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**A COMPREHENSIVE INTEGRATED MODELLING FRAMEWORK
FOR THE OPTIMIZATION OF TRANSIT OPERATOR
WORKFORCE PLANNING AND MANAGEMENT**

by

KHALIL Y. FERZLI

B. Arch. - University of Texas, Austin, U.S.A, 1977

M.Sc. Community & Regional Planning - University of Texas, Austin, U.S.A, 1980

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
DOCTOR OF PHILOSOPHY

**DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
CARLETON UNIVERSITY
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FOR THE OPTIMIZATION OF TRANSIT OPERATOR
WORKFORCE PLANNING AND MANAGEMENT**

submitted by
KHALIL YOUSSEF FERZLI

in partial fulfilment of the requirements for the degree of
DOCTOR OF PHILOSOPHY IN ENGINEERING

Chair, Department of Civil Engineering

Thesis Supervisor

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December, 1992

To my beloved wife Diana
and two wonderful sons Nizar and Ziad
thank you for your love and patience

ABSTRACT

Transit operator workforce planning and management processes consist of a set of interrelated tasks involving the determination of: 1) size of the transit operator workforce, 2) annual vacation schedules for operators, 3) timing and sizing of hiring decisions, 4) weekly work schedules for available operators, and 5) daily report times for extra-board (reserve) operators.

The most important attribute of the decision-oriented methodological framework developed in this thesis is its focus on the relationship between strategic and operational level decisions, specifically the relationship between the estimation of open work cover requirements at the strategic level, and extra-board assignment decisions at the operational level. The similarities between these two problems have suggested that, in order to develop reliable estimates of open work cover requirements at the strategic level, the peculiarities of the operational problem had to be incorporated into strategic decisions. The incorporation of aspects of demand into the modelling framework has resulted in considerable contributions to transit operator workforce planning and management.

First, among these contributions is the ability of the open work cover models, developed in this research, to: 1) identify the cost and reliability levels associated with alternative daily open work cover patterns, and 2) develop cover policies which minimize the cost associated with a given level of reliability. A sense of the magnitude of the savings -- about 20 percent in open work cover costs, or 2 percent of total operator costs - which may be realized by these models, is illustrated in the Case Study.

Second, the modelling framework has provided a mechanism by which higher level decisions (e.g., workforce size) are supported by lower level (vacation, tactical and operational) decisions, thus ensuring that the benefits of optimization at the strategic level are not dissipated by sub-optimal decisions at lower levels of the decision hierarchy.

Third, by considering the duration aspects and temporal pattern of open work, these models have offered the opportunity to solve, simultaneously, the problem created by

morning trippers and improve the efficiency of reserve operators.

Fourth, the performance measures developed to evaluate open work cover, hiring and operational policies have provided a firm basis for informed decision making at all levels. This in turn reduces the uncertainty underlying the overstaffing problem in most transit agencies -- estimated at about 7.2 percent of the operator workforce in U.S. transit properties (MacDorman and Associates, 1984).

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GLOSSARY

ADFTC

The average cost of one full-time operator work day (including fringe benefits), calculated by dividing annual operator costs by the total annual operator work days.

AHOTC

Average hourly overtime cost.

Attrition

The depletion of the workforce via retirement, promotions and change of jobs.

Block

A collection of scheduled trips serviced by a single vehicle for a day of operations.

Cover Decisions

The determination of open work cover requirements.

Cover Operators

The pool of operators retained for the cover of open work.

Daily Cover Policy

The set of assignment decisions of reserve operators to open work over time periods of day.

Daily Open Work

All open work with the exception of attrition.

Day Type

A day type describes a group of days, in a timetable, with similar service schedules (scheduled runs and trippers) and daily patterns of open work. An example

is Mondays of the Fall timetable .

Extra-Board

The pool of reserve operators assigned to open work and some trippers.

Fringe benefits

Fringe benefits include insurance and employer-paid pensions, sick leave pay, vacation and holiday pay.

Open Work

Work without assigned cover, which usually results from operator absenteeism and attrition, vehicle breakdowns and accidents, and minor service changes during the course of the service plan.

Operating Day

The portion of the day over which transit services are provided.

Operator work day

A full day's work for one operator, usually 8 hours.

Operating Expenses

Total expenses incurred in the operation of the transit services.

Overtime

Work performed by operators in excess of their contractual work duties -- usually 8 hours per day and 40 hour per week.

Overtime Premium

Compensation paid to operators for overtime work. Overtime premiums are usually valued at one and a half times the regular wage rate.

Regular Operators

Operators assigned to runs.

Regular Pay (wages)

Compensation paid to operators for performing their

contractual work duties.

Regular Work

Work duties specified by the labour contract -- usually 8 hours, 5 days a week for full-time operators, and less than or equal to 36 hours per week for part-time operators.

Reserve Operators

Operators assigned to work (open work) that is temporarily unfilled.

Rostering

The scheduling of drivers to runs.

Run

A grouping of trips, or partial trips constituting a full day's work, which is usually cut into work shifts (pieces).

Service Plan

A document listing the number, timing and location of scheduled runs and trippers for various time periods of the year (timetables).

Sign-up

The choosing of work by operators, usually in the order of seniority. The driver signifies his/her willingness to accept a certain grouping of assignments (runs and/or reserve duties) by his/her signature opposite these assignments on a posted list.

Split Run

A day's work consisting of two or three pieces of work separated by an unpaid break.

Straight Run

A straight piece of work separated by a paid break

(usually 15 to 30 minutes).

Trip

A one-way movement of a transit vehicle in service between the terminal (end) points of route.

Tripper

A short piece of work (usually less than 3 hours) that can not be incorporated into a run.

Work Rules

Detailed provisions governing job security, selection of work, durations of the work day and week, holidays, and the functions to be performed by different categories of drivers.

Timetable

A segment of the service plan (usually 2 to 6 months) with unique service characteristics.

Chapter I

INTRODUCTION

Workforce planning and management is the set of processes by which labour requirements are estimated, acquired, trained, maintained, and put to work. These decisions take on special importance in industries where the contribution of labour costs to total production costs is proportionately high. In such cases, small variations in estimated labour requirements or assignment decisions tend to produce significant impacts on total system costs and reliability.

Transit properties fall into this category. In the United States, transit operator wages and fringe benefits, in 1989, amounted to 5.6 billion dollars, about 35 percent of total operating costs (Transit Fact Book, 1990). The breakdown of operating costs in Canadian transit properties resembles to a large extent that of their U.S. counterparts. In 1991, Ottawa's OC Transpo spent little over 60 million dollars on operator wages and fringe benefits, about 39 percent of its annual operating costs of 155.5 million dollars. Although considerably smaller than OC Transpo, the Société De Transport De l' Outaouais (STO) - - Hull, Quebec -- also spent about 39 percent of its 1991 operating budget on operator wages and fringe benefits.

Growing transit subsidies during the seventies and early eighties, paired with high levels of operator absenteeism -- three times the U.S. national average (Perin, 1984) -- have generated interest in the development of analytical models for transit operator

workforce planning and management. In order to make up for the shortages resulting from operator absenteeism, transit companies tend to hire reserve operators. Transit operator costs and service reliability are highly sensitive to the size of this reserve workforce. A relatively large reserve workforce, usually, entails high operator costs and a high level of service reliability. Conversely, a small reserve workforce, usually entails lower operator costs and a low level of service reliability.

Given, the sensitivity of transit managers to the reliability issue, because of its impact on the public's perception, patronage, and support of transit services, transit managers have tended to oversize their reserve (extra-board) workforce as a precaution against service cancellations. A 1982 survey of twenty five U.S. transit systems, with over 100 vehicles, have found that on average U.S. transit properties did utilize 7.3 percent more operators than would be required under optimal conditions (MacDorman & Associates, 1984). These findings apply, as well, to Canadian transit systems. Accordingly, a reduction of only two percent in operator costs would result in over one million dollars in savings for a transit property of the size of OC Transpo.

1.1 TRANSIT OPERATOR WORKFORCE PLANNING

Transit systems play an important role in the provision of passenger transportation services in urban areas. The reliability and cost of these systems are highly sensitive to operator planning and management decisions. The primary objective of these decisions is to ensure that scheduled transit services (runs and trippers) are delivered according to plan. Figure 1.1 shows transit operator workforce planning in the context of the transit service planning process, its inputs and role in this process.

FIGURE 1.1: TRANSIT SERVICE PLANNING PROCESS

The service planning process starts with the estimation of the magnitude and spatial pattern of the demand for transit. This information is used in conjunction with established policy headways, available vehicles, and the budget resources allocated to transit, to develop service frequencies and timetables for the various routes in the network. Planned capacities and frequencies are used with travel time distributions to schedule vehicles to various segments of the network. These vehicle blocks are then cut into pieces of work, which are used to create scheduled runs and trippers. Among the well known run cutting packages is the HASTUS system used at OC Transpo and at STO, and in some of the largest transit systems around the world -- such as those of Los Angeles and the City of Paris. The HASTUS system was developed at the Centre de Recherche sur les Transports of the Université de Montréal.

Scheduled runs and trippers are used in conjunction with open work to estimate operator requirements and costs for the service plan. These costs are then compared to available budget resources to determine the economic feasibility of the proposed service plan.

The transit operator workforce planning and management process is concerned with determining the most cost-effective staffing levels, hiring policy, work and vacation schedules for operators. This process consists of a set of interrelated planning and management procedures involving the determination of the following set of decisions:

1. The sizing of the transit operator workforce,
2. The scheduling of annual vacations,
3. The timing and sizing of hiring decisions,

4. The scheduling of operator work days to days of the week,
5. The assignment of daily report times to extra-board (reserve) operators.

In the case of properties with multiple garages the last task involves, as well, the assignment of reserve operators to garages.

The order by which these decisions are presented reflects the hierarchical structure of the transit operator planning and management process. In this hierarchy, higher level decisions are made first, and the outputs from these decisions serve as inputs to, and/or constraints on, lower level decisions. For example, workforce size decisions determine, in conjunction with work rules, annual vacation liabilities and vacation schedules. In turn, vacation schedules set a ceiling on the number of operators available for work on any given day of the service plan.

These decisions are, usually, grouped into three categories; strategic decisions (1 to 3), and tactical and operational decisions (4 & 5). Strategic decisions produce long-term impacts on service costs and reliability since they commit the company's resources, and affect operator availability for work, over relatively long periods of time -- several months to a year. These decisions are usually made at the service planning stage, and as part of the planning for various timetables.

Tactical and operational decisions are carried out as part of the rostering process, in which work duties and day-off schedules for the coming work period -- few weeks to a timetable -- are presented for pick-up by operators. In spite of their limited time scope, the impact of these decisions on service costs and reliability is as important as that of strategic decisions. Since without proper scheduling, even with the best strategic plans,

considerable resources are likely to be wasted both as surplus and shortage (unplanned overtime) costs. Furthermore, in case available overtime is not sufficient to cover all shortages, scheduled services would have to be cancelled thus negatively affecting service reliability.

Operator shortages and surpluses represent deviations from the ideal state of the system, in which surpluses and shortages are nil. Although these deviations cannot be eliminated altogether due to the stochastic and dynamic nature of demand, particularly open work (see Section 1.2.2), with proper planning and management procedures, surplus and shortage costs can be considerably reduced. The optimization of transit operator workforce planning and management is in essence the minimization of the surplus and shortage (unplanned overtime) costs associated with a particular level of service reliability.

1.2 INPUTS AND THEIR ATTRIBUTES

The inputs to transit operator workforce planning and management processes include: 1) scheduled runs and trippers, 2) open work, 3) supply options, and 3) work rules and regulations. The characteristics of these inputs and their interrelationships are the determinants which shape transit operator policies -- work force size and composition, hiring and vacation schedules, and weekly and daily operator assignment patterns.

Scheduled runs and trippers represent close to ninety percent of the demand for operators. The rest is due to the open work resulting from absenteeism and attrition among operators, vehicle breakdowns and accidents, and minor service changes over the course of a timetable. In spite of the relatively small contribution of open work (around

10 percent) to the overall demand for operators, it's probabilistic and dynamic aspects are the main source of the complexity underlying transit operator planning and management decisions.

1.2.1 Scheduled Runs and Trippers

Scheduled runs and trippers show considerable variability over time. This variability mirrors the pattern of demand for transit services, with its daily, weekly and seasonally peaking characteristics. Scheduled runs include straight and split runs. Straight-runs, are straight work shifts (pieces) separated by a short break (15 to 30 minutes). Split-runs consist of two or more pieces of work separated by a wide (unpaid) break. Both types of runs are full-time duties.

Trippers are short pieces of work (less than or equal to 3 hours) which cannot be incorporated into runs within work rule constraints. And hence they are assigned to full-time operators as overtime, in addition to their regular daily work, or combined with open work cover duties to create work for extra-board operators. Trippers are used primarily during the morning and afternoon hours to deal with the peaking in demand for transit services during these two periods. The same number of runs and trippers are, usually, scheduled to every week day (Monday to Friday) of every week in a timetable. Different schedules are used for Saturdays, Sundays and holidays to account for reduced demand during these days.

Figures 1.2 and 1.3 show the variability in the daily pattern of scheduled runs and trippers at STO during a typical week day of the fall timerable, 1990. These schedule patterns are typical for Canadian and U.S. Transit agencies. Accordingly, the demand for

FIGURE 1.2: START AND END TIMES OF SCHEDULED RUNS DURING A TYPICAL WEEK DAY AT STO -- FALL TIMETABLE 1990

FIGURE 1.3: START AND END TIMES OF TRIPPERS DURING A TYPICAL WEEK DAY AT STO -- FALL TIMETABLE 1990

operators is highest during the early morning hours, when the majority of scheduled daily runs and trippers start. This coincidence in the peaking of demand for operators creates a problem with respect to the cover of trippers, which are best covered as overtime by full-time operators, in addition to their regular daily assignments. As a result, some morning trippers would have to be covered by extra-board operators.

1.2.2 Open Work

Open work is caused largely by perturbations in the availability of operators for work (absenteeism) at various times of the day, and to a lesser extent by vehicle breakdowns and accidents, attrition and unanticipated changes in service schedules throughout the year. Hence, unlike the demand resulting from the service schedule (service runs and trippers) which may be assumed with some confidence, demand due to open work can only be known stochastically at best. Analysis of absenteeism among transit operators have shown significant variability in absence rates over months of the year, days of the week, and times of the day (Perin, 1983; Leahy & Schlegel, 1979).

While schedule demands (service runs and trippers) may be assumed to be independent of each other from one time period to another, open work at a given time period during the day is the sum of work becoming open during this time period and work remaining open from previous time periods, due to a shortage in the supply of operators during these time periods. Hence, open work is, as well, a function of the pattern of open work cover over times of the day. Unless otherwise specified, in this document "cover" refers to the operators assigned to open work. Figure 1.4 shows the interaction between open work and its cover over time periods of the day.

Another complicating aspect of open work is its variable duration, which ranges from less than one hour in some cases (tardiness), to several weeks in others (long term sickness, or service additions). Open work of small durations (four hours or less) can be effectively covered as overtime by operators in addition to their normal daily work duties.

FIGURE 1.4: INTERACTION BETWEEN OPEN WORK AND IT'S COVER OVER TIMES OF THE DAY

Open work of larger durations would have to be 1) covered by reserve operators as their sole daily work duties, 2) assigned as overtime to two or more operators, or 3) worked as overtime by operators on their day-off.

1.2.3 Supply Options

Three supply options available to management are full-time operators, part-time operators, and overtime. The choice among these options is usually influenced by their cost, availability, provisions of the labour agreement, and reliability consideration.

The cost of using part-time operators is considerably lower than that for full-time operators, particularly because of the fringe benefit accorded to full-time operators. Hence, transit agencies tend to favour the hiring of part-time operators because of their positive impact on labour efficiency. By 1983, 92 percent of transit agencies in the U.S. had obtained provisions to employ part-time operators. However, these gains in efficiency are considerably reduced by the concessions that are usually made to the operator unions in return for their acceptance of these provisions (Fielding, 1987).

Overtime is also favoured because of its relatively low cost compared to full-time operators. However, its use is usually restricted to the cover of trippers and uncovered open work (open work without reserve cover). This is largely due to its limited supply, about 3 to 5 percent of total work.

On any given day, overtime comes primarily from operators working on that day, and to a lesser extent from operator on their day-off. Hence, potential overtime at any particular time period, during the day, may be determined by finding the difference between the number of operators that have already reported to work on that day, and

those on active duty during that time period (see Figure 1.5). Hence, overtime is usually most abundant during the off-peak demand periods, when it is least needed. However, potential overtime is significantly higher during the afternoon peak period, compared to the morning peak period. This additional overtime comes from operators who have completed their assigned work duties for the day. During off-peak hours, overtime is mainly supplied by operators working split runs, who can work overtime during the break between their morning and afternoon work shifts.

FIGURE 1.5: TRANSIT OPERATOR WORK SCHEDULES ON A TYPICAL WEEK DAY AT STO -- FALL TIMETABLE, 1990.

Research into the relationship between uncovered open work -- work intended to be covered by overtime -- and service reliability have shown a strong positive relationship between the two . A case study of the relationship between uncovered open work and missed trips at the Massachusetts Bay Transit Authority (MBTA) has shown that beyond a certain threshold uncovered open work is usually translated into missed trips at the rate of one missed service hour for every four hours of uncovered open work (Shiftan and Wilson, 1991).

1.2.4 Work Rules and Regulations

The work rules, and regulations contained in the labour agreement have considerable impact on the size, composition, and utilization of the transit operator workforce. These regulations contain provisions concerning:

1. pay compensation and fringe benefits for full-time and part-time operators, and overtime,
2. vacation liability, holidays, and contractual absence,
3. proportion of full-time operators to part-time operators,
4. regulations concerning the permissibility of certain types of operators to perform certain kinds of jobs,
5. regulations concerning the length of the work day and work week for various operator classifications,
6. regulations concerning the right of full-time operators to consecutive days-off, and
7. pay compensation for sick leave, maternity leave, etc...

1.3 STATE OF THE ART

The two most critical steps in transit operator planning and management involve the estimation of open work cover requirements at the strategic level and the scheduling of these operators to cover duties over time periods of the day (operational decisions). Given the uncertainty concerning the demand resulting from open work and its considerable variability over time (weeks, days and times of day), and without the support of analytical models to predict the impact of open work cover decisions on service reliability, transit managers tend to aim for the highest level of cover attainable under budget constraints. Such high cover levels serve as a guarantee against service cancellations which, usually, result from shortages in the availability of operators.

At the strategic level, the common practice among transit agencies -- including OC Transpo and STO -- is to add on top of schedule requirements (i.e., number of drivers required to operate the scheduled runs estimated by the run cutting process) for a particular timetable, about 10 to 15 percent of these requirements for open work cover. The sum of schedule and open work requirements is then used to estimate vacation requirements.

The proportion of cover to schedule requirements is not based on any formal models. Instead it is based on the judgement and personal experience of the person or persons involved in decision making. Similar "rules of thumb" are used to assign reserve (extra-board) operators to cover duties on a daily basis -- e.g., one cover operator for every nine scheduled runs commencing at a particular time of day.

The cost and reliability of the transit operator workforce are highly sensitive to the

magnitude of cover decisions, which -- as stated earlier -- often tend to overestimate cover requirements as a precaution against service cancellations. In spite of that, shortages do occur even in the most overstaffed transit properties, due largely to the mismatch between the daily pattern of open work and the report times of reserve (extra-board) operators (MacDorman & Associates, 1987).

Growing transit operating expenses during the seventies and early eighties (primarily operator wages and fringe benefits), paired with increasing pressure for fiscal restraint, generated interest in the development of analytical models for the optimization of transit operator planning and management methods. The pioneering works in this area were undertaken by MacDorman and Associates, by commission from the Urban Mass Transportation Administration (MacDorman et al, 1982, 1984 and 1985). This effort culminated in the development of TOPDOG , a model for the optimization of strategic level decisions -- workforce size, hiring policy, and vacation schedules. The model uses incremental analysis to estimate the least cost operator work force (MacDorman et al, 1986).

TOPDOG has been criticized on several grounds, most important, among which are: 1) its assumption that full-time operators are always available to work overtime, 2) failure to consider the effect of attrition and promotions on hiring decisions, and 3) ignoring the issue of extra-board utilization. This last omission is arguably the most significant shortcoming of the model. Since, all the savings promised by the model cannot be realized unless optimal use is made of the extra-board at the daily level.

The only attempt, thus far, to develop a comprehensive modelling approach to transit

operator workforce planning was carried out at Massachusetts Institute of Technology (MIT), under the supervision of Professor Nigel Wilson. The focal point of this research was the Ph.D Thesis of Dr. Haris N. Koutsopoulos titled "A Methodology For Transit Operator Workforce Planning" (1987). The proposed modelling framework consists of three models -- strategic, tactical, and operational. These models have extended the work started by MacDorman by incorporating the effect of attrition on strategic level decisions, and by developing procedures for the assignment of operators to days of the week and time periods of the day.

The strategic model -- formulated as a nonlinear program with linear constraints -- determines simultaneously the most cost effective 1) workforce size, 2) hiring policy, and 3) vacation schedules, for two operator types (full-time and part-time operators) over the length of the service plan. The length of the week is used as the unit of analysis since it is the smallest time period for which vacation allocation and hiring decisions are made. The inputs to this model include: 1) schedule requirement by timetable; 2) open work; 3) labour agreement provisions pertaining to vacation requirement by operator classification, overtime premiums, fringe benefits, etc.; and 4) management's policy with respect to service reliability, promotions, and frequency of hiring decisions.

While, the strategic model determines the supply of operators, by job classification, for every week of the year, the tactical and operational models focus, respectively, on the allocation of available operators to days of the week and time periods of the day. The tactical model schedules available operators to work duties on various days of the week. This includes the assigning of operators to both scheduled runs and open work. The inputs

to this model are: 1) workforce size by job classification; 2) actual number of operators on vacation during each week of the timetable, or planning period; 3) scheduled runs and absenteeism levels by job classification for each day of the week; 4) operators who choose the single day vacation option; 3) amount of extra unscheduled work; and 4) work rule regulations with respect to days off.

The operational model focuses on the assignment of extra-board operators to unexpected open work during the day. The inputs to this model are: 1) distribution of open work by time of day, 2) work rule constraints, and 3) number of available extra-board operators.

This work has served as the basis for a number of models. The first of these models attempted to improve the strategic model, by incorporating into it the feedback from tactical and operational level decisions, and their interrelationships with the daily patterns of open work (Hickman et al, 1988). The second model dealt with the scheduling of the extra-board (Kaysi et al, 1990).

The last of these models was developed Yoram Shiftan (1991). This model, also, attempted to improve the solution to the strategic level problem, by decomposing it into two sub-problems: a single period problem and a multi-period problem. This approach simplifies the problem significantly in comparison to the formulation developed by Hickman et al (1988). This simplification makes it easy to solve the problem on micro computers using any of the commonly available linear programming packages, such as Lindo (Shiftan and Wilson, 1991).

This research has, also, investigated the relationships between overtime and

absenteeism on one hand, and missed trips and uncovered open work (open work left open to be covered by overtime) on the other hand. These two issues are important because of their impact on service cost and reliability. On the first issue, many including Leahy and Shlegel (1979), Brown (1981), Perry (1983), and Perin (1984) have suggested that widespread availability of overtime induces absences. However, there was no study to confirm or deny this hypothesis. On the second issue, Kaysi and Wilson (1990) had suggested a linear model for missed trips as a function of uncovered open work.

Based on data collected from Massachusetts Bay Transit Authority (MBTA), this study has found a strong relationship between missed trips and uncovered open work, and no relationship between absenteeism and the amount of overtime worked by an operator. The finding concerning the relationship between missed trips and uncovered open work was incorporated into the formulation of the strategic model.

In spite of the considerable contribution of these models to transit operator workforce planning and management, however by considering open work at the weekly level these models have overlooked two of the most important attributes, namely: 1) the variability in open work rates over time periods of the day, and 2) the interaction between cover decisions over time on one hand, and between these decisions and the daily pattern of open work on the other. These attributes of the problem are only discernable at the time of day level. Furthermore, these models have not dealt with the problem created by morning trippers. As demonstrated in the modelling framework developed in this research, these attributes of the problem have a significant impact on cover policies, their cost and reliability levels.

While open work at the weekly level may be considered, to some extent, to be independent from one week to the next -- given that a significant proportion of open work is of less than a week's duration, this is not the case with respect to open work at the daily level. Here the two parameters describing the state of the system at any time of day (the number of available extra-board operators, and the number of expected open runs) are determined by the history of the system. For example, extra-board operators available for work at time period t are the sum of reserve operators reporting for work at time period t , and those remaining without work from previous time periods because of the lack of a sufficient number of open runs. Similarly, open runs at time period t consist of runs that became open at time t and those remaining open from previous time periods, due to shortages in the supply of operators during these time periods. As a result of this, the decision to assign a given number of extra-board operators to a given time period t does not only affect the performance of the system with respect to the work that may become open at time (t) , but also with respect to all the work that may unexpectedly become open during the rest of the day. Hence, decisions of this nature can not be optimized for individual time periods without taking into consideration the interrelationships between open work and cover decisions over time periods of the day.

1.4 OPERATOR WORKFORCE PLANNING IN OTHER TRANSPORTATION SYSTEMS

The transit operator workforce problem has its parallel in other transportation systems which rely on individual operators for the control and guidance of their vehicles. Examples of such systems include intercity air and rail transport systems. Operator

workforce planning in these industries resembles very closely the planning and management process in transit systems, particularly with respect to the determination of reserve requirements and the scheduling of reserve operators over time.

