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Pinhole Single Photon Emission Computed Tomography

by

Tanya A. Hewitt

**A thesis submitted to the Department of Physics
in conformity with the requirements for
the degree of Master of Science**

**Ottawa-Carleton Institute for Physics
Department of Physics
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**The undersigned recommend to the Faculty of Graduate Studies and Research
acceptance of this thesis.**

Pinhole Single Photon Computed Tomography

Submitted by Tanya A. Hewitt, B.Sc.

**in partial fulfillment of the requirements
for the degree of Master of Science**

Dr. Barry McKee

Thesis Supervisor

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Date: 12th May 99

Abstract

Pinhole single photon emission computed tomography might be an advantageous technique to image objects (i.e. thyroid glands) that could benefit from the magnification gained from divergent ray geometry. This study assessed the response of two analytic reconstruction algorithms (Feldkamp – FDK - and Discrete Vertex Set - DVS) on data that were acquired using full-circle and half-circle acquisition geometry. Point, line and thyroid phantoms were imaged.

The reconstructions are encouraging – in most cases, the shapes, intensities and positions of the phantoms are preserved in their reconstructions. The reconstructions from the half-circle acquisitions show different artefacts (tails and undershoots) for the FDK and the DVS algorithms. The average resolution measured at different points on the axes is 5.3 ± 0.6 mm, the uniformity in most cases deviates by <10%, and cold spots in the thyroid phantom are identifiable in all reconstructions. This study shows promise for both the full- and half-circle pinhole SPECT techniques.

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Acronyms

COR – centre of rotation

DVS – Discrete Vertex Set reconstruction algorithm written by Fred Noo

**FDK – conebeam reconstruction algorithm written by Feldkamp, Davis and
Kress**

FOV – field of view

FWHM – full width at half maximum

HVL – half value layer

LRF – line response function

MRI – magnetic resonance imaging

PRF – point response function

QC – quality control

ROI – region of interest

SNR - signal to noise ratio

SPECT – single photon emission computed tomography

CHAPTER 1 INTRODUCTION

INTRODUCTION TO NUCLEAR MEDICINE

Medical imaging is an increasingly high profile aspect of health care; most people are familiar with modern x-ray, ultrasound or MRI machines. There is another player on the medical imaging team, who has a lower profile than the aforementioned, yet is no less impressive in its ability to diagnose disease. This imaging modality is nuclear medicine.

Most people are familiar with the idea of an anatomical image, be it from an x-ray film or a digital MRI image. The image is fairly easy for the layman to identify, as he will recognise familiar anatomical landmarks and be able to orient himself accordingly. Although one may need more training to recognise anatomy on an ultrasound image, it too is a modality focused principally upon getting a picture of the structures inside the body. Nuclear medicine, although a form of medical imaging, does not take these types of images.

The role nuclear medicine plays in this field is a complementary one to the more familiar anatomical imaging modalities. It can trace the *functionality* of an organ. An analogy to help explain the utility of nuclear medicine is the relationship of a heat-sensing image to an aerial photograph of a subdivision. An architect would be interested in obtaining various aerial photographs in order to know the structure of the houses. A fibreglass insulation vendor would be more interested in a heat-sensing image that could tell him how well the houses can

retain (or lose) heat, even though such an image would not have the structural detail of the aerial photograph. These images could not only identify which houses have a heat loss problem, but also the severity of the problem. A heat-sensing image is a functional image that can provide valuable information that simply cannot be obtained from a structural image. Such is the case with nuclear medicine imaging. The anatomical structures are not as well defined as with x-ray or MRI. However, the information obtained is functional.

The first stage in obtaining a nuclear medicine image is to introduce the radionuclide – an element that emits γ (gamma) rays due to an unstable nucleus. The radionuclide is most commonly injected into a patient, although it can be inhaled. The most common radionuclide used in nuclear medicine imaging is Tc-99m, which is a daughter of Mo-99. Other radionuclides include Tl-201, I-131, and Ga-67. The radionuclide is usually attached to some other molecule, called a vehicle, which can be used to target a particular organ for isolated study. The radionuclide and vehicle together are called the radiopharmaceutical, and there is a variety of radiopharmaceuticals in a nuclear medicine department. A different radiopharmaceutical is given for measuring kidney excretion, bone growth, blood flow, lung perfusion and ventilation, or thyroid function. In this way, a patient imaged for a gastric emptying study would be injected with a radiopharmaceutical whose vehicle targets the stomach, whereas a patient whose liver is under study would be injected with one whose vehicle targets the liver. Although these two organs are in proximity, the uniqueness of the target

localises the radionuclide differently, resulting in two very different looking scans.

As mentioned, there is a variety of scans that can be performed in nuclear medicine, even using the same radionuclide. They typically fall under three general image categories: planar (static), dynamic or tomographic. A planar image is the easiest of the three – the patient's (already radioactive) target organ is put in the field of view (FOV) of the camera, and the technologist acquires either a set number of counts (γ -ray interactions) or acquires for a set amount of time. Although only one image could constitute a static study, typically several planar views of the same organ are taken. For a dynamic image (for example kidney excretion), the camera is appropriately oriented, and the technologist simultaneously injects the radiopharmaceutical and begins the acquisition protocol, which consists of many time frames of acquisition. The time frames are often short at the beginning of the study and become longer as time progresses. The overall result is a set of images which can be played in a movie format (similar to a flipbook), and the path of the radiopharmaceutical can be traced over time.

The tomographic study is called SPECT, an acronym for Single Photon Emission Computed Tomography. Only a brief overview will be given here; much more detail is in the rest of the thesis. SPECT is fundamentally different from planar and dynamic studies, both of whose images are two-dimensional. The goal of a SPECT study is to generate a three-dimensional image. The

SPECT study is the result of an image reconstruction using as data a number of planar images taken in a circle around the object. This set of planar images (called projection images, or projections) is fed to a reconstruction algorithm, which uses all the planar images from the different camera positions to reconstruct the final image.

Although a SPECT study can be viewed as a three-dimensional surface rendition of an object or organ, this mode of viewing would give information on only the *surface* of the organ, whereas the physician is most interested in the uptake of a radiopharmaceutical *within* the organ. Thus, the three dimensional reconstruction of the organ is displayed as slices through the reconstruction, producing one of three sets of standard slices (coronal, sagittal or transverse).

All nuclear medicine studies are taken with a gamma camera, which is essentially a large thin crystal with photomultiplier tubes behind. Any gamma camera that can do SPECT studies must have rotational capability. In addition to the basic camera, one needs a collimator on top of the crystal, to help define the direction of detected radiation. The most common clinical SPECT studies employ the use of a parallel hole collimator, but this is not the only choice. There are various types of collimators available, and thus there are various opportunities for SPECT reconstruction. The focus of this thesis is on SPECT using a pinhole collimator, or pinhole SPECT.

1.2 PREVIOUS WORK DONE IN PINHOLE SPECT

The main advantage of using a pinhole collimator is to increase the magnification in the image, which serves to increase the resolution. Pinhole collimators were in use in the early days of gamma cameras. Hal O. Anger, the inventor of the gamma camera (otherwise known as an Anger camera) discussed the resolution and sensitivity properties of a pinhole collimator, which he foresaw being used for planar thyroid scans (Anger 1967). Planar pinhole imaging has involved the use of multi-pinhole collimators (Anger 1967, Vogel *et al* 1978), but is presently popular using one pinhole for planar views of the thyroid gland.

The idea of using a rotating gamma camera to collect and reconstruct divergent-ray projection images was published by Ronald J. Jaszczak *et al* in 1979, using multi-slice fanbeam collimators (Jaszczak *et al* 1979). This is probably the earliest reference to non-parallel hole SPECT. Naturally, the algorithms for reconstruction from divergent ray projection images (also called projections) are different from the conventional parallel hole SPECT algorithms – the conebeam reconstruction algorithm written by Feldkamp, Davis and Kress (referred to as the “FDK” algorithm in this thesis) is the earliest and the most popular (Feldkamp *et al* 1984). (In two relatively recent publications – Olsson *et al* 1990, Bahk *et al* 1998 – the authors used standard parallel ray SPECT algorithms to reconstruct from divergent ray projection images with little success). Jaszczak also used a conebeam collimator, to get a greater sensitivity advantage over a parallel hole SPECT system (Jaszczak *et al* 1986) – the reconstruction was

done using the FDK algorithm. Soon thereafter, a Swedish paper was published introducing a novel way to reconstruct objects from projection images collected with a pinhole collimator, called “divergent backprojection by successive normalised magnification” (Palmer and Wollmer, 1990). Each projection image was filtered and interpolated with many different magnification factors. The algorithm was excessively slow, however (3.5 h reconstruction time for 16 slices), and subsequent research in this direction was not found in the literature.

The group led by Jaszczak at Duke University became interested in ultra-high resolution, small-field-of-view pinhole SPECT (Jaszczak *et al* 1993). This system was quite sensitive to systematic error, and effort was expended upon centre-of-rotation correction modifications to both their filtered backprojection (modified FDK) and iterative reconstruction algorithms (Li *et al* 1994, Li *et al* 1995). A very good comprehensive paper was published in 1994 that summarises the experimental progress at the time, has many relevant formulae, comments and caveats concerning pinhole SPECT (Jaszczak *et al* 1994). It also included the small animal work of the group. The pinhole correction factor was another significant hurdle to such a high-resolution camera, which was investigated using different sized pinholes (Li *et al* 1995).

Other groups became interested in pinhole SPECT for small animal imaging (Erlandsson *et al* 1993, Weber *et al* 1993). In spite of the primary focus being the animal images, much attention was still paid to the FDK reconstruction algorithm and the various corrections that must be done for pinhole SPECT. Multi-

pinhole collimator SPECT was attempted by a Japanese group with encouraging results, although not much has been published on this technique since (Ischizu *et al* 1993, Ischizu *et al* 1995, Weber and Ivanovic 1995). Practical animal studies were also undertaken, from imaging tumors (Strand *et al* 1993), radiolabelling monoclonal antibodies (Strand *et al* 1993) to imaging myocardial infarcts (Yukihiro *et al* 1996).

The focus more recently has been in applying pinhole SPECT to patient imaging. A Belgian group began patient thyroid imaging using either a full or half circle of acquisition, with a tilted detector to get as close as possible to the thyroid, using the FDK algorithm (Wanet *et al* 1994). A year later, they reported on their clinical practice, and a second Belgian group did a quantitative study of nodule detection compared with echography (Wanet *et al* 1995, Stegen *et al* 1995). A clinical trial was published by the pioneer Belgian group, including thyroid phantom images, all with half a circle of acquisition (Wanet *et al* 1996). This group provided the reconstruction software to the nuclear medicine vendor Elscint, which has since been offering cameras with optional pinhole SPECT reconstruction capability. Other Elscint collaborators also published their clinical thyroid SPECT findings (Krausz *et al* 1997). The group at Duke University meanwhile assessed a breast phantom using pinhole SPECT (Scarfone *et al* 1997), and has since focused their attention on higher energy pinhole imaging. A French group became interested in applying their iterative algorithm with constraints, and chose to do thyroid phantoms and a clinical trial of thyroid nodule

sizing (Laurette *et al* 1998, Laurette *et al* in press). Vanhove *et al* (1998) have used an ordered subsets expectation maximization (OSEM) algorithm, imaging a thyroid phantom and a cervical spine phantom.

1.3 THIS PROJECT IN PERSPECTIVE

Although the field of pinhole tomography seems well studied, there appears to have been a lack of thorough characterisation of pinhole reconstruction. Jaszczak's group at Duke University did some characterisation in terms of sensitivity and resolution of their ultra-high resolution scanner, but most of the clinical trials did not seem to have similar endeavours prior to attempting patient scans from the pinhole SPECT technique. There is little attention paid to the reconstruction artefacts one might expect to encounter in pinhole SPECT reconstruction, which is of great importance to the nuclear medicine physician reading a scan. This is especially true in light of the acquisition being only half a circle (180°) for the thyroid gland, as one might expect some significant differences between a half and a full circle of acquisition.

As such, the aim of this project was to try to fill this apparent gap in the literature. Thorough characterisation of various phantoms was attempted, in order to have a good idea of the advantages and limitations of half-circle pinhole SPECT. Two analytic algorithms were used in this study. The algorithm by Feldkamp, Davis and Kress is the pioneering conebeam reconstruction algorithm, which most groups who have published in the field have used. The FDK

program used in this project was coded by Michel Defrise, a medical physicist at the Free University of Brussels in Belgium. The Discrete Vertex Set (in this thesis referred to as DVS) algorithm is the result of a Ph.D. project by his student Fred Noo (Noo 1998), which takes advantage of the decade of mathematical understanding since the FDK algorithm. Noo both derived the concept of the DVS algorithm and coded the special version of the program (reconstructing from circular acquisition paths) that was made available for use in this project.

The sample holder used was constructed by Jim Sliwka, a machinist in the Physics Department at Carleton University. Bob Doré, a machinist in the Bio-medical Engineering department at the Ottawa Hospital, Civic Campus, provided the laser mount and disk phantom.

Thus, the answers to two questions were sought.

1. What (if any) are the disadvantages, as seen in the reconstructions, from data that has been acquired with a half circle (180°) as compared to a full circle (360°) of projection images?
2. Is the newer DVS algorithm superior to the classic FDK algorithm?

As mentioned, phantoms were used as the source of radiation, and most of the data from these phantoms were reconstructed four ways – using one of the two reconstruction algorithms, and one of the two acquisition geometries. In this way, problems unique to a certain algorithm or a certain acquisition geometry could be elucidated.

The thesis proceeds by discussing the properties unique to a planar pinhole image in Chapter 2 complete with theory, methods, results and discussion. The bulk of the thesis is in reconstructed (tomographic) images – Chapter 3 - again self contained with its own theory, methods, results and discussion. A short conclusion finishes the thesis.

Some of the work in this thesis has been presented at three conferences (McKee *et al* 1997, Hewitt *et al* 1997, Hewitt *et al* 1998), and a paper is in press (Hewitt *et al*, 1999).

CHAPTER 2 PROJECTION IMAGES

2.1 THEORY

Projection images are planar images, which are typically taken around a circle. A single planar image is formed from the interaction of gamma rays (emitted from the patient) in the crystal of a gamma camera with angular acceptance restricted by a collimator. The planar image thus records the distribution of radiopharmaceutical in the FOV of the camera.

The patient is first injected with a radiopharmaceutical, which then distributes itself (according to its function) appropriately within the body. When the radionuclide decays, it emits a gamma ray, which can be in any direction from the point of emission, as depicted in **Figure 1**.

Figure 1. Depiction of a liver within a patient as the subject of a nuclear medicine study. The liver is the target of a radiopharmaceutical, which is emitting gamma rays in all directions.

