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***Redistribution of Longitudinal Moment in Straight
Continuous Concrete Slab-Steel Girder Composite
Bridges***

by
Afshin Esfandiari, B.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research in partial fulfillment
of the requirements for the degree of

Master of Applied Science

Department of Civil and Environmental
Carleton University
Ottawa, Ontario
August 2001

The Master of Applied Science in Civil Engineering is a joint program with the
University of Ottawa, administered by the Ottawa-Carleton Institute for Civil
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Concrete Slab-Steel Girder Composite Bridges***

Submitted by Afshin Esfandiari
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Chair, Department of Civil and Environmental
Engineering

Thesis Supervisor

Carleton University
Ottawa, Ontario
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ABSTRACT

The effect of loading and geometric parameters on longitudinal moment distribution transversely and longitudinally in continuous composite concrete slab on steel girder bridges in the entire range of loading up to failure was investigated. The load distribution for about 50 bridges with different loading cases or geometry were determined and compared with the relevant predictions of the CHBD (2000), and the AASHTO (1994) bridge codes in addition to those of some available methods in the literature.

The finite element program NONLACS was used as the analytical tool and only material nonlinearities were considered. The shear connector element in the program was modified to improve its formulation. Subsequently, the results of NONLACS were verified and compared well with experimental results.

The parameters studied included longitudinal truck location, number of lanes loaded, bridge width, number of girders, slab thickness, degree of interaction, and presence of diaphragms. This study revealed that at the ultimate limit state (ULS) among the preceding parameters only the number of lanes loaded and bridge width affect the transverse distribution of bridges significantly. Longitudinal moment redistribution at the ULS is noticeably influenced by most of the preceding parameters except for the diaphragms. Finally, plastic analysis of composite continuous bridges, treated as an equivalent beam, provides a reasonable estimate of their longitudinal moment redistribution capacity.

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LIST OF IMPORTANT NOTATIONS

A	area of cross section
$\{a\}$	flow vector
a, b	experimental constants of shear connectors
$[B]$	strain-displacement vector
B_x, B_y	rotation of the normals to the middle surface of the plane
b	half width of the bridge
C_f	correction factor (CHBD code)
$[D]$	elasticity matrix
D_d	distribution parameter
$[D]_p$	elasto-plastic constitutive matrix
D_x	longitudinal flexural rigidity per unit length
D_y	transverse flexural rigidity per unit width
D_{xy}, D_{yx}	longitudinal torsional rigidity per unit length
D_1, D_2	transverse torsional rigidity per unit width
$\{d\}, \{\delta\}$	vector of nodal displacements
(dM)	vector of unbalanced moments
$\{d\delta_i\}$	vector of incremental displacement due to incremental load
E	modulus of elasticity
E_c	secant modulus of elasticity of concrete at peak stress
E_0	initial tangent modulus of concrete

E_1, E_2	tangent moduli in the principle material directions
e_s	distance between the centroid of the girder and the middle plane of the slab
E_s	Young's modulus of elasticity of steel
E_s^*	strain hardening modulus of steel
F	shear force
F	width dimension factor (CHBD code)
$\{f\}$	load vector
F_m	amplification factor (CHBD code)
$\{F\}_b$	vector of nodal forces due to body forces
$\{F\}_s$	vector of nodal forces due to surface forces
$\{F\}_\epsilon$	vector of nodal forces due to initial strains
$\{F\}_\sigma$	vector of nodal forces due to initial stresses
$\{F\}_b^\epsilon$	element vector of nodal forces due to body forces
$\{F\}_s^\epsilon$	element vector of nodal forces due to surface forces
$\{F\}_\epsilon^\epsilon$	element vector of nodal forces due to initial strains
$\{F\}_\sigma^\epsilon$	element vector of nodal forces due to initial stresses
$\{F_i\}$	vector of unbalanced load for the i th iteration
f	yield function
f_c'	compressive cylinder strength of concrete

f_t'	uniaxial tensile strength of concrete
f_y	yield strength of steel
G	shear modulus
$\{g\}$	vector of body forces
H	strain hardening function
J_2', J_3'	invariants of the deviatoric stress tensor
I_G	flexural moment of inertia of girder cross-section
J_G	torsional moment of inertia of girder cross-section
$[K]$	global structure stiffness matrix
$[K]^e$	local element stiffness matrix
K_s	longitudinal stiffness parameter
$[K]_{pp}$	plane stiffness matrix
$[K]_{bb}$	bending stiffness matrix
$[K]_{pb}, [K]_{bp}$	coupling stiffness matrices
L	span of the bridge
M	total moment on a given section of a bridge
M_I	elastic moment at hinge I
M_{AASHTO}	maximum girder in a single girder due to AASHTO truck loading
M_G	girder moment
M_T	total longitudinal moment on a given section of a bridge

M_y	yield moment of girder
$M_{\theta I}$	moment calculated from moment-rotation relation
m_{RK}	redistribution of moment at hinge K
m_{IJ}	distribution moment coefficient
N	No. of trucks / No. of traffic lanes
N, N_G	number of girders
N, N_L	number of design lanes
n	modular ratio of girder and deck materials
$[N],[G],[H]$	shape function matrix
NL	number of traffic lanes
P	concentrated load
$\{p\}$	vector of surface forces
Q	plastic potential
$\{R\}$	vector of applied nodal forces
R_L, R_f	modification factor for multiple lane loading
S	girder spacing
$[T]$	transformation matrix
t_s	concrete slab thickness
u, v, w	displacements in x y and z directions
V^e	element volume
W_e	width of a design lane load in meters

α	torsional parameter of bridge
β	cracked shear constant
$\{\Delta f_i\}$	vector of incremental applied load
$\{\Delta \delta_i\}$	vector of total displacements due to incremental load
$\{\delta_i\}$	vector of total displacements due to total applied load
$\{\varepsilon\}$	vector of strains
$\left\{\bar{\varepsilon}\right\}$	vector of initial strains
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	normal and shear strains in plane x-y
ε_{cu}	concrete strain at ultimate strength
λ	shear slip
ν_c	Poisson's ratio of concrete
ν_s	Poisson's ratio of steel
$\{\sigma\}$	vector of stresses
$\left\{\bar{\sigma}\right\}$	vector of initial stresses
$\bar{\sigma}$	hydrostatic stress
$\sigma_x, \sigma_y, \tau_{xy}$	normal and shear stresses in plane x-y
σ'_x, σ'_y	deviatoric stresses
θ_x, θ_y	rotations normal to x-y plane
θ_J	rotation at hinge J

$\{x\}$

vector of curvature due to bending

1. INTRODUCTION

1.1 General

Composite bridges comprising a concrete slab on steel girders, with the slab being connected to the girders by means of shear connectors, are quite common in North America. Cost-effectiveness and ease of construction make these structures popular; consequently, they represent 25% of all bridges in Ontario.

In the past 50 years live loads have been increasing on bridges. Due to these higher permitted loads, older bridges, designed for smaller live loads, are often posted with load limitations, i.e. they may be restricted to certain types of traffic. The restriction may create detour and increase the time and cost of travel. The posted loads are, however, obtained from relatively simplified analysis which do not always reflect the true load-carrying capacity of these bridges. Therefore, in the interest of economy and safety, it is important to know the actual ultimate strength of composite bridges, and their moment redistribution capacity both longitudinally and transversely. The consequence of a more refined analysis may be a more accurate assessment of the load-carrying capacity of the structure which may obviate the need for restrictive posting and eschew the inconvenience and cost associated with detours, or in some cases, with the unnecessary retrofit of such bridges.

Although some existing codes, such as the Ontario Highway Bridge Design (OHBD/1991) code, have included moment redistribution in the transverse direction of simply supported composite bridges, no code has yet included transverse and longitudinal redistribution of moments in continuous composite bridges. Continuous composite

bridges are statically indeterminate structures in both the longitudinal and transverse directions. Therefore, when a part of a bridge section yields, the unyielded parts carry a greater proportion of the load compared to the amount shown by elastic analysis. For a ductile continuous beam, using limit analysis, up to 30% moment redistribution can be predicted. When the redistribution is taken into account, the ultimate load capacity of the bridge increases significantly compared to what one obtains based on elastic distribution of moments. This extra available strength is currently not considered by bridge design codes. Accordingly, in some cases, bridges are retrofitted to increase their strength, but the retrofit may be unnecessary if the actual strength of the bridge was correctly calculated. Normally, such calculations can only be made using advanced computer programs and are generally time-consuming. For design purpose, the process can be simplified by utilizing load distribution factors. Existing load distribution factors in design codes are based on elastic analysis, and are therefore inapplicable to inelastic stages of loading. In this thesis we will determine the inelastic load-distribution factors for concrete slab on steel girder composite continuous bridges. Both longitudinal and transverse moment redistribution factors in composite continuous bridges will be investigated.

1.2 Objectives and Scope

The objective of the present work is to determine how at various levels of load, including the ultimate limit state, live loads are distributed among the girders in slab-on-girder continuous composite bridges. It is important to point out that in this study, the ultimate limit state, or ultimate state for the sake of brevity, refers to the formation of the collapse mechanism in a bridge. Thus the ultimate load (or moment) is the minimum load

needed to form the collapse mechanism. More specifically, the answer to the following questions will be sought.

- (i) What is the affect of some key parameters on the redistribution of live loads in composite concrete-steel slab-girder continuous bridges?
- (ii) What is the magnitude of moment carried by the various girders at various levels of load, especially at ultimate load, and what is the extent of redistribution?
- (iii) Knowing the moment distribution, how do the predictions of the codes such as the Canadian Highway Bridge Design (CHBD/2000), and the American Association of State Highway and Transportation Officials (AASHTO/1994) codes, compare to the advanced analysis results and how can the provisions of these codes, especially the CHBD code, be modified to give a better estimate of moment distribution in composite continuous bridges?

To answer the above questions, the following procedure will be followed.

- (i) Key parameters affecting moment redistribution will be established and accordingly a number of bridges will be designed according to OHBD code provisions to investigate the effect of those parameters on moment redistribution at the elastic and inelastic stages.
- (ii) The designed bridges will be analyzed by the finite element method, both elastically and inelastically, and the distribution of moments in various girders in each bridge will be determined.
- (iii) Code predictions will be compared with the finite element results and ways of enhancing the code provisions will be investigated.

The choice of using a finite element program instead of experimental work is motivated by economic considerations. Since a number of parameters will be investigated in this study, many bridge models would be required for experimental investigation and therefore the cost, which is at least 10 thousand dollars for each bridge model, will be exorbitant. In contrast, finite element analysis, which gives accurate results as will be shown in the following chapters, costs a small fraction of the amount mentioned above. In addition to the economic reason, using finite element analysis allows users to model bridges with real dimensions and geometry, which is practically impossible in experimental works.

In this study because of the limitations of available computer facilities and because of the large number of parameters that will be investigated, the maximum number of girders is limited to 6 and the number of spans is limited to 3.

The analysis is limited to material nonlinearities; large deformations are not considered. However, compact steel girders are used for the bridges, which means that failure is expected to be initiated by yielding of materials prior to buckling of the girders. In addition, the concrete slab provides extra resistance against buckling, especially when it acts compositely with the steel girder.

2. LITRATURE REVIEW

2.1 General

As explained in Chapter 1, the purpose of this study is to find the longitudinal moment redistribution in composite continuous bridges in both the transverse and the longitudinal directions at various load levels. In this chapter, previous studies relevant to the subject of this study will be reviewed. It will be noticed in the following sections that a number of studies has been carried out regarding longitudinal moment distribution transversely, at both the elastic and the inelastic states, but very few studies are available with regard to longitudinal moment distribution in the longitudinal direction at ultimate state.

2.2 Previous Work

2.2.1 Transverse Distribution of Moments

Studies in this regard can be classified into two categories; elastic and inelastic (ultimate) distribution.

Elastic Distribution

Newmark (1948) developed an analytical procedure with simplified assumptions for transverse distribution of longitudinal moments in multi-girder bridges based on elastic theory. He analyzed a number of bridges and recommended the following relations for the transverse distribution of total longitudinal moment at any cross-section:

$$M_G = D_f M_T \tag{2.1}$$

$$D_f = \frac{S}{K} \quad (2.2)$$

where M_g is the design live load moment of a given girder at the section of interest, M_T is the maximum moment that can be produced in the same girder by a single line of wheels of the design truck, D_f is the distribution factor, S is the girder spacing and K is a constant. Newmark suggested $K = 1.676$ m.

Later other researchers using the orthotropic plate theory attempted to develop a more refined analytical procedure for bridge analysis. Bares and Massonet (1968) and Cusens and Pama (1975) developed the general solution for orthotropic plate theory.

Alternatively, other researchers used computer-based numerical procedures for finding the distribution of moments. As an example, Heins and Loony (1968) wrote a computer program for analyzing bridges. This program was based on the slope-deflection method and the slab was considered as a continuous orthotropic plate supported on flexible supports (Girders).

Bakht et al (1978) used the concept of orthotropic plate and grillage analogy to develop another simplified method for moment distribution in slab-girder bridges. In this research, an extensive parametric study was conducted, and as a result, it was concluded that the distribution factor of bridges is related to a torsional parameter α and a flexural parameter θ . These parameters are given by

$$\alpha = \frac{D_{xy} + D_{yx} + D_1 + D_2}{2(D_x D_y)^{0.5}} \quad (2.3)$$

$$\theta = \frac{b}{L} \left(\frac{D_x}{D_y} \right)^{0.25} \quad (2.4)$$

where b is half-width of the bridge, L is span of the bridge and the various rigidities are given by

$$D_x = \frac{E_G I_G}{S} + \frac{E_c t^3}{12} \quad (2.5)$$

$$D_y = \frac{E_c t^3}{12(1 - \nu_c^2)} \quad (2.6)$$

$$D_{xy} = \frac{G_G J_G}{S} + \frac{G_c t^3}{6} \quad (2.7)$$

$$D_{yx} = \frac{G_c t^3}{6} \quad (2.8)$$

$$D_1 = D_2 = \nu_c D_y \quad (2.9)$$

in which E , G and ν are the Young's modulus, the shear modulus and the Poisson's ratio respectively, t is the concrete slab thickness, S is the girder spacing, I_G and J_G are the flexural and torsional moment of inertia of the girder cross section. The subscript G refers to girder and c refers to the concrete slab. This method is the basis of the OHBD (1991) code provisions.

Later K^ostem (1984) used a linear finite element analysis for determining the transverse distribution of moments in composite bridges and as a result he proposed the following equation.

$$D_f = \left(2 \frac{N_L}{N_B} \right) + K_1 \left(\frac{S}{L} \right)^n + K_2 \quad (2.10)$$

where N_L , N_B , S , and L are the number of traffic lanes, number of girders, girder spacing and span of the bridge, respectively.

Kennedy and Grace (1983) carried out some experiments on continuous composite bridges. They concluded that use of diaphragms in such bridges enhance the distribution as well as the effectiveness of the plate theory in predicting the elastic response of a continuous composite bridge.

Zokaie et al (1991) introduced a new elastic load distribution factor for bridges as part of a National Cooperative Research in the US. This new factor is the basis of the pertinent AASHTO (1994) provisions and is based on experimental and analytical research. According to the latter study the transverse distribution of longitudinal bending moment for girder bridges is given by

$$D_f = 0.15 + \left(\frac{S}{3}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left[\frac{n(I + Ae^2)}{12Lt^3} \right]^{0.1} \quad (2.11)$$

where S is the girder spacing in *ft.* ($3.5 < S < 16$), L is the span length in *ft.* ($20 < L < 200$), n is the modular ratio of the beam and deck materials, I is the girder moment of inertia in *in.*⁴, A is the cross-sectional area of the girder, e is the distance from the neutral axis of the girder to the middle plane of the slab in *in.*, and t is the concrete slab thickness in *in.*.

