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System Identification Analysis of the Dynamic Monitoring Data of the Confederation Bridge

by

Mo Zhang

B. Eng. (Civil Engineering)
Beijing Polytechnic University, China, 1991

A thesis submitted to
The Faculty of Graduate Studies and Research
In partial fulfillment of the requirements

For the degree of
Master of Applied Science

**Department of Civil and Environmental Engineering
Carleton University, Ottawa
April, 2002**

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Administered by the Ottawa-Carleton Institute for Civil Engineering

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**System Identification Analysis of the Dynamic Monitoring
Data of the Confederation Bridge**

submitted by

Mo Zhang, B. Eng.

in partial fulfillment of the requirements for
The degree of Master of Applied Science

Chair, Department of Civil and
Environmental Engineering

Thesis Supervisor

Carleton University
Ottawa, Ontario, Canada
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ABSTRACT

The Confederation Bridge is one of the longest continuous bridges in the world built over seawater. Its construction represents a major engineering achievement in the design of long-span bridges. Since its completion in 1997, a comprehensive remote monitoring system installed in the bridge has been collecting data on the static and dynamic responses of the bridge. In this paper, a brief introduction of the dynamic monitoring system is presented. The system identification methods and modal analysis procedures in frequency domain and time domain employed in the analysis of the field monitoring data to extract the vibrational modal properties of the bridge are presented. Comparison of the frequency and time domain system identification methods shows that the Stochastic Subspace method in time domain is an effective tool for identification of the dynamic properties of full-scale bridges using the structural response data under ambient conditions. The system identification results obtained from the monitoring data can be used to calibrate and improve the FEM model of the bridge. System identification results show that the measured field data include noise and components of local vibration modes of the cross-section of the bridge, all of which tend to obscure the modal behavior of the bridge. Detailed noise reduction and data analysis of the field response data can lead to useful information on the modal properties of the structure.

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LIST OF SYMBOLS

A	state matrix in a discrete state-space model
B	input influence matrix in a discrete state-space model
A_c	state matrix in a continuous state-space model
B_c	input influence matrix in a continuous state-space model
C	output influence matrix in a state-space model
C_a	output influence matrices of acceleration
C_d	output influence matrices of displacement
C_q	controllability matrix of order q in a state-space model
C_v	output influence matrices of velocity
D	direct transmission matrix in a state-space model
H_1	estimator of frequency response function
H_2	estimator of frequency response function
$H_{p,q}$	block-Hankel matrix
H_{xy}	Frequency Response Function matrix
I	identified matrices
K	stiffness matrix
ζ	damping matrix
m	number of outputs
M	mass matrix
n_2	number of independent coordinates

σ_r	damping factor
O_p	observability matrix of order p in a state-space model
$PSD(f)$	power spectrum density function
γ_f	coherence function
r	number of inputs
$R_{xx}(\tau), R_{yy}(\tau)$	auto-correlation function
$R_{xy}(\tau), R_{yx}(\tau)$	cross-correlation function
$S_{xx}(\omega), S_{yy}(\omega)$	auto-power spectrum
$S_{xy}(\omega), S_{yx}(\omega)$	cross-power spectrum
$u(t)$	multiple input excitation function vector
Λ	diagonal matrix of eigenvalues
Ψ	matrix of eigenvectors
$\{\phi\}_r$	mode shape of the r-th mode
$v(k)$	measurement noise in a stochastic state-space model
$w(k)$	process noise in a stochastic state-space model
ω_r	damped natural frequency of the r-th mode
ω	displacement vector in a state-space model
$\dot{\omega}$	velocity vector in a state-space model
$\ddot{\omega}$	acceleration vector in a state-space model
W_1, W_2	weighting matrices for Singular Value Decomposition
x	state variable in a state-space model

$X(\omega)$	Fourier transform of the continuous time signal $x(t)$ at the analysis frequency ω
$X(k)$	discrete Fourier transform of the sampled signals $x(k)$
y	output vector in a state-space model
Y_k	Markov parameters
$Y(k)$	discrete Fourier transform of the sampled signals $y(k)$

Chapter 1

Introduction

1.1 Introduction

In recent years, the practice of structural engineering in Canada and other industrialized countries has gradually changed from building new structures to maintaining and upgrading existing ones. Because of the increasing number of aging infrastructures, this trend will continue and become even more important in Canada in the near future. Evaluation of the performance and assessment of the condition of the deteriorated or deficient structures is an integral part of the overall structural rehabilitation and upgrade scheme. In Canada, there is the need to detect in as early stage as possible structural damage or deterioration in the performance of structures such as buildings and bridges caused by the effect of corrosion resulting from the use of de-icing salt in winter, and in transportation infrastructures due to the increasing traffic volume and higher truck loads allowed on the highways. The primary means to detect structural damage are by periodic visual inspection, core sampling or other non-destructive techniques, such as acoustic or

magnetic. The conventional methods are often unreliable and can only detect the problem after the deterioration process is well underway and significant amount of damage has already occurred. In recent years, the detection of structural damage by means of vibration tests has been explored as an alternative to the conventional techniques. In the new approach, the deterioration process on structural damage is detected through changes in the structural properties, such as reduction in stiffness leading to changes in the modal vibration behaviour. Because the methodology relied on identification of changes in the vibration properties of the evaluated structures, accurate determination of the dynamic modal properties of the structure is therefore essential for the successful application of the technique as the basis for damage detection and structural evaluation. Although the general use of the dynamic vibration properties for structural evaluation is a relative new application in structural dynamics, the importance of having accurate information of the dynamic vibration properties of structures has been recognized for a long time. The design of large structures, such as long-span bridges, suspension bridges, and other complex structures, is often governed by the dynamic responses and vibration behaviour of the structures. The design must satisfy not only the criteria of structural integrity under extreme and critical loading conditions, but also maintaining high performance throughout the service life span of the structures. Therefore, even before reaching the stage of damage detection, a thorough and precise understanding of the structural dynamic properties of the structures under the field and operating conditions is essential.

Generally, structural evaluation includes building analytical models of the evaluated system, which is commonly by the finite element method. The purpose of the analytical models is to predict the dynamic behaviour of the structure under study. It is known from experience that a finite element model without refinement, such as calibration by dynamic test results, is generally not accurate enough to reflect the true behaviour of the structure under the field operating conditions, and thus is not suitable for use directly in structural evaluation. Because assumptions are often made in the finite element model that may not truly reflect the actual structural conditions, experimental dynamic investigation of the structure is one methodology that can be applied to refine and verify the analytical models. System identification which also referred to as “modal testing” or “experimental modal analysis” is the process of identifying the modal properties of a structural system from measured response signals of the structure. The identified properties include the damping properties, the natural vibration frequencies, the vibration mode shapes and the stiffness etc. The response signals used in the system identification process are generally obtained from laboratory or field tests of actual full size structures. Using the experimental modal analysis results, the parameters of the structural model of the analyzed structure can be adjusted with reference to the identified modal properties prior to its use for predicting the responses of the structure to other loading conditions and more complex excitations. The system identification process using the structural monitoring data as the response signals for the determination of the actual dynamic properties of the structural system is illustrated in Fig. 1.1.

The modal analysis data of a structure while it is relatively new and in good condition can be used to establish a reference baseline for the continuing monitoring and evaluation of the structure for its performance. When there is a change in the structural property detected by the system identification process from the monitoring data, further investigation can be implemented to determine if there is any damage to the structure.

Many researchers have carried out studies of performing dynamic tests in order to determine the dynamic modal parameters of bridges (Ventura, et al, 1995; Wilson, et al, 1989; Paultre, et al, 1995; Abdel Wahab, et al, 1998; Beolchini, et al, 1997; Loh et al, 1998; Salawu, et al, 1995; Douglas, et al, 1982). Recent advances in system identification techniques can determine accurate by the natural vibration frequencies, and obtain good estimates of the vibration mode shapes and damping ratios of the structures. The majority of the excitation method employed in the modal testing of structures in the references stated above is by single- or multiple-input excitations, which generally relies on the use of a shaker or an instrumented impact hammer. For the tests, the actuators and their corresponding locations are selected to excite the structural modes of interest, while the sensors and the measurement locations are chosen to best capture the modes. In conventional modal analysis techniques, the exciting force is measured and known. For testing of large size bridges or bridges with heavy traffic, it is impractical to apply controlled excitation forces of required magnitude to induce significant or distinguishable responses by the structures. Therefore, tests using ambient excitations, such as those from wind, random traffic and sea currents, are carried out on large bridges and offshore structures as substitutes for the controlled vibration tests in the system identification

analysis. Comparatively, ambient vibration tests are easier to implement and form the basis for application in long term monitoring. Since the excitations are generally not known a priori or known in such details as the controlled vibration tests, the disadvantage of this approach is that more complex data processing and numerical techniques and algorithms are required for successful extraction of the dynamic modal properties information from the data in the system identification process. In some previous works on ambient vibration tests (Paultre, et al, 2000), the environmental loads that excite the structures as the source of the ambient-vibration responses of the structures are assumed to have white-noise characteristics.

When applied in the field of control theory in mechanical and aerospace engineering, the objective of the system identification procedures in the processing of the measured response signals is to construct a model of the mechanical or aerospace system for control design. In recent years, the techniques of control theories have been adapted for structural dynamic analysis of civil engineering structures such as buildings and bridges. In control application, the control synthesis and design tools require the construction of parametric system models, such as a state-space representation or a stochastic difference equation of the control system. When the identified system is a linear model in the state space representation, the eigenvalues and eigenvectors as determined from the eigensolution of the model completely characterize the dynamic characteristics and behaviour of the system. A detailed discussion of state-space model is presented in Chapter 2.

In civil engineering applications, one of the objectives of modal testing is to evaluate the performance of the structures and to detect any structural defects or deterioration processes in progress. The results of the experimental modal analysis of the evaluated system in the system identification are used to develop damage detection models of the structure. By comparing the system identification results with the baseline structural properties obtained while the structure is in good condition, a significant change in the mechanical properties of the structure and the location of the change may be identified. A continuous field monitoring system, such as the one implemented on the Confederation Bridge, together with advanced condition evaluation algorithms based on system identification techniques can provide bridge engineers a powerful and efficient tool to detect changes or deterioration in the condition of the structure so that immediate measures can be taken to correct the problems so as to ensure continuous high performance of the structure.

The increasing number of bridge failures in recent years has resulted in the interest and need of monitoring and testing of deficient structures on a routine basis. Theoretically, the damage of a structure can be detected and located by studying the changes in its dynamic characteristics before and after the significant event that caused the damage or by comparing the characteristic properties with the baseline values recorded when the structure was in good condition. There are still many gaps and obstacles in research which need to be overcome before accurate and robust algorithms to extract modal properties of the structure under actual operating environmental conditions for condition assessment of civil engineering structures. In the field, monitoring data are often

contaminated by noises. Furthermore, the problem is compounded due to the randomness nature of the ambient excitations and measurement errors in the sensors. The noise problems of ambient vibration tests and long-term structural monitoring are much more difficult to control under the operating conditions in the field of civil engineering structures than in typical aerospace or mechanical applications of modal testing measurements under controlled laboratory environment.

Many experimental studies have been conducted on the subject of damage detection (Salane, et al, 1990; Chang, et al, 1993; Alampalli, et al, 1997; Salawu, 1997; Filiatrault, et al, 1993). A common approach in damage detection is to identify structural damage based on differences in the dynamic properties of the structures, such as natural vibration frequency, damping, vibration mode shape and stiffness of the structure before and after a damage event, for example an earthquake. Results from some experimental studies show that the change in mode shape is a better indicator of the deterioration of the structural condition compared with those criteria based on frequency, damping and stiffness (Salane, et al, 1990). Among the studies on mode shape changes, the strain mode shapes (SMS) have been proven to be more sensitive to local damage than displacement mode shapes (DMS) (Chang, et al, 1993). The change of strain mode shape is obtained by the change of measured strain at different locations of the structure, i.e. SMS, after the structural damage. Previous studies have shown that it is generally very difficult to precisely identify the locations of damage in a real structure, even though changes in modal frequencies and mode shapes can give general assessment information about the possible existence of damage (Alampalli, et al, 1997).

The difficulties in damage detection together with other challenges in the numerical algorithms imply that further development and modification are necessary for the system identification techniques presently used in mechanical and aerospace applications for condition assessment of structures in civil engineering applications. Because under ambient excitations the monitoring response data may contain contributions from multiple vibration modes, this may affect the accuracy of the dynamic modal properties extracted for the individual modes by the system identification process. It is generally difficult to predict the accuracy of the system matrices or modal parameters identified from the test or monitoring data, because it depends on many factors, such as the characteristics of the data, the selection of the identification algorithm, etc. System identification errors may occur due to non-linearity behaviour of the system which more or less exists in all structural systems. Since there are many uncertainties in modal testing of complex structures, it is usually impossible or impractical to fully characterize the structure under study without error.

Although there are still many areas which require research in the relatively new field of damage detection and condition assessment of structures, many research works have been carried out and will continue to provide information and insight into the development of more accurate, reliable, robust and easily implementable methods for experimental modal analysis.

