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MODELING IMPLICATIONS OF CHANNEL LENGTH
REDUCTION IN MOS TRANSISTORS

by

David Hall Harper

A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree of

Doctor of Philosophy

Department of Electronics
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Carleton University
Ottawa, Canada
October, 1980

The undersigned recommend to the Faculty of Graduate Studies and
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ABSTRACT

The miniaturization of the MOST for very large scale integrated circuits, poses a challenge in both design and modeling. Rules for scaling down the MOST and their shortcomings are discussed. The implications of channel length reduction to MOST modeling are further examined.

This report contains a versatile, gradual channel, MOST model suitable for computer-aided-design application. Diffusion current is considered and the surface mobility dependence on the normal (oxide) field is properly included as a component of the basic current continuity relation. All derived conductances and capacitances are smooth functions of the terminal voltages throughout all modes of device operation including subthreshold.

The double-diffused MOST (DMOST) also demonstrates short channel characteristics. To investigate the internal operation of this device, a simple numerical model is presented. The operating mechanisms of the DMOST and the short channel MOST are found to be qualitatively similar.

Lastly, a novel short channel MOST modeling concept is evolved and a device model is further developed. This analysis captures several short channel electrostatic mechanisms within one electrostatic relation. In addition, this model accounts for the non-uniform doping present in a typical ion-implanted MOST and the surface mobility dependence on both normal and lateral electric fields.

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x

LIST OF ABBREVIATIONS

CAD	Computer-aided-design
CHE	Channel hot electron
CPU	Central processing unit
DIBL	Drain induced barrier lowering
DMOST	Double-diffused MOST
GCA	Gradual channel approximation
IC	Integrated circuit
LHS	Left hand side
LSI	Large scale integration
MOST	Metal oxide semiconductor transistor
RHS	Right hand side
SCMOST	Short channel MOST
SHE	Substrate hot electron
VLSI	Very large scale integration
VMOST	V-groove MOST
1-D	One dimensional
2-D	Two dimensional
3-D	Three dimensional

LIST OF SYMBOLS

C	Capacitance
C_g	Total gate capacitance
C_{gb}	Gate to substrate capacitance
C_{gd}	Gate to drain capacitance
C_{gs}	Gate to source capacitance
C_{jt}	Total junction capacitance
C_{ox}	Oxide capacitance per unit area
C_{oxt}	Total oxide capacitance of gate area
D	Diffusion coefficient
E	Electric field in y direction
E_c	Critical electric field
E_d	Surface electric field arising from the depletion charge
E_{ox}	Electric field across the oxide
E_s	Gate induced surface field
$\overline{E_{sd}}$	Average source to drain electric field
g_m	Transconductance
g_d	Drain conductance
g_s	Source conductance
I_d	Drain current
J_c	Current per unit channel width
J_{drift}	Drift current per unit channel width
$J_{diffusion}$	Diffusion current per unit channel width
J_n	Current density of electrons per unit area
k	Boltzmann's constant

LIST OF SYMBOLS (cont'd)

L	Channel length
L_e	Extrinsic debye length
L_g	Gate length
L_i	Intrinsic debye length
n	Electron density per unit volume
N	Impurity density of a uniformly doped substrate
N_c	Substrate impurity density at the silicon surface
n_i	Intrinsic carrier concentration
N_{ss}	Surface state density
P	Power
q	Electron charge
Q_m	Absolute value of mobile channel charge density
Q_d	Absolute value of depletion charge density
Q_{eff}	Effective oxide and interface charge density
Q_g	Charge density on the gate
T	Temperature
t_{ox}	Gate oxide thickness
T_p	Propagation delay
V_a	Externally applied potential
V_{bi}	Built-in potential
V_{ds}	Externally applied drain to source potential
V_{fb}	Flatband voltage
V_g	Effective gate potential
V_{gs}	Externally applied gate to source potential

LIST OF SYMBOLS (cont'd)

v_s	Saturated drift velocity
V_s	Surface potential
V_b	Internal substrate potential
V_{sb}	Externally applied source to substrate potential
V_t	Threshold voltage
W	Channel width
x_d	Depletion layer width
β	Gain constant
ϵ_{ox}	Permittivity of gate oxide
ϵ_{si}	Permittivity of silicon
λ	Integrating factor
μ	Mobility (low field unless otherwise designated)
μ_0	Low field mobility
ϕ_f	Fermi voltage
ϕ_{MS}	Metal semiconductor work function
ϕ_n	Quasi fermi voltage of electrons
ϕ_p	Quasi fermi voltage of holes

CHAPTER 1

INTRODUCTION

Over the past decade, the evolution of integrated circuit (IC) technology from small through medium to large scale integration (LSI) has steadily reduced the metal oxide semiconductor transistor (MOST) in size. At this time, n-channel, silicon gate MOSTs with 3.0 to 5.0 μm channel lengths are well within the fabrication capabilities of many commercial semiconductor manufacturers. Very large scale integration (VLSI) is the next step in the evolutionary ladder. In quantitative terms, VLSI refers to single integrated circuits containing more than 10^4 gates. It is also associated with the development of 64K random access memories and the 16 bit microprocessor. In terms of the MOST, VLSI will require 1.0 to 3.0 μm channel lengths in order to obtain the required circuit density. Submicrometer channel lengths are predicted for the future.

With new technological capabilities at arms reach and an identified need to advance existing state-of-the-art MOST technology, VLSI appears imminent. The steady improvement in yield and reliability have also made more complex integrated circuits economically feasible. For instance, lithographic techniques like one-to-one projection, and step and repeat reduction projection have virtually eliminated the circuit damage previously caused by conventional contact and proximity printing methods. In addition,

lithographic resolution to 1.0 μm line widths is now reproducibly achievable using deep ultra-violet and X-ray exposure techniques.

A further order of magnitude improvement in lithographic resolution can be obtained with direct electron beam writing methods. For a typical wafer however, the electron beam writing time is so long (usually several hours) that this method is only practical for mask making at this time. The use of ion-implantation instead of the conventional predeposition and drive-in allows much improved control of impurity concentrations. It also serves to minimize the number of high temperature fabrication steps and thus prevents migration of impurities in unwanted directions. Therefore it is possible to more accurately delineate source and drain junctions.

It is even possible to implant through oxide and silicon layers for threshold voltage tailoring or implant beneath the silicon surface for substrate doping adjustment. New etching techniques like RF plasma or reactive ion etching are capable of producing etch patterns with virtually none of the undercutting that is so difficult to control with conventional wet chemical etch methods. The proliferation of various IC design aids, circuit simulators and even fabrication modeling programs has substantially improved the efficiency of the IC design process. For VLSI, such tools become an absolute necessity. To a large extent, the usefulness of circuit simulators depends on the accuracy of the device models employed. This report is concerned with modeling of the short channel MOST (SCMOST).

The SCMOST for VLSI poses a challenge in both design and modeling. In order to maintain acceptable device characteristics,

the MOST must be reduced in size according to certain prescribed scaling rules. These rules and their shortcomings are discussed in chapter 2. Several mechanisms occur in SCMOSTs which are either unimportant or just do not happen in longer channel devices. These mechanisms which arise from channel length reduction are discussed in chapter 3. In this light, chapter 4 appraises existing SCMOST models. For the purpose of this report, it was necessary to investigate all analytic methods applicable to MOST modeling. A general overview of mathematical methods used for this purpose is presented in chapter 5 along with conclusions regarding the limitations of these methods.

Even with the steady infiltration of short channel devices into IC design, gradual channel theory still remains the backbone of MOST modeling efforts. With this fact in mind, a versatile MOST model based on gradual channel theory is presented in chapter 6. This analysis incorporates several refinements such as subthreshold current and the surface mobility dependence upon the normal electric field into a unified, mathematically self-consistent device model. Short channel MOST characteristics occur in the double-diffused MOST. To demonstrate the operating mechanisms of this device, a simple numerical model is presented in chapter 7. The operating mechanisms of the SCMOST and the DMOST are found to be surprisingly similar. Results and conclusions are presented which have not been disclosed by other authors. A SCMOST model is presented in chapter 8 which is representative of the device internal operation rather than just its terminal characteristics. A novel SCMOST model concept is developed which captures several short channel mechanisms within

one electrostatic relation. Numerical techniques are employed where analytic methods prove inadequate for SCMOST modeling purposes. The analysis also accounts for the non-uniform doping present in a typical ion-implanted MOST.

CHAPTER 2

SCALING THE MOST

In order to gain full advantage of higher resolution lithographic capabilities, the IC design engineer must come to grips with the problem of reducing the conventional MOST in size, without creating undesirable device characteristics in the process. Several techniques have been used in the past to accomplish this purpose. Increasing the substrate doping for instance, serves to reduce depletion layer widths in all directions. Reducing the oxide thickness yields tighter control of the surface potential with respect to the gate and at the same time minimizes lateral fringing field effects. To this end, a prescribed method to scale down the MOST has been put forward [1,2]. This scaling method, its implications and possible limitations will be discussed briefly.

A state-of-the-art, self-aligned, n-channel MOST typical of conventional fabrication and photolithographic techniques is illustrated in figure 2.1a. Presumably, this device is already optimized to minimize undesirable effects such as capacitances, high conductance in saturation, avalanche multiplication, several drain voltage breakdown mechanisms, punch-through and hot electron effects. The maximum drain voltage is typically ten volts and to reduce the threshold voltage variation with backbias, a substrate voltage of typically five volts is applied.

The scaling method consists simply of reducing all linear dimensions and bias levels by scaling parameter $K > 1$ while increasing

Figure 2.1 Demonstration of scaling for $K = 5$
 (a) Commercially available n-channel MOST for LSI
 (b) Scaled device structure for VLSI

the substrate doping in proportion to K. In the following, primed values refer to the scaled device while unprimed parameters refer to the original MOST. Specifically, the scaling rules are; $t'_{ox} = t_{ox}/K$, $L' = L/K$, $W' = W/K$, $x'_j = x_j/K$, $V'_{ds} = V_{ds}/K$, $V'_{gs} = V_{gs}/K$, $V'_{sb} = V_{sb}/K$ and $N' = KN$. A scaled version of the MOST illustrated in figure 2.1a, is shown in figure 2.1b for $K = 5$.

By direct application of these scaling rules, threshold voltage of the scaled device becomes

$$V'_t = (2\phi_f)' + \phi'_{MS} + \frac{\left(\frac{t_{ox}}{K}\right)}{\epsilon_{ox}} \left\{ -Q_{eff} + \sqrt{2q\epsilon_{si} (KN) \left[(2\phi_f)' + \left(\frac{V_{sb}}{K}\right) \right]} \right\} \approx \frac{V_t}{K} \quad (2.1)$$

where V_{sb} is the externally applied source to substrate bias, hence $V_{sb} > 0$. Assuming one dimensional (1-D) abrupt junction theory for illustrative purposes, the depletion layer width about the source and drain junctions are reduced as

$$W'_d = \left\{ \frac{2\epsilon_{si} \left[V'_{bi} + \left(\frac{V_{sb}}{K}\right) + \left(\frac{V_a}{K}\right) \right]}{q(KN)} \right\}^{1/2} \approx \frac{W_d}{K} \quad (2.2)$$

where $V_a = 0$ at the source and $V_a = V_{ds}$ at the drain. Similarly, the surface depletion layer width is also reduced by approximately K.

The conventional MOST drain current equation which assumes the gradual channel approximation (GCA) and takes into account substrate doping and lateral field dependant mobility, scales as follows.

$$L'_{eff} = \left[\left(\frac{L}{K}\right) + \left(\frac{V_{ds}}{K}\right) \frac{\mu}{v_s} \right] = \frac{L_{eff}}{K} \quad (2.3)$$

$$\beta' = \frac{\mu \epsilon_{ox} \left(\frac{W}{K}\right)}{\left(\frac{L_{eff}}{K}\right) \left(\frac{t_{ox}}{K}\right)} = K\beta \quad (2.4)$$

$$\alpha' = \frac{\left(\frac{t_{ox}}{K}\right)}{\epsilon_{ox}} \left[2q \epsilon_{si} (KN) \right]^{\frac{1}{2}} = \frac{\alpha}{\sqrt{K}} \quad (2.5)$$

Hence,

$$\begin{aligned} I'_d &= K\beta \left[\left(\frac{V_{gs}}{K}\right) + \frac{Q_{eff}}{\epsilon_{ox}} \left(\frac{t_{ox}}{K}\right) - \phi'_{MS} - (2\phi_f)' - \left(\frac{V_{ds}}{2}\right) \right] \left(\frac{V_{ds}}{K}\right) \\ &\quad - \frac{2}{3} \frac{\alpha}{\sqrt{K}} \left[\left(\frac{V_{ds}}{K}\right) + (2\phi_f)' + \left(\frac{V_{sb}}{K}\right) \right]^{\frac{3}{2}} \\ &\quad + \frac{2}{3} \frac{\alpha}{\sqrt{K}} \left[(2\phi_f)' + \left(\frac{V_{sb}}{K}\right) \right]^{\frac{3}{2}} \\ \frac{I_d}{K} & \end{aligned} \quad (2.6)$$

and so the drain current is also reduced by approximately K although the actual surface current density remains unaltered.

$$J'_c = \frac{I_d}{W'} = \frac{\left(\frac{I_d}{K}\right)}{\left(\frac{W}{K}\right)} = J_c \quad (2.7)$$

In equations (2.1), (2.2) and (2.6), $(2\phi_f - \phi_{MS})$ and V_{bi} actually increase with scaling by $\frac{KT}{q} \ln(K)$ while $(2\phi_f)$ increases by $2 \frac{KT}{q} \ln(K)$ and thus these quantities do not scale. This can be approximately compensated for by further adjustment of V_{sb} for example, to satisfy $V'_{sb} + (2\phi_f)' = (V_{sb} + 2\phi_f)/K$ provided that the resultant $V'_{sb} \geq 0$.

The power dissipation per MOST is reduced according to

$$P' = \left(\frac{V_{ds}}{K} \right) \left(\frac{I_d}{K} \right) = \frac{P}{K^2} \quad (2.8)$$

while the power dissipation per unit chip area remains constant since the device area has also been reduced by K^2 . The capacitance of source and drain diffusions to the substrate are also reduced via

$$C'_{jt} = \left\{ \frac{q \epsilon_{si} (KN)}{2 \left[V'_{bi} + \left(\frac{V_{sb}}{K} \right) + \left(\frac{V_a}{K} \right) \right]} \right\}^{\frac{1}{2}} \cdot \left(\frac{\text{area}}{K^2} \right) = \frac{C_{jt}}{K} \quad (2.9)$$

Similarly the total gate oxide capacitance is also reduced.

$$C'_{ox} = \frac{\epsilon_{ox}}{\left(\frac{t_{ox}}{K} \right)} \left(\frac{L}{K} \right) \left(\frac{W}{K} \right) = \frac{C_{ox}}{K} \quad (2.10)$$

For the simple case in which only the MOST is considered, the delay time of the scaled device is

$$T'_p = \frac{\left(\frac{C}{K} \right) \left(\frac{V}{K} \right)}{\left(\frac{I_d}{K} \right)} = \frac{T_p}{K} \quad (2.11)$$

Hence, the power delay product is

$$T'_p \cdot P' = \left(\frac{T_p}{K} \right) \left(\frac{P}{K^2} \right) = \frac{T_p \cdot P}{K^3} \quad (2.12)$$

Therefore much is to be gained by shrinking MOST dimensions.

The previous analysis also indicates that conventional MOST theory should remain applicable. There are, however, several mechanisms which do not scale in proportion to K such as the maximum electric field at the source and drain junctions.

$$E'_{\max} = \left\{ \frac{2q (KN) \left[V'_{bi} + \left(\frac{V_{sb}}{K} \right) + \left(\frac{V_a}{K} \right) \right]}{\epsilon_{si}} \right\}^{\frac{1}{2}} \approx E_{\max} \quad (2.13)$$

The average lateral electric field from source to drain

$$\overline{E'_{sd}} = \frac{\left(\frac{V_{ds}}{K}\right)}{\left(\frac{L}{K}\right)} = \overline{E_{sd}} \quad (2.14)$$

The electric field across the oxide

$$E'_{ox} = \frac{\left(\frac{V_{gs}}{K}\right) - \left(\frac{V_s}{K}\right)}{\left(\frac{t_{ox}}{K}\right)} = E_{ox} \quad (2.15)$$

where V_s refers to the surface potential at the point of interest along the channel. So even with scaled bias levels, the problems associated with high electric fields such as breakdown mechanisms, avalanche multiplication, hot electron effects and punch-through are certainly not diminished.

In subthreshold, the logarithmic slope of drain current with respect to gate voltage does not scale as desired.

$$S' = \frac{KT}{q} \ln(10) \left[1 + \frac{\left(\frac{Cd}{K}\right)}{\left(\frac{C_{ox}}{K}\right)} \right] = S \quad (2.16)$$

where Cd refers to the surface depletion capacitance. This nonscaling property of subthreshold current is of concern to dynamic MOS memories which require very low subthreshold source to drain leakage. Presuming the unscaled device threshold voltage has been optimized for maximum $(V_{gs} - V_t)$ and low subthreshold leakage at $V_{gs} = 0$ volts, then relation (2.16) implies this threshold voltage value will have to be maintained in the scaled device in order to achieve the same level of subthreshold leakage at $V'_{gs} = 0$ volts. Unfortunately, this further restricts the maximum $(V'_{gs} - V_t)$ available to drive the scaled device.

The extrinsic Debye length is often used as a measure of the distance over which a significant variation in potential can occur.

$$L'_e = \left[\frac{\epsilon_{si} kT}{q^2 (KN)} \right]^{1/2} = \frac{L_e}{\sqrt{K}} \quad (2.17)$$

Therefore the extrinsic Debye length does not scale in proportion to the device dimensions. This consideration plus the fact that V_{bi} , $2\phi_f$ and the subthreshold current do not scale at all point to substantial difficulty in the range of submicron size channel lengths where the maximum allowable bias levels will be less than one volt.

Thus far it has been assumed that all process tolerances, vertical dimensions and applied potentials would be scaled in proportion to the surface dimensions. This assumption, however, is certainly open to question. For instance, can gate oxide with thicknesses in the range of 20.0 nm or less be made in a manufacturing environment without an excessive increase in pin holes and other defects which severely degrade yield and reliability? Oxide thickness tolerances will now be in the range of 1.0 nm. Beyond these considerations, there is a lower limit to practical gate oxide thickness. Direct tunnelling of carriers through the SiO_2 barrier occurs for oxide thickness below 5.0 nm [3]. Some authors [3, 4] propose more conservative limits of minimum practical oxide thickness in the range of $t_{ox} = 9.4$ to 14.0 nm from oxide breakdown considerations:

Junction depths of 20.0 nm or less preclude the use of the standard phosphorous predeposition and drive-in, or phosphorous glass deposition and drive-in commonly used in n-channel, silicon gate, MOS technology. For this purpose, arsenic is a viable alternative. To minimize the number of high temperature processing steps, it will

require ion-implantation. There is the question of how shallow a diffusion can be made before contact resistance and sheet resistances become a source of difficulty.

Polysilicon as a fine line interconnect material might also have limited usefulness for VLSI due to its low conductivity. An alternative is to replace all polysilicon gates and interconnects with a composite layer consisting of doped polysilicon covered with a polycide such as MoSi_2 or WSi_2 . Such an approach offers an order of magnitude improvement in interconnect conductivity and appears compatible with existing silicon VLSI technology [5].

The tolerance on threshold voltage also reduces with scaling. To gain adequate control of the threshold voltage, ion-implantation of the channel will become a necessity.

The scaling of all potentials implies an equivalent reduction in signal voltages and noise margins. Without launching into a detailed discussion of noise, it suffices to say that the noise associated with interconnect resistances and the significant noise mechanisms associated with MOST operation do not scale.

In an effort to satisfy the aspirations of very large scale integration it seems reasonable to assume that compromises will be made between device performance and technological capability. Compromises of this kind can only serve to accentuate the problems pertinent to the short channel MOS transistor.

CHAPTER 3

THE CONSEQUENCES OF CHANNEL LENGTH REDUCTION

New high resolution lithographic techniques make possible considerable shrinking of MOSTs and integrated circuitry. With these new capabilities at arms reach, and an increasing awareness of the numerous advantages and applications of VLSI, the IC design engineer is strongly motivated to scale down the MOST.

There is doubt however, as to whether the scaling rules discussed in the previous chapter, will be or can be rigidly adhered to. The nonscaling properties of subthreshold current and noise for example, may make the scaling of applied potentials in accordance with other parameters of the device, impractical. These plus other nonscaling mechanisms may require the MOST operating voltages to be higher than those suggested by the scaling rules. Moreover, in an effort to achieve the highest possible circuit density and to fulfill all the expectations of VLSI, it is more than likely that the MOST will be pushed to its limits of operation.

Again the IC design engineer is confronted with several trade-offs. Increasing the bulk or implanted substrate doping and decreasing the junction depth and oxide thickness serves to contain potential drops to localized regions within the MOST. Tighter control of the surface potential with respect to the gate is also achieved. Unfortunately, high potential drops over small dimensions give rise to very high electric fields, associated breakdown

mechanisms and hot electron effects.

