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**LA THÈSE A ÉTÉ
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CHARACTERIZATION AND DESIGN OF MICROWAVE
CLASS C TRANSISTOR POWER AMPLIFIERS

by

Shamsur Rahman Mazumder, B.Sc. Engg., M.Eng.

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Faculty of Engineering
Carleton University
Ottawa, Ontario
Canada

November 1976

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The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis:

"Characterization and Design of Microwave Class-C Transistor Power Amplifiers"

submitted by Shamsur Rahman Mazumder, M.Eng., in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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December, 1976

"Leave the beaten track occasionally and
dive into the woods. You will be certain
to find something that you have never
seen before"

Alexander Graham Bell

ABSTRACT

The work, reported in this thesis, consists of two parts. First, a method of characterizing the effects of harmonics in a non-linear 2-port is developed. Using this method, the design of the optimum second harmonic terminations is achieved. Second, the nonlinearities in the fundamental frequency response of the 2-port (harmonics having been terminated) are characterized by using a new method of measuring the response of the 2-port under two simultaneously applied signals at the two ports. This method of characterization is used to design optimum Class C power amplifiers.

Two further applications of this new method are demonstrated. An application of the method is in measuring the large-signal S-parameters of a transistor under Class C conditions. The second application of the method is in generating the load-pull data of microwave power transistors.

A procedure of designing the matching networks on microstrip realizing the designed fundamental and the second harmonic impedances is given.

Finally, the predicted and the experimental performances of three Class C amplifiers are compared. Improvements in the performances of the three amplifiers, due to the proper choice of the second harmonic reactance terminations, are demonstrated.

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LIST OF PRINCIPAL SYMBOLS

a_i	incident wave variable at port i
b_i	reflected/transmitted wave variable at port i
$ a_i $	magnitude of a_i
$\angle \frac{a_2}{a_1}$	phase angle of a_2 with respect to a_1
BPF	bandpass filter
DF	describing functions
dB	decibels
d.c.	direct current
f_i	functional relationships
f	frequency in Hz
f_o	fundamental frequency in Hz
F_i	functional relationships
h	substrate thickness
G_p	power gain
Hz	Hertz
I_c	d.c. collector current in amperes
jx	reactance
jx_i	reactance at port i
K	stability factor
l, l_i	physical lengths of transmission line
m	a real number: $\left \frac{S'_{21}/S'_{12}}{S_{21}/S_{12}} \right $

mW	milliwatt
mA	milliampere
M	a real number: $\left \frac{S'_{41}/S'_{14}}{S_{41}/S_{14}} \right $
MDF	multiple input describing function
n	a real number: $ S'_{21}/S_{21} $
N	a real number: $ S'_{41}/S_{14} $
NL	non-linear
P_{dc}	d.c. power dissipated
P_{in}	power input
P_{out}	power output
r_{o2}	centre of constant gain circle
r_{s1}	centre of input stability circle
R_{LP}	equivalent parallel resistance at the output
R_{s1}	radius of input stability circle
R_{s2}	radius of output stability circle
R_{o2}	radius of constant gain circle
RF	radio frequency
R_c	wave impedance of free space
$S_{ij}, i, j=1,4$	S-parameters of a linear 4-port
$S_{ij}, i, j=1,2$	S-parameters of a linear 2-port
$S'_{ij}, i, j=1,2$	effective S-parameters of a non-linear 2-port
T_i	electrical length in degrees of transmission line i
V	volts
V_{CE}	collector-emitter d.c. voltage

V_{BE}	base-emitter d.c. voltage
V_{sat}	RF saturation voltage
v_{ph}	phase velocity
w	width of conductor
Y_0	characteristic admittance of a transmission line
Z_0	characteristic impedance of a transmission line
Z_S	impedance termination at the input port (source)
Z_L	impedance termination at the output port (load)
$Z(\omega_0)$	impedance at frequency ω_0
$Z(2\omega_0)$	impedance at frequency $2\omega_0$
Z_i	characteristic impedance of transmission line i
Γ_S	reflection coefficient of the termination at the input (source)
Γ_L	reflection coefficient of the termination at the output (load)
Γ_{MS}	reflection coefficient of the conjugate matched input termination
Γ_{ML}	reflection coefficient of the conjugate matched output termination
Γ_i	reflection coefficient of the second harmonic termination at port i ($i=3,4$)
ϵ_r	relative permittivity or dielectric constant
ϵ_{eff}	effective dielectric constant
η	conversion efficiency
λ_0	wavelength in free space
λ_g	guide wavelength
θ_i	angle of the reflection coefficient of the termination at port i

θ phase angle between the fundamental and the second harmonic signals

ω frequency in radians

ω_0 fundamental frequency in radians

$2\omega_0$ second harmonic frequency in radians

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The most significant implication of the advent of microwave power transistors is that it is now possible to build straight-through repeaters at microwave frequencies without going through complex processes of down-conversion, up-conversion and frequency multiplication through a long chain of varactor multipliers. In addition to the clear advantage of transistors with respect to the overall DC to RF conversion efficiency, the power transistor chain should lead to a considerable cost saving arising from a reduced number of components and the basic simplicity of the system.

Fig. 1.1 shows the state-of-the-art solid state power devices. The curves are taken from the high frequency portion of the data collected by Herold (1) to which are added the data points as of 1972 (2) and of 1976 (3-6). Herold (1) observed in 1965 that the power-frequency domain below curve A belonged to solid-state devices; the area between curves B and C was still contested between tube devices and solid-state devices; while the region above curve C was predicted to be quite safe for tube devices till 1985. But, in less than ten years, not only have the solid-state devices taken over the contested area but have a deep penetration into what was expected to be a safe haven for microwave tube devices. Transistor development is behind that of IMPATT and GUNN devices, but even transistors have already broken into tube territory above curve C

FREQ. (GHz)	PREDICTED POWER OUTPUT (5 ohm impedance)
4	10 watts
6	7 watts
8	6 watts
10	4.6 watts
12	4 watts

(b)

Fig. 1.1. (a) State-of-the-art microwave solid-state power devices.
 (b) Future of high power microwave transistors [2].

at frequencies below 4GHz as of 1972 and below 6GHz as of 1975. A complete 5 watt power transistor chain at 4GHz has been available since 1972 (2-4) and 1 watt at 10GHz has been reported in 1975 (6) and 1976 (5).

Commonly, microwave power transistors are operated at large-signal levels and are low impedance (high VSWR) devices. This leads to high sensitivity of the device response to the external matching impedances. Such high sensitive behaviour gives rise to degradation of device performance, makes the characterization problem difficult, and also makes matching networks design complicated.

At present, one major trend common to all of the major microwave transistor manufacturers is the internally-matched transistor (4,7) in which matching circuitry consisting of MOS capacitors and bonding wires is incorporated right into the package. Some of the advantages of internally-matched devices are (4,7):

- (i) Bandwidth is increased by a factor of 2.
- (ii) Collector efficiency is improved 5 to 10%. (This means lower junction temperatures and improved reliability.)
- (iii) Higher impedance level of these devices greatly simplifies the external matching required by circuit designers (hence reduced costs).

Microwave power transistor manufacturers do have a long way to go, however, before they start approaching theoretical limitations - based on the maximum energy transfer capability of high voltage transistors. "It is theoretically possible to deliver 157W of power per PF

(picofarad) of output capacitance from a 10GHz power transistor and 15.7W/PF at 100GHz." These estimates (3) are based on the maximum breakdown field and the scattered minimum velocity of the electrons in the semiconductor. These values, of course, are well beyond what anyone has achieved to date, but a 4W transistor at 12GHz is considered to be an achievable goal (2).

For microwave applications of power transistors, three operating modes for amplifiers are generally used which are: class A, class B and class C. These three modes of operation are established by using the proper bias arrangement. For example, in class A: the device conducts for the complete RF cycle. In class B, the device conducts only for half of the RF input cycle, whereas in the class C, the device conducts for only a portion less than half cycle of the RF input. Consequently, there are some basic differences among the three classes which limit their possible applications.

A class C amplifier offers the highest power output in terms of cost. It also offers the highest efficiency in terms of DC power input versus RF power output. However, under class C conditions, the device is highly non-linear and the dynamic range for a class C amplifier is very low (usually not more than 6dB).

A class B amplifier offers less power output and less efficiency compared to the class C conditions of the same device. Thus, for a given power output, the class B amplifier is more expensive as either larger or more transistors have to be used to produce the same power.

compared to class C. But, class B amplifiers have a larger dynamic range (typically 20dB) than the class C amplifier though, under class B, the device is also non-linear.

A class A amplifier is the best performing amplifier in terms of constant gain over a large dynamic range (typically 50dB). The same or equivalent transistor, as compared to its use in class C or class B amplifiers, will produce much less power output and will also be the least efficient. As a rough measure, the equivalent transistor operating in class A will produce, at saturation, only about one-fourth of the RF power output operating in class C (8).

Fig. 1.2 shows the trade-offs in dynamic range and saturated power output for the three classes of amplifiers (8). In each case, the same or equivalent transistors are used for the comparison. In essence, the costs are held constant and the performance is the variable parameter in the comparison. In Table 1.3, a summary of the capabilities of the three amplifier classes (8) is presented.

Power transistors used in power amplifiers are inherently large-signal (non-linear) devices. Qualitatively, it means that the device response is dependent on a number of parameters such as power level, circuit impedances, harmonic frequency impedances, etc. Thus, for a successful and accurate design of microwave power amplifiers, a non-linear model is necessary which is capable of predicting the important performance characteristics of a power amplifier with respect to power level, circuit impedances (at fundamental and harmonic frequencies) and power dissipation in the device, etc. Some of the important performance

Fig. 1.2. Comparison of dynamic range and saturated power output for classes A, B and C, using the same or equivalent transistors in each case [8].

AMPL. TYPE	RELATIVE COST *	AMPLIFIER CHARACTERISTICS: APPLICATIONS:
"C"	low	<ol style="list-style-type: none"> 1. CW signals 2. CW signals with phase or frequency modulation 3. Highly non-linear 4. Dynamic range less than 6dB
"B"	moderate	<ol style="list-style-type: none"> 1. CW signals 2. CW signals with phase or frequency modulation 3. AM signals where AM distortion can be tolerated 4. Non-linear 5. Dynamic range less than 20dB
"A"	high	<ol style="list-style-type: none"> 1. CW signals 2. Complex signals with any modulation 3. Third order intermod. product low 4. Linear for low power levels, but non-linear at high power levels 5. Dynamic range, typically 50dB

* Assuming microwave frequencies and maximum power output from a given transistor.

Table 1.3. Summary of the characteristics and types of applications of three classes of amplifier configurations (8).

characteristics of a power amplifier are: power output, DC to RF conversion efficiency, power gain and stability.

Compared to the transistor development, relatively little has been achieved on non-linear modelling of the device for circuit design. Presently available methods of characterization and amplifier design are mainly based on 'cut and try', 'equivalent linearization' and empirical or approximate device-physics models for computer-aided design, and invariably need significant final 'tuning'.

Broadly speaking, there are two approaches to device modelling, namely:

- (i) The physics oriented approach which has two forms, such as:
 - (a) purely device-physics models and
 - (b) equivalent-circuit models; and
- (ii) the 'black box' approach.

(i) Physics oriented approach

This approach is based on the physical mechanisms that lead to the device operating characteristics. In the purely device-physics models, the physical mechanisms of the device are represented in terms of the basic partial differential equations such as Poisson's equation and the diffusion equation, which govern the behaviour of the charge carriers in the semiconductor material. These equations are solved numerically with boundary conditions applied at the electrical terminals. The essential input quantities are the doping concentrations, the geometry and the applied bias. The output quantities are the electric field, the carrier concentrations and the current densities. From these,

the currents at the terminals and the signal delay times, etc. are calculated. In principle, all linear and non-linear effects can be explicitly included and the device behaviour can be calculated to any accuracy desired. However, in practice, the models have been generally limited to one dimension, since the solution of the basic equations in two or three dimensions has proved prohibitive in computer time and storage capacity (18).

Another form of the physics-oriented approach yields what is known as the 'equivalent-circuit models' (18). In this approach, the physical processes inside the device are not represented explicitly. Instead, the device is modelled by a network of lumped or distributed elements which represent, in an approximate manner, the various three dimensional mechanisms associated with device operation. In most cases, practical considerations require a simplified two- or one-dimensional model. Resistors, capacitors, diodes, dependent sources are commonly used in the models. The 'equivalent-circuit' models are more suitable for computer modelling and simulation because of the relatively reduced complexity required for good accuracy, compared to the explicit device-physics models.

In any case, the physics-oriented modelling approach yields circuit models with a large number of elements, some of which are to be determined by approximate curve-fitting of a set of experimental results. Modelling the parasitic effects also requires a large number of components, thus making the overall model a complicated one (53). Another

disadvantage of the physics oriented approach is that it is difficult to formulate a systematic procedure that would apply to a large variety of devices.

(ii) The 'black box' approach

This approach is based on the data measured from the device terminals. In contrast to the heavy reliance on physical principles as in the physics-oriented approach, only system- and circuit-theoretic principles are invoked in the 'black box' approach. While the physics-oriented approach facilitates device design and relating the device performances to the process parameters; for circuit design purposes, the 'black box' approach is found to be more convenient and economical in terms of design effort.

1.2 THESIS OBJECTIVE

The aim of this thesis is threefold. Firstly, to develop a systematic and accurate method of characterizing the non-linear effects in microwave transistors when operating under class C conditions.

Secondly, to develop a design approach for class C power amplifiers based on this method of characterization. The design approach should enable one to predict the important performance characteristics of the power amplifier.

Finally, to demonstrate the effectiveness of the characterization and design approach by constructing a number of microwave transistor class C amplifiers.

Throughout this work, the 'black box' approach, in which the transistor under class C conditions is used as a non-linear 2-port, will be adopted.

CHAPTER 2

REVIEW OF CURRENT METHODS

2.1 INTRODUCTION

Although extensive characterization and design procedures have been developed for transistors operating at small-signal levels (9,10), no similar work has been done for transistors operating in class C modes. Since the operating characteristics of large-signal devices are a function of both the signal level and the circuit impedances, most designs have been based on 'cut and try', 'equivalent linearization' or empirical design (15,23) techniques.

In this Chapter, a brief review of two of the current experimental approaches, for microwave transistor power amplifiers, will be presented. The two methods to be reviewed are:

- (i) large-signal S-parameters method and
- (ii) load-pull characterization method.

At the end, the strategy of a non-linear characterization and design approach to be presented in this thesis will be described.

2.2 LARGE-SIGNAL S-PARAMETERS METHOD

2.2.1 Principle

This method is based on 'equivalent linearization' or 'describing function' technique, widely used in non-linear control systems (11).

The general theory of 'describing function' states that (11): "a non-linearity N , subject to n input signals of known form, is replaced by n different quasi-linear approximations N_i , one for each input.

These quasi-linear approximators are chosen such that the differences between the sum of their outputs and the output of the non-linearity N is minimized in the least square sense. The impulse responses of these quasi-linear approximators are related to the describing functions in the same manner as the impulse responses of linear circuits are related to their transfer functions." The above statements are illustrated in Fig. 2.1.

The scope of utilization of describing function technique can be illustrated by different types of 'describing functions' (11) such as:

- (i) Describing function (DF) or single-sinusoid-input describing function;
- (ii) Multiple-sinusoid-input describing function (MDF) and
- (iii) Random-input describing function, etc.

The particular type of describing function to be used is dictated by the 'form' of the actual input to the non-linearity (i.e. $x(t)$ in Fig. 2.1).

The characterization assuming single-sinusoid-input describing function is the basis of the 'large-signal S-parameters method'. Thus, in the large-signal S-parameters method, at a particular bias, frequency and input power level (or current through the device), the non-linear device is approximated by a linear one; and it is characterized by S-parameters (12-15). Thus, the design may now be carried out by using the standard linear amplifier design approach (9,10,16) using S-parameters. The definitions of the standard S-parameters used to characterize a linear 2-port are shown in Fig. 2.2. The measurement of the four S-parameters is accomplished by using the standard system using the Network

The approximators N_i 's are chosen so that:

$$\epsilon = \sum_j [y(t_j) - y_a(t_j)]^2$$

is minimum.

Fig. 2.1. General principle of describing function technique [11].

a_1, b_1 's are wave variables (26,27)

The linear 2-port is characterized by:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (2-1)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (2-2)$$

Definition of S_{ij} 's

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2 = 0}$$

$$S_{12} = \left. \frac{b_1}{a_1} \right|_{a_1 = 0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2 = 0}$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1 = 0}$$

$a_1 = 0$, means port 1 is terminated at the characteristic impedance.

Note: (i) while measuring S_{11}, S_{21} ; signal is applied at port 1 and port 2 is terminated at the characteristic impedance.

(ii) while measuring S_{12}, S_{22} ; signal is applied at port 2 and port 1 is terminated at the characteristic impedance.

Fig. 2.2 Definition of 4 S-parameters used to Characterize a Linear 2-port.

Analyzer (19,20) as shown in Fig. 2.3(b).

2.2.2 Assumptions and Limitations

There are two basic assumptions inherent in the large-signal S-parameters method, such as:

- (i) the device is only 'slightly' non-linear and also
- (ii) the large-signal S-parameters are not a function of the reference impedance of measurement.

Some of the limitations and disadvantages of the large-signal S-parameters method are briefly noted in the following:

- (a) Though the measurement of the four S-parameters is now standard (19,20), there are difficulties in measuring the two parameters S_{12} and S_{22} in the cases of class B and class C bias conditions.

To understand these difficulties, we may recall (Fig. 2.2) that while measuring the two parameters S_{12} and S_{22} the signal is to be applied at the output port and the input port being terminated at the characteristic impedance (usually 50 ohms). When this is done, the d.c. condition of a class B or class C transistor is no longer the same as that of the intended amplifier. Thus, the parameters S_{12} and S_{22} , measured by the standard S-parameters method, are not realistic ones. Müller (12) suggested a method to measure S_{12} and S_{22} of transistors under class C conditions (for $f < 1\text{GHz}$) by the following procedure. The incident and reflected waves are measured twice with two slightly different values of the load admittance. Thus, using the two linear equa-

(a)

(b)

Fig. 2.3. (a). Test setup for load-pull data.
 (b). Standard S-parameter measurement system schematic.

tions, shown in Fig. 2.2, the two parameters S_{12} and S_{22} are expressed (12) as:

$$S_{12} = \frac{b'_1 - b_1}{a'_2 - a_2} \quad (2.3)$$

and

$$S_{22} = \frac{b'_2 - b_2}{a'_2 - a_2} \quad (2.4)$$

where the primed parameters correspond to the case when the load admittance is perturbed, and a_1 is kept the same in both load admittance cases. Since the perturbation in the load admittance has to be necessarily small, this approach will require the phase measuring device to have a good angle resolution (less than 0.2 degrees). In any case, the calibration procedure and the measurement procedure are quite involved (12).

- (b) Though the 'S-parameters design method' (9,10) easily yields gain and stability information, it does not, however, predict the most important performance parameters of a power amplifier, such as power output, conversion efficiency, etc. Design for specified power output was reported (14,15) with limited success and by the use of some approximate relations (such as

$$P_{out} = \frac{(V_{cc} - V_{sat})^2}{2R_L},$$

whose validity is generally not acceptable.

(c) The result of the design will be poor when the device is 'significantly' non-linear (in other words, if the harmonic content is 'high'). This is due to the fact that the theory of single-sinusoid-input describing function fails (11) in such situations. Consequently, considerations of the multiple-sinusoid-input describing function becomes a necessity.

2.3. LOAD-PULL CHARACTERIZATION METHOD

2.3.1 Principle

This technique was originally developed for the interstage matching between a varactor multiplier and a transistor power stage (23) and was later applied to the broadband optimization of class C and class A transistor power amplifiers (7,22). Conceptually, the method is an extension of the technique of 'tuning' the active device in a working circuit (Fig. 2.3(a)) to a specific operating condition (for example, power input, power output, current through the device, etc.); then removing the tuning networks, and using a network analyzer system (Fig. 2.3(b)) to characterize the tuners alone. From a large number of measured data, constant-power output, constant-current, constant-gain, etc. locii are plotted on the Smith chart. From such information, the 'optimum' terminations of the amplifier are chosen.

2.3.2 Limitations

This method is presently considered to be the most elaborate one, and known variously as 'load-pull characterization' (22,23) or 'power-load contours' (24) method. However, there are some limitations and

disadvantages of this method, as noted below.

- (i) The primary objective of maintaining a condition of constant power output or constant current or constant gain, over a range of load impedances sufficient to produce a closed contour on a Smith chart, is an overwhelming task when attempting to use manual measurement techniques. The same effort at several discrete frequencies, for broadband characterization, can be prohibitive. An automatic impedance tuning apparatus coupled with a self-optimizing search algorithm (24) can remove the burden of the required effort but may not be economical.
- (ii) Since the performance characteristics of a non-linear device are dependent on harmonic impedance terminations, the 'tuners' have to be of a complicated type (25) to enable wide variation of the harmonic terminations to achieve optimum performance.
- (iii) In addition to the fact that the experiment is laborious, if extreme care is not taken, 'connection-disconnection errors' are introduced; since every time a required operating condition is established, the tuners are to be disconnected for characterization by a separate measurement system.
- (iv) This method is incapable of predicting stability since no information on the reverse transmission through the device is obtained.

2.4 A NOVEL METHOD PROPOSED IN THIS THESIS

It has been established, thus far, that for a successful, systematic and accurate design of microwave transistor power amplifiers, a

non-linear characterization and design approach is essential but currently not available. Such an approach should be capable of predicting the important performance characteristics of a power amplifier with respect to the power level, circuit impedances and power dissipation in the device.

It is observed (54) that in a microwave transistor operated as a highly non-linear device (such as under class C conditions), the fundamental frequency response of the device is influenced significantly by the harmonic frequency impedance terminations. Thus, a method of characterizing the effects of harmonics in a non-linear 2-port has been developed. The details of this method are presented in Chapter 3. This method enables one to study the effects of harmonic terminations on the fundamental frequency parameters and provides an approach to designing the 'proper' harmonic terminations. The possibility of using this approach to design transistor frequency multipliers is also investigated.

Once the non-linear device is terminated in the proper harmonic impedances, the next step is to characterize the non-linear response of the device at the fundamental frequency of operation. For this purpose, a very general method of characterizing the non-linearity in the fundamental frequency response of a non-linear 2-port has been developed, as will be presented in Chapter 4 in detail. This method is based on a simple approach by which the load impedance of the 2-port is 'simulated' practically by applying a signal (at the same frequency as that of the input signal), at the output port. This signal can be conveniently

derived from the input signal of the 2-port. By such a technique, it is possible to eliminate the necessity of 'tuners' which are indispensable in the conventional load-pull characterization method. This method has resulted in a number of outcomes. These outcomes will be explored in detail in Chapter 4. However, the most important outcome is that this principle of simulating an impedance electrically, makes it possible to generate the load-pull data of a transistor in an easy and fast manner, without necessitating any complicated or laborious experimental procedures.

The above two methods, in combination, form a systematic approach to the characterization and design of microwave transistor class C amplifiers.

The results of amplifier design, employing this approach, using a number of transistors are presented in this thesis.

The principle of the complete approach necessarily requires that the matching networks be capable of presenting the designed fundamental and second harmonic frequency impedances at the device terminals. For the purpose of realizing such matching networks on microstrip, a topology similar to the elliptic function filters, is used for convenience.

2.5 THESIS ORGANIZATION

The contents of this thesis are organized in the following manner.

In Chapter 1, state-of-the-art information, different types of power amplifiers and the requirements of a desirable method of non-linear characterization and design of microwave transistor power amplifiers are discussed.

In Chapter 2, some of the current approaches are reviewed briefly, and the strategy of a method proposed in this thesis is given.

In Chapter 3, a method of characterizing the effects of harmonics in a non-linear 2-port is proposed. A method of analysis to study the effects of second harmonic reactance terminations is described. Considerations for optimal design of the second harmonic terminations are discussed. Results of a number of microwave transistors are presented. The possibility of using this approach to design frequency multipliers is also discussed.

In Chapter 4, a method of characterizing the non-linearity in the fundamental frequency response of a non-linear 2-port is proposed. The application of this method to design optimum power amplifiers is given for a number of microwave transistors. It is shown that this method provides a very realistic approach to measuring the large-signal S-parameters, especially S_{12} and S_{22} , of a transistor under class C conditions. It is also shown that the principle of this method provides a very fast and easy method of generating the load-pull data for microwave power transistors.

In Chapter 5, a method of designing microstrip matching networks, which present some specified fundamental and second harmonic frequency impedances to the device terminals is given.

In Chapter 6, a comparison of the predicted and experimental performances of a number of power amplifiers, is presented.

Finally, the conclusions and topics of further investigations are discussed in Chapter 7.

CHAPTER 3

CHARACTERIZATION OF THE EFFECTS OF HARMONICS IN A NON-LINEAR 2-PORT

3.1 INTRODUCTION

In the following, a non-linear 2-port (Fig. 3.1) whose port variables are expressed in terms of power waves (26,27) will be considered. Thus, corresponding to a single frequency incident wave a_i (excitation), the reflected/transmitted waves (response) b_j will contain harmonic components. For amplifier design, any non-harmonically related components that may arise from 'parametric instabilities' or sub-harmonic generations (28) will not be considered, as these are deleterious to amplifier performance and should not be present in correctly designed amplifiers (28).