Tarhini and Fredrick (1992) used the finite element method to conduct a parametric study of elastic moment distribution in composite bridges and developed the following equation for the distribution factor

$$D_f = .00013L^2 - 0.021L + 1.25\sqrt{S} - \frac{(S+7)}{16} \quad (2.12)$$

where L is the span length (*ft*) and S is the spacing of the girders (*ft*).

Ebeido and Kennedy (1996) used the finite element method to conduct an extensive parametric study on prototype continuous composite skew bridges. They verified their

finite element model by results from tests on three continuous composite bridges. Based on their results, they deduced empirical formulas for both distribution factors of mid-span and support moments. Finally, Mabsout et al (1999) verified the formula developed by Zokaie (1992) by finite element analysis.

Ultimate Distribution

The elastic moment redistribution studies were reviewed basically for historical interest because the concern of the present study is redistribution at the ultimate limit state. The studies in the latter area are few, particularly as they relate to longitudinal redistribution.

Heins and Kuo (1975) used the finite difference method to determine the load distribution factor of bridges at ultimate state. They assumed both concrete and steel as elastic-perfectly plastic materials. They used two methods to calculate the distribution factor at the ultimate limit state:

Method 1: In this method D_f is determined based on the fraction of the total moment on the entire section and is given by

$$D_f = \frac{2 \left(\text{Max. } \frac{M_y}{M} \right) M_y}{N \times M_{AASHTO}} \quad (2.13)$$

where

M = total longitudinal moment on a given section of the bridge

M_y = yielding moment of girder

N = No. of trucks / No. of traffic lanes

M_{AASHTO} = maximum moment in a single girder due to AASHTO truck loading

Method 2: In this method D_f is determined based on fraction of a line of truck wheels and is given by

$$D_f = \frac{\left(\text{Max. } \frac{M_y}{M} \right) (2 \times NL)}{\sum \frac{M}{M_y}} \quad (2.14)$$

where

NL = No. of traffic lanes

$\sum \frac{M}{M_y}$ = Sum of moment ratios for all girders across the bridge

Based on experimental results, Fan (1974) proposed the following equation for moment distribution factor at ultimate.

$$D_f = \frac{2 \times \text{Number of traffic lanes}}{\text{Number of Girders}} \quad (2.15)$$

Using the finite differences method, Fu et al (1976) verified the equation proposed by Fan (1974). They also stated that diaphragms do not have significant effect on transverse moment redistribution in bridges.

Cheung et al (1986) carried out a parametric study for transverse moment distribution in composite bridges at ultimate state. They tested a bridge model and verified the results of the finite element program ANSYS by achieving good comparison between the experimental and the analytical results. Subsequently, they analyzed a number of bridges and concluded that the distribution parameters given by OHBD (1983) are 15% to 17% conservative. They also stated that the distribution of moment at the ultimate state is different from that in the elastic state and that diaphragms have no significant effect on the transverse distribution of moments.

Later Razaqpur and Nofal (1988) verified their program NONLACS by comparing its analytical results with the experimental result of the bridge model which they built for this purpose. They carried out a parametric study and stated that for simply supported composite bridges, 6% redistribution of moment can be allowed at ultimate state.

Soliman and Kennedy (1990) used the finite element program ABAQUS (Hibbit et al 1989) to investigate the effects of bridge continuity, aspect ratio and welded and bolted diaphragms on the load distribution factor of composite bridges. They concluded that distribution factors of two-span bridges are not significantly different from those of simple span bridges. They also stated that bridges with welded diaphragms are stronger than those with bolted diaphragms and the aspect ratio of the bridge has a significant effect on load distribution factor at ultimate state. This study was limited to two-span bridges and also does not report the longitudinal moment redistribution of such bridges.

2.2.2 Longitudinal Moment Distribution

Dishongh and Galambos (1992) introduced the IMR (Inelastic Moment Redistribution) procedures for determining the inelastic moment redistributions and residual deformations in continuous bridges. IMR procedure makes use of conjugate beam method to relate inelastic hinge rotations to redistribution of moments and the resulting displacements. The following steps are involved in this method.

1. The elastic moment envelope for the load level to be analyzed is determined.
2. Moment-rotation relation for all possible plastic hinges is determined by sectional analysis. The hinges are usually formed at the supports and under the concentrated loads created by the passing truck.

3. The distribution moment coefficient m_{IJ} , which is the rotation of the hinge J due to a unit rotation of hinge I , is calculated by the conjugate beam method.
4. Each hinge is unlocked and its rotation is calculated while all the other hinges are held locked at their previously attained values. Then it is written that

$$M_I = \sum_{J=1}^n m_{IJ} \theta_J + M_{\theta I} \quad (2.16)$$

in which M_I is the elastic moment at hinge I from the elastic moment envelop, m_{IJ} is the redistribution coefficient, θ_J is rotation of hinge J , $M_{\theta I}$ is the moment calculated from moment-rotation relation derived in step 2, and n is the number of possible plastic hinges.

5. Step 4 is repeated until none of the calculated rotations change value compared with the previous cycle.
6. Using the rotations calculated in step 5, redistribution of moment at any hinge is determined by the following formula.

$$m_{RK} = \sum_{J=1}^n m_{KJ} \theta_J \quad (2.17)$$

where m_{RK} is redistribution of moment at hinge K .

7. Using the conjugate beam method with redistribution moments and calculated rotations, the associated residual deformations are computed.

This method assumes that a multi girder bridge behaves as a simple beam and all girders experience plastic hinges at the same longitudinal location, which may not be always true. This is because it essentially assumes that irrespective of the live load position, there is no transverse load distribution. The implication of this assumption is

that at a critical section, plastic hinge develops at all the girders at the same section. With that assumption, IMR may not predict the moment redistribution of a bridge properly.

2.2.3 Collapse Load

Rowe (1962) extended the yield line theory for composite bridges and later some attempts were made by Sawko and Saha (1967), Reddy and Hendry (1969), Lowe and Flint (1971), McCarth and Melham (1988) to predict the collapse load of these structures.

Kennedy and Soliman (1992) developed an analytical procedure based on yield line theory and proposed two formulas for determining the collapse load of continuous composite bridges under symmetric and asymmetric loading cases. They verified their formulas with some experimental tests and extended the proposed formulas for AASHTO truck loading.

Later Kennedy and Helba (1994) used the yield line theory for continuous composite skew bridges. They proposed two relations for the collapse load of these bridges under eccentric and concentric loading and compared their results with the experimental results of bridge models they had built for this purpose. Finally, they adjusted their formula for AASHTO truck loading.

Theoretically, the yield line theory is a reasonable method for predicting the collapse load of structures. However, it is practically difficult to determine the longitudinal and transverse moment distribution by this method.

2.3 Nonlinear Analysis

One of the analytical methods, which considers the elasto-plastic behavior of materials and can be used to determine the ultimate strength of slab-girder bridges, is

grillage analogy. In this method the bridge is idealized as an equivalent grillage and the idealized structure is analyzed by using the plastic hinge method. This method is well described by Sawko (1964) and Jaeger and Bakht (1982).

Yield line theory is another method, which can be used for ultimate strength of the structure and was extended to composite bridges by Rowe (1962). In this method the composite bridge is transformed to an equivalent orthotropic plate based on elastic theory and analyzed by using the yield line theory. Kennedy and Soliman (1992) based on yield line theory developed another method to compute the ultimate load of composite bridges. In this method the real ultimate moments of different parts of bridge are used along the yield lines and no equivalent bridge is used.

Finite difference and finite element methods are two numerical methods for nonlinear analysis. These methods rely on computers because a large number of equations need to be solved during the analysis. Szilard (1974) has given the complete formulation of finite differences method for plates. Heins and Looney (1969) also modeled composite bridges using the finite difference method. Since the present study will be carried out by means of the finite element method, the finite element formulation will be discussed in the next chapter.

2.4 Code Approaches

Currently the two important codes for bridge design in North America are the Canadian Highway Bridge Design (CHBD/2000), and the American Association of State Highway and Transportation Officials (AASHTO/1994) codes. Neither code at present permits redistribution of longitudinal moments in the longitudinal direction. In their latest version, distribution of longitudinal moment in the transverse direction is determined by

$$M_G = D_f M_T \quad (2.18)$$

where M_G is the design live load moment for a girder, M_T is the maximum moment that can be produced in the same girder by a single truck load, and D_f is the distribution factor. Distribution factor D_f is given in each of the above codes as follows.

CHBD code:

$$D_f = \frac{F_m n R_L}{N} \quad (2.19)$$

where

n = Number of design lanes

R_L = Modification factor for multiple lane loading

N = Number of girders

F_m = Amplification factor given by

$$F_m = \frac{S N}{F \left(1 + \frac{\mu C_f}{100} \right)} \geq 1.05 \quad (2.20)$$

in which

$$\mu = \frac{w_e - 3.3}{0.6} \leq 1 \quad (2.12)$$

w_e = Width of design lane in meters

S = Center-to center girder spacing in meters

C_f = Correction factor in %

F = width dimension factor

F and C_f are obtained from the specified tables in the code. For the bridges which have more than four design lanes, F is calculated from the following.

$$F = F_4 \frac{n R_L}{2.80} \quad (2.22)$$

where F_4 is the value of F for four-design lane bridge.

AASHTO code:

If only one design lane is loaded D_f is given by

$$D_f = 0.06 + \left(\frac{S}{4300} \right)^{0.4} \left(\frac{S}{L} \right)^{0.3} \left(\frac{K_g}{L t_s^3} \right)^{0.1} \quad (2.23)$$

If two or more design lanes are loaded D_f is given by

$$D_f = 0.075 + \left(\frac{S}{2900} \right)^{0.6} \left(\frac{S}{L} \right)^{0.2} \left(\frac{K_g}{L t_s^3} \right)^{0.1} \quad (2.24)$$

In the above equations S is the spacing of the girders (mm), L is the span of the bridge (mm), t_s is the depth of concrete slab (mm), and K_g is the longitudinal stiffness parameter given by

$$K_g = n (I + A e_g^2) \quad (2.25)$$

in which A is the area of the girder cross-section (mm^2), n is the modular ratio of girder and deck materials, I is the moment of inertia of the girder (mm^4), and e_g is the distance between the centroid of the girder and middle plane of the slab (mm).

In the present study we will investigate whether the code equations can be applied to inelastic moment redistribution transversely. Furthermore, we will determine the

significance of transverse versus longitudinal moment redistribution at the ultimate limit state in continuous multi-girder slab-girder concrete-steel bridges.

3. FINITE ELEMENT PROGRAM

3.1 General

As mentioned in Chapter 1, the finite element method will be used for the analyses in this study. The program used for this purpose is called NONLACS (Nonlinear analysis of concrete and steel) developed in the Department of Civil and Environmental Engineering at Carleton University (Razaqpur & Nofal 1988). NONLACS has benefited from program FELARC developed at the University of Calgary (Ghoniem and Ghali 1982). The latter has used the fundamental structure and some material models of programs NARCS and NOTACS, developed at the university of California at Berkeley. While the Calgary and Berkeley programs can only analyze concrete structures, NONLACS can be used for concrete, steel and composite structures.

The choice of using NONLACS in this research was made due to the fact that this program includes all the elements and material models necessary for analyzing composite bridges, and as will be shown in the next chapter, is fully capable of capturing the detailed behavior of composite bridges.

NONLACS can analyze any structure which can be represented by an assemblage of thin shell elements such as beams, plates, shells, folded plates, box girders and shear walls. This program can also include time-dependent behavior of materials such as creep and shrinkage. In its latest version, large displacements have been included by Razaqpur and Vasilescu (2000), but it does not account for shear deformations normal to the plate and shell surfaces nor for bond slip at the interface of the concrete and steel

reinforcement. However, these effects are small in composite bridges and can be neglected.

In this chapter, the basic finite element equations, finite elements and material models in addition to nonlinear procedure employed in NONLACS will be discussed.

3.2 Basic Equations of Finite Element

The basic finite element equations solved by NONLACS, which are based on minimum potential energy, can be written as

$$[K]\{d\} = \{R\} + \{F\}_b + \{F\}_s + \{F\}_{\epsilon} + \{F\}_{\sigma} \quad (3.1)$$

where for any structure

$[K]$ = Stiffness matrix

$\{d\}$ = Vector of nodal displacements

$\{R\}$ = Vector of applied loads

$\{F\}_b$ = Vector of nodal forces due to body forces

$\{F\}_s$ = Vector of nodal forces due to surface forces

$\{F\}_{\epsilon}$ = Vector of nodal forces due to initial strains

$\{F\}_{\sigma}$ = Vector of nodal forces due to initial stresses

Elements of equation (3.1) can be obtained from

$$[K] = \sum_e [T]^T [K]^e [T] \quad (3.2)$$

$$\{F\} = \sum_e [T]^T \{F\}^e \quad (3.3)$$

$$[K]^e = \int [B]^T [D] [B] dV^e \quad (3.4)$$

$$\{F\}_b^e = \int [N]^T \{g\} dV^e \quad (3.5)$$

$$\{F\}_s^e = \int [N]^T \{p\} dS^e \quad (3.6)$$

$$\{F\}_{\bar{\epsilon}}^e = \int [B]^T [D]^T \{\bar{\epsilon}\} dV^e \quad (3.7)$$

$$\{F\}_{\bar{\sigma}}^e = - \int [B]^T \{\bar{\sigma}\} dV^e \quad (3.8)$$

in the above equations $\{g\}$ and $\{p\}$ are vectors of body and surface forces, respectively;

$\{\bar{\epsilon}\}$ and $\{\bar{\sigma}\}$ are vectors of initial strains and initial stresses ,respectively; $[N]$ is the

shape function of the element; $[T]$ is a transformation matrix which relates components of

a vector in the local coordinate system to its components in the global coordinate system;

$[D]$ is the constitutive matrix of the element, which is derived from material models and

will be discussed later; superscript e stands for element; and $[B]$ is the strain-displacement matrix .

3.3 The Employed Finite Elements

The finite elements used in NONLACS are bar element, shear connector element, four node plane quadrilateral element, four node plate bending quadrilateral element and four node facet shell element. The existing shear connector element was modified for the purpose of this study, but the other elements remain the same.

3.3.1 Bar Element

The bar element is a two node element with two degrees of freedom as shown in Fig. 3.1. It is the standard truss element, which can be used to model any uniaxial

member such as reinforcing bars, links or prestressed tendons. Its shape function, strain-displacement and stiffness matrices are as follows:

$$[N] = [N_1 \quad N_2] \quad (3.9)$$

$$[B] = \frac{1}{L} [-1 \quad 1] \quad (3.10)$$

$$[K] = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \quad (3.11)$$

where

$$K_1 = \frac{EA}{L} \quad (3.12)$$

$$N_1 = 1 - \frac{x}{L} \quad (3.13a)$$

$$N_2 = \frac{x}{L} \quad (3.13b)$$

and E is the modulus of elasticity, A is area of the cross section, x is distance from left node, and L is the length of the element.

Fig. 3.1. Bar Element

3.3.2 Shear Connector Element

Shear connectors, as shown in Fig. 3.2(a), can be modeled as an element with five degrees of freedom where the sixth degree of freedom is constrained by making it axially rigid. To derive the stiffness matrix for these elements, a relationship between shear slip and shear forces acting on the ends of the element is required. One such relationship was proposed by Yam and Chapman (1968) based on their test data, and is shown in Fig. 3.3 and represented by Eq. 3.14.

$$F = a(1 - e^{-b\lambda}) \quad (3.14)$$

where F is the shear force acting on either end of the bar in one of the two orthogonal directions (kN); λ is the corresponding relative displacement or slip in the direction of F (mm), Fig. 3.2 (b); a and b are experimental constants depending on the connector geometry and strength; and e is base of natural logarithm. For the type of connectors used in the current study, Yam and Chapman suggested a and b to be 30 kN and 4.72 /mm.