On the other hand, decreasing the substrate doping while increasing the oxide thickness and junction depth allows voltage drops to occur over larger regions of the MOST. As a result, maximum electric field strengths are reduced. However, the depletion regions surrounding the source and drain widen and can even merge. The tight control of surface potential by the gate is diminished. Lateral fringing fields penetrate well into the channel from both source and drain regions. The channel potential now becomes a complicated two dimensional (2-D) electrostatics problem. The position and value of the potential minimum in the channel is now influenced not only by the gate but also by the drain potential. Since this potential minimum is the major barrier to current flow in a SCMOST, its behaviour with respect to bias conditions strongly influences the device terminal characteristics including the threshold voltage. Thus the threshold voltage becomes a function of drain voltage as well as gate voltage. The lowering of the channel potential minimum with drain voltage is often referred to as drain induced barrier lowering (DIBL) [6]. This will be discussed in detail in section 2 of this chapter. Another mechanism which results from the merging of source and drain depletion regions is punch-through. This mechanism is characterized by the formation of a subsurface channel and subsequent subsurface current flow.

3.1 Hot Carrier Effects

Hot electrons can arise in regions of an n-channel MOST where

electric fields reach approximately 10^5 V cm^{-1} . Such carriers have enough energy to surmount the SiO_2 energy barrier. When a sufficient number of hot carriers are involved, a measurable gate current can result. This mechanism is used to advantage in some reprogrammable read-only memories. For logic circuitry, this small gate current is usually of little concern. However, the subsequent trapping of emitted hot electrons in the oxide causes an undesirable transconductance degradation and/or a permanent positive shift in threshold voltage.

There are two important hot electron effects which occur in n-channel MOSTs. These are the substrate hot electron (SHE) effect where electrons originating from the substrate become 'hot' in transit to the silicon surface, and the channel hot electron (CHE) effect where the primary source of electrons is the surface channel current carriers [7].

The electrons which cause the SHE effect, originate from the edge of the surface depletion region as diffusion current. While initially under the influence of low electric fields, the electrons undergo elastic scattering with acoustical phonons with the result that the low field, constant mobility, drift velocity relation $v_d = \mu \cdot E$ applies. When an electron reaches a point in the surface depletion layer where the electric field is approximately 10^4 V cm^{-1} , optical phonon emission and inelastic scattering processes prevail. This limits the average electron energy and saturated drift velocity results. When the field reaches approximately 10^5 V cm^{-1} , electrons gain more energy between scattering events than they lose when scattered. These electrons are no longer in equilibrium with the

crystal lattice. Their energy with respect to the conduction band edge increases. For electrons under the influence of electric fields slightly greater than 10^5 V cm^{-1} , impact ionization becomes an additional energy loss mechanism.

Emission of electrons from the silicon over the SiO_2 barrier is assisted by another mechanism which can be explained with reference to figure 3.1. Due to the high field normal to the silicon surface, the SiO_2 barrier is virtually reduced to zero for electrons at point X_c . Note that high electric fields at the surface are necessary to keep X_c small. The probability that an electron at X_c can reach the surface without undergoing a collision is given by the 'lucky' electron emission model [7] as

$$P_r = A \exp(-X_c/\lambda) \quad (3.1)$$

where A is a normalization constant, and λ is related to the optical phonon scattering mean free path λ_p , and the impact ionization mean free path λ_i by

$$\frac{1}{\lambda} = \frac{1}{\lambda_i} + \frac{1}{\lambda_p} \quad (3.2)$$

For electron emission, one can conclude that a high surface field is required in order to keep X_c small and the resultant emission probability high. This simple logic however, ignores several important factors. It neglects Schottky lowering of the potential peak and tunnelling through the peak which both serve to enhance emission. It also ignores the fact that electrons closer to the surface than X_c have a good probability of surmounting the SiO_2 barrier simply because they are 'hot' with energy greater than the conduction band edge. For such hot carriers, the requirement of a very high surface field is not as great.

Figure 3.1 Band diagram for demonstration of hot electron emission. ϕ_b is the barrier height at the silicon surface.

It is perhaps interesting, at this point, to estimate the maximum surface electric field in the scaled device illustrated in figure 2.1b. In keeping with the suggested scaled bias levels, the maximum surface to substrate potential is $(2\phi_f)' + V_{sb}' + V_{ds}' = (0.7 + 1.0 + 2.0) = 3.7$ volts. Using the standard 1-D calculation, the surface electric field is $1.7 \times 10^5 \text{ V cm}^{-1}$. This field strength is sufficient for the generation and emission of hot electrons.

In the case of CHEs, there are two conditions which favour emission over the SiO_2 energy barrier. Firstly, the MOST bias conditions must be such that CHEs are generated. In long channel MOSTs, this only occurs in devices with a high substrate doping, operating in strong saturation. For short channel devices with scaled up substrate doping and strong source to drain electric fields, CHEs are generated in all modes of device operation. The effect, however, is accentuated in saturation. The scaled down oxide thickness and radius of junction curvature also causes a concentration of electric field lines and attendant creation of CHEs. Secondly, the electric field across the gate oxide must not repel hot electrons from the gate. This implies that the gate voltage should be at least equal to or greater than the surface potential at the point along the channel where emission is likely. In the linear mode of operation, this condition is fulfilled along the entire length of the channel. In saturation however, the surface field is reversed at the drain where the 'hottest' electrons exist. Consequently, the point of peak emission is some distance away from the drain where the combination of attractive gate field and CHE generation is most favourable.

There are other factors relevant to high fields in MOSTs worth mention. Considering the scaled device of figure 2.1b, a crude estimate of the average lateral source to drain electric field is

$$\overline{E}_{sd} = \frac{V_{ds}}{L} = \frac{2V}{10^{-4} \text{ cm}} = 2 \times 10^4 \text{ V cm}^{-1} \quad (3.3)$$

Using typical values, the critical field at which drift velocity saturates is

$$E_c = \frac{v_s}{\mu_o} = \frac{6.5 \times 10^6 \text{ cm sec}^{-1}}{650 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}} = 10^4 \text{ V cm}^{-1} \quad (3.4)$$

This implies that at maximum bias levels, one can expect carrier transport to be limited by saturated drift velocity over a major portion of the MOST channel. Borrowing from conventional MOST theory [8], the effective channel length is $L_{eff} = L + \Delta L$ where

$$\Delta L = \frac{V_{ds}}{E_c} = \frac{2.0 \text{ V}}{10^4 \text{ V cm}^{-1}} = 2.0 \text{ } \mu\text{m} \quad (3.5)$$

So even if the actual channel length were reduced to submicrometer dimensions, the terminal characteristics would be indicative of a device with $L > 2.0 \text{ } \mu\text{m}$. Similarly with $V_{ds} = 1.0 \text{ volt}$, the effective channel length would be at least $1.0 \text{ } \mu\text{m}$ due to the effect of saturated drift velocity.

The creation of SHEs and CHEs together with the probable occurrence of impact ionization is another reason for concern particularly in scaled MOSTs. Within the silicon, any n-channel MOST forms a lateral npn bipolar structure. In long channel devices, the p-region is sufficiently long that the gain of this parasitic bipolar transistor is much less than unity. Unfortunately, the prime objective of

scaling is to reduce the length of the p-region as much as possible, and thereby significantly increase the beta of this parasitic bipolar. Holes created by impact ionization and further increased in numbers by avalanche multiplication will be attracted to the negatively biased substrate contact. If conductivity through the substrate is poor, so that the potential of the substrate in the vicinity of the channel is not sufficiently reverse biased with respect to the source, parasitic bipolar action can occur. If it does, then the surplus holes directly contribute to the base current. A dramatic increase in source to drain current then results. A family of drain current versus drain voltage curves for a VMOST showing evidence of parasitic bipolar action [9] is shown in figure 3.2. One could conclude that in order to suppress this effect, it is necessary to maintain a reasonable substrate bias and/or start with a substrate of sufficiently good conductivity.

3.2 The Lateral Variation of Depletion Charge

The depletion charge as 'seen' from the silicon surface varies from source to drain in all SCMOSTs in the following manner. It has a maximum somewhere near the middle of the channel and decreases near both the drain and source. This realization has significant implications to the basic mechanisms associated with current flow in SCMOSTs. After all, the constraints imposed by charge neutrality and current continuity are interrelated even in conventional MOST theory.

This lateral variation in depletion charge occurs for several reasons. It can occur due to a direct variation in substrate doping laterally from source to drain. For instance, the lateral diffusion of

Figure 3.2 I_d versus V_{ds} characteristics of an implanted VMOST used for PROM applications showing parasitic bipolar action [9].

source and drain impurities during high temperature fabrication processes serves to compensate the substrate impurities near the source and drain metallurgical junctions.

In another class of MOSTs, the lateral variation in substrate doping is intentional. This class includes the double-diffused MOST (DMOST) and the V-groove MOST (VMOST) as illustrated in figures 3.3 and 3.4 respectively. These devices exhibit short channel characteristics as a direct result of the channel impurity profile. Furthermore, their channel doping profile is tailored to spread the majority of source to drain potential over a large portion of the channel and thereby minimize maximum electric field strength. Consequently, these devices find immediate application wherever short channel gain and/or high voltage operation is required. The DMOST has the advantage that it can be fabricated using conventional planar technology and photolithographic techniques but with the result that its gate length is long by VLSI standards (usually $> 10.0 \mu\text{m}$). When reduced in size, the DMOST is most commonly referred to as the diffusion self-aligned MOST (DSA MOST). Its voltage advantage places it in strong contention with the SCMOST for VLSI applications [10]. Like the SCMOST, this general class of devices is characterized by a lateral variation in depletion charge (as 'seen' from the silicon surface) with a maximum in the active portion of the channel. To first order approximation, this depletion charge is independent of the 2-D charge sharing mechanisms that plague the SCMOST. For this reason, the effects of this characteristic lateral depletion charge variation were initially investigated using the DMOST as a case in point [11]. These results will be discussed in chapter 7.

In SCMOSTs, a strong lateral variation in depletion charge

Figure 3.3 Cross sectional view of a typical DMOST.

Figure 3.4 Cross sectional view of a typical VMOST structure.

(as 'seen' from the silicon surface) occurs primarily due to the close proximity of the source and drain. In long channel devices, where the channel length is very much greater than an extrinsic Debye length, it is justifiable (moreover analytically convenient) to assume that all electric field lines which originate in the surface depletion region terminate on the gate. In SCMOSTs on the other hand, a major portion of electric field lines which originate in the depletion region terminate on the source and drain. Figures 3.5a and 3.5b illustrate this point for a MOST with a relatively long channel ($L = 5.0 \mu\text{m}$) by VLSI standards. In addition these figures lead one to question the validity of the long standing GCA used in virtually all field-effect transistor models. The GCA after all presupposes that all electric field lines are approximately normal to the silicon surface.

If the substrate doping is reduced to minimize hot carrier effects, charge sharing mechanisms worsen and DIBL is accentuated. Electric field lines penetrate far into the depletion region from both source and drain. Hence the depletion charge (as 'seen' from the silicon surface) is diminished toward both ends of the channel. For the case where $V_{ds} = 0$, the resulting surface potential is no longer constant in the lateral direction. Rather it has a potential minimum midway between the source and drain where the depletion charge (as 'seen' from the silicon surface) is a maximum. The value and position of this potential minimum changes with bias conditions. With increasing drain bias, this potential minimum or barrier moves toward the source. This characteristic of surface potential in SCMOSTs is illustrated in figure 3.6 and compared with the surface potential distribution of a longer channel length device.

Figure 3.5 Computer calculated potential distribution of a MOST with $N = 2 \times 10^{15} \text{ cm}^{-3}$, $t_{\text{ox}} = 100.0 \text{ nm}$, $L = 5.0 \text{ } \mu\text{m}$ and $V_{\text{gs}} = 3.75 \text{ V}$ [12]. In (a) $V_{\text{ds}} = 1.75 \text{ V}$ while in (b) $V_{\text{ds}} = 6.75 \text{ V}$. The electric field lines are drawn in as dashed lines.

Figure 3.6 Comparison of the surface potential distribution for $V_{\text{gs}} = 1.8 \text{ V}$ shown for a short versus a longer channel MOST [6]. DIBL is demonstrated for the SCMOST.

The surface potential distribution shown in figure 3.6 illustrates another anomaly of the SCMOST which becomes a source of analytic difficulty. According to conventional MOST theory, the channel current from source to drain is primarily drift current when $V_{gs} > V_t$ [13]. If the diffusion current is also taken into account, the equation of current continuity for a n-channel MOST is used in the form [14, 15]

$$I_d = W \left[\mu Q_m - D \frac{dQ_m}{dV_s} \right] \frac{dV_s}{dy} \quad (3.6)$$

where Q_m is the absolute value of mobile channel charge. At the potential minimum of a SCMQST, $\frac{dV_s}{dy} = 0$. Then according to equation (3.6) $I_d = 0$. Therefore equation (3.6) is incapable of explaining current flow at the potential minimum. Furthermore, since $\frac{dQ_m}{dV_s} < 0$ under all circumstances and since $\frac{dV_s}{dy} < 0$ between the potential minimum and the source, equation (3.6) predicts $I_d < 0$. Only in the drain section of the channel does equation (3.6) correctly predict that $I_d > 0$. The premise that $\frac{dV_s}{dy} < 0$ near the source is further substantiated by the direction of electric field lines within the MOST shown in figures 3.5a and 3.5b. Adjacent the source at the silicon surface, the electric field lines are inclined backward toward the source and not toward the drain. So $\frac{dV_s}{dy} < 0$ in this portion of the channel.

Of course current must flow in the same direction and have the same magnitude throughout the length of the channel (neglecting recombination, generation and impact ionization effects). At the potential minimum where $\frac{dV_s}{dy} = 0$, the channel current must be totally

diffusion current since the drift current is obviously zero. Near

the source, where the electric field opposes current flow, diffusion current again must be the dominant current carrying mechanism. These conclusions bring to light an important realization. The current equation which forms the basis for all conventional MOST theory (i.e. equation (3.6) or a subset of it), is incapable of describing the true current flow mechanisms of the SCMOST.[†]

Conventional MOST theory is valid for a long channel MOST chiefly because its channel length is very much greater than an extrinsic debye length. This means that to good approximation, the various regions of a long channel MOST act independantly of each other and are therefore separable analytically. Moreover, each region is usually associated with a 1-D mechanism which is relatively free of 2-D interactions with other regions. So a 1-D analytic treatment in each region is usually representative of the true mechanisms associated with that section of the device.

In SCMOSTs the ratio of channel length to extrinsic Debye length is substantially reduced with the result that the electrostatics becomes truly 2-D in nature. The depletion charge sharing for instance, is dependant upon the 2-D boundary conditions and 2-D interaction of such factors as substrate and drain potentials, junction depth and curvature, channel length, oxide thickness, the screening effect of the surface current carriers, and spatial distribution of depletion impurities. This later factor is particularly important if the substrate has been ion-implanted to tailor the threshold voltage or to prevent punch-through.

[†] A more applicable form of the current continuity equation is given by equations (7.2) through (7.5).

3.3 Punch-Through

Originally in long channel MOSTs, the term 'punch-through' was associated with the following concept. If the source to drain spacing is sufficiently small and the substrate resistivity high enough, the drain depletion region will spread with increasing drain bias and eventually touch the source depletion region. With a further increase in drain voltage, mobile majority carriers from the source are injected into the depletion channel and swept toward the drain by high electric fields [16]. Hence punch-through could conceptually occur and was often demonstrated throughout all modes of MOST operation. Generally the punch-through condition was identified as an anomalous triode-like rise in drain current with increasing drain voltage. In order to demonstrate punch-through in a 10 μm to 20 μm channel length MOST, it was necessary that the substrate resistivity be in the order of $10^4 \Omega \text{ cm}$.

The advent of the SCMOST has somewhat altered this concept. Firstly, the individual limits of the source, drain and channel depletion regions are no longer well defined. Secondly, surface barrier lowering and the subsequent formation of a surface channel due to increased drain bias is already categorically defined in terms of DIBL brought about by depletion charge sharing. Thirdly, as long as the current carrying channel remains at the surface over a reasonable portion of the channel length, the effects that result from electric field lines penetrating into the depletion region from the drain are explained in terms of depletion charge sharing. It remains then that punch-through in SCMOSTs refers to the formation of a subsurface channel and subsequent

subsurface current flow. Although recently, this mechanism has been referred to as bulk DIBL [6].

Punch-through is an extreme form of depletion charge sharing where the majority of electric field lines originating in the depletion region terminate on the drain. A computed potential distribution for a n-channel SCMOST operating in the punch-through mode is shown in figures 3.7a and 3.7b for $V_{ds} = 0$ and 2.0 volts respectively. These figures show the formation of a subsurface channel in each case and a subsurface potential minimum at the saddle point in each distribution. Since $V_{gs} = -1.0$ volt ($V_{gs} - V_t \approx -1.5$ volts), carriers are repelled from the surface and the formation of a surface channel is inhibited. As the drain voltage is raised from 0 to 2.0 volts, the subsurface potential minimum increases in value and is also shown to move toward the source.

If the gate voltage is increased or drain voltage reduced this subsurface channel will move toward the silicon surface and eventually become a surface channel. Figures 3.8a and 3.8b illustrate this transition for a SCMOST with $V_{ds} = 7.0$ and 12.0 volts respectively when the gate voltage is held constant at $V_{gs} = 0.5$ volts. For $V_{ds} = 12.0$ volts, a current carrying channel exists at approximately 1.0 μm below the silicon surface throughout the entire distance between source and drain. When the drain voltage is reduced to 7.0 volts, the current carrying channel moves to the surface near the source but still remains below the surface near the drain. With a further decrease in drain voltage, this device will revert to total surface channel behaviour. Thus the 2-D electrostatics determines whether a surface or subsurface channel forms. These are therefore, competing mechanisms

(a)

(b)

Figure 3.7 Internal potential distribution of a SCMOST with $V_{gs} - V_t = -1.5$ V. The formation of a subsurface channel is illustrated (a) for $V_{ds} = 0$ V and (b) for $V_{ds} = 2.0$ V [17].

(a)

(b)

(c)

Figure 3.8 Subthreshold behaviour of a $1.8 \mu\text{m}$ channel length MOST. The normalized current density distribution for $V_{GS} = 0.5$ is shown for (a) $V_{DS} = 7.0 \text{ V}$ and in punch-through (b) where $V_{DS} = 12.0 \text{ V}$. The low level current/voltage characteristics are plotted in (c). Points A and B correspond to figures (a) and (b) respectively [6].

when $V_{gs} < V_t$. Figure 3.8a also implies that there is no well defined point of transition between these two modes of operation.

The SCMOST terminal characteristics differ distinctly in each mode of operation. At point A of figure 3.8c, where the channel of the SCMOST is for the most part at the surface, the logarithmic slope of drain current with respect to drain voltage is smaller (approximately 3.0 volts per decade) and characteristic of DIBL. At point B, where punch-through has occurred, the subsurface channel current has a much reduced dependance on gate voltage and a significantly greater dependance on drain voltage.

Figure 3.9 shows drain current as a function of gate voltage for seven MOSTs with channel lengths ranging from 5.0 to 0.8 μm . For the 5.0, 3.0 and 2.0 μm channel length devices, the logarithmic slope of subthreshold current is approximately given by equation (2.16). Thus these characteristics agree with subthreshold theory for long channel devices which assumes that the subthreshold diffusion current flows at the silicon surface. For shorter channel devices, a considerable increase in the 'tail current' is evident. The reduced dependance of drain current on gate voltage is characteristic of subsurface channel behaviour or the punch-through condition.

For gate voltages above threshold, both modes of operation can take place at the same time. The drain current versus drain voltage characteristics of a SCMOST showing punch-through behaviour when $V_{gs} > V_t$ are shown in figure 3.10a. For this device, the computed transverse potential distribution (approximately at the potential minimum) is illustrated in figure 3.10b for $V_{gs} = 1.0$ volt. For $V_{ds} = 0$ to 1.0 volt, the transverse potential distribution is such that

Figure 3.9 Log (I_{ds}) versus V_{gs} characteristics of various channel length MOSTs [17]. For all devices $N = 10^{15} \text{ cm}^{-3}$, $t_{ox} = 50.0 \text{ nm}$, $x_j = 0.33 \text{ μm}$, $V_{sb} = 0 \text{ V}$ and $V_{ds} = 2.0 \text{ V}$.

(a)

(b)

Figure 3.10 Punch-through behaviour at gate voltages above threshold [17]. This SCMOST has $L = 1.0 \mu\text{m}$, $N = 10^{15} \text{cm}^{-3}$ and $x_j = 0.33$. The I_D versus V_{DS} characteristics are plotted in (a) while (b) is the corresponding potential distribution vertical to the surface approximately at the potential minimum for $V_{GS} = 1.0 \text{V}$.

only a surface channel can exist. For $V_{ds} = 5.0$ through 10.0 volts, the potential is substantially reduced at a depth of 0.5 μm from the surface allowing the formation of a subsurface channel. In addition, the surface potential is also decreased slightly due to surface DIBL. Hence both surface and subsurface channels exist simultaneously and both contribute to the total drain current. With a further increase in drain voltage, the punch-through current will dominate.

While virtually every physical parameter of the SCMOST affects punch-through behaviour to some extent, the most important parameters are channel length, substrate doping and junction depth. For a specific VLSI design, one must assume that the channel length is reasonably fixed. The object then is to suppress punch-through behaviour for a given channel length device.