Any changes in the terminations of the non-linear 2-port, which alter the impedance to harmonic waves while maintaining the same fundamental wave impedance, result in changing values of harmonic wave components and, most importantly, in different values of the fundamental wave component. As a result, circuit designs that are based only on fundamental frequency impedance and ignore the significant effects of harmonic frequency impedances often lead to less than optimum performance. The major cause of such difficulty may be eliminated if the effects of harmonic impedances on the fundamental frequency performance are characterized.

Incident waves:

$$a_i = A_i e^{j\omega t} ; i = 1, 2$$

Reflected/Transmitted waves:

$$b_k = B_k e^{j\omega t} + \sum_{n=2}^{\infty} B_{kn} e^{jn\omega t} ; k = 1, 2$$

Fig. 3.1. A non-linear 2-port whose port variables are expressed in terms of power waves, $a_i, b_j; i, j = 1, 2$.

In this Chapter, a method of characterizing the effects of harmonics in a non-linear 2-port is proposed. This method is based on the theory of 'multiple-sinusoid-input describing function' (11) and conceptually is similar to the method followed by Brackett (29) for characterizing the second harmonic effects in IMPATT diode oscillators. Brackett's approach is, however, based on an equivalent circuit model rather than on measured parameters.

For simplicity, it will be assumed that only the second harmonic frequency terminations have significant effects on the fundamental frequency response of the non-linear 2-port. The second and third harmonic contents in a number of transistors under class C conditions were measured as shown in Fig. 3.2(b). The increasing current (I_C) levels were established by increasing the input RF drive power. It is clear that the second harmonic components are higher than the third harmonics and thus are expected to have more significant effects on the fundamental frequency response of the devices (63).

The theory of the proposed method is developed in Section 3.2.1. To measure the parameters, a measurement system employing two-sinusoid-input signal is described in Section 3.2.3. A method of analysis to study the effects of second harmonic reactance terminations on the fundamental frequency parameters is developed in Section 3.2.4. Based on the results of this analysis, some important considerations in designing the optimum second harmonic reactances are discussed in Section 3.2.6. Finally, the possibility of using this approach to design frequency doublers is investigated in Section 3.3.

(a)

Fig. 3.2. Second and third harmonic contents with respect to the fundamental power (0dB) for three transistors using the setup in (a).

3.2 CHARACTERIZATION OF THE SECOND HARMONIC EFFECTS IN A NON-LINEAR 2-PORT

3.2.1 Theory: Second Harmonic Effects

The proposed method is based on the theory of multiple-sinusoid-input describing function techniques (11) as briefly described in Section 2.2. The approach is to consider a non-linear 2-port to be equivalent to a linear 4-port as shown in Fig. 3.3, such that:

- Port 1: corresponds to input port at the fundamental frequency;
- Port 2: corresponds to output port at the fundamental frequency;
- Port 3: corresponds to input port at the second harmonic frequency;
- Port 4: corresponds to output port at the second harmonic frequency.

The above consideration is based on the assumption that each of the four reflection/transmission parameters of the non-linear 2-port is considered to consist of 4 parts. For example, the input reflection coefficient $\frac{b_1}{a_1}$ at the fundamental frequency of the non-linear 2-port consists of:

- (i) input reflection coefficient (S_{11}) due to the fundamental frequency signal at the input port (all other ports being terminated at the characteristic impedance, say 50 ohms);
- (ii) contribution (through reverse transmission) due to the fundamental frequency signal at the output port (i.e. S_{12});
- (iii) contribution (through 'coupling' or 'interaction') due to the second harmonic signal at the input port (effect of

Fig. 3.3. Equivalent linear 4-port representation of a non-linear 2-port (to represent the effects of second harmonic impedances on the fundamental frequency response).

second harmonic termination at the input port) i.e. S_{13} ,

- (iv) contribution (through 'coupling' or 'interaction') due to the second harmonic signal at the output port (effect of second harmonic termination at the output port) i.e. S_{14} .

Consequently, the linear 4-port can be described by the following 4 linear equations or 16 S-parameters S_{ij} ; $i, j = 1-4$, where the parameters S_{ij} ; $i=1,2, j=3,4$ and S_{ij} ; $i=3,4, j=1,2$ may be termed 'coupling parameters', shown by dotted boxes in (3.2) :

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + S_{14}a_4 \quad (3.1(a))$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + S_{24}a_4 \quad (3.1(b))$$

$$b_3 = S_{31}a_1 + S_{32}a_2 + S_{33}a_3 + S_{34}a_4 \quad (3.1(c))$$

$$b_4 = S_{41}a_1 + S_{42}a_2 + S_{43}a_3 + S_{44}a_4 \quad (3.1(d))$$

or in matrix form:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \boxed{S_{13}} & \boxed{S_{14}} \\ S_{21} & S_{22} & \boxed{S_{23}} & \boxed{S_{24}} \\ \boxed{S_{31}} & \boxed{S_{32}} & S_{33} & S_{34} \\ \boxed{S_{41}} & \boxed{S_{42}} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad (3.2)$$

It will be remembered that the power wave components a_1, b_1, a_2 and b_2 are at the fundamental frequency whereas the wave components a_3, b_3, a_4 and b_4 are at the second harmonic frequency.

We now consider that the second harmonic ports 3 and 4 are

terminated such that the reflection coefficients are $\Gamma_3 = \frac{a_3}{b_3}$ and $\Gamma_4 = \frac{a_4}{b_4}$ respectively. Thus, substituting $\frac{a_3}{b_3} = \Gamma_3$ and $\frac{a_4}{b_4} = \Gamma_4$ in (3.1), we can obtain:

$$b_1 = S'_{11}a_1 + S'_{12}a_2 \quad (3.3(a))$$

$$b_2 = S'_{21}a_1 + S'_{22}a_2 \quad (3.3(b))$$

where S'_{ij} ; $i, j=1, 2$ may be termed 'effective S-parameters' of the non-linear 2-port (Fig. 3.3) and they can be expressed as a function of Γ_3 , Γ_4 and S_{ij} ; $i, j=1-4$ (i.e. the linear 4-port S-parameters) as shown in (3.4):

$$S'_{11} = S_{11} + \frac{S_{14}S_{41}}{\frac{1}{\Gamma_4} - S_{44}} + \frac{\left[S_{13} + \frac{S_{14}S_{43}}{\frac{1}{\Gamma_4} - S_{44}} \right] \left[S_{31} + \frac{S_{34}S_{41}}{\frac{1}{\Gamma_4} - S_{44}} \right]}{\frac{1}{\Gamma_3} - S_{33} - \frac{S_{34}S_{43}}{\frac{1}{\Gamma_4} - S_{44}}} \quad (3.4(a))$$

$$S'_{12} = S_{12} + \frac{S_{14}S_{42}}{\frac{1}{\Gamma_4} - S_{44}} + \frac{\left[S_{13} + \frac{S_{14}S_{43}}{\frac{1}{\Gamma_4} - S_{44}} \right] \left[S_{32} + \frac{S_{34}S_{42}}{\frac{1}{\Gamma_4} - S_{44}} \right]}{\frac{1}{\Gamma_3} - S_{33} - \frac{S_{34}S_{43}}{\frac{1}{\Gamma_4} - S_{44}}} \quad (3.4(b))$$

$$S'_{21} = S_{21} + \frac{S_{24}S_{41}}{\frac{1}{\Gamma_4} - S_{44}} + \frac{\left[S_{23} + \frac{S_{24}S_{43}}{\frac{1}{\Gamma_4} - S_{44}} \right] \left[S_{31} + \frac{S_{34}S_{41}}{\frac{1}{\Gamma_4} - S_{44}} \right]}{\frac{1}{\Gamma_3} - S_{33} - \frac{S_{34}S_{43}}{\frac{1}{\Gamma_4} - S_{44}}} \quad (3.4(c))$$

$$S'_{22} = S_{22} + \frac{S_{24}S_{42}}{\frac{1}{\Gamma_4} - S_{44}} + \frac{\left[S_{23} + \frac{S_{24}S_{43}}{\frac{1}{\Gamma_4} - S_{44}} \right] \left[S_{32} + \frac{S_{34}S_{42}}{\frac{1}{\Gamma_4} - S_{44}} \right]}{\frac{1}{\Gamma_3} - S_{33} - \frac{S_{34}S_{43}}{\frac{1}{\Gamma_4} - S_{44}}} \quad (3.4(d))$$

or, in general, for $i, j=1, 2$,

$$S'_{ij} = S_{ij} + \frac{S_{i4}S_{4j}}{\frac{1}{\Gamma_4} - S_{44}} + \frac{\left[S_{i3} + \frac{S_{i4}S_{43}}{\frac{1}{\Gamma_4} - S_{44}} \right] \left[S_{3j} + \frac{S_{34}S_{4j}}{\frac{1}{\Gamma_4} - S_{44}} \right]}{\frac{1}{\Gamma_3} - S_{33} - \frac{S_{34}S_{43}}{\frac{1}{\Gamma_4} - S_{44}}} \quad (3.5)$$

or

$$S'_{ij} = \frac{a_{ij}\Gamma_3 + b_{ij}\Gamma_4 + c_{ij}\Gamma_3\Gamma_4 + S_{ij}}{d\Gamma_3 + e\Gamma_4 + f\Gamma_3\Gamma_4 + 1} \quad (3.6)$$

where

$$a_{ij} = S_{i3}S_{3j} - S_{ij}S_{33} \quad (3.6(a))$$

$$b_{ij} = S_{i4}S_{4j} - S_{ij}S_{44} \quad (3.6(b))$$

$$c_{ij} = S_{ij}S_{33}S_{44} + S_{i3}S_{34}S_{4j} + S_{i4}S_{43}S_{3j} - S_{ij}S_{34}S_{43} - S_{i3}S_{3j}S_{44} - S_{i4}S_{4j}S_{33} \quad (3.6(c))$$

$$d = -S_{33} \quad (3.6(d))$$

$$e = -S_{44} \quad (3.6(e))$$

$$f = S_{33}S_{44} - S_{34}S_{43} \quad (3.6(f))$$

and S_{ij} 's are the linear 4-port S-parameters in (3.2).

Thus, the non-linear 2-port (Fig. 3.3) is characterized by four 'effective' S-parameters S'_{ij} ; $i, j=1, 2$, which are functions of the second harmonic terminations Γ_3 (at the input port) and Γ_4 (at the output port) as shown in (3.6). Now, if the 16 S-parameters, S_{ij} ; $i, j=1-4$, of the linear 4-port are known, then the effects of the second harmonic terminations on the fundamental frequency parameters S'_{ij} 's can be studied by using (3.5) or (3.6). An analysis for this purpose is developed in Section 3.2.4. The considerations, based on this analysis, which lead to the design of optimum second harmonic terminations will also be discussed. However, in the following section, it is shown how the 16 S-parameters S_{ij} 's can be measured.

3.2.2 Measurement Conditions for the Equivalent 4-Port S-Parameters

The measurement conditions for the 16 S-parameters S_{ij} 's in (3.2) can be derived from (3.1) as shown in Table 3.4. It is observed from these conditions that the 4 parameters S_{ij} ; $i, j=1, 2$ can be measured by operating the 2-port under fundamental frequency signal input conditions only, and the 4 parameters S_{ij} ; $i, j=3, 4$ can be measured by operating the 2-port under second harmonic frequency signal input conditions only; whereas to measure the remaining 8 parameters (the 'coupling' parameters S_{ij} ; $i=1, 2, j=3, 4$ and $i=3, 4, j=1, 2$), the 2-port is to be operated under two-sinusoidal-signal input conditions. However, it is possible to measure all the 16 parameters by operating the 2-port under two-sinusoidal-signal-input conditions. For example, let us consider the ratio $\frac{b_2}{a_1}$ under $a_2=0$ and $a_4=0$ (in Table 3.4). It is

$$\left. \frac{b_1}{a_1} \right|_{\substack{a_2=0, a_3=0 \\ a_4=0}} = S_{11} \equiv \bar{S}_{11},$$

$$\left. \frac{b_1}{a_2} \right|_{\substack{a_1=0, a_3=0 \\ a_4=0}} = S_{12} \equiv \bar{S}_{12}$$

$$\left. \frac{b_2}{a_1} \right|_{\substack{a_2=0, a_3=0 \\ a_4=0}} = S_{21} \equiv \bar{S}_{21},$$

$$\left. \frac{b_2}{a_2} \right|_{\substack{a_1=0, a_3=0 \\ a_4=0}} = S_{22} \equiv \bar{S}_{22}$$

$$\left. \frac{b_3}{a_3} \right|_{\substack{a_1=0, a_2=0 \\ a_4=0}} = S_{33} \equiv \bar{S}_{33},$$

$$\left. \frac{b_3}{a_4} \right|_{\substack{a_1=0, a_2=0 \\ a_3=0}} = S_{34} \equiv \bar{S}_{34}$$

$$\left. \frac{b_4}{a_3} \right|_{\substack{a_1=0, a_2=0 \\ a_4=0}} = S_{43} \equiv \bar{S}_{43},$$

$$\left. \frac{b_4}{a_4} \right|_{\substack{a_1=0, a_2=0 \\ a_3=0}} = S_{44} \equiv \bar{S}_{44}$$

$$\left. \frac{b_1}{a_1} \right|_{\substack{a_2=0 \\ a_4=0}} = S_{11} + S_{13} \frac{a_3}{a_1} \equiv \bar{S}_{11} + \bar{S}_{13} \left| \frac{a_3}{a_1} \right|$$

$$\left. \frac{b_1}{a_2} \right|_{\substack{a_1=0 \\ a_3=0}} = S_{12} + S_{14} \frac{a_4}{a_2} \equiv \bar{S}_{12} + \bar{S}_{14} \left| \frac{a_4}{a_2} \right|$$

$$\left. \frac{b_2}{a_1} \right|_{\substack{a_2=0 \\ a_4=0}} = S_{21} + S_{23} \frac{a_3}{a_1} \equiv \bar{S}_{21} + \bar{S}_{23} \left| \frac{a_3}{a_1} \right|$$

$$\left. \frac{b_2}{a_2} \right|_{\substack{a_1=0 \\ a_3=0}} = S_{22} + S_{24} \frac{a_4}{a_2} \equiv \bar{S}_{22} + \bar{S}_{24} \left| \frac{a_4}{a_2} \right|$$

Table 3.4. Measurement conditions for the equivalent 4-port S-parameters. The measured parameters are indicated by bars. ($a_i=0$ is established by terminating the i th port at the characteristic impedance, say 50 ohms.)

$$\left. \begin{aligned} \frac{b_3}{a_3} &= s_{33} + s_{31} \frac{a_1}{a_3} \\ a_2=0 \\ a_4=0 &\equiv \bar{s}_{33} + \bar{s}_{31} \left| \frac{a_1}{a_3} \right| \end{aligned} \right\}$$

$$\left. \begin{aligned} \frac{b_3}{a_4} &= s_{34} + s_{32} \frac{a_2}{a_4} \\ a_1=0 \\ a_3=0 &\equiv \bar{s}_{34} + \bar{s}_{32} \left| \frac{a_2}{a_4} \right| \end{aligned} \right\}$$

$$\left. \begin{aligned} \frac{b_4}{a_3} &= s_{43} + s_{41} \frac{a_1}{a_3} \\ a_2=0 \\ a_4=0 &\equiv \bar{s}_{43} + \bar{s}_{41} \left| \frac{a_1}{a_3} \right| \end{aligned} \right\}$$

$$\left. \begin{aligned} \frac{b_4}{a_4} &= s_{44} + s_{42} \frac{a_2}{a_4} \\ a_1=0 \\ a_3=0 &\equiv \bar{s}_{44} + \bar{s}_{42} \left| \frac{a_2}{a_4} \right| \end{aligned} \right\}$$

Table 3.4. (cont'd)

Measurement conditions for the equivalent 4-port S-parameters. The measured parameters are indicated by bars. ($a_i=0$ is established by terminating the i th port at the characteristic impedance, say 50 ohms.)

observed that the ratio $\frac{b_2}{a_1} = S_{21} + S_{23} \frac{a_3}{a_1}$, describes a circle on the $\frac{b_2}{a_1}$ plane as a function of the phase angle between a_3 and a_1 (i.e. $\angle \frac{a_3}{a_1}$, the phase angle between the fundamental and the second harmonic frequency signals). Thus, the centre of this circle corresponds to the parameter S_{21} and the radius of the circle gives the magnitude of $S_{23} \frac{a_3}{a_1}$. Now, if the values of a_1 and a_3 (including the phase angle between them) are known, the parameter S_{23} can be calculated. The magnitudes of a_1 and a_3 can be calculated from the corresponding power measurements, since $P_1 = |a_1|^2$ and $P_3 = |a_3|^2$. However, the phase angle between a_1 and a_3 does not seem to be a measurable quantity. In any case, it is shown in the following that under certain conditions the need to know the phase angle between a_1 and a_3 (also between a_2 and a_4) may be avoided. For this purpose, let us consider any arbitrary phase angle θ between a_1 and a_3 (also between a_2 and a_4). Thus, if we consider $\angle \frac{a_3}{a_1} = \angle \frac{a_4}{a_2} = \theta$, then all the 8 'coupling' parameters can be measured with this phase angle θ included in them. For example, in the above case of the ratio $\frac{b_2}{a_1} = S_{21} + S_{23} \frac{a_3}{a_1}$, under the conditions $a_2=0$ and $a_4=0$, the 'coupling' parameter that can be measured is $\bar{S}_{23} \equiv S_{23} e^{j\theta}$. Hence, the desired parameter (S_{23}) in terms of the measured parameter ($\bar{S}_{23}$) is given by $S_{23} = \bar{S}_{23} e^{-j\theta}$.

Following the above procedure for other 'coupling' parameters using the measurement conditions given in Table 3.4, the 4x4 S-parameter matrix shown in (3.2) becomes:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (3.7(a))$$

$$= \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{13}e^{-j\theta} & \bar{S}_{14}e^{-j\theta} \\ \bar{S}_{21} & \bar{S}_{22} & \bar{S}_{23}e^{-j\theta} & \bar{S}_{24}e^{-j\theta} \\ \bar{S}_{31}e^{j\theta} & \bar{S}_{32}e^{j\theta} & \bar{S}_{33} & \bar{S}_{34} \\ \bar{S}_{41}e^{j\theta} & \bar{S}_{42}e^{j\theta} & \bar{S}_{43} & \bar{S}_{44} \end{bmatrix} \quad (3.7(b))$$

where $\bar{S}_{ij}$'s are the measured parameters and the phase angle θ is unknown.

At this point, it is very important to note that the 4 effective S-parameters S'_{ij} (in 3.5) of the non-linear 2-port are independent of the phase angle θ . This can be observed by taking the parameters from (3.7(b)) and substituting in (3.5). For example, using (3.7(b)) in (3.5), we get:

$$S'_{ij} = \bar{S}_{ij} + \frac{\bar{S}_{14}\bar{S}_{4j}}{\frac{1}{\Gamma_4} - \bar{S}_{44}} + \frac{\left[\bar{S}_{13} + \frac{\bar{S}_{14}\bar{S}_{43}}{\frac{1}{\Gamma_4} - \bar{S}_{44}} \right] \left[\bar{S}_{3j} + \frac{\bar{S}_{34}\bar{S}_{4j}}{\frac{1}{\Gamma_4} - \bar{S}_{44}} \right]}{\frac{1}{\Gamma_3} - \bar{S}_{33} - \frac{\bar{S}_{34}\bar{S}_{43}}{\frac{1}{\Gamma_4} - \bar{S}_{44}}}$$

which is the same as (3.5) except that S_{ij} 's are replaced by $\bar{S}_{ij}$'s.

This implies that it is not necessary to know the phase angle θ between a_1 and a_3 (and also between a_2 and a_4). However, it must be remembered

that all the 8 'coupling' parameters must be measured at the same arbitrarily chosen phase angle θ . Now, the question remains, however, as to what arbitrary phase angle θ should be chosen. It was considered that the phase angle should be so adjusted so as to result in the most 'positive' effects of the second harmonic. For example, while measuring the ratio $\frac{b_2}{a_1}$ under $a_2=0$ and $a_4=0$, the phase angle θ is adjusted to the value which corresponds to the largest possible magnitude of $\frac{b_2}{a_1}$, and corresponding to this θ all the other coupling parameters are to be measured.

The measurement procedures, employing the circular locus principle, for all the 16 S-parameters are illustrated in Fig. 3.5.

A measurement system, which was used to measure the equivalent 4-port S-parameters, is described in the following section.

3.2.3 Measurement System for the Equivalent 4-Port S-Parameters

As mentioned in the last section, the 16 S-parameters of the equivalent linear 4-port are to be measured by operating the non-linear 2-port under two-sinusoidal-signal input conditions. A two-sinusoidal-signal source, with adjustable amplitude and phase of both fundamental and second harmonic signals, was realized in a manner shown in Fig. 3.6(a). The complete reflection/transmission measurement system, employing the two-frequency source, is shown in Fig. 3.6(b). The measurement system is calibrated by a procedure similar to the standard S-parameters measurement (19,30-34). The frequency of measurement (either fundamental frequency ω_0 or the second harmonic frequency $2\omega_0$,

Fig. 3.5. Measurement procedures for the sixteen 4-port S-parameters S_{ij} 's, by employing the circular locus principle.

(a)

Fig. 3.6. (a) Realization of a 2-sinusoidal-signal source.
 (b) Measurement system to measure the reflection/transmission parameters under 2-sinusoidal-signal input conditions.

as the case may be) is selected by using the tunable bandpass filter (BPF) shown in Fig. 3.6(b).

The arrangements of the measurement system in Fig. 3.6(b) for measuring the different parameters S_{ij} 's are summarized in a tabulated form as shown in Table 3.7.

3.2.4 Analysis of the Effects of Second Harmonic Reactance Terminations

In order to realize the maximum possible contribution, due to the effects of the second harmonics, to the fundamental frequency response of the non-linear 2-port, clearly the second harmonic terminations are to be 'reactive'; so that no harmonic power is dissipated in the terminations. Thus, let us consider the second harmonic terminations (reflection coefficients) to be $\Gamma_3 = e^{j\theta_3}$ at port 3 (i.e. input port) and $\Gamma_4 = e^{j\theta_4}$ at port 4 (i.e. output port). Consequently, the four 'effective' S-parameters in (3.6) can be expressed as:

$$S'_{ij} = \frac{a_{ij}e^{j\theta_3} + b_{ij}e^{j\theta_4} + c_{ij}e^{j(\theta_3+\theta_4)} + S_{ij}}{de^{j\theta_3} + ee^{j\theta_4} + fe^{j(\theta_3+\theta_4)} + 1} \quad (3.8)$$

where a_{ij} , b_{ij} , c_{ij} , d , e and f are given by (3.6(a)-(f)) in terms of S_{ij} 's, i.e. the measured 4-port S-parameters.

Now, to study the effects of the second harmonic terminations (θ_3, θ_4) on the fundamental frequency parameters (S'_{ij}), let us define the following three conditions to be applied to (3.8):

$$\text{Condition 1: } |S'_{ii}| < 1; i=1,2, \text{ i.e.} \quad (3.9)$$

PARAMETER S_{ij}	BPF at:	REFLECTION OR TRANSMISSION MEASUREMENT	MEASURE: LOCUS OF $b_1 \frac{1}{a_1}$	CENTRE CORRESPONDS TO: S_{ij} Fig. 3.5	RADIUS PROVIDES: S_{ij} Fig. 3.5.
S_{11} & S_{13}	ω_0	reflection	$\left. \frac{b_1}{a_1} \right _{\substack{a_2=0 \\ a_4=0}} = S_{11} + S_{13} \frac{a_3}{a_1}$	S_{11}	S_{13}
S_{22} & S_{24}	ω_0	reflection	$\left. \frac{b_2}{a_2} \right _{\substack{a_1=0 \\ a_3=0}} = S_{22} + S_{24} \frac{a_4}{a_2}$	S_{22}	S_{24}
S_{33} & S_{31}	$2\omega_0$	reflection	$\left. \frac{b_3}{a_3} \right _{\substack{a_2=0 \\ a_4=0}} = S_{33} + S_{31} \frac{a_1}{a_3}$	S_{33}	S_{31}
S_{44} & S_{42}	$2\omega_0$	reflection	$\left. \frac{b_4}{a_4} \right _{\substack{a_1=0 \\ a_3=0}} = S_{44} + S_{42} \frac{a_2}{a_4}$	S_{44}	S_{42}

Table 3.7. Summary of the arrangements of the measurement system in Fig. 3.6(b) to measure the 16 S-parameters of the equivalent linear 4-port. (cont'd)

PARAMETER S_{ij}	BPF at:	REFLECTION OR TRANSMISSION MEASUREMENT	MEASURE: LOCUS OF $b_i \frac{a_j}{a_i} ; i \neq j$	CENTRE CORRESPONDS TO: S_{ij} Fig. 3.5.	RADIUS PROVIDES: S_{ij} Fig. 3.5
$S_{21} \text{ \& } S_{23}$	ω_0	transmission	$\frac{b_2}{a_1} \Big _{\substack{a_2=0 \\ a_4=0}} = S_{21} + S_{23} \frac{a_3}{a_1}$	S_{21}	S_{23}
$S_{12} \text{ \& } S_{14}$	ω_0	transmission	$\frac{b_1}{a_2} \Big _{\substack{a_1=0 \\ a_3=0}} = S_{12} + S_{14} \frac{a_4}{a_2}$	S_{12}	S_{14}
$S_{43} \text{ \& } S_{41}$	$2\omega_0$	transmission	$\frac{b_4}{a_3} \Big _{\substack{a_2=0 \\ a_4=0}} = S_{43} + S_{41} \frac{a_1}{a_3}$	S_{43}	S_{41}
$S_{34} \text{ \& } S_{32}$	$2\omega_0$	transmission	$\frac{b_3}{a_4} \Big _{\substack{a_1=0 \\ a_3=0}} = S_{34} + S_{32} \frac{a_2}{a_4}$	S_{34}	S_{32}

Table 3.7. Summary of the arrangements of the measurement system in Fig. 3.6(b) to measure the 16 S-parameters of the equivalent linear 4-port.

$$(a) \quad |S'_{11}| < 1 \quad (3.9(a))$$

$$(b) \quad |S'_{22}| < 1 \quad (3.9(b))$$

Condition 2:

$$n = \left| \frac{S'_{21}}{S_{21}} \right|; \quad n > 1 \quad (3.10)$$

Condition 3:

$$m = \left| \frac{S'_{21}/S'_{12}}{S_{21}/S_{12}} \right|; \quad m > 1 \quad (3.11)$$

where $|x|$ indicates the 'magnitude' of x .

