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**A COMBINED ANALOGUE AND DIGITAL
PULSE COMPRESSION SYSTEM
USING LARGE TIME BANDWIDTH PRODUCT SIGNALS
FOR USE IN SYNTHETIC APERTURE RADAR**

by

PUSHKAR E. GODBOLE, BEng

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of**

Master of Engineering

Department of Electronics

**Carleton University
Ottawa, Ontario
5 July 1989**

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submitted by Pushkar E. Godbole, BEng
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the degree of Master of Engineering

Prof. J.S. Wight
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ABSTRACT

Pulse compression is widely used in modern radar systems primarily because it facilitates the use of long duration low power pulses in place of short high power pulses. Pulse compression processing can be accomplished in either an analogue or a digital manner. Modern, analogue pulse compression devices are capable of processing signals with time bandwidth (TBW) products in excess of 5000, however they lack the flexibility associated with digital pulse compression techniques. Due to the excessive signal processing requirements, digital pulse compression systems are usually limited to TBW products in the order of 100. The purpose of this work is to examine the possibility of implementing pulse compression in two stages using a Surface Acoustic Wave analogue compressor for the first stage and digital compression techniques in the second stage. Development of a two stage pulse compression system for use in high resolution tactical Synthetic Aperture Radar is considered.

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LIST OF SYMBOLS

<u>SYMBOL</u>	<u>MEANING</u>
dB	Decibel
dBm	Decibel Referenced to 1 Milliwatt
$F[.]$	Fourier Transform
$F^{-1}[.]$	Inverse Fourier Transform
F_c	Center Frequency
GHz	Giga Hertz
Hz	Hertz
λ	Wavelength
μ	Linear FM Rate
μS	Micro Seconds
MHz	Mega Hertz
nS	Nano Seconds
S	Seconds
$\text{Sinc}(.)$	$\text{Sin}(.)/(.)$
T_p	Sampling Period

LIST OF ABBREVIATIONS

<u>ABBREVIATION</u>	<u>MEANING</u>
A/D	Analogue to Digital
AFG	Arbitrary Function Generator
D/A	Digital to Analogue
DRFM	Digital Radio Frequency Memory
DSP	Digital Signal Processing
FFT	Fast Fourier Transform
FM	Frequency Modulation
FSK	Frequency Shift Keying
IDT	Inter Digital Transducer
IFFT	Inverse Fast Fourier Transform
LFM	Linear Frequency Modulation
PPF	Pulse Repetition Frequency
PRI	Pulse Repetition Interval
RAC	Reflective Array Compressor
RF	Radio Frequency
SAR	Synthetic Aperture Radar
SAW	Surface Acoustic Wave
SNR	Signal to Noise Ratio
SSB	Single Side Band
TBW	Time Bandwidth
TBWP	Time Bandwidth Product

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CHAPTER 1

1.1 PULSE COMPRESSION IN SYNTHETIC APERTURE RADAR

Pulse compression is widely used in modern radar systems. Its primary advantage is that it allows the use of long duration, low power pulses in place of short high power pulses which facilitates the use of low power transmitters. The generation of high power short duration pulses present formidable difficulties in transmitter design. The problem becomes even more challenging when the constraints of transmitter coherence, size, weight, and power consumption are considered. Such constraints often limit the performance of airborne/spaceborne systems. Despite the additional signal processing complexity, the clear advantages of pulse compression have led to its widespread use.

Modern high resolution Synthetic Aperture Radars (SAR's) have the additional constraint of requiring raw radar data to be available in a digital format in order to implement the azimuth compression via Digital Signal Processing (DSP) techniques. Early SAR's used optical techniques for azimuth compression whereas today's state-of-the-art systems employ digital techniques. The requirement for coherent digital data, which is not unique to SAR, often limits the choice of pulse compression techniques which can be utilized.

The obtainable compression gain or pulse compression ratio of a particular signal is proportional to the product of its time duration and frequency bandwidth

or its Time Bandwidth (TBW) product where the bandwidth is a measure of the range of frequencies contained in a single pulse. A monochromatic rectangular pulse has a TBW product of approximately one. Radars which use pulse compression typically utilize pulse compression ratios in the order of 100. The signal processing complexity required to implement pulse compression is dependent on the TBW product of the signal.

Pulse compression can be implemented using either digital or analogue techniques. The former allow flexibility in waveform selection, but are hardware intensive due to the considerable DSP and data storage requirements. Signal processing considerations are even more critical in SAR's due to the substantial DSP requirements associated with SAR azimuth compression processing. In digital pulse compression systems the signal is quantized prior to compression. Since the quantization noise does not compress, the dynamic range is not limited by the quantization hardware, provided of course, that sufficient compression gain is available. Digital pulse compression systems are usually restricted to relatively low TBW product signals because of the excessive DSP hardware requirements.

By contrast, analogue pulse compression devices can efficiently compress signals with TBW products in the thousands. Surface acoustic wave (SAW) devices are particularly well suited to radar applications by virtue of their operating characteristics. SAW devices capable of processing very large (up to 10000) TBW product signals have been produced^[5,13,19]. Although SAW devices are efficient, and

have a high dynamic range, they lack the flexibility associated with digital pulse compression systems and are usually based on a single fixed coding such as Linear Frequency Modulation (LFM). The use of analogue pulse compression in systems where digital data is required results in limiting of the dynamic range due to quantization. The maximum Signal to Noise Ratio (SNR) and hence dynamic range will be limited by the noise introduced during quantization.

High resolution radars operate with signal bandwidths up to about 500 MHz. Current state-of-the-art quantization systems operating at these frequencies are based on flash A/D converters and are generally limited to 8 bits of resolution. In practice, the number of effective bits available after 8 bit quantization is usually between 5 and 7 due to the non ideal performance of the quantization hardware^[6]. The number of effective bits is reduced even further when the signal is scaled to less than full scale in order to reduce the probability of saturation.

The type of radar and its role will influence the design of the range pulse compression system. Land and ship based radars for example are usually not subject to severe size and weight constraints permitting the use of high power transmitters and large signal processing facilities. By contrast, SAR's operate from either aircraft or satellites where the size, weight, and power consumption of the system are critical. The traditional role of SAR's has been that of ground mapping applications where radar returns are stored and processed in non real time. Because of its high resolution and long range capability, there has recently been considerable interest

in the use of SAR's in tactical roles. Operating in a tactical environment imposes a number of unique requirements on the radar such as the capability of processing returns in real time. In addition, the system should have the ability to operate to some degree in a hostile jammer environment. Maximizing the dynamic range of the system is of concern when a target classification capability is required. The system should also be as compact as possible so that it can be carried in small agile aircraft without displacing weapons payloads or other sensors.

It would be advantageous for a SAR pulse compression system to be able to efficiently compress very large (approximately 5000) TBW product signals but still retain the high dynamic range capability and flexibility associated with digital pulse compression. One possibility is to implement the compression in two stages where the first stage compression processing is done by an analogue device followed by digital techniques in the second stage. With careful design such a system could retain the advantages of both digital and analogue systems.

1.2 THESIS AIM AND OBJECTIVES

The aim of this thesis is to collect data on a two stage pulse compression system for use in SAR which can accommodate signals with TBW products in the order of 5000. The system would use a SAW Reflective Array Compressor (RAC) device for the first stage of compression. Information on such a system is not readily available in the open literature, due in part to the relatively recent appearance of hardware

capable of achieving such performance. Motivation for the research stems from the desire of the Airborne Radar section of the Defence Research Establishment Ottawa to develop a compact high resolution spotlight mode SAR based on a low peak power transmitter capable of operating in a tactical environment.

The aim of the research is summarized by means of the following thesis objectives:

- a. Design and build a system capable of generating arbitrarily coded expanded pulses with bandwidths in excess of 200 MHz and TBW products greater than 5000.
- b. Develop a compression system capable of evaluating the two stage compression process using a SAW RAC device for the first stage of compression.
- c. Evaluate various signal coding methods and determine those which are most suitable to the two stage compression process.
- d. Compare various means of implementing the second stage of compression and determine their signal processing requirements.
- e. Evaluate the effectiveness of the system in a hostile environment in the presence of tone, noise, and repeater jammer interference.

1.3 THESIS ORGANIZATION

This chapter discussed pulse compression systems for SAR and outlined the purpose of the thesis. Chapter 2 reviews conventional pulse compression theory and hardware and also examines the unique requirements of SAR's. Chapter 3 covers the theoretical aspects of two stage pulse compression systems which utilize LFM devices for the first stage of compression and the development of the complete signal generation and compression system. In chapter 4 the performance of the two stage pulse compression system is evaluated and chapter 5 consists of a concluding discussion followed by recommendations for further work.

CHAPTER 2 PULSE COMPRESSION THEORY

2.1 BACKGROUND

The development of radar pulse compression was motivated by the requirement for increased system performance. Early radars, such as those employed in World War II, had the conflicting requirements of increased detection performance and finer target resolution. The limited peak power capabilities of transmitters precluded the use of short duration high power pulses. By using longer pulses it was possible to take advantage of the greater average power capabilities of early transmitters; however the loss in resolution was often unacceptable.

The concept of using a long pulse with a linearly swept carrier frequency on transmission combined with a receiver filter possessing a linear delay versus frequency characteristic was thought of by a number of individuals^[3,8] at approximately the same time. These linear FM radars were also named "chirp" radars in reference to the sound made by a LFM signal at audio frequencies.

The fundamental principle behind all pulse compression systems is the fact that range resolution and accuracy are functions of signal bandwidth and not simply pulse length. An increase in range resolution requires an increase in bandwidth and not necessarily a reduction in pulse length. In a rectangular monochromatic pulse the signal bandwidth is approximately equal to the inverse of the pulse length. Shortening the pulse length provides higher resolution because of the increase in bandwidth.

Pulse compression systems exploit the fact that techniques other than shortening of pulse length can be employed to increase signal bandwidth. Consider the complex form of an arbitrary signal as shown in {2.1} where f_c is the carrier frequency and $A(t)$, $\phi(t)$ are the amplitude and phase modulation functions respectively. In a radar without pulse compression, $A(t)$ is a pulsed envelope, and $\phi(t)$ is zero. Signal bandwidth can be increased by introducing some form of modulation on the functions $A(t)$ (Amplitude Modulation) and $\phi(t)$ (Phase Modulation).

$$s(t) = A(t)e^{j(f_c t + \phi(t))} \quad \{2.1\}$$

Amplitude modulation other than the periodic pulsed envelope is usually not employed because it is not an effective means of spreading the signal bandwidth^[15] and it reduces pulse energy for a given peak power level. Another difficulty with AM is that it is difficult to obtain amplitude linearity throughout the radar system. The use of saturating amplifiers, for example, would distort or eliminate any amplitude modulation. The vast majority of pulse compression techniques exclusively involve modulation of the phase function.

2.2 MATCHED FILTERING AND PULSE COMPRESSION

For any given signal in the presence of white stationary noise a filter which maximizes the peak signal to mean noise power is called a matched filter. All pulse

compression filters are based on the matched filter concept. The impulse response of a matched filter for a given signal $s(t)$ is defined in the time and frequency domains by equations {2.2} and {2.3} where k and d are arbitrary constants.

$$h(t) = k[s^*(d-t)] \quad \{2.2\}$$

$$H(f) = k[S^*(f)] \quad \{2.3\}$$

In the time domain the matched filter impulse response is the time reversed conjugate (with arbitrary scaling and delay) of the original signal. In the frequency domain the matched filter is simply the scaled conjugate of the original signal. From the point of view of maximizing signal to noise ratio, matched filtering, by definition, represents the optimum processing technique.

When a signal is the input to its own matched filter the output of the filter is the autocorrelation function of the original signal. This is shown in equation {2.4} where an arbitrary signal $[s(t)]$ is convolved with the impulse response of its matched filter $[s^*(-t)]$ with the resulting autocorrelation function given by $R_{ss}(t)$. The frequency domain equivalent of {2.4} shown in equation {2.5} where it is seen that the autocorrelation function is the Fourier transform of the signal's energy spectral density $|S(f)|^2$.

$$[s(t)] * [s^*(-t)] = \int_{-\infty}^{\infty} s(\tau) s^*(-(t-\tau)) d\tau = \int_{-\infty}^{\infty} s(\tau) s^*(\tau-t) d\tau = R_{ss}(t) \quad \{2.4\}$$

$$S(f)S^*(f) = |S(f)|^2 = F[R_{ss}(\tau)] \quad \{2.5\}$$

The fact that the compressed pulse shape is described by a signal's autocorrelation function (for a matched filter receiver) is of use in selecting pulse compression waveforms. Since a signal's autocorrelation function is related to its spectral magnitude according to equation {2.5}, the shape of a signal's magnitude spectrum can be used to identify signals with desirable compressed pulse characteristics. In particular, for a signal to have low sidelobe levels the magnitude spectrum (and therefore magnitude squared) should have a smooth envelope which tapers towards the edges. The spectrum should also be free of discontinuities and cyclic ripples in order to avoid paired echo sidelobes as discussed in section 2.5.

2.3 THE AMBIGUITY FUNCTION

In most radar systems, targets of interest are often moving with respect to the radar. As a result the target return will have an inherent Doppler shift in addition to being delayed. In order to obtain a more complete picture of a waveform's properties the effects of Doppler shifts on a signal's autocorrelation function must be considered. The ambiguity function represents the output of a matched filter receiver as a function of Doppler frequency shift as well as time delay. The general form of the ambiguity function is shown in equation {2.6} where τ represents time delay and θ represents Doppler shift. Equations {2.7} to {2.9} indicate some of the

properties of the ambiguity function.

$$x(\tau, \theta) = \int_{-\infty}^{\infty} s(t) s^*(t - \tau) e^{j2\pi\theta t} dt \quad \{2.6\}$$

$$x(\tau, \theta) = x^*(-\tau, -\theta) \quad \text{symmetry about the origin} \quad \{2.7\}$$

$$|x(0, 0)|^2 \geq |x(\tau, \theta)|^2 \quad \text{maximum value} \quad \{2.8\}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x(\tau, \theta)|^2 d\tau d\theta = |x(\tau, \theta)|^2 \quad \text{constant volume} \quad \{2.9\}$$

The ambiguity function is useful in determining the suitability of a particular waveform for a particular application. The primary function of the radar will determine the target parameters which are to be measured as well as the resolution and interval over which the measurement is to be taken. The ambiguity function describes the ability of a particular signal to measure range and Doppler frequency as well as the interaction between the two measurements.

The distance or time interval at which two targets can be resolved is a function of the signal's spectral envelope (magnitude). It is also true that the Doppler frequency interval at which two targets can be resolved is a function of the envelope of the signal in the time domain. In general a wider envelope function in one domain will permit greater resolution in the other domain. It is interesting to note

that in theory pulse compression waveforms have the potential of providing high resolution in range as well as Doppler frequency since they can have a wide signal bandwidth as well as a long time duration.

2.4 PULSE COMPRESSION WAVEFORMS

It is clear from {2.1} that in theory the possible choice of techniques for bandwidth spreading is limitless. Because of practical considerations only a few pulse compression waveforms have come into widespread use in radar. In addition to a waveform's ambiguity function, there are a number of application-dependent properties which will influence waveform selection. In particular, the ease with which a waveform can be generated and compressed are often the main criteria for waveform selection. The discussion on pulse compression waveforms will be restricted to those waveforms with a constant envelope since it is usually desirable to maximize pulse energy for a given peak power level.

2.4.1 Linear Frequency Modulation

Linear FM was one of the first signals to be used in pulse compression systems and is still one of the most widely used. The phase function of the LFM signal {2.10} consists of a quadratic phase coefficient μ which results in a linear frequency versus delay characteristic. The term μ is also known as the sweep rate or FM rate.

One of the main reasons for the popularity of the LFM waveform has to do with the ease of fabrication of analogue pulse compression filters based on dispersive delay lines. The analogue pulse compression devices with the largest pulse compression ratios are usually based on LFM signals.

