma^2}(\mathbf{J}_1 + \gamma^2 \mathbf{J}_2)\end{aligned}\tag{2.19}$$

where γ is a frequency-dependent coefficient determined from (2.20) and G is obtained from (2.21). For (2.19) to be valid, $\mathbf{J}_1$ and $\mathbf{J}_2$ must be normalized according to (2.22) instead of the normalization commonly performed on the eigencurrents in (2.2).

$$\gamma = \frac{\lambda_1 - \lambda_b}{G} = \frac{\lambda_a - \lambda_2}{G}\tag{2.20}$$

$$G = \min\left(\frac{|\lambda_1 - \lambda_2|}{2}\right)\tag{2.21}$$

$$\begin{aligned}\mathbf{J}_1^T \mathbf{R} \mathbf{J}_1 &= \gamma^2 + 1 \\ \mathbf{J}_2^T \mathbf{R} \mathbf{J}_2 &= \gamma^{-2} + 1\end{aligned}\tag{2.22}$$

2.5 Conclusion

The concepts discussed in this chapter related to the TCM will be used to analyze a bow-tie antenna in chapter 3 and a capacitively fed crossed bow-tie antenna in chapter 4. In chapter 3, the bow-tie structure was chosen to familiarize one's self with CMA due to its simplicity and similarities in appearance to the crossed bow-tie structure that will be analyzed in chapter 4. The eigencurrents from the different characteristic modes present on the bow-tie will be used to provide insight on how to manipulate characteristic attributes such as the modal significance.

In chapter 4, CMA will be performed on a capacitively fed crossed bow-tie structure to describe the response of the antenna with a few fundamental modes. The analysis of the structure will be shown to be complex with many characteristic modes, some of which are coupling with one another according to CMT. The characteristic modes will be decoupled into auxiliary modes and their eigenvalues, eigencurrent, and modal admittance will be discussed.

Chapter 3

Characteristic Mode Analysis of Bow-tie Antenna

3.1 CMA of Bow-tie Antenna

A dipole antenna is one of the most fundamental antennas with it being discussed extensively in any antenna textbook. The bandwidth of a dipole can be increased by widening the end of the dipole's arms (increasing the flare angle) to create a bow-tie antenna [21]. In this chapter, a simple bow-tie antenna is modified to achieve an even wider bandwidth by using CMA to provide guidance.

The geometry of the bow-tie antenna to be analyzed can be seen in Fig. 3.1 with the dimensions stated in Table 3.1. The conductors, shown in orange, are modeled as a PEC and the feed is located at the center, shown in green. The simulation software used to perform the analysis is CST Studio Suite 2020 [22]. CMA results were obtained using the integral equation solver with a 1 V source as the excitation. Driven results were obtained using the frequency domain solver with a port referenced to 50Ω .

Figure 3.1: Bow-tie antenna.

Name	Variable	Dim. [mm]
length	l	100
width	w	50
feed length	fl	3
feed width	fw	4

Table 3.1: Bow-tie antenna dimensions.

The driven reflection coefficient of the bow-tie antenna is shown in Fig. 3.2. The modal significance, MS_n , and modal weighting coefficient, a_n , from the CMA performed on the structure is shown as well. Only the first two odd modes, CM₁ and CM₃, are shown as they are the ones excited by the feed placed at the center of the bow-tie structure.

Figure 3.2: Characteristic attributes for characteristic modes excited by the port and the driven reflection coefficient of the bow-tie antenna.

In section 2.2, it was stated that a mode is at resonance when its modal significance equals one. In Fig. 3.2, this occurs for the first mode near 1 GHz and for the third mode near 4.4 GHz. For mode 1, the maximum a_n occurs at the same frequency as modal resonance. While for mode 3, the maximum a_n occurs near 3.8 GHz, a couple hundred MHz lower than the modal resonance. This is interesting and suggests the eigencurrent for mode 3, $\mathbf{J}_3$, couples to the excitation better below the modal resonance. This was seen in Fig. 2.1(c) as well as the V_n^i of the PEC sheet decreased with frequency.

The maximums for a_n of the excited modes line up with the minimums for the driven reflection coefficient. This is expected as the modal admittance is directly related to a_n as expressed in (2.12). Notice no minimum for the reflection coefficient occurs between the CM₁ and CM₃, where the modal resonance of CM₂ would be. This

is because a_2 (not shown in Fig. 3.2) is zero at all frequencies due to CM₂ having an eigencurrent null at the location of the excitation.

The eigencurrent distributions on the bow-tie structure is shown in Fig. 3.3 for CM₁ and CM₃ near their respective modal resonances. The eigencurrent for mode 1, $\mathbf{J}_1$, has a half wavelength current distribution and the eigencurrent for mode 3, $\mathbf{J}_3$, has a three half wavelength current distribution [23]. For both modes, an eigencurrent maximum occurs at the center of the structure where the excitation is located, enforcing why a_n for these modes is large. The phase of $\mathbf{J}_3$ undergoes a 180° phase shift near the center of the structure causing current nulls to appear on each arm offset from the center on the x-axis by 14.5 mm. At the same location, the magnitude of the eigencurrent for $\mathbf{J}_1$ is not at maximum but is significant.

Figure 3.3: Modal eigencurrent distribution on bow-tie structure for a) $\mathbf{J}_1$ at 1 GHz and b) $\mathbf{J}_3$ at 4 GHz. Eigencurrents are normalized to their respective maximums and have units of dBA/m.

3.2 Modification of Bow-tie Antenna using CMA.

The eigencurrent maximum's and minimum's for the excited modes shown in Fig. 3.3 can be used as a guide to manipulate the modal resonances. Stated in the previous section, a current null for $\mathbf{J}_3$ is located close to the current maximum of $\mathbf{J}_1$. By placing stubs at the null perpendicular to the longitudinal axis (x-axis), the modal resonance of CM₃ can be electrically lengthened without affecting CM₁ by increasing the length of the stub. A similar method was applied to a slotline antenna in [24] by placing stubs at the nulls of its electric field and in [25] by placing stubs at the current nulls of a dipole's 5th order mode to shift its resonance close to the 3rd order mode. The bow-tie antenna with stubs can be seen in Fig. 3.4 with its dimensions stated in Table 3.2. Once again, the conductors are modeled as a PEC and the feed is located at the center.

Name	Variable	Dim. [mm]
length	l	100
width	w	50
feed length	fl	3
feed width	fw	4
stub length	sl	60
stub width	sw	6
stub offset	so	14.5

Figure 3.4: Bow-tie antenna with stubs.

Table 3.2: Bow-tie antenna with stubs dimensions.

A parametric sweep is performed where the stubs are lengthened to 40 mm and 60 mm to view the effect the stubs have on modal resonance. The modal significance of CM_1 and CM_3 for the sweep is shown in Fig. 3.5. As expected, the modal resonance of CM_3 reduced significantly as the length of the stub was increased while the modal resonance of CM_1 was not affected. From a stub length of zero (i.e. no stubs) to a stub length of 60 mm, the modal resonance of CM_3 decreased by 1.8 GHz while the modal resonance of CM_1 decreased by 25 MHz.

Figure 3.5: Modal significance and reflection coefficient for bow-tie antenna with no stubs and at stubs lengths of 40 mm and 60 mm.

Figure 3.6: Characteristic attributes for the characteristic modes excited by the port and the driven reflection coefficient of the bow-tie antenna with 60 mm stubs.

In Fig. 3.6, the modal significance, modal weighting coefficient for the excited modes and driven reflection coefficient are shown for the case where the stub length is 60 mm. Compared to the bow-tie with no stubs in Fig. 3.2, all the parameters related to CM_3 have shifted down in frequency considerably with the parameters for CM_1 being relatively unchanged. The reflection coefficient minimums in Fig. 3.2, which were located at 1 GHz and 3.8 GHz for the first order mode and the third order mode respectively have started to merge into each other in Fig. 3.6. Now, they are located at 975 MHz and 2.3 GHz forming a much wider impedance bandwidth due to the dual resonance.

The term impedance bandwidth is usually defined as the frequency range where the reflection coefficient is less than 0.33 [21]. In the previous paragraph, the bow-tie with stubs antenna was stated to have a large bandwidth even though in the middle of the bandwidth the reflection coefficient is 0.6, which is larger than the 0.33 guideline. Instead, what is used here to define a wide bandwidth is the emergence of a loop in the reflection coefficient on a Smith chart due to the dual resonance. In Fig. 3.7, a loop in the reflection coefficient appears on the trace corresponding to the bow-tie with stubs. The loop is not present on the trace corresponding to just the bow-tie. By optimizing the dimensions of the bow-tie with stubs, the loop can be moved

Figure 3.7: Reflection coefficient displayed on smith chart for bow-tie antenna with 60 mm stubs and without stubs. Circle represents 0.5 GHz and square represents 5 GHz.

closer to the center of the chart such that the reflection coefficient is less than 0.33 between the resonances. The focus of the analysis performed in this chapter was to demonstrate how characteristic modes can be manipulated rather than optimization, therefore, the optimization was not performed.

The excited eigencurrents, $\mathbf{J}_1$ and $\mathbf{J}_3$, on the bow-tie structure with stubs are shown near their respective modal resonances in Fig. 3.8. Similar to the case without stubs, the half wavelength current distribution is present on $\mathbf{J}_1$ and the three half wavelength current distribution is present on $\mathbf{J}_3$. Both modes have current that flows on the stubs.

