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**A LAW-OF-THE-WALL SHIFT CORRELATION FOR
DETERMINISTICALLY AND STOCHASTICALLY ROUGH
SURFACES**

by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment
of the requirements for the degree of

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Abstract

This thesis deals with the aerodynamic characterization of stochastic and deterministic roughness for purposes of computational fluid dynamics (CFD) calculation of flow over rough surfaces. This is accomplished by developing a method for prediction of the law-of-the-wall intercept for flow over isotropic surface roughness, based on correlations with statistical parameters that characterize roughness geometric features. Previous work correlating flow behavior was only applicable to deterministically rough surfaces (surfaces comprised of regular arrays of simply shaped roughness elements). This work attempts to find a correlation using parameters that can be applied to both deterministically and stochastically (randomly) rough surfaces. Five stochastically rough surfaces were tested in a wind tunnel by the author and the data from these tests, together with data from the literature for deterministic roughness, was used to test various combinations of characterizing parameters. It was found that by using a simple roughness-height distribution parameter together with a suitably defined roughness height, a fairly good correlation could be obtained. The uncertainty level of the

correlation corresponds to about $\pm 30\%$ error uncertainty in the calculated skin friction co-efficient. By adding a slope and blockage parameter to the correlation, it was found that the uncertainty could be reduced to about $\pm 20\%$. It is recommended that modified definitions of slope and blockage parameters should be explored to further reduce the margin of error.

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List of Symbols

A	Slope of log-law of defect law, law of the wall, law of the wall shift curve
A_f	Projected frontal area of the roughness element
A_s	Plan surface area of the rough wall
A_w	Wetted area of the upstream portion of the roughness element
b_m	Average roughness element width

B	Intercept of log law region of law of the wall plot
B_{smooth}	Intercept of log law region of law of the wall plot for a smooth surface
B_{exp}	Intercept of log law region of law of the wall plot for a experimental surface
C	Intercept of log law region of law of the wall intercept shift curve
C_f	Skin friction coefficient
C_{P0}	Stagnation (Total) pressure coefficient
C_{PS}	Static pressure coefficient
C_q	Dynamic pressure coefficient
d	Displacement Height
D	Intercept of log law region of defect law
H	Shape factor
k	Roughness height
k_{mean}	Mean roughness height
k_{rms}	RMS roughness height
k_s	Equivalent sand roughness height
k_s^+	Non-dimensional Equivalent sand roughness height
k^+	Non-dimensional sand roughness height
P_0	Stagnation (Total) pressure
P_s	Static pressure
P_{C1}	Wind tunnel settling chamber static pressure
P_{C2}	Wind tunnel contraction static pressure
r_{mean}/k	Roughness distribution parameter

$r_{rms/k}$	Roughness distribution parameter
$r_{rms/mean}$	Roughness distribution parameter
R	Air gas constant
Re	Reynolds Number
T	Temperature
u^+	Non-dimensional velocity
U	Local velocity
U_{ref}	Reference velocity
U_c	Free Stream velocity
U^*	Friction Velocity
x	Longitudinal distance along test surface and along wind tunnel test section
X	Roughness distribution parameter
y	Vertical distance from surface (fluid flow)
Y	Roughness slope parameter
y^+	Non-dimensional distance above zero datum
z	Vertical distance from surface (surface metrology)
Z	Roughness blockage parameter

Greek symbols

δ	Boundary Layer height
δ^*	Displacement height

θ	Momentum thickness
τ_w	Shear stress at the wall
ρ	Fluid density
μ	Fluid dynamic viscosity
ν	Fluid kinematic viscosity
β	Pressure gradient parameter
Λ	Waigh's spacing parameter
λ_D	Dvorak's spacing parameter
λ	Simpson's spacing parameter
φ	Wooding's shape parameter
α	Fraction of roughness height used to define surface strip for projection calculations

Chapter 1

Introduction

This thesis deals with the prediction of the aerodynamic effects of surface roughness. This is an area of significant importance to designs whose performance is directly affected by fluid dynamics, such as turbomachinery efficiency, or the drag coefficients of wings. Thus, it is important to find a means to predict the effects of roughness so that they can be accounted for during the design process. This roughness effect modeling will take the form of empirical relations as done in previous work. This enables application in Computational Fluid Dynamics (CFD) programs with minimal requirement for computational resources.

Early attempts at characterization of roughness effects dealt mainly with deterministic roughness (regular arrays of simply shaped roughness elements) primarily due to the ease of characterizing this class of roughness, with little being done with stochastic roughness (random roughness). The present work attempts to correlate the geometric parameters of stochastic roughness with its aerodynamic effects in a zero pressure gradient environment. Several stochastically rough surfaces were tested in a wind tunnel to provide data for the correlation. The correlation is also extended to include standard sand roughness and deterministic roughness.

Chapter 2 of this thesis is a literature review outlining the fundamentals of turbulent boundary layer behavior in flow over smooth and rough surfaces. Chapter 3 outlines the experimental methods and equipment used for measurement and testing of several stochastic roughness specimens along with data reduction methods. Chapter 4 briefly describes the tested surfaces and discusses the parameters chosen to characterize the surfaces for aerodynamic purposes. Chapter 5 presents the results of the experiments and correlation efforts together with a discussion of the findings.

Chapter 2

Behavior of Flow over Smooth and Rough Surfaces

2.1 Boundary Layers

When fluid flows over any surface, frictional effects occur due to interaction between the fluid and the surface. Because of this friction, a thin layer of the fluid near the surface is retarded. In this thin layer of fluid, the relative velocity increases from zero at the surface to the free stream value at some distance away from the surface, corresponding to external frictionless flow. See Figure (2.1)

Figure 2.1: The Turbulent and Laminar Boundary-Layer Profile in a Zero Pressure Gradient. Reproduced from Schetz (1984)

This thin layer of fluid is known as the boundary layer. U is the local velocity, U_e is the free stream velocity, y is the distance away from the surface, and δ is the boundary-layer thickness – the distance between the surface and the point at which local velocity becomes equal to free stream velocity. Depending on the streamwise distance away from the start of the boundary layer (the point where the surface and fluid begin to interact with one another), the boundary layer characteristics can change significantly. In order to simplify the boundary layer analysis, it is convenient to consider a condition of fully developed flow. Fully developed flow is a condition where the boundary layer is in a dynamic equilibrium such that the velocity distribution, when suitably non-

dimensionalized, does not change in the streamwise direction [Townsend (1954)]. Thus the fully developed boundary-layer profile, when non-dimensionalized by the free stream velocity and the boundary-layer thickness becomes independent of streamwise location. Unfortunately, the state of fully developed flow only occurs in very particular instances, and is seen very rarely. However, this somewhat ideal state is frequently approached, at least approximately, in practical situations.

Boundary layers can be classified into two groups –laminar boundary layers, and turbulent boundary layers. Simply, flow in a laminar boundary layer is very smooth and orderly. Laminar flow tends to occur in a low Reynolds number environment. In the turbulent boundary layer, fluid particles follow very erratic paths and the velocity fluctuates at any point in the flow. Turbulent boundary layers tend to occur in high Reynolds number flow. This thesis will only concentrate on the incompressible turbulent boundary layer.

Strictly speaking, a fully developed turbulent boundary layer would only occur when the Reynolds number approaches infinity. The Reynolds number (Re) is a ratio of the inertial forces to the viscous forces in the flow. But in the case of zero pressure gradients with constant roughness heights and spacing, an approximately fully developed flow can be achieved as long as the flow is far downstream of any change in the surface roughness. Garthshore and de Croos (1977) determined this distance to be about 350 times the roughness height, while Antonia and Luxton (1971) found fully developed flow to be satisfied at a point where $x/\delta > 20$, where x is the streamwise distance from the start

of the roughness, and δ is the local boundary-layer thickness. These two criteria by Gartshore and de Croos (1977) and Antonia and Luxton (1971) were used as guidelines to fully developed flow in this study

2.2 Boundary-Layer Parameters

The importance of the boundary-layer parameters is that they can be used to define any particular boundary layer. These parameters are quantitative measures of boundary-layer characteristics.

Boundary-Layer Thickness (δ)

The boundary-layer thickness, δ , is defined as the smallest distance away from the wall where the local velocity equals the free stream velocity. Experimentally, the boundary-layer thickness at a particular point on a surface is defined as the height where the local velocity first reaches 99.5% of the free stream velocity, as scatter in the data near the boundary-layer edge makes it difficult to consistently determine where the local velocity reaches exactly the free stream velocity value.

Displacement Thickness (δ^)*

The displacement thickness is an integral length scale of the boundary layer related to the mass flux deficit due to skin friction or shear stress at the surface. It is defined as the distance by which the external potential flow is displaced outwards due to the decrease in velocity in the boundary layer [Schlichting(1979)]. Its definition is

$$\delta^* = \int_0^{\delta} (1 - U/U_e) dy \quad (2.1)$$

Momentum Thickness (θ)

The momentum thickness is an integral length scale of the boundary layer related to the momentum flux deficit due to skin friction. Its definition is

$$\theta = \int_0^{\delta} U/U_e (1 - U/U_e) dy \quad (2.2)$$

Shape Factor (H)

The shape factor is a ratio between the displacement thickness and the momentum thickness, and used to describe the shape of the boundary-layer velocity profile. Its definition is

$$H = \delta^* / \theta \quad (2.3)$$

For a wide range of pressure gradients, shape factors of turbulent boundary layers are generally in the range 1.2 – 2.5 [Clauser(1956)], with the higher values of H occurring for strong adverse pressure gradients.

2.3 Turbulent Boundary Layers on Smooth Surfaces

When first looking at a fully developed turbulent velocity profile plotted against y/δ (see Figure 2.2), one begins to see how large a velocity gradient occurs across the lower half of the boundary layer – especially in comparison to that of a laminar boundary layer. For any laminar boundary layer on a flat plate with a zero pressure gradient, velocity profiles normalized by the free stream velocity (U/U_e) plotted against y/δ will all

collapse onto a single curve, regardless of fluid, Reynolds number or any other characteristic of the flow. Unfortunately, in the case of turbulent boundary layers, one is not so fortunate to be able to rely on these variables to produce a universal curve. Because of this, it is necessary to find new scaling variables in order to effectively collapse the turbulent boundary layer data.

2.3.1 The Velocity-Defect Law

It was found by Von Karman (1930) that when the local velocity (expressed as a velocity defect in the form $1-U/U_e$) in the boundary layer is normalized by a variable proportional to $1/\sqrt{C_f}$ (where C_f is the skin-friction coefficient), the velocity curves in the outer region of the turbulent boundary layer collapse onto one curve. This normalizing variable is known as the friction velocity (U^*) defined as

$$U^* = \sqrt{\tau_w / \rho} \quad (2.4)$$

Where τ_w is the shear stress at the wall and ρ is the density of the fluid. From this we find

$$U^*/U_e = \sqrt{(\tau_w / (\rho U_e^2))} = \sqrt{C_f/2} \quad (2.5)$$

The *defect law* for turbulent boundary layers can be written as

$$(U-U_e)/U^* = \text{fn} [y/\delta] \quad (2.6)$$

Velocity data for turbulent boundary layers is plotted in this format in Figure 2.2. It can be seen that the data points collapse into a single curve in the outer region of the boundary layers. Equation (2.6) is known as the *velocity-defect law*.

Figure 2.2: Defect-law Plot. Reproduced from Schetz (1984)

The defect law's ability to collapse curves begins to fail near the wall when y/δ approaches zero because the near-wall behavior is directly affected by wall shear stress and fluid viscosity, while flow in the outer region, because it's highly turbulent, does not depend directly on fluid viscosity, as shear stress in the outer region is dominantly due to turbulent mixing, and is only indirectly affected by wall shear. Clauser (1956) found the outer region exists approximately between $0.10 < y/\delta < 1.00$, and the inner region at $0 < y/\delta < 0.1$. It would later be found that an overlap region between the two regions exist between about $0.05 < y/\delta < 0.15$ (to be discussed in the next section). Because of the differences between the inner and outer region, the defect law is unsuited to collapse both

the inner and outer regions of the turbulent boundary layer. The defect law has excellent ability to collapse velocity profiles only in the outer region of turbulent boundary layers in zero pressure gradients, or in very particular non-zero pressure gradient distributions.

2.3.2 The Law-of-the-wall

Now that the outer layer of the boundary layer can be collapsed onto a single curve, the problem of collapsing the inner region of the boundary layer arises. It can be argued that in the inner layer, since wall shear is so dominant there, the boundary layer height is of no direct significance. By normalizing the y variable and the velocity, we get

$$y^+ = yU^*/\nu \quad (2.7)$$

$$u^+ = U/U^* \quad (2.8)$$

Using these variables, the law-of-the-wall can be written as

$$u^+ = f_n [y^+] \quad (2.9)$$

Doing further analysis of the innermost portion for the wall layer, one can argue that the velocity fluctuations are strongly damped since at the wall, velocity fluctuations in any direction are zero due to the no slip condition. Since all the velocity fluctuations are damped, then turbulence cannot exist, meaning the flow near the wall is laminar. This near-wall laminar flow is known as the laminar sublayer. The velocity in the laminar sublayer obeys the relation

$$u^+ = y^+ \quad (2.10)$$

Note that this is consistent with the more general law-of-the-wall, (Equation (2.9)).

Figure 2.3: Law-of-the-wall Plot. Reproduced from Schetz (1984)

From Figure 2.3, one can see that the upper limit for which the laminar sublayer exists is at about where $y^+ = 7$. Now that the inner and outer regions have been defined through the law of the wall and the defect law, it is of significant importance to consider the overlap region (also known as the log-law region) where the inner and outer regions of the turbulent boundary layer overlap.

In this overlap region, both similarity equations (Equations (2.6) and (2.9)) must apply. For the outer region:

$$(U - U_c) / U^* = f(y / \delta) \quad (2.11a)$$

And from the inner region

$$U/U^* = g(yU^*/\nu) \quad (2.11b)$$

Re-organizing and equating (2.11a) and (2.11b), we get the overlap region

$$U/U^* = f(y/\delta) + U_e/U^* = g[(y/\delta)(\delta U^*/\nu)] \quad (2.11c)$$

Taking a look at (2.11c), one must first note that U_e/U^* and $\delta U^*/\nu$ are constants, in the sense that they are not a function of y . Because of this, we can then surmise that any multiplicative function inside of the function g , must have the same effect as an additive function inside of f . The only function that exhibits such attributes is the logarithmic function. Thus, in the overlap region, (2.11a) and (2.11b) become

$$U/U^* = A \log(yU^*/\nu) + B \quad (2.12a)$$

and

$$(U - U_e)/U^* = A \log(y/\delta) + D \quad (2.12b)$$

where $A=5.62$, $B = 5.0$, and $D = -2.58$ (White(1991))

2.4 Roughness Effects on the Turbulent Boundary Layer

When trying to describe the effect of roughness, the traditional method of characterizing its effect is through the roughness height parameter ' k '. Although ' k ' alone is insufficient to show the total effects of the roughness (as will be outlined later in the chapter), it is the starting point in describing the effects of roughness on flow. It should be noted that all roughness elements discussed in this thesis are assumed to have a height (k) that is small compared to the boundary-layer thickness δ , such that the direct effects

are confined to the law of the wall region. As before, when trying to apply any type of variable to any of the plots mentioned above, it must be of a non-dimensional form. The method used to non-dimensionalize the y co-ordinate is also used on k. Thus, a suitable non-dimensional form of the roughness height is

$$k^+ = kU^*/\nu \quad (2.13)$$

Flow over rough walls can be characterized into three regimes. These regimes are:

$k^+ < 5$	hydraulically smooth
$5 \leq k^+ \leq 70$	transitionally rough
$k^+ > 70$	fully rough

For roughness that is immersed in the laminar sublayer, it can be expected that any effects or disturbances made will be damped out (as previously discussed). Thus, the influence of the roughness is quite minimal and this is known as hydraulically smooth. The hydraulically smooth boundary layer is dependent only on fluid viscosity ν and prevails when $k^+ < 5$. Once k^+ begins to extend past 10, it is found that the skin-friction and thus the shear velocity increases as a result of roughness effects. The effect that this will have on the log-law plot is to shift the overlap region downwards (see Figure 2.4).

Figure 2.4: Log-law Shift due to Roughness. Modified from Schlichting (1979)

In the transitionally-rough regime, where $5 \leq k^+ \leq 70$, flow is dependent on both roughness height and viscous effects. Once k^+ is greater than 70, the flow is in the fully-rough regime. The wall shear stress, shear velocity, and the characteristics of the flow in general, then depend only on roughness height and are independent of viscosity. Thus, in the fully-rough regime, we can expect that

$$U/U^* = \text{fn} [y/k] \quad (2.14)$$

This implies that in the fully-rough regime, the downward shift, ΔU , of the log-law plot (due to the increase in U^*), has the form

$$\Delta U/U^* = A \log(kU^*/\nu) + C \quad (2.15)$$

When combined with Equation (2.12), we obtain, for the fully-rough regime;

$$U/U^* = A \log (yU^*/\nu) + B - \Delta U/U^* \quad (2.16)$$

$$= A \log (y/k) + B - C \quad (2.17)$$

which is consistent with Equation (2.14).

It should be noted that although various researchers have supported the theory of these three flow regimes, there has been some disagreement on the limits assigned to them – where they start, where they end. This brings up the point that the roughness height alone is insufficient for describing the roughness features. Effects such as the spacing of elements must be accounted for as it has been found to have a significant influence on roughness effects, as one would expect. This is one of the main reasons why there is some discrepancy in the limits of the flow regimes, but brings out the fact that more variables for describing the roughness must be included in the analysis. It should be noted that the slope of the log-law region is not subject to change from roughness effects. The $\Delta U/U^*$ in Equation (2.16) corresponds to the downward shift of the log-law plot towards the origin.

Note that Equation (2.17) implies that for fully-rough conditions, the aerodynamic effects of any roughness can be described by two parameters – the roughness height k and a roughness parameter C . The value of C depends on the shape of the roughness elements and the spacing between them, and will be referred to as the *roughness-configuration constant* in this thesis. It is expected to have a constant value for any particular roughness configuration.

Various attempts to determine the form of the roughness-configuration constant, C , have been made, but none have been able to successfully capture the effects of stochastic roughness. On the other hand, notable advances in correlating C for deterministic roughness have been made. This thesis is focused on determining C for both deterministic and stochastic roughness in the fully-rough regime.