Here, as well, the workforce planning process relies on "rules of thumb" rather than on analytical models. This is the case at Air Canada which maintains -- according to industry sources -- a pilot reserve workforce of between 12 to 15 percent in excess of its schedule requirements. Reserve pilots are placed on call, at major hub airports: 1) to cover any shortages resulting from operator absenteeism and attrition, and 2) to work extra flights which are not part of the regular service schedule.

Based on extensive review of the literature, and telephone conversations with Air Canada and CP Rail personnel -- the following conclusions were reached concerning operator workforce planning in transport services, other than urban transit:

1. the structure of the operator planning process in these industries is similar, in its general outline, to that of the transit industry,
2. there appears to be very little or no transfer of knowledge among various competitors and systems,
3. the existing methodologies for operator workforce planning and management rely, primarily, on personal experience and judgement, and
4. the cover of open work is usually based on intuitive assessment of historical records of absenteeism, without the benefit of comprehensive mathematical models.

1.5 OBJECTIVES OF THIS RESEARCH

The research carried out in this field has provided ample evidence of the need for --

and the benefits which may be attained from the development of analytical models for transit operator workforce planning and management. Among these benefits is the ability to develop better estimates of transit operator requirements, and to assess accurately the costs and reliability levels associated with alternative workforce plans.

This research aims to develop a comprehensive and integrated modelling framework for the optimization of the transit operator workforce planning and management decisions, namely:

1. transit operator workforce size,
2. the annual vacation schedules of operators,
3. magnitude and frequency of hiring decisions,
4. assignment of operator work duties to days of the week, and
5. assignment of daily report times to extra-board operators.

The proposed modelling framework is intended to serve as a planning aid to transit managers at all levels of the managerial hierarchy. Its objectives are to:

- improve the accuracy with which transit operator requirements are estimated, minimize transit operator costs and improve service reliability,
- permit management to consider and evaluate a wider range of service options,
- free managerial resources for work on other tasks and projects, and
- improve the understanding of transit operator planning and management processes.

Toward this end, the proposed modelling framework has attempted to incorporate into the planning process all the dynamic and probabilistic aspects of the system discussed above, namely:

- the dynamic aspects of the demand for operators resulting from the temporal variability in the patterns of scheduled runs, trippers, and open work,
- the stochastic variability in the magnitudes of open work and attrition rates,
- the interdependence between open work and its cover over time periods of the day,
- the variability in the duration of open work and trippers, and
- the dynamic and stochastic variability in the availability of overtime cover.

The incorporation of these aspects of the system are key to the achievement of the optimization objective.

1.6 A BRIEF DESCRIPTION OF THIS DOCUMENT

In addition to this chapter, this document consists of five chapters. Chapter 2 outlines the modelling framework developed in this research; its underlying assumptions and the interrelations among its various models -- strategic, tactical and operational -- and their contributions to the state of art. As well, it describes briefly each of these models, their objectives, inputs and solution procedure. The detailed mathematical formulations for these models are presented in chapters 3 and 4.

The strategic group of models are developed in Chapter 3. These models estimate consecutively the optimal; 1) transit operator workforce size, 2) annual vacation schedules for operators, and 3) hiring policy. The first of these models -- the workforce size optimization procedure -- is by far the most important, because of its considerable impact on service cost and reliability, either directly or via its inputs to lower level decisions. The main building block of this model is the dynamic programming procedure used to estimate the cover requirements for daily open work of various durations. This procedure

incorporates into the decision process all the dynamic and probabilistic aspects of open work, and its interdependence and interaction with cover decisions over time periods of the day.

The tactical and operational models are presented in Chapter 4. These two models deal, consecutively, with the assignment of operators to days of the week and to time periods of the day. The operational model is the more complex of the two. This model involves the scheduling of reserve (extra-board) operators to cover unexpected open work at various time periods of the day. This problem is similar to the open work cover problem at the strategic level. As a result, the dynamic programming procedure developed for the scheduling of extra-board operators is similar to that used for the estimation of open work cover requirements at the strategic level. In fact, the open work cover model may be used for scheduling purposes, as is demonstrated in the case study presented in Chapter 5.

Given the importance of these dynamic programming procedures to the optimization of open cover decisions at the strategic and operational levels, a case study is presented in Chapter 5, to demonstrate the effectiveness of this group of models, and their sensitivity to changes in the magnitude of inputs namely, cancelled trip costs and open work rates.

The last chapter (Chapter 6) highlights the contributions of this research to transit operator work force planning, and presents recommendations for future research in this area, particularly with respect to the implementation of the proposed modelling framework.

Chapter II

METHODOLOGICAL FRAMEWORK

2.1 INTRODUCTION

The proposed modelling framework consists of five models designed to optimize the set of strategic, tactical and operational decisions involved in transit operator workforce planning and management. These models are:

1. Workforce size model,
2. Vacation model,
3. Hiring model,
4. Tactical model, and
5. Operational model.

Figure 2.1 shows the structure of the decision process optimized by these models and the interrelationships among its various decisions and their primary inputs -- scheduled runs, and trippers, open work and attrition.

This methodology is structured such that higher level decisions receive the support of lower level decisions, thus ensuring that the benefits of optimization at higher levels of the decision hierarchy are not dissipated due to sub-optimal policies at lower levels. It is important to note in this regard the role of the scheduling models (Vacation, Tactical and Operational models) in maximizing the availability of operators for work over time -- weeks, days and times of day.

FIGURE 2.1: DECISION STRUCTURE

The corner stones of these models are the dynamic programming procedures used for the optimization of daily open work cover, hiring decisions, and the assignment of daily report times to extra-board operators. The optimization of these tasks is the main source of complexity in transit operator workforce planning and management. This is largely due to: 1) the probabilistic and dynamic aspects of open work, 2) its variable duration, and 3) its interaction with supply decisions over time.

Dynamic programming is particularly suited for the representation of these dynamic and probabilistic aspects of the problem. A common feature of these dynamic programming procedures is the division of the planning period into a number of stages (smaller time periods). At every stage, the system could be in any one of a number of states, which represent the range of supply options considered over the planning period. The transition into these states is affected by the set of decisions made at a particular stage. The subject of these decisions is the number of operator days or weeks to be assigned to the stage.

The set of consecutive decisions leading to a particular state-stage combination constitutes a policy. The decisions in a particular policy are interrelated in the sense that a decision in one stage results in surplus and / or shortages, which affect levels of supply (operator availability for work) and demand (level of open work) during the following stage. These surpluses and shortages are the means by which the decisions in a particular policy interact over time, and they affect both the cost of decisions, as well as the cumulative cost and reliability of their associated policies.

At every stage, the objective of the decision process is to determine the minimum cost

policies leading to various states. At the final stage, the state which attains the desired service reliability level is then selected. Figure 2.2 shows the general structure of these dynamic programming procedures and the interrelationships among decisions over time. Accordingly, the outputs of one decision are the inputs to another decision. These outputs/inputs include the surplus, shortage, cost and reliability levels resulting from the set of decisions leading to the decision under consideration.

A general description of the objectives, inputs, and optimization methodology for each of the five models included in this modelling framework, is presented in the following sections. Detailed mathematical formulations of these models are presented in Chapters 3 and 4.

FIGURE 2.2: GENERAL STRUCTURE OF DYNAMIC PROGRAMMING PROCEDURES

2.2 WORKFORCE SIZE

2.2.1 General

The objective of the workforce size model is to estimate the minimum cost transit operator work force size required to maintain a certain level of reliability throughout the service plan. This model is by far the most important among the models included in the proposed modelling framework. This task is carried out several times in the year, at the service planning stage and at the beginning of every service plan.

Assuming that all scheduled runs have either full-time or part-time operators assigned to them and that the proportion of full-time and part-time work is predetermined by the labour agreement or management decision, workforce size optimization is in essence the optimization of cover requirements for open work. The general features of the dynamic programming procedure developed for this purpose are outlined in the introduction to this chapter.

Given a particular daily pattern of open work, optimization involves the determination of the daily cover pattern which minimizes the discrepancy between these two patterns. This discrepancy is a function of the magnitude and pattern of daily cover. Since all the utilization policies associated with a particular daily cover option (number of operator days) have similar direct costs -- daily cover level multiplied by the cost of one operator day of cover -- and different shortage and surplus costs, hence the minimum cost cover policy associated with a given daily cover option is that which minimizes the sum of surplus and shortage costs.

Since reliability is negatively related to shortages (high reliability correspond to low

shortages and vice versa), hence increasing the cost assigned to shortages versus surpluses increases the desirability of policies with low shortages. To illustrate this point let us compare the minimum cost utilization patterns of 9 operator days over a four hour test period (discussed in Chapter 5). All things being equal, a change in shortage costs from \$25/hour in Test 1 to \$500/hour in Test 5 of the Case Study, has resulted in a considerable change in the assignment pattern and reliability level of the minimum cost cover policy.

In Test 1 the minimum cost operator utilization pattern over the four hour period was "0, 4, 5, 0" with a reliability level of 99.13 percent. This assignment pattern was changed in Test 5 to "2, 5, 2, 0" which has a reliability level of 99.55 percent. This policy is more effective than the minimum cost policy associated with 11 operator days in Test 1 with its 99.5 percent reliability (see Tables A.1.1 and A.5.1 in Appendix A).

2.2.2 Inputs

The inputs to the workforce optimization procedure include the following demand and cost factors as well as functional, contractual and managerial constraints:

- service runs and trippers,
- hourly open work rates,
- daily and hourly compensation rates for full-time operators,
- hourly compensation rates for part-time operators,
- overtime premiums,
- hourly overtime availability rates,
- proportion of part-time and full-time operator days,

- proportion of overtime to total work,
- contractual daily and weekly work hours for full-time operators,
- contractual daily and weekly work hours for part-time operators,
- maximum permitted daily work hours (including overtime), for full-time operators,
- number of work days per week for full-time operators,
- number of work days per week for part-time operators,
- number of vacation weeks per year for various operator classifications, and
- reliability level.

Figure 2.3 shows the contributions of these inputs to the various decisions involved in the workforce size estimation process.

2.2.3 Optimization Methodology

In order to incorporate fully the effects of the above inputs into the decision process, particularly, those of open work, a multi-stage estimation procedure was developed. Figure 2.4 shows the structure of this approach in which the service plan is broken down into a number of timetables. Each timetable has a number of day types with similar service schedules (runs and trippers). These day types correspond to various days of the week and holidays (which usually have similar service patterns as Sundays).

The estimation of annual operator requirements starts at this level. This is carried out in two stages. The first and most important of which involves the estimation of daily open work cover requirements. At this level all the complex issues in transit operator workforce planning are dealt with, namely: 1) the availability of overtime, 2) the dynamic and probabilistic aspects of open work, 3) its variable duration and 4) interaction with

FIGURE 2.3: WORKFORCE ESTIMATION PROCESS

* Annual operator costs with the exception of hiring costs.

FIGURE 2.4: STRUCTURE OF THE WORKFORCE SIZE OPTIMIZATION PROCEDURE

cover decisions over time periods of the day.

Open work cover is estimated in two steps. This breakdown is intended to account for the duration aspects of open work. At the first step, the cover requirements for open work greater than or equal to five hours in duration are estimated. Since, the average duration of open work of this type is about 7 hours (STO open work data), its cover duties are assumed to constitute a full day's work. At the second step, the cover requirements for open work less than five hours in duration are estimated. Because of its relatively short duration, this type of open work can be combined with trippers to constitute a full day's work. A dynamic programming procedure is developed for the optimization of each of these two policies. The details of these two models are presented in Chapter 3.

The analysis of open work data from STO has indicated that open work greater than five hours in duration constitutes, respectively, 80 percent of open work hours and 60 percent of open work pieces. On the other hand, open work less than five hours in duration constitutes, consecutively, 20 percent of open work hours (about 2 percent of total demand) and 40 percent of open work pieces. The same analysis has shown that trippers constitute about 5 percent of total demand. Hence, there are enough trippers to be used as complementary pieces for open work of less than five hours, and to absorb all the overtime (about 3 to 5 percent) that can be reliably supplied. The implication of this finding is that, at the strategic level, overtime is considered only with respect to the cover of trippers. This decision is consistent with practice, which tends to limit the number of hours that an operator can work on any given day to less than twelve hours.

Having determined open work cover requirements and costs for a particular day type,

these estimates are then added to scheduled runs and their costs, to determine consecutively the total number of operator days required for that particular day type, and their costs. The same procedures are repeated for every day type in each timetable of the service plan.

In order to estimate operator requirements and costs for a particular timetable, operator requirements and costs for every day type are multiplied by the number of similar days in that timetable, and then added across all day types to give the total number of operator days required during the timetable. This procedure is repeated for every timetable in the service plan. The sum of the operator requirements for each of these timetables and their associated costs represent the total operator requirements (operator work days) and costs for the service plan, with the exception of the operator requirements and costs of hiring decisions.

Workforce size is then estimated by dividing the total number of operator work days required for the service plan by the average number of contractual work days per operator per year. In the case of two operator classification (full-time and part-time), provisions of the labour agreement with respect to this issue is used to determine the numbers of full-time and part-time operators.

The following is a step by step outline of the tasks described in this section:

Step 1 : Estimate the number of operator work days and costs required for the cover of daily open work greater than or equal to five hours in duration, during a particular day type.

Step 2: Estimate the number of operator work days and costs required for the cover of

trippers and daily open work less than five hours in duration. These costs also include the cost of overtime cover.

Step 3 : Multiply scheduled runs by the average daily operator cost (including wages, fringe benefits, and vacation pay) to determine the daily operator costs of scheduled runs.

Step 4 : Add scheduled runs and their costs, for a particular day type, to daily open work cover and its costs, for the same day type, to determine the number of operator work days required for that day type, and their associated costs.

Step 5 : Multiply the number of operator days and costs from step 4 by the number of days of this particular day type in the timetable, (eg. Mondays).

Step 6 : Repeat steps 1 through 5 for each day type in a particular timetable to estimate the number of operator work days required for the timetable and their associated costs.

Step 7 : Repeat steps 1 through 6 for every timetable in the service plan to estimate the total number of operator work days required for the service plan and their associated costs.

Step 8 : Estimate work force size by dividing the total number of operator work days for the service plan by the average number of contractual work days per operator per year.

Step 9 : In case two operator classifications are used, apply provisions of the labour agreement, to determine the proportions of full-time and part-time operators.

2.3 VACATION MODEL

2.3.1 General

This model has two objectives: 1) to satisfy the vacation requirements of the service plan, and 2) to maximize the availability of operators for work throughout the service plan. Since operators tend to take their vacations in pieces of weeks, the week is used as the time unit for scheduling vacations.

During any one week throughout the service plan operators are either scheduled to work, or be on vacation. Along these lines the total number of operator weeks in the system (work force size multiplied by 52) may be divided into vacation weeks and work weeks. While the total number of available vacation weeks must be equal to total vacation requirements, their schedules to various weeks of the year may be adjusted in accordance with the desire to maximize the availability of operators for work.

2.3.2 Inputs

The source of inputs to this model are work rules and regulations, and the workforce size optimization procedure and these inputs include:

- average number of work days per week for full-time operators,
- average number of work days per week for part-time operators,
- number of vacation weeks per year for various operator classifications,
- estimated operator work day requirements for various day types in every timetable, and
- estimated work force size.

The last two of these inputs are outputs of the workforce size optimization process.

2.3.3 Optimization Methodology

In order to maximize the availability of operators for work, the vacation weeks scheduled to a particular week during a given timetable are subordinated to work requirements during that week. As a result, weeks with high work requirements receive fewer vacation weeks than weeks with low work requirements.

This point is illustrated Figure 2.5, which shows the interrelationships between weekly work requirements and vacation schedules. In contrast to the common practice in which the number of vacation weeks is constant throughout a given timetable, the vacation schedules produced by this model are sensitive to the variations in the demand for operators from one week to another within the same timetable. Accordingly, weekly work requirements as high as the total number of operators in the system (workforce size) are permissible during weeks of exceptionally high demand. As is the case during seasonal festivities (Winterlude in Ottawa), when additional transit services are usually provided. Demand levels in excess of the workforce level may be accommodated, to some extent, by arranging hiring decisions such that a part of, or all, new hirees for a given year become available for work during the period or week of high demand.

The critical step in this procedure involves the estimation of operator work weeks required during every week of the service plan. This is carried out by summing the daily work requirements (operator work days) for all days of the week, and then dividing it by the average weekly work days per operator. Vacation weeks are then estimated by subtracting the estimated operator work weeks required for every week in the service plan

FIGURE 2.5: INTERRELATIONSHIPS BETWEEN VACATIONS AND WORK REQUIREMENTS

from the estimated workforce level. Then the total number of scheduled vacation weeks is checked against total vacation requirements to ensure that these requirements are met. These two values should be equal. Figure 2.6 shows the general structure of the vacation scheduling model. This is followed by a step by step outline of the scheduling process.

FIGURE 2.6: STRUCTURE OF THE VACATION SCHEDULING MODEL

Step 1 : Determine the total number of operator work days required during every week of the service plan by adding the operator work days required for all day types in the week, based on the estimates of step 4 in the workforce sizing model.

- Step 2 :** Estimate the average number of work days per week for the average operator based on work rules and the labour agreement.
- Step 3 :** Estimate the number of operator work weeks required during every week of the service plan, by dividing the results from Step 1 by those from Step 2.
- Step 4 :** Estimate the number of vacation weeks to be scheduled to every week of the year by subtracting the operator work weeks required during every week from the estimated work force size.
- Step 5 :** Estimate the total number of vacation weeks required for the service plan, based on workforce size and composition, operator classifications, and the labour agreement.
- Step 6 :** Compare the total number of vacation weeks resulting from Step 4, to total vacation requirements for the service plan. If the total number of vacation weeks is greater than or equal to annual vacation liabilities, then end procedure. If not, increment workforce size by one and repeat steps 4 to 6 until the above condition is met.

The common practice is that uniform vacation weeks are provided throughout a given timetable. Such conditions may be accommodated by the model, at a cost (loss in efficiency), by subordinating work requirements to vacation requirements. This policy would result, as well, in a uniform level of operator work weeks being supplied throughout the timetable. These uniform work weeks can be determined by dividing available work weeks for the time table by the number of weeks in that timetable.

2.4 HIRING MODEL

2.4.1 General

The demand for new operators results from two sources: 1) changes in the service plan, and 2) attrition. This model is built on the assumption that vacation requirements would be subordinated to work requirements, and hence all vacation provisions will be filled as scheduled. Operators interested in changing their vacation schedules would do so by trading vacation spots with other operators.

Since the demand resulting from service changes has already been accounted for in the workforce size model, the objective of this model is to determine the most cost-effective hiring policy to maintain the estimated workforce level in the face of attrition. Since the hiring policy is intended to communicate to the personnel department the needs of the transit agency for new operators, then it is appropriate to construct the hiring policy in terms of the number of new operators required at various time periods throughout the year. Such policy may be easily translated into a hiring plan by incorporating into these decisions the lead time required for advertisement and training.

Attrition exhibits the same dynamic and probabilistic characteristics as daily open work. The demand due to attrition at a given time period during the year is the sum of demand resulting from attrition at that time period, and as attrition-related shortages from previous time periods. For example, a vacancy created by an operator leaving the system remains open as long as a replacement is not in place. Hence, open work due to attrition is a function of hiring decisions at previous time periods.

However, unlike open work cover, the horizon for hiring decisions is the year rather

than the day. Furthermore, given the low frequency of attrition, the smallest time period for which this type of demand may be meaningfully evaluated is the week.

2.4.2 Inputs

Figure 2.7 shows the various inputs to hiring decisions. In addition to attrition rates, the inputs to the hiring model include a number of cost factors, contractual and managerial constraints. These inputs are:

- estimated workforce level,
- attrition rates among operators at various time periods (few weeks to a month) during the year,
- service reliability level,
- average weekly costs of a new operators (including wages and fringe benefits),
- advertisement and processing cost per hiring decision (order), and
- average training cost per operator.

2.4.3 Optimization Methodology

The system to be optimized at this level is the number of new operators required over the course of one year to cover the open work resulting from attrition among operators. A probabilistic dynamic programming model was developed for this purpose. In this model, the year is divided into an equal number of stages (time periods) each consisting of the same number of weeks. At each stage, a set of decisions are made on how to attain a particular state. These decisions affect the magnitude of the system (number of new operators hired up to and including the present stage), its cost and reliability.

Figure 2.7: HIRING POLICY

The decision which minimizes cumulative system costs is then chosen. The system costs associated with a particular state-stage combination consist of the cumulative costs of all the decisions included in the minimum cost policy leading up to that state-stage combination. At the final stage, the system configuration which attains the desired level of reliability at least cost is then chosen. The details of this model are presented in Chapter 3.

2.5 TACTICAL MODEL

2.5.1 General

The purpose of this model is to schedule work duties and days-off to days of the week for selection (sign-up) by available operators. Work duties consist of two types of work: regular and reserve (extra-board) work duties. Regular work duties correspond to scheduled runs. Extra-board work duties are intended to cover the demand created by absenteeism, attrition, vehicle breakdowns, accidents and some trippers. In order to maximize the availability of operators for work, the task of assigning operators to work duties is given precedence over the scheduling of days-off.

Figure 2.8 shows the interrelationship between work and day-off schedules over days of the week. Days with high work requirements (Monday to Friday) have very few off days scheduled to them with the bulk of the day-off requirements usually scheduled to the weekend when the demand for work is at its lowest level.

The frequency with which this activity is carried out varies considerably from one transit agency to another. In some agencies work schedules are made only once at the beginning of the timetable. While in others, they are updated every 2 to 3 weeks. The

FIGURE 2.8: DAILY OPERATOR SCHEDULES

latter policy permits management to incorporate more recent information on the state of demand and hence improve the match between demand and work schedules.

2.5.2 Inputs

The inputs to this model are largely determined by strategic level decisions; workforce levelling process, vacation schedules, and hiring policy. These inputs are:

- workforce level at the beginning of the scheduling period,
- daily work requirements throughout this period,
- vacation schedules,
- hiring policy,
- attrition rates, and
- the average number of work days per operator per week.

Figure 2.9 shows how these inputs influence work and day-off schedules.

2.5.3 Optimization Methodology

The proposed scheduling methodology consists of five steps. The first four steps are used for the scheduling of operator work duties to days of the week. While, the fifth step is used to determine daily day-off schedules.

Step 1 : Estimate available operator weeks for every week of the target period.

Step 2 : Estimate available operator work weeks for every week of scheduling period, by subtracting from the available operator weeks, during a particular week, the number of vacation weeks scheduled to that week.

FIGURE 2.9: WORK SCHEDULING PROCESS

Step 3 : Estimate available work days for every week by multiplying the available operator work weeks for that week by the average number of work days per operator per week (usually 5 days).

Step 4 : Allocate available work days to days of the week in proportion to the estimated daily work requirements determined in the workforce size model.

step 5 : Estimate daily day-off schedules by subtracting from the available operator work weeks the daily work schedules determined in Step 4.

2.6 OPERATIONAL MODEL

2.6.1 General

Work duties are divided into regular and reserve duties. While the report times for regular work duties are known in advance, since they correspond to the start of scheduled runs, the report times for reserve work duties have to be determined based on the observed patterns of open work. The objective of this model is to determine the most cost-effective assignment pattern for extra-board operators to time periods of the day, i.e. that which minimizes the sum of surplus and shortage costs associated with a particular level of reliability.

In large transit agencies, with multiple garages, the operational problem also involves the assignment of reserve operators to various garages. Given the ability of the operational model to estimate the cost and reliability levels associated with cover levels of various magnitudes, it would be relatively easy to determine the assignment pattern to garages which maximizes reliability. Such a problem may be formulated as dynamic programming

procedure with the garages representing the stages and assignment levels as decisions.

2.6.2 Inputs

The inputs to the reserve scheduling process include the following supply, demand and cost factors:

- available reserve cover on a given day,
- available overtime at various time periods of the day,
- hourly open work rates for open work greater than five hours,
- hourly open work rates for open work less than or equal to five hours,
- scheduled trippers,
- overtime premiums, and
- reliability level.

Figure 2.10 shows the interaction between inputs and decisions at various stages of the reserve scheduling process.

2.6.3 Optimization Methodology

The assignment of daily report times to reserve operators is carried out in three steps. The first two steps involve the scheduling of reserve operators to the cover of open work. A probabilistic dynamic programming model was developed for the optimization of each of these two tasks. The structures of these two models resemble to a large extent the two corresponding models used for the estimation of open work cover requirements at the strategic level. The details of these two models are presented in Chapter 4.

FIGURE 2.10: RESERVE SCHEDULING PROCESS

The first step involves the assignment of reserve operators to open work greater than or equal to five hours in duration. The remaining reserve operators are then assigned to open work of less than five hours in duration, in addition to some trippers. Since open work of this duration constitutes less than a full day's work, morning and afternoon work pieces of this type will be matched with each other and trippers to maximize the utilization of extra-board operators. In case any reserve operators are left, these operators will then be assigned to tripper pairs created from morning and afternoon trippers.

2.7 CONTRIBUTIONS OF THIS RESEARCH

Previous models have considered open work at the daily or weekly levels thus overlooking two of its most important aspects: 1) its variable duration, and 2) the interaction between its daily pattern and cover decisions (policies) over time periods of the day. As demonstrated in the Case Study (Chapter V), the nature of these two patterns and their interaction have significant impact on cover requirements, their cost and reliability levels.