In order to form an image from the isotropic emission of gamma rays, a collimator is put on the surface of the gamma camera crystal, which allows only

select gamma rays to enter dependent upon their direction of travel. The equation expressing the projection image from a parallel hole collimator is

$$R(u, v) = \int_{-\infty}^{\infty} f(x, y, z) dz \quad \text{Equation 1}$$

where $f(x, y, z)$ is the distribution of activity in the patient. This is depicted in **Figure 2**.

Figure 2. A gamma ray (red arrow) travels through the liver and the rest of the patient's body, reaches the detector and adds to the distribution of gamma ray counts from the liver. Shown is a y-z slice through the liver, and a slice through the projection R. The v co-ordinate of the projection image is perpendicular to the plane shown.

$R(u, v)$ denotes the projection image, using the projection co-ordinates (u, v)

This is a line integral through z , which is perpendicular to the surface of the detector. That is, there is no way to distinguish the z dimension in this projection image – all the planes through the patient with z as their normal are superimposed in this projection image.

Ideally, the projection image reflects the detection of only those gamma rays that travel directly from their emission location to the detector face. However, gamma rays might deviate from their original paths due to interactions with dense media. Attenuation is the process describing signal loss (in general) due to both absorption and scattering. In nuclear medicine imaging, the absorption coefficient of tissue at the energy of the gamma rays is such that the gamma rays are rarely absorbed; scatter is much more common. One refers to attenuation as the reduction in the number of gamma rays detected due to scatter directed away from the detector, and to scatter as the detection of gamma rays scattered towards the detector for which one does not know the origin. As such, attenuation leads to an image for which one does not know the true number of gamma rays emitted in the direction of the detector. Scatter can skew the distribution of detected true events in the image, as forward scattered events can count as true events, due to the energy resolution of a gamma camera being limited to about 10%. There are analytic formalisms and iterative methods that attempt to correct for attenuation and scatter in the data. However, because of their size, the phantoms used in this project were not subject to much attenuation or scatter, and thus neither correction was performed.

In SPECT acquisition, the camera rotates, but stops at specified angles to collect gamma rays (counts) which form the projection images – this type of acquisition is called step-and-shoot mode. Projection images comprise the data that are fed to the reconstruction algorithm to form a three dimensional image.

The collimator used will have specific geometric response characteristics, affecting the intensity (also known as sensitivity or efficiency) and resolution. These response characteristics are different for different types of collimators; described below are those unique to a pinhole collimator.

2.1.1 PINHOLE COLLIMATORS

A pinhole collimator on a gamma camera crystal acts similarly to a pinhole in a shoe box camera – the object is both magnified and inverted in the final image, as seen in **Figure 3**.

Figure 3. Depiction of a radioactive line and point being imaged using a pinhole collimator, where an inversion and magnification of the objects take place in the image (incident upon the crystal). The diameter of the pinhole aperture is d , the diameter of the base of the collimator is D and the aperture to base distance is a , all of which are fixed distances. The aperture to object distance is b , which is variable. The triangle underneath the pinhole collimator represents a cross-section of the imaged volume, while its base is the FOV at the magnification chosen. The magnification is a / b .

The distance b is the distance between the centre of the pinhole to the centre of the Field of View (FOV). The FOV is a circle at the base of the cone ex-

tending from the pinhole. For a single static projection, anything in the FOV can be detected through the pinhole. If the object is moved closer to the pinhole (b decreases), the image will be magnified even more, although the FOV will get correspondingly smaller. As the object is moved further away (b increases), the reverse will be true. The distance between the object and the pinhole also affects two important properties of the image: resolution and intensity.

2.1.1.1 Resolution and Geometrical Efficiency

Resolution

The resolution of a camera refers to its ability to distinguish two objects in an image (to *resolve* the objects in the image). It can be calculated by imagining two point sources in the object plane, distance b from the pinhole, and their resultant “disks” in the image plane being just tangent to one another.

Figure 4. Method to find the resolution of a pinhole collimator via similar triangles. R_g is the distance in the object plane between two point sources. The disks are images of the points, spatially localised by Anger logic.

One can see from **Figure 4** that the triangle with base d and the triangle with base Rg are similar, thus

$$\frac{Rg}{d} = \frac{a+b}{a}$$

$$Rg = \frac{d(a+b)}{a}$$

Equation 2.

The geometric resolution of a pinhole collimator is a linear function of b ; the closer the object is to the pinhole (b small), the closer points can get in the object plane to make their disks tangent in the image plane, and the better the resolution. Conversely, the resolution degrades as the objects are taken further away from the pinhole (b large).

Geometrical Efficiency

The geometrical efficiency (sensitivity) of a collimator is the largest factor in determining the intensity of an image.

Figure 5. Finding the geometrical efficiency of a pinhole collimator.

Radioactive objects emit radiation isotropically. Thus, at a distance b away

from the object, one could envision a spherical shell of radiation surrounding the object, area $4\pi b^2$ (see **Figure 5**). However, the pinhole has area $\pi(\frac{d}{2})^2$. The fraction of the sphere of radiation that the pinhole lets through is the *on-axis geometric efficiency*:

$$\text{on-axis geometric efficiency} = \frac{\pi\left(\frac{d}{2}\right)^2}{4\pi b^2} = \frac{d^2}{16b^2} \quad \text{Equation 3}$$

Equation 3 is valid if $b > d / 2 \tan \alpha$, where α is half the cone angle of the pinhole collimator. With the efficiency of a pinhole collimator being proportional to $1 / b^2$, it is very sensitive to the distance the object is away from the pinhole. Very close to the pinhole, the efficiency is very high, but drops off very quickly as the distance b increases.

Resolution and Efficiency

The effect of both geometric resolution and efficiency changing with distance from the pinhole collimator in accordance with equations 2 and 3 is shown in **Figure 6**.

It is obvious that the distance of the object from the pinhole is critical in determining the intensity of the image – the higher the efficiency, the more counts detected. Resolution is also affected, although not as severely. It would seem advantageous to image as close to the pinhole as possible; however, there is a trade-off because at a small distance from the pinhole the available FOV (section 2.1.1) becomes very small.

One important note concerning a divergent ray collimator (such as a pinhole collimator) is that the response in the projection image is shift variant. That is, the response (such as the image intensity) is dependent upon where the object is, making the interpretation of the image more difficult. In a shift-invariant system (a gamma camera with a parallel hole collimator approximates one), the

Figure 6. Theoretical resolution and geometric efficiency of a pinhole collimator, 3.175 mm diameter pinhole, varying the distance from the pinhole.

response is position independent, and the interpretation is consequently simpler.

2.1.1.2 Planar Variation

All that has been presented in section 2.1.1.1 is specific to activity directly in line with the pinhole. Naturally, most of the activity imaged will be distributed throughout the FOV. The distance between any point in the FOV (other than the centre) and the pinhole is *greater* than b .

From **Figure 7**, it is clear that r , being the hypotenuse of the triangle with orthogonal sides l and b , is larger than b by a factor of $1 / \cos\psi$, and thus the geometrical efficiency changes from that in Equation 3. The radius of the sphere of radiation in this case is r . In addition, the pinhole aperture, which looks circular if in line with the pinhole, will have one of its dimensions reduced when looking at it off axis. What was d becomes $d\cos\psi$. Thus the general expression for the pinhole geometric efficiency including off axis response is

$$\text{pinhole geometric efficiency} = \frac{d(d \cos \psi)}{16r^2} = \frac{d^2 \cos \psi}{16\left(\frac{b}{\cos \psi}\right)^2} = \frac{d^2 \cos^3 \psi}{16b^2} \quad \text{Equation 4.}$$

Figure 7. The non-centred pinhole response. A point is offset from the centre by a distance l , its angle of deviation from the normal of the pinhole is ψ , and its distance to the pinhole becomes r . As well, the effective aperture is decreased by a factor of $\cos\psi$.

The efficiency for activity located in the FOV is decreased, compared to the activity at the centre of the FOV, by a factor of $\cos^3\psi$. This geometric efficiency

describes the fraction of activity from a point source that reaches the detector through the pinhole.

If the detector is divided into square pixels and the object into layers of voxels, the probability P_{pv} of activity in a layer of voxels to be mapped to a pixel on the detector can be evaluated. From **Figure 8**, it is seen that P_{pv} is independent of ψ and proportional to b^2 / a^2 , the reduced area of the pixel at the perpendicular distance b . Thus, the pixel / voxel response is

$$\text{pixel / voxel response} = \frac{d^2 \cos^3 \psi}{16b^2} P_{pv} = \frac{a^2 d^2 \cos^3 \psi}{16} \quad \text{Equation 5}$$

Figure 8. The mapping P_{pv} from a pixel Δu on the detector to a row of voxels in the object. The larger triangle formed between a pixel on the detector plane and the pinhole is similar to the smaller triangle formed by the portion of the object in a voxel the pixel detects and the pinhole. The base of the smaller triangle is just the pixel extent divided by the magnification, a / b . The probability of activity in this layer of voxels to be mapped to the pixel is thus proportional to b^2 / a^2 .

2.1.2 SYSTEM RESOLUTION

As seen in section 2.1.1.1, the pinhole collimator affects the resolution of the

image detected. There is another significant component to the detected resolution: the intrinsic resolution of the gamma camera. To detect a gamma ray, it must enter the crystal and initiate a scintillation, which is in turn amplified by the photomultiplier tubes at the back of the crystal. This process of detection will inherently degrade the resolution of the object being imaged. These two independent resolution-blurring functions together result in a system resolution that can be measured on the detector plane.

2.1.2.1 Intrinsic Resolution

The intrinsic resolution (the resolution of the camera without a collimator) is modelled as a Gaussian function,

$$f(x) = Ae^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \quad \text{Equation 6}$$

where A is the amplitude, $\bar{x}$ is the mean, and σ is the standard deviation of the distribution.

To measure this for a gamma camera, one needs to choose the source carefully. A single point source in front of a crystal would simply flood the crystal with counts, while using a typical collimator would not result in a measurement of the pure intrinsic response.

2.1.2.2 Modelling the Pinhole Collimator (Geometric) Resolution

The pinhole is a finite circle in a tapered metal insert (sintered tungsten).

The shape of a point imaged through a pinhole aperture on the detector plane (assuming no penetration through the sides of the pinhole) is a disk (see 2.1.1.1). This disk, which is magnified on the detector plane, characterises the geometric resolution of the collimator. It can be represented as a two dimensional circ function, essentially a disk with unit intensity. The circ is defined as

$$\begin{aligned} f(r) &= \{1 \text{ for } |r| < r_o\} \\ &= \{0 \text{ for } |r| > r_o\} \end{aligned} \quad \text{Equation 7}$$

where r_o is the radius of the pinhole projected onto the detector plane.

It has been stated that the overall system resolution of a gamma camera with a pinhole collimator is the quadrature sum of the two resolutions (Anger 1967, Strand *et al* 1993, Jaszczak *et al* 1994). In the latter reference, Jaszczak mentions that this is not accurate, since this is true only if the two resolutions are Gaussian functions, which the pinhole aperture resolution is not. The most accurate method to model the overall system resolution is to convolve the intrinsic resolution with the geometric resolution. However, since we are interested in the width of the overall response, finding its variance is sufficient. Since variance is additive under convolution (Bracewell 1986) and the variance of the intrinsic resolution function is known (given the standard deviation output from the fit (sections 2.1.2.1 and 3.2.4.2)), we need to find the variance of the collimator resolution function - the circ function, defined in Equation 7. The formula for finding the variance σ^2 of a two-dimensional function $f(x,y)$ about the x axis is Equation 8.

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 f(x, y) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy}$$

Equation 8

For the circ function defined in Equation 7, this works out to

$$\sigma^2 = \frac{r_0^2}{4}$$

Equation 9

With this calculation and the measurement of the intrinsic resolution variance, one should be able to predict the measured system variance, enabling a prediction of the system resolution as a FWHM.

2.2 METHODS

2.2.1 ORIENTATION

The orientation defined in **Figure 9** will be used throughout this thesis.

Figure 9. Orientation. The gamma camera is depicted at left, the head orbiting the axis of rotation. At right are shown the circle of acquisition with degree markings (blue), and the three orthogonal axes.

The three orthogonal axes are depicted in the right of **Figure 9**. These represent the following:

- 1 – Axial direction (along the axis of rotation)

- 2 – Horizontal Transverse direction (from wall to wall)
- 3 – Vertical Transverse direction (from floor to ceiling)

2.2.2 PINHOLE RESPONSE

A disk (8.8 cm inner diameter, 10.1 cm outer diameter, 0.8 cm inner height, 1.4 cm total height, filling port on the perimeter) was used to measure the planar response variation. It was filled with Tc-99m, and placed so that the normal of the disk was pointing into the pinhole (magnification = 2.1 - see **Figure 4**; $a = 210$ mm, $b = 100$ mm) when the camera was at the 0° position. The matrix size was 128×128 , 3.07 mm / pixel side. One million counts were collected.

Activity whose distance to the pinhole changes throughout a tomographic study (any activity not along the axis of rotation) will be subject to the large geometrical efficiency variation of a pinhole collimator (section 2.1.1.1).

To investigate this intensity variation, a point (a small pom-pom soaked with Tc-99m activity) was placed 3 cm offset from the centre, along the horizontal transverse axis. The magnification was set at 2.1. Projection images were taken at 2.8° increments around a circle, 128 in all. A region of interest (ROI) was then drawn on selected projection images, capturing all the counts detected from the point source. These total counts were then plotted against projection angle.

2.2.3 INTRINSIC RESOLUTION

As mentioned in section 2.1.2.1, the source to measure the intrinsic resolution of a gamma camera must be carefully chosen. Nuclear Diagnostics, a Swedish company providing the software used clinically in the Nuclear Medicine department, supplied a linearity phantom. It is a circular mask of lead with precisely machined 1 mm wide parallel slits (through the 3mm thick lead) 15 mm apart. It is used for making linearity maps on older cameras. It was placed as close as possible to the crystal of the gamma camera. Using a source of nearly parallel gamma rays (a point source high above), the narrow lines were imaged (no gamma rays penetrated the lead between the slits). This was done with the slits parallel to first the horizontal transverse axis and then the axis of rotation in two separate scans. Count profiles (shown in **Figure 31**) were then generated across the images, and the central profile (from the middle of the FOV) was extracted. The profile in the u direction was fitted with a Gaussian model, and the variance was noted. A similar calculation was done for the v direction, and the overall intrinsic variance is the average of the variances in the u and v directions. This provides a good approximation to the intrinsic resolution of the camera because the 1 mm lines are narrow when compared with the intrinsic resolution.

