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**An Investigation Into the Effect of Aerodynamic Cross-Coupling
on Aircraft Motion at High Angles of Attack**

by
Abdulhamid Ashur Ghmmam

A thesis submitted to
the Faculty of Graduate Studies and Research
in the partial fulfilment of
the requirement for the degree of
Master of Engineering

Department of Mechanical and Aeronautical Engineering

**Carleton University
Ottawa, Ontario
Canada
March, 1987**

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ABSTRACT

High angle of attack aerodynamics and the effect on aerodynamic stability derivatives for high performance fighter aircraft is discussed. Mathematical modeling for aircraft motion under asymmetric flight conditions is also discussed, and equations describing the aircraft motion, for more general flight conditions than straight and level flight, are presented.

The need for a reliable method to solve the full non-linear equations of motion, and the application of numerical techniques is discussed. A suitable computer program was written and implemented. The motion time history of a fighter aircraft after certain control inputs, is presented with special emphasis on the effect of cross-coupling stability derivatives.

The results show that the effects of dynamic cross-coupling moment derivatives due to pitching are very significant, while effects of cross-coupling derivatives due to yawing are much less important.

Local linearization of some aerodynamic derivatives is a suitable approach for taking into consideration the non-linear variation of some stability derivatives with the angle of attack.

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List Of Symbols

b	-Wing span (m)
$\bar{c}$	-Wing mean aerodynamic chord (m)
I	-Moment or product of inertia (kg m^2)
m	-Mass(kg)
S	-Gross wing area (m^2)
c_{rt}	-Wing-root chord (m)
c_{tp}	-Wing-tip chord (m)
S_t	-Horizontal tail area (m^2)
l_t	-Distance between center of pressure of horizontal tail and aircraft's C.G (m)
S_F	-Fin area (m^2)
l_F	-Distance between fin aerodynamic center and aircraft's C.G (m)
Z_F	-Distance between fin aerodynamic center and body x-axis (m)
c_a/c	-Ratio of aileron chord to wing chord
τ_a	-Aileron effectiveness
c_r/c	-Ratio of rudder chord to vertical tail chord
τ_r	-Rudder effectiveness
c_e/c	-Ratio of elevator chord to horizontal tail chord
τ_e	-Elevator effectiveness
a_w	Wing lift curve slope (per radian)
a_t	-Horizontal tail lift curve slope (per radian)
a_F	-Vertical tail lift curve slope (per radian)
F	-Force(N)
L	-Aerodynamic rolling moment
M	-Aerodynamic pitching moment, or Mach number
N	-Aerodynamic yawing moment
g	-Acceleration of gravity
h	-Altitude(m)
$\bar{q}$	-Dynamic pressure (N/m^2)
T	-Thrust(N)
ρ	-Density of air (kg/m^3)

V	-Free stream velocity(m/s)
Φ	-Aircraft bank angle (deg or rad)
Φ_A	-Aerodynamic roll angle(deg or rad)
α	-Angle of attack (deg or rad)
β	-Angle of sideslip (deg or rad)
$\Delta\alpha$	-Difference between α and α_T (deg or rad)
θ, ϕ, ψ	-Euler angular rotations in, pitch, roll, and yaw, respectively (deg or rad)
u, v, w	-Body axes linear velocities (m/sec)
p, q, r	-Body axes angular velocities (rad/sec)
x, y, z	-Body-fixed axes system
C_{ip}	-Dynamic derivatives with respect to roll rate parameter $\partial C_i / \partial (pb/2V), i=1, m, n$
C_{iq}	-Dynamic derivatives with respect to pitch rate parameter $\partial C_i / \partial (qc/2V), i=1, m, n$
C_{ir}	-Dynamic derivatives with respect to yaw rate parameter $\partial C_i / \partial (rb/2V), i=1, m, n$
$C_{i\alpha}$	-Static derivative with respect to angle of attack $\partial C_i / \partial \alpha, i=1, m, n$
$C_{i\dot{\alpha}}$	-dynamic derivative with respect to α -rate of change parameter $\partial C_i / \partial (\dot{\alpha}c/2V), i=1, m, n, z$
$C_{i\beta}$	-Static derivative with respect to angle of sideslip $\partial C_i / \partial \beta, i=1, m, n, y$
$C_{i\dot{\beta}}$	-Dynamic derivative with respect to β -rate of change parameter $\partial C_i / \partial (\dot{\beta}b/2V), i=1, m, n, y$
$C_{i\delta_a}$	-Static derivative with respect to aileron deflection $\partial C_i / \partial \delta_a, i=1, n, y$
$C_{i\delta_e}$	-Static derivative with respect to elevator deflection $\partial C_i / \partial \delta_e, i=m, x, z$
$C_{i\delta_r}$	-Static derivative with respect to rudder deflection $\partial C_i / \partial \delta_r, i=1, n, y$
C_x, C_y, C_z	-Basic force coefficients with respect to x, y, z body axes respectively (Aerodynamic force component)/ $\bar{q}S$

C_l	-Rolling moment coefficient, $L/(qSb)$
C_m	-Pitching moment coefficient, $M/(qSc)$
C_n	-Yawing moment coefficient, $N/(qSb)$
X, Y, Z	-Earth fixed orthogonal axes with origin at origin of flight and with Z axis pointing vertically down.
γ_v	-Vertical flight path angle with respect to horizontal plane
γ_H	-Horizontal flight path angle with respect to vertical plane

Subscripts

(a dot), refers to rate of change with respect to time
 (0), refers to value at time zero
 (T), refers to value at trim (reference) condition
 (x,y,z), refers to x,y,z body axes

Abbreviations

A/C

Aircraft

C.G

Center of gravity

Chapter 1

INTRODUCTION

The high performance modern military aircraft often operates at high angles of attack during close-in air-to-air combat, where extreme maneuvers are performed. In such engagements and maneuvers the aircraft must be capable of controlled flight. Fighter aircraft which are not designed to maneuver at high angles of attack, could suffer a loss of control and could experience serious problems regarding handling qualities. The military aircraft sometimes also flies at moderate angle of attack but at non-zero angle of sideslip, which is equivalent to flying at high angle of attack as far as the behavior of the flow on the aircraft is concerned. Such flight conditions give rise to very complex aerodynamic phenomena, that have to be well understood and dealt with.

In short, the most important flow phenomena associated with high angle of attack are the formation and asymmetric shedding of forebody vortices, and the formation and break down of wing leading edge asymmetric vortices (vortex burst). The interaction between these two asymmetric flows complicates the picture even more. This problem occurs mainly on pointed-nose aircraft such as high performance fighters. The phenomenon as a whole is very complicated, and it is dependent, to a large extent, on the geometry of the aircraft.

The effect of asymmetric flow behavior at high angles of attack on the dynamic stability derivatives can be summarized in the following points:

- 1) Strong non-linear variations of stability derivatives with the angle of attack.
- 2) The importance of cross-coupling stability derivatives.
- 3) The existence of time dependent parameters such as those due to the rate of change of the angles of attack and sideslip.

Because of numerous effects of asymmetric flow at high angle of attack flight conditions,

lateral aerodynamic reactions may develop as a result of a pitching maneuver and longitudinal forces and moments as a result of a lateral maneuver. Also because the reactions take place both in-phase and out of-phase with the primary motion, static and dynamic cross-coupling derivatives will occur. In the presence of such cross-coupling effects, the longitudinal and the lateral groups of the equations of motion of the aircraft have to be solved simultaneously, differing from the traditional way of solving them separately at least when linearization is possible. Numerical techniques is one approach to solve the full non-linear equations of motion using digital computer. Other methods using hybrid computers have been used also. (Ref. 8)

For correct prediction of the motion of an aircraft at high angle of attack, certain cross-coupling stability derivatives have to be included in the equations of motion. In the past there was a tendency to neglect the strong non-linear variations of some stability derivatives (traditional and cross-coupling) and give them one fixed value; this was mainly because the analysis of aircraft motion history were applied to small and moderate angles of attack, where non-linearities and time-dependent effects are negligible and aerodynamic cross-coupling does not exist. In recent years this approach is no longer applicable, and the non-linearities have to be taken in to account. One way of doing this is by linearizing the equations around a certain angle of attack, where the strong variations take place with relatively small change in that parameter.

It is inevitable that more work be done to improve the mathematical models and the methods to predict the dynamics of aircraft at high angle of attack flight regime. In recognition of this need, several research institutions around the world are working on research programs designed to provide means of correct prediction of motion for modern fighter configurations and missiles at high angles of attack.

The dynamic stability study documented in this thesis was conducted at Carleton University using the main frame computer facilities available in CP-6 computing system. The primary goal of this work was to look further into techniques for analyzing the aircraft motion at high angles of attack, and, to examine the sensitivity of the aircraft motion to

certain perturbations from steady state as well as the relative importance of some cross-coupling stability derivatives.

To meet the objective a mathematical model simulating aircraft motion had to be set up. The model was applied to a hypothetical high performance fighter airplane. The information flow diagram was done in a block form (see Fig. 3.1) to show how different blocks interact with each other. Then the relevant computer program had to be developed in a subroutine form, such that each subroutine could be checked and debugged separately. The computer variables were used as long words whenever possible so that they are easy to relate to the engineering variables. The Fortran programming language was used. The numerical integration was performed using a fourth order RUNGE-KUTTA method with a time step of 0.025 seconds, over a time duration of 8 seconds.

Aircraft motion time histories, after certain perturbations in the control surface deflections for initially straight and level and for (2g) turning flight, are presented with emphasis on the effect of cross-coupling derivatives.

Chapter 2

mathematical modeling

2.1 Background

In order to place the role of dynamic stability analysis in perspective and demonstrate the bases for accurate analytical prediction of aircraft motion at extreme maneuvers, it is useful to understand the basic aerodynamic concepts and other topics related to the subject. In this thesis we will only consider the case of high performance modern military aircraft; the same principles of analysis could be applied to other areas of interest, like missiles and space shuttle, since they are also subjected to flying at high angles of attack, or with a combination of moderate angle of attack and non-zero angle of sideslip.

2.1.1 High angle of attack aerodynamics

To achieve tactical superiority in the air, modern military fighter aircraft are often designed to be capable of high performance maneuvers. To perform such maneuvers the aircraft must be able to operate under controlled flight at high angle of attack. Experience has shown that fighter aircraft configurations not designed for flight at high angle of attack may exhibit severe degradation in terms of stability and control characteristics in such flight conditions, which make them extremely susceptible to inadvertent loss of control, departure, and spins from which recovery may be difficult or some times impossible (Ref. 7). Military aircraft also fly at moderate angle of attack and small angle of sideslip, where flow may become highly asymmetric and significant non-linear effects take place. These aerodynamic phenomena have become the focus of studies and analysis (theoretically and experimentally), because of their role in aircraft dynamics.

The irregular and asymmetric flow phenomena associated to flying at high- α (see Fig. 2.1) vary significantly with that parameter. For a flying aircraft, when the angle of attack changes from small to moderate, flow separation over different components of the aircraft, such as wings, forebody and tail surfaces, start to occur. As the angle of attack increases

further, the cross flow around the fuselage becomes of more dominant effect and begins to sweep the boundary layer to the leeward side of the body, and then, gradual rolling up into a pair of vortices occurs. With even more increase in the angle of attack these vortices, which were formed on the body, become asymmetric. At about the same time the vortices of the wing leading edge are formed, and they become unstable, and then eventually break down, at some location on the wing chord, to form what is known as vortex burst. Some times the flow over the wing and the body vortices may interact with each other, which increases the complexity of the flow field even more.

From discussion above we conclude that there are essentially two important phenomena associated with high- α , 1) the formation and asymmetric shedding of forebody vortices and 2) the formation and asymmetric bursting of wing leading-edge vortices. It has become known that these flow phenomena have their direct impact on the dynamic stability parameters of the aircraft. In fact, these phenomena can happen in both steady and oscillatory flows, but when the element of time is introduced in the case of oscillating motion, the effect on dynamic stability is more evident, since the various vortices change their lateral positions as a function of the angle of attack, which itself is a function of time. The effect of high- α on the dynamic stability derivatives, then can be stated as:

- *Large non-linear variations of stability derivatives with the angle of attack:*

Figure 2.2a, shows the variation of the subsonic damping derivatives in pitch ($C_{m\dot{\alpha}} + C_{m\dot{\alpha}}$) and yaw ($C_{nr} - C_{n\dot{\beta}} \cos \alpha$) for a wing-body configuration, for a range of angle of attack from 0 to 40°, while Figure 2.2b, shows the low-subsonic cross derivatives ($C_{np} + C_{n\dot{\beta}} \sin \alpha$), and ($C_{lr} - C_{l\dot{\beta}} \cos \alpha$) for a swept wing fighter aircraft. The magnitude and the suddenness of the variation with the angle of attack are quite clear. In reality if it happens that the angle of attack about which the oscillation takes place is in the region of non-linearity, then a large effect of amplitude can be expected, since the stability derivatives undergo large changes even with small corresponding changes in the angle of attack. Similar non-linearities for other stability derivatives are also exist (although not shown here).

- *Significant static and dynamic aerodynamic cross-coupling between lateral and longitudinal degrees of freedom:*

This is the second important effect of high- α on dynamic stability, where aerodynamic cross-coupling exists as a result of asymmetric flow; this causes the occurrence of secondary

lateral aerodynamic forces and moments as a response to a primary pitching maneuver, and, longitudinal reactions as a response to a primary lateral maneuver. These reactions, because of various time lags, take place in-phase and out-of-phase with the primary motion. Therefore, they will give rise to both static and dynamic cross-coupling effects which leads to introducing separate groups for static and dynamic cross-coupling derivatives.

A model which resembles an aircraft situated in flow of Mach number ($M=0.7$) at an angle of sideslip of 10° is shown in Figure 2.3. It is obvious that the flow separation lines move from one side of the body to the other side as the angle of attack is increased from 12.5° to 17.5° ; this indicates the corresponding movement of one of the forebody vortices. If the model were oscillating in pitch around an equilibrium angle of attack (say 15°) with an amplitude of $2 - 3^\circ$, then the vortex would be oscillating laterally, causing lateral aerodynamic reactions, such as yawing and rolling moments and side force, as functions of the angle of attack. These reactions are both in-phase and out-of-phase with the model motion because of the time lag involved during the movement of the vortex.

Once the flow becomes asymmetric due to highly dissimilar flows over the wings of the aircraft....etc, the other type of cross-coupling should also exist, i.e longitudinal forces and moments due to a lateral motion, such as static and dynamic pitching moments due to yawing. All these cross-coupling effects are represented by their corresponding stability derivatives, as for example, pitching moment derivative due to yawing C_{m_r} and rolling moment derivative due to pitching C_{l_q} , separating them from the traditional cross derivatives for example yawing moment derivatives due to rolling C_{n_p} , or rolling moment derivative due to yawing C_{l_r} .

Non-linear variation of some of the cross coupling derivatives is shown in Figure 2.3b. The non-linear behavior of the dynamic cross-coupling moment derivatives due to pitching ($C_{n_q} + C_{n\dot{\alpha}}$) and ($C_{l_q} + C_{l\dot{\alpha}}$), and due to yawing ($C_{m_r} - C_{m\beta} \cos \alpha$), can be easily seen at high- α , or even low angle of attack in case of yawing, where sideslip angle is non-zero.

• *Various time-dependent and hysteresis effects:*

The derivatives of various aerodynamic reactions due to angular velocities (p, q, r) represent the quasi-steady effects. In addition to these, we must consider the purely unsteady effects existing in oscillating flow such as derivatives due to the time rate of change of aerodynamic angles, $\dot{\alpha}$ or $\dot{\beta}$. These derivatives form part of the dynamic results obtained during

standard wind-tunnel experiments of oscillations around a fixed axis, where the output is always an expression composed of more than one derivative, such as $(C_{m\dot{q}} + C_{m\dot{\alpha}})$. In the past the effects of $\dot{\alpha}$ and $\dot{\beta}$ were usually ignored, or dealt with by a simple correction. This simplification could be justified since the error introduced by such a procedure was often small at low angles of attack (Ref. 32).

The case with high angles of attack, however, is much more complex, since the effects of $\dot{\alpha}$ and $\dot{\beta}$ become substantial and can no longer be ignored or corrected by a simple formula. Figure 2.4 presents a comparison between the purely rotary derivatives and the composite derivatives obtained from experiments of a body oscillating about a fixed axis for two fighter aircraft. The difference between the derivatives for each case can be ascribed to the unsteady effects due to the rate of change of the aerodynamic angles with respect to time, which is $\dot{\beta}$ in this case.

As a first approximation, derivatives due to the rate of change of aerodynamic angles with time, are aerodynamically equivalent to derivatives due to translational acceleration in the same plane of motion, i.e. effects due to $\dot{\alpha}$ are the same as the effects of translational acceleration in the vertical plane, and those of $\dot{\beta}$ are equivalent to translational acceleration in the horizontal plane of motion. This gives them high interest for aircraft designs which use direct-sideforce or direct-lift controls; it also makes it possible to determine them experimentally using translational oscillations in the desired plane of motion.

Flow phenomena at high angles of attack, such as asymmetric vortex shedding and vortex burst, are sensitive to variations in α and β which are responsible for aerodynamic hysteresis effects, e.g. the location of vortex burst in an aircraft wing is affected by α -hysteresis, and the variation of lateral aerodynamic coefficients are affected by β -hysteresis. Because of the presence of such aerodynamic hysteresis effects the dynamic aerodynamic derivatives of an aircraft can be expected to be dependent on both the amplitude and the frequency of the oscillation. For better understanding of high angle of attack aerodynamics, see the paper by Orlik-Ruckemann (Ref. 32).

2.1.2 Configuration dependence

The aerodynamic phenomena discussed in the above paragraphs, are not only dependent on the angles of attack and sideslip, and their rates of change, but are also very

sensitive to aircraft geometry. Such phenomena usually happen with long-pointed nose (high fineness ratio) fighter aircraft. They are sensitive to any change in the aircraft shape especially the nose where the body vortices are formed.

There have been some techniques introduced to ensure a symmetrical shedding of forebody vortices, such as forebody STRAKES, altering the geometry of the nose (shark nose), and helical trips on the forebody. Any of these methods has its advantages, such as giving a symmetrical shedding pattern, increasing the dynamic directional stability, distributing the shed vorticity along the forebody, but they have some disadvantages also, for example the working range is affected by the angle of attack, and by the rate of rotation in roll and increase of the bank angle (Ref. 32).

Figure 2.5a shows the point of vortex burst as a function of both the angle of attack and the sweep back angle of delta wings as obtained in water tunnel tests (Ref. 42), and figure (2.5 b,c) illustrates the effect of strakes on some stability derivatives, and the nose shape on dynamic directional stability of the body respectively.

Different aircraft components with different planforms can have a wide range of effects on stability derivatives; more of these are shown in Figure 2.6 a,b.

2.1.3 Modelling

All the above-mentioned effects have to be included when it comes to the problem of modelling the aircraft motion at high angles of attack. Mathematical models used in the past to describe the aircraft flight history were applied mainly at small to moderate angles of attack, where non-linearities are small, time-dependent effects are not significant and aerodynamic cross-coupling is nonexistent. More sophisticated mathematical modelling must be used to deal with one of the difficult problems in aircraft flight dynamics, that is the formulation of models with sufficient accuracy to cover the wide range of maneuvers typical of modern aircraft, and takes care of different problems when flying at high angle of attack, such as, a) non-linearities, b) aerodynamic cross-coupling, c) time-dependent and hysteresis effects, d) strong configuration dependence, which means the need for determining a complete set of stability parameters for a particular configuration.

At high angle of attack flight conditions, as we have seen, aerodynamic cross-coupling is a dominant phenomenon which suggests that the two modes (lateral and longitudinal)

of aircraft motion are strongly coupled and this requires a six-degree-of-freedom motion analysis, since the separation of the aircraft equations of motion into two independent groups can no longer be applied as it is in the case of symmetric flight conditions. Both hybrid and digital computers have been used to solve the two sets of equation of motion simultaneously and the results were essentially the same (Ref. 15). In the present case a digital computer was used to perform the numerical integration. More information about mathematical modelling is presented in the following sections in this chapter.

The simulation of aircraft motion has become a very important tool in the development, testing and operational phases of modern fighter aircraft programs. This is done by integration of the non-linear equations of motion, after some perturbation starting from a known flight condition (e.g straight level flight, or a coordinated turn of load factor $=n$) and recording the aircraft flight history for some time after the perturbation. Details of this method will be seen in the upcoming chapters.

The system of equations required to perform a dynamic study contains, (a) the proper equations of motion, (b) the external loading equations, (c) the auxiliary equations, (d) the necessary physical characteristics for the assumed aircraft, (e) the initial condition equations, and, (f) the set of stability and control derivatives. The equations presented in the following sections are given without going in to the derivations from the basic principles, as this can be found in many books on flight mechanics (such as Ref. 10). Symbols and parameters are defined in the list of symbols at the front of this thesis.

2.2 EQUATIONS OF MOTION

This set of non-linear first order differential equations, represents the aircraft translational and rotational motion referred to the (x, y, z) body-fixed system of axes that are rigidly attached to the aircraft and therefore move with it, Fig.2.7. The expressions for the equations represent Newton's laws for translation along and rotation about each axis. It is assumed that:

- i) the aircraft is a single rigid body with xz a plane of symmetry
- ii) the earth is a stationary plane in inertial space, and
- iii) the motion of the aircraft consists of small disturbances from a reference condition of steady, symmetric flight.

