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Effect of Second-Phase Particle Clustering on Aluminum-Silicon Alloy Sheet Formability

by

A. Keith Pilkey

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Doctor of Philosophy
in Mechanical Engineering

**Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute
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July 15, 1997**

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and Research acceptance of the thesis

**Effect of Second-Phase Particle Clustering on
Aluminum-Silicon Alloy Sheet Formability**

submitted by A. Keith Pilkey
in partial fulfilment of the requirements for
the degree of Doctor of Philosophy
in Mechanical Engineering

A handwritten signature in black ink, appearing to read 'M.J. Worswick', followed by a long horizontal line that ends in a sharp upward-pointing hook.

M.J. Worswick, Thesis Supervisor

A handwritten signature in black ink, appearing to read 'Robert Bell', followed by a long horizontal line that ends in a sharp upward-pointing hook.

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
July 15, 1997

Abstract

The effect of second-phase particle clustering on the formability of four model aluminum-silicon (Al-Si) alloy sheets is evaluated using quantitative stereology and mechanical testing methods. Results indicate that the size, aspect ratio, orientation, and spatial distribution of first-order clusters in the Al-Si alloys under study have a significant impact on forming limits through their intrinsic promotion of void nucleation and growth.

An image analysis software package, developed during the course of this work, is employed to acquire and process large-scale high-resolution digital images of the Al-Si microstructures. Explicit measures of second-phase particle clustering at several orders of magnitude are extracted from digitized particle fields with the aid of spatial tessellation and dilational counting techniques. Distinct second-phase particle clustering characteristics are evident in each Al-Si alloy sheet, pertaining to the overall degree of clustering as well as the preferential orientation for high aspect ratio first-order clusters.

Strain localization behaviour in the Al-Si alloys is investigated on the basis of stress-strain data and forming limit diagrams (FLD's), obtained from uniaxial tensile and hemispherical punch tests, respectively. A general divergence in forming limits with increasing tensile minor strains is observed, indicating the predominant role of void damage in the onset of strain localization. Void damage rates are known to increase directly with stress triaxiality. The FLD data also reveals varying levels of forming limit anisotropy between the four Al-Si alloy sheets, reflecting the noted trends for preferential orientation of first-order clusters.

Void damage within undeformed and deformed Al-Si microstructures is detected and associated with adjacent second-phase features to facilitate correlational studies. These analyses confirm that voids are primarily concentrated within first-order clusters in the Al-Si microstructures.

The impact of first-order cluster orientation on forming limit anisotropy is validated through numerical simulation. A simulation of voids evolving within first-order cluster fields of the Al-Si alloy sheets produces anisotropic in-plane behaviour that is consistent with experimental trends.

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To Rachel,
whose love, support, and understanding, constitute my lifeblood;

and to my parents,
whose countless sacrifices are embodied in this achievement.

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List of Symbols

σ	true (or Cauchy) stress
$\bar{\sigma}$	effective stress
σ_y	yield stress
$\bar{\sigma}_M$	local flow stress in the matrix
ε	true (or logarithmic) strain
$\bar{\varepsilon}$	effective strain
$\varepsilon^e, \varepsilon^p$	elastic and plastic strain
$\bar{\varepsilon}_M^p$	local effective plastic strain in the matrix
$\sigma_{ij}, \varepsilon_{ij}$	stress and strain in tensor notation (i.e. summation applies over repeated indices)
e	engineering strain
E, G	Young's (elastic) and shear moduli
m	strain rate sensitivity coefficient
n	strain hardening coefficient
ρ	strain path
r	normal anisotropy parameter

r_0, r_{45}, r_{90} normal anisotropy parameters at angles of 0°, 45°, and 90° with respect to the sheet rolling direction

Biaxial Stretching Model

Φ	Gurson-Tvergaard yield function
q_1, q_2, q_3	Tvergaard coefficients
S	net stress
Ψ	damage effect parameter
a_1, a_2	void axis dimensions
$a_1^{\infty}, a_2^{\infty}$	global void axis dimensions
f	porosity (or void volume fraction)
κ	void aspect ratio
κ^{∞}	global void aspect ratio
K_{ϑ}	global porosity scaling factor
L_1, L_2	sheet dimensions
n_{ϑ}	number of voids in the sheet
p_0, p_f	initial and final void nucleation probabilities
ϑ	void volume
V_{foc}	first-order cluster volume

Chapter 1

Introduction

The majority of research into aluminum alloy sheet formability is currently motivated by increasing requirements for automobile fuel efficiency. It is recognized that higher fuel efficiencies are readily achieved through the advent of lighter vehicle components, with aluminum alloy sheet providing the potential for significant weight saving over traditional steels used in body panels and frames [1,2]. However, the formability of aluminum alloy sheet is generally inferior to that of steels [3,4], demanding improved formability before gaining widespread acceptance in the automotive industry. Such improvements are being pursued through a better understanding of failure mechanisms in aluminum alloys and their limiting effect on sheet formability.

An intrinsic microstructural feature of most aluminum alloys is the presence of a constituent second-phase particle population. The addition of second phases is vital for

achieving modest levels of strength and formability [5,6]. Hence, the mechanical properties of aluminum alloys are generally optimized through careful tailoring of several key second-phase particle parameters, including size, shape, composition, and spatial distribution [6,7]. Although these requisite second phases are generally beneficial, particle populations inevitably contain harmful subsets which initiate void damage during applied deformation. Dating back to the pioneering work of Tipper in 1948 [8], it is well recognized that the ductile fracture mechanism of void nucleation, growth, and coalescence can severely limit material ductility. In sheet, the deleterious influence of void damage often results in reduced formability due to premature strain localization. Although the removal of void-initiating particles provides a simple solution to this problem [9], such an approach is not considered feasible in commercial aluminum alloys. Given that common sources of detrimental particles are recycled products, an expected increase in the use of scrap aluminum dictates that future alloys tolerate higher numbers of undesirable inclusions to avoid excessive purification costs [2]. Therefore, further research is needed to establish the sensitivity of aluminum alloy sheet formability to the physical attributes and spatial distribution of second-phase particles.

Since the early findings of Tipper [8], a great deal of fundamental research into void nucleation, growth, and coalescence has been conducted, as evidenced by several extensive ductile fracture reviews [9-12]. However, the review of Thompson [12] notes that the bulk of contemporary void damage research is theoretical in approach. While a number of numerical and analytical models are proving effective for predicting strain distributions in deforming sheets, the onset of strain localization and failure is still reliant upon experimental forming limits. This numerical shortcoming suggests that more sophisticated experimental methods are necessary to fully understand the role of evolving void damage in

sheet formability predictions. Thompson [12] also points out that ductile fracture models commonly consist of single voids or periodic void arrangements, ignoring the stochastic nature of void damage events caused by clustering of second phases in real microstructures. Even though the spatial distribution of void damage is generally considered critical for localization and failure behaviour [6], few studies have thoroughly investigated this relationship, particularly in an experimental programme. As a result, it is imperative that an experimental study of void damage effects in aluminum alloy sheet formability explores the impact of second-phase particle clustering on the proliferation of voids.

The preceding text contends that the formability of aluminum alloy sheet is influenced by definitive features of constituent second-phase particle populations. This correspondence is derived from the pivotal impact of second phases on the ductile fracture mechanism of void nucleation, growth, and coalescence, which often leads to strain localization and failure in deforming sheet. As a result, it is proposed that appropriate research methods are needed to effectively measure and model the spatial variation of void damage and its dependence upon the physical attributes and spatial distribution of second-phase particles. The validity of these statements is substantiated by the following synopsis of documented research in three fundamental areas: sheet formability, ductile fracture, and particle clustering. Upon completion of this review, an outline of the current research programme is presented and rationalized.

1.1 Sheet Formability

In practice, sheet formability refers to the ability of a sheet to be stamped without discernable necking or fracture. Sheet formability research concerns the quantification of formability behaviour, through the generation of forming limit diagrams (FLD's), as well as the identification and influence of significant microstructural parameters.

1.1.1 Forming Limit Diagram (FLD)

A forming limit diagram (FLD) represents a locus of limiting in-plane strains measured from sheet specimens loaded along a wide range of strain paths. A typical FLD is depicted Figure 1.1. The concept of an FLD was first proposed by Keeler [13] for biaxial stretching, denoted by the right-hand side of the FLD where positive minor strains are realized. The FLD was later expanded by Goodwin [14] to include drawing conditions corresponding to negative minor strains. Consequently, the FLD is sometimes referred to as a Keeler-Goodwin diagram. A regular grid pattern, such as a periodic array of circles or squares, is applied to the sheet surface prior to deformation to allow for direct measurement of in-plane strains. Each strain measurement is subject to a rigid set of classification criteria, which vary somewhat amongst researchers, in order to differentiate between safe and necked grids. The resulting boundary that separates safe and necked regions on the FLD characterizes the formability of the tested sheet. Mechanical loading for the generation of an FLD is usually provided by a hemispherical punch press, while the diverse range of minor strains is achieved from appropriate variations in the width of the sheet blank. Namely, as the width to length ratio of sheet specimens drops from unity, the resulting

strain paths shift from biaxial stretching to plane strain and eventually to drawing, as noted in Figure 1.1. For equi-biaxial stretching, a hydraulic bulge test is often adopted since frictional forces between the hemispherical punch and deforming sheet prohibit the attainment of truly equi-biaxial conditions.

In general, sheet formability behaviour can be rationalized by studying the macroscopic phenomena which precipitate failure, namely the onset of diffuse and ultimately local necking conditions. It is widely believed that prior to the formation of a diffuse neck, deformation is governed by elemental plastic properties, with crystallographic texture being a dominant feature [15]. On the other hand, once diffuse necking has arisen, the subsequent local necking that leads directly to failure is believed to be primarily dependent upon microstructural inhomogeneities [15]. In the case of aluminum alloys, hard second phases and their associated voids comprise the bulk of potentially critical inhomogeneities. Given that forming limits are obtained from visible and/or palpable necks in a sheet specimen, it follows that FLD curves are actually associated with local necking criteria. This affinity is clearly indicated in Figure 1.2.

The emergence of diffuse and local necking conditions is effectively explained by the fundamental concepts of plasticity theory from Hill [16,17]. These principles are introduced here in the context of simple uniaxial tension, and then applied accordingly to biaxial sheet deformation. In a uniaxial tensile test, straining throughout the specimen gauge length remains homogeneous until a maximum principal load, F , is obtained, as indicated by:

$$dF = 0. \quad (1.1)$$

This critical condition of equation 1.1 essentially indicates the onset of diffuse necking, and is universally applicable as:

$$\frac{d\sigma_I}{d\varepsilon_I} = \sigma_I \quad (1.2)$$

where σ_I is a major principal stress and ε_I is a major principal strain. Equation 1.2 is commonly referred to as the Considère criterion. In regard to sheet forming, equation 1.2 can be re-written in terms of a critical strain. Applying the maximum load principle of equation 1.1, the major principal strain at the onset of diffuse necking, $(\varepsilon_I^*)_d$, is constant for all possible strain paths at:

$$(\varepsilon_I^*)_d = n \quad (1.3)$$

where n is the strain hardening coefficient. The formulation of equation 1.3 assumes that the material obeys the power-law hardening model of Hollomon [18], defined as:

$$\bar{\sigma} = K \bar{\varepsilon}^n \quad (1.4)$$

where $\bar{\sigma}$ is effective stress, $\bar{\varepsilon}$ is effective strain, and K is a constant. Swift [19] proposed an alternative locus of major principal diffuse necking strains, $(\varepsilon_I^*)_{Swift}$, that vary as a function of imposed constant strain path, ρ , given by:

$$(\varepsilon_I^*)_{Swift} = \frac{2n(1 + \rho + \rho^2)}{(\rho + 1)(2\rho^2 - \rho + 2)} \quad (1.5)$$

The constant strain path, ρ , in equation 1.5 is defined by the following ratio of major, ε_I , and minor, ε_2 , principal strains:

$$\rho = \frac{\varepsilon_2}{\varepsilon_I} \quad (1.6)$$

It is evident in Figure 1.2 that the difference between the diffuse necking criteria of Hill [16] and Swift [19] is relatively small, referring to equations 1.3 and 1.5, respectively.

Applying geometric considerations, Hill [17] deduced that the occurrence of local necking is indicated by a cessation of straining along the length of the potential neck, such that:

$$d\varepsilon_{2'} = 0 \quad (1.7)$$

where $\varepsilon_{2'}$ is the strain parallel to the neck. This behaviour is illustrated in Figure 1.3. For uniaxial tension, the local necking condition of equation 1.7 corresponds to:

$$\frac{d\sigma_1}{d\varepsilon_1} = \frac{\sigma_1}{2}. \quad (1.8)$$

For loading of a sheet along a constant strain path, ρ , the critical major principal strain for local necking, $(\varepsilon_1^*)_l$, becomes:

$$(\varepsilon_1^*)_l = \frac{n}{1+\rho}; \quad \rho \leq 0. \quad (1.9)$$

As noted in equation 1.9, the critical necking strain formulation is only applicable to the negative strain paths lying on the left-hand side of the FLD. This restriction ensues from the fact that equation 1.9 predicts the formation of a neck at an angle, θ , with respect to the principal axis, defined by:

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{-\rho}} \right) \quad (1.10)$$

Hence, positive strain paths are inadmissible in this context, meaning that it is impossible to obtain a direction of zero strain increment in a biaxially stretching sheet according to plasticity theory. In reality though, localized necking is prevalent under these conditions.

Therefore, a revised form of equation 1.7 is adopted to signal the onset of local necking in biaxial stretching, whereby the incremental strain path deviates towards a plane strain condition [20], such that:

$$\frac{d\varepsilon_2}{d\varepsilon_1} = 0. \quad (1.11)$$

The condition defined by equation 1.11 is commonly achieved in theoretical formulations by introducing a microstructural inhomogeneity into the sheet, modelled as a band of reduced thickness. This so-called defect band is oriented transversely to the direction of major principal loading, thus providing a physical framework for strain localization. Adoption of a reduced-thickness band to induce local necking under biaxial stretching conditions was first suggested by Marciniak and Kuczynski [21,22]. This strategy, dubbed a Marciniak-Kuczynski or M-K analysis, employs the model geometry depicted in Figure 1.4. An M-K instability analysis asserts that the most deleterious inhomogeneity governs strain localization.

Implementation of an M-K model requires that the magnitude of the initial defect, f_0 , be quantified according to:

$$f_0 = \frac{t^b}{t^a} \quad (1.12)$$

where t is a sheet thickness, and the superscripts b and a designate parameter values outside and inside the defect band, respectively. Equilibrium conditions must be satisfied between the band and the surrounding bulk, requiring that:

$$\sigma_1^a t^a = \sigma_1^b t^b \quad (1.13)$$

where σ_1 is a major principal stress, noting that the major principal direction in an M-K model is perpendicular to the length of the band. Strain compatibility must also be maintained, such that the minor principle strains, ε_2^a and ε_2^b , are given by:

$$\varepsilon_2^a = \varepsilon_2^b. \quad (1.14)$$

For a chosen set of boundary conditions, the forming limit is signalled by a large enough deviation in strain path, according to equation 1.11, or sufficient intensification of major principal strain within the band, such that:

$$\frac{d\varepsilon_1^a}{d\varepsilon_1^b} - 0 \quad (1.15)$$

Figure 1.5 displays a typical set of strain paths and forming limits predictions from an M-K analysis. While the inhomogeneity defect is physically represented by a band of reduced thickness in the model, it has been emphasized that the M-K approach is applicable to all possible forms of microstructural inhomogeneities [20,22,23]. Common sources include non-uniform spatial distributions of particles and their associated voids. Successful applications of M-K analyses have been reported by several researchers [20,23-25], although explicit correlation of real microstructural inhomogeneities to the initial defect parameter, f_0 , is often omitted. In one investigation, Jain *et al.* [26] used surface roughness measurements to calibrate a literal interpretation of the reduced-thickness inhomogeneity, adopting a formulation from Parmar *et al.* [27] to account for surface roughening as a function of accumulating strain. In another series of formulations, Jalinier and co-workers [28,29] studied particle damage with an M-K type analysis by establishing probability functions for particle cracking and decohesion in relation to volume fraction.

1.1.2 Microstructural Parameters

In the preceding review of general FLD behaviour, it was noted that a transition in microstructural behaviour coincides with the attainment of the Considère instability for diffuse necking, stipulated in equation 1.2. Specifically, deformation prior to the Considère criteria is believed to be dictated by crystallographic plasticity, while behaviour following this condition is dominated by microstructural inhomogeneities, such as second-phase particles [15]. Insight into these deformation regimes is provided by several measurable parameters, including normal anisotropy, strain hardening, and strain rate sensitivity. The following section outlines the relevance of these parameters, and a selection of secondary ones, in the context of aluminum alloy sheet formability.

Normal Anisotropy

In sheet formability, normal or through-thickness anisotropy provides an indication of a material's ability to resist thinning. Normal anisotropy, r , is measured as a ratio of strains in a sheet specimen subjected to uniaxial tensile testing [30], corresponding to:

$$r = \frac{\varepsilon_w}{\varepsilon_t} \quad (1.16)$$

where ε_w and ε_t are the principal strains in the respective width and thickness directions. The strains in equation 1.16 are measured prior to diffuse necking, typically at 15% elongation. Therefore, r values strongly reflect crystallographic texture [15], indicating that polycrystalline slip is isotropic when $r=1$. As a result, normal anisotropy is usually measured in three fundamental tensile tests, producing r_0 , r_{45} , and r_{90} , where the

subscripts indicate the angle between the principal tensile loading direction and the sheet rolling direction. These measures are often used to gauge the extent of in-plane textural effects on sheet forming behaviour.

By definition, normal anisotropy values can be incorporated into uniaxial necking predictions. Assuming that r remains constant, the diffuse necking condition in equation 1.2 can be rewritten for uniaxial tension to reflect the normal anisotropy, such that:

$$\frac{d\sigma_l}{d\varepsilon_l} = \frac{r+1}{2} \sqrt{\frac{3}{r^2+r+1}} \sigma_l \quad (1.17)$$

Similarly, the local necking condition of equation 1.8 is altered for uniaxial tension to produce:

$$\frac{d\sigma_l}{d\varepsilon_l} = \frac{\sigma_l}{r+1} \quad (1.18)$$

Although the parameter r is defined under uniaxial conditions, several researchers have postulated correlations between values of r and overall magnitudes of forming limit strains, reporting a vast range of relationships. Hecker [3] attributes the reduced formability of aluminum alloys in comparison to low-carbon steels to significant differences in their respective values of r . Typically, $r < 1$ in aluminum alloys, while in steels $1 < r < 2.5$. Graf and Hosford [25] contend that the influence of normal anisotropy on the drawing side of an FLD is minimal, while stretch forming limits are often reduced by very high levels of r . In sharp contrast with both of these conclusions, Barlat and co-workers [31-35] have argued that r is a poor indicator of right-hand side behaviour on an FLD. Alternatively, Barlat *et al.* demonstrate that stretch forming limits are a direct manifestation of the stress ratio between equi-biaxial and plane strain yielding.

The effect of normal anisotropy on in-plane sheet deformation was first modelled by Hill [36], who incorporated r into a quadratic anisotropic yield function. However, researchers have since reported that Hill's quadratic criterion is overly sensitive to the parameter r [25], and is unsuitable for materials with $r < 1$ [20], such as aluminum alloys. To address this deficiency, Hill derived subsequent non-quadratic criteria [37-39] which produce yield and forming limit predictions in closer agreement with experimental data [40]. Alternative non-quadratic yield criteria have been proposed by Barlat and co-workers [33,41,42] for modelling aluminum alloy sheet. These latter criteria from Barlat *et al.* have led to very promising numerical predictions [43,44], and hence, are certain to receive greater attention in future aluminum alloy sheet formability research.

Strain Hardening

Strain hardening is accomplished by effective pinning of mobile dislocations. As previously indicated by equation 1.4, strain hardening is often modelled by the power-law relationship, where a higher n corresponds to superior strain hardening ability. Given that critical strains for both diffuse and local necking are a direct function of n (see equations 1.3 and 1.9), it follows that an increased strain hardening capacity results in improved overall formability. On a microstructural level, an elevated strain hardening rate suppresses local necking by rendering localized regions more resistant than the surrounding bulk to further deformation. Consequently, a prolonged period of uniform strain can be sustained in a deforming sheet [45].

It is important to recognize that quoted values of n for aluminum alloys tend to be over-predictions because these materials do not strictly follow a power-law hardening rule [3,26,46,47]. Instead, the minimal hardening abilities of aluminum alloys are better reflected by the saturated flow stress concept of a Voce equation [48], defined as:

$$\bar{\sigma} = K_1 - (K_1 - K_2) \exp(-N\bar{\epsilon}) \quad (1.19)$$

where K_1 , K_2 , and N are material constants. Nonetheless, strain hardening is commonly quantified in terms of n throughout the research literature.

The strong impact of strain hardening behaviour on the occurrence of both the diffuse and local necking instabilities has been established by Ghosh [4], as illustrated schematically by the typical stress-strain response in Figure 1.6a. Based on experimental observations, Lloyd *et al.* [46] have verified this relationship, ascertaining that the inferior formability of aluminum alloys is a consequence of the relatively quick decay in hardening rate as the Considère criterion of equation 1.2 is approached. On the contrary for steels, a gradual decline in hardening rate is observed immediately prior to the diffuse instability. In aluminum alloys, the rapid drop in hardening rate is attributed to extensive sub-grain refinement caused by high rates of dynamic recovery. Lloyd *et al.* have reported that the addition of solute atoms, such as Mg [47] and Mn [46], partially alleviates this dramatic decrease in strain hardening rate by inhibiting the cell formation mechanism of dynamic recovery. By comparison, second-phase particles are generally far less effective for improving strain hardening ability, depending upon their size and coherency. For example, micron-sized particles are of little benefit in terms of strain hardening [49]. Even in the case of sub-micron particles, enhanced strain hardening rates are often only realized at small strains, particularly for incoherent particles [47].

Strain Rate Sensitivity

Similar to strain hardening, strain rate sensitivity can also be modelled with a power-law hardening relationship of the form:

$$\bar{\sigma} = K \dot{\bar{\epsilon}}^m \quad (1.20)$$

where K is a constant, $\bar{\sigma}$ is the effective stress, $\dot{\bar{\epsilon}}$ is the effective strain rate, and m is the strain rate sensitivity coefficient. Higher values of m indicate a higher resistance to the formation of local necks, generally leading to elevated forming limits.

The influence of strain rate sensitivity on post-Considère stress-strain behaviour is depicted schematically in Figure 1.6b, from Ghosh [4]. Ghosh established this effect by comparing strain distributions in failed sheet specimens of aluminum alloys and steels possessing similar values of n but differing m coefficients. A very gradual decay in strains as a function of increasing distance from failed necks was noted in steel specimens ($m > 0$), whereas a sharp drop in strains was observed in the aluminum alloys ($m < 0$), signifying severe localization in the latter case. Ghosh concluded the inferior formability of aluminum alloys, in comparison to steels, is directly related to their characteristically low strain rate sensitivity coefficients.

Negative strain rate sensitivities in aluminum alloys have been attributed to several microstructural features, a common one being the presence of free solutes of Mg. At low strain rates, dislocations are readily pinned by Mg atoms because the diffusion rate of Mg in aluminum exceeds that of the moving dislocations, even at room temperature. As strain rates increase, however, Mg atom diffusion rates are insufficient in comparison to the faster moving dislocations, causing a significant reduction in the effectiveness of this pinning

process [52]. Although this problem can be partially alleviated by raising the forming temperature to increase the diffusion rate of Mg [45], elevated temperatures are generally avoided with aluminum alloys since matrix strain hardening rates are dramatically compromised [5]. An additional disadvantage of free Mg solutes in aluminum alloy sheet formability is the promotion of dynamic strain aging, also known as the Portevin-Le Chatelier effect [53] or Luder banding. This mechanism entails the alternate pinning and releasing of dislocations by mobile Mg solutes as plastic straining proceeds, manifested by a fluctuating stress-strain response and a visibly uneven finish on the sheet surface.

Another microstructural event that has been linked to negative m coefficients is the shearing of coherent second-phase particles by moving dislocations [54-57]. During deformation, strain hardening is realized through the formation of dislocation loops around coherent particles. Because of the high particle/matrix interfacial strength of coherent phases, it is possible for dislocations to pass right through these particles. Hence, relatively brittle coherent particles are prone to shearing, an occurrence which relieves the back stresses generated by their surrounding dislocation loops. This stress relief process results in a local softening effect. Since the shearing of second-phase particles is found to occur more readily at higher strain rates, a negative strain rate sensitivity is produced.

Secondary Parameters

Although less prominent than the three aforementioned parameters, several other microstructural properties influence sheet forming limits. These factors include sheet thickness, grain size, and surface roughness, all of which demonstrate inter-related effects.

It is well recognized that an increase in sheet thickness alone can raise forming limits [58], but the added weight of a thicker gauge seldom represents an acceptable design solution. Numerical results by Mellor and associates [27, 59] suggest that the ratio between grain size and sheet thickness is a significant parameter, with low ratios promoting higher stretching limit strains. An appropriate explanation for this trend is that the rotation of relatively large grains during deformation causes considerable roughening of the sheet surface. In turn, experimental results confirm that evolving surface roughness plays a role in failure initiation [60] and that failure at free surfaces tends to be accelerated [61]. It should be noted that a certain degree of uniform surface roughness is considered beneficial for maintaining an even distribution of lubricant between the tooling and sheet during deformation [45]. Another reason for avoiding a large-grained microstructure is that fine grains inherently retard microband formation and linking [49,62], thus preventing premature through-thickness shearing of the sheet.

1.2 Ductile Fracture

Ductile failure in the current work encompasses the events leading up to and following the formation of an unstable neck, namely failure through plastic strain localization. In Section 1.1.1, it was established that the onset of necking coincides with the attainment of a diffuse plastic instability condition, as indicated by the Considère criterion of equation 1.2. It was also stated that diffuse plastic instability is a necessary but insufficient condition for final failure, requiring the subsequent localization of necking. In terms of microstructural phenomena, it is generally accepted that the deformation behaviour

prior to diffuse necking is dominated by crystallographic plasticity, while the localization of strain leading to fracture is governed by microstructural inhomogeneities [15].

According to Ashby *et al.* [63], the establishment and localization of a neck can be attributed to three distinct mechanisms of ductile failure: plastic failure, shear fracture, and ductile fracture. These mechanisms are illustrated in Figure 1.7. Plastic failure is attained by extensive plastic slip, in the absence of sufficient microstructural inhomogeneities, eventually reducing the neck to a chisel point. A failure caused by the formation of a macroscopic shear band across a developing neck is termed a shear fracture. In many cases, full extension of the shear band is facilitated by the development of planar sheets of microvoids in regions of intense plastic slip. The third failure category, ductile fracture, is the focus of the current research programme due to its intrinsic dependence on second-phase particles and its frequent occurrence in sheet forming failures [63,64]. The operation of this mechanism is characterized by the nucleation of microvoids at material discontinuities, such as particles and grain boundary triple points, followed by a period of stable void growth. As voids continue to grow and deformation begins to localize, void-void interactions eventually cause neighbouring voids to coalesce until a critically-sized flaw ultimately triggers final failure. It is worth noting that considerable strain localization is often sustained prior to the onset of global coalescence.

This section provides a brief overview of ductile fracture research, the subject of several comprehensive reviews [9-12]. The sub-sections deal with the component mechanisms of void nucleation, growth, and coalescence. Although these three stages of void evolution are addressed separately, it is important to clarify that they are not mutually exclusive events. For example, new voids continue to nucleate while existing voids grow.

Since the current research programme is focused on second-phase particle effects, only void damage evolving from hard particles is considered, with references comprising both experimental and theoretical findings. The theoretical analyses are based on a diverse variety of fundamental approaches, including dislocation theory, slip-line and velocity field analyses, energy criteria, stability theory, and continuum mechanics. Following this synopsis of component ductile fracture mechanisms, the formulation of a constitutive model which facilitates prediction of the macroscopic response of a porous solid throughout its entire deformation history is presented. This model, originally derived by Gurson [65,66] and subsequently refined by Tvergaard [67,68], is utilized extensively in ductile fracture research. It is recognized that ductile fracture mechanisms operating in real microstructures are inherently stochastic in nature, a direct consequence of inhomogeneous spatial distributions of second-phase particles. Specifically, second-phase particle clusters are known to have a strong influence on each of the component stages of void nucleation, growth and coalescence. Particle clustering research is of primary interest to the present study, and therefore, is left for separate discussion in a later section. As a final note, the current research programme deals with the proliferation of voids under quasi-static bulk deformation conditions, meaning that void damage associated with the advancement of a crack tip or with the propagation of a stress wave is considered beyond the scope of this thesis.