An interesting property of the LFM waveform is its symmetry between the time and frequency domains. When the TBW product of the LFM signal exceeds 100, the signal spectrum can be approximated by equation {2.11}. Comparison of equations {2.10} and {2.11} shows that a LFM signal in the time domain with a real envelope function $A(t)$ and a quadratic phase term μt^2 has a spectrum with an identical envelope function and quadratic phase term (f^2/μ) . The time frequency symmetry of the LFM waveform provides a simple means of reducing compressed pulse sidelobes. Weighting of the time domain envelope results in a similar weighting of the signal's frequency spectrum which can be used to reduce sidelobe levels at the expense of lowering pulse energy. The exact frequency spectrum of a LFM signal with envelope function $\text{rect}(t/T)$ is derived in appendix 1.

$$s(t) = A(t)e^{j\pi\mu t^2} \quad \{2.10\}$$

$$S(f) = \frac{1}{\sqrt{|\mu|}} A\left[\frac{f}{\mu}\right] e^{-j\pi f^2/\mu \pm j\pi/4} \quad \{2.11\}$$

As the TBW product of the LFM signal increases, the signal's spectral shape will become more rectangular. In particular, the drop off at the band edges becomes

steeper with the result that a smaller fraction of the signal's energy is out of band. This is very useful in systems where the signal is quantized at baseband since it permits sampling the signal close to the Nyquist rate with minimal aliasing effects. The near rectangular spectral shape results in the signal having an autocorrelation function which is closely approximated by a $\text{Sin}(x)/x$ function. The high sidelobe levels of this function (-13.2 dB) may be objectionable in most applications and frequency domain weighting techniques are often used to reduce sidelobe levels.

2.4.2 Non Linear Frequency Modulation

Non LFM refers to a general class of continuous phase codings in which the sweep rate is not restricted to a constant as with LFM. This may include wave forms such as QFM where the phase function includes a cubic term and Cos FM where the FM rate follows a sinusoidal function. The main advantage of nonlinear FM waveforms in general is that the autocorrelation function will have low sidelobes even though the transmitted pulses have a constant amplitude envelope. Sidelobe reduction with nonlinear FM waveforms is implemented without the loss in SNR associated with windowing techniques however nonlinear FM waveforms usually have reduced Doppler tolerance.

2.4.3 Binary Phase Coded Waveforms

Binary phase coded waveforms are signals where the phase function of the waveform in {2.1} alternates between the values of 0 and π . The transmitted pulse is usually divided into n equal sub pulses and the value of the phase function is determined by a binary sequence of length n .

For radar applications it is important that a binary sequence have a well behaved autocorrelation function since spectral windowing techniques are not effective in reducing sidelobes. This is due to the fact that the sidelobes of a binary sequence are attributed to the fine structure of the spectrum rather than the gross shape of the spectrum.

For a given length, the shape of the autocorrelation function of a sequence depends on the content of the sequence. Barker codes for example have side lobe levels of $1/n$ for an n length code. Unfortunately Barker codes are limited to a maximum length of 13. A computer search of binary codes of length up to 40 was carried out^[12]. It is interesting to note that the best codes had peak sidelobe levels which were only marginally better than that of a 13 bit barker code ($1/13$ versus $3/40$ for example) although the code lengths were significantly greater.

Another class of binary sequences known as maximal length sequences or M sequences have low peak sidelobe levels and no theoretical limit on sequence length.

M sequences have periodic autocorrelation functions with sidelobe levels in the order of $1/n$ where n is the sequence length. It is however the aperiodic autocorrelation function which is of interest in most radar applications and its peak sidelobe levels are higher. Actual peak sidelobe levels of the aperiodic autocorrelation function depend on the particular sequence. For example, levels of -18 to -19.8 dB for $N=127$ and -21.3 to -22.6 dB for $N=255$ (instead of -21 dB and -24 dB for the periodic autocorrelation functions) have been reported^[17]. It is worth noting that the autocorrelation function of an M sequence consists of a narrow main peak of width $2/n$ and very small (of the order of $1/n$) close in sidelobes. This can be seen in the computed autocorrelation function of a 127 bit M sequence shown in Figure 2.2.

2.4.4 Polyphase Coded Waveforms

Polyphase coded wave forms are discrete phase coded wave forms where phase shifts are not restricted to 0 or π . Clearly this describes a very broad class of waveforms. One group of polyphase codes known as Frank codes are of interest in radar because of their low sidelobe levels. Frank codes, and similar polyphase codes are based on a N phase alphabet where N^2 is the code length and pulse compression ratio. Frank codes have peak sidelobe levels in the order $1/(\pi N)^2$ which is substantially better than that of binary phase codes of comparable length. The phase jumps of a Frank code are actually a quantized approximation to the quadratic phase of a LFM waveform. This is also true of the Barker codes discussed previously.

Additional codes which are similar to Frank codes (P1,P2,P3,P4) but with greater tolerance to band limiting effects and Doppler shifts have also been developed^[11].

For comparison purposes the autocorrelation functions for each of the 4 types of waveforms discussed have been computed and are shown in Figures 2.1 to 2.4. Each waveform has a TBW product of approximately 100 except for the M sequence which has a TBW product of 127. The non linear Cosine FM waveform (Figure 2.4) has an FM rate which follows a raised cosine squared function $((\cos(X)+1)^2 + 4)/5$. The first sidelobe of its autocorrelation function has a level of -24 dB and the second sidelobe has level of -21.5 dB.

Figure 2.1 - LFM Autocorrelation Function, TBWP=100

Figure 2.2 - 127 Element M Sequence Autocorrelation Function

Figure 2.3 - 100 Element Frank Code Autocorrelation Function

Figure 2.4 - Cosine FM Autocorrelation Function

2.5 SIDELobe REDUCTION TECHNIQUES

As mentioned earlier, it is often the case that the autocorrelation function of a pulse compression waveform will have objectionably high side lobes which accompany the main response. In most radars, and certainly in high resolution SAR's, it is desirable to reduce peak sidelobes below -30 db with respect to the main lobe. Since it may not be practical to use a waveform whose autocorrelation function has such low sidelobe levels additional processing techniques must be employed. In general this implies a deviation from ideal matched filtering and a corresponding decrease in SNR.

In addition to the peak sidelobe levels, the total energy contained in all sidelobes with respect to the main lobe is also of interest. It is of particular concern in radars where a continuum of target returns are integrated or compressed. Any given target must compete with the sidelobes from adjacent targets. The Integrated Sidelobe Ratio (ISLR) is often used in comparing waveforms and is the ratio of the integrated areas of all of the sidelobes of a signal's autocorrelation function compared to the main lobe area. In most cases reducing the peak sidelobe levels will also lead to a proportional decrease in ISLR.

In general all sidelobe reduction techniques involve modifying the energy spectral density of the receiver matched filter output to a desirable shape. Sidelobe reduction techniques can be divided into two categories based on how the spectral

shape is modified. Techniques which modify the gross spectral shape can be referred to as windowing techniques while techniques which modify the fine spectral structure can be categorized under paired echo sidelobe reduction. The distinction between these techniques is somewhat arbitrary but is convenient for discussion purposes. It should be pointed out that paired echo sidelobe reduction techniques are also applicable to reducing non-monochromatic spectral ripples. Systems which require sidelobe levels of approximately -40 dB will often require a combination of techniques. It worth mentioning that since the transfer function of the complete radar system is the product of the individual subsystem transfer functions, any subsystem can cause an increase in sidelobe levels.

2.5.1 Sidelobe Reduction By Spectral Windowing

Windowing techniques involve multiplying the magnitude of the frequency spectrum by an appropriate weighting function in order to reduce time sidelobes. The technique is analogous to amplitude weighting of the aperture illumination function of antenna to reduce sidelobes in the antenna radiation pattern and the same weighting functions are applicable.

The use of windowing techniques will result in a broadening of the main lobe and an decrease in SNR. Choice of a particular weighting function will be a compromise between required sidelobe levels and the acceptable reduction in resolution due to pulse broadening as well as the reduction in SNR. The rate at

which the sidelobes decay as the distance from the main lobe is increased is also an important property of a particular weighting function. In some applications it may be acceptable to have high close in sidelobes provided that the far out sidelobes decay rapidly as is the case with the $\cos^x$ type weighting functions.

The previous discussion on weighting functions assumes that the signal spectrum prior to application of the weighting function is rectangular (constant amplitude) in shape. In general the spectral shape will not be perfectly rectangular resulting in sidelobes which are higher than expected. An example of this effect is shown in Figure 2.5 where the spectrum of a linear FM waveform with a TBW product of 100 is windowed with a Blackman weighting function after ideal matched filtering. The Fresnel ripples of the LFM waveform result in higher than expected sidelobe levels. These sidelobes, also known as Fresnel sidelobes^[19], are largest in the area around $\pm \tau/2$ where τ is the expanded pulse length. This is shown in Figure 2.6 where the peak sidelobes are much higher than the -60 dB expected from a Blackman weighting.

Figure - 2.5 Magnitude Spectrum - LFM with Blackman Window

Figure 2.6 - Compressed Pulse (One Side), LFM with Blackman Window

2.5.2 Paired Echo Sidelobe Reduction

Paired echo sidelobes are caused by cyclic variations in the frequency spectrum of a signal. This is a result of the relationship between the time and frequency domains whereby cyclic variations in one domain cause impulse functions in the other domain. The position of the impulse functions is determined by the cyclic rate of the fluctuations. Frequency domain ripple will not always result in distinct paired echoes. In the spectrum of a low TBW product LFM waveform for example, the ripples are non-monochromatic and result in a broad pedestal shaped sidelobe structure surrounding the main response of the compressed pulse. However, any technique which reduces the spectral ripple structure will also be effective at reducing both types of sidelobes. For discussion purposes, the application of paired echo sidelobe reduction techniques is extended to the general case of reducing any type of sidelobe structure which is caused by frequency domain ripple.

A cyclic amplitude variation with a rate of X cycles/second in the time domain causes impulses in the frequency domain at $\pm X$ Hz. Similarly, cyclic amplitude variations at a rate of Y cycles/Hz in the frequency domain will cause impulses (sidelobes) in the time domain located at $\pm Y$ seconds with respect to the main response. Close in paired echoes are caused by low frequency (in the frequency domain) ripple and far out paired echoes from high frequency ripple. As few as two ripple cycles across the signal bandwidth will cause the appearance of paired echoes and they can be caused phase ripples as well as amplitude ripples.

Sidelobes which are caused by spectral ripple are particularly troublesome since the windowing techniques discussed in the last section are not effective in reducing sidelobe levels. An amplitude ripple of 0.25 dB peak to peak in the magnitude spectrum of a signal will cause paired echo sidelobes at -30.5 dB after ideal matched filtering. Reduction of paired echo sidelobes to levels of less than -30 dB places stringent requirements on radar system components.

One method of reducing spectral ripple sidelobes involves the use of an inverse filter. For an arbitrary signal as specified in equation {2.13} where $|S(f)|$ is the spectral magnitude including amplitude ripple and $r(f)$ is the phase ripple, the inverse filter is one with an inverse magnitude and conjugate phase characteristic. After inverse filtering the signal will have a constant amplitude spectrum and will be free of phase ripples. A weighting function would be used in conjunction with the inverse filter to shape the signal spectrum and reduce sidelobes to the desired level. The complete pulse compression filter matched to the signal in equation {2.13} is given in equation {2.14} where $W(f)$ is an arbitrary weighting function. It is worth noting that this inverse matched filter consists of the ideal matched filter multiplied by $|W(f)|/(|S(f)|^2)$. This can be implemented by starting with the ideal matched filter and performing a magnitude inversion followed by multiplication by the windowing function.

$$|S(f)|e^{j2\pi(\phi(f) + r(f))} \quad \text{arbitrary signal} \quad \{2.13\}$$

$$\frac{W(f) e^{-j2\pi(\phi(f) + r(f))}}{|S(f)|} \quad \begin{array}{l} \text{inverse matched filter} \\ \text{with window} \end{array} \quad \{2.14\}$$

Implementation of the inverse matched filter is not as straight forward as equation {2.14} suggests. For the filter to be defined at all points the signal spectrum must be non zero within the signal bandwidth. It might not be possible to use an inverse matched filter when target returns are Doppler shifted. If Doppler shifts are substantial and the spectrum ripple frequency is high, the inverse filter may reinforce rather than cancel the ripples. Another difficulty arises if the spectral ripples are random in nature. In the case where the ripple structure changes from pulse to pulse, a new matched filter is required for each pulse. Because of the difficulties associated with the implementation of the inverse filter it is preferable to avoid its use by reducing or eliminating the source of the ripple if possible.

2.6 ANALOGUE PULSE COMPRESSION DEVICES

Analogue pulse compression devices are extremely efficient in terms of their signal processing capability when compared to similar DSP systems. Of the various analogue technologies which are available, Surface Acoustic Wave (SAW) dispersive delay lines have come to be the preferred choice for radar pulse compression applications. In particular, their operating frequencies, signal bandwidths, and dispersion times are well suited to typical radar system requirements. Because of the relatively slow propagation speeds of surface waves (1-6 KM/S compared to 300,000

Km/s for free space propagation) relatively large delays can be incorporated on small substrates. SAW dispersive delay lines can be broken down into two categories based on device operation as discussed below.

2.6.1 SAW IDT Devices

SAW Inter Digital Transducer (IDT) devices are based on transducer structure of overlapping apodized metal fingers placed on a piezo electric substrate which is usually quartz or Lithium Niobate. In the IDT type device, when an electrical signal is applied to the transducer, surface waves are generated in the substrate material because of the piezoelectric coupling. Dispersion is controlled by adjusting the position and length of the interdigital electrodes within the transducer.

As can be seen from Figure 2.7 the electrode spacing of the dispersive output transducer changes from end to end. When the electrode spacing is of the order of $\lambda/2$ the transducer is resonant and maximum energy is coupled out. Because of the varied electrode spacing different parts of the transducer are resonant to different wavelengths and hence frequencies. In the SAW IDT device of Figure 2.7 the lower frequencies are resonant at the far end of the transducer and are therefore delayed more than the high frequencies. Note that both transducers in the SAW IDT device can be made to be dispersive. It is also possible to introduce spectral windowing by controlling the overlap or length of the fingers. The maximum TBW products of typical SAW IDT devices are usually in the order of 500 and device insertion losses in the range of -25 to -40 dB.

INTERDIGITAL TRANSDUCER SAW DEVICE

Figure 2.7 - SAW IDT Device

SAW REFLECTIVE ARRAY COMPRESSOR DEVICE

Figure 2.8 - SAW RAC Device

2.6.2 SAW RAC Devices

The other main type of SAW dispersive delay line is the SAW Reflective Array Compressor (RAC) device which is based on a reflecting array structure. In the SAW RAC device, dispersion is achieved by controlling the path length that different frequencies must take in propagating from input to output. A typical SAW RAC configuration is shown in Figure 2.8. The signal propagates through the device in a U shaped pattern after making two reflections. Other reflector configurations are available but the 2 reflection U shape is the most widely used. The principle behind the operation of the SAW RAC device is the fact that the reflectors are frequency dependent. By controlling the spacing between the reflectors different parts of the array reflect different frequencies. When the spacing between reflector elements is of the order of $\lambda/2$, where λ is the surface waves wavelength, maximum energy is reflected. Inter element spacing can be varied to provide the desired frequency verses delay characteristic.

The reflector elements are usually formed by either depositing metal strips or dots on the substrate or by etching grooves in the substrate. In the case of etched grooves reflectivity can be controlled by adjusting groove depth. This technique can be used to correct or modify the devices amplitude spectrum^[24,5]. Corrections to the phase of the device are carried out by depositing a shaped metal film on the area between the reflector arrays as shown in Figure 2.8. Plating the surface of a piezoelectric changes the velocity of the surface wave. Adjusting the width of the

plated region will control the phase as a function of frequency.

Quartz and Lithium Niobate are the two most widely used substrates for SAW devices. Large TBW product SAW RAC devices are usually fabricated on a Lithium Niobate substrate since its high coupling properties help to minimize device insertion loss. Because of the high temperature coefficient of lithium niobate, devices which use it as substrate material are often ovenized if a high degree of device stability is necessary. Since the SAW RAC device achieves dispersion by the reflection of surface waves it is not necessary for the portion of the substrate which contains the reflecting grooves to be piezoelectric.