2.5 Displacement Height

Unfortunately, one of the problems when measuring boundary layers in rough flows is determining where the origin of the y co-ordinate is. The origin of the nominal surface is difficult to determine for rough walls due to the unevenness of the surface. For deterministic roughness that has a well defined wall, the displacement height is used to determine the effective origin (See Figure 2.5):

$$y = y_0 - d \quad (2.18)$$

Figure 2.5: Displacement Height for Deterministic Roughness. Reproduced and Modified from Waigh (1996)

A common method of determining the origin is used in Schlichting(1936) and O'Loughlin(1964) where the volume-average height (also known as the melt down height) of the surface is used. The problem with this lies in roughness that has little volume – e.g. thin plates normal to the flow direction. The surfaces with this roughness presented the problem of having a very low volume-average height, which was not indicative of the displacement height. Koloseus (1962) made the suggestion that it would be more realistic in such instances to include an estimate of the volume of separation pocket behind each element. The problem with this approach is that this would have to be applied to all types of roughness, such as 2-D, 3-D cubical, or 3-D spherical which all have substantially different effects on trailing separation pockets, so a new problem would arise in just distinguishing the types. The problem with the methods mentioned above is that they are a measure of the roughness geometry, not of the effect of the roughness geometry. One of the most widely used methods of determining the origin of the y coordinate is through a graphical means used by Clauser (1956). He noted that the straight line portion of the logarithmic law-of-the-wall plot becomes curved if y is shifted by a constant amount. Thus, when reducing the experimental data, the origin is shifted until the logarithmic portion becomes straight. The problem with this is that there is still some ambiguity in terms of the level of accuracy with which this method can be implemented. Jackson (1981) has argued that the origin for y should be at the level of the center of pressure of the form or pressure drag on the roughness elements, that is, at the level of the average momentum 'sink.' This method was not used due to the difficulty of implementing it in practice.

2.6 Roughness Spacing Flow Regimes

When analyzing the variables used in the determining the roughness-configuration constant 'C' in Equation (2.17), it is important to understand that C should reflect those characteristics of the roughness that the roughness height (k) cannot capture. Morris (1955) suggested that rough walled flows can be divided into 3 types:

Isolated-roughness flow: In this flow, roughness elements are sufficiently far from one another such that their wakes do not affect or interact with one another. Thus, roughness elements may be treated in an isolated manner. This is also known as the sparse regime.

Wake-interference flow: In this flow, elements are sufficiently close to one another to result in interaction of their wakes. This is also known as the dense regime.

Skimming flow: Here, elements are so close together that flow passing over the elements form stationary vortices in the small spaces between the elements, effectively 'filling' the spaces. Losses are primarily due to fluid entrainment into stationary vortices.

2.7 Current Approaches to Modeling Roughness Effects

There are currently two general methods of formulating roughness models – the classical correlation approach and the discrete-element approach. The correlation approach attempts to reduce the problem to a correlation involving various geometric and statistical parameters of the roughness. A correlation between these parameters and the

roughness-configuration constant C of Equation (2.17) is sought. One of the most common applications of the correlation approach is the equivalent sand approach based on work by Nikuradse (1933). Nikuradse made various experiments involving closely-packed sand-grain roughness, and Schlichting (1936) proposed to correlate other roughness types to the sand roughness curves. This is known as the 'equivalent sand roughness' for the given surface. Dvorak (1969), Simpson (1973) and Dirling (1973) have made significant contributions to the correlation approach with deterministic roughness. Little success has been achieved with stochastic roughness.

While the correlation approach is completely based on empirical correlations, the discrete-element approach considers the physical characteristics of the roughness elements in the solution of the boundary-layer equations. This method was also proposed by Schlichting. He suggested that the total drag on a rough surface is the sum of skin friction and form drag on the roughness elements with consideration to blockage effects of each individual element. Blockage is effectively a measure of the fraction of space the roughness element occupies thus blocking the flow. As the flow moves over and around the element, form drag occurs resulting in a loss of momentum in the flow. The focus of the present thesis is towards the correlation approach in the sense that correlations will be based on empirical data for the surface and flow parameters.

2.7.1 The Correlation Approach

Dvorak was able to find a spacing parameter that was able to distinguish and quantify the difference between a densely and sparsely-spaced roughness regime in

deterministic roughness. By using the spacing parameter on transverse square bar roughness (i.e. 2-D):

$$\lambda_D = L/s \quad (2.19)$$

where 'L' is the distance between the roughness elements (also known as roughness 'pitch'), and 's' is the streamwise dimension of the roughness element. Dvorak found that the densely-spaced roughness regime is considered to exist when $\lambda_D \leq 4.7$, while the sparsely-spaced roughness regime occurs when $\lambda_D > 4.7$.

Figure 2.6: Dvorak's Correlation Between λ_D and C . Reproduced from Dvorak (1969)

The explanation of the trend seen in Figure 2.6 is as follows. The densely-spaced roughness has a decreasing effect on the constant term of the roughness-configuration constant 'C' as λ_D decreases when $\lambda_D \leq 4.7$. This is consistent with the 'wake-interference flow' proposed by Morris as the flow approaches 'skimming flow.' Here

roughness elements become closer and closer as the spacing parameter decreases, reducing the amount of space between roughness, creating stationary vortices and reducing the effect of roughness. It can be expected that for spacing values of λ_D near 1, that the effect of the roughness becomes minimal as flow is considered to be ‘skimming’. For the sparse regime, as λ_D increases, the effect of roughness on C also decreases due to the fact that the wakes of the individual elements interact less and there are fewer elements to exert drag on the flow. As the spacing gets higher, the flow will become an Isolated-Roughness Flow.

Simpson (1973) argued that the effect of the roughness should correlate with its frontal area, rather than with the plan area as used by Dvorak. He therefore introduced a new spacing parameter based on the projected frontal area of the roughness element (A_f) and the plan surface area of the wall (A_s). (See Figure 2.7). He proposed that the spacing parameter should be defined as:

$$\lambda = A_s / A_f \quad (2.20)$$

Although the collapse of the data was not very good, there was enough of a correlation to warrant a connection between roughness effects and Simpson’s spacing formula. This would later be explored by Waigh (1996).

Figure 2.7: Simpson's Correlation Between λ and C . Reproduced from Simpson (1973)

Although the spacing parameter, λ , was a good start in characterizing deterministic roughness, there was still a fair amount of scatter in the correlation. Wooding (1972) attempted to improve existing spacing parameters by introducing a parameter that would be characteristic of the shape of the roughness elements, thus differentiating a cube element from a cylindrical element for instance. However, his approach was adapted into a non-dimensionalized roughness parameter that was multiplied directly with the roughness height, k , instead of being applied to a roughness-configuration constant that is an additive function to $\text{Alog } k^+$ (see Figure 2.8).

Figure 2.8: Wooding's Correlation Between U_c/U^* and Shape Parameter. Reproduced from Wooding (1973)

Wooding found a correlation for U_c/U^* to be:

$$U_c/U^* = 2.87 \ln (\delta / ((k/\lambda)\phi)) - 2.05 \quad (2.21)$$

Where δ is the boundary-layer height, λ is the spacing parameter as mentioned from Simpson, and ϕ is a measure of the aspect ratio of the roughness element equal to k/s , where k is the roughness element height, and s is the streamwise dimension of the

roughness element. This relation by Wooding led to a fairly good collapse of his data for values of $\delta/((k/\lambda)\phi)$ between 30 to 2000, but did show some deviation with regard to some of the 2-D and cylinder type roughness elements as seen in Figure 2.8.

Waigh (1996) worked on implementing a combined spacing and element shape parameter to describe the effects of roughness on C . Using the spacing parameter

$$\Lambda = \lambda(k/b_m)^a (A_w/A_f)^b \quad (2.22)$$

where λ is the same as Simpson's spacing parameter, k/b_m is the spanwise aspect ratio of the roughness element, and A_w/A_f is the element bluntness proposed by Dirling (1973), A_w is the wetted area of the upstream portion of the roughness element, A_f is the frontal area of the element, and b_m is the mean width of the element (See Figure 2.9 for an illustration of the parameters). The constants a and b are dependent on the spacing regime in question.

By applying regression analysis to all suitable data available to him, Waigh found that for the dense spacing regime

$$C = 10.50 \log (\lambda(k/b_m)^{0.87} (A_w/A_f)^{0.44}) - 7.59 \quad (2.23)$$

and for the sparse spacing regime

$$C = -5.75 \log (\lambda(k/b_m)^{0.55} (A_w/A_f)^{1.38}) + 5.78 \quad (2.24)$$

Figure 2.9: Diagram of roughness parameters. Reproduced from Waigh (1996)

Figure 2.10: Waigh's correlation between λ_{eff} and C . Reproduced from Waigh (1996)

The accuracy of these correlations can be seen from Figure 2.10. As one can see, the collapse is very good – especially in comparison to previous works in the same area. Unfortunately, the problem with all these correlations is that they only pertain to deterministic roughness. This type of roughness application is quite limited from an

engineering standpoint. When looking at surfaces suffering from caked on deposits, or corrosion, the roughness that they provide is of a stochastic nature. Due to the random and non-geometric nature of stochastic roughness, it is not possible to apply Waigh's correlation to this type of roughness.

Stochastic roughness has no spacing regimes *per se*, as there are no well-defined elements to apply a spacing parameter to. There have been various attempts at finding a correlation for stochastic roughness (Chuah et al (1982) and Acharya (1986). They determined that the roughness height alone measured as a total roughness height from peak to valley or as an average surface height (also known as a meltdown height), was not sufficient for characterizing the roughness effects. Ferrari (2002) attempted to find a correlation between various geometric roughness parameters and the roughness-configuration constant 'C,' but had limited success. This thesis is an extension of Ferrari's work, and its goal is to find what characterization parameters are necessary in order to describe the roughness sufficiently, and correlate them to their effect on the boundary layer flow. It is also hoped that the correlations can be adapted in such a manner where that can be used for both deterministic and stochastic type roughness.

Chapter 3

Experimental Description and Data Reduction

Velocity profiles in boundary layers were measured along several rough surfaces in Carleton University's low speed closed circuit wind tunnel. This data was used to deduce skin-friction coefficients (C_f) for each of the surfaces, which would then be used to find the shear velocity (U^*). Further processing of the data allowed the determination of the roughness-configuration constant from Equation (2.17). A complete description of the apparatus, methods and data reduction techniques will be presented in this chapter.

Figure 3.2: Test Section

The test section was constructed of plywood with a plexiglass side-wall for viewing purposes on one side, and an access door on the other side. The ceiling of the test section had an adjustable slope to minimize any streamwise pressure gradient due to boundary layer growth (See Figure 3.2) The ceiling was adjusted by a screw locking mechanism employed on two arms attached to the ceiling panel. An aluminum sheet metal lip was attached to the upstream end of the adjustable ceiling and came in contact with the inside of wind tunnel to guide airflow into the test section. All test surfaces were mounted on a 30"x 72"x 7/16" medium density fiber (MDF) board which was attached to the floor of the test section. The test section was mounted on a stand with adjustable legs

allowing the test surfaces to be aligned directly in line with the bottom of the wind tunnel contraction exit. The test section was also tuft tested to ensure that no significant separation occurred anywhere within it (especially near the front of the test section where the flow is guided onto the adjustable ceiling by the metal lip).

The ceiling also has a static pressure tap located above each of the probing stations – 5 along the centerline in the stream wise direction, and 2 off the centerline (see Figure 3.3). The static pressure taps have a 1/64” inner diameter, are burr free and square edged. Brass adapters connect the pressure taps to the pressure transducers using R-3603 Tygon tubing (3/32” inner diameter, 5/32” outer diameter). It was determined, using Benedict (1997) that the maximum error from the static pressure tap size and alignment was less than 1% of the dynamic pressure.

Figure 3.3: Surface Layout (all dimensions in millimeters)

3.1.2 Traverse Gear and Probe

A Pitot probe was used along with a traverse gear to measure velocity profiles at various measuring stations. The traverse gear was mounted underneath the test section on a mounting block with guide rails attached to it. A stepper motor with a lead screw was used to drive the probe mounting block. See Figure 3.4. A five-wire type stepper motor utilizing a 1.8° degree step was used in this experiment. This corresponded to a vertical step of 0.00635mm. The lead screw is brass with a $\frac{1}{4}$ " diameter with a pitch of 20 revolutions per inch. The traverse gear was controlled by the SCANNER program (to be discussed later) which read an input file containing the traverse points to be measured. Backlash was not a factor in these experiments because all test cases required traverses only in one direction.

Figure 3.4: *Traverse Gear (Reproduced from Ferrari, 2002)*

The Pitot probe was mounted on the probe mounting block which was moved by the lead screw. The probe was constructed from two cylindrical brass tubes (see Figure 3.5). A smaller tube (outer diameter 1/16", inner diameter 1/32" inner diameter, 2" in length) was bent into shape and inserted into a 1/8" opening drilled into the larger brass tube (outer diameter 1/8", inner diameter 3/32", 12" in length). The mouth of the probe was then flattened slightly and sanded smooth such that the center of the Pitot probe mouth/opening was as close as possible to the wall when in contact with the wall. The connection between the two tubes was bonded and sealed with solder. The top of the large brass tube was also sealed with solder. The Pitot probe was then leak tested with success to ensure that there were no leaks.

Figure 3.5: Pitot tube and mouth dimensions (in mm)

3.1.3 Pressure Transducer and Data Acquisition System

A computer controlled Pressure Systems Inc (PS9010) Intelligent pressure scanner was used for pressure measurements for boundary layer traverses and at static pressure taps. The system is comprised of a pressure transducer and a data acquisition system. It features 16 silicon DH200 pressure transducers – 8 absolute (310 kpa range) and 8 differential (± 5 kpa). The PS9010 uses a built-in 12 bit sequential analog to digital converter, and transmits the data to the computer through a serial RS-232 interface port. The unit is powered by an external 15 volt DC power supply. It is also supplied with nitrogen gas at 100 psi, which is used to provide pressure for the pneumatic manifold which is used to re-zero the calibration offsets. In the present experiments, 6 differential channels were used in the pressure gradient measurements along the test section (5 static pressure taps and P_{C1} – refer to Figure 3.1) while 3 differential channels were used during boundary layer traverses.

Calibrations were done for all 8 differential channels. A sample calibration graph is shown in Figure 3.6. The pressure measurements are accurate to ± 2 pa with a 95% level of confidence when re-zeroing the offsets every 50 sample points. (Mahallati (2003)). A sampling rate of 16 samples/sec for each channel was used (fixed by the PS9010 system) with a sample size of 300 for each point (as used by Ferrari, 2002), giving a sampling time of 18.75 seconds, as Mahallati found that 200 samples for each point is the minimum number required to properly capture the mean flow, as determined by taking a running average of the flow velocity measurements. This would later be supported by the limited amount of scatter found in the velocity profiles measured, as an

insufficient sample size would result in large amounts of scatter. The PS9010 unit was reset and re-zeroed before each traverse and re-zeroed every half hour during testing which corresponded to almost 50 data points

Figure 3.6: Sample Calibration Graph

3.1.4 Computer and Software Setup

An Intel 486 33MHz PC was used for data storage and control of the PS9010 unit and the stepper motor, which was controlled by a program ('SCANNER') written by A.Mahallati. The software uses an ASCII input file of the boundary-layer heights to be taken, along with presets if the lead screw pitch, and the degree step of the stepper motor. See Appendix A for an example traverse input file.

The software also controls the pressure transducer calibration operation, along with system resetting and re-zeroing, and the display of the pressure values during traverse runs. Three data files are output after each boundary layer traverse. The first file shows the average pressure differentials for each channel at each traverse point. The second output file gives the static, dynamic and stagnation pressure coefficients of all the channels at each traverse point. The third output file shows a record of the entire traverse, showing probe movement, time, sampling time... etc. See Appendix B for an example.

3.1.5 Miscellaneous Apparatus

The tunnel air temperature was measured with a portable digital thermocouple wire probe located four inches from the downstream end of the test section. Its accuracy was verified to be accurate within 1° C against other thermometers known to be accurate. The measured temperatures were used to calculate air density and kinematic viscosity.

3.2 Boundary Layer Experiments and Data Reduction

Eight surfaces were tested with traverses taken at stations 2 through 5 on the centerline of the test section (see Figure 3.3). Station 1 was omitted because it was too close to the front of the test section and the boundary layer at that point would not be in dynamic equilibrium, as discussed earlier in Section 2.1. Flow two-dimensionality was checked for three of the test surfaces (see Figure 3.7) by measuring velocity profiles at an off centerline measurement station. See Table 3.1 for a summary of tests.

Figure 3.7: Sample Two-Dimensionality Check

surface	centerline traverse	off-centerline traverse
Coarse Shingle	x	
Coarse Spackle	x	x
Fine Shingle	x	
Fine Spackle	x	x
Tar Paper	x	
Vinyl	x	
White Formica	x	
Smooth	x	x

Table 3.1: Experimental Test Matrix

3.2.1 Wind Tunnel Settings and Test Sections

The tunnel airspeed was set to approximately 55 m/s to give a Reynolds number of approximately 4 million per meter. It should be noted that for Reynolds number dependency tests, additional tests at station 3 (Figure 3.4) were done at 45m/s and 35m/s. (See Figure 3.8). Minor variations occurred in the free stream Reynolds number (almost 2% over large traverses) primarily due drift in the wind tunnel temperature – as the wind tunnel ran, the temperature of the air flow would slowly increase. Fortunately, it was determined that this change in temperature would have negligible effects on the results. This was checked through various test cases, where the Reynolds numbers was varied by about 5%, and noting that the measured boundary-layer parameters showed no significant change (less than 1%). Temperature measurements were done at 15 minute intervals during the boundary layer traverses and temperatures between the intervals were linearly interpolated for completeness. These temperature readings were used to calculate air density (ρ) and air dynamic viscosity (μ). Air density was calculated using the ideal gas law:

$$\rho = p/(RT) \quad (3.1)$$

where $R = 287\text{kJ/kgK}$ is the air gas constant, p is the absolute tunnel static pressure, and T is the tunnel temperature. The dynamic viscosity (μ), was calculated through Sutherland's Law:

$$\mu = \mu_0(T/T_0)^{3/2}(T_0+110)/(T+110) \quad (3.2)$$

where $\mu_0 = 1.7894 \times 10^{-5} \text{ Pa-s}$ and $T_0 = 288.16 \text{ K}$.

Figure 3.8: Sample Velocity Dependence test

3.2.2 Boundary Layer Measurements

The vertical interval between the sample points in the boundary layer traverse was varied with respect to the height within the boundary layer. Near-wall values were subject to the strongest shear and as such experienced the greatest gradients in velocity. Because of this, steps needed to be smaller to accurately capture the boundary layer in this region. Step sizes of 0.095mm were used for the first 2.5 mm of the boundary layer. As the distance away from the wall increased, the changes in velocity became smaller, and as such larger step sizes could be used. From the range of 2.5mm to 10.5mm, a step size of 0.191mm was used, and from 10.5mm to 20.5mm, as step size of 0.286mm was chosen. All heights past 20.5mm used a sample spacing of 0.476mm. The total number of test

points tested is dependent on the thickness of the boundary layer estimated from preliminary low resolution boundary layer traverses. After the entire boundary layer traverse was completed, a measurement was taken at the highest distance available from the traverse gear (at 123.99mm) to sample the local free stream conditions. This was later used to find the local dynamic pressure coefficient C_q (see Section 3.2.4). See Appendix A for a sample input file.