For $\mathbf{J}_1$, the currents on each stub are self contained since they oppose each other directly on the stub. Thus cancelling out and minimizing the impact the stubs have on the first mode. For $\mathbf{J}_3$, current minimums still occur in the middle of the bow-tie offset from the center by 14.5 mm. However, the currents that flow in the y-direction from the two opposite phases add together as they travel along the stub. Thus, moving the minimum that was present at the edge of the bow-tie to the end of the stub. Therefore, the electrical length of CM_3 increased as the currents need to travel a longer distance along the length of the stub before changing in phase by 180° . The currents on one of the stubs oppose the currents on another stub, minimizing the impact the stubs have on the third mode other than reducing the resonance frequency.

Figure 3.8: Modal eigencurrent distribution on bow-tie structure with 60 mm stubs for a) $\mathbf{J}_1$ at 1 GHz and b) $\mathbf{J}_3$ at 2.4 GHz. Eigencurrents are normalized to their respective maximums and have units of dBA/m.

The far-field radiation pattern for CM_1 and CM_3 for the bow-tie and the bow-tie with stubs is shown in Fig. 3.9. For the bow-tie without stubs, the radiation pattern for CM_1 and CM_3 is what would be expected from a half wavelength and three half wavelength current distribution. With the addition of the stubs, the radiation pattern for CM_1 is unaffected. This result reassures that the current on the stub for CM_1 was self contained and cancelled out. For CM_3 , the overall radiation pattern stays similar with the addition of the stubs as the three lobes are present in the case with and without the stubs. The difference is the main lobe becomes larger for the case with the stubs, making the side lobes smaller.

Figure 3.9: Far-field radiation pattern of CM_1 and CM_3 for the bow-tie and the bow-tie with stubs in a) E-plane (z-x plane) and b) H-plane (z-y plane).

3.3 Fabrication and Measurements

The bow-tie antenna and the bow-tie antenna with stubs were fabricated using copper tape on a very thin sheet of FR4. A UFL co-axial cable was used as an ICE to feed the antennas. The fabricated antennas can be seen in Fig. 3.10 and have the same dimensions as those stated in Table 3.1 and Table 3.2 respectively. The FR4 sheet is used to provide a rigid material for the copper to be placed on rather than to act as a dielectric load on the antenna.

Figure 3.10: Images of a) fabricated bow-tie and b) fabricated bow-tie with stubs.

The reflection coefficient for the fabricated bow-tie and bow-tie with stubs were measured using an Agilent 8753ES S-parameter network analyzer. The measured results can be seen in Fig. 3.11. The dashed lines represent the simulated results and the solid lines are the measured results. For both cases, the measured results follow the simulated results closely. With the addition of the stubs, the third order resonance decreased in frequency while the first order resonance remained unchanged. For the bow-tie antenna, the first order resonance occurs at 975 MHz for the measured results and at 1 GHz for the simulated results. The third order resonance occurs at 3.6 GHz for measured results and at 3.8 GHz for the simulated results. For the bow-tie antenna with stubs, the first order resonance occurs at 950 MHz for the measured results and at 975 MHz for the simulated results. The third order resonance occurs at 2.3 GHz for the measured results and the simulated results.

3.4 Conclusion

In this chapter, insight was obtained on how to modify the modal resonance of characteristic modes by visualizing the eigencurrent. Stubs were placed on a bow-tie structure at eigencurrent nulls for the third order mode close to an eigencurrent maximum for the first order mode. The eigencurrent nulls on the third order mode

Figure 3.11: Measured and simulated reflection coefficient for bow-tie antenna with and without stubs.

were moved from the edge of the bow-tie to the end of the stub, increasing the path distance of the eigencurrent and reducing the modal resonance of CM_3 . The eigencurrent distribution on the bow-tie and the modal resonance for CM_1 was relatively unaffected by the addition of the stubs. Eigencurrent present on the stubs for CM_1 was self contained and opposed each other directly on the stub. The bow-tie antennas with and without stubs were fabricated and the measured reflection coefficient results correlated well with the simulated results.

In chapter 4, a complex antenna structure with features that physically resemble a bow-tie is analyzed using CMA. Similar characteristic attributes will be looked at to those in chapter 3, with the addition of another parameter, modal admittance. The focus of the analysis will be to decompose the complex structure into a set of modes with fundamental responses rather than modifying the structure. Coupling between modes will be uncovered which can be described using CMT.

Chapter 4

Capacitively Fed Crossed Bow-tie Antenna

4.1 CMA of Crossed Bow-tie Antenna

In this chapter, a structure modeled after Tallysman's VeroStar™ antenna, seen in Fig. 4.1, is termed a capacitively fed crossed bow-tie antenna and analyzed using CMA. The VeroStar™ was designed to be compact and have a wide bandwidth, low axial ratio (AR), small gain roll-off from zenith to horizon, and a tight phase center variation (PCV). These design constraints allow the antenna to support the entire GNSS spectrum, receive signals from low elevation angles with excellent multipath mitigation, and provide high levels of positional precision and accuracy [26].

Figure 4.1: Image of Tallysman's VeroStar™ high precision GNSS antenna [27].

When compared to other commercially available GNSS antennas, seen in Table 4.1, the exceptional performance of the antenna is shown. The complicated structure

of the antenna allows it to meet the strict performance requirements, however, the complexity makes it more difficult to understand the underlying mechanisms contributing to the response of the antenna. The goal of this chapter is to analyze a similar complex structure using CMA to understand how the currents flow on the structure over the entire bandwidth.

Table 4.1: Comparison of VeroStar™ average performance over bandwidth to other commercial GNSS antennas [26].

Parameter	VeroStar™	Other antennas
AR at horizon	2 dB	6 dB
PCV	< 2.9 mm	< 4.5 mm
Low angle gain roll-off	7 dB	11 dB

The proposed antenna structure, a capacitively fed crossed bow-tie antenna, can be seen in Fig. 4.2. The dimensions of the structure are stated in Table 4.2. The antenna structure consists of a circular ground plane, two dipoles, and two pairs of triangular elements which closely resemble a bow-tie antenna. The two dipoles are arranged at right angles with each other and placed horizontally above the ground plane. The triangular elements are located directly over each arm of the dipoles such that they capacitively couple with the dipoles [26]. The triangular elements are angled to allow for the wide end to be close to the ground plane and the narrow end to be at the center of the structure above the dipoles. All of the components are modeled as PECs and a port is located at the center of each dipole. The simulation software used to perform the analysis is CST Studio Suite 2020 [22]. CMA results were obtained using the integral equation solver with 1 V sources as the ports. Driven results were obtained using the frequency domain solver with the ports referenced to 50 Ω .

Table 4.2: Capacitively fed crossed bow-tie antenna dimensions.

Name	Variable	Dim. [mm]	Name	Variable	Dim. [mm]
ground dia.	gd	110	dipole length	dl	64
ground spacing	gs	1	dipole width	dw	5
bottom width	bw	45	dipole height	dh	10.5
bow-tie height	bh	30	feed gap	fg	2
bow-tie length	bl	99	top width	tw	2
angled length	al	6	top spacing	ts	3

The eigenvalues for the first sixteen characteristic modes for the structure are shown in Fig. 4.3. The modes are sorted from most resonant to least resonant at 1.85 GHz. I.e. CM₁ is labelled as such due to it's eigenvalue being the closest value to zero at 1.85 GHz, CM₂ is labelled as such due to its eigenvalue being the second closest value to zero at 1.85 GHz, etc... A frequency of 1.85 GHz for mode sorting was chosen because 1.85 GHz is near the center of the antenna's impedance bandwidth as seen

Figure 4.2: Capacitively fed crossed bow-tie antenna. a) Top view b) 3-D view c) x-z plane view.

from the driven reflection coefficient in Fig. 4.4. By performing mode sorting near the center of the antenna's bandwidth, all characteristic modes of relevance should have a mode number that is low enough such that it's eigenvalue is displayed in Fig. 4.3.

The large amount of modes displayed in Fig. 4.3 express that the CMA of the structure is very complex as many characteristic modes are close to resonance within the bandwidth of the antenna. The eigenvalues, λ_2 , λ_8 , and λ_{10} , shown as dashed lines, are identical to the eigenvalues, λ_1 , λ_9 , and λ_{11} respectively. The similarities between these modes is due to the symmetry of the structure and the modes only differ by a 90° rotation in their eigencurrent about the structure.

The characteristic modes that are relevant to the analysis of the antenna are the modes that are excited by the 1 V sources at the center of each dipole. These modes will be the modes that contribute the most to the overall surface current and field distribution of the antenna and will have significant modal weighting coefficients as described in (2.9). The MWCs for characteristic modes excited by port 1 can be seen in Fig. 4.4 for modes with an a_n maximum greater than 0.3. Any other modes with an a_n maximum lower than 0.3 were deemed not significant relative to the modes with larger a_n values.

In Fig. 4.4, the modal weighting coefficients show there are three modes excited by port 1. They are CM_1 , CM_9 , and CM_{11} . If instead port 2 was analyzed, it would be seen that CM_2 , CM_8 , and CM_{10} would have large a_n values which would look identical to the MWC curves for port 1. Due to the similarities in the eigenvalues

Figure 4.3: Eigenvalues for first sixteen characteristic modes on crossed bow-tie structure. Characteristic modes are sorted at 1.85 GHz.

and MWCs between the excited modes of both ports, only port 1 will be analyzed as the results will be the same (albeit rotated 90°) if they were obtained for port 2. Therefore, the number of characteristic modes to be analyzed has been reduced from the sixteen shown in Fig. 4.3 to three. The eigenvalues for the three modes are displayed in Fig. 4.9.