Using Eq. 3.14 and assuming that shear connectors only allow slip at the interface of the two surfaces joined by them, the stiffness matrix can be determined as follows:

$$[K] = \begin{bmatrix} k_1 & 0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & 0 & k_1 & 0 \\ 0 & 0 & -k_2 & 0 & k_2 \end{bmatrix} \quad (3.15)$$

where k_1 and k_2 are the shear stiffness coefficients in the two orthogonal directions in the plane of the connector cross-section, given by

$$k_1 = \frac{\partial F_1}{\partial \lambda_1} = abe^{-b\lambda_1} \quad (3.15a)$$

Fig. 3.2. Shear Connector Element

Fig. 3.3. Typical Load Slip Curve for Shear Connector

$$k_2 = \frac{\partial F_2}{\partial \lambda_2} = abe^{-b\lambda_2} \quad (3.15b)$$

where F_1 and F_2 are the two components of shear force acting on a connector cross-section and λ_1 and λ_2 are the corresponding slips (Fig. 3.2b).

As stated earlier, the bar is assumed to be axially rigid, which is enforced in the finite element program by imposing equal axial displacement at the two ends of the shear connector.

In using the above stiffness matrix, there is another problem, which should be taken into consideration. As illustrated in Fig. 3.2(b) lateral forces acting on a shear connector equilibrate each other ($\sum F_x = 0$, $\sum F_y = 0$), but there are unbalanced moments $F_1 L$ and $F_2 L$ acting on the connector. This means that the element is not in equilibrium, therefore, it produces extra moments on the structure and violates the equilibrium of the system. This problem leads to over estimation of stresses and earlier prediction of failure of the structure in finite element analysis.

To overcome this problem, the unbalanced moments due to shear connectors should be reversed and applied as unbalanced load at the end of every iteration to restore the equilibrium. These unbalanced moments can be determined by the following equations.

$$(dM_1)_i = (K_1)_{i-1} (d\lambda_1)_i L \quad (3.16a)$$

$$(dM_2)_i = (K_2)_{i-1} (d\lambda_2)_i L \quad (3.16b)$$

where $(dM_1)_i$ and $(dM_2)_i$ are unbalanced moments for i th iteration, $(K_1)_{i-1}$ and $(K_2)_{i-1}$ are stiffness of shear connector corresponding to the results of previous iteration

and $(d\lambda_1)_i, (d\lambda_2)_i$ are incremental slips of shear connector. The total slip and incremental slip of each shear connector is determined using

$$\lambda_1 = u_4 - u_1 \quad (3.17a)$$

$$\lambda_2 = u_3 - u_5 \quad (3.17b)$$

$$d\lambda_1 = du_1 - du_4 \quad (3.18a)$$

$$d\lambda_2 = du_3 - du_5 \quad (3.18b)$$

where u_i and du_i are respectively the total displacement and incremental displacement of the i th degree of freedom.

3.3.3 Refined Plane Quadrilateral Element (RQUAD4)

RQUAD4 is a four node membrane finite element with three degrees of freedom (DOF) per node as shown in Fig. 3.4 and is fully developed by Aziz (1988) and McNeal and Harder (1988). Razaqpur and Nofal (1988) tested the performance of this element in many cases and found that the results were satisfactory. Note that the DOF of this element comprise the two components of inplane displacement and a rotation normal to the plane of the element. As pointed out by Aziz (1988) the rotation ω is not the true rotation, but is a relative rotation. Aziz discusses why the true rotation as a DOF can not be used.

The displacement function of this element can be derived from the eight node plane quadrilateral element QUAD8, Fig. 3.5. Using the side shape function given in Fig. 3.6 for four sides of RQUAD4 element, the displacement of these two elements can be related as follows.

$$\{d\} = [T] \{d^*\} \quad (3.19)$$

where $[T]$ is a 16×12 transformation matrix, $\{d\}$ is the displacement vector of the degrees of freedom of QUAD8 and $\{d^*\}$ is the displacement vector of the degrees of freedom of RQUAD4. In the expanded form Eq. 3.19 can be rewritten as

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \\ u_6 \\ v_6 \\ u_7 \\ v_7 \\ u_8 \\ v_8 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0.5 & 0 & b_1 & 0.5 & 0 & -b_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & a_1 & 0 & 0.5 & -a_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & b_2 & 0.5 & 0 & -b_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & a_2 & 0 & 0.5 & -a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & b_3 & 0.5 & 0 & -b_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & a_3 & 0 & 0.5 & -a_3 \\ 0.5 & 0 & -b_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 & b_4 \\ 0 & 0.5 & -a_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & a_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \\ u_2 \\ v_2 \\ w_2 \\ u_3 \\ v_3 \\ w_3 \\ u_4 \\ v_4 \\ w_4 \end{Bmatrix} \quad (3.20)$$

where $a_i = \frac{(x_i - x_{i+1})}{8}$ and $b_i = \frac{(y_i - y_{i+1})}{8}$ for $i=1,2,3$; and $a_4 = \frac{(x_1 - x_4)}{8}$ and

$b_4 = \frac{(y_4 - y_1)}{8}$. x_i and y_i are coordinates of nod i of RQUAD4 element.

Using the transformation matrix and the principle of virtual work, the stiffness matrix $[K^*]$ and equivalent nodal force matrix $[F^*]$ of RQUAD4 can be determined from the following equations.

$$[K^*] = [T]^T [K] [T] \quad (3.21)$$

$$\{F^*\} = [T]^T \{F\} \quad (3.22)$$

In the above equations, $\{F\}$ is the equivalent nodal force matrix of QUAD8 and $[K]$ is the stiffness matrix of QUAD8, which can be derived as

$$[K] = \int [B]^T [D] [B] dV \quad (3.23)$$

where $[B]$ is the strain-displacement matrix, $[D]$ is the constitutive matrix and V is the volume of QUAD8 element.

Substituting $[K]$ from Eq. (3.23) in Eq. (3.21), Eq. (3.21) can be rewritten as

$$[K^*] = \int [B^*]^T [D] [B^*] dV \quad (3.24)$$

where

$$[B^*] = [B][T] \quad (3.25)$$

Since element QUAD8 is described in details by Owen and Hinton (1980), it will not be elaborated on here any further. For complete details of formulating RQUAD4, reference can be made to Razaqpur and Nofal (1988).

3.3.4 Improved Discrete Kirchhof Quadrilateral Plate Bending Element (IDKQ)

IDKQ is a 12 degree of freedom bending element with four nodes. The degrees of freedom, as shown in Fig. 3.7, are one displacement and two rotations normal to the plane of the element. This element is fully developed by Chinniah (1985). He used the shape functions of nine node Lagrange element, N_i , and replaced them with the

substitute shape functions, $\bar{N}_i$. These shape functions are given by

$$N_i = a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^2\eta + a_8\xi\eta^2 + a_9\xi^2\eta^2 \quad (3.26)$$

$$\bar{N}_i = b_1 + b_2\xi + b_3\eta + b_4\xi^2 + b_5\xi\eta + b_6\eta^2 + b_7\xi^2\eta + b_8\xi\eta^2 \quad (3.27)$$

where ξ and η are the natural coordinates of a given point of the element, a_i are the Lagrange shape function coefficients, and b_i are the substitute shape function coefficients.

The unknown coefficients of the substitute shape function are obtained in terms of known coefficients of the Lagrange shape function by using the least square smoothing procedure as follows:

$$\frac{\int_A (N_i - \bar{N}_i)^2 d\xi d\eta}{\partial b_j} = 0 \quad (j=1 \dots 8) \quad (3.28)$$

in which N_i and $\bar{N}_i$ are the Lagrange and the substitute shape functions, respectively, and b_j is the substitute shape function coefficients.

Chinniah assumed the following constraints for the center and middle points of IDKQ.

$$B_x = \frac{\partial w}{\partial x} \quad (3.29)$$

$$B_y = \frac{\partial w}{\partial y} \quad (3.30)$$

where B_x and B_y are the rotation of the normals to the undeformed middle surface of the plane in the x - z and y - z planes, w is the normal displacement, and x and y are local coordinates.

Using the above constraints, B_x and B_y of any given point can be related to the vector of nodal displacements, $\{\delta\}$ as follows:

$$B_x = \sum_{i=1}^{12} G_i \delta_i = [G]^T \{\delta\} \quad (3.31)$$

Fig. 3.4. Four Node Refined Plane Quadrilateral Element

Fig. 3.5. Eight Node Plane Quadrilateral Element

Fig. 3.6. Side Shape Function of RQUAD4

Fig. 3.7. Quadrilateral Plate Bending Element (IDKQ)

$$B_y = \sum_{i=1}^{12} H_i \delta_i = [H]^T \{\delta\} \quad (3.32)$$

where $[G]$ and $[H]$ are the new shape functions that can be expressed in terms of substitute shape functions.

Using the new shape functions the strain-displacement matrix, $[B]$, is given by

$$[B] = [\{B_1\}, \dots, \{B_{12}\}] \quad (3.33)$$

in which

$$\{B_i\} = \begin{Bmatrix} \frac{\partial G_i}{\partial x} \\ \frac{\partial H_i}{\partial y} \\ \frac{\partial G_i}{\partial y} + \frac{\partial H_i}{\partial x} \end{Bmatrix} \quad (3.34)$$

For more details about finite element formulation of the IDKQ element reference can be made to Aziz (1988).

3.3.5 Quadrilateral Facet Shell Element (QFSE)

The quadrilateral facet shell element is the combination of the RQUAD4 and the IDKQ elements. This element has four nodes and 24 degrees of freedom, 6 per node comprising 3 displacements and 3 rotations and is formulated based on coupling the bending and inplane actions. The following equations are used to couple these two actions.

$$w(x, y) = w_0(x, y) \quad (3.35)$$

$$u(x, y) = u_0(x, y) - z \frac{\partial w}{\partial x} \quad (3.36)$$

$$v(x, y) = v_0(x, y) - z \frac{\partial w}{\partial y} \quad (3.37)$$

where u, v, w are displacement in x, y, z axis directions respectively, u_0, v_0, w_0 are the displacements of the middle or reference plane of the shell. Differentiating the above equations, we obtain the strain components

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \quad (3.38)$$

or

$$\{\varepsilon\} = \{\varepsilon_0\} - z\{\chi\} \quad (3.39)$$

where the inplane strain $\{\varepsilon_0\} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}$, the curvature $\{\chi\} = \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$, and z is the

distance of the point at which strains are calculated to the reference plane.

From basic theory,

$$\{\varepsilon\} = [B]_p \{d\}_p \quad (3.40)$$

$$\{\chi\} = [B]_b \{d\}_b \quad (3.41)$$

where $[B]_p, [B]_b$ are the inplane and the bending strain-displacement matrices and $[d]_p, [d]_b$ are inplane and bending displacement vectors, respectively.

Substituting $\{\chi\}$ and $\{\varepsilon\}$ from Eq. 3.40 and 3.41 in Eq. 3.39, strains are given by

$$\{\varepsilon\} = ([B]_p - z[B]_b) \begin{Bmatrix} d_p \\ d_b \end{Bmatrix} = [B] \{d\} \quad (3.42)$$

Therefore, for the shell element

$$[B] = ([B]_p - z[B]_b) \quad (3.43)$$

The stiffness matrix of the element is given by

$$[K] = \int [B]^T [D] [B] dV \quad (3.44)$$

where $[D]$ is the constitutive matrix.

Substituting $[B]$ from Eq. 3.43 in Eq. 3.44, we get

$$[K] = \int \begin{bmatrix} [B]_p^T [D] [B]_b & -z[B]_p^T [D] [B]_b \\ -z[B]_b^T [D] [B]_p & z^2[B]_b^T [D] [B]_b \end{bmatrix} dV \quad (3.45)$$

or

$$[K] = \begin{bmatrix} [K]_{pp} & [K]_{pb} \\ [K]_{bp} & [K]_{bb} \end{bmatrix} \quad (3.46)$$

where the submatrices can be determined by

$$[K]_{ij} = \int [B]_i^T [D] [B]_j dV \quad (3.46a)$$

in which i, j stand for p, b , respectively.

Dividing the thickness of element into n layers, $[D]_{ij}$ can be obtained as follows:

$$[D]_{pp} = \int [D] dz = \sum_{i=1}^n (z_{i+1} - z_i) [D]_i \quad (3.47)$$

$$[D]_{pb} = [D]_{bp} = - \int z [D] dz = -\frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [D]_i \quad (3.48)$$

$$[D]_{bb} = \int z^2 [D] dz = -\frac{1}{3} \sum_{i=1}^n (z_{i+1}^3 - z_i^3) [D]_i \quad (3.49)$$

where $[D]_i$ is the constitutive matrix of layer i , which depends on the material properties of the layer and will be discussed next and z_{i+1}, z_i are the z coordinates of the top and bottom surface of layer i , respectively.

Using Eq. 3.39 and following the same procedures as used to derive the stiffness matrix, the equivalent nodal forces due to initial strain $\{F\}_{\bar{\epsilon}}$ and initial stress $\{F\}_{\bar{\sigma}}$ are given by

$$\{F\}_{\bar{\epsilon}} = \iint \begin{bmatrix} [B]_p^T \left\{ \bar{\sigma}_{\bar{\epsilon}} \right\}_p \\ [B]_b^T \left\{ \bar{\sigma}_{\bar{\epsilon}} \right\}_b \end{bmatrix} dx dy \quad (3.50)$$

$$\{F\}_{\bar{\sigma}} = - \iint \begin{bmatrix} [B]_p^T \left\{ \bar{\sigma}_{\bar{\sigma}} \right\}_p \\ [B]_b^T \left\{ \bar{\sigma}_{\bar{\sigma}} \right\}_b \end{bmatrix} dx dy \quad (3.51)$$

where $\left\{ \bar{\sigma}_{\bar{\epsilon}} \right\}$ and $\left\{ \bar{\sigma}_{\bar{\sigma}} \right\}$ are the vectors of stress resultants due to initial strain and initial stress, i.e.

$$\left\{ \bar{\sigma}_{\bar{\epsilon}} \right\}_p = \int [D] \left\{ \bar{\epsilon} \right\} dz = \sum_{i=1}^n (z_{i+1} - z_i) [D]_i \left\{ \bar{\epsilon} \right\}_i \quad (3.52)$$

$$\left\{ \bar{\sigma}_{\bar{\epsilon}} \right\}_b = -z \int [D] \left\{ \bar{\epsilon} \right\} dz = -\frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [D]_i \left\{ \bar{\epsilon} \right\}_i \quad (3.53)$$

$$\left\{ \bar{\sigma}_{\bar{\sigma}} \right\}_p = \int [D] \left\{ \bar{\sigma} \right\} dz = \sum_{i=1}^n (z_{i+1} - z_i) [D]_i \left\{ \bar{\sigma} \right\}_i \quad (3.54)$$

$$\left\{ \bar{\sigma}_{\bar{\sigma}} \right\}_b = -z \int [D] \left\{ \bar{\sigma} \right\} dz = -\frac{1}{2} \sum_{i=1}^n (z_{i+1}^2 - z_i^2) [D]_i \left\{ \bar{\sigma} \right\}_i \quad (3.55)$$

In which $\left\{ \bar{\epsilon} \right\}_i$, $\left\{ \bar{\sigma} \right\}_i$ are vectors of initial strain and initial stress for layer i .

