INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

UMI

A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor MI 48106-1346 USA
313/761-4700 800/521-0600

NOTE TO USERS

The original manuscript received by UMI contains pages with indistinct print. Pages were microfilmed as received.

This reproduction is the best copy available

UMI

COMPUTER-AIDED NONLINEAR ANALYSIS OF SIMPLE BEAMS AND ONE WAY SLABS REINFORCED BY FRP AND/OR STEEL BARS

by

Fathi Abushoglin, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment
of the requirements for the degree of

Master of Engineering

Department of Civil and Environmental Engineering

The Master of Engineering Program in Civil and Environmental Engineering
is a joint program with the University of Ottawa, administered
by The Ottawa-Carleton Institute
for Civil Engineering

Carleton University
Ottawa, Ontario, Canada

November, 1997

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-32391-9

Department of Civil and Environmental Engineering

The undersigned recommend to the faculty of Graduate Studies and Research
acceptance of the thesis

**Computer-Aided Nonlinear
Analysis Of Simple Beams And One Way Slabs
Reinforced By FRP And/or Steel Bars**

submitted by

Fathi Abushoglin, B.Eng.

in partial fulfillment of the requirements for the degree of
Master of Engineering.

A. M. Khan

Chair, Department of Civil and
Environmental Engineering

A. Ghani Najari

Thesis Supervisor

Carleton university
Ottawa, Ontario, Canada
November 1997

Abstract

An interactive computer-aided analysis program that runs on a microcomputer was developed to perform the detailed nonlinear flexural analysis of simply supported beams and one way slabs, based on first principles. The program is written in Visual Basic and operates within Windows 95 environment. The member may be reinforced with steel and/or fibre reinforced plastics (FRP). The program enables one to study the complete flexural response of such members, at the serviceability and ultimate limit states, including their time-dependent behavior due to creep and shrinkage. It uses the Hogenstad's parabolic relationship for concrete, an elasto-plastic relationship for steel and a linear elastic relationship for FRP to model the relevant constitutive laws. Concrete creep and shrinkage are modelled according to the relations recommended by the American Concrete Institute while a simple relationship is employed to capture the creep behaviour of FRP. The program develops the full moment-curvature response and the calculation of maximum deflection at any load stage by integrating the curvature along the beam. The beam section may be rectangular, T or I-shape or any arbitrary shape with a vertical axis of symmetry. The cross section is divided into layers along its height in order to find the correct stress distribution. Comparison of the results from the program with available examples and experimental data shows the program to be capable of producing reliable and accurate results within the confines of the assumptions and theories upon which it is based.

Finally, parametric studies are conducted to find the influence of many geometric and loading patterns upon the behavior of FRP reinforced members under flexure.

ACKNOWLEDGEMENT

I would like to acknowledge the professional guidance and help of my thesis supervisor Dr. A. Ghani Razaqpur. His constant guidance, never faltering patience, and encouragement during every step of this thesis is much appreciated.

Financial support for this research was received from the Libyan Secretariat of Education through the Canadian Bureau for International Education (CBIE). The assistance from both agencies is appreciated.

Table of Contents

Introduction	1
1.1 General	1
1.2 Objectives and scope	2
Literature Review	4
2.1 General	4
2.2 Basic mechanical properties of FRP	7
2.2.1 Inorganic fibers.	8
2.2.2 Organic fibers	9
2.2.3 Comparison of FRP's properties	10
2.3 Existing analytical procedures and related material	13
2.4 Previous work on the behavior of FRP reinforced concrete structures	22
2.5 Computer programming languages	30
2.5.1 General	30
2.5.2 Engineering programming languages	30

2.6	Choosing the programming language	32
2.6.1	General approach to programming engineering problems	32
2.6.2	Visual Basic	35
2.6.2.1	General	35
2.6.2.2	Structure of Visual Basic	35
2.6.2.3	Features of Visual Basic	36
	Analysis of reinforced concrete sections	37
3.1	Introduction	37
3.2	Constitutive relationships.	39
3.3	Flexural response using layered section	42
3.4	Determinations of moment-curvature relation	45
3.5	Non-time dependent analysis	49
3.5.1	Uncracked cross section	49
3.5.2	Cracked cross section	53
3.6	Time dependent analysis.	55
3.6.1	Method of analysis	58
3.7	Prediction of deflections	61
3.7.1	General	61

3.7.2	Effective moment of inertia method	62
3.7.3	Tension stiffening	63
3.7.4	Calculations of deflections from curvatures	67
	The computer program - Wizard	72
4.1	Introduction	72
4.2	Program Structure	73
4.2.1	Main Menu	74
4.2.2	Sub-menus	77
4.3	Graphical Interface Features	88
	Numerical examples and verifications	91
5.1	General	91
5.2	Verification of Moment-Curvature Relationship	92
5.3	Verification of short-term response of reinforced beams	98
5.4	Verification of long-term deflections of simply supported beams	101
5.5	Analysis of FRP reinforced slabs by Wizard	106
	Parametric study	111
6.1	General	111

6.2	Materials and beam sections	112
6.3	The beam reinforcement	114
6.4	Selection of span to depth ratio	116
6.5	Moment-curvature analysis	117
6.6	Ultimate moment capacity	121
6.7	Load-deflection analysis	122
6.8	Effect of creep of FRP bars	125
6.9	The reinforcement ratio vs. (ρ /d)	127
	Conclusion and recommendation	139
7.1	Summary	139
7.2	Conclusions	140
7.3	Recommendations	141
	References	142

LIST OF FIGURES

2.1: Stress-strain relationship of typical fiber materials [JSCE, 1993]	12
2.2: Stress-strain relationship for concrete of different compressive strengths [Portland Cement Association (1979)].	20
2.3: Time-dependent concrete strain	21
3.1: Stress-strain relationship (a) for concrete	41
3.2: Flexural analysis using layered section (a) Layered section (b) Strain distribution (c) Stresses and forces.	44
3.3: Moment-curvature relationship of flexural member.	45
3.4: Uncracked cross-section (a) Transformed section (b) Initial strain	52
3.5: Cracked cross-section (a) Strain (b) Stress	54
3.6: $M - \psi$ relationship (a) A reinforced concrete member in flexure (b) Moment-curvature relationship for the member in (a), [Ghali and Favre, (1986)].	66

3.7: Determination of curvature and deflections (a) Cross section (b) Deflected shape (c) Curvature diagram.	70
3.8: Moment, curvature and deflections of beams (a) Deflected shape (b) Bending moment diagram (c) Curvature diagram	71
4.1: Main menu screen	76
4.2: Main menu structures and items. (a) Non-Time dependent analysis	80
4.3: Main menu structure and items (b) Time-dependent analysis .	81
4.4: Main menu structure and items (c) File, section properties plot, Help and Exit menus	82
4.5: Typical data input screen for concrete and cross section properties.	83
4.6: Typical data input screen for FRP reinforcement	84
4.7: Typical data input screen for steel.	85
4.8: Typical data input screen for loads	86
4.9: Typical output screen for deflection analysis	87

4.10: Common Dialog box for selecting data files	90
5.1: Beams cross sections (a) By Alsayed et al (b) By Tezuka et al	95
5.2: Moment-curvature curves (Theoretical vs. experimental for beams tested by Alsayed et al (1996)).	96
5.3: Comparison of predicted moment-curvature relationship by program Wizard with the experimental results of Tezuka et al (1995)).	97
5.4: The beam details tested by Razaqpur and Ali(1995) (a) Elevation (b) Cross section	99
5.5: Load-Deflection curve: Theoretical vs. experimental for the beam tested by Razaqpur and Ali (1995).	100
5.6: Beam details (Brown and Bartholomew,1993) (a) Sectional elevation (b) Beam cross section	103
5.7: Comparison of predicted and observed short term load-midspan deflection curve for the beam tested by Brown and Bartholomew (1993).	104
5.8: Deflection vs. Time curves (theoretical vs. experimental by Brown and Bartholomew (1993) for FRP and steel	

reinforced beams)	105
5.9: Bridge deck slab (DH) by Schmeckpeper (1992)	
(a) Side view (b) Plan	108
5.10: Load-deflection curves (Wizard predictions vs.	
experimental results by Schmeckpeper(1992).	109
5.11: Midspan longitudinal strains vs. applied load for a slab	
strip. (Wizard predictions vs. experimental results by	
Schmeckpeper (1992)).	110
6.1: Beams cross sections used in the parametric study	
(a) R-section (b) T-section (c) Load distribution.	113
6.2: Moment-curvature relation of rectangular cross sections	
reinforced with (a) steel. (b) GFRP. (c) CFRP.	119
6.3: Moment-curvature relation of T-cross sections	
reinforced with (a) Steel. (b) GFRP. (c) CFRP.	120
6.4: Variation of ultimate moment capacity for a typical rectangular	
section reinforced with (a) Steel (b) GFRP (c) CFRP.	129
6.5: Variation of ultimate moment capacity for a typical T-section	
reinforced with (a) Steel (b) GFRP (c) CFRP	130

6.6: Effect of reinforcement ratio on the load-deflection behavior of a simply supported R-section beams reinforced with (a) Steel. (b) GFRP. (c) CFRP.	131
6.7: Effect of reinforcement ratio on the load-deflection behaviour of a simply supported T-section beams reinforced with (a) Steel. (b) GFRP. (c) CFRP.	132
6.8: Effect of span/depth ratio on the load-deflection behavior of a simply supported R-section beams reinforced with (a) Steel. (b) GFRP. (c) CFRP.	133
6.9: Effect of span/depth ratio on the load-deflection behavior of a simply supported T-section beams reinforced with (a) Steel. (b) GFRP. (c) CFRP.	134
6.10: Effect of concrete creep on the time-dependent maximum deflection of a simply supported R-beams reinforced with (a) Steel (b) GFRP (c) CFRP.	135
6.11: Effect of concrete creep on the time-dependent maximum deflection of a simply supported T-beams reinforced with (a) Steel (b) GFRP (c) CFRP.	136

6.12: FRP ultimate creep coefficient vs. maximum deflection (FRP with creep)/maximum deflection (No creep in FRP) (a) GFRP (b) CFRP reinforced beam.	137
6.13: C_u/d versus reinforcement ratio ($\frac{\rho}{\rho_b}$) in rectangular and T-sections.	138

LIST OF TABLES

Table No.

1.0	Composition of glass fibres	8
3.1	Quantitative values of parameters used for time-dependent properties of concrete. [from Naaman (1982)]	57
5.1	Material properties of Alsayed et al beams	92
5.2	Material properties of CFRP reinforcements (Tezuka et al)	93
6.1	Material properties used in the analysis	112
6.2	Minimum thickness of beams and one way slabs	117

Chapter 1

Introduction

1.1 General

Fibre Reinforced Plastics (FRP) are new reinforcing materials for concrete structures. Due to the differences that exist between the mechanical properties of FRP and steel, FRP reinforced structures require their own special considerations in design. While steel has the advantage of a long yield plateau and a high modulus, FRP behaves linear-elastically up to failure, but it has its own structural advantages. It has superior strength, low density and high durability. However, FRP reinforced concrete structures have very little ductility due to the linear elastic stress-strain relationship of FRP up to failure. To evaluate strength and serviceability requirements for steel reinforced concrete structures, the designer is guided by the applicable codes, but when the full response, including the long-term behaviour of FRP reinforced structures is needed, the situation is different. First, because the existing codes are currently restricted to only steel reinforced concrete, they cannot be used. Techniques used to predict the

response of steel reinforced structures cannot be applied directly to FRP reinforced concrete unless certain adjustments are made. Secondly, the majority of the available computer programs for FRP reinforced structures are still limited to some aspects of the design process rather than the determination of full detailed response. It is therefore, apparent that the need exists for an integrated, windows-based program which can take advantage of the modern computer capabilities in order to analyze, based on first principles, the full response of FRP reinforced structures.

1.2 Objectives and scope

The objective of this thesis is to develop a computer-aided analysis and design program for concrete beams reinforced with FRP and/or steel bars. The program is written in Visual Basic for Windows 95 characterized by ease of use and includes a good graphical user interface.

Elastic and inelastic analysis procedures, which include the time-dependent effects of creep and shrinkage in both the concrete and the FRP will be considered. The program will use first principles, applicable constitutive laws for the concrete and the reinforcement and the requirements of equilibrium and compatibility. The design and analysis routines of the program will apply under service load and ultimate load conditions to simply supported reinforced concrete beams and one way slabs. Loads will be limited to gravity loads. Beam cross-sections may have any solid non-standard shape with one axis of symmetry and with the

resultant applied loads lying in the plane of symmetry. The scope of the thesis will be restricted to the study of the flexural behaviour of these beams. In pursuing the previous objectives, a literature review will be conducted to provide sufficient background information with respect to the features of the program. Such topics will include a general background of FRP materials and previous work related to the use of FRP in concrete structures. Also, a general review of the computer programming languages for engineering applications will be presented.

Chapter 2

Literature Review

2.1 General

The replacement of steel by FRP in reinforced concrete structures offers a solution to the corrosion problems that occur due to exposure to aggressive environments, use of de-icing salts or other deleterious chemicals. Therefore, the use of FRP's in concrete structures can lead to longer service life of a structure and can result in lower maintenance cost due to avoidance of damage caused by corrosion. On the other hand, steel reinforcement cannot be easily replaced by any other material, including FRP's. The attractive characteristics of steel such as high strength, high modulus of elasticity and ductility, make it difficult to completely replace steel by FRP. Only the sensitivity of steel to corrosion seems to undermine its universal appeal, for even the epoxy-coated steel bars could not offer a satisfactory solution. For example, Rasheeduzzafar et al (1992) and Keeslar and Powers (1988) reported in their studies on corrosion of steel that the epoxy-coated steel bars still undergo significant corrosion.

In those situations where corrosion could be a problem, FRP may be an eco-

nomie and appropriate material due to its excellent resistance to both active and passive corrosion. These materials, however, possess a number of other excellent attributes such as:

- High tensile strength.
- Low density (Light weight).
- Relatively high modulus of elasticity.
- Non-conductive magnetic characteristics.
- Energy absorption capabilities.

It should be pointed out that different types of FRP materials behave differently in reinforced concrete structures due to differences in their properties. The high strength and stiffness of carbon fibres (CFRP), as compared to glass (GFRP) or aramid (AFRP) fibres, can be used to increase the uncracked stiffness, cracking stress and strain. Within the same type of FRP material, the composition of FRP material endows the composite with its specific properties, and consequently, its area of application. For instance, glass fibres of the type E-glass have inadequate resistance to the alkalis present in portland cement whereas the alkali-resistant glass has better characteristics due to the presence of zirconium oxide in its composition. It is also possible to produce a mixture of fibres, in an attempt to optimize the benefits derived from the individual fibres. Win Somboonsong et al (1996) has considered combinations of low modulus, aramid fibre (AFRP), with high modulus fibres, Carbon fibre (CFRP). The result of the combination is a new ductile hybrid FRP with unique bi-linear or tri-linear response and modulus of elasticity similar to that of steel reinforced bar.

Examples of applications of FRP reinforcement in reinforced concrete structures include non-prestressed reinforced beams [Razaqpur and Ali (1995)], as tendons for prestressed strands [Thakurta et al (1994)], in non-conductive environments [Erki et al (1992)] and in strengthening and rehabilitation of highway bridges [Saadatmariesh and Ehasani (1991)].

However, the development of the computer program in this thesis requires knowledge of those topics that are relevant to computer-aided analysis and design of concrete structures. Such topics are:

1. Review of some previous work on FRP reinforced concrete structures with respect to :
 - Flexural strength and bond of FRP reinforcement to concrete
 - Short & long-term deflections of FRP reinforced concrete structures.
2. Proposed analytical procedures and related material.
3. Selection of a suitable programming language.

It should be mentioned that, to our knowledge, there is no computer aided program currently available for FRP reinforced concrete structures. Therefore, topics related to computer programming will be limited to general overview on computer programming languages, selection of a suitable programming language and recommended steps for writing programs.

2.2 Basic mechanical properties of FRP

The mechanical properties of interest here are those properties associated with the ability of the material to resist applied forces and/or deformation. Generally, the three primary constituents of FRP reinforcement are:

- Inorganic and organic reinforcing fibres such as carbon, glass, aramid and polyvinyl alcoholic.
- Thermosetting polymer resins such as epoxies, vinylesters and polyesters.
- Inorganic fillers such as calcium carbonate, clay and alumina trihydrate.