1.2.1 Void Nucleation

The predominance of heterogeneous void nucleation from hard phases in particle-containing alloys has been well established, originating from the early findings of Tipper

[8], Puttick [69,70] and Rogers [71]. Similarly, the significance of the void nucleation stage in the overall ductile fracture process is also widely recognized, stemming from the work by Edelson and Baldwin [72]. Their data, presented in Figure 1.8, implicitly demonstrates this fact by showing that ductility is inversely related to the volume fraction of particles, presumably a result of the increased number of nucleated voids. While these early discoveries were greeted with widespread acceptance, subsequent research into the various particle characteristics and local criteria which govern void nucleation is far less definitive.

Experimentation

Nucleation of voids around second-phase particles can occur by one of two means: particle fracture or particle/matrix decohesion. According to experimental observations of spheroidized steel, smaller particles tend to debond from the matrix while larger ones are more prone to fracture [73-75]. In equiaxed particles, this behaviour is believed to be caused by the presence of very low-level particle impurities. It has been suggested that the likelihood of an internal impurity flaw increases directly with particle size [73,74], while the probability of a surface flaw decreases with increasing particle size [76,77] (i.e. the surface area per unit volume is lower in a larger particle). Several studies have verified that trace impurities segregate to second-phase particles during typical heat treatment processes [76-79]. In spheroidized steel, however, the larger particles are predominantly MnS inclusions while the smaller ones are carbide precipitates, suggesting that the particle size effect is also caused by the lower particle/matrix interfacial strength of the less coherent MnS inclusions [74].

Regardless of the specific mode of void initiation, there is general agreement in the literature regarding the effect of particle size on void nucleation. The vast majority of experimenters have demonstrated that voids first form at larger particles as strain accumulates, based on behaviour exhibited in aluminum alloys [80-82] as well as in spheroidized steels [73,83-86]. Conversely, Goods and Brown [87] have argued the opposite size effect, citing data from a selection of studies in a Cu-SiO₂ system [88,89] and spheroidized steel [90,91]. They conclude that the experimental evidence in disagreement with their claim is confounded by the non-equiaxed shapes and internal flaws characteristic of most real particles. It is worth noting, however, that the authors of these last two studies [90,91] attribute their findings to nearest-neighbour spacing effects. Nonetheless, the vast majority of experimental data indicates that voids first nucleate from larger particles as strains accumulate.

In addition to particle size, particle shape is also important in promoting void formation. It has been found in both aluminum alloys [81,82] and spheroidized steels [73,75,84,93-95] that particles with high aspect ratios oriented in the tensile straining direction fracture at lower strains than equiaxed particles, due to a fibre-loading effect. It has also been shown that a high aspect ratio particle which is oriented in the direction of an applied tensile load can develop considerable normal interfacial stresses at its ends, causing voids to initiate by decohesion [95,96]. Because of such directional behaviour, Sailors [97] concluded that macroscopic fracture anisotropy is directly dependent upon particle anisotropy, demonstrating that the effective volume fraction in the fracture plane of interest is the determining factor in ductility response. Rolled products are particularly susceptible to such anisotropic fracture properties, as evidenced by Figure 1.9 from Pickering [98].

The early initiation of voids at large or high aspect ratio particles, followed by void nucleation events around smaller equiaxed particles at higher strains, has led many researchers to consider dual sets of particle populations [28,84,99]. Kwon and Asaro [99] designated these two distinct classes of particles as Type I and Type II. Type I particles are characterized by one of three low-strain void initiation events: decohesion of large particles; fracture of large elongated particles; and decohesion between closely spaced particles. The onset of void nucleation from Type I particles is assumed to coincide with yielding. For rolled alloy microstructures, such as sheet, voids are commonly present in the fully annealed state as a consequence of continual fragmentation of as-cast second-phase structures during fabrication [100,101]. For Type II particles, LeRoy *et al.* [84] ascertained that void nucleation commences at strains ranging from 0.47 to 0.62 for a series of spheroidized steels. Their results are drawn from inflection points in experimental data relating the number of voids and the area fraction of voids to applied strain, evident in Figure 1.10. LeRoy *et al.* point out that the dramatic increases in the number of voids (Figure 1.11a) and void volume fraction (Figure 1.11b) which signify the onset of Type II void initiation are further enhanced by a 5:1 ratio of Type II to Type I particles in their spheroidized steel microstructures. Extending this idea of two distinct void nucleation stages, some researchers have suggested that a continuous transformation in particle characteristics which favour void nucleation is more appropriate [6,94,102,103]. Embury and co-workers [6,102,103] have visualized this concept as a nucleation front moving through a distribution of prominent void-nucleating particle features, in conjunction with accumulating levels of applied strain. A schematic example, with void nucleation dictated by particle size, is illustrated in Figure 1.12.

In keeping with experimental evidence of dual particle populations, Broek [81] contends that the primary role of larger particles is to dictate the location of final failure by establishing local strain variations, while the influence of smaller particles becomes increasingly more predominant during completion of the ductile fracture mechanism. Experiments by Lloyd [104], using a SiC particulate-reinforced 6061 aluminum alloy composite, have revealed that the application of a stress-relieving heat treatment to a partially-strained tensile specimen effectively restores the entire fracture strain of the material. In fact, substantial strains to fracture are exhibited by specimens initially deformed to within 1.5% of the expected fracture strain. Given that the dislocation density in the matrix is greatly reduced by this prescribed heat treatment, while the state of void damage remains unaffected, these findings further support the above notion of Broek [81]. Voids initiated early in the fracture process appear to establish the location of failure, inherently governing the distribution of subsequent strain leading to localization. It is conceivable that the predominance of smaller particles emerges after localization, as suggested by Broek, when catastrophic void coalescence becomes imminent.

Theory

Theoretical calculations for the conditions required to nucleate a void at a hard second phase are based on local conditions of energy, stress and/or strain. The pioneering work in this area was published by Gurland and Plateau [105], who considered principles of energy, and by Ashby [106], who applied dislocation theory to establish critical levels of normal stress at the particle/matrix interface.

Gurand and Plateau [105] were the first to propose that void nucleation due to particle fracture or decohesion was driven simply by the following balance of free energy:

$$\Delta E_{el} + \Delta E_s \leq 0 \quad (1.21)$$

where ΔE_{el} represents the release of elastic strain energy from the particle and ΔE_s is the corresponding increment of free energy associated with the nucleated void surfaces. Utilizing the energy approach of equation 1.21, Gurand and Plateau determined that the critical stress for nucleating a particle-sized crack, σ_N , is given by:

$$\sigma_N = \left(\frac{\gamma E}{q^2 d} \right)^{1/2} \quad (1.22)$$

where γ is the surface energy of the crack, E is the Young's modulus of the particle, and q is a constant which combines the relative rigidity of particle and matrix with the appropriate stress concentration factor. Values for surface strain energy, γ , vary according to the location of the potential crack, with:

$$\gamma = \begin{cases} \gamma_m + \gamma_p - \gamma_{mp} & , \text{ fracture at interface} \\ 2\gamma_p & , \text{ fracture in particle} \end{cases} \quad (1.23)$$

where γ_m , γ_p , and γ_{mp} refer to the respective surface energies of the matrix, particle, and particle/matrix interface. Subsequent work by Brown and Stobbs [107] utilized the energy principle of equation 1.21 to obtain a critical strain criterion for void nucleation of:

$$\varepsilon_N = \frac{3\gamma}{\mu_p b} \quad (1.24)$$

where ε_N is the critical strain, γ is the free surface energy, b is the Burgers vector, and μ_p is the shear modulus of the particle. Goods and Brown [87] have since shown that equation 1.24 overestimates the nucleation strain since it assumes complete decohesion of

the particle from the matrix. Their analysis of the void nucleation process, depicted in Figure 1.13, accounts for decohesion of only the caps at the particle poles, effectively scaling equation 1.24 to produce:

$$\varepsilon_N = \frac{1 - \cos \theta}{4 \sin^2 \theta} \left(\frac{3\gamma}{\mu_p b} \right) \quad (1.25)$$

where θ is denoted in Figure 1.13. Slight modifications to equation 1.25 have recently been suggested by a number of researchers who have found that the location of nucleated voids is not precisely at the pole. Instead, voids form on particle surfaces at a slight angle from the principal axis, with predicted values ranging between 15° and 55° [86,108]. As well, it has been noted that separation usually occurs at only one end of the particle [109].

Although energy-based criteria evolving from the principle of equation 1.21 are necessary conditions for void nucleation, Brown and Stobbs [110] noted that a potential reduction of the free energy in the system is insufficient for promoting decohesion of the particle from the matrix. In fact, Tanaka *et al.* [111] demonstrated that the energy criterion for void formation is already attained at yielding for particles greater than $250 \text{ \AA}$ in diameter. Therefore, in addition to a favourable free energy transfer, the normal stresses between the particle and matrix must exceed a critical value, corresponding to the interfacial strength. By applying the elastic inclusion theory of Eshelby [112-114], Ashby [106] was the first to rationalize void nucleation behaviour as a direct consequence of excessive normal stresses being generated at the particle/matrix interface. His model is based on the concept that dislocations are punched out from a particle/matrix interface to act as a relief mechanism for the local stress concentration around a particle. Under an electron microscope, Ashby verified the existence of a special form of dislocations, called prismatic loops, at the poles of second-phase particles, where the poles are defined as the ends of a

particle experiencing the highest applied tensile stress. He speculated that prismatic loops continually form and glide away from particle poles to accommodate the elastic mismatch between particles and their surrounding matrix. However, as each prismatic loop is formed, a corresponding tensile stress, σ_T , is generated at the particle/matrix interface, given by:

$$\sigma_T = \frac{\alpha \varepsilon d}{l} \quad (1.26)$$

where α is a proportionality constant, ε is the strain, d is the particle size, and l is the distance between the particle and the loop. According to this model, a void forms when σ_T reaches a critical level, denoted by σ_N , which is specific to a given particle/matrix interface. Therefore, equation 1.26 can be rewritten in terms of a critical nucleation strain, ε_N , as:

$$\varepsilon_N = \frac{\sigma_N l}{\alpha d} \quad (1.27)$$

Of particular note, the inverse dependency of ε_N on particle size in equation 1.27 agrees with the experimental evidence that larger particles nucleate voids at lower strains. Subsequent work by Hirsch and Humphreys [115,116] concluded that larger particles nucleate voids first because they are unable to generate prismatic loops at sufficient rates. Argon *et al.* [117] have suggested that Ashby's model is applicable only for small particles where diffuse stress relaxation is possible due to the absence of dislocations in the immediate surrounding matrix. For larger particles, however, cavitation is independent of particle size but instead governed by diffuse slip and dislocation interactions between neighbouring particles. Alternatively, their reasoning for premature void nucleation around larger particles is based upon the simple fact that particles are not uniformly distributed as a function of volume fraction. Recognizing that a diffuse stress relief mechanism requires

space to operate, which is directly related to the particle size, a large particle is more likely to be encroached upon by an adjacent particle. In other words, for particle volume fraction to remain uniform, larger particles must be surrounded by larger particle-free zones, which is seldom the case. The link between particle size and accumulated strain is consistent with the theory alluded to earlier and depicted in Figure 1.12, suggesting that void nucleation is properly modelled by a theoretical front moving through a particle size distribution, from largest to smallest as plastic strain accumulates [94,102,103].

Employing a similar approach to that of Ashby [106], which relates the dislocation density associated with the build-up of interfacial stresses around a particle to the amount of accumulated strain, Brown and Stobbs [107] developed the following strain criterion for void nucleation:

$$\varepsilon_N = \frac{1}{30} \left(\frac{\sigma_N}{\alpha \mu_m} \right)^2 \frac{r}{b} \quad (1.28)$$

where σ_N is the critical local stress, r is the particle radius, μ_m is the shear modulus of the matrix, and α is a proportionality constant of 1/7. It is important to note that the direct dependency of void nucleation strain on particle size in equation 1.28 conflicts with the bulk of experimental evidence as well as with the dislocation model of Ashby [106].

Irrespective of the relationship between particle size and nucleation strain, Goods and Brown [87] also considered the relationship between local levels of stress and strain around a particle. From equation 1.28, it is evident that:

$$\varepsilon_N^{1/2} \propto \sigma_N. \quad (1.29)$$

The critical local stress for nucleation, σ_N , represents a combination of the intrinsic particle/matrix interfacial strength, σ_I , and the hydrostatic component of applied stress, σ_H , such that:

$$\sigma_N = \sigma_I - \sigma_H \quad (1.30)$$

Applying the principles of equations 1.29 and 1.30, Goods and Brown [87] produced the following expression:

$$\varepsilon_N^{1/2} = K(\sigma_I - \sigma_H) \quad (1.31)$$

where K is a constant based on particle volume fraction and particle size. The validity of equation 1.31 has been confirmed by experimentation [84,90,118].

1.2.2 Void Growth

In general, the stable growth of voids in a straining material, and the various parameters which influence this behaviour, is reasonably well understood for isolated voids. This state of agreement is supported by both experimental and theoretical findings. Nevertheless, the role of interactions between closely-spaced voids is readily recognized as being significant, but still requires additional study.

Experimentation

Experimental investigations have identified a number of key void growth variables, including strain hardening rate and stress state. Based on void data collected from a series

of maraging steels aged to various strengths, it has been shown that the rate of void expansion increases directly with yield strength [119,120]. This outcome is attributed to the combined effect of a lower strain-hardening rate and a higher level of applied stress in a steel of greater strength. Cox [93] has shown that the influence of stress state on void growth is specifically related to the level of stress triaxiality, which affects overall void growth rates as well as the evolution of void shapes. In his work, it was discovered that voids grow more quickly and maintain more equiaxed shapes in the highly triaxial region at the centre of a tensile specimen neck, as compared to the voids experiencing minimal triaxialities near the specimen surface. Figure 1.14 depicts a schematic representation of this relationship from Sun Jun [121]. An exponential dependence between growth rate and tensile stress triaxiality has been documented by many experimenters [122-127]. Accordingly, it has also been demonstrated that dilational void growth is stunted in tensile specimens by the superposition of hydrostatic pressure [102].

Theory

A substantial portion of void growth research has been conducted using theoretical models. While early models relied on analytical means to investigate isolated voids in infinite media, more recent initiatives have adopted numerical schemes, particularly finite element methods (FEM), to study periodic arrays of voids. One of the first analytical void growth models was developed by McClintock [128], who derived an approximate solution for the growth of an elliptical through-thickness hole in an infinite plate, depicted in Figure 1.15. McClintock's analysis predicts growth in terms of average void radius, R , as:

$$R = R_0 \exp \left\{ \frac{\bar{\epsilon}_M^p \sqrt{3}}{2(1-n)} \sinh \left[\frac{\sqrt{3} (1-n)}{2} \frac{\sigma_3 + \sigma_1}{\bar{\sigma}_M} \right] + \frac{\epsilon_3^p + \epsilon_1^p}{2} \right\} \quad (1.32)$$

where R_0 is the initial average void radius, $\bar{\epsilon}_M^p$ is the effective plastic strain in the matrix, $\bar{\sigma}_M$ is the matrix flow stress, n is the strain hardening coefficient, and σ_3 , σ_1 , ϵ_3^p and ϵ_1^p are remote (or macroscopic) components of in-plane stresses and plastic strains. The principal in-plane directions are denoted by x_3 and x_1 in Figure 1.15. For spheroidized steels, Sowerby and Chandrasekaran [129] have found that equation 1.32 provides close agreement with experimental data [84,102]. Kwon and Asaro [99] had similar success predicting Type II void behaviour (defined in Section 1.2.1) in a dual particle population, by appropriately modifying the plastic strains in equation 1.32 to account for nucleation strain. Their favourable results are presented in Figure 1.16. On the other hand, it has also been reported that McClintock's model is unsuitable for maraging steels [74,119], under-predicting void growth behaviour.

A subsequent analytical model by Rice and Tracey [130] considered the three-dimensional geometry in Figure 1.17. Their solution predicts the early growth behaviour of an initially spherical void of radius R^0 in an infinite block of rigid perfectly-plastic material. The rate of void growth in the principal directions, $\dot{R}_3$ and $\dot{R}_1$, are governed by:

$$\dot{R}_3 = [D\dot{\epsilon}_3^p + (1+E)\dot{\epsilon}_3^p]R^0 \quad (1.33a)$$

$$\dot{R}_1 = \left[D\dot{\epsilon}_1^p - \left(\frac{1+E}{2} \right) \dot{\epsilon}_1^p \right] R^0 \quad (1.33b)$$

where $\dot{\epsilon}_3^p$ and $\dot{\epsilon}_1^p$ are principal plastic strain rates. The two parameters, D and $1+E$, are referred to as dilational and extensional growth factors, respectively. These factors are size

and shape ratios which relate evolving void geometry to that of the surrounding solid block, and are defined as follows:

$$D = 0.842 \sinh \left(\frac{3\sigma_{hyd}}{2\bar{\sigma}_M} \right) + 0.008L \cosh \left(\frac{3\sigma_{hyd}}{2\bar{\sigma}_M} \right) \quad (1.34)$$

$$1 + E = \begin{cases} \frac{5}{3} & , \text{ low triaxialities} \\ 2 & , \text{ high triaxialities} \end{cases} \quad (1.35)$$

where σ_{hyd} is the remote hydrostatic stress, $\bar{\sigma}_M$ is the matrix flow stress, and L is the Lode parameter [131] dependent upon remote plastic strain rates. It is important to note that the multiplier of 0.842 in equation 1.34 differs from the 0.558 of the original Rice and Tracey paper, reflecting a calculation error discovered by Huang [132]. From equations 1.34 and 1.35, it is evident that increasing levels of triaxiality promote exponential increases in dilational void growth rate, while having minimal effect on the rate of extensional void growth. Based on the experimental findings of Cox [93], this direct but weak relationship between triaxiality and void shape development put forward by Rice and Tracey is inapplicable to entire deformation histories. Hence, LeRoy *et al.* [84] extended the early growth restriction of the Rice and Tracey model by modifying the extensional growth factor so that it decays as a function of plastic strain. This upgraded model produced reasonable agreement with evolving void population measurements collected from uniaxial tensile specimens of spheroidized steels.

While Rice and Tracey [130] limited their analysis to just the onset of void growth in a rigid perfectly-plastic solid, Budiansky *et al.* [133] proposed a similar three-dimensional model which estimated void growth over a much larger range of deformation and also accounted for matrix strain hardening. The strong impact of triaxiality on the

dilational response in the Budiansky *et al.* model parallels that of Rice and Tracey, while the extensional growth behaviour is largely contradictory. According to Budiansky *et al.*, an increased level of triaxiality promotes extensional growth in the direction perpendicular to the tensile axis. In other words, void shapes become oblate in highly triaxial stress states because the elevated transverse restraint effectively produces necking in the incompressible inter-void ligaments. Of further interest, the dilational growth response is unaffected by raised levels of initial porosity while the void oblation phenomenon is considerably enhanced. Such tendencies have been confirmed by Worswick and Pick [134], employing a three-dimensional FEM unit cell to represent a periodic array of spherical voids in a porous solid. Their relevant findings, in relation to those of Rice and Tracey [130] and Budiansky *et al.* [133], are included in Figures 1.18 and 1.19. In Figure 1.19, it is evident that the solutions of Budiansky *et al.* as well as Worswick and Pick capture the suppressant influence of strain hardening on dilational void growth, consistent with experimental observations noted earlier [119,120]. As well, Figure 1.20 from Worswick and Pick is included to illustrate that the three cited theoretical models under-predict actual values of dilational growth measured from real materials with $n \approx 0.2$ [122-126]. It should be noted that Huang's correction to Rice and Tracey's dilational growth factor in equation 1.34 is not accounted for in Figure 1.18. Therefore, the true curve lies closer to the experimental values, although it is still conservative given that $n=0$ in this model.

A serious limitation of the above theoretical models is that they only consider voids which are initially spherical, thus failing to examine the intrinsic growth of voids with inhomogeneous shapes and/or orientations, such as those commonly occurring in rolled sheet [100]. Lee and Mear [135,136] constructed a theoretical model to assess the effect of

void aspect ratio on yielding in a porous material, discovering that both void aspect ratio and void orientation relative to the tensile axis influence void expansion rate. Consequently, their model exhibits anisotropic yielding because the material compliance varies with direction relative to the elongated void axis. Similar observations have been reported by other numerical investigators [137,138]. FEM results from Worswick and Pick [134] under uniaxial loading conditions demonstrate that extensional void growth rates diminish with plastic strain, indicating that elongated voids aligned with the tensile axis are expected to experience less extensional growth than comparable spherical ones.

1.2.3 Void Coalescence

Conceptually, void coalescence appears to be a relatively straightforward process, whereby adjacent voids link together to precipitate separation of the final fracture plane. In practice though, it is easily the least understood phase of ductile fracture. This deficiency is a direct result of the swiftness with which final failure appears to be triggered by the attainment of favourable global void coalescence conditions. As well, long-range coalescence events often seem restricted to one critical fracture path, particularly in sparse particle fields, with physical evidence of substantial coalescence conspicuously absent in regions immediately adjacent to the final fracture surface [99]. Added to this problem is the fact that void coalescence is commonly preceded by a regime of considerable strain localization. Given that dilational and extensional void growth is very sensitive to stress triaxiality, the exact state of void evolution just prior to coalescence is effectively masked by the abrupt stress state changes in a developing neck. Despite the many inherent

obstacles which impede the progress of void coalescence research, a number of meaningful investigations are documented in the literature.

Experimentation

Based on metallographic examinations, two distinct forms of void coalescence events are commonly exhibited. In some instances, failure of the inter-void ligament is facilitated by the formation of a shear band between closely-spaced voids [71,74]. In turn, these regions of intense shear often spawn a plane of small secondary voids which ultimately severs the ligament, a mechanism called void sheeting. The other conventional means of void coalescence entails stable void growth, equivalent to stable necking of the inter-void ligament, until complete impingement of adjacent voids is realized [74,121,139]. A schematic of void impingement under varying degrees of stress triaxiality was previously illustrated in Figure 1.14 from Sun Jun [121]. Of further interest, Low [140] has demonstrated that these particular modes are not exclusive to a given material or stress state, discovering occurrences of void sheeting and void impingement within 20 microns of one another in an aluminum alloy tensile specimen.

It follows from the intrinsic nature of void coalescence, which relies heavily upon the stability of inter-void ligaments, that the coalescence path is very sensitive to macroscopic stress state. Studies by Geltmacher *et al.* [141], using sheet specimens with through-thickness holes, have demonstrated that coalescence paths are preferentially oriented perpendicular to the direction of principal tensile load. Therefore, transverse linkages are predominant under uniaxial loading while no orientation preference is exhibited

in the presence of equi-biaxial loading, as seen in Figure 1.21. From these results, Geltnacher *et al.* contend that equi-biaxial loading is more efficient for producing fracture because a greater number of potential linkage sites are available. Extending this principle, they also conclude that equi-biaxial loading produces lower fracture strains, as shown in Figure 1.22 from work by Bourcier and Koss [142] with commercially pure titanium sheets. It is evident that the right-hand side of the fracture limit diagram in Figure 1.22, although differing somewhat from a forming limit diagram, is at odds with local necking trends discussed earlier and with other published data for particle-containing sheets [100]. Figure 1.22 implies that void damage is an overwhelming factor in stretching situations and that catastrophic coalescence proceeds swiftly. However, the fracture locus in this particular case is most certainly confounded by the strongly oriented crystallographic textures measured by Bourcier and Koss [142] in their sheet, a direct consequence of titanium's highly anisotropic hexagonal close-packed crystal structure. Without further data to establish the influence of void damage on local stress and strain states prior to the emergence of a localized neck, recalling the characteristic stretching strain paths of Figure 1.5, it appears that the efficient coalescence theory of Geltnacher *et al.* becomes less applicable as deformation proceeds and local conditions deviate from the equi-biaxial case. It is inconceivable that strains to fracture in particle-containing sheets can be predicted simply from coalescence phenomena related to the initial remote loading conditions. Instead, accurate modelling requires careful consideration of the various factors affecting local strain paths, such as the deviation towards plane strain prior to localization.

Another variation of the catastrophic void coalescence criterion contends that final failure occurs when the evolving void volume fraction reaches a certain minimum threshold. Critical values of coalescence-inducing porosity are typically obtained from

metallographic samples of regions directly beneath the fracture surface. In the literature, void volume fractions attributed to the onset of void coalescence range between three and five percent [85,94,99,102,143], primarily obtained from spheroidized steel specimens. Many of these investigations also report that the threshold void volume fraction is lowered by a corresponding elevation in stress triaxiality caused by notched tensile specimens [85,99,143]. The findings of Brownrigg *et al.* [102] are at odds with this last relationship, however, indicating that the porosity level precipitating final fracture in a simple tensile specimen of spheroidized steel is unaffected by varying amounts of superimposed pressure. This relationship is evident in Figure 1.23a. Figure 1.23b reveals that the expected suppression of void growth rate caused by elevated levels of hydrostatic pressure is suitably counteracted by the nucleation of additional voids over an extended period of stable plastic straining. In cases where considerable numbers of voids exist prior to deformation, it follows that the void volume fraction at failure is likely to be sensitive to hydrostatic pressure since void nucleation is negligible in the overall void evolution process. Spitzig [144] has proven this fact using porous iron compacts by demonstrating that either an increased level of initial porosity or a decreased magnitude of applied hydrostatic pressure causes higher void volume fractions in failed specimens.

Theory

Given the experimental evidence showing that individual void coalescence events range between complete impingement and unstable shearing of the inter-void ligament, it is not surprising that theoretical approaches for void coalescence are varied. Early analytical work by McClintock [128], which focused mainly on void growth behaviour, simply

assumed that impingement brought on final fracture. Soon thereafter, Brown and Embury [145] speculated that a void length to spacing ratio of unity signalled coalescence, as shown in Figure 1.24. Even though Garber [146] has pointed out that the two-dimensional nature of Brown and Embury's geometric criterion overlooks the three-dimensional reality of coalescence (see Figure 1.25), this apparent shortcoming does not detract from its overall merit. In the field of quantitative stereology, two-dimensional representations are commonplace. The direct correlation between microstructural features of metallographic planes and bulk material characteristics are well established [147,148]. In fact, LeRoy *et al.* [84] achieved reasonable predictions for ductile fracture under uniaxial loading by incorporating a critical void length to spacing ratio of 1.4 into their model.

As an alternative to basic geometric considerations, Thomason has developed two-dimensional [149-151] and three-dimensional [152,153] coalescence models based on the attainment of unstable plastic flow within inter-void ligaments. Figures 1.26 and 1.27 illustrate the corresponding void ligament geometries considered by Thomason. His models stipulate that global void coalescence, indicative of failure, is fully characterized by a geometric constraint factor equation. For two-dimensional analyses, the following expression is employed [151]:

$$\frac{\sigma_l'^c}{2k} = 1.49 \left(1 + \frac{w-b}{a} \right)^{1/3} - 0.91 \quad (1.36)$$

where $\sigma_l'^c$ is the incipient limit load stress, k is the yield shear stress of the matrix, and a , b , and w are dimensional parameters specified in Figure 1.26. Applying equation 1.36, final failure of the inter-void ligament is predicted when the remote stress, σ_l , reaches the incipient limit-load stress, $\sigma_l'^c$. It is important to note that the incipient limit-load stress decays with applied strain, and therefore, requires the implementation of an appropriate

void growth model to predict evolving void sizes and shapes. Thomason's three-dimensional geometric constraint equation for void coalescence is of the form:

$$\frac{\sigma_I^{I^C}}{\bar{\sigma}_M} = 0.1 \left(\frac{d}{a} \right)^{-2} + 1.2 \left(\frac{b+d}{b} \right)^{-1/2} \quad (1.37)$$

where $\bar{\sigma}_M$ is the matrix flow stress and dimensions a , b , and d are dimensional parameters denoted Figure 1.27. As with the two-dimensional criterion, failure of the inter-void ligament is signalled by a remote stress equal to the incipient limit-load stress, $\sigma_I^{I^C}$.

As a final point on void coalescence, it is worth noting that each of the above theoretical criteria contain a profound weakness. This weakness stems from the implication that final fracture is completely represented by the onset of coalescence at one characteristic dimension, most often considering a single pair of voids. Given the non-uniform spatial arrangement of actual particle populations, it is evident that this assumption ignores the strong possibility for several successive void coalescence events, each at progressively larger magnitudes of scale, preceding a catastrophic chain-reaction to fracture. All told, the current state of void coalescence theories, which base predictions on average void features and loading conditions, underscores the need for a better understanding of stochastic particle clustering effects and their proper integration into void damage models.

1.2.4 Gurson-Tvergaard Model

Theoretical investigations of ductile fracture behaviour are conducted on two levels: models of individual void nucleation, growth, and coalescence stages; and integrated models of overall constitutive and failure behaviour. Naturally, it follows that many

integrated ductile fracture models are simply an amalgamation of component models. However, one integrated formulation that is uniquely self-contained is the Gurson-Tvergaard model for constitutive softening.

