Sensitivity analysis and experimental validation of plunger-type wavemakers modelled with a steady flow

by

Stephanie Lowell

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Stephanie Lowell

Abstract

The inclusion of a steady flow in a water channel is a critical requirement for recreating accurately scaled ocean environments in a laboratory. In this thesis, the effect of a uniform flow on the theoretical model and experimental performance of a plunger-type wavemaker has been investigated. Through a variance-based global sensitivity analysis, the influence of all input parameters on the output variance of the theoretical model was investigated. The analysis determined that the wave frequency had the highest influence on the wavemaker model. For a uniform flow, the first order and total effect sensitivity indices were estimated as $0.74 \pm 0.30\%$ and $6.84 \pm 0.16\%$, respectively. Although the sensitivity of the model to the flow was relatively low compared to the wave frequency, there exists an impact due to the interaction of the flow parameter with the remaining model parameters. Therefore, it was established that the inclusion of the flow in the plunger-type wavemaker model is essential for application of the model to an experimental system. To investigate the performance of an experimental plunger-type wavemaker, an ultrasonic sensor was used to measure the generated wave profile for a variety of testing conditions based on the results of the sensitivity analysis. It was observed that as the difference between the experimental results and the theoretical model increased, both frequency and flow increased. Therefore, a corrected model was suggested and the operational range of the experimental system was established.

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Nomenclature

Greek Symbols

β	Wedge angle [deg]
δ	Uncertainty in given parameter
∇	Laplace gradient
ϵ	Total surface elevation error [m]
ζ	Correction factor
η	Position of the free surface [m]
η_i	Free surface elevation of incident wave [m]
η_r	Free surface elevation of reflected wave [m]
η_u	Free surface elevation for each sensor u [m]
θ	Wave absorber angle [deg]
π	Mathematical constant ($\pi \cong 3.14$)
σ	Standard deviation
σ_c	Conductivity of water [S/m]
ϕ, Φ	Velocity potential [m ² /s]
ϕ_i	Incident wave phase [rad]
ϕ_r	Reflected wave phase [rad]
ω	Angular Wave frequency [rad/s]

Roman Symbols

a	Wave amplitude [m]
a_a	Acceleration of actuator [m/s ²]
a/s	Wavemaker amplitude to stroke ratio
A_i	Wave amplitude of the i^{th} mode [m]
$\mathbf{A}'$	Input matrix
b	Mean wedge width [m]
b_1, b_2	Ishigami coefficients
$\mathbf{B}$	Input matrix
c_{ij}	Beach absorber matrix
C	Celerity [m/s]
$C(t)$	Bernoulli term
d	Wire diameter [m]
d_μ	Mean wedge depth [m]
d'_μ	Mean wedge clearance [m]
D	Distance between gauge wires [m]
$\mathbf{D}$	Input matrix
$e_u(t)$	Surface elevation error for sensor u [m]
$E(\cdot)$	Expectation or mean of argument ($\cdot$)
$E_{X_{\sim i}}$	Expectation or mean taken over all parameters except $X_{\sim i}$
f	Wave frequency [Hz]
F_j	Wave absorber matrix
g	Gravitational Acceleration [m/s ²]
$\mathbf{G}$	Monte Carlo matrix
$\mathbf{G}_H$	Monte Carlo matrix
h	Water height [m]

H	Wave height [m]
H_i	Incident water height [m]
H_s	Significant wave height [m]
H_r	Reflected wave height [m]
$\mathbf{H}$	Monte Carlo matrix
i	Index number
i_m	Imaginary number
j	Index number
k	Wavenumber [rad/m]
k_i	Incident wavenumber [rad/m]
k_p	Progressive wavenumber [rad/m]
k_n	Standing wavenumber [rad/m]
k_r	Reflected wavenumber [rad/m]
L	Wave length [m]
m	Index number
M	Node points
n	Index number/Number of waves
$\vec{n}$	Normal vector
N_B	Number of bootstrap samples
N_s	Number of Monte Carlo samples
N_u	Number of sensors u
p	Number of model parameters
$P(\cdot)$	Probability of $(\cdot)$ occurring
q	Index number
r	Reflection coefficient
R	Resistance of the water [Ω]

R_c	Camera rotation matrix
s	Stroke amplitude [m]
S_i	First order sensitivity indices
S_{Ti}	Total effect sensitivity indices
t	Time [s]
t_c	Camera translation vector
T	Wave period [s]
T'	Wave period in moving reference frame [s]
T_m	Time duration for beach absorber measurements [s]
u	Index number for ultrasonic sensors
U	Water current [m/s]
v	Speed of sound [m/s]
V	Model variance
V_i	First order partial variance
V_{Ti}	Total effect partial variance
x, y, z	Cartesian coordinates
x_c, y_c, z_c	Camera pixel coordinates
x_w, y_w, z_w	Camera world coordinates
x_i	Fixed value of model parameter X_i
x_u	Position of sensor u [m]
X_i	Model parameters
X_j	Wave absorber vector components
Y	Model output
z_u	Distance measured by ultrasonic sensor [m]
z_w	Depth of wave gauge

Chapter 1

Introduction

An accurate measurement of wave climate is important to both marine engineering and oceanography. It is essential for properly describing a sea state, as well as providing information on wave height, period, and directionality. To study advanced hydrodynamics for marine engineering applications such as ship motion and the launch and recovery of tow-bodies, laboratory water channels provide an environment in which applications can be studied under scaled conditions. An image of the water channel at Carleton University is displayed in Fig. 1.1. The channel in Fig. 1.1 is equipped with the ability to produce a steady and uniform flow through the test area, allowing for direct observations of fluid dynamics through the side of the tank. However, the ability to extend to further applications such as the study of waves and ocean environments is limited, since the tank is not equipped with a built-in wavemaker. Wavemakers provide an effective method for generating and studying various water waves in a controlled laboratory setting. Therefore, installation of wavemaker and experimental validation of the system is essential in order to study future hydrodynamics applications in marine engineering. Of the existing wavemakers, plunger-types are the only wavemaker which allow for flow across the boundary; and as such, the

Figure 1.1: A photograph of the laboratory water channel at Carleton University showing the main test area.

inclusion of a plunger system would complement the pre-existing equipment for generating flow through the test area. However, research has focused on waves generated without the influence of a steady flow and considerable experimental validation for plunger systems without flow is limited in literature.

In order to study the performance of a plunger-type wavemaker, dependable tools and methods must be properly installed in the water channel. *Therefore, the first objective of this thesis is to develop and investigate the equipment required to measure the free surface elevation of the water over time in a laboratory water channel.* During wave generation, three sensors will be implemented to measure the wave profile. The first sensor is a conductive wave gauge to be developed and calibrated to rest partially submerged in the water. Second, an ultrasonic sensor will be studied for placement directly above the water. The third sensor is a camera to be implemented and calibrated sitting horizontal and external to the water channel. With the three

sensors in place, the free surface of the water in the channel can be measured and cross-referenced for accuracy across multiple systems. The first objective will also investigate the impact of a wave absorber, which will act to absorb incident waves at the end of the water channel and decrease the amount of reflective energy moving back towards the wavemaker. With the appropriate equipment in place to support the experimental validation of the wavemaker, the model which governs the theoretical waves can be explored.

The theoretical model of a plunger-type wavemaker has been developed for the still water case [Wu88] and flow case [McP19]; however, further research into the model and the parameters on which it is dependent is limited. *Therefore, the second objective of this thesis is to quantify the influence of the input parameters on the output of both the conductive wave gauge and the wavemaker models through variance-based global sensitivity analyses for design applications.* The model's dependency on its parameters can provide insight towards the strengths and weaknesses of the wavemaker model and a basis for the design of future plunger-type wavemakers. Current experimental research with plunger systems is often limited to a single set of test parameters without the inclusion of flow. However, plunger-type wavemakers are advantageous as can be easily installed in a water channel that was not originally built for wave generation. *Therefore, the third objective of this thesis is to explore the design aspects of a plunger-type wavemaker and experimentally validate the wavemaker's transfer function for water with flow.* The wavemaker's ability to generate regular waves in the water channel will be explored through comparison of the experimental and theoretical model. The experimental validation of the wavemaker theory in the presence of flow will be crucial to the field and the operational range of the wavemaker more clearly defined for the water channel in use. As such, a full understanding the plunger-type wavemaker's model and physical system will be more clear

and accessible.

The key contributions of the research presented in this thesis are:

1. The development of a conductive wave gauge for the water channel test facility at Carleton University.
2. The development and evaluation of a wave absorber to reduce the amount of wave energy reflectance in the water channel.
3. An investigation of the theoretical model of the plunger-type wavemaker through application of a variance-based global sensitivity analysis.
4. An experimental validation of the inclusion of moving water in the pre-existing theory for plunger-type wavemakers.
5. An investigation into correction methods for the wavemaker theory to better reflect the experimental results.
6. The determination of the operational range of the experimental plunger-type wavemaker for the water channel at Carleton University.

1.1 Organization

The following thesis has been organized into 7 remaining chapters. Chapter 2 introduces the background knowledge on the topics covered in the thesis while Chapter 3 provides information on the current laboratory equipment both previously available and developed for the water channel. Chapter 4 describes the theory associated with performing a variance-based global sensitivity analysis for a given model with the results of the sensitivity analyses for the conductive wave gauge and the theoretical

wavemaker model discussed in Chapter 5. The design of the experimental plunger-type wavemaker and the procedure for evaluating its performance is presented in Chapter 6. In Chapter 7, the results of the wavemaker experimental tests are discussed, including an evaluation of the operating range of the wavemaker. Finally, conclusions are reported in Chapter 8 along with recommendations for future work.

Chapter 2

Background and Literature Review

The work conducted in this thesis draws on the development of key concepts in hydrodynamics and wavemaker theory; therefore, this chapter reviews the literature that was consulted in order to complete the objectives of the thesis research. Section 2.1 outlines the modelling of regular waves with and without the inclusion of water flow. Section 2.2 discusses the requirement for wave absorbers in a water channel and the theory for separating incident waves from reflected waves with respect to the first objective. In Section 2.3, an overview of wavemakers is provided along with the theory which describes the plunger-type wavemaker in the presence of flow in relation to the second objective. The applicability of sensitivity analyses, associated with the third objective of the thesis work, is discussed in Section 2.4. Finally, a chapter summary is provided in Section 2.5.

2.1 Modelling Regular Waves

Evaluating the performance and design of the plunger-type wavemaker is dependent on the surface waves that are produced. For the experimental work in this thesis, it is desired that the wavemaker can produce regular waves in both quiescent water and

in the presence of a flow. In the following subsections, the theory for regular waves is described along with the modifications that must be introduced to include the effects of a uniform flow.

2.1.1 Regular Waves Without Flow

Regular waves produced in a water channel without the influence of a water flow are governed by small amplitude wave theory under the simplifying assumption that the fluid is incompressible and irrotational [DD84]. The small amplitude water wave problem is thus formulated as a boundary value problem such that a velocity potential ϕ exists which satisfies the continuity equation,

$$\nabla \cdot \nabla \phi = 0, \quad (2.1)$$

where the divergence of the gradient ∇ leads to the Laplace equation, which must hold throughout the fluid [DD84]. Solving the boundary value problem is dependent on the boundary conditions set by the water channel. Fig. 2.1 illustrates the main boundaries in the two-dimensional boundary value problem for regular waves. The surface elevation of the water in the channel is represented by $\eta(x, t)$ for a position x and time t relative to the plane $z = 0$. The still level water depth is given by h and the parameters L and H represent the wavelength and height of the regular wave. The channel can thus be described by four boundary conditions: the bottom boundary condition (BBC), the kinematic free surface boundary condition (KFSBC), the dynamic free surface boundary condition (DFSBC), and the periodic lateral boundary condition (PLBC).

The BBC describes the lower boundary of the water channel in Fig. 2.1 where the no flow condition applies. For an impermeable, horizontal bottom, the BBC is

Figure 2.1: Schematic of the two-dimensional boundary value problem for regular water waves. The boundaries include the bottom boundary condition (BBC), the kinematic free surface boundary condition (KFSBC), the dynamic free surface boundary condition (DFSBC), and the periodic lateral boundary condition (PLBC).

mathematically defined on $z = 0$ by [DD84],

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = 0. \quad (2.2)$$

At the free surface of the water, represented by the upper boundary in Fig. 2.1, the fluid is free to deform under the influence of forces. Therefore, there exists both a KFSBC and a DFSBC. The KFSBC ensures that there is no flow across the fluid interface at $z = h$ such that [DD84],

$$-\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} - \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x}. \quad (2.3)$$

Since the interface between the air and the water is “free”, the water surface cannot support variations in pressure and will instead respond in order to maintain uniform

pressure. Therefore, the DFSBC prescribes the uniform pressure distribution on the air-water boundary $z = h$ and can be mathematically expressed as,

$$-\frac{\partial\phi}{\partial t} + \frac{1}{2} \left[\left(\frac{\partial\phi}{\partial x} \right)^2 + \left(\frac{\partial\phi}{\partial z} \right)^2 \right] + \frac{p_\eta}{\rho} + g\eta = C(t), \quad (2.4)$$

where p_η is constant and usually taken as the gauge pressure, $p_\eta = 0$ [DD84]. The parameter g is the gravitational acceleration and $C(t)$ is referred to as the Bernoulli term and constant for steady flows. Finally, there exists conditions along the lateral boundaries. For waves that are periodic in space and time, the PLBC can be used to constrain the flow exiting the downstream lateral boundary to re-enter the upstream lateral boundary. The spatial PLBC $\phi(x, t)$ is expressed as [DD84],

$$\phi(x, t) = \phi(x + L, t). \quad (2.5)$$

Similarly, the temporal PLBC $\phi(x, t)$ is represented by [DD84],

$$\phi(x, t) = \phi(x, t + T), \quad (2.6)$$

where T is the period of the regular wave. For other configurations of the lateral boundaries, the appropriate boundary condition must be applied. For example, the presence of a wave absorber requires the application of either a kinematic boundary condition or a radiation condition such that only outgoing waves occur at infinity. On the other hand, if the wave motion occurs as the result of a prescribed disturbance, such as a wavemaker, then the lateral boundary at $x = 0$ must be defined by a kinematic boundary condition unique to the type of the wavemaker.

For the water wave boundary value problem representing waves that are periodic in space and time propagating over a horizontal bottom, Dean and Dalrymple [DD84]

proposed a linearized solution to the Laplace equation expressed as the velocity potential ϕ ,

$$\phi = \frac{Hg \cosh(k[h+z])}{2\omega \cosh(kh)} \cos(kx) \sin(\omega t), \quad (2.7)$$

where $\omega = 2\pi/T$ is the angular frequency of the wave and $k = 2\pi/L$ is the corresponding wave number. The relationship between ω and k can be described by the dispersion relationship,

$$\omega^2 = gk \tanh(kh). \quad (2.8)$$

Rearranging this dispersion relation such that,

$$\frac{\omega^2 h}{gkh} = \tanh(kh), \quad (2.9)$$

the terms on both sides of the equation are expressed as a function of kh for a particular value of $\omega^2 h/g$. Therefore, when the dispersion equation (2.9) is satisfied there exists a single solution or equivalently, one value of k for given values of ω and h . Finally, the surface elevation of the water $\eta(x, t)$ shown in Fig. 2.1 is determined by retaining the linear terms of the DFSBC in (2.4) such that [DD84],

$$\eta(x, t) = \left(\frac{1}{g} \frac{\partial \phi}{\partial t} \right)_{z=0} = \frac{H}{2} \cos(kx) \cos(\omega t). \quad (2.10)$$

The wave described by the surface elevation in (2.10) represents a standing wave, as it does not propagate in any direction. To represent waves that move through the medium, referred to as progressive waves, the velocity potential in (2.7) can be

expressed as [DD84],

$$\phi = -\frac{Hg \cosh(k[h+z])}{2\omega \cosh(kh)} \sin(kx - \omega t). \quad (2.11)$$

The surface elevation corresponding to the progressive wave velocity potential is thus [DD84],

$$\eta(x, t) = \left(\frac{1}{g} \frac{\partial \phi}{\partial t} \right)_{z=0} = \frac{H}{2} \cos(kx - \omega t). \quad (2.12)$$

The speed of the progressive wave is expressed by the celerity $C = L/T$. The general dispersion relation describes the way in which a field of propagating waves consisting of multiple frequencies would separate due the different celerities of the frequency components. Therefore, noting that $\omega = 2\pi/T$ and $k = 2\pi/L$, the celerity can be expressed as $C = \omega/k$ and the dispersion relation takes the form,

$$C^2 = \frac{L^2}{T^2} = \frac{g}{k} \tanh(kh). \quad (2.13)$$

In shallow water, the dispersion equation in relation to the celerity, reduces to the form $C = \sqrt{gh}$, such that the speed is solely determined by the water depth. For deep water, kh is large and the wavelength is expressed as $L_0 = gT^2/2\pi$ where the zero subscript denotes a deep water value. Deep water is thus defined for where $h/L_0 > 0.5$ and the dispersion relation is expressed as [DD84],

$$C = \frac{L_0}{T} \tanh(kh) \approx \frac{g}{\omega}. \quad (2.14)$$

In deep water, the wave period is not dependent on the water depth; therefore, waves of constant period will slow down as they enter shallow water. It is important to also

consider water waves propagating on a flow, such as the waves on ocean currents, in order to accurately reproduce the free surface of the ocean at the laboratory scale.

2.1.2 Regular Waves with Flow

For the following analysis, it is assumed that the water flow is uniform over depth and horizontal distance and flowing in the same direction as the waves. The velocity potential which satisfies the Laplace equation for a uniform flow U and a progressive is expressed by Dean and Dalrymple [DD84] as,

$$\phi = -Ux + A \cosh(k[h + z]) \cos(kx - \omega t), \quad (2.15)$$

where A is an unknown coefficient that can be determined from solving the DFSBC. The form of the solution satisfies the BBC while guaranteeing the periodicity of the wave in space and time. Satisfying the KFSBC using the velocity potential in (2.15) leads to a modified version of the dispersion relation [DD84],

$$\omega^2 = Uk + \sqrt{gk \tanh(kh)}. \quad (2.16)$$

As in the no flow case, the dispersion relation can be expressed in terms of the celerity C such that [DD84],

$$(C - U)^2 = \frac{g}{k} \tanh(kh). \quad (2.17)$$

For convenience, a reference frame that moves with the flow is adopted such that the solution to the boundary value problem in the no flow condition can still be used. To relate the moving reference frame to the stationary frame, the wavelength must be equal in both systems; therefore the period T' relative to the moving frame relates to

the period T in the stationary frame by [DD84],

$$T' = \frac{T}{(1 - \frac{U}{C})}. \quad (2.18)$$

For shallow water, the dispersion relation in (2.17) reduces to the form [DD84],

$$C = U + \sqrt{gh}. \quad (2.19)$$

Since the celerity is independent of wavelength in the shallow water case, it is simply increased by the magnitude of the uniform flow. On the other hand, for deep water, the solution to the dispersion relation in (2.17) with regards to C is determined using a quadratic solution and replacing the wavenumber k with ω/C [DD84], such that,

$$C = \left(U + \frac{g}{2\omega} \right) + \sqrt{\frac{Ug}{\omega} + \frac{1}{4} \left(\frac{g}{\omega} \right)^2}. \quad (2.20)$$

The definition of the celerity in (2.20) thus provides the wave speed for a progressive wave in the presence of a uniform flow $U \geq 0$. Application of the modified dispersion relation is critical for accurately representing ocean waves in a laboratory water channel. The accuracy of the generated waves in the laboratory environment will also be dependent on limiting the amount of wave reflection within the water channel.

2.2 Wave Absorption and Reflection

A common problem with wave generation in short water channels is the interference of the incident waves with waves that have been reflected back from the end of the channel. The reflected waves propagate in the opposite direction of the incident waves causing unpredictable constructive and destructive interference. Therefore, to reduce

the wave reflection, wave absorbers or beaches are placed at the end of the water channel. A number of studies have been conducted to analyze the interaction of the water with the wave absorber structure [Lim14]. In particular, Requejo et al. [RVL72] determined that for a permeable structure with a wall shape, the reflections and dissipation strongly depend on the wave steepness and relative width since these two parameters determine the location of the reflection. More recently, Lim [Lim14] conducted a series of experiments on sloping-wall-type wave absorbers. The study found that the reflectance strongly depends on the angle of the wave absorber, with smaller angles between the bottom of the channel and the wave absorber leading to a lower reflectance value. Gadelho et al. [GLS⁺15] also studied the performance of a wave absorber, observing that the reflection is a function of both the wavelength and the wave amplitude. Both of the aforementioned studies [Lim14, GLS⁺15] also determined that the reflectance increases when the period of the wave increases.

The reflection coefficient is determined by separating the incident waves from the reflected waves in the wave record measured simultaneously at several locations in the channel. Typically, the surface elevation is measured using two or more wave gauges. Goda and Suzuki [GS76] developed a method based on the use of two gauges however, it does not give a solution when $\lambda/L = n/2$ for $n = 0, 1, 2, \dots, \infty$ where λ is the distance between the gauges, and L is the wavelength. The method is also sensitive to non-linear wave interactions, signal noise, and measurement error [SPP01]. On the other hand, the method developed by Mansard and Funke [MF80] and Park et al. [POC92] is less sensitive to these phenomena and has no limitation in its application range of frequency and wavelength. To extend the method proposed by [POC92] for application to waves propagating on a flow, Suh et al. [SPP01] developed a technique to separate the incident and reflected waves using a least squares method. The method also has the advantage of working for both regular and irregular waves and simply

reduces to the method in [POC92] when no flow is present. Therefore, to determine the reflection in the water channel at Carleton University due to the wave absorber, the method developed by [SPP01] will be followed.

2.2.1 Separating Incident and Reflected Waves

A regular wave profile is formed by the superposition of the waves incident on the wave absorber with a height H_i and the wave reflected back from the absorber with height H_r . The incident waves propagate in the positive x -direction with the surface elevation $\eta = 0$ representing the still water height. The wave profile of the incident $\eta_i(x, t)$ and reflected $\eta_r(x, t)$ waves are expressed as [SPP01],

$$\eta_i(x, t) = \frac{H_i}{2} \cos(k_i x - \omega t + \phi_i), \quad (2.21)$$

$$\eta_r(x, t) = \frac{H_r}{2} \cos(k_r x + \omega t + \phi_r), \quad (2.22)$$

where k_i and k_r represent the wavenumbers of the incident and reflected waves, respectively, ω is the absolute angular frequency in the stationary frame of reference. The phase angles of the incident and reflected waves are given by ϕ_i and ϕ_r , respectively. The flow rate in the water channel is taken into consideration within the dispersion relations for each wavenumber given by [SPP01],

$$\omega = k_i U + \sqrt{g k_i \tanh(k_i h)}, \quad (2.23)$$

$$\omega = -k_r U + \sqrt{g k_r \tanh(k_r h)}, \quad (2.24)$$

where g is the gravitational acceleration and h is the water height.

The wave measurements are taken at two locations in front of the wave absorber, the position of which is denoted by x_1 and x_2 for the respective measurement sensors.

The surface elevation given at x_1 for example can be expressed as [SPP01],

$$\eta_1(t) = \frac{H_i}{2} \cos(k_i x_1 - \omega t + \phi_i) + \frac{H_r}{2} \cos(k_r x_1 + \omega t + \phi_r) + e_1(t), \quad (2.25)$$

in which $e_1(t)$ is the error due to signal noise, non-linear wave interactions, and measurement error recorded by the sensor at x_1 . A similar equation exists for the surface elevation at x_2 described by $\eta_2(t)$. Equation (2.25) can be rewritten as [SPP01],

$$\eta_1(t) = X_1 \cos(\omega t - k_i x_1) + X_2 \cos(\omega t + k_r x_1) + X_3 \sin(\omega t - k_i x_1) - X_4 \sin(\omega t + k_r x_1) + e_1(t), \quad (2.26)$$

where X_j for $j = 1, \dots, 4$ are unknown coefficients that are expressed in terms of the height and phase of the incident and reflected waves given as [SPP01]:

$$X_1 = \frac{H_i}{2} \cos \phi_i; \quad (2.27)$$

$$X_2 = \frac{H_r}{2} \cos \phi_r; \quad (2.28)$$

$$X_3 = \frac{H_i}{2} \sin \phi_i; \quad (2.29)$$

$$X_4 = \frac{H_r}{2} \sin \phi_r. \quad (2.30)$$

To determine the unknown coefficients, a least squares method is used assuming that the waves are measured in the two locations for a duration of T_m . The total squared error ϵ^2 in the measurement of the surface elevation can be expressed as [SPP01],

$$\epsilon^2 = \int_0^{T_m} [e_1(t)]^2 dt + \int_0^{T_m} [e_2(t)]^2 dt. \quad (2.31)$$

By minimizing the total squared error with respect to each unknown coefficient, the

heights and phases of the incident and reflected waves can be determined resulting in a system of linear equations given by,

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}. \quad (2.32)$$

Here the elements of the c matrix are expressed in terms of the trigonometric functions on the right hand side of (2.26) and the elements F_j for $j = 1, \dots, 4$ in terms of the same functions and the surface elevation measured by each ultrasonic sensor. The exact definitions for the c_{ij} and F_j elements are provided in Appendix A.

