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Problem-specific priming in division and multiplication

By

Brenda L. Smith

**A dissertation submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of**

Doctor of Philosophy

Department of Psychology

Carleton University

Ottawa, Ontario

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Brenda Smith

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Chair

External Examiner

Thesis Supervisor

Carleton University
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Abstract

How do people solve single-digit division problems? According to the independent representation hypothesis, division and multiplication facts are represented separately in memory. In contrast, according to the common representation hypothesis, people solve division problems by accessing their multiplication knowledge. In particular, they may recast a division problem such as $72 \div 8$ as the related multiplication problem (e.g., $8 \times _ = 72$) to access the answer. On this view, solving a problem in one operation (e.g., 8×9) should affect the subsequent solution of the corresponding problem in the other operation (e.g., $72 \div 8$). If division and multiplication are represented separately in memory, however, then no such cross-operation priming should occur. In two experiments, adults solved two blocks of single-digit multiplication or division problems. In the second block, half of the problems were related to those solved in the first block. Problem-specific priming was indicated by better performance on the related problem set relative to the unrelated set.

In Experiment 1 ($n = 96$), significant problem-specific priming was observed both within operations (e.g., when multiplication problems were solved in both blocks 1 and 2) and across operations (e.g., when division problems were solved in block 1 and multiplication problems were solved in block 2). In Experiment 2 ($n = 160$), two different explanations for the observed priming from division-to-multiplication were tested. If participants check their division answers by referencing multiplication (i.e., post-response checking), then reducing the time between problems (the inter-trial interval) or providing accuracy feedback should reduce priming. Alternatively, if participants access their multiplication knowledge to solve division problems (i.e., recast

$72 \div 8$ as $8 \times _ = 72$), then forcing participants to respond quickly may reduce division-to-multiplication priming. In Experiment 2, only participants who were instructed to respond quickly showed reduced division-to-multiplication priming. The results of these experiments support the view that people often use multiplication knowledge to solve division problems.

Acknowledgments

Twelve years ago, almost to the day of my dissertation defence, my world fell apart. Although the break-up of my marriage was devastating to me, I realised that my two children ultimately and undeservedly would suffer more. At this point, I decided to go to university and work towards improving my life. To do this, my children had to make sacrifices. Nicole and Richard had to cope with a mom who was too busy to properly celebrate birthdays that coincided with exam schedules. They made pots of coffee and peanut butter sandwiches when I was writing essays and too busy to make dinner. They thrived with a mother who was often just a voice on the telephone. Most importantly, they believed in me. Of everything I have done, they are the very best. This dissertation is nothing without them.

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Finally, I have some unsolicited advice for those of you who are entering or suffering through your undergraduate thesis or graduate school. Surround yourself with excellent people, as I did. Some of it does rub off! Treat these people well because what goes around, comes around. Tell yourself that you love what you are doing and that you are a lucky person, even when you don't feel that way. Oddly enough, I have found that when I did this, I started to love what I was doing and I turned out to be a very fortunate person. All the best to everyone and remember, nothing good comes easy!

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Problem-Specific Priming in Division and Multiplication

The cognitive processing of basic arithmetic seems to be relatively simple and straightforward compared to the cognitive processing of words or ideas. For example, the correct answer to " $2 + 2$ " is always "4", regardless of who you are, where you were educated, or what language you speak. Basic arithmetic is introduced at similar times by formal educational systems throughout the world (Geary, 1996). In cultures where mathematics is formally taught, Arabic, base-10 notation is almost invariably used (Zhang & Norman, 1995) and proficiency in basic arithmetic is considered a fundamental skill that must be mastered as the basis for more complicated mathematical concepts (Geary, 1996). Beyond the consistency of arithmetic across cultures and educational systems, arithmetic is interesting because arithmetic facts are simple and invariant expressions of logical relations that form the basis of more complex mathematical concepts. Potentially, studying the processing of arithmetic facts will facilitate theories of how simple elements of knowledge in memory are combined to express more complicated thoughts.

There is an assumption that because arithmetic is uncomplicated, the cognitive processing of basic facts will be fairly straightforward. For example, most theories of arithmetic cognition are based on the notion that arithmetic facts are stored in an associative network in long-term memory (Ashcraft, 1992, 1995; Ashcraft & Battaglia, 1978; Campbell, 1995; Campbell & Clark, 1988, 1992; Campbell & Oliphant, 1992; Dehaene, 1992; Geary, Widaman, & Little, 1986; Miller, Perlmutter, & Keating, 1984). For example, Baroody (1994) noted that all fact-retrieval theories are based on the expectation that an association between an arithmetic problem (e.g., $5 + 6$) and the

correct answer (i.e., 11) will develop over the hundreds or thousands of exposures to a problem during learning. The strength of the association between a particular problem and answer depends on various factors, such as the number of problem/answer pairings and the number of times the arithmetic problem is paired with an incorrect answer (e.g., $5 + 6 = "12"$). If a problem is not practiced, or if it is frequently answered incorrectly, then the problem/correct answer association will be weak.

According to fact-retrieval models, errors are made on basic arithmetic problems because individuals make mistakes during the solution process or during learning. For example, a child may accidentally count a finger twice while counting on their fingers to solve $5 + 6$, and incorrectly answer "12". Thus, both 12 and 11 become associated as potential answers to $5 + 6$. Individuals may make errors because problems are physically similar, for example sharing operands and/or operation symbol. For example, individuals may incorrectly answer $8 \times 6 = 56$, the correct answer to the problem 8×7 that shares an operand (i.e., 8) and an operation symbol (i.e., $\times$) with the presented problem. Errors also occur because problems are cognitively similar. For example, individuals commonly make an error by answering $6 \times 9 = 56$. Fifty-six is similar in numerical magnitude to the correct answer 54.

The observation that similar error patterns are made by many individuals is used as support for the assumption that errors reveal important features of the underlying organization of arithmetic knowledge (Ashcraft, 1992, 1995, for a review of common arithmetic errors). For example, many theorists assume that problems that are frequently confused are more closely associated in memory than problems that are rarely confused. However, although some problems are more closely associated in memory than other

problems, in fact-retrieval models each arithmetic problem is a separate and independent entity. Baroody (1994) notes that many fact-retrieval models specify that even commutative pairs of problems (e.g., $2 + 3$ and $3 + 2$) are represented separately in memory.

Most theorists assume that all arithmetic problems are stored in an associative network, regardless of operation. One of the reasons for this assumption is that performance patterns observed in addition are also observed in multiplication (Ashcraft & Battaglia, 1984; Miller et al., 1984). For example, in both addition and multiplication, individuals are slower and make more errors to problems with large operands (e.g., 8×9 or $7 + 6$) than small operands (2×4 or $3 + 5$). The positive correlations between latencies and errors and operand size are known as the problem-size effect. As well, both addition and multiplication problems involving ties (e.g., $3 + 3$ or 4×4 , or the same operand twice) are solved more quickly and with fewer errors than non-tie problems of a similar size (Miller et al., 1984; Parkman & Groen, 1971). The similarity of performance patterns across addition and multiplication is consistent with the notion that similar cognitive processes may be involved in solving addition and multiplication problems. Also, patterns in the type of errors made in arithmetic production tasks suggest that addition and multiplication knowledge are closely associated in memory.

Sometimes individuals will incorrectly answer a problem in one operation (addition or multiplication) with the correct answer involving the same operands in the other operation (e.g., answering "8" to " 4×4 "). Cross-operation confusions occur in both children and adults' arithmetic performance (Ashcraft, 1992, 1995; Siegler, 1988; Miller & Paredes, 1990). Some theorists suggest such confusions may result from an

attempt to retrieve a correct answer from memory and instead accessing information that is stored in close association with the correct information (Ashcraft, 1992, 1995; Siegler, 1988, Miller & Paredes, 1990). For example 4×4 is physically similar to the problem $4 + 4$, suggesting that operand similarity is an important feature of the organization of arithmetic facts.

The view that addition and multiplication are associated in long-term memory is not surprising because the two operations are linked conceptually (i.e., a theoretical mathematical link) and practically (e.g., how addition and multiplication are taught). For example, multiplication is conceptually related to arithmetic as multiplication represents repeated additions (e.g., 4×3 represents $4 + 4 + 4$). As well, for North American children, addition and multiplication are practically linked because children are often initially taught to multiply using repeated addition (Geary 1994, 1996; Siegler, 1988). Even adults sometimes report solving multiplication problems by referencing their knowledge of addition facts (LeFevre, Bisanz, et al., 1996). However, the relation among “basic” facts in other operations has received little attention. For example, subtraction is the inverse of addition, but relatively few studies have examined the cognitive processing of subtraction (Geary, Frensch, & Wiley, 1993; Siegler, 1989). Until recently, even fewer empirical studies had examined the relation between multiplication and division performance.

The lack of interest in the cognitive representation of division facts and how they relate to multiplication is not surprising. Basic division is rare in real world practice. For example, dividing a lunch bill between four people rarely involves an amount that is evenly divisible by four. The uncommon status of basic division is mirrored by a lack of

educational emphasis on learning basic division. For example, in the primary mathematics text Quest 2000 (recommended for use in Ontario schools), basic division is introduced well after multiplication has been established and very few lessons in the text are devoted to teaching basic division. Typically, the text lessons stress the relation between the basic division problem (e.g., $48 \div 6$) and the related multiplication problem (i.e., $6 \times 8 = 48$) and uses reference to the multiplication fact to solve the division problem. In the Quest 2000 arithmetic curriculum, children progress rapidly from basic division to solving division problems with remainders (e.g., $38 \div 4 = 9$ with a remainder of 2) and eventually on to long division (e.g., division where the answer is not a whole digit). Interestingly, long division is a process that does not require specific knowledge of basic division facts and instead requires multiplication and subtraction knowledge.

The lack of real world practicality and educational resources devoted to teaching division facts suggests that division problems may not be represented in memory as independent entities and that solving basic division problems involves referencing the related multiplication problem. However, studies examining the relation between division and multiplication knowledge in normal adults have lead some researchers to conclude that division problems are stored in memory independently from multiplication problems (Campbell, 1997, 1999; LeFevre & Morris, 1999; Rickard & Bourne, 1996; Rickard, Healey, & Bourne, 1994). The notion of independence between multiplication and division problems seemingly conflicts with the educational emphasis on exploiting the conceptual relation between the two operations. Accordingly, in the experiments detailed in this thesis, I will explore the conceptual relation between division and multiplication.

Neuropsychological evidence of the relation between division and multiplication

A relation between basic division and multiplication knowledge has been observed in the patterns of cognitive loss and preservation among individuals with acquired brain injuries (Hittmair-Delazer, Semenza, & Denes, 1994; Cipolotti & de Lacy Costello, 1995). For example, Hittmair-Delazer et al. (1994) reported the case of a 45-year-old brain injured patient who exhibited a parallel loss and preservation for related division and multiplication problems. When the individual was rehabilitated on the lost multiplication facts, the rehabilitation transferred to a recovery of the related division problems. The pattern of loss and subsequent recovery of division and multiplication facts is consistent with the notion that the ability to solve division problems involves referencing the related multiplication problem (Campbell, 1997; Rickard & Bourne, 1996).

In contrast, Cipolotti and de Lacy Costello (1995) observed a patient with acalculia who had largely intact multiplication knowledge but was seriously impaired in the ability to solve division problems. The independence between this patient's multiplication and division knowledge could be cautiously interpreted as evidence supporting the notion that multiplication and division facts are stored as independent representations in memory. However, without evidence supporting a double dissociation (i.e., an individual with preserved division knowledge and loss of multiplication knowledge), the loss of division in the individual observed by Cipolotti and de Lacy Costello is also consistent with the notion that solving division involves reference to multiplication. The specific loss of the ability to solve division problems could result from the loss of the ability to use the referencing process, rather than from the loss of

division knowledge per se. As such, the neuropsychological evidence has not unequivocally supported either (a) that division problems are stored in independent representations or (b) that solving division problems involves referencing multiplication knowledge. Other researchers have sought to determine the relation between multiplication and division facts in memory among normal adults.

Cross-operation transfer: Training studies

Rickard et al. (1994) and Rickard and Bourne (1996) conducted the first series of studies looking at the relation between basic division and multiplication in skilled adults. The underlying rationale of these experiments was the prediction that if solving some arithmetic problems (e.g., division) involves accessing or referencing a common representation (i.e., the related multiplication problem), then practice on one problem should benefit performance on all the problems that are solved by accessing the same representation. In Experiment 1, Rickard et al. tested whether practice on a set of problems resulted in transfer to related problems. Participants practiced two sets of problems: A set of standard multiplication problems (e.g., $4 \times 7 = \underline{\quad}$) and a set of problems that are expressed as a multiplication problems, but in fact require that the participants perform basic division (e.g., $9 \times \underline{\quad} = 45$). Practice occurred over three, one-hour sessions. In a fourth session, the participants were tested on five types of problems, the same problems they practiced (*no-change problems*), problems in which the order of operands was reversed from the practice problems (*order-change problems*, from 4×7 to $7 \times 4 = \underline{\quad}$), problems that were in the reciprocal operation from the practiced problems (*operation-change problems*, from 4×7 to $4 \times \underline{\quad} = 28$), and problems that were both in the reciprocal operation and in the reversed order from the practice problems (*operation-*

plus-order-change problems, from 4×7 to $7 \times __ = 28$). Rickard et al. suggested that by examining the features shared by the problems solved during the training sessions and the types of problems shared in the benefit of training, the key features constituting the internal representation of basic arithmetic facts could be identified.

Rickard et al. (1994) found that in the test session, participants were fastest in the no-change condition for both multiplication and division. For the multiplication problems, benefit of training also occurred for the order-change problems. For example, if the participant practiced 4×7 , benefit of training occurred not only 4×7 but also for 7×4 . There was no benefit of training for operation-change problems (e.g., $4 \times __ = 28$), or operation- plus order-change problems (e.g., $7 \times __ = 28$). For the problems formally requiring division (e.g., $4 \times __ = 28$), only the no-change problems benefited from training (e.g., $4 \times __ = 28$). Practicing specific division problems did not benefit performance on any other problems.

In a second experiment, Rickard et al. addressed whether the novel format of the problems formally requiring division (i.e., the unknown before the equal sign and the lack of the formal division sign) may have interfered with the transfer between order-change and operation-change conditions. They controlled the location of the unknown for both multiplication and division problems and also included problems using the division operation symbol (e.g., $__ = 4 \times 7$ and $__ \div 4 = 7$). A pattern of performance was found that was similar to the pattern in Experiment 1. Benefit of training was greatest for problems on the test that were identical to the practiced problems. When the problem formally required multiplication to be solved, operand-change problems received benefit from training. However, operand-change problems did not benefit from training with

problems formally using division. Rickard et al. concluded that the lack of a benefit of training was consistent with the notion that multiplication and division problems are stored in independent representations in memory. Additionally, both operand orders of a multiplication problem (e.g., 4×8 and 8×4) share a common representation but each operand order of a division problem (e.g., $48 \div 8$ and $48 \div 6$) is stored in independent representations.

In Experiment 1 of a subsequent study, Rickard and Bourne (1996) replicated Rickard et al., but during the testing session included a set of *new problems*, $3 \times 8 = ___$, or $3 \times ___ = 24$) that had not been practiced in the previous sessions. The inclusion of new problems allowed a better control of general transfer effects (i.e., a general benefit of training to all multiplication or division performance). In Experiment 2 of the same study, Rickard and Bourne observed benefit of transfer across presentation formats (e.g., four x seven and 4 x 7). In both experiments, they replicated the pattern of benefit from training observed in Rickard et al. (1994). As well, similar patterns of benefit from training also occurred across presentation formats. They concluded further support for the notion that multiplication and division are represented independently in memory, that the multiplication representation was insensitive to operand order, but the division representation was specific to a particular operand order. As well, the cognitive representations for multiplication and division problems were insensitive to presentation format (e.g., word or digit). On the basis of these results, Rickard et al. proposed the identical elements model.

The identical elements model. The identical elements model is based on the concept that there are distinct and sequential stages in arithmetic for the perceptual

encoding, cognitive processing required to determine an answer, and motor responses needed to generate the answer response (Rickard & Bourne, 1996; Rickard et al., 1994). In the identical elements model, abstract, semantic representations of arithmetic facts are independent of presentation modality. Therefore, the same representation of an arithmetic fact is accessed regardless of whether the arithmetic problem is heard, seen in Arabic format, or seen in number-word format. A common, abstract, and modality independent, internal representation for a multiplication problem consist of three essential elements. One element corresponds to each digit, and another to the formal (and not expressed) operation. For example, both $_\div 4 = 7$ and $4 \times 7 = __$ access the internal representation [4, 7, multiplication]. The order of the elements is not a feature of the internal representation so that 7×4 and 4×7 access a common internal representation. Changes in the format (e.g., order, or expressed operation) affect only the perceptual encoding stage and not the calculation stage of processing. Notably, in the identical elements model, division and multiplication facts are stored independently in memory.

The suggestion that division and multiplication facts are stored independently in memory is not consistent with Hittmair-Delazer et al.'s observations of cross-operation benefit to re-introduced problems. Rickard et al. (1994) and Rickard and Bourne (1996) noted that their study did not preclude the possibility that children who are learning division or less-skilled adults may use multiplication knowledge to solve basic division problems. They suggested that division problems might initially be learned by accessing a common representation and cautioned that the observation of complete independence between division and multiplication may have be due to the training paradigm used in their experiments. For example, it was possible that individual representations for

division problems could have been created or existing representations strengthened, during the series of training trials. Accordingly, performance patterns observed in subsequent studies examining different types of transfer between operations are more consistent with the notion that division and multiplication problems may, in part, share common representations.

Priming between division and multiplication

Campbell (1997) examined the relation between division and multiplication in a production task. Participants were asked to verbally solve alternating basic multiplication (e.g., 4×7) and division problems (e.g., $28 \div 4$). Unlike Rickard et al.'s (1994) or Rickard and Bourne's (1996) training studies, participants were tested in a single session. Campbell suggested that exposing participants to problems over one session would reduce the likelihood that they would alter their pre-existing internal organization of multiplication and division representations in memory. Accordingly, by examining the pattern of performance in adults who are not specially trained, a more naturalistic reflection of how division and multiplication knowledge is cognitively organized could be established.

Campbell (1997) suggested that a relation between multiplication and division problems could be gauged by a phenomenon called error priming (Arbuthnott, 1996; Arbuthnott & Campbell, 1996; Campbell, 1987, 1991, 1994; Campbell & Clark, 1989; Campbell & Tarling, 1996). Error priming is the observation that errors by people solving basic arithmetic problems under speeded conditions are strongly influenced by preceding problems that share similar cognitive features. Positive error priming is the observation that errors are often the correct answer to a problem that was solved in the

preceding 2 to 10 trials and that share features with the incorrectly answered problem (e.g., an operand, operation). Negative error priming is the observation that an error is rarely the answer to a correctly answered problem from the immediately preceding trial. Campbell (1991) concluded that error priming is due to residual cognitive activation from retrieving an answer in memory, rather than from answer production processes. For example, naming a number that is a potential problem answer (e.g., "48") does not cause error priming. Solving a problem (e.g., 6×8) results in error priming (Campbell, 1991). Similarly, Campbell and Tarling (1996) concluded that error priming is not due to encoding processes because encoding of a problem occurs in answer verification tasks (e.g., $6 \times 8 = 48$, "true" or "false"), but answer verification does not result in error priming in subsequent production trials.

Campbell (1995) suggested that correctly solving an arithmetic problem strengthens the association between the problem's components (e.g., operands, operation, and answer) in memory. Campbell termed the process of strengthening the association "unitization". In speeded conditions involving repeated presentation of arithmetic problems; unitization interferes with the solution of subsequent problems that share features with the unitized representation. For example, correctly solving $4 \times 6 = 24$ increased the likelihood that 24 will be stated incorrectly as an answer to problems such as 6×3 (shares the operand "6" and the operation "x" with the previous problem), 7×4 (shares the operand "4" and the operation "x" with the previous problem), and $4 + 6$ (shares both operands with the previous problem). Campbell (1997) proposed that if division and multiplication problems share cognitive features, then error priming should be observed between problems that share similar cognitive features. Alternatively, if

division and multiplication are stored in independent representations, error priming should not be observed in related cross-operation problems.