In the context of ductile fracture, the concept of constitutive softening refers to the decrease in load-carrying capacity of a material as a direct result of an evolving void volume fraction. In the Gurson-Tvergaard model, porosity content directly influences plastic flow behaviour through a pressure-sensitive yield criteria developed by Gurson [65,66]. The yield function is derived from an upper-bound solution for the average stresses acting on the outer surface of a rigid perfectly-plastic sphere containing a concentric spherical void. Reflecting a subsequent modification by Tvergaard [67,68], the Gurson-Tvergaard yield locus, Φ , facilitates the determination of the local matrix flow stress, $\bar{\sigma}_M$, arising from a remote applied stress, σ_{ij} , in a voided material, according to:

$$\Phi = \left(\frac{\sigma_{eq}}{\bar{\sigma}_M} \right)^2 + 2fq_1 \cosh \left(q_2 \frac{\sigma_{kk}}{2\bar{\sigma}_M} \right) - 1 - q_3 f^2 = 0 \quad (1.38)$$

where σ_{eq} is the remote equivalent stress and f is the current porosity (or volume fraction of voids). The remote equivalent stress, σ_{eq} , is analogous to an effective stress, defined as:

$$\sigma_{eq} = \sqrt{\frac{3}{2} \sigma_{ij}' \sigma_{ij}'} \quad (1.39)$$

where σ_{ij}' is the deviatoric component of stress. From equations 1.38 and 1.39, it is evident the Gurson-Tvergaard yield criterion reduces to the von Mises condition in the absence of porosity. The coefficients q_1 , q_2 , and q_3 , in equation 1.38 were introduced by Tvergaard [67,68] to achieve better flow predictions at low void volume fractions. While values of $q_1=1.5$, $q_2=1$, and $q_3=q_1$ were proposed in this initial work, Becker *et al.* [154] and Worswick [155] found that settings of $q_1=1.25$, $q_2=0.95$, and $q_3=q_1^2$ produce closer

agreement with corresponding FEM solutions. Validation of the Gurson-Tvergaard model has also been established through direct comparisons with mechanical test specimens of known initial porosity, fabricated using powder metallurgy techniques [154,156].

To produce anisotropic yielding due to inhomogeneous void shapes, Nagaki *et al.* [157,158] proposed a modified form of the Gurson-Tvergaard yield function, employing the damage tensor concept of Murakami and Ohno [159]. The unit cell studied by Nagaki *et al.* is shown in Figure 1.28. Specifically, these alterations entail the replacement of each macroscopic principal stress, σ_i , with a net principal stress, S_i , to reflect the effective void area fraction in each principal plane. This transformation is carried out using a damage effect parameter for each principal direction, Ψ_i , defined by:

$$\Psi_i = (1 - c\Omega_j)^{-1} \quad (1.40)$$

where c is a scalar that accounts for stress concentrations, and Ω_j denotes the void area density in the principal plane which lies normal to Ψ_i . Each principal net stress, S_i , is then constructed from the corresponding principal remote stress, σ_i , by applying the appropriate damage effect multiplier, according to:

$$S_i = \Psi_i \sigma_i \quad (1.41)$$

Nagaki *et al.* [157,158] utilized equations 1.40 and 1.41 in a Gurson-Tvergaard model for a sheet containing an inhomogeneous arrangement of voids and subject to equi-biaxial stretching. Their configuration is illustrated in Figure 1.29. Figure 1.30 shows that the resulting yield function generally agrees with FEM predictions. However, Nagaki *et al.* [158] did not obtain satisfactory results with this modified form of the Gurson-Tvergaard

model from a second study involving uniaxial loading in the presence of superimposed pressure.

In addition to a yield function, the Gurson-Tvergaard model also includes formulations to incorporate the three component stages of ductile fracture. Based on experimental evidence, the rate at which porosity evolves, $\dot{f}$, is dictated by the rate at which new voids are nucleated, $\dot{f}_{nucleation}$, and existing ones grow, $\dot{f}_{growth}$, such that:

$$\dot{f} = \dot{f}_{nucleation} + \dot{f}_{growth} \quad (1.42)$$

If void nucleation is assumed to follow a normal distribution with plastic strain, as suggested by Chu and Needleman [24], the rate of void nucleation, $\dot{f}_{nucleation}$, is given by:

$$\dot{f}_{nucleation} = A \dot{\bar{\epsilon}}^p \quad (1.43a)$$

$$A = \frac{f_N}{S_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\epsilon}_M^p - (\bar{\epsilon}_M^p)_N}{S_N} \right)^2 \right] \quad (1.43b)$$

where $\bar{\epsilon}_M^p$ and $\dot{\bar{\epsilon}}_M^p$ are the effective plastic strain in the matrix and its corresponding rate, $(\bar{\epsilon}_M^p)_N$ and S_N are the respective mean and standard deviation of the effective plastic matrix strains at which particles nucleate voids, and f_N is the volume fraction of void-nucleating particles. Since the original analysis of Gurson assumes that the matrix material is incompressible, the void growth rate, $\dot{f}_{growth}$, is determined exclusively by the rate of remote plastic dilational strain, $\dot{\epsilon}_{kk}^p$, according to:

$$\dot{f}_{growth} = (1-f) \dot{\epsilon}_{kk}^p \quad (1.44)$$

To simulate the expected acceleration in constitutive softening associated with the onset of void coalescence, Tvergaard and Needleman [160] proposed that the current porosity, f , in equation 1.38 be replaced by an effective porosity parameter, f^* . Based on the belief that void coalescence is induced by the attainment of a critical void volume fraction, f_c , the effective porosity, f^* , becomes:

$$f^* = \begin{cases} f & , f < f_c \\ f_c + \frac{f_u^* - f_c}{f_f - f_c} (f - f_c) & , f \geq f_c \end{cases} \quad (1.45)$$

where f_f is the porosity at fracture and $f_u^* = 1/q_1$. In essence, equation 1.45 artificially elevates void volume fraction during the coalescence phase until a level corresponding to failure is obtained. At this point, the effective strength of the material, tantamount to the equivalent stress, σ_{eq} , in equation 1.38, is driven to zero. For their analysis, Tvergaard and Needleman [160] selected values of $f_c=0.15$ and $f_f=0.25$, citing appropriate experimental [145] and theoretical [161] justifications. Further studies have demonstrated that f_c is dependent upon the initial void volume fraction [154,162], as seen in Figure 1.31.

It is important to recognize that successful implementation of the Gurson-Tvergaard model requires the appropriate seeding of several parameters. Notwithstanding the calibration coefficients of equation 1.38, the availability of suitable experimental data is necessitated by the material-dependent variables comprising the void nucleation and coalescence equations of 1.43 and 1.45. Consequently, Thomason [163] has argued that the modification of Tvergaard and Needleman [160], which artificially simulates void coalescence, exemplifies the overall inadequacy of the Gurson-Tvergaard model for predicting ductile fracture. It is his belief that the Gurson-Tvergaard yield function is based

on a ‘weak’ dilational plastic flow field because it encompasses average behaviour. On the other hand, Thomason’s incipient limit-load formulation, introduced in Section 1.2.3, is based on the attainment of plastic instability in the inter-void ligament where failure is expected to occur. Therefore, Thomason believes that it represents a ‘strong’ formulation, thus requiring no further adjustment. Zhang and Niemi [164] have demonstrated that it is possible to combine the Gurson-Tvergaard and Thomason models, adopting the Gurson-Tvergaard yield surface until it is overcome by Thomason’s incipient limit-load condition. This compromise produces reasonable results, while avoiding use of the contentious coalescence formulation in equation 1.45.

In spite of these objections from Thomason [163], the Gurson-Tvergaard model is easily adapted to a wide range of theoretical analyses. For example, several authors have successfully employed Gurson-Tvergaard yield functions in M-K analyses of biaxial sheet forming [23,24]. As well, Micari and co-workers [165-167] have used tensile test data to calibrate the void nucleation and coalescence parameters in the Gurson-Tvergaard model, reporting reliable predictions for limiting punch depths in deep drawing of square boxes from sheet.

1.3 Particle Clustering

The fact that spatial distributions of real particle populations are inherently inhomogeneous has serious implications with regard to each component stage of the ductile fracture mechanism. As alluded to in the preceding ductile fracture review, interactions between closely-spaced particles often lead to enhanced void damage rates by way of

reduced nucleation strains, accelerated levels of dilational and extensional growth, and higher probability paths for coalescence. To date, the concept of particle clustering has been ignored in most theoretical models, where ductile fracture predictions are inhibited by assumptions of periodic geometries and average behaviour. On the experimental side, particle clustering data collected from sample specimens has generally been lacking in sophistication and statistical significance. However, the recent advent of image analysis techniques in the field of quantitative stereology has provided a powerful tool to properly address many of these issues. Consequently, this section first provides a synopsis of particle clustering experimentation and theory as it pertains to ductile fracture, followed by a survey of the spatial tessellation techniques commonly used to acquire second-phase particle clustering measures from actual microstructures.

1.3.1 Ductile Fracture

The significance of particle clusters in promoting void nucleation at low strains was first reported by Argon and Im [90] who observed such a trend in spheroidized steels. This behaviour lead to speculation that the constraint of closely spaced particles inhibits local plastic flow in the matrix, thereby inducing void formation. Argon *et al.* [117] incorporated the findings of Argon and Im [90] into a theoretical model which uses dislocation theory to rationalize void nucleation as a function of particle size and neighbour spacing. This analytical study proposes that the non-uniformity of particle volume fraction density in real microstructures brings about premature initiation of voids at larger particle features. As a result, Argon *et al.* [117] implicitly identify particle clusters as preferential void nucleation sites, a conclusion supported by many other researchers [94,99,109].

In the context of void growth, particle clustering *per se* has received little attention in the literature. Instead, research in this area is limited to the nature of interactions between adjacent pairs of voids. In general, studies have shown that a void induces meaningful accelerations in the growth rate of its neighbours provided they are suitably proximal, with a distance of roughly one void diameter often quoted. Specifically, enhanced lateral growth is obtained at the expense of the inter-void ligament, behaviour supported by considerable experimental evidence [141,168,169]. On the other hand, a numerical investigation by Hom and McMeeking [170] exhibited no such amplification effect. Similar studies involving adjacent voids which are distinctly different in size have shown that the normalized growth rate of the smaller void is further enhanced [124,171,172].

The bulk of experimental investigation into the role of particle clusters during void coalescence has been undertaken by Koss and a variety of co-workers [141,173-175]. In each of these studies, sheet and/or plate specimens containing regular and random arrangements of through-thickness holes were fabricated to simulate voids in a matrix. These specimens were then loaded in uniaxial tension to investigate the coalescence pattern of neighbouring holes and the ensuing fracture behaviour. Analytical work by Kushner and Needleman [176] has shown that this two-dimensional simplification is suitable for investigating coalescence behaviour and overall ductility. In the first of these novel experiments, Dubensky and Koss [173] determined that coalescence of adjacent voids is very sensitive to their relative spacing and orientation, with linkage preferentially occurring between closely-spaced holes aligned transverse to the direction of the applied uniaxial load. As well, their tests indicate that coalescence of just two holes is insufficient for promoting fast fracture. Instead, failure by unstable coalescence requires the linkage of at

least three holes. Together, these observations demonstrate that a critical amount of local void damage is required to induce failure and that this damage emanates from clusters of voids. On a macroscopic scale, this behaviour is manifested by the relative fracture strains obtained in specimens with different spatial distributions of holes. For a given area fraction of holes, Dubensky and Koss discovered that sheets with uniformly-spaced voids exhibit two to three times the ductility of specimens containing holes in a random arrangement, shown in Figure 1.32. Figure 1.32 also illustrates that the fracture strain of a specimen with a random distribution of holes is sharply reduced by a corresponding drop in minimum allowable inter-void spacing, S_m . This last outcome indicates that a greater degree of clustering has a more detrimental effect on ductility, recognizing that the potential for void clustering increases as S_m decreases, clearly visible in Figure 1.33. These void clustering effects have been substantiated experimentally by other researchers [141,174,177,178]. Magnusen *et al.* [175] also constructed a two-dimensional numerical model to simulate the linkage behaviour of random void distributions under uniaxial tension. Using an elliptical representation for the region of elevated strains around an isolated hole, as illustrated in Figure 1.34, void strain fields were dilated in conjunction with applied strains to predict coalescence events between neighbouring voids based on the overlap of their strain fields. The resulting evolution of void damage, shown in Figure 1.35, echoes the above threshold of roughly three linked voids to precipitate long-range coalescence. Overall, Magnusen *et al.* effectively confirm the deleterious nature of void clusters which are indigenous to random distributions, in agreement with numerical findings reported elsewhere [177,178].

The idea that a specific magnitude of local void damage is required to precipitate strain localization under multi-axial loading has also been asserted by Huang [179], using

the concept of a critical particle cluster. This work employs an analytical scheme previously developed by Huang and Hutchinson [180] for plastic shear localization, based upon a two-dimensional M-K model geometry (see Section 1.1.1). The defect band is of width λ and contains an inhomogeneous distribution of particles. For calibration purposes, the particle distribution within the band is represented by an individual cluster, modelled as an n -sided polygon. The average inter-particle distance, d_{avg} , in an n -sided polygon cluster with side length d is given by:

$$d_{avg} = \frac{d}{(n-1)[1 - \cos(\pi/n)]} \quad (1.46)$$

Clusters are modelled as n -sided polygons because d_{avg} is found to be relatively insensitive to the exact arrangement of particles within a cluster for given values of n and particle volume fraction. Huang notes that the four alternative cluster geometries depicted in Figure 1.36 have d_{avg} values within 8% of the solution from equation 1.46 for the 6-sided polygon (i.e. hexagon). Incorporating a Gurson-Tvergaard constitutive formulation with strain-controlled nucleation into an FEM model, Huang predicted localization strains for individual particle clusters as a function of d_{avg}/λ for a wide range of n , where $n=2, 3, \dots, 8$. These predictions are summarized in Figure 1.37. Applying the concept of a critical particle cluster, the localization strain for a random arrangement of particles within the defect band is believed to be equal to the localization strain of its most damaging cluster, referring to the cluster with characteristics, n and d_{avg}/λ , that produce the lowest localization strain in Figure 1.37. To validate this approach, Huang obtained rigorous strain localization predictions for several random particle distributions within the defect band and related them to the localization strain of their most critical cluster. The correspondence between these two strain prediction methods is evident in Figure 1.38.

While the outcome presented in Figure 1.38 is quite promising, it is important to recognize that this technique does not consider clustering effects beyond that of particle clusters. For example, the potential influence of clusters of particle clusters is ignored. For application to real microstructures, it is conceivable that the notion of a critical cluster needs to be extended to include a critical subset of clusters that reflect the definitive dimensions and spatial distribution of clustering within second-phase particle populations. This proposed extension of Huang's critical cluster theory coincides with the findings of Spitzig *et al.* [126] which note that global strain localization is induced by the combined effect of void damage evolving from particle clusters in several isolated regions.

For the sake of completeness, it must be mentioned that theoretical work by Melander [181], Melander and Stahlberg [182], and Thomason [151] has proposed that smaller fracture strains are realized in porous solids containing uniform void distributions as opposed to random ones. These conclusions are a contradiction of the documented research outlined above, with the respective authors providing no experimental evidence to support their claims. In fact, close inspection of these analyses reveals model behaviour which is not reflected in the overall conclusions. In his analysis of random void fields, Melander [181] demonstrates that fracture strains decrease with increasing variability of void distribution, indicating that non-uniform arrangements produce lower ductilities. However, citing earlier work from Thomason [149] which predicts a lower fracture strain for a uniform arrangement of voids, Melander concludes that the uniform case is the most harmful. Since Thomason himself has more recently condemned his earlier work [149] as unsound and overly conservative [150], it follows that Melander's investigation actually supports the notion of void clusters inducing premature ductile fracture. In a separate study, Thomason [151] assessed the effect of non-uniform void spacings on ductile

fracture strains by comparing the response of the two void fields depicted in Figure 1.39. Relatively speaking, Thomason [151] demonstrates that the non-uniform void spacings of Figure 1.39a withstand higher levels of applied strain before fracturing, concluding that an inhomogeneous spatial distribution of particles leads to improved ductility. However, careful inspection of Figure 1.39a reveals that the non-uniform void spacings actually correspond to a homogeneous spatial distribution as a function of void area fraction (i.e. the area fraction of each void and its surrounding void-free region is constant in each sub-cell). Continuing this observation, the spatial distribution of void volume fraction in Figure 1.39b is inhomogeneous, while Thomason refers to it as a uniform arrangement. Therefore, the increased ductility associated with Figure 1.39a is expected because the void field has a constant volume fraction. Recalling the pioneering work of Argon *et al.* [117] in Section 1.2.1, it was rationalized that void damage is exacerbated near large or clustered particles because these regions possess inherently higher local particle volume fractions. In essence, Thomason [151] bases his conclusion on uniformity of void spacing, whereas uniformity of void volume fraction is perhaps a more appropriate parameter. By recognizing this distinction, Thomason's predictions demonstrate that a more homogeneous distribution of voids, according to void volume fraction, increases failure strains. Such an outcome, implicating the harmful nature of void clusters, agrees with the existing body of literature.

As a final point, the preceding formulations of Melander [181,182] and Thomason [151] also serve to highlight an underlying flaw of most particle clustering models. In these particular studies, the average constraint factor of all participating ligaments is used to establish one overall failure condition, rather than assessing plastic instability criteria for individual inter-void ligaments and considering the proliferation of stable void coalescence

through the structure. By adopting a simple average, the failure condition is dominated by the largest inter-void ligaments. Such a simplistic approach effectively defeats the purpose of examining an inhomogeneous situation. In reality, the smallest inter-void ligaments are likely to fail first, effectively altering loading conditions around neighbouring ligaments. Hence, failure is better represented by a sequence of stable coalescence events eventually spawning global instability, a concept not reflected in these models or the vast majority of theoretical ductile fracture research.

1.3.2 Spatial Tessellations

Spatial tessellation techniques are regularly applied in many research fields of mathematics and science to establish degrees of order in otherwise random arrangements of finite bodies. By definition, a spatial tessellation sub-divides two- or three-dimensional space into a network of cells, called a Voronoi diagram [183], such that each cell is uniquely related to the corresponding field of features. This means that an unlimited number of algorithms exist to spatially tessellate a given distribution of features, as evidenced by the comprehensive text of Okabe *et al.* [184]. In metallurgical research though, the vast majority of investigators have adopted the Dirichlet tessellation [185] to delineate prominent spatial properties in microstructures, such as clustering of second-phase particles.

The manner in which Dirichlet tessellations are constructed from point features stipulates that each tessellated cell contains all regions of the space closest to the feature contained therein. For bodies of finite size, the respective centroids are usually taken as the

points of interest. Physically speaking, a Dirichlet tessellation cell represents an n -sided polygon constructed from a set of right bisectors spanned between one point and all of its near-neighbours, as is illustrated in Figure 1.40. Figure 1.41 depicts a Dirichlet tessellation for an entire particle field.

In an earlier discussion regarding the validity of two-dimensional void coalescence criteria in three-dimensional solids, it was explained that such simplifications are widely accepted in microstructural investigations. Research has shown that microstructural features, such as particle or void volume fraction, can be adequately represented by data acquired from a sufficient number of metallographic planes [147,148]. Naturally, sound metallographic practices and image processing techniques must be employed to ensure the accuracy of such measures [186]. By adhering to these principles, the degree of particle clustering in a given microstructure can be extracted from its corresponding two-dimensional spatial tessellation. This strategy stems from the fact that certain geometric attributes of the tessellation cells act as strong indicators of clustering and/or periodicity in the coinciding particle distribution [187-191]. Several examples of these correlations are summarized in Table 1.1 from metallographic studies by Burger *et al.* [189]. It is worth noting that the use of coefficients of variance, as opposed to the standard deviations referred to in Table 1.1, facilitates unbiased comparison of clustering between different particle populations, especially ones with markedly dissimilar volume fractions [190,191]. A coefficient of variance is simply a standard deviation normalized by its mean. Several researchers have successfully correlated clustering parameters from Table 1.1 with macroscopic mechanical properties, such as fracture toughness [192], as well as with microscopic characteristics, including interfacial stresses around fibre reinforcements [193]. Furthermore, anisotropic spatial features of the distribution have been readily

extracted from Dirichlet tessellation data, with high tessellation cell aspect ratios being indicative of banding [187,188], for example.

The exact location of particle clusters can also be determined directly from Dirichlet tessellations by calculating local particle area fractions of individual cells. Elevated levels of local particle area fraction distinguish clustered regions. Spitzig *et al.* [126] applied this technique to locate the relative position of particle clusters in porous iron specimens, revealing that the inhomogeneous band geometry adopted for their corresponding numerical model constitutes an inappropriate physical representation. It has also been suggested by Brownrigg *et al.* [102] that spatial tessellations effectively define coalescence, whereby the growth of each void in the fracture plane is limited by some critical area fraction of its tessellation cell. On a global scale, this concept is analogous to the final coalescence criterion described in section 1.2.3, where failure is ultimately induced by the attainment of a critical void volume fraction. From microscopic examinations of test specimen fracture surfaces, Brownrigg *et al.* arrived at a critical Dirichlet tessellation cell area fraction of about 0.9 for coalescence.

A slight variation of the Dirichlet tessellation is also possible by employing a particle dilation algorithm to skeletonize the particle population, as implemented by Shehata [191] and illustrated in Figure 1.42. This tessellation scheme entails dilation of the particles at constant rates until they impinge upon one another, with the skeletonized field corresponding to the respective boundaries of impingement. Conceptually, an equivalent tessellation is obtained by incrementally removing a single width of matrix pixels from around each particle until a shell of matrix material remains, a process known as matrix erosion. It follows that the relationships compiled in Table 1.1 are equally applicable to

particle dilation (or matrix erosion) tessellations. There are several advantages to the particle dilation strategy in comparison to the Dirichlet construction, including its easy application to discretized binary images which represent an integral part of automated image analysis systems. As well, particle shapes are strongly reflected in the resulting tessellation cells, a correspondence that is significant in studies of anisotropic properties caused by high aspect ratio particle populations, such as those typically present in rolled sheet.

While Dirichlet and particle dilation tessellations clearly provide a functional method for comparing relative degrees of clustering, their applicability is limited to clusters of particles, referred to herein as first-order clusters. Consequently, the nature of second-order clustering (i.e. clusters of first-order clusters) and clustering at successively higher orders requires a more sophisticated spatial tessellation scheme. The adoption of dilational counting techniques, as proposed by Shehata [191], permits an explicit assessment of clustering at several orders of magnitude. Furthermore, this strategy is a straightforward extension of the particle dilation tessellation algorithm. To obtain dilational counting data, the number of objects which merge (or agglomerate) during each dilation increment is recorded, with the resulting frequency plot providing several measures of key spatial distribution characteristics. An example of the dilation process and three fundamental classifications of dilational counting data are included in Figures 1.43 to 1.45 from Shehata [191]. Of particular interest in the current research programme, Figure 1.45c illustrates the manner in which successive frequency peaks in dilational counting data indicate multiple orders of clustering in the particle distribution. While the first prominent peak is expected in all cases, due to the merging of particles with their near-neighbours to form first-order clusters, it is really the presence and location of subsequent local maxima which reveals the nature of particle clustering as a whole. For ordered or periodic arrays (Figure 1.45b),

secondary merging events are not exhibited because the spacing of near neighbours is common throughout the structure. In heavily clustered arrangements (Figure 1.45c), ordering is observed at a number of distinct magnitudes of scale. Between these two extremes, random distributions (Figure 1.45a) are inherently defined by a reduced prominence in the first-order clustering peak which is further obscured by an immediate transition into merging events of higher orders. Owing to the nature of material fabrication processes, it is expected that the spatial distribution of most second-phase particle populations comprises a clustered component superimposed on a random background [188].

Although spatial tessellation techniques have proven effective for establishing measures of second-phase particle clustering, their application to predictive numerical formulations is rare. One such effort by Ghosh and various co-workers [194-196] entails the development of a Voronoi cell finite element method (VCFEM) which facilitates direct integration of a Dirichlet tessellation into a finite element model. The foundation of this work encompasses the formulation of an n-sided polygonal element to represent individual Dirichlet tessellation cells. Because of the generic form of these elements, Ghost *et al.* have reported several problems with the derivation of their model, including incompatible inter-element displacements and rank deficiencies in the stiffness matrix. As a result, successful application of the VCFEM formulation has thus far been limited to elastic [197] and small deformation elastic-plastic [198] analyses of local stresses in particle-containing microstructures. Additional refinement of the VCFEM model is reported to be ongoing, in an attempt to model large-strain deformations [198].

1.4 Current Research

Throughout the preceding review, it is evident that significant research deficiencies exist in the fields of sheet formability, ductile fracture, and particle clustering. Some notable issues that have yet to be resolved include:

- i) explicit characterizations of second-phase particle clustering at several orders of magnitude in real microstructures;
- ii) *in situ* measures of void damage, related directly to the physical attributes and spatial distribution of definitive second-phase features.
- iii) theoretical models to predict trends in sheet formability based on the evolution of void damage within actual particle fields.

The overall purpose of this thesis is to assess the influence of second-phase particle clusters on the formability of aluminum alloy sheet. It is believed that this task can be accomplished by thoroughly addressing each of the three failings listed above.

The experimental programme outlined herein examines four distinct aluminum-silicon (Al-Si) alloy sheets, specifically fabricated for this research by Alcan KDRC. A binary Al-Si alloy was chosen to accentuate second-phase particle effects and to provide relatively consistent behaviour within the second phase and the matrix. Namely, the Si produces hard, incoherent particles of homogeneous composition, while the matrix is relatively free of complex solutes.

Second-phase particle populations in the four Al-Si microstructures are characterized with the aid of conventional image analysis and stereological techniques. The

acquisition of large-scale high-resolution digital images enables several explicit clustering measures to be extracted from the Al-Si particle fields. Sheet formabilities are established by standard uniaxial tensile and hemispherical punch tests, providing stress-strain and forming limit data, respectively. From a stereological investigation of deformed test specimens, the occurrence of void damage in the Al-Si microstructures is directly correlated with distinguishing second-phase particle clustering features. On the basis of these correlations, the effect of second-phase particle clustering on sheet formability is implicitly evaluated. To validate the various stereological and experimental findings, an analytical model is presented which demonstrates that second-phase particle clustering has a predominant impact on the sheet formability of the four Al-Si alloys under study.

In addition to the research contained in this thesis, parallel work by Pilkey *et al.* [199], Thomson [200] and Worswick, Pilkey and Thomson [201] has been completed within the broader scope of this research programme. These complementary investigations include the characterization of second-phase particle clustering in a preliminary set of model aluminum alloy sheets [199], parametric FEM modelling of void nucleation and growth within particle clusters [200], and a semi-analytical simulation of void damage proliferation in deforming Al-Si microstructures [201]. Consequently, relevant findings from these studies are referenced in this document.

The following text commences with the acquisition and measurement of second-phase particle clustering in the undeformed Al-Si microstructures, comprising Chapter 2. Chapter 3 details the mechanical testing programme used to determine the sheet formability and related material properties of the Al-Si alloys. Chapter 4 presents the void damage data acquired from hemispherical dome specimens and its association with particle clustering

measures. Predictions of biaxial forming trends are discussed in Chapter 5, based on an analytical formulation that utilizes the particle field and void damage data from the four Al-Si sheets. Chapter 6 contains supplementary discussion as well as commentary related to future direction for this research programme. Thesis conclusions and recommendations are reported in Chapter 7.

Chapter 2

Second-Phase Particle Clustering

In order to establish the nature of second-phase particle clustering in the Al-Si alloy sheets under study, stereological characterization of the four Al-Si microstructures has been conducted with the aid of image analysis. The results of this microstructural characterization are presented in this chapter, commencing with a description of the quantitative stereology techniques employed to acquire digital second-phase particle fields and extract explicit measures from these particle populations. The subsequent discussion focuses on interpretation of distinguishing particle clustering characteristics in the Al-Si microstructures and their anticipated role in the evolution of void damage. Although particle clustering is of primary concern in this thesis, the physical attributes of individual particles are also considered, acknowledging their relevance in void nucleation. The nature of void damage in the undeformed Al-Si sheets is detailed in Chapter 4.

2.1 Sheet Composition and Fabrication

The four Al-Si alloy sheets fabricated for this research programme contain nominal compositions of either 2% or 4% Si. Two types of sheet were produced for each Si content level, one with approximately 100 ppm Sr in the melt to act as a particle refining agent. The Sr addition sufficiently altered the final particle populations to provide four distinct distributions. These four sheets are referred to herein as Al-2%Si unrefined, Al-2%Si Sr-refined, Al-4%Si unrefined and Al-4%Si Sr-refined. Their measured compositions are documented in Table 2.1.