Generally for deep drain and source junctions, the major portion of laterally radiating electric field lines radiate well below the surface and therefore are not greatly influenced by the gate potential. In comparison to shallow junctions, deep junctions are more likely to bring about the formation of a subsurface channel. It is advisable then to minimize junction depths as much as possible to lessen the likelihood of punch-through. This advice after all is implied by the scaling rules.

An overall increase in substrate doping raises the channel length to Debye length ratio and thereby reduces the electrostatic interaction between the drain and channel depletion regions. As a result, both punch-through and DIBL effects are reduced. Unfortunately an overall increase in substrate doping increases junction capacitances, hot carrier effects and alters the threshold voltage. A more attractive

alternative is ion-implantation of impurities beneath the surface. The implant depth should be approximately where the subsurface channel would otherwise form. If the implant is sufficiently beneath the surface, the effect on threshold voltage should be minimal. Since the source and drain junctions would only 'see' high substrate doping around their periphery, capacitances should not drastically increase. While hot carrier emission over the SiO_2 barrier is still a reason for concern, the relatively small increase in surface doping that results from a deep implant has less drastic consequences than an overall increase in substrate doping.

CHAPTER 4

EXISTING SCMOST MODELS

4.1 Conventional MOST Theory

Over recent years, conventional MOST theory has been applied to ever decreasing channel length devices. Therefore classical, gradual channel, MOST models are in contention as SCMOST models. In SCMOSTs however, several troublesome effects occur which either do not happen or are not significant in longer channel devices. Some of these are; 2-D depletion charge sharing causing DIBL and the attendant threshold voltage dependence on drain potential, extremely high electric fields giving rise to saturated drift velocity over a major portion of the channel, and the dominance of diffusion current in the channel between the source and the potential minimum. Perhaps it is not surprising that conventional MOST theory inadequately describes SCMOST behaviour.

A question comes to mind though. With the steady decrease in MOST dimensions over recent years, why has conventional MOST theory held out for so long? Part of the answer is that to date, no simple analytic solution has been put forward which encompasses all the aforementioned mechanisms. The analytic difficulties which arise will be discussed in chapter 5. Another reason is the simplicity of conventional MOST theory, both conceptual and analytic. For instance, computer-aided-design (CAD) aids generally require

that device models be expressed in the simplest possible mathematical terms. There is another reason however.

The 1-D, gradual channel, constant mobility MOST model [18]

$$I_d = \frac{\mu_o C_{ox} W}{L} \left\{ \left[V_{gs} + \frac{Q_{eff}}{C_{ox}} - \phi_{MS} - 2\phi_f - \frac{V_{ds}}{2} \right] V_{ds} - \frac{2}{3 C_{ox}} (2q \epsilon_{si} N)^{1/2} \left[(V_{ds} + 2\phi_f)^{3/2} - (2\phi_f)^{3/2} \right] \right\} \quad (4.1)$$

when used in conjunction with a drain section model [19] to account for widening of the drain space charge region in saturation, yields remarkably accurate terminal characteristics for devices with channel lengths as small as 3.0 μm [20]. To support this statement, the preceding 1-D MOST model is compared with measured I_d versus V_{ds} characteristics of a 3.0 μm channel length, n-channel MOST in figure 4.1a. In addition, this figure contains the results from a 2-D, numerical MOST simulation [21] assuming constant electron mobility and the 2-D simulation results assuming the field dependant mobility expression for electrons [22].

$$\mu = \frac{\mu_o}{\left[1 + \left(\frac{\mu_o |E|}{v_s} \right)^2 \right]^{1/2}} \quad (4.2)$$

Note the accuracy of the constant mobility, 1-D MOST model. For $V_{gs} = 4.0$ volts, it yields the best agreement with the measured device characteristics. Perhaps a more surprising observation is that the constant mobility 2-D simulation yields the poorest agreement.

Figure 4.1b shows the surface electric field which arises from the depletion charge as predicted by conventional 1-D MOST

Figure 4.1 a I_d versus V_{ds} characteristics for a 3.0 μm channel length MOST [20] predicted by

- constant mobility, 1-D, gradual channel theory
- constant mobility 2-D analysis
- △ field dependant mobility, 2-D analysis
- from device measurement

Figure 4.1 b Electric field arising from the depletion charge as predicted by — field dependant 2-D analysis and ○ constant mobility, 1-D, gradual channel MOST theory [20].

theory.

$$E_d(y) = \frac{1}{\epsilon_{si}} \sqrt{2q \epsilon_{si} N V_s(y)} \quad (4.3)$$

This is compared with the more realistic results obtained from the field dependant mobility, 2-D simulation. The latter results indicate that the depletion charge as 'seen' from the surface diminishes toward both the source and drain with a maximum approximately midway in the channel. Aside from the actual distribution of depletion charge, this figure shows that 1-D MOST theory over estimates the amount of depletion charge which electrostatically interacts with the current carrying channel.

The accuracy of the 1-D, gradual channel, constant mobility MOST model occurs because depletion charge sharing and the reduction of mobility with high lateral fields are to some extent compensating mechanisms. According to 1-D MOST theory

$$I_d = W \mu Q_m \frac{dV_s}{dx} \quad (4.4)$$

$$Q_m = C_{ox} (V_{gs} - \Phi_{MS} + \frac{Q_{eff}}{C_{ox}} - V_s) - Q_d \quad (4.5)$$

A reduction in μ due to high lateral fields causes a decrease in I_d , but this is compensated for by a decrease in depletion charge Q_d which causes an increase in I_d . This logic also explains why the constant mobility, 2-D simulation yields such poor agreement with the measured MOST characteristics in figure 4.1a. In addition, it explains why classical, gradual channel, MOST models have not overwhelmingly adopted the simple expression for lateral field dependant mobility [8]. Conventional MOST models fail when applied to SCMOSTs because these mechanisms no longer compensate. One can

conclude then that any viable SCMOST model must account for the effects of both depletion charge sharing and mobility reduction due to high lateral electric fields.

4.2 Threshold Voltage Models

In SCMOSTs, a lowering of threshold voltage occurs due to the close proximity of source and drain regions and associated 2-D charge sharing. If a source to drain bias is applied, a further decrease in threshold voltage results due to DIBL. Of all electrical parameters of a MOST, the turn-on voltage is perhaps the most important. For this reason, it has been the primary focus of attention. The hope is that a good short channel threshold voltage formula could be used to extend the validity of classical MOST models to shorter channel length devices.

Although the electrostatics of depletion charge sharing is truly 2-D, the basis of virtually all SCMOST threshold voltage models is the partitioning of depletion charge into regions associated with the source, drain and channel. Such a partitioning scheme is illustrated in figure 4.2. The limits of each region are usually determined with the aid of 1-D analysis with further emphasis on geometrical simplicity. The channel depletion charge which is not claimed by either the source or drain is average over the entire channel length. By this means, a bias and geometry dependant expression for threshold voltage is achieved.

All threshold voltage models consider the proximity effect of source and drain junctions and reduce to the conventional

Figure 4.2 A SCMOST depletion charge partitioning scheme for bulk charge and threshold voltage calculation [14] shown for two different sets of bias conditions.

threshold voltage expression for long channel lengths. Generally a uniform substrate doping is assumed although several authors maintain that an ion-implanted impurity distribution can be accounted for by averaging techniques.

The following four models deal only with the case where $V_{ds} = 0$ and hence do not consider the effects of DIBL. All four take the substrate bias into account. The first threshold voltage model by Chaney and Kotch [23] considers rectangular junctions of finite depth. Two other models assume deep source and drain junctions [24, 25] so that the effect of junction depth can be ignored. Yau [26] deals primarily with cylindrical junctions but extends this to the case of infinite radius where the junctions become vertical with infinite depth.

The threshold voltage dependence on drain voltage is considered by Dang [27] but only for the saturation mode of operation. His analysis does not include the effect of substrate bias. Like Yau, Dang deals with cylindrical junctions and vertical junctions of infinite depth. The model by Lee [28] has the widest applicability. He considers rectangular junctions of finite depth along with the effects of drain to source and substrate bias. His expression for threshold voltage is rather lengthy however.

Troutman and Fortino [29] take a distinctly different approach. The conceptual basis of their model is the 2-D penetration of lateral fields into the channel region and how this affects the virtual cathode (potential minimum). The effects of source to drain and substrate bias are taken into account. Their expression for threshold voltage is simple but unfortunately requires the

predetermination of two penetration coefficients. Determination of these coefficients requires either 2-D simulation or measurement of each device under consideration.

4.3 Complete SCMOST Models

To the best knowledge of this author, there exists only three SCMOST models which could be construed as different from classical MOST theory.

Dang's SCMOST model [14] attacks two major mechanisms inherent to SCMOSTs, the reduction in surface mobility due to high lateral electric fields and the decrease in depletion charge as 'seen' from the silicon surface. His model uses the lateral field dependant mobility expression

$$\mu = \frac{\mu_0}{1 + \frac{\mu_0 |E|}{V_s}} \quad (4.6)$$

and it is properly integrated along the channel. This expression is more representative of hole rather than electron mobility. The more correct expression for electron mobility [22] leads to analytic difficulties as will be discussed in the next chapter. The GCA is assumed to facilitate solution of the current equation. With this conventional simplification, application of the charge neutrality condition yields the 1-D equation for channel charge

$$Q_m = C_{ox} (V_{gs} - V_{fb} - V_s) - K \ 2q\epsilon_{s1} N(V_s + V_{sb}) \quad (4.7)$$

This differs from its classical counterpart in only one respect, the depletion charge reduction factor $K(V_{gs}, V_{ds}, V_{sb})$. As in threshold voltage models, the factor K is derived from depletion

charge partitioning analysis. Dang's equation of current continuity includes diffusion current but is applied in the classical form

$$I_d = W \left[\mu Q_m - D \frac{dQ_m}{dV_s} \right] \frac{dV_s}{dy} \quad (4.8)$$

The depletion charge distribution assumed in Dang's model is merely a reduced version of the classical depletion charge distribution. Thus it is not representative of the true depletion charge distribution as 'seen' from the silicon surface in SCMOSTs. This model then contains no mechanism to analytically create either the condition in the channel near the source where $\frac{dV_s}{dx} < 0$ or the potential minimum. Both these conditions are characteristic of the true surface potential in SCMOSTs (see figure 3.6). It is in the channel towards the source and at the potential minimum where the diffusion current is of importance (see section 3.2). Since the mechanisms inherent to these regions of the channel have not been captured, and since this model is not applicable in subthreshold, the inclusion of diffusion current in Dang's model is for the most part superfluous. The equation for drain current in the triode mode is

$$I_d = \frac{\mu W C_{ox}}{L \left(1 + \frac{\mu V_{ds}}{v_s L} \right)} \left\{ \left[V_{gs} - V_{fb} + \frac{kT}{q} - 2\phi_f - \frac{V_{ds}}{2} \right] V_{ds} - \frac{K}{C_{ox}} (2q\epsilon_{si}N)^{\frac{1}{2}} \left\{ \frac{2}{3} \left[(V_{ds} + 2\phi_f + V_{sb})^{\frac{3}{2}} - (2\phi_f + V_{sb})^{\frac{3}{2}} \right] + \frac{kT}{q} \left[(V_{ds} + 2\phi_f + V_{sb})^{\frac{1}{2}} - (2\phi_f + V_{sb})^{\frac{1}{2}} \right] \right\} \right\} \quad (4.9)$$

This formula significantly differs from conventional MOST theory only with respect to coefficient K and the last two terms in square

brackets which account for the diffusion current. In saturation Dang uses his own drain section model. In general, this SCMOST model shows reasonable agreement with the measured I_d versus V_{ds} characteristics of a $1.0 \mu\text{m}$ channel length device. Some discrepancies are evident in both triode and pentode mode which Dang attributes to inaccuracies in his K factor and the assumed GCA.

The hot electron MOST model of Hoeffinger, Sibbert and Zimmer [30] exploits the premise that current carriers are for the most part 'hot' along the channel of a typical SCMOST. They feel that this is the overriding mechanism in SCMOSTs which satisfy the criterion $\frac{V_{ds \text{ max}}}{L} > \frac{v_s}{\mu}$. Using this argument they resurrect the out dated MOST concept of pinch-off field (which originates from junction field-effect transistor theory). The analysis presented is essentially a derivation of all electrical parameters in terms of this pinch-off field. The list of parameters is rather complete though and these are expressed in very simple mathematical terms. Their model contains drain section analysis applicable to the saturation mode of operation. The model requires a threshold voltage formula to account for the proximity effect of source and drain regions and DIBL. Factors such as substrate doping, substrate bias and junction depth do not enter into their derivation. The pinch-off field and one other major fitting parameter must be calculated from electrical measurement of each device under consideration. When this model is fitted to the I_d versus V_{ds} characteristics of a $2.0 \mu\text{m}$ channel length MOST, the agreement is good.

Sun and Moll presented an analytic SCMOST model at the 36th Annual Device Research Conference. Unfortunately the conference

abstract [31] does not contain sufficient information for a detailed appraisal. From the description given, this model appears conceptually similar to Dang's [14] with the diffusion current excluded. The model contains a lateral field dependant mobility expression. The depletion charge is partitioned using the trapezoid approximation. The reduced depletion charge is used to calculate the threshold voltage and bulk charge in active operation. A simple drain section model is employed in saturation. The authors claim agreement within 2.0% of the measured I_D versus V_{DS} characteristics of a 3.0 μm channel length MOST.

The SCMOST models presented in this section are merely extended versions of classical MOST theory. They all assume the GCA. All analysis is one-dimensional. All models employ the classical form of the current continuity equation which is incapable of capturing the true SCMOST current mechanisms. None of these models apply to subthreshold operation which becomes increasingly important in shorter channel length devices. The models of Dang, and Sun et al. differ from classical theory only with respect to the depletion charge reduction factor which is calculated using charge partitioning analysis. Although an overall reduction of depletion charge is accomplished by this means, its distribution is classical and therefore not representative of the true depletion charge distribution as 'seen' from the silicon surface of a SCMOST. Hoeffinger et al. on the other hand, do not even consider depletion charge in their model. Unaltered classical MOST theory has demonstrated remarkable accuracy for the 3.0 μm channel length device of section 4.1, even though the assumptions of constant

mobility and the GCA are invalid for SCMOSTs. There is reason to speculate that a representative short channel threshold voltage formula if used in conjunction with a good long channel model such as presented in chapter 6, might yield results comparable in accuracy to the SCMOST models discussed in this section.

CHAPTER 5

THE LIMITATIONS OF ANALYTIC METHODS

The general scope of mathematical methods is virtually limitless. For instance, numerical methods can be applied to a multitude of multi-dimensional problems with highly irregular boundary conditions. The only obvious limitation is computer time and memory capacity. In the case of MOSTs, several 2-D numerical models exist [21, 32-35]. These require in the order of 10^4 memory locations. Computer computation time is typically from a minute to the major portion of an hour depending on the level of sophistication of the simulation and the bias conditions. Recently, a 3-D finite element MOST simulation has been reported [36]. The authors point out though that 3-D simulation requires $T_2 \cdot P_1^2$ more CPU time than an equivalent 2-D simulation where T_2 is the time required for 2-D simulation and P_1 is the number of planes extending into the third dimension.

The scope of analytic methods is by contrast much more bounded. Multi-dimensional solutions only exist for a limited number of 'nicely' defined classical problems. Generally, the governing partial differential equation must be at least linear and the boundary conditions must be very simply defined in terms of the coordinate system. The boundary condition constraint can sometimes be alleviated by conformal mapping techniques but at the cost of greatly increased complexity. However, in nonequilibrium

semiconductor theory, the governing partial differential equations are highly nonlinear.

Even within the bounds of 1-D differential equations there are severe handicaps. If a differential equation is not linear or if it can not be put into an exact or separated form or if it can not be put into the form of a few classically studied equations, then there is no known analytic method of solution [37]. The only viable recourse then is numerical methods. Further limitations are imposed on analytic semiconductor device models intended for CAD. Electrical parameters must be expressed in simple mathematical terms. A slight increase in accuracy can not be justified if it requires an order of magnitude increase in computation time. For this reason an electrical parameter can not be expressed in integral form or as an infinite series. In short, the equations which frame a device model must be solvable in closed form. The remainder of this chapter is devoted to relevant analytic methods and specific difficulties related to modeling of short channel MOS devices.

The basis of virtually all analytic MOST models is the drift current equation.

$$\frac{I_d}{W} = \mu Q_m \frac{dV_s}{dy} \quad (5.1)$$

(Attention will be restricted to n-channel devices, p-channel analysis will require the appropriate change of sign or signs.)

If in addition, diffusion current is considered, the current continuity equation is applied in the form [14, 15]

$$\frac{I_d}{W} = \left[\mu Q_m - D \frac{dQ_m}{dV_s} \right] \frac{dV_s}{dy} \quad (5.2)$$

Given an expression for $Q_m(V_g)$ and a single boundary condition, the solution of equation (5.1) or (5.2) is the surface potential distribution. Then if one fixes $y = y_d$, an equation relating $V_s(y_d)$ and I_d is obtained. All analytic MOST models which apply in the triode or saturation mode are derived in this manner, although considerable finesse may be used to arrive at the boundary conditions. In analytic subthreshold models, on the other hand, the channel charge is so negligibly small that it is not considered in the surface potential calculation. Hence the solution of equations like (5.1) or (5.2) are generally not required.

Only a limited number of analytic expressions for channel charge are found in classic gradual channel MOST models. All are derived from a simple 1-D application of the charge neutrality condition in the direction normal to the silicon surface. In these expressions, several constants often appear, such as; $2\phi_f$, V_{fb} , ϕ_{MS} , V_{sb} , V_t , V_p , etc. which are used to adjust the reference of either gate voltage or surface potential. To eliminate redundancy, the expressions in this chapter are written in terms of an effective gate voltage V_g , and with the surface potential referenced to the substrate. Attention will be further restricted to expressions which apply to a uniformly doped substrate. To the best knowledge of this author, the following is a complete list of analytic channel charge expressions which have been used in classical MOST models.

$$Q_m = C_{ox} (V_g - V_s) \quad (5.3)$$

$$Q_m = C_{ox} (V_g - V_s) - \sqrt{2q\epsilon_{si} N V_s}, \text{ Ihantola [18]} \quad (5.4)$$

$$Q_m = C_{ox} (V_g - V_s) - \sqrt{2q\epsilon_{si} N (V_s - \frac{kT}{q})}, \text{ Kennedy [38]} \quad (5.5)$$

$$Q_m = \frac{C_{ox}^2 (V_g - V_s)^2 - (2q\epsilon_{si} N V_s)}{C_{ox} (V_g - V_s)}, \text{ El-Mansy [39]} \quad (5.6)$$

Substitution of any of the preceding expressions for Q_m into either current equation (5.1) or (5.2) results in a first order differential equation in the separated form.

$$R(y) dy = S(V_s) dV_s \quad (5.7)$$

A solution can then be achieved by straight forward integration methods. For all combinations of the preceding Q_m expressions and current equations (5.1) or (5.2), the required integration in V_s is not only integrable but integrable in closed form.

A variation on conventional MOST analysis is to introduce a lateral field dependant expression for mobility such as

$$\mu = \frac{\mu_o}{1 + \frac{\mu_o}{v_s} \left| \frac{dV_s}{dy} \right|} \quad (5.8)$$

When this is substituted into say equation (5.1), the result is simply

$$\frac{I_d}{W} dy = \mu_o \left[-\frac{I_d}{Wv_s} + Q_m \right] dV_s \quad (5.9)$$

Again for any preceding Q_m expression, this equation is in separated form and integrable in closed form. If expression (5.4) were used, the familiar result [8] follows. It is known however, that equation (5.8) is more characteristic of hole mobility than electron mobility [22].

Electron mobility is better described by

$$\mu = \frac{\mu_o}{\left[1 + \left(\frac{\mu_o}{v_s} \frac{dV_s}{dy} \right)^2 \right]^{1/2}} \quad (5.10)$$

Substituting this into current equation (5.1) and using Q_m expression (5.4), one obtains

$$\frac{I_d}{W} dy = \mu_0 \left[-\left(\frac{V_d}{W V_s}\right)^2 + C_{ox} (V_g - V_s)^2 - 2C_{ox} (V_g - V_s) \sqrt{2q\epsilon_{si} N V_s} + (2q\epsilon_{si} N V_s) \right]^{1/2} dV_s \quad (5.11)$$

which again is in separated form but the required integration in V_s is not integrable in closed form. One can of course expand the right hand side (RHS) of this equation into a series and then perform integration term by term. The resulting series equation for I_d would be so much more complex than the familiar field dependant MOST model [8] that for CAD, it simply is not worth the effort. This may explain why such a derivation has never appeared in literature.

It is also known that surface mobility decreases with increasing surface (or normal) electric field strength [40]. To account for this nature of surface mobility, one might adopt a simple representative expression such as

$$\mu = \frac{\mu_0}{1 + Z(V_g - V_s)} \quad (5.12)$$

Since the surface field changes along the channel from source to drain, surface mobility must also change along the channel. To properly capture this mechanism, the surface mobility dependence on the surface field must be considered in the basic current continuity relation. For example, using equations (5.1), (5.5) and (5.12), the resulting drift current equation would appear as

$$\frac{I_d}{W} dy = \mu_0 \left[\frac{C_{ox} (V_g - V_s) - \sqrt{2q\epsilon_{si} N (V_s - \frac{kT}{q})}}{1 + Z (V_g - V_s)} \right] dV_s \quad (5.13)$$

Although equation (5.13) can be solved in closed form, such a solution has never been reported. Using equation (5.13) as a basis, a versatile long channel MOST model is developed in chapter 6. To further extend its validity into the subthreshold range, diffusion current analysis is also included as an integral component of the model. Good agreement with measured device characteristics is demonstrated for all modes of device operation (subthreshold, triode and saturation).