When taking a closer look at a_1 , a_9 , a_{11} and the driven reflection coefficient in Fig. 4.4, the same trend is seen as with the bow-tie antenna discussed in section 3.1 where a_n is large when Γ is small. Both a_1 and a_9 share a maximum near 1.5 GHz with a_1 sharing another maximum with a_{11} near 2 GHz. The first mode is wideband as a_1 is large over the entire band. Therefore, CM₁ should contribute to the overall current distribution of the antenna over the whole bandwidth. CM₉ and CM₁₁ are more narrowband with a_9 being large at lower frequencies and a_{11} being large at higher frequencies. Therefore, the current distribution on the antenna at the lower band will primarily be the sum of $\mathbf{J}_1$ and $\mathbf{J}_9$ while the current distribution on the antenna at the higher band will primarily be the sum of $\mathbf{J}_1$ and $\mathbf{J}_{11}$.

The eigencurrent distribution on the structure for CM₁ and CM₉ can be seen in Fig. 4.5 at 1.3 GHz, 1.5 GHz, and 1.7 GHz. The same is shown in Fig. 4.7 for CM₁ and CM₁₁ at 1.9 GHz, 2.1 GHz, and 2.3 GHz. Stated in the previous paragraph, these modes will make up the majority of the current distribution on the antenna in

Figure 4.4: Modal weighting coefficient for characteristic modes excited by port 1 and the driven reflection coefficient of the crossed bow-tie antenna.

the lower part and upper part of the bandwidth respectively. The frequency points were chosen such that they were spaced apart, within the bandwidth of the antenna and not encroaching on the modes in the other band (lower or higher). Also, 1.5 GHz and 2.1 GHz were specifically chosen as eigenvalue crossing avoidances occurs at those frequencies. This will be discussed later in section 4.2.

The eigencurrent distributions are normalized with respect to their maximum value at each frequency. Eigencurrents with a magnitude less than 1% of the maximum (less than -40 dB) are not shown on the plots to improve clarity. The red arrows overlaid on each plot show the general direction of the eigencurrents near where each arrow is placed.

4.1.1 Eigencurrents for Excited Modes in the Lower Frequency Band

In Fig. 4.5, the eigencurrent distribution on the dipole and the triangular elements are the same at each frequency for both modes, $\mathbf{J}_1$ and $\mathbf{J}_9$. The current on the dipole

travels in the $-y$ direction and has a half wavelength distribution with a maximum in the middle where the port is. The current direction and magnitude does not change with frequency. Notice that only the dipole corresponding to port 1 is shown because the dipole corresponding to port 2 has minimal current flowing on it for either mode. If instead the eigencurrent for the symmetric modes, $\mathbf{J}_2$ and $\mathbf{J}_8$, were shown, the current distribution on the structure would be rotated 90° and there would be current present on the port 2 dipole rather than the port 1 dipole.

The current on the triangular elements directly above the port 1 dipole have a half wavelength distribution. If viewed over the entire bow-tie shape, the two elements are in-phase and a local minimum occurs between them. Since there is not a 180° phase shift in the middle, the distribution over the bow-tie is not a full wavelength distribution. The majority of the current is along the long edges of the triangle with a maximum in the middle of the edge. The currents travel in the $-y$ direction on the elements, in-phase with the current on the dipole. Unlike the dipole, the magnitude of the current on these elements relative to the maximum decreases with frequency.

Figure 4.5: Eigencurrent distribution for CM_1 and CM_9 at 1.3 GHz, 1.5 GHz, and 1.7 GHz. Values are normalized to their respective maximums.

The current on the triangular elements orthogonal to the port 1 dipole have a circular distribution in the clockwise direction if viewed from a $+x$ reference point. The circular distribution on the orthogonal triangular elements will not be a focus

and will not be analyzed further. This is because these elements are not directly excited by the port 1 dipole and the currents flowing on the elements are thought to be due to coupling with adjacent conductors. At 1.7 GHz, the magnitude of the current on these elements is insignificant and is not shown.

The difference in the eigencurrent between CM_1 and CM_9 occurs on the ground plane which can distinctly be seen by observing both modes at 1.3 GHz. For $\mathbf{J}_9$ at 1.3 GHz, the majority of the currents on the ground plane flow around the outer edge in the $+y$ direction. Current nulls occur underneath the base of the bow-tie shape parallel to the port 1 dipole and current maximums occur underneath the orthogonal bow-tie shape. The eigencurrent distribution on the ground plane for CM_9 is the same as the first characteristic mode for a circular sheet as seen in Fig. 4.6 [12]. For $\mathbf{J}_1$ at 1.3 GHz, the currents on the ground plane are directly underneath the dipole and travel in the $+y$ direction, opposing the currents on the dipole. There is no eigencurrent distribution for a circular sheet that is similar to the distribution seen on the ground plane for CM_1 . Therefore, the ground plane currents in $\mathbf{J}_1$ are only present to oppose the dipole current.

Figure 4.6: Normalized eigencurrent distribution for first few modes of a circular sheet. Symmetric modes are not shown. Images obtained from [12].

As frequency increases, the ground plane currents on both modes start to look similar. At 1.5 GHz, $\mathbf{J}_9$ shows a mix of $\mathbf{J}_1$ and $\mathbf{J}_9$ with current on the ground plane opposing the dipole in the middle and current on the outer edge. At 1.7 GHz, both modes show ground plane currents on the outer edge and in the middle opposing the dipole. The mixing of the ground plane currents is due to the coupling between the two modes and will be explored further in section 4.2.

To summarize $\mathbf{J}_1$ and $\mathbf{J}_9$ in the lower band, the eigencurrent distribution on the dipole and the parallel triangular elements are the same for both modes at each frequency. The dipole and each parallel triangular element have a half wavelength distribution and are all in-phase. The ground plane current for $\mathbf{J}_1$ only opposes the dipole while the ground plane currents for $\mathbf{J}_9$ have a distribution the same as the first mode of a circular sheet. As frequency increases, the two modes couple and the ground plane current of the modes mesh together.

4.1.2 Eigencurrents for Excited Modes in the Higher Frequency Band

The excited eigencurrents in the higher band are $\mathbf{J}_1$ and $\mathbf{J}_{11}$. They are analyzed at 1.9 GHz, 2.1 GHz, and 2.3 GHz as seen in Fig. 4.7. The distribution on the port 1 dipole and the parallel triangular elements are the same on both modes at each frequency. Explained in section 4.1.1, the port 2 dipole and the eigencurrents on the orthogonal triangular elements are not of interest and will not be discussed.

The current on the port 1 dipole travels in the $-y$ direction and has a half wavelength distribution with a maximum in the middle. The current direction and magnitude does not change with frequency. The current on the triangular elements parallel to the port 1 dipole have a half wavelength distribution. The currents on the two triangular elements are in-phase and travel in the $+y$ direction, anti-phase with the dipole. At 1.9 GHz, the magnitude of the current on the triangular elements is small relative to the maximum, but as frequency increases so does the magnitude of the current on the elements. The direction of the current on the elements does not change with frequency.

Similar to the excited modes of the lower band, the difference in the eigencurrent between $\mathbf{CM}_1$ and $\mathbf{CM}_{11}$ in the higher band occurs on the ground plane. For $\mathbf{J}_1$ at 1.9 GHz, the majority of the eigencurrent on the ground plane has a distribution similar to the first characteristic mode for a circular sheet. The current nulls are located underneath each parallel triangular element and current maximums are located underneath each orthogonal triangular element. Current flows on opposite edges of the ground plane in the $-y$ direction. Some current exists beneath and opposing the dipole current. For $\mathbf{J}_{11}$ at 1.9 GHz, little current is flowing on the ground plane. The current that is present is located beneath and opposing the current on the dipole. As frequency increases, $\mathbf{CM}_1$ and $\mathbf{CM}_{11}$ start to couple and the ground plane currents for both modes have current around the outer edge of the ground plane and in the middle opposing the dipole.

To summarize $\mathbf{J}_1$ and $\mathbf{J}_{11}$ in the higher band, the eigencurrent distribution on the dipole and the parallel triangular elements are the same for both modes at each frequency. The current on the dipole are anti-phase with the currents on the parallel triangular elements which are in-phase with each other. The current distribution on the dipole and triangular elements is a half wavelength distribution. The ground plane current for $\mathbf{J}_1$ has the same distribution as the first mode of a circular sheet while the ground plane current for $\mathbf{J}_{11}$ opposes the dipole current. As frequency increases, the ground plane eigencurrents of the two modes mesh together due to the modes coupling.

Figure 4.7: Eigencurrent distribution for CM_1 and CM_{11} at 1.9 GHz, 2.1 GHz, and 2.3 GHz. Values are normalized to their respective maximums.

4.1.3 Discussion of Eigencurrents in Lower Band and Higher Band

There are a lot of similarities between the excited modes of the lower band and higher band. For every instance, the current on the dipole and triangular elements have a half wavelength distribution. For the dipole this makes sense as the port acts as an ICE and the first mode of a thin sheet is excited just as in Fig. 2.1.

For the triangular elements, it is less intuitive because when viewing the entire structure it is easy to visualize the triangular elements as a bow-tie antenna. That is why in this thesis, the structure was named a crossed bow-tie structure. However, this would suggest the lower order eigencurrent distributions for a bow-tie antenna would be present. These eigencurrent distributions can be seen in Fig. 4.8. The first mode of a bow-tie has a maximum in the middle and can be ruled out. The second mode of a bow-tie has a minimum in the middle but the current on either side are anti-phase. The structure in question has the currents in-phase on either side and thus the second mode can be ruled out as well. Instead, the triangular elements need to be thought as just that and separate from a bow-tie. Each element has a

half wavelength distribution where the dipole acts as a CCE to excite the current minimum (voltage maximum) that occurs at the narrow end of the triangle. Since the currents over the whole dipole are in-phase, the currents on the two triangular elements are forced in-phase as well.