The fabrication of each sheet entailed direct-chill (DC) casting into 95 x 225 x 380 mm ingots. Following casting, the ingots were scalped and then heated to 450°C for one hour to prepare them for hot rolling. Hot rolling reduced the ingot thickness to 5 mm. A final gauge of 1.6 mm was achieved by cold rolling. Homogenization and stress relief of the rolled microstructure was provided by a one-hour annealing process at 350°C.

Recognizing that the underlying objective of the current research program is to investigate second-phase particle effects on sheet formability, particle populations were examined in the metallographic plane coincident with the plane of the sheet. Typical in-plane views of the four Al-Si microstructures, at low and high magnifications, are depicted in the micrographs of Figures 2.1 through 2.4. In these micrographs, the banded distribution of second phases is evident (although much more prominent in the digital images of Section 2.4). Namely, successive rolling passes have caused fragmentation and alignment of second phases in such a manner that the microstructure appears to contain alternating bands of particle-rich and particle-free zones. In addition to the second-phase particle features, the grain structure of the four Al-Si alloys was also examined. Grains are

generally equiaxed in all three principal sheet planes, with a considerable size variation noted from centreline (large) to surface (small) in each sheet. Average equiaxed grain size measurements were obtained from the larger centreline grains and are summarized in Table 2.2. It is evident that the Al-2%Si alloys contain larger grains, especially in the unrefined sheet. Given the possibility for premature failure due to very high grain size to sheet thickness ratios, explained in Section 1.1.2, the implication of large grains in the Al-2%Si unrefined alloy is addressed during the interpretation of mechanical testing results in Chapter 3.

2.2 Metallography

The accuracy and precision of quantitative stereology is heavily dependent upon the quality and consistency of metallographic specimen preparation, as alluded to in Section 1.3.2. In the current research programme, the need to acquire large-scale high-resolution metallographic fields further stipulates that specimen surfaces be virtually free of scratches. In an effort to maintain high standards of quality and repeatability, the preparation of metallographic specimens was carried out using a Buehler Economet III semi-automatic grinding/polishing system with a Whirlimet attachment, as well as a Buehler Vibromet I vibratory polisher. The adopted grinding and polishing procedure consisted of the following steps:

- i) grinding, 200 rpm, Diamond Grit #400 SiC paper, water lubricant,
by hand, until flat and at desired sheet depth;

- ii) grinding, 200 rpm, Buehler #600 Microcut paper, water lubricant, by hand, 2 minutes;
- iii) polishing, 150 rpm, Buehler Metadi 6 micron diamond suspension, Buehler Metcloth, Struers Blue DP-Lubricant, 15 KPa, 6 minutes;
- iv) polishing, 150 rpm, Buehler Micropolish B 0.05 micron alumina, Buehler Microcloth, water lubricant, 15 KPa, 5 minutes;
- v) vibratory polishing, 10 rpm, Struers OP-S colloidal silica suspension (10:1 water dilution), Buehler Mastertex cloth, 5 KPa, 2 hours.

Bakelite specimen mounts were regularly measured with digital calipers in steps (i) and (ii) to determine depth of penetration into the sheet and to ensure that metallographic surfaces remained flat and square to the sidewall. All in-plane sheet specimens were ground sufficiently to obtain final polished metallographic planes within $\pm 10 \mu\text{m}$ of the sheet centreline. All polished microstructures were left unetched for stereological examinations. This facilitated separate detection of voids (black), Si particles (grey), and the Al matrix (light-grey) according to their digitized intensities, commonly referred to as grey-scale values or 'grey-scales'.

2.3 Quantitative Stereology

Standard quantitative stereology practices were employed to identify prominent particle clustering features in the four Al-Si sheets. These adopted methods fall into two basic categories: image acquisition and image processing.

2.3.1 Image Acquisition

Image acquisition generally involves the conversion of optical microscope fields of view into digitally discretized representations. For the current work, digital images of metallographic samples were acquired using an IPC 486/33 personal computer equipped with a PCVISIONplus 640 x 480 pixel, 8-bit (256 grey-scale) frame grabber, a Sony 1/3 inch CCD camera, and an Olympus PMEB metallograph. The metallograph has a trinocular head, which allows simultaneous specimen viewing through the microscope eyepieces and the camera connector.

Extraction of particles and voids from digital grey-scale images was accomplished using a software package developed as part of this research programme. This software is specifically designed to reduce or correct errors commonly incurred during the digital discretization process, including non-uniform illumination in the metallographic field of view as well as grey-scale fluctuations due to video signal noise. Non-uniform field illumination is effectively reduced by applying correction kernels to each pixel of an acquired image, where correction kernels are calculated from local grey-scale intensity gradients arising in a particle-free calibration image. The adverse effect of video noise is

minimized through an image averaging algorithm, where four frames are acquired in rapid succession and then averaged to reduce grey-scale fluctuations.

For each digital grey-scale image, a corresponding ternary image is generated and stored by the acquisition software. These ternary images represent an encoded form of the microstructural features, whereby each pixel is assigned the designation of void, particle, or matrix. Ternary encoding of metallographic field is carried out using multiple thresholds of the image grey-scales. Threshold values are automatically calculated for each image using inherent characteristics of the grey-scale frequency data. A sample log grey-scale histogram is depicted in Figure 2.5, where 0 is black and 255 is white. The bimodal nature of the histogram corresponds to the grey-scales of the particles (low value peak) and the matrix (high value peak). The void-particle threshold is set at the first inflection point of the low peak while the particle-matrix threshold is positioned at the midpoint of the two peaks. Additional discussion regarding void detection is provided in Section 4.1. A grey-scale image and its extracted particle and void fields are displayed in Figures 2.6 and 2.7. Ternary void-particle-matrix images are separated into binary particle and void fields for subsequent processing. These binary images are stored using a compressed file format.

Given that the current research programme proposes to establish clustering measures at several levels of magnitude, the acquisition of large-scale particle fields is imperative. In direct conflict with this size requirement, the proper discretization of individual microstructural features demands a reasonable degree of accuracy, and therefore, a relatively high magnification. It is worth noting that the majority of stereological investigations referenced in Section 1.3.2 are plagued by an inability to satisfy both of

these requirements. Therefore, a digital merging scheme was developed through the course of this work to solve this compromising situation.

To obtain a large-scale particle field while maintaining high resolution, multiple overlapping images are collected and then digitally merged during the subsequent processing stage to reconstruct a single image. A 10 x 10 grid of images is obtained from each metallographic sample with a nominal overlap between fields of 15-20%. This approach results in a final merged image size of roughly 5500 x 4000 pixels. For the current work, a 50X objective was employed in combination with green light, corresponding to a resolving power of 0.42 μm . Resolving power, d , is defined as:

$$d = 0.61 \frac{\lambda}{N.A.} \quad (2.1)$$

where λ is the wavelength of illuminating light and $N.A.$ is the numerical aperture of the objective lens. In the configuration described above, the values for λ and $N.A.$ are 546 nm and 0.80, respectively. The resulting digital image pixel resolution is 0.1862 μm , enabling sufficient discretization of the raw image. In terms of the large-scale image dimensions, the 5500 x 4000 pixel field corresponds to a physical size of approximately 1024 x 745 μm .

Of final note, regardless of the substantial effort made to avoid image artifacts, undesirable features are ultimately captured by the image acquisition process. Fortunately, these features tend to be very small, often the remnants of scratches incurred prior to final polishing, and therefore, can be effectively eliminated from the encoded digital images based on size. In the current work, a minimum allowable particle or void size of 5 square pixels, equivalent to 0.1734 μm^2 , was employed. Although this simple approach also

removes actual features, the significance of these tiny elements in the context of void damage is deemed negligible.

2.3.2 Image Processing

A custom-built image processing software package was developed during the course of this work to extract meaningful measures of particle clustering from 10 x 10 grids of digitally acquired images. Overlapping particles fields were digitally merged and then spatially tessellated on a Silicon Graphics Iris Unix workstation. Given the large size of images involved, the in-house software employs a bit-encoding technique to minimize memory allocation while maintaining reasonable processing run-times.

Digital merging involves piecing together a series of individual overlapping images into one large-scale image. The large-scale particle image is constructed one image at a time, progressing in sequence from the upper left corner of the grid to the bottom right. The relative position of each image is established by minimizing the mismatch error caused by its inclusion in the large-scale image. A logical 'exclusive OR' (XOR) is applied to the overlapped regions of adjacent images to produce this measure of mismatch. For determining optimal image positions, to within one pixel dimension (i.e. 0.1862 μm), a mismatch error is obtained for every possible overlap position, within a range of prescribed limits. Once the large-scale particle image has been constructed, the associated void field is simply generated using the same 'best-fit' positions. This ensures that the relative positions of particles and voids, each originally acquired from a common digital image, are maintained.

There are a number of spatial tessellation techniques available in the literature to analyze microstructural features. However, the matrix erosion tessellation employed by Shehata [191] was adopted for the current work because of its many benefits for investigating second-phase particle clustering, as outlined in Section 1.3.2. During preliminary trials with this algorithm though, it was discovered that the particle dilations are prone to directional biases because of the discrete nature of digital images. These artifacts evolve from the dilation rule applied to each pixel. Figure 2.8 demonstrates that a dilation pass involving all eight possible pixel neighbours, called an eight-connected dilation, causes square-shaped features to emerge. Repeated dilations of this type ultimately lead to a spatial tessellation with cell boundaries oriented at 45° angles with respect to the Cartesian axes. Alternatively, a dilation pass which excludes the four diagonal neighbours of each particle pixel enhances dilational rates along the horizontal and vertical axes. This four-connected approach produces the undesirable outcome of Figure 2.9. Therefore, the matrix erosion algorithm created for the current work utilizes an alternating four and eight-connected dilation scheme to obtain the less biased spatial tessellation depicted in Figure 2.10. It is also worth noting that the ill-defined tessellation cells of particles positioned near the image edge are omitted from subsequent data calculations, a loss which is minimal in light of the large-scale images acquired in the current research programme.

2.4 Results and Discussion

Employing the image processing techniques described above, large-scale particle fields (5500 x 4000 pixels; 1024 x 745 μm) were obtained from the four Al-Si microstructures, using metallographic specimens in the plane of the sheet. Application of

the proposed matrix erosion algorithm to each of these large-scale images produces the spatial tessellations displayed in Figures 2.11 through 2.14. These figures have been cropped to a size of 4800 x 3600 pixels (894 x 670 μm) with the rolling (or longitudinal) sheet direction oriented vertically on the page. The presence of particle clustering on several scales is evident in all cases. From visual inspection of Figures 2.11 to 2.14, the banded nature of the microstructure is the most prominent feature, a reflection of the as-cast structure and subsequent rolling process. These bands, corresponding to macroscopic particle clusters of very high aspect ratio, are preferentially oriented in the longitudinal direction. It appears that the degree of banding is generally enhanced by the sparser 2% Si content and is particularly noticeable in the Al-2%Si Sr-refined sheet of Figure 2.12.

To establish objective measures of particle population features from the fields contained in Figures 2.11 to 2.14, several particle and tessellation cell attributes are available for extraction and interpretation. For example, it was noted in Section 1.3.2 that the relative degree of particle clustering in two particle fields can be determined by comparing certain spatial tessellation parameters, such as the mean and standard deviation of the tessellation cell areas [189-191]. Previous work with the Al-Si alloys under study [199], however, has demonstrated that the dilational counting approach of Shehata [191] provides more detailed clustering information. Consequently, various dilational counting techniques were utilized in the current programme to identify the distinguishing second-particle clustering characteristics of the Al-Si alloys.

Although visual inspection of the four Al-Si microstructures seems to indicate a distinct difference at the level of macroscopic particle clustering caused by banding, a thorough examination of definitive second-phase particle features requires a series of

comparisons at a number of dimensional scales. Therefore, the ensuing discussion of prominent microstructural features is divided into three feature classifications: particles, first-order clusters, and higher order clustering.

2.4.1 Particles

A number of particle attributes have been extracted from the images of Figures 2.11 through 2.14 and tabulated in Table 2.3 to facilitate direct comparison between the Al-Si alloys. Table 2.3 also includes theoretical Si volume fractions, which are calculated using the analyzed weight percentages of Table 2.1 and by considering the relative densities of Al and Si as well as the portion of Si in solution. It is evident from the Al-4%Si data that the measured area fractions are a good representation of these expected volume fractions. In the case of the Al-2%Si alloys though, it appears that the tiny microstructural features removed during the image acquisition process cause a considerable reduction in detected area fraction. As alluded to in Section 2.3.1, the omission of these very small features is not considered to be significant in terms of void damage.

In Figures 2.15 through 2.18, the absolute and normalized distributions of the Si particle populations are presented, grouped according to size and aspect ratio. Aspect ratio values are obtained by modelling each particle as an ellipse with equivalent second moments of area. To illustrate alignment tendencies, particles with major axis aligned more closely to the longitudinal direction are assigned aspect ratios greater than 1. Particles with aspect ratios in excess of 2 or less than 1/2 are referred to herein as high aspect ratio particles. The histograms in Figures 2.15 and 2.16 illustrate that the Sr refinement causes

an increase in the number of particles with a general decrease in their size. For particle aspect ratio, Figures 2.17 and 2.18 reveal that the relative number of high aspect ratio particles aligned in the rolling direction is reduced by the addition of Sr, particularly in the Al-2%Si Sr-refined microstructure. Figure 2.19 demonstrates that there is a strong correlation between particle size and aspect ratio. Due to the continual fragmentation of second phases from an as-cast dendritic structure during rolling, particle aspect ratios decrease directly with size.

It is evident from the above results that distinct differences in size and aspect ratio exist between the Al-Si particle populations. In relation to ductile fracture, the void nucleation research of Section 1.2.1 indicates that particles which are large in size or possess high aspect ratios have greater potentials for initiating voids. Therefore, particles with either of these distinguishing attributes must be considered key microstructural features. In the case of high aspect ratio particles though, it must also be recognized that the void nucleation probability varies with orientation to the principal tensile axis, although Figure 2.19 suggests that the majority of high aspect ratio particles are also large in size. For the remaining second-phase particles which are small and possess aspect ratios near unity, the theory of Argon *et al.* [117] suggests that their significance in void damage evolution is dependent upon their proximity to neighbouring particles, as determined by measures of particle clustering.

2.4.2 First-Order Clusters

As stated in Section 1.3.2, dilation counting techniques are beneficial for particle clustering investigations because they furnish explicit clustering measures at several

magnitudes of scale. Furthermore, these measures are easily tabulated during the construction of a matrix erosion spatial tessellation. The foremost representation of this data entails a histogram of inter-particle dilational spacing (IPDS) frequencies. Recalling that the matrix erosion tessellation algorithm involves repeated particle dilation passes, equivalent to matrix erosion passes, the number of distinct features that disappear from the particle field during each dilation pass is recorded as a frequency. The disappearance of a feature occurs when it agglomerates with another feature (i.e. dilating neighbours come in contact). At the start of the spatial tessellation process, each particle represents a feature. As the process progresses, agglomerated particles become a single feature. By recognizing that each particle dilation pass is of characteristic length in a digitized image, the agglomeration frequencies can be plotted against dilational distance to produce an IPDS frequency histogram. Appropriate interpretation of the resulting IPDS data is then required to establish prominent aspects of clustering in the particle fields.

IPDS data for the four model Al-Si alloys is plotted in Figure 2.20. In each case, the frequency values rise sharply to a peak at an IPDS slightly beyond 1 μm and then drop off at varying rates for higher IPDS readings. The occurrence of this first peak is coincident with the majority of particles dilating to touch their nearest neighbour. Agglomerations of particles which have formed by this peak are defined herein as first-order clusters. The average number of particles in these first-order clusters is between 2.4 and 2.5 in each of the four Al-Si microstructures. It is likely that the majority of first-order clusters result from particles which are broken into two or three pieces late in the rolling schedule. The sharpness of the first IPDS peak is due in part to the uniformity of particle spacing in first-order clusters, but more so to the prominence of first-order clusters in the overall clustering of the microstructure. Namely, Figure 2.20 shows that the first-order

cluster peaks in the Al-4%Si alloys decay in a monotonic fashion for higher IPDS values, indicating that the spatial arrangement of particles is relatively uniform with an average spacing near the first-order peak. On the other hand, the first-order cluster peak is less distinct in the Al-2%Si alloys, signifying a greater degree of higher order clustering.

In the context of void damage, the significance of closely spaced particles in the initiation of voids is well established, as stated in Section 1.2.1. First-order clusters represent preferential sites for void nucleation due to their constraint on plastic flow in the surrounding matrix. In fact, voids are found to already exist in many of the first-order clusters of the undeformed material, presumably due to the considerable plastic deformation and particle fracture incurred during rolling. The void data collected from the ternary images of the undeformed microstructures is presented and discussed in Chapter 4.

Given that the first-order clusters represent primary sites for void nucleation, their physical characteristics are likely to be relevant with respect to ductile fracture. First-order clusters are shown in Figures 2.21 through 2.24 as agglomerated features from the partially dilated Al-Si particle fields. These fields have been dilated up to the first-order IPDS peak and all non-agglomerated particles have been removed. Histograms of the size and aspect ratio of these clusters are presented in Figures 2.25 through 2.28, while Figure 2.29 illustrates that the correlation between size and aspect ratio of first-order clusters is relatively weak compared to that previously shown for particles. Figures 2.25 and 2.26 demonstrate that the trends in first-order cluster size follow those of the particle size data, where features are generally larger in Al-4%Si alloys relative to the Al-2%Si alloys and the Sr refinement causes an overall decrease in size. The aspect ratios of the first-order clusters, presented in Figures 2.27 and 2.28, also correspond to the particle data, with

noticeable divergence between the two Al-2%Si sheets. High aspect ratio first-order clusters are strongly oriented in the longitudinal direction for the Al-2%Si unrefined alloy while preferential alignment is generally not exhibited by the Al-2%Si Sr-refined microstructure. In the Al-4%Si sheets, the tendency for longitudinal alignment of first-order clusters possessing high aspect ratios is moderate in both cases, with the unrefined microstructure showing slightly more preference. Overall, these differences in clustering orientation are of particular significance because they indicate that the banded nature of the Al-Si microstructures, previously noted from visual observations of Figures 2.11 to 2.14, is not related to the particle clustering at the scale associated with void nucleation. The effect of preferentially orientated first-order clusters on sheet deformation is examined more closely in the discussion of Chapter 3.

In addition to the physical attributes of size, shape, and orientation, the spatial distribution of first-order clusters themselves is believed to have a meaningful influence on ductile failure behaviour. Documented research suggests that the void growth and coalescence events leading to strain localization and final fracture are enhanced in regions of closely spaced voids. In turn, these voids are likely to originate from first-order clusters. Therefore, an evaluation of second and higher orders of particle clustering in the Al-Si sheets is necessary.

2.4.3 Higher Order Clustering

The nature of particle clustering at higher orders is denoted by peaks in the IPDS frequency data at progressively larger dilational spacings. However, an inherent problem

with IPDS histograms of Figure 2.20 is that the frequency data is dominated by nearest-neighbour merging events. Furthermore, many of these nearest-neighbour agglomerations occur at IPDS values well beyond the first-order clustering peak, meaning that their influence on void initiation and subsequent evolution is substantially diminished. As a result, a modified form of the IPDS plot is warranted to filter out these insignificant events. By adopting the principle that void nucleation occurs predominantly within first-order clusters, and that subsequent growth and coalescence is focused around these features, the spatial distribution of first-order clusters can be obtained through the construction of an agglomerated IPDS histogram. Although it was mentioned in Section 2.4.1 that large and/or high aspect ratio particles are also likely to promote void nucleation, they are omitted from the following agglomerated IPDS analyses. The implications of this decision are addressed in the discussion of Chapter 5.

Agglomerated IPDS data was collected for the Al-Si microstructures by performing dilational passes on images comprised of first-order clusters, referring to Figures 2.21 through 2.24. The resulting agglomerated IPDS frequencies are presented in Figure 2.30. It is evident that the prominent peaks in the agglomerated IPDS data are faintly visible in the IPDS frequencies of Figure 2.20. Applying the same principle used to extract first-order clusters in Section 2.4.2, second-order clusters correspond to the partially tessellated features which have merged by the first agglomerated IPDS frequency peak. As a result, second-order clusters comprise clusters of first-order clusters. It is apparent from the normalized data of Figures 2.31 and 2.34 that the physical attributes of second-order clusters are similar to those of the first-order clusters in all four Al-Si microstructures. Namely, Figures 2.31 and 2.32 depict a lower number of second-order clusters in the 2% Si content alloys, as well as the unrefined microstructures. In the Al-2%Si unrefined sheet,

very few second-order clusters are present. Once again, preferential alignment of high aspect ratio second-order clusters in the longitudinal direction is strong in the Al-2%Si unrefined alloy and weak in the Al-2%Si Sr-refined microstructure. This outcome demonstrates that the macroscopic second-phase particle banding caused by the rolling stages of sheet fabrication is not reflective of clustering at the scale of second-order clusters. In the Al-Si alloy sheets, second-order clusters are expected to influence void growth and coalescence. Therefore, the impact of macroscopic banding in the Al-Si microstructures is expected to be negligible in the context of void nucleation, growth, and coalescence.

In regard to the relationship between feature size and aspect ratio, Figure 2.35 seems to indicate that the strength of this correlation diminishes as the order of clustering increases, on the basis of a comparison with Figures 2.19 and 2.29. The implication of this reduced correlation is explored in the void damage results of Chapter 4.

Generally speaking, agglomerated IPDS data also provides insight regarding the potential manner in which void damage proliferates through the microstructure. By comparison, Figure 2.30 shows that complete agglomeration of first-order clusters occurs rather quickly in the two Al-4%Si microstructures, signifying that the spatial distribution of these features is relatively uniform. Therefore, the spatial variability of void damage is expected to be minimal in the Al-4%Si alloys because their void nucleation sites are quite evenly spaced. The importance of this distinction is two-fold. First, the difference in local and global void damage rates is relatively small. Assuming that failure originates from the most detrimental flaw, failure in the Al-4%Si sheets is essentially dictated by average behaviour. The second benefit of spatial uniformity in void nucleation is that void

interactions which induce accelerated growth and void coalescence behaviour are less probable. In sharp contrast with these trends for the Al-4%Si alloys, the agglomerated frequencies for the Al-2%Si microstructures decay over a much larger IPDS range in Figure 2.31, due in part to the sparseness of these particle fields as well as the prominence of clustering at higher orders of magnitude. This behaviour is particularly noticeable in the Al-2%Si unrefined alloy, where IPDS frequencies decline very gradually with several local peaks, each corresponding to a successively higher order of clustering. Overall, this demonstrates that the degree of clustering in the Al-Si microstructures is substantially increased by a lower Si content and moderately decreased by the presence of Sr. From a ductile fracture standpoint, the highly non-uniform spatial distribution of first-order clusters in the two Al-2%Si alloys suggests that void damage is focused within a few selected areas as deformation progresses. Consequently, the severity of void damage in a 'worst-case' region is certain to increase because of this spatial variability. In terms of strain localization and final failure, the prominence of higher order clusters in the Al-2%Si unrefined microstructure suggests that the expansion of void damage is readily promoted with each increment of strain. Namely, a substantial number of new potential void growth and/or coalescence interactions are made available as these zones of concentrated void damage increase in magnitude. As a result of these distinct differences in agglomerated IPDS data, the influence of void damage evolution on sheet formability is expected to vary significantly between the four Al-Si alloys.

Chapter 3

Mechanical Testing

The mechanical properties of the four Al-Si alloy sheets were investigated using two standard methods: uniaxial tensile tests and hemispherical punch tests. The uniaxial tensile testing facilitates a comparison of deformation behaviour between the Al-Si alloys, based on strain hardening, diffuse instability, local instability, and ductility data. To evaluate the Al-Si sheet formabilities, forming limit diagrams (FLD's) were obtained from hemispherical punch specimens deformed along a wide range of strain paths. As a result, this chapter examines the mechanical properties of the four Al-Si alloy sheets in relation to their distinguishing microstructural features. The relative effect of plastic anisotropy and void damage on the observed trends in forming limits is rationalized in terms of the uniaxial tensile behaviour and microstructural characteristics, including the various second-phase particle clustering measures identified in Chapter 2.

3.1 Uniaxial Tensile Tests

The uniaxial tensile tests were conducted at the Metals Technology Laboratory of the Canadian Centre for Mineral and Energy Technology (CANMET-MTL). A total of 36 tests were commissioned, employing the specimen geometry specified by ASTM Standard E8 [202], shown in Figure 3.1. Uniaxial tensile specimens were machined from each of the four Al-Si alloy sheets in three orientations, corresponding to angles of 0°, 45°, and 90° with respect to the rolling direction. Three repetitions of each configuration were produced. Tensile tests were performed at a nominal displacement rate of 3.0 mm/min, with measures of load, gauge length and gauge width acquired at regular intervals.

3.2 Hemispherical Punch Tests

The hemispherical punch tests and surface strain grid measurements were conducted at Carleton University. A total of eight FLD's were generated, corresponding to tests with major principal loads applied in both the longitudinal and transverse direction for each of the four Al-Si alloy sheets. A schematic of the loading and sheet directions for longitudinal and transverse formability tests is included in Figure 3.2. While hydraulic bulge testing was also completed during the course of this work to investigate sheet formability under truly balanced biaxial conditions, instrumentation difficulties encountered during these tests prevented reliable data from being collected.

3.2.1 Test Matrix

Hemispherical punch tests were performed using a 101.6 mm diameter hemispherical punch press, depicted in Figures 3.3 and 3.4 from Lamontagne [203]. To conduct a test with this apparatus, a specimen blank is placed between the upper and lower clamping dies. Through the action of Enerpac hydraulic clamping cylinders, the top clamping die is lowered onto the sheet to engage the lock bead. The lock bead secures the sheet for subsequent deformation by the punch, with the clamping cylinders maintaining a total force of roughly 400 kN throughout the remainder of the test. The punch is lowered by means of an MTS servo-hydraulic controller/actuator. A custom triangular wave generator provides constant punch speed, while a digital camera mounted beneath the fixture enables necking to be visually detected. Load and displacement data are collected from the load cell and linear variable differential transducer (LVDT), respectively, with the aid of a Nicolet Pro30 two-channel 12-bit digital oscilloscope.

In the current study, a total of 60 punch tests were completed. For each of the four Al-Si alloy sheets, punch tests were performed along five distinct strain paths. Three repeat trials were carried out for each of these test cases. To attain the five desired strain paths, three different specimen widths were adopted: a 25.4 mm x 203.2 mm dogbone geometry for drawing strains, a 127.0 mm x 203.2 mm rectangular blank for plane strain, and a 203.2 mm x 203.2 mm square blank for stretching strains. Drawing and stretching strains refer to states with negative and positive minor strains, respectively. A schematic of the dogbone specimen used to obtain drawing limits is depicted in Figure 3.5. Five different strain paths were generated from three specimen geometries by conducting drawing and stretching tests under both dry and lubricated conditions. Lubrication between

the punch and sheet was provided by a layer of petroleum jelly sandwiched between 0.1 mm gauge polyethylene sheets. For plane strain tests, an aluminum spacer was necessary to reduce the penetration of the lock bead into the sheet, thus preventing premature specimen failure at the lock bead itself.

For the hemispherical punch tests, a punch speed of 12 mm/min. was employed. To assist in the determination of critical necking strains for the FLD's, the initial specimen of each configuration was tested until visible necking occurred. The two remaining repeat specimens were then deformed to punch heights within 1-2 mm of this observed necking level in order to produce incipient necks. In many cases, these incipient necks could not be detected visually but were readily palpable on the specimen surface.

3.2.2 Strain Measurement Procedure

Strain measurements were facilitated by a uniform array of square grids silk-screened onto the Al-Si sheet surfaces prior to testing. The nominal dimension of the undeformed grid is 2.79 mm x 2.79 mm. This grid specification corresponds to that used by Alcan KRDC [204].

The strain measurement system employed for this research comprises a hand-held video probe with image acquisition and processing software, developed at Carleton University. The video probe is constructed from a Panasonic miniature CCD camera, with a hollow cone extending forward from the lens. This cone is brought in contact with the specimen surface during image acquisition to ensure that the camera angle is normal to the surface. Digital images are acquired and processed via a PCVISIONplus 640 x 480, 8-bit

(256 grey-scale) frame-grabber and an IPC 486/33 personal computer. A custom software package enables the user to locate individual surface grid vertices on an acquired digital images, either manually or automatically. The automatic process utilizes an edge-detection algorithm for this task. Manual measurements were collected in the current study to minimize precision error. Deformed grid vertices are related to undeformed calibrations for determining displacement derivatives, and subsequently, major and minor principal strains. These calculated strains represent average readings for the grid.