The contributions of this research to the state of art stem from: 1) its original approach to the estimation of open work cover requirements, and 2) the development of a methodological framework to ensure that the above mentioned aspects of open work and its cover are incorporated into transit planning and management decisions. This methodology is structured such that the benefits of optimization at higher levels of the decision hierarchy are not dissipated due to sub-optimal decisions at lower levels. This objective is realized through: a) the subordination of vacation and day-off schedules to

weekly and daily work requirements, and b) the use of uniform procedures for the estimation of demand requirements for various decisions. The combined effect of these methodological and analytical contributions would result in a significant reduction in the level of uncertainty underlying the overstaffing problem in most transit agencies, and hence affect a significant reduction in transit operator costs.

The estimation of cover requirements is the main source of uncertainty in transit operator planning and management. Daily open work cover estimates influence policies at all levels of the decision hierarchy -- workforce size, vacation, tactical and operational decisions -- thus compounding the effect of miscalculations at this level. The ability of the open work cover models, developed in this research, to measure the reliability associated with alternative cover policies provide a firm basis for informed decision making at all levels of the decision hierarchy. Furthermore, the uniformity in the procedures used to estimate work requirements at various levels of the decision hierarchy improves the coordination among various decision units and ensures compatibility between their decisions.

The breakdown down of open work by duration has two advantages. On the one hand, it provides a basis for providing better estimates of cover requirements, by enabling us to maximize the amount of work assigned to cover operators. On the other hand, it provides the opportunity to solve the problem of morning trippers by combining some of these trippers with open work of short duration.

While at the daily or weekly level, it is very difficult to estimate the magnitude of overtime of the appropriate duration which may be used to cover open work at various

time periods of the day, this task is considerably easier at the time of day level, where the daily patterns of overtime of various durations can be plotted against open work patterns to determine those durations of open work which may be best covered by overtime.

Last but not least, the ability of the models to evaluate quickly and accurately the cost implications of service changes expands the range of options which may be considered by management at the service planning stage and during the planning for various timetables.

Chapter III

STRATEGIC MODELS

3.1 INTRODUCTION

In addition to this introduction, this chapter consists of three sections. The first of these sections (Section 3.2) presents the workforce size model, its underlying assumptions and mathematical formulation. The objective of this model is to estimate the most cost-effective workforce size required for a particular service plan, or timetable. At this level all the complex issues in transit operator workforce planning and management are considered, namely the dynamic and probabilistic aspects of open work and overtime availability.

Transit operator workforce size is estimated at the service planning stage and during the planning for various timetables. The outputs of this process have considerable impact on: 1) total system costs (operator wages, and fringe benefits and overtime), 2) transit service reliability, and 3) the range of options considered by other models.

The vacation model is presented in section 3.3. The objective of this model is to maximize the availability of operators for work throughout the service plan. All the inputs to this model are determined by the workforce size model. The hiring model is presented in Section 3.4. The objective of this model is to determine the most cost-effective hiring policy required to maintain estimated operator levels in the face of attrition.

3.2 WORKFORCE SIZE MODEL

3.2.1 General

The structure of the proposed workforce size estimation procedure was influenced by the desire to capture the dynamic, stochastic and duration aspects of open work, and to represent its interaction with cover decisions over time periods of the day. The failure to model these interrelationships would bias the estimation of reserve requirements and hence workforce size.

Open work in a given time period of the day is the sum of work which becomes open during that period and work remaining open from previous time periods because of the lack of sufficient cover during these periods. The same is true for open work cover. Assuming that open work is sufficiently long to constitute a full day's work, open work cover available at any time period during the day is the sum of open work cover scheduled to report at that period and all the open work cover which did not find work during the preceding time periods. These relationships are only discernable at the daily level, and hence, open work cover decisions have to be optimized at the daily level.

Figure 2.4 shows the structure of the proposed transit operator workforce size optimization procedure. This optimization methodology is structured along the major break points in the service plan -- the division of the service plan into a number of timetables, and the division of each timetable into a number of day types with unique service characteristic (scheduled service runs and trippers).

The estimation of annual operator requirements proceeds along these lines from the daily level up. At the daily level, daily operator requirements for every day type in each timetable of the service plan are estimated. These estimates are, then, used to determine operator requirements for each timetable, which are then used to estimate operator requirements for the service plan.

The estimation of daily operator requirements is carried out in two phases. The cover requirements for open work and trippers are estimated first. At this phase, all the complex issues in the estimation process are considered, namely the dynamic and probabilistic aspects of open work, its variable duration, and its interaction with cover decisions over time periods of the day. At the second phase, open work cover estimates are added to scheduled service runs to determine daily service requirements and costs.

3.2.2 Open work and Tripper Cover Requirements

Open work cover requirements are estimated in two steps. First, cover requirements for open work greater than or equal to five hours in duration are estimated. This is followed by the estimation of cover requirements for open work less than five hours, in conjunction with some trippers.

This breakdown is intended to ensure that full day cover duties are created. As indicated in Chapter II open work greater than or equal to five hours in duration has an average length of little over seven hours. On the other hand, open work duties of similar length may be created by combining open work pieces of less than five hours with each other and with trippers of the longest duration.

It is important to note here that the true optimum may require different splits than the two categories considered here. Such categories would have to be determined after careful analysis of the daily patterns of open work, trippers and overtime of various durations. However, the primary objective in determining the number of open work categories of various durations should be the maximization of the number of daily work hours assigned to individual cover operators.

The analysis of a sample of open work data, from STO, has indicated that open work greater than or equal to five hours in duration represents close to 80 percent of total open work hours (about 8 percent of total demand), and about 60 percent of all open work duties. The same analysis has shown that open work less than five hours in duration represents around 20 percent of total open work hours (about 2 percent of total demand), and about 40 percent of all open work duties.

Open work greater than or equal to five hours in duration is caused, primarily, by operator absenteeism. Since the majority of operators start their work during the morning hours, this type of open work tends to be concentrated in and around the morning peak travel period -- about 67 percent of this type of open work duties (STO open work sample data). Open work of less than five hours duration is caused largely by vehicle breakdowns and accidents. Such open work tends to be concentrated in and around the afternoon peak travel period -- about 75 percent of this type of open work duties (STO open work sample data). This phenomenon may be explained by the fact that vehicle breakdowns and accidents tend to increase with long service hours and under congested road conditions.

Open work is either covered by reserve (extra-board) operators, or as overtime by full-time operators, in addition to their regular work duties. Overtime is more economical as a means of open work cover, because operators working overtime are paid only for time spent on the job, while reserve operators are guaranteed full day's pay regardless of the number of active work hours. As well, overtime hourly compensation rates are, usually, lower than full-time hourly compensation rates (including fringe benefits). However, the use of overtime is limited by its availability.

Given the typical daily work profile, overtime is least available when it is most needed, i.e., during periods of high demand, particularly the morning peak-travel period. Analysis of overtime data at the Massachusetts Bay Transit Authority (MBTA) has shown that the overtime corresponding to reliability levels of 99.1 to 99.4 percent represents between 2 and 2.5 percent of total work (Hickman et al, 1988, and Shiftan et al, 1991). Since full-time operators at the MBTA represent only 60 percent of MBTA's operator workforce, it may be concluded that the overtime which can be reliably used in transit systems employing only full-time operators is in the order of 3 to 5 percent of total work. This is about the amount of work required to cover trippers, which represent about 5 percent of the total demand for operators (STO service data, Fall timetable 1990).

To avoid confusing the terms of the two open work cover models, terms used in the model for "estimating cover requirements for open work greater than or equal to five hours" will be superscripted by "1". While the terms used in the model for "estimating cover requirements for open work less than five hours" will be superscripted by "2".

3.2.2.1 Estimating Cover Requirements for Open Work Greater Than or Equal to Five Hours

System Definition

The system to be optimized, at this step, is the number and assignment pattern of operator days required on a particular day for the cover of open work greater than or equal to five hours in duration. Since most transit companies tend to limit their extra-board staff to full-time operators, the system to be optimized, in this step, will be the number of full-time operator days required for this type of cover duties.

Demand

The demand on this system at any time period "t" during the day, is the sum of uncovered open work from the previous time period "t-1", and open work arising during the current period "t". Open work generated at time period "t" is a function of: 1) the number of scheduled runs commencing at time period "t", and 2) the open run probability density function for this time period. While, the number of scheduled runs commencing at time period "t" may be easily obtained from the service plan, the probability distribution of open runs, for the same period, have to be estimated based on historical records of the operational performance of the system during similar time periods.

Any run scheduled to begin at time period "t" has a probability " $P(OW^t_i)$ " of becoming open, equivalent to the expected open work rate for that period " $EOWR^t_i$ ", and $1 - P(OW^t_i)$ chance of being filled as scheduled. Let us assume that the probability of any run becoming open during time period "t" is independent of whether other scheduled runs

became open during that period. Also, given that "SR_t" scheduled runs start at time (t) -- each with the same probability of becoming open -- it follows that the probability of exactly "OW_t¹" runs, out of "SR_t" scheduled runs, becoming open during period "t", may be reasonably approximated by the binomial distribution (Kaysi & Wilson, 1990);

$$P(OW_t^1) = \binom{SR_t}{OW_t^1} (EOWR_t^1)^{OW_t^1} (1 - EOWR_t^1)^{SR_t - OW_t^1} \quad (3.1)$$

or the Poisson distribution;

$$P(OW_t^1) = \frac{\exp^{-EOW_t^1} * (EOW_t^1)^{OW_t^1}}{(OW_t^1)!} \quad (3.2)$$

where,

OW_t¹: Open work of type "1" and magnitude "1" starting at time period "t".

P(OW_t¹): The probability that open work of magnitude "1" and duration "1" will occur at time period "t".

SR_t: Scheduled runs starting at time period "t".

EOWR_t¹: Expected open work rate of type "1" at time period "t".

EOW_t¹: Expected open work of type "1" at time period "t".

EOW_t¹ and EOWR_t¹ are estimated in Equations 3.3 and 3.4.

While the binomial distribution is better suited to estimate the probability of open work of this duration (type "1") during the morning peak travel period when the majority of daily runs are scheduled to start, the Poisson distribution is better suited for estimating the probability of this type of open work during off-peak periods, because; 1) few or no

additional runs are scheduled to start at these time periods, and 2) expected open work of this duration is very small. These suggestions are based on general observations. However, in practice the nature of these distributions must be determined from historical data.

Model Structure

In order to be able to model the above attributes of demand, namely 1) the dynamic and probabilistic aspects of open work, and 2) its interaction with cover decisions over time periods of the day, a probabilistic dynamic programming procedure was developed. In this model, the operating day is partitioned into a number of stages (time periods) of one hour duration. The states of the system at any particular stage are known with certainty, while the stage's demand is a random variable. The states of the system represent the range of alternatives (open work cover options) evaluated by the model for their cost and reliability values. All open work during a stage is assumed to occur at the beginning of the stage.

At every stage " t ", a set of decisions " D_t " are made concerning the number of operator days to be allocated to this time period, i.e., the number of cover operators required to report to work at the beginning of this time period. These decisions are influenced by the state of the system at the beginning of the period - the number of operators assigned to the system, over all the previous time periods, which influence the state of the system by changing its magnitude, cost and reliability levels.

The objective of these decisions is to minimize system cost -- the cumulative cost of all decisions up to and including the current time period. At the final stage of the decision

process, alternative system designs (open work cover options) are evaluated with respect to their cost as well as their reliability levels, and the system which attains the desired level of reliability at least cost is then selected. Figure 3.1 is a network representation of this decision process showing the interrelationships between the decisions and states over time periods (stages) of the day.

The following is a detailed description of the proposed open work cover optimization procedure; its inputs, stages, states, decisions, and performance measures. This model uses four performance functions to measure the efficiency and reliability of alternative open work cover policies. These measures are:

- the cumulative (total) cost function,
- the cumulative surplus-shortage cost function,
- the cumulative surplus-shortage function, and
- the reliability function.

The first three functions measure the efficiency of the system, while the reliability function measures its effectiveness.

Inputs: The inputs to this model include service and operational data, and cost and reliability factors. These inputs are:

- scheduled service runs (SR_t) commencing at the beginning of various time periods of the day,
- expected open work rates ($EOWR^1_t$), or expected open work (EOW^1_t), for open work greater than or equal to five hours in duration at various time periods of the day,

FIGURE 3.1: NETWORK REPRESENTATION OF THE DAILY OPEN WORK OPTIMIZATION MODEL

Note: The numbers in the circles represent the magnitude of the states, i.e. the cumulative number of operator-days assigned the system up to and including a particular stage.

- average daily full-time operator cost (ADFTC),
- average hourly overtime cost (AHOTC),
- cancelled service costs (CTC), and
- reliability level.

Scheduled service runs, average daily full-time operator costs, average hourly overtime cost, and the reliability level can be easily determined from the service plan, the labour agreement, composition of the operator workforce, and management policy.

Management's attitude toward service reliability is reflected in the manner by which it values cancelled services. In this regard "CTC" may be viewed as a penalty placed on shortages. The higher this penalty the more desirable are the policies which minimize shortages.

Since reliability is negatively related to shortages, hence the incorporation of this term into the objective function provides a mechanism by which we can determine for every cover option the minimum cost cover policy (cover pattern) which maximizes reliability. As is demonstrated in the Case Study presented in Chapter V, high "CTC" values tend to produce, for the same supply level, cover patterns which result in considerable improvements in system efficiency and reliability. A starting value of \$25 for "CTC" is used in Test 1 -- the Case Study -- because it gives equal weight to shortages versus surpluses, which are costed at the rate of the average hourly overtime cost. Progressively higher values are used in Tests 2-5 to show how changes in these values affect the reliability levels of the optimum (minimum cost) cover policies associated with alternative open work cover options.

Although these policies may be slightly more expensive than other policies of the same magnitude, however the gains in reliability resulting from their assignment patterns make these options more effective than the cover policies generated under lower "CTC" values for much larger and more expensive daily cover options. This point is well illustrated in the Case Study and by the example presented on Page 29 of Chapter II.

Expected open work rates and expected open work have to be estimated from operational data (Figures 1.1 and 2.1), which usually record open work by cause by time of day (Appendix D). Expected open work of type "1" (EOW^1_i), greater than or equal to five hours in duration is estimated for a given time period of a particular day type in a timetable, by dividing the sum of open work of this duration, for all similar time periods - by timetable, day type, and time of day -- of the previous service plan (eg., recorded open work of this duration during the fifth service hour of all Mondays) by the number of days of that type in that service plan. These are then multiplied by the ratio of the service runs scheduled to commence at this time period in the service plan under review, to the service runs scheduled to this time period during the previous service plan.

$$EOW^1_i = \frac{\sum_{d=1}^{Nd} OW^1_i}{Nd} * \frac{SR^c_i}{SR^p_i} \quad (3.3)$$

Expected open work rates of type "1" ($EOWR^1_i$) are estimated by dividing expected open work at a particular time period of the previous service plan by the number of service runs scheduled for this time period in the previous service plan.

$$EOWR_t^I = \frac{\sum_{d=1}^{Nd} OW_t^I}{Nd * SR_t^P} \quad (3.4)$$

OW_t^I : Open work of type "I" starting at time period "t".

d: Day type "d".

Nd : Number of days of type "d" in a given timetable.

SR_t^c : Scheduled service runs starting at time period "t", of day type "d", in a particular timetable of the coming service plan.

SR_t^p : Scheduled service runs starting at time period "t", of day type "d", in a particular timetable of the previous service plan.

Stages ($t=1..n1$): For the purposes of this model the operating day has been divided into a number ($n1$) of one hour intervals (stages). However, since the work likely to become open during the last four hours will be less than five hours in duration, the number of stages over which daily open work cover -- for open work of this type -- is to be optimized consists of all time periods in the operating day with the exception of the last four hours.

States (S^I_t): At any time period (stage) during the day the state of the system represents the number of full-time operator days assigned to this type of open work cover up to and including that time period. As shown in Figure 3.1, at any stage the system may be in one of a number of states. These states represent the range of options to be evaluated by the model. Each of these states has associated with it a multitude of assignment policies -- decision sets.

Since the optimal number of operator days required for this type of work is not likely to exceed by far -- not more than twice -- expected open work for the day, an upper limit may be set on the magnitude of states which the system may assume at any particular stage. This constraint is expressed as follows:

$$0 \leq S_t^I \leq \beta * EOW_d^I \quad (3.5)$$

where,

$$1 < \beta \leq 2 \quad (3.6)$$

and, " EOW_d^I " is expected open work greater than or equal to five hours for day type "d".

At any one stage, the state of the system serves a dual role. On the one hand it represents the output of the decision process up to and including that stage. On the other hand it serves as an input to the next stage of the decision process (Figure 2.2).

The input state at stage 1 is zero.

$$S_0^I = 0 \quad (3.7)$$

Decisions (D_t): At every stage, a set of decisions is made on the number of full-time operator days required for this type of cover. These decisions are influenced by the state of the system in the previous stage. Since decisions are always positive and given constraints on the magnitude of states (Equation 3.5), the range of decisions associated with a particular stage-state combination is determined by the difference between input and output states.

$$0 \leq D_t \leq S_t^I - S_{t-1}^I \quad (3.8)$$

where " S_{t-1}^I " and " S_t^I " represent, consecutively, the input and output states.

Furthermore, since the optimal policy is very likely to be such that open work cover allocated to a particular stage closely approximates the demand for that stage. The number of operators assigned to any state-stage combination may also be constrained as follows:

$$0 \leq D_t \leq \gamma EOW_t^I \quad (3.9)$$

where,

$$1 \leq \gamma \leq 2 \quad (3.10)$$

At any stage, the most restrictive of constraints 3.8 and 3.9 would apply. These two equations enhance the efficiency of the proposed dynamic programming procedure by reducing the amount of computations that would have to be performed. However, these limits should be set such that the optimum is not affected.

Transition Function: The decision associated with a particular stage-state combination transforms the system by increasing the number of operator work days allocated to this type of cover, on a given day:

$$S_t^I = S_{t-1}^I + D_t \quad (3.11)$$

Cumulative Total Cost Function: Cost is the criterion by which the system's efficiency is evaluated. The total cost function is the means by which the overall cost associated

with a particular policy, or set of decisions, leading to a particular state-stage combination is determined.

$$F_{t+1}(S_{t+1}^I) = EC_{t+1}(S_t^I, D_{t+1}) + F_t(S_t^I) \quad (3.12)$$

where,

$F_{t+1}(S_{t+1}^I)$: Cumulative cost of the set of decisions -- allocation policy, or assignment pattern -- leading to state " S_{t+1}^I " at stage " $t+1$ ".

EC_{t+1}^I : Expected cost of the decision " D_{t+1} ", at stage " $t+1$ ", leading from state " S_t^I " at stage " t " to state " S_{t+1}^I " at stage " $t+1$ ". This decision is included in the above set of decisions.

$F_t(S_t^I)$: Cumulative cost of the set of decisions leading to state " S_t^I " at stage " $t+1$ ". This set of decisions is a subset of the decision set leading to state " S_{t+1}^I ".

Given the numerous paths (policies) leading to a particular stage-state combination, the cost of the system associated with such a state varies considerably according to which of these paths is followed.

The decision cost associated with a particular state-stage combination consists of three parts: a deterministic term and two probabilistic terms. The deterministic term represents the direct cost of the operator work days allocated to a particular stage. The two probabilistic terms represent surplus and shortage costs.

$$EC_{t+1}(S_t^I, D_{t+1}) = DC(D_{t+1}) + ESURC_{t+1}(S_t^I, D_{t+1}) + EShC_{t+1}(S_t^I, D_{t+1}) \quad (3.13)$$

$DC(D_{t+1})$: Direct cost of decision " D_{t+1} ".

$ESurC_{t+1}(S_t^1, D_{t+1})$: Expected surplus cost of decision " D_{t+1} ", at stage " $t+1$ ", given that the input state is " S_t^1 ".

$EShC_{t+1}(S_t^1, D_{t+1})$: Expected shortage cost of decision " D_{t+1} ", at stage " $t+1$ ", given that the input state is " S_t^1 ".

Direct decision costs are determined by multiplying the number of operator work days allocated at a particular time period (stage), by average daily full-time operator costs.

(3.14)

$$DC(D_{t+1}) = ADFTC * D_{t+1}$$

Average daily full-time operator costs (ADFTC) include fringe benefits and vacation pay.

While direct decision costs at a particular stage are entirely independent of the state of the system, expected shortage and surplus costs are also influenced by the pattern of decisions leading to that stage and their interactions with open work patterns over time periods of the day. Although, shortage and surplus costs are unavoidable because of the stochastic nature of open work, unreasonably high surplus and / or shortage costs are indications of inefficiencies in the system.

Surplus costs occur when the number of operator days available at a given time period during the day is larger than the demand for open work cover during that period. The number of operator work days available at a particular time period of the day is the sum of operator work days allocated to that time period and the expected operator surplus from the preceding time period. Similarly, demand at a particular time period consists of open work becoming open during that time period and expected operator shortages from the previous time period.

Consistent with the assumption that open work cover duties are sufficiently long to constitute a full day's work for full-time cover operators in order to be able to use surplus operators for full-time cover duties at later stages these operators must be compensated for the extra time they spend in the system between the time they report to work and the time they are assigned a run. This extra time, or surplus time as referred to in this cost function, is actually an overtime coming from cover operators. Hence, expected operator surplus time will be valued at overtime rates.

$$ESurC_{t+1} = ESur_{t+1} * AHOTC \quad (3.15)$$

where,

$$ESur_{t+1} = \sum_{i=0}^{AS_{t+1} - Esh_t} P(OW_t^i | t+1) * (AS_{t+1} - AD_{t+1}) \quad (3.16)$$

and

$$AD_{t+1} = OW_t^1 + ESh_t \quad (3.17)$$

$$AS_{t+1} = D_{t+1} + ESur_t \quad (3.18)$$

$$AS_{t+1} \geq AD_{t+1} \quad (3.19)$$

$$AS_{t+1} - Esh_t \geq 0 \quad (3.20)$$

$ESur_{t+1}$: Expected surplus associated with a particular decision-state combination at stage "t+1".

$ESurC_{t+1}$: Expected surplus cost associated with a particular decision-state combination at stage "t+1".

AHOTC: Average hourly overtime cost.

AD_{t+1} : Actual demand at time period "t+1" , given that the open work occurring at this time period is of magnitude " OW_t^1 ".

AS_{t+1} : Actual operator supply at the beginning of stage "t+1".

$P(OW_t^1/t+1)$: The probability that open work of magnitude "i" and duration "l" will occur at stage "t+1".

$ESur_t$: Expected surplus from stage "t".

ESh_t : Expected shortages from stage "t".

The expected surpluses ($ESur_t$) and shortages (ESh_t) associated with a particular state " S_t " at stage "t" incorporate into a given assignment decision " D_{t+1} " at stage "t+1", the effect of all the decisions leading to " S_t " which are members of the same supply policy as " D_{t+1} ".

Having decided to use the limited supply of overtime available in the system (transit operator workforce) for the cover of trippers and some unexpected open work of less than five hours duration, shortages in this model would result in cancelled trips. As discussed earlier, cancelled transit services may be assigned any one of a range of values starting with overtime rates as its lower limit.

$$EShC_{t+1} = ESh_{t+1} * CTC \quad (3.21)$$

where,

$$ESh_{t+1} = \sum_{x > AS_{t+1} - ESh_t}^{SR_{t+1}} P(OW_t^I / t+1) * (AD_{t+1} - AS_{t+1}) \quad (3.22)$$

and,

$$AD_{t+1} \geq AS_{t+1} \quad (3.23)$$

SR_{t+1} : Scheduled service runs commencing at time period "t+1".

CTC: Cost assigned to one hour of cancelled bus service.

Figure 3.2 illustrates the relationship between supply decisions and open work on one hand, and surplus and shortage costs on the other hand.

Cumulative Surplus-Shortage Cost Function:

The cumulative surplus-shortage cost (CSSC) associated with a particular state-stage combination is the sum of the cumulative surplus cost (CSurC) and cumulative shortage cost (CShC) of the set of decisions (cover policy) leading to that state-stage combination.

These cost are estimated as follows:

$$CSSC_{t+1}(S_{t+1}^I) = CSurC_{t+1}(S_{t+1}^I) + CShC_{t+1}(S_{t+1}^I) \quad (3.24)$$

$$CSurC_{t+1}(S_{t+1}^I) = ESurC_{t+1}(S_t^I, D_{t+1}) + CSurC_t(S_t^I) \quad (3.25)$$

2

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FIGURE 3.2: PROBABILISTIC COSTS OF DECISIONS

$$CShC_{t+1}(S_{t+1}^1) = EShC_{t+1}(S_t^1, D_{t+1}) + CShC_t(S_t^1) \quad (3.26)$$

$$CSurC_t(S_t^1) = EShC_t(S_{t-1}^1, D_t) + CSurC_{t-1}(S_{t-1}^1) \quad (3.27)$$

$$CShC_t(S_t^1) = EShC_t(S_{t-1}^1, D_t) + CShC_{t-1}(S_{t-1}^1) \quad (3.28)$$

$CSurC_{t+1}(S_{t+1}^1)$: Cumulative surplus cost of a particular decision set leading to a given state " S_{t+1}^1 " at stage " $t+1$ ".

$CSurC_t(S_t^1)$: Cumulative surplus cost of the decision set leading to state " S_t^1 " at stage " t ", which is a subset of the decision set leading to state " S_{t+1}^1 " at stage " $t+1$ ".