2.2.4 SYSTEM RESOLUTION

A point was imaged in the centre of the FOV with magnification 2.1 on a

256 x 256 projection plane (1.54 mm / pixel side) to ascertain the empirical system resolution projected on the detector plane. The variance in the u direction was calculated using Equation 10, after summing 15 adjacent rows of pixels surrounding the point to yield $f(u)$.

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} u^2 f(u) du}{\int_{-\infty}^{\infty} f(u) du} \quad \text{Equation 10}$$

A similar calculation was done in the v direction. The variances of the two directions were then averaged together to yield a total system variance. This was compared with the modelled system resolution, which is the sum of the measured variance of the intrinsic resolution and the computed variance of the pinhole aperture function (section 2.1.2.2).

2.3 RESULTS

2.3.1 PINHOLE RESPONSE

The count profile taken of the disk imaged with the gamma camera head at 0° is shown in **Figure 10**.

There is a trend to the data suggesting a decrease away from the centre. The solid curve is the result of a model fit of A and n using Equation 11.

$$A \cos(\arctan(\frac{x}{y}))^n \quad \text{Equation 11}$$

In Equation 11, A is the amplitude, x is the distance across the imaged disk (on the projection plane), y is the fixed distance from the pinhole to the crystal (204 mm), and n is the power to which the cosine term is raised. (Note that $\arctan(x/y)$ is the angle ψ described in section 2.1.1.2.) The curve fit gave a value of $n = 2.9 \pm 0.2$, in agreement with the predicted $\cos^3\psi$ response.

Figure 10. A count profile across the diameter of the disk and the fitted curve. The data are Poisson distributed; the error bars are $\sqrt{N}$.

The intensity variation of an off-centred point over 360° of projection is shown in **Figure 11**.

The model and data agree well, illustrating the large variation in intensity for the same isolated off axis point in a full circle of projection images. This illustrates the very large dependence of the image intensity on the object's distance from the pinhole, one of the challenges for a divergent ray reconstruction algorithm.

Figure 11. Plot of the number of normalised counts taken of a point offset 3 cm perpendicular to the axis of rotation, and a $\cos^3 \psi$ model, where r, ψ are changing continually with projection angle. The Poisson error bars on the data are smaller than the size of the points.

2.3.2 RESOLUTION

The intrinsic variance was found as described in section 2.2.3. The calculated variance of the pinhole aperture function was found using the radius of the disk ($r_o = 4.92$ mm) projected on the detector plane in Equation 9. The measured system variance was calculated as described in section 2.2.4. The results of the system resolution are shown in Table 1.

Modelled			
Intrinsic (Measured)	Pinhole Aperture (calculated)	Sum of Intrinsic and Aperture	
7.5	6.1	13.6	
			13.2

Table 1. Comparing the modelled and empirical system variances. All numbers are in units of mm^2 . The left three columns are involved in the calculation of the modelled variance, while the right column is the calculated variance of the measured point's projection image.

In comparing the measured variance of 13.2 mm^2 to the predicted 13.6 mm^2 , there is good agreement. This implies, *if the system resolution could be characterised as a Gaussian distribution*, the FWHM corresponding to this measured variance is 8.8 mm on the projection plane, equivalent to 4.2 mm in the object plane at magnification 2.1. The FWHM corresponding to the intrinsic resolution of the camera is 6.5mm. With this magnification, objects separated by more than 4.2 mm could be resolved at the image plane.

2.4 DISCUSSION

The quality of a SPECT reconstruction will depend on the imaging characteristics, such as intensity and resolution, of its input projection images. The intensity response in a single planar projection seems well understood as a combination of the inverse square intensity drop off and the oblique pinhole aperture. These effects become apparent when viewing the intensity variation between projection images for something localised on the horizontal transverse axis. This variation is unique to a divergent ray collimator, such as a pinhole collimator. It poses a challenge to an analytic reconstruction algorithm, and helps account for why divergent ray imaging is more complex than parallel hole imaging.

The divergent ray reconstruction algorithms used in this project were written for data collected from a conebeam collimator, and the oblique aperture part of the $\cos^3\psi$ is not accounted for. Thus, the algorithm (or, as was done in this

case, the projections) must be corrected for $\cos\psi$ to allow projection images acquired with a pinhole collimator to be reconstructed properly by the conebeam reconstruction algorithms FDK or DVS.

It is reassuring that the predicted variance agrees well with the measured. The corresponding FWHM of 4.2 mm indicates the approximate system resolution to be expected from this pinhole aperture and this intrinsic resolution. The effect of doing a tomographic reconstruction will not preserve this object resolution in the reconstructed image – it will blur, and thus get larger.

CHAPTER 3 RECONSTRUCTED (TOMOGRAPHIC) IMAGES

3.1 THEORY

In order to determine the radiopharmaceutical uptake distribution in a patient, a nuclear medicine image is taken. One has a choice of which type of image to obtain (planar, flow or SPECT), but if the localised activity distribution within a three-dimensional organ is sought, SPECT is the most appropriate. SPECT provides images that can be sliced post-reconstruction to reveal such information.

Imaging a patient using a SPECT technique employs the use of a gamma camera with rotational capabilities. The patient is positioned so that the organ of interest is near the centre of the circle that the head of the camera traces out during its acquisition. At equally spaced angular intervals, the camera stops rotating and takes a planar picture called a projection image (Equation 1). At the end of the first (acquisition) phase of the study, one is left with a number of projection images taken at discrete places around a circle. The final SPECT image results from a two or three-dimensional reconstruction algorithm, the choice of which depends upon which collimator was employed to take the projection images. A 2D reconstruction algorithm can reconstruct slices of the object from isolated rows from all the projection images (i.e. parallel hole collimation in the projection images), whereas a 3D algorithm uses the entirety of the projection images to reconstruct one slice (i.e. pinhole collimation).

The concept of tomography is validated by the Central Slice Theorem, a fundamental relationship between a single projection image and the Fourier transform of the reconstructed object. The one-dimensional Fourier transform of a row of pixels from a SPECT projection image is equal to a slice through the two-dimensional Fourier transform of the object. This theorem confirms that there is sufficient data in the projection images taken around a circle to reconstruct a three-dimensional representation of the object. The sampling in the Fourier (or frequency) space is non-uniform – to compensate, a band-limited ramp filter in frequency space is used. The filter suppresses signal at the low spatial frequencies and favours higher frequencies. One method of reconstruction proceeds as follows: find the 1D Fourier transforms of the projection images, build up the 2D Fourier transform of the object, filter and inverse 2D Fourier transform. However, this is not a common method. Because of the convolution theorem, which states that multiplication in the Fourier transformed space is equivalent to convolution in image space, the above procedure can be done in real space. Although this process is properly referred to as “convolution back-projection” (two real space operations), it is also commonly referred to as “filtered backprojection”.

3.1.1 FILTERED BACKPROJECTION ALGORITHMS

The basic concept of filtered backprojection is first to filter the projection images in Fourier space, then “backproject”, or smear, them each in turn over

image space. The most common collimator used for SPECT acquisition has parallel holes, and the corresponding filtered backprojection method is the simplest of those presented here. Fan beam collimators are also available, though not as common, and require a different reconstruction algorithm due to the different geometry. Both parallel hole and fanbeam reconstruction algorithms are 2D reconstruction algorithms, while the Feldkamp, Davis and Kress (FDK) algorithm is a 3D algorithm, based on the 2D-fanbeam algorithm. The following summary was taken from various textbooks and course notes (Kak and Slaney 1988, Clack and Zeng 1994, Barrett and Swindell 1981).

3.1.1.1 Parallel Hole Collimation

The geometry for parallel hole acquisition is shown in **Figure 12**.

Figure 12. Depiction of the acquisition geometry of a SPECT study done with parallel hole collimation. The (x,y) axes are fixed, and the detector rotates by angle θ . Line integrals of activity through the object (which is placed somewhere near the origin - here as $f(x,y)$) - are collected in the projection image, R_θ , whose row co-ordinate is u .

Although SPECT projection images are two-dimensional planar images, the algorithms here are for only one row of data in the projections, whose coordinate will be named u .

The equation for parallel hole filtered back projection (Kak and Slaney, p 64) is:

$$f(x, y) = \int_0^\pi \int_0^\infty R_\theta(u) h(u - u') du d\theta \quad \text{Equation 12}$$

This can be seen as distinct processes:

1. Filter the projections

$$Q_\theta(u) = R_\theta(u) \otimes h(u) \quad \text{Equation 13}$$

where $\otimes$ denotes convolution, $R_\theta(u)$ is the projection as a function of u for the rotation angle θ , $h(u)$ is the inverse Fourier transform of the band-limited ramp filter, $F^{-1}(|\omega|k(\omega))$ where u, ω are conjugate variables, and $k(\omega)$ is the band-limiting filter.

2. Backproject the filtered projections

$$f(x, y) = \int_0^\pi Q_\theta(u) d\theta \quad \text{Equation 14}$$

This copies the same value of $Q_\theta(u)$ for $u = x\sin\theta + y\cos\theta$ for a given value of θ . That is, each line parallel to the dotted line in **Figure 12** has the same value of $Q_\theta(u)$ copied along them. The image is built up by smearing back the filtered projections for every value of θ in this way.

This is the basic filtered back projection algorithm, which can be used to reconstruct parallel hole slices. The algorithm can be further refined to avoid the lengthy convolution computation.

Because the projection images are identical at opposite sides of the circle (ignoring attenuation, scatter, distance-dependent collimator blurring, and noise), one need rotate around the object for only 180° in order to collect a complete (sufficient) set of projections.

3.1.1.2 Fanbeam Collimation

A fanbeam collimator looks like a parallel hole collimator at first glance. The difference is in how the holes are angled. In one dimension of the fan beam collimator, the rows of holes are angled towards a focal point, in line with the centre of the detector. This fan is repeated for all rows of the detector such that there is a line of foci for the collimator. The object of interest is placed at the centre of rotation, somewhere between the detector face and the line of foci. The fan beam collimator being considered is one with equally spaced collimator holes, as opposed to equiangular rays from the focus point. Again, all that will be considered in the algorithm is the reconstruction of one slice, or one fan. The geometry is shown in **Figure 13**.

The equation for fan beam tomography (Kak and Slaney p 90) is shown in Equation 15.

Figure 13. Geometry of a fanbeam acquisition. The angle of rotation is θ ; the focus is at F , and its distance from the centre of rotation is D . The point to be reconstructed has co-ordinates (r, ϕ) , and the projection is $R_\theta(u)$. The distance along the central ray from F to (r, ϕ) is depicted as C . The object, placed near the centre of rotation, is omitted from the diagram for clarity.

$$f(r, \phi) = \frac{1}{2} \int_0^{2\pi} \frac{1}{B^2} \int_{-\infty}^{\infty} R_\theta(u) h(u - u') \frac{D}{\sqrt{D^2 + u^2}} du d\theta \quad \text{Equation 15}$$

Equation 15 can also be split into distinct steps:

1. Scale the projections by the cosine of the incidence angle

$$R'_\theta(u) = R_\theta(u) \frac{D}{\sqrt{D^2 + u^2}} \quad \text{Equation 16}$$

where D is the perpendicular distance from the focus to the centre of rotation, u is the projection coordinate, $R_\theta(u)$ is the projection at rotation angle θ .

This pre-weighting reflects the different geometry between acquisition along parallel lines and lines from a fan. This pre-weighting can be generalised to three dimensions, and is typical of many conebeam reconstruction algo

rithms.

2. Filter the scaled projections

$$Q_{\theta}(u) = R_{\theta}'(u) \otimes \frac{1}{2} h(u) \quad \text{Equation 17}$$

where $h(u)$ is the inverse 1D Fourier transform of the band limited ramp filter.

3. Perform a weighted back projection

$$f(r, \phi) = \int_0^{2\pi} \frac{1}{B} Q_{\theta}(u) d\theta \quad \text{Equation 18}$$

where $B=C / D$, $Q_{\theta}(u)$ is the filtered projection, and (r, ϕ) are the polar coordinates of the point to be reconstructed. The backprojection is done along the *fanbeam* lines; u identifies the ray of the fan that passes through (r, ϕ) at angle θ . It is possible that $Q_{\theta}(u)$ might not be known for all u desired, in which case interpolation is necessary.

It is also possible to reconstruct from fan beam acquisition by rebinning the fan beam geometry to the equivalent parallel hole system. Although it can lead to a faster reconstruction, it can also be less accurate.

In this case, to guarantee a complete (sufficient) set of projections (excluding attenuation and collimator performance effects), one need acquire $180^{\circ} +$ the fan beam angle (the angle between the extreme rays). However, the weighting of the rays is no longer uniform, and a special weighting scheme must be used (Parker 1982). This can be seen if one rebins the divergent rays as if to have come from a parallel hole collimator.

3.1.1.3 FDK Algorithm

The algorithm named after Feldkamp, Davis and Kress is a generalization of the standard fanbeam algorithm to 3D (Feldkamp *et al* 1984). A conebeam collimator is the natural extension of a fanbeam collimator; all the holes are angled to meet at one focus point instead of a focus line. The geometry of the acquisition is shown in **Figure 14**.

The FDK formula for reconstruction from a conebeam collimator (Kak and Slaney, p 106) is:

$$f(t, s, z) = \frac{1}{2} \int_0^{2\pi} \frac{D^2}{(D-s)^2} \int_{-\infty}^{\infty} R_\theta(u, \zeta) h(u'-u) \frac{D}{\sqrt{D^2 + \zeta^2 + u^2}} du d\theta \quad \text{Equation 19}$$

This can be seen as the following processes:

1. Scale the projections by the cosine of the incidence angle, in this case including the third (height) dimension:

$$R'_\theta(u, \zeta) = R_\theta(u, \zeta) \frac{D}{\sqrt{D^2 + \zeta^2 + u^2}} \quad \text{Equation 20}$$

where D is the central ray of the tilted fan (distance from collimator to focal point), u is the distance along the rotated axis (t) for the ray of the point of interest, and ζ is the height of the tilted fan.

2. Filter the projections at the elevation (independently done for each value of ζ)

$$Q_\theta(u, \zeta) = R'_\theta(u, \zeta) \otimes \frac{1}{2} h(u) \quad \text{Equation 21}$$

Figure 14. The geometry of conebeam acquisition. The fixed axes (x,y) are transformed to the rotating co-ordinate axes (t,s) which become the standard axes in the reconstruction equation. The cone beam collimator would be in the z-t plane, focused at F. One point in the object has co-ordinates (s,t,z), and is situated on a ray from the tilted fan defined by the central ray D and the ray from the projection displacement u converging at F. The height of this tilted fan is ζ . The rotation angle relative to the fixed axes is not shown in the rotated co-ordinate system.

where again $h(u)$ is the inverse 1D Fourier transform of the band limited ramp filter. The filtering is applied in only the transaxial direction.