3.2.3 Strain Measurement Errors

It is recognized that the above strain calculation inherently introduces error into the strain measures by ignoring sheet curvature. In the context of the proposed research though, this error is not expected to confound comparisons between the Al-Si alloys because curvature is essentially constant for any given strain state. However, the strain measurement procedure is inherently susceptible to a number of user and equipment errors, including variations in silk-screened grid arrays, improper focusing of grid images, optical and/or digital distortions, and inaccurate location of grid vertices. Therefore, an estimate of the collective effect of these systematic precision errors is of interest and can be obtained with the aid of fundamental sampling statistics [205].

Repeated strain measurements on a single grid reveal that the strain readings conform to a normal distribution. Therefore, a confidence interval for a single engineering strain reading, e , is defined as:

$$e \pm t \sqrt{\frac{s^2}{N}} \quad (3.1)$$

where t is the Student's t -distribution parameter for the selected confidence interval and sampling degrees of freedom, s^2 is the sample variance and N is the sample size. The sampling degrees of freedom correspond to $N-1$, while the sample variance, s^2 , is specified by:

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (e_i - \bar{e})^2 \quad (3.2)$$

where e_i is an individual engineering strain measure and $\bar{e}$ is the sample mean. In the current study, twenty independent measures of undeformed grids yielded a 95% confidence interval of ± 0.009 for any one measure of engineering strain. It is important to note that this error simply estimates the variability of strain measurement precision. The overall confidence of forming limit data is also influenced by the subjectivity of safe/necked classification procedures for individual grids, an effect which is not readily quantified.

3.2.4 Safe/Necked Classification

In the current work, a binary safe/necked classification approach was adopted. Recognizing that this classification process is quite subjective, a great deal of effort was made to maintain consistency from specimen to specimen in order to facilitate a meaningful comparative analysis of the FLD data. To avoid excessive bias, each measured grid was subjected to several classification passes and the sequence in which specimens were examined was changed between each pass.

Safe grids are defined herein as those which do not contain the non-uniform strain field associated with a neck. Consistent application of this rule is difficult because the

extent of this non-uniform strain field varies for different necks. For instance, the boundary of safe grids is farther removed from a well-developed neck than from an incipient one. As well, drawing strain paths produce more diffuse necks than do plane strain or stretching conditions. Furthermore, the extent of non-uniform strain on either side of a given neck is different, with a smaller gradient zone occurring closest to the top of the dome. This last disparity is a consequence of the close proximity between necks and the punch-sheet contact point. Namely, the punch applies a frictional constraint on deformation close to the top of the dome. On a positive note, the silk-screened grids inherently delineate strain gradients because cracks in the ink widen as a function of increasing strain. All told, relatively repeatable classifications were obtained throughout.

3.3 Results

In this section, a synopsis of the data collected from uniaxial tensile tests and hemispherical punch tests is given. Interpretation of the observed mechanical behaviour, as it pertains to ductile fracture, is provided in the ensuing discussion.

3.3.1 Stress-Strain Curves

The complete set of stress-strain curves from the uniaxial tensile tests are presented in Figures 3.6 through 3.17. Instrumentation failures encountered during two of these tests explain the missing curves in Figures 3.6 and 3.16. From Figures 3.6 to 3.17, it is evident that the yield strengths and strain hardening rates are quite consistent amongst the

three repeated trials. While there is some variation in fracture strains between repetitions, the range is modest considering that the Al-Si alloy sheets were produced using laboratory-scale casting and rolling facilities. In the analysis of uniaxial tensile data, a 0.2% true strain offset was employed to determine yield stresses, while strain hardening coefficients were estimated from least-squares fits of stress-strain curves, employing the power-law hardening relationship in equation 1.4. For these calculations, Young's modulus was assigned a handbook value of 70 GPa [206]. Table 3.1 summarizes the resulting yield stresses and strain hardening coefficients for the Al-Si alloys, indicating a slight elevation in yield strength and strain hardening ability with increased Si content. As mentioned in Section 1.1.2, n tends to over-predict strain hardening abilities in aluminum alloys. However, this measure provides an adequate basis for comparison in the current study.

By selecting the intermediate stress-strain curves from each of the three repeat trials in Figures 3.6 to 3.17, direct comparisons in uniaxial tensile behaviour can be made for the Al-Si alloys. Figures 3.18 through 3.21 illustrate specimen orientation effects, revealing a strong divergence in strain to fracture between the three tested directions of the Al-2%Si unrefined sheet. This orientation effect is not as prominent in the other three alloys. From Figures 3.22 to 3.24, it is evident that the Sr-refinement process generally improves ductility in the transverse direction, at the expense of longitudinal ductility. For the Al-2%Si alloys, a substantial improvement in transverse strains to fracture is caused by the addition of Sr.

Gauge width and thickness data collected during uniaxial tensile tests also facilitates measures of normal anisotropy, as characterized by r values for the Al-Si alloys. This data is compiled in Table 3.2 for the three characteristic specimen orientations of each Al-Si

sheet, quoted at 15% gauge elongation. In all cases, the r_{45} values are elevated. Overall, the four alloys exhibit similar levels of normal anisotropy. The most noticeable divergence is that of the Al-2%Si Sr-refined alloy, which has a reduced r_0 compared to the other three sheets. Consequently, the r value is identical in both the longitudinal and transverse directions for the Al-2%Si Sr-refined sheet.

The strain hardening derivatives, $d\sigma/d\varepsilon$, are also included in the stress-strain plots of Figures 3.6 through 3.24 to give a general impression of diffuse and local necking behaviour. The predicted diffuse necking strains of the Al-Si alloy sheets are compiled in Table 3.3, as defined by the modified Considère criterion of equation 1.17. These predicted figures show that diffuse instability occurs at virtually identical levels of strain in all cases. It is also worth noting that the diffuse necking strains in Table 3.3 are very similar to the Considère strains obtained from equation 1.2 over the range of normal anisotropies exhibited by the Al-Si alloys. Therefore, the cross-over strain of the stress and derivative curves in Figures 3.6 to 3.24 is a reasonable indicator of diffuse instability. All told, the occurrence of diffuse necking does not appear to be greatly affected by normal anisotropy in the Al-Si alloys. Applying the modified local necking criterion of equation 1.18, uniaxial local necking strains can also be predicted. These predictions, compiled in Table 3.4., reveal a significant difference in anticipated local necking behaviour for the longitudinal and transverse orientations of the Al-2%Si unrefined alloy sheet. It is important to recognize, however, that the stress data used to calculate local necking predictions from equation 1.18 is somewhat questionable because the specimen gauge is undergoing non-uniform deformation prior to the onset of local necking. Presumably, a more reliable measure of failure behaviour is ductility, evaluated on the basis of failure strains in Table 3.5. The failure strain data indicates that the degree of failure anisotropy in

the Al-2%Si unrefined alloys is substantial, while the Al-4%Si unrefined sheet also exhibits a noticeable level of in-plane failure anisotropy. The emergence of more definitive in-plane anisotropies as deformation conditions progress from diffuse necking to final failure, in Tables 3.3 through 3.5, suggests that occurrence of failure is sensitive to the level of stress triaxiality. It is well established that stress triaxialities increase within a developing neck. The implication of a stress triaxiality effect, in the context of failure mechanisms, is elaborated upon in the upcoming discussion.

3.3.2 Forming Limit Diagrams (FLD's)

The FLD's constructed for the longitudinal and transverse directions of the four Al-Si alloy sheets are depicted in Figures 3.25 through 3.32, where longitudinal and transverse loading configurations are defined in Figure 3.2. The truncated forming limit curve in Figure 3.25 indicates that longitudinal necking could not be obtained beyond positive minor engineering strains of about 0.15 in the Al-2%Si unrefined alloy sheet because the transverse necking limit is profoundly low. In other words, premature transverse failures severely restrict the height of near-equi-biaxial domes in this particular alloy.

In agreement with the stress-strain ductility results in Table 3.5, the most prominent characteristic of the FLD data is the degree of in-plane forming limit anisotropy exhibited by the Al-Si unrefined alloys in Figures 3.33 and 3.35. Furthermore, the gap in longitudinal versus transverse behaviour is seen to progressively widen with larger tensile minor strains. As noted in the uniaxial tensile results, this diverging trend supports the sensitivity of failure behaviour to levels of stress triaxiality. Contrary to the unrefined

microstructures, formability in the Sr-refined sheets is virtually isotropic, as is illustrated in Figures 3.34 and 3.36. In fact, the degree of in-plane isotropy in the Al-2%Si Sr-refined sheet is so evenly balanced that separate equi-biaxial hydraulic bulge tests produced a longitudinal primary neck in one case and a transverse primary neck in another. Figures 3.37 and 3.38 underscore the differing formabilities of the four Al-Si alloys, indicating that the presence of Sr in these alloy sheets leads to isotropic forming limits and superior overall formability. Also evident in Figures 3.37 and 3.38 is the fact that forming limits increase directly with Si content in the unrefined sheets, while no benefit of this kind is realized in the Sr-refined alloys.

3.4 Discussion

From the uniaxial tensile and formability data, it is evident that the four Al-Si alloy sheets possess distinct mechanical properties. To evaluate the role of second-phase particle populations in these formability results, it is necessary to rationalize the observed behavioural trends in terms of microstructural features which promote failure. For this purpose, two basic classifications of microstructural features are considered. The first encompasses plastic anisotropy, while the second entails the proliferation of void damage.

3.4.1 Plastic Anisotropy

The potential influence of plastic anisotropy or crystallographic texture on the formability of the four Al-Si sheets can be implicitly established or discounted by

examining the characteristics of the relevant microstructural parameters described in Section 1.1.2. These signifying properties include normal anisotropy, diffuse necking strains, and grain size to sheet thickness ratios. Although direct measures of crystallographic texture and dislocation structures undoubtedly yield more explicit information regarding polycrystalline slip, these methods were deemed unnecessary in light of the relatively strong evidence provided by the available indicators.

The general state of crystallographic texture, and hence, the operation of crystallographic slip, is manifested in measures of normal anisotropy. It is reasoned that directional variations in normal anisotropy are dependent upon in-plane textural anisotropy while remaining independent of preferential orientation in second phases [15]. It is evident from Table 3.2 that the normal anisotropies do not vary greatly between the four Al-Si sheets, with one noticeable deviation in the r_0 value of the Al-2%Si Sr-refined alloy. In the Al-2%Si Sr-refined sheet, the values of r_0 and r_{90} indicate that the directional trends in normal anisotropy and in-plane formability closely resemble one another. This correspondence is not evident in the other three Al-Si sheets though, where r_0 values are slightly higher than r_{90} in every case but relative levels of longitudinal versus transverse formability are wide ranging. Therefore, it seems that the in-plane crystallographic textures of the Al-Si alloy sheets do not account for the observed variations in forming limit behaviour.

Diffuse necking behaviour is also considered a good indicator of crystallographic behaviour as well. It is widely accepted that plastic deformation is dominated by crystallographic slip up to the point of the Considère diffuse instability condition, after which microstructural inhomogeneities control the onset of local necking and final failure.

The similarities in diffuse necking strains compiled in Table 3.3 suggest that polycrystalline slip in the three measured orientations is achieved by comparable means in each of the four Al-Si microstructures. Again, this outcome suggests that the variations in forming limit data between the Al-Si sheets are not a result of differing mechanisms of crystallographic slip.

The last consideration, involving the ratio of grain size to sheet thickness, actually encompasses elements of crystallographic plasticity as well as microstructural inhomogeneities. Hence, this parameter is relevant to both diffuse and local necking instabilities. This dual classification stems from the potential for large grains to rotate during deformation, thereby causing a significant roughening of the sheet surface. While the rotation of grains provides favourable orientations for operational slip systems, a roughened surface inherently represents a field of reduced thickness inhomogeneities. The rate of surface roughening increases directly with grain size, thereby inducing through-thickness shearing in severe circumstances [27,59]. Regardless of the proper categorization of this effect, the possible influence of excessively large grains on the formability of the Al-Si alloy sheets must be addressed. It was previously noted in Table 2.2 that the through-thickness grain size of the Al-2%Si unrefined alloy is the largest of the four Al-Si alloys, while mechanical testing revealed that this particular sheet also exhibits the poorest formability. However, this apparent relationship is not reflected in the Al-2%Si sheet, which exhibits tremendous formability despite the fact it contains the second-largest grain size. Generally speaking, the mechanical test results suggest that the effect of surface roughness on sheet formability is minimal for the Al-Si alloys under study. While the sheets are all visibly scored in the rolling direction, their in-plane formabilities range from virtually isotropic to highly anisotropic.

3.4.2 Void Damage

In the Al-Si alloys under study, second-phase particles represent prominent microstructural inhomogeneities which intrinsically promote void nucleation, growth, and coalescence as a failure mechanism. Therefore, the potential role of void damage in the observed mechanical testing behaviour must be evaluated in light of the various second-phase particle populations contained within the Al-Si sheets. In Chapter 2, several distinguishing second-phase particle clustering features were identified, and hence, are referred to in the ensuing discussion.

The close correspondence between local necking and forming limits, and the pivotal role of microstructural inhomogeneities in the emergence of local necks are both relationships supported by ductile fracture research, as outlined in Section 1.1. According to the local necking strain thresholds in Table 3.4, little difference in local necking behaviour is exhibited by three of the four Al-Si alloys under uniaxial tension. The premature transverse localization of the Al-2%Si unrefined alloy is the only notable exception in this data. From a void damage standpoint, the strong and direct dependence of void growth rates on stress triaxiality is widely accepted (see Section 1.2.2). As a consequence, the extent of void nucleation, growth and coalescence prior to local necking is limited due to the relatively low levels of hydrostatic stress. Therefore, it is not surprising that the divergent characteristics of the four Al-Si particle populations are not clearly reflected in either the local necking data of Table 3.4 or the associated forming limits at compressive minor strains in Figures 3.37 and 3.38. Instead, distinct differences in failure behaviour begin to emerge as levels of stress triaxiality increase, as indicated by the failure strains of Table 3.5 or stretch formability data.

By definition, the progression from drawing to stretching strain paths requires remote stress states with increasing levels of stress triaxiality. It has been noted that void damage rates also rise in accordance with stress triaxiality. Therefore, the prominence of evolving void damage on strain localization in the Al-Si sheets is evidenced by the formability trends which emerge as imposed minor strains increase, moving from left to right on an FLD. The FLD's in Figures 3.37 and 3.38 demonstrate that the longitudinal stretching limits of the four Al-Si alloys have similar slopes, while the transverse slopes are very dissimilar. The divergence of behaviour with increasing triaxiality in Figure 3.38 supports a failure mechanism involving void damage. It is also apparent that the level of forming limit anisotropy in the four Al-Si alloys, depicted in Figures 3.33 through 3.36, is similar to the degree of preferential orientation for high aspect ratio first-order clusters, shown in Figure 2.28. Namely, the Al-2%Si unrefined alloy exhibits highly anisotropic formability, while it contains first-order clusters with the highest degree of preferential alignment in the longitudinal direction. Reiterating the argument of Chapter 2, that first-order clusters represent definitive microstructural features for void nucleation and growth, it appears that the formability behaviour of the Al-Si alloy sheets can be rationalized in terms of their distinct first-order cluster attributes.

The importance of preferential orientation of first-order clusters with high aspect ratios in relation to void damage rates has been demonstrated by Thomson [200] using a three-dimensional FEM unit cell model. In these studies, first-order clusters have been modelled as a linear arrangement of three particles, depicted in Figure 3.39. The geometry of Figure 3.39 corresponds to the stringers of broken particles typically associated with the Al-Si alloy first-order clusters, alluded to in Chapter 2. Regardless of loading direction, voids initiate at the particle poles in contact with inter-particle ligaments. However, overall

void damage rates are enhanced in first order clusters with their major axes lying perpendicular to the direction of major loading. Under these conditions, void nucleation is initially suppressed until sufficient tensile stresses develop in the inter-particle ligaments. Once nucleation occurs though, voids rapidly expand at the expense of the inter-void ligament. The opposite is observed when major loading is aligned with the major cluster axis. In other words, direct tensile stresses at the particle poles produce early void nucleation, but subsequent growth is minimal because the inter-void ligament is effectively shielded from further deformation by the remaining particles within the cluster. Therefore, major loading applied perpendicular to the first-order cluster axis induces a higher overall void damage rate, as is plotted in Figure 3.40. Different rates of void damage caused by the direction of major principal loading are manifested in the Al-Si sheets by the diminished transverse formability of the unrefined alloys. In these microstructures, the greatest preference for longitudinal orientation of high aspect ratio first-order clusters is found. The FEM analysis of Thomson [200] also indicates that the disparity in directional void damage rates of Figure 3.40 increases directly with stress triaxiality. This amplification of void damage explains the widening gap in formability with elevating components of minor strain. All together, these observations support a strong relationship between first-order cluster features and sheet formability in the Al-Si sheets, resulting from the nucleation and growth of voids.

One final aspect of the Al-Si formability data which pertains to void damage is the insignificant impact of Si content. The classical results of Edelson and Baldwin [72] assert that limit strains are a direct consequence of second-phase volume fractions and the associated number of void nucleation sites. In the current investigation, it has been suggested that void damage is closely associated with first-order clusters in the Al-Si

microstructures. In the four Al-Si sheets, however, the relationship between the number of first-order clusters (see Figure 2.25) and measured formabilities does not obey this classical theory. Instead, it seems that the spatial arrangement of voids, emanating from first-order clusters of the Al-Si microstructures, effectively overwhelms the influence of bulk porosity on the ensuing forming limits. Based on the agglomerated IPDS frequencies of Figure 2.30, it is proposed that the heavier degree of higher order clustering in the Al-2%Si alloys results in higher local porosities because voids are associated with first-order clusters. Furthermore, clustering of first-order clusters enhances void growth and coalescence rates. In the Al-2%Si unrefined alloy, accelerated void damage rates exacerbate the differences between longitudinal and transverse behaviour caused by preferentially oriented first-order clusters. On the other hand, the weak preference for alignment of first-order clusters in the Al-2%Si Sr-refined microstructure means that the intensification of void damage has little bearing on in-plane anisotropy. The notion that clustering of first-order clusters overwhelms the anticipated effect of bulk particle volume fraction on sheet formability further supports the definitive nature of first-order clusters in the four Al-Si alloys.

Chapter 4

Void Damage

The influence of various second-phase particle features on the evolution of void damage was evaluated by applying the quantitative metallography methods of Chapter 2 to deformed hemispherical punch specimens from the four Al-Si alloy sheets. In the resulting stereological data, individual voids are directly associated with their neighbouring particles. This correspondence facilitates an investigation of the relationship between void damage and physical characteristics of second-phase features, including the size and aspect ratio of both particles and first-order clusters. The stereological data also provides insight into the role of second-phase particle clustering on the evolution of voids, encompassing both first- and second-order clusters in the Al-Si microstructures under study. Unfortunately, accurate measures of void sizes and shapes could not be obtained, meaning that the interpretation of void damage data is primarily focused on void nucleation behaviour, as

indicated by the location of voids within the particle population. On the basis of qualitative metallographic observations, however, indications of void growth behaviour are noted and discussed.

4.1 Stereological Measurements

In the following sections, the metallographic and image processing procedures required to extract void damage data from undeformed specimens are outlined. A presentation of the statistical methods used to correlate void occurrences with definitive second-phase features is also included.

4.1.1 Metallographic Specimens

Stereological void data was collected from undeformed and deformed Al-Si microstructures to investigate the evolution of void damage with accumulating strain. Once again, metallographic specimens were prepared in the plane of the sheet to facilitate measurement of second phases in the plane of principal strains. Each deformed specimen was taken at a strain state just below the forming limit of a near-equi-biaxial dome. These near-equi-biaxial conditions represent the most balanced biaxial stretching paths obtained from the sheet formability testing methods employed in this research. These highly biaxial strain states were specifically chosen because they produce the most divergent orientation effects in formability, and presumably, the most prominent evolution of void damage. The stretch specimen for the longitudinal orientation of the Al-2%Si unrefined alloy has a

significantly lower minor strain than the other alloys since transversely aligned necks cannot be obtained in this particular sheet under near-equi-biaxial conditions, as explained in Section 3.3.2.

4.1.2 Void Detection

The metallographic preparation and image processing procedures employed in the void damage studies were identical to those outlined in Sections 2.2 and 2.3, respectively, with one additional step. Due to the shallowness of voids in the in-plane metallographic view, an excessive amount of light is reflected from the bottom of larger voids. Consequently, the contrast between voids and the surrounding matrix is very low. Close visual inspection of the optical micrograph in Figure 4.1 reveals the presence of several voids. However, the vast majority of these voids cannot be fully delineated by the multiple thresholding technique of Section 2.3.1 which relies upon distinct grey-scale intensities between component features. To partially alleviate this problem, two digital images were acquired from each metallographic field, with the first image focused on the particles and the second one focused on the voids. While particle fields were extracted exclusively from the first image, void fields were obtained from both images and then combined through the application of a logical OR. Figure 4.2 consists of the particle and void fields extracted from the metallographic image in Figure 4.1. Particles with adjacent voids were flagged for subsequent correlational analyses. While the repeatability of void detection is vastly improved by the acquisition of a second image, void perimeters remain incomplete, prohibiting repeatable measures of void sizes and shapes from being collected in the current research programme.

4.1.3 Measurement Confidence

The direct association of voids with particles in the Al-Si microstructures allows several correlations between void damage and prominent second-phase features to be evaluated. This task is accomplished with the aid of statistical confidence intervals assigned to various stereological measures.

In the stereological data, two different particle classifications are employed: particles located within first-order clusters versus particles not located within first-order clusters, and particles associated with voids versus particles not associated with voids. These classifications are both binomial in nature. Therefore, the probability of any one event occurring can be estimated by a normalized frequency, $\hat{p}$, given as:

$$\hat{p} = \frac{Y}{N} \quad (4.1)$$

where Y is the number of event occurrences and N is the total number of samples. A statistically significant difference between two independent measures of normalized frequency, $\hat{p}_1$ and $\hat{p}_2$, is satisfied by the absence of zero within the following confidence interval:

$$(\hat{p}_1 - \hat{p}_2) \pm z \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{N_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{N_2}} \quad (4.2)$$

where z is the z -distribution parameter for the selected confidence interval, and N_1 and N_2 are the sample sizes of the two independent measures. For the void damage analysis, a simplification of equation 4.2 is adopted to allow graphical evaluations of potential

correlations. In this modified approach, the confidence interval of each normalized frequency, $\hat{p}$, is calculated as:

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1 - \hat{p})}{N}} . \quad (4.3)$$

From equation 4.3, the statistical significance of dissimilar values is indicated by the degree of overlap in their respective confidence intervals. Although this approach does not produce exact measures of statistical significance for the chosen level of confidence, it provides a sound basis for a comparative study.

In the following presentation of results, error bars denote 95% confidence intervals calculated from equation 4.3. It is also recognized that the confidence interval defined by equation 4.3 is formulated for large sample sizes. Therefore, normalized frequency values obtained with less than ten samples are excluded from the stereological results.

4.2 Results and Discussion

Overall, the stereological data revealed the presence of considerable void damage in the undeformed Al-Si alloy sheets, presumably incurred during rolling, while deformed microstructures exhibited a moderate increase in the number of voids. In the following text, the interpretation of void damage data is carried out by correlating the occurrence of voids with distinguishing characteristics of their adjacent second phases. These defining characteristics include the size, aspect ratio, and clustering of second-phase features; parameters which are frequently tied to void damage behaviour in ductile fracture literature.

In this context, the term 'second-phase feature' refers to both individual particles as well as first-order clusters.

4.2.1 Size of Second-Phase Features

In general, the strong relationship between second-phase feature size and void occurrence in the four Al-Si alloys is exhibited by the stereological data. This fact is illustrated by Figures 4.3 to 4.5, where normalized void damage frequencies (equation 4.1) show the fraction of particles which possess neighbouring voids. Figure 4.3 contains measurements from the undeformed Al-Si microstructures, while Figures 4.4 and 4.5 indicate the levels of void damage in deformed microstructures at biaxial stretch forming limits. Figure 4.4 facilitates a comparison between unrefined and Sr-refined microstructures, while Figure 4.5 re-plots the same data in terms of longitudinal versus transverse forming limits. From these graphs, the affinity of voids for larger particles is clearly demonstrated. The fact that error bars increase directly with particle size is simply a reflection of the fact that there are fewer large particles. A comparison of Figure 4.3 with Figures 4.4 and 4.5 shows that the number of voids is increased by applied strain. For both Si content levels, it is also evident that the Sr refinement or major loading direction has relatively minimal effect on the occurrence of voids according to particle size. The only exceptional curves are those of the Sr-refined sheets at the longitudinal forming limit, where a reduced level of void damage is noted, particularly in the Al-2%Si Sr-refined alloy. Such an outcome implies that the isotropic formability of the Sr-refined sheets stems from a lower average void damage rate under major straining in the longitudinal direction. However, it is important to recognize that void nucleation or growth rates are not explicitly

indicated by this data. Figures 4.3 to 4.5 simply identify the state of void damage at undeformed and forming limit conditions.

Stereological void data has also been considered in relation to first-order clusters. Figures 4.6 through 4.8 plot void occurrences as a function of first-order cluster size. Similar to the particle size effect, voids are preferentially associated with larger first-order clusters in undeformed (Figure 4.6) and deformed (Figures 4.7 and 4.8) microstructures. As well, the extent of void damage for each Si content level does not vary greatly from unrefined to Sr-refined sheets or from longitudinal to transverse forming limits. Although a reduction of void damage is again observed in the Sr-refined alloys at their longitudinal forming limits, it is not as prominent as the difference observed in the above particle data. Given the rather small magnitudes of these noted variations, in relation to the diverse range of sheet formabilities between the four Al-Si alloys, it seems that their strain localization and failure behaviour is not explained by the total number of voids nucleated in these sheets. Instead, anisotropic formabilities are believed to result from directional variations in overall void damage rates, resulting from accelerated void growth under conditions in which major loads are applied perpendicular to the major axes of high aspect ratio first-order clusters. This directional void damage effect has been demonstrated by the FEM models of Thomson [200], as explained in Section 3.4.2. While Figure 4.3 to 4.8 indicate that both individual particles and first-order clusters exhibit similar size dependencies in their association with voids, the predominance of first-order clusters in the initiation of void damage is investigated further in Section 4.2.3.

4.2.2 Aspect Ratio of Second-Phase Features

Stereological measurements reveal that the aspect ratio of particles and first-order clusters is also a distinguishing characteristic in terms of void damage. Figures 4.9 through 4.11 depict these trends for individual particles in the four Al-Si alloy sheets, whereas Figures 4.12 to 4.14 plot void damage data as a function of first-order cluster aspect ratio. Figures 4.9 and 4.12 show that the distribution of voids as a function of particle aspect ratio in undeformed microstructures is consistent amongst each Si content pairing. As deformation proceeds towards the forming limit, Figure 4.11 demonstrates that transverse loading conditions promote the initiation of voids at high aspect ratio particles possessing longitudinal orientations. Such a trend is evident in all four Al-Si microstructures, while the opposite orientation effect is less prominent, presumably due to a reduced number of high aspect ratio particles oriented transversely. For individual particles, existing void nucleation theories contend that voids are more easily initiated from high aspect ratio particles when major loads are aligned with the major particle axis. The discrepancy between the current results and documented research implies that the majority of void-nucleating particles in Figure 4.11 are contained within first-order clusters, adding further support to the primary role of first-order clusters in the evolution of voids. Figures 4.12 through 4.14 show that voids emanating from first-order clusters exhibit a moderate affinity towards high aspect ratio clusters, with virtually no preference for orientation. This apparent lack of an orientation effect suggests that the occurrence of voids within first-order clusters at forming limits is primarily dictated by size, rather than aspect ratio in the Al-Si alloy sheets.

The relative importance of second-phase feature size over aspect ratio for the evolution of voids in the Al-Si alloy sheets is further indicated by a comparison of unrefined and Sr-refined microstructures. In Figures 4.10 and 4.13, the Sr-refined features lag behind the unrefined ones in their affinity for voids at all of the forming limits. Based on the size effect asserted in Section 4.2.1, it follows that this widespread reduction in the Sr-refined data stems from the fact that smaller features are present in each of the aspect ratio categories. For example, particles and first-order clusters with high aspect ratios tend to be the largest features of a given particle field, as noted in Section 2.4.1. In the case of the Sr-refined alloys, second-phase features are generally smaller than those found in the unrefined microstructures. Therefore, the average size in the high aspect ratio classification for the Sr-refined alloys is certain to be lower than the comparable unrefined value. This strong and direct relationship between void occurrence and feature size accounts for the disparity between unrefined and Sr-refined data in Figures 4.10 and 4.13. Such an outcome supports the contention that feature size is a more predominant factor with regard to void damage in the Al-Si sheets, while feature aspect ratio is indirectly linked to void damage through its correlation with feature size. The primary role of feature size is further supported by the relatively weak trends exhibited in Figures 4.9 through 4.14. Only very high aspect ratios produce elevated probabilities for association with voids, with four-fold increases in a few extremes cases. On the other hand, ten-fold elevations in void occurrence are common over a typical range of feature sizes in all four Al-Si microstructures. However, it should be clarified that these observations do not detract from the anticipated role of first-order cluster aspect ratios on directional void damage rates in the Al-Si alloy sheets.

As a point of clarification, even though the aspect ratio of particles and first-order clusters has been deemed a secondary parameter for void nucleation in the Al-Si microstructures, the stereological data does not reveal void damage rates and the influence of high aspect ratio first-order clusters on sheet formability. As alluded to in Section 3.4.2, Thomson [200] has shown that void nucleation is delayed in high aspect ratio clusters subject to major loads in a direction perpendicular to their major axes, with the subsequent acceleration of void growth resulting in higher overall void damage rates for this configuration. Furthermore, it is believed that the diminished impact of first-order aspect ratio on void nucleation in the Al-Si microstructures is a manifestation of the brittle, weakly-bonded Si particles. The aspect ratio of first-order clusters is conceivably more significant for the nucleation of voids from coherent second phases, where the strength of particles and particle/matrix interfaces are high.

4.2.3 Clustering of Second-Phase Features

In the four Al-Si alloy sheets, the clustering of second phases at two orders of magnitude have been related to the proliferation of voids: first-order clusters and second-order clusters. First-order clusters are defined as clusters of individual particles, while second-order clusters represent clusters of first-order clusters.

Thus far, the definitive nature of first-order clusters in the four Al-Si alloys has been implicitly asserted through the interpretation of several observed stereological and experimental trends. To provide explicit evidence of the primary role of first-order clusters on the nucleation of voids in the Al-Si alloys, correlations between void occurrence and first-order clusters have been explored. These results are graphed in Figures 4.15 through

4.18 as a function of particle size. In each case, the void data from Section 4.2.1 has been re-plotted as the bottom curve to indicate the relative levels of void damage throughout the selected range of particle sizes. Meaningful insight into void nucleation behaviour is gained from Figures 4.15 to 4.18 by a comparison of the top and middle curves. For any given particle size, the top curve signifies the probability for particle membership in a first-order cluster provided that a void is adjacent to the particle, while the middle curve denotes the probability for particle inclusion in a first-order cluster regardless of particle affinity with a void. Therefore, a statistically significant difference between these two measures demonstrates that particles of the given size require membership in a first-order cluster to substantially improve their potential for nucleating a void. From Figures 4.15 to 4.18, it is evident that smaller particles are much more likely to initiate voids if they are contained within first-order clusters. For larger particles, their association with void damage is not dependent upon their inclusion in a first-order cluster. Recalling the particle size data of Table 2.3, it appears that the threshold size at which inclusion in a first-order cluster becomes important is roughly one standard of deviation in excess of the mean particle size for each of the Al-Si alloys. These thresholds are marked on the graphs of undeformed strain states in Figures 4.15 through 4.18. This critical size estimate is adopted in subsequent analyses to define large isolated particles (i.e. large particles located outside first-order clusters). Overall, the results presented in Figures 4.15 to 4.18 signify that the void damage incurred during sheet fabrication and the subsequent formability testing is associated with first-order clusters and large isolated particles.

A comparison of the void damage data from undeformed and forming limit states in each of Figures 4.15 to 4.18 can also be used to deduce certain void nucleation and growth phenomena in the Al-Si sheets. The foremost of these observations entails the fact that

higher strains cause a decrease in the maximum particle size exhibiting a statistically significant correlation between cluster inclusion and void occurrence. This response indicates that continued void nucleation from individual particles with accumulating strain is preferential to the largest ones which have not yet initiated a void. Such an observation is in agreement with the work of Embury *et al.* [6,102,103] which proposes that the nucleation of voids progresses from larger to smaller particles as deformation proceeds. In general, the increased likelihood for individual particles to nucleate voids with the accumulation of strain is manifested by a drop in the upper curve for the forming limit graphs. This downward shift at the forming limit, evident in Figures 4.15 through 4.18, signifies a reduced importance for inclusion in a first-order cluster at elevated strain states. It is found that higher levels of strain cause the middle curve to move downward as well. Presumably, such an occurrence is due to the considerable expansion of voids within first-order clusters as a result of plastic flow. In other words, void growth amongst closely spaced particles tends to spread them out into a more uniform spatial arrangement, effectively erasing some first-order clusters. The accelerated growth of voids in first-order clusters is also implicitly supported by Figure 4.19, where stereological data from an additional Al-4% Si unrefined specimen is presented for direct comparison. This second specimen, which has been strained beyond the longitudinal forming limit, reveals that void nucleation has essentially ceased prior to the attainment of the forming limit, implying that the growth of voids is more predominant in this deformation regime. The significance of void growth is further reflected in Figure 4.19 by a continuing decline in the number of particles situated within first-order clusters at elevated strain states. Metallographic observations confirm that enhanced void growth occurs in first-order clusters of the Al-Si microstructures. However, quantitative void growth data was not obtained.

Based on the interdependence of feature size and aspect ratio noted in Section 4.2.3, it follows that the definitive nature of first-order clusters in the Al-Si alloys is also reflected by plots of void occurrence as a function of particle aspect ratio. These graphs are presented in Figures 4.20 to 4.23. For each of the Al-Si microstructures, it is found that particles with aspect ratios near unity require inclusion in first-order clusters to significantly increase their chances of nucleating a void. Once again, this outcome stems from the fact that smaller particles tend to have low aspect ratios. In relation to larger particles, only very high aspect ratio particles appear to initiate voids in isolation, with the threshold aspect ratio decreasing as strain accumulates. All told, these trends substantiate the pivotal influence of first-order clusters as well as feature size on void damage behaviour in the Al-Si alloy sheets.

As a final indication of the strong correlation between first-order clusters and evolving void damage, the voids captured by each dilation pass of the spatial tessellation process are plotted in Figures 4.24 to 4.26. For both the undeformed and deformed Al-Si microstructures, the majority of voids are included within the close spacings of first-order clusters. Beyond this first-order scale, a diminishing number of voids are added to the agglomerating features which correspond to higher order clusters. The general behaviour in Figures 4.24 to 4.26 is consistent with the theory of Argon *et al.* [117] which argues that the size and relative spacing of second-phase features are both determining factors in void nucleation. For the Al-Si microstructures, these two characteristics are simultaneously maximized within the range of first-order clusters, an effect elaborated upon in Chapter 6. It is also evident from a comparison of undeformed and deformed data in Figures 4.24 through 4.26 that the void damage amongst higher order clusters becomes more prominent as strain accumulates. This shift towards larger spacings is likely a direct result of

accelerated void growth within first-order clusters, a phenomenon noted earlier to explain the lower number of first-order clusters in deformed Al-Si microstructures. The percentage of void damage attributed to first-order clusters in each Al-Si sheet is summarized in Table 4.1. Table 4.1 also includes void data for large particles located outside of first-order clusters, referred to as large isolated particles. To reiterate, large isolated particles refer to particles located outside of first-order clusters with a size that exceeds the sum of the mean and standard deviation for the corresponding particle population, based on the data compiled in Table 2.3. These calculated size thresholds are estimates of minimum particle sizes which initiate voids regardless of their inclusion in first-order clusters. The totals in Table 4.1 illustrate that between 70-80% of void damage is encompassed by these two distinguishing second-phase features. However, the overall impact of first-order clusters is undoubtedly heightened by the potential for accelerated void growth caused by interactions of closely spaced particles. On the other hand, the significance of large isolated particles on void growth and coalescence behaviour is dependent upon their membership in second-order clusters.

In relation to second-order clusters, the declining rate of void association beyond the magnitude of first-order clusters in Figures 4.24 to 4.26 is manifested by a lack of correlation between second-order clusters and void occurrences, depicted in Figures 4.27 through 4.30. Although the generally low number of second-order clusters in the Al-2%Si microstructures inherently increases the variability of void data, particularly in the undeformed cases, there is little evidence of a correlation between void damage and second-order clusters in Figures 4.27 through 4.30. Therefore, clustering at the level of second orders is considered irrelevant for void nucleation behaviour in the four Al-Si alloy sheets. Instead, these features are likely to be influential in terms of the subsequent growth and

coalescence of voids. As noted earlier, extensive void growth is observed within first-order clusters, establishing the potential for growth and coalescence interactions between these primary nucleation sites. Qualitative metallographic results, however, reveal no evidence of void coalescence between first-order clusters until localized necking is thoroughly established, corresponding to strain states well beyond forming limits. This observation suggests that the impact of second-order clusters on sheet formability is primarily within the realm of void growth.

Chapter 5

Biaxial Stretching Model

From the stereological and experimental investigations of the preceding chapters, several distinguishing characteristics of void damage in the four Al-Si sheets have been employed as a basis for relating second-phase particle clustering to sheet formability. Some notable observations pertaining to second-phase particle clustering include:

- i) the varying degrees of second-phase particle clustering in the Al-Si microstructures, particularly at higher orders, ranging from heavily clustered in the Al-2% alloys to moderately clustered in the Al-4% sheets;
- ii) the distinct tendency for alignment of high aspect ratio first-order clusters parallel to the rolling direction in unrefined particle

populations, while far less preference is exhibited in Sr-refined fields;

- iii) the strong correlation between first-order clusters and void damage, including voids present in the undeformed microstructure as well as those initiated during deformation.

In terms of sheet formability, the most predominant experimental finding is the dramatic proportion of stretching anisotropy exhibited by unrefined alloy sheets, whereas the Sr-refined microstructures have proven virtually isotropic in this regard. Anisotropic behaviour is manifested in the unrefined sheets by severe reductions in stretch forming limits under major transverse loads. Notwithstanding the overall detriment to sheet formability, an anisotropic in-plane response is generally considered unacceptable for a commercial forming application. As a result, it is of primary interest to further investigate the factor which lead to this anomalous deformation behaviour.

Thus far, stereological void data has been used to implicitly establish a link between sheet formability and the size, aspect ratio, orientation, and spatial distribution of first-order clusters in the Al-Si alloy sheets. In particular, these arguments contend that the divergent in-plane anisotropies exhibited by the Al-Si sheets are accounted for by measured differences in preferential orientation of first-order clusters because void damage is concentrated within first-order clusters. Therefore, an analytical model is developed here to pursue validation of this proposed dependence and to gain additional insight into other microstructural influences on stretching behaviour. It is important to emphasize that the adopted numerical formulation is not purported to be a rigorous predictive model, but rather an analytical demonstration of the direct correspondence between void damage evolving

within first-order particle clusters and forming limit behaviour on the right-hand side of an FLD.

Given that strain localization along stretching strain paths is precipitated by a shift towards incremental plane strain (equation 1.11), this model is designed to calculate strain path deviations in a deforming elemental sheet. The formulation is based upon the Gurson-Tvergaard constitutive softening model [65-68] reviewed in Section 1.2.4, applying the modification of Nagaki *et al.* for anisotropic void fields [157,158]. Void damage is assumed to nucleate at first-order clusters as a function of cluster size and level of effective plastic strain, employing a scheme similar to that proposed by Embury *et al.* [6,102,103] and illustrated in Figure 1.12.

General operation of the proposed model entails the seeding of voids into first-order clusters of biaxially stretching sheets, utilizing the acquired second-phase particle fields, clustering measures, and void data of Chapters 2 and 4. Predicted plastic strain paths are then compared with observed trends in sheet formability from Chapter 3.

5.1 Model Configuration

The biaxial stretching model is designed to represent a field of in-plane voids nucleating within first-order particle clusters and evolving in accordance with applied biaxial stretch loads. Based on the extent of void damage observed in Al-Si microstructures deformed to the forming limit, the nucleation and growth of voids is restricted to the level of first-order clusters. Hence, this analysis focuses on the relationship between first-order clusters and sheet formability, neglecting the influence of

isolated particles and higher order clustering. With regard to void coalescence, it is considered a non-factor in the forming limit behaviour of the Al-Si alloy sheets. As alluded to in Section 4.2.3, metallographic observations revealed no physical evidence of coalescence between first-order clusters in localized Al-Si microstructures.

5.1.1 Geometry

The model geometry is based upon a voided sheet of unit thickness, as shown in Figure 5.1. The voids themselves are directly associated with first-order cluster fields acquired from undeformed in-plane microstructures, as detailed in Section 5.3.1 on void nucleation. Each individual void is idealized as an elliptical hole with aspect ratio, κ , given by:

$$\kappa = \frac{a_1}{a_2} \quad (5.1)$$

where a_1 and a_2 are the principal elliptical dimensions shown in Figure 5.1. The overall aspect ratio of void damage is represented by a single elliptical void in an initially square sheet, as illustrated in Figure 5.2. The aspect ratio of this globally representative void, κ^∞ , is obtained from a sum of individual void dimensions, according to:

$$\kappa^\infty = \frac{\sum_{i=1}^{n_v} (a_1)_i}{\sum_{i=1}^{n_v} (a_2)_i} \quad (5.2)$$

where n_v is the number of voids. The principal elliptical dimensions of the global void, a_1^{∞} and a_2^{∞} , are then calculated in a manner which reflects the total area of the void field, such that:

$$a_1^{\infty} = \sqrt{\frac{4\kappa^{\infty}}{\pi t} \sum_{i=1}^{n_v} \vartheta_i} ; \quad a_2^{\infty} = \frac{a_1^{\infty}}{\kappa^{\infty}}. \quad (5.3)$$

where ϑ is the volume of a void and t is the sheet thickness. It is important to note that the summation approach of equation 5.2 signifies that the spatial arrangement of void damage is not considered beyond the scope of first-order clusters. Once again, the model seeks to examine the effect of preferentially oriented first-order clusters on the overall compliance of a deforming sheet, and therefore, does not attempt to include the impact of higher order clustering.

Based on the configuration of the voided sheet in Figure 5.2, the damage effect parameter, Ψ_i , of equation 1.32 is established as:

$$\Psi_i = \left[1 - \left(\frac{a_j^{\infty}}{L_j} \right) \right]^{-1} \quad (5.4)$$

where L_j is the sheet dimension in the direction perpendicular to Ψ_i . For the current investigation, the stress concentration parameter, c , of equation 1.32 is set to unity because the underlying analytical objective is to simply establish relative levels of in-plane anisotropy. A similar justification was put forth by Nagaki *et al.* [157,158] for a comparable study with this damage tensor formulation.

5.1.2 Loading

Deformation in the model sheet is induced by the application of direct principal stress rates, as is shown in Figure 5.2. The ratio of principal stress rates is kept constant throughout each deformation trial. Although these load-controlled boundary conditions are not truly representative of sheet forming operations, they are easily applied to the constitutive equations of the model and provide a suitable framework for investigating in-plane anisotropy.

The principal net stresses, S_i , are determined directly from the damage effect parameter, Ψ_i , following the relationship of equation 1.33, repeated here as:

$$S_i = \Psi_i \sigma_i \quad (5.5)$$

where σ_i is a principal applied stress.

5.2 Constitutive Equations

The elastic-plastic constitutive equations adopted in this numerical formulation are derived from fundamental principles of continuum mechanics.

5.2.1 Elastic Equations

The elastic component of deformation is modelled using Hooke's law for linear isotropic elastic solids, given by:

$$d\varepsilon_{ij}^e = \frac{1}{2G} d\sigma_{ij} + \left(\frac{1}{3K} - \frac{1}{2G} \right) \frac{1}{3} d\sigma_{kk} \delta_{ij} \quad (5.6)$$

where ε_{ij}^e is the elastic component of the strain tensor, σ_{ij} is the applied stress tensor, G and K are the respective shear and bulk moduli, and δ_{ij} is the Kronecker delta.

5.2.2 Plastic Equations

The development of plastic constitutive relations for this analysis comprises a derivation of the Prandtl-Reuss equations. These equations effectively relate an applied stress increment to its resulting strain increment through the elementary principles of a yield function, hardening rule, and flow rule.

The yield function, Φ , incorporated into the stretching sheet model is that of the Gurson-Tvergaard constitutive softening model, is written in terms of the net stress tensor, S_{ij} , defined in equation 5.5, such that:

$$\Phi = \left(\frac{S_{eq}}{\bar{\sigma}_M} \right)^2 + 2fq_1 \cosh \left(q_2 \frac{S_{kk}}{2\bar{\sigma}_M} \right) - 1 - q_3 f^2 = 0 \quad (5.7)$$

where S_{eq} is the equivalent net stress, $\bar{\sigma}_M$ is the local matrix flow stress and f is the current level of porosity. The Tvergaard coefficients [67,68] are specified with the settings of Becker *et al.* [154] and Worswick [155], such that $q_1=1.25$, $q_2=0.95$, and $q_3=q_1^2$. The equivalent net stress, S_{eq} , is defined by:

$$S_{eq} = \sqrt{\frac{3}{2} S_{ij}' S_{ij}'} \quad (5.8)$$

where S_{ij}' is the deviatoric component of net stress.

Employing a fundamental approach of plasticity theory, a set of plastic constitutive equations are derived from the flow potential of equation 5.7 by invoking the principle of normality [65,66], such that:

$$d\varepsilon_{ij}^p = \frac{1}{H} \left(\frac{3S_{ij}'}{2\bar{\sigma}_M} + \alpha \delta_{ij} \right) \left(\frac{3S_{ij}'}{2\bar{\sigma}_M} + \alpha \delta_{kl} \right) dS_{kl} \quad (5.9)$$

where α and H are defined as:

$$\alpha = \frac{f}{2} q_1 q_2 \sinh \left(q_2 \frac{S_{kk}}{2\bar{\sigma}_M} \right) \quad (5.10a)$$

$$H = \frac{h}{1-f} \beta^2 - \left[3\alpha \bar{\sigma}_M (1-f) + \frac{A \bar{\sigma}_M}{1-f} \beta \right] \left[q_1 \cosh \left(q_2 \frac{S_{kk}}{2\bar{\sigma}_M} \right) - q_3 f \right] \quad (5.10b)$$

$$\beta = \left[\left(\frac{S_{eq}}{\bar{\sigma}_M} \right)^2 + \alpha \frac{S_{kk}}{\bar{\sigma}_M} \right] \quad (5.10c)$$

In equation 5.10, A is a void nucleation parameter specified by:

$$A = \frac{1}{L_1 L_2 d\bar{\varepsilon}^p} \sum_{i=1}^{n_v} (d\vartheta_{nucleation})_i \quad (5.11)$$

where $d\vartheta_{nucleation}$ is an increment in void volume caused due to nucleation, defined in Section 5.3.1. For equation 5.10, the hardening parameter, h , is obtained from the following relationship:

$$h = \frac{d\bar{\sigma}_M}{d\bar{\varepsilon}_M^p} \quad (5.12)$$

where $\bar{\sigma}_M$ is the flow stress in the matrix and $\bar{\epsilon}_M^p$ is the effective plastic strain in the matrix. Invoking the principle of work equivalence, the effective plastic strain in the matrix, $\bar{\epsilon}_M^p$, is determined directly from:

$$d\bar{\epsilon}_M^p = \frac{S_{ij}}{(1-f)\bar{\sigma}_M} d\epsilon_{ij}^p \quad (5.13)$$

where ϵ_{ij}^p is the plastic strain tensor. To evaluate equation 5.11, an expression for strain hardening behaviour must first be established. Assuming that the matrix material obeys power-law hardening, the stress-strain behaviour in uniaxial tension is of the form:

$$\frac{\bar{\sigma}_M}{\sigma_y} = \left(\frac{\bar{\epsilon}_M}{\epsilon_y} \right)^n \quad (5.14)$$

where σ_y and ϵ_y are the respective principal stress and strain at yield. By considering the effective strain as a simple sum of elastic and plastic components, equation 5.13 is mathematically manipulated to produce the following differential:

$$\frac{d\bar{\sigma}_M}{d\bar{\epsilon}_M^p} = E \left[\frac{1}{n} \left(\frac{\bar{\sigma}_M}{\sigma_y} \right)^{\frac{1}{n}-1} - 1 \right]^{-1} \quad (5.15)$$

where E is Young's modulus. Evaluation of equations 5.12 and 5.15, employing the uniaxial tensile test data of Table 3.1, produces a hardening parameter, h , as a function of matrix flow stress, $\bar{\sigma}_M$.

5.3 Void Damage

Central to the operation of this biaxial stretching model is the evolution of void damage and its overwhelming influence on the principal compliances of the deforming sheet. Therefore, it is imperative that the nucleation, growth and coalescence of voids

proceeds in a manner which is truly representative of the model Al-Si alloy sheets studied in this programme.

To incorporate void damage rates into the numerical formulation, the governing equation for porosity, f , is taken from the Gurson-Tvergaard model (equation 1.34), but expressed as a sum of individual voids, such that:

$$df = K_v \frac{1}{L_1 L_2} \sum_{i=1}^{n_v} [(d\vartheta_{nucleation})_i + (d\vartheta_{growth})_i] \quad (5.16)$$

where $d\vartheta_{nucleation}$ and $d\vartheta_{growth}$ are incremental changes in the volume of individual voids caused by nucleation and growth. In equation 5.16, K_v is a global porosity scaling factor which can be altered in separate predictions to investigate the influence of bulk porosity.

5.3.1 Void Nucleation

It is assumed that the stereological void nucleation data of Chapter 4 provides a sound basis for proper seeding of voids into the model. Given that roughly 60% of void damage in the Al-Si microstructures is associated with first-order clusters and that first-order clusters are found to enhance void growth, the proposed model contends that void nucleation be driven by these features, as opposed to individual particles. The rate of void nucleation is governed by both the level of effective plastic strain and the size of first-order clusters. Again, this concept is one suggested by Embury *et al.* [6,102,103], whereby nucleation progresses from larger to smaller particles as a function of effective plastic strain. In the current study though, the availability of measured void nucleation data from Section 4.2 allows the progression of nucleation events from larger to smaller features to be

driven by calculated probabilities, linking void nucleation to first-order clusters through cluster size. These probabilities are plotted in Figures 5.3 through 5.5 for the Al-Si alloys, corresponding to the undeformed state, the longitudinal forming limit, and the transverse forming limit, respectively. In Figures 5.3 to 5.5, the error bars delineate a 95% confidence interval for binomial data (see Section 4.1.3), with linear ‘least-squares’ regressions fit to the upper bounds of confidence. The upper bound values are adopted in the current modelling strategy to reflect a worst-case scenario. They are collectively graphed in Figure 5.6 and summarized parametrically in Table 5.1. In the absence of data for intermediate strains, void nucleation probabilities are assumed to vary linearly with effective plastic strain in the model.

The probability values of Table 5.1 are incorporated into the numerical model by nucleating voids as a fraction of every first-order cluster, where the fraction itself is a function of the void nucleation probability, p , for the given cluster. Therefore, the initial increment of void volume nucleated by each first-order cluster, $(d\mathcal{V}_{nucleation})_0$, is specified as:

$$(d\mathcal{V}_{nucleation})_0 = V_{foc} p_0 \quad (5.17)$$

where V_{foc} is the volume of the first-order cluster, and p_0 is the initial void nucleation probability for the cluster. Values of p_0 for equation 5.17 are calculated by employing the first-order cluster size, V_{foc} , in the appropriate probability relationship from Table 5.1. The aspect ratio of each nucleated void, κ , is equated to that of its associated first-order cluster to determined principal elliptical dimensions for use in equation 5.2.

During deformation, continued nucleation of voids is realized through the constant variation of void nucleation probabilities with effective plastic strain in the matrix, producing an increment in void volume, $d\vartheta_{nucleation}$, given by:

$$d\vartheta_{nucleation} = V_{foc} dp \quad (5.18)$$

In the biaxial stretching model formulation, it is assumed that the aspect ratio of individual voids, κ , is unaltered by the application of equation 5.18. However, the global void aspect ratio, κ^* , adjusts accordingly to equation 5.2. Incremental void nucleation probabilities for equation 5.18, dp , are obtained using the following linear interpolation scheme:

$$dp = \frac{(p_f - p_o)}{(\bar{\epsilon}_M^p)_f} d\bar{\epsilon}_M^p \quad (5.19)$$

where p_o and p_f are initial and final void nucleation probabilities and $(\bar{\epsilon}_M^p)_f$ is the effective plastic strain at the final nucleation condition in Table 5.1,. From the Gurson-Tvergaard constitutive equations, the increment of effective plastic strain in the matrix, $d\bar{\epsilon}_M^p$, is calculated on the basis of energy equivalence as:

$$d\bar{\epsilon}_M^p = \frac{S_{ij}}{(1-f)\bar{\sigma}_M} d\epsilon_{ij}^p \quad (5.20)$$

Inherent in equations 5.18 and 5.19 is the fact that the proposed nucleation strategy partially nucleates voids at all clusters, rather than inducing complete nucleation at a fraction of potential first-order cluster sites. This latter criterion is common to more elegant and sophisticated approaches for investigating stochastic behaviour, such as repeated trials with a Monte Carlo style void nucleation strategy. Such techniques were considered unnecessary for the present study in light of the general trends being sought for in-plane compliance.

5.3.2 Void Growth

While comprehensive experimental data facilitates a representative algorithm for void nucleation, such is not the case for void growth. As explained in Chapter 4, conventional metallographic and stereological methods failed to produce reliable measures of void growth from deformed sheet microstructures. Because of this deficiency, void growth in this numerical analysis is simulated through equation 1.36 from the Gurson-Tvergaard model, applied directly to individual voids, such that:

$$d\vartheta_{growth} = (L_1 L_2 - \vartheta) d\varepsilon_{kk}^p \quad (5.21)$$

where ε_{ij}^p is the plastic remote strain tensor. In the proposed formulation, it is assumed that the void growth dictated by equation 5.21 does not affect the aspect ratio of individual voids, κ . Instead, global void damage aspect ratio, κ^∞ , implicitly develops through the relationship described by equation 5.2. The FEM solutions of Thomson [200], which predict the effect of void aspect ratio on growth, were not considered in this analysis.

5.3.3 Void Coalescence

For the mechanism of void coalescence, metallographic observations alluded to in Section 4.2 suggest that coalescence events between first-order clusters only occur in the Al-Si sheets at strain states beyond forming limits. Therefore, void coalescence criteria are not included in the biaxial stretching model.

5.4 Numerical Integration

The general operation of the biaxial stretching model entails a time-integration of the formulated constitutive equations, utilizing the first-order cluster data extracted from the undeformed Al-Si microstructures in Chapter 2 in conjunction with the void nucleation probabilities of Table 5.1. Selected material parameters from Table 3.1 are also required in the numerical formulation.

For the sake of mathematical convenience, a simple forward integration method without iteration is adopted for the current analysis, even though the constitutive equations are susceptible to numerical instabilities at higher strains. These instabilities arise as increased levels of porosity exhaust the strain hardening capacity of the material, leading to strain softening. Hence, rigorous calculations with the Gurson-Tvergaard model usually employ complex numerical integration schemes to overcome this problem. For example, a backwards-Euler [207] and a ‘sub-increment, elastic predictor-normal corrector’ scheme [208] have both proven effective in this regard. Nonetheless, forward integration is considered a suitable solution for the current analytical exercise since appreciable levels of deformation are achieved before the constitutive equations become intractable.

Recalling the plastic instability principles outlined in Section 1.1.1, the tendency for local necking under biaxial stretching is indicated by a deviation in strain path towards plane strain (equation 1.11). In Figure 5.7, the right-hand side FLD data from Section 3.3.2 is re-plotted on a common set of axes for each Al-Si alloy sheet to combine longitudinal and transverse stretching limits. This general FLD configuration is illustrated in Figure 5.8, effectively combining the longitudinal and transverse FLD configurations of Figure 3.2. By adopting this composite representation in Figure 5.7, in-plane stretching

anisotropy can be easily explained in terms of deviating strain paths. Figure 5.9 schematically illustrates the expected strain paths in an isotropic sheet, while a typical response for an anisotropic sheet is shown in Figure 5.10. It is evident from a comparison of Figure 5.7 with Figures 5.9 and 5.10 that the deviation of equi-biaxial and near-equi-biaxial strain paths dictates the degree of in-plane anisotropy present in the four Al-Si alloy sheets. Therefore, the biaxial stretching model implicitly demonstrates anisotropic in-plane behaviour by predicting deviations in strain path as a function of applied strains.

5.5 Results and Discussion

The in-plane deformation response of the Al-Si alloys is investigated in this section by examining the results from two types of numerical simulations. The first series of calculations is performed along an equi-biaxial stress path to assess the overall tendency in each sheet for strain path deviation towards longitudinal or transverse necking. The second series predicts necking preferences caused by near-equi-biaxial stress paths.