Solving for the X_j coefficients allows for the determination of the incident and reflected wave phases by [SPP01],

$$\phi_i = \tan^{-1} \frac{X_3}{X_1}, \quad (2.33)$$

$$\phi_r = \tan^{-1} \frac{X_4}{X_2}, \quad (2.34)$$

and the wave heights by [SPP01],

$$H_i = \frac{2(X_1 + X_3)}{\cos \phi_i + \sin \phi_i}, \quad (2.35)$$

$$H_r = \frac{2(X_2 + X_4)}{\cos \phi_r + \sin \phi_r}. \quad (2.36)$$

The reflection coefficient r is thus represented by the ratio of the reflected wave height to the incident wave height,

$$r = \frac{H_r}{H_i}, \quad (2.37)$$

where a reflection less than one indicates that the incident waves are larger than the waves reflected off the back of the tank or wave absorber. On the other hand, a reflection coefficient larger than one indicates an undesirable situation in which the reflected wave heights are larger than the incident wave heights. The incident waves are generated in a water channel through the use of wavemakers. Therefore, the next section reviews the different types of common wavemakers along with the theoretical model for plunger-type wavemakers in the presence of flow.

2.3 Wavemakers

Water waves can be produced in a laboratory channel through the use of mechanical wavemakers, of which two main types exist: wall-type and plunger-type. The moveable wall-type wavemakers generate waves through oscillatory motion in the direction of the wave propagation. Such wavemakers are generally immersed in water and include flexible and rigid flap-type wavemakers and piston-type wavemakers. Flexible flap-type wavemakers are advantageous in channels of limited length as they are able to produce waves with stable and correct motions immediately in front of the flap [Cha69]. Rigid flap-type wavemakers are best suited for generating deep water waves, but if shallow water waves are required, a long channel is necessary for the correct wave profile to develop. On the other hand, piston-type wavemakers are suited for generating shallow water waves [Cha69]. Although application of flap- and piston-type wavemakers are versatile, they require a substantial amount of submerged mechanical components. In addition, these wavemakers do not typically allow for flow across the lateral boundary; as a consequence, the inclusion of flow in the generated wave profile is prohibitive. For the moveable wall-type wavemakers, analytical solutions for determining the surface elevation and wavemaker transfer

Figure 2.2: Schematic of a triangular plunger-type wavemaker in a laboratory water channel with water flow U .

function are provided by Biésel and Suquet [BS51] and Dean and Dalrymple [DD84].

The plunger-type wavemaker forms waves by displacing the water through vertical oscillations of a rigid body. Although a variety of cross-sectional forms are possible, the plunger generally takes the shape of a wedge with a triangular cross-section, or with one side parabolically contoured with a vertical back [Cha69]. A schematic of a plunger-type wavemaker with a triangular-shaped cross section positioned in a water channel with flow U is provided in Fig. 2.2. The specific application of wavemaker theory to a wedge-shaped wavemaker was established by Wang [Wan74] who showed that the ratio of the water wave amplitude to the plunger stroke amplitude will be dependent on the wavenumber and the mean width of the wedge at the still water level. By using the boundary collocation method, Wu [Wu88] developed a semi-analytical method to study waves generated by a plunger-type wavemaker that demonstrated the required inclusion of water height in the wavemaker model. The wave profiles

produced by a plunger-type wavemaker were studied by Ellix and Arumugam [EA84] which Wu [Wu88, Wu91] referenced for comparison to the theoretical model; however, both theory and experiment were only applied to the still water case and did not include the effect of a steady flow. The plunger-type wavemaker theory was advanced by McPhee [McP19], who proposed that the uniform flow in a water channel be taken into account, but experimental validation was outside the scope of the study. Unlike the moveable-wall wavemakers, the motion of the plunger allows for flow across the lateral boundary while also not requiring any submerged mechanical components. A disadvantage of the plunger-type wavemaker is the requirement for relatively long wave channels to achieve wave stabilization. However, in McPhee's [McP19] study on the design of the plunger-type wavemaker, it was determined that it is theoretically possible to achieve stable waves in the water channel at Carleton University.

Simulations of the plunger-type wavemaker have been developed in order to predict the generated wave profile in the water channel. As part of a wavemaker renewal project, Mikkola [Mik07] applied an unstructured finite volume method to develop a time accurate simulation of a wedge-shaped plunger. The performance of the plunger was compared for three different wedge angles in terms of the height and quality of the generated wave for a variety of stroke amplitudes and oscillation periods. Mikkola [Mik07] concluded that a wedge with a 35 degree angle was most appropriate for their water channel however, the simulation was only conducted for the no-flow case. Similarly, Gadelho et al. [GLS⁺15] developed a computational fluid dynamics (CFD) model to study the performance of a wedge-shaped wavemaker using a Navier-Stokes model to simulate the interaction of the wedge with the water. The work also investigated the performance of a wave absorber and observed that the performance of the wave absorber decreased for large wave periods with the wave height having an effect on the calculated coefficient of reflection. Kashiwagi [Kas96]

also presented a numerical solution for the wavemaker problem; the numerical results of the work along with those found by [GLS⁺15] were compared to a limited number of experimental results but neither the numerical or experimental work include water flow. Therefore, to address the gap in knowledge, it is critical to study the effect of flow on the theoretical and experimental performance of the plunger-type wavemaker.

2.3.1 Plunger-type Wavemaker Theory

A schematic of the plunger-type wavemaker problem for a wedge with a triangular cross-section in a laboratory water channel is displayed in Fig. 2.3, where x represents the horizontal axis and z is the vertical axis. The angle β is measured between the

Figure 2.3: Schematic of an plunger-type wavemaker in a laboratory water channel with flow U . The channel is defined by four conditions. The far-field radiation condition (RC), bottom boundary condition (BBC), kinematic boundary condition (KBC), and combined free surface boundary condition (CFSBC) which includes the kinematic free surface boundary condition (KFSBC) and dynamic free surface boundary condition (DFSBC).

vertical z axis and the hypotenuse of the wedge, while the mean wedge depth d_μ is measured at the still water height h . The clearance between the bottom of the water channel and the wedge is denoted d'_μ . Since d_μ and d'_μ represent average distances measured during wave generation, both parameters are independent of time. The vector normal to the surface of the wedge is represented by $\vec{n}$ while the position of the free surface of the water is represented by $\eta(x, t)$ at position x and time t relative to the plane $z = h$. Forward waves are produced by the vertical displacement s of the plunger as it oscillates in the water column. Thus, the water wave length and height are controlled by the oscillation frequency and fluid displacement of the plunger. The oscillating motion allows for flow across the lateral boundary throughout the duration of the operation. The magnitude of the water flow U is positive along the x -axis when moving the same direction as the waves produced by the wavemaker. The water channel in Fig. 2.3 can be defined by four boundary conditions and a radiation condition (RC) that ensures only the existence of progressive waves at large distance from the plunger [Wu88]. Similar to the water wave boundary value problem presented in Section 2.1.1, the BBC describes the horizontal plane along $z = 0$ where the no flow condition applies. The combined free surface boundary condition (CFSBC) describes the free surface of the water which includes the KFSBC and the DFSBC. The KFSBC ensures that there is no flow across the free surface from the water to the air, while the DFSBC constrains a uniform pressure distribution across the free surface boundary. The final boundary condition is a kinematic boundary condition (KBC) that exists along the inclined surface of the plunger wedge and the clearance defined by d'_μ .

The transfer function between the wave amplitude a and the stroke amplitude of the plunger s provides an important metric for wavemaker design. For a known stroke amplitude, the wave amplitude can be determined by examining the water

wave profile $\eta(x, t)$ produced by the wavemaker. Under the assumption of classic hydrodynamics, the fluid flow is inviscid, incompressible, and irrotational such that the elements of the fluid do not undergo a net rotation. Thus, the problem can be defined by small amplitude wave theory where there exists a velocity potential ϕ that satisfies the continuity equation,

$$\nabla \cdot \nabla \phi = 0, \quad (2.38)$$

where the gradient ∇ leads to the Laplace equation. Solving for the linear wave profile therefore becomes a boundary value problem constrained by the four boundary conditions and radiation condition shown in Fig. 2.3. The BBC at $z = 0$ is mathematically defined by [Wu88],

$$\left(\frac{\partial \phi}{\partial z} \right)_{z=0} = 0. \quad (2.39)$$

Similarly, the CFSBC at $z = h$ is expressed by [Wu88],

$$\left(\frac{\partial \phi}{\partial z} - \frac{\omega^2}{g} \phi \right)_{z=h} = 0, \quad (2.40)$$

where ω is the wave frequency and g is the acceleration due to gravity. Assuming no leakage around the wedge, the KBC along the wavemaker and the clearance section is respectively defined by [Wu88],

$$\left(\frac{\partial \phi}{\partial n} \right)_{d'_\mu \leq z \leq h} = s\omega \sin(\beta) \cos(\omega t), \quad (2.41a)$$

$$\left(\frac{\partial \phi}{\partial x} \right)_{0 \leq z < d'_\mu} = 0. \quad (2.41b)$$

When $\beta = 0$, the condition in (2.41a) is equal to that of (2.41b); therefore, a single

equation can be used to describe the KBC for $\beta \geq 0$ and $0 < z < h$. Combining (2.41a) and (2.41b), the modified KBC on the wavemaker at $x = (z - d'_\mu) \tan(\beta)$ is expressed as [Wu88],

$$\frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial z} \tan(\beta) = s\omega \tan(\beta) \cos(\omega t). \quad (2.42)$$

One form of the velocity potential $\phi(x, z)$ which solves the Laplace equation and also satisfies the BBC, CFSBC, and RC is given by [Wu88],

$$\phi(x, z) = A_0 \cosh(k_p z) e^{i_m k_p x} + \sum_{n=1}^{\infty} A_n \cos(k_n z) e^{-k_n x}, \quad (2.43)$$

where $n = 1, 2, \dots, \infty$, i_m is the imaginary number, A_0 and A_n are unknown coefficients of the velocity potential whose values are to be determined, and the wavenumbers k_p and k_n represent progressive and standing waves, respectively [Wu88]. Equation (2.43) is constrained by the dispersion relationship of both wavenumbers k_p and k_n defined by [Wu88],

$$\omega^2 = g k_p \tanh(k_p h), \quad (2.44)$$

$$\omega^2 = -g k_n \tanh(k_n h). \quad (2.45)$$

For the no flow condition, the dispersion relations in (2.44) and (2.45) hold true; however, water waves in a uniform flow U experience a Doppler shift. Therefore, as initially proposed by McPhee [McP19], the dispersion relation used to constrain the velocity potential for the plunger-type wavemaker boundary value problem must be modified to account for the flow rate U . By shifting the frequency ω , the modified

dispersion relation for the progressive wavenumber k_p can be expressed as [DD84],

$$(\omega - k_p U)^2 = g k_p \tanh(k_p h). \quad (2.46)$$

From the definition of phase velocity where the celerity C is equal to ω/k_p , (2.46) can be rewritten as [DD84],

$$(C - U)^2 = \frac{g}{k_p} \tanh(k_p h). \quad (2.47)$$

Note that the dispersion relation in (2.47) for the progressive wavenumber is the same as the dispersion relation in (2.17). Thus, the solution to (2.47) in terms of C is given by the celerity in (2.20) and the progressive wavenumber can be determined analytically by [DD84],

$$k_p = \frac{\omega}{C} = \frac{\omega}{\left(U + \frac{g}{2\omega}\right) + \sqrt{\frac{Ug}{\omega} + \frac{1}{4}\left(\frac{g}{\omega}\right)^2}}. \quad (2.48)$$

Adopting the reference frame that moves with the flow, a modified wave frequency ω' can be defined by,

$$\omega' = \frac{2\pi}{T'}, \quad (2.49)$$

where T' is period in the moving reference frame, such that a flow $U \geq 0$ is taken into consideration for the wavemaker boundary value problem. Equation (2.49) also allows for the determination of the standing wavenumber k_n in the presence of flow by rewriting (2.45) in terms of ω' and rearranging such that,

$$\frac{\omega'^2 h}{k_n h g} = -\tanh(k_n h). \quad (2.50)$$

By isolating the dimensionless value $k_n h$ on both sides of (2.50), the intersection of each term provides a means of obtaining the standing wavenumber for a given water depth h .

With the dispersion constraint modified to include the effect of flow, substituting the velocity potential solution defined by (2.43) into the modified KBC expressed in (2.42) yields an expression from which A_0 and A_n can be determined [Wu88],

$$\begin{aligned} & \left[i_m A_0 k_p \cosh(k_p z) e^{i_m k_p (z - d'_\mu) \tan(\beta)} - \sum_{n=1}^{\infty} A_n k_n \cos(k_n z) e^{-k_n (z - d'_\mu) \tan(\beta)} \right] - \\ & \left[A_0 k_p \sinh(k_p z) e^{i_m k_p (z - d'_\mu) \tan(\beta)} - \sum_{n=1}^{\infty} A_n k_n \sin(k_n z) e^{-k_n (z - d'_\mu) \tan(\beta)} \right] \tan(\beta) \\ & = s\omega \tan(\beta). \end{aligned} \quad (2.51)$$

To solve for the coefficients A_0 and A_n , a semi-analytical solution using the boundary collocation method is applied [Wu88]. For the wavemaker problem, the boundary is defined as the KBC in Fig. 2.3 which includes the inclined surface of the wedge and the clearance between wedge bottom and channel bottom d'_μ . The boundary is divided into segments of equal length with two node points at each end of a segment totalling M node points. To satisfy (2.51) at all M node points, it is expressed in matrix form as [Wu88],

$$\mathbf{B}\mathbf{A}' = \mathbf{D}, \quad (2.52)$$

where $\mathbf{B}$ is a $m \times n$ matrix, $\mathbf{A}'$ is a $n \times 1$ matrix, and $\mathbf{D}$ is a $m \times 1$ matrix with the number of node points M representing the number of matrix rows m and total number of waves n representing the number of matrix columns. The elements of

matrices $\mathbf{B}$, $\mathbf{A}'$, and $\mathbf{D}$ are expressed as [Wu88],

$$B_{m1} = k_p h \left(i_m \cosh \left[k_p h \left(\frac{z}{h} \right)_m \right] - \tan(\beta) \sinh \left[k_p h \left(\frac{z}{h} \right)_m \right] \right) e^{i_m k_p h \tan(\beta) \left(\frac{z-d'_\mu}{h} \right)_m}, \quad (2.53)$$

$$B_{mn} = -k_{n-1} h \left(\cos \left[k_{n-1} h \left(\frac{z}{h} \right)_m \right] - \tan(\beta) \sin \left[k_{n-1} h \left(\frac{z}{h} \right)_m \right] \right) e^{-k_{n-1} h \tan(\beta) \left(\frac{z-d'_\mu}{h} \right)_m}, n \neq 1, \quad (2.54)$$

$$A'_1 = \frac{A_0}{s\omega h}, \quad (2.55)$$

$$A'_n = \frac{A_{n-1}}{s\omega h}, \quad n \neq 1, \quad (2.56)$$

$$D_m = \tan(\beta), \quad (2.57)$$

where B_{m1} represents a single progressive wave and B_{mn} represents the standing waves. Equation (2.52) is solved using a least squares method to minimize the sum of quadratic error by multiplying both sides by the Hermitian transpose of $\mathbf{B}$ denoted $\mathbf{B}^T$ such that,

$$\mathbf{B}^T \mathbf{B} \mathbf{A}' = \mathbf{B}^T \mathbf{D}. \quad (2.58)$$

Wu [Wu88] suggests using a finite number of waves by setting $n = 16$ such that the approximate solution to (2.58) includes one progressive wave and 15 standing waves. The RC ensures that the standing waves will decay with distance from the wavemaker such that terms in (2.43) relating to k_n will be negligible. Thus, taking the real part

of the first term in (2.43), the velocity potential ϕ reduces to [Wu88],

$$\phi = \text{Re} [A_0 \cosh(k_p z) e^{i_m(k_p x - \omega t)}]. \quad (2.59)$$

The linear wave profile of the free surface $\eta(x, t)$ where only the progressive wave exists is expressed as [Wu88],

$$\eta(x, t) = \frac{1}{g} \frac{\partial \phi}{\partial t} = \text{Re} \left[\frac{-i_m \omega A_0}{g} \cosh(k_p h) e^{i_m(k_p x - \omega t)} \right], \quad (2.60)$$

where the amplitude of the wave a is defined by [Wu88],

$$a = \left| \frac{-i_m \omega A_0}{g} \cosh(k_p h) \right|. \quad (2.61)$$

By dividing both sides of (2.61) by the stroke amplitude s , the theoretical plunger-type wavemaker model which relates the water wave amplitude a to the stroke amplitude s is derived to be [Wu88],

$$\frac{a}{s} = \left| -i_m A'_1 k_p h \sinh(k_p h) \right|, \quad (2.62)$$

where A'_1 is defined by (2.55). However, since the definition of A'_1 is dependent on the unknown coefficient A_0 , it is determined by solving (2.58). Equation (2.62) is the key output equation for the plunger-type wavemaker model.

The wavemaker model is non-linear and requires six input parameters including the water flow U . The water flow, along with the wave frequency ω , will impact the value of k_p . The progressive wavenumber is also required to determine A'_1 ; therefore, A'_1 will be dependent on U and ω , as well as additional parameters including the water height h , mean wedge depth d_μ , wedge angle β , and the number of node points

M used to describe the theoretical KBC. Although (2.62) has the same form of the wavemaker model as presented by Wu [Wu88], its inclusion of the flow causes an effect on the output of the model. Fig. 2.4 plots the effect of an applied uniform flow U flowing in the direction of the propagating wave on the amplitude ratio a/s . The “flow model” represents the wavemaker model defined by (2.62) which includes

Figure 2.4: The ratio of the wave amplitude to stroke amplitude for a uniform flow as predicted by the wavemaker model which incorporates U and the standard model which does not include U . The values of the remaining input parameters were fixed at $\omega = 3.2\pi$ rad/s, $\beta = 25.7^\circ$, $d_\mu = 12$ cm, $h = 0.583$ m, and $M = 200$.

flow, while the “standard model” excludes the influence of flow. The remaining input parameters have been fixed where $\omega = 3.2\pi$ rad/s, $\beta = 25.7^\circ$, $d_\mu = 12$ cm, $h = 0.583$ m, and $M = 200$. Since the standard model does not account for the flow rate, the

value of a/s remains constant for all values of U . On the contrary, the flow model demonstrates that for a constant stroke amplitude s , the amplitude of the waves a propagating from the wavemaker will be smaller if a steady flow is included. As the flow rate is increased, the wave amplitude will further decrease under a constant stroke amplitude. The behaviour of the relationship in Fig. 2.4 is consistent for the wavemaker model; however, the exact value of a/s will also be dependent on the values of the input parameters which include the flow U , wave frequency ω , wedge angle β , mean wedge depth d_μ , water height h , and node points M . The relative importance of each input parameter on the output of the plunger-type wavemaker model can be determined through a variance-based global sensitivity analysis.

2.4 Sensitivity Analysis

A sensitivity analysis aims to quantify the relative importance of the input parameters on the outcome of a mathematical model. The input parameters are those which are not deterministic and are independent of one another. Various methods exist to determine the sensitivity of the mathematical model; the use of each method is dependent on the computational expense and the complexity of the model, among other constraints. Derivative methods are based on the partial derivative of the model taken at an operating point in the space of the input [SRA⁺08]. In derivative methods, only one variable is explored at a time to examine small perturbations in the model; thus, the derivative method is often referred to as a local method of sensitivity analysis. Although the local method can be more straight forward to implement for simple models, it only provides information on a parameter's individual influence on the model, rather than analyzing the entire space of the input parameters.

For a full exploration of the input space for a given model, global sensitivity analysis (GSA) methods exist including regression and variance based methods. Regression analysis involves fitting a linear regression to a set of model outputs for a series of Monte Carlo simulations [SRA⁺08]. The sensitivity of the output to each input parameter is thus quantified by the regression coefficients. The regression analysis can account for the entire input space but, the accuracy is limited by the degree of non-linearity of the model. For a model whose output is strongly, non-linearly dependent on the inputs, the regression analysis will fail to provide meaningful results. On the other hand, variance-based methods allow for the sensitivity analysis of models with any degree of non-linearity [SRA⁺08]. As such, variance based methods have the advantage of being model-independent. Often, regardless of the form of the model, the sensitivity analysis can be too computationally expensive. In such circumstances, additional methods have been developed to overcome the computational constraint including emulators, high-dimensional model representations, and Fourier amplitude sensitivity tests [SRA⁺08]. For the purposes of the sensitivity analysis performed in this thesis, the variance-based global sensitivity method will be used since it can fully explore the input parameter space of the mathematical model and is applicable to any model, regardless of its linearity.

Variance-based global sensitivity analysis (VBGSA) makes use of sensitivity indices S as a measure of the influence a parameter has on a model including both first- and higher-order effects. Therefore, the goal of the sensitivity analysis is to determine the sensitivity indices for each input parameter of the mathematical model. Currently, VBGSA is used across many fields with the main focus on design optimization applications [Rob18, WXZ⁺15, JVB⁺17, IDD06, HBH⁺09]. Specifically, in the marine engineering field, sensitivity analysis has been instrumental for applications such as ship design [ZLZF16, LFLZ17] and modelling ship motion [CLS⁺20, WZXR14]. The initial

concept for VBGSA was developed for use in chemistry by Cukier et al. [CFS⁺73]. The method for the analysis was referred to as the Fourier Amplitude Sensitivity Test; however, the method was difficult for practitioners to execute and did not allow for the computation of higher-order indices at the time. As such, Russian mathematician I.M. Sobol' generalized the method to provide a straightforward Monte Carlo-based implementation of the concept.

To begin Sobol's method, let the scalar output of a model Y be expressed as a function of the model's input parameters X_i where $i = 1, 2, \dots, p$ and it is assumed that each parameter is uniformly distribution in $U(0, 1)$. To analytically compute the sensitivity indices, Sobol' considered the expansion of a generic non-linear function $Y = f(X)$ into a finite number of terms expressed as [Sob93],

$$f(X) = f_0 + \sum_i f_i(X_i) + \sum_i \sum_{q>i} f_{iq}(X_i, X_q) + \dots + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p), \quad (2.63)$$

where each term is square integrable over the p -dimensional unit hypercube $\Omega = \{X \mid 0 \leq x_i \leq 1; i = 1, \dots, p\}$ and x_i represents a fixed value of the parameter X_i . The constant term f_0 is the mean of the output of f while $f_i(X_i)$ is the contribution of X_i on f . Similarly, $f_{iq}(X_i, X_q)$ are the second-order terms representing the contribution of the interaction between X_i and X_q where $\{q \in \mathbb{Z} \mid i < q \leq p\}$ on the output of f . Higher-order interaction effects are also included up to the final term $f_{1,2,\dots,p}(X_1, X_2, \dots, X_p)$ which accounts for any residual higher-order effects. It is important to note that $f(X)$ in (2.63) is not an approximation of the model through a series decomposition but rather an exact expansion containing 2^p terms.