For comparison, other MOST models assume mobility is constant to solve the current equation. Then the mobility term μ is simply replaced with an expression of the form

$$\mu = \frac{\mu_0}{1 + Z (V_{gs} - V_t)} \quad (5.14)$$

to account for the normal field dependence of surface mobility [14, 41], even though it is not physically correct. Such an approach can not account for the fact that the normal field (and therefore mobility) changes along the channel for source to drain.

The DMOST and VMOST (see figures 3.3 and 3.4) are intentionally fabricated with a variation in impurity density along the channel. Both devices exhibit SCMOST characteristics as a direct result of this impurity variation. Assuming the GCA, the depletion charge along the channel can be represented by

$$Q_d = \sqrt{2q\epsilon_{si} N_c(y) V_s} \quad (5.15)$$

For the DMOST, $N_c(y)$ is the difference between two complementary error functions while in the VMOST, $N_c(y)$ is more gaussian. One could also argue that the true depletion charge as 'seen' from the

surface of a SCMOST could be represented as a modified version of the classic gradual channel depletion charge distribution or specifically

$$Q_d = \xi(y) \sqrt{2q\epsilon_{si} N V_s} \quad (5.16)$$

where N is constant and $\xi(y)$ is a dimensionless function of position and bias conditions. In general $\xi(y) \leq 1$ would have a maximum approximately midway in the channel and diminish toward both the source and drain. Hence $\xi(y)$ could be parabolic. Mathematically, expressions (5.15) and (5.16) have the same general form so the following analysis will deal primarily with the former case although the conclusions will be applicable to both cases. Clearly this common mathematical form is relevant to the analytic study of several MOS devices. The channel charge then can be written as

$$Q_m = C_{ox} (V_g - V_s) - \sqrt{2q\epsilon_{si} N_c(y) V_s} \quad (5.17)$$

In the DMOST, diffusion current plays an important role [11] and one would expect the same to be true of the VMOST. It is also important in the SCMOST as discussed in section 3.2. Hence diffusion current must be considered in the current continuity equation to capture the true device operating mechanisms. Since Q_m now has direct dependence on position through $N_c(y)$, an alteration to current equation (5.2) is required [11].

$$\frac{I_d}{W} = \left[\mu Q_m - D \frac{\partial Q_m}{\partial V_s} \right] \frac{dV_s}{dy} - D \frac{\partial Q_m}{\partial y} \quad (5.18)$$

For any representative $N_c(y)$, substitution of equation (5.17) into (5.18) leads to a differential equation which is not separable, nor exact, nor can it be made exact by any known method, nor is it linear in either variable, nor is it in any classical form. In an

effort to simplify the analysis, one could neglect the diffusion current by using equation (5.1) instead of (5.18). The result is much simpler.

$$\frac{I_d}{W} dy = \mu \left[C_{ox} (V_g - V_s) - \sqrt{2q\epsilon_{si} N_c(y) V_s} \right] dV_s \quad (5.19)$$

Again for any representative $N_c(y)$, equation (5.19) is not separable, nor is it exact, nor is it linear in either variable, nor is it of classical form. However, an inexact equation of this type

$$R(y, V_s) dy = S(y, V_s) dV_s \quad (5.20)$$

can often be made exact (hence solvable) if multiplied by a suitable integrating factor λ . The resulting equation would then satisfy the exactness test.

$$\frac{\partial}{\partial V_s} [\lambda R] = - \frac{\partial}{\partial y} [\lambda S] \quad (5.21)$$

Unfortunately integrating factors can only be determined in certain special cases. Expansion of equation (5.21) yields

$$\lambda = - \left[\frac{S \frac{\partial \lambda}{\partial y} + R \frac{\partial \lambda}{\partial V_s}}{\frac{\partial R}{\partial V_s} + \frac{\partial S}{\partial y}} \right] \quad (5.22)$$

If the required λ happens to be independent of V_s then equation (5.22) can be simplified to

$$\frac{d\lambda}{\lambda} = - \frac{1}{S} \left(\frac{\partial R}{\partial V_s} + \frac{\partial S}{\partial y} \right) \quad (5.23)$$

and it follows that

$$\lambda = \exp \left\{ \int_y - \frac{1}{S} \left(\frac{\partial R}{\partial V_s} + \frac{\partial S}{\partial y} \right) dy \right\} \quad (5.24)$$

Similarly if λ happens to be independent of y

$$\lambda = \exp \left\{ \int_{V_s} -\frac{1}{R} \left(\frac{\partial S}{\partial y} + \frac{\partial R}{\partial V_s} \right) dV_s \right\} \quad (5.25)$$

To be a valid integrating factor, it must of course substantiate the premise of being either independant of y in the case of equation (5.24) or independant of V_s in the case of equation (5.25).

Returning to equation (5.19), one finds that an integrating factor can only be calculated if $N_c(y)$ has the form $N_c(y) = (a_0 + a_1 y)^2$. Unfortunately this is not very representative of either the DMOST or the VMOST since this distribution has a minimum at the point where $N_c(y) = 0$. In the SCMOST case, this translates to the form $\xi(y) = (a_0 + a_1 y)$ which clearly is not parabolic as hoped for. Nonetheless given $N_c(y) = (a_0 + a_1 y)^2$, an integrating factor can be calculated using equation (5.25),

$$\begin{aligned} \lambda &= \exp \left\{ \int_{V_s} \frac{W\mu a_1}{I_d} (2q\epsilon_{si} V_s)^{\frac{1}{2}} dV_s \right\} \\ &= \exp \left\{ \frac{2W\mu a_1}{3I_d} (2q\epsilon_{si})^{\frac{1}{2}} V_s^{\frac{3}{2}} \right\} \end{aligned} \quad (5.26)$$

Then equation (5.19) takes the exact solvable form.

$$\begin{aligned} \frac{I_d}{W} \exp \left\{ \frac{2W\mu a_1}{3I_d} (2q\epsilon_{si})^{\frac{1}{2}} V_s^{\frac{3}{2}} \right\} dy &= \mu \left[C_{ox} (V_g - V_s) - \right. \\ &\left. (2q\epsilon_{si} V_s)^{\frac{1}{2}} (a_0 + a_1 y) \right] \exp \left\{ \frac{2W\mu a_1}{3I_d} (2q\epsilon_{si})^{\frac{1}{2}} V_s^{\frac{3}{2}} \right\} dV_s \end{aligned} \quad (5.27)$$

Part of the solution is obtained by integrating the LHS with respect to y . No complications arise here because the LHS is independant of y . The remainder of the solution is obtained by integrating the RHS with respect to V_s . Unfortunately the integration of

$$\int_{V_s} \mu C_{ox} (V_g - V_s) \exp \left\{ \frac{2W\mu a_1}{3 I_d} (2q\epsilon_{si})^{1/2} V_s^{3/2} \right\} dV_s \quad (5.28)$$

can not be performed in closed form. This should not seem surprising, after all the well known and much simpler function $\exp(V^2)$ can not be integrated in closed form either. It appears then that this method of attack is incapable of producing a viable CAD model of the VMOST, DMOST or SCMOST for a number of reasons. Any attempt to expand the integrand of integral (5.28) into a series would lead to an extremely complex solution. Furthermore the solution would contain transcendental functions from the remainder of the solution. Moreover, the assumptions of constant surface mobility and negligible diffusion current can not be justified. The impurity distribution $N_c(y) = (a_0 + a_1 y)^2$ is not representative of either the DMOST or VMOST and in the case of the SCMOST, $\xi(y) = a_0 + a_1 y$ does not capture the parabolic form required.

In some SCMOSTs, an increase in gate oxide thickness near the source and drain can result from the lateral penetration of thermal oxidation processes beneath the silicon gate [42]. This then is another case where a parameter, specifically the oxide capacitance, takes on a y dependence. If one assumes the GCA, constant mobility and neglects diffusion current for simplicity, the current equation can be written as

$$\frac{I_d}{W} dy = \mu \left\{ C_{ox}(y) (V_g - V_s) - \sqrt{2q\epsilon_{si} N} V_s \right\} dV_s \quad (5.29)$$

Unless $C_{ox}(y)$ is constant, this equation is not in solvable form. Further investigation reveals that an integrating factor can only be calculated if C_{ox} takes the form $C_{ox}(y) = (a_0 + a_1 y)$ which would

imply $t_{ox} = \epsilon_{ox} (a_0 + a_1 y)^{-1}$. This inverse relation might be representative of the 'bird's beak' oxide thickness variation observed in each half of the channel of some SCMOSTs [42]. Along this train of thought, perhaps a two section model could be derived. Using equation (5.25), the required integrating factor is

$$\lambda = \exp \left\{ - \frac{W C_{ox} a_1}{I_d} \left(V_g V_s - \frac{V_s^2}{2} \right) \right\} \quad (5.30)$$

and the current equation takes the exact solvable form

$$\begin{aligned} \frac{I_d}{W} \exp \left\{ - \frac{W \mu C_{ox} a_1}{I_d} \left(V_g V_s - \frac{V_s^2}{2} \right) \right\} dy \\ = \mu \left\{ (a_0 + a_1 y) (V_g - V_s) - \sqrt{2q\epsilon_{s1} N V_s} \right\} \\ \cdot \exp \left\{ - \frac{W \mu C_{ox} a_1}{I_d} \left(V_g V_s - \frac{V_s^2}{2} \right) \right\} dV_s \end{aligned} \quad (5.31)$$

Unfortunately equation (5.31) can not be solved in closed form and thus will not yield a solution suitable for CAD. Moreover the validity of such a solution is very much in doubt because of the assumptions necessary to simplify the analysis.

There is a MOST for which this integrating factor method of attack does yield a solution in relatively simple closed form. The device is a long channel MOST with a resistive gate [43]. The device boasts linear I_d versus V_{ds} characteristics if the resistive gate is biased so that $V_g(y) = (y/L) V_{ds} + V_{go}$ where V_{go} is the applied input potential with respect to the source. The solution is contained in appendix A. This device however, appears to be of limited usefulness because it requires an external operational amplifier to bias the resistive gate.

The preceding analysis demonstrates the level of analytic difficulty encountered in modeling such devices as the DMOST, VMOST and SCMOST, although it only represents the most straight forward method of attack. Other methods have been attempted with generally less success. For instance, in any of the preceding differential equations or variations of them, numerous substitutions can be made but none have been found which allow a closed form solution. First order perturbation theory has been investigated. This technique is applicable to very stable solutions where a slight deviation in any parameter can be approximately accounted for using linearization methods. Unfortunately in MOSTs nearing saturation, the solution of the current equation is extremely unstable. At saturation the solution is completely unstable. Thus first order perturbation theory is drastically inaccurate. Conformal mapping techniques might also seem applicable. For instance, the DMOST impurity profile could be mapped so that it is constant along the channel in a transformed coordinate system. Presumably then classical MOST theory could be applied. This presumption however, is incorrect because classical MOST theory even if transformed can not describe the mechanisms inherent to the DMOST (see section 3.2). One might also consider framing the current continuity equation in terms of the electron quasi Fermi level instead of surface potential in the hope that a simpler form of the current equation could be derived. Such 1-D analysis has been performed numerically [44] for long channel MOSTs. However the expression for $Q_m(\phi_n)$ is in a complicated integral form which prohibits analytic solution except in subthreshold where simplifying approximations can be made [45, 46]. Analytically

then, it appears one is forced to solve the current equation in terms of surface potential. Another approach to modeling the SCMOST is presented in chapter 8. Again the current equation does not solve analytically. Other techniques are suggested to facilitate a solution.

It is clear that simple CAD models for the aforementioned devices and particularly the SCMOST can not be obtained using the usual straight forward method. There may be an analytic means of capturing the SCMOST mechanisms in compact mathematical form. At this time however, such means are not obvious to this author.

There is every reason to believe that the potential market for VLSI along with the promised world-wide financial investment (in the order of 10^9 dollars over the next five years), will make VLSI a reality. As circuit complexity grows, CAD aids will become increasingly important. The penalty of an IC design error will be much more costly and much less tractable. Despite this motivation, a truly representative SCMOST model has not appeared in literature. One can only conclude that others in the field are encountering analytic difficulties similar to those outlined in this chapter.

Two options remain. Attempts can be made to modify existing analytic theory however inapplicable, in the hope that somehow its validity can be extended. This would serve to delay the inevitable allowing more time for further investigation. Alternatively one can resort to techniques which lie beyond the bounds of analytic methods. Numerical analysis, for instance, can be utilized to study the device mechanisms. There is always the slim chance that better understanding of the device might precipitate

some simplification that would allow analytic solution.

Numerical device models are important tools in their own right. When analytic models are either nonexistent or too inaccurate, numerical models can provide the required answers. They can also be used to test simpler device models. To date, numerical models have not been employed in CAD simulators because of the computing time and memory capacity required. Moreover for long channel MOSTs, analytic models are quite adequate. Generally, the aim of numerical models has been accuracy. To this end, they have been made as rigorous as numerical and computing capabilities will allow. It seems then, that there is a widening gap between the accuracy of analytic SCMOST models and their numerical counterparts. There are cases where a compromise between the two would be useful. For instance, in linear circuitry, some node voltages are critically dependant on device parameters. For SCMOSTs, the available device models often are too inaccurate for this purpose while numerical models are either too costly or just not available to the IC design engineer. Consequently, longer channel MOSTs are often used instead. SCMOST subthreshold current is an important parameter for dynamic memory design. At the moment it can only be determined accurately by direct measurement or numerical simulation. It appears then that there may be a market for a hybrid model using primarily analytic methods in conjunction with numerical analysis where analytic methods fail to capture the device mechanisms. Such an approach is taken in chapter 8 to model the SCMOST.

Alternatively, an analytic MOST model could be written in terms of several fitting parameters. Given the physical geometry

of a device, efficient numerical techniques could be employed to calculate these fitting parameters. Numerical computation would only take place once, at the beginning of each CAD session.

Fitting parameters for frequently used geometries, could be stored in permanent memory. A scheme of this type might prove even more efficient than existing device models. Such an approach to device modeling however, will not be addressed in this report.

A MOST model based on gradual channel theory is presented in the next chapter. It was developed with the intention of incorporating several refinements such as subthreshold diffusion current and the mobility dependance on the normal field, into a unified, comprehensive, mathematically self-consistent, long channel MOST model suitable for CAD. The mathematical development is intentionally rigorous and so some simplifying approximations may be possible. Drain region analysis and saturated drift velocity effects are not considered because these mechanisms have been adequately modeled by others for the purposes of long channel devices. Moreover, the appropriate modifications can easily be incorporated into the following MOST model.

CHAPTER 6

A REFINED LONG CHANNEL MOST MODEL

6.1 Introduction

Classical MOST models have been derived on the premise that surface mobility is a constant [13, 18, 47, 48], whereas in reality, surface mobility is known to be dependant on the surface (or normal) field. Over the normal gate voltage range of a typical <100> silicon n-channel MOST, the surface mobility at room temperature can decrease by as much as 50% from threshold to strong inversion. This effect is often manifested in the device terminal characteristics as a decrease in slope of output conductance with increasing gate voltage and as a decrease in linear mode transconductance with increasing gate bias. There exists two schools of thought regarding the mechanism responsible for this apparent surface mobility behaviour; surface scattering and quantum mechanical considerations.

Surface scattering is itself divisible into two components; specular scattering and diffuse scattering. Specular scattering is an elastic collision process in which mobile carriers are reflected from the Si-SiO₂ interface. Under such a collision process, the carrier momentum in the direction perpendicular to the interface is reversed. Since no energy is lost and the momentum in the direction of current flow is preserved, specular scattering has no effect on surface mobility provided that the effective mass of current carriers

remains unaffected [49].

Diffuse scattering, on the other hand, describes a random scatter process. A surface field in the direction to attract carriers to the surface tends to perturb the path of mobile carriers between collisions such that their trajectories become more inclined toward the surface. Although surface scattering is an ever present mechanism in MOS devices, the number of collisions with the surface while traversing the channel length increases with increasing surface field strength. The subsequent decrease in average mean-free-path of mobile carriers (between random scatter events) can directly explain a decrease in carrier mobility. Theoretical predictions based entirely on diffuse scattering [50] however, demonstrate poor agreement with experiment [51].

Quantum mechanical considerations are relevant whenever an atomic particle is confined to dimensions smaller than its wavelength in free space. Such is the case for electrons in the surface channel of an MOST, especially in strong inversion. The simultaneous solution of Poisson's equation and Schrödinger's equation has been carried out in 1-D for the surface inversion layer of an MOS device [52]. When compared to the more classical solution (which assumes Maxwell - Boltzmann statistics and no quantum mechanical effects), one finds that the average distance of carriers from the surface differs by less than 3.0 nm between the two solutions. This amount of quantum mechanical repositioning of the channel is negligible in comparison to typical oxide thicknesses (100 nm) and typical surface depletion widths (600 nm) of an MOS device. Therefore, the decrease in slope of MOST output conductance with increasing gate

bias can not possibly be explained in terms of quantum mechanical repositioning of the channel.

The dominant quantum mechanical effect can be explained in terms of quantization of the inversion-layer carrier wave function in the direction perpendicular to the silicon surface [53, 54]. In the surface inversion layer, the carrier wavefunction in the direction perpendicular to the surface is small in comparison to its corresponding wavelength in bulk silicon. The distribution of quantization levels and electrons among these quantization levels is influenced directly by the width of the surface channel which in turn is a function of the surface field strength. In silicon, (where carrier energy is not quadratically related to momentum) this changes the effective mass of mobile carriers and subsequently alters the value of surface mobility.

To date, no unified quantitative (quantum mechanical, diffuse scattering) theory has been advanced. It is clear though, that both mechanisms correlate surface mobility with the strength of electric field in the direction perpendicular to the silicon surface.

The main objective of this chapter is to derive a MOST model with a surface field dependant mobility. An empirical expression for surface mobility is employed. Its degree of agreement with measured surface mobility data will be demonstrated. Since the surface field changes along the channel from source to drain, the surface mobility must also change along the channel. To properly capture this mechanism, the surface mobility dependance on the surface field is included as a component of the basic current continuity equation. It is believed that this is the first time

surface field dependant mobility has been properly incorporated into an analytic MOST model.

In dynamic logic and dynamic memory circuits subthreshold leakage often sets the lower limit for the clock speed or the refresh interval. For this reason subthreshold diffusion current analysis is included in the model derivation. This diffusion current analysis can also be incorporated into any other drift current MOST model to achieve complete drain current characterization over all modes of device operation without the traditional problem of matching solutions at threshold. Furthermore it will be demonstrated that this diffusion current analysis is in agreement with the work of others [45, 46, 55] over the range of bias conditions where their analytic subthreshold models are valid.

In computer-aided-design network analysis programs, it is necessary that all device terminal currents are expressed as continuous functions of the terminal voltages. It is preferable that the first derivatives of all device electrical parameters are continuous as well. The current, conductances and capacitances derived in the subsequent sections are continuous and smooth functions over all modes of device operation.

Another premise of classical MOST models is that the potential at the source end of the channel, V_1 is fixed (i.e. $V_1 = 2\phi_F$). At the drain end of the channel, a similar assumption is commonly made (i.e. $V_2 = 2\phi_F + V_{ds}$). These assumptions are part of the reason why subthreshold characteristics can not be achieved from conventional MOST models. This assumption is not made for the model presented here; rather in the following section the channel

boundary potentials, V_1 (adjacent the source) and V_2 (adjacent the drain), are derived. This solution is a necessary requirement for the subthreshold diffusion current analysis. A similar solution for subthreshold current has been proposed by other authors [15, 56]. However, the diffusion current analysis of El-Mansy and Boothroyd [56] differs particularly with respect to the expression employed for mobile charge while Brews [15] did not consider the effect of substrate bias.

6.2 The Channel Boundary Potentials

The most direct route to a solution for the boundary potentials at each end of the channel, is to consider the equation of charge neutrality as derived by Pao and Sah [44] in terms of normalized effective potentials for a n-channel MOST.

$$(U_g - U_s) = \frac{C_D}{C_{ox}} \left\{ e^{\frac{U_s - \xi - U_f}{kT}} + e^{\frac{U_f - U_s}{kT}} + (U - 1) e^{\frac{U_f}{kT}} - (U_s - e^{-\xi - U_f}) e^{\frac{-\xi - U_f}{kT}} \right\} \quad (6.1)$$

For all active modes of device operation (where the surface potential is greater than the fermi potential for electrons), this equation can be simplified as shown in appendix B. When written in more common notation the result is

$$(V_g - V_s)^2 = G \frac{kT}{q} \left\{ e^{\frac{q}{kT} (V_s - \phi_n - \phi_f)} - (V_s - V_b - \frac{kT}{q}) \right\} \quad (6.2)$$

where V_g is the effective gate potential, V_s is the surface potential which must be solved for, V_b is the internal bulk potential, and ϕ_n is the electron fermi potential set by either the source or drain depending on which case is being considered. The bulk Fermi voltage

is

$$\phi_f = \frac{kT}{q} \ln \left(\frac{N}{n_i} \right) \quad (6.3)$$

$$\text{and } G = \frac{(2q\epsilon_{si} N)^{1/2}}{C_{ox}} \quad (6.4)$$

At this point, none of the potentials V_s , V_g , V_b , or ϕ_n in equation (6.2) have any specific reference voltage. It is customary though, to set $V_b = -V_{sb}$ where V_{sb} is the externally applied source to substrate bias. This establishes an internal reference for all other internal potentials. It then follows that

$$V_g = V_{gs} - \phi_{MS} + \frac{Q_{eff}}{C_{ox}} \quad (6.5)$$

It also follows that the electron quasi Fermi potential at the source ϕ_{n1} is

$$\phi_{n1} = \phi_f \quad (6.6)$$

and the corresponding electron quasi Fermi potential at the drain ϕ_{n2} is

$$\phi_{n2} = \phi_f + V_{ds} \quad (6.7)$$

Several terms in equation (6.2) are identifiable. The gate charge is proportional to $(V_g - V_s)$. The term $(V_s - V_b - \frac{kT}{q})$ is proportional to the bulk charge and the exponential accounts for the presence of mobile charge at the inverted silicon surface.