The secondary constituents are agents (coupling agents, accelerators, mold release) and additives (stabilizers, flame retarders, absorbers, modifiers).

FRP materials, when used as reinforcing materials, usually bond together with binding agents such as resins and cement, resulting in a new composition and appearance. The mechanical properties of FRP depend on the cross-section of the material, method of processing and type and content of binding agent.

In all cases, the purpose of using FRP reinforcement is to control the brittle behaviour of cement-based materials and to maximize the use of enhanced reinforced concrete structures by FRP materials. FRP is attractive as reinforcement for concrete in aggressive environments due to its high chemical resistance and strength.

In the following sections, some important FRP materials and their mechanical properties will be presented. This includes glass, carbon, aramid, polypropylene and polyvinyl alcohol fibres.

2.2.1 Inorganic fibres.

- **Glass fibres**

The most common glass fibres manufactured and used in the construction market are E-glass fibre and alkali-resistant glass fibre. E-glass fibre is marked by low alkali content and hence have poor resistance to the alkalis while the alkali-resistant glass fibre has improved alkali-resistant characteristics due to the presence of 16% of zirconium oxide in its composition. Table 2.1 [Fordyce and Wodehouse (1983)] shows the composition of the two types of glass fibres. In general, glass fibres have high tensile strength and excellent resistance against alkalis and salts provided the sustained stress is not very high. In addition, glass fibres have the advantage of being potentially available in a large variety of forms such as chopped strands, mats, rovings, ropes and woven fabric.

Table 1: Composition of glass fibres

Component	E-glass (%)	Alkali-resistant glass (%)
SiO ₂	55	71
AlO ₃	15	1
B ₂ O ₃	7	--
CaO ₂	7	--
Alkalis (Na ₂ O, K ₂ O etc)	2	11
ZrO ₂	--	16
LiO ₂	--	1

- **Carbon fibres**

Carbon fibres are commercially available in two types; the PAN (polyacrylic nitril) and the pitch carbon. The PAN fibre is classified as high strength or high Young's modulus and the pitch fibre as ordinary or high Young's modulus fibre. PAN-based fibres have higher ultimate strain, whereas pitch-based fibres have higher stiffness. Compared to other types of fibres, carbon fibre has a superior Young's modulus, low elongation and high susceptibility to impact. Carbon fibre also provides excellent resistance against alkalis and salts.

2.2.2 Organic fibres

Organic fibres include aramid fibres, polyvinyl alcoholic, polypropylene and others. There are at least three types of aramid available commercially; Kevlar, Twaron and Technora. The aramid fibres are characterized by excellent tensile strength and modulus of elasticity compared to other fibres. These fibres also have good resistance against alkalis and heat. The aramid fibres, which are organic in nature, still have poor resistance to ultraviolet rays, i.e. resistance against weather, and thus the deterioration in the strength of these fibres should be taken into account when used in an adverse environment for long periods of time.

Other examples of organic fibres are polypropylene, polyethylene, nylon and polyvinyl alcohol fibres. Although polypropylene, polyethylene and nylon are resistant to most chemicals, they can make very small contribution to the strength of concrete. The ability to absorb energy is of great importance to the

behaviour of these fibres. Among these fibres, the polypropylene has the most successful commercial application so far. The common forms of these fibres are smooth monofilament, twisted, fibrillated and tridimensional mat. Although the polyvinyl alcohol fibre has a relatively low tensile strength and Young's modulus, it is light, has a good stretch quality, and it remains stable in the presence of alkalis. Generally, the major shortcomings of plastic fibres are low modulus of elasticity and low melting point.

2.2.3 Comparison of FRP's properties

Continuous fibre is a general term for fibres used to reinforce concrete. In general, the mechanical properties of fibre reinforcing materials (FRP) vary from that of non-prestressed steel in several ways. The tensile strength of fibres is generally greater than the tensile strength of ordinary steel, but the modulus of elasticity of fibres is lower than that of steel. Because of this fact, i.e. higher strengths and lower Young's modulus, the elastic strain of the concrete member becomes large resulting in larger deflection and crack widths than in conventional steel reinforced concrete members. FRP reinforcement does not yield, therefore, if the reinforcement is arranged in more than one layer, failure will be initiated by rupturing of the highest strained reinforcement material. Thus, the structural depth as applied to steel which is from the compression face to the centroid of the steel reinforcement cannot be applied to FRP reinforcement. Also, because FRP reinforcement does not have an inelastic range, the failure mode either due to the rupture of FRP's or concrete crush is brittle whereas in

steel brittle failure can be avoided due to its ductility.

Figure 2.1 [JSCE (1993)] illustrates the stress-strain relationship of some typical fibres. Fig.2.1 is based on tests conducted on 15 different fibres with different shapes, lengths and testing methods. Note that the carbon fibres tend to have higher elastic modulus and lower strain (around 1%) compared to other fibres. This indicates that carbon fibres behave more brittle than aramid or glass fibres. Due to this fact, many studies have proposed a number of methods to enhance the brittle mode of the carbon fibre such as using different types of FRP (e.g. CFRP and GFRP or CFRP and polypropylene) within the same member, e.g [Razaqpur and Ali (1995)].

Using FRP materials as compression reinforcement is not so common due to their lower strength in compression. A study by Kobayashi et al (1988) showed that the compressive strength of continuous fibre reinforcing materials made from glass fibres is equal to or higher than their tensile strength and that of continuous fibre reinforcing materials made from carbon and glass fibres, about 80% of their tensile strength. However, when used as compression reinforcement, the useful strain limit is governed by the maximum strain in concrete which is generally of the order of 0.3-0.5%. Given the relatively low modulus of most FRP's and considering a reasonable reinforcement ratio, the contribution of FRP to the compressive strength would not be very high.

Figure 2.1: Stress-strain relationship of typical fibre materials [JSCE, 1993].

2.3 Existing analytical procedures and related material

Because concrete is used to resist compression, its compressive stress-strain relationship influence the response of reinforced concrete members. Typically concrete behaves linearly up to a stress of $0.3 f'_c$, with f'_c being its compressive strength, beyond that it exhibits a nonlinear response. Thus evaluation of the full response of reinforced concrete members requires the use of non-linear analysis. The actual stress-strain relationship for a range of concrete strengths as reported by the Portland Cement Association (1979) is given in Figure 2.2.

As can be seen from this figure, the stress-strain relationship of concrete is non-linear. The first portion of the curve to about 30-40% of the ultimate strength can essentially be considered linear for all practical purposes. After approximately $0.70 f'_c$, the material loses a large portion of its stiffness, thereby exhibiting increased nonlinearity. At ultimate load, cracks parallel to the direction of loading become distinctly visible, and most concrete cylinders (except those with very low strengths) fail suddenly shortly thereafter [Nawy (1990)]. It also can be seen that as the concrete strength increases, the initial concrete stiffness increases with a reduction in both the ductility and the failure strain.

May be the most common model for concrete stress-strain curve is the Hognes-tad's stress-strain curve. It consists of two portions, parabolic and linear portions. Collins and Mitchell (1990) used the simple parabola model in their

computer program "Response" for the determination of the response of prestressed concrete structures. However, a full nonlinear stress-strain model has been adopted in this thesis.

The response of flexural members can be predicted by using the equilibrium and compatibility conditions. In this technique, in ordinary concrete beams and slabs the concrete strain distribution is defined by two variables and the moment and axial load acting across the section is determined from the equilibrium equations. The layer by layer approach to evaluate the sectional forces was used by Collins and Mitchell (1990). This approach is appropriate for any cross sectional shape provided that a microcomputer is used. This approach, as used also in this thesis, will be described in details later. Codes, such as ACI or CSA, use the stress-block factors in order to determine the forces acting on a cross section. The factors are determined so that the magnitude and location of the resultant forces are the same in the equivalent uniform stress distribution as in the actual distribution [Collins and Mitchell (1990)]. However, for a width of concrete cross section varying along the height, the stress-block factors cannot be applied.

Young's modulus of elasticity is affected by many factors such as the strength of the concrete, the quality of the coarse aggregates, the water-cement ratio, age of concrete, moisture and temperature. In the absence of data about those factors, the expressions given by the ACI or CSA code can be applied to determine the values of Young's modulus. These equations are:

$$E_c = 0.043 w_c \sqrt{f'_c} \quad (2.1)$$

$$E_c = 5000 \sqrt{f'_c} \quad (2.2)$$

where, w_c is the density of concrete ($kg\ m^3$) and f'_c is the compressive strength of the concrete (MPa). Carrasquillo et al (1981) found that equation (2.2) overestimates the values of f'_c for concrete strengths greater than 41. Instead, they proposed the following equation:

$$E_c = 3320 \sqrt{f'_c} + 6900 \quad (2.3)$$

The cracking of concrete occurs at relatively low stresses. This is because the concrete is very weak in tension. Because of the difficulty in testing the concrete in tension, the cracking strength is usually determined from an indirect test, either by the split strength or by bending test. In the bending test, the modulus of rupture, which represents the concrete tensile strength, is determined. The ACI code gives the following equation for determination of the modulus of rupture for normal weight concrete.

$$f_{cr} = 0.6 \sqrt{f'_c} \quad (2.4)$$

The creep of concrete and the different methods of predicting the creep are well described by Gilbert (1988), and therefore, the review of this subject will be brief. Under sustained load, if stress and temperature remain constant, the increase of strain with time is caused by creep and shrinkage. Shrinkage is a reduction in concrete volume caused principally by the loss of water during the drying process, and hence shrinkage strain is independent of sustained stress. On the other hand, creep is produced by sustained stress. The resulting excessive strains frequently increase the deflection and curvatures of the concrete member.

In addition, creep causes a redistribution of stresses between concrete and the bonded reinforcement in the concrete cross section. In a reinforced concrete column, for example, as the concrete contracts either due to the effect of creep and/or shrinkage, the reinforcement is compressed. To satisfy the equilibrium condition, the increase in the compressive force in the reinforcement should be balanced by an equal and opposite tensile force on the concrete. Thus, the compressive stress in the concrete diminishes with time while the stress in the reinforcement increases rapidly.

The variation of creep and shrinkage with time is illustrated in figure 2.3. The initial loading is known as the elastic strain while the additional strain within time, t , is called the creep strain. The ratio of creep strain to elastic strain is the creep coefficient, ϕ . The magnitude of creep strain in beams is usually small compared to the maximum strain at the ultimate load condition. On the other hand, stresses developed by shrinkage are generally reduced by the effect of creep of concrete. However, in most cases, creep and shrinkage are considered two important phenomena in stress analysis.

To account for the effects of creep and shrinkage, several different techniques are available for time analysis of concrete structures. This includes the age-adjusted effective modulus method, the rate of creep method and the improved Dischinger method. Some of these methods are easy to use, others are much more complicated. What should be known about the methods of time analysis is that these methods are approximate and based on simplifying assumptions, thus none of them can be perfectly accurate. This is because the data used by these

methods depend on the material properties tested in the laboratory for a period of time, but reliable data measured in the laboratory can vary largely from those in the real structure. Moreover, materials differ also in their properties within the same structural member. Therefore, reliable data of the materials is the most important factor for analysis of creep and shrinkage. Neville et al (1983) pointed out that there remains a serious lack of reliable test data for both the development of creep with time and the final creep coefficient for different values of effective thickness.

A brief study on the different models for estimation of creep and shrinkage coefficients shows that the different methods vary widely among each other. Hilsdorf and Müller (1979) compared a number of methods such as CEB-FIP, ACI 209-1978 and Bazant and Panula (1978). They remarked on the differences in these methods particularly in the early ages of loaded concrete at low relative humidities. They found higher estimates of creep provided by the CEB models whilst the ACI-209 and the Bazant-Panula models give the smallest values of creep coefficient. The highest estimates of shrinkage were given by ACI-209 and Bazant-Panula models, whereas the CEB model provides the smallest estimate. It was also pointed out that the most complicated methods are not necessarily more accurate than the simpler ones. However, a number of other studies point out that the CEB-FIP method predicts the creep of some Australian concrete the best. Such investigations were conducted by Bakoss et al (1982) and Roper and Bott (1979).

The methods for time analysis vary from the simplest to more complicated ones.

The Faber's effective modulus method (EMM) is the simplest of all methods for the creep and shrinkage analysis. In this method the creep strain at time, t , depends only on the current stress and does not take the aging of concrete into account. Thus, for a decreasing stress history creep is underestimated whereas for an increasing stress history the creep is overestimated. Therefore, the EMM is suitable only for cases in which the concrete stress is constant in time or when the effects of concrete aging is negligible.

The age-adjusted effective modulus method (AEMM) has overcome the shortcomings of the effective modulus method by considering the age of concrete in the analysis. This method was developed by Bazant (1972) and it was found that this method is suitable for most applications provided that the aging coefficient is carefully calculated from the assumed creep coefficient.

The rate of creep method (RCM) was developed by Glanville (1930) and Whitney (1932). The RCM method assumes that the rate of change of creep with time is independent of the age of concrete at loading. In this method, it is assumed that better agreement with actual behaviour of the creep-time curve caused by a specified stress applied at any age is defined by parallel creep curves. According to this method, as time increases, both the rate of change of creep and the rate of change of strain approach zero. It does not consider the fact that old concrete still creeps. Thus, this method underestimates the creep for an increasing stress history. On the other hand, creep strains are overestimated for a decreasing stress history. In addition, RCM does not consider the prediction of creep recovery.

Improved Dischinger Method (IDM) [Gilbert (1988)] is another form of RCM where the creep recovery is considered. Therefore, when unloading occurs, IDM predicts the long term more accurately than RCM.

As a result of the above, the ACI Committee 209 (1982) model for estimation of creep and shrinkage coefficient and the age-adjusted effective modulus method (AEMM) for creep and shrinkage analysis have been adopted in this thesis. Beside the other advantages of these methods, as explained, they are easy to apply and suitable for computer implementation.

Figure 2.2: Stress-strain relationship for concrete of different compressive strengths [Portland Cement Association (1979)].

Figure 2.3: Time-dependent concrete strain

2.4 Previous work on the behaviour of FRP reinforced concrete structures

A number of studies have been conducted to study the behaviour of FRP reinforced concrete structures. This includes the bond between FRP bars and concrete, strength and serviceability, including both short and long-term. Faza and Ganga Rao (1992) conducted a study on the bond strength, the pre- and post cracking behaviour, load-deflection and modes of failure of glass fibre (GFRP) reinforced beams. Twenty four cantilever beams were subjected to flexure and twelve pull-out specimens were tested. Normal and high concrete strengths ranging from 29 to 69 MP_a were used. The study showed that in order to take full advantage of the high strength of FRP reinforcement, high concrete strength should be utilized. The bond failure of smooth bars occurred prior to reaching the ultimate strength. Deflections of GFRP reinforced beams were greater than those of steel reinforced beams by about 4 times. Large deflections with GFRP reinforced beams was mainly due to the lower modulus of elasticity of GFRP bars (48 GPa) than those of steel (200 GPa). Due to the low modulus of elasticity, the stiffness of sections upon cracking diminishes substantially, resulting in higher deflections.

Soudki et al (1997) studied the bond performance of carbon fibre (CFRP) bars to concrete. Bond stress-slip relationship and the effect of low temperature environment on the bond behaviour was investigated. The experimental study consisted of 6 hinge beams and 27 pull-out specimens. The beams and specimens

were subjected to low temperature, ambient temperature and freeze and thaw. The test results indicated that the bond-slip response was a bilinear relationship. The effect of low temperature on the bond strength was small. The bond stresses in flexure were greater than those obtained from the pull-out specimens. This may be due to the effect of anchors and confinement on the reinforcement in pull-out specimens. The reduction in bond strength due to freeze thaw and low temperature on pull-out specimens for 10d embedment length was greater than specimens with 15d embedment length, where d is the bar diameter. Finally, the beam specimens subjected to freeze and thaw and low temperature had slightly less bond strength than those specimens kept at room temperature.