Boundary layer traverses were performed as follows. The probe was aligned with the flow direction to an accuracy of $\pm 1^\circ$ with the help of a protractor aligned along the centerline of the test section – well within acceptable limits for a Pitot probe's yaw accuracy. Using the controlling software SCANNER, the probe was manually positioned on the surface 'wall' with a small amount of flex (rebound on the pitot probe pushing the mouth of the probe upwards a small amount). The flex would be used to help locate the y-datum of the of the boundary layer data (to be explained later in Section 3.2.4).

For small-scale roughness surfaces (relative to probe mouth size), the summits of the roughness were sufficiently close together to constitute a wall. For surfaces where the roughness scale was large enough to allow the pitot probe mouth to fall between summits, the probe mouth was placed on the valleys at the base of the roughness elements (this was only necessary for the spackle surfaces). The traverse was started on an automatic survey run using the appropriate input file. A wait time between each sampling point of approximately 6 seconds was used (as recommended by Mahallati, 2003) in order for the disturbances due to probe movement to dissipate and allow for any

settling time needed for dynamic responses in the probe from the movement. This wait time was deemed acceptable for two reasons. First, the pressure lines from the static taps to the data acquisition system were roughly the same length as in Mahallati's experiments. Second, the mouth of the pitot tube used in this thesis is much larger than the pitot tube used in Mahallati (2003). These two factors would reduce the required wait time, making the 6 second wait time more than adequate. During each run, the pressure transducers were re-zeroed roughly every 50 traverse sample points.

3.2.3 Flow 2-D Check

As noted in Section 3.2.1, boundary layer traverses were made for select surfaces testing two dimensionality. Two-dimensional flow is one of the assumptions made in the usage of the momentum integral method (to be discussed later in Section 3.2.6) used to help find the skin-friction coefficient (C_f). Two-dimensional flow was considered satisfied when, for a given surface, the boundary-layer parameters (δ , δ^* , and θ) were equal to within 1% at the tested span-wise stations. The boundary-layer profiles were also plotted on the same graph, and visually inspected to ensure that the profiles overlapped one another. See Figure 3.7. It was found that two-dimensionality was satisfied for all surfaces tested.

3.2.4 Boundary Layer Data Analysis

Velocity Profile Determination

Referring to Figure 3.1, the static pressure tap P_{C1} is located at the inlet to the contraction of the wind tunnel where the velocity is approximately zero, while P_{C2} is

located at the end of the contraction. The Pitot tube measures the stagnation pressure P_o at a particular height. The static pressure taps are located directly above the Pitot probe measuring the static pressure P_s .

P_{C2} was used as the reference pressure for all three differential channels used in the boundary layer traverses. Thus one channel measured $P_o - P_{C2}$, another channel measured $P_s - P_{C2}$, and the last channel measured $P_{C1} - P_{C2}$. The controlling software SCANNER calculated the stagnation pressure (C_{po}) and static pressure (C_{ps}) coefficients defined by

$$C_{po} = (P_o - P_{C2}) / (P_{C1} - P_{C2}) \quad (3.3)$$

$$C_{ps} = (P_s - P_{C2}) / (P_{C1} - P_{C2}) \quad (3.4)$$

The dynamic pressure coefficient C_q relates local dynamic pressure at the probing station to the pressure difference $P_{C1} - P_{C2}$. The reference velocity, denoted by U_{ref} , is defined as

$$U_{ref} = (2(P_{C1} - P_{C2})/\rho)^{0.5} \quad (3.5)$$

The local dynamic pressure coefficient (C_q) can be measured to find the true local free stream velocity:

$$\begin{aligned} C_q &= (\frac{1}{2}\rho U_c^2) / (\frac{1}{2}\rho U_{ref}^2) \\ &= U_c^2 / U_{ref}^2 \\ &= (P_o - P_s) / (P_{C1} - P_{C2}) \quad \text{at free stream} \\ &= C_{po} - C_{ps} \quad \text{at free stream} \end{aligned} \quad (3.6)$$

Combining all the above, the normalized velocity calculation becomes

$$U/U_e = ((C_{po} - C_{ps}) / C_q)^{0.5} \quad (3.7)$$

y-datum determination

As mentioned in Section 3.2.2, the probe mouth was initially bent up from the surface due to the flex from bringing the probe towards the surface while in contact with the surface itself. This gave a higher total pressure reading than the actual surface value. To correct for this, the lowest velocity measured during a traverse is taken as the point where the probe mouth is at the surface. This method was used for its repeatability to find velocity values at the lowest point. This is also important in accurately finding the skin friction (to be discussed later in the section). This method of finding the lowest velocity along with the method of finding the y-datum by adjusting the datum to give a straight log-law region (as mentioned in Section 2.5) gives the ability to consistently and accurately produce boundary-layer plots. It should be noted that probe mouth size and displacement effect errors associated with small streamline deflections did not need to be accounted for, as the shifting of the y-datum to produce straight log-law plots would offset these effects. The reasoning behind this is that, regardless of displacement and probe mouth errors, the datum will be adjusted accordingly to produce a straight line in a log-law plot. Thus in effect, the errors associated with probe mouth and displacement effects are accounted for.

3.2.5 Boundary Layer Parameter Calculations

Two other boundary-layer parameters (outlined in chapter 2) were also calculated. Using a computerized data reduction method in Microsoft Excel, a numerical method of

calculating the displacement thickness (δ^*) and momentum thickness (θ) was implemented using a simple discretized numerical integration scheme as follows:

$$\delta^* = \sum_{i=1}^{\delta/\Delta y} (1 - (U/U_{e,i} + U/U_{e,i-1})/2) \Delta y \quad (3.8)$$

$$\theta = \sum_{i=1}^{\delta/\Delta y} ((U/U_{e,i} + U/U_{e,i-1})/2)(1 - (U/U_{e,i} + U/U_{e,i-1})/2) \Delta y \quad (3.9)$$

3.2.6 Skin-Friction Coefficient Calculation

The skin-friction coefficient

$$C_f = \tau_w / (\frac{1}{2} \rho U_c^2) \quad (3.10)$$

was required to calculate the friction velocity U^* , which is in turn used to calculate u^+ , y^+ and k^+ , as shown in chapter 2. The following methods were used in the current work to calculate C_f .

The Clauser Plot

The Clauser plot (Clauser, 1956) is an expanded law-of-the-wall plot. Starting with the law of the wall (Equation (2.12)) and substituting the friction velocity (Equation (3.10)), we get

$$U/U_c = [A \log y U_c (C_f/2)^{0.5} / \nu + B] [C_f/2]^{0.5} \quad (3.11)$$

Equation (3.11) gives a plot of normalized velocity versus Reynolds number (Re_y) for different values of C_f in the log-law region. See Figure 3.9.

Figure 3.9: The Clauser Plot Based on Experimental Smooth-Wall Data for Known Values of C_f . Modified from Bauman (1999)

To use the Clauser plot, the experimental boundary-layer velocity profile is plotted in terms of U/U_e vs. Re_y . When the experimental points overlap a particular C_f curve from Equation (3.11), it indicates that the experimental velocity profile has the same C_f as the C_f used to generate the overlapped C_f curve. Unfortunately, the Clauser plot only works well for smooth-wall flow but can be used for flows in mild to moderate pressure gradients since the law of the wall is insensitive to moderate pressure gradients.

Defect-Law plot

The defect-law plot method has been successfully used by Kind and Lawrysyn (1991) to determine U^* and C_f for rough-surface flows. It is similar to the Clauser plot

except instead of using the log-law equation, the defect law is used. Starting with the defect law (Equation (2.6)), and substituting the friction-velocity equation (Equation (3.10)) and rearranging, the equation becomes

$$U/U_c = [A \log y/\delta + D] (C_f/2)^{0.5} + 1 \quad (3.12)$$

This gives a plot of normalized velocity versus a normalized boundary-layer height for various values of C_f in the log-law region (see Figure 3.10).

Figure 3.10: Defect-law Plot Based on Experimental Data (Curved Lines) and Defect-Law Equations (Equation 3.12) for Known Values of C_f ($\beta=0$). Reproduced from Ferrari, 2002

To use the defect-law plot, the experimental boundary-layer profile is plotted in terms of U/U_c vs y/δ . When the experimental plots match the given profiles, they share the same C_f . The defect law can be used for both smooth and rough surfaces but is not applicable for test cases in different pressure gradients because the defect law varies in correspondence to the pressure gradients. Fortunately, due to the experimental set up in this thesis, all test conditions were essentially in zero pressure gradient. This method was chosen as the primary means of finding the skin-friction coefficient in this thesis.

Momentum Integral Equation

The 2-D momentum integral method, developed by Von Karman

$$C_f/2 = d\theta/dx + (2+H)\theta/U_c (dU_c/dx) \quad (3.13a)$$

is used to find the skin-friction coefficient. In zero pressure gradient where $dU_c/dx = 0$, Equation (3.13a) becomes

$$C_f/2 = d\theta/dx \quad (3.13b)$$

By calculating the momentum thickness at every streamwise station and finding its change in the streamwise direction, the value $d\theta/dx$ can be estimated at each interval and used to find C_f . This method is sensitive to departures from two dimensionality of the flow. Also, it can be difficult to accurately determine $d\theta/dx$ from the experimental data, because the calculation of $d\theta/dx$ is based only on four stations, because it was found that the boundary-layer flow at station 1, was not in equilibrium (to be discussed in Section

5.1). For these reasons, this method was used just to estimate the skin-friction coefficient as a check for the Defect-Law method.

Pressure Gradient Parameter

Although efforts were made to minimize any pressure gradient developments in the test section, it was still important that any residual pressure gradients that could not be negated were accounted for to ensure that their effects were negligible. Recalling Equation (3.6) where $U_e^2 = U_{ref}^2 C_q$ at free stream, the pressure gradient is defined as

$$\begin{aligned}
 dp/dx &= -\frac{1}{2}\rho \frac{dU_e^2}{dx} \\
 &= -\rho U_e \frac{dU_e}{dx} \\
 &= -\rho U_{ref}^2 \sqrt{C_q} \frac{d\sqrt{C_q}}{dx} \\
 &= -\frac{1}{2}\rho U_{ref}^2 \frac{dC_q}{dx} \\
 &= -(P_{C1}-P_{C2}) \frac{dC_q}{dx}
 \end{aligned}$$

The defect-law pressure gradient parameter (β) defined by Clauser (1956) is

$$\begin{aligned}
 \beta &= (\delta^*/\tau_w) dp/dx \\
 &= -(\delta^*/(C_f C_q)) dC_q/dx
 \end{aligned} \tag{3.14}$$

The effect of the pressure gradient can be seen in Figure 3.11.

Figure 3.11: Effect of β on the Defect Law (Reproduced from Herring and Norbury (1967))

It should be noted that the magnitude of β for all the wind tunnel experiments conducted never exceeded 0.03, showing that the pressure gradient had little effect on the velocity profile measurements.

3.3 Curve Fitting Techniques

3.3.1 Intercept Shift Calculations

Figure 3.12 gives an example of the intercept shift of the law-of-the-wall plot due to roughness. The intercept of the smooth-wall log-law region in the law-of-the-wall plot (B_{smooth}) minus the intercept of the log-law portion of the experimental plot gives the shift

$$\Delta U^+ = B_{\text{smooth}} - B_{\text{exp}} \quad (3.15)$$

Figure 3.12: Example Intercept Shift of the Law-of-the-Wall Plot due to roughness

3.3.2 k^+ , C intercept Calculation

The roughness Reynolds number k^+ is calculated using (2.13). Once the intercept shifts and roughness Reynolds numbers are calculated, the C-intercept can be found using Equation (2.15).

$$\Delta U^+ = A \log k^+ + C \quad (2.15)$$

3.3.3 Correlation Model

The correlation model used is based on finding a combination of roughness parameters that can characterize the surface roughness. As discussed in Chapter 2, the log-law intercept shift (ΔU^+) is a function of both the roughness Reynolds number (k^+) and roughness geometry as represented by the roughness-configuration constant 'C' in

Equation (2.17). The objective is to identify and combine suitable roughness parameters and to correlate them with 'C'. Six forms of C were tested:

$$C_1 = (aX^b + cY^d + eX^f + g) \quad (3.16a)$$

$$C_2 = (aX^bY^d + cZ^e + f) \quad (3.16b)$$

$$C_3 = (aX^bY^cZ^d + e) \quad (3.16c)$$

$$C_4 = a \log(bX^c + dY^e + fX^g) + h \quad (3.16d)$$

$$C_5 = a \log(bX^cY^d + eZ^f) + g \quad (3.16e)$$

$$C_6 = a \log(bX^cY^dZ^e) + f \quad (3.16f)$$

Where X,Y,Z represent a combination of roughness spacing, blockage and slope parameters (see Chapter 4) and a,b,c... etc. represent constants to be determined by regression analysis.

3.3.4 Model Coefficient Determination

The coefficients a,b,c.. etc were found using a Matlab curve-fitting program written by the author (see Appendix C), that was used to vary the coefficients to find a correlation between the shift points and the roughness characterization parameters. A least squares method was employed in the curve fitting algorithm. Initially all coefficients were tested from a range between boundary values of 8 to -8 by increments of 1. If it was found that the value of any of the coefficients was within 3 of the boundary values, the boundary values for the particular coefficient was shifted such that the range of integers tested was kept the same but is centered on the newly found coefficient. Once this process of curve fitting was completed, the results were examined. Correlations with

large amounts of scatter were discarded through visual inspection, and the remainder of the correlations were run through the curve fitting program once more. The boundary values of each of the coefficients in the second correlation run are set at 1 above and below the found coefficient value at increments of 0.1 effectively giving the curve fit an extra degree of accuracy.

3.3.5 Relation To Equivalent Sand Roughness

One of the methods used for representing roughness effects is the Prandtl - Schlichting curve for close-packed sand-grain roughness. It is used in various CFD programs where roughness geometries are expressed as an equivalent sand roughness (k_s). Thus it is necessary to find the relation between the equivalent sand roughness, k_s , and the roughness-configuration constant, C . For the fully-rough regime the velocity-shift equation for closely-packed sand roughness is (Grigson, 1984)

$$\Delta u^+ = A \log k_s^+ - 4.0 \quad (3.17)$$

That is, C has a value of -4.0 for standard-close-packed sand roughness. The value k_s is equal to the size of the sand-grains and is only applicable for closely-packed sand-grain roughness. Thus the value ' k ' as discussed in Section 2.4 will have the same value as ' k_s .' By equating the velocity-shift equation for standard-sand roughness and Equation 2.15, we can obtain

$$\begin{aligned} A \log k_s^+ - 4.0 &= \Delta u^+ = A \log k^+ - C \\ k_s/k &= 10^{(4.0 + C)/A} \end{aligned} \quad (3.18)$$

Chapter 4

Surface Metrology

Previous studies involving roughness effects on the boundary layer have primarily involved deterministic roughness where surface roughness has been easily definable due to its regular arrays of simply shaped roughness elements. Because this thesis will deal with stochastic (random) roughness, surface roughness will be more difficult and complex to define. As such, new characterization parameters applicable to both stochastic and deterministic roughness must be developed through surface metrology studies. This chapter will introduce concepts that are applicable to this thesis from the field of surface metrology, discuss the characterization parameters developed in this study, and look at the test surfaces used in this study

4.1 Defining Roughness

In the field of surface metrology, surface features can be divided into various different categories according to the *scale of observation*. For instance, what may seem smooth from a large scale of observation, can appear rough from a much smaller scale of observation. Another consideration is how the actual surface differs from the intended surface. The intended surface is known as the *form* of the surface, and is not considered to be roughness. Any large-scale deviations from the form are known as the *form error*. This could take the form of a large depression in the surface for instance. Reducing the scale of observation, the next feature of note is the *waviness* of the surface, and is also considered too large scale to be roughness. *Lay* is another feature of interest, which refers to any predominant direction of the surface texture, which is usually determined by the particular production method used to manufacture the surface. The texture of a surface would be defined as all deviations that are shorter in wavelength than form error. Further reducing the scale of observation, we finally come across *roughness*, which is considered the smallest wavelength of deviations from the nominal surface. See Precision Devices' webpage for more details (www.predev.com). From this, we can see that surface features can be defined according to the magnitude of the features.

Normally, surfaces must be filtered for the larger scale deviations from the nominal surfaces in order to find the surface roughness, but fortunately this was not required for the present surfaces as all tested roughness surfaces were manufactured on a smooth flat board. Thus, all of the present rough surfaces are void of any form error, or waviness. Also, all the rough surfaces sampled are isotropic, which means that the

roughness parameters do not vary with direction. Thus, there is no lay or directionality in the roughness. Although all test surfaces were stochastic, data from homogeneous deterministic roughness experiments was also used in this thesis during the correlation phase.

Figure 4.1: Surface Characterization (Modified reproduction from www.predev.com)

4.2 Measurement Techniques

4.2.1 Measurement Considerations

Sample Size

Ideally, it would be best to scan the whole surface in question from which surface parameters could be calculated. But due to budgetary and resource constraints, it is difficult to implement this as it can be very expensive to scan such a large area, and can also take significant computing resources to analyze the data. Thus it would be best to use a minimum sample size large enough to represent the surface. If this is achieved, then the descriptive parameters measured from the sample surface will not differ significantly

from parameters measured from a bigger sample size. Unfortunately, it is difficult to determine this minimum sample size, as it would require scans, and re-scans of the surfaces, resulting in increased costs. Engineering judgment and experience were used to choose a sample size that was thought to be adequate in providing a full representation of the surface in question.

Vertical, Lateral Resolution

The *vertical resolution* of a scan is the smallest change in height that can be measured. The *lateral resolution* of a scan is the smallest lateral step size that can be measured. The required resolution will depend on the size of the roughness. If it is large-scale roughness that is of interest, a lower resolution can be used. On the other hand, if a very fine, small-scale roughness is of interest, a higher resolution must be used in order to accurately capture the surface. Often it is the vertical and lateral resolution requirements that dictate the scanning method chosen (See Section 4.2.2).

4.2.2 Surface Measurement Methods

Contact Stylus Profiling

Contact Stylus Profiling is the oldest method used to measure surface profiles. A stylus is dragged across the surface, and its vertical motion is converted digitally through a transducer. The main advantage of this method is its simplicity. Unfortunately, because there is physical contact between the stylus and the surface, this method can be destructive to fragile surfaces. This method is not ideal for fine small-scale roughness as the resolution is limited by the size of the stylus tip.

Optical Interferometry

Optical Interferometry uses phase-shifted interference fringe patterns between two waves of light to measure changes in height in the surface. One wave reflects off the surface of interest, while the other wave remains unaffected. The resulting interference pattern between the two waves allows the measurement of any changes in height in the surface. This method enables the measure of very small surface features (down to the nanometer scale), depending on the wavelength of light used in the scan. Since this is a non-contact measuring technique, it can be used on fragile surfaces without causing damage.

