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BURST-TRAPPING CODES FOR THE
LAND MOBILE DATA CHANNEL

By

Eddy N. Hum, B Sc. (Eng)

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
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ABSTRACT

Digital land mobile channel is subject to severe multipath fading. As a result, the bit error rate performance is usually unsatisfactory. One way to lower the bit error rate is to use error control codes. Because of the fading, errors occur in burst as well as in random. Codes that are designed to correct either random or burst errors alone are ineffective. A technique called Burst-Trapping is capable of adaptively correcting both random and burst errors. In this thesis, the performance of this technique over the digital land mobile channel is investigated. The performance of this technique is compared to interleaved BCH codes.

An innovative approach is used to generate the error pattern files for the performance evaluation of the Burst-Trapping technique and the BCH codes. This approach uses a real time mobile channel simulator. Addition to the study of the Burst-Trapping technique, this thesis also studies the burst error statistics of the digital land mobile channel.

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LIST OF SYMBOLS

C	systematic block code used for encoding the BT code
C_1, C_2	systematic block codes used for encoding the GBT codes
(n, k, d_m)	linear block codes with block length = n , information length = k and minimum distance = d_m
t	number of correctable errors of the C code
$t+d$	number of detectable errors of the C code
t_1	number of correctable errors of the C_1 code
t_1+d_1	number of detectable errors of the C_1 code
d_{1m}	minimum distance of the C_1 code
t_2	number of correctable errors of the C_2 code
t_2+d_2	number of detectable errors of the C_2 code
d_{2m}	minimum distance of the C_2 code
I_i	the i 'th information block
I_i^m	the m 'th portion of I_i

C_i	codeword of I_i
P_i	parity of C_i
B_i	transmitted BT or GBT codeword
Q_i	parity of B_i
$\bar{I}_i^*$	equivalence of I_i plus channel errors
$\bar{I}_i^{*m}$	equivalent of I_i^m plus channel errors
$\bar{B}_i^*$	received BT or GBT codeword
$\bar{Q}_i^*$	parity of $\bar{B}_i^*$
$\bar{I}_i$	decoded information block
$\bar{I}_i^m$	the m'th portion of $\bar{I}_i$
B_i'	codeword in the C or Cl codes plus channel errors
P_i'	parity of B_i'
$\bar{P}_i^*$	parity computed using the C or Cl encoder and $\bar{I}_i$
b	$(b-1)/b$ = code rate of the BT code
f	folding factor
L	bit or block interleaving degree
$P_B(\geq b)$	probability that the burst interval last for b bits or more
$P_R(\geq b)$	probability that the burst repetitive interval last for b bits or more
$P(\geq m/n, n)$	probability that the percentage of error is greater than or equal to m/n in a given block length of n

m_B	mean value of burst interval
m_F	mean value of error free interval
m_R	mean value of burst repetitive interval
σ_B	standard deviation of burst interval
σ_F	standard deviation of error free interval
σ_R	standard deviation of burst repetitive interval
M	number of consecutive correct bits in between error bits
N_T	number of data bit in the time interval where the vehicle has travelled a distance of $\lambda/2$
λ	wavelength
v	velocity

CHAPTER 1.0

INTRODUCTION

1.1 Introduction

In recent years, the transmission of digital information over land mobile fading channel has received considerable attention because of the growing demand for the exchange of data through such channel. One area of growing demand is the cellular mobile radio telephone system which has to serve an ever increasing user demand for applications such as computer data exchange and digital voice transmission. In the cellular system, up to 11% of the channel capacity is occupied by digital data for the purpose of channel assignment and control functions [1]. Other areas of rapid growth for data transmission over mobile radio channel are transmission of data to portable terminals, automatic vehicle location systems and computerized dispatch systems.

The design of a data transmission system over a mobile radio channel is a difficult task. When the mobile receiver moves through a stationary field pattern which has field strength distributed randomly with position, the received signal energy undergoes rapid changes. This

effect is termed as multipath fading and is considered to be the most serious impairment of the mobile radio channel. The multipath fading gives rise to a rapid variation of the signal strength. From time to time, the signal strength will undergo a rapid fade of short duration, down to about 30 to 40 dB below the average signal level [2]. The rate and the duration of these fades depend on the speed of the mobile receiving unit. During these fades, system thermal noise and other impairments will give rise to a high probability of transmission errors. Because of the deep fades, error will occur in burst as well as in random. Uncoded channel bit error rate in the order of 1 to 10 percent are typical [3]. There are many techniques for lowering the bit error rate, among them is the use of error control coding.

The use of error control coding to combat channel errors in a mobile radio channel has been advocated for years.

Because errors occur in burst as well as in random, codes that are designed to correct either random or burst errors alone will be ineffective in a mobile radio channel. The common technique used to correct both random and burst errors is to use bit interleaving or concatenation [4].

The bit interleaving technique uses a random error correcting code together with an interleaver at the

transmitting end. The interleaver performs the re-arrangement of the data sequence in a pre-determined manner such that data bits belong to the same codeword are separated by L bits, where L is called the bit interleaving degree. The process of the interleaver can be visualized as putting L codewords in rows and reading them out in columns. At the receiving end, a de-interleaver performs the reverse process of the interleaver before the received data is passed to the decoder. For an error burst that lasts for L bits, each codeword will see only one bit error due to this burst. The bit interleaving, in essence, is a method of dispersing the burst errors in such a way that these errors are shared among the interleaved codewords. The concatenation technique uses two codes, an outer code and an inner code. At the transmitting end, the outer code operates on the incoming data stream. The inner code operates on the output of the outer code. At the receiving end, the corresponding decoders are connected in reverse order, i.e., the inner decoder is followed by the outer decoder. When properly selected, the inner code and the outer code together can correct both random and burst errors. While the bit interleaving and concatenation techniques are effective in correcting both random and burst errors, there are other techniques called Burst-Trapping techniques which are designed to adaptively correct both random and burst errors as well.

Figure 1.1 shows the three types of techniques that can be used to correct burst and random errors. The bit interleaving technique, as shown in Figure 1.1(a), can be considered as a form of randomizing the channel errors and the interleaver does not involve adding additional data to the data produced by Encoder #1. The concatenation technique, as shown in Figure 1.1(b), has similar effect as the bit interleaving in the sense that channel errors are randomized. However, as compared to the interleaver, Encoder #2 adds additional data in the form of parity check bits to the data produced by Encoder #1. In general, the bit interleaving technique is easier to implement than concatenation because it involves using one decoder instead of two. Any channel that possesses burst errors is said to be a channel with memory. In information theory, memory increases the information capacity [27]. This added capacity due to memory is referred to as memory capacity in here. For both the bit interleaving and the concatenation techniques, this memory capacity is not exploited because these techniques attempt to make the channel memoryless through randomization. The burst-trapping technique, as shown in Figure 1.1(c), is a technique that attempts to exploit this memory capacity. As will be described, the burst-trapping technique decodes adaptively according to the channel errors and no attempt is made to randomize

(a) Bit Interleaving

(b) Concatenation

(c) Burst-Trapping

Figure 1.1 Functional Block Diagram Illustrating the Techniques that can be Used to Correct Burst and Random Errors.

these errors. As will be seen, this technique does not introduce additional data to the data produced by Encoder #1. Moreover, this technique can be considered as a form of time diversity. In term of complexity, the burst-trapping and the bit interleaving techniques are approximately the same. Both techniques use one encoder and one decoder. Additionally, they do not add additional data to the data produced by the encoders. For both the bit interleaving and the concatenation techniques, both block codes and convolutional codes can be used. For the burst-trapping technique, only linear block codes can be used. In this thesis, the class of primitive BCH codes which are cyclic linear block codes are selected for the performance study of the burst-trapping techniques over the land mobile fading channel. The BCH codes are selected because they are widely used and because both the encoding and the decoding can be implemented with ease. Additionally, the BCH codes satisfy the constraints set forth by the burst-trapping techniques.

The Burst-Trapping techniques or the Burst-Trapping (BT) codes were first introduced by S. Y. Tong [5] in 1969. These codes have the characteristics of adaptively correcting both random and burst errors. Both simulation and experimental results of these codes over the telephone

channel have indicated that the performance of these codes, when compared to interleaved block codes, offers significant improvement [6, 7]. The telephone channel is known to have both random and burst errors. In the subsequent years, the capability of the Burst-Trapping codes was enhanced [8]. The enhanced Burst-Trapping codes are called the Generalized Burst-Trapping (GBT) codes. Field test result over the telephone channel has also indicated that the Generalized Burst-Trapping codes offer significant system improvement [9].

Motivated by this significant performance gain and realizing that both the telephone channel and the land mobile channel possess burst and random errors, the aim of this thesis is to investigate whether this performance gain can also be achieved in a land mobile channel. Although the BT and the GBT codes have been studied by many researchers [31-36], virtually nothing is known about their performance over the land mobile channel. Are these codes suitable for use in the land mobile channel? The answer is provided by this thesis. Additional to providing this answer, this thesis also evaluates the performance of interleaved BCH codes over the land mobile channel. This gives the foundation for judging the suitability of using the BT and the GBT codes over the land mobile channel.

The investigation of the performance of the BT, the GBT and the BCH codes over the land mobile channel requires the generation of error files. In this thesis, a simulation model is developed to generate these error files. This model is innovative in the sense that it makes use of a real time mobile channel simulator. Experimental data obtained from a previous field test experiment is also used throughout this thesis to validate the findings.

Additional to the study of the performance of these three classes of codes over the land mobile fading channel, the burst error statistics of this channel are also analyzed. The aim is to give a further insight into the understanding of the error characteristics of the land mobile channel and to explain some of the findings discovered in this thesis.

1.2 Thesis Organization

Chapter 2 gives a brief overview of the land mobile channel characteristics. The model of multipath fading is discussed. Detailed descriptions of the land mobile channel characteristics may be found in the references.

Chapter 3 discusses the simulation model and the method of generating the error pattern files for the performance

evaluation of the BT, the GBT and the BCH codes. The experimental data files used to validate the findings are also discussed.

Chapter 4 describes the encoding and the decoding techniques of the Burst-Trapping codes. A simple example is studied first and followed by the formal discussion.

Chapter 5 describes the encoding and the decoding techniques of the Generalized Burst-Trapping codes. A simple example is considered. The aim is to bring out the major characteristics of these codes and the difference between these codes and the Burst-trapping codes.

Chapter 6 discusses the design of the coding parameters and the method of evaluating the coding performance.

Chapter 7 discusses the simulation results using both the simulated and the experimental data. It begins with the discussion of the burst error statistics. The performance of the BT, the GBT and the BCH codes are then presented and compared.

Chapter 8 summarizes the results and conclusions, with suggestions for future works.

CHAPTER 2.0

THE LAND MOBILE CHANNEL

2.1 Introduction

The land mobile radio channel has been studied for years. A large volume of literatures describing the characteristics of this channel now exists. Among them, [2, 23, 24, 26] are good references. The aim of this chapter is to give a qualitative description of this channel. Attention is paid to the various sources of errors and the techniques to combat them. Multipath propagation is identified as the major source of errors and it is discussed in length.

2.2 Multipath Propagation

The multipath propagation is considered to be the most serious impairment and the major source of error producing mechanism in the land mobile channel. There are two components associated with the multipath propagation. The first is multipath fading and is generally referred to as Rayleigh fading even though the fading characteristics may

not be Rayleigh distributed. The second is shadowing and is generally referred to as Lognormal fading even though the fading characteristics may not be lognormal distributed. The multipath fading is due to the transmitted signal arrives at the mobile receiver through multiple transmission paths. These multiple paths come from reflection, refraction and scattering of the transmitted signal by objects surrounding the mobile receiver. Because of these multiple paths, the received signal is a composite signal consisting of many components. Depending on the location in space, these components can add constructively creating a strong signal or they can add destructively creating a signal dropout. In essence, these multipath components create a spatial standing wave pattern, with maxima and minima representing the peaks and the troughs of the signal amplitude. As the mobile receiver moves through this spatial pattern, the received signal energy fluctuates with the locations of the maxima and the minima. These fluctuations are spatially related, with signal dropouts occur on the order of one-half wavelength apart. Signal dropouts can be as high as 30 to 40 dB below the local mean value [2]. At 820 MHz, signal dropouts occur approximately every 18 cm. For a moving vehicle at a speed of 40 km/hr, the dropouts occur approximately every 16 msec. Hence, the multipath fading causes rapid signal variations for a moving mobile

receiver. The second component of the multipath propagation is the shadowing. It is the variation of the mean signal value due to the general topography in the vicinity of the mobile receiver.

The generally accepted statistical model for the multipath fading is Rayleigh even though many field test experiments suggest the channel may well be described by Rician, Nakagami or other distributions [14, 17]. For the shadowing, the widely accepted model is lognormal even though a host of distributions have been reported [17, 18, 19]. Appendix D contains a list of relevant distribution functions for the land mobile channel. The model of a Rayleigh fading channel is shown in Figure 2.2.1. In this model, it is assumed that the amplitude, phase and angle of arrival of the multipath components are randomly distributed, and that the received signal is the sum of many such components. Since each multipath component is a complex random variable, using the central limit theorem, it is easy to show that the composite signal at the mobile receiver is a complex random variable, with the real part and the imaginary part being gaussian distributed random variables. The result is then the composite signal has a Rayleigh distributed amplitude and a uniformly distributed phase. A sample plot of the Rayleigh distributed fading envelope is shown in Figure 2.2.2.

Figure 2.2.1 Rayleigh Fading Model

Figure 2.2.2 Sample of Rayleigh Fading Envelope

There are many techniques proposed to combat the multipath fading. One method is antenna diversity. It uses more than one antenna located some distance apart such that the fading processes at each antenna are sufficiently decorrelated, making the probability of simultaneous fades low. This method has the advantage of conserving transmitted power and bandwidth, but requires extra hardware in the form of antennas and combining circuits. Error control coding is another method to combat the occurrences of errors during fades. Both block codes and convolutional codes can be used, although most researches have focused on using block codes [20, 21]. Error control schemes utilizing forward error correction and retransmission are also effective in combating the transmission errors [10]. This technique is known as hybrid ARQ.

2.3 Other Channel Impairments

There are other channel impairments besides the multipath fading, but their effects are considered secondary. They include shadowing, ignition noise, interference and random FM. The effect of shadowing can be resolved by properly locating the base stations such that shadow areas are independently distributed from each base station. Ignition

noise is generated by the vehicle ignition systems in the form of electromagnetic interference. This type of interference is of short duration and can be controlled by proper shielding and the use of high quality receiver. Interference in the form of adjacent and co-channel interferences can be quite acute in cellular mobile radio systems due to the large number of frequency reuses and the close proximity of adjacent channels. Solutions to these problems include careful system planning, responsible spectrum management and frequency allocations, power level emission control and the use of proper equipments. Finally, the random FM is caused by the rapid phase de-correlation during multipath fading and is responsible for inducing an irreducible bit error rate floor at high signal to noise ratio [2, 23, 24, 25]. This irreducible error floor can be lowered by antenna diversity. The effect of random FM is of secondary importance when compared to multipath fading at signal to noise ratio below approximately 30 dB [23].

CHAPTER 3.0

GENERATION OF ERROR PATTERN FILES

3.1 Introduction

This chapter describes the method of generating the error pattern files for the evaluation of the coding performance. An error pattern file contains the bit by bit description of the error process of the channel. When a data is received in error, a '1' is entered into the error pattern file. When a data is received correctly, a '0' is entered. Hence, an error pattern file is a sequence of '0' and '1', where a '0' indicates no error and a '1' indicates an error.

There are two types of error pattern files used in this thesis. One type is obtained by a simulation model. The other type is obtained from a field test experiment [30]. The aim of using the field test data is to validate the findings obtained using the simulated data.

3.2 Simulated Data

There are generally two simulation approaches to generate the error pattern files for the Rayleigh fading land mobile

channels. One approach is the use of a stochastic model such as Gilbert-Elliott model [27]. Another approach is to generate a Rayleigh fading process and subsequently obtain the error pattern files from this process. In this thesis, the second approach is used because such a Rayleigh fading process is readily available for the generation of the error pattern files.

A Rayleigh fading process can be generated using computer routines [28]. However, if a real time mobile channel simulator is available, the Rayleigh fading process can be obtained directly from the output of the simulator and the time consuming approach of using computer routines can be avoided. Such a real time channel simulator is available at the Department of Systems and Computer Engineering at Carleton University. This simulator was designed and built by R. A. Goubran [29] during his Ph.D. study. Descriptions and design details of this simulator can be found in Chapter 4 of [29].

The real time mobile channel simulator used in this thesis accepts four types of system parameters as inputs. These parameters are channel frequency, vehicular speed, line of sight ratio and shadowing. These parameters are entered via front panel keyboard and the entered values are also

displayed. The outputs of the simulator are the baseband in-phase and quadrature-phase gaussian distributed analog signals which can be combined to obtain the Rayleigh fading process.

Figure 3.2.1 shows the approach of how the error pattern files are obtained using the real time mobile channel simulator. The approach can be divided into four main steps. The first step is to obtain two gaussian distributed data files. The second step is to obtain a Rayleigh distributed data file from the two gaussian distributed data files. The third step is to assign signal to noise ratio to the Rayleigh distributed data. The last step is to use a modulation bit error rate curve to generate the error pattern files. The details of generating the error pattern files are now given as follows.

The mobile channel simulator is first set up by entering the system parameters via the front panel keyboard. The system parameters are shown in Table 3.2.1. The channel frequency is set at 820 MHz. Two vehicular speeds are considered. They are 40 km/hr and 100 km/hr, representing typical city and highway speeds. For a Rayleigh fading channel, both the line of sight ratio and shadowing are set to zero. Additionally, the data transmission rate under

Figure 3.2.1 Generation of Error Pattern files.