$ESurC(S_t^1, D_{t+1})$: Expected surplus cost of decision " D_{t+1} " at stage " $t+1$ ", given that the input state is " S_t^1 ".

$CShC_{t+1}(S_{t+1}^1)$: Cumulative shortage cost of a particular decision set leading to a given state " S_{t+1}^1 " at stage " $t+1$ ".

$CShC_t(S_t^1)$: Cumulative shortage cost of the decision set leading to state " S_t^1 " at stage " t ", which is a subset of the decision set leading to state " S_{t+1}^1 " at stage " $t+1$ ".

$EShC(S_t^1, D_{t+1})$: Expected shortage cost of decision " D_{t+1} " at stage " $t+1$ ", given that the input state is " S_t^1 ".

Expected shortage and expected surplus costs are determined according to Equations 3.15 and 3.21.

Cumulative Surplus-Shortage Function:

Unlike the two previous functions this performance measure does not incorporate cost elements. Instead it measures the cumulative surplus and shortages (CSS) associated with alternative cover policies.

$$CSS_{t+1}(S^1_{t+1}) = CSur_{t+1}(S^1_{t+1}) + CSh_{t+1}(S^1_{t+1}) \quad (3.29)$$

$$CSur_{t+1}(S^1_{t+1}) = ESur_{t+1}(S^1_t, D_{t+1}) + CSur_t(S^1_t) \quad (3.30)$$

$$CSh_{t+1}(S^1_{t+1}) = ESh_{t+1}(S^1_t, D_{t+1}) + CSh_t(S^1_t) \quad (3.31)$$

$$CSur_t(S^1_t) = ESur_t(S^1_{t-1}, D_t) + CSur_{t-1}(S^1_{t-1}) \quad (3.32)$$

$$CSh_t(S^1_t) = ESh_t(S^1_{t-1}, D_t) + CSh_{t-1}(S^1_{t-1}) \quad (3.33)$$

$CSur_{t+1}(S^1_{t+1})$: Cumulative surplus associated with a particular set of cover decisions leading to a given state " S^1_{t+1} " at stage " $t+1$ ".

$CSur_t(S^1_t)$: Cumulative surplus associated with a particular set of cover decisions leading to a given state " S^1_t " at stage " t ".

$ESur(S^1_t, D_{t+1})$: Expected surplus due decision " D_{t+1} " at stage " $t+1$ ", given that the input state is " S^1_t ".

$CSh_{t+1}(S^1_{t+1})$: Cumulative shortage associated with a particular set of cover decisions

leading to a given state " S_{t+1}^1 " at stage " $t+1$ ".

$CS_h(S_t^1)$: Cumulative shortage associated with a particular set of cover decisions leading to a given state " S_t^1 " at stage " t ".

$ESh(S_t^1, D_{t+1})$: Expected shortage due decision " D_{t+1} " at stage " $t+1$ ", given that the input state is " S_t^1 ".

Expected shortages and expected surpluses are determined according to Equations 3.16 and 3.22.

Reliability Function: The reliability function is the means by which the effectiveness of the system is measured. For the purposes of this model, reliability is defined as the proportion of covered services to total scheduled services, multiplied by one hundred. Since covered services are simply the difference between scheduled and cancelled service hours (shortages), and since open work of this duration represents, only, a portion of all open work hours of all durations (about 80 percent), hence the reliability level associated with a particular system design (policy) can be measured as follows:

$$RL(S_{n1}^1) = (1 - \frac{\sum_{t=1}^{n1} ESh_t}{\sigma * \sum_{t=1}^{n1} SSH_t}) * 100 \quad (3.34)$$

$$\sigma = \frac{EOW_{Td}^1}{EOW_{Td}^1 + EOW_{Td}^2} \quad (3.35)$$

$$SSH_t = SSH_{t-1} + SR_t - ESR_t \quad (3.36)$$

$RL(S_{n1}^1)$: The reliability level associated with a particular open work cover option.

$n1$: Number of one hour time periods (stages) in the day.

EOW_{Td}^1 : Expected open work of type "1", greater than or equal to five hours in duration, during day type "d" of timetable "T".

EOW_{Td}^2 : Expected open work of type "2", less than five hours in duration, during day type "d" of timetable "T".

SR_d : Scheduled service runs for day type "d".

SSH_t : Scheduled service hours during time period "t".

SSH_{t-1} : Scheduled service hours during time period "t-1".

SR_t : Service runs starting during stage "t".

ESR_t : Service runs ending at time period "t".

The reliability function may also be represented as follows:

$$RL(S_{n1}^1) = \left(1 - \frac{CSh_{t-1}(S_{t-1}^1) + ESh_t(S_{t-1}^1, D_t)}{\sigma * (CSSH_{t-1} + SSH_t)}\right) * 100 \quad (3.37)$$

where,

$$CSh_{t-1}(S_{t-1}^1) = CSh_{t-2}(S_{t-2}^1) + ESh_{t-1}(S_{t-2}^1, D_{t-1}) \quad (3.38)$$

$$CSSH_{t-1} = CSSH_{t-2} + SSH_{t-1} \quad (3.39)$$

and,

$CSh_{t,1}(S^1_{t,1})$: Cumulative shortages associated with state " $S^1_{t,1}$ " at stage "t-1".

$CSh_{t,2}(S^1_{t,2})$: Cumulative shortages associated with state " $S^1_{t,2}$ " at stage "t-2".

$CSSH_{t,1}$: Cumulative scheduled service hours at time period "t-1".

$CSSH_{t,2}$: Cumulative scheduled service hours at time period "t-2".

Optimization Procedures

The optimization of system performance is concerned with determining the cover policy which minimizes the cost associated with a particular level of reliability. This objective is achieved by determining for each cover option the policy (cover pattern) which minimizes either total system costs, or the the sum of surplus and shortage costs, and then by selecting from among these minimum cost policies the policy which attains a predetermined level of reliability set by management

In either case, costs are minimized at every stage of the decision process, such that the state of the system at every stage is optimal. That is, it represents the least cost allocation pattern from among all the possible utilization patterns for this particular option.

In terms of the cumulative system cost function, the optimization strategy attempts, at every stage, to sort out -- from all the decisions leading to a particular state -- the decision which minimizes cumulative system costs. These costs are then entered into the costs of the decisions stemming from this state at the next stage.

$$F_{t,i}(S^1_{t,i}) = \text{Min}_{D_{t,i}} \{ EC_{t,i}(S^1_{t,i}, D_{t,i}) + F_i(S^1_{t,i}) \} \quad (3.40)$$

The elimination of inferior decision sets proceeds in this fashion until the final stage. At this stage, the states of the system are then compared and the state which attains the

desired level of reliability (set by management) is then selected.

Since the direct costs associated with a particular state is directly proportional to the magnitude of the state (system), irrespective of the policy (set of decisions) leading to that state, it is sufficient in this case to minimize the sum of surplus and shortage costs in order to determine the optimal decision set associated with a given state-stage combination.

$$F_{t+1}(S_{t+1}^I) = \text{Min}_{D_{t+1}} \{ \text{CSSC}_{t+1}(S_{t+1}^I) + \text{DC}(S_{t+1}^I) \} \quad (3.41)$$

Stochastic costs are determined by the cumulative shortage-surplus cost function (CSSC) according to Equation 3.24. On the other hand, direct system costs ($\text{DC}(S_{t+1}^I)$) are determined by multiplying the number of full-time operator days (S_{t+1}^I) allocated to this type of cover -- up to and including stage "t+1" -- by the average daily full-time operator cost (ADFTC).

$$\text{DC}(S_{t+1}^I) = S_{t+1}^I * \text{ADFTC} \quad (3.42)$$

3.2.2.2 Estimating Cover Requirements for Open Work Less Than Five Hours

System Definition

The system to be optimized, at this step, is the number and pattern of full-time operator days required on a particular day type, for the cover of open work less than five hours in duration.

From an economic point of view, open work of such duration may best be covered by

regular operators working overtime, for the same reasons explained in Section 3.2.2 (Page 53, 3rd paragraph). However, due to the limited availability of overtime, especially during the morning peak-travel period -- when the demand for overtime is highest, and when the majority of regular operators reporting to work on that day are usually actively performing their regular work duties -- reserve operators are used to cover this type of open work in addition to some trippers.

Demand

This type of open work is caused largely by vehicle breakdowns and accidents which tend to be concentrated in and around the afternoon peak travel period. This may be explained by the higher probability of vehicle breakdowns and accidents accompanying long operating hours and congested traffic conditions.

With the exception of its duration, this type of open work exhibits the same dynamic and stochastic characteristics common to all open work. Hence, the probability of open work of various magnitudes may be determined by using Equations 3.1 and 3.2. Similarly, open work at any time period during the day is the sum of work which became open during this time period, and the open work remaining from previous time periods because of a shortage in the supply of operators during that time period.

Model Structure

The structure of this model is similar in most aspects to the previous model. Here as well, open work cover decisions are optimized over a number of stages representing time periods of the day. At every stage, inferior decision sets are eliminated, thus reducing the

amount of computation needed to arrive at the optimal design policy. The optimal system design is represented by the decision set which minimizes the cumulative open work cover costs associated with a particular level of reliability.

The primary differences between the two models are related to the duration aspects of open work and the problem of creating full-time work duties out of this type of open work. In order to maximize the use of operators assigned to this type of cover, these work duties would have to be combined with each other and with trippers of the longest duration to make full-time operator work duties. This arrangement assumes that some reserve operators would have to work split shifts as is the case with the operators scheduled to split runs.

Since operators cannot be assigned overlapping pieces of work, the optimization of cover requirements for this type of work must be carried out in two phases. Accordingly, the cover requirements for the morning and afternoon open work would be optimized separately. In either phase, open work pieces would be considered as full day work duties. Their cover will be optimized in the same manner as that for open work greater than or equal to five hours in duration.

Having done that, then the optimal cover requirements for the morning and afternoon periods will be compared. The largest of the two would be selected to represent the optimal daily cover requirements for this type of open work. Since the magnitude of this type of open work during the afternoon period is far greater than that for the morning period, afternoon open work in excess of that for the morning period would have to be matched with morning trippers to create full-time work duties for reserve operators.

The matching of some of these trippers with afternoon open work serves a dual purpose. On one hand it enhances the utilization of operators. On the second hand, it solves the problem of morning tripper cover by reducing the amount of work which has to be covered by overtime. This arrangement is particularly useful in solving the problem of covering the morning trippers (about 75 percent of all trippers), which correspond to periods of low overtime availability.

Given the similarities between this model and the one used for the optimization of the cover requirements for open work greater than or equal to five hours, in order to avoid repetition, only the changes from the previous model will be noted in the following sections.

Inputs: The inputs to this model are the same as those for the previous model, with the exception of open work rates and trippers. These inputs include:

- expected open work rates ($EOWR^2$), or expected open work (EOW^2), of type "2" -- open work less than five hours in duration -- at various time periods of the day,
- morning trippers, and
- afternoon trippers.

Expected open work and expected open work rates of this duration are estimated from operational data, by the same methods (Equations 3.3 and 3.4) used to estimate these values for open work greater than or equal to five hours in duration.

Stages ($t=1..n2$): For the purposes of this model the operating day has been divided into a number of one hour stages. Since, this type of open work may occur at any time period

of the day, "n2" represents the number of operating hours in the day. The morning stages include all operating hours between the start of the service and mid-day (1..s). The afternoon stages including all operating hours between mid-day and the end of service (s+1..n2).

States (S_t^2): The state of the system during stage "t" is the cumulative number of operator days assigned to the system up to and including stage "t". Given the stated condition concerning the matching of morning trippers with afternoon open work, the states of the system during the afternoon stages should be constrained to the following ranges:

$$0 < PMS_t^2 < \sum_{i=1}^s AMT_i^2 \quad (3.43)$$

The morning states may also be constrained to the limits of afternoon trippers, since it is very unlikely that the magnitude of open work during this period would exceed that the number of afternoon trippers.

$$0 < AMS_t^2 < \sum_{i=s+1}^{n2} PMT_i^2 \quad (3.44)$$

where,

AMS_t^2 : State of the system " S_t^2 " during the morning stage "t" (t=1..s).

PMS_t^2 : State of the system " S_t^2 " during the afternoon stage "t" (t=s+1..n).

AMT_t^2 : Morning trippers at stage "t".

PMT_t^2 : Afternoon trippers at stage "t".

The number of operator days required for the day is the same as that for the part of

the operating day with the highest cover requirements -- the afternoon time period.

$$S_{n2}^2 = PMS_{n2}^2 \quad (3.45)$$

S_{n2}^2 : Optimal daily cover requirements for open work of type "2", less than five hours in duration.

Decision (D_i): The same conditions which constrain the range of the values for the morning and afternoon states also apply to the range of values which may be assumed by decisions. These decisions are constrained as follows:

$$0 < PMS_{i-1}^2 + D_i^2 < \sum_{s=1}^s AMT_i^2 \quad (3.46)$$

$$0 < AMS_{i-1}^2 + D_i^2 < \sum_{s=1}^{n2} PMT_i^2 \quad (3.47)$$

Which are equivalent to (Equations 3.43 and 3.44), since,

$$AMS_i^2 = AMS_{i-1}^2 + D_i^2 \quad (3.48)$$

and

$$PMS_i^2 = PMS_{i-1}^2 + D_i^2 \quad (3.49)$$

Tripper Costs: Tripper costs are determined in the following manner. First, the number of morning trippers required for matching with afternoon open work are determined. These trippers are then subtracted from the morning trippers to determine the remaining morning

trippers. The remaining morning trippers are then added to afternoon trippers to determine the total number of trippers, which would have to be covered by overtime.

$$ReqAMT_{Td} = PMS_{n2}^2 - AMS_s^2 \quad (3.50)$$

$$RemAMT_{Td} = AMT_{Td} - ReqAMT_{Td} \quad (3.51)$$

$$TRemT_{Td} = RemAMT_{Td} + PMT_{Td} \quad (3.52)$$

Then the total length of these trippers is added up and multiplied by the average hourly overtime cost to determine the remaining trippers cost.

$$TRemTC_{Td} = \sum_{i=1}^{K_{Td}} RTL_i * AHOTC \quad (3.53)$$

$ReqAMT^{Td}$: Morning trippers required for matching with afternoon open work.

AMS_s^2 : Optimal cover requirements for morning open work.

PMS_{n2}^2 : Optimal cover requirements for afternoon open work.

$RemAMT_{Td}$: The number of morning trippers remaining without cover on day type "d" of timetable "T".

$TRemT_{Td}$: The total number of trippers remaining without cover on day type "d" of timetable "T".

$TRemTC_{Td}$: The total cover cost of the remaining trippers scheduled to day type "d" of timetable "T".

RTL_i : The duration (in hours) of the remaining tripper "i".

Reliability Function: The reliability function, in this model, takes into account the proportion of open work of this type to open work demand. This is accounted for in the denominator by "1 - σ ", where σ is the ratio of expected open work hours greater than or equal to five hours, for the day, to expected open work hours of all durations (see Equation 3.32).

$$RL(S_{n2}^2) = (1 - \frac{\sum_{i=1}^{n2} ESh_i}{(1 - \sigma) * \sum_{i=1}^{n2} SSH_i}) * 100 \quad (3.54)$$

3.2.3 Daily Operator Requirements and Costs

Having estimated the cover requirements and costs of daily open work and trippers, the next step in this optimization procedure involves the determination of operator requirements and costs for all daily demand including scheduled runs as well. The number of operator days required on a particular day type are determined by adding the number of service runs, scheduled to that day type, to the number of operator days required for the cover of open work and trippers, on that day.

$$EODR_{Td} = SR_{Td} + S_{n1Td}^1 + S_{n2Td}^2 \quad (3.55)$$

Similarly, average daily operator costs are the sum of the costs of covering open work and trippers, and the cost of scheduled service runs.

However, since the average work duration for both types of open work is shorter than the eight hours for which operators are paid for, cover operators can be expected to wait - without compensation -- on average a length of time equivalent to the difference

between 8 hours and the average duration of open work of a given type. This is accounted for by subtracting from the cost of open work cover an amount equal to the product of the magnitude of the optimum cover policy for each type, or its associated surplus -- which ever is smaller -- multiplied by this difference and the average hourly over time cost. Accordingly, daily operator requirements and costs are determined as follows:

$$EODC_{Td} = SRC_{Td} + F_{n1Td}^a(S_{n1Td}^1) + F_{n2Td}^a(S_{n2Td}^2) + TRemTC_{Td} \quad (3.56)$$

where,

$$F_{n1Td}^a(S_{n1Td}^1) = S_{n1Td}^1 * (8 - AOWD^1) * AHOTC \quad (3.57)$$

if,

$$CSur(S_{n1Td}^1) \geq S_{n1Td}^1 \quad (3.58)$$

otherwise,

$$F_{n1Td}^a(S_{n1Td}^1) = ESur(S_{n1Td}^1) * (8 - AOWD^1) * AHOTC \quad (3.59)$$

The cost of cover for open work of short durations is adjusted in the same manner.

$EODR_{Td}$: Estimated operator work days required for a day of type "d", in timetable "T".

$EODC_{Td}$: Estimated operator cost for an average day of type "d", in timetable "T".

SR_{Td} : Scheduled service runs on day type "d" of timetable "T".

S_{n1Td}^1 : Optimal number of operator days required for the cover of open work greater than or equal to five hours in duration, on day type "d" of timetable "T".

S_{n2Td}^2 : Optimal number of operator days required for the cover of open work less than five hours in duration, on day type "d" of timetable "T".

$F_{n1Td}^a(S_{n1Td}^1)$: Adjusted cost of the optimal cover policy for open work greater than or equal to five hours in duration, on day type "d" of timetable "T".

$F_{n2Td}^a(S_{n2Td}^2)$: Adjusted cost of the optimal cover policy for open work less than five hours in duration, on day type "d", of timetable "T".

$F_{n1Td}(S_{n1Td}^1)$: Cost of the optimal cover policy for open work greater than or equal to five hours in duration, on day type "d" of timetable "T".

AOWD¹: Average duration of open work of type "1".

TRemTC_{Td} has been defined on Page 86.

In transit companies which employ part-time as well as full-time operators, it is best that service runs (work duties) are created, specifically, for these two operator categories at the run cutting stage. Such practice would enhance the efficiency of the transit operator workforce by improving the match between operators and their work assignments. In this case, scheduled run costs are estimated as follows:

$$SRC_{Td} = (SFTOD_{Td} * ADFTC) + (SPTOD_{Td} * ADPTC) \quad (3.60)$$

$$SR_{Td} = SFTOD_{Td} + SPTOD_{Td} \quad (3.61)$$

$$\alpha = \frac{SPTOD_{Td}}{EODR_{Td}} \quad (3.62)$$

where,

SFTOD_{Td}: Scheduled full-time operator days on day type "d", of timetable "T".

$SPTOD_{Td}$: Scheduled part-time operator days on day type "d", of timetable "T".

$ADFTC$: Average daily cost of a full-time operator including all benefits.

$ADPTC$: Average daily cost of a part-time operators including all benefits.

SRC_{Td} : Scheduled runs cost for day type (d), of timetable (T).

In transit agencies which employ only full-time operators, scheduled service costs are determined by multiplying the number of scheduled service runs, on a particular day type, by the average daily full-time operator cost.

$$SRC_{Td} = SR_{Td} * ADFTC \quad (3.63)$$

3.2.4 Workforce Size

The estimation of workforce size is influenced by the number of operator classifications -- full-time and part-time, or full-time only -- and in case two operator types are used, by the proportion of part-time work to total work, as specified in the labour agreement or by management.

3.2.4.1 One Operator Type

In transit agencies which employ only full-time operators, a constant work force size for the year may be estimated as follows:

$$YODR = \sum_{T=1}^M \sum_{d=1}^D EODR_{Td} * N_d \quad (3.64)$$

$$EOL = \frac{YODR}{AYFTOWD} \quad (3.65)$$

where,

M: The number of timetables in a particular service plan.

D: The number of day types in a particular timetable.

N_d : The number of days of type "d", in a particular timetable.

YODR: The number of operator days required per service plan (year).

EOL: Estimated operator level required throughout the year.

AYFTOWD: Average yearly full-time operator work days required by the labour contract.

As well a workforce level may be estimated for each timetable by dividing the estimated operator work days required during a particular timetable by the average full-time operator work days ($AFTOWD_T$) required by the labour contract from operators during that timetable.

$$ODR_T = \sum_{d=1}^D EODR_{Td} * N_d \quad (3.66)$$

$$EOL_T = \frac{ODR_T}{AFTOWD_T} \quad (3.67)$$

where,

EOL_T : Required workforce level during timetable "T".

ODR_T : Operator work days required during timetable "T".

3.2.4.2 Two Operator Types

The measure used to specify the proportion of part-time to total work has a direct impact on the method used for estimating work force size and composition. In case this proportion is specified as a percentage of total work days, as in Equation 3.59, then the annual full-time and part-time operator days, and the corresponding work force level, can be estimated as follows:

$$YFTODR = \sum_{T=1}^M \sum_{d=1}^D (EODR_{Td} - SPTOD_{Td}) * N_d \quad (3.68)$$

$$YPTODR = \sum_{T=1}^M \sum_{d=1}^D SPTOD_{Td} * N_d \quad (3.69)$$

$$EFTOL = \frac{YFTODR}{AYFTOWD} \quad (3.70)$$

$$EPTOL = \frac{YPTODR}{AYPTOWD} \quad (3.71)$$

$$EOL = EFTOL + EPTOL \quad (3.72)$$

where,

YFTODR: Yearly full-time operator days required during the service plan.

YPTODR: Yearly part-time operator days required during the service plan.

EFTOL: Estimated full-time operators required throughout the service plan.

EPTOL: Estimated part-time operators required throughout the service plan.

AYFTOWD: Average full-time operator work days per year.

AYPTOWD: Average part-time operator work days per year.

SPTOD_{Td}: Scheduled part-time operator days on day type "d", of timetable "T".

Here, as well, workforce size for each timetable may be estimated. This can be carried out along the same lines as that for the single operator type, by appropriately accounting for this change in conditions.

3.2.5 Annual Operator Costs

Annual operator costs are estimated by adding up the estimated daily operator costs for all days in the service plan. These estimates include all costs, with the exception of hiring costs. These costs are estimated in Section 3.4.

$$YOC = \sum_{T=1}^M \sum_{d=1}^D EODC_{Td} * N_d \quad (3.73)$$

where,

YOC: Annual operator costs, with the exception of hiring costs.

EODC_{Td}: Estimated operator days cost for a day of type "d", in timetable "T".

Since among other things, transit service plans have to meet constraints on budget resources, annual operator cost estimates are particularly important because they determine the economic feasibility of alternative service plans.

3.3 VACATION MODEL

In addition to satisfying annual vacation requirements, the purpose of this model is to develop vacation schedules that maximize the availability of operators for work throughout the service plan. This objective is attained by subordinating vacation requirements to work requirements.

Since, operators during any week of the service plan are either on their annual vacation or scheduled for work, the critical step in this procedure involves the estimation of operator work weeks required during every week of the service plan (Equation 3.73). Having done that, optimal vacation schedules for the service plan (year) can be determined by subtracting the estimated operator work weeks required during every week, of every timetable, in the service, plan from the estimated workforce size (EOL). the following is a detailed presentation of this model.

$$EVW_{Tw} = EOL - EOWWR_{Tw} \quad (3.74)$$

where,

$$EOL \geq EOWWR_{Tw} \quad (3.75)$$

$$EOWWR_{Tw} = [\sum_{T=1}^M \sum_{w=1}^W \sum_{D=1}^n EODR_{Td} * N_{Twd}] / AWWD \quad (3.76)$$

$$AVW \geq RVW \quad (3.77)$$

$$AVW = \sum_{T=1}^M \sum_{w=1}^W EVW_{Tw} \quad (3.78)$$

EVW_{Tw} : Estimated vacation weeks during week "w", of timetable "T".

$EOWWR_{Tw}$: Estimated operator work weeks required for week "w" of timetable "T".

$AWWD$: Average work days per operator work week, full-time and part-time operators included.

N_{Twd} : Number of type "d" days in week "w" of timetable "T".

AVW : Available vacation weeks during the service plan.

RVW : Required vacation weeks during the service plan.

Similarly, the number of vacation weeks scheduled during any week of a particular timetable may be estimated by subtracting the estimated operator work weeks required during every week of that timetable from the estimated workforce level during that timetable (EOL_T).

Vacation requirements for the service plan are estimated from workforce size (EOL), on the basis of operator seniority. If available vacation weeks are less than annual vacation requirements, then the estimated workforce size should be incremented by one. This should be enough to satisfy Equation 3.74, since the difference between these values should usually be very small. Such small differences may occur as a result of rounding errors.

The estimated workforce size is a ceiling on the demand that can be accommodated

by the system. Slightly higher demand levels, over a short period (a week or two) of time, may be accommodated by configuring the hiring policy (Section 3.4) to allow a sufficient number of new operators to start work at the beginning of such a period. However, it is highly unlikely that service levels beyond workforce will be planned into the system.

3.4 HIRING MODEL

3.4.1 General

The objective of this model is maintain, throughout the service plan, estimated operator levels -- determine by the Workforce Model -- at minimum cost, while satisfying the reliability criterion set by management.