Another study by El-Namawy (1997) was carried out to determine the effect of parameters such as shear span to depth ratio (L/h), cover and lap splices on the bond strength of GFRP reinforced concrete beams. The experimental work was conducted on 18 simply supported beams reinforced with 10mm and 12mm diameter of GFRP bars. The experimental tests indicated that as the ratio (L/h) decreases, the bond strength of GFRP increases whereas the strength increases as the concrete cover increases. It was also reported that the experimental results agreed well with their proposed theoretical model for calculating the bond stress with and without lap splices.

A study on the effect of deformed and plain FRP bars on the bond response of FRP reinforced concrete slabs was conducted by Jerrett and Ahmed (1995). Three concrete slabs with embedment lengths of 152mm, 305mm and 427mm were tested. Test results showed that the deformed bars have greater ability to

maintain the initial free end slip than the plain bar. Also, the tests showed that as the slab thickness increased the bond strength also increased.

Nanni et al (1995) conducted an experimental work to study the parameters that control the bond performance. FRP rods were subdivided into two groups. The first group consisted of commercially available FRP rods and conventional steel reinforcing bars. The second group were made of the following material systems, glass-vinylester (GV), carbon vinylester (CV) and carbon -epoxy (CE). The embedment lengths were 5db and 10db. Based on the test results, they concluded:

- The bond of FRP rods tend to be controlled by strength and mechanical action of deformations on the rod surface rather than adhesion and friction.
- The finish and composition of the resin-rich surface layer of an FRP rod affect the development of bond strength due to adhesion and friction. Smaller diameter rods have higher pull-out strength than larger diameter rods.
- All machined rods exhibit the same failure mechanism. Shear off of rings followed by sliding. This failure mechanism is controlled by the resin (epoxy-based systems are stronger than vinyl ester-based systems).
- The strength of concrete appears to have an insignificant influence on bond strength and failure mechanism of FRP rods.
- Concrete failure was not experienced in any case of FRP rod pull-out. For steel reinforcing bars, the concrete between the lugs crushed.

A study on bond creep behaviour of FRP rods embedded in concrete was conducted by Hattori et al (1995). Two types of FRP materials, twisted carbon fibre

cable (7.5mm) and deformed aramid fibre bar (6mm), were tested for 8000 hours by pull-out creep test. Test results showed that the bond slip behaviour of the twisted carbon fibre cable and deformed aramid fibre bar, especially in the preliminary stage of pull-out, was affected by the mechanical properties of the resins. Also, deformed aramid fibre bar exhibited larger free end slip than the deformed mild steel bar under the same condition of the average bond stress in the bond length of 64mm.

The preceding studies have shown that most FRP's have generally good bond characteristics, therefore in the present study, it will be assumed that throughout the loading range, full bond exists between the FRP reinforcement and the surrounding concrete.

Kawakami et al (1992) tested a series of slabs made of silica fume mortar and reinforced with AFRP or CFRP meshes in an attempt to take full advantage of the high strength of FRP reinforcement and the high strength and durability of silica fume mortar. Twelve FRP reinforced slabs and two steel reinforced slabs were tested. As expected, FRP content had no effect before cracking since, in this state, the concrete alone governs the response. The tests also showed that the CFRP reinforced slabs had more brittle response than the AFRP reinforced slabs. Deflections of steel reinforced slabs were lower than those of CFRP and AFRP reinforced slabs by two and three times, respectively. It was also found that as the amount of FRP in the cross section of the slab increased, the deflection decreased.

Nawy et al (1971) conducted a study on twenty beams reinforced with different

reinforcement ratios of steel and GFRP bars. The test results showed that steel reinforced beams had less cracks than the beams reinforced with fibre glass. Failure of the majority of GFRP reinforced beams was due to concrete crushing. The moment at the initial cracking was not affected by the type of reinforcement. The inelastic deflections of GFRP reinforced concrete beams were approximately 3 times greater than those of steel reinforced concrete beams.

Studies about the creep and shrinkage of FRP reinforced concrete structures have been limited. Creep in conventional reinforced concrete members is caused by the creep of concrete. The creep of steel reinforcement is negligible while the creep of FRP's reinforcement may be important because they are made of polymeric materials. Any creep in FRP's reinforcement will aggravate the larger deformations caused by the FRP's low elastic modulus. Because most FRP reinforcement may lose tensile strength and fail by stress rupture under sustained loads, it is important to evaluate the creep strain in FRP.

Rahman and Kingsley (1995) studied the creep behaviour of FRP grids. The FRP reinforcement they tested is known as NEFMAC. The NEFMAC they tested is a mixture of E-glass and carbon or carbon only. Test results showed that creep of both types of NEFMAC had negligible effect on the concrete reinforced with NEFMAC subjected to exposed air and under a sustained 10000 hour stress up to 75% of the ultimate strength. Also, creep strain and residual tensile strength were not affected by temperature rise up to 50 °C.

In another study, Uomoto et al (1995) conducted experiments on three types of FRP materials, aramid fibre (AFRP), glass fibre (GFRP) and carbon fibre

(CFRP). Tensile strength, creep behaviour and fatigue strength of the FRP materials were evaluated. 100 specimens were tested for each case. Sustained load was applied to the rods by using a servo mechanical test machine from 70 to 90% of the mean tensile strength for AFRP, 66.7 to 94% for GFRP and 96.4 to 101.2% for CFRP. Test results showed that the creep behaviour depends on the type of FRP used and the applied stress. Creep decreased with an increase in applied stress and at the same applied stress, the time to failure was longer for CFRP, AFRP and GFRP, respectively. The effect of bond creep on the long term deflection of reinforced concrete beams using FRP was also discussed. They concluded that the effect of compressive creep of concrete in the compression zone mainly controlled the development of the deflection while the bond creep had less significant effect on the long-term deflection of the beam reinforced with a twisted carbon fibre cable or a deformed aramid fibre bar.

Schemeckpeper (1992) conducted an experimental work on a number of beams reinforced with FRP grids. The mode of failure of the beams was due to either shear or crushing of concrete. He observed the tested slabs to behave similar to overreinforced steel reinforced beams. Deflections of FRP reinforced concrete beams were almost two times those of steel reinforced concrete beams. Using ACI equations to predict the deflections of cracked sections resulted in lower values than the experimental equations which means that ACI code underestimates deflection of FRP reinforced beams after the cracking of the reinforced beams occurs.

Kage et al (1995), in their experimental study on continuous fibre concrete

beams have also confirmed the non-suitability of the ACI equation for predicting the short term post-cracking deflections of FRP reinforced concrete members.

The current ACI and CSA approaches recommend that creep deflection be calculated by multiplying the creep coefficient by the initial deflection. In FRP reinforced concrete members, creep deflections may be also affected by the variations of stresses in the FRP.

This point has been considered by Brown and Bartholomew (1993). In their study, long-term deflection of GFRP reinforced simply supported concrete beams was investigated. The study was conducted at ambient temperatures on four reinforced beams, two beams with GFRP and the other two beams with steel reinforcement. It was shown that in order to use creep calculations techniques applicable to steel reinforced concrete beams, the ultimate creep of FRP should involve the initial deflection and the difference in stress levels in compression zone. It was also shown that GFRP reinforced beams had initial deflection of 3.76 times and concrete compression stressed of 2.24 times larger than that of steel reinforced concrete member.

This literature review reveals that using first principles and the requirements of equilibrium, compatibility and the constitutive laws of materials, the flexural response of FRP reinforced concrete structures can be determined. On the other hand, the existing code oriented equations may not be suitable for FRP due to lack of an yield plateau. The assumption of full bond will make the analysis of these structures simple. Deflection calculations cannot be performed using the

so-call effective moment of inertia method recommended by the ACI and CSA codes for steel reinforced concrete. In addition, consideration of creep in the FRP and the difference in concrete stresses under service loads in FRP versus steel reinforced concrete structures would make it difficult to apply the empirical equations of the code to determine either the immediate or long term deflection of FRP reinforced members. The current study is undertaken with the foregoing considerations in mind.

2.5 Computer programming languages

2.5.1 General

Selecting a suitable programming language for a particular application requires that the designer has at least general background knowledge about available languages. Many languages when selected and used properly will result in saving time and efforts for the designer.

The aim of this section is to review the general approach to programming engineering problems. The main steps in the process are defined and discussed with particular emphasis on the importance of choosing a programming language for a particular application. In addition, features provided by languages such as FORTRAN, Pascal, C-language and Basic will be examined.

2.5.2 Engineering programming languages

There are many programming languages used for engineering purposes. BASIC, C, FORTRAN and Pascal are some of the most prevalent. The FORTRAN language, short for Formula Translation language, has become one of the most suitable languages for scientific and engineering applications during the last few decades. The structured version of FORTRAN is known as FORTRAN 77 and FORTRAN 90. Structured programming language is preferred because it is easy to understand, debug and modify. In structured programming language, pro-

grams are designed from the main program to the lowest level subprograms. Nowadays, almost all programming languages have become structured. Examples of structured programming languages are C-language, Pascal and Visual Basic. Like FORTRAN, Pascal was found to be suitable for teaching programming as a logical and systematic spline, thus encouraging well-structured and well-organized programs [Wilson and Clark, (1988)]. C-language is flexible, very powerful and the most popular language for professionals. The programming language Basic is still considered the simplest and easiest programming language to learn. Visual Basic has been developed to allow for easy Windows applications. Visual Basic has the ability to create user-interface features easily and quickly. C++, Visual C++, Smalltalk and object oriented versions of Pascal are object oriented programming languages. These languages are very powerful and can provide good tools for professional applications. In object oriented programming (OOP), functional flow is replaced by message passing between objects which cause changes in object state [Graham, (1991)]. Encapsulation and inheritance are the main features behind the object oriented programming languages. Encapsulation (or information hiding) refers to the practice of including within an object everything it needs, and furthermore doing this in such a way that no other object need ever be aware of this internal structure [Graham, (1991)]. Inheritance (or classification hierarchy) is used to inherit properties and procedures from superior classes to lower-level classes [Graham, (1991)]. Class is the description of objects having the same structure and behaviour. The advantage of inheritance is that lower level classes and objects may inherit shared proper-

ties and methods eliminating the need to store them in every instance.

2.6 Choosing the programming language

Selecting a suitable programming language for this thesis was the first step. The need for Windows-based programming language with good user-graphical interface eliminated FORTRAN, Pascal and C languages from our choice. On the other hand, although object oriented programming languages are very powerful Windows-based languages, their applications in civil engineering is still not common. Besides, they are not easy to use. Visual Basic has good user-graphical interface, efficient, simple and easy to use and thus, it was selected to be the programming language for the computer program developed in this study. It should be emphasized, however, that the principle of objective of the thesis is not the development of the most effective program, but of a program which can effectively analyze FRP reinforced members and which can be easily applied by the user.

2.6.1 General approach to programming engineering problems

Fenner (1974) discussed the main steps which a programmer must follow when writing suitable computer software to solve a typical problem. The steps are:

1. Define the problem.

Identification of needs, goals and limitations of the computer applications is the first step. The need for a computer program which can save time and money for the programmer and which leads to more productivity will certainly help the

programmer with what to accomplish with the program. Defining the unknowns, types and sources of input data is also important. Identifying the goals of a program will assist in developing the algorithms and mathematical model. Calculation of moment-curvature in reinforced concrete beams, for example, may represent one of the program goals which should be defined. Limitations of the program will predict the behaviour of the solutions and the accuracy of computations.

2. Establish a suitable mathematical model.

A mathematical model is an abstract and often approximate representation of the problem at hand [Jewell and Thomas (1990)]. Within the model, limitations and any assumptions used must be identified. Units used in a particular equation should be consistent and input data should be in such a way that output be of reasonable approximation of the actual system.

3. Check the numerical solution.

The accuracy of the computations and any approximations used in the computer problem should be checked. Errors should be carefully monitored and any systematic errors should be either eliminated or accounted for. Furthermore, the selected convergence criterion or criteria should reflect the desired degree of accuracy, consistent with the accuracy of the input data.

4. Build an algorithm.

The algorithm is a series of detailed of steps used to solve the problem [Jewell and Thomas, 1990]. To build an algorithm one must understand the general procedures of the computer program.

5. Select a suitable computer programming language.

Selecting a suitable programming language depends on many considerations. Spencer (1973) mentioned some important consideration such as:

The application requirement: will the application be developed for engineering or business purposes, for large or small purposes, etc.

Features of the programming language: features are graphic capabilities, adequate memory, user-friendly interface (menus, icons, windows, ...), etc.

The power of the language: represents the facilities provided by the language with respect to the ease of use, speed and support of concept of structured programming. Features provided by a well structured program have become a necessity for the majority of applications. Features, such as, sequences, decisions, loops and subroutines can effectively help in understanding and modifying the computer program.

6. Write the program code.

Write the code for the main and sub programs. Comments, definitions and a list of variables should be also written to make the program easy to understand and modify.

7. Test the program.

Check the result of the computations at every step of the program in all subroutines.

8. Write a user's manual.

Writing a user's manual will certainly help a user how to effectively use the program.

2.6.2 Visual Basic

2.6.2.1 General

For a number of reasons, the program "Wizard" developed in this thesis has been written in Visual Basic. The need for Windows-based program was the priority for this research in order to take full advantage of the latest Windows capabilities. Visual Basic is the quickest and easiest way to create powerful applications for Microsoft Windows operating systems, and is designed to make user-friendly programs easier to develop. Command buttons, labels, text boxes or any other standard control can be easily used. The properties window is used to change how objects look and react such as fonts, size, color, ... etc. Visual Basic is not restricted only to simple programs but can also be used to create more advanced programs. In additions, Visual Basic is a complete structured programming language and has features that are found in most other modern programming languages. Moreover, the finished applications can be a true executable file

2.6.2.2 Structure of Visual Basic

Applications written in Visual Basic can contain several different types of files. The files can be summarized as follows:

- Form module: the code is stored in a single form.

- Standard module: it is used to implement a common code for different forms.
- Class module: contains only the code, these modules provide reusable modules for objects of the application.

2.6.2.3 Features of Visual Basic

Visual Basic helps one to create powerful, full-featured applications. Graphical user interface can be created by drawing objects in graphical format. The properties of the objects can be set to refine their appearance and behaviour. To make this interface react with the user, program code is written to implement the events that occur in the interface. With dynamic data exchange (DDE) and object linking and embedding (OLE), data transfer from other applications, such as Microsoft Word or Excel Spreadsheet become relatively automatic and invisible to the user. Other features include additional custom controls and the help compiler for adding a help system to an application.

In view of the preceding advantage of Visual Basic, it is adopted for the development of the computer program in this thesis.

Chapter 3

Analysis of reinforced concrete sections

3.1 Introduction

In this chapter, the short and long-term behaviour of concrete members subjected to flexure is analyzed. The short and long-term analysis deals with both uncracked and cracked cross-sections. The age-adjusted effective modulus method (*AEMM*) is adopted for creep and shrinkage analysis. For cracked sections, the effect of tension stiffening is also considered in the analysis.

In all analyses, strain distribution over the depth of the section is assumed to vary linearly, i.e. plane sections are assumed to remain plane. Concrete in the tension zone is assumed to be ineffective in resisting internal forces acting on cracked section. It is also assumed that complete bond exists between the reinforcement and the concrete. Concrete compressive stresses rarely exceed one half of the maximum concrete strength $0.5 f_c'$ under service loads. Since creep is a serviceability limit state requirement, for creep and shrinkage analysis, the stresses in concrete are assumed to be linearly distributed. For sign convention, compressive forces, strains and stresses are assumed positive while tensile

forces, strains and stresses are considered negative. Positive bending moments and the corresponding curvatures are those which cause tensile stresses in the bottom face of the beam cross-section.

In all the analyses, the top surface of the cross-section is taken as the reference surface. The expressions for stress, strain and curvature are developed with respect to this reference surface which does not change with time.

3.2 Constitutive relationships.

The degree of accuracy in the analysis and design of concrete structures partly depends on how the actual stress-strain relationships of the constitutive materials are represented. Hence good representation of material behaviour along with precise methods of analysis are likely to provide greater insight into the behaviour of a structure and to yield more economical and safer design. Thus, the program Wizard utilizes the strain compatibility procedures together with a nonlinear stress-strain relationship for concrete, a linear elastic relationship for FRP and an elasto-plastic strain-hardening relationship for steel. Fig. 3.1 shows the adopted stress-strain relationship for concrete, steel and FRP's, where,

f_c'' = concrete cylinder strength.