1.2 Objectives and Outline of Thesis

In light of the recent advances and new research findings in experimental modal analysis, the objectives of the thesis are to develop a methodology for the processing the monitoring data obtained from the Confederation Bridge, to extract the dynamic structural properties of the bridge in the field, and to compare the field system identification results with those from finite element models (FEM). The research works from the basis for future research on condition assessment and damage detection of the bridge.

Different system identification techniques are available for experimental modal analysis of structural systems after several decades of research and practice. It is beyond the scope of this thesis to give in-depth evaluations or comparisons of all the different methods. A brief overview of the common system identification techniques is presented in a later chapter of the thesis.

This thesis is divided into five chapters. Chapter 1 presents a brief introduction on the present status of system identification. Chapter 2 gives a more detailed explanation on the common modal analysis methods used in recent applications. The methods in frequency domain are discussed in the first part of Chapter 2. The stochastic subspace method, a modal analysis method in time domain adopted to analyze the response monitoring data from the Confederation Bridge, is discussed in the second part of Chapter 2. Following the discussion on the modal analysis methods, the case study of the Confederation Bridge is presented in Chapters 3 and 4, wherein the results of modal

analysis of the ambient traffic induced vibration test data are presented, respectively.

Finally, a summary and conclusions are presented in Chapter 5.

Fig. 1.1 Schematic Representation of the System Identification Problem

Chapter 2

Experimental Modal Analysis Methods

2.1 Introduction of Modal Testing

Experimental testing can provide accurate information on the behaviour and structural properties of complex structures. In experimental modal testing, the dynamic responses and sometimes also the vibration forces of the structure are measured when the structure is subjected usually to a known source of excitation. To carry out the experimental modal analysis using the measured test data accurate measurements of the vibration data and advanced numerical techniques for data processing and data analysis are necessary.

The development of modal testing methodologies has been evolving since the 1940s from the dynamic analysis of aircraft structures with the development of “Resonance Testing” procedures, which later formed the basis for the determination of structural natural frequencies and damping levels (Ewins, 1984). There is available extensive

literature on the subject of many different approaches for modal testing and data analysis (Juang, et al, 1988). A brief introduction of the common modal analysis methods is presented herein. However, it is beyond the scope of this thesis to give complete details of all the developments specifically developed for special applications in the field. The focus of the discussion here on the experimental modal analysis methods is to give an overview of the system identification techniques suitable for future applications in structural evaluation and damage detection..

At the early stage of the development, the time consuming modal testing experiments were conducted using analog laboratory equipment. The data were analyzed by hand calculations. Later on because of significant improvements in electronics and computer technologies, digital modal testing and analysis techniques were developed which replaced the analog approach as the standard in the 1970's.

For conventional modal tests, the trend on the source of excitation employed for modal testing gradually shifts from single-input to multiple-input with the increases and improvements in measurement and computation capacities. In the single input method, the frequency response of the system is measured, and then the vibration mode shapes and other modal parameters can be extracted from the frequency response data. With the development of digital Fourier analysis techniques, the single input method, compared with other methods, is very easy to implement and requires less computation effort than the multiple-input technique. The shortcoming of the single-input method is that it becomes inadequate and inaccurate when the modes of a structure are close to each other (Vold, et al, 1983) or the exciter was installed on the

node of certain mode. Closely spaced modes can be better identified from the response data recorded with multiple-input excitations by the multiple reference modal identification algorithms.

Since around 1980 the multiple-input random excitation method has become popular for experimental modal analysis. One of the commonly adopted modal testing procedures in the 1980's is the multiple-input sine dwell method, which extracts each mode by measuring the responses at selected points on the test structure. The damping property is determined by measuring the decay rates of the responses after the excitation is removed. It is estimated that about half of the laboratory modal tests in North America were conducted using the multiple-input excitation techniques in the 1990's, while about 30% of the tests were carried out using single-input random excitation because of its simple and fast advantages (Juang, et. al, 1988).

Theoretically, a single exciter at one location is able to provide all the necessary information to determine the dynamic properties of a structure. However, when the exciter is placed near the node of an important structural mode during the test, the particular structural mode may not be sufficiently excited and thus becomes difficult to identify from the data. Therefore, multiple exciter locations are often more reliable to extract information of all the important vibration modes.

Forced vibration responses and free vibration responses are two successive measurement stages in a vibration test. Data analysis of the dynamic response data from the two stages yields different types of results on the behaviour and properties of the test structure. The Frequency Response Function (FRF) is obtained from the

analysis of the forced vibration responses, whereas the natural vibration frequencies and damping factors are usually determined from the free vibration part of the test data.

In recent years, modal tests of bridges and buildings have been carried out using ambient conditions such as wind and random traffic loads, as the sources of excitation. This type of modal testing technique has attracted significant interest and research efforts in recent years because of increasing interest on performance monitoring and assessment of critical or deficient structures in civil engineering. The sources of the ambient excitation forces in modal testing of buildings and bridge structures may be from wind, sea currents, heavy traffic etc. Due to the large mass in a large size structure, the application of artificial controlled vibration excitation is difficult and impractical because of high cost and technical difficulties. The use of controlled external forced excitations is thus not suitable for experimental modal analysis applications in long-term monitoring of usually complex and large size structures. The continuous monitoring of the responses and behavior of the Confederation Bridge is an example of this type of modal testing application. The ambient vibration responses of the bridge are mainly caused by strong wind, water waves, ice impact, heavy traffic, and earthquakes. Modal analysis of the vibration responses from ambient conditions has practical significance for long-term monitoring and condition assessment of structures.

2.2 Experimental Modal Analysis Methods

Experimental modal analysis procedures can be generally divided into two categories: frequency and time domain methods. The basic principle of the frequency domain system identification approach is to extract the dynamic properties of the structure by nonparametric spectral analysis. The distribution of power over the frequency band of a finite number of records of data sequences are determined by calculating of the data using Fast Fourier Transform (FFT) algorithms and Power Spectral Density (PSD). On the other hand, in time-domain system identification methods, it is assumed that the response signals satisfy a selected model. The procedures in the time domain methods are to determine the parameters of the approximate system model. When the assumed model is a close approximation to the reality, the parametric time-domain approach generally can give more accurate estimates than the nonparametric frequency-domain identification procedures (Stoica, et al, 1997).

In the early 1980s, experimental modal testing techniques have been developed for multi-degree-of-freedom (MDOF) in both time and frequency domains. The response data at different locations caused by multiple exciters are processed simultaneously. The modal parameters can then be estimated and the vibration mode shapes are computed. In general, methods based on the MDOF time-domain approach are the most commonly adopted techniques for modal parameter identification applications since the early 1990's.

In modal identification analysis, methods in either time or frequency domain can yield good results for the system identification problems of simple small structures. However, this is not the case for complex large size structures. In system identification analysis of large complex structures as those often encountered in civil engineering applications, accurate results can only be obtained after repeated testing. In comparison, the accuracy and consistency of the system identification results obtained are not as good as those that can be expected of simple structures. Therefore, no one specific method is applicable for all type of structures in system identification practices. Consequently, the selection of the most appropriate approach for the particular application and modification of the identification numerical techniques for the particular conditions are often required.

There are shortcomings in the frequency domain methods due to the limitations associated with the numerical procedures of fast Fourier transform (FFT), such as low frequency resolution, leakage, and aliasing etc. Although some improvements in the techniques can avoid or reduce the computing errors in FFT, the preference of system identification approach has gradually shifted to the time domain approach which can generally yield more accurate results for the type of application commonly encountered in civil engineering. The time-domain method has been used in control engineering applications of system identification algorithm in the past 20 years. The time-domain based system identification numerical procedures has become the dominant choice of techniques in control engineering applications because the time-based methods provide parametric system models, such as a state-space

representation, stochastic difference equations for the control synthesis and design tools in the control field applications. The concept of constructing parametric system models is relatively new in civil engineering applications of system identification of buildings and bridges.

Discussions on the frequency domain methods and the time domain approaches are presented in the following sections. The time-domain based technique of the stochastic subspace method is presented in details.

2.2.1 Frequency Domain Methods

The identification techniques in frequency domain include aspects of frequency response measurements and direct spectral analysis of the time response data sequences. In early modal testing during the 1940's to 1960's, modal analysis methods using frequency response function employ various curve-fitting techniques to extract modal parameters from the measured data. The curve-fitting or circle-fitting method of decomposing FRF into the constituent modes was introduced by Kennedy and Pancu (Kennedy, et al, 1947), which set an important milestone in the early stage of the development of experimental modal analysis methods.

The Single Degree-of-Freedom (SDOF) curve-fit procedure is a commonly most widely used curve-fitting approach which matches or curve-fits a circle to the measured data points to identify the appropriate modal parameters by making use of the circular nature of the modulus/phase polar plot (the Nyquist plot) of the frequency response function of a SDOF system. The shortcoming of this method is mainly its poor performance for system identification problem of structures with closely-spaced

modes, which lacks an obvious circular behaviour on the Nyquist plot. The alternative approach is the multi-degree-of-freedom (MDOF) curve-fit algorithm which solves the problem by accepting the simultaneous influence of more than one mode and thus can provide more accurate estimates of the modal properties but with higher cost because of the increased computation required.

The development of the FFT algorithm by Cooley and Tukey (Cooley, et al, 1965) had made possible direct digital spectral analysis of the time history data from practical engineering problems. Since its introduction in the 1960's, the FFT algorithm has been established as the most widely adopted and dominant numerical technique in frequency domain modal analysis. In spectral analysis, the power spectral density (PSD) function of the free- or forced-response data blocks is computed by the FFT algorithm, as opposed to measuring the FRF directly in modal testing. The mode shapes and natural frequencies of a structure can be identified from the peaks of the Fourier spectrum of the recorded vibration response data. For systems with small damping values, the damped and undamped frequencies are approximately the same. Therefore, the natural frequencies of the structure can be reasonably estimated from the peaks of the amplitude spectrum or the PSD function of the FFT results of the recorded response data.

One of the basic modal analysis methods of the frequency domain approach to analyze ambient vibration response data is the coherence technique. According to studies on modal testing and analysis of bridges, the coherence technique has better performance compared with some time-domain methods, such as the data-dependent

system (DDS) method (Abdel Wahab, et al, 1998). The basic theory and algorithm of the coherence technique are illustrated in details in a later section of the chapter.

2.2.2 Time Domain Methods

With the advances in computer technology, faster and more powerful computers have made it a practical approach to conduct modal analysis of vibration response data by numerical procedures in the time domain, which are generally considered too costly in term of computation in the past. The time-domain modal analysis methods are generally categorized according to the choices of the estimated system model and the identification criterion. The time-domain based modal analysis algorithms for MDOF systems have the clear advantage of the capability to analyze wideband data consisting of the participation from many modes.

Some widely used system analysis methods in time domain, such as the time series method (Huang, 1999) and the data-dependent system (DDS) method (Pandit, 1991), of which the typical estimated system models may include the auto-regressive model (AR), the auto-regressive moving average model (ARMA), and the state-space model, are suitable for conducting modal analysis of the monitoring data generated by ambient vibration because the time domain methods can directly use the recorded time-history response data without the measurement of the input forces. The AR model was developed in the 1920's and the ARMA model was developed later from the AR theory, all of which were applied to structural mechanics problems during the 1970's (Gersch, 1974). The input signal is assumed to be Gaussian white noise for

the AR model, whereas the ARMA model only requires that the response data are from a stationary process in the modal analysis.

The DDS methodology is an approach for time series analysis which has two major differences from other time series methods (Pandit, 1983). The DDS methodology does not need to use sample correlations or sample spectra evaluated from data for determining the modal orders. It assumes the data are uniformly sampled from a system governed by differential equations. The DDS methodology generally works well in modal analysis and system identification applications in science and engineering.

In the AR and ARMA models, the assumption of single-input-single-output (SISO) in the modal analysis procedure leads to easy and fast numerical algorithms for the processing and analysis of ambient vibration data. But the SISO approach may not be accurate for estimating the frequencies, damping and mode shapes of higher modes, and the modal analysis results may be different for each measuring location. Therefore, the use of simultaneous measured signals from all the measurement locations at the same time becomes more attractive. The auto-regressive vector model (ARV) and the auto-regressive moving average vector model (ARMAV) models are developed in which the responses of the system at multiple locations are expressed in a vector.

The methods for estimating the coefficients or parameters of the AR and ARMA models include the traditional least square approach, the maximum likelihood method (ML), the instrumental variable method, etc. The structural properties such as natural

frequencies and damping ratios can be obtained directly from knowledge of the ARMA model parameters or from the determined ML estimates of the ARMA parameters (Gersch, 1976).

The least-square error method has been adopted to solve a wide range of problems since its basic concept was formulated by Gauss in the early 1800's. The least-square procedure gives the best fit to experimental data based on the criterion of minimizing the error-squares function. The main advantage of the least square algorithm is that it requires very little information on the statistics of the data, and is simple to implement. For linear systems and Gaussian noise data, the maximum likelihood approach yields the same results in the parameter calculations as the least-square approach (Giordano, et al, 1985). The principal disadvantage of the least-square approach is that it does not always provide the best performance due to insufficient statistical information of the data.