The expansion of $f(X)$ in (2.63) is not unique; for a given model f there could be infinite choices for its terms. However, Sobol' proved that if each term except for f_0

has zero mean such that [Sob93],

$$\int f(x_i) dx_i = 0, \quad (2.64)$$

then all the terms of the expansion are orthogonal in pairs such that [Sob93],

$$\int f(x_i) f(x_q) dx_i dx_q = 0. \quad (2.65)$$

As a result of the condition expressed in (2.64), the individual terms can be calculated using the expectations of the model output Y such that [SAA⁺10],

$$f_0 = E(Y), \quad (2.66a)$$

$$f_i = E_{X_{\sim i}}(Y \mid X_i) - E(Y), \quad (2.66b)$$

$$f_{iq} = E_{X_{\sim iq}}(Y \mid X_i, X_q) - f_i - f_q - E(Y), \quad (2.66c)$$

and similarly for higher orders where $E(\cdot)$ is the expectation or mean of the argument $(\cdot)$ and $E_{X_{\sim i}}(\cdot)$ is the mean of the argument $(\cdot)$ taken over all parameters except X_i . Similarly, $E_{X_{\sim iq}}(\cdot)$ is the mean of the argument $(\cdot)$ taken over all parameters except X_{iq} . The expressions in (2.66a)-(2.66c) originate from the following integral definitions of the respective expectation terms. The term $E(Y)$ is unconditional so it is expressed as the multi-dimensional integral of $f(X)$ over all elements in X in the distribution $U(0, 1)$ [Rob18],

$$\begin{aligned} E(Y) &= \int_0^1 f(X) dX \\ &= \int_0^1 f_0 + \sum_i f_i(X_i) + \sum_i \sum_{q>i} f_{iq}(X_i, X_q) + \dots \\ &\quad + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX \end{aligned}$$

$$\begin{aligned}
&= \int_0^1 f_0 dX + \sum_i \int_0^1 f_i(X_i) dX + \sum_i \sum_{q>i} \int_0^1 f_{iq}(X_i, X_q) dX + \dots \\
&\quad + \int_0^1 f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX \\
&= f_0 \int_0^1 dX \\
&= f_0.
\end{aligned} \tag{2.67}$$

On the other hand, $E_{X_{\sim i}}(Y \mid X_i)$ is conditioned on each element X_i and expressed as the integral of $f(X)$ over the elements of X which do not include X_i [Rob18],

$$\begin{aligned}
E_{X_{\sim i}}(Y \mid X_i) &= \int_0^1 f(X) dX_{\sim i} \\
&= \int_0^1 f_0 + \sum_i f_i(X_i) + \sum_i \sum_{q>i} f_{iq}(X_i, X_q) + \dots \\
&\quad + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX_{\sim i} \\
&= \int_0^1 f_0 dX_{\sim i} + \sum_i \int_0^1 f_i(X_i) dX_{\sim i} + \sum_i \sum_{q>i} \int_0^1 f_{iq}(X_i, X_q) dX_{\sim i} \\
&\quad + \dots + \int_0^1 f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX_{\sim i} \\
&= f_0 \int_0^1 dX_{\sim i} + f_i(X_i) \int_0^1 dX_{\sim i} + \sum_{q \neq i} \int_0^1 f_q(X_q) dX_{\sim i} \\
&= f_0 + f_i(X_i).
\end{aligned} \tag{2.68}$$

Finally, $E_{X_{\sim iq}}(Y \mid X_i, X_q)$ is conditioned on the second-order interaction of X_i and X_q and is expressed as [Rob18],

$$\begin{aligned}
E_{X_{\sim iq}}(Y \mid X_i, X_q) &= \int_0^1 f(X) dX_{\sim iq} \\
&= \int_0^1 f_0 + \sum_i f_i(X_i) + \sum_i \sum_{q>i} f_{iq}(X_i, X_q) + \dots
\end{aligned}$$

$$\begin{aligned}
& + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX_{\sim iq} \\
& = \int_0^1 f_0 dX_{\sim iq} + \sum_i \int_0^1 f_i(X_i) dX_{\sim iq} + \sum_i \sum_{q>i} \int_0^1 f_{iq}(X_i, X_q) dX_{\sim iq} \\
& \quad + \dots + \int_0^1 f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) dX_{\sim iq} \\
& = f_0 \int_0^1 dX_{\sim i} + f_i(X_i) \int_0^1 dX_{\sim iq} + f_q(X_q) \int_0^1 dX_{\sim iq} \\
& \quad + \sum_{r \neq i, r \neq q} \int_0^1 f_r(X_r) dX_{\sim iq} + f_{iq}(X_i, X_q) \int_0^1 dX_{\sim iq} \\
& \quad + \sum_{r \neq i} \sum_{s>r} \int_0^1 f_{rs}(X_r, X_s) dX_{\sim iq} + \sum_{s>i, s \neq q} \int_0^1 f_{is}(X_i, X_s) dX_{\sim iq} \\
& = f_0 + f_i(X_i) + f_q(X_q) + f_{iq}(X_i, X_q). \tag{2.69}
\end{aligned}$$

Similar analytical expressions can be determined for higher-order interactions; however, only the expectations of the total, first-, and second-order terms have been given for illustrative purposes.

The variance of each term in the expansion of $f(X)$ in (2.63), which lead directly to the sensitivity indices, are thus determined by taking the variance of (2.66a)-(2.66c) as follows [SAA⁺10],

$$V(f_0) = V(E(Y)) = 0, \tag{2.70a}$$

$$V(f_i(X_i)) = V_{X_i}(E_{X_{\sim i}}(Y \mid X_i)), \tag{2.70b}$$

$$V(f_{iq}(X_i, X_q)) = V_{X_i X_q}(E_{X_{\sim iq}}(Y \mid X_i, X_q)) - V_{X_i}(E_{X_{\sim i}}(Y \mid X_i)) - V_{X_q}(E_{X_{\sim q}}(Y \mid X_q)), \tag{2.70c}$$

where, using the orthogonality condition expressed in (2.65) [Rob18],

$$V(E(Y)) = \int_0^1 (E(Y) - E(E(Y)))^2 dX$$

$$\begin{aligned}
&= \int_0^1 (f_0 - E(f_0))^2 dX \\
&= \int_0^1 (f_0 - f_0)^2 dX \\
&= 0,
\end{aligned} \tag{2.71}$$

$$\begin{aligned}
V_{X_i}(E_{X \sim i}(Y \mid X_i)) &= \int_0^1 (E_{X \sim i}(Y \mid X_i) - E(E_{X \sim i}(Y \mid X_i)))^2 dX_i \\
&= \int_0^1 ((f_0 + f_i(X_i)) - E(f_0 + f_i(X_i)))^2 dX_i \\
&= \int_0^1 (f_0 + f_i(X_i) - f_0)^2 dX_i \\
&= \int_0^1 (f_i(X_i))^2 dX_i \\
&= V(f_i(X_i)),
\end{aligned} \tag{2.72}$$

$$\begin{aligned}
V_{X_i X_q}(E_{X \sim iq}(Y \mid X_i, X_q)) &= \int_0^1 (E_{X \sim iq}(Y \mid X_i, X_q) - E(E_{X \sim iq}(Y \mid X_i, X_q)))^2 dX_{iq} \\
&= \int_0^1 ((f_0 + f_i(X_i) + f_q(X_q) + f_{iq}(X_i, X_q)) \\
&\quad - E(f_0 + f_i(X_i) + f_q(X_q) + f_{iq}(X_i, X_q)))^2 dX_{iq} \\
&= \int_0^1 (f_0 + f_i(X_i) + f_q(X_q) + f_{iq}(X_i, X_q) - f_0)^2 dX_{iq} \\
&= \int_0^1 (f_i(X_i) + f_q(X_q) + f_{iq}(X_i, X_q))^2 dX_{iq} \\
&= \int_0^1 f_i(X_i)^2 dX_i + 2 \int_0^1 f_i(X_i) f_q(X_q) dX_{iq} \\
&\quad + 2 \int_0^1 f_i(X_i) f_{iq}(X_i, X_q) dX_{iq} + \int_0^1 f_q(X_q)^2 dX_q \\
&\quad + 2 \int_0^1 f_q(X_q) f_{iq}(X_i, X_q) dX_{iq} + \int_0^1 f_{iq}(X_i, X_q)^2 dX_{iq} \\
&= V(f_i(X_i)) + V(f_q(X_q)) + V(f_{iq}(X_i, X_q)),
\end{aligned} \tag{2.73}$$

and similarly for higher order terms. The unconditional variance of the output $V(Y)$ can also be determined by [Rob18],

$$\begin{aligned}
V(Y) &= \int_0^1 (Y - E(Y))^2 dX \\
&= \int_0^1 Y^2 dX - 2E(Y) \int_0^1 Y dX + E(Y)^2 \int_0^1 dX \\
&= \int_0^1 Y^2 dX - E(Y)^2 \\
&= \int_0^1 \left(f_0 + \sum_i f_i(X_i) + \sum_i \sum_{q>i} f_{iq}(X_i, X_q) + \dots \right. \\
&\quad \left. + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p) \right)^2 dX - E(Y)^2 \\
&= \int_0^1 \left(f_0^2 + \sum_i f_i(X_i)^2 + \sum_i \sum_{q>i} f_{iq}(X_i, X_q)^2 + \dots \right. \\
&\quad \left. + f_{1,2,\dots,p}(X_1, X_2, \dots, X_p)^2 \right) dX - E(Y)^2 \\
&= f_0^2 \int_0^1 dX + \sum_i \int_0^1 f_i(X_i)^2 dX + \sum_i \sum_{q>i} \int_0^1 f_{iq}(X_i, X_q)^2 dX + \dots \\
&\quad + \int_0^1 f_{1,2,\dots,p}(X_1, X_2, \dots, X_p)^2 dX - E(Y)^2 \\
&= \sum_i \int_0^1 f_i(X_i)^2 dX + \sum_i \sum_{q>i} \int_0^1 f_{iq}(X_i, X_q)^2 dX + \dots \\
&\quad + \int_0^1 f_{1,2,\dots,p}(X_1, X_2, \dots, X_p)^2 dX \\
&= \sum_i V(f_i(X_i)) + \sum_i \sum_{q>i} V(f_{iq}(X_i, X_q)) + \dots \\
&\quad + V(f_{1,2,\dots,p}(X_1, X_2, \dots, X_p)). \tag{2.74}
\end{aligned}$$

Dividing both sides of the expression for $V(Y)$ in (2.74) by the term $V(Y)$ provides

a summation of normalized variances given by,

$$\frac{V(Y)}{V(Y)} = \sum_i \frac{V(f_i(X_i))}{V(Y)} + \sum_i \sum_{q>i} \frac{V(f_{iq}(X_i, X_q))}{V(Y)} + \dots + \frac{V(f_{1,2,\dots,p}(X_1, X_2, \dots, X_p))}{V(Y)}, \quad (2.75)$$

where each term represents a corresponding sensitivity index. The result is a summation of the sensitivity indices adding to unity expressed as [SAA⁺10],

$$1 = \sum_i S_i + \sum_i \sum_{q>i} S_{iq} + \dots + S_{1,2,\dots,p}, \quad (2.76)$$

where S_i is first-order sensitivity index of parameter X_i , S_{iq} is a second-order sensitivity index representing the interaction between parameters X_i and X_q , and so forth until $S_{1,2,\dots,p}$ which is the highest-order sensitivity index representing the interaction between all input parameters. Therefore, Sobol's theory provides a basis from which to determine the sensitivity indices for a mathematical model. However, for complex models such as the ones studied in this thesis, computation of the indices through Sobol's analytical method can be computationally prohibitive. Therefore, numerical methods, namely Monte-Carlo sampling, will be explored in Chapter 5 for estimating the sensitivity measures of the wave gauge model and the plunger-type wavemaker model.

2.5 Summary

In this chapter, the motivation and theory for producing regular waves in a laboratory water channel in the presence of flow was discussed. In order to understand the impact of flow on the regular waves, small amplitude wave theory was presented initially without the inclusion of flow. It was shown that in order to incorporate a uniform

flow into the wave theory, the dispersion relations require modification. The use of wave absorbers was also discussed in order to produce the desired wave profiles by reducing the amount of wave reflection in the water channel. The amount of reflection can be determined by separating the incident waves from the reflected waves using two or more measurement tools to be investigated through the first objective.

The wavemaker of interest in this thesis is the plunger-type wavemaker. To fulfil the third objective, a review of the theoretical and experimental uses of plunger-type wavemakers was provided in this chapter. The model which describes the relationship between the plunger system's stroke amplitude and the generated wave amplitude was presented which takes into consideration a uniform flow equal to or greater than 0. In order to further explore the theoretical model of the wavemaker and the influence of the model's input parameters, a review of sensitivity analysis methods was discussed. It was determined that a variance-based global sensitivity analysis is appropriate for the models investigated in this thesis; therefore, the underlying theory for the analysis was presented in relation to the third objective. With the background theory of this thesis covered, the previously proposed wavemaker model can be expanded and experimentally validated for the first time.

Chapter 3

Laboratory Equipment

The experimental work in thesis will be conducted in a laboratory water channel. In order to determine the change in water elevation due to the motion of the plunger-type wavemaker, the development of experimental laboratory equipment to measure the surface of the water was necessary. Therefore, this chapter will outline the design of the water channel along with its associated equipment in relation to the first objective of the thesis. Section 3.1 provides an overview of the water channel at Carleton University. The measuring equipment developed and installed in the tank is discussed in Section 3.2. In Section 3.3, the design of a wave absorber for decreasing reflectance in the water channel is discussed. A summary of the chapter is presented in Section 3.4.

3.1 Water Channel

The water channel at Carleton University is a flow visualization water channel, consisting of closed circuit for the study of marine engineering applications and hydrodynamics phenomena. The water channel, whose schematic is shown in Fig. 3.1, can

contain up to 5000 gallons of water and consists of four main sections: inflow delivery, contraction, test, and discharge [Rol14]. In the inflow delivery section, water is

Figure 3.1: Schematic of the water channel at Carleton University demonstrating the inflow, contraction, test, and discharge sections as well as the location of the wavemaker.

circulated at a flow rate up to 2800 gallons/minute to provide up to a 0.305 m/s (12.0 in/s) uniform flow [Rol14]. The water flows from the inflow and through the honeycomb structure to the contraction section which has an area ratio of 6:1. The specific geometry has been chosen by the manufacturer to provide the minimum contraction length such that the water has ideal velocity distribution, turbulence reduction, and avoidance of local separation and vorticity development [Rol14]. Following the contraction, the water enters the test section which consists of tempered glass on three sides and has internal dimensions of 0.610 m (24.0 in) in width, 0.914 m (36.0 in) in height, and 1.829 m (72.0 in) in length. The glass's high thermal conductivity allows it to efficiently conduct heat away from any nearby heat sources such as the ones used for photography or lighting conditions [Rol14]. A velocity sensor is located in

the channel's plumbing circuit and is calibrated to provide a digital readout of the test section velocity on the control panel. The fluid velocity can be adjusted by the user in increments of 0.1 in/s. Finally, the water is discharged at the end of the tank, using a geometry to insure that no flow is developed that would propagate upstream into the test section area. From the discharge section, the water flows downward into the plumbing system to be recirculated to the inflow delivery section.

The water channel provides a unidirectional fluid velocity which can be used to represent a flow during the generation of waves. The wavemaker, whose location is shown in Fig. 3.1, is mounted upstream of the test section so that the surface elevation of the water can be measured in the accessible test area. For the plunger-type wavemaker theory, only fully developed progressive waves are desired in the area of interest. McPhee [McP19] determined for distances that are double or triple the water depth h of the water channel, standing waves can be considered negligible compared to progressive waves. Thus, sensors for measuring the surface elevation should be placed at a distance of $x \geq 2h$ relative to the positioning of the wavemaker where x is the distance from the wavemaker.

3.2 Sensors for Measuring Water Elevation

There are several available types of gauges used to measure changes in the surface elevation of the water during the generation of waves in a water channel. For the experiments conducted for this thesis, the sensors chosen include a conductive wave gauge, a camera, and an ultrasonic sensor. Figure 3.2, which is not to scale, displays the relative location of each piece of hardware in the water channel, along with the positioning of a plunger-type wavemaker and a wave absorber developed for generating and absorbing waves, respectively. The conductive wave gauge, camera, and

Figure 3.2: A schematic of the water channel with the mounted ultrasonic sensor, wave gauge, camera, wave absorber, and wavemaker.

ultrasonic sensor are aligned such that each sensor measures the surface elevation of the water at the same distance x from the wavemaker. The camera is positioned outside of the tank at a vertical height similar to the height of the water h . Meanwhile, both the wave gauge and the ultrasonic sensor are located within the width of the water channel. The boundary due to the walls of the water channel is assumed to be semi-infinite; therefore, the wave gauge and ultrasonic sensor are placed in the middle of the channel to avoid wall effects. In the following subsections, the specifications and details pertaining to each measurement sensor is discussed.

3.2.1 Conductive Wave Gauge

Conductive wave gauges operate by measuring the resistance of the water between two separate wires which have been submerged at the point of interest. The resistance of the water is proportional to the immersion depth of the gauge, thus allowing for the measurement of any changes in the water level. Due to their simplistic design, conductive wave gauges are often used in laboratory water channels for measuring the rapidly changing water surface during wave generation [LKW82, TL87].

The requirement for multiple gauges in applications such as separating incident and reflective waves has lead to an increase in commercially available, but expensive gauges [DSDI18, PRIS09, SPP01]. Therefore, as a low cost solution, a conductive wave gauge was developed for the purposes of this thesis.

The constructed wave gauge, shown in Fig. 3.3, consists of two stainless steel wires held at a fixed distance apart by the plastic structure. Each wire is separately

Figure 3.3: Photograph of the conductive wave gauge developed for measuring the surface elevation of the water.

connected to a circuit such that a signal sent through one wire can be read by the second wire due to the water acting as a resistor. Thus, the performance of the wave gauge will be dependent on the electrical conductivity of the water. For example, highly ionized water like salt water will be more conductive than that of distilled water. The electrical resistivity R measured by two electrodes is related to the electrical

conductivity σ_c by the expression [MLB19],

$$R = \frac{D}{A\sigma_c}, \quad (3.1)$$

where D is the distance between the electrodes and A is the surface area of the electrodes. Therefore, as the submerged depth z_w of the wave gauge wires change, the resistance of the water R measured across the wires fluctuates through the relationship,

$$R = \frac{D}{\pi d z_w \sigma_c}, \quad (3.2)$$

where D becomes the distance between the two wires and d is the diameter of each wire. As a signal is passed through one wire, the voltage readout of the second wire is recorded as a function of time using a National Instruments DAQ system and LabVIEW. The complete circuit diagram for the conductive wave gauge can be found in Appendix B. Using a reference point on the wave gauge, the surface elevation can be deduced from the change in wire depth. Theoretically, the voltage output could be used to determine the resistance of the water; however, since the water channel does not have a method for measuring the water conductivity, the depth of the gauge cannot be directly determined from its relation to the resistance in (3.2). A calibration of the wave gauge, which relates the measured voltage to the depth of the wires in the water is therefore necessary.

The relationship between the wire depth and the output voltage was determined through a static calibration. Using an actuator positioned vertically above the tank, the voltage corresponding to a range of known depths was measured over a period of 20 s. A point midway up the wire of the gauge was set as a reference point of zero such that changes in the wire depth about that point directly signified the change

Figure 3.4: Relationship between the water depth and the voltage measured by the wave gauge for various flow rates.

the water elevation. Figure 3.4 plots the depth verses voltage calibration curves of the gauge for five significant flow rates. Based on the calibrations in Fig. 3.4, the relationship between the output voltage and the gauge depth for the developed wave gauge is quadratic; however, there is a dependency on the flow rates in the water channel. Without the presence of flow, the calibration follows a quadratic trend but there is an overall shift in the voltage. As the flow rate increases, the shift in voltage appears to converge. The discrepancy can be attributed to the design of the wave gauge and the water in the channel. Since fresh water is being used, its conductivity is relatively lower than that of oceanic water; as such, there is an accumulation of charge on the first wire that can not be dispersed quickly enough into the water. To

accommodate for the phenomenon, a square wave voltage signal is sent to the first wire. When a flow is present, the combination of the flow velocity and the square wave signal alleviates the build up of charge; however, when no flow is present, some voltage accumulation still occurs resulting in a divergent calibration dependent on the flow rate. The resulting calibration is therefore also dependent on the water conductivity which in turn is dependent on a number of factors including the ion content of the water and the water temperature. To ensure that the ion content of the water is uniformly distributed, a flow rate should be in the water channel prior to using the wave gauge. Regardless, since the conductivity can not be measured in the water channel, a constant conductivity cannot be ensured day-to-day. Due to the nature of the wave gauge, it was concluded that the gauge should not only be calibrated for each flow rate used in the experiments, but also each day that testing is conducted.

Aside from the design constraints, use of conductive wave gauges can be disadvantageous in small water channels. The gauge operates based on the requirement for it to be in direct contact with the water. As a result, the structure of the gauge disrupts the surface of the water. If such disturbances are reflected by the walls of the channel, the overall measured wave profile will be distorted. Therefore, sensors which do not require contact with the water would be more advantageous. One example of such a sensor is a camera, which can be mounted to the side of the tank and measure the wave profile from a horizontal point of view.

3.2.2 Camera

Cameras provide an indirect method for determining the wave profile in a water channel. Through calibration, the recorded pixels can be transformed into world coordinates, allowing a visual recording of both spacial and time domains. The camera

used herein is the ZED stereo vision camera from Stereolabs [Ste20]. The stereoscopic properties of the camera system allow for a wide range of applications including spatial detection, motion tracking, and depth perception. The stereo vision system has the potential for determining the wave height through depth perception by placing it above the tank; however, that application is outside the scope of this study. Instead, for the current work, only a single camera is used to determine the wave profile as a function of time by placing the system outside the channel at a height similar to that of the water.

A schematic of the stereo vision camera is shown in Fig. 3.5 with the two circles representing each camera lens. Each camera is backside illuminated with high low-

Figure 3.5: Photograph of the ZED stereo vision camera.

light sensitivity, making it advantageous for use with the indoor water channel and artificial lighting. The default depth range for the system is 0.5 m to 20 m (1.64 ft to 65 ft), but can be adjusted according to the application. The resolution of the camera and video frame rate can also be varied by the user, along with various imaging properties such as the exposure time, brightness, contrast, and saturation.

The system is aligned so that the left camera has the conductive wave gauge centred in its field of view. In doing so, the wave profile is measured by the camera at the same position along the water channel as the gauge. An LED light, which switches on when waves are being generated, was also placed in the field of view of the camera in order to align the timing of the recorded videos with signals measured by the wave gauge and the ultrasonic sensor.

For each image recorded by the camera, the changing surface of the water will be in terms of pixel coordinates relative to the camera frame. Therefore, it is critical to transform the camera's pixel coordinates into real world coordinates. The world coordinates $[X_w \ Y_w \ Z_w]$ are related to the pixel coordinates $[x_c \ y_c \ z_c]$ through the expression,

$$[x_c \ y_c \ z_c] = [X_w \ Y_w \ Z_w]R_c + t_c, \quad (3.3)$$

where R_c is the rotation matrix of the camera and t_c is the translation vector of the camera, both of which must be determined through a calibration of the camera. The camera is calibrated using a checkerboard pattern of known size placed on the side glass of the channel where the camera record the wave profile. Multiple images of the checkerboard are taken with the camera in its fixed position and using the single camera calibration tool in MATLAB, the camera's intrinsic properties are determined along with the rotation matrix R_c and translation vector t_c of the camera with respect to the checkerboard.

The desired wave profile is determined in a post-process by applying edge detection to each frame since the water surface creates a defined line along the side of the tank. An example of the edge detection applied to a single camera frame is shown in Fig. 3.6 along with the original image.

Figure 3.6: A single image frame (left) photographed with the camera with the corresponding edge detection (right).

The edge detection of the wave gauge structure and wires at $X_w = 243$ mm provides a simple reference point for the desired measuring location of the wave profile. However, at that coordinate, the wave profile is indistinguishable from the gauge wires; therefore, the measurement of the wave profile is taken at $X_w = 244$ mm. Since the wave profile will oscillate vertically at $X_w = 244$ mm, the range in Y_w is limited to $60 \text{ mm} < Y_w < 140 \text{ mm}$ in order to isolate the water surface from the other edge features found in each frame. Using the frame rate and noting when the LED switches on, the desired wave profile can be constructed as a function of time in world coordinates. Thus, the signal as measured by the camera can be compared to that measured by the conductive wave gauge and the ultrasonic sensor.