ϵ_o = concrete strain corresponding to f_c'' .

f_c' = concrete stress.

ϵ_c = concrete strain.

f_y = yield strength of steel.

ϵ_y = yield strain of steel.

ϵ_{SH} = steel strain at the beginning of strain hardening.

ϵ_{SU} = steel ultimate strain.

f_{SU} = steel ultimate strength.

ϵ_{JU} = FRP's strain at rupture.

f_{fU} = FRP's stress at rupture.

The stress-strain relationship for concrete shown in Fig.3.1 (a) is due to Hognestad (1951) which comprises primarily two segments. The ascending portion of the curve is represented by a parabola which has the following equation [Hognestad (1951)]

$$f_c = f_c'' \left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right] \quad (3.1)$$

ε_o can be calculated from,

$$\varepsilon_o = \frac{n}{(n-1)} \left(\frac{f_c''}{E_c} \right) \quad (3.2)$$

where E_c = concrete modulus of elasticity = $5000 \sqrt{f_c'}$

n = curve-fitting factor

For normal weight concrete, n can be calculated [Collins and Mitchell (1990)] from

$$n = 0.8 + \frac{f_c''}{17} \quad (3.3)$$

The descending part of the curve is linear and has the following equation

$$f_c = f_c'' [1 - 100 (\varepsilon_c - \varepsilon_o)] \quad \text{with } \varepsilon_c \leq \varepsilon_{cu} \quad (3.4)$$

Note ε_{cu} should be an input value. The value 0.0038 in Fig. 3.1 is for illustration purpose.

Figure 3.1: Stress-strain relationship (a) for concrete
(b) for non-prestressed steel (c) for FRP

3.3 Flexural response using layered section

In all analyses, program Wizard evaluates the resisting moment, forces, strains, stresses and curvatures of concrete members at any load level based on the layered section approach. This approach has the advantage of being appropriate and effective for cases such as:

1. Cross sections with constant or varied widths.
2. Sections with any stress distribution through the depth
3. Any disposition of reinforcement and combinations of reinforcement types.

To illustrate the main ideas of this approach, consider the reinforced concrete section in Fig. 3.2 with the given strain distribution. First, the cross-section is divided into a number of rectangular layers, n . Each layer, i , has the width of the section at that level and a depth equal to (h/n) . Due to the plane section remain plane assumption, the concrete strain distribution is defined by two variables, usually by either the top and the bottom strains or by top strain and the depth of the compression zone, c .

The strain in each idealized layer is equal to the actual strain at the centre of the layer. The value of the average strain in each layer, ϵ_i can be expressed in terms of top strain, ϵ_t , and the depth of the compression zone, c as follows:

$$\varepsilon_i = \frac{\left[n \frac{c}{h} - i + 0.5 \right]}{n \frac{c}{h}} \times \varepsilon_t \quad (3.5)$$

The corresponding stress, σ_i in each layer is determined from the relevant stress-strain relationship of the materials in the i^{th} layer. The force in each layer can then be calculated by multiplying the area of the layer, A_i by the stress, σ_i . Steel or FRP reinforcement can also be represented as thin layers with thickness t where t is determined by dividing the area of the given reinforcement layer by the width of the web at the location of the same layer. The contribution of the layer i to the resisting moment of the section can be calculated by multiplying the force of the layer by the distance from the reference axis to the centre of the layer.

It should be pointed out that as the layer becomes thinner the force in the layer and consequently the resultant forces will be more accurately calculated.

Figure 3.2: Flexural analysis using layered section (a) Layered section
(b) Strain distribution (c) Stresses and forces.

3.4 Determinations of moment-curvature relation

The basic definition of curvature of a reinforced concrete member (Fig. 3.3) can be related to the rotation of the same member per unit length with the following expression

$$\text{Curvature} = \psi = \frac{1}{R} = \frac{\epsilon_t}{c} = \frac{\epsilon_r}{(d-c)} \quad (3.6)$$

where ψ = the curvature or the rotation per unit length of a member.

R = the radius of curvature measured to the neutral axis.

ϵ_t = the top concrete strain.

ϵ_r = the strain in the tension reinforcement.

c = the depth of compression zone.

d = the depth of the tension reinforcement.

Figure 3.3: Moment-curvature relationship of flexural member.

Curvature can also be defined by the usual elastic equation:

$$\psi = \frac{M}{EI} \quad (3.7)$$

EI is the flexural rigidity of the cross section. The flexural rigidity of a member remains constant as long as the materials behave elastically. As moments increase, concrete cracks and flexural rigidity diminishes gradually. The behaviour of the cracked section depends mainly on the amount of reinforcement. For ductile reinforced concrete member, i.e. under-reinforced steel concrete members, the $M - \psi$ curve may be nonlinear up to the yielding point of the steel. After yielding, a large increase in curvatures takes place at approximately constant bending moment. The moment increases slightly to the peak value due to the increase in lever arm in the concrete section. In the case of over-reinforced unconfined concrete members or concrete members reinforced with FRP bars, the $M - \psi$ curve is basically linear and failure is initiated by the crushing of concrete.

To determine the complete moment-curvature response of a flexural member, as in program Wizard, the curvatures and moments corresponding to various values of top concrete strains need to be determined. An iterative process in conjunction with a layered section analysis is used. The main steps of the process can be summarized as follows:

1. Select a starting value for the top concrete strain, ϵ_t .
2. Assume an initial value for the depth of the compression zone, c .
3. For a layered section, Fig. 3.4, of n layers numbered from the top with

each layer having a depth of h/n , the average strain in layer, i , is

$$\epsilon_i = \frac{\left[n \frac{c}{h} - i + 0.5 \right]}{n \frac{c}{h}} \times \epsilon_t \quad (3.8)$$

4. Determine the stress in each concrete layer corresponding to the average concrete strain in the same layer. This is done using the stress-strain relationship of the concrete shown in Fig. 3.1.

5. Determine the stress in each reinforcement layer corresponding to the strain assumed in Step 2.

For steel the stress-strain relationship utilized in the program Wizard is given by Fig. 3.2 where,

$$\sigma_s = E_s \epsilon_s \quad (\text{for } 0 < \epsilon_s \leq \epsilon_y) \quad (3.9)$$

$$\sigma_s = f_y \quad \left(\text{for } \epsilon_y < \epsilon_s \leq \epsilon_{SH} \right) \quad (3.10)$$

$$= f_y + \left(\epsilon_s - \epsilon_{SH} \right) \times (\sigma_u - \sigma_y) / \left(\epsilon_u - \epsilon_{SH} \right) \quad \left(\text{for } \epsilon_{SH} < \epsilon_s \right) \quad (3.11)$$

where σ_s is the steel stress.

For FRP reinforcing bars,

$$\sigma_f = E_f \times \epsilon_f \quad (3.12)$$

where σ_f , ϵ_f and E_f are the stress, strain and elastic modulus of FRP.

6. Find the concrete compressive force for each layer where,

$$N_i = \text{Concrete compressive force in layer } i = f_c \times A_i$$

A_i is the area of layer i

7. Find the net concrete compressive forces by summing up the forces in all the concrete layers, found in Step 5.

$$N = \sum_{i=1}^n N_i \quad (3.13)$$

8. Calculate the force $(T_r)_i$ in reinforcement layer i where

$$(T_r)_i = (\sigma_r \times A_r)_i \quad (3.14)$$

where $(T_r)_i$, σ_r and A_r are the force, stress and area of the reinforcement layer.

9. Calculate the resultant force T of the reinforcement layers.

$$T = \sum_{i=1}^n (T_r)_i \quad (3.15)$$

10. Check equilibrium of internal forces

if $N < T$, increase c

if $N > T$, decrease c

Repeat Steps 2-9 for different c values until (N) and (T) are essentially equal within an acceptable range. Once equilibrium is achieved, the value of c and the concrete top strain, ϵ_t , are used to find the corresponding curvature and moment.

$$Curvature = \psi = \frac{\epsilon_t}{c} \quad (3.16)$$

The final bending moment, M_i , is the summation of the product of layer forces by their distance from the reference line to the centre of each layer i .

11. Increase the value of the top concrete strain by a fixed value and repeat Steps 1-10 each time to calculate points on the $M - \psi$ curve.

3.5 Non-time dependent analysis

The purpose of the analysis is to determine the concrete and steel stresses in the section under a given applied moment and/or axial stress.

3.5.1 Uncracked cross section

It is assumed here that the stresses and strains caused by the applied moment, M_s , are within the elastic range. The elastic analysis is usually performed on the transformed section.

Consider the section in Fig. 3.4 subjected to a normal force, N_s and a bending moment, M_s . The area of the reinforcement (steel or FRP) at each reinforcement level is transformed into an equivalent area of concrete.

The steps of analysis can be reviewed in view of the preceding sections as follows:

The strain and stress of each layer of the section at depth, y_i from the top face are

$$\epsilon_i = \epsilon_t - y_i \psi_s \quad (3.17)$$

$$\sigma_i = \frac{E}{\bar{E}_{ref}} \epsilon_i = \frac{E}{\bar{E}_{ref}} (\epsilon_t - y_i \psi_s) \quad (3.18)$$

where E_{ref} is the modulus of elasticity of a reference section, E_i is the modulus of elasticity of the particular layer material. ψ_s is the curvature obtained from the short term analysis (initial curvature).

The resultant axial force N_s is found by integrating the stress block over the depth of the section.

$$N = \int \sigma_i dA = \int E_{ref} (\epsilon_t - y_i \psi_s) \frac{E_i}{E_{ref}} dA_i \quad (3.19)$$

$$N = E_{ref} [\epsilon_t A - \psi_s B] \quad (3.20)$$

where, A is the area of the transformed section and equals

$$A = \sum_{i=1}^n \frac{E_i}{E_{ref}} A_i \quad (3.21)$$

and B is the first moment of the section with respect to the reference plane.

$$B = \sum_{i=1}^n \frac{E_i}{E_{ref}} B_i = \sum_{i=1}^n \frac{E_i}{E_{ref}} y_i dA_i \quad (3.22)$$

The resultant moment, M_s can be calculated by integration of the first moment of the transformed area about the top face of the section.

$$M = - \int \sigma_s y_i dA_i \quad (3.23)$$

$$M = -E_{ref} \int (\epsilon_t + y_i \psi_s) \frac{E_i}{E_{ref}} y_i dA_i \quad (3.24)$$

$$M = -E_{ref} [\epsilon_t B + \psi_s I] \quad (3.25)$$

where

I is the second moment of the transformed area about the top face of the section.

$$I = \sum_{i=1}^n \frac{E_i}{E_{ref}} I_i = \sum_{i=1}^n \frac{E_i}{E_{ref}} y_i^2 dA_i \quad (3.26)$$

The preceding equations may be written in compact form using matrix notation as follows:

$$\begin{bmatrix} N \\ M \end{bmatrix} = E_{ref} \begin{bmatrix} A & B \\ B & I \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \psi_s \end{bmatrix} \quad (3.27)$$

$$\begin{bmatrix} \epsilon_t \\ \psi_s \end{bmatrix} = \frac{1}{E_{ref}} \begin{bmatrix} A & B \\ B & I \end{bmatrix}^{-1} \begin{bmatrix} N \\ M \end{bmatrix} \quad (3.28)$$

$$\begin{bmatrix} \epsilon_t \\ \psi_s \end{bmatrix} = \frac{1}{E_{ref} (AI - B^2)} \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} \quad (3.29)$$

Figure 3.4: Uncracked cross-section (a) Transformed section (b) Initial strain

3.5.2 Cracked cross section

If the tensile stress at a point reaches the tensile strength of concrete, cracks occur and concrete is assumed to no longer carry any tensile stresses across cracks. In the cracked section, any tensile stresses are assumed to be carried only by the reinforcement. Concrete, as stated earlier, is assumed to be ineffective in the cracked zone.

The method of analysis used by program Wizard for fully cracked section under short-term loads is presented here.

Consider the fully cracked reinforced concrete section shown in Fig. 3.5.

The non-linear portion of the concrete stress-strain relationship, as discussed earlier, will be used and the calculations are based on the layered section concept. The applied stresses in concrete are given by the following expressions.

$$f_c = 0 \quad (\text{for } y_i > c) \quad (3.30)$$

$$f_c = f_c'' [1 - 100 (\epsilon_i - \epsilon_o)] \quad (\text{for } y_i \leq x) \quad (3.31)$$

$$f_c = f_c'' \left[\frac{2\epsilon_i}{\epsilon_o} - \left(\frac{\epsilon_i}{\epsilon_o} \right)^2 \right] \quad (\text{for } x < y_i < c) \quad (3.32)$$

where f_c'' is the concrete cylinder strength, ϵ_o is the strain corresponding to f_c'' ,

f_c is the concrete stress, ϵ_i is the concrete strain of layer i .

The stress in the reinforcement is computed based on the stress-strain relationship described in Section 3.4.

The compression and the tensile forces, moments, curvatures are then determined using the same technique as in Section 3.4.

Figure 3.5: Cracked cross-section (a) Strain (b) Stress

3.6 Time dependent analysis.

The time-dependent effects considered in the current study include creep and shrinkage in concrete and creep in FRP reinforcement. The effect of these phenomena on redistribution of stresses and deflection are considered.

The approach of ACI Committee 209 (1982) for predicting the creep and shrinkage coefficients has been used in program Wizard. Accordingly, the creep coefficient is given by

$$\phi(t, t_a) = \frac{(t - t_a)^{0.6}}{1 + (t - t_a)^{0.6}} \phi(t_\infty, t_a) K_{CH} K_{CA} K_{CS} \quad (3.33)$$

where $\phi(t, t_a)$ = creep coefficient (ratio of creep strain to initial elastic strain).

t = time in days after application of load

t_a = concrete age in days at first loading.

$\phi(t_\infty, t_a)$ = Ultimate creep coefficient at time infinity caused by a unit stress applied at age, t_a . Its value ranges from 1.30 to 4.15, with an average value of 2.35 [Park and Paulay (1975)].

K_{CA} = Age of loading coefficient.

K_{CH} = Relative humidity coefficient.

K_{CS} = Member size coefficient.

The coefficients K_{CA} , K_{CH} and K_{CS} are given in Table 3.1 based on the Committee's recommendation.

The unrestrained shrinkage strain at any time, t is given by

$$\epsilon_{sh}(t, t_a) = \frac{t}{b+t} K_{SH} K_{SS} \epsilon_{sh}(t_{\infty}) \quad (3.34)$$

where the constant $b = 35$ for moist cured concrete or 55 for steam cured concrete

K_{SH} = Correction factor for relative humidity.

K_{SS} =Correction factor for member size.

$\epsilon_{sh}(t_{\infty})$ = Ultimate shrinkage coefficient. Its value ranges from 0.000415 to 0.00107 with an average value of 0.00080 for moist-cured concrete or 0.00073 for steam-cured concrete [Park and Paulay(1975)].

Table 3.1: Quantitative values of parameters used for time-dependent properties of concrete.[from Naaman(1982)]

Property	Values of constants	
	Moist-cured concrete	Steam-cured concrete
Young's modulus(E_s)	$b=4$ $c=0.85$	$b=1$ $c=0.95$
Shrinkage Strain (ϵ_{sh})	$40\% \leq H \leq 80\%$ $b=35$ $t \geq 7 \text{ days}$ $K_{SH}=1.40 - 0.01H$ $K_{SS}=1.14-0.09(V/S)$ $40\% \leq H \leq 100\%$ $b=35$ $t \geq 7 \text{ days}$ $K_{SH} = 3 - 0.03 H$ $K_{SS}=1.14-0.09(V/S)$	$40\% \leq H \leq 80\%$ $b=55$ $t \geq 1 \text{ to } 3 \text{ days}$ $K_{SH}=1.40-0.01H$ $K_{SS}=1.14-0.09(V/S)$ $80\% \leq H \leq 100\%$ $b=55$ $t \geq 1 \text{ to } 3 \text{ days}$ $K_{SH} = 3 - 0.03 H$ $K_{SS}=1.14-0.09(V/S)$
Creep coefficient (ϕ)	$t, t_A \geq 7 \text{ days and } H \geq 40\%$ $K_{CA} = 1.25 t_A^{-0.118}$ $K_{CH} = 1.27 - 0.0067 H$ $K_{SS}=1.14-0.09(V/S)$	$t, t_A \geq 1 \text{ to } 3 \text{ days and } H \geq 40\%$ $K_{CA} = 1.25 t_A^{-0.118}$ $K_{CH} = 1.27 - 0.0067 H$ $K_{SS}=1.14-0.09(V/S)$

3.6.1 Method of analysis

Bresler and Selna's (1964) approach for the time analysis of the concrete members, known as relaxation method, was used in program Wizard. Additional details of the method can be found in Ghali and Favre(1986) and Gilbert (1988). This approach can be summarized as follows:

1. Let the free creep strain of the concrete member occur.
2. If the reinforcement is in tension, tensile creep tends to elongate the reinforcement and increase the tensile stresses in the reinforcement. Shrinkage, on the other hand tends to shorten the reinforcement and thus reducing its tensile stress.