3.4 Material Models

To determine the constitutive matrix $[D]$, we need to define the constitutive laws of the materials being analyzed. The materials involved in this study are concrete, steel plate and reinforcing bars. In the following, the constitutive models used for them are described.

3.4.1 Concrete

The uniaxial and biaxial stress-strain relationship of concrete, shown in Fig. 3.8, is used in NONLACS. The compression part consists of two part which are represented respectively by

$$\sigma = \frac{E_0 \varepsilon}{1 + \left(\frac{E_0}{E_{sc}} - 2\right) \frac{\varepsilon}{\varepsilon_{cu}} + \left(\frac{\varepsilon}{\varepsilon_{cu}}\right)^2} \quad (\text{Saenz 1964}) \quad (3.56)$$

$$\sigma = \sigma_c \left(\frac{\varepsilon}{\varepsilon_{cu}}\right)^{\left(1 - \frac{\varepsilon}{\varepsilon_{cu}}\right)} \quad (\text{Smith-Young 1955}) \quad (3.57)$$

where by reference to Fig. 3.8

E_{sc} = secant modulus of elasticity at peak stress;

E_0 = initial tangent modulus;

σ = stress;

σ_c = compressive strength

ε = strain

ε_{cu} = strain at peak stress.

Concrete is assumed to behave as stress-induced orthotropic material under biaxial stresses. This means that initial tangent moduli of the two principal directions are

functions of $\frac{\sigma_1}{\sigma_2}$, where σ_1 and σ_2 are major and minor principle stresses, respectively.

In this model the tangent modulus of each direction is different unless $\frac{\sigma_1}{\sigma_2} = 1$.

The biaxial model used in NONLACS was proposed by Darvin and Pecknold (1977) and is used to relate increments of stress to increments of strain as

$$\{\Delta\sigma\} = [D] \{\Delta\varepsilon\} \quad (3.58)$$

where $\{\Delta\sigma\}$ is a vector of incremental stresses, $\{\Delta\varepsilon\}$ is the corresponding vector of incremental strains and $[D]$ is the constitutive matrix given by

$$[D] = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & \frac{1}{4}(E_1 - E_2 - 2\nu\sqrt{E_1E_2}) \end{bmatrix} \quad (3.59)$$

in which ν = Poisson's ratio; and E_1 and E_2 are the tangent moduli in principal material directions, which are given by

$$E_i = \frac{E_0(1-r^2)}{\left[1 + \left(\frac{E_0}{E_s} - 2\right)r_i + r_i^2\right]^2} \quad (3.60)$$

where

$$r_i = \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \quad (3.61)$$

$$\varepsilon_{iu} = \sum \Delta\varepsilon_{iu} = \sum \frac{\Delta\varepsilon_i}{1-\nu\alpha n} \quad (3.62)$$

in which $\alpha = \frac{\sigma_j}{\sigma_i}$; and $n = \frac{E_i}{E_j}$, where j = the conjugate direction to i .

The concrete strength in biaxial and combined stress field is determined from the failure envelope of Kupfer and Gerstle (1973) shown in Fig. 3.10. If the principal tensile stress at any point equals or exceeds the tensile strength of the concrete, concrete will be assumed to be cracked and the constitutive matrix will be reduced to

$$[D] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \beta G \end{bmatrix} \quad (3.63)$$

where G =elastic shear modulus; and β =shear relation factor with a value less than 1. This factor approximately accounts for aggregate interlock and dowel action and is assumed to be 0.1 in this study. If strain at any point equals or exceeds $\varepsilon_{\max}$, concrete assumed to be crushed.

After concrete cracks, it can still transfer tension between the cracks. This behavior of concrete, which is called tension stiffening, is treated in NONLACS by assuming the descending branch in the tensile stress-strain relation, Fig.3.9, for concrete after cracking.

3.4.2 Steel Plate

The theory of plasticity is used for modeling steel plates in NONLACS. The model adopted here is based on so-called incremental theory of plasticity, which is well established, Hill (1950) and Owen and Hinton (1980), and will be discussed only briefly. In this theory incremental strain is obtained by summing the elastic and plastic increments as in Eq. 3.64.

$$(d\varepsilon)_{ij} = (d\varepsilon_e)_{ij} + (d\varepsilon_p)_{ij} \quad (3.64)$$

in which $d\varepsilon_e$ =elastic strain increment vector; $d\varepsilon_p$ =plastic strain increment vector; and are given by

Fig. 3.8. Uniaxial Stress Strain Relationship of Concrete

Fig. 3.9. Tensile Stress-Strain Relationship of Concrete

Fig. 3.10. Failure Envelope of Concrete

$$(d\varepsilon_e)_{ij} = \frac{d\sigma_{ij}}{2G} + \frac{(1-2\nu)}{E} \delta_{ij} d\sigma_{aa} \quad (3.65)$$

$$(d\varepsilon_p)_{ij} = d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \quad (3.66)$$

where G is the shear modulus, also referred to as the *Lame'* constant, σ_{ij} is the deviatoric stress given by Eq. 3.70, δ_{ij} is the Kronecker delta, Q is the plastic potential, $d\lambda$ is a constant and $d\sigma_{aa}$ is the incremental hydrostatic stress given by

$$\sigma_{aa} = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}) \quad (3.67)$$

As explained by Hill (1970), Q is assumed to be a function of the second and third deviatoric stress invariants, J_2' , J_3' which are given by

$$J_2' = \frac{1}{2} \sigma_{ij}' \sigma_{ij}' \quad (3.68)$$

$$J_3' = \frac{1}{3} \sigma_{ij}' \sigma_{jk}' \sigma_{ki}' \quad (3.69)$$

where the deviatoric stress vector σ_{ij}' is given by

$$\sigma_{ij}' = \sigma_{ij} - \sigma_{aa} \delta_{ij} \quad (3.70)$$

Since the yield function, f , is also a function of J_2' and J_3' , Q is assumed to be equal to f (associated plasticity), therefore, Eq. 3.66 can be rewritten as

$$(d\varepsilon_p)_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (3.71)$$

For yield function the Von-Mises criterion is considered in this study. This yield criterion is based on the assumption that if

$$\sqrt{3}(J_2')^{\frac{1}{2}} \geq \sigma_y \quad (3.72)$$

then plastic flow occurs, where σ_y is the uniaxial yield strength of the material.

The constant $d\lambda$ is given by Owen and Hinton (1980) as

$$d\lambda = \frac{\{a\}^T \{d\sigma\}}{H + \{d_D\}^T \{a\}} \quad (3.73)$$

where

$$\{a\} = \frac{\partial f}{\partial \sigma_{ij}} \quad (3.74)$$

$$\{d_D\} = [D] \{a\} \quad (3.75)$$

$$H = \frac{E_s E_s^*}{E_s - E_s^*} \quad (3.76)$$

in which E_s, E_s^* are the Young's and strain hardening moduli of steel as shown in Fig.

3.11.

Using Von-Mises yield criterion and substituting $d\varepsilon_e$ and $d\varepsilon_p$ from Eq. 3.65 and 3.71 into Eq. 3.65, this model leads to the following finite element formulation.

$$\{d\sigma\} = [D]_{ep} (d\varepsilon) \quad (3.77)$$

in which $\{d\sigma\}$ = incremental stress; $\{d\varepsilon\}$ = incremental strain; and $[D]_{ep}$ is the elasto-plastic constitutive matrix given by

$$[D]_{ep} = \frac{E_s}{S} \begin{bmatrix} \sigma_y'^2 + 2R & & & \text{Symmetry} \\ -\sigma_x' \sigma_y' + 2\nu R & \sigma_x'^2 + 2R & & \\ \frac{\sigma_x' + \nu \sigma_y'}{1+\nu} \tau_{xy} & -\frac{\sigma_y' + \nu \sigma_x'}{1+\nu} \tau_{xy} & \frac{T}{2(1+\nu)} + \frac{2H}{9E} (1-\nu) \sigma'^2 & \end{bmatrix} \quad (3.78)$$

where

$$\begin{bmatrix} R \\ S \\ T \end{bmatrix} = \begin{bmatrix} \frac{2H}{9E} \bar{\sigma}^2 + \frac{\tau_{xy}^2}{1+\nu} \\ T + 2(1-\nu^2)R \\ \sigma_x'^2 + 2\nu\sigma_x'\sigma_y' + \sigma_y'^2 \end{bmatrix} \quad (3.79)$$

In which $\bar{\sigma}$ = hydrostatic stress; and σ_x' and σ_y' = deviatoric stresses given by

$$\bar{\sigma} = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \quad (3.80)$$

$$\sigma_x' = \sigma_x - \bar{\sigma} \quad (3.81)$$

$$\sigma_y' = \sigma_y - \bar{\sigma} \quad (3.82)$$

In this formulation $\sigma_x, \sigma_y, \sigma_z$ are normal stresses and τ_{xy} is the shear stress in the x-y plane.

3.4.3 Reinforcing Bars

The bilinear stress strain relationship shown in Fig. 3.11 is used for reinforcing bars. The material parameters f_y, E_s, E'_s , and ϵ_{su} shown in this figure, are respectively the yield stress; the Young's modulus; the strain-hardening modulus and the ultimate strain.

3.5 Nonlinear Analysis Method

The incremental-iterative procedure, Fig. 3.12, is used for nonlinear analysis in this study. In this method, the load vector $\{f\}$ is subdivided into a number of increments, N , such that

$$\{f\} = \sum_{i=1}^N \{\Delta f_i\} \quad (3.83)$$

These load increments are applied one after another and their effects are added together to find the final response of the structure. Each load step is solved by an iterative

Fig. 3.11. Stress-Strain Relationship for steel

Fig. 3.12. Incremental-Iterative Nonlinear Analysis Method

procedure. In this procedure, the unbalanced load of the k th iteration in the i th load increment, $\{F_k^i\}$, which remains at the end of the previous iteration, is computed by

$$\{F_k^i\} = \sum_{j=1}^i \{\Delta f_j\} - \{F_{k-1}^e\} \quad (3.84)$$

where $\{\Delta f_j\}$ is the j th incremental applied load and $\{F_{k-1}^e\}$ is the equilibrated load after the previous iteration. The incremental displacement $\{d\delta_k^i\}$ corresponding to $\{F_k^i\}$ is then computed by

$$\{d\delta_k^i\} = [K_{k-1}]^{-1} \{F_k^i\} \quad (3.85)$$

where $[K_{k-1}]$ is the stiffness matrix corresponding to the results of previous iteration. The total displacement due to the incremental load after k th iteration is given by

$$\{\Delta \delta_i\} = \sum_{j=1}^k \{d\delta_j^i\} \quad (3.86)$$

Iterations are continued until the unbalanced load $\{F_k^i\}$ becomes very small and nearly zero, then the total displacement $\{\delta_i\}$ due to the total applied load at the end of i th load increment is determined by

$$\{\delta_i\} = \sum_{j=1}^i \{\Delta \delta_j\} \quad (3.87)$$

In addition to the method discussed here, NONLACS employs other procedures such as incremental method, initial stiffness iterative method and incremental-iterative method. Since these methods are not used in this study, they will not be discussed here.

Finally, it is important to mention that NONLACS allows use of either the displacement or the force convergence criterion, and the Euclidean norm (Bathe 1982) is used to check convergence.

4. VERIFICATION OF THE PROGRAM

4.1 General

The applicability of the program NONLACS described in Chapter 3 will be verified by means of analyzing a number of composite beam and bridge models that have been tested by others. Although this program was verified by Razaqpur and Nofal (1988), a modification was introduced here in the modeling of the shear connector element as described in Chapter 3, which needs verification.

In this chapter, the experimental results of Yam and Chapman's simply supported beam (1968), Yam and Chapman's continuous beam (1972) and Razaqpur and Nofal's bridge model (1988) will be compared with computed results of NONLACS.

4.1 Imperial College Simply Supported Beam

Yam and Chapman tested a number of simply supported composite beams at Imperial College in 1968. One of their beams, which had the loading and geometry shown in Fig. 4.1 and material properties shown in Table 4.1, was analyzed by the current version of NONLACS. The beam was simply supported on a 5.5 m span and consisted of a 304 mm deep I-section attached to a 152 mm thick and 1220 mm wide concrete slab by means of stud shear connectors. The material properties in Table 4.1 were taken from Yam and Chapman's work whenever possible. A number of properties were not, however, specified in their work and had to be assumed as indicated in Table 4.1. The assumed values are reasonable based on general information in the literature. The finite element idealization of the beam is illustrated in Fig. 4.2. The finite element mesh

Elevation

Cross Section

Fig. 4.1. Imperial College Simply Supported Beam Details

Table 4.1. Material properties of Imperial College simply supported beam

Material Constants	Concrete	Reinforcing Steel	Steel Plate	
f_y (MPa)	-	260	260	Given
E_s (MPa)	-	200000	200000	Given
E_s^* (MPa)	-	5000	5000	Given
f'_c (MPa)	50	-	-	Given
ϵ_{max}	0.0035	-	-	Assumed
ϵ_{cu}	0.002	-	-	Assumed
Connector Details	$\frac{1}{2} \times 2"$ Headed studs No. of connectors 100 (Triangular distribution)

Imperial College Simply Supported Beam Elevation

Imperial College Continuous Beam Elevation

Fig. 4.2. Finite Element Idealization of Imperial College Beams

Finite element 1: unbalanced moment is included
 Finite element 2: unbalanced moment is not included

Fig. 4.3. Load –Mid Span Deflection Diagram of Imperial College Simply Supported Beam

Fig. 4.4. Concrete-Steel Interface Slip of Imperial College Simply Supported Beam at P=448 kN

consists of 18 elements along the span and 10 in the cross-section (Fig.4.2). The steel girder web and flange elements were idealized by a single layer, while the concrete slab was divided into 10 layers. The steel reinforcement in the bottom of the slab was modeled by a smeared steel layer and the slab and girder were connected by shear connector elements as indicated in Fig. 4.2. The applied load was divided into 10 increments and the beam was analyzed over the entire loading range up to failure.

The results of the analysis are illustrated in the form of the load-mid span deflection curve, Fig. 4.3, and slip distribution at load 448 kN, Fig. 4.4. Two load-deflection curves are shown in Fig. 4.3, one curve includes the effect of removing the unbalanced moment in the shear connectors, the other does not. These are designated as finite element 1 and 2, respectively in the figure. It is evident that finite element 1 analysis agrees more closely with the experimental results. The finite element results predicted shear slip at the concrete-steel interface along the length of the beam at $P=448$ kN; this compares well with the experimental results (Fig. 4.4). Ultimate load determined by the finite element method is 510 kN which is 2% different from the experimental ultimate load of 500 kN.

4.2 Imperial College Continuous Beam

Yam and Chapman reported the experimental data of testing a number of continuous composite beams in 1972. The beam analyzed in their study had the loading and geometry shown in Fig. 4.5 and the material properties mentioned in Table 4.2. The beam had two spans of 3.355 m and consisted of 152 mm deep I-section attached to a 60 mm thick and 920 mm wide concrete slab by means of shear connectors. Material properties shown in Table 4.2 were taken according to Yam and Chapman's report in addition to

Elevation

Cross Section

Fig. 4.5. Imperial college continuous beam details

Table 4.2. Material Properties of Imperial College Continuous Beam

Material Constants	Concrete	Reinforcing Steel	Steel Plate	
f_y (MPa)	-	270	270	Given
E_s (MPa)	-	200000	200000	Given
E_s^* (MPa)	-	5000	5000	Given
f'_c (MPa)	47.6	-	-	Given
ϵ_{max}	0.035	-	-	Assumed
ϵ_{cu}	0.002	-	-	Assumed
Connector Details	$\frac{3}{8} \times 2$ Headed studs, two rows at 5.72 inch			

some assumed properties, which were not reported by them, and had to be assumed based on general information in the literature. The finite element idealization of the beam is shown in Fig. 4.2. As shown in that figure, the finite element mesh is exactly the same as that for the Imperial College simply supported beam except one support is added in the middle.