More similarities occur when focusing on the ground plane currents. For both frequency bands, one mode's eigencurrent looks similar to the first mode of a circular sheet while the other mode's eigencurrent opposes the dipole. The ground eigencurrents merge together as the modes in each frequency band couple.

Figure 4.8: Eigencurrent distribution near modal resonance for a) $\mathbf{J}_1$ on a bow-tie, b) $\mathbf{J}_2$ on a bow-tie, and c) $\mathbf{J}_2$ on a triangular element. Eigencurrents are normalized to their respective maximums and have units of dBA/m.

Differences are noticed when oddities are considered. CM_1 , which is an excited mode in both frequency bands is not the same in the lower band and the higher band. At 1.3 GHz, the ground current on $\mathbf{J}_1$ only opposes the dipole and the current magnitude on the triangular elements is large. As frequency increases, the magnitude of the current on the triangular elements decreases and the ground current moves to the outer edge of the circular sheet. At higher frequencies, the current direction on the triangular elements changes in phase by 180° then increases in magnitude. Also, the ground current are now mostly present on the outer edge just as with the first mode of a circular sheet. Therefore, CM_1 should not be considered the same mode in both bands as it has coupled and exchanged characteristics with the other excited modes, CM_9 and CM_{11} .

The change in phase on the triangular elements in CM_1 is another sign of coupling. For synchronous coupling, which was discussed in section 2.4, a lower frequency mode exhibits an in-phase characteristic while a higher frequency mode exhibits an anti-phase characteristic. In the lower band for both CM_1 and CM_9 , the current on the dipole and the triangular elements are in-phase. Then in the higher band for CM_1 and CM_{11} , the current on the triangular elements are anti-phase with the dipole. The mixing of characteristics due to eigenvalue crossing avoidance will be discussed in the next section while the coupling reminiscent of synchronous coupling will be explored at the end of the chapter.

4.2 Eigenvalue Crossing Avoidance Between the Excited Modes

The CMA of the structure was shown to be complex in Fig. 4.3 due to the large number of characteristic modes present. By viewing the MWC relating to port 1 and ignoring symmetric modes relating to port 2, the number of characteristic modes of relevance reduced greatly from sixteen to three. The eigenvalues of the three modes, λ_1 , λ_9 , and λ_{11} , are shown in Fig. 4.9.

In Fig. 4.9, the characteristic mode with the lowest frequency modal resonance, CM_9 , is resonant around 1.4 GHz and is close to resonance when below 1.5 GHz. Shown in the highlighted grey region at 1.5 GHz, λ_9 changes direction suddenly increasing its eigenvalue significantly and now follows a trajectory similar to the path λ_1 was taking had it not changed direction at 1.5 GHz as well. The eigenvalue for CM_9 changes direction again near 1.8 GHz causing its eigenvalue slowly decrease with frequency afterwards.

Figure 4.9: Eigenvalues for characteristic modes excited by port 1. Highlighted regions show eigenvalue crossing avoidance's between modes.

The modal resonance for CM_1 occurs near 1.8 GHz and is close to resonance from 1.5 GHz to 2.1 GHz. At both these frequencies which are highlighted in grey and pink

respectively, λ_1 changes direction suddenly. After 1.5 GHz, λ_1 follows the trajectory of the path λ_9 was following before 1.5 GHz until 2.1 GHz where a more subtle change in trajectory occurs. After 2.1 GHz, λ_1 starts to follow a path akin to λ_{11} . The modal resonance for CM_{11} occurs at a frequency higher than 2.5 GHz but is near resonance after it changes direction at 2.1 GHz.

All three of the excited modes experience a sudden change in their respective eigenvalue curve over frequency. Highlighted in grey, the eigenvalues for CM_1 and CM_9 seem to avoid crossing each other near 1.5 GHz and change direction to follow the expected path of the other mode. Highlighted in pink, the eigenvalues for CM_1 and CM_{11} experience a more subtle change in direction at 2.1 GHz. Once again, the eigenvalues of the characteristic modes avoid each other and swap trajectories.

This phenomenon is called eigenvalue crossing avoidance and is proposed and discussed in [16]. The relationship between the two mode's eigenvalues follows CMT as shown in (2.16) and (2.18) in section 2.4. Hence, the two modes can be stated as coupling together near the crossing avoidance. The phenomenon occurs when the eigenvalues of two modes avoid each other and exchange trajectories. A mode that experiences more than one crossing avoidance is termed a multiply coupled mode. CM_1 would be an example of this as its eigenvalue avoids crossing with the eigenvalue of CM_9 and CM_{11} .

The two modes exchanging characteristics through coupling is easiest to identify using but not unique to the eigenvalues and is shown to occur for the eigencurrents as well. Near the crossing avoidance, the eigencurrents of the two coupled modes mesh together and become a mix of both. In section 4.1, this is seen occurring on the ground plane currents in Fig. 4.5 for $\mathbf{J}_1$ and $\mathbf{J}_9$ and in Fig. 4.7 for $\mathbf{J}_1$ and $\mathbf{J}_{11}$.

Another instance of crossing avoidance most likely occurs between CM_9 and CM_{16} near 1.8 GHz. CM_{16} is not shown in Fig. 4.9 but can be seen in Fig. 4.3. This crossing avoidance will not be analyzed as it occurs when the eigenvalues for both modes are large and the modal weighting coefficients are small. Therefore, while these modes are coupling their effect on the overall surface current is small.

One of the strengths of CMA is it allows for the surface current on a structure to be broken down into a set of more basic current (or field) distributions that are easier to analyze. However, since the excited characteristic modes are coupling the eigencurrent distributions are no longer basic. Instead, they are the weighted sum of the eigencurrents from the coupled modes just like how the overall surface current is the weighted sum of the eigencurrent from the characteristic modes. It is difficult to individually distinguish what the coupled characteristic modes are contributing to the overall surface current. Instead, what the coupled modes contribute together is seen. Fortunately, Schab et al. developed a method in [16] to decouple these characteristic modes into auxiliary modes. The auxiliary modes do not experience an eigenvalue crossing avoidance and the eigencurrents do not mesh together.

4.2.1 Decoupled Auxiliary Modes

The method Schab et al. developed to decouple the characteristic modes experiencing eigenvalue crossing avoidance's is summarized in section 2.4. The complete process can be found in [16]. Discussed in the previous section, there are two instances of eigenvalue crossing avoidance's shown in Fig. 4.9. One instance highlighted in gray at 1.5 GHz between CM_1 and CM_9 and another instance highlighted in pink at 2.1 GHz between CM_1 and CM_{11} . Therefore, the decoupling procedure is performed twice.

The decoupled modes, called auxiliary modes, are denoted with superscripts, a and b . The auxiliary modes and their respective attributes are obtained from the attributes of two modes experiencing a crossing avoidance. Therefore, the characteristic modes the auxiliary modes are obtained from will be expressed using subscripts, n, m . I.e. the auxiliary eigenvalues obtained from CM_1 and CM_9 would be $\lambda_{1,9}^a$ and $\lambda_{1,9}^b$. Likewise, the auxiliary eigenvalues obtained from CM_1 and CM_{11} would be $\lambda_{1,11}^a$ and $\lambda_{1,11}^b$. The eigenvalues for the characteristic modes excited by port 1 and their respective auxiliary eigenvalues can be seen in Fig. 4.10.

Figure 4.10: Eigenvalues for characteristic modes excited by port 1 and the auxiliary eigenvalues from the crossing avoidance between CM_1 , CM_9 and CM_1 , CM_{11} . Highlighted region shows an artefact occurring due to a multiply coupled mode.

In Fig. 4.10 at the instances of crossing avoidance between the coupled characteristic modes, the auxiliary eigenvalues continue along their expected path and cross. At 1.5 GHz, $\lambda_{1,9}^a$ and $\lambda_{1,9}^b$ are both equal to zero and cross such that they continue along their expected path. At 2.1 GHz, $\lambda_{1,11}^a$ and $\lambda_{1,11}^b$ are both equal to zero and cross such that they continue along their expected path. Interestingly, both auxiliary mode pairs are resonant at the same frequency. However, this is not an implication of anything significant and rather is a product of $\text{CM}_{n,m}^a$ being modally resonant over a large bandwidth for both auxiliary pairs. Therefore, when $\lambda_{n,m}^a$ and $\lambda_{n,m}^b$ do cross, it is likely to occur when both auxiliary modes are resonant.

The auxiliary modes that would have large Q factors and hence a sharp resonance, $\text{CM}_{1,9}^b$ and $\text{CM}_{1,11}^b$, have eigenvalue curves that do not display any oddities and act as expected with the crossing avoidance removed. For $\text{CM}_{1,9}^a$ and $\text{CM}_{1,11}^a$, the eigenvalue curves act as expected near the crossing avoidance of the two modes they were calculated from. However, $\lambda_{1,9}^a$ and $\lambda_{1,11}^a$ exhibit crossing avoidances with the eigenvalues of the characteristic modes they were not calculated from, λ_{11} and λ_9 respectively. Also, highlighted in red at 1.8 GHz, $\lambda_{1,9}^a$ and $\lambda_{1,11}^a$ asymptotically approach each other eventually separating when far enough from 1.8 GHz. At first, this may look like another instance of a crossing avoidance. However, this asymptote and the crossing avoidances occurring with the unused characteristic modes is more likely artefacts of CM_1 being multiply coupled causing $\lambda_{1,9}^a$ and $\lambda_{1,11}^a$ to be composed of the same part of λ_1 when in between 1.5 GHz and 2.1 GHz.