The restraining forces due to concrete creep are

$$\Delta N_{creep} = \bar{E}_e \phi (A_c \epsilon_s - B_c \psi_s) \quad (3.35)$$

$$\Delta M_{creep} = \bar{E}_e \phi (I_c \psi_s - B_c \epsilon_s) \quad (3.36)$$

where A_c , B_c and I_c are the uncracked concrete area, first moment of area and second moment of area, excluding the reinforcement area.

ϕ is the creep coefficient of concrete.

ϵ_s and ψ_s are the concrete top strain and curvature both derived from short-term analysis (subscript s denotes short term analysis).

$\bar{E}_e$ is the age-adjusted effective modulus, given by

$$\bar{E}_e = \frac{E_c}{1 + \eta \phi} \quad (3.37)$$

where

E_c is the elastic modulus of concrete

η = concrete aging coefficient, which is assumed to vary between 0.46 and 1.0.

In this program η is assumed equal to 0.8.

The restraining forces due to the shrinkage are calculated by

$$\Delta N_{sh} = \bar{E}_e \varepsilon_{sh} A_c \quad (3.38)$$

$$\Delta M_{sh} = -\bar{E}_e \varepsilon_{sh} B_c \quad (3.39)$$

Similarly, the restraining forces due to the creep of FRP reinforcement ΔN_f and

ΔM_f can be determined by

$$\Delta N_f = \phi_f (\varepsilon_t - d_f \psi_s) A_f E_f \quad (3.40)$$

$$\Delta M_f = -\phi_f (\varepsilon_t - d_f \psi_s) A_f d_f E_f \quad (3.41)$$

where

ϕ_f = Creep coefficient of FRP reinforcement

d_f = distance of FRP reinforcement from the reference fibre.

A_f = Area of FRP reinforcement.

Total restraining forces are

$$\Delta N = \Delta N_{creep} + \Delta N_{sh} + \Delta N_f \quad (3.42)$$

$$\Delta M = \Delta M_{creep} + \Delta M_{sh} + \Delta M_f \quad (3.43)$$

3. To restore equilibrium, remove the applied restraining forces ΔN and ΔM by applying equal and opposite force and moment to the entire section.

The change in strain and curvatures are then given by

$$\Delta \epsilon_o = \frac{\bar{B}_e \Delta M + \bar{I}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e^2)} \quad (3.44)$$

$$\Delta \psi = \frac{\bar{A}_e \Delta M + \bar{B}_e \Delta N}{\bar{E}_e (\bar{A}_e \bar{I}_e - \bar{B}_e^2)} \quad (3.45)$$

where $\bar{A}_e$, $\bar{B}_e$ and $\bar{I}_e$ are the area, first moment of area and moment of inertia, respectively, of the age adjusted transformed section.

4. The final concrete and reinforcement stresses are then given by:

For concrete,

$$\sigma_c(t) = \sigma_c(t_a) + \Delta \sigma_c(t, t_a) \quad (3.46)$$

For reinforcement:

$$\sigma_r(t) = \sigma_r(t_a) + \Delta \sigma_r(t, t_a) \text{ where} \quad (3.47)$$

$\sigma_c(t_a)$ and $\sigma_r(t)$ are the initial concrete and reinforcement stresses, respectively.

$\Delta \sigma_c(t, t_a)$ = The change in concrete stress

$$= -\bar{E} [\phi \epsilon_i + \epsilon_{sh} - (\Delta \epsilon_o - y_i \Delta \psi)] \quad (3.48)$$

$\Delta \sigma_r(t, t_a)$ = The change in reinforcement stress

$$= E_r (\Delta \epsilon_o - d_i \Delta \psi) \quad (3.49)$$

3.7 Prediction of deflections

3.7.1 General

In design it is important that the deflection of the concrete structure remain acceptable under normal service conditions. Large deflections are generally not acceptable. Damage to non-structural elements such as partitions or glass can be caused by excessive deflections. In corrosive environments, excessive deflection can produce large cracks which, in turn, lead to faster corrosion of steel reinforcement. For these reasons, most codes, such as the ACI and CSA Standard codes, specify limits to the maximum deflections using the minimum depth to span ratio.

The present section is concerned with the methods of calculation of deflections under short and long-term loading. The ACI approach based on the estimation of the effective moment of inertia along the concrete member is briefly discussed. Method of analysis in which curvatures are known at various sections is discussed in more detail. The effect of the tensile stresses in the concrete between cracks is also discussed.

3.7.2 Effective moment of inertia method

When flexural cracking develops, the effect of concrete in tension zone gradually diminishes and flexural rigidity (EI) decreases accordingly. The method developed by Branson (1977) and adopted by ACI code for estimation of instantaneous deflection due to load on a cracked section is based on determination of the effective moment of inertia. Effective moment of inertia, I_e , is then assumed constant along the full length of the member.

The effective moment of inertia, I_e , lies between the moment of inertia of uncracked concrete cross section and the moment of inertia of cracked cross-section. The expression for I_e is given by:

$$I_e = I_g \left[\frac{M_{cr}}{M_a} \right] + I_{cr} \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] \quad (3.50)$$

where

I_g = Moment of inertia of the uncracked gross concrete section.

I_{cr} = Moment of inertia of the cracked cross-section about the centroid axis.

M_{cr} = Moment at first cracking.

M_a = Maximum moment in the concrete member due to the load for which the deflection is calculated.

The long-term deflection can be estimated by multiplying the initial short-term deflection, Δ_i , by a multiplier factor, λ

$$\Delta_t = \lambda \Delta_i \quad (3.51)$$

where

$$\lambda = \frac{T}{1 + 50\rho'}$$

ρ' = The compression reinforcement ratio calculated at mid-span.

$T = 1.0$ for loading time period of 3 months.

$= 1.2$ for 6 months.

$= 1.4$ for 12 months.

$= 2$ for 5 years or more.

For FRP reinforced concrete structures, Razaqpur and Ali (1995) and Faza (1991) showed in their experimental work that the computation of deflections according to ACI approach is not applicable to FRP reinforced concrete members. This is mainly due to the behaviour of high strength low modulus FRP reinforcing bars. Therefore, considerations should be given to more fundamental methods, such as the moment-curvature method. This method as has been utilized by the program Wizard and will be discussed in more detail later in this chapter.

3.7.3 Tension stiffening

Consider a reinforced concrete member shown in Fig. 3.6 and subjected to a bending moment, M . Before the primary cracks occur in concrete, tensile stresses are fully resisted by concrete, (state 1). When the tensile strength in concrete exceeds the cracking stress, all the tensile stresses at the cracking loca-

tions will be resisted by the reinforcement (state 2). In this case the highest tensile stresses in the reinforcement will be at crack locations while the tension in the concrete is zero. Between consecutive cracks, bond between the concrete and reinforcement tends to restrain the elongation of the reinforcement and, therefore, a part of the reinforcement tensile stresses at the crack locations are introduced into the surrounding concrete. At this stage, the curvatures will vary along the length of the concrete member as shown in the figure with higher values at crack locations.

The tensile stresses in the concrete between the cracks are known as "tension stiffening". Tensile stresses in concrete between cracks stiffens the concrete member and thus deflections will be overestimated if the effect of tension stiffening is not included in the analysis.

Program Wizard uses the CEB approach for modeling the tension stiffening. At an applied moment, M , the effective curvature is expressed as

$$\psi_e = (1 - \zeta) \psi_1 + \zeta \psi_2 \quad (3.52)$$

where

$$\zeta = 1 - \beta_1 \beta_2 \left(\frac{M_{cr}}{M_d} \right) \quad (\text{for } M \geq M_{cr}) \quad (3.53)$$

or

$$\zeta = 1 - \beta_1 \beta_2 \quad (\text{for } M < M_{cr}) \quad (3.54)$$

where curvatures ψ_1 and ψ_2 correspond to the two states mentioned earlier and ψ_e is the effective curvature. Coefficients β_1 and β_2 account for the bond of the

reinforcing bars and for long-term loads or cycles of load.

Equation (3.54) indicates that when the applied moment (M_a) does not exceed

the cracking moment (M_{cr}), $\left(\frac{M_{cr}}{M_a} \right)$ should be taken as unity in equation (3.53).

Figure 3.6: $M - \psi$ relationship (a) A reinforced concrete member in flexure
 (b) Moment-curvature relationship for the member in (a),
 [Ghali and Favre, (1986)].

3.7.4 Calculations of deflections from curvatures

The response of a beam under flexure can be determined by using the governing equation of bending which relates the moment at any section to its corresponding curvature as

$$\frac{M}{EI} = \psi = \frac{d^2 w}{dx^2} \quad (3.55)$$

in which EI represents the flexural rigidity of the beam, w is the deflection, x is the axial coordinate and ψ is the curvature.

Integrating Eq. (3.55) gives

$$\theta = \frac{dw}{dx} = \int \frac{M}{EI} dx = \int \psi dx \quad (3.56)$$

where θ is the beam rotation

Integrating Eq. (3.56) gives the deflection w

$$w = \int \psi x dx \quad (3.57)$$

Equations (3.56) and (3.57) are in fact the mathematical expressions of the first and second moment area theorems. The principles of first and second moment area theorems can be explained by referring to Fig. 3.7.

The change of slope between any two points along the beam is equal to the area under the curvature diagram between these two points. Therefore, the change of slope between points C and B , in Fig. 3.7 is the area A_{CB} of the curvature diagram between these two points. The deviation of point C from a tangent drawn

at point B is equal to the first moment of the area A_{CB} under the curvature diagram between points C and B , taken about point C .

In order to find deflections using the preceding equations, the computer program Wizard utilizes numerical integration. For the given beam in Fig. 3.8, the steps of analysis can be explained as follows:

1. The beam is divided into a number of sections of equal length, Δx .
2. Calculate the moments at these sections
3. Use the moment-curvature diagram (Fig. 3.8) of the beam to determine the curvatures (Fig. 3.8(c)) corresponding to moments at these sections.
4. Calculate the deflection of B from the tangent drawn at point A ,

$$= \sum_{i=1}^n \left[\frac{(\Psi_i + \Psi_{i+1})}{2} \Delta x \right] (L - (i - 0.5) \times \Delta x) \quad (3.58)$$

where $\left[\frac{(\Psi_i + \Psi_{i+1})}{2} \Delta x \right]$ represents the area of each segment under the curvature, and

$\left(L - i \times \Delta x + \frac{\Delta x}{2} \right)$ represents the moment arm of segment i from its centre to the right support, B

5. Calculate the slope at left support, A

$$\theta_1 = \text{Slope at A} = \frac{\delta_{BA}}{L} \quad (3.59)$$

6. The slope at any other section can be calculated as follows

$$\theta_{i+1} = \theta_i - \frac{(\psi_i + \psi_{i+1})}{2} \times \Delta x \quad (3.60)$$

7. The deflection can then be easily calculated using the changes of slope between segments as follows,

$$\delta_{i+1} = \sum_{i=2}^n \delta_{i-1} + (\theta_{i-1} + 4\theta_i + \theta_{i+1}) \times \frac{\Delta x}{3} \quad (3.61)$$

Program Wizard uses Simpson's rule to perform the integration. This is indi-

cated by the expression, $(\theta_{i-1} + 4\theta_i + \theta_{i+1}) \times \frac{\Delta x}{3}$

Deflection at $i=2$ is given by the following expression

$$\delta_2 = (\theta_1 + \theta_2) \Delta x / 2 \quad (3.62)$$

Figure 3.7: Determination of curvature and deflections (a) Cross-section (b) Deflected shape (c) Curvature diagram.

Figure 3.8: Moment, curvature and deflections of beams (a) Deflected shape
(b) Bending moment diagram (c) Curvature diagram

Chapter 4

The computer program - Wizard

4.1 Introduction

The computer program Wizard developed in this thesis is aimed towards the analysis and design of simply supported reinforced concrete beams or slabs. The reinforcement may be up to five layers of steel bars and/or FRP bars located in the tension or in the compression zone. The cross-section may have any arbitrary solid shape with vertical symmetry. The concrete cross-section and reinforcement may also have up to five different types of material properties. Loading patterns can be any combinations of distributed and /or concentrated loads with maximum number of loads for each type being 10. The flexural response of simply supported beams or slabs can be easily estimated by program Wizard both for short and long-term analysis. The formulations are based on the requirements for equilibrium and strain compatibility in the concrete and in the reinforcement. Wizard allows the designer to specify materials, dimensions and properties all in S.I. units.

Wizard is written in Visual Basic programming language and operates in Win-

dows 95 (32 bits). The execution of commands is simple and straightforward and can be done either by a mouse or by keyboard functions.

Wizard does not allow the user to proceed to the next step in the problem if the design parameters are not properly defined. The results can be directed to the printer and/or on screen. Next, the program, its structures and various units will be presented.

4.2 Program structure

The program Wizard consists of a number of different modules. Each module performs a specific function accessed through the options available from the main menu, sub-menus or control objects (such as command buttons, text boxes,... etc.). In most cases, the sequence of procedures executed in the program is controlled by events that the user creates, rather than by a predetermined sequence of procedures. Each control object contains a number of independent instructions that are activated by the events such as button clicks or mouse movements.

The program uses a number of other general procedures, defined as a Sub or Function, which are invoked only when explicitly called by a program instruction. In addition, the program uses a number of libraries known as Dynamic Link Libraries (DDL). DDLs are external functions written in C-language, Visual C++, Turbo Pascal or Delphi.

Each event procedure is limited to a specific form in which the event procedure is defined while the general procedures are defined in the general section of any form or code module.

All procedures contained in the instructions declare variables, arrays, data types and constants in different ways. Depending on the scope of each variable, declarations can be put at the form, the module or the procedure level. Variables declared within a procedure are local to that procedure.

The key words used in declarations are Dim, ReDim, Public, Static, Const and type. Public (or global) is used to give a global declaration of a variable to every procedure in every form and code modules. Dim to declare variables at the module level. Static declares variables inside a procedure for local scope. ReDim has a specific importance for declaring dynamic arrays. With ReDim instruction, the size of arrays can be dynamically modified during run time. The program can then modify the dimensions of the arrays as many times as required at run time. As a result, the use of the memory will not be wasted for unnecessary array elements.

Figures 4.1, 4.2(a), 4.2(b), 4.3(c) show the menus used in the program. These menus and all other graphical interface units with their functions will be described in the next sections.

4.2.1 Main menu

The Main menu has six icons (Fig.4.1) which provide pop-up menu commands.

These icons are:

- **File:** provides the options for loading, saving or editing a new data file or editing an existing file.

- **Non-time dependent analysis:** calculates the ultimate moment capacity of the section and short term-deflection of concrete member.
- **Section properties:** plots diagrams for cross-section and material stress-strain relation for concrete, steel or FRP reinforcements.
- **Time-dependent analysis:** Calculates the long-term deflection and stresses in the member.
- **Help:** contains information about the program and how to use it.
- **Exit:** quit the execution of the program.

