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Serviceability and Strength of Concrete Parking Structures Reinforced by Fibre-Reinforced Plastics

by

DAGMAR ŠVECOVÁ, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Department of Civil and Environmental Engineering

The Master of Engineering Program
in Civil and Environmental Engineering
is a joint program with the University of Ottawa,
administered by The Ottawa-Carleton Institute for Civil Engineering

Carleton University
Ottawa, Ontario, Canada
September 1994

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ISBN 0-612-08966-5

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Department of Civil and Environmental Engineering

The Undersigned recommend to the Faculty of Graduate
Studies and Research acceptance of the thesis

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in partial fulfillment of the requirements for
the degree of Master of Engineering

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Carleton University
Ottawa, Ontario, Canada
September 1994

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Acknowledgments

I would like to acknowledge the professional guidance and help of my advisor, Dr. A. Ghani Razaqpur. His moral support throughout the time when we were working on this project is much appreciated.

Financial support, without which my studies here would not have been possible was received from Dr. Otto J. Svec, The Faculty of Graduate Studies, and Dr. A. Ghani Razaqpur, and I am grateful to all of them.

Special thanks go to Dr. Neal Holtz for his help whenever there was a problem with our computer facilities.

These acknowledgements would not be complete without expressing my thanks to my fellow students, especially Mostafa Nofal, for acquainting me with the finite element program NONLACS.

Abstract

The latest surveys of existing concrete parking structures in Canada and in some parts of the United States show major deterioration in these structures due to corrosion of steel reinforcement caused primarily by deicing salts. The use of non-corrosive reinforcement instead of steel reinforcement has been suggested as a possible solution to the problem. Fibre-reinforced plastics (FRP) are non-corrosive and high strength, but some of their mechanical properties, such as brittleness and relatively low modulus of elasticity when compared to steel may create problems insofar as deflections and ductility of FRP reinforced concrete structures are concerned. This thesis investigates the complete behaviour of two-way slab systems reinforced with FRP materials and subjected to monotonically increased loading.

The investigation deals with the performance of two-way FRP reinforced slab systems, with particular emphasis on flat slab structures. The FRP's considered in the study included carbon, glass and aramid fibre reinforced plastic reinforcement. Specifically, the load-deflection behaviour throughout the entire range of loading, stresses in the concrete and in the reinforcement under service loads, as well as at ultimate, are determined and compared with similar structures reinforced with steel. The nonlinear finite element method was employed to perform the analyses.

Results of the analysis show that any of the preceding FRP's can be used without exceeding the current deflection requirements in the Code. The issue of ductility is more complex, but FRP reinforced slabs undergo significant amount of deflection prior to failure. Overall the study supports the feasibility of replacing steel by FRP in two-way slabs such as flat slab/plate structures.

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Chapter 1

Introduction

1.1 General

Structural design should meet three main requirements: safety, serviceability, and economy. For parking structures all of them are of special importance because they operate in a very harsh environment. This thesis will examine the safety and serviceability of parking structures reinforced with fibre reinforced plastic reinforcement (FRP), and the comparison of such structures with similar structures reinforced with conventional steel reinforcement. The issue of economy is not within the scope of the study. In this chapter the background for the study is discussed and the objectives of the research and the methods to be used are described.

1.2 Problem Definition

There are two main reasons why parking structures suffer from deterioration: first, due to volume and temperature changes cracking occurs in slabs and affects their structural performance. This problem can be partially solved by designing expansion joints in structures. The second cause of deterioration is corrosion of steel reinforce-

ment embedded in concrete. Parking garages in downtown areas of large cities serve frequent traffic, which in cold regions brings in a large quantity of road salt carried by vehicles. A lot of parking structures built over the last 25 years started to suffer from corrosion of steel reinforcement after they were around 10 years old (Walker 1986). There have been a lot of effort expended towards prevention of corrosion of steel caused by road salts, but all of them appear to have furnished only partial solution to this problem.

Corrosion of reinforcing steel is understood to be an electrochemical process (Bennet 1991). Due to corrosion, steel rebar volume expands and therefore high tensile stresses occur around the reinforcement. If these stresses exceed the tensile strength of concrete, cracking develops between bars, and later leads to delaminations. In order for corrosion to start, it needs an anode and a cathode, embedded in an electrolyte. Only if these three components are present together with free negative ions, electrochemical reaction will occur. Unfortunately this environment is often found in concrete, which has high content of chloride ions, coming from sources such as deicing salts. Whereas only 0.6 to 0.8 kg of chloride ion per cubic meter of concrete is sufficient to promote corrosion (Clear and Hay 1973), in some cases it was found that the steel rebars were exposed to salt concentrations of 9 kg per cubic meter or even higher (Bennet 1991). Major deteriorations of concrete parking structures are in such cases unavoidable.

According to a survey by Litvan and Bickley (1989), "almost all garages built until very recently by conventional methods will require rehabilitation", which is mainly due to the fact that chloride concentration in concrete has reached quite high levels. A total of 215 garages in the areas of Toronto, Ottawa, and Montreal were surveyed in order to assess delamination, half-cell potential, depth of cover, and conditions of structural components. Additional testing on cores taken out of the system determined the chloride content, the compressive strength, and the permeability of concrete. Most of the surveyed garages needed some repair work. The rate of deterioration was found to be 2.3 %, while it was found to be 2.1 % for bridge decks

according to a similar study by Cady and Weyers (1984). However, it was concluded by Litvan and Bickley (1989) that durable garages can be built, especially when high quality dense concrete is used, together with adherence to the Canadian Standards Association (CSA) requirements for reinforcement cover, drainage and adequate waterproofing system. The only cause of delamination was established to be corrosion of reinforcement because neither cracking, nor compressive strength of concrete were found to have any effect on it. In the study, no problems with large deflections were reported. Accordingly, steel corrosion could be considered the principal cause of all malfunctioning and deterioration of parking structures.

The same conclusions can be drawn from the study conducted by Tiche and Van Volkinburg (1989), where the main cause of deterioration of 13 years old parking structure was reported to be corrosion of reinforcement. Another parking structure (ACI 1989) suffering from deterioration was a 700-car parking structure with a medical office building above it. This structure suffered major deterioration, and had to be repaired with the repair cost much higher than the initial construction cost. The Foster-Winter parking garage in Detroit suffered due to large amounts of road salts which had penetrated the concrete surface and corroded the top mat of reinforcing steel.

In order to solve this problem, much effort has been devoted towards developing efficient methods to prevent corrosion, or to stop the already initiated corrosion process. These methods will be mentioned later in this thesis. Some practical guidance for design of durable parking structures can be also found in Chrest et al (1989), and in the ACI (1986) report on parking structures. The intent of existing technologies which aim to ensure durable parking structures is to protect the reinforcing steel from the elements which lead to its corrosion. To date none of the proposed solutions have proven fully effective. Consequently, in recent years the idea of replacing reinforcing steel by fibre reinforced plastic reinforcement has been proposed and researched. Replacement of steel with fibre reinforced plastics appears to be a feasible solution for this problem. These relatively new materials have excellent corrosion resistance.

Their tensile strength is very high, but their compressive strength is much lower. Therefore, they are usually not placed in locations where they may be stressed in compression. Of course, there is rarely need for compression reinforcement in slabs and therefore the compressive strength of such reinforcement is not of practical significance in this case.

Steel reinforcement and concrete are very compatible materials. This compatibility includes bond and thermal expansion. Bond performance of FRP reinforcement is reported to be the same as that of steel and it can even be enhanced using different surface treatments (i.e. sand blasting), but the coefficients of thermal expansion of concrete and FRP are not in the same range. Another drawback of some FRP materials is their relatively low modulus of elasticity. Therefore, replacing steel reinforcement of high modulus of elasticity with FRP may lead to potentially large deflections, especially after cracking. FRP reinforcement has been used so far mainly for prestressing bridge decks. Mostly because of their low modulus, high cost and possible fire damage susceptibleness, FRP reinforcement has not been used as ordinary reinforcement in floor slabs. In this thesis the issues pertaining to the deflection, ductility and strength of FRP reinforced parking structures will be studied in order to establish the feasibility of the complete replacement of steel reinforcement by FRP. The issue of fire resistance is not within the scope of the current study.

1.3 Objectives and Scope

Cast-in-place parking structures are mostly flat plate or flat slab type of structures. While they may be either reinforced or prestressed, reinforced concrete is more common. For the latter reason and due to time constraints, this investigation focuses on reinforced concrete flat plate/slab parking structures. Such structures essentially consist of slabs and columns. Since FRP reinforcement has a rather small compressive strength, it cannot be used as reinforcement in columns. This signifies that in the

current study the design of columns will not be altered nor will their behaviour be investigated.

Serviceability limit state is mostly of interest in this work. A comparison of the behaviour of similar steel and FRP reinforced flat slab/plate parking structures will be made in order to find the limitations of FRP reinforced systems insofar as deflections are concerned, but the systems will also be compared for their ductility and strength. The study will include glass fibre reinforced plastic (GFRP) and carbon fibre reinforced plastic (CFRP) reinforced concrete flat slab/plate structures, in addition to conventionally reinforced concrete structures of the same kind.

These structures will be designed using the existing ultimate strength design provisions of CSA Code for buildings (CSA 1984). The deflection requirements will be satisfied by using the appropriate span/depth ratio for conventional parking structures. The designed structures will be analyzed by the nonlinear finite element method which can account for the full nonlinear behaviour of concrete, including cracking, and for the yielding of the steel. The structures will be subjected to monotonically increasing load up to failure. Some modifications will be made to an existing finite element program to account for the behaviour of FRP reinforcement in the analysis.

To assess the suitability of the nonlinear finite element method for the analysis of FRP reinforced concrete structures, a number of FRP reinforced concrete beams and slabs will be first analyzed. These structures have been tested in the laboratory by other investigators and their deformations and strength have been reported. The beams and slabs will be selected in such a manner as to include the many different types of FRPs used for reinforcing, prestressing and for strengthening. Satisfactory comparison between the laboratory test results and the finite element simulations will ensure that the method of analysis adopted in the current investigation is suitable for studying the complete response of FRP reinforced concrete parking structures.

The analysis results will be used to determine the complete load deflection curve, the stresses in the concrete and the steel at each load level, the ultimate strength, and

the failure mode of each structure. Since deflection control is of particular interest in the current study, the slab deflection under service loads will be determined and will be compared to CSA requirements.

Finally, this study does not address long term deflections due to creep and shrinkage nor does it address issues related to fatigue and/or thermal incompatibility.

Chapter 2

State of the Art

2.1 General

It was stated in the previous chapter that the principal objective of the present investigation is to determine the feasibility of substituting conventional steel reinforcement in concrete by high strength FRP reinforcement. More specifically, the feasibility of using FRP to reinforce concrete parking structures is the focus of the study. Since FRP reinforcement for concrete is a relatively recent development, its applications have been quite limited. Most existing research and applications have been aimed at beams and girders in bridges. Nevertheless, in this chapter some existing FRP materials and their properties as well as the basic features of parking structures and the available methods for their corrosion prevention and remedy will be discussed. In addition, the basic code requirements for the design of parking structures will be highlighted. The latter information will be used in the subsequent chapters to see whether FRP reinforced concrete structures could satisfy those requirements.

2.2 Parking Structures

A wide ranging discussion concerning concrete parking structures and their design, construction, maintenance, and repair can be found in Chrest et al (1989). Parking structures are basically divided into two main groups: open parking structures, and closed parking garages. Of course, most of the problems arising from corrosion of reinforcing steel apply to both types. Only slight differences exist in the requirements for these two types: generally, for the open structures, which are more affected by the severe weather conditions, quality of material, and protection against corrosion are more important than for the closed structures. However, the closed garages are also affected by the aggressive environment created by exhalants. Therefore, in the latter type of parking structure ventilation requirements need to be imposed.

Although the National Building Code (1985) stipulates criteria which separate open structures from closed structures, from the point of view of the present investigation the differences between the two types of structures are not important because both have the same basic structural system and are nearly equally susceptible to the damaging effects of deicing salts. Consequently, henceforth no attempt will be made to distinguish between them, and the term parking structure will be applied in its generic sense.

2.2.1 Structural Systems Used for Parking Structures

Structural systems that are commonly used in concrete parking structures may be divided into two main groups

- cast-in-place concrete system
- precast concrete system

Both systems can be either conventionally reinforced, or prestressed. The most common structural framing systems for parking structures are classified as follows (Shah 1980)

1. Cast-in-place conventionally reinforced concrete

- flat slab or flat plate
- one-way slab and beams
- one-way joists

2. Cast-in-place post-tensioned concrete

- one-way slab and beams
- two-way flat slab or waffle slab

3. Precast prestressed concrete

- double T's
- hollow-core slabs

From the durability point of view, some of these systems are more suitable than the others, but even the performance of the less suitable systems can be improved if some additional protection is provided. This protection involves using protective membranes, sealing surface of concrete and many others. This will increase the initial cost of the structure, but the final cost may be reduced by better structural performance and durability of the system.

Every system mentioned above has its own advantages and disadvantages. For example, in conventionally reinforced cast-in-place systems the effect of shrinkage and temperature is not as important as in the prestressed concrete systems. But the former cannot be used for very long spans (maximum span is usually 8 m) because they will become rather stocky. Another common problem of those systems is that they crack under service loads, and due to their continuity, cracks will develop on the top

surface of slabs around columns or in column strips. In combination with abrasion forces due to traffic and easy intrusion of moisture and salt into the cracks, this will lead to early deterioration of the slab.

One-way joist system is likely to malfunction due to cracking in the thin deck. Also, where large differences in thicknesses of the slab are present, this may cause additional stresses during curing of concrete which will lead to cracking (Shah 1980). Even though cracking by itself is not the main reason for malfunctioning of parking structures, it makes it easy for aggressive substances to penetrate the concrete and to harm the steel reinforcement. Water combined with salt often easily penetrates concrete through cracks where it reacts with the steel reinforcement, and starts the corrosion process. According to basic knowledge about corrosion, corroded steel expands in volume, which leads to spalling of the concrete. In fully prestressed systems, cracking is usually not allowed while in partially prestressed systems cracking may be allowed under live load only. For precast prestressed decks, detailing of joints between prefabricated elements needs special care because they provide potential avenues for leakage and possible damage. Generally, there is no top surface cracking in prefabricated members because contrary to cast-in-place systems these are usually simple span members without negative moment regions.

From the broad spectrum of slab systems, the abovementioned systems are most likely to be used for the purpose of parking structure because they allow for long clear spans and relatively easy maintenance. But the choice needs to be very carefully made because it will affect the service life performance of the parking structure. For reasons stated earlier our focus will be the behaviour of FRP reinforced flat plate/slab structures and its comparison with similar steel reinforced structures.

2.2.2 Improving Durability of Parking Structures

Parking structures have been very often compared to bridges because of the traffic environment they serve in, and also due to the fact that they are exposed to the changes of weather practically on a daily basis. Because of this hostile environment, special care is needed to design a durable structure. According to the State-of-the-Art Report on Parking Structures (ACI 362R-85) attention has to be paid mainly to the freezing and thawing resistance, corrosion resistance, water-cement ratio, and cover of reinforcement. This requires that special attention be paid in the design of concrete mixture, type of reinforcement, and the selection of the structural system.

Currently there are several ways to improve performance of concrete in aggressive environment. They are usually based on the fact that if concrete has low permeability, it is more resistant to attack by different chemicals either from the air or water. Therefore, inhibitors are used to reduce concrete permeability. Inhibitors basically lock themselves to the concrete so their performance is theoretically expected to last during the whole life of the structure (Berk 1991). Some of the inhibitors very commonly used are superplasticizers, or microsilica. Durning and Hicks (1991) suggested using microsilica to increase concrete resistance to chemicals. A two year study of the effect of microsilica on the improvement of chemical resistance of concrete was performed. They reported that the deterioration of concrete can be characterized by two mechanisms: dissolution of material from cement paste, and expansion of material within concrete. Adding microsilica into concrete reduces its permeability and therefore also the possibility of attack by aggressive chemicals. Significant abrasive forces may partially or completely diminish the effect of silica. The importance of good quality concrete with low water-cement ratio for improving concrete durability has been emphasized in the literature. Table 2.1 summarizes the principal concrete ingredients and their effect on concrete quality (Chrest 1989).

So far only interior protection of concrete using different inhibitors was discussed. However, there exist another type of protection system, which is fixed on the concrete

Table 2.1: Concrete Ingredience (Chrest 1989)

Ingredient	Requirement	Reason for Use
Cement	≥ 6 sacks	Strength
Water	$w/c \leq 0.40$	Durability
Aggregate	Varies	Strength, durability, pumpability
Chlorides	≤ 0.06 % for P/S, otherwise ≤ 0.15 %	Durability
Thiocyanates	Zero	Durability
Entrained air	Varies with climate	Freeze-thaw resistance, workability
Water reducers	Varies	Durability, workability
Superplasticizers	Slump ≤ 6 "	Durability, workability
Accelerators	Varies, but chloride free	Cold weather concreting
Retarders	Varies	Hot weather concreting
Corrosion inhibitors	Varies	Durability
Fly ash	≤ 25 %	Economy, handling, low heat
Silica fume	≤ 7 %	Strength, durability, impermeability
Latex	Varies	Impermeability

Table 2.2: Exterior Protection Systems (Chrest 1989)

System	Usual application
Sealants	Waterproof control, construction, and isolation joints
Sealers	Keep out chlorides
Membranes	Keep out water and chlorides; bridge cracks
Overlays	Durability protection

surface. The exterior protection systems include the following (Chrest 1989): sealants, membranes, and overlays. All of these systems are very commonly used in practice. Table 2.2 summarizes the current techniques of exterior protection (Chrest 1989).

The main concern of engineers in steel-reinforced concrete structures exposed to aggressive environment is corrosion of steel reinforcement embedded in concrete. The basic requirements of adequate concrete cover which are sufficient for regular reinforced concrete structures has to be increased for parking structures in order to protect the reinforcement. Very often another type of protection, epoxy coating of reinforcing steel bars is used. Under ideal conditions, the corrosion resistance of epoxy coated bars has been reported to be excellent (ACI 362R-11), but in practice the quality of the coating and the handling of the reinforcement on the site produces less than ideal conditions which lead to premature deterioration. Scratches in coating give corrosion opportunity to propagate. Finally, the fire behaviour of this reinforcement is still questionable (D'Arcy 1988).

Improving the environment for steel in concrete, either by using internal protection via inhibitors, or external protection with sealers, overlays, or other methods, improves durability. The major cause of premature corrosion is considered to be chloride ions (ACI 1993). They may be present in cement paste or in aggregates, but mostly they can be found as compounds of deicing salts (D'Arcy 1988). The only system known

until now, which can protect structure from corrosion is cathodic protection. If it is applied before corrosion starts, it gives a good chance that corrosion will never start (Chrest 1989). This procedure is though considered very costly. In reality, none of the existing methods can provide perfect protection for steel. Consequently, the only sure method for avoiding corrosion is the removal of the steel reinforcement from concrete and its replacement by a non-corrosive material.

Some available fibre-reinforced plastic reinforcement are produced precisely because of their superior resistance to attack by many chemicals, including alkalis that cause corrosion of steel in concrete. In the following sections some important FRP materials and their properties will be presented. Following that presentation, it will be discovered that while FRP materials are non-corrosive, they also have some mechanical properties that are inferior compared to steel. The latter fact necessitates the need for careful investigations in order to establish whether FRP can indeed replace steel in concrete structures.

2.3 FRP as a Reinforcing Material for Concrete

Fibre reinforced plastic is not a new material in engineering. It has been used for decades in mechanical and aerospace engineering, and very often for military purposes. It has many desirable properties, for example it is non-corrosive, non-magnetic, non-conductive, and very light in weight. Within the last 15 years, there has been some increase in the usage of this material for reinforcing concrete, with the possibility of strengthening existing structures, particularly wood and concrete structures. A brief review of the material properties of FRPs used for reinforcing concrete and recent reports from research in this area of concrete structures as well as some case studies which show the performance of these materials in service will be presented in this section.

2.3.1 Properties of Fibres and Matrices for FRP

FRPs are composite materials consisting of two main components, a high strength, high stiffness reinforcing fibre material and a matrix/resin. There are different types of fibres used in FRPs, but the most frequently used ones are carbon, glass, aramid, and polyvinyl alcohol fibre.

According to Hollaway (1989) two types of *carbon fibres* are available: synthetic fibres known as polyacrylonitrile or PAN, which are similar to fibres used for making textiles, and pitch based fibres, obtained by the destructive distillation of coal. The manufacturing process of carbon fibre consists of carbonization, which is preheating PAN fibre under tension in air at 250° C in order to avoid shrinkage, and gain strength. This process also changes its colour from white to yellow, to brown, and finally to black. The stiffness of the carbon filament can be increased further by heating it up to 2 200° C. There are three grades of carbon fibre (Hollaway 1978)

- Type I is the stiffest with the highest modulus of elasticity (more than 310 GPa). Its tensile strength was found to be from 1.5 to 2.1 GPa, and strain at failure reaches values from 0.6 to 0.7 %
- Type II is the strongest fibre with tensile strength from 2.4 to 2.8 GPa, but its modulus of elasticity is lower than for type I (from 220 to 240 GPa), while the ultimate strain is higher (from 0.8 to 0.9 %)
- Type III is the one with the smallest modulus of elasticity which ranges from 175 to 205 GPa, and its strength is between 1.9 and 2.6 GPa. This fibre has the highest strain at failure, 1.0 %, and is therefore sometime referred to as the high strain fibre.

Glass fibres are produced by rapid drawing molten glass from an electrically heated furnace through platinum bushings at high speed (Hollaway 1989). There are five types of glass fibre:

- E-glass of low alkali content. It is used mainly with polyester and epoxy resins. It has been used for more than 50 years, however at the beginning some problems were reported about its performance within thermosetting resins at high moulding pressures (Parkyn 1970). However, its performance with polyester type resins is favourable.
- Z-glass has very high resistance to alkali attack. It is usually used for reinforcing cements, mortars, and concretes.
- A-glass of high alkali content is gradually going out of production. This is due to the fact that it does not have a good long term performance under moisture conditions. Therefore it is being replaced by E-glass.
- S-glass has extra high modulus and high strength. It has been used mostly in aircraft components and missile casings.
- C-glass has very high corrosion resistance to acids and is used mostly in chemical plants.

The term *aramid fibre* comes from its chemical bases as aromatic polyamide. For the purpose of FRPs, Kevlar 49, Twaron, and Technora are used. Technora does not belong to the aromatic polyamide group, it is aromatic polyetheramide fibre. Kevlar 49 fibres have a linear stress strain behaviour, but when under bending, they exhibit a high degree of yielding on the compression side (Mallick 1988). Therefore Kevlar 49 composites have superior performance among other fibres' composites. Aramid fibres can be used by themselves, or combined with glass, or carbon fibres in hybrid form. This fibre is fire resistant (up to 425° C), has good fatigue, and chemical resistance. *Polyvinyl alcohol fibres* have very high strength. They are manufactured by polymerization of polyvinyl alcohol of high degree, and rolled to provide strength and elasticity to the fibres (JSCE, 1993).

The mechanical properties of fibres are summarized in the Table 2.3 (JSCE 1993, Gill 1972). All of them are characterized with a linear stress strain relationship up

Table 2.3: Mechanical Properties of Fibres Used for FRPs

			Tensile Strength [MPa]	Young's Modulus [GPa]	Elongation [%]
Carbon Fibre	PAN	High Strength	3500	200 - 240	1.3 - 1.8
		High Modulus	2500 - 4000	350 - 650	0.4 - 0.8
	Pitch	Ordinary	780 - 1000	38 - 40	2.1 - 2.5
		High Modulus	3000 - 3500	400 - 800	0.4 - 1.5
Aramid Fibre	Kevlar 49		2800	130	2.3
	Twaron		2800	130	2.3
	Technora		3500	74	4.6
Glass Fibre	E-Glass		3500 - 3600	74 - 75	4.8
	S-Glass		4900	87	not supplied
	Alkali Resistant Glass		1800 - 3500	70 - 76	2.0 - 3.0
Polyvinyl Alcohol Fibre	High Strength		2300	61	5.0

to failure. Some of them were/are used for reinforcing mortars, but because of their small abrasion resistance, this usage is diminishing. More efficient use for fibres was found in combination with resins, creating continuous fibre reinforced plastics. They have shape of regular reinforcing bars, or rectangular cross section. Two dimensional meshes, as well as three dimensional meshes are available, and also other different shapes, as for example sheets, and grids (see Fig. 2.2 on page 20). In the following, the properties of matrices and the manufacturing process will be described.