The maximum-likelihood method constructs the likelihood function which is a function of the data and unknown parameters. The estimation of the unknown parameters is to determine the parameter that maximizes the likelihood function. The ML method can also obtain a measure of the statistical reliability of the parameter estimates, such as the coefficient of variation, the ratio of the standard deviation to the mean of the parameter estimate. Gersch et al (1973) demonstrated a ML method in providing estimates of the structural parameters by constructing an ARMA discrete time series model for random vibration data.

Previous experience on the application of the DDS technique on modal analysis of monitoring data obtained from ambient vibrations shows that it is difficult to determine accurately the vibration mode shapes beyond the first mode from the ambient vibration test data (Abdel Wahab et al, 1998).

The state-space representation of a mechanical system is widely used in both the classical structural dynamics problems and in the field of control engineering. In recent developments, the state-space representation has become popular in modal analysis applications. For the case of ambient vibration test, the modal analysis techniques employed for system identification must take into account that the input forces are generally not known or measured. If the output of a system can be uniquely specified by the input together with certain initial conditions and times, the system is deterministic. Otherwise it is stochastic, which the non-unique response to input signals is supposed that the system has a random noise input in addition to the possible control inputs. The field measured data are usually contaminated with noise. Therefore, stochastic modeling and system realization methods are suitable for modal analysis of the monitoring data caused by ambient vibrations. The procedures of system realization involve the computation of the state matrix parameters which satisfy the state-space model. A detailed discussion on the stochastic realization method via the state-space model is presented in the following section.

For the Confederation Bridge monitoring project, a number of system identification techniques, mainly the Coherence technique and the stochastic subspace method, are

considered for the evaluation of the dynamic field monitoring data under different dynamic loading conditions.

2.3 Coherence Technique

In frequency domain, a commonly adopted method to analyze ambient vibration test data is the coherence technique. In this approach, the measured time signals are transformed into components in the frequency domain. Since the input force is not usually measured, one channel of data is chosen as the reference to correlate the response measurements. The general theory of the coherent technique is that the natural vibration frequencies of the structure can be identified from the locations of the peaks in the auto- and cross-spectra density plots, and the relative displacements of the vibration mode shapes are evaluated from the ratios of the amplitudes of the PSD curves between the calculated and the reference stations. The sign of the relative displacement term in the mode shape vector is determined by the phase relationship between the selected station and the reference station responses. The coherence functions between response data channels are computed to examine the influence of noise and non-linear response of the bridge in the vibration modes (Paultre et al. 1995).

The experimental modal analysis techniques in frequency domain have been developed in conjunction with the developments of digital spectral analysis and the discrete Fourier transforms numerical techniques. The Fourier transform procedure

converts signals in the time domain to the frequency domain. Based on the Fourier integral, the Fourier transform of the continuous time signals takes the form of

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (2.1)$$

which gives the Fourier transform $X(\omega)$ of the continuous time signal $x(t)$ at the analysis frequency ω . Since the vibration responses of the test structure are sampled at discrete intervals not continuously, the Discrete Fourier Transform (DFT) $X(k)$ of the sampled signals $x(k)$ is calculated instead by the expression

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-i2\pi kn/N} \quad (2.2)$$

where n is the running sample index and N is the total number of samples. The DFT of the response data is calculated by the FFT algorithm, which reduces the enormous computer processing time to practical level, to the order of $N \log_2 N$ operations. The FFT algorithm improves the accuracy of the results and reduces the less round-off error due to significantly low computation operations than before.

The auto- and cross-correlation function of each pair of data channels are calculated to give the auto- and cross-power spectra of the data channels. The auto-correlation function of the vibration response at a sensor measurement location of the test on monitoring structures, considered here as a random process $x(t)$, is defined as the average value of the product $x(t)x(t+\tau)$.

$$\begin{aligned} R_{xx}(\tau) &= E[x(t)x(t+\tau)] \\ &= \frac{1}{N} \sum_{k=0}^{N-1} x(k)x(k+\tau) \end{aligned} \quad (2.3)$$

The cross-correlation functions between two different random processes or time sequence functions $x(t)$ and $y(t)$ are defined as

$$\begin{aligned} R_{xy}(\tau) &= E[x(t)y(t+\tau)] = \frac{1}{N} \sum_{k=0}^{N-1} x(k)y(k+\tau) \\ R_{yx}(\tau) &= E[y(t)x(t+\tau)] = \frac{1}{N} \sum_{k=0}^{N-1} y(k)x(k+\tau) \end{aligned} \quad (2.4)$$

Assuming the time sequence functions are stationary, the relationships between R_{xy} and R_{yx} have the properties:

$$\begin{aligned} R_{xy}(\tau) &= E[x(t-\tau)y(t)] = R_{yx}(-\tau) \\ R_{yx}(\tau) &= E[y(t-\tau)x(t)] = R_{xy}(-\tau) \end{aligned} \quad (2.5)$$

The Fourier transforms of $R_{xx}(\tau)$ and $R_{xy}(\tau)$ give the auto-power spectrum or mean square spectral density S_{xx} and cross-power spectrum S_{xy} , respectively, as follows

$$\begin{aligned} S_{xx}(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i\omega\tau} d\tau \\ S_{xy}(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-i\omega\tau} d\tau \end{aligned} \quad (2.6)$$

Because the computation of the correlation functions R_{xx} , R_{yy} and R_{xy} is usually very time-consuming, another approach is to compute the Fourier transform of the time history sequences first and then multiply the corresponding spectrum to obtain the spectral density. For each measured channel of the discrete signal data, the auto- and cross-power spectrum functions at the frequency f can be determined as follows:

$$\begin{aligned}
S_{xx}(f) &= X(f)X^*(f) \\
S_{xy}(f) &= X(f)Y^*(f) \\
S_{yy}(f) &= Y(f)Y^*(f)
\end{aligned} \tag{2.7}$$

where the symbol * denotes complex conjugate.

The auto and cross-power spectrum density functions can be obtained by normalizing the auto- and cross-spectrum functions, respectively, by the bandwidth Δf of the time history sequence $x(t)$ and taking the limit $\Delta f \rightarrow 0$

$$PSD(f) = \lim_{\Delta f \rightarrow 0} \frac{S_{\Delta f}}{\Delta f} \tag{2.8}$$

where $S_{\Delta f}$ can be either S_{xx} or S_{xy} .

To minimize the influence of noise in the response data, the average of the spectra derived from multiple data blocks is calculated in order to cancel un-correlated noise signals and to facilitate the detection of the natural frequencies of all the vibration modes excited by the ambient vibration. The correct way to compute the average spectrum values is to obtain multiple blocks of time history test data by performing multiple experiments or dividing a long time sequence into multiple segments.

$$\begin{aligned}
\bar{S}_{xx}(f) &= \frac{1}{N} \sum_{i=0}^N S_{xx}^{(i)}(f) = \frac{1}{N} \sum_{i=0}^N X^{(i)}(f)X^{*(i)}(f) \\
\bar{S}_{xy}(f) &= \frac{1}{N} \sum_{i=0}^N S_{xy}^{(i)}(f) = \frac{1}{N} \sum_{i=0}^N X^{(i)}(f)Y^{*(i)}(f)
\end{aligned} \tag{2.9}$$

where the notation “—” denotes the average taken over multiple sets of data at frequency f .

In model analysis of bridge responses, the PSD functions calculated from different sets of response data in the vertical direction are combined and averaged into one single function, which is then used to identify the vertical vibration modes. The same approach is applied to the PSD functions in the horizontal directions to identify the transverse and longitudinal modes of vibration.

After evaluating the Fourier transform X_f of the input or reference response channel x and Y_f of the output or calculating response channel data y by the FFT algorithm, the frequency response functions (FRF) and the coherence between the pair of measured channel data can be evaluated to identify the natural vibration frequencies of the structure.

The Frequency Response Function (FRF) matrix H_{xy} expresses the frequency domain relationship between the inputs and the outputs of a linear time-invariant system. If X and Y are the Fourier transforms of the system input and output respectively, the FRF matrix H_{xy} can be defined as follows

$$Y(f) = H_{xy}(f)X(f) \quad (2.10)$$

For the ambient vibration case, one channel of the response data is selected as the reference time sequence data and used as the input signal, denoted by X_f while other channels of time history data are taken as the output signals Y_f . The above relationship describes an ideal noise-free system. Since it is rare to have a free system in engineering practices, therefore various estimators are used to give estimates of H_{xy} from the measured input and output data. Two of the most

commonly used forms are the H_1 and H_2 estimators, in which H_1 assumes that there is no noise in the input data and H_2 assumes no noise in the output data. The FRF matrix is given by the ratio of the average cross- and auto-power spectrum functions, where

$$H_1(f) = \frac{\bar{S}_{yx}(f)}{\bar{S}_{xx}(f)} \quad (2.11)$$

The other estimator H_2 is composed as:

$$H_2(f) = \frac{\bar{S}_{yy}(f)}{\bar{S}_{yx}(f)} \quad (2.12)$$

A coherence function γ_f is defined as the ratio of the FRF estimators H_1 and H_2 as follows

$$\gamma_f^2 = \frac{\bar{S}_{xy} \times \bar{S}_{xy}^*}{\bar{S}_{xx} \times \bar{S}_{yy}} \quad (2.13)$$

The coherence function is a real number scalar function which varies between 0 and 1. A value close to 1.0 shows that the FRF estimators H_1 and H_2 are close to each other, which indicates a strong linear relationship of the output data channel with the reference channel at the frequency f . The coherence function is effective to distinguish the influence of noise from the structural natural frequencies in experimental modal analysis of bridge responses. The peaks in the PSD curves corresponding to the above-mentioned interferences should have low values in the coherence curve. Hence, the structural natural vibration frequencies can be identified

from the peaks in the coherence curve with high coherence values, which normally range between 0.8 to 1.0.

Beside the method of averaging and the use of the coherence function to identify the effect of noise in the modal analysis process, the procedure of filtering can also be employed to isolate and limit the noise problem in vibration measurements. The vibration modes in the range of frequency of interest can be separated out or isolated from the total response data by filtering.

From the PSD curves, the dominant natural vibration frequencies of a structure can be identified from the strength of the response signal. Since the location of the sensor may coincide with the location of the node of a vibration mode, which results that the PSD curve does not exhibit a noticeable peak at the frequency of the particular vibration modes. In this case, the PSD curves from sensors at other locations and other set of modal tests have to be investigated to determine the natural frequencies of the missing modes.

After the natural frequencies are identified, the vibration mode shapes can be determined from the ratios of the relative modal amplitudes at the various measurement locations. The relative vertical displacements of the mode shapes are calculated from the amplitude ratios of the PSD curves of the output channels to the reference channel in the vertical direction. The displacements in the transverse and longitudinal directions can also be determined in similar fashion. The phase angle of the ratio is computed to determine the direction of the motion which is given by the sign of the ratio obtained from the relative phase angle. If the phase angle is close to

0°, the modal amplitude ratio was considered positive because the two degrees-of-freedom are in phase. Similarly, when the phase angle is near 180°, the modal amplitude ratio is considered negative. However, the damping of the structure may have influence on the phase angle that makes it not exact 0° or 180°. After all the modal amplitude ratios with respect to a reference degree-of-freedom are computed, they are combined to form the potential mode shape vector.

Another important structural property determined in the system identification process, is the damping ratio. The modal damping can be calculated using the half-power bandwidth method (Chopra, 1995, Abdel Wahab, 1996).

Since the amount of damping controls the shape of the frequency response curve, the damping ratio can be estimated from the characteristics of the curve (Clough and Penzien, 1993). The half-power bandwidth method is a convenient method to estimate the modal damping ratio from the spectrum of the data by the following expression

$$\xi_i = \frac{\omega_2 - \omega_1}{2\omega_i} = \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1} = \frac{f_2 - f_1}{f_2 + f_1} \quad (2.14)$$

where ω_1 and ω_2 are the frequencies on the two sides of the spectral peak, ω_i is the frequency at the half-power amplitude which is equal to $1/\sqrt{2}$ times the resonant peak value, as shown in Figure 2.1.

Due to the low frequency resolution, leakage and aliasing problem in frequency domain methods, it is usually difficult to locate the exact location of the peak in PSD.

thus there is error in the peak value which can cause error in the damping value obtained by the half-power bandwidth method.

Evaluations of the test data obtained from ambient vibrations, such as wind or heavy traffic, have shown that the coherence technique can achieve better results than the ARV model, particularly when the ambient vibrations arise from vehicles passing over the bridge (Abdel Wahab and De Roeck, 1998).

Based on the coherence algorithm, the natural frequencies of the Confederation Bridge have been identified from the peaks of the coherence curve and the relative amplitudes of the power spectral density (PSD) functions of the ambient vibration monitoring data. The system identification results from the ambient vibration data of the Confederation Bridge, such as wind and heavy traffic, by the coherence technique are presented in Chapter 4.

2.4 Stochastic Realization Method

2.4.1 Introduction

Convenient for computer implementation, the state-space method has become an important and most often used tool for system analysis and design. In the state-space method, a lower-order system model is employed to approximate the behaviour of a high-order system. The state-space method can give a close and lower-order approximation to the measured input-output relation, and can be employed to identify the dynamic parameters of the system.