It will be demonstrated that equation (6.2) is in exact agreement with conventional MOST theory at threshold. Employing the standard approximation for surface potential, $V_s = 2\phi_f$, at the source where $\phi_n = \phi_f$, equation (6.2) reduces to the accepted threshold form of the charge neutrality equation

$$C_{ox} (V_g - V_s) = \sqrt{2q\epsilon_{si} N (V_s - V_b)} \quad (6.8)$$

where the gate charge (LHS) is equal to the standard expression for bulk charge (RHS).

One can also derive a number of partial derivatives from equation (6.2) which will be useful later in the conductance and capacitance analysis. These are

$$\frac{\partial V_s}{\partial V_g} = \frac{2 (V_g - V_s)}{A} \quad (6.9)$$

$$\frac{\partial V_s}{\partial \phi_n} = \frac{G^2 \cdot E}{A} \quad (6.10)$$

$$\frac{\partial V_s}{\partial V_b} = \frac{G^2}{A} \quad (6.11)$$

$$\text{where } A = 2(V_g - V_s) + G^2 (E + 1) \quad (6.12)$$

$$E = \exp \left\{ \frac{q}{kT} (V_s - \phi_n - \phi_f) \right\} \quad (6.13)$$

In summary, equation (6.2) is solved twice. Once at the source where $\phi_{n1} = \phi_f$ to obtain V_1 , the potential in the channel adjacent the source, plus three partial derivatives. Equation (6.2) is solved again at the drain where $\phi_{n2} = \phi_f + V_{ds}$ to obtain V_2 and the corresponding three partial derivatives.

It is not possible to solve equation (6.2) explicitly but a solution can be achieved very efficiently using Newton - Raphson iteration. In subthreshold, a solution to double precision accuracy can be obtained with only two or three iterations. Under the worst bias conditions, this method never requires more than six iterations to achieve a solution for surface potential accurate to fourteen significant figures.

6.3 Diffusion Current

6.3.1 In Subthreshold

Once the boundary voltage V_1 and V_2 have been determined, subthreshold diffusion current characteristics follow almost directly. Assuming solely diffusion current, the current continuity equation takes the form

$$I_{d \text{ diffusion}} = -W \cdot D \int_{x=0}^{x_d} \frac{dn}{dy} dx = -W \cdot D \frac{dQ_m}{dy} \quad (6.14)$$

where Q_m is the absolute value of mobile charge (cm^{-2}). Integrating with respect to y along the channel from source to drain, one obtains an equation of a form frequently encountered in bipolar transistor theory.

$$I_{d \text{ diffusion}} = \frac{W}{L} \cdot D [Q_{m1} - Q_{m2}] \quad (6.15)$$

From application of the gradual channel charge neutrality equation

$$C_{ox} (V_g - V) = Q_m + C_{ox} G \left(V - V_b - \frac{kT}{q} \right) \quad (6.16)$$

The mobile charge at the source Q_{m1} , and at the drain Q_{m2} , can be expressed as

$$Q_{m1} = C_{ox} \left[(V_g - V_1) - G \left(V_1 - V_b - \frac{kT}{q} \right) \right] \quad (6.17)$$

$$Q_{m2} = C_{ox} \left[(V_g - V_2) - G \left(V_2 - V_b - \frac{kT}{q} \right) \right] \quad (6.18)$$

Despite the apparent simplicity of the foregoing diffusion current analysis, it is in agreement with seemingly more complex analytic subthreshold models [45, 46, 55] as will be demonstrated.

From equations (6.2) with $V_s = V_1$ and (6.17), the mobile charge at the source can be expressed as

$$Q_{m1} = C_{ox} \cdot G \left\{ \left\{ \frac{kT}{q} \exp \left[\frac{q}{kT} (V_1 - \phi_{n1} - \phi_f) \right] - \left(V_1 - V_b - \frac{kT}{q} \right) \right\}^{\frac{1}{2}} - \left(V_1 - V_b - \frac{kT}{q} \right)^{\frac{1}{2}} \right\} \quad (6.19)$$

At this point, it is necessary to assume that Q_m is negligible in comparison with the bulk charge. In subthreshold, this is entirely justifiable. This assumption however, is the reason that analytic (subthreshold) diffusion current models [45, 46, 55] have validity only in subthreshold. By application of the Taylor series expansion to equation (6.19) in this limit where

$$\frac{kT}{q} \exp \left[\frac{q}{kT} (V_1 - \phi_{n1} - \phi_f) \right] \ll \left(V_1 - V_b - \frac{kT}{q} \right) \quad (6.20)$$

the mobile charge at the source end of the channel is

$$Q_{m1} = C_{ox} \cdot G \frac{\frac{kT}{q} \exp \left[\frac{q}{kT} (V_1 - \phi_{n1} - \phi_f) \right]}{2 \sqrt{V_1 - V_b - \frac{kT}{q}}} \quad (6.21)$$

Using the same mathematical method

$$Q_{m2} = C_{ox} \cdot G \frac{\frac{kT}{q} \exp \left[\frac{q}{kT} (V_2 - \phi_{n2} - \phi_f) \right]}{2 \sqrt{V_2 - V_b - \frac{kT}{q}}} \quad (6.22)$$

Since in subthreshold the mobile charge is negligible in comparison to the bulk charge at all points along the channel, it follows that the solution to the charge neutrality equation is essentially the same at either end of the channel. Hence $V_1 = V_2$. Using this relation and equations (6.6), (6.7), (6.15), (6.21) and (6.22), one finds

that in subthreshold, this analysis reduces to

$$I_{d \text{ diffusion}} = W L_1 \frac{q n_i D}{L} \frac{\exp\left[\frac{q}{kT} (V_1 - 1.5\phi_f)\right]}{\sqrt{\frac{q}{kT} (V_1 + V_{sb} - \frac{kT}{q})}} \cdot \left[1 - \exp\left(-\frac{q}{kT} V_{ds}\right) \right] \quad (6.23)$$

where L_1 is an intrinsic Debye length given by

$$L_1 = \sqrt{\frac{\epsilon_{si}}{2q n_i} \frac{kT}{q}} \quad (6.24)$$

Equation (6.23) is identical to the analytic expression derived by Masuhara [55] in 1974. If V_{sb} is set equal to zero, equation (6.23) is also identical to the analytic expression developed by Barron [45] in 1972 and later put forward by Brews [15] in 1978. Both Barron and Brews considered only the case of zero substrate bias. A similar expression was also developed by Troutman [46]. It has the form

$$I_d = W L_1 \frac{q n_i D}{L} \frac{\exp\left[\frac{q}{kT} (V_1 - 1.5\phi_f)\right] - \exp\left(-\frac{q}{kT} .5\phi_f\right)}{\sqrt{\frac{q}{kT} (V_1 + V_{sb} - \frac{kT}{q})}} \cdot \left[1 - \exp\left(-\frac{q}{kT} V_{ds}\right) \right] \quad (6.25)$$

Troutman's expression differs from equation (6.23) only with respect to the second exponential in the numerator. The purpose of the second exponential is to turn I_d off when the band bending parameter equals unity (or $V_1 = \phi_f$). In practical MOS devices however, space charge and surface generation currents dominate long before the second exponential becomes significant. Therefore it can be looked upon as a theoretical refinement and not of practical significance.

6.3.2 Above Threshold

Above threshold, equation (6.15) remains valid, but conceptually, it takes on a slightly different meaning as will be discussed. In the more active modes of device operation, the total channel current anywhere along the channel can be written as the sum of drift and diffusion current where the ratio of drift to diffusion current changes continuously from source to drain.

$$I_d = I_{\text{drift}}(y) + I_{\text{diffusion}}(y) \quad (6.26)$$

$$= W\mu Q_m \frac{dV_s}{dy} - WD \frac{dQ_m}{dy} \quad (6.27)$$

The gradual channel approximation (which is the basis for virtually all analytic MOST models) allows Q_m to be expressed as a function of V_s and independent of y . This spacial independence of Q_m allows one to write equation (6.27) in the form of a separated differential equation.

$$I_d dy = W \left[\mu Q_m - D \frac{dQ_m}{dV_s} \right] dV_s \quad (6.28)$$

The next step in any MOST model derivation is usually to integrate the current continuity equation from $y = 0$ to L . By so doing

$$I_d = \frac{W}{L} \int_{V_s=V_1}^{V_2} \left[\mu Q_m - D \frac{dQ_m}{dV_s} \right] dV_s \quad (6.29)$$

but this is equivalent to the integration of equation (6.26) with respect to y .

$$I_d = \frac{1}{L} \int_{y=0}^L I_{\text{drift}}(y) dy + \frac{1}{L} \int_{y=0}^L I_{\text{diffusion}}(y) dy \quad (6.30)$$

Thus I_d is equal to the sum of average drift and average diffusion current along the channel.

$$I_d = \overline{I_{\text{drift}}(y)} + \overline{I_{\text{diffusion}}(y)} \quad (6.31)$$

Furthermore, comparison of equations (6.29) and (6.31) shows

$$\overline{I_{\text{drift}}(y)} = \frac{W}{L} \int_{V_S=V_1}^{V_2} \mu Q_m dV_S \quad (6.32)$$

$$\overline{I_{\text{diffusion}}(y)} = -\frac{W}{L} D \int_{V_S=V_1}^{V_2} \frac{dQ_m}{dV_S} dV_S = \frac{W}{L} D [Q_{m1} - Q_{m2}] \quad (6.33)$$

Equation (6.31) (hence (6.29)) may not appear to explicitly enforce current continuity. However, this point of conceptual difficulty can be overcome when it is realized that one degree of freedom still remains, viz., the spacial distribution of V_S with respect to y . It is not necessary to know a specific $V_S(y)$ which guarantees current continuity 'a priori' because the right hand side of equation (6.28) exhibits spacial invariance. Therefore I_d can be determined without knowledge of the $V_S(y)$ which satisfies current continuity. Once I_d has been determined (from say equation (6.29)), there always exists a $V_S(y)$ which allows current continuity and satisfies the boundary conditions $V_S(0) = V_1$ and $V_S(L) = V_2$. The inverse relation $y(V_S)$ can always be determined from an integrated version of the particular current continuity equation under

consideration. In the case of equation (6.29)

$$y = \frac{W}{I_d} \int_{V_s=V_1}^{V_s(y)} \left[\mu Q_m - D \frac{dQ_m}{dy} \right] dV_s \quad (6.34)$$

Since the right hand sides of equations (6.33) and (6.15) are identical, one might also suspect that this analysis forces $I_{\text{diffusion}}(y)$ to be constant with respect to y . It has been clearly demonstrated that $I_{\text{diffusion}}(y)$ varies along the channel [44]. Specifically, for a MOST in saturation, $I_{\text{diffusion}}(y)$ increases rapidly in value approaching the high field drain region where the gradient of Q_m is large and carriers have not yet attained saturated drift velocity. Using equations (6.26), (6.27) and (6.28) one can show

$$I_{\text{diffusion}}(y) = -D \frac{dQ_m}{dV_s} \frac{I_d}{\left[\mu Q_m - D \frac{dQ_m}{dV_s} \right]} \quad (6.35)$$

Hence this analysis (equation (6.29) or (6.31)) clearly does not imply $I_{\text{diffusion}}(y)$ is constant along the channel except, of course, in subthreshold where

$$\mu Q_m \ll -D \frac{dQ_m}{dV_s} \quad (6.36)$$

for all y from $y = 0$ to L .

Therefore equations (6.29), (6.31), (6.32) and (6.33) are justifiable on the basis that a $V_s(y)$ always exists which not only assures current continuity but also embraces the correct interplay between drift and diffusion current along the channel. Thus justified, equation (6.31) brings to light a more important

realization. The first term of equation (6.31), as given by equation (6.32), is the mathematical form used to derive drain current in virtually all existing drift current MOST models. The second term of equation (6.31), as given by equation (6.33), is I_d diffusion as given by equation (6.15). Thus equation (6.31) implies that one can simply add I_d diffusion as predicted by equation (6.15) to I_d drift as predicted by most other drift current MOST models to obtain complete and continuous drain current characterization from subthreshold through all other modes of device operation. Moreover, there is no need to match drift and diffusion current solutions at threshold because the diffusion current analysis presented here remains valid at all bias levels above threshold since it accounts for the presence of mobile charge carriers in the solution to the boundary potentials.

6.4 Surface Mobility {

The complete MOST model to follow includes drift current as well as the diffusion current analysis of the previous section. The drift current analysis assumes a mobility of the form

$$\mu = \frac{\mu_0}{1 + \theta \cdot E_s} \quad (6.37)$$

where μ_0 is the low field (or zero field) mobility, and E_s is the surface (or normal) field just inside the silicon. For a MOS device,

$$E_s = \frac{C_{ox}}{\epsilon_{si}} (V_g - V_s) \quad (6.38)$$

The surface field coefficient θ has a typical value of $10^{-6} \text{ cm V}^{-1}$

for a $\langle 100 \rangle$ n-channel MOST, but can be adjusted to accommodate a range of MOS technologies.

To demonstrate that equation (6.34) has the correct general tendencies, it is compared with experimentally measured surface mobility data [57] in figure 6.1. These data were taken from a range of $\langle 100 \rangle$ n-channel devices with different ion-implant doses. In practice equation (6.34) can be fitted within 4% of measured mobility data over the full range of surface fields encountered in normal MOST operation.

This theory can be further modified to account for saturated drift velocity effects. However, Hoeneissen and Mead [8] have already demonstrated how this effect can be accounted for in the final expression for MOST drain current. The relevant modification to mobility could be adopted and carried through the model derivation, but this would be simply a duplication of their work. Therefore in an effort to minimize mathematical complexity while demonstrating the features of this theory, saturated drift velocity effects will not be considered in the following model derivation.

One might also argue that the diffusion coefficient should be treated as a surface field dependant parameter in the same manner as surface mobility. Diffusion current, however, is only of primary importance in subthreshold. Moreover, for any fixed value of V_{sb} , the change in surface field strength over the entire subthreshold bias range is relatively small, or at least not large enough to significantly alter the value of the diffusion coefficient. With this justification, the diffusion coefficient will be assumed constant with great saving in mathematic complexity of the analysis

Figure 6.1 Mobility function (equation (6.34))—compared with mobility data [57] measured from several ion-implanted, $\langle 100 \rangle$, n-channel MOSTs. For the mobility function, $\mu_{01} = 800 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}$ and $\theta = 1.07 \cdot 10^{-6} \text{ cm V}^{-1}$.

to follow. If one further assumes the Einstein relation at threshold then the value of the diffusion coefficient is given by

$$D = \frac{kT}{q} \mu_t \quad (6.39)$$

where the threshold value of surface mobility μ_t is obtained from equations (6.8) and (6.37) as

$$\mu_t = \frac{\mu_0}{1 + \frac{\theta}{\epsilon_{si}} \sqrt{2q\epsilon_{si} N (2\phi_f - V_b)}} \quad (6.40)$$

6.5 Drain Current and the Conductances

In n-channel MOS transistors, source-drain current arises due to the flow of electrons in the y direction parallel to the silicon surface. At any point within the surface inversion layer, the electron current density can be written as the sum of drift and diffusion current components:

$$J_n(x,y) = -q \left[\mu_n \frac{dV_s}{dy} + D \frac{dn}{dy} \right] \quad (6.41)$$

Integrating across the channel in the x direction, the total source-drain current at any point from source to drain is

$$I_d = -W \int_{x=0}^{x_d} J_n(x,y) dx \quad (6.42)$$

$$= W \left[\mu Q_m \frac{dV_s}{dy} - D \frac{dQ_m}{dy} \right] \quad (6.43)$$

Inherent to the previous integration is the justifiable assumption that $\frac{dV_s}{dy}$ is constant in the x direction. It also assumes that D and μ are constant with respect to x, which is not rigorously

2

correct. However, in view of the surface scattering and quantum mechanical considerations as discussed earlier, it is reasonable to consider D and μ as weighted averages over the surface channel from this point on. Further assuming the gradual channel approximation and uniform substrate doping, it follows that the absolute value of mobile charge can be expressed as

$$Q_m = C_{ox} \left[(V_g - V_s) - G \sqrt{V_s - V_{bx}} \right] \quad (6.44)$$

$$\text{where } V_{bx} = V_b + \frac{kT}{q} \quad (6.45)$$

Surface mobility as given by equation (6.37), can be written as

$$\mu = \frac{\mu_0}{1 + Z (V_g - V_s)}$$

$$\text{where } Z = \theta \frac{C_{ox}}{\epsilon_{si}} \quad (6.47)$$

Then the integration of the current continuity equation (6.43) along the channel from $y = 0$ to L can be written in the form of a definite integral with respect to surface potential.

$$I_d = \int_{V_s=V_1}^{V_2} f(V_s) dV \quad (6.48)$$

where integrand is

$$f(V_s) = \frac{\mu_0 C_{ox} W}{L} \left\{ \frac{(V_g - V_s) - G \sqrt{V_s - V_{bx}}}{1 + Z(V_g - V_s)} + \frac{D}{\mu_0} \left[1 + \frac{G}{\sqrt{V_s - V_{bx}}} \right] \right\} \quad (6.49)$$

After carrying out the integration, the resulting expression for drain current is

$$I_d = \frac{\mu_o C_{ox} W}{L Z} \left\{ V_s \left(1 + \frac{D}{\mu_o} Z \right) + \frac{\ln (1 + Z (V_g - V_s))}{Z} \right. \\ \left. + G \left[\sqrt{V_s - V_{bx}} \left(2 + \frac{D}{\mu_o} Z \right) - r(V_s) \sqrt{\frac{1}{Z} + (V_g - V_{bx})} \right] \right\} \quad \begin{matrix} V_2 \\ V_1 \end{matrix} \quad (6.50)$$

$$\text{where } r(V_s) = \ln \left\{ \frac{\sqrt{\frac{1}{Z} + (V_g - V_{bx})} + \sqrt{V_s - V_{bx}}}{\sqrt{\frac{1}{Z} + (V_g - V_{bx})} - \sqrt{V_s - V_{bx}}} \right\} \quad (6.51)$$

The right hand side of equation (6.50) must be evaluated at the boundary voltages V_1 and V_2 as indicated. The predicted $I_d - V_{ds}$ characteristics are demonstrated in figure 6.2 for a range of gate bias levels. For this demonstration and the other device characteristics plotted in figures 6.3 through 6.10, the device data given in table 6.1 have been assumed. As is evident from figure 6.2, the drain conductance in saturation is zero because this model in its present form does not account for channel length modulation. One of a number of drain section models [19, 58-60] could be adopted to account for this mechanism however.

A good check of any MOST model, to plot the square root of saturated drain current against gate voltage (with $V_{gs} = V_{ds}$) for a number of different substrate bias levels. When MOS transistors are measured in this manner, the result is a family of virtually straight lines which do not converge with increasing gate bias. The results predicted by this analysis are shown in figure 6.3.

The subthreshold drain current characteristics are plotted down to 10^{-16} amperes in figure 6.4. In subthreshold, the logarithmic slope of I_d is virtually constant. As V_{sb} is decreased from five

Table 6.1 Data for Long Channel MOST Model Demonstration
(figures 6.2 to 6.13)

n-channel, n-doped polysilicon gate MOST

Parameter	Value	Units
W	100	μm
L	100	μm
t_{ox}	100	nm
μ_0	850	$\text{cm}^2 \text{V}^{-1} \text{sec}^{-1}$
θ	$8.0 \cdot 10^{-7}$	cm V^{-1}
N	$4.0 \cdot 10^{15}$	cm^{-3}
N_{ss}	$5.0 \cdot 10^{10}$	cm^{-2}

Figure 6.2 I_d versus V_{ds} characteristics predicted by equation (6.50).

Figure 6.3 Square root of drain current in saturation, equation (6.50), as a function of gate voltage for several different substrate bias voltages.

Figure 6.4 Subthreshold characteristics for several different substrate bias levels as predicted by equation (6.50).

to zero volts, the logarithmic slope is shown to decrease due to the increasing depletion capacitance.

The ratio of I_d diffusion (equation (6.15)) to I_d is plotted in figure 6.5 for increasing V_{gs} . It is evident that drift current dominates in saturation, while in subthreshold, I_d arises almost entirely from diffusion current transport. The transition between these two extremes, however, is not abrupt. Moreover, this figure implies that some error would result if the diffusion current above threshold was neglected, particularly for V_{gs} in the range of $V_t < V_{gs} < V_t + 0.5$ volts.