The hiring policy consists of two parts; a) a decision on when to hire, and b) another concerning the optimal number of operators to be hired at each hiring point. An important consideration, here, is the lead time required for advertising and the training of new operators. Lead time can vary depending on the state of the applications pool, and the qualifications of the applicants. Furthermore, since the hiring policy is intended primarily to communicate to the personnel department the needs of the transit agency for new operators, it is appropriate to construct the hiring policy in terms of the number of new operators required to join the operator workforce at various time periods during the year. Such a plan may be easily translated (in time) into a hiring policy, without any additional costs, by incorporating the lead time.

3.4.2 System Definition

The system to be optimized by this model is the number of new operators to be hired during the service plan to replace outgoing operators, and the times at which various groups of these operators have to join the operator workforce on active duties. This information would, then, serve as the basis for the development of hiring and training programs.

3.4.3 Demand

Demand is represented by attrition, which exhibits both dynamic and stochastic variability over time periods of the year. As is the case with open work, the demand resulting from attrition at a given time period during the year, is the sum of the demand created by attrition during this time period, and uncovered demand from the previous time period due to a shortage in the supply of operators.

Attrition information is gathered as part of operational data, which also includes information on daily open work. Because of its relatively small magnitude, such information is usually aggregated by week or month, depending on the size of the workforce. Let us assume for the sake of discussion that the probability of attrition of a certain magnitude at a particular time period of the year (month or week) can be approximated by the poisson distribution.

$$P(ATTR_i/t) = \frac{\exp^{-ATTR_i} * ATTR_i^{ATTR_i}}{ATTR_i} \quad (3.79)$$

where,

$P(ATTR_i|t)$: Probability of attrition of magnitude "i" at a given time period "t", of the service plan.

$ATTR_i$: Expected attrition at time period "t".

$ATTR_i$: Attrition of magnitude "i" at time period "t".

3.4.4 Model Structure

The structure of this model is similar to that of the models used to optimize the cover requirements for daily open work. This problem is also modelled as a probabilistic dynamic program in which the states of the system at a particular stage are known with certainty, while the stage's demand is a random variable. The following is a detailed description of various aspects of this model.

3.4.4.1 Inputs

Figure 2.7 shows the various inputs to hiring decisions. These inputs include:

- estimated workforce level,
- attrition rates among operators at various time periods during the day (weeks or months).
- reliability level,
- average weekly costs of a new operators (including wages and fringe benefits),
- cancelled trip costs,
- advertisement and processing cost per hiring decision, and
- average training cost per operator.

3.4.4.2 Stages (t=1..n):

In this model, the year is divided into "n" equal stages (time periods). Since the week is the smallest time period for which attrition rates may be meaningfully represented, each of these time periods consist of an integer number of weeks.

3.4.4.3 States (S_t)

At any stage of the decision process, the states of the system represent the sum of all new operators hired up to and including that stage. These states also represent the range of hiring options to be investigated by the model for their cost and reliability. Since, it is reasonable to assume that the number of new operators required at any stage should not greatly exceed expected attrition for the whole year, the range of hiring options (states) to be evaluated by the model may be constrained as follows:

$$0 \leq S_t \leq \delta * EYAtt \quad (3.80)$$

where,

$$1 < \delta \leq 2 \quad (3.81)$$

and EYAtt is the expected yearly attrition.

Here as well we have to distinguish between two set of states; input states and output states. Input states represent the information about the state of the system up to the current stage, i.e., the number of operators hired thus far. The output states incorporate the effect of the decisions made during the current stage on the state of the system.

3.4.4.4 Decision (HD_t)

Decisions represent the number of new operators required to start work at the beginning of a given time period during the year. Since the number of operators hired at any stage cannot greatly exceed the expected annual attrition, the range of these decisions are as well constrained by the limits put on state values.

$$0 \leq HD \leq \delta * EYAtt \quad (3.82)$$

Furthermore, the magnitude of decisions should not exceed the difference between their input and output states.

$$0 \leq HD_t \leq S_t - S_{t-1} \quad (3.83)$$

where " S_{t-1} " and " S_t " are, respectively, the input and output states.

3.4.4.5 Transition Function

The decisions at every stage have the effect of transforming the state of the system in the following manner.

$$S_t = S_{t-1} + HD_t \quad (3.84)$$

3.4.4.6 Cost Function

The cost function is one measure of performance used to evaluate alternative hiring policy. This function measures the system's efficiency, while the reliability function (Section 3.4.4.7) measures its effectiveness. The costs associated with a particular state-

stage combination reflect the costs of various policies (decision sets or paths) leading to that state. The cost of any one of these policies is determined as follows:

$$F_t(S_t) = EC_t(S_{t-1}, HD_t) + F_{t-1}(S_{t-1}) \quad (3.85)$$

$F_t(S_t)$: Cumulative cost of all hiring decisions up to and including stage "t".

$EC_t(S_{t-1}, HD_t)$: Expected cost of the hiring decision "HD_t", at stage "t", given that the input state is "S_{t-1}".

The decision cost associated with a particular state-stage combination consists of three parts; a deterministic term, and two probabilistic terms. The deterministic term represents the direct hiring cost function (DHC), at a particular stage. The two probabilistic terms represent the surplus costs (ESurC) and shortage costs (EshC) associated with a particular decision-state-stage combination.

$$EC_t(S_{t-1}, HD_t) = DHC_t(S_{t-1}, HD_t) + ESurC_t(S_{t-1}, HD_t) + EShC_t(S_{t-1}, HD_t) \quad (3.86)$$

$$DHC_t(S_{t-1}, HD_t) = OrC(HD_t) + TrC(HD_t) + (HD_t * NW_t * ANOWC) \quad (3.87)$$

$$ESurC_t(S_{t-1}, HD_t) = \sum_{i=0}^{AS_t - ESh_{t-1}} P(ATTR_t/t) * (AS_t - AD_t) * NW_t * ANOWC \quad (3.88)$$

given that,

$$AD_t = ATTR_t + ESh_{t-1} \quad (3.89)$$

$$AS_t = HD_t + ESUR_{t-1} \quad (3.90)$$

$$AS_t \geq AD_t \quad (3.91)$$

$$AS_t - Esh_{t-1} \geq 0 \quad (3.92)$$

and,

$$EShC_t(S_{t-1}, HD_t) = \sum_{t > AS_t - ESh_t} P(ATTR_t | t) * \quad (3.93)$$

$$(AD_t - AS_t) * NW_t * CTC$$

$$AD_{t-1} \geq AS_{t-1} \quad (3.94)$$

DHC_t: Direct cost of the hiring decision "HD_t", given that the input state is "S_{t-1}".

OrC(HD_t): Advertisement and processing cost (order cost) associated with the hiring decision.

TrC: Training cost.

ESurC_t: Expected surplus cost during stage "t", given an input state and a hiring decision.

EShC_t: Expected shortage cost during stage "t", given an input state and a hiring decision.

NW_t: Number of weeks in stage "t".

ANOWC: Average weekly cost of new operators.

ATTR_i: Attrition of magnitude "i" during stage "t".

P(..): Probability notation.

CTC_t: Cost of cancelled trips at stage "t".

AD_t : Actual demand at time period "t", given that the attrition occurring at this time period is of magnitude ATTR_i.

AS_t : Actual operator supply at the beginning of stage "t".

ESur_t : Expected surplus operator work days from stage "t".

ESh_t : Expected shortages from stage "t".

The last two terms "ESur_{t,1}" and "ESh_{t,1}" incorporate into the decision process, at stage "t", the effect of supply decisions during the preceding stages, which are members of the same supply policy, and their interaction with attrition patterns over time periods of the year.

3.4.4.7 Reliability Function

The reliability function is the means by which system effectiveness is measured. The reliability level associated with a given set of decisions (policies) leading to a particular state-stage combination can be determined as follows:

$$RL(S_t) = (1 - \frac{\sum_{i=1}^n ESh_i * NW_i}{\sum_{i=1}^n \sum_{j=1}^{NW_i} EOWWR_j}) * 100 \quad (3.95)$$

where,

$$ESh_t = \sum_{i > AS_t - ESh_t}^{EOL} P(ATTR_i/t) * (AD_t - AS_t) * NW_t \quad (3.96)$$

$$EOWWR_i = \sum_{d=1}^D EODR_{id} * N_{id} / AWWD \quad (3.97)$$

and,

$RL(S_n)$: Reliability of a particular system configuration, or hiring policy.

ESh_t : Expected shortage during stage (t).

n: Number of time periods t in the year

NW_t : Number of weeks in time period t.

$EOWWR_i$: Estimated operator work weeks during week i of time period t.

$EODR_{id}$: estimated operator work days required on day type d of week i.

N_{id} : Number of days of type d during week i.

$AWWD$: Average work days per operator work week.

3.4.4.8 Optimization Procedure

The hiring policy is optimized in the same manner as the daily cover policy, that is by determining for each hiring level the minimum cost policy, and then by selecting from among these minimum cost policies the one which attains a predetermined level of reliability.

Chapter IV

TACTICAL AND OPERATIONAL MODELS

4.1 INTRODUCTION

Tactical and operational models are interrelated parts of the work scheduling process. These two models are influenced by strategic level decisions in two ways. On the one hand, the workforce size, vacation and hiring decisions determine in conjunction with attrition the inputs to these models, i.e. the availability of operators for work during any week of the timetable. On the other hand, while the Tactical Model relies on the Daily Model (Workforce Size Model) to estimate the pattern of demand for operators during a particular week of a timetable, the Operational Model uses a dynamic programming procedure similar to the Open Work Cover Model (Workforce Size Model -- Sections 3.2.2.1 and 3.2.2.2) to determine the most cost-effective daily assignment pattern for reserve operators without known work on a particular day.

The relationship of these models to strategic level decisions and models is well illustrated in Equations 4.1-4.4 and in the example presented in Section 4.2.4. This example shows the use of these formulations for the determination of weekly work and days-off schedules for operators. It, as well, illustrates the use of the model for maximizing the number of consecutive days-off.

4.2 TACTICAL MODEL

4.2.1 General Description

The purpose of this model is to create weekly work schedules for operators -- other than those taking their annual vacation leave -- which maximize the availability of operators for work during days of the week. These schedules are usually created for available operators at the beginning of a given timetable, and updated thereafter every 2 to 3 weeks to reflect the changes in operator supply and demand. Accordingly, groupings of daily operator work assignments and days-off are posted for pick-up by operators according to seniority. Operators indicate their willingness to work individual groupings by signing their names opposite to their choices.

The objective of the model is achieved by subordinating day-off requirements on any particular day to work requirements on that day. The critical step in this model involves the estimation of work requirements for every day of every week in the coming work period. As mentioned earlier, this information is provided by the Daily Model (Equation 3.55 -- Workforce Size Model) which is used to estimate daily operator work requirements. These estimates are then used to schedule operator work days and days-off to days of the week (Section 4.2.2).

However, instead of estimating work requirements for a particular day type of the average week in a timetable, Equation 3.55 will be used to estimate unique work requirements for each day in every week. This task would require the running of the

Workforce Size Model with inputs which incorporate into the estimation process recent information about open work. Accordingly, expected open work and expected open work rates for the coming work period would be estimated using Equations 4.12 and 4.13, instead of Equations 3.3 and 3.4.

Operator days-off for each day of the week are determined by subtracting the number of daily operator work duties from available operator supply for the week. Figure 2.8 shows the interrelationship between work and day-off schedules over days of the week. Accordingly, week days -- Monday to Friday -- with high work requirements have few days-off assigned to them, compared to Saturdays and Sundays where the bulk of days-off are scheduled.

4.2.2 Scheduling of Operator Work Days to Days of the Week

The inputs to this model are determined primarily by strategic level decisions -- workforce size, vacations schedules, and hiring policy. These inputs include the following:

- workforce level at the beginning of the scheduling period,
- daily work requirements throughout this period,
- vacation schedules,
- hiring policy,
- attrition rates, and
- the average number of work days per operator per week.

Figure 2.9 shows the interrelationships between these inputs and their impact on daily work and day-off schedules.

The assignment of operator work days to a particular day of a given week in the

scheduling period is carried out as follows. First, available operator weeks for every week in the scheduling period are determined from available information on: 1) expected workforce size at the beginning of every week of the scheduling period, 2) expected attrition rates for every week, and 3) the number of new operators expected to begin work at the beginning of every week.

$$AOW_{TW} = AOL_{TW} (1 - ATTR_{TW}) + NOW_{TW} \quad (4.1)$$

Second, available operator work weeks during a particular week are determined by subtracting scheduled vacation weeks from available operator weeks during that week.

$$AOWW_{TW} = AOW_{TW} - SVW_{TW} \quad (4.2)$$

The final step in this procedure involves the scheduling of available operator work days to various days of the week, in proportion to the ratio of estimated operator days required during each day to total work requirement for the week.

$$SOWD_{TWd} = AOWW_{TW} * \frac{EODR_{TWd}}{\sum_{d=1}^n EODR_{TWd}} \quad (4.3)$$

Where,

AOW_{TW} : Available operator weeks at week "W" of timetable "T".

AOL_{TW} : Available operator level at beginning of week "W" of timetable "T".

$ATTR_{TW}$: Attrition rate during week "W" of timetable "T".

NOW_{TW} : New operators entering the system at beginning of week "W" of timetable "T".

$AOWW_{TW}$: Available operator work weeks at week "W" of timetable "T". This is equivalent to the number of operators available for work during that week.

AOWWD: Average operator work days per operator work week.

SVW_{TW} : Vacation weeks scheduled to week "W" of timetable "T".

$SOWD_{TWd}$: Operator work days scheduled to a particular day "d", of week "W", of timetable "T".

$EODR_{TWd}$: Estimated operator work days required on a particular day "d", of week "W" of time table "T".

As mentioned in the introduction to this model " $EODR_{TWd}$ " is estimated using Equation 3.55, after making the appropriate modifications in the inputs to the models used to estimate this term.

4.2.3 Weekly Day-Off Schedules

The number of permissible days-off on any given day of the week can be easily determined by subtracting the number of available operator work weeks for that week from the operator work days scheduled for that day.

$$DaysOff_{TWd} = AOWW_{TW} - SOWD_{TWd} \quad (4.4)$$

Where, " $DaysOff_{TWd}$ " represents the number of permissible days-off on a given day of type "d" during week "W" of timetable "T". Permissible days-off are used to create weekly day-off schedules to be presented for "pick-up" in conjunction with the weekly work schedules. Given the desirability of consecutive day-off pairs, weekly day-off schedules are usually designed to maximize the number of consecutive days-off.

The maximum number of day-off pairs assigned to any two consecutive days in the week (mon.-tues., Tues.-Wed., Wed.-Thur., etc...) is limited by the smallest number of

permissible days-off on any one of the two members of the pair. For example, if the permissible number of days-off on monday is 40 and that on tuesday is 45, then the maximum number of day-off pairs which may be assigned to the "mon.-tues." pair is 40. In this case, the remaining days-off on tuesday are "5".

Given the above, the determination of an optimal day_off schedule during a particular week of the scheduling period may be carried out as follows:

Step 1 : Determine the number of permissible days-off for every day in the week.

Step 2 : Find and select the consecutive day pair with one of its members having the lowest number of permissible days-off for the week.

Step 3: Arrange day pairs sequentially (example: Tues.-Wed., Wed.-Thur.,etc), such that the day pair with the lowest number of permissible days-off is at the head of the list.

Step 4 : Explore all weekly day-off schedules which correspond to day-off values for the head pair ranging from 0 to the maximum number of day-off pairs which may be assigned to that pair.

Step 5 : Select the weekly day-off schedule which maximizes the number of consecutive days-off, or conversely minimizes the number of single days-off.

Assuming that the day-off pair "Mon.-Tues." has a member (Monday) with the lowest number of permissible days-off compared to all other days of the week, an initial schedule of day-off pairs for the week can be determined in the following manner:

$$DoffPair_{M,T} = DaysOff_M \quad (4.5)$$

$$RemDoff_T = DaysOff_T - DoffPair_{M,T} \quad (4.6)$$

If,

$$RemDoff_T \leq DaysOff_w \quad (4.7)$$

then,

$$DoffPair_{T,w} = RemDoff_T \quad (4.8)$$

and,

$$RemDoff_w = DaysOff_w - DoffPair_{T,w} \quad (4.9)$$

Otherwise,

$$DoffPair_{T,w} = DaysOff_w \quad (4.10)$$

and,

$$RemDoff_T = DaysOff_T - DoffPair_{T,w} \quad (4.11)$$

where,

$DaysOff_M$: Number of days-off scheduled to Monday.

$DaysOff_T$: Number of days-off scheduled to Tuesday.

$DaysOff_w$: Number of days-off scheduled to Wednesday.

$DoffPair_{M,T}$: Number of day-off pairs scheduled to Monday and Tuesday.

$RemDoff_T$: Number of remaining days-off on Tuesday.

$DoffPair_{T,w}$: Number of day-off pairs scheduled to Tuesday and Wednesday.

$RemDoff_w$: Number of remaining days-off on Wednesday.

This procedure is carried out for every day and day-off pair until all days and day-off pairs for the week are considered. Having done that, then the number of day-off pairs assigned to "Mon.-Tues." are decremented by one and the above process is repeated. After all day-off pairs for "Mon.-Tues." are exhausted, then the weekly day-off schedule with the least number of single days-off is selected as the optimal weekly day-off schedule.

4.2.4 Example

Let us assume that a transit agency has 36 operators available for work at the beginning of a given week in its current timetable. By contract each operator is committed to working 5 days a week, 8 hours a day. Five of these operators are scheduled to their annual vacation leave during that week. No new operators are expected to take up driving duties at the beginning of this week, and no operators are expected to leave or quit work.

Using the Daily Model, it was estimated -- based on schedule information (runs and trippers) and historical as well as recent open work data -- that 24 operator work days will be required on every day of the work week -- Monday through Friday -- versus 16 and 12 operator work days for Saturday and Sunday, respectively.

Given the above, determine: 1) daily work and day-off schedules which maximize the availability of operators for work, and 2) the weekly pattern of consecutive day-off pairs which minimizes the number of single days-off.

The first step involves determining the number of operators available for work during the week ($AOWW_{TW}$). Using Equation 4.2, this is determined by subtracting vacation requirements from the available operator weeks.

$$AOWW_{TW} = 36 - 5 = \underline{31 \text{ operator work weeks}}$$

The second step involves the determination of the number of operator work days to be scheduled to each day in the week ($SOWD_{TWd}$). Using Equation 4.3, the number of operator work days to be scheduled to Monday is determined as follows:

$$SOWD_{TWmo} = (5 \times 31 \times 24) / [(24 \times 5) + 16 + 12] = 25.4$$

= 25 operator work days

Similarly, the number of operator work days to be scheduled to Saturday is:

$$SOWD_{TWsa} = (5 \times 31 \times 16) / [(24 \times 5) + 16 + 12] = 16.8$$

= 17 operator work days

and the number of operator work days to be scheduled to Sunday is:

$$SOWD_{TWsu} = (5 \times 31 \times 12) / [(24 \times 5) + 16 + 12] = 12.6$$

= 13 operator work days

The third step involves the determination of the number of permissible days-off for every day of the week. This is determined by subtracting operator work days scheduled to various days of the week from the number of available operator work weeks.

Hence, the number of permissible days-off on each day of the work week (Mon.-Fri.)
is equal to:

$$31 - 25 = \underline{6 \text{ days-off}}$$

for Saturday,

$$31 - 17 = \underline{14 \text{ days-off}}$$

and for Sunday,

$$31 - 13 = \underline{18 \text{ days-off}}$$

The last step involves the determination of the weekly pattern of consecutive day-off pairs which minimizes the number of single days-off, i.e. maximizes the number of consecutive days-off. The results of the application of the procedure developed in Section 4.2.3 are presented below.

PATTERN OF WEEKLY DAY-OFF PAIRS							Single Days-Off
Su.-Mo	Mo.-Tu	Tu.-Wd	Wd.-Th.	Th.-Fr.	Fr.-Sa.	Sa.-Su.	
6	0	6	0	6	0	12	2
5	1	5	1	5	1	13	0
4	2	4	2	4	2	12	2

The highlighted row represents the pattern of day-off pairs, which maximizes the number of consecutive pairs for the week as is indicated by the "0" at the single day-off column.

4.3 OPERATIONAL MODEL

4.3.1 General

When operators select their daily work assignments during "sign-ups", they choose between two types of work duties; service runs (regular work), and extra-board reserve duties. A requirement of this process is that operators know the start times for their daily work assignments. While the report times for regular work duties are predetermined by the run cutting process, the report times for extra-board operators are usually determined by the scheduler based on past experience and knowledge of open work patterns during various days of the week, and various time periods of the day.

The purpose of this model is to develop daily assignment patterns for available reserve operators -- without known work on a particular day -- which minimizes operator surpluses and shortages. Given the dynamic and probabilistic aspects of open work and it's interaction with assignment decisions over time periods of the day, and in order to incorporate these attributes of the problem into the decision process, a two step assignment procedure was developed. The underlying reasons for this breakdown are the same as those presented in Section 3.2.2.

The first step involves the assignment of reserve operators to open work cover duties greater than or equal to five hours in duration. The second step aims to find the optimal daily assignment pattern for the remaining extra-board operators to open work less than five hours in duration, in addition to some trippers. Since open work of this duration

constitutes less than a full day's work. morning and afternoon open work pieces of this type will be matched with each other and with trippers, when necessary, to maximize the utilization of reserve operators. In case any reserve operators are left, these operators will then be assigned to tripper pairs.

Unlike cover requirements -- at the strategic level -- which rely on operational data that are at least one year old, the operational model has the opportunity to utilize, in addition to historical data, information as recent as the previous week's open work data. While at the strategic level, all open work is considered to be unknown, at the operational level a significant portion of open work is known in advance -- long term absenteeism, and service changes -- and hence may be assigned to reserve operators outside the model. Furthermore, while open work due to attrition is treated separately at the strategic level, at this level, open work includes, by definition, all unexpected open work.

Given the uncertainty concerning the magnitude and incidence pattern of daily open work, these two assignment models have been formulated as probabilistic dynamic programming procedures in which the states of the system are known while demand is a random variable.

To avoid confusing terms of the two assignment models, terms used in the first model will be superscripted by "1", and terms of the second model will be superscripted by "2". The details of these two models are presented below.

4.3.2 Assignment of Extra-board Operators to Open Work Greater Than or Equal to Five Hours in Duration

4.3.2.1 System Definition

The system to be optimized by this model is the daily assignment pattern of reserve operators to open work greater than or equal to five hours in duration. The optimal assignment pattern is that which attains the desired level of reliability, set by management, at least cost.

4.3.2.2 Demand

The demand on the system is represented by open work greater than or equal to five hours in duration resulting largely from absenteeism and attrition among operators. Open work at any time period during the day is the sum of work becoming open during that time period and uncovered open work remaining from previous time periods, because of a shortage in the availability of operators during these time periods. Here, as well as in the open work cover estimation model, the probability of open work of a given magnitude -- during a particular time period of the day -- may be approximated by either the binomial or Poisson distributions.

4.3.2.3 Model Structure

This model is structured along the same lines as the dynamic programming procedure used at the strategic level for the estimation of reserve requirements for open work greater than or equal to five hours in duration (Section 3.2.2.1). However, instead of estimating cover requirements for a particular day type, the Operational Model aims to find the

optimal daily assignment pattern for available operators without known work duties on a particular day.

Here as well, the operating day is subdivided into a number of equal stages. At each stage, the system could be in any one of a number of states, each representing the number of reserve operators remaining in the system, that is the number of operators who may be assigned to reserve cover duties at later stages. The states at any particular stage are determined by assignment decisions up to and including the current stage. The decisions at any particular stage are influenced by the state of the system in the previous stage (input state), which limits the range of decision options for that stage. The number of states at any stage is determined by the size of the extra-board. Figure 4.1 is a network representation of the decision process. It resembles Figure 3.1, except with respect to the magnitude of the initial state which in the present model represents the size of the extra-board.

Optimization in means the minimizing the discrepancy between the pattern of open work -- greater than or equal to four hours in duration -- and the assignment patterns of reserve operators. The gap between these patterns may be measured in one of two ways; either as the sum of surpluses and shortages, or as the sum of surplus and shortage costs.

This model uses three performance functions to measure the efficiency and reliability of the assignment process. These measures are:

- the cumulative surplus-shortage function,
- the cumulative surplus-shortage cost function, and
- the reliability function.

FIGURE 4.1: NETWORK REPRESENTATION OF THE EXTRA-BOARD ASSIGNMENT MODEL

The cumulative surplus-shortage function is a special case of the cumulative surplus-shortage cost function, i.e it gives equal weight to surpluses and shortages. Similar results are produced by the cumulative surplus-shortage cost function when equal unit costs are assigned to shortages and surpluses. This point is illustrated in Test 1 of the Case Study, where the minimum surplus-shortage policy and the minimum surplus-shortage cost policy have identical cover patterns, and cost and reliability levels (see Table A.1.2 and Figures A.1.3 - A.1.4 in Appendix A).

These performance functions are similar in every aspect to those of the open work cover model presented in Section 3.2.2.1. Hence, the following text will only present those formulations that are unique to this optimization procedure, namely those which define inputs, states, decisions and the transition function.

Inputs

In general, the inputs to this decision process are similar to those used for the estimation of open work cover requirements at the strategic level. The primary differences concern the nature of open work and the time interval over which it is determined. While expected open work rates at the strategic level are based on information that is at least one year old, open work rates for this model may include information as recent as the previous week's open work data. Furthermore, open work at this level also incorporates the effects of attrition, which is considered separately at the strategic level in the Hiring Model. These inputs are:

- scheduled service runs (SR_t) commencing at the beginning of various time periods "t" of the day,

- expected open work rates ($EOWR^1$), or expected open work (EOW^1) of type "1" -- open work greater than or equal to five hours in duration -- at various time periods "t" of the day,
- average daily full-time operator cost (ADFTC),
- average hourly overtime cost (AHOTC),
- cancelled service cost, and
- reliability level.