Condition 1 determines what values of θ_3 and θ_4 , if any, will cause 'negative conductance activity' (NCA) (56) in the device. If the magnitude of $|S'_{ii}|$ is greater than unity, then the input ($i=1$) and/or output ($i=2$) impedances of the non-linear 2-port will possess negative real parts. In amplifier design, the negative conductance activity is avoided, if possible, because transistors having NCA will necessarily require lossy matching networks or sophisticated embedding networks (55).

Condition 2 is defined to obtain an indication of any increase in the transducer gain ($n>1$) of the 2-port.

However, the application of Condition 2 may result in the increase of the magnitude of S'_{12} , the reverse transmission coefficient, thus enhancing the chances of potential instability and making 'tunability' more difficult. The Condition 3 is designed to counteract this possibility. It can be seen that to derive favourable contribution

from the effects of the second harmonic, it is necessary that both m and n are greater than unity and as large as possible.

Now, using the expression for S'_{ij} in (3.8), the three conditions (3.9)-(3.11) become:

Condition 1:

$$|S'_{ii}| = \left| \frac{a_{ii}e^{j\theta_3} + b_{ii}e^{j\theta_4} + c_{ii}e^{j(\theta_3+\theta_4)} + S_{ii}}{d_{ii}e^{j\theta_3} + e_{ii}e^{j\theta_4} + f_{ii}e^{j(\theta_3+\theta_4)} + 1} \right| < 1 \quad (3.12)$$

Condition 2:

$$\left| \frac{S'_{21}}{S_{21}} \right| \equiv n = \left| \frac{a_{21}e^{j\theta_3} + b_{21}e^{j\theta_4} + c_{21}e^{j(\theta_3+\theta_4)} + S_{21}}{S_{21}(d_{21}e^{j\theta_3} + e_{21}e^{j\theta_4} + f_{21}e^{j(\theta_3+\theta_4)} + 1)} \right| \quad (3.13)$$

Condition 3:

$$\left| \frac{S'_{21}/S'_{12}}{S_{21}/S_{12}} \right| \equiv m = \left| \frac{a_{21}e^{j\theta_3} + b_{21}e^{j\theta_4} + c_{21}e^{j(\theta_3+\theta_4)} + S_{21}}{S_{21}/S_{12}(a_{12}e^{j\theta_3} + b_{12}e^{j\theta_4} + c_{12}e^{j(\theta_3+\theta_4)} + 1)} \right| \quad (3.14)$$

where a_{ij} , b_{ij} , c_{ij} , d , e and f are given by (3.6(a)-(f)).

Since the three expressions in (3.12)-(3.14) are of a similar form, we can treat the three conditions together. For this purpose, let us define a function in the following form:

$$F(\theta_3, \theta_4) = \left| \frac{Ae^{j\theta_3} + Be^{j\theta_4} + Ce^{j(\theta_3+\theta_4)} + K_1}{De^{j\theta_3} + Ee^{j\theta_4} + Fe^{j(\theta_3+\theta_4)} + K_2} \right| \leq 1 \quad (3.15)$$

where the coefficients A, B, C, K_1, D, E, F and K_2 corresponding to the three conditions are given in a tabulated form in Table 3.8.

The expression in (3.15) can be converted into a convenient form as shown below:

$$\begin{aligned} & (\alpha_1 \cos \theta_3 + \beta_1 \sin \theta_3 + v_1) \cos \theta_4 \\ & + (\alpha_2 \cos \theta_3 + \beta_2 \sin \theta_3 + v_2) \sin \theta_4 \\ & + (\alpha_3 \cos \theta_3 + \beta_3 \sin \theta_3 + v_3) \geq 0 \end{aligned} \quad (3.16)$$

or

$$H_1 \cos \theta_4 + H_2 \sin \theta_4 + H_3 \geq 0 \quad (3.17)$$

where

$$H_i = \alpha_i \cos \theta_3 + \beta_i \sin \theta_3 + v_i, \quad i=1,2,3 \quad (3.18(a))$$

$$\alpha_1 = 2 \operatorname{Real}(G_1 + G_2) \quad (3.18(b))$$

$$\beta_1 = 2 \operatorname{Imag}(G_1 + G_2) \quad (3.18(c))$$

$$v_1 = 2 \operatorname{Real}(G_3) \quad (3.18(d))$$

$$\alpha_2 = 2 \operatorname{Imag}(G_2 - G_1) \quad (3.18(e))$$

$$\beta_2 = 2 \operatorname{Real}(G_1 - G_2) \quad (3.18(f))$$

$$v_2 = 2 \operatorname{Imag}(G_3) \quad (3.18(g))$$

$$\alpha_3 = 2 \operatorname{Real}(G_4) \quad (3.18(h))$$

CONDITION EQN. (3-9) to (3-11)	A	B	C	K_1	D	E	F	K_2
1(a) Eqn. (3-9(a))	a_{11}	b_{11}	c_{11}	S_{11}	d	e	f	1.0
1(b) Eqn. (3-9(b))	a_{22}	b_{22}	c_{22}	S_{22}	d	e	f	1.0
2 Eqn. (3-10)	a_{21}	b_{21}	c_{21}	S_{21}	ndS_{21}	neS_{21}	nfS_{21}	nS_{21}
3 Eqn. (3-11)	a_{21}	b_{21}	c_{21}	S_{21}	$\frac{ma_{12}S_{21}}{S_{12}}$	$\frac{mb_{12}S_{21}}{S_{12}}$	$\frac{me_{12}S_{21}}{S_{12}}$	$\frac{mS_{21}}{S_{12}}$

NOTE: a_{ij} , b_{ij} , c_{ij} , d, e, f are given by Eqn. (3-6) in terms of the measured 4-port S-parameters S_{ij} 's.

Table 3.8. Coefficients of Eqn. (3-15).

$$\beta_3 = 2 \operatorname{Imag}(G_4) \quad (3.18(i))$$

$$v_3 = |D|^2 + |E|^2 + |F|^2 + |K_2|^2 - |A|^2 - |B|^2 - |C|^2 - |K_1|^2 \quad (3.18(j))$$

$$G_1 = D^* \cdot E - A^* \cdot B \quad (3.18(k))$$

$$G_2 = F^* \cdot K_2 - C^* \cdot K_1 \quad (3.18(l))$$

$$G_3 = D \cdot F^* + E^* \cdot K_2 - A \cdot E^* - B^* \cdot K_1 \quad (3.18(m))$$

$$G_4 = D^* \cdot K_2 + E \cdot F^* - A^* \cdot K_1 - B \cdot C^* \quad (3.18(n))$$

where * denotes 'complex conjugate' and the complex quantities A, B, C, D, E, F, K₁ and K₂ correspond to the coefficients in (3.15) which are shown in Table 3.8.

By observing (3.6), (3.15) and (3.18), we find that the real coefficients α_i , β_i , v_i , ($i=1,2,3$) in the inequality expression (3.16) are known in terms of the 16 measured S-parameters S_{ij} , $i,j=1-4$.

We can now solve (3.17) by assuming a value of θ_3 (0° to 360°) and finding the range of θ_4 (0° to 360°) for which (3.17) is satisfied. Consequently, we can generate 'constant-amplitude locii' on θ_3 vs θ_4 plane, corresponding to the three conditions given in (3.9) to (3.11). For condition 1, locii on which $|S'_{ii}|=1$ are to be generated. For condition 2, locii for different values of $n>1$ may be generated. Similarly, for condition 3, locii for different values of $m>1$ may be generated. The effects of second harmonic terminations (i.e. $\Gamma_3 = e^{j\theta_3}$ and $\Gamma_4 = e^{j\theta_4}$) can now be ascertained by observing the 'constant amplitude' locii on

θ_3 vs θ_4 plane. Results for a number of transistors are presented in the next section.

3.2.5 Results: Effects of Second Harmonic Terminations

For the purpose of demonstrating the approach, proposed in this Chapter, a number of microwave transistors under Class C conditions were used as non-linear 2-ports. The equivalent linear 4-port S-parameters were measured by following the measurement approach described in Section 3.2.2 and by using a measurement system as shown in Fig. 3.6. To study the effects of the second harmonic reactance terminations on the fundamental frequency parameters, the 'constant amplitude' locii for the three conditions, described in Section 3.2.4, were generated on the θ_3 vs θ_4 plane. The results are shown in Figs. 3.9. The reactance values corresponding to θ_3 and θ_4 can be determined by using a Smith chart (or by using the relation $jx = j50 \cot(\theta/2)$ ohms). A brief discussion of the results of each of the transistors will now be made.

(a) Device No. 1

(i) Transistor Type: HP35821E.

(ii) The equivalent linear 4-port S-parameter matrix ($Z_0 = 50$ ohms)

($f_0 = 2\text{GHz}$, $V_{CE} = 20\text{V}$, $V_{BE} = 0.0$, $I_C = 24\text{mA}$)

$$[S_{ij}] = \begin{bmatrix} .575 \angle -145 & .1165 \angle 60 & .1396 \angle 35 & .0073 \angle 60 \\ .565 \angle 77 & .95 \angle -70 & .5933 \angle 77 & .0472 \angle 110 \\ .411 \angle -140 & .0153 \angle 50 & 1.6 \angle 40 & .18 \angle 50 \\ .351 \angle -120 & .017 \angle -104 & .76 \angle -120 & 0.9 \angle 76 \end{bmatrix}$$

(3.19)

Fig. 3.9(a). Constant amplitude loci for HP35821E.

$\Gamma_3 = e^{j\theta_3}$, $\Gamma_4 = e^{j\theta_4}$: Reflection coefficients of second harmonic reactance terminations at the input port and the output port respectively.

Fig. 3.9(b). Constant amplitude loci for HP35821B.

$\Gamma_3 = e^{j\theta_3}$, $\Gamma_4 = e^{j\theta_4}$: Reflection coefficients of the second harmonic reactance terminations at the input and output ports respectively.

Fig. 3.9(c). Constant amplitude loci for 2SC1255.

$\Gamma_3 = e^{j\theta_3}$, $\Gamma_4 = e^{j\theta_4}$: Reflection coefficients of the second harmonic reactance terminations at the input and output ports respectively.

(iii) Constant amplitude loci: (Fig. 3.9(a))

The constant amplitude loci, corresponding to the three conditions given in (3.9) to (3.11) by using the 4-port S-parameters (in (3.19)) were generated as shown in Fig.

3.9(a). The regions (in θ_3 vs θ_4 plane) inside which $|S'_{ii}|$ is greater than unity are shown. The constant m and n curves are fairly close to each other, implying that the effects of the harmonic reactances on the parameter $|S'_{12}|$ are not significant. This can be observed by studying the expressions for m and n. The regions of interest, where m and n are 'large', are bounded approximately by $\theta_3 = 290^\circ - 360^\circ$ and $\theta_4 = 0-180^\circ$, $260-280^\circ$ and $290^\circ - 360^\circ$.

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) The equivalent linear 4-port S-parameter matrix ($Z_0 = 50$ ohms)

($f_0 = 2\text{GHz}$, $V_{CB} = 20\text{V}$, $V_{BE} = 0.0$, $I_C = 25\text{mA}$)

$$[S_{ij}] = \begin{bmatrix} .39 \angle 166 & .03 \angle 45 & 0.1354 \angle -14 & .0735 \angle 45 \\ .39 \angle -80 & 1.05 \angle -84 & 0.0698 \angle -80 & .2013 \angle 96 \\ .0959 \angle -42 & .0025 \angle 120 & .45 \angle 138 & .10 \angle 120 \\ .203 \angle 130 & .0185 \angle 80 & .52 \angle 130 & 1.05 \angle -100 \end{bmatrix} \quad (3.20)$$

(iii) Constant amplitude loci (Fig. 3.9(b))

The constant amplitude loci for the transistor, having the 4-port S-parameters as in (3.20) are shown in Fig. 3.9(b).

For this transistor, the parameter $|S'_{22}|$ is greater than unity for all reactance terminations. The parameter $|S'_{11}|$ is greater than unity inside the regions shown by the curves $|S'_{11}|=1$. By observing the constant amplitude curves for m and n , we find the regions of interest where m and n are 'large', as bounded approximately by $\theta_3=60^\circ-200^\circ$ and $\theta_4=80^\circ-100^\circ$.

(c) Device No. 3

- (i) Transistor Type: NEC28C1255.
- (ii) The equivalent linear 4-port S-parameter matrix ($Z_0=50$ ohms)
($f_0=2\text{GHz}$; $V_{CE}=15\text{V}$, $V_{BE}=0.0$, $I_C=70\text{mA}$)

$$[S_{ij}] = \begin{bmatrix} .6016 \angle 150 & .095 \angle 250 & .146 \angle -30 & .0286 \angle 250 \\ .510 \angle -55 & .811 \angle 220 & .199 \angle -55 & .0998 \angle 40 \\ .1563 \angle 191 & .0119 \angle -25 & .62 \angle 11 & .325 \angle -25 \\ .326 \angle 20 & .0278 \angle -60 & .96 \angle 20 & .745 \angle 120 \end{bmatrix} \quad (3.21)$$

- (iii) Constant amplitude locii (Fig. 3.9(c))

The constant amplitude locii for the transistor No. 3 are given in Fig. 3.9(c). The parameters $|S'_{ii}|$, $i=1,2$ are greater than unity for the second harmonic reactance terminations corresponding to the regions inside the curves marked $|S'_{ii}|=1$. We also observe that the constants m and n curves are fairly close to each other. This implies that the parameter $|S'_{12}|$ is not significantly influenced by the

second harmonic reactance terminations. The regions of interest are bounded approximately by $\theta_3 = 95^\circ - 280^\circ$ and $\theta_4 = 210^\circ - 250^\circ$.

3.2.6 Important Considerations for Designing Optimum Second Harmonic Reactance Terminations

There are a number of important considerations to be taken into account while attempting to choose the 'optimum' second harmonic reactance terminations using the 'constant amplitude' locii given in Figs. 3.9.

One of the important considerations is that the second harmonic reactance terminations (θ_3 and θ_4) should not cause negative conductance activity in the device. To ensure this, the second harmonic reactance terminations (i.e. $jx_3 = j50 \cot(\theta_3/2)$ at the input port and $jx_4 = j50 \cot(\theta_4/2)$ at the output port) should be chosen to satisfy the condition 1 in (3.9). In other words, θ_3 and θ_4 should be chosen from the regions in the θ_3 vs θ_4 plane of Fig. 3.9, where $|S'_{ii}| < 1$; ($i=1,2$).

The next important consideration in choosing θ_3 and θ_4 is to ensure that the corresponding values of n and m (defined by conditions 2 and 3 in (3.10) and (3.11) respectively) are as large as possible. By observing the 'constant amplitude' locii (shown in Figs. 3.9) for n and m , the regions having large n and m may be identified. In this connection, it is, however, important to study the sensitivity of the values of m and n with respect to θ_3 and θ_4 . This is essential because of the limitations of practical realization of the second harmonic reactance corresponding to θ_3 and θ_4 . For this reason, it may often be necessary

to choose relatively smaller values of m and n . To elaborate this consideration, a set of curves showing the variation of m and n with respect to θ_3 (and θ_4 as a parameter) may be generated (by using (3.13) and (3.14) for a particular region of interest. Such a set of curves may be termed as 'sensitivity curves' and can be drawn for any particular region of interest, chosen from the θ_3 vs θ_4 plane shown in Figs. 3.9.

For the three transistors for which the 'constant amplitude locii' are presented in Figs. 3.9, the 'sensitivity curves' for some regions of interest are shown in Figs. 3.10. A brief discussion on each of the three sets of 'sensitivity curves' (in Figs. 3.10) are given below.

(a) Device No. 1.

(i) Transistor Type: HP35821E

(ii) Sensitivity curves: (Fig. 3.10(a))

The sensitivity curves showing the variation of n with θ_3 (and θ_4 as a parameter) for some regions of θ_3 vs θ_4 plane in Fig. 3.9(a) are shown in Fig. 3.10(a).

By studying these curves, we observe that by choosing $\theta_3 = 332^\circ \pm 5^\circ$ and $\theta_4 = 308^\circ \pm 10^\circ$ we can ensure a minimum value of $n = 1.75$. Other ranges of θ_3 and θ_4 may be considered. However, this depends on the tolerance with which the reactances are realized.

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) Sensitivity curves: (Fig. 3.10(b))

Fig. 3.10(a). Sensitivity curves for HP35821E.

Fig. 3.10(b). Sensitivity curves for HP35821B.

Fig. 3.10(c). Sensitivity curves for 2SC1255.

The sensitivity curves for this device are shown in Fig. 3.10(b). The ranges of θ_3 and θ_4 were chosen by observing the constant amplitude loci for this transistor as given in Fig. 3.9(b). From these sensitivity curves, we see that if $\theta_3 = 140^\circ \pm 4^\circ$ and $\theta_4 = 92^\circ \pm 4^\circ$ then the minimum values of $m = 1.38$ and $n = 1.55$ may be assured.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255

(ii) Sensitivity curves: (Fig. 3.10(c))

The sensitivity curves for this transistor are shown in Fig. 3.10(c). The ranges of θ_3 and θ_4 are chosen by observing the constant amplitude loci for this transistor as given in Fig. 3.9(c). By observing the sensitivity curves we see that if $\theta_3 = 220^\circ \pm 10^\circ$ and $\theta_4 = 226^\circ \pm 4^\circ$ then we can ensure a minimum value of m and n to be approximately 1.5.

Thus, it is usually possible to decide upon a reasonable range of θ_3 and θ_4 in which certain minimum values of m and n (greater than unity) are ensured. By studying the sensitivity curves (in Figs. 3.10) a number of possible choices of θ_3 and θ_4 are provided in a tabulated form as shown in Table 3.11.

So, using the 'constant amplitude' loci shown in Figs. 3.9 and following the considerations described in this section, the second harmonic reactance terminations (corresponding to θ_3 and θ_4) may be

DEVICE	θ_3^* DEGREES ± 5	θ_4^* DEGREES ± 5	RANGE OF m AND n			
			m (3-11)		n (3-10)	
			Min. =	Max. =	Min. =	Max. =
No. 1	332	308	1.81	2.0	1.75	1.95
HP35821E	338	308	1.83	2.02	1.78	1.97
From	340	302	1.88	2.20	1.83	2.17
Fig. 3.10(a)	308	255	1.60	1.64	1.56	1.60
and	110	294	1.25	1.36	1.25	1.36
Fig. 3.9(a)						
No. 2	140	92	1.32	1.60	1.47	>3.0
HP35821B	10	92	1.01	1.14	1.22	1.38
From	40	92	1.05	1.18	1.26	1.47
Fig. 3.10(b)	60	92	1.08	1.20	1.29	1.55
and	80	92	1.12	1.23	1.32	1.67
Fig. 3.9(b)	100	92	1.17	1.26	1.34	1.87
No. 3	220	225	1.45	1.58	1.48	1.58
NEC25C1255	200	225	1.41	1.75	1.45	1.73
From	140	232	1.5	2.6	1.5	2.6
Fig. 3.10(c)	232	224	1.45	1.54	1.45	1.54
and						
Fig. 3.9(c)						

* Corresponding second harmonic reactances are:

$$jx_3 = j50 \cot \left(\frac{\theta_3}{2} \right) \Omega \text{ (at input port)}$$

$$jx_4 = j50 \cot \left(\frac{\theta_4}{2} \right) \Omega \text{ (at output port).}$$

Table 3.11. A summary of the optimum second harmonic reactance terminations for the three transistors under Class C operation.

chosen to 'optimize' the effects of the second harmonics on the fundamental frequency response of the transistors.

3.3 APPLICATION OF THE METHOD TO FREQUENCY DOUBLERS

An observation of the theory developed in Section 3.2.1 reveals a possibility of applying this method to characterize the effects of the second harmonic reactance at the input port and the fundamental frequency reactance termination at the output port of a transistor frequency doubler. A theory for this can be developed by a slight modification of the one presented in Section 3.2.1. This modification is described in the next section.

3.3.1 Theory: Modification for Frequency Doublers

To modify the theory in Section 3.2.1, so as to make it applicable to a frequency doubler, we consider that the second harmonic port 3 (at the input) and the fundamental frequency port 2 (at the output) of Fig. 3.3, are terminated by reflection coefficients $\Gamma_3 = \frac{a_3}{b_3}$ and $\Gamma_2 = \frac{a_2}{b_2}$ respectively. Thus, substituting $\frac{a_3}{b_3} = \Gamma_3$ and $\frac{a_2}{b_2} = \Gamma_2$ in (3.1), we can obtain:

$$b_1 = S'_{11}a_1 + S'_{14}a_4 \quad (3.22(a))$$

$$b_4 = S'_{41}a_1 + S'_{44}a_4 \quad (3.22(b))$$

where S'_{ij} , $i, j=1$ and 4 can be expressed in terms of Γ_3 and Γ_2 in the following form:

For $i, j=1$ and 4 :

$$S'_{ij} = S_{ij} + \frac{S_{i2}S_{2j}}{\frac{1}{\Gamma_2} - S_{22}} + \frac{S_{i3} + \frac{S_{i2}S_{23}}{\frac{1}{\Gamma_2} - S_{22}}}{\frac{1}{\Gamma_3} - S_{33}} \frac{S_{3j} + \frac{S_{32}S_{2j}}{\frac{1}{\Gamma_2} - S_{22}}}{\frac{1}{\Gamma_2} - S_{22}} - \frac{S_{32}S_{23}}{\frac{1}{\Gamma_2} - S_{22}} \quad (3.23)$$

or

$$S'_{ij} = \frac{a_{ij}\Gamma_3 + b_{ij}\Gamma_2 + c_{ij}\Gamma_3\Gamma_2 + S_{ij}}{d\Gamma_3 + e\Gamma_2 + f\Gamma_3\Gamma_2 + 1} \quad (3.24)$$

where

$$a_{ij} = a_{i3}a_{3j} - S_{33}S_{ij} \quad (3.24(a))$$

$$b_{ij} = S_{i2}S_{2j} - S_{22}S_{ij} \quad (3.24(b))$$

$$c_{ij} = S_{ij}S_{22}S_{33} + S_{i3}S_{32}S_{2j} + S_{i2}S_{23}S_{3j} \quad (3.24(c))$$

$$- S_{ij}S_{32}S_{23} - S_{i3}S_{3j}S_{22} - S_{i2}S_{2j}S_{33}$$

$$d = -S_{33} \quad (3.24(d))$$

$$e = -S_{22} \quad (3.24(e))$$

$$f = S_{33}S_{22} - S_{32}S_{23} \quad (3.24(f))$$

It is worthwhile to note here that the expressions in (3.23) and (3.24(a)-(f)) can also be obtained directly from (3.5) and (3.6(a)-(f)) simply by substituting 2 for 4 (and vice-versa) in the subscripts of S_{ij} 's and Γ_i . This implies that if we interchange the rows 2 and 4

and also columns 2 and 4 in the 4-port S-parameter matrix in (3.2), then all the expressions derived in the previous sections of this chapter become applicable to a frequency doubler.

3.3.2 Effects of the Second Harmonic Reactance at the Input and the Fundamental Frequency Reactance Termination at the Output Port of a Frequency Doubler

Following a similar argument presented in Section 3.2.4, it is assumed that the second harmonic at the input port is terminated reactively (at a corresponding reflection coefficient of $\Gamma_3 = e^{j\theta_3}$ say) and the fundamental component at the output is also terminated reactively (at a corresponding reflection coefficient of $\Gamma_2 = e^{j\theta_2}$ say). Thus, the four 'effective' S-parameters (S'_{ij} ; $i, j = 1$ and 4 in (3.24), characterizing the frequency doubler become:

For $i, j = 1$ and 4 :

$$S'_{ij} = \frac{a_{ij}e^{j\theta_3} + b_{ij}e^{j\theta_2} + c_{ij}e^{j(\theta_3+\theta_2)} + S_{ij}}{de^{j\theta_3} + ee^{j\theta_2} + fe^{j(\theta_3+\theta_2)} + 1} \quad (3.25)$$

where a_{ij} , b_{ij} , c_{ij} , d , e and f are given by (3.24(a)-(f)).

We now consider three conditions similar to those described in (3.9) to (3.11) that is:

$$\text{Condition 1: } |S'_{ii}| < 1; \quad i = 1 \text{ and } 4, \text{ i.e.} \quad (3.26)$$

$$(a) \quad |S'_{11}| < 1 \quad (3.26(a))$$

$$(b) \quad |S'_{44}| < 1 \quad (3.26(b))$$

Condition 2:

$$N = \left| \frac{S'_{41}}{S_{41}} \right| ; N > 1 \quad (3.27)$$

Condition 3:

$$M = \left| \frac{S'_{41}/S'_{14}}{S_{41}/S_{14}} \right| ; M > 1 \quad (3.28)$$

Thus, using (3.25)-(3.28) and following the similar procedure described in Sections 3.2.4 and 3.2.6, we can generate the 'constant amplitude' loci and the 'sensitivity curves'. By studying these curves, we may now choose the second harmonic reactance termination at the input port (i.e. θ_3) and the fundamental frequency reactance termination at the output port (i.e. θ_2) for the desired frequency doubler. Results for the three transistors, under consideration, will be shown in the next section.

3.3.3 Results: For Frequency Doublers

The 'constant amplitude loci' and the 'sensitivity curves', obtained by applying the modified theory for frequency doublers, to the three transistors whose 4-port S-parameters have been given in Section 3.2.5, will be given in this section. For simplicity, the results of each device are discussed separately.

(a) Device No. 1

(1) Transistor Type: HP35821E

4-port S-parameters are given in (3.19).

(ii) Constant amplitude locii: (Fig. 3.12(a))

The constant amplitude locii for this transistor on the θ_3 vs θ_2 plane are shown in Fig. 3.12(a). The parameter $|S'_{11}|$ is greater than unity inside the region bounded by the curves marked $|S'_{11}|=1$. The parameter $|S'_{44}|$ is less than unity inside the region bounded by the curves shown as $|S'_{44}|=1$. It can be observed that the value of M is quite insensitive to θ_2 for some values of θ_3 (such as $\theta_3=200^\circ$).

(iii) Sensitivity curves: (Fig. 3.12(b))

The sensitivity curves for this transistor were generated by using (3.28) and (3.25) as shown in Fig. 3.12(b). There exists a value of $\theta_3=200^\circ$ for which the value of M is very insensitive to θ_2 . However, to take care of the necessary tolerance in θ_3 , we can choose $\theta_3=170^\circ \pm 5^\circ$ and $\theta_2=170^\circ \pm 20^\circ$ by which a minimum value of $M=4.5$ can be assured. Corresponding to this choice of θ_3 and θ_2 , the value of n remains approximately constant at 1.36.

(b) Device No. 2

(i) Transistor Type: HP35821B

4-port S-parameters are given in (3.20).

(ii) Constant amplitude locii: (Fig. 3.13(a))

The constant amplitude locii for this transistor are shown in Fig. 3.13(a). For this transistor, the parameter $|S'_{11}|$ is less than unity everywhere in the θ_3 vs θ_2 plane, whereas

Fig. 3.12(a): Constant amplitude loci (for frequency doubler) using HP35821E.

$\Gamma_3 = \frac{V_3}{V_2}$, $\Gamma_2 = \frac{V_2}{V_1}$: Reflection coefficients of the second harmonic reactance at the input port and the funda-

Fig. 3.12(b). Sensitivity curves (for frequency doubler) for HP35821E.

Fig. 3.13(a). Constant amplitude loci¹⁰ (for frequency doubler) for HP35821B.

Fig. 3.13(b). Sensitivity curves (for frequency doubler) for HP35821B.

the parameter $|S'_{44}|$ is greater than unity everywhere in the same plane. By observing the constant M and N locii, we may select a region of interest (where both M and N are large) as the one bounded by $\theta_3 = 30^\circ - 170^\circ$ and $\theta_2 = 86^\circ - 110^\circ$.

(iii) Sensitivity curves: (Fig. 3.13(b))

The sensitivity curves for this device were generated by using (3.25), (3.27) and (3.28) as shown in Fig. 3.13(b).

By observing these sensitivity curves, a choice of $\theta_3 = 120^\circ \pm 10^\circ$ and $\theta_2 = 100^\circ \pm 5^\circ$ can be made. This choice of θ_3 and θ_2 ensures a minimum value of M and N to be approximately 1.2.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255

4-port S-parameters are given in (3.21).

(ii) Constant amplitude locii: (Fig. 3.14(a))

Fig. 3.14(a) shows the constant amplitude locii on θ_3 vs θ_2 plane. It is observed from these locii that the parameter $|S'_{44}|$ is greater than unity in the region enclosed by the $|S'_{44}|=1$ curve. The parameter $|S'_{11}|$ is less than unity everywhere on the θ_3 vs θ_2 plane. The regions of interest, where the values of M and N are large, are bounded by $\theta_3 = 85^\circ - 220^\circ$ and $\theta_2 = 0 - 130^\circ$; $\theta_3 = 85^\circ - 170^\circ$ and $\theta_2 = 170 - 360^\circ$.

Fig. 3.14(a). Constant amplitude loci (for frequency doubler) for HP28C1255.

(iii) Sensitivity curves: (Fig. 3.14(b))

The sensitivity curves for this device are shown in Fig. 3.14(b). The curves for N are not shown because it is quite insensitive to θ_3 and θ_2 for the ranges shown in Fig. 3.14(b). For example, N is approximately constant at 1.27 for $\theta_2 = 80-100^\circ$ and $\theta_3 = 80-190^\circ$. From the sensitivity curves for M , if we choose $\theta_3 = 145^\circ \pm 5^\circ$ and $\theta_2 = 90^\circ \pm 8^\circ$, then the value of M is assured to lie between 7.5 and 10.0. Another choice of $\theta_3 = 135^\circ \pm 4^\circ$ and $\theta_2 = 80^\circ \pm 10^\circ$ may assure the value of M to lie between 8.2 and 9.5. These two choices are shown by shaded areas in Fig. 3.14(b).

3.4 SUMMARY AND REMARKS

3.4.1 Summary

A method of characterizing the effects of harmonics in a non-linear 2-port has been proposed in this chapter. This method is based on the theory of multiple-input describing function techniques (11). The implementation of the method requires the measurement of the S-parameters under multiple-sinusoidal-signal input conditions.

A detailed theory for characterizing the effects of the second harmonics on the fundamental frequency response has been developed. A measurement system to measure the S-parameters under two-sinusoidal-signal-input conditions is described. A detailed analysis to study the effects of the second harmonic reactance terminations is shown. This

Fig. 3.14(b). Sensitivity curves (for frequency doubler) for 28C1255.

analysis uses some parameters (such as n and m as defined in (3.10) and (3.11)), as indicators of the 'favourable' effects of the second harmonic reactance terminations of the non-linear 2-port. An approach to choosing the optimum second harmonic reactance terminations, based on a 'sensitivity' study of the parameters m and n , is illustrated. Detailed results using three transistors under Class C conditions as non-linear 2-ports are presented. The necessary modifications and the corresponding results for treating the case of a transistor frequency doubler are also included in this chapter.

3.4.2 Remarks

The analysis in Section 3.2.4 is given for purely reactive second harmonic terminations. But practically, pure reactances (the magnitude of the corresponding reflection coefficient being unity) are not realizable. However, realization of a reactance having the magnitude of the corresponding reflection coefficient greater than 0.98 is possible. If required, the results corresponding to the actual 'impure' reactances can be obtained by taking into account the actual magnitude of the reflection coefficient of the reactance terminations in the analysis of Section 3.2.4.

By observing the 'constant amplitude' loci (in Fig. 3.9), it is possible to determine the significance of the effects of the second harmonic. Since the indicators m and n are normalized parameters, the larger the values of these, the more significant are the effects. But,

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since $|S'_{ii}|$ is not normalized, to ascertain the effects on S'_{ii} , it is necessary to compare this with S_{ii} (i.e. to the value when the second harmonics are terminated in the characteristic impedances).

The usefulness of the method of characterizing the effects of harmonics in a non-linear 2-port may be realized in two respects. First, it has been shown that if the effects of harmonics are significant, then some harmonic reactance terminations may cause negative conductance activity (NCA) in transistors. So, it is helpful to know the harmonic reactance terminations which will not cause NCA. Secondly, if the harmonic effects are significant, instead of dissipating the harmonic powers in the terminations, it may be possible to increase the fundamental frequency performance (say increased gain and/or efficiency, etc.) by properly choosing the harmonic reactance terminations.

Once the optimum harmonic terminations are chosen, it now remains to develop a method of characterizing the non-linearity in the fundamental frequency response of the non-linear 2-port. In the next chapter, a novel method for this purpose is proposed.

CHAPTER 4

A NEW 'BLACK BOX' APPROACH TO THE CHARACTERIZATION OF A NON-LINEAR 2-PORT

4.1 INTRODUCTION

In this chapter, a new 'black box' approach to the characterization of a general non-linear 2-port is proposed. The principle of this approach consists of measuring the responses of the 2-port by simultaneously applying two inputs at the two ports. Using the principle, a measurement technique at microwave frequencies is illustrated in Section 4.3. A procedure for applying the technique to predict optimum performances of microwave transistor Class C power amplifiers is given in Section 4.5. Two significant applications of this method are demonstrated in Section 4.7 of this chapter. It is shown that this method provides a more realistic method of measuring the large-signal S-parameters (especially for transistors under Class C conditions) than the ones currently in use (Chapter 2). It is also shown that this technique can be used to generate load-pull data, for microwave power transistors, in a considerably faster and easier manner than can be done by the conventional method of using tuners (Chapter 2).

4.2 THEORY: CHARACTERIZATION OF A NON-LINEAR 2-PORT

Let us consider a non-linear 2-port whose port variables are in terms of wave variables (26) as shown in Fig. 4.1. It will be assumed in the following that the 2-port is already terminated in some

Non-linear functional relationships:

$$\frac{b_1}{a_1} = F_1(|a_1|, |a_2|, < \frac{a_2}{a_1}) \quad (4-3(a))$$

$$\frac{b_2}{a_2} = F_2(|a_1|, |a_2|, < \frac{a_2}{a_1}) \quad (4-3(b))$$

$$I_c = F_3(|a_1|, |a_2|, < \frac{a_2}{a_1}) \quad (4-3(c))$$

Fig. 4.1. A non-linear 2-port with the port variables in terms of power waves (a_1 's incident wave, b_1 's reflected/transmitted waves).

'optimum' harmonic impedances, for example, by the approach described in Chapter 3. Consequently, the variables, a_i 's and b_i 's in Fig. 4.1, are considered as phasor quantities at the fundamental frequency of operation. Thus, the non-linear 2-port can be characterized by two functional relationships among the port variables, such as:

$$b_1 = f_1(|a_1|, |a_2|, \angle \frac{a_2}{a_1}) \quad (4.1(a))$$

$$b_2 = f_2(|a_1|, |a_2|, \angle \frac{a_2}{a_1}) \quad (4.1(b))$$

where $|a_i|$ indicates magnitude of a_i and $\angle \frac{a_2}{a_1}$ indicates the phase angle of a_2 with reference to a_1 . It may be mentioned here that according to the definition of the wave variables (26), $|a_1|^2$ is equal to the power carried by the wave a_1 and similarly $|b_1|^2$ is the power carried by the wave b_1 .

In the case of a non-linear device (such as a transistor under Class C conditions), the d.c. operating condition (e.g. I_c , the collector current) is also a non-linear function of the port variables. Thus, for complete characterization of the non-linear device, another functional relationship is to be considered, such as:

$$I_c = f_3(|a_1|, |a_2|, \angle \frac{a_2}{a_1}) \quad (4.2)$$

Since the ratio of two wave variables is conveniently measured at microwave frequencies using a Network Analyzer, the above three relationships will be considered in the following form:

$$\frac{b_1}{a_1} = F_1(|a_1|, |a_2|, \langle \frac{a_2}{a_1} \rangle) \quad (4.3(a))$$

$$\frac{b_2}{a_2} = F_2(|a_1|, |a_2|, \langle \frac{a_2}{a_1} \rangle) \quad (4.3(b))$$

$$\text{and } I_c = F_3(|a_1|, |a_2|, \langle \frac{a_2}{a_1} \rangle) \quad (4.3(c))$$

If the quantities $\frac{b_i}{a_i}$; $i=1,2$ and I_c , as a function of the three variables $|a_1|$, $|a_2|$ and $\langle \frac{a_2}{a_1} \rangle$, are measured then the non-linear 2-port is completely characterized by the three relationships in (4.3).

It may be mentioned here that in the case of a linear 2-port the relationships in (4.1) are approximated by linear functions of the form:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (4.4(a))$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (4.4(b))$$

This is well known as the standard S-parameters method of characterizing a linear 2-port (20,26) and the parameters S_{ij} ; $i,j=1,2$ are known as the S-parameters. Consequently, such simplification allows us to measure the S-parameters (S_{ij} 's) by applying the superposition theorem to the response of the linear 2-port. But, in the determination of the functions F_i 's in (4.3) characterizing a non-linear 2-port, it is essential to measure the responses under simultaneous application of the two inputs a_1 and a_2 . In the next Section (4.3), a measurement system at microwave frequencies, using a Network

Analyzer (HP8410), is described which is capable of measuring the relationships in (4.3) by employing the principle of two simultaneous inputs to the 2-port.

4.3 MEASUREMENT SYSTEM TO CHARACTERIZE A NON-LINEAR 2-PORT

The system for measuring the relationships (4.3) consists of simultaneously applying the two inputs a_1 and a_2 to the 2-port, and measuring the parameters $\frac{b_i}{a_i}$ by a 'reflection' type of measurement using a Network Analyzer. However, it is essential that the signals b_i and a_i be of the same frequency and be coherent to enable their ratio measurement by the Network Analyzer (34). The measurement system is shown in Fig. 4.2, in which the input signals a_1 and a_2 are derived from the same oscillator. The variable attenuators and the phase shifter are used to enable variation of the parameters $|a_1|$, $|a_2|$ and $\angle \frac{a_2}{a_1}$. The incident (a_i) and 'reflected' waves (b_i) are separated by using the dual directional couplers and their ratio $\frac{b_i}{a_i}$ is measured by using a network analyzer. The calibration of the reference plane of measurement is performed by using a 'short' and the phase angle ($\angle \frac{a_2}{a_1}$) between a_1 and a_2 may be measured by calibrating with a standard 'through' (or 'joining the two ports').

Thus, using the oscillator output control, variable attenuators and the phase shifter, the parameters $|a_1|$, $|a_2|$ and $\angle \frac{a_2}{a_1}$ can be varied to establish the relationships (4.3). The parameters $|a_1|$ and $|a_2|$ can be calculated by measuring the corresponding powers, since $|a_i|^2 = P_i$ (Fig. 4.2).

Note:

- (i) To measure $\frac{b_1}{a_1}$: connect C to Reference and D to Test.
- (ii) To measure $\frac{b_2}{a_2}$: connect A to Reference and B to Test.
- (iii) To measure $\frac{a_2}{b_2}$: connect B to Reference and A to Test.
- (iv) $P_1 = |a_1|^2$, $P_2 = |a_2|^2$.

Fig. 4.2. Measurement system to characterize a non-linear 2-port.

4.4 GENERAL DESIGN APPROACH FOR A NON-LINEAR 2-PORT

Once the functional relationships (4.3), characterizing a non-linear 2-port, are known (either in tabulated form or in some empirical form obtainable from the tabulated data), it now remains to design the terminations of the 2-port to achieve some specified optimum performance characteristics of the 2-port. In general, this analysis is apparently possible either graphically or by computer-aided optimization methods. A general analysis, clearly, will require an enormous amount of data to be collected, which may be laborious but not difficult by using the measurement system described in Section 4.3. For such situations, computer-controlled automated measurements using the principle presented in this chapter may be considered. In any case, the accuracy of such analysis improves with the amount of judiciously chosen data.

However, for some specific cases, such as for transistor power amplifier designs, the design procedure can be simplified by systematizing the measurement steps. The sequence of measurement steps should be so devised as to achieve some specified optimum performance characteristics (for example, power output or efficiency) of the power amplifier. Such a sequence of measurement steps will be developed in the next section.

4.5 AN APPROACH TO THE DESIGN OF POWER AMPLIFIERS

In this section, a sequence of systematic measurement steps, while using the measurement system of Section 4.3 to measure the

relationships in (4.3), will be developed. It will be seen that such a sequence of measurement steps will enable one to achieve amplifier designs for optimum power output or d.c. to RF conversion efficiency.

To facilitate a logical development of the sequence of measurement steps, let us study some of the performance parameters of a power amplifier in terms of the terminal wave variables of the 2-port. For example, the power input to the 2-port is given by (26):

$$P_{in} = |a_1|^2 - |b_1|^2 \quad (4.5(a))$$

or

$$P_{in} = |a_1|^2 \left[1 - \left| \frac{b_1}{a_1} \right|^2 \right] \quad (4.5(b))$$

Similarly, power output from the 2-port (i.e. power delivered to the load) is given by (26):

$$P_{out} = |b_2|^2 - |a_2|^2 \quad (4.6(a))$$

or

$$P_{out} = |a_2|^2 \left[\left| \frac{b_2}{a_2} \right|^2 - 1 \right] \quad (4.6(b))$$

where a_i 's and b_i 's are defined in Section 4.2. Consequently, the power gain of the 2-port is defined as:

$$G_p = \frac{P_{out}}{P_{in}} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2} \quad (4.7)$$

Again, if the d.c. voltage across the transistor is V_c and the collector current is I_c , then the d.c. power dissipated in the device is:

$$P_{dc} = V_c I_c \quad (4.8)$$

Thus, the d.c. to RF conversion efficiency is expressed as:

$$\eta = \frac{P_{out}}{P_{dc}} = \frac{|b_2|^2 - |a_2|^2}{V_c I_c} \quad (4.9)$$

Some important observations will now be made from the expressions (4.5)-(4.9) of the performance characteristics of a power amplifier, as follows.

Observation 1:

Since P_{in} and P_{out} are to be positive, from (4.5) and (4.6), we observe that:

$$\left| \frac{b_2}{a_2} \right| > 1 \quad (4.10)$$

and

$$\left| \frac{b_1}{a_1} \right| < 1 \quad (4.11)$$

Also, P_{out} is large if $\left| \frac{b_2}{a_2} \right|$ is large for a constant $|a_2|$.

Observation 2:

For a particular P_{out} , the d.c. to RF conversion efficiency η is inversely proportional to the d.c. collector current I_c so long as V_c is kept constant.

From the above two observations and referring to the expressions (4.5) to (4.9), we can now develop a sequence of measurement

steps which will lead to the design of a power amplifier for optimum power output or conversion efficiency, as follows:

Step 1:

- (i) The network analyzer (in Fig. 4.2) is connected to measure the parameter $\left| \frac{b_2}{a_2} \right|$.
- (ii) Set $|a_1|$ and $|a_2|$ to some arbitrary values by adjusting the variable attenuators.
- (iii) Vary $\frac{a_2}{a_1}$ by varying the phase shifter and observe the variation of the parameter $\frac{b_2}{a_2}$ on the network analyzer display unit. Also, observe the collector current I_c .
Set the phase shifter to a position which corresponds to the maximum value of $\left| \frac{b_2}{a_2} \right|$ greater than unity (according to Observation 1, (4.10)). If it is not possible to obtain $\left| \frac{b_2}{a_2} \right| > 1$ for any position of the phase shifter, then choose a different value of $|a_2|$ by adjusting the corresponding variable attenuator, and repeat (iii) of this step. The choice of $|a_1|$ depends on what current (I_c) level is desired.
- (iv) While performing this step of measurement, the variation of I_c may also be observed as the phase shifter is varied in (iii). According to the Observation (2), it is desired, if possible, to set the phase shifter to a position corresponding to the minimum value of I_c so that the conversion efficiency is high.

It may not be usually possible to find one position of the phase shifter corresponding to both the requirements in (iii) and (iv). However, depending on the requirement of maximum power output or conversion efficiency, the procedure in (iii) or (iv) is adopted respectively.

Thus, the parameter $\frac{b_2}{a_2}$ (both magnitude and angle) is measured corresponding to some $|a_1|$, $|a_2|$ and $\angle \frac{a_2}{a_1}$ having been adjusted for maximum power output or conversion efficiency of the amplifier. The parameters $|a_1|$ and $|a_2|$ are calculated by measuring the corresponding powers, since $|a_1|^2 = P_1$ and $|a_2|^2 = P_2$. The load reflection coefficient of the amplifier is thus given by $\Gamma_L = \frac{a_2}{b_2}$.

Step 2:

- (i) Leaving the position of the attenuators and the phase shifter as in Step 1, the network analyzer (in Fig. 4.2) is now connected to measure $\frac{b_1}{a_1}$.
- (ii) According to Observation 1 in (4.11), if $|\frac{b_1}{a_1}|$ is less than unity then the magnitude and angle of $\frac{b_1}{a_1}$ is measured. If, however, it is found that $|\frac{b_1}{a_1}|$ is greater than unity, then it is necessary to repeat Step 1 with a different set of $|a_1|$ and $|a_2|$.

Thus, a successful operation of Steps 1 and 2 yields a set of values of the parameters I_c , $|a_1|$, $|a_2|$, $|\frac{b_i}{a_i}|$

and $\angle \frac{b_i}{a_i}$; $i=1,2$ which are noted to form a table of data.

A set of these parameters constitutes a complete design of the power amplifier. It may be noted that the reflection coefficient (Γ_S) of the input termination of the amplifier is given by $\Gamma_S^* = \frac{b_1}{a_1}$ (where * means complex conjugate).

Step 3:

Repeating Steps 1 and 2, we can generate a number of designs for different current (I_C) levels. Thus, a large table of data can be compiled containing the values of the parameters I_C , $|a_1|$, $|a_2|$, $|\frac{b_i}{a_i}|$ and $\angle \frac{b_i}{a_i}$, $i=1,2$.

Step 4:

Using the values in the table of data of Step 3, we can now calculate the different performance parameters of the power amplifier by using (4.5) to (4.9). These results may be tabulated in the form of a 'design table'.

Step 5:

From the 'design table' completed in Step 4, the case which corresponds to the optimum values of power output (P_{out}), conversion efficiency (η) and power gain (G_p) may be selected. In this connection, some trade-offs among the parameters P_{out} , η and G_p may be necessary according to the particular requirements, for example, in some situations, it may be desirable to optimize power output (P_{out}) or conversion efficiency (η).

Thus, by following the sequence of steps described above, we can achieve optimum design of power amplifiers by using the principle of measurement proposed in this chapter. The results of the designs of a number of microwave transistors will be presented in the next section.

4.6 . RESULTS: MICROWAVE TRANSISTOR CLASS C POWER AMPLIFIERS

To illustrate the proposed power amplifier design theory, a number of transistors under Class C conditions were used as a non-linear 2-port. The harmonics were terminated reactively by the method described in Chapter 3. The tables of data and the corresponding design tables for the transistors were generated by following the sequence of steps described in Section 4.5 and using the measurement system as described in Section 4.3. These tables of data and the 'design tables' are presented in Tables 4.3-4.5. A brief discussion of the results of each transistor is made in the following.

(a) Device No. 1

(i) Transistor Type: HP35821E.

(ii) Second harmonic reactance terminations

The second harmonic reactance terminations for this transistor were chosen corresponding to $\theta_3 = 338^\circ$ and $\theta_4 = 308^\circ$ as designed in Chapter 3, Table 3.11. We may recall here that θ_3 and θ_4 correspond to the angles of reflection coefficient of the corresponding reactance

NO.	I_C (mA)	$ a_1 ^2 = P_1$ (mW)	$ a_2 ^2 = P_2$ (mW)	$\left \frac{b_2}{a_2} \right $	$\angle \frac{b_2}{a_2}$ (degrees)	$\left \frac{b_1}{a_1} \right $	$\angle \frac{b_1}{a_1}$ (degrees)
1	15	70	145	1.27	-70	.55	-148
2	20	70	110	1.48	-70	.52	-147
3	20	80	180	1.33	-72	.52	-152
4*	20	90	125	1.47	-68	.55	-153
5	25	80	150	1.43	-80	.52	-150
6	25	100	240	1.30	-70	.53	-154
7	25	90	200	1.35	-71	.50	-153
8	30	100	200	1.34	-73	.51	-153

Table 4.3(a). Table of data for device no. 1 (HP35821E).

NO.	I_C (mA)	$P_{dc} = V_{CC} I_C$ (mW) (4-8)	P_{in} (mW) (4-5)	P_{out} (mW) (4-6)	G_p (dB) (4-7)	$\eta(\%)$ (4-9)	$\Gamma_S = \left(\frac{b_1}{a_1} \right)^*$	$\Gamma_L = \frac{a_2}{b_2}$
1	15	300	48.83	88.87	2.6	29.6	.55 $\angle$ 148	.79 $\angle$ 70
2	20	400	51.07	130.94	4.09	32.7	.52 $\angle$ 147	.68 $\angle$ 70
3	20	400	58.37	138.40	3.75	34.6	.52 $\angle$ 152	.75 $\angle$ 72
4*	20	400	62.78	145.1	3.64	36.28	.55 $\angle$ 153	.68 $\angle$ 68
5	25	500	58.37	156.74	4.29	31.35	.52 $\angle$ 150	.70 $\angle$ 80
6	25	500	71.91	165.60	3.62	33.1	.53 $\angle$ 154	.77 $\angle$ 70
7	25	500	67.50	164.50	3.87	32.9	.50 $\angle$ 153	.74 $\angle$ 71
8	30	600	73.99	159.12	3.33	26.5	.51 $\angle$ 153	.75 $\angle$ 73

Table 4.3(b). Design table for device no. 1 (HP35821E).