Saw RAC devices are characterized by very large TBW products with typical values in the range of 500-10,000^[5,7,19]. Devices in this category have considerable insertion losses with typical values of -45 to -60 dBm. SAW RAC devices are relatively free of second order effects^[24] and can typically be fabricated with .5 dB peak to peak magnitude ripple and phase errors in the order of a few degrees rms depending on the devices TBW product^[7,24]. With the exception of exotic technologies such as superconductive pulse compressors, SAW RAC devices have the greatest processing capabilities of current technologies for radar pulse compression applications^[13].

2.7 PULSE COMPRESSION CONSIDERATIONS FOR SAR

The selection of pulse compression waveforms and techniques is dependent on the requirements of the particular radar system. Signal parameters for high resolution airborne SAR systems operating in side looking and squinted mode will be discussed.

To begin with a more precise definition of the term high resolution is required. Clearly any such definition is only a guideline and will change with time as newer technologies evolve. In the case of SAR systems, high resolution systems will be defined as those system with signal bandwidths greater than 40 MHz with typical values in the range of 40 to 400 MHz. Systems in this category will have range resolutions in the order of 12 ft to 1.2 ft and similar azimuth resolutions.

SAR's exploit the fact that returns from targets in different azimuth positions within the beam have different Doppler shifts. In the side looking case (SAR with the beam pointed perpendicular to the vehicle motion) the azimuth returns have a quadratic phase or LFM coding and targets in the center of the beam have zero Doppler shift. Azimuth resolution is improved by compressing or focusing the azimuth returns in a manner analogous to range compression of LFM returns. It is worth noting that in a SAR the total compression gain of the system is the product of the range and azimuth compression gains.

The fact that received signals in a SAR have an inherent Doppler shift must be considered when selecting waveforms for range pulse compression. Ideally the shape of the autocorrelation function of the transmitted waveform should remain constant over the expected Doppler frequency range. In terms of the ambiguity surface this implies a knife edge along the Doppler axis extending over the Doppler bandwidth.

Clearly the expected range of Doppler frequency shifts will influence the range compression processing and waveform selection. The general expression for the Doppler shift (f_d) seen by a platform in horizontal motion over a plane surface is given by {2.15}^[21] where X, Y, H are the 3 dimensional coordinate points of the target, f_c is the carrier frequency, v the platform velocity in the X direction, and c is the speed of light. The expected Doppler bandwidth (BW_d) for a side looking SAR is given by {2.16}^[21] where β_{az} is the azimuth beamwidth.

$$f_d = \frac{2vXf_c}{c\sqrt{X^2 + Y^2 + H^2}} \quad \{2.15\}$$

$$BW_d = \frac{2vf_c\beta_{az}}{c} \quad \{2.16\}$$

In the case of side looking airborne SAR's typical values for maximum Doppler shift will be in the order of $f_c \cdot 10^{-7}$ (based on a 2 degree azimuth beamwidth and an aircraft velocity of 1500 KM/H). An X Band ($f_c = 10$ GHz) SAR with these specifications will encounter Doppler shifts with a maximum value of 1000 Hz.

Nyquist sampling considerations require that the pulse repetition frequency (PRF) be greater than the doppler bandwidth (for complex baseband sampling) in order to avoid aliasing effects. In general the expected Doppler shifts in high resolution systems are very small compared to the signal bandwidth. If the SAR described above had a bandwidth of 200 MHz then the maximum Doppler shift in sidelooking mode would be less than 0.0005% of the signal's bandwidth. This simplifies the task of finding waveforms with desirable ambiguity properties since the expected values of Doppler shift will have little effect on the signals autocorrelation function. It is worth noting that in a satellite based system the platform velocities are greater and beamwidths are narrower. The net effect of these changes is a greater Doppler bandwidth than that of a typical airborne SAR with the same centre frequency.

The previous discussion on Doppler shifts has implicitly assumed that the Doppler shift from a particular scatterer is constant over the signal bandwidth. Equation {2.15} clearly shows that this will not be the case since the magnitude of the Doppler shift is proportional to the instantaneous frequency. This implies that the received signal spectrum is not simply shifted but also dispersed. This dispersion causes the receivers cross-range compression filter to be mismatched. For proper processing a Doppler dependent matched filter would be required. The effect is more pronounced in large TBW product systems however in most operating regimes the effect is negligible^[15].

2.7.1 Spotlight Mode SAR's - Range Compression Considerations

Some SAR's have the capability of squinting or spotlighting the beam in a direction other than broadside. This has the effect of changing the centre frequency and reducing the SAR's Doppler bandwidth. The shift in centre frequency is of particular concern in the design of the range pulse compression system. As the beam squints towards the direction of platform motion the Doppler centre frequency approaches $2vf_c/c$. The reduced Doppler bandwidth usually precludes operating with the beam close to the direction of motion. Consequently the expected Doppler centre frequency will be less than $2vf_c/c$, however the maximum Doppler frequency which must be accommodated, will still be approximately 1 to 2 orders of magnitude greater than that of the side looking case. As a result the Doppler tolerance requirements for pulse compression waveforms in squint mode SAR's are more stringent than that of sidelooking SAR's. However, even when squinting the beam forward, the effect of the doppler shift is negligible for most high resolution systems.

CHAPTER 3

TWO STAGE PULSE COMPRESSION SYSTEM DESIGN

In developing a two stage pulse compression system there are a number of considerations which will influence the design. First of all, the type of signal coding for the analogue compression stage must be selected. The main function of this stage is to provide efficient processing of large TBW product signals and since the analogue signal processors with the highest performance are based on dispersive delay lines, linear FM (or possibly nonlinear FM) is the defacto choice. The choice of signal codings associated with the second (digital) stage of compression will be driven by the system requirements and is an area which will be investigated in depth in chapter 4. The relative weights of the first and second stage codes must also be determined. While the maximum second stage code TBW product will be influenced by the digital processing capability of the system, it is also of interest to determine if there is an optimal choice of second stage compression ratio.

The motivation for using a two stage pulse compression system is to allow use of very large TBW product signals and yet retain to some degree the advantages of a digital pulse compression system. The two main characteristics of a digital pulse compression system which are of interest are:

- a) that the dynamic range of the signal is not limited to that of the A/D converters as is the case when a signal is quantized after analogue

compression; and

b) that from a signal processing point of view, the choice of transmitted waveform is arbitrary and can be changed to provide protection against repeater/deception jammers.

For a typical airborne application, the TBW product of the analogue code is limited by the capability of the analogue compressor, to approximately 5000. In a similar manner, the TBW product of the digital code would be in the order of 100 because of the DSP hardware requirement. The relative size of these two codes implies that approximately 98% of the signal processing is done in an analogue fashion and 2% is accomplished digitally. It is of interest to determine how much of an effect the relatively small digital code can have in influencing the overall system performance.

The only real drawback of a digital pulse compression system for radar is its hardware efficiency. Digital processing of signals with time bandwidth products in the order of 5000 by one estimate^[13] would occupy a rack of equipment and consume over 100 watts of power. By comparison, analogue pulse compression devices such as the SAW RAC device can perform the equivalent processing function with a single chip or component. Comparisons of the relative computational speeds of analogue and digital signal processors^[13,18] indicate that SAW RAC devices are 3-4 orders of magnitude more powerful than current programmable array signal processor systems commonly used in radar signal processing applications.

In essence, digital and analogue pulse compression systems represent the two ends of the spectrum while a combined system retains, to some extent, the capabilities of both.

While it may be possible to implement the two stage pulse compression concept in any radar, its application is more suitable in certain types of radars than others. Modern SAR's for example have a requirement for digitized data and consequently the quantization hardware will already be a part of the receiver regardless of the type of pulse compression system utilized. In such systems implementing a digital or a hybrid (combined analogue and digital) pulse compression capability can be accomplished with minimal modification to the receiver circuitry and mainly impacts the signal processing workload. In a system where the SAR processing is done in non real time this implies that more computer time will be required to process the returns. In real time systems however, an increase in processing hardware will be required. It is interesting to note that in an existing system employing digital compression, a hybrid (two stage) capability could be achieved with the addition of an analogue compressor and modification of the digital compression algorithms. By incorporating a two stage pulse compression capability, this existing SAR could be modified to use a low power long pulse transmitter without any additional signal processing hardware.

3.1 PARTIAL COMPRESSION OF CODED LFM SIGNALS

As a signal moves through a pulse compression network, the time bandwidth of the signal is gradually reduced, and when the $TBW=1$ the signal is fully compressed. In most systems the bandwidth of the signal remains constant and it is only the time duration which is reduced from the expanded pulse length to the minimum value of approximately $1/BW$.

In the hybrid pulse compression system, the expanded pulse can be considered to be the product of two components; a linear FM component which is compressed by the SAW RAC device, and an arbitrary secondary code which is digitally compressed. The complex form of an arbitrarily coded LFM signal is given in equation 3.1 where $c(t)$ is the secondary code and $l(t)$ is the LFM component. The two codes are considered to both have the same duration τ , which is the expanded pulse length. In {3.1} $c'(t)$ is used to identify the secondary code component other than the $\text{rect}(\cdot)$ envelope function. The time bandwidth of the combined signal is the sum of the individual TBW products as shown in equation {3.2}. Since the signals have the same time duration, the ratio of their TBW products is simply the ratio of their bandwidths $\Delta f_c/\Delta f_l$.

A coded LFM signal is given by:

$$c(t)l(t) = [\text{rect} \left[\frac{t}{\tau} \right] c'(t)] e^{j\pi\mu t^2} \quad \{3.1\}$$

$$\begin{aligned} TBWP_T &= TBWP_c + TBWP_l \\ &= \tau[\Delta f_c + \mu\tau] \end{aligned} \quad \{3.2\}$$

The impulse response of the SAW is given by:

$$h(t) = \text{rect}\left[\frac{t}{\tau}\right] l^*(-t) = \text{rect}\left[\frac{t}{\tau}\right] e^{-j\pi\mu t^2} \quad \{3.3\}$$

During the first stage of compression the signal is partially compressed by the SAW device which is a matched filter for the LFM portion of the composite signal in {3.1}. This implies that the impulse response of the SAW device ($h(t)$) will be the time reversed conjugate of the LFM signal as shown in equation {3.3}. Notice that due to the time symmetry of the LFM waveform {3.3} is simply the conjugate of the LFM signal. A mathematical expression for the partially compressed output of the SAW device can be obtained by convolving the composite signal in equation 3.1 with the impulse response of the SAW device {3.3}. This is shown below using the definition of convolution given in equation {3.4} and the final result is shown in equation {3.5}.

$$X(t) * Y(t) = \int_{-\infty}^{\infty} X(\alpha) Y(t-\alpha) d\alpha \quad \text{definition of convolution} \quad \{3.4\}$$

$$\begin{aligned} [c(t) l(t)] * h(t) &= \int_{-\infty}^{\infty} [c(\alpha) l(\alpha)] h(t-\alpha) d\alpha \\ &= \int_{-\infty}^{\infty} [c(\alpha)] \text{rect}\left[\frac{t-\alpha}{\tau}\right] e^{j\pi\mu\alpha^2} e^{-j\pi\mu(t-\alpha)^2} d\alpha \\ &= e^{-j\pi\mu t^2} \int_{-\infty}^{\infty} [c(\alpha)] \text{rect}\left[\frac{t-\alpha}{\tau}\right] e^{j2\pi\mu t\alpha} d\alpha \end{aligned} \quad \{3.5\}$$

It is of interest to note that the integral of equation {3.5} is in the form of a scaled inverse Fourier transform of the secondary code. The result of {3.5} is in fact based on a well known transformation^[7] called the chirp transformation. The essence of this transformation is the fact that a Fourier transform is equivalent to premultiplication by a chirp followed by convolution with a chirp followed by post multiplication with a chirp. This relationship can be verified by starting with the definition of the Fourier transform as given in {3.6} and using the linear relationship between time and frequency in an LFM waveform with the final result shown in {3.7}

Starting with the definition of the Fourier Transform given by:

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt \quad \{3.6\}$$

Let $f = \mu\tau$ (LFM relationship) therefore

$$S(\mu\tau) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi\mu\tau t} dt$$

Now substituting $-2\tau t = (t-\tau)^2 - t^2 - \tau^2$ gives:

$$\begin{aligned} S(\mu\tau) &= \int_{-\infty}^{\infty} s(t) e^{-j2\pi\mu[1/2(t-\tau)^2 - t^2/2]} e^{-j2\pi\mu\tau^2/2} dt \\ &= e^{-j2\pi\mu\tau^2/2} \int_{-\infty}^{\infty} [s(t) e^{-j2\pi\mu t^2/2}] e^{j2\pi\mu(t-\tau)^2/2} dt \end{aligned}$$

But $(t-\tau)^2 = (\tau-t)^2$, and using the definition of convolution given in (3.4) we get:

$$\begin{aligned} S(\mu\tau) &= e^{-j2\pi\mu\tau^2/2} \int_{-\infty}^{\infty} [s(t) e^{-j2\pi\mu t^2/2}] e^{j2\pi\mu(\tau-t)^2/2} dt \\ &= [e^{-j2\pi\mu\tau^2/2}] \cdot [(s(t) \cdot e^{-j2\pi\mu t^2/2}) * [e^{j2\pi\mu t^2/2}]] \end{aligned} \quad \{3.7\}$$

The significance of equations {3.5} and {3.7} is the fact that the partially compressed SAW output is approximated by the Fourier transform of the secondary code. To determine the exact form of the SAW output the scaling factor $\mu\tau$ and the postmultiplication chirp must be taken into account. If the secondary code is scaled such that it occupies a bandwidth of $\mu\tau$ in the frequency domain then the inverse Fourier transform of this signal (after post multiplication by a chirp) will represent the time domain output of the SAW with proper units of time. This result is easily verified by considering the case when no secondary coding is used namely $c'(t) = 1$ and therefore $c(t) = \text{rect}(t/\tau)$. By equation {3.6}, the output of the SAW would be approximated by the inverse Fourier transform of a rectangular window over a bandwidth $\mu\tau$. This is clearly a sinc(x) function which is the expected result, since the absence of a secondary code implies that the SAW fully compresses the signal. For the complete Fourier transform relationship of equations {3.6}, {3.7} to hold, the SAW output must be multiplied by a chirp. In the absence of this chirp, only the signal magnitudes will follow the Fourier relationship and the phase functions will not.

The Fourier relationship is useful in assessing the relative merits of one secondary code over another. In general, the partially compressed SAW output should be as time constrained as possible in order to minimize the amount of data which must be sampled and processed. This implies that secondary codes which have a bandlimited frequency spectrum are preferable since they will also have a time limited SAW output. An example of the Fourier transform relationship is shown in Plate 3.1 which is the partially compressed SAW output of a 100 MHz LFM signal that has been multiplied by a 13 bit Barker code. For comparison, the magnitude of the Fourier transform of a 13 bit Barker code is shown in Figure 3.1.

Plate 3.1 - 100 MHz Barker Coded
Partially Compressed LFM Signal

Figure 3.1 - Fourier Transform (Magnitude) of a 13 Bit Barker Code

Plate 3.2 - 100 MHz LFM Signal
Fully Compressed by SAW Device

Plate 3.3 - 100 Mhz LFM Signal Fully Compressed by
SAW Device, Detailed Sidelobe Structure

3.2 PROPERTIES OF PARTIALLY COMPRESSED SIGNALS

In designing a two stage pulse compression system some portion of the total TBWP is associated with each of the two codes. It is useful to determine the effect of changing the relative weights of the two codes on overall system performance. The parameters of particular interest are the peak value and time extent of the SAW output.