A number of methods may be used to estimate expected open work and expected open work rates. At one extreme, open work rates and expected open work may be estimated from historical records, in the same manner used to estimate these values for the open work cover models at the strategic level (Inputs, Section 3.2.2.3). At the other extreme, only open work records for the past 3 to 4 weeks may be used to estimate expected open work rates. A balanced approach to this problem is to incorporate into the estimation process historical as well as recent open work data. While historical data reflect long term tendencies of the system to produce open work, recent information reflects current trends in absence behaviour and accident patterns. The combining of these two factors -- historical and recent open work data -- to estimate expected open work rates ($EOWR^1$) and expected open work (EOW^1) for the operational model may be carried out as follows:

$$EOW_t^1 = ((1 - \delta) * hEOW_t^1) + (\delta * rEOW_t^1) \quad (4.12)$$

$$EOWR_t^1 = ((1 - \delta) * hEOWR_t^1) + (\delta * rEOWR_t^1) \quad (4.13)$$

with

$$0 \leq \delta \leq 1 \quad (4.14)$$

The value assigned to " δ " reflects the relative weight given to recent open work information versus historical trends. The higher these values, the greater is the weight assigned to recent trends versus historical ones. A value of "0.5" means that equal weights are being assigned to both recent and historical trends. Such may be the case under steady state conditions. However, in case new policies are being implemented which are expected to positively affect open work rates, higher values for " δ " may be used.

EOW_t^1 : Expected open work of type "1", greater than or equal to five hours, during time period "t".

$EOWR_t^1$: Expected open work rate of type "1", greater than or equal to five hours, during time period "t".

$hEOW_t^1$: Expected open work during time period "t", based on historical open work data of type "1".

$hEOWR_t^1$: Expected open work rate during time period "t", based on historical open work data of type "1".

$rEOW_t^1$: Expected open work during time period "t", based on recent open work data of type "1".

$rEOWR_t^1$: Expected open work rate during time period "t", based on recent open work data of type "1".

"hEOW_t¹" and "hEOWR_t¹" are estimated in the same manner by which expected open work and expected open work rates are estimated for the strategic model (Equations 3.3 and 3.4). "rEOW_t¹" and "rEOWR_t¹" are estimated as follows:

$$rEOW_t^1 = \frac{\sum_{w=1}^{nw} OW_{wdt}^1}{nw} \quad (4.15)$$

$$rEOWR_t^1 = \frac{\sum_{w=1}^{nw} OW_{wdt}^1}{\sum_{w=1}^{nw} SR_{wdt}} \quad (4.16)$$

where,

OW_{wdt}¹: Open work of type "1" during time period "t" of day type "d" of week "w".

SR_{wdt}: Scheduled runs commencing during time period "t" of day type "d" of week "w".

nw: Number of weeks (about 3 to 4 weeks) during the recent time period, which serve as the basis for estimating "rEOW_t¹".

d: Day type "d".

t: Time period "t" of day "d"

Stages (t=1..n1)

For the purposes of this model the operating day has been subdivided into a number of one hour intervals (stages). However, since the work likely to become open during the last four hours of the operating day is less than five hours in duration. Hence, the number of stages (n1) over which the assignment of extra-board operators -- to open work greater than or equal to five hours in duration -- is to be optimized consists of all time periods

in the operating day with the exception of the last four hours.

States (S^i)

The states of the system at a particular stage represent the number of extra-board operators available for assignment to open work cover duties during the following stages. Here as well, we must distinguish between input and output states. The output states at a particular stage are the input states to the next stage. The output states at a particular stage "n" represent the number of extra-board operators remaining in the system. The magnitude of these states are determined by the optimal sets of extra-board assignment decisions associated with them.

$$S_t^i = S_0^i - \sum_{i=1}^{t-n} D_i \quad (4.17)$$

where,

$$S_0^i = AEB \quad (4.18)$$

and,

S_t^i : represents the state of the system (output state) at stage "t".

D_i : Represents the range of assignment decisions at stage "t" leading to a particular output state " S_t^i ".

S_0^i : The initial state of the system, which represents the number of extra-board operators to be assigned to reserve cover duties on a particular day.

AEB: Available extra-board operators on a particular day.

Given that the magnitude of states at any stage cannot exceed the number of available extra-board operators, the range of values that the states of the system can assume at any

stage varies between zero and "AEB".

$$0 \leq S_t^I \leq AEB \quad (4.19)$$

Decisions (D_t)

At the beginning of every stage and for every state, the model evaluates a range of options (decisions) with respect to the number of reserve operators to be assigned to that particular time period. The magnitude of decisions at a particular stage may be limited by the difference between output (S_t^I) and input (S_{t-1}^I) states.

$$0 \leq D_t \leq S_t^I - S_{t-1}^I \quad (4.20)$$

However, since it is very unlikely that open work during any given time period would exceed a certain percentage of scheduled runs (SR_t) for that period, it would also be appropriate to limit the magnitude of decisions at any stage during the day to a percentage " α " (not to exceed 20 percent) of scheduled runs for that particular stage.

$$0 \leq D_t \leq \alpha * SR_t \quad (4.21)$$

At any stage, the most restrictive of Equations 4.15 and 4.16 would apply. These two constraints further improve the efficiency of the proposed dynamic programming procedure by reducing the amount of computation that has to be performed in order to determine the optimal reserve assignment policy to this type of daily open work.

Transition Function

The decision associated with a particular stage-state combination transforms the system

by reducing the number of extra-board operators available for assignment at the following stage by the number of reserve operators assigned to that stage.

$$S_{t+1}^I = S_t^I - D_t \quad (4.22)$$

where S_{t+1} represents the state of the system at stage "t+1".

4.3.2.4 Optimization Procedures

Systems performance may be optimized by minimizing either the sum of surpluses and shortages or their cumulative costs.

Surplus-Shortage Minimization

In this procedure, the cumulative surplus-shortage (CSS) criterion is used to sort out, at every stage, the decision sets (assignment patterns) which minimizes the sum of operator surpluses and shortages associated with various state options.

$$CSS_t(S_t^I) = \text{Min.} D_t \{ ESS_t(S_{t-1}^I, D_t) + CSS_{t-1}(S_{t-1}^I) \} \quad (4.23)$$

$$CSS_{t-1}(S_{t-1}^I) = \text{Min.} D_{t-1} \{ ESS_{t-1}(S_{t-2}^I, D_{t-1}) + CSS_{t-2}(S_{t-2}^I) \} \quad (4.24)$$

The elimination of inferior options proceeds in this fashion from one stage to the next. At the final stage, the states of the system and their associated assignment policies are compared with respect to their cumulative surpluses and shortages and the system configuration which attains the desired level of reliability at minimum "CSS" value is then selected as the optimal daily assignment pattern to this type of open work.

4.3.3 Assignment of Extra-board Operators to Open Work Less Than Five Hours

4.3.3.1 System Definition

The system to be optimized by this model is the daily assignment pattern of the remaining extra-board operators to open work of less than five hours. The optimal assignment pattern is that which minimizes the sum of surpluses and shortages or their cumulative costs.

4.3.3.2 Demand

The demand on this system is represented by open work less than five hours in duration. This type of open work is caused largely by vehicle accidents and breakdowns. It represents, consecutively, about 20 percent of total open work hours and 40 percent of open work pieces. A sample of open work data from STO has indicated that 75 percent of open work pieces of this type tend to occur in the afternoon.

Open work at any time period during the day is the sum of the work becoming open during that time period, and that remaining open from previous time periods, due to a shortage in the availability of operators during these time periods. Here as well, the probability of open work of a given magnitude during a particular time period of the day may be approximated by the binomial or Poisson distributions.

4.3.3.3 Model Structure

In order to ensure that reserve operators assigned to this type of open work during the afternoon time periods would have the required complimentary open work pieces during the morning time periods and vice versa, the assignment of reserve operators to morning

and afternoon open work cover duties would be carried out in two separate steps.

In the first step, the remaining reserve operators (extra-board operators not assigned to open work greater than or equal to five hours in duration) would be assigned to cover open work less than five hours during the morning time periods. At the second step the same number of extra-board operators would be assigned to this type of cover duties during the afternoon time periods. The estimated cover requirements for the morning and afternoon time periods are then compared, and the highest of these two values (afternoon open work cover requirement) is then selected as the daily reserve cover requirement for this type of open work.

In case the number of available extra-board operators is greater than cover requirements for this type of open work, then the remaining extra-board operators after this phase of the assignment process will be assigned to matching morning and afternoon tripper pairs. On the other hand, if the available extra-board operators are less than the required open work cover, the balance would be supplied by overtime. One of the advantages of this model is that overtime requirements can be predicted well in advance, thus permitting management to make the necessary arrangements to ensure that the required overtime is made available.

However, given the methods used for the estimation of open work cover requirements (Sections 3.2.2.2 and 3.2.2.3), it is very unlikely that the extra-board size would either significantly exceed, or fall short of the requirements of open work cover. Figure 2.10 has shown the overall structure of the extra-board assignment process, and the interrelationships among its various phases.

Since the matching requirements for afternoon open work is much higher than morning open work of this type, a number of morning trippers equivalent to the difference between open work cover requirements for the afternoon and morning time periods would have to be set aside as complimentary work pieces to some of the afternoon open work. The remaining morning and afternoon trippers are left to be covered by overtime.

The model used for optimizing the assignment of reserve operators to morning and afternoon open work of this type (type 2) is very similar to the dynamic programming procedure used at the strategic level, for the estimation of cover requirements for open work less than five hours in duration (Section 3.2.2.2).

Here as well, optimization means the minimization of the discrepancy between the daily patterns of extra-board report times and the incidence pattern of open work. The gap between supply and demand is measured in one of two ways; either as the sum of surpluses and shortages, or as their cumulative costs.

Given the similarity between this and the previous model (Section 4.3.2), and in order to avoid repetition, only those aspects which are unique to this model will be detailed.

Inputs

The primary differences between the inputs to this model and those of the previous model (Section 4.3.2) pertain to the nature of open work, and the inclusion of trippers as complimentary pieces to open work, particularly:

- expected open work rates ($EOWR^2_t$), or expected open work (EOW^2_t) of type "2" -- open work less than five hour in duration -- during stage "t",
- morning trippers, and

- afternoon trippers.

All other inputs are the same as in the previous model (Inputs, Section 4.3.2.3).

Stages ($t=1..n2$)

Since open work of this duration may occur at any time period during the operating day, hence the number of stages ($n2$) over which assignment decisions are to be made includes all hours in the operating day. The morning stages include all operating hours between the start of the service and noon ($1..s$). The afternoon stages include all operating hours between noon and the end of service ($s+1..n2$).

States (S^2)

At every stage, the states represent the number of extra-board operators available for assignment to cover duties during the following time periods. That is the number of reserve operators available at the start of this decision process minus the number of operators assigned to the system up to and including the present stage.

Given the requirement that morning and afternoon open work of this type be matched to create full day work duties, hence morning and afternoon states must be distinguished such that the cover requirements for morning and afternoon open work may be determined separately. The period with the highest optimum cover requirements (afternoon period) determines the magnitude of this type of cover for the whole day. However, the assignment of report times (two per day for each extra-board operator, a morning and an afternoon report time) would be determined by the optimum assignment pattern to the morning as well as the afternoon time periods. The initial states for the

morning and afternoon time periods are determined as follows:

$$AMS_0^2 = AEB - S_{n1}^1 \quad (4.25)$$

$$PMS_s^2 = AEB - S_{n1}^1 \quad (4.26)$$

These initial states determine the range of values the states of the system may assume at any particular stage.

$$0 \leq AMS_i^2 \leq AMS_0^2 \quad (4.27)$$

$$0 \leq PMS_i^2 \leq PMS_s^2 \quad (4.28)$$

where

AEB: Number of extra-board operators available for assignment to open work of all durations on a particular day.

S_{n1}^1 : Number of extra-board operators assigned to open work cover duties of type "1" during a particular day.

AMS_0^2 : Number of extra-board operators available for assignment to open work cover duties of type "2" during the morning time periods.

PMS_s^2 : Number of extra-board operators available for assignment to open work cover duties of type "2" during the afternoon time periods.

Here as well we must distinguish between input and output states. At a particular stage, the states of the system represent the output of the decision process up to and

including that stage. These same states are the input states to the next stage of the decision process.

Decisions (D_t)

Decisions refer to the number of reserve operators to be assigned to a particular stage. The magnitude of these decisions is constrained by the difference between input and output state values. In the case of morning assignment decisions, this constraint may be expressed as follows:

$$0 \leq D_t \leq (AMS_t^2 - AMS_{t-1}^2) \quad (4.29)$$

where

$$AMS_t^2 = AMS_{t-1}^2 - D_t \quad (4.30)$$

and

AMS_{t-1}^2 : Input state to the decision process at stage "t" during the morning period.

AMS_t^2 : Output state to the decision process at stage "t" during the morning period.

The same constraints apply to afternoon assignment decisions, with the exception that the states in this case refer to afternoon input and output states.

$$0 \leq D_t \leq (PMS_t^2 - PMS_{t-1}^2) \quad (4.31)$$

where

$$PMS_t^2 = PMS_{t-1}^2 - D_t \quad (4.32)$$

and

PMS_{t-1}^2 : Input state to the decision process at stage "t" during the afternoon period.

PMS^2_t : Output state to the decision process at stage "t" during the afternoon period.

Reliability Function

The reliability of extra-board assignment patterns produced by this model are measured in the same manner as the reliability levels of the cover patterns for open work of type "2" at the strategic level (Equation 3.57).

4.3.3.4 Optimization Method

Extra-board assignment patterns to this type of open work are optimized, separately, for the morning and afternoon time periods, by minimizing either the sum of surpluses and shortages or their cumulative costs. The period with the highest cover requirements represents the daily reserve requirements for this type of open work. Since afternoon open work of type "2" is greater than its morning counterpart, the morning trippers would be required to complement some afternoon open work. The magnitude of these trippers is equivalent to the difference between the states associated with the optimum afternoon and morning extra-board assignment patterns.

$$ReqAMT = PMS^2_{n2} - AMS^2_s \quad (4.33)$$

where

AMS^2_s : State of the system associated with the optimal assignment pattern of extra-board operators to open work of type "2" during the morning time period.

PMS^2_t : State of the system associated with the optimal assignment pattern of extra-board operators to open work of type "2" during the afternoon time period.

ReqAMT: Morning trippers required for matching with some afternoon open work.

Given the above, the work duties assigned to this group of extra-board operators would include both morning and afternoon shifts. The report times to these work shifts would correspond to extra-board assignment patterns to morning and afternoon open work of this duration, and to the start times of the morning trippers required to supplement afternoon open work in excess of its morning counterpart.

If the optimal assignment pattern of the remaining extra-board operators fails to produce the desired reliability level set by management, then the number of extra-board operators available for this type of open work cover will be incremented by one and the assignment process repeated until the desired level of reliability is attained. The additional operator hours required would be acquired at overtime premium rates.

Chapter V

CASE STUDY

5.1 INTRODUCTION

The two most critical steps in transit operator planning and management involve: 1) the estimation of open work cover requirements at the strategic level, and 2) the assignment of daily report times to extra-board reserve operators at the operational level. This objective was accomplished, in this thesis, through the development of a number of dynamic programming procedures, which incorporate into the decision process the dynamic and probabilistic aspects of open work, and its interaction with cover decisions over time periods of the day. This case study is intended to show:

- the effectiveness of this group of models in capturing the dynamic and probabilistic aspects of open work, and its interaction with cover decisions overtime,
- the impact of daily open work patterns on cover costs and reliability,
- the impact of daily cover patterns on cover cost and reliability, and
- the impact of the changes in shortage costs on cover patterns.

Shortage costs are affected by the values assigned to "CTC" (cost of one cancelled hour of bus service) which is used to incorporate into the objective function -- the cumulative surplus-shortage cost function -- the reliability concerns of management. The higher these values the higher is the concern over reliability.

This term provides management with a control mechanism by which it can influence the outputs of these models, i.e. operator utilization patterns (policies) and their associated costs and reliability levels.

Given the similarities between the various dynamic programming procedures presented in Chapters 3 and 4 -- aside from minor differences which reflect the changes in the tasks they are designed to accomplish -- the capabilities of these models can be demonstrated through the application of tests to only one of these models, namely that for "estimation of open work cover requirements for open work greater than or equal to five hours" (Workforce Size Model, Section 3.2.2.1). Hereafter, this model would be referred to as the "Open Work Cover Model".

5.2 STRUCTURE OF PRESENTATION

This demonstration involves the estimation of optimal open work cover requirements (operator days) -- for open work greater than or equal to five hours in duration -- during the first four hours of a typical Monday of the Fall timetable, 1990, at the Societe' De Transport De l' Outaouais (STO), Hull, Quebec. It consists of seven tests. The inputs to these tests are based on actual operational and cost data collected from STO for that same period. The details of these inputs are presented with each test. The length of the test period (four stages) and the magnitude of inputs to each test (particular, open work) are considered appropriate for demonstrating the purposes stated at the introduction to this chapter.

The first five tests demonstrate: 1) the importance of the interaction between open work and its cover over time periods of the day, and 2) the effect of the changes in

shortage costs on the schedule patterns of the minimum cost policies associated with alternative cover options, their costs and reliability levels.

An important factor in this regard is the value assigned to one hour of cancelled bus service (CTC). Since management would be at least willing to pay overtime rates to cover open work resulting from a shortage in the availability of reserve cover, hence a "CTC" value equal to the average hourly overtime pay rate (AHOTC) was used in Test 1. Under this assumption, the minimum surplus-shortage cost patterns should be equal to the minimum surplus-shortage patterns, because in both cases surpluses and shortages receive equal weights. This expectation is confirmed by the results of Test 1, in which the optimal cover policy for the study period according to both criteria is the same (see Table A.1.2 and Figures A.1.3 - A.1.4 in Appendix A).

Tests 6 and 7 show the sensitivity of the model to small variations in the magnitude and pattern of daily open work. Test 7 demonstrate the effect of changes in the pattern of open work on the cover requirements, their costs and reliability levels.

In order to carry out these tests, a computer program was developed by the author. This program is written in the Pascal Language, runs on micro computers, and incorporates the system performance functions presented in Section 3.2.2.1, namely:

- the cumulative total cost function (Equation 3.12),
- the cumulative surplus-shortage cost function (Equation 3.24),
- the cumulative surplus-shortage function (Equation 3.29), and
- the system reliability function (Equation 3.37).

The structure of this program is described in Appendix C.

5.2.1 Inputs

Inputs are divided into two groups: cost variables and operational data. Cost variables include:

- average daily full-time cost, including fringe benefits and vacation pay (ADFTC),
- average hourly overtime cost (AHOTC), and
- cost assigned to one hour of cancelled bus service (CTC).

Operational inputs are period specific and include:

- scheduled service runs commencing at various time periods of the day (SR), and
- expected open work rates at various time periods of the day (EOWR).

These inputs are the same for Tests 1-5, with the exception of "CTC" which is assigned progressively higher values from one test to the next. The inputs to Tests 6 and seven are the same as those of Test 5, except open work rates which are changed to show the effect of changes in the magnitude and pattern of these values on cover requirements.

5.2.2 Outputs

The outputs for each test are presented in four tables and six figures. The tables and figures for various tests are grouped in Appendix "A". In order to ensure consistency throughout this presentation and to facilitate the comparison between the results of various tests, tables and figures are indexed by one letter and two numbers. The letter refers to Appendix "A", the first number refers to the test, while the second indicates the position of the table or figure with respect to other tables or figures in the same test. For example,

Table A.2.4 refers to Table 4 of test 2.

The first table in each test shows the actual system costs and reliability levels associated with the optimum daily utilization patterns of alternative open work cover options (states). The second table presents for each of these optimum open work cover designs:

- cumulative surplus (CSUR -- Equation 3.30),
- cumulative shortage (CSH -- Equation 3.31),
- cumulative surplus cost (CSURC -- Equation 3.25),
- cumulative shortage cost (CSHC -- Equation 3.26),
- cumulative surplus-shortage (CSS -- 3.29), and
- cumulative surplus-shortage cost (CSSC -- 3.24).

The third and fourth tables present a stage by stage analysis of the optimal open work cover designs according to the "CSS" and "CSSC" criteria.

The first and second figures present, consecutively, cost and reliability comparisons of the optimum open work cover designs presented in the first table. The third and fourth figures compare these same assignment patterns with respect to their cumulative surplus-shortage and cumulative surplus-shortage costs. The last two figures show, consecutively, network representations of the optimal system design according to the "CSS" and "CSSC" criteria.

Except in Test 1, where "CTC" value is equal to average hourly overtime cost (AHOTC), optimal open work cover designs according to the "CSSC" criterion tend to produce higher levels of reliability compared to those based on the "CSS" criterion. The

underlying cause of this behaviour is the higher penalty cost assigned to shortages via higher "CTC" values, which makes high reliability systems more attractive because of their low shortage levels, and hence lower cumulative surplus-shortage costs.

It is important to distinguish between the "CTC" value used to calculate the total system costs presented in the first group of tables (Tables A.X.1), and that used in the cumulative surplus-shortage cost function (objective function) as a basis for computing cumulative shortage costs (CSHC) and cumulative surplus-shortage costs (CSSC) presented in second group of tables (Tables A.X. 2) and for comparison and selection among alternative cover policies and options. In the first case, "CTC" is always evaluated as equal to the average hourly overtime cost (AHOTC). In the second case "CTC" is increased from one test to the next to reflect the concern for reliability on the part of management.

5.3 TEST 1

5.3.1 Inputs

The inputs to this test represent, respectively, actual operational and cost data for the test period and the fall timetable, 1990, at STO. With the exception of the "CTC" value and expected open work rates ($EOWR_t$), these inputs are the same for each of the tests presented in this case study. These inputs are:

- $EOW_d^1 = 9$ open work duties (greater than or equal to five hours in duration during the first four hours of the operating day), or 7.2 percent of the runs commencing during this four hour period.

- ADFTC = \$230.00 / operator day (cost of one operator day including fringe benefits and vacation pay).
- AHOTC = \$25.00 / hour (average hourly overtime cost).
- CTC = AHOTC = \$25.00 / hour (penalty cost for one hour of cancelled bus service).
- $\sigma = 0.8$ (proportion of open work hours of this duration to total open work hours).

PERIOD SPECIFIC INPUTS	STAGE 1	STAGE 2	STAGE 3	STAGE 4
EOWR _i ¹	0.085	0.075	0.065	0.090
SR _i	6	42	64	13

"EOWR_i¹" and "SR_i" refer, consecutively, to open work rates and the number of runs scheduled to start at various stages.

5.3.2 Outputs

The outputs of this test are presented in three tables (Tables A.1.1 - A.1.3) and five figures (Figures A.1.1 - A.1.5). Table A.1.1 lists for every cover option (Column 1), the optimum cover policy (Columns 2 to 5) -- which minimizes the sum of surplus and shortage costs -- its associated system cost (Column 6) and reliability level (Column 7). Hence, management would be able to select the system design which best achieves its targeted level of reliability. For example, for a reliability of 99.4 percent, the optimal open work cover design under these conditions is Option 10 (0, 4, 6, 0).

Table A.1.2 lists for these same minimum cost open work cover policies surplus-shortage and surplus-shortage cost information. This information is particularly important

for the determination of the most efficient cover policy, which is defined in this model as that which minimizes the discrepancy between the daily pattern of open work and its corresponding cover pattern. In this model, this discrepancy is measured as the sum of surplus and shortage costs (CSSC).

Since "CTC" and "AHOTC" are assigned equal values (\$25) in this test, hence the optimal overall cover policy is that which minimizes the sum of surpluses shortages (CSS) as well as the sum of surplus and shortage costs (CSSC). This policy is associated with the cover Option 9 and it has a schedule pattern of "0, 4, 5, 0". This policy, its cost, reliability, surplus and shortage, and surplus and shortage cost values are highlighted in Tables A.1.1 and A.1.2. Table A.1.3 presents a stage by stage analysis of this policy.

Figures A.1.1 and A.1.2 present, consecutively, a graphic comparison of the system costs and reliability levels of various optimum open work cover policies. Similarly, Figures A.1.3 and A.1.4 present graphic comparisons of the same policies with respect to their impact on cumulative surpluses and shortages and their costs.

These figures show that up to a supply level of 9 operator days the system is capable of realizing significant improvements in efficiency by increasing workforce size. This is explained by the decline in "CSS" and "CSSC" values (Figures A.1.3 and A.1.4), which underlay the lower than normal change in system costs -- less than the average daily full-time operator cost of \$230 -- accompanying the incremental increase in supply. Past this point "CSS" and "CSSC" values start to increase, accompanied by a steeper rise in system costs, over \$230 per additional operator day (Figure A.1.1). These observations are supported by Figure A.1.2, where the increase in reliability values drop drastically past

Option 9, with almost no change accounted for past Option 10.

Figure A.1.5 is a network representation of the optimal open work cover policy according to the "CSS" and CSSC" criteria. This policy is highlighted by the thick line, and it corresponds to Table A.1.3.