Mobile Channel Simulator Set up		
	#1	#2
Frequency (MHz)	820	820
Vehicular Speed (km/hr)	40	100
Line of Sight Ratio	0	0
Shadowing (dB)	0	0

Data Rate = 4800 bits/sec

Modulation = Non-Coherent Binary Frequency

Shift Keying

Table 3.2.1: Simulation Model System Parameters

consideration is 4800 bit/sec and the modulation is assumed to be non-coherent binary frequency shift keying (NBFSK). The design of the above system parameters is based on the field test experiment. They are chosen such that results using the simulated data and results using the experimental data can be compared.

After the mobile channel simulator is set up, one of the outputs is sampled by a 12-bit analog to digital converter (A/D) at a sampling rate of 1.2 kHz. The sampled data is gaussian distributed data and is stored under a file name on the computer disk for later processing. Two such data files are collected for each simulator set up. Each file contains a total of 300,000 samples of data. Sampling at a rate of 1.2 kHz, each data file corresponds to a measurement time of 4 min 10 sec. Because the output of the simulator is bandlimited to the maximum doppler frequency [29], anti-aliasing filter between the simulator and the A/D is not required. With a channel frequency of 820 MHz, the maximum doppler frequency at a vehicular speed of 100 km/hr is equal to $v/\lambda = 76$ Hz, where v is the vehicular speed and λ is the wavelength. At a vehicular speed of 40 km/hr, the maximum doppler frequency is 30 Hz.

For each simulator set up, let the first data file be called file X and the second data file be called file Y.

Both file X and file Y are gaussian distributed envelope data. From these two files, the Rayleigh distributed envelope data, z, is obtained as follows.

$$z = \sqrt{x^2 + y^2} \quad (3.2.1)$$

where x = envelope data from file X

y = envelope data from file Y

The Rayleigh distributed envelope data is stored under file Z. Each sample in data file Z is linearly interpolated to give three additional samples. The result is a data file R containing 1,200,000 samples. The result is equivalent to sampling the Rayleigh distributed signal at a rate of 4.8 kHz with a measurement time of 4 min*10 sec. The values in data file R are digitized A/D values.

The next step is to determine the S/N ratio of each sample in the data file R. From the data file R, the root mean square (rms) value is first calculated. A S/N ratio is then assigned to the rms value. Since the mean square value is the average power of a signal, the assigned S/N ratio defines the average S/N ratio of the Rayleigh distributed envelope data. The rms value together with the assigned S/N ratio are used to calculate the S/N ratio of each sample in the data file R as follows.

$$10 \log s = 20 \log (z/r) + (d \text{ in dB}) \quad (3.2.2)$$

where s = S/N ratio of the sample to be determined

d = assigned S/N ratio of the rms value

z = value of the sample from file R

r = rms value from file R

For a given d in dB, the S/N ratio s of each sample z is calculated using equation (3.2.2). The value of s is used to determine the bit error rate. For the NBFSK modulation, the channel bit error rate in the presence of Gaussian noise is given by [16]

$$P_e = 0.5 \exp (-s/2) \quad (3.2.3)$$

where s = signal to noise ratio

Using equations (3.2.2) and (3.2.3), the bit error probability P_e for each sample z in the data file R is calculated. For each calculated P_e , a uniformly distributed random number generator with value between zero and one is used to generate the channel bit error as follows.

If (the random number $> P_e$) then

channel = no bit error (represented by a 0)

else

channel = one bit error (represented by a 1).

Since there are 1,200,000 samples in data file R, the result obtained above is an error pattern file containing 1.2 Mbit of 0 or 1. This error pattern file describes the error process of the Rayleigh fading channel when data is transmitted at a rate of 4800 bit/sec with a S/N ratio = d. For each vehicular speed, there are 7 error pattern files generated. These files correspond to S/N ratio of 0, 5, 10, 15, 20, 25 and 30 dB. This range covers the typical S/N ratio experienced in a land mobile environment.

The ~~above method~~ of generating the error pattern files has the following advantages. First of all, using the real time mobile channel simulator, error pattern files can be obtained with ease. Any changes to the system parameters such as channel frequency, vehicular speed, line of sight ratio and shadowing can be done by simply entering the parameters into the simulator and no software changes are required. The above method can also be used to generate error pattern files for other type of modulation. The only change required is equation (3.2.3). The limitation of the above method, however, is the number of samples that can be collected during each data acquisition. The existing data acquisition system requires data to be first stored in the system memory and then transfers the data to computer disk at the end of data acquisition. Hence the number of samples is limited by the available system memory. If the

acquisition system can be modified such that data can be transferred directly to computer disk during data acquisition, then the number of samples can be increased substantially. With the existing data acquisition system, the total number of data in each error file is 1.2 Mbits.

3.3 Experimental Data

In the spring of 1983, a field test experiment [30] was conducted by the Department of Systems and Computer Engineering at Carleton University to measure the channel characteristics of the land mobile channel at 820 MHz. The field test experiment consisted of using a commercial FM transmitter with the base antenna located at the top of the MacKenzie Engineering Building. Pseudo-random data sequences were transmitted at a data rate of 4800 bit/sec. The mobile receiving equipments consisted of a commercial FM transceiver with the mobile antenna mounted on the top of the mobile unit. A 7-track analog FM tape recorder acted as the storage medium during the field test. Recorded information consisted of IF envelope, signal at the limiter/discriminator output, received data, received data clock, reference data clock, start/stop signal, vehicular speed pulses, and voice comments to identify each measurement. The recorded information was later analyzed in

the laboratory. The field test experiment was conducted over four geographical regions in the vicinity of the greater Ottawa area. These four regions were downtown Ottawa, downtown Hull, Queensway and Residential Ottawa (south).

The experimental error pattern files used in this thesis are obtained from a previous study [13]. The error pattern files collected in [13] are stored on computer disks and are in a format suitable for computer analysis. Table 3.3.1 shows the error pattern files selected for this thesis. These error pattern files are for the geographical regions of Ottawa, Hull and Queensway. For each geographical region, there are more than one error pattern files and the statistical result is obtained by averaging over these files.

The fading characteristics of Ottawa, Hull and Queensway have been studied by [17]. The results of [17] indicate that the channel characteristics of Ottawa and Hull are Rician distributed, while that of Queensway is Nakagami distributed. The reported line of sight ratio is between 1.0 to 1.2. The scattering is higher in the Ottawa and the Hull areas when compared to the Queensway.

Location	File Record Number	Total Number of Files	Total Number of Data (bits)
Ottawa	MDI01.01 to MDI01.09	9	107,892
Hull	MDI09.01 to MDI09.09	9	107,892
Queensway	MDI12.01 to MDI12.23	23	266,568

Table 3.3.1: Experimental Error Pattern Files Selected
for this Thesis.

CHAPTER 4.0

DESCRIPTION OF BURST-TRAPPING (BT) CODES

4.1 Introduction

The principle characteristic of the BT codes is that information appears twice in the data stream without changing the overall code rate. The information is first encoded using standard block encoding technique which generates parity digits. The same information is added to the parity digits of later blocks before transmission. Because the information appears twice in the data stream, lost information due to channel errors can be recovered from the duplicated sources. Since duplicated information is added to the parity digits of later blocks, the overall code rate is not changed.

In order to better understand the BT codes, a simple example is presented first. After mastering the concept, the formal encoding and decoding techniques are then described.

4.2 Example of Encoding the BT Codes

Figure 4.2.1 shows an example of encoding a rate 2/3 BT code.

Let C be an (n, k, d_m) linear systematic block code

where n = length of code word,

k = length of information block,

d_m = minimum distance of C ,

k/n = code rate = 2/3.

Assume the decoder is designed to correct up to t errors and detect up to $t+d$ errors per block where t and d satisfy

$$2t + d + 1 \leq d_m \quad (4.2.1)$$

Let I_i be the i th information block of length k bits,

I_i^1 be the first $k/2$ bits of I_i ,

I_i^2 be the second $k/2$ bits of I_i ,

C_i be the codeword associated with I_i ,

P_i be the parity digits of the codeword C_i .

Let the symbol $|$ denotes concatenation of blocks of digits.

For examples,

$$I_i = I_i^1 | I_i^2 ;$$

Figure 4.2.1 Encoding a rate 2/3 BT code.

$$C_i = I_i^1 \mid I_i^2 \mid P_i.$$

As shown in Figure 4.2.1, the encoding rule of the BT codes is as follows:

1. encode I_i to give C_i using the C code,

$$C_i = I_i^1 \mid I_i^2 \mid P_i ;$$
2. compute $Q_i = P_i + I_{i-1}^1 + I_{i-2}^2$ where addition is modulo-2 bit by bit addition;
3. transmit $B_i = I_i^1 \mid I_i^2 \mid Q_i.$

Clearly, the BT codeword B_i carries more than one source of information, namely the information of I_i , I_{i-1}^1 and I_{i-2}^2 . Moreover, the overall code rate is not changed since duplicated information digits are added to the parity digits of the codeword C_i .

4.3 Example of Decoding the BT Codes

Let the following notations be used to represent the received data at the decoder,

$$B_i = I_i^1 \mid I_i^2 \mid Q_i$$

where B_i^* is the equivalent of B_i plus errors,
 I_i^{1*} is the equivalent of I_i^1 plus errors,
 I_i^{2*} is the equivalent of I_i^2 plus errors,
 Q_i^* is the equivalent of Q_i plus errors.

At the decoder, there are three modes of operation, they are:

- Random Error Decoding mode,
- Burst-Trapping Decoding mode,
- Blind Faith Decoding mode.

4.3.1 Example of Random Error Decoding Mode for the BT Codes

Figure 4.3.1 shows the operations of the BT decoder in the random decoding mode for the rate 2/3 BT code. The decoder starts with the random decoding mode when it first receives B_0^* , which is the first data block in the transmission. Note that B_0^* is a codeword in the C code plus channel errors. In the random decoding mode, the block B_0^* is first decoded using the C decoder. Let

$$\bar{I}_0 = \bar{I}_0^1 \mid \bar{I}_0^2$$

Figure 4.3.1 Example Showing the rate 2/3 BT code in Random Decoding Mode.

Figure 4.3.2 Example Showing the rate 2/3 BT code in Burst-Trapping Mode.

denotes the decoded information block of B_0^* using the C decoder. As long as the C decoder thinks that there are t or fewer errors in block B_0^* , it will feed $\bar{I}_0^1$ to the next block B_1^* and decode B_1^* in the random decoding mode by first computing

$$P_1' = Q_1^* + \bar{I}_0^1 ;$$

$$B_1' = \bar{I}_1^1 \mid \bar{I}_1^2 \mid P_1' .$$

Block B_1' is now a codeword in the C code plus errors and can be decoded using the C decoder. Again, as long as the C decoder thinks there are t or fewer errors in block B_1' , it will decode B_1' to give $\bar{I}_1$ and continue to decode the next block B_2^* in the random decoding mode by first computing

$$P_2' = Q_2^* + \bar{I}_1^1 + \bar{I}_0^2 ;$$

$$B_2' = \bar{I}_2^1 \mid \bar{I}_2^2 \mid P_2' .$$

Again B_2' is a codeword in the C code plus errors and can be decoded using the C decoder. As long as the C decoder thinks there are t or fewer errors in block B_2' , the next block B_3^* will be decoded in the same manner as block B_2^* . Decoding continues in the random decoding mode as long as the C decoder thinks that each B_i' has t or fewer errors. Clearly, the principle characteristic of random error decoding is to first remove the effects of pass information that was added to the parity portion of B_1^* and then decode B_1' using the C decoder.

Errors can be classified into three types, namely, correctable errors, uncorrectable and undetectable errors and uncorrectable but detectable errors. Both correctable errors and uncorrectable and undetectable errors in B_1' will cause the C decoder to think that there are t or fewer errors in B_1' . In the event that the type of errors in B_1' is uncorrectable and undetectable, the C decoder will make decoding errors and the incorrectly decoded result will be passed onto subsequent blocks. Even in the absence of channel errors, the incorrectly decoded result can cause subsequent blocks to be decoded incorrectly. The effect of decoding error will be studied in more detail later.

At any time during the random error decoding mode, when the C decoder decides that there are more than t errors in block B'_i , the BT decoder will enter into the burst-trapping decoding mode and attempt to recover $\bar{I}_i$ from the duplicated sources.

4.3.2 Example of Burst-Trapping Decoding Mode for the BT Codes

Figure 4.3.2 shows the operation of the BT decoder in the burst-trapping decoding mode for the rate $2/3$ BT code. The BT decoder will operate in the burst-trapping mode whenever the C decoder detects but cannot correct the errors contained in B'_i . In the burst-trapping mode, the lost information contained in B'_i is recovered from the duplicated sources.

Assume the B'_2 is detected to be in error and is uncorrectable by the C decoder. When this happens, $\bar{I}_2$ cannot be recovered from B'_2 . However $\bar{I}_2$ can be recovered from blocks B_3^* and B_4^* which contain the information $\bar{I}_2$. As shown in Figure 4.3.2, the recovery of $\bar{I}_2$ in the burst-trapping mode is as follows:

- (i) decode blocks B_3^* and B_4^* using blind faith decoding to obtain $\bar{I}_3$ and $\bar{I}_4$ respectively,

(ii) compute the parities $\bar{P}_3^*$ and $\bar{P}_4^*$ by encoding $\bar{I}_3$ and $\bar{I}_4$ using the C encoder,

(iii) compute $\bar{I}_2^1 = \bar{Q}_3^* + \bar{P}_3^* + \bar{I}_1^2$

$$\bar{I}_2^2 = \bar{Q}_4^* + \bar{P}_4^* + \bar{I}_3^1,$$

(iv) finally $\bar{I}_2$ is recovered. $\bar{I}_2 = \bar{I}_2^1 \mid \bar{I}_2^2$.

Blocks $\bar{B}_3^*$ and $\bar{B}_4^*$ are called guard space blocks of $\bar{B}_2^*$. The guard space blocks of block $\bar{B}_i^*$ are those blocks that contain the duplicated information of $\bar{I}_i$. When a block, say $\bar{B}_i^*$ is decoded in the burst-trapping mode, all the guard space blocks of $\bar{B}_i^*$ will be decoded in the blind faith mode. In order to enter into the burst-trapping mode, the C decoder must be designed to have error detection capability.

4.3.3 Example of Blind Faith Decoding Mode for the BT Codes

In the blind faith decoding mode, block $\bar{B}_i^*$ is decoded to give $\bar{I}_i$ as follows:

let $B_i^* = I_1^1 | I_1^2 | Q_i^*$

then $\bar{I}_i = I_1^1 | I_1^2$.

Hence, blind faith decoding is simply the process of taking the information portion of the received block B_i^* as the decoded information. If B_i^* contains any channel errors, it is most likely that the decoded $\bar{I}_i$ will be incorrect.

Since in the burst-trapping decoding mode, successful decoding relies on correctly decoded information from the guard space blocks, it is obvious that if the guard space blocks contain any channel errors, the burst-trapping decoding technique will fail. If this is the case, the error correcting capability of the BT code is exceeded.

After block B_2^* is decoded by burst-trapping decoding and blocks B_3^* and B_4^* are decoded by blind faith decoding, the BT decoder returns to random error decoding mode for block B_5^* . It should be noted that any error propagation due to false decoding in the random decoding mode will stop at block B_5^* . It is because information decoded prior to and including block B_2^* will no longer be passed to block B_5^* and any subsequent blocks. Hence, any error propagation will stop after the BT decoder enters into the burst-trapping decoding mode.

4.4 Encoding the BT Codes in General

The encoding and the decoding techniques described in Sections 4.2 and 4.3 for the special case of rate $2/3$ BT codes can be expanded to include codes with different code rates. Moreover, the BT codes can be block interleaved to increase the burst error correcting capability. In this section, the encoding technique of the BT codes in general is described.

In Section 4.2, the parity Q_i of the rate $2/3$ BT code was defined as follows:

$$Q_i = P_i + I_{i-1}^1 + I_{i-2}^2.$$

Assume now Q_i is redefined as follows:

$$Q_i = P_i + I_{i-L}^1 + I_{i-2L}^2.$$

The rate $2/3$ BT code is said to be block interleaved to degree L . Figure 4.4.1 shows an example of encoding a rate $2/3$ BT code with block interleaving degree $L = 2$. Figure 4.4.1 should be compared to Figure 4.2.1 which has block interleaving degree $L = 1$.

Figure 4.4.1 Encoding a rate 2/3 BT code with Block Interleaving Degree $L=2$.

Any systematic block code, C , whose code rate R satisfies

$$R = (b-1)/b \quad (4.4.1)$$

where b is a positive integer

can be used to encode the BT codes. Codes with other rates can always be shortened such that the final code rates satisfy (4.4.1).

Using the same notations as in Section 4.2, the encoding rule of a rate $R = (b-1)/b$ BT code with block interleaving degree L is as follows:

1. let I_{iL} be the information block which can be written as

$$I_{iL} = I_{iL}^1 \mid I_{iL}^2 \mid \dots \mid I_{iL}^{b-1},$$

2. encode I_{iL} to give C_{iL} using the systematic block code C ,

$$C_{iL} = I_{iL}^1 \mid I_{iL}^2 \mid \dots \mid I_{iL}^{b-1} \mid P_{iL},$$

3. compute

$$Q_{iL} = P_{iL} + I_{(i-1)L}^1 + I_{(i-2)L}^2 + \dots + I_{(i-b+1)L}^{b-1}$$

4. transmit

$$B_{iL} = I_{iL}^1 \mid I_{iL}^2 \mid \dots \mid I_{iL}^{b-1} \mid Q_{iL}.$$

Clearly, B_{iL} carries more than one source of information.

Besides I_{iL} , each B_{iL} carries information from the previous $b-1$ interleaved blocks. On the other hand, each I_{iL} is duplicated by spreading I_{iL} to the subsequent $b-1$ interleaved blocks. Interleaved blocks are those blocks that have a separation of L blocks in between them, where L is the block interleaving degree. For example, the previous $b-1$ interleaved blocks of B_{iL} are $B_{(i-1)L}$, $B_{(i-2)L}$, ..., $B_{(i-b+1)L}$. The subsequent $b-1$ interleaved blocks of B_{iL} are $B_{(i+1)L}$, $B_{(i+2)L}$, ..., $B_{(i+b-1)L}$.

4.5 Decoding the BT Codes in General

Using the same notations as in Section 4.3, let

$$\bar{B}_{iL} = \bar{I}_{iL} \mid \bar{Q}_{iL}$$

$$= \bar{I}_{iL}^* | \bar{I}_{iL}^2 | \dots | \bar{I}_{iL}^{*(b-1)} | \bar{Q}_{iL}^*$$

be received word of $\bar{B}_{iL}$.

As have been discussed in Section 4.3, the BT decoder has three modes of operation, namely, random error decoding mode, burst-trapping decoding mode and blind faith decoding mode. When a block $\bar{B}_{iL}^*$ is received, block $\bar{B}_{iL}^*$ will be decoded by the BT decoder to give $\bar{I}_{iL}$ in one of these three modes.

4.5.1 Random Error Decoding Mode for the BT Codes

The primary method of decoding is the random error decoding mode. If none of the previous $b-1$ interleaved blocks are decoded by burst-trapping decoding, then the BT decoder will attempt to decode block $\bar{B}_{iL}^*$ using the random error decoding in which the decoder computes

$$P'_{iL} = \bar{Q}_{iL}^* + \bar{I}_{(i-1)L}^1 + \bar{I}_{(i-2)L}^2 + \dots + \bar{I}_{(i-b+1)L}^{(b-1)}$$

$$\bar{B}'_{iL} = \bar{I}_{iL}^* | P'_{iL}.$$

Note that P'_{iL} is computed by feeding information from the previous $b-1$ interleaved blocks to $\bar{Q}_{iL}^*$. Following the

standard terminology, the information from the previous $b-1$ interleaved blocks are said to be fed back to block B_{iL}^* .

B_{iL}^* is now a codeword in the C code plus errors and can be decoded using the C decoder to give $\bar{T}_{iL}$. If the C decoder thinks there are t or fewer errors in B_{iL}^* , it will decode B_{iL}^* to give $\bar{T}_{iL}$ and continue to decode the next interleaved block in random error decoding mode.

Both correctable errors and uncorrectable and undetectable errors will cause the decoder to think there are t or fewer errors in B_{iL}^* . Uncorrectable and undetectable errors in B_{iL}^* will result in false decoding and error propagation. Error propagation due to false decoding in the random decoding mode will be studied in more detail later.

On the other hand, if the C decoder detects but cannot correct the errors in B_{iL}^* , it will attempt to recover $\bar{T}_{iL}$ using the burst-trapping decoding method.

4.5.2 Burst-Trapping Decoding Mode for the BT Codes

The BT decoder enters the burst-trapping decoding mode when the C decoder detects but cannot correct the errors in B_{iL}^* . In the burst-trapping decoding mode, the decoding of B_{iL}^{**} to

give $\bar{I}_{iL}$ is delayed until all the subsequent $b-1$ interleaved blocks are received. Decoding of block $\bar{B}_{iL}^*$ in burst-trapping mode is then as follows:

- (i) decode all the subsequent $b-1$ interleaved blocks of $\bar{B}_{iL}^*$ using blind faith decoding. Let the decoded information be

$$\bar{I}_{(i+1)L}, \bar{I}_{(i+2)L}, \dots, \bar{I}_{(i+b-1)L}.$$

- (ii) Compute the parities of the decoded information $\bar{I}_{(i+1)L}, \bar{I}_{(i+2)L}, \dots, \bar{I}_{(i+b-1)L}$ using the C encoder. Let the respectively computed parities be

$$\bar{P}_{(i+1)L}^*, \bar{P}_{(i+2)L}^*, \dots, \bar{P}_{(i+b-1)L}^*.$$

- (iii) Compute the information segments of $\bar{I}_{iL}$ as follows:

$$\begin{aligned} \bar{I}_{iL}^1 &= \bar{Q}_{(i+1)L}^* + \bar{P}_{(i+1)L}^* \\ &\quad + \bar{I}_{(i-1)L}^2 + \bar{I}_{(i-2)L}^2 + \dots + \bar{I}_{(i-b+2)L}^{b-1} \end{aligned}$$

$$\bar{I}_{iL}^2 = \bar{Q}_{(i+2)L}^* + \bar{P}_{(i+2)L}^*$$

$$+ \bar{I}_{(i+1)L}^1$$

$$+ \bar{I}_{(i-1)L}^3 + \bar{I}_{(i-2)L}^4 + \dots + \bar{I}_{(i-b+3)L}^{b-1}$$

$$\bar{I}_{iL}^3 = \bar{Q}_{(i+3)L}^* + \bar{P}_{(i+3)L}^*$$

$$+ \bar{I}_{(i+2)L}^1 + \bar{I}_{(i+1)L}^2$$

$$+ \bar{I}_{(i-1)L}^4 + \bar{I}_{(i-2)L}^5 + \dots + \bar{I}_{(i-b+4)L}^{b-1}$$

⋮

$$\bar{I}_{iL}^{b-1} = \bar{Q}_{(i+b-1)L}^* + \bar{P}_{(i+b-1)L}^*$$

$$+ \bar{I}_{(i+b-2)L}^1 + \bar{I}_{(i+b-3)L}^2 + \dots + \bar{I}_{(i+1)L}^{b-2}$$

(iv) Finally, the decoded information $\bar{I}_{iL}$ is

$$\bar{I}_{iL} = \bar{I}_{iL}^1 \mid \bar{I}_{iL}^2 \mid \dots \mid \bar{I}_{iL}^{b-1}$$

All the subsequent $b-1$ interleaved blocks of B_{iL}^* are the guard space blocks of B_{iL}^* . As have been discussed before, if any of these guard space blocks contains errors, the decoding in the burst-trapping mode will fail and $\bar{T}_{iL}$ cannot be recovered correctly. Again, decoding error in the burst-trapping mode does not cause error propagation. Moreover, any error propagation due to false decoding in the random decoding mode will be terminated when the decoder enters into the burst-trapping mode.

Block interleaving the BT codes increases the burst error correcting capability of the codes. For a block interleaving degree of L blocks, burst errors that last for L blocks can be corrected by the burst-trapping method provided that all the guard space blocks are error free.

Since the decoding of B_{iL}^* in burst-trapping mode cannot start until all the subsequent $b-1$ interleaved blocks are received, the decoding delay of the BT code is $[(b-1)L + 1]$ blocks or $[(b-1)L + 1]n$ bits, where n is the block length.

4.5.3 Blind Faith Decoding Mode for the BT Codes

When a block B_{iL}^* is decoded by burst-trapping decoding, all the guard space blocks which are the subsequent $b-1$

interleaved blocks will be decoded by blind faith decoding. Conversely speaking, if any one of the previous $b-1$ interleaved blocks of B_{iL}^* is decoded by burst-trapping decoding, then block B_{iL}^* will be decoded by blind faith decoding.

In blind faith decoding, the block B_{iL}^* is decoded as follows:

$$\begin{aligned} \text{let } B_{iL}^* &= \bar{I}_{iL}^* \mid Q_{iL}^* \\ \text{then } \bar{I}_{iL}^* &= \bar{I}_{iL}^* \end{aligned}$$

After all the guard spaces blocks are decoded by blind faith decoding and at the end of burst-trapping decoding, the BT decoder returns to random error decoding mode.

4.6 Summary of Error Correcting Capability of the BT Codes

The error correcting capability of the BT codes can be summarized as follows:

The BT codes are capable of correcting

- (i) in the random mode, all blocks with t or fewer errors.

Should a block have more than t errors, it is still correctable if

- (ii) these errors are detectable with the C decoder and
- (iii) the block is preceded by $b-2$ interleaved blocks that were correctly decoded and
- (iv) the block is followed by $b-1$ interleaved blocks each of which contains no error.

4.7 Error Propagation of the BT Codes

It was pointed out that decoding error in the random decoding mode could cause additional decoding errors even after all the channel errors have stopped. Should this effect continue forever, the result would be catastrophic. The purpose of this section is to show that the error propagation due to false decoding is quite limited and more specifically, an upper bound on the error propagation is derived. The derivation is similar to [5] but the approach taken here is easier to understand and the result is simpler than [5].

Assume for the moment, the block interleaving degree is one. Further assume that after block B_i^* , all the channel errors have stopped. Let all the subsequent blocks after block B_i^* be divided into sets of $b-1$ blocks each. The first set contains blocks $B_{(i+1)}^*, B_{(i+2)}^*, \dots, B_{(i+b-1)}^*$. The second set contains blocks $B_{(i+b)}^*, B_{(i+b+1)}^*, \dots, B_{(i+2b-2)}^*$ and so on.

Now consider the first set of $b-1$ blocks. Decoding errors in the first set can only affect the second set of $b-1$ blocks. Each block can make at most t decoding errors because the code is designed to correct up to t errors. So the maximum number of errors e_1 that the first set of $b-1$ blocks can pass onto the second set of $b-1$ blocks is

$$e_1 = (b-1) t.$$

Now consider the second set of $b-1$ blocks. To commit a decoding error, a block must have at least $t+d+1$ errors because the code is designed to detect up to $t+d$ errors. So the maximum number of blocks n_2 in the second set that can make decoding errors is

$$\begin{aligned} n_2 &= \frac{e_1}{t + d + 1} \\ &= (b-1) \frac{t}{t + d + 1} \end{aligned}$$

Now consider the third set of $b-1$ blocks. The maximum number of errors the second set can give the third set is $t n_2$. The maximum number of blocks n_3 in the third set that can make decoding errors is

$$\begin{aligned} n_3 &= \frac{t n_2}{t + d + 1} \\ &= (b-1) \left[\frac{t}{t + d + 1} \right]^2 \end{aligned}$$

Continue in this fashion, it is easy to see that the maximum number of blocks n_i in the i 'th set that can make decoding errors is

$$n_i = (b-1) \left[\frac{t}{t + d + 1} \right]^{i-1}$$

The maximum number of blocks N that can be affected by error propagation is then

$$\begin{aligned} N &= (b-1) + n_2 + n_3 + \dots + n_i + \dots \\ &= (b-1) \left\{ 1 + \left[\frac{t}{t + d + 1} \right] + \left[\frac{t}{t + d + 1} \right]^2 + \dots \right\} \\ &= (b-1) \left[\frac{t + d + 1}{d + 1} \right] \end{aligned} \tag{4.7.1}$$

For a BT code with block interleaving degree L , the maximum number of erroneous blocks due to error propagation is simply NL .

Note that in the derivation of (4.7.1), a series of highly unlikely events has been assumed. Hence, it is highly unlikely that an undetected error will propagate more than a few blocks. For the case of a rate $1/2$ BT codes, it is easy to show that the maximum number of erroneous blocks that can be affected by error propagation is one. For a rate $2/3$ BT codes, the maximum is three blocks.

CHAPTER 5.0

DESCRIPTION OF GENERALIZED BURST-TRAPPING (GBT) CODES

5.1 Introduction

One limitation of the burst-trapping codes is that after a burst is detected, an error free guard space must follow to ensure its correction. This condition, however, cannot be satisfied in most cases if the channel background random error rate is high.

An enhanced method called generalized burst-trapping (GBT) is designed to overcome this limitation. The characteristic of the GBT codes is similar to that of the BT codes in that the information appears twice in the data stream. At the receiving end, one of these two versions of information is recovered by the decoder. The GBT codes have the added capability of correcting errors in the guard space as compared to the BT codes. However, this added capability is obtained at the expense of a longer guard space or a lower rate and a small increase of equipment complexity.

Because the GBT codes and the BT codes have similar encoding and decoding procedures, only a simple example is given here. The purpose is to illustrate the concept and to bring out the principle difference between these two codes. A formal description of encoding and decoding the GBT codes can be found in [8].

5.2 Example of Encoding the GBT Codes

The encoding of the GBT codes uses two linear systematic block codes C1 and C2. The relationship between C1 and C2 must satisfy the condition that C1 is a subcode of C2. Let C1 and C2 be denoted as

$$C1 = (n_1, k_1, d_{1m})$$

$$C2 = (n_2, k_2, d_{2m})$$

where n_1, n_2 = length of codeword

k_1, k_2 = length of information block

d_{1m}, d_{2m} = minimum distance.

Assume that the C1 decoder is designed to correct up to t_1 errors and detect up to $t_1 + d_1$ errors. Similarly, the C2 decoder is designed to correct up to t_2 errors and detect up to $t_2 + d_2$ errors.

Let $g_1(x)$ be the generating polynomial of the C1 code,
 $g_2(x)$ be the generating polynomial of the C2 code,
 $c_1(x)$ be a codeword in the C1 code,
 $c_2(x)$ be a codeword in the C2 code.

If $g_1(x)$ is divisible by $g_2(x)$, i.e.

$$g_1(x) = g_2(x) g_3(x) \quad (5.2.1)$$

where $g_3(x)$ is a non-zero polynomial

then C1 is a subcode of C2.

The sum of $c_1(x)$ and $c_2(x)$ is a codeword in C2. It is because

$$\begin{aligned} c_1(x) + c_2(x) &= a_1(x)g_1(x) + a_2(x)g_2(x) \\ &= a_1(x)g_2(x)g_3(x) + a_2(x)g_2(x) \\ &= [a_1(x)g_3(x) + a_2(x)]g_2(x). \end{aligned}$$

Because $c_1(x) + c_2(x)$ is divisible by $g_2(x)$, $c_1(x) + c_2(x)$ is a codeword in the C2 code. Because C1 is a subcode of C2, d_{1m} is greater than d_{2m} and hence the error correcting

capability of the C1 code is greater than that of the C2 code.

Assume for now that C1 and C2 exist and that they can be used in the following example to encode a rate 1/2 GBT code. Example of how to find the C1 and the C2 codes will be described later. As shown in Figure 5.2.1, the encoding rule of the rate 1/2 GBT code is as follows:

1. encode I_i to give C_i using the C1 code,

$$C_i = I_i^1 \mid I_i^2 \mid P_i;$$

2. encode $I_{(i-1)}^1$ and $I_{(i-2)}^2$ to give $U_{(i-1)}^1$ and $U_{(i-2)}^2$ using the C2 code,

$$U_{(i-1)}^1 = I_{(i-1)}^1 \mid V_{(i-1)}^1;$$

$$U_{(i-2)}^2 = I_{(i-2)}^2 \mid V_{(i-2)}^2;$$

3. compute

$$Q_i = P_i + U_{(i-1)}^1 + U_{(i-2)}^2;$$

4. transmit

$$B_i = I_i^1 \mid I_i^2 \mid Q_i.$$

Since B_i is obtained by adding a codeword from C1 and two codewords from C2, B_i is a codeword in C2. As will be seen later, it is because of this special property that B_i is a codeword in C2 that the GBT codes can correct errors in the guard space.

Clearly, the encoding technique of the GBT codes is similar to that of the BT codes with the exception that the feedback information $I_{(i-1)}^1$ and $I_{(i-2)}^2$ are encoded first using the C2 code before they are added to the parity portion of the codeword from the C1 code. In the BT codes, the feedback information is not encoded.

5.2.1 Example of Finding the C1 and the C2 Codes

In the example shown in Figure 5.2.1, both C1 and C2 are rate 1/2 codes. Moreover, the information is divided into two segments for encoding using the C2 code. This puts additional condition on finding C1 and C2 because they have to satisfy $k_1/k_2 = 2$. The ratio k_1/k_2 is called the folding factor, f . The folding factor represents the

Figure 5.2.1 Example of Encoding a rate 1/2 GBT code.

Figure 5.3.1 Example of Decoding a rate 1/2 GBT code in Random Decoding Mode.

number of segments the information is divided for encoding using the C2 code. The example of finding C1 and C2 below is for the case of $f = 2$.

Consider the following two BCH codes, B1 and B2

$$B1 = (31, 21, 5)$$

$$B2 = (31, 26, 3).$$

Both the generating polynomials $g_1(x)$ and $g_2(x)$ of B1 and B2 are obtained by factorizing the same polynomial $X^{31}-1$. Hence, $g_1(x)$ and $g_2(x)$ satisfy equation (5.2.1) and B1 and B2 form the subcode relationship. More specifically, B1 is a subcode of B2.

The next step is to shorten the B1 and the B2 codes in order to satisfy the condition that $k_1/k_2 = 2$. Shortening a code is simply the process of reducing the information bits before encoding by the same generating polynomial. For example, an (n, k, d_m) code can be shortened to an $(n-b, k-b, d_m)$ code by dropping b information bits. Shortening a code does not change the minimum distance because the same generating polynomial is used. For the B1 and the B2 codes, they can be shortened to give C1 and C2 as follows:

$$C1 = (20, 10, 5)$$

$$C2 = (10, 5, 3).$$

The C1 and the C2 obtained above can now be used to encode the rate 1/2 GBT code as shown in Figure 5.2.1. Following the same approach, other pairs of C1 and C2 can be obtained by finding a suitable pair of B1 and B2 from the BCH table. Appendix C contains a complete list of C1 and C2 pairs that can be used to encode the rate 1/2 GBT code with folding factor = 2 and block size up to 1020 bits.

5.3 Example of Decoding the GBT Codes

Similar to the BT decoder, the GBT decoder has three modes of operations, they are:

- Random Error Decoding mode,
- Burst-Trapping Decoding mode,
- Good Faith Decoding mode.

Each of these decoding mode operates in a very similar fashion as that of the BT codes. The normal mode of operation is the random error decoding mode in which the information that was added to the parity is first removed and then decoded using the C1 decoder. When the C1 decoder

decides that there are more errors than it can correct, it enters into the burst-trapping mode. In the burst-trapping mode, the lost information is recovered from the guard space blocks. Guard space blocks are decoded by good faith decoding using the C2 decoder.

5.3.1 Example of Random Error Decoding Mode for the GBT Codes

Figure 5.3.1 shows the operation of the GBT decoder in the random error decoding mode for the rate 1/2 GBT code.

The GBT decoder starts with random error decoding mode when it first receives the first block $\bar{B}_0^*$. Note that although $\bar{B}_i^*$ is a codeword in the C2 code plus channel errors, $\bar{B}_0^*$ is a codeword in the C1 code plus channel errors. Block $\bar{B}_0^*$ is first decoded by the C1 decoder. Similar to that of the BT codes, as long as the C1 decoder thinks that there are t_1 or fewer errors in block $\bar{B}_0^*$, it will decode $\bar{B}_0^*$ to give

$$\bar{I}_0 = \bar{I}_0^1 \mid \bar{I}_0^2$$

and continue to decode the next block $\bar{B}_1^*$ in the random decoding mode. To decode $\bar{B}_1^*$, the information segments of $\bar{I}_0$ is first encoded using the C2 encoder to give

$$\bar{U}_0^1 = \bar{I}_0^1 \mid \bar{V}_0^1$$

The GBT decoder then computes

$$P_1' = \bar{Q}_1^* + \bar{U}_0^1,$$

$$B_1' = \bar{I}_1^1 \mid \bar{I}_1^2 \mid P_1'.$$

Block B_1' is now a codeword in the C1 code plus errors and can be decoded by the C1 decoder. Again, as long as the C1 decoder thinks there are t_1 or fewer errors in B_1' , it will decode B_1' to give

$$\bar{I}_1 = \bar{I}_1^1 \mid \bar{I}_1^2$$

and continue to decode the block B_2^* in the random decoding mode. Decoding of block B_2^* is similar to block B_1^* . Assuming that the GBT decoder continues to decode in the random decoding mode, when block B_i^* is received, it will be decoded as follows:

- (i) encode the information segments $\bar{I}_{(i-1)}^1$ and $\bar{I}_{(i-2)}^2$ using the C2 encoder

$$\bar{U}_{(i-1)} = \bar{I}_{(i-1)} \mid \bar{V}_{(i-1)},$$

$$\bar{U}_{(i-2)}^2 = \bar{I}_{(i-2)}^2 \mid \bar{V}_{(i-2)}^2,$$

- (ii) add the encoded information segments to the parity portion of B_i^* ,

$$P_i' = Q_i^* + \bar{U}_{(i-1)} + \bar{U}_{(i-2)}^2,$$

$$B_i' = \bar{I}_1^* \mid \bar{I}_1^2 \mid P_i',$$

- (iii) decode B_i' using the C1 decoder.

As long as the C1 decoder thinks that there are t_1 or fewer errors in block B_i' , the C1 decoder will decode B_i' to give $\bar{I}_i$ and continue to decode the next block $B_{(i+1)}^*$ in the random decoding mode.

Clearly, the random decoding mode of the GBT codes is similar to that of the BT codes with the exception that the pass information is first encoded using the C2 encoder before it is added to Q_i^* . Similar to the BT codes, each time when B_i' is decoded, the C1 decoder may make decoding error. When this happens, the incorrectly decoded result

will pass onto the subsequent blocks. Even when channel errors have stopped to occur, the incorrectly decoded result may cause subsequent blocks to be decoded incorrectly. The effect of error propagation due to false decoding for the GBT codes will be studied in more detail later.

In the random decoding mode, when the C1 decoder detects but cannot correct the errors in block B_i , the GBT decoder will enter into the burst-trapping decoding mode and attempt to recover $\bar{I}_i$ from the duplicated sources.

5.3.2 Example of Burst-Trapping Decoding Mode for the GBT Codes

Figure 5.3.2 shows the operation of the GBT decoder in the burst-trapping decoding mode for the rate 1/2 GBT code.

Assume that at block B_2 , the C1 decoder decides that there are more than t_1 errors. In this case, the GBT decoder will attempt to recover $\bar{I}_2$ from blocks B_3^* and B_4^* which contain the information of $\bar{I}_2$. As shown in Figure 5.3.2, the recovery of $\bar{I}_2$ in the burst-trapping mode is as follows:

Figure 5.3.2 Example of Decoding a rate 1/2 GBT code in Burst Trapping mode.

(i) decode blocks $\bar{B}_3^*$ and $\bar{B}_4^*$ to give $\bar{I}_3$ and $\bar{I}_4$ using good faith decoding,

(ii) compute the parities $\bar{P}_3^*$ and $\bar{P}_4^*$ by encoding $\bar{I}_3$ and $\bar{I}_4$ using the C1 encoder,

(iii) encode $\bar{I}_1^2$ and $\bar{I}_3^1$ using the C2 encoder to give

$$\bar{U}_1^2 = \bar{I}_1^2 \mid \bar{V}_1^2,$$

$$\bar{U}_3^1 = \bar{I}_3^1 \mid \bar{V}_3^1,$$

(iv) compute

$$\bar{U}_2^1 = \bar{I}_2^1 \mid \bar{V}_2^1 = \bar{Q}_3^* + \bar{P}_3^* + \bar{U}_1^2,$$

$$\bar{U}_2^2 = \bar{I}_2^2 \mid \bar{V}_2^2 = \bar{Q}_4^* + \bar{P}_4^* + \bar{U}_3^1,$$

(v) finally $\bar{I}_2$ is recovered from $\bar{U}_2^1$ and $\bar{U}_2^2$

$$\bar{I}_2 = \bar{I}_2^1 \mid \bar{I}_2^2.$$

Similar to the BT codes, when a block is decoded in the burst-trapping mode, all the guard space blocks will be

decoded in the good faith mode. The guard space blocks are those blocks that contain the duplicated information. For example, Blocks $\bar{B}_3^*$ and $\bar{B}_4^*$ are the guard space blocks of $\bar{B}_2^*$. The number of guard space blocks is equal to the folding factor.

It should be noted that once the GBT decoder enters into the burst-trapping decoding mode, any previously decoded information will no longer be fed back to any blocks subsequent to the guard space blocks. Hence, any error propagation due to false decoding in the random mode will stop to propagate once the GBT decoder enters into the burst-trapping mode. In order to enter into the burst-trapping mode, the C1 decoder must be designed to have error detection capability.

5.3.3 Example of Good Faith Decoding Mode for the GBT Codes

In the good faith decoding mode, block $\bar{B}_i^*$ is decoded to give $\bar{I}_i$ using the C2 decoder. As can be seen, the major difference between the GBT and the BT codes is that the received word $\bar{B}_i^*$ can be decoded by the C2 decoder to remove guard space errors.

As long as the number of errors in each guard space block is t_2 or fewer, all the guard space errors will be corrected before the guard space information is used for burst-trapping decoding. Hence, even in the presence of guard space errors, the GBT decoder is still capable of decoding correctly in the burst-trapping mode. If any one of the guard space blocks has more than t_2 errors, then the error correcting capability of the GBT decoder is exceeded. However, if the error pattern is detectable by the C2 decoder, the uncorrectable blocks can be flagged as erroneous.

Similar to the BT codes, the GBT codes can be block interleaved to increase the burst error correcting capability. Because decoding in the burst-trapping mode cannot start until all the guard space blocks are decoded by good faith decoding, the overall coding delay of the GBT code is $[fL + 1]$, where f is the folding factor and L is the block interleaving degree. ?

5.4 Summary of Error Correcting Capability of the GBT Codes

The error correcting capability of the GBT codes can be summarized as follows:

The GBT codes are capable of correcting

- (i) in the random mode, all blocks with t_1 or fewer errors.

Should a block have more than t_1 errors, it is still correctable if

- (ii) these errors are detectable with the C1 decoder and
- (iii) the block is preceded by $f - 1$ interleaved blocks that were correctly decoded and
- (iv) the block is followed by f interleaved blocks each of which contain t_2 or fewer errors.

Should a block contain more than t_2 errors in one of the subsequent f interleaved blocks, the erroneously decoded blocks may still be flagged as uncorrectable if the error pattern is detectable by the C2 decoder.

5.5 Error Propagation of the GBT Codes

For the GBT codes, the effect of error propagation is similar to that of the BT codes. However, the analysis is

far more complicated due to the fact that the information is first encoded before it is fed back and added to subsequent blocks. In the event that a decoding error is made in the random decoding mode, it has been shown [8] that if the following conditions are true, at most one block of error propagation can occur:

$$(i) \quad (e + t_1) < d_{2m} \quad (5.5.1)$$

where e is the number of channel errors in the block affected by error propagation due to decoding error,

(ii) the next f blocks contain no more than t_2 errors each.

The proof of the above theorem can be found in [8] and it requires the formal discussion of encoding and decoding the GBT codes.

Since t_1 and t_2 and d_{2m} are the design parameters of the C1 and the C2 codes, by proper choice of C1 and C2, the above conditions can be easily met and error propagation is not a problem. It should be emphasized that when a block is detected to be in error, decoding will begin in the burst-trapping mode and there can be no more error propagation.

The condition of having one block detected to be in error is not a stringent requirement and can be easily met by increasing the error detecting capability of the Cl code. In short, error propagation is not a problem with the GBT codes.

CHAPTER 6.0

DESIGN OF CODING PARAMETERS

6.1 Introduction

This chapter describes the design of the coding parameters that are used for evaluating the performance of the BT, the GBT and the BCH codes. All the codes considered in here are rate $1/2$ codes. Furthermore, the folding factor of the GBT codes is set to two. All the block codes that are used to encode the BT and the GBT codes are BCH codes.

6.2 Coding Parameters for the GBT Codes

For the rate $1/2$ GBT codes with folding factor = 2 and block size up to 1020 bits, Appendix C tabulates all the possible pairs of C1 and C2 codes that can be used. Table 6.2.1 gives the four pairs of C1 and C2 codes that are selected for encoding the GBT codes.

Since the C2 codes are used for correcting guard space errors, they are designed to have full error correcting capability with no detection. The C1 codes must be designed to have error detecting capability. Two designs are considered. One is to design C1 to have full error

C1	C2
(20, 10, 5)	(10, 5, 3)
(64, 32, 9)	(32, 16, 5)
(252, 126, 29)	(126, 63, 15)
(1020, 510, 115)	(510, 255, 53)

Table 6.2.1: The C1 and the C2 Codes Selected for Encoding the GBT Codes.

Design	C1	t_1	t_1+d_1	C2	t_2	t_2+d_2
1	(20, 10, 5)	0	4	(10, 5, 3)	1	1
2	(20, 10, 5)	1	3	(10, 5, 3)	1	1
3	(64, 32, 9)	0	8	(32, 16, 5)	2	2
4	(64, 32, 9)	2	6	(32, 16, 5)	2	2
5	(252, 126, 29)	0	29	(126, 63, 15)	7	7
6	(252, 126, 29)	7	21	(126, 63, 15)	7	7
7	(1020, 510, 115)	0	114	(510, 255, 53)	26	26
8	(1020, 510, 115)	26	88	(510, 255, 53)	26	26

Table 6.2.2: Coding Parameters for the GBT Codes.

2

MicroD

detecting capability with no correction. The other is to design C1 to have the same error correcting capability as the C2 with the remaining capability for detection.

There are two reasons to design the C1 codes to have the same error correcting capability as the C2 codes. One reason is that the design satisfies condition (5.5.1) whenever each block contains no more than t_2 errors and hence error propagation can be minimized. The other reason is motivated by the fact that the actual hardware implementation of the GBT decoder can be simplified [9].

The block interleaving degree L of the GBT codes is varied from 1 to 10 in order to evaluate the burst error correcting capability. For $L = 10$, the overall coding delay is $\lceil fL + 1 \rceil = \lceil 2 \times 10 + 1 \rceil = 21$ blocks.

Table 6.2.2 summarizes all the design parameters for the GBT codes.

6.3 Coding Parameters for the BT Codes

For the rate $1/2$ BT codes, any rate $1/2$ BCH codes can be used as the C codes for encoding. However, in order to compare the performance with GBT codes, the C codes should be the same as the C1 codes used for encoding the GBT

codes. Table 6.3.1 gives the four selected C codes for encoding the BT codes. Three of the selected C codes are the same as those C1 codes for encoding the GBT codes.

The C codes must be designed to have error detecting capability. Two designs are given to the C codes. One design is to have full error detecting capability. The other design is to have correcting and detecting capability. For comparison purposes, the error correcting and detecting capabilities are designed to be the same as those designed for the C1 codes. The exception is the (6, 3, 3) code which is designed to have full error detection only.

The block interleaving degree L of the BT codes is varied from 1 to 20 such that the results can be compared to the GBT codes. For $L = 20$, the overall coding delay is $[(b-1)L + 1] = [(2-1)20 + 1] = 21$ blocks.

Table 6.3.2 summarizes all the design parameters for the BT codes.

6.4 Coding Parameters for the BCH Codes

Since the BCH codes are used to encode the BT and the GBT codes, the performance of those selected BCH codes is also.

C
(6, 3, 3)
(20, 10, 5)
(64, 32, 9)
(252, 126, 29)

Table 6.3.1: The C Codes Selected for Encoding the BT Codes.

Design	C	t	t + d
1	(6, 3, 3)	0	2
2	(20, 10, 5)	0	4
3	(20, 10, 5)	1	3
4	(64, 32, 9)	0	8
5	(64, 32, 9)	2	6
6	(252, 126, 29)	0	28
7	(252, 126, 29)	7	21

Table 6.3.2: Coding Parameters for the BT Codes.

analyzed and results are compared to the BT and the GBT codes. All the codes are designed to have full error correcting capability with no detection. Moreover, bit interleaving is applied to the BCH codes such that the results can be compared to the BT and the GBT codes with block interleaving. Bit interleaving degree L is varied from 1 to 21 blocks. For $L = 21$, the overall coding delay is 21 blocks.

Table 6.4.1 summarizes all the design parameters for the BCH codes.

6.5 Measurements of System Performance

The system performance is measured in terms of block error rate instead of bit error rate. The block error rate measurement is preferred because of the following reasons.

- Over the land mobile channel, data are usually transmitted in packets and some hybrid control schemes [10] are used. When a packet is received in error, regardless of how many error bits are in the packet, the corrupted packet will be discarded and retransmitted. For this type of systems, block error rate measurement is more meaningful and indicative than bit error rate measurement.

Design	BCH	t	t + d
1	(6, 3, 3)	1	1
2	(20, 10, 5)	2	2
3	(64, 32, 9)	4	4
4	(252, 126, 29)	14	14
5	(1020, 510, 115)	57	57

Table 6.4.1: Coding Parameters for the BCH Codes.

- Block error rate measurement simplifies the decoding algorithms and allows more parameters to be analyzed.

- Comparative system performance in terms of block error rate should give a good indication of the comparative system performance in terms of bit error rate.

Appendix B gives the simulation algorithms that are used to measure the block error rate performance of the BT, the GBT and the BCH codes. For the BT and the GBT codes, the simulation gives upper bound and lower bound results.

CHAPTER 7.0

PERFORMANCE RESULTS

7.1 Introduction

This chapter presents the simulation results obtained using both the simulated data and the experimental data. The first part of this chapter discusses the burst error statistics of the simulated Rayleigh fading channel and compares the results to those obtained using the experimental data. Both the bit error rate and the block error rate of the simulated channel are also discussed and the results are compared to the theoretical bit error rate of the Rayleigh fading and the non-fading channels. Finally, the performance of the BCH, the BT and the GBT codes over the Rayleigh fading channel are discussed and the results are compared.

7.2 Uncoded Channel Performance

For the land mobile channel, because of deep fading, errors occur in burst as well as in random. To characterize a channel with burst errors, it is necessary to analyze the burst error statistics. The burst error statistics of the

experimental data have been studied and the results are reported in [3, 13]. The work in here is to determine the burst error statistics of the simulated data and compare the results to those of the experimental data.

Figures 7.2.1 explains the definitions of burst interval, error free interval and burst repetitive interval. A burst interval is defined as the interval that begins with an error bit, ends with an error bit and does not contain M or more consecutive correct bits in between any two adjacent error bits. An error free interval is defined as the interval in between two burst intervals. A burst repetitive interval is defined as the interval between the beginnings of two consecutive burst intervals. Hence, within a burst interval, the maximum number of consecutive correct bits is $M - 1$. Moreover, the minimum length of an error free interval is M bits.

The choice of M is critical because all burst error statistics are dependent on M . If M is too large, multiple bursts can be counted as one single burst. Value of M as small as one [14] and as large as 50 [15] have been used. The works reported in [3] using the experimental data have indicated that error free runs of 10 bits or longer rarely exist in a burst and the value of M used in [3] is 10. For

Figure 7.2.1 Definitions of Burst Interval, Error Free Interval and Burst Repetitive Interval, where "0" Represents Correct Bit and "1" Represents Error Bit.

comparison purposes, the value of M adopted in this thesis is also 10.

For the simulated and the experimental data, the following burst error statistics are analyzed:

- (i) burst interval distribution,
- (ii) burst repetitive interval distribution,
- (iii) percentage of errors in a given block length distribution.

7.2.1 Burst Interval Distribution

The burst interval statistics are shown in the form of cumulative distribution function $P_B(\geq b)$ which is defined as the probability that the burst interval will last for b bits or more. Figures 7.2.2(a) and 7.2.2(b) show the distribution $P_B(\geq b)$ of the simulated data at the vehicular speeds of 40 and 100 km/hr respectively, with the S/N as a parameter. Figure 7.2.2(c) shows the distribution $P_B(\geq b)$ of the experimental data, with the geographical region as a parameter.

SPEED = 40 KM/HR

Figure 7.2-2(a) Burst Interval Distributions
at Speed = 40 km/hr.

Figure 7.2.2(b) Burst Interval Distributions
at Speed = 100 km/hr.

Figure 7.2.2(c) Burst Interval Distribution of Ottawa, Hull and Queensway.

Tables 7.2.1 to 7.2.3 give the mean, m_B , and the standard deviation, σ_B , of the burst interval statistics.

Additionally, Tables 7.2.1 and 7.2.2 also give the mean and the standard deviation of the error free interval and the burst repetitive interval statistics for the simulated data.

From the distributions $P_B(>b)$ of the simulated data shown in Figures 7.2.2(a) and 7.2.2(b), the following observations can be made.

- When S/N increases, the burst interval decreases. This result is as expected. At low S/N, the burst intervals can be as long as hundreds of bits. At high S/N, the burst intervals are typically confined to within tens of bits.
- For the same S/N, the burst interval at 40 km/hr is longer than that at 100 km/hr. This result is also as expected because at a lower speed, the duration of fade is longer, consequently the burst interval lasts longer.

From Tables 7.2.1 and 7.2.2, the following observations can be made.

	40 km/hr			100 km/hr		
S/N (dB)	m_B (bits)	m_F (bits)	m_R (bits)	m_B (bits)	m_F (bits)	m_R (bits)
0	91	14	106	88	13	101
5	30	25	54	23	21	44
10	16	56	72	9	38	48
15	10	132	143	5	87	91
20	6	305	311	3	251	254
25	3	797	801	2	897	899
30	2	2497	2499	1	3267	3269

Table 7.2.1: Mean Values of the Burst Interval (m_B), the Error Free Interval (m_F) and the Burst Repetitive Interval (m_R) for the Simulated Data.

	40 km/hr			100 km/hr		
S/N (dB)	σ_B (bits)	σ_F (bits)	σ_R (bits)	σ_B (bits)	σ_F (bits)	σ_R (bits)
0	126	7	126	113	5	113
5	41	21	45	32	12	33
10	21	54	57	13	34	34
15	12	126	126	6	90	90
20	7	291	291	4	263	263
25	4	788	788	2	895	895
30	3	2468	2468	1	3165	3165

Table 7.2.2: Standard Deviations of the Burst Interval (σ_B), the Error Free Interval (σ_F) and the Burst Repetitive Interval (σ_R) for the Simulated Data.

- Both m_B and σ_B decrease with increasing S/N. Both m_B and σ_B at 40 km/hr are larger than those at 100 km/hr. These results are in agreement with those observed above.
- The ratio of m_B at 40 km/hr and m_B at 100 km/hr changes with S/N. The ratio is smaller at low S/N, indicating that the burst interval is less sensitive to vehicular speed at low S/N. The ratio of σ_B at 40 km/hr and σ_B at 100 km/hr gives the same result.
- When S/N decreases, σ_B increases faster than m_B , indicating that the boundary of error bursts is more difficult to define at low S/N. This observed result is also reported in [3] using the experimental data.

From Figure 7.2.2(c) and Table 7.2.3 which give the burst interval statistics of the experimental data, the following observations can be made.

- The burst interval distributions $P_B(\geq b)$ of the experimental data have similar patterns as those of the simulated data, indicating that their burst interval statistics are similar.

Ottawa		Hull		Queensway	
m_B (bits)	σ_B (bits)	m_B (bits)	σ_B (bits)	m_B (bits)	σ_B (bits)
8.5	9.0	10.4	14.0	9.8	13.8

Table 7.2.3: Mean (m_B) and Standard Deviation (σ_B) of the Burst Interval for the Experimental Data.

- Taking the assumption that the Ottawa and the Hull measurements were conducted at a constant speed of 40 km/hr, and that of Queensway at 100 km/hr, it is possible to estimate the signal to noise ratio of these measurements by comparing the distribution curves $P_B(>b)$ and the mean and standard deviation statistics to those of the simulated data. The comparison indicates that the signal to noise ratios of the Ottawa, the Hull and the Queensway measurements are approximately in the neighbourhood of 20 dB, 15 dB and 10 dB respectively. These estimated results are in general agreement with those reported in [3].

7.2.2 Burst Repetitive Interval Distribution

The burst repetitive interval statistics are shown in the form of cumulative distribution function $P_R(>b)$ which is defined as the probability that the burst repetitive interval will last for b bits or more. Figures 7.2.3(a) and 7.2.3(b) show the distributions $P_R(>b)$ of the simulated data at the vehicular speeds of 40 km/hr and 100 km/hr respectively, with the S/N as a parameter. Figure 7.2.3(c) shows the distribution $P_R(>b)$ of the experimental data, with the geographical region as a parameter. The mean, m_R , and the standard deviation, σ_R , of the burst repetitive

SPEED = 40 KM/HR

Figure 7.2.3(a) Burst Repetitive Interval
Distributions at Speed = 40 km/hr.

SPEED = 100 KM/HR

Figure 7.2.3(b) Burst Repetitive Interval
Distributions at Speed = 100 km/hr.

Figure 7.2.3(c) Burst Repetitive Interval Distribution of Ottawa, Hull, and Queensway.

interval for the simulated data are given in Tables 7.2.1 and 7.2.2.

From Figures 7.2.3(a) and 7.2.3(b) and Tables 7.2.1 and 7.2.2 which give the burst repetitive interval statistics of the simulated data, the following observations can be made.

- Except for the case of $S/N = 0$ dB, when S/N increases, the burst repetitive interval increases. At high S/N , the burst repetitive intervals can be as long as thousands of bits. At low S/N , the burst repetitive intervals are generally confined to within hundreds of bits.
- At $S/N = 0$ dB, the burst repetitive intervals are longer than those at $S/N = 5$ and 10 dB. The reason is due to long burst interval at $S/N = 0$ dB. From Tables 7.2.1 and 7.2.2, it can be seen that at $S/N = 0$ dB, the mean burst interval, m_B is substantially larger than the mean burst repetitive interval, m_R , at $S/N = 5$ and 10 dB. Consequently, the mean burst repetitive intervals are longer at 0 dB than those at 5 and 10 dB.
- Except for the case of $S/N = 25$ and 30 dB, for the same S/N , the burst repetitive interval at 40 km/hr is longer

than that at 100 km/hr. This result is as expected because at a lower speed, both the fade and the non-fade intervals are longer.

- For the case of $S/N = 25$ and 30 dB, the burst repetitive interval at 100 km/hr is longer than that at 40 km/hr. This result is opposite to that observed above. The reason, according to the opinion of the author, is the result of some slight discrepancy between the simulated data files at 100 km/hr and at 40 km/hr. As will be seen later, the bit error rate of the simulated data at 100 km/hr is lower than that at 40 km/hr, indicating that there is some slight discrepancy between these two data files.

- When dealing with Rayleigh fading channel, it is generally assumed that the burst interval and the error free interval are uncorrelated. The results given in Tables 7.2.1 and 7.2.2 show that the above assumption is quite valid. Since

$$R = B + F$$

where R , B and F are the random variables of the burst repetitive interval, the burst interval and the error free interval respectively,

B and F are uncorrelated if and only if

$$\sigma_R^2 = \sigma_B^2 + \sigma_F^2 \quad (7.2.1)$$

Within statistical accuracy, it can be seen that equation (7.2.1) is generally satisfied by all the results given in Table 7.2.2. The results given in Table 7.2.1 also satisfy $m_R = m_B + m_F$.

From Figure 7.2.3(c) which gives the burst repetitive interval statistics of the experimental data, the following observations can be made.

- For Ottawa and Hull, the distribution patterns are similar to those of the simulated data, although they do not seem to have a perfect fit.
- For Queensway, the distribution pattern is substantially different from those of the simulated data, indicating that the channel characteristic of Queensway is substantially different from that of the simulation model.

7.2.3 Percentage of Errors in a Given Block Length Distribution

The probability of finding m errors in a block length of n bits, $P(m,n)$, is a useful parameter for designing error

control coding. Because the distribution of $P(m,n)$ is an absolute count of the number of errors, a more useful parameter is the probability of finding m/n errors in a block length of n , $P(m/n,n)$. The cumulative distribution function $P(>m/n,n)$ is the probability of finding the percentage of errors equal to or exceeding m/n in a given block length n .

The significance of $P(>m/n,n)$ is illustrated as follows. For a given (n,k,d_m) code, assume it is designed to correct t number of errors. Since t/n is the percentage of errors this (n,k,d_m) code can correct, the block error rate performance of this code is given by $P(>m/n,n)$ with $m/n = (t+1)/n$.

For the simulated data, Figures 7.2.4(a) to 7.2.4(g) show the distributions $P(>m/n,n)$ for the various S/N at 40 km/hr, with the block length n as a parameter. Figures 7.2.5(a) to 7.2.5(g) show the distributions $P(>m/n,n)$ for the various S/N at 100 km/hr, with the block length n as a parameter. For the experimental data, Figures 7.2.6(a) to 7.2.6(c) show the distributions $P(>m/n,n)$ for the three geographical regions, with the block length n as a parameter.

SPEED = 40 KM/HR

S/N = 0 dB

Figure 7.2.4(a) The $P(\geq m/n, n)$ Distributions at Speed = 40 km/hr and S/N = 0 dB.

SPEED = 40 KM/HR

S/N = 5 dB

BLOCK SIZE = 6 BITS
 BLOCK SIZE = 20 BITS
 BLOCK SIZE = 64 BITS
 BLOCK SIZE = 252 BITS
 BLOCK SIZE = 1020 BITS

Figure 7.2.4(b) The $P(\geq m/n, n)$ Distributions at Speed = 40 km/hr and S/N = 5 dB.

SPEED = 40 KM/HR

S/N = 10 dB

- BLOCK SIZE = 6 BITS
- BLOCK SIZE = 20 BITS
- · - · - BLOCK SIZE = 64 BITS
- BLOCK SIZE = 252 BITS
- BLOCK SIZE = 1020 BITS

Figure 7.2.4(c) The $P(\geq m/n, n)$ Distributions at
Speed = 40 km/hr and S/N = 10 dB.

SPEED = 40 KM/HR

S/N = 15 dB

Figure 7.2.4(d) The $P(\geq m/n, n)$ Distributions at
Speed = 40 km/hr and S/N = 15 dB.

SPEED₁ = 40 KM/HR

S/N = 20 dB

Figure 7.2.4(e) The $P(\geq m/n, n)$ Distributions at
Speed = 40 km/hr and S/N = 20 dB.

SPEED = 40 KM/HR

S/N = 25 dB

———— BLOCK SIZE = 6 BITS
----- BLOCK SIZE = 20 BITS
- · - · - BLOCK SIZE = 64 BITS
- - - - - BLOCK SIZE = 252 BITS
———— BLOCK SIZE = 1020 BITS

Figure 7.2.4(f) The $P(\geq m/n, n)$ Distributions at
Speed = 40 km/hr and S/N = 25 dB.

SPEED = 40 KM/HR

S/N = 30 dB

*
 ————— BLOCK SIZE = 6 BITS
 - - - - - BLOCK SIZE = 20 BITS
 - · - · - BLOCK SIZE = 64 BITS
 - - - - - BLOCK SIZE = 252 BITS
 - - - - - BLOCK SIZE = 1020 BITS

Figure 7.2.4(g) The $P(\geq m/n, n)$ Distributions at
 Speed = 40 km/hr and S/N = 30 dB.

SPEED = 100 KM/HR

S/N = 0 dB

————— BLOCK SIZE = 6 BITS
- - - - - BLOCK SIZE = 20 BITS
- . - . - BLOCK SIZE = 64 BITS
- - - - - BLOCK SIZE = 252 BITS
- - - - - BLOCK SIZE = 1020 BITS

Figure 7.2.5(a) The $P(\geq m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 0 dB.

SPEED = 100 KM/HR

S/N = 5 dB

————— BLOCK SIZE = 6 BITS
----- BLOCK SIZE = 20 BITS
- · - · - BLOCK SIZE = 64 BITS
- - - - - BLOCK SIZE = 252 BITS
————— BLOCK SIZE = 1020 BITS

Figure 7.2.5(b) The $P(>m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 5 dB.

SPEED = 100 KM/HR

S/N = 10 dB

BLOCK SIZE = 6 BITS
 BLOCK SIZE = 20 BITS
 BLOCK SIZE = 64 BITS
 BLOCK SIZE = 252 BITS
 BLOCK SIZE = 1020 BITS

Figure 7.2.5(c) The $P(\geq m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 10 dB.

SPEED = 100 KM/HR

S/N = 15 dB

Figure 7.2.5(d) The $P(>m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 15 dB.

SPEED = 100 KM/HR

S/N = 20 dB

Figure 7.2.5(e) The $P(\geq m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 20 dB.

SPEED = 100 KM/HR

S/N = 25 dB

————— BLOCK SIZE = 6 BITS
- - - - - BLOCK SIZE = 20 BITS
- . - . - BLOCK SIZE = 64 BITS
- - - - - BLOCK SIZE = 252 BITS
- - - - - BLOCK SIZE = 1020 BITS

Figure 7.2.5(f) The $P(\geq m/n, n)$ Distributions at
Speed = 100 km/hr and S/N = 25 dB.

SPEED = 100 KM/HR

S/N = 30 dB

Figure 7.2.5(g) The $P(\geq m/n, n)$ Distributions at Speed = 100 km/hr and S/N = 30 dB.

OTTAWA

Figure 7.2.6(a) The $P(\geq m/n, n)$ Distributions of Ottawa.

Figure 7.2.6(b) The $P(\geq m/n, n)$ Distributions of Hull.

QUEENSWAY

Figure 7.2.6(c),- The $P(\geq m/n, n)$ Distributions of Queensway.

From Figures 7.2.4(a) to 7.2.4(g) and Figures 7.2.5(a) to 7.2.5(g) which give the distributions $P(>m/n, n)$ of the simulated data, the following observations can be made.

- When S/N increases, the probability of finding m/n or more errors decreases.
- When block size n increases, the $P(>m/n, n)$ distribution becomes steeper.
- For the same S/N, the distribution curves intersect each other. Let the m/n value at which two distribution curves intersect be called the threshold value of these two curves. The threshold value decreases when S/N increases.
- For the same S/N, the probability of finding m/n or more errors is lower for a larger n than for a smaller n , provided that the m/n ratio exceeds the threshold value.
- For the same S/N, the probability of finding m/n or more errors is lower at 100 km/hr than at 40 km/hr, provided that the m/n ratio exceeds the threshold value.

There are several implications that can be drawn from the last two observations.

- The second last observation above indicates that given two codes which are designed to correct t/n errors, the one with a larger block size will perform better than the one with a smaller block size, provided that the t/n ratio exceeds the threshold value. Loosely speaking, the result indicates that for a Rayleigh fading channel, a code with a larger block size will perform better.
- The second last observation also indicates that given two codes which are designed to achieve the same block error rate performance, the one with a larger block size can have a higher code rate than the one with a smaller block size, provided that the t/n ratio exceeds the threshold value. Loosely speaking, a code with a larger block size will give a higher system throughput because the code rate can be made higher.
- The last observation above indicates that a given block code will give a better performance at a higher vehicular speed than at a lower vehicular speed, provided that the t/n ratio exceeds the threshold value. Loosely speaking, the result indicates that for a coded system, the system performance will improve when the

vehicular speed goes up. It should be stressed in here that the bit error rate of the 100 km/hr data files is lower than that of the 40 km/hr data files. Although the result appears to indicate that a code will perform better at a higher speed, it is uncertain whether the improvement observed in here is totally because of the higher speed or may be because of the lower bit error rate.

From Figures 7.2.6(a) to 7.2.6(c) which give the distributions $P(\gamma_m/n, n)$ of the experimental data, the following observations can be made.

- For Ottawa and Hull, the distribution patterns are similar to those of the simulated data. In particular, the Ottawa and Hull distributions resemble those shown in Figures 7.2.4(e) and 7.2.4(d) which give the distributions of the simulated data at $S/N = 20$ and 15 dB respectively. These observed S/N values are in agreement with those observed previously.
- For Queensway, the distribution patterns are substantially different from those of the simulated data, indicating that the channel characteristic of Queensway is substantially different from that of the

simulation mode. This observed result has also been seen earlier.

7.2.4 Error Rate of the Uncoded Channel

Using the simulated data files, the uncoded channel block error rates (BLER) are measured for block sizes of 1, 6, 20, 64, 252 and 1020 bits. For block size of 1 bit, the BLER is the bit error rate. The measured BLER are shown in Figures 7.2.7(a) and 7.2.7(b) together with the theoretical bit error rates for the non-fading and the Rayleigh fading channels.

For non-coherent binary frequency shift keying (NBFSK) modulation, the average bit error rate in the presence of additive white gaussian noise (AWGN) is given by [16]

$$P_{\text{AWGN}}(s) = \frac{1}{2} \exp\left(-\frac{s}{2}\right) \quad (7.2.2)$$

where s is the signal to noise ratio.

For the Rayleigh fading channel, the average bit error rate using NBFSK Modulation is given by [16]

$$P_{\text{RAYLEIGH}}(s) = \frac{1}{2+s} \quad (7.2.3)$$

Equations (7.2.2) and (7.2.3) are the theoretical bit error rates for the non-fading and the Rayleigh fading channels

SPEED = 40 KM/HR

— RAYLEIGH FADING CHANNEL (SIMULATED)
- - - RAYLEIGH FADING CHANNEL (THEORETICAL)
- · - NON-FADING CHANNEL (THEORETICAL)

Figure 7.27(a) Uncoded Channel Error Rate at Speed = 40 km/hr.

SPEED = 100 KM/HR

— RAYLEIGH FADING CHANNEL (SIMULATED)
- - - RAYLEIGH FADING CHANNEL (THEORETICAL)
- - - NON-FADING CHANNEL (THEORETICAL)

Figure 7.2.7(b) Uncoded Channel Error Rate at Speed = 100 km/hr.

respectively. They are plotted together with the measured BLER as shown in Figures 7.2.7(a) and 7.2.7(b). From Figures 7.2.7(a) and 7.2.7(b), the following observations can be made.

- The bit error rate of the fading channel is much higher than that of the non-fading channel, indicating that system performance will be substantially degraded in a land mobile channel environment.
- When block size increases, the BLER increases. When S/N increases, the BLER decreases.
- For both 40 km/hr and 100 km/hr, the measured bit error rates are slightly lower than the theoretical bit error rate. The discrepancy is due to some slight dc offset during data acquisition using the A/D converter. An examination of the mean value of the gaussian distributed data files indicates that the mean is not truly zero in each file. Because of the small dc offset, which gives a Rician distributed channel with a small line of sight component, the resultant bit error rate is lower than the theoretical bit error rate. The dc offset is higher for the 100 km/hr data files, resulting in a lower bit error rate when compared to that of the 40 km/hr. Within equipment accuracy and in

light of the small discrepancy, the data generated by the simulation model do approximate the Rayleigh distributed fading channel closely.

Comparison of the BLER between the two vehicular speeds indicates that for the same block size, the BLER curves are approximately the same and are within approximately 2 dB of each other. This observation appears to suggest that the uncoded channel block error rate is not sensitive to vehicular speed. There is a cross over of BLER performance when the S/N increases. When S/N increases from 0 to 30 dB, the 100 km/hr BLER is higher and then becomes lower than the 40 km/hr BLER. The reason for this cross over of BLER performance may be due to the lower bit error rate of the 100 km/hr data files.

7.3 BCH Coded Channel Performance

Using the simulated data, the block error rate results of the BCH codes are shown in Figures 7.3.1(a) to 7.3.1(b), and Figures 7.3.2 and 7.3.3. Additional results including results using the experimental data are shown together with the results of the BT and the GBT codes and they will be discussed later.

SPEED = 40 KM/HR

Figure 7.3.1(a) Performance Result of the BCH Codes at 40 km/hr with Block Size as Parameter.

SPEED = 100 KM/HR

Figure 7.3.1(b) Performance Result of the BCH Codes at Speed = 100 km/hr with Block Size as Parameter.

Speed = 40 Km/Hr

Figure 7.3.2 Performance Result of the BCH Codes at Speed = 40 km/hr with Delay as Parameter.

Speed = 100 Km/Hr

Figure 7.3.3 Performance Result of the BCH Codes at Speed = 100 km/hr with Delay as Parameter.

Figures 7.3.1(a) and 7.3.1(b) show the block error rate results of the BCH codes at 40 km/hr and 100 km/hr, with the block size as a parameter. These results are measured with no interleaving applied. Figures 7.3.2 and 7.3.3 show the results at 40 km/hr and 100 km/hr for the four different block sizes of 6, 20, 64 and 252 bits, with the bit interleaving degree, L , as a parameter. For $L = 1$, the BCH code is not interleaved and the coding delay is one block. For an interleaving degree L , the coding delay is L blocks.

From Figures 7.3.1(a) and 7.3.1(b), the following observations can be made.

- There is a cross over of block error rate performance when the block size is increased. Below a certain S/N ratio, the BCH code with a smaller block size performs better. Above that S/N ratio, the BCH code with a larger block size performs better. This result comes as no surprise. As have been observed previously with the $P(\geq m/n, n)$ distributions, it has been stipulated that code with a larger block size will perform better as long as the t/n ratio exceeds the threshold value. When S/N increases, the threshold value decreases, consequently the t/n ratio will exceed the threshold

value after the S/N is increased above a certain value.

Table 7.3.1 gives the t/n ratio of the BCH codes.

Comparison of the t/n ratio with the threshold value shown in the $P(>m/n, n)$ distributions indicates that the cross over of block error rate performance occurs after the t/n ratio exceeds the threshold value.

- For the same BCH code, there is a cross over of block error rate performance between the two vehicular speeds. Below a certain S/N ratio, the performance at the 40 km/hr is better. Above that S/N ratio, the performance at the 100 km/hr is better. This result, again, is attributed to the relationship between the t/n ratio and the threshold value of the $P(>m/n, n)$ distributions. As have been discussed before, this cross over of block error rate performance may also be because of the lower bit error rate of the 100 km/hr data files.

From Figures 7.3.2 and 7.3.3, the following observations can be made.

- For the same BCH code, there is a cross over of block error rate performance when the bit interleaving degree is increased. Below a certain S/N ratio, the BCH code with a lower interleaving degree performs better. Above

that S/N ratio, the BCH code with a higher interleaving degree performs better. Careful study has revealed that as long as the t/n ratio exceeds the uncoded channel bit error rate, code with a higher interleaving degree will tend to have a better performance. Table 7.3.2 gives the uncoded channel bit error rate. Comparison of the t/n ratio to the uncoded channel bit error rate indicates that the cross over of performance occurs after the t/n ratio exceeds the uncoded channel bit error rate. The reason for this cross over can be further explained as follows. With interleaving, the characteristic of a bursty channel approaches that of a random channel. After the bursty channel is randomized with interleaving, the percentage of error in a given block is approximately equal to the uncoded channel bit error rate. If the percentage of error is less than t/n , the code will correct these errors and result in an overall lower block error rate. However, if the percentage of error is more than t/n , there will be more blocks containing uncorrectable errors than before the randomization, and consequently, the overall block error rate goes up.

n (bits)	t (bits)	t/n
6	1	0.1667
20	2	0.1000
64	4	0.0625
252	14	0.0556
1020	57	0.0559

Table 7.3.1: The t/n Ratio of the BCH Codes, Where t is the Number of Errors the Code can Correct and n is the Block Size.

S/N (dB)	BER	
	40 km/hr.	100 km/hr.
0	0.3321	0.3333
5	0.1912	0.1916
10	0.8118×10^{-1}	0.7992×10^{-1}
15	0.2840×10^{-1}	0.2578×10^{-1}
20	0.8959×10^{-2}	0.6697×10^{-2}
25	0.2462×10^{-2}	0.1535×10^{-2}
30	0.5892×10^{-3}	0.3575×10^{-3}

Table 7.3.2: Uncoded Channel Bit Error Rate

7.4 BT Coded Channel Performance

Using the simulated data and the experimental data, the block error rate performance of the BT codes is analyzed. The results are shown together with the results of the BCH codes such that the relative performance between these two codes can be compared.

The performance of the BT and the BCH codes is compared on the basis of equal overall coding delay. For the BCH codes, bit interleaving degree L gives an overall coding delay of L blocks. For the BT codes which are rate $1/2$ codes, block interleaving degree L gives an overall coding delay of $L + 1$ blocks. The performance results of the BT codes are given in the form of upper and lower bounds.

The design parameters for the BT codes have been discussed in Chapter 6.3. For the C code that is designed to have full error detecting capability, the BT code is called BT code with no correction. For the C code that is designed to have correcting and detecting capability, the BT code is called BT code with correction. The BCH codes under comparison are the same as the C codes used for encoding the BT codes.

Figure 7.4.1 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and Delay = 3 Blocks.

Speed = 40 Km/Hr

Delay = 9 Blocks

Figure 7.4.2 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and Delay = 9 Blocks.

Speed = 40 Km/Hr

Delay = 15 Blocks

Figure 7.4.3 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and Delay = 15 Blocks.

Figure 7.4.4 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and Delay = 21 Blocks.

Speed = 100 Km/Hr

Delay = 3 Blocks

Figure 7.4.5 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and Delay = 3 Blocks.

Figure 7.4.6 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and Delay = 9 Blocks.

Figure 7.4.7 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and Delay = 15 Blocks.

Figure 7.4.8 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and Delay = 21 Blocks.

Figure 7.4.9 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and S/N = 10 dB.

Figure 7.4.10 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and S/N = 15 dB.

Figure 7.4.11 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and S/N = 20 dB.

Figure 7.4.12 Performance Result of the BT and the BCH Codes at Speed = 40 km/hr and S/N = 25 dB.

Figure 7.4.13 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and S/N = 10 dB.

Figure 7.4.14 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and S/N = 15 dB.

Figure 7.4.15 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and S/N = 20 dB.

Figure 7.4.16 Performance Result of the BT and the BCH Codes at Speed = 100 km/hr and S/N = 25 dB.

Figure 7.4.17 Performance Result of the BT and the BCH Codes at Location = Ottawa.

Figure 7.4.18 Performance Result of the BT and the BCH Codes at Location = Hull.

Figure 7.4.19 Performance Result of the BT and the BCH Codes at Location = Queensway.

Figures 7.4.1 to 7.4.16 show the performance results of the BT and the BCH codes using the simulated data. Figures 7.4.17 to 7.4.19 show the performance results of these two codes using the experimental data.

Figures 7.4.1 to 7.4.8 show the performance of the BT and the BCH codes when the S/N ratio is increased. The performance shown in these figures are for the coding delay of 3, 9, 15 and 21 blocks and for the vehicular speed of 40 and 100 km/hr. From these figures, the following observations can be made.

- For the BT codes with no correction, when the block size is increased, the performance of these codes decreases. This result comes as no surprise. It is because the performance of the BT codes with no correction relies on error free guard space blocks for correcting errors. In the presence of both burst and random errors, when the block size is increased, the probability of having error free guard space blocks decreases. Consequently, the performance of the BT codes with no correction decreases with increasing block size.
- For the BT codes with correction, there is a cross over of block error rate performance when the block size is

increased. This cross over effect is similar to that of the BCH codes discussed previously. Below a certain S/N ratio, the BT code with a smaller block size performs better. Above that S/N ratio, the BT code with a larger block size performs better. The reason can be attributed to the error correcting capability of the C codes. The cross over occurs at a much higher S/N ratio than that of the BCH codes. It is because the error correcting capability of the C codes is less than that of the BCH codes.

- For the block size of 64 and 252 bits, the BT codes with correction perform better than the BT codes with no correction. This result implies that error propagation is not a problem with the BT codes. It is because the probability of error propagation is higher when the error detecting capability of the C code is reduced. However, the result indicates that even when the error detecting capability of the C code is reduced, the overall result is an improvement instead of degradation, implying that error propagation is not a problem with the BT codes. The above result also indicates that a Rayleigh fading channel does not produce burst errors alone.

- For the same BT code, there is a cross over of block error rate performance between the two vehicular speed as the S/N ratio is increased. The result is similar to that of the BCH codes discussed earlier. However, the cross over of the BT codes occurs at a much higher S/N ratio than that of the BCH codes. As have been discussed before, the lower bit error rate of the 100 km/hr data files may contribute to this cross over of block error rate performance.
- Comparison of the BT codes to the BCH codes indicates that the BCH codes perform better than the BT codes. The difference in performance goes up when block size goes up. For block size = 6 bits, the performance between these two codes is about the same.

Figures 7.4.9 to 7.4.16 show the performance of the BT and the BCH codes when the bit and the block interleaving degrees are increased. The performance shown in these figures are for the S/N ratio of 10, 15, 20 and 25 dB and for the vehicular speed of 40 and 100 km/hr. From these figures, the following observations can be made.

- For the BT codes with block size of 64 and 252 bits, the results indicate that there are no significant changes in performance when the block interleaving degree is

increased. It should be noted that because of statistical accuracy, curves that have a lower Block error rate will fluctuate more than those that have a higher block error rate.

- Let T be the time interval the vehicle has travelled a distance of $\lambda/2$, where λ is the wavelength. Let N_T be the number of data transmitted over the time interval T . At 40 km/hr, $N_T = 79$ bits. At 100 km/hr, $N_T = 39$ bits.
- For the BT codes with block size of 6 and 20 bits, the results indicate that the performance cycles up and down when the block interleaving degree is increased. The cycling effect is reduced when the block interleaving degree is high. Moreover, the separation between each cycle is equal to N_T bits, where N_T is as defined above. To elaborate more, Figure 7.4.9(b) shows the performance of the BT codes for the block size of 20 bits and at the speed of 40 km/hr. From this figure, the peaks of these curves occur at the delay of 6 and 10 blocks. The separation of these peaks is 4 blocks, or $4 \times 20 = 80$ bits. For the case of 100 km/hr and block size of 6 bits, Figure 7.4.13(a) shows that the peaks occur at the delay of 7 and 13 blocks. The separation of these peaks is 6 blocks, or $6 \times 6 = 36$ bits. Hence,

the separation between these cycles is approximately equal to N_T . Because the performance cycles over a period of $\lambda/2$, this result also indicates that fading occurs at a rate of every $\lambda/2$.

- The results observed above indicate that if the block size is larger than N_T bits, the performance of the BT codes will not change with block interleaving. On the other hand, if the block size is smaller than N_T bits, the performance will cycle up and down at a rate equal to N_T bits when the block interleaving degree is varied. Moreover, if the block size is smaller than N_T bits, the performance of the BT codes will change with the vehicular speed. Hence, these results suggest that the choice of block size for the BT codes should be as large as possible. However, the previous result has indicated that as the block size goes up, the performance of the BT codes can go down. Hence, care must be exercised when choosing the block size for the BT codes.

- For the performance curves that cycle up and down, the location of the peaks and the troughs occurs at a fixed interleaving degree and does not change with S/N ratio. This result indicates that the mean and the standard

deviation of the burst statistics are not applicable in designing the block interleaving degree for the BT codes.

- When the bit interleaving degree is increased, the performance results of the BCH codes agree with what have been discussed before. Moreover, the results indicate that the BCH codes perform better than the BT codes. Except for the case of $S/N = 10$ dB, the difference in performance between these two codes increases when the coding delay is increased.

Figures 7.4.17 and 7.4.19 show the performance of the BT and the BCH codes using the experimental data. From these figures, which show the performance as a function of coding delay, the following observations can be made.

- For Ottawa and Hull, the performance results are similar to those obtained using the simulated data. More specifically, for the BCH codes, the performance improves as the bit interleaving degree is increased. The improvement is higher for a larger block size. For the BT codes with block size of 64 and 252 bits, the performance does not seem to improve very much when the block interleaving degree is increased.

- Both the results of Ottawa and Hull indicate that for the block size of 252 bits, the BT code with correction performs better than the BT code with no correction. This result is in agreement with that obtained using the simulated data.
- The results of Ottawa indicate that for the block size of 64 and 252 bits, the BCH codes perform better than the BT codes. The results of Hull indicate that for the block size of 252 bits, the BCH code also performs better than the BT codes. These results are in agreement with those obtained using the simulated data.
- For Queensway, the performance results are totally different. The results indicate that the BT codes perform better than the BCH codes. The result comes as a little surprise. However, it has been seen that the channel characteristics of Queensway are substantially different from those of the simulated data and from those of Ottawa and Hull. Because of the difference in channel characteristics, the results of Queensway are not contradicting any results obtained previously.

7.5 GBT Coded Channel Performance

Using the simulated data and the experimental data, the performance of the GBT codes is analyzed. Similar to those of the BT codes, the performance results are shown together with the BCH codes such that the relative performance between the BCH codes as well as the BT codes can be compared.

Similar to that of the BT codes, the performance of the GBT and the BCH codes is compared on the basis of equal overall coding delay. For the GBT codes, which are rate $1/2$ codes with folding factor of two, block interleaving degree L gives an overall coding delay of $2L + 1$ blocks. The performance results of the GBT codes are given in the form of upper and lower bounds.

The design parameters for the GBT codes have been discussed in Chapter 6.2. For the C1 code that is designed to have full error detecting capability, the GBT code is called the GBT code with no correction. For the C1 code that is designed to have correcting and detecting capability, the GBT code is called the GBT code with correction. The BCH codes under comparison are the same as the C1 codes used for encoding the GBT codes.

Figures 7.5.1 to 7.5.16 show the performance results of the GBT and the BCH codes using the simulated data. Figures 7.5.17 to 7.5.19 show the performance results of these two codes using the experimental data. It should be noticed that the block sizes for the GBT codes are 20, 64, 252 and 1020 bits. The block sizes for the BT codes are 6, 20, 64 and 252 bits. When the GBT and the BT codes are compared, the figures are offset by one position.

Figures 7.4.1 to 7.4.8 show the performance of the GBT and the BCH codes when the S/N ratio is increased. The performance shown in these figures are for the coding delay of 3, 9, 15, and 21 blocks and for the vehicular speed of 40 and 100 km/hr. From these figures, the following observations can be made.

- There is a cross over of performance between the GBT codes with different block sizes when the S/N ratio is increased. The cross over effect is similar to that of the BCH codes discussed previously. Below a certain S/N ratio, the GBT code with a smaller block size performs better. Above that S/N ratio, the GBT code with a larger block size performs better. This result can be attributed to the error correcting capability of the C1 and the C2 codes used for encoding the GBT codes. The

Speed = 40 Km/Hr

Delay = 3 Blocks

Figure 7.5.1 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and Delay = 3 Blocks.

Speed = 40 Km/Hr

Delay = 9 Blocks

Figure 7.5.2 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and Delay = 9 Blocks.

Speed = 40 Km/Hr

Delay = 15 Blocks

Figure 7.5.3 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and Delay = 15 Blocks.

Speed = 40 Km/Hr

Delay = 21 Blocks

Figure 7.5.4 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and Delay = 21 Blocks.

Figure 7.5.5 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and Delay = 3 Blocks.

Figure 7.5.6 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and Delay = 9 Blocks.

Speed = 100 Km/Hr Delay = 15 Blocks

Figure 7.5.7 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and Delay = 15 Blocks.

Figure 7.5.8 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and Delay = 21 Blocks.

Speed = 40 Km/Hr

S/N = 10 dB

Figure 7.5.9 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and S/N = 10 dB.

3 of/de 3

MicroD

Speed = 40 Km/Hr

S/N = 15 dB

Figure 7.5.10 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and S/N = 15 dB.

Speed = 40 Km/Hr

S/N = 20 dB

Figure 7.5.11 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and S/N = 20 dB.

Speed = 40 Km/Hr

S/N = 25 dB

Figure 7.5.12 Performance Result of the GBT and the BCH Codes at Speed = 40 km/hr and S/N = 25 dB.

Speed = 100 Km/Hr S/N = 10 dB

Figure 7.5.13 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and S/N = 10 dB.

Speed = 100 Km/Hr S/N = 15 dB

Figure 7.5.14 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and S/N = 15 dB.

Speed = 100 Km/Hr S/N = 20 dB

Figure 7.5.15 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and S/N = 20 dB.

Speed = 100 Km/Hr S/N = 25 dB

Figure 7.5.16 Performance Result of the GBT and the BCH Codes at Speed = 100 km/hr and S/N = 25 dB.

Figure 7.5.17 Performance Result of the GBT and the BCH Codes at Location = Ottawa.

Figure 7.5.18 Performance Result of the GBT and the BCH Codes at Location = Hull.

QUEENSWAY

Figure 7.5.19 Performance Result of the GBT and the BCH Codes at Location = Queensway.

C2 codes are used for correcting errors in the guard space blocks.

- For block sizes of 252 and 1020 bits, the GBT codes with correction perform better than the GBT codes with no correction. This result is similar to that obtained for the BT codes and indicates that error propagation is not a problem with the GBT codes. This result further confirms that the Rayleigh fading channel contains both burst and random errors.
- Within statistical accuracy, comparison of the same GBT code at the two vehicular speeds indicates that there is a cross over of performance when the S/N ratio is increased. The cross over effect is similar to that of the BCH and the BT codes discussed previously. As have been discussed before, the lower bit error rate of the 100 km/hr data files may also contribute to this cross over of block error rate performance.
- Comparison of the GBT codes to the BCH codes indicates that the BCH codes perform better than the GBT codes.
- Comparison of the GBT codes to the BT codes indicates that there is a cross over of performance when the block

size is increased. For the block size of 20 bits, the BT codes perform better than the GBT codes. For the block size of 64 bits, their performance is about the same. For the block size of 252 bits, the GBT codes perform better than the BT codes. Table 7.5.1 summarizes the relative performance of the BCH, the BT and the GBT codes.

Figures 7.5.9 to 7.5.16 show the performance of the GBT and the BCH codes when the bit and the block interleaving degrees are increased. The performance shown in these figures are for the S/N ratio of 10, 15, 20 and 25 dB and for the vehicular speed of 40 and 100 km/hr. From these figures, the following observations can be made.

- For the GBT codes, there appears to be no significant changes in performance when the block interleaving degree is increased. Additionally, the cycling effect as seen in the BT codes is not prominent in here. The reason is most likely because of the error correcting capability of the C2 codes which acts to suppress this effect.
- When the bit interleaving degree is increased, the performance of the BCH codes agrees with what has been

Codes	Block Size (bits)		
	20	64	252
BCH	GOOD	GOOD	GOOD
BT	AVERAGE	AVERAGE	POOR
GBT	POOR	AVERAGE	AVERAGE

Table 7.5.1: Summary of the Relative Performance of the BCH, the BT and the GBT Codes in a Rayleigh Fading Channel.

Codes	Block Size (bits)		
	20	64	252
BCH	POOR	POOR	POOR
BT	GOOD	AVERAGE	AVERAGE
GBT	AVERAGE	AVERAGE	GOOD

Table 7.5.2: Summary of the Relative Performance of the BCH, the BT and the GBT Codes Using the Queensway Data Files.

discussed before. Additionally, the results indicate that the BCH codes perform better than the GBT codes.

Figures 7.5.17 to 7.5.19 show the performance results of the GBT and the BCH codes using the experimental data.

From these figures, which show the performance as a function of coding delay, the following observations can be made.

- The results of Ottawa and Hull are similar to those obtained using the simulated data. More specifically, for the BCH codes, the performance improves when the interleaving is increased. For the GBT codes, the performance does not seem to change much when the interleaving is increased. For the block size of 1020 bits, the result indicates that the GBT code with correction performs better than the GBT code with no correction. Additionally, the results also indicate that the BCH codes perform better than the GBT codes.
- Once again, the results of Queensway are different from those obtained using the simulated data. The results indicate that the GBT codes perform better than the BCH codes. The reason is attributed to the channel characteristics of Queensway being substantially

different from that of the simulation model and those of Ottawa and Hull. Comparison of the GBT code to the BT codes indicates that there is a cross over of performance when the block size is increased. For block size of 20 bits, the BT codes perform better than the GBT codes. For block size of 64 bits, their performance is about the same. For block size of 252 bits, the GBT codes perform better than the BT codes. Table 7.5.2 summarizes the relative performance of the BCH, the BT and the GBT codes using the Queensway data files.

CHAPTER 8.0

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORKS

8.1 Conclusions

The channel characteristics of the simulated Rayleigh fading channel have been analyzed. The burst error statistics have been shown to be similar to those of Ottawa and Hull. This indicates that the simulation model and the method of generating the error pattern files approximate closely the Rayleigh fading land mobile channel.

The performance of the BCH, the BT and the GBT codes have been studied using both the simulated data and the experimental data. Coding parameters such as block size, interleaving degree, error correcting and detecting capability of the BT and the GBT codes are analyzed under different vehicular speeds and S/N ratios. The simulation results have indicated that the BCH codes outperform the BT and the GBT codes in a Rayleigh fading channel.

For the BCH codes, the simulation results have indicated that a code with a larger block size will give a better coding performance as long as the t/n ratio is maintained

above an acceptable value. This notion of using a larger block size in a Rayleigh fading channel appears to be in contravention with the general concept that a larger block size will give a poorer performance. The simulation results have also indicated that for the BCH codes, the use of bit interleaving is advisable only if the uncoded channel bit error rate is less than the t/n ratio.

Although, the simulation results have indicated that as long as the t/n ratio is maintained above an acceptable value, the BCH codes will perform better when the vehicular speed is increased, it is uncertain whether this performance increase is due to the vehicular speed or may be because of the lower bit error rate of the 100 km/hr data files. Hence, it is inconclusive to say whether the BCH codes will perform better when the vehicular speed is increased.

Although the BT and the GBT codes have the capabilities of adaptively correcting both random and burst errors, the simulation results have indicated that their capabilities are inferior to the BCH codes in the presence of a Rayleigh fading channel. The major reason that the BT and the GBT codes cannot perform better can be attributed to the high background random error rate of the Rayleigh fading channel. It is because both the BT and the GBT codes rely

on guard space blocks for correcting bursts. In the presence of high background random error rate, the probability of having uncorrectable guard space blocks is high, consequently the burst error correcting capability is lowered. Simulation results have indicated that for block size greater than 64 bits, the GBT codes will perform better than the BT codes. Furthermore, both the BT codes with correction and the GBT codes with correction will perform better than those with no correction. Results have also indicated that there is no significant performance improvement when the block interleaving degree is increased.

Except for the Queensway data files, the simulation results using the experimental data agree quite well with those using the simulated data. The simulation using the Queensway data files gives opposite results. The reason is attributed to the channel characteristics of Queensway being substantially different from that of the simulation model and those of Ottawa and Hull. The results using the Queensway data files have indicated that the performance of the BT and the GBT codes perform better than the BCH codes.

8.2 Recommendations For Future Works

Most of the works in this thesis has concentrated on the performance of the BT and the GBT codes in a Rayleigh fading land mobile channel. Although the results have indicated that the BT and the GBT codes are inferior to the BCH codes, the results of Queensway have indicated that apart from a Rayleigh fading channel, the BT and the GBT codes can have a better performance. Consequently, it is interesting to find out the performance of the BT and the GBT codes under other fading environments. With the existing system set up using a real time mobile channel simulator, it would be extremely easy to investigate the performance of these codes under a Rician fading condition with different degree of line of sight ratio, shadowing, and vehicular speed.

With the number of coding parameters, this research has only been able to investigate the performance of the BT and GBT codes with a code rate of $1/2$ and a folding factor of 2. To get more insight, the performance of these codes with other code rate and folding factor would be of interest.

Finally, the analysis of the coding performance has focussed on block error rate measurements. It is interesting to find out whether the comparative performance obtained in this research agree with the comparative performance obtained in terms of bit error rate measurements.

APPENDIX A

DESCRIPTION OF BCH CODES

A.1 Introduction

This chapter is to review the basic material of block coding, and more specifically, the algebraic coding theory which leads to the description of Bose-Chaudhuri-Hocquenghem (BCH) codes. The description of BCH codes can also be found in many good references [4, 11, 12].

The BCH codes are linear cyclic codes and can be constructed by very simple encoding rules. These codes are multiple errors correcting codes with a well-defined minimum distance. Efficient decoding algorithms have also been developed for the class of BCH codes. In order to describe the BCH codes, the algebraic coding theory has to be presented first.

A.2 Introduction to Algebra

This section introduces the basic definitions that will be used to define linear block codes.

Group

Let G be a set,

$+$ be an operator within G between any two elements.

The set G is called a Group if it satisfies the following four properties:

1. Closure.

For every a, b in the set G ,

$$c = a + b$$

is also in the set G .

2. Associative Law.

For every a, b, c in the set G ,

$$a + (b + c) = (a + b) + c.$$

3. Identity.

There is an element e called the identity element that satisfies

$$a + e = e + a = a$$

for every a in the set G .

4. Inverses.

If a is in the set G , then there exists an element b in G such that

$$a + b = b + a = e.$$

Element b is called the inverse of a and vice versa.

If the group G satisfies the following additional property, it is called an abelian group.

5. Commutative Law.

For any a, b in G ,

$$a + b = b + a.$$

Ring

Let R be a set,

$+$ and $*$ be two operators within R between any two elements.

The set R is called a Ring if it satisfies the following four properties:

1. The set R is an abelian group under the $+$ operator.

2. Closure.

For any a, b in R ,

$$c = a * b$$

is also in R .

3. Associative Law.

For any a, b, c in R ,

$$a * (b * c) = (a * b) * c.$$

4. Distributive Law.

For any a, b, c in R ,

$$a * (b + c) = (a * b) + (a * c)$$

$$(a + b) * c = (a * c) + (b * c)$$

If the ring R satisfies the following additional property, it is called a commutative ring.

5. Commutative Law.

For any a, b in R ,

$$a * b = b * a.$$

Note that the $+$ operator within the ring R is always commutative, regardless of whether the ring R is commutative or not.

Field

Let F be a set,

$+$ and $*$ be two operators within F between any two elements.

The set F is called a field if it satisfies the following properties:

1. The set F is a ring under the $+$ and $*$ operators.
2. Excluding the identity element under the $+$ operator, the set F is an abelian group under the $*$ operator.

Hence, a field has all the properties of a ring and a ring has all the properties of a group.

Subfield and Extension Field

Let E and F are fields under the 'same' $+$ and $*$ operators. If F is a subset of E , then F is called a subfield of E and E is called an extension field of F . The $+$ and $*$ operators of F are the inherited $+$ and $*$ operators of E .

Galois Field

The number of elements in a field can be infinite or finite. A field with a finite number of elements is called a finite field or a Galois field. A Galois field with q elements is denoted by the label $GF(q)$.

Galois field $GF(q)$ exists if and only if q satisfies the following:

$$q = p^m \quad (A.2.1)$$

where q = number of elements in the Galois field

p = prime number

m = arbitrary positive integer

Equation (A.2.1) states that the number of elements in a Galois field must be a power of prime number. If q does not satisfy (A.2.1), then Galois field $GF(q)$ does not exist.

The smallest Galois field is $GF(2)$, a field with two elements. These two elements are commonly labelled as 0 and 1, where the 0 is the identity element under the + operator, and the 1 is the identity element under the * operator.

Vector Space

Let V_S be a set of n -tuple vectors,

F be a field,

$+$ be an operator within V_S between any two
— vectors,

* be an operator between an element from V_S and an element from F .

The elements of V_S are called vectors and the elements of F are called scalars.

Let V_1 and V_2 be two n -tuple vectors from V_S ,

$$V_1 = (a_{n-1}, a_{n-2}, \dots, a_0)$$

$$V_2 = (b_{n-1}, b_{n-2}, \dots, b_0)$$

where a_i and b_i , $i = 0$ to $n-1$, are elements from F .

Let c be an element from F . The operators $+$ and $*$ are then defined as follows:

$$V_1 + V_2 = (a_{n-1} + b_{n-1}, a_{n-2} + b_{n-2}, \dots, a_0 + b_0)$$

$$c * V_1 = (c * a_{n-1}, c * a_{n-2}, \dots, c * a_0)$$

It should be noted that the $+$ operator on the left hand side refers to the operation between vectors. The $+$ operator on the right hand side refers to the operation between field elements. However, the dual meaning of the $+$ operator is clear from its context.

Similarly, the $*$ operator on the left hand side refers to the operation between field element and vector. The $*$

operator on the right hand side refers to the operation between field elements. Again, the dual meaning of the $*$ operator is clear from its context.

The set V_S is called a vector space over a field F if it satisfies the following properties:

1. V_S is an abelian group under the $+$ operator.

2. Distributive Law I.

For any vectors V_1 and V_2 , and any scalar c ,

$$c * (V_1 + V_2) = (c * V_1) + (c * V_2).$$

3. Distributive Law II.

For any vector V , and any scalars c_1 and c_2 ,

$$1 * V = V$$

$$(c_1 + c_2) * V = (c_1 * V) + (c_2 * V)$$

where 1 is the identity element in F under the $*$ operator.

4. Associative Law.

For any vector V , and any scalars c_1 and c_2 ,

$$(c_1 * c_2) * V = c_1 * (c_2 * V)$$

It should be noted that a vector space can be constructed using a field with infinite or finite number of elements.

Vector Subspace

Let V_S be a vector space over a field F , with the $+$ and $*$ operators,

V_{SS} be a nonempty subset of V_S .

If V_{SS} is also a vector space over the field F with the same $+$ and $*$ operators, then V_{SS} is called a vector subspace or simply a subspace.

A.3 Introduction to Cyclic Codes

Linear Block Code

When a vector space is constructed using Galois field, it is labelled as $[GF(Q)]^n$, where $GF(q)$ denotes the Galois field with q elements and n denotes the length of the n -tuple vector.

A linear block code of length n is a subspace of the vector space $[GF(q)]^n$. A special case of important interest is $q=2$, the binary linear code. Example of binary linear code is BCH code. When q is greater than 2, the linear code is nonbinary. Example of nonbinary code is Reed-Solomon code.

Cyclic Codes

Cyclic codes are linear block codes which have the following additional property:

If $C = (c_{n-1}, c_{n-2}, \dots, c_1, c_0)$ is a codeword

then $C_1 = (c_0, c_{n-1}, c_{n-2}, \dots, c_1)$ is also a codeword.

Note that C_1 is obtained by a cyclic shift of the elements of C .

It is obvious that the set of all cyclic codes is a subset of the set of all linear block codes. The class of cyclic code is considered to be an important class of linear block codes. It is because many properties of cyclic codes can be described using Galois field. Using the Galois field as a mathematical tool, many good cyclic codes, such as BCH codes and Reed-Solomon codes, have been found. Moreover, a number of efficient encoding and decoding algorithms have been devised for cyclic codes.

Polynomial Representation of Cyclic Codes

Recall that cyclic code is a subspace of a vector space. Elements in a cyclic code are called codewords, which are

vectors. In dealing with cyclic codes, however, it is convenient to associate codewords with polynomials.

Let v be a vector of length n ,
 $v(x)$ be a polynomial of degree $n-1$.

The association between v and $v(x)$ is defined as follows:

$$v = (c_{n-1}, c_{n-2}, \dots, c_1, c_0) \quad (\text{A.3.1})$$

$$v(x) = c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \dots + c_1x + c_0 \quad (\text{A.3.2})$$

The coefficients of $v(x)$ are elements of $GF(q)$. Addition and multiplication of polynomials follow the usual addition and multiplication rules of polynomials with the exception that addition and multiplication of coefficients are carried out in $GF(q)$.

When the coefficients of $v(x)$ are elements of $GF(q)$, $v(x)$ is said to be a polynomial over $GF(q)$. For the sake of clarity and as a matter of notation, the polynomial $v(x)$ given by (A.3.2) will be written in the form of a vector, i.e.

$$v(x) = (c_{n-1}, c_{n-2}, \dots, c_1, c_0) \quad (\text{A.3.3})$$

It is understood that the notation of (A.3.3) means $v(x)$ is a polynomial over $GF(q)$ in the form of (A.3.2). It should

be noted that a polynomial of degree $n-1$ corresponds to a vector of length n .

Generation of Cyclic Codes

For each positive integer value of n , the polynomial $(X^n - 1)$ can be factorized into product terms of prime polynomials, i.e.

$$\begin{aligned} X^n - 1 &= g_1(x) g_2(x) \dots g_m(x) \\ &= g(x) h(x) \end{aligned} \quad (\text{A.3.4})$$

where $g_i(x)$ = prime polynomial, $i=1, \dots, m$

$g(x)$ = product term of one or more $g_i(x)$'s

$h(x)$ = product term of the remaining $g_i(x)$'s

The degree of $g(x)$ and $h(x)$ is dependent on which prime polynomials, $g_i(x)$'s, are used. However, both $g(x)$ and $h(x)$ have degrees less than $n-1$. A cyclic code of block length n can be generated by using $g(x)$ as the generator polynomial, where $g(x)$ is obtained using (A.3.4).

Let $i(x)$ = information polynomial of degree $k - 1$,

$g(x)$ = generator polynomial of degree $n - k$,

codeword $c(x)$ of degree $n - 1$ is obtained by multiplying $i(x)$ and $g(x)$,

$$c(x) = i(x) g(x). \quad (\text{A.3.5})$$

The set of codewords generated by (A.3.5) is called a (n,k) code, where n is the length of the codeword and k is the length of the information vector. The ratio of k/n is called the code rate.

From (A.3.4) and (A.3.5), it can be seen that any polynomial that divides $(X^n - 1)$ can be used as a generator polynomial to define a cyclic code of block length n . For a given generator polynomial $g(x)$, the corresponding $h(x)$ that satisfy (A.3.4) is called the parity check polynomial.

Codeword $c(x)$ generated by (A.3.5) satisfies the following:

$$[c(x)] \bmod g(x) = 0 \quad (\text{A.3.6})$$

where $\bmod$ means the remainder after dividing $c(x)$ by $g(x)$.

Cyclic Codes in Systematic Form

Given any information polynomial $i(x)$,

$$i(x) = (i_{(k-1)}, i_{(k-2)}, \dots, i_0) \quad (\text{A.3.7})$$

it is possible to encode $i(x)$ to give a codeword $c(x)$ in the following form:

$$c(x) = (i_{(k-1)}, i_{(k-2)}, \dots, i_0, p_{(n-k-1)}, p_{(n-k-2)}, \dots, p_0) \quad (A.3.8)$$

where p_i , $i=0$ to $n-k-1$, are called the parity symbols.

Cyclic codes that have the form of (A.3.8) are called cyclic codes in systematic form. Cyclic codeword $c(x)$ generated by (A.3.5) is not in systematic form. Cyclic codeword $c(x)$ in systematic form can be generated as follows:

$$c(x) = x^{(n-k)} i(x) + p(x) \quad (A.3.9)$$

where $p(x) = -[x^{(n-k)} i(x)] \bmod g(x)$

The systematic encoding rule (A.3.9) and the non-systematic encoding rule (A.3.5) produce the same set of cyclic codewords. The difference between them is that the association between $i(x)$ and $c(x)$ is different. Cyclic codeword in systematic form also satisfies (A.3.6), it is because

$$\begin{aligned} [c(x)] \bmod g(x) &= [x^{(n-k)} i(x) + p(x)] \bmod g(x) \\ &= [x^{(n-k)} i(x)] \bmod g(x) + [p(x)] \bmod g(x) \\ &= [x^{(n-k)} i(x)] \bmod g(x) \\ &\quad - [[x^{(n-k)} i(x)] \bmod g(x)] \bmod g(x) \\ &= [x^{(n-k)} i(x)] \bmod g(x) \\ &\quad - [x^{(n-k)} i(x)] \bmod g(x) \\ &= 0 \end{aligned}$$

A.4 Introduction to BCH Codes

Construction of Extension Fields

For a Galois field $GF(q)$ with q elements, it is possible to construct an extension field $GF(q^m)$ with q^m elements, where m is any positive integer. To construct $GF(q^m)$ from $GF(q)$, it is only necessary to find a prime polynomial of degree m over $GF(q)$.

Let $p(x) = (p_m, p_{m-1}, \dots, p_0)$ be a prime polynomial.

The coefficients of $p(x)$ is from $GF(q)$ and the degree of $p(x)$ is m . It is known that prime polynomial $p(x)$ exists for every positive degree of m . Extension field $GF(q^m)$ of $GF(q)$ is then constructed as follows:

Let $a(x)$ be any polynomial over $GF(q)$,

construct

$$r(x) = [a(x)] \bmod p(x)$$

The set of all $r(x)$'s is the extension field $GF(q^m)$.

Note that all the polynomials in the extension field $GF(q^m)$ have degrees less than m . Equivalently, elements in the

extension field can be viewed as vectors over $GF(q)$. The length of the vector is m and the coefficients are elements in $GF(q)$, hence there are q^m vectors in $GF(q^m)$.

Let $v_1(x)$ and $v_2(x)$ be two elements in the extension field $GF(q^m)$,

$$v_1(x) = (a_{m-1}, a_{m-2}, \dots, a_0)$$

$$v_2(x) = (b_{m-1}, b_{m-2}, \dots, b_0)$$

where a_i and b_i , $i = 0$ to $m-1$, are elements of $GF(q)$.

The $+$ and $*$ operators between two elements in the extension field $GF(q^m)$ is defined as:

$$v_1(x) + v_2(x) = (a_{m-1} + b_{m-1}, a_{m-2} + b_{m-2}, \dots, a_0 + b_0)$$

$$v_1(x) * v_2(x) = [v_1(x) * v_2(x)] \bmod p(x)$$

The $+$ operator on the left hand side represents addition between two elements in the extension field $GF(q^m)$. The $+$ operator on the right hand side represents addition between two elements in $GF(q)$. The dual meaning of the $+$ operator is clear from its context.

The $*$ operator on the left hand side represents multiplication between two elements in the extension field

$GF(q^m)$. The $*$ operator on the right hand side represents multiplication of two polynomials over $GF(q)$. The dual meaning of the $*$ operator is clear from its context.

Primitive Element

An interesting property of Galois field is that there exist an element called primitive element, h , in $GF(q)$ that every element in $GF(q)$ except the zero can be expressed as a power of h .

Let h be the primitive element of $GF(q)$.

$$GF(q) = \{0, h, h^2, h^3, \dots, h^{(q-1)}\}$$

It is known that primitive element of any $GF(q)$ exists and is unique.

Minimal Polynomial

Any polynomial $v(x)$ including prime polynomial over $GF(q)$ can be factorized into linear terms over $GF(q^m)$, i.e.

$$v(x) = (x - b_1)(x - b_2) \dots (x - b_s)$$

where b_i , $i = 1$ to s are elements in $GF(q^m)$.

It is clear that $v(b_1) = v(b_2) = \dots = v(b_s) = 0$.

It can be shown that for every element b_i in $GF(q^m)$, there is a polynomial $m(x)$ over $GF(q)$ such that $m(b_i) = 0$. The smallest degree of $m(x)$ that satisfies $m(b_i) = 0$ is called a minimal polynomial of b_i . Minimal polynomial exists for every element in $GF(q^m)$. Note that for two different elements, b_1 and b_2 , in $GF(q^m)$, the minimal polynomials of b_1 and b_2 may be the same.

Construction of BCH Code

To construct BCH code over $GF(q)$ of length n , where $n = q^m - 1$, with t error correcting capability, the generator polynomial $g(x)$ is chosen as follows:

1. Choose a prime polynomial over $GF(q)$ of degree m and construct an extension field $GF(q^m)$.
2. Let h be the primitive element of $GF(q^m)$.
Find all the minimal polynomials of $h, h^2, \dots, h^{2t}$.
Let these minimal polynomials be $f_1(x), f_2(x), \dots, f_{2t}(x)$.
3. $g(x) = \text{LCM}\{f_1(x), f_2(x), \dots, f_{2t}(x)\}$
where LCM stands for least common multiple.

Decoding the BCH code

The description of decoding the BCH codes is beyond the scope of this thesis. Interested readers should refer to those references cited at the beginning of this appendix.

APPENDIX B

DESIGN OF SIMULATION ALGORITHMS

B.1 Introduction

This chapter describes the design of simulation algorithms that were used to evaluate the performance of the BT, the GBT, and the BCH codes.

B.2 Simulation Algorithms for the BT and the GBT Codes

In order to understand the simulation algorithms, it is useful to review the error correcting capabilities of the BT and the GBT codes first. The design of the simulation algorithms is based on their error correcting capabilities.

B.2.1 Summary of Error Correcting Capability

From the discussions in Sections 4 and 5, the error correcting capabilities of the BT and the GBT codes can be summarized as follows:

Both the BT and the GBT codes are capable of correcting

(i) in the random mode, all blocks with e_1 or fewer errors.

Should a block with more than e_1 errors, it is still correctable if

(ii) these errors are detectable with the B decoder and

(iii) the block is preceded by $g - 1$ interleaved blocks that were correctly decoded and

(iv) the block is followed by g interleaved blocks each of which contains e_2 or fewer errors.

For the BT codes, the B decoder = the C decoder, $e_1 = t$, $e_2 = 0$ and $g = b - 1$ as described in Section 4.

For the GBT codes, the B decoder = the C1 decoder, $e_1 = t_1$, $e_2 = t_2$ and $g = f$ as described in Section 5.

B.2.2 Implementation of the BT and the GBT Algorithms

The notations given in Section B.2.1 are used in the following description. Furthermore, let e_3 denotes the number of errors the B decoder is designed to detect.

The design of the simulation algorithms is simplified by the use of error tracers. The tracers record the result of each block decoded by the BT or the GBT decoder at any given time. There are two error tracers called burst tracer and false tracer. They are used to record the following events:

- if the block is correctable then both tracers are set to 0,
- if the block is uncorrectable but detectable then the burst tracer is set to 1 and the false tracer is set to 0,
- if the block is uncorrectable and undetectable then the burst tracer is set to 0 and the false tracer is set to 1.

Each time when one block of channel errors is read from the error file, the corresponding tracers are updated as follows:

- (i) if the previous g interleaved blocks are burst free, as indicated by the respective burst tracers = 0, then

update the tracers in random decoding mode,

else

update the tracers in burst-trapping mode.

In the random decoding mode, the tracers are updated as follows:

(ii) if the previous g interleaved blocks are free of false decoding, as indicated by the respective false tracers = 0, then

error count = channel errors

else

error count = channel errors + feedback errors,

(iii) if error count $< e_1$ then

set both tracers to 0,

if $e_1 < \text{error count} < e_3$ then

set burst tracer = 1 and false tracer = 0,

if error count $> e_3$ then

for upper bound analysis,

set burst tracer = 0 and false tracer = 1,

for lower bound analysis,

set burst tracer = 1 and false tracer = 0.

In the burst-trapping mode, the tracers are updated as follows:

- (iv) if channel errors $\leq e_2$ then
 set both tracers to 0,
 else
 for upper bound analysis,
 set burst tracer = 0 and false tracer = 1,
 for lower bound analysis,
 set burst tracer = 1 and false tracer = 0.

After the tracers are updated, the block error rate performance are measured as follows. For each block,

- (v) if the burst tracer = 0 and the false tracer = 1,
 then block error count is increased by one,
- (vi) if the burst tracer = 1 and the false tracer = 0
 then
 for all the preceding $g - 1$ and following g
 interleaved blocks, if any one of the burst
 or false tracers = 1, then block error count
 is increased by one,
- (vii) for all other cases, block error count remains
 unchanged.

Note that error rate measurement cannot start until g interleaved blocks are decoded and the tracers are updated.

The number of feedback errors due to false decoding in step (ii) is designed as follows:

- For the BT codes, the maximum number of feedback errors is $(b-1)t$. For the rate $1/2$ BT codes under consideration, $b=2$, and hence the feedback errors = t .
- For the GBT codes, the actual number of feedback errors is difficult to estimate unless the actual encoding and decoding processes of the C1 and C2 codes are implemented. However, since the feedback data are encoded using the C2 encoder, in the event of a false decoding, the falsely encoded feedback codeword would most likely be at a distance of d_{2m} from the actual codeword. Hence, it is reasonable to say the feedback errors = d_{2m} . For the rate $1/2$ GBT codes under consideration, the feedback errors = d_{2m} .

B.3 Simulation Algorithms for the BCH codes

The block error rate performance of the BCH codes is measured as follows.

If channel errors $> e_1$ then block error count is increased by one

where

e_1 is the number of errors the BCH code is designed to correct.

For interleaving degree L , the channel errors are first de-interleaved before the number of errors is counted.

De-interleaving is the process of first putting the L error blocks in rows and reading them out in columns.

APPENDIX C
The C1 and the C2 Tables for the GBT Codes

Primitive BCH Codes		Shortened BCH Codes	
B1	B2	C1	C2
(31, 21, 5)	(31, 26, 3)	(20, 10, 5)	(10, 5, 3)
(63, 51, 5)	(63, 57, 3)	(24, 12, 5)	(12, 6, 3)
(63, 39, 9)	(63, 51, 5)	(48, 24, 9)	(24, 12, 5)
(127, 113, 5)	(127, 120, 3)	(28, 14, 5)	(14, 7, 3)
(127, 99, 9)	(127, 113, 5)	(56, 28, 9)	(28, 14, 5)
(127, 85, 13)	(127, 106, 7)	(84, 42, 13)	(42, 21, 7)
(127, 71, 19)	(127, 99, 9)	(112, 56, 19)	(56, 28, 9)
(255, 239, 5)	(255, 247, 3)	(32, 16, 5)	(16, 8, 3)
(255, 223, 9)	(255, 239, 5)	(64, 32, 9)	(32, 16, 5)
(255, 207, 13)	(255, 231, 7)	(96, 48, 13)	(48, 24, 7)
(255, 191, 17)	(255, 223, 9)	(128, 64, 17)	(64, 32, 9)
(511, 493, 5)	(511, 502, 3)	(36, 18, 5)	(18, 9, 3)
(511, 475, 9)	(511, 493, 5)	(72, 36, 9)	(36, 18, 5)
(511, 457, 13)	(511, 484, 7)	(108, 54, 13)	(54, 27, 7)
(511, 439, 17)	(511, 475, 9)	(144, 72, 17)	(72, 36, 9)
(511, 421, 21)	(511, 466, 11)	(108, 90, 21)	(90, 45, 11)
(511, 403, 25)	(511, 457, 13)	(216, 108, 25)	(108, 54, 13)
(511, 385, 29)	(511, 448, 15)	(252, 126, 29)	(126, 63, 15)
(511, 367, 33)	(511, 439, 17)	(288, 144, 33)	(144, 72, 17)
(511, 349, 39)	(511, 430, 19)	(324, 162, 39)	(162, 81, 19)
(511, 331, 43)	(511, 421, 21)	(360, 180, 43)	(180, 90, 21)
(511, 313, 47)	(511, 412, 23)	(396, 198, 47)	(198, 99, 23)
(511, 295, 53)	(511, 403, 25)	(432, 216, 53)	(216, 108, 25)
(511, 277, 57)	(511, 394, 27)	(468, 234, 57)	(234, 117, 27)
(511, 259, 61)	(511, 385, 29)	(504, 252, 61)	(252, 126, 29)
(1023, 1003, 5)	(1023, 1013, 3)	(40, 20, 5)	(20, 10, 3)
(1023, 983, 9)	(1023, 1003, 5)	(80, 40, 9)	(40, 20, 5)
(1023, 963, 13)	(1023, 993, 7)	(120, 60, 13)	(60, 30, 7)
(1023, 943, 17)	(1023, 983, 9)	(160, 80, 17)	(80, 40, 9)
(1023, 923, 21)	(1023, 973, 11)	(200, 100, 21)	(100, 50, 11)
(1023, 903, 25)	(1023, 963, 13)	(240, 120, 25)	(120, 60, 13)
(1023, 883, 29)	(1023, 953, 15)	(280, 140, 29)	(140, 70, 15)
(1023, 863, 33)	(1023, 943, 17)	(320, 160, 33)	(160, 80, 17)
(1023, 843, 37)	(1023, 933, 19)	(360, 180, 37)	(180, 90, 19)
(1023, 823, 41)	(1023, 923, 21)	(400, 200, 41)	(200, 100, 21)
(1023, 803, 45)	(1023, 913, 23)	(440, 220, 45)	(220, 110, 23)
(1023, 783, 49)	(1023, 903, 25)	(480, 240, 49)	(240, 120, 25)
(1023, 763, 53)	(1023, 893, 27)	(520, 260, 53)	(260, 130, 27)
(1023, 743, 57)	(1023, 883, 29)	(560, 280, 57)	(280, 140, 29)
(1023, 723, 61)	(1023, 873, 31)	(600, 300, 61)	(300, 150, 31)
(1023, 703, 65)	(1023, 863, 33)	(640, 320, 65)	(320, 160, 33)
(1023, 683, 69)	(1023, 853, 35)	(680, 340, 69)	(340, 170, 35)
(1023, 663, 73)	(1023, 843, 37)	(720, 360, 73)	(360, 180, 37)
(1023, 643, 77)	(1023, 833, 39)	(760, 380, 77)	(380, 190, 39)
(1023, 623, 81)	(1023, 823, 41)	(800, 400, 81)	(400, 200, 41)
(1023, 603, 85)	(1023, 813, 43)	(840, 420, 85)	(420, 210, 43)
(1023, 583, 89)	(1023, 803, 45)	(880, 440, 89)	(440, 220, 45)
(1023, 563, 93)	(1023, 793, 47)	(920, 460, 93)	(460, 230, 47)
(1023, 543, 97)	(1023, 783, 49)	(960, 480, 97)	(480, 240, 49)
(1023, 523, 101)	(1023, 773, 51)	(1000, 500, 101)	(500, 250, 51)
(1023, 503, 105)	(1023, 763, 53)	(1040, 520, 105)	(520, 260, 53)

APPENDIX D

LIST OF RELEVANT DISTRIBUTION FUNCTIONS FOR THE LAND MOBILE CHANNEL

D.1 Rayleigh Distributed Probability Density Function

$$f_r(x) = \begin{cases} \frac{x}{a^2} \exp\left(-\frac{x^2}{2a^2}\right) & ; x \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

where a = mean value

D.2 Rician Distributed Probability Density Function

$$f_r(x) = \begin{cases} \frac{x}{b^2} \exp\left(-\frac{(x^2+a^2)}{2b^2}\right) \cdot I_0\left(\frac{ax}{b^2}\right) & ; x \geq 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

where

$I_0(x)$ = modified Bessel function of zeroth order
 a = amplitude of the line of sight component
 b^2 = mean power of the scattered waves

D.3 Nakagami Distributed Probability Density Function

$$f_r(x) = \begin{cases} \frac{2}{\Gamma(a)} \cdot \left(\frac{a}{b^2}\right) \cdot x^{2a-1} \cdot \exp\left(-\frac{ax^2}{b^2}\right) & ; x \geq 0, a \geq \frac{1}{2} \\ 0 & ; \text{elsewhere} \end{cases}$$

where $\Gamma(a)$ = Gamma function

D.4 Lognormal Distributed Probability Density Function

$$f_r(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\beta} x^{-1} \exp\left(-\frac{(\ln x - \alpha)^2}{2\beta^2}\right) & ; x > 0, \beta > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

where $\ln x$ = natural logarithm of x .

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