The equations of motion for an aircraft, referred to the body-fixed axes are then as follows:

$$\dot{u} = \sum F_x/m - qw + rv \quad (2.2.1)$$

$$\dot{v} = \sum F_y/m - ru + pw \quad (2.2.2)$$

$$\dot{w} = \sum F_z/m - pv + qu \quad (2.2.3)$$

$$\dot{p} = [\sum L - (I_x - I_y)qr + I_{xz}(\dot{r} + pq)]/I_x \quad (2.2.4)$$

$$\dot{q} = [\sum M - (I_x - I_z)rp + I_{xz}(r^2 - p^2)]/I_y \quad (2.2.5)$$

$$\dot{r} = [\sum N - (I_y - I_z)pq + I_{xz}(\dot{p} - qr)]/I_z \quad (2.2.6)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (2.2.7)$$

$$\dot{\phi} = p + q\sin\phi \tan\theta + r\cos\phi \tan\theta \quad (2.2.8)$$

$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta \quad (2.2.9)$$

Knowing that a (dot) bears the usual meaning of rate of change with respect to time, (u, v, w) represent the linear velocity components along the three body axes (x, y, z) respectively, while (p, q, r) are the angular velocities around the same axes. The three angles (θ, ϕ, ψ) are the vehicle Euler angles which determine the orientation of the aircraft at any time. θ is given by a rotation about the y -axis, and is also known as pitch angle. ϕ is given by a rotation about the x -axis and is called roll angle, and ψ is generated by a rotation about z -axis giving yaw angle. In these equations $\sum F_x, \sum F_y, \sum F_z$ are the externally applied forces on the aircraft, and $\sum L, \sum M$, and $\sum N$ represent the three externally applied moments about the three body axes x, y, z . For better understanding of the angles and the directions of different velocities the reader is advised to refer to Figure 2.7. A complete derivation can be found in any text on aircraft dynamics. See Reference 10 for example.

2.3 LOADING EQUATIONS

The external loading on the aircraft consists mainly of three types of forces and associated moments, forces due to gravity (weight), forces due to the power plant (thrust) and, aerodynamic forces as a result of the airplane motion in the air. The determination and description of these forces is a vital subject. It is obvious that the estimation of the first

two types is not difficult, while the situation is unlikely to be the same for the last group of forces and moments, primarily because they are due to aerodynamic effects. A great deal of total effort in aerodynamic research has been devoted towards the exploration of means, experimentally and theoretically, to determine the stability derivatives needed for applications to flight mechanics.

For brevity we will explain only the basic concept of how the aerodynamic reactions are described, although more details could be found in other places, eg. Reference 10. The method for specifying aerodynamic force and moment relations is that developed by Bryan (see Ref. 10). It is based on the hypothesis that aerodynamic reactions can be expanded with respect to the instantaneous values of the disturbance velocities u, v, w, p, q, r and their time derivatives. Expansions for forces and moments are obtained in the form of a Taylor series in these variables which is linearized by neglecting the higher order terms. It is usually assumed that we have a symmetric aircraft in a condition initially of a symmetric flight. This will reduce the number of variables needed to describe the aerodynamic reactions. But in general this is not a necessary assumption and any terms omitted can be restored to the equations. This model has been subjected to many modifications to include other effects such as non-linearities. Recently the concept of linear aerodynamic indicial functions and superposition, to describe the relationship between the aerodynamic reaction and the motion variables, have been developed, to improve the modeling of the aerodynamics of today's high performance aircraft. For more information see papers by Tobak and by Schiff (Refs. 46,47).

In conclusion, the aerodynamic forces and moments are described as a function of the state variables (e.g. angle of attack, sideslip.....etc.). Using the linear aerodynamic theory every aerodynamic reaction can be expressed in the form of stability derivatives (such as $C_{x\alpha}, C_{l\beta}$etc.) multiplied by the corresponding variables.

Making use of the assumptions made in section 2.2, and referring again to Fig.2.7, the equations describing the external forces ($\sum F_x, \sum F_y, \sum F_z$) and moments ($\sum L, \sum M, \sum N$) on the aircraft are given as below.

$$\sum F_z = T - mg \sin \theta + (C_z + C_{z\delta_s} \delta_s) \bar{q} S \quad (2.3.1)$$

$$\sum F_y = mg \cos \theta \sin \phi + (C_{y\beta} \beta + C_{y\delta_a} \delta_a + C_{y\delta_r} \delta_r) \bar{q} S \quad (2.3.2)$$

$$\begin{aligned} \sum F_x = mg \cos \theta \cos \phi + [(C_{xT} + C_{x\alpha}(\alpha - \alpha_T) + C_{xq}(q\bar{c}/2V) + C_{x\dot{\alpha}}(\dot{\alpha}\bar{c}/2V) \\ + C_{x\delta_s} \delta_s] \bar{q} S \end{aligned} \quad (2.3.3)$$

$$\begin{aligned} \sum L = [C_{lT} + C_{l\alpha}(\alpha - \alpha_T) + C_{l\beta} \beta + C_{lp}(pb/2V) + C_{lq}(q\bar{c}/2V) + C_{l\dot{\alpha}}(\dot{\alpha}\bar{c}/2V) \\ + C_{lr}(rb/2V) + C_{l\dot{\beta}}(\dot{\beta}b/2V) + C_{l\delta_a} \delta_a + C_{l\delta_r} \delta_r] \bar{q} S b. \end{aligned} \quad (2.3.4)$$

$$\begin{aligned} \sum M = [C_{mT} + C_{m\alpha}(\alpha - \alpha_T) + C_{m\beta} \beta + C_{mp}(pb/2V) + C_{mq}(q\bar{c}/2V) + C_{m\dot{\alpha}}(\dot{\alpha}\bar{c}/2V) \\ + C_{mr}(rb/2V) + C_{m\dot{\beta}}(\dot{\beta}b/2V) + C_{m\delta_s} \delta_s] \bar{q} S \bar{c} \end{aligned} \quad (2.3.5)$$

$$\begin{aligned} \sum N = [C_{nT} + C_{n\beta} \beta + C_{n\alpha}(\alpha - \alpha_T) + C_{np}(pb/2V) + C_{nr}(rb/2V) + C_{n\dot{\beta}}(\dot{\beta}b/2V) \\ + C_{nq}(q\bar{c}/2V) + C_{n\dot{\alpha}}(\dot{\alpha}\bar{c}/2V) + C_{n\delta_a} \delta_a + C_{n\delta_r} \delta_r] \bar{q} S b \end{aligned} \quad (2.3.6)$$

The above summations F_z, F_y, F_x and L, M, N are the forces and moments (about the CG) on the aircraft, in each of the six degrees of freedom.

2.4 AUXILIARY EQUATIONS

Equations for the rate of change of the aircraft position ($\dot{X}, \dot{Y}, \dot{Z}$) with respect to a fixed point on the earth, and equation for angles of attack, sideslip, and, their rates of change ...etc, are required to complete the analysis.

$$\begin{aligned}\dot{X} = & u \cos \psi \cos \theta + v(\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi) \\ & + w(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi)\end{aligned}\quad (2.4.1)$$

$$\begin{aligned}\dot{Y} = & u \sin \psi \cos \theta + v(\sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi) \\ & + w(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi)\end{aligned}\quad (2.4.2)$$

$$\dot{Z} = -u \sin \theta + v \cos \theta \sin \phi + w \cos \theta \cos \phi \quad (2.4.3)$$

$$\alpha = \arctan(w/u) \quad (2.4.4)$$

$$\beta = \arctan(v/u) \quad (2.4.5)$$

$$\dot{\alpha} = (u\dot{w} - \dot{u}w)/(u^2 + w^2) \quad (2.4.6)$$

$$\dot{\beta} = (u\dot{v} - \dot{u}v)/(u^2 + v^2) \quad (2.4.7)$$

$$\gamma_H = \arctan(\dot{Y}/\dot{X}) \quad (2.4.8)$$

$$\gamma_V = \arctan[-\dot{Z}/(\dot{X}^2 + \dot{Y}^2)^{1/2}] \quad (2.4.9)$$

$$V = (u^2 + v^2 + w^2)^{1/2} \quad (2.4.10)$$

$$\Phi_A = \arctan(v/w) \quad (2.4.11)$$

Fig 2.8 shows the horizontal and vertical flight path angles γ_H and γ_V .

2.5 INITIAL CONDITION EQUATIONS

A sensitivity study requires the use of suitable initial conditions for a more general state of flight than straight and level flight; such a state may require aerodynamic trimming around several axes. There is a substantial number of parameters that have to be included in such initial conditions.

Two initial, or reference, states of flight were used in the present work :

- 1) straight and level.
- 2) steady turning at constant load factor, in a horizontal plane.

The expressions used to determine the nominal values of the parameters just before the perturbation occurs for both straight and level and a steady turning at a constant load factor flight, were as follows:

$$\bar{q}_0 = nmg / [-C_x(\sin \alpha_0 \tan \alpha_0 + \cos \alpha_0)S] \quad (2.5.1)$$

$$T = (-C_x \tan \alpha_0 - C_z)\bar{q}_0 S \quad (2.5.2)$$

$$\rho_0 = 1.2253e^{-(h_0/10150)} \quad (2.5.3)$$

$$V_0 = (2\bar{q}_0/\rho_0)^{1/2} \quad (2.5.4)$$

$$\Phi_0 = \arccos(1/n) \quad (2.5.5)$$

$$\theta_0 = \arcsin(\sin \Phi_0 \sin \beta_0 + \cos \Phi_0 \sin \delta \cos \beta_0) \quad (2.5.6)$$

$$\psi_0 = \arcsin[(\sin \Phi_0 \sin \delta \cos \beta_0 - \cos \Phi_0 \sin \beta_0) / \cos \theta_0] \quad (2.5.7)$$

$$\phi_0 = \arcsin[(\sin \Phi_0 \cos \beta_0 - \cos \Phi_0 \sin \delta \sin \beta_0) / \cos \theta_0] \quad (2.5.8)$$

$$\dot{\psi}_0 = g \sin \phi_0 / [(\cos \Phi_0 \cos \alpha_0 + \tan \alpha_0 \sin \alpha_0)V_0] \quad (2.5.9)$$

$$p_0 \equiv -\dot{\psi}_0 \sin \theta_0 \quad (2.5.10)$$

$$q_0 = \dot{\psi}_0 \cos \theta_0 \sin \phi_0 \quad (2.5.11)$$

$$r_0 = \dot{\psi}_0 \cos \theta_0 \cos \phi_0 \quad (2.5.12)$$

$$C_{xT} = C_x - C_{xq}(q_0 \bar{c} / 2V_0) \quad (2.5.13)$$

$$C_{lT} = -C_{lp}(p_0 b / 2V_0) - C_{lq}(q_0 \bar{c} / 2V_0) - C_{lr}(r_0 b / 2V_0) \quad (2.5.14)$$

$$C_{mT} = -C_{mq}(q_0 \bar{c} / 2V_0) - C_{mr}(r_0 b / 2V_0) \quad (2.5.15)$$

$$C_{nT} = -C_{np}(p_0 b / 2V_0) - C_{nr}(r_0 b / 2V_0) - C_{nq}(q_0 \bar{c} / 2V_0) \quad (2.5.16)$$

where $\delta = \arctan(\tan \alpha_0 \cos \beta_0)$.

For an equilibrium angle of attack of (33°) , and altitude of 9960 m, consistent with the assumed initial velocity (V_0) and dynamic pressure ($\bar{q}_0$) for the hypothetical aircraft of section 2.6, using the above equations, the set of parameter values for straight and 2 g turning unperturbed flight are as follows:

parameter	straight level flight(n=1)	coordinated turn flight(n=2)	units
$\bar{q}_0$	3414	6828	N/m^2
T	100400	200800	N
ρ_0	0.4593	0.4593	Kg/m^3
V_0	121.9	172.4	m/s
Φ_0	0.0	60.0(1.047)	deg(rad)
θ_0	33(0.576)	15.8(0.2758)	deg(rad)
Φ_0	0.0	64.17(1.1199)	deg(rad)
ψ_0	0.0	29.35(0.5123)	deg(rad)
$\dot{\psi}_0$	0.0	5.644(0.09851)	deg/s(rad/s)
p_0	0.0	-1.537(-0.02683)	deg/s(rad/s)
q_0	0.0	4.888(0.08531)	deg/s(rad/s)
r_0	0.0	2.367(0.04131)	deg/s(rad/s)
C_{LT}	-0.841	-0.82531	
C_{IT}	0.0	-0.001237	
C_{mT}	0.0	0.008303	
C_n	0.0	-0.000205	

2.6 THE PHYSICAL CHARACTERISTICS

The aircraft chosen for simulation was a hypothetical one, but it was assumed that it possessed physical characteristics representative of a high performance military fighter aircraft (Refs. 8,11). The characteristics necessary to perform the aircraft motion simulation were the following :

b	$=11.89 \text{ m}$	
$\bar{c}$	$=4.88 \text{ m}$	
I_z	$=36605 \text{ kgm}^2$	
I_y	$=162700 \text{ kgm}^2$	
I_x	$=183000 \text{ kgm}^2$	
I_{xz}	$=6780 \text{ kgm}^2$	
mass(m)	$=17510 \text{ kg}$	
S	$=50.2 \text{ m}^2$	
c_{rl}	$=7.16 \text{ m}$	
c_{tp}	$=1.2 \text{ m}$	
S_t	$=16 \text{ m}^2$	
l_t	$=7.85 \text{ m}$	(estimated)
S_F	$=6.27 \text{ m}^2$	
l_F	$=7.1 \text{ m}$	(estimated)
z_F	$=2 \text{ m}$	(estimated)
c_a/c	$=0.24$	(estimated)
τ_a	$=0.21$	(estimated)
c_r/c	$=0.164$	(estimated)
τ_r	$=0.18$	(estimated)
c_e/c	$=0.45$	(estimated)
τ_e	$=0.4$	(estimated)
a_w	$=1.2 \text{ per radian}$	(estimated)
a_t	$=4.7 \text{ per radian}$	(estimated)
a_F	$=4.0 \text{ per radian}$	(estimated)

These estimated parameters had to be assumed depending upon the available data, or calculated using other designated parameters and charts for flap effectiveness (Refs. 3, 10, 41).

Figure 2.9 shows a three-view sketch for the hypothetical airplane with some, but not all, dimensions.

2.7 STABILITY AND CONTROL DERIVATIVES

As seen in section 2.3, aerodynamic reactions are described in terms of their gradients with respect to motion variables that are evaluated at the reference flight condition. For instance the aerodynamic rolling moment is expressed as:

$$L = L_p.p + L_r.r + \dots \text{other terms}$$

The symbol L_p stands for $(\frac{\partial L}{\partial p})_0$, similarly M_r stands for $(\frac{\partial M}{\partial r})_0$, and so on, such quantities being called the stability derivatives of the aircraft. The subscript zero indicates that the derivative is to be evaluated at the reference flight condition. In practice, stability derivatives are usually put into nondimensional form, and the equations of motion are treated nondimensionally. Aerodynamic stability derivatives include static derivatives (the variation of a given reaction with aerodynamic angles α or β) and dynamic derivatives. The various dynamic derivatives include those due to pure rotation (p, q, r) such as the damping derivatives (C_{mp}, C_{nr}, C_{lp}), and cross-coupling derivatives ($C_{mr}, C_{lq}, \dots \text{etc}$). Derivatives due to time rate of change of aerodynamic angles ($\dot{\alpha}, \dot{\beta}$) are also included to account for effects of oscillating motion at high angles of attack.

The set of stability derivatives consists of basic force coefficients, aerodynamic derivatives and, control derivatives. The traditional aerodynamic derivatives were based on low speed NASA experiments on an aircraft configuration representative of the hypothetical aircraft that is used in this study (Refs. 11, 6). The cross-coupling derivatives were based on data available in References 30, 31; these data were based on a similar, but simplified, aircraft configuration at higher speed. This inconsistency had to be accepted, since no other data for aerodynamic cross-coupling for the same configuration and speed are available. The cross-coupling derivative C_{mp} , and all dynamic force derivatives, except C_{sq} and $C_{s\dot{\alpha}}$, were assumed to be zero, because their values were not known. The pure rotary derivatives, such as, C_{lq} , C_{nq} and the acceleration derivatives such as, $C_{l\dot{\alpha}}$, $C_{n\dot{\alpha}}, \dots \text{etc}$, were given nominal values in agreement with their combined values e.g. $(C_{lq} + C_{l\dot{\alpha}})$ measured from oscillation around a fixed body axis. It was assumed that both derivatives contribute equally to the combined value (Ref. 8). Some derivatives were locally linearized around the designated equilibrium angle of attack of (33°) . As mentioned previously, this is because of the highly non-linear behavior of these derivatives in this angle of attack regime. To do this local

linearization the range of $\Delta\alpha$ had to be kept to about $\pm 3^\circ$. Some control derivatives were estimated using the aircraft physical characteristics and the methods available in the literature (Refs. 3, 10, 41); this is illustrated in Appendix(A). The definitions of traditional and cross-coupling moment derivatives in their non-dimensional form are given below.

Traditional moment derivatives

$$C_{l\beta} = \partial C_l / \partial \beta$$

$$C_{l\dot{\beta}} = \partial C_l / \partial (\dot{\beta}b/2V)$$

$$C_{lp} = \partial C_l / \partial (pb/2V)$$

$$C_{lr} = \partial C_l / \partial (rb/2V)$$

$$C_{m\alpha} = \partial C_m / \partial \alpha$$

$$C_{m\dot{\alpha}} = \partial C_m / \partial (\dot{\alpha}\bar{c}/2V)$$

$$C_{mq} = \partial C_m / \partial (q\bar{c}/2V)$$

$$C_{n\beta} = \partial C_n / \partial \beta$$

$$C_{n\dot{\beta}} = \partial C_n / \partial (\dot{\beta}b/2V)$$

$$C_{np} = \partial C_n / \partial (pb/2V)$$

$$C_{nr} = \partial C_n / \partial (rb/2V)$$

Cross-Coupling moment derivatives

$$C_{l\alpha} = \partial C_l / \partial \alpha$$

$$C_{l\dot{\alpha}} = \partial C_l / \partial (\dot{\alpha}\bar{c}/2V)$$

$$C_{lq} = \partial C_l / \partial (q\bar{c}/2V)$$

$$C_{m\beta} = \partial C_m / \partial \beta$$

$$C_{m\dot{\beta}} = \partial C_m / \partial (\dot{\beta}b/2V)$$

$$C_{mp} = \partial C_m / \partial (pb/2V)$$

$$C_{mr} = \partial C_m / \partial (rb/2V)$$

$$C_{n\alpha} = \partial C_n / \partial \alpha$$

$$C_{n\dot{\alpha}} = \partial C_n / \partial (\dot{\alpha}\bar{c}/2V)$$

$$C_{nq} = \partial C_n / \partial (q\bar{c}/2V)$$

Similarly the force derivatives are defined as follows:

$$C_{y\beta} = \partial C_y / \partial \beta$$

$$C_{z\alpha} = \partial C_z / \partial \alpha$$

$$C_{zq} = \partial C_z / \partial (q\bar{c}/2V)$$

$$C_{z\dot{\alpha}} = \partial C_z / \partial (\dot{\alpha}\bar{c}/2V)$$

The force and moment derivatives due to different controls are defined by:

Aileron derivatives

$$C_{y\delta_a} = \partial C_y / \partial \delta_a$$

$$C_{l\delta_a} = \partial C_l / \partial \delta_a$$

$$C_{n\delta_a} = \partial C_n / \partial \delta_a$$

Elevator derivatives

$$C_{z\delta_e} = \partial C_z / \partial \delta_e$$

$$C_{s\delta_e} = \partial C_s / \partial \delta_e$$

$$C_{m\delta_e} = \partial C_m / \partial \delta_e$$

Rudder derivatives

$$C_{y\delta_r} = \partial C_y / \partial \delta_r$$

$$C_{l\delta_r} = \partial C_l / \partial \delta_r$$

$$C_{n\delta_r} = \partial C_n / \partial \delta_r$$

Finally the set of aerodynamic and control derivatives used in the simulation of the hypothetical aircraft were as follows:

$$C_z = -0.04$$

$$C_y = 0.0$$

$$C_x = -0.841$$

$$C_{l\alpha} = 0.0$$

$$C_{l\beta} = -0.2$$

$$C_{lp} = -0.3$$

$$C_{lq} = 0.5 - 30\Delta\alpha$$

$$C_{l\dot{\alpha}} = 0.5 - 30\Delta\alpha$$

$$C_{lr} = 0.25$$

$$C_{l\beta} = -0.3$$

$$C_{m\alpha} = -0.6$$

$$C_{m\beta} = -0.1$$

$$C_{mp} = 0.0$$

$$C_{mq} = -7 - 120\Delta\alpha$$

$$C_{m\dot{\alpha}} = -7 - 120\Delta\alpha$$

$$C_{mr} = 0.1 - 25\Delta\alpha$$

$$C_{m\beta} = -0.1 - 25\Delta\alpha$$

$$C_{y\beta} = -0.8$$

$$C_{n\alpha} = 0.05$$

$$C_{n\beta} = 0.25$$

$$C_{np} = 0.2$$

$$C_{nq} = 0.5 - 60\Delta\alpha$$

$$C_{n\dot{\alpha}} = 0.5 - 60\Delta\alpha$$

$$C_{nr} = -0.15 - 6\Delta\alpha$$

$$C_{n\beta} = 0.2 - 6\Delta\alpha$$

$$C_{z\alpha} = -2$$

$$C_{zq} = -13$$

$$C_{z\dot{\alpha}} = -13$$

$$C_{y\delta_a} = 0.0$$

$$C_{l\delta_a} = -0.0143$$

$$C_{n\delta_a} = 0.00214$$

$$C_{z\delta_e} = 0.0$$

$$C_{z\delta_e} = -0.599$$

$$C_{m\delta_e} = -0.963$$

$$C_{y\delta_r} = 0.101$$

$$C_{l\delta_r} = 0.017$$

$$C_{n\delta_r} = -0.0604$$

Chapter 3

COMPUTER SIMULATION

3.1 Choice of computer program

It has been mentioned that the simulation of aircraft motion by solving the six-degree-of-freedom equations of motion is a vital step in aircraft development programs. By solving the full equations of motion, allowing non-linear and cross-coupling terms, we are able to predict the pattern of aircraft motion after some perturbation. Furthermore, this allows us to test the motion sensitivity to any change in any of the aerodynamics derivatives or other parameter of interest. This brought up the need, some years ago, to find reliable methods for solving the aircraft full-equations of motion in the high angle of attack flight regime.

In summary, sensitivity studies are usually carried out by programming on a computer, as complete a set as possible of equations of motion pertinent to a particular configuration and flight condition. This is done by inserting a set of stability derivatives, including the non-linear effects, by varying those parameters in a predetermined manner, and by observing the response of the variables of motion to some form of disturbance applied to the set of equations. The system of equations required to do the sensitivity study was described in CHAPTER 2.

To conduct the study, equations of motion and other related mathematical expressions have to be programmed in such a way that the flow of events in the real flying situation is simulated. This requires a computer program that could handle the relatively large number of parameters, constants and variables, and manipulate them between different sets of equations in the proper time at the right position.

Some consideration was given to using the CP-6 library simulation packages, but after checking them, it was found that, most of them were intended for specific uses. In fact only one simulation language was intended for general purpose. This is called SIMUL8, and is designed primarily to solve systems of differential equations. This library subroutine has no capacity to deal with large scale problems, such as the subject matter of this thesis,

mainly because 1) each block was only designed to do one job, e.g. integrator, summer, multiplier and etc., 2) initialization for different integrators has to be given on line, and 3) the number of parameters that could be defined for each block can not be more than three. It can be seen, because of these limitations, that this simulation package is only useful in small scale problems.

In recognition of the large number of parameters, equations, and expressions necessary to perform the aircraft motion simulation, the author adopted the general approach of writing his own computer program to fulfill the requirements of a complete study.

3.2 Numerical integration

The computer program was written in the FORTRAN computer programming language. Since there are large number of parameters, the computer variables were chosen so as to facilitate the recognition of different parameters by other users. For example:

$C_{m\alpha}$	CMALPHA
$\sum F_x$	SUMFX
$\bar{c}$	CDASH
$\alpha_T(\text{radians})$	ALPHATR
$C_{n\delta_r}$	CNDELRL

A complete set of parameters and corresponding FORTRAN variables are found in Appendix (B).

Computer simulation of aircraft motion involves a numerical integration of the motion variables $\dot{u}$, $\dot{p}$, $\dot{\theta}$, etc. There are many techniques for numerical integration, such as the PREDICTOR CORRECTOR method, the RUNGE-KUTTA method and others. The fact that every method requires certain initial conditions to start with, led the author to choose the RUNGE-KUTTA technique for numerical integration. The equations of motion, as known, are given in terms of rate of change of motion variables with time. Thus the RUNGE-KUTTA method is quite suitable, since it requires only the value of the function and its first derivative at the start time.

Partial instability can occur with any numerical integration technique. But this problem could be eliminated by using a higher order method, and also by using a small step

size for integration. Therefore a fourth order RUNGE-KUTTA method with a step size $\Delta T = 0.025$ seconds, was used. The RUNGE-KUTTA technique and the final formula for integration are given in Appendix (C).

3.3 Information flow diagram

To prepare the equations of aircraft motion for computer simulation, different sets of equations have to be put into groups, each in one separate block. The relationship between different blocks can then be drawn by inspecting each block individually and finding out what parameters are needed for that block and from where they must come. This defines the necessary flow of information required for the simulation. Figure 3.1 shows the information flow diagram for aircraft motion simulation in block form. This diagram is helpful in building up the required computer program. It is clear that the blocks are closely related to each other. A large number of parameters have to be transferred from one block to the others, and there is always a feed back after each integration step.

For the purposes of discussion, assume that at time T_1 , we initiate some disturbance in the angle of attack (α), sideslip (β), or control setting, then from block (3) the initial conditions could be found. These will remain the same, since the trim angle of attack or sideslip is the same. Block (2) gives the stability and control derivatives, then block (4) provides the required loading. Block (5) determines the motion variables $\dot{u}, \dot{v}, \dots, \dot{\psi}$ at time T_1 , then integrates them to give $u, v, \dots, \psi$ at time $T_1 + \Delta T$. Block (6) then, computes the other complementary parameters, such as $\dot{X}, \dot{Y}, \dot{Z}$, which are integrated to give the aircraft position, where the aircraft orientation θ, ϕ, ψ was found from block (2). From block (6) we also obtain $\beta, \alpha, \dot{\beta}$, and $\dot{\alpha}$ to feed them back to find the new set of aerodynamic derivatives, which are functions of α , and the new loading to repeat the operation again for another time step ΔT .

3.4 Program Subroutines

In recognition of the nature of the problem the computer program had to be built in a subroutine form, which means each set (block) of equations or expressions was programmed in a separate subroutine, so that each subroutine could be checked and debugged separately, after providing the necessary input data.

The computer program consists of a main program and a number of subroutines. The main part of the program controls the sequence of operations performed for each simulation manipulates different parameters between different subroutines, and extracts the desired aircraft motion history parameters to put them in the form of output. To avoid mixing between parameters, each common block carries the same parameters wherever this block exists, except where some parameters are not needed. Then they are simply replaced by symbols that do not exist anywhere else in the program such as $P_1, P_2, \dots$ etc. A complete listing of the computer program is given in Appendix D. Below, the different subroutines in the "FLIGHT" program are described.

1) Subroutine AIRCRAFT

This subroutine has the physical characteristics of the hypothetical fighter aircraft. It also contains the trim angles of attack and sideslip, and it reads the input for the initial value for the angles of attack α_0 , sideslip β_0 , and control perturbations δ_a , δ_e , and δ_r . These angles are converted to radians. The common blocks used to transfer data from this subroutine are SET1 to SET6. The control perturbation angles read by subroutine AIRCRAFT represent an impulse function. To change the perturbation to a step function, the user must go to subroutine REINITIAL, which will be discussed later.

2) Subroutine BASIC

Subroutine BASIC comprises the aerodynamic stability derivatives and the control derivatives. It also reads the aerodynamic derivatives for the current case of study. The derivative/s for which the sensitivity of the aircraft motion is going to be examined is multiplied by certain constants that could be given any value, and varied from one case to another. Note that for the first time step we put $\Delta\alpha = 0$ to compute the stability derivatives at the reference flight state, since some of these derivatives are used later to find the trim condition parameters such as $C_{LT}, C_{mT}, \dots$ etc. The output of this subroutine is obtained from blocks BASIC1 to BASIC6.

3) Subroutine INITIAL

This subroutine calculates the initial condition parameters necessary for the simulation.

These parameters are functions of the angle of attack, sideslip, and load factor, which are provided by subroutine AIRCRAFT. Common block INITIAL1 is the output of the subroutine.

4) Subroutine START

Subroutine START has the starting parameters, some of which are already found by subroutine INITIAL, others are found in terms of the new initial angle of attack or sideslip. This subroutine initialize the parameters at time zero, and also defines the size of the time step that will be used for numerical integration. Common blocks OLD1 to OLD5 are the subroutine output.

5) Subroutine LOADING

As the name applies, this subroutine finds out the loading forces and moments on the aircraft $\sum F_x, \sum F_y, \dots, \sum N$. The output is written in block LOADING1.

6) Subroutine MOTION

The motion variables $\dot{u}, \dot{v}, \dots, \dot{\psi}$ are calculated using subroutine MOTION, then the RUNGE-KUTTA method is used for numerical integration to find the motion parameters at time $(T + \Delta T)$

7) Subroutine AUXILIARY

The expressions found in block (6) in the information flow diagram are solved in this subroutine. Then the numerical integration is used to find the aircraft position (X, Y, Z) with respect to a fixed point at the origin where the perturbation occurred. This subroutine also finds the instantaneous values of the angles of attack, sideslip, and their rates of change. The subroutine output is found in common block AUXILIARY1.

8) Subroutine NEW

This subroutine is designed to update the values of different parameters, so the new values can be used for the following step in the loop. The angles of attack, sideslip, and also some stability derivatives which are functions of α , are updated. It also computes the horizontal and vertical flight path angles, and the aerodynamic roll angle Φ_A .

9) Subroutine REINITIAL

This is a very small subroutine, it contains only the three control perturbation angles given the value zero. For holding the control perturbation throughout the simulation (ie. a step perturbation), simply put a comment sign in front of each line.

Figure 3.2 illustrates the flow chart for the computer program, and shows how different subroutines communicate with each other.

3.5 FEATURES OF THE COMPUTER PROGRAM

To facilitate the use of the computer program, which was designed to perform the aircraft motion simulation, the author tried to make it as easy to follow as possible. The most important features of the program are summarized below;

- 1) Any common block, carrying the same name, has the same parameters wherever this block exists. This is to avoid confusion by using the same block name for different parameters at different places.
- 2) Starting conditions for motion simulation, such as the angle of attack, sideslip, control perturbation, load factor, and the constants representing the chosen stability derivatives for study, are given as ON LINE data. This is done in order to provide flexibility to the user to choose the case he wants in every simulation. For example, for $\delta_r=10^\circ$, we can have load factor $n=1$, or 2, and at the same time we have the desired stability derivatives, which can be changed as many times as required, (note that in the present program it is limited to seven cases).
- 3) All angles that have to be fed on line are given in degrees, so the user can have physical sense of what the situation is, e.g $\alpha_0=36$, $\beta_0=-2$, $\delta_a=10$ etc. These parameters are

provided by the user after the parameter prompt with the question mark appears on the screen.

3.6 GUIDELINES FOR THE PROGRAM USER

For the program FLIGHT to be used for the sensitivity study for aircraft motion, the user is advised to carry out the following steps:

- a) Determine the study cases and what parameters have to be changed or input to the computer. This could be easily done if a diagram is drawn representing the study for different cases as in Figure 3.3.
- b) In the present program, the control perturbations are considered as an impulse input. The length of the time for the impulse input, can be determined by using a statement in FLIGHT after CALL NEW, by making COUNT.LT.12 for 0.3 second and LT.24 for 0.6 seconds. To change them to a step input, the user has only to put a comment sign in front of the three lines defining δ_a , δ_e , and δ_r in subroutine REINITIAL, so the control perturbations maintain the same value assigned initially.
- c) The constants A1,A2,B1,B2,C1,C2,D1 and, D2, are to be predetermined depending on the stability derivatives that are going to be examined. In the FLIGHT program, A1,A2,B1,B2 are assigned to the stability derivatives to be studied, C1,C2 for the rest of the cross-coupling stability coefficients, and finally, D1,D2 are given to the remaining locally linearized traditional derivatives. This is done to allow the possibility of involving traditional derivatives in the study. For example, if we take the case of C_{nq} and $C_{n\dot{\alpha}}$, then the study cases and the corresponding constants will be as follows;
 - case #1: constant stability coefficients with C_{nq} and $C_{n\dot{\alpha}} = 0$, $A1=B1=0$, $C1=1$, $C2=0$, $D1=D2=1$
 - case #2: constant coefficients all with nominal values, $A1=B1=1$, $C1=1$, $A2=B2=0$, $C2=0$, $D1=D2=1$
 - case #3: constant coefficients with C_{nq} and $C_{n\dot{\alpha}}$ doubled, $A1=B1=2$, $C1=1$, $A2=B2=0$, $C2=0$, $D1=D2=1$
 - case #4: locally linearized coefficients with C_{nq} and $C_{n\dot{\alpha}} = 0$, $A1=B1=0$, $A2=B2=0$, $C1=C2=1$, $D1=D2=1$

- case #5: locally linearized coefficients all with nominal values, $A1=B1=1, A2=B2=1, C1=C2=1, D1=D2=1$
- case #6: locally linearized coefficients with $C_{n\eta}$ and $C_{n\alpha}$ doubled, $A1=B1=2, A2=B2=2, C1=C2=1, D1=D2=1$
- case #7: all cross-coupling derivatives =0, $A1=B1=0, A2=B2=0, C1=C2=0, D1=D2=1$

Note that these study cases could be changed by changing the coefficients and, also increased to more than seven cases. Finally, the user has to make some changes in subroutines BASIC, START, and NEW to specify the desired stability derivatives and then put different constants at the proper places.

Chapter 4

AIRCRAFT MOTION TIME HISTORIES

In this chapter the results of the computer study for aircraft response to certain control inputs are presented in graphical form. Both straight and level ($N=1$) flight, and coordinated turn ($N=2$) flight are considered. The initial condition parameters for an equilibrium angle of attack of 33° and angle of sideslip zero were discussed and given in chapter 2, section 2.5, for both flight cases.

The method is to start the simulation with the initial angles of attack and sideslip the same as in the equilibrium case, then to perturb the control setting for some time to initiate an impulse perturbation. The length of perturbation time depends on the control surface being disturbed; for rudder perturbations, it is 0.6 second, while for elevator perturbation 0.3 seconds, after which we record the aircraft motion history over a time of 8 seconds from the start of the simulation.

The effect of different stability derivatives on the aircraft motion is introduced by choosing the desired stability derivative/s and giving it different values. The coefficients are given values of zero, the nominal value, or double the nominal value; the nominal values for all stability derivatives were given in chapter 2, section 2.7. This can be easily done since the computer program was designed to accept these values as on line data. This is done with the set of stability derivatives considered either to be constant or locally linearized. If constant, the constant part of the stability derivative/s has to be changed for each study case, while in the case of locally linearized stability derivatives, both the constant part, and the slope have to be made zero, nominal, or doubled. For treating different stability derivatives we will use the same terminology as in Reference 8. For example, C_{nq} , $C_{l\dot{\alpha}}$etc, are referred to as dynamic cross-coupling derivatives, $C_{m\beta}$, $C_{n\alpha}$, as static cross-coupling derivatives, C_{lr} , $C_{l\dot{\beta}}$etc, as traditional cross derivatives, and Φ_A as aerodynamic roll angle....etc.

This sensitivity study will be carried out with the emphasis on the effect of cross-coupling aerodynamic derivatives, such as C_{lq} , $C_{l\dot{\alpha}}$, C_{nq} , $C_{n\dot{\alpha}}$, and C_{mr} , $C_{m\dot{\beta}}$. The effect

of some cross derivatives will also be examined, for example, the effect of C_{lr} , $C_{l\dot{\beta}}$, as well as that of traditional derivatives such as C_{nr} , $C_{n\dot{\beta}}$. Some stability derivatives are usually found in pairs as a result of wind tunnel experiments with oscillation about a fixed body axis, as discussed in chapter 2, section 2.7. These will be considered as one study case, such as the combination (C_{lq} and $C_{l\dot{\alpha}}$).

The choice of the control input depends upon the stability derivative/s that is going to be studied. If the case is for (C_{lq} and $C_{l\dot{\alpha}}$), then the perturbation would be in the elevator setting, and the aircraft response in β or Φ_A would be presented. If it is for (C_{mr} and $C_{m\dot{\beta}}$) then the rudder setting must be disturbed, and the aircraft response in α or $\dot{\alpha}$ is recorded. For the sake of analysis more and different motion parameters will be presented. The sign convention for aerodynamic control surface perturbations is as follows :

Elevator perturbation δ_e	down	+ve
δ_e	up	-ve
Rudder perturbation δ_r	left	+ve
δ_r	right	-ve
Aileron perturbation δ_a	right down, left up	+ve
δ_a	right up, left down	-ve

The simulation was performed for constant thrust (both magnitude and direction), and several runs had to be made for each study case to choose the magnitude of control perturbation, so that the change in the angle of attack from the trim condition, $\Delta\alpha$, was not more than $\pm 3^\circ$, in the 8 seconds period studied, in order to ensure that local linearization of stability derivatives, at the vicinity of the trim angle of attack, is valid.

The motion variables chosen to be presented as aircraft motion time histories, such as the aerodynamic roll angle (Φ_A), vertical flight path angle (γ_V), and horizontal flight path angle (γ_H)...etc, were defined in chapter 2, section 2.4. For better understanding of γ_H and γ_V , it would be helpful to refer to Figure 2.8.

Finally, the results of digital computer simulation are presented in Figures 4.1 to 4.17. Figures 4.1, and 4.2 show the effect of changing (C_{lq} and $C_{l\dot{\alpha}}$), on aircraft motion, after elevator perturbation, for straight and level and 2g turning, initial flight conditions,

respectively. Figures 4.4, 4.5 show some effects of changing the cross-coupling derivative combination (C_{nq} and $C_{n\dot{\alpha}}$) under the same conditions. Figure 4.3 shows the effects of C_{lq} and $C_{l\dot{\alpha}}$ separately for an initial condition of 2g turning flight, while Fig. 4.6 shows the effect of treating C_{nq} and $C_{n\dot{\alpha}}$ separately for the same initial condition.

Some effects of the static cross-coupling derivative $C_{n\alpha}$, for initial 2g turning flight, are presented in Fig. 4.7. Figures 4.8 and 4.9 show the angular rates, after some perturbation in the elevator angle, for initially straight and level flight and 2g turning flight respectively.

Figure 4.10 shows the effect of changing (C_{mr} and $C_{m\dot{\beta}}$), for initial condition of straight and level flight, following rudder perturbation, while Fig. 4.11 shows the effect of the same combination of derivatives for the other initial flight condition. Figure 4.12 shows some effects of $C_{m\beta}$ for initial 2g turning flight.

The effect of the cross derivative combination (C_{lr} and $C_{l\dot{\beta}}$) for 2g turning initial flight condition is shown in Fig. 4.13. Figure 4.14 shows some effects of (C_{nr} and $C_{n\dot{\beta}}$) for the same initial condition. Figures 4.15, 4.16 show the angular rates after rudder perturbation, for straight and level flight, and 2g turning flight initial conditions respectively.

The last figure 4.17 shows the angular rates for the initial condition of 2g turning flight following either elevator or rudder perturbation. In this study case, all cross-coupling derivatives are zero, with the rest of derivatives locally linearized at their nominal values.

Chapter 5

DISCUSSION

Some sample results from using the computer study are presented in Figures 4.1 to 4.17. They give results of an investigation of the aircraft response to control perturbation. The sensitivity of the aircraft motion to some aerodynamic derivatives, is examined, with the emphasis on the effect of cross-coupling stability derivatives. In most cases the aircraft flight history is presented for both initially straight and level flight, and 2g turning flight. A computer graphics subroutine was used to draw the curves.

5.1 Sensitivity assesment

By doing a quick sensitivity analysis, one can find out the relative importance of different stability derivatives, and hence the need for the inclusion or omission of these derivatives in the aircraft equations of motion. The magnitude of any cross-coupling derivatives can be compared to the well established traditional derivatives in the appropriate equation of motion. As in Ref. 31, for example, for a captive aircraft model, the rolling moment due to the rate of yaw and rate of pitch is given by;

$$\begin{aligned} & (L_r - L_{\dot{\beta}} \cos \alpha) r + (L_q + L_{\dot{\alpha}}) q \\ &= (\bar{q} S b / 2V) [(C_{l_r} - C_{l_{\dot{\beta}}} \cos \alpha) b r + (C_{l_q} + C_{l_{\dot{\alpha}}}) \bar{c} q] \end{aligned}$$

Now taking the ratio of the cross-coupling term $(C_{l_q} + C_{l_{\dot{\alpha}}})$ to the traditional cross derivative term $(C_{l_r} - C_{l_{\dot{\beta}}} \cos \alpha)$, we get

$$\begin{aligned} & \frac{\bar{c} q}{b r} \frac{(C_{l_q} + C_{l_{\dot{\alpha}}})}{(C_{l_r} - C_{l_{\dot{\beta}}} \cos \alpha)} \\ &= \frac{\bar{c} q}{b r} \frac{(C_{l_q} + C_{l_{\dot{\alpha}}})}{(C_{l_r} - C_{l_{\dot{\beta}}} \cos \alpha)} \end{aligned}$$

For the present aircraft configuration $\frac{\bar{c}}{b} = \frac{4.88}{11.89} = 0.41$, or $\frac{b}{\bar{c}} = 2.436$

Substituting different values for different parameters, we have;

$$\begin{aligned} (0.41) \frac{q}{r} \frac{(0.5 + 0.5)}{(0.25 + 0.3 \cos 33)} \\ = 0.82 \frac{q}{r} \end{aligned} \quad (5.1.1)$$

Similarly, for the yawing moment terms, the ratio is

$$\begin{aligned} \frac{\bar{c} q}{b r} \frac{(C_{nq} + C_{n\alpha})}{(C_{nr} - C_{n\beta} \cos \alpha)} \\ = 0.41 \frac{q}{r} \frac{(0.5 + 0.5)}{(-.15 - 0.2 \cos 33)} \\ = 0.41 \frac{q}{r} (-3.147) \\ = -1.29 \frac{q}{r} \end{aligned} \quad (5.1.2)$$

Finally for pitching moment equation, we have;

$$\begin{aligned} \frac{b r}{\bar{c} q} \frac{(C_{mr} - C_{m\beta} \cos \alpha)}{(C_{mq} + C_{m\dot{\alpha}})} \\ = (2.436) \frac{r}{q} \frac{(0.1 + 0.1 \cos 33)}{(-7 - 7)} \\ = -0.032 \frac{r}{q} \end{aligned} \quad (5.1.3)$$

Now, for simplicity, if we assume that the rate of pitch (q) is equal to the rate of yaw (r), then 5.1.1 to 5.1.3 give the ratio of the cross coupling derivatives to the traditional derivatives for each case. They vary from about 3% to 130%. The minimal percentage is for the cross-coupling stability derivative in pitching moment, from which we can expect that, the aircraft response due to any change in cross-coupling derivatives in pitch, is less significant than the response due to any change in cross-coupling derivatives in yaw or roll. In the general case if (q) is not equal to (r), then these ratios will change; some will increase, some will decrease. This depends on the values of p and r , but in any case they hold some degree of significance. The reader must be cautioned that the above analysis is somewhat speculative, and must be confirmed by full analysis using the full set of the equations of motion, as is done in this thesis. Moreover, the ratios of 5.1.1 to 5.1.3 of course depend directly on the values of the cross-coupling derivatives.

5.2 Results of sensitivity study

The motion variables, shown as time histories, depend on the values of the derivative under investigation, on the characteristics of the aircraft as represented by values of the various other parameters, and on the flight condition. In the following sections we will discuss the effect of some stability derivatives on the motion of the hypothetical modern military aircraft, defined in chapter 2, sections 2.6, following some perturbation in the control setting. Nominal values for stability derivatives were discussed and presented in chapter 2, section 2.7.

5.2.1) Effect of (C_{l_q} and $C_{l_{\dot{\alpha}}}$)

Histories for aerodynamic roll angle Φ_A and horizontal flight path angle γ_H , for straight and level flight, were presented in Figure 4.1. Histories for Φ_A and the vertical flight path angle γ_V , were shown in Figure 4.2 for 2g turning flight.

It is quite clear from those figures that the motion is sensitive, to a noticeable degree, to (C_{l_q} and $C_{l_{\dot{\alpha}}}$). The extent of sensitivity depends upon the direction of perturbation, and the nature of the derivative. For negative elevator perturbation, for the case of straight and level flight, the constant coefficients have more effect as far as Φ_A is concerned. It does not seem to be the case for positive elevator perturbation. There the difference in Φ_A behavior is much more pronounced, this is probably because of the combined effect of the nature of derivative being considered, and the sign of q , $\dot{\alpha}$, and $\delta\alpha$. To examine this effect let us go through the following argument :

Take the case of -ve elevator perturbation;

		constant coefficients	product
$\delta_e(-ve)$	$\dot{\alpha}(+ve)$	$C_{l_{\dot{\alpha}}} = 0.5(+ve)$	$C_{l_{\dot{\alpha}}} \cdot \dot{\alpha}(+ve)$
	$q(+ve)$	$C_{l_q} = 0.5(+ve)$	$C_{l_q} \cdot q(+ve)$
	$\Delta\alpha(+ve)$		

With constant coefficients both products $C_{l_{\dot{\alpha}}} \cdot \dot{\alpha}$ and $C_{l_q} \cdot q$ are positive, this means less stability in roll.

	locally linearized	product
$\dot{\alpha}(+ve)$	$C_{l\dot{\alpha}} = 0.5 - 30\Delta\alpha(less + ve)$	$C_{l\dot{\alpha}}.\dot{\alpha}(less + ve)$
$q(+ve)$	$C_{lq} = 0.5 - 30\Delta\alpha(less + ve)$	$C_{lq}.q(less + ve)$
$\Delta\alpha(+ve)$		

In case of locally linearized coefficients the products $C_{l\dot{\alpha}}.\dot{\alpha}$ and $C_{lq}.q$ are less positive if compared to constant coefficients case, this might indicate more stability in roll. Obviously the opposite effect could happen with positive elevator perturbation. Therefore, in case of initially straight and level flight the motion is more stable in roll with either $\delta_e(-ve)$ and locally linearized coefficients, or $\delta_e(+ve)$ and constant coefficients. In fact, this is exactly what Fig. 4.1 shows.

For positive elevator perturbation, when using the set of stability derivatives as constants, the maximum amplitude is about 0.015 radians. This occurs at about 2 seconds. For locally linearized derivatives, (i.e function of the angle of attack), the maximum amplitude is around 0.03 radians. The effect on γ_H is much less than on Φ_A .

For 2g turning flight condition, and for the same direction of perturbation, Figure 4.2 show that Φ_A is different in magnitude, and direction in some cases for the same set of stability derivatives. The two directions of control perturbation give totally different figures. This suggests the strong effect of cross-coupling terms included in the equations of motion. When both derivatives are zeroed, doubled or given the nominal value, the behavior of Φ_A and γ_V is also changing, in some cases not only in amplitude but in sign as well.

For negative elevator input, the vertical flight path angle γ_V does not change much with the set of constant coefficients. The case of locally linearized derivatives gives noticeable changes in γ_V . Figure 4.2b shows that γ_V varies between 0.02 to 0.15 radians with corresponding gain of altitude of about 9 to 70 meters respectively. For positive elevator input and for the same set of stability derivatives (locally linearized), the loss of altitude ranges from 3 to 34 meters (Fig. 4.2d). Note that when we use the nominal set for stability derivatives we allow for the combined effect of all the derivatives at the same time. In almost all cases the first 4 seconds of the simulation are important for Φ_A , while the last 4 seconds are for either γ_V or γ_H .

Figure 4.3 shows the effect of treating C_{lq} and $C_{l\dot{\alpha}}$ separately. The amplitude of Φ_A , and the sign in some cases, are changing. The effect is seen also in γ_V for both positive and negative control perturbation. For $\delta_c = -3^\circ$, and for $C_{lq} = 0$ and $C_{l\dot{\alpha}}$ double the nominal value, γ_V was about 0.035 radians with a related increase in altitude of 19 meters. For C_{lq} doubled and $C_{l\dot{\alpha}} = 0$, γ_V became 0.15 radians after 8 second of simulation and the aircraft gained a total of 70 meters in altitude. When giving all stability derivatives their nominal values, γ_V was about 0.09 radians and the aircraft altitude increased by about 40 meters.

Again, with positive elevator input, γ_V and Φ_A displayed totally different response from those for negative input. With $C_{lq} = 0$ and $C_{l\dot{\alpha}} =$ twice nominal, γ_V was about 0.01 with altitude gain of 12 meters. When $C_{lq} =$ twice nominal and $C_{l\dot{\alpha}} = 0$, γ_V had a value of -0.14 and the aircraft lost 52 meters of its original altitude, by the end of simulation. This might suggest the need for treating C_{lq} and $C_{l\dot{\alpha}}$ separately and not considering them in combination.

5.2.2 Effect of (C_{nq} and $C_{n\dot{\alpha}}$)

The general trend in the case of (C_{nq} and $C_{n\dot{\alpha}}$) is nearly the same as with (C_{lq} and $C_{l\dot{\alpha}}$), with Φ_A replaced by β . For the same perturbation and set of derivatives, the effect of (C_{nq} and $C_{n\dot{\alpha}}$) on γ_V , for the case of 2g turning flight, is more than the effect due to (C_{lq} and $C_{l\dot{\alpha}}$). This is understandable, since C_{nq} and $C_{n\dot{\alpha}}$ have the same sign as that of C_{lq} and $C_{l\dot{\alpha}}$, but with $\Delta\alpha$ coefficient doubled.

For both straight and 2g turning flight (Fig. 4.4 to 4.5) the amplitude of β is less than Φ_A . The effect of C_{nq} and $C_{n\dot{\alpha}}$ on horizontal flight path angle γ_H is very small. Probably the greatest effect is with locally linearized derivatives and positive elevator perturbation (Fig. 2.4d). For both derivatives zeroed, after 8 seconds, the aircraft deviated 17 meters in $-ve$ y-direction. For all stability derivatives with nominal values the deviation became 24 meters. Finally, when both derivatives were doubled it reached 34 meters from the original flight path.

Generally for the straight and level initial flight condition, we noticed that the horizontal flight path angle γ_H and hence Y (Y-component of aircraft position) are increasing with time. It is not clear what will happen to them if the simulation time is increased to more than 8 seconds, e.g., 20 seconds. Up to this point the continuous increase in Y

makes us suspicious about some instability in the aircraft spiral mode. Sometimes the use of uncoupled modes of motion could help analyze the aircraft motion, and its stability with respect to different modes.

We know that the lateral characteristic equation for the aircraft motion is given in the form :

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

The λ 's, are the eigenvalues, A, B, C, D, and E are constants, where the last coefficient (E) is very important in determining the spiral mode stability, i.e if E is negative, then the spiral mode is unstable. The coefficient E is given by $E = C_{L_0}[C_{l\beta}C_{nr} - C_{n\beta}C_{lr}]$. Substituting for the parameters, for the present aircraft configuration, and knowing that C_{l_0} is the lift coefficient (positive), we get :

$$E = C_{L_0}[(-0.2)(-0.15) - (0.25)(0.25)]$$

$$E = C_{L_0}[0.03 - 0.0625]$$

$$E = C_{L_0}[-0.0325]$$

Therefore, E is negative meaning that the aircraft does have an unstable spiral mode as suspected. An unstable spiral mode means increases in ψ , ϕ , and γ_H for any rate of roll, which in this case developed from the elevator perturbation, as a result of cross-coupling effects.

For the initial condition of 2g turning flight, the change in γ_V was remarkably high in the case of locally linearized derivatives, for both negative and positive aerodynamic control perturbation (Fig 4.5 b,d). With negative elevator perturbation, when the two derivatives were zeroed γ_V showed little change and the aircraft remained almost at the reference altitude. But when both of them were doubled, γ_V reached a value of nearly 0.2 radians (11.5°) with associated gain in altitude of about 95 meters. Positive perturbation gave a magnitude of .02 radians for zeroed coefficients and -0.12 radians for doubled coefficients. The related change in altitude was an increase of 10 meters and a decrease of 46 meters, respectively.

When C_{n_q} and $C_{n_{\dot{\alpha}}}$ were treated separately (Fig. 4.7), they showed different effects on β and γ_V . Higher β amplitude occurred with C_{n_q} zeroed and $C_{n_{\dot{\alpha}}}$ doubled, where γ_V had

less magnitude. We should note here, that γ_V is sometimes changing sign. For example, for +ve perturbation when $C_{nq}=0$ and $C_{n\dot{\alpha}}$ = twice nominal, γ_V was in the neighborhood of 0.05 radians. In this case altitude increased by 26 meters. When C_{nq} = twice nominal and $C_{n\dot{\alpha}}=0$, γ_V had a value of $-.18$ radians with a loss of 67 meters in altitude. Again giving C_{nq} and $C_{n\dot{\alpha}}$ unequal magnitudes would give different results.

5.2.3 Effect of $C_{n\alpha}$

The effect of the static stability derivative, $C_{n\alpha}$, is shown in Figure 4.7, for initial 2g turning flight, for positive and negative elevator perturbation. The effect on β is moderate, while the effect on γ_V is significant. For positive control perturbation, γ_V changed from -0.13 to about 0.017 radians, for the cases of zeroed and doubled derivative, respectively. For negative elevator perturbation, γ_V was 0.155 radians for $C_{n\alpha} = 0$, and 0.028 radians for $C_{n\alpha}$ double the nominal value. Generally $C_{n\alpha}$ has a considerable effect of the vertical flight path angle.

5.2.4 Effect of (C_{mr} and $C_{m\dot{\beta}}$)

In this case we examine the effect of aerodynamic cross-coupling on the longitudinal aircraft motion, by introducing lateral perturbations of the rudder angle. Figures 4.10 and 4.11 include the effect of (C_{mr} and $C_{m\dot{\beta}}$) for both flight conditions. The result indicates that C_{mr} and $C_{m\dot{\beta}}$ have little effect on the aircraft motion. For straight and level initial flight, for doubling of C_{mr} and $C_{m\dot{\beta}}$ or zeroing them, in comparison with the response when all stability derivatives are given the nominal values, α showed little change, and γ_H remained the same. Exactly the same thing happened with 2g turning flight. Small changes in α but no changes in γ_V were observed, i.e in both cases the response was very close to the nominal case. This clearly indicates the minimal effect C_{mr} and $C_{m\dot{\beta}}$ have on the aircraft motion.

The most significant effect is the overall effect of perturbing the rudder. For straight and level flight, a positive perturbation of 10° for 0.6 seconds made the aircraft drift away from the original flight path by 42 meters. Opposite perturbation gave approximately the opposite effect. For 2g turning flight, positive rudder perturbation increased the altitude

by 117 meters, where the opposite perturbation made the aircraft lose about 150 meters of altitude.

The overall response in the angle of attack shows that α is oscillating for the first few seconds and then continues to increase in case of positive rudder perturbation, or decrease in case of negative rudder perturbation. It might be a long time before α comes back to the reference steady state.

5.2.5 Effect of $C_{m\beta}$

The sensitivity of the aircraft motion to the static cross-coupling derivative $C_{m\beta}$ was examined. The results are presented in Figure 4.12 for the turning flight condition with the set of locally linearized stability derivatives. $C_{m\beta}$ showed some effect on the angle of attack response, but negligible effect on γ_V . The maximum change in the angle of attack from the trim value is observed for the case when $C_{m\beta}$ doubled with both positive and negative rudder perturbation up to about 4 seconds from the input. Generally $C_{m\beta}$ has more effect on α than (C_{mr} and $C_{m\dot{\beta}}$), which indicates a higher degree of significance for this static cross-coupling derivative.

5.2.6 Effect of (C_{lr} and $C_{l\dot{\beta}}$)

For the sake of comparison and completeness, it was considered desirable to assess the aircraft motion sensitivity to some of the traditional derivatives, such as C_{lr} , $C_{l\dot{\beta}}$ and C_{nr} , $C_{n\dot{\beta}}$ under the same conditions where cross-coupling derivatives showed a substantial effect. The results for (C_{lr} and $C_{l\dot{\beta}}$) are shown in Figure 4.13 for 2g turning flight. Φ_A showed some oscillation with decreasing amplitude up to the time where the simulation is stopped, with both derivatives zeroed. With nominal and doubled derivatives, Φ_A showed less response. This is, in fact, the same for when the set of derivatives is constant or locally linearized; in other words, both γ_V and Φ_A showed the same response when either set was used.

In relative terms, the effects of (C_{lr} and $C_{l\dot{\beta}}$) on Φ_A are not markedly greater than the effect of the corresponding cross-coupling derivatives. The effects on γ_V are considerably higher.

5.2.7 Effect of (C_{nr} and $C_{n\dot{\beta}}$)

The effects of these well-established derivatives on the aircraft motion are presented in Figure 4.14 for the initial case of 2g turning flight. It is interesting to note that in comparison with (C_{lr} and $C_{l\dot{\beta}}$) these derivatives have a smaller effect on both β and γ_V . The maximum amplitude for β , with negative rudder perturbation for locally linearized derivatives, is about 0.035 radians. In all cases varying (C_{nr} and $C_{n\dot{\beta}}$) had negligible effect on β , but moderate effect on γ_V .

5.2.8 Angular rates

The angular rates, p , q , r , $\dot{\alpha}$, and $\dot{\beta}$, are presented in Figs. 4.8, 4.9, 4.15, and 4.16 for the two flight conditions with different control perturbations, for the nominal set of locally linearized stability derivatives. They are also presented for the case where the effect of cross-coupling is eliminated, by making all cross-coupling stability derivatives equal to zero (Fig. 4.17).

Figure 4.8 shows the angular rates for straight and level initial flight, for both positive and negative control perturbation. The rates are higher for the case of positive elevator perturbation. In this case p , r , $\dot{\beta}$, and $\dot{\alpha}$ have higher amplitudes. Basically the same pattern of behavior is happening with 2g turning flight (Fig. 4.9), except all the rates are magnified. Again for positive perturbation $\dot{\beta}$ showed considerable oscillation for the first 4 seconds.

For rudder perturbations, Figure 4.15 presents the angular rate for straight flight, for both directions of perturbation, the rate of roll (p) and the rate of change of sideslip angle $\dot{\beta}$, are high compared to the rate of yaw (r). The rate of pitch (q) and $\dot{\alpha}$ are relatively small in both cases. The same thing can be said about the angular rates for 2g turning flight. (see Fig. 4.16)

The last figure (4.17) presents the angular rates for the 2g turning flight initial condition, with all cross-coupling aerodynamic derivatives equal to zero, with the set of locally linearized derivatives. Effects of both elevator and rudder perturbations are presented. If this figure is compared to figures 4.8, 4.9, 4.15, and 4.16, as shown in Figure 4.18, we can clearly see the difference in angular rates due to the elimination of any cross-coupling effect.

For negative and positive elevator perturbations (Fig. 4.17 a,b) p , r , and $\dot{\beta}$ are almost zero. Similarly for rudder perturbation (Fig. 4.17 c,d) q and $\dot{\alpha}$ are negligible, almost zero, compared to p , r and $\dot{\beta}$. Comparison of the angular rates with and without cross-coupling effects for the initially 2g turning flight is found in Figure 4.18. This strongly suggests the need for including the cross-coupling terms in the aircraft equations of motion, and solving the entire set of equations simultaneously instead of dealing with two separate groups of equations.

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

An investigation of the importance of including the aerodynamic cross-coupling terms in the equations of motion of a modern fighter aircraft, flying at high angle of attack, was performed. The investigation was done for both straight and level and 2g turning, reference flight conditions, using a six-degree-of-freedom digital computer simulation written by the author. The findings of the study were as follows :

- 1) In most cases, large effects were observed when taking into account the variations of stability derivatives with angle of attack by using local linearization.
- 2) Most dynamic cross-coupling moment derivatives had a large effect on the aircraft motion time histories, compared to the effect of the corresponding, well-established, aerodynamic derivatives. The most important of these cross-coupling derivatives are C_{lq} , $C_{l\dot{\alpha}}$, C_{nq} , and $C_{n\dot{\alpha}}$. This result is generally in agreement with the simplified sensitivity assessment of the relative magnitudes of some cross-coupling terms.
- 3) The effect of the derivatives of the pitching moment due to yawing, C_{mr} and $C_{m\dot{\rho}}$, were found to be much less significant than the derivatives due to pitching (in item 2). Once again, this agrees with the simplified sensitivity analysis. However, the order of magnitude of these derivatives was compared to the damping in pitch, which is usually large. If it happens that the damping in pitch is smaller, then a larger effect of C_{mr} and $C_{m\dot{\rho}}$ might be expected.
- 4) With elevator perturbation, and constant stability derivatives, the results for Φ_A , were comparable for both initial flight conditions, with slightly higher amplitude in the case of 2g turning flight. The differences were much larger with locally linearized stability derivatives. The amplitudes for both Φ_A and β , for 2g turning flight, were nearly double the amplitudes for straight and level flight.

For both initial flight conditions when the rudder was perturbed, the response in the angle of attack did not show noticeable difference when changing the set of stability derivatives from constant to locally linearized. Generally, the direction of rudder perturbation played a dominant role in the subsequent time history for the angle of attack. Negative rudder perturbation made α oscillate for the first 4 seconds, and then to continue decreasing after that. Opposite perturbation gave the same effect in the opposite direction.

5) In the case of initial 2g turning flight, when investigating the separate effects of the pure rotary and of the acceleration derivatives on the relevant time histories, it was concluded that :

- i) when examining the effect of rolling motion derivatives on aerodynamic roll angle (Φ_A), C_{lq} was found to be more significant than $C_{l\dot{\alpha}}$. The same effect was observed on γ_v .
- ii) When examining the effect of yawing motion derivatives on the angle of sideslip (β), the effect of $C_{n\dot{\alpha}}$ was found to be much larger than the effect of C_{nq} . The effect is opposite on γ_v .

It is clear in both cases, that simply giving the same magnitude to both the pure rotary and the acceleration derivatives in a derivative combination or giving them an arbitrary value, could lead to significant errors in the predicted aircraft motion.

6) When perturbing the elevator, the cross-coupling effects due to pitching caused noticeable change in rates of yaw, roll, and sideslip. Rudder perturbation gave less significant effects on rates of pitch and angle of attack.

When none of the cross-coupling terms (C_{lq} , $C_{l\dot{\alpha}}$, C_{nq} , $C_{n\dot{\alpha}}$, $C_{n\alpha}$, C_{mr} , $C_{m\dot{\beta}}$, and $C_{m\beta}$) were included, the rates p , r , and $\dot{\beta}$ remain essentially unaffected by elevator perturbation, and the rates q and $\dot{\alpha}$, remain unaffected by rudder perturbation. Therefore, the inclusion or omission of the cross-coupling terms could lead to significant changes in the predicted aircraft motion.

7) Finally, digital computer simulation using suitable numerical techniques is a very convenient way of predicting aircraft motion, especially considering the cost and speed in implementation of any changes in the aircraft physical and/or aerodynamic characteristics.

The above conclusions apply to flight at high angle of attack or at any other asymmetric flow condition. They are based on data obtained for two different configurations flying at different speeds. It is, however, believed that the investigated ranges of the various derivatives and their rates of change with the angle of attack are representative of the aerodynamic characteristics of a modern fighter aircraft, flying in an asymmetric flight condition.

6.2 RECOMMENDATIONS

In order to permit better and more accurate prediction of aircraft motion at high angle of attack, it is recommended that;

- i) A complete set of aerodynamic derivatives for the specific configuration at a suitably-chosen flight condition, must be obtained.
- ii) In an asymmetric flight condition, careful attention must be given to the dynamic cross-coupling and acceleration derivatives since they influence the aircraft motion to about the same extent as some of the traditional dynamic derivatives.
- iii) A comprehensive wind tunnel study for different configurations at different flight conditions is needed to properly interpret the aerodynamic flow phenomena responsible for the behavior of some of the derivatives. This would assist in finding an analytical description of the variation of these derivatives with angle of attack. Availability of a suitable analytical description would allow deviations of more than 3° in the angle of attack, from the reference condition, in similar studies done in the future.

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FIGURES

Fig. 2.1

Some aerodynamic phenomena associated with oscillatory flight at high angle of attack (Ref. 32)

Fig.2.2a

Sub-sonic damping derivatives for a wing-body configuration at $M=0.7$ (Ref.31)

Fig.2.2b

Sub-sonic cross derivatives for a swept wing fighter aircraft at $M=0.7$ (Ref. 12)

$\alpha = 12.5^\circ, \beta = 10^\circ$

$\alpha = 14^\circ, \beta = 10^\circ$

$\alpha = 17.5^\circ, \beta = 10^\circ$

Fig. 2.3a

Surface flow on schematic aircraft configuration at $M=0.7$ (Ref. 31).

Fig. 2.3b

variation of cross-coupling derivatives with the angle of attack (Ref. 33)

Fig. 2.4

Comparison between purely rotary and oscillating derivatives, due to time rate of change of angles of attack and sideslip, here for $\dot{\beta}$ (Ref. 42)

Fig. 2.5a

Vortex burst point from water tunnel tests (Ref. 42)

Fig.2.5b

Strake effect on stability derivatives (Ref. 43)

Fig. 2.5c

Effect of nose shape and leading edge extensor on dynamic directional stability(Ref. 43)

Fig. 2.6a
Effect of wing planform (Ref. 38)

Fig. 2.6b
Effect of vertical tail (after Ref. 21)
Fig. 2.6 Effect of aircraft geometry

Fig. 2.7

Body system of axes and related angles, positive directions indicated by arrows

Vertical flight path angle γ_v , vertical plane

Horizontal flight path angle γ_H , horizontal plane

Fig.2.8

Fig. 2.9

Three-view sketch for the hypothetical fighter airplane, dimensions are in feet (meters) (after Ref. 11)

Fig. 3.2 FLOW CHART FOR PROGRAM FLIGHT

Fig. 3.3 Study case schematic diagram

a) Constant Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	$(C_{lq} \text{ And } C_{l\dot{\alpha}})$
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

Fig. 4.1, Effect Of $(C_{lq} \text{ And } C_{l\dot{\alpha}})$ For Initially Straight And Level Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	(C_{lq} And C_{la})
—△—	Zero
—□—	Nominal
—○—	Twice Nominal

c) Constant Coefficients; $\delta_c = +3^\circ$ For 0.3 Sec.

Legend	(C_{lq} And $C_{l\dot{\alpha}}$)
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

d) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	(C_{lq} And $C_{l\alpha}$)
- Δ	Zero
- $\square$	Nominal
- $\diamond$	Twice Nominal

a) Constant Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	$(C_{l\dot{q}} \text{ And } C_{l\dot{\alpha}})$
—△—	Zero
—□—	Nominal
—○—	Twice Nominal

Fig. 4.2, Effect Of $(C_{l\dot{q}} \text{ And } C_{l\dot{\alpha}})$ For Initial 2g Turning Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	(C_{lq} And $C_{l\dot{\alpha}}$)
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

f) Constant Coefficients; $\delta_c = +3^\circ$ For 0.3 Sec.

Legend

($C_{l\dot{q}}$ And $C_{l\dot{\alpha}}$)

—△—

Zero

—□—

Nominal

—◇—

Twice Nominal

d) Locally Linearized Coefficients; $\delta_c = +3^\circ$ For 0.3 Sec.

Legend	$(C_{lq}$ And $C_{la})$
	Zero
	Nominal
	Twice Nominal

a) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	C_{lq}	$C_{l\alpha}$
$\triangle$	Zero	Twice Nominal
$\square$	Nominal	Nominal
$\circ$	Twice Nominal	Zero

Fig. 4.3 More Effects for $C_{lq}, C_{l\alpha}$ For Initial 2g. Turning Flight.

.....Cont

b) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	$C_{l\dot{\gamma}}$	$C_{l\dot{\alpha}}$
—△—	Zero	Twice Nominal
—□—	Nominal	Nominal
—◇—	Twice Nominal	Zero

a) Constant Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend $(C_{nq} \text{ And } C_{n\dot{\alpha}})$

$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

Fig. 4,4, Effect Of $(C_{nq} \text{ And } C_{n\dot{\alpha}})$ For Initially Straight And Level Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_c = -3^\circ$ For 0.3 Sec.

c) Constant Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And $C_{n\dot{\alpha}}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

d) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And C_{na})
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

a) Constant Coefficients; $\delta_c = -3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And C_{na})
—△—	Zero
- - -	Nominal
- - -	Twice Nominal

Fig. 4.5, Effect Of (C_{nq} And C_{na}) For Initial 2g Turning Flight.

Cont

b) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And C_{na})
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

c) Constant Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And $C_{n\dot{\alpha}}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

d) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	(C_{nq} And $C_{n\alpha}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

a) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	C_{nq}	C_{na}
—△—	Zero	Twice Nominal
-□-	Nominal	Nominal
-◇-	Twice Nominal	Zero

Fig. 4.6 More Effects for C_{nq} , C_{na} For Initial 2g Turning Flight.

..... Cont.

b) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	C_{nq}	C_{na}
$\triangle$	Zero	Twice Nominal
$\square$	Nominal	Nominal
$\diamond$	Twice Nominal	Zero

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a) Locally Linearized Coefficients; $\delta_e = -3^\circ$ For 0.3 Sec.

Legend	C_{na}
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

Fig. 4.7, Effect Of C_{na} For Initial 2g Turning Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	C_{na}
—△—	Zero
—□—	Nominal
---◇---	Twice Nominal

Legend	$p, q, \text{ And } r$
$\triangle$	p
$\square$	q
$\diamond$	r

Legend	$\alpha \text{ And } \beta$
$\triangle$	α
$\square$	β

a) Locally Linearized Coefficients, With Nominal Values; $\delta_e = -3^\circ$ For 0.3 Sec.

Fig. 4.6 Angular Rates, For Initially Straight And Level Flight

.....Cont.

b) Locally Linearized Coefficients, With Nominal Values; $\delta_e = +3^\circ$ For 0.3 Sec.

a) Locally Linearized Coefficients, With Nominal Values; $\delta_e = -3^\circ$ For 0.3 Sec.

Fig. 4.9 Angular Rates, For Initial 2g Turning Flight

.....Cont.

b) Locally Linearized Coefficients, With Nominal Values; $\delta_e = +3^\circ$ For 0.3 Sec.

a) Constant Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Fig. 4.10, Effect Of $(C_{mr} \text{ And } C_{m\dot{\beta}})$ For Initially Straight And Level Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend	(C_{mr} And $C_{m\dot{\theta}}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

c) Constant Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{mr} And $C_{m\beta}$)
$\triangle$	Zero
$\square$	Nominal
$\circ$	Twice Nominal

d) Locally Linearized Coefficients; $\delta\alpha = -10^\circ$ For 0.6 Sec.

Legend	($C_{m\dot{r}}$ And $C_{m\dot{\beta}}$)
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

a) Constant Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend	(C_{mr} And $C_{m\beta}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice-Nominal

Fig. 4.11, Effect Of (C_{mr} And $C_{m\beta}$) For Initial 2g Turning Flight.

Cont.

b) Locally Linearized Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend	($C_{m\dot{r}}$ And $C_{m\dot{\theta}}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

c) Constant Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{mr} And $C_{m\dot{\theta}}$)
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

d) Locally Linearized Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend

(C_{m_r} And $C_{m_{\dot{\theta}}}$)

—△—

Zero

—□—

Nominal

—◇—

Twice Nominal

a) Locally Linearized Coefficients; $\delta_r = +10^0$ For 0.6 Sec.

Legend	$C_{m\beta}$
$\triangle$	Zero
$\square$	Nominal
$\circ$	Twice Nominal

Fig. 4.12, Effect Of $C_{m\beta}$ For Initial 2g Turning Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_r = -10^0$ For 0.6 Sec.

Legend	$C_{m\beta}$
—Δ—	Zero
—□—	Nominal
—○—	Twice Nominal

a) Constant Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Fig. 4.13, Effect Of $(C_{lr} \text{ And } C_{l\beta})$ For Initial 2g Turning Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend	(C_{lr} And $C_{l\beta}$)
△	Zero
□	Nominal
◇	Twice Nominal

c) Constant Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{lr} And $C_{l\delta}$)
—Δ—	Zero
—□—	Nominal
—◇—	Twice Nominal

d) Locally Linearized Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{lr} And $C_{l\beta}$)
—△—	Zero
—□—	Nominal
—◇—	Twice Nominal

a) Constant Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend	$(C_{nr} \text{ And } C_{n\beta})$
$\triangle$	Zero
$+$	Nominal
$\circ$	Twice Nominal

Fig. 4.14, Effect Of $(C_{nr} \text{ And } C_{n\beta})$ For Initial 2g Turning Flight.

.....Cont.

b) Locally Linearized Coefficients; $\delta_r = +10^\circ$ For 0.6 Sec.

Legend

(C_{nr} And $C_{n\beta}$)

Δ

Zero

$\square$

Nominal

$\diamond$

Twice Nominal

c) Constant Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{nr} And $C_{n\beta}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

d) Locally Linearized Coefficients; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend	(C_{nr} And $C_{n\dot{\theta}}$)
$\triangle$	Zero
$\square$	Nominal
$\diamond$	Twice Nominal

Legend $p, q, \text{ And } r$

$\triangle$ p

$\square$ q

$\diamond$ r

Legend $\alpha \text{ And } \beta$

$\triangle$ α

$\square$ β

a) Locally Linearized Coefficients, With Nominal Values; $\delta_r = +10^0$ For 0.6 Sec.

Fig. 4.15 Angular Rates, For Initially Straight And Level Flight

.....Cont.

b) Locally Linearized Coefficients, With Nominal Values; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend

 $p, q, \text{ And } r$ $\triangle$ p $\square$ q $\diamond$ r

Legend

 $\alpha \text{ And } \beta$ $\triangle$ α $\square$ β

a) Locally Linearized Coefficients, With Nominal Values; $\delta_r = +10^\circ$ For 0.6 Sec.

Fig. 4.16, Angular Rates, For Initial 2g Turning Flight

.....Cont.

b) Locally Linearized Coefficients, With Nominal Values; $\delta_r = -10^\circ$ For 0.6 Sec.

Legend α And β

$\triangle$ $\dot{\alpha}$

$\square$ $\dot{\beta}$

Legend $p, q, \text{ And } r$

$\triangle$ p

$\square$ q

$\diamond$ r

a) All Cross-Coupling Derivatives = 0, Locally Linearized Coefficients, Nominal Values; $\delta_e = -3^\circ$ For 0.3 Sec.

Fig. 4.17 Angular Rates For Initial 2g Turning Flight, With All Cross-Coupling Derivatives = 0

Cont

b) All Cross-Coupling Derivatives = 0, Locally Linearized Coefficients, Nominal Values; $\delta_e = +3^\circ$ For 0.3 Sec.

Legend	α And β
$\triangle$	$\dot{\alpha}$
$\square$	$\dot{\beta}$

Legend	p , q , And r
$\triangle$	p
$\circ$	q
$\square$	r

c) All Cross-Coupling Derivatives = 0, Locally Linearized Coefficients, Nominal Values; $\delta_r = +10^0$ For 0.6 Sec.

Legend α And β

$\triangle$ α

$\square$ β

Legend p , q , And r

$\triangle$ p

$\square$ q

$\diamond$ r

d) All Cross-Coupling Derivatives = 0, Locally Linearized Coefficients, Nominal Values; $\delta_r = -10^0$ For 0.6 Sec.

Legend

Cross-coupling

Cross-coupling

Legend

No Cross-Coupling

No Cross-Coupling

 α And β α β

Legend

Cross-coupling

Cross-coupling

Cross-coupling

Legend

No Cross-Coupling

No Cross-Coupling

No Cross-Coupling

p, q, And r

p

q

r

a) Locally Linearized Coefficients, Nominal Values; $\delta_c = -3^\circ$ For 0.3 Sec.

Fig. 4.18 Comparison of Angular Rates For Initial 2g Turning Flight, With and Without Cross-Coupling Effects.

Cont

Legend		Legend		α And β
$\triangle$	Cross-coupling	—	No Cross-Coupling	α
$\square$	Cross-coupling	- - -	No Cross-Coupling	β

Legend		Legend		p , q , And r
$\triangle$	Cross-coupling	—	No Cross-Coupling	p
$\square$	Cross-coupling	- - -	No Cross-Coupling	q
$\diamond$	Cross-coupling	- . - .	No Cross-Coupling	r

b) Locally Linearized Coefficients, Nominal Values; $\delta_c = +3^\circ$ For 0.3 Sec.

Cross-coupling

[illegible]

No Cross-Coupling

 $\dot{O}$

i

$\triangle$

Cross-coupling

Cross-coupling

— — — — —

No Cross-Coupling

No Cross-Coupling

P

9

7

c) Locally Linearized Coefficients, Nominal Values; $\delta_r = 10^6$ For 0.6 Sec.

Legend

—△—

Cross-coupling

-□-

Cross-coupling

Legend

——

No Cross-Coupling

No Cross-Coupling

 α And β α β

Legend

—△—

Cross-coupling

-□-

Cross-coupling

-○-

Cross-coupling

Legend

——

No Cross-Coupling

No Cross-Coupling

No Cross-Coupling

p, q, And r

p

q

r

d) Locally Linearized Coefficients, Nominal Values; $\delta_r = -10^4$ For 0.6 Sec.

Appendix A

ESTIMATION OF CONTROL DERIVATIVES

The purpose of the following analysis is to obtain a quick and simple estimation for some control derivatives. See (Ref. 3)

-Aileron derivatives

Figure A.1

Wing-Aileron Arrangement

Figure A.1 shows a wing-aileron arrangement in general case. Using a strip theory, the additional lift (dL) on the strip (RT), chord c , width dy , due to star board aileron deflection down.

$$dL = \frac{1}{2} \rho V^2 c dy a_2 \delta_a \quad (A.1)$$

Where a_2 is the lift curve slope for the aileron, then the rolling moment due to starboard aileron $= 1/2 \rho V^2 \delta_a \int_{y_1}^{y_2} a_2 c y dy$.

Then the rolling moment due to starboard down and port up aileron is:

$$l^*_{\delta_a} \delta_a = -\rho V^2 \delta_a \int_{y_1}^{y_2} a_2 c y dy \quad (A.2)$$

then $C_{l\delta_a} = \frac{l^*_{\delta_a}}{1/2 \rho V^2 S b} = \frac{-1}{S b/2} \int_{y_1}^{y_2} a_2 c y dy$, which gives

$$C_{l\delta_a} = \frac{-1}{S b/2} \int_{y_1}^{y_2} a_2 C_y dy \quad (A.3)$$

$C_{l\delta_a}$ depends upon the location of the aileron (y_1 and y_2), also a_2 can be found in terms of wing lift curve slope and the aileron effectiveness (r_a) which is a function of c_a/c , therefore;

$$C_{l\delta_a} = \frac{-1}{S b/2} r_a \int_{y_1}^{y_2} a c y dy \quad (A.4)$$

Here we need to find (c) as a function of c_{tp} , c_{rt} , and y, therefore

$$c = c_{rt} - \frac{2y}{b}(c_{rt} - c_{tp}) \quad (A.5)$$

For an aileron position between 44.5% and 67% of half wing span, and $c_a/c=0.24$ we have $y_1=0.445b/2$, $y_2=0.67b/2$, and $r_a=0.21$, see (ref. 41)

Now substituting for different parameters in equation (A.4) we get:

$$\begin{aligned} C_{l\delta_a} &= \frac{-0.21a}{50.2 \times \frac{11.89}{2}} \int_{0.445b/2}^{0.67b/2} \left[7.16 - \frac{2}{11.89}(7.16 - 1.2)y \right] y dy \\ &= -7.036 \times 10^4 a \int_{0.445b/2}^{0.67b/2} (7.16y - 1.002y^2) dy \\ &= -7.036 \times 10^4 a \left[\frac{7.16}{2} y^2 - \frac{1.002}{3} y^3 \right]_{0.445b/2}^{0.67b/2} \end{aligned}$$

substituting for the limits of the integration to get;

$$C_{l\delta_a} = -0.0119a$$

For $a_w=1.2$, $C_{l\delta_a} = -0.0143$

Similarly for the yawing moment derivative $C_{n\delta_a}$

$$C_{n\delta_a} = \frac{r_a}{S b/2} \int_{y_1}^{y_2} \left(\frac{\partial C_D}{\partial \alpha} \right) C_y dy$$

$$= 0.0199 \left(\frac{\partial C_D}{\partial \alpha} \right) C_Y dy$$

$\left(\frac{\partial C_D}{\partial \alpha} \right)$ can be assumed as (15% a), see (Ref. 41), therefor

$$C_{n\delta_a} = 0.00214$$

For side force coefficient $C_{Y\delta_a}$, there is no formula available, therefor for simplicity we will assume it zero.

-Elevator Derivatives

$C_{x\delta_e} = -\frac{S_t}{S} r_e \left(\frac{\partial C_D}{\partial \delta_e} \right)_t$, this is usually negligible.

$$C_{x\delta_e} = 0.0$$

$$C_{z\delta_e} = -a_t r_e \frac{S_t}{S}, \text{ for } c_e/c = 0.45, r_e = 0.4$$

$$C_{z\delta_e} = -0.4 \cdot \frac{16}{50.2} \cdot 4.7$$

$$= -0.599$$

$C_{m\delta_e} = -a_t r_e \frac{S_t}{S} \frac{L_t}{c}$, which gives.

$$C_{m\delta_e} = -4.7 \times \frac{7.85}{4.88} \times \frac{16}{50.2} \times 0.4$$

$$= -0.963$$

-Rudder derivatives

$C_{Y\delta_r} = a_F r_r \frac{S_F}{S}$, for $c_r/c = 0.164$, $r_r = 0.18$

$$C_{Y\delta_r} = 4.5 \times 0.18 \times \frac{6.27}{50.2}$$

$$= 0.101$$

Similarly $C_{n\delta_r} = -a_{Pr} \frac{S_r l_r}{S} \frac{l_r}{b}$

$$\begin{aligned} C_{n\delta_r} &= -4.5 \times 0.18 \times \frac{6.27}{50.2} \times \frac{7.1}{11.89} \\ &= -0.0804 \end{aligned}$$

For rolling moment coefficient, $C_{l\delta_r} = a_{Pr} \frac{S_r z_r}{S} \frac{z_r}{b}$

$$\begin{aligned} C_{l\delta_r} &= 4.5 \times 0.18 \times \frac{6.27}{50.2} \times \frac{2}{11.89} \\ &= 0.017 \end{aligned}$$

Appendix B

COMPUTER VARIABLES

This Appendix illustrates different parameters with the corresponding FORTRAN variables.

PARAMETER	COMPUTER VARIABLES
mass	MASS
S	S
b	B
$\bar{c}$	CDASH
I_x	IX
I_y	IY
I_z	IZ
I_{xz}	IXZ
S_F	SF
Z_F	ZF
S_t	ST
l_t	LT
a_w	AW
a_t	AT
a_F	AF
α	ALPHA
β	BETA

$\alpha(\text{rad.})$ $\beta(\text{rad.})$ h $\Delta\alpha$ δ_a δ_e δ_r C_z C_y C_x $C_{l\beta}$ C_{lp} C_{lq} $C_{l\alpha}$ $C_{l\dot{\alpha}}$ $C_{m\alpha}$ $C_{m\beta}$ C_{mq} $C_{m\dot{q}}$ $C_{m\dot{\alpha}}$ C_{mr} $\dot{C}_{m\beta}$ $C_{\dot{x}\alpha}$ $C_{n\alpha}$ $C_{n\beta}$ $C_{n\dot{q}}$ $C_{n\dot{\alpha}}$ C_{nr} C_{np} $C_{n\beta}$

ALPHAR

BETAR

ALT

DELALPHA

DELA

DELE

DELR

CX

CY

CZ

CLBETA

CLP

CLQ

CLALPHA

CLALPHAD

CMALPHA

CMBETA

CMP

CMQ

CMALPHAD

CMR

CMBETAD

CZALPHA

CNALPHA

CNBETA

CNQ

CNALPHAD

CNR

CNP

CNBETAD

C_{xq}	CZQ
$C_{x\delta_a}$	CXDELE
$C_{y\delta_a}$	CYDELA
$C_{y\delta_r}$	CYDELR
$C_{z\delta_a}$	CZDELE
$C_{l\delta_a}$	CLDELA
$C_{l\delta_r}$	CLDELR
$C_{m\delta_r}$	CMDELE
$C_{n\delta_a}$	CNDELE
$C_{n\delta_r}$	CNDELR
$\bar{q}$	QDASH
θ	THETA
ϕ	PHI
ψ	PSI
P	P
q	Q
r	R
T	T
ρ	RO
Φ_{hi}	PHIB
V	VA
$\sum F_x$	SUMFX
$\sum F_y$	SUMFY
$\sum F_z$	SUMFZ
$\sum L$	SUML
$\sum M$	SUMM
$\sum N$	SUMN

u	U
v	V
w	W
u	UD
v	VD
w	WD
p	PD
q	QD
r	RD
θ	THETAD
ϕ	PHID
ψ	PSID
X	XD
Y	YD
Z	ZD
α	ALPHAD
β	BETAD
γ_H	GAMMAH
γ_V	GAMMAV
Φ_A	AEROPHI
ΔT	DELT

Appendix C

RUNGE-KUTTA METHOD AND FINAL FORMULA FOR NUMERICAL INTEGRATION

In general case if we have $y' = f(x, y)$, and at point X , $y(X) = Y$, then

$$y(X + h) = y(x) + \frac{1}{6}[K_0 + 2K_1 + 2K_2 + K_3]$$

h is the increment in the independent variable, where

$$K_0 = hf(X, Y)$$

$$K_1 = hf\left(X + \frac{1}{2}h, Y + \frac{1}{2}K_0\right)$$

$$K_2 = hf\left(X + \frac{1}{2}h, Y + \frac{1}{2}K_1\right)$$

$$K_3 = hf(X + h, Y + K_2)$$

In our case let $U(T = 0) = U_0$ and $U(T = 0)' = U'_0$, therefor

$$K_0 = hU'_0$$

$$K_1 = h\left(U'_0 + \frac{1}{2}hU'_0\right)$$

$$= hU'_0 + \frac{h^2}{2}U'_0$$

$$K_2 = h\left(U'_0 + \frac{1}{2}h + \frac{h^2}{4}U'_0\right)$$

$$= hU'_0 + \frac{h^2}{2}U'_0 + \frac{h^3}{4}U'_0$$

$$K_3 = h\left(U'_0 + hU'_0 + \frac{h^2}{2}U'_0 + \frac{h^3}{4}U'_0\right)$$

$$= hU'_0 + h^2U'_0 + \frac{h^3}{2}U'_0 + \frac{h^4}{4}U'_0$$

This gives, $U(T+h) = U_0 + \frac{1}{6}[hU'_0 + 2hU'_0 + h^2U'_0 + 2hU'_0 + h^2U'_0 + \frac{h^3}{3}U''_0 + hU'_0 + h^2U'_0 + \frac{h^3}{2}U'_0 + \frac{h^4}{4}U'_0]$

$$= U_0 + \frac{U'_0}{6}[h + 2h + h^2 + 2h + h^2 + \frac{h^3}{2} + h + h^2 + \frac{h^3}{2} + \frac{h^4}{4}]$$

$$= U_0 + \frac{U'_0}{6}[6h + 3h^2 + h^3 + \frac{h^4}{4}]$$

Therefore, $U(T+h) = U(T) + U'(T)[h + \frac{h^2}{2} + \frac{h^3}{3} + \frac{h^4}{24}]$, where (h) is the step size.

This is called a fourth-order process with an error of order of (h^5)

Appendix D

PROGRAM FLIGHT

Appendix D contains a complete listings of FLIGHT program. The features of the program and guide lines for program user were given in chapter 3. To remind the reader, the number of study cases is defined by the parameters "DERIVATIVE" and "CASE", the study conditions are defined by the parameter "FLIGHT" in this case FLIGHT is 4.

The computer variable "COUNT" is given to the number of computations done in 8 seconds of simulation, it depends on the time step; $COUNT = \frac{8}{0.025} + 1$, where 0.025 is the time step and the (1) is to count for time zero.

PROGRAM FLIGHT

 DISCREPTION :
 PROGRAM FLIGHT WAS PREPARED AS PART OF A MASTERES THESIS RESEARCH
 DEALING WITH NUMERICAL TECHNEQUES FOR AIRCRAFT MOTION SIMULATION
 ,THATS TO PREDICT AND ANALYSIS THE AIRCRAFT MOTION AND,THE EFFECT
 OF AERODYNAMIC CROSS-COUPLING AT HIGH ANGLES OF ATTACK.
 THE SIMULATION TIME WAS CHOSEN TO BE 8 SECONDS, WITH STEP SIZE OF
 0.025 SECONDS,THIS MEANS 10 UPDATES PER SECOND OR 320 CALCULATIONS
 IN 8 SECONDS PLUS THE VALUES OF DIFFERENT PARAMETERS AT TIME ZERO.

AUTHOR: A. GHMMAM

DATE: SEP. 1986

REAL TEMPP(321),TEMPO(321),TEMPR(321),TEMPHETA(321)
 REAL TEMPPSI(321),TEMPT(321),TEMPPHI(321)

REAL TEMPALPHA(321),TEMPBETA(321),TEMPALPHAD(321)
 REAL TEMPGAMMAH(321),TEMPGAMMAV(321),TEMPBETAD(321)

REAL TEMPX(321),TEMPY(321),TEMPZ(321),TEMPPAEROPHI(321)
 REAL TEMPOD(321),TEMPOVD(321),TEMPOD(321)
 REAL MASS,IX,IY,IZ,IXZ,LT,LF

INTEGER COUNT

THESE ARE THE COMMON STATMENTS USED TO, MANIPULATE THE PARAMETERS
 BETWEEN THE DIFFERENT SUBROUTINES,AND BETWEEN THE SUBROUTINES AND
 THE MAIN PROGRAM.

COMMON/SET1/MASS,S,B,CDASH,IX,IY,IZ,IXZ

COMMON/SET2/SF,ZF,LF,ST,LT

COMMON/SET3/AW,AT,AF

COMMON/SET4/ALPHA0,BETA0,FACTOR,ALTQ,DELALPHA,ALPHAOR,BETAOR

COMMON/SET5/ALPHATR,BETATR

COMMON/SET6/DELA,DELE,DELR,DELAOLD,DELEOLD,DELROLD

COMMON/BASIC1/CX,CY,CZ,CLBETA,CLP,CLQ,CLALPHA,CLALPHAD,
 -CLR,CLBETAD,CYBETA,CZALPHAD

COMMON/BASIC2/CMALPHA,CNBETA,CMF,CMQ,CMALPHAD,CNR,
 -CMBETAD,CZALPHA

COMMON/BASIC3/CNALPHA,CNBETA,CNQ,CNALPHAD,CNR,CNF,CNBETAD,CZQ

COMMON/BASIC4/A1,A2,B1,B2,C1,C2,D1,D2

COMMON/BASIC5/CXDELE,CYDELA,CYDELR,CZDELE

COMMON/BASIC6/CLDELA,CLDELR,CMDELE,CNDELA,CNDELR

COMMON/INITIAL1/QDASHO,THETAO,PHIO,CZT,CLT,FO,QO,RO,CHT,CNT,T
-,VAO,PSIO,ROO,PHIRO

COMMON/LOADING1/SUMFX,SUMFY,SUMFZ,SUML,SUMH,SUMN

COMMON/MOTION1/UD,VD,WD,PD,QD,RD,THETAD,PHID,PSID

COMMON/OLD1/UOLD,VOLD,WOLD,FOLD,QOLD,ROLD,THETAOLD,
-PHIOLD,PSIOLD,PDOLD,RDOLD

COMMON/AUXILIARY1/XD,YD,ZD,ALPHAD,BETAD,ALPHA,BETA
-,GAMMAH,GAMMAV,AEROPHI

COMMON/OLD2/XOLD,YOLD,ZOLD

COMMON/OLD3/ALPHAOLD,BETAOLD,ALPHAOLD,BETAOLD,VAOLD,QDASHOLD

COMMON/OLD4/TIME,DELT

COMMON/NEW1/UNEW,VNEW,WNEW,FNEW,QNEW,RNEW,THETANEW,PHINEW,PSINEW

COMMON/NEW2/XNEW,YNEW,ZNEW

THE FOLLOWING ARE THE MOTION VARIABLES, CHOSEN TO BE PRESENTED
AS AN OUTPUT FOR AIRCRAFT MOTION SIMULATION.

4 FORMAT('1',19X,'TIME'8X,'F'9X,'Q'9X,'R'7X,'THETA'5X,'PHI'8X
-, 'PSI')

44 FORMAT(20X,'SEC.'2X,'RAD/SEC.'2X,'RAD/SEC.'2X,'RAD/SEC.'1X,
-'RAD.'5X,'RAD.'7X,'RAD.'///)

5 FORMAT('1',18X,'TIME'6X,'ALPHA'5X,'BETA'6X,'ALPHAD'4X,'BETAD'5X,
-'GAMMAH'4X,'GAMMAV'4X,'AEROPHI')

55 FORMAT(19X,'SEC.'7X,'RAD.'6X,'RAD.'5X,'RAD/SEC.'2X,'RAD/SEC.'3X,
-'RAD.'6X,'RAD.'6X,'RAD.'///)

6 FORMAT('1',23X,'TIME'12X,'X'15X,'Y'14X,'Z')

66 FORMAT(24X,'SEC.'12X,'M'15X,'M'14X,'M'///)

8 FORMAT('1')

DETERMINE THE NUMBER OF STUDY CASES (+VE AND -VE DELR,N-1,2 ETC.)

DO 103 FLIGHT=1,4

126

CALL AIRCRAFT

ASSIGN THE TEST CASES (HOW MANY TIMES THE EFFECT OF CERTAIN STABILITY DERIVATIVE IS GOING TO EXAMINED)

DERIVATIVE =6

CASE =0

102 CASE =CASE+1

CALL BASIC

CALL INITIAL

CALL START

DETERMINE HOW MANY CALCULATIONS ARE REQUIRED, DEPENDING ON THE SIZE OF THE TIME STEP AND A CHOSEN TIME INTERVAL OF '8' SECONDS

COUNT=0

101 COUNT=COUNT+1

CALL LOADING

CALL MOTION

CALL AUXILIARY

OPEN A TEMPORARY FILE FOR EACH VARIABLE YOU WANT TO PRINT OUT AT THE END OF EACH RUN.

TEMPUD(COUNT)=UD

TEMPVD(COUNT)=VD

TEMPWD(COUNT)=WD

TEMPT(COUNT) =TIME

TEMPP(COUNT) =POLDI

TEMPPQ(COUNT) =QOLD

TEMPPR(COUNT) =ROLD

TEMPTHETA(COUNT) =THETAOLD

TEMPPHI(COUNT) =PHIOLD

TEMPPSI(COUNT) =PSIOLD

TEMPAEROPHI(COUNT)=AEROPHI

TEMPALPHA(COUNT) =ALPHAOLD

TEMPBETA(COUNT) =BETAOLD

TEMPALPHAD(COUNT) =ALPHADOLD

TEMPBETAD(COUNT) =BETADOLD

TEMPGAMMAH(COUNT) =GAMMAH

TEMPGAMMAV(COUNT) =GAMMAV

TEMPX(COUNT)=XOLD

TEMPY(COUNT)=YOLD
TEMPZ(COUNT)=ZOLD
CALL NEW

129

THE FOLLOWING STATEMENT WILL DETERMINE THE DURATION FOR IMPULSE
CONTROL PERTURBATION, FOR 0.3 SECONDS USE COUNT.LT. 12. FOR 0.6
USE COUNT .LT. 24, WHICH MEANS 4 ITERATIONS FOR EACH 0.1 SECOND

IF(COUNT.LT.12)GO TO 101

CALL REINITIAL

IF(COUNT.LT.321)GO TO 101

WRITE(1,4)
WRITE(1,44)
DO 51 COUNT=1,321,4
WRITE(1,20)(TEMPT(COUNT),TEMPF(COUNT),TEMPQ(COUNT),TEMPR(COUNT),
-TEMPHETA(COUNT),TEMPPHI(COUNT),TEMPPSI(COUNT))
FORMAT(15X,7F10.5)
51 CONTINUE

WRITE(2,5)
WRITE(2,55)
DO 52 COUNT=1,321,4
WRITE(2,21)(TEMPT(COUNT),TEMPALPHA(COUNT),TEMPBETA(COUNT),TEMP
-ALPHAD(COUNT),TEMPBETAD(COUNT),TEMPGAMMAH(COUNT),TEMPGAMMAV(COUNT)
-TEMPAEROPHI(COUNT))
FORMAT(15X,8F10.5)
52 CONTINUE

WRITE(3,6)
WRITE(3,66)
DO 53 COUNT=1,321,4
WRITE(3,22)(TEMPT(COUNT),TEMPX(COUNT),TEMPY(COUNT),TEMPZ(COUNT))
FORMAT(15X,4F15.5)
53 CONTINUE

WRITE(4,8)

PRINT*, "*****"

IF(CASE.LT.DERIVATIVE)THEN
GO TO 102
ELSE
ENDIF

CONTINUE

STOP

SUBROUTINE AIRCRAFT

THIS SUBROUTINE HAS SOME OF THE GEOMETRIC DATA FOR THE HYPOTHETICAL AIRCRAFT, THE ASSIGNED INITIAL ANGLES OF ATTACK AND SIDESLIP AND THE CONTROL PERTURBATION ANGLES.

COMMON/SET1/MASS,S,B,CDASH,IX,IY,IZ,IXZ

COMMON/SET2/SF,ZF,LF,ST,LT

COMMON/SET3/AW,AT,AF

COMMON/SET4/ALPHA0,BETA0,FACTOR,ALTO,DELALPHA,ALPHAOR,BETAOR

COMMON/SET5/ALPHATR,BETATR

COMMON/SET6/DELA,DELE,DELR

REAL MASS,S,B,CDASH,IX,IY,IZ,IXZ,SF,ZF,LF,ST,LT
-,FACTOR,ALTO,ALPHAT,BETAT,ALPHATR,BETATR,DELALPHA

PRINT*, 'THE ANGLE OF ATTACK ='

READ*,ALPHA0

PRINT*, '*****'

PRINT*, 'THE ANGLE OF SIDESLIP ='

READ*,BETA0

PRINT*, '*****'

PRINT*, 'THE LOADFACTOR ='

READ*,FACTOR

PRINT*, '*****'

PRINT*, 'THE PERTURBATION IN ALPHA ='

READ*,DELALPHA

IN CASE OF STEP FUNCTION, BEFORE THE RUN, GO TO SUBROUTINE REINITIAL TO MAKE THE REQUIRED CHANGES FOR THE CONTROL PERTURBATION ANGLES.

PRINT*, '*****'

PRINT*, 'THE PERTURBATION OF AILERON IS:'

READ*,DELA

PRINT*, '*****'

PRINT*, 'THE PERTURBATION OF ELEVATOR IS:'

READ*,DELE

PRINT*, '*****'

PRINT*, 'THE PERTURBATION OF RUDDER IS:'

READ*,DELR

PRINT*, '*****'

PRINT*, 'NO CONTROL PERTURBATION DENOTED BY '0', IMPULSE '1', STEP '2'
READ*,CONTROL PERTURBATION FUNCTION

PRINT*, '*****'

PI =3.14159

MASS =17510

S =50.2

R =11.89

CDASH =4.88

IX =36610

IY =162700

IZ =183000

IXZ =6780

SF =6.27

ZF =2.0

LF =7.1

ST =16

LT =7.85

AW =1.2

AT =4.7

AF =4.0

PUT THE DESIGNATED ANGLES OF ATTACK AND SIDESLIP.
AND CONVERT THEM TO RADIAN.

ALPHAT =33

BETAT =0

ALPHATR =ALPHAT*PI/180

BETATR =BETAT*PI/180

ALTO =9960

CONVERT THE INITIAL ANGLES OF ATTACK, SIDESLIP AND CONTROL PERTURBA

ALPHAOR =ALPHA0*PI/180

BETAOR =BETA0*PI/180

DELALPHA =DELALPHA*PI/180

DELA =DELA*PI/180

DELE =DELE*PI/180

DELR =DELR*PI/180

RETURN

END

SUBROUTINE BASIC

THIS SUBROUTINE CALCULATES THE STABILITY DERIVATIVES, AND ALSO
READS THE CONTROL DERIVATIVES EVALUATED IN APPENDIX A.

COMMON/SET1/MASS,S,B,CDASH,IX,IY,IZ,IXZ

COMMON/SET2/SF,ZR,LF,ST,LT

COMMON/SET3/AW,AT,AF

COMMON/SET4/ALPHA0,BETA0,FACTOR,ALTO,DELALPHA,ALPHAOR,BETAOR

COMMON/BASIC1/CX,CY,CZ,CLBETA,CLP,CLQ,CLALPHA,CLALPHAD,
-CLR,CLBETAD,CYBETA,CZALPHAD

COMMON/BASIC2/CMALPHA,CMBETA,CMP,CMQ,CMALPHAD,CMR,
-CMBETAD,CZALPHA

COMMON/BASIC3/CNALPHA,CNBETA,CNQ,CNALPHAD,CNR,CNF,CNBETAD,CZQ

COMMON/BASIC4/A1,A2,B1,B2,C1,C2,D1,D2

COMMON/BASIC5/CXDELE,CYDELA,CYDELR,CZDELE

COMMON/BASIC6/CLDELA,CLDELR,CNDELE,CNDELA,CNDELR

REAL CX,CY,CZ,CLBETA,CLP,CLQ,CLALPHAD,CLR,CLBETAD,CYBETA,
-CZALPHAD,CMALPHA,CMBETA,CMP,CMQ,CMALPHAD,CMR,CMBETAD,CZALPHA,
-CNALPHA,CNBETA,CNQ,CNALPHAD,CNR,CNF,CNBETAD,CZQ,CLALPHA
-CXDELE,CYDELA,CYDELR,CZDELE,CLDELA,CLDELR,CNDELE,CNDELA,CNDELR
REAL MASS,IX,IY,IZ,IXZ,LF,LT

NOW EVALUATE THE STABILITY DERIVATIVES

PRINT*, 'THIS CASE IS TO STUDY THE EFFECT OF CNQ & CNALPHAD'

PRINT*, 'THE FOLLOWING IS THE LEGEND FOR DIFFERENT STUDY CASES'

PRINT*, '*****'

PRINT*, 'CASE #1 CONST. COEFFICIENTS WITH CNQ & CNALPHAD=0'

PRINT*, 'CASE #2 CONST. COEFFICIENTS ALL WITH NOMINAL VALUES'

PRINT*, 'CASE #3 CONST. COEFFICIENTS WITH CNQ & CNALPHAD DOUBLED'

PRINT*, 'CASE #4 LOCALLY LINEARIZED COEFFS. WITH CNQ & CNALPHAD=0'

PRINT*, 'CASE #5 LOCALLY LINEARIZED COEFFS. ALL WITH NOMINAL VALUE'

PRINT*, 'CASE #6 LOCALLY LINEARIZED COEFFS. CNQ & CNALPHAD DOUBLED'

PRINT*, 'CASE #7, ALL CROSS-COUPLED STABILITY DERIVATIVES=0'

PRINT*, '*****'

PRINT*, 'THIS IS CASE #'

READ*, CASE

PRINT*, '*****'

PRINT*, 'THE CONSTANTS A1,A2,B1,B2,C1,C2,D1&D2 DETERMINE THE CASE'

```

PRINT*, '*****'
PRINT*, 'A1 ='
READ*, A1
PRINT*, 'A2 ='
READ*, A2
PRINT*, 'B1 ='
READ*, B1
PRINT*, 'B2 ='
READ*, B2
PRINT*, 'C1 ='
READ*, C1
PRINT*, 'C2 ='
READ*, C2
PRINT*, 'D1 ='
READ*, D1
PRINT*, 'D2 ='
READ*, D2

```

FOR THE FIRST TIME YOU PUT DELALPHA=0, SINCE LATER, THESE STABILITY DERIVATIVES WILL BE USED TO FIND THE TRIM CONDITIONS

DELALPHA=0

```

CX      =-0.04
CY      =0.0
CZ      =-0.841

```

```

CLALPHA =0
CLBETA  =-0.2
CLP      =-0.3
CLQ      =0.5*C1-30*C2*DELALPHA
CLALPHA=0.5*C1-30*C2*DELALPHA
CLR      =0.25
CLBETAD =-0.3

```

```

CYBETA  =-0.8
CZALPHA=-13

```

```

CMALPHA =-0.6
CMBETA  =-0.1*C1
CMP      =0
CMQ      =-7*D1-120*D2*DELALPHA
CMALPHA=-7*D1-120*D2*DELALPHA
CMR      =0.1*C1-25*C2*DELALPHA
CMBETAD =-0.1*C1-25*C2*DELALPHA

```

CZALPHA =-2

```

CNALPHA =0.05*C1
CNBETA  =0.25
CNQ      =0.5*A1-60*A2*DELALPHA
CNALPHA=0.5*A1-60*A2*DELALPHA
CNR      =-0.15*D1-6*D2*DELALPHA
CNP      =0.2
CNBETAD =0.2*D1-6*D2*DELALPHA

```

CZQ =-13

CXDELE =0
 CYDELA =0
 CYDELK =0.101
 CZDELF =-0.599
 CLDELA =-0.0143

CLDELK =0.017
 CMDELE =-0.963
 CNDELA =0.00214
 CNDELP =-0.0604
 RETURN
 END

SUBROUTINE INITIAL

THIS SUBROUTINE CALCULATES THE INITIAL CONDITIONS WHEN THE
 PERTURBATINS ARE STILL ZERO (TRIM CONDITION)

COMMON/BASIC1/CX,G1,CZ,G2,CLP,CLQ,G4,G5,CLR

COMMON/BASIC2/Z1,Z2,Z3,CMQ,Z4,CMR

COMMON/BASIC3/W1,W2,CNQ,W3,CNR,CNF,W4,CZQ

COMMON/SET1/MASS,S,B,CDASH

COMMON/SET4/ALPHA0,BETA0,FACTOR,ALTO,DELALPHA,ALPHAOR,BETAOR

COMMON/INITIAL1/QDASH0,THETA0,PHI0,CZT,CLT,FO,QO,RO,CMT,CNT,T
 -,VAO,PSIO,ROO,PHIRO

COMMON/SETS/ALPHATR,BETATR

REAL QDASH0,THETA0,PHI0,CZT,CLT,FO,QO,RO,CMT,CNT,T
 -,ROO,VAO,PHIRO,PSIOD,DEL,G,PSIO,MASS

G =9.81

VALUE1 =TAN(ALPHATR)*COS(BETATR)
 DEL =ATAN(VALUE1)

QDASH0 =FACTOR*MASS*G/(-CZ*(SIN(ALPHATR)*TAN(ALPHATR)
 +COS(ALPHATR))*S)

T =(-CZ*TAN(ALPHATR)-CX)*QDASH0*S

ROO =1.2253*EXP(-ALTO/10150)

VAO =(2*QDASH0/ROO)**0.5

VALUE2 =1/FACTOR

PHIBO =ACOS(VALUE2)

VALUE3 =SIN(PHIBO)*SIN(BETATR)
 VALUE4 =COS(PHIBO)*SIN(DEL)*COS(BETATR)
 VALUE5 =VALUE3+VALUE4
 THETA0 =ASIN(VALUE5)
 VALUE6 =SIN(PHIBO)*SIN(DEL)*COS(BETATR)
 VALUE7 =COS(PHIBO)*SIN(BETATR)
 VALUE8 =COS(THETA0)
 VALUE9 =(VALUE6-VALUE7)/VALUE8

PSIO =ASIN(VALUE9)

VALUE10 =SIN(PHIBO)*COS(BETATR)
 VALUE11 =COS(PHIBO)*SIN(DEL)*SIN(BETATR)
 VALUE12 =COS(THETA0)
 VALUE13 =(VALUE10-VALUE11)/VALUE12
 PHIO =ASIN(VALUE13)

VALUE14 =COS(PHIO)*COS(ALPHATR)
 VALUE15 =TAN(THETA0)*SIN(ALPHATR)
 VALUE16 =(VALUE14+VALUE15)*VA0
 PSIOD =Q*SIN(PHIO)/VALUE16

P0 =-PSIOD*SIN(THETA0)
 Q0 =PSIOD*COS(THETA0)*SIN(PHIO)
 R0 =PSIOD*COS(THETA0)*COS(PHIO)

CZT =CZ-CZQ*(Q0*CDASH/(2*VA0))
 CLT =-CLP*(P0*B/(2*VA0))-CLQ*(Q0*CDASH/(2*VA0))
 -CLR*(R0*B/(2*VA0))
 CMT =-CMQ*(Q0*CDASH/(2*VA0))-CMR*(R0*B/(2*VA0))
 CNT =-CNP*(P0*B/(2*VA0))-CNR*(R0*B/(2*VA0))-CNQ*
 (Q0*CDASH/(2*VA0))

RETURN
 END

SUBROUTINE START

THIS SUBROUTINE CALCULATES THE STARTING VALUES FOR LINEAR VELOCITIES
 , ANGULAR VELOCITIES, DISPLACEMENTS, ETC., WHEN THE PERTURBATION JUST
 BEGINS

COMMON/INITIAL1/QDASH0, THETA0, PHIO, CZT, CLT, P0, Q0, R0, CMT, CNT, T
 -, VA0, PSIO, R00, PHIBO

COMMON/OLD1/UOLD, VOLD, WOLD, FOLD, ROLD, ROLD, THETAOLD, PHIOLD
 -, PSIOld, PDOLD, RDOLD

COMMON/OLD2/XOLD, YOLD, ZOLD

COMMON/OLD3/ALPHAOLD, BETAOLD, ALPHADOL, BETADOLD, VAOLD, QDASHOLD

COMMON/OLD4/TIME, DELT

COMMON/SET4/ALPHA0, BETA0, FACTOR, ALTO, DELALPHA, ALPHAOR, BETAOR

COMMON/SET5/ALPHATR, BETATR

COMMON/SET6/DELA, DELE, DELR, DELAOLD, DELEOLD, DELROLI

COMMON/BASIC1/K1, K2, C2, K4, K5, CLQ, K6, CLALPHAD

COMMON/BASIC2/L1, L2, L3, CMQ, CMALPHAD, CMR, CMBETAD

COMMON/BASIC3/M1, M2, CNQ, CNALPHAD, CNR, M3, CNBETAD

COMMON/BASIC4/A1, A2, B1, B2, C1, C2, D1, D2

DELALPHA = ALPHAOR - ALPHATR

CLQ = $0.5 * C1 - 30 * C2 * DELALPHA$ CLALPHAD = $0.5 * C1 - 30 * C2 * DELALPHA$ CMQ = $-7 * D1 - 120 * D2 * DELALPHA$ CMALPHAD = $-7 * D1 - 120 * D2 * DELALPHA$ CMR = $0.1 * C1 - 25 * C2 * DELALPHA$ CMBETAD = $-0.1 * C1 - 25 * C2 * DELALPHA$ CNQ = $0.5 * A1 - 60 * A2 * DELALPHA$ CNALPHAD = $0.5 * A1 - 60 * A2 * DELALPHA$ CNR = $-0.15 * D1 - 6 * D2 * DELALPHA$ CNBETAD = $0.2 * D1 - 6 * D2 * DELALPHA$ VALUE1 = $TAN(ALPHAOR) * COS(BETAOR)$ DEL = $ATAN(VALUE1)$ VALUE2 = $1 / FACTOR$ PHIBO = $ACOS(VALUE2)$ VALUE3 = $SIN(PHIBO) * SIN(BETAOR)$ VALUE4 = $COS(PHIBO) * SIN(DEL) * COS(BETAOR)$ VALUE5 = $VALUE3 + VALUE4$ THETA0 = $ASIN(VALUE5)$ VALUE6 = $SIN(PHIBO) * SIN(DEL) * COS(BETAOR)$ VALUE7 = $COS(PHIBO) * SIN(BETAOR)$ VALUE8 = $COS(THETA0)$ VALUE9 = $(VALUE6 - VALUE7) / VALUE8$ PSIO = $ASIN(VALUE9)$ VALUE10 = $SIN(PHIBO) * COS(BETAOR)$ VALUE11 = $COS(PHIBO) * SIN(DEL) * SIN(BETAOR)$ VALUE12 = $COS(THETA0)$ VALUE13 = $(VALUE10 - VALUE11) / VALUE12$ PHIO = $ASIN(VALUE13)$ UOLD = $VAO * COS(BETAOR) * COS(ALPHAOR)$ VOLD = $VAO * SIN(BETAOR)$ WOLD = $VAO * COS(BETAOR) * SIN(ALPHAOR)$ VAOLD = $(UOLD ** 2 + VOLD ** 2 + WOLD ** 2) ** 0.5$ FOLD = FO QOLD = QO ROLD = RO

THETAOLD =THETA0
 PHIOLD =PHI0
 PSIOLD =PSI0

QDASHOLD =QDASH0

XOLD =0
 YOLD =0
 ZOLD =0

ALPHAOLD =ALPHA0R
 BETAOLD =BETA0R
 DELAOLD =DELA
 DELEOLD =DELE
 DELROLD =DELR

ALPHADOL =0
 BETADOLD =0

INITILYZE TIME AND DEFINE THE INTEGRATION STEP SIZE(DELT).

TIME =0
 DELT =0.025

PDOLD =0
 RDOLD =0

RETURN
 END

SUBROUTINE LOADING

THIS SUBROUTINE CALCULATES THE LOADING FORCES & MOMENTS ON THE A/C
 REAL SUMFX,SUMFY,SUMFZ,SUML,SUMM,SUMN

COMMON/BASIC1/CX,CY,CZ,CLBETA,CLP,CLQ,CLALPHA,CLALPHA0,CLR
 ,CLBETAD,CYBETA,CZALPHA0

COMMON/BASIC2/CMALPHA,CMBETA,CMF,CMQ,CNALPHA0,CMR,CMBETAD,CZALPHA

COMMON/BASIC3/CNALPHA,CNBETA,CNQ,CNALPHA0,CNR,CNF,CNBETAD,CZQ

COMMON/BASIC5/CXDELE,CYDELA,CYDELR,CZDELE

COMMON/BASIC6/CLDELA,CLDELR,CMDELE,CNDELA,CNDELR

COMMON/SET1/MASS,S,B,CDASH

COMMON/OLD1/UOLD,VOLD,WOLD,POLD,ROLD,ROLD,THETAOLD,
 -PHIOLD,PSIOLD

COMMON/SET6/H1,H2,H3,DELAOLD,DELEOLD,DELROLD

COMMON/SET5/ALPHATR,BETATR

COMMON/OLD3/ALPHAOLD,BETAOLD,ALPHADOL,BETADOLD,VAOLD,QDASHOLD

COMMON/INITIAL1/QDASHO,R1,R2,CZT,CLT,R3,R4,R5,CMT,CNT,T

COMMON/LOADING1/SUMFX,SUMFY,SUMFZ,SUML,SUMM,SUMN

REAL MASS

G = 9.81

P = POLD

Q = QOLD

R = ROLD

THETA = THETAOLD

PHI = PHIOLD

ALPHA = ALPHAOLD

DELA = DELAOLD

DELE = DELEOLD

DELR = DELROLD

BETA = BETAOLD

ALPHAD = ALPHADOL

BETAD = BETADOLD

ALPHAT = ALPHATR

VA = VAOLD

QDASH = QDASHOLD

SUMFX = T-MASS*G*SIN(THETA)+(CX+CXDELE*DELE)*QDASH*S

SUMFY = MASS*G*COS(THETA)*SIN(PHI)+(CYBETA*BETA+CYDELA*DELA
+CYDELR*DELR)*QDASH*S

SUMFZ = MASS*G*COS(THETA)*COS(PHI)+(CZT+CZALPHA*(ALPHA-ALPHAT)
+CZQ*(Q*CDASH/(2*VA))+CZALPHAD*(ALPHAD*CDASH/(2*VA))
+CZDELE*DELE)*QDASH*S

SUML = (CLT+CLALPHA*(ALPHA-ALPHAT)+CLBETA*BETA+CLP*(P*B/(2*VA))
+CLQ*(Q*CDASH/(2*VA))+CLALPHAD*(ALPHAD*CDASH/(2*VA))
+CLR*(R*B/(2*VA))+CLBETAD*(BETAD*B/(2*VA))+CLDELA*DELA
+CLDELR*DELR)*QDASH*S*B

SUMM = (CMT+CMALPHA*(ALPHA-ALPHAT)+CMBETA*BETA+CMF*(F*B/(2*VA))
+CMQ*(Q*CDASH/(2*VA))+CMALPHAD*(ALPHAD*CDASH/(2*VA))+CMR
*(R*B/(2*VA))+CMBETAD*(BETAD*B/(2*VA))+CMDELE*DELE)
*QDASH*S*CDASH

SUMN = (CNT+CNBETA*BETA+CNALPHA*(ALPHA-ALPHAT)+CNF*(F*B/(2*VA))
+CNR*(R*B/(2*VA))+CNBETAD*(BETAD*B/(2*VA))+CNQ*
(Q*CDASH/(2*VA))+CNALPHAD*(ALPHAD*CDASH/(2*VA))+CNDELA
*DELA+CNDELR*DELR)*QDASH*S*B

RETURN
END

SUBROUTINE MOTION

THIS SUBROUTINE CALCULATES THE MOTION VARIABLES AT TIME (T),
,AND THEN,USES THE RUNGE-KUTTA METHOD OF NUMERICAL INTEGRATION
TO FIND THESE MOTION VARIABLES AT TIME (T+DELT)

COMMON/SET1/MASS,S,B,CDASH,IX,IY,IZ,IXZ

COMMON/LOADING1/SUMFX,SUMFY,SUMFZ,SUML,SUMM,SUMN

COMMON/OLD1/UOLD,VOLD,WOLD,POLD,QOLD,ROLD,THETAOLD,PHIOLD
,PSIOLD,PDOLD,RDOLD

COMMON/OLD4/TIME,DELT

THE FOLLOWING IS THE SUBROUTINE OUTPUT

COMMON/MOTION1/UD,VD,WD,PD,QD,RD,THETAD,PHID,PSID

COMMON/NEW1/UNEW,VNEW,WNEW,FNEW,QNEW,RNEW,THETANEW,PHINEW,PSINEW

REAL UD,VD,WD,PD,QD,RD,THETAD,PHID,PSID

REAL UNEW,VNEW,WNEW,FNEW,QNEW,RNEW,THETANEW,PHINEW,PSINEW
REAL MASS,IX,IY,IZ,IXZ

Q = QOLD
P = POLD
R = ROLD

W = WOLD
V = VOLD
U = UOLD

PHI = PHIOLD
THETA = THETAOLD

UD = SUMFX/MASS - Q*W + R*V
VD = SUMFY/MASS - R*U + P*W
WD = SUMFZ/MASS - P*V + Q*U

PD = (SUML - (IZ - IY)*Q*R + XIX*(RDOLD + P*Q))/IX
QD = (SUMM - (IX - IZ)*R*P + XIX*(R**2 - P**2))/IY
RD = (SUMN - (IY - IX)*P*Q + IXZ*(PDOLD - Q*R))/IZ

THETAD = (Q*COS(PHI) - R*SIN(PHI))
PHID = P + Q*SIN(PHI)*TAN(THETA) + R*COS(PHI)*TAN(THETA)
PSID = Q*SIN(PHI)/COS(THETA) + R*COS(PHI)/COS(THETA)

PDOLD = PD

RDOLD =RD

NOW PERFORM THE NUMERICAL INTEGRATION

UNEW =UOLD+UD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
VNEW =VOLD+VD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
WNEW =WOLD+WD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

PNEW =POLD+PD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
QNEW =QOLD+QD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
RNEW =ROLD+RD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

THETANEW =THETAOLD+THETAD *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
PHINEW =PHIOLD+PHID *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)
PSINEW =PSIOLD+PSID *(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

RETURN
END

SUBROUTINE AUXILIARY

THIS SUBROUTINE CALCULATES THE AUXILIARY VARIABLE NECESSARY TO
COMPLETE THE ANALYSIS, AND USES THE RUNGE-KUTTA METHOD TO FIND THE
AIRCRAFT POSITION(X,Y,Z)

COMMON/OLD1/UOLD,VOLD,WOLD,POLD,QOLD,ROLD,THETAOLD,
PHIOLD,PSIOLD

COMMON/OLD2/XOLD,YOLD,ZOLD

COMMON/MOTION1/UD,VD,WD

COMMON/OLD4/TIME,DELT

COMMON/AUXILIARY1/XD,YD,ZD,ALPHAD,BETAD,ALPHA,BETA
GAMMAH,GAMMAV,AEROPHI

COMMON/NEW2/XNEW,YNEW,ZNEW

COMMON/NEW1/UNEW,VNEW,WNEW

REAL XD,YD,ZD,ALPHAD,BETAD,ALPHA,BETA
REAL VALUE1,VALUE2,VALUE3,VALUE4,VALUES
REAL XNEW,YNEW,ZNEW,GAMMAH,GAMMAV,AEROPHI

U =UOLD
V =VOLD
W =WOLD

THETA =THETAOLD
PHI =PHIOLD
PSI =PSIOLD

```

C      XD      =U*COS(PHI)*COS(THETA)+V*(COS(PHI)*SIN(THETA)*SIN(PHI)-
-      SIN(PHI)*COS(PHI))+W*(COS(PHI)*SIN(THETA)*COS(PHI)
-      +SIN(PHI)*SIN(PHI))
C
C      YD      =U*SIN(PHI)*COS(THETA)+V*(SIN(THETA)*SIN(PHI)*SIN(PHI)
-      +COS(PHI)*COS(PHI))+W*(SIN(PHI)*SIN(THETA)*COS(PHI)
-      -COS(PHI)*SIN(PHI))
C
C      ZD      =-U*SIN(THETA)+V*COS(THETA)*SIN(PHI)+W*COS(THETA)
-      *COS(PHI)

```

```

C      VALUE1  =U      *XD
C      ALPHA   =ATAN(VALUE1)

```

```

C      VALUE2  =(-V      *UD      )/COS(ALPHA)
C      ALPHA   =ATAN(VALUE2)

```

```

C      ALPHA   =(U      *UD      +UD      *W      )/(U      *U      +W      *W      )

```

```

C      ALPHA   =(U      *VD      +VD      *W      )/(U      *U      +W      *W      )

```

```

C      VALUE3  =(YD/XD)

```

```

C      GAMMA   =ATAN(VALUE3)

```

```

C      VALUE4  =(-ZD/((XD**2+YD**2)**0.5))

```

```

C      GAMMA   =ATAN(VALUE4)

```

```

C      VALUE5  =(V      *W      )

```

```

C      AEROPHI =ATAN(VALUE5)

```

```

C      NOW PERFORM THE NUMERICAL INTEGRATION

```

```

C      XNEW    =XOLD+XD*(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

```

```

C      YNEW    =YOLD+YD*(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

```

```

C      ZNEW    =ZOLD+ZD*(DELT+DELT**2/2+DELT**3/6+DELT**4/24)

```

```

C      RETURN
C      END

```

```

C      *****

```

```

C      SUBROUTINE NEW

```

```

C      *****

```

```

C      THIS SUBROUTINE IS TO UPDATE THE VALUES OF DIFFERENT
C      PARAMETERS TO PROCEED TO THE NEXT STEP IN THE LOOP.

```

```

C      COMMON TO OLD NEW, UNEW, VNEW, WNEW, PNEW, QNEW, RNEW, THE TANew, PHINew, PSINew
C      COMMON TO OLD NEW, YNEW, ZNEW

```


COMMON/OLD3/ALPHAOLD,BETAOLD,ALPHADOL,BETADOLD,VAOLD,QDASHOLD
COMMON/AUXILIARY1/F1,F2,F3,ALPHAD,BETAD,ALPHA,BETA
COMMON/OLD4/TIME,DELT
COMMON/SET5/ALPHATR,BETATR
COMMON/OLD1/UOLD,VOLD,WOLD,FOLD,QOLD,ROLD,THETAOLD,PHIOLD,PSIOLD
COMMON/SET4/ALPHAO,BETAO,FACTOR,ALTO,DELALPHA,ALPHAOR,BETAOR
COMMON/BASIC1/CX,CY,CZ,CLBETA,CLP,CLQ,CLALPHA,CLALPHAD
-,CLR,CLBETAD,CYBETA,CZALPHAD
COMMON/BASIC2/CMALPHA,CMBETA,CMF,CMQ,CMALPHAD,CNR,CMBETAD,CZALPHA
COMMON/BASIC3/CNALPHA,CNBETA,CNQ,CNALPHAD,CNR,CNF,CNBETAD,CZQ
COMMON/BASIC4/A1,A2,B1,B2,C1,C2,D1,D2
COMMON/OLD2/XOLD,YOLD,ZOLD
COMMON/SET6/DELA,DELE,DELR,DELAOLD,DELEOLD,DELROLD
COMMON/SET1/MASS,S

UOLD =UNEW
VOLD =VNEW
WOLD =WNEW
FOLD =FNEW
QOLD =QNEW
ROLD =RNEW

THETAOLD =THETANEW
PSIOLD =PSINEW
PHIOLD =PHINEW

FIND THE INSTANTINOUS PERTURBATION IN THE ANGLE OF ATTACK
DELALPHA =ALPHA-ALPHATR

XOLD =XNEW
YOLD =YNEW
ZOLD =ZNEW

TIME =TIME+DELT

ALPHAOLD =ALPHA
BETAOLD =BETA

ALPHADOL =ALPHAD
BETADOLD =BETAD

VAOLD =(UOLD**2+VOLD**2+WOLD**2)**0.5

ALT =ALTO-ZNEW

RO =1.2253*EXP(-ALT/10150)

QUASHOLD =0.5*RO*(VAOLD**2)

THE VALUES OF SOME LOCALLY LINEARIZED STABILITY DERIVATIVES HAVE
TO BE UPDATED, SINCE THEY ARE FUNCTION OF THE ANGLE OF ATTACK.

CLQ =0.5*C1-30*C2*DELALPHA

CLALPHAD=0.5*C1-30*C2*DELALPHA

CMQ =-7*D1-120*D2*DELALPHA

CMALPHAD=-7*D1-120*D2*DELALPHA

CMR =0.1*C1-25*C2*DELALPHA

CMBETAD =-0.1*C1-25*C2*DELALPHA

CNQ =0.5*A1-60*A2*DELALPHA

CNALPHAD=0.5*A1-60*A2*DELALPHA

CNR =-0.15*D1-6*D2*DELALPHA

CNBETAD =0.2*D1-6*D2*DELALPHA

RETURN

END

SUBROUTINE REINITIAL

COMMON/SET6/DELA,DELE,DELR,DELAOLD,DELEOLD,DELROLD

THE FOLLOWING VALUES WILL MAKE CONRTOL PERTURBATION AN IMPULSE
FUNCTION. FOR CHANGING THEM TO SEP PERTURBATION, PUT A COMMENT
SIGN IN FRONT OF EACH LINE.

DELAOLD =0
DELEOLD =0
DELROLD =0

RETURN
END

Appendix E

SAMPLE COMPUTER OUTPUT

Some of the computer output is presented for the study case of ($C_{n\dot{\alpha}}$ and $C_{n\dot{\alpha}}$) with initial 2g turning flight. This sample also has the input data, note that perturbation in elevator angle, and the initial angles of attack and sideslip, are all given in degrees. By changing the constants A1 to D2 we can chose the case as desired.

14:04 MAR 16 '87 *T

DRIBBLE ON @ 14:00 03/16/87

! *L

THE ANGLE OF ATTACK =

?33

THE ANGLE OF SIDESLIP =

?0

THE LOADFACTOR =

?2

THE PERTURBATION IN ALPHA =

?0

THE PERTURBATION OF AILERON IS:

?0

THE PERTURBATION OF ELEVATOR IS:

?3

THE PERTURBATION OF RUDDER IS:

?0

NO CONTROLE PERTURBATION DENOTED BY '0', IMPULSE '1', STEP '2'

?1

THIS CASE IS TO STUDY THE EFFECT OF CNQ & CNALPHAD

THE FOLLOWING IS THE LEGEND FOR DIFFRENT STUDY CASES

CASE #1 CONST. COEFFICIENTS WITH CNQ & CNALPHAD=0

CASE #2 CONST. COEFFICIENTS ALL WITH NOMINAL VALUES

CASE #3 CONST. COEFFICIENTS WITH CNQ & CNALPHAD DOUBLED

CASE #4 LOCALLY LINEARIZED COEFFS. WITH CNQ & CNALPHAD=0

CASE #5 LOCALLY LINEARIZED COEFFS. ALL WITH NOMINAL VALUE

CASE #6 LOCALLY LINEARIZED COEFFS. CNQ & CNALPHAD DOUBLED

CASE #7, ALL CROSS-COUPPLING STABILITY DERIVATIVES=0

THIS IS CASE #

?1

THE CONSTANTS A1, A2, B1, B2, C1, C2, D1 & D2 DETERMINE THE CASE

A1 =

?0

A2 =

?0

B1 -

70

B2 =

70

C1 =

71

C2 =

70

D1 =

71

D2 =

70

TIME SEC.	P RAD/SEC.	Q RAD/SEC.	R RAD/SEC.	THETA RAD.	PHI RAD.	PSI RAD.
.00000	-.02684	.08536	.04133	.27580	1.11986	.51232
.10000	-.02941	.03626	.04134	.27497	1.11933	.513048
.20000	-.03795	.00134	.04045	.27215	1.11711	.515938
.30000	-.04948	-.02722	.03802	.26800	1.11303	.522500
.40000	-.05895	.00694	.03379	.26404	1.10769	.52543
.50000	-.06086	.03944	.02842	.26204	1.10246	.52874
.60000	-.05734	.06841	.02258	.26199	1.09794	.53471
.70000	-.05116	.09275	.01694	.26376	1.09450	.54296
.80000	-.04498	.11180	.01212	.26707	1.09220	.55303
.90000	-.04088	.12537	.00860	.27159	1.09087	.56449
1.00000	-.04015	.13364	.00669	.27692	1.09013	.57688
1.10000	-.04313	.13712	.00649	.28268	1.08955	.58983
1.20000	-.04931	.13651	.00788	.28849	1.08867	.60300
1.30000	-.05750	.13270	.01056	.29406	1.08713	.61610
1.40000	-.06617	.12657	.01408	.29916	1.08471	.62891
1.50000	-.07374	.11901	.01794	.30364	1.08137	.64128
1.60000	-.07892	.11081	.02166	.30744	1.07724	.65311
1.70000	-.08089	.10265	.02485	.31059	1.07256	.66434
1.80000	-.07941	.09505	.02724	.31314	1.06765	.67497
1.90000	-.07478	.08837	.02873	.31519	1.06287	.68502
2.00000	-.06779	.08283	.02935	.31689	1.05851	.69454
2.10000	-.05945	.07849	.02924	.31833	1.05478	.70361
2.20000	-.05090	.07533	.02861	.31964	1.05161	.71230
2.30000	-.04317	.07325	.02771	.32090	1.04960	.72069
2.40000	-.03706	.07210	.02675	.32218	1.04806	.72888
2.50000	-.03304	.07171	.02592	.32350	1.04704	.73694
2.60000	-.03125	.07192	.02535	.32488	1.04633	.74494
2.70000	-.03150	.07258	.02509	.32633	1.04573	.75297
2.80000	-.03337	.07356	.02513	.32783	1.04508	.76106
2.90000	-.03629	.07473	.02542	.32939	1.04424	.76926
3.00000	-.03962	.07600	.02587	.33098	1.04314	.77759
3.10000	-.04279	.07729	.02639	.33260	1.04177	.78607
3.20000	-.04537	.07854	.02689	.33426	1.04015	.79470
3.30000	-.04705	.07969	.02729	.33595	1.03837	.80347
3.40000	-.04774	.08069	.02756	.33768	1.03651	.81237
3.50000	-.04749	.08153	.02768	.33945	1.03466	.82136
3.60000	-.04650	.08218	.02768	.34128	1.03291	.83043
3.70000	-.04502	.08262	.02758	.34315	1.03131	.83955
3.80000	-.04335	.08286	.02744	.34507	1.02990	.84871
3.90000	-.04175	.08291	.02731	.34702	1.02868	.85787
4.00000	-.04043	.08278	.02722	.34898	1.02763	.86703
4.10000	-.03951	.08251	.02722	.35096	1.02671	.87618
4.20000	-.03903	.08213	.02731	.35292	1.02587	.88530
4.30000	-.03895	.08168	.02749	.35485	1.02508	.89440
4.40000	-.03917	.08119	.02775	.35675	1.02428	.90347
4.50000	-.03955	.08069	.02805	.35860	1.02347	.91251
4.60000	-.03996	.08023	.02838	.36041	1.02262	.92153
4.70000	-.04027	.07981	.02869	.36218	1.02174	.93054
4.80000	-.04040	.07946	.02896	.36391	1.02084	.93953
4.90000	-.04030	.07918	.02917	.36560	1.01996	.94850
5.00000	-.03997	.07897	.02933	.36727	1.01910	.95747
5.10000	-.03945	.07883	.02942	.36893	1.01829	.96643
5.20000	-.03880	.07874	.02947	.37058	1.01756	.97540
5.30000	-.03811	.07870	.02948	.37224	1.01690	.98436
5.40000	-.03744	.07868	.02947	.37389	1.01634	.99332

5.50000	-.03287	.07868	.02947	.37555	1.01585	1.0022
5.60000	-.03643	.07869	.02948	.37721	1.01543	1.0112
5.70000	-.03615	.07870	.02951	.37888	1.01506	1.0202
5.80000	-.03600	.07869	.02957	.38054	1.01474	1.0292
5.90000	-.03598	.07868	.02965	.38221	1.01444	1.0382
6.00000	-.03603	.07866	.02976	.38386	1.01416	1.0472
6.10000	-.03612	.07863	.02988	.38551	1.01389	1.0562
6.19999	-.03621	.07858	.03001	.38714	1.01362	1.0652
6.29999	-.03624	.07854	.03014	.38877	1.01337	1.0742
6.39999	-.03622	.07848	.03025	.39038	1.01313	1.0833
6.49999	-.03611	.07843	.03035	.39198	1.01292	1.0923
6.59999	-.03594	.07837	.03044	.39358	1.01273	1.1014
6.69999	-.03571	.07831	.03051	.39516	1.01258	1.1104
6.79999	-.03544	.07825	.03058	.39674	1.01246	1.1195
6.89999	-.03516	.07818	.03063	.39831	1.01239	1.1286
6.99999	-.03489	.07812	.03068	.39987	1.01236	1.1376
7.09999	-.03465	.07805	.03073	.40142	1.01238	1.1467
7.19999	-.03444	.07797	.03079	.40296	1.01243	1.1558
7.29999	-.03428	.07790	.03085	.40450	1.01251	1.1649
7.39999	-.03415	.07783	.03092	.40602	1.01262	1.1739
7.49999	-.03406	.07775	.03099	.40754	1.01276	1.1830
7.59999	-.03399	.07768	.03107	.40904	1.01292	1.1921
7.69999	-.03393	.07761	.03115	.41053	1.01310	1.2012
7.79999	-.03387	.07755	.03123	.41201	1.01330	1.2103
7.89999	-.03380	.07749	.03131	.41348	1.01352	1.2194
7.99999	-.03371	.07743	.03138	.41493	1.01376	1.2285

TIME SEC.	ALPHA RAD.	BETA RAD.	ALPHA RAD/SEC.	BETA RAD/SEC.	GAMMA RAD.	GAMMA RAD.	AEROPHI RAD.
.00000	.57596	.00000	.00000	.00000	.00000	.00000	.00000
.10000	.57377	-.00003	-.03944	-.00088	.01019	.00012	-.00012
.20000	.56727	-.00030	-.07750	-.00491	.02017	.00012	-.00081
.30000	.55769	-.00099	-.10347	-.00966	.02992	.00000	-.00231
.40000	.54838	-.00200	-.07871	-.01218	.03920	-.00037	-.00434
.50000	.54255	-.00293	-.04535	-.00903	.04844	-.00074	-.00600
.60000	.53987	-.00334	-.01567	-.00181	.05769	-.00107	-.00849
.70000	.53990	-.00302	.00923	.00717	.06697	-.00132	-.00547
.80000	.54211	-.00192	.02871	.01575	.07633	-.00148	-.00297
.90000	.54594	-.00018	.04257	.02214	.08576	-.00152	-.00061
1.00000	.55082	.00191	.05102	.02508	.09528	-.00145	.00467
1.10000	.55625	.00403	.05461	.02403	.10488	-.00126	.00855
1.20000	.56178	.00384	.05408	.01918	.11455	-.00096	.01162
1.30000	.56704	.00705	.05031	.01139	.12428	-.00056	.01346
1.40000	.57176	.00751	.04421	.00201	.13404	-.00007	.01383
1.50000	.57577	.00716	.03665	-.00741	.14383	.00050	.01274
1.60000	.57896	.00608	.02841	-.01542	.15363	.00114	.01043
1.70000	.58131	.00445	.02018	-.02090	.16342	.00183	.00726
1.80000	.58287	.00254	.01251	-.02326	.17319	.00257	.00371
1.90000	.58370	.00059	.00578	-.02248	.18293	.00335	.00024
2.00000	.58392	-.00114	.00023	-.01901	.19264	.00414	.00275
2.10000	.58366	-.00247	-.00406	-.01367	.20232	.00496	.00495
2.20000	.58305	-.00370	-.00713	-.00744	.21196	.00579	.00623
2.30000	.58220	-.00481	-.00909	-.00130	.22157	.00662	.00656
2.40000	.58120	-.00343	-.01010	.00394	.23114	.00746	.00606
2.50000	.58016	-.00289	-.01034	.00773	.24068	.00831	.00495
2.60000	.57913	-.00212	-.00999	.00979	.25019	.00916	.00348
2.70000	.57817	-.00126	-.00919	.01015	.25967	.01001	.00192
2.80000	.57731	-.00045	-.00807	.00904	.26913	.01088	.00048
2.90000	.57657	.00021	-.00675	.00694	.27857	.01175	.00064
3.00000	.57598	.00067	-.00533	.00428	.28798	.01264	.00136
3.10000	.57553	.00088	-.00389	.00157	.29738	.01354	.00166
3.20000	.57522	.00089	-.00250	-.00080	.30676	.01446	.00158
3.30000	.57505	.00072	-.00122	-.00255	.31613	.01540	.00122
3.40000	.57500	.00045	-.00009	-.00352	.32548	.01636	.00068
3.50000	.57506	.00013	.00087	-.00372	.33482	.01734	.00011
3.60000	.57519	-.00016	.00162	-.00326	.34415	.01834	-.00041
3.70000	.57539	-.00039	.00217	-.00235	.35347	.01936	-.00080
3.80000	.57564	-.00053	.00251	-.00121	.36278	.02041	-.00101
3.90000	.57590	-.00057	.00265	-.00008	.37208	.02147	-.00105
4.00000	.57617	-.00053	.00261	.00086	.38137	.02255	-.00093
4.10000	.57642	-.00042	.00243	.00147	.39065	.02366	-.00071
4.20000	.57665	-.00028	.00213	.00171	.39992	.02477	-.00045
4.30000	.57684	-.00014	.00176	.00159	.40919	.02591	-.00020
4.40000	.57699	-.00002	.00134	.00118	.41844	.02706	.00000
4.50000	.57710	.00005	.00093	.00058	.42768	.02822	.00010
4.60000	.57717	.00004	.00054	.00007	.43691	.02940	.00010
4.70000	.57721	.00002	.00021	.00066	.44613	.03059	.00001
4.80000	.57721	-.00006	-.00007	.00111	.45534	.03179	-.00015
4.90000	.57719	-.00017	-.00026	.00135	.46453	.03300	-.00036
5.00000	.57715	-.00028	-.00039	.00138	.47371	.03422	-.00057
5.10000	.57711	-.00039	-.00046	-.00121	.48289	.03546	-.00077
5.20000	.57706	-.00048	-.00046	-.00090	.49205	.03670	-.00091
5.30000	.57702	-.00054	-.00043	-.00051	.50120	.03795	-.00100
5.40000	.57698	-.00056	-.00037	-.00010	.51034	.03921	-.00103

5.50000	.57695	-.00055	-.00029	.00025	.51947	.04048	-.0009
5.60000	.57692	-.00051	-.00021	.00052	.52859	.04176	-.0009
5.70000	.57691	-.00046	-.00013	.00066	.53770	.04305	-.0008
5.80000	.57690	-.00040	-.00006	.00069	.54680	.04434	-.0007
5.90000	.57689	-.00035	.00000	.00060	.55590	.04564	-.0006
6.00000	.57690	-.00030	.00005	.00044	.56499	.04695	-.0005
6.10000	.57690	-.00028	.00008	.00023	.57407	.04827	-.0005
6.19999	.57691	-.00027	.00011	.00001	.58314	.04959	-.0005
6.29999	.57693	-.00028	.00013	-.00018	.59220	.05092	-.0005
6.39999	.57694	-.00030	.00014	-.00032	.60126	.05226	-.0005
6.49999	.57696	-.00033	.00015	-.00040	.61031	.05360	-.0006
6.59999	.57697	-.00037	.00016	-.00041	.61935	.05495	-.0006
6.69999	.57699	-.00040	.00016	-.00037	.62838	.05631	-.0007
6.79999	.57700	-.00043	.00016	-.00028	.63741	.05767	-.0008
6.89999	.57702	-.00045	.00015	-.00018	.64644	.05903	-.0008
6.99999	.57703	-.00046	.00015	-.00007	.65545	.06040	-.0008
7.09999	.57705	-.00046	.00013	.00003	.66447	.06177	-.0008
7.19999	.57706	-.00045	.00012	.00009	.67347	.06315	-.0008
7.29999	.57707	-.00044	.00010	.00013	.68247	.06453	-.0008
7.39999	.57708	-.00043	.00009	.00014	.69146	.06591	-.0007
7.49999	.57709	-.00042	.00007	.00011	.70045	.06730	-.0007
7.59999	.57710	-.00041	.00005	.00007	.70944	.06869	-.0007
7.69999	.57710	-.00041	.00004	.00002	.71842	.07008	-.0007
7.79999	.57710	-.00041	.00003	-.00003	.72739	.07147	-.0007
7.89999	.57711	-.00041	.00002	-.00008	.73636	.07287	-.0007
7.99999	.57711	-.00042	.00001	-.00011	.74532	.07427	-.0007

TIME SEC.	X M	Y M	Z M
.000000	.000000	.000000	.000000
.100000	17.45503	.06690	-.00091
.200000	34.90706	.31037	-.000318
.300000	52.35839	.72662	-.000470
.400000	69.81348	1.30997	-.000229
.500000	87.27619	2.05555	.00667
.600000	104.74624	2.96319	.02185
.700000	122.22214	4.03346	.04230
.800000	139.70146	5.26739	.06658
.900000	157.18104	6.66639	.09291
1.000000	174.65713	8.23150	.11928
1.100000	192.12570	9.96434	.14363
1.200000	209.58255	11.86589	.16396
1.300000	227.02354	13.93695	.17833
1.400000	244.44474	16.17799	.18511
1.500000	261.84250	18.58916	.18268
1.600000	279.21362	21.17027	.16976
1.700000	296.55527	23.92087	.14525
1.800000	313.86502	26.84029	.10824
1.900000	331.14087	29.92771	.05804
2.000000	348.38110	33.18221	-.00588
2.100000	365.58426	36.60282	-.08391
2.200000	382.74902	40.18855	-.17630
2.300000	399.87433	43.93849	-.28323
2.400000	416.95905	47.85166	-.40481
2.500000	434.00223	51.92716	-.54113
2.600000	451.00281	56.16413	-.69226
2.700000	467.95969	60.56173	-.85829
2.800000	484.87183	65.11915	-1.03935
2.900000	501.73810	69.83562	-1.23558
3.000000	518.55728	74.71034	-1.44718
3.100000	535.32813	79.74256	-1.67439
3.200000	552.04932	84.93153	-1.91749
3.300000	568.71942	90.27648	-2.17680
3.400000	585.33710	95.77666	-2.45264
3.500000	601.90076	101.43131	-2.74538
3.600000	618.40897	107.23967	-3.05538
3.700000	634.86020	113.20095	-3.38300
3.800000	651.25287	119.31437	-3.72863
3.900000	667.58545	125.57912	-4.09260
4.000000	683.85638	131.99439	-4.47525
4.100000	700.06415	138.55935	-4.87692
4.200000	716.20728	145.27314	-5.29789
4.300000	732.28424	152.13491	-5.73845
4.400000	748.29361	159.14376	-6.19887
4.500000	764.23401	166.29881	-6.67939
4.600000	780.10403	173.59914	-7.18023
4.700000	795.90234	181.04382	-7.70161
4.800000	811.62763	188.63193	-8.24373
4.900000	827.27860	196.36252	-8.80677
5.000000	842.85394	204.23465	-9.39092
5.100000	858.35239	212.24736	-9.99635
5.200000	873.77271	220.39968	-10.62321
5.300000	889.11368	228.69066	-11.27166
5.400000	904.37402	237.11932	-11.94185

5.50000	919.55252	245.68468	-12.63392
5.60000	934.64801	254.38577	-13.34800
5.70000	949.65918	263.22153	-14.08424
5.80000	964.58484	272.19101	-14.84275
5.90000	979.42383	281.29321	-15.62366
6.00000	994.17493	290.52713	-16.42708
6.10000	1008.83691	299.89175	-17.25314
6.19999	1023.40866	309.38602	-18.10192
6.29999	1037.88898	319.00891	-18.97355
6.39999	1052.27670	328.75937	-19.86811
6.49999	1066.57071	338.63632	-20.78570
6.59999	1080.76987	348.63873	-21.72640
6.69999	1094.87305	358.76553	-22.69029
6.79999	1108.87915	369.01559	-23.67746
6.89999	1122.78708	379.38788	-24.68796
6.99999	1136.59570	389.88129	-25.72186
7.09999	1150.30399	400.49472	-26.77921
7.19999	1163.91083	411.22705	-27.86007
7.29999	1177.41516	422.07721	-28.96448
7.39999	1190.81598	433.04407	-30.09248
7.49999	1204.11227	444.12650	-31.24409
7.59999	1217.30298	455.32336	-32.41935
7.69999	1230.38712	466.63355	-33.61828
7.79999	1243.36371	478.05588	-34.84089
7.89999	1256.23178	489.58923	-36.08720
7.99999	1268.99036	501.23245	-37.35721

 THIS CASE IS TO STUDY THE EFFECT OF CNQ & CNALPHAD
 THE FOLLOWING IS THE LEGEND FOR DIFFRENT STUDY CASES

 CASE #1 CONST. COEFFICIENTS WITH CNQ & CNALPHAD=0
 CASE #2 CONST. COEFFICIENTS ALL WITH NOMINAL VALUES
 CASE #3 CONST. COEFFICIENTS WITH CNQ & CNALPHAD DOUBLED
 CASE #4 LOCALLY LINEARIZED COEFFS. WITH CNQ & CNALPHAD=0
 CASE #5 LOCALLY LINEARIZED COEFFS. ALL WITH NOMINAL VALUE
 CASE #6 LOCALLY LINEARIZED COEFFS. CNQ & CNALPHAD DOUBLED
 CASE #7, ALL CROSS-COUPPLING STABILITY DERIVATIVES=0

 THIS IS CASE #
 ?5

 THE CONSTANTS A1,A2,B1,B2,C1,C2,D1&D2 DETERMINE THE CASE

 A1 =
 ?1
 A2 =
 ?1
 B1 =
 ?1
 B2 =
 ?1
 C1 =
 ?1
 C2 =
 ?1
 D1 =
 ?1
 D2 =
 ?1

TIME SEC.	P RAD/SEC.	Q RAD/SEC.	R RAD/SEC.	THETA RAD.	PHI RAD.	PSI RAD.
.00000	-.02684	.08536	.04133	.27580	1.11986	.51232
.10000	-.02940	.03630	.04088	.27496	1.11932	.52048
.20000	-.03933	-.00150	.03784	.27225	1.11707	.52433
.30000	-.05834	-.02922	.03008	.26847	1.11257	.52477
.40000	-.08260	.00132	.01738	.26537	1.10556	.52428
.50000	-.09508	.03232	.00596	.26480	1.09708	.52610
.60000	-.09208	.06354	-.00097	.26658	1.08855	.53034
.70000	-.07437	.09402	-.00149	.27028	1.08162	.53723
.80000	-.04626	.12187	.00478	.27525	1.07772	.54696
.90000	-.01478	.14454	.01658	.28074	1.07768	.55395
1.00000	.01202	.15945	.03136	.28599	1.08148	.57465
1.10000	.02759	.16495	.04621	.29036	1.08821	.59163
1.20000	.02926	.16107	.05889	.29344	1.09633	.60952
1.30000	.01904	.14967	.06835	.29501	1.10415	.62736
1.40000	.00233	.13380	.07465	.29511	1.11041	.64434
1.50000	-.01454	.11664	.07831	.29391	1.11453	.66000
1.60000	-.02660	.10070	.07986	.29164	1.11664	.67417
1.70000	-.03131	.08744	.07965	.28860	1.11737	.68694
1.80000	-.02859	.07743	.07787	.28510	1.11753	.69852
1.90000	-.02005	.07063	.07477	.28142	1.11789	.70915
2.00000	-.00822	.06666	.07068	.27780	1.11904	.71905
2.10000	.00422	.06503	.06606	.27442	1.12125	.72846
2.20000	.01497	.06523	.06145	.27141	1.12456	.73755
2.30000	.02243	.06679	.05731	.26880	1.12878	.74649
2.40000	.02578	.06929	.05403	.26659	1.13357	.75542
2.50000	.02501	.07238	.05184	.26472	1.13856	.76448
2.60000	.02072	.07575	.05081	.26310	1.14338	.77374
2.70000	.01399	.07914	.05085	.26165	1.14771	.78330
2.80000	.00608	.08236	.05175	.26025	1.15138	.79318
2.90000	-.00176	.08526	.05324	.25885	1.15432	.80339
3.00000	-.00845	.08774	.05502	.25738	1.15657	.81393
3.10000	-.01326	.08975	.05681	.25582	1.15826	.82477
3.20000	-.01582	.09126	.05839	.25415	1.15960	.83584
3.30000	-.01613	.09230	.05961	.25239	1.16079	.84709
3.40000	-.01451	.09288	.06039	.25056	1.16204	.85846
3.50000	-.01150	.09306	.06074	.24867	1.16350	.86989
3.60000	-.00776	.09290	.06072	.24674	1.16528	.88133
3.70000	-.00396	.09246	.06044	.24480	1.16741	.89272
3.80000	-.00065	.09182	.06001	.24284	1.16988	.90404
3.90000	.00176	.09105	.05955	.24087	1.17260	.91526
4.00000	.00307	.09023	.05913	.23888	1.17549	.92638
4.10000	.00326	.08943	.05883	.23686	1.17841	.93740
4.20000	.00249	.08874	.05866	.23480	1.18127	.94831
4.30000	.00103	.08818	.05862	.23270	1.18399	.95915
4.40000	-.00079	.08780	.05869	.23055	1.18650	.96994
4.50000	-.00265	.08762	.05882	.22836	1.18879	.98068
4.60000	-.00426	.08764	.05898	.22613	1.19088	.99141
4.70000	-.00543	.08783	.05912	.22386	1.19280	1.00214
4.80000	-.00605	.08817	.05923	.22157	1.19460	1.01290
4.90000	-.00613	.08862	.05929	.21927	1.19634	1.02368
5.00000	-.00575	.08913	.05931	.21696	1.19807	1.03450
5.10000	-.00504	.08967	.05931	.21466	1.19985	1.04536
5.20000	-.00417	.09020	.05930	.21235	1.20168	1.05627
5.30000	-.00329	.09068	.05930	.21005	1.20359	1.06721
5.40000	-.00254	.09109	.05935	.20774	1.20557	1.07819

5.50000	-.00201	.09143	.05944	.20541	1.20759	1.08920
5.60000	-.00173	.09168	.05959	.20306	1.20964	1.10023
5.70000	-.00169	.09185	.05979	.20069	1.21169	1.11128
5.80000	-.00184	.09196	.06003	.19827	1.21371	1.12234
5.90000	-.00210	.09203	.06030	.19582	1.21568	1.13341
6.00000	-.00239	.09207	.06058	.19331	1.21760	1.14448
6.10000	-.00263	.09210	.06086	.19077	1.21947	1.15556
6.19999	-.00276	.09214	.06112	.18817	1.22129	1.16664
6.29999	-.00275	.09220	.06135	.18554	1.22307	1.17772
6.39999	-.00258	.09228	.06155	.18287	1.22483	1.18880
6.49999	-.00228	.09239	.06172	.18016	1.22658	1.19989
6.59999	-.00187	.09252	.06188	.17742	1.22834	1.21099
6.69999	-.00141	.09268	.06202	.17466	1.23012	1.22210
6.79999	-.00093	.09286	.06216	.17186	1.23192	1.23322
6.89999	-.00048	.09305	.06231	.16904	1.23373	1.24434
6.99999	-.00008	.09324	.06247	.16619	1.23556	1.25549

RIBBLE
OFF @ 14:04 03/16/87

END

22-06-88

FIN