The state of an object or system is defined as the property of the system which relates the input or reference data to the output or response data such that knowledge of the input for $t \geq t_0$ and the state at time $t = t_0$ completely determine the output for $t \geq t_0$. The state space is the set of all the state variables $x(t)$. The system realization process is the procedure of constructing a state-space representation using the experimental data. Because there can be many different ways to express the same input-output relation, the state-space representation is not unique. The order of the state-space representation depends on the desired level of accuracy of the approximation. A minimum realization is a model with the smallest state space dimension that has the same input-output relations for the specified degree of accuracy (Ho, 1965).

The Hankel matrix approach is a reliable algorithm for the state-space approximation. The Hankel matrix uses the Markov parameters in system realization. The Hankel matrix approach is actually to find another Hankel matrix which has a lower rank in order to correspond to a lower order system and is a reasonably close approximation to the original matrix. The advantage of using the Hankel matrix in system realization is because the rank of the matrix reflects the order of the corresponding system in the ideal environment. In the experimental modal analysis, the environment is not noise free. Therefore the Hankel matrix criterion alone is not very effective due to the contamination of noise and lack of information on the closeness of the matrix to a lower rank matrix. To improve the system realization process, Kung (1979) has presented an algorithm employing the singular value decomposition (SVD) technique to account for the influences of the noise. The SVD displays a set

of non-zero singular values corresponding to the rank of the matrix. It can provide important information about the approximation error in the low order matrix. Essentially, Kung's method is to find a Hankel matrix of a lower rank which is a close approximation to the original matrix.

Of a stochastic system, the input under certain initial conditions do not necessary lead to unique set of output or response data due mainly to the effect of random noise input in modal analysis of ambient vibrations. In recent years, realization methods for stochastic systems have been developed for extraction of modal parameters using only the output or response data. The general idea of the stochastic subspace methods is to conduct modal analysis via stochastic state-space model employing the singular value decomposition technique, which is illustrated in the following sections.

2.4.2 State-space Model and Identification Method (Juang, 1994)

The basic equation of motion of a finite-dimensional linear system can be expressed as a set of second-order differential equations

$$\mathbf{M}\ddot{\boldsymbol{\omega}} + \boldsymbol{\zeta}\dot{\boldsymbol{\omega}} + \mathbf{K}\boldsymbol{\omega} = \mathbf{f}(\boldsymbol{\omega}, t) \quad (2.15)$$

where $\mathbf{M}$, $\boldsymbol{\zeta}$, $\mathbf{K}$ are the mass, damping and stiffness matrices while $\boldsymbol{\omega}$, $\dot{\boldsymbol{\omega}}$, $\ddot{\boldsymbol{\omega}}$ represent displacement, velocity and acceleration vectors, respectively, and $\mathbf{f}(\boldsymbol{\omega}, t)$ is the vector of the forcing functions at specific locations. Equation (2.15) can be rewritten in the state-space form as follows

$$\dot{\mathbf{x}} = \mathbf{A}_c \mathbf{x} + \mathbf{B}_c \mathbf{u} \quad (2.16)$$

where

$$\begin{aligned} A_c &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\boldsymbol{\zeta} \end{bmatrix}, \quad B_c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_2 \end{bmatrix} \\ \mathbf{x} &= \begin{bmatrix} \boldsymbol{\omega} \\ \dot{\boldsymbol{\omega}} \end{bmatrix}, \quad \mathbf{f}(\boldsymbol{\omega}, t) = \mathbf{B}_2 \mathbf{u}(t) \end{aligned} \quad (2.17)$$

The state variable $\mathbf{x}$ is composed of the displacement vector $\boldsymbol{\omega}$ and its derivatives. In Equation (2.17), A_c is a $2n_2$ by $2n_2$ state matrix, where n_2 is the number of independent coordinates, $\mathbf{B}_2$ is an n_2 by r input influence matrix characterizing the locations and type of inputs of which r is the number of inputs, and $\mathbf{u}(t)$ is the multiple input excitation function vector of dimension r .

For the sensor measurement, a matrix output equation in which the output vector $\mathbf{y}$ is expressed in terms of the dynamic system responses obtained from the site monitoring data can be written as follows

$$\mathbf{y} = \mathbf{C}_a \ddot{\boldsymbol{\omega}} + \mathbf{C}_v \dot{\boldsymbol{\omega}} + \mathbf{C}_d \boldsymbol{\omega} \quad (2.18)$$

where $\mathbf{C}_a$, $\mathbf{C}_v$ and $\mathbf{C}_d$ are the output influence matrices of acceleration, velocity and displacement, respectively. The size of the measurement vector $\mathbf{y}$ equals to the m outputs. The acceleration vector $\ddot{\boldsymbol{\omega}}$ in Equation (2.18) can be replaced by the solution from Equation (2.15) as follows

$$\mathbf{y} = \mathbf{C}_a \mathbf{M}^{-1} [\mathbf{B}_2 \mathbf{u} - \boldsymbol{\zeta} \dot{\boldsymbol{\omega}} - \mathbf{K} \boldsymbol{\omega}] + \mathbf{C}_v \dot{\boldsymbol{\omega}} + \mathbf{C}_d \boldsymbol{\omega} \quad (2.19)$$

It can again be simplified as follows

$$y = Cx + Du \quad (2.20)$$

where

$$C = [C_d - C_a M^{-1} K \quad C_v - C_a M^{-1} \zeta] \quad D = C_a M^{-1} B_2 \quad (2.21)$$

where C is an $m \times n$ output influence matrix of which $n = 2n_2$ for the state vector x , and D is an $m \times r$ direct transmission matrix.

Equation (2.16) and Equation (2.20) comprise a continuous-time state-space model.

In civil engineering applications, it is more practical and convenient to obtain the discrete-time signals in structural testing or monitoring. By using the analog-to-digital converter, the output signal which is originally a continuous-time function is converted to discretized signals or pulse-function signals. To obtain a discrete-time state-space model, the state parameter vector x can be solved first from Equation (2.16) with the initial condition $x(t_0)$ at time $t = t_0$ (Pandit, 1991) as follows

$$x(t) = e^{A_c(t-t_0)} x(t_0) + \int_{t_0}^t e^{A_c(t-\tau)} B_c u(\tau) d\tau \quad (2.22)$$

when $u(t)$ is the input function. The first term of Equation (2.22) is due to the homogeneous solution of $x(t)$ and the second term is due to the forcing function. The discrete-time representation is sampled at equally spaced time interval of Δt . A new state matrix A can be defined by the parameters of the deterministic part of $x(t)$ as shown in Equation (2.23). Similarly, the input influence matrix B can also be defined by the parameters of the stochastic part of Equation 2.22 as follows

$$\begin{aligned} \mathbf{A} &= e^{\mathbf{A}_c \Delta t} \\ \mathbf{B} &= \int_0^{\Delta t} e^{\mathbf{A}_c \tau'} d\tau' \mathbf{B}_c \end{aligned} \quad (2.23)$$

Consequently, the state variable $\mathbf{x}$ can be expressed as follows

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k); \quad k = 0, 1, 2, \dots \quad (2.24)$$

where the notation of $x(k) = x(k \Delta t)$ is adopted.

Together with the matrix output equation from Equation (2.20), a discrete-time state-space model can be constructed which forms the basis for the system identification of linear, time-invariant, dynamic systems.

A stochastic state-space model can be constructed by adding noise processes to the equations of the linear system. When the input is zero, the m -vector outputs $\mathbf{y}_k$ can be expressed in terms of the n -vector states $\mathbf{x}_k$ by the equations:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{w}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \end{aligned} \quad (2.25)$$

where $\mathbf{x}(k)$ represents the state vector of dimension r , $\mathbf{y}(k)$ is the output vector of dimension m ; and $\mathbf{w}(k)$ denotes for the process noise while $\mathbf{v}(k)$ represents the measurement noise. Both $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are taken as zero-mean, uncorrelated random vector sequences.

An accurate model of a system should ensure that all the system states of interest can be controlled and/or observed. If a state $\mathbf{x}(k)$ of a system can be reached from any initial state of the system in a finite time interval by some control action, the system is called controllable. On the other hand, if knowledge of the input $\mathbf{u}(k)$ and output $\mathbf{y}(k)$ over a finite time interval can completely determine the state $\mathbf{x}(p)$ at the given sample

time p ($0 < k \leq p$), the system is called observable. According to controllability and observability theory, the observability matrix O_p of order p and the controllability matrix C_q of order q can be defined as follows

$$O_p = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix}; \quad C_q = [G \quad AG \quad \dots \quad A^{q-1}G] \quad (2.26)$$

where p and q are the given sample time; $G = E[x_{k-1}y_k^T]$ and E represents the expectation on the mean value.

The realization of the system now involves the computation of A and C which satisfy the discrete-time model presented in Equation (2.25). There is an infinite number of realization for each system with the same response output for any particular input. Hence, the Minimum Realization is defined as the one which yields the smallest state-space dimensions among all the realized systems that have the same input-output relations. The eigenvalues obtained from the minimum realization give the system modal parameters.

Assuming the state matrix A of order n has a complete set of linearly independent eigenvectors ($\psi_1, \psi_2, \dots, \psi_n$) and the corresponding eigenvalues ($\lambda_1, \lambda_2, \dots, \lambda_n$), the dynamics of the system are then completely characterized by the eigenvalues and the eigenvectors of the A matrix. Let Λ be the diagonal matrix of eigenvalues and Ψ be the matrix of eigenvectors:

$$\begin{aligned} \mathbf{A} &= \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n] \\ \mathbf{\Psi} &= [\psi_1, \psi_2, \dots, \psi_n] \end{aligned} \quad (2.27)$$

From the definition of eigenvalue and eigenvector:

$$\mathbf{A}\psi = \psi\lambda \quad (2.28)$$

In combined matrix form,

$$\begin{aligned} \mathbf{A}[\psi_1, \psi_2, \dots, \psi_n] &= [\lambda_1\psi_1, \lambda_2\psi_2, \dots, \lambda_n\psi_n] \\ &= [\psi_1, \psi_2, \dots, \psi_n] \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \\ &= [\psi_1, \psi_2, \dots, \psi_n] \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n] \end{aligned}$$

or

$$\mathbf{A}\mathbf{\Psi} = \mathbf{\Psi}\mathbf{\Lambda} \quad (2.29)$$

The eigenvalue decomposition of $\mathbf{A}$ is given by

$$\mathbf{A} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{-1} \quad (2.30)$$

The complex eigenvectors and eigenvalues in Equation (2.28) are complex conjugate pairs. The diagonal matrix $\mathbf{\Lambda}$ contains the information of modal damping ratios and damped natural frequencies, which are the real and imaginary parts of the eigenvalues. In order to obtain accurate results of the modal parameters, the matrix $\mathbf{\Lambda}$ obtained from the discrete-time data acquisition method needs to be transferred to the continuous-time domain, denoted as $\mathbf{\Lambda}_c$. According to Equation (2.23), the discrete

eigenvalues λ_r on the diagonal of Λ can be transformed into continuous eigenvalues μ_r .

Since

$$\lambda_r = e^{\mu_r \Delta t}$$

then

$$\mu_r = \sigma_r + i\omega_r = \frac{1}{\Delta t} \ln(\lambda_r) \quad (2.31)$$

where σ_r represents the damping factor, and ω_r is the damped natural frequency of the r -th mode.

The damping ratio ξ_r of the r -th mode can be obtained by

$$\xi_r = -\frac{\sigma_r}{\sqrt{\omega_r^2 + \sigma_r^2}} \quad (2.32)$$

The mode shape $\{\phi\}_r$ of the r -th mode at the sensor locations is determined from the system eigenvectors $\{\psi\}_r$ of Ψ :

$$\{\phi\}_r = C\{\psi\}_r \quad (2.33)$$

The above procedures show that once A and C are determined, the natural frequency, damping and mode shape can then be easily obtained. The procedure to calculate A and C from the monitoring response data is presented in the following section.

2.4.3 The Stochastic Realization Problem

A very promising approach for the stochastic realization is the Hankel matrix approach which makes use of the Hankel matrix formed from the Markov parameters

(Juang, 1994). The Markov parameters are commonly used as the basis for identifying mathematical models for linear dynamic systems. The Markov parameter can be defined by the solution of the output $y(k)$ from Equations (2.20) and (2.24) with zero initial condition

$$\begin{aligned}
 \mathbf{x}(0) &= \mathbf{0}, \\
 \mathbf{y}(0) &= \mathbf{D}\mathbf{u}(0), \\
 \mathbf{x}(1) &= \mathbf{B}\mathbf{u}(0), \\
 \mathbf{y}(1) &= \mathbf{C}\mathbf{B}\mathbf{u}(0) + \mathbf{D}\mathbf{u}(1), \\
 \mathbf{x}(2) &= \mathbf{A}\mathbf{B}\mathbf{u}(0) + \mathbf{B}\mathbf{u}(1), \\
 \mathbf{y}(2) &= \mathbf{C}\mathbf{A}\mathbf{B}\mathbf{u}(0) + \mathbf{C}\mathbf{B}\mathbf{u}(1) + \mathbf{D}\mathbf{u}(2), \\
 &\vdots \\
 \mathbf{x}(k) &= \sum_{i=1}^k \mathbf{A}^{i-1} \mathbf{B}\mathbf{u}(k-i), \\
 \mathbf{y}(k) &= \sum_{i=1}^k \mathbf{C}\mathbf{A}^{i-1} \mathbf{B}\mathbf{u}(k-i) + \mathbf{D}\mathbf{u}(k)
 \end{aligned}$$

When the input variable is a pulse function, $u_i(0)=1$ ($i=1, 2, \dots, r$) and $u_i(k)=0$ ($k=1,2,\dots$), the results can be assembled into a sequence of pulse-response matrix Y_k or Markov parameters with dimension m by r as follows

$$Y_0 = \mathbf{D}, Y_1 = \mathbf{C}\mathbf{B}, Y_2 = \mathbf{C}\mathbf{A}\mathbf{B}, \dots, Y_k = \mathbf{C}\mathbf{A}^{k-1}\mathbf{B} \quad (2.34)$$

When the ambient vibration response is used to solve the realization problem, some outputs are chosen as reference vector since there is no input in this case. Thus the Markov parameters Y_k can be obtained as the correlation of output vector with the selected reference (input) vector as follows

$$Y_k = E[\mathbf{y}_{k-m} \{\mathbf{y}_m\}_{ref}^T] \quad (2.35)$$

where $\{y_m\}_{ref}$ is a N_{ref} output vector which is selected to act as reference.