Program Wizard

Figure 4.1: Main menu screen

4.2.2 Sub-menus

Figures 4.2(a), 4.2(b) and 4.2(c) show all the sub-menus written to the main menu. These sub-menus are:

- **File:** contains sub-menus for loading, saving or editing a new data file. Figures 4.3, 4.4 and 4.5 show the graphical interface units for data input or editing purposes for concrete, steel, and FRP reinforcements.
- **Non-time dependent analysis:** This icon contains the following sub-menus.
- **Ultimate moment capacity**

This option contains the following:

- ***Full response***

Calculates the full moment-curvature response of a given reinforced concrete cross-section. To find the full response, the program increases the top strain from zero until the maximum strain specified by the user. Then, the program stores and displays the range of strains, curvatures, and moments in a grid. Also, another four menus appear for saving, print and display moment-curvature diagram graphically. To return to the main menu, close icon should be selected.

- ***At specified top strain***

This command calculates the moment-curvature response at a user specified top strain. The user is prompted to enter a value for top concrete strain, then the values of moments, curvatures and depth of compression zone are displayed on the

screen

- **Serviceability analysis**

This menu includes two sub-menus, deflection calculation and finding stresses in concrete and the reinforcement for a given moment. These are further explained below.

- ***Deflection calculation***

Calculates short-term deflection of the reinforced concrete member. Selecting this menu icon leads to another screen for data entry. Fig. 4.6 shows the relevant input data for deflection calculation. Once the data is entered, the program calculates the moment, shear force, curvature and deflection at various sections along the span. The number of sections is specified by the user as indicated in Fig. 4.6. The results of the analysis are then displayed on another screen (Fig. 4.7). Using the latter screen, moment, shear force, curvature and deflection variation along the beam can be plotted either on the screen or sent to a printer.

- **Time-dependent analysis**

This menu option contains the following icons

- ***Creep and shrinkage data***

This option allows a user to the input data which is required for creep and shrinkage time-dependent analysis. Two options are available to enter data; either by “direct values” or using “the ACI code formula”. Direct values can be used to enter pre-determined value of creep and shrinkage coefficients. The

option “using ACI code formula” requires from a user data that could be used in the suggested equations of the ACI in order to calculate the creep and shrinkage coefficients.

- ***Deflection Calculations***

This option lets a user enter data that is needed for calculating deflections due to sustained loads. Deflections are determined at the specified beam sections. It also permits the user to plot deflection and curvature due to sustained and total loads.

- ***Stresses for a given moment***

This option calculate stresses in concrete and the reinforcement for a given moments. The user is prompted to enter the moment caused due to combined temporary and sustained loads and due to sustained load only.

Figure 4.2: Main menu structures and items. (a) Non-Time dependent analysis

Figure 4.3: Main menu structure and items (b) Time-dependent analysis

Figure 4.4: Main menu structure and items (c) File, section properties plot, Help and Exit menus

1

1	35	29580.4	1.819756E-	3.549648	29.75	0.0038
---	----	---------	------------	----------	-------	--------

1

1	350	350	580	1
---	-----	-----	-----	---

50

Figure 4.5: Typical data input screen for concrete and cross section properties.

			1
1	40000	600	

			1
1	520	2732	1

OK

Figure 4.6: Typical data input screen for FRP reinforcement

1					
1	200000	400	400	0.01	0.02

1			
1	520	2756	1

Figure 4.7: Typical data input screen for steel.

		6000	
		100	
		1	
		1	

1	20	6000	0
---	----	------	---

2400		
------	--	--

1	50000	3000
---	-------	------

Figure 4.8: Typical data input screen for loads

Bending moments, shear forces and deflections along the beam span

155440
0
-155440
4640
180310400
2320
15.71244
2320

1	0	155440	0	0	0	0	0
2	46.4	152331.2	7140292	4.204277E-08	1.223803E-05	0.4713981	
3	92.8	149222.4	1.413634E+07	8.323619E-08	2.422882E-05	0.942721	
4	139.2	146113.6	2.098813E+07	1.235803E-07	3.597239E-05	1.41385	
5	185.6	143004.8	2.769568E+07	1.63075E-07	4.746871E-05	1.884728	
6	232	139896	3.425898E+07	2.017204E-07	5.871781E-05	2.355241	
7	278.4	136787.2	4.067803E+07	2.395164E-07	6.971967E-05	2.825334	
8	324.8	133678.4	4.695283E+07	2.764631E-07	8.04743E-05	3.294897	
9	371.2	130569.6	5.308338E+07	3.125604E-07	9.09817E-05	3.76388	
10	417.6	127460.8	5.906969E+07	3.478084E-07	1.012419E-04	4.232176	
11	464	124352	6.491175E+07	3.82207E-07	1.112548E-04	4.699738	
12	510.4	121243.2	7.060955E+07	1.728314E-06	2.558149E-04	5.165991	
13	556.8	118134.4	7.616319E-07	7.083388E-06	7.689719E-04	5.628902	

Figure 4.9: Typical output screen for deflection analysis

4.3 Graphical interface features

The graphical interface features of program Wizard offer an easy, effective and a flexible way for user interaction utilities. In addition to the menus discussed in previous section, features such as file directory handling, graphs, error trapping, help and data input are presented in this section.

- **File and directory handling**

Tasks such as loading, editing or saving a data file is facilitated by using the Common Dialog control shown in Fig.4.7. Common Dialog is a custom control and provides a standard set of dialog boxes for common Windows operations, such as selecting a file. Common Dialog box is an example for external functions (DDLs) which is used by the program Wizard. Then, by writing a suitable instruction for opening a file, saving or retrieving data can be executed well with the Common Dialog box. It displays directory listings and default path on which the files are to be stored or opened. Moreover, files can be renamed or deleted in a simple way.

- **Help**

The program uses a short message for each parameter used in the program. To display a help message, one simply places the mouse pointer at the required parameter and then press on the “F1” key. Help is also provided through the main menu option called “Help”.

- **Error trapping**

With error trapping, problems can easily be corrected, and then execution would be allowed to proceed. In program wizard, if any mistake happens with respect to format during data entry, a remedial message from the program will prompt the user for the correct input format.

- **Printer control**

Because program Wizard is basically a Windows-based program, it does support hard copy output. Visual Basic can connect files to a device driver which allows output to be sent to the default printer specified by Windows. Routines to check the status of the printer has been constructed to facilitate the printing process.

Figure 4.10: Common Dialog box for selecting data files

Chapter 5

Numerical examples and verifications

5.1 General

In this chapter the accuracy and validity of the program “Wizard” is demonstrated through comparison of its predicted results with available experimental data obtained from the literature. The comparison is made for some concrete beams reinforced by steel or FRP’s bars which were subjected to either short or long term loading. The load-deflection curves, moment-curvature relation and long term deflection are compared to verify the program.

It should be noted that the problems selected can cover only some aspects of the program Wizard, the full range of the program applicability are not covered by these examples.

5.2 Verification of Moment-Curvature Relationship

Determination of the moment-curvature relationship is an important part of this study. The degree of accuracy of non-linear analysis depends on the accurate prediction of the moment-curvature relationship of sections. Here the experimental moment-curvature relationship of concrete sections reinforced by GFRP and CFRP bars are compared with the predictions of the computer program Wizard.

Three beams were tested by Alsayed et al (1996). The beams were 2900 *mm* long by 200 x 260 *mm* in cross section and reinforced with tension reinforcement (4 ϕ 12.7 *mm* GFRP bars) at an effective depth of 210.7 *mm*. Further details are given in Fig.5.1(a). All beams were tested by two point loads, each placed at 100 *mm* from the middle of the 2700 *mm* span. We select beam I of this series for analysis here.

The relevant material properties of this beam are shown in Table 5.1.

Table 5.1: Material properties of Alsayed et al beams

Concrete compressive strength at 28 days	31.3 MPa
GFRP Reinforcement	
Ultimate stress for ϕ 12.7 <i>mm</i>	740 MPa
Elastic modulus	50 GPa

Fig 5.2 shows the experimental and the predicted moment-curvature relationship of this beam. As can be seen from this figure, the theoretical results agree well with the experimental results. Note that in a displacement control test, that is hydraulic actuators, the reported experimental response is not possible because the machine cannot maintain a constant load while the stiffness of the beam drops due to cracking. In accurately recorded experiments, the predicted response is expected.

Tezuka et al carried out their investigation on two span continuous concrete beams reinforced with Aramid FRP (AFRP) or Carbon FRP (CFRP) used as pre-stressing tendon or flexural reinforcement. The reinforcement was placed in the tension and the compression zones, as shown in Fig. 5.1 (b). The beam reinforced with CFRP is selected here for comparison with the results of program Wizard. The beam details are shown in Fig.5.1 (b). The CFRP bars were braided in shape and their mechanical properties are shown in Table 5.2. The concrete compressive strength was 40 MPa.

Table 5.2: Material properties of CFRP reinforcements (Tezuka et al)

Type of Reinforcement	Nominal Diameter (mm)	Sectional Area (mm ²)	Tensile Strength (MPa)	Young's Modulus (GPa)
CFRP - ϕ 10.5	10.5	55.7	1900	140

The experimental and predicted moment-curvature curves are shown in Fig. 5.3. As shown in this figure, there is some difference between the experimental and

theoretical values of the ultimate moment and the curvature. Tezuka et al also observed this difference when they compared their theoretical calculations with the experimental results. They concluded that the difference could be due to the underestimating of the section rigidity at the middle support of the continuous beam under investigation. It is important to point out that this beam was statically indeterminate, therefore the moment at the middle support depends on the magnitude of the positive moment at midspan. It is, however, significant to notice that the predicted moment-curvature response immediately after cracking (i.e. the drop in moment immediately after cracking) is consistent with the observed behaviour for this beam.

(a)

(b)

Figure 5.1: Beams cross sections (a) By Alsayed et al (1996) (b) By Tezuka et al (1995)

Figure 5.2: Moment-curvature curves (Theoretical vs. experimental for beams tested by Alsayed et al (1996))

Figure 5.3: Comparison of predicted moment-curvature relationship by program Wizard with the experimental results of Tezuka et al (1995)).

5.3 Verification of short-term response of reinforced beams

The experimental results of this example were taken from the work of Razaqpur and Ali (1995) for CFRP (Carbon Fibres) reinforced beams under flexure. The beam is an inverted T-beam of 4.2 *m* length and overall depth of 400 *mm* as shown in Fig. 5.4. The compressive strength and Young's modulus of concrete are 80.3 *MPa* and 44.7 *GPa*, respectively. The beam contained 2-8 *mm* diameter deformed CFRP bars with a total area of 98.6 mm². The elastic modulus and the ultimate strength of CFRP bars were 147 *GPa* and 2316 *MPa*. The beam was tested for the entire range of loading up to failure using quasi-static monotonic loading.

Fig. 5.5 shows the load versus midspan deflection curve of the beam. It can be noticed that the Wizard's prediction and the experimental results are almost identical within the elastic range. Beyond the elastic range, the difference in displacements is relatively small.

The ultimate capacity and the maximum deflection of the beam from the experimental work are 119 *KN* and 60 *mm*, respectively. These values agree well with the corresponding values of 124 *KN* and 58.6 *mm* predicted by the Wizard program.

(a)

(b)

Note: all dimensions are in mm.

Figure 5.4: The beam details tested by Razaqpur and Ali(1995) (a) Elevation (b) Cross section

Figure 5.5: Load-Deflection curve: Theoretical vs. experimental for the beam tested by Razaqpur and Ali (1995).

5.4 Verification of long-term deflections of simply supported beams

The following example was taken from an experimental study reported by Brown and Bartholomew (1993) on the long-term behaviour of four simply supported beams. Two beams were reinforced with GFRP reinforcement and two beams were reinforced with non-prestressed steel bars. The beams are 100 *mm* wide by 150 *mm* high and reinforced with 2-9.5 *mm* diameter rebars, as shown in Fig. 5.6(b). They were tested under two point loads, each of 3.87 *kN* on a 685 *mm* span length as shown in Fig. 5.6(a).

The following design data is provided: the concrete compressive at 28 days = 35 *MPa*. The FRP bars are linear elastic up to failure with tensile strength = 550 *MPa* and elastic modulus = 41.4 *MPa*. Steel bars are Grade 60 deformed bars (f_y = 400 *MPa*).

The beams were moist cured for seven days and then tested at ambient temperature over an 8 month period. The creep coefficients of concrete and FRP reinforcement are 2.35 and 0.495, respectively. The ultimate shrinkage coefficient can be taken as 0.0008 in case of moist cured concrete [Park and Paulay (1975)]. Test results showed that the GFRP reinforced beams had an initial average midspan deflection of 9.20 *mm* and 2.4 *mm* for steel reinforced beams. The Wizard program showed good agreement with experimental results; it gave 9.8 *mm* for GFRP reinforced beams and 2.6 *mm* for steel reinforced beams. Figures 5.7 and 5.8 compare the experimental and predicted load versus midspan deflection and

time versus maximum deflection, respectively. As can be seen from these figures, results obtained by the Wizard program agree well with the experimental results.

(a)

(b)

Figure 5.6: Beam details (Brown and Bartholomew, 1993) (a) Sectional elevation (b) Beam cross section

Figure 5.7: Comparison of predicted and observed short term load-midspan deflection curve for the beam tested by Brown and Bartholomew (1993).

Figure 5.8: Deflection vs. Time curves (theoretical vs. experimental by Brown and Bartholomew (1993) for FRP and steel-reinforced beams)

5.5 Analysis of FRP reinforced slabs by Wizard

The following example was taken from the research work conducted by Schmeckpeper (1992). The objective of his study was to determine the flexural behaviour of FRP reinforced bridge deck slabs. In this section, the test results of bridge deck-DH will be compared with the predictions of the computer program Wizard.

The concrete bridge deck slab was 220 *mm* thick, 1.22 *m* wide and 3.05 *m* long as shown in Fig.5.9. It was supported on structural steel sections, providing a simple span length of 2.4 *m*. The applied load was distributed over a 250 x 630 *mm* area at the centre of the slab.

The concrete and FRP properties as reported by Schmeckpeper are:

Concrete compression strength, $f_c' = 34.50 \text{ MPa}$.

Concrete elastic modulus, $E_c = 27545 \text{ MPa}$.

Total area of FRP, $A_f = 4800 \text{ mm}^2$.

Ultimate stress of FRP, $f_f = 630 \text{ MPa}$.

FRP elastic modulus, $E_f = 41200 \text{ MPa}$

FRP ultimate strain = 0.015

Fig. 5.10 shows the load versus deflections at the centre of the deck. The stress versus strain diagram for the deck slab at midspan concrete and at FRP reinforcement are shown in Fig.5.11. The strain in both FRP reinforcement and concrete is linear up to the ultimate load of the test slabs. As can be seen from these figures, test results are in reasonable agreement with the results of program Wiz-

ard. The preceding examples illustrate the degree of accuracy that can be achieved by the analytical methods adopted in the current program. They also demonstrate that the procedures have been correctly implemented.

Figure 5.9: Bridge deck slab (DH) by Schmeckpeper (1992)

(a) Side view (b) Plan

Figure 5.10: Load-deflection curves (Wizard predictions vs. experimental results by Schmeckpeper(1992).

Figure 5.11: Midspan longitudinal strains vs. applied load for a slab strip.
(Wizard predictions vs. experimental results by Schmeckpeper (1992)).

Chapter 6

Parametric study

6.1 General

In this chapter, the effect of some of the main parameters that have influence on the flexural behaviour of reinforced concrete beams will be studied with the aid of the computer program Wizard. One of the reasons for the development of this program is to facilitate the study of the detailed response of FRP reinforced beams in an expeditious manner. We shall investigate the following responses and how they are influenced by certain beam characteristics.

- Moment-curvature relation.
- Depth of neutral axis: c .
- Ultimate moment capacity.
- Short-term deflections.
- Long-term deflections.
- The parameters whose influence will be studied include the reinforcement ratio (ρ), the span/depth ratio, the creep of FRP and the section type (rectangular versus T-section).

6.2 Materials and beam sections

The beams under consideration were designed according to the provisions of CSA Standard (CSA 1994). The dimensions of the beams were selected as shown in Fig. 6.1. The material properties of both concrete and the reinforcements are summarized in Table 6.1. Clearly, other beam sizes can be studied later for the sake of completeness, but the objective of the present study is mainly to demonstrate the efficiency of the program Wizard in providing detailed results for the in-depth response of FRP reinforced members under flexure.