By definition, drain conductance g_d is the first derivative of drain current with respect to drain voltage. Physically though, the drain sets the quasi Fermi level for electrons and the surface potential in the channel adjacent the drain adjusts itself accordingly. To capture this mechanism mathematically, the drain conductance must be expressed as the product of two derivatives

$$g_d \equiv \frac{dI_d}{dV_d} = \frac{dI_d}{d\phi_{n2}} = \frac{dI_d}{dV_2} \cdot \frac{\partial V_2}{\partial \phi_{n2}} \quad (6.52)$$

$$\text{where } \frac{dI_d}{dV_2} = f(V_2) \quad (6.53)$$

and $f(V_2)$ is equation (6.49) evaluated at V_2 . The second derivative is given by equation (6.10) and it has already been evaluated as part of the solution to the boundary voltages. This output conductance is plotted in figure 6.6 against V_{gs} with $V_{ds} = 0$ volts. The slope is an ever decreasing function of V_{gs} due to the decreasing surface mobility with increasing gate voltage.

In the same manner, a conductance with respect to source bias

Figure 6.5 Percentage $I_{diffusion}$, equation (6.15), to total drain current, equation (6.50).

Figure 6.6 Output conductance, equation (6.52), as a function of gate voltage for $V_{ds} = 0$ volts.

can be derived.

$$g_s \equiv \frac{dI_d}{dv_{\text{source}}} = \frac{dI_d}{d\phi_{nl}} = \frac{dI_d}{dv_1} \cdot \frac{\partial v_1}{\partial \phi_{nl}} \quad (6.54)$$

$$\text{where } \frac{dI_d}{dv_1} = -f(V_1) \quad (6.55)$$

and the second partial derivative is given by equation (6.10).

The transconductance can be derived in a manner to reflect the change in boundary voltages with gate bias.

$$g_m \equiv \frac{dI_d}{dv_g} = \frac{\partial I_d}{\partial v_g} + \frac{\partial I_d}{\partial v_1} \cdot \frac{\partial v_1}{\partial v_g} + \frac{\partial I_d}{\partial v_2} \cdot \frac{\partial v_2}{\partial v_g} \quad (6.56)$$

Of all the above partial derivatives, only one remains to be determined.

$$\frac{\partial I_d}{\partial v_g} = \frac{\mu_o C_{ox} W}{L} \left\{ \left[\frac{1}{1 + Z(V_g - V_s)} + G \frac{Z\sqrt{V_s - V_{bx}}}{1 + Z(V_g - V_s)} - \frac{r(V_s)}{2\sqrt{\frac{1}{Z} + (V_g - V_{bx})}} \right] \right\} \quad (6.57)$$

The plotted g_m as shown in figure 6.7, has a slight decrease in slope in the saturation mode with increasing gate bias. The transconductance in the linear mode is an ever decreasing function of V_{gs} . Both these effects result from the decreasing surface mobility with increasing gate bias. Like all other electrical parameters, g_m is a continuous and smooth function throughout all modes of MOST operation.

A conductance can also be derived with respect to substrate bias.

$$g_b \equiv \frac{dI_d}{dv_b} = \frac{\partial I_d}{\partial v_1} \cdot \frac{\partial v_1}{\partial v_b} + \frac{\partial I_d}{\partial v_2} \cdot \frac{\partial v_2}{\partial v_b} + \frac{\partial I_d}{\partial v_b} \quad (6.58)$$

Figure 6.7 Transconductance predicted by equation (6.56) as a function of gate voltage.

Figure 6.8 Conductance g_b predicted by equation (6.58) for several drain bias voltages.

$$\text{where } \frac{\partial I_d}{\partial V_b} = \frac{\mu_o C_{ox} W}{L} G \left\{ -\frac{D}{\mu_o 2 \sqrt{V_s - V_{bx}}} + \frac{r(V_s)}{2 \cdot Z \sqrt{\frac{1}{Z} + (V_g - V_{bx})}} \right\} \quad \begin{matrix} V_2 \\ V_1 \end{matrix} \quad (6.59)$$

In figure 6.8, g_b is plotted against gate voltage for a number of drain bias levels. The linear mode decrease in g_b with increasing V_{gs} is due to the decreasing surface mobility with increasing surface field. The decreasing slope of g_b in saturation is indicative of decreasing capacitive coupling between the substrate and drain end of the channel.

6.6 The Gate Capacitances

The current which must flow into the gate terminal of an MOST in response to an incremental change in bias conditions, can be described in terms of several gate capacitances. In order to evaluate these capacitances, it is first necessary to have an expression for total gate charge. This can be achieved by simply integrating the D field along the channel and multiplying the result by channel width.

Hence

$$Q_g = W \int_{y=0}^L C_{ox} (V_g - V_s) dy = W \int_{V_s=V_1}^{V_2} C_{ox} (V_g - V_s) \frac{dy}{dV_s} dV_s \quad (6.60)$$

From the current continuity equation (6.41), one can obtain

$$\frac{dy}{dv_s} = \frac{\mu_o C_{ox} W}{I_d} \left\{ \frac{(V_g - V_s) - G\sqrt{V_s - V_{bx}}}{1 + Z(V_g - V_s)} + \frac{D}{\mu_o} \left[1 + \frac{G}{2\sqrt{V_s - V_{bx}}} \right] \right\} \quad (6.61)$$

Letting

$$Fq(V_s) = W \mu_o C_{ox} (V_g - V_s) \frac{dy}{dv_s} \quad (6.62)$$

Then Q_g can be expressed as a definite integral with respect to surface potential.

$$Q_g = \int_{V_s=V_1}^{V_2} Fq(V_s) dv_s \quad (6.63)$$

and after performing the integration,

$$Q_g = \frac{W^2 C_{ox}^2 \mu_o}{I_d} \left\{ V_s \left[\left(\frac{D}{\mu_o} + \frac{1}{Z} \right) (V_g - \frac{V_s}{2}) - \frac{1}{Z^2} \right] - \frac{\ln[1 + Z(V_g - V_s)]}{Z^3} \right. \\ + G \left[\sqrt{V_s - V_{bx}} \left[\frac{D}{\mu_o} (V_g - \frac{V_{bx}}{2}) - \left(\frac{D}{\mu_o} + \frac{2}{Z} \right) \frac{(V_s - V_{bx})}{3} - \frac{2}{Z^3} \right] \right. \\ \left. \left. + \frac{r(V_s) \sqrt{\frac{1}{Z} + (V_g - V_{bx})}}{Z^2} \right] \right\} \quad (6.64)$$

The total gate capacitance C_g is commonly defined as the first derivative of gate charge with respect to gate voltage. By appropriate application of the chain rule,

$$C_g \equiv \frac{dQ_g}{dV_g} = \frac{\partial Q_g}{\partial V_1} \cdot \frac{\partial V_1}{\partial V_g} + \frac{\partial Q_g}{\partial V_2} \cdot \frac{\partial V_2}{\partial V_g} + \frac{\partial Q_g}{\partial V_g} - \frac{g_m}{I_d} \cdot Q_g \quad (6.65)$$

$$= -Fq(V_1) \cdot \frac{\partial V_1}{\partial V_g} + Fq(V_2) \cdot \frac{\partial V_2}{\partial V_g} + \frac{\partial Q_g}{\partial V_g} - \frac{g_m}{I_d} \cdot Q_g \quad (6.66)$$

The terms $F_q(V_1)$ and $F_q(V_2)$ are given by equation (6.62) evaluated at V_1 and V_2 respectively. All other partial derivations have been previously determined except

$$\frac{\partial Q_g}{\partial V_g} = \frac{W^2 C_{ox}^2 \mu_o}{I_d} \left\{ V_s \left(\frac{D}{\mu_o} + \frac{1}{Z} \right) - \frac{1}{Z [1 + Z(V_g - V_s)]} \right. \\ \left. + G \left[\frac{D}{\mu_o} \sqrt{V_s - V_{bx}} - \frac{\sqrt{V_s - V_{bx}}}{Z [1 + Z(V_g - V_s)]} + \frac{r(V_s)}{2Z^2 \sqrt{\frac{1}{2} + (V_g - V_{bx})}} \right] \right\} \quad (6.67)$$

Equation (6.66) is defined as long as a current carrying channel exists. Consequently C_g can be plotted down to the weakest range of subthreshold operation as demonstrated in figure 6.9. For this device $V_t = 0.4$ volts. In strong inversion, C_g approaches the oxide capacitance value. Throughout all modes of device operation, C_g is a smooth, and continuous function.

For computer-aided-design applications, C_g must be broken down into three components; the gate to source capacitance C_{gs} , the gate to drain capacitance C_{gd} and the gate to bulk capacitance C_{gb} . By taking the appropriate derivatives of total gate charge,

$$C_{gs} = - \frac{dQ_g}{dV_{source}} = - \frac{\partial Q_g}{\partial V_1} \cdot \frac{\partial V_1}{\partial \phi_{n1}} + \frac{g_s}{I_d} \cdot Q_g \quad (6.68)$$

$$= F_q(V_1) \cdot \frac{\partial V_1}{\partial \phi_{n1}} + \frac{g_s}{I_d} \cdot Q_g \quad (6.69)$$

$$C_{gd} = - \frac{dQ_g}{dV_d} = - \frac{\partial Q_g}{\partial V_2} \cdot \frac{\partial V_2}{\partial \phi_{n2}} + \frac{g_d}{I_d} \cdot Q_g \quad (6.70)$$

Figure 6.9 Total gate capacitance predicted by equation (6.66) as a function of V_{gs} for several drain bias voltages.

$$= -F_q(V_2) \cdot \frac{\partial V_2}{\partial \phi_{n2}} + \frac{g_d}{I_d} \cdot Q_g \quad (6.71)$$

$$C_{gb} \equiv -\frac{dQ_g}{dV_b} = -\frac{\partial Q_g}{\partial V_1} \cdot \frac{\partial V_1}{\partial V_b} - \frac{\partial Q_g}{\partial V_2} \cdot \frac{\partial V_2}{\partial V_b} - \frac{\partial Q_g}{\partial V_b} + \frac{g_b}{I_d} \cdot Q_g \quad (6.72)$$

$$= F_q(V_1) \cdot \frac{\partial V_1}{\partial V_b} - F_q(V_2) \cdot \frac{\partial V_2}{\partial V_b} - \frac{\partial Q_g}{\partial V_b} + \frac{g_b}{I_d} \cdot Q_g \quad (6.73)$$

where,

$$\frac{\partial Q_g}{\partial V_b} = \frac{w^2 C_{ox}^2 \mu_o}{I_d} \cdot G \left\{ \left(\frac{1}{Z} - \frac{D}{2\mu_o} \right) \sqrt{V_s - V_{bx}} + \frac{D}{\mu_o} \frac{(V_g - V_{bx})}{2\sqrt{V - V_{bx}}} \right. \\ \left. \frac{r(V_s)}{2Z^2 \sqrt{\frac{1}{Z} + (V_g - V_{bx})}} \right\} \quad (6.74)$$

V_2
 V_1

These capacitance functions are demonstrated in figure 6.10 for $V_{ds} = 1.0$ volt and $V_{sb} = 1.0$ volt. C_{gb} is shown to be a decreasing function of gate voltage. Deep in subthreshold, C_{gb} accounts for all the gate capacitance. In the limit of strong surface inversion where the surface inversion layer screens the gate from the bulk, C_{gb} approaches zero. C_{gs} rises from zero just below threshold and attains a maximum of approximately $(2/3) C_{ox}$ at the transition from saturation to the linear mode. In the linear mode, C_{gs} is shown to be a monotonically decreasing function of V_{gs} . C_{gd} rises from zero at the transition from saturation to the linear mode of device operation. In the limit of strong inversion, both C_{gd} and C_{gs} converge to one-half C_{ox} .

Figure 6.10 Gate to source capacitance C_{gs} , gate to drain capacitance C_{gd} , and gate to substrate capacitance C_{gb} as predicted by equations (6.69), (6.71) and (6.73) respectively for $V_{ds} = 1.0$ volt.

6.7 Comparison with Experiment

The output conductance, transconductance and gate capacitance predicted by this analysis is compared with experimentally measured data in figures 6.11, 6.12 and 6.13. These measured MOST characteristics were reported by El-Mansy and Boothroyd [56] along with the device information listed in table 6.2. In order to calibrate the mobility function of this analysis (equation (6.37)), values of θ and μ_0 were chosen to achieve a reasonable match between the theoretical and experimental output conductance at $V_{ds} = 0$ volts. In figure 6.13, the theoretical gate capacitance has been adjusted upwards to account for source and drain overlap capacitance and other parasitic capacitance present in the experimental C_g data.

Good agreement between theory and experiment is also demonstrated in figure 6.14 for current levels ranging over 6 orders of magnitude. The experimental data was reported by Troutman and Chakravarki [46] for a $L = 8.0 \mu\text{m}$, n-channel MOST. For this comparison, their values of $t_{ox} = 57.0 \text{ nm}$, $N = 5.6 \times 10^{15} \text{ cm}^{-3}$, $L = 8.0 \mu\text{m}$ and $W = 102.0 \mu\text{m}$ were employed. A value of $\mu_0 = 800 \text{ cm}^2 \text{ V}^{-1} \text{ sec}^{-1}$ was selected to match the high current region for $V_{sb} = 0$ volts, and a typical value of the surface field coefficient, $\theta = 10^{-6} \text{ cm V}^{-1}$ was assumed.

6.8 Conclusion

A versatile MOST model has been presented and experimentally verified. Unlike the more classical approach, the surface mobility

Table 6.2 Data for Device D-2

n-channel, metal gate MOST [†]

Parameter	Data from [†] Processing	Data Used in Calculation	Units
N	$(7 \text{ to } 10) \cdot 10^{15}$	$7 \cdot 10^{15}$	cm^{-3}
t_{ox}	105 to 110	110	nm
W	100	100	μm
L	97.5	97.5	μm
μ_0	—	630	$\text{cm}^2 \text{V}^{-1} \text{sec}^{-1}$
θ	—	$1.04 \cdot 10^{-6}$	cm V^{-1}

[†] Data reported by El-Mansy and Boothroyd [56].

Figure 6.11 Comparison of experimental measured MOST output conductance [56] with theory, equation (6.52).

Figure 6.12 Comparison of experimentally measured MOST transconductance [56] with theory, equation (6.56).

Figure 6.13 Comparison of measured MOST gate capacitance [56] with theory, equation (6.66).

Figure 6.14 MOST current characteristics in subthreshold predicted by theory, equation (6.50) compared with experimental measurement [46].

dependance on the normal field is assumed at the onset of the model derivation and properly taken into account in the basic current continuity relation. Close agreement between the mobility function and mobility data has been demonstrated. Moreover, the mobility function can be adjusted to accomodate a range of MOST technologies. Diffusion current analysis is included as an integral part of the model. However, this diffusion current analysis can be incorporated into most other drift current MOST models to obtain continuous drain current characterization down to the weakest range of subthreshold operation without the traditional problem of matching solutions at threshold. Expressions are derived for I_d , g_m , g_d , g_s , g_b , C_g , C_{gs} , C_{gd} and C_{gb} . These expressions are shown to be continuous and smooth functions of the MOST terminal voltages from subthreshold through the linear and saturation modes. All model parameters are defined in terms of common process and device characterization data. These features make this MOST model particularly suitable for CAD applications.

CHAPTER 7

DIFFUSION CURRENT IN THE DMOST

The DMOST exhibits SCMOST characteristics as a direct consequence of the nonuniform doping along its channel. Although the gradient of the channel impurity distribution profoundly influences DMOST operation, it has been largely ignored in the interests of achieving simple analytic models for CAD. As an example, one DMOST model treats the device as a uniformly doped, enhancement SCMOST in series with a longer channel, depletion MOST [41, 61].

Another model considers the DMOST simply as a uniformly doped, enhancement SCMOST in series with a resistor [62]. In such models, it is often necessary to use a value of substrate doping in the SCMOST which not only contradicts the threshold voltage calculation but which is, significantly less than average doping in the corresponding short channel region of the DMOST. This is necessary in order to obtain satisfactory agreement with the measured device characteristics.

Numerical 1-D simulations of the DMOST have been performed [63, 64]. While these take the varying impurity profile into account, they consider only drift current and thus neglect diffusion current. Recently, a 2-D simulation was reported and used to demonstrate the rise of DMOST gate capacitance above the oxide capacitance value [65]. The following analysis will show that the gradient of the channel impurity profile gives rise to diffusion current which is important to the overall device operation.

In the DMOST, the source and drain diffusions are generally much deeper than the channel depletion region at least near the source. For simulation purposes then it is reasonable to assume that the substrate doping is uniform in the direction perpendicular to the silicon surface and varies only in the lateral y direction. Thus substrate doping will be considered solely y dependant. Since the substrate potential and the flatband voltage are direct functions of substrate doping, both are y dependant as well. Typically, the impurity density changes by two orders of magnitude along the channel with a maximum between 10^{16} and 10^{17} cm^{-3} near the source. On this basis alone, surface mobility can be expected to change by at least 20% along the channel. Thus mobility is also y dependant. Furthermore, current carriers can approach saturated drift velocity in the graded impurity portion of the channel even under moderate bias condition. This mechanism serves to limit channel current in the presence of high lateral fields. Consequently a lateral field and doping dependant representation of surface mobility is essential for accurate DMOST simulation.

7.1 Device Structure.

The DMOST chosen for analysis is illustrated in figure 3.3. This structure is typical of an n-channel silicon gate DMOST intended for low voltage digital applications. An oxide thickness of 100.0 nm and a bulk substrate doping of $0.5 \times 10^{15} \text{ cm}^{-3}$ are assumed.

When appropriately biased, an n-channel DMOST exhibits short channel MOST characteristics due to a high concentration of p-type

impurities in the channel adjacent the source. This is achieved by the successive diffusion of p and n-type impurities from the source contact window. It is known that diffusion of a single impurity type from a stationary contact window results in a complementary error function distribution in the lateral direction [66]. For the DMOST, double diffusion case, the following representative double complementary error function distribution was chosen.

$$N_c(y) = 2.0 \times 10^{19} \operatorname{erfc}\left(\frac{y}{.25}\right) + 7.0 \times 10^{16} \operatorname{erfc}\left(\frac{y}{1.25}\right) + 0.5 \times 10^{15} \text{ cm}^{-3} \quad (7.1)$$

where y is the distance in micrometers along the channel from the source contact window. This impurity profile, as shown in figure 7.1, has a p-n metallurgical junction at $y = 0.56 \mu\text{m}$ and a maximum of $N_{c \text{ max}} = 2.9 \times 10^{16} \text{ cm}^{-3}$ at $y = 0.68 \mu\text{m}$. Beyond $y = 3.0 \mu\text{m}$, the distribution is essentially uniform. Although the physical distance between source and drain contact windows is $10.0 \mu\text{m}$, lateral diffusion of source and drain impurities reduces the actual channel length to $8.8 \mu\text{m}$. Assuming $Q_{\text{eff}} = 0$ and $N = N_{c \text{ max}}$, the standard threshold voltage calculation yields $V_t = 2.3$ volts.

7.2 Analysis of the DMOST

To study the effects of the lateral impurity variation, a simple numerical model was developed based on the following equations of current continuity.

$$J_{\text{channel}} = J_{\text{drift}} + J_{\text{diffusion}} \quad (7.2)$$

Figure 7.1 DMST channel impurity profile.

$$J_{\text{drift}} = \mu Q_m E \quad (7.3)$$

$$J_{\text{diffusion}} = D \frac{dQ_m}{dy} = D \left[\frac{\partial Q_m}{\partial y} - \frac{\partial Q_m}{\partial V_s} \cdot E \right] \quad (7.4)$$

$$E = - \frac{dV_s}{dy} \quad (7.5)$$

$$D = \frac{kT}{q} \mu \quad (7.6)$$

The partial derivative of Q_m with respect to y in equation (7.4) represents a significant departure from conventional MOST theory. As will become evident, this term dominates the diffusion current in the graded portion of the DMOST channel.

The following simple expression for surface mobility was developed for ease of implementation in initial value problems of the type described here,

$$\mu = \frac{\mu_0}{\sqrt{1 + \frac{N_c}{(N_c/350) + 3.0 \times 10^{16}} + \frac{\mu_0 |E|}{v_s}}} \quad (7.7)$$

Equation (7.7) is essentially a composite of other expressions for mobility which have either electric field dependence [8] or doping dependence [40]. Like the mobility expression of Scharfetter and Gummel [67], equation (7.7) allows carriers to reach the same value of saturated drift velocity v_s regardless of the doping level. Unlike their expression, v_s can be independently set. For this analysis, $v_s = 6.0 \times 10^6 \text{ cm sec}^{-1}$ [68] and the low field value of mobility $\mu_0 = 850 \text{ cm}^2 \text{ v}^{-1} \text{ sec}^{-1}$.