The system realization process involves the determination of the Hankel matrix for the multi-input multi-output case, which consists of the Markov parameters Y_k . For $p \geq q$, $H_{p,q}$ is defined as the block-Hankel matrix as follows

$$H_{p,q} = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_q \\ Y_2 & Y_3 & \cdots & Y_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ Y_p & Y_{p+1} & \cdots & Y_{p+q-1} \end{bmatrix} \quad (2.36)$$

Substituting the Markov parameters from Equation (2.34) to Equation (2.36), the Hankel matrix is decomposed into two matrices with the following factorization property

$$H_{p,q} = O_p C_q \quad (2.37)$$

The matrix decomposition reduces the matrix to a simpler canonical form. The Singular Value Decomposition (SVD) technique is employed for the decomposition. Two user-defined invertible weighting matrices W_1 and W_2 of size pm and qm , respectively, are necessary for SVD. Pre-and post multiplying the Hankel matrix with W_1 and W_2 and performing a SVD on the Hankel matrix yields:

$$W_1 H_{p,q} W_2^T = U \Sigma V = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & [0] \\ [0] & [0] \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T \quad (2.38)$$

where S_1 is composed of n positive singular values in decreasing order as follows

$$\mathbf{S}_1 = \text{diag}[\sigma_1, \sigma_2, \dots, \sigma_n] \quad \text{with} \quad \sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n > 0 \quad (2.39)$$

and the left singular vectors $[U_1]$ are the n columns of U and the right singular vectors $[V_1]$ are the n columns of V .

According to (2.37) the weighted Hankel matrix is also equal to

$$\mathbf{W}_1 \mathbf{H}_{p,q} \mathbf{W}_2^T = \mathbf{W}_1 \mathbf{O}_p \mathbf{C}_q \mathbf{W}_2^T \quad (2.40)$$

Therefore, the observability matrix can be determined from equations (2.39) and (2.41) as follows

$$\mathbf{O}_p = \mathbf{W}_1^{-1} \mathbf{U}_1 \mathbf{S}_1^{-1/2} \quad (2.41)$$

Through the approach of SVD on the Hankel matrix which is composed of the monitoring response data, the observability matrix is obtained thus the state matrix $\mathbf{A}$ and output influence matrix $\mathbf{C}$ can be determined, with which the structural dynamic properties can be identified.

The system matrices can be easily estimated by using the shift structure of $\mathbf{O}_p$ as follows:

$$\mathbf{C} = \{\text{first block of row } [\mathbf{O}_p]\} \quad (2.42)$$

At the same time, $\mathbf{A}$ can be determined according to the definition of observability matrix as follows

$$[\hat{\mathbf{O}}_{p-1}] = [\mathbf{O}_{p-1}] \mathbf{A} \quad (2.43)$$

where $[\mathbf{O}_{p-1}^\uparrow]$ is the matrix that is upper shifted by one block row and $[\mathbf{O}_{p-1}]$ is obtained by deleting the last block row of $[\mathbf{O}_p]$.

The choice of weighting matrix give rise to different stochastic subspace identification methods. The identified weighting matrices which establishes the Balance Realization Method is selected to conduct modal analysis for the Confederated Bridge

$$\mathbf{W}_1 = \mathbf{I} \qquad \mathbf{W}_2 = \mathbf{I} \qquad (2.44)$$

The above procedure is the basic theory for system realization through SVD. The actual systems may have a larger or even an infinite order. In this case, the Hankel matrices in Equation (2.38) are composed of “empirical” correlations

$$[\hat{\mathbf{R}}_k] = \frac{1}{M} \sum_{m=0}^M \{v_{m+k}\} \{v_m\}_{ref}^T \qquad (2.45)$$

where M is the number of data samples.

The SVD decomposition is then conducted with the Hankel matrix filled with empirical correlations

$$[\mathbf{W}_1][\hat{\mathbf{H}}_{p,q}][\mathbf{W}_2]^T = \begin{bmatrix} \hat{\mathbf{U}}_1 & \hat{\mathbf{U}}_2 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{S}}_1 & [0] \\ [0] & \hat{\mathbf{S}}_2 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_1^T \\ \hat{\mathbf{V}}_2^T \end{bmatrix} = [\hat{\mathbf{U}}_1][\hat{\mathbf{S}}_1][\hat{\mathbf{V}}_1]^T + [\hat{\mathbf{U}}_2][\hat{\mathbf{S}}_2][\hat{\mathbf{V}}_2]^T \qquad (2.46)$$

where

$$\begin{aligned} \hat{\mathbf{S}}_1 &= \text{diag}(\sigma_1 \cdots \sigma_n), \quad \sigma_1 \geq \sigma_2 \cdots \geq \sigma_n \geq 0 \\ \hat{\mathbf{S}}_2 &= \text{diag}(\sigma_{n+1} \cdots \sigma_{pR_{nw}}), \quad \sigma_{n+1} \geq \sigma_{n+2} \cdots \geq \sigma_{pR_{nw}} \geq 0 \end{aligned} \qquad (2.47)$$

As shown in the basic Hankel matrix, if the rank of the Hankel matrix is r , then the singular value $(\sigma_{n+1}, \dots, \sigma_{pRresp})$ should be zero. If the singular values $(\sigma_{n+1}, \dots, \sigma_{pRresp})$ are very small and close to zero, it shows that the matrix is not too far from a r -rank matrix. Therefore, if it is assumed that the singular values $(\sigma_{n+1}, \dots, \sigma_{pRresp})$ represents the noise part, then it suggests the noise free system has an r -th order representation. When the noise level is much lower than the signal level, identification of a model with order n is done by truncating the singular values. When the $[S_2]$ is omitted, the observability matrix can then be expressed as follows

$$[\hat{O}_p] = [W_1]^{-1} [\hat{U}_1] [\hat{S}_1]^{-1} \quad (2.48)$$

The remaining steps of the algorithm are similar to those in Equation (2.42) to Equation (2.44) with theoretical quantities replaced by empirical ones.

2.5 Other Methods

A disadvantage of the Fourier analysis is that frequency information can only be extracted for the entire duration of the time series data. If there is an impulse that happened during the measurement, its point of measurement on the time axis is lost after the Fourier transform. A method to alleviate this problem is to select a shorter time period or a relatively narrow observation period, i.e. a fixed length time window, to compute the frequencies in the selected length of time, for the FFT algorithm, i.e. short-time Fourier transform (STFT) (Chui, 1992). The time or observation window

then slides along the entire time signal to obtain a series of spectra in the time-frequency relationship.

There is also a disadvantage in this technique in which the same time window is used throughout the analysis with the selected frequency resolution ($\Delta f = 1/\Delta T$). If the vibration period T is too small, the low frequency component may be missed. This means the signal component can be analyzed with either good time resolution or good frequency resolution, but it is impossible to have good resolution for both at the same time.

The above disadvantage can be overcome by the wavelet analysis that provides an alternative way by breaking a signal down into its constituent parts in such a way that the addition of all the wavelets accurately reproduces the original signal (Newland, 1993). Instead of windowing the Fourier transforms as the STFT does, the wavelet transform windows the signal directly. The use of the scaling factor to dilate or contract the basic wavelet results in an analysis window that is narrow at high frequencies and wide at low frequencies, which makes it impossible to have good resolution for both time and frequency at the same time. To make the calculation simple and easier, the fast wavelet transform (FWT) decomposes a signal into components differing in size by a factor of two (Hubbard, 1998). The wavelet analysis is a relatively new subject and a popular topic in the research of time-frequency analysis technique, and much research are currently being carried out to explore its great potential for applications in modal analysis.

Integral transformation of various kinds of data series is a major part of signal processing and the common transforms include Fourier, Laplace, Hilbert, Z-, Hartley and wavelets, etc. Loh and Lee (1998) adopt the empirical mode decomposition method and the Hilbert transform of the non-stationary data obtained from a vehicle-induced vibration test and successfully identified the vehicle-bridge interaction. The detail theory of Hilbert transform can be found in the reference material (Hahn, 1996).

Figure 2.1: Definition of half-power bandwidth method.

Chapter 3

Dynamic Monitoring Of The Confederation Bridge

3.1 Introduction of the Confederation Bridge Project

3.1.1 Introduction

The Confederation Bridge at Cape Tormentine in eastern Canada, connecting Prince Edward Island with mainland New Brunswick, was opened to traffic in the summer of 1997. The total length of the bridge is 12.9 km, and it consists of 20 approach spans and 45 main spans of 250 m each at a typical height of 40.8 m above the mean sea level, as shown in Fig. 3.1. The main spans of the bridge were constructed by connecting 192 m double cantilever main girders of variable depth with two alternative types of 60 m drop-in spans. A drop-in span is rigidly connected to two double cantilever main girders to form a rigid frame. Two adjacent rigid frame units are connected by a simply supported drop-in span resulting in the bridge

superstructure having an alternating rigid frame and simply supported girder span of 250 m each.

The span superstructure of the bridge is a single-cell post-tensioned concrete box girder. The cross-section of the main span is a trapezoidal shape, as shown in Fig. 3.2, with a total deck width of 12 m. The depth of the single cell post-tensioned reinforced concrete box girder varies from 4.5 m at the mid-span to 14.5 m at the pier supports.

Each bridge pier is constructed as a 600 mm thick hollow shaft with a varying cross section changing from a rectangular section of 5 m by 10 m at the top of the shaft to an 8 m octagonal cross-sectional shape at the top of the conical ice shield in the middle height of the pier, and a conical shell with a ring-shaped footing at the bottom on the seabed, as shown in Fig. 3.3.

Detailed information about the design and configuration of the Confederation Bridge can be found in the reference (Tadros 1997).

The service life of the Confederation Bridge is designed to be 100 years under very severe environment. Therefore, the design criteria for this long span bridge are different from the common practices in bridge design, such as those covered in the Canadian Highway Bridge Design Code (CAN/CSA-S6-2000). For the design of the Confederation Bridge, the load and resistance factors must be derived from basic principles using probabilistic and reliability techniques. One of the design criteria of the Confederation Bridge is that the collapse of any single span must not lead to the collapse of other spans by a domino effect.

In order to ensure continuous high long-term performance of the bridge and to establish a database necessary for the condition evaluation of the bridge in the future, a comprehensive remote field monitoring and research program has been implemented to collect data on the short- and long-term deformations and material properties, thermal effects, ice loads, dynamics responses due to traffic loads, wind and earthquakes, and corrosion of the bridge. Details of the overall monitoring program have been described in the paper by Cheung et al (1997).

This paper focuses on the experimental modal analysis of the Confederation Bridge using the monitoring data of the dynamic responses induced by ambient and other transient load vibrations.

3.1.2 Dynamics Monitoring

The monitoring system has been designed to have high degree of reliability in system operation to obtain data critical for better understanding of the dynamic response behaviour of the Confederation Bridge due to wind, earthquake and traffic. In order to minimize the cost of monitoring and obtain data which can be correlated with the behaviour of the other areas in the monitoring program, the dynamic monitoring instruments are installed at different locations of the bridge between Piers 30 and 33, as shown in Fig. 3.2, in the same common instrumented sections of the bridge as the other areas.

Overall, the dynamic monitoring system consists of a network of 76 accelerometers, 6 dynamic tilt-meters, and 8 displacement transducers across the expansion joints of the instrumented spans to obtain information on the dynamic behavior and the spatial

effects in the dynamic responses of the bridge. The data from all areas are measured using a common time reference, so that the load effects of one area on the response of the bridge can be separated out from those of the others.

The dynamic responses of the main girder are measured at 13 sections as shown in Fig. 3.2. Three accelerometers are installed at each section to measure the torsional, lateral transverse and vertical transverse vibration responses of the main girder. The acceleration responses of the pier shaft in both the longitudinal and transverse directions are measured at the top and ice-shield levels of the pier shaft. The accelerometers V_N and V_S in Fig. 3.2 refer to V_1 and V_2 in the following plots. In addition, the rotation or tilt of the pier shaft at the top and ice shield levels of Piers 31 and 32 are also measured by dynamic tilt-meters. The relative sliding movements at the bearing supports of the instrumented drop-in spans are measured by dynamic displacement transducers.