3. Perform a weighted backprojection over the 3D reconstruction grid

$$f(s, t, z) = \int_0^{2\pi} \frac{D^2}{(D-s)^2} Q_\theta(u, \zeta) d\theta \quad \text{Equation 22}$$

where $u = \frac{Dt}{D-s}$ and $\zeta = \frac{Dz}{D-s}$, and $\frac{D}{(D-s)^2}$ is the 3D analog of B in fan-

beam geometry.

The basic idea is to collect projections in cone beam geometry, then treat the different rows of pixels in the projection image in two different steps. To do

the ramp filtering, one first pretends that each row of pixels came from a fan beam collimator at that height, and applies the filter as one would in fan beam reconstruction. Then, in the backprojection stage, one reverts back to the 3D geometry, and backprojects the filtered projections along tilted fans, with all rays converging to one focus point.

The FDK algorithm is an approximate algorithm, as the sequence described above does not reconstruct the object exactly for tilted fans. The plane of the focus is reconstructed exactly, as there is no tilt to the fan beams ($\zeta=0$), and the reconstruction for this plane is the exact fanbeam formula (for full-circle acquisition and Parker weighted 180° +cone angle acquisition).

Nonetheless, FDK is a common reconstruction algorithm for a few reasons. First, it is fast and robust, two attributes that many of its exact 3D reconstruction rivals cannot easily claim. Second, it is quite simple, and easy to code and implement on a computer. Third, it is very well studied, and its theory is thoroughly understood. It is the most widely used/studied 3D reconstruction algorithm, in spite of yielding an approximate reconstruction.

When published by Feldkamp *et al.* in 1984, the theory of 3D reconstruction was still in its infancy. Since then, an entire field of applied mathematics has grown, which has seen the introduction of many new analytic 3D-reconstruction algorithms, and a very elaborate theory. It is in this theory that the condition for complete sampling is arrived at, which will be explained in the next section.

3.1.2 RADON RECONSTRUCTION

3.1.2.1 Radon Space and Tuy's Sufficiency Condition

In 2D tomography, Radon space and projection space are one and the same. A line integral through a location in object space maps to a point in projection space, and the set of all line integrals through a location in object space maps to a circle in projection space. This is shown in **Figure 15**.

Figure 15. Depiction of the measurements in object space (left column) and their equivalent representations in 2D Radon space (right column) for one projection line (top row) and all projection lines (bottom row) through a point P at (s, θ) . A single projection line ℓ (rotation angle not shown) through P maps to a point at polar co-ordinate (s, θ) in projection space, whereas the set of *all* projection lines maps to a circle with diameter from the origin to (s, θ) .

In 3D tomography, projection space and Radon space are different, although 3D Radon space is analogous to 2D Fourier space. Analogous relationships exist between the measurement in object space and the equivalent representation in Radon space, as depicted in **Figure 16**.

In order to reconstruct something exactly using 3D geometry, one must fill

Figure 16. Depiction of object space (left column) and Radon space (right column) for one measured plane (top row) and all measured planes (bottom row) through a point x at (s, θ) . The single plane is labelled P with normal ϕ , and maps to a single point in 3D Radon space. The set of planes (rotation angle not shown) maps to a sphere in 3D Radon space whose diameter is defined by the origin and (s, θ) .

all of the Radon space that corresponds to the FOV of interest, and then apply the inverse Radon transform into image space. The region of Radon space that is filled depends intrinsically upon the acquisition path (of the pinhole) used by the camera. The only acquisition paths presented in this thesis are circles and portions of circles. The cross-section of Radon space for a circular path is shown in **Figure 17**.

As previously mentioned, the volume corresponding to the FOV of the object being imaged should be completely sampled in the Radon domain to assure exact reconstruction. However, the only point imaged along the z -axis is the origin - all other points along this axis are in the *shadow zone*, and thus are not sampled. It is easy to see from **Figure 17** that acquisition with a circular path will

Figure 17. A cross-section of Radon space corresponding to the set of projection images taken with a circular path. Spherical shells tangent to the origin in Radon space represent all the planes that pass through a given point; the torus of these spheres corresponds to a circular path about the origin, as shown. The darkest area within the torus is unsampled, called the shadow zone.

not fill the shadow zone near the origin, and thus will not sample Radon space completely for a central FOV.

A path that does fill the shadow zone is one that satisfies Tuy's condition [Tuy, 1983]. If all planes that intersect the support of the object (to be reconstructed) simultaneously and non-tangentially intersect the acquisition path, then exact reconstruction is possible. This is not usually met by a single circle of acquisition. Any object that does not lie in the plane of the circle can have a plane parallel to the plane of the circle that will intersect the object, and will not intersect the acquisition path (see **Figure 18**).

The Tuy path is achievable in principle – any path that ensures any plane drawn through the object simultaneously intersects the path qualifies. Tuy paths

Figure 18. Demonstration of why a single circle of acquisition does not satisfy Tuy's sufficiency condition. The red circle shown represents the path of the pin-hole during SPECT acquisition. In the situation depicted at left, the blue plane intersects both the object and the acquisition path (the thyroid and the circle), while in the case depicted at right, the blue plane intersects the object, but does not intersect the acquisition path. It is because planes such as those depicted at right exist for acquisition with a single circle that this path does not satisfy Tuy's sufficiency condition.

include helices, circles with lines (parallel to the normal of the circle), and two orthogonal circles.

However, in clinical applications, even a full circle of acquisition may be difficult to achieve, let alone a Tuy path. For example, in the case of the thyroid gland, the patient's shoulders provide obstruction to the acquisition path, and images from the inferior side of the patient (toward the back of the neck) are severely attenuated by spine and muscle. The data set becomes even more incomplete if the acquisition is limited to only half a circle of acquisition, as the number of planes that can be drawn through the object which do not intersect the acquisition path increases.

An alternate way of viewing the shadow zone (unsampled Radon space) is

to consider an isolated point in object space. To fully image this point, *all* the plane integrals (the integral of the functional representation of the object on a plane) that contain the point must be known. Any plane integrals that are missing are part of the shadow zone.

The treatment of the shadow zone in 3D reconstruction is of importance in determining the outcome of the reconstruction. The FDK backprojection algorithm fills the shadow zone with zeroes, and thus the reconstruction is expected to be less accurate the farther one is from the plane of the circular acquisition path. The Discrete Vertex Set algorithm is more sophisticated in its treatment of the shadow zone.

3.1.2.2 *The Discrete Vertex Set algorithm*

The Discrete Vertex Set (DVS) algorithm was written by Fred Noo for his Ph.D. thesis [Noo 1998, Noo *et al*, 1997]. It is an elegant approach to the problem of difficult acquisition geometry. Most reconstruction algorithms are linked to the idea of a continuous path, as has been discussed thus far. In some cases, this continuous path might be very difficult to achieve. (In reality, since projections are taken at discrete places on a path, the raw data is inherently discrete, and no set of projections can strictly satisfy Tuy's sufficiency condition.) The DVS algorithm need not assume projections were on a path – any random set of arbitrarily oriented projections can be used. If the co-ordinates of all the projections are fed to DVS, it will do the best it can to reconstruct the ob-

ject. This flexibility of the DVS algorithm was not used in this project. The DVS algorithm was used to reconstruct projection data taken from circular paths, and paths of half a circle of acquisition.

The DVS Method

DVS is classified as a Radon reconstruction method. That is, the strategy of reconstruction is to build up the Radon domain in the region of the FOV in order to use an inverse Radon transform to obtain the reconstruction. This is fundamentally different from a backprojection approach, such as that used in the FDK algorithm. Radon methods involve various stages. First, the projection images need to be obtained in a divergent ray geometry. Secondly, these must be transformed into the Radon domain. Thirdly, the filled Radon domain in the region of the FOV must be inverted back to object space.

The terminology used in the DVS algorithm is different than that of the FDK. The projection line integrals are defined here as (Noo *et al* 1997, p 1310)

$$g(u, v, \underline{a}_i) = \int_0^{\infty} f(\underline{a}_i + t\underline{\alpha}) dt \quad \text{Equation 23}$$

where f is the functional representation of the object, (u, v) are the coordinates on the projection plane, $\underline{a}_i$ is the vertex point defining the centre of the projection, and $\underline{\alpha}$ is the vector from $\underline{a}_i$ to (u, v) . This is depicted in **Figure 19**.

Once the projections have been acquired, one must transform them into Radon space. The 3D Radon transform $(Rf(\underline{\theta}, \ell))$ associates a plane integral to each position (defined by radial position ℓ and direction $\underline{\theta}$) in the object. The

Figure 19. Depiction of the 3D acquisition geometry, where $\underline{a}_i$ is the vertex point (focal point), D_i is the distance from $\underline{a}_i$ to the detector plane, $\underline{\alpha}$ is the vector of interest (from $\underline{a}_i$ to (u,v)), $L(s,\mu)$ is a line on the detector which makes an angle μ with a projection axis and lies a distance s from the origin. The line integral is along $\underline{\alpha}$, and all the line integrals for each location (u,v) comprise the projection image $g(u,v,\underline{a}_i)$.

different formulae used to achieve this difficult task are called intermediate functions. The intermediate function used by the DVS algorithm is known as Grangeat's formula. Grangeat's formula relates $r(s,\mu)$ - a function of the projections - to the first derivative of Radon space. The function $r(s,\mu)$ is equal to the sum of the conebeam projections $g(u,v,\underline{a}_i)$ scaled by $\frac{D_i}{\sqrt{u^2 + v^2 + D_i^2}}$ along

$L(s,\mu)$. Grangeat's formula (Noo et al 1997, p 1311) is

$$R' f(\underline{\theta}, l) = \frac{s^2 + D_i^2}{D_i^2} \frac{\partial}{\partial s} r(s, \mu) \quad \text{Equation 24}$$

where the variables on the right hand side are described above, and

$$\underline{\theta} = \frac{D_i \cos \mu \underline{e}_u + D_i \sin \mu \underline{e}_v + s \underline{e}_w}{\sqrt{s^2 + D_i^2}}, \quad l = \underline{\theta} \cdot \underline{a}_i \quad \text{Equation 25}$$

where $\underline{e}_u, \underline{e}_v, \underline{e}_w$ are the basis vectors of the projection plane. The essential steps are to (i) pointwise scale the projections $g(u, v, \underline{a}_i)$, (ii) find $r(s, \mu)$ in the detector plane, and (iii) differentiate in the s direction and apply a final scaling. This yields Radon samples $R'f(\underline{\theta}, \ell)$ for the planes defined by $\Pi(\underline{\theta}, \ell) = \{x \mid x \cdot \underline{\theta} = \ell\}$ in object space.

The final step in Radon reconstruction is to invert the data in the Radon domain to object space. In the DVS algorithm, this is achieved using Marr's algorithm, which can invert either $Rf(\underline{\theta}, \ell)$ or $R'f(\underline{\theta}, \ell)$ through a type of filtered back-projection. One caveat worth mentioning is that each projection fills a subset of the Radon domain. These subsets, depending on the acquisition path, may overlap and leave gaps, causing irregular sampling of the Radon domain. However, the Marr algorithm requires uniform sampling of the Radon domain. One of the responsibilities of the DVS algorithm is to interpolate and rebin the data so that the Radon domain is uniformly sampled for the Marr algorithm to proceed.

The DVS Contribution

The goal is to fully sample the appropriate region of Radon space, that is, to have all the plane integrals of the object. Using Grangeat's formula, from a projection, one gets all the plane integrals containing a vertex point. Since all the planes containing the vertex points are not necessarily the same plane integrals as those through the object, the DVS algorithm tries to approximate the

planes it needs with planes it already has. A sphere approximating the object is chosen, and a central point P is chosen. The plane chosen will have the smallest angle with the plane desired, and will thus be a good estimate. This is depicted in **Figure 20**.

Figure 20. Depiction of the interpolation process used in the DVS algorithm. P is a point on the sphere of interest, which is also common to the planes Π (the desired plane) and $\tilde{\Pi}$ (the best measured substitute for the missing plane). The angle between the normals of the planes is minimal with respect to the angles between θ and all the other choices for $\tilde{\theta}$.

Because of its attempt to fill out Radon space, it seems a reasonable hypothesis to suggest that the DVS algorithm might have superior reconstructions when compared to those of FDK. This comparison was a focus of the project.

3.2 METHODS

3.2.1 OVERVIEW

The primary goal of the experiments was to assess the resolution and uniformity of the Feldkamp (FDK) and Discrete Vertex Set (DVS) reconstruction algorithms for both full- and half-circle acquisition using a pinhole collimator. Phantoms (section 3.2.2.7) were aligned (section 3.2.2.3) and imaged, with corrections (sections 3.2.2.8.1, 3.2.2.8.2), using a rotating gamma camera (section 3.2.2.1). The acquisition was a full-circle SPECT study, and the half-circle acquisition was extracted from the full-circle. Reconstructions of both acquisition geometries were done using both algorithms. The analysis done depended on the phantom. The point phantom provided both a Point Response Function (PRF), and the ability to take quantitative resolution measurements. Count distribution profiles (section 3.2.4.1) and region of interest (ROI) analysis (section 3.2.4.3) was done of the points (one pixel wide) in HERMES (section 3.2.3.2) to find their FWHM resolution using a Gaussian model (section 3.2.4.2). The line phantom was used to quantitatively characterise the uniformity; count profiles of summed slices were taken to include the entire line, resulting in a count distribution profile. The Picker thyroid phantom, used to quantitatively characterise contrast and Signal to Noise (SNR) ratio, was analysed in Qdisplay (section 3.2.3.1) using ROIs. A count distribution profile, one pixel wide, was also taken

of the nodules in the Picker thyroid phantom, and the Full Width at Half Maximum (FWHM) was assessed (section 3.2.4.4).

3.2.2 APPARATUS

3.2.2.1 Camera

To image the distribution of gamma-emitting radioactive isotopes, a gamma or Anger camera is used. In this study, a Siemens ZLC3700 was used, an older camera which has SPECT capabilities, shown in **Figure 21**.

Figure 21. The Siemens ZLC 3700 gamma camera with the pinhole collimator mounted, rotating about a sample on the sample holder.