5.3.3 Comments

The outputs of this test highlight important aspects of the model, particularly those related to the interaction between open work and its cover over time periods of the day. Figure A.1.1 shows the tendency of the model to satisfy the demand for early time periods before it proceeds to assign cover to the following time periods. Since surplus operators from a given time period are likely to find work during the following time periods, hence by scheduling operators to early time periods the system is able to ensure high reliability during these periods at a small surplus cost. For example, up to a daily supply level of four operator days all cover have been scheduled to the second stage. No operators are assigned to the first stage because under these cost conditions such a decision would result in higher surplus-shortage costs compared to the optimum pattern.

5.4 TEST 2

5.4.1 Inputs

With the exception of "CTC", the inputs to this test are the same as those for Test 1. "CTC" is assigned a value of \$100.00.

5.4.2 Outputs

The outputs of this test are presented in four tables (Tables A.2.1 - A.2.4) and six figures (Figures A.2.1 - A.2.6) which have similar format as those for the previous test. The only exceptions are Table A.2.4 and Figure A.2.6, which present a stage by stage analysis and a network representation of optimal open work cover policy according to the "CSSC" criterion.

5.4.3 Comments

This test highlights the impact of the changes in cover patterns on system cost and reliability. The increase in "CTC" has affected a general shift in the optimum cover patterns in the direction of early time periods, reflected in the rescheduling of one operator day from Stage 3 to Stage 1 (Table A.2.1)

This change in policies has resulted in a general increase in system reliabilities of 0.3 percent (Table A.2.1) affected by a general decline in cumulative shortages (Table A.2.2). As a result of that it is possible to attain a higher level of reliability with 9 operator days and a cover pattern of "1, 4, 4, 0" than it is possible with 10 operator days and a cover pattern of "0, 4, 6, 0". A comparison of these two patterns show a saving of about \$258 or 10 percent in cover cost for open work greater than or equal to five hours in duration during the first four hours of the day (Tables A.1.1 - A.2.1).

The general increase in cumulative shortage costs, due to higher "CTC" values has pushed down the minimum cumulative surplus-shortage cost (CSSC) policy to Option 10 (Table A.2.2 and Figure A.2.4). However, in spite of the general increase in cumulative surplus (CSUR) and cumulative surplus-shortage (CSS) values, the optimum policy with

the minimum surplus-shortage value is still that of Option 9. This may be explained by the fact that Option 9 is equal to expected open work duties of this duration for the study period.

This test as well as Tests 3 to 5 demonstrate the effectiveness of the "Open Work Cover Model" in representing the interaction among cover decision and open work patterns over time. These interactions are responsible for the shifts in cover policies in response to the increase in shortage costs versus surplus costs.

5.5 TEST 3

5.5.1 Inputs

The inputs to this test are the same as those for Test 2, with the exception of the "CTC" value, which has been increased to \$200 per hour -- double it's previous value.

5.2.2 Outputs

The presentation of the outputs for this test follows the same format as that for the two previous tests, i.e., they are presented in four tables (Tables A.3.1 - A.3.4) and six figures (Figures A.3.1 - A.3.6).

5.5.4 Comments

The additional increase in "CTC" values has further shifted to the left the schedule patterns of the optimal policies associated with alternative cover options (Table 5.3.1). In comparison to Test 2, this change in pattern is reflected in the rescheduling of one operator day from Stage 3 to Stage 2. Hence, the options affected by this change are those with operator days assigned to Stage 3 in Test 2 -- Options 6 to 14. However,

Options 1 through 5 have not changed either with respect to their schedule patterns, or reliability levels (Table A.3.1).

The increase in "CTC" values has also resulted in a further divergence between the optimum policy with the minimum cumulative surplus-shortage (CSS) and that with the minimum cumulative surplus shortage costs (CSSC). These changes are evidenced from the comparison of Table A.2.1 and Figures A.2.1 - A.2.6, with Table A.3.1 and Figures A.3.1 - A.3.6, consecutively.

The effect of this change in policy from Test 2 is not as dramatic as that from Test 1 to Test 2. In Test 2 a gain of 0.29 percent in system reliability over Test 1 was realized compared to a gain of only 0.06 percent for this over Test 2. This indicates a limit with respect to the improvements in reliability which may be attained as a result of changes in schedule patterns.

5.6 TEST 4

5.6.1 Inputs

In this test, "CTC" is further increased to \$300 per hour. All other inputs remain unchanged from their original values.

5.6.2 Outputs

The results of this test are presented in Tables A.4.1 - A.4.4 and Figures A.4.1 - A.4.6. These outputs further demonstrate the sensitivity of the model to the changes in "CTC" values.

5.6.3 Comments

The effect of this increase of "CTC" value points in the same direction as the two previous tests, i.e. toward a shift in optimal schedule patterns in the direction of Stage 1. As well, this test reinforces the conclusion made earlier concerning the limits on the improvement in reliability which may be attained as a result of the scheduling of more operators to early time periods.

The comparison of the improvements in reliability levels versus the increases in system costs from one test to the next indicates a diminishing rate of return on investment, as we proceed to increase "CTC" values from Test 2 to Test 4. While, it took only 13 dollars to realize an improvement of about 0.29 percent in system reliability in Test 2, the schedule patterns of Tests 3 and 4 have resulted in considerably lower improvements in reliability levels -- 0.06 and 0.04 percent, respectively -- for costs of 16 and 21 dollars, respectively. In relative terms this amounts to \$45 for an increase of 1 percent in reliability from Test 1 to Test 2, compared to \$267 for Test 3, and \$525 for Test 4.

5.7 TEST 5

5.7.1 Inputs

In this test, "CTC" is assigned a value of \$500 per hour. The other inputs remain unchanged from their original values.

5.7.2 Outputs

The results of this test are presented in Tables A.5.1 - A.5.4 and Figures A.5.1 - A.5.6.

These outputs further demonstrate the sensitivity of the model to the changes in "CTC" values.

5.7.3 Comments

The 0.03 percent increase in reliability over the previous test further reinforces the finding concerning the limit with respect to the improvements that can be realized as a result of shifts in schedule patterns.

Further tests of the model with higher "CTC" values have resulted in no change in schedule patterns. This indicates that the shortages that can be eliminated by shifts in schedule pattern have already been diminished, which makes it difficult to affect significant changes in cumulative shortage costs -- to offset the corresponding increases in surplus costs -- to cause further shifts in schedule patterns to the left. This conclusion is supported by the results shown in Table A.5.2 which shows a cumulative shortage (CSH) level of 0.17 hours for the minimum surplus-shortage cost policy (Option 11).

However, a comparison of Option 9 in this test with Option 11 in Test 1 shows an overall saving of \$455 or 20 percent in cover costs for open work of this type over the four hour test period. These savings have resulted from the improvements in cover patterns which made Option 9 in this test more effective than Option 11 in Test 1. These improvements in the cover pattern were made possible by: 1) taking into account the interaction between cover decisions and open work patterns over time, and the incorporation of reliability concerns into the objective function via the "CTC" term.

5.8 TEST 6

5.8.1 Inputs

The purpose of this test, as well as Test 7, is to demonstrate the sensitivity of the model to changes in the magnitude and pattern of open work. With the exception of open work rates, the inputs to this test are the same as those for Test 5, which is chosen as a pivot case for comparison with these two tests. In this test, expected open work rates are reduced by 5 percent for all time periods (stages). These values are:

STAGES	1	2	3	4
EOWR_t	0.08	0.07	0.06	0.085

5.8.2 Outputs

The results of this test are presented in Tables A.6.1 - A.6.4 and Figures A.6.1 - A.6.6. These outputs show considerable change over the results of Test 5. These changes affect schedule patterns, optimal policies, system reliability and costs.

5.8.3 Comments

The reduction in open work rates has resulted in across the board increase in the reliability levels associated with various cover options (Table A.6.1). These results are consistent with expectations. Since, a low demand means that higher levels of reliability can be attained with low levels of supply -- compare the level of reliability attained with 8 operator days in this test (99.7 percent -- Table A.6.1), to the level of reliability attained with 9 operator days in the Test 5 (99.6 percent -- Table A.5.1).

5.9 TEST 7

5.9.1 Inputs

Except for open work rates, the inputs to this test are the same as those for Test 5. Instead of the variable open work rates from one stage to another (Test 5). In this test, expected open work rates are similar for all time periods. However, expected open work rates for the day (0.072) are similar for both tests.

STAGES	1	2	3	4
EOWR_t	0.072	0.072	0.072	0.072

5.8.2 Outputs

The results of this test are presented in Tables A.7.1 - A.7.4 and Figures A.7.1 - A.7.6. These outputs show considerable changes over the schedule patterns, reliability levels and costs associated with similar options in Test 5.

5.9.3 Comments

Although expected open work for the entire test period is similar for Tests 5 and 7, however, the results of these two tests are entirely different. This is the case with respect to minimum surplus-shortage cost schedule patterns of various cover options, their system costs, reliability levels, shortages and surpluses and their costs (compare Tables A.7.1 and A.7.2 with Tables A.5.1 and A.5.2). Hence, it is important to consider, as part of the open work cover estimation process, the variability in open work rates over time period of the day. The failure to do so would result in a discrepancy between estimated cover levels

and actual needs.

5.10 CONCLUSION

The tests presented in this case study have provided sufficient evidence of the capabilities of the dynamic programming procedures developed in this thesis, particularly those used for: 1) the estimation of open work cover requirements, and 2) the assignment of daily report times to extra-board operators. As explained throughout this document, this group of models play a critical role in the proposed modelling framework, as means of incorporating into transit workforce planning and management processes -- at the strategic and operational levels -- the dynamic and probabilistic aspects of open work and its interaction with cover decisions over time periods of the day. Based on the results of these tests, the following conclusions can be made.

The first and most important of these conclusions is that the schedule patterns of reserve operators have significant impacts on their costs and reliability levels. This is clearly illustrated by the results of Tests 1 through 5. A comparison of the cost and reliability levels associated with the minimum surplus- shortage cost policies for Option 9 in Test 5 and Option 11 in Test 1 shows that 9 operators with a "2, 5, 2, 0" schedule pattern and a cost of \$2245 are more effective than 11 operators with a "0, 4, 7, 0" schedule pattern and a cost of \$2693. This entails a saving of \$455 or 20 percent in cover costs for open work of this type over the four hour test period. However, it should be noted that the optimum cover policy associated with Option 11 in Test 1 is still better than a cover policy that would result from the application of "rules of thumb", which for example would assign 1 cover operator for every 10 scheduled runs.

The second conclusion is that the system model is capable of producing optimal cover policies which maximizes the reliability associated with alternative cover options. These policies are affected by the assignment of more weight to shortages versus surpluses. This is attained by increasing the value of "CTC", which is used in this model to incorporate into the objective function the reliability concerns of management. As "CTC" values increase, from one test to the next, shortage costs assume increasing weight in comparison to surplus costs, thus driving the system toward solutions with relatively small, or no, schedule (open work cover allocation pattern) related shortages. This, in turn, makes it difficult to affect significant changes in cumulative shortage costs -- to offset the corresponding increases in surplus costs -- to cause further shifts in schedule patterns to the left.

The implication of this is that there is a limit to the improvements in reliability which may be attained from the optimization of schedule patterns. The comparison of the improvements in reliability levels from one test to the next indicates a decreasing rate of improvement accompanying the increase in "CTC" values -- 0.29 percent for Test 2, 0.06 percent for Test 3, 0.04 percent for Test 4, and 0.03 percent for Test 5.

The last comment concerns the effect of open work patterns on the patterns, costs and reliability levels of the optimum policies associated with alternative cover options. A comparison of the results of Tests 5 and 7 shows significant impacts on cover costs and reliability levels due to these change.

Chapter VI

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Transit operator workforce planning and management processes consist of a set of interrelated tasks involving the determination of:

1. Size of the transit operator workforce,
2. Annual vacation schedules for operators,
3. Timing and sizing of hiring decisions,
4. Weekly work schedules for available operators, and
5. Daily report times for extra-board (reserve) operators.

In the case of properties with multiple garages the last task involves, as well, the assignment of reserve operators to garages.

The two most important steps in this process involve the estimation of cover requirements at the strategic level, and the assignment of daily report time to extra-board (reserve) operators at the operational level.

This chapter consists of two main sections. The first section (6.1) summarizes the contributions of this research to transit operator workforce planning and management, as is embodied in the modelling framework developed in this thesis. Section 6.2 presents recommendations with respect to the implementation of this model and further research.

6.1 SUMMARY AND CONCLUSIONS

The central organizing principle of the modelling framework developed in this research is the desire to incorporate into transit operator workforce planning and management decisions the dynamic and probabilistic aspects of open work and its interaction with cover decisions over time periods of the day. This objective is accomplished by decomposing the problem, along decisions lines, into stages such that the problem of open work is dealt with separately. In addition to facilitating the solution of the open work cover problem, this construction is responsible for the flexibility, modularity, and responsiveness of the modelling framework to the temporal variability (seasonal, weekly, and daily) in demand conditions.

A central feature of this modelling framework are the dynamic programming procedures developed for the optimization of open work cover requirements and hiring decisions at the strategic level, and daily report times for reserve operators at the operational level. With the exception of few minor differences, these optimization procedures are similar in structure and process.

A common feature of these models is the division of the planning period into a number of stages (smaller time periods). At every stage, a supply decision is made which affects the state of the system. The subject of these decisions is the number of operator days or weeks assigned to a specific stage. While the states of the system are determined by the decision process, demand (open work and attrition) is a random variable. The objective of the decision process, in every case, is to minimize the discrepancy between demand and supply patterns, as measured by the various performance functions discussed

as part of these models.

Figure 2.1 shows the structure of this modelling framework, and the interrelationships among its various models and their inputs. These models are the:

1. Workforce size model,
2. Vacation model,
3. Hiring model,
4. Tactical model, and the
5. Operational Model.

These models are presented in detail in Chapters II-IV. A summary of each of these models and its contributions are presented below.

6.1.1 Workforce Size model

The most important attribute of this methodological framework is its focus on the relationship between strategic and operational level decisions, specifically the relationship between the estimation of open work cover requirements at the strategic level and extra-board assignment decisions at the operational level. The similarities between these two problems have suggested that in order to develop reliable estimates of open work cover requirements at the strategic level, the peculiarities of the operational problem had to be incorporated into strategic decisions regarding this issue, namely:

- the variability in open work cover requirements over days of the week,
- the dynamic and stochastic variability in open work rates over time periods of the day,
- the interdependence between open work and its cover over time periods of the day,
- the duration aspect of open work and trippers, and their relationships, and

- the dynamic and stochastic variability in the availability of overtime over time periods of the day.

The incorporation of these aspects into the Workforce Size Model has had a decisive effect in the shaping this model. This is reflected in the multi-stage estimation procedure presented in Figure 2.4, in which the service plan is broken down into a number of timetables. Each timetable has a number of day types with similar service schedules (runs and trippers). These day types correspond to various days of the week, and holidays (which usually have similar service patterns as Sundays).

The estimation of annual operator requirements starts at the daily level. This is carried out in two phases. The first and most important of which involves the estimation of daily open work cover requirements, which is estimated in two steps. The objective of this breakdown is to ensure that cover duties of sufficient length are created.

At the first step, the cover requirements for open work greater than or equal to five hours in duration are estimated. Since, the average duration of open work of this type is about 7 hours (STO open work data), its cover duties are assumed to constitute full day's work. At the second step, the cover requirements for open work less than five hours in duration are estimated. Because of its relatively short duration, this type of open work is combined with trippers to constitute a full day's work.

Having determined open work cover requirements and costs for a particular day type, these estimates are then added to scheduled runs and their costs, to determine consecutively the total number of operator days required for that particular day type, and their costs. The same procedures are repeated for every day type in each timetable of the

service plan.

In order to estimate operator requirements and costs for a particular timetable, operator requirements and costs for every day type are multiplied, consecutively, by the number of similar days in that timetable, and then added for all day types to give, respectively, the total number of operator days and costs required during the timetable. This procedure is repeated, as well, for every timetable in the service plan. The sum of the operator requirements for each of these timetables and their associated costs represent the total operator requirements (operator work days) and costs for the service plan, with the exception of the operator requirements and costs of hiring decisions.

Workforce size is then estimated by dividing the total number of operator work days required for the service plan by the average number of contractual work days per operator per year. In the case of two operator classification (full-time and part-time), provisions of the labour agreement with respect to this issue are used to determine, consecutively, the number of full-time and part-time operators.

The Workforce Size Model and its various sub-models have made considerable contributions to transit operator work force planning. First, they have incorporated into the decision process, at this level: 1) the dynamic and probabilistic aspects of open work, 2) its duration aspect, and 3) its interaction with cover decisions over time periods of the day. The incorporation of these aspects of the problem into the decision process have created a firm basis for informed decision making, by providing reliable estimates of the impacts of cover decisions on service cost and reliability.

The case study presented in Chapter 5 has shown that considerable savings may be

attained from the open work cover optimization procedures alone. For example, a saving of \$455, or 20 percent of cover costs, was realized over a four hour period, as a result of better cover schedule patterns. Equally important is the information generated by this model on the cost and reliability impacts of various cover policies. This information is particularly important because it reduces the uncertainty associated with this problem, thus eliminating the main cause of overstaffing in most transit agencies.

Given a reserve cover of 10 percent of the workforce size, a 20 percent reduction in open work cover costs represents about 2 percent of total operator costs. In the case of a transit property of the size of OC Transpo this would amount to over one million dollars in annual savings.

A second important contribution of this methodology is its innovative approach to the problem created by morning trippers. By combining trippers with open work of short durations, the utilization of reserve operators is maximized and the magnitude of morning trippers is reduced to levels compatible with available overtime during the morning peak travel period. This solution, as well, eliminates with no loss to efficiency the uncertainty associated with the determination of the magnitude and pattern of overtime availability over time periods of the day, which exhibits characteristics that are similar to these for open work.

The third contribution concerns the ability of this methodology to accommodate the changes in demand levels over time periods of the day, days of the week, and between timetables. This attribute of the model is particularly important for the Vacation and Tactical models which rely on the Daily Model (Equation 3.55) to develop estimates of

work requirements during the various weeks and days of a given timetable.

6.1.2 Vacation Model

This model plays a critical role in the overall modelling framework by ensuring that operator availability is maximized throughout the service plan. This objective is achieved by subordinating vacation requirements to weekly work requirements. The critical step in this procedure involves the estimation of the operator work weeks required during every week of the service plan (Equation 3.73). In case of variable weekly work requirements as is the case during seasonal festivities (Winterlude in Ottawa), variable vacation schedules are produced by the model (Figure 2.5).

This model is also capable of accommodating work rule constraints, which in many cases dictate that a uniform number of vacation weeks be provided throughout a particular timetable. Such constraints may be satisfied at a cost (loss in efficiency) by subordinating work requirements to vacation requirements. Such a policy would, as well, result in a uniform level of operator work weeks being supplied throughout the timetable.

6.1.3 Hiring Model

The structure of this model is similar to that used for the estimation of open work cover requirements (Workforce Size Model). However, instead of daily open work, demand in this case consists of attrition. The dynamic programming procedure developed for this purpose incorporates into the hiring process the dynamic and stochastic attributes of attrition and its interaction with hiring decisions over time periods of the year.

The length of these time periods constitute the second main difference between this

model and the open work cover models. Given the relatively low frequency of attrition, the time span over which meaningful attrition patterns may be identified is usually in the order of weeks and months rather than hours and days.

Given the similarities between this model and the open work cover model, hence it is reasonable to conclude that savings in hiring costs proportional to those produced by the open work cover model may be produced by this model.

6.1.4 Tactical Model

This model serves a common purpose with the Vacation and Operational Models, that is to maximize the availability of operators for work over time. While the Vacation and Operational models are intended to maximize, respectively, operator availability during the various weeks of a timetable, and the time periods of a particular day, the Tactical model is intended to create weekly work schedules for operators -- other than those taking their annual vacation leave -- which maximize their availability for work during days of the week.

This objective is achieved by subordinating day-off requirements on any particular day to work requirements on that day. The critical step in this model involves the estimation of work requirements for every day of every week in the coming work period (2 to 3 weeks). This information is provided by Equation 3.55 (Workforce Size Model), which estimates daily operator work requirements. These estimates are then used to schedule operator work days to days of the week (Section 4.2.2).

Another objective of this model is maximize the number of consecutive day-off combinations. A special procedure is developed for this purpose as described in Section

4.2.3. This procedure is particularly relevant in situations where consecutive days-off are required by the labour agreement.

6.1.5 Operational models

The objective of this model is to develop daily assignment patterns for extra-board operators which minimizes the discrepancy between the report times of these operators and the incidence pattern of open work. This optimization procedure uses a dynamic programming formulation similar to that used for the estimation of cover requirements (Workforce Size Model). As well, it incorporates into the decision process: 1) the dynamic and probabilistic aspects of open work, 2) its variable duration, and 3) its interaction cover decisions over time periods of the day.

However, while at the strategic level numerous open work cover options are analyzed to determine optimal daily cover requirements, the task here involves the determination of the optimal assignment policy (schedule pattern) for available extra-board operators on a particular day. Otherwise, this model is similar in every aspect to the open work cover model discussed as part of the Workforce Size Model. Hence, all the comments made concerning that model do apply to this model, as well.

6.2 RECOMMENDATIONS

These recommendations are intended to point out areas where further research is needed: 1) in support of the modelling framework developed in this thesis, and 2) as means toward better understanding of the problems and issues related to transit operator workforce planning and management.

Although sufficient evidence is provided in the Case Study which indicates that significant savings can be achieved from the application of this modelling framework, however, it is not possible to ascertain the full contributions of these models to transit operator workforce planning and management without a full application to a transit property. In this respect, the two most important issues are those of phasing and data requirements.

Given the structure of the modelling framework and the interrelationships among its various models, the most plausible implementation scheme is that which implements the Workforce Size Model first, followed by the Vacation Model, Hiring Model, Tactical and Operational Models. This order of implementation is necessitated by the critical role that the Workforce Size Model and sub-models play in the overall modelling framework.

However, the most taxing effort in this regard is the development of a data base for input to various models. The two most important elements of this data base are open work and attrition rates. The collection, analysis and classification of these inputs is an ongoing requirement of this decision process.

In this thesis, the distribution of daily open work is assumed to follow either a binomial or a Poisson distribution. However, the objective function is capable of accommodating other probability distribution(s) which may appropriately describe the incidence pattern of open work at various time periods of the day. Given the sensitivity of the open work cover models to the shape of this function, the distribution of open work at various times periods of the day must be investigated. This task can be facilitated, greatly, by the availability of an organized data base.

As well, the relationship between historical and recent open work patterns must be investigated as a means of predicting open work patterns for future time periods. The results of such an investigation are particularly important to the development of inputs to the operational model.

Another area which requires investigation, in this regard, is the variability in open work rates over the weeks of a given timetable. This information is particularly useful, at the tactical level, as it indicates the need for variable supply patterns (operator availability) over these time periods (weeks).

In the present formulation, the open work cover problem is solved in two steps: one for estimating cover requirements for open work greater than or equal to five hours, and another for estimating cover requirements for open work less than five hours. However, it may be best to provide cover, only, for open work of long durations (over four hours, with some of this work being matched with trippers), while leaving open work of short durations to be covered by overtime. Such a solution may be best, given the spread of this type of work throughout the day. Hence, further investigation of the magnitude and temporal pattern of open work of various durations is required.

While at the strategic level, open work cover requirements may be estimated for the entire system, at the operational level the problem created by multiple garages must be addressed in an appropriate manner. This involves the estimation of the number of reserve operators to be assigned to various garages on any particular day. The solution to this problem is facilitated by the ability of the operational model to estimate the cost and reliability levels associated with various supply options. With this information, it would

be relatively easy to determine the allocation pattern to garages which optimizes the performance of the system, i.e., maximizes its reliability.