Due to its positioning to the side of the water channel, the use of the camera does have some limitations. Since the boundary of test section of the water channel is comprised of glass, the camera is able to observe the generated waves propagating through that area. However, if the wave profile was desired in another section of the channel, one where visual access from the side is not available, use of the camera would

not be applicable. In addition, while the wave gauge measures the water surface in the middle of the channel, the camera is measuring the surface at the edge of the channel. The presence of walls or finite boundaries exerts a slowing effect on the velocity of the water; therefore, the wave profile as measured by the camera may be biased due to wall effects. Considering the disadvantages and limitations to both the wave gauge and camera, an ideal sensor would be one that can does not require direct contact with water and can also be mounted vertically above the water surface. Therefore, the third sensor that was investigated for measuring the surface of the water was an ultrasonic sensor.

3.2.3 Ultrasonic Sensor

An ultrasonic sensor determines distances by measuring the time between the emission of a high frequency sound wave and the reception of the wave upon reflection off an object or target. The distance of the object z_u relative to the sensor is determined by the expression,

$$z_u = \frac{tv}{2}, \quad (3.4)$$

where t is the total time between emission and reception and v is the speed of the sound wave. An ultrasonic sensor is advantageous for the measurement of water waves because the sound waves can reflect off of a liquid surface regardless of the transparency, while also being independent of light, smoke, and dust. In addition, there is no contact between the sensor and the water, therefore it will not interfere with the generated wave profile. A photograph of the ultrasonic sensor used herein is displayed in Fig. 3.7. The ultrasonic sensor is mounted above the tank with the emitter pointing directly down towards the water surface. The sensor, manufactured

Figure 3.7: Photograph of the ultrasonic sensor.

by SICK, outputs an analog signal between 1 V and 5 V corresponding to a default detection range of 0.12 m to 1.30 m [SIC18]. The sensor also has the ability to be taught a scaled detection limit in order to increase the resolution of the output signal. Since the generated waves will be relatively small compared to the maximum default limit, the detection limit was scaled to a range of 0.1275 m to 0.3259 m. The calibration of the ultrasonic sensor is displayed in Fig. 3.8, which plots the relationship between the analog voltage output and the new scaled limit. The distance measurements in the calibration are negated due to the orientation of the ultrasonic with respect to the water surface. Since a higher voltage indicates a further distance from the sensor, crests in the voltage signal will also correspond to crests in the water. The calibration provides the profile of the generated waves as distance measurements with respect to the location of the sensor. To determine the exact amplitude of

Figure 3.8: Calibration curve for the ultrasonic sensor. Negative values indicate a further position from the ultrasonic.

the waves, the ultrasonic signal is offset by the distance between the sensor and the water surface. Although the ultrasonic is at a fixed height relative to the bottom of the water channel, its distance to the water surface varies due to fluctuations in the water height h . Therefore, the offset is determined post-process for each measured wave profile. Due to the ultrasonic sensor's dependency on the speed of sound in air, the calibration is also effected by environmental conditions such as temperature and humidity. However, since the laboratory which holds the water channel is temperature controlled, it was deemed that unlike the conductive wave gauge, daily calibration was unnecessary.

The voltage signal from the ultrasonic sensor is measured as a function of time

into LabVIEW using a National Instruments DAQ. The sensor's time measurement coincides with that of the conductive wave gauge as well as the total wave generation time; however, the sensor has a response time of 0.08 s [SIC18]. As such, a lag exists between the measured signal and the true signal. By taking the lag into account post-process, the performance of the ultrasonic sensor can be compared to the wave profiles measured by the wave gauge and camera. The non-intrusive nature of the ultrasonic sensor theoretically makes it an ideal instrument for measuring the change in the water surface elevation in a laboratory water channel. However, since the true wave profile is unknown, a cross-check between the three sensors should provide sufficient information about the wave profile as well as any further limitations or advantages to use of each sensor.

An additional phenomena in the water channel during wave generation is reflectance. Following the theory outlined in Section 2.2 for separating incident and reflected waves, a minimum of two sensors are required at different positions x in the water channel. Therefore, a second ultrasonic sensor, which has a similar calibration to the first one, will be placed further down the water channel away from the wavemaker. As such, the reflectance in the water channel can be determined without interfering with the generated wave profile. In order to minimize the reflection in the water channel, a wave absorber will be positioned in the discharge section.

3.3 Wave Absorber

A photograph of the wave absorber, whose base structure had been previously constructed by McPhee [McP19], is displayed in Fig. 3.9 with its slope pointing towards the test section. The face of the absorber is comprised of perforated PVC board which has been mounted on two sets of rails such that the angle θ between the absorber and

Figure 3.9: Photograph of the wave absorber installed at the end of the water channel with attached floating rope.

the bottom of the channel can be varied. Greslou and Mahe [GM54] reported that a lower inclined angle results in greater efficiency of the wave absorber which was verified through a series of experimental tests on the design of sloping-type wavemakers performed by Lim [Lim14]. Therefore, the inclined angle was set to 20.2° , as that is the smallest achievable angle based on the geometry of the channel and absorber. The ability to attenuate wave energy by subaquatic vegetation is well recognized and studies based on artificial vegetation is extensive [SMP⁺11]. Therefore, to further dissipate incident wave energy, white floating rope was attached to the face of the wave absorber for the experimental work in this thesis.

The performance of the wave absorber and its impact on the reflection in the water channel was investigated through a set of preliminary tests. To verify the use of the wave absorber, three scenarios were considered: (1) no wave absorber, (2)

wave absorber without rope, and (3) wave absorber with rope. Following the theory for determining the reflection coefficient r outlined in Section 2.2, two ultrasonic sensors were used to measure the wave profile at two separate locations in the water channel. For each scenario, the reflection coefficient was determined for a constant wave frequency of 1.2 Hz and constant wavemaker stroke amplitude of $s = 0.04$ m across five significant flow rates ranging from 0.000 m/s to 0.305 m/s. Three trials for each test were also conducted in order to establish repeatability. The reflection coefficients are summarized in Table 3.1 with respect to each wave absorber scenario. A reflection coefficient less than one indicates that the incident waves are larger

Table 3.1: The reflection coefficient r for three different scenarios involving the wave absorber. Scenario 1 has no wave absorber, Scenario 2 has a plain wave absorber, and Scenario 3 has a wave absorber with attached floating rope installed in the water channel.

		0.000 m/s	0.076 m/s	0.152 m/s	0.229 m/s	0.305 m/s
Scenario 1	Trial 1	0.29	0.40	1.53	1.13	0.57
	Trial 2	0.31	0.41	1.05	1.07	0.46
	Trial 3	0.32	0.34	0.63	1.02	0.92
Scenario 2	Trial 1	0.22	0.33	0.36	0.98	0.69
	Trial 2	0.21	0.39	0.28	1.06	1.20
	Trial 3	0.20	0.35	0.39	1.10	0.62
Scenario 3	Trial 1	0.21	0.19	0.44	1.02	0.62
	Trial 2	0.19	0.37	0.22	1.03	0.74
	Trial 3	0.19	0.30	0.20	1.06	0.91

than the reflected waves, with values approaching zero signifying very little reflection within the water channel. On the other hand, a reflection coefficient greater than one indicates the reflected waves have a larger height than that of the incident waves. In general, with reference to Table 3.1, the presence of the wave absorber either with or without rope results in a smaller reflection coefficient compared to no wave absorber in the water channel. Therefore, the wave absorber performs as expected by lowering the amount of wave energy reflected into the test section. Furthermore,

when considering the average reflection coefficient across each trial set, adding the floating rope also results in a lower reflection coefficient for each flow rate. On the other hand, as the flow rate increases, the reflection coefficient also tends to increase in all three scenarios, as does the variability between the trials. A possible explanation for the variability in the reflection is due to the increased turbulence at the higher flow velocities. As such, it is difficult to produce repeatable results at higher flow rates. Therefore, it is concluded that the wave absorber should be included in the water channel to reduce wave reflection into the test section, especially for high flow rates. In addition, the inclusion of the floating rope is deemed appropriate to further dissipate the wave energy at the end of the water channel.

In their study on the design of a sloping-wall-type wave absorber in a water channel, Lim [Lim14] reported that increasing the wave period increased the reflectance. Gadelho et al. [GLS⁺15] also found a similar trend when studying a flap-type wave absorber in a laboratory water channel. Therefore, a preliminary set of tests was also performed to investigate the wave absorber's dependency on wave frequency. For this set of tests, only the wave absorber with floating rope was considered. For a flow velocity of $U = 0.000$ m/s and constant wavemaker stroke amplitude of $s = 0.02$ m, the reflection in the water channel was determined for frequencies of 0.8 Hz, 1.2 Hz, 1.6 Hz, 2.0 Hz, and 2.4 Hz. The resulting reflection coefficients are plotted in Fig. 3.10 as a function of the dimensionless parameter kb , where k is the wavenumber and b is the mean width of the plunger-type wavemaker as it generates the wave profile. At $kb = 0.15$, which corresponds to the lowest tested frequency of $f = 0.8$ Hz, the reflection coefficient $r = 0.75$ is larger than all other determined values of r except for $kb = 0.59$ ($f = 1.6$ Hz). However, there is no observable trend between the frequency and the reflectance overall in Fig. 3.10 which agrees with the results reported by

Figure 3.10: The reflection coefficient for the wave absorber with rope for varying wave frequencies given as the dimensionless parameter kb .

both Lim [Lim14] and Gadelho et al [GLS⁺15]. Despite no clear dependency between the reflectance and the frequency, it is important to note that none of the resulting reflection coefficients are greater than one. Therefore, as desired, the incident waves at the tested frequencies will be larger than the reflected waves.

In addition, the previous test, whose results are presented in Table 3.1, demonstrated that for no included flow, calculation of the reflection coefficient is repeatable. As such, the reflection coefficients determined for each tested frequency are representative of the actual reflectance in the water channel and may suggest some desired operating points for the wavemaker. For example, the lower reflectance of $r = 0.21$ at both $kb = 0.34$ and $kb = 0.93$ indicates that it would be beneficial to generate waves

at $f = 1.2$ Hz and $f = 2.0$ Hz, respectively, compared to the other frequencies for $U = 0.000$ m/s. On the other hand, at $kb = 0.59$, the reflection coefficient $r = 0.93$ indicates that the reflected waves are nearly the same height as the incident waves which, due to any constructive or deconstructive interference between the waves, is undesirable while generating uniform regular waves. While the determined reflection coefficients in Fig. 3.10 are representative for $U = 0.000$ m/s, the values may increase with increasing flow while the repeatability is less reliable. Therefore, it is concluded that a more in depth study, which will be discussed in Chapter 7, would be more appropriate for evaluating the efficiency of the wave absorber. In doing so, the influence of additional parameters, such as the wavemaker stroke amplitude s and mean depth b , on the reflection coefficient can also be investigated.

3.4 Summary

In this chapter, the experimental equipment required for the water channel was discussed. An overview of the water channel was presented, which outlined the flow of water through the various sections of the channel. In particular, the test section of the water channel offers a known flow velocity which is ideal for studying and observing the generation of a desired wave profile. To evaluate the performance of the wavemaker during wave generation, sensors must be used to measure the surface of the water. Therefore, in relation to the first objective, three different sensors were discussed in this chapter which included a conductive wave gauge, a camera, and an ultrasonics sensor. The conductive wave gauge was developed as a low cost solution which relates the resistance measured across two wires submerged in the water to a change in the water height. Although wave gauges are commonly used sensors, the developed gauge requires direct contact with the water which interferes with the

generated wave pattern. In addition, a daily calibration is required of the wave gauge due to its dependency on both the conductivity of the water and the flow rate. Unlike the wave gauge, the camera is positioned outside the water channel and visually records the side profile of the generated waves. The camera used herein is a stereo vision camera; however, only a single lens was required for the experimental work in this thesis. Due to its fixed position, the camera only requires a single calibration but, in general, the area of interest must be observable from the side of the channel and only the water line along the wall of the channel can be measured. Therefore, the use of an ultrasonic sensor was also discussed, which makes use of high frequency sounds waves to determine the wave profile. The ultrasonic sensor is mounted above the water channel so it neither interferes with the wave pattern nor requires the side of the channel to be visible. Since the true wave profile is unknown, comparison of the signals measured by each sensor will provide a metric for evaluating the design of the plunger-type wavemaker while also exhibiting the advantages of each sensor.

In order to dissipate the generated wave energy before it is reflected by the end of the water channel, the use of a wave absorber was investigated. It was determined that the inclusion of the wave absorber decreases the reflection in the water channel but, the addition of floating rope onto the face absorber decreases it further. The dependency of the reflection coefficient on both flow rate and frequency was discussed in a preliminary study. It was observed that as the flow increased, the efficiency of the wave absorber decreased, as did the repeatability of the tests due to the increase in random noise introduced at higher flows. On the other hand, no observable trend could be determined with respect to the frequency for a single flow rate; however, lower reflection coefficients for specific frequencies could suggest the operating points for the wavemaker. With the experimental equipment defined, attention can now focus on the design considerations and limitations of the wavemaker.

Chapter 4

Variance-Based Global Sensitivity Analysis Theory

A sensitivity analysis of a mathematical model determines the influence of the input parameters on the model's output which can provide insight to the design of the model and its applications. Due to the complexity of the plunger-type wavemaker model, a variance-based global sensitivity analysis was chosen. To achieve the second objective of this thesis, the sensitivity indices pertaining to each input parameter of the model must be determined. Therefore, this chapter introduces the general methodology to numerically estimate a model's sensitivity indices. The theory for numerically estimating the sensitivity indices in a variance-based global sensitivity analysis is presented in Section 4.1 followed by a validation of the code using the Ishigami function in Section 4.2. Finally, a summary of the findings is given in Section 4.3. Parts of this chapter are also available in the work completed by Lowell and Irani [LI20].

4.1 Sensitivity Indices

Variance-based global sensitivity analysis makes use of sensitivity indices as a metric of determining the relative importance of the input parameters. For the sensitivity analysis, a model can be represented by the form,

$$Y = f(X_1, X_2, \dots, X_p), \quad (4.1)$$

where the output Y is a scalar and X_i represents the model's input parameters where $i = 1, 2, \dots, p$. The first order sensitivity index S_i for a given parameter X_i is the ratio of the partial variance V_i taken over X_i to the overall variance of the model $V(Y)$ and can be expressed as,

$$S_i = \frac{V_i}{V} = \frac{V_{X_i}(E_{X_{\sim i}}(Y|X_i))}{V(Y)}, \quad (4.2)$$

where E is the inner expectation evaluated for the output Y , conditioned on the parameter X_i being fixed to some value x_i^* while the elements of $X_{\sim i}$ are varied. Here $X_{\sim i}$ represents the set of parameters X that does not include X_i . V_{X_i} is the outer variance of the inner expectation while $V(Y)$ is the variance of the output when all parameters X are varied. The first order indices S_i represented by (4.2) are the independent contribution of the variance of each input parameter, but do not account for the interactions between multiple factors, or higher order effects. For example, the variance of the output for the second order effects of X_i and X_q where $\{q \in \mathbb{Z} \mid i < q \leq p\}$, gives a measure of the variance attributed to the independent effects of varying X_i and X_q , as well as the interaction effects of varying both parameters simultaneously. The sensitivity index S_{iq} represents the second order

effect between X_i and X_q and is determined by,

$$\begin{aligned}
 S_i + S_q + S_{iq} &= \frac{V_{X_i X_q}(E_{X_{\sim i} X_{\sim q}}(Y|X_i, X_q))}{V(Y)} \\
 S_{iq} &= \frac{V_{X_i X_q}(E_{X_{\sim i} X_{\sim q}}(Y|X_i, X_q))}{V(Y)} - S_i - S_q \\
 S_{iq} &= \frac{V_{X_i X_q}(E_{X_{\sim i} X_{\sim q}}(Y|X_i, X_q))}{V(Y)} - \frac{V_{X_i}(E_{X_{\sim i}}(Y|X_i))}{V(Y)} - \frac{V_{X_q}(E_{X_{\sim q}}(Y|X_q))}{V(Y)}.
 \end{aligned} \tag{4.3}$$

Higher interactions, up the p^{th} order also exist; therefore, to account for all interactions due to p input parameters, a total of $(2^p - 1)$ sensitivity indices are required, the sum of which are equal to unity such that,

$$\sum_{i=1} S_i + \sum_{i=1} \sum_{q>i} S_{iq} + \sum_{i=1} \sum_{q>i} \sum_{r>q} S_{iqr} + \dots + S_{123\dots p} = 1, \tag{4.4}$$

where S_{iqr} represents the combined effect of X_i , X_q , and X_r where $\{r \in \mathbb{Z} \mid i < q < r \leq p\}$ and so forth for higher indices until $S_{123\dots p}$, which represents the sensitivity index due to combined effect of all parameters. The total number of sensitivity indices can very quickly become large with increased number of input parameters. For example, in analysing the wave gauge model, which has four input parameters, $(2^4 - 1) = 15$ sensitivity indices exist. On the other hand, for the plunger-type wavemaker model, which has six input parameters, the number of indices raises to $(2^6 - 1) = 63$. As the number of input parameters increases for a given model, the number of required indices can become computationally prohibitive. Therefore, total effect sensitivity indices S_{Ti} [HS96] are calculated to account for the first and higher order interactions of parameter X_i and are given by,

$$S_{Ti} = \frac{E_{X_{\sim i}}(V_{X_i}(Y|X_{\sim i}))}{V(Y)} = 1 - \frac{V_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))}{V(Y)}, \tag{4.5}$$

where $V_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))$ is the first order effect of $X_{\sim i}$. Thus, subtracting $V_{X_{\sim i}}(E_{X_i}(Y|X_{\sim i}))$ from $V(Y)$ gives the contribution of all terms in the variance decomposition which include X_i such that,

$$S_{T_i} = S_i + \sum_{q \neq i} S_{iq} + \sum_{q \neq i} \sum_{r > q} S_{iqr} + \dots + S_{123\dots p}. \quad (4.6)$$

Using the approach of total effect sensitivity indices, only $2p$ sensitivity indices are required, S_i and S_{T_i} for $i = 1, \dots, p$. Therefore, for the wave gauge model, there will be a total of $2 \times 4 = 8$ sensitivity indices to examine rather than 15. Similarly, the plunger-type wavemaker will now only require a total of $2 \times 6 = 12$ sensitivity indices instead of 63.

For complex models, computation of the sensitivity indices through the analytical methods outlined in Section 2.4 can be computationally prohibitive. Therefore, numerical methods, namely Monte-Carlo sampling, will be explored for estimating the first order and total effect sensitivity indices of the models in this thesis.

In the Monte Carlo sampling based approach, a set of N_s samples are generated for each input parameter X_i from a uniform distribution of each parameter. The samples can be presented in the form of a matrix as,

$$X = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \dots & x_1^{(p)} \\ x_2^{(1)} & x_2^{(2)} & \dots & x_2^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_s}^{(1)} & x_{N_s}^{(2)} & \dots & x_{N_s}^{(p)} \end{bmatrix}, \quad (4.7)$$

where $x_j^{(i)}$ represents a single sample of the parameter $X^{(i)}$ for $i = 1, 2, \dots, p$ and $j = 1, 2, \dots, N_s$. Evaluating the model f for all $X_j = [x_j^{(1)}, \dots, x_j^{(p)}]$ produces an

$N_s \times 1$ vector of realizations of the model output Y given as,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_s} \end{bmatrix}. \quad (4.8)$$

The value of N_s therefore indicates the number of model evaluations, and must be reasonably large in order to obtain reliable results. Although the large value of N_s can lead to computational expensive simulations, it allows for the unconditional variance $V(Y)$ to be estimated as,

$$V(Y) \approx \frac{1}{N_s} \sum_{j=1}^{N_s} f(X_j)^2 - f_0^2, \quad (4.9)$$

where f_0 is the mean of the model output approximated by,

$$f_0 \approx \frac{1}{N_s} \sum_{j=1}^{N_s} f(X_j). \quad (4.10)$$

Calculation of (4.9) determines the denominator in both the first order (4.2) and total effect (4.5) sensitivity indices; however, resolving the numerators of these expressions are far more complex. For example, to calculate the variance of a first order sensitivity index, N_s function evaluations are required per inner expectation $E_{X_{\sim i}}$. N_s inner expectations are then required per outer variance calculation, resulting in a total of $N_s \times N_s$ function evaluations. Since N_s must initially be sufficiently large, the computational cost of a complex model may be too expensive.

To reduce the computational cost, Saltelli [Sal02] proposed a more efficient Monte Carlo sampling method. For the Monte Carlo analysis, $\mathbf{G}$ and $\mathbf{H}$ are two independent

matrices whose elements are comprised of g_{ji} and h_{ji} , where $j = 1, 2, \dots, N_s$,

$$\mathbf{G} = \begin{bmatrix} x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(p)} \\ x_2^{(1)} & x_2^{(2)} & \cdots & x_2^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_s}^{(1)} & x_{N_s}^{(2)} & \cdots & x_{N_s}^{(p)} \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} x_{N_s+1}^{(1)} & x_{N_s+1}^{(2)} & \cdots & x_{N_s+1}^{(p)} \\ x_{N_s+2}^{(1)} & x_{N_s+2}^{(2)} & \cdots & x_{N_s+2}^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{2N_s}^{(1)} & x_{2N_s}^{(2)} & \cdots & x_{2N_s}^{(p)} \end{bmatrix}. \quad (4.11)$$

Based on the sampling matrices $\mathbf{G}$ and $\mathbf{H}$, the unconditional variance $V(Y)$ in (4.9) can be estimated by,

$$V(Y) = \frac{1}{N_s} \sum_{j=1}^{N_s} (f(\mathbf{G})_j)^2 - \frac{1}{N_s} \sum_{j=1}^{N_s} f(\mathbf{G})_j f(\mathbf{H})_j, \quad (4.12)$$

where $f(\cdot)_j$ is the model evaluation of the j^{th} row of each matrix. Without requiring any further sampling, a third matrix is constructed denoted $\mathbf{G}_{\mathbf{H}}$ which is equal to $\mathbf{G}$ but with the i^{th} column replaced by the i^{th} column from $\mathbf{H}$ such that,

$$\mathbf{G}_{\mathbf{H}} = \begin{bmatrix} x_1^{(1)} & \cdots & x_{(N_s+1)}^{(i)} & \cdots & x_1^{(p)} \\ x_2^{(1)} & \cdots & x_{(N_s+2)}^{(i)} & \cdots & x_2^{(p)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{N_s}^{(1)} & \cdots & x_{N_s}^{(i)} & \cdots & x_{N_s}^{(p)} \end{bmatrix}. \quad (4.13)$$

Thus, the normalized first order sensitivity index S_i for each input parameter is numerically estimated by [SAA⁺10],

$$S_i = \frac{V_i}{V(Y)} = \frac{\frac{1}{N_s} \sum_{j=1}^{N_s} f(\mathbf{H})_j (f(\mathbf{G}_{\mathbf{H}}^{(i)})_j - f(\mathbf{G})_j)}{\frac{1}{N_s} \sum_{j=1}^{N_s} (f(\mathbf{G})_j)^2 - \frac{1}{N_s} \sum_{j=1}^{N_s} f(\mathbf{G})_j f(\mathbf{H})_j}, \quad (4.14)$$

where the form of the numerator has been determined by Saltelli *et al.* [SAA⁺10]

as the most accurate estimate of $V_i = V_{X_i}(E_{X_{\sim i}}(Y|X_i))$. In the numerator of S_i in (4.14), the scalar product of $f(\mathbf{H})f(\mathbf{G})$ is the mean of the output model while the influence of the parameter on the model output is dependent on $f(\mathbf{H})f(\mathbf{G}_{\mathbf{H}}^{(i)})$. In the latter term, $f(\mathbf{H})f(\mathbf{G}_{\mathbf{H}}^{(i)})$, the model evaluation Y corresponding to each row of $\mathbf{H}$ is multiplied by the value of Y from $\mathbf{G}_{\mathbf{H}}$. In comparison to $\mathbf{H}$, each of the parameters in $\mathbf{G}_{\mathbf{H}}$ except X_i are resampled while the specific values of X_i are fixed. Therefore, if X_i is a non-influential parameter, the high and low values of $f(\mathbf{H})$ and $f(\mathbf{G}_{\mathbf{H}}^{(i)})$ will be randomly multiplied by one another [SRA⁺08]. On the other hand, if X_i is an influential parameter, the high values of $f(\mathbf{H})$ will be preferentially multiplied by high values of $f(\mathbf{G}_{\mathbf{H}}^{(i)})$ [SRA⁺08]. Thus, the resulting scalar product of $f(\mathbf{H})f(\mathbf{G}_{\mathbf{H}}^{(i)})$ will increase along with the value of V_i . In a similar manner, the total effect sensitivity index S_{Ti} can be estimated by [Jan99, SAA⁺10],

$$S_{Ti} = \frac{V_{Ti}}{V(Y)} = \frac{\frac{1}{2N_s} \sum_{j=1}^{N_s} \left(f(\mathbf{G})_j - f(\mathbf{G}_{\mathbf{H}}^{(i)})_j \right)^2}{\frac{1}{N_s} \sum_{j=1}^{N_s} \left(f(\mathbf{G})_j \right)^2 - \frac{1}{N_s} \sum_{j=1}^{N_s} f(\mathbf{G})_j f(\mathbf{H})_j}, \quad (4.15)$$

where a similar reasoning can be applied for the form of the numerator as with S_i in (4.14). The value of S_{Ti} can also be used to directly differentiate between influential and non-influential parameters. For instance, a total effect index of zero indicates that a parameter can take on any value without significantly impacting the model output.