For wind monitoring, the wind speed and direction at the instrumented span and at the navigation span are measured by anemometer units. Generally, the wind speed at the navigation span which is the highest point of the bridge is expected to be the highest at the bridge site.

3.1.3 Data Acquisition And Transmission

To process and store the data collected from the monitoring sensors, a complex data acquisition system consisting of a multi-unit central networked computer system and a number of data loggers has been specifically designed for the Confederation Bridge

monitoring project. The data acquisition system is installed in a data control room inside the main box girder above Pier 31.

The data logger system consists of low speed and high speed units, in which the high speed data loggers are for data collection of ice forces, traffic loads and dynamic responses, while the low speed loggers are for collection deformation and thermal stresses data.

The data loggers are programmed to operate in two modes for data monitoring and collection: time-averaged and event triggered burst mode. In time-averaged mode, the loggers store only the time-averaged statistical data (mean, variance, minimum and maximum) at fixed interval of time. When a significant event occurs, the burst mode is initiated to start collecting time-history data at a sampling rate of 133 Hz. About 30 seconds of pre-triggered data and 60 seconds of post-triggered data are also collected. The time history data are analyzed to determine the characteristics of the bridge responses during extreme events such as ice impact, earthquake, strong wind or heavy traffic.

The communication between the data loggers and the control computer in the data control room inside the bridge, and the transmission of the collected field monitoring data to receiving on-shore gateway computers located in the bridge operating building are handled by fiber-optic links. From the on-shore computer which serves as the on-site data management and communication node of the monitoring system, the monitoring data are transmitted to Ottawa via high speed data links for detailed analysis and research.

3.2 Modal Analysis of the Monitoring Data

3.2.1 Ambient Vibration Responses

The sampling rate used for taking measurements of the ambient vibration responses of the bridge affects the frequency resolution achieved in the frequency domain system identification analysis. Adopting a data sampling rate that is too low may lead to contamination of the sampled data in the frequency domain due to folding and aliasing problems of the discrete Fourier transform procedures. To eliminate the folding and aliasing problems in the discretization of the data, the sampling frequency must be chosen at least twice the frequency of the highest vibration modes involved in the overall responses. The sampling rates of 133.3 and 166.7 Hz are adopted in the collection of the monitoring data of the Confederation Bridge. In the data collection process, the duration of data collection for system identification or experimental modal analysis of the structural behaviour can affect the resolution and reliability of the frequency domain results, such as the natural vibration frequencies and mode shapes.

Figs. 3.4 to 3.10 show the typical acceleration time history responses triggered by wind and heavy traffic at different locations. The duration of 60 seconds for each set of demonstration data includes 30 seconds of pre-triggered data and 30 seconds of post-triggered data. The time history vibration responses of the bridge at locations 5,

8, 9, 10, 12, 13 and 14 are plotted in the vertical V1 and V2 directions and in the transverse T direction.

The acceleration time history plot of a typical wind triggered event shows a lower amplitude in all three directions than the response triggered by heavy traffic. Since the sample data is recorded in a late winter evening with less traffic than the normal daytime condition, the influence of the vehicle traffic on the dynamic response of the bridge may be considered as small. Comparing the characteristics of the acceleration response data recorded from wind triggered and traffic induced events, as shown in Figs 3.4 to 3.6 and Figs 3.7 to 3.10, respectively, it is obvious there is no traffic induced vibration event during the duration of the time history data shown of the wind event. The low frequency periodic characteristic in the acceleration time history plots from the wind events is clearly noticeable.

Figs. 3.7 to 3.10 display the time history responses triggered by heavy traffic recorded after midnight of September 12, 2000. The acceleration time history plots show that the amplitude of the response is generally higher than that of the wind induced response, especially for the response in the vertical directions V1 and V2. Generally, the behaviour has a single peak in the acceleration time history plot when the structure is excited by a single vehicle. According to the investigations by FFT spectral analysis of the monitoring data, the ambient excitation vibrations caused by wind and traffic excite the first 10 vibration modes of the bridge structure.

3.2.2 Modal Analysis in Frequency Domain

The modal analysis of the measured traffic and wind vibration data is carried out by both frequency and time domain methods. The coherence technique is adopted for the frequency domain analysis and a more advanced time domain method, the stochastic subspace method, is used to identify the dynamic properties of the bridge. The results of modal analysis by both methods are presented in this chapter.

When a structure is subjected to the excitation caused by the ambient vibration with white-noise characteristics, its response is strongest near the frequencies at which the structure resonates. This can be detected from the peaks in the PSD functions computed from the acceleration records. The coherence functions of the measured data are computed to determine the influences of noises and non-linear behaviour in the responses of the bridge on the extracted modes.

Under ambient condition, the local vibration modes which involve primarily the local distortion or deformation of the box girder cross section of the main girder of the bridge, can be significant in amplitude compared to that of the global modes. The structural frequencies tend to be obscured in the FFT amplitude spectrum. This can be improved by using the power spectral density (PSD) functions in the coherence method of frequency domain system identification analysis procedure (Ventura et al. 1995). Figs. 3.11 to 3.20 show the PSD of the wind triggered responses in the T, V1, V2 directions at different locations on the monitored spans. Similarly, the PSD functions of the time history responses triggered by heavy traffic are presented in Figs. 3.21 to 3.30.

Comparing the PSD functions determined from wind and traffic induced vibration responses, it is noted that wind load can excite the structural vibration modes in both the vertical and transverse directions, whereas the traffic load mainly excites the vertical modes. This is due to the difference in the load direction that the traffic load mainly acts in the vertical direction whereas the wind load is generally horizontal.

Another important characteristic shown in the PSD functions calculated from the traffic responses is that there are significant peaks in the frequency range of 10 to 14 Hz. According to analytical results of the vibration behaviour of the bridge, the frequency spread is only about 5 Hz for the first 40 vibration modes of the bridge. To determine the significance of the observed behaviour in the measured traffic induced vibration responses, the coherence between corresponding pairs of data at different locations is examined.

To reduce the effect of noise on the analysis of the data, the averages of the PSD functions are calculated from multiple records of data. Figs. 3.31 to 3.34 show the average PSD functions at location 5 and location 9 calculated from 10 sets of traffic response data recorded from February 8 to March 1, 2000. The cross-power spectra of the responses at location 5 and location 9 are also calculated and plotted in Figs. 3.35 and 3.36.

The coherence function is computed according to Equation (2.13) using the averaged auto-spectrum and averaged cross-spectrum. In the study here, a peak in the coherence function above the value 0.8 is considered to represent the correlating frequency of a vibration mode. The coherence between the corresponding vertical

data at the same locations (5V1 and 5V2, 9V1 and 9V2) are shown in Figs. 3.37 and 3.38. Because the responses are very similar at the same cross section, dominant peaks in the frequency range of 0 – 4 Hz and 10 – 14 Hz can be found in both of these two coherence plots. On the other hand, the coherence for the pair of sensors at different locations (5V1 and 9V1, 5V2 and 9V2) presented in Figs. 3.39 and 3.40 has very low coherence values in the frequency range 10 - 14 Hz when compared to the dominant peak values in Fig. 3.31 to 3.34 at the corresponding frequency range. The coherence results show that correlated vibration behavior of the main girder occurs mainly between 1-4 Hz, which indicates that the frequencies in the range of 10 – 14 Hz are the results of local vibration modes of the bridge cross section. The above observation is confirmed by analytical results obtained from finite element method for the cross section at the same locations.

3.2.3 Modal Analysis in Time Domain

From the results of the modal analysis by the PSD and coherence techniques, one of the disadvantages is low resolution in the Fourier transform and coherence functions, especially when the modes are closely spaced and coupled in the frequency range of interest. The stochastic subspace method in time domain analysis can solve this problem and obtain more information on the vibration modes which may be easily missed by the coherence approach.

The stochastic subspace method is applied to a single set of data for each calculation. The Hankel matrix representing the correlation relationship of the pairs of sequences has the characteristics on minimizing the influence of noise in the system

identification calculation if the noise in the two channels of data selected for correlation calculation are not correlated.

To reduce the interference of high frequency modes and the amount of calculation, a low-pass filter of 15 Hz is applied to eliminate the high frequency components in the data for the analysis here. At the current stage of research, the two vertical response records on each cross-section of sensor location are averaged to eliminate the torsional response components. Therefore a wire geometric model of the bridge is used to illustrate the mode shapes. The results are compared to the mode shapes obtained from finite element analysis using the equivalent beam model.

The results by the stochastic subspace method in the time domain was calculated with the assistance of the software CADA-X from LMS company, which can handle multiple channels of data at the same time and large amount of computing involved in the decomposition of the Hankel Matrix. The modal analysis results of the mode shapes are illustrated by a 3 dimension geometric model of the bridge.

A total of 22 vibration modes have been identified from different sets of ambient vibration (wind and heavy traffic) data in the frequency range of 0 – 5.33 Hz. Table 3.1 lists the experimentally identified modes, in which 15 are pure vertical bending modes, 5 are coupled transverse-vertical modes and 2 are pure transverse bending modes. The vertical modes of vibration of the bridge are excited significantly by random traffic and can be identified quite readily. This is a particular advantage because secondary effects associated with the vertical modes contributed significantly to the observed lateral motions. Heavy traffic on the bridge induced peak vertical

accelerations of the deck of up to 0.02 g. Table 3.1 shows that the dynamic response of the bridge is characterized by the presence of many closely spaced and coupled modes, dominated by vertical and coupled vertical-transverse modes below 6 Hz, which are also indicated in the results by the coherence technique.

From the list, it is noted the modes are closely spaced. The 22 vibration modes identified between 0.33 to 5.33 Hz are not evenly distributed in the period range that makes it difficult to identify by the system identification methods in the frequency domain. The closely spaced distribution of the spectral peaks in the frequency plot leads to clusters of peaks within a narrow frequency range, which presents difficulties in the system identification because of low spectral resolution. The stochastic subspace method in time domain analysis can significantly improve the results in this aspect.

It is more difficult to have accurate estimates of damping from response data induced by ambient vibration than to determine the modal frequencies and mode shapes. This is mainly because in frequency domain identification, the small-valued damping ratios are sensitive to the non-stationary nature of the ambient vibrations, the averaging time duration considered in the data analysis and the physical measurement of the bandwidth of the spectral peaks in the calculation by half-power bandwidth method. The damping ratios are comprised of both structural and aerodynamic damping. The former refers to energy dissipation due mainly to relative motion of structural members at interfaces and joints, and to material behaviour. The latter is

principally dependent upon wind velocity, dimensions of the structure and dynamic properties of the bridge. The discussion of damping will be presented in Chapter 4.

This chapter presents the modal analysis results from the field monitoring data of the response induced by ambient vibration including wind and heavy traffic. The modal analysis results of the controlled traffic test and the comparison with the theoretical analysis results which are obtained by Finite Element Analysis will be illustrated in Chapter 4.

Figure 3.1. General diagram of the Confederation Bridge.

Figure 3.2. Accelerometers installed between Piers 30 and 33 of the Confederation Bridge.

Figure 3.3. Cross-section of the bridge pier.

Figure 3.4. Measured time history responses triggered by wind recorded at location 5 in V1, V2, T directions and location 8 in V1 direction (Date: Feb. 15, 2001, 22:17:05).

Figure 3.5. Measured time history responses triggered by wind recorded at location 9 in V2, T directions and location 10 in T direction (Date: Feb. 15, 2001, 22:17:05).

Figure 3.6. Measured time history responses triggered by wind recorded at location 13 in V1, T directions and location 14 in V2, T directions (Date: Feb. 15, 2001, 22:17:05).

Figure 3.7. Measured time history responses triggered by heavy traffic recorded at location 5 in T, V1, V2 directions and location 8 in V1 directions (Date: Sept. 12, 2000, 01:22).

Figure 3.8. Measured time history responses triggered by heavy traffic recorded at location 8 in V2 direction; location 9 in V1, V2 directions and location 10 in T direction. (Date: Sept. 12, 2000, 01:22).

Figure 3.9. Measured time history responses triggered by heavy traffic recorded at location 10 in V1, V2 directions and location 12 in T, V1 directions (Date: Sept. 12, 2000 01:22).

Figure 3.10. Measured time history responses triggered by heavy traffic recorded at location 12 in V2 directions and location 14 in T, V1 directions (Date: Sept. 12, 2000, 01:22).

Figure 3.11. PSD of time history responses triggered by wind at location 5 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.12. PSD of time history responses triggered by wind at location 6 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.13. PSD of time history responses triggered by wind at location 7 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.14. PSD of time history responses triggered by wind at location 8 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.15. PSD of time history responses triggered by wind at location 9 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.16. PSD of time history responses triggered by wind at location 10 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.17. PSD of time history responses triggered by wind at location 11 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.18. PSD of time history responses triggered by wind at location 13 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.19. PSD of time history responses triggered by wind at location 14 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.20. PSD of time history responses triggered by wind at location 15 in T, V1, V2 directions. (Date: Feb. 15, 2001, 22:17:05).