The time extent or shape of the SAW output can readily be determined by the Fourier transform relationship between the secondary code and the SAW output given in equation {3.5}. For a given family of secondary codes (LFM,Biphase,etc.) an increase in signal bandwidth will result in a corresponding increase in the width of its Fourier transform. This in turn implies that the time extent or width of the partially compressed pulse at the SAW output will increase linearly as the secondary code bandwidth is increased. In the Fourier transform relationship of {3.5} the frequency axis is scaled by the factor $\mu\tau$ which is also the primary code bandwidth. Because of these two relationships we find that the width of the partially compressed pulse is proportional to the ratio $\Delta f_2/\Delta f_1$ for a given code family where Δf_2 is the secondary code bandwidth and Δf_1 the primary code bandwidth. This relationship is given in equation {3.8} where τ_p is the partially compressed pulse width. The constant "k" is dependent on code family and definition of pulse width.

$$\tau_p = k \left[\frac{\Delta f_2}{\Delta f_1} \right] = kTBWP_2/TBWP_1 \quad \{3.8\}$$

The above relationship indicates that for a given second stage TBW product, the time extent of the saw output should increase linearly with secondary code TBW product. Verification of the above relationship is shown in Figure 3.3 (page 52) which is a plot of partially compressed pulse width versus second stage TBW product. The results include computed (simulated) values, as well as experimentally measured values obtained by using the software and hardware developed for evaluating the two stage compression process (discussed in detail in Section 3.3). The computed data was obtained by mathematically generating LFM signals of various TBW products from 5000 to 5500 and convolving them with the simulated impulse response signal of a SAW device whose FM rate was matched to the signal with a TBW product of 5000. The convolution was accomplished by taking 16k FFT's of both signals, generating the product, and taking the IFFT. All of the LFM test signals had the same time duration to allow comparison of partially compressed pulse amplitudes. In the case of the experimentally measured data, signals as described above were generated in hardware and partially compressed using the SAW RAC device described in Section 3.3.3. All signals had an expanded pulse length of 24.5 μ s and bandwidths between 204.2-224.6 MHz.

The computed as well as measured values verified the linear relationship between second stage TBW product and partially compressed pulse width. The experimental points however tended to be lower (indicating a narrower compressed pulse) than the computed values. This is attributed to the fact the experimentally generated expanded pulse was tapered at the ends due to band limiting filters in the

circuit. After partial compression the 3 dB width of the signal will be narrower than the ideal computed signal. The last experimental data point corresponding a $TBWP=5500$ does not follow the linear trend. Although included in the graph, the point is considered to be invalid since the bandwidth of the signal (224.6 MHz) significantly exceeds the bandwidth of the SAW device (214.5 MHz).

The other important characteristic of the partially compressed pulse is its peak amplitude and this can also be determined from the Fourier transform relationship of equation {3.5}. It is useful to consider the case when the signal is fully compressed by the SAW device and the secondary code is simply the rectangular envelope of length τ_{ex} . In this case the secondary code has a TBW product of one and a bandwidth of $1/\tau_{ex}$. Now consider the case where the TBW product of the secondary code is increased from 1 by introducing some form of phase modulation under the constant amplitude envelope. Since the pulse length is still the same the increase in the bandwidth of the secondary code is proportional to the increase in TBW product as shown in {3.9}.

$$\Delta f_2 = TBWP_2 / \tau_{ex} \quad \{3.9\}$$

Equation {3.9} shows that a secondary code with a TBWP of X will have a bandwidth which is X times the minimum possible bandwidth of $1/\tau_{ex}$. For an phase coded rectangular expanded pulse of amplitude A, and length τ_{ex} , the total signal energy is given by $A^2\tau_{ex}$. In general the pulse energy is given by the area under the

square of the signal envelope. Notice that the pulse energy is unaffected by any phase modulation under the rectangular envelope. Regardless of the TBW product of the secondary code, the signal energy remains constant. It is also true that the relative pulse energy is the same for partially and fully compressed pulses. However, during the second stage of compression the peak power level is increased by a factor equal to the second stage TBW product ($TBWP_2$). As a result the peak amplitude of a partially compressed pulse should be inversely related to the square root of the secondary code TBW product. This is shown in equation {3.10} where A_p is the partially compressed pulse peak amplitude, A_f is the fully compressed peak amplitude, and k is a constant that depends on the type or family of the second stage code as well as definition of pulse width.

$$A_p \approx \frac{kA_f}{\sqrt{TBWP_2}} \quad \{3.10\}$$

When the second stage TBW product is small (less than 25), the secondary code type will significantly influence the peak level. For example, a biphase code with a TBW product of 10 and a $\text{sinc}(x)$ spectral shape, will result in a partially compressed pulse with a $\text{sinc}(x)$ shape and a width 10 times that of a fully compressed pulse. Since a fully compressed pulse also has a $\text{sinc}(x)$ shape, the value of k in equation {3.10} would be one, and the partially compressed pulse should have a peak amplitude which is $1/\sqrt{10}$ that of a fully compressed pulse. If the -3.92 dB points are used to define the pulse width ($\pm \pi/2$ values of $\text{sinc}(x)$ function), then an exactly rectangular pulse has a value of $k = 1$. Most practical waveforms will have peaks and

valleys in the partially compressed pulse and as a result the value of k will be greater than 1.

Verification of equation {3.10} is shown in Figure 3.2 (page 52) where the computed and experimentally measured values of peak amplitudes of partially compressed LFM pulses are plotted. Note the close agreement between the both sets of data and equation {3.10}. For the computed data a value of $k = 1.18 \pm 0.02$ was measured and $k = 1.49 \pm 0.08$ for the experimental data. The higher peak values for the experimental data are probably due to the fact that the peaks in the actual pulses are more pronounced than those of the ideal simulated pulses.

The significance of equation {3.10} is the fact that the peak value of the partially compressed pulse drops very quickly as the second stage TBW product is increased from 1. For example, an increase in secondary code length from 1 to 50 reduces peak amplitude from 1.0 to approximately .18 while an increase from 100 to 200 only results in a decrease from .12 to .084 .

The decrease in peak amplitude of partially compressed pulses is an important consideration in trying to maximize the dynamic range of the signal. In a system where the signal is quantized after being fully compressed, the dynamic range of the signal is limited by the quantization noise. In a two stage system the signal is quantized at the reduced peak amplitude (after partial compression) and after digital compression, the signal achieves its full peak amplitude. Since the quantization

noise does not compress, the signal to quantization noise ratio increases by an amount equal to the second stage TBW product. As a result, a relatively small secondary code TBW product can result in a significant improvement in dynamic range. For example, a second stage code with a TBW product of 16 will result in peak amplitude reduction of approximately $1/4$ which corresponds to two bits or approximately 12 dB of additional dynamic range for a given quantization system.

Figures 3.2, and 3.3 (page 52) also indicate that there is an optimum secondary code bandwidth from the point of view of increasing dynamic range. Although increasing the TBW product of the secondary code to 200 results in a significant improvement in dynamic range (23 dB approximately), further increases have a diminishing effect. On the other hand, the partially compressed pulse width (and processing effort) increase linearly with secondary code TBW product. Based on these findings it is clear that second stage codes with TBW products in the range of 25-200 are optimum from the point of view of maximizing the dynamic range while minimizing the digital processing requirements.

Equation {3.10} also shows that the decrease in peak amplitude is not dependent on the TBW product of the first stage (analogue) code. Experimental verification of this can be seen by comparing Plate 3.1 (page 43) which is the SAW output of a 100 MHz LFM signal coded with a 13 bit Barker code with Plate 3.2 (page 44) which shows a fully compressed 100 Mhz LFM signal (TBW=1230) with no secondary code. Notice that even with the reduced analogue code TBW product,

as compared to the experimental data of Figure 3.2 which uses an analogue code $TBW = 5000$, the peak value of the partially compressed pulse (excluding spikes) is $96/380 = .27$ (or $124/380$ if spikes are measured). This value is comparable with $1/\sqrt{13} = .277$ as predicted by equation {3.10} for a $\text{sinc}(x)$ function where $k = 1$. The spikes in waveform cause the shape to deviate from a $\text{sinc}(x)$ function and as a result, the actual peak value is approximately .326 that of the fully compressed pulse indicating $k = 1.18$.

This effect explains why pulse compression systems which use large TBW product signals seem to be highly sensitive to system mismatches. When the signal is mismatched with respect to the pulse compressor the pulse is partially compressed (or over compressed) and the TBW product of the residual component is greater than one. The residual component is equivalent to the secondary code in the previous analysis. As an example, a LFM signal with a $TBW = 5000 = \mu \tau^2$ will have residual component with a TBW product of 5 for a 0.1% change in μ and this results in the peak amplitude reducing by more than half. For a similar effect on a signal with a $TBW = 100$ a 5.0% change in the value of μ is required.

Figure 3.2 - Peak Amplitude of Partially Compressed LFM Pulses

Figure 3.3 - Pulse Width of Partially Compressed LFM Pulses

3.3 ARBITRARY SIGNAL GENERATION

In order to evaluate the hybrid pulse compression system the capability of generating a wide variety of waveforms is required. For this reason a digital signal generation technique was chosen. By using a system based on a D/A converter, it is possible to mathematically specify the points of a discrete waveform in a completely arbitrary manner. For the signals to be representative of typical high resolution SAR waveforms, the system must be capable of generating signals with bandwidths in excess of 200 MHz which requires a D/A system with a sampling rate of 400 Mega samples per second. This implies that data values must be transferred from memory to the D/A converter once every 2.5 nS. Systems operating at these speeds represent state-of-the-art technology and are difficult to obtain, particularly if 8 or more bits of resolution are desired.

One means of overcoming the bandwidth limitation of the D/A converters is by generating a complex baseband signal. Using this technique, a signal of bandwidth of Δf can be generated with two D/A converters each with a bandwidth of $\Delta f/2$. This permits doubling the system bandwidth by using two channels in parallel. Once the signal is generated it must be translated to a higher frequency for eventual transmission. The two main techniques for achieving this are single sideband (SSB) modulation and mixing and filtering. The SSB modulator can be considered as half of a complex mixer which translates a complex (two channel) base band signal to a real signal at some RF offset frequency. The other technique involves starting with

a real signal and mixing with a real signal and using filters to suppress the unwanted side bands. The SSB modulation technique has the advantage of not requiring filters with sharp roll off to adequately suppress the unwanted sideband. On the other hand, for SSB modulation to effectively cancel the unwanted sideband good phase and amplitude balance between modulator components is necessary. While it may be possible to fine tune the components to achieve the necessary balance at a particular frequency maintaining the balance over a wide bandwidth is significantly more difficult.

It might not always be possible or desirable to generate a base band signal with the required bandwidth and other techniques must be used to expand the bandwidth. Wideband frequency doublers provide a means of increasing signal bandwidth and can be considered as a mixer where the two input signals are the same. This has the effect of squaring the input, with the result that phase functions are added and amplitude functions squared as shown in equation 3.11. The net result (excluding the DC term) is a signal at twice the carrier frequency ($2\omega_c$) and double the bandwidth.

$$[A(t)\cos(\omega_c t + \phi(t))]^2 = A^2(t)/2[\cos(2\omega_c + 2\phi(t)) + 1] \quad \{3.11\}$$

One of the difficulties of using a doubler is the fact that in addition to doubling (x2) the doubler output will include the x1, x3, x4 etc. products of the input signal. This is shown in Figure 3.4 (page 56) where the response of a doubler to a CW signal at 100 MHz is shown. One of the other difficulties with the doubler is the

output response is frequency sensitive. This is shown in Figure 3.5 (page 56) where the doubler output to a signal with a bandwidth of 50 MHz and centre frequency of 250 MHz is shown. Another consideration in using doublers as shown in equation {3.11} is that the envelope function of the signal ($A(t)$) is squared, so any amplitude ripples in the signal will increase in size. The net result is that magnitude ripples of negligible amplitude which exist in the signal prior to doubling, may lead to paired echoes of objectionable size after doubling (or quadrupling).

Figure 3.4 - Frequency Doubler Output Spectrum for a 100 MHz Input

Figure 3.5 - Doubler Output for a 225-275 MHz Swept Input Signal

3.3.1 Circuit Description

A number of signal generation techniques were evaluated and the system which achieved the best performance utilized complex baseband signal generation as well as doublers to increase the signal bandwidth. A complete block diagram of pulse expansion and compression hardware is shown in Figure 3.6 (page 60) and the local oscillator/clock generation system is shown in Figure 3.7 (page 61). The system was designed around the SAW RAC device which had a centre frequency of 450 MHz. A photograph of pulse generation/compression facility is shown in Plate 3.3 (page 62).

An arbitrary function generator (AFG) (LeCroy Model 9100) was used as the signal source. The AFG which is essentially a D/A and memory system is capable of generating waveforms at 200 million samples/second into 1 channel or 100 million samples/second into two channels with 8 bit resolution. In either case the maximum achievable bandwidth is 100 MHz. Waveform data was transferred to the AFG over a IEEE-488 GPIB interface from a personal computer. Both channels of the AFG were used to form an SSB modulator which generated a real signal with a centre frequency of 225 MHz and a maximum bandwidth of 100 MHz. Two doublers were cascaded with amplifiers and filters in between to permit generating signals with bandwidths in excess of 200 MHz, and also to avoid operating the D/A converters at close to the Nyquist rate. The system was however designed such that the $\times 2$ signal was at the appropriate centre frequency ($f_c = 450$ MHz) to allow the signal to

go directly into the SAW device if desired. In the circuit of Figure 3.6 (page 60) this implies that the middle portion of the circuit is circumvented. After quadrupling, the signal had a centre frequency of 900 MHz and a maximum bandwidth of 400 MHz. At this point, the signal was at the desired bandwidth but had to be mixed down to the SAW's centre frequency (450 MHz) and this was done by mixing with a 1.35 GHz carrier frequency. The reason for generating the signal at such a high carrier frequency and then mixing down was to prevent the signal bands of the unwanted mix products of the doublers from overlapping with the desired signal band.

After passing through the SAW RAC device the signal was mixed to baseband using a quadrature detector and the I and Q components were sampled using a Model 945 LeCroy Digital Oscilloscope. The oscilloscope was capable of sampling both channels at 400 Mega samples per second and had a 40 K word acquisition memory which allowed a maximum sampling window of 100 μ S (at this sampling rate). Captured waveforms were transferred to a personal computer over a GPIB interface similar to the one connected to the AFG.

As can be seen from Figure 3.6 a large number of attenuators were used to minimize reflections between adjacent components. Mixer and doubler outputs were found to be particularly sensitive to reflections. As a consequence a large number of amplifiers were also required. To minimize delivery time the required amplifiers were purchased as discrete components, mounted on circuit boards, and packaged on site. The same is true of the frequency doublers and Quadrature detectors. All

required circuit boards were designed using the P-CAD electronic component design system.

Spectral plots of signals generated by the circuit can be found in Figures 3.8, and 3.9 (page 63). Figure 3.8 shows a 50 MHz LFM signal at 225 MHz. Because of the Max/Min data display capability of the HP 8566B spectrum analyzer it is possible to see the uncanceled side band at approximately -30 db with respect to the desired signal. Figure 3.9 is a plot of the same signal after quadrupling with a bandwidth of 200 MHz and a centre frequency of 450 MHz. Unwanted signal components in this signal are approximately -25 db below the desired signal.

As can be seen in Figure 3.7 (page 61), all clocks and local oscillator signals used in the circuit were generated from a single source and consequently are coherent. Since all local oscillator signals are multiples of the 225 MHz signal from the frequency synthesizer the centre frequency of the generated signal can be adjusted by changing this one signal. The digital scope reference clock was not made to be coherent with the remainder of the circuit in order to better represent the situation in an actual system. If the digital scope is synchronized with the generation circuit, pulses would be sampled at exactly the same points each time and this is clearly unrealistic since in an actual radar there is no control over the time of arrival of radar returns.