Results shown in Fig. 4.6 to 4.8. Fig. 4.6 compare the experimental and computed results of the deflected shape of the beam. We notice good agreement between the two sets of results. Figure 4.7 shows the experimental result and the corresponding finite element prediction of the variation of slip along concrete-steel interface at load $P=108$ kN. We see good comparison between the two sets of results.

Figure 4.8 compares the experimental and analytical results of strain along the bottom flange of the girder from the left support to the center line at load $P=108.5$ kN. Here again the finite element results agree closely with the experimental data.

4.3 Bridge Model

This $\frac{1}{3}$ scale bridge model was built and tested by Razaqpur and Nofal (1988). It was the model of a bridge prototype designed according to the Ontario Highway Design Code (OHBD 1983) recommendations. The bridge prototype is a two-lane bridge, 6.24m wide and 18.00m long. It has three W840 $\times$ 170 compact steel beams spaced at 1.86m. The concrete slab has a total thickness of 182mm. Figures 4.9 and 4.10 show the geometry and loading of the bridge model and Table 4.3 shows its material properties. The material strength values were obtained by Razaqpur and Nofal from ancillary tests performed on concrete cylinders and steel coupons. The bridge is simply supported with

Fig. 4.6. Deflected Shape of Imperial College Continuous Beam at $P=108.5$ kN

Fig. 4.7. Concrete-Steel Interface Slip of Imperial College Continuous Beam at $P=108.5$ kN

Fig. 4.8. Strain along the Bottom Flange of Imperial College Continuous Beam at $P=108.5$ kN

Fig. 4.9. Geometry of Bridge Model

Fig. 4.10 Loading of Bridge Model

Table 4.3 Material properties of Bridge Model

Material Constants	Concrete	Reinforcing Steel	Steel Plate	
f_y (MPa)	-	400	300	Given
E_s (MPa)	-	200000	200000	Given
E_s^* (MPa)	-	0	0	Given
f'_c (MPa)	40	-	-	Given
ϵ_{max}	0.0035	-	-	Assumed
ϵ_{cu}	0.002	-	-	Assumed
Connector Details	15 mm $\times$ 60 mm Headed studs, two rows at 150 mm			

three girders (3 W250x39) on 6 m span. The supports are roller at one end and hinged at the other and the slab is 70 mm thick and 2060 mm wide.

The loading scheme as described by Razaqpur and Nofal was intended to simulate the OHBDG truck in a modified form. The modified truck combines the two lines of wheels into a single line. Furthermore, the two 70 kN axle loads are combined into a single axle. The reason for these modifications was to reduce the number of loading points, which in the current model would be rather close to each other if the axle configuration in the code was used. The actuator loads were applied through 83mm×200mm steel plattens, placed above the central girder flange on the concrete slab.

4.3.1 Numerical Results and Comparison with Experimental Data

In the following the different experimentally measured strains, deflections and calculated moments will be presented together with the corresponding predictions of the finite element program NONLACS using the mesh shown in Fig. 4.11. As shown in Fig 4.11, the finite element mesh consists of 20 elements along the span and 24 in the cross section. The steel girder web and flange elements were idealized by a single layer, while the concrete slab was divided into 10 layers. The steel reinforcement in the top and the bottom of the slab was modeled by two smeared steel layers and the slab and girder were connected by shear connector elements. The applied load was divided into 20 increments and the bridge was analyzed over the entire loading range up to failure.

4.3.1.1 Load-deflection of girders

The experimental load-maximum deflection curves of the three steel girders are plotted in Fig. 4.12, together with the corresponding finite element predictions. Good comparison between the experimental and the finite element results can be noticed. The

Elevation

Cross Section

Fig. 4.11 Finite Element Idealization Of Bridge Model

(a) Girder 1

(b) Girder G2

(c) Girder G3

Fig. 4.12 Load-Maximum Deflection Curve of Bridge Model

ultimate load predicted by finite element is 800 kN, which is 4% higher than the experimental ultimate load of 766 kN. This experimental ultimate load, as described by Razaqpur and Nofal, could also be a bit more. They reported that the test was stopped due to the fact that the actuator stroke was exhausted, albeit large deformations and a visible plastic hinge had formed in the central beam. As a result, they believed that the bridge could still carry a small amount of extra load.

Figure 4.13 shows the deflected shape of the three beams at different stages of loading. The load values shown in these graphs are the gross load of the two actuators. Good agreement between the experimental and analytical values can be noticed.

4.3.1.2 Strain variation in girders flanges

The variation of the longitudinal strain along the top and the bottom flanges of the steel girders are plotted in Fig. 4.14. The strains are plotted for the different levels of loading in the latter figure. By considering the values in the bottom flanges, it is obvious from Fig. 4.14 that the tensile strains in the bottom flange have exceeded five times the yield strain at the section of maximum moment which implies that the bridge is close to failure. Since the strain comparisons are quite favorable, one would expect that the moment field predicted by finite element analysis to be a close representation of the actual moments in the bridge.

4.3.1.3 Strain variation in the concrete

The variation of the longitudinal strain on the top surface of the concrete, is plotted in Fig. 4.15. The strains are plotted for different levels of loading, together with their

(a) Girder G1

(b) Girder G2

(c) Girder G3

Fig. 4.13 Variation of Deflection along the Length of Bridge Model Girders

(a) Girder G1

(b) Girder G2

(c) Girder G3

Fig. 4.14. Variation of Strain along the Bottom Flange of Bridge Model Girders

(a) Girder G1

(b) Girder G2

(c) Girder G3

Fig. 4.15. Variation of Strain along the Top Flange of Bridge Model Girders

numerically values obtained from finite element. In this case too, the comparison between the two sets of strains is favorable.

It can be noticed that the maximum strain in the concrete has reached 0.002, which is the strain corresponding to maximum concrete stress. Thus, the concrete at the section of maximum moment and on the axis of the central girder has reached its maximum stress. This was verified by visual observations, which showed the concrete in this region to be severely distressed. Along the axes of the outer girders, however, the maximum measured strain is less than 0.002. Hence these girders may still be capable of carrying more load.

4.3.1.4 Moment variation in the girders

Moments cannot be directly measured but can be calculated using the measured values. To calculate the experimental moment, Razaqpur and Nofal used the experimentally measured strain profiles in conjunction with the stress-strain relationships of concrete and steel. The steel was assumed to be elasto-perfectly-plastic while for concrete the parabolic stress-strain curve proposed by Saenz (1964) was used. The moments were calculated by dividing the beam into a number of layers. The force in each layer was calculated and the moment resultant of these forces was obtained at the location of the geometric centroid of the girder. Since the finite element program does not output moments directly, the same procedure was used for the finite element calculations. In the case of finite element results, the layer stresses obtained from the output were used to obtain the longitudinal moment at any section.

Figure 4.16 shows the variation of longitudinal bending moments in the three girders at various stages of loading. Good comparison is observed between the measured and calculated moments for all the girders and at all stages of loading.

4.3.1.5 Distribution parameter, D_d , of the girders

The distribution parameter, D_d , is a parameter which is used to determine the design moment, M_G , for any girder in the OHBD code by the following equation.

$$M_G = \frac{S}{D_d} \times \left[\frac{M_T}{N \times R_f} \right] \quad (4.1)$$

where M_T is the total moment at a given section of the bridge, S is the girder spacing, N is the number of wheels on the same section and R_f is a reduction factor due to number of loaded lanes according to OHBD code. Equation 4.1 can be rewritten as

$$D_d = S \times \left[\frac{M_T}{M_G \times N \times R_f} \right] \quad (4.2)$$

Using Eq. 4.2, the computed D_d variation for the three girders is plotted in Fig. 4.17. The corresponding experimental values, which were calculated by Razaqpur and Nofal using the above equation, are also plotted in the same graph. It should be mentioned that in plotting this figure, the maximum girder moments were used. The experimental and numerical values are in good agreement. We notice that as the D_d graph of girder G2 rises, those of girders G1 and G3 drop in value. This means that girder G2 attracts less load while the other two girders resist a higher portion of the total applied load. This is called redistribution of load and happens after yielding of girder G2. This bridge shows

(a) Girder G1

(b) Girder G2

(c) Girder G3

Fig. 4.16. Longitudinal Moment Variation in the Bridge Model Girders

Fig. 4.17. Variation of Distribution Parameters with Increased Loading

16% redistribution of moment in the transverse direction.

Based on the favorable comparison between the experimental data of the analyzed beams and bridges and the corresponding finite element results of the same structures, it can be stated that the finite element program NONLACS, using the elements and material models described in Chapter 3, can capture the detailed behavior of composite bridges in the entire range of loading up to failure, and can be used for finding the ultimate moment distribution in continuous multi-girder composite bridges.

5. PARAMETRIC STUDY

5.1 Selection of Parameters

Some of the parameters which have been found important for moment distribution in composite bridges according to Heins and Kuo (1975), Bakht and Jaeger (1985), Razaqpur and Nofal (1988), and Kennedy and Soliman (1990) are: truck position and number of lanes loaded, bridge width, girder spacing, number of girders, relative rigidity of slab to girders, degree of interaction, and diaphragms.

In this chapter the effects of these parameters on ultimate moment distribution of composite bridges will be investigated for two-span and three-span bridges. The geometry of the analyzed bridges has been selected using the OHBD (1991) code provisions. For example, to use the OHBD simplified method of slab design for composite bridges; slab thickness should be at least 225 mm. As a result, in all cases slab thickness is 225mm unless the effect of slab thickness is being investigated. Furthermore, for all the bridges the length of the external and the internal spans are kept constant, being equal to 25m and 30 m, respectively. The latter span ratio of 1.2 is determined from optimization of moment distribution by elastic analysis. The parameters studied here include:

1. Longitudinal truck position
2. Number of loaded lanes and transverse location of trucks
3. Bridge width/span ratio
4. Number of girders (Girder spacing)
5. Slab thickness (Slab/ girder stiffness ratio)

6. Degree of interaction
7. Presence of transverse diaphragms

5.2 Design of Bridges

The analyzed bridges in this study were designed in accordance with OHBD 1991 provisions. It should be mentioned that the new CHBD (2000) (Canadian Highway Bridge Design) code had not been published when this work started and therefore could not be used for the purpose of the present investigation.

To identify the bridges, a mnemonic system will be used, consisting of a number, followed by two letters and ending with another number. The first number stands for the number of spans, the letters denote the relevant parameter being considered and the last number represents the order of a particular bridge in each set. For example, 2NG3 is a two span bridge, which is analyzed for the effect of number of girders, and is the third bridge analyzed in this set.

The material properties for all bridges were assumed as follows: concrete strength 40 MPa, Young's modulus 28×10^3 MPa, and Poisson's ratio 0.15. For the girder steel yield stress 300 MPa, Young's modulus 200×10^3 MPa, no strain hardening, and Poisson ratio 0.30. For the reinforcing steel, yield stress 400 MPa, Young's modulus 200×10^3 MPa, and no strain hardening. For shear connectors $a=30$ kN and $b= 4.74$ 1/mm, unless otherwise noted. All information about the geometry of analyzed bridges is shown in Table. 5.1 and 5.2 for two-span and three-span bridges, respectively.

Table 5.1. Geometry of Two-Span Bridges Analyzed

Bridge	No. Of Girders	Girder Type	Slab thickness (mm)	Girder spacing (mm)	Initial stiffness of shear connectors(kN/mm)	Diaphragms
2NL1	4	W610x415	225	3500	142	-
2NL2	4	W610x415	225	3500	142	-
2BW1	4	W530x300	225	2000	142	-
2BW2	4	W530x300	225	2250	142	-
2BW3	4	W530x331	225	2500	142	-
2BW4	4	W530x331	225	2750	142	-
2BW5	4	W530x370	225	3000	142	-
2BW6	4	W530x370	225	3375	142	-
2BW7	4	W610x415	225	3500	142	-
2NG1	4	W610x415	225	3500	142	-
2NG2	5	W530x370	225	2800	142	-
2NG3	6	W530x331	225	2333	142	-
2ST1	4	W610x415	175	3500	142	-
2ST2	4	W610x415	225	3500	142	-
2ST3	4	W610x415	275	3500	142	-
2DI1	4	W610x415	225	3500	0	-
2DI2	4	W610x415	225	3500	71	-
2DI3	4	W610x415	225	3500	142	-
2DI4	4	W610x415	225	3500	850	-
2TD1	4	W610x415	225	3500	142	-
2TD2	4	W610x415	225	3500	142	S
2TD3	4	W610x415	225	3500	142	S-M

Notes:

- (1) All deck slabs $f'_c = 40$ MPa
- (2) All girder steel yield stress = 300 MPa
- (3) All slab reinforcement yield stress = 400 MPa
- (4) All reinforcement 0.3% top and bottom both directions
- (5) S = Support diaphragms provided , S-M = Support and Mid-span diaphragms provided

Table 5.2. Geometry of Three-Span Bridges Analyzed

Bridge	No. Of Girders	Girder Type	Slab thickness (mm)	Girder spacing (mm)	Initial stiffness of shear connectors(kN/mm)	Diaphragms
3NL1	4	W610x455	225	3500	142	-
3NL2	4	W610x455	225	3500	142	-
3BW1	4	W530x331	225	2000	142	-
3BW2	4	W530x331	225	2250	142	-
3BW3	4	W530x370	225	2500	142	-
3BW4	4	W530x370	225	2750	142	-
3BW5	4	W630x415	225	3000	142	-
3BW6	4	W630x415	225	3375	142	-
3BW7	4	W610x455	225	3500	142	-
3NG1	4	W610x455	225	3500	142	-
3NG2	5	W610x415	225	2800	142	-
3NG3	6	W530x370	225	2333	142	-
3ST1	4	W610x455	175	3500	142	-
3ST2	4	W610x455	225	3500	142	-
3ST3	4	W610x455	275	3500	142	-
3DI1	4	W610x455	225	3500	0	-
3DI2	4	W610x455	225	3500	71	-
3DI3	4	W610x455	225	3500	142	-
3DI4	4	W610x455	225	3500	850	-
3TD1	4	W610x455	225	3500	142	-
3TD2	4	W610x455	225	3500	142	S
3TD3	4	W610x455	225	3500	142	S-M

Notes:

- (1) All deck slabs $f'_c = 40$ MPa
- (2) All girder steel yield stress = 300 MPa
- (3) All slab reinforcement yield stress = 400 MPa
- (4) All reinforcement 0.3% top and bottom both directions
- (5) S = Support diaphragms provided , S-M = Support and Mid-span diaphragms provided

5.3 Calculation Procedure and Presentation of the Results

All designed bridges were analyzed by NONLACS using the finite element models described in Chapter 3. The program outputs stresses and strains at the integration points. Using these quantities at both the elastic and nonlinear stages, the moment variation in each girder was calculated along the span. Subsequently, the distribution parameter and the longitudinal moment redistribution at the critical sections were determined using Eq. 4.2, and the following equation,

$$R = \frac{M_{Nonlinear} - M_{Elastic}}{M_{Elastic}} \quad (5.1)$$

in which $M_{Nonlinear}$ is the moment acting on the critical sections of a bridge at failure based on nonlinear finite element analysis. The moment $M_{Elastic}$ is the moment corresponding to the same load, but obtained from elastic analysis.

In this study transverse redistribution is represented by the distribution parameter while longitudinal redistribution at ultimate is directly calculated and shown as percentage of the elastic moment. The longitudinal redistribution is shown for a section, i.e. the maximum positive and the maximum negative moment sections. Transverse moment redistribution is shown for each girder at the maximum positive and maximum negative moment sections. Note that girders at each section are numbered sequentially as G1, G2, etc., with G1 being the left-most girder in the section.