Figure 3.21. PSD of time history responses triggered by traffic at location 5 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.22. PSD of time history responses triggered by traffic at location 6 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.23. PSD of time history responses triggered by traffic at location 7 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.24. PSD of time history responses triggered by traffic at location 8 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.25. PSD of time history responses triggered by traffic at location 9 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.26. PSD of time history responses triggered by traffic at location 10 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.27. PSD of time history responses triggered by traffic at location 11 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.28. PSD of time history responses triggered by traffic at location 13 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.29. PSD of time history responses triggered by traffic at location 14 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.30. PSD of time history responses triggered by traffic at location 15 in T, V1, V2 directions. (Date: Sept. 12, 2000, 01:21:38).

Figure 3.31. PSD of V_N direction at location 9

Figure 3.32. PSD of V_S direction at location 9

Figure 3.33. PSD of V_N direction at location 5.

Figure 3.34. PSD of V_S direction at location 5

Figure 3.35. Cross-power Spectrum at location 5 and location 9 (5V1 & 9V1)

Figure 3.36. Cross-power Spectrum at location 5 and location 9 (5V2 & 9V2)

Figure 3.37 Coherence for 5V1 & 5V2

Figure 3.38. Coherence for 9V1 & 9V2

Figure 3.39. Coherence for 5V1 & 9V1

Figure 3.40. Coherence for 5V2 & 9V2

Mode	Frequency (Hz)	Damping (%)	Direction	Response Data
1	0.33	3.85	Transverse	Wind
2	0.47	3.92	Transverse	Wind
3	0.61	6.97	Vertical	<i>Traffic</i>
4	0.65	1.81	Vertical	Wind, <i>Traffic</i>
5	0.79	8.15	Vertical	Wind, <i>Traffic</i>
6	0.85	1.66	T/V	Wind, <i>Traffic</i>
7	0.96	1.88	T/V	Wind
8	1.23	6.13	T/V	Wind
9	1.34	2.33	T/V	Wind
10	1.54	1.35	T/V	Wind, <i>Traffic</i>
11	1.81	1.01	Vertical	Wind, <i>Traffic</i>
12	2.77	1.47	Vertical	Wind, <i>Traffic</i>
13	2.84	2.43	Vertical	Wind, <i>Traffic</i>
14	3.00	2.22	Vertical	<i>Traffic</i>
15	3.21	1.24	Vertical	<i>Traffic</i>
16	3.42	1.18	Vertical	<i>Traffic</i>
17	3.88	1.32	Vertical	Wind
18	4.50	1.21	Vertical	Wind, <i>Traffic</i>
19	4.65	1.82	Vertical	Wind
20	4.95	1.62	Vertical	<i>Traffic</i>
21	5.12	2.27	Vertical	Wind
22	5.33	0.44	Vertical	Wind

Table 3.1. Experimentally identified modes induced by ambient vibration including wind and heavy traffic.

Chapter 4

Truck traffic Tests And Modal Analysis Results

4.1 Introduction of the Traffic Test

The monitoring data obtained from the Confederation Bridge include dynamic responses under ambient conditions and other transient load excitations caused by traffic, wind, sea current, earthquake, and ice impact in the winter. Due to the complexity of the natural environment, there are many uncertainties in the effects and in the interactions of these different excitations on the structural responses. In order to capture more accurately the dynamic modal properties of the bridge and also to have a better understanding of the interactions of these different dynamic effects in the interpretation of the monitoring data, controlled truck traffic vibration tests were conducted in December 2000 on the Confederation Bridge. The traffic tests were conducted with support and technical assistance from Public Works and Government

Services Canada, and the staff at the bridge site from the Strait Crossing Bridge Ltd. The test was conducted from 09:54 PM of December 13 to 04:37 AM of December 14 on the Confederation Bridge. According to the test plan, there were 17 different cases including static and dynamic loading tests. Table 4.1 lists the detailed information on all the 17 cases. Figs 4.1 to 4.6 present the typical diagrams for the test cases, where NB demotes the New Brunswick direction and PEI for the Prince Edward Island direction.

Cases 1, 2 and 9 are static loading tests of which one or two trucks are placed on the specified locations on the instrumented spans to measure the response of the bridge to the static load by the strain gages. Cases 1 to 8 and 10 to 17 are different moving load combinations of one or two trucks passing from Span 28 to Span 35 and in the opposite direction at 3 different constant speeds (40 km/h, 60 km/h and 80 km/h). The constant speed is maintained over the test spans after acceleration over the distance of four spans. The duration of data collection for each test is 120 seconds which is longer than the normal triggered data collection time in order to obtain more detailed information of the responses than normal operation of the long-term monitoring system of the bridge.

After static and dynamic loading tests, two 5-minute ambient responses of the bridge were recorded when there was no traffic on the bridge. The wind data at the time of the test were also recorded. The measured results represent the rare occasion of silent responses of the bridge without the interference of traffic under calm wind condition.

In order to obtain more detailed information on the local vibration responses of the bridge, additional accelerometers were installed at location 15. The cross-sectional diagram of the installation locations of the additional accelerations at location 15 is shown in Fig. 4.7. The results of the modal analysis of the local vibration responses based on the data obtained at this location are illustrated in a later section of this chapter. A total of 10 strain gages were installed at locations 12 and 15 for the static load test.

4.2 Analysis of Traffic Test Monitoring Data

4.2.1 Time History Response Data

The typical acceleration time histories of duration 120 seconds at locations 5, 6, 7, 8, 10, 12, 14 and 17 in the vertical direction (V_N) are shown in Figs. 4.8 to 4.10. The vibration response data of Case 7 in the controlled traffic test are selected as an example to illustrate the vibration responses caused by a single truck passing from Pier 28 to Pier 35 at a constant speed of 80 km/h with no other traffic on the bridge during this test.

It is about from the 77th to the 103rd second that the truck passed through the 3 spans (Pier 30-33) where the accelerometers are installed. The time history plot shows that the acceleration at the simply supported drop-in span is larger than the other part of the structure. Because the vibration is induced by heavy traffic load, the time history response in the vertical direction is generally bigger than that in the transverse or horizontal direction. The peak vertical acceleration of the deck can reach up to 0.02g.

The wind information during the traffic load tests was also collected. The wind data are shown in Fig. 4.11. The X and Y components of the wind data represent the longitudinal and transversal directions of the wind, respectively. It is noted that the low X components of a maximum of 25 km/h, which is significantly lower than the trigger level set for extreme wind event, indicates that the wind perpendicular to the bridge was not strong during the test. Therefore, this set of data can be regarded as response induced from pure traffic excitation.

After the closed traffic test, the “pure” ambient vibration responses of the bridge under no traffic and calm wind condition were also recorded. This set of response data is very valuable for comparison with the modal analysis results under normal operation environment presented in Chapter 3. The structural response time histories at locations 5, 8, 9, 10, 13 and 14 in the vertical and transverse directions are shown in Figs. 4.12 to 4.14. It can be seen from these plots that the response is very stable and the vibration amplitude is significantly smaller than that of the responses triggered by heavy traffic and wind. The wind information during the recording period of the ambient vibration responses is shown in Fig. 4.15. It is noted that the perpendicular component of the wind to the bridge was not significant during the test, due to that the amplitude of the wind component speed is lower than 25 km/h.

4.2.2 Modal Analysis of The Monitoring Data

The field monitoring data of the controlled truck traffic tests are also analyzed using the time domain sub-space method. The results are presented in Table 4.2. Comparing with the results of the ambient vibration with no traffic interference, many

more vibration modes are excited during the controlled traffic tests of different combinations of dynamic loads. The frequency distribution of the extracted 20 modes is also similar to that from the modal analysis of other dynamic response data obtained under other situations.

Using the time domain sub-space method, modal analysis of the test response data is carried out for the test case of ambient vibration condition of no traffic on the bridge. The modes identified from the field data are listed in Table 4.3. The results show that only 10 modes are reliably extracted. Compared to the vibration responses excited by truck traffic, fewer modes are excited under the ambient environment when there is no traffic and under calm wind condition. The frequency distribution of the modes is similar to the results obtained from modal analysis of the wind and random traffic excited responses discussed in Chapter 3.

Combining the modal analysis results in the range of 0 to 6 Hz from different environmental and loading conditions, including vibration induced by strong wind, random heavy traffic, controlled truck traffic vibration tests and “pure” ambient vibration tests, a total of 28 experimentally vibration modes are identified, as listed in Table 4.4. Among these modes, 2 of them are pure transverse bending modes, 6 are coupled transverse-vertical modes and 20 are vertical bending modes. It can be seen that the bridge has closely spaced vertical modes between 0 to 6 Hz. It is recalled that there are local vibration modes between 10 to 15 Hz identified by the coherence technique results.

Table 4.2 also shows the damping ratios for the modes extracted from the monitoring data by the sub-space time domain method. The vibration modes damping ratios are plotted against the vibration frequencies in Fig. 4.16. The general trend as shown in the plot is that the damping ratio decreases with the increase of modal frequency. Because it is difficult to obtain accurate estimates of the damping property from ambient vibration data due to the sensitivity of the damping ratio to the ambient vibration level and the effect of noise in the data acquisition and data processing, the damping ratios for the lower frequency modes are not as reliable as those of higher frequency modes. The estimates of the damping for the structural modes below 2 Hz can be as high as 6 to 8% (Table 4.2), which is similar to the damping values calculated by the half-power bandwidth method.

4.3 Comparison with Analytical Results

The extracted mode shapes by the sub-space methods are determined and plotted with the assistance of the CADA-X software. Figs. A.01 to A.28 illustrate the 28 mode shapes in plan, elevation and 3-Dimension view plans. As a comparison, the mode shapes of the first 50 modes (0 – 6.2 Hz) obtained from analytical models (Finite Element Method) are also presented in Figs. A.29 to A.78. Figs. A.79 to A.83 display the mode shapes for the local vibration at location 15, and the analytical mode shapes for local vibration are shown in Figs. A.84 to A.86, respectively.

To reduce the amount of computation required in the processing of large amount of monitoring data, the modal analysis is conducted using an equivalent beam or wire

model of the bridge where the response data obtained from the two top corners of the bridge cross-section are averaged to give the average vertical response data. The mode shapes of the modal analysis results by the wire model are illustrated from Fig. A.01 to Fig. A.28. The equivalent beam model works well for the pure vertical modes when the transverse and torsional components in the responses are small. One typical mode shape of the wire model is shown in Fig. A.87.

A difficulty in the analysis of the higher vibration modes is that the number of sensors installed along the bridge span is still relatively limited and not sufficient in number to provide detailed information to construct the complicated mode shapes of the higher modes.

Comparing the experimentally identified mode shapes with those obtained from the finite element analysis results, some well correlated modes can be found. The typical well correlated modes include Mode 1 (0.33 Hz) vs. FEM Mode 2 (0.3 Hz); Mode 3 (0.61 Hz) vs. FEM Mode 10 (0.63 Hz); Mode 4 (0.65 Hz) vs. FEM Mode 9 (0.59 Hz); Mode 14 (2.77 Hz) vs. FEM Mode 25 (2.4 Hz). Generally, the lower modes correlate better than the higher modes and with less frequency shift. Besides the above mentioned limited number of sensors, the modeling assumption in the FEM model is also a contributing factor to the discrepancies between the set of results in modal properties.

The mode shapes of the local vibration responses at location 15 as extracted from the controlled traffic tests are well correlated with the results from the finite element method, in which Modes 3, 4, 5 correspond to Modes 1, 2, 3 from the FEM modal

respectively. This further proves that these modes are local modes instead of structural modes which has been suggested by the coherence method.

It can be found that there is not an exact correlation of all the modes predicted by the finite element model with those found from the monitoring data. This is about the same with the studies of other bridges where modes appear in the experimental data but do not appear in the analytic analysis, or vice versa. The encouraging side of this is that there are still lots of agreements and it is our task to modify the analytical procedures and models to reduce the disagreements with modal analysis results from monitoring data.

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Date (Dec, 2000)	13	13	13	13	13	13	13	13	14	14	14	14	14	14	14	14	14
No. of Truck	1	1	1	1	1	1	1	1	2*	2*	2*	2*	2*	2**	2**	2**	2**
Truck Speed (km/h)	S	S	40	40	60	60	80	80	S	40	40	60	60	40	40	60	60
Direction	NB	PEI	NB	PEI	NB	PEI	NB	PEI	NB	NB	PEI	NB	PEI	NB	PEI	NB	PEI

Note: 1. NB – New Brunswick direction; PEI – Prince Edward Island direction
 2. * - two trucks moving side by side; ** - two trucks moving in one line and one following another.
 3. S – static loading test.