5.5.1 Equi-Biaxial Stretching

Under equi-biaxial loading, the biaxial sheet model was applied to each of the four Al-Si alloys. In each trial, the sheet was seeded with the appropriate longitudinal and transverse nucleation conditions from Table 5.1. This initial analysis was carried out with the global porosity scaling factor of equation 5.16 set to one (i.e. $K_g = 1$), meaning that a first-order cluster with a void nucleation probability of 1.0 initiates a void comprising the

entire size of the cluster. While such a high setting undoubtedly produces an excessive amount of porosity, it facilitates a preliminary comparison of in-plane anisotropies. The resulting deviations in strain path are depicted in Figure 5.11, where the percent deviation in strain path is taken with respect to the first plastic strain increment after yielding. Engineering strains are employed to maintain consistency with the FLD format. The signs of strain path deviations in Figure 5.11 are important because they indicate the direction of potential necking, as was shown schematically in Figures 5.9 and 5.10. Consequently, the negative strain path changes in Figure 5.11 indicate a tendency in all four Al-Si alloys to form necks running parallel to the longitudinal direction under equi-biaxial conditions. This trend is reflected in the experimental FLD data, compiled in Figure 5.7, which shows that equi-biaxial strain paths generally deviate towards transverse forming limits. Furthermore, the relative levels of in-plane anisotropy revealed by the model are also similar to the experimental results in Figure 5.7. Both unrefined alloys show a strong preference for premature transverse failure, while the tendency for the Al-2%Si Sr-refined sheet to fail in any particular direction is much weaker. In Section 3.3.2, it was noted that the in-plane isotropy of the Al-2%Si Sr-refined alloy is particularly strong, exhibiting primary necks in both principal directions in separate equi-biaxial hydraulic bulge test specimens.

One inconsistency between the predictions of Figure 5.11 and the experimental data of Figure 5.7 is that the Al-4%Si alloys actually exhibit moderately higher degrees of in-plane anisotropy than expected. For example, the strain path deviations predicted for the Al-4%Si unrefined sheet are slightly more divergent than those of the Al-2%Si unrefined alloy. By considering the respective levels of volume fraction and aspect ratio in the evolving void populations, shown in Figures 5.12 and 5.13, it appears that the in-plane

anisotropy in both of the Al-4%Si sheets is overly enhanced by higher values of bulk porosity. A straightforward explanation for this discrepancy suggests that a lower global porosity scaling factor, K_g , is applicable for the Al-4%Si alloys, implying that the size ratio between voids and associated first-order cluster is lower in these microstructures. Unfortunately, the inability to accurately measure void sizes in the Al-Si microstructures in the current research programme leaves the validity of this argument in question. Nonetheless, it is believed that the most plausible explanation for this anomalous behaviour stems from the omission of higher order clustering in the current model formulation. While the addition of higher order clustering to the biaxial stretching model is expected to produce a minimal elevation in overall growth rates, the impact on spatial variability of damage is likely to be substantial. It is reasoned here that the consideration of void damage and plastic deformation at appropriate local regions within the sheet would ultimately produce better representations of 'worst-case' behaviour which precipitates strain localization in more heavily clustered microstructures, such as those of the Al-2%Si alloys. This point is left to further discussion in Chapter 6.

To assess the influence of bulk porosity on strain path deviations, two additional sets of calculations were performed using the void nucleation settings of Table 5.1, but with values of 0.1 and 0.5 assigned to K_g . The sensitivity of strain path deviations to bulk porosity is evident in Figures 5.14 and 5.15. Regardless of global porosity, however, the same anisotropic trends are still present at low void volume fractions, even when the void volume fraction remains below 0.01 throughout the deformation history, as is the case when K_g is 0.1.

5.5.2 Near-Equi-Biaxial Stretching

The near-equi-biaxial stress path employed for this set of predictions produces an initial plastic engineering strain path, ρ_e , of about 0.9. This nominal strain path is typical of the most balanced plastic stretching ratios generated in square sheet blanks during hemispherical punch testing. As mentioned in Chapter 3, frictional constraint between the punch and sheet prohibits truly equi-biaxial strain paths from being applied. Two load cases are investigated for each of the four Al-Si alloy sheets, corresponding to major principal loading in the longitudinal and transverse directions. Nucleation conditions are selected from Table 5.1 accordingly.

The simulated deformation response under near-equi-biaxial conditions is presented in Figure 5.16. It is important to recognize that the constant near-equi-biaxial stress paths induced by the model do not produce constant strain paths, except in the equi-biaxial case. Therefore, strain path deviations were also obtained for identical first-order cluster fields and nucleation schemes in each case, but with the global void aspect ratio, κ^∞ , fixed at one. These curves are included in Figure 5.16 to denote the isotropic baseline for comparison purposes. Once again, the model reveals in-plane anisotropies for the Al-Si sheets that are relatively consistent with the stretching FLD data. Similar to the equi-biaxial results though, the anisotropic trends in the Al-4%Si alloys are generally excessive in comparison to the Al-2%Si results, an inconsistency attributed to the neglected effect of higher order clustering. The incorporation of higher order clustering effects into numerical forming limits models is addressed in Chapter 6.

Applying the same approach as with the equi-biaxial stretch modelling, a subsequent set of calculations were compiled with K_ϕ settings of 0.1 and 0.5. Figures

5.17 and 5.18 depict the strain path deviations and porosity levels from these near equi-biaxial trials. The most prominent feature in Figure 5.17 is the strong potential exhibited by the unrefined sheets for transverse strain path deviations under longitudinal loading, a tendency that is exacerbated with increased levels of void damage. Figure 5.19 shows that the possibility of forming abnormally oriented necks decays as the initial plastic strain path recedes from an equi-biaxial state. As a consequence, these predictions suggest that a changeover from longitudinal to transverse necking occurs at an intermediate plastic strain path in the unrefined sheets. This outcome further supports the correspondence between the in-plane forming limits of the unrefined sheets in Figure 5.7 and the anisotropic schematic depicted in Figure 5.10. In fact, this type of behaviour is so prominent in the Al-2%Si unrefined alloy sheet that premature necks aligned in the longitudinal direction prevent longitudinal engineering strain paths, ρ_e , above 0.4 from producing the expected transverse orientation of necks (see Section 3.3.2). Of final note from Figure 5.19, the strain path deviations of all four Al-Si alloys begin to coincide as the applied strain path ratio decreases towards plane strain. This occurrence is caused by the imposed ratio of principal strains overwhelming the anisotropy of emerging void damage, consistent with the narrowing differences in stretch forming limits between the Al-Si sheets as minor strains decrease in Figure 5.7.

Chapter 6

Discussion and Future Direction

The effect of second-phase particle clustering on the formability of four Al-Si alloy sheets has been highlighted in this thesis through its direct relation with the evolution of voids in deforming microstructures. Stereological and experimental results indicate that the formability behaviour of the Al-Si sheets is sensitive to the size, aspect ratio, orientation, and spatial distribution of first-order clusters. While these findings comprise the major contribution of this research, a numerical investigation was also undertaken. In general, the biaxial stretching model presented in Chapter 5 confirms that the forming limits of the Al-Si sheets can be explained in terms of the noted second-phase particle clustering characteristics. However, a few inconsistencies in model behaviour raise some fundamental questions regarding the prediction of sheet formability from digitally acquired

particle fields. These outstanding issues are grouped into three topics for discussion in this chapter: strain localization, local damage formulation, and critical particle sub-fields.

6.1 Strain Localization

Overall, the biaxial stretching model provides a meaningful demonstration of in-plane sheet deformation behaviour by predicting deviations in plastic strain paths. Given the small magnitudes of these deviations, however, their direct association with forming limits is unclear, recalling the incremental plane strain localization condition of equation 1.11. In essence, these rather subtle deformation responses are a direct manifestation of the biaxial stretching model configuration. Namely, the pertinent void damage and deformation variables are calculated on a global scale in the proposed model, meaning that their values are relatively dilute in comparison to those concentrated within the neck of a strain localization model, such as the Marciniak-Kuczynski approach [21,22] outlined in Section 1.1.1. The physical framework of an M-K analysis facilitates forming limit predictions through the localization of strain within a band of weakened material.

Extension of the biaxial stretching model into an M-K analysis is achieved by representing the microstructural inhomogeneity of the defect band with a digitally acquired first-order clusters field. Specifically, the defect band consists of a random first-order cluster field, while the surrounding bulk contains a comparable area fraction of equiaxed first-order clusters in a uniform spatial distribution. By adopting the anisotropic form of the Gurson-Tvergaard constitutive softening model [65-68,157,158], anisotropic yield behaviour within the defect band leads to the localization of strains, thus differing from the

reduced thickness effect of a classical M-K implementation. Since the M-K configuration places the defect band under a combined set of load and displacement boundary conditions, an iterative algorithm is required to integrate the constitutive equations.

While adoption of an M-K approach is likely to produce strain localization, the modelling results of Chapter 5 suggest that the accuracy of forming limit predictions for the Al-Si alloy sheets is largely dependent upon representative modelling of spatial variations in deformation caused by clustering of second-phase particles. Therefore, the relationship between evolving void damage and in-plane deformation must be rigorously evaluated at a local level throughout the sheet, indicating the need for a local damage formulation. The following section examines a possible strategy for incorporating a local damage formulation into a numerical model.

6.2 Local Damage Formulation

In the biaxial stretching model, an anisotropic form of the Gurson-Tvergaard constitutive softening model [65-68,157,158] was utilized to link in-plane deformation response with the size and shape of voids evolving within the sheet. Since the constitutive calculations were carried out on a global scale, the anticipated harm of locally accelerated void damage rates, due to local elevations in porosity level and void-void interactions, was not reflected in the predicted results for the more clustered Al-Si microstructures. It is believed that this modelling inconsistency highlights the need for a local damage formulation which simultaneously accounts for in-plane deformation as well as the proliferation of voids. The implementation of such a numerical method is explored in this

section by first introducing the concept of damage percolation, followed by a proposed methodology for coupling a parallel sheet deformation calculation.

Simulating the proliferation of void damage within a particle field is often referred to as a damage percolation model. In this type of computational scheme, probabilistic and geometric criteria are employed to simulate the ductile fracture mechanisms of void nucleation, growth and coalescence events, providing a tool for investigating macroscopic failure trends. To illustrate the application of a damage percolation model, a brief synopsis is presented here from a study by Worswick, Pilkey and Thomson [201].

Worswick *et al.* developed a semi-analytical damage percolation model to simulate void damage evolving within the four Al-Si alloy sheets under examination in this thesis. The Al-Si microstructures were modelled in the plane of the sheet using digitally acquired particle fields. The nucleation of voids was assumed to occur within first-order clusters, progressing from the largest to smallest in size as a function of applied strain. This approach follows the nucleation theory of Embury *et al.* [6,102,103] depicted in Figure 1.12. FEM solutions from Thomson [200] provided the basis for void growth rates, while void coalescence events were predicted with a modified form of the geometric criterion proposed by Brown and Embury [145]. Figure 6.1 includes an example of predicted void damage from this modelling scheme, depicting two successive strain states just prior to failure. Each ellipse in Figure 6.1 represents an individual particle. Particles which have not met the nucleation criterion are coloured either yellow or brown, depending upon their location inside or outside a first-order cluster, respectively. Magenta is used to signify particles which have nucleated a void, while red is indicative of particles included in a crack. Black ellipses are used to denote cracks, which form as a result of void coalescence

between adjacent first-order clusters. In these simulations, Worswick *et al.* found that the linking of three cracks typically triggers catastrophic failure, as is evident in Figure 6.1. Three separate cracks from Figure 6.1a have linked together to form one crack in Figure 6.1b that is large enough to induce a catastrophic chain-reaction of void coalescence events. This critical level of coalescence is similar to the experimental and computational observations of Koss and co-workers [141,173-175], who discovered that the coalescence of three through-thickness holes precipitated final failure in voided sheets, as described in Section 1.3.1.

Although the findings of Worswick *et al.* [201] indicate that a damage percolation approach is useful for investigating ductile fracture, certain limitations were observed. In general, predicted failure strains were not found to coincide with experimental trends exhibited by the Al-Si alloys, suggesting that an improved representation of void damage behaviour is necessary. It is evident that the model would benefit from the utilization of stereological void damage data. For example, the void nucleation probabilities obtained in this thesis facilitate the advent of a Monte Carlo style algorithm to dictate void initiation events. A Monte Carlo technique would also allow the effect of spatial variations in void nucleation to be studied more closely through repeated modelling trials. However, in addition to limitations of the adopted void nucleation, growth, and coalescence criteria, a further deficiency in the damage percolation model of Worswick *et al.* stems from the fact that the sheet's in-plane compliance is not considered. In other words, constant strain paths are applied throughout the deformation history, even though the aspect ratios of evolving voids are expected to produce anisotropic variations in both local and remote stress-strain behaviour. It is conceivable that this problem can be alleviated by the

implementation of a second model to predict sheet deformation, run in parallel with the damage percolation model.

A possible approach for this parallel calculation involves the development of a finite element method (FEM) model. Since local parameters can be incorporated into an FEM model at the scale of individual elements, it is possible to map the local void volume fractions of a damage percolation model directly onto an FEM mesh. By adopting element constitutive equations which facilitate anisotropic yielding from inhomogeneous void shapes, such as the modified Gurson-Tvergaard formulation [65-68,157,158] presented in Chapter 5, an FEM model can be used to determine local in-plane deformations on the basis of local porosity levels and far-field loading conditions. The resulting element strains are then available to the percolation model for updating void nucleation, growth, and coalescence behaviour. This bilateral transfer of state variables allows alternating increments of deformation and void damage to be calculated for each element, effectively maintaining a local and global coupling between these two phenomena under applied loads.

Having proposed a rigorous modelling approach to locally couple void damage and deformation behaviour, including the adoption of a Monte Carlo strategy that necessitates repeated trials, it is apparent that the size of a modelled particle field has a substantial bearing on computational complexity. Therefore, the ability to obtain predictions from smaller particle fields is desirable. A guideline for selection of a representative particle sub-field is examined in the following section.

6.3 Critical Particle Sub-Fields

In the biaxial stretching model, the size of the input first-order cluster field was arbitrarily chosen, corresponding to a 10 x 10 grid of digitally merged binary images. Because of the relatively large number of second-phase particles in these fields, the resulting global calculations were deemed to be representative of average behaviour in the Al-Si alloy sheets. However, the findings of Worswick *et al.* [201] and Koss *et al.* [141,173-175] have suggested that a critical scale of void damage precipitates final failure. These observations are consistent with Huang's [179] concept of a critical cluster, introduced in Section 1.3.1, which demonstrates a direct correlation between the predicted localization strain for a random particle field and that of its most detrimental particle cluster. Upon review of Huang's study, it has been ascertained that the size of the critical sub-field is likely to depend upon the intrinsic nature of particle clustering in the random field. Namely, selection of a representative sub-field must consider the relevance of various second-phase particle clustering orders in relation to the evolution of void damage. It is believed that the availability of second-phase particle clustering measures and stereological void damage data from the present research facilitates an approach for selecting a critical sub-field. This procedure involves the construction of a separate spatial representation for a given second-phase particle field, comprising expected void damage rates due to the mechanisms of nucleation, growth, and coalescence.

Evidence of spatial variations in void damage rates within the Al-Si particle fields has been reported in the stereological results of Chapter 4. Briefly, these findings contend that:

- i) void nucleation is strongly correlated with large second-phase features as well as the close relative particle spacings characteristic of first-order clusters; however, voids are not initiated by particle clustering beyond this first-order scale;
- ii) void growth appears to be enhanced at the level of first-order clusters;
- iii) void coalescence between several first-order clusters is presumed to precipitate final failure.

In qualitative terms, these behavioural descriptions relate the void damage mechanisms of nucleation, growth and coalescence to characteristic particle clustering orders within the Al-Si microstructures. Recognizing that particle clustering orders are distinguished by their inter-particle dilational spacing (IPDS) values, it is possible to produce a graphical representation of void damage rates in terms of IPDS. A schematic of this kind is illustrated in Figure 6.2, indicating the relative spacings which are likely to cause significant interactions between features in the Al-Si alloys, in accordance with the observations listed above. The void damage rate relationships in Figure 6.2 are represented by gamma probability functions, noting that Argon *et al.* [117] adopted gamma probability functions to model void nucleation probabilities in terms of relative particle spacing. Gamma curves reflect the compromising relationship between larger features versus smaller relative particle spacings and their combined effect on void nucleation. Namely, high IPDS values correspond to large agglomerated features, while low IPDS values signify small relative particle spacings. Therefore, the potential for void nucleation is maximized at intermediate levels of IPDS. It is also worth noting that the nucleation curve in Figure 6.2

is consistent with the void nucleation data of Figures 4.24 to 4.26, showing the number of voids enveloped by each IPDS agglomeration event. As for the void damage rate curves attributed to growth and coalescence in Figure 6.2, their shapes are more arbitrary in nature, based on the assumption that void growth and coalescence behaviour is more significant at the scale of first- and second-order clusters, respectively.

The significance of the three relationships depicted in Figure 6.2 is that they provide a basis for assigning anticipated void damage rates to clustered regions within a second-phase particle population. In the Al-Si particle fields, for example, first-order clusters are designated as areas for high void damage rates due to nucleation and growth, whereas regions comprising second-order clusters are assigned high void damage rates on the basis of coalescence. Consequently, a spatial representation of anticipated void damage rates can be constructed for any given particle field to identify local regions of potentially critical behaviour. The definition of critical behaviour is dependent upon the nature of the predictive model. In the Al-Si alloy sheets under study, metallographic observations suggest that voids primarily nucleate and grow prior to the onset of strain localization at the forming limit. Hence, it follows that critical sub-fields for forming limit predictions require regions of high void nucleation and growth rates. According to the void nucleation and growth trends in Figure 6.2, an appropriate size for these critical sub-fields is between that of first- and second-order clusters in the Al-Si alloys. On the other hand, predictive modelling of failure must consider the additional mechanism of void coalescence, suggesting a critical sub-field beyond the magnitude of second-order clusters in the Al-Si sheets.

As a final note, it is recognized that the effectiveness of this selection approach is directly dependent upon the three void damage rate functions depicted in Figure 6.2. For demonstration purposes, the relative amplitudes and shapes of these curves have been arbitrarily chosen. In reality, proper ‘calibration’ of these functions requires a series of rigorous numerical modelling trials to compare predictions from large random fields and their potentially critical sub-fields. Although such an undertaking is deemed beyond the scope of this thesis, the discussion herein provides guidance for the selection of critical sub-fields in future strain localization and failure predictions.

Chapter 7

Conclusions and Recommendations

The principal objective of this thesis is to establish the effect of second-phase particle clustering on Al-Si alloy sheet formability. To achieve this goal, contemporary methods from the fields of quantitative metallography, image analysis and mechanical testing were combined to develop a unique experimental programme. The resulting set of research tools provides a reliable means for collecting statistically significant samples of stereological data from particle-containing microstructures. In the current investigation, explicit measures of distinguishing second-phase features were extracted from four Al-Si microstructures, indicating the nature of particle clustering at several orders of magnitude. Coincident detection and association of voids with individual particles allowed correlations between void damage and definitive second-phase features to be evaluated. Overall, the

size, aspect ratio, orientation and spatial distribution of first-order clusters were found to influence sheet formability in the four Al-Si alloys.

Together, the creation of state-of-the-art metallographic image analysis tools and the stereological and experimental analysis of the Al-Si alloy sheets comprise the major contribution of this work to the research fields of sheet formability, ductile fracture, and particle clustering. It is believed that these achievements address the first two research deficiencies enumerated in Section 1.4, recognizing the need for:

- i) explicit characterizations of second-phase particle clustering at several orders of magnitude in real microstructures;
- ii) in-situ measures of void damage, related directly to the physical attributes and spatial distribution of definitive second-phase features.

The development of a biaxial stretching model has also been completed during the course of this study, relating the measured proliferation of void damage in the Al-Si alloy microstructures to in-plane deformation. Although the sole purpose of this numerical analysis was to investigate forming limit trends, the applicability of its constitutive formulation to more rigorous predictive models has also been noted. Therefore, the implementation of this numerical scheme is regarded as a preliminary step in satisfying the third research deficiency of Section 1.4, requiring:

- iii) theoretical models to predict trends in sheet formability based on the evolution of void damage within actual particle fields.

In the following section, the major findings which comprise the current research contribution are presented. Future directions for this work are outlined in Section 7.2.

7.1 Conclusions

The relationship between second-phase particle clustering and sheet formability in the four Al-Si alloys has been indicated by a number of prominent observations detailed in this thesis. These significant and interrelated results are grouped into four topical classifications: second-phase particle clustering, mechanical testing, void damage, and numerical modelling.

7.1.1 Second-Phase Particle Clustering

The nature of second-phase particle clustering in the four Al-Si sheets under study has been established using quantitative metallography and image processing methods. The dilational counting scheme of Shehata [191] was adopted to extract explicit clustering measures during the construction of spatial tessellations from digitally acquired particle fields. Clusters of second-phase particles, referred to as first-order clusters, were identified with the aid of inter-particle dilational spacing (IPDS) data. To evaluate clustering at higher orders of magnitude, a modified implementation of the IPDS algorithm was developed, dubbed an agglomerated IPDS. The significant results provided by these stereological analyses include:

- i) First-order clusters, comprising an average of 2.5 particles, are present in each of the Al-Si microstructures. These clustering features are directly attributed to the fragmentation of as-cast second-phases during rolling. Spatial distribution of first-order clusters ranges from heavily clustered at several successive orders to relatively uniform spatial arrangements beyond first-order clustering, according to the sequence: Al-2%Si unrefined, Al-2%Si Sr-refined, Al-4%Si unrefined, and Al-4%Si Sr-refined. This outcome reveals that a more substantial reduction in clustering intensity is achieved through the higher 4% Si content as opposed to the addition of Sr.
- ii) The preferential orientation of first-order clusters possessing high aspect ratios is seen to differ considerably between the four Al-Si alloys. The overall tendency for longitudinal alignment of major first-order cluster axes progresses from strong to weak in the following manner: Al-2%Si unrefined, Al-4%Si unrefined, Al-4%Si Sr-refined, and Al-2%Si Sr-refined. The observed rank of Al-Si alloys indicates that the adoption of a Sr refinement process during sheet fabrication lessens the introduction of this rolling direction effect into high aspect ratio first-order clusters. In fact, the orientation of first-order clusters is virtually isotropic in the Al-2%Si Sr-refined microstructure.

7.1.2 Mechanical Testing

Mechanical testing of the Al-Si alloy sheets was conducted to determine strain localization behaviour under a diverse range of imposed strain paths. Stress-strain curves

were obtained from uniaxial tensile tests while hemispherical punch tests facilitated the generation of forming limit diagrams (FLD's) from surface strain data. The principal findings of this mechanical test programme are as follows:

- i) Relatively small differences in diffuse necking strains, strain hardening rates and normal anisotropy values are found between the four Al-Si alloys. This correspondence suggests that crystallographic slip is achieved through comparable means in each of the Al-Si sheets, thereby indicating that differences in formability behaviour are not likely caused by disparate crystallographic textures. Similar in-plane variations of normal anisotropy amongst the Al-Si alloy sheets also support this conclusion.
- ii) Uniaxial stress-strain behaviour indicates that local necking and fracture strains differ amongst the Al-Si sheets, providing evidence of distinct microstructural inhomogeneities within each of these alloys. On FLD's, divergences in forming limit strains widen directly as a function of tensile minor strain, consistent with the expected influence of void damage emanating from second-phase particles. It is well established that void damage rates are accelerated by elevated levels of hydrostatic stress.
- iii) Overall levels of formability in the Al-Si alloy sheets vary tremendously. Superior formability is exhibited by both Sr-refined sheets, coupled with an isotropic in-plane response. In fact, the FLD's of these two alloys virtually coincide, displaying very little effect of Si content on formability. On the contrary, the forming limits of the unrefined alloys are diminished and anisotropic, with severe reductions in formability occurring when major loading is applied in the transverse

direction. The unrefined sheets also experience a dramatic reduction in formability for the lower 2% Si volume fraction. From these test results, it is ascertained that void damage concentrated within first-order clusters is highly influential on in-plane deformation compliance and the resulting formability of the four Al-Si sheets. Namely, the anticipated detriment of higher particle volume fraction on formability in the Al-4%Si alloys, caused by an increased number of nucleated voids, appears to be overwhelmed by the clustered spatial distribution of second-phase particles in the Al-2%Si sheets. Furthermore, measured levels of anisotropy in stretch forming limits are similar to the relative strengths for preferential orientation of high aspect ratio first-order clusters in the Al-Si microstructures. This apparent correlation is consistent with the FEM modelling results of Thomson [200] which predict elevated void damage rates within high aspect ratio first-order clusters when major loads are applied perpendicular to the major axis of the cluster.

7.1.3 Void Damage

The evolution of void damage in the Al-Si microstructures was investigated on the basis of stereological data collected from undeformed and deformed sheet specimens. For this study, deformed samples were obtained at strain states just below near-equi-biaxial forming limits, where the largest disparities in forming limits are exhibited by the four Al-Si alloys. Near-equi-biaxial strain states also correspond to the highest levels of applied stress triaxiality, and therefore, are presumed to induce the greatest degree of void damage in the Al-Si microstructures. To evaluate potential correlations between the occurrence of void damage and the definitive characteristics of second-phase features, individual voids

were detected and assigned to adjacent particles. From this study, meaningful insight has been gained in the following areas:

- i) The potential for void initiation is strongly dependent upon the size of both individual particles as well as first-order clusters. A positive and nearly linear relationship between void frequency and the size of associated second-phase features is obtained for each Al-Si microstructure. Consistent void damage trends are observed as a function of particle size within each Si content pairing, supporting the theory of Argon *et al.* [117] which attributes the likelihood for void nucleation to both feature size and bulk particle volume fraction. The aspect ratio of second-phase features is also related to void nucleation, although less prominently than size. It is believed that the apparent affinity of voids for high aspect ratio features in the Al-Si sheets is indirectly formed through the close correlation between size and shape of features. Larger features tend to possess higher aspect ratios. Further indicating the predominance of feature size in the Al-Si alloys, the impact of major loading orientation on the occurrence of voids is found to be minimal. However, it is also recognized that the secondary effect of aspect ratio in this study is likely a manifestation of the incoherent Si particle populations. In general, void nucleation phenomena in the Al-Si alloys sheets are minimized because the brittle, weakly-bonded Si particles introduce an extensive degree of void damage into the as-rolled microstructure.
- ii) Statistical correlations between void damage and individual particle characteristics prove that membership in a first-order cluster causes a substantial increase in the amount of void damage adjacent to small and/or low aspect ratio particles. Void

nucleation potentials for particles in excess of a certain threshold size and/or aspect ratio are unaffected by inclusion in first-order clusters. Therefore, void damage in both the undeformed and deformed Al-Si microstructures primarily nucleates from particles which are either large in size, possess high aspect ratios, or are included in first-order clusters. This noted subset of second-phase features is commonly linked to preferential void initiation throughout the ductile fracture literature. However, qualitative metallographic observations in the current investigation indicate that void growth rates within first-order clusters are considerably higher than those associated with isolated particles. Therefore, first-order clusters are considered a predominant second-phase feature with respect to void damage proliferation in the Al-Si alloy sheets.

7.1.4 Numerical Modelling

A numerical model was developed to investigate the impact of first-order clusters on the biaxial stretch forming limits of the Al-Si sheets. The proposed model formulation utilizes the Gurson-Tvergaard constitutive softening model [65-68], with a modification proposed by Nagaki *et al.* to facilitate anisotropic yielding [157,158]. By considering the effective void area fraction in each principal direction, significant deviations in plastic strain paths were obtained in sheets of elliptical voids nucleated from first-order cluster fields of the Al-Si alloys. The relative magnitudes of these strain path deviations were then used to rationalize observed levels of anisotropy in stretch forming limits. Although a truly predictive sheet formability model was not pursued, the adopted numerical formulation provided a viable framework for calculating in-plane deformation compliances from

digitized particle fields and void nucleation data. Several stereological and experimental trends are supported by this numerical analysis, with the foremost entailing:

- i) Numerical predictions demonstrate that the preferential orientation of first-order clusters in the four Al-Si sheets has a profound impact on their respective levels of stretching anisotropy. The strength of this relationship adds further support to the definitive nature of first-order clusters in the Al-Si alloys under study.
- ii) Increased levels of bulk porosity are found to exacerbate anisotropic in-plane behaviour, resulting in slight over-predictions for the degree of stretching anisotropy in the Al-4%Si alloys relative to their Al-2%Si counterparts. The noted discrepancy between predicted and experimental trends is attributed to the omission of clustering effects beyond the first order from the model formulation. It is believed that predictions based on smaller spatial dimensions, such as second-order clusters, are needed to better reflect experimental results by capturing the critical scale of concentrated void damage in the more heavily clustered Al-2%Si microstructures. In general, this anomaly in predicted behaviour accentuates the requirement for careful consideration of length scales in numerical modelling of strain localization caused by void damage.