For the purpose of producing a reinforcing material, the fibres need to be embedded in

Table 2.4: Properties of Matrices Used for FRPs (Hollaway 1978)

Material Properties		Tensile Strength [MPa]	Young's Modulus [GPa]	Coefficient of linear expansion ($10^{-6}/^{\circ}\text{C}$)
Thermosetting	Polyester	45 - 90	2.50 - 4.00	100 - 110
	Epoxy	90 - 110	3.00 - 7.00	45 - 65
	Phenolic	45 - 59	5.50 - 8.30	30 - 45
Thermoplastic	PVC	58	2.40 - 2.80	50
	ABS	17 - 62	0.69 - 2.82	60 - 130
	Nylon	48 - 83	1.03 - 2.76	80 - 150

matrices. Commonly used matrices are epoxy, polyester, vinylester, and bisphenol-A. When matrices are unreinforced, they have poor mechanical properties as summarized in Table 2.4 (Hollaway 1978). Resins can be divided into two groups: thermoplastic, and thermosetting. Thermoplastics cannot be reset by heating while thermosets can be reheated to reshape them. For the purpose of continuous fibre reinforced plastics, thermosetting matrices are used more often, but they have to be chemically and thermally compatible with the fibres.

The matrices have the following functions (Hollaway 1989)

1. To bind the fibres together as a single unit and to protect them from surface damage
2. To transfer stresses to the fibres by bond and/or friction
3. To separate the fibres.

A common technique of manufacturing FRPs is *pultrusion*. A schematic representation of this technique can be seen in Figure 2.1. Continuous strands can be produced

Figure 2.1: Pultrusion Technique (Hollaway, 1989)

with this method. The continuous strands of reinforcing material are taken from creels, through resin tank, where they are saturated with resin, and then they go through a number of wiper rings into the mouth of a heated die. The speed of pulling through the die is predetermined by the curing time needed. To ensure good bond with concrete, the surface of the bars is usually braided, or sand coated.

2.3.2 Continuous Fibre Reinforced Plastics

Continuous fibre reinforced plastics can be classified by material or by shape. The classification according to Japan Society of Civil Engineers (JSCE 1993) can be seen in Figure 2.2. Mechanical properties of continuous fibre reinforced plastics depend on the type of fibres, content of fibres used (which varies from 35 to 65 %), and manufacturing process. Standard mechanical properties of some FRPs are summarized in Table 2.5 (JSCE 1993, Popov 1990, Hollaway 1978). For comparison, properties of steel and concrete are also included.

All materials used for structural design are characterized by their stress-strain relationship. Prior to design, properties such as bond, resistance to chemicals and to mechanical attacks, fatigue, thermal expansion characteristics must also be considered. Therefore some of those properties for FRPs will be briefly discussed now.

Stress-strain relationship for all FRPs is linear up to failure. The tensile strength

Figure 2.2: Classifications of Continuous Reinforcing Material

Table 2.5: Mechanical Properties of Continuous Fibre Reinforced Plastics

Type of Fibre		Tensile Strength [MPa]	Young's Modulus [GPa]	Ultimate Strain [%]	Coefficient of Linear Expan. ($10^{-6} / ^\circ\text{C}$)
Carbon	PAN	1500 - 2000	120 - 150	1.2 - 1.6	0.60 - 1.00
	Pitch	800 - 1800	80 - 150	0.5 - 1.2	approx. 0.70
Aramid	Kevlar 49	1300	66	2.2	- 5.71
	Twaron	1360	63	2.2	- 2.00
	Technora	1800	54	3.6	- 3.00
Glass		750 - 1600	35 - 53	2.0 - 3.2	9.00 - 10.00
Polyvinyl Alcohol		600 - 780	28 - 37	3.0	4.40
Mild Steel		370 - 700	210	12.0	12.00
Concrete		2 - 5	30	0.4	10.80

of FRPs is much higher than the tensile strength of steel. This could be considered as an advantage for design of high strength slender structural members. However, in combination with low modulus of elasticity in comparison to steel, and small ultimate strain in tension, design of FRP reinforced members to satisfy serviceability requirements for deflections may be troublesome. Increasing the ductility of FRP reinforced structures may also be a problem to be solved by future research.

Some effort to increase ductility of FRP rebars, and by implication structures reinforced with them, has been already made by combining different types of fibres within a reinforcing bar (JSCE 1993). The idea was inspired by the different toughness of fibres, which in combination with each other were expected to increase the ductility of the reinforcement bar, because the more brittle fibres would break first followed by the fibres with larger ultimate strain. Nevertheless, the combination of different fibres, as for example carbon and aramid fibres, have not increased ductility compared to that of steel. Another feasible solution for increase of ductility can be combination of different FRP bars within one structural member. This may increase ductility because if carbon fibre bars and glass fibre bars are used, then after the carbon fibre

bar ruptures, the glass fibre bar will be capable of carrying some additional load due to the fact that its ultimate strain in tension is higher. This possibility should be studied experimentally.

FRP reinforcement is rarely suggested as a compressive reinforcement. Due to its excellent tensile strength, it is usually used as tensile reinforcement. However, recent research work (JSCE 1993, Challal 1992) reported that the compressive strength of FRPs is around 80 % of its tensile strength. These results were reported using different testing methods, therefore prior to using FRP as a compressive reinforcement, additional standard testing should be performed.

Another problem arises from the fact that the coefficient of thermal expansion for FRPs varies from negative values to $9 - 10 \times 10^{-6}/^{\circ}\text{C}$. Favourable behaviour of ordinary reinforced concrete is partly due to the fact that both steel and concrete have very similar values of coefficient of thermal expansion. Of all the FRP materials only glass fibres have comparable coefficient of thermal expansion to that of concrete. As indicated in Table 2.5, aramid fibres are the only type with negative thermal expansion coefficient. Therefore, testing behaviour of AFRP reinforced concrete under low temperatures is necessary. Different thermal properties of FRP and concrete may lead to loss of bond, and additional thermal stresses within the structure.

The tensile strength of FRPs, and their modulus of elasticity was found to be affected by high temperatures. If temperatures exceed 60°C , every additional 10°C increase in temperature will induce a decrease in tensile strength of 50 to 100 MPa, and in Young's modulus of 1 to 2 GPa (JSCE 1993). Therefore, in the design this has to be accounted for by determining safety factors for the tensile strength (JSCE 1993), or designing with adequate concrete cover for FRP rebars.

This summary of properties of continuous fibre reinforced plastic materials does not include properties of prestressing tendons, which are out of the scope of this thesis. For more information about properties of tendons, and/or anchorages, as well as for more detailed findings from research in the area of FRP reinforced concrete, refer to

2.3.3 Durability of FRP Materials

The purpose of this thesis is to examine the performance of FRP reinforced concrete flat slabs. One possible application for this floor system, which is investigated in this thesis, is in parking structures, where large spacious areas are needed. Because of the harsh environment in those structures, steel does not show good performance due to corrosion. FRPs are basically non-corrosive, therefore their applicability for parking structures seems feasible. However, parking structures are not only subjected to a corrosive environment, but they also experience such influences as high temperature variations, vibrations, and freezing and thawing processes during their service life. Therefore, susceptibility of reinforcing material to these conditions should be known.

First the chemical resistance of FRPs will be discussed. Fibres are usually known for their good chemical resistance. Table 2.6 (JSCE 1993) gives an idea about their performance in acid, alkali, or organic solutions. The acids used include hydrochloric acid, sulfuric acid, and nitric acid. Alkalies included sodium hydroxide, and brine. Organic solutions considered were acetone, benzene, and gasoline. Carbon fibres were found to have the best performance of all tested fibres.

However, in general the durability of FRPs is controlled by the durability of resins in which the fibres are embedded. Unfortunately, resins are usually less durable than fibres. Resins used for anti-corrosion FRP include phenol type epoxy resins, polyester resins, and novolak vinylester type resin. One of the reasons of deterioration of resins is water saturation. Deterioration in plastics manifests in three forms: surface reaction, layer formation, and intrusion (JSCE 1993). They may be collectively referred to as corrosion of resin, and the first two types may influence bond between concrete and FRPs. All three progress usually very slowly. The surface reaction type is expected to grow about 1.32 mm in 10 years, and about 6.6 mm in 50 years (JSCE

Table 2.6: Chemical Resistance of Fibres (JSCE 1993)

			Acid Resistance	Alkali Resistance	Organic Solvent Resistance
Carbon	PAN	High Strength	good	excellent	excellent
		High Modulus	excellent	excellent	excellent
	Pitch	Ordinary	excellent	excellent	excellent
		High Modulus	excellent	excellent	excellent
Aramid	Kevlar 49		poor	good	excellent
	Technora		good	good	good
Glass	E-Glass		poor	fair	excellent
	Alkali Resistant Glass		not supplied	good	not supplied
Polyvinyl Alcohol Fibre			good	good	good
Steel Fibre			poor	excellent to sodium poor brine resistance	excellent

1993). According to the same source, the corrosion layer formation is expected to grow 0.37 mm in 10 years, or about 0.82 mm in 50 years, and the third type, intrusion, is believed to be even slower. This will definitely change with change of conditions, as for example high temperatures, or stresses.

As reported by Kubota et al. (1991) the flexural strength of prestressed concrete beam reinforced with FRP tendons and submerged in an alkali solution of pH13, at a temperature of 60° C was decreased by up to 60 % after 13 months. This may indicate some problems in a long term performance of FRP prestressing tendons, however, due to many types of FRPs, more tests of tendons from various fibres would be necessary before any general conclusions can be drawn.

Some tests were performed on FRP rebars subjected to alkali environment, very similar to that in concrete (Yamashita et al 1989, Ichikawa et al 1991). The bars were immersed in alkali solution for nine months under tension of 60 % of their tensile strength. CFRP rebars showed a decrease in strength of about 5 %, while aramid fibre bars strength decreased by 20 % or more. Tests of immersing different fibre bars into alkaline solution without applying stress showed no deterioration in carbon fibre bars, minor deteriorations in aramid fibre bars, and the largest deterioration in glass fibre bars. The rate of deterioration was found to be faster with increased tension and temperature.

Parking structures are very often exposed to cycles of freezing and thawing. Due to the different expansion properties of FRPs and concrete, this may create a possible problem. Research work conducted by Tokuda and Majima (1991) showed that the maximum bond stress is nearly constant regardless of the number of freeze and thaw cycles, and the report concludes that there will be no problem in behaviour of FRP reinforced members due to freezing and thawing cycles. However, Fukushima et al. (1991) concluded from the bending tests on beams reinforced with CFRP and/or AFRP bars that the bending strength of aramid fibre bars tend to decrease about 20 % after 30 cycles of freezing and thawing. More testing on performance of actual

structural members will be needed.

It is generally known that performance of FRP reinforced structures in aggressive environment is superior to ordinary reinforced concrete. However, different types of fibres, as well as binders, performs differently under different conditions. Therefore, the type of FRP reinforcement has to be chosen for the intended usage with due regard to the results of the limited number of tests of these materials available. The large variety of FRP materials now available with different fibres, fibre/matrix ratio, surface conditions, and manufacturing procedure is an advantage with regards to the large number of choices a designer has, however, this also creates difficulties in standardizing methods of testing in order to obtain necessary material properties for design.

2.3.4 Previous Research on FRP Reinforced Concrete

FRPs have been used for reinforcing, prestressing, and strengthening of concrete structures. Therefore, past research work will be reviewed with those categories in mind. This review is not intended to be comprehensive. For comprehensive reviews, reference may be made to Mufti et al (1992), JSCE (1993), Razaqpur and Kashif (1992).

Bending and bond behaviour of glass fibre plastic reinforced concrete beams was studied by Faza in 1991. His experimental work included tests of twenty seven beams subjected to bending, twenty four cantilever specimens subjected to bond forces, and twelve pull-out specimens. The beams were cast from concrete with different strengths ranging from 29 to 69 MPa. Glass fibre plastic rebars were of different sizes as well as types (smooth, ribbed, and sand coated). The pre- and post-cracking behaviour, load-deflection and stress variations, and modes of failure of FRP reinforced beams were studied. An increase of ultimate moment capacity by 90 %, as well as increased ductility was reported when high strength concrete ($f'_c = 52$ MPa) was reinforced with

high strength rebars. The increase was relative to normal strength concrete ($f'_c = 34.5$ MPa) and using the same type and amount of GFRP reinforcement (2 # 3 GFRP rebars). When smooth bars were used, bond failure occurred prior to reaching the ultimate moment capacity. Deflections of GFRP reinforced beams were around four times larger than deflections of similar steel reinforced beam after cracking. This was mainly due to the fact that GFRP had a modulus of elasticity of 48 GPa compared to the 200 GPa modulus of steel. Therefore, after cracking the flexural stiffness of the beam decreased substantially, and the deflections increased rapidly. The difference between using regular and sand coated bars affected the crack distribution, and also increased the ultimate moment capacity. Contrary to this finding, Nagasaka et al. (1990) reported only slight improvement of flexural behaviour of beams reinforced with either braided, or sand coated bars. This difference might be attributed to the fact that while Faza was testing beams reinforced with glass fibre rods, Nagasaka's beams were reinforced with carbon fibre reinforced rods.

Schmeckpeper (1992) tested a series of beams reinforced with an FRP mesh, known as NEFMAC. All tested beams failed either in compression or shear. They behaved similar to over-reinforced beams with mild steel reinforcement. The deflections of these beams after cracking were almost twice the deflections of steel reinforced beams. Comparison of deflections computed according to ACI Code and the experimental deflections was performed. After cracking occurred, the calculated deflections were only half of experimental deflections. Thus the use of ACI equations would underestimate deflections throughout the whole range of loading after cracking.

Glass fibre rebars are known for their good bond properties. A study of development length of GFRP reinforced beams was reported by Daniali (1992). Due to the low modulus of elasticity of GFRP rebars, serviceability of the beams was affected by large deflections. In beams reinforced with larger profile (8 mm) GFRP rebars, bond failure occurred.

FRP reinforcement has excellent corrosion resistance, however, it is seldom used for

reinforcing concrete beams because of its small modulus of elasticity, which causes deflections after cracking to be large. Even though this is a drawback for ordinary reinforced concrete, for prestressed concrete this seems to be an advantage because prestressing losses are smaller. The stress-strain behaviour of FRP rebars is very similar to prestressing tendons. In combination with its excellent corrosion resistance, it is the main reason for the broad research efforts in this area of FRP application.

Naaman et al. (1992) tested partially prestressed concrete beams using carbon fibre composite strands, known under the trade mark CFCC. Two T-beams were designed and the beams were tested under third-point loading to study their flexural behaviour and strength. The following parameters were investigated: curvature, load-deflection characteristics, cracking pattern, strain in non-prestressed reinforcement at the critical section, ultimate strength, and slip of the strands. The load-deflection response of the beams was found to be trilinear with decreasing slope. The conventional methods of analysis were used for partially prestressed beams and yielded satisfactory results. In deflection calculations conventional beam theory underestimated deflections of prestressed concrete beams.

Sumitomo Construction Co., Ltd of Japan is the producer of AFRP rods with trade mark name Technora. In 1992 they reported several tests in order to investigate the bond and bending behaviour of beams pre-tensioned with Technora aramid rods (Sumitomo 1992). Three types of tests were conducted: bonding length test, bending fatigue test, and static loading test. Beams used for these tests were 2 m long, and prestressed solely with Technora rods. This fact has to be pointed out because research is mostly focusing on combination of FRP tendons and steel reinforcement in the form of stirrups (Naaman 1992). The main reason for this is to avoid catastrophic failure of the specimens. Two different series of test were performed, with different prestressing forces. The reported failure mode was compressive failure of the concrete. In addition to the above mentioned tests one large scale test was performed. The beam tested was post-tensioned, and it underwent both static loading and bending fatigue test. The beam was 10.5 m long, and was reinforced with a bundle of nineteen AFRP

rods of 6 mm diameter with a combined tensile strength of 80 tons. Sumitomo report shows that the experimental values of ultimate moment and deflection at ultimate load were predicted by simple calculations with very high accuracy.

The largest number of experimental work has been conducted in the area of strengthening steel reinforced concrete beams (Ritchie et al. 1984, Meier et al. 1989, McKenna and Erki 1994, Tonen Corp. 1993). This application utilizes all the advantages of FRP material, including light weight, non-corrosiveness and high tensile strength. Strengthening of under-strengthened or damaged structures by means of externally bonded steel plates is a well established practice. However, steel plates used for this purpose are very heavy and difficult to handle, and the selfweight of a structure repaired in this manner can increase substantially. Meier et al. (1989) reported that 94 kg of steel plates for strengthening of one particular structure were replaced by 4.5 kg of CFRP laminate. This reduced the weight of repaired structure, as well as it reduced labour cost for repair works, and started a new era in strengthening of structures.

Meier and Kaiser (1992) studied the flexural behaviour of twenty six beams with 2 m span, and one beam with 7 m span. These beams were reinforced with steel bars and later strengthened with CFRP laminate, bonded by means of epoxy to the bottom surface of the beams. One conclusion drawn from these tests, which was also confirmed by others (McKenna 1992, Tonen 1992), is that the application of FRP sheets has to be performed very carefully because peeling-off of the strengthening sheet might lead to premature failure of the beams. The following failure modes were observed in the latter tests:

- tensile failure of the CFRP laminate, which occurred suddenly
- failure of concrete in the compression zone of the beam
- peeling-off of the laminate due to uneven surface of the beam
- peeling-off of the laminate due to shear cracking which causes relative vertical

displacement of the beam

Also the prestressing of the sheets was mentioned, as a significant contribution towards the improvement of serviceability of structures.

Tonen Corporation of Japan (1992), which is producer of a carbon fibre sheet, known as FORCA TOW SHEET (FTS), conducted tests on twelve concrete beams strengthened with one, two or three plies of FTS. Three types of FTS were tested: FTS-C1-30, FTS-C4-30, and FTS-C5-30. They have different tensile strengths, ranging from 1.8 GPa to 3.5 GPa. Most of the tested beams underwent premature failure due to peel-off. Failure of the beams strengthened with one or three plies of FTS-C4-30 was reported to be due to rupture of FTS. Two reference beams were tested, both of them did not have strengthening sheets, and they failed in compression. The sheets were bonded either on the bottom of the tested beams, or they were bonded to the sides and bottom of the beams. In some beams, the ends of the sheets were clamped to the bottom of the beams. A substantial increase in ultimate load (up to 280 %) for the beam with FTS on all three sides was found. Overall the results of the experiments indicated the advantage of using several plies of FTS to increase the ultimate strength compared to one or two plies. It has to be mentioned though that the ductility of the beams decreased and the problem of debonding of the sheets was not fully resolved.

Results from tests performed on slabs reinforced with FRPs are rarely reported in the literature. One of the few reported testing of slabs reinforced with FRP is that by Kawakami et al (1992). They tested silica fume mortar slabs reinforced with either AFRP or CFRP meshes. The combination of very high strength and very high durability silica fume mortar with high strength reinforcement is very interesting from the point of view of developing high strength and durable structural members. A total of twelve FRP reinforced slabs and two steel reinforced slabs as reference specimens were tested. Particular attention was paid to the occurrence of cracking. It was found that the cracking load was not strongly affected by FRP content, but this is expected because the initiation of cracking is affected mainly by the tensile strength

of concrete. The effect of FRP reinforcement was remarkable after cracking took place. CFRP slabs were found to have more brittle response than AFRP reinforced slabs. The ultimate load increased with the increase of FRP content. After cracking, deflections of CFRP reinforced slabs were twice the deflections of the corresponding steel reinforced slabs. Deflections of AFRP reinforced slabs were three times the steel reinforced slab deflections. Deflections decreased with increase in the amount of FRP reinforcement in the cross section.

Schmeckpeper (1992) tested the performance of slabs reinforced with NEFMAC grid. All slabs exceeded the design load before failure. Failure was brittle as indicated by the linear load deflection relationship. Reported failure was due to diagonal shear-tension or due to concrete compression failure.

Most of the applications of FRP discussed so far have been either for prestressing or strengthening purposes. The main reason for this is the relatively low modulus of elasticity of FRP, which is expected to cause large deflections (Faza 1991, Schmeckpeper 1992). More testing of slabs reinforced with FRP is needed because as shown by the work of Kawakami et al. (1992) deflection problems can be diminished by increasing the FRP reinforcement ratio. Nevertheless, ductility of FRP reinforced members is low, and little has been reported about procedures to increase ductility.

2.3.5 Case Studies of FRP Reinforced Structures

FRPs can be used either as reinforcement in concrete to replace steel and/or prestressing tendons, or very often they are used to strengthen existing structures. The majority of applications to date have been in bridges and bridge related structures. Bridges reinforced with FRP tendons have been built in Canada, Japan, Germany, and the USA.

Prestressed concrete bridges have been built with FRP tendons, including glass (Ulenbergstrasse road Bridge, Düsseldorf), carbon (Beddington Trail Bridge, Calgary), and

aramid (Sumitomo Road Bridge, Japan). Although aramid tendons have been used, there is no indication of problems with the fact that aramid has a negative thermal expansion coefficient. Another application of prestressed concrete was for a floor slab, where prestressing tendons were made of aramid fibres (Sugita et al. 1993). In this case also no problems with deflections were reported. It has to be mentioned that the spans used for this floor slab were relatively short.

Further information on existing structures reinforced with FRP can be found in a report by Strabag Bau (1992). Some applications include structures which have been functioning without problems for more than 10 years. Among them is the Ulenbergstrasse road bridge in Düsseldorf, which was prestressed with Polystall glass fibre tendons and is reported to behave normally under everyday traffic. Several pedestrian bridges, antenna systems, and many other structures have also been built without any reported problems.

Future research needs to investigate the possibility of reinforcing two-way slabs with FRP reinforcement due to the wide range applications of two-way slabs in buildings and other structures. More information on existing applications of FRP in concrete structures can be found in Razaqpur and Kashif (1992), JSCE (1993), and McKenna and Erki (1994). Most of these applications are important but are not directly related to the subject matter of the present research.

2.4 Design Requirements for Two-Way Slab Systems

CSA Standard A23.3-M84 contains specific provisions for the design of two-way slab systems in Chapter 13 (Two-Way Slab Systems), but also the requirements of Chapter 9 (Strength and Serviceability Requirements) should be satisfied when designing two-way slabs.

Two-way slab systems have different behaviour than one-way systems. The actions in longitudinal direction are coupled with those in the transverse direction, therefore capturing the behaviour of two-way systems is more complex. First of all structural design has to meet serviceability requirements. These are associated with performance of structures under specified service loads. Among these requirements are limited deflections and small vibrations. Only deflections will be discussed further because dynamic analysis is beyond the scope of this thesis.

Deflections depend on different factors, i.e. support conditions, and the overall layout of structure, and load level. For FRP reinforced structures, cracking appears to be an even greater contributor to deflections than in steel reinforced concrete because of the relatively low modulus of elasticity of most FRPs. Therefore pre- and post-cracking behaviour of FRP reinforced structures has been a problem of interest for researchers as mentioned earlier. This thesis is dealing with deflection control according to CSA Standard A23.3-M84 which uses span to depth ratio for controlling deflections of structural elements. However, these empirical limits have been established for steel-reinforced concrete, therefore it is one of the objectives of the present investigation to establish whether they can be applied to FRP reinforced concrete slabs.

