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**Aerodynamic Design of an Experiment for Flow Measurements
in the Gap Region of a Two-Element Airfoil in a Small
Wind Tunnel**

by
François-Léon Bélanger
B.Eng. Université du Québec à Chicoutimi, 1992

**A thesis submitted to the faculty of Graduate
Studies and Research in partial fulfilment
of the requirements for the degree of**

Master of Aerospace Engineering

**Department of
Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
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**Aerodynamic Design of an Experiment for Flow Measurements
in the Gap Region of a Two-Element Airfoil in a Small
Wind Tunnel**

François-Léon Bélanger

1995

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**Department of Mechanical
& Aerospace Engineering**

Abstract

Aircraft designs are strongly affected by low-speed performance requirements and so by aerodynamic characteristics of the high-lift devices. The complexity of high-lift flow phenomena makes them difficult to predict. Detailed measurements of flow behaviour are required to enable development of suitable turbulence models for use in predicting behaviour of high-lift devices.

The aerodynamic design of an experiment to enable measurement of flow field parameters in the gap region of a two-element high-lift airfoil is outlined in this thesis. Computer simulations using a linear vortex panel method program model the flow around a multi-element airfoil. The images of the airfoil placed directly above and below simulates the wind tunnel floor and ceiling. All elements (airfoil and images) are modeled in the inviscid computation. There is no boundary layer computation performed on the high-lift airfoil.

The design appears to be feasible. However, flow separation from the wind tunnel floor present some difficulties. To ensure steady flow without oscillation, boundary layer computations are performed on the wind tunnel floor and ceiling. Two approaches are investigated to keep the boundary layer attached to the wind tunnel floor: suction and accelerating the flow by placing a "hump-like" shape on the floor.

A conceptual mechanical design of the wind tunnel test section is also presented in this thesis.

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List of Symbols

A, B, C,D	Coefficients in the Relation for Induced Velocity
b	Right Hand Side of Eqn 3.5
C_D	Drag Coefficient; - $C_D = \frac{Drag}{1/2\rho V_\infty^2 c}$
C_L	Lift Coefficient; - $C_L = \frac{Lift}{1/2\rho V_\infty^2 c}$
C_{Lmax}	Maximum Lift Coefficient
C_p	Pressure Coefficient; - $C_p = \frac{P-P_{REF.}}{1/2\rho V_\infty^2}$
C_{pp}	Peak Pressure Coefficient (pressure peak located at the lower surface trailing edge due to the flap presence)
c	Chord Length
e	Constant Error Term
G	Gap height (vertical distance between flap and airfoil trailing edge)
l	Panel Length
K	Influence Coefficients Matrix
M	Mach Number
N	Panel Number (inviscid computation)
OH	Overhang (horizontal distance for which the airfoil trailing edge overhang the flap leading edge)
Re or RN	Reynolds Number

r	Distance (radius) Between Junction and Control Point
S	Scaling Length
U, U_{∞}	Free Stream Velocity
U_e	Boundary Layer Tangential External Velocity
u	Tangential Component of Velocity
v	Normal Component of Velocity
X/c	Non-dimensional X axis
Y/c	Non-dimensional Y axis
Z/c	Non-dimensional Z axis
X_{pp}	Peak Pressure Coefficient Location on the X Axis

Greek Symbols:

α	Angle of Attack
Γ	Circulation
γ	Vorticity
δ	Angle of Deflection
Θ	Momentum Thickness
θ	Angle
μ	Doublet strength
ν	Kinetic viscosity
Π	Parameter
ρ	Fluid Density

σ **Source Strength**

Φ **Angle**

Subscripts:

SP   **Spoiler**

i   **Control Point**

m   **Vortex Panel**

F   **Flap**

Chapter 1

INTRODUCTION

1.1 Background

The influence of high-lift airfoil configurations in aeronautical engineering is well known. Aircraft operational features are strongly affected by low-speed performance and, so, by aerodynamic characteristics of the high-lift devices. An increased understanding of high-lift systems is needed to design tomorrow's transport aircraft. An increase of the maximum lift coefficient with minimum weight increase, leading to a larger payload capacity and shorter take-off and landing distance, is a highly desirable design goal.

The complexity of high-lift flow phenomena makes it difficult to predict the lift coefficient (C_L), the drag coefficient (C_D) and to model the inviscid/viscous flow interaction. The complex flow field is particularly hard to predict, especially where the wake of an upstream airfoil element mixes with the boundary layer developing on a downstream element. The analysis requires an appropriate method to simulate the relevant flow mechanism. Present high-lift flow simulation models are based on panel methods or on solution of the Reynolds Averaged Navier-Stokes equations (RANS).

As mentioned by K. de Cock (1993), a consequence of the complexity of the geometries of interest is a strong pressure for fast turn-around time of the high-lift system

analysis in high-lift system design work. This turn-around time requirement partially explains the heavy usage of panel methods in airfoil design studies.

Any model that attempts to predict viscous effects requires a turbulence model. This is true of methods which use boundary layer analysis and of methods which solve the full Navier-Stokes equations. Turbulence models are only approximations and this results in uncertainty and inaccuracy in the flow prediction.

Turbulence models must rely heavily on experimental data and experiments which simulate conditions existing on high-lift airfoils are needed. In an attempt to develop a suitable turbulence model for high-lift devices, one must design an experimental wind tunnel set-up, taking into account the instrumentation required, to provide detailed measurements of the flow behaviour. Those measurements should reflect realistic flow fields, if computational models are to be developed by comparisons with them.

1.2 Scope of the Thesis

This thesis investigates the possibility of obtaining realistic flow field measurements of high-lift type flows, in an existing small closed circuit wind tunnel located at Carleton University. The test section cross-section dimensions are 20 by 30 inches, limiting the dimensions of an experimental set-up and the space available between two airfoil elements (gap). Detailed flow field measurements, including turbulence parameters, are desired, especially in the gap region of a multi-element airfoil. It is important to have as large a model as possible, both to obtain reasonably high Reynolds number and to obtain good spatial resolution in the measurements. This thesis develops a design for an

experiment for obtaining the desired type of measurement.

In such a small wind tunnel with a reasonably large model, the effect of wall interference would hinder the reproduction of a true free stream condition. In the proposed experiment, however, there is no need to reproduce free stream conditions since only a flow representative of that over any high-lift airfoil is needed. Nevertheless, the solid wall effect, especially the ceiling and the floor of the test section, must be considered in the design of the experimental set-up. Flow calculations are carried out to arrive at a geometric configuration suitable for the desired experiments. To ensure an appropriate configuration that will give a realistic pressure distribution in the experimental set-up, the airfoil, the floor and the ceiling are represented using an inviscid panel method. The method of images is used to make the floor and the ceiling a streamline of the flow. The boundary layers developing on the floor and on the ceiling are neglected at this point in the design, but they are considered later in the analysis.

An experimental configuration that produces a typical pressure distribution is easily found by using an inviscid computation. The problem is to prevent the boundary layer on the floor of the wind tunnel test section from separating. Hence, for the tunnel floor a boundary layer analysis was carried out and 'bump' shapes were developed, the purpose of the bump being to prevent separation of the floor boundary layer. Several approaches to predict and control the boundary layer development on the wind tunnel floor have been investigated, and are presented in this thesis.

The effects of the tunnel sidewalls were essentially neglected in the thesis work. The assumption is that it will be possible to obtain two-dimensional flow in the mid-span

region of the model by use of trial-and-error techniques to find ways of minimizing end effects.

1.3 Overview of Method of Design

The main program used in the design of the experimental set-up consisted of a vortex panel method which could represent multi-element airfoils. This program was an extension of an existing program which could compute flow over a single-element airfoil. The work to extend the program was done as part of the present thesis work. The original single-element airfoil program was developed in the early seventies by Kind (1973). The theory and the development of the technique are presented later, in Chapter 3 of this thesis.

The modification enables the computation of the potential flow around a group of up to ten elements, predicting the pressure distribution and the lift coefficient of each element. This enables the use of images so that the test section floor and ceiling are streamlines of the flow.

A linear-vortex panel method creates an overdetermined system of equations. The number of equations is the sum of the number of panels plus the number of elements times two (2 eqn. Kutta condition, see section 3.2), while the number of unknowns is equal to the number of panels plus the number of elements. Different approaches to solving the overdetermined system of linear equations are also discussed in this thesis.

As mentioned earlier, the design process included a boundary layer analysis. The

boundary layer behaviour is predicted with the use of two independent programs. The first is a program calculating the development of a two-dimensional turbulent boundary layer over rough surfaces in pressure gradients. It used the method of Dvorak (1969). The second one uses a modified skin friction law (Stevenson, 1963) to calculate the development of the turbulent boundary layer over a porous surface. Both programs use Head's (1958) entrainment method, the momentum integral equation and a skin friction law. The skin friction laws are different in these two programs and they will be discussed in detail. The program using the method of Dvorak (1969)(Rough Wall Boundary Layer Program) has been in use at Carleton University for many years and it served, together with Stevenson's correlation, to validate the other code (Porous Wall Boundary Layer Program) that was written for the purpose of this thesis.

1.4 Scope of the Design Work

The thesis work included mechanical design of the experimental set-up. The materials choice, the configuration of the set-up and the type of measurements considered are presented in Chapter 4.

The experimental set-up has to suit the hardware (data acquisition system). The choice of which type of measurements and the way those measurements are to be taken needs to be determined at an early stage of the design. Even if the full implementation of the data acquisition system is not a part of this thesis, some decisions had to be made to carry on the design.

Chapter 2

REVIEW OF PREVIOUS WORK

2.1 High-Lift Airfoil Testing

Even though the first patent for a high-lift system was granted to G.V. Lachmann in February 1918, the physical principle underlying its operation was not understood until 1972. In his classical paper, A.M.O. Smith (1972) showed that the slot does not perform as a boundary layer device, but instead that it works because it manipulates the inviscid pressure distribution.

From the data of Foster, Ashill and Williams (1972), Fig. 2.1, it can be seen that the trailing edge of the slat is in the high velocity region of the main element leading-edge; because of this, the pressure rise on the slat (between its suction peak and its trailing edge) is reduced. The same phenomena happen at the trailing edge of the main element due to the leading edge of the flap. The main element leading-edge suction peak is reduced by a downwash induced by the slat and the same mechanism operates near the leading-edge of the flap. The overall pressure rise is split up into a number of relatively small pressure rises, making the possible overall pressure rise and the maximum lift coefficient relatively large.

**Figure 2.1 - Flow Over Two-Dimensional Wing with Slat and Slotted Flap
(Taken from Ashill and Williams, 1972)**

An interesting feature shown in Fig. 2.1, is the viscous boundary layer profile at the trailing edge of the main flap. It can be seen that if two elements are too close, the boundary layer on the aft element will merge with the wake of the upstream element to form a thick boundary layer that will provoke separation.

One of the most recent published papers on the subject of high-lift devices was by Alemdaroglu (1992) concerning an experimental investigation of flow around a multi-element airfoil. That experimental investigation aimed to obtain detailed information

about the flow in and around the flap and the slat regions. Measurements were made using pitot probes, hot-wire anemometer and laser Doppler velocimeter. The Reynolds number varied from 0.5 to 0.8×10^6 in this experiment. An interesting feature of this experimental investigation is the dimension of the wind tunnel used (22 in. X 28 in. and a stream wise length of 44 in.), similar to the wind tunnel dimensions used for this thesis. The tunnel had no facilities for side wall boundary layer control. The three-element airfoil used was the two dimensional LB-572 high-lift model, with a chord length of 12 in. with the elements stowed. Alemdaroglu's discussion of the results for flap-well and slat cavity measurement is focused on the effect of overhang (OH/c) and gap (G/c) ratio on separated flow.

Alemdaroglu found that the effect of modifying the flap overhang ratio from 4% to 5% created an abrupt change in the pressure distribution on the upper surface of the main element. Figure 2.2a shows the effect of flap overhang on the surface pressure distribution. It was suggested that the sudden change could be attributed to a shift of the transition location on the upper surface on the main element. Figure 2.3 indicates the influence of overhang on the position of peak pressure on the flap-well ceiling ($X_{pp}=X/c=[0.80,0.84]$) and on the corresponding peak pressure coefficient (C_{pp}) for three different gaps. It can be seen in Fig. 2.4 that the X_{pp} is inversely proportional to the overhang, indicating that X_{pp} moves almost the same distance as the displacement of the flap leading edge. It is important to note that the peak pressure coefficient, C_{pp} , remains unaffected and is practically constant with changing overhang. Also, note that the main airfoil's trailing edge pressure decreases considerably as the overhang is increased in such

a way that the pressure gradient between the peak pressure and the gap exit is maintained nearly constant.

Figure 2.2 - Surface pressure distribution,
a - effect of flap overhang
b - effect of flap gap
(Taken from Alemdaroglu, 1992)

Figure 2.3 - Effects of Overhang on the Dividing Streamline and Pressure inside the Flap-Well $\delta_{\text{flap}} = 10^\circ$, $G/c = 2\%$. (Taken from Alemdaroglu, 1992)

Figure 2.4 - Position and the Value of the Peak Pressure on the Flap-well Ceiling Surface as Function of OH , $\delta_{\text{flap}} = 10^\circ$. (Taken from Alemdaroglu, 1992)

An investigation on the effect of gap on the separated flap-well flow showed that the dividing streamline and the flap move in opposite directions. As the gap increases (i.e. the flap is lowered), the dividing streamline moves up, Fig. 2.5. This is due to the fact that as the gap is increased more fluid from below the main airfoil moves into the flap-well and passes through the gap, thereby the dividing streamlines are pushed farther away from the flap leading edge to allow for an increased mass flow rate. Also note, in Fig 2.5, that the stagnation streamline moves with the flap. With decreasing gap (flap moving up), the shear layer edge will move towards the flap which will reduce the potential core going through the gap.

Figure 2.5 - Effect of gap on the dividing streamline and pressure inside the flap well, $\delta_{\text{sep}} = 10^\circ$, $OH = 4\%$. (Taken from Alemdaroglu, 1992)

Another recent experimental program on high-lift airfoils by Valarezo et al. (1992), was carried out at high Reynolds number to develop performance concepts for high-lift aircraft applications. These investigations added to the existing high-lift database with relevant results at near-flight Reynolds number conditions. The experimental program was conducted by the NASA Langley Research Centre and Douglas Aircraft Company. It was conducted at the NASA Langley Low Turbulence Pressure Tunnel (LTPT) in Hampton Virginia.

The Langley LTPT is a single return, closed-throat wind tunnel and can be operated at up to 10 atmospheres pressure, thus allowing very high Reynolds number capability. The studies were performed at a chord Reynolds number as high as 16 million. The test section is 3 feet wide by 7.5 feet high by 7.5 feet long. To promote two-dimensional flow over the model in view of its low aspect ratio and strong wall interference, a side-wall boundary layer control system (BLC) was installed. The BLC system employed the differential pressure between the test section and the atmosphere to provide suction of the side-wall boundary layer through porous endplates. Most of the results were obtained at 0.20 Mach number and a chord Reynolds number of 9 million. These conditions are representative of full-scale wing conditions for low-speed stall.

The effects due to spoiler deflection on a three-element airfoil were investigated while the flap gap and overhang were held constant to isolate the effects, Fig. 2.6 and 2.7. It was found that drooping the spoiler (5° or 10°) decreased the maximum lift capability and stall angle. However, drooping the spoiler did increase the lift of the airfoil at pre-stall angles of attack. This is a consequence of increased aft camber of the main element.

Besides generating more lift at pre-stall incidence, drooping the spoiler changed the character of the stall, leading to sharper stall. The Reynolds number effects on the drooped spoiler configuration were similar to those obtained with the undeflected spoiler. Maximum lift increased with increasing Reynolds number. Two key observations were made from drooping the spoiler. First, the increased loading on the main element leads to reduced loading on the flap (downwash) and increased loading on the slat (upwash). Second, increased loading on the slat leads it to stall soon after the main element stalls.

Figure 2.6 - Airfoil geometries tested in the LTPT
(Taken from Valarezo et al. ,1992)

Figure 2.7 - Effect of spoiler droop on a three-element airfoil,
a - on maximum Lift, b - on stall angle
 $\delta_{slat} = 30^\circ$, $\delta_{flap} = 30^\circ$
(Taken from Valarezo et al. ,1992)

Small trailing-edge wedges were attached to both a single and a three-element airfoil, Fig. 2.8. Trailing edge wedge application provided a significant increase in C_l and $C_{L_{max}}$ on the single-element airfoil and a substantial increase on each element of the three-element airfoil, Fig 2.9 and 2.10. Reynolds number effects on the three-element trailing edge wedge application were significant.

Figure 2.8 - 30° wedge on single and three-element airfoils
(Taken from Valarezo et al. ,1992)

Figure 2.9 - Effect of a 30 wedge on the lift
a - single element airfoil, b - of a three-element airfoil
 $\delta_{\text{max}} = 30^\circ$, $\delta_{\text{flap}} = 30^\circ$ (Taken from Valarezo et al. ,1992)

Finally, a new advanced single segment flap concept developed at Douglas Aircraft was tested, Fig 2.10. The new shape led to significantly reduced flow separation for the single-segment landing flap case. The airfoil profile drag was reduced by 29% over the design baseline. As shown in Figure 2.11 the new shape was not as Reynolds-number sensitive as the baseline flap at higher Reynolds numbers tested.

Figure 2.10 - Advanced Flap Shape
(Taken from Valarezo et al. ,1992)

Figure 2.11 - Effect of Reynolds Number on Maximum Lift
 $\delta_{max} = 30^\circ$, $\delta_{rep} = 30^\circ$
(Taken from Valarezo et al. ,1992)

Over the years, many experiments have been done using the GA(W)-1 airfoil, making it a perfect airfoil for which to compare experimental and computational data. The studies were presented in NASA contractor report form, which are more complete than most journal publications. The studies covered: flow separation, the development of high-lift system, slot gap variations and cove shape modifications. These studies were done by: Wentz and Seetharam (1974), Wentz and Seetharam (1977) and Wentz and Ostowari (1983).

Focusing in this thesis on flow separation and gap effect, the most recent publication provides an overview of the experiment done on the two-element GA(W)-1 airfoil with 30 percent chord Fowler flap by Wentz and Ostowari (1983). Three conditions were investigated, corresponding to pre-stall, stall ($C_{L_{max}}$) and post-stall conditions. For those experiments the Reynolds number was 2.2×10^6 and the Mach number was 0.13. Force, surface pressure, total pressure and split-film turbulence measurements were made. Results were compared with theoretical values when available.

It was found that with a narrow gap (0.02C), as compared to the optimum gap (0.03C), there is a larger region of high turbulence with intermittent reversal over the flap at the $C_{L_{max}}$ condition, and at post stall angle of attack. The wide gap (0.05C) had separated regions over the flap at all three angles of attack, and produced lower lift at pre-stall and at the $C_{L_{max}}$ condition. It is interesting to note that all three gaps produce lower lift than viscous theory predicts, but the discrepancy is greatest for large gap case.

Cove shape studies showed that both a standard and sharp-lip cove have a region of separation and reattachment ahead of the slot, while a blended cove shape has no

separation at all, Fig. 2.12. Despite the difference in cove shape and in local flow, they all produce very similar velocity profiles at the flap slot exit, and all of them produced lower lift than theory predicts.

a - Standard Cove Airfoil

b - Blended Cove Airfoil

c - Sharp-Lip Cove Airfoil

Figure 2.12 - Geometry For Coves
(Taken from Wentz and Ostowari, 1983)

The work of Alemdaroglu (1992), of Valarezo et al. (1993) and on the GA(W)-1 airfoil provides examples of how relatively small configuration changes in the gap region between two elements of a multi-element airfoil can have substantial effects on the performance of the airfoil. Reliable turbulence models are needed if analysis methods are to be able to predict these effects.

