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**LA THÈSE A ÉTÉ
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PRELIMINARY TESTING OF A TWISTED DELTA PROPELLER
FOR TRANSONIC CRUISE SPEED

by

MALCOLM D: IMRAY

A thesis submitted to the Faculty of Graduate Studies and
Research in partial fulfillment of the requirements for the
Degree of Master of Engineering.

DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, CANADA

September, 1986

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research, acceptance of the thesis: "Preliminary Testing of a Twisted Delta Propeller for Transonic Cruise Speed", submitted by Imray, M.D., in partial fulfillment of the requirements for the degree of Master of Engineering.

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ABSTRACT

A transonic propeller design proposed by Ribner has been tested at low speed. A simple, low-cost method of propeller construction is evaluated. A rigid foam core was constructed to the correct shape and covered with graphite fibre and epoxy to achieve the necessary strength and rigidity. Measurements consist of balance and wake survey data. Tests of the fixed pitch propeller model for a range of advance ratios revealed design point thrust and power coefficients more than double those predicted. Propulsive efficiency near design point is encouraging. Further development of theory to include three-dimensional effects is necessary to improve predictions.

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LIST OF SYMBOLS.

AR	Aspect Ratio
B	Number of blades
b	Local chord
b_o	Axial length
C_D	Section drag coefficient
C_L	Section lift coefficient
C_p	Power coefficient; Pressure coefficient
C_Q	Torque coefficient
C_T	Thrust coefficient
c	Sonic velocity
D	Tip span (propeller diameter)
D_e	Effective drag
H	Local total pressure coefficient
I(BC)	Rolling moment parameter
J	Advance ratio
k	Camber parameter
L	Measured rolling moment
M	Freestream Mach number
M_n	Incident Mach number normal to leading edge
M_R	Resultant Mach number
$\bar{M}$	Average resultant Mach number
m	Mass flow rate
N	Propeller rotational speed in revolutions per minute.
n	Propeller rotational speed in revolutions per second
P	Pressure, Power
P_D	Power to overcome profile drag
p	Pitch
Q	Torque
Q_o	Torque for uncambered propeller
q	Freestream dynamic pressure
R	Tip half-span (propeller radius)

r	Local half-span
S	Surface Area; Side force
S_{wt}	Weighted surface area
s	Normalized radial position
T	Thrust
T_i	Ideal thrust
T_S	Thrust due to leading edge suction
t	Section semi-thickness
u	Local axial velocity (aft of propeller trailing edge)
V	Flight velocity
V_f	Final average fluid velocity
V_n	Velocity component normal to blade surface; Incident velocity component normal to leading edge
V_R	Resultant velocity
V_T	Wind tunnel freestream velocity
y_v	Local radial velocity (aft of propeller trailing edge)
w	Local swirl velocity (aft of propeller trailing edge)
X	Measured axial force
x	Axial coordinate
Y	Measured side force
y	"Spanwise" coordinate
z	"Thickness" coordinate
α	Angle due to pitch; Local angle of attack
β	Local pitch angle
η	Propulsive efficiency
η_f	Froude efficiency
η_i	Ideal propulsive efficiency
θ	Mean surface helicoid angle
θ_o	Mean surface helicoid angle for uncambered propeller
λ	Sweep angle
ρ	Density
τ	Fraction of radius that is not cambered
ϕ	Local resultant velocity angle
ϕ	Local mean surface slope
ω	Propeller rotational speed in radians per second

CHAPTER I - INTRODUCTION

Thrust for an aircraft is provided by imparting momentum to a quantity of air. The rate of change of fluid momentum by some device on the aircraft is equivalent to the thrust provided. Consider the idealized device shown in Figure 1.1. The Froude efficiency is obtained by considering the momentum equation in one dimension and assuming no pressure gradients exist. The net thrust is simply:

$$T = m(V_f - V) \quad (1.1)$$

The useful output power is the product of thrust and aircraft velocity. The input power is equal to the output power plus that required to accelerate the air to its final velocity. The ratio of output to input power is the ideal propulsive efficiency.

$$\begin{aligned} \eta_f &= \frac{m(V_f - V) V}{[m(V_f - V) V + m/2 (V_f + V) (V_f - V)]} \quad (1.2) \\ &= \frac{2}{[1 + V_f/V]} \end{aligned}$$

Note that to achieve a desired value of thrust a quantity of fluid must be accelerated to some final velocity greater than its initial velocity. This can be done by providing a small mass of fluid with a large velocity increase or a large mass of fluid with a small velocity increase. Equation (1.2) illustrates that

Figure 1.1 Idealized Thrust Device

the latter option is desirable for high efficiency.

Large civil and military transport aircraft require engines of high power-to-weight ratio. The gas turbine is the only suitable engine for such an application and has proven successful in many forms. Initially the gas turbine was used to drive a conventional propeller. The propeller, however, does not retain its efficiency at speeds much above Mach 0.5 due to the transonic drag rise experienced along the outer portion of the propeller blades. Turbojets were introduced to reach high subsonic cruise speeds of Mach 0.8 in addition to providing low cabin noise levels.

For long range aircraft and as fuel costs increase, the specific fuel consumption of propulsion systems must be reduced. This is achieved largely through the increase in propulsive efficiency afforded by the bypass or turbofan engine. The large fan accelerates a relatively large mass of air through a small velocity increase and provides a moderate pressure increase. Turbofan engines enjoy other advantages such as low near-field (cabin) and far-field (airport environment) noise levels. Furthermore, the fan shroud or duct provides a measure of safety in the event of fan blade loss. The design philosophy of further increasing fan size is limited by other considerations including: difficulty providing reverse thrust; high Mach numbers experienced by the fan blade tips; drag of the large duct surrounding the fan.

Recent research efforts for improving propulsive efficiency

have focused on the development of fans with variable pitch swept blades and no duct. These projects are known variously as prop-fan, unducted fan (UDF), and ultra-bypass engines (UBE) among others [1,2]. The objective is to combine the high propulsive efficiency of a turboprop with the cruise speed of a turbofan. Variable pitch blades provide a mechanism for reverse thrust to reduce landing runs in addition to allowing optimum blade pitch angle to be set for each phase of flight including take-off, climb, cruise, and descent. The blades are swept to reduce the detrimental effects of high tip Mach number at cruise conditions. This geometry introduces large bending moments at the hub due to centrifugal force. Furthermore, the airfoil sections used for the blades are very thin. The combination of such structural problems and the advent of the turbojet engine precluded the development of high speed propellers in the past.

The present drive to improve fuel economy and recent advances in materials technology, particularly in composites, have encouraged NASA, Hamilton Standard, General Electric and others to pursue the swept blade prop-fan concept [2]. The Boeing Commercial Airplane Company proposes building a prop-fan powered airliner to be in service in the 1990's [3]. Despite this, a number of problems remain. The most pressing of these continues to be that of structural integrity. Furthermore, the majority of proposed prop-fan configurations require a reduction gearbox. This requirement may be difficult to meet in terms of size and weight penalties when fulfilling the power requirements. Finally,

there is a two-fold problem with noise. First, cabin noise must be comparable to that of existing turbofan powered designs for passenger acceptance. Second, the far-field noise levels must meet current and proposed U.S. regulations or face restricted operations at many locations around the world.

The subject of this thesis is an alternative proposal for a propeller designed to deliver high efficiency at high subsonic cruise Mach number. The concept is essentially that of a delta wing appropriately twisted and cambered for use as a propeller. It was first proposed by Ribner in 1947 [4]. One of the key features of the twisted delta concept is that virtually all the centrifugal loads are carried radially. Bending moments at the blade root due to centrifugal force are eliminated allowing greater confidence in the integrity of the propeller.

Lacking a satisfactory analytical method for predicting the performance of this propeller, a wind tunnel test program is necessary. The first part of the program is to conduct low speed tests to ascertain aerodynamic behaviour without compressibility effects. The design method described by Ribner in references [4] and [6] are employed and the resulting performance predictions are compared with test results. This provides a basis for future theoretical development and testing. In addition, this project examines the viability of a simple, low-cost construction technique. The method uses a foam core with an epoxy/graphite covering. The capability to accurately produce a complex shape without special tools and jigs is assessed. If successful, such a

method can reduce the cost and time associated with a wind tunnel test program. Description of the design, fabrication, and low speed testing of a twisted delta propeller model constitutes the balance of this thesis.

CHAPTER II - THEORY

Before describing the test of the propeller, it is important to understand the fundamentals for predicting its performance and the limitations of any such predictions. The following sections paraphrase theory development for the twisted delta propeller. Many details are beyond the scope of this work and are not described herein. Such details are available in the references cited.

2.1 Propeller With No Twist

The theory behind the twisted delta propeller is based on the triangular wing theory of Jones [5]. The cornerstone of Jones' theory is that the flow is two-dimensional in planes perpendicular to the direction of flight; the patterns are, of course, influenced by local conditions. Ribner [4] argues that this property is retained even when a long flat triangular airfoil is rotating about its axis of symmetry. A surface velocity potential describing the flow shown in Figure 2.1 at any plane orthogonal to the propeller surface can be written as in equation (2.1).

$$\phi = \pm \frac{1}{2} \omega y \sqrt{r^2 - y^2} \quad (2.1)$$

Figure 2.1 2-D Flow Pattern

Figure 2.2 Spanwise Distribution of Normal Force

The surface pressure distribution of equation (2.2) can be derived from equation (2.1).

$$\frac{\Delta P}{q} = 2\pi \frac{Rx}{b_o^2 J} \cot \eta \quad (2.2)$$

$$\text{where } \cos \eta = \frac{y}{r}$$

Integrating the pressure distribution from leading edge to trailing edge yields the spanwise distribution of normal force described in equation (2.3) and illustrated in Figure 2.2.

$$\frac{dN}{dy} = \rho V \omega y \sqrt{R^2 - y^2} \quad (2.3)$$

Torque is computed simply by integrating the product of spanwise normal force distribution and moment arm with respect to span as in equation (2.4).

$$Q = \int_{-R}^R y \left(\frac{dN}{dy} \right) dy = \frac{\pi}{128} \rho D^4 \omega V \quad (2.4)$$

Since the propeller described to this point is flat, the normal force derived above has no axial component. The only source of thrust must be at the leading edge where, according to the pressure distribution of equation (2.2), there exists infinite negative pressure. In reference [4] Ribner considers a cylinder of elliptical cross section over whose surface the pressure distribution is continuous and may be integrated to compute the suction force per unit length. The minor axis of the ellipse is allowed to approach zero length to obtain the solution for a flat plate. Provided the vertex angle of the triangular planform propeller is sufficiently small, the use of a

cylindrical approximation is valid and is indeed the basis for considering a two dimensional velocity potential in the first instance. The vertex angle must be finite, however, if there is to be a component of the leading edge suction parallel to the axis giving thrust, the magnitude of which may be obtained by integrating along the span.

$$T_i = \frac{\pi}{256} \rho \omega^2 D^4 \quad (2.5)$$

Equation (2.5) gives the ideal thrust due to leading edge suction and does not account for profile drag.

It is important to note here that this theory is restricted to small vertex angle and local angle of attack since it is based on a locally two-dimensional velocity potential and small disturbance, or slender body, theory. In terms of the propeller this places an upper limit on aspect ratio and a lower limit on advance ratio. Ribner prescribes limits of applicability on aspect ratio and propeller advance ratio:

$$AR = \frac{b^2}{S} < \frac{1}{\sqrt{1 - M^2}} \quad (2.6)$$

$$J = \frac{V}{nD} > 6$$

2.2 Propeller With Helical Twist

A substantial increase in thrust produced by the propeller can be realized by making use of the pressure distribution acting normal to its surface. This is achieved by incorporating helical

twist. The amount of twist is restricted by the limitations of the theory discussed in the previous article to preserve locally two-dimensional flow. The twist must be kept small such that the relative rotation of flow patterns between adjacent axial positions is small. In other words, the propeller will be of high pitch. Consider Figure 2.3 in which the spanwise pressure distributions at two axial positions are shown at the same instant in time for a high pitch twisted delta propeller. The two differ in magnitude and angular orientation by amounts proportional to the vertex angle (aspect ratio) and twist (pitch), respectively. Provided these changes are gradual, the flow can be treated by slender body theory.

By considering the pitch geometry and velocity vectors on a typical surface element, Ribner [4] derives an expression for the velocity component normal to the blade surface. This appears in equation (2.7) in terms of the advance ratio and pitch.

$$V_n = \omega y (1 - \frac{J}{P/D}) \quad (2.7)$$

Notice that if the advance ratio and the pitch-diameter ratio are equal, the velocity normal to the propeller surface is zero, corresponding to zero angle of attack. This suggests that, neglecting profile drag, the propeller will neither absorb any torque nor produce any thrust.

For the flat airfoil $V_n = \omega y$. Ribner argues, therefore, that using $\omega(1 - \frac{J}{P/D})$ in place of ω corrects the previously used velocity potential for use with a twisted propeller. The torque obtained

Figure 2.3 Spanwise Pressure Distributions for Two Adjacent Axial Stations

by making this substitution is:

$$Q = \frac{1}{256} \rho \omega^2 D^5 J \left(1 - \frac{J}{P/D}\right) \quad (2.8)$$

Similarly, the thrust due to suction becomes:

$$T_s = \frac{\pi}{256} \rho \omega^2 D^4 \left(1 - \frac{J}{P/D}\right) \quad (2.9)$$

The component of normal force in the axial direction is derived by considering the pressure on a small surface element and the surface geometry associated with the pitch. Substituting the known velocity potential corrected for pitch and then integrating over the surface area yields equation (2.10) for thrust due to normal force.

$$T_n = \frac{\pi}{128} \rho \omega^2 D^4 \frac{J}{P/D} \left(1 - \frac{J}{P/D}\right) \quad (2.10)$$

Total ideal thrust is the sum of contributions due to leading edge suction and normal force.

$$T_i = \frac{\pi}{256} \rho \omega^2 D^4 \left[1 - \left(\frac{J}{P/D}\right)^2\right] \quad (2.11)$$

Defining the ideal efficiency as the ratio of useful power output to power required gives equation (2.12).

$$\eta_i = \frac{T_i V}{Q \omega} = \frac{1}{2} \left(1 + \frac{J}{P/D}\right) \quad (2.12)$$

As anticipated, thrust and torque vanish as the angle of attack reaches zero while the efficiency approaches a maximum governed by profile drag. This implies that a lightly loaded twisted delta propeller will yield high efficiency just as a lightly loaded

conventional propeller does.

2.3 Modifications For A Practical Propeller

Considerable modification of the theory is required before a practical propeller can be designed. These modifications violate some of the limitations discussed above but reflect real-world constraints on propeller design and operation.

The original theory was developed for a wing on which tip and apex are travelling with the same velocity. The resultant velocity of any point on a propeller operating at typical cruise advance ratio will vary considerably from hub to tip. This effect may be corrected for by using a technique of varying the sweep angle according to the change in resultant velocity such that the incident velocity normal to the leading edge remains constant. Ribner [4] derives a differential equation giving the necessary relationship.

$$\left(\frac{V}{V_n}\right)^2 = \left(\frac{V}{V_R}\right)^2 + \left(\frac{dx}{dr}\right)^2 \quad (2.13)$$

The modified planform obtained by solving equation (2.13) is illustrated in Figure 2.4.

The infinite pressure peak at the leading edge is only possible for an ideal fluid with no viscosity. For a real fluid the flow will separate at the leading edge. To prevent this from occurring the airfoil should be cambered. A cambered section will

Figure 2.4 Modified Planform to Account for Velocity Variation

Figure 2.5 Spanwise Section with Camber

compromise the structural advantage offered by the twisted delta. The camber, therefore, is restricted to a small region near the leading edge and does not introduce large bending moments. A typical spanwise section with camber is shown in Figure 2.5 and may be described mathematically by equation (2.14).

$$z = 0 \quad ; \quad -ra < y < ra$$

$$z = k(y - ra)^2 \quad ; \quad -a < y < -ra \quad (2.14)$$

$$z = -k(y - ra)^2 \quad ; \quad ra < y < a$$

In reference [6] Ribner develops an expression for the velocity normal to the surface of such a section in much the same way as is done to obtain equation (2.7). An additional term in z is required to include the effect of the cambered edge. A velocity potential which will induce this velocity distribution is expressed as a trigonometric series from which the pressure distribution is deduced. All that remains is to determine the geometry which will yield a pressure coefficient of zero at the leading edge. In this way, Ribner [6] derives a camber parameter k which can be evaluated by equation (2.15) for use with equation (2.14).

$$k = \frac{r^2 (1 - \frac{J}{P/D})}{JDC \cos \epsilon (2\epsilon - \sin 2\epsilon)} \quad (2.15)$$

The resulting spanwise pressure distribution is modified such that the infinite peak at the leading edge is replaced by a finite one somewhat inboard as illustrated in Figure 2.6. The magnitude and width of the peak are directly related to the

Figure 2.6 Spanwise Load Distribution With and Without Camber

fraction of radius that is cambered.

Altering the pressure distribution also reduces the torque and thrust developed. The reduction can be estimated by comparing the integrated values for torque with and without camber as in equation (2.16).

$$\frac{Q(r)}{Q_0} = \left[1 - \frac{2\epsilon - \frac{4}{3} \sin 2\epsilon + \frac{1}{6} \sin 4\epsilon}{2\epsilon - \sin 2\epsilon} \right] \quad (2.16)$$

where $\epsilon = \cos^{-1} r$

The reduction factor of equation (2.16) is applied to torque and thrust equations (2.8) and (2.11), respectively.

The propeller designer must compromise between the conflicting requirements for low wave drag and low skin friction. A highly swept design has greater surface area and skin friction but lower wave drag than a moderately swept planform of the same span. Reducing the sweep angle sufficiently for a practical propeller increases the vertex angle and may violate the aspect ratio restrictions. This effect is accounted for by using the methods of Ribner [6] in which the rolling moment stability derivative is corrected for aspect ratio and Mach number. In its original form this correction is restricted to high pitch and small variation of Mach number with radius. The method is applied, however, by estimating an effective sweep angle based on the incident velocity normal to the leading edge and an average resultant Mach number. With these, a rolling moment parameter $I(BC)$ is estimated and applied to thrust and torque equations.

The last remaining modification to the theory is that of

optimizing the geometry and design operating point in terms of the advance-pitch-diameter ratio, $(\frac{J}{P/D})$. Recalling that as the advance and pitch-diameter ratios approach equality, thrust vanishes while torque reaches some minimum value governed by the profile drag, it can be argued that an optimum ratio exists. The value is determined by including the effect of profile drag in the calculation of efficiency. The power required to overcome profile drag is estimated as in equation (2.17) in which S_{wt} is the blade area corrected to include the effect of rotation.

$$P_D = \frac{1}{2} \rho v^3 C_D S_{wt} \quad (2.17)$$

This is subtracted from the useful power to obtain equation (2.18) for the propulsive efficiency.

$$\eta_p = \frac{T_i V - P_D}{Q \omega} \quad (2.18)$$

Differentiating equation (2.18) with respect to the advance-pitch-diameter ratio and setting the result equal to zero fixes the optimum operating point.

$$\left(\frac{J}{P/D}\right)_{opt} = 1 - \frac{4\sqrt{2} J}{\pi^{3/2}} \frac{C_D S_{wt}}{D^2 I(BC)} \left(\frac{Q_o}{Q(\tau)}\right) \quad (2.19)$$

The greatest difficulty in evaluating equation (2.19) is selecting an appropriate value for the drag coefficient. This problem is discussed further in the next chapter. Equation (2.19) allows the designer to choose advance ratio arbitrarily. For practical purposes the choice is generally limited to the cruise range of 2.5 to 3.5 according to reference [2]. This is clear

violation of Ribner's suggested minimum of 6.

Problems of reverse thrust and variable pitch are not addressed in this work though they must be acknowledged. Ribner proposed a possible solution in reference [4]. Without variable pitch the speed of rotation must vary with forward speed to keep the propeller from operating off its design advance ratio. No theory has been proposed for predicting off-design behaviour or performance.

CHAPTER III - MODEL DESIGN

This section deals with the detailed design of a model of the twisted delta propeller suitable for a low speed wind tunnel test. It is expected that useful data regarding the flow field, thrust, drag, torque and efficiency may be gathered at low speed. Specifically, the test consists of operating a two blade propeller at a Mach number up to 0.112 in a continuous flow wind tunnel of the National Research Council (NRC) of Canada in which the test section measures 2 metres by 3 metres.

The design procedures follow closely those of Ribner in references [4] and [6]. A FORTRAN program was developed to perform all the calculations necessary to predict thrust, torque, and efficiency in addition to computing the coordinates describing the shape of the propeller given certain input design parameters. Many of the assumptions used to develop the theory are violated as explained in the previous chapter. It is felt, however, that making use of the theory to obtain some prediction of performance will yield some insight into the validity of its application and the extent to which it must be further developed. The program, entitled HSPROP, consists nominally of two parts: performance predictions; detailed calculation of the shape.

3.1 Performance Predictions

Program HSPROP is written in FORTRAN 77 to run on Carleton University's Honeywell level 66 computing system. The user provides a group of input parameters governing the propeller design operating point. These are: cruise Mach number; normal incident Mach number; propeller diameter; hub diameter; advance ratio; and camber parameter. The normal incident Mach number specified will govern the sweepback angle of the leading edge. The leading edge camber parameter specifies the uncambered radius fraction. These inputs are read from a user-created data file associated with unit number five. Ambient air conditions are specified within the program and are approximately those of sea level, standard day. The pertinent constants such as cruise velocity, sonic velocity, rotational speed and tip Mach number are calculated directly from the inputs.

The next step is to solve the differential equation (2.13) governing the leading edge sweep. Equation (2.13) is derived on the basis of zero angle of attack. This implies that the advance ratio and the pitch-diameter ratio are equal. There is a small error inherent in this approximation but Ribner [4] argues that it can be neglected. It is convenient for the purposes of the program to rewrite equation (2.13) in terms of quantities known to the program at this point. The velocity can simply be replaced by the corresponding Mach number:

$$\left(\frac{M}{M_n}\right)^2 = \left(\frac{M}{M_R}\right)^2 + \left(\frac{dx}{dr}\right)^2 \quad (3.1)$$

The normal incident Mach number (M_n) and the cruise Mach number (M) are specified by the user. The resultant Mach number (M_R) of any point on the propeller is a function of the local radius as follows:

$$M_R^2 = M^2 + \left(\frac{r\omega}{c}\right)^2 \quad (3.2)$$

The form of equation (2.13) solved in the program is:

$$\frac{dx}{dr} = \sqrt{\left(\frac{M}{M_n}\right)^2 - \frac{M^2}{M^2 - (r\omega/c)^2}} \quad (3.3)$$

The solution for this equation can be obtained using various numerical methods. Program HSPROP uses a Runge-Kutta method for which SUBROUTINE RUNGE was written by White [7]. For ten discrete values of the radius the solution for $X(r)$ is returned by the subroutine. The accuracy of using only ten points was confirmed by doubling the number and noting that there was no change in the solution. The subroutine is called upon again to find the solution for X at the hub radius which may not correspond with one of the original discrete radial positions.

To reduce the severe load that would be experienced by a thin pointed tip, the trailing edge of the propeller is extended by a small arbitrary amount. This results in a constant span section which, according to the theory of Jones [5], will not contribute to the thrust. In fact, this region will contribute to thrust due to the twist incorporated in the design but it must be kept small to minimize drag. The new trailing edge is made straight and radial due to its thinness.

The nominal surface area of the propeller is estimated using SUBROUTINE INTGRT, an integration routine developed by the author. This subroutine calls on another, SUBROUTINE LAGRAN, to fit a second order Lagrangian polynomial through three adjacent points. The integral of the polynomial is evaluated between any two of the included points and added to the result obtained from the previous intervals. Having obtained the surface area, the aspect ratio of the propeller can then be calculated and compared with the suggested limit. As explained previously, the limit is generally exceeded for a practical propeller.

To account for the effect of profile drag a weighted blade area is defined in reference [4]. This quantity reflects an effective increase in the blade area due to its rotation. The axial blade chord at each radial station is increased by a factor of $(V_R/V)^3$. The result is then integrated from hub to tip using SUBROUTINE INTGRT.

The correction for Mach number and aspect ratio is estimated using the procedure outlined in Appendix A of reference [6]. The correction is based on the rolling moment stability derivative analysis by Ribner and Malvestuto [8]. A suitable average value of the resultant Mach number must be chosen in such a way that it reflects the greater contribution of the tips to thrust and torque. To finally evaluate the correction term $I(BC)$ Ribner consults Figure 4 of reference [8]. For the purposes of program HSPROP a linear approximation to the applicable curve is used:

$$I(BC) = -0.1454(\overline{BC}) + 1.0163 \quad (3.4)$$

The purpose of camber is to eliminate the theoretical pressure peak at the leading edge. In doing so the torque and thrust are reduced by a factor related to the camber parameter, τ . The camber induced reduction factor is calculated per equation (2.16). The optimum advance-pitch-diameter ratio is then calculated using equation (2.19).

Having obtained all the necessary preliminary quantities, the thrust, torque, drag, and efficiency can now be estimated using the equations of references [4] and [6] and listed below as equations (3.5).

$$Q = \frac{\pi \rho D^4 \omega V(1 - \frac{J}{P/D})}{128} \frac{Q(\tau)}{Q_0} I(BC)$$

$$T_i = \frac{\pi}{256} \rho \omega^2 D^4 [1 - (\frac{J}{P/D})^2] \frac{Q(\tau)}{Q_0} I(BC) \quad (3.5)$$

$$D_e = C_D \frac{1}{2} \rho V^2 S_{wt}$$

These results and the corresponding inputs are then output by the program to a file associated with FORTRAN unit number six.

3.1.1 Input and Output

It is appropriate at this point to explain and justify the selection of certain constants within the program and the inputs specified by the user. The first of these is the advance ratio, J . The normal cruise advance ratio for a propeller of this class

lies in the range 2.5 to 3.5 [2] defined in equation (3.6) in the usual way. The present project is conducted for a design advance ratio of $J = 4$. The choice is arbitrary but reflects the desire to keep J fairly high in the interests of preserving the validity of the theory.

$$J = \frac{V}{nD} \quad (3.6)$$

The model size to be tested is a choice based on several factors. A large model is easier to build accurately, operates at a more realistic Reynolds number and has proportionally thicker airfoil sections than a small model. Large models, however, need large wind tunnels or cause excessive tunnel wall interference and take considerable power to drive. The model diameter of 0.6096 metres (2 feet) is selected considering the available power and wind tunnel size.

Hub size is chosen on the basis of the drive motor size. The drive motor is mounted directly on the six-component force and moment balance and a fairing is used to enclose both motor and balance. A diameter of 0.1143 metres fair the hub smoothly with the rest of the apparatus.

The input cruise Mach number is essentially an independent variable whose choice reflects attainable running condition in the wind tunnel. In this case the value of 0.112 is used. The choice of normal incident Mach number is one of trial and error. A low value reduces wave drag at the expense of increased skin friction and profile drag. The choice can also be made in

reverse; the apex angle may be specified and the corresponding value of M_n found by hand calculation for $r=0$. In the present case the sweep angle at the apex is chosen to be 20 degrees resulting in a normal incident Mach number of 0.106. The program returns the values of apex sweep angle and tip sweep angle.

The camber parameter, τ , specifies the fraction of the radius that remains purely radial. Ribner [6] suggested that this region should be raked in the opposite sense to the leading edge camber to offset bending moments induced but this feature is not included in the present design. The choice of τ is arbitrary and simply a matter of judgement in this design although a stress analysis would perhaps provide a basis for optimizing the choice against aerodynamic requirements. The present design uses a value of τ equal to 0.9 while Ribner used 0.925 in his sample [6].

When accounting for the aspect ratio and Mach number by using the rolling moment stability derivative, an average resultant Mach number, $\bar{M}_R$, must be estimated. Since most of the load is carried near the propeller tip, the resultant Mach number at 90% radius was chosen arbitrarily to represent the average. The value of the rolling moment parameter $I(BC)$ is not particularly sensitive to the choice of $\bar{M}_R$.

Within the program an estimate of the drag coefficient is required to calculate the profile drag. Ribner used 0.0086 in reference [6] but did not elaborate on how he obtained it. The airfoil section selected for this design is based on the NACA 16-006. The drag coefficient is available in graphical form in

reference [9] and the value chosen is 0.01. This is not regarded with great confidence by the author since it is based on a two-dimensional airfoil section test. The plane of two-dimensional flow for an airfoil section is orthogonal to that assumed for the twisted delta propeller. The choice of this airfoil is detailed in section 3.2 below.

Figure 3.1(a) shows the appropriate form of the input file for the program. The first four numbers in line one are: initial specified diameter; increment for diameter increase; final specified diameter; and hub diameter. All these dimensions are in metres. The fifth and final number is the camber parameter, τ . Line two specifies initial and final advance ratios and cruise Mach numbers in addition to the increment by which each is to be increased. The program carries out performance calculations for each combination of advance ratio, propeller diameter and cruise Mach number. To choose a particular design point one need only modify the input file such that the initial and final values are the same. For such a case the increments are not important but must still be present for the READ statement in the program. The final entry in line two is the normal incident Mach number which must never be greater than the cruise Mach number.

The output contains all the pertinent input data plus the calculated performance based on the present theory. Thrust and power are given in terms of the usual non-dimensional coefficients defined in equations (3.7). In addition, estimated horsepower is listed along with efficiency per equation (3.8).

$$C_T = T / \rho n^2 D^4 \quad C_P = P / \rho n^3 D^5 \quad (3.7)$$

0.6096,0.0001,0.6096,0.1143,0.900
 3.1415927,0.50,3.1415927,0.111972,0.01,0.111972,0.105716

Figure 3.7(a) Sample Input File for PROGRAM HSPROP

PROPELLER DESIGN POINT

D	DH	TAU	J	M	MN	C	RO
.6096	.1143	.9000	3.142	.1120	.1057	.342.0	1.2182
RPM	MT	LAMDAR	LAMDAT	AR	ARLIM		
1200.	.1584	20.0	45.2	5.373	1.006		
P/D	P	CP	CT	ETA			
3.770	.420	.382	.101	.833			

Figure 3.7(b) Sample Performance Output File from PROGRAM HSPROP

$$n = \frac{C_T}{C_P} J \quad (3.8)$$

30

Figure 3.1(b) illustrates a sample performance output file.

3.2 Defining and Describing the Shape

Mathematically describing the three-dimensional surface of this propeller is sufficiently complex to warrant a separate article. The difficulty arises not with the helicoid mean surface or swept leading edge but with the airfoil thickness and the cambered region. Defining the mean surface without thickness or camber is, however, the first step.

Program HSPROP begins the process of calculating coordinates if the user specifies he wants them in response to a prompt from the program. Since the input file may specify several design points, and output of coordinates for each occupies considerable disk space, it is recommended that a single design point be chosen before generating any coordinates. For instance, having obtained the performance data for a propeller at several advance ratios for a given Mach number and propeller size, one should modify the input file to choose one design point for which the surface coordinates are needed.

3.2.1 The Mean Surface Helicoid

Using cylindrical coordinates to describe the mean surface helicoid enables computation of a unique function of $\theta_0(x)$. The

helicoid angle θ_0 is independent of radius, r , since all sections are radial except in the cambered region. The function $\theta_0(x)$ is determined by the propeller pitch and is described as follows:

$$\theta_0(x) = 2\pi \frac{x-x_0}{p} \quad (3.9)$$

The constant x_0 is arbitrarily chosen such that the trailing edge of the propeller corresponds to $\theta_0 = 0$. The ordinate triplets defining points on the helicoid mean surface are easily obtained by evaluating the following equations for various values of r .

$$y(x, r) = r \cos \theta_0(x) \quad (3.10)$$

$$z(x, r) = r \sin \theta_0(x)$$

The mean surface in the cambered region is dependent on the camber parameter, r . The intermediate parameter, k , must first be calculated according to equation (3.11).

$$k = \frac{\pi^2 (1 - \frac{J}{P/D}) \tan(\lambda_T)}{DJ \cos \epsilon (2\epsilon - \sin 2\epsilon)} \quad (3.11)$$

$$\lambda_T = \text{tip sweep angle}$$

The decision to use the tip sweep angle is made in the interests of achieving the greatest accuracy where the radius, and camber, are maximums. Having obtained k , the helicoid angle is then modified in the cambered region according to (3.12) and becomes a function of both r and x .

$$\theta(x, r) = \theta_0(x) - k(r - 2ra + \frac{r^2 a^2}{r_0}) \quad (3.12)$$

The ordinate triplets are again simply defined by converting from cylindrical to cartesian coordinates.

$$\begin{aligned} y(x, r) &= r \cos \theta(x, r) \\ z(x, r) &= r \sin \theta(x, r) \end{aligned} \quad (3.13)$$

3.2.2 Upper and Lower Surfaces

In defining the upper and lower surfaces of the propeller blade, an appropriate airfoil section must be chosen. The conflicting requirements determining such a selection are for thin sections conducive to high critical speeds yet thick enough to carry the loads imposed by the high speed of rotation. The NACA 16 series appears to offer a good compromise for these constraints in that the position of maximum thickness is well aft such that a shallow favourable pressure gradient exists over a large portion of the blade surface while the trailing edge angle is relatively large implying adequate material is available to support the loads experienced in the region of maximum radius. The 16-series sections have small leading edge radii thus reducing the mass and the resulting bending moments in the cambered portion of the present propeller. According to Abbott [9], these sections have been successfully applied for conventional propellers. A description of the airfoil section is given in reference [9] as a table of thickness versus chordwise

position. This table is used as an input file associated with unit number eight for use by PROGRAM HSPROP.

In general terms the method used to determine surface coordinates is as follows:

- 1) A mean surface coordinate whose ordinate triplet is known is chosen.
- 2) The true chord length of the constant radius line on which the point lies is calculated based on the propeller geometry (pitch).
- 3) The chord fraction of the chosen point is calculated.
- 4) The airfoil semi-thickness at the chosen point is estimated by interpolating between known values listed in the airfoil input file.
- 5) The local mean surface slope in the radial direction is calculated and the upper and lower surface ordinate triplets can then be obtained.

3.2.3 Calculating True Chord

The true chord at any radius is somewhat greater than the difference of leading and trailing edge x-wise positions due to the pitch. Consider the simplified diagram showing a chord line in Figure 3.2. The local pitch angle, α , can be calculated by evaluating the change in z with respect to x from the leading edge to the trailing edge. This quantity is related to the

Figure 3.2 Chord Geometry With Pitch.

Figure 3.3 Local Mean Surface Slope

helicoid angle, θ_0 , as follows:

$$EL(x) = r \sin \theta_0(x) \quad (3.14)$$

$$\alpha(r) = \tan^{-1} \left(\frac{EL(x)}{x_{TE} - x_{LE}} \right)$$

The true chord is then:

$$CHORD(r) = \frac{x_{TE} - x_{LE}}{\cos \alpha(r)} \quad (3.15)$$

The chord fraction for each point on the mean surface is calculated similarly by equation (3.16).

$$CFRAC(x, r) = \frac{(x - x_{LE}) / \cos \alpha(r)}{CHORD(r)} \quad (3.16)$$

Each chord fraction is then compared with those listed in the airfoil descriptive input file. SUBROUTINE LAGRAN is used to fit a quadratic polynomial through three adjacent points in the input file enclosing the point considered. The corresponding semi-thickness is easily interpolated using the quadratic polynomial.

In the cambered region the process is complicated by the absence of data describing the leading edge and the chord lengths. Due to the gradual curvature of the leading edge these numbers are estimated using linear interpolation. Although more precision could be obtained by once again invoking SUBROUTINE RUNGE to solve equation (2.13) for the required leading edge positions, the difference is negligible.

To compute the ordinate triplets describing the surface, the local mean surface slope, ϕ , is required. This is illustrated

in Figure 3.3. Clearly, $\phi = \theta_0$ except in the cambered region where the intermediate camber parameter k is used per equation (3.17).

$$\phi = \theta_0(x) - 2k(r - r_a) \quad (3.17)$$

where $r_a < r < a$

Referring again to Figure 3.3 the surface coordinates are:

$$\begin{aligned} y_u(x, r) &= y(x, r) - t \sin \phi \\ z_u(x, r) &= z(x, r) + t \cos \phi \\ y_l(x, r) &= y(x, r) + t \sin \phi \\ z_l(x, r) &= z(x, r) - t \cos \phi \end{aligned} \quad (3.18)$$

The method described above enables complete description of radial sections of the propeller shown in Figure 3.4. These sections form the basis for the fabrication techniques employed to build the wind tunnel model.

Figure 3.4 Radial Sections of Propeller

CHAPTER IV - BUILDING THE PROPELLER

When considering means of building the wind tunnel model to be tested it was felt that some inaccuracies could be tolerated if the cost could be kept low and a method of composite construction could be evaluated. For many years a popular method of construction used by amateur aircraft builders has been that of a foam core with epoxy and fibre coating. The foam merely describes the shape while the combination of epoxy and fibreglass, Kevlar, or graphite fibres provides the strength. No female molds are required since the workpiece is to be built only once, hence this method is well suited to prototype construction. Furthermore the cost is low, compared with having a metal propeller blade milled, and does not require specialized machinery or technicians. The greatest problem with foam and fibre construction lies with the difficulty of fabricating sections thin enough for propeller application.

The materials chosen for the present propeller include: Klegecell foam; unidirectional H110 graphite fibre cloth; lightweight fibreglass cloth; CIBA-GEICY 3456 two-component epoxy. The foam is a very rigid small cell type. Rigidity is essential for making thin airfoil sections of a propeller blade. The choice of graphite cloth is dictated by its low cost compared

with Kevlar and the recognition that virtually all loads are carried in the radial direction. The 3456 epoxy is recommended by the supplier for use with the foam and fibre selected.

4.1 Building The Foam Core

The construction method for the foam core is based on radial blade sections described in the previous chapter. Two successive radial sections are oriented at some small angle to each other and may be used as templates to shape the material between them. Based on this concept, balsa templates are made from full scale drawings of the radial sections. The foam is supplied in sheets of 12.7 millimetres thickness. Template spacing, or axial distance between each radial section, is not constant. It is necessary, therefore, to cut oversize sections of foam and then reduce the thickness of each to its required value. Cutting the foam is most easily accomplished with a thin nichrome wire heated electrically. The apparatus used to reduce foam thickness is shown in Figure 4.1. A thin nichrome wire is heated using AC line current whose voltage is stepped down by a potentiometer to an appropriate level determined by trial and error. The wire is held taut at a fixed distance above the level of the workbench by removable table vises. Wire height can be adjusted by releasing the tension. This simple apparatus is quite successful in creating smooth plane surfaces.

Figure 4.1 Apparatus to Reduce Foam Section Thickness

Figure 4.2 Jig for Aligning Templates

Having cut all sections to the correct thickness, the next task is to shape them to match their respective templates. The most critical part of this operation is template alignment. A simple jig, illustrated in Figure 4.2, performs this task. The jig employs a vertical centrepost to align the template origins and fix the foam in place. The bottom template is secured to the jig by pins such that its centreline corresponds to a known datum. The foam is then lowered onto the centrepost followed by the top template. Angular alignment of the top template is achieved with the aid of a string attached to the centrepost and pulled taut. The string and template are then aligned with a mark indicating the correct angle with respect to the datum line and bottom template. The entire assembly of foam sandwiched between two templates is pinned together and removed from the jig. Shaping the section is done by using a hand-held battery powered hot wire foam cutter shown in Figure 4.3. Assembling the foam core is completed by glueing adjacent radial sections to each other after removing the balsa templates. The complete core is illustrated in Figure 4.4.

4.2 Applying The Graphite Fibre Cloth

The unidirectional graphite fibre cloth is supplied in a roll of 15.2 centimetres width. For handling, the fibres are held in place with a loose weave of nylon. A quantity of epoxy is

Figure 4.3 **Templates; Foam; Foam Cutter**

Figure 4.4 **Completed Foam Core**

mixed according to the manufacturer's specifications and spread over the foam surface followed by the fibre cloth. Pressure is applied to the cloth using a flexible spatula to remove excess epoxy from between foam and cloth, and to insure that epoxy penetrates between the fibres. More epoxy is then spread over the surface and forced into the cloth with the spatula.

A narrow strip of fibre cloth is applied axially to the centre section of the propeller, as shown in Figure 4.5, for reinforcement. Two pieces must be used since this material cannot be bent around sharp corners. Thickness is not a concern in this region since it is to be covered by the cylindrical hub.

The balance of blade surface area is covered using a single layer of cloth. Each side requires two strips since the blade root chord exceeds cloth width. Achieving an evenly rounded leading edge is difficult due to the rigidity of the cloth. As shown in Figure 4.5, extra length is permitted to overhang the leading edge on both surfaces. The excess material is manually removed with coarse sandpaper after the epoxy dries. The leading edge contour is then built up using layers of epoxy sanded between applications.

4.3 Mounting Provision

At this stage it is necessary to prepare for installing the propeller on the drive motor to be used in the wind tunnel.

Figure 4.5 Centre Section Reinforcement

Figure 4.6 Motor Shaft Detail

Figure 4.6 illustrates the motor shaft configuration. The two main tasks that the mounting system must accomplish are those of transferring torque and sustaining a thrust or drag load. A third requirement is that the propeller be easy to install and remove from the motor.

Torque is transferred by a steel sleeve which mates with the keyed motor shaft. A thin steel shaft is used to extend the motor shaft and is tapped to fit on to its threaded portion. These items are shown in Figure 4.7. To accommodate the torque hub, a slot is cut into the propeller blade near its trailing edge as shown in Figure 4.8. The hub is attached to the blade with epoxy and further secured with layers of graphite fibre cloth as in Figure 4.9. Throughout this operation the shaft extension is used to align the hub, through which it must pass when the propeller is mounted on the motor.

To install the propeller one threads the shaft extension on to the motor shaft and slides the propeller rearwards over the shaft extension until the torque hub engages the motor shaft. A self locking nut is used on the end of the shaft extension to retain the propeller. With this configuration the structural integrity of the one-piece propeller blade is preserved.

Figure 4.7(a) Drawing of Machined Steel Components

Figure 4.7(b) Photograph of Machined Steel Components

Figure 4.8, Slot in Blade for Torque Transfer Hub

Figure 4.9 Installing Torque Transfer Hub

4.4 Trailing Edge; Spinner and Hub

Since foam core construction cannot be made thin enough to make a sharp trailing edge, another method is required. A solid epoxy trailing edge is fabricated by using a wedge of mylar plastic as a mold. Each mold is simply taped to the blade surface, sealed with silicon at its inboard end, and filled with epoxy resin poured in from the tips. The resulting trailing edge is smooth and sharp.

The next task is to construct a spinner and hub to conform with the drive motor fairing. A constant diameter hub is assumed in the design program and is easy to fabricate. The size is determined by the motor fairing and the root chord length of the propeller. Figure 4.10 shows how it is built up using layers of fibreglass cloth and epoxy over a wooden form. After curing, the ends are sanded plane. To cut slots for the blade, two profiles of the airfoil section are drawn at the appropriate pitch angle and locations on a sheet of paper which is wrapped around the wooden form as in Figure 4.11. The form is then reinserted into the hub and temporarily attached to it. A small high speed cutting disc is used to remove material for the blade slots. The hub is then attached to the blade using fibreglass cloth and epoxy.

Figure 4.10 Building the Hub

Figure 4.11 Blade Cutouts in Hub

Figure 4.12 Spinner and Hub

The spinner, shown in Figure 4.12, is made on a wood lathe and is of arbitrary paraboloid shape. Its weight is reduced by drilling material out of its centre. It is attached to the hub using self tapping wood screws.

4.5 Surface Finish

The final step in construction is that of surface finish. Due to the nature of graphite cloth the surface is very rough and must be filled. The first part of this process is to eliminate the major hills and valleys using automotive body filler. After the filler hardens, the surface is sanded smooth and the procedure is repeated if necessary. In this way the strength of the graphite fibres is not compromised. The second part of the process is to cover the entire blade surface with a layer of fibreglass cloth and epoxy. Fine sandpaper is used to obtain the final surface finish.

4.6 Balancing

The completed assembly is statically balanced to prevent severe eccentric loading. This is accomplished by mounting the propeller on its drive motor and observing it freewheeling slowly. Putty and body filler are placed inside the hub until no

bias is observed as the propeller comes to rest.. This procedure, combined with low propeller weight of approximately ten Newtons, effectively eliminates problems of dynamic imbalance.

4.7 Summary

The accuracy of the finished product is not of the high standard one would expect from machining or using molds. The airfoil sections, for instance, are somewhat thicker than intended due to the author's inexperience in working with these materials and the consequent uncertainties in the thickness of composite layers. All the main features of the concept, however, are incorporated in an inexpensive propeller suitable for low speed wind tunnel testing. The cost of materials is summarized as follows:

Foam and Fibreglass cloth:	\$32.00
Graphite Fibre Cloth:	\$40.00
Epoxy:	\$56.00
Machining: (Torque hub; Shaft extension; Spinner; Wooden Hub form)	\$175.00
Miscellaneous Materials: (Glue; Silicon; Balsa; etc)	\$15.00
TOTAL	\$318.00

The construction time is sixty to seventy hours although subsequent propellers could be built in less time based on the experience gained with the first one.

CHAPTER V - WIND TUNNEL TESTING

5.1 General Description

The facility used for the tests described herein is the National Research Council (NRC) of Canada low speed wind tunnel located in building M-2 on the Montreal Road campus of NRC in Ottawa, Canada. This test was conducted under the direction of R.H. Wickens in cooperation with the author in November, 1985.

The wind tunnel is a continuous flow type driven by a 1000 horsepower electric motor and has a nominal working section of 2 metres by 3 metres (6 feet by 9 feet). It is capable of continuous steady-state operation up to approximately 300 ft/sec. at essentially sea level ambient conditions. The speed can be set directly or in terms of dynamic pressure. Peripheral equipment includes a digital computer for data acquisition and processing and a control system for the variable frequency propeller drive motor. The large main balance located under the floor of the working section was not used for this test due to the small magnitudes of forces involved. A pedestal located on the main balance was used to mount the small six-component balance and propeller drive motor as shown in Figure 5.1. The balance is a product of Task Research Corporation and is fully

calibrated by NRC. A fairing enclosing both drive motor and balance is mounted "off balance". The unfaired drive motor is shown in Figure 5.1 with the complete assembly shown faired and ready to run in Figure 5.2.