The mobile channel charge density is given by

$$Q_m = C_{ox} (V_{gs} - V_{fb} - V_{sb} - (V_s - V_b)) - \sqrt{2q\epsilon_{si}N_c (V_s - V_b - \frac{kT}{q})} \quad (7.8)$$

In this analysis the surface potential V_s is measured with respect to the source instead of the substrate because the internal substrate potential V_b is itself a function of the channel impurity density. Using Boltzman statistics, the flatband voltage V_{fb} and substrate potential are defined as

$$V_{fb} = -\frac{kT}{q} \ln \left\{ \frac{N_c N_g}{n_i^2} \right\} - \frac{Q_{eff}}{C_{ox}} \quad (7.9)$$

$$V_b = -V_{sb} - \frac{kT}{q} \ln \left\{ \frac{N_c N_{sr}}{n_i^2} \right\} \quad (7.10)$$

where N_g , N_{sr} and n_i represent the n-type impurity density in the poly-silicon gate, the impurity density at the source surface and the intrinsic carrier concentration respectively. The quantity V_{sb} is the externally applied source to substrate bias (hence $V_{sb} \geq 0$). Since a uniform sheet of oxide charge Q_{eff} merely causes a negative shift in V_{gs} , it will be simply neglected for the purposes of this analysis. The channel charge density Q_m as written in equation (7.8) has direct dependance on $N_c(y)$ and indirect dependance on $N_c(y)$ through V_{fb} and V_b .

The numerical simulation was implemented as an initial value problem beginning at the source end of the channel ($y_{sc} = 0.6 \mu m$). At this point, the initial channel potential was determined by numerical solution of the equation

$$\begin{aligned} & C_{ox}^2 [V_{gs} - V_{fb} - V_{sb} - (V_s - V_b)]^2 \\ & = (2q\epsilon_{si} N_c) \left\{ (V_s - V_b - \frac{kT}{q}) + \left(\frac{n_i}{N_c} \right)^2 \frac{kT}{q} \exp \left[\frac{q}{kT} (V_s - V_b - V_{sb}) \right] \right\} \end{aligned} \quad (7.11)$$

which is derived in the appendix B. At the drain, V_{ds} can be calculated from

$$V_{ds} = V_s - V_b - V_{sb} - \frac{kT}{q} \ln \left(\frac{N_c}{2\epsilon_{si} k T n_i^2} \right) - \frac{kT}{q} \ln \left\{ C_{ox}^2 [V_{gs} - V_{fb} - V_{sb} - (V_s - V_{sb})]^2 - (2q\epsilon_{si} N_c) (V_s - V_b - \frac{kT}{q}) \right\} \quad (7.12)$$

This relation is easily obtained from equation (B.16) of appendix B. For normal DMOST operation, the substrate is electrically connected to the source, so for this analysis $V_{sb} = 0$ in equation (7.9), (7.10), (7.11) and (7.12). The numerical solution was achieved using the second order Euler predictor-corrector method with variable step size (maximum step = 0.1 μm).

7.3 Simulation Results

Short channel properties arise when the DMOST is operated at gate voltages close to threshold. Simulation results in this mode where $V_{gs} = 2.6$ volts are shown in figures 7.2 through 7.5.

The progression from triode to short channel saturation is demonstrated using five channel current levels ranging from zero to $J_c = 12.6537 \text{ mA cm}^{-1}$. All currents shown are normalized with respect to channel width.

The channel potential as plotted in figure 7.2 shows that the majority of channel potential drop occurs just after the point of maximum doping in the channel, $N_{c \text{ max}}$. This result is, at odds with

Figure 7.2 DMOST surface potential distribution for $V_{gs} = 2.6$ volts.

two circuit element DMOST models which imply that the greatest channel electric fields should always arise near the end of the graded region, where $y = 3.0 \mu\text{m}$. Beyond $y = 3.0 \mu\text{m}$, the almost linear voltage profile is indicative of strong triode mode behaviour over the uniformly doped portion of the channel.

An important consequence of the simulation is the demonstration that the channel potential varies along the DMOST channel even when $J_c = 0$. The channel potential reaches a minimum at the point where the channel impurity density reaches its maximum. At this point, V_b attains its minimum value and the high doping further reduces the potential drop across the surface depletion layer. Specifically, it is the inclusion of the diffusion component that allows this result. Otherwise, a zero channel current would imply

$$0 = J_c = J_{\text{drift}} = \mu Q_m E \quad (7.13)$$

and a constant surface potential would result. This is clearly not the case in the DMOST. This argument can also be applied to show that the electric field along the channel can never be positive unless the diffusion current is considered in the current equations.

Actually when $J_c = 0$, the electric field strength on either side of $N_c \text{ max}$ exceeds 10^3 V cm^{-1} as shown in figure 7.3. At $N_c \text{ max}$ where the gradient of the impurity profile is zero, the electric field is also zero. At the source end of the channel where there is a high positive gradient in the channel impurity profile, the electric field opposes channel current flow for all channel current levels shown. This is an indication that diffusion current is the mechanism responsible for transport of carriers into the channel from the source and not drift current.

Figure 7.3 Electric field along the DMOST channel for $V_{gs} = 2.6$ volts.

In conventional MOST theory, the point of maximum electric field strength in the channel is stationary and coincides with the point of saturation in the channel. For short channel DMOST operation, this point is shown to move with changing bias conditions. At $J_c = 0$, $(-E)_{\max}$ occurs at $y = .77 \mu\text{m}$ while at the onset of saturation where $J_c = -12.653 \text{ mA cm}^{-1}$, $(-E)_{\max}$ occurs at $y = 1.19 \mu\text{m}$. It is also evident that the electric field does not decrease abruptly after the occurrence of $(-E)_{\max}$. Rather, for channel currents of $J_c = 0$ through $-12.653 \text{ mA cm}^{-1}$, the majority of short channel potential drop occurs after $(-E)_{\max}$ and not before it.

The high electric fields which occur over the graded portion of the DMOST channel give rise to drift current. When $J_c = 0$, this drift current is counteracted by an equal and opposite diffusion current. For $J_c = 0$, diffusion currents exceeding 10 mA cm^{-1} are shown on both sides of $N_{c \max}$ in figure 7.4. At $N_{c \max}$, both the drift and the diffusion currents are zero. At the source end of the channel and for all current levels shown, the ratio $J_{\text{diffusion}} / J_c > 1$. Therefore, the drift current near the source opposes current flow and it is diffusion current which transports carriers into the channel from the source. This situation is analogous to low level current transport across a p-n junction in which the quasi Fermi levels remain virtually constant in the presence of extremely high but opposing drift and diffusion currents.

Beyond the point where $(-E)_{\max}$ occurs and where a negative gradient in N_c exists, the diffusion current opposes J_c . For $J_c = -11.0 \text{ mA cm}^{-1}$, the diffusion current reaches a maximum of 8.1 mA cm^{-1} at $y = 1.35 \mu\text{m}$, and so drift current exceeds J_c by 74%. Similarly

Figure 7.4 Diffusion current along the DMOST channel for $V_{gs} = 2.6$ volts.

high levels of opposing diffusion current are also evident for the other current levels. Thus the inclusion of the diffusion current in the continuity equations has the net effect of predicting higher drift currents over the negatively graded portion of the channel. Thus for a given channel current, higher electric fields result. Stated inversely, this diffusion current tends to reduce the drain current for a given set of bias conditions as does velocity saturation of mobile carriers.

The channel charge density, for all current levels shown in figure 7.5, reaches a minimum $Q_{m\min}$, at the point where the diffusion current in figure 7.4 equals zero. For channel current levels $J_c = -11.0 \text{ mA cm}^{-1}$ through $-12.6537 \text{ mA cm}^{-1}$, $Q_{m\min}$ occurs at virtually the same point as $(-E)_{\max}$ shown in figure 6.3. This unfortunately is not true at lower current levels as comparison of figures 7.3 and 7.5 for $J_c = 0$ reveals. At the onset of saturation where $J_c = 12.6537 \text{ mA cm}^{-1}$, $Q_{m\min} = 3.7 \times 10^{-9} \text{ coul cm}^{-2}$ while the limiting value determined by velocity saturation of carriers is $|J_c/v_s| = 2.1 \times 10^{-9} \text{ coul cm}^{-2}$. Numerical simulations of the type described here become intractable in the limit of absolute saturation, but it is reasonable to conclude that for all practical purposes; carriers attain saturated drift velocity over the graded portion of the channel when the DMOST is biased in the short channel saturation mode.

The channel electric field, for the higher current level of $J_c = -27.8 \text{ mA cm}^{-1}$ and gate voltage of $V_{gs} = 2.9, 3.0$ and 4.0 volts, is plotted in figure 7.6. At $V_{gs} = 2.9$ volts, the channel electric field is shown to exceed 10^4 V cm^{-1} at the drain end of the channel as well as over the negatively graded portion of the DMOST channel.

Figure 7.5 Inversion charge density Q_m along the DMOSF channel for $V_{gs} = 2.6$ volts.

Figure 7.6 Electric field along the DMOST channel for $J_c = -27.8 \text{ mA cm}^{-1}$.

Thus saturation is about to occur simultaneously at two locations along the channel. As the gate voltage is lowered from $V_{gs} = 4.0$ to 2.9 volts, the position of the first $(-E)_{max}$ is again shown to move slightly down channel, but at the onset of saturation $(-E)_{max}$ still remains a substantial distance away from the end of the graded region (where $y = 3.0 \mu m$).

The channel potential as plotted in figure 7.7 shows that the majority of channel potential drop occurs over the uniformly doped portion of the channel at this current level. Beyond $y = 3.0 \mu m$, the potential profile is typical of conventional MOST theory.

The interplay between drift and diffusion current as described with respect to short channel operation qualitatively applies to the results shown in figure 7.8 for $J_c = -27.8 \text{ mA cm}^{-1}$. The rise in diffusion current at the drain end of the channel, evident for $V_{gs} = 2.9$ volts, is typical of normal MOS transistors approaching saturation [44]. For $V_{gs} = 4.0$ volts, the diffusion current opposes 32% of the net channel current at $y = 1.2 \mu m$. Thus drift current must exceed J_c by 32% at this point. Comparison of this value with the much higher value determined from the short channel simulation results leads to the following conclusion. As J_c is increased, the significance of the diffusion current diminishes since it then comprises a smaller fraction of the total channel current.

The DMOST transconductance in saturation $g_{m \text{ sat}}$ and the drain to source conductance g_d for $V_{ds} = 0$, were determined using the following incremental relations.

Figure 7.7 DMOST surface potential distribution for $J_c = -27.8 \text{ mA cm}^{-1}$.

Figure 7,8 Diffusion and drift current along the DMOST channel for $J_c = -27.8 \text{ mA cm}^{-1}$.

$$\frac{g_d}{W} = \left. \frac{1}{W} \frac{dI_d}{dV_{ds}} \right|_{V_{ds}=0} \approx - \left. \frac{J_c}{V_{ds}} \right|_{V_{ds} \leq 50 \text{ mV}} \quad (7.14)$$

$$\begin{aligned} \frac{g_m \text{ sat}}{W} &= \left. \frac{1}{W} \frac{dI_d \text{ sat}}{dV_{gs}} \right|_{V_{gs2}-V_{gs1} \leq 50 \text{ mV}} \\ &= - \frac{J_c \text{ sat}(V_{gs2}) - J_c \text{ sat}(V_{gs1})}{V_{gs2} - V_{gs1}} \end{aligned} \quad (7.15)$$

Each $J_c \text{ sat}(V_{gs})$ in equation (7.15) was determined by incrementing J_c until the unrealistic condition $J_c/Q_c > v_s$ occurred. Using this value and the previous value as limits, a binary search was then conducted. When two values of J_c were sequentially found that differed less than 0.005%, the highest was selected as $J_c \text{ sat}(V_{gs})$.

The results are shown in figures 7.9 and 7.10 for four DMOSTs which differed only in gate length, $L_g = 5, 7, 10$ and $20 \mu\text{m}$. In each case the same impurity profile, equation (7.1), was used. These had actual source to drain channel lengths of $L = 3.8, 5.8, 8.8$ and $18.8 \mu\text{m}$ respectively.

When $V_{ds} \approx 0$ and $V_{gs} \gg V_t$, the mobile channel charge greatly exceeds the depletion charge everywhere along the channel and so the actual impurity profile becomes virtually irrelevant. From first order MOST theory

$$\left. \frac{1}{W} \frac{dg_d}{dV_g} \right|_{V_{ds}=0} = \frac{\mu_0 C_{ox}}{L} \quad (7.16)$$

For all gate lengths shown in figure 7.9, the slope (g_d/W) taken at $V_{gs} > 4.5$ volts is constant and agrees with equation (7.16) in each case. As V_{gs} decreases, the slope of (g_d/W) is shown to increase approaching threshold and decreases to zero in the subthreshold

Figure 7.9 Calculated DMOST output conductance when $V_{ds} = 0$ volts for several gate lengths.

Figure 7.10 Calculated DMOST transconductance in saturation for various gate length devices. The arrows designate the gate voltages at which the point of saturation shifts from the graded portion of the channel to the drain.

region. As the gate length is increased, the resultant slope of (g_d/W) at threshold decreases. This implies that the impurity profile assumed yields subthreshold currents which are large enough to cause a non-negligible potential drop across the uniformly doped portion of the channel.

Substantial subthreshold behaviour of $g_{m \text{ sat}}$ is also evident in figure 7.10. At low gate voltages where the DMOST is in the short channel mode, neither figure 7.9 nor figure 7.10 show any extended region of linearity that is predicted by simpler DMOST models [41, 61, 62]. However, in each case a point of inflection always occurs in g_d and $g_{m \text{ sat}}$ at or just above threshold. The arrows in figure 7.10 mark the gate voltages at which the graded portion of the channel reverts to triode mode operation and short channel saturation ceases.

To determine how the DMOST is affected by a change in the length of the graded region, the doping profile was scaled by making the substitution $y = (y_{sc} + y^*/\beta_c)$ in equation (7.1). For comparison, $N_c(y^*)$ is shown in figure 7.11 for the five values of the expansion factor β_c used in the analysis. In all cases, $N_{c \text{ max}} = 2.9 \times 10^{16} \text{ cm}^{-3}$ and $L = 8.8 \text{ } \mu\text{m}$.

The corresponding numerically determined results for $g_{m \text{ sat}}$ are shown in figure 7.12. In long channel operation where $V_{gs} \gg V_t$, $g_{m \text{ sat}}$ is the same for all values of β_c . In short channel operation, the maximum value of $g_{m \text{ sat}}$ is shown to decrease with increasing β_c . The slope of $g_{m \text{ sat}}$ at threshold also decreases with expansion of the impurity profile. The relationship between the slope of $g_{m \text{ sat}}$ at threshold and β_c^{-1} however, is not linear. Simple DMOST models [41, 61, 62], on the other hand, imply that this relation should be linear.

Figure 7.11 Various channel impurity distributions assumed for transconductance calculation. When the expansion factor $\beta_c = 1.0$, the impurity distribution is identical to the distribution of figure 7.1. For all cases, $L = 8.8 \mu\text{m}$.

Figure 7.12 Calculated transconductance in saturation for the channel impurity distributions of figure 7.11.

Table 7.1 Calculated Effective Gain Factor for Short Channel DMOST Operation

expansion factor β_c	normalized gain factor $\alpha = \frac{1}{W} \frac{d g_m \text{ sat}}{d V_{gs}}$ (mhos cm^{-1} volts $^{-1}$)	$\alpha \cdot \beta_c$
0.333	0.669	0.223
0.667	0.373	0.249
1.0	0.299	0.299
1.5	0.214	0.321
2.0	0.166	0.332

As shown in table 7.1, the product of the slope of g_m sat at threshold and β_c decreases as the impurity profile is compressed.

7.4 Conclusion

A simple numerical method for simulation of the DMOST has been presented. Although the analysis does not consider 2-D depletion charge sharing, it does demonstrate the effects of the varying impurity profile on DMOST operation. The simulation results show that diffusion current arises as a consequence of the gradient in the impurity profile. Carriers are injected into the channel from the source primarily by diffusion current. High electric fields exist along the DMOST channel even when the net channel current is zero. Over the negatively graded portion of the channel, diffusion current opposes the net channel current. This allows substantially higher lateral fields to exist, giving rise to higher source to drain potential for given channel current. This logic explains why anomalously low values of substrate doping must be assumed in simpler DMOST models in order to obtain agreement with the measured device characteristics. One would expect similar results in any MOST which has a depletion charge maximum in the active portion of the channel between the source and drain.

CHAPTER 8

A SCMOST MODEL USING NUMERICAL TECHNIQUES

8.1 Introduction

8.1.1 Implications of Depletion Charge Sharing

Conventional MOST analytic models deal with the MOS depletion layer as a 1-D electrostatic problem. In each case it is assumed that all the electric flux lines run normal to the silicon surface and terminate on the gate. While this may be a reasonable approximation for long channel devices, it is seriously in error for SCMOSTs.

In short channel devices, a substantial portion of the flux lines, which originate from the unneutralized depletion charge, terminate on the source or drain instead of on the gate. Thus depletion charge is shared between three electrodes. The divergence of this lateral flux is sufficiently large to invalidate the underlying assumption of gradual channel MOST theory, viz

$$\frac{\partial^2 V}{\partial y^2} \ll \frac{\partial^2 V}{\partial x^2} \quad (8.1)$$

where x is the direction normal to the silicon surface. On a more physical level, this means that less depletion charge is 'observed' everywhere along the silicon surface of a SCMOST. This effect is non-uniform however, and generally increases with proximity to the source or drain. This lateral variation of apparent depletion charge is compensated for electrostatically by a redistribution of current

carrying inversion charge and surface potential. The 2-D nature of this mechanism is not properly captured in the depletion charge partition schemes discussed in chapter 4 chiefly because the depletion charge is treated as a lumped quantity. Therefore such analysis is incapable of describing the true depletion charge distribution as 'observed' from the silicon surface of a SCMOST.

The SCMOST model presented in this chapter, on the other hand, characterizes the distribution of depletion charge as 'observed' at the silicon surface and its electrostatic interaction with the current carrying surface inversion layer.

In terms of the device behaviour, the most obvious effects of depletion charge sharing are, a general decrease in the logarithmic slope of the subthreshold current versus gate voltage characteristics, a decrease in substrate bias sensitivity, an overall increase in current at all bias levels, as well as a threshold voltage dependence upon channel length, junction depth and drain bias. A more detailed discussion of SCMOST behaviour is contained in chapter 3.

The implications of depletion charge sharing to device modeling are perhaps most evident from a direct examination of Poisson's equation. If for the moment, the substrate doping N is assumed uniform, then Poisson's equation for a completely depleted surface space charge layer of a n-channel device is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{qN}{\epsilon_{si}} \quad (8.2)$$

Thus within the surface depletion layer one could write

$$\frac{\partial^2 V}{\partial x^2} = \frac{qN_x(x,y)}{\epsilon_{si}} \quad (8.3)$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{qN_y(x,y)}{\epsilon_{si}} \quad (8.4)$$

where $N = N_x(x,y) + N_y(x,y)$ for conservation of charge. If, for a given device structure and set bias conditions, the division of N between distribution $N_x(x,y)$ and $N_y(x,y)$ was known, the solution to Poisson's equation would follow quite simply. Clearly any charge which contributes to $N_y(x,y)$ does not enter into the calculation for either voltage or electric field in the x direction. Thus a solution in the x direction along any line of constant y should be achievable by simple integration of $\frac{\partial^2 V}{\partial x^2} = N_x(x,y)$. It is important to note that the second derivative of potential must be non-zero in order to account for depletion charge. The mere presence of a (constant) electric field does not influence the distribution of either $N_x(x,y)$ or $N_y(x,y)$.

From equation (8.2) and (8.3) one can also obtain

$$N_x(x,y) = N - \frac{\epsilon_{si}}{q} \frac{\partial^2 V}{\partial y^2} \quad (8.5)$$

Along any line perpendicular to the surface, one might hypothesize that $\frac{\partial^2 V}{\partial y^2}$, hence $N_x(x,y)$ in equation (8.5), is governed by two separate influences. Near the surface inversion layer, the value of $\frac{\partial^2 V}{\partial y^2}$ is dictated by the voltage requirements of current continuity. Beneath the surface, $\frac{\partial^2 V}{\partial y^2}$ is influenced by the divergence of electric flux from the source and drain.

In reality of course, the precise distribution of $N_x(x,y)$ or $N_y(x,y)$ can only be determined from a two dimensional solution of Poisson's equation. Within the depletion region, such a solution

would have to satisfy peripheral boundary conditions at the source, the drain, the silicon surface, and at the depletion charge boundary $x_d(y)$. However, the position of the depletion charge boundary $x_d(y)$ is itself a function of the solution to Poisson's equation. Moreover, for surface channel MOST operation, the surface boundary voltage distribution and normal component of the surface electric field are both dependant on the solution to the current continuity equation which in turn is a function of the solution to Poisson's equation.

Even if the quantity $N_x(x,y)$ were known, further complications would arise. For instance, as a first step toward a device model, one might invoke 1-D charge neutrality across the oxide in order to obtain an expression for the mobile charge density.

$$Q_m = C_{ox}(V_g - V_s) - \int_{x=0}^{x_d(y)} \frac{qN_x(x,y)}{\epsilon_{si}} dx \quad (8.6)$$

where $x_d(y)$ must be obtained as a solution to

$$(V_s - V_b) = \int_{x=0}^{x_d(y)} xN_x(x,y) dx \quad (8.7)$$

If the x dependance of $N_x(x,y)$ is complicated, the simultaneous solution for Q_m in terms of V_s might not be achievable by analytic means. Furthermore, any direct y dependance in Q_m (through $N_x(x,y)$) virtually excludes the possibility of an analytic solution to the current continuity equation for reasons discussed in chapter 5.