2.2 Overview of Panel Methods

The first method to analyze flows past bodies of realistic two-dimensional geometry was the "Douglas Neumann" method devised by Smith and Hess (1966). It was based on a constant-strength distribution of sources and vortices on straight-line panels. In this method the source strength σ_i varies around the airfoil surface, but remains constant on each panel. The vortex strength γ provides the circulation and is constant over the whole airfoil. Since that method was first introduced, many variants have been developed. All elementary flow solutions (or singularities) and their combination have been explored over the years. Most of these approaches are discussed in detail in books on aerodynamics, see: Anderson (1991), Katz and Plotkin (1991) and Moran (1984).

Another variant is Morino's and Kuo's (1974) potential-based panel method, a very powerful method capable of treating a wide range of potential flow problems. The normal component of $\nabla\phi$ is known (i.e. zero) on the body surface, where ϕ is either the total or the perturbation potential. Discretization with the application of the Green's identity yield a set of algebraic equations. It should be mentioned that Maskew (1982) claims that the use of the perturbation potential yields better-conditioned algebraic

equations.

Panel methods can also be used to model the wall interference effect, Mokry (1978). For two-dimensional flow, it is assumed that the flow region is bounded by two parallel, infinite wind tunnel walls (ceiling and floor), and that if porous, the walls behave in such a way that the normal component of velocity is proportional to the pressure drop across the wall (Darcy's law).

Three-dimensional panel methods enable modelling of an entire aircraft with its wake. Methods using low-order singularity distributions (e.g. uniform or linear) and plane panels have been formulated as have so called higher-order methods (e.g. quadratic distributions of singularity strength, curved panels). There was considerable discussion on whether the significantly less expensive so-called Low Order Panel Methods could be of similar accuracy as the High-Order Panel Methods, see: Maskew (1982), Youngren, et al. (1983), Carmichael and Erickson (1981) and, Smith and Woodward (1987).

The philosophy of panel methods is to make the airfoil contour a streamline of the flow. It is possible to approximate the contour as a streamline by using panels or sheets of singularity distribution (or distribution of elementary solution, e.g. vortices or doublets) placed on the contour to form a closed polygon, as shown in Fig. 3.1. Each panel of the polygon has an assigned singularity-strength value, whether it is the potential (ϕ_i), the doublet strength (μ_i) or the vorticity (γ_i). The required strengths of these elementary solutions of Laplace's equation are obtained by enforcing the zero normal flow condition on the airfoil boundary. Applying the Green theorem, which is the basis of the method, in discretized form gives a set of linear algebraic equations.

The choice of approach in a panel method is based on the type of application (complexity) and the type of results expected (velocity or potential). The vortex-based method has the advantage of being physically simpler to interpret, as discussed below. The doublet-based methods are usually more efficient for three-dimensional cases. They require less computing for a given level of accuracy because doublet strength can be treated as a scalar quantity while vorticity has direction and must be treated as a vector quantity. Vorticity is equal to the gradient of the doublet strength. Doublet methods thus have substantial advantages in three-dimensional flow applications. However, these advantages vanish in two-dimensional flow cases, since the vorticity is always in the spanwise direction. In potential based methods the velocity has to be determined by differentiation.

A source based method is mostly useful for non-lifting bodies (since it produces no circulation). A surface source distribution used with another type of singularity distribution (vortices or doublets) can be set to equal a prescribed normal component of velocity to model the displacement effect of boundary layer growth along the body surface. This approach is often used in an iterative procedure to model the inviscid/viscous interaction phenomenon.

An important characteristic of these methods is the rate of convergence toward the inviscid exact solution. The error associated with any of these methods is found to decrease roughly in proportion to $1/N$, where N is the number of panels used to represent the body.

/ A physical approach to developing a two-dimensional vortex panel method is to

recognize that there is zero velocity or uniform potential inside the airfoil contour. This follows from Green's theorem. The local vorticity strength (γ_i) must therefore be equal to the local tangential velocity on the exterior surface of the airfoil. As mentioned, in the exact solution the contour of the airfoil is a streamline, and the exterior velocity varies smoothly and continuously. To avoid discontinuity between each panel, it is preferable to have linear variation of vorticity strength, rather than uniform strength, with a continuous value at the junction points between adjacent panels. A quadratic variation will simply add computational time without a great improvement in accuracy.

Chapter 3

THE LINEAR VORTEX PANEL METHOD

3.1 Fundamentals of the Linear Vortex Panel Method

A linear vortex panel method was used in the present work. The arrangement of panels and nomenclature are shown in Fig 3.1. A vortex panel is formed of an infinite number of straight vortex filaments side by side, as shown in Fig. 3.2, where the strength of each filament is infinitesimally small. A vortex filament ds induces an infinitesimally small velocity at any control point i . Eqn 3.1 gives the velocity components induced by filament ds .

$$\begin{aligned} du &= -\frac{\gamma d\theta}{2\pi} \\ dv &= -\frac{\gamma}{2\pi} \frac{dr}{r} \end{aligned} \quad (3.1)$$

The velocity at i induced by an entire vortex sheet is the integration of equation 3.1 over the overall length L_m , Kind (1973). The integration of equation 3.1, assuming a linear variation of γ , yields equation 3.2, where A_{im} , B_{im} , C_{im} and D_{im} are coefficients defined by Eqn 3.3.

$$\begin{aligned} u_{im} &= A_{im}\gamma_m + B_{im}\gamma_{m+1} \\ v_{im} &= C_{im}\gamma_m + D_{im}\gamma_{m+1} \end{aligned} \quad (3.2)$$

Figure 3.1 - Vortex panel method nomenclature

Figure 3.2 - Velocity induced at control point i by vortex panel m .

$$\begin{aligned}
A_{im} &= \frac{1}{2\pi l_m} \{ (\theta_{im} - \theta_{i,m+1}) (l_m - r_{im} \cos \theta_{im}) - \ln \left(\frac{\sin \theta_{i,m+1}}{\sin \theta_{im}} \right) r_{im} \sin \theta_{im} \} \\
B_{im} &= \frac{1}{2\pi l_m} \{ (\theta_{im} - \theta_{i,m+1}) r_{im} \cos \theta_{im} + \ln \left(\frac{\sin \theta_{i,m+1}}{\sin \theta_{im}} \right) r_{im} \sin \theta_{im} \} \\
C_{im} &= \frac{1}{2\pi l_m} \{ \ln \left(\frac{r_{im}}{r_{i,m+1}} \right) (l_m - r_{im} \cos \theta_{im}) + (\theta_{im} - \theta_{i,m+1}) r_{im} \sin \theta_{im} + l_m \} \\
D_{im} &= \frac{1}{2\pi l_m} \{ \ln \left(\frac{r_{im}}{r_{i,m+1}} \right) r_{im} \cos \theta_{im} - (\theta_{im} - \theta_{i,m+1}) r_{im} \sin \theta_{im} - l_m \}
\end{aligned} \tag{3.3}$$

If the vorticity γ is made continuous at the junction points, it can be shown, using this result, that there is no velocity singularity at the junction points if the panels are coplanar. The coefficients are computed by the FORTRAN subroutine XKS, for more detail see appendix A.

Applying the boundary condition of no flow normal to any vortex sheet control point i , yields equation 3.4.

$$\sum_{n=1}^K u_{in} \sin(\theta_n - \phi_i) + \sum_{n=1}^K v_{in} \cos(\phi_n - \phi_i) = -U_\infty \sin(\alpha - \phi_i) \tag{3.4}$$

The velocity components u_{in} and v_{in} are given by equation 3.2. Rearranging and making the substitution of equation 3.2 into equation 3.4 for all N control points yields a set of N equations in $N+1$ unknowns, $\gamma_1, \gamma_2, \dots, \gamma_{N+1}$:

$$\sum_{n=1}^{N+1} K_{in} \gamma_n = -U_\infty \sin(\alpha - \phi_i) \quad i=1, 2, \dots, N \tag{3.5}$$

where,

$$K_{jn} = A_{jn} \sin(\phi_n - \phi_j) + B_{j,n-1} \sin(\phi_{n-1} - \phi_j) \\ + C_{jn} \cos(\phi_n - \phi_j) + D_{j,n-1} \cos(\phi_{n-1} - \phi_j)$$

The K_{jn} are known as the influence coefficients, with $B_{i,0} = D_{i,0} = 0$ and $A_{i,N+1} = C_{i,N+1} = 0$.

Since the boundary condition is only applied at a finite number of control points, the polygon of vortex sheets can only approximately coincide with the airfoil contour streamline of the flow solution.

3.2 Choice of a Suitable Kutta Condition

In steady flow over a given airfoil at a given angle of attack, nature adopts that particular value of circulation (Γ) which results in the flow leaving smoothly at the trailing edge. This requirement is introduced into the theoretical analysis by use of a so-called Kutta condition.

Two versions of the Kutta condition are available.

- 1 - make the trailing edge a stagnation point

$$\gamma_1 = \gamma_{N+1} = 0 \quad (3.6)$$

- 2 - enforce equal velocity on the upper and lower surfaces at the trailing edge

$$\gamma_1 = \gamma_{N+1} \quad (3.7)$$

The stagnation condition at the trailing edge involves two equations, which creates an overdetermined system of equations. When the equal-velocity-condition equation is

used the resulting system of $N+1$ unknowns can be solved directly.

The effects of the various Kutta conditions are discussed in section 3.6 "Validation Checks of the Vortex Panel Method Program" where the results of the test calculations are analyzed.

3.3 Pressure Coefficient Distribution and Lift Based on Circulation

Once the values γ_i of the vortex strength distribution have been obtained, the tangential velocity is known ($u_{i \text{ external}} = \gamma_i$). The sheet vorticity strength is equal to the local jump of tangential velocity across the sheet. The velocity inside the airfoil contour should be equal to zero, as already mentioned, by virtue of the Green's theorem. Therefore the local exterior velocity is equal to the local sheet strength.

Using the Bernoulli theorem, the pressure coefficients are evaluated as:

$$C_p = 1 - \gamma^2 \quad (3.8)$$

The circulation Γ around the airfoil is the integral of the vorticity around the contour:

$$\Gamma = \oint \gamma ds \quad (3.9)$$

The Kutta-Joukowski theorem gives the lift (L) based on bound circulation and

is expressed for computational purposes as:

$$C_L = \frac{\rho U \Gamma}{\frac{1}{2} \rho U^2 c} = \frac{2\Gamma}{Uc} \quad (3.10)$$

where C_L is the lift coefficient.

3.4 Solver Considerations

The procedure to solve a linear set of equation is quite simple. The system of the form $[K][\gamma]=[b]$ is rearranged, the inverse of matrix $[K]$ is multiplied by the matrix $[b]$, thus solving for $[\gamma]$ in the equation $[\gamma]=[b][K]^{-1}$.

The matrix $[K]$ is formed from the influence coefficients (Eqn. 3.6), the column matrix $[\gamma]$ is the unknown vorticity strength (γ_m) and the column matrix $[b]$ is the right hand side of Equation 3.5. Using the fact that the matrix K is positive definite, one can use to great advantage Crout's or Choleski factorisation scheme. The matrix solution is performed by the FORTRAN subroutine LEQT1F, see Appendix A.

This approach demands that the number of equations is equal to the number of unknowns. This is true when the Kutta condition of equal velocity at the trailing edge is used. However, the use of Kutta condition of stagnation point at the trailing edge creates a set of $N+1$ Equations for N unknowns. One simplistic approach is to discard one equation, the boundary-condition equation representing a control point that must be located at an unstrategic location on the airfoil contour, eg. about mid-chord on the lower surface. This approach gives suitable results and will be compared to a more elaborate method.

One way to deal with an overdetermined system is the least-squares method (Noble 1965). The idea is as to find the γ_m 's so to minimize

$$E^2 = \sum_{i=1}^{N+1} \left(\sum_{m=1}^{N+1} K_{im} \gamma_m - b_i \right)^2 \quad (3.11)$$

this requires

$$\begin{aligned} \frac{\partial E^2}{\partial \gamma_m} &= 0 \\ &= 2 \sum_{i=1}^{N+1} \left(\sum_{m=1}^N K_{im} \gamma_m - b_i \right) K_{im} \end{aligned} \quad (3.12)$$

and so determine the γ_m 's so as to satisfy

$$\sum_{m=1}^N \left(\sum_{i=1}^{N+1} K_{im} K_{ik} \right) \gamma_m = \sum_{i=1}^{N+1} b_i K_{ik} \quad (3.14)$$

which is a determinate system.

The extra summations are time consuming. An alternative way to deal with an over-determined system was introduced by Bristow (1977). A constant error term "e" is added to each of the governing equations, obtaining

$$\sum_{m=1}^N K_{im} \gamma_m + e = b_i \quad \text{for } i=1, \dots, N+1 \quad (3.14)$$

where i varies from 1 to N+1

Since e is unknown, the equations contain N+1 unknowns and are thus determinate.

There are many other methods to solve overdetermined systems of equations. Mokry (1978) compared with an exact solution the results obtained using three of them: 1- Least Squared, 2- Minimization of the L_1 Norm of the Residuals, 3- Maximum residual

(L_∞ or Chebyshev Norm). The differences between the solutions are insignificant from an engineering point of view. However, in the case of ill-conditioned systems due to nearly coinciding boundary elements, Mokry (1990) specified that the condition on which the least squares is based, is worse than the original equation. It is also specified that the least squares solution offers a better economy when repeated computations are required, such as in iterative coupling of potential flow and boundary layer.

3.5 Modification of the Linear Vortex Panel Method for Multi-Element Airfoils

As mentioned earlier, the existing panel program was only suitable for single-element airfoils. The computer program was modified to enable solution of multi-element airfoils as well.

As shown in Figure 3.3 the origin of the Cartesian global axis system is located at the leading edge of the main element. Each element leading edge is the origin of a local auxiliary axis system in which the polygon representing the element is defined. The input data for the main element is in the global axis system and the angle of attack, α , is defined with respect to the global x axis. The input data file also contains the location of the origin and the orientation of each auxiliary axis system. A non-dimensional scaling length S/c (where c is the main element chord length), for each auxiliary element enables easy changing of the relative size of an element. All auxiliary-element coordinates are transformed into the global x,y axis system by the program before the main computations begin.

Figure 3.3 - Multi-element airfoil in cartesian system of axis

The upper-surface trailing-edge coordinate for each element is the first coordinate for that element, because coordinate input for each element must start at its trailing-edge and go counter clockwise around the element, stopping at the lower surface trailing-edge. Therefore, the last input coordinate for an element is also the trailing-edge coordinate (i.e. same cartesian value if the trailing edge is sharp, or slightly different if the trailing-edge has finite thickness).

The program calculates panel lengths (l) and panel orientation (Φ). This computation has to be done for all the panels forming an element (polygon). However, the jump from a junction point on one element to a junction point on the next element does not define a panel. This has to be recognized when processing the coordinates and assembling the equations. This was the main change required to extend the existing program to handle multiple element airfoils. Thus, the input data file has to precisely follow the specified format, see Appendix A. The extension includes the computation of the pressure coefficient distribution and of the lift based on bound circulation for each element. A sample input data file and the results are included in Appendix B.

Figure 3.4 shows how the image method was used to simulate the wind tunnel walls. The symmetrical lines between two airfoils correspond to streamlines which simulate the wind tunnel walls. The theoretical normal velocity on these streamlines is equal to zero, just as it would be on an actual wall. The three airfoil/flap arrangements are solved in a matrical system, as detailed above, with the upper and lower airfoil/flap arrangement as auxiliary elements of the main element airfoil in the middle. Once the system solved, the velocity is computed at some stations (or points) located on the lines

**Figure 3.4 - Airfoil configuration set-up for simulation
(all six elements are modeled with panels)**

of symmetry. The tangential velocity can be assumed as the boundary layer external tangential velocity (U_e). The normal velocity provides the opportunity to verify the inviscid computation, comparing the computed results (usually small, often negligible) to the theoretical one (zero).

3.6 Validation Checks of the Vortex Panel Method Program

The program validation work included comparisons with inviscid exact solution and other approximate inviscid solutions. The program results were analyzed/compared for a row of circular cylinders, for the Williams (1971) airfoil and for blades in cascade.

Many cases involving circular cylinders were analyzed. The cylinders were formed of eight panels each. An interesting arrangement is to place nine cylinders in a row. The results should reflect the symmetry of this arrangement. The pressure coefficient distribution and the lift (zero) do in fact respect the geometrical symmetry, as shown in Fig. 3.5.

The Williams airfoil is the only two-element airfoil for which there is an exact inviscid solution (as far as the author is aware). The pressure coefficient distribution obtained with the vortex panel method (62 panels per element) is compared with the exact pressure coefficient distribution in Fig. 3.6. Agreement is good; the Kutta conditions used are the stagnation condition (C_p is equal to unity at the trailing edge) and equal velocity. The lift coefficients generated by the Williams airfoil are tabulated in table 3.1.

Figure 3.5 - C_p distribution of nine circular cylinders in row (8 panels)

Figure 3.6 - C_p distribution for Williams airfoil

	Main Airfoil	Flap	Total
Exact [Williams, 1971]	2.1853	0.6242	2.8095
Vortex Panel			
$\gamma_1 = \gamma_{N+1} = 0$	2.1026	0.7266	2.8292
$\gamma_1 = \gamma_{N+1}$	2.1080	0.7275	2.8355
Error (%)			
$\gamma_1 = \gamma_{N+1} = 0$	3.78	16.40	0.70
$\gamma_1 = \gamma_{N+1}$	3.54	16.55	0.92

TABLE 3.1
Exact and Computed Lift Coefficient for the Williams' Airfoil,
 $\alpha = 0^\circ$

The results obtained for GA(W)-1 airfoil pressure distribution were in good agreement for small angles of attack ($\alpha < 5^\circ$) with the results obtained by Wentz and Ostowari (1983) experiments.

Another validation check was to compare results of the vortex panel program with an existing program that deals with cascades of blades [Kind, 1992]. The test cases included two angles of attack and four different blade configurations (the number of blades varying between three to nine). The blades had the NACA 4412 profile shape. The vortex panel program MINVAIRF provided reasonable results for the blade placed in the middle of the cascade. This blade and its companions were simply treated as airfoil

elements in the program. Fig. 3.7 compares the pressure coefficient distribution for a blade having an angle of attack of -2.766° in a cascade of nine blades.

3.7 Results Obtained Solving an Overdetermined System of Equations

A small investigation was conducted on the Williams airfoil to determine if it was necessary to solve the overdetermined system of equations. A subroutine created by Abdelmalek of the National Research Council, Mokry (1978) for solving the overdetermined system using a least squares and pseudo-inverse approach was used.

Figure 3.8 compares the pressure coefficient distribution obtained with a least squares solution, pseudo-inverse solution and a determined system of equations (boundary-condition dropped at one control point) to the exact solution. The approximated solutions follow the exact curve (quite) closely except at the gap. It can be shown, from a zoom of the gap region, Fig. 3.9, that the overdetermined system solutions provide a solution very similar to that of the determined system, the curves being almost on top of one another. The stagnation point Kutta condition was used in all cases.

The lift coefficients are presented in table 3.2. This shows that the lift coefficients generated by the overdetermined and the determined solutions are virtually the same. It can be concluded that it is not necessary to use an overdetermined system of equations to obtain good accuracy.

	Exact (Williams, 71)	Least Square	Pseudo- Inverse	determined System
Main-Element	2.1853	2.1033	2.1033	2.1080
Flap	0.6242	0.7266	0.7266	0.7275
Total	2.8095	2.8299	2.8299	2.8355

Table 3.2
Williams Airfoil Lift Coefficient,
for Various Matrix Solutions Methods.
 $\alpha = 0.0^\circ$

Figure 3.7 - C_p distribution of a cascade blade (NACA 4412)

Figure 3.8 - Williams airfoil C_p distribution

Figure 3.9 - Williams airfoil C_p distribution in the gap/overhang region

Chapter 4

DESIGN OF EXPERIMENTAL SET-UP

4.1 Choosing a General Set-Up

The objective of the experiment is to reproduce realistic two-dimensional shear flows typical of those in and downstream of a slot on a multi-element high-lift airfoil. Therefore, tunnel wall interference does not matter as long as there is no flow separation on the wind tunnel walls and the pressure distribution remains typical of high-lift airfoils.

By maximizing the chord length, it is possible to achieve a Reynolds number up to 3×10^6 in the available wind tunnel. The wind tunnel has a maximum velocity of 200 mph (90 m/s). The kinematic viscosity (ν) is assumed constant at the value for room temperature and atmospheric pressure. A chord length of 29 in. (74 cm) would then correspond to a Reynolds number (Re) of 3×10^6 .

4.1.1 Slat and "Bump Shape" Geometry

The original idea was to use a "bump" to accelerate the flow on the wind tunnel floor and a movable slat to simulate the high-lift-airfoil flow field, as shown in Fig. 4.1. The "bump" needed to be designed with a small slope to avoid flow separation. That

simplistic approach was rejected because the boundary layer growth on the "bump" does not adequately represent the boundary layer growth on the main element of a high-lift airfoil. The boundary layer growth of the main element starts at the leading stagnation point, as was shown in Fig. 1.1. The boundary layer growth for the "bump" starts with the wind tunnel test section and when it reaches the slat the boundary layer is much thicker than it is supposed to be.

Figure 4.1 - Bump & Slat Configuration

The Reynolds number achievable with this approach would be based on the slat length, creating a problem for comparing results with the literature. The typical slat design includes a high curvature of the lower surface, where a region of separated flow is formed. This region is not predictable by a two-dimensional vortex panel method. Modelling that type of flow field requires more advanced techniques, such as Reynolds Averaged Navier Stokes.

4.1.2 Type of Airfoil

There are many high-lift devices in existence. The choice of one for an experimental set-

up is constrained by the available test facility.

The GA(W)-1 airfoil with flap is one type of high-lift device, as mentioned earlier, for which many experiments have been performed. Many publications, whether on experiments or computer simulations, Wentz and Seetharam (1974), Wentz and Seetharam (1977), Wentz and Ostowari (1983), Nelson, Zing and Johnston (1992), provide results for the two or three element version of the GA(W)-1 airfoil. That large bank of available data enables a comparison of results.

The work of this thesis was on the two-element version, leaving the option to add the third-element (slat) in a future effort without any major modifications.

4.1.3 Size of the Airfoil, Reynolds Number Considerations

Realistic airfoil Reynolds numbers (Re) have a value of 10^6 or more. Chord lengths from 10 in. to 35 in. would correspond to Re of 1.04×10^6 to 3.64×10^6 for the available maximum wind speed of 200 mph.

Other considerations are the flap gap and overhang, which are generally proportional to the chord length, making flow field measurements particularly tricky in that region for small airfoil models. A big model limits the angles of attack to about $\pm 4^\circ$ and creates strong adverse pressure gradients on the floor and ceiling of the wind tunnel. However, a big airfoil model allows more interior space for pressure tubes, as detailed in section 4.5.

The chord length of the airfoil was chosen at 25 in. with the flap retracted, giving a Reynolds number of 2.6×10^6 . The flap chord length is 7.5 in. The overall high-lift

airfoil design parameters were fixed at: flap deflection 35° , overhang $OH = 0.3$ in. and gap $G = 0.75$ in., as shown in Fig. 4.2.

After an observation of the airfoil profile, an educated guess lead to believe that a minor modification of the lower surface of the main element was needed to ensure attached flow on the airfoil surface. The curvature of the S shape near the trailing edge of the main element, Fig.4.3, was modified to avoid separation or at least to minimize the size of the separation bubble usually located there.

Figure 4.2 - Airfoil design parameters

Figure 4.3 - Modification of the GA(W)-1 airfoil

4.2 Boundary Layer Analysis

4.2.1 Predicting Boundary Layer Growth on Solid Walls

The program used to predict the boundary layer growth on the wind tunnel floor and ceiling was based on Dvorak's publication (1969). This existing program combined the momentum integral equation with an auxiliary equation for the shape factor, Head (1958) to predict the growth of the turbulent boundary layer over rough surfaces in pressure gradient. The skin friction law includes correlations for the effect of roughness height and spacing.

The wind tunnel wall surfaces were assumed smooth. Therefore, the boundary layer growth was only affected by the pressure gradients created by the presence of the airfoil. The inviscid velocity U_e along the wind tunnel walls was computed using the vortex panel method program. Separation was predicted on the floor just ahead of the main element leading edge, for all angles of attack. On the other hand, the flow was predicted to separate on the ceiling only for angles of attack greater than 3° .

The strong adverse pressure gradient located upstream of the main element leading-edge retards the flow until separation occurs. Table 4.1 presents the boundary layer behaviour on the wind tunnel floor, for airfoil angle of attack $\alpha=0^\circ$. It can be seen that the shape factor (H), the momentum thickness (Θ) and the displacement thickness are rising while the friction coefficient (C_f) is diminishing accordingly. Appendix B contains computed results for other cases.

Reducing the model chord showed no real improvement in the predicted boundary layer behaviour. Also an attempt with a different airfoil profile, Williams (1971) airfoil,

produced similar results.

Therefore, to keep the Reynolds number high, somehow the boundary layer has to be controlled. Adding roughness to the wall would simply increase the boundary layer growth rate, resulting in even earlier separation. It was decided to investigate the use of suction, with porous walls, to prevent or delay separation from the wind tunnel floor.

Predicted Boundary Layer Behaviour on the Wind Tunnel Floor, Without Suction
Reynolds Number Based on Airfoil
Chord Length : 2.6×10^6

<u>X/c</u>	<u>U/U_{ref}</u>	<u>H</u>	<u>Θ/c</u>	<u>δ'/c</u>	<u>C_f</u>
<u>-0.80</u>	0.98096	1.5000	0.01000	0.0150	0.00139
<u>-0.75</u>	0.97375	1.5064	0.01030	0.0155	0.00137
<u>-0.70</u>	0.96508	1.5155	0.01066	0.0162	0.00133
<u>-0.65</u>	0.95466	1.5284	0.01111	0.0170	0.00129
<u>-0.60</u>	0.94213	1.5462	0.01167	0.0180	0.00124
<u>-0.55</u>	0.92707	1.5710	0.01239	0.0195	0.00116
<u>-0.50</u>	0.90896	1.6054	0.01333	0.0214	0.00108
<u>-0.45</u>	0.88721	1.6509	0.01458	0.0241	0.00097
<u>-0.40</u>	0.86115	1.7213	0.01630	0.0281	0.00083
<u>-0.35</u>	0.83006	1.8433	0.01875	0.0346	0.00065
<u>-0.30</u>	0.79328	2.1248	0.02245	0.0477	0.00038
<u>-0.25</u>	***** The	flow has	separated		*****

Table 4.1
Leading edge of the main element is at $x/c = 0.0$

4.2.2 Predicting Boundary Layer Growth on Porous Wall With Suction

In an attempt to predict and control the boundary layer development on a porous wall, a FORTRAN program was developed. The program used the momentum integral equation, Head's entrainment equation (1958) and a modified logarithmic law of the wall to estimate skin friction.

A porous wall with a suction system could hardly be implemented with precision at low cost. The intent is to use the difference of pressure between the wind tunnel interior and the room atmospheric pressure. The wind tunnel floor is subject to positive pressure ($C_p > 0.0$) created by the lower surface of the airfoil and this should provide the suction required to keep the boundary layer attached.

4.2.2.1 Skin Friction Law

The non-zero suction velocity (v_w) at the wall adds a strong streamwise convective acceleration $v_w(\delta u/\delta y)$ to the near-wall boundary layer ($v_w < 0$, suction; $v_w > 0$, blowing). If zero pressure gradient flow (flat plate) is assumed, the integral momentum equation very near the wall becomes, White (1991)

$$\rho v_w \frac{\delta u}{\delta y} \approx \frac{\delta \tau}{\delta y} \quad \text{or} \quad \tau \approx \tau_w + \rho v_w u \quad (4.1)$$

Stevenson (1963) matched eqn. 4.1 to an "eddy viscosity" model for turbulent shear stress and derived a modified logarithmic law of the wall with suction or blowing:

$$\frac{2}{v_w^+} [(1 + v_w^+ u^+)^{\frac{1}{2}} - 1] = \frac{1}{k} \ln(y^+) + B \quad (4.2)$$

where $k=0.41$ and $B=5.0$ are from the Coles and Hirst correlation (1968).

u^+ and v_w^+ are non-dimensionalized velocity components (tangential and normal) defined as: $u^+ = u/u^*$, $v_w^+ = v_w/u^*$. The parameter $u^* = (\tau_w/\rho)^{1/2}$ has units of velocity and is called the wall friction velocity. The typical value of v_w^+ is small, less than 0.1. Therefore using a binomial series expansion of the $(1 + v_w^+ u^+)^{1/2}$ term, i.e. $v_w^+ \rightarrow 0$, in eqn 4.2:

$$\lim_{v_w^+ \rightarrow 0} \left\{ \frac{2}{v_w^+} \left[1 + \frac{1}{2} v_w^+ u^+ \dots - 1 \right] \right\} = \frac{1}{k} \ln(y^+) + B \quad (4.3)$$

In the usual logarithmic law the effect of v_w is "lost". Keeping one more term in the above binomial expansion provides that necessary term:

$$u^+ = \frac{u}{u^*} = \frac{\frac{1}{k} \ln y^+ + B}{1 - 0.25 u^+ v_w^+} \quad (4.4)$$

To get a relation for the overall velocity profile the Cole's law of the wake (1956) is added to eqn.4.4.

$$u^+ = \frac{u}{u^*} = \frac{\frac{1}{k} \ln y^+ + B}{1 - 0.25 u^+ v_w^+} + \frac{2\Pi}{k} \sin^2\left(\frac{\pi}{2} \frac{y}{\delta}\right) \quad (4.5)$$

Equation 4.5 can be written for the outer edge of the boundary layer, where $y/\delta=1$ and $u=U_e$. Making the appropriate substitution, that gives a skin friction law for porous walls:

$$\sqrt{\frac{2}{C_f}} = \frac{U_o}{u^*} = \frac{\frac{1}{k} \ln\left(\frac{\delta U_o}{\nu}\right) + B}{1 - 0.25 \left(\sqrt{\frac{2}{C_f}} \frac{\nu_w}{u^*}\right)} + \frac{2\Pi}{k} \quad (4.6)$$

The Cole's wake parameter Π can be determined by a Π - β correlation, where β is the Clauser (1954) pressure-gradient parameter, defined by:

$$\beta = \frac{\delta^*}{\tau_w} \frac{dp}{dx} = \frac{\delta^*}{\tau_w} \left(-\rho U_o \frac{dU_o}{dx} \right) \quad (4.7)$$

Das (1987) suggested the following polynomial correlation:

$$\beta = -0.4 + 0.76\Pi + 0.42\Pi^2 \quad (4.8)$$

Assigning a value to ν_w^* , it is then possible to solve eqn. 4.6 for C_f at any station by an iterative process. The momentum thickness (Θ) and the shape factor (H) have to be determined at each station as well. Equation 4.6 is solved together with the momentum integral equation (eqn. 4.9) and the Head's entrainment equations (see below) to obtain Θ , H and C_f . A Runge-Kutta integration is used to advanced the solution in the x direction.

4.2.2.2 The Momentum Integral Equation

As in the skin friction law, the momentum integral equation must include the ν_w term. Equation 4.9 is the momentum integral relation of Kármán (1921) including the non-zero wall velocity term.

where ν_w is positive for blowing, negative for suction.

$$\frac{d\Theta}{dx} = \frac{C_f}{2} - \frac{\Theta}{U_o} \frac{dU_o}{dx} (2+H) + \frac{v_w}{U_o} \quad (4.9)$$

4.2.2.3 Head's Entrainment Equation for Turbulent Flow

The entrainment equation (4.10) introduces two more functions $F(H_{\delta-\delta^*})$ and $G(H)$, Head (1958). Those empirical functions have been numerically defined as follows by Dvorak (1969):

$$\frac{1}{U_o} \frac{d}{dx} [U_o (\delta - \delta^*)] + v_w = F(H_{\delta-\delta^*}) \quad (4.10)$$

$$F(H_{\delta-\delta^*}) = \exp[-3.512 - 0.617 \ln(H_{\delta-\delta^*} - 3)] + v_w \quad (4.11)$$

where v_w has been added because its effect on the entrainment, and

$$G(H) = 3.3 + \exp[0.4667 - 2.722 \ln(H - 0.6789)]$$

for $H \leq 1.6$, or (4.12)

$$G(H) = 3.3 + \exp[0.4383 - 2.064 \ln(H - 0.6789)]$$

for $H > 1.6$.

4.2.2.4 Validity of the Porous Wall Boundary Layer Program

The results of the program for porous wall (PWBL) were compared to those of the Dvorak based program, for $v_w = 0$. When $v_w = 0$ the expressions in the program (PWBL) reduce to those for impermeable walls. On the other hand, for the Dvorak based program,

the flow was aerodynamically smooth, having the parameter $U_\tau k/\nu < 5.0$, Dvorak (1969). Figure 4.4 shows two hypothetical cases, one with slightly favourable pressure gradient (run # 1) and one with strong adverse pressure gradient (run # 2). It can be seen that the results of PWBL compare closely with those of the Dvorak method, indicating that PWBL works correctly at least for non-porous walls. When used to predict the boundary layer behaviour on the wind tunnel floor, both programs, the Dvorak based program and the program for porous walls, predicted flow separation at the same station, near the leading edge of the airfoil.

The typical range of v_w^+ is ± 0.06 . Figure 4.5 shows the effect of suction and blowing on the law of the wall (the sublayer is not shown) and compares the Stevenson (1963) correlation, eqn. 4.2, with the power series approximation, eqn 4.4. The discrepancies in the blowing regime (blowing v_w) between the Stevenson correlation and eqn 4.4 are great, but of no-importance since only the suction regime was of interest. For impermeable wall, $v_w^+ = 0$, eqn 4.4 reduces to the Stevenson correlation. When small suction is added, $v_w^+ = -0.02$, eqn 4.4 follows the Stevenson correlation quite closely. The difference between the Stevenson correlation and eqn 4.4, when moderate and high suction are introduced ($v_w^+ = -0.04$ and $v_w^+ = -0.06$), were assumed satisfactory to obtain preliminary results for the boundary layer behaviour.

4.2.2.5 Accelerating the Flow Below the Airfoil With a "Bump"

Suction provided some interesting results. The velocity U_e deceleration rate was reduced and the overall boundary layer behavior improved accordingly. However, the boundary

Figure 4.4 - Comparison of Dvorak and PWBL

Figure 4.5 - Law of the wall for porous wall

layer would not stay attached to the tunnel floor and there was no significant improvement of the boundary layer behaviour. Even with very high suction ($v_w^+ = -0.18$) the predicted boundary layer would not stay attached to the wind tunnel floor. To help the boundary layer to stay attached, the velocity U_e on the floor should be prevented from decreasing abruptly.

The aim is to modify the flow, so the velocity U_e will not decrease too abruptly between computing stations when approaching the airfoil. For that purpose a simple sinusoidal-like "bump" was placed on the wind tunnel floor in the inviscid simulation. Figure 4.6 shows the experimental set-up obtained for the GA(W)-1 airfoil ($\alpha = 0.0$) and different "bump" configurations. The slow velocity at the leading edge of the bump, due to the unrealistic stagnation points in the inviscid flow, was manually modified to enable the boundary layer computation to continue past the start of the bump without separation. A small separation bubble might actually form at the bump leading edge but this would not have a major influence on the flow pattern and would be acceptable.

The results were encouraging. Table 4.2 shows the boundary layer behaviour on the floor and on the bump until separation occurs. In the light of these results the sine-like bump was modified by stretching it and placing a second sine like bump on top of the first one.

This approach worked very well until the flap trailing edge was approached. At this point all changes to the floor contour had to be small because any changes in the gap height between the flap and the bump influence the mass flow rate passing under the airfoil.

The advantages of the bump approach are its feasibility and low cost. On the other hand, the disadvantages are that every airfoil/flap arrangement will require a different bump. Figure 4.7 shows the C_p distribution obtained for the GA(W)-1 airfoil ($\alpha = 0.0$) with and without the final bump (Fig. 4.6). It is clear that the presence of the bump changes the C_p pressure distribution around the airfoil and the flap by increasing the loading.

Predicted Boundary Layer Behaviour on the Wind Tunnel Floor,
Floor Having a Sine Shape "Bump".
Reynolds Number Based on Airfoil
Chord Length : 2.6×10^6

X/c	U/U_{ref}	H	Θ/c	δ^*/c	C_f
<u>-0.80</u>	1.0672	1.500	0.0100	0.0150	.162E-02
<u>-0.70</u>	1.0511	1.514	0.0106	0.0161	.137E-02
<u>-0.60</u>	1.0232	1.549	0.0117	0.0182	.794E-03
<u>-0.50</u>	0.9727	1.639	0.0141	0.0231	.433E-04
<u>-0.40</u>	0.9031	1.850	0.0186	0.0344	.532E-06
<u>-0.34</u>	0.8710	2.035	0.0215	0.0436	.132E-06
<u>-0.28</u>	0.9065	1.779	0.0184	0.0328	.266E-02
<u>-0.22</u>	0.9163	1.726	0.0178	0.0307	.266E-02
<u>-0.16</u>	0.9024	1.760	0.0188	0.0332	.416E-03
<u>-0.10</u>	0.8724	1.881	0.0215	0.0403	.160E-05
<u>-0.04</u>	0.8342	2.201	0.0257	0.0565	.671E-07
*** The flow has separated ***					

Table 4.2
Leading edge of the main element is at $x/c = 0.0$

Figure 4.6 - Different bump configurations

Figure 4.7 - Effect of bump on C_p distribution

4.3 Materials Considerations

The two-element airfoil must be built with materials having specific characteristics such as: smooth surface finish, light but stiff (to avoid excessive trailing edge deflection or oscillation), easily machinable, no or low moisture absorption. With these materials characteristics in mind, the mechanical design of the test section must also take into account accessibility (flow measurement), external support and minimum cost.

The mechanical design was done simultaneously with the attempt to predict/control the boundary layer with suction. Therefore, the lift was found using a C_p pressure distribution having a peak and averaged C_p pressure lower than the one obtained in presence of the bump, as it was shown in Fig 4.7. That should be borne in mind before building the wind tunnel test section.

The concept is to use the test section structure to provide external support. The main airfoil and the flap are held together by metal side plates, as shown in the technical drawings (Fig. 4.8 a to f).

The mechanical design computations are presented in Appendix C. The bolts (no. 8 and 9) will have to support the airfoil/flap loads and the weights of the side plates, transmitting the load to the external wind tunnel structure. Traditional metal materials are quite heavy (high density). Therefore, to minimize the overall weight modelling board materials were selected (Ren Shape 450 and MB-100, described further below). The airfoil trailing edge deflection was determined using a cantilever beam-like approximation with lift force as the only force acting on the airfoil.

Figure 4.8 0

Figure 4.8c

Figure 4.8 d

Figure 4.8 f

The materials Ren Shape 450 and MB-1000 are modelling materials (wood-like product) free of grain. They have a lower density than Aluminium ($\rho = 0.66 \text{ g/cm}^3$ compared to $\rho = 2.7 \text{ g/cm}^3$), reducing the weight of the main element considerably. These products are: easy to machine, have excellent surface finish, have low moisture absorptivity and are durable. Their Young's modulus ($E \approx 320 \text{ MPa}$) is smaller than that of Aluminium ($E = 71 \text{ GPa}$), which would result in larger deflection of the trailing edge.

Therefore, the main element will be made of two materials, as indicated in Fig. 4.8. The trailing edge, requiring a stiffer material, will be in aluminium, while the rest of the main element will be of MB-1000. The flap due to its sharper trailing edge will be machined entirely out of aluminium.

The test section structure will be of aluminium channels. The side walls, floor and ceiling will be composed of plywood and plexiglas.

Due to the limitations of the Science Technology Centre machine that would be used for manufacturing the airfoil, the design span had to be reduced from 30 to 25 in. This had the effect of reducing the test section dimensions to 25 x 20 in. Therefore, the test section must be equipped with venturi like side walls that will further contract the flow leaving the wind tunnel contraction. This will have the advantage of higher attainable flow speed, but may increase difficulties due to end effects.

4.4 Equipment and Instrumentation

Before proceeding with the experimental part of the research, one will have to install measuring instrument and a data acquisition system. This section of the thesis expresses

some thoughts and findings of the author on these matters. No detailed design has been developed for instrumentation or data acquisition.

As mentioned earlier, the experimental design is conducted for a closed circuit wind tunnel with a test section of 20 x 30 in. (0.5 x 0.76 meter). A summary of the wind tunnel characteristics is presented in Table 4.3. A breather vent located at the end of the test section will maintain nominally atmospheric pressure within the tunnel test section.

Wind Tunnel and Test Section Characteristic

Cross Sectional Area	500 in. ²
Collector Area	500 - 600 in. ²
Contraction	Ratio
Wind Tunnel	12.0
Test Section	1.2
Maximum Wind Speed	295 ft/s
Turbulence Intensity	0.25 % †

Table 4.3

† Measured in standard 20 x 30 in. test section

The pressure taps distributed around the airfoil (see Fig. 4.8 top view) and a pitot tube mounted on a traversing gear mechanism will measure the boundary layer pressures. Pressure signals will be transmitted by Tygon tubing to pressure transducers. A Sciometrics Labmate Data Acquisition system could digitize the signals from the transducers and relay them to a computer.

Precise longitudinal and vertical location of the pitot tube mouth are essential. The airfoil curvature and the slot gap makes it difficult to have the pitot tube mouth perpendicular to the surface on which the boundary layers develop. The same problem will occur if one wants to measure the velocity with a hot wire anemometer. Therefore, the same traversing gear mechanism can be use for both measurements. A schematic of a possible instrumentation set-up is shown in Fig. 4.9.

Figure 4.9 Wind Tunnel Apparatus Schematic

Chapter 5

Conclusions and Recommendations for Further Study

The aerodynamic design of an experiment to enable measurement of flow field parameters in the gap region of a two-element high-lift airfoil has been outlined. The design appears to be feasible but separation of flow from the wind-tunnel floor may present difficulties. A conceptual mechanical design is also presented.

The inviscid computation method used for the design of the proposed experiment, provided the required pressure distributions with satisfactory accuracy. There is no need to improve that part of the design methodology, for example by using a combination of vortex and source singularities that would simulate the boundary layer growth on the airfoil. The prediction of the two merging boundary layers is not required for the design process. The ultimate goal of developing of a suitable turbulence model would require more advanced techniques to model the entire boundary layer behaviour.

The use of bumps on the wind tunnel floor appears to be a feasible approach to eliminate the flow separation from the floor.

The presence of the bump on the floor of the wind tunnel test section substantially modifies the load on the airfoil. However, the shape of the inviscid pressure distribution is much the same, except under the airfoil, where there was stagnating flow. Now with the presence of the bump accelerating the flow the pressure coefficient (C_p) decreases

from 1 to around 0.6. The influence of the bump on the position of the pressure peak on the flap-well ceiling is not clear. That should be investigated before building the test section. It might affect the region of interest.

The presence of the bump limits the angle of attack and the bump has to be redesigned for every change of configuration: gap height, overhang, angle of attack. Therefore, the possibility of improving the computer program for porous wall boundary layer analysis should be pursued. The power series expansion approximation (eqn 4.3) could be replaced by Stevenson's law of the wall (eqn 4.2) in the program. The difficulty of implementing a wind tunnel floor with suction would remain. However, in spite of the difficulties resulting from the implementation of suction, it might be worthwhile.

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Appendix A

Computer Codes

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INVAIRF INVISCID INCOMPRESSIBLE FLOW SOLUTION FOR ARBITRARY AIRFOILS

This program calculates the solution of the inviscid incompressible flow over a single element or multi-element airfoil of arbitrary section using plane vortex panels of linearly varying strength to represent the airfoil. It calls various subroutines for input, output, and calculation processes. Subroutine 'LEQT1F' performs matrix inversion and matrix multiplication. A maximum of 500 panels can be used.

Two Kutta condition are available: equal velocities on the upper and lower surface at the trailing edge

supervised by Prof. R.J.Kind CARLETON UNIVERSITY

revised by Francois L. Belanger : May 30th,94

*****

*.....PROGRAM MAIN

PROGRAM minvairf

INTEGER I,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1
CHARACTER*25 INFILE,OUTFILE

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)

KUTTA=1

*.....Reading in the file names

```

WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
8  WRITE(*,*)'What is the name of the data file to be read from?'
   READ(*,1) INFILE
   WRITE(*,*)
   WRITE(*,*) 'What will be the name of the output file?'
   READ(*,1) OUTFLE
   WRITE(*,*)
1  FORMAT(A25)

   IF(INFILE.EQ.OUTFLE)THEN
       WRITE(*,*)
       WRITE(*,*)'***** The input file has the same name of the output
file. *****'
       WRITE(*,*)
       GO TO 8
   END IF

2  OPEN(UNIT=1,ERR=7,FILE=INFILE,STATUS='OLD')
4  OPEN(UNIT=3,ERR=9,FILE=OUTFLE,STATUS='OLD')
   GOTO 10
7  WRITE(*,*)'ERROR, DATA FILE TO BE READ FROM DOES NOT EXIST'
   GOTO 8
9  OPEN(UNIT=3,FILE=OUTFLE,STATUS='NEW')

*.....Initialize option indicators
10 REWIND 1
   REWIND 3
   DO 11 I=1, 10
       POSTN(I)=0
11 CONTINUE
   PI=3.1415926536
   CALL INPUT

100 CALL FLOCAL
   IF (POSTN(7) .EQ. 1) CALL OUTPUT
   IF (POSTN(7) .EQ. 1) CALL FLOCAL

   CALL OUTPUT
   POSTN(1)=1

```



```
CALL INPUT
IF (POSTN(8) .EQ. 1) GO TO 100
```

```
STOP
END
```

SUBROUTINE INPUT

```
*****
```

```
*
```

```
*   This subroutine controls the reading in of data
*   into the program. It also allows user selection
*   of output options desired.
```

```
*
```

```
*****
```

```
INTEGER I,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1
```

```
COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)
```

```
INTEGER II,III,J,VEL
REAL X1,Y1
REAL ALPHAA,ANG
```

```
*.....Skip initial section if returning to INPUT
```

```
IF (POSTN(1) .EQ. 1) GOTO 5
```

```
1  WRITE(*,*) 'The number of elements, the reference chord length'
  WRITE(*,*) 'of the main element and signed orientation of local'
  WRITE(*,*) 'axis and position coordinate of origin axis system'
  WRITE(*,*) 'of each auxiliary elements of the airfoil and also the'
  WRITE(*,*) 'number of junction pts, the non-dim. scaling length,'
  WRITE(*,*) 'the coordinate of junction pts for of each elements'
  WRITE(*,*) 'should be input from a data file as described in'
```

```

WRITE(*,*) 'INFO*AIRFOIL. '

REWIND 1
III=1
II=0
NJUTOT=0
NPAN=0
2  READ (1,100) NEL, CHORD
100 FORMAT (I2,2x,F8.5)
   D(1) = 0.0
   XO(1) = 0.0
   YO(1) = 0.0
c  IF(NEL.EQ.1) GO TO 4
   READ (1,101) (D(I),XO(I),YO(I), I=1,NEL)    !if I=1 D=0.. I=2,NEL
101 FORMAT(F8.4,2x,F8.5,2x,F8.5)
4  DO 10 I=1,NEL
   IF (I.EQ.1) THEN
     READ (1,102) NJU(1),S(1)
102   FORMAT (I3,2x,F8.5)
     II=NJU(1)
     READ (1,103) (X(J),Y(J), J=1,II)
103   FORMAT(F8.5,2x,F8.5)
   ELSE
     READ (1,104) NJU(I),S(I)
     III=III+NJU(I-1)
     II=II+NJU(I)
104   FORMAT (I3,2x,F8.5)
     READ (1,105) (X(J),Y(J), J=III,II)
105   FORMAT(F8.5,2x,F8.5)
   END IF
10  CONTINUE

```

*.....Transformation of all auxiliary axis systems into the global (x,y)

*.....axis system

```

DO 15 I=1,NEL

   D(I)=D(I)*PI/180.0
15  CONTINUE
   III=1
   II=0
   DO 20 I=1,NEL
     II=II+NJU(I)
     DO 40 J=III,II

```

2 of/de 2

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```

      IF((X(J).EQ.0.0).AND.(Y(J).GT.0.0))THEN
        ANG = PI/2.
        GO TO 9
      ELSE IF((X(J).EQ.0.0).AND.(Y(J).LT.0.0))THEN
        ANG = - PI/2.
        GO TO 9
      ELSE IF((X(J).EQ.0.0).AND.(Y(J).EQ.0.0))THEN
        ANG = 0.0
        GO TO 9
      END IF
      ANG=ATAN2(Y(J),X(J))
9      X1=X(J)
      Y1=Y(J)
      X(J)=XO(I)*S(1)+SQRT(X1*X1+Y1*Y1)*COS(D(I)+ANG)*S(I)
      Y(J)=YO(I)*S(1)+SQRT(X1*X1+Y1*Y1)*SIN(D(I)+ANG)*S(I)
40     CONTINUE
      III=III+NJU(I)
20    CONTINUE

```

*.....The following statements compute the total number of junction
 *.....points and the total number of panels.

```

      NJUTOT=0
      NPAN=0
      NJUTOT=II
      NPAN=NJUTOT-NEL
      WRITE(*,*)
      WRITE(*,*)
3     WRITE(*,*) 'Please enter the angle of attack in degrees.'
      READ (5,106) ALPHA
106  FORMAT (1F8.3)
      ALPHA=ALPHA*PI/180
      WRITE(*,*)
      WRITE(*,*) 'Please enter the airfoil section name. '
      READ (5,107) NAME1
107  FORMAT (A8)

      WRITE(*,*)
      WRITE(*,*) 'What Kutta condition do you want?'
      WRITE(*,*) '  1) equal velocities at trailing edge'
      WRITE(*,*) '  2) trailing edge a stagnation point.'
      WRITE(*,*) 'Enter 1 or 2 (equal velocities assumed).'
      READ(5,108) KUTTA

```

*.....Selecting how the system will be resolved.

```

IF (KUTTA .EQ. 2 ) THEN
  WRITE(*,*)
  WRITE(*,*) 'The overdetermined system of linear equations '
  WRITE(*,*) 'can be solved by rewriting one equation over '
  WRITE(*,*) 'another (1) or by using an overdetermined solver (2).'
  WRITE(*,*) 'Enter your choice, 1 or 2.'
  WRITE(*,*)
  READ(5,108) POSTN(11)
  IF ( POSTN(11).EQ. 2 ) THEN
    WRITE(6,*) ' '
    WRITE(6,*) ' IENTRY : DEPENDING ON VALUE THE FOLLOWING IS
DONE'
    WRITE(6,*) '   = 0 FACTORIZATION ONLY'
    WRITE(6,*) '   = 1 LEAST SQUARE SOLUTION'
    WRITE(6,*) '   = 2 PSEUDO INVERSE'
    WRITE(6,*) '   = 3 LEAST SQUARE SOLUTION & PSEUDO INVERSE'
    READ (5,*) IENTRY
    END IF
  END IF
END IF

```

*.....Selection of output options

*.....If returning, can select same options as before

```

IF (POSTN(9) .EQ. 1 ) GO TO 99

```

```

WRITE(*,*)
WRITE(*,*) 'Do you wish to calculate and display normal and '
WRITE(*,*) 'internal tangential velocities at the control '
WRITE(*,*) 'points on the airfoil contour? '
WRITE(*,*) 'Enter 1 for YES or 0 for NO. '
READ (5,108) POSTN(2)

```

```

WRITE(*,*)
WRITE(*,*) 'Do you wish to display the initial matrix ?'
WRITE(*,*) 'Enter 1 for YES or 0 for NO. '
READ (5,108) POSTN(3)

```

```

IF (POSTN(11) .EQ. 2 ) GO TO 99
WRITE(*,*)
WRITE(*,*) 'Do you wish to display the inverted matrix and '
WRITE(*,*) 'right hand sides? Enter 1 for YES or 0 for NO. '
READ (5,108) POSTN(4)

```

```

99  WRITE(*,*)
    WRITE(*,*) 'Calculations in progress, please wait'
    WRITE(*,*) ' approximately 15 minutes (80286/87 processor).'
    RETURN

```

*.....To continue or exit program

```

5   WRITE(*,*)
    WRITE(*,*) 'Calculations are complete, solutions are on file.'
    WRITE(*,*)

    WRITE(*,*)
    WRITE(*,*) 'Would you like to have the velocity components '
    WRITE(*,*) 'calculate somewhere else then on the countour. '
    WRITE(*,*) 'If yes, you should provide a file having the X,'
    WRITE(*,*) 'Y and the angles (just as a serie of panels) '
    WRITE(*,*) 'Enter 1 for YES or 0 for NO. '
    READ (5,108)VEL

```

```

    IF(VEL.EQ.1)THEN
        CALL CVEL
        WRITE(*,*)
        WRITE(*,*)
        WRITE(*,*) 'Would you like to do another calculation '
        WRITE(*,*) 'of velocity for a different file?'
        WRITE(*,*) 'Enter 1 for YES or 0 for NO. '
        READ (5,108)VEL
        IF(VEL.EQ.1) GOTO 5
    END IF

```

```

    WRITE(*,*)
    WRITE(*,*)
    WRITE(*,*) 'Would you like to do another calculation? '
    WRITE(*,*) 'Enter 1 to continue or 0 to stop. '

```

```

    READ (5,108) POSTN(8)
    IF (POSTN(8) .NE. 1) RETURN

```

*.....By reactivating the following statements, more
 *.....than one airfoil contour may be tested in the
 *.....same run provided they are in the same datafile.

```

*.....WRITE(*,*) 'Do you want to use the same airfoil? '
*.....WRITE(*,*) 'Enter 1 for YES or 0 for NO. '

```

*.....READ (5,108) POSTN(10)

WRITE(*,*)

WRITE(*,*) 'Do you wish to use the same output options? '

WRITE(*,*) 'Enter 1 for YES or 0 for NO. '

READ (5,108) POSTN(9)

IF (POSTN(10) .EQ. 0) THEN

GO TO 1

ELSE

GO TO 3

END IF

108 FORMAT (I11)

END

SUBROUTINE FLOCAL

*****

*

* This subroutine carries out the calculations
* necessary to find the pressure coefficients around
* the airfoil at the stated angle of attack.

* It calls three other subroutines;

* -XKS to find parts of the influence coefficients
* -TRMATRIX arrange the matrice to be solvable
* -LEQTIF subroutine to solve matrices A,B

*

*****

INTEGER I,POSTN,M,MM,IEL,NJUTOT

INTEGER NPAN,NEL,NJU,KUTTA,IENTRY

REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA

REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D

CHARACTER*8 NAME1

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),

* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,

* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,

* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,

* XO(10),YO(10),S(10),D(10)

INTEGER J,II,U,JJ,ISKIP,MTE,MMTE,JTE,IER

REAL GAM,V,VN,XX,YY,XK20,XK40,PHM,PHM1,SCLGAM

*.....Skip initial section if returning to FLOCAL

```
IER = 0
IF (POSTN(7) .EQ. 1 ) GO TO 7
```

*.....The following statements calculate coordinates of the
 *.....control points, the angles PH(I), and the length of
 *.....each panel.

```
J=0
IEL=1
ISKIP=NJU(1)
DO 1 I=1, NJUTOT-1
  IF(I.EQ.ISKIP)THEN          ! The code must skip over
    IEL=IEL+1                ! non-existent control point
    ISKIP=ISKIP+NJU(IEL)     ! and panel
    GO TO 1
  END IF
  J=J+1                      ! J is the panel index number
  XI(J) = 0.5*(X(I+1)+X(I))
  YI(J) = 0.5*(Y(I+1)+Y(I))
  XX = X(I+1)-X(I)
  YY = Y(I+1)-Y(I)
  PH(J) = ATAN2(YY,XX)
  XL(J) = SQRT(XX*XX+YY*YY)
1  CONTINUE
```

*.....The following statements compute the influence
 *.....coefficients for all the vortex panels and set up the
 *.....matrix of boundary condition equations.

*.....There is one equation for each control point, I, and
 *.....a contribution in each equation for each panel, M

*.....The variables XK(1) to XK(4) are the engineering
 *.....variables A,B,C, and D respectively. They are
 *.....evaluated by the subroutine XKS.

```
II = NPAN
DO 3 I=1, II
  IEL=1
  MTE=NJU(1)
  DO 4 M=1, NJUTOT
```


- *....The code must skip over non-existing panels (MM panel
- *....number)

MM = M - IEL + 1

- *....The followin statements are a special procedure at
- *....the end of last panel and start of first panel on each
- *....airfoil element

```

PHM=PH(MM)
PHM1=PH(MM-1)
IF (M.EQ.MTE) THEN                ! For the last jct pts
    IEL=IEL+1                      ! of each element
    MTE=MTE+NJU(IEI)
    XK(1)=0.
    XK(3)=0.
    PHM=0.
    GO TO 2
END IF

```

CALL XKS

```

IF (M .EQ. MTE-NJU(IEI)+1) THEN    ! For the first jct pts
    XK20=0.                        ! of each element
    XK40=0.
    PHM1=0.
END IF

```

- *....The next lines give the influence coefficients.
- *....These form the left hand side of the first NPAN equations.

```

2    A(I,M) = XK(1)*SIN(PHM-PH(I))+XK20*SIN(PHM1-PH(I))
*      +XK(3)*COS(PHM-PH(I))+XK40*
*      COS(PHM1-PH(I))
      XK20 = XK(2)
      XK40 = XK(4)
1    CONTINUE
3    CONTINUE

```

- *....The right hand sides of the first NPAN equations are
- *....computed next.

```

U=NPAN
DO 6 I=1,U

```

```

      BR(I,1) = SIN(PH(I)-ALPHA)
6    CONTINUE

```

```

      POSTN(7) = 1

```

*.....Subroutine TRMATRIX is called to transform the matrix A
 *.....ans BR in a way that they are solvable by LEQT1F or ELULS.

```

      CALL TRMATRIX

```

```

      IF (POSTN(3) .EQ. 0) GO TO 7
      RETURN
7    CONTINUE

```

*.....IER is an error parameter, if IER=129 then
 *.....matrix A is algorithmically singular.

```

      IF (IER .EQ. 129) POSTN(5) = 1

```

*.....Subroutine LEQT1F is called to solve the boundary
 *.....condition equations. The routine returns the coefficients
 *.....of the inverse matrix and the solution of the equations.

```

      IF (POSTN(11) .EQ. 2 ) GO TO 22
      CALL LEQT1F(IER)
      GO TO 222

```

*.....Subroutine LEQT1F is called to solve the boundary
 *.....condition equations. The routine returns the coefficients
 *.....of the inverse matrix and the solution of the equations.

```

22  CALL ELULS
     write(6,*)'salut4'

```

*.....The following statements integrate the bound vorticity
 *.....around the airfoil contour to determine the total
 *.....circulation around the airfoil.

```

222  SCLGAM = 0.0

```

```

      GAM = 0.0
      IEL=1
      JTE= NJU(1)

```

```

JJ=JTE-1
DO 8 J = 1,NPAN

```

*.....I and I+1 are the indices of the junction points and vortex
 *.....strengths at the end of panel J.

```

      I=J+IEL-1
      GAM = GAM+XL(J)*(BR(I,1)+BR(I+1,1))
      IF (J. EQ. JJ) THEN
        IF(IEL.EQ.1)THEN          ! This part is use
          CLGAMMA(IEL) = -GAM      ! to find the lift
          GO TO 11                ! coefficient based
        END IF                    ! on bound circulation
        CLGAMMA(IEL) = -GAM-SCLGAM ! for each element
11      SCLGAM = SCLGAM + CLGAMMA(IEL)
        IEL = IEL + 1
        JTE = JTE + NJU(IEL)
        JJ=JTE-IEL
      END IF
8    CONTINUE

```

*.....Lift coefficient based on bound circulation

```

      CLGAM = -GAM
      IF (POSTN(2) .EQ. 0) GO TO 99

```

*.....Calculation of velocity at a control points I.

```

DO 9 I=1, NPAN

```

*.....Free stream tangential velocity at a control points I.

```

      V = COS(ALPHA-PH(I))

```

*.....Free stream normal velocity at a control points I.

```

      VN = SIN(ALPHA-PH(I))

```

*.....Calculation of internal tangential and normal velocity

*.....induced by the vortex panels at the control points.

```

      IEL=1
      MMTE=NJU(1)
      DO 10 MM=1, NPAN

```

*....BR are the vortex strength at the end of panel MM.

```

      IF (MM. EQ. MMTE) THEN
        IEL = IEL+1
        MMTE = MMTE+NJU(IEI)-1
      END IF

```

*.....M and M+1 are the indices of the junction points and vortex

*.....strengths at the end of panel MM.

```

      M = MM+IEL-1
      CALL XKS
      V = V+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*COS(PH(MM)-PH(I))-
*      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*SIN(PH(MM)-PH(I))
      VN = VN+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*SIN(PH(MM)-PH(I))+
*      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*COS(PH(MM)-PH(I))

```

*.....Summation of velocities produced by each panel at control points I.

```

      VNN(I) = VN
      VTAN(I) = V
10    CONTINUE
9     CONTINUE
99    POSTN(7) = 2
      RETURN
      END

```

SUBROUTINE XKS

*****

*

* This subroutine calculates XK(1) to XK(4) which
 * represent the engineering variables A,B,C, and D.
 * The influence coefficients consist of simple
 * combinations of these coefficients.

*

*****

```

      INTEGER I,POSTN,M,MM,IEL,NJUTOT
      INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
      REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
      REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
      CHARACTER*8 NAME1

```

```

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)

```

```

REAL XX,YY,RA,XX1,YY1,RB,THA,THB
REAL C1,C2,C3,C4,C5,C6,C7

```

```

XX = XI(I)-X(M)
YY = YI(I)-Y(M)

```

```

*.....Radius to control point from one end of panel
RA = SQRT (XX*XX+YY*YY)

```

```

XX1= XI(I)-X(M+1)
YY1= YI(I)-Y(M+1)

```

```

*.....Radius to control point from other end of panel
RB = SQRT (XX1*XX1+YY1*YY1)

```

```

*.....Angles between panel and radii to control point
*.....MM is the index value of the panel between junction points
*.....X(M), Y(M),and X(M+1), Y(M+1).

```

```

THA = ATAN2(YY,XX)-PH(MM)
THB = ATAN2(YY1,XX1)-PH(MM)

```

```

*.....The next statement causes the calculation on the inside
*.....of the contour.

```

```

IF (ABS(ABS(THA-THB)-PI) .LE. 0.00001) THEN
  THA = 0.0000
  THB = PI
END IF

```

```

*.....The next two statements ensure that THA and THB
*.....always have the same sign so that their difference
*.....is always less than pi.

```

```

IF (THA .LT. 0.0 .AND. THB .GT. 0.0) THB = THB-2.*PI
IF (THA .GT. 0.0 .AND. THB .LT. 0.0) THA = THA-2.*PI

```

```

C1 = RA*SIN(THA)/(2.*PI*XL(MM))
IF (ABS(SIN(THA)) .LE. 0.001 .OR. ABS(SIN(THB)) .LE. 0.001)
* THEN
    C5 = 0.0
    GO TO 1
ENDIF

C5 = C1*ALOG((SIN(THB)/SIN(THA)))
1 C4 = C1*(THB-THA)
C6 = RA*COS(THA)/XL(MM)
C2 = 1.0-C6
C3 = C6/(2.0*PI)
C7 = ALOG(RA/RB)

XK(1) = C2*(THA-THB)/(2.0*PI)-C5
XK(2) = C3*(THA-THB)+C5
XK(3) = C7*C2/(2.0*PI)-C4+1.0/(2.0*PI)
XK(4) = C3*C7 +C4-1.0/(2.0*PI)

RETURN

END

SUBROUTINE OUTPUT

*****
*
*   This subroutine controls all the outputs of the
*   program system according to the user options chosen
*   in the INPUT subroutine
*
*****

INTEGER I,N,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,

```

```
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)
```

```
INTEGER II,I,J
REAL ALPHA A,XOC,YOC,CPI,SUM,X1,Y1,ANG
```

```
*.....Skip initial section if returning to subroutine
*.....or if user selected not to display initial matrix.
```

```
      N=NJUTOT
      IF ((POSTN(11).EQ.2).AND.(IENTRY.GT.0)) N=NJUTOT+NEL
      II=POSTN(7)
      GOTO (1,3,5), II
1      L = N+1
      WRITE (3,110)
110  FORMAT (////,' MATRIX TO BE INVERTED WITH RIGHT HAND SIDES ')
      DO 10 I=1, N
          write(3,*)'I=' ,I
          WRITE (3,111) (A(I,M),M=1,NJUTOT), BR(I,1)
10   CONTINUE
111  FORMAT ( , 3F10.4)
      RETURN
3   CONTINUE
```

```
*.....Give error warnings if errors or inaccuracies indicated
*.....by the subroutine LEQT1F.
```

```
      IF (POSTN(5) .EQ.1) THEN
          WRITE(3,114)
          WRITE(3,115)
      ENDIF
114  FORMAT ('***** TERMINAL ERROR *****')
115  FORMAT ('The matrix is algorithmically singular.')
```

```
*.....Skip next section if not selected by user.
```

```
      IF (POSTN(4) .EQ. 0) GO TO 5
      WRITE(3,118)
118  FORMAT ( /, 'Inverted matrix with right hand side.')
```

```
      DO 99 I=1, N
          write(3,*)'I (inv)= ',I
          WRITE (3,111) (A(I,M), M=1,NJUTOT), BR(I,1)
99   CONTINUE
      WRITE (3,103)
```

```

WRITE(*,*)
WRITE(*,*)
WRITE (3,104) (BR(I,1), I=1,N)
103 FORMAT ( /, 'Non-D, Vortex strengths at junction points.')
104 FORMAT (5F10.5)
5 CONTINUE

```

*.....The output file characteristics are presented in the
 *.....following form.

```

WRITE (3,128) NAME1
128 FORMAT ( //, 'The section name is: ',A8)
WRITE (3,126) CHORD
126 FORMAT ('The chord length is: ',1F8.5)
IF(KUTTA.EQ.1)THEN
  WRITE(3,*)'The Kutta condition in this computation ; '
  WRITE(3,*)' equal velocities at trailing edge'
ELSE
  WRITE(3,*)'The Kutta condition in this computation ; '
  WRITE(3,*)'the trailing edge is a stagnation point.'
END IF
WRITE (3,106) CLGAM
106 FORMAT ('Lift coefficient based on circulation = ',F9.5)
ALPHA A = ALPHA * 180./PI
WRITE (3,125) ALPHA A
125 FORMAT ('Angle of attack (degrees) = ',F8.3)
WRITE (3,124)
124 FORMAT ('Junction point coordinates X, Y, and pressure', /,
  *'coefficients based on vortex strengths at junction points.')
WRITE(3,*)
WRITE(3,130)CLGAMMA(1)
130 FORMAT(' Main element' /, '(lift coefficient based on circulation',
  *' = ',F9.5,' )')
WRITE(3,*)
WRITE(3,123)
WRITE(3,*)
123 FORMAT ('      X/C      Y/C      CP ')

```

*.....Transforming the global axis system back in local axis
 *..... systems for each element.

```

OPEN(UNIT=7)

```



```

c   WRITE(7,*)NJUTOT
    DO 133 I=1,NJUTOT
      WRITE(7,201)X(I), 1.-BR(i,1)**2
c   WRITE(6,*)'Salut'
133 CONTINUE
    CLOSE(UNIT=7)

    IEL = 1
    SUM = NJU(1)
    II = NJUTOT
    DO 12 J=1, II
      X1=X(J)-XO(IEL)*S(1)
      Y1=Y(J)-YO(IEL)*S(1)
      IF((X1.EQ.0.0).AND.(Y1.GT.0.0))THEN
        ANG = PI/2.
        GO TO 9
      ELSE IF((X1.EQ.0.0).AND.(Y1.LT.0.0))THEN
        ANG = - PI/2.
        GO TO 9
      ELSE IF((X1.EQ.0.0).AND.(Y1.EQ.0.0))THEN
        ANG = 0.0
        GO TO 9
      END IF
      ANG=ATAN2(Y1,X1)
9     XOC =SQRT(X1*X1+Y1*Y1)*COS(ANG-D(IEL))/S(IEL)
      YOC =SQRT(X1*X1+Y1*Y1)*SIN(ANG-D(IEL))/S(IEL)
      CPJ = 1.-BR(J,1)**2
      CP(J)=CPJ
c     X(J)=XOC

```

*.....Pressure coefficients at junction points

```

      WRITE (3,122)J, XOC, YOC, CPJ
122   FORMAT (I3,2X,F8.5,3X,F8.5,7X,F9.5)

      IF(J.EQ.(SUM))THEN
        IF(J.EQ.NJUTOT) GO TO 12
        IEL=IEL+1
        WRITE(3,*)
        WRITE(3,131)IEL,CLGAMMA(IEL)
131   FORMAT(' Element # ',I3/, ' (lift coefficient based on '
        *'circulation = ',F9.5,')')
        WRITE(3,*)
        WRITE (3,123)

```

```

      WRITE(3,*)
      SUM = SUM+NJU(IEL)
    END IF

```

```

12  CONTINUE

```

*.....Skip section if not selected by user

```

      IF (POSTN(2) .EQ. 0) GO TO 7

```

*.....Output the velocities

```

      WRITE (3,121)
      WRITE (3,120) (VNN(I), I=1, NPAN)
121  FORMAT ( /, ' NORMAL VELOCITY RATIO AT CONTROL POINTS ')
120  FORMAT (5E12.3)
      WRITE (3,108)
      WRITE (3,107) (VTAN(I), I=1,NPAN)
108  FORMAT ( /, ' INTERNAL TANGENTIAL VELOCITY RATIO AT ',
      * 'CONTROL POINTS ')
107  FORMAT (5E12.3)

```

```

7  CONTINUE

```

*.....Files fort.2 and fort.4 are to be use for the transformation

*..... of the coordinate and the plotting of the results by GNUPLOT.

```

      OPEN(UNIT=2)
      OPEN(UNIT=4)
      WRITE(2,*)NJUTOT
      DO 13 I=1,II
        WRITE(2,201)X(I),CP(I)
        WRITE(4,201)X(I),Y(I)
13  CONTINUE

```

```

201  FORMAT(E11.5,' ',E11.5)

```

```

      CLOSE(UNIT=2)
      CLOSE(UNIT=4)

```

```

      RETURN
      END

```

SUBROUTINE TRMATRIX

*****

*

* This subroutine place the added equations in a
 * way that they are solvable by LEQT1F. A test is also
 * perform to verify if the matrix is singular.

*

*****

INTEGER I,N,POSTN,M,MM,IEL,NJUTOT
 INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
 REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
 REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
 CHARACTER*8 NAME1

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
 * BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
 * POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
 * IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
 * XO(10),YO(10),S(10),D(10)

INTEGER IER,L,J,NN,NJ,NP

*.....The next statements form the (NJU-1)th and (NJU)th equations
 *..... for each element these express the Kutta condition.

*.....We get the Kutta condition of equal velocity on upper and
 *..... lower surfaces at the trailing edge.

*.....As the NJUth equation we equate the vortex sheet strength on
 *..... the lower surface at the trailing to the negative of that on
 *..... the upper surface for each airfoil element.

NP=NPAN
 NJ=NJUTOT
 DO 555 IEL = NEL,1,-1
 NN = NJ-NJU(IEL)+1
 DO 55 M=1,NJUTOT
 A(NJ,M) = 0.0 ! Add an eqn by element, all jct pts
 55 CONTINUE
 BR(NJ,1) = 0.0
 A(NJ,NN) = 1.0 ! Add an eqn by element, first jct pts
 A(NJ,NJ) = 1.0 ! Add an eqn by element, last jct pts

IF((IEL.EQ.1).AND.(KUTTA.EQ.1))GO TO 54
 IF ((IEL.EQ.1).AND.(POSTN(11).EQ.2).AND.(IENTRY.GT.0))GO TO 50

DO 54 I= NJ-1,NJ-NJU(IEL),-1

IF((POSTN(11).EQ.1).AND.(KUTTA.EQ.2).AND.(I.EQ.(NJ-7)))THEN
 NP=NP-1 ! The NJ-7 eqn is discarded to make room for
 END IF ! the kutta condition equation.

J=NP

DO 53 M=1,NJUTOT

53 A(I,M)=A(J,M) ! All eqns of the IEL element
 BR(I,1)=BR(J,1) ! are move downward to make room
 NP=NP-1 ! to place the next added eqn where
 54 CONTINUE ! it belongs.

IF(KUTTA.EQ.2) NP=NP+1

- *.....This Kutta condition makes the trailing edge a stagnation point.
- *.....The (NJU-1)th equation for this Kutta condition over-writes the
- *..... previously formed (NJU-1)th equation.

IF ((POSTN(11).EQ.2).AND.(IENTRY.GT.0))GO TO 50

IF (KUTTA.EQ.2) THEN

DO 5 M=1, NJUTOT

A(NN,M) = 0.0 ! First ctrl pts, all jct pts
 5 CONTINUE
 A(NN,NN)=1.0 ! First ctrl pts, first jct pts
 BR(NN,1) = 0.0
 50 END IF

NP = NP+1

NJ = NJ-NJU(IEL)

555 CONTINUE

- *.....Solving for an over-determined system of (NJU)th equation.
- *.....Transforming the matrix by putting the added eqn. into place.
- *.....There are IEL eqn. added to the matrice due to the stagnation
- *..... Kutta condition, each eqn. should be place before the first eqn.
- *..... of the element they are added for.

IF((POSTN(11).EQ.1).OR.(IENTRY.EQ.0).AND.(KUTTA.EQ. 1)) GOTO 111

```

NJ=1
DO 22 IEL=1,NEL
  DO 25 J=1,NJUTOT
    A(NJUTOT+IEL,J)=0.0
    IF(J.EQ.NJ)THEN
      A(NJUTOT+IEL,J)=1.0
    END IF
25  CONTINUE
    BR(NJUTOT+IEL,1)=0.0
    NJ = NJ+NJU(IEL)
22  CONTINUE

```

*.....Check to see if it is still an invertible matrix.

```

111 CONTINUE
DO 20 L=1,N

  IF (A(L,L).EQ.0.0) THEN
    IER=129
    RETURN
  ELSE
    IER = 0
  END IF
20  CONTINUE

```

```

RETURN
END

```

SUBROUTINE LEQT1F(IER)

*****

*

* This subroutine finds the inverse of matrix A,
 * stores the inverse matrix back into A and multi-
 * plies it by matrix B, thus solving for C in the
 * equation AC=B. Solution C is then stored back
 * into the matrix B (fortran array BR).

*

*****

```

INTEGER I,N,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY

```

```

REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1

```

```

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)

```

```

INTEGER IER,L,J,K
REAL SUM,C(501,501)

```

*.....matrix inversion performed here

```

N = NJUTOT
DO 60 L=1,N

```

```

    A(L,L)=1./A(L,L)
    DO 40 I=1,N
        IF(I.EQ.L) GO TO 40
        A(I,L)=A(I,L)*A(L,L)
        DO 30 J=1,N
            IF(J.EQ.L) GO TO 30          !      ??????
            A(I,J)=A(I,J)-A(I,L)*A(L,J)
30      CONTINUE
40      CONTINUE

```

```

    DO 60 J=1,N
        IF(J.EQ.L) GO TO 60
        A(L,J)=-A(L,L)*A(L,J)
50      FORMAT(1X,F6.1)
60      CONTINUE

```

*.....matrix multiplication performed here

```

    DO 90 I=1,N
        SUM=0.
        DO 91 K=1,N
            SUM=SUM+A(I,K)*BR(K,1)
91      CONTINUE
        C(I,1)=SUM
90      CONTINUE

```

*.....Solution C replaced into B

```

      DO 95 I=1,N
        BR(I,1)=C(I,1)
95    CONTINUE
      RETURN
      END

```

SUBROUTINE CVEL

```

*****

```

```

*
*   This subroutine finds the velocity component at
*   any points around the countour of the airfoil if
*   those points are distributed as a serie of panels.
*   Therefore a file containing the X,Y and the angle
*   VPH should provide these informations at this
*   point.
*

```

```

*****

```

```

      INTEGER I,POSTN,M,MM,IEL,NJUTOT
      INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
      REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
      REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
      CHARACTER*8 NAME1
      CHARACTER*25 INFILE,OUTFLE

```

```

      COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
*   BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
*   POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
*   IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
*   XO(10),YO(10),S(10),D(10)

```

```

      INTEGER NU,MMTE
      REAL VELN(500),VELT(500),VPH(500),V,VN

```

*.....Reading in the file names

```

      WRITE(*,*)
      WRITE(*,*)
      WRITE(*,*)
8    WRITE(*,*)'What is the name of the data file to be read from?'
      READ(*,1) INFILE
      WRITE(*,*)

```

```

WRITE(*,*) 'What will be the name of the output file?'
READ(*,1) OUTFLE
WRITE(*,*)
1  FORMAT(A25)

2  OPEN(UNIT=11,ERR=7,FILE=INFILE,STATUS='OLD')
4  OPEN(UNIT=13,ERR=9,FILE=OUTFLE,STATUS='OLD')
   GOTO 11
7  WRITE(*,*)'ERROR, DATA FILE TO BE READ FROM DOES NOT EXIST'
   GOTO 8
9  OPEN(UNIT=13,FILE=OUTFLE,STATUS='NEW')

11 READ (11,*) NU
   DO 10 I=1,NU
       READ (11,*) XI(I),YI(I),VPH(I)
c     WRITE(6,*) XI(I),YI(I),VPH(I)
* 103  FORMAT(F8.5,2x,F8.5)

10  CONTINUE

*.....Calculation of velocity at a control points I.
      WRITE(6,*)'NPAN (vel)=' ,NPAN
      DO 15 I=1, NU

*.....Free stream tangential velocity at a control points I.

      V = COS(ALPHA-VPH(I))

*.....Free stream normal velocity at a control points I.

      VN = SIN(ALPHA-VPH(I))

*.....Calculation of internal tangential and normal velocity
*.....induced by the vortex panels at the control points.

      IEL=1
      MMTE=NJU(1)

      DO 20 MM=1, NPAN

*....BR are the vortex strengthat the end of panel MM.

      IF (MM. EQ. MMTE)THEN

```



```

      IEL = IEL+1
      MMTE = MMTE+NJU(IEL)-1
    END IF

```

*.....M and M+1 are the indices of the junction points and vortex
 *.....strengths at the end of panel MM.

```

      M = MM+IEL-1
      CALL XKS
      V = V+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*COS(PH(MM)-VPH(I))-
*      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*SIN(PH(MM)-VPH(I))
      VN = VN+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*SIN(PH(MM)-VPH(I))+
*      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*COS(PH(MM)-VPH(I))

```

*.....Summation of velocities produced by each panel at control points I.

```

      VELN(I) = VN
      VELT(I) = V
20    CONTINUE
15    CONTINUE

```

*.....Output file for this subroutine

```

      WRITE(13,*)
      WRITE(13,*) 'The results velocities for the file called ',INFILE
      WRITE(13,*)

      DO 30 I=1,NU
        WRITE(13,103)XI(I),YI(I),VELN(I),VELT(I)
103    FORMAT('X=',F9.5,2x,' Y=',F7.3,2x,' VELN=',F7.5,2x,
*      ' VELT=',F7.5)
        WRITE(12,*)XI(I)+0.8000,VELT(I)
30    CONTINUE

      WRITE(*,*)
      WRITE(*,*) 'The results for the different velocities are now'
      WRITE(*,*) 'in the file called ',OUTFILE
      WRITE(*,*)

      REWIND 11
      CLOSE(UNIT=11)
      CLOSE(UNIT=13)

```

RETURN
END

SUBROUTINE ELULS

*****

* SUBROUTINE ELULS

*

*

DR.N.N.ABDELMALEK, NRC

*

*****

C INPUT DATA

C A : MATRIX OF THE SYSTEM

C M : NUMBER OF ROWS IN A (COLUMNS IN APSUDO)

C N : NUMBER OF COLUMNS IN A (ROWS IN APSUDO)

C BR : THE RIGHT HAND SIDE IN $AX = B$

C IEXMPL : THE NUMBER OF LEAST SQUARE SOLUTIONS TO BE
CALCULATED

C EPS : ALLOWED TOLERANCE. S.T. IF $|X| < EPS$ THEN X IS SET TO 0

C IENTRY : DEPENDING ON VALUE THE FOLLOWING IS DONE

C = 0 FACTORIZATION ONLY

C = 1 LEAST SQUARE SOLUTION

C = 2 PSEUDO INVERSE

C = 3 LEAST SQUARE SOLUTION & PSEUDO INVERSE

C IDIM : THE ASSIGNED SIZE OF MATRICES.

C NOTE:MAX SIZE FOR INTERNAL MATRICES IS SET = 38

C

C OUTPUT DATA

C IRNK : CALCULATED RANK OF A

C APSUDO : PSEUDO INVERSE OF A T

C L : THE UNIT LOWER TRIANGULAR MATRIX

C U : THE UPPER TRIANGULAR MATRIX WHOSE ROWS ARE

C SCALED SUCH THAT ITS DIAGONAL ELEMENTS ARE ALL ONES

C DIAG : THE DIAGONAL ELEMENTS OF MATRIX U BEFORE SCALING

C IR : AN M-VECTOR CONTAINING THE ROW PERMUTATION

C OF MATRIX A

C IC : AN N-VECTOR CONTAINING THE COLUMN PERMUTATION

C OF MATRIX A

C AERR : VALUE OF $\|A \cdot AINV \cdot A - A\| / \|A\|$

C XS : THE LEAST SQUARE SOLUTIONS

C RES : RESIDUALS IN THE LEAST SQUARE SOLUTION $RES = B - AX$

C RERR : NORM OF THE RESIDUAL VECTOR

C

C THIS SUBROUTINE IS IN SINGLE PRECISION. TO CHANGE IT TO

C DOUBLE PRECISION REPLACE THE FOLLOWING TWO STATEMENTS BY
C THE SINGLE STATEMENT

C IMPLICIT REAL*8(A-H,L,O-Z)

C

*****

INTEGER I,N,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)

INTEGER IEXMPL,IDIM,IRNK,J,IOUT,KP,KDP,IOUT1
INTEGER IRNK1,IRNKK,K,KP1,IEND,IFIRST,ISECND,IRI,IM1,II
INTEGER IP1,KK

DOUBLE PRECISION SM,SMA,SMB
REAL L,U,IR,IC,B,XS,RES,APSUDO,XR,RERR,EPS,AERR,E,G,GH
REAL PIVOT,DIAG,YY,W,TEMP,T,V,CON,SUMA,SUMC,ERR

DIMENSION L(501,501),U(501,501),IR(501),IC(501),
1 B(501),XS(501,1),RES(501,1),ERR(501,501),
2 APSUDO(501,1),XR(501),RERR(1),
3 T(501,501),V(501,501),YY(501),W(501),TEMP(501),DIAG(501)

*.....Temporary initialisation

IEXMPL = 1
IDIM = 501
EPS = 0.0
N = NJUTOT
M = NJUTOT+NEL

IF(((IENTRY.EQ.1).OR.(IENTRY.EQ.3)).AND.(IEXMPL.GT.0)) GO TO 1
IF(IENTRY.EQ.0) GO TO 1
IF(IENTRY.EQ.1) RETURN
IENTRY = 2

*.....Solving the overdetermined system

```

write(6,*)'salut3'
1 DO 2 J=1,N
  IC(J)=J
  DO 2 I=1,M
    U(I,J)=A(I,J)
2   ERR(I,J)=A(I,J)

  DO 4 I=1,M
    IR(I)=I
    DO 3 J=1,M
3     L(I,J)=0.0
4     L(I,I)=1.0
    IOUT=1
5 IF(IOUT.GT.M.OR.IOUT.GT.N) GO TO 15
    KP=IOUT
    KDP=IOUT
    DO 6 I=IOUT,M
      DO 6 J=IOUT,N
        E=U(I,J)
        IF(E.LT.0.0) E=-E
        G=U(KP,KDP)
        IF(G.LT.0.0) G=-G
        IF(E.LE.G) GO TO 6
        KP=I
        KDP=J
6     CONTINUE
    E=U(KP,KDP)
    IF(E.LT.0.0) E=-E
    IF(E.LT.EPS) GO TO 15
    IF(IOUT.EQ.KDP) GO TO 8
    J=IC(IOUT)
    IC(IOUT)=IC(KDP)
    IC(KDP)=J
    DO 7 I=1,M
      GH=U(I,IOUT)
      U(I,IOUT)=U(I,KDP)
7     U(I,KDP)=GH
8 IF(IOUT.EQ.KP) GO TO 11
    J=IR(IOUT)
    IR(IOUT)=IR(KP)
    IR(KP)=J
    DO 9 J=1,N
      GH=U(IOUT,J)

```

```

      U(IOUT,J)=U(KP,J)
9    U(KP,J)=GH
      IF(IOUT.EQ.1) GO TO 11
      IOUT1=IOUT-1
      DO 10 J=1,IOUT1
        GH=L(IOUT,J)
        L(IOUT,J)=L(KP,J)
10   L(KP,J)=GH
11   PIVOT=U(IOUT,IOUT)
      IF(IOUT.EQ.M) GO TO 14
      KP1=IOUT+1
      DO 13 I=KP1,M
        GH=U(I,IOUT)/PIVOT
        L(I,IOUT)=GH
        DO 12 J=IOUT,N
12    U(I,J)=U(I,J)-GH*U(IOUT,J)
        U(I,IOUT)=0.0
13   CONTINUE
14   IOUT=IOUT+1
      GO TO 5
15   IRNK=IOUT-1
      DO 16 I=1,IRNK
16    DIAG(I)=U(I,I)
        IF(IRNK.EQ.M) GO TO 19
        IRNK1=IRNK+1
        DO 18 I=IRNK1,M
          DO 17 J=IRNK,N
17     U(I,J)=0.0
18    CONTINUE
19   IRNKK=IRNK
      IF(IRNK.EQ.M) GO TO 22
      DO 21 I=1,IRNK
        DO 21 J=1,IRNK
          SM=0.0
          DO 20 K=1,M
            SMA=L(K,I)
            SMB=L(K,J)
20     SM=SM+SMA*SMB
21     APSUDO(I,J)=SM
      CALL CHOL(APSUDO,T,IRNK,IDIM)
22   IF(IRNK.EQ.N) GO TO 27
      DO 24 I=1,IRNK
        U(I,I)=1.0
        K=I+1

```

```

      DO 23 J=K,N
23    U(I,J)=U(I,J)/DIAG(I)
24    CONTINUE
      DO 26 I=1,IRNK
      DO 26 J=1,IRNK
        SM=0.0
        DO 25 K=1,N
          SMA=U(I,K)
          SMB=U(J,K)
25      SM=SM+SMA*SMB
26      APSUDO(I,J)=SM
      CALL CHOL(APSUDO,V,IRNK,IDIM)
27 IF(IENTRY.EQ.0)THEN
      DO 70 I=1,N
70    BR(I,1) = XR(I)
      RETURN
    END IF

```

C

C END OF L-U FACTORIZATION.

C

```

      IEND=M
      IF(IENTRY.NE.2) IEND=IEXMPL
28 DO 64 IFIRST=1,IEND
      IF(IENTRY.NE.2) GO TO 30
      DO 29 ISECND = 1,M
29    B(ISECND)=0.0
      B(IFIRST)=1.0
      GO TO 32
30    DO 31 I=1,M
      IRI=IR(I)
31    B(I)=BR(IRI,IFIRST)
32    IF(IRNKK.NE.M) GO TO 35
      YY(1)=BR(1,1)
      IF(IRNKK.EQ.1) GO TO 43
      DO 34 I=2,IRNKK
      IM1=I-1
      SM=-B(I)
      DO 33 K=1,IM1
        SMA=L(I,K)
        SMB=YY(K)
33      SM=SM+SMA*SMB
      GH=SM
34    YY(I)=-GH
      GO TO 43

```

```

35 DO 37 I=1,IRNKK
    SM=0.0
    DO 36 K=1,M
        SMA=L(K,I)
        SMB=B(K)
36    SM=SM+SMA*SMB
37    TEMP(I)=SM
    W(1)=TEMP(1)/T(1,1)
    IF(IRNKK.EQ.1) GO TO 40
    DO 39 I=2,IRNKK
        IM1=I-1
        SM=-TEMP(I)
        DO 38 K=1,IM1
            SMA=T(I,K)
            SMB=W(K)
38    SM=SM+SMA*SMB
        GH=SM
39    W(I)=-GH/T(I,I)
40    YY(IRNKK)=W(IRNKK)/T(IRNKK,IRNKK)
    IF(IRNKK.EQ.1) GO TO 43
    DO 42 II=2,IRNKK
        I=IRNKK-II+1
        IP1=I+1
        SM=-W(I)
        KK=II-1+IP1-1
        DO 41 K=IP1,KK
            SMA=T(K,I)
            SMB=YY(K)
41    SM=SM+SMA*SMB
        GH=SM
42    YY(I)=-GH/T(I,I)
43    IF(IRNKK.NE.N) GO TO 46
    XR(IRNKK)=YY(IRNKK)/U(IRNKK,IRNKK)
    IF(IRNKK.EQ.1) GO TO 56
    DO 45 II=2,IRNKK
        I=IRNKK-II+1
        IP1=I+1
        SM=-YY(I)
        KK=II-1+IP1-1
        DO 44 K=IP1,KK
            SMA=U(I,K)
            SMB=XR(K)
44    SM=SM+SMA*SMB
        GH=SM

```

```

45  XR(I)=-GH/U(I,I)
    GO TO 56
46  DO 47 I=1,IRNK
47  YY(I)=YY(I)/DIAG(I)
    W(1)=YY(1)/V(1,1)
    IF(IRNKK.EQ.1) GO TO 50
    DO 49 I=2,IRNKK
      IM1=I-1
      SM=-YY(I)
      DO 48 K=1,IM1
        SMA=V(I,K)
        SMB=W(K)
48      SM=SM+SMA*SMB
      GH=SM
49  W(I)=-GH/V(I,I)
50  TEMP(IRNKK)=W(IRNKK)/V(IRNKK,IRNKK)
    IF(IRNKK.EQ.1) GO TO 53
    DO 52 II=2,IRNKK
      I=IRNKK-II+1
      IP1=I+1
      SM=-W(I)
      KK=II-1+IP1-1
      DO 51 K=IP1,KK
        SMA=V(K,I)
        SMB=TEMP(K)
51      SM=SM+SMA*SMB
      GH=SM
52  TEMP(I)=-GH/V(I,I)
53  DO 55 I=1,N
      SM=0.0
      DO 54 K=1,IRNKK
        SMA=U(K,I)
        SMB=TEMP(K)
54      SM=SM+SMA*SMB
55  XR(I)=SM
C PERMUTE X.
56  DO 57 I=1,N
      K=IC(I)
57  W(K)=XR(I)
    IF(IENTRY.EQ.2) GO TO 62
    DO 58 I=1,N
58  XS(I,IFIRST)=W(I)
    DO 60 J=1,M
      SM=-BR(J,IFIRST)

```



```

        DO 59 K=1,N
            SMA=A(J,K)
            SMB=W(K)
59      SM=SM+SMA*SMB
60      RES(J,IFIRST)=-SM
        SM=0.0
        DO 61 K=1,M
            SMA=RES(K,IFIRST)
61      SM=SM+SMA*SMA
        CON=SM
        RERR(IFIRST)=CON**0.5
        GO TO 64
62      DO 63 ISECND = 1,N
            IRI=IR(IFIRST)
63      APSUDO(ISECND,IRI)=W(ISECND)
64      CONTINUE
        IF(IENTRY.EQ.1)THEN
            DO 71 I=1,N
71          BR(I,1) = XS(I,1)
            RETURN
        END IF
        IF(IENTRY.EQ.2) GO TO 65
        IENTRY = 2
        IEND = M
        GO TO 28
65      CALL MNORM(ERR,IDIM,SUMA)
        DO 67 I=1,M
            DO 67 J=1,M
                SM=0.0
                DO 66 K=1,N
                    SMA=A(I,K)
                    SMB=APSUDO(K,J)
66          SM=SM+SMA*SMB
67          T(I,J)=SM
            DO 69 I=1,M
                DO 69 J=1,N
                    SM=-A(I,J)
                    DO 68 K=1,M
                        SMA=T(I,K)
                        SMB=A(K,J)
68          SM=SM+SMA*SMB
69          V(I,J)=SM
        CALL MNORM(V,IDIM,SUMC)
        AERR=SUMC/SUMA

```

```

IF(IENTRY.EQ.2)THEN
  DO 72 I=1,N
    XR(I)=0
    DO 72 J=1,M
72      XR(I)=XR(I)+APSUDO(I,J)*BR(J,I)

```

```

    DO 73 I=1,N
73      BR(I,1) = XR(I)
  END IF

```

```

  RETURN
  END

```

C

```

  SUBROUTINE MNORM(ERR,IDIM,SUM)

```

```

  *****

```

```

  INTEGER IDIM,J
  DOUBLE PRECISION SM,SMA
  REAL ERR,SUM
  DIMENSION ERR(IDIM,IDIM)

```

```

  INTEGER I,POSTN,M,MM,IEL,NJUTOT
  INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
  REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
  REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
  CHARACTER*8 NAME1

```

```

  COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
  * BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
  * POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
  * IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
  * XO(10),YO(10),S(10),D(10)

```

```

  SM=0.0
  DO 1 J=1,NJUTOT
    DO 1 I=1,NJUTOT+NEL
      SMA=ERR(I,J)
1    SM=SM+SMA*SMA
  SM=SM**0.5
  SUM=SM
  RETURN
  END

```

C

```

  SUBROUTINE CHOL(Q,ELT,IRNK,IDIM)

```

*****

C   CHOLSKY DECOMPOSITION OF POSITIVE DEFINITE SYMMETRIC
MATRIX

C   IMPLICIT REAL*8(A-H,O-Z)

INTEGER IRNK,IDIM,IM1,L,LP1,K
REAL Q,ELT,GH,C,CONST,SMB,RGH
DOUBLE PRECISION SM,SMA
DIMENSION Q(IDIM,IDIM),ELT(IDIM,IDIM)

INTEGER I,POSTN,M,MM,IEL,NJUTOT
INTEGER NPAN,NEL,NJU,KUTTA,IENTRY
REAL X,Y,XI,YI,PH,XL,A,BR,VTAN,VNN,ALPHA
REAL CLGAM,PI,XK,CP,CHORD,CLGAMMA,XO,YO,S,D
CHARACTER*8 NAME1

COMMON X(0:501),Y(0:501),XI(501),YI(501),PH(0:501),XL(0:501),
* BR(501,1),A(501,501),VTAN(500),VNN(500),ALPHA,
* POSTN(11),CLGAM,M,MM,I,PI,NAME1,XK(4),CP(502),CHORD,KUTTA,
* IEL,NJUTOT,NPAN,NEL,NJU(10),CLGAMMA(10),IENTRY,
* XO(10),YO(10),S(10),D(10)

L=1
C=Q(1,1)
ELT(1,1)=C**0.5
IF(IRNK.LE.1) RETURN
1 LP1=L+1
ELT(LP1,1)=Q(1,LP1)/ELT(1,1)
IF(L.EQ.1) GO TO 4
DO 3 I=2,L
IM1=I-1
CONST=-Q(I,LP1)
SM=CONST
DO 2 K=1,IM1
SMA=ELT(I,K)
SMB=ELT(LP1,K)
2 SM=SM+SMA*SMB
GH=SM
3 ELT(LP1,I)=-GH/ELT(I,I)
4 DO 5 I=1,L
5 ELT(I,LP1)=0.0
CONST=-Q(LP1,LP1)
SM=CONST
DO 6 K=1,L

```
      SMA=ELT(LP1,K)  
6  SM=SM+SMA*SMA  
   RGH=SM  
   ELT(LP1,LP1)=(-RGH)**0.5  
   L=L+1  
   IF(L.LT.IRNK) GO TO 1  
   RETURN  
   END
```

```

C*****
C
C      POROUS WALL BOUNDARY LAYER PROGRAM
C
C      This program calculates the developement of two-dimensional
C      turbulent boundary layers over porous surfaces in pressure
C      gradients .
C
C      Input variables:
C      -----
C
C      REYNO : the Reynolds number  $U_{ref} \cdot C / \nu$  , where C is a
C              reference length, and  $\nu$  is the viscosity
C      THETA : the momentum thickness,normalized by C
C      X      : the streamwise position, normalized by C
C      DELTA : the boundary layer thickness,normalized by C
C      CFO    : an initial guess of the skin-friction coefficient
C      U      : the non-dimensional velocity at the outer edge of
C              the boundary layer,normalized by UREF.
C
C      The calculations and output can be done in dimensioned form
C      simply by choosing C=unit length , and UREF=unit velocity.
C
C*****

```

PROGRAM MAIN

```

REAL U(100),UREF,HCONV(100),THETA(100),THETAN
REAL X,DELS,HDELS,DU,DX,CFI,VIS
REAL C,REYNO,CFN
REAL THETAO,HDELSON,DXP,HCONVN,HDELSN,DELSN
INTEGER I,N,NM1

```

```

DIMENSION X(100),DELS(100),HDELS(100),DU(100
1),DX(100),CFI(100)

```

```

CHARACTER*30 OUTFLE,INFILE

```

```

REAL CFO,Vwp,HCONVO,DELSON,U1,U2,DUP

```

REAL CFOO
EXTERNAL CFOO

```

WRITE(*,'(//////////)')
WRITE(*,*) 'PLEASE ENTER THE OUTPUT FILE NAME'
READ(*,11) OUTFLE
11  FORMAT(A30)
    OPEN(UNIT=3,FILE=OUTFLE,ERR=31,STATUS='NEW')
    REWIND(3)
    GOTO 32
31  OPEN(UNIT=3,FILE=OUTFLE,STATUS='OLD')
    REWIND(3)
32  WRITE(*,*) 'PLEASE ENTER THE INPUT FILE NAME'
    READ(*,35) INFILE
35  FORMAT(A30)
    OPEN(UNIT=1,FILE=INFILE,ERR=34,STATUS='OLD')
    REWIND(1)
    GOTO 36
34  WRITE(*,*)
    WRITE(*,*) '*** INPUT FILE NOT FOUND, TRY AGAIN. ***'
    WRITE(*,*)
    GOTO 32

C
C   Open output file that will be used to make graph
C
36  READ (1,*) N
    DO 88 I=1,N
        READ(1,*) X(I), U(I)
88  CONTINUE
    READ(1,*) C,UREF,HCONV(1),THETA(1),CFO,REYNO
101  CONTINUE
C
C   C is the reference length
C
    VIS=1.0/REYNO

    WRITE (6,*)
    WRITE (6,*) ' The normal velocity at the wall has to be '
    WRITE (6,*) ' enter at this point (reminder : Vw+<0 suction,'
    WRITE (6,*) ' Vw+>0 blowing) ( e.g. 0.06<Vw+<-0.06 )'
    WRITE (6,*)
    READ (5,*)Vwp

    WRITE(*,*)

```

```

WRITE (*,22)
WRITE (3,22)
22  FORMAT('*** POROUS WALL BOUNDARY LAYER CALCULATION')
WRITE (3,33) C,UREF,Vwp
WRITE (*,33) C,UREF,Vwp
33  FORMAT('  C=',F8.2,'  UREF=',F5.1,'  Vw+= ',F6.4)
WRITE (3,97) REYNO
WRITE (*,97) REYNO
97  FORMAT ('REYNOLDS NUMBER =',F16.0)

```

NM1 = N-1

```

C
C  N is the number of data points
C  to calculate the values of DU and DX
C
DO 60 I=1,NM1
  DU(I)=(U(I+1)-U(I))/(X(I+1)-X(I))
  DX(I)=X(I+1)-X(I)
60  CONTINUE
C
C  To calculate the initial value of H delta-deltastar, equal to
C  G[Hconv]
C
IF (HCONV(1).GT.1.6) GO TO 61
HDELS(1)=3.3+EXP(0.4667-2.722*LOG(HCONV(1)-0.6798))
GO TO 62
61  HDELS(1)=3.3+EXP(.4383-3.064*LOG(HCONV(1)-0.6798))
62  CONTINUE

```

DELS(1)=HDELS(1)*THETA(1)

```

C
C  to find Cf (skin friction coefficient) for the first data point
C
CFI(1)= CFOO(CFO,HCONV(1),THETA(1),VIS,U(1),DU(1),Vwp,HDELS(1))
c  write(6,*)' CFI(1) = ',CFI(1)
DO 200 I=1,N

```

IF(I.EQ.N) GO TO 200

```

C
C  The 'old' or initial values to be used to find the values
C  at the next station are taken as the values at the present
C  station
C

```

```

CFO=CFI(I)
HCONVO=HCONV(I)
THETAO=THETA(I)
U1=U(I)
U2=U(I+1)
HDELISO=HDELS(I)
DELISO=DELS(I)
DUP=DU(I)
DXP=DX(I)

```

```

C
C   The subroutine RUNGEK is called to integrate the differential
C   equations to find values for Hconv, theta, delta-deltastar,
C   H delta-delatstar, and Cf, at the next station.
C

```

```

IF (HCONVO.GT.3.0) GO TO 736

```

```

CALL RUNGEK(CFO,HCONVO,THETAO,VIS,U1,U2,HDELISO,
1 DELSO,DUP,HCONVN,THETAN,DELSN,HDELSN,CFN,DXP,Vwp)

```

```

c   write(6,*)' Cfi('i,') = ',CFI(i)
c   write(6,*)' HDELS = ',HDELS(I)
c   write(6,*)

```

```

C
C   updating of initial values
C

```

```

736  HCONVO=HCONVN
    THETAO=THETAN
    HDELISO=HDELSN
    DELISO=DELSN
    HCONV(I+1)=HCONVN
    THETA(I+1)=THETAN
    DELS(I+1)=DELSN
    HDELS(I+1)=HDELSN
    CFI(I+1)=CFN

```

```

200  CONTINUE

```

```

C
C   The subroutine OUTPUT is called to print the calculated values
C

```

```

CALL OUTPUT (N,X,U,HCONV,THETA,DELS,CFI,HDELS,VIS,Vwp)
CONTINUE

```

```

STOP
END

```



```

C*****
C
C      FUNCTION SUBROUTINE CFOO
C
C      This function returns the value of Cf, which is obtained
C      through a simple relaxation method ( CFO=(Cfnew+Cfold)/2.0).
C
C*****

```

```

      REAL FUNCTION CFOO(CFO,HCONVO,THETAO,VIS,UP,DUP,Vwp,HDELSO)

      REAL VIS,CFO,Vwp,HCONVO,THETAO,UP,DUP,CFN,HDELSO
      REAL FNOO
      EXTERNAL FNOO

10   CFN = FNOO(CFO,HCONVO,THETAO,VIS,UP,DUP,Vwp,HDELSO)

      IF(ABS(CFN-CFO).LT.0.000001) GO TO 20
c    write(6,*)' CFO = ',CFO,' CFN = ',CFN
c    CFO = (CFN+CFO)/2.0
      CFO = CFN
      GO TO 10

20   CFOO = CFN
      RETURN
      END

```

```

C*****
C
C      FUNCTION SUBROUTINE FNOO
C
C      This function returns the value of Cf for the input values
C      of the Cf guess,Hconv,theta,Vw+.
C
C*****

```

```

      REAL FUNCTION FNOO(CFO,HCONVO,THETAO,VIS,UP,DUP,Vwp,HDELSO)

      REAL VIS,CFO,Vwp,HCONVO,THETAO,UP,DUP,HDELSO
      REAL PIE,LAMBDA,DELTA,K1,K2,K3

      REAL FN1
      EXTERNAL FN1

```

```

      PIE = FN1(DUP,HCONVO,THETAO,UP,CFO)
c      write(6,*)' PIE = ',PIE
      LAMBDA = SQRT(2.0/CFO)

      DELTA = (HDELISO+HCONVO)*THETAO           ! Using Head Shape factor
c      DELTA = THETAO*HCONVO*0.41*LAMBDA/(1+PIE) ! F. White eqn (6-48)

      K1=(ALOG(UP*DELTA/VIS)+ALOG(1.0/LAMBDA))/0.41+5.0
      K2=1.0-0.25*(LAMBDA*Vwp)
      K3 = K1/K2+2.0*PIE/0.41

      FNOO = 2.0/(K3*K3)

      RETURN
      END

```

```

C*****
C
C      FUNCTION SUBROUTINE FN1
C
C      This function returns the value of PIE using Clauser's
C      parameter BETA and Das (1987) polynomial correlation. It (PIE)
C      represents Coles wake parameter used in the skin-friction law
C      for porous surface.
C
C*****

```

```

      REAL FUNCTION FN1(DUP,HCONVO,THETAO,UP,CFO)

```

```

      REAL HCONVO,THETAO,DUP,UP,CFO
      REAL BETA

```

```

C
C      Clauser's parameter
C
      BETA = -2.0*HCONVO*THETAO*DUP/(UP*CFO)
C
C      Das polynomial correlation
C
c      write(6,*)' u = ',UP
c      write(6,*)' du = ',DUP
c      write(6,*)' DEIS = ',DELISO
c      write(6,*)' beta = ',BETA

```

```

IF(BETA.LT.-0.4)BETA=-0.4
IF(BETA.LT.-0.4)
+ WRITE(6,*)'Beta is out of range, Beta =',Beta,'< -0.4'
FN1= (-0.76+SQRT(0.76**2-4.0*0.42*(-BETA-0.4)))/(2.0*0.42)

RETURN
END

```

```

C*****
C
C      FUNCTION SUBROUTINE FN2
C
C      This funtion returns the value of F(H delta-deltastar), equation
C      A1 of Dvorak's paper modified to sustain a change in the
C      entrainment rate due to suction (Vw+<0) or blowing (Vw+>0)
C      in the porous wall.
C
C*****

```

```

REAL FUNCTION FN2(HDELS,Vwp,CFO,Up)
REAL HDELS,Vwp,CFO,Up

FN2=EXP(-3.512-0.617*LOG(HDELS-3.0))+Vwp*SQRT(CFO/2.0)*Up
RETURN
END

```

```

C*****
C
C      FUNTION SUBROUTINE FN3
C
C      This function returns the value of Hconv, given the input
C      value of H delta-deltastar, based on equations A2 and A3 of
C      Dvorak's paper.
C
C*****

```

```

REAL FUNCTION FN3(HDELS)
REAL HDELS

IF (HDELS.LE.3.305) HDELS=3.305

```

```

      IF(HDELS.GT.5.30) GO TO 10
      FN3=0.6798+EXP((0.4383-LOG(HDELS-3.3))/3.064)
      RETURN
10   FN3=0.6798+EXP((0.4667-LOG(HDELS-3.3))/2.722)
      RETURN
      END

```

```

C*****
C
C      SUBROUTINE RUNGEK
C
C      This subroutine contains the Runge-Kutta integration of the
C      momentum integral equation and Head's entrainment equation
C      to get the values of Hconv,theta,delta-deltastar,Hdelta-
C      deltastar, and Cf at the next station.
C
C*****

```

```

      SUBROUTINE RUNGEK(CFO,HCONVO,THETAO,VIS,U1,U2,HDELSON,
      IDELSON,DU,HCONVN,THETAN,DELSN,HDELSN,CFN,DX,Vwp)

```

```

      REAL CFO,HCONVO,THETAO,VIS,U1,U2,HDELSON,DELSN,DU,HCONVN
      REAL THETAN,DELSN,HDELSN,CFN,DX,UI,DTHETO,Vwp,DDELSON
      REAL HDELS1,DELS1,THETA1,HCONV1,CF1,THETA2,DTHET1,DELS2
      REAL HCONV2,CF2,THETA3,DTHET2,DELS3,DDELS2,HCONV3,CF3
      REAL DDELS1,HDELS2,HDELS3,DTHET3,DDELS3

```

```

      REAL FN2,FN3,CFOO
      EXTERNAL FN2,FN3,CFOO

```

```

      UI=(U1+U2)/2.0

```

```

      DTHETO=(CFO/2.0-(HCONVO+2.0)*(THETAO/UI)*DU+Vwp*SQRT(CFO/2.0))*DX
      DDELSON=(FN2(HDELSON,Vwp,CFO,UI)-(DELSN*DU/UI))*DX

```

```

      THETA1=THETAO+0.50*DTHETO
      DELS1=DELSN+0.50*DDELSON
      HDELS1=DELS1/THETA1

```

```

c   write(6,*)' HDELS1 = ',HDELS1,' THETA1 = ',THETA1,' DELS1 = ',DELS1

```

```

c   write(6,*)

```

```

      HCONV1=FN3(HDELS1)

```

```

      IF(HCONV1.GE.4.50) HCONV1=4.5

```

```

      CF1=CFOO(CFO,HCONV1,THETA1,VIS,U1,DU,Vwp,HDELS1)

```

```

      DTHET1=(CF1/2.0-(HCONV1+2.0)*(THETA1/UI)*DU+Vwp*SQRT(CFO/2.0))*DX

```

```

DDELS1=(FN2(HDELS1,Vwp,CFO,UI)-(DELS1*DU/UI))*DX

THETA2=THETA0+0.5*DTHET1
DELS2=DELSON+0.5*DDELS1
HDELS2=DELS2/THETA2
c  write(6,*) ' HDELS2 = ',HDELS2,' THETA2 = ',THETA2,' DELS2 = ',DELS2
c  write(6,*)
HCONV2=FN3(HDELS2)
IF(HCONV2.GE.4.50) HCONV2=4.5

CF2=CFOO(CF1,HCONV2,THETA2,VIS,UI,DU,Vwp,HDELS2)
DTHET2=(CF2/2.0-(HCONV2+2.0)*(THETA2/UI)*DU+Vwp*SQRT(CFO/2.0))*DX
DDELS2=(FN2(HDELS2,Vwp,CFO,UI)-(DELS2*DU/UI))*DX

THETA3=THETA0+DTHET2
DELS3=DELSON+DDELS2
HDELS3=DELS3/THETA3
c  write(6,*) ' HDELS3 = ',HDELS3,' THETA3 = ',THETA3,' DELS3 = ',DELS3
c  write(6,*)
HCONV3=FN3(HDELS3)
IF(HCONV3.GE.4.50) HCONV3=4.5
CF3=CFOO(CF2,HCONV3,THETA3,VIS,U2,DU,Vwp,HDELS3)
DTHET3=(CF3/2.0-(HCONV3+2.0)*(THETA3/U2)*DU+Vwp*SQRT(CFO/2.0))*DX
DDELS3=(FN2(HDELS3,Vwp,CFO,U2)-(DELS3*DU/U2))*DX

THETAN=THETA0+(DTHETO+2.0*DTHET1+2.0*DTHET2+DTHET3)/6.0
DELSN=DELSON+(DDELSON+2.0*DDELS1+2.0*DDELS2+DDELS3)/6.0
HDELSN=DELSN/THETAN
c  write(6,*) ' HDELSn = ',HDELSn,' THETAN = ',THETAN,' DELSN = ',DELSN
c  write(6,*)
HCONVN=FN3(HDELSN)
IF(HCONVN.GE.4.50) HCONVN=4.5
CFN=CFOO(CF3,HCONVN,THETAN,VIS,U2,DU,Vwp,HDELSN)

RETURN
END

```

```

C*****
C
C      SUBROUTINE OUTPUT
C
C      This subroutine controls the output of the calculated values
C

```

C*****

SUBROUTINE OUTPUT (N,X,U,HCONV,THETA,DELS,CFI,HDELS,VIS,Vwp)

INTEGER I,N,J

REAL X,U,HCONV,THETA,DELS,CFI,HDELS,DELTA,DELTASTAR,VIS,Vwp

DIMENSION X(100),U(100),HCONV(100),THETA(100),

1 DELS(100), CFI(100),DELTASTAR(100),DELTA(100),HDELS(100)

open (unit=20)

open (unit=21)

write(20,*)'# File illustrating the law of the wall. '

c write(21,*)'# File illustrating the law of the wall. '

c write(21,*)'# Using the correlation of Stevenson, 1963'

c write(21,*)'# Y+ U+ Vw+ '

WRITE(3,*)

WRITE(*,*)

WRITE(*,5)

WRITE(3,5)

WRITE (*,*) ' X/C U/Uref Hconv THETA/C DELS/C DELTA/

1C CF Vw '

WRITE(3,*) ' X/C U/Uref Hconv THETA/C DELS/C DELTA/

1C CF Vw '

J=0

DO 10 I=1,N

IF(HCONV(I).GT.3.0.AND.J.EQ.0) THEN

WRITE (3,3)

WRITE (*,3)

J=1

END IF

3 FORMAT(' ***THE FLOW HAS SEPARATED *** ')

DELTA(I)=THETA(I)*(HCONV(I)+HDELS(I))

DELTASTAR(I)=DELTA(I)-DELS(I)

WRITE (*,2) X(I),U(I),HCONV(I),THETA(I),DELTASTAR(I),DELTA(I),

1 CFI(I),sqrt(CFI(I)/2.)*Vwp*U(I)

WRITE (3,2) X(I),U(I),HCONV(I),THETA(I),DELTASTAR(I),DELTA(I),

1 CFI(I),sqrt(CFI(I)/2.)*Vwp*U(I)

write(20,*)DELTA(I)*U(I)/(VIS*sqrt(2.0/CFI(I))),sqrt(2.0/CFI(I))

c write(21,*)DELTA(I)*U(I)/(VIS*sqrt(2.0/CFI(I))),sqrt(2.0/CFI(I))

10 CONTINUE

WRITE(3,4)

```
WRITE(*,4)
4  FORMAT(/)
5  FORMAT(/)
2  FORMAT(/,F8.4,F9.5,F9.4,F9.5,F9.4,F9.4,2x,E10.5,2x,F9.4)
```

```
close (unit=20)
close (unit=21)
```

```
RETURN
END
```

Appendix B

Sample Input/Output

Input file for Invairf.f

(Input file, Fig. 3.4 case)

6	1.23024		# of element, and total chord length
0.0	0.00	-0.10	Angle and coord. of 1 st element
-35.0000	0.98800	-0.12211	" " " 2 nd element
0.0000	0.00000	0.90000	
35.0000	0.98800	0.92211	
0.0000	0.00000	-0.70000	
35.0000	0.98800	-0.67789	" " " 6 th element
76	1.00000		# of junction points and chord length of 1 st element
1.00000	-0.0007		junction points coordinates
0.95000	0.0129		
.			
.			
.			
0.97500	0.0010		
1.00000	-0.0044		
32	1.00000		# of junction points and chord length of 2 nd element
0.30000	-0.00612		junction points coordinates
0.27500	0.00100		
.			
.			
.			
0.27500	-0.00400		
0.30000	-0.00612		
76	1.00000		# of junction points and chord length of 3 rd element
1.00000	0.0007		junction points coordinates
0.95000	-0.0129		
.			
.			
.			

Output file from Invairf.f

(output file, Fig 3.4 case)

The section name is: GA(W)-1

The chord length is: 1.23024

The Kutta condition in this computation ;
equal velocities at trailing edge

Lift coefficient based on circulation = -2.83474

Angle of attack (degrees) = 0.000

Junction point coordinates X, Y, and pressure
coefficients based on vortex strengths at junction points.

Main element

(lift coefficient based on circulation' = 3.45534)

	X/C	Y/C	CP
1	1.00000	-0.00070	-1.70588
2	0.95000	0.01290	-1.63161
3	0.92500	0.01960	-1.64389
4	0.90000	0.02640	-1.72241
5	0.87500	0.03320	-1.82668
6	0.85000	0.03990	-1.92848
7	0.82500	0.04650	-2.05439
8	0.80000	0.05290	-2.17027
9	0.77500	0.05910	-2.29145
17	0.57500	0.09670	-3.34322
.	.	.	.
.	.	.	.
.	.	.	.
70	0.85000	0.01810	0.89871
71	0.87500	0.01670	0.88778
72	0.90000	0.01330	0.83502
73	0.92500	0.00950	0.74308
74	0.95000	0.00530	0.49468
75	0.97500	0.00100	-0.23290
76	1.00000	-0.00440	-1.70588

Element # 2
 (lift coefficient based on circulation = 0.81806)

	X/C	Y/C	CP
77	0.30000	-0.00612	0.16795
78	0.27500	0.00100	0.29553
79	0.25000	0.00530	0.14265
80	0.22500	0.00950	-0.08335
81	0.20000	0.01330	-0.34124
82	0.17500	0.01650	-0.67941
83	0.15000	0.01840	-0.94134
84	0.12500	0.01950	-1.37081
85	0.10000	0.01900	-1.81103
.	.	.	.
.	.	.	.
.	.	.	.
103	0.17500	-0.00580	0.94854
104	0.20000	-0.00360	0.93882
105	0.22500	-0.00250	0.92455
106	0.25000	-0.00260	0.90092
107	0.27500	-0.00400	0.86172
108	0.30000	-0.00612	0.16785

NORMAL VELOCITY RATIO AT CONTROL POINTS

0.808E-06	0.102E-05	0.224E-05	0.104E-05	-0.152E-05
-0.179E-05	-0.122E-05	-0.145E-05	-0.337E-06	0.177E-05
-0.115E-05	0.102E-05	-0.317E-08	-0.176E-05	0.114E-05
0.158E-05	0.206E-06	-0.779E-06	0.113E-05	0.221E-05
-0.528E-06	0.177E-06	0.258E-05	0.291E-05	0.273E-05
0.265E-05	0.110E-05	-0.361E-05	-0.148E-05	-0.165E-05
-0.178E-05	0.108E-05	0.234E-05	-0.419E-05	0.936E-05
0.487E-06	0.314E-06	0.476E-05	-0.215E-05	-0.159E-05
0.639E-06	0.200E-05	0.289E-06	-0.240E-05	-0.210E-06
0.133E-05	-0.383E-06	-0.288E-06	-0.544E-06	0.186E-05
-0.102E-05	0.325E-06	-0.403E-06	-0.193E-05	-0.188E-05
-0.188E-05	-0.257E-05	-0.851E-06	-0.203E-05	-0.699E-06
0.126E-05	-0.716E-08	-0.530E-06	-0.421E-06	-0.148E-05
-0.130E-05	0.736E-06	0.282E-05	-0.205E-06	0.145E-05
-0.527E-06	-0.357E-06	-0.197E-05	-0.175E-05	-0.149E-06
0.368E-05	0.109E-04	0.446E-05	-0.208E-05	0.300E-05
0.163E-06	-0.187E-05	0.283E-05	0.321E-05	0.430E-05
0.276E-05	-0.604E-05	0.499E-06	0.840E-06	0.204E-05
-0.529E-05	-0.526E-06	-0.618E-05	-0.229E-05	0.523E-05
-0.214E-05	-0.235E-05	0.571E-06	-0.700E-05	-0.167E-05

INTERNAL TANGENTIAL VELOCITY RATIO AT CONTROL POINTS

0.737E-02	0.391E-03	-0.175E-02	-0.127E-02	-0.719E-03
-0.388E-04	0.481E-03	0.618E-03	0.766E-03	0.202E-02
0.269E-02	0.340E-02	0.446E-02	0.442E-02	0.487E-02
0.518E-02	0.319E-02	0.938E-02	0.809E-02	0.722E-02
0.722E-02	0.778E-02	0.864E-02	0.112E-01	0.414E-02
0.597E-02	0.704E-02	0.115E-01	0.164E-01	0.296E-01
0.136E-01	0.284E-01	0.540E-01	0.573E-01	0.175E-01
0.705E-01	0.570E-01	0.912E-02	0.236E-01	0.165E-01
0.945E-03	-0.410E-03	0.333E-03	-0.118E-02	-0.131E-02
-0.137E-02	-0.122E-02	-0.107E-02	-0.216E-02	-0.206E-02
-0.182E-02	-0.185E-02	-0.189E-02	-0.206E-02	-0.300E-02
-0.897E-03	-0.109E-02	-0.171E-02	-0.481E-02	-0.869E-02
-0.259E-02	-0.922E-02	-0.433E-02	0.846E-02	0.702E-02
0.363E-02	0.353E-02	0.354E-02	0.192E-02	0.298E-02
0.295E-02	0.116E-02	-0.280E-02	-0.988E-02	0.363E-01
0.189E+00	0.172E-01	0.106E-01	0.913E-02	0.107E-01
0.995E-02	0.113E-01	0.165E-01	0.202E-01	0.223E-01
0.352E-01	0.178E-01	0.345E-01	0.369E-01	0.133E+00
-0.162E+01	0.425E-01	-0.840E+00	0.209E-01	0.119E-01

Input for Invairf.f

(Input file containing points where the velocity is required)

72**# of points****Coordinates in the global axis system and it orientation****-10 -0.4 0.0****-9 -0.4 0.0****-8 -0.4 0.0****-7 -0.4 0.0****-6 -0.4 0.0****-5 -0.4 0.0****-4 -0.4 0.0****-3 -0.4 0.0****-2 -0.4 0.0****2.25 -0.4 0.0****2.50 -0.4 0.0****2.75 -0.4 0.0****3.00 -0.4 0.0****4.00 -0.4 0.0****5.00 -0.4 0.0****6.00 -0.4 0.0****7.00 -0.4 0.0****8.00 -0.4 0.0****9.00 -0.4 0.0****10.00 -0.4 0.0**

Output file from Invairf.f

(Output velocity on the wind tunnel floor, Fig. 3.4)

The results velocities for the file called floor.dat

X=-10.00000	Y= -0.400	VELN=-.02099	VELT=1.00125
X= -9.00000	Y= -0.400	VELN=-.02305	VELT=1.00166
X= -8.00000	Y= -0.400	VELN=-.02568	VELT=1.00207
X= -7.00000	Y= -0.400	VELN=-.02891	VELT=1.00255
X= -6.00000	Y= -0.400	VELN=-.03296	VELT=1.00337
X= -5.00000	Y= -0.400	VELN=-.03824	VELT=1.00458
X= -4.00000	Y= -0.400	VELN=-.04536	VELT=1.00646
X= -3.00000	Y= -0.400	VELN=-.05487	VELT=1.00940
X= -2.00000	Y= -0.400	VELN=-.06656	VELT=1.01343
X= -1.00000	Y= -0.400	VELN=-.06766	VELT=0.99955
X= -0.90000	Y= -0.400	VELN=-.06493	VELT=0.99196
X= -0.70000	Y= -0.400	VELN=-.05600	VELT=0.96509
X= -0.50000	Y= -0.400	VELN=-.04099	VELT=0.90896
X= -0.30000	Y= -0.400	VELN=-.01958	VELT=0.79329
X= -0.20000	Y= -0.400	VELN=-.00814	VELT=0.70155
X= -0.10000	Y= -0.400	VELN=0.00100	VELT=0.59230
X= 0.00000	Y= -0.400	VELN=0.00537	VELT=0.49286
X= 0.10000	Y= -0.400	VELN=0.00522	VELT=0.43472
X= 0.20000	Y= -0.400	VELN=0.00336	VELT=0.41586
X= 0.40000	Y= -0.400	VELN=0.00049	VELT=0.40973
X= 0.60000	Y= -0.400	VELN=-.00159	VELT=0.37905
X= 0.80000	Y= -0.400	VELN=-.00449	VELT=0.31298
X= 1.00000	Y= -0.400	VELN=-.00504	VELT=0.25687
X= 1.25000	Y= -0.400	VELN=0.00408	VELT=0.50535
X= 1.50000	Y= -0.400	VELN=0.03275	VELT=0.86041
X= 2.00000	Y= -0.400	VELN=0.06473	VELT=0.99900
X= 3.00000	Y= -0.400	VELN=0.06725	VELT=1.01602
X= 5.00000	Y= -0.400	VELN=0.04619	VELT=1.00721
X= 6.00000	Y= -0.400	VELN=0.03893	VELT=1.00501
X= 8.00000	Y= -0.400	VELN=0.02932	VELT=1.00279
X= 10.00000	Y= -0.400	VELN=0.02344	VELT=1.00179

Input file for PWBL.f**(Hypothetical case with favourable pressure gradient)**

5
0.1776 1.0
0.3680 1.008
0.5601 1.014
0.7514 1.020
0.9426 1.035
1.830 50.0 1.38 0.000694 0.0030 5500000

# of stations**non dim: X(i) U(I)**

C, Uref, Hconv, ND Theta, Cf, REYNO
^Z

Output file from PWBL.f**(Results of input file previously mentioned)******* POROUS WALL BOUNDARY LAYER CALCULATION****C= 1.83   UREF= 50.0   Vw= 0.0000**
REYNOLDS NUMBER = 5500000.

X/C	U/Uref	Hconv	THETA/C	DELS/C	DELTA/C	CF	Vw+
0.1776	1.00000	1.3800	0.00069	0.0010	0.0062	.33042E-02	0.0000
0.3680	1.00800	1.3758	0.00098	0.0013	0.0087	.31128E-02	0.0000
0.5601	1.01400	1.3694	0.00125	0.0017	0.0113	.29742E-02	0.0000
0.7514	1.02000	1.3626	0.00150	0.0020	0.0137	.28804E-02	0.0000
0.9426	1.03500	1.3500	0.00170	0.0023	0.0159	.28725E-02	0.0000

Appendix C

Other Design Computations

Force acting on the airfoil

Lift force (Main element)

- From code if $\alpha = 0^\circ$, $C_l = 3.455$ and if $\alpha = 5^\circ$
 $C_l = 4.714$

Looking for the worse case, $C_l = 4.714$

- From code $C_l = \frac{L'}{\frac{1}{2} \rho U_\infty^2 C}$, where $U_\infty = 1.0$
 $C = 1.0$
and $\rho = 1.0$

In the wind tunnel, being conservative $U_\infty = 250 \frac{\text{ft}}{\text{sec}}$ from design $C = 25$ in. and since incompressible flow, ρ is assumed ρ_{air} at Temp = 15° and Patm.

$$\rho = 0.00264 \frac{\text{slug}}{\text{ft}^3}$$

$$C_l = 4.75 = \frac{L'}{\frac{1}{2} \rho U_\infty^2 C} = \frac{L'_w}{\frac{1}{2} \rho U_w^2 C_w}$$

$$L'_w = 4.75 \times \frac{1}{2} \times 0.00264 \frac{\text{slug}}{\text{ft}^3} \times 250^2 \left(\frac{\text{ft}}{\text{sec}} \right)^2 \times \frac{25 \text{ in}}{12}$$

$$L'_w = 816.4 \frac{\text{lb}}{\text{ft}}$$

Using the same procedure for the flap
(from code $C_i = 0.85$)

$$L'_{wf} = 0.85 \times \frac{1}{2} \times 0.00264 \frac{\text{slug}}{\text{ft}^3} \times 250^2 \left(\frac{\text{ft}}{\text{s}}\right)^2 \times \frac{25 \text{ ft}}{12}$$

$$L'_{wf} = 292.2 \text{ lb/ft}$$

Note : - all dimension are nondimensional by the main element chord length.

For airfoil $C_i/C_o > 20$, so for preliminary calculation, C_o has been neglect so then C_m .

Location of L' on the airfoil is commonly place at $C/4$ from the leading edge. But since C_o and C_m are neglect, the C_p distribution is used to determinate the location of L' .

By inspection of graphs, L' is located at $x = 0.4c$

Then for the wind tunnel airfoil 10 inches from the leading edge.

- Effect on the T.E (trailing edge)

Flexural strength is found using the C_p distribution around the airfoil.

$$P_L = S(Y_1 + Y_2) = 2T \text{ where } Y_i = \sqrt{1 - C_p}$$

Thus,

$$C_L = 5 (\sqrt{1 - C_{p1}} + \sqrt{1 - C_{p2}})$$

It was decided to study 3 cases on the main element at $x/c = 0.85$, $x/c = 0.80$ and $x/c = 0.70$.

$$\begin{array}{lll} x/c = 0.85 \text{ gives a } C_L \approx 0.420 & L' = 144.4 \text{ lb/ft} \\ x/c = 0.80 \text{ gives a } C_L \approx 0.59 & L' = 206.25 \text{ lb/ft} \\ x/c = 0.70 \text{ gives a } C_L \approx 0.972 \approx 1 & L' = 343.75 \text{ lb/ft} \end{array}$$

By inspection of the C_p vs x graph, it has been decided to locate the force at mid distance between the inspection face and the trailing edge.

Note $\Delta C_L \approx \Delta C_p \times \Delta x$, the previous value can be verified with the C_p distribution, and these values are smaller than the ^{one} obtain doing an integration.

Case $x/c = 0.85$ or $x = 21.25''$

$$M_0 - L' \times (1.875/12) = 0 \Rightarrow M_0 = 56.41 \text{ lbf ft}$$

The bending stress $\sigma = \frac{My}{I}$ with I varying with x

$$I = \frac{bh^3}{12}$$

$b = \text{const}$ $h = f(x)$, the inspection at 3 stations is required
 $x = 21.25''$, $x = 22.0''$ and $x = 22.5''$

$$\sigma = \frac{My}{I} = \frac{M \frac{h}{2}}{\frac{bh^3}{12}} = \frac{6M}{bh^2}$$

$$\sigma_{x=21.25} = \frac{6 \times 56.41 \text{ lbf ft}}{\left(\frac{30}{12} \times \left(\frac{0.53}{12}\right)^2\right) \text{ ft}^2} = 69.4 \times 10^3 \text{ psi} \approx 482 \text{ psi}$$

$$\sigma_{x=22} = \frac{6 \times 33.85}{\left(\frac{30}{12} \times \left(\frac{0.45}{12}\right)^2\right)} = 57.76 \text{ psi} \approx 401 \text{ psi}$$

$$\sigma_{x=22.5} < \sigma_{x=21.25}$$

$$\text{Worse case } \sigma_{x=21.25''} = 482 \text{ psi}$$

- The shear stress

for rectangular section $\tau_{(xy)_{max}} = \frac{3V}{2bh}$

$$\tau_{max}_{x=21.25} = \frac{3 \times 361 \text{ lb}}{2 \times 30 \times 0.53 \text{ in}} = 34 \text{ psi}$$

$$\tau_{max}_{x=22} = \frac{3 \times 361 \text{ lb}}{2 \times 30 \times 0.45 \text{ in}} = 40 \text{ psi}, \tau_{max}_{x=22} \approx 50 \text{ psi}$$

Combining stress and even with maximal stress theory (Mohr circles), the σ_{max} is well below S_y of model board materials.

Looking for deflection, an approximation is obtained assuming the slope constant.

$$\delta = \frac{+L}{EI} [3ax^2 - x^3 + (x-a)^3] \text{ where } x = 3.75 \text{ in}, a = 1.875 \text{ in}, L = 361 \text{ lb}$$

$$E = 46000 \text{ psi} \quad I = \frac{bh^3}{12} = \frac{30 \times 0.53^3}{12} = 0.37 \text{ in}^4$$

$$\delta = 0.117 \text{ in or } 3 \text{ mm}$$

Too high

Using Aluminum $E = 10.3 \text{ Mpsi}$ and $\delta = 5.2 \times 10^{-4} \text{ in. or } 0.13 \text{ mm}$

Case $x/c = .80$ or $x = 20''$

Same kind of diag. then for $x/c = 0.85$

$$M_o = L \times 2.5/12 = 107.3 \text{ ft ft}$$

As previously done, 2 stations are investigate, $x = 20''$ and $x = 21.5''$

Bending stress

$$\sigma_x = \frac{6M}{bh^2}$$

$$x = 20''$$

$$\sigma_x = \frac{6 \times 107.3}{\left(\frac{30}{12} \times \left(\frac{0.9}{12}\right)^2\right)} = 46 \times 10^3 \text{ psf} \approx 318 \text{ psi}$$

$$x = 21.5''$$

$$\sigma_x = 79 \times 10^3 \text{ psf} \approx 550 \text{ psi}$$

Shear stress

$$\tau_{xy, \max} = \frac{3V}{2bh}$$

$$x = 20''$$

$$\tau_{xy} = \frac{3 \times 515}{2 \times 30 \times 0.9} \approx 28.1 \text{ psi}$$

$$x = 21.5''$$

$$\tau_{xy} = \frac{3 \times 515}{2 \times 30 \times 0.53} \approx 49 \text{ psi}$$

Combining both stresses, the results is still quite small compare to the tensile strength of Pen-Shape $S_u = 2660 \text{ psi}$. The deflection will be a better criteria in this case too.

- Deflection at the trailing edge

$$v = \frac{L}{6EI} [3ax^2 - x^3 + (x-a)^3]$$

$$\text{where } a = 2.5 \text{ in}$$

$$x = 5 \text{ in}$$

$$E = 46000 \text{ psi}$$

$$I = 187 \text{ in}^4$$

$$v = 0.08 \text{ in} \approx 2 \text{ mm}$$

if h is evaluate at $x = 21.5$ ($h = 0.53 \text{ in}$)

$$v = 0.394 \text{ in} \approx 10 \text{ mm} \quad \text{impossible (to break before)}$$

Using Aluminum the deflection is reduce to:

$$v = 3.5 \times 10^{-4} \text{ in} \approx 0.009 \text{ mm}$$

Case $\frac{x}{c} = 0.70$ or $x = 17.5$ in

At this station h is much bigger, reducing chance of breaking and deflection.

From previous analysis the airfoil trailing must be in Aluminium

Bolts Computation

Approximation of airfoil weight

Rough estimate of airfoil volume.

$$Vol = 30 \text{ in} \times (16 \times 4 + 2 \times 4 - 1 \times 8) = 1920 \text{ in}^3$$

$$= 31, \times 10^4 \text{ cm}^3 \quad \begin{array}{l} \text{estimate} \\ \text{of T.E.} \end{array} \quad \begin{array}{l} \text{hole in} \\ \text{airfoil} \end{array}$$

Using model board (ben-Shape) product ($S = 0.669 \frac{\text{kg}}{\text{in}^3}$)

$$Mass = S \times Vol = 20.77 \text{ kg} \quad (46 \text{ lb})$$

$$W_a = 46 \text{ lb} \approx 207.7 \text{ N}$$

- Using Aluminium ($\rho = 2.7 \text{ g/cm}^3$)

$$\text{Mass} = 85 \text{ kg}$$

$$W_a = 188 \text{ lb} \approx 850 \text{ N}$$

Note - Over four times the model board weight.

The gravity center has been determined at 10 in. from the pivot bolt

The side plates and the flap must also be considered.

- Weight of side plates

$$\text{Vol} = \frac{1}{4}'' \times 10'' \times 32 = 80 \text{ in}^3 \approx 1311 \text{ cm}^3$$

for Aluminium ($\rho = 2.7 \text{ g/cm}^3$)

$$\text{Mass} = 2.7 \text{ g/cm}^3 \times 1311 \text{ cm}^3 = 3.54 \text{ kg}$$

$$W_a = 7.8 \text{ lb}$$

for Carbon Steel ($\rho = 7.86 \text{ g/cm}^3$)

$$\text{ratio of Density } \rho_{sc}/\rho_{al} \approx 3$$

Therefore it is advantageous to use Aluminium as material for the side plates.

Weight of the flap

Flap volume appr.

$$Vol = 30" \times 1" \times 5" = 150 \text{ in}^3 = 2460 \text{ cm}^3$$

using Aluminium (Density of I. E.)

$$Mass = 2.7 \frac{\text{g}}{\text{cm}^3} \times 2460 \text{ cm}^3 = 6.6 \text{ kg}$$

$$W_a = 15 \text{ lb}$$

Design of Holding Bolt

where subscript a is for main airfoil, p is for side plates and f for flap.

In the analysis the bolts will have to support 60% of the weight (instead of 50%) to compensate for any asymmetry possible.

- Case with no lift

$$\sum F_x = 0 \rightarrow R_{1x} = 0 \quad R_{2x} = 0 \quad C_0 \text{ neglected}$$

$$\sum F_y = 0 \rightarrow R_{1y} + R_{2y} = 252 + (100 + 15) \times 0.6 = 321 \text{ lb}$$

$$\sum M_i = 0 \rightarrow (100 \times 10 + 15 \times 28) \times 0.6 + 252 \times 16 = R_{2y} \times 14$$

$$R_{2y} = 349 \text{ lb} \quad R_{1y} = -28 \text{ lb}$$

- Case with lift

$$\sum F_y = 0 \quad R_{1y} + R_{2y} = 252 + (100 + 15) \times 0.6 - (2040 - 730) \times 0.6$$

$$R_{2y} = -1314 \text{ lb} \quad R_{1y} = -27 \text{ lb}$$

From these two cases

$$R_{1y} = -28 \text{ lb} \quad R_{2y} = -1314 \text{ lb}$$

The bolt #1 (near the leading edge) is a pivot, it cannot sustain any moment. A shear force is acting on that bolt, for that reason A.S.T.M. bolt is recommended, because they design to deal mostly with structure (loaded in shear).

$$Z = F/A < \frac{SP}{SF} \quad \text{where, } SP \text{ is minimum proof strength and } SF \text{ stand for security factor.}$$

$$\frac{28}{\frac{\pi \times .25^2}{4}} = \frac{33 \times 10^3}{S.F.}$$

S.F. = 60 sure enough
with ASTM: A307 $\frac{1}{4}$ "

bolt #2

$$\frac{1314}{\frac{\pi \times 0.25^2}{4}} = \frac{33 \times 10^3}{S.F.}$$

S.F. = 20 with A 307 $\frac{1}{4}$ "

All other bolts have been choose by inspecting other test sections.

The test section itself is only the modification of a previous design.

END

1 6 - 1 1 - 9 5

FIN