The drive motor is a variable frequency unit nominally rated for 20 horsepower at 11,000 rpm. Experience with the motor during this test and in the past have shown it to be limited by the temperature of the bearings despite water cooling. Consequently, the available speed and power for this test were approximately 1400 rpm and 1.3 horsepower, respectively. No blockage or interference corrections are necessary due to the low ratio (0.06) of propeller disc area to test section area.

5.1.1 Balance Testing

The first part of the test consisted of collecting data with the Task Research balance. Figure 5.3 is a schematic diagram showing the mounting arrangement of the motor and balance and the forces and moments measured. Note that since the motor and balance axes are not colinear any side force must be measured and the resulting moment added to measured rolling moment to compute

Figure 5.1 Balance and Motor without Fairing

Figure 5.2 Propeller Ready to Run

Figure 5.3 Motor and Balance Installation Schematic

torque. The data acquisition computer calculates thrust and torque according to equations (5.1).

$$\text{Thrust } T = (X - X_T) k_X$$

$$\text{Side Force } S = (Y - Y_T) k_Y \quad (5.1)$$

$$\text{Torque } Q = (L - L_T) k_L + S Z$$

Note that X_T , Y_T , L_T are balance tare values and k_X , k_Y , k_L are balance calibrations in lb_f/mv and $\text{lb}_f\text{-ft}/\text{mv}$, respectively.

To verify channel connections and balance calibration, checks were performed for thrust and side force measurements. These were done by simply suspending a known weight from a string attached to the motor drive shaft and routed over a pulley so as to apply an axial or transverse load as required.

Rotational speed was measured digitally by counting the number of electrical pulses induced by two magnets on the motor shaft. Thus 40 pulses per second is 20 revolutions per second. In addition, working section dynamic pressure and total temperature were recorded.

The computer reduces data according to equations (5.2) for presentation in dimensionless form.

$$\text{Thrust Coeff. } C_T = \frac{T}{\rho n^2 D^4}$$

$$\text{Power (hp)} P = \frac{2\pi n Q}{550}$$

$$\text{Power Coeff. } C_P = \frac{P}{\rho n^3 D^5} \quad (5.2)$$

$$\text{Advance Ratio } J = \frac{V}{nD}$$

Balance data were gathered for a propeller speed of 1200 rpm and wind speeds ranging from zero to 155 ft/sec. For each case, data were collected a minimum of three times to check repeatability.

5.1.2 Wake Survey

The second part of the test program consisted of performing a wake survey using a five-hole probe. For this purpose a traverse rig was erected vertically in the tunnel to support a probe facing axially upstream from a location just aft of the propeller trailing edge. The probe and traverse rig arrangement are illustrated in Figure 5.4. Probe position is controlled from outside the tunnel while the propeller and wind tunnel are running.

The five-hole probe consists of five pitot tubes arranged as shown in Figure 5.5. The total pressure detected by each pitot tube is determined by local flow direction due to their different beveled leading edge orientations. By recording the pressure sensed by all five pitot tubes on the probe plus free stream dynamic pressure, the local flow speed and direction can be deduced. A complete description of the probe used in this test and its calibration is available in reference [10].

The test was run for a single operating point and consisted of taking data at 1/2-inch radial intervals starting close to the

Figure 5.4 Wake Survey Traverse Rig Schematic

Figure 5.5 Five-Hole Probe Detail

fairing and stepping to well outside the propeller arc. The data recorded include reference dynamic pressure, probe pressure coefficients and probe position. Data processing according to the calibration equations of reference [10] produced final output of: probe position; probe pressure coefficients; axial, radial, tangential and resultant velocity coefficients; radial and tangential flow angles; total total pressure coefficient. Further post-processing by the author using the methods of Reid [11] is described in section 5.2.2.

5.2 Interpretation of Data

5.2.1 Balance Data

Data collected using the Task Research balance are listed in Table 5.1. Repeatability was excellent for all but one case. Variations observed during the static thrust run can be explained by the tendency for the propeller to drive the tunnel, albeit at a velocity too low to be measured. For the highest advance ratio tested, rotational speed was 1230 rpm. For each advance ratio the data are averaged and the propulsive efficiency is calculated using equation (5.3).

$$\eta = \frac{C_T}{C_P} J \quad (5.3)$$

TABLE 5.1 BALANCE DATA

RPM	Vel. (ft/sec)	C _T	Power (HP)	C _P	J
1200	0.0	0.207	1.001	0.924	0.00
1200	0.0	0.203	1.000	0.922	0.00
1200	0.0	0.205	1.002	0.924	0.00
1200	0.0	0.211	1.035	0.951	0.00
1200	0.0	0.210	1.036	0.952	0.00
1200	0.0	0.214	1.045	0.957	0.00
1200	0.0	0.221	1.040	0.953	0.00
1200	41.7	0.240	1.081	0.990	1.04
1200	41.7	0.241	1.079	0.989	1.04
1200	41.7	0.239	1.074	0.984	1.04
1200	41.7	0.240	1.081	0.990	1.04
1200	80.0	0.257	1.080	0.990	2.00
1200	80.1	0.256	1.079	0.989	2.00
1200	80.1	0.257	1.081	0.991	2.00
1200	80.1	0.258	1.079	0.989	2.00
1200	80.2	0.257	1.078	0.988	2.00
1200	100.8	0.281	1.084	0.994	2.52
1200	100.9	0.280	1.084	0.994	2.52
1200	100.7	0.279	1.080	0.990	2.51
1200	100.8	0.279	1.082	0.993	2.52
1200	100.7	0.277	1.080	0.991	2.51
1200	100.6	0.279	1.083	0.993	2.51
1200	115.8	0.266	1.077	0.989	2.89
1200	115.8	0.261	1.078	0.988	2.89
1200	115.8	0.262	1.079	0.990	2.89
1200	115.9	0.264	1.075	0.985	2.89
1200	126.2	0.248	1.096	1.009	3.15
1200	126.2	0.248	1.093	1.006	3.15
1200	126.2	0.248	1.099	1.011	3.15
1200	126.1	0.248	1.091	1.004	3.15
1200	126.1	0.250	1.097	1.009	3.15
1200	126.0	0.248	1.088	1.000	3.15
1200	138.9	0.233	1.078	0.990	3.47
1200	139.4	0.232	1.078	0.990	3.48
1200	139.4	0.233	1.080	0.992	3.48
1200	139.3	0.234	1.078	0.991	3.48
1200	139.4	0.232	1.078	0.990	3.48
1200	139.2	0.233	1.076	0.988	3.48
1200	139.5	0.234	1.079	0.990	3.48

(continued next page)

TABLE 5.1 BALANCE DATA ... (continued)

RPM	Vel. (ft/sec)	C_T	Power (HP)	C_P	J
1200	145.8	0.221	1.059	0.972	3.64
1200	145.5	0.223	1.064	0.976	3.64
1200	145.5	0.222	1.058	0.971	3.63
1230	154.2	0.198	1.093	0.932	3.76
1230	154.1	0.203	1.087	0.927	3.76
1230	154.2	0.201	1.079	0.920	3.76
1230	154.3	0.197	1.088	0.927	3.76

Plotting C_T , C_p , and η versus J results in Figure 5.6. The general form of these curves is similar to that observed for conventional propellers of fixed pitch. Also shown in Figure 5.6 are the points of maximum attainable efficiency for a two blade propeller with ideal circulation distribution and no profile drag. These are estimated according to the method of Theodorsen [12] based on the experimental disk loadings and advance ratios.

Large discrepancies exist between predicted and measured thrust and torque. Near the design advance ratio, thrust and power coefficients are approximately 2.6 times greater than those predicted using PROGRAM HSPROP. Clearly, the slender body theory can not be applied to this three dimensional case even as a first approximation. This result is not entirely unexpected in view of the many violations of limits placed on the theory. The most severe of these is the aspect ratio which exceeds the suggested limit by a factor of five.

One encouraging result is observed in the propulsive efficiency. Though slightly lower than predicted at design point, it is approximately equal to the prediction at the observed optimum advance ratio of 3.6. Recall that selection of optimum design advance-pitch-diameter ratio is also based on two dimensional flow. Furthermore, it is essentially a function of the profile drag which is a quantity difficult to estimate. This problem is discussed in Chapter Three in connection with airfoil selection. Despite these problems and the question of pitch accuracy of the model, the observed optimum point is close to the

Efficiency; Thrust Coefficient; Power Coefficient

Figure 5.6 Summary of Balance Data

design value. The high propulsive efficiency is remarkable when one considers the large surface area compared with a conventional propeller. Since this test was conducted for wholly subsonic flow, the benefits of sweep in delaying the onset of transonic drag rise were not observed. Had the subsonic efficiency proved to be low, however, one could hardly expect that it would be improved for transonic conditions except, perhaps, for a Reynolds number effect. Good low speed efficiency is a necessary though insufficient prerequisite to good high speed efficiency.

5.2.2 Wake Survey Data

Raw wake survey data are listed in Tables 5.2 and 5.3. Table 5.2 lists radial position of the probe and the five pressure coefficients. The average reference dynamic pressure for the run is also given in terms of wind tunnel working section velocity. The data in Table 5.3 are the results of post-processing by the data acquisition computer using the probe calibration factors and the equations of reference [10]. They include radial and swirl flow angles followed by the resultant, radial, yaw, and axial velocity coefficients, and local total pressure coefficient.

TABLE 5.2 WAKE SURVEY PRESSURE COEFFICIENTS

Position of Probe (in.)	Cp1	Cp2	Cp3	Cp4	Cp5
2.75	0.217	0.375	0.645	0.459	0.991
3.25	0.277	0.383	0.655	0.518	1.027
3.75	0.299	0.393	0.654	0.526	1.028
4.25	0.313	0.409	0.658	0.533	1.030
4.75	0.326	0.419	0.663	0.533	1.042
5.25	0.339	0.429	0.653	0.529	1.042
5.75	0.350	0.434	0.654	0.532	1.047
6.25	0.367	0.440	0.651	0.532	1.058
6.75	0.371	0.447	0.640	0.531	1.067
7.25	0.384	0.449	0.649	0.535	1.062
7.75	0.386	0.454	0.635	0.525	1.058
8.25	0.401	0.452	0.632	0.524	1.064
8.75	0.410	0.459	0.621	0.524	1.061
9.25	0.421	0.458	0.610	0.515	1.056
9.75	0.427	0.456	0.602	0.505	1.049
10.25	0.439	0.454	0.589	0.504	1.053
10.75	0.442	0.446	0.566	0.485	1.033
11.25	0.446	0.419	0.517	0.453	0.997
11.75	0.460	0.412	0.470	0.443	0.971
12.25	0.477	0.417	0.472	0.457	0.973
14.25	0.483	0.419	0.467	0.454	0.978

Tunnel reference velocity = 123.5 ft/sec

Propeller speed = 1200 RPM

TABLE 5.3 PROCESSED WAKE SURVEY DATA

Probe Radial Posit. (in.)	Radial Flow Angle (Deg) α	Swirl Flow Angle (Deg) β	Resultant Velocity Coeff. v/V_T	Swirl Vel. Coeff. w/V_T	Radial Vel. Coeff. yv/V_T	Axial Vel. Coeff. u/W_T	Local Total Press. Coeff. H
2.75	7.1	2.2	1.057	0.133	0.042	1.047	0.995
3.25	6.2	3.1	1.059	0.115	0.057	1.051	1.031
3.75	5.9	3.0	1.050	0.109	0.056	1.043	1.031
4.25	5.8	2.9	1.043	0.107	0.053	1.036	1.033
4.75	5.6	2.7	1.047	0.104	0.050	1.041	1.045
5.25	5.3	2.4	1.045	0.098	0.045	1.040	1.044
5.75	5.1	2.3	1.045	0.095	0.044	1.039	1.049
6.25	4.8	2.2	1.051	0.088	0.041	1.046	1.060
6.75	4.5	2.0	1.059	0.083	0.038	1.055	1.068
7.25	4.5	2.0	1.048	0.083	0.038	1.044	1.064
7.75	4.2	1.8	1.048	0.078	0.033	1.045	1.059
8.25	3.9	1.7	1.051	0.073	0.033	1.048	1.065
8.75	3.6	1.6	1.048	0.067	0.030	1.045	1.062
9.25	3.2	1.4	1.045	0.060	0.026	1.043	1.056
9.75	3.0	1.2	1.042	0.056	0.023	1.040	1.049
10.25	2.6	1.2	1.046	0.048	0.022	1.045	1.053
10.75	2.1	0.9	1.039	0.040	0.018	1.038	1.034
11.25	1.2	0.7	1.029	0.023	0.014	1.028	0.997
11.75	0.0	0.5	1.014	0.002	0.011	1.014	0.971
12.25	-0.1	0.7	1.009	0.002	0.014	1.009	0.973
14.25	-0.3	0.6	1.013	0.007	0.012	1.013	0.978

Further post processing of these data enables computation of thrust and torque coefficients for comparison with balance measured quantities. In addition, section lift and drag coefficients can be estimated. The methods used are those of Reid [11].

Each radial blade element is considered in isolation. Elemental torque is the product of mass rate and change of tangential velocity due to the element. The mass rate is related to axial velocity. Combining these, the torque can be expressed in terms of velocity components in non-dimensional form as in equation (5.4).

$$\frac{dC_Q}{dy} = \frac{\pi y^2 J^2}{4} \left(\frac{u}{V_T} \right) \left(\frac{w}{V_T} \right) \quad (5.4)$$

Elemental thrust estimation is based on the static pressure rise due to the element without any change in local axial velocity. To estimate the static pressure rise the swirl dynamic pressure must be subtracted from the local total pressure coefficient. The elemental thrust coefficient is:

$$\frac{dT}{dy} = \frac{\pi y J^2}{4} \left[H - 1 - \left(\frac{w}{V_T} \right)^2 \right] \quad (5.5)$$

Consider Figure 5.7 in which an isolated blade element is shown operating at angle of attack. With all the velocity components and geometry known, section lift and drag coefficients may be estimated using equations (5.6).

$$C_L = \frac{4}{BJ^2 (b/D) \left[\left(\frac{\pi s w}{J V_T} \right)^2 + \left(\frac{u}{V_T} \right)^2 \right]} \left[\frac{dC_T}{ds} \cos \phi + \frac{2}{s} \frac{dC_Q}{ds} \sin \phi \right]$$

$$C_D = \frac{4}{BJ^2 (b/D) \left[\left(\frac{\pi s w}{J V_T} \right)^2 + \left(\frac{u}{V_T} \right)^2 \right]} \left[\frac{2}{s} \frac{dC_Q}{ds} \cos \phi - \frac{dC_T}{ds} \sin \phi \right] \quad (5.6)$$

Equations (5.4) through (5.6) are used in a FORTRAN program called WAKE to perform the post-processing calculations. PROGRAM WAKE reads in the necessary data from file WAKDAT and writes the results to a file associated with FORTRAN unit 6. The results include spanwise gradient of thrust and torque coefficients and section lift and drag coefficients as functions of radial position. In addition, PROGRAM WAKE computes integrated coefficients using a trapezoidal scheme of integration.

Figure 5.8 shows how spanwise thrust and torque gradients vary with radius. For the advance ratio tested, the integrated thrust and torque coefficients obtained by the wake survey are approximately fifteen percent below those measured using the balance. There are two explanations for this. The five-hole probe was located approximately thirty millimetres aft of the propeller trailing edge. Therefore, due to the radial inflow velocity component, the pressure coefficients detected by the probe are not those induced by the blade section corresponding to probe position. Secondly, any thrust and torque effects due to the large hub and spinner cannot be accounted for in this wake survey but are, of course, buried in the balance data.

The integrated lift coefficient of 0.47 is similar to typical values observed for conventional propellers [11]. The section drag coefficients must be discounted due to the observed, and not unexpected, scatter. The integrated drag coefficient is at least of the same order of magnitude as the assumed value used in the prediction, though nothing more can be said for it.

Figure 5.8 Spanwise Variation of Torque and Thrust

It is important to note that due to time constraints, a calibration check for the five-hole probe was not performed and it is suspected that the probe axis may not have been parallel with the test section centreline. It is evident from the pressure coefficient data of Table 5.2 that far outside the propeller arc the probe is either misaligned or the streamlines are significantly curved. In any case the total pressure coefficient in this region should certainly be unity, though such is not the case. Consequently, the data were "zero-shifted" for processing by an amount equal to the discrepancies noted well outside the propeller wake. The validity of such a technique is questionable, at best. Running the tunnel without the propeller mounted would have enabled a calibration and alignment check.

5.3 Experimental Uncertainties

There are a number of sources of error in the experiment about which some comment should be made. These include uncertainties in the model and the experimental data.

The final shape of the completed propeller was not accurately determined due to lack of a suitable method. It is believed that the design airfoil shape is reasonably correct but that the pitch may not be. The rigidity of the foam core during the epoxy curing period is not known. An error in pitch is one suspected cause for the measured maximum net propeller

efficiency to have occurred at an advance ratio somewhat greater than design.

Calibration of both the balance and the five-hole probe provide uncertainties in the experimental data. Unfortunately, there was neither the time nor the means to perform in-situ calibrations during the test. Reliance was therefore placed upon calibrations previously performed by the staff of the NRC.

Both sets of data are influenced by wind tunnel velocity measurement. This is obtained using total pressure and temperature transducers in the settling chamber, a correction factor for the screens and contraction, a static pressure tap in the working section and a reading of current barometric pressure.

According to the manufacturer, the balance has less than 0.25% coupling between any two components. Furthermore, the thrust limits are +150 lb_f with torque limits of +1500 in.lb_f. The low coupling and adequate balance sensitivity in the load ranges encountered during this test have been verified by NRC according to R. H. Wickens in private communication with the author.

When a propeller is tested in a solid wall wind tunnel the gross test section velocity is affected in two ways. First, the usual effect of blockage due to the presence of a model and its associated support structure tends to locally accelerate the flow to satisfy continuity. Secondly, a propeller produces a wake of increased velocity and therefore causes the tunnel velocity to decrease, again to satisfy continuity. This propeller

wake blockage is in the opposite sense to that of a conventional wake from, say, a wing model. These two effects of solid and wake blockage therefore tend to offset each other. As previously stated, there were no wall interference or blockage corrections made for this test. This is subsequently justified by estimating the corrections as described by Pope and Rae [13] based on the test geometry and loadings. These estimates suggest that the tunnel velocity is altered by less than 1%. This is further substantiated by applying the method of Pankhurst [14].

One further area of uncertainty is the back pressure in the propeller hub. This may indeed have a significant effect on the measured thrust due to the large hub size relative to the propeller diameter. To measure this a pressure transducer should be mounted on the front face of the motor though this was not feasible for this test.

CHAPTER VI - CONCLUSIONS

The major conclusion to be drawn from this experiment is that the present theory inadequately predicts the performance of the twisted delta propeller. Balance and wake survey data are in reasonable agreement and show convincingly that the prediction grossly underestimates both thrust and torque. The notable exception to the poor predictions is that of propulsive efficiency. Good low speed efficiency suggests that the potential exists for good high speed efficiency. This can only be verified by a test conducted at sufficiently high Mach number. The disagreement between predicted and observed optimum advance ratio can be attributed to construction inaccuracies in pitch and the basis for selection of optimum pitch-diameter ratio which uses an assumption of negligible angle of attack.

The discrepancy between balance and wake survey results is most probably due to the presence of the large hub and spinner. It is not possible to quantify these effects satisfactorily although one can speculate that both torque and thrust are increased due to the presence of hub and spinner. Hub skin friction and blade-hub interference could conceivably contribute sufficient torque to match the wake results with the balance results. Similarly, the pressure distribution over a large

spinner could make up the difference in thrust coefficient. In addition, recall that the wake survey results are in error due to radial inflow and the raw data may be in error due to probe misalignment or calibration.

Initially, future work should be directed at improving the performance prediction of a twisted delta propeller operating at low speed. This could likely be accomplished using a vortex lattice method. It may be possible to then incorporate some compressibility corrections to extend the theory for transonic propeller operation. Though not strictly valid, one can argue that the portion of propeller blade operating close to Mach number unity is very small and therefore will not introduce large error. Certainly such a method contains less approximation than the present slender body analysis. Hanson [15,16] has taken essentially this approach and reports good agreement for some prop-fan configurations.

To follow up this effort, a transonic test using a fixed pitch propeller of small diameter is recommended. Building and testing a very small propeller requiring relatively little power, most likely provided by an air turbine, would be sufficient to gain some measure of the benefits of the twisted delta in terms of propulsive efficiency.

For the low speed test the construction method employed proved to be cheap, robust and sufficiently accurate. To improve accuracy would require more sophisticated jigs and an improved epoxy/fibre layup method such as vacuum bags. Even with these

improvements, it is unlikely that a propeller thin enough for transonic testing, with a diameter of say thirty centimetres, can be built accurately using this technique. It is certainly a viable method for quickly constructing somewhat thicker bodies for wind tunnel testing, as it is ideally suited to a "one-off" project. The reason for this is that there are no molds to fabricate. This is indeed the very reason for its popularity with amateur aircraft builders.

Although this test has demonstrated a number of shortcomings with the present theory, it has nevertheless shown that the twisted delta propeller has the potential to offer high propulsive efficiency at transonic speeds while reducing the structural difficulties associated with swept propeller blades.

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PROGRAM HSPROP

```
* THIS PROGRAM PERFORMS PRELIMINARY DESIGN POINT CALCULATIONS FOR
* A HIGH CRUISE MACH NUMBER FIXED PITCH PROPELLER OF TRIANGULAR
* PLANFORM. THE DESIGN CONDITIONS CAN BE VARIED ACCORDING TO MACH
* NUMBER, ADVANCE RATIO, PROPELLER DIAMETER, AMBIENT CONDITIONS,
* AND DETAILED CHARACTERISTICS OF THE PROPELLER. IT IS IMPORTANT
* TO RECALL THAT THE THEORETICAL BASIS FOR THIS PROGRAM IS APPROX-
* IMATE ONLY. IT IS BASED ON TWO PAPERS BY RIBNER: "A TRANSONIC
* PROPELLER OF TRIANGULAR PLANFORM", NACA TECHNICAL NOTE 1303,
* 1947; "A TRANSONIC PROPELLER OF TRIANGULAR PLANFORM (AN
* EXTENSION)", UTIAS TECHNICAL NOTE, 1978. THE PROGRAM IS INTENDED
* TO ASSIST IN THE DESIGN OF A PROPELLER MODEL FOR TESTING IN A
* WIND TUNNEL.
```

```
*
      IMPLICIT REAL(A-H), INTEGER(I), REAL(J-Z)
      REAL X(12), R(12), S(10), BWT(11), IBC, XH(1), F(1), YX(1)
      REAL Y(12,12), Z(12,12), THETA(12), RC(5), YC(11,5), ZC(11,5)
      REAL EL(12), ALPHA(12), CHRD(12), AFLX(17), AFLY(17)
      REAL YU(12,12), ZU(12,12), YL(12,12), ZL(12,12)
      REAL XCAM(5), ALPHAC(5), CHRDC(5), DEG(12)
      REAL YUC(11,5), ZUC(11,5), YLC(12,5), ZLC(12,5)
      DATA PI/3.14159265/

* ASK THE USER IF HE WANTS THE ORDINATES CALCULATED
PRINT*, 'DO YOU WANT TO CALCULATE COORDINATES ON THIS RUN?'
PRINT*, '(TYPE 0 FOR NO; 1 FOR YES)'
READ*, REPLY

* DESIGN POINT AMBIENT CONDITIONS
P=1.018E05
T=291.17
GASR=287
GAMMA=1.4
RO=P/(GASR*T)

* READ IN THE PROPELLER CONSTANTS: DIAMETER-D; HUB DIAMETER-DH;
* CAMBER PARAMETER-TAU; ADVANCE RATIO-J; MACH NUMBER-M; INCIDENT
* NORMAL MACH NUMBER-MN)
      READ(5,15) DIN, DDEL, DFIN, DH, TAU
      15 FORMAT(5F10.6)
      READ(5,16) JIN, JDEL, JFIN, MIN, MDEL, MFIN, MN
      16 FORMAT(7F10.6)

* INITIALIZE THE VALUES FOR D, J AND M
      D=DIN
      J=JIN
      M=MIN

* DERIVED CONSTANTS (SONIC SPEED - C; CRUISE SPEED - V; ROTATIONAL
* SPEEDS: REVS/SEC - N; REVS/MIN - RPM; RADS/SEC - OMEGA)
      C=(GAMMA*GASR*T)**0.5
      100 V=M*C
      N=V/(J*D)
      RPM=N*60
      OMEGA=N*2*PI

* TIP MACH NUMBER
      VT=(V**2+(D/2*OMEGA)**2)**0.5
      MT=VT/C
```

```

* SWEEP ANGLE - VARIES WITH RADIUS TO MAINTAIN A CONSTANT VALUE
* OF THE NORMAL INCIDENT MACH NUMBER
* COMPUTE THE ROOT AND TIP SWEEP ANGLES ONLY.
  LAMDAR=180/PI*((M/MN)**2-1)**0.5
  LAMDAT=180/PI*((M/MN)**2-(1/(1+(D/2*OMEGA/(C*M))**2)))**0.5
*
* DEFINE THE DISCRETE RADIAL POINTS IN ARRAY R(I)
  R(1)=0.0
  DO 10 I=2,11
10  R(I)=(I-1)*0.1*D/2
  R(12)=R(11)
*
* USE SUBROUTINE RUNGE TO NUMERICALLY SOLVE THE DIFFERENTIAL EQUATION
* DESCRIBING THE CHORD AS A FUNCTION OF RADIUS. (SEE SUBROUTINE DOC-
* UMENTATION FOR DESCRIPTION).
  I=1
  X(1)=0.0
  IN=1
  IM=0
  RX=0.
  DR=0.1*D/2
  YX(1)=0.0
20  IF (RX-(D/2))21,21,22
21  CALL RUNGE(IN,YX,F,RX,DR,IM,IK)
  IF (IK,EQ,1) THEN
    F(1)=((M/MN)**2-(1/(1+(RX*OMEGA/(C*M))**2)))**0.5
    GOTO 21
  ELSE
* FILL ARRAY X(I) AS VALUES OF X ARE RETURNED FROM THE SUBROUTINE.
    I=I+1
    X(I)=YX(1)
    GOTO 20
  ENDIF
*
* NOW ADD THE TRAILING EDGE EXTENSION. THIS IS A FIXED FRACTION OF
* THE TIP RADIUS.
22  XTE=0.025*D
  X(12)=X(11)+XTE
*
* USE SUBROUTINE INTGRT TO CALCULATE THE AREA INCLUDING THE HUB
* FOR USE IN CALCULATING THE ASPECT RATIO
  CALL INTGRT(X,R,11,S)
  AREA=0
  DO 30 I=1,10
30  AREA=AREA+2*S(I)
  AREA=2*X(12)*R(12)-AREA
*
* COMPUTE THE ASPECT RATIO
  AR=D**2/AREA
* COMPARE THIS WITH THE SUBSONIC LIMITING ASPECT RATIO FOR THE DESIGN
* MACH NUMBER. (THIS IS THE LIMIT OF APPLICABILITY FOR THE SLENDER
* BODY THEORY).
  ARLIM=1/(1-M**2)**0.5
*
* CALCULATE THE WEIGHTED BLADE AREA - SWT
* FIRST, CALCULATE THE QUANTITY BWT(R)
  DO 40 I=1,11
40  BWT(I)=(1+(R(I)*OMEGA/D)**2)**1.5*(X(12)-X(I))
*
* MUST ESTABLISH THE LOCATION OF THE HUB RADIUS WRT ARRAY R(I)

```



```

50 I=1
   IF((DH/2).GT.R(I))THEN
     BWT(I)=0
     I=I+1
     GOTO 50
   ENDIF
R(I)=DH/2

*
* NEED TO CALL SUBROUTINE RUNGE AGAIN TO DETERMINE THE VALUE OF THE
* CHORD AT THE HUB RADIUS.
  RX=R(I-1)
  IN=1
  IM=0
  DR=R(I)-R(I-1)
  XH(1)=X(I-1)
55 CALL RUNGE(IN,XH,F,RX,DR,IM,IK)
   IF(IK.EQ.1)THEN
     F(1)=((H/MN)**2-(1/(1+(RX*OMEGA/(C*M))**2)))**0.5
     GOTO 55
   ELSE
* NOW REDEFINE X(I) TO CORRESPOND TO THE NEWLY CALCULATED HUB CHORD.
     X(I)=XH(1)
   ENDIF
  BWT(I)=(1+(R(I)*OMEGA/V)**2)**1.5*(X(12)-X(I))
* NOW USE SUBROUTINE INTGRT TO CALCULATE SWT
  CALL INTGRT(BWT,R,11,S)
  SWT=0
  DO 60 I=1,10
60  SWT=SWT+2*S(I)

*
* EVALUATION OF ROLLING MOMENT PARAMETER - I(BC), INCLUDING
* EFFECTIVE SLEEP ANGLE. THE MEAN VALUE OF MACH NUMBER USED
* IS THAT CORRESPONDING TO 90% RADIUS SINCE MOST OF THE LOADING
* OCCURS NEAR THE BLADE TIP.
  MBAR=(V**2+(0.9*OMEGA*D/2)**2)**0.5/C
  CEBAR=(H/MBAR)*1/TAN(MN/MBAR)
  BBAR=(ABS(MBAR**2-1))**0.5
  BCBAR=BBAR*CEBAR
  IBC=-0.1454*BCBAR+1.0163

*
* CAMBER PARAMETERS (IE. TORQUE CORRECTION)
  EPS=ACOS(TAU)
  IF(TAU.EQ.1)THEN-
    QOQ=1.
  ELSE
    QOQ=1.-((2*EPS-(4./3)*SIN(2*EPS)+(1./6)*SIN(4*EPS)))/
    (2*EPS-SIN(2*EPS))
  ENDIF

*
* NOW CALCULATE THE OPTIMUM PITCH/DIAMETER RATIO - P/D
* NEED ESTIMATE OF DRAG COEFFICIENT CD. ESTIMATED CD=0.0100
  CD=0.0100
  JPD=1-1.0159*J*(CD*SWT/(D**2*IBC*QOQ))**0.5
  POPT=J/JPD*D
  POD=POPT/D

*
* CAN NOW ESTIMATE THE PERFORMANCE OF THE PROPELLER
* POWER ABSORPTION - P (HP); POWER COEF. - CP; THRUST COEF. - CT;
* EFFICIENCY - ETA; TORQUE - Q (N-M.); IDEAL THRUST - TI (N);
* EFFECTIVE DRAG - DE (N); THRUST - T (N)

```

```

Q=PI*RO*D**4*OMEGA*V*(1-JPD)*IBC*QQO/128
P=Q*OMEGA/746
TI=PI*RO*OMEGA**2*D**4*(1-JPD**2)*QQO*IBC/256
DE=CD*0.5*RO*V**2*SWT
T=TI-DE
ETA=T*V/(Q*OMEGA)
PCOEF=(Q*OMEGA)/(RO*(RPM/60.0)**3.*D**5.)
TCOEF=T/(FO*(RPM/60.0)**2.*D**4.)

```

```

*
* WRITE TO OUTPUT FILE IN UNIT 6
  WRITE(6,23)
* 23 FORMAT(/'PROPELLER DESIGN POINT')
  WRITE(6,24)
* 24 FORMAT(/,' D',8X,'DH',7X,'TAU',6X,'J',7X,'M',7X,'MN',7X,'C',
    07X,'RO')
  WRITE(6,25)D,DH,TAU,J,M,MN,C,RO
* 25 FORMAT(F6.4,3X,F6.4,3X,F6.4,3X,F5.3,3X,F6.4,2X,F6.4,3X,F6.1,
    03X,F6.4)
  WRITE(6,34)
* 34 FORMAT(/,' RPM',8X,'MT',6X,'LAMDAR',4X,'LAMDAT',4X,'AR',
    06X,'ARLIM')
  WRITE(6,35)RPM,MT,LAMDAR,LAMDAT,AR,ARLIM
* 35 FORMAT(F7.0,5X,F6.4,5X,F4.1,5X,F4.1,4X,F6.3,3X,F6.3)
  WRITE(6,44)
* 44 FORMAT(/,' P/D',7X,'P',9X,'CP',8X,'CT',8X,'ETA')
  WRITE(6,45)POD,P,PCOEF,TCOEF,ETA
* 45 FORMAT(F6.3,3X,F8.3,2X,F7.3,3X,F6.3,3X,F7.3)

```

```

* IF(REPLY)300,300,200
* NOW CALCULATE THE CARTESIAN COORDINATES FOR THE HIGH SPEED PROP.
* THE X COORDINATES ARE CONTAINED IN THE ARRAY X(I). THE Y AND Z
* COORDINATES ARE CONTAINED IN THE ARRAYS Y(I,J) AND Z(I,J) WHERE
* I REPRESENTS THE LOCAL RADIAL POSITION AND J REPRESENTS THE LOCAL
* X-WISE POSITION. THE ORIGIN IS LOCATED AT THE APEX OF THE PROP
* WITH THE Y AXIS POINTING TOWARDS THE TRAILING EDGE.

```

```

* COMPUTE THE HELICOID ANGLES SETTING THETA=0 AT THE TRAILING EDGE.

```

```

200 DO 140 I=1,12
  THETA(I)=2*PI*(X(I)-X(12))/POFT

```

```

140 DEG(I)=THETA(I)*180.0/PI

```

```

* WRITE THE HELICOID ANGLES (IN DEGREES) TO OUTPUT FILE ON UNIT 7

```

```

  WRITE(7,142)
* 142 FORMAT(/'MEAN SURFACE HELICOID ANGLES (DEGREES)')
  DO 143 I=1,12

```

```

143 WRITE(7,144)I,DEG(I)

```

```

144 FORMAT(I2,4X,F5.1)

```

```

* COMPUTE THE CARTESIAN COORDINATES OF THE MEAN SURFACE..

```

```

DO 145 I=1,12
  Y(I,1)=0.0

```

```

145 Z(I,1)=0.0

```

```

DO 150 I=2,12
  IYZ=2/

```

```

160 IF(IYZ.LE.I)THEN

```

```

  Y(I,IYZ)=R(IYZ)*COS(THETA(I))
  Z(I,IYZ)=R(IYZ)*SIN(THETA(I))

```

```

  IYZ=IYZ+1

```

```

  GOTO 160

```

```

  ENDF

```

```

150 CONTINUE

```

2

OF/DE

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

```

* NOW WRITE THE COORDINATES TO AN OUTPUT FILE ON UNIT 7
* WRITE(7,155)
155 FORMAT(/'MEAN SURFACE COORDINATES (NO CAMBER)')
DO 170 I=2,12
  WRITE(7,180)X(I)
180 FORMAT(3X,'X=',E10.4,8X,'Y',13X,'Z',13X,'R')
  DO 190 IYZ=2,I
    WRITE(7,195)Y(I,IYZ),Z(I,IYZ),R(IYZ)
195 FORMAT(18X,E10.4,4X,E10.4,4X,E10.4)
170 CONTINUE

* COMPUTE THE COORDINATES IN THE CAMBERED REGION IF ANY.
* IF(TAU.EQ.1)GOTO 295
  WRITE(7,210)
210 FORMAT(/'MEAN SURFACE COORDINATES IN CAMBERED REGION')
* CALCULATE THE CONSTANT K NEEDED TO COMPUTE CAMBER CURVATURE.
* (NOTE THAT THE VALUE OF SWEEP ANGLE USED IS THAT AT THE TIP.)
  K=PI**2*(1-JPD)*TAN(LAMDAT)/(DJ*COS(EPS)*(2*EPS-SIN(2*EPS)))
* FOR EACH X STATION COMPUTE Y AND Z AT 5 POINTS IN THE CAMBERED
* REGION.
  DO 220 I=2,12
    RC(1)=TAU*R(I)
    RC(2)=0.25*(R(I)-RC(1))+RC(1)
    RC(3)=0.50*(R(I)-RC(1))+RC(1)
    RC(4)=0.75*(R(I)-RC(1))+RC(1)
    RC(5)=R(I)
    WRITE(7,230)X(I)
230 FORMAT(3X,'X=',E10.4,8X,'YC',12X,'ZC')
    DO 240 IC=1,5
      THETAC=THETA(I)-K*(RC(IC)-2*RC(1)+RC(1)**2/RC(IC))
      YC(I,IC)=RC(IC)*COS(THETAC)
      ZC(I,IC)=RC(IC)*SIN(THETAC)
      WRITE(7,250)YC(I,IC),ZC(I,IC)
250 FORMAT(18X,E10.4,4X,E10.4)
240 CONTINUE
220 CONTINUE

* CALCULATE THE SURFACE COORDINATES OF THE PROP WITHOUT CAMBER.
* CALCULATE THE TRUE CHORD LENGTHS FOR EACH RADIAL STATION.
295 DO 310 I=1,12
  EL(I)=2*R(I)*SIN(THETA(I)/2)
  ALPHA(I)=ATAN(EL(I)/(X(12)-X(I)))
310 CHRDI=(X(12)-X(I))/COS(ALPHA(I))

* READ IN THE AIRFOIL COORDINATES FROM THE DATA FILE ON UNIT 8.
  READ(8,320)(AFLX(I),AFly(I),I=1,17)
320 FORMAT(2F7.3)

* FOR EACH EXISTING MEAN SURFACE POINT CALCULATE SEMI-THICKNESS BY
* INTERPOLATING BETWEEN KNOWN POINTS IN THE DATA FILE. THIS REQUIRES
* COMPUTING THE CHORDWISE POSITION OF EACH MEAN SURFACE POINT.
  DO 330 I=1,12
    IYZ=1
400 IF(IYZ.EQ.12)THEN
  CFRAC=0.0
  GOTO 405
ENDIF
  CFRAC=((X(I)-X(IYZ))/COS(ALPHA(IYZ)))/CHRD(IYZ)*100.0

```

```

* CFAC IS THE CHORDWISE POSITION OF THE MEAN SURFACE POINT DESCRIBED
* BY THE ORDINATE PAIR (I,IYZ) EXPRESSED IN PERCENT LOCAL CHORD.
* COMPARE CFAC TO AIRFOIL CHORD VALUES LISTED IN AFLX(I).
405 IAFLX=1
340 IF(AFLX(IAFLX+2).LT.CFAC)THEN
    IAFLX=IAFLX+1
    GOTO 340
ENDIF
* INTERPOLATE BETWEEN KNOWN POINTS TO COMPUTE LOCAL SEMI-THICKNESS
* USING SUBROUTINE LAGRAN.
350 IF((IAFLX+2).GT.(17))THEN
    IAFLX=IAFLX-1
    GOTO 350
ENDIF
    CALL LAGRAN(AFLX,AFLY,IAFLX,A,B,C)
    T=(A*CFAC**2+B*CFAC+C)/100.0*CHRD(IYZ)
* NOTE:VALUE OF T IS DIVIDED BY 100 TO CONVERT FROM PERCENT THICKNESS.
* NEED LOCAL MEAN SURFACE SLOPE PHI TO COMPUTE COORDINATES.
* (IN THE UNCAMBERED CASE THIS IS THE SAME AS THETA).
    PHI=THETA(I)
* NOW COMPUTE THE SURFACE COORDINATES.
    YU(I,IYZ)=Y(I,IYZ)-T*SIN(PHI)
    ZU(I,IYZ)=Z(I,IYZ)+T*COS(PHI)
    YL(I,IYZ)=Y(I,IYZ)+T*SIN(PHI)
    ZL(I,IYZ)=Z(I,IYZ)-T*COS(PHI)
    IYZ=IYZ+1
    IF(IYZ.LE.I)THEN
        GOTO 400
    ELSE
        IYZ=1
    ENDIF
330 CONTINUE
    YU(12,12)=0.0
    ZU(12,12)=0.0
    YL(12,12)=0.0
    ZL(12,12)=0.0
*
* NOW WRITE THE COORDINATES TO AN OUTPUT FILE ON UNIT 7
    WRITE(7,410)
410 FORMAT(/'UPPER SURFACE COORDINATES (NO CAMBER)')
    DO 420 I=1,12
        WRITE(7,430)X(I)
430 FORMAT(3X,'X=',E10.4,7X,'YU',13X,'ZU')
        DO 440 IYZ=1,I
            WRITE(7,450)YU(I,IYZ),ZU(I,IYZ)
450 FORMAT(18X,E10.4,4X,E10.4)
420 CONTINUE
*
    WRITE(7,460)
460 FORMAT(/'LOWER SURFACE COORDINATES (NO CAMBER)')
    DO 470 I=1,12
        WRITE(7,480)X(I)
480 FORMAT(3X,'X=',E10.4,7X,'YL',13X,'ZL')
        DO 490 IYZ=1,I
            WRITE(7,500)YL(I,IYZ),ZL(I,IYZ)
490 FORMAT(18X,E10.4,4X,E10.4)
500
470 CONTINUE
*
* COMPUTE SURFACE COORDINATES IN THE CAMBERED REGION.

```

* THIS IS DONE IN THE SAME MANNER AS FOR THE BASIC HELICOID (UNCAMBERED) EXCEPT THAT THE CHORD AT EACH STATION MUST BE APPROXIMATED.
 * THIS IS DONE BY A LINEAR INTERPOLATION.

```

*
  DO 510 I=2,12
    RC(1)=TAU*R(I)
    RC(2)=0.25*(R(I)-RC(1))+RC(1)
    RC(3)=0.50*(R(I)-RC(1))+RC(1)
    RC(4)=0.75*(R(I)-RC(1))+RC(1)
    RC(5)=R(I)
  * DETERMINE THE FIRST KNOWN VALUE OF R INBOARD OF RC(1)
    IYZ=12
    520 IF(R(IYZ).GT.RC(1))THEN
      IYZ=IYZ-1
      GOTO 520
    ENDIF
  *
  * USE LINEAR INTERPOLATION TO EVALUATE THE CHORD LENGTHS AT EACH
  * VALUE OF RC.
    DO 530 IC=1,5
      540 IF(RC(IC).LE.R(IYZ+1))THEN
        XCAM(IC)=X(IYZ)+(RC(IC)-R(IYZ))/(R(IYZ+1)-R(IYZ))
        @ *(X(IYZ+1)-X(IYZ))
        CHRDC(IC)=CHRD(IYZ)-(RC(IC)-R(IYZ))/(R(IYZ+1)-R(IYZ))
        @ *(CHRD(IYZ)-CHRD(IYZ+1))
        @ ALPHAC(IC)=ALPHA(IYZ)+(RC(IC)-R(IYZ))/(R(IYZ+1)-R(IYZ))
        @ *(ALPHA(IYZ+1)-ALPHA(IYZ))
        IF(X(I).EQ.X(12))THEN
          CFRAC=0.0
          GOTO 545
        ENDIF
        CFRAC=((X(I)-XCAM(IC))/COS(ALPHAC(IC)))/CHRDC(IC)*100.0
        ELSE
          IYZ=IYZ+1
          GOTO 540
        ENDIF
      545 IAFLX=1
      560 IF(AFLX(IAFLX+2).LT.CFRAC)THEN
        IAFLX=IAFLX+1
        GOTO 560
      ENDIF
      570 IF((IAFLX+2).GT.(17))THEN
        IAFLX=IAFLX-1
        GOTO 570
      ENDIF
      CALL LAGRAN(AFLX,AFLY,IAFLX,A,B,C)
      T=(A*CFRAC**2+B*CFRAC+C)/100.0*CHRDC(IC)
  * THE MEAN SURFACE SLOPE DIFFERS FROM THETA IN THE CAMBERED REGION.
      PHI=THETA(I)-2*K*(RC(IC)-RC(1))
      YUC(I,IC)=YC(I,IC)-T*SIN(PHI)
      ZUC(I,IC)=ZC(I,IC)+T*COS(PHI)
      YLC(I,IC)=YC(I,IC)+T*SIN(PHI)
      ZLC(I,IC)=ZC(I,IC)-T*COS(PHI)
    530 CONTINUE
    510 CONTINUE
  *
  * NOW WRITE THE COORDINATES TO AN OUTPUT FILE ON UNIT 7
    WRITE(7,610)
    610 FORMAT(/'UPPER SURFACE COORDINATES CAMBERED REGION (RADIALS)')
    DO 620 I=2,12

```

```

WRITE(7,630)X(I)
630 FORMAT(3X,'X=',E10.4,6X,'YUC',11X,'ZUC')
      DO 640 IC=1,5
640   WRITE(7,650)YUC(I,IC),ZUC(I,IC)
650   FORMAT(18X,E10.4,4X,E10.4)
620 CONTINUE
*
      WRITE(7,660)
660 FORMAT(/'LOWER SURFACE COORDINATES CAMBERED REGION (RADIALS)')
      DO 670 I=2,12
        WRITE(7,680)X(I)
680   FORMAT(3X,'X=',E10.4,6X,'YLC',12X,'ZLC')
        DO 690 IC=1,5
690    WRITE(7,700)YLC(I,IC),ZLC(I,IC)
700    FORMAT(18X,E10.4,4X,E10.4)
670 CONTINUE
*
* REPEAT THE CALCULATIONS FOR THE NEXT DESIGN POINT (IE. CHANGE THE
* VALUE OF D, J OR M).
300 IF(J.LT.JFIN)THEN
      J=J+JDEL
      GOTO 100
    ELSE
      J=JIN
    ENDIF
    IF(M.LT.MFIN)THEN
      M=M+MDEL
      GOTO 100
    ELSE
      M=MIN
    ENDIF
    IF(D.LT.DFIN)THEN
      D=D+DDEL
      GOTO 100
    ENDIF
    STOP
  END
*****
SUBROUTINE INTGRT(F,V,N,R)
* PERFORMS INTEGRATION OF FUNCTION F WITH RESPECT TO INTERVALS
* DESCRIBED BY ARRAY V FOR THE NUMBER OF POINTS SPECIFIED BY N.
* THE METHOD CALLS ON SUBROUTINE LAGRAN TO FIT A QUADRATIC POLY-
* NOMIAL THROUGH THREE VALUES OF F. THE INTEGRAL IS THEN EVALUATED
* SEPARATELY FOR THE TWO INTERVALS OVER WHICH THE QUADRATIC IS
* DESCRIBED. THE SECOND AND THIRD POINTS ARE THEN USED AS THE FIRST
* AND SECOND POINTS OF THE NEXT SET OF THREE DESCRIBING ANOTHER
* POLYNOMIAL. THE FINAL INTEGRAL FOR THE OVERLAPPING INTERVAL IS
* THEN THE AVERAGE OF THE VALUES OBTAINED FROM EACH POLYNOMIAL. (THIS
* IS NOT POSSIBLE FOR THE FIRST AND LAST INTERVALS).
      REAL F(10),V(10),R(10),A,B,C
      INTEGER I,N
      R(1)=0.
      DO 10 I=1,N-2
        CALL LAGRAN(V,F,I,A,B,C)
* SINCE THE POLYNOMIAL IS OF THE FORM AX**2+BX+C THE INTEGRAL IS
* SIMPLY (A/3)X**3+(B/2)X**2+CX
        R(I)=0.5*(R(I)+(A/3.*(V(I+1)**3.-V(I)**3.))+B/2.*(V(I+1)**2.
        -V(I)**2.))+C*(V(I+1)-V(I)))
        R(I+1)=A/3.*(V(I+2)**3.-V(I+1)**3.))+B/2.*(V(I+2)**2.-V(I+1)
        **2.))+C*(V(I+2)-V(I+1))

```

```

10 R(N-1)=R(I+1)
   R(1)=2.*R(1)
* THE ARRAY R NOW CONTAINS THE RESULTS FOR EACH INTERVAL 1 TO (N-1)
   RETURN
END
*****
SUBROUTINE LAGRAN(X,Y,I,A,B,C)
* THIS SUBROUTINE CALCULATES THE COEFFICIENTS A,B,C FOR A LAGRANGE
* QUADRATIC POLYNOMIAL OF THE FORM  $F(X)=AX^2+BX+C$  TO BE USED FOR
* INTERPOLATION OF THE VALUES OF THE ARRAY B1(X) BETWEEN KNOWN VALUES
* Y(I),Y(I+1),Y(I+2) AT LOCATIONS X(I),X(I+1),X(I+2) RESPECTIVELY.
   REAL A,B,C,D1,D2,D3,X(10),Y(10)
* FIRST CALCULATE THE DENOMINATOR FOR EACH TERM
   D1=(X(I)-X(I+1))*(X(I)-X(I+2))
   D2=(X(I+1)-X(I))*(X(I+1)-X(I+2))
   D3=(X(I+2)-X(I))*(X(I+2)-X(I+1))
* NOW CALCULATE A,B,C AND RETURN CONTROL TO CALLING PROGRAM
   A=Y(I)/D1+Y(I+1)/D2+Y(I+2)/D3
   B=-1.*Y(I)*(X(I+1)+X(I+2))/D1-Y(I+1)*(X(I)+X(I+2))/D2-
   Y(I+2)*(X(I)+X(I+1))/D3
   C=Y(I)*X(I+1)*X(I+2)/D1+Y(I+1)*X(I)*X(I+2)/D2+
   Y(I+2)*X(I)*X(I+1)/D3
   RETURN
END
*****
SUBROUTINE RUNGE(N,Y,F,X,H,M,K)
* THIS ROUTINE PERFORMS RUNGE-KUTTA CALCULATIONS BY GILLS METHOD.
* THE METHOD ESSENTIALLY INTEGRATES A FIRST-ORDER EQUATION.
* ARGUMENTS N TO K ARE DESCRIBED AS FOLLOWS IN ORDER OF APPEARANCE.
* N=NUMBER OF D.E.'S TO BE SOLVED
* Y=ARRAY OF N DEPENDENT VARIABLES-INITIAL VALUES SET BY USER
* F=ARRAY OF N DERIVATIVES OF THE VARIABLES Y-USER MUST PROVIDE
*   AN EXPRESSION IN MAIN PROGRAM FOR CALCULATION OF EACH F(I).
* X=INDEPENDENT VARIABLE-INITIALIZED BY THE USER
* H=STEP SIZE DX-SET BY USER
* M=INDEX USED IN THE SUBROUTINE WHICH MUST BE SET TO ZERO BY
*   THE USER BEFORE THE FIRST CALL
* K=INTEGER FROM SUBROUTINE USED TO DETERMINE WHETHER THE
*   SUBROUTINE NEEDS A NEW CALCULATED VALUE OF THE DERIVATIVES
*   OR IS FINISHED WITH THE CURRENT VALUE OF THE INDEPENDENT
*   VARIABLE
* SEE "VISCOUS FLUID FLOW" BY FRANK M. WHITE APPENDIX C FOR COMPLETE
* DESCRIPTION OF THE SUBROUTINE.
*
   DIMENSION Y(1),F(1),Q(1)
   M=M+1
   GOTO (1,4,5,3,7),M
1  DO 2 I=1,N
2  Q(I)=0
   A=0.5
   GOTO 9
3  A=1.707107
4  X=X+0.5*H
5  DO 6 I=1,N
   Y(I)=Y(I)+A*(F(I)*H-Q(I))
6  Q(I)=2.*A*H*F(I)+(1.-3.*A)*Q(I)
   A=0.2928932
   GOTO 9
7  DO 8 I=1,N

```



```
8 Y(I)=Y(I)+H*F(I)/6.-Q(I)/3.  
  M=0  
  K=2  
  GOTO 10  
9 K=1  
10 RETURN  
  END
```

```

PROGRAM WAKE
* POST PROCESSES DATA FROM WAKE SURVEY OF TWISTED DELTA PROF.
REAL Y(20),B(20),S(20),WVT(20),H(20),UVT(20)
REAL DCTDS(20),DCQDS(20),FI(20),CLS(20),CDS(20)
REAL RPM,REVS,J,CTINT,CQINT,CLINT,CDINT,VTUNNL,RP
REAL TERM1,TERM2,TERM3,TERM4,BNO,PI
DATA PI/3.141593/
BNO=2.
RP=12.0
RPM=1200.
REVS=RPM/60.
VTUNNL=123.514
J=VTUNNL/(REVS*RP/6)
CTINT=0.0
CQINT=0.0
CLINT=0.0
CDINT=0.0
WRITE(6,5)
5  FORMAT(/' PT      Y      S=Y/R      B/D      DCTDS'
   &,' DCQDS      FI      CLS      CDS')
DO 10 I=1,20
READ(5,20)Y(I),B(I),WVT(I),H(I),UVT(I)
20  FORMAT(5F6.3)
B(I)=B(I)/(2*RP)
S(I)=Y(I)/RP
DCTDS(I)=PI*0.25*J**2.*S(I)*(H(I)-1.-WVT(I)**2.)
DCQDS(I)=PI*0.25*J**2.*S(I)*S(I)*UVT(I)*WVT(I)
FI(I)=ATAN(UVT(I)/((PI*S(I)/J)-WVT(I)))
IF(B(I).EQ.0.0)THEN
TERM1=0.0
ELSE
TERM1=4./((BNO*J**2.*B(I))
ENDIF
TERM2=DCTDS(I)*COS(FI(I))+DCQDS(I)*SIN(FI(I))*2./S(I)
TERM3=((PI*S(I)/J)-WVT(I))**2.+UVT(I)**2.
CLS(I)=TERM1*TERM2/TERM3
TERM4=DCQDS(I)*COS(FI(I))*2./S(I)-DCTDS(I)*SIN(FI(I))
CDS(I)=TERM1*TERM4/TERM3
IF(I.GT.1)THEN
CTINT=CTINT+(S(I)-S(I-1))*DCTDS(I-1)
CTINT=CTINT+0.5*(DCTDS(I)-DCTDS(I-1))*(S(I)-S(I-1))
CQINT=CQINT+(S(I)-S(I-1))*DCQDS(I-1)
CQINT=CQINT+0.5*(DCQDS(I)-DCQDS(I-1))*(S(I)-S(I-1))
CLINT=CLINT+(S(I)-S(I-1))*CLS(I-1)
CLINT=CLINT+0.5*(CLS(I)-CLS(I-1))*(S(I)-S(I-1))
CDINT=CDINT+(S(I)-S(I-1))*CDS(I-1)
CDINT=CDINT+0.5*(CDS(I)-CDS(I-1))*(S(I)-S(I-1))
ENDIF
WRITE(6,30)I,Y(I),S(I),B(I),DCTDS(I),DCQDS(I),FI(I),
&CLS(I),CDS(I)
30  FORMAT(/I2,2X,F6.2,2X,F6.3,2X,F6.3,2X,F8.4,2X,F8.4,2X,F8.4,
&2X,F8.4,2X,F8.4)
10  CONTINUE
WRITE(6,40)CTINT,CQINT,CLINT,CDINT
40  FORMAT(/' INTEGRATED COEFFICIENTS ARE:  CT=' ,F6.4, '      CQ=' ,F6.4,
&'      CL=' ,F6.4, '      CD=' ,F6.4)
WRITE(6,60)J
60  FORMAT(/' ADVANCE RATIO:  J=' ,F5.3)
STOP
END

```

Appendix III Surface Coordinates

MEAN SURFACE HELICOID ANGLES (DEGREES)

1	-29.2
2	-27.5
3	-26.0
4	-23.7
5	-21.4
6	-18.8
7	-16.0
8	-12.9
9	-9.6
10	-6.1
11	-2.4
12	.0

MEAN SURFACE COORDINATES (NO CAMBER)

X= .1078E-01	Y	Z	R
	.2703E-01	-.1409E-01	.3048E-01
X= .2085E-01	Y	Z	R
	.2740E-01	-.1335E-01	.3048E-01
	.5138E-01	-.2502E-01	.5715E-01
X= .3534E-01	Y	Z	R
	.2791E-01	-.1225E-01	.3048E-01
	.5233E-01	-.2297E-01	.5715E-01
	.8373E-01	-.3675E-01	.9144E-01
X= .4999E-01	Y	Z	R
	.2838E-01	-.1112E-01	.3048E-01
	.5321E-01	-.2085E-01	.5715E-01
	.8513E-01	-.3337E-01	.9144E-01
	.1135E+00	-.4449E-01	.1219E+00
X= .6641E-01	Y	Z	R
	.2885E-01	-.9837E-02	.3048E-01
	.5409E-01	-.1845E-01	.5715E-01
	.8655E-01	-.2951E-01	.9144E-01
	.1154E+00	-.3935E-01	.1219E+00
	.1442E+00	-.4919E-01	.1524E+00
X= .8454E-01	Y	Z	R
	.2930E-01	-.8396E-02	.3048E-01
	.5494E-01	-.1574E-01	.5715E-01
	.8790E-01	-.2519E-01	.9144E-01
	.1172E+00	-.3358E-01	.1219E+00

	.1465E+00	-.4198E-01	.1524E+00
X= .1043E+00	.1758E+00	-.5037E-01	.1829E+00
	Y	Z	R
	.2971E-01	-.6804E-02	.3048E-01
	.5571E-01	-.1276E-01	.5715E-01
	.8913E-01	-.2041E-01	.9144E-01
	.1188E+00	-.2722E-01	.1219E+00
	.1486E+00	-.3402E-01	.1524E+00
	.1783E+00	-.4083E-01	.1829E+00
	.2080E+00	-.4763E-01	.2134E+00
X= .1254E+00	Y	Z	R
	.3005E-01	-.5076E-02	.3048E-01
	.5635E-01	-.9517E-02	.5715E-01
	.9016E-01	-.1523E-01	.9144E-01
	.1202E+00	-.2030E-01	.1219E+00
	.1503E+00	-.2538E-01	.1524E+00
	.1803E+00	-.3045E-01	.1829E+00
	.2104E+00	-.3553E-01	.2134E+00
	.2404E+00	-.4060E-01	.2438E+00
X= .1478E+00	Y	Z	R
	.3031E-01	-.3225E-02	.3048E-01
	.5683E-01	-.6047E-02	.5715E-01
	.9093E-01	-.9675E-02	.9144E-01
	.1212E+00	-.1290E-01	.1219E+00
	.1515E+00	-.1612E-01	.1524E+00
	.1819E+00	-.1935E-01	.1829E+00
	.2122E+00	-.2257E-01	.2134E+00
	.2425E+00	-.2580E-01	.2438E+00
	.2728E+00	-.2902E-01	.2743E+00
X= .1714E+00	Y	Z	R
	.3045E-01	-.1270E-02	.3048E-01
	.5710E-01	-.2381E-02	.5715E-01
	.9136E-01	-.3809E-02	.9144E-01
	.1218E+00	-.5079E-02	.1219E+00
	.1523E+00	-.6349E-02	.1524E+00
	.1827E+00	-.7618E-02	.1829E+00
	.2132E+00	-.8888E-02	.2134E+00
	.2436E+00	-.1016E-01	.2438E+00
	.2741E+00	-.1143E-01	.2743E+00
	.3045E+00	-.1270E-01	.3048E+00
X= .1866E+00	Y	Z	R
	.3048E-01	.0000E+00	.3048E-01
	.5715E-01	.0000E+00	.5715E-01
	.9144E-01	.0000E+00	.9144E-01
	.1219E+00	.0000E+00	.1219E+00
	.1524E+00	.0000E+00	.1524E+00
	.1829E+00	.0000E+00	.1829E+00
	.2134E+00	.0000E+00	.2134E+00
	.2438E+00	.0000E+00	.2438E+00
	.2743E+00	.0000E+00	.2743E+00
	.3048E+00	.0000E+00	.3048E+00
	.3048E+00	.0000E+00	.3048E+00

MEAN SURFACE COORDINATES IN CAMBERED REGION

X= .1078E-01

YC	ZC
.2432E-01	-.1269E-01
.2499E-01	-.1305E-01
.2565E-01	-.1343E-01
.2630E-01	-.1384E-01
.2694E-01	-.1427E-01

X= .2085E-01	YC	ZC
	.4624E-01	-.2252E-01
	.4751E-01	-.2318E-01
	.4874E-01	-.2392E-01
	.4993E-01	-.2474E-01
	.5108E-01	-.2563E-01
X= .3534E-01	YC	ZC
	.7536E-01	-.3307E-01
	.7741E-01	-.3409E-01
	.7937E-01	-.3531E-01
	.8124E-01	-.3673E-01
	.8301E-01	-.3834E-01
X= .4999E-01	YC	ZC
	.1022E+00	-.4004E-01
	.1049E+00	-.4133E-01
	.1078E+00	-.4299E-01
	.1100E+00	-.4500E-01
	.1123E+00	-.4736E-01
X= .6641E-01	YC	ZC
	.1298E+00	-.4427E-01
	.1333E+00	-.4578E-01
	.1366E+00	-.4787E-01
	.1397E+00	-.5053E-01
	.1426E+00	-.5375E-01
X= .8454E-01	YC	ZC
	.1582E+00	-.4534E-01
	.1625E+00	-.4702E-01
	.1665E+00	-.4953E-01
	.1703E+00	-.5288E-01
	.1738E+00	-.5705E-01
X= .1043E+00	YC	ZC
	.1872E+00	-.4287E-01
	.1922E+00	-.4464E-01
	.1970E+00	-.4756E-01
	.2015E+00	-.5164E-01
	.2056E+00	-.5685E-01
X= .1254E+00	YC	ZC
	.2164E+00	-.3654E-01
	.2223E+00	-.3832E-01
	.2279E+00	-.4163E-01
	.2332E+00	-.4646E-01
	.2381E+00	-.5279E-01
X= .1478E+00	YC	ZC
	.2455E+00	-.2612E-01
	.2522E+00	-.2782E-01
	.2587E+00	-.3148E-01
	.2649E+00	-.3707E-01
	.2707E+00	-.4460E-01
X= .1714E+00	YC	ZC
	.2741E+00	-.1143E-01
	.2816E+00	-.1296E-01
	.2891E+00	-.1691E-01
	.2963E+00	-.2328E-01
	.3031E+00	-.3205E-01
X= .1866E+00	YC	ZC
	.2743E+00	.0000E+00
	.2819E+00	-.1213E-02
	.2895E+00	-.4851E-02
	.2970E+00	-.1091E-01
	.3042E+00	-.1939E-01

UPPER SURFACE COORDINATES (NO CAMBER)

X= .0000E+00	YU	ZU
	.0000E+00	.0000E+00
X= .1078E-01	YU	ZU
	.1161E-02	.2227E-03
	.2703E-01	-.1409E-01
X= .2085E-01	YU	ZU
	.1486E-02	.3052E-02
	.2844E-01	-.1122E-01
	.5138E-01	-.2502E-01
X= .3534E-01	YU	ZU
	.1711E-02	.3899E-02
	.2933E-01	-.9008E-02
	.5343E-01	-.2047E-01
	.8373E-01	-.3675E-01
X= .4999E-01	YU	ZU
	.1777E-02	.4535E-02
	.2995E-01	-.7120E-02
	.5456E-01	-.1740E-01
	.8610E-01	-.3089E-01
	.1135E+00	-.4449E-01
X= .6641E-01	YU	ZU
	.1717E-02	.5034E-02
	.3042E-01	-.5236E-02
	.5552E-01	-.1426E-01
	.8773E-01	-.2603E-01
	.1163E+00	-.3679E-01
	.1442E+00	-.4919E-01
X= .8454E-01	YU	ZU
	.1534E-02	.5352E-02
	.3074E-01	-.3388E-02
	.5628E-01	-.1106E-01
	.8910E-01	-.2102E-01
	.1182E+00	-.3004E-01
	.1472E+00	-.3939E-01
	.1758E+00	-.5037E-01
X= .1043E+00	YU	ZU
	.1237E-02	.5402E-02
	.3089E-01	-.1662E-02
	.5683E-01	-.7854E-02
	.9017E-01	-.1588E-01
	.1198E+00	-.2312E-01
	.1494E+00	-.3053E-01
	.1789E+00	-.3827E-01
	.2080E+00	-.4763E-01
X= .1254E+00	YU	ZU
	.8494E-03	.5030E-02
	.3088E-01	-.2133E-03
	.5715E-01	-.4801E-02
	.9092E-01	-.1073E-01
	.1209E+00	-.1608E-01
	.1509E+00	-.2153E-01
	.1809E+00	-.2716E-01
	.2108E+00	-.3311E-01
	.2404E+00	-.4060E-01
X= .1478E+00	YU	ZU
	.4244E-03	.3989E-02
	.3073E-01	-.6863E-03
	.5724E-01	-.2196E-02

	.9133E-01	-.5906E-02
	.1214E+00	-.9229E-02
	.1519E+00	-.1261E-01
	.1822E+00	-.1609E-01
	.2125E+00	-.1974E-01
	.2427E+00	-.2370E-01
	.2728E+00	-.2902E-01
X= .1714E+00	YU	ZU
	.8293E-04	.1989E-02
	.3054E-01	.7054E-03
	.5718E-01	-.4100E-03
	.9144E-01	-.1837E-02
	.1219E+00	-.3107E-02
	.1523E+00	-.4380E-02
	.1828E+00	-.5669E-02
	.2133E+00	-.6991E-02
	.2437E+00	-.8400E-02
	.2741E+00	-.1002E-01
	.3045E+00	-.1270E-01
X= .1866E+00	YU	ZU
	.0000E+00	.1119E-03
	.3048E-01	.1058E-03
	.5715E-01	.1006E-03
	.9144E-01	.9347E-04
	.1219E+00	.8631E-04
	.1524E+00	.7804E-04
	.1829E+00	.6839E-04
	.2134E+00	.5714E-04
	.2438E+00	.4409E-04
	.2743E+00	.2906E-04
	.3048E+00	.1190E-04
	.0000E+00	.0000E+00

LOWER SURFACE COORDINATES (NO CAMBER)

X= .0000E+00	YL	ZL
	.0000E+00	.0000E+00
X= .1078E-01	YL	ZL
	-.1161E-02	-.2227E-02
	.2703E-01	-.1409E-01
X= .2085E-01	YL	ZL
	-.1486E-02	-.3052E-02
	.2637E-01	-.1547E-01
	.5138E-01	-.2502E-01
X= .3534E-01	YL	ZL
	-.1711E-02	-.3899E-02
	.2649E-01	-.1549E-01
	.5123E-01	-.2547E-01
	.8373E-01	-.3675E-01
X= .4999E-01	YL	ZL
	-.1777E-02	-.4535E-02
	.2681E-01	-.1512E-01
	.5186E-01	-.2431E-01
	.8417E-01	-.3584E-01
	.1135E+00	-.4449E-01
X= .6641E-01	YL	ZL
	-.1717E-02	-.5034E-02
	.2728E-01	-.1444E-01
	.5267E-01	-.2263E-01
	.8536E-01	-.3299E-01
	.1145E+00	-.4191E-01

X= .8454E-01	.1442E+00	-.4919E-01
	YL	ZL
	-.1534E-02	-.5352E-02
	.2787E-01	-.1340E-01
	.5360E-01	-.2042E-01
	.8671E-01	-.2935E-01
	.1162E+00	-.3712E-01
	.1458E+00	-.4457E-01
	.1758E+00	-.5037E-01
X= .1043E+00	YL	ZL
	-.1237E-02	-.5402E-02
	.2853E-01	-.1195E-01
	.5458E-01	-.1766E-01
	.8809E-01	-.2495E-01
	.1179E+00	-.3132E-01
	.1478E+00	-.3751E-01
	.1777E+00	-.4338E-01
	.2080E+00	-.4763E-01
X= .1254E+00	YL	ZL
	-.8494E-03	-.5030E-02
	.2923E-01	-.9938E-02
	.5556E-01	-.1423E-01
	.8940E-01	-.1972E-01
	.1195E+00	-.2453E-01
	.1496E+00	-.2923E-01
	.1798E+00	-.3375E-01
	.2100E+00	-.3795E-01
	.2404E+00	-.4060E-01
X= .1478E+00	YL	ZL
	.4244E-03	-.3989E-02
	.2989E-01	-.7136E-02
	.5642E-01	-.9898E-02
	.9053E-01	-.1344E-01
	.1208E+00	-.1657E-01
	.1512E+00	-.1964E-01
	.1815E+00	-.2261E-01
	.2119E+00	-.2541E-01
	.2422E+00	-.2790E-01
	.2728E+00	-.2902E-01
X= .1714E+00	YL	ZL
	.8293E-04	-.1989E-02
	.3037E-01	-.3245E-02
	.5702E-01	-.4351E-02
	.9128E-01	-.5781E-02
	.1217E+00	-.7051E-02
	.1522E+00	-.8317E-02
	.1826E+00	-.9568E-02
	.2131E+00	-.1079E-01
	.2436E+00	-.1192E-01
	.2740E+00	-.1283E-01
	.3045E+00	-.1270E-01
X= .1866E+00	YL	ZL
	.0000E+00	-.1119E-03
	.3048E-01	-.1058E-03
	.5715E-01	-.1006E-03
	.9144E-01	-.9347E-04
	.1219E+00	-.8631E-04
	.1524E+00	-.7804E-04
	.1829E+00	-.6839E-04
	.2134E+00	-.5714E-04

.2438E+00	-.4409E-04
.2743E+00	-.2906E-04
.3048E+00	-.1190E-04
.0000E+00	.0000E+00

UPPER SURFACE COORDINATES CAMBERED REGION (RADIALS)

X= .1078E-01	YUC	ZUC
	.2462E-01	-.1211E-01
	.2524E-01	-.1261E-01
	.2583E-01	-.1314E-01
	.2640E-01	-.1369E-01
	.2694E-01	-.1427E-01
X= .2085E-01	YUC	ZUC
	.4673E-01	-.2152E-01
	.4795E-01	-.2240E-01
	.4909E-01	-.2338E-01
	.5013E-01	-.2446E-01
	.5108E-01	-.2563E-01
X= .3534E-01	YUC	ZUC
	.7593E-01	-.3176E-01
	.7804E-01	-.3295E-01
	.7995E-01	-.3447E-01
	.8162E-01	-.3628E-01
	.8301E-01	-.3834E-01
X= .4999E-01	YUC	ZUC
	.1028E+00	-.3844E-01
	.1056E+00	-.4003E-01
	.1083E+00	-.4196E-01
	.1106E+00	-.4439E-01
	.1123E+00	-.4736E-01
X= .6641E-01	YUC	ZUC
	.1304E+00	-.4244E-01
	.1341E+00	-.4430E-01
	.1375E+00	-.4677E-01
	.1405E+00	-.4982E-01
	.1426E+00	-.5375E-01
X= .8454E-01	YUC	ZUC
	.1588E+00	-.4331E-01
	.1633E+00	-.4538E-01
	.1674E+00	-.4835E-01
	.1711E+00	-.5217E-01
	.1738E+00	-.5705E-01
X= .1043E+00	YUC	ZUC
	.1877E+00	-.4072E-01
	.1931E+00	-.4291E-01
	.1980E+00	-.4634E-01
	.2024E+00	-.5093E-01
	.2056E+00	-.5685E-01
X= .1254E+00	YUC	ZUC
	.2168E+00	-.3438E-01
	.2230E+00	-.3659E-01
	.2288E+00	-.4048E-01
	.2340E+00	-.4580E-01
	.2381E+00	-.5279E-01
X= .1478E+00	YUC	ZUC
	.2457E+00	-.2413E-01
	.2529E+00	-.2622E-01
	.2596E+00	-.3038E-01
	.2657E+00	-.3651E-01
	.2707E+00	-.4460E-01

X= .1714E+00	YUC	ZUC
	.2741E+00	-.1002E-01
	.2821E+00	-.1180E-01
	.2897E+00	-.1612E-01
	.2969E+00	-.2289E-01
	.3031E+00	-.3205E-01
X= .1866E+00	YUC	ZUC
	.2743E+00	.0000E+00
	.2819E+00	-.1213E-02
	.2895E+00	-.4851E-02
	.2970E+00	-.1091E-01
	.2743E+00	-.1939E-01

LOWER SURFACE COORDINATES CAMBERED REGION (RADIALS)

X= .1078E-01	YLC	ZLC
	.2402E-01	-.1326E-01
	.2475E-01	-.1348E-01
	.2547E-01	-.1373E-01
	.2620E-01	-.1399E-01
	.2694E-01	-.1427E-01
X= .2085E-01	YLC	ZLC
	.4575E-01	-.2352E-01
	.4707E-01	-.2397E-01
	.4839E-01	-.2447E-01
	.4972E-01	-.2502E-01
	.5108E-01	-.2563E-01
X= .3534E-01	YLC	ZLC
	.7478E-01	-.3439E-01
	.7677E-01	-.3523E-01
	.7879E-01	-.3615E-01
	.8086E-01	-.3718E-01
	.8301E-01	-.3834E-01
X= .4999E-01	YLC	ZLC
	.1015E+00	-.4164E-01
	.1042E+00	-.4264E-01
	.1068E+00	-.4401E-01
	.1095E+00	-.4560E-01
	.1123E+00	-.4736E-01
X= .6641E-01	YLC	ZLC
	.1292E+00	-.4610E-01
	.1325E+00	-.4727E-01
	.1358E+00	-.4898E-01
	.1390E+00	-.5124E-01
	.1426E+00	-.5375E-01
X= .8454E-01	YLC	ZLC
	.1576E+00	-.4736E-01
	.1617E+00	-.4865E-01
	.1656E+00	-.5072E-01
	.1695E+00	-.5359E-01
	.1738E+00	-.5705E-01
X= .1043E+00	YLC	ZLC
	.1867E+00	-.4501E-01
	.1914E+00	-.4636E-01
	.1961E+00	-.4879E-01
	.2006E+00	-.5234E-01
	.2056E+00	-.5685E-01
X= .1254E+00	YLC	ZLC
	.2160E+00	-.3871E-01
	.2215E+00	-.4006E-01
	.2269E+00	-.4284E-01

	.2323E+00	-.4712E-01
	.2381E+00	-.5279E-01
X= .1478E+00	YLC	ZLC
	.2453E+00	-.2811E-01
	.2516E+00	-.2943E-01
	.2578E+00	-.3258E-01
	.2641E+00	-.3764E-01
	.2707E+00	-.4460E-01
X= .1714E+00	YLC	ZLC
	.2740E+00	-.1283E-01
	.2812E+00	-.1411E-01
	.2884E+00	-.1770E-01
	.2957E+00	-.2366E-01
	.3031E+00	-.3205E-01
X= .1866E+00	YLC	ZLC
	.2743E+00	.0000E+00
	.2819E+00	-.1213E-02
	.2895E+00	-.4851E-02
	.2970E+00	-.1091E-01
	.2743E+00	-.1939E-01

NACA 16 - 006 Airfoil Coordinates

x/c y/100c

0.0,0.0
 1.25,0.646
 2.5,0.903
 5.0,1.255
 7.5,1.516
 10.0,1.729
 15.0,2.067
 20.0,2.332
 30.0,2.709
 40.0,2.927
 50.0,3.000
 60.0,2.917
 70.0,2.635
 80.0,2.099
 90.0,1.259
 95.0,0.707
 100.0,0.060
 0.176

Appendix IV Program WAKE Output

PT	Y	S=Y/R	R/D	DCTDS	DCQDS	FI	CLS	CDS
1	2.75	.229	.268	.0053	.0529	1.4715	.3324	.0293
2	3.25	.271	.261	.0881	.0635	1.4147	.3485	.0099
3	3.75	.313	.253	.1048	.0779	1.3681	.3909	.0003
4	4.25	.354	.245	.1251	.1000	1.3255	.4444	.0121
5	4.75	.396	.237	.1773	.1219	1.2856	.4924	.0025
6	5.25	.438	.228	.1965	.1400	1.2425	.5212	.0158
7	5.75	.479	.219	.2352	.1625	1.2032	.5679	.0192
8	6.25	.521	.209	.3033	.1785	1.1645	.5945	.0061
9	6.75	.563	.197	.3648	.1977	1.1298	.6319	.0238
10	7.25	.604	.186	.3737	.2257	1.0928	.6963	.0099
11	7.75	.646	.174	.3791	.2420	1.0577	.7204	.0323
12	8.25	.688	.161	.4381	.2567	1.0248	.7683	.0118
13	8.75	.729	.147	.4527	.2632	.9901	.7982	.0165
14	9.25	.771	.132	.4488	.2615	.9565	.8138	.0243
15	9.75	.813	.116	.4332	.2694	.9257	.8614	.0573
16	10.25	.854	.099	.4860	.2540	.8975	.9342	.0110
17	10.75	.896	.082	.3866	.2281	.8650	.9023	.0511
18	11.25	.938	.063	.1516	.1337	.8275	.5511	.1435
19	11.75	.979	.043	-.0293	.0072	.7888	.0251	.0764
20	12.25	1.021	.000	-.0077	.0008	.0958	.0000	.0000

INTEGRATED COEFFICIENTS ARE: CT= .2131 CU= .1339 CL= .4658 CD= .0182

ADVANCE RATIO: J=3.088

END

08.08.87

FIN