Appendix A

TEST RESULTS

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Test 5	193 - 199
Test 6	200 - 206
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TABLE A.1.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 1

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	525	91.0
1	0	1	0	0	681	92.3
2	0	2	0	0	843	93.4
3	0	3	0	0	1014	94.5
4	0	4	0	0	1196	95.5
5	0	4	1	0	1377	96.4
6	0	4	2	0	1561	97.2
7	0	4	3	0	1751	97.9
8	0	4	4	0	1952	98.6
9	0	4	5	0	2174	99.1
10	0	4	6	0	2425	99.4
11	0	4	7	0	2694	99.5
12	0	4	8	0	2970	99.5
13	0	4	9	0	3248	99.5
14	0	4	10	0	3528	99.5

**TABLE A.1.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 1**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	525	525
1	0.0	18.0	18.0	0	451	451
2	0.0	15.3	15.3	1	382	382
3	0.2	12.7	12.9	5	319	324
4	0.6	10.5	11.0	15	262	276
5	0.6	8.5	9.1	15	212	227
6	0.7	6.6	7.2	17	164	181
7	0.9	4.8	5.7	22	120	142
8	1.3	3.2	4.5	32	80	112
9	2.2	2.0	4.2	54	50	104
10	3.6	1.4	5.0	90	35	125
11	5.4	1.2	6.5	134	29	164
12	7.3	1.1	8.4	182	27	210
13	9.3	1.1	10.3	232	27	258
14	11.3	1.1	12.3	282	26	308

FIGURE A.1.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 1

FIGURE A.1.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 1

FIGURE A.1.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES --TEST 1

FIGURE A.1.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES --TEST 1

**TABLE A.1.3: STAGE BY STAGE ANALYSIS OF THE OPTIMUM OPEN
WORK COVER POLICY ACCORDING TO THE "CSS" AND
"CSSC" CRITERIA -- TEST 1**

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	0	0.0	0.5	0.5	13	89.4
2	4	0.6	1.1	1.6	961	97.6
3	5	1.9	1.5	3.4	2154	98.9
4	0	2.2	2.0	4.2	2174	99.1

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**FIGURE A.1.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSS" AND
"CSSC" CRITERIA -- TEST 1**

TABLE A.2.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 2

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	525	91.0
1	1	0	0	0	685	92.4
2	1	1	0	0	847	93.6
3	1	2	0	0	1018	94.7
4	1	3	0	0	1201	95.7
5	1	4	0	0	1394	96.6
6	1	4	1	0	1577	97.4
7	1	4	2	0	1767	98.2
8	1	4	3	0	1966	98.9
9	1	4	4	0	2187	99.4
10	1	4	5	0	2438	99.7
11	1	4	6	0	2706	99.8
12	1	4	7	0	2982	99.8
13	1	4	8	0	3261	99.8
14	1	4	9	0	3540	99.9

**TABLE A.2.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 2**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	2098	2098
1	0.6	17.6	18.2	15	1762	1777
2	0.6	14.9	15.5	16	1485	1501
3	0.8	12.3	13.1	20	1233	1253
4	1.2	10.1	11.2	29	1005	1035
5	1.8	7.9	9.7	45	794	839
6	1.9	6.0	7.9	47	600	647
7	2.1	4.2	6.3	52	419	471
8	2.5	2.6	5.0	62	256	319
9	3.4	1.3	4.7	84	132	216
10	4.8	0.7	5.5	120	71	191
11	6.6	0.5	7.0	164	47	211
12	8.5	0.4	8.9	212	38	251
13	10.5	0.4	10.8	262	36	297
14	12.5	0.4	12.8	311	35	346

FIGURE A.2.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 2

FIGURE A.2.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 2

FIGURE A.2.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES --TEST 2

FIGURE A.2.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES --TEST 2

TABLE A.2.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 2

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	1	0.6	0.1	0.7	247	98.0
2	4	1.8	0.3	2.1	1203	99.2
3	4	3.1	0.8	3.9	2167	99.4
4	0	3.4	1.3	4.7	2187	99.4

TABLE A.2.4: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 2

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	1	0.6	0.1	0.7	247	98.0
2	4	1.8	0.3	2.1	1203	99.2
3	5	3.8	0.6	4.4	2410	99.6
4	0	4.8	0.7	5.5	2438	99.7

**FIGURE A.2.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSS"
CRITERION -- TEST 2**

FIGURE A.2.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 2

TABLE A.3.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 3

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	525	91.0
1	1	0	0	0	685	92.4
2	1	1	0	0	847	93.6
3	1	2	0	0	1018	94.7
4	1	3	0	0	1201	95.7
5	1	4	0	0	1394	96.6
6	1	5	0	0	1595	97.4
7	1	5	1	0	1785	98.2
8	1	5	2	0	1983	98.9
9	1	5	3	0	2204	99.5
10	1	5	4	0	2454	99.8
11	1	5	5	0	2722	99.9
12	1	5	6	0	2998	99.9
13	1	5	7	0	3277	99.9
14	1	5	8	0	3556	99.9

**TABLE A.3.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 3**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	4196	4196
1	0.6	17.6	18.2	15	3524	3539
2	0.6	14.9	15.5	16	2970	2986
3	0.8	12.3	13.1	20	2466	2486
4	1.2	10.0	11.2	29	2011	2040
5	1.8	7.9	9.7	45	1588	1633
6	2.7	5.9	8.6	67	1188	1255
7	2.9	4.1	7.0	72	821	893
8	3.3	2.5	5.7	82	492	573
9	4.2	1.2	5.4	104	238	342
10	5.6	0.6	6.2	140	114	254
11	7.4	0.4	7.7	184	65	249
12	9.3	0.2	9.5	232	47	280
13	11.3	0.2	11.5	282	42	323
14	13.25	0.2	13.5	331	40	371

FIGURE A.3.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 3

FIGURE A.3.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 3

FIGURE A.3.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 3

FIGURE A.3.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 3

TABLE A.3.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 3

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	1	0.6	0.1	0.7	247	99.0
2	5	2.6	0.2	2.8	1449	99.6
3	3	3.9	0.7	4.6	2183	99.5
4	0	4.2	1.2	5.4	2204	99.5

TABLE A.3.4: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 3

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	1	0.6	0.1	0.7	247	98.0
2	5	2.6	0.2	2.8	1449	99.6
3	5	5.5	0.3	5.8	2675	99.3
4	0	7.4	0.3	7.7	2722	99.9

**FIGURE A.3.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSS"
CRITERION -- TEST 3**

**FIGURE A.3.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSSC"
CRITERION -- TEST 3**

TABLE A.4.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 4

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	525	91.0
1	1	0	0	0	685	92.4
2	2	0	0	0	868	93.7
3	2	1	0	0	1039	94.7
4	2	2	0	0	1221	95.7
5	2	3	0	0	1414	96.6
6	2	4	0	0	1616	97.5
7	2	4	1	0	1805	98.3
8	2	4	2	0	2004	99.0
9	2	4	3	0	2224	99.5
10	2	4	4	0	2474	99.8
11	2	4	5	0	2743	99.9
12	2	4	6	0	3019	99.9
13	2	4	7	0	3298	99.9
14	2	4	8	0	3577	100

**TABLE A.4.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 4**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	6294	6294
1	0.6	17.6	18.2	15	5286	5301
2	1.5	14.8	16.3	38	4429	4468
3	1.7	12.2	13.9	43	3674	3716
4	2.1	10.0	12.0	52	2990	3042
5	2.7	7.8	10.6	68	2356	2424
6	3.6	5.8	9.4	90	1756	1846
7	3.8	4.0	7.8	95	1206	1301
8	4.2	2.4	6.6	105	712	816
9	5.1	1.1	6.2	127	331	458
10	6.5	0.5	7.0	163	145	308
11	8.3	0.2	8.5	207	71	278
12	10.2	0.1	10.4	255	45	300
13	12.2	0.1	12.3	304	37	341
14	14.2	0.1	14.3	354	34	388

FIGURE A.4.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 4

FIGURE A.4.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 4

FIGURE A.4.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 4

FIGURE A.4.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 4

TABLE A.4.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 4

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.0	1.5	498	99.8
2	4	3.5	0.1	3.6	1470	99.8
3	3	4.8	0.6	5.4	2204	99.6
4	0	5.1	1.1	6.2	2224	99.5

TABLE A.4.4: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 4

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.0	1.5	478	99.8
2	4	3.5	0.1	3.6	1470	99.8
3	5	6.4	0.2	6.6	2696	99.9
4	0	8.3	0.2	8.5	2743	99.9

FIGURE A.4.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 4

FIGURE A.4.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 4

TABLE A.5.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 5

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	525	91.0
1	1	0	0	0	685	92.4
2	2	0	0	0	868	93.7
3	2	1	0	0	1039	94.7
4	2	2	0	0	1221	95.7
5	2	3	0	0	1414	96.6
6	2	4	0	0	1616	97.5
7	2	5	0	0	1827	98.3
8	2	5	1	0	2026	99.0
9	2	5	2	0	2246	99.5
10	2	5	3	0	2496	99.8
11	2	5	4	0	2764	99.9
12	2	5	5	0	3040	100
13	2	5	6	0	3317	100
14	2	5	7	0	3598	100

**TABLE A.5.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 5**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	10490	10490
1	0.6	17.6	18.2	15	8811	8826
2	1.5	14.8	16.3	38	7382	7420
3	1.7	12.3	14.0	43	6123	6165
4	2.1	10.0	12.0	52	4984	5036
5	2.7	7.9	10.6	68	3927	3995
6	3.6	5.8	9.4	90	2927	3016
7	4.7	4.0	8.7	117	1994	2112
8	5.1	2.3	7.4	127	1166	1293
9	6.0	1.0	7.0	150	525	674
10	7.4	0.4	7.8	185	212	397
11	9.2	0.2	9.4	230	87	317
12	11.1	0.1	11.2	278	44	321
13	13.1	0.1	13.1	327	29	356
14	15.1	0.0	15.1	377	24	402

FIGURE A.5.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 5

FIGURE A.5.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 5

FIGURE A.5.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 5

FIGURE A.5.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 5

**TABLE A.5.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION
-- TEST 5**

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.0	1.5	498	99.8
2	5	4.4	0.0	4.5	1721	99.9
3	2	5.7	0.5	6.2	2225	99.6
4	0	6.0	1.0	7.0	2246	99.5

**TABLE A.5.4: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION
-- TEST 5**

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.0	1.5	498	99.8
2	5	4.4	0.0	4.5	1721	99.9
3	4	7.3	0.1	7.5	2717	99.9
4	0	9.2	0.2	9.4	2764	99.9

FIGURE A.5.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 5

**FIGURE A.5.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSSC"
CRITERION -- TEST 5**

TABLE A.6.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 6

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	488	91.6
1	1	0	0	0	644	93.2
2	2	0	0	0	821	94.6
3	2	1	0	0	990	95.8
4	2	2	0	0	1172	96.9
5	2	3	0	0	1371	97.8
6	2	4	0	0	1584	98.6
7	2	4	1	0	1789	99.3
8	2	4	2	0	2020	99.7
9	2	4	3	0	2276	99.9
10	2	4	4	0	2547	99.9
11	2	4	5	0	2825	100
12	2	4	6	0	3104	100
13	2	4	7	0	3384	100
14	2	4	8	0	3664	100

**TABLE A.6.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 6**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	19.5	19.5	0	9763	9763
1	0.7	15.9	16.5	16	7947	7964
2	1.8	12.7	14.5	44	6341	6385
3	2.2	9.8	12.0	55	4888	4943
4	2.9	7.2	10.0	72	3603	3675
5	3.8	5.0	8.8	95	2506	2601
6	5.0	3.2	8.2	125	1580	1705
7	5.5	1.7	7.2	136	848	984
8	6.4	0.7	7.1	161	374	535
9	7.9	0.3	8.2	199	150	348
10	9.8	0.1	9.9	244	67	311
11	11.7	0.0	11.8	293	40	333
12	13.7	0.0	13.8	342	32	374
13	15.7	0.0	15.7	392	30	422
14	17.7	0.0	17.7	442	29	471

FIGURE A.6.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 6

FIGURE A.6.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 6

FIGURE A.6.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 6

FIGURE A.6.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 6

TABLE A.6.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 6

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.1	1.5	499	99.8
2	4	4.6	0.6	4.7	1497	99.9
3	1	5.5	0.7	6.2	1764	99.5
4	0	5.5	1.7	7.2	1789	99.3

TABLE A.6.4: STAGE BY STAGE ANALYSIS OF THE OPTIMUM OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 6

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.5	0.0	1.5	499	99.8
2	4	4.6	0.0	4.7	1497	100
3	4	7.8	0.1	8.0	2499	99.9
4	0	9.8	0.1	9.9	2547	99.9

FIGURE A.6.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION -- TEST 6

FIGURE A.6.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 6

TABLE A.7.1: COMPARISON OF OPTIMUM DAILY OPEN WORK COVER POLICIES -- TEST 7

OPEN WORK COVER OPTIONS (Oper. days)	CORRESPONDING DAILY SCHEDULES				SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
	Stg 1	Stg 2	Stg 3	Stg 4		
0	0	0	0	0	424	91.0
1	1	0	0	0	682	92.5
2	2	0	0	0	861	93.9
3	2	1	0	0	1029	95.1
4	2	2	0	0	1210	96.2
5	2	3	0	0	1405	97.2
6	2	4	0	0	1612	98.1
7	2	4	1	0	1807	98.8
8	2	4	2	0	2016	99.4
9	2	4	3	0	2253	99.7
10	2	4	4	0	2514	99.9
11	2	4	5	0	2787	99.9
12	2	4	6	0	3064	100
13	2	4	7	0	3343	100
14	2	4	8	0	3623	100

**TABLE A.7.2: COMPARISON OF SURPLUS AND SHORTAGE DATA FOR
OPTIMUM OPEN WORK COVER POLICIES -- TEST 7**

OPEN WORK COVER OPTIONS (Oper. days)	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	CSURC (Dollars)	CSHC (Dollars)	CSSC (Dollars)
0	0.0	21.0	21.0	0	10475	10476
1	0.7	17.4	18.1	17	8697	8714
2	1.8	14.2	16.0	45	7112	7157
3	2.2	11.3	13.5	55	5666	5721
4	2.9	8.7	11.6	71	4372	4444
5	3.7	6.5	10.2	93	3235	3328
6	4.8	4.5	9.3	120	2237	2357
7	5.1	2.8	7.9	128	1387	1515
8	5.6	1.4	7.0	141	706	847
9	6.7	0.6	7.3	167	308	476
10	8.3	0.3	8.5	207	133	340
11	10.1	0.1	10.3	253	67	320
12	12.1	0.1	12.2	302	43	345
13	14.1	0.1	14.1	351	35	386
14	16.0	0.1	16.1	401	32	433

FIGURE A.7.1: COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 7

FIGURE A.7.2: RELIABILITY COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 7

FIGURE A.7.3: SURPLUS-SHORTAGE COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 7

FIGURE A.7.4: SURPLUS-SHORTAGE COST COMPARISON OF OPTIMUM OPEN WORK COVER POLICIES -- TEST 7

**TABLE A.7.3: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSS" CRITERION
-- TEST 7**

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.6	0.0	1.6	500	99.9
2	4	4.6	0.1	4.7	1496	99.9
3	2	5.6	0.7	6.3	1998	99.5
4	0	5.6	1.4	7.0	2016	99.4

**TABLE A.7.4: STAGE BY STAGE ANALYSIS OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION
-- TEST 7**

STAGE	DEC.	CSUR (Hrs)	CSH (Hrs)	CSS (Hrs)	SYSTEM COST (Dollars)	SYSTEM REL. (Percent)
1	2	1.6	0.0	1.6	500	99.9
2	4	4.6	0.1	4.7	1496	99.9
3	5	8.1	0.1	8.2	2734	99.9
4	0	10.1	0.1	10.2	2787	99.9

**FIGURE A.7.5: NETWORK REPRESENTATION OF THE OPTIMAL OPEN
WORK COVER POLICY ACCORDING TO THE "CSS"
CRITERION -- TEST 7**

FIGURE A.7.6: NETWORK REPRESENTATION OF THE OPTIMAL OPEN WORK COVER POLICY ACCORDING TO THE "CSSC" CRITERION -- TEST 7

Appendix B

A SAMPLE OF SERVICE AND OPERATIONAL DATA

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A SAMPLE OF ABSENTEEISM DATA AT "STO", FALL TIMETABLE, 1990

DRIVER	DATE	R/O	STATUS	CAUSE	DURATION
8283354	90-09-04	1	B	43	08:00
8283431	90-09-04	1	B	64	08:00
8283439	90-09-04	1	B	64	08:00
8283541	90-09-04	1	B	64	08:00
8283596	90-09-04	1	B	54	02:06
8284334	90-09-05	1	B	51	01:30
8284354	90-09-05	1	B	43	08:00
8284431	90-09-05	1	B	64	08:00
8284439	90-09-05	1	B	64	08:00
8284541	90-09-05	1	B	64	08:00
8284596	90-09-05	1	B	54	02:06
8285431	90-09-06	1	B	64	08:00
8285439	90-09-06	1	B	64	08:00
8285517	90-09-06	1	B	21	05:08
8285517	90-09-06	1	B	54	02:52
8285541	90-09-06	1	B	64	08:00
8285596	90-09-06	1	B	54	02:06
8286378	90-09-07	1	B	64	08:00
8286431	90-09-07	1	B	64	08:00
8286439	90-09-07	1	B	64	08:00
8286476	90-09-07	1	B	56	01:12
8286541	90-09-07	1	B	64	08:00
8286576	90-09-07	1	R	21	08:00
8289355	90-09-10	1	B	32	08:00
8289431	90-09-10	1	B	64	08:00
8289569	90-09-10	1	B	51	01:30
8289596	90-09-10	1	B	54	02:06
8290355	90-09-11	1	F	32	08:00
8290579	90-09-11	1	P	21	08:00
8290579	90-09-11	1	B	51	02:15
8290596	90-09-11	1	B	54	02:06
8291355	90-09-12	1	B	32	08:00
8291431	90-09-12	1	B	63	08:00
8291458	90-09-12	1	R	51	02:10
8291579	90-09-12	1	B	21	08:00
8291579	90-09-12	1	B	51	02:15
8291596	90-09-12	1	B	54	08:00

A SAMPLE OF OPEN WORK RECORDS AT "STO", FALL TIMETABLE, 1990

DATE	START TIME	END TIME	CAUSE	DRIVER No.	PIECE No.
90/09/03	05:40	09:30	54	3063	5
90/09/03	10:31	12:31	54	3230	27
90/09/03	12:55	19:49	54	3230	27
90/09/03	13:24	17:20	54	3063	5
90/09/03	16:20	24:17			40
90/09/03	17:20	25:07	32	3233	47
90/09/03	18:05	24:07	24	3242	51
90/09/03	24:16	26:19	24	3242	51
90/09/04	05:55	07:57			9027
90/09/04	05:05	09:12	24	468	8
90/09/04	05:47	11:58	24	3027	34
90/09/04	05:40	09:19	64	431	27
90/09/04	05:46	09:13	64	439	33
90/09/04	06:25	08:02			9007
90/09/04	06:20	09:35	24	553	78
90/09/04	06:25	10:19	24	3041	84
90/09/04	06:40	09:05	25	545	102
90/09/04	06:35	12:25	32	3055	97
90/09/04	06:06	13:44	33	376	62
90/09/04	06:18	09:13	43	354	73
90/09/04	06:10	08:06	51	3192	9006
90/09/04	06:25	08:21	51	3236	9008
90/09/04	06:28	08:03	54	3193	88
90/09/04	06:30	11:28	73	3133	90
90/09/04	06:35	10:24	73	3031	95
90/09/04	07:05	12:57	21	3164	121
90/09/04	07:10	08:48	25	3175	124
90/09/04	07:05	08:42	64	541	120
90/09/04	07:06	11:49	81	3099	123
90/09/04	12:51	19:06	25	3175	124
90/09/04	12:30	18:38	54	3189	116
90/09/04	14:55	18:25	24	3041	84
90/09/04	14:55	18:35	25	545	102
90/09/04	14:57	17:35	64	439	33
90/09/04	15:50	17:53	10	332	9022

A SAMPLE OF CANCELLED TRIP RECORDS AT "STO", FALL 1990

DATE	START TIME	END TIME	DRIVER No.	CAUSE	PIECE No.
90/09/05	06:42	07:10	3157	21	106
90/09/07	06:55	08:29			9020
90/09/10	05:31	06:45	568	10	21
90/09/10	05:55	08:10			9001
90/09/13	05:55	08:10	579	51	9001
90/09/13	06:05	08:12	3039	51	9004
90/09/13	06:32	08:30			9011
90/09/28	05:55	08:10			9001
90/10/03	06:27	08:12			9009
90/10/04	05:35	08:33	3091	21	23
90/10/05	12:54	13:30	3073	81	89
90/10/10	05:55	08:10	579	51	9001
90/10/10	07:02	07:22	3173	21	118
90/10/17	12:40	15:30	3073	32	112
90/10/17	18:46	18:55	3073	32	112
90/10/18	05:52	08:06			9006
90/10/19	18:00	18:21	3073	32	89
90/10/25	06:35	08:37	3064	54	96
90/10/26	16:20	16:30	3040	21	4
90/11/01	06:32	08:30			9011
90/11/02	19:02	19:04	541	64	120
90/11/06	06:57	08:42	3073	81	112
90/11/12	06:35	08:05	569	51	9013

**DAILY SERVICE SCHEDULES (SCHEDULED RUNS) AT "STO",
FALL TIMETABLE, 1990**

WEEK DAY		SATURDAY		SUNDAY	
Departure Time	Sch. Runs	Departure Time	Sch. Runs	Departure Time	Sch. Runs
4:00	3	4:30	1	5:00	2
4:30	3	5:00	3	5:30	5
5:00	13	5:30	6	6:00	6
5:30	29	6:00	14	6:30	7
6:00	40	6:30	6	7:00	1
6:30	24	7:00	4	8:30	1
7:00	13	8:00	1	9:00	2
10:30	4	8:30	3	9:30	1
11:00	8	9:00	1	10:00	1
11:30	2	9:30	1	10:30	1
12:00	2	10:00	3	11:30	2
14:30	1	10:30	5	12:00	1
15:00	4	11:00	1	13:30	1
15:30	6	12:00	2	14:00	4
16:00	4	12:30	1	14:30	1
16:30	4	13:00	1	15:30	2
17:00	6	13:30	1	16:00	2
17:30	2	14:00	3	16:30	4
		14:30	1	17:00	5
		15:30	1	17:30	1
		16:00	4	18:00	1
		16:30	4		
		17:00	8		
		18:00	1		
TOTAL	168		76		51

**A TYPICAL WEEK DAY SCHEDULE OF TRIPPERS AT "STO",
FALL TIMETABLE, 1990**

DEPARTURE TIME	ARRIVAL TIME	DURATION
5:55	8:10	2H15
5:55	7:57	2H02
5:57	8:07	2H10
5:57	8:07	2H10
6:00	8:10	2H10
6:05	8:11	2H06
6:05	8:12	2H07
6:05	8:28	2H23
6:10	8:06	1H56
6:25	8:02	1H37
6:25	8:21	1H56
6:27	8:12	1:45
6:30	8:40	2H10
6:32	8:30	1H58
6:33	8:03	1H30
6:35	8:05	1H30
6:35	8:15	1H40
6:40	8:13	1H33
6:42	8:27	1H45
6:45	8:22	1H37
6:53	8:28	1H35
6:55	8:29	1H34
6:55	8:26	1H31
6:58	8:23	1H25
15:30	17:33	2H03
15:50	17:53	2H03
15:55	17:45	1H50
16:00	18:03	2H03
16:05	17:35	1H30
20:25	21:54	1H29
20:45	21:55	1H10
20:45	21:23	1H38

**DAILY REPORT TIMES OF EXTRA-BOARD OPERATORS AT "STO",
FALL TIMETABLE, 1990**

MONDAY	TUESDAY	WEDNESDA	THURSDAY	FRIDAY	SATURDAY	SUNDAY
04:30	04:30	04:30	04:30	04:30	06:30	07:00
04:55	04:55	04:55	04:55	04:55	07:30	PM01
05:10	05:10	05:10	05:10	05:10	PM01	PM02
05:25	05:25	05:25	05:25	05:25	PM02	PM03
05:50	05:50	05:50	05:50	05:50	PM03	PM04
06:00	06:00	06:00	06:00	06:00	PM04	PM05
06:02	06:02	06:02	06:02	06:02	PM05	PM06
06:04	06:04	06:04	06:04	06:04	PM06	PM07
06:05	06:05	06:05	06:05	06:05	PM07	PM08
06:15	06:15	06:15	06:15	06:15	PM08	
06:20	06:20	06:20	06:20	06:20	PM09	
06:25	06:25	06:30	06:25	06:25		
06:30	06:30	06:35	06:30	06:30		
06:35	06:35	06:40	06:45	06:40		
06:40	06:40	06:45	06:50	06:50		
06:45	06:45	06:50	07:05	07:05		
06:50	06:50	07:05	07:15	07:15		
06:55	07:05	07:15	07:20	07:20		
07:00	07:15	07:20	07:25	09:00		
07:05	07:20	09:00	07:30	PM01		
07:15	09:00	PM01	07:35	PM02		
07:20	PM01	PM02	09:00	PM03		
07:25	PM02	PM03	PM01	PM04		
07:30	PM03	PM04	PM02	PM05		
07:35	PM04	PM05	PM03	PM06		
07:40	PM05	PM06	PM04	PM07		
09:00	PM06	PM07	PM05	PM08		
PM01	PM07	PM08	PM06	PM09		
PM02		PM09	PM07			
PM03			PM08			
PM04			PM09			
PM05			PM10			
PM06						
PM07						
PM08						

Appendix C

A BRIEF DECRYPTION OF THE COMPUTER PROGRAM

This is a brief description of the computer program used to compute the optimum open work cover policies presented in the Case Study.

In addition to the input, initialization and output procedures, this computer program consists of two main modules. The first module (FDC) is used to compute the surplus, shortage, cost and reliability values associated with alternative decisions. The surplus, shortage, cost and reliability levels associated with a particular decision-state-stage combination include, as well, those same values for the minimum cumulative surplus-shortage cost policy of the input state to this decision, as was explained in Section 3.2.2.1.

The second module compares the range of decisions leading from various input states to a particular state-stage combination, and selects the decision which minimizes the cumulative surplus-shortage costs for the set of assignment decisions (which include the assignment decision to this stage) over all stages up to and including this stage.

The flow chart on the next page presents in a diagrammatic form the structure of the computer program and the interrelationships among its various procedures. The following key explains the terms used in this diagram.

$D_{i,s,t}$: A record of surplus, shortage, cost and reliability values for a particular decision and its associated cover policy.

FLOW CHART

$DC_{i,s,t}$: Cumulative surplus-shortage costs associated with a particular decision.

S_t : A record of surplus, shortage, cost and reliability values for the minimum cumulative surplus-shortage cost policy associated with a particular state-stage combination.

SC_t : Cumulative surplus-shortage costs associated with a particular state-stage combination.

i : Magnitude of decision, or the number of operator days assigned to a particular stage.

t : Magnitude of state, or cover level for the planning (study) period.

S : State limit, the maximum cover level considered for the planning period.

T : Number of stages (time periods) in the study period.

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