Campbell (1997) observed that multiplication and division performance were strongly related. When participants solved division problems, latencies were generally slower, but highly correlated with latencies for multiplication problems. As well, performance patterns typically observed in multiplication performance were also present in division. For example, as is typically observed in multiplication, tie- and five-operand (e.g., $36 \div 6 = 6$ and $15 \div 3 = 5$) division problems were solved faster than non-tie and non-five operand problems of a corresponding problem size (Campbell & Graham, 1985; LeFevre, Bisanz et al., 1996; Miller et al., 1984). Campbell noted that the problem-size effect in division was similar to the problem-size effect found in multiplication. Also, the type of errors made by participants suggested that division and multiplication share cognitive features.

Campbell (1997) examined the pattern of error priming and noted that correctly solving division problems (e.g., $\underline{56} \div 7 = 8$ or $18 \div 3 = \underline{6}$) primed errors in subsequent multiplication problems (e.g., $7 \times 9 = \underline{56}$ or $2 \times 4 = \underline{6}$). However, correctly solving multiplication problems (e.g., $2 \times 4 = \underline{8}$ or $7 \times 8 = \underline{56}$) did not prime errors in division (e.g., $63 \div 7 = \underline{8}$). Campbell concluded that cross-operation error priming was asymmetrical. Error priming occurred from division problems to multiplication errors, but not from multiplication problems to division errors. Campbell proposed that pattern of error priming was most consistent with the idea that division and multiplication problems are independently represented in memory.

Campbell (1997) noted that if a related multiplication and division problem access the same representation, then error priming would be symmetrical. Alternatively, if division and multiplication problems are stored in independent representations in memory, then the presentation of a problem in one operation could have minimal impact on the solution of a subsequent problem in the other operation. Campbell proposed that the absence of error priming between multiplication problems and division errors was incompatible with the idea of a common representation. Rather, he suggested that the asymmetrical pattern of error priming was most consistent with the hypothesis that division and multiplication are represented independently in memory. However, participants exploit the conceptual relation between the two operations and the well-learned status of multiplication to verify the accuracy of their division answers. The checking process after a division problem is solved (i.e., post-response checking) is the source of division-to-multiplication error priming.

According to Campbell (1997), post-response checking is a strategic reaction to the speeded conditions of the experiment. Solving division problems is harder than solving multiplication problems. Therefore, participants may sometimes check that their division answer was correct (or verify that the answer was incorrect) by accessing the representation for related multiplication problem. This accessing of multiplication information results in unitization of the multiplication representation and increases the probability of error priming for subsequent multiplication problems. Campbell noted that post-response checking must occur *after* the division response is made because if participants checked *before* they produced the answer, the error rate for division would be

similar to the error rate for multiplication. Instead, the error rate for division was almost twice the error rate for multiplication (16% vs. 9%).

Campbell (1997) proposed that asymmetrical priming occurs because participants would not check their multiplication responses by referencing the related division information. Participants are more confident in their ability to correctly solve multiplication problems than they are in their ability to correctly solve division problems. Campbell suggested that post-response checking did not occur in the training studies performed by Rickard et al. (1994) and Rickard and Bourne (1996) because the need to check division responses was eliminated by providing accuracy feedback after each trial. By preventing post-response checking, the related multiplication problem was never accessed during the solution process and as a result, the benefit of training did not extend to the related multiplication problem.

Campbell (1999) explored the relation between multiplication and division latencies using a priming-probe design. He examined the impact that solving a specific multiplication or division problem (the prime) had on the solution latency of a subsequent related problem (the probe). Prime and probe problems were separated by either four or six intervening problems (i.e., lags). In the experiment, Campbell prevented post-response checking by reducing the inter-trial interval. He suggested that by reducing the time between trials, participants would not have time to check their division answers after they responded. According to the identical elements model (Rickard et al., 1994; Rickard & Bourne, 1996), there should be positive latency priming between identical prime and probe problems (e.g., $28 \div 4$ and $28 \div 4$, or 6×7 and 6×7) and between inverted forms

of multiplication problems (e.g., 6×7 and 7×6). However, no priming should occur between inverted division problems (e.g., $48 \div 8$ or $48 \div 6$) or across operations.

Campbell (1999) observed positive within-operation priming. For example, solving 4×7 primed both 4×7 and 7×4 and solving $28 \div 4$ primed subsequent solution of the same problem, $28 \div 4$. Campbell observed less, but significant amounts of priming between inverted division problems (e.g., $28 \div 4$ and $28 \div 7$) in contrast to Rickard et al. (1994; Rickard & Bourne, 1996). Campbell also observed asymmetrical cross-operation priming. Notably, solving a particular multiplication problem (e.g., 4×7) primed the solution latencies of a subsequent related division problem (e.g., $28 \div 4$) whereas solving a division problem (e.g., $48 \div 8$) did not prime the related multiplication problem (i.e., 6×8). The asymmetrical priming observed in this experiment is opposite to the division-to-multiplication error priming that was observed by Campbell (1997). Campbell (1999) concluded that the short inter-trial interval successfully prevented participants from engaging in post-response checking, thereby eliminating division-to-multiplication priming. However, he suggested that the multiplication-to-division priming observed in this experiment was consistent with the notion that participants sometimes referenced their multiplication knowledge when they solved division problems.

Campbell (1999) proposed that participants exploit their superior knowledge of multiplication to solve a division problem when they have recently solved the related multiplication problem (i.e., in the past 4 to 6 trials). The use of the multiplication information when it has been recently accessed is the source of multiplication-to-division priming. Campbell suggested that because division is not as efficient or as accurate as multiplication. Thus, participants do not similarly use their knowledge of division to

solve a multiplication problem, even if they have recently solved the division problem. The term "cognitive convenience" is used here to describe the phenomenon whereby participants solve division problems by accessing the related multiplication representation because of situational factors that ensure the multiplication knowledge is easier to access and/or is more likely to be accurate than the representation for the division problem.

A situational account for asymmetrical priming

Campbell (1999) concluded that the observation of priming between division and multiplication depends on situational factors. When participants are in speeded response conditions but have time to check their division answers by accessing the related multiplication fact between trials, division-to-multiplication priming is observed (i.e., post-response checking). In situations where division and multiplication problems are mixed together, participants will use a recently accessed multiplication fact to solve a subsequent division problem and multiplication-to-division priming will be observed. The notion that skilled adults can strategically exploit the conceptual relation between operations to improve their performance has been observed in other research. For example, educated adults may refer to addition knowledge to solve multiplication problems (LeFevre, Bisanz, et al., 1996). Geary et al. (1993; Daley, 1997) observed that participants sometimes reported referring to their knowledge of addition to solve subtraction problems.

Strategic approaches to solving basic arithmetic problems may result from an attempt to reduce errors by accessing better-learned arithmetic problems (LeFevre et al., in press). The notion of solution procedures involving reference to related operations,

however, differs from the cognitive convenience hypothesis in the degree to which the solution procedure is a feature of the cognitive organization of the arithmetic knowledge. The solution-procedure hypothesis, proposed by LeFevre, Bisanz, et al. (1996; LeFevre et al., in press) is based on the notion that basic arithmetic solution procedures are associated with a problem during learning (Siegler & Shipley, 1995) and are stable over time (LeFevre et al., in press; Smith-Chant & LeFevre, 2001). Accordingly, solution strategies are a feature of the cognitive representation of the problem's solution in memory (Siegler & Shipley, 1995). Alternatively, the cognitive convenience notion is based on the notion that arithmetic strategy use is the use of available information that is easier or more reliable than accessing the cognitive representation of a problem in memory. As such, solution procedures are not viewed as a feature of the organization of the arithmetic problem in memory.

To explore the procedures used by individuals when they solve division problems, LeFevre and Morris (1999) asked adult participants to provide introspective reports of how they arrived at a division or multiplication answer. In LeFevre and Morris, adults solved all the simple multiplication and division problems from 1×2 to 9×9 and from $2 \div 1$ to $81 \div 9$. Problems were presented in two blocks grouped by operation. The order of presentation was counterbalanced across participants (division/multiplication, multiplication/division). The instructions given to participants weighted speed and accuracy equally. Immediately after solving a problem, half of the participants were asked to provide a description of how they solved the problem.

Consistent with Campbell (1997, 1999), LeFevre and Morris (1999) observed that latency patterns (i.e., the problem-size effect, ties effect, fives effect) were similar for

multiplication and division, with the problem-size effect being slightly larger for division. They also replicated the observation of asymmetrical cross-operation priming. LeFevre and Morris found that participants who solved a block of division problems were faster at solving the subsequent block of multiplication problems than the participants who solved multiplication problems in the first block. However, participants who solved multiplication problems in the first block were not faster at solving division problems than the participants who solved division in the first block. The asymmetric pattern of cross-operation facilitation noted by LeFevre and Morris corresponded to the asymmetrical division-to-multiplication error priming observed by Campbell (1997). However, rather than attributing the presence of division-to-multiplication priming to post-response checking, LeFevre and Morris noted that participants often reported that they were making use of the conceptual relation between division and multiplication to solve the division problems. The solution procedure these participants used was termed “recasting”.

LeFevre and Morris (1999) observed that on large division problems, participants often reported that they “recast” division problems into a multiplicative form. For example, to solve $56 \div 8$, participants reported that they mentally reordered the question into a multiplication format (e.g., $8 \times __ = 56$). Then, they would activate the multiplication problem (i.e., $8 \times 7 = 56$) to retrieve the answer “7”. LeFevre and Morris found that division-to-multiplication priming occurred mainly for large multiplication problems which corresponded with the observation that participants were much more likely to report that they used a recasting solution method with larger division problems than with smaller division problems. LeFevre and Morris concluded that small

multiplication and division facts are stored in independent representations in long-term memory. However, larger and less well-learned division facts share common representations with large multiplication facts in memory. They proposed that recasting is the process by which participants mentally reorganize the division problem to allow access to the multiplication representation (and hence the answer) in memory.

Both Campbell (1999) and LeFevre and Morris (1999) observed asymmetrical latency priming between multiplication and division. However, Campbell observed multiplication-to-division priming and LeFevre and Morris observed division-to-multiplication priming. Interestingly, in spite of the inconsistency in the direction of priming, both researchers concluded that participants accessed their knowledge of specific multiplication problems to solve the corresponding division problems, albeit in different manners. In Campbell (1999), participants solved division and multiplication problems in mixed blocks. In contrast, participants in LeFevre and Morris (1999) solved a block of problems in one operation before solving a block of problems in the other operation. Theoretically, it is possible that the differences between the directions of the priming are due to the mixed operation versus blocked operation design. When problems are blocked, the time between the multiplication problem and the corresponding division problem may be too long to be cognitively convenient, thereby eliminating multiplication-to-division priming. Alternatively, the division-to-multiplication priming observed by LeFevre and Morris could be attributed to experimental features that promoted recasting in their experiment that were not present in Campbell's experiment.

The opportunity for participants to use recasting to solve division problems might have differed across experiments. Campbell (1997, 1999) and LeFevre and Morris

(1999) differed in the emphasis placed on speed and accuracy in the instructions provided to participants. Speeding generally results in faster latencies and higher error rates (Smith-Chant & LeFevre, 2001). It is possible that in LeFevre and Morris, the balanced emphasis on speed and accuracy encouraged participants to use a recasting solution procedure and access their multiplication knowledge *before* they stated an answer. The ability to recast not only would promote the observation of division-to-multiplication priming but would also promote more accurate performance. Accordingly, LeFevre and Morris noted that the division error rate in their experiment was not dramatically different from the error rate for multiplication (i.e., 11% and 7%). In contrast, the emphasis placed on speeded responses in Campbell (1999) deterred participants from taking the time *before* they responded to recast during the solution of a division problem, which eliminated division-to-multiplication priming. Instead, participants may have been limited to checking the accuracy of their answers after they responded. Accordingly, post-response checking would promote the observation of multiplication-to-division priming and the error rate for division would remain high (i.e., 16%).

Limitations of previous studies

Researchers have employed different methodologies when exploring the relation between division and multiplication and the observation that these differences are associated with the direction of priming is problematic. Notably, the issue of how the previous studies address the cognitive organization of multiplication and division in adult memory is confounded with the possibility that the studies promote the use of different information or solution processes. For example, training participants on division and/or multiplication problems may alter the pre-existing organization of arithmetic information

in memory which limits the utility of using training studies to explore the natural relation between multiplication and division in memory (Campbell, 1997; Rickard et al., 1994; Rickard & Bourne, 1996).

In Campbell (1997), altering the natural organization of multiplication and division in memory was avoided by testing participants in a single session. However, Campbell explored the relation between multiplication and division using the error priming method. The observation of error priming across division and multiplication may have been limited by the outstanding differences in the range and magnitude of the answers in division and multiplication. Notably, in Campbell's study, division answers were always numbers ranging from 2 to 9. However, multiplication errors could range from 6 to 72. Because possible errors to basic division problems were much more constrained than possible errors to basic multiplication problems, the observation of cross-operation error priming could have been limited. Accordingly, error priming may offer only a limited view into the relation between the two operations.

Observations of cross-operation priming may be confounded not only by the repeated presentation of a problem over many sessions, but also by the repeated presentation of a problem in a single session. For example, in Campbell (1999), participants repeatedly observed each division or multiplication problem three times in each of 12 blocks (36 presentations) during the hour-long experiment. If the impact of solving a specific multiplication or division problem does not dissipate quickly, then the repeated exposure of the same problem during a single session would complicate identifying which presentation of a problem was contributing to the priming effect. For example, although Campbell (1999) did not find evidence of positive priming over lags

longer than six problems, there was an unusual pattern of negative latency priming over longer lags. The presence of negative priming over long lags may indicate that solving a specific problem was influencing performance on related problems beyond four to six lags.

LeFevre and Morris's (1999) experiment is limited in that it was not designed to examine cross-operation priming. Rather, the study was designed to examine self-reported solution procedures in multiplication and division. Their observation of cross-operation priming was a result of block counterbalancing rather than an experimental manipulation. Further, because participants solved all the basic multiplication and division problems, LeFevre and Morris were unable to conclusively determine that solving a specific problem in one operation primed the solution of the related problem in the other operation. The division-to-multiplication benefit could be attributed to a form of practice effect caused when participants were exposed to more difficult division problems prior to solving multiplication problems. Because of this limitation, LeFevre and Morris termed the cross-operation benefit they observed as a general "facilitation" effect rather than problem-specific "priming".

Summary. Baroody (1994) suggested that the ability to condense a large amount of information (i.e., the division table) into a single heuristic (i.e., reference multiplication) is beneficial because it is cognitively economical. Solving division problems by referencing the related multiplication representation would eliminate the extensive practice required to memorize another complete table of arithmetic facts. The notion of a common representation underlying both division and multiplication is also more consistent with the educational experience of North American students. However,

the argument for common representations of division and multiplication is seriously hampered by the asymmetrical and inconsistent nature of cross-operation priming.

One of the major limitations of the previous studies derives from the difficulty in controlling the effects of practice and multiple presentations in arithmetic priming-probe experiments. The possible number of problems that can be used as stimuli is limited in basic arithmetic (i.e., a total of 100 facts per operation), especially considering that experimenters must also control for problem difficulty (i.e., the problem size effect) and other effects when selecting problems (i.e., ties effects, zero- and one-rules; see Ashcraft, 1992 for a review of common effects in arithmetic). The limited number of arithmetic stimuli leads to repeated use of the same problems in experiments. In mixed operation experiments, problem repetition leads to difficulties in determining which of the previous exposures has contributed to the priming effect. Alternatively, in blocked operation experiments, practice and order of presentation effects make it difficult to differentiate the specific impact of solving a problem on the related problem versus the more general impact of completing one arithmetic task prior to another.

The methodology designed for the current series of experiments allows for problem-specific priming to be observed in a within-subject experiment while controlling confounding caused by multiple presentations of specific problems. This series of experiments was designed to address issues arising from the possible contribution of methodological limitations of the previous studies to the appearance of asymmetrical priming and to more directly test predictions based on the common and independent representation hypotheses.

A new paradigm to examine priming

The paradigm used in the present research is a modification of a priming-probe design. Participants first solve a small set of division or multiplication problems. Next, they solve a second set of problems consisting of (a) problems related to the first set and (b) a set of novel problems that are unrelated to the first set. The novel problem set and the related problem set are designed to be of equal difficulty (i.e., balanced in latencies and errors). Accordingly, performance differences between the novel and related sets can be attributed to the previous presentation of problems. Positive priming would be indicated by an overall reduction in the latencies and errors for the related problems relative to the novel problems.

This paradigm allows problem-specific priming to be identified because it contrasts performance on problems that are related to previously presented problems relative to problems that are novel. If there is only general cross-operation facilitation and not problem-specific priming (i.e., solving a group of division problems generally makes solving any multiplication problem easier), then the novel and related sets will not be differentially benefited by exposure to the first set. Likewise, confounding caused by repeated exposure to the same problems over many trials is prevented. Problems are viewed only once in the first set, and only once in the second set. Thus, there is no ambiguity as to which previous presentation of a problem contributed to the priming effect.

Experiment 1

This experiment served as a test of the paradigm that was developed to examine problem-specific priming between problems. The paradigm used in the experiment is based on the assumption that participants should be faster and make fewer errors the second time they access the cognitive representation used to solve a particular problem in a short period of time (i.e., 20 minutes or so). Second, this experiment contrasted predictions based on the common and independent representation hypotheses. If related division and multiplication problems share a common representation, then solving a specific problem (e.g., 6×8) should benefit the solution of any problem that shares that representation, including related problems in the inverse operation (e.g., $48 \div 8$). Alternatively, solving a problem in one operation would not benefit the solution of the related problem if division and multiplication problems were stored in independent representations.

In this experiment, the priming that occurs within operation (e.g., between 6×8 and 6×8) was contrasted with priming that occurs across operations (e.g., between $48 \div 8$ and 6×8). For the within-operation conditions, the problems in the first block (e.g., 6×8) were repeated in the second block (i.e., 6×8). The latencies and errors made to the repeated problems were contrasted with the latencies and errors to a set of novel problems in the same operation (e.g., 7×9). In the cross-operation conditions, during the second block of problems, participants solved problems that were related but in the inverse operation (e.g., $48 \div 8$) of the problems presented in the first block (i.e., 6×8). Thus, the degree to which participants benefited from solving a problem twice could be contrasted with the degree to which participants benefited from previously solving the

related problem. If multiplication and division problems share a common representation, then problem-specific priming should occur both within- and cross-operations.

Alternatively, if multiplication and division problems are represented in independent representations, then problem-specific priming should only occur within- and not cross-operations.

Method

Participants

Ninety-six undergraduate psychology students enrolled in a first-year psychology course at Carleton University (36 males and 60 females) participated in the experiment for course credit. Participants volunteered for the study by signing up for an appointment time on the experimental credit notification message board located in the foyer outside of the Psychology lecture hall.

Materials

Modified Priming-Probe Task. Participants were asked to verbally solve arithmetic problems that were presented on a computer monitor. The problems were presented in two blocks. The problem sets for the first block were created by dividing all the possible problems from 2×3 to 9×8 and excluding ties, 0-operand and 1-operand problems into two equivalent sets ($n = 28$, see Appendix A). Tie problems (e.g., 2×2 or 5×5) were excluded because reaction times for these problems are faster than to problems with equal magnitude, suggesting that the cognitive process used to solve tie problems may differ than the processes used to solve other problems (Ashcraft, 1992). Similarly, zero- and one-operand problems (e.g., 4×0 or 6×1) were excluded because most researchers assume that many people might solve these problems using rules (i.e.,

anything times 0 is 0, anything times 1 is itself; Ashcraft 1992, 1995). After omitting ties, zero- and one-operand problems, 56 problems remained for inclusion.

To create two equivalent groups, the 56 problems were divided into two groups ($n = 28$). Each of the groups were matched to be equivalent in average latencies using the latencies reported by LeFevre, Bisanz et al. (1996). The mean latencies reported for each multiplication problem in Experiments 1 and 2 were averaged (Appendix, LeFevre, Bisanz, et al., 1996). The overall average reaction time for each set of problems was balanced to be equal as possible ($M = 1375$ ms for list A vs. $M = 1374$ ms for list B). The sets were matched to have the same number of large (product > 25) and small (product < 25) problems in each. The sets were further constrained in that each problem (e.g., 3×2) and its complement (e.g., 2×3) appeared in the same list. This constraint was to eliminate confusion about which form of the problem (e.g., 2×3 or 3×2) potentially shares a common representation with the related problems (e.g., $6 \div 2$ or $6 \div 3$). Once two equivalent lists were created for multiplication, the multiplication problems were replaced with the related division problems to create division lists. Two separate orders were made for each list.