In order to calculate the redistribution factor, one must first determine the moment diagram for each girder using the stresses computed by finite element. Figure 5.1 shows the typical moment diagrams for the external and internal girders of bridge 2BW1. All the moment diagrams in Fig. 5.1 correspond to the ultimate load determined by nonlinear

External Girder

Internal Girder

Fig. 5.1. Longitudinal Moment Diagram of Bridge 2BW1

analysis. Therefore, the difference between the two moment diagrams is engendered by the assumption of elastic versus inelastic (nonlinear) behavior in the analysis. It is important to point out that in these bridges, the concrete slab does not significantly contribute to the negative moment capacity of the bridge while it makes appreciable contribution to its positive moment capacity. Therefore, the positive moment section has a much larger capacity than the negative moment section, as indicated in Fig. 5.1. The difference between the elastic and nonlinear moment diagrams show the amount of redistribution of moments due to material nonlinearity. When calculating the longitudinal moment redistribution according to Eq. 5.1, we use the total moment in all girders at the section of interest while when calculating transverse moment redistribution according to Eq. 4.2, we use the moment in the individual girders at each section.

To investigate whether the distribution factors proposed by AASHTO (1994) and CHBD (2000) codes are applicable to ultimate state of continuous composite bridges, the relevant predictions of these codes are compared with the finite element results. Both codes give distribution factor per lane while the presented graphs based on OHBD (1991) show the distribution parameters per one line of wheels. Therefore, to unify the presentation here, the results of the codes are converted to distribution parameters by the following equation:

$$D_d = \frac{S}{2D_f} \quad (5.2)$$

where D_d is distribution parameter, D_f is distribution factor given by the codes, and S is spacing of the girders.

Predictions of a modified form of Fan's (1974) equation are also compared with the finite element results. As indicated in Chapter 2, Fan proposed Eq. 2.15 for ultimate

distribution factor of composite bridges. Converting this distribution factor to distribution parameter and incorporating the effect of multilane loading reduction factors, Fan's equation can be rewritten as:

$$D_d = \frac{S \times \text{Number of Girders}}{2 \times \text{Number of Lanes} \times R_f} \quad (5.3)$$

where D_d is the distribution parameter, R_f is a reduction factor due to multilane loading, and S is the spacing of the girders. Equation 5.3 is referred to as Fan's Equation in the presented graphs. The choice of comparing Fan's equation results with the finite element results follows the work of Razaqpur and Nofal (1988) wherein they showed that this equation gives better results compared with Hein's Equation (2.14), which is the other available expression for this purpose.

5.4 Parametric Study and Discussion of the Results

In the following parametric study, we present the effect of various parameters mentioned in Section 5.1 on both the longitudinal and transverse moment redistribution at all load levels until failure. The transverse section of interest are the section of maximum positive and maximum negative moment according to elastic analysis. These sections are at the supports and under the third axle from the front of the OHBD truck load. Although the distribution parameters for all load stages are shown, in fact there are three distinct load stages that are of great interest in design. These are the elastic stage, the yield initiation stage in the steel girders and the ultimate or failure state. We will compare the distribution parameters for each girder at elastic and ultimate states. To avoid excessive number of graphs, the latter information will be presented in compact form in tables at

the end of this chapter (Table 5.3 to 5.6). Observations regarding the bridge behavior at the girder yielding initiation state will also be made.

The load level in the presented graphs are expressed in terms of the OHBD truck. The truck load means the total weight of all the OHBD trucks on the bridge times the reduction factor. For example, for a bridge with two loaded lanes, 3.2 truck means $3.2 \times 2 \times 0.9 = 5.76$ times one truck of OHBD 1991 code, in which 0.9 is the reduction factor for two lanes loaded according to the OHBD 1991 code.

5.4.1 Longitudinal Truck Position

As elastic analysis shows, longitudinal truck position has a significant effect on the magnitude and location of the maximum moment acting on a bridge, and the locations of interest are the ones which create the maximum moment at critical sections or lead to the lowest ultimate load for the bridge. In the present parametric study, all trucks are located on the same span of the continuous bridge. This causes high moment at the mid-span while it does not cause any reduction in the support moments of the loaded span. Therefore, under the latter load condition, following the formation of the first plastic hinge, the positive moment section will be able to attract the least amount of extra moment beyond the level of its maximum elastic value. Consequently, the bridge will show the least ultimate strength compared to other loading cases. For example, treating a bridge as beam, hand calculation shows that the collapse load of a two-span continuous bridge with two trucks on the same span is half of the collapse load of the same bridge with one truck on each span.

For the sake of reducing computation, all trucks are located on the same longitudinal section of the bridge. This loading configuration is the one that creates the largest

moments at critical sections of the loaded span (supports, maximum positive moment section), which moments are at most 3% smaller than the maximum possible moments at these sections due to any truck location. The selected truck load configuration is shown for both two-span and three-span bridges in Fig. 5.2.

5.4.2 Number of Lanes Loaded and Transverse Location of the Trucks

In this section the effect of different transverse loading cases is investigated. It is obvious that the maximum moment on the cross-section of a bridge is higher when all lanes are loaded, however, girders of a bridge may resist a different portion of the total moment under different loading cases. As a result, it is not obvious which loading case is the most critical one. The objective of this part is not only to determine the behavior of these bridges under complicated loading configurations, which are hard to capture by available hand calculation methods, but also to find out the most critical loading case which will be used when investigating the effect of other parameters. In this section, two-span bridge 2NL1 and three-span bridge 3NL1 are analyzed under 8 possible loading cases as shown in Fig. 5.3.

Figures 5.5 and 5.6 show the variation of distribution parameters of all the girders of Bridges 2NL1 and 3NL1 under loading Case 1 at different loading stages. The first part of these graphs represents the elastic state in which the distribution parameters remain unchanged with increasing load. Upon advent of nonlinearity, the distribution parameter of the most loaded girder (G1) at the maximum positive moment section starts to decrease while that of the maximum negative moment section remains almost unchanged.

Fig. 5.2. Truck Longitudinal Locations

Fig. 5.3. Loading Cases for Bridges 2NL1, 3NL1 (Case 1 to 4)

Fig. 5.4. Loading Cases for Bridges 2NL1, 3NL1 (Case 5 to 8)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.5. Girder Distribution Parameters of Bridge 2NL1 (Load Case1)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.6. Girder Distribution Parameters of Bridge 3NL1 (Load Case1)

This shows that under such loading when the first girder experiences nonlinearity at the support section, the dominant redistribution is longitudinal from the section of maximum negative moment to the section of maximum positive moment in the same girder while the other girders do not absorb much extra moment at that section. In Figs. 5.5 and 5.6 these changes are not clear for the directly loaded girder due to the scale of the graph, however, it can be easily noticed by the increase in the value of the distribution parameter of the least loaded girder which shows that while the moment at the section increases, the least loaded girder does not absorb much extra moment. Notice also that afterwards the distribution parameter of the latter girder at both the maximum positive moment section and the maximum negative moment section decreases, which implies that moments at both critical sections of the girders start to redistribute transversely and this goes on until failure. For example, Bridge 2NL1 (case1) behaves linearly until 2.8 truck load, thereafter it mostly redistributes moments longitudinally until 3.7 truck load, after which the redistribution is mainly transverse until failure at 4.6 truck. Bridge 2NL1 shows 12% and 20% change in distributions parameters of girder G1 from elastic to ultimate at the maximum positive and negative moment sections, respectively, while the corresponding values for Bridge 3NL1 are 6% and 12%, respectively.

Variations of the girder distribution parameters for bridges 2NL1 and 3NL1 under load Case 2, where trucks are positioned along the centerline of Girder G2, are shown in Figs. 5.7 and 5.8, respectively. The same behaviour as load Case 1 is noticed, except the most loaded girder is G2 in these cases. Nonlinearity coincides with the decrease in the value of the distribution parameter of girder G2 at the maximum positive moment section while

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.7. Girder Distribution Parameters of Bridge 2NL1 (Load Case2)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.8. Girder Distribution Parameters of Bridge 3NL1 (Load Case2)

that of the maximum negative section does not change significantly due to longitudinal redistribution. Subsequently at both sections, mainly due to transverse redistribution, the distribution parameters start to converge to their ultimate values. Under this loading case the distribution parameters of the directly loaded girders of bridges 2NL1 and 3NL1 show almost 13% and 8% change, respectively, from the elastic to the ultimate state at both critical sections.

Figures 5.9 to 5.12 illustrate the distribution parameter variations with load for the same bridges under load Cases 3 and 4. In all the figures there is a reduction in the value of positive distribution parameter of the directly loaded girders, i.e. G1 and G4 in Bridge 2NL1, and G2 and G3 in Bridge 3NL1, right after these girders experience nonlinearity. However, there is no such reduction at the maximum negative moment sections, which implies the dominance of longitudinal redistribution. Subsequently, the descending branch for the most loaded girders at both sections appear which represents the dominance of transverse redistribution. It is surprising that in all four cases when the most loaded girders begin to yield at the support, mainly longitudinal moment redistribution occurs till the maximum positive sections of the same girders reach their maximum moment capacity and only thereafter moments starts to redistribute transversely in the maximum positive moment section to the adjacent girders. Such high redistribution does not occur at the section of maximum negative moment. As a result, in all the figures the ultimate distribution parameters of the bridges are closer to each other in value at maximum positive moment section than at the maximum negative moment section. The latter behaviour is interesting given that initial yielding occurs at the

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.9. Girder Distribution Parameters of Bridge 2NL1 (Load Case3)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.10. Girder Distribution Parameters of Bridge 2NL1 (Load Case4)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.11. Girder Distribution Parameters of Bridge 3NL1 (Load Case3)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.12. Girder Distribution Parameters of Bridge 3NL1 (Load Case4)

maximum negative moment section. For example, the distribution parameters of Bridge 2NL1 at ultimate under loading Case 4 are 3.39 m and 4.57 m for girders G1 and G2 at the maximum negative moment section while the corresponding values for the section of maximum positive moment are 3.95 m and 3.83. Both bridges show almost 20% to 15% difference between the elastic and ultimate distribution parameters of the most loaded girders at maximum positive moment section and 17% to 12% at the maximum negative moment section under load Cases 3 and 4, respectively.

Figures 5.13 and 5.14 show the variation of distribution parameters with load for bridges 2NL1 and 3NL1 under load Case 5. This load case demonstrates both the longitudinal and transverse redistribution branches. As can be observed in these figures, both bridges have the same distribution parameters for the two loaded girders (G1 and G2), which girders reached their ultimate capacity while the other girders did not. The difference between the elastic and the ultimate distribution parameter values for the most critical girder (G1) of these bridges is almost 35% and 40% for the maximum positive moment section and the maximum negative moment section, respectively. These values indicate a significant level of redistribution in the loaded girders.

The variations of distribution parameters with load for load cases 6, 7 and 8 are shown in Figs. 5.15 to 5.20. These figures show a different behaviour compared with the other loading cases. In the latter figures, one observes that nonlinearity coincides with increase in the value of the distribution parameters of the most loaded girders both at the maximum positive and the maximum negative moment sections while the distribution parameter of the least loaded girder decreases and this trend continuous until failure. This implies that upon commencement of inelasticity, redistribution occurs both longitudinally

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.13. Girder Distribution Parameters of Bridge 2NL1 (Load Case5)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.14. Girder Distribution Parameters of Bridge 3NL1 (Load Case5)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.15. Girder Distribution Parameters of Bridge 2NL1 (Load Case6)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.16. Girder Distribution Parameters of Bridge 2NL1 (Load Case7)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.17. Girder Distribution Parameters of Bridge 2NL1 (Load Case8)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.18. Girder Distribution Parameters of Bridge 3NL1 (Load Case6)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.19. Girder Distribution Parameters of Bridge 3NL1 (Load Case7)

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.20. Girder Distribution Parameters of Bridge 3NL1 (Load Case8)

and transversely and the extra longitudinal moment is not absorbed by only some of the girders but by all of the girders. As an example, Bridge 2NL1 (Load Case 8) experiences nonlinearity at 2.8 truck load and from this point onward the values of the distribution parameters at both critical sections tend to converge toward a single ultimate value at failure, with failure occurring at 4.0 truck load. In all three loading cases, full transverse moment redistribution has occurred which means that all the girders take equal part in carrying the moment at the ultimate limit state. The greatest change of 33% in the value of the distribution parameter of the critical girder from elastic to ultimate state occurs under load Case 6 for both bridges. The same difference is only 5% under load Case 8.

Figures 5.21 and 5.22 show the variation of ultimate distribution parameter with the loading case. As can be noticed, Case 8, in which all the design lanes of the bridges are loaded, shows the least value of 2.5 m for both two-span and three-span bridges at the ultimate state. The variations of the longitudinal moment redistribution at ultimate state for different loading cases are shown in Figs. 5.23 and 5.24 for sections of maximum positive and maximum negative moment, respectively. According to these figures, Case 8 shows the maximum longitudinal moment redistribution while Case 1 shows the least longitudinal moment redistribution. Longitudinal redistributions at maximum negative moment sections vary from 13 to 40%.

Figure 5.25 shows the variation of the ultimate bridge load versus the loading case. Again, load Case 8 has the least ultimate load of 4.0 truck load for both two-span and three-span bridges, therefore, is the most critical loading case while Case 1, in which only the external lane of a bridge is loaded is the second critical loading case. It should be

(a) External Girder

(b) Internal Girder

Fig. 5.21. Variation of Distribution Parameter with Load Cases for Two-Span Bridges at Ultimate

(a) External Girder

(b) Internal Girder

Fig. 5.22. Variation of Distribution Parameter with Load Cases for Three-Span Bridges at Ultimate

(a) Maximum Positive Moment section

(b) Maximum Negative Moment section

Fig. 5.23. Variation of Longitudinal Moment Redistribution with Load Cases for Two-Span Bridges at Ultimate

(a) Maximum Positive Moment section

(b) Maximum Negative Moment section

Fig. 5.24. Variation of Longitudinal Moment Redistribution with Load Cases for Three-Span Bridges at Ultimate

(a) Two-Span Bridges

(b) Three-Span Bridges

Fig. 5.25. Variation of Ultimate Load with Load Cases

mentioned that the effect of multilane loading reduction factor based on CHBD (2000) is included in these graphs.

5.4.3 Bridge Width

As reviewed in Chapter 2, Bakht et al (1978) found that the distribution parameters of bridges are related to their torsional and flexural parameters α and θ given by Eq. 2.3 and 2.4, respectively. The quantity θ (Eq. 2.4) is directly proportional to bridge width; therefore, it can have significant effect on the distribution characteristics of bridges in the elastic state. However, its effect in the nonlinear states may be different and needs to be investigated.

Bridges analyzed here for the effect of bridge width varied in width from 8 m to 14 m. These bridges are designated as 2BW1 to 2BW7 and 3BW1 to 3BW7 for two span and three span bridges, respectively. Since in the previous section the critical loading case was when all design lanes were loaded, these bridges were subjected to the latter loading case. Note that the span length of the bridges analyzed here is kept constant; therefore, the bridge width variation represents the variation in the aspect ratio of the slab.

Figures 5.26 to 5.37 illustrate the variation of the distribution parameters of all the girders of Bridges 2BW1 to 2BW6 and 3BW1 to 3BW6 with load, while Bridges 2BW7 and 3BW7 are exactly the same as Bridges 2NL1 and 3NL1, therefore, reference can be made to Figs. 5.17 and 5.20 for the latter bridge graphs. These figures show full transverse redistribution of moments at the ultimate limit state, which means all the girders of each bridge take the same amount of moment. Transverse moment redistribution of these bridges varies from 1% to 6%. This implies that even in the elastic state, moments in the different girders are not significantly different. These figures also show that upon initiation of yielding the distribution parameters of all the girders

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.26. Girder Distribution Parameters of Bridge 2BW1

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.27. Girder Distribution Parameters of Bridge 2BW2

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.28. Girder Distribution Parameters of Bridge 2BW3

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.29. Girder Distribution Parameters of Bridge 2BW4

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.30. Girder Distribution Parameters of Bridge 2BW5

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.31. Girder Distribution Parameters of Bridge 2BW6

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.32. Girder Distribution Parameters of Bridge 3BW1

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.33. Girder Distribution Parameters of Bridge 3BW2

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.34. Girder Distribution Parameters of Bridge 3BW3

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.35. Girder Distribution Parameters of Bridge 3BW4

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.36. Girder Distribution Parameters of Bridge 3BW5

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.37. Girder Distribution Parameters of Bridge 3BW6

start to converge toward the final distribution parameters at the ultimate state. This shows that longitudinal distribution in these bridges does not increase the moment in only some of the girders, instead all the girders at the maximum positive moment section take part in redistribution once yielding of some girders at the maximum negative moment section commences.

Figures 5.38 and 5.39 illustrate the variation of ultimate distribution parameters with bridge width for external and internal girders. These parameters vary from 2.21 m for 2BW1 and 3BW1 to 2.82 m for 2BW6 and 3BW6. The relevant predictions of the CHBD and the AASHTO code and the modified form of Fan's equation are shown in the same graphs. The CHBD code predictions are on the safe side and close to the finite element values for bridges 8 to 11 m wide (aspect ratio 2.27 to 3.125) while they become conservative for bridges 12 to 14 m wide (aspect ratio 2.08 to 2.14). In contrast, the AASHTO code results are not on the safe side for bridge 3BW4, which is three-span and 11 m wide. It gives better results for bridges 8 and 14 m wide while it becomes conservative for the other analyzed bridges. Fan's equation, as shown in these figures, shows good comparison with the finite element results.

Variations of the ultimate longitudinal moment redistribution with bridge width are shown in Fig. 5.40 and 5.41. The amount of moment redistributions varies from 33% to 45 %, which is more significant than the maximum 6% transverse redistribution. The relevant results of hand calculations are also shown in the same graphs. Hand calculations were carried out by treating each bridge as a continuous beam and using plastic analysis. The two sets of results are in good agreement because when all the lanes are loaded, the ultimate distribution parameters of the different girders are almost the same at the critical

(a) External Girder

(b) Internal Girder

Fig. 5.38. Variation of Distribution Parameter with Bridge Width for Two-Span Bridges

(a) External Girder

(b) Internal Girder

Fig. 5.39. Variation of Distribution Parameter with Bridge Width for Three-Span Bridges

(a) Maximum positive moment section

(b) Maximum Negative moment Section

Fig. 5.40. Variation of Longitudinal Moment Redistribution with Bridge Width for Two-Span Bridges at Ultimate

(a) Maximum Positive moment Section

(b) Maximum Negative moment Section

Fig. 5.41. Variation of Longitudinal Moment Redistribution with Bridge Width for Three-Span Bridges at Ultimate

sections. This means that plastic hinges form in all the girders at the same longitudinal location. This condition corresponds to the same collapse mechanism as in an equivalent beam.

5.4.4 Number of Girders (Girder Spacing)

The number of girders does not change either the torsional or the flexural parameters α , and θ of bridges, which means that this parameter is not known to have a significant effect on the distribution parameters in the elastic state. However, it is not evident if it is also the case at the ultimate state. Six bridges were analyzed for this parameter. They are referred to as 2NG1 to 2NG3 for two-span bridges and 3NG1 to 3NG3 for three-span bridges. All the bridges have the same width of 14 m while the number of girders varies from 4 to 6. For the same reason explained in Section 5.4.3, all four-design lanes of the bridges are loaded.

Figs. 5.42 and 5.43 show the variation of the distribution parameters of each set of bridges at the section of maximum positive moment and maximum negative moment. We observe that ultimate distribution parameters of the girders of both the two-span and the three-span bridges are the same and equal to 2.5 m even though their distribution parameters are different by up to 15% in the elastic state. This means that the maximum possible transverse moment redistribution has occurred in all cases. This transverse redistribution varies from 2% to 7% for two span bridges and three span bridges, respectively, which is not significant. In these graphs the distribution parameters of the most loaded girders decrease to their ultimate value at both critical sections, which means that at ultimate longitudinal moment is redistributed over the whole maximum positive moment section.

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.42. Girder Distribution Parameters of Bridge 2NG1 to 2NG3

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.43. Girder Distribution Parameters of Bridge 3NG1 to 3NG3

In Figs. 5.44 and 5.45 the variation of ultimate distribution parameter with the number of girders as predicted by finite element analysis is shown and compared with the AASHTO and CHBD codes predictions as well as with Fan's equation. According to finite element results, the number of girders does not affect the distribution parameters of bridges analyzed here. The CHBD code also gives the same distribution parameter irrespective of the number of girders, but its results are conservative. In contrast, the AASHTO code results differ with the number of girders, but they are closer to the finite element results, compared with those of the CHBD code. Fan's equation gives the closest results to the finite element values.

Figures 5.46 and 5.47 compare the finite element variation of ultimate longitudinal moment redistribution of critical sections with hand calculation results using equivalent beams for two-span and three span-bridges. The longitudinal moment redistribution varies from 35% to 40 % at the maximum negative moment section for two-span bridges while it varies from 38% to 45% for three-span bridges, which is significantly higher than the transverse redistribution of 6%. Good comparison can be noticed between the finite element and the hand calculation results.

5.4.5 Slab Thickness (Slab/Girder Stiffness Ratio)

This parameter is investigated by analyzing two sets of beams, one for two-span and one for three-span bridges. These bridges are designated 2ST1 to 2ST3 and 3ST1 to 3ST3 and loaded over all the lanes. The geometry of the bridges in each set is the same except that the thickness of the slab is varied, using 175, 225, and 275 mm, respectively.

Figures 5.48 and 5.49, respectively, show the distribution parameters of critical sections of two-span and three-span bridges at different load stages up to failure. As expected, the thicker the slab, the better the distribution in the elastic state; however, all the bridges

(a) External Girder

(b) Internal Girder

Fig. 5.44. Variation of Distribution Parameter with Number of Girders for Two-Span Bridges

(a) External Girder

(b) Internal Girder

Fig. 5.45. Variation of Distribution Parameter with Number of Girders for Three-Span Bridges

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.46. Variation of Longitudinal Moment Redistribution with Number of Girders for Two-Span Bridges at Ultimate

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.47. Variation of Longitudinal Moment Redistribution with Number of Girders for Three-Span Bridges at Ultimate

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.48. Girder Distribution Parameters of Bridge 2ST1 to 2ST3

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.49. Girder Distribution Parameters of Bridge 3ST1 to 3ST3

have the same distribution parameter of 2.5m for all the girders at ultimate load. In the elastic state the distribution parameters of these bridges vary by up to 4% with the slab thickness, and the ultimate load of each set increases by up to 15% with increasing the slab thickness from 175 mm to 225 mm.

Figures 5.50 and 5.51 illustrate the variation of ultimate distribution parameter of the external and the internal girders of the bridges. The relevant prediction of the CHBD and AASHTO codes in addition to Fan's equation are also shown in the same graphs. According to finite element results, slab thickness has no influence upon distribution parameter. The CHBD code is on the safe side and it also does not predict a different distribution parameter for different slab thickness. In contrast, the AASHTO results are close to the finite element results in some cases, but are not on the safe side in other cases.

Variation of longitudinal moment redistribution with slab thickness at critical sections is shown in Fig. 5.52 and in Fig.5.53. The results show that more longitudinal moment redistribution occurs as the slab increases. This redistribution varies from 20% to 45% and 30% to 53% at the maximum negative moment section for the two-span and the three-span bridges, respectively. Hand calculation results, using an equivalent beam model are also shown in these figures. Both finite element and hand calculation show increased longitudinal moment redistribution with increased slab thickness. Hand calculations give comparable results with those obtained by finite element analysis.

(a) External Girder

(b) Internal Girder

Fig. 5.50. Variation of Distribution Parameter with Slab Thickness for Two-Span Bridges

(a) External Girder

(b) Internal Girder

Fig. 5.51. Variation of Distribution Parameter with Slab Thickness for Three-Span Bridges

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.52. Variation of Longitudinal Moment Redistribution with Slab Thickness for Two-Span Bridges at Ultimate

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.53. Variation of Longitudinal Moment Redistribution with Slab Thickness for Three-Span Bridges at Ultimate

5.4.6 Degree of Interaction

The behavior of composite members depends upon the degree of interaction among their constitutive parts. The degree of interaction signifies the extent to which the concrete slab and steel girder act as one member to resist the applied loads. Lack of interaction means, the slab and girder act as independent components while full interaction means that they act as one composite member. In composite bridges, interaction occurs by friction at the interface of the steel girder and the concrete slab or by means of shear connectors. Friction is not reliable for interaction because it deteriorates at small load levels. Therefore, the degree of interaction depends on the strength and stiffness of the connectors that join the slab to the girder. There are different types of shear connectors, but the shear studs used in this study are the most prevalent type.

The CHBD and AASHTO codes assume that there is full interaction between the slab and the girders. In other words, they assume that no relative displacement occurs at the interface of steel and concrete which is rare and probably impossible. However, hand calculations show that a composite section capable of developing 50% interaction has an ultimate moment of 94% of that of full interaction. On the other hand, zero interaction reduces the ultimate capacity of a beam by up to 50% compared with full interaction. As a result, it should be investigated whether the assumption of full interaction is reasonable or not.

Bridges 2DI1 to 3DI4 and 3DI1 to 3DI4 are analyzed for this parameter. Each set of three is assumed to have shear connectors with initial stiffness value of 0, 71, 142, and 850 kN/mm. The 850 kN/mm represents full interaction, and 142 kN/mm is the initial

stiffness suggested by Yam and Chapman (1972). Once again all the design lanes are assumed loaded.

Figures 5.54 to 5.55 show the distribution parameters of all the girders of these bridges at the critical sections in the whole range of loading up to failure. In all cases, except cases with zero interaction, full redistribution of moment occurs and all girders of a bridge have the same distribution parameter at the ultimate limit state. In bridges with zero interaction, ultimate load is significantly lower than the others, however, for bridges whose connectors have an initial stiffness of 71 kN/mm, which is 50% of the initial stiffness suggested by Yam and Chapman (1972), there is 9% reduction in ultimate load capacity compared to bridges with full interaction while this reduction is almost zero for bridges with initial stiffness of 142 kN/mm. As these figures show, even when the initial stiffness of shear connectors is 71 kN/mm, the ultimate distribution parameter is the same as for full interaction, and even in the elastic state distribution parameters of bridges with shear connectors which have an initial stiffness of 71 kN/mm is 4% different from the bridges with full interaction. This confirms that the codes assumption of full interaction for composite bridges is reasonable.

Figures 5.56 and 5.57 illustrate the variation of longitudinal moment redistribution with degree of interaction. Longitudinal moment redistribution of bridges with initial stiffness of 142 kN/mm is very close to that of bridges with full interaction, and is only 2% different. For bridges with shear connectors of initial stiffness 71 kN/mm the longitudinal moment redistribution is about 20% less than the bridges with full interaction for the both two-span and the three-span bridges. For the bridges with zero degree of interaction, which is generally unrealistic, the longitudinal moment redistribution is insignificantly affected by the initial stiffness of shear connectors.

(a) Distribution parameters at maximum positive moment section

(b) Distribution parameters at maximum negative moment section

Fig. 5.54. Girder Distribution Parameters of Bridge 2DI1 to 2DI4

(a) Distribution Parameter at Maximum Positive Moment

(b) Distribution Parameter at Maximum Negative Moment

Fig. 5.55. Girder Distribution Parameters of Bridge 3DI1 to 3DI4

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.56. Variation of Longitudinal Moment Redistribution with Degree of Interaction for Two-Span Bridges at Ultimate

(a) Maximum Positive Moment Section

(b) Maximum Negative Moment Section

Fig. 5.57. Variation of Longitudinal Moment Redistribution with Degree of Interaction for Three-Span Bridges at Ultimate

5.4.7 Transverse Diaphragms

As explained in Chapter 2, some researchers, such as Fu et al. (1976), found the effect of transverse diaphragms to be insignificant on transverse distribution parameter while some others (Ebeido and Kennedy 1996) have stated that transverse diaphragms have significant effect on the transverse distribution factor and ultimate capacity of bridges.

To investigate this parameter, six bridges were analyzed which are designated as 2TD1 to 2TD3 and 3TD1 to 3TD3. Each set of three have the same geometry except that 2TD1 and 3TD1 have no diaphragms while 2TD2 and 3TD2 have diaphragms at the supports and 2TD3 and 3TD3 have diaphragms at the supports and the mid-spans. These six bridges were analyzed while all design lanes were loaded.

Figs. 5.58 and 5.59 compare the distribution parameter of each set of bridges over the whole range of loading. As we can notice, the support diaphragms have enhanced the distribution ability of bridges 2TD2 and 3TD2 at maximum negative moment section, but it has less effect on the mid-span distribution. On the other hand, the mid-span and support diaphragms of bridges 2TD3 and 3TD3 improved the distribution characteristics of these bridges both at the supports and mid-span sections. In the analyzed bridges, diaphragms improve distribution in elastic state by up to 2%, which is not significant. At ultimate state, as shown in these figures, all bridges have the same distribution parameter of 2.5 m. Finite element analysis gives exactly the same longitudinal redistribution for the bridges with and without diaphragms.

It is clear from this parametric study that all parameters which affect moment redistribution at elastic state do not equally affect the distribution at the ultimate state. Furthermore, in continuous bridges, longitudinal redistribution is significantly more important than transverse redistribution. If in design advantage is taken of this beneficial

(a) Distribution Parameters at Maximum Positive Moment section

(b) Distribution Parameter at Maximum Negative Moment

Fig. 5.58. Elastic Girder distribution parameters of bridges 2TD1 to 2TD3

(a) Distribution Parameters at Maximum Positive Moment section

(b) Distribution Parameters at Maximum Negative Moment section

Fig. 5.59. Elastic Girder distribution parameters of bridges 3TD1 to 3TD3

redistribution, a much more realistic estimate of the actual ultimate capacity of the bridge can be obtained, the consequence of which is reduction in unnecessary posting of some bridges and retrofit of other bridges. Since building design codes permit redistribution of moments at ultimate, there is no reason why bridge codes would not do the same.

The parametric study also revealed that critically loaded girders distribute their moment initially longitudinally to their positive moment section and the positive moment section distributes this load transversely to the other girders at the same section. This behaviour may be partly expected because the concrete slab at the negative moment section is cracked and therefore is not able to effectively distribute the load transversely to the other girders at this section. On the other hand, the undamaged slab in the positive moment section efficiently distributes the load transversely.

The results of the present moment redistribution analyses are summarized in Tables 5.3 to 5.6. Tables 5.3 and 5.4 show the distribution parameters of all the analyzed two-span bridges at the sections of maximum positive and maximum negative moment, respectively. The corresponding results for three-span bridges are given in Tables 5.5 and 5.6, respectively. In these tables, the values of distribution parameters of the girders in each of the analyzed bridges at the elastic and ultimate limit state are given. The indicated values are for the critical load case, which corresponds to full loading of all lanes, except for the cases in which a number between parentheses is given in column 1 of the tables. In the latter cases, the indicated number represents the relevant load case shown in Figs. 5.3 and 5.4.

Table 5.3. Distribution Parameters of Two-Span Bridges at Maximum Positive Moment Section

Bridge	Distribution parameters (m)											
	Elastic						Ultimate					
	G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6
2NL1(1)	3.02	6.49	17.68	33.55	-	-	3.39	6.35	14.12	20.83	-	-
2NL1(2)	6.06	5.03	7.32	14.05	-	-	6.05	5.71	6.94	11.51	-	-
2NL1(3)	3.08	5.28	5.28	3.08	-	-	3.78	4.00	4.00	3.78	-	-
2NL1(4)	4.71	3.31	3.31	4.71	-	-	3.95	3.83	3.83	3.95	-	-
2NL1(5)	2.25	3.14	5.75	10.89	-	-	3.03	3.09	4.75	6.11	-	-
2NL1(6)	2.20	2.55	3.19	4.75	-	-	2.91	2.91	2.91	2.93	-	-
2NL1(7)	2.38	3.05	3.59	2.91	-	-	2.91	2.91	2.93	2.91	-	-
2NL1 (8)	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2BW1	2.23	2.21	2.21	2.23	-	-	2.22	2.22	2.22	2.22	-	-
2BW2	2.51	2.49	2.49	2.51	-	-	2.50	2.50	2.50	2.50	-	-
2BW3	2.79	2.76	2.76	2.79	-	-	2.78	2.78	2.78	2.78	-	-
2BW4	2.28	2.31	2.31	2.28	-	-	2.29	2.29	2.29	2.29	-	-
2BW5	2.41	2.60	2.60	2.41	-	-	2.50	2.50	2.50	2.50	-	-
2BW6	2.71	2.92	2.92	2.71	-	-	2.81	2.81	2.81	2.81	-	-
2BW7	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2NG1	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2NG2	2.33	2.66	2.53	2.66	2.33	-	2.50	2.50	2.50	2.50	2.50	-
2NG3	2.55	2.54	2.40	2.40	2.54	2.55	2.50	2.50	2.50	2.50	2.50	2.50
2ST1	2.36	2.65	2.65	2.36	-	-	2.50	2.50	2.50	2.50	-	-
2ST2	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2ST3	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-
2DI1	2.30	2.74	2.74	2.30	-	-	2.33	2.70	2.70	2.33	-	-
2DI2	2.35	2.67	2.67	2.35	-	-	2.50	2.50	2.50	2.50	-	-
2DI3	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2DI4	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2TD1	2.39	2.62	2.62	2.39	-	-	2.50	2.50	2.50	2.50	-	-
2TD2	2.41	2.60	2.60	2.41	-	-	2.50	2.50	2.50	2.50	-	-
2TD3	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-

Table 5.4. Distribution Parameters of Two-Span Bridges at Maximum Negative Moment Section

Bridge	Distribution parameters (m)											
	Elastic						Ultimate					
	G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6
2NL1(1)	2.73	7.97	18.92	37.64	-	-	3.29	7.15	12.65	20.56	-	-
2NL1(2)	6.69	4.37	7.15	18.87	-	-	6.48	4.93	7.31	12.95	-	-
2NL1(3)	2.82	6.26	6.26	2.82	-	-	3.29	4.76	4.76	3.29	-	-
2NL1(4)	5.48	3.02	3.02	5.48	-	-	4.57	3.39	3.39	4.57	-	-
2NL1(5)	2.15	3.13	5.75	14.07	-	-	3.03	3.03	4.01	8.36	-	-
2NL1(6)	2.19	2.53	2.96	4.75	-	-	2.90	2.91	2.92	2.93	-	-
2NL1(7)	2.31	3.07	3.92	2.80	-	-	2.91	2.91	2.93	2.91	-	-
2NL1 (8)	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2BW1	2.28	2.17	2.17	2.28	-	-	2.22	2.22	2.22	2.22	-	-
2BW2	2.56	2.44	2.44	2.56	-	-	2.50	2.50	2.50	2.50	-	-
2BW3	2.80	2.76	2.76	2.80	-	-	2.78	2.78	2.78	2.78	-	-
2BW4	2.24	2.35	2.35	2.24	-	-	2.29	2.29	2.29	2.29	-	-
2BW5	2.42	2.59	2.59	2.42	-	-	2.50	2.50	2.50	2.50	-	-
2BW6	2.72	2.91	2.91	2.71	-	-	2.81	2.81	2.81	2.81	-	-
2BW7	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2NG1	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2NG2	2.19	2.69	2.67	2.69	2.19	-	2.50	2.50	2.50	2.50	2.50	-
2NG3	2.55	2.53	2.42	2.42	2.53	2.55	2.50	2.50	2.50	2.50	2.50	2.50
2ST1	2.38	2.63	2.63	2.38	-	-	2.50	2.50	2.50	2.50	-	-
2ST2	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2ST3	2.47	2.53	2.53	2.47	-	-	2.50	2.50	2.50	2.50	-	-
2DI1	2.29	2.75	2.75	2.29	-	-	2.33	2.70	2.70	2.33	-	-
2DI2	2.35	2.66	2.66	2.35	-	-	2.50	2.50	2.50	2.50	-	-
2DI3	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2DI4	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2TD1	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
2TD2	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-
2TD3	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-

Table 5.5. Distribution Parameters of Three-Span Bridges at Maximum Positive Moment Section

Bridge	Distribution parameters (m)											
	Elastic						Ultimate					
	G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6
3NL1(1)	3.16	6.00	16.73	34.79	-	-	3.34	6.52	14.12	20.83	-	-
3NL1(2)	5.82	5.29	7.04	14.56	-	-	5.93	5.75	7.00	11.60	-	-
3NL1(3)	3.20	4.95	4.95	3.20	-	-	3.78	4.00	4.00	3.78	-	-
3NL1(4)	4.64	3.35	3.35	4.64	-	-	3.95	3.83	3.83	3.95	-	-
3NL1(5)	2.29	3.14	5.51	10.99	-	-	3.10	3.16	4.42	6.10	-	-
3NL1(6)	2.26	2.51	3.21	4.58	-	-	2.91	2.91	2.91	2.93	-	-
3NL1(7)	2.41	3.01	3.14	3.10	-	-	2.91	2.91	2.93	2.91	-	-
3NL1 (8)	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3BW1	2.24	2.21	2.21	2.24	-	-	2.22	2.22	2.22	2.22	-	-
3BW2	2.52	2.48	2.48	2.52	-	-	2.50	2.50	2.50	2.50	-	-
3BW3	2.79	2.77	2.77	2.79	-	-	2.78	2.78	2.78	2.78	-	-
3BW4	2.27	2.31	2.31	2.27	-	-	2.29	2.29	2.29	2.29	-	-
3BW5	2.36	2.65	2.65	2.36	-	-	2.50	2.50	2.50	2.50	-	-
3BW6	2.66	2.99	2.99	2.66	-	-	2.81	2.81	2.81	2.81	-	-
3BW7	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3NG1	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3NG2	2.35	2.64	2.54	2.64	2.35	-	2.50	2.50	2.50	2.50	2.50	-
3NG3	2.54	2.52	2.44	2.44	2.52	2.54	2.50	2.50	2.50	2.50	2.50	2.50
3ST1	2.38	2.63	2.63	2.38	-	-	2.50	2.50	2.50	2.50	-	-
3ST2	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3ST3	2.46	2.54	2.54	2.46	-	-	2.50	2.50	2.50	2.50	-	-
3DI1	2.30	2.74	2.74	2.30	-	-	2.35	2.69	2.69	2.35	-	-
3DI2	2.34	2.68	2.68	2.34	-	-	2.50	2.50	2.50	2.50	-	-
3DI3	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3DI4	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3TD1	2.43	2.58	2.58	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3TD2	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3TD3	2.45	2.55	2.55	2.45	-	-	2.50	2.50	2.50	2.50	-	-

Table 5.6. Distribution Parameters of Three-Span Bridges at Maximum Negative Moment Section

Bridge	Distribution parameters (m)											
	Elastic						Ultimate					
	G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6
3NL1(1)	2.89	7.22	17.33	34.65	-	-	3.23	8.45	12.92	21.00	-	-
3NL1(2)	6.48	4.61	6.73	19.44	-	-	4.48	4.93	7.31	12.95	-	-
3NL1(3)	2.84	6.15	6.15	2.84	-	-	3.33	4.68	4.68	3.33	-	-
3NL1(4)	5.40	3.04	3.04	5.40	-	-	4.57	3.39	3.39	4.57	-	-
3NL1(5)	2.21	3.14	5.40	13.89	-	-	3.06	3.06	3.99	8.01	-	-
3NL1(6)	2.26	2.49	2.95	5.31	-	-	2.91	2.91	2.91	2.93	-	-
3NL1(7)	2.35	3.04	3.64	2.92	-	-	2.91	2.91	2.93	2.91	-	-
3NL1 (8)	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3BW1	2.25	2.19	2.19	2.25	-	-	2.22	2.22	2.22	2.22	-	-
3BW2	2.55	2.45	2.45	2.55	-	-	2.50	2.50	2.50	2.50	-	-
3BW3	2.81	2.74	2.74	2.81	-	-	2.78	2.78	2.78	2.78	-	-
3BW4	2.26	2.32	2.32	2.26	-	-	2.29	2.29	2.29	2.29	-	-
3BW5	2.33	2.69	2.69	2.33	-	-	2.50	2.50	2.50	2.50	-	-
3BW6	2.66	2.99	2.99	2.66	-	-	2.81	2.81	2.81	2.81	-	-
3BW7	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3NG1	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3NG2	2.21	2.68	2.64	2.68	2.21	-	2.50	2.50	2.50	2.50	2.50	-
3NG3	2.55	2.53	2.42	2.42	2.53	2.55	2.50	2.50	2.50	2.50	2.50	2.50
3ST1	2.35	2.67	2.67	2.35	-	-	2.50	2.50	2.50	2.50	-	-
3ST2	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3ST3	2.48	2.53	2.53	2.48	-	-	2.50	2.50	2.50	2.50	-	-
3DI1	2.28	2.77	2.77	2.28	-	-	2.34	2.70	2.70	2.34	-	-
3DI2	2.35	2.66	2.66	2.35	-	-	2.50	2.50	2.50	2.50	-	-
3DI3	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3DI4	2.40	2.60	2.60	2.40	-	-	2.50	2.50	2.50	2.50	-	-
3TD1	2.39	2.61	2.61	2.39	-	-	2.50	2.50	2.50	2.50	-	-
3TD2	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-
3TD3	2.43	2.57	2.57	2.43	-	-	2.50	2.50	2.50	2.50	-	-

6. SUMMARY AND CONCLUSIONS

6.1 Summary

A study based on extensive nonlinear finite element analysis was undertaken to investigate the effect of different parameters on the longitudinal moment distribution in the transverse and the longitudinal directions in continuous composite concrete slab on steel girder bridges.

A detailed literature review showed that with respect to ultimate moment distribution in composite bridges, especially in the longitudinal direction, there is lack of research and a study needed to be carried out to find its significance with respect to the ultimate load capacity of such bridges. The latter furnishes the motivation and constitutes the basis for the work described in this thesis.

Because of its detailed nonlinear analysis capabilities, with particular reference to composite bridges, the finite element program NONLACS was chosen for the analyses in this study. The shear connector element of this program was modified to improve its formulation and to ensure that equilibrium was satisfied, both in the bridge and in the shear connector. Subsequently, the modified formulation of the shear connector element was verified by comparing the analytical results of NONLACS with available experimental results of composite beams and a bridge model tested by other investigators.

Thereafter, a detailed parametric study was conducted to determine the effect of some geometric and loading parameters on moment distribution of bridges in the whole range of loading up to failure. The studied parameters were longitudinal truck location,

transverse truck location and number of lanes loaded, bridge width, number of girders, slab thickness, degree of interaction, and presence of transverse diaphragms. Bridges analyzed in this work were 8 m to 14 m wide, with slab thickness of 175 to 275 mm. The bridges analyzed had between 4 and 6 girders. To simulate the effect of composite action between the slab and the girders, the strength and the spacing of the shear connectors were determined according to the OHBD (1991), but the initial stiffness of the stud shear connectors was varied from 0 to 850 kN/mm. However, in most cases it was assumed to be 142 kN/mm, a value which has been suggested by Yam and Chapman (1972) for shear studs similar to those used in the current study. The parametric study revealed that the number of girders, slab thickness, and transverse diaphragms do not have a significant effect on transverse distribution of moment at ultimate limit state. On the other hand, transverse location of the truck and bridge width can change the distribution parameters of bridges by up to 120% and 30%, respectively. In addition, longitudinal redistribution is significant and varied from 23% to 45% at the maximum negative moment section. The relevant predictions of CHBD (2000) and AASHTO (1994) codes, as well as the modified form of Fan's equation (5.3) and hand calculation results were compared with the finite element results. The investigation revealed that the ultimate and elastic distribution parameters of continuous composite bridges are different, and depending on geometry and loading, they can be different by as little as 2% or as much as 40 %. Furthermore, the CHBD and AASHTO codes can not accurately predict the distribution parameters of bridges in all cases while the predictions of Fan's equation (1974) are in reasonable agreement with the nonlinear finite element analysis results for the critical loading cases. More importantly, while longitudinal moment redistribution is quite

significant in continuous composite bridges, neither code includes any provisions for such redistribution.

6.2 Conclusions

1. The nonlinear finite element method is an accurate analytical procedure to predict the behavior of composite bridges in the entire loading range up to failure.

2. The load distribution parameter in composite bridges is affected by the level of loading. This effect is more prominent when all lanes are not loaded and the load is eccentrically applied. According to the results of finite element analysis, distribution parameters of the critical girders can change by up to 40% in eccentric loading from elastic to ultimate limit state while it changes a maximum of 6% when all lanes are loaded.

3. Distribution parameter is influenced by the transverse location of the truck and the number of lanes loaded, and it can change by up to 120%. However, in general the governing loading case is the one in which all the design lanes are loaded.

4. When all design lanes of a bridge are loaded, generally full transverse redistribution of moment occurs in the girders. In other words, under full nonlinear or inelastic condition, all the girders at the section of maximum moment equally participate in resisting the applied load.

5. Bridge width has a marked influence upon moment distribution and up to 30% change in the value of distribution parameter can occur when bridge width is increased while the span is kept constant. On the other hand, number of girders and slab thickness do not have any significant affect.

6. The CHBD (2000) code gives good results for distribution parameters of bridges with aspect ratio ranging from 2.27 to 3.75, but it yields conservative estimate for smaller aspect ratios.

7. The AASHTO (1994) code results are generally more conservative than the CHBD (2000) code. In a few cases, the AASHTO results are not on the safe side.

8. Eq. (5.3), which is a modified form of Fan's Equation (2.14), predicts distribution parameter for continuous composite bridges well.

9. Longitudinal moment redistribution at ultimate is significant and varies from 23% to 45% at the maximum negative section.

10. For the critical loading case, plastic analysis of composite continuous bridges treated as an equivalent beam provides a reasonably good estimate of the moment redistribution in the longitudinal direction.

11. Non-composite bridges have very poor transverse and longitudinal moment redistribution characteristic; however, a reasonable number of shear connectors will improve the load distribution substantially.

12. Transverse diaphragms enhance the distribution characteristics of bridges by almost 4% in the elastic state; however, it does not seem to have any effect at the ultimate state.

6.3 Recommendations for Future Work

As described in Chapter 1, the current study was restricted to continuous composite bridges with compact sections. The following future work is recommended with regard to ultimate moment distribution in

- Skew bridges

- Curved bridges
- Bridges with non-compact girders
- Precast concrete girder bridges

Also, experimental investigations are required to verify the applicability of the nonlinear finite element program NONLACS to the above cases.

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