7.2 Recommendations

The research programme detailed in this thesis has demonstrated an ability to rationalize the sheet formability behaviour of four Al-Si alloys in terms of void damage evolution within their respective second-phase particle populations. The strength of this

work lies in the methods developed to extract explicit second-phase particle clustering measures and void damage data from the Al-Si microstructures. The resulting stereological data furnished statistically significant correlations between void damage and various second-phase features, revealing the definitive nature of first-order clusters in the Al-Si sheets. Numerical modelling studies also benefited from the availability of these stereological measures. Consequently, it is believed that reasonable contributions have been made to three deficient areas of research, as outlined in Section 7.1. To facilitate continued progress, recommendations for future research are presented in this section.

Although two-dimensional metallographic views intrinsically reflect microstructural characteristics of the third direction, the acquisition of three-dimensional particle and void fields should be considered. By preparing metallographic planes at regular intervals, it is possible to reconstruct a three-dimensional digital representation from a series of two-dimensional images. Provided that the penetration depth between successive two-dimensional images is less than the average particle size, a mismatch error would indicate relative positioning in the plane of the sheet, similar to the approach adopted for digital merging in the current work. Assumptions of feature shapes in the third dimension are required to completely resolve individual particles and voids. Upgrading the matrix erosion tessellation algorithm to handle three dimensions is relatively straightforward, given the pixel-based nature of the algorithm. The vastly increased memory requirements are likely to restrict the volume of material that can be digitally represented. However, current results indicate that clustering beyond the second or third order is insignificant in the Al-Si microstructures, meaning that the number of images in the overlapping grid can be dramatically reduced to compensate for additional resolution in the third direction. In spite

of the viability of computational requirements, the success of such an undertaking undoubtedly relies upon the daunting metallographic component.

To gain a more complete understanding of void growth and coalescence behaviour in the Al-Si sheets, void shapes and sizes need to be accurately determined. Notwithstanding the advent of three-dimensional representations outline above, stereological data should be collected in the two remaining metallographic planes. Due to the increased depths of voids in these alternative directions, it is likely that repeatable measures of void shapes and sizes will result. Presumably, the distinguishing aspects of second-phase particle clustering in the four Al-Si alloy sheets will also be evident in this supplementary stereological data. Hence, the role of first and second-order clusters in void growth and coalescence can be examined more rigorously than was possible in the current study.

From a numerical modelling perspective, the eventual development of a fully predictive forming limit model is paramount. It is evident from the biaxial sheet model results and associated discussion that future work in this area entails implementation of a Marciniak-Kuczynski analysis [21,22] to provide a physical framework for strain localization. It is also imperative that the proposed numerical scheme incorporates the spatial distribution of first-order clusters. The concept of a coupled model has been introduced for this purpose, where damage percolation and FEM calculations run in parallel to obtain local measures of evolving void populations and deformation, respectively. To properly couple the effect of void sizes and shapes on in-plane sheet compliance, the element constitutive equations must accommodate anisotropic behaviour. The anisotropic form of the Gurson-Tvergaard constitutive softening equations [65-68,157,158], adopted

for the biaxial stretching model, seem to be suitable for this task. In terms of length scales for the FEM model, the predominance of first-order clusters in the Al-Si alloy sheets suggests an element size based on this feature. For the inhomogeneous M-K defect band, a sub-field of particles is necessary to capture the significant influence of spatial variability in void damage caused by higher order clustering. A sub-field comprised of the most detrimental second or third-order cluster is likely to be representative of a critical defect in the Al-Si microstructures, since void damage interactions do not occur beyond this magnitude, particularly in the context of strain localization. The use of a relatively small particle field also minimizes the computational complexity of each modelling prediction, allowing the stochastic aspect of void nucleation to be studied through repeated Monte Carlo trials. For the selection of definitive particle sub-fields, it is believed that the proposed concept of a spatial representation for void damage rates will prove to be a useful pre-processing tool.

As a final point, it is recognized that the Al-Si alloy sheets fabricated for the current research programme have provided an effective model material for developing the investigative methods presented in this thesis. The intrinsic characteristics of the relatively brittle, weakly-bonded Si particle populations, however, have biased the observed trends in void damage behaviour. In particular, the importance of void nucleation is generally diminished by the tremendous extent of void damage in the as-rolled Al-Si microstructures. Therefore, it is imperative that future work considers a near-commercial or commercial aluminum alloy system, where the evolution of voids from more tenaciously bonded second phases is certain to differ from that of the Al-Si alloys. The initiation of voids in commercial aluminum alloy sheets typically occurs at higher levels of strain and over a wider range of strains, implying that void nucleation has a greater impact on strain

localization. Given the predominant role of first-order clusters in nucleating voids, it is conceivable that the failure behaviour in many of these sheets is sensitive to the degree of second-phase particle clustering.

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		RELATIVE SENSITIVITY TO	
		PERIODICITY	CLUSTERING
1.	NEAREST-NEIGHBOUR DISTANCE		
	i) average value	WEAK	WEAK
	ii) standard deviation	WEAK	WEAK
2.	NEAR-NEIGHBOUR DISTANCE		
	i) average value	VERY WEAK	VERY WEAK
	ii) standard deviation	STRONG	WEAK
3.	NUMBER OF SIDES PER CELL		
	i) average value	NO SIGNIFICANT DEPENDENCE	
	ii) standard deviation	WEAK	MILDLY STRONG
4.	CELL AREA		
	i) average value	NO DEPENDENCE	NO DEPENDENCE
	ii) standard deviation	WEAK	STRONG
5.	CELL ASPECT RATIO		
	i) average value	WEAK	STRONG
	ii) standard deviation	WEAK	STRONG

Table 1.1: Sensitivity of various spatial tessellation parameters to the degree of periodicity or clustering in a particle distribution [189].

Element (wt. %)	Alloy			
	Al-2%Si unrefined	Al-2%Si Sr-refined	Al-4%Si unrefined	Al-4%Si Sr-refined
Al	97.93	97.68	95.64	95.60
Si	2.05	2.29	4.32	4.34
Sr	---	94 ppm	---	57 ppm

Table 2.1: Elemental sheet compositions by weight percent.

Alloy	Average Equiaxed Grain Size (μm)		
	longitudinal	transverse	in-plane
Al-2%Si unrefined	43	44	37
Al-2%Si Sr-refined	30	35	34
Al-4%Si unrefined	26	27	27
Al-4%Si Sr-refined	24	28	29

Table 2.2: Average equiaxed grain sizes in principal sheet planes.

Alloy	Number of Particles	Particle Size		Measured Area Fraction	Theoretical Volume Fraction
		Mean (μm^2)	Standard Deviation (μm^2)		
Al-2%Si unrefined	6181	2.16	2.18	0.0176	0.0229
Al-2%Si Sr-refined	8686	1.92	1.96	0.0220	0.0257
Al-4%Si unrefined	11441	3.24	3.29	0.0486	0.0492
Al-4%Si Sr-refined	16368	2.33	2.32	0.0500	0.0494

Table 2.3: Particle characteristics from in-plane particle fields of Figures 2.11 to 2.14.

Alloy	0°		45°		90°	
	σ_y (MPa)	n	σ_y (MPa)	n	σ_y (MPa)	n
Al-2%Si unrefined	39.6	0.24	39.1	0.24	38.6	0.25
Al-2%Si Sr-refined	38.9	0.25	38.2	0.25	37.9	0.25
Al-4%Si unrefined	40.4	0.26	39.9	0.26	40.9	0.26
Al-4%Si Sr-refined	39.6	0.28	40.4	0.26	40.9	0.26

Table 3.1: Yield stresses, σ_y , and strain hardening coefficients, n , at angles of 0°, 45°, and 90° with respect to the rolling direction.

Alloy	r_0	r_{45}	r_{90}
Al-2%Si unrefined	0.68	0.79	0.61
Al-2%Si Sr-refined	0.56	0.75	0.56
Al-4%Si unrefined	0.68	0.80	0.60
Al-4%Si Sr-refined	0.65	0.85	0.59

Table 3.2: Normal anisotropies, r , at angles of 0° , 45° , and 90° with respect to the rolling direction.

Alloy	$(\epsilon_I^*)_d$		
	0°	45°	90°
Al-2%Si unrefined	0.24	0.23	0.23
Al-2%Si Sr-refined	0.25	0.24	0.25
Al-4%Si unrefined	0.24	0.24	0.25
Al-4%Si Sr-refined	0.24	0.24	0.25

Table 3.3: Predicted diffuse necking strains, $(\epsilon_I^*)_d$, at angles of 0°, 45°, and 90° with respect to the rolling direction, from stress condition of equation 1.17.

Alloy	$(\epsilon_l^*)_l$		
	0°	45°	90°
Al-2%Si unrefined	0.35	0.30	0.26
Al-2%Si Sr-refined	0.33	0.33	0.35
Al-4%Si unrefined	0.33	0.31	0.31
Al-4%Si Sr-refined	0.32	0.34	0.32

Table 3.4: Predicted local necking strains, $(\epsilon_l^*)_l$, at angles of 0°, 45°, and 90° with respect to the rolling direction, from stress condition of equation 1.18.

Alloy	$(\epsilon_l)_f$		
	0°	45°	90°
Al-2%Si unrefined	0.40	0.34	0.29
Al-2%Si Sr-refined	0.37	0.39	0.40
Al-4%Si unrefined	0.39	0.35	0.34
Al-4%Si Sr-refined	0.37	0.39	0.36

Table 3.5: Fracture strains, $(\epsilon_l)_f$, at angles of 0°, 45°, and 90° with respect to the rolling direction.

Alloy	Deformation State	Void Damage (%)		
		First-Order Clusters	Large Isolated Particles	Total
Al-2%Si unrefined	undeformed	60.6	18.3	78.9
	longitudinal	50.5	20.6	71.1
	transverse forming limit	53.5	17.9	71.4
Al-2%Si Sr-refined	undeformed	64.4	16.4	80.8
	longitudinal forming limit	55.9	17.4	73.3
	transverse forming limit	59.2	16.3	75.5
Al-4%Si unrefined	undeformed	65.1	18.1	83.2
	longitudinal forming limit	59.9	17.5	77.4
	transverse forming limit	54.8	19.4	74.2
Al-4%Si Sr-refined	undeformed	78.2	11.3	89.5
	longitudinal forming limit	71.2	11.7	82.9
	transverse forming limit	66.6	13.9	80.5

Table 4.1: Percentage of void damage associated with first-order clusters and large isolated particles.

Alloy	p_0		p_f			$(\bar{\epsilon}_M^p)_f$
	slope	p -intercept	major direction	slope	p -intercept	
Al-2%Si unrefined	0.0156	-0.0717	longitudinal	0.0202	-0.0293	0.498
			transverse	0.0231	-0.0494	0.600
Al-2%Si Sr-refined	0.0128	-0.0353	longitudinal	0.0198	-0.0762	0.837
			transverse	0.0201	-0.0431	0.863
Al-4%Si unrefined	0.0077	-0.0101	longitudinal	0.0128	-0.0076	0.750
			transverse	0.0106	0.0909	0.708
Al-4%Si Sr-refined	0.0082	-0.0201	longitudinal	0.0113	-0.0325	0.837
			transverse	0.0119	0.0234	0.864

Table 5.1: Void nucleation parameters from linear probability functions of Figure 5.6.

Figure 1.1: Schematic of a forming limit diagram (FLD), depicting sheet blank geometries for strain paths in uniaxial, plane strain and equi-biaxial tension.

Figure 1.2: Comparison of experimental forming limit diagram for low-carbon steel [3] with diffuse and local necking theories for $n=0.25$ (adapted from Hosford and Caddell [15]).

Figure 1.3: Development of: (a) diffuse neck; (b) local neck. Local necking coordinate system shown in (c) [15].

Figure 1.4: Marciniak-Kuczynski (M-K) model geometry [21,22], employing a defect band of reduced thickness [15].

Figure 1.5: Deviating strain paths within the defect band of an M-K analysis [15].

Figure 1.6: Schematic plots of true stress, σ , and strain hardening rate, $d\sigma/d\epsilon$, as functions of true strain, ϵ , illustrating the effect of (a) strain hardening and (b) strain rate sensitivity on diffuse and local necking conditions [4].

**(a) PLASTIC
FAILURE**

**(b) SHEAR
FRACTURE**

**(c) DUCTILE
FRACTURE**

Figure 1.7: Mechanisms of ductile failure according to Ashby *et al.* [63] (from Worswick [155]).

Figure 1.8: Effect of second-phase particle volume fraction on ductility [72].

Figure 1.9: Ductility of steel containing sulphides elongated in the longitudinal direction [98] (from Hosford and Caddell [15]).

Figure 1.10: Comparison between experimental data and model predictions for void area fractions in a spheroidized steel under uniaxial tension. Predicted curves are based on dual populations of void-initiating particles [84].

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Figure 1.12: Conceptual progression of a void nucleation front from larger to smaller particles with increasing levels of effective strain [6,102,103].

Figure 1.13: Nucleation of void 'caps' through decohesion at the 'poles' of a particle [87] (from Worswick [155]).

Figure 1.14: Void growth and coalescence behaviour under (a) low (b) medium and (c) high levels of stress triaxiality [121].

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Figure 1.28: Unit cell for anisotropic yielding due to an ellipsoidal void (from Nagaki *et al.* [157, 158]).

(a)

(b)

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Figure 1.32: Elongations to failure for 7075 Al plate specimens containing a 5% area fraction of holes in either regular or random arrangements [173].

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Figure 1.37: Predicted relationships between localization strains and average particle spacing for n -particle clusters [179].

Figure 1.38: Predicted localization strains for randomly arranged particle fields versus their most detrimental n -particle cluster [179].

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Figure 1.40: Construction of a Dirichlet tessellation cell for a given point, based on perpendicular bisectors from its near-neighbours [189].

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Figure 1.42: A particle dilation (or matrix erosion) tessellation for a field of particles with varied sizes and shapes.

Figure 1.43: Schematic representation of particle merging (or agglomeration) events produced by successive particle dilations [191].

Figure 1.44: Point fields depicting (a) ordered, (b) random and (c) clustered spatial distributions [191].

Figure 1.45: Normalized dilational counting frequencies obtained from point fields depicted in Figure 1.44 [191].

Figure 2.1: Optical micrographs of Al-2%Si unrefined alloy at low and high magnifications. In-plane sheet view: longitudinal (rolling) direction is horizontal.

Figure 2.2: Optical micrographs of Al-2%Si Sr-refined alloy at low and high magnifications. In-plane sheet view: longitudinal (rolling) direction is horizontal.

Figure 2.3: Optical micrographs of Al-4%Si unrefined alloy at low and high magnifications. In-plane sheet view: longitudinal (rolling) direction is horizontal.

Figure 2.4: Optical micrographs of Al-4%Si Sr-refined alloy at low and high magnifications. In-plane sheet view: longitudinal (or rolling) direction is horizontal.

Figure 2.5: Log grey-scale histogram illustrating multiple thresholds for segmentation of ternary void/particle/matrix images.

Figure 2.6: Grey-scale digital image of deformed Al-4%Si alloy microstructure (640 x 480 pixels). [Note: print quality is 128 pseudo-grey-scales]

Figure 2.7: Binary images of particle and void fields extracted from grey-scale image of Figure 2.6.

Figure 2.8: Partially dilated and fully tessellated particle field from an eight-connected dilation algorithm.

Figure 2.9: Partially dilated and fully tessellated particle field from a four-connected dilation algorithm.

Figure 2.10: Partially dilated and fully tessellated particle field from an alternating four- and eight-connected dilation algorithm.

Figure 2.11: Large-scale high-resolution spatial tessellation for Al-2%Si unrefined alloy particle field. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.12: Large-scale high-resolution spatial tessellation for Al-2%Si Sr-refined alloy particle field. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.13: Large-scale high-resolution spatial tessellation for Al-4%Si unrefined alloy particle field. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.14: Large-scale high-resolution spatial tessellation for Al-4%Si Sr-refined alloy particle field. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.15: Histograms of particle size.

Figure 2.16: Normalized histograms of particle size.

Figure 2.17: Histograms of particle aspect ratio. Aspect ratios greater than unity indicate particles oriented longitudinally.

Figure 2.18: Normalized histograms of particle aspect ratio. Aspect ratios greater than unity indicate particles oriented longitudinally.

Figure 2.19: Average particle size as a function of aspect ratio. Aspect ratios greater than unity indicate particles oriented longitudinally.

Figure 2.20: Normalized frequencies for inter-particle dilational spacing (IPDS).

Figure 2.21: Large-scale high-resolution first-order clusters in Al-2%Si unrefined alloy. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.22: Large-scale high-resolution first-order clusters in Al-2%Si Sr-refined alloy. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.23: Large-scale high-resolution first-order clusters in Al-4%Si unrefined alloy. In-plane sheet view: longitudinal (rolling) direction is vertical.

Figure 2.24: Large-scale high-resolution first-order clusters in Al-4%Si Sr-refined alloy. In-plane sheet view: longitudinal (or rolling) direction is vertical.

Figure 2.25: Histograms of first-order cluster size.

Figure 2.26: Normalized histograms of first-order cluster size.

Figure 2.27: Histograms of first-order cluster aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 2.28: Normalized histograms of first-order cluster aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 2.29: Average first-order cluster size as a function of aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 2.30: Normalized frequencies for agglomerated inter-particle dilational spacing (agglomerated IPDS).

Figure 2.31: Histograms of second-order cluster size.

Figure 2.32: Normalized histograms of second-order cluster size.

Figure 2.33: Histograms of second-order cluster aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 2.34: Normalized histograms of second-order cluster aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 2.35: Average second-order cluster size as a function of aspect ratio. Aspect ratios greater than unity indicate clusters oriented longitudinally.

Figure 3.1: Uniaxial tensile specimen.

Figure 3.2: Principal loading and sheet directions for longitudinal and transverse formability tests.

Figure 3.3: Schematic of Carleton University hemispherical punch press (from Lamontagne [203]).

Figure 3.4: Close-up of draw bead configuration in Figure 3.3 (from Lamontagne [203]).

Figure 3.5: Dogbone specimen for drawing strain paths in hemispherical punch tests.

Figure 3.6: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-2%Si unrefined alloy in the longitudinal orientation.

Figure 3.7: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-2%Si unrefined alloy in the transverse orientation.

Figure 3.8: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-2%Si unrefined alloy in the 45° orientation.

Figure 3.9: Stress, σ , and hardening rate, $d\sigma/d\epsilon$, as a function of strain, ϵ , for the Al-2%Si Sr-refined alloy in the longitudinal orientation.

Figure 3.10: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-2%Si Sr-refined alloy in the transverse orientation.

Figure 3.11: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-2%Si Sr-refined alloy in the 45° orientation.

Figure 3.12: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-4%Si unrefined alloy in the longitudinal orientation.

Figure 3.13: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-4%Si unrefined alloy in the transverse orientation.

Figure 3.14: Stress, σ , and hardening rate, $d\sigma/d\epsilon$, as a function of strain, ϵ , for the Al-4%Si unrefined alloy in the 45° orientation.

Figure 3.15: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-4%Si Sr-refined alloy in the longitudinal orientation.

Figure 3.16: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-4%Si Sr-refined alloy in the transverse orientation.

Figure 3.17: Stress, σ , and hardening rate, $d\sigma/d\varepsilon$, as a function of strain, ε , for the Al-4%Si Sr-refined alloy in the 45° orientation.

Figure 3.18: Comparison of stress and hardening rate as a function of strain in three orientations for the Al-2%Si unrefined alloy.

Figure 3.19: Comparison of stress and hardening rate as a function of strain in three orientations for the Al-2%Si Sr-refined alloy.

Figure 3.20: Comparison of stress and hardening rate as a function of strain in three orientations for the Al-4%Si unrefined alloy.

Figure 3.21: Comparison of stress and hardening rate as a function of strain in three orientations for the Al-4%Si Sr-refined alloy.

Figure 3.22: Comparison of stress and hardening rate as a function of strain in the longitudinal orientation for the four Al-Si alloys.

Figure 3.23: Comparison of stress and hardening rate as a function of strain in the transverse orientation for the four Al-Si alloys.

Figure 3.24: Comparison of stress and hardening rate as a function of strain in the 45° orientation for the four Al-Si alloys.

Figure 3.25: Forming limit diagram for the Al-2%Si unrefined alloy in the longitudinal configuration of Figure 3.2.

Figure 3.26: Forming limit diagram for the Al-2%Si unrefined alloy in the transverse configuration of Figure 3.2.

Figure 3.27: Forming limit diagram for the Al-2%Si Sr-refined alloy in the longitudinal configuration of Figure 3.2.

Figure 3.28: Forming limit diagram for the Al-2%Si Sr-refined alloy in the transverse configuration of Figure 3.2.

Figure 3.29: Forming limit diagram for the Al-4%Si unrefined alloy in the longitudinal configuration of Figure 3.2.

Figure 3.30: Forming limit diagram for the Al-4%Si unrefined alloy in the transverse configuration of Figure 3.2.

Figure 3.31: Forming limit diagram for the Al-4%Si Sr-refined alloy in the longitudinal configuration of Figure 3.2.

Figure 3.32: Forming limit diagram for the Al-4%Si Sr-refined alloy in the transverse configuration of Figure 3.2.

Figure 3.33: Comparison of FLD curves in the two principal orientations for the Al-2%Si unrefined alloy.

Figure 3.34: Comparison of FLD curves in the two principal orientations for the Al-2%Si Sr-refined alloy.

Figure 3.35: Comparison of FLD curves in the two principal orientations for the Al-4%Si unrefined alloy.

Figure 3.36: Comparison of FLD curves in the two principal orientations for the Al-4%Si Sr-refined alloy.

Figure 3.37: Comparison of FLD curves in the longitudinal orientation for the four Al-Si alloys.

Figure 3.38: Comparison of FLD curves in the transverse orientation for the four Al-Si alloys.

Figure 3.39: FEM unit cell of Thomson [200], comprising a linear cluster of three Si particles in an aluminum matrix. Due to symmetry, only 1/8th of the cell is modelled.

Figure 3.40: Predicted void damage rates within linear Si particle cluster of Figure 3.39 under uniaxial tension. Major cluster axis is longitudinally oriented (from Thomson [200]).

Figure 4.1: Optical micrograph of deformed Al-4%Si unrefined alloy microstructure.

Figure 4.2: Binary images of particle and void fields extracted from digital image of microstructure shown in Figure 4.1.

Figure 4.3: Normalized frequencies for void association with particles in undeformed Al-Si microstructures. Comparison between unrefined and Sr-refined alloys as a function of particle size.

Figure 4.4: Normalized frequencies for void association with particles in deformed Al-Si microstructures at forming limits. Comparison between unrefined and Sr-refined alloys as a function of particle size.

Figure 4.5: Normalized frequencies for void association with particles in deformed Al-Si microstructures at forming limits. Comparison between longitudinal and transverse major loading as a function of particle size.

Figure 4.6: Normalized frequencies for void association with first-order clusters in undeformed Al-Si microstructures. Comparison between unrefined and Sr-refined alloys as a function of cluster size.

Figure 4.7: Normalized frequencies for void association with first-order clusters in deformed Al-Si microstructures at forming limits. Comparison between unrefined and Sr-refined alloys as a function of cluster size.

Figure 4.8: Normalized frequencies for void association with first-order clusters in deformed Al-Si microstructures at forming limits. Comparison between longitudinal and transverse major loading as a function of cluster size.

Figure 4.9: Normalized frequencies for void association with particles in undeformed Al-Si microstructures. Comparison between unrefined and Sr-refined alloys as a function of particle aspect ratio.

Figure 4.10: Normalized frequencies for void association with particles in deformed Al-Si microstructures at forming limits. Comparison between unrefined and Sr-refined alloys as a function of particle aspect ratio.

Figure 4.11: Normalized frequencies for void association with particles in deformed Al-Si microstructures at forming limits. Comparison between longitudinal and transverse major loading as a function of particle aspect ratio.

Figure 4.12: Normalized frequencies for void association with first-order clusters in undeformed Al-Si microstructures. Comparison between unrefined and Sr-refined alloys as a function of cluster aspect ratio.

Figure 4.13: Normalized frequencies for void association with first-order clusters in deformed Al-Si microstructures at forming limits. Comparison between unrefined and Sr-refined alloys as a function of cluster aspect ratio.

Figure 4.14: Normalized frequencies for void association with first-order clusters in deformed Al-Si microstructures at forming limits. Comparison between longitudinal and transverse major loads as a function of cluster aspect ratio.

Figure 4.15: Normalized frequency data for Al-2% unrefined alloy, showing effect of particle size and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.16: Normalized frequency data for Al-2% Sr-refined alloy, showing effect of particle size and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.17: Normalized frequency data for Al-4% unrefined alloy, showing effect of particle size and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.18: Normalized frequency data for Al-4% Sr-refined alloy. showing effect of particle size and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.19: Normalized frequency data for Al-4% unrefined alloy, showing effect of particle size and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at and beyond longitudinal forming limit.

Figure 4.20: Normalized frequency data for Al-2% unrefined alloy, showing effect of particle aspect ratio and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.21: Normalized frequency data for Al-2% Sr-refined alloy, showing effect of particle aspect ratio and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.22: Normalized frequency data for Al-4% unrefined alloy, showing effect of particle aspect ratio and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.23: Normalized frequency data for Al-4% Sr-refined alloy, showing effect of particle aspect ratio and particle membership in first-order clusters (FOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.24: Additional voids associated with agglomerated particle features for each dilation pass of a particle dilation (or matrix erosion) algorithm. Applied to undeformed Al-Si microstructures.

Figure 4.25: Additional voids associated with agglomerated particle features for each dilation pass of a particle dilation (or matrix erosion) algorithm. Applied to deformed Al-Si microstructures at longitudinal forming limits.

Figure 4.26: Additional voids associated with agglomerated particle features for each dilation pass of a particle dilation (or matrix erosion) algorithm. Applied to deformed Al-Si microstructures at transverse forming limits.

Figure 4.27: Normalized frequency data for Al-2% unrefined alloy, showing effect of particle size and particle membership in second-order clusters (SOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.28: Normalized frequency data for Al-2% Sr-refined alloy, showing effect of particle size and particle membership in second-order clusters (SOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.29: Normalized frequency data for Al-4% unrefined alloy, showing effect of particle size and particle membership in second-order clusters (SOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 4.30: Normalized frequency data for Al-4% Sr-refined alloy, showing effect of particle size and particle membership in first-order clusters (SOC's) on void nucleation. Deformed microstructures at longitudinal and transverse forming limits.

Figure 5.1: Elliptical voids in a sheet of unit thickness.

Figure 5.2: Biaxial stretching model representation for void field in Figure 5.1, comprising a single elliptical void in a square sheet of unit thickness.

Figure 5.3: Void nucleation probabilities as a function of particle size at effective strain of zero. Linear fit on upper bounds of confidence.

Figure 5.4: Void nucleation probabilities as a function of particle size under major longitudinal loading at noted effective strains. Linear fit on upper bounds of confidence.

Figure 5.5: Void nucleation probabilities as a function of particle size under major transverse loading at noted effective strains. Linear fit on upper bounds of confidence.

Figure 5.6: Composite of void nucleation probabilities from Figures 5.3 to 5.5.

Figure 5.7: Experimental stretch forming limits of the Al-Si alloys. Longitudinal and transverse FLD data is plotted on a common axis according to the configuration illustrated in Figure 5.8.

Figure 5.8: Principal loading and sheet directions for common FLD axis employed in Figure 5.7.

Figure 5.9: Schematic of deviating strain paths in a sheet exhibiting isotropic stretch formability.

Figure 5.10: Schematic of deviating strain paths in a sheet exhibiting anisotropic stretch formability.

Figure 5.11: Predicted deviations in plastic engineering strain paths under an equi-biaxial remote stress path. Percent deviations taken with respect to plastic strain path at first yield.

Figure 5.12: Porosity, f , in modelling predictions of Figure 5.11.

Figure 5.13: Global void aspect ratios, κ^* , in modelling predictions of Figure 5.11.

Figure 5.14: Predicted deviations in plastic engineering strain paths under an equi-biaxial remote stress path with varying levels of global porosity. Percent deviations taken with respect to plastic strain path at first yield.

Figure 5.15: Porosity, f , in modelling predictions of Figure 5.14.

Figure 5.16: Predicted deviations in plastic engineering strain paths under a near-equibiaxial remote stress path. Percent deviations taken with respect to plastic strain path at first yield.

Figure 5.17: Predicted deviations in plastic engineering strain paths under a near-equibiaxial remote stress path with varying levels of global porosity. Percent deviations taken with respect to plastic strain path at first yield.

Figure 5.18: Porosity, f , in modelling predictions of Figure 5.17.

Figure 5.19: Predicted deviations in plastic engineering strain paths under three near-equibiaxial remote stress paths. Percent deviations taken with respect to plastic strain path at first yield.

Figure 6.1: Void damage percolation model, depicting two successive equi-biaxial stretching states immediately prior to final failure (from Worswick, Pilkey and Thomson [201]).

Figure 6.2: Schematic representation for expected influence of IPDS on void damage rates attributed to nucleation, growth, and coalescence mechanisms in the Al-Si alloys.