A total of $N_s(p+2)$ model evaluations are thus required to compute the first order and total effect sensitivity indices for p independent input parameters. Compared to the N_s^2 evaluations using the original Monte Carlo method, the sampling method developed by [Sal02] significantly decreases the required computational expense. The accuracy of Monte Carlo analysis can also be taken into account by determining the error associated with the estimation of S_i in (4.2) and S_{Ti} in (4.5). We can now

examine the uncertainty in the sensitivity indices.

Since the sensitivity of both models to their input parameters were determined within a reasonable computational time, the use of emulators and other computationally more efficient methods was deemed unnecessary.

4.1.1 Index Uncertainty

The uncertainty in the estimation of each sensitivity index can be determined by calculating its respective confidence interval. In this thesis, 95% confidence intervals on the first order variance V_i and total effect variance V_{Ti} have been calculated using a numerical method [SRA⁺08] based on the form of the numerators in (4.14) and (4.15). Since the solution presented by Saltelli *et al.* [SRA⁺08] is based on previous forms of the estimators for S_i and S_{Ti} , the uncertainty equations have been modified for the present study. The variance of a given distribution σ^2 is given as [Gri95],

$$\sigma^2 = \langle (\Delta x)^2 \rangle, \quad (4.16)$$

where σ is the standard deviation, the brackets $\langle \cdot \rangle$ represent the mean of the inner value, and Δx is the deviation of a given scalar term x from the average defined by,

$$\Delta x = x - \langle x \rangle. \quad (4.17)$$

The variance can be expressed in terms of the probability of x occurring $P(x)$ by,

$$\begin{aligned} \sigma^2 &= \langle (\Delta x)^2 \rangle = \sum (\Delta x)^2 P(x) \\ &= \sum (x - \langle x \rangle)^2 P(x) \\ &= \sum (x^2 - 2x\langle x \rangle + \langle x \rangle^2) P(x) \end{aligned}$$

$$= \sum x^2 P(x) - 2\langle x \rangle \sum x P(x) + \langle x \rangle^2 \sum P(x). \quad (4.18)$$

For a uniform probability distribution, the following identities hold true [Gri95],

$$\langle x \rangle = \sum x P(x), \quad (4.19)$$

$$\langle x^2 \rangle = \sum x^2 P(x), \quad (4.20)$$

$$1 = \sum P(x). \quad (4.21)$$

Equation (4.18) can thus be rewritten as,

$$\begin{aligned} \sigma^2 &= \langle x^2 \rangle - 2\langle x \rangle \langle x \rangle + \langle x \rangle^2 \\ &= \langle x^2 \rangle - \langle x \rangle^2. \end{aligned} \quad (4.22)$$

Taking the square root of (4.22) results in the standard deviation who's 95% confidence interval can be determined by multiplying by $1.96/\sqrt{N_s}$. Thus, the modified 95% confidence intervals for the first order δV_i and total effect δV_{Ti} variances are given by,

$$\begin{aligned} \delta V_i &= \frac{1.96}{\sqrt{N_s}} \left(\frac{1}{N_s} \sum_{j=1}^{N_s} \left[f(\mathbf{H})_j (f(\mathbf{G}_{\mathbf{H}}^{(i)})_j - f(\mathbf{G})_j) \right]^2 - \right. \\ &\quad \left. \left[\frac{1}{N_s} \sum_{j=1}^{N_s} f(\mathbf{H})_j (f(\mathbf{G}_{\mathbf{H}}^{(i)})_j - f(\mathbf{G})_j) \right]^2 \right)^{1/2}, \end{aligned} \quad (4.23)$$

$$\begin{aligned} \delta V_{Ti} &= \frac{1.96}{\sqrt{N_s}} \left(\frac{1}{2N_s} \sum_{j=1}^{N_s} \left[\left(f(\mathbf{G})_j - f(\mathbf{G}_{\mathbf{H}}^{(i)})_j \right)^2 \right]^2 - \right. \\ &\quad \left. \left[\frac{1}{2N_s} \sum_{j=1}^{N_s} \left(f(\mathbf{G})_j - f(\mathbf{G}_{\mathbf{H}}^{(i)})_j \right)^2 \right]^2 \right)^{1/2}. \end{aligned} \quad (4.24)$$

Similarly, the confidence interval for the unconditional variance δV is expressed as,

$$\delta V = \frac{1.96}{\sqrt{N_s}} \left(\frac{1}{N_s} \sum_{j=1}^{N_s} \left[(f(\mathbf{G})_j)^2 \right]^2 - \left[\frac{1}{N_s} \sum_{j=1}^{N_s} (f(\mathbf{G})_j)^2 \right]^2 \right)^{1/2}. \quad (4.25)$$

Applying error propagation theory to S_i (4.14) and S_{Ti} (4.15), the uncertainty associated with the first order δS_i and total effect δS_{Ti} sensitivity indices therefore take the following forms,

$$\delta S_i = \left[\left(\frac{\delta V_i}{V} \right)^2 + \left(\frac{V_i \delta V}{V^2} \right)^2 \right]^{1/2}, \quad (4.26)$$

$$\delta S_{Ti} = \left[\left(\frac{\delta V_{Ti}}{V} \right)^2 + \left(\frac{V_{Ti} \delta V}{V^2} \right)^2 \right]^{1/2}. \quad (4.27)$$

To verify the uncertainty calculations determined with the numerical method, a bootstrap resampling method was also used to calculate 95% confidence intervals on the sensitivity indices. The N_s model evaluations associated with each sensitivity index are sampled N_B times with replacement, where $N_B = 2000$ for the bootstrap resampling in this thesis. For each set of resamples, the sensitivity index is recalculated, yielding an overall distribution of bootstrap estimates for each index. Using the empirical bootstrap method, the 95% confidence intervals can be determined from the new distribution of each first order and total effect sensitivity index represented by the variables $\delta_B S_i$ and $\delta_B S_{Ti}$. Matlab was used to apply the Monte Carlo analysis and estimate the sensitivity indices for the models presented in this thesis. A validation of the code is thus presented in next section.

4.2 Code Validation - Ishigami Function

The sensitivity indices of the Ishigami function have been analytically determined and thus, the performance of the global sensitivity analysis implemented in the current thesis can be validated. The Ishigami function is given as,

$$f(X_i) = \sin(X_1) + b_1 \sin^2(X_2) + b_2 X_3^4 \sin(X_1), \quad (4.28)$$

where the parameter $b_1 = 7$ and b_2 is typically equal to either 0.1 or 0.05, depending on the literature [HS96,SL99]. In this thesis, the fixed parameters have been set to $b_1 = 7$ and $b_2 = 0.05$. The uniform distribution of each input parameter X_i ranges from $-\pi \leq x_i \leq \pi$. The Ishigami function is a benchmark for sensitivity analyses since all indices can be computed analytically and there is peculiar result in that $S_3 = 0$ yet S_{T3} cannot be neglected. Thus, the purpose of the Ishigami function example is to examine how the sensitivity estimators represented by S_i (4.14) and S_{Ti} (4.15) perform using the Monte Carlo computation compared to the exact analytical values. The total variance V of the model and its partial variances can be obtained from the relations [SRA⁺08]:

$$V = \frac{a^2}{8} + \frac{b\pi^4}{5} + \frac{b^2\pi^8}{18} + \frac{1}{2}, \quad (4.29)$$

$$V_1 = \frac{b\pi^4}{5} + \frac{b^2\pi^8}{50} + \frac{1}{2}, \quad (4.30)$$

$$V_2 = \frac{a^2}{8}, \quad (4.31)$$

$$V_3 = 0, \quad (4.32)$$

$$V_{12} = 0, \quad (4.33)$$

$$V_{13} = \frac{b^2\pi^8}{18} - \frac{b^2\pi^8}{50}, \quad (4.34)$$

$$V_{23} = 0, \quad (4.35)$$

$$V_{123} = 0. \quad (4.36)$$

Table 4.1 provides the results of the sensitivity analysis obtained through the Monte Carlo sampling method using $N_s = 50000$, along with the exact values and the uncertainty associated with each estimated sensitivity index. The sensitivity

Table 4.1: Estimated and exact sensitivity indices for the Ishigami model with error on the estimated indices.

Sensitivity Index		Exact S [%]	Estimated S [%]	δS [%]	$\delta_B S$ [%]
First Order	S_1	21.85	21.50	± 1.18	± 1.21
	S_2	68.69	69.36	± 2.21	± 2.28
	S_3	0.000	-00.60	± 0.63	± 0.63
Total Effect	S_{T1}	31.31	30.77	± 1.17	± 0.75
	S_{T2}	68.69	68.79	± 1.94	± 1.28
	S_{T3}	9.46	9.41	± 0.33	± 0.23

analysis for the Ishigami function reveals that S_2 , which is the first order sensitivity index for parameter X_2 , has the largest individual influence on the output variance of the function. Since S_2 is approximately equal to S_{T2} in the estimation, there is no interaction between X_2 and the remaining parameters. On the other hand, the contrast between S_1 and S_{T1} signifies that X_1 does have some interaction with the other parameters. Since it has already been established that X_2 does not interact with the other parameters, X_1 must be interacting with X_3 . Finally, the first order sensitivity index for X_3 is estimated as $-0.60\% \pm 0.63\%$. Theoretically, S_i and S_{Ti} must be greater than or equal to zero for the input parameters of any given model; however, since the sensitivity indices from the variance based sensitivity analysis are estimated, negative values close to zero can occur. In the context of S_3 , the negative value is simply a result of the approximation and its error places the estimation within range of its exact value of 0. $S_3 = 0$ indicates that X_3 has no influence on the output variance of the Ishigami function; however, since $S_{T3} > 0$, the parameter does

interact with the other parameters in the model which agrees with previous results determined by examining X_1 . The estimated values for the sensitivity indices are all in good agreement with the exact values determined analytically. Therefore, the authors' are confident that the methods used herein for the sensitivity analysis of the wave gauge and wavemaker are valid.

Comparing the sensitivity index uncertainty methods, the first order sensitivity errors are in good agreement between the two methods; however, the total effect indices are overestimated by the numerical method δS_{Ti} compared to the bootstrap averages $\delta_B S_{Ti}$. To verify the accuracy of the uncertainty methods, Fig. 4.1 displays the convergence of the first order and total effect sensitivity index uncertainties for each of the input parameters in the Ishigami function. The rate of convergence for the numerical and bootstrap method agree for both the first order δS_i and total effect δS_{Ti} uncertainties. For the total effect error, the overestimation by the numerical method is observed throughout the duration of the convergence. Similar results were observed in [JVB⁺17] for the uncertainty estimate of S_{Ti} ; however, the form of S_{Ti} (4.15) was presented differently. One advantage that was found to using the numerical method over the bootstrapping method was a decrease in the computational cost of determining the error of the sensitivity indices. In the numerical method, the pre-existing model evaluations are used to directly compute the uncertainty on the sensitivity indices using S_i (4.26) and S_{Ti} (4.27). Although the bootstrap method also does not require any further model evaluations, the N_s realizations for each input parameter must be resampled N_B times. For the bootstrap confidence interval to be sufficiently accurate, a large value of N_B must be chosen which can add to the computational expense, especially for large N_s . Therefore, despite the inconsistency in the total effect uncertainty, the numerical method will be used as a computationally

Figure 4.1: Convergence of the numerical and bootstrap sensitivity index uncertainty for the Ishigami model's first order (top) and total effect (bottom) sensitivity indices.

efficient guide for determining the uncertainty in the sensitivity indices for the models presented in the following sections.

4.3 Summary

The methodology for estimating the sensitivity indices in a variance-based global sensitivity analysis was described. Using Monte Carlo simulations, numerical expressions for the first order and total effect sensitivity indices were presented. Two methods for calculating the uncertainty in the sensitivity index estimates were also included in the chapter; the first based on the variance in the numerical estimators, and the

second using a bootstrapping method. To validate the developed MATLAB code, an example sensitivity analysis was performed with the Ishigami function. The results of the analysis demonstrated that the estimated sensitivity indices agreed with the exact indices that were determined analytically within the bounds of the calculated uncertainty. Comparing the two uncertainty methods, it was concluded that due the lower computational cost, the numerical uncertainty method is more advantageous for the sensitivity analyses performed herein. With the numerical estimators established and the code validated, the sensitivity indices for both the conductive wave gauge model and the plunger-type wavemaker model can be determined.

Chapter 5

Sensitivity Analysis Results

Using the numerical estimators presented in the previous chapter, the first order and total effect sensitivity indices for the mathematical model's used in this thesis can be determined. In doing so, the influence of the model's input parameters on the variance of its output can be established in order to fulfil the second objective. The results of the global sensitivity analysis are presented in Section 5.1 for the conductive wave gauge model. In Section 5.2, the sensitivity analysis is applied to the theoretical plunger-type wavemaker for a general case, followed by a specific case that pertains the wavemaker developed for the water channel in Carleton University. A summary of the findings is discussed in Section 5.3. The results in this chapter are also available in the work completed by Lowell and Irani [LI20].

5.1 Sensitivity Analysis of Wave Gauge Model

The following section presents the results of the variance-based global sensitivity analysis applied to the conductive wave gauge model. To recall, the wave gauge constructed for use in the water channel at Carleton University is modelled by [MLB19],

$$R = \frac{D}{\pi d z_w \sigma_c}, \quad (5.1)$$

where R is the measured electrical resistance of the water, D is the distance between the two wires, d is the diameter of each wire, z_w is the depth of the wires in the water, and σ_c is the conductivity of the water. For the sensitivity analysis, Table 5.1 outlines the uniform distribution chosen for each parameter based on the geometry of the wave gauge. The distribution range for the distance between the wires D and

Table 5.1: The uniform distributions of each input parameter for the conductive wave gauge model.

Input Parameter	Minimum	Maximum
D	0.0018 [m]	0.0022 [m]
d	3.64×10^{-4} [m]	4.44×10^{-4} [m]
z_w	0.056 [m]	0.256 [m]
σ_c	0.01152 [S/m]	0.02952 [S/m]

the diameter of the wires d was chosen based off the mean value of each parameter along with a 10% uncertainty buffer on either side of the value. The mean values for D and d are represented by the measured values of the physical gauge where the distance between the wires is 0.0020 m and the diameter of each wire is 4.04×10^{-4} m. The range of the wire depth is based on the length of wire that could potentially be exposed to the water where the zero-point on the gauge is taken at a length of 0.156 m of wire. Finally, the range in water conductivity is based on the estimated experimental range of conductivity for the water equal to 0.0144 - 0.0246 S/m with

a 20% buffer on either side.

The first order and total effect sensitivity indices and their associated numerical error are presented in Table 5.2 as percentages of the total output variance of the conductive wave gauge model. The wire depth z_w was determined by the sensitivity

Table 5.2: Percent sensitivity indices and error for the conductive wave gauge model.

Parameter	S_i [%]	δS_i [%]	S_{Ti} [%]	δS_{Ti} [%]
D	1.01	± 0.30	1.40	± 0.08
d	1.08	± 0.30	1.42	± 0.08
z_w	65.33	± 4.06	71.70	± 3.78
σ_c	24.00	± 1.88	29.49	± 1.60

analysis to have the highest influence on the output variance of the wave gauge model. The first order index S_{z_w} accounts for $65.33 \pm 4.06\%$ of the variance in the model output while the total effect of z_w was determined to account for $71.70 \pm 3.78\%$. The conductivity of the water has the second highest impact on the wave gauge with a first order index equal to $24.00 \pm 1.88\%$ and total effect index equal to $29.49 \pm 1.60\%$. On the other hand, the output variance of the model is not greatly impacted by either the wire diameter d or the wire distance D whose total effect sensitivity indices were $1.42 \pm 0.08\%$ and $1.40 \pm 0.08\%$. The high first order values for the wave gauge parameters also indicate that variance in the output of the model is not greatly due to interactions between the parameters but, rather it is the individual contribution of each parameter.

From a design standpoint, it is important to identify that the conductivity of the water has the second highest impact on the measured resistance. Water conductivity is impacted by both salinity and temperature changes, neither of which are actively monitored in the water channel used for the experimental work. Therefore, to account for any unknown changes in the conductivity, the wave gauge should be calibrated

each day of testing to avoid erroneous result. The high dependency on the wire depth also indicates that the depth measured during the calibration should be as precise as possible. On the other hand, since the influence of the wire distance is low, small fluctuations in the distance that may occur due to turbulent flow of the water can be disregarded.

5.2 Sensitivity Analysis of Wavemaker Model

The following chapter section presents the results of the variance-based global sensitivity analysis applied to the theoretical plunger-type wavemaker model. The model for the wavemaker which includes the effect of a steady flow is given by its amplitude to stroke ratio a/s expressed as [McP19],

$$\frac{a}{s} = \left| -i_m A'_1 k_p h \sinh(k_p h) \right|, \quad (5.2)$$

where i_m is the imaginary number, A'_1 is a coefficient determined in the semi-analytical analysis, k_p is the progressive wavenumber, and h is the still water height of the water. As shown in the derivation of the model in Section 2.3.1, the model is dependent on the flow U , wave frequency ω , wedge angle β , wedge mean depth d_μ , water height h , and number of nodes points M . Plunger-type wavemakers are versatile as the wavemaker can be outfitted with different plunger sizes and shapes, depending on the desired wave profiles to be replicated. To account for various plunger sizes and flow rates, the global sensitivity analysis was conducted using a broad range for each input parameter's sample distribution in the Monte Carlo analysis. The uniform distribution for the six input parameters U , ω , β , d_μ , h , and M are outlined in Table 5.3. The range of 50-400 for the number of boundary node points M is based on the range used by

Table 5.3: Uniform parameter distributions for plunger-type wavemaker.

Parameter	Minimum	Maximum
U	0.0 [m/s]	2.5 [m/s]
ω	0.2π [rad/s]	10π [rad/s]
β	10 [Deg]	60 [Deg]
d_μ	0.01 [m]	0.30 [m]
h	0.30 [m]	2.00 [m]
M	50	400

Wu [Wu88], who examined the change in wavemaker transfer function a/s for $M = 50, 100, 200$, and 400 . The analysis completed by Wu [Wu88] concluded that $M = 200$ was an appropriate choice to achieve accurate results with the wavemaker model. The remaining parameter distribution ranges represent values that are typically observed in laboratory environments, incorporating various geometries of both the wedge and water channel along with a broad range of wave frequencies ω and flow rates U . Using the sample distributions presented in Table 5.3, the sensitivity analysis was conducted on the new wavemaker model and the first order and total effect sensitivity indices were determined for each parameter along with the uncertainty on each estimate from S_i (4.26) and S_{Ti} (4.27).

The sensitivity indices and their associated numerical error for the general case are presented in Table 5.4 as percentages of the total output variance of the plunger-type wavemaker model. The frequency ω was determined by the sensitivity analysis

Table 5.4: Percent sensitivity indices and error for the plunger-type wavemaker model.

Parameter	S_i [%]	δS_i [%]	S_{Ti} [%]	δS_{Ti} [%]
U	0.74	± 0.30	6.84	± 0.16
ω	28.47	± 0.81	44.86	± 0.99
β	16.21	± 0.61	25.90	± 0.56
d_μ	16.90	± 0.60	24.21	± 0.55
h	0.99	± 0.19	2.45	± 0.10
M	0.02	± 0.03	0.06	± 0.00

to have the highest impact on the output variance of the wavemaker model. The first order effect S_ω accounts for $28.47 \pm 0.81\%$ of the uncertainty in the model, while its total effect sensitivity index $S_{T\omega}$ is equal to $44.86 \pm 0.99\%$. The output variance of the wavemaker model is also greatly impacted by both the wedge's angle β and mean depth d_μ whose total effect sensitivity indices were $25.90 \pm 0.56\%$ and $24.21 \pm 0.55\%$, respectively. Comparatively, the height of the water h has a low impact on the output variance of the wavemaker. The first order sensitivity for h was estimated to be $S_h = 0.99 \pm 0.19\%$ and its total effect sensitivity $S_{Th} = 2.45 \pm 0.10\%$. However, the number of boundary node points M has the lowest sensitivity indices where $S_M = 0.02 \pm 0.03\%$ and $S_{TM} = 0.06 \pm 0.00\%$. Therefore, M has the least amount of influence on the output variance of the wavemaker model.

For the presented parameter distributions, the flow U independently accounts for $0.74 \pm 0.30\%$ of the output variance of the wavemaker model as given by the first order sensitivity index S_U . The total effect of the flow, which includes the first order index and the interaction of U with the other parameters accounts for $6.84 \pm 0.16\%$ of the model's output variance. Despite the relatively low influence of U , the experimental testing in Section 4 demonstrated an observable decrease in the wave amplitude to stroke amplitude ratio a/S for increasing flow. From Table 5.4, it can be concluded that the effect was mostly due to the total interaction of U with the other input parameters determined by $S_{TU} - S_U = 6.10 \pm 0.34\%$. Since the majority of the impact of the flow rate on the wavemaker amplitude ratio is due to the higher order effects in S_{TU} , inclusion of the water flow as an uncertain input parameter in the wavemaker model is essential.

To demonstrate the influence of the flow on the other wavemaker parameters, the sensitivity analysis was rerun for three fixed values of U equal to 0, 1, and 2 m/s. Fig. 5.1 displays the first order sensitivity indices S_i of the input parameters ω , β , d_μ , h ,

and M for each fixed value of U . As the fixed value of the flow is increased from 0

Figure 5.1: The first order sensitivity indices of the remaining input parameters when flow is not included in the sensitivity analysis.

m/s to 1 m/s to 2 m/s, the first order sensitivity index for the frequency S_ω increased from 8.32% to 34.97% to 41.07%. The first order indices also increased for β and d_μ , where for $U = 2$ m/s, the influence of the parameters inverted resulting in the mean wedge depth having a larger influence on the model the wedge angle. The sensitivity of the wavemaker model to the water height h only has a small change relative to the previously mentioned parameters; however, there is still an observable decrease of 1.34% in the sensitivity index as the flow increases. Therefore, the interaction between the flow U and water height h is relatively low compared to the frequency, mean wedge depth, and wedge angle. On the contrary, the first order sensitivity

indices for the node points M remain constant for each flow rate at approximately zero.

The total effect sensitivity indices S_{Ti} for ω , β , d_μ , h , and M are displayed in Fig. 5.2 for the three fixed values of U . For the three respective values of U , the frequency

Figure 5.2: The total effect sensitivity indices of the remaining input parameters when flow is not included in the sensitivity analysis.

ω had the largest deviation in the total effect sensitivity index $S_{T\omega}$ which increased from 20.90% to 48.53% to 52.19%. Similar trends as the first order sensitivity indices are also observed for the remaining parameters. By neglecting the effect of the flow, the dependency of the wavemaker model on the remaining input parameters shifts with fluctuations in the flow. As the flow increases, the output model becomes more influenced by the wave frequency. Therefore, small errors in the measurement of the

frequency could lead to large errors in the wave amplitude to stroke amplitude ratio a/s whereas for low flow rates, the uncertainty in the frequency measurement would not impact the output as much. Thus, even though the sensitivity indices for the flow U in Table 5.4 are low compared to ω , β , and d_μ , the inclusion of flow and its specific value must be considered in the wavemaker model.

In regards to the other input parameters, a notable observation is the sensitivity indices for the node points M . For each of the fixed flow velocity cases in Fig. 5.1 and Fig. 5.2, there is little fluctuation in both S_M and S_{TM} with the average values across the three cases equal to 0.09% and 0.06%, respectively. As observed with S_3 of the Ishigami function in Section 4.2, a sensitivity index close to zero indicates that the parameter has no influence on the uncertainty of the model. From the first order sensitivity index of M in Table 5.4 and the fixed current cases, it can be concluded that the number of node points does not individually impact the output variance of the wavemaker model. A similar conclusion can be drawn with respect to the total effect sensitivity index indicating that the number of node points M can be deemed a non-influential parameter and fixed to a deterministic value. An additional observation is the low sensitivity indices for the water height h in Table 5.4 where $S_h = 0.99 \pm 0.19\%$ and $S_{Th} = 2.45 \pm 0.10\%$, and in Fig. 5.1 and Fig. 5.2 where the maximum indices are $S_h = 1.23 \pm 0.13\%$ and $S_{Th} = 3.07 \pm 0.11\%$. Through application of the boundary collocation method, Wu [Wu88] concluded that the water height was a very important parameter in the plunger-type wavemaker model. However, the sensitivity analysis of the model displayed that for a general distribution value of the input parameters, the water height had the lowest impact of the influential parameters on the wave amplitude to stroke amplitude ratio a/s of the wavemaker. Although the height of the water still needs to be measured and included in the model for hydrodynamic considerations, any uncertainty in the experimental measurement will

not greatly impact the output of the theoretical model.

From a design standpoint, the sensitivity analysis revealed that for the generation of waves by a plunger-type wavemaker, the uncertainty in the model will be governed by the uncertainty in the wave frequency. Therefore, to lower the uncertainty in the amplitude to stroke ratio a/s for experimental testing, reducing the uncertainty measurement in the frequency should be prioritized. Reducing the wave frequency uncertainty is especially critical when creating irregular wave spectra in a water channel, since the desired profile is dependent on a range of frequencies. The dimensions and positioning of the wedge should also be measured with reduced uncertainty as those parameters have the second and third highest influences on the output of the model. On the other hand, although the water height and flow will influence the amplitude ratio, their lower sensitivity indices indicate that uncertainty in their measurements will not generate large uncertainties in the model.

The specific sensitivity measurements of the analysis for the wavemaker model are also dependent on the uniform distribution range chosen for each input parameter. Although it is expected that the general trend of the sensitivity indices would be similar to the results presented in this section, the sensitivity analysis was rerun for the distribution of parameters that pertain to the experimental wavemaker used in the water channel in the next section.

5.2.1 Specific Case

To consider the specific design of the physical wavemakers constructed for this thesis, the sensitivity analysis was applied to the plunger-type wavemaker model where the range for each parameter distribution was determined by the physical limitations of the equipment and size of the wedges. The uniform distribution of each uncertain

Table 5.5: Uniform parameter distributions for the specific wavemaker case.

Parameter	Minimum Range	Maximum Range
U	0.000 [m/s]	0.305 [m/s]
ω	0.2π [rad/s]	6π [rad/s]
β	20 [Deg]	50 [Deg]
d_μ	0.06 [m]	0.18 [m]
h	0.18 [m]	0.86 [m]

input parameter is presented in Table 5.5 for the specific case. The water channel at Carleton University can accommodate flow rates U up 0.305 m/s and a maximum water height h of 0.86 m. The minimum range for the water height is limited by the maximum mean depth of the wedge. Since the experimental testing included multiple wedges, the range in d_μ was chosen to account for wedges with a mean depth between 0.06 m and 0.18 m and a wedge angle β ranging from 20° to 50° . The range for the frequency ω is bounded by the operational limitations of the actuator used to oscillate the wedge. Since the influence of the node points M was determined to be insignificant in the general case, it is not included in the current sensitivity analysis but has been instead set to a deterministic value of 200 as recommended by Wu [Wu88].

The first order S_i and total effect S_{Ti} sensitivity indices and their respective uncertainties for the specific case are presented in Table 5.6 as percentages of the unconditional variance for the wavemaker model. For the specific case, the water flow U

Table 5.6: Percent sensitivity indices and error for the specific wavemaker case.

Parameter	S_i [%]	δS_i [%]	S_{Ti} [%]	δS_{Ti} [%]
U	0.87	± 0.14	1.28	± 0.03
ω	77.59	± 1.66	83.37	± 1.66
β	10.05	± 0.48	13.66	± 0.30
d_μ	5.67	± 0.36	7.63	± 0.17
h	0.04	± 0.15	1.46	± 0.06

had a first order sensitivity index $S_i = 0.87 \pm 0.14\%$ and a total effect sensitivity index $S_{Ti} = 1.28 \pm 0.03\%$. The wave frequency ω was revealed to be the most influential input parameter on the output variance of the wavemaker model. The first order sensitivity index for ω was estimated to be $S_i = 77.59 \pm 1.66\%$ while the total effect sensitivity index was estimated as $S_{Ti} = 83.37 \pm 1.66\%$. The second most influential parameter was the angle of the wedge β with $S_{Ti} = 13.66 \pm 0.30\%$, followed by the mean wedge depth d_μ with $S_{Ti} = 7.63 \pm 0.17\%$. The water height h had the lowest impact on the model for the first order sensitivities, with $S_{Ti} - S_i = 1.42 \pm 0.16\%$ indicating that most of its impact on the output variance of the model is due to interactions of h with the other parameters.

As predicted, the sensitivity analysis for the specific case followed a similar trend compared to the general case; however, the wavemaker model is not as influenced by the flow in the specific case and the output variance depends more heavily on the frequency ω . The sensitivity analysis also helps to identify where discrepancies between the theory and experiment originate. For example, inconsistencies between the theoretical a/s and the experimentally measured values for the experimental work in the water channel can mostly be attributed to uncertainty in the frequency rather than the uncertainty in the other parameters. Therefore, with respect to the design of the wavemaker, the uncertainty in the frequency should be reduced as much as possible to decrease uncertainty in the amplitude to stroke ratio a/s . In addition, since the wedge angle β had the second highest sensitivity in the wavemaker model, the wedge should be designed such that the angle remains constant. Bending of the inclined edge of the wedge due to forces between it and the water during wave generation should be avoided by using rigid materials and re-enforcing the inside of the wedge. The wavemaker should also be designed such that the mean depth of the wedge can be both easily and precisely established since it is the third most influential

parameter. On the other hand, uncertainty in both the flow and the water height will not have as much impact on a/s due to their low sensitivity indices determined in the analysis. The low impact of the water height is an important observation for the experimental work as a fixed height is difficult to regulate in the water channel. Therefore, measurement of the water height should be recorded each day of testing. Finally, since the flow velocity has a low influence on the output of the model, use of the built-in control panel reading is sufficient for the experimental work in this thesis as opposed to the use of more precise external instruments for measuring flow.

5.3 Summary

In this chapter, a sensitivity analysis was first performed with the model for the conductive wave gauge which showed that the model is most dependent on the depth of the wires in the water. In addition, since the model is also influenced by the water conductivity, the wave gauge must be calibrated for each day of testing. The sensitivity analysis was also applied to the wavemaker theory which relates the wave amplitude to the stroke amplitude a/s of a plunger-type wavemaker in the presence of a uniform flow. For a both a broad and specific distribution range of the input parameters, the sensitivity analysis provided results regarding the influence of all input parameters as well as their interactions with one another within the wavemaker model. By determining the sensitivity of the wavemaker model to its input parameters, the theoretical design of the plunger-type wavemaker can be analyzed for application to plungers in an experimental water channel. The sensitivity analysis established that the number of node points required to define the kinematic boundary on the wavemaker is a non-influential parameter, such that uncertainty in the output of the wavemaker model will not be effected by changing its value. On the other

hand, the analysis determined that the wave frequency has the largest influence on the output variance of the wavemaker model. Therefore, hydrodynamic applications dependent on plunger-type wavemakers should take caution when executing and determining the wave frequency for the model in order to achieve a desired wave profile. Compared to the wave frequency, the flow has a low impact on the wavemaker model; however, its influence cannot be neglected. It was shown that by fixing the flow in the wavemaker model to a set magnitude, the influence of each parameter changes as the value of the fixed flow increases for both the first order and total effect sensitivity indices. Therefore, to accurately recreate a scaled ocean environment in a laboratory water channel, the uniform flow must be measured and taken into consideration for the plunger-type wavemaker model.

Chapter 6

Experimental Design of the Plunger-type Wavemaker

The plunger-type wavemaker used herein operates by oscillating a triangular-shaped wedge in the water in order to generate waves. Due to the oscillating motion, plunger-type wavemakers are advantageous over other types of wavemakers since plunger systems require less submerged equipment and allow for a current to flow across the lateral boundary. The desired wave pattern is thus dependent on the design of the wavemaker along with the parameters which govern its movement. Therefore, to contribute to the second objective, this chapter will outline the design and initial testing of the plunger-type wavemaker. Section 6.1 provides an overview of the experimental wavemaker and its operating limitations. In Section 6.2, verification of the wavemaker's ability to generate regular waves is presented. The parameters to be varied during the experimental tests of the wavemaker are outlined in Section 6.3, followed by an evaluation of the sensors for measuring the wave profile in Section 6.4. Finally, a summary of the chapter is provided in Section 6.5.

6.1 Wavemaker Design

Building on the conceptual design suggestions of McPhee [McP19] for a plunger-type wavemaker, the experimental wedges for the wavemaker used herein are displayed in Fig. 6.1. The wavemaker is supported by a vertical structure built with extruded

Figure 6.1: Photograph of the experimental wedges used for the plunger-type wavemaker.

t-slotted aluminium members. A linear actuator is fixed to the structure with its actuating rod pointing vertically down towards the water’s surface. The triangular-shaped wedge is attached to the end of the rod through a carriage system which is equipped with guide rails to maintain the horizontal position of the wedge in the water channel as it oscillates. The wedge is the only component of the wavemaker which makes contact with the water; therefore, the wedge can be efficiently interchanged with wedges of other sizes as well as the wave gauge for its calibration requirements. The motion of the wavemaker is governed by the displacement of the actuating rod. The linear actuator, manufactured by Thomson [Tho21], is accompanied by a Kollmorgen servo motor [Kol21] and controlled by a voltage signal through a National Instruments DAQ

system. Using LabVIEW, the setpoint of the actuator and its oscillating motion for a desired wave profile are controlled while simultaneously recording the voltage signals measured by the conductive wave gauge and ultrasonic sensor. Since the movement of the wavemaker is based on an input voltage signal, it is possible to create irregular waves within the water channel as well based on a desired wave spectrum; however, the generation of irregular waves is outside the scope of this thesis.

There are pre-existing factors that must be taken into consideration for the initial design and operating range of the wavemaker. The actuator has a maximum acceleration of 6 m/s^2 which must be taken into consideration with respect to the frequency and the stroke amplitude of the sinusoidal wave. The frequency f of the wavemaker is related to the stroke amplitude s of the actuator through the expression,

$$f = \frac{1}{2\pi} \sqrt{\frac{a_a}{s}}, \quad (6.1)$$

where a_a is the acceleration of the actuator. Substituting the maximum acceleration of the actuator into (6.1) provides an upper limit on the frequencies and corresponding stroke amplitudes that can be achieved with the wavemaker. The height of the wedge also limits the available stroke amplitude of the wavemaker. The limitations imposed by the actuator and the size of the experimental wedge are displayed in Fig. 6.2 in relation to the wavemaker's stroke amplitude and frequency; the solid curve represents the expression for the frequency-amplitude relation in (6.1) while the dashed line represents the maximum stroke amplitude that can theoretically be achieved by the initial wedge size. Since the initial wedge has a height of 22 cm, its maximum stroke amplitude is 11 cm; however, to avoid fully submerging the wedge or having the wedge break contact with the water, the stroke amplitudes should be much less than 11 cm. The intersection of the actuator and the wedge limits shown in Fig. 6.2 creates an

Figure 6.2: The general operating limits of the plunger-type wavemaker in relation to its frequency and stroke amplitude.

initial range in which to operate the wavemaker with respect to both frequency and stroke amplitude. Further limitations may exist depending on the performance of the wavemaker which must be explored through experimental testing. Therefore, it is critical to first verify that the experimental design of the plunger-type wavemaker can generate regular waves.

6.2 Initial Wavemaker Testing

In order to confirm the wavemaker is operational and able to produce waves within the water channel, the wave profiles generated during preliminary tests were analyzed. Fig. 6.3 demonstrates the wave profile as measured by the ultrasonic sensor for waves

generated with no flow (top) and with a flow of 0.305 m/s (bottom). In both the no

Figure 6.3: Initial measurement of the wave profile generated by the wavemaker for flow rates of 0.000 m/s (top) and 0.305 m/s (bottom). The waves were produced by wedge A with $f = 1.2$ Hz, $s = 2$ cm, and $d_\mu = 12$ cm.

flow and flow cases, the wavemaker was oscillated at a frequency of 1.2 Hz, with a stroke amplitude of 2 cm for 60 s. The mean depth of the plunger was also constant between the cases at approximately 12 cm. The no flow wave profile in Fig. 6.3 has a maximum recorded peak of 10.22 mm while the maximum for the flow case is 8.32 mm. Comparing the two wave profiles, it is evident that the flow impacts the amplitude of the waves. When flow is included, the amplitude of the produced wave decreases compared to when there is no flow in the water channel. The result agrees with the trend predicted by the theoretical model with respect to the introduction of flow. Fig. 2.4 demonstrated that for a constant stroke amplitude the wave amplitude

should decrease with increasing flow U . Therefore, the experimental wavemaker is behaving in a similar manner to the model. The 0.305 m/s case also exemplifies that the higher flow rates introduces more noise to the measured signal. As such, it may not be ideal to operate at such flow rates, especially for small stroke amplitudes.

From the sensitivity analysis of the theoretical model for the plunger-type wavemaker, it was shown that the desired wave profile is not only dependent on the flow U , but on other parameters as well, namely, the frequency. Therefore, an additional initial test was performed for a different frequency and a flow velocity of $U = 0.000$ m/s. Fig. 6.4 displays the wave profile measured by the ultrasonic sensor for a wavemaker frequency of 1.6 Hz. The generated wave in the given example, Fig. 6.4, is

Figure 6.4: Example of the wave profile generated by the wavemaker at a frequency of 1.6 Hz. The waves were produced by wedge A with $U = 0.000$ m/s, $s = 2$ cm, and $d_\mu = 12$ cm.

considerably different from the previous example, Fig. 6.3, despite the stroke amplitude and mean depth of the plunger being constant and only the frequency having shifted by 0.4 Hz. Excluding the transient period at the beginning of the wave generation, for the wave profile in Fig. 6.4, the maximum recorded wave amplitude was 12.38 mm while the minimum wave amplitude was 5.14 mm. The waves in Fig. 6.4 therefore have more variation in regards to its amplitude, fluctuating considerably throughout the 60 s duration of the test as compared to the $f=1.2$ Hz wave profile in the top plot of Fig. 6.3. For evaluating the performance of the wavemaker, a steady state period is desired; therefore, while the tests will be conducted for 60 s, only the period of time between 15 - 25 s will be considered in the post-process analysis. As such, the transient data at the beginning of the wave generation will also be removed from the analysis. The given example demonstrates that the flow is not the only parameter that effects the generated wave profile, similar to the results of the sensitivity analysis. Instead, the flow, frequency, mean wedge depth, wedge angle, and stroke amplitude parameters need to be considered for wave generation and their impact on the experimental design of plunger system. Therefore, to fully evaluate the plunger-type wavemaker and determine its applicable operating range, a series of experimental tests were performed which explores the effect of varying each input parameter deemed influential in the sensitivity analysis.

6.3 Summary of Experimental Tests

Experimental work with plunger-type wavemakers in literature is often limited to testing only a single frequency or stroke amplitude and without the inclusion of flow [GLS⁺15, Mik07]. Based on the theoretical model, there are five parameters which can be varied experimentally including the flow rate, wedge angle, mean wedge depth,

frequency, and stroke amplitude. Therefore, it is critical to consider the influence of each parameter in the experimental testing. The design of the wavemaker will be explored by measuring the transfer function of the generated regular waves produced by three different wedge-shaped plungers to address the wedge angle parameter. Fig. 6.5 outlines the geometry of the three respective wedges where A is the original wedge constructed for the wavemaker by McPhee [McP19], and wedges B and C were constructed in order to observe the effect of the wedge angle β on the generation of the waves in the water channel. Wedge B has an inner angle of 35.0° and wedge

Figure 6.5: The geometry of three triangular cross-section wedges for experimental testing of the plunger-type wavemaker.

C has an inner angle of 45.0° . Along with the wedge angle, the sensitivity analysis of the wavemaker model showed that the frequency, mean wedge depth, and flow parameters interacted the most with one another. Although the sensitivity analysis showed that the water height interacts with the other parameters in the wavemaker

model as well, it was not tested as an experimental parameter since it is difficult to regulate the water height in the water channel. Therefore, the impact of frequency, mean wedge depth, and flow were tested for each wedge. In addition, to evaluate an experimental a/s , various stroke amplitudes were also tested. Fig. 6.6 displays the range of tests performed as a function of frequency and stroke amplitude in relation to the operational range of the wavemaker. For each wedge, the 20 tests

Figure 6.6: The range in experimental tests with regards to frequency and stroke amplitude along with the limits of the wedge and actuator.

shown in Fig. 6.6 were repeated at mean wedge depths of 10 cm, 12 cm, and 14 cm which in turn were repeated at five significant flow rates of 0.000 m/s (0.00 in/s), 0.076 m/s (3.00 in/s), 0.152 m/s (6.00 in/s), 0.229 m/s (9.00 in/s), and 0.305 m/s (12.00 in/s). In total, 900 experimental tests were performed with the plunger-type

wavemaker. The test points in Fig. 6.6 were chosen in order to cover the majority of the initial operating range of the wavemaker based on the physical limitation of the smallest wedge and the acceleration limit for the actuator. The mean depths were chosen such that hypothetically, a large enough wave would still be generated for the range in tested stroke amplitudes. The range in the flow values was chosen based on the maximum available flow rate for the water channel. As such, the impact of the flow, frequency, wedge angle, wedge mean depth, and stroke amplitude were investigated experimentally to evaluate the performance of the wavemaker and the system's limitations. A full list of the tests performed on a single wedge is provided in Appendix C.

From the wavemaker model, for the same wedge mean depth, a wedge with a larger angle β will theoretically create larger amplitude waves due to the increase in the mean wedge width b . For the output wave amplitude to be the same regardless of the wedge angle, the mean width of the wedge must be constant. In doing so, the performance of the three wedges can be directly compared. Fig. 6.7 demonstrates the relationship between the wedge angle β and mean wedge depth d_μ for a constant wedge width. To maintain a consistent wedge width with wedge A, the mean depth d_μ of wedge B and wedge C must be adjusted according to the inner angle of the β by,

$$d_\mu = \frac{b}{\tan \beta}. \quad (6.2)$$

Thus, to compare the output of the wedges, the tests for wedge B and C were repeated but with the mean wedge depth adjusted to match the 0.100 m, 0.120 m, and 0.140 m mean depths of wedge A. The exact values of the updated wedge depths are displayed

Figure 6.7: An example of the mean depths d_μ required for each wedge such that the mean width of the wedges in still water are equal.

Table 6.1: The mean wedge depths of the second two wedges corresponding with the mean wedge depth of the first wedge.

Mean Depth 25.7° [m]	Mean Depth 35° [m]	Mean Depth 45° [m]
0.100	0.0687	0.0482
0.120	0.0825	0.0578
0.140	0.0962	0.0674

in Table 6.1. The six corresponding mean depths were tested at each of the significant flow rates for the same frequencies and stroke amplitudes outlined in Fig. 6.6. Not all stroke amplitudes can be achieved at the new mean wedge depths; therefore, the 0.04 m and 0.05 m stroke amplitudes tests were omitted from the wedge comparison tests.

6.4 Sensor Performance and Initial Testing Results

For each of the conducted experimental tests, the generated wave profile was simultaneously measured by the conductive wave gauge, the camera, and the ultrasonic sensor. Since the true elevation of the water is unknown, cross-checking the three sensors provides a more reliable measurement of the free surface; however, it is desired that only one sensor be required to evaluate the performance of the wavemaker. Thus, prior to analysing the wave profile with respect to the performance of the wavemaker, the operating ranges and performance of the sensors were compared.

An example of a generated wave as measured by all three sensors is shown in Fig. 6.8 over a time range of 10 s. All three sensors were able to measure the generated wave pattern in the water channel throughout the duration of the test. In general, the measured wave profile from each sensor is similar; however, there is some loss in information with the camera measurement at $t = 17.0$ s and $t = 20.3$ s specifically. The theoretical frequency of the given example wave was 1.2 Hz. Using Matlab to analyze the peaks in the measured wave profile, the percent difference between the theoretical frequency and the measured frequency from the wave gauge, camera, and ultrasonic was 0.53%, 0.35%, and 0.08%, respectively. Thus, all three instruments are in well agreement with the theoretical frequency. With respect to the measured amplitude, the average amplitude over the 10 s range was compared between each sensor. The percent difference in amplitude between the wave gauge and camera was 17.47% while the wave gauge and ultrasonic had a percent difference of 9.37%. Finally, the percent difference in average amplitude between the camera and the ultrasonic was 8.13%. Although the results for the displayed example in Fig. 6.8 show that all

Figure 6.8: The wave profile as measured by the conductive wave gauge, camera, and ultrasonic sensor. The profile was generated by wedge B with $f = 1.2$ Hz, $U = 0.000$ m/s, $d_\mu = 14$ cm, $s = 2$ cm.

three sensors are in agreement, it is important to consider the performance of the sensors across the range of the experimental parameters under consideration.

The difference in the theoretical frequency and the frequency measured by each sensor is also an indicator of the sensor's performance as it shows the operating range of each sensor. The frequency of the generated waves does not depend on the accuracy of the theoretical model but instead should simply be equal to the frequency at which the wavemaker oscillates. The percent difference between the theoretical frequency and the measured frequency by each sensor is presented in Fig. 6.9 for the various experimental test cases. For simplicity with processing the camera data, only wedge A at a mean depth of 14 cm will be considered for representing the performance

Figure 6.9: Percent difference between the measured frequency of each sensor and the theoretical frequency of the waves. The wave profiles analyzed here were generated by wedge A with $d_\mu = 14$ cm.

comparison between the sensors. Each section in Fig. 6.9, separated by a vertical black line, represents the frequency at which the tests were performed while the x-axis of the plot indicates the stroke amplitude of the wavemaker. For these results, only a single mean depth is displayed. The five different markers thus represent the various flow rates which range from 0.000 m/s to 0.305 m/s. All three sensors tend to perform better in terms of the frequency measurement in the 1.2 Hz and 1.6 Hz frequencies while higher frequencies tend to have larger percent differences, especially for the maximum flow of 0.305 m/s (+). Overall, the average percent difference between the measured frequency of the camera and theoretical frequencies was 6.53%, with the

camera having more difficulties at measuring the high frequency, high flow cases. On the hand, the wave gauge has an average percent difference in frequency of 10.49% but does not operate as well at the lowest frequency of 0.8 Hz or in the high frequency range. Finally, the ultrasonic had an average percent difference of 5.25% with the theoretical frequency. The ultrasonic tends to be the most reliable across the range of frequencies and flow rates with most outlying cases occurring at the high flow rate and may be due to increased noise from which the wave profile cannot be accurately measured. Overall, all three sensors operate within an acceptable range and there is not a significant impact due to the stroke amplitude at lower frequencies.

By determining the limitations and comparing the performance of each sensor, the design of the wavemaker can be investigated based on the measurement from a single sensor. The advantages and disadvantages to the use of each sensor is summarized in Table 6.2. Based on the ultrasonic sensor's reliability across a wider range of frequencies, as well as its non-contact method of measuring the water, it was concluded that the ultrasonic sensor is best suited for measuring the wave profile for evaluating the performance of the wavemaker and assessing its operational range. Therefore, for the results of the wavemaker tests and investigation, only the wave profiles as measured by the ultrasonic sensor will be considered.

6.5 Summary

In this chapter, the design of the experimental plunger-type wavemaker was presented. The main body of the wavemaker is comprised of an actuated wedge in the water. As such, the wavemaker is limited by the size of the water channel as well as the acceleration limit of the actuator. To ensure that the wavemaker is operating as expected, an example of the wave profile as measured by the ultrasonic was presented

Table 6.2: Summary of the comparison between each tested sensor.

Sensor	Advantages	Disadvantages
Wave Gauge	<ul style="list-style-type: none"> • Can measure any wave steepness. • Average error in frequency: 10.49%. 	<ul style="list-style-type: none"> • Requires direct contact with water. • Requires daily calibration. • Unreliable at low and high end of frequency range.
Camera	<ul style="list-style-type: none"> • No contact with water. • Provides visual information. • Can measure wave steepness. • Average error in frequency: 6.53%. 	<ul style="list-style-type: none"> • Requires the point of interest to be visible from side of tank. • Unreliable for high flow, high frequency. • Calibration is dependent on fixed position.
Ultrasonic	<ul style="list-style-type: none"> • No contact with water. • Only requires single calibration • Average error in frequency: 5.25%. 	<ul style="list-style-type: none"> • Measures the highest points within area of emitted sound.

and the general impact of flow was demonstrated which agrees with the theoretical model. The ability for the wavemaker to generate the desired wave profile was also shown to be dependent on the frequency. Therefore, to determine the experimental operating range, the design of the experiment to test each significant wavemaker parameter from the sensitivity analysis was presented. The dimensions for three different wedges were presented in order to investigate the impact of wedge angle, mean wedge depth, frequency, stroke amplitude, and flow for a total of 900 tests. The

impact of each wedge on the symmetry of the generated waves will also be explored through a set of 420 secondary tests. By comparing the sensors used to measure the profile of the generated wave, it was shown that the ultrasonic measures the frequency of the wave most accurately compared to the theoretical wave frequency. Therefore, from these results, it was determined that for evaluating the performance and design of the plunger-type wavemaker, the ultrasonic sensor is the most advantageous sensor due to its flexibility in measuring and its accuracy. In the following chapter, the results of the experimental tests are presented and discussed.

Chapter 7

Experimental Wavemaker Results

This chapter presents and discusses the results of the 1320 tests performed with the plunger-type wavemaker in relation to the third objective of the thesis. In section 7.1, the experimental amplitude to stroke ratio a/s from 900 tests are evaluated and compared to the value predicted by the theoretical plunger-type wavemaker model. As such, the influence of flow rate, wedge angle, wedge mean depth, wave frequency, and stroke amplitude on the performance of the wavemaker are investigated. The reflection of waves in the water channel are investigated in Section 7.2 based on the wave profiles measured in the first set of tests. To accommodate for the discrepancies observed between the experimental and theoretical results, Section 7.3 presents two methods for correcting the plunger-type wavemaker model. In Section 7.4, a comparison of the three experimental wedges is conducted which makes use of the remaining 420 tests. In this second set of tests, the dependency on the wedge angle is removed, allowing for an investigation on the performance of the wedge design for various flow rates. Finally, a summary of the chapter is presented in Section 7.5.

7.1 Amplitude to Stroke Ratio

The output of the theoretical plunger-type wavemaker model is the wave amplitude to wavemaker stroke amplitude ratio a/s . Therefore, a/s is the key metric for evaluating the performance and operational range of the experimental wavemaker. To compare the values of a/s as measured experimentally and predicted by the model, the following analysis will consider the percent difference expressed by,

$$\% \text{ Difference} = \frac{|\text{Experimental} - \text{Theoretical}|}{(\text{Experimental} + \text{Theoretical})/2} \times 100\%, \quad (7.1)$$

where “experimental” refers to the measured a/s and “theoretical” corresponds to the value as predicted by the wavemaker model. As determined in Section 6.4, only the wave profiles measured by the ultrasonic sensor were considered for comparison to the model. An example wave profile measured by the ultrasonic sensor as a function of time for wedge A ($\beta = 25.7^\circ$) is displayed in Fig. 7.1 along with the theoretical wave for the no flow (top) and 0.076 m/s flow case (bottom). 7.1 in comparison to the theoretical wave profile. For both cases, the wavemaker has a frequency of 1.2 Hz, a set stroke amplitude of 2 cm, and a mean depth of 10 cm. The experimental a/s is dependent on the wave amplitude measured by the ultrasonic sensor and the experimental stroke amplitude of the wavemaker. In this thesis, the wave amplitude was determined in a post-process analysis using MATLAB. After converting the voltage signal recorded by the ultrasonic sensor to a distance measurement through the sensor’s calibration, the wave profile was shifted to have a zero mean. To determine the average amplitude over the steady state time interval from 15 - 25 s, the amplitude of both wave crests and troughs are added together and divided by the total number of

Figure 7.1: The measured and theoretical wave profile for generated waves with and without flow.

waves. On the other hand, for the stroke amplitude, a discrepancy was observed between the theoretical and experimental stroke amplitude dependent on the frequency of the wavemaker. Table 7.1 displays the percent difference between the theoretical stroke amplitude and the true stroke amplitude for the various tested frequencies. The % differences in Table 7.1 all represent a reduction in the stroke amplitude from the theoretical amplitude. As an example, for a set stroke amplitude of 10.00 mm at a frequency of 0.8 Hz, the true stroke amplitude of the wavemaker was 9.58 ± 0.01 mm while at a frequency of 2.4 Hz, the true stroke amplitude was 6.52 ± 0.01 mm. Therefore, for simplicity when referencing the various test cases, the theoretical stroke amplitudes will be used as opposed to the true stroke amplitudes of the wavemaker.

Table 7.1: % difference between the theoretical and true wavemaker stroke amplitudes as a function of frequency.

Frequency [Hz]	Stroke Amplitude Difference [%]
0.8	4.22
1.2	9.66
1.6	16.82
2.0	25.35
2.4	34.83

As such, the experimental a/s was determined by dividing the experimental average wave amplitude by the experimental wavemaker stroke amplitude.

The reduction in stroke amplitude is also taken into consideration for the theoretical a/s such that the experimental amplitude can be directly compared to the theoretical wave amplitude. For the no flow case in Fig. 7.1, the theoretical amplitude of the generated waves was 6.74 mm while the average measured amplitude was 7.26 ± 0.23 mm. For the flow case, the theoretical amplitude was 6.15 mm while the average measured amplitude 4.59 ± 0.23 mm. As such, in the no flow case, the model underestimated the generated wave amplitude while in the flow case, the theoretical amplitude is overestimated. However, the impact of flow can be observed in both the theoretical and experimental wave profiles. Therefore, it is important that the inclusion of flow be investigated for the experimental plunger-type wavemaker.

The results of the sensitivity analysis revealed that the frequency has the most influence on the output variance of the model. As such, each of the influential parameters in the sensitivity analysis were investigated in the experimental testing of the wavemaker including the flow rate U , frequency ω , wedge angle θ , and wedge mean depth d_μ . Since the tests provide an experimental wave amplitude, various stroke amplitudes were considered in addition to the GSA parameters. The first set of 900 tests outlined in Chapter 4 were then analyzed and a/s for each test was determined for comparison to the theoretical model. Note that in determining the theoretical

Figure 7.2: % difference in a/s as a function of the flow rate and frequency measured by the ultrasonic sensor and predicted by the model for wedge B.

a/s , the experimental values of each input parameter were used as an input to the semi-analytical solution. Fig. 7.2 displays the a/s % difference between the measured and theoretical for the tests performed with wedge B. Each point in the surface plane is averaged over the three mean depths and up to five stroke amplitudes. The surface demonstrates that as the frequency and the flow both increase, the average percent difference also increases. The observed trend is not unique to wedge B, but was observed with all three wedges. Similar diagrams for wedge A and wedge C are available in Appendix B. For wedge B, the maximum percent difference was 112.7% which occurred at a frequency of 2.4 Hz and flow rate of 0.305 m/s. On the other hand, the minimum percent difference was 5.0%, observed at 0.8 Hz and 0.000 m/s

Figure 7.3: Comparison of the a/s % difference for three different mean depths for wedge C with $U = 0.076$ m/s. Each point is averaged over the tested stroke amplitudes for a given frequency.

flow rate. According to the sensitivity analysis, the mean depth also had an impact on the output of the model; therefore, the results of Fig. 7.2 were broken down into the respective mean depths as well.

Fig. 7.3 displays the impact the mean depth has on the a/s percent difference as a function of the frequency ω for wedge C at a flow rate of 0.076 m/s with each symbol representing the three different mean depths of 10 cm, 12 cm, and 14 cm. Comparing the results of each mean depth, there is no discernable trend or dependency on the mean depth in Fig. 7.3. Similar results are observed for the other flow rates as well, but in general, an increase in frequency leads to an increase difference between

the theoretical and experimental results. For reference, the results of the mean depth percent difference for the other flow rates for all three wedges are provided in Appendix B.

To determine the operational range and assess the performance of the wavemaker, observing the results of all 900 tests was critical. To do so, the percent difference is plotted as function of the dimensionless parameter kb , where k is the wavenumber and b is the mean width of the plunger wedge. The wavenumber k is given as [DD84],

$$k = \frac{\omega}{\left(U + \frac{g}{2\omega}\right) + \sqrt{\frac{Ug}{\omega} + \frac{1}{4}\left(\frac{g}{\omega}\right)^2}}, \quad (7.2)$$

where $\omega = 2\pi f$ is the wave frequency, U is the flow rate, and g is the gravitational acceleration. The mean width of the plunger b can be determined based on the geometry of the triangular cross-section of the wedge by,

$$b = d_\mu \tan \beta, \quad (7.3)$$

where d_μ is the mean depth of the plunger wedge and β is the inner angle of the wedge with respect to the vertical. As such, the dimensionless parameter kb accounts for each of the sensitivity analysis parameters: the frequency f , flow rate U , mean wedge depth d_μ , and wedge angle θ . Fig. 7.4 displays the a/s percent difference for wedge A as a function of kb , separated by the five tested flow rates. Each point represents a single test, described by the parameters outlined in Chapter 6, including each mean depth as well as the multiple stroke amplitudes. Averaging over all tests for each wedge, the average percent difference between the model and the experimental results is 38.20% for wedge A. The maximum % difference occurs at a flow rate of 0.305 m/s for $kb = 0.86$ and is 142.26% while the minimum % difference at $kb = 0.14$

Figure 7.4: a/s percent difference for wedge A as a function of kb for the five tested flow rates.

is 0.01% for a flow rate of 0.076 m/s. Similar results for wedges B and C are displayed in Fig. 7.5 and Fig. 7.6, respectively.

The overall percent difference between the model and experiment for wedge B was 33.24% with the maximum difference occurring for a flow rate of 0.305 m/s at $kb=0.89$ and equal to 125.39%. The minimum % difference for wedge B is observed in the no flow case (0.000 m/s) at $kb=0.22$ and equal to 0.09%. Similarly, the percent difference observed throughout the tests performed with wedge C had an average of 44.85%. The maximum difference of 142.28% is at $kb=1.53$ for $U=0.305$ m/s while the minimum difference of 0.21% is at $kb=3.25$ for $U=0.000$ m/s. In general, wedge B performs most closely to the predicted model; however, there is still a large gap between theory and experimental with the model overestimating a/s in 795 of the 900 experimental tests. The percent difference tends to increase with increasing kb value, especially at the higher flow rates. For a single flow rate, kb is mostly dependent on the frequency ω ; thus, the observed increase in deviation between theory and experiment can be attributed to the increase in frequency. The discrepancy may be due to the decrease in reliability of the ultrasonic sensor at higher frequencies; therefore, it would be ideal to operate the wavemaker at lower frequencies when possible for any flow rate. The increase in percent difference due to the flow rate further suggests that using lower flow rates will result in waves that can be more closely predicted by the model. By operating a lower flow rates, the increased noise due to turbulence is limited as well. Further error may be due to external factors within the water channel and hydrodynamics phenomena that are not taken into account in the theoretical model such as leakage around the wedge and reflection of the waves off the walls of the water channel. As such, the reflection in the water channel was examined and quantified as a potential source of error for the wavemaker model.

Figure 7.5: a/s percent difference for wedge B as a function of kb for the five tested flow rates.

Figure 7.6: a/s percent difference for wedge C as a function of kb for the five tested flow rates.

7.2 Wave Reflection

It was shown in Section 3.3 through a preliminary study that the reflection of waves in the water channel tends to increase with increased flow rate. In addition, the repeatability of the reflection coefficient is less reliable at high flow rates as well. The reflection coefficient may also provide a secondary metric for determining the operational points of the wavemaker. Therefore, the reflection coefficient was determined for the first set of 900 tests performed with the experimental wavemaker. In doing so, the effects of frequency, flow rate, mean depth, stroke amplitude, and wedge angle could be investigated. As previously mentioned, a reflection coefficient less than one is desired since it indicates that the incident waves are higher in amplitude than the reflected waves with values approaching zero corresponding to pure incident waves and no reflection due to the energy absorption of the wave absorber or beach. The reflection coefficients determined for tests performed with wedge A are displayed in Fig. 7.7 as a function of the dimensionless parameter kb . The solid horizontal line represents a reflection coefficient equal to one for simplicity in observing the results. For a flow of 0.076 m/s, there are four data points at $kb=0.43$ that lie outside the range of reflection coefficients shown equal to 4.17, 4.81, 6.78, and 13.66. The four points have been omitted in order to show the majority of the results with appropriate resolution. Similarly, the reflection coefficients for wedges B and C are presented in Fig. 7.8 and Fig. 7.9, respectively. Due to the range shown in Fig. 7.8, one data point at $kb=0.75$ equal to 7.24 is not displayed in the 0.076 m/s flow case. Similarly, for wedge C in Fig. 7.9, two data points have been omitted for the 0.076 m/s flow at $kb=1.07$ and $kb=1.25$ equal to 7.86 and 8.97, respectively. The omitted data points for all three wedges correspond to a frequency of 1.6 Hz as well as the maximum

Figure 7.7: The reflection coefficient for each test performed with wedge A.

measured reflection coefficients.

For the majority of the tests, the reflection coefficient remains below one which is the desired result; however, the impact of the reflection is still significant. For the no flow (0.000 m/s) and 0.076 m/s flow cases, there is no observable trend in the reflection coefficient as a function of kb , which is consistent with the initial reflectance testing discussed in Section 3.3. On the other hand, at the higher flow rates, the

Figure 7.8: The reflection coefficient for each test performed with wedge B.

reflection tends to increase with increasing kb which is observed for all three wedges. Since there is little variation when considering the impact of mean depth and stroke amplitude, it was concluded that the dependency of the reflection can be mostly attributed to the frequency parameter within kb . As such, operating in the low frequency range will result in a reflection coefficient closer to zero. However, the large values of the reflection coefficient at a flow rate of 0.076 m/s indicate that a potential

Figure 7.9: The reflection coefficient for each test performed with wedge C.

resonant frequency occurs in the water channel at $f=1.6$ Hz which should be avoided in future experimental tests. Further investigation into the resonance phenomenon was outside the scope of the current thesis. Overall, the results of the reflectance tests agree with trends observed in the comparison between the theoretical model and the experimental a/s . Operating in the flow frequency, low flow rate area will create an environment in the water channel in which the wave absorber is the most

efficient. However, to correct for the general influence of the reflection, along with other unknown hydrodynamic phenomena, correction factors were investigated for the theoretical model.

7.3 Model Corrections

To accommodate for the discrepancies observed in the amplitude to stroke ratio between the experimental and theoretical values, two different methods were explored to apply a correction to the model. In doing so, it is desired that the model will better represent the experimental results that were observed. The methods include a case specific correction and an operational correction, the results of which will be presented in the following subsections. Since the majority of the experimental tests indicated that the model was over-predicting the amplitude of the generated waves, both methods will work to decrease the value of a/s in the model.

7.3.1 Case Specific Correction

The first method for correcting the model will be referred to as the case specific correction and is based on the standard deviation of a given wave profile. For the experimental results to match that of the model, they should both have the same standard deviation. For cases where the standard deviations are not the same, the model can be scaled to match the distribution of the experimental case by a correction factor ζ given by,

$$\zeta = \frac{\sigma_e}{\sigma_m}, \quad (7.4)$$

where σ_e is the standard deviation of the experimental wave profile and σ_m is the standard deviation of the theoretical wave profile. The correction factor is determined based on the normalized output z_e of the experimental wave profile. Normalizing the experimental data to have a zero mean and a standard deviation of one can be achieved using the expression,

$$z_e = \frac{x_i - x}{\sigma_e}, \quad (7.5)$$

where x_i is a data point and x is the average of all points. The theoretical wave profile z_{me} normalized with respect to the experimental can be achieved using the expression,

$$z_{me} = \frac{x_i - x}{\sigma_e}, \quad (7.6)$$

where the closer that z_{me} is to a value of one, the closer the theoretical wave profile matches that of the experimental. Therefore, to force the model to match the experimental, the normalized model needs to have a standard deviation of one, denoted as z_m and given by,

$$z_m = \frac{x_i - x}{\sigma_m}. \quad (7.7)$$

Scaling the theoretical data to match that of the experimental, denoted by $x_{i-scaled}$ can therefore be accomplished by,

$$x_{i-scaled} = (x_i - x)\zeta. \quad (7.8)$$

Similarly, inverting the correction factor would result in a scaling of the experimental to match the distribution of the theoretical wave profile. Applying the correction factor on a case-by-case basis; meaning one applies the individual correction factor to their respective test case, the average percent difference between experiment and theory reduces to 3.30%, 2.86%, and 2.47% for wedges A, B, and C, respectively. However, the correction can only be applied post-process and the correction factor is dependent on the experimental test. Instead, a generalized correction factor, which is dependent on the kb factor was applied to the model.

To determine the generalized correction factor, a benchmark case was used taking into consideration the results of wedge B, at a single mean depth of 12 cm. In doing so, validation could be examined for higher and lower wedge angles and mean depths. The results of the benchmark case were then applied to the other mean depths of wedge B as well as all cases for wedges A and C. Analyzing the correction factors for the benchmark case as a function of kb , a linear trend was observed creating a correction factor ζ_{kb} following the expression,

$$\zeta_{kb} = -0.1015kb + 0.8285. \quad (7.9)$$

As such, the correction factor ζ_{kb} can be applied to the a/s model for any value of kb . Fig. 7.10 displays the a/s percent difference between the experimental results for wedge A and the corrected version of the model with respect to kb . The overall percent difference between the corrected model and the experimental results for wedge A was 26.59%. The maximum difference is observed for a flow rate of 0.305 m/s with $kb=0.86$, similar to the uncorrected case, but is now equal to 125.85%. Similar plots exist for wedges B and C which are displayed in Fig. 7.11 and Fig. 7.12,

Figure 7.10: The a/s percent difference between the case specific corrected model and the experimental test cases for wedge A.

respectively. The overall percent difference between the corrected model and the experimental results for wedge B was 22.59%. The maximum difference with the correction applied still occurs at a flow of 0.305 m/s and $kb=0.89$, but has a value of 105.21%. Similarly, the overall percent difference observed with wedge C between the case specific model and the experimental results was 27.19%. For wedge C, the

Figure 7.11: The a/s percent difference between the case specific corrected model and the experimental test cases for wedge B.

maximum percent difference with the corrected model occurs at $kb=1.53$ and is equal to 119.79% for a flow rate of 0.305 m/s. As a result of the benchmark experimental tests used to develop the model correction in (7.9), the correction factor ζ_{kb} will be less than one for values of kb greater than -1.69. Based on the range of parameters explored in the experimental tests, $0.10 \leq kb \leq 3.25$; therefore, the correction factor will always work to decrease the value of a/s in the theoretical model. Since the majority of test

Figure 7.12: The a/s percent difference between the case specific corrected model and the experimental test cases for wedge C.

cases (795/900) are overestimated by the model, a decrease in the model a/s is desired; however, the remaining 105 tests are negatively effected as a result. Overall, while the case specific correction decreases the average percent difference of the experimental tests, the correction factor ζ_{kb} is unique to the experimental equipment and any unaccounted for errors due to hydrodynamic phenomena in the water channel. As such, the method may limited in its applicability to other plunger systems. Therefore,

a second method for correcting the model was explored which applies a common bias to the model regardless of the specific test case.

7.3.2 Operational Correction

A second method for correcting the model was investigated which is referred to as the operational correction. The operational correction instead applies a straight bias to the normalized model. The model is normalized such that a given wave profile will have a zero mean and a standard deviation of one. To determine the bias which should be applied to model, the value of the bias that would lower the overall percent difference in the same benchmark case previously used. The average percent difference between experimental results of the benchmark case and the model is displayed in Fig. 7.13 as a function of the percent decrease applied to the normalized model. For no change in the model, the average percent difference in the experimental results of the benchmark case is 29.55%; however, a local minimum exists by decreasing the amplitude of the normalized model by 21% (x-axis in Fig. 7.13), causing the average percent difference decreases to 19.44% (y-axis in Fig. 7.13). Therefore, it would be beneficial to decrease the normalized model for all cases and all wedges by 21%. Fig. 7.14 displays the percent difference in a/s as measured by the sensor and predicted by operational corrected model. After applying the decrease to the model, the data is denormalized and compared to the measured a/s by the ultrasonic sensor. For wedge A the average percent difference in a/s between the corrected model and experimental results for all flow rates was 28.30%. The maximum % difference still occurs for a flow of 0.305 m/s with $kb=0.86$, but is equal to 129.64%. For wedge B, the average percent difference in a/s was 23.43% with the maximum value observed at a flow of

Figure 7.13: Average decrease in the difference between the experimental and theoretical a/s for a range of decreases in the normalized model.

0.305 m/s for $kb=0.89$ and equal to 110.05%. The average percent difference in a/s between the operational model and the experimental results observed for wedge C was 28.57%. The maximum \$ difference for wedge C also occurs at a flow of 0.305 m/s with $kb=1.53$ and is equal to 129.57%. As a result, the operational correction to the model has a similar effect on the a/s percent difference as the case specific corrected model despite the difference in method. Similar to the first method, the theoretical a/s is decreased for all test cases such that some tests result in an increased a/s percent difference. However, applying a standard bias to the model rather than the correction factor ζ_{kb} is more applicable to plunger systems external to the experimental work performed in this thesis. Therefore, the operational correction is a more appropriate

Figure 7.14: Percent difference in a/s between the measured and operational correction model for wedge A.

method for correcting the theoretical model. With a correction applied to the model, conclusions regarding the experimental results and the operating range of the plunger-type wavemaker can be addressed.

Figure 7.15: Percent difference in a/s between the measured and operational correction model for wedge B.

7.3.3 Summary and Conclusions

Table 7.2 compares the original results to those determined by applying the two different corrections to the model. Overall, application of a correction to the theoretical plunger-type wavemaker model is beneficial to lowering the a/s percent difference between the model and experimental results. While the case specific correction has a

Figure 7.16: Percent difference in a/s between the measured and operational correction model for wedge C.

Table 7.2: Comparison between the original a/s percent difference and the two corrected models.

Wedge	Original [%]	Case Specific Correction [%]	Operational Correction [%]
A	38.20	26.59	28.30
B	33.24	22.59	23.43
C	44.85	27.19	28.57

larger impact on the average percent difference, the results are comparable with that of the operational method. The operational method is not constrained by the value of kb and application of a straight bias is more applicable to external plunger systems; therefore, it is more appropriate for correcting the model such that the experimental a/s can be more accurately predicted.

With a correction to the model established, the operational range of the wavemaker can be evaluated based on the results. In general, running the wavemaker in the low frequency, low flow (or no flow) range will generate waves that can be more easily predicted by the corrected model. With respect to the frequency, values lower than 2.0 Hz will more advantageous for reducing the error between the model and experimental outcome. Additionally, the ultrasonic sensor was shown to become more accurate in its measurement of frequency for frequencies less than 2.0 Hz. Since the sensitivity analysis of the wavemaker model revealed that uncertainty in the frequency has the largest impact on uncertainty in the model, reliability in its measurement is critical. In regards to the flow, rates lower than 0.229 m/s will also be beneficial for the model prediction since there will be less noise in the measured signal due to turbulence. Considering the results of the reflection analysis, operating at lower frequencies and lower flow rates will also increase the efficiency of the wave absorber, allowing for more steady and repeatable waves to be generated in the water channel. However, a combination of $U = 0.076$ m/s and $f = 1.6$ Hz should be avoided due to the high amount of reflection that occurs under those circumstances.

The analysis of the experimental tests indicated that the impact of the mean wedge depth and wavemaker stroke amplitude was not significant compared to the frequency and flow rate. Nevertheless, for applications that require operating near the maximum limits of the frequency and flow parameters, larger mean depths and stroke amplitudes will be advantageous for generating the predicted waves due to the

signal-to-noise ratio. Therefore, the range of operational values for both the mean depth and stroke amplitude remains limited only by the geometry of the wedge. Finally, with respect to the wedge angle, wedge B had the lowest average a/s percent difference in both the original experimental results and with the applied correction models. However, since the three wedges produce different wave profiles for the same set of parameters, evaluating the impact of the wedge angle required an additional set of tests.

7.4 Wedge Comparison

For the same test case parameters (mean depth, frequency, stroke amplitude and flow rate), each wedge will produce a wave with a different amplitude due to the difference in wedge angle. The wave profile for an example case as produced by wedge A and wedge C is compared in Fig. 7.17 for an interval of 10 s. As predicted, the wave amplitude for wedge C is larger than that of wedge A in Fig. 7.17. For a larger angle, the width of the wedge at a 10 cm mean depth will be larger than that of a smaller wedge at the same mean depth. Therefore, to compare the performance of the three wedges, an additional set of tests were performed on wedge B and wedge C where the width of the wedge was reconfigured so as to match the width of wedge A at its mean depths of 10 cm, 12 cm, and 14 cm. In doing so, for the same flow, frequency, and stroke amplitude, each wedge should produce the same wave profile. Fig. 7.18 demonstrates the waves produced by each wedge with (bottom) and without (top) the presence of flow. For the no flow case in Fig. 7.18, the theoretical wave amplitude is 6.74 mm. The average wave amplitude measured wedge A, B, and C are 7.26 mm, 8.83 mm, and 7.41 mm, respectively. Similarly, the theoretical wave amplitude for

Figure 7.17: The wave profile produced by wedge A (solid) and wedge C (dashed) for $f = 1.2$ Hz, $d_\mu = 10$ cm, $s = 2$ cm, and $U = 0.000$ m/s.

the 0.076 m/s case was 6.15 mm whereas the average amplitude generated by wedges A, B, and C were 4.59 mm, 5.19 mm, and 4.13 mm, respectively. In both cases, all three wedges are comparable in terms of the amplitude of the waves produced with all three wedges overestimating the model in the no flow case but underestimating the model in the flow case. A preliminary analysis of the additional 420 tests was performed comparing the experimental and theoretical wave profiles; however, the results did not indicate that one wedge was more advantageous over another. To fully compare the performance of the wedges, there are various additional metrics which can be considered; for example, the vertical wave symmetry, horizontal symmetry, wave amplitude variability, and phase shift. In general, a wedge with a larger angle will

Figure 7.18: The wave profiles produced by each wedge in comparison to the theoretical wave profile for no flow (top) and flow (bottom). The waves were generated with $f = 1.2$ Hz and $s = 2$ cm

be able to generate higher amplitude waves for smaller stroke amplitudes; therefore, wedge C would be more advantageous to use with varying hydrodynamic applications. However, to choose the most suitable wedge for a specific application would require a much more involved statistical analysis of the results which was outside the scope of the thesis.

7.5 Summary

In this chapter, the results of the experimental testing for the plunger-type wavemaker were presented. First, a comparison between the a/s as measured by the ultrasonic

and predicted by the theoretical model was investigated considering the effects of flow rate, frequency, mean wedge depth, wedge angle, and stroke amplitude. Averaging the results of the tests performed with each experimental wedge, the percent difference between experiment and theory was determined to be 38.20%, 33.24%, and 44.85% for wedge A, B, and C, respectively. The reflection in the water channel for each of the first 900 tests was also investigated as a possible source of error. While the majority of the test cases has reflection coefficients less than one, it was observed that for a flow of 0.076 m/s and frequency of 1.6 Hz, all three wedges produced waves with significantly high reflection coefficients. Since the model does not take into consideration phenomena such as wave reflection, two different methods for correcting the theoretical model were investigated. The case specific correction scaled the model by a correction factor ζ_{kb} dependent on the dimensionless parameter kb . The correction lead to a reduction in the average a/s difference to 26.59%, 22.59%, and 27.19% for wedge A, B, and C, respectively. The operational correction applied a consistent decrease of 21% to the normalized model, regardless of kb . In doing so, the average a/s difference reduced to 28.30%, 23.43%, and 28.57% for the respective wedges A, B, and C. Since both methods resulted in similar results, it was concluded that the second method is more appropriate for correcting the model due to the limited applicability of the case specific correction. With the correction applied, a comparison between three wedges was presented using an additional set of 420 tests which eliminated the wedge angle dependency. Preliminary analysis of the wedge comparison tests demonstrated that the wave profiles are comparable regardless of wedge angle; however, a full statistical analysis of the results was outside the scope of the thesis. Overall, it was determined that operating the wavemaker at flow rates under 0.229 m/s and frequencies under 2.0 Hz will generate waves that can be more accurately predicted by the corrected model.

Chapter 8

Conclusions and Future Work

The major objectives of this thesis were:

1. To develop and investigate the equipment required to measure the free surface elevation of the water over time in a laboratory water channel.
2. To explore the design aspects of a plunger-type wavemaker and experimentally validate the wavemaker's transfer function for water with flow.
3. To quantify the influence of uncertain input parameters on the output of both the conductive wave gauge and the wavemaker models through variance-based global sensitivity analysis.

In this chapter, the main results and conclusions from each chapter are summarized in relation to the objectives and contributions of the thesis. A discussion of potential areas for future work is also included along with a list of publications based on the work in this thesis.

8.1 Conclusions

CHAPTER 1: INTRODUCTION

Chapter 1 introduced the requirement for a plunger-type wavemaker in the water channel at Carleton University. The chapter also discussed the deficiency of research involving the wavemaker model and experimental work with regards to the inclusion of a steady flow.

CHAPTER 2: BACKGROUND AND LITERATURE REVIEW

Chapter 2 provided a review of the literature and background information required to fulfil the objectives of the thesis. Initially, it was shown that in order include the impact of a steady flow in the theory for regular waves, the dispersion relations required modification. The use of wave absorbers, which reduce the amount of wave reflection in the channel was also discussed. It was shown that the reflection in the channel can be quantified by separating the incident waves from the reflected waves by measuring the wave profile with two or more sensors. To support the second and third objectives, a review of the theoretical and experimental uses of plunger-type wavemakers was provided in this chapter. The model which describes the relationship between the plunger system's stroke amplitude and wave amplitude was presented which takes into consideration a steady flow ≥ 0 . Finally, through a review of sensitivity analysis methods, it was determined that a variance-based sensitivity analysis was most appropriate for evaluating the influence of the input parameters in the wavemaker model.

CHAPTER 3: LABORATORY EQUIPMENT

Chapter 3 presented a detailed discussion on the laboratory equipment required for the water channel in order to fulfil the first objective of the thesis. In a water channel,

the most commonly used sensor is that of a wave gauge; however, commercially available gauges are expensive. Therefore, as the first contribution of the thesis, *a conductive wave gauge was developed as a low cost solution for measuring the surface elevation of the water*. In this chapter, the design and calibration of the wave gauge was presented. To cross-check the data recorded by the gauge, two additional sensors were investigated for use in the channel including a camera and an ultrasonic sensor. The camera was mounted outside the water channel to visually record the wave profile from a horizontal position while the ultrasonic sensor was positioned vertically above the water. Since the true elevation of the water is unknown, comparison of the signals measured by each signal provides a metric for evaluating the design of the wavemaker while also exhibiting the advantages of each sensor.

As the second contribution of this thesis, *the use of a wave absorber, to reduce the amount of wave energy reflection in the water channel, was developed and evaluated through some preliminary tests*. It was determined that the inclusion of the absorber decreases the reflection in the channel but, the addition of floating rope to the face of the absorber can decrease it further. Furthermore, it was observed that as the flow rate increased, the efficiency of the wave absorber decreased, as did the repeatability of the tests due to the increase in random noise introduced at higher flows.

CHAPTER 4: VARIANCE-BASED GLOBAL SENSITIVITY ANALYSIS THEORY

In Chapter 4, the numerical method for conducting a variance-based global sensitivity analysis was described in detail. It was shown that by using Monte Carlo simulations, numerical expressions exist that estimate the first order and total effect sensitivity indices. Furthermore, by using a numerical method, the uncertainty in the sensitivity index estimates can be efficiently determined. This chapter also provided a validation of the MATLAB developed to implement the sensitivity analysis.

CHAPTER 5: SENSITIVITY ANALYSIS RESULTS

Chapter 5 presented the results of the sensitivity analysis for both the conductive wave gauge model and wavemaker model in order to fulfil the second objective of the thesis. The analysis of the wave gauge showed that the model is most dependent on the depth of the wires in the water and that calibration of the gauge should be performed regularly to accommodate for the unknown conductivity of the water. Next, the sensitivity indices for the input parameters in the plunger-type wavemaker model were presented and discussed for both a broad and specific distribution range of the input parameters. The analysis established that the number of nodes points required to define the kinematic boundary on the wavemaker is a non-influential parameter. On the other hand, the analysis determined that the wave frequency has the largest influence on the wavemaker model. Therefore, for hydrodynamic applications dependent on plunger systems, the error in the frequency measurement should be reduced as much as possible in order to achieve a more accurate wave profile. On the other hand, although the flow parameter had a low impact on the wavemaker, its influence and inclusion in the model cannot be neglected due to its interactions with the remaining influential parameters. Completing the third objective in this chapter resulted in the third contribution of the thesis, *an investigation of the theoretical model of the plunger-type wavemaker through application of a variance-based global sensitivity analysis*. Since a sensitivity analysis has not been previously applied to the plunger model, the results of this thesis allow for the theoretical design of the wavemaker to be taken into consideration for application to experimental plunger systems. The work in this chapter also lead to a publication of the sensitivity analysis of the plunger-type wavemaker in Global OCEANS 2020 [LI20].

CHAPTER 6: EXPERIMENTAL DESIGN OF THE PLUNGER-TYPE WAVE-MAKER

In Chapter 6, the design of the wavemaker system was described and preliminary tests showing the generation of wave profiles with and without flow were discussed. As a result, it was observed that there is an impact in the experimental a/s which must be taken into consideration with the plunger model. The design of the experiment for evaluating the operational range of the plunger system was also provided in this chapter. In addition, a quantitative comparison between each of the sensors was provided which demonstrated that the ultrasonic sensor is the most advantageous tool for use in the water channel.

CHAPTER 7: EXPERIMENTAL WAVEMAKER RESULTS

In Chapter 7, the results of 1320 wavemaker tests were presented and discussed. First, for the first set of 900 tests, a comparison between the a/s as predicted by the model and measured by the ultrasonic sensor was analyzed considering the effects of flow rate, frequency, mean wedge depth, wedge angle, and stroke amplitude. The average percent difference between experiment and theory was determined to be 38.20%, 33.24%, and 44.85% for wedge A, B, and C, respectively. It was observed that as both frequency and flow rate increase, the a/s percent difference also increased. Since experimental work with plunger systems is limited to small numbers of tests that do include the effects of a steady flow, this work lead to the fourth contribution of the thesis, *an experimental validation of the inclusion of moving water in the pre-existing theory for plunger-type wavemakers*. To quantify potential sources of error between the model and experimental results, the reflection in the water channel was determined for the set of 900 tests. While the majority of the test cases had reflection coefficients less than one, it was observed that for a flow of 0.076 m/s and frequency

of 1.6 Hz, all three wedges produced waves with high reflection coefficients.

Since, the plunger model does not take wave reflection into consideration, the work lead to a fifth contribution, *an investigation into correction methods for the wavemaker theory to better reflect the experimental results*. Two different methods for correcting the model were explored in this chapter; however, it was determined that the second method, which applied a 21% decrease to the normalized wavemaker model, was more applicable to external plunger systems. By applying the correction, the average a/s percent difference reduced to 28.30%, 23.43%, and 28.57% for wedges A, B, and C, respectively. To directly compare the performance of the wedges, an additional 420 experimental tests were run and analyze; however, a preliminary analysis suggested that the wave profiles are comparable regardless of wedge angle. Taking into consideration the observed trends, it was concluded that in order for the generated waves to be more accurately predicted by the corrected model, the wavemaker should be operated at flow rates under 0.229 m/s and frequencies under 2.0 Hz. Along with the physical limitations of the wedge, the sixth contribution was therefore, *the determination of the operational range of the experimental plunger-type wavemaker for the water channel at Carleton University*. With an operational range in place and the validation of the inclusion of flow in the model, the plunger-type wavemaker can be more efficiently used for future applications.

8.2 Future Work

This thesis presented an in depth analysis of the plunger-type wavemaker model and experimental system. While a considerable amount of experimental work was conducted in this thesis, there are aspects of the research which could be further explored in future work.

To reduce error in the measurement of the generated wave profiles, further investigation into the design and application of the sensors is required. For example, a study on the effect of wave steepness and its influence on each sensor, particularly the ultrasonic sensor, may provide further insight into the decrease in reliability in the higher frequency range. Additionally, refinement to the design of the conductive wave gauge could eliminate the dependency of its calibration on flow; thus, increasing the gauge's accuracy and efficiency. Similarly, further analysis on the performance of the wave absorber will allow for a more in-depth understanding of the reflectance results observed. Particularly, a study focused on the large reflection coefficients observed for a flow of 0.076 m/s and frequency of 1.6 Hz would be most beneficial to the work.

With respect to the experimental wavemaker, research to quantifying and reducing the amount of leakage around the wedge during wave generation would be beneficial to its design. Further analysis comparing the experimental wedges could also provide interesting knowledge regarding their performance. With improvements applied to the wavemaker as well as the sensors, the experimental validation could be extended to irregular waves. Thus, wave patterns governed by spectra such as the Pierson-Moskowitz and Bretschneider can be studied into order to further expand the range of applications with which the water channel can be used.

8.3 List of Publications

The following paper was published based on the work presented in this thesis:

1. Lowell S., Irani R. A. (2020) Sensitivity analysis of plunger-type wavemakers with water current, IEEE Global OCEANS 2020: Singapore - U.S. Gulf Coast, Virtual, October 5-20th.

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Appendix A

Wave Absorber Matrix Expressions

The following expressions are the elements for the matrix equation (2.32) required for the wave absorber theory in Section 2.2.1. The c_{ij} elements are given as follows [SPP01],

$$\begin{aligned} c_{11} &= \sum_{u=1}^{N_u} \int_0^{T_m} \cos^2(\omega t - k_i x_u) dt \\ &= \sum_{u=1}^{N_u} \left(\frac{T_m}{2} + \frac{1}{4\omega} [\sin(2\omega T_m - 2k_i x_u) + \sin(2k_i x_u)] \right), \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} c_{12} &= \sum_{u=1}^{N_u} \int_0^{T_m} \cos(\omega t - k_i x_u) \cos(\omega t + k_r x_u) dt \\ &= \sum_{u=1}^{N_u} \left(\frac{T_m}{2} \cos(k_i x_u + k_r x_u) + \frac{\sin(2\omega T_m)}{4\omega} \cos(k_i x_u - k_r x_u) + \right. \\ &\quad \left. \frac{1 - \cos(2\omega T_m)}{4\omega} \sin(k_i x_u - k_r x_u) \right), \end{aligned} \quad (\text{A.2})$$

$$c_{13} = \sum_{u=1}^{N_u} \int_0^{T_m} \cos(\omega t - k_i x_u) \sin(\omega t - k_i x_u) dt$$

$$= \sum_{u=1}^{N_u} \frac{1}{4\omega} [\cos(2\omega T_m - 2k_i x_u) + \cos(2k_i x_u)], \quad (\text{A.3})$$

$$\begin{aligned} c_{14} &= \sum_{u=1}^{N_u} \int_0^{T_m} -\cos(\omega t - k_i x_u) \sin(\omega t + k_r x_u) dt \\ &= \sum_{u=1}^{N_u} \left(-\frac{T_m}{2} \sin(k_i x_u + k_r x_u) - \frac{\sin(2\omega T_m)}{4\omega} \sin(k_i x_u - k_r x_u) - \right. \\ &\quad \left. \frac{1 - \cos(2\omega T_m)}{4\omega} \cos(k_i x_u - k_r x_u) \right), \end{aligned} \quad (\text{A.4})$$

$$c_{21} = c_{12}, \quad (\text{A.5})$$

$$\begin{aligned} c_{22} &= \sum_{u=1}^{N_u} \int_0^{T_m} \cos^2(\omega t + k_r x_u) dt \\ &= \sum_{u=1}^{N_u} \left(\frac{T_m}{2} + \frac{1}{4\omega} [\sin(2\omega T_m + 2k_r x_u) - \sin(2k_r x_u)] \right), \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} c_{23} &= \sum_{u=1}^{N_u} \int_0^{T_m} \cos(\omega t + k_r x_u) \sin(\omega t - k_i x_u) dt \\ &= \sum_{u=1}^{N_u} \left(-\frac{T_m}{2} \sin(k_i x_u + k_r x_u) - \frac{\sin(2\omega T_m)}{4\omega} \sin(k_i x_u - k_r x_u) + \right. \\ &\quad \left. \frac{1 - \cos(2\omega T_m)}{4\omega} \cos(k_i x_u - k_r x_u) \right), \end{aligned} \quad (\text{A.7})$$

$$c_{24} = \sum_{u=1}^{N_u} \int_0^{T_m} -\cos(\omega t + k_r x_u) \sin(\omega t + k_r x_u) dt$$

$$= \sum_{u=1}^{N_u} \frac{1}{4\omega} [\cos(2\omega T_m + 2k_r x_u) - \cos(2k_r x_u)], \quad (\text{A.8})$$

$$c_{31} = c_{13}, \quad (\text{A.9})$$

$$c_{32} = c_{23}, \quad (\text{A.10})$$

$$\begin{aligned} c_{33} &= \sum_{u=1}^{N_u} \int_0^{T_m} \sin^2(\omega t - k_i x_u) dt \\ &= \sum_{u=1}^{N_u} \left(\frac{T_m}{2} - \frac{1}{4\omega} [\sin(2\omega T_m - 2k_i x_u) + \sin(2k_i x_u)] \right), \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} c_{34} &= \sum_{u=1}^{N_u} \int_0^{T_m} -\sin(\omega t - k_i x_u) \sin(\omega t + k_r x_u) dt \\ &= \sum_{u=1}^{N_u} \left(-\frac{T_m}{2} \cos(k_i x_u + k_r x_u) + \frac{\sin(2\omega T_m)}{4\omega} \cos(k_i x_u - k_r x_u) + \right. \\ &\quad \left. \frac{1 - \cos(2\omega T_m)}{4\omega} \sin(k_i x_u - k_r x_u) \right), \end{aligned} \quad (\text{A.12})$$

$$c_{41} = c_{14}, \quad (\text{A.13})$$

$$c_{42} = c_{24}, \quad (\text{A.14})$$

$$c_{43} = c_{34}, \quad (\text{A.15})$$

$$\begin{aligned}
c_{44} &= \sum_{u=1}^{N_u} \int_0^{T_m} \sin^2(\omega t + k_r x_u) dt \\
&= \sum_{u=1}^{N_u} \left(\frac{T_m}{2} - \frac{1}{4\omega} [\sin(2\omega T_m + 2k_r x_u) - \sin(2k_r x_u)] \right), \tag{A.16}
\end{aligned}$$

where N_u is the total number of sensors and $u = 1, 2, \dots, N_u$. Similarly, the F_j are defined by [SPP01],

$$F_1 = \sum_u^{N_u} \int_0^{T_m} \cos(\omega t - k_i x_u) \eta_u(t) dt, \tag{A.17}$$

$$F_2 = \sum_u^{N_u} \int_0^{T_m} \cos(\omega t + k_r x_u) \eta_u(t) dt, \tag{A.18}$$

$$F_3 = \sum_u^{N_u} \int_0^{T_m} \sin(\omega t - k_i x_u) \eta_u(t) dt, \tag{A.19}$$

$$F_4 = \sum_u^{N_u} \int_0^{T_m} -\sin(\omega t + k_r x_u) \eta_u(t) dt. \tag{A.20}$$

Appendix B

Wave Gauge Circuit Diagram

This appendix provides the circuit diagram of the conductive wave gauge. The circuit for the wave gauge consists of two operational amplifiers and a source of voltage provided by an external power supply. The voltage signal is passed through the first operational amplifier and the signal is sent to the wave gauge which is at a fixed position in the water. The signal passes from one wire, through the water which acts as a resistor and is picked up by the second wire which is connected to ground. The overall voltage signal is then sent through a second operational amplifier and the voltage difference between it and the ground is read through a National instruments DAQ into the developed LabVIEW program as a function of time.

Figure B.1: Circuit diagram for the conductive wave gauge.

Appendix C

Table of Experimental Tests

The following appendix provides a list of the experimental tests performed with the plunger system. Table C outlines the mean wedge depths, stroke amplitudes, and frequencies to be varied for each of the three experimental wedges and five flow rates.

Table C.1: Outline of the regular plunger-type wavemaker experimental tests for a single wedge and flow velocity.

Test No.	Mean Wedge Depth [m]	Stroke Amplitude [m]	Frequency [Hz]
01	0.100	0.01	0.80
02	0.100	0.01	1.20
03	0.100	0.01	1.60
04	0.100	0.01	2.00
05	0.100	0.01	2.40
06	0.100	0.02	0.80
07	0.100	0.02	1.20
08	0.100	0.02	1.60
09	0.100	0.02	2.00
10	0.100	0.02	2.40

continued ...

... continued

Test No.	Wedge Depth [m]	Stroke Amplitude [m]	Frequency [Hz]
11	0.100	0.03	0.80
12	0.100	0.03	1.20
13	0.100	0.03	1.60
14	0.100	0.03	2.00
15	0.100	0.03	2.40
16	0.100	0.04	0.80
17	0.100	0.04	1.20
18	0.100	0.04	1.60
19	0.100	0.05	0.80
20	0.100	0.05	1.20
21	0.120	0.01	0.80
22	0.120	0.01	1.20
23	0.120	0.01	1.60
24	0.120	0.01	2.00
25	0.120	0.01	2.40
26	0.120	0.02	0.80
27	0.120	0.02	1.20
28	0.120	0.02	1.60
29	0.120	0.02	2.00
30	0.120	0.02	2.40
31	0.120	0.03	0.80
32	0.120	0.03	1.20
33	0.120	0.03	1.60

continued ...

... continued

Test No.	Wedge Depth [m]	Stroke Amplitude [m]	Frequency [Hz]
34	0.120	0.03	2.00
35	0.120	0.03	2.40
36	0.120	0.04	0.80
37	0.120	0.04	1.20
38	0.120	0.04	1.60
39	0.120	0.05	0.80
40	0.120	0.05	1.20
41	0.140	0.01	0.80
42	0.140	0.01	1.20
43	0.140	0.01	1.60
44	0.140	0.01	2.00
45	0.140	0.01	2.40
46	0.140	0.02	0.80
47	0.140	0.02	1.20
48	0.140	0.02	1.60
49	0.140	0.02	2.00
50	0.140	0.02	2.40
51	0.140	0.03	0.80
52	0.140	0.03	1.20
53	0.140	0.03	1.60
54	0.140	0.03	2.00
55	0.140	0.03	2.40
56	0.140	0.04	0.80

continued ...

...continued

Test No.	Wedge Depth [m]	Stroke Amplitude [m]	Frequency [Hz]
57	0.140	0.04	1.20
58	0.140	0.04	1.60
59	0.140	0.05	0.80
60	0.140	0.05	1.20

Appendix D

Experimental Results

This appendix provides the additional figures from the experimental results in Section 7.1. The a/s percent difference between the model and experimental results is provided in Fig. D.1 and Fig. D.2 as a function of the flow rate U and frequency f . Each data point in the surface planes is averaged over three mean depths and up to five stroke amplitudes. The maximum and minimum percent difference is also shown and represented by a (+) and (o) marker, respectively. For wedge A, the maximum percent difference occurs at $f=2.4$ Hz and $U=0.305$ m/s, and is equal to 112.41%, while the minimum percent difference occurs at $f=0.8$ Hz and $U=0.076$ m/s, and is equal to 6.55%. For wedge C, the maximum percent difference occurs at $f=2.4$ Hz and $U=0.305$ m/s, and is equal to 120.21%. The minimum percent difference for wedge C occurs at $f=1.2$ Hz and $U=0.076$ m/s, and is equal to 11.77%.

Figure D.1: a/s percent difference as a function of the flow rate and frequency measured by the sensor and predicted by the model for wedge A.

Figure D.2: a/s percent difference as a function of the flow rate and frequency measured by the sensor and predicted by the model for wedge C.