Laser Triangulation

Laser Triangulation senses changes in height by measuring a light that is reflected from the surface in question. A transmitter (usually a laser diode) emits a light to the target area. A receiving element then detects the reflected light through an optical lens system. If the position of the target changes (i.e. the surface height changes), the position of the projected spot on the receiving element will change as well. The positional change of the light on the receiving element can then be measured and from this, the target's change in position can be found. This technique allows for detailed scans with resolutions down to the micrometer scale, and is applicable for the roughness scales that are of interest in this thesis. See Figure 4.2

Figure 4.2: Laser Triangulation Schematics (Reproduced from XYZ RGB, 2003)

4.3 Roughness Parameters

4.3.1 Considerations

Roughness characterization parameters are used to represent the various surfaces giving a numerical means to distinguish one surface from another. The challenge was finding parameters that were of importance to not only defining the rough surface, but also in influencing the roughness effects on the airflow. There was also the issue of finding the variables that would concisely describe the surfaces, reducing any redundant parameters, and minimizing the factors of influence during the correlation phase of the thesis. This last point would prove to be critical for two reasons. For each additional

parameter added, the number of combinations with other parameters during the correlation phase of the data regression increases exponentially and this could prove problematic with available computer resources (computing time being the primary area of concern). Also, with each additional surface parameter, a new variable is added into the correlation formula (in the form of an exponent or constant). See Section 3.3.3. Each additional variable would require more data points from experimental data in order to produce an accurate correlation. For example, there are far more possible combinations with eight variables from five roughness parameters than say four variables with two roughness parameters. One could possibly generate a good correlation with eight variables with the use of random roughness parameters due to the flexibility of using of too many variables. The final consideration was to find roughness parameters that would be applicable to both stochastic and deterministic rough surfaces. Most correlations available are only useful for deterministically rough surfaces, and not stochastic surfaces.

4.3.2 Roughness Parameter Definitions

Raw ASCII data files (see Appendix B) were imported into a Fortran analysis program (see Appendix D) written by the author, which was used to calculate various roughness parameters characterizing the surface height and configuration, including element shape and spacing, of the roughness. The various parameters are described here.

Roughness Height 'k'

The most basic and important roughness parameter that has been used in all deterministic and stochastic roughness studies has been 'k,' the measure of the height of

the roughness elements (see Section 2.4). In the case of uniform deterministic roughness, it is defined simply as the distance between the top and the bottom of the roughness elements. Because deterministic roughness elements are easily defined, 'k' is easily measured on a consistent basis.

For stochastic roughness, it is far more difficult to measure this parameter since each of the roughness elements, even though similar, are not identical. A revised roughness height definition would ideally give values consistent with the 'k' values defined previously in studies of deterministic roughness, but also would be applicable to stochastic roughness. This would leave open the possibility of integrating results for stochastic roughness with the substantial body of information available for deterministic roughness. It was then realized that if the roughness height could be defined as the range between the highest point on the surface and the lowest point on the surface then the value of 'k' would remain unaffected for deterministic roughness, yet be applicable towards stochastically rough surfaces. The problem with this definition is that any occasional random spike or drop in the surface height of the sample would give misleading values that do not accurately represent the bulk of the roughness sample.

With this in mind, a filtered form of the roughness height range is needed. By ordering all the sample surface heights from highest to lowest in a data bank and deleting the highest 1% and lowest 1% of all values, the roughness height 'k' can be found by subtracting the lowest remaining height value from the highest remaining height value. In doing so, any rare random surface features that are not indicative of the norm are

effectively filtered out. See Figure 4.3. In conjunction with this definition, we define the zero datum for height ‘z’ to be at the level the of lower deletion height, that is the lowest point of the filtered values.

Figure 4.3: *The difference between k' (unfiltered height definition) and k (new height definition) when applied to stochastic roughness.*

At the same time, because such a small fraction of the surface is deleted, the filtered roughness height value should only differ marginally if at all from the unfiltered value for deterministic roughness. It should be noted that this definition of ‘k’ is not applicable to very sparse roughness-configurations because the one percent filtration could eliminate too many, or even all of the roughness elements if these cover only a few percent or less of the surface. In the application to roughness in the sparse regime, a smaller fraction of cut-off points during the filtering process may be warranted. It was found that all stochastic roughness samples used in the present work were in the dense regime, thus this issue did not arise.

Blockage Parameter

This is one of the parameters used to represent the shape of the roughness. The idea of a blockage parameter originated from previous works using the discrete-element approach. As mentioned in Section 2.7, blockage is a measure of the spanwise fraction of

space occupied by the roughness elements blocking the flow, forcing the flow to move over and around the element. The blockage factor is defined as the percentage of element width associated with element height above $0.7k$ on a given spanwise traverse. See

Figure 4.4.

Figure 4.4: *Blockage Parameter Example*

For instance, for the particular configuration shown in Figure 4.4, one fourth of the total profile lies above $0.7k$. Thus the blockage parameter for this surface would be $L / 4L = 0.25$. The reason that the 70 percent value was chosen was because fuller roughness elements should have a significantly higher blockage value than say a sharper element (i.e. a cubical roughness element vs. a conical element), assuming that the roughness height is the same. The easiest way to distinguish elements is by a measure of how the height of the element is distributed, and it can be seen that fuller roughness elements would have a higher distribution of points near the top half of the element versus the bottom half. It was also decided that the half way cut off point of $0.5k$ would be unsuitable as a blockage parameter because it would not be sensitive enough to the distribution of points near the top of the element. On the other hand, a cut off point at $0.7k$ would be a more suitable choice without getting too close to the top of the element, and was thus chosen. Thus the blockage is expressed as

$$\text{Blockage} = \left[\sum_{i=1}^n g(i) \right] / n \quad (4.1)$$

$$\begin{aligned} \text{where} \quad & g(i) = 1 \text{ if } z(i) \geq 0.7k \\ & g(i) = 0 \text{ if } z(i) < 0.7k \end{aligned}$$

From the above definition, $g(i)$ is a Boolean function dependent on $z(i)$ which is the local surface height at a particular point. 'k' is the roughness height as defined previously, and 'n' is the total number of points on the surface – defined by the area of the sample surface and the horizontal and vertical scan resolution. Note that Equation (4.1) assumes that the transverse spacing between the data points is constant. Although this definition works well with stochastic roughness, its application to deterministic roughness is poor. Since the stochastic roughness is assumed to be isotropic, none of the parameters, including blockage, will change with the direction of the traverse or with the streamwise location of the traverse. But since deterministic roughness is distributed in an orderly fashion, any traverses in directions or locations which lie in between the rows of roughness elements, will result in blockage values of zero. Thus a revised definition was adopted. The revised definition of the blockage factor is defined as the maximum value given by Equation (4.1) for any spanwise traverse of the roughness sample. For example, if three spanwise traverses are made on a surface, and Equation (4.1) gives values of 0.4, 0.1, and 0.0 for the individual traverses, then the blockage factor for that particular surface is 0.4, since it is the maximum spanwise blockage measurement for any spanwise traverse. This definition is also applicable to stochastic roughness, since the maximum blockage should equal the average blockage of the entire surface.

The main reason why the blockage parameter was introduced is because it is a parameter that will have an appreciable magnitude even in the event low volume (thin) roughness elements are used – e.g. roughness in the form of flat plates normal to the flow. When this type of element is used, some of the mean surface parameters introduced below (e.g. average height, average slope (in particular directions), average RMS height) will reduce to extremely small values. The blockage parameter gives a measure of the effect of roughness elements without being dependent on the volume of the roughness.

Figure 4.5: Staggering

The blockage parameter also has the ability to distinguish between staggered and non-staggered deterministic roughness (Figure 4.5), as staggered surfaces will have lower blockage values for the same number and size of elements in an array. Currently, no

deterministic roughness correlation has the ability to successfully distinguish between staggered and non-staggered rough surfaces.

One area of concern is the problem of diagonal staggering. One might expect rough surfaces 'a' and 'c' from Figure 4.5 to have similar effects on air flow (assuming spacing and element size are identical). Unfortunately, the blockage parameter would show a higher value for surface 'a'. If a reasonable roughness correlation is found with a heavy dependence on the blockage parameter, diagonal staggering may become an issue. One potential way to address this problem is to use the maximum average blockage parameter measured from a projected surface profile from a thin strip across the test surface, instead of using a measure from a single traverse. One could set the width 'b' of this strip to be a function of the roughness height. That is,

$$b = \alpha k$$

where α is a constant yet to be determined. Unfortunately, more staggered deterministic roughness is needed to determine the validity of this suggested solution, and to confirm if, in fact, diagonal staggering would have any affect on flow. The current work will utilize the maximum average blockage parameter from a single traverse as it is the simpler of the two and can be used to determine if there is any indication of a correlation between blockage and 'C'. If a correlation does exist, further work can be done to explore more refined versions of the blockage parameter.

Average-absolute Slope Parameter

An average-absolute 2-D slope parameter was selected to give an indication of the ratio between the wetted surface area over the plan area. It is argued that for any given element shape, the average slope value for a given 'k' would vary only with an increase in the concentration or number of roughness elements – in effect increasing the wetted surface to plan area ratio. The reasoning behind this is that although local slope values at various points may be different for various roughness shapes, the average value of the slope will reflect the overall change in height (rise) over a particular distance (run). Thus, if the height of the roughness elements remains fixed, the slope parameter will only be affected by a change in the run, which effectively reflects the concentration of roughness elements. This parameter also can be looked upon as an indication of the fragmentation of the surface, as it will be able to distinguish between one large element and several small elements bunched together in a group of similar size to the large element, for example. This parameter is believed to be of significance to the correlation process, and it was used in some of the regression analysis. The defining equation for the absolute average slope parameter for spanwise traverses is

$$\text{abslope} = \left(\sum_{i=1}^n | (z_i - z_{i-1}) / \Delta x | \right) / n \quad (4.2)$$

where 'z' is the surface height, 'Δx' is the step size between height measurement points with a value between 0.25k – 0.33k (to be explained later), and 'n' is the total number of local slope calculations. Because this thesis deals only with isotropic stochastic and deterministic roughness, the average-absolute slope will yield the same

results whether it is measured in the streamwise or spanwise direction provided that the sample size 'n' is sufficiently large. Note that for deterministic roughness, it is essential that that sample includes a sufficient number of distinct traverses to obtain an average representative of the surface as a whole, rather than of just one row of elements. In future work, where the roughness may be non-isotropic, it may be necessary to differentiate between streamwise and spanwise slope values. In fact, if the roughness elements become non-symmetric, an absolute slope may not be applicable as it may be necessary to differentiate between a positive (frontal) slope, and a negative (back) slope.

It should also be noted that the average-absolute slope is dependent on the resolution (Δx) used in the slope calculation. The higher the resolution used, the higher the absolute average slope becomes due to the fact that higher resolution would be more sensitive to small scale roughness that is superimposed on the larger elements. Since the average-absolute slope should reflect the concentration of the major roughness elements, it is not appropriate to use the highest resolution available. But, on the other hand, if the resolution is too low, the slope calculation will be unable to suitably define the surface. Using a set step size for all the surfaces would be inappropriate since the roughness element sizes and concentrations of surfaces would vary significantly from surface to surface. It was then decided to define the step size as a fraction of the roughness height parameter 'k'. After some trial calculations, and using engineering judgment, it was decided that the resolution used in the slope calculation would be approximately one quarter of the roughness height ($0.25k$). The reasoning for this choice was that it was believed to be fine enough to adequately capture the roughness elements of interest, yet

coarse enough to be minimally affected by small-scale roughness. Actual resolutions used varied between 0.25k – 0.33k as some surfaces were unable to produce a 0.25k resolution. The maximum interpolated variation in the slope values calculated for step sizes between 0.25k and 0.33k was approximately 7%.

Maximum-Average-Absolute Slope

This parameter is defined as the highest value given by Equation (4.2) for any spanwise traverse for a particular surface. This parameter is used not only as an indication of the roughness element concentration (similar to the Average-absolute Slope), but also as a means to distinguish between staggered and non-staggered deterministic roughness elements. This parameter was formulated after initial testing showed that many correlations using the average-absolute slope did not consistently distinguish between staggered and non-staggered roughness. Thus, the maximum-average-absolute slope was used in an attempt to obtain a better correlation. This value should only differ from the average-absolute slope for the deterministic samples, as the stochastic surfaces are essentially isotropic, and their values of average-absolute slope should be essentially the same for any given traverse. The concern regarding diagonal staggering (previously discussed when presenting the blockage parameter) also exists for this parameter, and a similar solution using a projected profile from a strip of width αk may be applicable here as well, but more testing is needed to determine this.

Distribution Parameters

Another way to look at the spacing of roughness elements on a surface is to examine the total height distribution over the whole surface. The simplest and most common parameter that addresses this issue is the mean height ' k_{mean} '. This is calculated by the following relation:

$$k_{\text{mean}} = 1/n \left(\sum_{i=1}^n z_i \right) \quad (4.3)$$

where ' n ' is the total number of points, and ' z ' is the surface height at point ' i '. ' k_{mean} ' can be visualized as the 'meltdown' height. When the mean height is normalized by the roughness height ' k ', the corresponding parameter ' $r_{\text{mean}/k}$ ' gives an indication of the height distribution – see Equation (4.4).

$$r_{\text{mean}/k} = \left(\sum_{i=1}^n z_i \right) / (kn) \quad (4.4)$$

where ' n ' is the total number of points, z is the surface height at point ' i ', and ' k ' is the roughness height parameter. A higher ' $r_{\text{mean}/k}$ ' would signify that a larger fraction of the surface is higher. This could indicate that there is a higher concentration of roughness elements, or that the size (not height) of the roughness elements is larger (or both).

Another means of looking at the distribution of height on the surface is to use the RMS (root mean squared) height instead of the mean height. The RMS height is calculated through

$$k_{\text{rms}} = \left((1/n) \sum_{i=1}^n z_i^2 \right)^{0.5} \quad (4.5)$$

The corresponding normalized variable k_{rms}/k (named ' r_{rms}/k ') would essentially accomplish the same function as ' r_{mean}/k ,' only be more sensitive to the higher heights. See Equation (4.6).

$$r_{rms}/k = ((1/n) \sum_{i=1}^n z_i^2)^{0.5}/k \quad (4.6)$$

Because of this, ' r_{rms}/k ' would be more sensitive to more blunt roughness elements (cubical) as opposed to sharp elements (conical) than ' r_{mean}/k '. Since the mean height and the RMS height could possibly be of significant importance, the two were also normalized against one another to create ' r_{rms}/r_{mean} ' (equal to k_{rms}/k_{mean}) which could serve as another possible distribution parameter (See Equation (4.7)). It was later found that these three distribution parameters could be used as a basis to distinguish between the dense and sparse roughness regimes (to be discussed later in chapter 5).

$$r_{rms}/r_{mean} = ((1/n) \sum_{i=1}^n z_i^2)^{0.5} / ((\sum_{i=1}^n z_i)/n) \quad (4.7)$$

Fourier Transform

It should also be noted that an attempt to describe the surfaces through the use of Fourier Transforms was made. Simply put, this method attempts to characterize surface maps through a summation of waves of various magnitudes and wavelengths. After some preliminary research on this subject, it was decided that this approach was unnecessarily complex for the present purposes. Although this subject was not further explored, it may be an area of interest for future work in this area.

4.4 Experimental Surfaces

The experimental surfaces used in this thesis were chosen to represent a wide range of stochastically rough surfaces – from very rough, to mildly rough, as well as a smooth surface for validation purposes. Six of the surfaces including the smooth surface were also tested by Ferrari (2002). This section will describe the surfaces tested along with calculated surface parameters. Calculated surface parameters for deterministic roughness can be found in Appendix F.

4.4.1 Surface Metrology Preparation

Five of the six stochastically rough surfaces were tested in the wind tunnel and are the same ones tested by Ferrari (2002). The surface scanning device used for these is the CyberOptics CyberScan 212 system (www.cyberoptics.com) that utilizes the laser triangulation method. Ferrari (2002) arranged for the scanning of these surfaces. For more details on the surface scanner, refer to Ferrari (2002). The sixth surface scanned was the closely-packed sand roughness prepared by the author, as described in Section 4.4.2. The author arranged for scanning of this surface using the XYZ RGB system (www.xyzrgb.com), which also uses the laser triangulation method. As mentioned earlier, relatively small samples from each surface were used to represent the surface as a whole.

Table 4.1 summarizes the surface scan parameters for each of the sample surfaces.

	scan area (mm x mm)	lateral res (μm)	vertical res (μm)	total points
Fine Shingle	8x8	25	3.8	103041
Coarse Shingle	12x12	40	10.2	90601
Fine Spackle	20x20	100	10.2	40401
Tar Paper	8x8	20	2	160801
Formica	10x10	40	3.8	63001
Sand Roughness	35x75	110	110	218368
Smooth	-	-	-	-

Table 4.1: Surface scan parameters

Two additional surfaces of Vinyl and Coarse Spackle were also tested but were not used in the study. It was found that the Vinyl surface was covered in a clear coating. The surface scanning device would not have been able to detect this coating and would actually be measuring the surface underneath the coating, giving inaccurate readings on the surface. The Coarse Spackle surface was not used because it was too rough. It was found that the roughness elements gave large enough local disturbances that affected the boundary layer traverses, as the readings were found to differ depending where the pitot tube mouth was and whether the mouth was directly behind a roughness element.

4.4.2 Description of Surfaces

Coarse Shingle

This surface is used in roofing, and is composed of coarse-grit crushed slate grains embedded in tar. Unlike roofing shingles, this sheet was a single large continuous surface. This manufacturer of this particular surface is EMCO Building Products Group, and the model number for this surface is 'RR 401 CSA Spec A 123.2-M Type M.' This surface was purchased from a local hardware store and was bonded to a MDF (Medium Density Fiber) board using water-based contact cement. Figure 4.6 is a shaded relief overhead view of the surface, giving representation closely similar to an actual photograph of the surface, and Figure 4.7 contains a standard meshed surface map as well as a color contoured map of the surface. Table 4.2 gives a summary of the various surface-characterization parameters calculated for this surface.

Figure 4.6: Shaded Relief Overhead View of the Coarse Shingle Surface (all dimensions in mm) (Ferrari, 2002)

height, k (mm)	$r_{\text{mean}/k}$ Eqn (4.4)	$r_{\text{rms}/\text{mean}}$ Eqn (4.7)	$r_{\text{rms}/k}$ Eqn (4.6)	blockage Eqn (4.1)	slope Eqn (4.2)	max slope
0.86	0.58	1.07	0.63	0.33	0.42	0.42

Table 4.2: Characterization Parameters of the Coarse Shingle Surface

Figure 4.7: *Meshed Surface Map (top) and Color Contour Map (bottom) of the Coarse Shingle Surface (all dimensions in mm) (Ferrari, 2002)*

Fine Shingle

This surface is used in roofing, and is composed of fine grains of grit embedded in tar. Unlike roofing shingles, this sheet was a single large continuous surface. The manufacturer of this particular surface is EMCO Building Products Group, and the model

number of this surface is 'RR 303 CSA Spec A 123.2-M Type S.' This surface was purchased from the a local hardware store and was bonded to a MDF board using water-based contact cement. Figure 4.8 is a shaded relief overhead view of the surface, giving representation closely similar to an actual photograph of the surface, and Figure 4.9 contains a standard meshed surface map as well as a color contoured map of the surface. Table 4.3 gives a summary of the various surface-characterization parameters calculated for this surface.