Table 4.1 Controlled Truck Traffic Vibration Tests Case Specification

Mode No.	Frequency (Hz)	Damping (%)	Direction
1	0.33	3.85	Transverse
2	0.47	3.92	Transverse
3	0.65	1.81	Vertical
4	0.96	1.88	T/V
5	1.47	4.08	T/V
6	1.54	1.35	T/V
7	1.68	4.15	Vertical
8	1.81	1.01	Vertical
9	2.77	1.47	Vertical
10	2.84	2.43	Vertical
11	3.00	2.22	Vertical
12	3.05	1.32	Vertical
13	3.21	1.24	Vertical
14	3.42	1.18	Vertical
15	3.68	1.8	Vertical
16	3.88	1.32	Vertical
17	4.50	1.21	Vertical
18	4.95	1.62	Vertical
19	5.12	2.27	Vertical
20	5.33	0.44	Vertical

Table 4.2 Modal Analysis Result For Controlled Traffic Test Monitoring Data

Mode No.	Frequency (Hz)	Damping (%)	Direction
1	0.47	3.92	Transverse
2	0.65	1.81	Vertical
3	0.96	1.88	T/V
4	1.34	2.33	T/V
5	1.68	4.15	Vertical
6	1.81	1.01	Vertical
7	2.84	2.43	Vertical
8	3.42	1.18	Vertical
9	3.97	2.45	Vertical
10	4.71	2.05	Vertical

Table 4.3 List of Mode Extracted From Ambient Vibration Response Without Traffic Interference

Mode No.	Frequency (Hz)	Damping (%)	Direction	Response Data
1	0.33	3.85	Transverse	Wind
2	0.47	3.92	Transverse	Wind
3	0.61	6.97	Vertical	Traffic
4	0.65	1.81	Vertical	Wind, Traffic
5	0.79	8.15	Vertical	Wind, Traffic
6	0.85	1.66	T/V	Wind, Traffic
7	0.96	1.88	T/V	Wind, Traffic
8	1.23	6.13	T/V	Wind, Traffic
9	1.34	2.33	T/V	Wind
10	1.47	4.08	T/V	Traffic
11	1.54	1.35	T/V	Wind, Traffic
12	1.68	4.15	Vertical	Traffic
13	1.81	1.01	Vertical	Wind, Traffic
14	2.77	1.47	Vertical	Wind, Traffic
15	2.84	2.43	Vertical	Wind, Traffic
16	3.00	2.22	Vertical	Wind, Traffic
17	3.05	1.32	Vertical	Traffic
18	3.21	1.24	Vertical	Traffic
19	3.42	1.18	Vertical	Traffic
20	3.68	1.80	Vertical	Traffic
21	3.88	1.32	Vertical	Traffic
22	3.97	2.45	Vertical	Wind, Traffic
23	4.50	1.21	Vertical	Wind, Traffic
24	4.65	1.82	Vertical	Wind, Traffic
25	4.71	2.05	Vertical	Wind, Traffic
26	4.95	1.62	Vertical	Wind, Traffic
27	5.12	2.27	Vertical	Wind, Traffic
28	5.33	0.44	Vertical	Wind, Traffic

Table 4.4 Modal Analysis Results For Ambient Vibration And Traffic Test Response

Figure 4.1 Traffic Test Case 1: Static Loading, 1 truck

Figure 4.2 Traffic Test Case 2: Static Loading, 1 truck

Figure 4.3 Traffic Test Case 9: Static Loading, 2 trucks

Figure 4.4 Traffic Test Case 3: Dynamic Loading, 1 truck

Figure 4.5 Traffic Test Case 11: Dynamic Loading, 2 trucks

Figure 4.6 Traffic Test Case 16: Dynamic Loading, 2 trucks

Figure 4.7 Traffic Test: Additional accelerometers at location 15

Figure 4.8 Measured time history responses in traffic test Case 7 recorded at location 5 in V1, V2, T directions and location 8 in V1 direction.

Figure 4.9 Measured time history responses in traffic test Case 7 recorded at location 9 in V2, T directions and location 10 in T direction.

Figure 4.10 Measured time history responses in traffic test Case 7 recorded at location 13 in V1, T directions and location 14 in V, T directions.

Figure 4.11 Wind Information for Traffic Test Case 7

Figure 4.12 Measured time history responses in ambient vibration test after the traffic test recorded at location 5 in V1, V2, T directions and location 8 in V1 direction.

Figure 4.13 Measured time history responses in ambient vibration test after the traffic test recorded at location 9 in V2, T directions, location 10 in T direction and location 13 in V1 direction.

Figure 4.14 Measured time history responses in ambient vibration test after the traffic test recorded at location 14 in V1, T directions.

Figure 4.15 Wind Information for Ambient Vibration Test

Figure 4.16 Relation of Damping Ratio and Frequency

Chapter 5

Summary and Conclusion

The modal analysis results from this study demonstrate that fundamental dynamic characteristics of large and long span bridges can be determined from ambient vibration monitoring data. This technique has important significance for structures such as the Confederation Bridge because of its relatively low cost for the monitoring and no interference of the normal operation of the structures.

There are many system identification methods at present, but many are not suitable for structural monitoring applications. More research and studies are necessary to develop more reliable modal analysis methods for system identification problems, especially for processing of structural monitoring data under different operation conditions. The study of the dynamic behaviour of the Confederation Bridge gives a valuable insight of this problem on the processing of field monitoring data to extract structural dynamic properties. Based on the analysis results, the findings can be summarized as follows.

1. Stochastic Subspace method has been shown an effective tool for identification of the dynamic properties of full-scale bridges using the structural response data under ambient conditions of wind and traffic induced vibrations. The analysis results show the complex dynamic behaviour of the Confederation Bridge.
2. There are some disadvantages of the frequency domain modal analysis methods in comparison to other system identification techniques. But the frequency domain methods are generally easier and simpler to implement comparing with time domain methods. Therefore, frequency domain methods, such as the coherence method, can be used as an important numerical tool for preliminary modal analysis of the structure to get an insight on the behaviour and complexity of the system identification problem.
3. The typical sources of dynamic loads of the structure in monitoring problem are truck traffic and strong wind. Compared with wind load, vehicle traffic loads usually excite more significantly the vertical modes. Consequently, the vertical vibration modes of the bridge are more easily and reliably identified from the traffic induced vibration response monitoring data.
4. With the realization results of the mode shapes of the bridge, it is suggested that the sensors can be moved to different locations along the bridge to better capture the details of the higher modes.
5. As expected, there is not an exact correlation but still very good agreement between the vibration modal properties and behaviour predicted by the finite

element model and the results from the monitoring data. The system identification results obtained from the monitoring data can be used to calibrate and improve the FEM model of the bridge.

6. The results on damping properties obtained in this study show that more research is needed to improve the accuracy and reliability of damping estimates from the field monitoring data. It is found in the present study that lower modes have relatively higher damping values than high modes. The validity and accuracy of these high damping properties need to be further investigated by additional analysis of the measured data.
7. Controlled truck traffic vibration tests can provide valuable information on the dynamic response characteristics of the structure. It is observed that the controlled truck traffic can excite more vibration modes than that can be extracted from typical ambient vibration conditions of normal traffic and typical wind conditions.
8. System identification results show that the measured field data have noise components from local vibration modes of the cross-section of the bridge, which tend to obscure the modal behavior of the bridge. Detailed noise reduction and data analysis of the field response data can lead to useful information on the modal properties of the structure.

The development of computer technology has significantly improved the capability of modal testing in the past four decades and increased the reliability of the system identification results. Furthermore, monitoring response data can now be used to

refine finite-element models. Accurate prediction models and system identification results from monitoring of the structure are essential for the development of the technologies for damage detection and performance prediction of the structure.

There are other different approaches to carry out structural performance monitoring and damage surveillance using technologies in the high frequency band, e.g. acoustic signals. Some of these approaches are developed mainly for specific kind of structures, such as pre-stressed and post-tensioned structures, where the dynamic response from a breakage of a pre-stressing wire is detected by acoustic sensors. After primary filtering process in which the spurious acoustic signals caused by ambient activities, such as traffic or impacts, are filtered out, the data can then be processed to determine the time, location and classification of the specific structural failure event.

Experimental, modal testing or system identification is a new area in structural engineering. There are still many uncertainties and knowledge gaps which need further research. With the development of computer capability, new data processing techniques and improvement of data acquisition hardware, such as super sensor, amplifier and even wireless technologies, modal testing can be applied to more complex problems in engineering practices, such as damage detection and structural control.

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Appendix:

**Mode Shape Figure From System
Identification and Finite Element
Method**

Figure A.01 Mode 1, 0.33 Hz.

Figure A.02 Mode 2, 0.47 Hz.

Figure A.03 Mode 3, 0.61 Hz.

Figure A.04 Mode 4, 0.65 Hz

Figure A.05 Mode 5, 0.79 Hz.

Figure A.06 Mode 6, 0.85 Hz.

Figure A.07 Mode 7, 0.96 Hz.

Figure A.08 Mode 8, 1.23 Hz.

Figure A.09 Mode 9, 1.34 Hz.

Figure A.10 Mode 10, 1.48 Hz.

Figure A.11 Mode 11, 1.54 Hz.

Figure A.12 Mode 12, 1.68 Hz.

Figure A.13 Mode 13, 1.81 Hz.

Figure A.14 Mode 14, 2.77 Hz.

Figure A.15 Mode 15, 2.84 Hz.

Figure A.16 Mode 16, 3.00 Hz.

Figure A.17 Mode 17, 3.05 Hz

Figure A.18 Mode 18, 3.21 Hz.

Figure A.19 Mode 19, 3.42 Hz.

Figure A.20 Mode 20, 3.68 Hz.

Figure A.21 Mode 21, 3.88 Hz.

Figure A.22 Mode 22, 3.97 Hz.

Figure A.23 Mode 23, 4.50 Hz.

Figure A.24 Mode 24, 4.65 Hz.

Figure A.25 Mode 25, 4.71 Hz.

Figure A.26 Mode 26, 4.95 Hz.

Figure A.27 Mode 27, 5.12 Hz.

Figure A.28 Mode 28, 5.33 Hz.

Figure A.29 FEM result: Mode 1, 0.29 Hz

Figure A.30 FEM result: Mode 2, 0.3 Hz

Figure A.31 FEM result: Mode 3, 0.34 Hz

Figure A.32 FEM result: Mode 4, 0.37 Hz

Figure A.33 FEM result: Mode 5, 0.42 Hz

Figure A.34 FEM result: Mode 6, 0.45 Hz

Figure A.35 FEM result: Mode 7, 0.46 Hz

Figure A.36 FEM result: Mode 8. 0.49 Hz

Figure A.37 FEM result: Mode 9, 0.59 Hz

Figure A.38 FEM result: Mode 10, 0.63 Hz

Figure A.39 FEM result: Mode 11, 0.74 Hz

Figure A.40 FEM result: Mode 12, 0.81 Hz

Figure A.41 FEM result: Mode 13, 0.81 Hz

Figure A.42 FEM result: Mode 14, 0.84 Hz

Figure A.43 FEM result: Mode 15, 0.96 Hz

Figure A.44 FEM result: Mode 16, 1.1 Hz

Figure A.45 FEM result: Mode 17, 1.1 Hz

Figure A.46 FEM result: Mode 18, 1.2 Hz

Figure A.47 FEM result: Mode 19, 1.5 Hz

Figure A.48 FEM result: Mode 20, 1.5 Hz

Figure A.49 FEM result: Mode 21, 1.5 Hz

Figure A.50 FEM result: Mode 22, 1.6 Hz

Figure A.51 FEM result: Mode 23, 2.2 Hz

Figure A.52 FEM result: Mode 24, 2.3 Hz

Figure A.53 FEM result: Mode 25, 2.4 Hz

Figure A.54 FEM result: Mode 26, 2.5 Hz

Figure A.55 FEM result: Mode 27, 2.8 Hz

Figure A.56 FEM result: Mode 28, 2.8 Hz

Figure A.57 FEM result: Mode 29, 3 Hz

Figure A.58 FEM result: Mode 30, 3 Hz

Figure A.59 FEM result: Mode 31, 3.1 Hz

Figure A.60 FEM result: Mode 32, 3.2 Hz

Figure A.61 FEM result: Mode 33, 3.3 Hz

Figure A.62 FEM result: Mode 34, 3.3 Hz

Figure A.63 FEM result: Mode 35, 3.5 Hz

Figure A.64 FEM result: Mode 36, 3.6 Hz

Figure A.65 FEM result: Mode 37, 3.9 Hz

Figure A.66 FEM result: Mode 38, 4 Hz

Figure A.67 FEM result: Mode 39, 4.3 Hz

Figure A.68 FEM result: Mode 40, 4.4 Hz

Figure A.69 FEM result: Mode 41, 4.5 Hz

Figure A.70 FEM result: Mode 42, 4.5 Hz

Figure A.71 FEM result: Mode 43, 4.5 Hz

Figure A.72 FEM result: Mode 44, 5.1 Hz

Figure A.73 FEM result: Mode 45, 5.2 Hz

Figure A.74 FEM result: Mode 46, 5.5 Hz

Figure A.75 FEM result: Mode 47, 5.5 Hz

Figure A.76 FEM result: Mode 48, 5.6 Hz

Figure A.77 FEM result: Mode 49, 5.8 Hz

Figure A.78 FEM result: Mode 50, 6.2 Hz

Figure A.79 Mode 1 for local vibration

Figure A.80 Mode 2 for local vibration

Figure A.81 Mode 3 for local vibration

Figure A.82 Mode 4 for local vibration

Figure A.83 Mode 5 for local vibration

Figure A.84 Mode 1 for local vibration by FEM

Figure A.85 Mode 2 for local vibration by FEM

Figure A.86 Mode 3 for local vibration by FEM

Figure A.87 Modal Analysis Result (Mode 3, 0.61 Hz) Illustrated In Square Cross-section Model