The second block trials consisted of all the problems from the first block plus the novel problems from the set of problems the participants had not solved. Problems from the two sets were semi-randomly mixed together. The list was organized to avoid having problems with identical answers adjacent to one another (e.g., 8×3 or 6×4 , $12 \div 3$ or $8 \div 2$) and large and small problems were somewhat equally interspersed throughout the list. Two different orders of the second block problems were created. To prevent

potential list or order effects, complete counterbalancing was used (list A or B, order 1 or 2).

Apparatus. Problems were presented on a Samsung model SM 125FA7 monitor connected to an IBM-type 80286 computer. A Sharpe LM6 microphone which was interfaced with a SoundBlaster I/O board recorded individual responses with millisecond accuracy via a Digitek I/O board.

Arithmetic Fluency Test. Participants completed the addition and the subtraction-multiplication sub-tests of the French Kit as a measure of arithmetic fluency (French, Ekstrom, & Price, 1963). Collecting an independent measure of arithmetic fluency is useful for examining skill-related differences in performance. Each sub-test of this pencil-and-paper task consists of two pages of problems. Participants were asked to read the instructions on a covering page before each of the two sub-tests that described the task and the scoring procedure. They were directed to solve as many problems on a page as they could in a two-minute limit. The addition sub-test consisted of three 2-digit numbers arranged in a column. The multiplication problems consisted of a 2-digit problem multiplied by a single digit. A total score on the French Kit reflects the sum of the correctly solved problems on all four pages. Participants were asked to complete practice problems on the cover page before beginning. Total time to complete the French Kit was about 10 minutes.

Questionnaire of non-retrieval strategy use. Participants were asked to indicate where they received their education, first language, major in university (e.g., arts or sciences) and attitudes towards mathematics and English (see Appendix B). Participants were also asked to indicate how frequently they use non-retrieval processes when solving

basic multiplication, division, subtraction, and addition facts. As well, participants were asked to indicate the degree to which they use procedures such as derived facts, counting, rules, tricks (e.g., songs, rhymes, etc.), checking, and other procedures when solving basic arithmetic problems. This information allows potential pre-existing differences between participants due to confidence in mathematics, culture, education, language, and non-retrieval strategy use to be identified. As well, information gathered in the questionnaire is combined with information gathered in other studies ongoing in the lab to assist in creating a base of information about common arithmetic strategies and experiences (LeFevre et al., in press). Total time to complete the survey was approximately 10 minutes.

Procedure

Participants were tested in a quiet room with an experimenter present. Participants were told that simple multiplication or division problems would appear on the computer screen. They were instructed to answer the problem by speaking loudly and clearly into the microphone. Participants were instructed to answer the mathematical problem as quickly and as accurately as possible but were told “occasional errors are normal when people go fast.” The participant and the experimenter viewed separate monitors. The participants’ monitor was partially screened with a black frame surrounding the periphery so that the numerical answer entered by the experimenter could not be seen by the participant.

The participants were asked to press a button located in front of them to begin the experiment and were again reminded to reply aloud as quickly and as accurately as possible. Problem operands appeared as digits and were displayed horizontally in the

centre of the screen using yellow characters against a black background. An operation sign ($\times$ or $\div$) separated the digits. On each trial, a diamond shaped symbol that served as a fixation point was presented in the centre of the screen for one second. After this initial presentation, the diamond flashed off for 250 ms and back on for 250 ms intervals. On what would have been the third presentation of the flashing diamond, an arithmetic problem was presented on the screen (e.g., 5×6), with the operation sign ($\times$ or $\div$) replacing the position of the asterisk. Each problem remained on the computer screen until the participant had given a verbal response or until fifteen seconds had elapsed. Timing on each trial began at the presentation of the problem and continued until the participant responded. Once the participant responded, the experimenter entered the stated answer, or coded the trial as invalid and the next trial was initiated. Between the first and second blocks, the participants were given the opportunity to rest.

Before the experimental trials began, participants solved six practice problems consisting either of all multiplication problems, all division problems, or a mixture of problems in both operations. The type of problems presented during the practice corresponded to the experimental condition to which the participant was assigned (i.e., all multiplication, all division, first multiplication then division, or first division then multiplication problems). The practice problems were selected from the tie, zero- and one-operand problems that were omitted from the stimulus lists for the current experiment. No feedback concerning accuracy or reaction times was provided to participants at any point during either the practice or the first or second block trials. Once the practice problems were solved, the experiment began. After solving the two blocks of problems, participants completed the pencil-and-paper arithmetic fluency task

and the questionnaire on non-retrieval strategy use. Total time to complete the experiment was approximately 45 minutes.

Results

The median reaction times for the small and large problems in the experimental trials were computed for each of the problem sets (block 1 trials, block 2 primed and unprimed trials). Trials were included in the calculation if the response was valid and correct. Trials were considered invalid if there was a failure of the voice key, a non-answer vocalization by the participant (e.g., um, er, pfff), a failure of the participant to respond within the 15-second time limit allowed for responses, or if the reaction time was less than 500 ms. A trial also was scored as invalid if the participant started to say one answer, but changed their answer to another number before they finished stating the first (e.g., stating “forty....fifty-six”). In total, 520 trials (6%) were coded as invalid.

Unless otherwise noted, all assumptions underlying ANOVAs (homogeneity of variance, sphericity) were met and the univariate tests yielded results identical to the multivariate tests. In every analysis described here, list effects (differences between list A and list B) and order effects (differences between order 1 and order 2 for each of the lists) did not approach significance. As well, list and order did not interact meaningfully with other variables. The absences of set and order effects suggest that the stimulus lists used in the experiment (e.g., List A, orders 1 and 2 and List B, orders 1 and 2) were balanced in difficulty. Thus, differences between the two problem sets can be attributed to the influence of preceding experimental trials rather than to pre-existing set differences. List and order effects were not included in the subsequent analyses for Experiments 1 and 2.

Prior to analyzing the results to determine if problem-specific priming occurred both within- and cross-operations, seven participants were excluded from analyses. One was excluded because of extremely slow latencies on the experimental trials (+3 standard deviations from their group's overall median reaction time in the first block of trials). The other six were excluded because they validly and correctly answered fewer than half of the large problems ($n = 14$) in the first block of problems. Omitted participants were evenly dispersed among the experimental conditions, with 2 omitted participants in each of the within-operation priming groups (both multiplication-to-multiplication and division-to-division) and 2 omitted participants in the multiplication-to-division priming group. A single participant was omitted in the division-to-multiplication priming group.

The remaining participants ranged in age from 17 to 51 years with a mean of 22.9 ($SD = 7.9$ years). The demographic characteristics of the participants in each of the experimental condition (e.g., age, location of primary and secondary education, first language) can be found in Appendix C. There were no significant differences among the experimental conditions on any of the demographic variables ($ps > .10$). Table 1 contains a summary of the number of participants and the minimum, maximum, and mean French Kit score for each experimental group. There were no significant differences between groups in arithmetic fluency.

First Block Trials

The latencies and errors made by participants in the first block were analysed to determine if the patterns observed in the first block of multiplication or division trials yielded similar patterns to other studies examining multiplication and division. The percentage of errors and latencies were analysed in a 2 (problem size: small, large) by 2

(operation of problems: multiplication, division) ANOVA with repeated measures on the first factor. Appendix D provides the mean percentage of errors and latencies for multiplication and division first block problems for each problem type and problem size and Table 2 contains the summary of the ANOVA analyses.

Percentage of errors. Participants made more errors on large than on small problems (14% vs. 3%). The observation that errors increased with problem-size is a standard observation in arithmetic cognition. There was a trend for participants to make more errors when they solved division problems than when they solved multiplication problems (10% vs. 8%). The observation that participants are more likely to make errors when solving division problems than when solving multiplication problems is consistent with observations of an effect of operation described by Campbell (1997, 1999). However, the actual percentage of errors made in each operation and the difference between error rates in division and multiplication in the current experiment is more consistent with the pattern of errors observed by LeFevre and Morris (1999; 11% for division and 7% for multiplication) than with the pattern observed by Campbell (1997; 16% division, 9% multiplication).

The higher percentage of errors for division in Campbell (1997) may be attributable to the mixed-block design and the emphasis on speeded response in the instructions provided to participants used in his experiment. When participants are told to emphasize speed, they tend to respond more quickly and make more errors (Smith-Chant & LeFevre, 2001). As well, participants may be more likely to make errors in mixed operation experiments because the need to attend to the operation of the problem in a mixed operation design may increase the difficulty of the task.

Solution latencies. Participants solved small problems more quickly than large problems (954 ms vs. 1295 ms), a pattern known as the problem-size effect. As well, participants solved multiplication problems more quickly than division problems (954 ms vs. 1206 ms), replicating the operation effect observed by Campbell (1997, 1999). The absence of a problem-size by operation interaction indicated that the increase in latencies across problem-size was similar for multiplication and division. The overall pattern of division and multiplication performance in the first block is consistent with performance on multiplication and division in other studies in which participants were slower to solve division problems than multiplication problems (Campbell, 1997; 1999; LeFevre & Morris, 1999). The lack of a problem-size by operation interaction in the current experiment is inconsistent with LeFevre and Morris's (1999) observation that the increase in latencies associated with problem-size is greater for division than for multiplication. However, the absence of an observable problem-size by operation interaction is consistent with Campbell (1997, 1999).

Second Block Trials

The median latencies and the percentage of errors for the second block of problems were calculated for each participant by trial type (primed, unprimed) by problem size (small, large). The mean latencies and percent errors for each experimental group are listed in Appendix E. Each dependent measure was analyzed in a 2 (experimental condition: within-operation, cross-operation) by 2 (operation in 2nd block: multiplication, division) by 2 (trial type: primed, unprimed) by 2 (problem size: small, large) ANOVA with repeated measures on the last two factors. Table 3 contains a summary of the ANOVAs.

Percentage of errors. As is typically observed in mathematical cognition, there was a significant effect of problem size. Participants were more likely to make errors on larger problems than smaller problems (15% vs. 4%). Participants also made fewer errors when solving problems that had been primed than they did when solving problems that were not primed (8% vs. 11%). The observation that participants made fewer errors to primed problems than to unprimed problems suggests that exposure to a problem during the first block, either in its identical or related form, significantly reduced the likelihood that participants would make an error when solving the problem during the second block. Thus, problem-specific priming occurred for errors across the first and second blocks.

There was also a significant 3-way interaction among problem size, priming, and experimental condition, as shown by the error rates in Table 4. In the within-operation conditions, the effect of priming differed across problem-size. Small primed problems were solved significantly more accurately than small, unprimed problems. However the effect of priming was not significant for large problems. In the cross-operation conditions, the effect of priming also differed across problem-size, but the pattern was the reverse observed in the within-operation conditions. Across-operations, there was no effect of priming in the errors made for small problems. However, participants were significantly less likely to make an error when solving large primed problems than large unprimed problems.

The pattern of errors made in the second block of the current experiment is consistent with the observation by LeFevre and Morris (1999) of greater cross-operation facilitation for large problems than for small problems. LeFevre and Morris suggested

that large multiplication and related division problems share representations in common. Thus, solving a large problem would benefit subsequent solution of the problem regardless of operation. However, small division problems may be represented independently from the related multiplication problem. Thus, solving a small division or multiplication problem would only benefit the subsequent solution of the identical problem but not the related problem in the inverse operation.

Solution latencies. Generally, latencies to primed and unprimed problems were slower than latencies for the corresponding operation and problem size in block one (see Appendix D and E). The slower performance for problems in the second block suggests either participants experienced general interference effect from exposure to problems in the first block or they were experiencing fatigue in the second block. The notion that the slower latencies are caused by an overall fatigue effect is not supported by the observation that performance was generally slower for all problems. Rather, the difference between block one and two latencies for small problems is less than the difference for large problems. The presence of interference effects is more difficult to assess. However, the interference effect caused by exposure to the first block problem should not differ for unprimed problems and primed problems. Accordingly, in the current experiment the effect of priming is gauged by the relative difference between primed and unprimed problems in the second block. Additionally,

In Figure 1, the confidence intervals were calculated using the Mean Square Error term from the 4-way interaction using the procedure outlined by Loftus and Masson (1994). Participants solved small problems significantly more quickly than large problems (986 ms vs. 1296 ms). This finding replicates the problem-size effect that is

typical in research on mathematical cognition. The effect of operation previously observed in other studies (Campbell, 1997, 1999; LeFevre & Morris, 1999) was replicated as well. Participants who solved multiplication in the second block were faster than participants who solved division (1051 ms vs. 1231 ms). There was an operation by problem-size interaction, similar to that observed by LeFevre and Morris. For participants who were solving multiplication, the difference in latencies to small and large problems was significant (922 ms vs. 1179 ms). However, for participants who were solving division, the problem-size effect was larger than that observed for multiplication (1051 ms vs. 1412 ms).

As predicted, participants solved primed problems faster than unprimed problems (1073 ms vs. 1209 ms). The observation of a significant effect of priming for both errors and latencies supports the hypothesis that solving problems in the first block influenced performance on either identical or related problems in the second block. The observation of a significant effect of priming validates the use of this paradigm as a methodology that is sensitive to problem-specific priming. The effect of priming was modified by both problem-size and experimental type, as indicated by significant 2-way interactions.

The problem-size by priming interaction reflected the effect of priming was significant for small problems (939 ms vs. 1033 ms for primed and unprimed, respectively). However, the effect of priming was even greater for large problems (1207 ms vs. 1385 ms for large primed and unprimed, respectively). Thus, the participants' solution of both large and small problems benefited from priming, however, large problems benefit more. The effect of priming also differed for each experiment type. The effect of priming was significant in the cross-operation conditions (1142 ms vs. 1227

ms, for primed and unprimed, respectively). However, the effect of priming was even greater in the within-operation conditions (1005 ms vs. 1191 ms, for primed and unprimed respectively).

There was a near significant trend for a problem-size by operation by priming effect ($p = .06$). The mean latencies for this 3-way interaction are described in Table 5. The effect of priming was significant for both small and large problems for both participants who were solving division and for those solving multiplication. Overall, large problems benefited more from priming than small problem. However, there was greater priming for participants who were solving large division problems than there was for participants who were solving large multiplication problems. The observation of greater priming for participants who solved division problems in the second block did not interact with experiment type. This suggests the pattern for greater priming when participants solved division than when participants solved multiplication was similar for the within- and cross-operation conditions.

Initially, the greater priming for large problems in the multiplication-to-division conditions relative to large problems in the division-to-multiplication condition could be interpreted as asymmetrical priming across operations. However, the greater priming in conditions where participants were solving division may have occurred because division problems are typically solved more slowly than multiplication problems and there is more opportunity for priming to benefit performance. Multiplication problems are practiced more than division problems and as a result, multiplication latencies may be closer to a minimum than division latencies, allowing more opportunity for priming to benefit division problems than there is for priming to benefit multiplication problems. Consistent

with this interpretation, there was a trend for more priming for large problems in the division-to-division condition than for large problems in the multiplication-to-multiplication condition. Thus, attributing the greater priming of large problems in multiplication-to-division than in division-to-multiplication solely to asymmetrical priming across operations is not supported.

Summary

In this experiment, problem-specific priming was observed both within and across operations. However, within-operation priming was significantly greater than cross-operation priming. As well, significantly more priming occurred in conditions where participants were solving division problems than in conditions where they were solving multiplication problems. The greater priming for division problems, even in within-operation conditions, can account for the observation that there was significantly more priming in the multiplication-to-division condition than in the division-to-multiplication condition. More importantly, cross-operation priming was significant in both the division-to-multiplication and multiplication-to-division conditions. Hence, asymmetrical priming (i.e., significant priming in one direction but not the other) did not occur in the current experiment.

Discussion

The observation of more priming in the within-operation conditions than in the cross-operation conditions suggests that solving a problem in a first block of problems makes it easier to solve the related problem in a subsequent block. However, though solving a related problem in a previous block benefits performance, the benefit is not as great as that of solving the same problem a second time in the within-operation

conditions. The identical elements model offers an explanation for greater within-operation priming than cross-operation priming (Rickard & Bourne, 1997; Rickard et al., 1994).

Rickard et al. proposed that problem-specific priming occurs at two stages: physical encoding and calculation. When a problem is presented for the second time (i.e., in the within-operation condition), physical encoding of the problem is benefited because of its physical similarity to the previous presentation of the problem. Priming occurs at the calculation stage when a problem accesses a common representation that has been recently activated to solve the same (within-operation) or related (between-operation) problem. Thus, a problem that was physically identical and shared a common representation at the calculation stage with a recently solved problem (e.g., 6×7 and 6×7) would be more primed than a problem that was physically different but shared a common representation (e.g., 6×7 and $42 \div 6$). Priming at encoding and calculation stages would explain why within-operation priming is greater than cross-operation priming. Identical problems benefit from priming at both the encoding and calculation stages. Related problems benefit from only from priming at the calculation stage. However, the notion that related problems can benefit from priming at any stage of processing is not consistent with the identical elements model.

Rickard et al. (1996, Rickard & Bourne, 1994) proposed that all the elements (i.e., both operands and the formal operation used to solve the problem) must be in the presented problem in order to access a specific representation. This is the reason the model was termed the identical elements model. Accordingly, division problems do not share enough elements with the related multiplication problem to access the

representation. However, if the cross-operation priming observed in the current experiment is a result of a common representation for division and multiplication problems, it is possible that a division problem could share enough elements to access the related multiplication representation. If the degree of match required among the elements is relaxed, the identical elements model would provide a concise explanation for greater within- than cross-operation priming and account for cross-operation priming.

LeFevre and Morris (1999) and Campbell (1999) observed that the impact of previous exposure to problems in one operation (i.e., division or multiplication) had an asymmetrical impact on performance on subsequent problems in the inverse operation. LeFevre and Morris observed greater priming between division-to-multiplication problems than between multiplication-to-division problems in a blocked design experiment. In contrast, Campbell (1999) observed greater multiplication-to-division priming than division-to-multiplication priming in a mixed block, speeded design experiment. In the current experiment, both multiplication and division performance were significantly and equally benefited by previously solving related problems in the other operation. However, both LeFevre and Morris and Campbell noted that although the effect of priming was not symmetrical, there was a trend for cross-operation priming in both directions. It is possible that the modified priming probe paradigm developed for these experiments is a more powerful method to isolate problem-specific priming.

The observation of bi-directional cross-operation priming in the current experiment is consistent with the common representation hypothesis. If solving a division problem involves accessing the representation for a multiplication problem, then the effect of priming should occur regardless of whether the direction of priming is from

multiplication-to-division or from division-to-multiplication. Accordingly, multiplication and division performance benefit equally from previous exposure to related problems in the inverse operation. In all the previous studies, cross-operation priming was asymmetrical.

The lack of multiplication-to-division facilitation in LeFevre and Morris (1999) could be explained by noting that the study was not designed to examine cross-operation priming. Rather, LeFevre and Morris noticed the facilitating effect of solving division problems on performance on a subsequent block of multiplication problems as a significant order effect in the counterbalancing. Also, LeFevre and Morris noted that there was evidence of a trend for multiplication-to-division priming, but the trend missed being significant. However, the explanation that the study was not specifically designed to address problem-specific priming cannot be used to justify the observation of asymmetrical priming in Campbell's (1997, 1999) experiments.

Campbell (1997, 1999) suggested that independent representations exist for division and multiplication. Asymmetrical priming between multiplication and division was attributed to participants exploiting the conceptual relation between multiplication and division or available information in memory (i.e., post-response checking, Campbell, 1999 and cognitive convenience, Campbell, 1997). These explanations for asymmetry between multiplication and division priming in two different experiments could be adapted to explain the presence of symmetrical priming in the current experiment. For example, in the current experiment, participants were not provided with feedback and the inter-trial interval was relatively long. Therefore, participants could have engaged in post-response checking which would promote the observation of division-to-

multiplication priming. As well, participants could have also been exploiting the availability of the more reliable multiplication representation to solve the related division problem rather than accessing their less-reliable division representation. Thus, the observation of multiplication-to-division priming could have resulted from cognitive convenience. However, the notion that participants were exploiting multiplication information in a blocked operation design is problematic.

In the present experiment, problems were blocked. If cognitive convenience is the source of multiplication-to-division priming, the convenience associated with the recent activation of the multiplication problem occurred across blocks and over the rest period between the first and second block. Considerably more than 4-6 trials intervened between the presentation of the multiplication problem in the first block and presentation of the related division problem in the second block. Campbell (1999), however, found evidence that the effect of priming lasted only 4-6 trials. Possibly, the longer lasting effect of priming in the present experiment could be attributed to the blocked design, and relative novelty of each problem presented in this experiment.

In Campbell (1997), participants viewed large mixed blocks of multiplication and division problems. Over blocks, the same problems were presented many times (number of trials per problem = 36). In this mixed block design, the benefit of activating a related problem (e.g., 6×8) on solving a subsequent problem (e.g., $48 \div 8$) may be short-lived because the repeated activation of similar problems could have increased interference in the associative network of problems. In contrast, in the current experiment, there were only 28 problems in the first block. Problems were only presented once in the first block and only once in the second block. Thus, the activation of the first presentation of the

related problem may be more cognitively salient than in an experimental design where related and unrelated problems have been repeatedly activated across blocks. As well, interference effects are constant for both the primed and unprimed problems. The possibility that the repeated presentation of the same problem in a single session may increase network interference, suggests that in mixed blocks, cross-operation error priming should be greater in the earlier blocks of problems than in later blocks.

The lack of significant division-to-multiplication priming in Campbell (1999) presents a problem for the common representation hypothesis. Notably, if post-response checking is not the source of the priming and participants access a common representation to solve multiplication and division, then why was division-to-multiplication priming not observed in that experiment? One possible explanation is that in Campbell's experiment, participants may have been prevented from recasting because of the instructions emphasizing the need to respond quickly. On this view, participants reduced the time allotted to solve the problem. If recasting is a necessary step in solving a division problem, the multiplication representation may not have been accessed. The failure to access the common representation would explain the lack of division-to-multiplication facilitation and also the relatively high error rates for division problems observed by Campbell (16%, 1997; 17%, 1999).

Experiment 1 showed that problem-specific priming occurs both between division and multiplication problems and that cross-operation priming is not as great as within-operation priming. However, the experiment was not able to substantiate or refute either the common or independent representation hypotheses. Accordingly, Experiment 2 was

designed to address specific predictions arising from the common and independent representation hypotheses.

Experiment 2

The first goal of this experiment was to replicate bi-directional cross-operation priming observed in Experiment 1 because evidence for cross-operation priming has been inconsistent across studies. Accordingly, the division-to-multiplication and multiplication-to-division conditions were repeated in Experiment 2. The replication conditions also provide a baseline for comparing the influence of other experimental conditions on cross-operation priming. Because problem-specific within-operation priming has been observed in previous experiments (Rickard & Bourne, 1996; Rickard et al., 1994) and because within-operation priming is predicted by both the independent and common representation hypotheses, the within-operation priming conditions were omitted.

In Experiment 2, predictions based on the common representation and independent representation hypotheses were contrasted. The common representation hypothesis is the view that multiplication and division share a common representation in memory and that this representation is based on multiplicative relations. Recasting is the process whereby division problems are put into a multiplicative form in order to better "match" the common representation. Cross-operation priming derives from the repeated activation of the common representation by problems that access that representation during the solution process.

The independent representation hypothesis is the view that division and multiplication are represented independently in memory. However, although division and multiplication problems have independent representations, the association between a division problem and answer in memory is weaker (i.e., less unitized) than the association

between the multiplication problem and answer. Where possible, participants will exploit the stronger representation (i.e., multiplication) either to check that their division answer was correct (i.e., post-response checking), or to solve a division problem where task-related characteristics (i.e., cognitive convenience), such as recently solving the multiplication problem.

Post-response checking and recasting occur at different stages of arithmetic processing. Campbell (1999) proposed that participants access their knowledge of a multiplication fact *after* they have solved the related division problem. Therefore, post-response checking should be influenced by experimental factors that affect the period *after* a response is made. For example, Campbell (1999) concluded that reducing the inter-trial interval (the time between the recording of the spoken answer given by the participants and the presentation of the next problem) prevented post-response checking reduced division-to-multiplication priming. The short interval between trials was not sufficient to allow participants to check their division answers. Similarly, Campbell (1997) proposed that the presence of accuracy feedback after participants responded was the explanation for the absence of division-to-multiplication benefit of training in Rickard et al. (1994) and Rickard and Bourne (1996). There is no need to check the accuracy of a division response if the participant is informed about the accuracy of an answer.

Alternatively, recasting is a process that occurs *before* an answer is accessed in memory. Whether participants mentally reorganize division problems to access a common representation should be influenced by experimental conditions that reduce the time *before* an answer response is recorded. One such manipulation may be to emphasize speeded responses. Participants who solve arithmetic problems where there is an

emphasis on speed generally respond more quickly than when speed and accuracy are equally emphasised. Participants also abandon slower solution procedures in favour of faster "guessing" (Smith-Chant & LeFevre, 2001). The abandonment of a relatively slow recasting procedure under speeded conditions might explain the absence of division-to-multiplication priming in Campbell (1999). Campbell (1999) used instructions that emphasized the need to respond quickly over the need to be accurate. Allowing less time to encode and cognitively process division problems may discourage participants from recasting division problems into a form that would access the appropriate multiplication representation to solve the problem. If participants fail to access the multiplication representation, they should make more errors and show less priming when they subsequently solve the related multiplication problems.

In summary, post-response checking should be eliminated, or significantly reduced, by reducing the time between trials (the inter-trial interval) or by providing accuracy feedback. Alternatively, increasing the requirements to respond quickly may discourage participants from recasting division problems. To contrast these two hypotheses, participants were placed in one of four conditions during the first block of problems. In the first condition, participants solved the initial set of problems with a relatively *short inter-trial interval*. Potentially, if the time between the participants' response and the next trial is very short, post-response checking may be discouraged or prevented. In the second condition, participants were given *accuracy feedback* immediately after solving a problem during the first block of problems. Both visual and auditory accuracy feedback was provided. After the participants responded and the experimenter entered the response, participants heard a pleasant tone if they responded

correctly, or a distinct and less pleasant tone if they responded incorrectly. They also received visual confirmation of their answer status (i.e., the word "correct!" or the answer was shown if incorrect). Theoretically, if participants are given accuracy feedback during division trials, they should not use post-response checking and therefore division-to-multiplication priming should be greatly reduced relative to the amount of priming observed in the replication condition.

In the third condition, predictions arising from the recasting hypothesis were tested. Participants were given instructions to respond quickly. Theoretically, if participants are *speeded* they will be discouraged from taking the time to recast division problems into a multiplicative form. Smith-Chant and LeFevre (2001) found that speeding participants on multiplication problems reduced the frequency with which they used slow derived-fact procedures (e.g., to solve 6×8 , first solve related problem, 5×8 , and add 8 to get the answer "48") and increased the frequency of guessing. As outlined previously, preventing recasting should result in generally faster latencies and higher error rates relative to the replication conditions. If recasting is more likely for large problems than small problems, speeding participants should affect the solution of large problems more than small problems.

The fourth condition served as a control for the provision of feedback to participants. Immediately after solving a problem, participants in the fourth condition heard a tone identical to the correct response tone used in the feedback condition. Concurrently, the words "NEXT TRIAL" appeared on the screen (to parallel the visual feedback given to participants in the feedback condition). However, the tone in this control condition did not indicate the status of the answer and no accuracy feedback was

given to participants. The inclusion of filler information during the inter-trial interval addressed the possibility that intervening, but unrelated stimuli present during the inter-trial interval could have an impact on cross-operation facilitation. Because the fourth condition is a control for the feedback condition, there are no theoretically based predictions as to whether or why relatively meaningless information presented briefly between trials should affect either post-response checking or recasting. Accordingly, this condition termed the *filled ITI* condition, should be comparable to the replication condition.

The third goal of this experiment was to contrast factors that influence division-to-multiplication priming with those that influence multiplication-to-division priming. According to the post-response checking hypothesis, providing feedback or reducing the inter-trial interval should not influence multiplication-to-division priming because multiplication-to-division priming occurs when participants take advantage of the recent activation of a multiplication fact to solve a division problem. The convenience of a particular multiplication problem should not be influenced by the provision of feedback or inter-trial interval manipulations. Alternatively, the common representation hypothesis leads to predictions that speeding participants may cause them to abandon slower solution procedures, such as recasting. If recasting is prevented, it is less likely that the presented division problem will successfully activate the related multiplication representation in memory. As a result, division-to-multiplication priming will be reduced.

All of the experimental manipulations in Experiment 2 were manipulations for only the first block of trials. During the second block, participants solved problems under

conditions identical to Experiment 1, regardless of the conditions that were present in the first block. Specifically, during the second block of trials for all conditions, participants were instructed to equally balance speed and accuracy when responding, the inter-trial interval was the same as that used in Experiment 1 and the replication condition, and participants did not receive accuracy feedback. By using a standard experimental format for the second block, the source of performance differences can be isolated to the differences experienced by participants during the first block performance. Further, latency and error differences between the experimental groups when solving the second block of problems cannot be attributable to differences in the experimental format during the second block.

Method

Participants

A total of 160 undergraduate psychology students enrolled in a first year psychology course at Carleton University (69 males and 91 females) participated in the experiment for course credit. Participants volunteered for the study by signing up for an appointment time on the experimental credit notification message board located in the foyer outside of the Psychology lecture hall.

Materials

Facilitation Task. Participants verbally solved arithmetic problems that were presented on a computer display. Stimuli were the same as those used in Experiment 1. In the replication condition, the cross-operation conditions from Experiment 1 were duplicated. However, the other experimental conditions involved modifications to the first block of trials. In the *short ITI* condition, the inter-trial interval was reduced such

that the diamond signalling the presentation for the next trial was initiated immediately after the participant responded. The experimenter entered the participant's response during the interval (between trials) when the diamond was flashing. This procedure reduced the time between trials by over 1,500 ms relative to the other conditions (see Appendix F). No accuracy feedback was provided, instructions equally emphasized responding quickly and accurately, and no information was presented to participants between trials.

In the *feedback* condition, once the experimenter entered the answer response given by the participant, accuracy feedback was provided. If the participant was correct, a pleasant series of tones was sounded and the word "correct!" appeared on the screen. If the participant was wrong, an unpleasant tone was sounded and the correct answer appeared on the screen. For example, if the participant incorrectly answered 6×8 , the unpleasant tone sounded while the correct answer "48" appeared on the screen. The inter-trial interval was not modified from Experiment 1, instructions equally emphasized responding quickly and accurately, and nothing other than feedback concerning accuracy was presented to participants between trials.

In the *speeded* condition, participants were provided with instructions to respond quickly, told that errors were typical with speeded response, and were encouraged to ignore errors (see Appendix G for full transcript of instructions). After the practice and immediately prior to beginning the first block of problems, participants were reminded to respond quickly. The inter-trial interval was not modified from Experiment 1, no accuracy feedback was provided, and nothing was presented to participants between trials.

The *filled ITI* condition was included as a control condition for the feedback condition. After the participant responded with the answer, a tone (corresponding to a correct tone in the feedback condition) was sounded and the words “Next trial” appeared on the screen. Accordingly, the inter-trial interval was not modified from Experiment 1, instructions equally emphasized responding quickly and accurately, and no feedback concerning accuracy was presented to participants between trials.

Apparatus. Same as Experiment 1.

Arithmetic Fluency Test. As in Experiment 1, participants completed the addition and the subtraction-multiplication sub-tests of the French Kit described in Experiment 1 as a measure of arithmetic fluency (French et al., 1963).

Questionnaire of non-retrieval strategy use. Participants completed a slightly modified version of the questionnaire of non-retrieval strategy use used in Experiment 1. The questionnaire was modified after analyses examining questions from this measure indicated that some questions that did not yield theoretically useful information and were omitted (LeFevre et al., in press). Total time to complete the survey was approximately 10 minutes.

Procedure

Participants were tested in a quiet room with an experimenter present. Participants were told that simple multiplication or division problems would appear on the computer screen and they were instructed to answer the problem by speaking loudly and clearly into the microphone. Participants were instructed to answer the mathematical problems according to the instructions on the screen. The instructions provided to participants outlined the speed/accuracy requirements as specified by the experimental condition to

which they had been randomly assigned. A short practice was provided with the presentation format consistent with the experimental condition (replication, short ITI, feedback, speeded, filled ITI). The experimental session lasted 45 minutes.

As in Experiment 1, before the experimental trials began, participants solved six practice problems consisting either of all multiplication problems, all division problems, or a mixture of problems in both operations corresponding to the experimental condition to which the participant was assigned. However, the practice session replicated the conditions of the first block experimental trials to which the participant had been assigned (i.e., replication, short ITI, feedback, speeded, filled ITI). Once the practice problems were solved, the experiment began. In between the first and second blocks, the participants were given the opportunity to take a break. Prior to beginning the second block, participants completed a short practice in which the practice problems were presented using the experimental format in Experiment 1. After solving the two blocks of problems, participants completed the pencil-and-paper arithmetic fluency task and the Questionnaire of Non-retrieval Strategy Use.

Results

The median reaction times for the small and large problems in the experimental trials were computed for each of the problem sets (block 1 trials, block 2 primed and unprimed trials). Trials were included in the calculation if the response was valid and correct. Trials were considered invalid if there was a failure of the voice key, a non-answer vocalization by the participant (e.g., um, er, pfff), a failure of the participant to respond with the 15-second time limit allowed for responses, or if the reaction time was less than 500 ms. A trial also was scored as invalid if the participant started to say one

answer, but changed their answer to another number before they finished stating the first (e.g., stating “forty.... fifty-six”). In total 865 trials (6%), were coded as invalid.

Prior to analyzing the results to determine if problem-specific priming occurred in each condition, 15 participants were excluded from further analyses. One was excluded because of extremely slow latencies on the first block trials ($+3$ standard deviations from overall mean reaction time). Another 11 participants were excluded because they validly and correctly answered fewer than half of the large problems ($n = 14$) in the first block. A further three participants were omitted because they correctly answered fewer than half of the large problems ($n = 14$) in the unprimed conditions of the second block. Omitted participants were generally evenly dispersed among the experimental conditions (see Appendix J or Table 7. The demographic characteristics of the participants in each of the experimental conditions (e.g., age, location of primary and secondary education, first language) are described in Appendix H for the division-to-multiplication groups and in Appendix I for the multiplication-to-division groups. There were no significant differences between the individuals in each of the conditions on any of the demographic variables ($ps > .10$).

First Block Trials

The percentage of errors and median latencies for the first block trials were analyzed in separate 2 (problem size: small, large) by 2 (operation: multiplication, division) by 5 (experiment type: replication, short ITI, feedback, speeded, filled ITI) ANOVAs with repeated measures on the first factor. Unless otherwise noted, all assumptions underlying ANOVAs (homogeneity of variance, sphericity) were met and the univariate tests yielded identical results as the multivariate tests. A summary of the

ANOVA analysis can be found in Table 6. A summary of the mean latencies and errors can be found in Appendix J. A summary of the error latencies for each condition can be found in Appendix K.

Percentage of errors. Both the problem-size and operation effects typically observed in arithmetic production tasks were observed among the pattern of errors in the first block. Participants made significantly more errors when solving large problems than small problems (16% vs. 4%). As well, participants who solved division problems made significantly more errors than those who solved multiplication problems (12% vs. 8%). However, participants were not more likely to make errors in a particular experimental condition and experimental condition did not interact with any other variables. The lack of effect of experimental condition on errors was unexpected. Specifically, participants in the speeded conditions were expected to make more errors than participants in the other conditions. The lack of an effect of instructional emphasis on errors might be attributable to the between-groups experimental design. Notably, statistical comparisons involving totally between-group comparisons are less powerful than mixed or within-group comparisons. The combination of a relatively small percentage of trials that result in errors and the lower statistical power may have contributed to the failure to observe significant between-group differences.

Solution latencies. As is typically observed in arithmetic studies, participants solved small problems more quickly than large problems (1057 ms vs. 1447 ms). As well, participants who solved multiplication in the first block were significantly faster than the participants who solved division (1108 ms vs. 1397 ms). However, latencies did

not differ across experimental conditions, nor did the experimental condition the participants were in significantly interact with any of the other variables ($ps > .10$).

The absence of any differences among experimental conditions was not predicted. At the very least, it was expected that participants in the speeded condition should respond faster than participants in the other conditions. The absence of a significant effect of speeded instructions either in latencies and errors could be attributable to the between-group design of the current experiment. For example, in Smith-Chant and LeFevre (2001), the significant effect of speeding participants using instructional emphasis was observed as a within-subject factor. Between-group comparisons are not as powerful as within-group comparisons. If the participants in the speeded conditions were less skilled than participants in the other groups, then the subtle effect of speeding may have been subsumed by the between-group differences in skill. In accord with this possibility, the group means on the measure of arithmetic fluency (i.e., French Kit scores) for both the multiplication-to-division and division-to-multiplication speeded groups are lower than almost all of the other group means. Table 7 describes the mean French Kit scores for each group. Although a one-way ANOVA analysis examining the omnibus difference between groups on the French Kit was not significant, $F(9, 136) = 0.67, p = .75, MSE = 88,553$, the 95% confidence intervals described in Figure 2 indicate that the speeded groups were below the mean on the French Kit. Even a subtle difference in skill between-groups could have confounded the observation of a significant effect of speeded instructions on first block latencies.

Summary. The lack of an observable impact of each experimental condition, particularly the speeded condition, on the percentage of errors made by participants or on

the median latencies may be due to confounding between-group differences combined with a lack of power in the totally between-group comparisons. For example, there are a relatively small number of participants in each group and there were only 28 trials in the first block of problems. Although the small number of participants and trials provides an experimental design powerful enough to identify the effect of problem-size, operation, or priming (see Experiment 1 and following), the number of participants in the current experiment may not provide enough power to identify totally between-group differences, such latency and percentage of error differences. Individual variability in basic arithmetic can be quite dramatic, and finding subtle effects on performance can be difficult as a result. Fortunately, the effect of priming in the second block trials is a within-participant factor measured as latency or error savings for each participant on problems they solved that were related to the first block problems in relation to their performance on problems that are unrelated to the first block problems.

Second Block Trials

The mean latencies for each experimental group on the second block trials are described in Appendix L and the percentage errors are described in Appendix M. Unless otherwise stated, alpha was set $p = .05$ and assumptions underlying the ANOVA (i.e., homogeneity of variance, sphericity, proportional cells) were met. The hypothesis examining the prediction of symmetrical priming and the hypotheses contrasting the influence of short ITI, feedback, and speeded on the effect of priming were tested with specific analyses. All figures are shown with error bars representing the 95% confidence intervals (Loftus & Masson, 1994).

Experiment 2A: Replication of Symmetrical Cross-operation Priming

The goal of Experiment 2A was to determine if the symmetrical priming from division-to-multiplication and multiplication-to-division observed in Experiment 1 was replicated in the current experiment. Accordingly, a 2 (problem size: small, large) by 2 (priming: primed, unprimed) by 2 (block 2 operation: multiplication-to-division, division-to-multiplication) ANOVA was used to comparing the percentage of errors and median latencies. The results of the analyses are summarized in Table 8. The latency savings (i.e., unprimed - primed) for each experimental group and problem size are shown in Figure 3.

Percentage of errors. As expected, the effect of problem size was significant: Participants were significantly less likely to make errors when solving small problems than large problems (5% vs. 13%). Additionally, participants who were solving division problems in the second block were somewhat ($p < .10$) more likely to make an error than those who solved multiplication (11% vs. 7%). Although the effect of priming failed to reach significance, there was a trend for participants to make more errors when solving unprimed problems than they were to primed problems. However, though participants in the division-to-multiplication group replicated the pattern of fewer errors to primed problems that was observed in Experiment 1, the multiplication-to-division group actually made more errors when solving large, primed problems than they did when solving unprimed problems.

Latencies. The latency analysis indicated that participants solved small problems more quickly than large problems (1063 ms vs. 1406 ms), an observation that is consistent with the standard observation of the problem size effect in basic arithmetic. As

well, participants who solved multiplication in the second block were significantly faster than participants who solved division in the second block (1134 ms vs. 1335 ms). The current experiment successfully replicated the priming effect observed in Experiment 1: Participants solved primed problems more quickly than unprimed problems (1195 ms vs. 1274 ms). However, the pattern of cross-operation priming was asymmetrical.

A significant interaction between priming and operation indicated that participants in the division-to-multiplication condition solved primed problems significantly faster than unprimed problems (1069 ms vs. 1200 ms) whereas participants in the multiplication-to-division condition were not significantly faster to solve primed problems than unprimed problems (1321 ms vs. 1348 ms). According to Figure 3, for the multiplication-to-division condition the effect of priming was only significant for large problems and even then, the effect of priming was significantly less than for large problems in the division-to-multiplication conditions. Initially, the failure to replicate bi-directional priming could be because chance factors or an experimental confound account for the appearance of symmetrical priming in Experiment 1. Upon more careful consideration, however, a more appropriate conclusion would be that the multiplication-to-division replication group in the current experiment is exceptional and the failure to replicate may reflect a sampling bias in that condition.

The replication group in the multiplication-to-division condition differs from many of the other experimental groups on the French Kit (see Figure 2). The mean for the replication group multiplication-to-division condition was over 90 whereas the other groups had means ranging from the mid-seventies to the lower-eighties (i.e., 75 to 83). According to the 95% confidence interval, the replication group had a significantly

higher French Kit score than all the other groups. The notion that the replication group in the multiplication-to-division condition may be more uniformly of high skill than the other groups is supported by the latency data.

The mean latency and errors for the groups outlined in Appendix M indicated that the latencies and errors for the replication group in the multiplication-to-division condition were consistently different from the other groups. For example, the latencies and percent errors were consistently either the lowest, or among the lowest of the multiplication-to-division group means. More alarming, the replication group's standard deviation was consistently low relative to the deviation observed among the other groups. In fact, the replication group in the multiplication-to-division condition had standard deviations for the latencies that were often less than half of the standard deviations observed on the same variable for the other groups. The relative low error rate and fast latencies, combined with reduced variability is consistent with the notion that the replication group in the multiplication-to-division condition may have been more uniformly high skilled than the other groups.

The lack of a significant priming effect could be attributed to between-group differences in skill. If high-skilled participants were relatively efficient in their arithmetic performance (i.e., fast and accurate), then they would not benefit as much from priming as low-skilled participant who are slower and less accurate in their performance. On this view, the reduced benefit of priming for high skilled individuals could be attributable to floor effects (i.e., less room for benefit). As well, high skilled participants could experience less of a cross-operation benefit because they are less likely to use a recasting solution procedure to solve a division problem (LeFevre and Morris, 1999), or

to feel as though they need to check their response after answering the problem (Campbell, 1997). If high skill participants were less likely to access the related multiplication representation in memory via recasting or post-response checking, then they would experience less cross-operation priming. Thus, between-group differences in skill would confound the effect of priming in the replication, multiplication-to-division group.

Finally, empirical support for the notion that the multiplication-to-division replication group was atypical is provided by the pattern of performance observed across operations and experimental conditions. In the current experiment, the experimental manipulations are designed to eliminate or reduce division-to-multiplication priming (i.e., recasting during the first block, post-response checking). Thus, between-group variability in the degree of priming is expected among the division-to-multiplication experimental groups. In contrast, variations in the amount of latency savings associated with priming for the multiplication-to-division experimental groups are not predicted by any of the theoretical perspectives. Thus, the observation that there appears to be considerable between-group differences in the multiplication-to-division conditions and in the case of the replication group, negative savings (i.e., a cost for priming) for large problems is especially surprising. The observation of more between-group variability in the multiplication-to-division conditions than in the division-to-multiplication conditions could imply that multiplication-to-division priming is sensitive to the experimental factors and/or that the replication group in the multiplication-to-division condition is atypical relative to the other groups.

To explore if between-group differences in skill could explain why participants in the replication group in the multiplication-to-division condition did not benefit significantly from priming, a hierarchical multiple-regression analysis was conducted to see the relative contribution of specific variables in predicting the latencies for primed problems. Separate analyses were conducted for the multiplication-to-division and division-to-multiplication groups. In this regression, the latencies to unprimed problems were always entered in the first step. By accounting for the variance in primed problem latencies predicted by the unprimed latencies in the first step, the residual latencies (or difference between the two) corresponded to the effect of priming. The variables included in the regression analysis were variables that were significantly correlated with the latencies for primed and unprimed problems among all the demographic, French Kit, and questionnaire information that was gathered during the experiment. The variables included were the French Kit score, and items from the questionnaire on non-retrieval procedure use. Specifically, items indicating the participants' use of division solution strategies, skill at basic mathematics, math anxiety, math avoidance, and perceived emphasis of fact learning in elementary school. These variables were selected because they were significantly correlated with either latencies to large primed/unprimed problems or the French Kit score. Tables 9 and 10 contain the correlation matrix for the variables of interest. The results of the regression analyses are summarized in Table 11.

For the multiplication-to-division group, the only variable that predicted priming was the French Kit score. The unstandardized beta coefficient for the French Kit variable was -2.4, indicating that high skilled participants showed less priming. For example, someone with a French kit score of 90 would show 24 ms less priming than someone

with a French kit score of 80. In contrast, among the division-to-multiplication groups, only math avoidance was a significant predictor of latencies for large primed problems after the latencies for large unprimed problems had been entered. This suggests that experience with math may be a factor in the degree to which participants show priming from division problems in the first block to related multiplication problems in the second block. The unstandardized beta coefficients indicated that in the division-to-multiplication condition, higher ratings of math avoidance were associated with an increased difference between primed and unprimed problems.

The observation that skill was a significant predictor of a positive priming effect in the multiplication-to-division condition supports the notion that multiplication-to-division priming is sensitive to between-group differences in skill. As a result, the observation that the multiplication-to-division replication group may be more uniformly high skilled relative to the other groups could account for the lack of a significant effect of priming in that condition. Alternatively, math avoidance, the factor associated with more priming in the division-to-multiplication condition, was more evenly distributed among all the groups (see Table 12). Thus, between-group differences in math avoidance are not confounding observations of division-to-multiplication priming in the current experiment.

Unfortunately, the participants in the multiplication-to-division replication condition are not equivalent to the other experimental groups in arithmetic skill is a serious concern for this experiment. All of the experimental hypotheses are based on the comparisons of the experimental groups to the replication condition. If the replication conditions are inappropriate baselines for the comparisons because of the skill confound

then the relative reduction of priming in the experimental conditions relative to the "standard" condition cannot be determined.

The experiment included the filled ITI condition as a control for the feedback condition. None of the changes to the experimental methodology of the filled ITI condition were linked theoretically with either the recasting or post-response checking hypotheses. Because it served as a control condition, the only difference between the filled ITI group and the replication group was that the filled ITI condition featured the presentation of a tone and the introduction "next trial" prior to the presentation of the focal point (i.e., flashing diamond). There was no reason to expect that including meaningless information between trials should affect post-response checking or recasting. The filled ITI and replication groups were equivalent in the length of the ITI, instructions given to participants, and absence of accuracy feedback. As a result, if the filled ITI group in the division-to-multiplication condition were not significantly different than the replication group in the division-to-multiplication condition, then the decision to replace or combine the replication groups with the filled ITI groups in both the division-to-multiplication and the problematic multiplication-to-division conditions would be validated.

Comparison of the Division-to-Multiplication Replication and Filled ITI Groups

To determine if the replication and filled ITI groups in the division-to-multiplication condition were equivalent on latencies and errors, a 2 (problem size: small, large) by 2 (priming: primed, unprimed) by 2 (ITI: filled, unfilled) ANOVA was conducted with repeated measures on the first two factors. The results of the ANOVA analyses are summarised in Table 13.

The analyses indicated that participants in the replication and filled ITI groups were faster and less likely to make errors when solving small problems than they were when solving large problems (970 vs. 1306 ms and 2% vs. 13%). As well, participants were significantly faster and made fewer errors when solving primed problems than they when solving unprimed problems (1081 vs. 1194 ms and 6% vs. 9%, respectively).

There were no differences between the replication and filled ITI groups. The observation that in the multiplication-to-division condition the filled ITI group was not significantly different from the replication group in either latencies or errors reflects that the filled ITI group's performance was equivalent to the replication group's performance.

Summary

The confirmation that the replication and filled ITI groups are equivalent in the division-to-multiplication condition is an important step to deciding how best to create a baseline comparison group to replace the atypical multiplication-to-division replication group. One option could be to simply replace the atypical group (i.e., the replication group in the multiplication-to-division condition) with the corresponding filled ITI group. Alternatively, the filled ITI groups could be used to replace both replication groups (i.e., in both the division-to-multiplication and multiplication-to-division directions), in the interest of symmetry. Finally, the filled ITI and replication groups could be combined into a new, larger group that would serve as the new baseline comparison group. This option is appropriate, provided that the assumptions underlying repeated measures ANOVA are not violated (especially homogeneity of variance). By combining the groups, the restricted variability of the replication group would be addressed by the addition of more participants and without the loss of data points associated with omitting

a group. As well, the power of the ANOVA would be improved due to a larger value in the degrees of freedom for the error term.

To be cautious, all of the preceding analyses were attempted and fortunately, the analyses yielded similar results. The analyses using the combined replication and filled ITI groups are described here. This option was selected because it resulted in more powerful analyses (i.e., higher degrees of freedom), particularly when separate analyses were calculated for each hypothesis (i.e., recasting and post-response checking).

Although the increase in power associated with combining the replication and filled ITI groups did not affect comparisons among the division-to-multiplication groups, it did yield one difference that was not significant in the other options for comparisons involving the multiplication-to-division group. In the subsequent description of results, any comparisons that were significant only for the more powerful test (but not for the other, less-powerful options) are noted. The combined filled ITI and replication groups were re-named as the "standard" groups. The mean latencies and errors for the division-to-multiplication and multiplication-to-division standard groups are included in Appendices K and L.

Comparison of division-to-multiplication and multiplication-to-division standard groups

To determine if symmetrical priming occurred from division-to-multiplication and multiplication-to-division, the two standard groups were compared in a 2 (problem-size: small, large) by 2 (priming: primed, unprimed) by 2 (operation: multiplication-to-division, division-to-multiplication) ANOVA, with repeated measures on the first two factors was calculated for latencies and errors. Table 14 provides a summary of the

results and Figure 4 describes the latency savings for each of the groups for large and small problems, with the 95% confidence intervals.

Percentage of errors. As is typically observed, participants made significantly more errors when solving large problems than when they solved small problems (4% vs. 14%). The effect of priming was also significant. Participants made significantly fewer errors when solving primed problems than when solving unprimed problems (8% vs. 10%). There was a non-significant trend ($p = .07$) for participants to make more errors when solving division than when they solved multiplication problems, as was observed in Experiment 1 (8% vs. 10%). However, the pattern of errors for the division-to-multiplication and multiplication-to-division groups did not interact with the effect of priming, supporting the notion that the effect of priming was symmetrical for errors.

Solution latencies. Consistent with the expected problem-size effect, participants solved small problems faster than large problems (1062 ms vs. 1404 ms). Also, the typical operation effect was observed. Participants solving division in the second block were significantly slower than those solving multiplication (1329 ms vs. 1138 ms). As predicted, the effect of priming was significant (1184 ms vs. 1283 ms for primed and unprimed, respectively). As observed in Experiment 1, the effect of priming interacted with problem-size. Although the difference between primed and unprimed small problems was significant (1038 ms vs. 1086 ms), the difference between primed and unprimed large problems was significantly greater (1329 ms vs. 1480 ms).

The effect of priming did not differ for the division-to-multiplication and multiplication-to-division standard groups. The effect of priming was significant for large problems in both conditions. Significant priming was observed for small problems

in the division-to-multiplication condition, but not the multiplication-to-division condition, although there was a trend for participants to take longer to solve unprimed problems. The lack of a significant difference between the standard groups replicated the observation of bi-directional priming observed in Experiment 1.

Experiment 2B: Comparison of the Short ITI and Standard Conditions

The latencies and percentage of errors for the standard and short ITI groups were compared in 2 (problem size: small, large) by 2 (priming: primed, unprimed) by 2 (operation: multiplication-to-division, division-to-multiplication) by 2 (ITI: short, long) ANOVA, with repeated measures on the first two factors. This comparison is a test of the post-response checking hypothesis. According to Campbell (1999), reducing the inter-trial interval between the participant's response and the initiation of the next trial (i.e., the focal diamond) should prevent post-response checking because participants have less time between the trials to reference the related multiplication problem and verify the answer. Accordingly, division-to-multiplication priming should either be eliminated or dramatically reduced as observed in a reduction in the difference between primed and unprimed latencies and errors. Table 15 contains the results of the ANOVA analyses and Figure 5 describes the latency savings (i.e., unprimed - primed).

Percentage of errors. The analysis of the percentage of errors indicated that participants were more likely to make errors when solving large problems than when they solved small problems (15% vs. 4%). As well, there was a trend for participants to make more errors when solving unprimed problems than when they solved primed problems (10% vs. 9%, $p = .09$). However, there were no significant differences between the short ITI and standard groups in either the multiplication-to-division or division-to-multiplication conditions. The pattern of errors is not consistent with the prediction that reducing the ITI will reduce the effect of priming for errors.

Solution latencies. Participants solved small problems more quickly than large problems (1040 ms vs. 1426 ms), replicating the typical problem size effect. Participants

solved primed problems significantly faster than they solved unprimed problems (1173 ms vs. 1292 ms). The effect of priming significantly interacted with problem size: Participants showed greater priming on large than on small problems (187 vs. 51 ms).

The four-way interaction (see Figure 5) reflected that in all four experimental groups, participants significantly benefited from priming when solving large problems. However, participants in the short ITI, multiplication-to-division group benefited from priming on large problems significantly more than the other groups. In contrast, the benefit of priming on small problems was not significant only for the short ITI group in the division-to-multiplication condition. The observation that priming was not significant for small problems with the aforementioned group may be a reflection of power, as the trend was for primed problems to be solved faster than unprimed problems.

Summary. The short ITI condition was not associated with less division-to-multiplication priming. Thus, the prediction that a shorter ITI would discourage participants from post-response checking and thereby reduce the observation of division-to-multiplication priming was not supported in the current study. There are two potential explanations for the observation that priming was not significantly reduced. The first possibility is that post-response checking is not the source of division-to-multiplication priming. Campbell reduced the ITI in his experiment to eliminate division-to-multiplication priming. However, a shorter ITI was not associated with reduced division-to-multiplication priming in the current experiment. Possibly, the lack of an effect of division-to-multiplication priming may have been attributable to other aspects of his experiment, such as the greater emphasis on speed than accuracy in the instructions given to participants. The possibility that speeding, and not reduced ITI, was the source of

reduced division-to-multiplication priming will be explored later in the comparisons between speeded conditions and standard conditions.

Second, it is possible that the manipulation to reduce the ITI in the current experiment did not sufficiently reduce the time between the answer statement and the beginning of the next trial. If the ITI was not sufficiently short, post-response checking may not have been prevented. However, the mean ITI for the short ITI conditions was more than 1,500 ms shorter than the ITI in the other conditions (see Appendix F). Even if the reduced ITI in the current experiment was insufficient to totally eliminate post-response checking, there should have at least been a reduction. In this comparison, there wasn't even a trend for reduced division-to-multiplication priming for the short ITI group. It may be that reducing the ITI is not the optimal way to reduce post-response checking because some participants may be able to check their responses even with a relatively short ITI. Alternatively, providing people with accuracy feedback consistently and uniformly removes the need to check a stated response.

Experiment 2C: Comparison of the Feedback and Standard Conditions

According to the post-response checking hypothesis, providing accuracy feedback on every trial should eliminate the need for participants to check their answers after responding. Accordingly, division-to-multiplication priming should be significantly reduced or eliminated with feedback whereas multiplication-to-division priming should not be influenced. The latencies and percentage of errors for the standard and feedback groups were compared in a 2 (problem size: small, large) by 2 (priming: primed, unprimed) by 2 (operation: division-to-multiplication, multiplication-to-division) by 2 (feedback type: none, feedback) ANOVA with repeated measures on the first two factors. Table 16 contains the results of the ANOVA analyses and Figure 6 describes the latency savings for the groups.

Percentage of errors. The analysis of the errors indicated that participants made significantly more errors when they solved small problems than when they solved large problems (15% vs. 4%). As well, there was a trend for participants to make more errors when they solved unprimed problems than when they solved primed problems, although the trend failed to reach significance (10% vs. 9%, $p = .15$). However, there were no significant differences in errors between feedback groups. Hence, the prediction that providing participants with feedback would specifically eliminate division-to-multiplication priming was not supported by the analysis of errors.

Solution latencies. As is always expected, participants were significantly faster when solving small problems than they were when solving large problems (1085 ms vs. 1462 ms). Consistent with previous comparisons, participants who solved multiplication problems in the second block were significantly faster than participants who solved

division problems in the second block (1195 ms vs. 1352 ms). As well, participants were significantly faster when solving primed problems than they were when solving unprimed problems (1229 ms vs. 1318 ms). The effects of priming and problem size interacted such that for small problems, participants were significant faster when solving primed problems than they were to solve unprimed problems (1064 ms vs. 1107 ms). However, for large problems the difference between primed and unprimed problems was significantly greater (1395 ms vs. 1528 ms).

There was qualified support for the prediction that accuracy feedback would result in less priming for the division-to-multiplication condition. However, as seen in Figure 6, the 95% confidence intervals indicated that the effect of priming for each feedback group did not significantly differ from the corresponding standard group in that condition (division-to-multiplication and multiplication-to-division). Thus, overall participants in the feedback condition showed less priming. However, the reduction in priming was not specific to the division-to-multiplication condition as predicted by the post-response checking hypothesis.

Summary. If feedback significantly reduces post-response checking in both the division-to-multiplication and multiplication-to-division conditions, then post-response checking could not occur in manner proposed by Campbell (1999). Campbell (1999) suggested that post-response checking occurred because the strength of association between a division problem and the correct answer are relatively weak in memory. In order to determine if their stated division answer was correct, participants reference the related multiplication problem to check their answer. However, Campbell specified that preventing post-response checking would not impact multiplication-to-division priming

because participants would not be inclined to check the accuracy of their multiplication answer by accessing the weaker division representation. Rather, multiplication-to-division priming results from cognitive convenience factors, which were not manipulated in the current experiment. Assuming that providing participants with accuracy feedback eliminates post-response checking, the observation that both division-to-multiplication and multiplication-to-division priming was reduced suggests that post-response checking contributes to cross-operation priming regardless of direction.

Experiment 2D: Comparison of the Speeded and Standard Groups

According to the proposals outlined in the introduction to Experiment 2, recasting is the process by which participants reorganize a division problem to better "match" the internal (i.e., multiplicative) representation of the problem. If instructing participants to respond quickly during the first block of problems encourages them to abandon the relatively slow recasting procedure on some or most trials, then these participants should show less of an effect of priming when they subsequently solve multiplication problems. Thus, participants in the division-to-multiplication speeded condition should show less priming than participants in the multiplication-to-division speeded condition and less priming than participants in both the standard conditions. To test this hypothesis, the latencies and percentage of errors for the standard and speeded groups were compared in a 2 (problem size: small, large) by 2 (priming: primed, unprimed) by 2 (operation: division-to-multiplication, multiplication-to-division) by 2 (instruction type: balanced, speeded) ANOVA with repeated measures on the first two factors. Table 17 contains the results of the ANOVA analyses and Figure 7 describes the latency savings for each group.

Percentage of errors. As is typically observed, participants were significantly more likely to make errors when solving large problems than when solving small problems (14% vs. 4%). Although the main effect of priming was not significant, there was a significant interaction between priming and experiment type. Participants in the standard conditions made significantly fewer errors when solving primed problems than when solving unprimed problems (8% vs. 10%). Participants who solved the first block of problems with speeded instructions did not show an effect of priming in the errors

(10% vs. 9% for primed and unprimed, respectively). The significant 4-way interaction reflected that participants in the division-to-multiplication speeded condition were actually significantly more likely to make errors when solving large *primed* problems than when they solved large unprimed problems. The observation that participants were more likely to make errors on large primed problems is the opposite of the effects of priming observed in all other experiments and suggests that speeded instructions eliminated the effect of priming in errors. The pattern of priming for latencies supported the observation that participants in the speeded group of the division-to-multiplication condition did not benefit from priming.

Solution latencies. As expected, participants were faster when solving small problems than they were when solving large problems (1127 ms vs. 1570 ms). As well, participants were also significantly faster when solving primed problems than they were when solving unprimed problems (1300 ms vs. 1397 ms). As in earlier analyses, the effect of priming varied with problem size. Participants were significantly faster to solve small primed problems than unprimed problems (1106 ms vs. 1148 ms), but the difference between large primed and unprimed problems was significantly larger (1493 ms vs. 1646 ms). There was also a significant difference between the instructional groups. Participants in the standard groups were significantly faster to solve problems than participants in the speeded groups (1233 ms. vs. 1463 ms). The observation that participants in the standard group were faster to solve problems may have been a result of the reduced benefit of priming for the speeded group in the division-to-multiplication condition.

According to the 95% confidence intervals shown on Figure 7, there was significantly less priming for large problems in the speeded group relative to the standard group in the division-to-multiplication condition. Further, small problems were not significantly primed for the speeded group in the division-to-multiplication condition. The reduction of priming in the speeded group was limited to the division-to-multiplication condition. Thus, speeded instructions are associated with a significant reduction in division-to-multiplication priming as predicted.

Summary. The observation that participants in the speeded group in the division-to-multiplication condition showed less priming for large problems than participants in the standard group is consistent with the predictions based on the recasting hypothesis. Speeded instructions in the initial block may encourage participants to abandon a recasting solution process when solving the division problems. By failing to recast the division problem, these participants may not have accessed the appropriate multiplication representation and as a result, showed less of a latency benefit when solving the related multiplication problem in the second block. The observation of a significant priming effect in the speeded multiplication-to-division condition is consistent with the hypothesis that recasting during the second block of division problems is not affected by speeding the first block of multiplication problems.

General Discussion

The results of both experiments support the notion that for most people, division and multiplication knowledge is stored in a common representation in memory and that recasting is the process by which people reorganise a division problem to better access the related multiplication information. Experiment 1, problem-specific priming in division and multiplication occurred both within and across operations. Although cross-operation priming was not as great as the priming that occurs within operations, the effect of priming was significant both from multiplication-to-division and from division-to-multiplication. In Experiment 2, having participants solve division problems under instructions to respond quickly significantly reduced division-to-multiplication priming. The observation of reduced division-to-multiplication priming is consistent with the notion that speeding participants prevents recasting, a process by which participants re-organize a division problem to better access a common representation based on multiplicative relations. However, manipulations designed to reduce post-response checking (i.e., providing feedback or reducing the ITI), did not specifically reduce division-to-multiplication priming. As briefly outlined in the discussion for Experiment 1, the identical elements model could be extended to address the pattern of cross-operation priming observed in the current experiments.

Rickard et al. (1994; Rickard & Bourne, 1996) proposed that problem-specific priming could occur at two stages of processing; encoding and calculation. According to the model, each of these stages is independent of the other. Priming at the encoding stage occurs when information to be processed is perceptually similar or identical to previously encoded information. When participants solve the same problem twice (e.g., within-

operation condition), there is a benefit at the encoding stage because the physical features of the problem are the same (e.g., 3×4 and 3×4). This encoding benefit does not occur with related problems (e.g., between-operation condition) because related problems do not share many physical features. For example, only the “4” is shared in the related problems 3×4 and $12 \div 4$.

Priming at the calculation stage occurs when information to be processed shares cognitive features with previously encoded information. In the identical elements model (Rickard et al., 1994; Rickard & Bourne, 1996), the internal representation of a particular arithmetic fact consists of the two operands and the formal operation required to solve the problem (e.g., 6, 8, multiplication). This representation is then associated with an answer in memory (i.e., 48). To explain why the benefit of training to a problem in one operation does not transfer to a benefit for related problems or to the inverse form of division problems (e.g., 48, 6, division; 48, 8, division), Rickard et al. (1994; Rickard & Bourne, 1996) proposed that related problems are stored independently (e.g., “6, 8, multiplication”; “48, 6, division”). They concluded that all of the components of the representation must be present in order for problems to access the same representation. However, if the internal representation were altered to include the two operands, the multiplication operator, and the answer (e.g., 6, 8, multiplication, and answer 48; see Campbell, 1995), then all the elements to solve both the division and multiplication forms of a particular problem would be present in the representation.

If the internal representation of a problem consisted of all the elements required to solve both the multiplication and division form of the problem, then recasting could be the process by which a division problem was reorganized to better match the internal

representation (LeFevre & Morris, 1999). For example, by recasting the division problem (e.g., $48 \div 6$) into a multiplication format ($6 \times _ = 48$), the division problem is organized into the mental format that more closely resembles the multiplicative format of the representation (e.g., 6, 8, multiplication, and answer 48). The common representation is based on multiplicative relations because multiplication problems are presented earlier and practiced extensively compared to division. To solve the problems 6×8 or $48 \div 6$, the task is to identify the missing element of the arithmetic fact. For example, in the multiplication problem, the product "48" must be identified. In a division problem $48 \div 6$, it is the operand "8" that must be identified.

Modifying the identical elements model

Recasting allows the elements of the division problem to more closely match the elements in the multiplication representation. For example, $48 \div 6$ shares only the "48" and "6" with the multiplication representation "6, 8, multiplication, answer 48". By recasting the problem to " $6 \times _ = 48$ " the problem now shares the first operand, the operation, and the identification that 48 is the answer with the multiplication representation. However, according to the identical elements model (Rickard et al., 1996, Rickard & Bourne, 1994) both operands and the formal operation must be present in a problem in order to activate an internal representation. If the identical elements model was modified to allow a presented problem to activate an internal representation based on the number of "matching" elements shared by the problem and the representation, then the model could be reconciled with the common representation hypothesis.

To explain greater priming for smaller problems than larger problems, LeFevre and Morris (1999) suggested that participants have independent representations for small

division and multiplication problems and access common representations for large problems. However, in the present experiment, cross-operation priming was consistently observed for both small and large problems, which supports the notion that even small division problems may lack independent representations in long-term memory. However, LeFevre and Morris also noted that participants reported less use of recasting when solving small division problems. One possible explanation to mediate the observation of latency priming for small division problems in the current experiment, with a reduction of reported recasting on small problems observed by LeFevre and Morris, could be that for many individuals the ability to recast small problems is well learned and obligatory. Thus, they are not consciously aware of the recasting process. Another explanation for the reduction of reported recasting for small division problems could be that the representations for small problems are so well learned and the elements of the representation are so strongly associated together that reorganization via recasting is not necessary.

Recasting reorganizes a division problem to better match the internal (multiplicative) representation in the matching elements model. However, if the association between the elements in the representation were very strong, then presentation of two of the elements in the multiplication representation (i.e., the answer "6" and an operand 2) in the division problem ($6 \div 2$) could be sufficient to correctly activate the representation for 2×3 . If recasting can be bypassed for very well learned problems, then one would predict that the bypassing process could result in an increased possibility that the wrong multiplication node would be activated when more than one multiplication representation shares elements with the division problem. For example, the division problem $6 \div 2$ would not

only activate the multiplication representation [2, 3, multiplication, and answer 6] but it might also activate the multiplication representation [2, 6, multiplication, and answer 12]. Thus, a common error to $6 \div 2$ would be 12. The activation of the incorrect representation would be more common for problems where the first division operand was less than 10 and therefore more likely to match a potential operand in basic multiplication.

The role of post-response checking

As proposed in Experiment 2, the possibility that post-response checking occurs after both multiplication and division problems are solved is supported by anecdotal observations made during these and previous studies. Participants in multiplication production tasks often realize that they have made an error after (or even while) they respond. The realization that an error has been made after the response occurs suggests that participants activated the correct response at some point after the incorrect answer was selected as the answer. The activation of the correct multiplication representation after the response has been made could occur either by continuing the solution process that generated the incorrect answer, using an answer verification process to check the accuracy of the response, or by implementing a new solution process that generated a new candidate answer.

The notion that participants would generate a new candidate answer by implementing a new solution process seems unlikely because it would be slow. Participants may have checked their answer by verifying the truth of the multiplication fact they created by their response. If the participant answered "56" to the presented problem " 6×8 ", they could realize their stated answer was incorrect by verifying " $6 \times 8 =$

56" is false. However, Campbell and Tarling's (1996) finding that answer verification is not associated with error priming weakens the idea that post-response checking involves an answer verification task. Alternatively, if the process of activating an arithmetic answer problem continued even after a candidate answer was selected and stated as the answer, it is possible that the solution procedure could continue to activate the correct answer after the premature answer was stated. The continued activation of a multiplication representation occurs even after a candidate answer is selected and, as supported in Experiment 2, is only circumvented by providing participants with accuracy feedback. Circumventing the continuation of the answer activation process would only eliminate unitization of the elements of the representation associated with post-response checking process and not the unitization associated with the initial activation of the candidate answer.

The possibility that post-response checking is a process occurring during both division and multiplication production tasks is consistent with a common representation hypothesis and is not consistent with the notion that division problems are represented independently in memory. In the current experiments, both division-to-multiplication and multiplication-to-division priming occurred. If basic division and multiplication problems were represented independently in memory, then the continuation of the activation of a specific answer in one operation would not benefit subsequent activation of a problem that did not share that representation. The notion that post-response checking is a source of priming in both multiplication and division predicts that eliminating the process, such as when participants are provided with feedback, should reduce even within-operation priming.

The effect of guessing on cross-operation priming

The observation that in the speeded condition participants in the division-to-multiplication condition actually made significantly more errors when they solved primed problems than when they solved unprimed problems has interesting implications for models of arithmetic cognition. The tendency for participants to make significantly more errors to multiplication problems that were related to the division problems in the first set suggests that solving the related division problem significantly increased the likelihood that the participant would make an error on the subsequent related multiplication problem. This observation could be explained by an increased use of guessing by participants solving division problems under speeded conditions in place of recasting as a solution process.

Increased guessing under speeded conditions has been observed in other experiments. For example, Smith-Chant and LeFevre (2001) noted that participants who solved multiplication problems with instructions to equally weight speed and accuracy sometimes reported using slow solution procedures, such as deriving the answer by accessing $8 \times 5 = 40$ and adding 8 to get 48. Slower solution procedures are more common with larger problems than smaller problems. Smith-Chant and LeFevre found that when these same participants were asked to solve multiplication problems with instructions emphasizing speeded responses, they tended to abandon slow strategies and indicate that they guessed the answer. Guessing was associated with relatively long latencies and with very high error rates (i.e., 57% for trials solved by guessing in speeded conditions).

Siegler (1988) observed that children sometimes reported that they "guessed" an answer in basic multiplication. Based on the latencies, absence of overt strategies, and pattern of errors, he suggested that guessing was different than retrieving the most highly associated (or peaked) candidate among the answers associated with the problem.

Rather, he suggested that guessing occurs when a child implements an answer retrieval process (i.e., using the network of associations among the multiplication problems) and a single answer fails to be activated to the point of being selected. When this happens, Siegler suggested that the network of associations is reset and a new process is implemented. Using the reset process, the individual selects the most highly activated answer in the entire system. Sometimes, this answer will be the correct answer. More often, the answer will be the answer to a recently presented problem that is somewhat related to the presented problem. This conceptualization accounts for the observation that guesses are frequently incorrect, have mean latencies that are longer than those associated with direct retrieval, and are identified as being different from "just knowing" or retrieving the answer by participants (Siegler, 1988; Smith-Chant & LeFevre, 2001).

In the current experiment, if participants were more likely to abandon recasting and guess the answer to larger division problems under speeded conditions, the guessing process may have weakened the association between the answer and the related multiplication problem in memory. For example, when solving $48 \div 8$, a participant in speeded conditions might guess "7". This could result in weakening the correct association between $8 \times \underline{6}$ and 48 in memory by increasing the association between $8 \times \underline{7}$ and 48 in memory. Alternatively, a proposal by Siegler and Shipley (1995) could be modified to explain the asymmetrical errors in the second block associated with increased

guessing in the first block. Siegler and Shipley suggested that each time a solution procedure is used to successfully generate a correct response, it strengthens the association between that solution *procedure* and the problem in memory. For example, if by guessing to solve $48 \div 8$, the correct response was generated, participants might have reinforced the association between "guessing" as a solution procedure and 8×6 in memory. Thus, when presented with 8×6 in the second block of problems, participants might have been more likely to use guessing as a solution procedure.

Siegler and Shipley (1995) suggested that generation of an incorrect answer would weaken the association between a problem and a solution procedure. Thus, the strengthening of an association between a problem and guessing as a solution procedure would occur only if the guessing process resulted in the correct answer. In other studies, guessing is associated with a high rate of errors (Siegler, 1988; Smith-Chant & LeFevre, 2001). Accordingly, a switch to guessing as a solution procedure in the first block of division problems might lead to the expectation that participants should make more errors when solving the first block division problems. This was not the case in the current experiment. However, it is possible that the limited potential answers in basic division may allow participants to maintain a relatively high degree of accuracy even with guessing.

The possible division answers in the current experiment ranged from 2 to 9 (i.e., $6 \div 3$ to $72 \div 8$), whereas the possible multiplication answers ranged from 6 to 72 (also in Smith-Chant & LeFevre, 2001). Not only is the range of possible answers quite restricted in division, but also the magnitude of the operands in the division problem might offer participants a constraint on potential division answers. For example, a participant with

moderate skill in basic arithmetic would be able to determine that guessing the answer to $72 \div 8$ would involve a relatively large number (i.e., 8, or 9) and thereby increase the likelihood of selecting the correct answer. If participants who guessed the answers to division problems were able to maintain a relatively high degree of accuracy, Siegler and Shipley (1995) suggest that the success of the guessing procedure in division would serve to further reinforce the subsequent use of the guessing procedure for any problem that shares the same representation in memory. Likewise, guessing the answer to multiplication problems would not result in the re-enforcement of a guessing solution procedure because the low success rate of the solution procedure would weaken the association between the problem and the guessing procedure in memory.

Summary

The idea that individuals solve basic division problems by accessing their knowledge of related multiplication problems is appealing on many levels. First, sharing a common representation is consistent with the real world status of division both educationally and practically. Children do not practice or rote memorize "basic division" the way that they practice or drill basic addition and multiplication problems. Even the other inverse operation, subtraction, is practiced and used more educationally and practically. Potentially, the greater use of subtraction may encourage individuals to develop independent representations for subtraction problems rather than rely on their addition knowledge. However, Daley (1997; see also LeFevre et al., in press) noted that even adults sometimes report solving subtraction problems by referencing their knowledge of addition. The status of basic subtraction as an independent operation could be explored using the modified priming-probe methodology designed here. If the greater

utility and occurrence of subtraction results in the development of independent representations, then cross-operation priming between addition and subtraction should not occur.

The observation that individuals solve basic division problems using heuristics, rules, solution procedures and semantic information poses a challenge to models that maintain that arithmetic problems are solved via an associative network view. First, network associative models do not provide an explanation as to why introspective reports of adult performance indicate that people use solution procedures. Campbell (1995) proposed that solution procedures other than direct retrieval may reflect the activation of semantically related information that is associated with the problem in memory, rather than reflect actual solution procedures. However, reports of solution procedures correspond to latency data. For example, Smith-Chant and LeFevre (2001) observed that when participants said they derived the answer from another problem, the mean latencies were slower than when they said they "just knew" the answer but faster than when they said they solved the problem using repeated addition.

From a developmental perspective, rule-based models, such as Siegler (1988, Siegler & Shipley, 1995), are consistent with the notion that strategies associated with children's arithmetic performance could persist as a feature of adult performance (Smith-Chant & LeFevre, 2001). In contrast, associative network models do not mediate adult and child performance. As a result, there is a dichotomy between the explanations offered by associative network models and developmental models (LeFevre, Bisanz, et al., 1994; Smith-Chant & LeFevre, 2001). For example, in most developmental models, the problem-size effect occurs because children use slower solution procedures to solve

large problems than they do for small problems (Siegler, 1988). However, the same effect in most associative network models is proposed to arise solely from patterns of activation and inhibition (Campbell, 1995).

Another attractive feature of the common representation hypothesis is that it is consistent with the neuropsychological literature on the association between preservation and loss of both operations after brain trauma. Additionally, the common representation hypothesis also can account for potential differences between trained and untrained individuals (Rickard & Bourne, 1996; Rickard et al., 1994). Although division and multiplication representations may share common representations, this does not preclude that independent representations could not be developed after specific training. For example, individuals who may start out solving division problems by referencing multiplication problems could eventually begin to develop direct associations between the problem and answer. Thus, prior to specific training on division problems, participants would show evidence of cross-operation priming. However, the effect of priming should be reduced as the training encourages the development of independent representations.

In addition to the appeal of a common representation underlying related multiplication and division problems, the results of the current experiments suggest that experimental manipulations have implications for arithmetic production tasks. Latencies and errors to problems are affected by features such as the emphasis on speed versus accuracy in the instructions, the length of the ITI, the provision of accuracy feedback, and the skill level of the participants. The observation that even minor methodological differences influence arithmetic performance suggest that the process of solving

arithmetic problems is complex, easily altered, and variable across individuals. The sensitivity of arithmetic to individual and experimental differences belies the assumption that the mathematical invariance of basic arithmetic is reflected by a corresponding invariance in how different individuals organize basic arithmetic in memory. However, the observation that even simple cognitive tasks can be richly diverse provides some insight into the potential complexity of the cognitive processing of information such as language or emotion.

References

- Arbuthnott, K. (1996). To repeat or not to repeat: Repetition facilitation and inhibition in sequential retrieval. *Journal of Experimental Psychology: General*, 125, 261-283.
- Arbuthnott, K., & Campbell, J.I.D. (1996). Effects of operand order and problem repetition on error priming in cognitive arithmetic. *Canadian Journal of Experimental Psychology*, 50, 182-195.
- Ashcraft, M.H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, 44, 75-106.
- Ashcraft, M.H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition*, 1, 3-34.
- Ashcraft, M.H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in basic addition. *Journal of Experimental Psychology: Human Learning and Memory*, 4, 527-538.
- Baroody, A.J. (1994). An evaluation of evidence supporting fact-retrieval models. *Learning and Individual Differences*, 6, 1-36.
- Campbell, J.I.D. (1991). Conditions of error priming in number-fact retrieval. *Memory & Cognition*, 19, 197-209.
- Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition*, 53, 1-44.
- Campbell, J.I.D. (1995). Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition*, 1, 121-164.
- Campbell, J.I.D. (1997). On the relation between skilled performance of simple division and multiplication. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 23, 1140-1159.
- Campbell, J.I.D. (1999). Division by multiplication. *Memory & Cognition*, 27, 791-802.

- Campbell, J.I.D., & Clark, J.M. (1988). An encoding-complex view of cognitive number processing: Comment on McCloskey, Sokol, & Goodman (1986). *Journal of Experimental Psychology: General*, 117, 204-214.
- Campbell, J.I.D., & Clark, J.M. (1989). Time course of error priming in number-fact retrieval. Evidence for excitatory and inhibitory mechanisms. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 15, 920-929.
- Campbell, J.I.D., & Clark, J.M. (1992). Cognitive number processing: An encoding-complex perspective. In J.I.D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 457-491). North Holland: Elsevier.
- Campbell, J.I.D., & Graham, D.J. (1985). Mental multiplication skill: Structure, process, and acquisition. *Canadian Journal of Psychology*, 39, 388-366.
- Campbell, J.I.D., & Oliphant, M. (1992). Representation and retrieval of arithmetic facts: A network model and simulation. In J.I.D. Campbell (Ed.) *The nature and origins of mathematical skills* (pp. 331-364). North Holland: Elsevier.
- Campbell, J.I.D. & Tarling, D. (1996), Retrieval processes in arithmetic production and verification. *Memory & Cognition*, 24, 156-172.
- Cipolotti, L., & de Lacy Costello, A. (1995). Selective impairment for simple division. *Cortex*, 31, 433-449.
- Daley, K.E. (1997). *Adults' subtraction: A comparison of self-report and no-self-report conditions*. Unpublished research, Carleton University, Ottawa, Ontario.
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44, 1-42.
- French, J.W., Ekstrom, R.B., & Price, I.A. (1963). *Kit of reference tests for cognitive factors*. Princeton, NJ: Educational Testing Service.
- Geary, D.C. (1994). *Children's mathematical development*. Washington, DC: American Psychological Association.
- Geary, D.C. (1996). The problem-size effect in mental addition: Developmental and cross-national trends. *Mathematical Cognition*, 2, 63-93.

- Geary, D.C., Frensch, P.A., & Wiley, J.G. (1993). Simple and complex mental subtraction: Strategy choice and speed-of-processing differences in younger and older adults. *Psychology & Aging, 6*, 474-483.
- Geary, D.C., Widaman, K.F., & Little, T.D. (1986). Cognitive addition and multiplication: Evidence for a single memory network. *Memory & Cognition, 14*, 478-487.
- Hittmair-Delazer, M., Semenza, C., & Denes, G. (1994). Concepts and facts in calculation. *Brain, 117*, 715-728.
- LeFevre, J., Bisanz, J., Daley, K.E., Buffone, L., Greenham, S.L., & Sadesky, G.S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General, 125*, 284-306.
- LeFevre, J., & Morris, J. (1999). More on the relation between division and multiplication in simple arithmetic: Evidence for mediation of division solutions via multiplication. *Memory & Cognition, 27*, 803-812.
- LeFevre, J., Smith-Chant, B.L., Hiscock, K., Daley, K.E., & Morris, J. (in press). Young adults' strategic choices in simple arithmetic: Implications for the development of mathematical representations. In A.J. Baroody and A. Dowker (Eds.) *The development of arithmetic concepts and skills: Recent research and theory*. Mahwah, NJ: Erlbaum.
- Loftus, G.R., & Masson, M.E.J. (1994). Using confidence intervals in within-subjects designs. *Psychonomic Bulletin & Review, 1*, 476-490.
- Mauro, D. (1999). *Simple division and multiplication: More evidence for recasting*. Unpublished Honour's Thesis, Carleton University, Ottawa, Ontario.
- McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44*, 107-157.
- Miller, K. & Paredes, D. (1990). Starting to add worse: Effects of learning to multiply on children's addition. *Cognition, 37*, 213-242.

- Miller, K., Perlmutter, M., & Keating, D. (1984). Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 10, 46-60.
- Ottawa-Carleton District School Board (1998). *Early Start: Identification and intervention. Mathematics support document for the Kindergarten Program*. Ottawa, Ontario.
- Parkman, J., & Groen, G. (1971). Temporal aspects of simple addition and comparison. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 20, 1139-1153.
- Rickard, T.C. & Bourne, L.E. (1996). Some tests of an identical elements model of basic arithmetic skills. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 22, 1281-1295.
- Rickard, T.C., Healy, A.F., & Bourne, L.E. (1994). On the representation of arithmetic facts: Operand order, symbol, and operation transfer effects. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 20, 1139-1153.
- Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skills. *Journal of Experimental Psychology: General*, 106, 250-264.
- Siegler, R.S. (1989). Mechanisms of cognitive development. *Annual Review of Psychology*, 40, 353-379.
- Siegler, R.S., & Shipley, C. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modelling* (pp.31-76). Hillsdale, NJ: Erlbaum.
- Smith-Chant, B. L. & LeFevre, J. (2001). *Doing as they are told and telling it like it is: Self-reports in mental arithmetic*. Manuscript submitted for publication.
- Zhang, J., & Norman, D. (1995). A representational analysis of numerical systems. *Cognition*, 57, 271-295.

Table 1

Minimum, maximum, and mean French Kit Score and Number of Participants (N) in each Experimental Condition in Experiment 1

	N *	French Kit		
		Minimum	Maximum	Mean
Within-operation				
Multiplication	22	43	122	80.8
Division	22	53	140	95.6
Cross-operation				
Multiplication-Division	22	57	136	91.1
Division-Multiplication	23	57	128	90.9

Note. N sizes reflect the number of participants in each group once outliers were omitted.

Table 2*Summary of 2 (problem size: small, large) by 2 (operation: multiplication, division)**ANOVA of Percentage of Errors and Latencies First Block Division and Multiplication**Trials in Experiment 1*

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Block 1 Operation (Oper)	1	5.44*	.021	7.52**	.080
Error (Oper)	87	(177.88)		(9157,859.17)	
Within Participants					
Problems Size (Psize)	1	72.60***	.455	158.05***	.645
Psize x Operation	1	0.34	.004	0.25	.003
Error (Psize)	87	(24.77)		(32,904.29)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p< .06. *p<.05. **p<.01. ***p<.001.

Table 3

Summary of 2 (problem size: small, large) by 2 (experimental condition: within-operation, between operation) by 2 (operation in 2nd Block: multiplication, division) by 2 (priming: primed, unprimed) for ANOVA of Percentage of Errors and Latencies Second Block Trials in Experiment 1

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Exp. Condition (Exp Cond)	1	0.35	.004	2.36	.027
Block 2 Operation (B2 Oper)	1	0.07	.001	10.27***	.108
Exp Cond x B2 Oper	1	0.10	.001	0.04	.000
Error (Exp Cond x Oper)	85	(157.02)		(282,385.71)	
Within Participants					
Problem Size (Psize)	1	99.17***	.538	176.37***	.675
Psize x Exp Cond	1	0.00	.000	0.57	.007
Psize x B2 Oper	1	2.93	.033	5.02*	.056
Psize x Exp Cond x B2 Oper	1	0.30	.004	0.03	.000
Error (Psize)	85	(115.60)		(48,281.06)	
Priming (Prime)	1	7.63**	.082	86.64***	.510
Prime x Exp Cond	1	0.01	.000	12.48***	.128
Prime x B2 Oper	1	2.34	.027	1.46	.017
Prime x Exp Cond x B2 Oper	1	1.61	.019	0.54	.006
Error (Prime)	85	(66.55)		(18,379.67)	
Psize x Prime	1	0.54	.006	8.54**	.091
Psize x Prime x Exp Cond	1	5.94*	.065	0.67	.008
Psize x Prime x B2 Oper	1	1.05	.012	3.59 [§]	.041
Psize x Prime x Exp Cond x B2 Oper	1	1.81	.021	0.39	.000
Error (Psize x Prime)	85	(55.55)		(18,107.24)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p < .06. *p < .05. **p < .01. ***p < .001.

Table 4

*Mean Percent Errors for problem size, problem type, and experimental
condition 2nd Block in Experiment 1*

Experimental Condition	Problem Type	Problem Size	
		Small	Large
Within-Operation	Unprimed	6.2	16.2
	Primed	2.6	15.2
Cross-Operation	Unprimed	3.5	17.5
	Primed	3.5	12.4

Table 5

Median latencies for each problem size, problem type, and experimental operation in the

2nd Block in Experiment 1

2nd Block Operation	Problem Type	Problem Size	
		Small	Large
Multiplication	Unprimed	974	1246
	Primed	870	1113
Division	Unprimed	1093	1523
	Primed	1008	1302

Table 6

Summary of 2 (problem size: small, large) by 2 (operation: multiplication, division) by 5 (experiment type: replication, short ITI, feedback, speeded, filled ITI) ANOVA of Percentage of Errors and Latencies First Block Division and Multiplication Trials in Experiment 2

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Block 1 Operation (B1 Oper)	1	5.44*	.038	22.31***	.141
Experiment Type (Exp Type)	4	0.90	.026	0.53	.015
B1 Oper x Exp Type	4	0.69	.020	0.70	.020
Error (Oper)	136	(122.57)		(271,388.02)	
Within Participants					
Problems Size (Psize)	1	142.27***	.511	130.31***	.489
Psize x B1 Oper	1	0.08	.001	2.79	.020
Psize x Exp Type	4	0.70	.020	0.45	.013
Psize x B1 Oper x Exp Type	4	0.49	.014	1.21	.034
Error (Psize)	136	(72.52)		(84,600.36)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p< .06. *p<.05. **p<.01. ***p<.001.

Table 7

Minimum, maximum, and mean French Kit Score and Number of Participants (N size) in each Experimental Condition in Experiment 2

	N size*	French Kit		
		Minimum	Maximum	Mean
Multiplication-to-Division:				
Replication	15	47	138	90.4
Short ITI	14	49	114	75.3
Feedback	16	60	133	81.1
Speeded	14	60	106	76.3
Filled ITI	15	53	130	81.7
Division-to-Multiplication:				
Replication	14	39	152	77.0
Short ITI	12	31	185	83.0
Feedback	16	45	125	72.3
Speeded	14	32	134	75.2
Filled ITI	16	44	121	83.4

Note. N sizes reflect the number of participants in each group once outliers were omitted.

Table 8

Analysis of Variance Comparing the Multiplication-to-Division and Division-to-Multiplication Replication Groups on Percent Errors and Latencies in 2nd Block of Experiment 2

<i>Source</i>	Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Block 2 Operation (B2 Oper)	1	2.94 [§]	.098	4.81*	.151
Error (B2 Oper)	27	(114.96)		(24,1676.68)	
Within Participants					
Problems Size (Psize)	1	20.17***	.428	75.06***	.735
Psize x B2 Oper	1	0.00	.000	0.37	.013
Error (Psize)	27	(96.93)		(45,617.11)	
Priming (Prime)	1	1.12	.040	13.09***	.327
Prime x B2 Oper	1	0.88	.032	5.69*	.174
Error (Prime)	27	(43.97)		(13,647.91)	
Psize x Prime	1	0.22	.008	2.91 [§]	.097
Psize x Prime x B2 Oper	1	0.93	.033	0.08	.003
Error (Psize x Prime)	27	(79.67)		(19,929.31)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p< .10. *p<.05. **p<.01. ***p<.001.

Table 9

*Correlations between Latencies on Large Primed Problems and Other Variables
for Multiplication-to-Division Groups in Experiment 2*

Variable :	Luprim	Fkit	Divs	Basic	Anx	Avoid	Learn
Large Primed (Lprim)	.83**	-.55**	.16	-.44**	-.19	-.22	-.27*
Large Unprimed (Luprim)	--	-.49*	.10	-.42**	-.27	-.26*	-.29*
French Kit (Fkit)		--	-.11	.36	.33	.21	.23
Division strategy (Divs) ¹			--	.02	-.01	.09	.01
Basic Math Skill (Basic) ¹				--	.60**	.59**	.58**
Math Anxious (Anx) ¹					--	.65**	.67**
Math Avoidant (Avoid) ¹						--	.66**
Fact Learning (Learn) ¹							--

¹From the Questionnaire of Arithmetic Procedure Use Questionnaire

*p<.05. **p<001.

Table 10

*Correlations between Latencies on Large Primed Problems and Other Variables
for Division-to-Multiplication Groups in Experiment 2*

Variable :	Luprim	Fkit	Divs	Basic	Anx	Avoid	Learn
Large Primed (Lprim)	.84**	-.53**	.21	-.37**	-.19	-.10	-.11
Large Unprimed (Luprim)	--	-.55**	.24*	-.36*	-.19	-.20	-.19
French Kit (Fkit)		--	-.16	.45**	.29*	.30*	.13
Division strategy (Divs) ¹			--	-.11	.03	-.22	.26*
Basic Math Skill (Basic) ¹				--	.46**	.48**	.12
Math Anxious (Anx) ¹					--	.52**	-.03
Math Avoidant (Avoid) ¹						--	-.12
Fact Learning (Learn) ¹							--

¹From the Questionnaire of Arithmetic Procedure Use Questionnaire

*p<.05. **p<.001.

Table 11
*Summary of Final Model Regression Analysis for Variables Predicting
 Large Primed Problem Latencies in Experiment 2*

Multiplication-to-Division			
Variable	B	SE B	β
Latencies for Large Unprimed	.60	.06	.74***
French Kit Score	-2.42	1.45	-.12*
Emphasis of fact retrieval in school	-36.35	20.94	-.11
Division strategy use	36.25	29.05	.08
Basic skill in math	-38.17	28.42	-.11
Math anxiety	15.72	19.79	.07
Math avoidance	3.10	19.66	.01
Division-to-Multiplication			
Variable	B	SE B	β
Latencies for Large Unprimed	.67	.07	.79***
French Kit Score	-1.48	1.61	-.08
Division strategy use	29.88	28.18	.07
Emphasis of fact retrieval in school	11.93	38.52	.02
Basic skill in math	-51.80	33.98	-.13
Math anxiety	-20.67	26.42	-.06
Math avoidance	59.81	25.09	.20*

*p = .05. ***p = .001.

Table 12

Mean self-rating of Math Avoidance in each Experimental Condition for Division-to-Multiplication and Multiplication-to-Division Groups in Experiment 2

Direction of Priming:	Condition:	Math Avoidance	
		Mean	Std. Deviation
Multiplication-to-Division	Replication	5.2	2.2
	Short ITI	4.9	1.3
	Feedback	5.6	1.8
	Speeded	4.6	2.2
	Filled ITI	5.1	2.0
	<i>*Standard Condition</i>	<i>5.1</i>	<i>2.1</i>
Division-to-Multiplication	Replication	5.4	1.8
	Short ITI	4.4	1.8
	Feedback	5.1	1.8
	Speeded	5.3	1.8
	Filled ITI	4.9	1.7
	<i>*Standard Condition</i>	<i>5.1</i>	<i>1.9</i>

***Combined Replication and Filled ITI groups.**

Table 13

Analysis of Variance Comparing the Division-to-Multiplication Filled ITI and Replication Groups on Percent Errors and Latencies in 2nd Block of Experiment 2

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
ITI Type (ITI)	1	0.28	.604	0.00	.000
Error (ITI)	28	(26.73)		(399,168.91)	
Within Participants					
Problems Size (Psize)	1	37.31***	.571	73.08***	.723
Psize x ITI	1	1.51	.051	0.66	.023
Error (Psize)	28	(84.93)		(46,107.64)	
Priming (Prime)	1	4.20*	.130	22.02***	.440
Prime x ITI	1	0.00	.000	0.57	.020
Error (Prime)	28	(41.98)		(17,094.48)	
Psize x Prime	1	1.11	.038	3.62	.115
Psize x Prime x ITI	1	0.08	.003	0.01	.001
Error (Psize x Prime)	28	(34.21)		(19,659.89)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p< .06. *p<.05. **p<.01. ***p<.001.

Table 14

Analysis of Variance Comparing Standard Groups in the Multiplication-to-Division and Division-to-Multiplication Conditions in Experiment 2

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Operation (Oper)	1	3.32	.050	5.68*	.089
Error (Oper)	58	(124.13)		(386,296.92)	
Within Participants					
Problems Size (Psize)	1	62.77***	.520	109.52***	.653
Psize x Oper	1	0.55	.001	0.07	.001
Error (Psize)	58	(98.28)		(64,364.00)	
Priming (Prime)	1	4.63*	.074	32.21***	.357
Prime x Oper	1	0.14	.002	0.43	.007
Error (Prime)	58	(55.54)		(18,483.93)	
Psize x Prime	1	0.28	.005	7.83**	.119
Psize x Prime x Oper	1	2.59	.043	0.02	.000
Error (Psize x Prime)	58	(68.14)		(20,188.14)	

Note. Values enclosed in parentheses represent mean square errors.

^sp < .06. *p < .05. **p < .01. ***p < .001.

Table 15

Analysis of Variance Comparing the Multiplication-to-Division and Division-to-Multiplication Short ITI and Standard Groups on Percent Errors and Latencies in 2nd

Block of Experiment 2

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
ITI Type (ITI)	1	2.71 [§]	.032	2.27	.027
Operation (Oper)	1	1.41	.017	0.00	.000
ITI x Oper	1	0.03	.000	0.78	.009
Error (ITI x Oper)	82	(152.40)		(464,331.70)	
Within Participants					
Problems Size (Psize)	1	80.60***	.496	102.17***	.555
Psize x ITI	1	0.29	.004	0.08	.001
Psize x Oper	1	0.17	.002	1.30	.016
Psize x ITI x Oper	1	0.08	.001	0.26	.003
Error (Psize)	82	(101.67)		(105,787.43)	
Priming (Prime)	1	3.04 [§]	.036	42.16***	.340
Prime x ITI	1	0.00	.000	0.54	.007
Prime x Oper	1	0.53	.006	1.09	.013
Prime x ITI x Oper	1	0.16	.002	1.86	.022
Error (Prime)	82	(50.97)		(24,231.86)	
Psize x Prime	1	0.05	.001	11.29***	.121
Psize x Prime x Oper	1	1.44	.017	0.07	.001
Psize x Prime x ITI	1	0.70	.008	0.66	.008
Psize x Prime x Oper x ITI	1	0.39	.005	0.02	.000
Error (Psize x Prime)	82	(64.01)		(29,506.82)	

Note. Values enclosed in parentheses represent mean square errors.

[§]p< .06. *p<.05. **p<.01. ***p<.001.

Table 16

Analysis of Variance Comparing the Standard and Feedback Groups on Percent Errors and Latencies in 2nd Block of Experiment 2

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Feedback Type (Feed)	1	0.57	.006	4.76*	.051
Operation (Oper)	1	0.81	.009	1.23	.014
Feed x Oper	1	1.30	.015	0.22	.002
Error (Feed x Oper)	88	(160.16)		(436522.24)	
Within Participants					
Problems Size (Psize)	1	87.17***	.498	143.97***	.621
Psize x Feed	1	2.12	.024	2.05	.023
Psize x Oper	1	0.15	.002	1.20	.013
Psize x Feed x Oper	1	1.42	.016	1.34	.015
Error (Psize)	88	(107.25)		(82282.10)	
Priming (Prime)	1	2.07	.023	29.98***	.254
Prime x Feed	1	0.05	.001	0.58	.007
Prime x Oper	1	0.91	.010	0.51	.006
Prime x Feed x Oper	1	0.04	.000	0.00	.000
Error (Prime)	88	(62.35)		(21629.69)	
Psize x Prime	1	0.30	.003	6.25**	.006
Psize x Prime x Oper	1	0.03	.000	0.07	.001
Psize x Prime x Feed	1	1.35	.015	0.13	.002
Psize x Prime x Oper x Feed	1	2.84	.031	0.01	.000
Error (Psize x Prime)	88	(70.51)		(26836.85)	

Note. Values enclosed in parentheses represent mean square errors.

^sp < .06. *p < .05. **p < .01. ***p < .001.

Table 17*Analysis of Variance Comparing the Standard and Speeded Groups on Percent**Errors and Latencies in 2nd Block of Experiment 2*

<i>Source</i>	Percent Errors			Latencies	
	<i>df</i>	<i>F</i>	η^2	<i>F</i>	η^2
Between Participants					
Instruction Type (Inst)	1	0.43	.005	8.20**	.089
Operation (Oper)	1	0.44	.005	1.77	.021
Inst x Oper	1	1.84	.021	1.10	.013
Error (Inst x Oper)	84	(129.49)		(492,304.89)	
Within Participants					
Problems Size (Psize)	1	83.23***	.498	156.51***	.651
Psize x Inst	1	1.79	.021	0.22	.003
Psize x Oper	1	0.56	.007	8.00**	.087
Psize x Inst x Oper	1	1.19	.014	0.51	.006
Error (Psize)	84	(111.96)		(95,472.46)	
Priming (Prime)	1	0.42	.005	22.99***	.215
Prime x Inst	1	3.26 [§]	.037	0.30	.003
Prime x Oper	1	1.13	.013	0.01	.000
Prime x Inst x Oper	1	2.21	.026	1.22	.014
Error (Prime)	84	(54.44)		(31,381.02)	
Psize x Prime	1	0.04	.001	9.67**	.103
Psize x Prime x Oper	1	0.08	.001	0.54	.006
Psize x Prime x Inst	1	0.17	.002	0.04	.001
Psize x Prime x Oper x Inst	1	4.71*	.053	0.34	.004
Error (Psize x Prime)	84	(63.19)		(23,949.44)	

Note. Values enclosed in parentheses represent mean square errors.[§]p< .06. *p<.05. **p<.01. ***p<.001.

Figure 1

Latency savings for within- and cross-operation groups in Experiment 1

Figure 2

Experiment 2: Mean French Kit Scores with 95% Confidence Intervals

Figure 3

Symmetrical priming: Comparison of replication groups in Experiment 2

Figure 4

Symmetrical priming: Comparison of standard groups in Experiment 2

Figure 5

Comparison of Latency savings for short ITI and standard groups in Experiment 2

Figure 6

Comparison of Latency savings for feedback and standard groups in Experiment 2

Figure 7

Comparison of Latency savings for speeded and standard groups in Experiment 2

Appendix A

Problem Sets for Multiplication and Division Initial Trials with Mean Reaction Times (Error Rates) from LeFevre et al. (1996)

Multiplication		Division		
List A	List B	List A	List B	
1	2 x 3 = 6	2 x 5 = 10	6 ÷ 2 = 3	10 ÷ 2 = 5
2	3 x 2 = 6	5 x 2 = 10	6 ÷ 3 = 2	10 ÷ 5 = 2
3	2 x 4 = 8	2 x 7 = 14	8 ÷ 2 = 4	14 ÷ 2 = 7
4	4 x 2 = 8	7 x 2 = 14	8 ÷ 4 = 2	14 ÷ 7 = 2
5	2 x 6 = 12	3 x 5 = 15	12 ÷ 2 = 6	15 ÷ 3 = 5
6	6 x 2 = 12	5 x 3 = 15	12 ÷ 6 = 2	15 ÷ 5 = 3
7	3 x 4 = 12	2 x 8 = 16	12 ÷ 3 = 4	16 ÷ 2 = 8
8	4 x 3 = 12	8 x 2 = 16	12 ÷ 4 = 3	16 ÷ 8 = 2
9	2 x 9 = 18	7 x 3 = 21	18 ÷ 2 = 9	21 ÷ 3 = 7
10	9 x 2 = 18	3 x 7 = 21	18 ÷ 9 = 2	21 ÷ 7 = 3
11	3 x 6 = 18	6 x 4 = 24	18 ÷ 3 = 6	24 ÷ 4 = 6
12	6 x 3 = 18	4 x 6 = 24	18 ÷ 6 = 3	24 ÷ 6 = 4
13	4 x 5 = 20	3 x 8 = 24	20 ÷ 4 = 5	24 ÷ 3 = 8
14	5 x 4 = 20	8 x 3 = 24	20 ÷ 5 = 4	24 ÷ 8 = 3
15	3 x 9 = 27	7 x 4 = 28	27 ÷ 3 = 9	28 ÷ 4 = 7
16	9 x 3 = 27	4 x 7 = 28	27 ÷ 9 = 3	28 ÷ 7 = 4
17	5 x 7 = 35	5 x 6 = 30	35 ÷ 5 = 7	30 ÷ 5 = 6
18	7 x 5 = 35	6 x 5 = 30	35 ÷ 7 = 5	30 ÷ 6 = 5
19	4 x 9 = 36	4 x 8 = 32	36 ÷ 4 = 9	32 ÷ 4 = 8
20	9 x 4 = 36	8 x 4 = 32	36 ÷ 9 = 4	32 ÷ 8 = 4
21	5 x 8 = 40	5 x 9 = 45	40 ÷ 5 = 8	45 ÷ 5 = 9
22	8 x 5 = 40	9 x 5 = 45	40 ÷ 8 = 5	45 ÷ 9 = 5
23	6 x 7 = 42	6 x 8 = 48	42 ÷ 6 = 7	48 ÷ 6 = 8
24	7 x 6 = 42	8 x 6 = 48	42 ÷ 7 = 6	48 ÷ 8 = 6
25	9 x 6 = 54	7 x 9 = 63	54 ÷ 6 = 9	63 ÷ 7 = 9
26	6 x 9 = 54	9 x 7 = 63	54 ÷ 9 = 6	63 ÷ 9 = 7
27	7 x 8 = 56	8 x 9 = 72	56 ÷ 7 = 8	72 ÷ 8 = 9
28	8 x 7 = 56	9 x 8 = 72	56 ÷ 8 = 7	72 ÷ 9 = 8
Mean = 1375		Mean = 1374		

Note: The Mean latencies for the multiplication lists A and B represent the mean of the combined latencies observed for each problem in Experiment 1 and 2 of LeFevre et al.

(Appendix, 1996)

Appendix B

Questionnaire of non-retrieval strategy use.

QUESTIONNAIRE

GENERAL BACKGROUND

1. From which province did you receive your education (or country if not Canada)?

Elementary: _____

Secondary: _____

Post-Secondary: _____

2. Age: _____

3. Sex: male female

PROCEDURE USE

There are many ways to solve simple arithmetic problems. For each of the problems, please explain how you would **usually** solve it.

Problem	Explanation
1. 9×7	
2. $20 \div 4$	
3. $13 - 6$	
4. $8 + 7$	
5. $72 \div 8$	
6. 9×6	
7. $11 - 2$	
8. $9 + 5$	

1. How do you solve problems like 7×6 , 9×5 , 3×9 ? Do you switch the numbers around so that the smaller number is placed first? Do you always do this? When did you learn to do this?

2. Overall, please estimate what proportion of the time you solve simple multiplication problems in ways *other than* by automatically remembering the answers?

never rarely sometimes often always

3. Overall, please estimate what proportion of the time you solve simple addition problems in ways *other than* by automatically remembering the answers?

never rarely sometimes often always

4. Overall, please estimate what proportion of the time you solve subtraction problems in ways *other than* by automatically remembering the answers?

never rarely sometimes often always

5. Overall, please estimate what proportion of the time you solve division problems in ways *other than* by automatically remembering the answers?

Never rarely sometimes often always

MATHEMATICS

Mathematics Education

Highest level math course taken in high school: _____

Grade: _____

Math or related courses in University/College: _____

Grade: _____

1. Do you remember using a calculator when doing simple arithmetic? If so, when did you start using a calculator?

2. How would you rate your level of basic mathematical skill (e.g., skill at calculation)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

3. How would you rate your level of mathematical skill in more complex areas of mathematics (e.g., calculus, algebra)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

4. Do you ever actively AVOID situations involving mathematics?

1	2	3	4	5	6	7
always try to avoid			sometimes try to avoid			never try to avoid

5. How often do you find that situations involving mathematics make you nervous?

1	2	3	4	5	6	7
almost always			sometimes			almost never

6. How important do you feel that mathematics will be in the development of your career?

1	2	3	4	5	6	7
not important			moderately			very important

7. How difficult was mathematics for you in high school?

1	2	3	4	5	6	7
extremely difficult			moderately difficult			not at all difficult

8. How much emphasis was placed on learning basic arithmetic facts (e.g., $3 + 4$, 5×9) by your teachers in elementary school?

1	2	3	4	5	6	7
little emphasis			moderate emphasis			strong
emphasis						

9. How much emphasis was placed on rote learning (or memorizing) basic arithmetic facts by your teachers in elementary school?

1	2	3	4	5	6	7
little emphasis			moderate emphasis			strong
emphasis						

LANGUAGE (Spoken)

1. What languages do you speak?
English ____ French ____ Other _____
2. Which do you consider your first language (i.e., most proficient speaking)?
English ____ French ____ Other _____
3. If you did French immersion in school, please answer the following questions:
 - (a) You were in French immersion from grades _____ to _____
 - (b) When did you start doing math in French? _____
 - (c) When did you stop doing math in French? _____

LANGUAGE (Reading & Writing)

Language Education

Highest level language/English course taken in high school: _____

Grade: _____

Language/English related course in University/College: _____

Grade: _____

1. How would rate your level of basic reading skill (e.g., reading an instructional manual for information)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

2. How would you rate your skill in reading complex materials (e.g., reading a novel for an upper-year English course)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

3. How would you rate your spelling skills?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

4. How would you rate your basic written communication skills (e.g., summarizing a set of notes for a friend or writing a letter)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

5. Overall, how would you rate your written language skill (e.g., writing a paper for a university course)?

1	2	3	4	5	6	7
very low skill			moderate skill			very high skill

6. Do you ever actively AVOID situations involving reading?
1	2	3	4	5	6	7
always try
to avoid			sometime try
to avoid			never try
to avoid
7. Do you ever actively AVOID situations requiring writing?
1	2	3	4	5	6	7
always try
to avoid			sometimes try
to avoid			never try
to avoid
8. How often do you find that situations requiring that you "read for meaning" make you nervous?
1	2	3	4	5	6	7
almost always			sometimes			almost never
9. How often do you find that situations involving writing make you nervous?
1	2	3	4	5	6	7
almost always			sometimes			almost never
10. How important do you feel that language skills (reading and writing) will be in the development of your career?
1	2	3	4	5	6	7
not important			moderately			very important
11. How difficult was English for you in high school?
1	2	3	4	5	6	7
extremely
difficult			moderately
difficult			not at all
difficult

Appendix C

Demographic Characteristics for Each Experimental Condition in Experiment 1

	Operation in 1 st Block				Overall
	Multiplication:		Division:		
	Within	Cross	Within	Cross	
Age (Years)					
Mean	21.0	23.9	23.5	23.3	22.9
Range	18-33	18-49	18-41	17-51	17-51
Sex					
% Males	45.5	63.6	27.3	26.1	40.4
1 st Language					
% English	77.3	77.3	77.3	78.3	77.5
Location of Elementary Education					
% North America	73.0	77.3	72.7	69.6	73.0
Location of Secondary Education					
% North America	86.4	90.9	77.3	87.0	85.4

Appendix D

Mean Percentage of Errors and Solution Latencies (Standard Deviations)

for First Block Trials in Experiment 1.

<i>Operation of Initial Trials</i>			
Problem Size	Multiplication ^a	Division ^b	Overall
<i>Percentage of Errors</i>			
Small	2.0 (4.0)	4.7 (5.4)	3.3 (4.9)
Large	13.6 (11.4)	14.8 (11.9)	14.2 (11.6)
<i>Solution Latencies</i>			
Small	879 (136)	1028 (235)	954 (205)
Large	1207 (353)	1384 (427)	1296 (400)

^a n=44

^b n=45

Appendix E

Number of Invalids, Percentage of Errors, and Median Latencies (Standard Deviation)

For problem types in the 2nd Block of Experiment 1.

	Operation in 1 st Block of Problems			
	Multiplication		Division	
	Within	Cross	Within	Cross
<i>Percent errors:</i>				
Small: unprimed	5.33 (7.34)	5.57 (6.92)	7.06 (7.22)	1.33 (2.98)
primed	2.17 (3.52)	5.03 (4.91)	2.95 (4.24)	1.94 (3.33)
Large: unprimed	16.87 (13.87)	18.88 (11.84)	15.44 (12.54)	15.93 (14.53)
primed	15.90 (15.64)	9.29 (9.58)	14.53 (12.77)	15.88 (12.39)
<i>Solution Latencies:</i>				
Small: unprimed	957 (252)	1107 (206)	1078 (180)	992 (240)
primed	827 (138)	1070 (241)	946 (194)	918 (203)
Large: unprimed	1228 (315)	1545 (464)	1501 (396)	1266 (401)
primed	1058 (280)	1400 (389)	1203 (300)	1174 (407)

Appendix F

Mean Inter-trial Interval (ms) for Experiment 2, Block 1 Trials

	Mean ITI	Standard Deviation
Multiplication-to-Division:		
Replication	4263	633
Short ITI	2717	527
Feedback	5842	371
Speeded	4361	506
Filled ITI	5784	443
Division-to-Multiplication:		
Replication	4166	347
Short ITI	2576	388
Feedback	5814	373
Speeded	4291	392
Filled ITI	5762	331

Appendix G

Transcript of Instructions for Modified Priming-Probe Task

Speeded Condition:

"In this experiment, you will solve simple arithmetic problem. Each problem will be presented in the centre of the screen. Please say the answer to each problem into the microphone. Speak clearly and answer as quickly as possible. (Participant was prompted to press the middle key to go to the next screen)

"On each trial, you will see a diamond in the center of the computer screen. The diamond will begin to flash. The flashing diamond will be replaced by an arithmetic problem, such as:

$$1 \times 2 =$$

"Answer this problem by speaking loudly and clearly into the microphone. (Participant was prompted to press the middle key to go to the next screen)

"After you have solved each problem, you may receive accuracy feedback. If you are in the feedback condition, when you answer a problem correctly, you will hear a pleasant tone and you will see the word **CORRECT! appear in the centre of the screen. If you answer a problem incorrectly, you will hear an unpleasant tone and the correct **ANSWER** will appear in the middle of the screen. (Participant was prompted to press the middle key to go to the next screen)**

"Remember, you should answer the arithmetic problem as quickly as you can. You will make some mistakes. Occasional errors are normal when people go fast, so do **NOT be overly concerned if you make a mistake. **IT IS IMPORTANT THAT YOU RESPOND AS QUICKLY AS POSSIBLE** (Participant was prompted to press the middle key to go to the next screen)**

"The first few problems will allow you to practice. Please remember to respond as **QUICKLY as possible!" Participant was instructed to press the middle button when they were ready to start.**

Balanced Instructions

"In this experiment, you will solve simple arithmetic problems. Each problem will be presented in the centre of the screen. Please say the answer to each problem into the microphone. Speak clearly and answer as accurately and quickly as possible. (Participant was prompted to press the middle key to go to the next screen)

"On each trial, you will see a diamond in the center of the computer screen. The diamond will begin to flash. The flashing diamond will be replaced by an arithmetic problem, such as:

$$1 \times 2 =$$

"Answer this problem by speaking loudly and clearly into the microphone. (Participant was prompted to press the middle key to go to the next screen)

"After you have solved each problem, you may receive accuracy feedback. If you are in the feedback condition, when you answer a problem correctly, you will hear a pleasant tone and you will see the word **CORRECT!** appear in the centre of the screen. If you answer a problem incorrectly, you will hear an unpleasant tone and the correct **ANSWER** will appear in the middle of the screen. (Participant was prompted to press the middle key to go to the next screen)

"Remember, you should answer the arithmetic problem as accurately and quickly as you can. You will make some mistakes. Occasional errors are normal when people go fast, but please try to avoid making any mistakes. **IT IS IMPORTANT THAT YOU RESPOND AS ACCURATELY AND QUICKLY AS POSSIBLE.**

"The first few problems will allow you to practice. Please remember to respond as **ACCURATELY** and **QUICKLY** as possible!" Participant was instructed to press the middle button when they were ready to start.

Note: Instructions appeared centred in the screen viewed by the participant, with lines spaced to facilitate easy reading. A box, with the button used to progress to the next screen was placed in front of the participant.

Appendix H

Demographic Characteristics for Each Experimental Condition in Experiment 2, Division-to-Multiplication Conditions

	Replication ¹	Short ITI ²	Feedback ³	Speeded ⁴	Filled ITI ⁵	Overall ⁶
<i>Age</i>						
Mean	21.2	21.0	22.4	21.4	21.5	21.5
Range	18-39	18-39	18-43	18-32	18-41	18-43
<i>Sex</i>						
% Males	43	58	38	27	50	43
<i>1st Language</i>						
% English	79	75	81	50	81	74
<i>Elementary Education</i>						
% North America	86	83	81	71	94	83
<i>Secondary Education</i>						
% North America	86	100	81	86	94	89
<i>French Kit Score</i>						
Mean	77.0	83.0	72.3	75.2	83.4	78.3
Range	39-152	31- 185	45-125	32-134	44- 121	31-185

¹n=14, ²n=12, ³n=16, ⁴n=14, ⁵n=16, ⁶N=72

Appendix I

Demographic Characteristics for Each Experimental Condition in Experiment 2,

Multiplication-to-Division Conditions

	Replication ¹	Short ITI ²	Feedback ³	Speeded ⁴	Filled ITI ⁵	Overall ⁶
<i>Age</i>						
Mean	20.2	20.8	20.5	21.1	21.3	20.8
Range	18-22	19-37	19-25	18-30	18-35	18-37
<i>Sex</i>						
% Males	47	57	38	43	47	46
<i>1st Language</i>						
% English	67	86	75	93	73	78
<i>Elementary Education</i>						
% North America	73	86	81	100	73	82
<i>Secondary Education</i>						
% North America	87	93	94	100	93	93
<i>French Kit Score</i>						
Mean	90.4	75.3	81.1	76.3	81.7	81.1
Range	47-138	49-114	60-133	60-106	53-13	47-138

¹n=15, ²n=14, ³n=16, ⁴n=14, ⁵n=15, ⁶N=74

Appendix J

*Mean Number of Invalids, and Percentage of Errors, and Latencies for 1st Block Trials in
Experiment 2*

1 st Block Operation	Condition	% Errors		<i>Solution Latencies</i>	
		Small	Large	Small	Large
Division:	Replication	2.2	12.7	1092	1455
	Short ITI	9.3	19.3	1179	1800
	Feedback	6.2	18.9	1191	1489
	Speeded	4.3	18.2	1205	1748
	Filled ITI	6.4	17.6	1200	1609
Multiplication:	Replication	1.5	13.3	968	1270
	Short ITI	5.2	12.2	881	1163
	Feedback	2.3	15.2	882	1276
	Speeded	1.5	13.3	989	1306
	Filled ITI	1.4	16.8	989	1352

Appendix K

Mean Error Latencies for 1st Block Trials in Experiment 2

1 st Block Operation	Condition	Solution Latencies	
		Small	Large
Division:	Replication	1084	1437
	Short ITI	1531	2254
	Feedback	2446	2511
	Speeded	1444	2349
	Filled ITI	1527	2329
Multiplication:	Replication	1025	1947
	Short ITI	961	1378
	Feedback	1976	2076
	Speeded	1306	2079
	Filled ITI	1372	1814

Appendix L

Second Block Means (Standard Deviations) for Primed and Unprimed Latencies for each Experimental Condition for Division-to-Multiplication (Divi-to-Mult) and Multiplication-to-Division (Mult-to-Divi) Groups

Operation	Experimental Condition	Small Problems		Large Problems	
		Primed	Unprimed	Primed	Unprimed
Mult-to Divi	Replication	1180 (222)	1169 (194)	1462 (422)	1527 (357)
	Short ITI	1015 (197)	1099 (264)	1323 (459)	1591 (677)
	Feedback	1120 (271)	1139 (218)	1568 (537)	1676 (701)
	Speeded	1201 (281)	1247 (243)	1620 (432)	1830 (473)
	Filled ITI	1093 (256)	1172 (243)	1404 (435)	1624 (585)
	<i>*Standard</i>	1137 (239)	1171 (216)	1576 (479)	1311 (406)
Divi-to-Mult	Replication	911 (126)	990 (178)	1227 (266)	1409 (354)
	Short ITI	966 (222)	989 (221)	1349 (542)	1525 (853)
	Feedback	1058 (222)	1116 (302)	1356 (381)	1477 (498)
	Speeded	1147 (289)	1173 (313)	1696 (785)	1793 (791)
	Filled ITI	965 (280)	1013 (273)	1223 (475)	1363 (557)
	<i>*Standard</i>	940 (220)	1002 (230)	1385 (476)	1537 (530)

**Combined Replication and Filled ITI groups, n=30*

Appendix M

*Second Block Means (Standard Deviations) for Primed and Unprimed Percent Errors for
each Experimental Condition for Division-to-Multiplication (Divi-to-Mult) and
Multiplication-to-Division (Mult-to-Divi) Groups*

Operation	Experimental Condition	Small Problems		Large Problems	
		Primed	Unprimed	Primed	Unprime
Mult-to-Divi	Replication	5.1 (6.0)	7.6 (7.4)	15.7 (12.6)	13.5 (13.
	Short ITI	6.3 (5.9)	7.0 (7.6)	15.9 (14.4)	17.6 (10.
	Feedback	7.1 (9.3)	4.6 (5.2)	12.2 (14.6)	15.5 (14.
	Speeded	5.4 (5.5)	4.2 (5.5)	12.0 (14.9)	15.6 (9.
	Filled ITI	1.4 (3.0)	6.9 (7.6)	15.1 (14.1)	16.2 (14.
	<i>*Standard</i>	3.3 (5.0)	7.3 (7.4)	15.4 (13.2)	14.8 (13.
Divi-to-Mult	Replication	2.1 (3.5)	3.8 (6.5)	9.5 (7.9)	12.8 (11.
	Short ITI	4.0 (6.3)	3.0 (5.1)	14.5 (11.8)	16.6 (15.
	Feedback	3.2 (5.9)	3.5 (6.0)	17.1 (13.2)	17.7 (14.
	Speeded	3.3 (4.0)	2.2 (4.9)	20.2 (12.6)	15.1 (14.
	Filled ITI	1.3 (2.8)	2.3 (4.3)	12.3 (10.8)	16.1 (11.
	<i>*Standard</i>	1.7 (3.2)	3.0 (5.4)	11.0 (9.5)	14.6 (11.

*Combined Replication and Filled ITI groups, n=30