Figure 4.8: *Shaded Relief Overhead View of the Fine Shingle Surface (all dimensions in mm) (Ferrari, 2002)*

height, k (mm)	$r_{\text{mean/k}}$ Eqn (4.4)	$r_{\text{rms/mean}}$ Eqn (4.7)	$r_{\text{rms/k}}$ Eqn (4.6)	blockage Eqn (4.1)	Slope Eqn (4.2)	max slope
0.58	0.52	1.06	0.55	0.14	0.61	0.61

Table 4.3: *Characterization Parameters of the Fine Shingle Surface*

Figure 4.9: *Meshed Surface Map (top) and Color Contour Map (bottom) of the Fine Shingle Surface (all dimensions in mm) (Ferrari, 2002)*

Fine Spackle

This surface is produced by applying small-scale wall spackle paint to MDF board. The make and model of the paint is 'BEHR Premium Plus Sand Wall Texture – Sand Finish.' The paint is latex based and applied directly to the MDF board with a

small-scale spackle roller ‘Bennett, Heavy Texture Stucco Roller.’ Both items were purchased from the local hardware store. Figure 4.10 is a shaded relief overhead view of the surface, giving representation closely similar to an actual photograph of the surface, and Figure 4.11 contains a standard meshed surface map as well as a color contoured map of the surface. Table 4.4 gives a summary of the various surface-characterization parameters calculated for the surface.

Figure 4.10: Shaded Relief Overhead View of the Fine Spackle Surface (all dimensions in mm) (Ferrari, 2002)

height, k (mm)	$r_{\text{mean}/k}$ Eqn (4.4)	$r_{\text{rms}/\text{mean}}$ Eqn (4.7)	$r_{\text{rms}/k}$ Eqn (4.6)	blockage Eqn (4.1)	Slope Eqn (4.2)	max slope
0.84	0.38	1.16	0.44	0.08	0.21	0.21

Table 4.4: Characterization Parameters of the Fine Spackle Surface

Figure 4.11: *Meshed Surface Map (top) and Color Contour Map (bottom) of the Fine Spackle Surface (all dimensions in mm) (Ferrari, 2002)*

Tar Paper

Tar paper is used in construction applications and is comprised of a matrix of fibers and tar. The manufacturer of this surface is EMCO Building Products Group, and the model number is 'BP RF403 #15 Asphalt Felt Plain.' It was acquired from a local

hardware store and attached to a MDF board using a water-based contact cement. Figure 4.12 is an shaded relief overhead view of the surface, giving a close representation to an actual photograph of the surface, and Figure 4.13 contains a standard meshed surface map as well as a color contoured map of the surface. Table 4.5 gives a summary of the various surface-characterization parameters calculated for the surface

Figure 4.12: *Shaded Relief Overhead View of the Tar Paper Surface (all dimensions in mm) (Ferrari, 2002)*

Height, k (mm)	$r_{\text{mean}/k}$ Eqn (4.4)	$r_{\text{rms}/\text{mean}}$ Eqn (4.7)	$r_{\text{rms}/k}$ Eqn (4.6)	blockage Eqn (4.1)	slope Eqn (4.2)	max slope
0.23	0.43	1.13	0.49	0.14	0.32	0.32

Table 4.5: *Characterization Parameters of the Tar Paper Surface*

Figure 4.13: *Meshed Surface Map (top) and Color Contour Map(bottom) of the Tar Paper Surface (all dimensions in mm) (Ferrari, 2002)*

Formica

Formica is a brand of plastic laminate veneer primarily used in decorative surface applications commercially and residentially (e.g. countertops). The model used is

‘Wilsonart 4604-50 Pearl Sand, Premium #50 Finish.’ It was purchased from a local hardware store and attached to a MDF board using a water-based contact cement. Figure 4.14 is an shaded relief overhead view of the surface, giving a close representation to an actual photograph of the surface, and Figure 4.15 contains a standard meshed surface map as well as a color contoured map of the surface. Table 4.6 gives a summary of the various surface-characterization parameters calculated for the surface.

Figure 4.14: *Shaded Relief Overhead View of the Formica Surface (all dimensions in mm) (Ferrari, 2002)*

Height, k (mm)	$r_{\text{mean}/k}$ Eqn (4.4)	$r_{\text{rms}/\text{mean}}$ Eqn (4.7)	$r_{\text{rms}/k}$ Eqn (4.6)	blockage Eqn (4.1)	slope Eqn (4.2)	max slope
0.13	0.53	1.09	0.58	0.23	0.06	0.06

Table 4.6: *Characterization Parameters of the Formica Surface*

Figure 4.15: *Meshed Surface map (top) and Color Contour Map (bottom) of the Formica Surface (all dimensions in mm) (Ferrari, 2002)*

Closely Packed Sand-Grains

This surface is a uniform coating of closely packed silica sand-grains on a flat piece of high-density particleboard 3 x 6 inches in size. The sand-grains were run through a pair of sieves to ensure that the sand-grains were of similar size (approximately 0.94mm in diameter). The sand was attached to the particle board by applying a very fine coat of Krylon Interior/Exterior paint (1501 Glossy White) from an aerosol can. The board was then left to dry for 5 minutes to minimize moisture in the paint to reduce the chance of the paint running. The particleboard was placed lightly into a box of the silica sand (painted side down) and left for 10 minutes before taking the surface out of the box and leaving it to dry. Since silica sand-grains were used, there was a slight translucency of the sand-grains that resulted in some difficulty of the surface scanning device in accurately capturing the surface. This problem was solved by a very light application of spray paint to the surface, thus removing the translucency. There was no pooling of the paint on the surface, and it was determined that the paint would have no significant effect on the true surface measurements. It should be noted that the method used to prepare this sample is similar to that used in Nikuradse (1933).

This surface was not subjected to wind tunnel testing, since there are already correlations in the literature between the sand-grain size and the roughness-configuration constant for closely packed sand-grain roughness (see Section 3.3.5 for details). Figure 4.16 is meshed surface map of the closely packed sand surface and Table 4.7 gives a summary of the various surface-characterization parameters calculated for the surface.

Figure 4.16: Wire Mesh Representation of CloselyPacked Sand-grain Surface

Height, k (mm)	$r_{\text{mean}/k}$ Eqn (4.4)	$r_{\text{rms}/\text{mean}}$ Eqn (4.7)	$r_{\text{rms}/k}$ Eqn (4.6)	blockage Eqn (4.1)	slope Eqn (4.2)	max slope
0.98	0.58	1.03	0.63	0.32	0.54	0.54

Table 4.7: Characterization Parameters for the Closely-Packed Sand-grain Surface

It should be noted that the value shown in Table 4.7 for the sand roughness height, k , was the value found from the surface scan data using the height definition given in Section 4.3.2. It is in good consistency with the directly measured average sand-grain size (0.94mm as mentioned earlier) that was found by measuring the length occupied by 60 sand-grains lined up against one another along a straight edge. The measured length was then divided by the number of sand-grains used in the measurement (60), which calculates the average size of the sand-grain.

Chapter 5

Results

The experimental results and data regression analysis are presented in this chapter. The results of the boundary layer analysis are presented in Section 5.2 and the regression analysis is presented in Section 5.3.

5.1 Preliminaries

Before any of the results are presented, a note must be made on the applicability of the results from the first, second, and last probing stations. Although the first probing

station is 31 cm downstream of the front of the test section, the lip on the adjustable ceiling extends eight inches (20cm) from the front of the test section. See Figure 5.1.

Figure 5.1: Adjustable Lip in Test Section

Because of this, the effective start of the test section lies at the point where the lip meets the adjustable ceiling, since the region that lies directly beneath the lip would be subject to a significant static pressure change and free stream velocity variation. Thus, the distance of the measurement stations from the effective start of the test section ('x') are 20cm shorter than the distance measured from the front of the test section. This is important in determining the validity of the measurements from each particular station number. Antonia and Luxton (1971) found that the readjustment of the boundary layer to an equilibrium state requires approximately twenty boundary-layer thicknesses from the start of the roughness. Comparing this ratio to measured values from experimental data (See Table 5.1) we find that two of the five test surfaces (Coarse Shingle, Fine Spackle)

may not be in equilibrium at Station 2. Since Station 2 is close enough to be in question, station 1 is obviously not in equilibrium, and is of no use in this study.

	δ (mm)		δ (mm)	
	Station 2 (x=118mm)	x/ δ	Station 3 (x=476mm)	x/ δ
Fine Shingle	18.6	25.2	26.2	31.8
Coarse Shingle	20.4	23.1	27.9	29.8
Fine Spackle	20.8	22.6	26.8	31.4
Tar Paper	15.9	29.5	21.2	39.2
Formica	15.1	31.1	19.5	42.7

Table 5.1: Boundary Layer Equilibrium

Experimental data showed that station 5 for each of the test surfaces, exhibited moderate amounts of scatter indicating that some mild unsteadiness may be occurring towards the rear of the test section. This may have been caused by the boundary layer on the ceiling separating at the exit of the test section, where there is a step between the adjustable ceiling and the test section ceiling. It should also be noted that station 5 is about 9cm upstream of the bleeder slot (see Figure 5.1 and Figure 3.3) at the end of the test section, and the flow may be affected by the influx of air into this bleeder slot. Thus the results of station 5 should also be interpreted with caution.

The pressure gradient parameter (β) calculation is also subject to some error.

Recalling Equation (3.14):

$$\beta = -(\delta^*/(C_f C_q)) dC_q/dx \quad (3.14)$$

dC_q/dx was calculated from a single test trial and it can be expected that the gradient should not change with wind tunnel speed. On the other hand, values of δ^* , C_f and C_q had to be determined individually at each station using boundary layer traverses resulting in the values being evaluated using separate traverses. Since is difficult to have each test

case at the exact same Reynolds number, it is necessary to determine the sensitivity of δ^* , C_f , and C_q to the Reynolds number. Testing showed that the variables in Equation (3.14) changed by less than 1%, over a difference in free stream velocity of 12 m/s (approximately a 20% change in Reynolds number). Observing that the free stream velocity varied less than 2 m/s between test cases at different stations, it was decided that the effect on ' β ', which is non dimensional, would be negligible. It should also be noted that ' β ' was not used in any calculations, but just as an indication of the mildness of any streamwise pressure gradients in the test section.

5.2 Boundary Layer Results

The methodology used to calculate the boundary layer results will be covered in Section 5.2.1. Section 5.2.2 will present the boundary-layer profile, skin-friction coefficient ' C_f ', law-of-the-wall shift, and the roughness-configuration constant ' C '. Data from the smooth surface test case will also be presented for verification purposes.

5.2.1 Methodology

A rough estimate of the skin-friction coefficient is first found using the momentum integral equation (Equation 3.13b). This was used to give an initial estimate of the friction velocity U^* , which can be used to create a preliminary logarithmic log-law plot. The displacement height for each profile is then adjusted to make the overlap region in the law-of-the-wall plot a straight curve, as described in Section 2.5. A new skin-friction coefficient is then found by applying the Defect-law method discussed in Section 3.2.6, and used to calculate a revised U^* . This is applied back to the logarithmic law-of-the-wall plot, and any plots that do not have a straight overlap region in the plot are re-

adjusted once again. This iterative procedure continues until the skin- friction coefficients converge, resulting in a straight curve in the overlap region for each curve.

5.2.2 Smooth Surface

Table 5.2 lists all the boundary-layer parameters for the smooth surface. ‘x’ (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using three different methods – the Momentum Integral Method, the Clauser Plot, and the Defect-law Plot.

Smooth						
station	x (mm)	δ (mm)	δ^* (mm)	Θ (mm)	H	C_q
2						
3	1032	16.4	2.4	1.7	1.4	1.07
4	1394	22.8	3.2	2.3	1.4	1.07
5	1724	25.8	3.9	2.8	1.4	1.07

station	Beta	C_f (MIM)	C_f Clauser	C_f Defect	U_{ref}
2					
3	-0.014	-	0.0032	0.0030	38.6
4	-0.003	0.0031	0.0030	0.0030	38.5
5	0.002	0.0030	0.0029	0.0029	38.5

Table 5.2: Smooth Surface Boundary Layer Results

The values of ‘H’ confirm that the boundary layer is turbulent, and they are also consistent with expected values of ‘H’. For smooth-wall boundary layers, ‘H’ usually has a value close to 1.4 (White, (1991)), which is seen in the data. Station 2 data for this particular surface was unavailable for analysis due to the data file being corrupted. The importance of this data is that it shows that the skin-friction coefficients calculated from all three methods are consistent with one another, and this supports their validity.

Figure 5.2: Normalized Smooth Surface Boundary-Layer Profiles

Figure 5.3: Clauser Plot for Smooth Surface Velocity Profiles

5.2.3 Formica

Table 5.3 lists all the boundary-layer parameters for the Formica test surface. ‘x’ (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using two different methods – the Momentum Integral Method, and the Defect-law Plot. The corresponding friction velocity, U^* , roughness Reynolds number, along with roughness-configuration constant are also listed.

Formica station	x (mm)	δ (mm)	δ^*	θ	H	C_q	U_{ref}
2	676	15.1	2.2	1.6	1.4	1.20	61.1
3	1032	19.7	2.7	2.1	1.3	1.18	61.1
4	1394	23.7	3.6	2.6	1.4	1.17	60.7
5	1724	28.4	4.3	3.1	1.4	1.19	60.0

station	β	C_f MIM	C_f def	U^*	k^+	$A \log k^+$	$\Delta U/U^*$	C
2	-0.007		0.0030	2.60	20.5	7.4	0.7	-6.7
3	-0.030	0.0028	0.0031	2.61	20.6	7.4	1.7	-5.7
4	-0.002	0.0030	0.0031	2.57	20.2	7.3	2.1	-5.2
5	-0.002	0.0029	0.0031	2.50	19.7	7.3	2.4	-4.9

Table 5.3: Formica Surface Boundary Layer Results

Figure 5.5: Defect-Law Plot for Formica Surface Velocity Profiles

5.2.4 Fine Shingle

Table 5.4 lists all the boundary-layer parameters for the Fine Shingle test surface. 'x' (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using two different methods – the Momentum Integral Method, and the Defect-law Plot. The corresponding friction velocity, U^* , roughness Reynolds number, along with roughness-configuration constant are also listed.

F. Shingle							
station	x (mm)	δ (mm)	δ^*	θ	H	C_q	U_{ref}
2	676	18.6	4.0	2.5	1.6	1.21	54.5
3	1032	26.2	5.3	3.4	1.6	1.18	56.2
4	1394	34.3	6.6	4.3	1.5	1.17	54.3
5	1724	39.4	7.4	5.0	1.5	1.18	56.2

station	β	C_f MIM	C_f def	U^*	k^+	$A \log k^+$	$\Delta U/U^*$	C
2	-0.037		0.0060	3.28	115.3	11.6	9.5	-2.1
3	-0.021	0.0053	0.0056	3.23	113.5	11.5	9.5	-2.1
4	-0.005	0.0050	0.0053	3.02	106.2	11.4	9.5	-1.9
5	+0.001	0.0037	0.0048	3.07	107.9	11.4	9.0	-2.4

Table 5.4: Fine Shingle Surface Boundary Layer Results

Figure 5.8: Log-Law Plot for Fine Shingle Surface Velocity Profiles

5.2.5 Coarse Shingle

Table 5.5 lists all the boundary-layer parameters for the Coarse Shingle test surface. 'x' (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using two different methods – the Momentum Integral Method, and the Defect-law Plot. The corresponding friction velocity, U^* , roughness Reynolds number, along with roughness-configuration constant are also listed.

C. Shingle							
station	x (mm)	δ (mm)	δ^*	θ	H	C_q	U_{ref}
2	676	20.4	4.3	2.7	1.6	1.21	54.4
3	1032	27.9	5.8	3.7	1.6	1.19	55.9
4	1394	35.7	7.1	4.6	1.5	1.19	55.3
5	1724	43.3	8.3	5.6	1.5	1.18	54.6

	β	C_f MIM	C_f def	U^*	k^+	$A \log k^+$	$\Delta U/U^*$	C
2	-0.032		0.0062	3.36	175.1	12.6	9.9	-2.7
3	-0.018	0.0056	0.0060	3.33	173.6	12.6	10.3	-2.3
4	-0.001	0.0050	0.0056	3.19	166.3	12.5	10.3	-2.2
5	0.004	0.0059	0.0054	3.08	160.5	12.4	10.1	-2.3

Table 5.5: Coarse Shingle Surface Boundary Layer Results

5.2.6 Fine Spackle

Table 5.6 lists all the boundary-layer parameters for the Fine Spackle test surface. 'x' (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using two different methods – the Momentum Integral Method, and the Defect-law Plot. The corresponding friction velocity, U^* , roughness Reynolds number, along with roughness-configuration constant are also listed.

F. Spackle							
station	x (mm)	δ (mm)	δ^*	θ	H	C_q	U_{ref}
2	676	20.56	4.20	2.67	1.56	1.19	52.7
3	1032	26.65	5.53	3.48	1.59	1.20	49.5
4	1394	35.47	7.10	4.57	1.55	1.17	54.2
5	1724	43.08	8.40	5.51	1.52	1.18	53.3

station	β	C_f MIM	C_f def	U^*	k^*	$A \log k^*$	$\Delta U/U^*$	C
2	-0.032		0.0054	2.98	150.8	12.2	9.0	-3.24
3	-0.018	0.0045	0.0050	2.71	137.1	12.0	8.7	-3.31
4	-0.001	0.0060	0.0046	2.79	141.2	12.1	8.6	-3.48
5	-0.004	0.0057	0.0044	2.72	137.6	12.0	9.0	-3.02

Table 5.6: Fine Spackle Surface Boundary Layer Results

Figure 5.12: Log-Law Plot for the Fine Spackle Surface Velocity Profiles

5.2.7 Tar Paper

Table 5.7 lists all the boundary-layer parameters for the Tar Paper test surface. 'x' (as illustrated in Figure 5.1) is the streamwise distance measured from the start of the roughness at the test section entrance. The skin-friction coefficient C_f is calculated using two different methods – the Momentum Integral Method, and the Defect-law Plot. The corresponding friction velocity, U^* , roughness Reynolds number, along with roughness-configuration constant are also listed.

Tar Paper station	x (mm)	δ (mm)	δ^*	θ	H	C_q	U_{ref}
2	676	15.93	2.76	1.86	1.48	1.19	59.8
3	1032	21.22	3.47	2.50	1.39	1.17	60.3
4	1394	26.71	4.42	3.16	1.40	1.17	60.6
5	1724	32.17	5.04	3.65	1.38	1.17	58.8

	β	C_f MIM	C_f def	U^*	k^+	$A \log k^+$	$\Delta U/U^*$	C
2	-0.032		0.0042	2.92	40.7	9.0	4.7	-4.35
3	-0.018	0.0036	0.0039	2.85	39.7	9.0	4.7	-4.29
4	-0.001	0.0037	0.0037	2.79	38.9	8.9	4.7	-4.23
5	0.004	0.0029	0.0036	2.68	37.4	8.8	4.6	-4.24

Table 5.7: Tar Paper Surface Boundary Layer Results

Figure 5.14: Log-Law Plot for Tar Paper Surface Velocity Profiles

5.2.8 Assessment of Boundary Layer Results

5.2.8.1 Fully-Rough Flow Analysis

The roughness Reynolds numbers (k^+) for each surface at stations 3, and 4 are listed in Table 5.8 along with their corresponding roughness-configuration constant 'C'. The roughness-configuration constant 'C' used for each of the surfaces in the correlations was the average of the two stations for each surface, unless otherwise stated.

	Station 3		Station 4		Average	
	k^+	C	k^+	C	C	$\Delta U/U^*$
Coarse Shingle	173.6	-2.29	166.3	-2.18	-2.23	10.3
Fine Shingle	113.5	-2.05	106.2	-1.89	-1.97	9.5
Fine Spackle	137.1	-3.31	141.2	-3.48	-3.4	8.7
Tar Paper	39.7	-4.35	38.9	-4.23	-4.26	4.7
Formica	20.6	-5.68	20.2	-5.25	-5.47	1.9

Table 5.8: *Law-of-the-wall Shift Data Summary*

From the results for k^+ in Table 5.8, we see that both the Tar Paper and Formica surfaces have k^+ values below 70 – the approximate accepted lower limit of the fully-rough regime (See Section 2.4). Thus, they may be in the transitionally-rough regime. Referring to Figure 5.15, we see that although the transitionally-rough regime is traditionally defined to exist when the value of k^+ is between 5 and 70 (where the data curves in Figure 5.15 tend to be curved), some of the lines in the plot are straight for values of k^+ well below 70. Superimposing the data points found for the Tar Paper and Formica surfaces onto Figure 5.15, we find the Tar Paper data point is in the region where the curves tend to be straight. From this, one can argue that the Tar Paper is in fact in the Fully-Rough regime. This is further supported by the Tar Paper experimental log-law plots (Figure 5.14) where the overlap regions collapse.

Figure 5.15: Effect of Roughness on the Law-of-the-wall Intercept (Modified from Dvorak (1969))

On the other hand, the Formica data point is not convincingly in this straight line region. However, if we extrapolate into the straight-line region, we can determine the roughness-configuration constant 'C' that the surface would yield if the Reynolds number of the flow were high enough to result in fully-rough flow. Using the points $k^+ = 100$ and $\Delta U/U^* = 6$, we insert these values into Equation (2.15) and find that $C = -5.24$ for fully-rough flow, as opposed to our measured value of $C = -5.47$ which was in the transitionally-rough regime. Thus the value of $C = -5.24$ was used in the regression analysis for the Formica data point.

5.2.8.2 Spacing Regime Analysis

Looking at the work done by Simpson (1973) and Dvorak (1969), rough surfaces can be categorized into two spacing regimes – a sparse and a dense spacing regime. Unfortunately, due to the little work that has been done with stochastic roughness in the past, there is no way to categorize stochastic surfaces according to deterministic spacing definitions. Thus a revised definition was needed to identify which spacing regime the rough surfaces belong to. Without this, it would be very difficult to produce any correlation that would be accurate. For instance, if two of the five tested sample surfaces belonged to the sparse regime while the other surfaces belong to the densely-spaced roughness regime, it would be impossible to find an accurate correlation.

Examining the deterministic roughness variables (extracted from Hosni (1993), O'Loughlin (1954), and Coleman (1984)) in Table 5.9, we can distinguish which spacing regime each surface belongs to by examining their corresponding spacing parameter, λ , originally defined by Simpson (see Section 2.7.1) where $\lambda = 4.7$ separates the two spacing regimes.

By comparing the distribution parameters ($r_{rms/k}$, $r_{mean/k}$, $r_{rms/mean}$) of the deterministic roughness between the densely-spaced surfaces and the sparsely-spaced surfaces, one can see that there is a significant difference between the two spacing regimes (densely-spaced surfaces are in bold). ' $r_{rms/k}$,' and ' $r_{mean/k}$,' tend to have higher values for densely-spaced roughness, whereas ' $r_{rms/mean}$ ' tends to have lower values.

Comparing these values to those for the stochastic surfaces, one can see that the stochastic surfaces, according to all three distribution parameters, consistently show that they are in the dense spaced roughness regime. Because of this, one can assume that all the stochastic surfaces used in the present experiments are in the densely-spaced regime.

Sample	roughness type	sparse	dense	C	λ	$r_{rms/k}$	$r_{mean/k}$	$r_{rms/mean}$
HOSNI 1	hemipheres	x		-0.42	10.2	0.32	0.13	2.38
HOSNI 2	hemipheres	x		-3.92	40.7	0.16	0.03	4.76
O'LOUGHLIN 1	cube	x		-1.55	9.1	0.33	0.11	3.00
O'LOUGHLIN 2	cube		x	-1.36	4.0	0.50	0.25	2.00
O'LOUGHLIN 3	cube		x	-3.01	2.3	0.66	0.44	1.50
O'LOUGHLIN 4	cube		x	-6.54	1.3	0.84	0.70	1.20
O'LOUGHLIN 5	cube (staggered)		x	-0.94	4.0	0.50	0.25	2.00
O'LOUGHLIN 6	cube (staggered)		x	-3.54	2.3	0.66	0.44	1.50
O'LOUGHLIN 7	cube (staggered)		x	-5.95	1.3	0.84	0.70	1.20
COLEMAN 1	spheres	x		-8.27	121.2	0.08	0.01	11.57
COLEMAN 2	spheres	x		-5.21	30.3	0.15	0.03	5.42
COLEMAN 3	spheres	x		-0.76	7.6	0.31	0.11	2.80
COLEMAN 4	spheres		x	-0.60	2.7	0.51	0.31	1.66
COLEMAN 5	spheres		x	-5.41	1.1	0.80	0.75	1.06
COLEMAN 6	spherical segements	x		-13.01	106.8	0.11	0.02	6.14
COLEMAN 7	spherical segements	x		-11.40	60.1	0.15	0.03	4.71
COLEMAN 8	spherical segements	x		-6.18	26.7	0.23	0.07	3.21
COLEMAN 9	spherical segements		x	-3.10	3.7	0.61	0.53	1.85
Coarse Shingle	stochastic	-	-	-2.23	-	0.44	0.38	1.16
Fine Shingle	stochastic	-	-	-1.96	-	0.62	0.58	1.07
Fine Spackle	stochastic	-	-	-3.39	-	0.55	0.52	1.06
Tar Paper	stochastic	-	-	-4.26	-	0.49	0.43	1.13
Formica	stochastic	-	-	-5.24	-	0.53	0.57	1.09
Sand	stochastic	-	-	-4.00	-	0.66	0.58	1.03

Table 5.9: Spacing Parameter Values for Various Roughness Samples

Further investigation of the distribution parameters shows that ' $r_{rms/mean}$ ' may be the least valid of the three. By plotting each of the 'C' values against the corresponding distribution parameters, we find that the plots involving ' $r_{rms/k}$ ' and ' $r_{mean/k}$ ' exhibit a trend seen in studies by Simpson (1973), Dvorak (1969) and Waigh (1996) from deterministic

roughness studies, where the 'C' tends to rise as spacing becomes wider (in the dense regime) and falls as the spacing becomes wider (in the sparse regime). See Figures 5.16 to 5.19. The plots based on the distribution parameter ' $r_{rms/mean}$ ' did not exhibit this, as points in the dense regime did not follow any apparent trend. See Figures 5.20 and 5.21. From this, one could argue that either $r_{mean/k}$, or $r_{rms/k}$ may in fact be a good equivalent to the spacing parameter λ used by Dvorak or Simpson to distinguish between the dense and sparse regimes, but applicable to both stochastic and deterministic roughness.

Figure 5.16: Plot of C vs. $r_{rms/k}$ on a Linear Scale

Figure 5.17: Plot of C vs. r_{rms}/k on a Logarithmic Scale

Figure 5.18: Plot of C vs. r_{mean}/k on a Linear Scale

Figure 5.19: Plot of C vs. r_{mean}/k on a Logarithmic Scale

Figure 5.20: Plot of C vs. $r_{\text{rms}}/\text{mean}$ on a Linear Scale

Figure 5.21: Plot of C vs. $r_{rms}/mean$ on a Logarithmic Scale

5.3 Correlation Results

As mentioned previously in Section 3.3.3, five different correlation models were tested, each using one of two distribution parameters (X_i), one of two slope parameters (Y_j), and the one blockage parameter (Z). The following correlation models were tested:

$$C_{L1ij} = a \log(bX_i^c Y_j^d Z^e) + f \quad (5.1a)$$

$$C_{L2ij} = a \log(bX_i^c Y_j^d + eZ^f) + g \quad (5.1b)$$

$$C_{L3ij} = a \log(bX_i^c + eY_j^d Z^f) + g \quad (5.1c)$$

$$C_{L4ij} = a \log(bY_j^d + eX_i^c Z^f) + g \quad (5.1d)$$

$$C_{L5ij} = a \log(bX_i^c + dY_j^e + fZ^g) + h \quad (5.1e)$$

$$C_{N1ij} = (a X_i^b Y_j^c Z^d + e) \quad (5.1f)$$

$$C_{N2ij} = (a X_i^b Y_j^d + cZ^e + f) \quad (5.1g)$$

$$C_{N3ij} = (a X_i^b + cY_j^d Z^e + f) \quad (5.1h)$$

$$C_{N4ij} = (a Y_j^d + cX_i^b Z^e + f) \quad (5.1i)$$

$$C_{N5ij} = (a X_i^b + cY_j^d + eZ^f + g) \quad (5.1j)$$

where a,b,c... etc. are used as constants, coefficients, or exponents. The subscripts i, j denote which candidate distribution and slope parameters were used – i.e:

$$\begin{aligned} X_1 &= r_{\text{mean}}/k & Y_1 &= \text{average-absolute slope} \\ X_2 &= r_{\text{rms}}/\text{mean} & Y_2 &= \text{maximum-average-absolute slope} \\ X_3 &= r_{\text{rms}}/k \end{aligned}$$

Only a single candidate blockage parameter, defined by Equation 4.1, was used. It should be noted that not all of the available data points were used in the regression analysis. Four points (one sand roughness, and the three staggered data points from O’Loughlin (1954)) were kept out of the regression analysis and were subsequently used as test points to test the correlations for accuracy. The results are summarized in Tables 5.9, 5.10, 5.11, and 5.12 (Plots of each of these results can be seen in Appendix E). Along with each of the test cases, the RMS (Root Mean Squared) error for all the points used in the correlation equation (err1) is included, the RMS errors for the four test points (err2), and the RMS error for all the points, including the four test points (err3). The RMS error is defined as

$$\text{err} = ((\sum e_i^2)/n)^{0.5} \quad (5.2)$$

where ‘err’ is the RMS error, and ‘n’ is the total number of data points. ‘e_i’ is the error for data point ‘i’ and is defined as

$$e_i = C_{\text{actual}} - C_{\text{predicted}} \quad (5.3)$$

where C_{actual} is the experimental value of C for a particular data point, and C_{predicted} is the value predicted by the correlation.

	Adjustable Variables in Correlation								RMS errors		
	a	b	c	d	e	f	g	h	err1	err 2	err 3
L111	-1.0	0.4	9.0	-1.4	0.6	-6.5			1.19	0.94	1.12
L211	4.5	1.2	-1.2	0.6	-7.3	7.3	-4.0		0.69	2.00	1.22
L311	2.0	1.4	-2.6	-0.8	-5.8	1.4	-4.6		0.97	1.17	1.03
L411	0.8	-2.4	-8.0	9.5	4.5	1.6	-5.0		1.02	0.67	0.93
L511	-1.6	3.0	3.3	0.6	11.5	-4.5	8.0	-4.3	0.97	0.58	0.88
L121	-1.0	10.5	-11.5	-1.0	2.0	-4.0			1.35	1.54	1.41
L221	3.0	1.6	2.3	0.8	-14.5	8.8	-3.2		0.74	2.40	0.96
L321	-1.8	3.0	-5.5	10.0	11.0	5.8	-3.0		1.01	1.24	1.08
L421	2.5	-2.0	3.8	5.0	5.8	0.0	-5.8		1.03	0.77	0.96
L521	3.3	1.3	4.5	-0.4	7.8	3.2	0.0	-5.8	1.02	0.71	0.94
L131	-1.8	4.8	14.0	-1.2	-1.4	-6.0			1.05	0.50	0.93
L231	3.5	2.5	-2.8	1.0	-17.0	6.5	-5.3		0.77	1.94	1.22
L331	-6.3	49.5	6.8	0.0	1.4	-0.6	1.2		1.04	0.89	1.00
L431	13.6	2.2	4.8	0.2	-3.5	-0.2	-5.3		0.79	1.50	1.04
L531	12.5	-5.5	3.5	1.8	0.4	2.0	0.2	-5.8	0.72	1.23	0.90

Table 5.10: *Logarithmic Correlations and Corresponding RMS Errors using the Average-Absolute Slope Parameter*

From Table 5.10, it can be seen that the C_{L511} (Figure 5.22) correlation produced the best results. It was expected that the L5 series would produce the best correlations considering that this family uses the largest number of variables (i.e. constants, coefficients, and exponents) in the curve fit. However, because of its relative complexity it was decided that unless the L5 series showed a substantial improvement in correlation, it would not be recommended for use. Comparing the L5 series errors with the best L1, L2, L3 and L4 correlations, it was found that the L5 series did not show a substantial improvement and is thus not recommended for use. The next best curve fits are C_{L131} (Figure 5.23) and C_{L411} (Figure 5.24) which also give good curve fits. But since the C_{L131} correlation uses one less variable than C_{L411} (six as opposed to seven), it was recommended as the best correlation to be considered from the logarithmic correlations using the average-absolute slope parameter.

Figure 5.22: Predicted Value of Roughness-Configuration Constant ' C_{L511} ' vs. Measured Values of ' C ' for Deterministic and Stochastic Roughness where

$$C_{L511} = -1.6 \log(3.0 r_{\text{mean}/k}^{3.3} + 0.6 \text{slope}_{\text{avg}}^{11.5} - 4.5 \text{blockage}^{8.0}) - 4.3$$

Figure 5.23: Predicted Value of Roughness-Configuration Constant ' C_{L131} ' vs. Measured Values of ' C ' for Deterministic and Stochastic Roughness where

$$C_{L131} = -1.8 \log(4.8 r_{\text{rms}/k}^{14.0} \text{slope}_{\text{avg}}^{-1.2} \text{blockage}^{-1.4}) - 6.0$$

Figure 5.24: Predicted Value of Roughness-Configuration Constant ' C_{L411} ' vs. Measured Values of ' C ' for Deterministic and Stochastic Roughness where

$$C_{L411} = 0.8 \log(-2.4 \text{ slope}_{\text{avg}}^{9.5} - 4.5 r_{\text{mean/k}}^{-8.0} \text{ blockage}^{1.6}) - 5.0$$

	Adjustable Variables in Correlation								RMS errors		
	a	b	c	d	e	f	g	h	err1	err 2	err 3
L112	1.4	2	-5.8	-0.4	0.6	-6			1.17	0.74	1.07
L212	4	0.8	-1.2	0.2	-1	1.4	-3.8		1.08	1.10	1.09
L312	3.8	0.6	-1.8	0.6	-0.4	0.6	-4.3		1.07	0.86	1.01
L412	4	-3.6	-1.6	0.8	5.8	0.2	-7.5		1.05	0.8	0.99
L512	-3	1	0	-1	1	6	5	-4	0.87	2.26	1.41
L122	-2	5	-6	0	1	-4			1.31	1.28	1.30
L222	-5	2	-2	0	6	6	-2		1.03	1.87	1.33
L322	-5	2	-2	0	6	6	-2		1.03	1.87	1.33
L422	-9	1	-2	2	6	0	3		1.2	1.95	1.45
L522	-3	1	0	-1	1	9	6	-4	0.78	2.34	1.41
L132	-3.2	3.6	7	0	-1	-5.2			1.08	0.82	1.01
L232	-6.3	49.5	6.8	0	1.4	-0.6	1.2		1.04	0.89	1.00
L332	-6.3	49.5	6.8	0	1.4	-0.6	1.2		1.04	0.89	1.00
L432	2.2	-0.6	-5.5	1.8	0.8	1.2	-4		1.16	0.89	1.10
L532	-3	1	0	-1	1	6	5	-4	0.87	2.26	1.41

Table 5.11: Logarithmic Correlations and Corresponding RMS Errors using the Maximum-Average-Absolute Slope Parameter

Comparing the results between Table 5.10 and 5.11, one can see that the correlations from Table 5.10 using the average-absolute slope parameter produce better results than the correlations using the maximum-average-absolute-slope parameter. This indicates that the average-absolute-slope parameter gives better correlations. It should be noted that various correlations in Table 5.11 exhibited relatively large amounts of scatter, especially in the four test points. Consequently no second pass regression analysis was done to refine the values of the adjustable variables.

Thus, one may be able to conclude that the maximum-average-absolute-slope parameter is inferior to the average-absolute slope as a parameter for use in a logarithmic correlation scheme, and attempts to find logarithmic correlations utilizing the maximum-average-absolute slope parameter need not be pursued.

	Adjustable Variables in Correlation							RMS errors		
	a	b	c	d	e	f	g	err1	err 2	err 3
N111	-8.5	2	0	0	-1			1.26	0.67	1.12
N211	-3.4	1	-23.5	-0.4	9.8	-0.4		0.78	1.99	1.25
N311	0.2	-2	-0.6	10.8	11.8	-4.2		1.07	1.11	1.08
N411	-0.3	-1.4	0.7	6.8	0.3	-4.6		0.99	0.51	0.88
N511	0.2	-2	-0.4	6.6	1.8	0.6	-5	1.01	0.69	0.93
N121	1.7	1.4	0.2	-0.3	-6.2			1.36	1.59	1.43
N221	-4.3	-0.9	-20	-0.2	8	1.6		0.68	2.28	1.35
N321	-7.5	-0.7	-3.8	2.6	5.3	3.6		1.02	1.62	1.22
N421	-0.7	-0.9	-6.5	4.8	0	2.5		1.04	0.69	0.95
N521	2	1.2	-0.6	5.5	0.8	1.8	-5.8	1.02	1.61	1.22
N131	1.2	-2.8	0.4	0.2	-6			1.32	1.04	1.25
N231	1.4	-2.4	-20.8	1	9	-5.4		0.64	2.01	1.20
N331	-19	4.3	8.3	0.3	0.1	6.8		0.7	0.94	0.78
N431	-0.3	-5	1.2	6.8	1.1	-4		0.95	0.97	0.96
N531	-13	3.8	5.3	0.6	-10	7.5	-5	0.69	1.42	0.96

Table 5.13: Non-Logarithmic Correlations using the Average-Absolute Slope Parameter

	Adjustable Variables in Correlation							RMS errors		
	a	b	c	d	e	f	g	err1	err 2	err 3
N112	-8.5	2	0	0	-1			1.26	0.67	1.12
N212	0.2	-1.8	-4.4	2	3	-3.2		0.91	1.66	1.17
N312	0.8	-1.2	-1.8	0.2	3.8	-5		1.09	1.14	1.10
N412	-0.8	-5.6	2	3.6	11	-3.2		0.68	1.69	1.07
N512	-5	1.5	2	0.6	-3.8	3	-2.2	1.04	1.43	1.16
N122	-5	2.4	-0.2	0.2	-0.8			1.27	1.45	1.32
N222	0.4	-2.8	-4.2	-1.2	3	-3.2		1.02	1.7	1.25
N322	1.4	-1.6	-2.6	-0.2	2.6	-4.6		1.08	1.72	1.30
N422	-0.8	-20.8	3.8	-4	21	-3.2		1.02	1.69	1.25
N522	-6	0.6	-1.8	0.4	-4.6	2.8	1.6	1.05	1.99	1.39
N132	-12.8	13	-1.8	-1.2	-2.6			1.04	1.23	1.10
N232	-17	-5.6	3.6	0	0.6	-3.8		0.82	1.33	0.99
N332	-20	-6.6	2.6	-0.4	0.2	-3.8		0.75	1.33	0.95
N432	-1	6.6	0.2	-1.6	2.8	-3.2		0.91	1.53	1.12
N532	-27.1	-9.7	-2	1.2	-4	3.8	-3.5	0.69	1.16	0.85

Table 5.14: Non-Logarithmic Correlations using the Maximum-Average-Absolute Slope Parameter

Comparing the correlations between the non-logarithmic correlations using the average-absolute slope (Table 5.13) and the maximum-average-absolute slope (Table 5.14), one can see that the correlations using the maximum-average-absolute slope parameter generally give correlations where accuracy is poor compared to the average-absolute slope. Thus on the basis of both the logarithmic and non-logarithmic correlations, one can conclude that the maximum-average-absolute slope parameter is inferior when compared to the average-absolute slope.

Examining Table 5.13, attention must first be drawn to correlation C_{N111} . By using just the distribution variable $r_{\text{mean}/k}$ along with one constant, one coefficient, and one exponent, it was able to produce a fairly reasonable correlation. See Figure 5.25.

Figure 5.25: Predicted Value of Roughness-Configuration Constant ' C_{N111} ' vs. Measured Values of ' C ' for Deterministic and Stochastic Roughness where

$$C_{N111} = -8.5 r_{mean/k}^2 - 1.0$$

The best non-logarithmic correlation (and best overall) correlation found was the C_{N331} correlation (See Figure 5.26). Since C_{N331} uses $r_{rms/k}$ rather than $r_{mean/k}$ as the distribution parameter, this was somewhat surprising, considering the level of accuracy that the correlation based only on $r_{mean/k}$ exhibits in C_{N111} in Figure 5.25.

Figure 5.26: Predicted Value of Roughness-Configuration Constant ' C_{N331} ' vs. Measured Values of ' C ' for Deterministic and Stochastic Roughness where

$$C_{N331} = -19.0 r_{rms/k}^{4.3} + 8.3 slope_{avg}^{0.3} blockage^{0.1} + 6.8$$

Overall, it is apparent that $r_{mean/k}$ and $r_{rms/k}$ are the best of the distribution parameters tested, while $r_{rms/mean}$ is the worst. This was expected, as discussed earlier in Section 5.8.2.2, and it is suggested that $r_{rms/mean}$ be removed for any future considerations as a distribution parameter. The recommended correlation out of the 60 various correlation attempts is C_{N331} , as it provides the best accuracy. It is somewhat surprising that the non-logarithmic correlations provided the best results, since the majority of previous work involving deterministic roughness has indicated that some type of logarithmic correlation would be best. Results in this study indicate a non-logarithmic relation between the roughness-configuration constant ' C ' and the chosen characterization parameters for deterministic and stochastic roughness.

Extending these results to the sparse-spacing regime, the distribution parameters $r_{\text{mean}/k}$ and $r_{\text{rms}/k}$, along with the average-absolute slope parameter, and the blockage parameter were correlated with the available sparse deterministic data. It was found that the best correlation was presented by

$$C_{\text{sparse}} = -3.6 r_{\text{mean}/k}^{1.2} \text{blockage}^{-3} + 4.9$$

giving an RMS error of 1.55. Unfortunately, there is insufficient data to calculate the error uncertainty in C_f .

Figure 5.27: Predicted Value of Roughness-Configuration Constant ' C_{sparse} ' vs. Measured Values of ' C ' for Deterministic Roughness where $C_{\text{sparse}} = -3.6 r_{\text{mean}/k}^{1.2} \text{blockage}^{-3} + 4.9$

Recalling that $r_{\text{mean}/k}$, alone, was fairly successful at correlating for C (see correlation C_{N111} in Figure 5.27), it is of interest to note the level of success $r_{\text{mean}/k}$ will be able to achieve, by itself, in the sparse-spacing regime. Removing the use of any blockage, and slope parameters, it was found that $r_{\text{mean}/k}$, alone, gave fairly poor

correlation to C in the sparse-spacing regime (See Figure 5.28), as a RMS error of 2.52 was found in this correlation. In comparison to Figure 5.27, one can see that the use of the blockage parameter, alongside with the $r_{\text{mean}/k}$ distribution parameter shows significant improvement over using just $r_{\text{mean}/k}$.

Figure 5.28: Predicted Value of Roughness-Configuration Constant ' C_{sparse}' ' vs. Measured Values of ' C ' for Deterministic Roughness using just $r_{\text{mean}/k}$ where $C_{\text{sparse}}' = 81 r_{\text{mean}/k} - 10.5$

Figure 5.29: Example Correlation Illustrating Correlation Success Between Stochastic and Deterministic Roughness

The most common problem found in the correlations is illustrated in Figure 5.28. The correlation shown is clearly insensitive to some characteristic of the stochastic roughness samples since it predicts similar values of C for all the stochastic samples (approximately -3.4 in this particular example) while the actual values vary substantially (from -5.2 to -2). At the same time, the correlation seems to work relatively well for the deterministic roughness data set. A possible explanation for this is that the definition of the blockage parameter may be unsuitable. Since any changes in the cut off point would have considerably less effect on the blockage parameter value for most of the deterministic data points used in this correlation (due to their bluntness), one may be led to suspect that the blockage parameter may need adjustment, and cut off points such as 0.8, 0.6, and 0.4 should be explored in future work. These cut off points should increase

the magnitude of the blockage parameter giving it greater effect in the correlation schemes. Also, use of projected surface profiles may be a suitable alternative means of calculating the roughness parameters in question, as discussed in Section 4.3.2.

The slope parameter may also need to be revised. As the measured slope parameters are considerably smaller for stochastic roughness compared to those found for the deterministic roughness, it may be necessary to recalculate the slopes by using smaller step sizes. It may be possible that the current step size used is too large for our purposes and is not sensitive enough to capture surface features effectively. Since the roughness parameters for the deterministic roughness samples are calculated in such a manner where a change in resolution would have no effect on the parameters, it would keep the correlation points involving the deterministic roughness to remain the same, while the stochastic parameters will be subject to change.

There is insufficient evidence to determine which distribution parameter, $r_{\text{mean}/k}$ or $r_{\text{rms}/k}$, is more appropriate, for use as a general spacing parameter similar to those of Simpson (1973) or Dvorak (1969) but applicable to both deterministic and stochastic surfaces. By referring to Figure 5.29 we can see that the two roughness spacing regimes are divided around $r_{\text{mean}/k} = 0.2$, whereas in Figure 5.30, they are divided at $r_{\text{rms}/k} = 0.4$. Although, experimentally, $r_{\text{rms}/k}$ has been shown to be slightly more successful when used in conjunction with other characterization parameters, it is apparent that a correlation using only $r_{\text{mean}/k}$ has significantly less scatter than one using only $r_{\text{rms}/k}$.

Figure 5.30: Plot of C vs. $r_{mean/k}$ on a Linear Scale

Figure 5.31: Plot of C vs. $r_{rms/k}$ on a Linear Scale

In fact, further reflection on the relationship between the distribution parameter $r_{mean/k}$, blockage parameter and slope parameter shows that together, they give the ability

to define geometrically similar surfaces. For two surfaces to be geometrically similar to one another, the shape of their roughness elements as well as the element spacing, in proportion to the roughness element height, must be the same. For example, if element A on surface A' is the same shape as element B on surface B', but element B is twice as large as element A, the two surfaces are geometrically similar if the spacing between the elements on surface B' is twice as large as the spacing on surface A'. See Figure 5.31 for an example

Figure 5.32: Example of Similar Surfaces

For geometrically similar surfaces, the slope parameter, the blockage parameter and the distribution parameter, r_{mean}/k , would be identical for each the surfaces. If we have the roughness-configuration constants for any one of these, we can find the log-law shift ($\Delta U/U^*$) for any other geometrically similar surfaces using only the roughness Reynolds number, k^+ . Although it may seem that such surfaces are rare, one need only look at the

example of closely-packed sand roughness to see its application, where closely-packed sand surfaces are considered to be similar to one another. Referring back to the equivalent sand roughness equation (Equation (3.17)), it was found that the roughness-configuration constant was equal to -4.0. The basis of this equivalent sand-grain roughness is the assumption that all sand-grains are similar in shape (geometrically similar) and are closely packed (where each sand-grain is considered to be in contact with the surrounding sand-grains which are relatively equal in size). As long as these two conditions are maintained on any sand roughness surface, the surfaces are considered to be similar. This results in the roughness-configuration constant equaling -4.0 (recall Section 3.3.5), enabling us to predict the log-law shift ($\Delta U/U^*$) for any other closely-packed sand surface using only the roughness Reynolds number, k^+ . By utilizing the parameters $r_{\text{mean}/k}$, slope, and blockage, we can identify new families of similar surfaces making it easier to find their corresponding C_f , as long as the roughness-configuration constant is known for a similar surface.

Because of the prospect of similarity applications and other reasons mentioned above, the author still favors the use of the $r_{\text{mean}/k}$ distribution parameter over $r_{\text{rms}/k}$ even though the best correlation uses $r_{\text{rms}/k}$. There is also significant evidence to suggest that both the slope and blockage parameter are applicable in correlation schemes, even though their definitions may need to be revised.

5.4 Error Analysis

The measured values of the roughness height (k), for each stochastic surface, are expected to vary by less than $\pm 5\%$. This variation is associated with place-to-place variation of the roughness over the surface, affecting the local surface sample. This value was estimated by dividing sample surface scans into 4 sections, measuring the roughness height for each of the sections, followed by a comparison of the values, and use of a factor of 2.5 to account for the local nature of the sample. It should be noted that some of the variation may be due to the change in the effective sample size when calculating the roughness height.

The uncertainty associated with measuring $\Delta U/U^*$ from the law-of-the-wall plot is ± 0.05 , which is relatively small compared to the measured values $\Delta U/U^*$ (up to 3% uncertainty in the most extreme cases). The uncertainty associated with the skin friction coefficient, C_f , is ± 0.0001 (approximately 2-3% depending on the surface) which is primarily due to scatter in the boundary-layer data. This scatter resulted in some uncertainty in the application of the Defect-Law method when finding C_f . Combining the maximum uncertainty of the roughness height, and the skin friction coefficient, the errors translate to a variation of about ± 0.1 in the measured values of the roughness-configuration constant, C . The variation in the value of C was calculated by inserting the uncertainties of C_f , k , and $\Delta U/U^*$ into Equation (2.15) and calculating the resulting change in C .

The estimated uncertainty in the values of C_f predicted from using the C_{N111} and C_{N331} correlations are calculated by finding the values of C_f that correspond to the nominal values of C given by the correlation, and to the (nominal $\pm$ uncertainty) value. The C_f values are found from the skin-friction law which is found by combining Equations (2.12b) and (2.17). Isolating U/U^* in both equations and equating, we get the equation

$$A \log y/k + B - C = A \log y/\delta + D + U_e/U^* \quad (5.4)$$

Recalling that $U^* = U_e (C_f/2)^{0.5}$ and substituting into Equation (5.4), we find that

$$C_f = (A \log \delta/k + B - C - D)^2/2 \quad (5.5)$$

The maximum error on a single point given by C_{N111} was 1.9, while C_{N331} gave a maximum error of 1.4. Thus the estimated uncertainty to be expected from C_{N111} is ± 2 (with a small margin for safety), while C_{N331} is expected to yield results with an uncertainty level of ± 1.5 . Employing this uncertainty range on each of experimental C values and the C_f that corresponds to the upper and lower limits of C it was found that C_{N111} yielded a maximum C_f error of approximately 30% while C_{N331} yielded an error of 20%.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

From the experimental and analytical work, the following conclusions can be drawn.

Boundary layer development along five stochastically rough surfaces was measured in a wind tunnel. Using the Defect Law along with the Momentum Integral Equation, the skin-friction coefficients were successfully found for the tested surfaces. The flow over four of the five test surfaces was found to be in the fully-rough regime, with the White Formica surface being in the transitionally rough regime.

Roughness-configuration parameters suitable for both stochastic (irregular) and deterministic (regular) roughness, were defined. The parameters can be evaluated from laser-scan data for stochastically rough surfaces or directly from the geometry of deterministic surfaces. Candidate parameters were assessed for suitability in correlating the aerodynamic effects caused by stochastic and deterministic roughness. It was found that a combination of the distribution parameter ' $r_{mean/k}$ ', a blockage parameter, and the average-absolute slope parameter produced reasonable results for both stochastic and deterministic roughness. Aerodynamic effects are represented by the roughness-configuration constant, C , which appears in the logarithmic law of the wall for fully-rough surfaces.

The best correlation found for the roughness-configuration constant in the densely-spaced roughness regime is

$$C = 6.8 r_{mean/k}^{-1.4} \text{blockage}^{0.3} - 0.3 \text{slope}_{avg}^{0.7} - 4.6$$

It was found that the distribution parameter $r_{mean/k}$ may be used as an alternative to the spacing parameter ' λ ' derived by Simpson (1973), which is limited to deterministic roughness. ' $r_{mean/k}$ ' has been shown to be applicable to both deterministic and stochastic roughness, but is limited to isotropic roughness. It was found that the dense and sparse spacing regimes are bounded by $r_{mean/k} = 0.2$.

A correlation that uses only the distribution parameter $r_{\text{mean}/k}$ was found to be almost as good as the aforementioned, more complex correlation. This simpler correlation, also valid only for the densely-spaced regime is

$$C = -8.5 r_{\text{mean}/k}^2 - 1.0$$

6.2 Recommendations for Future Research

Further work is required to find a correlation that is more accurate, less restrictive, and includes non-isotropic roughness. The current correlation is not applicable to roughness that is not symmetric to both the x and y axis – such as roughness consisting of flat plates normal to the flow direction. A wider range of stochastic roughness samples should be tested with a greater variation of surface types to ensure the validity of the correlations for a broad range of roughness configurations.

The blockage parameter should be investigated for lower cut off values – such as 0.6k or 0.5k. Both slope parameters and the blockage parameter can also be calculated from projected profiles from a strip of the surface (as mentioned in Section 4.3.2) and this should be investigate as well.

Further testing should be done for stochastic surfaces in the sparse spacing regime. Now that we have a means for defining a stochastic surface in the sparse regime, work can be done to find a relation applicable to both stochastic and deterministic roughness in the sparse regime using the same roughness characterization parameters.

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Appendix A

Sample Traverse Input File

Below is a sample traverse input file. The first number is the number of traverse points. The left column holds the x position values (zero in this case since only a vertical traverse is required) and the right column contains the y position values (in mm). Values near the start and the end of the traverse are shown. All other values in the middle were omitted due to the large number of points.

```
148
0.000 0.09525
0.000 0.1905
0.000 0.28575
0.000 0.381
0.000 0.47625
0.000 0.5715
0.000 0.66675
0.000 0.762
0.000 0.85725
0.000 0.9525
0.000 1.04775
0.000 1.143
0.000 1.23825
0.000 1.3335
0.000 1.42875
0.000 1.524
0.000 1.61925
.....
0.000 26.57475
0.000 27.051
0.000 27.52725
0.000 28.0035
0.000 28.47975
0.000 28.956
0.000 29.43225
0.000 29.9085
0.000 30.38475
0.000 30.861
0.000 31.33725
0.000 31.8135
0.000 32.28975
0.000 139.7
```

Appendix B

Sample 'SCANNER' Program Raw Output File

Below is a sample of the output file containing the average pressure differentials for each channel used. The columns contain, from left to right, the x position, the y position (in mm), $P_S - P_{C2}$, $P_0 - P_{C2}$, and $P_{C1} - P_{C2}$ (in Pa).

0.00000	0.09525	-63.700142	251.087860	924.211975
0.00000	0.19050	-64.159782	243.973938	924.266663
0.00000	0.28575	-63.915611	242.873749	924.577576
0.00000	0.38100	-64.183746	237.079010	924.189270
0.00000	0.47625	-63.961979	236.664612	924.475830
0.00000	0.57150	-64.065918	224.998825	924.230957
0.00000	0.66675	-64.417549	217.566177	924.026367
0.00000	0.76200	-63.744884	203.225662	924.953674
0.00000	0.85725	-64.600235	196.342285	924.136230
0.00000	0.95250	-64.226295	185.571686	925.236877
0.00000	1.04775	-64.520767	181.531097	923.636475
0.00000	1.14300	-64.367729	183.041061	923.320129
0.00000	1.23825	-63.979439	185.475281	923.896667
0.00000	1.33350	-65.195114	179.838104	923.664368
0.00000	1.42875	-64.703438	182.554184	923.584167
0.00000	1.52400	-65.153183	179.817566	923.281860
0.00000	1.61925	-64.731392	180.540176	924.165283
0.00000	1.71450	-64.378754	175.522202	923.581543
0.00000	1.80975	-65.599960	175.168945	923.050110
0.00000	1.90500	-64.471916	177.162552	922.276367
0.00000	2.00025	-65.162308	177.921478	922.283325
0.00000	2.09550	-64.935440	176.075455	922.403015
0.00000	2.19075	-65.054527	174.678619	922.017944
....				
0.00000	26.57475	-66.150269	896.959473	904.334351
0.00000	27.05100	-65.837296	896.282410	904.233337
0.00000	27.52725	-65.812675	898.131592	904.120422
0.00000	28.00350	-65.736664	898.229553	904.179993
0.00000	28.47975	-65.879059	898.130859	903.762939
0.00000	28.95600	-65.564079	898.593567	903.979980
0.00000	29.43225	-65.961319	899.814087	904.696228
0.00000	29.90850	-65.324577	899.534363	903.866882
0.00000	30.38475	-65.904449	900.311768	904.777832
0.00000	30.86100	-65.792366	900.484253	905.261536
0.00000	31.33725	-66.324059	901.672424	906.067078
0.00000	31.81350	-66.305779	901.158630	905.492188
0.00000	32.28975	-66.675400	901.459717	905.765747
0.00000	139.70000	-66.902008	898.959045	905.883728

Below is a sample of the output file containing the average pressure differentials for each channel used. The first number is the number of data points. The columns contain, from left to right, the x position, the y position (in mm), C_{PS} , C_{PO} , and C_{PPC1} (Which is always equal to 1) (in Pa).

148

0.00000	0.09525	-0.068929	0.271661	1.000000
0.00000	0.19050	-0.069423	0.263962	1.000000
0.00000	0.28575	-0.069135	0.262691	1.000000
0.00000	0.38100	-0.069456	0.256525	1.000000
0.00000	0.47625	-0.069192	0.256001	1.000000
0.00000	0.57150	-0.069323	0.243434	1.000000
0.00000	0.66675	-0.069718	0.235458	1.000000
0.00000	0.76200	-0.068924	0.219702	1.000000
0.00000	0.85725	-0.069908	0.212461	1.000000
0.00000	0.95250	-0.069422	0.200566	1.000000
0.00000	1.04775	-0.069860	0.196545	1.000000
0.00000	1.14300	-0.069720	0.198235	1.000000
0.00000	1.23825	-0.069257	0.200749	1.000000
0.00000	1.33350	-0.070590	0.194702	1.000000
0.00000	1.42875	-0.070062	0.197661	1.000000
....				
0.00000	26.09850	-0.073293	0.989268	1.000000
0.00000	26.57475	-0.073153	0.991856	1.000000
0.00000	27.05100	-0.072816	0.991213	1.000000
0.00000	27.52725	-0.072797	0.993383	1.000000
0.00000	28.00350	-0.072708	0.993424	1.000000
0.00000	28.47975	-0.072897	0.993775	1.000000
0.00000	28.95600	-0.072531	0.994052	1.000000
0.00000	29.43225	-0.072916	0.994613	1.000000
0.00000	29.90850	-0.072277	0.995214	1.000000
0.00000	30.38475	-0.072846	0.995068	1.000000
0.00000	30.86100	-0.072681	0.994730	1.000000
0.00000	31.33725	-0.073207	0.995157	1.000000
0.00000	31.81350	-0.073231	0.995221	1.000000
0.00000	32.28975	-0.073616	0.995253	1.000000
0.00000	139.70000	-0.073857	0.992364	1.000000

Below is a sample of the output file containing a record of the traverse. The first several lines may be entered into the scanner program (the defaults are shown here), but are of little consequence for our needs, as 'SCANNER' does not utilize them in any of measurements here. The zero offsets are shown where the PSI system was re-zeroed.

Date: 10-28-2003
Time: 10:36:21
Data Acquisition: Automatic
Data Mode: RUN
Test Section: TUNNEL
Ambient Pressure: 750.1 mm Hg
Ambient Temperature: 23.0 deg C
Active Channels: 4 6 8
Samples per channel: 300
Units: Pa
Zero Offsets: -3.443593 35.531662 -0.037400
Probe Point 1 => (0.00000, 0.09525), Moving: 6.980 sec., Sampling: 19.059 sec.,
I/O: 0.000 sec., Total: 26.039 sec.
Probe Point 2 => (0.00000, 0.19050), Moving: 7.031 sec., Sampling: 19.059 sec.,
I/O: 0.000 sec., Total: 26.090 sec.
Probe Point 3 => (0.00000, 0.28575), Moving: 7.031 sec., Sampling: 19.063 sec.,
I/O: 0.000 sec., Total: 26.094 sec.
...
Probe Point 140 => (0.00000, 28.95600), Moving: 7.090 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 26.148 sec.
Zero Offsets: -3.505824 34.836323 -0.150361
Probe Point 141 => (0.00000, 29.43225), Moving: 7.090 sec., Sampling: 19.063
sec., I/O: 0.000 sec., Total: 26.152 sec.
Probe Point 142 => (0.00000, 29.90850), Moving: 7.078 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 26.137 sec.
Probe Point 143 => (0.00000, 30.38475), Moving: 7.090 sec., Sampling: 19.051
sec., I/O: 0.000 sec., Total: 26.141 sec.
Probe Point 144 => (0.00000, 30.86100), Moving: 7.090 sec., Sampling: 19.063
sec., I/O: 0.000 sec., Total: 26.152 sec.
Probe Point 145 => (0.00000, 31.33725), Moving: 7.078 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 26.137 sec.
Probe Point 146 => (0.00000, 31.81350), Moving: 7.094 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 26.152 sec.
Probe Point 147 => (0.00000, 32.28975), Moving: 7.141 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 26.199 sec.
Probe Point 148 => (0.00000, 139.70000), Moving: 42.070 sec., Sampling: 19.059
sec., I/O: 0.000 sec., Total: 61.129 sec.
Total time: 4332.51953125 sec.

Appendix C

Sample Matlab Program used to find Roughness Correlations


```

%      k991 rmeank per70 slope rmsk rmsslope
r_pa = [0.83512  0.380496  0.0802    0.21045  0.36797  0.27865;
        0.85928  0.583046  0.3271    0.42154  0.53700  0.58517;
        0.58127  0.520137  0.1402    0.60698  0.32076  0.83145;
        0.23446  0.43359   0.1405    0.31765  0.11461  0.43331;
        0.12965  0.52588   0.2319    0.06120  0.07411  0.00000;
        12.7    0.25      0.5       0.5      6.35    0.38255;
        12.7    0.44      0.66332   0.88     8.42423  0.51148;
        12.7    0.7       0.83666   1.4      10.62    0.64635;
        4.1     0.3073829  0.6      0.72771  2.09346  2.52321;
        2.6     0.527764   0.66667   0.54436  1.593    0.71112;
    ]

%      FSP   CSH   FSH   TAR   Wform O25 O44 O70 C27 C11 C71
AlogK = [0 0 0 0 0 0 0 0 0 0]
dUstar = [-3.396   -2.23   -1.967   -4.26   -5.24   -1.36   -3.01   -6.54   -0.6   -3.1]

% dUstar-AlogK-corr=error
% least squares sum((dUstar-alogK)-corr)^2
as=-8, bs=-8, cs=-8, ds=-8, es=-8, fs=-8 %starting variable entry
a=as, b=bs, c=cs, d=ds, e=es, f=fs, Err1=100, Err2 = 100, n=0
while a<(8+0.01)
    while b<(8+0.01)
        while c<(8+0.01)
            while d<(8+0.01)
                while e<(8+0.01)
                    while f<(8+0.01)
                        q1 = a*(r_pa(1,2)*r_pa(1,1)/r_pa(1,5))^b*r_pa(1,3)^c+d*(r_pa(1,4))^e;
                        q2 = a*(r_pa(2,2)*r_pa(2,1)/r_pa(2,5))^b*r_pa(2,3)^c+d*(r_pa(2,4))^e;
                        q3 = a*(r_pa(3,2)*r_pa(3,1)/r_pa(3,5))^b*r_pa(3,3)^c+d*(r_pa(3,4))^e;
                        q4 = a*(r_pa(4,2)*r_pa(4,1)/r_pa(4,5))^b*r_pa(4,3)^c+d*(r_pa(4,4))^e;
                        q5 = a*(r_pa(5,2)*r_pa(5,1)/r_pa(5,5))^b*r_pa(5,3)^c+d*(r_pa(5,4))^e;
                        q6 = a*(r_pa(6,2)*r_pa(6,1)/r_pa(6,5))^b*r_pa(6,3)^c+d*(r_pa(6,4))^e;
                        q7 = a*(r_pa(7,2)*r_pa(7,1)/r_pa(7,5))^b*r_pa(7,3)^c+d*(r_pa(7,4))^e;
                        q8 = a*(r_pa(8,2)*r_pa(8,1)/r_pa(8,5))^b*r_pa(8,3)^c+d*(r_pa(8,4))^e;
                        q9 = a*(r_pa(9,2)*r_pa(9,1)/r_pa(9,5))^b*r_pa(9,3)^c+d*(r_pa(9,4))^e;
                        q10 = a*(r_pa(10,2)*r_pa(10,1)/r_pa(10,5))^b*r_pa(10,3)^c+d*(r_pa(10,4))^e;
                        FSP = (q1)+f;
                        CSH = (q2)+f;
                        FSH = (q3)+f;
                        TAR = (q4)+f;
                        VIN = (q5)+f;
                        O25 = (q6)+f;
                        O44 = (q7)+f;
                        O07 = (q8)+f;
                        C27 = (q9)+f;
                        C37 = (q10)+f;
                        Err2=((dUstar(1))-FSP)^2+((dUstar(2))-CSH)^2+((dUstar(3))-
                        FSH)^2+((dUstar(4))-TAR)^2+((dUstar(5))-VIN)^2 +((dUstar(6))-O25)^2 +((dUstar(7))-O44)^2
                        +((dUstar(8))-O07)^2 +((dUstar(9))-C27)^2 +((dUstar(10))-C37)^2;

                        if Err2<Err1
                            Err1=Err2;

```

```

FSP1=FSP;
CSH1=CSH;
FSH1=FSH;
TAR1=TAR;
VIN1=VIN;
aa=a
bb=b
cc=c
dd=d
ee=e
ff=f
Err1=Err1
end

    f=f+1;
end
    e=e+1;
    f=fs;
end
    d=d+1;
    e=es;
end
    c=c+1;
    d=ds;
end
    b=b+1;
    c=cs;
end
    a=a+1,
    b=bs;
end
display(aa),display(bb),display(cc),display(dd),display(ee),display(ff)
display(FSP1),display(CSH1),display(FSH1),display(TAR1),display(VIN1)
display(Err1)

```

Appendix D

Sample Fortran Program used to Calculate Roughness Parameters

```

Program input72ac
implicit none
real, dimension(40401):: surfx, surfy, surfz, surfxo, surfy, surfzo, surfzo2
!*****

integer, dimension (1)::
garbage,fill,maxsize,counter,mc,nc,cc,ccl,hnumber,lnumber,hdiv,ldiv
integer, dimension (1)::nx,hnu,lnu,hnuc,lnuc,sn,px,
xnum,ynum,cx,cx2,xnuml,xstep,sigmaount,co,rcount
real, dimension(1)::
k955,high,low,hnsum,lsum,peaknum,vallnum,rmean,rmeank,k70,per70,surfsum,ms,xslo
pesq,rmsslope
real, dimension(1)::a,b,col,sigma, sigmammu,sigmammut,
pxslopesum,nxslopesum,xslope,paveslope,naveslope,pxl,nxl,surfsumsq,rmsk
real, dimension(10) :: res,dx,dy
open (unit=1,file='input72ac.yun',status='new')
open (unit=2,file='C:\Documents and Settings\Administrator\Desktop\Ferrari Test
Data\Fine Spackle\fspx.yun')
open (unit=3,file='C:\Documents and Settings\Administrator\Desktop\Ferrari Test
Data\Fine Spackle\fspx.yun')
open (unit=4,file='C:\Documents and Settings\Administrator\Desktop\Ferrari Test
Data\Fine Spackle\fspz.yun')
read (unit=2,FMT=999) surfx
read (unit=3,FMT=999) surfy
read (unit=4,FMT=999) surfz

! This section removes all garbage values from the input file
garbage(1) = 1
fill(1) = 1
write (*,*) fill(1)
maxsize(1) = 40401 !*****

1 if (surfz(garbage(1)).gt.1000) go to 2
go to 3
2 garbage(1) = garbage(1) + 1
if (garbage(1).gt.maxsize(1)) go to 4
go to 1
3 surfx(fill(1)) = surfx(garbage(1)) !replaces garbage values
surfy(fill(1)) = surfy(garbage(1))
surfz(fill(1)) = surfz(garbage(1))
garbage(1) = garbage(1) + 1
fill(1) = fill(1) + 1
if (garbage(1).gt.maxsize(1)) go to 4
go to 1
4 maxsize(1) = fill(1) - 1
!defines cut off point of garbage

```

```

! This section sorts data from highest to lowest
  counter(1) = 0
  surfzo(maxsize(1)) = surfz(maxsize(1))
10  counter(1) = counter(1) + 1
    !number of cycles done
    mc(1) = maxsize(1)
    !serves as a marker in the ordered matrix
    nc(1) = maxsize(1) - counter(1)      !keeps track of which spot we are comparing
11  if (surfz(nc(1)).gt.surfzo(mc(1))) go to 12    !finds insertion point into o-matrix
    go to 30
12  mc(1) = mc(1) - 1
    if (mc(1).eq.nc(1)) go to 20
    go to 11
20  surfzo(mc(1))=surfz(nc(1))
    surfxo(mc(1))=surfx(nc(1))
    surfyo(mc(1))=surfy(nc(1))
    go to 10

30  cc(1) = 1
    cc1(1) = cc(1) + 1
    !shifts values higher than current value
31  surfzo(cc(1)) = surfzo(cc1(1))
    surfxo(cc(1)) = surfxo(cc1(1))
    surfyo(cc(1)) = surfyo(cc1(1))
    if (cc1(1).eq.mc(1)) go to 40
    cc(1) = cc(1) + 1
    cc1(1) = cc1(1) + 1
    go to 31

40  surfzo(mc(1)) = surfz(nc(1))
    surfxo(mc(1)) = surfx(nc(1))
    surfyo(mc(1)) = surfy(nc(1))
    if (counter(1).eq.maxsize(1)) go to 60
    go to 10

! This section is used to find the effective K parameter
60  high(1) = 0.99                      !*****
    low(1) = 0.01                       !*****
    hnumber(1) = (1-high(1))*maxsize(1)
    lnumber(1) = (1-low(1))*maxsize(1)

90  peaknum(1) = surfzo(hnumber(1))
    vallnum(1) = surfzo(lnumber(1))

```

```

k955(1) = peaknum(1) - vallnum(1)
nx(1)=1

!      This section finds the spacing parameters
!      ***** rezero the heights the lower 5% calculated before set to zero
110 sn(1) = 1
      surfsum(1) = 0
      surfsumsq(1) = 1
111  surfzo2(sn(1)) = surfzo(sn(1)) - vallnum(1)
      surfsum(1) = surfsum(1) + surfzo2(sn(1))
      surfsumsq(1) = surfsumsq(1) + surfzo2(sn(1))**2
      if (sn(1).eq.maxsize(1))go to 120
      sn(1) = sn(1) + 1
      go to 111
!      ***** rmean/k
120  rmean(1) = surfsum(1)/maxsize(1)
      rmean(1) = rmean(1)/k955(1)
      rmsk(1) = sqrt(surfsumsq(1)/maxsize(1))
      write (1,*) k955(1)
      write (unit=1,FMT=997) rmean(1)
      write (unit=1,FMT=997) rmean(1)
      write (unit=1,FMT=997) rmsk(1)
!      ***** percent of points above 70% of k995
      k70(1) = 0.5 * k955(1)
      sn(1) = 0
130  sn(1) = sn(1) + 1
      if (k70(1).lt.surfzo2(sn(1))) go to 130
      a(1) = sn(1)          ! placeholder for mixed mode division - change to real
      b(1) = maxsize(1)     ! placeholder for mixed mode division - change to real
      per70(1) = a(1)/b(1)
      write (unit=1,FMT=995) per70(1)

!      ***** this section finds frontal slope effects
      pxslopesum(1) = 0
      nxslpesum(1) = 0
      nx(1) = 0
      px(1) = 0
      xnum(1) = 197        !      ***** entry field for number of steps
      ynum(1) = 197        !      ***** entry field for number of steps
      dx(1) = 0.09998
      dy(1) = 0.09998
      xnum1(1) = xnum(1)
      co(1) = 1
      xslopesq(1) = 0
!

```

```

132  res(co(1)) = co(1)
      col(1) = co(1)          !real place holder since co is an integer
!    dx(co(1)) = 0.03999*col(1)
      co(1) = co(1) + 1
      if (co(1).gt.10) go to 134
      go to 132

134  dx(1) = 0.09998
      dx(2) = 0.09998*2.0
      dx(3) = 0.09998*3.0
      dx(4) = 0.09998*4.0
      dx(5) = 0.09998*5.0
      dx(6) = 0.09998*6.0
      dx(7) = 0.09998*7.0
      dx(8) = 0.09998*8.0
      dx(9) = 0.09998*9.0
      dx(10) = 0.09998*10.0

      rcount(1) = 1
135  cx(1) = 1
      cx2(1) = rcount(1) + 1
140  xslope(1) = (surfz(cx2(1)) - surfz(cx(1)))/dx(rcount(1))

!    loop structure to organize in to positive and negative slopes
      if (xslope(1).lt.0) go to 145
      go to 150
145  nxslopesum(1) = nxslopesum(1) + xslope(1)
      nx(1) = nx(1) + 1                      !      counter
      go to 160
150  pxslopesum(1) = pxslopesum(1) + xslope(1)
      px(1) = px(1) + 1                      !      counter

160  cx(1) = cx(1) + res(rcount(1))          !occurs in any change in Y
      cx2(1) = cx2(1) + res(rcount(1))
      xslopesq(1) = xslopesq(1) + xslope(1)**2
      if (surfy(cx2(1)) /= surfy(cx(1))) go to 165
      go to 140

165  xnum1(1) = xnum1(1) + xnum(1)
      cx(1) = cx(1)+1
      cx2(1) = cx2(1)+1
      if (cx2(1).gt.maxsize(1)) go to 170
      go to 160

170  rcount(1) = rcount(1) + 1              !cycles through different resolutions
      px1(1) = px(1)                        !real place holders

```

```

    nx1(1) = nx(1)
    paveslope(1) = pxslopesum(1)/px1(1)
    naveslope(1) = nxslopesum(1)/nx1(1)
    rmsslope(1) = sqrt(xslopesq(1)/(px1(1) + nx1(1)))
    write (1,*)
    write (unit=1,FMT=997) paveslope(1)
    write (unit=1,FMT=997) naveslope(1)
    write (unit=1,FMT=997) rmsslope(1)
    write (1,*)
    write (1,*)
    nx(1) = 0
    px(1) = 0
    nxslopesum(1)=0
    pxslopesum(1)=0
    xslopesq=0
    if (rcount(1).gt.10) go to 180
    go to 135

! calculation of standard deviation

180   sigmammu(1) = 0
      sigmacount(1) = 1
190   sigmammut(1) = (surfzo(sigmacount(1)) - rmean(1)*1.0)**2.0
      sigmammu(1) = sigmammu(1) + sigmammut(1)
      sigmacount(1) = sigmacount(1) + 1
      if (sigmacount(1).gt.maxsize(1)) go to 200
      go to 190
200   ms(1)=maxsize(1)
      sigma(1) = sqrt(sigmammu(1)/(ms(1) - 1.0))
      write (1,*) sigma(1)

994   format (I4)
995   format (F7.4)
996   format (I5)
997   format (E11.5)
998   format (3(E11.5,2x))
999   format (BN,f11.5)
close (1)
close (2)
close (3)
close (4)
end program input72ac

```


Appendix E

Correlation Results

The following section is a summary of all the plots from correlation results. Refer to Section 5.3 for notation.