The decoupling procedure laid out in [16] was not created to account for characteristic modes experiencing more than one crossing avoidance. However, the procedure can be applied to multiply coupled modes if there is enough frequency separation between the multiple crossing avoidances, which occurs for CM_1 . Basically, the decoupling procedure takes the coupled characteristic modes and gradually swaps their eigenvalues near the crossing avoidances. When on one side and far enough away from the crossing avoidance, the auxiliary eigenvalues are completely composed of one of the coupled mode's eigenvalues. On the other side of the crossing avoidance, the auxiliary eigenvalues are completely composed of the other coupled mode's eigenvalues. Therefore even though a crossing avoidance does not occur between $\text{CM}_{1,9}^a$ and $\text{CM}_{1,11}^a$ at 1.8 GHz, the eigenvalues of the two modes are equal and can be swapped seamlessly by using the decoupling procedure again on these auxiliary modes. Doing so produces modes, $\text{CM}_{a19,a111}^a$ and $\text{CM}_{a19,a111}^b$, that are auxiliary to the auxiliary modes. The eigenvalue of the later, $\lambda_{a19,a111}^b$ is identical to λ_1 and has eigencurrents of that are the same as well. Therefore, $\text{CM}_{a19,a111}^b$ will be assumed to be the same as CM_1 and ignored as its characteristics should already be account for in $\text{CM}_{a19,a111}^a$, $\text{CM}_{1,9}^b$, and $\text{CM}_{1,11}^b$.

After performing the decoupling procedure twice for each instance of crossing avoidance and once more to account for multiply coupling, the coupled modes, CM_1 , CM_9 , and CM_{11} are transformed into uncoupled auxiliary modes, $\text{CM}_{a19,a111}^a$, $\text{CM}_{1,9}^b$, and $\text{CM}_{1,11}^b$. The eigenvalues for all six of these modes are shown in Fig. 4.11, where

Figure 4.11: Eigenvalues for characteristic modes excited by port 1 and the auxiliary eigenvalues for the decoupled modes.

solid lines represent the characteristic modes and the dashed lines represent the auxiliary modes. In Fig. 4.11, $\lambda_{a19,a111}^a$ does not display any oddities and is extremely wideband as it is modally resonant for the entire frequency range. The Q factor of this mode would almost be equal to zero. $\lambda_{1,9}^b$ and $\lambda_{1,11}^b$ are the same as when they were discussed in relation to Fig. 4.10. When comparing the eigenvalues of the three characteristic modes to the auxiliary eigenvalues, it can be seen that the auxiliary eigenvalues are a very good representation of the original eigenvalues without any coupling effects. Therefore, the next step is to remove the coupling effects the crossing avoidances have on the eigencurrents of the excited characteristic modes.

4.2.2 Eigencurrents for Auxiliary Modes

With the auxiliary eigenvalues obtained, the eigencurrents of the characteristic modes, $\mathbf{J}_1$, $\mathbf{J}_9$, and $\mathbf{J}_{11}$, can be decoupled to obtain auxiliary eigencurrents, $\mathbf{J}_{a19,a111}^a$, $\mathbf{J}_{1,9}^b$, and $\mathbf{J}_{1,11}^b$. This is performed by calculating the frequency dependent coefficient, γ , from the coupled eigenvalues and the auxiliary eigenvalues using (2.20). Then the eigencurrents of the coupled characteristic modes are normalized using γ with (2.22)

such that the auxiliary eigencurrents can be calculated using (2.19).

The auxiliary eigencurrents, $\mathbf{J}_{a19,a111}^a$, $\mathbf{J}_{1,9}^b$, and $\mathbf{J}_{1,11}^b$, are shown at multiple frequencies within the bandwidth of the antenna in Fig. 4.12. At 1.5 GHz and 1.7 GHz, $\mathbf{J}_{a19,a111}^a$ is compared to $\mathbf{J}_{1,9}^b$ as those are frequency points where $\text{CM}_{1,9}^b$ is relevant. At 1.9 GHz and 2.1 GHz, $\mathbf{J}_{a19,a111}^a$ is compared to $\mathbf{J}_{1,11}^b$ as those are frequency points where $\text{CM}_{1,11}^b$ is relevant. $\text{CM}_{b19,b111}^a$ is modally resonant over the entire bandwidth, therefore, $\mathbf{J}_{b19,b111}^a$ is relevant at every frequency point. Each surface current plot is normalized with its respective maximum at that frequency and values less than 1% of the maximum are not shown to improve clarity. The red arrows overlaid on the plots show the general direction of the eigencurrent nearby. The arrow overlay on the triangular elements orthogonal to the port 1 dipole are not shown which was also done to improve clarity. The focus of the analysis will be relating to the port 1 dipole, parallel triangular elements and the ground plane.

In Fig. 4.12, it can be seen that the eigencurrent distribution stays similar over frequency as opposed to $\mathbf{J}_1$, $\mathbf{J}_9$, and $\mathbf{J}_{11}$ in Fig. 4.5 and Fig. 4.7. This is because the ground plane currents between the relevant auxiliary modes at each frequency point are not meshing together. Instead, the ground plane eigencurrent distribution in $\mathbf{J}_{a19,a111}^a$ are similar to the first mode of a circular sheet at each frequency point. While the ground plane eigencurrent in $\mathbf{J}_{1,9}^b$ and $\mathbf{J}_{1,11}^b$ are only present to oppose the dipole at each frequency point. On the ground plane in $\mathbf{J}_1$, $\mathbf{J}_9$, and $\mathbf{J}_{11}$, the circular sheet distribution and the current opposing the dipole are combined together with different weighting depending on the frequency.

Similar to $\mathbf{J}_1$, $\mathbf{J}_9$, and $\mathbf{J}_{11}$, all auxiliary eigencurrents have current on the dipole and triangular elements that have a half wavelength distribution. In the lower band at 1.5 GHz and 1.7 GHz, the current on the triangular elements are in phase with the current on the dipole. While in the higher band at 1.9 GHz and 2.1 GHz, the current on the triangular elements and the dipole are anti-phase. This makes sense for $\text{CM}_{1,9}^b$ and $\text{CM}_{1,11}^b$ as they are separate modes, with one exhibiting an in-phase characteristic where the other exhibits and anti-phase characteristic. As stated in section 4.1.3, the in-phase/anti-phase relationship is reminiscent of synchronous coupling. $\text{CM}_{a19,a111}^a$ is different because it is considered the same mode over the entire frequency range but its bow-tie eigencurrent experiences the swap from in-phase to anti-phase between 1.7 GHz and 1.9 GHz, thus, discrediting the idea that it is the same mode in both bands.

However, if it is assumed that the ground plane current is what defines $\text{CM}_{a19,a111}^a$ and the current on the bow-tie and dipole is what defines $\text{CM}_{1,9}^b$ and $\text{CM}_{1,11}^b$. Then $\text{CM}_{a19,a111}^a$ can be considered a single mode defined by the ground plane currents, which are similar over frequency, where the currents on the triangular elements and dipole mimic the relevant dipole/bow-tie mode at the frequency of interest. This assumption is reinforced by the eigenvalue for the first mode of a circular sheet being identical to $\lambda_{a19,a111}^a$ as seen in Fig. 4.13, both of which have similar eigencurrent

Figure 4.12: Eigencurrent distribution for $CM_{a19,a111}^a$ and $CM_{1,9}^b$ at 1.5 GHz, 1.7 GHz, and $CM_{a19,a111}^a$ and $CM_{1,11}^b$ at 1.9 GHz, 2.1 GHz. Values are normalized to their respective maximums.

distributions. The dimensions of the circular sheet have the same dimensions as the ground plane.

There are slight differences between $CM_{1,9}^b$ and $CM_{1,11}^b$ other than the bow-tie and dipole currents being in-phase or anti-phase. For $\mathbf{J}_{1,9}^b$, the current on the ground plane opposing the dipole current is more prominent than in $\mathbf{J}_{1,11}^b$. The other difference is relating to the magnitude of the current on the triangular elements. On average, the magnitude of the current on the triangular elements relative to the maximum is greater for $\mathbf{J}_{1,11}^b$ than for $\mathbf{J}_{1,9}^b$.

Figure 4.13: Plot comparing the eigenvalue for $CM_{a19,a111}^a$ to the eigenvalue for CM_1 of a circular sheet.

To summarize section 4.2, CM_1 , CM_9 , and CM_{11} were transformed into auxiliary modes, $CM_{a19,a111}^a$, $CM_{1,9}^b$, and $CM_{1,11}^b$ using the procedure developed in [16]. The eigenvalues relating to $CM_{a19,a111}^a$, $CM_{1,9}^b$, and $CM_{1,11}^b$ do not exhibit crossing avoidances and their eigencurrents are not a mix of multiple distributions. $\mathbf{J}_{a19,a111}^a$ is defined by the ground plane current which is the same as the eigencurrent of the first mode in a circular sheet. $\mathbf{J}_{1,9}^b$ and $\mathbf{J}_{1,11}^b$ are defined by the current distributions on the bow-tie and dipole. Where $\mathbf{J}_{1,9}^b$ has an in-phase bow-tie/dipole relationship with significant current on the ground plane opposing the dipole. $\mathbf{J}_{1,11}^b$ has an anti-phase bow-tie/dipole relationship with significant current on the triangular elements.

4.3 Modal Admittance and Reflection Coefficient

Input impedance and reflection coefficient are parameters that are not commonly shown when performing CMA even though they are important to antenna performance. This is due to two reasons. One reason is that CMA can be performed without an excitation source. In this case, input impedance is not important as there is no input. The other reason is when performing CMA, modal input impedance/admittance is not easily obtained from many simulation software. This may come as a surprise because (2.12) states that the input admittance is equal to the sum of the MWC

multiplied by the eigencurrent at the input. Both the eigencurrent and the MWC are easily obtained, however, the difficulty comes when trying to evaluating the eigencurrent at the input accurately enough to obtain a reasonable admittance value.

Figure 4.14: Image of the single bow-tie antenna structure used in FEKO simulation to obtain modal admittance.

All the simulations and results obtained up to this point have been from using CST Studio Suite [22], however, admittance is not easily obtained using this software. Modal admittance can be obtained using FEKO [28] and therefore FEKO will be used for this section. The model being analyzed, shown in Fig. 4.14, has the port 2 dipole and associated bow-tie removed to allow for a finer mesh without significantly increasing simulation time. The mesh is overlaid on the structure and is the small triangles outlined in white. In a complex structure like the one being analyzed, it is likely there will be errors in modal tracking, which is the ability to meaningfully link eigenvalues across frequency [11]. A finer mesh allows for the accuracy of the simulation and hence modal tracking to be increased.

In Fig. 4.15(a), the eigenvalues obtained in CST Studio from the crossed bow-tie structure are compared to the eigenvalues obtained from the FEKO simulation and associated structure. The eigenvalue for CM_{16} is included from the FEKO simulation as this mode can not be ignored like it was in section 4.1 due to its inductive contribution to the admittance. It is seen that the eigenvalues from both simulations are very similar. The only difference being the modal resonances and crossing avoidances occur at different frequencies. From the CST simulation to the FEKO simulation, the crossing avoidance between CM_1 and CM_9 shifts from 1.5 GHz to 1.43 GHz respectively. While the crossing avoidance between CM_1 and CM_{11} shifts from 2.1 GHz to 2.27 GHz respectively. The change in frequency is more likely due to the removal of the port 2 dipole and associated bow-tie rather than the different simulation software. According to (2.16), the decrease in modal resonance for CM_9 and the increase in modal resonance for CM_{11} indicates that the coupling coefficient, K , between these two modes is affected by the removed conductors. This will be discussed further in section 4.4.

Figure 4.15: a) Comparing eigenvalues from CST crossed bow-tie simulation (denoted as "c-") and FEKO single bow-tie simulation (denoted as "f-"). b) FEKO Single bow-tie eigenvalues and decoupled auxiliary eigenvalues.

In Fig. 4.15(b), the eigenvalues from the FEKO simulation that contribute to the modal admittance are shown. Just as in the CST simulation, these characteristic modes exhibit eigenvalue crossing avoidance, therefore, the decoupling procedure discussed in section 4.2.1 is performed. These auxiliary modes are shown in Fig. 4.15 as well. The auxiliary eigenvalues do not avoid each other and continue on their expected trajectory following the same trends shown in Fig. 4.11. The two plots in Fig. 4.15 show that the results from the FEKO simulation are similar to the results from the CST simulation and observations should be transferable between the two.

The admittances for the excited characteristic modes are shown on a Smith chart through the reflection coefficient in Fig. 4.16. The real and imaginary parts of the modal admittances are shown in Fig. 4.17(a) and Fig. 4.17(b) respectively. The reflection coefficient of each mode is calculated from the modal admittance using (4.1) where $Z = 1/Y$ and $Z_o = 50 \Omega$ [29].

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} \quad (4.1)$$

In Fig. 4.16 and Fig. 4.17, the sum of the modal admittances, ΣY_n , is the sum of the modal admittances, Y_1, Y_9, Y_{11}, Y_{16} , and some shunt capacitance, Y_{ho} . The shunt capacitance, Y_{ho} , is added to account for capacitance from higher order modes [14] and is approximately 0.5 pF at lower frequencies increasing linearly to 0.9 pF at higher frequencies. The susceptance added due to Y_{ho} is shown in Fig. 4.17(b). According to (2.12), the sum of the modal admittance is equal to the input admittance of the antenna, Y_{in} . This is shown to be the case as the traces corresponding to Y_{in} and ΣY_n in Fig. 4.18 and Fig. 4.19 are the same.

The Smith chart view of the modal admittances in Fig. 4.16 shows how each mode's admittance changes over frequency. The end of the trace with the circle represents 1.3 GHz and the end of the trace with the square represents 2.4 GHz. However, it is difficult to draw conclusions from this plot as each trace changes direction suddenly at some point. It is easier to draw conclusions when splitting the admittance into its real and imaginary components as seen in Fig. 4.17(a) and Fig. 4.17(b) respectively.

In the lower frequency band, the input admittance is dominated by Y_1 and Y_9 where both modes strongly contribute to the input conductance. At 1.43 GHz, the conductance of both modes are maximum and the slope of the input susceptance is steep. Before 1.43 GHz, the input susceptance is mostly influenced by Y_1 which is large and capacitive. After 1.43 GHz, Y_1 approaches a susceptance of zero while the susceptance of Y_9 becomes large, inductive and strongly influences Y_{in} . Higher order modes add capacitance which helps counteract the large inductive susceptance from Y_9 . In Fig. 4.15(b), the eigenvalue crossing avoidance between CM_1 and CM_9 occurs at 1.43 GHz and as such Y_1 and Y_9 are likely altered due to avoidance coupling just as the eigencurrent was. Interestingly, the crossing avoidance occurs near where Y_1 and Y_9 experience extremes in their conductance and susceptance.

Figure 4.16: Modal admittance on Smith chart for FEKO single bow-tie structure. Circle represents 1.3 GHz and square represents 2.4 GHz.

In the higher frequency band, the input conductance is the sum of Y_1 and Y_{11} with both modes contributing equally. The input susceptance is made up of many modes where the large capacitive susceptance of Y_{11} is being cancelled out by the large inductive susceptance of Y_{16} . The capacitance due to higher order modes counteracts

Figure 4.17: Modal a) conductance and b) susceptance for FEKO single bow-tie structure. The legend is the same for both plots.

any excess inductance from Y_{16} such that Y_{in} is slightly capacitive until the susceptance of Y_{11} starts to approach zero after 2.2 GHz. Then Y_{in} becomes inductive. Y_1 contributes little to the susceptance in the higher band before 2.2 GHz as it is near zero. After, 2.2 GHz, Y_1 provides a small amount of inductance. The eigenvalue crossing avoidance between CM_1 and CM_{11} occurs near 2.2 GHz and because of this their admittances are likely altered due to the coupling.

In this section, the characteristic modes that have large modal weighting coefficients, CM_1 , CM_9 , and CM_{11} , were shown to contribute significantly to the input admittance of the antenna. Other modes, such as CM_{16} , and higher order modes are shown to contribute susceptance/reactance to the input admittance even though these modes would have very small modal weighting coefficients and are not excited by the port. The input conductance is the sum of only the excited modes, while the input susceptance is the sum the excited modes and higher order modes. The modal admittance of the excited modes are likely exchanging characteristics due to eigenvalue crossing avoidances similar to what was seen in the eigencurrent.

4.3.1 Auxiliary Modal Admittance

In section 4.1, it was mentioned that the exchanging of characteristics due to eigenvalue crossing avoidances was not unique to eigenvalues and was occurring with the eigencurrent. In the previous section, it was stated that the exchanging of characteristics is likely occurring in the modal admittance as well. However, the method Schab et al. developed in [16] to decouple the characteristic modes only applied towards the eigenvalues and eigencurrent, not the modal admittance.

Re-arranging (2.12), the eigencurrent at the antenna input can be expressed as the modal admittance divided by the MWC as shown in (4.2).

$$\mathbf{J}_n(P) = \frac{Y_n}{a_n} \quad (4.2)$$

Where (4.2) can be substituted into (2.19) replacing $\mathbf{J}_1$ and $\mathbf{J}_2$. However to get Y_a or Y_b , the same substitution would need to be made with $\mathbf{J}_a$ or $\mathbf{J}_b$ which would require knowing the auxiliary modal weighting coefficients, a_a and a_b . Also, (2.19) is only valid when the eigencurrents are normalized according to (2.22). It does not make sense to perform this type of normalization on the admittance and therefore the simplification cannot be made. Instead the auxiliary modal admittance would have their own frequency dependent coefficients, α , β , γ , and δ , for both Y_a and Y_b as shown in (4.3). It is likely these coefficients would be different when trying to determine the auxiliary modal weighting coefficients as well.

$$\begin{aligned} \frac{Y_1}{a_1} &= \alpha \frac{Y_a}{a_a} + \beta \frac{Y_b}{a_b} \\ \frac{Y_2}{a_2} &= \gamma \frac{Y_a}{a_a} + \delta \frac{Y_b}{a_b} \end{aligned} \quad (4.3)$$

Expanding upon the work done by Schab et al. to obtain auxiliary modal weighting coefficients and auxiliary modal admittances is outside the scope of this thesis due to these reasons. Instead, an approximation is made to obtain auxiliary modal admittances based on how much the coupling eigencurrents contribute to the auxiliary eigencurrent which is related to γ . If γ is small then one mode is dominant in $\mathbf{J}_a$ and the other is dominant in $\mathbf{J}_b$. When γ is large, the main contributing mode switches to the other auxiliary mode. At the eigenvalue crossing avoidance, both coupling modes contribute equally.

The contribution the coupling eigencurrents have towards the auxiliary eigencurrents is shown in Table. 4.3 for CM₁ and CM₉. At 1.3 GHz, γ is small and $\mathbf{J}_9$ is the main contributor to $\mathbf{J}_{1,9}^a$ while $\mathbf{J}_1$ is the main contributor to $\mathbf{J}_{1,9}^b$. At 1.7 GHz, γ is large and $\mathbf{J}_1$ is the main contributor to $\mathbf{J}_{1,9}^a$ while $\mathbf{J}_9$ is the main contributor to $\mathbf{J}_{1,9}^b$. At 1.5 GHz where the crossing avoidance occurs, $\mathbf{J}_1$ and $\mathbf{J}_9$ both contribute close to 50% of $\mathbf{J}_{1,9}^a$ and $\mathbf{J}_{1,9}^b$.

Expressed in (2.20), only the eigenvalues are needed to obtain γ . Therefore, it can be obtained and used with the modal admittances. The $\mathbf{J}_b$ part of (4.3) is less affected by the normalization performed in (2.22) on the eigencurrents. Therefore, an approximation of the auxiliary modal admittances can be determined by using γ and an equation similar to the one to calculate $\mathbf{J}_b$ as shown in (4.4).

Table 4.3: Percent contribution the eigencurrent and modal admittance of CM₁ and CM₉ have towards their auxiliary mode's, CM_{1,9}^a and CM_{1,9}^b, eigencurrent and modal admittance.

Freq. [GHz]	γ	J₁ contrib. to J_{1,9}^a [%]	J₉ contrib. to J_{1,9}^a [%]	J₁ contrib. to J_{1,9}^b [%]	J₉ contrib. to J_{1,9}^b [%]
1.30	-0.08	24.71	75.29	97.99	2.01
1.50	-0.90	49.14	50.86	54.44	45.56
1.70	-6.17	88.65	11.35	17.00	83.00
Freq. [GHz]	γ	Y₁ contrib. to Y_{1,9}^a [%]	Y₉ contrib. to Y_{1,9}^a [%]	Y₁ contrib. to Y_{1,9}^b [%]	Y₉ contrib. to Y_{1,9}^b [%]
1.30	-0.16	13.46	86.54	99.63	0.37
1.43	-0.91	41.59	58.41	51.41	48.59
1.67	-6.17	87.17	12.83	0.47	99.53

$$\begin{aligned}
Y_a &= \frac{1}{1 + \gamma^2} (Y_2 + \gamma^2 Y_1) \\
Y_b &= \frac{1}{1 + \gamma^2} (Y_1 + \gamma^2 Y_2)
\end{aligned} \tag{4.4}$$

After performing (4.4), the contribution of Y_1 and Y_9 to $Y_{1,9}^a$ and $Y_{1,9}^b$ can be seen in Table 4.3 and compared to the eigencurrent. The admittance percent contributions are shown at values of γ similar to the γ values the eigencurrents are shown for. At the different γ values, the same trend is seen in the modal admittances and the eigencurrents. At a small γ value, Y_9 is the main contributor to $Y_{1,9}^a$ while Y_1 is the main contributor to $Y_{1,9}^b$. The opposite is seen at a large γ value and at 1.43 GHz, the location of the eigenvalue crossing avoidance, the modal admittance contribution is close to 50% for both modes in $Y_{1,9}^a$ and $Y_{1,9}^b$.

In Fig. 4.18 and Fig. 4.19, the auxiliary modal admittances, $Y_{a19,a111}^a$, $Y_{1,9}^b$, and $Y_{1,11}^b$, are shown. Fig. 4.19(a) and Fig. 4.19(b) show the real and imaginary admittance respectively. The sum of the auxiliary admittances, ΣY_{aux} , is the sum of $Y_{a19,a111}^a$, $Y_{1,9}^b$, $Y_{1,11}^b$, Y_{16} and Y_{ho} . The admittance from CM₁₆, the higher order modes, and the input admittance is the same as when shown in Fig. 4.16 and Fig. 4.17. It is seen that summing the auxiliary modal admittances, $Y_{a19,a111}^a$, $Y_{1,9}^b$, and $Y_{1,11}^b$, instead of the modal admittances, Y_1 , Y_9 , Y_{11} , produces a curve the same as Y_{in} . Therefore, the auxiliary admittances summing to Y_{in} and showing similar contribution percentages to the eigencurrent provides assurance that the approximation made to obtain the auxiliary admittances is reasonable.

On the Smith chart in Fig. 4.18, how the auxiliary admittances change over frequency is shown. Compared with the modal admittance shown in Fig. 4.16, it is easier to see trends in the admittance of the excited auxiliary modes. This is because

the effect of eigenvalue avoidance coupling is reduced in the auxiliary admittances. The admittance, $Y_{1,9}^b$, is susceptive for the entire frequency range where the mode is relevant, except for a brief period when the mode is resonant near 1.43 GHz. At resonance, $CM_{1,9}^b$ changes from being capacitive to inductive. The admittance, $Y_{1,11}^b$, is capacitive for most of the frequency range while $Y_{a19,a111}^a$ has very little susceptance and is almost purely conductive for the entire frequency range. Analyzing the real and imaginary admittances separately in Fig. 4.19a and Fig. 4.19b, the trends are similar to those stated relating to Fig. 4.17 with differences discussed previously.

Figure 4.18: Auxiliary modal admittance on Smith chart for single bow-tie. Circle represents 1.3 GHz and square represents 2.4 GHz.

The three auxiliary modes, $CM_{a19,a111}^a$, $CM_{1,9}^b$, and $CM_{1,11}^b$, that replace the excited characteristic modes, CM_1 , CM_9 , and CM_{11} , contribute significantly to the input admittance. The input conductance is the sum of only these auxiliary modes. While the input susceptance is affected by these modes, CM_{16} , and higher order modes. $CM_{1,9}^b$ and $CM_{1,11}^b$ are reactive modes with $CM_{1,9}^b$ being inductive and capacitive while $CM_{1,11}^b$ is mainly capacitive. The mode, $CM_{a19,a111}^a$, is mainly resistive with little reactance. In the next section, the two reactive modes, $CM_{1,9}^b$ and $CM_{1,11}^b$, will be shown to be directly related to the dipole and bow-tie conductors. The synchronous coupling between the conductors shift the resonance of $CM_{1,9}^b$ and $CM_{1,11}^b$ to provide the antenna with a wideband impedance bandwidth.

Figure 4.19: Auxiliary modal a) conductance and b) susceptance for single bow-tie. The legend is the same for both plots.

4.4 Discussion of Modal Coupling

In section 2.4 it was discussed how the characteristic modes in a structure can couple with each other following CMT. The coupling due to eigenvalue crossing avoidances was discussed extensively throughout this chapter. The other type of coupling where coupling characteristic modes exhibit an in-phase and anti-phase characteristic in the charge distribution was discussed by Borchardt and LaPointe in [14], [15] and [30]. This in-phase and anti-phase relationship is a phenomenon that was shown in section 4.2.2 to be occurring in the eigencurrents on the dipole and bow-tie conductors.

In an effort to understand this coupling, two structures, shown in Fig. 4.20, are analyzed using CMA in CST Studio. The structure in Fig. 4.20(a) consists of the port 1 dipole and the ground plane. The structure in Fig. 4.20(b) consists of the two triangular elements parallel to the port 1 dipole and the ground plane. In both structures, either the dipole or bow-tie is not present such that the analysis of each structure shows the decoupled response of the conductor that is present.

The auxiliary eigenvalues relating to the two structures in Fig. 4.20 are compared to the auxiliary eigenvalues of the crossed bow-tie structure in Fig. 4.21. The eigenvalues with the subscript _{dipole} are obtained from the structure in Fig. 4.20(a) and the eigenvalues with the subscript _{bowtie} are obtained from the structure in Fig. 4.20(b). All other eigenvalues are from the crossed bow-tie structure and are the same as those in Fig. 4.11. The auxiliary eigenvalue, λ^a , is identical between structures because it corresponds to the ground plane which is the same for every structure. In Fig. 4.13, this mode was shown to be the same as the first mode in a circular sheet. Since the eigenvalue for $\text{CM}_{a19,a111}^a$ does not change even if different conductors are placed above the circular ground plane, the mode can be stated as being decoupled.

Figure 4.20: Images of antenna structure with a) only port 1 dipole and ground plane and b) only single bow-tie and ground plane.

The other λ^b auxiliary modes are different and exhibit synchronous coupling. Synchronous coupling occurs when the decoupled resonant frequencies, ω_1 and ω_2 , are equal such that the coupled resonant frequencies, ω_+ and ω_- , are offset from the decoupled frequency by the coupling coefficient, K . This is shown in (4.5) by setting $\omega_1 = \omega_2 = \omega_o$ and plugging it into (2.16). Depending on the sign of K , one of the coupled modes will display an in-phase characteristic while the other coupled mode will display an anti-phase characteristic.

$$\omega_{\pm} = \omega_o \pm |K| \quad \text{or} \quad f_{\pm} = f_o \pm \frac{|K|}{2\pi} \quad (4.5)$$

In Fig. 4.21, the decoupled dipole and bow-tie eigenvalues, $\lambda_{\text{dipole}}^b$ and $\lambda_{\text{bowtie}}^b$, are both resonant at 1.8 GHz. Therefore, $f_{\text{dipole}} = f_{\text{bowtie}} = f_o = 1.8$ GHz. Their equivalent coupled eigenvalues, $\lambda_{1,9}^b$ and $\lambda_{1,11}^b$, are resonant at 1.5 GHz and 2.1 GHz respectively which is offset from f_o by 0.3 GHz. In Fig. 2.19 it was shown that the eigencurrents between the dipole and bow-tie are in-phase for $\text{CM}_{1,9}^b$ and anti-phase for $\text{CM}_{1,11}^b$. Therefore, K is negative and equal to -1.89×10^9 rad/s with $f_{1,9}^b = f_- = 1.5$ GHz and $f_{1,11}^b = f_+ = 2.1$ GHz. Since the trend of $\lambda_{1,9}^b$ is similar to the trend of $\lambda_{\text{dipole}}^b$, $\text{CM}_{1,9}^b$ will be termed the coupled dipole mode. For the same reasoning between $\lambda_{1,11}^b$ and $\lambda_{\text{bowtie}}^b$, $\text{CM}_{1,11}^b$ will be termed the coupled bow-tie mode. In section 4.3.1, it was mentioned that the coupling coefficient was affected by the port 2 dipole and the associated triangular elements. This was because in Fig. 4.15(b) with these conductors removed, the resonance of the dipole mode decreases and the resonance of the bow-tie mode increases. The increase in frequency separation meant the coupling coefficient increased as well.

The lower frequency mode, $\text{CM}_{1,9}^b$, being related to the dipole explains why its eigencurrent, $\mathbf{J}_{1,9}$, in Fig. 4.12 shows a larger current magnitude opposing the dipole on the ground plane compared to $\mathbf{J}_{1,11}$. $\text{CM}_{1,9}^b$ has a sharp resonance which switches from capacitive to inductive reactance just as a half wavelength dipole would. The higher frequency mode, $\text{CM}_{1,11}^b$, being related to the bow-tie explains why a larger

Figure 4.21: Auxiliary eigenvalues for the crossed bow-tie structure compared to auxiliary eigenvalues for the structure with only bow-tie and ground plane and for the structure with only dipole and ground plane.

current magnitude is seen on the triangular elements in $\mathbf{J}_{1,11}$ with less current opposing the dipole on the ground plane than in $\mathbf{J}_{1,9}$. The resonance of $\text{CM}_{1,11}^b$ is not as sharp as $\text{CM}_{1,9}^b$ due to the triangular elements being wider at the ends compared to the dipole.

Due to the synchronous coupling between the dipole and bow-tie conductors, the resonant frequency of the decoupled dipole and bow-tie modes shift by the coupling coefficient to a lower frequency dipole dominant in-phase mode and a higher frequency bow-tie dominant anti-phase mode. Thus, creating a dual resonance instead of a single resonance at 1.8 GHz. The coupling modes have a reactive impedance where one is inductive and the other is capacitive within the bandwidth of the antenna. The reactive modes in parallel with the resistive ground plane mode and the dual resonance creates the wide impedance bandwidth in the antenna.

4.5 Conclusion

The goal of this chapter was to comprehensively analyze the complicated antenna structure presented in Fig. 4.2. This was done using CMA as it allows the characteristics of a structure to be broken down into many characteristic modes. The characteristic modes needed to be deconstructed further into auxiliary modes where three dominant modes were determined; a ground plane mode, $CM_{a19,a111}^a$, a dipole mode, $CM_{1,9}^b$, and a bow-tie mode, $CM_{1,11}^b$. The ground plane mode's eigencurrent was the lowest frequency mode of a circular sheet and the admittance was primarily resistive. While the eigencurrent on the dipole and bow-tie modes were in-phase or anti-phase respectively with a half-wavelength distribution on each conductor. The admittance of these modes were reactive with the dipole mode being capacitive and inductive and the bow-tie mode being capacitive. The synchronous coupling between the dipole and the bow-tie structures create a dual resonance and hence a wide bandwidth.

Chapter 5

Conclusions and Future Work

5.1 Summary

In this thesis, characteristic mode analysis and coupled mode theory were used to understand the fundamental modes which make up the response of a bow-tie antenna and a capacitively fed crossed bow-tie antenna. In Chapter 3, CMA was performed on a basic bow-tie antenna to obtain the eigencurrent distribution the first two characteristic modes excited by the port. The location of eigencurrent maximums and minimums on these modes were used to provide guidance on how to manipulate the antenna to increase its bandwidth. The third order mode had an eigencurrent null in close proximity to an eigencurrent maximum on the first order mode. By placing perpendicular stubs at the location of the null, the third order mode was electrically lengthened, shifting its resonance down, without affecting the resonance of the first order mode. The third order mode's resonance was reduced such that it was close to the resonance of the first order mode to create a dual resonance and a wide bandwidth. The bow-tie antenna with and without stubs were fabricated and measured, where measured results correlated well with simulated results.

In Chapter 4, CMA was performed on a capacitively fed crossed bow-tie antenna. The analysis of the structure was complex with many characteristic modes. Through simplification, the relevant modes were reduced from sixteen to three. The three excited modes exhibited a phenomenon called eigenvalue crossing avoidance causing the parameters of the characteristic modes to mix with each other. A decoupling procedure created by Schab et al. [16] was performed on the coupling characteristic modes to obtain decoupled auxiliary modes with auxiliary eigenvalues and auxiliary eigencurrents. The procedure was approximated to allow it to be used on modal admittances. The three relevant auxiliary modes that make up the response of the antenna were termed the ground plane mode, the dipole mode, and the bow-tie mode. The ground plane mode's eigencurrent was the same as the lowest frequency mode

of a circular sheet and had an impedance that was purely resistive with little reactance. The ground plane mode was modally resonant for the entire bandwidth of the antenna. The dipole mode and the bow-tie mode were synchronously coupled which shifted the dipole mode's modal resonance down in frequency and the bow-tie mode's modal resonance up in frequency creating a dual resonance. Due to the synchronous coupling, the eigencurrent on the dipole and bow-tie elements were in-phase in the lower frequency range and anti-phase in the higher frequency range. The eigencurrent on each element had a half wavelength distribution. The modal admittance of the dipole mode was reactive with capacitance and inductance while the admittance of the bow-tie mode was just capacitive. The dual resonance and reactive admittance of the dipole and bow-tie modes in parallel with the resistive ground plane mode shows why this antenna structure is very wideband.

5.2 Contributions

The contributions from this thesis can be broken down into two parts.

1. Decomposition of a complex antenna structure into a set of modes with fundamental responses. CMA was demonstrated as a useful tool to understand the underlying mechanisms of an antenna. Three modes made up the response of a capacitively fed crossed bow-tie structure modeled after a high precision wideband GNSS antenna. CMT was shown to be relevant to antenna analysis through CMA. Coupling effects due to eigenvalue crossing avoidances were removed from the eigenvalues and eigencurrents. The decoupling method was expanded with approximation to include modal admittance. Synchronous coupling described the interaction between two of the excited modes. Their modal resonances were shifted in frequency by the coupling coefficient and their eigencurrents exhibited an in-phase/anti-phase relationship.
2. Modification of a simple bow-tie antenna through the use of CMA. A mode's eigencurrent nulls and maximums were demonstrated as a useful tool to manipulate the resonance of the mode. Stubs placed at an eigencurrent maximum were ignored by the respective mode. Stubs placed at an eigencurrent null affected the eigenvalues of the respective mode significantly.

5.3 Future Work

In this section, a few ideas to continue the work done in chapters 3 and 4 are stated. Chapter 3 was used more to familiarize one's self with characteristic mode analysis so

the results obtained were not optimized. The bow-tie with stubs antenna was wideband due to the dual resonance of the two modes, however, in Fig. 3.6 the reflection coefficient between the two resonances is around 0.6 which is large. Parameters such as the width of the stub, its placement along the x-axis and stub length could all be adjusted further to obtain a reflection coefficient that is below 0.33 for the entirety of the bandwidth. In Fig. 3.9, the far field pattern was shown and it was seen that the pattern for the first order resonance was different than the pattern for the third order resonance. A desirable trait for the far field pattern in a wideband antenna to stay constant over frequency, therefore, this could be looked into further. A possible solution is to angle the stub so it is parallel with the flare of the bow-tie. Then the eigencurrent on the stub for the third order mode would oppose the eigencurrent flowing from the wide end of the bow-tie to the center. Also, the current null on the stub would be moved closer to the current null at the wide end of the bow-tie. These changes might transform the far field pattern of the third order mode to be similar to the first order mode. Lastly, other structures could be analyzed and modified using the eigencurrent maximums and minimums similar to how the bow-tie antenna was modified.

In chapter 4, a capacitively fed crossed bow-tie antenna was extensively analyzed using CMA, however, not every parameter that is important to antennas was discussed. One of those parameters being the far field pattern. Due to the eigenvalue crossing avoidances that occur between the excited modes, the far field pattern would have a similar problem to the modal admittance where the decoupling procedure developed by Schab et al. [16] does not include them. Therefore, expanding the procedure to properly include modal admittance and far field pattern would be very useful. In Fig. 4.15(b), the removal of the port 2 dipole and associated triangular elements increased the coupling coefficient, K . Finding the mechanisms that alter K would provide specific insight into how the two modes, $CM_{1,9}^b$ and $CM_{1,11}^b$ are coupling together. This was done by Borchardt and LaPointe for a U-slot in [14] where the coupling coefficient was shown to be directly related to the slot width divided by the patch width. Another avenue would be to exchange out the dipole or bow-tie like resonators with a different resonator.

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