FIGURE 3.6 - HYBRID PULSE COMPRESSION TEST FACILITY

FIGURE 3.7 COHERENT FREQUENCY GENERATOR

Plate 3.4 - Pulse Generation/Compression Hardware

Figure 3.8 - 50 MHz Signal at a Center Frequency of 225 MHz

Figure 3.9 - 200 MHz Signal at a Center Frequency of 450 MHz

3.3.2 Discrete Signal Synthesis

Since the signal generation system is based on an arbitrary waveform generator, the problem of generating sophisticated waveforms becomes one of developing the appropriate software. The utility software which was supplied with the LeCroy 9100 AFG proved to be unsuitable for developing all but the most basic waveforms. As a result, software capable of generating a wide variety of waveforms such as M sequences, Frank codes, P1-P4 codes, random noise as well arbitrary polyphase and biphase codes was developed. In addition the software had the capability of combining (multiplying) signals and formatting the data in a format suitable for downloading to the AFG.

To generate the required complex base band signal, the real and imaginary components of the signal must be specified as discrete data points. Synthesis of these signals can be accomplished by starting with a continuous complex signal as shown in equation {3.12} and making the appropriate substitutions. In particular since a realizable signal must occupy a time period of $[0, \tau]$ instead of $[-\tau/2, \tau/2]$, t must be replaced with $t - \tau/2$. In addition since the signal is discrete the continuous time parameter t is replaced by $t = nT_p$ where T_p is the sampling period and in a similar manner $\tau = NT_p$ where N is the wave form duration in points. The function $\theta(t)$ is an arbitrary phase shift function such as that used to represent the phases of a biphase sequence.

$s(t) = e^{j2\pi\phi(t)}$ where

$$\phi(t) = f_c t + \frac{\mu}{2} t^2 + \frac{\beta}{3} + \theta(t) \quad \text{for } [-\frac{T}{2} \leq t \leq \frac{T}{2}] \quad \{3.12\}$$

Let $t = t - T/2$ and $t = nT_p$ for $[0 \leq n \leq N]$

$$\begin{aligned} \therefore \phi(n) = & [f_c T_p - \frac{\mu}{2} T_p^2 N + \frac{\beta}{4} T_p^2 N^2] n \\ & + [\frac{\mu}{2} T_p^2 + \frac{\beta}{2} T_p^3 N] n^2 + \frac{\beta}{3} T_p^3 n^3 + \theta(n) \end{aligned} \quad \{3.13\}$$

$$\text{for } f_c = \beta = \theta(n) = 0, \quad \phi(n) = [-\frac{\mu}{2} T_p^2 N] n + [\frac{\mu}{2} T_p^2] n^2 \quad \{3.14\}$$

The discrete phase function including third order phase terms but excluding constant phase terms is given in equation {3.13}. Notice that the arbitrary phase shift function $\theta(t)$ is simply replaced by $\theta(n)$ with the implication that phase shifts can only occur at the discrete sample points. Depending on the type of signal to be generated {3.13} can be simplified considerably. For example, to generate a symmetric base band LFM signal, equation {3.14} would be used. Once the inphase (I) and quadrature (Q) components ($\cos(2\pi\phi(n))$, $\sin(2\pi\phi(n))$) of the signal have been formed, the data points must be quantized to 8 bit (1 byte) values and stored in a file format suitable to the AFG.

Signals which were generated with the circuit of Figure 3.6 (page 60) were quadrupled in hardware and this had to be considered when synthesizing the signals in software. For continuous phase waveforms, the signal generated in software should

have $1/4$ of the desired bandwidth. For waveforms with discrete phase shifts, the size of the phase shift should be divided by 4. For example, to generate a 13 bit barker coded LFM waveform with a 200 MHz bandwidth, a 50 MHz LFM waveform with 45 degree phase transitions (instead of 180 degree transitions) was required. The capability of generating scaled phase shift codes was included in the signal processing software which was developed by the author.

When signals are generated in a discrete manner, the effects resulting from sample and hold circuits and operation close to the Nyquist rate must be considered. In most D/A system the output values are held constant or clamped in the time interval between the sample points by means of a sample and hold circuit. As shown in Figure 3.10 (page 68), this is equivalent to convolving the sample points with a rectangular window of unit magnitude and width T_P where T_P is the sampling period. In the frequency domain, this is equivalent to multiplying the signal by a sinc function with a null to null width spanning twice the sampling frequency ($2/T_P$ Hz). This effect is visible in Figure 3.11 where the spectrum of a 45 MHz ($.9 \times$ Nyquist frequency) baseband signal is shown. The high frequency components appear to be suppressed more than expected (from that of a Sinc function) and this is probably due to the fact that the unit has an output channel bandwidth of 100 MHz.

This effect is to some extent, desirable since it gives a taper to the frequency spectrum without reducing signal amplitude in the time domain and this can be a means of reducing signal sidelobes. The sinc function also helps to suppress the

duplicate high frequency components, resulting from discrete signal generation, which would otherwise have to be reduced by filtering.

Care must be taken to avoid aliasing effects when using digital techniques for signal generation. This particularly is a problem with signals that have significant spectral energy outside of the desired bandwidth. Biphase codes with their sinc shaped spectrum are an example of waveforms where this may be a problem. This will be of particular concern in the circuit of Figure 3.6 (page 60) since these high frequency components will increase the spurious components in the desired signal band after doubling and quadrupling of the signal. Fortunately in the hybrid pulse compression system the spectral content of the expanded pulse is primarily influenced by the dominant LFM component. As mentioned earlier, the rectangular spectrum of LFM make is suitable for operation close to the Nyquist rate and this applies equally for signal generation and quantization. Even if the adjacent spectral components do not overlap, suppressing the duplicate high frequency components becomes more difficult as signals are generated closer to the Nyquist frequency, since the bandwidth separating the desired and unwanted signal components decreases. As a result it is often desirable to operate as far below the Nyquist rate as practical.

TIME DOMAIN

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FREQUENCY DOMAIN

Figure 3.10 - Spectral Shaping Due to Sample and Hold Circuits

Figure 3.11 - Magnitude Spectrum, 0-45 MHz LFM Signal

3.3.3 SAW Compressor

The SAW RAC device is the key component in the two stage pulse compression system. Despite the fact that surface acoustic wave technology is fairly mature, obtaining large TBW product SAW RAC devices is difficult and costly. The SAW RAC which was used in the system of Figure 3.6 (page 60) was manufactured in France by Thompson CSF (Model number RD 450-214.5-26). The device had a bandwidth of 214.5 MHz and a dispersion of 26.0 μ S for a maximum TBW product of approximately 5500. A lithium niobate substrate material was used in this particular device and the LFM rate was found to be $+8.328 \times 10^{12}$ Hz/sec at room temperature (25 degrees celsius). The positive sign on the sweep rate indicates that higher frequencies are delayed more than low frequencies when propagating through the device.

The device was delivered in a hermetically sealed isotronics 64 pin dual inline package. Only four of the pins on the package were connected internally and the package appears to have been chosen because its length (3.5") can accommodate that of the substrate. To contain the device an aluminium box was machined and SMA connectors were used for external connections. The device was mounted on a circuit board with strip line connections between the package pins and the external connectors.

Because of the high insertion loss of the SAW RAC component (approximately -57 dB), it was necessary to provide a high degree of isolation between the input and output connectors on the package in order to minimize the signal component which propagates directly from the input to the output connectors without going through the lithium niobate substrate. To minimize this effect, the active pins on the device were physically trimmed and the input and output signal paths compartmentalized. In addition, the package to ground plane contact was sealed with a conducting silver metal paint which is commonly used to bond specimens in electron microscopy. With these measures it was possible to reduce the signal feed-through component to approximately -72 dB which is roughly 15 dB less than the desired signal component which propagates through the substrate. Although the feed-through component appears to be relatively large, when the signal compression gain is considered (37.4 dB for TBW = 5500) the feed-through component drops to less than -52 dB with respect to the desired signal.

It should also be mentioned that the feed-through component appears ahead of the compressed signal since it does not propagate through the delay medium. In this particular device, the feed-through signal will precede the compressed signal by 5 to 30 μ S depending on the characteristics of the expanded pulse. The performance of the SAW device was found to compare well with the manufacturers data. Plots of the SAW RAC devices magnitude spectrum can be found in Figures 3.12, and 3.13 (page 72). Although the overall shape of the spectrum is comparable with the 5.2 dB peak to peak ripple value reported by the manufacturer, the fine ripple structure can

be expected to cause paired echoes. In Figure 3.13 the markers indicate 4 ripple cycles over a 14 MHz bandwidth or $286 \cdot 10^9$ cycles/Hz which will cause paired echoes at ± 286 ns. From Figure 3.12 a slower ripple structure is also evident in range from 400 to 500 MHz . The rate of these slower ripples is approximately 7 cycles/50 MHz or 140 cycles/Hz which should cause paired echoes at ± 140 ns. As is evident from the compressed pulse in Plate 3.3 (page 44), the actual location of the paired echoes agrees very well with the values calculated above. In addition to these particularly large paired echoes the irregular magnitude spectrum of the SAW also adds a low level noise like component to the fully compressed pulse.

Figure 3.12 - SAW Magnitude Spectrum, 5 dB per Division

Figure 3.13 - SAW Magnitude Spectrum, 1 dB per Division

3.4 DIGITAL COMPRESSION PROCESSING

The partially compressed waveform at the SAW output was converted to a complex baseband signal by means of a quadrature detector and was then sampled and converted to digital data. The final step in the pulse compression processing is digitally compressing the signal by means of a suitable matched filter. Until this point, the discussion of pulse compression has dealt with generating an expanded pulse and then compressing this pulse. This is equivalent to the return signal from a single point target. In an actual radar the received waveform contains information from a continuum of targets. In particular, the radar return can be considered as the expanded pulse convolved with a target terrain profile. This return signal shown in equation {3.15} where $s(t)$ is the transmitted signal (expanded pulse) and $t(t)$ is the target profile.

$$\begin{aligned} [s(t)*t(t)]*s^*(-t) &= F^{-1}[S(f)T(f)S^*(f)] \\ &= F^{-1}[|S(f)|^2T(f)] = R_{ss}(t)*t(t) \end{aligned} \quad \{3.15\}$$

After ideal matched filtering with filter given by $s^*(-t)$ the final result is the target profile convolved with the autocorrelation function ($R_{ss}(t)$) of the transmitted signal. The target profile $t(t)$ is representative of the actual target but is scaled to double the length due to the two way path taken by the transmitted signal. For example, two targets separated by a slant range X will produce target returns which are separated by a distance of $2X$ (or by a time of $2X/c$ where c is the speed of

light).

As can be seen from equation 3.15, the pulse compression matched filtering can be accomplished in the time domain by means of a convolution or in the frequency domain by means of a multiplication. When signals are processed digitally, the relative efficiency of these two approaches can be assessed by comparing the number of required mathematical operations. In the case of the frequency domain multiplication technique, the majority of the overall processing effort is required to implement the forward and reverse Fourier transforms in order to translate signals between the time and frequency domains. In digital systems when an efficient algorithm such as the FFT is used to compute the discrete Fourier transform, the frequency domain technique becomes more efficient when the length of the convolution exceeds 64 real points or 32 complex points^[1].

The number of points required to digitally implement the convolution of equation (3.15) depends on the range extent of the radar return (length of $T(t) = N_t$) and the expanded pulse length (length of $S(t) = N_s$). To implement the convolution in one step requires that the number of points in the convolution be greater than $2N_s + N_t - 1$. Practical considerations, such as the size of N_t , may preclude implementing the convolution in one step. It is possible to reduce the number of points in the convolution, but this requires that the convolution be repeated a number of times. This can be done using the "overlap and save" technique, where some portion of each convolution is discarded, or the "overlap and add" technique,

where portions of subsequent convolutions are added, to obtain the final result.

Regardless of the technique used, whether it be time domain convolution, or frequency domain multiplication, the absolute minimum number of points will be the length of the impulse response of the matched filter $s^*(-t)$, which is also the length of the transmitted waveform $s(t)$. In a typical high resolution SAR, using large TBW product signals the length of $s(t)$, will significantly exceed 32 points and, as a result, matched filtering by means of frequency domain multiplication (fast convolution), will be more efficient. For example, an expanded pulse with a bandwidth of 200 MHz and length of 25 μ S if sampled at the Nyquist (slowest possible) rate, will have a matched filter impulse response consisting of 5000 complex points (10000 real points). It is of interest to note that the TBW product of a signal also represents the absolute minimum number of complex points that are required to digitally represent the signal. Thus, digital compression of signals with TBW products > 32 can be more efficiently accomplished by means of frequency domain multiplication.

When generating the second stage matched filter in the two stage pulse compression system, the partial compression effects of the SAW device must be taken into account. A general expression for the second stage matched filter can be obtained by starting with the expression for the complete matched filter. This is shown in equation {3.16} in the time and frequency domains, where $c(t)$ is the arbitrary second stage code, and $l(t)$ the LFM component as discussed earlier.

If the coded LFM waveform is given by:

$$s(t) = c(t)l(t)$$

Its ideal matched filter is as follows:

$$\begin{aligned} [c(-t)l(-t)]^* &= F^{-1} [C(f)^* L(f)^*] \\ &= F^{-1} [S^*(f)] \end{aligned} \quad \{3.16\}$$

The partially compressed saw output waveform is given by:

$$s(t) * h(t) = F^{-1} [S(f)H(f)] \quad \{3.17\}$$

Therefore the expression for the required second stage compression filter is:

$$F^{-1} [S^*(f)/H(f)] = [c(-t)l(-t)]^* / h(t) \quad \{3.18\}$$

After partial compression by the SAW device, the expression for the resulting coded LFM signal is given by equation {3.17} where $h(t)$ is the impulse response of the SAW device. By comparing equations {3.16} and {3.17}, an expression for the matched filter required to fully compress the signal can be determined and is given in equation {3.18}. Unfortunately direct evaluation of this expression may prove difficult. A more useful approach is to compare the expression for the SAW output with the required matched filter in the frequency domain and express the signal components in terms of their phase on magnitude functions as shown in equations {3.19} and {3.20}.

The partially compressed SAW output expressed in the frequency domain is:

$$S(f)H(f) = |S(f)| |H(f)| e^{j[\theta_s(f) + \theta_h(f)]} \quad \{3.19\}$$

The transfer function of the second stage compression filter is given by:

$$S^*(f)/H(f)$$

$$\begin{aligned}
 &= (|S(f)|e^{-j\theta_s(f)}) / (|H(f)|e^{j\theta_h(f)}) \\
 &= [|S(f)| |H(f)| e^{-j[\theta_s(f) + \theta_h(f)]}] / (|H(f)|^2) \quad \{3.20\}
 \end{aligned}$$

A comparison of equations {3.19} and {3.20} shows that the SAW output of a partially compressed pulse is the complex conjugate of the ideal matched filter multiplied by $(|H(f)|^2)$ where $(|H(f)|)$ is the magnitude spectrum of the analogue compressor. If $(|H(f)|)$ is sufficiently constant (rectangular spectrum) then the required matched filter can be generated by taking the complex conjugate of the SAW output. In terms of digital processing, this requires changing the sign of the imaginary data set in the frequency domain.

One method of calculating the matched filter is to use the Fourier transform relationship of equation {3.7}. Using this technique, the SAW output function is generated by taking an appropriately scaled inverse Fourier transform of the secondary code $c(t)$, and multiplying by a chirp of the form $e^{j\pi\mu t^2}$. Taking the complex conjugate of this function generates the approximate matched filter, as discussed previously.

3.4.1 Digital Adaptive Matched Filter

A more efficient means of generating and implementing the matched filter is to take advantage of the fact that the matched filter impulse response can be generated by feeding the transmitted waveform through the SAW device and sampling the output. Using this technique a matched filter can be generated without specific

knowledge of the transmitted waveform. With sufficient processing effort the matched filter can be made to adapt to the transmitted wave form on a pulse-to-pulse basis. One advantage of this technique is that the transmitted signal can be varied from pulse to pulse in a truly random fashion. For example, a random phase noise component of appropriate bandwidth could serve as the secondary code and the signal could still be compressed. Another benefit of using an adaptive matched filter is that any unintentional distortions of the signal, such as transmitter phase ripple, will automatically be compensated for.

A diagram of a basic radar system using the transmitter sampling technique is shown in Figure 3.14. For convenience two separate antennas have been shown and only the relevant system components are indicated. Also shown is the SAW output during one pulse repetition interval and a block diagram of the processing required to implement the digital adaptive matched filter.

The difficulty associated with implementing an adaptive matched filter will depend to some extent on the transmitted waveform characteristics. If for example, the magnitude spectrum of the signal is constant, the filter need only adapt to the phase function for the signal. The required size of FFT's will depend on the TBW product of the partially compressed waveform as well as the range extent of the return signal. The greatest difficulty in implementing the adaptive matched filter will be from realizing the inverse magnitude filtering function. This will be addressed further in chapter 4.

Figure 3.14 - Matched Filter Generation by Transmitted Waveform Sampling

CHAPTER 4

SYSTEM PERFORMANCE

An evaluation of the two stage pulse compression system was carried out using the hardware and software discussed in the previous chapter. In Section 4.1 the capability of the pulse compressor/expander system is discussed and limitations to performance are pointed out. An evaluation of various second stage codes is carried out in Section 4.2 and their relative merits are compared. In Section 4.3 the performance of the system in the tone and noise jammer environments is measured and the requirements for operation in the repeater jammer environment are discussed. Implementation considerations which are unique to the two stage pulse compression are covered in Section 4.4 . This includes a discussion of the signal processing requirements of the system as well as comments on the design and selection of system components.

4.1 EXPANDER/COMPRESSOR PERFORMANCE

The ability of the system to fully compress pulses was tested using the circuit of Figure 3.6 (page 60). Partially compressed pulses were down loaded to a personal computer from the Digital Storage Oscilloscope and compressed using the software mentioned in the last Section. The second stage pulse compression filter was generated using three different techniques. In the first case, the Fourier transform relationship between the secondary code and the SAW output of the expanded pulse

was used to compute the matched filter. The second method used the expanded pulse sampling technique. However, instead of using a single pulse, a number of pulses (100) were collected and averaged. In the third technique, only one pulse was sampled and used to form the matched filter. In an actual system using one of the first two techniques, the transmitted waveform must be known prior to transmission. This is not necessary in the last case since the processing can be based on a digital adaptive matched filter. Actual pulses which were generated with the circuit of Figure 3.6 and compressed using each of these techniques can be found in Figures 4.1 to 4.3 (pages 83,84). The same signal (200 MHz, TBWP = 4925.7, partially compressed pulse TBWP = 99.98) was used in all cases and inverse filtering and windowing techniques were not used. In the case when a single pulse was used to form the matched filter, a subsequent pulse was used to represent the target return and was compressed using the earlier pulse. This is a conservative approach and the performance of a real system should be better since inter-pulse instabilities will degrade the performance of the test system. This is due to the fact that in a real system, the same signal (pulse) is used to form the target return as well as the matched filter.

Notice that the paired echoes at ± 280 ns and, ± 140 ns from ripples in the SAW spectrum (as derived in Section 3.3.3) are evident in all cases but is more noticeable in the Figures 4.2 and 4.3 since $|H(f)|^2$ is present instead of just $|H(f)|$ where $|H(f)|$ is the rippled magnitude spectrum of the SAW device. In Figure 4.4 the signal of Figure 4.1 has been weighted with a Hamming window. Notice that the

2

OF/DE

2

MicroD

window function has almost no effect in reducing the paired echo sidelobes as expected. In Figures 4.5 and 4.6 (page 85) the signals of Figures 4.2 and 4.3 have been inverse magnitude filtered and windowed with a Hamming (-40 db sidelobes) weighting resulting in a significant improvement in sidelobe levels. Although all three methods of matched filter generation were effective at compressing the signals the techniques which were based on an inverse matched filter provided superior sidelobe reduction capabilities.

One of the advantages of the two stage pulse compression system is the fact that the dynamic range of system can be extended beyond that of the A/D converters. The total achievable dynamic range of the two stage pulse compression system is equal to the dynamic range of the quantizer plus the second stage compression ratio as shown in equation {4.1}. The signal to noise ratio of a quantizer is given in equation {4.2}^[14] where b is the number of bits in the quantizer. For second stage codes with TBW products in the range of 50-200 (17-23 db) a 4 bit quantizer (16 level) will be sufficient to permit a dynamic range in the order of 40 dB based on the relationship of equation {4.1}.

$$D_T = D_Q + TBWP_2 \quad \{4.1\}$$

$$SNR \text{ (dB)} = 6b - 1.24 \text{ dB} \quad \{4.2\}$$

Figure 4.1 Compressed Pulse Using Computed Matched Filter

Figure 4.2 Compressed Pulse - 100 pulse Averaged SAW Sampled Matched Filter

Figure 4.3 - Compressed Pulse, Single Pulse SAW Sampled Matched Filter

Figure 4.4 - Computed Matched Filter with Hamming Weighting

Figure 4.5 - 100 Pulse Averaged SAW Sampled Matched Filter with Inverse Filter and Hamming Weighting

Figure 4.6 As above with Single Pulse SAW Sampled Filter

By reducing the resolution requirements of the quantizers it is possible to operate with larger signal bandwidths. In general, by using fewer quantization levels higher operating speeds (sampling rates) can be achieved. An example of the I and Q components of a coarsely quantized (6 levels=2.6 bits) partially compressed pulse is shown Figure 4.7 (page 89) and the fully compressed pulse in Figure 4.8 . The signal had a 160 MHz bandwidth and a total TBWP =3315 and a second stage TBWP =122. In order to take advantage the full dynamic range capability of the system as discussed above the signal must be free from other noise sources. As mentioned earlier, expanded pulses generated using the circuit of Figure 3.6 (page 60) had unwanted signal components in the order of -25 db below the desired signal. These components did not however have a bandwidth and LFM rate equal to that of the desired signal. For example, the uncanceled sideband occupies the same bandwidth as the desired signal, however the LFM rate has the opposite sign. In a similar manner the unwanted doubler products may have the same LFM rate as the desired signal but occupy a different bandwidth. The net result is that the unwanted components do not compress or compress substantially less than the desired signal.

One of the most significant noise components in the signal is attributable to the magnitude and phase imbalance in the quadrature detector. The performance of this unit was evaluated by generating a number of continuous wave signals between 350-550 MHz and measuring the imbalance between the I and Q channels using a Hewlett Packard 8405A vector voltmeter. The drive level of the RF signals was also varied and the resulting phase imbalances are plotted in Figure 4.9 (page 90). Over

the signal bandwidth the balance between the two channels was in the order of ± 4.6 for the phase and $\pm 3\%$ for the amplitude with better performance at the lower drive levels. The mixers of the quadrature detector supported local oscillator drive levels of +10 dBm. Two quadrature detectors were fabricated and the figures which have been given are for the device with the better performance. A diagram showing the propagation of amplitude and phase errors generated by the quadrature detector components is found in Figure 4.10 (page 90).

The imbalances caused by the quadrature detector represent an additional noise component which will limit system performance. An approximate relationship for the sidelobe levels for LFM signals is given in equation {4.3}^[16] where δ is the phase imbalance, Δ the amplitude imbalance, and SL is the noise sidelobe level. Using this relationship, sidelobe levels of -42.8 db resulting from this noise component for a phase error of ± 5 degrees, an amplitude error of ± 1 db, and a compression gain of 100 for LFM signals have been reported^[16].

$$SL \text{ (dB)} = 10 \log \left[\frac{1 - 2(1+\Delta) \cos \delta + (1+\Delta)^2}{(TBWP) (1 + 2(1+\Delta) \cos \delta + (1+\Delta)^2)} \right] \quad \{4.3\}$$

The imbalance between the I and Q channels of the quadrature detector causes the presence of a conjugate signal component along with the desired signal. This is equivalent to the presence of the unsuppressed sideband in the SSB modulator during signal generator. During the final stage of compression, the conjugate component will spread rather than compress. In the case of LFM this causes the

presence of a uniform noise pedestal at a level given by equation {4.3}. The amplitude and phase errors of the quadrature detector which was used in the test circuit, can be expected to limit sidelobe reduction performance to approximately -40 to -45 dB.

Imaginary Component

Figure 4.7 - I and Q Components of Partially Compressed Signal

Figure 4.8 Fully Compressed Pulse

Figure 4.9 Quadrature Detector Phase Balance

Figure 4.10 Quadrature Detector Error Signals

4.2 SECOND STAGE CODE EVALUATION

The task of finding waveforms with desirable properties for use in the two stage pulse compression system is complicated by the fact that two codes are combined together. It is of importance to determine the properties of the combined waveform for various types of second stage codes in order to assess the utility of the two stage pulse compression system. Perhaps the most important property of a waveform is the shape of its compressed pulse. In the case where an ideal matched filter is used, this implies that the autocorrelation function of the waveform must have a desirable shape. Since this may not always be possible, the waveform should be such that additional filtering techniques such as windowing and inverse magnitude filtering can be employed to reduce the sidelobes of the compressed pulse.

Another important property of a combined waveform is that its spectrum be band limited or fairly well contained in order to permit sampling close to the Nyquist frequency without aliasing effects . Intuitively we can expect this to always be the case since the bandwidth of the LFM component is much larger than the bandwidth of second stage code. For example, in a system where the total TBW product of the signal is 5000 and the TBW product of the partially compressed pulse is 100, the bandwidth of the secondary code is 2% of the primary code (LFM) bandwidth. Since the signals are multiplied in the time domain, their individual frequency spectra are convolved in the frequency domain. Regardless of the spectral shape of the secondary code, the result after convolving with the rectangular LFM spectra will

still be approximately rectangular since the secondary code bandwidth is so small. The spectral shape of the secondary code is however important since it also represents the shape of SAW output, and hence the shape of the impulse response of the second stage compression filter. Because of the Fourier transform relationship between the secondary code and the saw output waveform (as discussed in Section 3.1), a secondary code with a band limited spectrum results in a time limited or compact filter impulse response which is desirable since it reduces the processing effort required to implement the second stage compression. Other properties such as randomness and Doppler tolerance, although application specific, should also be considered.

4.2.1 Autocorrelation Function Of Coded LFM Signals

The autocorrelation function of a signal can be determined by taking inverse Fourier transform of the square of its magnitude spectrum. For a coded LFM signal the autocorrelation function is defined by equation {4.4} where $C(f)$ and $L(f)$ are the secondary and primary codes respectively.

$$R_{ss}(t) = F^{-1} [|C(f) * L(f)|^2] = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} C(\alpha) L(f-\alpha) d\alpha \right]^2 e^{j2\pi ft} df \quad \{4.4\}$$

Determining the characteristics of the autocorrelation function from {4.4} for a particular secondary code within a code family (for example 13 bit Barker Biphase code) was found to be difficult and numerical methods were used to obtain the autocorrelation functions of combined signals. This was achieved by mathematically

generating the data points of a coded LFM signal ($S(t)=C(t)l(t)$), taking the FFT of the data set, multiplying each point by its complex conjugate ($S(f)S^*(f)=|S(f)|^2$) and then taking the IFFT of the data. Using this technique the autocorrelation functions of a number of waveforms were evaluated. Figures {4.11} to {4.13} (pages 96,97) show the autocorrelation functions of coded LFM signals with a variety of secondary codes. In all cases the LFM code had a TBW product of 1000.

A preliminary investigation of various secondary codes was carried out and one of the significant findings of this search was the fact that a secondary code with a desirable autocorrelation function does not necessarily yield a combined code with a desirable autocorrelation function. This is shown in Figures {4.11}, and {4.12} where a 100 element Frank code and a 13 bit Barker code have been combined with the above mentioned LFM code. Notice that the peak sidelobe levels are significantly higher than those of the individual LFM or secondary codes. For example, the peak sidelobe levels of the autocorrelation function of a 100 element Frank code are in the order of $1/31.4 = -29.9$ dB, and $1/13 = -22.3$ dB for a 13 bit Barker code. In Figure {4.13} a non optimal 13 bit biphase code (bit pattern 1001 0110 0011 0) was used. The autocorrelation function of this code by itself had peak sidelobe levels of -8.2 dB. Notice that the combined autocorrelation function of this code is similar to the 13 bit Barker coded LFM signal.

An assessment of the suitability of particular second stage codes can be made by considering the effect of the code on the combined signals magnitude spectrum.

As mentioned in Section 2.4.1, large TBWP linear FM waveforms have a direct correspondence between the time and frequency domains. In particular, each point along the time domain chirp contributes energy to a particular frequency within the signal bandwidth. Perturbations to the time domain waveform can be expected to cause some form of disturbance in the signal's frequency spectrum. The disturbance will occur at a frequency determined by the instantaneous frequency of the time domain chirp at the time of the perturbation.

Because of this effect, discontinuities in the secondary code will result in disturbances of the magnitude spectrum. An example of this effect can be seen in Figure {4.14} (page 98) which is the frequency spectrum of a LFM signal coded with a 13 bit Barker code. The actual binary pattern of the Barker code (reversed) has been included above the magnitude spectrum to help visualize the effect. Notice that nulls in the magnitude spectrum occur at the phase transitions. This effect can be explained by considering the phase shifts of the Barker code as phase advances or jumps. As a result the instantaneous frequency at the time of the phase jump is bypassed or omitted resulting in a null in the spectrum. This effect is not restricted to biphase codes and occurs in any code with phase discontinuities.

The nulls in the spectrum are of particular concern since they present a problem for inverse matched filtering. The presence of noise in the null area may cause the inverse filter to over compensate or under compensate the signal components in the area of nulls. An example of this is shown in Figure {4.15} (page 98) which is the

magnitude spectrum of a 13 bit Barker coded LFM pulse after inverse filtering. The signal was generated and partially compressed using the circuit described in Section 3.3.1 .

Another difficulty with attempting to inverse filter with the presence of spectral nulls, is the fact that the location of the nulls will change depending on the signals Doppler shift. In the case of biphasic codes, the nulls are steep and narrow and as a result, even a modest Doppler shift will cause severe signal distortion after inverse magnitude filtering. Without the use of an inverse filter the autocorrelation function of the signal is unusable for most applications. Based on the findings of Section 2.5 it is also clear that windowing techniques will not be effective in reducing sidelobes caused by the spectral nulls. The conclusion of the analysis is the fact that discontinuous phase coded LFM waveforms are unsuitable for use in the two stage pulse compression system. It may however be possible to make use of discrete phase shift codes by modifying the phase transition points so that phase transitions are less abrupt or by selecting codes where the maximum phase jumps between consecutive points are restricted.

Figure 4.11 - 13 Bit Barker Coded LFM Autocorrelation Function

Figure 4.12 100 Element Frank Polyphase Coded LFM Autocorrelation Function

Figure 4.13 - Non Optimal 13 Bit Biphase Coded LFM Autocorrelation Function

Figure 4.14 - 13 Bit Barker Coded LFM Magnitude Spectrum

Figure 4.15 - Inverse Magnitude Filtered Barker Coded LFM Spectrum

4.2.2 LFM Compatible Secondary Codes

The operational role of a particular radar will determine the required properties of the second stage code. In systems where protection against repeater jammers is required the waveform must possess some random characteristic. If simplicity and cost are a concern, then the waveform should be such that it can be generated and compressed with a minimum of hardware. The bandwidth and expected Doppler shifts will determine the required Doppler tolerance of the waveform. Even with the elimination of discontinuous phase shift codes, the choice of potential second stage codes is still considerable.

The most obvious choice for a secondary code is linear FM itself. Its rectangular spectral shape makes it ideal for bandlimited signal processing and also provides a time constrained SAW output function. Note that any shape other than rectangular results in a greater non zero portion along the abscissa axis for an equivalent effective bandwidth (3 db bandwidth for example). LFM also has good Doppler tolerance and can be readily generated using analogue hardware. In fact the primary motivation for using a secondary waveform other than LFM is to provide a pseudo random property to the waveform and to reduce the sidelobe levels of the autocorrelation function. As mentioned earlier, the secondary code has a small influence on the gross shape of the spectrum of the combined signal. As a result it will also have a minimal effect in reducing sidelobes by smoothing or shaping the spectrum. In some systems however, even a modest decrease in sidelobe levels

without a decrease in SNR may be significant. However, most high resolution systems will still need to make use of sidelobe reduction techniques (Section 2.5).

It is possible to obtain a random waveform based on LFM by changing the sweep rate on a pulse by pulse basis. This will however require changing the bandwidth and expanded pulse length in an inverse manner if the TBW product is to remain constant. Changing the transmitted pulse length will change the minimum range and changing the bandwidth will affect system resolution. The exact relationship is shown in equations {4.5}, and {4.6} where x is the percentage change in sweep rate.

The time bandwidth product of an arbitrary LFM signal is given by:

$$\mu \tau^2 = [(1 \pm x/100)\mu][\tau/\sqrt{1 \pm x/100}]^2$$

$$= \text{constant}$$

Therefore

$$\begin{aligned} \Delta f = \mu' \tau' &= (1 \pm x/100)/\sqrt{(1 \pm x/100)} \mu \tau \\ &= \sqrt{(1 \pm x/100)} \mu \tau \end{aligned} \quad \{4.5\}$$

and

$$\tau' = \tau/\sqrt{(1 \pm x/100)} \quad \{4.6\}$$

Notice that the sweep rate can be changed such that the SAW device over compresses as well as under compresses the signal during the first stage of compression as indicated by the sign of the change ($\pm x$). The relatively small change in resolution (and minimum range) would probably be acceptable in most systems. For example a 2% increase in μ will cause less than a 1% increase in resolution. A

jamming signal which is not at the correct sweep rate will not fully compress and the peak level of this signal can be determined using equation {3.10} (used to determine the peak value of partially compressed LFM signals). For example, if the sweep rate is mismatched by 2% a residual TBW product of approximately 100 ($2\% \times 8 \text{ MHz}/\mu\text{s} \times (25\mu\text{s})^2$) results. This implies the system will have a 20 db processing advantage over the jamming signal. One of the draw backs of this technique is the fact that changing the sweep rate in a continuous manner will cause the second stage TBW product to change and this will impact on the dynamic range of the system as well as the signal processing requirements. Another problem with this method is that the signal is random on an inter-pulse basis and lacks intra-pulse randomness. As a result a jammer need only observe the first few instants of the pulse in order to generate a suitable jamming pulse.

4.2.2.1 Non Linear FM

In systems using nonlinear FM waveforms, the FM rate is usually nonlinear in order to shape the magnitude spectrum of the signal and there by reduce sidelobe levels. In the case of the two stage pulse compression system the autocorrelation function will only be slightly improved because of the small secondary code bandwidth as mentioned earlier. It is possible to further reduce sidelobe levels by increasing the relative bandwidth of the secondary code however this will increase the second stage signal processing requirements because the length of the second stage matched filter impulse response is also increased.

Conventional nonlinear FM waveforms, although compatible with the two stage pulse compression system, are not suitable when the waveform must have a random characteristic. One method of achieving this is to encode the sweep rate with a discrete pseudo random function. Waveforms of this type were not found to be used in conventional radar pulse compression systems and were identified as "discrete Mu coded waveforms" by the author.

One example of a Mu coded waveform is generated by taking a binary sequence $\phi(t)$ and using it to modulate the sweep rate between two values as is shown in equation 4.7. The constant k will determine the extent to which the sweep rate is shifted above and below its mean value of μ . The term $\theta(t)$ in equation {4.7} is a phase compensation term which is synchronized to $\phi(t)$ in such a way that at

$$s(t) = e^{j2\pi(fct + \mu/2(1+k\phi(t))t^2 + \theta(t))} \quad \{4.7\}$$

the transition points of $\phi(t)$ the phase and frequency of the waveform are made to be piecewise continuous. Eliminating phase shifts or jumps is necessary to avoid nulls in the spectrum of the combined (with primary LFM code) waveform as discussed earlier. The signal processing software developed by the author included the capability of generating Mu coded waveforms with compensation for phase and frequency discontinuities.

In order to generate a family of codes with the same TBW product, a set of binary sequences where the number of 0's and 1's is constant should be chosen. The balance property of M sequences makes them suitable for this application. When the number of zeros and ones are balanced (equal or nearly equal for odd sequences) it also true that the number of zeros and ones is constant. As long as the one zero content is held constant, all codes will produce waveforms with the same bandwidth and time duration (and hence time bandwidth product). The signal bandwidth covered during one chip (1 bit of the binary code) will be greater during the high sweep rate than the low rate. If desired, the bandwidth covered during one chip width can be held constant by adjusting the time duration of each chip such that $\mu_0 t_0 = \mu_1 t_1$ or $\mu_0 / \mu_1 = t_1 / t_0$ where $\mu_i t_i$ ($i=0,1$) are the sweep rate and time duration associated with the two values of the binary code. A simple way of implementing this is to let $t_0 = 2t_1$ (and $\mu_1 = 2\mu_0$) and modify the binary sequences such that every 0 is replaces with 00 (for example 1011 becomes 10011). When the sweep rate is coded with the modified sequence, the frequency chips are the same width. An example of the computed magnitude spectrum of a discrete Mu coded waveform is shown in Figure 4.16 (page 107). This waveform has a bandwidth of 160 MHz, a time duration of 19.2 μ s, and the time domain signal has a constant amplitude envelope.

Discrete Mu coded waveforms will have an intra-pulse random characteristic since the sequence used to form the pulse is arbitrary. The autocorrelation function of these waveforms will be similar to LFM with the addition of paired echoes close to the main lobe due to the discrete steps of the magnitude spectrum. If the size of

the steps is small (value of k in equation 4.7 is small), then it may be possible to obtain a satisfactory autocorrelation function with windowing techniques alone however for greater sidelobe suppression the use of inverse magnitude filtering will also be required.

Instead of using a binary sequence to modulate the sweep rate it is possible to use any real function however care must be taken to ensure that the time bandwidth product of the set of codes is held constant. For example the modulation function could be a sinusoid with an arbitrary initial phase. This will result in a magnitude spectrum with large ripple structure. It is also possible to use a number of discrete sweep rates and this implies using a modulation function based on an alphabet greater than 2 (binary).

4.2.2.2 Frequency Shift Keying

Frequency Shift Keying (FSK) is commonly used in digital communications and can be a suitable secondary coding technique in the two stage pulse compression system because of the fact that FSK signals are phase continuous. An FSK signal is similar in concept to the discrete Mu coded waveforms discussed earlier except in this case the frequency is modulated by a binary signal. The equation for a coded LFM waveform using FSK for the secondary coding is given in equation {4.8}

$$S(t) = e^{j2\pi(fc(1 + k\phi(t)) + \mu t^2/2 + \theta(t))} \quad \{4.8\}$$

In the case of an FSK signal $\phi(t)$ is a binary sequence and as in equation {4.7} the constant k determines the level of modulation. The FSK signal has the effect of introducing peaks and valleys in the spectrum of the combined signal due in part to the time-frequency symmetry of large TBW product LFM signals. This can be explained by considering the fact that when the frequency is shifted upward some portion of the spectrum is skipped and a dip results. In a similar way, when the frequency is shifted backwards a portion of the spectrum is covered longer resulting in a peak. An example of the computed magnitude spectrum of an FSK coded LFM waveform is shown in Figure 4.17 (page 107). A 15 bit M sequence was used as the modulation function and the constant k in equation {4.8} was chosen such that frequency shifts of $1/\tau_s$ Hz occur at each transition, where τ_s is the subpulse duration ($\tau_s = \tau_{EX}/15$). Notice that the valleys are not as sharp and deep as the nulls caused by phase shift codes and can therefore be compensated by inverse magnitude filtering. The depth and width of the nulls can be controlled by adjusting the size of the frequency shifts as determined by the constant k in equation {4.8}.

A number of modifications to basic FSK waveform are possible. For example, it is possible to have only valleys in the spectrum by only using frequency advances. In this case the binary sequence would determine whether or not a frequency advance should occur rather than the direction of the shift. As in the case of M coded waveforms, it is possible to use any real function, and in the general sense, this corresponds to conventional frequency modulation. Because FSK waveforms can

be conveniently generated without digital signal generation techniques, it is possible to develop an analogue FSK coded LFM generator.

4.2.2.3 Noise as a Secondary Code

The possibility of using noise as a secondary code was eluded to in Section 3.4.1. The primary advantage of noise is the fact that it is truly random and can be easily obtained for example by amplifying thermal noise. The noise signal can be used to modulate any of the parameters of the LFM signal. For example a noise signal of suitable bandwidth and amplitude could modulate the frequency function resulting in an FM noise signal. In a similar manner a noise signal could be quantized to the values 0,1 and used as the input to an FSK coded LFM waveform. Regardless of the technique chosen, the phase of the noise function should be continuous to avoid spectral nulls as discussed earlier. Use of a random noise waveform as a secondary code requires that an adaptive matched filter be used since the exact nature of the signal is unknown until it is generated.

Figure 4.16 - Magnitude Spectrum of a μ Coded LFM Signal

Figure 4.17 - Magnitude Spectrum of a FSK Coded LFM Signal

4.3 HOSTILE ENVIRONMENT PERFORMANCE

The effectiveness of radar as a military sensor motivated developments in the area of electronic counter measures (ECM) in order to reduce or eliminate any tactical advantages provided by radar . ECM systems are usually categorized as denial or deception systems depending on their purpose. Denial systems reduce or eliminate the effectiveness of the radar while deception systems use techniques such as false target generation to confuse or deceive the radar. ECM systems in turn, motivated the development of Electronic Counter-Counter Measures (ECCM) techniques. The effectiveness of a particular ECM or ECCM technique is often difficult to predict because of the complexity of the interaction between the radar and the interfering system.

If a radar is to be used in a hostile environment, it should be incorporate ECCM techniques throughout the radar. The performance of the pulse compression system can be evaluated by generating various levels of interfering signals along with the desired signal. In general, pulse compression systems can be used to improve the performance of the radar in a jammer environment by using the fact the pulse compression waveform and the jamming waveform may be poorly correlated. During compression processing, the relative level of the desired signal increases with respect to the jammer signal.

4.3.1 Tone and Noise Jammer Performance

Tone and Noise jammers are two of the most common types of denial jammers mainly because of their simplicity. Tone jammers rely on the generation of discrete frequencies within the victims operating bandwidth, while noise jammers generate a broad band noise signal over the victims operating bandwidth. Both types of jammers commonly operate in pulsed and continuous modes and pulsed systems are usually capable of higher peak powers. If however the radar returns are gated, or observed over a specific narrow time (range) interval, a pulsed system may not be as effective since the jamming signal might not occur within the observation window, depending on the relative geometries and operating parameters of the radar and the jammer. This may be the case in a system such as a spotlight mode SAR operating over a relatively narrow range extent.

Since both the noise and tone jammer waveforms will always exhibit poor correlation with respect to the radar waveform, any type of pulse compression system will improve the system performance. Maximizing the time bandwidth product of the overall pulse compression system will maximize system performance. In the two stage pulse compression system, the entire TBW product (first and second stage) of the system is available for increasing the signal to jammer noise ratio. One of the concerns when operating in a hostile environment is to prevent the system components from being overwhelmed or saturated by the jamming signal. In terms of a pulse compression system with digital outputs, the main concern will be

saturation of the quantization hardware. In a digital pulse compression system this is more of a concern since the signals are quantized before compression and hence the quantization hardware must accommodate the full jammer signal amplitude without saturating. By contrast, in the two stage pulse compression system the relative level of the jammer signal is substantially reduced by the analogue compressor. This can be seen in Figure 4.18 (page 111) where the partially compressed pulse of a 200 MHz, 24.3 μ s signal is combined with a continuous wave tone with a peak amplitude of +15 db with respect to the desired signal. The signal was added to the output of the mixer with local oscillator frequency = 1350 MHz as shown in Figure 3.6 (page 60). Similar tests were carried out with tone and broad band noise signals with interference levels of +5 to +30 dB with respect to the desired signal. The noise signal was generated with an Hewlett Packard 346B noise source followed by 3 gain stages and band limiting filters. The fully compressed pulse in the presence of a +15 db tone signal (of Figure 4.18) is shown in Figure 4.19. In general, the jammer signal amplitude in the partially compressed pulse was reduced by a factor of approximately $\sqrt{\text{TBW}_1}$ with respect to the desired signal. The results demonstrate that the problem of quantizer saturation is much less of a concern in the two stage pulse compression system.

Figure 4.18 - Partially Compressed Pulse with +15 dB Tone Jammer

Figure 4.19 - Fully Compressed Pulse

4.3.2 Repeater Jammer Considerations

The return signal from a repeater type jammer is more difficult to deal with since the transmitted signal and jammer signal are essentially identical. A major factor influencing the effectiveness of a repeater jammer is how quickly the radar's repeated waveform can be retransmitted. In particular, if the radar and jammer parameters are such that the jammer signal can not be returned to the radar (after possible modification) within a single PRI, then the repeater jammer effectiveness can be reduced by altering the compression coding on a pulse to pulse basis.

Most repeater jammers are restricted by the requirement that the transmitter portion of the jammer be inoperative while the receiver is on. If this is the case, then using a long pulse can reduce the effectiveness of the jammer by increasing its return time. If for example the repeater jammer is located on the target of interest and the range extent of the partially compressed return signal is gated to less than the expanded pulse length then the repeated signal will not be able to interfere with the true target return. An example of such a scenario might be a spotlight mode SAR observing an unwilling target at sea. A system using a 25 μ S expanded pulse length can operate with a range extent in excess of 7.5 KM without interference from a repeater jammer co-located with or in close proximity to the target. Notice that in this situation even if the repeater jammer has a zero delay time it will not be able to interfere with the actual target return during that particular PRI unless the jammer employs additional techniques such as frequency shifting of the repeated

return. This causes the jammer signal to appear earlier at the SAW output since it takes a shorter path through the SAW. Clearly the relative geometries of the target, radar, and the jammer will influence the effectiveness of the jammer. Using a long large TBW product pulse also makes it more difficult for repeater jammers which capture, modify, and retransmit the radar signal to function. Systems such as those which are based on Digital RF Memories (DRFM) must be capable of storing and processing the entire waveform in a timely fashion. For waveforms with a TBW product of 5000 this may be difficult since it requires operating on a minimum of 10,000 real data points.

In a pulse compression system which is capable of changing waveforms on a pulse to pulse basis, as is required for protection against repeater jammers, the cross correlation properties of the various transmitted waveforms will influence the system performance. In the two stage pulse compression system only the secondary code will change between waveforms (pulses) and consequently the TBW product of the secondary code will limit the capability of the system to suppress repeater jammer interference. An example of the cross correlation function between two waveforms of the discrete Mu coded type, is shown in Figure 4.20 (page 115). The figure is representative of a fully compressed target return from a single point target in the presence of an interfering waveform of equal magnitude. One of the waveforms of Figure 4.20 used a binary pattern of 01011010 to discretely code the sweep rate while the other used a coding of 10100101. Both waveforms have a partially compressed pulse width of approximately $0.425 \mu\text{s}$ and a second stage TBW product

of 68 and the compression filter did not include windowing or magnitude inversion. Figure 4.21 (page 115) shows a similar result using two FSK type waveforms with binary codings generated from a 127 bit M sequence and its complement and an inverse filter was required to compensate for spectral peaks and valleys. The discrete Mu coded waveforms appear to exhibit better performance in terms of jammer rejection and processing requirements. Regardless of the type or family of waveforms chosen, the cross correlation function of the specific codes should be minimized. This is particularly important for consecutively transmitted codes since there is a high probability that they may interfere with each other because of the actions of repeater jammers.

4.4 SYSTEM IMPLEMENTATION CONSIDERATIONS

A number of possible techniques may be used for generating and compressing signals depending on the system requirements. In a basic system using only a LFM waveform, a digital signal generation system is not necessary and the signal can be generated using analogue means. The selection of a particular digital compression technique will also influence the choice of signal generation options. If for example, a pulse compression system using an adaptive matched filter is used, then the signal generator can consist of an analogue LFM generator combined with random noise or FSK signal generator. In the case of purely digital signal generation, the system should be capable of generating and randomly selecting from a number of possible waveforms.

The two main considerations in the implementation of the pulse compression system are whether the matched filter will be generated using the transmitted waveform sampling technique and if it is, will the system be based on an adaptive matched filter. The main advantage of the SAW sampling technique is that it provides a convenient means of compensating for amplitude and phase ripples such as those found in the magnitude spectrum of the SAW device used in Figure 3.6 (page 60). If an adaptive matched filter is not required, the transmitted waveform need not be sampled every waveform. This however implies that the matched filter for all possible waveforms must be stored in memory. This might not be desirable if a large number of waveforms are required and the address space of the signal processor is limited.

4.4.1 Signal Processing Requirements

The signal processing effort required to achieve the second stage compression processing will be influenced by the operational parameters of the radar system. The range extent over which a target is to be observed will determine the volume of data to be processed and PRF will determine the required throughput rate. The complexity of the signal processing will be greater in system which are based on a digital adaptive matched filter. The second stage TBW product and code type will also determine the minimum size of the impulse response of the second stage pulse compression filter.

An estimate of the signal processing overhead can be obtained by breaking down the processing into the number of multiplications and additions required over a single PRI. For calculation purposes a system using a signal with a total TBWP = 5000, a second stage compression ratio of 100, and a PRF = 1000 Hz will be used. The DSP requirements will be determined with and without the use of a digital adaptive matched filter. It is also assumed that the range extent of the signal is adjusted such that the complete return can be processed with a 1024 complex point FFT.

In a very basic system, the frequency domain matched filter including windowing and inverse magnitude filtering is assumed to have been precalculated and is available in memory. In this case the compression processing will consist of taking

the FFT of the target return, multiplying by the matched filter, and taking the inverse FFT to obtain the range compressed data.

The number of addition and multiplication operations required to implement the FFT algorithm depends on the radix of the algorithm. Table 4.1 shows the number of multiplications required for the radix 2,4,8 Cooley-Tukey FFT algorithms. For a $1024=2^{10}$ point complex FFT using a radix 2 algorithm we find that 16388 real multiplications and 28674 real additions are required. To implement the complete range compression processing in real time in a basic system, 36872 multiplications and 59,396 additions will have to be completed in one PRI.

It is useful to consider the processing in terms of one operation such as multiplication. By using the assumption that the time required to implement a multiplication is approximately twice that of an addition ($t_{\text{multiply}} \approx 2t_{\text{addition}}$), the number of operations can be expressed in terms of equivalent multiplications. Although this approximation is not accurate for all types of processors, it is a realistic guideline for an initial determination of processing requirements. Based on this assumption, we find that 66570 equivalent multiplications must be completed in one PRI and this corresponds to a multiply and accumulate time (MAC) of approximately 15 ns. Even if the additions are ignored completely, a MAC time of 27 ns is required. For processors which are capable of high level FFT operations, a 1K complex FFT must be completed in well below 500 μs . Using these times as a guideline, it is possible to obtain a rough estimate of the suitability of certain

processors. In particular, it is possible to eliminate those processors which may be too slow. For a more accurate estimate of processor suitability additional factors such as the memory access times and addressing space of individual processors should be considered.

A review of current DSP architectures^[9,10] reveals that single chip programmable digital signal processors are not powerful enough to meet the processing requirements of the example system described above. The newer single DSP chips such as the Texas Instruments TMS 320C30, the Motorola DSP 96002, and the AT&T DSP32C have MAC times in the range of 60-80 ns. Processors which utilize fixed point arithmetic are usually faster than those using floating point arithmetic however, the reduced dynamic range of fixed point systems may not be acceptable. The fastest single chip processor in terms of MAC time was found to be the AT&T DSP 16A chip with a MAC time of 33 ns.

To meet the signal processing requirements of the two stage pulse compression system it is necessary to turn to faster architectures such as array processor based systems or parallel architectures which combine a number of slower chips. One architecture which shows promise is based on the Honeywell Digital Array Signal Processor Chip (HDSP 66110) and the Programmable Array Controller (HDSP 66210). Delivery of these chips began in early 1989 and the devices are optimized for DSP algorithms based on the FFT. Using a basic configuration (one of each chip with supporting hardware) a 1k complex FFT time of 209.6 μ s has been reported by

the manufacturer. Multiple stages can be cascaded to achieve higher performance. A system using 5 cascaded stages is capable of implementing a 1024 point complex FFT in 20.96 μ s.

Implementing a pulse compression system based on a Digital Adaptive Matched Filter as discussed in Section 3.4.1 requires considerably more processing effort than the technique discussed above. In particular, the compression filter must be generated on a pulse-by-pulse basis and this requires an additional FFT, multiplication, conjugation, as well as computing the inverse magnitude as indicated in Figure 3.14 (page 79). The magnitude inversion is particularly difficult to implement because of the fact that most of the high speed DSP chips lack a divide instruction and as a result other techniques must be used.

For each complex ($I+jQ$) data point, magnitude inversion can be achieved by generating a data point with a magnitude of $1/\sqrt{I^2+Q^2}$ and identical phase. As is shown in equation {4.9} this can be accomplished by multiplying each of the I and Q components by the value $1/(I^2+Q^2)$.

If an individual data point is specified by:

$$I+jQ = \sqrt{I^2+Q^2} e^{j2\pi\arctan(Q/I)}$$

Then the magnitude inverted data point is given by:

$$\begin{aligned} [1/(I^2+Q^2)][I+jQ] &= \sqrt{I^2+Q^2}/(I^2+Q^2) e^{j2\pi\arctan(Q/I)} \\ &= (1/\sqrt{I^2+Q^2}) e^{j2\pi\arctan(Q/I)} \end{aligned} \quad \{4.9\}$$

As a result the problem of inverse magnitude filtering becomes one of calculating the value $1/(I^2+Q^2)$. One possible technique is to use an approximation to the function $1/X$ given in equation {4.10}. This approximation

$$1/x = \sum (1-x)^n \text{ for } 0 \leq n \leq \infty \quad \{4.10\}$$

is only valid for $X < 1$ however it may still be adequate over the range $0 < X < 2$ depending on the number of terms used. In Figure 4.23 (page 123) the approximation of equation {4.10} is plotted for various numbers of terms. To use the approximation data values could be scaled such that the magnitudes occupy the interval 0 to 1. This requires calculating the value $K(I^2+Q^2)$ instead of (I^2+Q^2) where K is a suitable constant. Notice that even if the values slightly exceed 1 the approximation is satisfactory depending on the number of terms used. Another advantage of using the approximation with a finite number of terms is that zero values result in a fixed large value rather than infinity.

Using the first 5 terms of equation {4.10} magnitude inversion can be implemented with 9 multiplications and 5 additions per complex point or 9216 multiplications 5120 additions for a 1024 point complex data set. It should be noted that magnitude inversion processing requirements are approximately half that of a 1024 point complex FFT. Implementing the complete adaptive matched filter processing operation will require 3 FFTs, 1 magnitude inversion, 1 complex multiplication, and 1 real multiplication. The conjugation can be included in the

complex multiplication operation by sign changes without any additional processing. In terms of basic operations implementing the complete adaptive matched filter operation for one PRI will require 64524 multiplications and 93,190 additions and it is worth noting that this is slightly less than double that which is required to implement the basic compression processing as discussed earlier. If the length of the target return is such that it must be processed in stages (using the overlap and add or overlap and save techniques discussed earlier) then the relative efficiency of the adaptive filter technique increases since the same compression filter is used a number of times. It appears to be feasible to implement the digital adaptive matched filter technique using the HDSP 66010 digital array signal processor system.

$N=2^B$	FFT ALGORITHM		
	BASE 2	BASE 4	BASE 8
Real Multiplications	$(2B-4)N + 4$	$(1.5B-4)N + 4$	$(1.33B-4)N + 4$
Real Additions	$(3B-2)N + 2$	$(2.75B-2)N + 2$	$(2.75B-2)N + 2$

Figure 4.22 - $1/X$ Approximation Functions

CHAPTER 5 CONCLUSION

5.1 SUMMARY

This thesis contains a comprehensive analysis of a combined analogue and digital (two stage) pulse compression system. The research was carried out at both the theoretical and experimental levels with good agreement between the two.

The work begins with a study of conventional pulse compression theory and techniques. It was determined that certain applications such as tactical SARs, required both the flexibility of digital pulse compression as well as the efficiency of analogue pulse compression techniques. An analysis of current analogue pulse compression hardware revealed that the SAW RAC devices had the greatest performance capabilities of available devices suitable for radar applications. The unique aspects of SAR pulse compression systems were examined and it was found that the Doppler shifts associated with airborne side looking SARs were negligibly small for high resolution systems. Even in the spotlight mode of operation, Doppler shifts were found to be insignificant when signal bandwidths of 40 Mhz or greater were used.

Next the theory of partially compressed coded linear FM signals was examined and the relationships between the partially compressed pulse width, the peak amplitude, and the second stage time-bandwidth product were derived. These relationships were later verified experimentally with very good agreement to the theory. The development of a facility capable of evaluating the two stage pulse

compression process was under taken. To allow evaluation of the system using a variety of waveforms the expanded pulse generation system was based on an arbitrary signal generator. In order to permit generating signals with bandwidths exceeding 200 MHz and time-bandwidth products in excess of 5000, a system of cascaded broadband frequency doublers was developed to quadruple the signal bandwidth. A SAW RAC device with a time-bandwidth product of 5570 was used as the analogue pulse compressor and due to the high insertion loss of the device care had to be taken to minimize the signal feed-through between the input and output ports of the device. To facilitate the generation and processing of complex waveform combinations, a software utility was developed by the author whereby sophisticated waveforms could be developed, analyzed, and down loaded to the arbitrary waveform generator for experimental evaluation.

The problem of efficiently computing the second stage pulse compression filter for arbitrarily coded LFM waveforms was examined and two techniques were developed. The first technique was based on the chirp transform and made use of the fact that the partially compressed pulses were related to the secondary code by means of a Fourier transform. In particular, it was determined that the second stage pulse compression filter could be synthesized by scaling the secondary code to occupy a bandwidth of $\mu\tau$, taking the inverse Fourier transform, and multiplying the result by a chirp with FM rate μ .

The second technique used the fact that the second stage compression filter could be obtained by feeding the transmitted pulse through the SAW device, sampling the output, and conjugating the phase. Using this technique it was possible to automatically compensate for the non-ideal phase performance of system components. The concept was extended further and the implementation of a digital adaptive matched filter was investigated so that prior knowledge of the transmitted waveform was not necessary and a truly random waveform could be employed.

An evaluation of various second stage codes was undertaken in order to determine which codes if any were unsuitable and to assess the relative merits of various codes. In general, it was found that the secondary codes with good autocorrelation functions did not necessarily yield combined (with LFM) codes with good autocorrelation functions, and that inverse filtering and windowing techniques were necessary to reduce sidelobes below -30 dB. A most significant finding was the fact that discrete phase shift codes were unsuitable because they resulted in nulls in the signal spectrum. The nulls in turn posed a problem for inverse filtering in the presence of noise.

A family of waveforms named discrete Mu coded waveforms by the author were developed as a means of providing a random property to the pulse compression waveforms, while retaining compatibility with the LFM primary code. In these waveforms the FM rate was varied in a random fashion between two values while maintaining phase continuity at the transition points. In addition frequency shift

keying was demonstrated to be a suitable secondary code. The performance of the system in a hostile environment was evaluated by injecting continuous wave tone and noise signals into the waveform prior to compression and it was shown that the two stage pulse compression system exhibited greater resistance to quantizer saturation than a comparable digital system.

Finally, an analysis of the second stage signal processing requirements was undertaken. It was found that current single chip programmable DSP architectures did not have sufficient processing power to implement real time pulse compression. However, it was shown that real time operation was possible using multichip array processor systems. The processing effort required to implement the compression using a digital adaptive matched filter was found to be slightly less than double that of a basic pulse compression system.

5.2 FUTURE WORK

Now that the feasibility of the two stage pulse compression system has been demonstrated, the development of functional radar system based on this concept can be undertaken. One area in which the author feels further work should be carried out is the selection of secondary codes. Using the FSK and discrete Mu coded waveform families it would be desirable to obtain specific code sets with minimal cross correlation functions for use against repeater type jammers. A further investigation to determine the existence of secondary codes which have desirable autocorrelation functions when combined with linear FM, would also be welcome.

In particular, the possibility of introducing a fine structure in the signal spectrum using FSK codes in order to improve the autocorrelation function should be explored. The development of an analogue Mu coded waveform generator should also be addressed since these waveforms were found to be well suited to the two stage pulse compression system.

The possibility of using other types of analogue compressors either individually or with a SAW RAC device should be examined. A SAW convolver for example could be used along with a SAW RAC device to supplement the waveform flexibility of the digital compression stage. Optimizing the design and development of a digital compression processor is also an area which merits further study and in particular the development of a compact processing system capable of real time range compression would be welcome.

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APPENDIX A

LFM FREQUENCY SPECTRUM

$$s(t) = \text{rect}\left(\frac{t}{T}\right) e^{j2\pi\frac{\mu}{2}t^2} \quad \text{LFM signal}$$

Using the Fourier transform to determine the frequency spectrum:

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt = \int_{-T/2}^{T/2} e^{j2\pi\left(\frac{\mu}{2}t^2 - ft\right)} dt$$

Completing the square in the exponential where

$$\begin{aligned} 2\pi\left(\frac{\mu}{2}t^2 - ft\right) &= \pi\mu\left(t^2 - \frac{2ft}{\mu}\right) = \pi\mu\left[\left(t^2 - \frac{2ft}{\mu} + \frac{f^2}{\mu^2}\right) - \frac{f^2}{\mu^2}\right] \\ &= \pi\mu\left(t - \frac{f}{\mu}\right)^2 - \frac{\pi f^2}{\mu} \end{aligned}$$

$$\therefore S(f) = \int_{-T/2}^{T/2} e^{j\left[\pi\mu\left(t - \frac{f}{\mu}\right)^2 - \frac{\pi f^2}{\mu}\right]} dt$$

With a change of variable such that:

$$z = \sqrt{2|\mu|}\left(t - \frac{f}{\mu}\right), \quad dt = dz/\sqrt{2|\mu|}, \quad \text{and}$$

$$z_2 = \sqrt{2|\mu|}\left(T/2 - \frac{f}{\mu}\right), \quad z_1 = \sqrt{2|\mu|}\left(T/2 + \frac{f}{\mu}\right)$$

we have

$$S(f) = \frac{e^{-j\frac{\pi f^2}{\mu}}}{\sqrt{2|\mu|}} \int_{-z_1}^{z_2} e^{j\frac{\pi}{2}z^2} dz = \frac{e^{-j\frac{\pi f^2}{\mu}}}{\sqrt{2|\mu|}} \left[\int_{-z_1}^{z_2} \cos\left(\frac{\pi}{2}z^2\right) dz \pm j \int_{-z_1}^{z_2} \sin\left(\frac{\pi}{2}z^2\right) dz \right]$$

with a positive sign corresponding to a positive sweep rate (μ).

The result is in the form of a Fresnel integral where

$$C(z) = \int_0^a \cos\left(\frac{\pi}{2} z^2\right), \quad S(z) = \int_0^a \sin\left(\frac{\pi}{2} z^2\right)$$

with properties $C(-z) = -C(z)$, and $S(-z) = -S(z)$

Therefore

$$S(f) = \frac{e^{-j\pi f^2}}{\sqrt{2}|\mu|} \{ C(z_1) + C(z_2) \pm j[S(z_1) + S(z_2)] \}$$

where

$$|S(f)| = \frac{([C(z_1) + C(z_2)]^2 + [S(z_1) + S(z_2)]^2)^{1/2}}{\sqrt{2}|\mu|} \text{ LFM Magnitude Spectrum}$$

and

$$\text{ARG}[S(f)] = -\frac{\pi f^2}{\mu} \pm \arctan[(S(z_1) + S(z_2)) / (C(z_1) + C(z_2))]$$

END

1 3 0 9 9 0

FIN