(* denotes the design chosen)

NO.	I_C (mA)	$ a_1 ^2 = P_1$ (mW)	$ a_2 ^2 = P_2$ (mW)	$\left \frac{b_2}{a_2} \right $	$\angle \frac{b_2}{a_2}$ (degrees)	$\left \frac{b_1}{a_1} \right $	$\angle \frac{b_1}{a_1}$ (degrees)
1	22	140	210	1.33	-65	.72	136
2	22	100	120	1.39	-60	.71	132
3	24	100	200	1.32	-69	.67	138
4	24	120	260	1.28	-70	.66	140
5*	26	130	205	1.35	-71	.71	142
6	26	140	230	1.32	-72	.72	143
7	27	150	230	1.30	-68	.68	138
8	30	150	310	1.24	-70	.69	138

Table 4.4(a). Table of data for device no. 2 (HP35821B).

NO.	I_C (mA)	$P_{dc} = V_C I_C$ (mW) (4-8)	P_{in} (mW) (4-5)	P_{out} (mW) (4-6)	G_p (dB) (4-7)	$\eta(\%)$ (4-9)	$\Gamma_S = \left(\frac{b_1}{a_1} \right)^*$	$\Gamma_L = \frac{a_2}{b_2}$
1	22	440	67.42	161.47	3.79	36.7	.72 $\angle$ -136	.75 $\angle$ 65
2	22	440	49.59	111.85	3.53	25.42	.71 $\angle$ -132	.72 $\angle$ 60
3	24	480	55.11	148.48	4.30	30.93	.67 $\angle$ -138	.76 $\angle$ 69
4	24	480	67.73	165.98	3.89	34.58	.66 $\angle$ -140	.78 $\angle$ 70
5*	26	520	64.47	168.61	4.18	32.43	.71 $\angle$ -142	.74 $\angle$ 71
6	26	520	67.42	170.75	4.04	32.84	.72 $\angle$ -143	.76 $\angle$ 72
7	27	540	80.64	158.70	2.94	29.39	.68 $\angle$ -138	.77 $\angle$ 68
8	30	600	78.59	166.66	3.26	27.78	.69 $\angle$ -138	.81 $\angle$ 70

Table 4.4(b). Design table for device no. 2 (HP35821B).

(* denotes the design chosen)

NO.	I_c (mA)	$ a_1 ^2 = P_1$ (mW)	$ a_2 ^2 = P_2$ (mW)	$\left \frac{b_2}{a_2} \right $	$\angle \frac{b_2}{a_2}$ (degrees)	$\left \frac{b_1}{a_1} \right $	$\angle \frac{b_1}{a_1}$ (degrees)
1	60	185	320	1.28	-55	.70	165
2*	70	220	500	1.28	-53	.65	162
3	75	210	450	1.30	-52	.67	158
4	80	260	500	1.31	-56	.63	160

Table 4.5(a). Table of data for device no. 3 (2SC1255).

NO.	I_c (mA)	$P_{dc} = V_c I_c$ (mW) (4-8)	P_{in} (mW) (4-5)	P_{out} (mW) (4-6)	G_p (dB) (4-7)	$\eta(\%)$ (4-9)	$\Gamma_S = \left(\frac{b_1}{a_1} \right)^*$	$\Gamma_L = \frac{a_2}{b_2}$
1	60	900	94.35	204.3	3.36	22.7	.70 $\angle$ -165	.78 $\angle$ 55
2*	70	1050	127.05	319.2	4.00	30.4	.65 $\angle$ -162	.78 $\angle$ 53°
3	75	1125	115.73	310.5	4.29	27.6	.67 $\angle$ -158°	.77 $\angle$ 52
4	80	1200	156.81	358.0	3.59	29.83	.63 $\angle$ -160	.76 $\angle$ 56

Table 4.5(b). Design table for device no. 3 (2SC1255).

(* denotes the design chosen)

terminations at the input port and the output port, respectively. Thus, the reactance terminations are $jx_3 = j50 \cot(\theta_3/2) = j257$ ohms and $jx_4 = j50 \cot(\theta_4/2) = -j102.5$ ohms.

(iii) 'Table of Data' and 'Design Table' (Table 4.3)

Following the power amplifier design approach, by adopting the systematic measurement steps described in Section 4.5 of this chapter, the 'table of data' was obtained as shown in Table 4.3(a). The corresponding 'design table' is shown in Table 4.3(b) which shows the calculated performance parameters of the power amplifier. Out of the eight designs given in the 'design table', a design may be chosen according to the requirement of optimum power output or power gain or conversion efficiency.

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) Second harmonic reactance terminations

The second harmonic reactance terminations for this transistor were chosen corresponding to $\theta_3 = 140^\circ$ and $\theta_4 = 92^\circ$ as designed in Chapter 3 (shown in Table 3.11).

This choice corresponds to the reactance terminations of $jx_3 = j50 \cot(\theta_3/2) = j18.2$ ohms at the input port and $jx_4 = j50 \cot(\theta_4/2) = j48.3$ ohms at the output port of the transistor.

(iii) 'Table of Data' and 'Design Table': (Table 4.4)

The 'table of data' and the 'design table' for this transistor were obtained by following the power amplifier design approach illustrated in Section 4.5. These are shown in Tables 4.4(a) and 4.4(b) respectively. An optimum design may now be chosen from the design table corresponding to maximum power output or power gain or conversion efficiency or alternatively, by making some trade-offs among these performance parameters.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255.

(ii) Second Harmonic Reactance Terminations

The designs of the second harmonic reactance terminations have been given in Chapter 3, Table 3.11. The reactance terminations corresponding to $\theta_3 = 220^\circ$ and $\theta_4 = 225^\circ$ were chosen. These correspond to reactances of $jx_3 = j50 \cot(\theta_3/2) \approx -j18.2$ ohms at the input port and $jx_4 = j50 \cot(\theta_4/2) \approx -j20.7$ ohms at the output port of the transistor.

(iii) 'Table of Data' and 'Design Table': (Table 4.5)

The 'table of data' and the corresponding 'design table' for this transistor are shown in Table 4.5. These were obtained by the procedure described in Section 4.5 of this chapter. An optimum design may now be chosen from the 'design table'.

4.7 FURTHER SIGNIFICANT APPLICATIONS OF THE 'BLACK BOX' METHOD

From a careful observation of the theory presented in Section 4.2 and the principle of the measurement system described in Section 4.3 of this chapter, we can derive two significant applications of the proposed 'black box' method. These two applications provide:

- (i) a new method for measuring the large-signal S-parameters of transistors under Class C conditions and
- (ii) a new method of generating the load-pull data for micro-wave power transistors.

The details of these applications and the corresponding results for a number of transistors will be presented in the following.

4.7.1 A New Method for Measuring the Large-Signal S-Parameters of a Transistor Under Class C Conditions

4.7.1(a) Principle

If the large-signal S-parameters method is valid for a transistor under Class C operation, the two functional relationships in (4.1) may be approximated by two linear equations such as:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad (4.12(a))$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (4.12(b))$$

where S_{ij} , $i, j=1, 2$ are known as the large-signal S-parameters at a particular bias, frequency and input power level of the transistor (11-15). The standard approach of accomplishing the measurement of

the four parameters S_{ij} 's is by establishing the following four measurement conditions:

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad (4.13(a))$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \quad (4.13(b))$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad (4.13(c))$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \quad (4.13(d))$$

The conditions $a_i=0$ are established by terminating the port i at the characteristic impedance (usually 50 ohms). As it was mentioned in Chapter 2, though the measurement of the S -parameters, by employing the conditions (4.13), is now standard; the two parameters S_{12} and S_{22} of a transistor under Class C conditions cannot be meaningfully measured by this method. This is because to measure these two parameters the signal is to be applied at the output port, the input port being terminated ($a_1=0$) by the characteristic impedance. By doing so, the input drive power of the Class C transistor is removed and so the voltages and currents flowing in the transistor are completely different from those of the intended amplifier. Hence, the

measurement of these two parameters (S_{12} and S_{22}) is not realistically performed by the standard 9-parameters method.

To demonstrate how the principle of the proposed 'black box' method presented in this chapter can be applied to meaningfully measure the four large-signal S-parameters (especially S_{12} and S_{22}), let us consider (4.12) in the following forms:

$$\frac{b_1}{a_1} = S_{11} + S_{12} \frac{a_2}{a_1} \quad (4.14(a))$$

$$\frac{b_1}{a_2} = S_{11} \frac{a_1}{a_2} + S_{12} \quad (4.14(b))$$

$$\frac{b_2}{a_1} = S_{21} + S_{22} \frac{a_2}{a_1} \quad (4.14(c))$$

$$\frac{b_2}{a_2} = S_{21} \frac{a_1}{a_2} + S_{22} \quad (4.14(d))$$

It should be noted that the variables in (4.14) are complex quantities. Thus, for a particular set of values of $|a_1|$ and $|a_2|$, each of the four quantities $\frac{b_i}{a_j}$; $i, j=1, 2$ describes a circle as a function of the phase angle $\angle \frac{a_2}{a_1}$ on the corresponding complex $\frac{b_i}{a_j}$ -plane. Also, the centres of the $\frac{b_i}{a_j}$ circular locii correspond to the parameters S_{ij} as shown in Fig. 4.6.

Thus, the four large-signal S-parameters S_{ij} 's can be measured, by using the measurement system shown in Fig. 4.2 and generating the

Fig. 4.6. Principle of the circular loci in $\frac{b_i}{a_j}$ - plane to measure the large-signal S-parameters. (Centre of b_i/a_j -locus corresponds to S_{ij} .)

circular locii for $\frac{b_i}{a_j}$'s, for particular values of $|a_1|$ and $|a_2|$ and by varying the phase shifter. The arrangements of the measurement system for measuring the particular $\frac{b_i}{a_j}$ -locus is shown in Fig. 4.7. For convenience, the locii can be obtained directly on a Smith chart by using an X-Y recorder,

4.7.1(b) Results: Large-Signal S-Parameters of Transistors Under Class C Conditions and Amplifier Design

Using the theory in Section 4.7.1(a) and the measurement procedures shown in Fig. 4.7, the large-signal S-parameters of a number of transistors under Class C conditions were measured. For convenience, the circular $\frac{b_i}{a_j}$ -locii were plotted on a Smith chart using an X-Y recorder and taking the 'horizontal' and 'vertical' outputs from the back of the Smith chart display in the network analyzer system. These S-parameters were then used to design amplifiers by adopting the method followed by Leighton et al (15) as shown in Appendix A. The results of the large-signal S-parameters and the corresponding amplifier design for a number of transistors under Class C conditions are given in the following.

(a) Device No. 1

(i) Transistor Type: HP35821E

(ii) The $\frac{b_i}{a_j}$ -locii (Fig. 4.8) and the large-signal S-parameters (4.15).

The $\frac{b_i}{a_j}$ -locii for this transistor were plotted using

the theory in Section 4.7.1(a) and with the measurement

system in Fig. 4.7. The locii for $\frac{b_1}{a_1}$ and $\frac{b_2}{a_2}$ are given in Fig. 4.8(a) and those for $\frac{b_1}{a_2}$ and $\frac{b_2}{a_1}$ are given in Fig. 4.8(b). In generating these locii, a_1 was chosen

Note: To measure the locus of:

- (i) $\frac{b_1}{a_1}$: Connect C to R; D to T; and vary the phase shifter.
- (ii) $\frac{b_1}{a_2}$: Connect A to R; D to T; and vary the phase shifter.
- (iii) $\frac{b_2}{a_1}$: Connect C to R; B to T; and vary the phase shifter.
- (iv) $\frac{b_2}{a_2}$: Connect A to R; B to T; and vary the phase shifter.

Fig. 4.7. Measurement system to measure the large-signal S-parameters.

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL

G9-756N82BSPR (Smith)

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL

G9-756 N82BSPR (Smith)

Fig. 4.8(b). Loci for $\frac{b_1}{a_2}$ and $\frac{b_2}{a_1}$ as functions of $\angle \frac{a_2}{a_1}$;

for different values of $|a_2|^2 = P_2$ mW.

----- $\frac{b_2}{a_1}$: (outer circle=4.0); $\frac{b_1}{a_2}$: (outer circle=1.0)

Device No. 1 (HP35821E)

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to establish the required bias current. The $\frac{b_i}{a_j}$ -locii for a number of values of $|a_2|^2$ were plotted. This was performed to observe the effect on the large-signal S-parameters (centres of $\frac{b_i}{a_j}$ -locii) of 'mismatch' at the output port. Some of the $\frac{b_i}{a_j}$ -locii are not circular (for example, $\frac{b_1}{a_1}$ and $\frac{b_2}{a_1}$ -locii), indicating that the characterization by the large-signal S-parameters is not accurate in this case. However, these can be approximated by a circle and the corresponding centre may be chosen as the corresponding S-parameter. Centres of the $\frac{b_i}{a_j}$ -locii were chosen to correspond to S_{ij} as shown in Fig. 4.8. The large-signal S-parameters, thus determined are given below:

$$(V_{CE}=20V, V_{BE}=0.0, I_C=24mA, f=2GHz)$$

$$\begin{aligned} S_{11} &= .57 \angle -107^\circ & S_{12} &= 0.125 \angle 40^\circ \\ S_{21} &= 1.012 \angle 63^\circ & S_{22} &= .715 \angle -78^\circ \end{aligned} \quad (4.15(a))$$

The corresponding S-parameters were also measured by using the standard method of S-parameter measurement (14) by applying the conditions (4.13) as shown below:

$$\begin{aligned} S_{11} &= .57 \angle -110^\circ & S_{12} &= .05 \angle 100^\circ \\ S_{21} &= 1.012 \angle 65^\circ & S_{22} &= .94 \angle -98^\circ \end{aligned} \quad (4.15(b))$$

(iii) Amplifier Design Using the S-Parameters in (4.15): Fig. (4.9)

The amplifier design was carried out by using the measured large-signal S-parameters given in (4.15(a)), by following the procedure used by Leighton et al. A summary of the procedure along with the values of the design parameters are shown in Appendix A. For comparison, the amplifier design is also carried out by using the large-signal S-parameters in (4.15(b)), which were measured by the standard method. The results of the two designs are shown in Fig. (4.9). The constant gain circles and constant power output circles are shown in solid lines for the design with the S-parameters (4.15(a)) measured by the new method.

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) The $\frac{b_i}{a_j}$ -locii (Fig. 4.10) and the large-signal S-parameters (4.16).

The $\frac{b_i}{a_j}$ -locii for this transistor are shown in Fig. 4.10.

The large-signal S-parameters S_{ij} 's (centres of $\frac{b_i}{a_j}$ -locii) are found as given in (4.16(a)).

($V_{CB}=20V$, $V_{BE}=0.0$, $f=2GHz$, $I_C=25mA$)

$$S_{11} = .64 \angle 142$$

$$S_{12} = .049 \angle 57$$

$$S_{21} = .728 \angle -90$$

$$S_{22} = .956 \angle -55$$

(4.16(a))

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G8-756 N82 B5PR (Smith)

RADIALLY SCALED PARAMETERS

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G9-756 N82BSPR (Smith)

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL

G9-756 N82BSPR (Smith)

Fig. 4.10(b). Loci for $\frac{b_1}{a_2}$ (---) and $\frac{b_2}{a_1}$ (----) as functions of $\frac{a_2}{a_1}$, for different values of $|a_2|^2 = P_2$ mW.
Device No. 2 (HP35821B)

For $\frac{b_1}{a_2}$: outer circle corresponds to 1.0.

For $\frac{b_2}{a_1}$: outer circle corresponds to 4.0.

The large-signal S-parameters were also measured by the standard method. They were found to be as below:

$$\begin{aligned} S_{11} &= .64 \angle 142 & S_{12} &= .03 \angle 102 \\ S_{21} &= .728 \angle -90 & S_{22} &= .98 \angle -30 \end{aligned} \quad (4.16(b))$$

(iii) Amplifier Design Using the S-Parameters in (4.16): (Fig. 4.11)

Amplifier designs using the large-signal S-parameters in (4.16) were carried out as shown in Appendix A.

Constant gain circles and constant power output circles are shown in Fig. 4.11. The solid curves show the design using the large-signal S-parameters measured by the new method.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255.

(ii) The $\frac{b_i}{a_j}$ -locii (Fig. 4.12) and the large-signal S-parameters (4.17).

The $\frac{b_i}{a_j}$ -locii for this device are shown in Fig. 4.12.

The large-signal S-parameters S_{ij} 's (centre of $\frac{b_i}{a_j}$ -locii) are found as shown in (4.17(a)).

$$(V_{CE}=15V, V_{BE}=0V, I_C=70mA, f=2GHz)$$

$$\begin{aligned} S_{11} &= .64 \angle -154 & S_{12} &= .14 \angle -33 \\ S_{21} &= .69 \angle -40 & S_{22} &= .86 \angle -55 \end{aligned} \quad (4.17(a))$$

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G9-756 N82BSPR (Smith)

RADIALLY SCALED PARAMETERS

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES

OAKVILLE

MONTREAL

G9-756 N82BSR (Smith)

Fig. 4.12(a). Loci for $\frac{b_1}{a_1}$ and $\frac{b_2}{a_2}$ as functions of $\angle \frac{a_2}{a_1}$, for

different values of $|a_2|^2 = P_2$ mW.

Device No. 3 (2SC1255)

For $\frac{b_1}{a_1}$: outer circle corresponds to 1.0.

For $\frac{b_2}{a_2}$: outer circle corresponds to 2.0.

$|a_2|^2 = 100, 150, 200, 250, 360$ mW

$|a_2|^2 = 100, 150, 250, 360$ mW

S_{11}

S_{22}

RADIALLY SCALED PARAMETERS

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G9-756 N82BSFR (Smith)

Fig. 4.12(b). Loci for b_1/a_2 (---) and b_2/a_1 (...) as functions of $\angle a_2/a_1$, for different values of $|a_2|^2 = P_2$ mW.

Device No. 3 (2SC1255).

For $\frac{b_1}{a_2}$: outer circle corresponds to 2.0.

For $\frac{b_2}{a_1}$: outer circle corresponds to 2.0.

RADIALLY SCALED PARAMETERS

TOWARD GENERATOR

TOWARD LOAD

CENTER

The large-signal S-parameters for this device by the standard method were also measured as shown below:

$$S_{11} = .64 \angle -156 \quad S_{12} = .03 \angle 88 \quad (4.17(b))$$

$$S_{21} = .69 \angle -42 \quad S_{22} = .96 \angle -93$$

(iii) Amplifier Design Using the S-Parameters in (4.17):

Fig. 4.13

Using the large-signal S-parameters in (4.17), amplifier design calculations are shown in Appendix A. The constant gain circles and the constant power output circle are shown in Fig. 4.13. The solid lines show the design, using the large-signal S-parameters measured by the new method.

4.7.2 A New Method of Generating the Load-Pull Data for Microwave Power Transistors

4.7.2(a) Principle

As mentioned in Chapter 2, the conventional method of generating the load-pull data for power transistors consists of presenting variable load and source terminations to the transistor under test through use of tuners. However, if we consider the theory in Section 4.2 and the principle of the measurement system shown in Fig. 4.2, we can visualize that this measurement system, in fact, simulates the impedances at the ports of the transistor under test. Such simulation

NAME	TITLE	DWG. NO.
		DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G9-756 N82BSPR (Smith)

Fig. 4.13. Amplifier design using the large-signal S-parameters
Device No. 3 (2SC1255).
—— Constant Gain (dB) circles using S-parameters in (4-17(a)).
--- Constant Gain (dB) circles using S-parameters in (4-17(b)).

of an impedance or reflection coefficient was verified by using the arrangement shown in Fig. 4.14. It was found that any reflection coefficient can be realized on the Smith chart display of the network analyzer by controlling the variable attenuators and the phase shifter. This method of realizing the terminating impedances is significantly faster than the conventional method of using tuners. This is because in the proposed method it requires only 'turning' the knobs of attenuators and phase shifter, as opposed to characterizing the adjusted tuners by a separate measurement system (Chapter 2).

A procedure for generating the load-pull data for power transistors, using the proposed principle, will now be given. It is usually desired to generate the load-pull data (i.e. constant power output (P_{out}), constant collector current (I_C), etc. curves on the Smith chart) of a transistor for a particular power input. For this purpose, we can use a fixed matching network (for example, a tuner) at the input port to provide a reasonable input match (e.g. input return loss greater than 20dB, say) and thus establish a particular power input to the transistor. Then, corresponding to this, input power (P_{in}), the load-pull data can be generated by using the measurement system shown in Fig. 4.15. The load impedance is simulated by injecting a signal a_2 at the same frequency as that of the input at the output port of the transistor. The impedance presented to the transistor at the output port corresponds to the reflection coefficient $\frac{a_2}{b_2}$ which can be measured by the network analyzer as shown in

Fig. 4.14. Simulation of a 'reflection coefficient'. $\frac{b}{a}$

Note:

- (i) The tuner is adjusted to make $P_{\text{reflected}} \approx 0$.
- (ii) The 'wave' b_2 is used as 'reference' signal and the ratio a_2/b_2 is measured by the network analyzer.
- (iii) a_2/b_2 is plotted on the Smith chart by varying the phase shifter, keeping a particular $|a_2|^2$ as constant.

Fig. 4.15. Measurement setup to generate load-pull data by the new method.

Fig. 4.15. So, it may be noted that in this case, measurement of $\frac{a_2}{b_2}$ (i.e. b_2 as reference signal and a_2 as test signal to the network analyzer) is preferred to measuring $\frac{b_2}{a_2}$ (which was adopted in Section 4.5). Thus, the variation of $\frac{a_2}{b_2}$ with respect to the phase of a_2 (i.e. position of the phase shifter in Fig. 4.15) for a particular value of $|a_2|^2$ ($=P_{in}$ at port 2) can be observed on the Smith chart display of the network analyzer. It is convenient to use an X-Y recorder to plot the variation of $\frac{a_2}{b_2}$ on a Smith chart, as a function of the phase of a_2 for a constant value of $|a_2|^2$ (the power input at the output port). It is also possible to obtain the constant I_c curves by noting the points for particular I_c while varying the phase of a_2 for different $|a_2|^2$.

Thus, using the measurement system in Fig. 4.15, we can generate two sets of curves on a Smith chart, such as:

- (i) constant current (I_c) curves and
- (ii) constant $|a_2|^2$ curves.

The advantage of getting the plots on the Smith chart is that it is possible to read the load impedances which correspond to the particular I_c or $|a_2|^2$.

It now remains to generate the constant power output curves by using the constant $|a_2|^2$ curves. This can be done using the expression for power output given in (4.6), from which we get:

$$P_{out} = |a_2|^2 \cdot \frac{1 - \left| \frac{a_2}{b_2} \right|^2}{\left| \frac{a_2}{b_2} \right|^2} \quad (4.18(a))$$

or

$$R(\text{say}) = \left| \frac{a_2}{b_2} \right| = \frac{1}{\sqrt{1 + \frac{P_{out}}{|a_2|^2}}} \quad (4.18(b))$$

Thus, to generate the constant P_{out} curves, we follow the following procedure:

- (i) consider a constant $|a_2|^2$ curve on the Smith chart
- (ii) assume a P_{out} for which the locus is desired
- (iii) then, calculate R using (4.18(b)) corresponding to the chosen $|a_2|^2$ and P_{out}
- (iv) find the points on the particular constant $|a_2|^2$ curve which are at a distance R from the centre of the Smith chart.

Results of applying the method, described in this section, to generate the load-pull data (constant current I_c , constant power output P_{out} curves on the Smith chart) for a number of transistors will be presented in the next section.

4.7.2(b) Results: Load-Pull Data for Transistors Under Class C Condition

Using the procedure described in Section 4.7.2(a) and using the measurement setup as shown in Fig. 4.15, the load-pull data for

a number of transistors were generated. The constant $|a_2|^2$ curves and the constant current (I_C) curves were plotted directly on the Smith chart by using an X-Y recorder. By using the constant $|a_2|^2$ curves, the constant P_{out} curves were then generated by the procedure described in Section 4.7.2(a). The results of the load-pull data for the three transistors are given in the following.

(a) Device No. 1

(i) Transistor Type: HP35821E.

(ii) Load-Pull Data (Fig. 4.16)

The load-pull data for this transistor are shown in Fig. 4.16. These were generated at an input power of 60mW. The input matching network was adjusted for a reasonable match (input return loss >20dB).

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) Load-Pull Data (Fig. 4.17)

The load-pull data were generated by using the procedure described in Section 4.7.2(a). These curves were generated for an input power level of 65mW, and they are shown in Fig. 4.17.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255.

(ii) Load-Pull Data (Fig. 4.18)

The load-pull data for this transistor are shown in Fig. 4.18. These correspond to an input power level of 120mW.

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE

CONSTANT P_{out}

CURVES. 100, 140, 160 mW

SUPPLIES
INTERNAL
 R (Smith)

Fig. 4.16. Load-pull data generated by the new method. Device No. 1 (HP35821E), $P_{in} = 60\text{mW}$, $\Gamma_S = .53/155^\circ$ i.e. $Z_S = 16 + j10\Omega$.

RADIALLY SCALED PARAMETERS

TOWARD GENERATOR

TOWARD LOAD

CENTER

NAME

TITLE

DWG. NO.

DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREALCONSTANT P_{out} 3-756N82BSPR (5mW)
120, 140, 160, 170mW

Fig. 4.17- Load-pull data generated by the new method. Device No. 2 (HP35821B). $P_{in} = 65\text{mW}$, $\Gamma_S = 7\angle -142$, i.e. $Z_S = 9.5 - j17\Omega$.

RADIALLY SCALED PARAMETERS

NAME	TITLE	DWG. NO.
		DATE

IMPEDANCE OR ADMITTANCE COORDINATES

CANADIAN CHARTS & SUPPLIES
OAKVILLE MONTREAL
G9-756 N82BSPR (Smith)

Fig. 4.18. Load-pull data generated by the new method. Device No. 3 (2SC1255), $P_{in} = 120mW$, $\Gamma_S = .65 \angle -160^\circ$ i.e. $Z_S = 10.75 - j8.5\Omega$

CONSTANT $|a_2|$ CURVES

- 500
- 470
- 450
- 410
- 360
- 300
- 270
- 240

RADIALLY SCALED PARAMETERS

4 SUMMARY AND REMARKS

4.8.1 Summary

A new 'black box' approach to the characterization of a non-linear 2-port has been proposed in this chapter. The theory for a non-linear 2-port in terms of wave variables is described and the basis of measuring the responses of the 2-port by simultaneously applying two signals at the two ports is established. A measurement system, at microwave frequencies, is shown which is capable of measuring the responses of a non-linear 2-port by using this principle. A sequence of measurement steps has been developed to achieve optimum design of transistor power amplifiers by using the measurement system. Results of optimum power amplifier designs for three transistors are given.

Two further significant applications of the principle of this 'black box' method are illustrated. First, it is shown that this method can be used to meaningfully measure the large-signal S-parameters of a transistor under Class C conditions. The large-signal S-parameters of three transistors under Class C operation, measured by the new method and the conventional method, are given. The results of the corresponding amplifier designs, using S-parameters, are also obtained. Second, it has also been demonstrated that the 'black box' method can be used to electrically simulate the load impedance of a transistor. And consequently, it has been found that this method can

be used to generate the load-pull data for power transistors in a considerably faster and easier manner compared to the conventional method of using tuners. The load-pull data for the three transistors, obtained by this method, are also given.

4.8.2 Remarks

So far, in this thesis, two methods have been proposed. First, a method of designing the optimum second harmonic reactance terminations of a transistor under Class C operation has been described in Chapter 3. Second, a 'black box' method has been proposed, in this chapter, which has been used to design the fundamental frequency terminations of the Class C amplifiers for optimum performance.

These two methods may thus be considered to form a systematic approach to the design of microwave transistor Class C power amplifiers. An application of these two methods yields the design of the fundamental and the second harmonic impedance terminations of the Class C amplifier for optimum performance. The next step is to build the amplifiers by realizing these design impedances.

The performances of the amplifiers, in which the matching networks were realized by using microstrip transmission lines, will be shown in Chapter 6. However, in the following Chapter, a method of designing the matching networks on microstrip, realizing the specified fundamental and second harmonic impedances, will be illustrated.

CHAPTER 5

DESIGN OF MATCHING NETWORKS

5.1 INTRODUCTION

In general, the design of matching networks involves finding a lossless network which transforms the characteristic impedance of a transmission line (usually 50 ohms) to a specified impedance at the terminals of a device. The specified impedance corresponds to some designed termination of the device for some required performances.

The matching networks for narrow-band applications may, usually, be realized with simple circuits. Because, in this case, it is required to realize an impedance match at a particular frequency (or a narrow-band of frequencies). However, often, a matching network is required to 'match' a specified range of impedances over a specified wide band of frequencies. Consequently, the degree of complexity of a matching network design depends on the requirement of the impedances to be matched.

In general, the design of matching networks for linear devices is relatively simpler than those required for non-linear devices. This is because the matching networks for non-linear devices should, in addition to matching the impedance at the frequency of operation, filter out the signals of undesired frequencies. Thus, the matching networks for Class C transistor power amplifier designs are usually of low-pass or band-pass type - the former being relatively simpler and more economical (7,62).

The matching networks are usually realized by using lumped elements and sections of transmission lines. However, when the frequency of operation is sufficiently high, the use of only transmission line sections as network elements becomes practical because of the short lengths required.

To briefly explain how a discrete circuit element can be approximately realized by using transmission line sections, reference is made to Fig. 5.1, which illustrates the exact T - and π - equivalent circuits of a length ℓ of a non-dispersive TEM transmission line (37). Also shown in Fig. 5.1 are the equivalent reactance (X) and susceptance (B) values of the networks when their physical length ℓ is small enough (so that the electrical length, $\omega\ell/v$, of the line is less than about $\pi/4$ radians, where ω is the frequency in radians and v is the velocity of propagation in the transmission line). By inspection of the equivalent circuits in Fig. 5.1, we see that a short length of line of high $-Z_0$ (where Z_0 is the characteristic impedance of the line) terminated at both ends by a relatively low impedance has an effect equivalent to that of a series inductance having a value of $L = Z_0 \frac{\ell}{v}$ henries. Similarly, a short length of low- Z_0 line terminated at either end by a relatively high impedance has an effect equivalent to that of a shunt capacitance having a value of $C = \frac{\ell}{Z_0 v}$ farads.

At microwave frequencies, the microstrip (41,49) transmission line may be conveniently used for designing the matching networks.

Z_0 = characteristic impedance.

(a)

$$\frac{X}{2} = Z_0 \tan \frac{\omega l}{2v} \approx Z_0 \frac{\omega l}{2v} \quad \left| \frac{\omega l}{v} < \frac{\pi}{4} \right|$$

$$B = Y_0 \sin \frac{\omega l}{v} \approx Y_0 \frac{\omega l}{v} \quad \left| \frac{\omega l}{v} < \frac{\pi}{4} \right|$$

(b)

$$X = Z_0 \sin \frac{\omega l}{v} \approx Z_0 \frac{\omega l}{v} \quad \left| \frac{\omega l}{v} < \frac{\pi}{4} \right|$$

$$\frac{B}{2} = Y_0 \tan \frac{\omega l}{2v} \approx Y_0 \frac{\omega l}{2v} \quad \left| \frac{\omega l}{v} < \frac{\pi}{4} \right|$$

(c)

Fig. 5.1 TEM transmission line equivalent circuits [37].

(a) TEM line

(b) Exact T-equivalent

(c) Exact π -equivalent circuit

Microstrip is a type of planar transmission line consisting of a single strip of conductor separated from a ground plane by a dielectric material as shown in Fig. 5.2. It is a simplified version of strip-line (38) (two ground planes and one strip of conductor in the middle) but is subject to more radiation and fringing fields. The propagation in microstrip is essentially by the TEM mode; the deviations from TEM mode arise because the dielectric properties of the material between the strip and the ground plane differ from those of the ~~air~~ above the strip. Based on extensive analysis and experimental data (35, 39, 42-52), design of microstrip circuits may be accomplished. A number of useful formulae that are presently used are described in Appendix B.

In the present context, the requirements of the matching networks are slightly different from the conventional ones. In the amplifier design approach presented in this thesis, the matching networks are required to present, at the device terminals, two specified impedances at two specified frequencies. For example, the method presented in Chapter 3 provides us with a design of the second harmonic reactance terminations for the amplifier; and the method of Chapter 4 provides us with a design of the fundamental frequency impedance termination of the transistor. Thus, for the predicted optimum performances of the amplifier, the matching networks must be capable of presenting the two designed impedances - one at the fundamental and the other at the second harmonic frequency of operation - at the device terminals.

(a)

(b)

Fig. 5.2 (a) Microstrip transmission line.
 (b) Electric and magnetic fields in the vicinity of a microstrip.

In this chapter, a convenient structure of matching network is considered and a method of analysis and design of such networks by using microstrip transmission lines is presented. Results of practical realizations of such networks on microstrip, using Duroid (39), are also presented.

5.2 DESIGN OF MATCHING NETWORKS ON MICROSTRIP

The requirement, mentioned at the end of Section 5.1, suggests that the structure of the matching networks should, if possible, be so chosen as to enable approximately independent variation of the two impedances (say, $Z(\omega_0)$ at fundamental and $Z(2\omega_0)$ at the second harmonic frequency) with respect to certain elements of the network. If this is possible, the number of variables in the design analysis can be reduced. In this respect, the elliptic-function (or Cauer parameters) type low-pass structures (36-38) may be considered for reasons stated in the following. A structure of such networks, in terms of discrete elements, is shown in Fig. 5.3(a). The discrete component design of filter networks of such structures is possible by synthesis methods or by published tabulated results (36).

Since this type of network structure uses series-resonant shunt branches, by judicious choice of the resonant frequencies ω_{aa} and ω_{ab} (Fig. 5.3(a)) of the shunt branches and the series inductances L_i 's, it is possible to design the two impedances $Z(\omega_0)$ and $Z(2\omega_0)$ in a fairly independent manner. For example, if ω_{aa} is chosen to be around $2\omega_0$, then the elements to the right of the corresponding

Fig. 5.3. (a) An elliptic-function (Cauer parameters) low-pass structure in terms of discrete components.
 (b) Microstrip version of (a):
 Z_i 's are characteristic impedances.
 T_i 's are the electrical lengths in degrees at the fundamental frequency.

shunt branch in Fig. 5.3(a) do not have significant influence on $Z(2\omega_0)$. In other words, to realize $Z(2\omega_0)$, one mainly needs to design only the elements L_2 , L_5 and C_6 in Fig. 5.3(a). After $Z(2\omega_0)$ is realized, now the other elements may be suitably chosen to realize $Z(\omega_0)$. The second shunt branch is used to attain large attenuation at the second harmonics. The capacitance C_9 is used to provide high attenuation at higher frequencies.

The structure of the microstrip version of the network in Fig. 5.3(a) is shown in Fig. 5.3(b). The L's and C's in Fig. 5.3(b) indicate the sections of microstrip transmission lines which approximate the specific elements in Fig. 5.3(a).

Thus, to design the required impedances $Z(\omega_0)$ and $Z(2\omega_0)$ the microstrip circuit structure, shown in Fig. 5.3(b), may be analyzed by using the transmission matrix (ABCD matrix) (57) formulation of the microstrip transmission line elements. It should, however, be noted that, in the designed structure, the lengths of the microstrip lines should be kept as short as possible and the impedance ratios as high as possible, mainly to maximize the frequency at which the first spurious response occurs (38). Of course, the minimum lengths are limited by the requirement of avoiding undesired coupling to other lines and the largest- Z_0 lines are limited by the etching process. For example, using the Duroid (39), highest- Z_0 of 80 ohms and lowest- Z_0 of 15 ohms are usually realizable.

To analyze the circuit in Fig. 5.3(b), a computer program was developed by employing the ABCD matrix formulation (57) of the

transmission line elements. The characteristic impedances Z_i 's and the electrical length T_i 's (at the fundamental frequency, ω_0) could be changed in an interactive manner through the use of some 'code' number to identify the different elements. The initial set of Z_i 's and T_i 's may be chosen arbitrarily. However, to cut down the design time, initial set of values were chosen corresponding to a design given in the published tabulated form (36) for discrete component versions. While using the interactive program, the response of the network around the fundamental and the second harmonic frequencies may be observed, every time the parameters Z_i and T_i of an element are changed. In this manner, a set of values of Z_i 's and T_i 's may be determined by satisfying some acceptable frequency sensitivity of the impedances $Z(\omega_0)$ and $Z(2\omega_0)$. Using these Z_i 's and T_i 's, the dimensions of the transmission lines can be calculated by using the formulae in Appendix B.

In general, the following sequence of procedure may be followed in analyzing the network in Fig. 5.3(b):

- (i) The elements 5-8 are chosen to obtain some desired large attenuation ($>40\text{dB}$, say) at the second harmonic frequency since the second harmonic termination is to be reactive.
- (ii) Vary the elements 1 and 2 to realize the desired second harmonic reactance, $Z(2\omega_0)$.
- (iii) Vary the elements 4, 9 and 10 (if necessary, also 3) to realize the desired fundamental frequency impedance, $Z(\omega_0)$.

It is possible to choose the elements 5-8 such that the second harmonic

reactances $Z(2\omega_0)$ are quite insensitive to the choice of the elements 3, 4, 9 and 10 in Fig. 5.3(b).

The filter circuits and the matching networks required for the three transistors, as specified by the amplifier design theory in Chapters 3 and 4, were designed by using the method described above. The theoretical and the experimental performance characteristics of these filter- and matching networks are shown in the next section.

5.3 RESULTS: FILTERS AND MATCHING NETWORKS

The power amplifier design theory in Chapter 4 presumes that the transistor has been terminated at the optimum second harmonic reactances designed by the method in Chapter 3. Thus, it is essential to design some filter networks which present the designed second harmonic reactances both at the input and output ports of the transistor. After the optimum terminations of the class C power amplifiers were designed in Chapter 4, it was then necessary to design the required matching networks realizing the designed fundamental frequency impedance and the second harmonic reactances for the transistor.

The filter and matching networks, for the three transistors used in this thesis, were designed by using the method described in Section 5.2, and the experimental performances of these networks are shown in the following.

(a) Device No. 1

(i) Transistor Type: HP35821E

(ii) Filters: (Fig. 5.4)

Input: (Fig. 5.4(a))

The second harmonic (4GHz) reactance corresponds

to $\theta_3 \approx 338^\circ$ in Table 3.11.

Output: (Fig. 5.4(b))

The second harmonic (4GHz) reactance corres-

ponds to $\theta_4 \approx 308^\circ$ in Table 3.11.

(iii) Matching networks for the amplifier: (Fig. 5.5)

Input Matching Network (Fig. 5.5(a))

The fundamental frequency impedance $Z_S(\omega_0) = 15.5 + j11.5$ ohms corresponding to $\Gamma_S = .55 \angle 153$ in the design table in Table 4.3(b).

The second harmonic reactance $Z_S(2\omega_0) \approx -j257$ ohms corresponds to $\theta_3 = 338^\circ$ in Table 3.11.

Output Matching Network (Fig. 5.5(b))

The fundamental frequency impedance $Z_L(\omega_0) \approx 28 + j66$ ohms corresponds to $\Gamma_L = .68 \angle 68$ in the design table in Table 4.3(b).

The second harmonic reactance $Z_L(2\omega_0) \approx -j102.5$ ohms corresponds to $\Gamma_4 = 308^\circ$ in Table 3.11.

(b) Device No. 2

(i) Transistor Type: HP35821B

(ii) Filters: (Fig. 5.6)

Input: (Fig. 5.6(a))

The second harmonic reactance corresponds to

 $\theta_3 = 140^\circ$ in Table 3.11.

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

Fig. 5.6(a). Input filter for the Device No. 2 (HP35821B).

$\Gamma_S(2\omega_0) = 1 \angle 140^\circ$, $Z_S(2\omega_0) = j18.25$ ohms

a: Theory
X: Experiment

a: Theory
X: Experiment

Fig. 5.6(b). Output filter for Device No. 2.
(HP35821B).
 $\Gamma_L(2\omega_0) = 1\angle 92^\circ$, $Z_L(2\omega_0) = j48.3$ ohms
a: Theory
X: Experiment

Output Filter: (Fig. 5.6(b)) .

The second harmonic reactance corresponds to

$\theta_4 = 92^\circ$ in Table 3.11.

(iii) Matching Networks for Amplifiers (Fig. 5.7)

Input Matching Network (Fig. 5.7(a))

The fundamental frequency (2GHz) impedance

$Z_S(\omega_0) \approx 10.3 - j16.6$ ohms corresponds to $\Gamma_S = .66 \angle -140$

in the design table in Table 4.4(b). The second

harmonic (4GHz) reactance $Z_S(2\omega_0) = j18.2$ ohms, corresponding to $\theta_3 = 140^\circ$ in Table 3.11.

Output Matching Network (Fig. 5.7(b))

The fundamental frequency impedance $Z_L(\omega_0) =$

$20 + j68$ ohms, corresponds to $\Gamma_L = .78 \angle 70$ in the design table in Table 4.4(b). The second harmonic reactance

$Z_L(2\omega_0) = j48.3$ ohms corresponds to $\theta_4 = 92^\circ$ in Table 3.11.

(c) Device No. 3

(i) Transistor Type: NEC2SC1255

(ii) Filters (Fig. 5.8)

Input (Fig. 5.8(a))

The second harmonic (4GHz) reactance corresponds to $\theta_3 = 220^\circ$ in Table 3.11.

Output (Fig. 5.8(b))

The second harmonic (4GHz) reactance corresponds to $\theta_4 = 225^\circ$ in Table 3.11.

Fig. 5.7(a). Input matching network for Device No. 2 (HP35821B).

$\Gamma_S(\omega_0) = .66 \angle -140^\circ$, $Z_S(\omega_0) = 10.3 - j16.6$ ohms

$\Gamma_S(2\omega_0) = .1 \angle 140^\circ$, $Z_S(2\omega_0) = j18.2$ ohms

e: Theory
X: Experiment

WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RESISTANCE COMPONENT (R)
REACTANCE COMPONENT (X)
RADIIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

(iii) Matching Networks for Amplifiers (Fig. 5.9)

Input Matching Network (Fig. 5.9(a))

The fundamental frequency impedance $Z_S(\omega_0) = 10.9 - j7.8$ ohms corresponds to $\Gamma_S = .65 \angle -162^\circ$ in the design table in Table 4.5(b). The second harmonic (4GHz) reactance $Z_S(2\omega_0) = -j18.2$ ohms corresponds to $\theta_3 = 220^\circ$ in Table 3.11.

Output Matching Network (Fig. 5.9(b))

The fundamental frequency (2GHz) impedance $Z_L(\omega_0) = 30 + j93$ ohms corresponds to $\Gamma_L = .78 \angle 53^\circ$ in the design table in Table 5(b). The second harmonic reactance $Z_L(2\omega_0) = -j20.7$ ohms corresponds to $\theta_4 = 225^\circ$ in Table 3.11.

5.4 SUMMARY AND REMARKS5.4.1 Summary

A procedure for realizing the matching networks for Class C transistor power amplifiers has been illustrated in this chapter. The results of realization of a number of filter and matching networks using the microstrip transmission lines are presented. It has been shown that by choosing a convenient network structure, it is possible to present two impedances (e.g. $Z(\omega_0)$, & the second harmonic reactance) at the transistor terminals by transforming a 50 ohm characteristic impedance.

IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

NETWORK	Z_1 Ω	T_1 deg	Z_2 Ω	T_2 deg	Z_3 Ω	T_3 deg	Z_4 Ω	T_4 deg	Z_5 Ω	T_5 deg	Z_6 Ω	T_6 deg	Z_7 Ω	T_7 deg	Z_8 Ω	T_8 deg	Z_9 Ω	T_9 deg
Z_{SF}	23	35	80	23.7	80	15	80	29.1	70	4.5	20	23.75	70	7.45	20	27	15	22
Z_{LF}	23	33	80	23.7	80	15	80	28.3	70	4.5	20	23.75	70	7.45	20	27	15	22
Z_{SM}	23	35	80	23.7	80	15	80	15	70	4.5	20	23.75	70	7.45	20	27	14	70
Z_{LM}	23	33	80	23.7	80	15	80	62	70	4.5	20	23.75	70	7.45	20	27	15	26
NETWORK	ω_1 mils	l_1 mils	ω_2 mils	l_2 mils	ω_3 mils	l_3 mils	ω_4 mils	l_4 mils	ω_5 mils	l_5 mils	ω_6 mils	l_6 mils	ω_7 mils	l_7 mils	ω_8 mils	l_8 mils	ω_9 mils	l_9 mils
Z_{SF}	84	393	13.5	281	13.5	178	13.5	345	17	53	99	258	17	88	99	294	139	243
Z_{LF}	84	370	13.5	281	13.5	178	13.5	336	17	53	99	258	17	88	99	294	139	243
Z_{SM}	84	393	13.5	281	13.5	178	13.5	178	17	53	99	258	17	88	99	294	150	774
Z_{LM}	84	370	13.5	281	13.5	178	13.5	736	17	53	99	258	17	88	99	294	139	288

Fig. 5.9(c). Filter and matching network designs for device no. 3. (First subscript denotes source (S) or load (L), and the second subscript denotes filter (F) or matching network (M), in the first column.)

The design of these matching networks was accomplished by writing an interactive type of computer program which employs the ABCD matrix formulation of the transmission line elements.

5.4.2 Remarks

For accurate practical realization of the matching networks on microstrip, it is required that the impedances of the designed network at the fundamental and the second harmonic frequencies are least sensitive to frequency. In other words, these two impedances are to be least sensitive with respect to small variations in the dimensions of the transmission line elements. Also to keep the circuit losses to a minimum, the total conductor area should be minimal.

These requirements emphasize the necessity of implementing an optimization routine along with the analysis program used in this chapter. However, for the designs presented in this chapter, the above requirements were met, as far as possible, by observing the response of the networks in an interactive manner. For this reason, the design of most of the circuits required a considerable amount of effort.

CHAPTER 6

COMPARISON OF THE PREDICTED AND EXPERIMENTAL RESULTS

6.1 INTRODUCTION

In this chapter, a comparison is made between the predicted results, presented in Chapters 3 and 4, and the experimental results obtained by using three microwave transistors.

It may be observed that the complete approach, presented in this thesis, consists of two parts: first, the method, in Chapter 3, of characterizing the effects of harmonics in a non-linear 2-port, enables one to design the optimum second harmonic reactance terminations of a Class C amplifier. Second, the method, in Chapter 4, of characterization and design of a general non-linear 2-port, provides us with the input and output terminations, at the fundamental frequency, for optimum performance of a Class C transistor amplifier. The amplifiers constructed on microstrip by realizing the second harmonic reactances and the fundamental frequency impedances, designed by these two methods will, hereafter, be referred to as the 'optimum amplifiers'. To ascertain the effects of the harmonics, amplifier performances were also measured by keeping the fundamental frequency terminations as those of the 'optimum amplifier', but by using some arbitrarily chosen second harmonic reactances which are different from those of the 'optimum amplifiers'.

The method of Chapter 4 was used in the measurement of the large-signal S-parameters of the transistors operating in Class C

conditions. The amplifier designs, using these S-parameters and the ones measured by the conventional method, have been given in Chapter 4. A summary of these designs has been given in Fig. A.2 of Appendix A. A comparison of the amplifier designs, using the large-signal S-parameters measured by the new method, with the designs of the 'optimum amplifiers' is given.

Finally, the load-pull data of the transistors, generated by using the principle of Chapter 4, are also used to compare the actual performances of the 'optimum amplifiers'.

For clarity, the comparison between the predicted and the experimental results of each transistor will be drawn separately, in the following sections.

6.2 PREDICTED AND EXPERIMENTAL PERFORMANCES OF THE AMPLIFIERS

6.2.1 Amplifiers Using Device No. 1

(a) Transistor Type: HP35821E

(b) 'Optimum Amplifier': Amplifier No. 1A

(i) Performances are shown in Figs. 6.1(a) and (b) by solid lines.

(ii) Second harmonic (4GHz) reactances:

Input port: $jx_3 = -j2.57 \text{ ohms}$

Output port: $jx_4 = -j102.5 \text{ ohms}$

(iii) Fundamental frequency (2GHz) impedances:

Input port: $15.8 + j11.3 \text{ ohms}$

Output port: $28.2 + j66.3 \text{ ohms}$

Fig. 6.1(a). Experimental performances of the Class C amplifiers using Device No. 1 (HP35821E). ($V_{CE}=20\text{V}$; $V_{BE}=0$, $f_o=2\text{GHz}$)

Fig. 6.1(b). Frequency response of the Class C amplifier using the Device No. 1 (HP35821E), $V_{CE}=20V$, $V_{BE}=0V$.
 $P_{in}=65mW$, $I_C=25mA$, at $f=2GHz$
 $P_{in}=65mW$, $I_C=28.3mA$, at $f=2GHz$

Fig. 6.1(c) Microstrip Amplifier using the Device no. 1, hp35821E.

(iv) Matching networks design realizing (ii) and (iii) is shown in Figs. 5.5(a) and (b).

(v) Filter networks design realizing (ii) is shown in Figs. 5.4(a) and (b).

(c) Amplifier with arbitrary second harmonic reactance terminations: Amplifier No. 1B

(i) Performances are shown in Figs. 6.1(a) and (b) by broken lines.

(ii) Second harmonic (4GHz) reactances:

Input: $jx_3 = -j18.2$ ohms

Output: $jx_4 = -j20.7$ ohms

(iii) Fundamental frequency (2GHz) impedances the same as b(iii).

(iv) Filter networks design realizing (ii) is shown in Figs. 5.8(a) and (b).

(v) Matching networks design realizing (ii) and (iii) by using the filters shown in Figs. 5.8(a) and (b) and a tuner.

(vi) Comparison of the predicted and the experimental performances of the optimum amplifier No. 1A:

As seen from Fig. 6.1(a), the power output

$P_o = 135\text{mW}$, power gain $G_p = 3.8\text{dB}$ and conversion

efficiency $\eta = 33\%$; the corresponding predicted

values from the design in Chapter 4 are $P_o = 145\text{mW}$,

$G_p = 3.64\text{dB}$, $\eta = 36.3\%$.

- (vii) Comparison of the 'optimum amplifier' No. 1A and the amplifier No. 1B (one with arbitrary second harmonic reactances).

The performances of the two amplifiers, 1A and 1B, are shown in Fig. 6.1(a). It is seen from these curves that the optimum amplifier 1A has about 19mW more power output, .65dB more power gain and about 9% more efficiency, than amplifier 1B.

6.2.2 Amplifier Using Device No. 2

(a) Transistor Type: HP35821B.

(b) 'Optimum Amplifier': Amplifier No. 2A

(i) Performances are shown in Figs. 6.2(a) and (b), by solid lines.

(ii) Second harmonic (4GHz) reactances:

Input port: $jx_3 = j18.2$ ohms

Output port: $jx_4 = j48.3$ ohms

(iii) Fundamental frequency (2GHz) impedances:

Input port: $Z_S = 10.1 - j17.2$ ohms

Output port: $Z_L = 19.8 + j68.1$ ohms

(iv) Matching networks design realizing (ii) and (iii) is shown in Figs. 5.7(a) and (b).

(v) Filter networks design realizing (ii) is shown in Figs. 5.6(a) and (b).

Fig. 6.2(a). Experimental performances of the Class C amplifiers using Device No. 2 (HP35821B), $V_{CB} = 20\text{V}$, $C_{EB} = 0\text{V}$.

Fig. 6.2(b). Frequency response of the Class C amplifiers using
 Device No. 2 (HP35821B), $V_{CB}=20V$, $V_{EB}=0V$,
 $P_{in}=60mW$, $I_C=25mA$, at $f=2GHz$
 $P_{in}=60mW$, $I_C=26.3mA$, at $f=2GHz$

(c) Amplifier with arbitrary second harmonic reactance

terminations: Amplifier No. 2B

(i) Performances are shown in Figs. 6.2(a) and (b) by broken lines.

(ii) Second harmonic (4GHz) reactances:

Input: $jx_3 = -j18.2 \text{ ohms}$

Output: $jx_4 = -j20.7 \text{ ohms}$

(iii) Fundamental frequency (2GHz) impedances are the same as b(iii).

(iv) Filter networks design realizing (ii) is shown in Figs. 5.8(a) and (b).

(v) Matching networks design, realizing (ii) and (iii), by using the filters shown in Figs. 5.8(a) and (b) and a tuner.

(vi) Comparison of the predicted and the experimental performances of the 'optimum amplifier' No. 2A.

As seen from Fig. 6.2(a), the power output, $P_o = 158 \text{ mW}$, power gain, $G_p = 4.0 \text{ dB}$ and the conversion efficiency $\eta = 30\%$; whereas, the corresponding values from the design in Chapter 4 are $P_o = 168.6 \text{ mW}$, $G_p = 4.12 \text{ dB}$ and $\eta = 32.43\%$.

(vii) Comparison of the 'optimum amplifier' No. 2A and the amplifier No. 2B (one with arbitrary second harmonic reactances).

The performances of the two amplifiers, 2A and 2B, are shown in Fig. 6.2(a). It is seen from these curves that the 'optimum amplifier' 2A has about 24mW more output power, 0.7dB more power gain and about 6% more conversion efficiency than the amplifier 2B.

6.2.3 Amplifier Using Device No. 3

(a) Transistor Type: NEC2SC1255.

(b) 'Optimum amplifier': Amplifier No. 3A

(i) Performances are shown in Figs. 6.3(a) and (b) by solid lines.

(ii) Second harmonic (4GHz) reactances:

Input: $jx_3 = -j18.2$ ohms

Output: $jx_4 = -j20.7$ ohms

(iii) Fundamental frequency (2GHz) impedances:

Input: $Z_S = 11.0 - j7.9$ ohms

Output: $Z_L = 30.3 + j93.5$ ohms

(iv) Matching networks design realizing (ii) and (iii) is shown in Figs. 5.9(a) and (b).

(v) Filter networks design realizing (ii) is shown in Figs. 5.8(a) and (b).

(c) Amplifier with arbitrary second harmonic reactance

terminations: Amplifier No. 3B

(i) Performances are shown in Figs. 6.3(a) and (b) by broken lines.

Fig. 6.3(a). Experimental performances of the amplifiers using Device No. 3 (2SC1255), $V_{CE} = 15\text{V}$, $V_{BE} = 0\text{V}$.

Fig. 6.3(b). Frequency response of the Class C amplifiers using Device No. 3 (2SC1255), $V_{CE} = 15V$, $V_{BE} = 0V$, $P_{in} = 120mA$, $I_C = 70mA$ at $f_o = 2GHz$, $P_{in} = 120mA$, $I_C = 76mA$ at $f_o = 2GHz$

(ii) Second harmonic (4GHz) reactances:

Input: $jx_3 = -j257$ ohms

Output: $jx_4 = -j102.5$ ohms

(iii) Fundamental frequency (2GHz) impedances are the same as b(iii).

(iv) Filter networks design realizing (ii) is shown in Figs. 5.4(a) and (b).

(v) Matching networks design realizing (ii) and (iii) by using the filters shown in Figs. 5.4(a) and (b) and a tuner.

(vi) Comparison of the predicted and the experimental performances of the 'optimum amplifier' No. 3A. As observed from Fig. 6.3(a), the power output $P_o = 300$ mW, power gain $G_p = 3.52$ dB and the conversion efficiency $\eta = 28.8\%$; whereas the corresponding values predicted by the design in Chapter 4 are $P_o = 319$ mW, $G_p = 4$ dB and $\eta = 30.4\%$.

(vii) Comparison of the 'optimum amplifier' No. 3A and the amplifier No. 3B (one with arbitrary second harmonic reactances).

The performances of the two amplifiers, 3A and 3B, are shown in Fig. 6.3(a). It is seen from these curves that the 'optimum amplifier' No. 3A has about 30 mW more output power, .5 dB more power gain and about 5.2% more conversion efficiency than amplifier 3B.

6.3 COMPARISON OF THE 'OPTIMUM AMPLIFIER' DESIGNS WITH THOSE BY THE LARGE-SIGNAL S-PARAMETER AND LOAD-PULL DATA

For a comparison of the 'optimum amplifier designs' with the designs by using the large-signal S-parameters and the load-pull data, we shall refer to Chapter 4.

The optimum amplifier designs using the three microwave transistors have been shown in Figs. 4.3 to 4.5. The designs using the large-signal S-parameters of the three transistors are shown in Figs. 4.9, 4.11 and 4.13. A summary of the large-signal S-parameters design is included in Table A.2 of Appendix A. The load-pull data for the three transistors are shown in Figs. 4.16 to 4.18.

From the above-mentioned figures, we can summarize the results of these designs as shown in Table 6.4.

From the summary (Table 6.4), we can observe that the designs from load-pull data compare well with the 'optimum designs'. It may be noted here that the load-pull data were generated only for one power input case. Thus, the 'optimum design' corresponding to approximately the same input power level is compared.

It is also observed that designs using the large-signal S-parameters do not compare very well with the 'optimum amplifier' designs. This, of course, is not surprising; because the large-signal S-parameters method is based on the linearization of the non-linear device. As can be observed from the plots of the $\frac{b_i}{a_i}$ -locii in Section 4.7.1, the S-parameters are not constant. However, the designs using the large-signal S-parameters measured by the new method

DEVICE NO.	DESIGN PARAMETERS	VALUES OF THE DESIGN PARAMETERS FOR VARIOUS DESIGNS			
		OPTIMUM AMPLIFIER DESIGNS	LARGE SIGNAL S-PARAMETERS		LOAD-PULL DATA BY THE NEW METHOD
			NEW METHOD	CONVENTIONAL METHOD	
Device no. HP35821E $V_{CE}=20V$ $V_{BE}=0.0$ $f=2GHz$ $I_C=25mA$ Refer to Figs.: 4.3(b) 4.9 4.16	P_{in} (mW)	62.78	95	95	60
	P_o (mW)	145.1	200	200	150
	G_p (dB)	3.64	4	4	3.98
	η (%)	36.28	40	40	37.5
	Γ_S	.55 $\angle$ 153	.575 $\angle$ 130	.62 $\angle$ 118	.545 $\angle$ 155
	Z_S	15.8+j11.3 Ω	16+j21 Ω	16+j28 Ω	15.3+j10.3 Ω
	Γ_L	.68 $\angle$ 68	.81 $\angle$ 65	.825 $\angle$ 72	.66 $\angle$ 65
	Z_L	28.2+j66.3 Ω	18+j75 Ω	13.5+j67 Ω	32.3+j68 Ω
Device no. HP35821B $V_{CB}=20.0V$ $V_{BE}=0.0$ $f=2GHz$ $I_C=25mA$ Refer to Figs.: 4.4(b) 4.11 4.17	P_{in} (mW)	64.47	92	90	65
	P_o (mW)	168.61	206	206	150
	G_p (dB)	4.18	4	4	3.63
	η (%)	32.43	41	41	28.85
	Γ_S	.71 $\angle$ -142	.705 $\angle$ -135	.62 $\angle$ 118	.72 $\angle$ -141
	Z_S	10.1-j17.2 Ω	10-j20 Ω	15.8+j28 Ω	9.2-j17 Ω
	Γ_L	.74 $\angle$ 71	.86 $\angle$ 73	.783 $\angle$ 49	.76 $\angle$ 69
	Z_L	19.8+j68.1 Ω	11+j66.6 Ω	34+j101 Ω	20.5+j69 Ω
Device no. NEC 2SC1255 $V_{CE}=15V$ $V_{BE}=0.0$ $f=2GHz$ Refer to Figs.: 4.5(b) 4.13 4.18	P_{in} (mW)	127.05	185	185	120
	P_o (mW)	319.2	300	300	300
	G_p (dB)	4	3.8	3.8	3.98
	η (%)	30.4	28.6	28.6	32.3
	Γ_S	.65 $\angle$ -162	.59 $\angle$ 133	.609 $\angle$ 161	.66 $\angle$ -160
	Z_S	11-j7.9 Ω	15.3+j20 Ω	12.5+j8 Ω	10.6-j8.3 Ω
	Γ_L	.78 $\angle$ 53	.79 $\angle$ 47	.845 $\angle$ 78.5	.76 $\angle$ 55
	Z_L	30.3+j93.5 Ω	35+j105 Ω	10.5+j60 Ω	30.2+j88 Ω

Table 6.4. Summary of class C amplifier designs by using the three transistors. The methods of design are presented in Chapter 4.

compare with the 'optimum amplifier' designs better than the ones using the S-parameters measured by the conventional method. It may also be mentioned that the power output predicted by the large-signal S-parameters method is crudely an approximate one. This limitation of the large-signal S-parameters method arises because of the fact that the RF saturation voltage (Appendix A) cannot be measured accurately. Also, the RF saturation voltage is a function of the impedance level at which it is measured (15).

6.4. DESIGN OF FREQUENCY MULTIPLIERS (DOUBLERS)

In Section 3.3, a method of designing the second harmonic reactance at the input and the fundamental frequency reactance at the output port of a frequency doubler was described. A method is now required to design the fundamental frequency impedance at the input and the second harmonic impedance at the output port to realize the frequency doubler.

Development of a method similar to the one reported in Chapter 4 for class C power amplifier designs has not yet been attempted. However, some attempts have been made to design the filter networks by using a structure topologically similar to the ones used in this thesis (Chapter 5). The attempts have not been very encouraging for reasons stated below.

A requirement of the frequency doublers is that the output filter - or matching network should be of a bandpass type. It has been found that the filters, meeting this requirement, necessitated

larger areas than could be accommodated on the measurement jig (4" x 4") available. It may, however, be possible to achieve some satisfactory designs by using an optimization routine along with the program used for designing the circuits in Chapter 5. Thus, it was decided that the design of frequency doublers be left as a topic of further investigation.

6.5 REMARKS

In this chapter, a comparison of the predicted and the experimental performances of the three Class C amplifiers has been presented.

It has been shown that it is possible to achieve an increase in the power output, gain, etc. of a Class C power amplifier by properly designing the second harmonic reactance terminations. The design of the reactances has been achieved by the method presented in Chapter 3.

It is seen from the experimental results of the designed amplifiers that the performances of Class C transistor power amplifiers at microwave frequencies can be predicted quite accurately by using the new method proposed in Chapter 4.

CHAPTER 7

SUMMARY, CONCLUSION AND FURTHER RESEARCH WORK

7.1 SUMMARY AND CONCLUSIONS

In this thesis, a systematic approach to the characterization and design of microwave transistor power amplifiers has been developed. A number of amplifiers, using microwave transistors in Class C operation, were designed and fabricated on microstrip. This design approach has been presented in two parts.

In the first part (Chapter 3), a method of characterizing the effects of the second harmonic terminations in a non-linear 2-port were described. This method is based on approximating a non-linear 2-port by a linear 4-port. A measurement system has been developed to measure the equivalent 4-port S-parameters by operating the 2-port under two-sinusoidal-signal input conditions. Consequently, the four effective S-parameters of the non-linear 2-port have been expressed as functions of the second harmonic terminations. A method of analysis was developed, by which the effects of the second harmonic reactance terminations on the fundamental frequency parameters of the non-linear 2-port, have been investigated. Some important considerations leading to the design of the second harmonic reactances to achieve improved fundamental frequency response, have been discussed. The results of applying this method to three microwave transistors under Class C operation have been presented. By studying the effects of the second harmonic in these transistors, a number of designed values of the second harmonic reactance terminations were determined.

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In the second part (Chapter 4), a novel method of characterization of the fundamental frequency response of a non-linear 2-port (harmonics terminated) has been presented. This method consists of measuring the response of the non-linear 2-port when two signals are applied simultaneously. These two input signals - one at the input port and the other at the output port - are of the same frequency. A measurement system has been developed, in which the two input signals were derived from the same oscillator. By applying this method, it was possible to essentially 'terminate' the non-linear 2-port electrically. It has been shown that, by adopting a sequence of judiciously devised measurement steps, this method is capable of being used for the design of optimum power amplifiers. Results of applying this method to design optimum power amplifiers, by using three microwave transistors in Class C operation, have been given.

Two applications of the method of measuring the response of a non-linear 2-port have been demonstrated. First, this method was used to measure the large-signal S-parameters of transistors operating in Class C condition. It has been shown that, by this new method, the large-signal S-parameters (especially the two parameters S_{12} and S_{22}) of a transistor under Class C operation can now be measured under conditions closely resembling those under which the device operates in the intended amplifier. A comparison of the amplifier designs using the large-signal S-parameters, measured by the new method and the ones measured by the conventional method, has been made. Second, it has been shown that, by applying this method, the load-pull data of a microwave power transistor can be generated in a considerably faster

and easier manner than can be done by the conventional method of using tuners. The results of applying the method to generate the load-pull data of the three transistors have been given.

Briefly, the power amplifier design theory in Chapters 3 and 4, provides a method of designing the required second harmonic reactance terminations and the fundamental frequency impedance terminations of a transistor, to achieve specified optimum performances of the power amplifier.

In Chapter 5, a method of designing a filter network or a matching network, using microstrip transmission lines, has been described. An elliptic function or Cauer-parameter type of low-pass structure was conveniently used to realize the filter and matching networks. Accurate computation of the microstrip line parameters were made by using a number of published results.

In Chapter 6, the measured performances of the Class C amplifiers, realized on microstrip, are presented. The performances of the corresponding amplifiers, with arbitrarily chosen second harmonic reactance terminations which were different from those predicted for the optimum amplifier, were also measured. It has been shown that, in the Class C amplifiers, it was possible to increase the power output, gain and conversion efficiency by properly designing the second harmonic reactances. It has also been shown that the experimental performances of the amplifiers, using three microwave transistors under Class C operation, were quite accurately predicted by the theory

developed in this thesis.

Some attempts were made, during the later part of this work, to develop a method of designing a transistor frequency doubler. The necessary modification of the theory in Chapter 3, to design the second harmonic reactance at the input port and the fundamental frequency reactance termination at the output port of a frequency doubler, has been shown in the later part of Chapter 3. The design and realization of suitable terminations for the frequency doubler could not be achieved because of difficulties encountered in obtaining the appropriate bandpass filter characteristics for the output port. These difficulties have been discussed and a possible solution using an optimization scheme has been suggested.

7.2 FURTHER RESEARCH WORK

In Chapter 3, two parameters, n and m , were used as 'indicators', of a possible improvement in the fundamental frequency performance of a Class C amplifier. These parameters are not, however, directly related to amplifier performance characteristics such as power output, power gain and conversion efficiency. Thus, a search for other 'indicators' or 'criteria' in terms of the four effective S-parameters (S'_{ij}) of the non-linear 2-port may be fruitful. If such an 'indicator' could be found, then it would be possible to design the second harmonic reactances to achieve a specified increase in the performance parameters P_o , G_p and η .

The extension of the theory in Chapter 3 to include the effects of higher harmonics is a possible topic of further research work.

For example, to include the effects of the third harmonic, the non-linear 2-port is to be approximated by a linear 6-port. To measure the 36 S-parameters of the linear 6-port, it is necessary to realize a three-sinusoidal-signal source, in which the amplitude and phase of the fundamental, second- and third harmonic signals are adjustable. The requirement of such a large number of measured parameters may make the approach quite unattractive. However, in some situations, the required number of measured parameters may be reduced significantly.

The possibility of designing the matching networks to realize specified fundamental and the second harmonic impedances, by using other types of network structures, should be investigated. In this respect, development and implementation of an optimization routine along with a circuit analysis program is recommended.

The development of the theory and design of transistor frequency multipliers in the light of the theory presented in this thesis should be undertaken urgently.

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APPENDIX A

POWER AMPLIFIER DESIGN USING THE LARGE-SIGNAL S-PARAMETERS

A.1 INTRODUCTION

In this Appendix, the procedure followed by Leighton et al (15) of designing a power amplifier using the large-signal S-parameters is summarized. The approach of the design is mainly that of small-signal amplifier design (9,10,16,21); in addition, the consideration of the 'RF saturation voltage' is taken into account. This is because the maximum available power output from a transistor is determined not only by the impedances and gain of the transistor, but also by its RF saturation voltage and the collector supply voltage (15).

The RF saturation voltage can be measured (15) by observing the variation of the output power with the collector supply voltage, while holding the input power constant. At low collector voltages, the output voltage swings from saturation to nearly twice the collector supply voltage and the output power increases with collector supply voltage. When the collector supply voltage is sufficiently high, the output voltage swing is insufficient to saturate the transistor. Under this condition, the gain is nearly independent of collector supply voltage. The saturation voltage can be determined from the power level and the collector supply voltage at which the output power ceases to increase with increased collector voltage. For example, if the collector voltage waveform is reasonably sinusoidal, then,

$$P_o = \frac{(V_{cc} - V_{sat})^2}{2R_{LP}} \quad (A.1)$$

where P_o is the power output, V_{cc} is the collector supply voltage, and R_{LP} is the parallel equivalent load resistance.

Thus, V_{sat} , the RF saturation voltage, can be calculated from (A.1) if the supply voltage V_{cc} is found at which P_o ceases to increase. The results of measurement of P_o vs V_{cc} for the three transistors, used in this thesis, under 50 ohms (R_{LP}) terminations, are shown in Fig. A.1. The RF saturation voltages calculated from these results are also shown in Fig. A.1.

The design steps for a power amplifier using the measured large-signal S-parameters and the RF saturation voltage of a transistor are given in the next section.

A.2 POWER AMPLIFIER DESIGN PROCEDURE (9,15,16)

The power amplifier design procedure using large-signal S-parameters may be summarized in the following steps:

Step 1: The RF saturation voltage and the large-signal S-parameters (S_{ij}) are measured, at a particular bias, frequency and power level.

Step 2: Calculate the RF stability factor, K.

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|} \quad (A.2)$$

where

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \quad (A.2(a))$$

RF Saturation Voltage: V_{sat}

$$P_{\text{out}} = \frac{(V_{CC} - V_{\text{sat}})^2}{2R_{LP}}, \quad R_{LP} = 50 \text{ ohms}$$

Device No. 1: $P_{\text{out}} \approx 22\text{mW}$, $V_{CC} \approx 8.5\text{V}$

$V_{\text{sat}} \approx 7.0\text{V}$

Device No. 2: $P_{\text{out}} \approx 20\text{mW}$, $V_{CC} \approx 9.5\text{V}$

$V_{\text{sat}} \approx 8.0\text{V}$

Device No. 3: $P_{\text{out}} \approx 25\text{mW}$, $V_{CC} \approx 2.5\text{V}$

$V_{\text{sat}} \approx 0.92\text{V}$

Fig. A.1.: Measurement of the RF saturation voltage of the three transistors used in this thesis.

Step 3: Plot the stability circles on the input and output plane
(on a Smith chart).

Input plane stability circle:

$$\text{Centre: } r_{S1} = \frac{c_1^*}{|s_{11}|^2 - |\Delta|^2} \quad (\text{A.3(a)})$$

$$\text{Radius: } R_{S1} = \frac{|s_{12}s_{21}|}{|s_{11}|^2 - |\Delta|^2} \quad (\text{A.3(b)})$$

Output plane stability circle:

$$\text{Centre: } r_{S2} = \frac{c_2^*}{|s_{22}|^2 - |\Delta|^2} \quad (\text{A.4(a)})$$

$$\text{Radius: } R_{S2} = \frac{|s_{12}s_{21}|}{|s_{22}|^2 - |\Delta|^2} \quad (\text{A.4(b)})$$

where

$$c_1 = s_{11} - \Delta s_{22}^* \quad (\text{A.5(a)})$$

$$c_2 = s_{22} - \Delta s_{11}^* \quad (\text{A.5(b)})$$

and

* means complex conjugate.

If $|K| > 1$ and $|\Delta| < 1$, then the device is unconditionally stable (27). Under this condition, the stability circles will lie outside the unit circle of the Smith chart. And so, in this case, the stability circles are not required to be plotted on the Smith chart. However, if this condition of unconditional stability is not satisfied, then it is essential to draw the

stability circles on the input and output plane. And the sign of amplifier terminations should correspond to the stable regions only. If the device is not unconditionally stable (i.e. if it is potentially unstable), then go to Step 5.

Step 4: If the device is unconditionally stable (i.e. if $|K| > 1$ and $|\Delta| < 1$), then the maximum gain can be achieved by conjugately matching the input and output.

The input and output terminations for conjugate matching are:

Reflection coefficient of input termination:

$$\Gamma_{MS} = c_1^* \left| \frac{B_1 \pm \sqrt{B_1^2 - 4|c_1|^2}}{2|c_1|^2} \right| \quad (\text{A.6(a)})$$

Reflection coefficient of output termination:

$$\Gamma_{ML} = c_2^* \left| \frac{B_2 \pm \sqrt{B_2^2 - 4|c_2|^2}}{2|c_2|^2} \right| \quad (\text{A.6(b)})$$

and the corresponding maximum power gain $G_{\max}$ is given by:

$$G_{\max} = \left| \frac{S_{21}}{S_{12}} \right| \left[K \pm \sqrt{K^2 - 1} \right] \quad (\text{A.6(c)})$$

where

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad (\text{A.6(d)})$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \quad (\text{A.6(e)})$$

In the relations (A.6(a)-(c)), the positive sign is used if $B_1 < 0$ (and $B_2 < 0$) and the negative sign is used when $B_1 > 0$

and $B_2 > 0$). Thus, if $G_{\max}$ is desired, the corresponding amplifier terminations Γ_{MS} and Γ_{ML} can be calculated.

Step 5: If power gain other than the maximum gain $G_{\max}$ is desired or if the device is potentially unstable, then the constant gain circles should be constructed on the output plane. The constant gain circles are given by:

$$\text{Centre: } r_{02} = \left(\frac{G}{1+D_2G} \right) c_2^* \quad (4.7(a))$$

$$\text{Radius: } R_{02} = \frac{\left[1 - 2K|S_{12}S_{21}|G + |S_{12}S_{21}|^2G^2 \right]^{1/2}}{(1+D_2G)} \quad (A.7(b))$$

where

$$D_2 = |S_{22}|^2 - |\Delta|^2$$

$$G = \frac{G_p}{G_o}$$

$$G_o = |S_{21}|^2$$

and

$$G_p = \text{the desired power gain (numeric).}$$

Step 6: Draw the constant power output circle:

The knowledge of RF saturation voltage of the transistor is now used to plot the constant power output circle on the output plane (on which the constant gain circles are plotted as in Step 5). By using (A.1), the value of R_{LP} is calculated

corresponding to a chosen V_{CC} and power output and the measured V_{sat} (as shown in Section A.1). The locus of load impedances having the parallel equivalent resistance (R_{LP}), is a constant conductance circle on the Smith chart. The centre of this constant conductance circle is located at $(1/1+R_{LP}, 0)$ on the Smith chart and the radius being $R_{LP}/1+R_{LP}$; (R_{LP} is normalized to the characteristic impedance, 50 ohms).

If a load impedance outside the constant conductance circle is selected, then the collector saturation will prevent the desired output power from being obtained. If a load impedance inside the circle is selected, the full RF power capability of the transistor will not be utilized and the peak collector current at the designed output power will be larger than necessary (15).

Step 7: The load impedance is now chosen corresponding to the intersection points of the constant power output circle (Step 6) and the constant power gain circles (Step 5) plotted on the Smith chart.

If the load impedance (Γ_L) is chosen, then the corresponding source impedance termination (Γ_S) is determined by using:

$$\Gamma_S = \left[\frac{S_{11} - \Gamma_L A}{1 - \Gamma_L S_{22}} \right]^* \quad (A.8)$$

As mentioned earlier, these choices of Γ_L and Γ_S should correspond to the stable regions identified by plotting the stability circles in Step 3.

A.3 DESIGN CALCULATIONS FOR THE THREE TRANSISTORS

By using the measured large-signal S-parameters discussed in Section 4.7 (Chapter 4) of this thesis, the design calculations for the three transistors are summarized in a tabulated form as shown in Table A.2. The plots of the constant power gain circles and the constant power output circles, etc. are shown in Section 4.7.

PARAMETERS		S-Parameters Measured by the New Method			S-Parameters Measured by the Standard Method		
		Device No. 1	Device No. 2	Device No. 3	Device No. 1	Device No. 2	Device No. 3
Large-Signal S-Parameters S_{ij}	S_{11}	.574/-107	.64/-142	.64/-154	.574/-110	.64/-142	.64/-156
	S_{12}	.125/-40	.049/-57	.14/-33	.05/-100	.03/-102	.03/-88
	S_{21}	1.012/-63	.728/-90	.69/-40	1.012/-65	.728/-90	.69/-42
	S_{22}	.715/-78	.956/-55	.86/-55	.94/-98	.98/-30	.96/-93
Stability Factor (Step 2)	k	1.24	1.04	1.24	.28	.66	.87
Stability Circles Centre: r_{si} Radius R_{si} i=1, input plane i=2, output plane (Step 3)	r_{s1}	1.82/-123	3.96/-104	5.83/-126	1.29/-105	2.75/-98	1.44/-173
	R_{s1}	.72	2.93	4.64	.57	1.99	.49
	r_{s2}	1.41/-86	1.07/-57	1.33/-50	1.02/-97	1.03/-31	1.03/-94
	R_{s2}	.35	.069	.275	.08	.039	.037
Output Power (P_o) Circle: Centre = r_p Radius = R_p (Step 6)	P_o	200mW	206mW	300mW	200mW	206mW	300mW
	r_p	.125/-180	.125/-180	.12/-180	.57/-97	.73/-31	.72/-94
	R_p	.875	.875	.88	.875	.875	.88
	R_p						
Gain Circles on Output Plane Gain = G (dB) Centre = r_{o2} Radius = R_{o2} Load Reflection Coefficient = Γ_L	G	3.0dB	3.6dB	3.0dB	3.0dB	3.6dB	3.0dB
	r_{o2}	.58/-86	.74/-57	.75/-50	.57/-97	.73/-31	.72/-94
	R_{o2}	.39	.26	.162	.43	.274	.28
	Γ_L	.805/-58	.862/-74	.78/-38	.82/-68	.785/-50	.84/-75
Source Reflection Coefficient = Γ_S L corresponds to intersection of the gain circles and the power output circle (Step 5, 7)	Γ_S	.515/-127	.70/-135	.665/-137	.605/-117	.679/-140	.60/-160
	G	4.0dB	4.0dB	3.4dB	4.0dB	4.0dB	3.4dB
	r_{o2}	.66/-86	.76/-57	.82/-50	.63/-97	.75/-31	.74/-94
	R_{o2}	.30	.24	.120	.38	.255	.26
	L	.81/-65	.86/-73	.785/-42	.825/-72	.783/-49	.843/-77
	S	.575/-130	.705/-135	.644/-136	.62/-118	.675/-140	.605/-160

Table A.2. Summary of the design calculations corresponding to the power amplifier design steps in Section A.2.

(cont'd)

PARAMETERS		S-Parameters Measured by the New Method			S-Parameters Measured by the Standard Method		
		Device No. 1	Device No. 2	Device No. 3	Device No. 4	Device No. 2	Device No. 3
	G	5.0dB	4.6dB	3.8dB	5.0dB	4.6dB	3.8dB
	Γ_{02}	.74/86	.79/57	.85/50	.68/97	.77/31	.76/94
	R_{02}	.20	.21	.06	.32	.228	.24
	Γ_L	.82/72	.858/71	.79/47	.83/76	.78/47.5	.845/78.5
	Γ_S	.635/129	.708/-134	.59/133	.635/118	.68/-139	.609/161

Table A.2. Summary of the design calculations corresponding to the power amplifiers design steps in Section A.2.

APPENDIX B

FORMULAE FOR ANALYSIS AND DESIGN OF 'MICROSTRIP CIRCUITS'

B.1 INTRODUCTION

In this Appendix, a number of useful formulae for designing microstrip circuits are listed. These formulae can be employed for accurate computation of the dimensions of the microstrip transmission lines from the knowledge of the characteristic impedances Z_0 's and the electrical lengths T_1 's.

B.2 APPROXIMATE DESIGN FORMULAE

If the parasitics due to 'end effects', 'impedance discontinuity' and 'dispersion' effects are neglected, the design of microstrip circuits may be accomplished by Wheeler's curves (47), or by using Wheeler's implicit equations (44,47) as given below:

For $\frac{w}{h} > 1$ (wide strip):

$$Z_0 = \frac{\sqrt{\frac{1}{\epsilon_r}} R_c}{.5 \left[\frac{w}{2h} + c' + \frac{\epsilon_r + 1}{2\pi\epsilon_r} \ln \frac{\pi e}{2} \left[\frac{w}{2h} + .94 \right] + \frac{\epsilon_r - 1}{2\pi\epsilon_r} \ln \frac{e\pi}{16} \right]} \quad (B.1)$$

For $\frac{w}{h} < 1$ (narrow strip):

$$Z_0 = \frac{R_c}{2\pi} \sqrt{\frac{2}{\epsilon_r + 1}} \left[\ln \frac{8h}{w} + \frac{1}{32} \left(\frac{w}{h} \right)^2 - \frac{1}{2} \frac{\epsilon_r - 1}{\epsilon_r + 1} \left(\ln \frac{\pi}{2} + \frac{1}{\epsilon_r} \ln \left(\frac{4}{\pi} \right) \right) \right] \quad (B.2)$$

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} \left[1 + \frac{\epsilon_r - 1}{\epsilon_r + 1} \cdot \frac{\ln\left(\frac{\pi}{2}\right) + \frac{1}{\epsilon_f} \ln\left(\frac{4}{\pi}\right)}{\ln\left(\frac{8h}{w}\right)} \right] \quad (\text{B.3})$$

$$v_{\text{ph}} = \frac{v_o}{\sqrt{\epsilon_{\text{eff}}}} \quad (\text{B.4(a)})$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{\epsilon_{\text{eff}}}} \quad (\text{B.4(b)})$$

where

R_c = wave impedance of free space = 120 ohms

Z_o = characteristic impedance of the microstrip transmission line

w = width of the microstrip conductor (Fig. 5.2)

h = height of the dielectric medium (Fig. 5.2)

ϵ_r = relative dielectric constant of the substrate (Fig. 5.2)

$e = 2.72$

$c' = \frac{h}{\pi} \ln(4)$ = edge correction for an infinitely wide microstrip transmission line

ϵ_{eff} = effective dielectric constant

v_{ph} = phase velocity of the wave

v_o = velocity of light

λ_o = free space wavelength

λ_g = guide wavelength in the microstrip.

To calculate the width (w) of the transmission line from the knowledge of the characteristic impedance (Z_o) and the substrate, the

following explicit formulae, (48) are useful:

For $\frac{w}{h} > 1$:

$$\frac{w}{h} \frac{\pi}{2} = \frac{R_c}{2\epsilon_r Z_0} - 1 + \ln\left(\frac{R_c}{\sqrt{\epsilon_r} Z_0} - 1\right) + \frac{\epsilon_r - 1}{2\epsilon_r} \left[\ln\left(\frac{R_c}{2\sqrt{\epsilon_r} Z_0} - 1\right) + 0.293 - \frac{0.157}{\epsilon_r} \right] \quad (B.5)$$

For $\frac{w}{h} < 1$:

$$\frac{2h}{w} = \frac{1}{4} e^a - \frac{1}{2} e^{-a}, \quad (B.6)$$

where

$$a = \sqrt{\frac{\epsilon_r + 1}{2}} \frac{Z_0}{60} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left(0.226 + \frac{0.12}{\epsilon_r} \right)$$

where the parameters w , h , Z_0 , ϵ_r , R_c are the same as those used in (B.1) and (B.3). It may also be mentioned that for thick dielectrics the effective dielectric constant (without dispersion) can be computed within 2% accuracy, by the following formulae (49):

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10h}{w} \right)^{-1/2} \quad (B.7)$$

However, for accurate microstrip circuit design, a number of effects should be accounted for. Some of the unavoidable effects are: effects of finite thickness of the microstrip conductor, dispersion effects, open end effects and impedance discontinuity effects, etc. A number of published useful formulae are listed below.

B.3 CORRECTION FORMULAE(i) Finite thickness of the conductor

An apparent increase in the width (Δw_t) of the conductor due to a thickness (t) of the conductor may be estimated (10,50) from:

$$\Delta w_t = \frac{t}{\pi} \left(\ln \frac{2h}{t} + 1 \right) \quad \text{for } \frac{w}{h} > \frac{1}{2\pi} \quad (\text{B.8(a)})$$

$$\Delta w_t = \frac{t}{\pi} \left(\ln \frac{4w}{t} + 1 \right) \quad \text{for } \frac{w}{h} < \frac{1}{2\pi} \quad (\text{B.8(b)})$$

(ii) Dispersion

Dispersion effects may be accounted for (43) by the empirical formula:

For $2 \leq \epsilon_r \leq 10$, $0.9 \leq w/h \leq 13$, $20 \text{ mils} \leq h \leq 120 \text{ mils}$

and $f_0 < f < 13 \text{ GHz}$;

$$\epsilon_{\text{eff}} = \epsilon_{\text{eff}_0} + 3 \times 10^{-6} (\epsilon_r^2 - 1) h \left[\frac{z_0}{x} \left(\frac{w'}{h} \right)^y \right]^{1/2} (f - f_0) \quad (\text{B.9})$$

where

w = width of the line in mils

w' = effective line width

$= w - \Delta w_t$, Δw_t from (B.8)

f = frequency in GHz

$x = y = 1$ for $w'/h < 4$

$x=3, y=2$ for $w'/h > 4$

$$f_0 = \frac{6.0}{(\epsilon_r - 1)^{1/4}} \sqrt{\frac{z_0}{h}} \quad (\text{B.10})$$

= frequency (GHz) below which dispersion may be neglected.

ϵ_{eff_0} = value of the effective dielectric constant from Wheeler's curves (47) for a given w/h and ϵ_r ($=\epsilon_{\text{eff}}$ for $f < f_0$).

(iii) End effects

Fringing field effects at open ends cause an apparent increase in the line width and line length (44-46). This effect may be accounted for by using the following formula:

$$\Delta l_c = \frac{1}{K} \cot^{-1} \left[\frac{4c+2w}{c+4w} \cot(Kc) \right] \quad (\text{B.11})$$

$$= c \left[\frac{c+4w}{4c+2w} \right] \quad \text{for } Kc < .3$$

where

$$K = \frac{2\pi}{\lambda_g}$$

λ_g = guide wavelength

$$= \frac{v_{\text{ph}}}{f} = \frac{\lambda_0}{\sqrt{\epsilon_{\text{eff}}}} \quad (\text{ref. B.4(b)})$$

$$c = \frac{2h}{\pi} \ln 2$$

Δl_c = apparent increase in length due to end effects.

For width correction Δw_c , the parameter w is replaced by l the length of the transmission line in (B.11). Thus, for a long line, $\Delta w_c \approx 2c$ from (B.11).

(iv) Impedance discontinuity effects

The impedance discontinuity effects may be accounted for by some series reactance and shunt susceptance (51,45) by using the following formulae:

Series reactance X_L (inductor) is given (51) by:

$$X_L = \frac{2Z_{01}w_1^*}{\lambda_g} \ln \left[\operatorname{cosec} \left(\frac{\pi}{2} \frac{w_2^*}{w_1} \right) \right] \quad (\text{B.12})$$

where

Z_{01} = characteristic impedance of the transformer section

Z_{02} = characteristic impedance of lines adjacent to the transformer section

$$w_n^* = \frac{hR_c}{Z_{0n}} \frac{1}{\sqrt{\epsilon_{\text{eff}}}} \quad n=1,2 \quad (\text{B.13})$$

= equivalent microstrip line width.

The shunt susceptance (capacitive) may be calculated by using the formula (B.11) for end effects. The susceptance corresponds to an apparent increase in the length Δl , which may be found by superposition of the apparent increases (45) Δl_h , for the high impedance line and Δl_l for the low impedance line, due to the end effects.

For example,

$$\Delta l = \Delta l_h - \Delta l_l = c \left[\frac{c+4w_h}{4c+2w_h} - \frac{c+4w_l}{4c+2w_l} \right] \quad (\text{B.14})$$

B.4 . REMARKS

The formulae given in this appendix were used to calculate the dimensions of the microstrip lines for realizing the matching networks shown in Chapter 5. A computer program (58) was used for this purpose. The characteristics of the material Duroid which was used to fabricate the microstrip circuits are shown in Table B.1.

However, while using Duroid, some of the correction formulae were not necessary at the frequency under consideration. Since f_0 in (B.10) is greater than 5GHz, the dispersion effects were neglected. However, the end effects and the impedance discontinuity effects were accounted for. This is necessary if some of the open ended lines are of short length and the impedance ratios are high ($>1:5$). Also, for large impedance steps, the correction formulae in (B.14) may be approximated by Δl_e , since it has been found (45,46) that the correction due to shunt susceptance corresponding to (B.14) is negligible compared to the series reactance given by (B.12) to account for the impedance discontinuity effects.

Table B.1., Microstrip data.

TYPE: DUROID* D-5870 (2 oz. copper-clad)

PROPERTY	VALUE
Overall thickness	320 μ m (12.6mils)
Dielectric substrate	
Material	Teflon
Reinforcement	Glass microfibre
Thickness (h)	250 μ m ($\approx$ 10mils)
Relative dielectric constant (ϵ_r)	2.35 at 1MHz
	2.35 at 10GHz
Dissipation factor ($\sigma/\omega\epsilon$)	0.0005 at 1MHz
	0.0012 at 10GHz
Surface resistivity (R_s)	$3 \times 10^{14} \Omega$
Thermal expansion coefficient:	
longitudinal direction	$2.9 \times 10^{-5} / ^\circ\text{C}$ (0-40°C)
transverse direction	$7.2 \times 10^{-5} / ^\circ\text{C}$ (0-40°C)
thickness direction	$18.0 \times 10^{-5} / ^\circ\text{C}$ (0-40°C)
Conductor	
Material	Copper (2oz./ft ²)
Thickness (t)	35 μ m (1.38 mils)
Conductivity (σ)	5.8×10^7 mhos/m
Skin depth at 2GHz	1.5 μ m
Surface resistivity (R_s)	$2.6 \times 10^{-7} \sqrt{f}$ ($\Omega/\square$)
Thermal expansion coefficient	$18 \times 10^{-6} / ^\circ\text{C}$
Strip conductor width (w)	Dependent on Z_0
Ground plane area	Extends over full circuit
Method of producing circuit patterns	Photo-etching (5:1 reduction)
Temperature at which circuit measurements were made.	25°C

* Tradename of Rogers Corp., Rogers, Connecticut

APPENDIX C

CALIBRATION OF THE MEASUREMENT SYSTEMS AND ANALYSIS OF THE SYSTEM ERRORS

C.1 INTRODUCTION

In this appendix, the standard procedure of calibration and correction of errors in a reflection/transmission measurement system using the network analyzer, are briefly described. In Section C.2, the calibration procedure is summarized and in Section C.3 a procedure for correcting the reflection/transmission parameters for system errors is given.

C.2 CALIBRATION PROCEDURES

In all the reflection/transmission measurement systems used in this work, the network analyzer (HP8410A) was used to measure the amplitude ratio and phase difference between the test and reference signal. The calibration and the measurement procedures were followed as specified in the manuals and application notes of the HP network analyzer system (34,59). The HP standard 'short' and 'through line' that come along with the transistor jig HP11608A, option 003, were used as calibration units.

Calibration for Reflection Measurements

To measure a reflection coefficient, it is necessary to establish the phase reference plane and a known amplitude level. These conditions can be achieved by using the 'short circuit' calibration unit.

- at the desired reference plane of measurement. After placing the 'short circuit' at the reference plane, the network analyzer display unit (either the gain-phase indicator or the polar display) is adjusted (59,60) to read a reflection coefficient of $1\angle 180^\circ$.

Calibration for Transmission Measurements

To calibrate the transmission measurement system, the 'through line' calibration unit is used. The through line unit is a 50 ohm microstrip transmission line (assumed lossless) of known length. If the 'through line' is assumed to have a 0 phase at the frequency of measurement, then the transmission measurement is calibrated by placing the 'through line' in place of the device and adjusting (59,60) the network analyzer display unit to read a transmission coefficient of $1\angle 0^\circ$. However, for accurate calibration of the reference plane at the device terminal, the phase angle of the transmission coefficient is offset by an angle corresponding to half of the electrical length of the 'through line' calibration unit.

C.3 CORRECTION OF REFLECTION/TRANSMISSION MEASUREMENTS

All the reflection/transmission measurements in this thesis were performed at single frequency. Though the single-frequency measurements are basically more accurate than the swept frequency measurements, still it is often necessary to correct the measurements for system errors. Some of the common sources of system errors are: 'residual' error, 'gain tracking' error and 'source match' errors, etc. (30).

A number of procedures are presently used to correct the system errors, especially in the measurement using the automatic network analyzer system. The most commonly used procedure (30-33,61) is to characterize the errors at each port by a 2-port error model as shown in Fig. C.1(a). The parameters $S_{ij}^{(k)}$ of the 2-port error models represent the following system errors:

For $i=1,2$

$S_{11}^{(i)}$ = residual errors = directivity errors of couplers,
test to reference channel isolation

$S_{12}^{(i)} S_{21}^{(i)}$ = gain tracking errors = tracking variation of reference
and test channels of the frequency converter of the
network analyzer. Tracking
variations of coupler
coefficients.

$S_{22}^{(i)}$ = source match errors = mismatch at connectors.
Internal mismatches down the main
line of directional couplers.

The parameters of the 2-port error model of each port can be determined by three reflection measurements at the port. For example, three measurements on port 1 can be performed as: (Fig. C.1(b)).

- (i) Reflection coefficient (R_{lm}) measurement on port 1 by terminating the output of the 2-port error model at 50 ohms standard termination. Thus,

$$R_{lm} = S_{11}^{(1)}$$

(C.1)

For standard loads:

(i) 50Ω , $\Gamma_L = 0$

(ii) 'short', $\Gamma_L = 1$

(iii) 'open', $\Gamma_L = \Gamma_L$

corresponding
'open-end
reactance'

$$\Gamma_{in} = S_{11}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)} \Gamma_L}{1 - S_{22}^{(1)} \Gamma_L}$$

Fig. C.1. (a) Error model for the two ports to correct for system errors.
(b) Measurement of the error parameters by three reflection measurements.
(c) Transmission error parameters measurement by joining the two ports.

- (ii) Reflection coefficient (R_{1s}) measurement on port 1 by terminating the output port of the 2-port error model at a standard 'short circuit' (Γ_{LO}). Thus,

$$R_{1s} = S_{11}^{(1)} - \frac{S_{12}^{(1)} S_{21}^{(1)}}{1 + S_{22}^{(1)}} \quad (C.2)$$

- (iii) Reflection coefficient (R_{10}) measurement on port 1 by terminating the output of the 2-port error model at an open circuit (Γ_{LO} , say, corresponding to the open end reactance (39) of a microstrip line). Thus,

$$R_{10} = S_{11}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)} \Gamma_{LO}}{1 - S_{22}^{(1)} \Gamma_{LO}} \quad (C.3)$$

Thus, solving (C.1) to (C.3), we can express the error parameters of port 1 as:

$$S_{11}^{(1)} = R_{1m} \quad (C.4)$$

$$S_{22}^{(1)} = \frac{R_{10} + R_{1s} \Gamma_{LO} - R_{1m} (1 + \Gamma_{LO})}{R_{10} - R_{1s}} \quad (C.5)$$

$$S_{12}^{(1)} S_{21}^{(1)} = \frac{R_{1s} - R_{1m}}{R_{1s} - R_{10}} \left\{ 2R_{10} - R_{1s} (1 - \Gamma_{LO}) - R_{1m} (1 + \Gamma_{LO}) \right\} \quad (C.6)$$

Similarly, the error parameters for port 2 can be expressed in terms of the three reflection coefficient measurements performed at port 2.

In general, we can write the error parameters as:

For $i=1,2$ ($i=1$ for port 1, $i=2$ for port 2)

$$S_{11}^{(i)} = R_{im} \quad (C.7)$$

$$S_{22}^{(i)} = \frac{R_{i0} + R_{is} \Gamma_{LO} - R_{im} (1 + \Gamma_{LO})}{R_{i0} - R_{is}} \quad (C.8)$$

$$S_{12}^{(i)} S_{21}^{(i)} = \frac{R_{is} - R_{im}}{R_{is} - R_{i0}} \left\{ 2R_{i0} - R_{is} (1 - \Gamma_{LO}) - R_{im} (1 + \Gamma_{LO}) \right\} \quad (C.9)$$

Now, to characterize the transmission errors through the two ports of the test unit, two transmission measurements are to be performed. These two measurements - one for forward transmission and the other for reverse transmission - are performed by joining the two ports using a 'through line' calibration unit as shown in Fig. C.1(c). For example:

- (i) Forward transmission coefficient (T_{21}) is measured by applying signal at port 1 with port 2 having been terminated at the characteristic impedance 50 ohms. Thus, from Fig. C.1(c), we obtain:

$$T_{21} = \frac{S_{21}^{(1)} S_{12}^{(2)}}{1 - S_{22}^{(1)} S_{22}^{(2)}} \quad (C.10)$$

- (ii) The reverse transmission coefficient (T_{12}) is measured by applying signal at port 2 and port 1 having been terminated at the characteristic impedance 50 ohms.

Thus, from Fig. C.1(c):

$$T_{12} = \frac{S_{21}^{(2)} S_{12}^{(1)}}{1 - S_{22}^{(1)} S_{22}^{(2)}} \quad (C.11)$$

Thus, from (C.10) and (C.11) using (C.8), we obtain:

$$S_{21}^{(1)} S_{12}^{(2)} = T_{21} \left[1 - \frac{R_{10} + R_{1s} - R_{1m}(1 + \Gamma_{LO})}{R_{10} - R_{1s}} \right] \cdot \left[\frac{R_{20} + R_{2s} - R_{2m}(1 + \Gamma_{LO})}{R_{20} - R_{2s}} \right] \quad (C.12)$$

$$S_{21}^{(2)} S_{12}^{(1)} = T_{12} \left[1 - \frac{R_{10} + R_{1s} - R_{1m}(1 + \Gamma_{LO})}{R_{10} - R_{1s}} \right] \cdot \left[\frac{R_{20} + R_{2s} - R_{2m}(1 + \Gamma_{LO})}{R_{20} - R_{2s}} \right] \quad (C.13)$$

Thus, the eight 'error parameters' are expressed in terms of the 8 measured parameters. Let us denote these error parameters as:

$$c_1^{(1)} = S_{11}^{(1)}; \quad c_2^{(1)} = S_{21}^{(1)} S_{12}^{(1)}; \quad c_3^{(1)} = S_{22}^{(1)}; \quad c_4^{(1)} = S_{21}^{(1)} S_{12}^{(2)} \quad (C.14)$$

$$c_1^{(2)} = S_{11}^{(2)}; \quad c_2^{(2)} = S_{21}^{(2)} S_{12}^{(2)}; \quad c_3^{(2)} = S_{22}^{(2)}; \quad c_4^{(2)} = S_{21}^{(2)} S_{12}^{(1)}$$

Now, using these 8 error parameters, the four corrected S-parameters (S_{ij} ; $i, j=1, 2$) of the unknown 2-port inserted between the two ports of the test setup (Fig. C.1(a)) can be expressed in terms of the four measured S-parameters S_{ij}^M ; $i, j=1, 2$. These expressions

can be derived (31) in the following explicit forms:

$$S_{11} = \frac{1}{D} (S_{11}^M - c_1^{(1)}) \{c_3^{(2)} (S_{22}^M - c_1^{(2)}) + c_2^{(2)}\} - c_3^{(2)} S_{21}^M S_{12}^M \quad (C.15)$$

$$S_{12} = \frac{1}{D} S_{12}^M c_4^{(1)} \quad (C.16)$$

$$S_{21} = \frac{1}{D} S_{21}^M c_4^{(2)} \quad (C.17)$$

$$S_{22} = \frac{1}{D} (S_{22}^M - c_1^{(2)}) \{c_3^{(1)} (S_{11}^M - c_1^{(1)}) + c_2^{(1)}\} - c_3^{(1)} S_{21}^M S_{12}^M \quad (C.18)$$

where

$$D = c_2^{(1)} + c_3^{(1)} (S_{11}^M - c_1^{(1)}) \quad c_2^{(2)} + c_3^{(2)} (S_{22}^M - c_1^{(2)}) - c_3^{(1)} c_3^{(2)} S_{21}^M S_{12}^M \quad (C.19)$$

Thus, the four measured S-parameters S_{ij}^M of a 2-port can be corrected for system errors by determining the error coefficients ($c_j^{(i)}$, $i=1,2$ and $j=1-4$); and using (C.15)-(C.19). The error coefficients (given by (C.14)) are calculated from the 8 measurements performed at the two ports as given by (C.7)-(C.9), (C.12) and (C.17).

Correction of 1-port Measurements

The three expressions (C.4)-(C.6) can be used to correct an 1-port reflection coefficient measurement. Referring to Fig. C.1(b), let the actual reflection coefficient be Γ_L . Then, the measured reflection coefficient Γ_{in}^M is given by:

$$\Gamma_{in}^M = S_{11}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)}}{\frac{1}{\Gamma_L} - S_{22}^{(1)}} \quad (C.20)$$

Thus, the actual reflection coefficient Γ_L is calculated by:

$$\Gamma_L = \frac{1}{S_{22}^{(1)} + \frac{S_{12}^{(1)} S_{21}^{(1)}}{\Gamma_{in}^M - S_{11}^{(1)}}}$$

or

$$\Gamma_L = \frac{c_3^{(1)}}{c_3^{(1)} + \frac{c_2^{(1)}}{\Gamma_{in}^M - c_1^{(1)}}}$$

where the c 's are given by (C.14) and they are calculated by using (C.4)-(C.6).

END

FIN