Table 6.1: Material properties used in the analysis

Material constant	Concrete	Steel	FRP
f_c'	35		
E_c	29580		
f_y		400	
E_s		200000	
f_{CFRP}			2500
E_{CFRP}			150000
f_{GFRP}			600
E_{GFRP}			40000

Figure 6.1: Beams cross sections used in the parametric study (a) Rectangular section (b) T-section (c) Load distribution.

6.3 The beam reinforcement

The reinforcement ratios considered in the present study were: $0.2 \rho_b$, $0.4 \rho_b$, $0.75 \rho_b$, $1.0 \rho_b$, $1.5 \rho_b$ and $2.0 \rho_b$, where ρ_b is the balanced reinforcement ratio. The balanced reinforced state of concrete beams reinforced with steel occurs when simultaneously the steel starts yielding and the concrete strain reaches its ultimate value. In the case of beams reinforced with FRP bars, the balanced condition occurs when concrete reaches its ultimate strain capacity and simultaneously the FRP reinforcement reaches its ultimate strain (rupture). The balanced reinforcement ratio of rectangular and T-sections with tension reinforcement are given by the ACI code as follows:

For rectangular section:

$$\bar{\rho}_b = \frac{0.85 \times f'_c \times \beta_1}{f_y} \times \left[\frac{0.0035 \times E_s}{0.0035 \times E_s + f_y} \right] \quad (6.1)$$

For T-section

$$\rho_b = \frac{b_w}{b} [\bar{\rho}_b + \rho_f], \text{ where} \quad (6.2)$$

$$\rho_f = \frac{0.85 \times f'_c \times (b_f - b_w) \times h_f}{b_w \times d \times f_y} \quad (6.3)$$

where,

β_1 = The stress block depth factor

f'_c = Compressive strength of concrete at 28 days.

f_y = yield strength of steel.

E_s = Modulus of elasticity of steel.

b_w = Beam web width

h_f = Flange thickness of the T-section.

d = Distance from extreme compression fibre to centroid of steel.

h_f = Flange thickness of the T-section.

From first principles, for FRP reinforced beams, the balanced reinforcement ratio can be easily determined which would give:

For rectangular section

$$\bar{\rho}_b = \frac{0.85 \times f'_c \times \beta_1}{f_{fU}} \times \left[\frac{0.0035 \times E_f}{0.0035 \times E_f + f_{fU}} \right] \quad (6.4)$$

For T-section

$$\rho_b = \frac{b_w}{b} [\bar{\rho}_b + \rho_f] \quad (6.5)$$

where

$$\rho_f = \frac{0.85 \times f'_c \times (b_f - b_w) \times h_f}{b_w \times d \times f_{fU}} \quad (6.6)$$

where,

f_{fU} = FRP's ultimate stress

E_f = FRP's modulus of elasticity.

We wish to emphasize that the preceding equations for FRP are based on the equivalent rectangular stress block parameters as provided for by ACI (1995). In the CSA Standard A23.3-94 the same parameters are determined slightly differently in order to make them applicable to both ordinary and high strength con-

crete, for low and medium strength concretes the two codes give essentially the same results. For the selected beams in the current study, the amount of steel, GFRP and CFRP reinforcements corresponding to rectangular (R)-section are 6891, 1366 and 301 mm^2 respectively. For T-section, they are 6534, 2116 and 466 mm^2 respectively.

6.4 Selection of span to depth ratio

The deflection of FRP reinforced concrete structures is one of the key parameters that govern their design. However, due to the high strength/modulus ratio of FRP's, compared to steel, it is not normally feasible to satisfy the deflection requirements using the customary span/depth ratios used for steel reinforced concrete. There are two simple, albeit not necessarily economical, ways to mitigate this problem. First, one could use higher reinforcement ratios. This would reduce deflections but will not permit the designer to take advantage of the high strength of FRP. The other alternative is to use smaller span/depth ratios.

We shall investigate the effect of the L/h (span/depth) on the maximum deflection of a number of rectangular and T-section. Although it is not expected that the present results alone will lead to definitive conclusions, nevertheless some insight will be gained with regard to the effect of the span/depth ratio on the maximum deflection. The chosen span to depth ratios are: $\frac{L}{6}$, $\frac{L}{8}$, $\frac{L}{10}$, $\frac{L}{12}$ and $\frac{L}{16}$. Note that although the effect of the span length alone is obvious, the effect

of h is not as obvious due to the effect of cracking on the moment of inertia of the section.

The ACI and CSA codes state that the serviceability requirements of maximum permissible deflections for beams or one way slabs reinforced with steel would be satisfied provided the member's depth is not less than the values given in Table 6.2. Beams and slabs which satisfy the specified limits are conservatively designed and are assumed to be free of deflection problems. However, these limits do not apply to FRP reinforced members.

Table 6.2: Minimum thickness of beams and one way slabs

Member	Simply supported
Solid one-way slabs	$L/20$
Beams or ribbed one-way slabs	$L/16$

6.5 Moment-curvature analysis

The full moment-curvature relationship for both rectangular and T-sections is studied. Figures 6.2 and 6.3 show the moment-curvature relation for different reinforcement ratios for rectangular and T-beam, respectively. Each figure shows the moment-curvature response of beams reinforced with different amounts of steel, GFRP or CFRP reinforcement.

As can be seen from these figures, beams reinforced with steel tend to be more ductile as the steel reinforcement ratio to balanced steel reinforcement ratio (ρ/ρ_b) becomes smaller. It is also clear from this figure that when $\rho > \rho_b$, the

steel reinforced beams behave as brittle members. In all cases, for both steel and FRP reinforced beams, as ρ/ρ_b approaches 1.0, the flexural capacity of the section increases almost linearly with ρ/ρ_b . However, for steel reinforced sections, the ultimate moment capacity increase for reinforcement ratios greater than ρ_b is relatively small. For FRP reinforced beams, however, there seems to be a steady and proportional increase in the ultimate moment capacity even for ρ values greater than ρ_b . Observe that moment-curvature relationship for beams reinforced with FRP bars is basically bi-linear up to 85% of ultimate moment. However, beyond that moment, especially for beams reinforced with $\rho > \rho_b$ there is a clear nonlinear portion in the moment-curvature response. Since the amount of reinforcement corresponding to ρ_b is usually very small in the case of FRP's, it is generally impractical to use such small amounts of reinforcement. Accordingly, FRP reinforced concrete members are mostly over-reinforced ($\rho > \rho_b$). Of course, the over-reinforcement means that these members will fail due to crushing of the concrete rather than the rupture of the of reinforcement. This is considered a desirable failure mode because rupture of FRP leads to complete disintegration of the member.

Finally, the results in Figures 6.2 and 6.3 do not show any significant difference between the behaviour of the rectangular and T-sections.

(a)

(b)

(c)

Figure 6.2: Moment-curvature relation of rectangular cross sections reinforced with (a) steel. (b) GFRP. (c) CFRP.

(a)

(b)

(c)

Figure 6.3: Moment-curvature relation of T-cross sections reinforced with (a) Steel (b) GFRP (c) CFRP.

6.6 Ultimate moment capacity

The variation of the ultimate moment capacity of rectangular and T cross sections are analyzed. Figures 6.4 and 6.5 show the variation of ultimate moment capacity with ρ/ρ_b for the analyzed beams. It is clear from these figures that in the compression failure region, i.e. $\rho > \rho_b$, the rate of moment increase with increased reinforcement ratio is relatively small. However, as noted earlier, the rate decreases dramatically for the steel reinforced sections while for the FRP reinforced sections the drop is smaller. For instance, in Fig. 6.4(a) which pertains to the steel reinforced sections, the moment increased from approximately 1100 KN.m to 1300 KN.m (18%) when ρ/ρ_b increased from 1.0 to 2.0 while in the case of CFRP (Fig. 6.4 (c)) it increased from 350 KN.m to 510 KN.m (around 46%) for identical change in the relative value of ρ . Furthermore, it should be appreciated that the amount of reinforcement corresponding to ρ_b in the steel reinforced sections is significantly higher than in the FRP reinforced sections. The stresses of the steel reinforcement and the internal lever arm decrease with increase in reinforcement ratio, i.e. there is little additional flexural strength to be gained by the increase in the reinforcement ratio above that for a balanced failure.

In the tension failure mode, $\rho < \rho_b$, the moment capacity does not increase linearly with reinforcement ratio. This is due to the fact that although the steel force increases linearly, there is a reduction in lever arm with increasing the

reinforcement content.

6.7 Load-deflection analysis

The load-deflection curves of the analyzed beams are shown in figures 6.6 and 6.7. Each figure illustrate the effect of reinforcement ratio on the deflected shape and maximum deflection of simply supported beams subjected to uniform load. The plotted results are for span/depth ratio of 12. As can be seen from these figures, the load-deflection relationship of under-reinforced steel beams ($\rho < \rho_b$) is basically trilinear. This relationship can be divided into three regions, as follows:

- Region I: represents the elastic behaviour of the uncracked beam (precracking stage). The load-deflection curve in this stage is essentially a straight line.
- Region II: Starts as the flexural cracking develops (postcracking stage). Because the flexural rigidity in this stage begins to diminish, the load-deflection curve is less steep than in the precracking stage segment.
- Region III: in this stage, the load-deflection relation is represented by a line which is significantly less steep than that in the previous two stages. This is because at yield the section moment of inertia dramatically decreases and in the case of over-reinforced steel beams ($\rho > \rho_b$), the load-deflection relationship changes gradually from trilinear to a complete nonlinear curve.

In the case of FRP due to the linear stress-strain relationship of FRP bars until rupture, the load-deflection relation, as can be seen from these figures, is com-

posed of essentially two regions. These regions correspond to precracking and postcracking stages. Comparing the load-deflection curves for rectangular sections in Fig. 6.6 with those for T-section in Fig. 6.7, we notice that there is little difference between the load-deflection response of the two types of sections. It is significant, however, to notice the dramatically larger deflection of the FRP reinforced beams compared to the steel reinforced beams. For instance, in Fig. 6.6(a) we can see that the beam with $\rho/\rho_b = 0.4$ has an ultimate load of approximately 80 KN.m and its deflection at 40 KN.m (assuming the service load to be half of the ultimate load) is approximately 13 mm. Among the GFRP reinforced beams, the beam with $\rho/\rho_b = 1.5$ has an ultimate load of approximately 80 KN.m. At 40 KN.m, this beam has a maximum deflection of approximately 60 mm. For the CFRP reinforced beam the deflection for the beam with $\rho/\rho_b = 1.5$ at 40 KN.m is almost 80 mm.

Clearly these large deflections are unacceptable. If the design were governed by the deflection limit, then the maximum service load for the CFRP and GFRP reinforced beams would not exceed 15% of their ultimate load capacity. Thus design based on deflection limit would render the high strength of the material useless. Thus ways must be found to overcome the problem of low modulus/strength ratio of FRP's.

One way of limiting deflections, without substantial increase in reinforcement, is to decrease the span/depth ratio for FRP reinforced beams. According to CSA standard A23.3-94 (1994) for simply supported reinforced concrete beams deflection would not be a problem if the span/depth ratio is less than or equal to

16. In Fig. 6.8 we plot the load-midspan deflection curves of the rectangular sections with span/depth ratio ranging from 6 to 16. Notice that the reinforcement ratio for the steel reinforced beams in Fig.6.8(a) was $0.4 \rho_b$ while for the GFRP and CFRP reinforced beams in Figs. 6.8(b) and (c) it was $2 \rho_b$. We notice a dramatic decrease in the value of maximum deflection with lower values of L/h . This can be clearly seen in Figs. 6.8 and 6.9 which show the deflected shape of the analyzed beams. Observe that for the GFRP and CFRP reinforced rectangular sections the maximum deflection decreases by approximately 30% and 40%, respectively, when L/h is reduced from 12 to 10. Thus one way of controlling deflection in FRP reinforced concrete members is to reduce their L/h ratio. For instance, for simply supported conventional reinforced concrete members an $L/h \leq 16$ is normally recommended [Pilai and Kirk (1988)]. According to Fig. 6.8(a) the maximum deflection of the beam with $L/h=16$ at $0.5q_u$, where q_u is the ultimate load, is equal to 25 mm. We assume the service load to be half of the ultimate load, hence the 25 mm is the service load maximum deflection. According to Fig. 6.8 (b), for a similar size GFRP reinforced beam to have 25 mm deflection at service load (i.e at 0.5 of its ultimate load), its $\frac{L}{h} \leq 8$. The same appears to be true for the corresponding CFRP reinforced beam. Hence, it may be concluded that a reasonable L/h for FRP reinforced members would be 50 % of the value for the corresponding steel reinforced beams. Note, however, that the reinforcement ratio for the FRP reinforced beams must be $\geq 2\rho_b$ while for

steel reinforced beams it could be $\geq 0.4\rho_b$.

Figure 6.9 shows that T-beams follow basically the same trend as the rectangular section. However, in the case of T-beam, the reduction in L/h leads to even higher proportional reduction in deflections than in rectangular beams. In accordance with these observations, it is recommended that deflection control in FRP reinforced concrete members be handled mainly through L/h ratio rather than provisions of excessive reinforcement.

The previous examples are not intended to deduce complete design recommendations, instead they are presented to demonstrate the type of analysis that can be performed using the program Wizard.

6.8 Effect of creep of FRP bars

Figures 6.10 and 6.11 show the change in the maximum deflection of reinforced beams with time. In case of FRP reinforced beams, the time-dependent deflections were calculated assuming first that FRP does not creep and then that it does. The sustained load on each analyzed beam was assumed to be 30% of the ultimate load and the span/depth ratio for the beams is 12. The reinforcement ratio values were $0.4 \rho_b$ for steel reinforced beams and $2.0 \rho_b$ for FRP reinforced beams. The latter ratios translate into 2756 mm^2 of steel, 2732 mm^2 GFRP and 602 mm^2 of CFRP. The other design parameters are:

Relative humidity = 60%

Ultimate concrete creep coefficient = 2.35

Ultimate FRP creep coefficient = 0.5

Ultimate shrinkage coefficient = 0.0008

Concrete aging factor = 0.8

The analysis was undertaken at time periods 7, 14, 28, 180, 365, 1095 and 1825 days.

It is clear from Figure 6.10 and 6.11 that deflections increase with time. The increase in deflections, however, depends on the reinforcement materials used, the value of sustained load (W_s is constant here and equals $0.3 W_u$), and on the parameters used in the analysis. However, for the assumed design conditions here, we can notice, for example that the long-term deflection of steel reinforced beams (rectangular sections) is about 21% more than the initial deflection while for the GFRP reinforced beams assuming that FRP bars do not creep it is about 10 %. On the other hand, when the GFRP is allowed to creep, then the time-dependent deflection after 1825 days would be 20 % higher than the initial deflection. The same trend can be observed for the CFRP where creep in the reinforcement further increases the time-dependent deflections. Therefore, it is important to include the effect of creep in the reinforcement on the final deflection of FRP reinforced members. Generally, we can notice that although the used ultimate creep coefficient of concrete is 4.7 times that of FRP's ultimate creep coefficient, the long-term deflection is more than the immediate deflection by about 10% for both cases (with and without FRP creep).

The relationship between FRP ultimate creep coefficient and the ratio of maximum deflection of FRP reinforced beam with creep to the maximum deflection

of FRP reinforced beam with no creep is shown in the Figure 6.12. It is clear that the relationship is completely linear. Hence for a creep coefficient different than the values used in the present analyses, the time-dependent change in deflection due to the creep of FRP can be estimated by using the linear relationships shown in Fig. 6.12.