The 37 photomultiplier tubes at the back of a scintillating crystal are used by the camera to detect gamma rays. Gamma cameras are both energy and position sensitive, allowing the detection of one (or more) radionuclides with spatial

localisation. The camera was “peaked” to detect the principal decay product of Tc-99m with a 20% window; the range of detectable energies spanned 126-154 keV, centred at 140 keV. An analog signal from a resistor network determines the position of a gamma ray interaction using Anger logic, although this would be done digitally on a newer camera. The pinhole collimator used (number 820-824458) is made of lead and is essentially a cone with a 22.1 cm diameter base (approximately half of the crystal’s diameter), and a 19.4 cm height to the insert. Set into the collimator is a pinhole insert, made of sintered tungsten (HVL 0.24 mm at 140 keV), with a 3.175 mm pinhole set 1 cm up, making the distance from the pinhole to the crystal 20.4 cm. This is depicted in **Figure 22**.

Figure 22. Schematic of the pinhole collimator with the insert. The entire extent of the lead shielding beyond the cone is not shown. The height depicted is that of the collimator – this does not include the mean crystal penetration depth.

The camera and pinhole collimator together constitute the tomograph, which was used to image throughout the project.

The analog signals from both x and y co-ordinates from the ZLC camera are fed to an acquisition box (HERMES Acquisition, Nuclear Diagnostics). The signal is digitised and stored on the acquisition computer, a Sparc20 workstation running SUN Solaris (version 2.5), from which the acquisition is controlled using HERMES software.

3.2.2.2 Computers

All reconstructions were done on a separate Sparc20 computer. Faster Pentium II computers are available in the department, also running Solaris 2.5, but Sparc architecture stores bytes as BIGENDIAN, and some programs (i.e. Qdisplay – section 3.2.3.1) could be run only under this architecture. The images were, however, stored on a Pentium computer, whose architecture stores bytes as LITTLEENDIAN.

3.2.2.3 Positioning

Knowing the axis of rotation of a SPECT camera is important for positioning the phantoms, and it must be known before a pinhole camera can be set at the desired magnification. Finding the axis of rotation on the ZLC camera is a tedious task, so a laser was installed on one wall of the room in line with the camera's axis of rotation.

The laser is glued to a front plate (**Figure 23**), with foam backing around its perimeter. The screws in the front plate screw directly in a back plate, sandwiching the drywall in between. The foam provides some resilience to the

mount, allowing the laser beam, with a fraction of a turn on one screw, to be substantially moved at the position of the pinhole focus. The laser was positioned to highlight the axis or Centre of Rotation (COR) with an accuracy of about 1 mm.

Figure 23. Schematic of the laser mounted in its bracket in the wall, in line with the axis of rotation of the ZLC camera. The red dot represents the laser beam, while the black cord underneath provides power.

3.2.2.4 Geometry

Although the distance from the pinhole to the detector plane is fixed (204 mm – a in **Figure 3**), the distance from the pinhole to the axis of rotation is not (b in **Figure 3**). This adjustable parameter determines the magnification of the object being imaged in the projection plane (section 2.1.1). This distance was kept constant for all imaging, and was set to one that would typically be used in the clinic when taking thyroid images. It was chosen to be 100 mm from pinhole to axis of rotation, so that along with the fixed distance (from pinhole to mean crystal depth) a magnification of 2.1 times was obtained (see **Figure 24**).

Figure 24. The geometry used for pinhole imaging, which yielded a magnification of 2.1. The height includes the mean penetration depth of the crystal.

Realising this chosen radius during an experiment was very challenging. Horizontal distances could be measured more accurately than vertical ones, so the camera was set using horizontal measurements (i.e. 90° and 270° orientations defined in section 2.2.1) which were confirmed (to approximately 1 mm) in the vertical orientation. Even with the laser beam defining the COR, this was still a tedious, time consuming process. However, a camera with a computer controlled head position would alleviate this problem.

3.2.2.5 Field of View of a Pinhole SPECT Study

The tomographic FOV is a cone, defined by b (section 2.1.1) and the radius of the planar FOV, rotated about the axis of rotation. The volume common to all cones is a sphere, and is the volume of interest for the tomographic reconstruction. The diameter of the sphere is given by:

$$\text{Diameter} = 2 \left(\frac{rb}{\sqrt{b^2 + r^2}} \right) = 2 \left(\frac{Db}{\sqrt{D^2 + 4a^2}} \right) \quad \text{Equation 26}$$

where r is the radius of a single projection's FOV, and the other variables are as defined in **Figure 3**. In this study, using the values listed in **Figure 22** and **Figure 24**, the diameter of this sphere was found to be 93.1 mm. If not confined to this sphere, objects would not be entirely within the FOV for all projection images, leading to truncation, which presents problems for the reconstruction.

3.2.2.6 Sample Holder

The cantilevered sample holder is depicted in **Figure 25**.

Figure 25. The sample holder, attached to a box (bottom right, brown). It is made of aluminium (grey) and lucite (clear) pieces. The degrees of freedom are shown. The laser beam shines along the axis of rotation as indicated.

In addition to the rotation of the circular sample plate, the entire assembly moves along the axis of rotation and the vertical transverse direction. It could

also pivot on the pin holding the Lucite bar to the aluminium bracket, which led to poorly controlled movement in the horizontal transverse direction. The smaller phantoms (points and lines) were on a Styrofoam block on the sample plate, and the thyroid phantom lay on the circular sample plate directly.

3.2.2.7 Phantoms

All the phantoms that were imaged in this project were filled with diluted $\text{Na}^+\text{TcO}_4^-$ as the radioactive compound and a drop of food colouring, using syringes and needles. Three different main phantoms were imaged: points, lines, and the Picker thyroid phantom.

Points

The points are aluminium zeolite molecular sieve beads, and are less than 2 mm in diameter. In this study, the beads ranged in activity from 0.9 MBq - 2.0 MBq. To characterise the point response function and FWHM resolution at different locations in the FOV, the beads were imaged in two different planes as depicted in **Figure 26**.

Lines

The lines are capillary tubes, 75 mm long, with an inner diameter of 1.15 ± 0.05 mm and a wall thickness of 0.20 ± 0.02 mm. They were plugged at both ends with Critoseal; the activity spanned approximately 7 cm. Three lines were imaged side by side, the centre one containing 11 MBq of activity, and the other two containing 11.5 MBq. The lines were used to assess the amplitude re-

sponse of the acquisition and reconstruction as a measure of uniformity. They were imaged in three ways: lying in the axial, horizontal transverse, and vertical transverse directions, as depicted in **Figure 27**. The information obtained was from only the central line.

Figure 26. The arrangement of seven points as seen from above (the positive end of the vertical transverse axis). The points are depicted in a) the horizontal plane (points along the axis of rotation and the horizontal transverse axis) and b) the vertical plane (points along the horizontal transverse axis and the vertical transverse axis).

Figure 27. The line sources as viewed from above. Capillary tubes were arranged in the a) axial direction, b) horizontal transverse direction and c) vertical transverse direction.

Figure 28. Depiction of the Picker thyroid phantom, showing three cold nodules, one hot nodule, and the insert. Activity is shaded, while plastic is clear. The insert is in the right hand lobe of the phantom, so that activity is on only the upper half of the right hand lobe (contrasted with throughout the left hand lobe), with the exception of the hot spot. The filling port is just below the large cold spot in the left hand lobe.

Thyroid Phantom

The Picker thyroid phantom (**Figure 28**) is a classic phantom, which was modified slightly by replacing metal screws with nylon ones, and securing the removable insert with silicone. It was the heaviest phantom imaged. Two different amounts of activity were used, 23 MBq and 2 MBq, the latter being more clinically realistic. The purpose of imaging this phantom was to quantify the quality of the image; contrast, signal-to-noise (SNR), percent background noise and lesion sizes were measured on the thyroid phantom reconstructions.

3.2.2.8 Acquisition

Using the HERMES acquisition system, corrections can be applied “on the fly” during acquisition. Two of these corrections were used in this study: uniformity and Centre of Rotation (COR).

3.2.2.8.1 Uniformity Correction

If a radioactive source illuminates a gamma camera crystal uniformly without a collimator (termed a “flood”), the image produced by the camera should have no intensity variation throughout the FOV. If the camera is not capable of reproducing a flat image, corrections are needed. On the ZLC, this correction is done by the acquisition software, external to the camera. If a high-count flood is acquired, the inverse of the image can serve as an intensity correction map, which the acquisition software can use to compensate subsequent acquisitions. Output from this HERMES QC program is shown in **Figure 29**.

3.2.2.8.2 Centre of Rotation

A SPECT camera rotates about an object, typically tracing out a circular path. There can be mechanical difficulties in doing this, depending on the design. The ZLC gamma camera's rotation is a rather complicated manoeuvre involving counterbalances and a gantry that pivots about its vertical axis. Thus, it is possible the camera does not rotate in a perfect circle. Corrections are needed to compensate for a non-circular orbit. The HERMES software has an “on the fly” COR correction, using a COR map derived from a single point source slightly offset from the centre of rotation, acquired at 64x64 matrix size, 64 images. Both a sinogram (projections column summed to create a single image - each projection is a row in the sinogram) and a compressed image (projections row summed to create a single image - each projection is a column)

Figure 29. A high count uncorrected flood (right hand images) and a subsequent corrected flood (left hand images). Note the difference in appearance, most evident in the “functional” image, bottom right - the pattern of the photomultiplier tubes is visible in the uncorrected image. The QC numbers (Integral and Differential Uniformity – essentially percentage difference of maximum and minimum pixel counts over their sum) are much lower once the map is applied (corrected image).

are made in the HERMES QC software (see **Figure 30**).

A sine curve is fitted to the sinogram, and the mean deviation (in pixels) is saved in a file to be applied to future acquisitions, the offset applied in the direction orthogonal to the axis of rotation (u direction). The offset was typically half a pixel width. This correction has two drawbacks: it is not projection-dependent, nor does it involve offsets in the direction of the axis of rotation (v direction).

Figure 30. The HERMES QC program for COR correction. The “X” represents the “u” direction (orthogonal to the axis of rotation), and “Y” represents the “v” direction (parallel to the axis of rotation). A sine wave is fitted to the sinogram (second column, top row), and the mean of the deviations from each projection image (second column, bottom row) serves as a correction, and is applied in only the u direction in subsequent acquisitions.

The former could correct, for example, systematic weight shifts as a function of projection angle during the rotation. The latter could correct an axis of rotation not parallel to the detector head (v direction) at 0° , or a tilt in v on the detector head between 0° and 180° . However, in spite of not incorporating corrections for the potential problems just described, based on the reconstructions achieved

in this project, the HERMES COR correction seemed adequate.

3.2.2.9 Data Collection

Once the correction maps were made, the scan could proceed. The sample was put on the sample holder, and, using the laser, centred as close to the centre of rotation as possible. The sample holder table was placed using the SPECT patient table inserts on the floor. The head, with the pinhole collimator mounted, was put to the 0° position, and the Pscope (direct camera interface) was set to acquire 128 views around a full circle in step-and-shoot mode. The HERMES acquisition acquired each projection image for 20 s on a 128×128 matrix, 3.07 mm / pixel side. The total study time included the transit time for the head to move between projection locations, when no imaging is done (approximately 3 seconds). At the end of the study, the information was saved on the central server in Interfile format, and the head of the camera was rotated in the opposite direction to unwind the cable.

3.2.3 RECONSTRUCTION

The acquired data are stored on the central server, accessible through the HERMES menu. The reconstruction process (not part of the HERMES environment) involves some pre-manipulation and the actual reconstruction using either the Feldkamp (FDK) or Discrete Vertex Set (DVS) conebeam reconstruction algorithm. More details on these steps are included in the Appendix. The FDK algorithm was reconstructed with a Hann apodizing filter, parameter 0.5

and cutoff at the Nyquist frequency. For each phantom imaged, four separate reconstructions were done: FDK-half circle, FDK-full circle, DVS-half circle, DVS-full circle.

3.2.3.1 *Qdisplay*

The format of all the image files was chosen to be compatible with the *Qdisplay* program, an image display program written at Queen's University by Phil Harvey. It is a versatile program, allowing various types of input, calculations on images, re-scaling, custom colour tables, display of a pre-selected number of images, etc. This program is able to display negative numbers and was used to view the low intensity artefacts (tails and undershoots).

3.2.3.2 *HERMES Static Display*

The Nuclear Diagnostics image display and processing package used by the Nuclear Medicine Department for clinical studies was also used in this project. The files were entered into the HERMES environment via a binary-to-Interfile conversion program. Once in the HERMES index, the user has a range of processing programs available, depending on the type of study. In this project, most of the reconstructed images were imported into HERMES as static images, in order to use the profile tool (section 3.2.4.1).

3.2.4 ANALYSIS

Following image reconstruction, four specific types of analysis were done.

To ascertain the FWHM resolution of the points, a Gaussian model was fit to the count profile, and the FWHM calculated from the Gaussian parameters. To assess the intensity response, an ROI was placed around each vertically oriented point, and count profiles were taken of line sources. To assess Signal to Noise Ratios (SNR) and contrast of the Picker thyroid phantom, a sized ROI was taken of each cold spot, and five ROIs were taken of the background. To find the size of the Picker thyroid phantom nodules, count profiles were fed to a Full Width at Half Maximum (FWHM) program.

3.2.4.1 Count Profiles

Profile analysis was done in the HERMES data-processing package entitled “static display” (see **Figure 31**). The reconstructed image file, displayed slice by slice, had adjacent slices summed (usually four), to ensure an adequate number of counts in the count profile. The profiles are restricted to be either horizontal or vertical. Reconstructions of the point phantoms and thyroid nodules had a one pixel wide count profile to assess resolution, whereas the line phantom had count profiles whose widths covered the width of the reconstructed image of the line. The data, written to a temporary file in the profile program, were saved elsewhere for later analysis. Profiles were taken in every possible orientation of all points; each point had three measured resolutions. This tool was invaluable for most of the analysis in this project.

3.2.4.2 Gaussian FWHM

The FWHM of a Gaussian distribution can be found by fitting a Gaussian model. Since the reconstructed points approximated Gaussian distributions in most cases, their count profiles were fitted to a Gaussian model in a statistics package (Systat, Systat Inc., Evanston Il, 1992), whose output parameters

Figure 31. An example of obtaining a profile from the HERMES static display. Shown at centre-left is the summed image slice (4 slices) with a vertical single pixel profile drawn through a point. At bottom is an interpolated graph of the counts in the profile, and at right is the data file, scaled counts per pixel. The data here are scaled by 256 to make full use of the colour table. This does not affect the FWHM values.

were used to find the FWHM. The equation that was given to the program was:

$$COUNTS = Ae^{-\frac{(PIXEL-\mu)^2}{2\sigma^2}} \quad \text{Equation 27}$$

where A represents the maximum amplitude (counts) of the distribution, μ represents the mean pixel value of the distribution, and σ represents the standard deviation of the distribution.

The program used a quasi-Newtonian minimisation method to find the parameters and errors. To verify that the Gaussian approximation was reasonable, a plot of the data overlaid with the estimated Gaussian was printed (Figure 32).

Figure 32. Finding the FWHM resolution using a Gaussian model. The points represent values from a count profile through a reconstructed point, and the solid line is the fitted Gaussian.

From a fitted σ value, a FWHM can be calculated with an error propagated from the fit, representing statistical error. The formulae to do this are

$$FWHM = 2\sigma\sqrt{2\ln 2} \quad \text{Equation 28}$$

$$err_{FWHM} = 2err_{\sigma}\sqrt{2\ln 2} \quad \text{Equation 29}$$

The assumptions in this include that the Gaussian model is valid, and that the standard deviations in the reconstructed data are random and uniform.

3.2.4.3 ROI analysis

Regions of Interest (ROIs) were used in the analysis of the reconstructed vertical points, and in the Picker thyroid phantom reconstructions. An ROI groups together a user defined selection of voxels (a voxel contains the smallest unit of reconstructed activity concentration), from which one can find the sum of the voxels' reconstructed activities, their average and their standard deviation. Here, reconstructed activity concentration values were not used, but the voxel values were treated as relative reconstructed intensities. The reconstructed points' ROIs were used to assess the points' relative intensities, while the thyroid phantom reconstruction ROIs were used to assess the contrast, SNR and percent background noise of the lesions (1 hot, 3 cold) to the (cold, hot) background.

The ROIs of the vertical points reconstructions were taken in HERMES, using the static display ROI tool. Five slices were summed, and a square ROI of the sum of 6x6x5 voxels was used to assess the reconstructed intensity. The results were normalised to the central point.

The ROIs in the thyroid phantom were defined on one slice, and their sizes varied depending on the size of the cold nodule. These are listed in **Table 2**. For each nodule size, only one lesion ROI was used, but 5 background ROIs

were used.

The quantities measured from the ROIs from the thyroid phantom reconstructions are defined as follows:

Nodule Size (mm) and Type	2x2x1	3x3x1	4x4x1	5x5x1
ROI Size (voxels)	4x4x1	4x4x1	3x3x1	2x2x1

Table 2. ROI sizes used in analysing the nodules of the Picker thyroid phantom reconstructions. Five identically sized ROIs were found of the background.

$$Contrast = \frac{|\mu_L - \overline{\mu_b}|}{\mu_b} \quad \text{Equation 30}$$

$$SNR = \frac{|\mu_L - \overline{\mu_b}|}{\sqrt{\sigma_L^2 + \sigma_b^2}} \quad \text{Equation 31}$$

$$\%background_noise = \frac{\sigma_b}{\mu_b} \times 100 \quad \text{Equation 32}$$

where μ_L is the average voxel value for the lesion ROI, and σ_L is the standard deviation of the voxel values in the lesion ROI. Each background ROI's mean and standard deviation are referred to as μ_b and σ_b respectively. However, since 5 background ROIs were measured for each lesion, $\overline{\mu_b}$ and σ_b are the mean and standard deviation respectively for the 5 background ROIs.

The reconstructed thyroid ROI analysis was done using the Qdisplay program (see **Figure 33**), by defining the top left and bottom right hand corners of the square ROI in a nodule or in a background region on one slice of the image. The statistics of the ROI were displayed; the mean and standard deviation

Figure 33. ROI analysis of the Picker thyroid phantom reconstructed image. The ROI is on the 12 mm cold spot (black square at bottom left), and the statistics are shown underneath the menu at the top left. Only the average and the standard deviation were recorded, but the number field identified the number of voxels in the ROI, which was helpful in ROI placement.

of the ROI were recorded.

3.2.4.4 Width at defined threshold program

A program was written to allow the user to define a threshold, and select the range of a count profile that was taken of either a hot spot or a cold spot. The program would then report on the width at that threshold for either type of spot. The abscissa at the threshold was found by linear interpolation of the abscissas corresponding to ordinates on either side of the edge ordinate. The

threshold in this analysis was set at 50%. That is, the width for a hot spot was the FWHM. The edge ordinate for a minimum (or valley) was found using

$$\text{Edge ordinate} = (\text{background-valley}) * 0.5 + \text{valley} \quad \text{Equation 33}$$

This method of finding the width is insensitive to the shape of the distribution, and was a quick check on resolution values found using the Gaussian fit. It was also used to find the sizes of the hot and cold nodules of the Picker thyroid phantom.

Many of the calculations used an Excel spreadsheet in addition to the analysis described.

3.3 RESULTS

The results from the tomographic reconstructions of the projection data of the phantoms are presented to encourage a four-way comparison. For each phantom, images were reconstructed from a) two different acquisition geometries: full- and half-circle (the data for the half-circle were extracted from the full-circle file), and b) two different reconstruction algorithms, the FDK and DVS. The images and analysis for the points, lines and thyroid phantom are presented.

All reconstructions yield a cube of data. In order to see the internal structure, the cube is sliced in one of three ways: coronal, sagittal or transverse (**Figure 34**). For example, slices from the axial line phantom reconstruction are shown in **Figure 35**. Although the entire cube is 64x64x64 voxels (1.53 mm / voxel side), many of the 64 slices for any given direction are empty so that only central slices are shown.

The point and line phantom reconstructions are presented similarly to **Figure 35**, with only the most intense slice shown for coronal, sagittal and transverse slices. The thyroid phantom reconstruction will be shown in only coronal slices, as these are the most informative for that phantom.

The four classes of reconstructions are the Feldkamp algorithm reconstructions for both full- and half-circle acquisitions, and the Discrete Vertex Set algorithm reconstructions for the same data. These are abbreviated as

Figure 34. The three dimensional reconstruction yields a cube; to see internally, one slices the cube to get *coronal*, *sagittal* or *transverse* slices. These are portrayed with reference to the axes.

Figure 35. Central slices from the full-circle FDK reconstruction of the axial line phantom. The top, middle and bottom rows are coronal, sagittal and transverse slices respectively. The coronal slices display the data most effectively for this phantom; both the sagittal and transverse slices would need more than the eight slices shown to convey the equivalent information. The intensities are mapped on a rainbow colour scale with a blue zero level.

FDK-full, FDK-half, DVS-full and DVS-half. Thus, a reconstruction using the FDK algorithm from a half circle of acquisition will be abbreviated as an FDK-half reconstruction. The intensities were set so that the colour scale is common to each set of images. These could be related to the imaged activity concentration (Bq/mm^3), but for this study were treated as relative numbers.

3.3.1 POINT PHANTOM RECONSTRUCTIONS

The 7-point phantom reconstructions are shown for the phantom lying in the horizontal plane (**Figure 36**) and the vertical plane (**Figure 37**).

3.3.1.1 Observations

3.3.1.1.1 Overall

It is worth noting that in most aspects the reconstructions depicted in **Figure 36** and **Figure 37** are good. The distance between the points is preserved, the intensities of the reconstructed points can be related to the activity imaged, and the shapes in the reconstructions approximate those of blurred points. The spatial resolution is adequate to resolve the points separated by 1.5 cm. However, the reconstructed images are shown with a colour scale that emphasises the low intensity artefacts. Low amplitude tails can be seen in all the reconstructed slices, although they are more pronounced in the half-circle reconstructions. They seem to point towards the centre top in the sagittal slices and in the transverse slices. They are most evident in the top half of the half-circle recon

Figure 36. Coronal (top row), sagittal (middle row) and transverse (bottom row) slices from the FDK-full (first column), FDK-half (second column), DVS-full (third column) and DVS-half (fourth column) reconstructions of the point phantom, imaged lying in the horizontal plane. The frames to the right and bottom of each slice are intensity profiles at the centre of each slice. The intensity is mapped into the colour scale shown at right, zero being a dark blue, and the negative values are cut off at a level of -10% of the peak intensity.

structions, for both algorithms.

3.3.1.1.2 FDK

Both full and half

There is a correlated pattern around the periphery of some of the slices.

FDK Half

The cross-sections of the reconstructed end points along the horizontal transverse axis are distorted from circular. They have tails pointing away from

the centre in the horizontal transverse direction, seen in the coronal slices, and in the transverse slices. The reconstructed points at the bottom of the vertical transverse axis have less intensity and poorer localisation than their full-circle counterparts (Figure 37).

Figure 37. Slices of the reconstructed point phantom imaged oriented in the vertical plane. The figure is arranged and the intensities are set identically as in Figure 36.

3.3.1.1.3 DVS

DVS Half

There are pronounced undershoots that can be seen around all point reconstructions. The cross-sections of the reconstructed points along the horizontal

transverse axis are markedly non-circular; they have an axial extent typical of the central point (**Figure 36**), small horizontal transverse extent, and large vertical transverse extent (**Figure 37**). The reconstructions of the vertical transverse points (**Figure 37**) all have similar intensities when compared with the full-circle reconstructions.

3.3.1.2 Analysis

The reconstructed images from the point phantoms were quantitatively assessed for spatial resolution as described in section 3.2.4.2. Values of the FWHM for each point in all three directions are shown in **Figure 38**. The measurements of the resolutions from the horizontal transverse points were made from the two arrangements of the phantom: the arrangement described **Figure 36**, and that described for **Figure 37**. It was decided that the measurements were equivalent, and thus only the data from the arrangement in **Figure 36** are shown.

3.3.1.2.1 Overall Resolution Results

The central point has a FWHM of 5.0 ± 0.3 mm in the axial direction, and 5.6 ± 0.2 mm in the two transverse directions, for both acquisitions and both algorithms (means and standard deviations). The difference between the algorithms' FWHM values for the central point is 0.1 mm, DVS being the larger. The four reconstructed points (both acquisitions, both algorithms) along the axis of rotation (**Figure 38**, top panels) have similar FWHM values - an average of

Figure 38. FWHM measurements from the point source phantom. The plots a, b and c display resolution information for the axial, horizontal transverse and vertical transverse directions respectively, from points along the axis of rotation (top panels), horizontal transverse axis (middle panels) and vertical transverse axis (bottom panels). Each graph has coloured points corresponding to the four reconstructions: FDK-full (red circles), FDK-half (blue squares), DVS-full (yellow triangles), and DVS-half (green diamonds). The error bars are statistical from the Gaussian fit, and probably underestimate the true error.

5.1 ± 0.3 mm. The four reconstructed points along both the horizontal transverse and vertical transverse axes (Figure 38 – middle and bottom panels)

have average FWHM values of 5.4 mm, with respective standard deviations of 0.4 mm and 0.5 mm. The overall resolution of the reconstructed points is 5.3 ± 0.6 mm.

Some trends are observed in the data – the horizontal transverse resolution for the horizontal transverse points (**Figure 38b**, middle panel) seems to follow an arc, with the extreme points having lower FWHM values than the middle points. The same general trend can be seen in the vertical transverse resolution for the vertical transverse points (**Figure 38c**, bottom panel), with the exception of the FDK-half reconstruction.

3.3.1.2.2 FDK

FDK Half

The FDK half-circle reconstructions of the points along the axis of rotation and horizontal transverse axis have low axial FWHM values (**Figure 38a**, top and middle panels). The reconstructions of the points below the horizontal transverse axis along the vertical transverse axis have high FWHM values (i.e. 7.1 mm at 40 mm, **Figure 38c**, bottom panel). Otherwise, the half-circle resolutions compare to those of FDK-full.

3.3.1.2.3 DVS

DVS Half

The DVS half-circle reconstructions of the non-centred points along the horizontal transverse axis have low horizontal transverse FWHM values (**Figure**

38b, middle panel). The horizontal transverse FWHM value is high (max. 7.2 mm) from points below the horizontal transverse axis along the vertical transverse axis (Figure 38b, bottom panel). As well, the vertical transverse FWHM value is high (max. 7.7 mm) from the non-centred points along the horizontal transverse axis (Figure 38c, middle panel).

3.3.1.3 Intensity Measurements

The largest intensity variation is for the points along the vertical transverse axis. Table 3 is a tabulation of the normalised intensity from these points, for each reconstruction. The error in the intensities is dominated by the error in the point activity measurements, estimated at 5%.

Position				
30 mm	107	124	105	103
0 mm	100	100	100	100
-20 mm	104	73	103	89
-40 mm	92	48	86	64

Table 3. Intensities of the point source reconstructions along the vertical transverse axis, as a percentage of the reconstructed intensity at the centre.

From Table 3, it is seen that the point source displaced along the positive end of the vertical transverse axis (closest to the half-circle pinhole trajectory) has enhanced counts for the FDK-half reconstruction. The intensity drops off significantly when displaced 40 mm from the centre, farthest from the half-circle

pinhole trajectory. The same effect is observed in the DVS-half reconstruction, though not as severely. The intensity loss for the same point in the full-circle reconstructions is much smaller. The intensity is measured at only discrete points using this method – using line sources is a more efficient method to obtain intensity variations along an axis.

3.3.2 LINE PHANTOM RECONSTRUCTIONS

Three lines were imaged parallel to each other in three different configurations: with the central line along the axis of rotation (**Figure 39**), the horizontal transverse axis (**Figure 40**) and the vertical transverse axis (**Figure 41**). Each figure displays coronal, sagittal and transverse slices in the top, middle and bottom rows, respectively.

3.3.2.1 Observations

3.3.2.1.1 Overall

The reconstructions represent the lines, their intensities and the distances between them accurately in most cases. Resolution is adequate to resolve the lines separated by 1 cm in most reconstructions and most orientations. Nonetheless, some problems are seen.

The oblique tails that seem to point towards the centre top of the sagittal and transverse slices are again most evident in the half-circle reconstructions. There is evidence of streaking artefacts in the transverse slices of **Figure 40**,

and undershoots are visible in all three figures. While the cross-sections of the reconstructed lines in **Figure 41** are circular, the half-circle reconstructions of the 2 cm offset line in **Figure 39** and both offset lines in **Figure 40** are non circular.

Figure 39. Reconstructions of three lines, imaged with the central line oriented along the axis of rotation, in the horizontal plane. The figure setup is the same as of **Figure 36**. The sagittal slice is of only the reconstructed central line.

3.3.2.1.2 FDK

Both full and half

The correlated pattern is evident in the periphery of the transverse slices of **Figure 39**.

Figure 40. Reconstructions of three lines, imaged with the central line oriented along the horizontal transverse axis, in the horizontal plane. The figure layout is the same as of **Figure 36**.

FDK Half

Evidence of tails along the horizontal transverse axis is apparent from the central line (sagittal slice) in **Figure 39** and all three lines (coronal slice) in **Figure 40**. The two vertical lines separated by 1 cm tend to merge towards the bottom of the transverse slice (**Figure 41**). There is a large intensity variation along the length of the line in **Figure 41**, which also can be seen from the profile of the central line.

Figure 41. Reconstructions of three lines, with the central line oriented along the vertical transverse axis. The layout is the same as of **Figure 36**.

3.3.2.1.3 DVS

Both full and half

A circular pattern in the background is visible in the background of the transverse slices for all orientations.

DVS Half

Undershoots are seen between the reconstructed lines in **Figure 39** and **Figure 41**, and at the ends of the lines in **Figure 40**. The reconstructed horizontal lines in **Figure 40** blur together at their ends. The reconstructed intensi-

ties vary along the length of the line in **Figure 41**, though not as severely as for the FDK-half reconstruction.

3.3.2.2 Analysis

The line phantoms were quantitatively assessed for uniformity as described in section 3.2.4.1. Summed profiles were taken of the central line in the three different orientations for all reconstructions, and plotted in **Figure 42**.

3.3.2.2.1 Overall Intensity Results

Axial variations in the summed intensity along the length of the line are very small for both full- and half-circle, both FDK and DVS reconstructions (<5% variation). The variation in the half-circle reconstructions is about 10% along the horizontal transverse axis and, in the case of FDK-half, over 50% along the vertical transverse axis.

3.3.2.2.2 FDK

FDK Half

The profile of the reconstructed line along the horizontal transverse axis does not reach zero at the end of the line – the profile remains at about 10% of the value at the centre beyond the extent of the line. The positive end of the horizontal transverse reconstructed line has a higher intensity than at the centre by about 10%. Along the vertical transverse axis, the profile has a skewed

shape: the intensity varies linearly from ~60% of the centre to ~120% of the centre from the bottom to the top of the reconstructed line (-ve to +ve mm).

Figure 42. Summed profiles through the central line along all three axes for all four reconstructions. The data points are as in the legend. The top plot is for the line along the axis of rotation; the middle plot, the horizontal transverse axis; the bottom plot, the vertical transverse axis. The intensity profiles are normalised to the intensity at the centre point.

3.3.2.2.3 DVS

Both full and half

The intensity profiles for the DVS reconstructions are very similar. In the horizontal transverse direction, DVS-half has a slightly higher intensity at the positive end of the line (5%), and the profiles differ by about 10% at the low end of the line along the vertical transverse direction.

3.3.3 THYROID PHANTOM RECONSTRUCTIONS

A Picker thyroid phantom was imaged in the horizontal plane (the plane in

which a patient's thyroid would be when imaged). The phantom was imaged filled with 23 MBq of activity and coronal image slices are shown in **Figure 43** (FDK-full and -half) and **Figure 44** (DVS-full and -half). It was also imaged with

Figure 43. Coronal slices of the FDK reconstructions of the thyroid phantom filled with 23 MBq of Tc-99m. The top two rows are from the FDK-full reconstruction; the bottom two, the FDK-half. The phantom reconstruction is sliced from top to bottom, with a slice separation of 4.7 mm. The slices from each scan are normalised to the scan's maximum, with negative values shown to a level of -10% of the maximum. The colour scale is shown at right.

2 MBq of activity, and summed coronal image slices of the half circle reconstructions are shown in **Figure 45**.

3.3.3.1 Observations (FDK)

The shape of the thyroid phantom is clearly recognisable from the slices. In both the FDK-full and FDK-half slices (**Figure 43**), the three cold spots (12 mm – bottom left, 9 mm – top right, 6 mm – top left) are visible (1st and 3rd rows, 3rd and 4th columns). The hot spot is also well visualised (2nd and 4th rows, 2nd column). The FDK-half slices seem noisier than their FDK-full counterparts, and show evidence of tails in the horizontal transverse direction (3rd row, 3rd column). Undershoots are seen in both reconstructions, although more prominently in FDK-half.

3.3.3.2 Observations (DVS)

The shape of the thyroid phantom is preserved in the reconstruction (**Figure 44**). The cold spots and the hot spot can be well visualised on both DVS reconstructions, in the same slices as in the FDK reconstructions (section 3.3.3.1). Undershoots are apparent, most prominently in DVS-half. The two reconstructions seem to have a comparable level of noise in the activity dominated slices. The vertical transverse extent of the thyroid phantom in the DVS-half reconstruction (as seen in the coronal slices) is greater than the same extent in the corresponding FDK-half slices.

Figure 44. Coronal slices of the DVS reconstructions, set up similarly as in Figure 43.

3.3.3.3 Observations

The summed slices from both FDK-half and DVS-half reconstructions of the thyroid phantom filled with 2 MBq are noisy (Figure 45). In spite of this, the three cold spots can be visualised (rows 1 and 3, 4th column), though with less confidence than in the higher activity phantom (Figure 43 & Figure 44). The hot spot can be well visualised (rows 2 and 4, 2nd column). Undershoots can be

seen, but at different places in the FDK-half than from the DVS-half reconstructions. Summed slices from the DVS-half reconstruction have a more smoothed appearance than those from the FDK-half reconstruction. Tails in the horizontal

Figure 45. Summed coronal slices of the FDK-half (top rows) and DVS-half (bottom rows) of the thyroid phantom filled with 2 MBq of Tc-99m. Each summed slice shown is the sum of three slices, making the distance between each summed slice 4.7 mm (the same as in **Figure 43** & **Figure 44**). The intensity was chosen to match the colour intensity of the 23 MBq reconstruction.

transverse direction are evident in the hottest FDK-half summed slice (1st row, column 4 of **Figure 43**).

3.3.3.4 Analysis

The thyroid phantom reconstructions were quantitatively assessed for contrast, signal-to-noise ratio, percent background noise of the cold spots, and cold and hot spot sizes. These results are shown in **Figure 46** and **Figure 47** for the 23 MBq activity filled phantom, and in **Figure 48** for the 2 MBq activity filled phantom.

Figure 46. The contrast, signal-to-noise and percent background noise values for the 6, 9 and 12 mm cold spot (lesion) sizes of the thyroid phantom filled with 23 MBq of activity. The measurements were made from single slices only. The results are shown for the four reconstructions, as identified in the legend.

Figure 47. Size measurement made from summed slices, of the cold and hot lesions of the 23 MBq phantom. The top panel shows size measurements in the horizontal transverse direction, while the bottom panel is for measurements in the axial direction. The four reconstructions are as described in the legend. The dotted line is apparent size = lesion size.

The signal in **Figure 46** is the cold spot, while the background is activity. The contrast (Equation 30) increases with size, to near unity with the 12 mm cold spot, for all four reconstructions (top panel of **Figure 46**). Indeed, there is little difference in the contrast numbers from the four reconstructions.

The signal-to-noise ratio (middle panel) is similar for the various cold spots (averaging 11.5 ± 1.7) with the exception of the FDK-half reconstructions.

The percent background noise (bottom panel) is lowest for the DVS-full reconstruction, and is similar for the DVS-half and FDK-full reconstructions. The FDK-half values were appreciably higher than the others.

The size estimates for the cold spots in the 23 MBq activity filled thyroid

phantom are shown in **Figure 47**. The 6 mm spot seems to be well measured in both axial and horizontal transverse directions, with the full-circle reconstructions' estimates higher than those from the half-circle reconstructions. The size estimates are accurate and consistent for the 9 mm cold spot, with the exception of the FDK-half reconstruction. The size estimates of the 12 mm cold spot are consistent but low, while the 12 mm hot spot is overestimated and less consistent in the horizontal transverse direction. A size estimate could not be ob-

Figure 48. Results of measurements done on the thyroid phantom filled with 2 MBq of activity, for the FDK-half and DVS-half reconstructions. The left plots (a) and right plots (b) are in the same format - with the exception of the scales - as **Figure 46** & **Figure 47** respectively.

tained for the 12 mm hot spot in the axial direction for the DVS-half reconstruction. Overall, good agreement is seen between the measurements of the horizontal transverse sizes and the axial sizes.

The size estimates for the cold spots in the 2 MBq activity filled thyroid phantom are shown in **Figure 48**. The contrast measurements (**Figure 48a**, top panel) do not follow the same trend as in **Figure 46**; there seems to be no predictable pattern in either the behaviour of FDK-half or DVS-half, or their relation to each other. The same observation applies to the signal-to-noise measurements (**Figure 48a**, middle panel). The percent background noise seems to increase with cold lesion size, and DVS-half outperforms FDK-half.

The size measurements of 6 mm and 9 mm are consistent for both the FDK-half and DVS-half reconstructions, but are more accurate in the axial direction. The 12 mm hot spot size estimates are reasonable, while the estimates of the 12 mm cold spot in the FDK-half reconstruction are very low.

3.3.4 TIMING

Timings were done on a Sparc20 SUN workstation for only one reconstruction with no competing processes running. The results are shown in **Table 4**.

It should be noted that the Sparc20 is a comparatively slow computer, and that the DVS algorithm might execute in a clinically realistic time (<2 min) on a faster computer.

Full-circle	Half-circle
1 min 28 sec	0 min 42 sec
29 min 24 sec	24 min 28 sec

Table 4. Reconstruction times of the FDK and DVS algorithms, for both full-circle acquisition (128 projections) and half-circle acquisition (64 projections).

Neither of these algorithms is iterative. The ratio between execution times for a full-circle reconstruction is 10, while for a half-circle reconstruction is 15, FDK being faster. The DVS code may not be fully optimised, although it is unlikely to execute in a faster time than FDK.

3.4 DISCUSSION

3.4.1 CHARACTERISATION - GENERAL

The point response function (PRF) of an imaging system is fundamental. If the system is shift-invariant, one can predict the output of an imaging system by convolving the input with the PRF. This is approximately the case for gamma cameras with parallel hole collimators. However, with a pinhole collimator, the rays detected are from a divergent geometry, and thus a planar image of a point will have a different intensity depending on its position from the pinhole, as seen in section 2.1.1.1. Ideally, an image taken with a pinhole tomograph satisfying the sufficiency condition (section 3.1.2.1) can provide a shift-invariant reconstruction. However, as neither full- nor half-circle acquisition satisfies the sufficiency condition, the resulting reconstructions will have deviations from the shift-invariant ideal (more so for the half-circle case). The focus in the Results and in this Discussion is on these deviations.

As mentioned in section 3.3.1.3, the reconstructed points can be used to measure the intensity response of this system at different positions. It is much easier to make this measurement from the line response function (LRF). The concentration of activity in the tube is assumed uniform; no correction need be done along its length.

The full characterisation of an imaging system at all points in imaging space would be a very time consuming procedure. We have instead measured the

imaging characteristics along the three principal axes: the axis of rotation, the horizontal transverse axis and the vertical transverse axis.

3.4.2 POINT RESPONSE FUNCTION

The reconstructions of the points are very good – the points are clearly recognisable, their intensities and positions well represented in most cases. The position dependent response in the projections is no longer apparent in the reconstruction. Overall, the reconstruction of the points was successful, for both full- and half-circle acquisition, using both FDK and DVS algorithms. The reconstructions, however, had tail and undershoot artefacts.

3.4.2.1 Tails

3.4.2.1.1 "Pinhole Tails"

The most prominent tails are those that are oblique, seen in the sagittal slices, especially in the top half of the half-circle reconstructions (**Figure 36** and **Figure 37**). These are probably tails that point to the centre of the plane of the trajectory of the pinhole, as shown in **Figure 49**. The pinhole traces out a circle, and the projections corresponding to a half-circle acquisition were taken from the upper half of the circle (mapped to the tops of these slices). These residual tails point toward the sampled region of the severely incomplete projection set.

There are also prominent tails seen in the transverse slices, which are likely due to the insufficient acquisition.

Figure 49. Depiction of a possible explanation of the tails seen in the sagittal slices. The half-circle pinhole path is represented by the dotted line, and a partial cone is formed by this path and a point on the axis of rotation. This cone is then cut by a sagittal slice, and the low amplitude tail (depicted in red) can be seen.

3.4.2.1.2 "FDK-half tails"

The tails pointing away from the centre in the horizontal transverse direction from non-centred points along the horizontal transverse axis from only the FDK-half reconstruction (**Figure 36 & Figure 37**, col 2) are called "FDK-half tails" in this thesis. These are likely due to the abrupt change in weighting at the ends of the half-circle acquisition. They will be discussed with relation to the line response function and the thyroid phantom reconstructions.

3.4.2.2 Undershoots

Undershoots are artefacts of the reconstruction process coupled with insufficient sampling, assigning negative values to the final image. The projection images from which the reconstructed images are derived are simply planar images with non-negative entries. When reconstructing using a filtered backprojection algorithm, the (necessary) ramp filter in Fourier space results in a sinc

function with negative side lobes in object space, which, for an object completely sampled, should exactly cancel with the back-projected positive contributions from the other projections. When incomplete sampling and / or noise are present, these negative contributions may remain in the reconstructed image as an undershoot (a non-physical negative region).

3.4.2.3 Resolution

The axial resolution for all the points, and all resolutions from the points on the axis of rotation are very good. The average FWHM of 5.3 mm could not be achieved using a parallel hole collimator at the same distance – a more typical FWHM value for a high resolution parallel hole collimator at a 10cm distance is nearly 9mm. The variation in the resolution from the full-circle reconstructions is not large, while the variation is greater in the half-circle reconstructions (section 3.3.1.2). In most cases, the variability in the resolution can be accounted for using properties of the PRF, and the acquisition geometry.

3.4.2.3.1 Undershoots Affecting Resolution

The resolution from points along the horizontal transverse axis reveals two trends. First, the DVS-half algorithm has the smallest horizontal transverse resolution for the three non-centred points along the horizontal transverse axis (**Figure 38b**, second panel) - the best measured. This is directly attributable to the non-circular shape of the reconstructed points along the horizontal transverse axis – a distortion with the short axis along the horizontal. Secondly, the

vertical transverse resolution from the DVS-half reconstruction of the points along the horizontal transverse axis (**Figure 38c**, second panel), is the largest FWHM value among the four reconstructions - the worst measured. This is attributable to their long axes pointing towards the vertical. The same applies for the DVS-half points along the negative vertical transverse axis, except that the vertical transverse resolution of these points is good (**Figure 38c**, last panel), whereas the horizontal transverse resolution is poor (**Figure 38b**, last panel). That the vertical transverse resolution for these remote points is relatively good must be due to the undershoots unique to the DVS reconstruction; the geometry of the acquisition would suggest a poorer resolution.

3.4.2.3.2 *Geometry affecting Resolution*

The FWHM resolution in the vertical transverse direction for the FDK half-circle reconstructed points along the negative vertical transverse axis is the opposite of DVS-half - the resolution is poor. These points are poorly localised in the image. The poor localisation may be because the half-circle used surrounded the positive end of the vertical transverse axis, but did not get nearly as close to the negative end of the vertical transverse axis (section 2.2.1). Since resolution is a linear function of the distance from the pinhole, one might expect those points closer to the pinhole to have better reconstructed resolution than those that do not get close to the pinhole.

3.4.3 LINE RESPONSE FUNCTION

3.4.3.1 Intensity

The full-circle reconstructions of the lines parallel to the axis of rotation (or "axial lines"- **Figure 39**) are excellent – both the images and the profiles of the slices are as expected. The half-circle reconstructions show only slight distortions from the expected circular shape of lines in cross-section (transverse slices). The summed profile intensity is excellent too, approaching the ideal case (**Figure 42**, top panel).

There are some deviations from ideal in the horizontal and transverse directions, however, as noted in section 3.3.2, which can be accounted for.

3.4.3.1.1 Geometry Affecting Intensity

The vertical transverse lines and points were inadequately sampled in the half-circle geometry. Since the sensitivity is a function of the distance from the pinhole $\frac{1}{b^2}$ (section 2.1.1.1), one expects the points closest to the pinhole in the projection images to have higher reconstructed intensity than those farther away. This effect combined with an incomplete orbit probably accounts for the skew on the profiles of the vertical transverse lines, and the decreased intensity of the points along this axis. The effect is greater in the FDK than the DVS case, suggesting that the DVS algorithm handles this severely incomplete sampling better than FDK.

The small tilt on the horizontal transverse summed intensity profile can also be accounted for by the "half circle" acquisition. Similar to a full-circle orbit being 360° less one projection angle step (to avoid doubly sampling the first projection image), the half-circle orbit is actually only 177.2° . This results in the half-circle sampling being asymmetric with respect to the horizontal transverse axis. The positive end of the horizontal transverse line is slightly favoured, as suggested by the tilt on the summed intensity profiles.

3.4.3.1.2 *Tails affecting Intensity*

Since the FDK-half tails are apparent on the slices shown of the horizontal transverse lines (**Figure 40**), it is not surprising that they are also apparent in the summed intensity profile (**Figure 42**). While all other reconstructions have zero intensity at the beginning and end of the profile (beyond the end of the line), the FDK-half horizontal transverse intensity stays appreciably above zero in this area due to the FDK-half tails.

3.4.3.1.3 *Resolution Affecting Intensity*

The DVS-half reconstruction of the lines oriented parallel to the horizontal transverse axis (**Figure 40**, last column) looks different from the other reconstructions. The reconstructed lines blur into each other at their ends, and the profile along the length of the central line confirms that the intensity is highest at the centre, but drops to either side. However, a uniform intensity is recovered in the summed intensity profile (**Figure 42**), with only a slight tilt, explained in sec-

tion 3.4.3.1.1. Thus, the blur is not due to lost counts, but degraded resolution. As mentioned in section 3.4.2.3.1, the non-centred points along the horizontal transverse axis, reconstructed with the DVS-half algorithm, have poor resolution in the vertical transverse direction. This is one of the two directions orthogonal to the line - this poor resolution no doubt results in the observed blurring.

3.4.4 THYROID PHANTOM

3.4.4.1 23 MBq of activity

The overall success of the reconstructions of the point and line phantoms is echoed in the reconstruction of the higher activity thyroid phantom. Indeed, the position of the thyroid gland in a patient lying supine (in the horizontal plane) is favourable for half-circle pinhole imaging. The loss in intensity and variation in resolution for activity distant from the horizontal plane in FDK-half reconstruction will not affect thyroid imaging.

Nonetheless, there are tails and undershoots in the thyroid phantom reconstructions, as one might expect from the PRF analysis. Since the thyroid phantom is an extended object, one might expect the cumulative effect of the PRF to significantly affect the reconstructed images. As can be seen in **Figure 43**, there is evidence of the FDK-half tails in coronal slices of the thyroid phantom, and there are undershoots in all four reconstructions (**Figure 43 & Figure 44**). The large vertical transverse extent of the DVS-half reconstruction as evidenced in the coronal slices is probably attributable to the poor vertical transverse

resolution from activity along the horizontal transverse axis. This might also account for the inability to obtain a 50% threshold measurement from the DVS-half reconstruction of the 12 mm hot spot.

The analysis included taking contrast, SNR and %background noise measurements (**Figure 46**). Contrast is expected to get better with increasing target size up to a threshold, above which everything is detected with certainty. The contrast increases linearly from the 6 mm to the 9 mm to the 12 mm cold spot, where the contrast is one (perfect). As such, one is confident about a 12 mm lesion, and would be less certain about a 6 mm one, although the 6 mm one is evident in the slices. The half-circle reconstructions perform equally as well as the full-circle, and there is little difference between the FDK and DVS reconstructions.

The DVS reconstructions have the same SNR for all three cold spot sizes. In addition, the DVS-half SNR values are just as good as their full-circle counterparts, but the same cannot be said of the FDK reconstructions. The FDK-half reconstruction has the poorest SNR, consistent with the observation that the noise in these images is greater than the other three reconstructions. The same trend is observed concerning the %background noise, which again shows the FDK-half to be the poorest reconstruction.

Edge definition is a challenging task in image reconstruction. The choice of what defines an edge is not a trivial one, although it is handled with a simple 50% threshold in this analysis. The expected trend is for small cold spots to "fill

in" with the surrounding high background, and for hot spots to spread out to the surrounding low background. The latter is observed in the case of the 12 mm hot spot, which is estimated as being larger than 12 mm for all four algorithms (Figure 47). However, the small cold spots are surprisingly well estimated, whereas the large cold spot is poorly estimated. Evidently, the 50% threshold chosen to define the edge is not optimum for both the small and the large cold spots. Perhaps size estimation for these reconstructions would benefit from a more exacting approach to the problem (i.e. an iterative technique with a size dependent threshold).

3.4.4.2 2 MBq of activity

The 2 MBq half-circle reconstructions are expectedly inferior to those from the higher activity thyroid phantom. The slices are noise dominated, which is reflected in the analysis. The contrast and SNR measurements show none of the expected trends due to the increased level of noise (Figure 48). The percent background noise is proportional to cold lesion size, and the DVS-half reconstruction outperforms the FDK-half. The size measurements are acceptable, with the exception of the estimate of the FDK-half large cold spot, where the noise in the image prevents this approach to edge definition from being accurate. In the smoother DVS-half reconstruction, this problem is not as apparent. Although the quantification of the 2 MBq phantom is far from ideal, the identification of even the smallest cold spot from the summed slices shows promise for

this pinhole SPECT technique in the clinic, even if restricted to a half-circle of acquisition and low activity.

3.4.5 MISCELLANEOUS

The DVS reconstructions have a smoother appearance than their FDK counterparts, evidenced especially in the 2 MBq thyroid phantom half-circle reconstructions. The process of interpolation in the DVS reconstruction is due to the use of derivative filters twice, and rebinning the data into plane integrals. The FDK algorithm does not have as much filtering; consequently, the same images look noisier.

As mentioned, the transverse slices of some DVS reconstructions have the hint of a circle in the background surrounding the activity. This is most likely due to the presumption of the activity being within a circle of a chosen radius (45 mm in these reconstructions), as this is the surface at which the interpolations are done (section 3.1.2.2).

In the central transverse slices of the FDK horizontal points reconstructions, there is evidence of higher activity in alternating voxels at the periphery (section 3.3.1.1.2). This is likely a binning artefact due to the poorer radial sampling in the region outside the useful FOV.

Filtered back projection techniques are subject to streak artefacts when the images are noisy. They can be seen in transverse slices, and seem more evi-

dent in the FDK reconstructions than in the DVS, possibly because of the interpolation occurring in the DVS algorithm.

CHAPTER 4 CONCLUSIONS AND FUTURE WORK

4.1 CONCLUSIONS

Pinhole SPECT is still a novel idea in nuclear medicine, but the technique shows promise for routine clinical use. A magnification factor is achievable for objects that extend over a small FOV, which serves to decrease the FWHM value (increase the spatial resolution), an advantage that parallel hole collimators do not have.

In this study, phantoms were imaged using both full- and half-circle acquisition data from a pinhole tomograph. These were reconstructed using two different algorithms and the system was characterised by qualitatively noting the reconstruction artefacts, and quantitatively measuring the uniformity and resolution along the three principal axes. Overall, the results are promising: relatively uniform response and an average resolution of 5.3 mm. However, there are less ideal results along particular directions, especially for the half-circle acquisition.

The results of the project should be able to answer the two questions posed at the very beginning:

1. What, if any, are the disadvantages of acquiring only a half circle (180°) as compared to a full circle (360°) of projection images?

Close to the axis of rotation there are remarkably few differences between full- and half-circle reconstructions. As one moves from this axis, the differ-

ences increase. The asymmetric horizontal transverse tails in the horizontal transverse direction in the FDK-half-circle reconstructions and the poor vertical transverse resolution in the DVS-half-circle reconstructions serve as examples of the more salient differences between full- and half-circle reconstruction. Tail and undershoot artefacts are more obvious in half-circle reconstructions over full-circle reconstructions, as expected. If it is possible to collect a full circle of projections, this is preferable to anything less, but the degradation with half-circle acquisition and reconstruction might be tolerable.

2. Is the newer DVS algorithm superior to the classic FDK algorithm?

The two algorithms are analytic, but have quite different mathematical backgrounds behind them. The FDK approach views the pinhole projections as a compilation of tilted fanbeams and backprojects accordingly. The DVS approach is to reconstruct in the Radon domain, interpolating for unknown plane integrals. The FDK algorithm is known to be approximate, whereas the DVS attempts to compensate for unknown data to get a better reconstruction. The images produced by the two algorithms are not as different as one might expect from the much different methods of analysis. In most respects, the reconstructions from the two algorithms are comparable. There are, however, exceptions where one algorithm's reconstruction outperforms the other. For example, the DVS-half reconstructions preserve the intensity along the vertical transverse axis much better than FDK-half. In contrast, the FDK half-circle reconstructions of points along the horizontal transverse axis have superior vertical transverse

resolution than DVS-half. The FDK executes in a clinically realistic time on a Sparc 20 whereas DVS does not, but migrating to a faster platform would alleviate this problem. In short, the DVS algorithm does not seem to be inherently superior to the older, approximate FDK algorithm for our full- or half-circle geometry. It must be noted, however, that this study did not take advantage of the DVS algorithm's inherent acquisition flexibility.

As such, there is no clearly superior algorithm for the acquisition geometry used in this study.

4.2 FUTURE WORK

As pinhole tomography does seem to be an advantageous method for imaging small FOV organs, it is worth finding an optimal reconstruction algorithm. There seems to be no clear choice between the two analytical algorithms studied here, although it is perhaps worth studying an iterative algorithm in a similar fashion, and comparing with these results. A tilt to the camera head (Wanet *et al* 1994), which reduces the obstruction due to the patient's shoulders, is worth investigating.

Testing the DVS algorithm using a more flexible geometry (i.e. a set of projections that are not necessarily on a path) might be worthwhile. A modern camera with a flexible head position under computer control would be necessary to undertake such a study. If this provided improved reconstructed images, it might have some clinical applications.

The eventual goal is for pinhole SPECT to be a viable option for imaging patients' thyroid glands. One might attempt a volunteer study to help establish patient positioning and an acquisition protocol. A clinical trial comparing pinhole SPECT and planar imaging would be important in establishing this technique as a valid one for use in the clinic.

Overall, I believe pinhole SPECT has promise, and I hope that the work done in this project will help establish pinhole tomography at the Nuclear Medicine Department of the Ottawa Hospital, Civic Campus as a routine diagnostic test for patients with thyroid disease. I hope too that it might be expanded to image cervical spines, breasts, shoulders, ankles, knees, and any other anatomy that may benefit from being imaged with pinhole SPECT.

APPENDIX

Pre-manipulation

A few operations are performed on the projections before they are fed to the reconstruction algorithms. First, the projections were decay corrected from the scan start time (available through the Interfile data header), and were corrected for the pinhole response of $\cos^3\psi$, as the versions of the algorithms used did not account for the distance dependent $\cos^2\psi$ correctly (section 2.2.2). The data were converted from short integer to floating point numbers, and preceded by a short header as required by both the reconstruction algorithms and Qdisplay. The projections were oriented such that the “u” direction (rows of the projection images) was parallel to the axis of rotation, 90° different from the clinical data, as required by the reconstruction algorithms. Partial (half) circle reconstructions were accommodated by selecting a continuous arc of projection images from a full circle of acquisition, as only full-circle imaging was done. Finally, all projection images were mirrored ($-x \leftrightarrow x$, $-y \leftrightarrow y$), effectively converting from pinhole to conebeam projection images by undoing the pinhole inversion.

FDK algorithm

All parameters are kept in a file named “cone_defs.h”. These were: distance from detector to pinhole, distance from pinhole to object, number of projections, matrix size of projections, voxel size, number of voxels, centres of both the projection plane and the reconstruction cube, a stretching parameter and a

ramp filter parameter. The reconstruction orbit was saved in a separate file; the code to generate it has a direction of rotation flag, set opposite to the acquired data to convert the pinhole geometry to that of conebeam. The reconstruction sequence is as follows: generate an orbit file, prepare the projection images, and generate a reconstruction. To make an orbit file, one types “*make_CB_orbit orbit_file_name*”, creating a binary file. To subsequently reconstruct, one types “*feldfft orbit_file_name projection_file_name reconstruction_file_name [mode hanning cutoff]*”. The mode specifies either backprojection or reconstruction; hanning specifies the Hann apodizing filter parameter used for the reconstructions (which was default set to 0.5); cutoff refers to the ramp filter, and was set at the Nyquist frequency. The resultant reconstructed image has a file structure similar to the projections (header, followed by transverse slices in floating point numbers).

The FDK algorithm was written to reconstruct from a full circle of projections, ensuring (ideally) exact reconstruction in the central transverse slice. Using only a half circle of projections cannot yield an exact reconstruction in the same slice, although if non-uniform weighting was done, the imaging properties measured here might be quite different.

Discrete Vertex Set algorithm

The DVS parameter file contains all the information that the FDK “*cone_defs.h*” file does, plus angular range, and parameters unique to the DVS

reconstruction (roi_radius, q, phi_positions and theta_positions). The DVS algorithm had an additional step in its reconstruction process: prior to reconstructing; one must generate a Radon file. Thus, to build an orbit (full or partial circle), one executes "make_short_scan *parameter_file*". (This orbit code was modified similarly to the FDK code to convert pinhole projection images to conebeam ones.) This same orbit file can be used by the FDK code, thus allowing partial circle reconstructions. Following this, one executes "rebin_single *parameter_file*", in order to create the Radon file, using both the orbit and projection files. One then executes "fbp_marr *parameter_file*", which creates the reconstruction, in the same format as the FDK reconstructed file.

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