2.4.1 Slab Thickness

Providing a sufficient thickness for a structural member is the first step for deflection control. The minimum thicknesses, or span/depth ratios specified in the Code were established based on experience gained from the design of ordinary reinforced and prestressed concrete structures. The same provisions will be used throughout this thesis and their applicability for FRP reinforced structures will be investigated.

In the case of two-way constructions, supported on walls or stiff beams, and/or columns, Clause 9.5.3 of CSA Standard A23.3-M84 applies. Calculated thickness for two-way slabs depends on their support conditions, namely whether there are

beams between supports or not. In the structures studied in this investigation no beams existed between supports, therefore provisions of Section 9.5.3.3 of CSA Standard A23.3-M84 were used. Clause 9.5.3.3.1 states that if the thickness, h , of a two-way slab is larger or equal to the thickness calculated from Eq. 2.1 below, the actual deflections of the slab need not to be calculated

$$h = \frac{l_n (800 + \frac{f_y}{1.5})}{36\,000} \quad (2.1)$$

where h = overall thickness of slab in millimetres

l_n = length in mm of clear span in long direction of two-way construction

f_y = specified yield strength of reinforcement in MPa

We notice that the thickness of the slab is dependent on the yield strength of reinforcement. When reinforcement with high ultimate strength such as FRP is used, Eq. 2.1 will lead to a rather large thickness. Therefore, the possibility of using the same thickness as a steel reinforced slab with $f_y = 400$ MPa will be investigated, and deflections of such systems will be compared to the limiting values given in the Code.

2.4.2 Design for Shear in Two-Way Slabs

Shear stresses in the vicinity of columns or other concentrated loads in flat plates/slabs are very high. Therefore, they usually govern the design of slab thickness. Two cases of shear failure may usually occur, namely beam shear, and punching shear. The more critical of those two would govern the design. The Code requirements for shear design in flat plate/slab structures are empirical and have been arrived at from extensive experimental data. The experiments have been conducted on steel reinforced concrete models. Since similar experiments have not been performed on FRP reinforced concrete, we will assume that the current provisions apply to FRP reinforced concrete. Such an assumption will be acceptable because

- the present study is primarily concerned with the bending or flexural behaviour of FRP reinforced slabs
- shear reinforcement is not commonly used in flat plate structures.

For completeness, the provisions of the code related to shear design of flat plate/slab structures are reviewed.

Beam Shear

In beam action slab fails across the entire width of the slab in the critical section like a beam (Park and Gamble 1980). The shear strength is usually checked by taking strips one metre wide in both directions. The critical section is considered to be at a distance d , which is effective slab depth, from the face of the support. The factored shear, V_f is given by (Pillai and Kirk 1988)

$$V_f = w_f \times 1 \times \left(\frac{l_n}{2} - d \right) \quad (2.2)$$

where w_f = factored load per unit area of the slab

l_n = clear span in transverse direction

d = effective depth

and must not exceed V_c given by the following equation

$$V_c = 0.2 \lambda \phi_c \sqrt{f'_c} b_w d \quad (2.3)$$

where V_c = shear carried by concrete in N

λ = factor to account for low density concrete

ϕ_c = material resistance factor

f'_c = compressive strength of concrete in MPa

b_w = web width in millimetres

d = effective depth in millimetres

It is not common practice to provide shear reinforcement in the form of stirrups in a slab. Consequently, the slab thickness is chosen such that it would resist the total applied beam shear. In view of this practice, the design of slabs for beam shear should not be affected by the change in the type of reinforcement material. But V_c needs to be carefully considered for FRP reinforced slabs because it is assumed to include shear transfer by other mechanism such as aggregate interlock and dowel action, and it is not known whether FRP reinforcement can mobilize those mechanisms.

Punching Shear

The punching shear failure is described as a failure in the local area around a concentrated load. A mass of concrete around the load separates from the slab in the shape of a cone. The consequences of such failure are serious because the load from the failed area is shifted to other areas of a flat slab, resulting in increased deflection and possible failure (Park and Gamble 1980). The punching shear is primarily affected by the following quantities: $\sqrt{f'_c}$, the ratio of the loaded area to the effective depth of the slab, c / d , the ratio of the side lengths of the loaded area (ASCE-ACI Committee 426 1974). According to Park and Gamble (1980), punching shear failure is independent of the steel index. However, the arrangement of steel in the column region is important to avoid catastrophic consequences of punching shear failure.

For calculation of the punching shear resistance, the critical section is located at a distance $d / 2$ from the periphery of the loaded area. The following equation is used to find the factored shear resistance of concrete in two-way action (Pillai and Kirk 1988)

$$V_c = 0.4 \lambda \phi_c \sqrt{f'_c} b_o d \quad (2.4)$$

where b_o = the perimeter of a critical section assumed located at distance $d/2$, and the periphery of loaded area = $2(c_1 + c_2 + 2d)$ for a column with cross section dimensions c_1 and c_2 . The factored shear force V_f is computed as the total downward load acting on an area around the column, and must be smaller than the factored shear resistance given by Eq. 2.4, unless shear reinforcement is provided. Because of the extensive amount of reinforcement in the column regions, and/or ease of construction, there is tendency to avoid provision of shear reinforcement in two-way slabs. If necessary drop panels are provided to reduce the applied stresses. In the design of parking structures in this thesis, shear reinforcement was avoided by providing drop panels because it was believed that the reinforcement for punching shear may be subjected to dowel action but there is not enough experimental data about the shear transfer capability of FRP bars by dowel action.

Shear Due to Transfer of Moments

Apart from a direct shear, there is also a portion of unbalanced moments at the connection of columns and slab which is being transferred by the eccentricity of shear about the centroid of critical section resisting the punching shear. This portion of the unbalanced moment is given in CSA Standard A23.3-M84 in Clause 13.3.3.3 as

$$\gamma_f = \frac{1}{1 + \frac{2}{3} \sqrt{\frac{c_1 + d}{c_2 + d}}} \quad (2.5)$$

where γ_f = fraction of unbalanced moment transferred by flexure at slab-column connections

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined in millimetres

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction moments are being determined in millimetres

d = distance from extreme compression fibre to centroid of tension reinforcement in millimetres.

The shear stress distribution at the critical section is then obtained by summation of stresses from the factored shear force and the unbalanced moment. It will be assumed that Eq. 2.5 applies to FRP reinforced slabs in part because none of the parameters within it is a function of the type or strength of the reinforcement.

2.4.3 Current Flexural Design Methods for Two-Way Slab Systems

In order to simplify the design of two-way slab systems supported on columns, CSA Standard A23.3-M84 permits the modelling of the actual three dimensional frame system as an assemblage of a system of orthogonal frames along the different column lines. The beams joining the columns are assumed to be wide beams of width equal to the distance between the centerlines of two parallel panels adjoining the given column line. There are two methods of analysis permitted by CSA A23.3-M84, the Direct Design Method (DDM), and the Equivalent Frame Method (EFM). They differ mostly in the way moments are being calculated. Full details of these methods may be found in any standard textbook on concrete design, as for example Pillai and Kirk (1988), MacGregor (1988). Details of the design procedures can be also found in Appendix A of this thesis.

Chapter 3

Nonlinear Analysis of FRP Reinforced Concrete Structures

3.1 General

In this chapter the finite element program NONLACS (NONLinear Analysis of Concrete and Steel), developed by Razaqpur and Nofal (1988) will be described as well as its ability to predict over the whole loading range the behaviour of structural concrete elements reinforced with FRP reinforcement. The applicability of this program to the analysis of FRP reinforced concrete structures will be established by analyzing a number of previously tested beams and slabs reinforced with FRP. The results of these analyses in the form of load-deflection curves, stresses and strains in concrete and FRP, will be compared with the corresponding experimental data. Where convenient the predicted behaviour according to conventional beam theory will also be presented.

3.2 Computer Program NONLACS

The program NONLACS was built on the program FELARC (Finite Element Layered Analysis of Reinforced Concrete), originally developed by Ghoneim and Ghali (1978). FELARC was enhanced by Razaqpur (1982), and Nofal (1988) adding new finite elements, shear connector element, and the capability to analyze steel structures and structures made of fibre reinforced plastics. In addition, new finite elements were added to model plane, plate bending and shell elements. As discussed by the authors of NONLACS, this program can be used for the analysis of any reinforced concrete, steel, composite concrete-steel, and FRP composite structures. The program has already been verified for the analysis of all the abovementioned types of structures. In the present work, verification of the applicability of the program for FRP reinforced concrete structures will be presented.

NONLACS contains membrane, plate bending, facet shell, axial bar, and shear connector elements. Only those elements that have been used for the analyses used in this thesis will be discussed. NONLACS accounts for material nonlinearity and nonlinearity due to a nonlinear shear stress-slip response at interfaces. For the analysis of structures based on theory of plasticity, any of the common yield criteria, i.e. Mohr-Coulomb, Von Misses, Tresca, or Druckler-Prager, may be utilized to conduct the full plastic analysis. In the case of FRP structures, the Tsai-Wu failure criterion is applied (Tsai and Wu 1971). The constitutive laws for concrete that are used in this program will be described in subsequent sections.

NONLACS employs an incremental-iterative technique to solve the system of nonlinear equations. Note, however, that this program cannot capture geometric nonlinearities, fatigue, and change in the bond stress between two surfaces, except by means of shear connectors joining interfaces.

3.2.1 The Finite Elements Involved in Analysis

NONLACS includes a two node bar element, a two node shear connector element, a four node plane quadrilateral element, a four node plate bending quadrilateral element, and a four node shell element obtained by combining plate bending and membrane elements. All these elements are presented by Nofal (1988). In all the cases analyzed in this thesis the facet shell element was used, which was obtained from the combination of a Refined QUADrilateral membrane element (RQUAD4) and an Improved Discrete Kirchhoff Quadrilateral plate bending element (IDKQ). Hence, only these three elements will be briefly described here.

Refined Quadrilateral Membrane Element

This element was originally developed by Razaqpur, and implemented by Nofal to analyze a series of structures. The element was found to provide highly accurate results for different types of stress fields (Razaqpur and Nofal 1990).

RQUAD4 has four corner nodes and three degrees of freedom per every node, namely two inplane displacements and one drilling degree of freedom (Figure 3.1 a). Its displacement functions were derived from the displacement functions of the eight node isoparametric element QUAD8. Its nodal displacements in a local coordinate system have quadratic variation normal to the element sides and linear variation parallel to the same side. The drilling degree of freedom is a quasi-rotation which gives the relative rotation between the two intersecting sides of the quadrilateral.

Quadrilateral Plate Bending Element

The improved discrete Kirchhoff quadrilateral plate bending element IDKQ, was developed by Chinniah and Kirkhope (1985). This element has four nodes and three degrees of freedom per node. These are two normal rotations θ_x , θ_y , and displacement w (see Figure 3.1 b). This element is derived from the eight node Mindlin plate bending element. By imposing the Kirchhoff hypothesis of zero shear at some nodes,

Figure 3.1: Elements (a) RQUAD4, and (b) IDKQ with their typical nodal degrees of freedom.

the number of degrees of freedom are reduced to 12. The element is highly accurate and was found to yield very good accuracy in many different structures (Chinniah 1988, Aziz 1990).

Quadrilateral Facet Shell Element

The facet shell element is obtained by combining the RQUAD4 and the IDKQ elements. It is an anisotropic shell, where inplane and bending actions are coupled. It has four nodes, which lie on the so-called reference surface, and six degrees of freedom

per node, as indicated in Figure 3.2. The element is divided through its thickness into several layers, and every layer is assumed to be in a state of plane stress. For plate bending problems, five to eight layers are generally sufficient to capture the stress gradient through the element thickness while for plane or membrane problems only one layer suffices. The layers can consist of different materials, and each layer may be reinforced with steel and/or fibre reinforced plastic reinforcement. Within a given layer the material properties may change, depending on the level and state of stress at the selected location. Finally, because the middle plane may not always be a plane of material symmetry, the shell element includes the coupling between inplane and plate bending actions.

3.2.2 Material Modelling

In the following section, the constitutive models for concrete, reinforcing steel, and FRP reinforcement that are employed in program NONLACS will be briefly described.

Concrete Model

Concrete is modelled as stress-induced orthotropic material with its stress-strain relationship in each direction shown in Figure 3.3. The curve in Figure 3.3 consists of two parts, Part I is represented by Saenz's equation (1964)

$$\sigma = \frac{E_o \epsilon}{1 + (\frac{E_o}{E_{sc}} - 2) \frac{\epsilon}{\epsilon_{cu}} + (\frac{\epsilon}{\epsilon_{cu}})^2} \quad (3.1)$$

The Part II is based on Smith-Young's equation

$$\sigma = \sigma_c \left(\frac{\epsilon}{\epsilon_{cu}} \right) e^{[1 - (\frac{\epsilon}{\epsilon_{cu}})]} \quad (3.2)$$

Note that Part I describes the ascending branch of the curve while Part II describes the descending or strain softening part. In the above equations and by reference to

Figure 3.2: Facet shell element with its typical degrees of freedom.

Figure 3.3: Uniaxial Compressive and tensile stress-strain relationship for concrete

Figure 3.3

E_o = initial tangent modulus

E_{sc} = secant modulus of elasticity at peak stress

ϵ = strain

ϵ_{cu} = strain at peak stress

σ = stress

σ_c = compressive strength

e = base of natural logarithm

It should be pointed out that σ_c is not necessarily equal to the uniaxial concrete strength, f'_c . The value of σ_c will depend on f'_c and on the ratio of the normal stresses

in the principal material directions. The actual strength in each of the two directions is established by using the preceding stress ratio in conjunction with the Kupfer and Gerstle (1973) failure criterion for concrete. Details of this procedure are given by Ghoneim (1978). The constitutive matrix $[D]$ is that given by Darwin and Pecknold (1977).

$$[D] = \frac{1}{1 - \nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1 E_2} & 0 \\ \nu\sqrt{E_1 E_2} & E_2 & 0 \\ 0 & 0 & 1/4 (E_1 - E_2 - 2\nu\sqrt{E_1 E_2}) \end{bmatrix} \quad (3.3)$$

where E_1 , and E_2 are the tangent moduli in the principal material directions, and ν is the Poisson's ratio. The moduli in principal directions are found from the stress-strain relationship

$$E_i = \frac{d\sigma_i}{d\epsilon_{iu}} = \frac{E_o(1 - r_i^2)}{[1 + (\frac{E_o}{E_i} - 2)r_i + r_i^2]^2} \quad (3.4)$$

where

$$r_i = \frac{\epsilon_{iu}}{\epsilon_{ic}} \quad \epsilon_{iu} = \sum \Delta\epsilon_{iu} = \sum \frac{\Delta\epsilon_i}{1 - \nu\alpha n}$$

where $\Delta\epsilon_i$ is the actual increment of strain in direction i , $\alpha = \frac{\sigma_i}{\sigma_j}$, and $n = \frac{E_i}{E_j}$, with j being a conjugate direction to i . Knowing the stress-strain relationship, and the constitutive matrix $[D]$, the relationship between the increments of stresses and strains can be established

$$\{\Delta\sigma\} = [D]\{\Delta\epsilon\} \quad (3.5)$$

For uncracked concrete the principal material directions coincide with the principal stress directions, and the constitutive matrix has the form shown in Equation 3.3. For cracked concrete these directions are parallel and normal to the direction of

cracks. The current state of concrete is checked by determining the magnitude of the principal stresses at Gauss integration points within each layer. When the principal stress equals or exceeds the tensile strength of concrete, cracking is assumed in the perpendicular direction to that stress, and in that case, Eq. 3.3 can no longer be used, and therefore the constitutive matrix is reduced to

$$[D] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \beta G \end{bmatrix} \quad (3.6)$$

where G is modulus of elasticity in shear, and β is a so-called shear retention factor with a value less than one usually taken as 0.1. The shear retention factor is used to simulate shear transfer across cracked surfaces. Although the phenomenon of shear transfer in cracked concrete is rather complex in the case of reinforced concrete, the problem can be reasonably approximated by this device of a reduced shear modulus as originally suggested by Suidan and Schnobrich (1973).

After the occurrence of a crack, concrete is assumed to have zero stiffness normal to the direction of the crack, but it is still capable to transfer tension between cracks. This effect is considered by including a descending branch in the bilinear stress-strain relationship for concrete in tension, Fig. 3.3. This, so-called tension-stiffening effect plays a major role in the calculation of deflections after cracking, as will be seen from the results in Section 3.3. The slope of the descending branch of this relationship can vary, depending on the modulus of elasticity of concrete and the tensile strength of concrete. The best results were achieved, using $10 \times \frac{f_t}{E_c}$.

The necessary input for the program is concrete strength in compression, the tensile strength of concrete, its initial modulus of elasticity, the strain corresponding to the maximum stress in compression, the ultimate strain in compression, the ultimate strain in tension, the shear constant, and the Poisson's ratio. In the cases when the self weight of the structure is an important factor, its unit weight has to be input.

Figure 3.4: Stress-strain relationship for steel

Steel Reinforcement

Reinforcing steel consisting of mild steel bars or prestressing tendons is modelled as elasto-plastic strain-hardening material. The stress-strain relationship used is illustrated in Figure 3.4. The basic input needed to describe that relationship is the yield stress, the Young's modulus, the strain-hardening modulus, and the ultimate strain. In the case of unloading and reloading, the Bauschinger's effect, i.e. the reduction in yield strength, is considered. In the analyses conducted in this thesis, full bond between steel and concrete interface was assumed.

Reinforcing bars are modelled as either element bars, or as smeared layers. Every

facet shell element can have up to four element bars. They can be treated as either reinforcing bars, or prestressing bars, once an initial stress/strain is specified. The smeared layers are specified by the coordinate of the middle of the layer, and their thickness. The thickness of the layer depends on the area and spacing of the reinforcement. This option of modelling reinforcement is usually used when dealing with slab reinforcement, or stirrups for beams. The option of a smeared reinforcement layer simplifies the input data and the amount of calculations needed. Note that reinforcing layers can carry only normal stresses in the directions of the reinforcement.

FRP Reinforcement

As discussed in Chapter 2, fibre reinforced plastic materials display linear elastic behaviour over the full stress range up to failure. Therefore, a simple linear-elastic stress-strain relationship was used for FRP reinforcement as shown in Figure 3.5. Since FRP reinforcement cannot carry significant compression, only the tensile behaviour was modelled. For unloading and reloading, the same linear-elastic relationship was used because FRP materials do not have a yielding plateau. Similarly as for steel reinforcement, full bond was assumed at the interface between FRP and concrete. The basic input is FRP modulus of elasticity, ultimate stress, and ultimate strain. As mentioned in the section describing steel reinforcement, FRP bars can be treated as element bars, or as smeared layers, and they may be either regular reinforcement or prestressed reinforcement.

3.3 Verification of Program NONLACS

A total of eight members, which had been previously tested in the laboratory by other investigators, were analyzed by NONLACS to determine its suitability for predicting the behaviour of structural elements reinforced with FRP reinforcement over the whole range of loading up to failure. These members include beams reinforced with FRP flexural reinforcement, beams strengthened with FRP sheets, beams prestressed

Figure 3.5: Stress-strain relationship for FRP

with FRP tendons, and slabs reinforced with FRP grids. The geometry, loading arrangements, material properties, as well as information about modelling of the analyzed members are given in the following sections.

3.3.1 Beams Reinforced with FRP Rebars

Faza and GangaRao (1991) tested twenty five rectangular beams to study the pre- and post-cracking behaviour of concrete beams reinforced with glass fibre bars. The beams were simply supported, approximately 3 m long, reinforced with glass-fibre bars, and subjected to four point bending (see Table 3.1). One of their beams, B # 10, was arbitrarily selected for the purpose of analysis here. The necessary material properties, given by the original investigators, can be found in the Table 3.2. The various symbols in the table signify the following quantities: for concrete, f'_c = compressive strength, f'_t = tensile strength, E_c = elastic modulus, ϵ_{cu} = ultimate strain, ν_c = Poisson's ratio; for steel, f_y = yield stress, E_s = elastic modulus, ϵ_{ult} = ultimate strain; for FRP, f_{FRP} = tensile strength, E_{FRP} = elastic modulus, and ϵ_{FRP} = ultimate strain. It may be noted that in many cases all the relevant material properties were not reported, hence where needed they were assumed based on known

values for the same type of material. Also, the notation N.A. (Not Applicable) was used where the particular parameter was not relevant, and the notation N.S. (Not Specified) was used when the necessary information was not given by the original investigators.

In the finite element modelling, only half of the beam was modelled. The finite element mesh included two elements per height, and ten elements per half span (see Figure 3.6a); finer mesh yielded the same results as the selected mesh. This mesh is typical for all the beams analyzed in this thesis. For all the analyses to be discussed in this chapter, the facet shell element was used. Only one concrete layer was used through the thickness while the flexural reinforcement was modelled as element bars, and the shear reinforcement as a smeared layer. Full bond was assumed between the reinforcement and concrete. In all the analyses in this chapter the shear-retention factor $\beta = 0.1$, and the maximum concrete strain in tension was assumed to be $10 \times \epsilon_{cr}$, where ϵ_{cr} is the cracking strain.

Comparison of Results

Only experimental load-deflection curves were supplied by the original investigators, therefore deflections will be considered. Load-deflection curves from NONLACS are compared in Fig. 3.7 to those calculated using the beam theory and to the corresponding experimental curves. The beam theory results were obtained from moment curvature relationship, using strain compatibility without tension stiffening.

It can be observed in the latter figure that the finite element results agree closely with the experimental data over the full range of loading in the experiment. On the other hand, the beam theory predicts excessive loss of stiffness following first cracking. The latter is mostly due to lack of tension stiffening in the beam model. Note that both the finite element method and the beam theory predict higher strength than the observed experimental value. The ultimate load from experiment was reported to be 30. kN, while NONLACS predicted it to be 36 kN, and the beam theory estimated it to be 38 kN. Converting these results into percentage of difference, with

Table 3.1: Geometry and Reinforcement of Beams Analyzed

Source	Elevation	Cross-Section
Faza and Gangarao (1992)		305 152 2 #3 FRP bars
Meier and Kaiser (1992)		150 250 4 #8 steel bars 0.3 x 200 mm CFRP laminate
Tonen Corporation (1993)		400 300 0.165 x 300 400 FTS (1 ply or 3 plies) 5 #13 steel bars
Kawakami (1992)		50 200 ϕ 3 / 60 mm CFRP, AFRP
Naaman (1992)		304.5 304.5 110.25 2-CFCC 1x7 2 #6
Sumitomo Corp. (1992)		200 100 6 #6 AFRP rods

Table 3.2: Material Properties of Analyzed Beams

Parameters [MPa]			Beam/Slab Designation							
			Faza	TONEN		Meier	Sumitomo	Naaman	Kawakami	
			B#10	9	10	B8	PCB	TC9	CFRP	AFRP
Published Parameters	Concrete	f'_c	34.47	27.0	27.0	42.8	68.8	42.0	106.6	106.6
		f_t	N.S.	N.S.	N.S.	N.S.	4.84	N.S.	7.61	7.61
		E_c	N.S.	N.S.	N.S.	N.S.	37 600.0	N.S.	33 400.0	33 400.0
	Reinforcement	f_y	N.A.	400.0	400.0	530.0	N.A.	400.0	N.A.	N.A.
		E_s	N.A.	209 000.0	209 000.0	210 000.0	N.A.	200 000.0	N.A.	N.A.
		ϵ_{ult}	N.A.	0.008	0.008	0.008	N.A.	0.008	N.A.	N.A.
		f_{FRP}	N.S.	2 980.0	2 980.0	963.0	5 090.0	2 092.0	1 226.0	1 324.0
		E_{FRP}	48 200.0	377 000.0	377 000.0	80 235.0	54 000.0	144 000.0	117 720.0	58 860.0
		ϵ_{FRP}	N.S.	N.S.	N.S.	0.0115	0.036	N.S.	0.011	0.022
Parameters Used for the Analysis	Concrete	f'_c	34.47	27.0	27.0	42.8	67.5	42.0	106.6	106.6
		f_t	2.00	2.50	2.50	2.70	3.0	3.82	6.20	6.20
		E_c	25 000.0	25 000.0	25 000.0	22 000.0	30 000.0	28 500.0	30 000.0	30 000.0
		ϵ_{cu}	0.003	0.003	0.003	0.003	0.0025	0.01	0.004	0.004
	Reinforcement	f_y	N.A.	400.0	400.0	530.0	N.A.	400.0	N.A.	N.A.
		E_s	N.A.	200 000.0	200 000.0	210 000.0	N.A.	200 000.0	N.A.	N.A.
		ϵ_{ult}	N.A.	0.008	0.008	0.008	N.A.	0.008	N.A.	N.A.
		f_{FRP}	490.0	2 980.0	2 980.0	963.0	2 000.0	2 092.0	1220.0	1324.0
		E_{FRP}	48 200.0	377 000.0	377 000.0	80 235.0	54 000.0	144 000.0	117 720.0	58 860.0
ϵ_{FRP}		0.01	0.0079	0.0079	0.012	0.037	0.015	0.01	0.022

Figure 3.6: Typical Finite Element Meshes for Analyzed Beams and Slabs

Figure 3.7: Load-Deflection Curve for Faza's Beam # 10

respect to the experimental results, NONLACS gave 19 %, and the beam theory 26 % higher ultimate load. The difference in ultimate load may be due either to loss of bond at high level of load or due to the effect of bending on the tensile strength of glass fibre rods. More experimental studies are needed to explain the departure of behaviour of concrete beams reinforced with FRP materials from the behaviour of conventional steel reinforced beams, and to establish the range of applicability of conventional methods of analysis to FRP reinforced concrete structures. It is significant to mention that the finite element method provides results that agree closely with the experimental data up to failure.

3.3.2 Beams Strengthened with Carbon Fibre Sheets

The TONEN Corporation of Japan (Tonen Corporation 1992) has been conducting research on concrete beams reinforced with proprietary FRP sheets, known by the trademark name of FORCA TOW SHEET. The sheets are unidirectional and are made of glass or carbon fibres. One of the anticipated applications of those sheets is in the area of strengthening and/or rehabilitation of existing reinforced concrete structures. Thus the TONEN experimental program involved a number of conventionally reinforced concrete beams strengthened by one or more plies of the sheet, epoxy bonded to the bottom of the beam. In the particular beams selected for the present analysis, the sheets were made of carbon fibres. Two beams were selected for analysis, designated as beam # 9 and beam # 10 by TONEN, with the beam # 9 being reinforced with one ply and the beam # 10 with three plies of CFRP sheet. Both beams were simply supported, had rectangular cross-sections, and were loaded in four-point bending. All material properties as supplied by the TONEN Corporation, are included in the Table 3.2, while the elevation and the cross-section of the beams can be seen in Table 3.1.

Meier and Kaiser (1992) also tested twenty six reinforced concrete beams strengthened with epoxy bonded 0.3 to 1.0 mm thick CFRP unidirectional laminates. One of

those beams was selected for analysis here and it is illustrated in Table 3.1, with the corresponding material properties listed in Table 3.2. The CFRP laminate was sand-blasted and the concrete surface was gridblasted to ensure sufficient bond between the sheet and the beam. Experimental work in the post-strengthening area of concrete structures has shown that it is very important to apply the strengthening sheets carefully and to assure that the concrete surface is even so that sufficient bond will be developed. Failure to do so leads to sheet peel off during loading, causing premature failure of the beam.

Both the Tonen and the Meier-Kaiser beams had conventional steel reinforcing bars as shown in Table 3.1. In the analysis, the CFRP sheet was modelled as uniaxial bar capable of carrying only tension, with full bond throughout the whole length of the beam and over the whole range of loading. Shear reinforcement was simulated by a smeared layer. In the case of Meier's beam, information regarding the properties of epoxy used to bond the CFRP sheet to concrete was available, therefore the epoxy was simulated as a smeared layer of thickness 2 mm. This layer was assumed to carry tension only and the epoxy was assumed to be linear elastic with its elastic modulus equal to 6 200.0 MPa, and its strength equal to 24 MPa.

Comparison of Results

Figure 3.8 shows the load-midspan deflection curve of the Tonen beam reinforced with only one ply of CFRP sheet, while Figure 3.9 depicts the strain variation in the CFRP sheet at midspan of the same beam. We notice rather close agreement between the experimentally measured deflections and the corresponding finite element predictions. The beam theory again overestimates the deflections because tension stiffening is not considered. For this beam, the predicted ultimate load is 184 kN, compared to the experimental value of 180 kN. If the service load for this beam is assumed to be 80 kN to 100 kN, then the predicted and measured deflections at that load level would differ by less than 10 %. In Fig. 3.9 the strains compare reasonably well albeit the finite element analysis overestimates the strain after first cracking. Although the precise

Figure 3.8: Load-Deflection Curve for Tonen's Beam # 9

cause for this overestimation is difficult to determine without further experimental data, it is conjectured that the presence of the epoxy that bounds the sheet to the soffit of the beam may impart additional tensile strength to the adjoining concrete. Also the presence of epoxy may affect the tension stiffening behaviour of the beam. Further experimental work would be needed to study the behaviour of the steel-epoxy-concrete interfaces. Note, however, that the differences disappear at higher level of loading which implies that tension-stiffening does not play a major role beyond a certain level of stress.

The comparison of experimental load-deflection curves of the Tonen beam # 10 with

Figure 3.9: Load-Strain Relationship for Tonen's Beam # 9

the results obtained from NONLACS and from the beam theory can be seen in Figure 3.10. The finite element results nearly exactly follow the experimental ones, but shows much higher ultimate load than the experimental ultimate load. The reported experimental ultimate load is 255 *kN*, while the finite element predicts a much higher ultimate load of 370 *kN*, i.e. 45 % higher than observed in the experiment. The beam theory predicted the same load to be 363 *kN* (12.3 % higher).

The lower experimental value signifies that most likely delamination of the sheets occurred during the experiment. It is the authors experience with the same sheets that beyond two layers of sheets, failure is governed by the limited interface shear capacity of the concrete. Regrettably, the Tonen report does not give the precise mode of failure. Since in the finite element program full bond is assumed, this mode of failure is not accounted for. One could account for this mode of failure by including shear interface elements in the program.

The delamination assumption is confirmed by the strain measurements for the preceding beam which are illustrated in Fig. 3.11. We notice that the strain in the FRP sheet at failure is less than 0.5 % which is only about 60 % of the ultimate strain of the same material. Before delamination, we can see that the finite element results agree closely with the experimentally measured strains.

Results from both the finite element analysis, and the beam theory are compared in Figure 3.12 to the experimental results of the beam tested by Meier and Kaiser. Very close agreement exists between the finite element results and the corresponding experimental data. The ultimate load from NONLACS was 48 *kN*, representing 6.6 % difference from the experimental value of 45 *kN*. The beam theory predicted the ultimate load to be 38.4 *kN*, which is 14.7 % less than the experimental value. The maximum deflection was predicted well by both theories, being 32.0 *mm*, 32.7 *mm*, and 32.7 *mm* from experiment, NONLACS, and the beam theory, respectively. The importance of tension stiffening can be seen clearly in this example. After cracking occurred, the slope of the load-deflection curve from the beam theory changed sud-

Figure 3.10: Load-Deflection Curve for Tonen's Beam # 10

Figure 3.11: Load-Strain Relationship for Tonen's Beam # 10

Figure 3.12: Load-Deflection Curve for Meier's Beam

denly, and started to follow the path of completely cracked section, whereas the actual beam behaved differently. After cracking starts to take place, conventional beam theory assumes that only the concrete in the compression part of the cross-section is transferring the stresses and the concrete in the tension part of cross-section is neglected. The good agreement for this beam may be attributed to the superior bonding of the laminate to the beam and to the modelling of the epoxy in the finite element analysis. We can notice that if the epoxy is ignored, then the results do not agree as well.

3.3.3 Beams Prestressed with FRP Strands

Sumitomo Construction Co., Ltd. investigated the performance of Technora aramid fibre reinforced plastic rods (AFRP) as tendons for pre-tensioned concrete beams. The investigation included test of bond performance, fatigue test, and two series of static loading test with different prestressing force in each series. One of their beams, tested in the static bending test in series A was selected for the purpose of analysis here. The beam was simply supported, had a span of 2 m, and was tested in four point bending. For details refer to Table 3.1, and Table 3.2, which provide the geometric and material properties, respectively.

Prestressing strands were modelled as element bars. NONLACS has the option to specify bar type as either being reinforcing, or prestressing bar. For the prestressing bar initial stress, or initial strain, in the bar can be specified. The option of specifying initial stress in the bar was used for this beam. The total prestressing force was applied in the first load step, while the applied point loads were applied in twenty load increments. Stirrups were modelled as a smeared layer. This experimental work was one of the few, where only AFRP reinforcement was used.

Test results for simply supported partially prestressed concrete T-beams were reported by Naaman et al (1992). The experimental beams were reinforced with carbon fibre composite strands, known by the trade name CFCC-Carbon Fibre Composite Cable, in addition to the steel reinforcing bars. Steel reinforcement is usually used to ensure that sudden brittle collapse of the beams will not occur. One of their beams designated by the original investigators as TC 9, was analyzed by NONLACS. The beam was loaded in four-point bending, as shown in Table 3.1. The material properties of the same beam are listed in Table 3.2, as reported by the original investigators.

The finite element mesh for this beam is shown in Figure 3.6 (b). Taking advantage of symmetry about two axes, only one quarter of the beam was modelled. The mesh for this beam consisted of seven elements per half of its web, and fourteen elements

per quarter of the top flange. Prestressing tendons were modelled as element bars with initial stress, steel reinforcement was modelled as reinforcing element bars, and stirrups as smeared layers. Prestressing force was applied to the beam in one load step, together with the self weight of the beam. Once again the applied concentrated loads were divided into twenty load steps and were applied incrementally. It should be pointed out that the load increments need not be equal and in the present analysis the size of the increments was gradually reduced as the load increased.

Comparison of Results

Results from the analysis of Sumitomo beam are shown in Figure 3.13. Before the occurrence of the first crack, favourable agreement between the experimental and NONLACS's load-deflection curve was found. After cracking, the theoretical curve shows a slightly stiffer response, but then at a somewhat higher load it agrees closely with the measured response. The experimental ultimate load reported by the investigators was 78.48 *kN* with the corresponding ultimate deflection being 16.5 *mm*. NONLACS predicted failure at 96 *kN*, with the corresponding deflection being 33.7 *mm*, which is 22.3 % higher than the experimental value. The deflection obtained from NONLACS at the load corresponding to the experimental ultimate load was 16.7 *mm*, representing a 1.8 % difference. The investigators reported "compressive failure of concrete", which was also observed in the analysis of results from NONLACS, where concrete started to crush at 56 *kN*. Therefore, it is assumed that due to initial crushing of concrete, the experiment was perhaps stopped. The nonlinear analysis shows that the beam could have carried an additional 22.3 % load before it reached its ultimate strength.

Load deflection curves for Naaman's beam TC 9 are shown in Figure 3.14. In the pre-cracking region, NONLACS gives very favorable results. After cracking, NONLACS predicts a stiffer response than the observed response. For the ultimate load, the experimental value is 21.5 *kN*, and the theoretical reported by Naaman et al, is 20.0 *kN*, which represents 7 % difference. NONLACS's ultimate load was 19 *kN*,

Figure 3.13: Load-Deflection Curve for Prestressed Sumitomo's Beam

Figure 3.14: Load-Deflection Curve for Prestressed Naaman's Beam

which is 11.5% lower than the experimental value. The ultimate deflections are 28.1 mm, 20.3 mm, 16.0 mm from the experimental, Naaman's, and NONLACS's analyses, respectively. Note, however, that Naaman's load-deflection curve shows a far stiffer behaviour than the experimental value. It is important to mention that in view of the insufficient experimental data about the properties of concrete and reinforcement for this beam, the differences are not entirely unexpected.

3.3.4 Slabs Reinforced with FRP Reinforcement

From the few available experiments performed on slabs reinforced with FRP reinforcement, two slabs were selected. These were tested by a group of researchers in Japan (Kawakami 1992). Their work was interesting for several reasons: they used AFRP, as well as CRRP meshes, and the slabs were not cast from ordinary concrete, but high strength silica fume mortar. The specimens were rectangular with a side length of 450 mm, width 200 mm and a thickness of 50 mm. Typical details are shown in Table 3.1, with the corresponding material properties listed in Table 3.2. Two slabs will be analyzed, one reinforced with two layers of a 60 × 60 mm CFRP mesh, the other reinforced with the same size mesh of AFRP. Both slabs were simply supported along two parallel edges and free on the remaining edges and they were loaded in four-point bending.

In the finite element analysis, only one quarter of the slabs was analyzed. The finite element mesh used for these slabs is shown in Figure 3.6. It consists of four shell elements. The FRP reinforcement was modelled as smeared layers, while the slab was divided into 4 layers through the thickness.

Comparison of Results

Experimental load-deflection curves, as reported by the original investigators, are compared to the finite element results in Figure 3.15. Favourable agreement was achieved. The cracking load was in both cases well predicted. The experimental ultimate load for the slab, reinforced with two layers of AFRP 60 × 60 mm mesh, was reported to be 18.65 kN. The ultimate load from finite element analysis was 20 kN, which is 7.2 % higher. The experimental ultimate deflection was 10.0 mm, versus the finite element value of 8.5 mm. The slab reinforced with two layers of CFRP meshes was reported to have an ultimate load of 18.3 kN. The finite element analysis estimated the ultimate load to be 18.0 kN, which is 1.6 % less than the experimental one. The maximum deflection of this slab was 5 mm versus 4.6 mm

behaviour of not only FRP reinforced conventional concrete structures, but also of structures cast from high strength concrete.

3.3.5. Slab Strip Specimen

When dealing with slab-column connections, most researchers avoid modelling columns. Very often in finite element analyses at location of columns just vertical displacements are restrained. Since the objective of the proposed investigation is the study of the behaviour of flat plate type structures, it was thought that analysis of a similar concrete structure and the favourable comparison of its results with the corresponding experimental data will enhance the credibility of program NONLACS and its applicability to flat plate/slab parking structures. Hence a slab strip, supported by columns with capitals, which was tested by Polak and Vecchio (1993) was modelled. Facet shell elements were used to model the columns, as well as the slab. Another purpose of this study was to ascertain whether four node facet shell elements can be used for modelling columns.

The experimental program involved two large scale slab strip specimens loaded under monotonically increasing line loads applied at the midspan. Figure 3.16 (b) illustrates typical dimensions of tested specimens, but the two specimens had different boundary conditions. For analysis here specimen TV-2 was selected. This specimen was 100 mm thick, 1500 mm wide slab built integrally with two 200 × 200 mm columns and 400 × 400 mm drop panels extending 100 mm below the slab surface. The material properties of the specimens are given in the Table 3.3. The specimen TV2 was restrained against horizontal displacements at both ends. The ends were also fixed against vertical displacements and the lower column bases were free to rotate, but fixed against horizontal and vertical displacements. The top ends of the upper columns were free.

Due to symmetry of the problem, only one half of the strip was modelled. The finite

a) Silica Fume Mortar Slab Reinforced with AFRP

b) Silica Fume Mortar Slab Reinforced with CFRP

Figure 3.15: Load-Deflection Curve for Slab Reinforced with AFRP and CFRP

Figure 3.16: Elevation of Slab Strip Specimen

Table 3.3: Material Properties of Analyzed Slab Strip Specimen

Parameter [MPa]	Concrete				Reinforcement	
	f_c	f_t	E_c	ϵ_o	f_y	E_s
Experimental	30.20	3.60	32200.00	0.00199	484.00	202500.00
Used for the Analysis	30.20	3.60	32200.00	0.00199	484.00	200000.00

element mesh consisted of 34 facet shell elements. The slab itself was modelled using a 4×7 grid of elements, while each column was modelled using three facet shell elements. The finite element mesh used is illustrated in Figure 3.17. In compliance with the original boundary conditions, all nodal points in the columns were considered to be free, except for the lower column base which was restrained against vertical and horizontal displacements. For the finite elements in slab, nodes at the edge perpendicular to the span and at the axis of symmetry were restrained, all other nodes were considered to be free. The edge beams were ignored in the model because it was felt that due to the vertical and horizontal restraints at the ends of the slab strip, they may not significantly affect the final response of the structure.

Comparison of Results

The original investigators reported only the load-slab midspan deflection curve, as illustrated in Fig. 3.18. The results from the finite element analysis are also shown in the same figure. The ultimate load reached in the experiment was 89.5 kN , while NONLACS predicted it to be 92.5 kN , which is 3 % difference from the actual ultimate load. Up to 90 % of the ultimate load, the finite element simulation is giving very good results. Thereafter though, the modelled slab is not as ductile as the experimental one. Overall it can be said that the facet shell element is able to predict the behaviour of this structural system with satisfactory results. Capturing the descending branch of load-deformation curve is beyond the capabilities of NONLACS.

It is significant to mention that from a practical point of view, deflection is a serviceability requirement while strength is an ultimate limit state requirement. The comparison of deflections at ultimate load does not have practical significance. Finally, NONLACS cannot predict the softening portion of the load-deflection curve, hence for practical purposes, the program predictions are considered to be quite reasonable and useful.

The results in this chapter have demonstrated that program NONLACS can capture

Section A-A:

Figure 3.17: Finite Element Mesh for Slab Strip Specimen

Figure 3.18: Load-Deflection Curve for the Slab Strip Specimen

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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the complete behaviour of FRP reinforced concrete beams and slab specimens, provided failure is not initiated by bond failure at the interface of the reinforcement and the concrete. The results also show that the assumptions of linear elastic behaviour and lack of compressive strength for FRP reinforcement are reasonable. Finally, the idealization of columns by shell finite elements would appear to yield satisfactory results. This latter aspect of the modelling would be further investigated in the following chapter.

Chapter 4

Simple Two-Way Slabs

4.1 General

In this chapter the behaviour of simple two-way slabs reinforced with FRP reinforcement will be investigated. Prior to the detailed analysis that will be performed, some important design considerations will be discussed. Subsequently, the effect of some material and geometric parameters, and of some analytic assumptions on the deformations and strength of FRP reinforced slabs will be investigated. The results of this study will then be utilized in the design and analysis of the parking structures that will be discussed in the next chapter.

4.2 Two-Way Slab Design

All reinforced concrete structures have to be designed to ensure ductile failure. Reinforcing steel is a ductile material with the ultimate strain reaching up to 12 %, a fact that assures us of enough warning from the structure in the form of large deformations before failure or collapse. This may not necessarily be the case if we replace steel with FRP materials. Their stress-strain relation usually exhibits elastic

behaviour up to failure, which is sudden, while their elongation is in a range from 1.0 to 4.0 % (in some cases even lower, depending on percentage as well as type of fibre used) and also their modulus of elasticity ranges from 35 GPa to about 150 GPa. On the other hand, the idea of replacing steel with FRP has been advanced because of its high tensile strength (0.7 GPa to 2.0 GPa, which is 5 times that of steel) as well as excellent corrosion resistance. The latter is an important benefit especially when we are dealing with structures exposed to a very aggressive environment, such as parking structures.

A good design has to take advantage of all the benefits of a material (high tensile strength, corrosion resistance), while at the same time it should mitigate the effects of its drawbacks (low ultimate strain, and low modulus of elasticity). Therefore, before the parking structures in the next chapter are designed, the behaviour of a group of simple two-way slabs reinforced with steel or FRP will be studied. The FRP used will be carbon-fibre and aramid-fibre reinforcement. These were chosen because they have very different material properties. While the carbon fibre has very high strength and relatively high modulus of elasticity (it can even exceed the modulus of elasticity for steel), the aramid fibre has a modulus of elasticity only 30 % of that for steel. If the reinforcing material with the modulus of elasticity as low as that of AFRP is going to be used, one would expect the deflections of the structure reinforced with it to be relatively large. But we need to investigate whether large deflections will occur at serviceability limit state. If unacceptably large deflections occurred at this limit state, then one would need to find ways of reducing them without excessive cost.

The two-way slabs in this chapter were designed according to the Appendix E of The Concrete Design Handbook (1984). The slabs analyzed were square slabs, simply supported on all four sides (see Figure 4.1). Although a slab of this type is rare in practice, nevertheless the relative performance of FRP reinforced slabs versus that of steel reinforced slabs of the same kind will show the significance of some important parameters on the overall behaviour of FRP reinforced slabs. The span of the slab was 5 m, the thickness of the slab was selected as the maximum of the thicknesses

Figure 4.1: Two-Way Slab Layout and Finite Element Model

calculated from the following equations

$$h_{min} = \frac{l_n (800 + \frac{f_y}{15})}{36\,000 + 5\,000 \beta (1 + \beta_s)} \quad (4.1)$$

$$h_{min} = \frac{l_n (800 + \frac{f_y}{15})}{36\,000} \quad (4.2)$$

$$h_{min} = \frac{PERIMETER}{140} \quad (4.3)$$

$$h_{min} = \frac{l_n}{30.7} \quad (4.4)$$

where l_n is the clear span of the slab, f_y is the yield stress of steel, β is a ratio of long to short clear spans, β_s is a ratio of continuous edge to perimeter of slab. The controlling minimum thickness was found from Equation 4.4 to be 150 mm. The designed moments were found from Table E - 2 of Appendix E of CSA Standard A23.3-M84, using the 2.39 kN/m² live load for parking structures. The steel reinforcement was designed according to the well-known procedures of CSA Standard A23.3-M84. For the design of FRP reinforced structures, there is no available standard in Canada or elsewhere yet. Therefore, in this analysis some assumptions were made prior to the design.

4.3 Design Considerations for FRP Reinforced Members

Let us first consider a steel reinforced rectangular section with depth of 500 mm and width 300 mm. Using the basic assumption of flexural theory, that is a plane section which was plane before bending remains plane after bending, and the strain compatibility principle, the well known formula for the balanced strain conditions can

be written. It may be recalled that under balanced condition the maximum strain in compression occurs in concrete simultaneously with the yield strain in the tension steel. Accordingly,

$$\left(\frac{c}{d}\right)_b = \frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_s} = \frac{E_s \epsilon_{cu}}{E_s \epsilon_{cu} + f_y} \quad (4.5)$$

where c is the depth of the neutral axis, d is the effective depth, ϵ_{cu} is the ultimate compressive strain in concrete, ϵ_s is the yield strain in steel, E_s is the modulus of elasticity of steel, and f_y is the yield strength of steel. For values of $\epsilon_{cu} = 0.003$, $E_s = 200 \text{ GPa}$, and $f_y = 400 \text{ MPa}$, $\frac{c}{d} = 0.6$. If we carry out the same calculation for CFRP with $E_{CFRP} = 120 \text{ GPa}$, $f_{CFRP} = 1.2 \text{ GPa}$, we obtain $\frac{c}{d} = 0.23$. Note that f_{CFRP} was substituted for f_y in Eq. 4.5 to obtain the latter $\frac{c}{d}$. For the AFRP used in these analyses $E_{AFRP} = 59 \text{ GPa}$, and $f_{AFRP} = 1.3 \text{ GPa}$, leads to $\frac{c}{d} = 0.109$, which is only one sixth of the neutral axis depth for steel reinforced section. As can be seen, the neutral axis has been shifted way up in the FRP section. In order to establish equilibrium, this will force concrete to produce high stresses/strains, and therefore failure is initiated by crushing of concrete rather than rupture of the FRP. This important characteristic will be further discussed after detailed analysis of the parking structures in the next chapter.

For FRP reinforced slabs in this chapter, the balanced condition was assumed to occur when $\epsilon_{cu} = 0.003$ and $\epsilon_{FRP} = 0.75 \epsilon_{FRP}^u$, where ϵ_{FRP}^u is the ultimate strain of FRP. Why only 75 % of ultimate strain rather than the full ultimate strain is explained below.

An analysis was performed to develop moment-curvature relationship for a cross section with various types and amount of reinforcement. The cross-section was assumed to be rectangular, 500 mm deep and 300 mm wide. The analysis included two sets of sections, one reinforced with steel, and the other with CFRP. Every set consisted of three strain profiles at ultimate state.

Figure 4.2: Depth of Neutral Axis, and Moment Curvature Relationship for Cross Sections with Different Reinforcement

For all the strain profiles the maximum strain in concrete was assumed to be 0.003. The strain in the reinforcement was assumed to be 0.002, 0.01, or 0.0075, respectively, as shown in Fig. 4.2. Using the strain compatibility method and the constitutive laws of the reinforcement and the concrete, the required steel and CFRP were designed. Notice, that the first strain profile corresponds to the so-called balanced condition in a steel reinforced section. The amount of steel and CFRP for the three strain profiles were calculated using the conditions of equilibrium, that is

$$T = C \quad (4.6)$$

which is for steel

$$A_s f_y = b \beta_1 c 0.85 f'_c \quad (4.7)$$

and for FRP

$$A_{FRP} f_{FRP} = b \beta_1 c 0.85 f'_c \quad (4.8)$$

where T is the force in reinforcement, C is the force in concrete, A_s is the area of steel, f_y is the yield strength of steel, A_{FRP} is the area of FRP, f_{FRP} is the ultimate strength of FRP, b is the width of the section, $\beta_1 c$ is the depth of the compression block of concrete. Because the right side of the equations remains constant, clearly one can see that the amount of FRP reinforcement would depend on the strength ratio of steel to FRP. Accordingly the reinforcement amounts are $A_{s1} = 5164 \text{ mm}^2$, $A_{s2} = 1986 \text{ mm}^2$, $A_{s3} = 2459 \text{ mm}^2$, $A_{FRP1} = 8607 \text{ mm}^2$, $A_{FRP2} = 662 \text{ mm}^2$, $A_{FRP3} = 1093 \text{ mm}^2$. The concrete was assumed to have an $f'_c = 30 \text{ MPa}$ in all the cases. Note that $E_s = 200 \text{ GPa}$ and $E_{CFRP} = 120 \text{ GPa}$. Notice that for the sections with strain profile 1, $(n\rho)_{steel} = (n\rho)_{FRP}$ where n is the modular ratio and ρ is the reinforcement ratio. Clearly the FRP reinforced section with the latter strain profile is uneconomical because despite the much larger strength of FRP we need more FRP reinforcement than steel reinforcement.

Using the strain compatibility method, a computer program was written to trace the moment-curvature relation of these sections as illustrated in Fig. 4.2 (a). We can

see that for strain profile 1, we obtain identical moment curvature relation for the steel and the corresponding CFRP section. Both sections appear to fail in a brittle manner, but as indicated earlier this strain profile produces an uneconomical section and is not recommended.

Profiles 2 and 3 require less CFRP reinforcement than the corresponding steel amounts, but the CFRP reinforced sections are more flexible and the beam may undergo larger deflections. Although profile 2 requires less CFRP, the maximum strain in the reinforcement in this case corresponds to the simultaneous failure of CFRP and concrete. Since failure of CFRP leads to the complete disintegration of the member, we would use profile 3 which indicates that the member failure would be initiated by the failure of concrete. While such failure mode would be unacceptable in ordinary reinforced concrete, for CFRPs it may be unavoidable unless more ductile FRPs become available.

The more flexible moment curvature diagrams of CFRP reinforced sections with profiles 2 or 3 signify that the neutral axis in the case of the latter sections moves rapidly upward compared to the corresponding steel reinforced beam (Fig. 4.2 b). Hence the flexural cracks propagate upward and reduce the effective moment of inertia, causing a reduction in stiffness.

Although the stiffness reduction in the moment-curvature diagram is quite dramatic, the deflections of beams do not necessarily increase in the same way as curvature increase in the moment-curvature diagram. The reason for this is that large curvatures only occur in sections of maximum moment but the deflection is a function of curvature over the entire length of the beam.

4.4 Parametric Study of Simply Supported Square Slabs

While modelling reinforced concrete members using NONLACS, certain material properties need to be input as data. There are three different materials to be modelled in the analyses to follow: concrete, steel, and FRP. The models for all of them were already discussed in Section 3.2.2. For concrete, the input includes compressive strength of concrete, tensile strength, modulus of elasticity, Poisson's ratio, strain at compressive strength, ultimate strain in compression, and maximum tensile strain. Different concrete standards provide different way of finding values for properties like the modulus of elasticity of concrete, tensile strength, as well as the ultimate strain values. It is important to know how certain assumptions regarding some of these properties influence the results.

In the following a number of simply supported square slabs were analyzed to study the effect of some parameters on the deformations and strength of the slab. These parameters were investigated for the case of the simple slab in order to reduce the number of computer runs for the parking structures. The results in this section will provide sufficient indication as to the influence of certain material and/or geometric properties of the slab on the results of the nonlinear finite element analysis by NONLACS.

4.4.1 Slab Design and Finite Element Modelling

Fig. 4.1 shows a $5\text{ m} \times 5\text{ m}$ square slab of 150 mm thickness. This slab was designed for a uniformly distributed load of 9.80 kN/m^2 in all the cases to be discussed in this chapter. The material properties used are given in Table 4.1. The slab was designed for the moments shown in Fig. 4.3. Hence an orthogonal mesh of steel reinforcement was used, with $A_{sx} = A_{sy} = 444\text{ mm}^2/\text{m}$. For the FRP reinforced

Table 4.1: Material Properties Used in the Analysis and Design of Simple Square Slab

AFRP	CFRP	Steel
$f_{AFRP} = 1\,324.35\text{ MPa}$	$f_{CFRP} = 1\,226.25\text{ MPa}$	$f_y = 400\text{ MPa}$
$E_{AFRP} = 58\,845\text{ MPa}$	$E_{CFRP} = 117\,720\text{ MPa}$	$E_s = 200\,000\text{ MPa}$
$\epsilon_{AFRP} = 0.022$	$\epsilon_{CFRP} = 0.011$	$\epsilon_{sy} = 0.002$

slabs $A_{CFRP,x} = A_{CFRP,y} = 125\text{ mm}^2/\text{m}$, and $A_{AFRP,x} = A_{AFRP,y} = 100\text{ mm}^2/\text{m}$. Note that the amount of AFRP and CFRP are respectively 23 % , and 28 % of the amount of steel in similarly reinforced slabs. In some cases the amount of FRP was increased to study its effect on the behaviour of the slab.

Due to symmetry, one quarter of the slab was modelled by a 4×4 mesh (see Fig. 4.1 b)) of the facet shell element discussed earlier. The slab was divided into five equal layers through its thickness while the reinforcement in each direction was modelled as a smeared layer. The load was applied in thirty increments and a maximum of 25 iterations were allowed for each load step.

4.4.2 Effect of Concrete Tensile Strength on Slab Deflections

Some parametric analyses were performed to compare the recommended tensile strength relation in CSA Standard A23.3-M84, with those recommended by the ACI Code (1983), and the EUROCODE 2 (1990). This parameter is important because it controls the occurrence of first cracking, which in the case of FRP reinforced structures is considered to be an important event because FRP has relatively low modulus of elasticity. This means that after cracking the deflections are expected to be larger than in a similar steel reinforced system.

The CSA Standard A23.3-M84 assumes the tensile strength of concrete to be given by

Figure 4.3: Design Moments for Two-Way Slab

$f_t = 0.6 \sqrt{f'_c}$, while the ACI Code recommends $f_t = 0.65 \sqrt{f'_c}$, and the EUROCODE 2 is more on the conservative side with $f_t = 0.3 f_{ck}^{2/3}$, where f_{ck} is the characteristic strength.

Using the preceding concrete tensile strengths, the load-deflection curves were plotted for the centre of the square slab analyzed by NONLACS. The results show that the only difference is the time of occurrence of the first crack. Because this difference is relatively small, if any, the CSA Standard A23.3-M84 recommendation for the tensile strength of concrete will be used in all subsequent analyses.

4.4.3 Effect of Tension Stiffening

In Section 3.2.2 the tension stiffening model for NONLACS was discussed. It is a bilinear model where the slope of the descending portion controls the rate at which cracked concrete releases its tensile stresses. Tension stiffening models account for the fact that concrete between two cracked sections is still capable of carrying tensile stresses. Failure to recognize this property normally leads to the predicted deflections being larger than the observed deflections. The tension stiffening property is a function of the bond characteristics of the reinforcement and the strength of concrete. While for steel reinforced concrete structures, the bilinear tension stiffening model has been found to be adequate (Ghoneim and Ghali 1982) for FRP reinforced concrete structures its adequacy needs to be investigated.

Here we will indirectly investigate the problem. We will analyze CFRP and AFRP reinforced slabs using a slope of either $-\frac{E_{co}}{4}$ or $-\frac{E_{co}}{9}$, for the descending branch, where E_{co} = initial elastic modulus of concrete, and the negative sign indicates a negative slope because of the descending branch. The preceding slopes correspond to maximum concrete tensile strains of $5\epsilon_{cr}$ and $10\epsilon_{cr}$, respectively, where ϵ_{cr} = cracking strain of concrete. At maximum tensile strain it is assumed that concrete is incapable of carrying any tension. For reference purpose, a similar steel reinforced

slab was analyzed.

Fig. 4.4 shows the load-midpoint deflection of the slab analyzed. The inset shows the full load-deflection curve while the enlarged figure shows the load-deflection curve up to a load slightly higher than the load causing yielding of steel.

In Figure 4.4, the effect of the tension stiffening, especially on the deflections after cracking, can be seen very clearly. In the case of steel reinforced slab, the strength of the section remained unchanged as expected, as well there were small differences in the deflections between the model assuming $5 \epsilon_{cr}$ and $10 \epsilon_{cr}$. For the FRP reinforced slab, reduction in strength was found with the steeper descending branch. As can be seen from Figure 4.4, the overall stiffness of the slab reinforced with FRPs was exhausted very soon after the first cracking occurred. The first cracking started at around 14.50 kN/m^2 and both the AFRP and the CFRP reinforced slabs reached failure at around 18 kN/m^2 , with the failure being sudden. It was not possible to estimate the value of the ultimate load once concrete on the bottom of the slab became extensively cracked because the stress in FRP jumped suddenly causing rupture. Because NONLACS cannot account for softening of the structure, we are unable to capture the load released by the structure.

It is surprising that there is no difference between the CFRP and the AFRP reinforced slabs for the two models with different amounts of maximum tensile strain in concrete. If this is proven by experiments, it means that one could use the low modulus high strength FRPs without much penalty as far as deflections are concerned. Today GFRP is not favoured due to its low modulus of elasticity, and the notion that large deflections will result after cracking.

It should be pointed out that when the same slabs were analyzed without allowing for tension stiffening, they failed abruptly and at a much lower load. However, lack of tension stiffening is not representative of the real behaviour of structures, therefore the results for the latter case were not plotted.

Figure 4.4: The Effect of Tension Stiffening on Load-Deflection Curve

4.4.4 Effect of Reinforcement Ratio

The purpose of the following analyses is to determine the effect of the amount of reinforcement on the deflections and strength of the slab.

Clause 10.5.1 of the Code specifies the minimum amount of reinforcement to be used in a design.

$$\rho_{min} = \frac{1.4}{f_y} \quad (4.9)$$

where ρ_{min} = the reinforcement ratio, and f_y = the yield strength of steel. This amount of reinforcement will ensure that after cracking takes place, the member will not disintegrate. Eq. 4.9 is valid for steel reinforced sections; for FRP reinforced sections, the minimum amount has not been established yet. For the purpose of the present analysis, we based the minimum amount of FRP on Eq. 4.9, substituting 75% f_{FRP} for f_y . This leads to the following relationship:

$$\rho_{min.FRP} = \frac{1.9}{f_{FRP}} \quad (4.10)$$

where f_{FRP} = the strength of FRP reinforcement. This will ensure that the member will not disintegrate after cracking, but will be able to carry some additional load.

Because for these slabs the design load was smaller than the cracking moment of the slab, this minimum area was used, which is considered to be the reason that both the CFRP and the AFRP reinforced slabs collapsed immediately after cracking occurred. To avoid this sudden failure, another set of slabs with double the area of FRP (twice the minimum reinforcement ratio) was analyzed. Doubling the area of FRP may be needed for practical reasons, i.e. the provision of sufficient FRP to maintain the requirements of maximum spacing of rebars in slabs. According to CSA A-23.3-M84 the latter spacing is $2 \times h_s$, where h_s is the slab thickness. For the slabs under consideration the maximum spacing would be 300 mm. The present Code requirements for the minimum area of reinforcement in slabs ($\rho_{min} = 0.002$) were established for steel reinforced slabs, which are more likely to suffer from corrosion, therefore the minimum reinforcement is provided in part to mitigate the deflections

effects of thermal and shrinkage cracks on the slab. The latter ρ_{min} leads to excessive amount of FRP reinforcement, hence it needs further scrutiny.

In view of those considerations a second set of slabs was reinforced with # 8 rebars, with spacing of 285 mm, which was twice the minimum area of reinforcement (the area needed to carry the design moment was even smaller than the minimum area was). Once again the maximum concrete tensile strains of $5\epsilon_{cr}$ and $10\epsilon_{cr}$ were used to analyze the latter slabs. The results of the analysis can be seen in Figure 4.5 which illustrate the load-midpoint deflection of the two slab sets. Notice that after extensive cracking took place, the same sudden loss of stiffness occurred, but with increased amount of reinforcement both the AFRP and the CFRP slabs were able to carry additional load before they collapsed completely. The ultimate load for the slabs with two different values of tensile strain were the same, which illustrates that tension stiffening does not affect ultimate strength of a section. We also notice that the tension-stiffening model has no influence on the deflections under service loads in the particular slabs here. Generally, however, this may not be the case. Since the $\epsilon_{t,max} = 10\epsilon_{cr}$ was used in the analysis of the beams and slabs in Section 3.3, and since the results of those analyses showed favourable comparison between the experimental and analytical results, we will use $\epsilon_{t,max} = 10\epsilon_{cr}$ in all subsequent analyses. It is expected that the adoption of this criterion will not have any significant effect on either the ultimate resistance or the deflections under service loads.

4.4.5 Effect of Drop Panel on the Deflections of Simple Slabs

In steel reinforced two-way slabs systems drop panels are usually used for maintaining the shear capacity of the slab around the column. The drop panels also increase the stiffness of the slab, thus reducing deflections. In order to reduce deflections in FRP-reinforced slabs, one could use drop panels. In order to quantify the effect of drop panels on deflections, similar slabs with and without drop panels were analyzed.

Figure 4.5: The Effect of Tension Stiffening on the Load-Deflection Response of Over-Reinforced FRP Reinforced Concrete Slabs

Although in a simply supported slab, the moment decreases from the center to the support, nevertheless the shear forces are higher near the support. Hence to get a feeling about the effect of drop panels on the deflections of slabs, the thickness of the previously analyzed slabs was increased by 50 mm over a distance of $\frac{l}{6}$ from supports, where l = span length. Once again the slabs were subjected to uniform load and the reinforcement consisted either of steel, CFRP, or AFRP.

Fig. 4.6 shows the load-midpoint deflection of the CFRP and AFRP reinforced slabs with drop panels and of the similar steel reinforced slab without drop panel. Notice that by providing the slab with drop panels, one could increase its moment capacity, and limit its deflections. In other words, the FRP reinforced slabs carry a larger moment than the steel reinforced slab and their deflections are small prior to failure. The large yield like plateau for the FRP reinforced slabs in Fig. 4.6 may not be a true indicator of the actual behaviour of these slabs and may be due to the analytical procedures and the interpretation of results of the analysis. In other words, the failure load of the slab is 29.53 kN/m^2 but the deflections corresponding to the failure load are not reliable when obtained from nonlinear finite element analyses. Hence provision of the drop panel may not increase the ductility, but it can control deflections and increase the moment capacity.

The effect of drop panels on slab deflections can be further examined by considering Fig. 4.7. Here the normalized midspan deflections of the slabs with different types of reinforcement are plotted against the applied load. The deflections are normalized by the corresponding deflections of similarly reinforced slabs without drop panels. This figure indicates that the provision of drop panel has nearly the same effect on the deflections of the slab, regardless of the type of reinforcement. In other words, deflections decrease with the provision of the drop panel and the magnitude of the reduction increases with increasing load level. Notice that at high load levels the reduction is of the order of 240%.

Finally, Figure 4.8 shows the deflected shape of the centreline of the slabs at a load

Figure 4.6: Load-Deflection Curves for Slabs with Drop Panel

Figure 4.7: Effect of Drop Panel on Center Deflections of Simple Square Slabs with Different Types of Reinforcement

level slightly less than the load causing yield in the steel reinforced slab without drop panel. One can again see that the provision of drop panel dramatically reduces the slab deflections within the entire slab section. Although the provision of drop panels for controlling deflections may not justify their cost, nevertheless it provides a means for achieving desirable deflections in FRP reinforced slabs. Drop panels may be needed for punching shear resistance, hence deflection control may be achieved as a by-product .

On the basis of the preceding analysis it was decided to provide the parking structures in the next chapter with drop panels.

Figure 4.8: Deflections in the Middle Section of the Slab Under the Load Just Prior to Steel Yielding

Chapter 5

Design and Analysis of Flat-Slab Parking Structures

5.1 General

In this chapter the design of three parking structures according to the current CSA Standard CAN3 - A23.3- M84 will be presented. These structures will be subsequently analyzed by the finite element method to investigate their full behaviour over the entire loading range up to failure. The three structures are denoted as A, B, and C and are designed to have identical layout but different types of reinforcement. The reinforcement types used are

- conventional mild steel
- carbon fibre reinforced plastic bars, known by the proprietary name LEADLINE
- glass fibre reinforced plastic bars, with the trademark name ISOROD.

The key properties of the given structures which will be investigated include stresses and deflections under service loads and the strength and mode of failure at the ulti-

mate limit state. In each case they will be subjected to monotonically increasing loads up to failure. We will subsequently compare the behaviour of FRP reinforced structures with the steel reinforced structures and investigate whether the FRP reinforced structures satisfy the strength and serviceability requirements of the Code.

5.2 Design of the Parking Structures

The three parking structures will have identical layout and dimensions and the only difference between them will be the type and amount of reinforcement. The complete design of these structures is given in Appendix A. In the following some of the salient assumptions and features of the designed structures will be described.

5.2.1 Geometry of the Structures

One of the following slab systems is usually considered for a parking structure (Aalami 1989)

1. flat plate
2. flat slab with square column capitals
3. flat slab with drop panels
4. banded slab
5. waffle slab
6. waffle slab with solid column lines

Systems 2, 3, and 4 are particularly common. The flat plates can be used for smaller loads and shorter spans (up to 6 m) while waffle slab and waffle slab with solid column lines are costly because they require a lot of complicated formwork. The span

Figure 5.1: A Schematic View of The Parking Structures Analyzed

length in a parking structure may vary from 8 m to 13.4 m, depending on the slab system used. The thickness of the slab depends mostly on whether the floor system is pretensioned or not. For pretensioned systems, the minimum slab thickness is 205 mm, for non-prestressed system, as the one designed in this thesis, the thickness of the slab depends on the length of the span, and geometry of the floor system.

The parking garages in this thesis are considered to be two storey flat slab structures with four bays of 6.4, 8, 8, and 6.4 m spans respectively (see Figure 5.1). For simplicity, the structures are assumed to be symmetric about two orthogonal axes in their plan. The slabs do not have either interior or edge beams.

The thickness of the slab was calculated according to CSA CAN3 - A23.3 - M84 Clause 9.5.3.3.1 Equation (9-9)

$$h_{min} = \frac{l_n (800 + \frac{f_y}{1.5})}{3600} \quad (5.1)$$

to be 230 mm. In the above equation, l_n is the clear span of the slab, and f_y is the yield strength of steel. This thickness was later reduced according to Clause 9.5.3.3.2 of the Code by 10 % to 215 mm, since drop panels were needed to resist punching shear. The drop panels were designed according to Clauses 13.4.7.2 and 13.4.7.3 of

the Code. The projection of the drop panel below the slab is 55 mm, and they are extended 1350 mm from the center line of the column on all four sides. All columns have cross section of 500 × 500 mm and are 3000 mm long (see Figure 5.2). Note that the same thickness of the slab and the drop panel was used for all three structures, regardless of the type of reinforcement.

5.2.2 Material Properties

The following material properties were used both in the design and the finite element analysis of the parking structures

1. Concrete

Compressive strength $f'_c = 30$ MPa

Modulus of elasticity $E_c = 5\,000 \sqrt{f'_c} = 27\,386$ MPa

Modulus of rupture $f_t = 0.6 \sqrt{f'_c} = 3.28$ MPa

2. Reinforcement

- *Steel*

Yield strength $f_y = 400$ MPa

Modulus of elasticity $E_s = 200$ GPa

- *Carbon fibre reinforced plastic*

Ultimate strength $f_u = 2\,260$ MPa

Modulus of elasticity $E_{FRP} = 147$ GPa

Ultimate strain $\epsilon_{ult} = 0.015$

- *Glass fibre reinforced plastic*

Ultimate strength $f_u = 690$ MPa

Modulus of elasticity $E_{FRP} = 42$ GPa

Ultimate strain $\epsilon_{ult} = 0.018$

Figure 5.2: The Elevation and the Plan of the Parking Structure

5.2.3 Preliminary Design

Reinforced concrete structures should be designed to satisfy the requirements for strength and serviceability. In the design, the provisions of Clause 13 and Clause 9 of the Code were applied. According to the Clause 9.1.1 the factored resistance, in terms of flexure, axial load, shear, and torsion, should be equal or larger than the effect of factored loads. Also the issue of ductility and type of failure is controlled by the reinforcement ratio. However, the latter provisions are specific to steel reinforced concrete, and are not necessarily applicable to FRP reinforced concrete. In the following an attempt will be made to describe the issues involved and the reason for the selection of the FRP reinforcement ratios in the design of structures analyzed in this study.

In the case of the parking structures designed here the loads were assumed to consist of only the dead and live gravity loads having the magnitudes shown in Table 1.13 of the National Building Code of Canada (NBC 1985). Accordingly, the superimposed dead load was 1.44 kN/m^2 and the live load was 2.39 kN/m^2 , which were distributed uniformly over the slab. Since the specified live load was less than 75 % of dead load, the factored moments were calculated assuming all spans to be loaded with both dead and live loads simultaneously.

The moment distribution method was used to find the factored moments. The flexural reinforcement was designed for the moments found in the analysis, distributed into the middle and column strips as specified in the Code (see Appendix A). The moments were quite small, therefore the amount of reinforcement needed was also small. In the middle strips only minimum reinforcement was used. For integrity of the structure, in structure A, i.e. the one with steel reinforcement, the positive moment reinforcement at the bottom was not curtailed. In the other two structures, reinforced with FRP reinforcement, the bottom reinforcement was curtailed at the appropriate locations because FRP generally cannot carry much compression. There is also no specified minimum FRP reinforcement ratio. The amount of required FRP in the present

design was calculated from the ultimate moment as follows

$$M_r = A_{FRP} f_{FRP} \left(d - \frac{a}{2} \right) \quad (5.2)$$

where the depth of the compression block, a , was calculated using the strain compatibility, assuming the maximum compressive strain in concrete at ultimate to be 0.003, and using 75 % of ultimate strain of FRP in tension. The reinforcement cover was taken to be the same for all three slabs, 20 mm, even though due to very high corrosive resistance of FRPs, the cover could be less. On the other hand fire protection may require larger cover because all FRPs are considered to be sensitive to high temperatures, but this issue is beyond the scope of the present investigation.

It was stated earlier that at ultimate the strain in FRP was assumed to be 75 % of its maximum strain capacity. The 75 % limit is arbitrary because higher strain values will equally induce concrete failure before FRP reaches its capacity. This could be a brittle failure. On the other hand if FRP is allowed to reach its capacity first, then it will rupture and the beam will collapse immediately thereafter, leading to the complete disintegration of the structure. For that reason compression failure of concrete was presumed to be a better choice. Note that in ordinary reinforced concrete, compression failure is allowed in columns.

In the steel reinforced concrete design, the type of failure is controlled by the amount of reinforcement used. The Code specifies a balanced reinforcement ratio, ρ_b , derived under the assumption of strain compatibility and assuming that when concrete reaches its ultimate compressive strain ϵ_{cu} , the yield strain in the tension steel reinforcement will occur simultaneously. If we use less reinforcement than given by ρ_b , the resulting failure will be ductile because steel would be allowed to stretch and create large curvatures in the section. If we use more reinforcement, the concrete will fail in compression before steel reaches its yield strength/strain, and this failure is known as brittle. The latter is usually the type of failure observed in FRP reinforced structures due to their brittle nature. If we were to find the balanced ratio for FRP reinforced section, we would assume that when concrete reached its ultimate compressive strain,

the ultimate strain in FRP would be reached simultaneously. For these conditions, under strain compatibility assumption the depth of the compression zone of concrete can be found as

$$c_b = \frac{0.003 E_{FRP}}{0.003 E_{FRP} + f_{FRP}} d \quad (5.3)$$

where

c_b = the depth of compression zone of concrete

d = the effective depth

f_{FRP} = the ultimate strength of FRP

E_{FRP} = the modulus of elasticity of FRP

The force in FRP under the same condition would be, T_b

$$T_b = A_{FRP} f_{FRP} = \rho_b b d f_{FRP} \quad (5.4)$$

and the force in the concrete would be, C_b

$$C_b = b \beta_1 c_b 0.85 f'_c = 0.85 \beta_1 b f'_c \frac{0.003 E_{FRP}}{0.003 E_{FRP} + f_{FRP}} d \quad (5.5)$$

Equating T_b to C_b gives

$$\rho_b = 0.85 \beta_1 \frac{0.003}{0.003 + \epsilon_{b,FRP}} \frac{f'_c}{f_{FRP}} \quad (5.6)$$

where

$\epsilon_{b,FRP}$ = maximum strain in FRP under balance condition

In more general form one could write

$$\rho_b = 0.85 \beta_1 \frac{1}{1 + \frac{\epsilon_{b,FRP}}{\epsilon_{bc}}} \frac{f'_c}{f_{FRP}} \quad (5.7)$$

in which ϵ_{bc} is the maximum strain in concrete under balance condition. It is evident that the balance reinforcement ratio is directly proportional to the ratio of concrete

strength to reinforcement strength. On the other hand, ρ_b decreases as the ratio of maximum strain in FRP to the maximum strain in concrete increases. Since FRPs have high strength and large maximum strain compared to the yield strength and strain of steel, their ρ_b is usually much smaller than that of steel.

For example, for CFRP $f_{CFRP} = 2260 \text{ MPa}$ and $\epsilon_{b,CFRP} = \epsilon_{max,CFRP} = 0.015$ gives

$$\rho_{b,CFRP} = 0.0019 \quad (5.8)$$

while for GFRP $f_{GFRP} = 690 \text{ MPa}$ and $\epsilon_{b,GFRP} = \epsilon_{max,GFRP} = 0.018$

$$\rho_{b,GFRP} = 0.0053 \quad (5.9)$$

compared to steel

$$\rho_{b,steel} = 0.038 \quad (5.10)$$

The above ratios for FRPs are quite small and in many cases may be smaller than $\rho_{min} = \frac{1.4}{f_y}$ for steel reinforced concrete. Of course, one could find ρ_{min} for FRPs as follows.

The force resisted by ρ_{min} immediately after cracking is

$$T = \rho_{min} b d f_y = \frac{1.4}{f_y} b d f_y = 1.4 b d \quad (5.11)$$

The amount of FRP needed to produce the same force would depend upon the level of stress allowed in the FRP. Since the minimum reinforcement is intended to prevent sudden failure following cracking, one cannot allow the FRP to reach its strength once the section cracks, for that would lead to sudden failure. In this thesis, we arbitrarily assume that the FRP can only reach 75 % of its strength after first cracking. Hence

$$0.75 f_{FRP} A_{FRP} = 1.4 b d \quad (5.12)$$

or

$$\rho_{min(FRP)} = \frac{1.4}{0.75 f_{FRP}} = \frac{1.9}{f_{FRP}} \quad (5.13)$$

This amount of reinforcement can be extremely small and in practice may lead to large spacing between the bars. A practical solution may be to produce FRP grids

similar to steel wire mesh in order to reduce the spacing between the bars. The latter particularly applies to very high strength FRPs such as carbon and aramid fibre reinforced plastic bars. For the purpose of the present investigation we have left these practical considerations aside and have assumed that the bar spacing is not of concern.

The shear strength of the slab was checked to verify that the chosen slab thickness was adequate, and that no reinforcement would be needed. Preliminary check of one way shear, as well as of punching shear was carried out. One way shear was within limits for this case. The punching shear was checked at the critical sections at $d/2$ from the column face and at a distance equal to half the effective depth within the drop panel. At the column-slab connection, check for shear due to the transfer of moments was made and was found to be within the limits.

Columns were designed for the minimum reinforcement, which is 5 % of A_g , where A_g is the gross area of the column. The reinforcing bars were assumed to be placed in the corners of the column section. Reference can be made to Appendix A for the full design of the structures.

5.3 The Finite Element Analysis

As stated earlier the objective of this investigation is to use the nonlinear finite element method in order to study the complete response of FRP reinforced flat slab structures under gravity loads. But to ensure that the chosen finite element program would yield satisfactory results, two different programs were used for the elastic analysis of the parking structure. The linear program SPAST (Ghali and Tadros 1980) was used to model the slab as plate bending elements and the columns as beam elements. The nonlinear program NONLACS was used to model the slab and the column as thin shell elements. As mentioned earlier, NONLACS does not contain beam finite elements.

5.3.1 Linear Elastic Analysis

The linear elastic finite element program SPAST (Ghali and Tadros 1980) will be used for the elastic analysis to verify that the modelling of the columns as shell elements gives satisfactory results. The program SPAST has three-dimensional beam finite elements which can be used to model the columns. Thus by comparing the results of SPAST with those of NONLACS, one could ascertain the validity of modelling columns as shell elements. It must be emphasized that the shell elements have all the necessary six degrees of freedom for describing the behaviour of a column.

The same structures will also be analyzed using the equivalent frame method (EFM) and the direct design method (DDM) in order to determine the difference between these approximate methods and the more refined finite element method. The latter comparison is made in order to assess the difference between the moments obtained from the Code recommended methods and those obtained from the finite element analysis. It may be recalled that the parking structures were designed for the moments and shear forces obtained by EFM as shown in Appendix A.

Only one floor of the building shown in Figure 5.1 was modelled. Taking advantage of the symmetry of the structure, only one quarter of the structure was modelled. The finite element mesh for analysis by program SPAST is shown in Figure 5.3. For later reference note that the origin of the coordinate system is situated at corner A and the x and y axes are in the plane of the slab as indicated while the z axis points downwards. The mesh consists of 225 plate bending elements, representing the slab, and nine beam elements representing the columns. The plate bending element is the same as the quadrilateral element in NONLACS which was described in Section 3.2. Furthermore, the mesh contains 265 nodes with each node possessing 6 degrees of freedom, three displacements and three rotations. In order to reduce number of elements used in the analysis, only the bottom columns were actually modelled, but the top columns were accounted for by doubling the cross-sectional area and moments of inertia of the modelled columns. This will not create any problems because the

Figure 5.3: Finite Element Mesh for SPAST Analysis

top and bottom columns are identical and therefore they have equal stiffness about any axis.

Since SPAST was used for linear analysis only, the reinforcement was not included in the model. The material properties for concrete used in the analysis can be found on page 101.

5.3.2 Nonlinear Analysis

The nonlinear analysis was performed by program NONLACS. The finite element mesh consisted of a 15×17 mesh of facet shell elements, representing the slab, and four shell elements representing each of the top and bottom columns as shown in Fig. 5.4. Each element within the slab was divided into five equal layers of 43 mm thickness, except for the elements representing the drop panels which were divided into six layers, with the first five layers being each 43 mm thick and the sixth layer

Figure 5.4: Finite Element Mesh for Nonlinear Analysis by Program NONLACS

being 55 mm.

The reinforcement in the slab was modelled as smeared layers with the thickness of each layer corresponding to the amount of reinforcement per unit length. The amount of reinforcement can be found in Tables A.4 to A.6 in Appendix A. These layers were situated at the appropriate distance from the middle plane of the slab.

The loads were applied in 30 load steps, but the self weight of the structure was applied in one load step. The increments of load were not even, smaller steps were chosen for the early stages of cracking and at ultimate load in order to capture the nonlinearity in the behaviour of concrete and steel.

To ascertain whether the NONLACS model yields satisfactory results, stresses from SPAST and NONLACS will be compared. Because SPAST is not capable of capturing

Figure 5.5: Moments M_x in the Frame 3-3 under a Load of 1 kN/m^2

nonlinearities in concrete, we had to choose small loads in NONLACS analysis in order to avoid cracking. Frame 3-3 (see Fig. 5.2) was chosen for this comparison as the most critical frame due to the fact that it takes load from the two adjacent largest bays of the floor slab system. When the structure was loaded with 1 kN/m^2 , it was uncracked and therefore the moments from the linear and nonlinear analysis could be compared. Favourable comparison was found, as can be seen in Figure 5.5, which shows the variation of longitudinal bending moments, M_x in the slab. Small discrepancies were found in the edge panel. They might be attributed to the differences between the two meshes. Furthermore, the total moments calculated according to SPAST, EFM, and DDM, respectively, are compared in Figure 5.6. Because all the above mentioned methods use linear elastic analysis, the moments were calculated under service loads used for the design of the parking structure. The favourable comparison in Fig. 5.5 shows that modelling columns by the shell element gives satisfactory results.

In Fig. 5.6 we observe that while the three methods provide similar moment distri-

FRAME 2-2

Figure 5.6: The Total Moments to be Distributed in the Middle and Column Strips of the Frame 2-2

bution, the actual magnitude of the moments differ from one method to the other. The negative moments over the interior columns are within 10 % of each other, while the positive moments are significantly different. Since EFM and DDM treat the actual three-dimensional frame as a two-dimensional frame, the above differences are expected. Of course, the moments given by EFM and DDM are generally higher than the ones given by finite element. This means that the slabs will have higher strength than their design strength.

5.4 Nonlinear Analysis Results and Discussion

In order to facilitate the presentation of the results that are to follow, reference may be made to Figure 5.7, which shows the layout of a quarter of the floor of the parking structure. Due to symmetry the results for this quarter are representative of those for the entire structure. Furthermore, we need to show the results for slab strips in only

Figure 5.7: Floor Plan of a Quarter of the Parking Structures Analyzed and the Various Sections

one direction because again due to symmetry the results for the orthogonal direction will be the same.

5.4.1 Deflection and Ductility

There are two approaches for checking deflections within the CSA Code. In one of them the depth of the member is chosen as a fraction of the span. If this thickness is provided, deflections do not have to be calculated. The other approach involves calculation of actual deflections and their comparison with the specified permissible values given in the Code.

For three-dimensional structures calculating deflections is rather complicated, therefore, the first approach is usually preferred. The Code provides the necessary equations according to which one can find the minimum thickness of the slab. Equation (9-9) of the Code uses the yield strength of the steel reinforcement as well as the span of the slab to determine the minimum thickness of the slab. When high strength FRP reinforcement is used, such as CFRP or GFRP, one may be tempted to replace the yield strength of the steel by the strength of the appropriate FRP, but such a procedure would yield a large thickness. Using such large thicknesses would produce uneconomical design. Hence one should actually modify the present Code equations for slab thickness to make them applicable to FRP reinforced slabs. In this study, the slab thickness was kept the same, regardless of the type of reinforcement. The intention was to check whether the type of reinforcement will have significant effect on the deflections of the slab.

Deflection control is a serviceability limit state requirement; therefore, we will plot deflections of the various sections of the slab under self-weight (DL), self-weight plus superimposed dead load (DL+SDL), and under the full service load DL+SDL+live load (DL+SDL+LL).

Figures 5.8 to 5.11 show, respectively, the slab deflections along the sections denoted as A-A, B-B, D-D and E-E in Fig. 5.7. These sections are along the centreline of the floor slab, the centreline of the interior and exterior bays, and along the external edge of the floor, respectively. The latter sections would be the regions where critical deflections may occur. The deflections along Section C-C, which coincides with the column lines, would be smaller than those along the foregoing sections. Notice that as expected, the greatest deflections occur along Section B-B, and the maximum deflection under full service load is approximately 8 mm.

As mentioned earlier, the thickness for FRP reinforced slabs was chosen the same as for the steel reinforced slab, even though Equation(9-9) of the CSA Code would lead to a thicker slab, therefore it is necessary to compare the deflections with the

Figure 5.8: Deflections Along Section A-A

Figure 5.9: Deflections Along Section B-B

Figure 5.10: Deflections Along Section D-D

Figure 5.11: Deflections Along Section E-E

maximum permissible values given by Table 9.2 of the Code. The parking structure under consideration was assumed to belong to the group “floor not supporting or attached to non-structural elements likely to be damaged by large deflections”, for which immediate deflection due to the specified live load cannot exceed the value given by $\frac{l_n}{360}$. In the parking structures under consideration, the largest deflection will occur in panel B2 (see Fig. 5.7) where the preceding relation will give approximately 22 mm deflection. The calculated maximum deflection from finite element analysis is less than 8 mm for any of the slabs analyzed. If we assume that the initial deflection is doubled by creep and shrinkage, still the total deflection will be less than the permissible value.

Note that the deflections in the FRP reinforced slabs are slightly larger than the corresponding deflections in the steel reinforced slabs, but the increase is of the order of 10 % only. Surprisingly, the maximum difference between the CFRP and the GFRP reinforced slabs is only about 4 %.

To further appreciate the evolution of deflections with increased load, Figs. 5.12 to 5.15 show the load versus deflection curve of the centre of panels A1, A2, B1, and B2, respectively. Each figure depicts the load-deflection curve for all three parking structures. From these curves we can notice that within the linear range the load-deflection responses of the three slab systems are practically the same. Furthermore, we observe that under service loads the slabs behaved linearly and that yielding did not begin before the load had reached more than twice the service load. Of course, in the case of FRP reinforced slabs, the nonlinearity is not due to yielding but either due to extensive cracking or high compressive stresses in concrete. Notice that once the nonlinear portion of the curve is reached, the FRP reinforced slabs tend to deflect significantly more than the steel reinforced slab, but limiting deflections in the post-yield region is of academic interest because this region falls outside the serviceability limit state.

While deflection control is a serviceability limit state requirement, the amount of

Figure 5.12: Central Deflection of Slab Panel A1

Figure 5.13: Central Deflection of Slab Panel A2

Figure 5.14: Central Deflection of Slab Panel B1

Figure 5.15: Central Deflection of Slab Panel B2

deflection prior to failure is also of interest. Large deflections prior to failure are indicative of a ductile behaviour. Observing the load-deflection curves for the three slab systems, there is little practical difference in their deflection values at failure. At first glance this would appear to be unexpected because FRPs are generally considered to be brittle while steel is rather ductile. In design, however, it is not possible to take advantage of the full ductility of steel because the ductility of the slab is limited by the strain capacity of concrete. Accordingly, generally the steel strain will not reach more than 0.20 % to 0.30 % when the concrete reaches its limiting value of 0.3 %. Since FRPs have ultimate strain of 1.5 % to 2.0 %, this means that their designed ductility of FRP reinforced slab systems can be the same as that of steel.

To verify the preceding assertion, we plot in Figs. 5.16 to 5.18 the strain in the reinforcement along sections A-A, B-B, and C-C at 90 % of the failure load. We observe that the strains in the two types of FRP are practically the same. In the regions of negative moments, where there is a concentration of high stresses/strains, strains in FRP are higher than strains in steel, which is due to the fact that FRPs have linear stress-strain relationship, very high strength and low modulus of elasticity. The relatively low strains in the reinforcement show that the behaviour of the section is controlled by the maximum strain in concrete. Therefore, despite the ability of steel to undergo large strains, it is unable to realize it because of the constraint imposed by the limited strain capacity of concrete.

Finally, it is important to mention that in reality steel reinforced sections will not completely disintegrate upon failure because despite the concrete failure the steel will carry some load by membrane action. In the case of FRP reinforced structures, the rupture of FRP may lead to the complete disintegration of the structure. This aspect of the behaviour of FRP reinforced systems needs to be further investigated. NONLACS cannot show this aspect of the behaviour of slabs because it does not account for geometric nonlinearity.

Figure 5.16: Strains in Steel, CFRP and GFRP Along Section A-A Under 90 % of Failure Load

Figure 5.17: Strains in Steel, CFRP and GFRP Along Section B-B Under 90 % of Failure Load

Figure 5.18: Strains in Steel, CFRP and GFRP Along Section C-C Under 90 % of Failure Load

5.4.2 Stresses in Concrete

Concrete of compressive strength of 30 *MPa*, and of tensile strength of 3.2 *MPa* was used for all three structures. In this section we will examine the following : occurrence of first cracking and its propagation, and the manner in which the concrete stresses vary with load. NONLACS provides the stresses in the middle of each layer, therefore, the extreme concrete fibre stresses were calculated by extrapolating the stresses from the closest concrete layer.

The first cracking occurred under self-weight in the negative moment regions on the top surface of the drop panels. The cracking started at nearly the same load in all three structures, consistent with the fact that cracking is initiated when the tensile strength of concrete is exceeded, and does not depend on the reinforcement type. With further loading the cracking pattern changed for the different reinforcement types.

When the structure is designed to be reinforced with FRP, which has very high strength, no specified yield strength, and a relatively low modulus of elasticity, for the concrete in compression to develop internal equilibrium with the monotonically increasing force in FRP, the neutral axis must shift higher than in a similar section reinforced with steel. When the steel yields, its strain is around 0.2 % and subsequently its stress does not increase (assuming no strain hardening), consequently the compression force in the concrete remains constant, but the stress increases due to the increase in the applied load. This means that the depth of the compression block decreases. This reduction occurs rather rapidly and leads to the eventual failure of the concrete. For the FRP reinforced structures, if the reinforcement ratio is below the balanced condition, then failure will occur upon rupture of the FRP. This type of failure will be sudden and will result in the complete disintegration of the structure. The current finite element analysis cannot distinguish between this type of failure and the type of failure where integrity of the structure is maintained.

All three structures were designed for the same load of 13.3 kN/m^2 , it was found by the finite element analysis that the ultimate load was 25.0, 25.5, and 29.0 kN/m^2 for GFRP, CFRP, and steel reinforced slabs, respectively. At those loads, large portions of the structures were excessively cracked and crushed. The crushing of concrete started to occur at 16 kN/m^2 for FRP reinforced slabs, and at 22 kN/m^2 for the steel reinforced slab. The steel slab carried a load 1.65 times larger than its design load. This could be, as stated earlier, due to the fact that in all the middle strips minimum reinforcement was used, even though the required reinforcement was less than minimum. Therefore, these sections were over-reinforced in the case of steel reinforcement. The FRPs were designed to have exactly the amount of FRP needed, therefore these systems failed at 1.2 times their design load.

The finite element analysis showed that the concrete in the floor slab was subjected to different stress states: biaxial tension, biaxial compression, and tension-compression. The bottom of the drop panel was usually under biaxial compression. This allowed the concrete in this areas to develop higher strength than its uniaxial compressive strength (Park and Paulay, 1974). This state of the stress also affects the development of the bond cracks, i.e. their occurrence is being delayed and the structure behaves more ductile. For those reasons all three structures carried higher loads than their designed loads. Some sections in the middle strips were found to be under combined compression and tension. Failure in this case occurs due to limited maximum tensile strain rather than stress (Park and Paulay, 1974). This can be seen in Figures 5.19 and 5.20 which show the strain distribution through the thickness of the slab at different locations along section A-A. We observe that the strains in CFRP and GFRP reinforced slabs are almost 10 % higher than the strains in the steel reinforced slab. Also the downward shifting of the neutral axis at 90 % of ultimate load in the CFRP and GFRP slabs is evident (see Fig. 5.20). Therefore, the maximum strain in compression will be reached earlier which will lead to early failure.

Let us now examine the variation of concrete stresses at the top and bottom surfaces of the slabs along their critical sections. These stresses will be plotted for each of

Figure 5.19: Strains in the Concrete in Section A-A

Figure 5.20: Strains in Cross-Section 1 and 2 from Fig. 5.19 at 90 % of Failure Load

the three slab systems individually. Figures 5.21 and 5.22 show, respectively, the variation of the longitudinal normal stresses σ_x , along the bottom and top surfaces of section A-A in the steel reinforced slab. Section A-A is along the column strip at the centre of the slab. The finite element analysis shows this section to be the critical section for all the slab systems. Similar stresses for other slab sections are shown in Appendix B. In these and in all subsequent similar figures, a positive stress indicates tension while a negative stress represents compression.

In Figs. 5.21 and 5.22 we observe that the maximum compression occurs in the bottom of the first interior column capital while the maximum tension occurs in the bottom of the middle strip near the slab centreline. These stresses follow the basic shape of the moment diagram, but it is significant to observe the large jump in stresses from a load of 1.5 times the service load, (13.30 kN/m^2), to 0.9 times the ultimate load, (26.1 kN/m^2). As expected, the stresses on the top surface of the slab are smaller (Fig. 5.22) than the compressive stresses in the bottom because the critical sections are under negative bending moment. The maximum concrete stress on the top surface of the slab is actually less than 15 MPa at 0.9 the ultimate load.

To better appreciate the state of stresses, in Figs. 5.23 and 5.24 the transverse normal stresses, σ_y , at the top and bottom surfaces of section A-A are plotted for the same slab. We notice in Fig. 5.23 that the maximum compression exceeds the uniaxial compressive strength of the concrete. This is possible because concrete in this region is subjected to biaxial compression. In the middle strip adjacent to the exterior column capital, the slab is subjected to a combined state of stress. It is well known that under combined stresses the concrete will be weaker than its cylinder strength. Figure 5.24 shows that the transverse normal stresses on the top surface of section A-A are all tension. A zero stress in this graph generally indicates cracking and release of stress. The combination of high transverse tensile stresses with relatively moderate longitudinal compressive stresses in the top of the middle strip near the center of the slab may cause concrete crushing in this region. In design this combined effect is not included because the slab system is analyzed as a grid of orthogonal frames. However,

Figure 5.21: Concrete Stress σ_x at the Bottom of Section A-A in the Steel Reinforced Slab

Figure 5.22: Concrete Stress σ_x at the Top of Section A-A in the Steel Reinforced Slab

in design only 60 % of the uniaxial compressive strength of concrete is actually utilized.

Next let us examine the stresses in Section A-A for the CFRP reinforced slab. Figures 5.25 and 5.26 show the longitudinal normal stresses in the bottom and top surfaces of Section A-A while Figs. 5.27 and 5.28 show the corresponding transverse normal stresses, respectively. By comparing the latter four figures with the corresponding figures for the steel reinforced slabs, we observe little difference in either the magnitude or the manner in which these stresses vary with the load or along the section. It would appear that the difference between the moduli of the CFRP and the steel does not significantly affect the magnitude or distribution of stresses in the concrete. The same conclusion was arrived at by comparing the stresses at other sections in the slab. The reader may refer to Appendix B for verification.

Finally, we consider the concrete stresses in the GFRP reinforced slab. Figures 5.29 and 5.30 show the longitudinal normal stresses on the top and bottom surfaces of section A-A while Figs. 5.31 and 5.32 show the corresponding transverse normal stresses. By comparing the stresses in Fig. 5.29 with the corresponding longitudinal normal stresses in the steel reinforced slab, we notice that the basic distribution of stresses is the same, but the magnitude of the stresses in the GFRP reinforced slab are higher at higher load levels. In particular, at 90 % of the ultimate load, which corresponds to a load of 22.5 kN/m^2 , the longitudinal compressive stresses in the GFRP slab are on the average 10 % to 25 % higher than in the steel reinforced slab. It is important to recall that the ultimate load for the GFRP slab was 25.5 kN/m^2 while for the steel reinforced slab it was 29.00 kN/m^2 . Hence the higher stresses are actually due to a lower load.

The foregoing higher stresses are expected because the GFRP must undergo substantial strain in order to be able to develop a sufficiently large stress. Such large strains would cause the neutral axis to shift closer to the extreme compressive fibre in concrete, causing a reduction in the available compressive area. It is significant to notice that while in an isolated beam such a shift may be quite dramatic, in large

Figure 5.23: Concrete Stress σ_y at the Bottom of Section A-A in the Steel Reinforced Slab

Figure 5.24: Concrete Stress σ_y at the Top of Section A-A in the Steel Reinforced Slab

Figure 5.25: Concrete Stress σ_z at the Bottom of Section A-A in the CFRP Reinforced Slab

Figure 5.26: Concrete Stress σ_x at the Top of Section A-A in the CFRP Reinforced Slab

Figure 5.27: Concrete Stress σ_y at the Bottom of Section A-A in the CFRP Reinforced Slab

Figure 5.28: Concrete Stress σ_y at the Top of Section A-A in the CFRP Reinforced Slab

Figure 5.29: Concrete Stress σ_x at the Bottom of Section A-A in the GFRP Reinforced Slab

Figure 5.30: Concrete Stress σ_x at the Top of Section A-A in the GFRP Reinforced Slab

Figure 5.31: Concrete Stress σ_y at the Bottom of Section A-A in the GFRP Reinforced Slab

Figure 5.32: Concrete Stress σ_y at the Top of Section A-A in the GFRP Reinforced Slab

slab system this does not appear to be the case. This may be in part due to the normally appreciable redistribution of stresses that can occur in continuous slabs. In other words, when a section begins to be highly stressed, the adjacent lightly stressed sections will absorb some of the load and partially relieve the heavily stressed section. Clearly, a full verification of the behaviour observed in the present analyses would require physical testing of an actual slab system.

For the comparison of the stresses in other sections of the GFRP reinforced slab, the reader is referred to Appendix B.

5.4.3 Stresses and Strains in the Reinforcement

In this section, the stresses and the strains in the reinforcement will be presented. It may be recalled that steel with a yield strength of 400 *MPa* and a modulus of elasticity of 200 *GPa* was used, and was placed in two or four layers. The CFRP had strength of 2 260 *MPa*, and modulus of elasticity of 147 *GPa*. This reinforcement was placed in two orthogonal layers only because FRPs are not suitable as compressive reinforcement. GFRP had a strength of 690 *MPa* and a modulus of elasticity of 42 *GPa*. The stresses will be plotted for the reinforcement along the critical section A-A.

Figures 5.33, 5.34 and 5.35 show the variation of the stresses in the steel, CFRP and GFRP, respectively, at different load levels. We notice in Fig. 5.33 that most of the top steel in the negative moment regions, i.e. in the column capitals, have either yielded, or are near yield under ultimate load. Even at 90 % of ultimate load the top steel in the interior column capitals have partially yielded. Notice, however, that in the middle strip near the free edge of the column the steel stresses are relatively small.

The corresponding stresses in the CFRP are shown in Fig. 5.34. We notice the significantly higher stresses in this reinforcement with the maximum value reaching almost 1 100 *MPa*. But despite this large magnitude, the latter stress is only 50 % of the strength of CFRP. This means that in CFRP reinforced slab system, failure was initiated by the crushing of the concrete rather than the rupture of the reinforcement. Unfortunately, unless very high strength concrete is used, it would be difficult to utilize the high strength of CFRP in ordinary reinforced concrete. If one calculated the balanced reinforcement ratio for CFRP, it would yield extremely small values, most likely less than the minimum reinforcement. In practice this would mean widely spaced bars which may not be acceptable. As stated previously, one should investigate to determine an optimum reinforcement strength in combination with a higher strength concrete. It is important to notice that in those regions where the steel had

Figure 5.33: Stresses in the Steel Reinforcement at Section A-A

Figure 5.34: Stresses in the CFRP Reinforcement at Section A-A

Figure 5.35: Stresses in the GFRP Reinforcement at Section A-A

not yielded, the CFRP and steel stresses are comparable.

As for the stresses in the GFRP, Figure 5.35 shows those stresses along section A-A. We notice that once again the maximum stress in the reinforcement reached only about 50 % of its strength. As stated before, in the case of the slab system reinforced with a low modulus material such as GFRP, failure will be initiated by the limited strain capacity of the concrete. One could see that quite clearly in Fig. 5.19, which shows that near ultimate load, the maximum concrete compressive strain is at its limit. Again this problem may be partially overcome by using high strength concrete.

To compare the magnitude and distribution of the stresses among the three types of reinforcement, Fig. 5.36 shows the stresses in the steel, CFRP and GFRP under full service load while Fig. 5.37 shows the stresses in the same reinforcement at ultimate. Figure 5.37 clearly shows the development of rather high stresses in the CFRP and GFRP at ultimate. Note on the other hand, the rather small stresses in the GFRP under service load conditions (Figure 5.36). This small stress is expected due to the small modulus of the GFRP and the fact that the strains are relatively small under service loads.

To further verify this point Fig. 5.38 shows the strain in the three types of reinforcement along section A-A under service load conditions. One can clearly see that the strains in all three reinforcement types are of a similar magnitude. Therefore GFRP with its low modulus of elasticity could not produce large stresses. We notice that at lower loads, representing serviceability state, the strain in all three reinforcement types is similar while the stress is largest in steel followed by CFRP and GFRP. However, at ultimate in the most heavily stressed areas of drop panels the largest stresses are in CFRP (see Fig. 5.37), due to the fact that steel has already yielded and cannot increase its stress any more and GFRP has smaller modulus of elasticity. Let us examine this point further.

Figure 5.16 on page 123 shows the strains in the three reinforcement types at the same section at 90 % of ultimate. It is significant to examine the latter strains carefully.

Figure 5.36: Comparison of Stresses in the Reinforcement at Section A-A Under Serviceability Loads

Figure 5.37: Comparison of Stresses in the Reinforcement at Section A-A Under Ultimate Loads

Notice that the maximum strain in the steel is only 0.003 which is one and a half times the yield strain, while the CFRP and GFRP undergo strains of 0.005 and 0.004, respectively. This shows that even though the steel can undergo much higher strains and is quite ductile, steel reinforced concrete structures cannot take advantage of the steel strain capacity due to the limited strain capacity of concrete. In fact it may be difficult to achieve strains of 1 % to 1.5 % in the reinforcement.

In practice steel reinforced slabs may be able to undergo very large deformations and they may be able to resist loads by membrane action, but in conventional design these attributes are not directly considered. Hence from the conventional design point of view, the limited strain capacity of FRPs compared to that of steel does not necessarily pose any problem and that the maximum deformations of FRP reinforced structures at failure may be comparable to that of steel reinforced structures, provided the FRP reinforced section is slightly over reinforced.

For the sake of completeness the reinforcement strain in other sections of the slabs are plotted in Appendix C. Those strains follow generally the same behaviour as those along section A-A.

In summary, the present nonlinear analyses have shown that in flat plate structures with column capitals, the deformations and stress of both the CFRP and GFRP reinforced slabs are quite similar to those of similar steel reinforced slabs. It would appear that the low modulus and limited strain capacity of FRPs do not create special problems. It is also evident that the large strain capacity of steel reinforcement cannot be utilized in reinforced concrete due to the limited strain capacity of concrete. It was also shown that for slabs satisfying the existing Code provisions with regard to thickness requirements, deflection under service loads is not a problem, regardless of the type of reinforcement.

Figure 5.38: Strains in Steel, CFRP and GFRP Along Section A-A Under Service Load

Chapter 6

Summary, Conclusions and Recommendations

6.1 Summary

This thesis presented the nonlinear finite element analysis of FRP reinforced concrete flat slab structures. The analysis was performed using the finite element program NONLACS. Due to the difference between the mechanical properties of FRP reinforcement and that of steel, some changes to the program were made. The suitability of the finite element program NONLACS for the analysis of FRP reinforced concrete members was demonstrated. Several FRP reinforced concrete beams and slabs, previously tested by other investigators, were modelled over the whole range of loading up to failure. The results of the analysis compared well with the available experimental data. This demonstrated the ability of the nonlinear finite element method to model FRP reinforced concrete structures.

Next a number of steel and FRP reinforced simple slabs were analyzed over their full loading range. The results showed that FRP reinforced slabs would be able to carry the same level of loads as steel reinforced slabs, and their deflections would not

be necessarily larger than those of steel reinforced slabs, but they will fail suddenly, without indicating large deflections. The analyses also showed that certain parameters in the finite element analysis, such as the tensile strength of concrete and the tension stiffening parameters do not have significant effect on the ultimate strength of the slabs.

Finally, a two storey flat slab parking structure was designed according to the CSA Standard. The structure was designed assuming either steel, CFRP, or GFRP as reinforcement. Each structure was subsequently analyzed by the program NONLACS to determine stresses and failure load. The analysis showed that continuous slabs even if reinforced with relatively brittle reinforcement such as GFRP and CFRP can undergo a great amount of deformation prior to failure. This may be in part due to the good moment redistribution characteristics of slabs. The analysis also showed that FRP reinforced slab will generally fail due to the limiting strain capacity of concrete. In view of those findings, it is feasible to use FRPs as reinforcement for parking structures.

6.2 Conclusions

The analysis performed in this investigation support the following conclusions:

1. The nonlinear finite element method can predict the deformations and stresses of FRP reinforced concrete structures reasonably well.
2. The program NONLACS can capture the full response of FRP reinforced concrete beams and slabs, provided failure is not initiated by loss of bond between the concrete and the reinforcement.
3. Program NONLACS can predict the strength and deformations of structures strengthened by FRP sheets and panels epoxied to concrete beams. The program cannot predict the premature peeling of FRP from the concrete. This inability

is partly because of lack of available experimental data about FRP-concrete interfacial strength.

4. Flat slab concrete parking structures with column capitals but without shear reinforcement when reinforced with either carbon fibre reinforced or glass fibre reinforced plastic bars have a maximum deflection under service loads smaller than the allowable deflection specified by the CSA Code. For the particular slabs that were analyzed, the maximum service load deflection was 8 mm compared to the permissible deflection of 17 mm . The maximum deflection was not significantly affected by the elastic modulus of the reinforcement.
5. The deflections of FRP reinforced slabs at higher load levels (i.e. larger than the service loads) may be larger than the corresponding deflections of similar steel reinforced slabs. However, since the large deflection will occur beyond the serviceability limit state, they will not constitute any practical problem.
6. FRP reinforced flat slab structures can reach their design ultimate load, but failure will be nearly always initiated by the failure of concrete, followed by the rupture of the FRP.
7. Although steel in isolation can undergo large strains up to 15 %, in normally reinforced concrete, the strain in the steel at ultimate does not usually exceed 1 % to 1.5 %. Further increase would require strains in concrete well beyond its limited strain capacity of about 0.4 %. Since most FRPs have maximum strain capacity of 1 % to 3 %, from practical point of view their strain capacity is almost the same as the usable strain capacity of steel. For that reason, the present analyses did not show much difference between the strength and deformations of FRP reinforced parking structures and the corresponding steel reinforced structures.
8. Column capitals can be used to avoid shear reinforcement in FRP reinforced parking structures and to reduce their deflections under service loads. However,

the amount of shear resisted by concrete in FRP reinforced slabs is not well established.

9. The thickness for FRP reinforced flat slabs may be chosen to be basically the same as the thickness of similar steel reinforced slab with $f_y = 400 \text{ MPa}$.
10. It is difficult to take advantage of the full strength of FRPs in reinforced concrete, even if very small reinforcement ratios are used. To be able to take advantage of their full strength, one needs to use either high strength concrete or prestress the structure. In present analyses, the FRP reinforcement reached only 50 % of its strength prior to failure while the steel reached yield in the critical moment regions.
11. Due to its high tensile strength, the area of FRP needed to carry the same ultimate moment as the steel reinforced section was only one third to one fifth of the corresponding area of steel.
12. For the slab systems analyzed here, the modulus of elasticity of FRP was found to have little influence on the deflections of two-way slab systems under service loads.
13. The behaviour of isolated simple slabs reinforced with FRP is noticeably different from the behaviour of FRP reinforced flat slab systems. Whereas isolated slabs will fail abruptly once the concrete reached its compressive strain capacity, flat slab structures will fail in a more ductile manner. This may be due to the stress redistribution capacity of continuous slabs.
14. This study showed that from the strength and serviceability load deflection points of view, it is possible to replace steel by either GFRP or CFRP as reinforcement in flat slab structures of the type analyzed in this investigation. The structures analyzed reached their design ultimate load without experiencing excessive deformations under service loads or showing too little deformation prior to failure.

6.3 Recommendations for Future Work

1. There is an urgent need for large scale tests to determine whether the type of reinforcement has any significant effect on the deflections of flat slab structures within the serviceability limit state.
2. Tests need to be performed on continuous slab systems reinforced with FRP to determine their moment redistribution capacity and their ductility.
3. Further analytical studies need to be performed on flat slabs, with or without column capitals, but with different span ratios and slab thickness to see if the type of reinforcement will have any significant effect on the slab deflections and strength.
4. Further analyses can be performed to study the effect of creep, shrinkage and thermal variations on the deformations and strength of FRP reinforced concrete parking structures.
5. The finite element program NONLACS needs to be modified in order to be able to predict failure due to debonding and reinforcement peel off.
6. Program NONLACS needs to be modified so that columns can be modelled as beam elements rather than shell elements. This would make the analysis simpler and would require less computer time.
7. Program NONLACS should be modified to include large deformations and the effect of membrane action on the behaviour of FRP reinforced slabs.

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Appendix A

Design of Two-Way Slab System By the Equivalent Frame Method

In the following section, the design of one floor of one of the parking structures analyzed in this thesis will be presented. The structure will be designed using the Equivalent Frame Method of CSA Standard A23.3-M84. For design examples of this type reference may be made to Pillai and Kirk (1988).

A.1 Design Parameters

A.1.1 Material Properties

Concrete

Compressive strength $f'_c = 30$ MPa

Modulus of elasticity $E_c = 5\,000 \sqrt{f'_c} = 27\,386$ MPa

Modulus of rupture $f_t = 0.6 \sqrt{f'_c} = 3.28$ MPa

Steel

Yield stress $f_y = 400$ MPa

Modulus of elasticity $E_s = 200$ GPa

Carbon-Fibre-Reinforced Plastic

For the design, LEADLINE, carbon fibre based reinforced plastic rods were used.

Ultimate strength $f_u = 2\,260$ MPa

Modulus of elasticity $E_{FRP} = 147$ GPa

Ultimate strain $\epsilon_{ult} = 0.015$

Glass-Fibre-Reinforced Plastic

Glass fibre bars ISOROD were used.

Ultimate strength $f_u = 690$ MPa

Modulus of elasticity $E_{FRP} = 42$ GPa

Ultimate strain $\epsilon_{ult} = 0.018$

A.1.2 Layout Dimensions and Load Arrangement

The layout of the parking structure is shown in Fig. A.1. It has four bays in each direction and for simplicity is made symmetrical about both planar axes. The interior bays have a span of 8.0 m while the exterior bays are 6.4 m long. The slab is supported on columns with dimensions $500 \times 500 \times 3\,000$ mm. This layout was chosen according to the recommendations of ACI publication on parking structures (SBM-3 1991).

Slab Thickness

According to CSA Standard A23.3-M84 the slab thickness may not be less than

$$h_{min} = \frac{l_n (800 + \frac{f_y}{1.5})}{36\,000} = 236.8 \text{ mm} \quad (A.1)$$

Figure A.1: Layout of the two-way slab

In accordance with Clause 9.5.3.3.2 of CSA, because of the presence of drop panels, the thickness may be reduced by 10 % to 215 mm while drop panels need to be extended 55 mm below the slab surface. The drop panels will be assumed to be square 2 700 × 2 700 mm around the interior columns, but 1 600 × 2 700 mm around the exterior columns.

Effective Depth

The following values for the effective depths will be used in all future calculations:

- $d_{1,slab} = 215 - 20 - 10/2 = 190 \text{ mm}$
- $d_{2,slab} = 215 - 20 - 10 - 10/2 = 180 \text{ mm}$
- $d_{1,dp} = 270 - 20 - 10/2 = 245 \text{ mm}$
- $d_{2,dp} = 270 - 20 - 10 - 10/2 = 235 \text{ mm}$

where subscripts 1 and 2 refer to the directions of the two orthogonal steel layers near the top and bottom surfaces of the slab. Because the effective depth in one direction will be less than in the orthogonal direction, the amount of reinforcement may be more in the direction with smaller effective depth. This means that despite the apparent symmetry of the structure, the reinforcement will not be symmetric, and thus the final stresses in the concrete and the reinforcement may not exhibit perfect symmetry.

Load Calculation

Slab selfweight	$1.25 \times (0.215 \times 2.4 \times 9.81)$	$= 6.327 \text{ kN/m}^2$
Superimposed dead load	1.25×1.44	$= 1.800 \text{ kN/m}^2$
Self weight of drop panel	$1.25 \times (0.055 \times 2.4 \times 9.81)$	$= 1.618 \text{ kN/m}^2$
Live load	1.5×2.39	$= 3.585 \text{ kN/m}^2$
<hr/>		
Total factored load on slab	6.327 kN/m^2	
	1.800 kN/m^2	
	3.585 kN/m^2	
	<hr/>	
	11.712 kN/m^2	
<hr/>		
Total factored load on drop panel	11.712 kN/m^2	
	1.618 kN/m^2	
	<hr/>	
	13.330 kN/m^2	

A.2 Preliminary Shear Capacity

The shear capacity will be checked using an approximate method to ensure that the previously determined slab thickness will be adequate for carrying the shear forces in the vicinity of columns. More detailed shear calculations will be performed after the equivalent frame analysis. The critical sections for shear are shown in Fig. A.2.

One-way shear

$$V_f = w_f \times l \times \left(\frac{l_n}{2} - d \right)$$

$$V_c = 0.2 \lambda \phi_c \sqrt{f'_c} b_w d \quad \text{If } V_f < V_c \text{ no shear reinforcement is needed.}$$

$$V_f = 13.331 \times 1.0 \times \left(\frac{8000}{2} - 190 \right) = 50.791 \text{ kN}$$

$$V_c = 0.2 \times 1.0 \times 0.6 \times \sqrt{30} \times 1000 \times 240 = 124.88 \text{ kN}$$

Figure A.2: The critical sections for one-way and two-way shear

$$V_f \ll V_c \quad \text{O.K.}$$

Punching shear

Interior column

$$V_f = w_f \times (l_n^2 - (b_c + d_s)^2)$$

$$V_r = 0.4 \times \lambda \phi_c \sqrt{f'_c} b_o d$$

$$V_f = 11.712 (8.0 \times 8.0 - (0.70)^2) = 743.9 \text{ kN}$$

$$V_r = 0.4 \times 1.0 \times 0.6 \times \sqrt{30} \times (4 \times 700) \times 200 = 736.0 \text{ kN}$$

$$1.2 \times V_f = 893.0 \text{ kN} \not\leq V_r = 736.0 \quad \text{Not O.K.}$$

Because the shear resistance in the vicinity of the interior column is smaller than the factored shear force, drop panels will be designed. According to the Code, the drop panel must extend a minimum of $l_n/6$ from the face of the column. Therefore, the following size of drop panel will be used: thickness below the slab 55 mm, planar dimensions $2\,700 \times 2\,700$ mm.

3 of/de 3

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

In the vicinity of drop panel

$$V_f = 11.712 \times (8.0^2 - 0.740^2) + 1.618 \times (2.7^2 - 0.740^2) = 754.0 \text{ kN}$$

$$b_o = 4 \times 0.740 = 2.96 \text{ m}$$

Average shear stress:

$$v_f = \frac{V_f}{A_c} = \frac{754.0 \times 10^3}{2960 \times 240} = 1.06 \text{ MPa}$$

$$v_r = \frac{V_c}{b_o d} = (1 + \frac{2}{\beta_c}) 0.2 \lambda \phi_c \sqrt{f'_c} \leq 0.4 \lambda \phi_c \sqrt{f'_c}$$

$$v_r = 0.4 \times 1.0 \times 0.6 \times \sqrt{30} = 1.31 \text{ MPa}$$

$$1.06 < 1.31 \quad \text{O.K.}$$

From the edge of drop panel:

$$V_f = 11.712 \times (8.0^2 - 2.89^2) = 651.7 \text{ kN}$$

$$v_f = \frac{651.7 \times 10^3}{(4 \times 2890) \times 190} = 0.30$$

$$v_f = 0.30 < v_r = 1.31 \quad \text{O.K.}$$

Exterior column

$$V_f = w_f \times (l_1 / 2 \times l_2 - (b_c + d_{dp})^2)$$

$$V_f = 13.331 (3.2 \times 8.0 - 0.740 \times 0.620) = 335.1 \text{ kN}$$

$$V_r = 0.4 \times 0.6 \times \sqrt{30} \times (2 \times (740 + 620)) \times 240 = 858.1 \text{ kN}$$

$$\frac{V_c}{V_f} = 2.5 > 1.6 \quad \text{O.K.}$$

Corner column

$$V_f = w_f \times ((l_1)^2 - b_o^2)$$

$$V_f = 13.331 (3.2^2 - 0.620^2) = 131.4 \text{ kN}$$

$$V_r = 0.4 \times 0.6 \times \sqrt{30} \times (2 \times (620 + 620)) \times 240 = 782 \text{ kN}$$

$$\frac{V_c}{V_f} = 5.92 > 2.0 \quad \text{O.K.}$$

Figure A.3: Column Properties for Equivalent Frame Analysis

A.3 Relative Stiffness Parameters of Equivalent Frame

Column Stiffness, K_c

Referring to Fig. A.3

$$\left. \begin{array}{l} H = 3000 \text{ mm} \\ H_c = 3000 - (162.5 + 107.5) = 2730 \text{ mm} \end{array} \right\} \quad H/H_c = 3.0 / 2.73 = 1.09$$

$$t_a = 162.5 \quad t_b = 107.5$$

$$\frac{t_a}{t_b} = 1.51 \quad \frac{t_b}{t_a} = 0.66$$

From Table 14.7 in Pillai and Kirk (1988):

$$\text{for } \left. \begin{array}{l} \frac{H}{H_c} = 1.1 \\ \frac{t_a}{t_b} = 1.5 \end{array} \right\} \begin{array}{l} K_{AB} = 5.235 \\ C_{AB} = 0.55 \end{array}$$

$$\text{for } \left. \begin{array}{l} \frac{H}{H_c} = 1.1 \\ \frac{t_a}{t_b} = 0.6 \end{array} \right\} \begin{array}{l} K_{BA} = 4.91 \\ C_{BA} = 0.59 \end{array}$$

$$K_c = K E I_c / H$$

$$I_c = 500^4 / 12 = 5.208 \times 10^9 \text{ mm}^4$$

$$K_{c, \text{below}} = 5.235 E_c 5.208 \times 10^9 / 3000 = 9.088 E_c \times 10^6$$

$$K_{c, \text{above}} = 4.91 E_c 5.208 \times 10^9 / 3000 = 8.524 E_c \times 10^6$$

Torsional Member

Because this slab does not have beams, torsional member is assumed to be the portion of slab having width equal to the column cross-section (500×270). The following is calculation of stiffnesses for the exterior frame 2-2.

$$C_{\text{ext}} = (1 - 0.63 \times \frac{270}{500}) \times \frac{270^3 \times 500}{3} = 2.1645 \times 10^9 \text{ mm}^4$$

$$C_{\text{int}} = C_{\text{ext}}$$

$$c_2 / l_2 = 500 / 7200 = 0.0694$$

$$K_t = \Sigma \frac{9 \times E_c \times C}{l_2 (1 - \frac{c_2}{l_2})^3}$$

$$K_{t,ext} = \frac{9 \times E_c \times (2.1645 \times 10^9)}{7200 (1 - 0.0694)^3} = 3.3572 \times 10^6 E_c$$

$$K_{t,int} = \frac{9 \times E_c \times (2.1645 \times 10^9)}{7200 (1 - 0.0694)^3} = 3.3572 \times 10^6 E_c$$

Equivalent Column Stiffness

$$K_{EC} = \frac{(\Sigma K_C) K_t}{\Sigma K_C + K_t}$$

In this case $K_{EC,ext} = K_{EC,int}$

$$K_{EC} = 2.8197 \times 10^6 E_c$$

Slab Stiffnesses and Fixed End Moment Coefficients

Refer to Fig. A.4 for loading details and the free body diagram of the slab.

$$C_{F1} = C_{N1} \quad C_{F1} = 500$$

$$C_{F2} = C_{N2} \quad C_{F2} = 500$$

$$C_{N1}/l_1 = 0.5/8.0 = 0.0625$$

$$C_{N2}/l_2 = 0.5/7.2 = 0.0694$$

Interpolating the values given in Table 14.6 in Pillai and Kirk (1988):

Stiffness factor, $K_{NF} = 5.979$

Carry over factor, $C_{NF} = 0.6$

FEM coefficient for uniformly distributed load $m_{NF,1} = 0.0933$

For additional load due to drop panel:

$$a = 0 \quad b - a = 0.17 \quad m_{NF,2} = 0.0339 \quad (\text{near end})$$

$$a = 0.8 \quad b - a = 0.2 \quad m_{NF,3} = 0.00186 \quad (\text{far end})$$

Figure A.4: Details of loading on the slab and drop panel

Moment of inertia of the slab section beyond the drop panel

$$I_s = 8000 \times 215^3 / 12 = 6.6256 \times 10^9 \text{ mm}^4$$

$$K_s = K_{NF} E_c I_s / l_1 = 4.952 \times 10^6 E_c$$

A.4 Equivalent Frame Analysis

Moment distribution factors for slab-beam members are:

At exterior supports columns

$$\frac{K}{\sum K} = 0.637$$

At interior supports columns

$$\frac{K}{\sum K} = 0.389$$

Since the specified live load (2.39 kN/m^2) is less than 75 % of dead load, the factored moments will be calculated using a single loading pattern, assuming full live and dead

load on all spans.

$$\text{UDL on slab} \quad 13.331 \times 7.2 = 84.326 \text{ kN/m}$$

$$\text{UDL on drop panel} \quad 1.618 \times \frac{0.17}{0.2} \times 2.7 = 3.713 \text{ kN/m}$$

Slab fixed end moments (FEM):

$$FEM_{NF} = \sum m_{NF} w l_1^2$$

$$\text{For long spans} \quad FEM_{NF}^{8.0} = 507.914 \text{ kN m}$$

$$\text{For short spans} \quad FEM_{NF}^{6.4} = 325.100 \text{ kN m}$$

We perform the moment distribution as usual to obtain the internal forces and moments.

Maximum Shear Forces long span

$$V_L = 343.2 - \frac{(M_L + M_R)}{8.0}$$

$$V_R = 343.2 + \frac{(M_L + M_R)}{8.0}$$

short span

$$V_L = 275.74 - \frac{(M_L + M_R)}{6.4}$$

$$V_R = 275.74 + \frac{(M_L + M_R)}{6.4}$$

Location of zero shear

$$x = \frac{V_L - 5.989}{84.33}$$

Maximum positive moment

$$+M_{max} = M_L + V_L(x) - 1.618 \times 2.7 \times 1.35 \times (x - 0.675) - 13.331 \times 7.2 \times x^2 / 2$$

Negative moment at column face

$$M_{CF}^L = M_L + V_L(0.25) - (88.695)(0.25)^2 / 2$$

$$M_{CF}^R = M_R - V_R(0.25) - (88.695)(0.25)^2 / 2$$

Table A.1: Equivalent Frame Analysis of Edge Frame 2-2

	A	B	B	C
$M_{Col.}$ (kN.m)	-114	469	-485	522
V_L, V_R (kN)	220.3	331.2	338.6	347.8
x (m)	2.54		3.94	
$+M_{max}$ (kN.m)	162.5		175.3	
M_{CF} (kN.m)	-61.7	388.9	-403.1	437.8
$\bar{M}$ (kN.m)	387.8		595.75	
M_o (kN.m)	369.6		595.6	
Correction Factor $M_o/\bar{M}$	0.9530		0.9997	
Design Moment M^- (kN.m)	58.8	370.6	403.0	437.7
Design Moment M^+ (kN.m)	154.9		175.3	

Because this structure satisfies both the requirements of EFM and DDM, the Code permits reduction of moments received from the frame analysis. According to the Clause 13.7.7.4 the analytical moments can be reduced such that sum of positive and average negative moments, $\bar{M}$, does not exceed the value of static design moment M_o , calculated according to Eq. A.2.

$$M_o = w_f \times l^2 / 8 \quad (A.2)$$

Static design moment

short span 369.6 kN m

long span 595.6 kN m

Table A.1 includes the results from the moment distribution method. Every column represents one span of the frame. Only one half of the frame was analyzed, due to the symmetry.

Table A.2: Equivalent Frame Analysis of Middle Frame 3-3

	A	B	B	C
$M_{Col.}$ (kN.m)	-154.4	508.75	-537	580
V_L, V_R (kN)	306	361	375	386
x (m)	3.2		3.9	
$+M_{max}$ (kN.m)	329		195	
M_{CF} (kN.m)	-81	421	-628	486
$\bar{M}$ (kN.m)	580		752	
M_o (kN.m)	410.3		661.4	
Correction Factor $M_o/\bar{M}$	0.7		0.8	
Design Moment M^- (kN.m)	57.2	297.9	551.7	427.4
Design Moment M^+ (kN.m)	232.8		171.4	

Distribution of Moments to Column Strip and Middle Strip

$$l_2/l_1 \text{ short span } 7.2/6.4 = 1.125$$

$$\text{long span } 7.2/8.0 = 0.9$$

$$\alpha_1 = 0 \quad (\text{no beam})$$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_{s,1}} = 0.181$$

Column Strip Share

Interior negative moment

$$l_2/l_1 = 0.9 \quad \alpha_1 = 0 \quad 75\%$$

Exterior negative moment

$$l_2/l_1 = 1.125 \quad \alpha_1 = 0 \quad \beta_t = 0.181 \quad 98\%$$

Positive moment

$$\alpha_1 = 0 \quad 60\%$$

Table A.3: Design Moments

Frame 2-2						
Total Fact. Mom. (kN.m)	-58.80	154.90	-370.60	-403.00	175.30	-437.70
Column Strip (kN.m)	-52.92	92.94	-277.95	-302.25	105.18	-328.27
Middle Strip (kN.m)	-5.88	61.96	-92.65	-100.75	70.12	-109.42
Frame 3-3						
Total Fact. Mom. (kN.m)	-57.20	232.80	-297.90	-551.70	171.10	-424.40
Column Strip (kN.m)	-56.20	139.70	-223.40	-413.80	102.80	-320.50
Middle Strip (kN.m)	-0.90	93.10	-74.50	-137.90	68.50	-106.80

Middle Strip Share

<u>Internal negative moment</u>	25%
<u>External negative moment</u>	2%
<u>Positive moment</u>	40 %

Table A.3 shows the design moments for both internal and external frames

A.5 Design of Flexural Reinforcement

A.5.1 Steel Reinforced Frames

The design of flexural reinforcement for both the edge frame and the middle frame is shown in the Table A.4. The reinforcement in the orthogonal direction was designed similarly.

Table A.4: Design of Flexural Reinforcement

Frame 2-2 Column Strip					
Moment (kN.m)	52.92	92.94	302.25	105.18	328.27
Strip width (mm)	2 700	3 600	2 700	3 600	2 700
Effective depth (mm)	245	190	245	190	245
$K_r = M_f \times 10^6 / bd^2$	0.665	0.715	1.865	0.809	2.025
$\rho \times 10^{-2}$ [Tab.5.4. Pillai et al.(1988)]	0.200	0.220	0.587	0.242	0.634
$A_s = \rho bd$ (mm ² / strip)	1 323	1 477	3 883	1 655	4 194
Frame 2-2 Middle Strip					
Moment (kN.m)	5.88	61.96	100.75	70.12	109.42
Strip width (mm)	4 500	3 600	4 500	3 600	4 500
Effective depth (mm)	190	190	190	190	190
$K_r = M_f \times 10^6 / bd^2$	0.665	0.665	0.665	0.665	0.674
$\rho \times 10^{-2}$ [Tab.5.4. Pillai et al.(1988)]	0.200	0.200	0.200	0.200	0.203
$A_s = \rho bd$ (mm ² / strip)	1 710	1 368	1 710	1 368	1 740
Frame 3-3 Column Strip					
Moment (kN.m)	56.20	139.70	413.80	102.80	320.50
Strip width (mm)	2 700	4 000	2 700	4 000	2 700
Effective depth (mm)	245	190	245	190	245
$K_r = M_f \times 10^6 / bd^2$	0.665	0.967	2.553	0.789	2.254
$\rho \times 10^{-2}$ [Tab.5.4. Pillai et al.(1988)]	0.200	0.304	0.830	0.239	0.721
$A_s = \rho bd$ (mm ² / strip)	1 323	2 310	5 490	1 816	4 768
Frame 3-3 Middle Strip					
Moment (kN.m)	0.90	93.10	137.90	68.50	106.80
Strip width (mm)	5 300	4 000	5 300	4 000	5 300
Effective depth (mm)	190	190	190	190	190
$K_r = M_f \times 10^6 / bd^2$	0.665	0.665	0.665	0.665	0.665
$\rho \times 10^{-2}$ [Tab.5.4. Pillai et al.(1988)]	0.200	0.200	0.200	0.200	0.200
$A_s = \rho bd$ (mm ² / strip)	2 597	1 520	2 597	1 520	2 597

Continuous bottom reinforcement for structural integrity

selfweight of slab + specified dead load	6.50 kN/m^2
equivalent selfweight of drop panel	0.33 kN/m^2
specified live load	<u>2.39 kN/m^2</u>
total specified load	9.21 kN/m^2

$$A_{sb} = \frac{0.5 (w_s \times 10^{-3})}{\phi_s f_y} l_s l_n$$

where w_s is the larger of total specified load calculated above, or $2 \times$ dead load. In this case, twice dead load governs

$$2 \times (6.50 + 0.33) = 13.66 \text{ kN/m}^2 > 9.21 \text{ kN/m}^2$$

$$A_{sb} = 1205.3 \text{ mm}^2$$

A.5.2 FRP Reinforced Frames

Two kinds of FRP were used as reinforcement: carbon-fibre reinforced plastics known as LEADLINE, and glass-fibre reinforced plastics called ISOROD. The material properties for these can be found on page 167.

In the design, the strain compatibility was assumed. When the ultimate strain in concrete in compression is reached, 75 % of the ultimate strain in FRP is reached.

$$a = 0.85 \times \frac{0.003 \times d}{0.003 + 0.75 \times \epsilon_{FRP}}$$

Under these conditions, and knowing the ultimate moment, the area of FRP was found. The following formula was used

$$A_{FRP} = \frac{M \times 10^6}{f_{FRP} (d - \frac{a}{2})}$$

Since the material resistance factors for FRP are not known, they were not included in the design. The design can be found in Table A.5 for both frames reinforced with

CFRP reinforcement, and in the Table A.6 for the frames reinforced with GFRP reinforcement.

A.6 Shear Stresses

Interior Column

Column B3

The shear stresses will be checked at the condition that not only the total specified load was applied on the structure, but also an additional load of 11 kN, as specified in the CSA Standard A23.3-M84. This load was not considered for the calculation of flexural reinforcement, because it does not increase the maximum moments very much. It is much more important in the calculations for shear stresses.

$$\text{Reaction } R = 372 + 376 + 11 = 759 \text{ kN}$$

$$\text{Unbalanced moment } M = 545 - 531.2 = 13.8 \text{ kN.m}$$

$$\text{Average effective depth } d = 240 \text{ mm}$$

$$V_f = 759 - (13.331 + 1.618) \times 0.74^2 = 751.7 \text{ kN}$$

$$\text{perimeter } b_o = 4 \times 740 = 2960 \text{ mm}$$

Moment transformed by shear is 40 % of unbalanced moment

$$M_{fv} = 0.4 \times 13.8 = 5.52 \text{ kN.m}$$

$$c_{AB} = (c_1 + d) / 2 = 370 \text{ mm}$$

$$J_c = \frac{(c_1 + d)d^3}{6} + \frac{(c_1 + d)^3 d}{6} + \frac{(c_1 + d)^2 (c_2 + d)d}{2} = 6.654 \times 10^{10} \text{ mm}^4$$

$$\frac{J_c}{c_{AB}} = \frac{66.541 \times 10^9}{370} = 179.84 \times 10^6 \text{ mm}^3$$

maximum shear stress:

$$v_{AB} = \frac{V_f}{b_o d} + \frac{M_{fv}}{J_c} c_{AB} = 1.09 \text{ MPa}$$

Table A.5: Design of CFRP Reinforcement

Frame 2-2 Column Strip					
Moment (kN.m)	52.92	92.94	302.25	105.18	328.27
Effective depth (mm)	245	190	245	190	245
a (mm)	43.84	34	43.84	34	43.84
A_{CFRP} (mm^2 / m)	123.49	279.65	705.31	316.49	766.04
Frame 2-2 Middle Strip					
Moment (kN.m)	5.88	61.96	100.75	70.12	109.42
Effective depth (mm)	190	190	190	190	190
a (mm)	34	34	34	34	34
A_{CFRP} (mm^2 / m)	17.7	186.44	303.16	210.99	329.26
Frame 3-3 Column Strip					
Moment (kN.m)	56.20	139.70	413.80	102.80	320.50
Effective depth (mm)	245	190	245	190	245
a (mm)	43.84	34	43.84	34	43.84
A_{CFRP} (mm^2 / m)	196.00	357.30	820.80	262.90	635.70
Frame 3-3 Middle Strip					
Moment (kN.m)	0.90	93.10	137.90	68.50	106.80
Effective depth (mm)	190	190	190	190	190
a (mm)	34	34	34	34	34
A_{CFRP} (mm^2 / m)	152.00	238.10	352.70	175.20	273.10

Table A.6: Design of GFRP Reinforcement

Frame 2-2 Column Strip					
Moment (kN.m)	52.92	92.94	302.25	105.18	328.27
Effective depth (mm)	245	190	245	190	245
a (mm)	37.86	29.37	37.86	29.37	37.86
A_{GFRP} (mm^2 / m)	399.12	903.89	2 279.58	1 022.93	2 475.86
Frame 2-2 Middle Strip					
Moment (kN.m)	5.88	61.96	100.75	70.12	109.42
Effective depth (mm)	190	190	190	190	190
a (mm)	29.37	29.37	29.37	29.37	29.37
A_{GFRP} (mm^2 / m)	57.19	602.59	979.85	681.95	1 064.21
Frame 3-3 Column Strip					
Moment (kN.m)	56.20	139.70	413.80	102.80	320.50
Effective depth (mm)	245	190	245	190	245
a (mm)	37.86	29.37	37.86	29.37	37.86
A_{GFRP} (mm^2 / m)	513.00	1 154.80	2 652.80	849.80	2 054.60
Frame 3-3 Middle Strip					
Moment (kN.m)	0.90	93.10	137.90	68.50	106.80
Effective depth (mm)	190	190	190	190	190
a (mm)	29.37	29.37	29.37	29.37	29.37
A_{GFRP} (mm^2 / m)	513.00	769.90	1 139.90	566.30	882.90

$$v_r = (0.4)(1.0)(0.6)\sqrt{30} = 1.31 \text{ MPa}$$

$$v_{AB} < v_r$$

O.K.

In the vicinity of interior drop panel

$$V_f = 751.7 \text{ kN}$$

$$b_o = 4 \times 2890 = 11560 \text{ mm}$$

$$c_{AB} = 2890 / 2 = 1445 \text{ mm}$$

$$J_c = 3.838 \times 10^{11}$$

$$v_{AB} = \frac{751.70}{11560 \times 190} + \frac{5.52 \times 10^6}{3.838 \times 10^{11}} \times 1445 = 0.363 \text{ MPa}$$

$$v_{AB} < v_r$$

O.K.

Exterior Column

Column A3

$$R = 240 + 11 = 255 \text{ kN}$$

$$V_f = 255 - (13.331 + 1.618) \times 0.74 \times 0.62 = 248.88 \text{ kN}$$

$$\text{perimeter } b_o = 2 \times 620 + 740 = 1980 \text{ mm}$$

$$M_{fv} = 0.4 \times 109.5 = 43.8 \text{ kN.m}$$

$$c_{AB} = \frac{(c_1 + d/2)^2}{2c_1 + c_2 + 2d} = 194 \text{ mm}$$

$$J_c = 21.65 \times 10^9 \text{ mm}^4$$

$$\frac{J_c}{c_{AB}} = \frac{21.65 \times 10^9}{194} = 111.599 \times 10^6 \text{ mm}^3$$

maximum shear stress:

$$v_{AB} = \frac{V_f}{b_o d} + \frac{M_{fv}}{J_c} c_{AB} = 0.916 \text{ MPa}$$

$$0.916 < 1.31$$

O.K.

In the vicinity of exterior drop panel

$$V_f = 248.88 \text{ kN}$$

$$b_o = 2 \times 1790 + 2890 = 6470 \text{ mm}$$

$$c_{AB} = \frac{1790^2}{2 \times 1600 + 2700 + 2 \times 190} = 510.20 \text{ mm}$$

$$J_c = 3.226 \times 10^{11}$$

$$v_{AB} = \frac{248.88}{6470 \times 190} + \frac{43.80 \times 10^6}{3.226 \times 10^{11}} \times 510.20 = 0.270 \text{ MPa}$$

$$v_{AB} < v_r$$

O.K.

This calculation ended the design of the external frame 2-2 of the parking structure.

The same procedures were followed for the internal frame 3-3.

Appendix B

Concrete Stresses from NONLACS

In the following section, the stresses along sections B-B, D-D, and E-E on the top and bottom surfaces of concrete will be shown for all three analyzed structures.

Figure B.1: Concrete Stresses σ_x at the Bottom of Section B-B in the Steel Reinforced Slab

Figure B.2: Concrete Stresses σ_x at the Top of Section B-B in the Steel Reinforced Slab

Figure B.3: Concrete Stresses σ_y at the Bottom of Section B-B in the Steel Reinforced Slab

Figure B.4: Concrete Stresses σ_y at the Top of Section B-B in the Steel Reinforced Slab

Figure B.5: Concrete Stresses σ_x at the Bottom of Section D-D in the Steel Reinforced Slab

Figure B.6: Concrete Stresses σ_x at the Top of Section D-D in the Steel Reinforced Slab

Figure B.7: Concrete Stresses σ_y at the Bottom of Section D-D in the Steel Reinforced Slab

Figure B.8: Concrete Stresses σ_y at the Top of Section D-D in the Steel Reinforced Slab

Figure B.9: Concrete Stresses σ_x at the Bottom of Section E-E in the Steel Reinforced Slab

Figure B.10: Concrete Stresses σ_x at the Top of Section E-E in the Steel Reinforced Slab

Figure B.11: Concrete Stresses σ_y at the Bottom of Section E-E in the Steel Reinforced Slab

Figure B.12: Concrete Stresses σ_y at the Top of Section E-E in the Steel Reinforced Slab

Figure B.13: Concrete Stresses σ_x at the Bottom of Section B-B in the CFRP Reinforced Slab

Figure B.14: Concrete Stresses σ_x at the Top of Section B-B in the CFRP Reinforced Slab

Figure B.15: Concrete Stresses σ_y at the Bottom of Section B-B in the CFRP Reinforced Slab

Figure B.16: Concrete Stresses σ_y at the Top of Section B-B in the CFRP Reinforced Slab

Figure B.17: Concrete Stresses σ_x at the Bottom of Section D-D in the CFRP Reinforced Slab

Figure B.18: Concrete Stresses σ_x at the Top of Section D-D in the CFRP Reinforced Slab

Figure B.19: Concrete Stresses σ_y at the Bottom of Section D-D in the CFRP Reinforced Slab

Figure B.20: Concrete Stresses σ_y at the Top of Section D-D in the CFRP Reinforced Slab

Figure B.21: Concrete Stresses σ_x at the Bottom of Section E-E in the CFRP Reinforced Slab

Figure B.22: Concrete Stresses σ_x at the Top of Section E-E in the CFRP Reinforced Slab

Figure B.23: Concrete Stresses σ_y at the Bottom of Section E-E in the CFRP Reinforced Slab

Figure B.24: Concrete Stresses σ_y at the Top of Section E-E in the CFRP Reinforced Slab

Figure B.25: Concrete Stresses σ_x at the Bottom of Section B-B in the GFRP Reinforced Slab

Figure B.26: Concrete Stresses σ_x at the Top of Section B-B in the GFRP Reinforced Slab

Figure B.27: Concrete Stresses σ_y at the Bottom of Section B-B in the GFRP Reinforced Slab

Figure B.28: Concrete Stresses σ_y at the Top of Section B-B in the GFRP Reinforced Slab

Figure B.29: Concrete Stresses σ_x at the Bottom of Section D-D in the GFRP Reinforced Slab

Figure B.30: Concrete Stresses σ_x at the Top of Section D-D in the GFRP Reinforced Slab

Figure B.31: Concrete Stresses σ_y at the Bottom of Section D-D in the GFRP Reinforced Slab

Figure B.32: Concrete Stresses σ_y at the Top of Section D-D in the GFRP Reinforced Slab

Figure B.33: Concrete Stresses σ_x at the Bottom of Section E-E in the GFRP Reinforced Slab

Stresses σ_x at the top surface of concrete at section H (GFRP)

Figure B.34: Concrete Stresses σ_x at the Top of Section E-E in the GFRP Reinforced Slab

Figure B.35: Concrete Stresses σ_v at the Bottom of Section E-E in the GFRP Reinforced Slab

Figure B.36: Concrete Stresses σ_y at the Top of Section E-E in the GFRP Reinforced Slab

Appendix C

Stresses and Strains in Reinforcement from NONLACS

Figure C.1: Stresses in the Steel Reinforcement in Section B-B

Figure C.2: Stresses in the Steel Reinforcement in Section D-D

Figure C.3: Stresses in the Steel Reinforcement in Section E-E

Figure C.4: Stresses in the CFRP Reinforcement in Section A-A

Figure C.5: Stresses in the CFRP Reinforcement in Section B-B

Figure C.6: Stresses in the CFRP Reinforcement in Section D-D

Figure C.7: Stresses in the CFRP Reinforcement in Section E-E

Figure C.8: Stresses in the GFRP Reinforcement in Section A-A

Figure C.9: Stresses in the GFRP Reinforcement in Section B-B

Figure C.10: Stresses in the GFRP Reinforcement in Section D-D

Figure C.11: Stresses in the GFRP Reinforcement in Section E-E

END

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FIN