6.9 The reinforcement ratio vs. (C_u/d)

Some researchers have suggested that to design beams reinforced with GFRP reinforcement, the value of $\frac{C_u}{d}$ should be in the range of 0.15 to 0.3 while for CFRP reinforced beams $\frac{C_u}{d}$ should be in the range of 0.25 to 0.5, where C_u is the depth of the compression zone at ultimate and d is the effective depth of the beam. To investigate the validity of these recommendations we will analyze a number of FRP reinforced beams using the program Wizard. Figure 6.13 shows the results of the analysis for GFRP and CFRP reinforced R-beams with reinforcement ratios ranging from 0 to 4%. The values of $\frac{\rho}{\rho_b}$ for the previously specified value of $\frac{C_u}{d}$, for GFRP reinforced beams are as follows:

For rectangular section, $0.8 \leq \frac{\rho}{\rho_b} \leq 3.5$

For T-section, $0.7 \leq \frac{\rho}{\rho_b} \leq 3$

Similarly, for the CFRP reinforced beams, the value of $\frac{\rho}{\rho_b}$ for the specified $\frac{C_u}{d}$ are as follows:

For rectangular section, $\frac{\rho}{\rho_b} > 4$

For T-section, $\frac{\rho}{\rho_b} > 3$

While $\frac{\rho}{\rho_b} = 0.8$ for GFRP and 0.7 for CFRP may satisfy the preceding recommendations, as shown earlier, beams reinforced with such low reinforcement ratios will undergo excessive and unacceptable deflection under service loads. Consequently, one should not use the lower bounds of the preceding recommendations. As indicated earlier reinforcement ratios less than the twice the balanced reinforcement ratio will most likely produce excessive deflection. Hence once again, deflection control should be achieved through the simultaneous reduction in the span/depth ratio and the increase in the reinforcement ratio. Using L/h (span/depth) ratios of 0.50 times those used for conventional steel reinforced beams and reinforcement ratios $\geq 2.0\rho_b$ may produce satisfactory deflection in most cases.

Figure 6.4: Variation of ultimate moment capacity for a typical rectangular section reinforced with (a) Steel (b) GFRP (c) CFRP

Figure 6.5: Variation of ultimate moment capacity for a typical T-section reinforced with (a) Steel (b) GFRP (c) CFRP

Figure 6.6: Effect of reinforcement ratio on the load-deflection behaviour of a simply supported rectangular section beams reinforced with (a) Steel (b) GFRP (c) CFRP.

Figure 6.7: Effect of reinforcement ratio on the load-deflection behaviour of a simply supported T-section beams reinforced with (a) Steel (b) GFRP (c) CFRP.

(a)

(b)

(c)

Figure 6.8: Effect of span/depth ratio on the load-deflection behaviour of a simply supported rectangular section beams reinforced with (a) Steel (b) GFRP (c) CFRP.

Figure 6.9: Effect of span/depth ratio on the load-deflection behaviour of a simply supported T-section beams reinforced with
(a) Steel (b) GFRP (c) CFRP.

(a)

(b)

(c)

Figure 6.10: Effect of concrete creep on the time-dependent maximum deflection of a simply supported R-beams reinforced with (a) Steel (b) GFRP (c) CFRP.

(a)

(b)

(c)

Figure 6.11: Effect of concrete creep on the time-dependent maximum deflection of a simply supported T-beams reinforced with (a) Steel (b) GFRP (c) CFRP.

(a)

(b)

Figure 6.12: FRP ultimate creep coefficient vs. maximum deflection

(FRP with creep) / maximum deflection (No creep in FRP).

(a) GFRP (b) CFRP reinforced beam.

Figure 6.13: C_u/d versus reinforcement ratio ($\frac{\rho}{\rho_b}$) in rectangular and

T-sections.

Chapter 7

Conclusions and recommendations

7.1 Summary

A general computer aided analysis program, called Wizard, was developed for the analysis of the full flexural response of concrete members reinforced with steel and/or FRP bars. Both the short and long-term flexural behaviour can be predicted by the aid of the program Wizard. The program for this thesis was structured in a modular form and was written in Visual Basic, operating within the Windows 95 environment. The modular form allows for any extension or enhancement of the program in the future. The user interface, such as menus, command buttons and file handling routines can facilitate such future development. A description of the program and a review of the programming languages were presented. Results of some experimental works were successfully compared against the results of the program Wizard. A parametric investigation was also conducted to demonstrate the program capabilities and to illustrate the utility of the program for the purpose of detailed analysis of FRP reinforced members under flexure.

7.2 Conclusions

1. The current code provisions for steel reinforced concrete cannot be applied to FRP reinforced concrete members.
2. Using basic principles of mechanics it is possible to predict the full behaviour of FRP reinforced beams under flexure, including the load-deflection response and ultimate strength.
3. Reliable results can be obtained by using detailed methods of analysis as incorporated in program Wizard.
4. The use of the moment-curvature integration for calculating the deflection of FRP reinforced concrete members gives accurate results.
5. Unlike steel reinforcement, FRP reinforcement undergo creep and it causes further increase in the time-dependent deflections of the beam. Hence its effect should be included in the design.
1. Programs such as “Wizard” can predict the full flexural response of the conventional reinforced concrete and/or FRP reinforced concrete beams, including their strength, deflections and load-deflection response.
6. Once verified, such programs, allow the user to conduct parametric studies in order to better understand the effect of various factors on the strength and deformations of concrete beams.

7. Languages such as Visual Basic enables the designer to include graphics, user interface routines and mathematical analysis routines in a simple manner, and to write the program for Windows environment.
8. Using highly structured programming languages facilitates the development of modular programs and addition of enhancements for future development.
9. The present results suggest that the span/depth ratio for FRP reinforced beams should not be greater than 0.5 the values used for similar size beams reinforced with steel.
10. To avoid excessive deflection, the reinforcement ratio of FRP reinforced beams should be at least equal to $2\rho_b$, where ρ_b is the balanced reinforcement ratio.

7.3 Recommendations

1. The capabilities of the program Wizard can be extended to include the following:
 - Shear and bond behaviour.
 - Crack spacing and width.
 - Design of indeterminate structures.
 - Design of composite structures.
2. The program Wizard should be used to conduct more detailed parametric studies in order to be able to deduce simple design guidelines.

References

1. ACI Committee 209, (1982) "Prediction of creep, shrinkage and temperature effects in concrete structures," ACI Publication SP-76, Detroit, Michigan., pp. 193-300.
2. AC I (1995) "ACI 318-95 Building Code requirements for structural concrete," American Concrete Institute, Farmington Hills, Michigan.
3. Ali, M. M. (1995) " A new concept in achieving ductility in FRP reinforced concrete members- analytical and experimental investigation," Master's Thesis, Department of Civil and Environmental Engineering, Carleton University, Ottawa, Canada.
4. Alsyaed, S., H., Al-Salloum, Y., A. , Almusallam , T., H. and Amjad, M., A. (1995) "Flexural behaviour of concrete elements reinforced by GFRP bars" Proceedings of Second International RILEM Symposium, Taerwe, L. (Editor), Ghent, E&FN SPON, London, pp. 219-226..
5. Bakis, C., E. (1993) "Fibre-reinforced plastics (FRP) reinforcement for concrete structures," A. Nanni (Editor), Elsevier Science Publishers B.V, Netherlands, p. 28.
6. Bakoss, S., L., Gilbert, R., I., Faulkes, K., A., and Pulmano, V., A. (1982) "Long-term deflections of reinforced concrete beams," Magazine of Concrete Research, Vol.34, No. 121, pp. 203-212.

7. Bazant, Z. P. (1972) "Prediction of concrete creep effects using age-adjusted effective modulus," ACI Journal, vol. 69, pp 212-217.
8. Brown, V., L., and Bartholomew, C., L. (1996) "Long-term deflections of GFRP-reinforced concrete beams," Proc. ICCI'96, Saadatmanesh, H. and Ehsani, M. R. (Editors), Tucson, AZ. PP.389-400.
9. Carasquillo, R., L., Nilson, A., H., and Slato, F., O. (1981) " Properties of high strength concrete subject to short-term loads, " ACI Journal, Vol. 78, No. 3, May-June, pp 171-178.
10. Collins, M., and Mitchell, D. (1991) " Prestressed Concrete Structures," Prentice-Hall, Inc. New Jersey, p. 63.
11. Collins, M., Mitchell, D., Felber, A., and Kuchma, D. (1990) "Response Version 1.0" Prentice-Hall, Inc., New Jersey.
12. CSA (1994) "CSA A23.3-94 Design of Concrete Structures," Canadian Standard Association, Rexdale (Toronto), Ontario, Canada.
13. El-Nawawy, O. (1997) "A study on the use of GFRP rebars in concrete buildings in the Arab world: bond and splices," International conference on engineering materials, Ottawa, Canada, pp.287-302
14. Erki, M. A., Johnson, G., E., Roll, F., Marran, K., and Sleeth, D. (1992) "Creep relaxation characteristics of a prestressed Kevlar cable/ FRP tube structural system," Proc. ACMBS-I (Neale and Labossiere Edit.) Sherbrooke,

Canadian Society for Civil Engineering, Montreal, pp. 445-454.

15. Faza S. S. (1991) "Bending and bond behaviour and design of concrete beams reinforced with fibre reinforced plastic rebars". Ph.D. dissertation, Department of Civil Engineering, West Virginia University, p. 115.
16. Faza, S. S., and GangaRao, H. (1991) "Pre- and post-cracking deflection behaviour of concrete beams reinforced with fibre-reinforced plastic rebars," Proc. ACMBS-I (Neale and Labossiere Edit.) Sherbrooke, Canadian Society for Civil Engineering, Montreal, pp. 151-160.
17. Fenner, R. T. (1974) "Computing for Engineers," The Macmillan Press LTD, London, pp 9-51
18. Fordyce, M. W. and Wodehouse, R. G. (1983) "GRC and Buildings," Butterworth and Co. Ltd., London.
19. Ghali, A. and Favre R. (1986) "Concrete Structures: Stresses and Deformations," J. W. Arrowsmith Ltd., Bristol, 352 pp.
20. Gilbert, R. I. (1988) "Time Effects in Concrete Structures," Elsevier science publisher B. V. , Netherlands, Vol.23., 321 pp.
21. Glanville, W. H. (1930) "Studies in reinforced concrete-III, the creep of flow of concrete under load," Building Research Technical paper No. 12, Department of Scientific and Industrial Research, London, p. 39.

22. Graham, Ian (1991), "Object Oriented Methods" Addison- Wesley Publishing Company, Inc. , U.K, pp.3-23.
23. Hattori, A., Inoue, S., Miyagawa, T. and Fujii, M. (1995) "A study on bond creep behaviour of FRP rebars embedded in concrete," Proceedings of Second International RILEM Symposium, Taerwe, L. (Editor), Ghent, E&FN SPON, London, pp. 172-179.
24. Hilsdorf, H.K., and Muller, H. S. (1979) "Comparison of Methods to Predict Time-Dependent Strains of Concrete," Institut fur Baustofftechnologie, Universitat Karlsruhe (TH), p. 91.
25. Hognestad , E. (1951) "A study of combined bending and axial load in reinforced concrete members," University of Illinois Engineering Experimental Station, bulletin series No. 399, p.128.
26. Jerrett, C., V., and Ahmed, S., H. (1995) "Bond tests of carbon fibre reinforced plastic rods," Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp.180-191.
27. Kage, T. , Masuda, Y., Tanano, Y. and Sato, K. (1995) "Long-term deflection of continuous fiber reinforced concrete beams," Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp. 251-258.

28. Keer, J.,G., and Hughes, D.,C (1982) "Polypropylene-reinforced cement sheeting for use as roofing and cladding elements," Proc. of 4th International conference on composite Materials, ICCM-IV, Vol. 2, pp. 1255-1262.
29. Keesler,R., J., and Powers, R., G. (1988) "Corrosion of epoxy-coated rebar-keys segmental bridge-Monroe County," Report No. 88-8a, Florida Department of Transportation, Materials Office, Gainesville, Florida.
30. Kawakami, M., Tokuda, H., Kagaya, M., Kobayashi, T. and Hayashi, T. (1992) "Mechanical behaviour of silica fume mortar slabs reinforced by AFRP and CFRP rods," JCA proceedings of cement & concrete, No.46, pp. 808-814.
31. Kobayashi,K. , Fukuda, M.,Fujisaki,T. and Kanjo, T. (1988) " Studies on usage of fiber reinforced composite material (NFM) to architectural structures. compressive strength of NFM," summaries of technical papers of annual meeting AIJ, pp. 815-816.
32. Machida, A. (Editor) (1993) " State of the art report on continuous fiber reinforcing materials," Research committee on continuous fiber reinforcing materials , JSCE (Japan Society of Civil Engineers), Tokyo., Japan.
33. Naaman, A.E. (1982) "Prestressed Concrete Analysis and Design- Fundamentals," McGraw-Hill Book Company, New York.
34. Nanny, A., Al-Zaharani, M. M., Al-Dulaijan,S.,U., Bakis, C., E., and

- Boothby, T. E. (1995) "Bond of FRP reinforcement to concrete-experimental results," Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp. 135-145.
35. Nawy, E. G. (1990) "Reinforced Concrete, a fundamental approach," 2nd Edition, Prentice hall, New Jersey.
36. Park, R., and Paulay, T. (1975) "Reinforced Concrete Structures," John Wiley & Sons, New York, NY, pp. 30-264.
37. Pillai, S. U. and Kirk, D. W. (1988) "Reinforced Concrete Design in Canada," McGraw-Hill Ryerson Limited, Toronto, pp.123-124.
38. Portland Cement Association (1979) "Design and control of concrete mixtures," 12th edition, PCA, Skokie, Ill., p.140.
39. Rahman, A., H., Kingsley, C., Y., and Crimi, J. (1995) "Behaviour of FRP grid reinforcement for concrete under sustained load," Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp. 190-99.
40. Rasheeduzzafar, A., Dakhil, F., Bada, M. and Khan, M. (1992) "Performance of corrosion-reinforcing steels in chloride-bearing concrete," ACI Materials Journal, Vol. 89, No. 5 pp. 439-448.
41. Razaqpur, A. G. (1997), Private Communication, Department of Civil and Environmental Engineering, Carleton University, Ottawa.

42. Roper, H. and Bott, G. (1979) "Recent concepts- their influence on code predictions of shrinkage and creep," School of Civil Engineering, University of Sydney, p.19
43. Saadatmanesh, H. and Ehsani, M. (Editors) (1996) "Fiber composites in Infrastructure: ICCI '96," Proc. 1st International conference on composites in infrastructure, Tucson, pp. 389-400.
44. Saadatmanesh H., and Ehsani, M. R. (1991) "RC beams strengthened with GFRP plates I: Experimental study," ASCE Journal of Structural Engineering, Vol. 117.
45. Schmeckpeper, E., R. (1992) " Performance of concrete beams and slabs reinforced with FRP grids," Ph.D. Dissertation, Department of Civil Engineering, University of New Hampshire.
46. Schneider, D. I. (1995) "An Introduction to Programming Using Visual Basic." Prentice-Hall, Inc., Englewood Cliffs, New Jersey
47. Somboonsong, W. , Ko, F. K. and Harris, H. G. (1996) " Ductile hybrid Fiber Reinforced Plastic (FRP) rebar for concrete structures: Design methodology," ACI Materials Journal, American Concrete Institute, Farmington Hills, MI..
48. Soudki, K., A. Stierner, R. M., and Green, M., F. (1997) " Bond of CFRP rods to concrete: low temperature durability," International Conference on

Engineering Materials, (Al-Manaseer, Nagataki and Joshi Editors), Ottawa, Canada, pp. 303-310.

49. Spencer, D. D. "Concepts and Programming Guide for Engineers," Howard W. Sons & Co., Inc., Indiana, USA, pp 88-89.
50. Tezuka, M., Ochiai M., Tottori, S. and Sato, R. (1995) " Experimental study on moment redistribution of continuous beams reinforced or pretensioned with fibre reinforced plastics." Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp387-394.
51. Thakurta, A. G., Abdelrahman, A. A. and Rizkalla, S. H. (1994) "First smart bridge in Canada," Developments in short and medium span bridge engineering , The Canadian Society for Civil Engineering, Halifax, Nova Scotia, August pp. 859-870.
52. Uomoto, T. , Nishimura, T. and Ohga, H. (1995) " Static and fatigue strength of FRP rods for concrete reinforcement," Proceedings of Second International RILEM Symposium, Taerwe, L., (Editor), Ghent, E&FN SPON, London, pp. 100-107.
53. Whitney, C. (1932) "Plain and reinforced concrete arches," ACI Journal, Vol. 28, pp. 479-519.

54. Wilson, L., B., and Clark, R. (1988) “ Comparative Programming Languages,” Addison- Wesley Publishing Co., U.K, pp. 49.

IMAGE EVALUATION TEST TARGET (QA-3)

APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved