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A COMPARISON OF THE
LINEAR EXPENDITURE SYSTEM
AND THE ROTTERDAM MODEL
USING CANADIAN CONSUMER
EXPENDITURE DATA

by

• DAVID B. CALDWELL

A thesis submitted to Carleton University in
partial fulfillment of the requirements for the
degree of Master of Arts in Economics

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SEPTEMBER 1979

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LINEAR EXPENDITURE SYSTEM
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DAVID B. CALDWELL

1979

The undersigned recommend to the Faculty of Graduate Studies acceptance of the thesis "A Comparison of the Linear Expenditure System and the Rotterdam Model using Canadian Consumer Expenditure Data" submitted by David B. Caldwell, B.A. Honours, in partial fulfilment of the requirements for the degree of Master of Arts.

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ABSTRACT

This paper compares the Linear Expenditure System and the Rotterdam Model.

It is argued that an adequate comparison of the additive version of the Rotterdam and the LES has not been made in the literature. Such a comparison is useful because both of these models are often applied empirically and it is important to know their relative strengths and weaknesses.

Both models are theoretically consistent with the demand theory restrictions of homogeneity, adding-up, symmetry and negativity. In addition, the LES and the additive version of the Rotterdam incorporate the restriction of group-wise additivity or strong separability.

The difference between the functional forms of the two models is due to different assumptions about the income flexibility of the marginal utility of money -- the LES allows it to vary with income and prices, while the additive Rotterdam assumes it constant.

The results of estimating the two models on Canadian consumer expenditure data indicate the following: There is apparently little to be gained from allowing the income flexibility of money to vary over time; the additivity restriction may be overly strong as the additive Rotterdam and the LES appear to fit the data less well than the Rotterdam without additivity; and, after allowing for loss of degrees of freedom, the additive Rotterdam is about the same as the LES on fit to the data.

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CHAPTER ONE

INTRODUCTION

Empirical analysis of demand using time-series data has a long history. Schultz published the pioneering work in 1938 and his methodology of single equation estimation is still in use today.¹

A significant advance took place in 1954 with Stone's publication of an empirical estimation of the Linear Expenditure System (LES).² This system is based on a particular utility function and hence the demand equations derived from it satisfy the demand theory restrictions of adding-up, homogeneity and symmetry.³ This was the first time that the theory of demand was fully utilized in empirical work.

A further major development occurred in 1965 when Theil published an article outlining a model which is now referred

¹ H. Schultz, The Theory and Measurement of Demand, (Chicago University Press, 1938).

² J. R. N. Stone, "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand", Economic Journal, 64, (1954): 511-527.

³ The LES is consistent with negativity and it also incorporates the assumption of additivity. See Sections 2.3 and 3.2 below.

to as the Rotterdam Demand System.⁴ The Rotterdam System was developed by applying demand theory restrictions to a set of equations in income and prices. As the restrictions are specified as linear constraints on the parameters, this model has led to considerable testing of the restrictions against the data.⁵ These tests indicate that the demand theory restrictions are rejected often enough to cause concern.⁶

With increasing use of the Rotterdam Model and the Linear Expenditure System, interest began to focus on their relative strengths and weaknesses. In 1969 Parks published the first theoretical and empirical comparison of

⁴ H. Theil, "The Information Approach to Demand Analysis", Econometrica, 33, (1965): 67-87.

⁵ A. Brown and A. Teaton, "Surveys in Applied Economics: Models of Consumer Behaviour", The Economic Journal, 82, (1972), p.1187, have noted that specifying any demand function involves restrictions on the data; hence tests of demand theory restrictions using a particular model, such as the Rotterdam, must be viewed as tests of the additional restrictions of demand theory.

⁶ To quote A.P. Barten, "The Systems of Consumer Demand Functions Approach: A Review", in Frontiers of Quantitative Economics, ed. M.D. Intriligator, Vol. IIIA, (North Holland-Amsterdam, 1977), p.51, "...one might summarize the findings as follows: homogeneity passes more easily for small systems than for large ones, where it is usually firmly rejected. Symmetry passes more easily than homogeneity, but it also meets with various rejections. Negativity, given symmetry and homogeneity, does not seem to be too limiting whenever its empirical validity is checked."

the two models.⁷ His results indicated that the Rotterdam Model dominated the LES in terms of fit; however he noted a number of unsatisfactory aspects of his comparison. First, the Swedish data he used was based on an industry classification rather than the normal consumer expenditure classification; and, second, the Rotterdam Model was constrained to adding-up, homogeneity and symmetry, but not additivity. Thus the additivity restriction incorporated in the LES could have caused its poorer performance.⁸

A second major comparison of these two models occurred in Deaton's 1974 article.⁹ He found that, using British data, the Rotterdam model, with various restrictions

⁷ R.W. Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms", Econometrica, 37, (1969): 629-650. Parks also included the Indirect Addilog Model in his comparison.

⁸ Additivity would not have caused the LES to perform less well if the data set used had been consistent with this restriction, but this was probably not the case, given the industry classification used. It should also be noted that A. Deaton, "The Analysis of Consumer Demand in the United Kingdom, 1900-1970", Econometrica, 42, (1974), p.342, criticized Parks' use of the "information inaccuracy" criteria to distinguish the fit of the two models, as the LES dependent variable was expenditure in level terms, while the Rotterdam predicts budget shares. This criteria should only be used when the dependent variables of the two models used are of a similar form. See H. Theil, Theory and Measurement of Consumer Demand, Vol.1; (North Holland-Amsterdam, 1975), p.244, for further discussion.

⁹ Deaton, "The Analysis of Consumer Demand in the United Kingdom". Deaton also included the Direct Addilog Model in his comparison.

other than additivity imposed, dominated the LES; however the LES was a little better than the additive version of the Rotterdam. As Deaton aggregated his data into groups which appear to satisfy the assumption of groupwise additivity fairly well, one cannot criticize his work on this account.¹⁰ As well, he transformed the LES so that it predicted budget shares, as does the Rotterdam, so that a valid comparison of their fit to the data could be made.

The problem with his comparison relates to his estimation of the LES. As he recognizes, his estimates of the subsistence parameters of the LES are not at all usual. Only two of nine parameters are greater than their standard error, and only one of these is more than twice its standard error.¹¹ Commenting on this, Deaton notes that his reformulation of the model "stretched [it] beyond its limits".¹²

¹⁰ Yet there exists some evidence of interactions between Deaton's commodity groups. Theil, Theory and Measurement of Consumer Demand, p.199, has shown that off-diagonal elements of the price matrix of the Rotterdam (not restricted to additivity) must be positive if a data set satisfies the additivity assumption. From Table III of Deaton's results there are three cases where coefficients are negative and more than twice their standard error.

¹¹ This refers to Deaton's estimates of the LES with intercepts.

¹² Deaton, "The Analysis of Consumer Demand in the United Kingdom", p.365.

Because of this estimation problem it is not clear whether the additive Rotterdam dominates the LES or vice-versa. It is also not clear whether additivity is more acceptable within the LES framework than within the Rotterdam framework.¹³

It is worth noting one more contribution to the literature. In 1975 Theil published a comparison of the LES and the Rotterdam Model using both Dutch and British data.¹⁴ Unfortunately neither the British nor the Dutch data groupings used to estimate the models appears to be a priori consistent with the assumption of groupwise additivity.¹⁵

Theil estimated the Rotterdam without groupwise additivity and, by examining the off-diagonal price coefficients, concluded that there was an interaction between the Durables and Remainder groups. Because of this he didn't fully impose the additivity restriction on the Rotterdam, but allowed one

¹³ Deaton rejected groupwise additivity for the Rotterdam based on its fit to the data.

¹⁴ Theil, Theory and Measurement of Consumer Demand. Theil included the Indirect Addilog Model in his comparison.

¹⁵ For example, in the Dutch data, pastry, chocolate, ice cream, etc., are not aggregated with the food group though they are likely to be specific substitutes or complements; while for the British data, fuel, light and water charges might well be complements of household goods but they are in different groups.

interaction (between Durables and Remainder). Thus in a comparison with the LES the Rotterdam had an advantage just as it did in Parks' study, though not to the same extent. Theil found that on the Dutch data the Rotterdam, with and without additivity, dominated the LES, but that on British data the LES held a slight edge over the additive Rotterdam though it did worse than the Rotterdam without this restriction.¹⁶

From the foregoing discussion one must conclude that a clear empirical comparison of the additive version of the Rotterdam Model and the Linear Expenditure System has not, as yet, taken place. It is the primary objective of this study to provide such a comparison. This is of interest from a theoretical point of view because, as Deaton has shown, the key difference between the functional forms of the additive Rotterdam and the LES is due to a different assumption about the stability of the income flexibility of the marginal utility of money,¹⁷ and this parameter has been claimed to be an indicator of the standard of living.¹⁸ Secondly, more work is needed on this type of model to ascertain whether the additivity restric-

¹⁶ Theil used the "information inaccuracy" to compare the fit of the models but he transformed the LES so that a valid comparison could be made.

¹⁷ Deaton, "The Analysis of Consumer Demand in the United Kingdom", p.346.

¹⁸ See Section 2.2, below, for a further discussion of the income flexibility of the marginal utility of money.

tion is useful in an empirical context. From a purely practical point of view, if it is desired to estimate a system of demand equations it is important to have a sufficient amount of evidence available about the relative ability of these models to fit the data so that one can make a rational choice about which functional form to choose. The LES and the additive version of the Rotterdam Model are convenient candidates as the number of price responses which have to be estimated are relatively few.

The rest of this study is devoted to a comparison of the Rotterdam Model and the LES. Chapter Two briefly reviews the theory of demand to provide a point of reference for the theoretical comparison of the Rotterdam and LES which is undertaken in Chapter Three. In Chapter Four the data and its aggregation are discussed and the method of estimation is outlined. Chapter Five presents the results of the estimation and a discussion of the models' relative performance. Chapter Six provides a brief overall summary.

CHAPTER TWO

DEMAND THEORY

The objective of this chapter is to give a very brief review of the basic results of demand theory to provide background in terms of concepts and notation for subsequent chapters. The results stated in this chapter will be of particular use when discussing the origins of the Rotterdam Model and in the comparison of substitution matrices of the Linear Expenditure System and the Rotterdam Model.¹

This chapter is organized as follows: Section 2.1 states the consumer problem of maximizing a utility function subject to a budget constraint; Section 2.2 discusses the four basic constraints on the demand system; Section 2.3 outlines the additional constraints that can be obtained from a specific type of separability of the utility function; and finally, Section 2.4 discusses some of the limitations of static demand theory.

¹ It should be emphasized here that it is not intended to give a complete review of demand theory -- reference should be made to Barten, "The Systems of Consumer Demand Functions Approach: A. Review"; Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour"; and Philips, Applied Consumption Analysis, (North Holland-Amsterdam, 1974), for a more comprehensive treatment.

2.1 STATEMENT OF THE CONSUMER PROBLEM

A single consuming unit is considered. An $n \times 1$ vector q of commodities and services is available to be purchased at the corresponding vector of prices p . The system of demand functions of the consuming unit is written,

$$(2.1) \quad q = q(m, p)$$

where m is total expenditure. The budget constraint is,

$$(2.2) \quad m = p'q.$$

Utility u is derived from these goods and services according to the utility function,

$$(2.3) \quad u = u(q).$$

The demand system (2.1) is the result of maximizing the utility function (2.3) subject to the budget constraint (2.2). The conditions that must be satisfied by the utility function to ensure consistent maximization are that it be strictly quasi-concave and monotonically increasing.²

In differential form the system can be written,

$$(2.4) \quad dq = q_m dm + q_p dp$$

² A utility function is strictly quasiconcave if points defined by a linear combination of two bundles of goods are preferred to either of these, and the consumer does not prefer one bundle of goods to the other.

where q_m is the vector of partial derivatives with respect to m and Q_p is the $n \times n$ matrix of partial derivatives with respect to p .

Similarly the budget constraint in differential form is,

$$(2.5) \quad dm = p' dq + q' dp$$

2.2 CONSTRAINTS ON THE DEMAND SYSTEM

ADDING-UP

Substituting (2.4) into (2.5) and allowing prices to change ($dp = 1$) but not total expenditure ($dm = 0$) gives,

$$(2.6) \quad p' Q_p + q' = 0$$

Alternatively, allowing total expenditure to change ($dm = 1$) and not prices ($dp = 0$) results in the condition,

$$(2.7) \quad p' q_m = 1$$

These two conditions, known as adding-up or aggregation conditions mean that changes in quantities bought, brought about by either price or total expenditure changes, must continue to exhaust the available total expenditure (i.e. satisfy the budget constraint).

HOMOGENEITY

Homogeneity is usually assumed when estimating demand equations as it means that a given increase in all prices and in nominal total expenditure will not affect the quantities of the goods demanded. Thus the demand equations are homogeneous of degree zero,

$$(2.8) \quad q = q(\alpha_m, \alpha_p)$$

for a positive α .

Euler's theorem allows this condition to be stated in terms of derivatives as,

$$(2.9) \quad q_m m + q_p p = 0.$$

Homogeneity has often been found not to be supported by the evidence when demand systems are estimated.³ This may, however, be a short-run phenomenon, or alternatively present only in times of strong uncertainty about price movements. In long-run equilibrium situations one would expect the condition to be fulfilled.

CONSTRAINTS FROM UTILITY THEORY

Maximizing (2.3) with respect to vectors q that satisfy the budget constraint yields,

³ Barten, "The Systems of Consumer Demand Functions Approach: A Review", p.51

$$(2.10) \quad u_q = \lambda p$$

where u_q is an $n \times 1$ vector of first order partial derivatives of the utility function with respect to q , and λ is a Lagrangean multiplier representing the marginal utility of income, i.e., $\lambda = \partial u / \partial m$.

To derive constraints on the derivatives of the demand functions we differentiate both sides of (2.10) with respect to m and p and get,

$$(2.11) \quad u \cdot q_m = \lambda_m \cdot p$$

$$(2.12) \quad u \cdot q_p = p(\lambda_p)' + \lambda \cdot I$$

where u is the Hessian matrix of the utility function; $\lambda_m = \partial \lambda / \partial m$ and $\lambda_p = \partial \lambda / \partial p$.

Combining the adding-up conditions (2.6) and (2.7) with (2.11) and (2.12) yields what Barten has called the Fundamental Matrix Equation of demand theory,

$$(2.13) \quad \begin{bmatrix} u & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} q_m & q_p \\ -\lambda_m & -\lambda_p' \end{bmatrix} = \begin{bmatrix} 0 & \lambda I \\ I & -q' \end{bmatrix}$$

Assuming u non-singular this equation's solution is,

$$(2.14) \quad q_m = (p' u^{-1} p)^{-1} u^{-1} p$$

$$(2.15) \quad q_p = (\lambda u^{-1} - q_m q_m' \phi m) - q_m q'$$

$$(2.16) \quad \lambda_p = -\lambda(q_m + (\phi m)^{-1} q)$$

$$(2.17) \quad \lambda_m = \lambda(\phi m)^{-1}$$

where

$$(2.18) \quad \phi = \frac{\lambda}{m}(p' u^{-1} p)$$

The inverse of the quantity ϕ was first introduced by Frisch who called it the "income flexibility of the marginal utility of money".⁴ From equation (2.17) we have,

$$(2.19) \quad \frac{1}{\phi} = \frac{\partial \lambda}{\partial m} \cdot \frac{m}{\lambda},$$

and this clearly shows why Frisch chose the name for this quantity that he did. In a static situation one would expect that increases in income would lead to a decline in the marginal utility of money; hence $\lambda_m < 0$; therefore ϕ is a negative quantity. Frisch suggested that a normal middle income population would have $\phi = -.5$ and that ϕ would fall within a range of $-.1$ to -10 . As all terms in (2.19) depend on income, ϕ may vary in value over time.

SYMMETRY

Equation (2.15) is the Slutsky decomposition of price responses into substitution and income effects. The substitution matrix K is obtained by compensating price changes for income loss so that utility does not change, i.e.,

⁴ See for example R. Frisch, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors", Econometrica, 27, (1959): pp.177-196.

$$(2.20) \quad Q_p + q_m q'_m = K = \lambda u^{-1} - q_m q'_m \phi_m.$$

K is symmetric if the Hessian is symmetric; and on this assumption we have the symmetry condition,

$$(2.21) \quad K = K'.$$

As demand equations sometimes represent the price coefficients as compensated substitution effects (eg., equations in real income and prices), it is useful to restate the adding-up condition (2.6) for variation in prices (income held constant) in terms of K as,

$$(2.22) \quad p'K = 0.$$

For the same reason it is useful to restate the homogeneity condition (2.9) as,

$$(2.23) \quad K_R = 0.$$

The term λu^{-1} in (2.20) has been referred to as the specific substitution effect as it arises from the direct interaction of goods as captured in the Hessian matrix of the utility function. The second term making up the K matrix is referred to as the general substitution effect as it arises due to competition between goods for the given level of total expenditure.

The specific substitution matrix is symmetric if the Hessian is symmetric, while the general substitution effect is symmetric by definition. Thus symmetry relates directly to the structure of the Hessian; it ensures that the modification in the change in utility due to an additional unit of one good, given an additional unit of another good, is the same regardless of which good is referred to first. For example, the change in utility due to an additional unit of bread, given an additional unit of butter, is identical to the change in utility due to an additional unit of butter given an additional unit of bread. If the modification in the change in utility is positive, the goods are complements; if it is negative they are substitutes.

NEGATIVITY

In order for demand equations to represent a maximum of the utility function, rather than a minimum, the principal diagonal elements of U must be negative. From equation (2.20) it is clear that the main diagonal elements of the first term of K are negative. In the absence of inferior goods ($q_m > 0$) the main diagonal elements of the second term are also negative due to the sign of ϕ . Hence the value of the second must be smaller than that of the first on the principal diagonal of K for the demand curves to be downward sloping. This is normally the case. However the Veblen effect (ostentatious consumption) allows principal diagonal elements in the Hessian to be positive.

The negativity condition in terms of K is written,

$$(2.24) \quad x'Kx < 0, \quad x \neq \alpha p$$

for some scalar α .

INTEGRABILITY

This is a condition on the demand functions in first difference form which ensures that a corresponding utility indicator can be constructed. By Young's theorem we must have,

$$(2.25) \quad \frac{\partial q_m}{\partial p} = \frac{\partial q_p}{\partial m}$$

This condition is satisfied by any demand system which is developed from a utility function, but not necessarily by systems derived by applying demand theory restrictions to some functional form.⁵

2.3 SEPARABILITY OF THE UTILITY FUNCTION-

Separability of the utility function is an assumption which is often made in empirical work as it is highly plausible and provides a justification for estimating demand systems using groups of commodities.⁶

⁵ See Section 3.1, below.

⁶ See Phelps, Applied Consumption Analysis, Chapter 3, for further discussion.

The consumer is depicted as following a two-stage maximization procedure. First, the total budget is allocated across broad commodity groups and, second, the expenditure for each group is allocated to elementary commodities within the group.

"Weak separability" occurs if the general utility function defined on elementary commodities q_i ,

$$(2.26) \quad u = \delta(q_1, q_2, \dots, q_n)$$

can be grouped,

$$(2.27) \quad u = \delta(q_{11}, \dots, q_{jm}; \dots; q_{n1}, \dots, q_{nv})$$

and rewritten as,

$$(2.28) \quad u = F(\delta_1(q_1), \delta_2(q_2), \dots, \delta_n(q_n))$$

where each q_j , ($j = 1, \dots, n$), is a vector of elementary commodities. Each $\delta_j(q_j)$ is called a branch utility function. The condition which must hold for (2.28) to be a valid representation of (2.26) is that the marginal rate of substitution between any two elementary commodities in one group must be independent of any commodity in another group. That is, for example,

$$(2.29) \quad \frac{\partial(u_{q_{1m}}/u_{q_{1j}})}{\partial q_{2k}} = 0$$

where $u_{q_{ij}}$ is the partial derivative of u with respect to q_{ij} .

"Strong separability" occurs if (2.28) can be rewritten as,

$$(2.30) \quad u = \delta_1(q_1) + \delta_2(q_2) + \dots + \delta_n(q_n) .$$

This form of the utility function has been called groupwise independent or block additive. It reduces to an additive utility function if there is only one elementary commodity in each group.

Empirical work using the Linear Expenditure System and the additive version of the Rotterdam Model is carried out on the assumption of strong separability and proceeds using grouped data. Therefore, for this assumption to be valid, it is important for elementary commodities making up each group not to be specific substitutes or complements of elementary commodities in other groups. If groups of goods are chosen so that there are no between-group interactions, then the off-diagonal elements of a Hessian defined on these groups are zero. From (2.20), this results in the off-diagonal structure of the K matrix being independent of specific interaction effects.

This assumption also provides a justification for using total expenditure in place of income as an independent variable in demand equations. First, the consumer is assumed to split his income by deciding what to save and what to spend;

the second stage of the problem is represented by the demand system approach, i.e., how to allocate total expenditure to the various goods and services available.

The assumption of block independence will be used in the models described in the following chapters. However in the estimation phase particular attention will be paid to the commodity composition of the groups to be estimated.

2.4, LIMITATIONS OF THE STATIC THEORY OF DEMAND

The static theory of demand described in previous sections has a number of limitations when used to explain actually observed behaviour. These can be conveniently broken down into three categories: aggregation, identification, and dynamics.

AGGREGATION

The theory of demand discussed above is in terms of a single consumer. However, available data on market behaviour is the result of actions of many consumers. For the theory to be directly applicable to available data a number of

strong restrictions must hold.⁷

Conceptually, the simplest way to avoid this problem is to assume that both the theory and data are applicable to a "representative" consumer. More sophisticated approaches have been discussed in the literature, such as conditions on the covariance matrix of residuals; however they will not be detailed here.

There is also the problem of aggregating elementary commodities into the categories which are actually used for estimation. This issue can be addressed through separability of the utility function as already discussed in Section 2.3.

IDENTIFICATION

Demand systems are estimated on the assumption that they are properly identified. This requires that an infinite supply is forthcoming at the given prices. As imports are one source of supply of consumer goods, this assumption may not be unreasonable. However, in practice one would expect this assumption to hold in varying degrees depending upon the commodities involved.

⁷ See Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour", pp.1167-1171, for a detailed discussion.

DYNAMICS

The theory of demand set out in previous sections is static; that is, it does not allow for past events to affect current behaviour. For instance, expectations of normal or "permanent" income may be a more appropriate determinant of the pattern of purchases than current income realized.

This idea is most highly developed in the theory of the consumption function. However, Houthakker and Taylor have argued that as current consumption is closely related to permanent income it is an appropriate independent variable to use in demand equations.⁸ As well, physical stocks of goods held or stocks of psychological habits may be important determinants of the pattern of current purchases.

Dynamics are most often ignored in the demand system approach; however some authors have attempted to include some dynamic effects. For example, Stone has argued that the parameters of the Linear Expenditure System may be subject to trends over time,⁹ while Pollack and Wales have suggested that

⁸ H.S. Houthakker and L.D. Taylor, *Consumer Demand in the United States, 1929-1970*, 2nd ed., (Harvard University Press, 1970).

⁹ See Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms", p.632.

committed consumption in the LES is related to past levels of consumption.¹⁰

¹⁰ R.A. Pollack and T.J. Wales, "Estimation of the Linear Expenditure System", Econometrica, 37, (1969): 611-628.

CHAPTER THREE

A THEORETICAL COMPARISON OF THE LINEAR EXPENDITURE SYSTEM AND THE ROTTERDAM MODEL

The objective of this chapter is to outline and compare the Linear Expenditure System and the Rotterdam Model. This comparison of the models' theoretical structure lays the basis for the empirical comparison undertaken in Chapter Four.

Both of these models are consistent with the theory of demand, yet their functional forms differ.¹ These differences reflect divergent assumptions embodied in the models and, in particular, the assumption about the stability of the income flexibility of the marginal utility of money.

This chapter is organized into three sections: the first outlines the theoretical development of the Rotterdam Model, the second discusses the Linear Expenditure System, and the third provides a comparison between the two.

¹ The Rotterdam Model provides one exception: as discussed in Section 3.1, it does not satisfy the condition of integrability.

3.1 DEVELOPMENT OF THE ROTTERDAM MODEL

Theil introduced the Rotterdam Model in an article showing the link between information theory and demand analysis.² Essentially it was an improvement of a model Barten had used in discussing 'almost additive' preferences.³

Following Brown and Deaton, the Rotterdam Model can be developed directly from demand theory.⁴ Substituting the definition of Q_p (in terms of K as given in (2.20)) into equation (2.4) yields,

$$(3.1) \quad dq = q_m [dm - q dp] + K dp.$$

Selecting the i^{th} equation and using the definition of a change in a variable in terms of logs ($dx = x d \log x$ for any arbitrary variable x) gives,

$$(3.2) \quad q_i d(\log q_i) = (q_i)_m [m d(\log m) - \sum_j^n p_j q_j d(\log p_j)] + p_1 k_{i1} d(\log p_1) + \dots + p_j k_{ij} d(\log p_j) + \dots + p_n k_{in} d(\log p_n).$$

Then multiplying through by $\frac{p_i}{m}$ gives,

$$(3.3) \quad \frac{p_i q_i}{m} d(\log q_i) = \{p_i (q_i)_m\} [d(\log m) - \sum_j^n \left(\frac{p_j q_j}{m}\right) d(\log p_j)] + \sum_j^n \frac{p_i k_{ij} p_j}{m} d(\log p_j).$$

² Theil, "The Information Approach to Demand Analysis".

³ A.P. Barten, "Consumer Demand Functions under Conditions of Almost Additive Preferences", *Econometrica*, 32, (1964): 1-38.

⁴ Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour", pp. 1189-1190.

Hence, each equation is of the form,

$$(3.4) \quad w_i d(\log q_i) = p_i(q_i)_m [d(\log m) - \sum_j w_j d(\log p_j)] + \sum_j \frac{p_i k_{ij} p_j}{m} d(\log p_j)$$

where $w_i = \frac{p_i q_i}{m}$. This simplifies to,

$$(3.5) \quad w_i d(\log q_i) = \beta_i d(\log Q) + \sum_j \pi_{ij} d(\log p_j)$$

where $\beta_i = p_i(q_i)_m$, $\pi_{ij} = \frac{p_i k_{ij} p_j}{m}$ and $d(\log Q) = [d(\log m) - \sum_j w_j d(\log p_j)]$. This is a typical equation of the Rotterdam Model.

For infinitesimal time periods the Rotterdam Model predicts the log-change in real expenditure on the i^{th} good weighted by its value share in total expenditure. Such a dependent variable is unusual but Theil points out that demand theory relates to the allocation of expenditure; therefore the change in value share is an interesting variable to study.

The identity,

$$(3.6) \quad d w_i = w_i d(\log q_i) + w_i d(\log p_i) - w_i d(\log m)$$

shows that the dependent variable in the Rotterdam Model -- the first term on the right -- is the only component of the change in share which is behaviourally determined. The dependent variable is explained by the log-change in total constant dollar expenditure and the log-change in all prices. The log-change in total constant dollar expenditure, $d(\log Q)$, is measured by $\sum_j w_j d(\log q_j)$ to ensure that it equals the sum of the dependent

variables. As explained by Theil, $\sum_j w_j d(\log q_j)$ is equal to $d(\log m) - \sum_j w_j d(\log p_j)$ except for a third order remainder term.⁵ The parameters of the model are β_i , the marginal propensity to spend on the i^{th} commodity, and the price coefficients π_{ij} . As the coefficients π_{ij} depend on prices and total expenditure they need not be constant, however it is one of the assumptions of the model that they will be sufficiently stable to be represented by constant parameters.

The Rotterdam Model is usually estimated in the form of equation (3.5). A number of authors have, however, added a constant term to the equations to capture changes in tastes over time.⁶ The constant terms serve an analogous rôle to a time trend in models which are not estimated in first difference form.

A flaw in the Rotterdam Model was noticed even prior to the publication of Theil's 1965 article, as McFadden in 1964 showed that the underlying preference system is of the Cobb-Douglas type and therefore implies constant budget shares over time.⁷ This condition is, of course, contradicted by

⁵ See Theil, Theory and Measurement of Consumer Demand, Vol.1, pp.22-23, for a discussion of the dependent variable; and pp.40-41 for a discussion of the total expenditure variable.

⁶ Examples are: Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms"; Deaton, "The Analysis of Consumer Demand in the United Kingdom".

⁷ See Theil, Theory and Measurement of Consumer Demand, Vol.1, pp.102-107.

casual empiricism. Subsequently, Barten defended the system by arguing that it should be viewed as the first terms in a Taylor expansion of arbitrary demand functions and that, provided there is not much variation in real income and prices, the model will perform satisfactorily.⁸ As Brown and Deaton have noted, however, variation in income and prices is needed to accurately derive income and price elasticities.⁹

Many applications of the model have taken place and it often fits the data better than other demand models -- including the LES -- which satisfy demand restrictions.¹⁰ Probably because of this and because it offers a very convenient way of testing the restrictions of demand theory the model is still popular.

ADDITIVITY IN THE ROTTERDAM MODEL

The assumption of additivity is not a necessary part of the Rotterdam Model, as it is in the LES. So, to make

⁸ A.P. Barten, "Maximum Likelihood Estimation of a Complete System of Demand Equations", European Economic Review, 1, (1969): 7-73.

⁹ Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour", p.1191.

¹⁰ For a recent discussion of comparative studies see Barten, "The Systems of Consumer Demand Functions Approach: A Review", pp.50-51.

a direct comparison between the two, additivity should be imposed on the Rotterdam Model to place the two models on the same footing. To do this we start by substituting the definition of k_{ij} from equation (2.20) into the definition of the price parameter π_{ij} .¹¹ This yields,

$$(3.7) \quad \pi_{ij} = v_{ij} - \phi \beta_i \beta_j$$

where v_{ij} is defined as

$$(3.8) \quad v_{ij} = \frac{p_i \lambda u_{ij} p_j}{m}$$

and u_{ij} is an element of the inverted Hessian. Hence v_{ij} is associated with the specific substitution effect.

Now, substituting equation (2.18) into (2.14) gives,

$$(3.9) \quad \phi q_m = \frac{\lambda}{m} u^{-1} p.$$

For commodity i , after multiplication by p_i , this gives,

$$(3.10) \quad \phi \beta_i = \frac{\lambda}{m} p_i \sum_j p_j u_{ij};$$

and from equation (3.8) it is clear that,

$$(3.11) \quad \phi \beta_i = \sum_j v_{ij}.$$

Returning to the price terms of (3.5), and using (3.7), we see that,

¹¹ See Theil, *Theory and Measurement of Consumer Demand*, Vol.1, p.28 and pp.43-48, for a discussion of groupwise additivity applied to the Rotterdam Model.

$$(3.12) \quad \sum_j \frac{p_i p_j^k}{m} d(\log p_j) = \sum_j v_{ij} d(\log p_j) \\ - \phi \beta_i \sum_j \beta_j d(\log p_j).$$

Additive preferences are easily imposed by noticing all u_{ij} , $i \neq j$, must be zero, so that all v_{ij} , $i \neq j$, must also be zero. And using the definition of $\phi \beta_i$, (3.12) can be written as,

$$(3.13) \quad \phi \beta_i [d(\log p_j) - \sum_j \beta_j d(\log p_j)]$$

which is used to replace the price terms in equation (3.5).

Hence the Rotterdam Model under additive preferences is written,

$$(3.14) \quad w_i d(\log q_i) = \beta_i d(\log Q) + \phi \beta_i [d(\log p_i) \\ - \sum_j \beta_j d(\log p_j)].$$

There is only the relative own price remaining in each equation and ϕ is viewed as a constant.

DEMAND THEORY RESTRICTIONS AND THE ROTTERDAM MODEL

The Rotterdam Model, with additivity imposed, can be shown to be consistent with the restrictions of demand

theory.¹² The form of (3.14) is clearly consistent with homogeneity as an equal percentage increase in all prices will cancel out due to the form of the price term in each equation and, hence, leave the quantity demanded unchanged.

Summing across equations yields,

$$(3.15) \quad \sum_i w_i d(\log q_i) = \sum_i \beta_i d(\log Q) + \phi \left[\sum_i \beta_i d(\log p_i) \right] \\ - \phi \sum_i \beta_i \left[\sum_j \beta_j d(\log p_j) \right]$$

so that if $\sum_i \beta_i = 1$ the adding-up condition is satisfied.

The additive version of the Rotterdam Model has all off-diagonal elements of the inverted Hessian equal to zero by definition. From equation (3.8) we see that on-diagonal elements of the specific substitution matrix (λu_{ii}) are,

$$(3.16) \quad \frac{v_{ii} \cdot m}{p_i^2}$$

And from equation (3.11) it is clear that,

$$(3.17) \quad \phi m = \frac{v_{ii}}{\beta_i} m ;$$

hence the general substitution effect of the Rotterdam Model is,

$$(3.18) \quad \frac{\beta_i \beta_j}{p_i p_j} \frac{v_{ii}}{\beta_i} m .$$

¹² Ibid., pp.199-201. The Rotterdam Model without additivity is also consistent with the restrictions of demand theory. However, the additive version is of most interest here as it provides a direct comparison with the LES.

Also from (3.17) the income flexibility is,

$$(3.19) \quad \phi = \frac{v_{ii}}{\beta_i}$$

and it is assumed constant in the Rotterdam Model. As the off-diagonal elements of K are equal to the general substitution effect, in equation (3.18), they are symmetric if,

$$(3.20) \quad \frac{v_{ii}}{\beta_i} = \frac{v_{jj}}{\beta_j} = \phi$$

and this is ensured by estimating ϕ as a parameter of the model.

Finally, combining the specific and general substitution effects of the Rotterdam Model we have,

$$(3.21) \quad \frac{v_{ii}}{p_i^2} = (1 + \beta_i) \phi$$

which is a typical element of the main diagonal of K and is negative if the price coefficient v_{ii} is negative. From the definition of v_{ii} in equation (3.20) we see that it should normally be negative as $\beta_i > 0$ and $\phi < 0$.

3.2 DEVELOPMENT OF THE LINEAR EXPENDITURE SYSTEM

The LES is a set of equations in prices and total expenditure which explains the level of expenditure on each commodity.

Klein and Rubin laid the basis for the system while working on constructing a constant utility index of the cost of living,¹³ and Samuelson made the model explicit by deriving the demand equations and giving the parameters an interpretation which has become standard.¹⁴ Further work on the utility function was contributed by Geary.¹⁵

The LES results from the maximization of the Klein-Rubin utility function,¹⁶

$$(3.22) \quad u = \sum_i \beta_i \log(q_i - \gamma_i),$$

subject to the budget constraint,¹⁷

$$(3.23) \quad m = \sum_i p_i q_i.$$

The utility function is additive and, hence, the marginal utility from each good is independent of other goods.

¹³ L. R. Klein and H. Rubin, "A Constant-Utility Index of the Cost of Living", The Review of Economic Studies, 15, (1947-48): 84-87.

¹⁴ P. A. Samuelson, "Some Implications of 'Linearity'", The Review of Economic Studies, 15, (1947-48): 88-90.

¹⁵ R. C. Geary, "A Note on 'A Constant-Utility Index of the Cost of Living'", The Review of Economic Studies, 18, (1949-50): 65-66.

¹⁶ See for example, Theil, Theory and Measurement of Consumer Demand, Vol. 1, pp. 4-5.

¹⁷ Alternatively the LES may be derived by applying the restrictions of adding-up, homogeneity and symmetry to a set of expenditure equations which are linear in total expenditure and all prices.

The Lagrangean function is written,

$$(3.24) \quad L = \sum_i \beta_i \log(q_i - \gamma_i) + \lambda(m - \sum_i p_i q_i)$$

And the partial derivatives (L_{q_i}) are,

$$(3.25) \quad \begin{aligned} L_{q_1} &= \beta_1(q_1 - \gamma_1)^{-1} - \lambda p_1 = 0 \\ &\vdots \\ L_{q_n} &= \beta_n(q_n - \gamma_n)^{-1} - \lambda p_n = 0 \\ L &= m - \sum_i p_i q_i = 0 \end{aligned}$$

Summing over the first n partial derivatives gives,

$$(3.26) \quad \lambda = \left(\sum_i \beta_i \right) (m - \sum_j p_j \gamma_j)^{-1}$$

Using (3.26) in the i^{th} partial derivative of (3.25) yields,

$$(3.27) \quad p_i q_i = p_i \gamma_i + \beta_i (m - \sum_j p_j \gamma_j)$$

where β_i now refers to the β_i of the utility function normalized to add to one.¹⁸

Equation (3.27) is typical of the system and the parameters to be estimated are γ_i and β_i . Samuelson called the γ_i committed expenditure which would be financed before other decisions took place,¹⁹ while total

¹⁸ If the utility function is written,
 $u = \sum_i \beta_i^* \log(q_i - \gamma_i),$
 then the demand equations are of the form (3.27) where,
 $\beta_i = \beta_i^* / \sum_i \beta_i^*.$

¹⁹ As various authors have noted, the usefulness of Samuelson's interpretation is reduced by the fact that a negative γ_i simply indicates that the good in question is price elastic.

expenditure less the sum of committed expenditure ($m' - \sum_j p_j Y_j$) is supernumerary income and is allocated by the constant marginal propensities to spend, β_i .

In 1954 Stone showed the model's consistency with demand theory, and applied it empirically to British data.²⁰ He judged the model a success and since that time it has enjoyed considerable popularity and has been estimated by many different authors on data from many different countries.

Alternative specifications of (3.27) have been used at times. For example Parks included time trends by making the assumptions,²¹

$$\beta_{it} = \beta_i^* + \beta_i^{**} \cdot t$$

(3.28)

$$Y_{it} = Y_i^* + Y_i^{**} \cdot t$$

And Pollack and Wales have attempted to add a dynamic element to (3.27) by assuming that committed expenditure is dependent on previous levels of consumption,²² i.e.,

²⁰ Stone, "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand".

²¹ Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms".

²² R.A. Pollack and T.J. Wales, "Estimation of the Linear Expenditure System".

$$(3.29) \quad \gamma_{it} = \alpha_0 + \alpha_1 q_{it-1}$$

Regardless of the exact form of (3.27) used, the LES has traditionally been estimated on highly aggregated commodity groups to avoid an obvious violation of the assumption of additive preferences embodied in the utility function. That is, the data groups must be assumed to satisfy strong separability.²³ Even so, there appears to have been little attention paid to data aggregation in estimating the LES in the past.²⁴

DEMAND THEORY RESTRICTIONS AND THE LES

The LES is capable of satisfying all the restrictions of demand theory.²⁵ Dividing equation (3.27) by p_i gives,

$$(3.30) \quad q_i = \gamma_i + \beta_i \left(\frac{m}{p_i} - \sum_j \gamma_j \frac{p_j}{p_i} \right)$$

and, hence, the homogeneity condition is satisfied since an equal percentage increase in all prices, and in m , will leave the quantity demanded unchanged.

²³ See Section 2.3 above.

²⁴ See Section 4.1 below for a further discussion of data aggregation in the context of estimating models with additivity imposed.

²⁵ See, for example, Stone, "Linear Expenditure Systems: An Application to the Pattern of British Demand".

Summing equation (3.27) over commodities, we get

$$(3.31) \quad \sum_i p_i q_i = m \sum_i \beta_i + (1 - \sum_i \beta_i) \sum_j p_j \gamma_j$$

and, as $\sum \beta_i = 1$, the adding-up condition is verified.

The inverted Hessian of the Klein-Rubin utility function has all off-diagonal elements equal to zero, and main-diagonal elements are,

$$(3.32) \quad u_{ii} = - \frac{(q_i - \gamma_i)^2}{\beta_i}$$

Using equation (3.30), this can be written,

$$(3.33) \quad \frac{\beta_i^2}{p_i^2} \frac{(m - \sum_j p_j \gamma_j)^2}{-\beta_i}$$

And using (3.26) the specific substitution matrix, λU^{-1} has main-diagonal elements of the form,

$$(3.34) \quad - \frac{\beta_i}{p_i^2} (m - \sum_j p_j \gamma_j)$$

with all off-diagonal elements equal to zero.

From the definition of λ ,

$$(3.35) \quad \lambda_m = \frac{-1}{(m - \sum_j p_j \gamma_j)^2}$$

and using equation (2.19) we see that,

$$(3.36) \quad \phi_m = \lambda / \lambda_m = -(m - \sum_j p_j \gamma_j)$$

From (3.36) it is clear that a typical element of the matrix of

general substitution effects from equation (2.20) is,

$$(3.37) \quad + \frac{\beta_i \beta_j}{p_i p_j} (m - \sum_j p_j \gamma_j) \gamma_j$$

As the off-diagonal elements of the specific substitution matrix are zero, and those of the general substitution matrix are symmetric, it is clear that the condition of symmetry is satisfied, i.e., the K matrix is symmetric.

Combining equations (3.34) and (3.37) we see that the main diagonal elements (i.e., $i = j$) of the substitution matrix are,

$$(3.38) \quad \frac{\beta_i}{p_i^2} (m - \sum_j p_j \gamma_j) (\beta_i - 1)$$

and, hence, the condition of negativity is satisfied if there are no inferior goods and supernumerary expenditure is positive. The restrictions of homogeneity and adding-up are imposed by the estimation procedure; symmetry holds by definition; but negativity can be violated.

Finally, as the assumption about the behaviour over time of the income flexibility of the marginal utility of money $(\frac{1}{\phi})$ provides one of the main contrasts between the LES and the Rotterdam model, its definition is shown below: from equation (3.36),

$$(3.39) \quad \phi = - (m - \sum_j p_j \gamma_j) / m .$$

The value of ϕ is negative when supernumerary income is positive; however, as the γ_j are estimated as constants, the absolute value of ϕ will decline over time as prices and real income rise.

3.3 A COMPARISON OF THE LINEAR EXPENDITURE SYSTEM AND THE ADDITIVE VERSION OF THE ROTTERDAM MODEL

As discussed in Section 3.2, the utility function underlying the LES is written,

$$(3.40) \quad U = \sum_i \beta_i \log(q_i - \gamma_i)$$

while application of the integrability condition to the Rotterdam Model reveals that it is based on a utility function of the form,

$$(3.41) \quad U = \sum_i \beta_i \log q_i$$

which is of the Cobb-Douglas type.

The obvious difference between the two utility

functions is the presence of the γ_i parameter in the LES.²⁶ It is therefore of interest to ascertain whether the γ_i are significantly different from zero.

The functional forms of the two models presented in Sections 3.1 and 3.2 appear quite different, even after the assumption of additive preferences is imposed on the Rotterdam Model. However, Deaton has shown that the LES can be written,²⁷

$$(3.42) \quad w_i d(\log q_i) = \beta_i d(\log Q) + \phi \beta_i [d(\log p_i) - \sum_j \beta_j d(\log p_j)].$$

which is identical to equation (3.14) for the Rotterdam Model.

This reveals that the two are in fact quite similar. The only difference relates to the assumptions about ϕ as Deaton has pointed out.²⁸ As shown in equation (3.39) the LES assumes that ϕ is negative and that its absolute value will decline over time as prices and real income rise.

²⁶ In 1969 Johansen formulated a general utility function of the form,

$$U = \sum \frac{\beta_i}{\alpha_i} \left(\frac{q_i}{\beta_i} \right)^{\alpha_i}$$

which contains the Klein-Rubin and Cobb-Douglas as special cases. This utility function also contains a number of other utility functions, which have been used in empirical applications, as special cases. See Barten, "The Systems of Consumer Demand Functions Approach: A Review", for a further discussion.

²⁷ Deaton, "The Analysis of Consumer Demand in the United Kingdom", p.346.

²⁸ Ibid.

The Rotterdam Model, on the other hand, assumes that ϕ is a constant over time, though it agrees on its sign. Both models yield empirical estimates of ϕ and these should provide an interesting comparison. It is also interesting to note that Frisch predicted that the absolute value of ϕ (though he dealt with its inverse) would decline as the level of welfare rose,²⁹ and this is in agreement with the LES, though not with the Rotterdam.³⁰

As described in Sections 3.1 and 3.2, both models are capable of satisfying the demand theory restrictions of homogeneity, adding-up, symmetry and negativity. In practice both models enforce the condition that the marginal propensities to spend add to one by the estimation procedure, and in both models it is possible for negativity to be violated. However negativity depends on the identical conditions $\beta_i > 0$ and $\phi < 0$ in each model.

Also, it should be noted that the marginal propensities to spend affect the price coefficients in both models and, as Brown and Deaton have noted, because of the

²⁹ R. Frisch, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors".

³⁰ See Theil, Theory and Measurement of Consumer Demand, Vol. 1, pp. 29-30.

large variation in income relative to prices in most data sets, income responses will tend to dominate price responses.³¹

It can be readily seen, from Sections 3.1 and 3.2, that the specific substitution effect of both models is equal to,

$$(3.43) \quad \frac{\beta_i}{p_i} \phi^m$$

and that the general substitution effect is equal to

$$(3.44) \quad \frac{\beta_i \beta_j}{p_i p_j} \phi^m.$$

Hence the only difference in the interactions captured is due to the divergent assumptions about ϕ described above.

From the foregoing it should be clear that the Linear Expenditure System and the additive Rotterdam Model are remarkably similar. The differences that do occur are due to divergent assumptions about parameters which can be estimated empirically. Thus evaluating the quality of the models as representations of consumer behaviour is largely an empirical exercise rather than a theoretical one. We now turn to this task.

³¹ Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour", p.1197.

CHAPTER FOUR

DATA AND ESTIMATION CONSIDERATIONS

The purpose of this chapter is to describe the data aggregation used to estimate the Rotterdam and LES models; to present the form of the models actually estimated; and to discuss estimation methods.

Canadian consumer expenditure data will be used for estimation and comparison of the models. This will be the first time a comparison of these models has been undertaken using Canadian data. The main issue when additive preferences are being considered is the aggregation used. Since no particular aggregation has become standard in the literature and, indeed, it appears that not much attention has been paid to data aggregation, this issue is discussed in some detail in Section 4.1.

The forms of the models presented in Chapter Three are not used for estimation. It is necessary to make some modifications to ensure that appropriate constraints are fulfilled; to ensure that the variance-covariance matrix of residuals

is not singular; and to avoid estimation problems such as autocorrelated residuals.

Both the Rotterdam and LES are systems of equations and therefore require a systems method of estimation. Also, as one is likely to have strong a priori expectations of contemporaneous correlation among the residuals of a set of demand equations, a generalized method of estimation is necessary to ensure efficiency of the estimates.. These issues are discussed in Section 4.2.

4.1 THE DATA

A breakdown of Canadian consumer expenditure into forty categories is available over the post-war period, 1947 to 1978, on an annual basis. To avoid the effects of possible disallocations associated with war-rationing of consumer goods the period used for estimation will begin in 1950.

Constant dollar consumer expenditure data is now based in 1971, however, until recently, it was based in 1961. When the base was changed, the 1971 data was extended backwards in time to 1947 and, to make the major aggregations consistent with the 1961 based data, adjustment items were included. As

it is desirable to have complete freedom in aggregating the 40 elementary commodities, the 1961 based data will be used since it is not clear how the adjustment items should be aggregated. This limits the data to the period 1950 to 1973. (A full description of the data sources used is given in Appendix A.)

As it is useful to forecast actual post-sample data with an estimated model, the estimation period will extend from 1950 to 1971. The period 1972-73 will be forecast ex-post. Estimation of the models will proceed in per capita terms to obtain elasticities based on income growth exclusively, rather than on population and income growth.

A change in definition which occurred in 1962 must be noted. In this year the government of Canada introduced a medicare program which effectively transferred much of personal expenditure on health to the government sector. This change was carried through in the presentation of consumer expenditure statistics, so that on a statistical basis consumer expenditure on health care was reduced relative to other components of expenditure as a result of institutional change rather than as a result of relative price changes. For this reason, to adequately capture interaction effects between health care and other goods as represented by the K matrix, government

expenditure on health care, after 1961, is added to private expenditure and total consumer expenditure is increased accordingly.

AGGREGATION OF THE DATA

The groups of goods to be used for estimating and comparing the additive Rotterdam and the Linear Expenditure System should satisfy, as closely as possible, the assumption of additive preferences.

Interactions between goods are captured by off-diagonal elements of the Hessian matrix, as explained in Chapter Two. Where an element is positive the relationship is complementary; where it is negative the goods are substitutes. An assessment of the forty categories of consumer expenditure in the Canadian accounts suggests that there may be a number of complementary and substitution relationships among them. Given an a priori specification of these interactions, the commodities can be aggregated into groups such that members of each group do not have specific interactions with members of other groups, but they may have interactions with other members of their own group. The result is a grouping which satisfies the assumption of additive preferences and this grouping should be suitable for comparing the two models.

As will be shown in Section 4.2, the symmetry constrained version of the Rotterdam Model can be used to empirically test the assumption of additivity for any grouping of data and this will be undertaken in Chapter Five.

AN A PRIORI SPECIFICATION OF INTERACTIONS

Table 1 shows postulated interactions between the forty categories of Canadian consumer expenditure. For example, *Food & non-alcoholic beverages* is assumed to be a specific substitute for *Expenditure on restaurants and hotels*; and *Alcoholic beverages* is assumed to have a complementary relationship with *Tobacco products*.

Using such interactions postulated a priori, the commodities in Table 1 could be aggregated into ten groups which would have internal interactions but no interactions with other groups. The resulting composition of these ten groups from such an exercise is shown in Table 2 in terms of their commodity numbers taken from Table 1.

Groups VI to X are small and to avoid dealing with a large number of groups they will be aggregated into a residual category. This does not violate the assumption of additive preferences as a set of groups showing additive preferences may

TABLE 1

A PRIORI SPECIFICATION OF INTERACTIONS
BETWEEN COMMODITY CATEGORIES

COMMODITY DESCRIPTION	SPECIFIC INTERACTIONS BY COMMODITY NUMBERS	
	Complements	Substitutes
1 Food & non-alcoholic beverages		37
2 Alcoholic beverages	3	37
3 Tobacco products		
4 Men's & boys' clothing	5, 6, 17, 34, 35, 36	
5 Women's & children's clothing	6, 17, 34, 35, 36	
6 Footwear & repair		
7 Gross imputed rent	13, 14, 15, 16	8, 9
8 Gross paid rent	13, 14, 15, 16	9
9 Other lodging		13, 14, 15, 16
10 Electricity	14	11, 12
11 Gas		12
12 Other Fuels		
13 Furniture, carpets, etc.	15	
14 Household appliances	30, 31, 32	18
15 S. D. household furnishings		
16 N. D. household supplies		
17 Laundry & dry cleaning		18
18 Domestic services		
19 Other household services		
20 Medical care	21, 22, 23	
21 Hospital care, etc.	22, 23	
22 Other medical care expenses	23	
23 Drugs & sundries		
24 New & used automobiles	25, 26, 27	28
25 Repairs & parts	26, 27	28
26 Gasoline, oil & grease	27	28
27 Other auto related serv.		28
28 Purchased transportation		40
29 Communications		
30 Recreation, sporting, etc. • equipment	32	
31 Books, newspapers, etc.	33	
32 Recreational services		
33 Education & cultural serv.		
34 Jewellery, watches & repairs		
35 Toilet articles & cosmetics	36	
36 Personal care		
37 Exp. on restaurants & hotels		
38 Financial, legal & other serv.		
39 Oper. exp. of non-profit organizations		
40 Net expenditure abroad		

NOTE: For any commodity, interactions are specified only with
**** commodities of a higher number to avoid specifying
symmetric relationships twice.

TABLE 2

TEN GROUPS SHOWING INDEPENDENT PREFERENCES

GROUP	DESCRIPTION	COMMODITY NUMBERS
I	Food	1, 2, 3, 37
II	Clothing	4, 5, 6, 17, 34, 35, 36
III	Housing	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19
IV	Medical	20, 21, 22, 23, + government exp. on hosp.
V	Transportation	24, 25, 26, 27, 28, 40
VI	Other	29
VII		30, 32
VIII		31, 33
IX		38
X		39

be aggregated in any manner and the resulting aggregation will still satisfy the assumption. One cannot, however, transfer some elements of one group to another. Thus we end up with six groups to be considered: Food, Clothing, Housing, Medical, Transportation and Other.

It is necessary to have price indexes for each of these groups to estimate the models. They are derived implicitly by aggregating current and constant (\$1961) data and dividing one by the other.

4.2 ESTIMATION CONSIDERATIONS¹

To make the demand models presented in Chapter Three useful for empirical estimation it is necessary to assume a stochastic element in each of, say, n equations which encompasses unspecified influences on demand as well as measurement errors in the data. The stochastic element in each equation is represented by a $T \times 1$ vector u with zero expectation.

$$(4.1) \quad E(u_i) = 0$$

The variance of the error term for each equation is assumed

¹ This section closely follows Brown and Deaton, "Surveys in Applied Economics: Models of Consumer Behaviour", pp.1211-1215.

to be constant; but the variance need not be constant across equations.

$$(4.2) \quad E(u_i u_i') = \sigma_{ii} I$$

A system of demand equations may be influenced by a number of factors which will have an effect on demand for all commodities considered, but which are not part of the specified system. Possibly chief among these would be expectations of prices of each commodity considered and the degree of confidence with which these expectations are held. Shifts in confidence levels or expectations may depress, or increase, consumer expenditure on various commodities and these shifts will be correlated according to the substitution effects.

Hence there is good reason to believe that the residuals in various equations will exhibit contemporaneous correlation. We will assume, however, that there is no auto-correlation in any of the equations.

$$(4.3) \quad E(u_i u_j') = \sigma_{ij} I$$

Given these assumptions the Zellner variant of Aitken estimation provides a more efficient method of estimating the demand system than OLS as long as all the independent

variables in each equation are not identical.² Where the i^{th} of n equations is,

$$(4.4) \quad y_i = X_i \beta_i + u_i,$$

the full system may be written as,

$$(4.5) \quad \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_n \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$$

or in more compact notation as,

$$(4.6) \quad y = X\beta + u.$$

The Aitken estimator of β is

$$(4.7) \quad b = (X' \Sigma^{-1} X)^{-1} X' \Sigma^{-1} y$$

where,

$$(4.8) \quad \Omega = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix}$$

and

$$(4.9) \quad \Sigma = \Omega \bullet I.$$

and $\bullet$ is the Kronecker product. The variance-covariance matrix of b is,

$$(4.10) \quad \text{var}(b) = (X' \Sigma^{-1} X)^{-1}.$$

The demand models to be estimated both have functional forms in which the parameters enter non-linearly.

² A. Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias", Journal of the American Statistical Association, 57, [1962]: 348-368.

However, the Zellner estimation procedure described above can still be used in the following manner: Consistent estimates of a subset of the parameters can be obtained from OLS applied to each equation in turn. These are used to transform the independent variables in a given model so that the remaining parameters enter linearly and can be estimated directly. These new efficient estimates of the second subset of the parameters are then used to transform the independent variables again. This procedure is applied iteratively until the parameter estimates stabilize and the likelihood function is maximized.³

SINGULARITY OF THE VARIANCE-COVARIANCE MATRIX OF RESIDUALS

The models to be estimated satisfy the adding-up conditions. As mentioned in Chapter Two, these conditions require that, summing over commodities, the income derivatives equal one and the matrix of price derivatives, K , is linearly dependent; hence the identity that the sum of expenditures equals total expenditure is verified.

For this condition to hold when actually estimating the equations it is clear that the error terms across equations must sum to zero, i.e.,

$$(4.11) \quad \sum_{i=1}^n u_{it} = 0$$

Because of this the rows in matrix (4.9) are linearly dependent. For example, using (4.11), and summing across the first row of Σ gives,

³ See, for example, Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms", pp.642-643.

$$(4.12) \quad \sum_t u_{1t}^2 + \sum_t u_{1t} u_{2t} + \dots + \sum_t u_{1t} u_{nt}$$

$$- \sum_t u_{1t} \left(\sum_i u_{it} \right) = 0.$$

Therefore the Σ matrix in (4.9) is singular and b in (4.7) cannot be obtained.

It can be noted that (4.11) implies that the observations in the system (4.6) are linearly dependent. To see this, gather the first observation ($t=1$) from each equation of (4.6), and where there are k independent variables in each equation, we obtain

$$(4.13) \quad y_{nx1}^* = X_{nxkn}^* \beta_{knx1} + u_{nx1}^*$$

Therefore, .

$$(4.14) \quad (y^* - X^* \beta) = u^*$$

and since $i'u = 0$ we also have

$$(4.15) \quad i'(y^* - X^* \beta) = 0.$$

This linear dependence of the observations indicates that there is redundant information in the system and therefore the n^{th} row in (4.14) can be dropped without reducing the information content of the system. As this is true for all $t=1, \dots, T$ observations, it suggests that the n^{th} equation can be dropped from the system. In fact this is the case as Theil has shown formally.⁴ Dropping the n^{th} equation and its associated residuals

⁴ H. Theil, Principles of Econometrics, (John Wiley & Sons, Inc., New York, 1971), Section G.7.

prevents the singularity of the matrix Σ and, hence, the estimator (4.7) can be used for the reduced system of $(n-1)$ equations.

ESTIMATION OF THE ROTTERDAM MODEL

The reason for estimating the basic version of the Rotterdam Model is that it can be used to check whether a grouping of data satisfies the assumption of additive preferences. It will be recalled from the discussion of Theil's work in Section 3.1 that additivity implies,

$$(4.16) \quad \pi_{ij} = -\phi \beta_i \beta_j, \quad i \neq j.$$

As the marginal propensities to spend are positive in the absence of inferior goods and, as ϕ is a negative quantity, all off-diagonal elements of the price-coefficient matrix should be positive. The model, as shown in equation (3.5), generates an estimate of the π_{ij} matrix and this can be used as an indicator of whether additive preferences provide a reasonable description of the data set being used.⁵

The Rotterdam Model must be modified before estimation takes place to ensure the demand theory restrictions.⁶

To impose the homogeneity condition, we substitute the relation

$$(4.17) \quad \pi_{in} = -\sum_{j=1}^{n-1} \pi_{ij} \text{ into equation (3.5) and, therefore,}$$

$$\bar{w}_{it} d(\log q_{it}) = \beta_i d(\log Q_t) + \sum_{j=1}^{n-1} \pi_{ij} x [d(\log p_{jt}) - d(\log p_{nt})].$$

⁵ Theil, "Theory and Measurement of Consumer Demand", pp.199-202, describes the use of this criterion.

⁶ Ibid., pp.185-186.

Hence a uniform increase in all prices does not affect the quantity demanded.⁷

As only five ($= n-1$) of six equations are used for estimation, the adding-up condition holds implicitly as β_6 can be obtained from the condition $\sum_{i=1}^6 \beta_i = 1$. Negativity is not imposed but can be tested for. Symmetry is imposed by the constrained estimation of the Zellner variant of Aitken estimation. The constraint is stated as $Rb=0$ where b is a 30×1 vector of the coefficients of the system. There are 10 pairings $\pi_{ij} = \pi_{ji}$ which must be ensured, each requiring a row of R ; hence, R is a 10×30 matrix. An F-test is used to determine if these restrictions are consistent with the data.

The problem of how to estimate the variances associated with the coefficients of the sixth equation remains. The solution lies in using the fact that observations associated with the n^{th} equation are linear combinations of the other $n-1$. This dependence in the context of the Rotterdam Model reduces to a linear dependence of the estimated coefficients. Knowledge of the linear dependence in the data can be used to generate the

⁷ It should also be noted that w_{it} has been replaced by $\bar{w}_{it} = \frac{w_{it-1} + w_{it}}{2}$. This is because the Rotterdam Model must be estimated with data measuring the period-to-period change in variables and, therefore, the budget share at the centre of the period of change should be used. This moving average of the budget shares also satisfies the condition $\sum_i \bar{w}_{it} = 1$.

See Ibid., pp.37-40, for further discussion.

missing elements of the full variance-covariance matrix.⁸

ESTIMATION OF THE ADDITIVE VERSION OF THE ROTTERDAM MODEL⁹

The additive version of the Rotterdam is estimated so that it can be compared to the LES; both incorporate the assumption of additive preferences as described in Chapter Three.

The additive version of the model is given in equation (3.14), and it was shown in Section 3.1 that this form of the model is capable of satisfying all demand restrictions. As far as estimation goes, the only restriction which must be imposed is $\sum_i \beta_i = 1$. Since the n^{th} equation must be dropped here, just as in the basic version of the model, this restriction can be handled in the same way, i.e., β_n is calculated from the restriction.

From (3.14), each equation to be estimated is of the form

$$(4.18) \quad \bar{w}_{it} d(\log q_{it}) = \beta_i d(\log Q_t) + \phi[\beta_i(d(\log p_{it}) - \sum_j^n \beta_j d(\log p_{jt}))].^{10}$$

⁸ Ibid., pp.198-199.

⁹ See Ibid., pp.199-207, for a detailed discussion of an estimation procedure for the additive version of the Rotterdam Model.

¹⁰ As in the basic version of the Model, $\bar{w}_{it}$ is substituted for w_{it} .

As the parameters enter in a non-linear fashion, an iterative procedure must be used for estimation. To begin, the expression within square brackets is calculated using the β_i from the symmetry constrained estimation of the basic Rotterdam (equation 4.17). This expression is then treated as an independent variable to obtain generalized least squares estimates of ϕ and each β_i associated with $d(\log Q_i)$, by the Zellner method described above.¹¹ Using these new estimates of the β_i , β_n is calculated from the adding-up condition and the expression within square brackets is re-evaluated. This procedure is carried out until the estimates of the β_i converge.

ESTIMATION OF THE LINEAR EXPENDITURE SYSTEM

The basic form of the LES is given in equation (3.27), however this form is not used for estimation. A number of econometric problems which have been found in other estimations of the model can be avoided if some modifications are made.

The LES, estimated in the form (3.27), usually exhibits strong positive autocorrelation of the residuals.¹² The Rotterdam Model often does not have this problem and the difference seems to be that the Rotterdam is stated in period

¹¹ Restricted estimation is employed to obtain a common estimate of ϕ from all equations.

¹² See Ibid., pp.236-243, for a detailed discussion of the estimation of the LES.

changes; hence first differences are taken of equation (3.27). Theil notes that estimating this version of the LES tends to produce residuals which are not homoscedastic. As the dependent variable refers to the change in expenditures, the dispersion of the residuals tends to be greater the larger are prices and income. He suggests that dividing by the average of total expenditure in each year, $m_t^* = [(m_t + m_{t-1}) \div 2]$, will correct this problem as the weight given to observations at the end of the sample period will be reduced. The resulting equation, apart from a third order remainder term, is equal to

$$(4.19) \quad x_{it} = \beta_i y_t + \gamma_i (1 - \beta_i) z_{it} - \sum_{k \neq i} \gamma_k (\beta_i z_{kt})$$

where,

$$x_{it} = \bar{w}_{it} d(\log p_{it}) + \bar{w}_{it} d(\log q_{it}) ,$$

$$y_t = d(\log Q_t) + \sum_{k=1}^n \bar{w}_{kt} d(\log p_{kt}) ,$$

and

$$z_{it} = \frac{\Delta p_{it}}{m_t^*} .$$

The parameters in (4.19) enter non-linearly just as they did for the additive version of the Rotterdam. Once, again an iterative estimation procedure must be adopted. As the form of the LES satisfies the demand restrictions no modifications need be made on this account. Only five equations are estimated to avoid the singularity problem, and the adding-up condition is used to derive the missing parameters.

To begin, Ordinary Least Squares is applied to the

form of the LES given in (4.19). This yields consistent estimates of five β_i (the coefficients of y_t) and five γ_i (derived from the coefficients of z_{it}). Equation (4.19) can now be rewritten as

$$(4.20) \quad x_{it} = \beta_i(y_t - \sum_{i=1}^5 \gamma_i^* z_{it}) + \gamma_i z_{it} + \gamma_6(-\beta_i^* z_{6t})$$

where β_i^* and γ_i^* denote the prior Ordinary Least Squares estimates.

Equation (4.20) is estimated by the Zellner variant of Generalized Least Squares to gain a second estimate of the five β_i^0 and γ_i .¹³ These new estimates are used to replace the β_i^* and γ_i^* in equation (4.20) and this procedure is continued until the parameter estimates converge.

¹³ Restricted estimation is used to obtain a common estimate of γ_6 from all equations.

CHAPTER FIVE

RESULTS OF THE ESTIMATION OF THE MODELS

The purpose of this chapter is to present and discuss the results obtained from the estimation procedures described in Chapter Four.

Results of the estimation of the Rotterdam Model are presented in Section 5.1; those for the LES are presented in Section 5.2; and a comparison of the two models is undertaken in Section 5.3.

5.1 ESTIMATION RESULTS FOR THE ROTTERDAM MODEL

BASIC VERSION OF THE ROTTERDAM

The symmetry constrained Rotterdam (equation 4.17) was estimated first in order to test the data grouping against the assumption of additive preferences. In doing this an F -statistic was also calculated to see if the null hypothesis associated with the symmetry constraints would be rejected. The value of the F -statistic is 3.64; and with degrees of freedom of 10 for the

numerator, and 75 for the denominator, the critical value of this statistic is slightly less than 2.63, which indicates that symmetry must be rejected. This is not an unusual result within the framework of the Rotterdam Model, and the violation is often traced to the weaker constraint of homogeneity.¹ Though symmetry is rejected, we follow the practice of other researchers and continue with the investigation.

Table 3 presents the symmetry constrained estimates of the Rotterdam Model. As measured by the correlation coefficient between the actual and estimated dependent variables, the fit of the Model is fairly good for the Food, Clothing, Housing and Transport groups, and mediocre for the Medical and Other groups.² As the Rotterdam Model does not appear to have been estimated on Canadian data before, there is nothing with which to directly compare these results.

Examination of the off-diagonal elements of the price matrix reveals that there are eight which have negative signs, however only two of these -- Housing in the Food equation and Other in the Medical equation -- have standard errors less than the point estimate. As there are no significantly negative

¹ See Barten, "The Systems of Consumer Demand Functions Approach: A Review", pp.51-53, for further discussion of the history of testing demand theory constraints using the Rotterdam framework.

² A. Buse, "Goodness of Fit in Generalized Least Squares Estimation", *The American Statistician*, 27 (1973)p.107, has suggested the correlation coefficient for this role in GLS estimation. However his preferred measure is a weighted (by the known variance-covariance matrix of residuals) version of the usual R^2 statistic. The weighting preserves the desirable properties of keeping the R^2 in a range of zero to one and having it monotonically related to the F-statistic.

TABLE 3

EFFICIENT ESTIMATES OF THE SYMMETRY CONSTRAINED ROTTERDAM

	π_{i1}	π_{i2}	π_{i3}	π_{i4}	π_{i5}	π_{i6}	β_i	λ
1. Food	-.1670 (.0357)	.0116 (.0150)	-.0360 (.0329)	.0201 (.0221)	.0990 (.0418)	.0723 (.0777)	.1149 (.0331)	.845 (1.79)
2. Clothing		-.0328 (.0136)	.0312 (.0250)	.0224 (.0145)	-.0040 (.0205)	-.0284 (.0309)	.0940 (.0182)	.865 (1.42)
3. Housing			-.0437 (.0755)	-.0133 (.0350)	-.0009 (.0510)	.0527 (.1137)	.2627 (.0447)	.786 (2.29)
4. Medical				.0064 (.0274)	-.0055 (.0297)	-.0301 (.0224)	.0872 (.0278)	.624 (1.28)
5. Transport					-.0868 (.0878)	-.0018 (.1190)	.2203 (.0594)	.800 (2.16)
6. Other						-.0747 (.1624)	.2209 (.0433)	.608 (1.90)

NOTE: The standard deviations are in brackets below each coefficient.
 **** The standard deviations for π_{i6} , $i = 1-6$, are estimated from the variance-covariance matrix of the coefficients of the Rotterdam Model constrained to adding-up and homogeneity, but not symmetry, due to the high cost of generating these estimates for the symmetry constrained version. The Durbin-Watson statistic is given in brackets below the correlation coefficients, λ , between the actual and estimated dependent variables.

off-diagonal coefficients, the a priori grouping of data appears to have produced aggregates which satisfy the assumption of additive preferences as far as the Rotterdam is concerned. Thus we are justified in estimating the additive version of the Rotterdam Model on this data set.

Before examining the results of the additive Rotterdam, however, the negativity condition should be checked. All on-diagonal coefficients of the price matrix are negative except for the *Medical* equation, and this point estimate is much less than its standard error. Therefore the negativity condition appears to be satisfied.

ADDITIVE VERSION OF THE ROTTERDAM

Table 4 presents the results of estimating the additive version of the Rotterdam Model. The fit is somewhat worse than for the symmetry-constrained version of the model for all groups except *Clothing*. As the *Clothing* group accounts for only about eleven percent of total expenditure, there appears to have been some loss due to the additivity restriction.³

³ Appendix A shows the budget shares of each group estimated, for selected years. Section 5.3, below, presents the Information Inaccuracy as a method of comparing each complete system estimated.

TABLE 4

EFFICIENT ESTIMATES OF THE ADDITIVE VERSION OF THE
ROTTERDAM MODEL

	ϕ	β_i	r
1. Food	-.3687 (.1139)	.1619 (.0254)	.774 (1.92)
2. Clothing	-.3687 (.1139)	.0815 (.0110)	.893 (1.60)
3. Housing	-.3687 (.1139)	.2800 (.0258)	.728 (2.04)
4. Medical	-.3687 (.1139)	.0764 (.0163)	.240 (.734)
5. Transport	-.3687 (.1139)	.2046 (.0396)	.773 (2.38)
6. Other	-.3687 (.1139)	.1956 (.0246)	.573 (1.57)

NOTE: The standard deviations are in brackets below
**** each coefficient. The Durbin-Watson statistic
is given in brackets below each correlation
coefficient between the actual and estimated
dependent variables.

The estimate of the income flexibility of the marginal utility of money ($1/\phi$) is -2.71 which is in line with other estimates of this parameter. For example, Deaton presents an estimate of -2.85 for British data,⁴ while Theil records estimates of -1.80 for Dutch data and -1.78 for British data.⁵ Frisch predicted that a value of -2 would correspond to the middle income portion of the population while -4 would indicate a somewhat poorer group.⁶

Negativity is satisfied in the additive Rotterdam if ϕ is negative and all β_i are positive; and this is the case.

5.2 ESTIMATION RESULTS FOR THE LINEAR EXPENDITURE SYSTEM

The results of estimating the LES are presented in Table 5. The model fits the data fairly well for the Food, Clothing and Other groups; not as well for the Housing and Transport groups; and very poorly for the Medical group.

As noted in Chapter Three, the difference between

⁴ Deaton, "The Analysis of Consumer Demand in the United Kingdom", p.359.

⁵ Theil, "Theory and Measurement of Consumer Demand", Vol.1, p.205.

⁶ Frisch, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors", p.18-9.

TABLE 5

EFFICIENT ESTIMATES OF THE LINEAR EXPENDITURE SYSTEM

	γ_i	β_i	r
1. Food	.1001 (.0089)	.1922 (.0202)	.914 (1.80)
2. Clothing	.0390 (.0063)	.0780 (.0093)	.898 (1.48)
3. Housing	.0727 (.0222)	.2801 (.0256)	.746 (1.84)
4. Medical	.0337 (.0099)	.0552 (.0160)	.113 (0.54)
5. Transport	.0103 (.0201)	.1882 (.0309)	.706 (2.41)
6. Other	.0357 (.0381)	.2063 (.0372)	.849 (1.43)

NOTE: (i) The standard deviations are in brackets below each coefficient. The Durbin-Watson statistic is given in brackets below each correlation coefficient between the actual and estimated dependent variables.

(ii) The γ_i are expressed in thousands of \$1961 per capita.

the Cobb-Douglas and the Klein-Rubin utility functions is the presence of the γ_i in the latter. The parameter estimates are all positive, less than the level of expenditure on the corresponding category; and for all groups except *Transport* and *Other*, these parameters are clearly different from zero, and hence the Cobb-Douglas, which the Rotterdam is based on, is invalidated.

Estimates of the income flexibility of the marginal utility of money, obtained from the LES (equation (3.39)), are shown in Table 6. They are negative throughout the period and decline in absolute value with time, which indicates that the standard of living was rising. This is as postulated by Frisch. He predicted that -2 would be characteristic of a middle income portion of the population and $-.7$ would be characteristic of the better-off portion.⁷ On this basis these estimates may be considered a little lower in absolute value than expected.⁸

The only demand theory restriction not automatically fulfilled by the LES is negativity, however the negative values for ϕ , combined with the positive values for the β_i , reported in Table 5, indicate that this condition is satisfied.

⁷ Ibid., p.189.

⁸ A.A. Powell, "Post-war Consumption in Canada: A First Look at the Aggregates", *Canadian Journal of Economics and Political Science*, 31, (1965), p.565, using a model very similar to the LES, found a value for ϕ of -1.6 .

TABLE 6

ESTIMATES OF THE INCOME FLEXIBILITY OF THE
MARGINAL UTILITY OF MONEY FROM THE LES

1950	-1.34
1955	-1.29
1960	-1.26
1965	-1.21
1970 .	-1.18

5.3 A COMPARISON OF THE PERFORMANCE OF THE LINEAR EXPENDITURE SYSTEM AND THE ROTTERDAM MODEL

Theil has proposed a statistic which can be used to compare the fit of complete systems of demand.⁹ The "information inaccuracy", I , is

$$I_t = \sum_i w_{it} \log \left(\frac{w_{it}}{\hat{w}_{it}} \right)$$

where w_{it} and $\hat{w}_{it}$ are, respectively, the actual and predicted budget shares of the i^{th} commodity. The information inaccuracy is calculated for each time period and the arithmetic average of these calculations is used to produce one value for the entire sample period.

Table 7 presents the information inaccuracy for the LES and the two versions of the Rotterdam Model discussed above. In the first sub-period presented, 1951-60, the symmetry-constrained version of the Rotterdam does slightly better than the LES which, in turn, does slightly better than the additive version of the Rotterdam. In the second sub-period, 1961-71, the ranking of the models is the same, but now the symmetry-constrained Rotterdam is somewhat better than the other two. For the sample period as a whole, the symmetry-constrained

⁹ For example, see Theil, Theory and Measurement of Consumer Demand, pp.168-180. M.B. McElroy, "Goodness of Fit For Seemingly Unrelated Regressions", Journal of Econometrics, 6, (1977): 381-387, has proposed an R^2 statistic which could also be used for this purpose.

TABLE 7

AVERAGE INFORMATION INACCURACIES OF THE DEMAND MODELS

Sample Period	Symmetry Constrained Rotterdam	Additive Rotterdam	Linear Expenditure System
1951-60	147.7	153.7	151.0
1961-71	140.6	212.6	211.0
1951-71	144.1	183.2	181.0
(1951-71)*	(178.0)	(194.3)	(200.1)
Post-Sample Period			
1972	155.7	36.4	46.3
1973	425.6	56.6	64.0

* The figures in brackets are corrected for loss of degrees of freedom.

NOTE: All entries are to be multiplied by 10^{-7} .

Rotterdam is the winner, showing an information inaccuracy approximately 80 percent as large as the other two models.

The situation is somewhat different when the information inaccuracies are corrected for loss of degrees of freedom as shown in the fourth row of Table 7.¹⁰ Now the LES does slightly worse than the two Rotterdam models. The symmetry constrained version of the Rotterdam retains its dominant position, but it is less strong relative to the additive version of the Rotterdam.

Over all the results presented in Table 7 are broadly consistent with the results of Parks, Deaton and Theil discussed in Chapter One, above. However, as noted there, these authors did not undertake a convincing, direct test of the additive version of the Rotterdam Model against the LES. Over the sample period as a whole, it has been found here that the LES is about equal to the additive version of the Rotterdam but that, after correction for loss of degrees of freedom, the LES appears to do slightly worse. However, it is not clear whether the difference between the two models corrected for degrees of freedom is significant. From the discussion in Chapter Three, this indicates that there is little, if anything, to gain by allowing ϕ to vary over a sample period of the length considered here.¹¹

¹⁰ Ibid., pp. 222-223.

¹¹ Of course, over a longer period, the standard of living of population might change sufficiently to necessitate allowing ϕ to vary.

The results presented here constitute additional, though possibly not very strong, evidence against the assumption of groupwise additivity. Even though the data groups used for estimation were carefully constructed to satisfy this assumption *a priori*, the fit of the models suggests that some between-group interactions may remain. It should be noted, however, that the poorer fit of the additive models appears to have been due to their lack of ability to explain expenditure on the *Medical* category. As much of this expenditure, over the last half of the sample period, was undertaken by the government, rather than by private individuals, this group might well be difficult to explain in terms of demand theory. This being the case, it might prove useful to re-estimate the models excluding the *Medical* group from consideration to obtain a more convincing test of the additivity restriction.

Finally, it is useful to compare the models on their ability to predict the post-sample period. The two bottom rows of Table 7 show the information inaccuracy of each of the models for the years 1972 and 1973. The additive version of the Rotterdam Model does somewhat better than the LES, and both additive models do much better than the symmetry-constrained Rotterdam. The particularly bad performance of the symmetry-constrained Rotterdam in 1973 may be due to the large relative price shifts -- due to the increase in the price of oil -- which took place in that year. Consumers probably did not react immediately to these shifts

but the symmetry-constrained Rotterdam would predict an equilibrium response. On the other hand, the additive models, being less price sensitive, would not predict large shifts in budget allocation due to a price shift. This example indicates that none of the static models considered here are adequate in the face of large and sudden relative price shifts.

CHAPTER SIX

SUMMARY

In this study a comparison of the Rotterdam Model and the Linear Expenditure System has been undertaken,

Such a comparison is important for two reasons: First, it provides evidence on the behaviour of the income flexibility of the marginal utility of money which has been claimed to be an indicator of the standard of living, and it also sheds light on whether the groupwise additivity restriction can be considered useful in empirical demand analysis. Second, it is useful to know which model appears to be more consistent with Canadian consumer expenditure data.

The basic results of demand theory were given in Chapter Two and these were used in Chapter Three to illustrate the development of the Rotterdam Model and to discuss the interactions associated with the substitution matrix of price responses captured by the two models. Chapter Three indicated that both the Rotterdam Model and the LES are theoretically consistent with the demand theory restrictions of homogeneity,

adding-up, symmetry and negativity. The LES and the additive version of the Rotterdam also incorporate the restriction of groupwise additivity or strong separability of the utility function. With regard to utility functions, the LES is based on the Klein-Rubin and the Rotterdam is based on the Cobb-Douglas. The Cobb-Douglas is not useful in the theory of demand, as it implies constant budget shares and, as a consequence, it has been argued in the literature that the Rotterdam should be viewed as a Taylor expansion of an arbitrary demand function. Chapter Three also indicated that the key difference between the functional forms of the additive Rotterdam and the LES is due to different assumptions about the income flexibility of money -- the additive Rotterdam assumes it constant while the LES allows it to vary with income and prices.

In Chapter Four it was pointed out that to adequately compare the additive Rotterdam Model and the LES a data set, which a priori satisfies the assumption of groupwise additivity, should be used. Forty categories of Canadian consumer expenditure data were aggregated into six groups which were deemed to be suitable for comparing the two models. It was also noted in Chapter Four that Zellner's seemingly unrelated regression technique is appropriate for estimating the models.

Chapter Five presented the results of estimating the models. It was found that the Canadian experience is broadly consistent with other investigations using British, Dutch and Swedish data. The Rotterdam Model without additivity seems to dominate both its additive version and the LES in terms of fit to the sample period data. The LES does about the same as the additive Rotterdam, but when loss of degrees of freedom is taken into account the LES appears to do slightly worse. Hence there does not appear to be any gain from allowing the income flexibility to vary over time. As the additive models do not appear to match the symmetry-constrained Rotterdam in fit to the data, this is additional evidence that there may be interactions between even highly aggregated commodity groups, and, if so, the additivity restriction may be regarded as only useful to a limited degree. As to choosing between the additive version of the Rotterdam and the LES, the answer is not clear cut. As already noted, the LES uses more parameters to achieve about the same fit but its utility function is more desirable than that of the Rotterdam.

APPENDIX A

The Canadian data used to estimate the models discussed in this paper are: Annual current dollar expenditures on forty categories of consumer goods obtained from CANSIM matrix 3325; annual constant (1961) dollar expenditure on the same forty categories obtained from CANSIM matrix 3326; and annual population figures taken from Population 1921-1971, Statistics Canada catalogue 91-512, occasional, and Economic Review, May 1977, Department of Finance, Canada, Table 1.

The percentage share in total expenditure of each category used for estimation is presented in the table below.

PERCENTAGE BUDGET SHARES OF THE SIX COMMODITY GROUPS
USED FOR ESTIMATION

	FOOD	CLOTHING	HOUSING	MEDICAL	TRANSPORT	OTHER
1951	36.0	13.5	25.3	4.4	11.2	9.7
1956	31.4	12.1	27.3	5.3	12.9	11.0
1961	30.2	11.4	27.5	6.6	12.3	12.1
1966	28.0	11.1	26.4	7.4	13.1	13.8
1971	26.6	9.8	27.4	6.9	12.5	17.0

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