More complication arises if the substrate doping is non-uniform in the direction perpendicular to the surface. As an example, if depletion charge sharing mechanisms are neglected for the moment, one

might consider the 1-D distribution $N_x(x,y) = N(x) = N_0 \exp(-\alpha x) + N_b$ as representative of the tail of a very shallow ion-implant through the oxide. Examination of equations (8.6) and (8.7) reveals, however, that an analytic solution for Q_m in terms of V_s is not possible for this distribution. One could further try a simple truncated series expansion such as $N_x(x,y) = N(x) = (N_b + N_0) - N_0 \alpha x + N_0 \alpha^2 x^2 / 2$, but even for this case a solution for Q_m in terms of V_s can not be achieved by direct analytic means.

8.1.2 Objectives of the Analysis

Despite rather formidable analytic obstacles, a primary objective of this chapter is to conceptually and quantitatively model the depletion charge sharing mechanism of a SCMOST. Often the essence of device modeling is to isolate the central dominant mechanisms of device operation from a myriad of less important detail and to capture those mechanisms in a simple concept or a representative mathematical form. While 2-D finite difference and finite element MOST simulations may provide a 'window' for observation of the device internal operation, these require exhaustive computer computational effort. Moreover, they generally do not lead directly to device modeling concepts. On the other hand, the failure of analytic methods to capture various mechanisms of the SCMOST operation not only handicaps SCMOST modeling efforts, but also inhibits the model evolution process. Therefore it is the intention of this author to use numerical methods wherever analytic methods fail to capture the mechanisms of the SCMOST.

In order to keep the analysis within manageable bounds, several simplifying assumptions have been made concerning the device operation. A n-channel device is considered in which both hole current and recombination currents are neglected. The current carrying channel is assumed to exist at the silicon surface as a charge sheet of infinitesimal depth. The interface charge density is considered constant and the surface depletion layer is assumed free of mobile carriers.

8.2 Depletion Region Analysis

The objective of this section is to capture (a) the 2-D effects of depletion charge sharing, (b) drain induced barrier lowering, (c) the threshold voltage dependence on device geometry and bias conditions, and (d) what is commonly referred to as drain section analysis within a single electrostatic concept which applies universally along the entire source to drain channel length under all bias conditions and modes of SCMOST operation. As presented, the analysis also accounts for (e) the non-uniform substrate doping of a typical ion-implanted SCMOST.

8.2.1 Potential within the Surface Depletion Region

In the SCMOST, the effective gate and bulk potentials, V_g and V_b , are defined in the conventional manner.

$$V_g = V_{gs} - \phi_{MS} + \frac{Q_{eff}}{C_{ox}} \quad (8.8)$$

$$V_b = -V_{sb} \quad (8.9)$$

The surface depletion region is considered to be divided into a number of rectangular cells ζ of depth x_{dk} and finite width. Within the k^{th} cell, as illustrated in figure 8.1, Poisson's equation takes the form

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{qN(x)}{\epsilon_{si}} \quad (8.10)$$

In general, the depletion charge $qN(x)$ is non-uniform with respect to x but constant with respect to y .

Since Poisson's equation is linear, the principle of superposition can be utilized. Hence one can obtain a solution for potential as the sum of several other solutions,

$$V - V_b = \psi_1 + \psi_2 + \psi_3 \quad (8.11)$$

provided that the resultant solution $(V - V_b)$ satisfies the required boundary conditions. Equation (8.11) is the foundation and conceptual basis for the following SCMOST model derivation. With each solution ψ will be associated a different electrostatic mechanism.

Specifically, ψ_1 will be chosen to satisfy the forcing function $qN(x)/\epsilon_{si}$ of equation (8.10). The solution ψ_2 will account for a second order perturbation of the surface potential, while ψ_3 will consider the effect of lateral penetration of source and drain electric flux.

The simplest form ψ_1 could assume in order to satisfy the forcing function of equation (8.10) and the anticipated boundary conditions is

$$\psi_1 = \int_{\gamma=0}^x \int_{\xi=0}^{\gamma} \frac{qN(\xi)}{\epsilon_{si}} d\xi d\gamma + C_1 x + C_0 \quad (8.12)$$

Figure 8.1 Schematic of mesh and cell organization.

This is the form of solution assumed in virtually all analytic gradual channel MOST models. In this analysis, however, it serves to eliminate the forcing function from equation (8.10) so that Ψ_2 and Ψ_3 need only satisfy the Laplace equation.

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = 0 \quad (8.13)$$

The second solution Ψ_2 must account for a second order perturbation of potential at the silicon surface. In other words, it must satisfy a surface boundary condition such as

$$\Psi_2(0,y) = \frac{(y - y_k)^2}{2} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \quad (8.14)$$

where y_k is the position of the mesh point within the k^{th} cell.

Boundary condition (8.14) forces the second derivative of Ψ_2 to be non-zero at least near the surface and thus it can bring about the sharing of charge between the x and y directions. Generally, analytic solutions to the Laplace equation are composed of harmonic functions. Therefore, it is more expedient to express this boundary condition (8.14) in the form of a harmonic function such as

$$\Psi_2(0,y) = \lim_{b \rightarrow 0} \left\{ \frac{1}{b^2} [1 - \cos(b(y-y_k))] \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \right\} \quad (8.15)$$

At the depletion boundary one further requires

$$\Psi_2(x_{dk}, y) = 0 \quad (8.16)$$

Boundary conditions (8.15) and (8.16) give rise to the unique solution

$$\Psi_2(x,y) = \lim_{b \rightarrow 0} \left\{ \frac{1}{b^2} \left[\left(1 - \frac{x}{x_{dk}}\right) - \cos[b(y-y_k)] \frac{\sinh[b(x_{dk} - x)]}{\sinh(bx_{dk})} \right] \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \right\} \quad (8.17)$$

In the limit as b approaches zero,

$$\cos [b(y-y_k)] \rightarrow 1 \quad (8.18)$$

$$\sinh (bx_{dk}) \rightarrow bx_d \quad (8.19)$$

$$\left(1 - \frac{x}{x_{dk}}\right) \sinh(bx_{dk}) - \sinh [b(x_{dk}-x)] \rightarrow \frac{b^3}{6} [x_{dk}^2(x_{dk}-x) - (x_{dk}-x)^3] \quad (8.20)$$

Thus along the line $y=y_k$,

$$\psi_2(x, y_k) = \left[\frac{x_{dk}^2(x_{dk}-x) - (x_{dk}-x)^3}{6 x_{dk}} \right] \cdot \left[\frac{\partial^2 v_s}{\partial y^2} \right]_{y=y_k} \quad (8.21)$$

Perhaps the most important observation from this result is

$$\left. \frac{\partial^2 \psi_2}{\partial y^2} \right|_{y=y_k} = - \left. \frac{\partial^2 \psi_2}{\partial x^2} \right|_{y=y_k} = \left(1 - \frac{x}{x_{dk}}\right) \left[\frac{\partial^2 v_s}{\partial y^2} \right]_{y=y_k} \quad (8.22)$$

Therefore, the second derivative of ψ_2 with respect to y varies linearly from the surface to x_{dk} , as one might have anticipated from the initial boundary conditions.

The remaining solution ψ_3 must satisfy the Laplace equation and account for the subsurface penetration of electric flux from the source and drain. The subsurface nature of this mechanism suggests a series solution of the form

$$\psi_3(x, y) = \sum_{m=1}^{\infty} \sin (P_m^* x) \left[A_m^* e^{-P_m^* (y-y_k)} + B_m^* e^{P_m^* (y-y_k)} \right] \quad (8.23)$$

$$\text{where } P_m^* = \frac{m\pi}{x_{dk}} \quad (8.24)$$

The factors within the square brackets serve to attenuate each component of the series as the distance from either the source or drain

increases. The rate of attenuation is extremely high for components of the series with high m integer values. One might reason, that the penetration of lateral electric flux in a typical SCMOST must not be too large, otherwise punch through will dominate all modes of device operation. Moreover, even near the source or drain it is expected that the magnitude of component $m = 1$ will be significantly larger than all components $m \geq 2$. Thus one could conclude that over the central active portion of the channel, the $m = 1$ component is by far the dominant member of the series. On this basis, Ψ_3 simplifies to

$$\Psi_3(x, y) = \sin(px) [Ae^{-p(y-y_k)} + Be^{p(y-y_k)}] \quad (8.25)$$

$$\text{where } p = \frac{\pi}{x_{dk}} \quad (8.26)$$

It is further necessary that the total lateral electric flux remains continuous across the boundary between adjacent cells. Letting additional subscripts k and $k + 1$ designate the solution Ψ_3 in two adjacent cells k and $k + 1$, then continuity of lateral flux requires

$$\int_0^{x_{dk}} \frac{\partial^2 \Psi_{3k}}{\partial y^2} \Big|_{y=y_k + H_{k+1/2}/2} \cdot dx = \int_0^{x_{dk}} \frac{\partial^2 \Psi_{3k+1}}{\partial y^2} \Big|_{y=y_{k+1} - H_{k+1/2}/2} \cdot dx \quad (8.27)$$

where $H_{k+1/2}$ is the spacing between y_k and y_{k+1} . It follows that the A coefficients A_k and A_{k+1} of cells k and $k+1$ are related as

$$A_{k+1} = A_k \exp \left[\frac{-H_{k+1/2}}{2} (p_k + p_{k+1}) \right] \quad (8.28)$$

$$\simeq A_k \left[1 - \frac{H_{k+1/2}}{2} (p_k + p_{k+1}) \right] \quad (8.29)$$

Hence for any cell $2 \leq k \leq \zeta$, the coefficient A_k is given by

$$A_k = A_1 \prod_{n=1}^{k-1} \left[1 - \frac{H_{n+1/2}}{2} (p_n + p_{n+1}) \right] \quad (8.30)$$

Similarly, the B_k coefficients can be derived as

$$B_k = B_\zeta \prod_{n=\zeta-1}^k \left[1 - \frac{H_{n+1/2}}{2} (p_n + p_{n+1}) \right] \quad (8.31)$$

The value of coefficients A_1 and B_ζ are determined from consideration of the source and drain boundary conditions as described in appendix C.

Finally, along the line $y = y_k$ within the k^{th} cell one has

$$\psi_3(x, y_k) = \sin(p_k x) [A_k + B_k] \quad (8.32)$$

where $p_k = \frac{\pi}{x_{dk}} \quad (8.33)$

Collecting solutions ψ_1 , ψ_2 , and ψ_3 , the total solution for $(V - V_b)$ along the line $y = y_k$ within the k^{th} cell is

$$\begin{aligned} V(x, y_k) - V_b &= \int_{\gamma=0}^x \int_{\xi=0}^{\gamma} \frac{qN(\xi)}{\epsilon_{si}} d\xi d\gamma + C_1 x + C_0 \\ &+ \left[\frac{x_{dk}^2 (x_{dk} - x) - (x_{dk} - x)^3}{6} \right] \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} + \sin(p_k x) [A_k + B_k] \end{aligned} \quad (8.34)$$

The coefficients C_1 and C_0 are determined by application of the remaining boundary constraint

$$\left. \frac{\partial V}{\partial x} \right|_{x=x_{dk}, y=y_k} = 0 \quad (8.35)$$

and it follows that

$$\begin{aligned}
V(x, y_k) - V_b &= \int_x^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} x dx - x \int_x^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} dx \\
&\quad - \frac{x_{dk}^2}{6} \left(1 - \frac{x}{x_{dk}}\right)^3 \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \\
&\quad - [A_k + B_k] [p_k (x_{dk} - x) - \sin(p_k x)]
\end{aligned} \tag{8.36}$$

Letting V_k designate the surface potential at $y = y_k$ then

$$V_k - V_b = \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} x dx - \frac{x_{dk}^2}{6} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} - \pi [A_k + B_k] \tag{8.37}$$

8.2.2 Surface Electric Field

The total surface field at $y = y_k$ is made up of contributions from the depletion region and the surface mobile charge density at $y = y_k$ which will be designated as the absolute value Q_k . Thus

$$E_{sk} = \frac{Q_k}{\epsilon_{si}} - \left. \frac{dV}{dx} \right|_{x=0, y=y_k} \tag{8.38}$$

and from equation (8.36) then

$$E_{sk} = \frac{Q_k}{\epsilon_{si}} + \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} dx - \frac{x_{dk}}{2} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} - \frac{2\pi}{x_{dk}} [A_k + B_k] \tag{8.39}$$

In order to solve equation (8.39), it is further necessary to express E_{sk} in terms of V_g and the local surface potential condition. Analytic MOST models invoke 1-D charge neutrality across the oxide to establish

† The electric flux arising from the term $\frac{Q_k}{\epsilon_{si}}$ is assumed normal to the silicon surface.

the following relationship

$$E_{sk} = \frac{C_{ox}}{\epsilon_{si}} (V_g - V_k) \quad (8.40)$$

The 2-D electrostatic considerations of appendix D demonstrate that a second order perturbation of surface potential alters this relation slightly with the result

$$E_{sk} = \frac{C_{ox}}{\epsilon_{si}} (V_g - V_k) - \frac{t_{ox} \epsilon_{ox}}{3\epsilon_{si}} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_K} \quad (8.41)$$

In practice, it has been found that the inclusion of the second term in equation (8.41) does not substantially influence the solution for either the surface potential distribution or drain current. This occurs chiefly because the third term of equation (8.39), which describes the corresponding effect in the depletion region, is typically over an order of magnitude larger in value and thus it dominates. For completeness though, equation (8.41) will be employed, which together with equation (8.39) yields the charge neutrality relation

$$0 = -\frac{C_{ox}}{\epsilon_{si}} (V_g - V_k) + \frac{Q_k}{\epsilon_{si}} + \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} dx \quad (8.42)$$

$$- R_k \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} - \frac{2\pi}{x_{dk}} [A_k + B_k]$$

$$\text{where } R_k = \left[\frac{x_{dk}}{2} - \frac{t_{ox} \epsilon_{ox}}{3\epsilon_{si}} \right] \quad (8.43)$$

8.3 Numerical Solution for Surface Potential

An overall flow diagram of the numerical scheme employed for SCMOST simulation is shown in figure 8.2. The method consists of alternately solving for the surface potential distribution V_k , the depletion depth distribution x_{dk} and the mobile charge distribution Q_k until the requirements of charge neutrality and current continuity are simultaneously satisfied. Equations (8.37) and (8.42) are used as the basis for solution of x_{dk} and V_k respectively. The method of solution for Q_k will be discussed in the next section. All solutions employ the same system of mesh points. A non-linear mesh is assumed of typically forty points distributed along the surface from source to drain. To minimize confusion, the spacing between any two adjacent mesh points, k and $k+1$, is written symbolically as $H_{k+1/2}$.

About any mesh point k , one can approximate the second derivative of surface potential in finite difference form as

$$\begin{aligned} \left. \frac{\partial^2 V_s}{\partial y^2} \right|_{y=y_k} &= \frac{\partial}{\partial y} \left[\frac{\partial V_s}{\partial y} \right]_{y=y_k} \\ &\approx \left[\frac{V_{k+1} - V_k}{H_{k+1/2}} - \frac{V_k - V_{k-1}}{H_{k-1/2}} \right] \left[\frac{2}{H_{k+1/2} + H_{k-1/2}} \right] \\ &= V_{k-1} \cdot HA_k - V_k \cdot HB_k + V_{k+1} \cdot HC_k \end{aligned} \quad (8.44)$$

$$\text{where } HA_k = \frac{2}{H_{k-1/2} (H_{k+1/2} + H_{k-1/2})} \quad (8.45)$$

$$HB_k = \frac{2}{H_{k+1/2} \cdot H_{k-1/2}} \quad (8.46)$$

$$HC_k = \frac{2}{H_{k+1/2} (H_{k+1/2} + H_{k-1/2})} \quad (8.47)$$

Figure 8.2 Flow diagram of iterative scheme for SCMOST simulation.

The solution for the depletion boundary distribution x_{dk} is obtained by application of the Newton method to equation (8.37). Letting

$$f_k = (V_k - V_b) + \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} x dx - \pi [A_k + B_k] - \frac{x_{dk}^2}{6} [V_{k-1} \cdot HA_k - V_k \cdot HB_k + V_{k+1} \cdot HC_k] \quad (8.48)$$

Then one requires the value x_{dk} which satisfies $f_k = 0$. Letting superscripts n and $n-1$ designate the n^{th} and $n-1$ iterations then

$$x_{dk}^n = \left[-f^{n-1} / \frac{df}{dx_{dk}} \right] + x_{dk}^{n-1} \quad (8.49)$$

$$\text{where } \frac{df}{dx_{dk}} = \frac{qN(x_{dk})}{\epsilon_{si}} - \frac{x_{dk}}{3} [V_{k-1} \cdot HA_k - V_k \cdot HB_k + V_{k+1} \cdot HC_k] \quad (8.50)$$

In practice, no more than two iterations are carried out for every new solution of V_k and Q_k .

The potential distribution V_k , is determined by application of the generalized Newton method to equation (8.42). Setting

$$F_k = -\frac{C_{ox}}{\epsilon_{si}} (V_g - V_k) + \frac{Q_k}{\epsilon_{si}} + \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} dx - R_k \cdot [HA_k \cdot V_{k+1} - HB_k \cdot V_k + HC_k \cdot V_{k+1}] - \frac{2\pi}{x_{dk}} [A_k + B_k] \quad (8.51)$$

then one requires the voltage distribution V_k which simultaneously satisfies all equations $F_k = 0$ for $K = 2$ to $\zeta - 1$. Although the present form of equation (8.51) implies that a solution for the V_k

distribution could be achieved by direct matrix inversion, such an approach would, for reasons to be discussed shortly, cause the overall iterative scheme to diverge rather than converge to a final solution. Instead a solution is obtained iteratively by solving the system of equations

$$\frac{\partial F_k}{\partial V_{k-1}} \Delta V_{k-1} + \frac{\partial F_k}{\partial V_k} \Delta V_k + \frac{\partial F_k}{\partial V_{k+1}} \Delta V_{k+1} = -F_k \quad (8.52)$$

in tridiagonal matrix form

$$[A] \cdot [\Delta V] = [-F] \quad (8.53)$$

by forward-backward substitution methods [69]. After the n^{th} iteration, the new solution for surface potential is obtained from

$$V_k^n = \Delta V_k + V_k^{n-1} \quad \text{for } 2 \leq k \leq \zeta - 1 \quad (8.54)$$

Complications arise however if equation (8.51) is employed in its present form. Specifically, the overall iterative alternate V_k , Q_k solution method will not converge. The source of difficulty is the term Q_k in equation (8.51). In brief, equation (8.51) has no mechanism by which to anticipate how the current continuity equation will alter the mobile charge distribution for any given solution ΔV_k . This gives rise to instability of the overall solution method. To overcome this difficulty, one notes that

$$Q_k \approx \exp \frac{q}{kT} (V_k - \phi_n) \quad (8.55)$$

Assuming ϕ_n is reasonably fixed by the constraints of current continuity and V_k^0 is the initial potential before iterative solution of equation (8.51), then Q_k must react as

$$Q_k = Q_k^0 \exp \left[\frac{q}{kT} (V_k - V_k^0) \right] \quad (8.56)$$

where Q_k^0 is the mobile charge determined by the previous solution to the current continuity equation. This is the reasoning behind the well-known Gummel algorithm [70]. Gummel, however, simply replaces the exponential with its Taylor expansion and ignores all terms with powers greater than unity. Substituting equation (8.56) into (8.51) (and dropping the superscript from Q_k^0) one obtains

$$F_k = -\frac{C_{ox}}{\epsilon_{si}} (V_g - V_k) + \frac{Q_k}{\epsilon_{si}} \exp \left[\frac{q}{kT} (V_k - V_k^0) \right] + \int_{x=0}^{x_{dk}} \frac{qN(x)}{\epsilon_{si}} dx \\ - R_k \cdot [HA_k \cdot V_{k-1} - HB_k \cdot V_k + HC_k \cdot V_{k+1}] \\ - \frac{2\pi}{x_{dk}} [A_k + B_k] \quad (8.57)$$

from which it follows

$$\frac{\partial F_k}{\partial V_{k-1}} = -R_k \cdot HA_k \quad (8.58)$$

$$\frac{\partial F_k}{\partial V_k} = \frac{C_{ox}}{\epsilon_{si}} + \frac{q}{kT} \frac{Q_k}{\epsilon_{si}} \exp \left[\frac{q}{kT} (V_k - V_k^0) \right] + R_k \cdot HB_k \quad (8.59)$$

$$\frac{\partial F_k}{\partial V_{k+1}} = -R_k \cdot HC_k \quad (8.60)$$

Equations (8.58), (8.59) and (8.60) yield a matrix which has very strong diagonal dominance and thus stability. The resulting overall iterative alternate solution method has been found to be convergent under all bias conditions.

At the source and drain boundaries, the boundary potentials V_1 and V_L are calculated assuming Boltzmann statistics from

$$V_1 - V_b = \frac{kT}{q} \ln \left\{ \frac{N_A N_b}{n_i^2} \right\} + V_{sb} \quad (8.61)$$

$$V_c - V_b = \frac{kT}{q} \ln \left\{ \frac{N_j N_b}{n_i^2} \right\} + V_{sb} + V_{ds} \quad (8.62)$$

where N_j and N_b are the junction doping and bulk substrate doping respectively.

8.4 Solution for the Mobile Charge Distribution

For normal MOST operation, the drain to source current is confined within a surface inversion layer over the majority of the distance between source and drain. Within this inversion layer, current flows parallel to the oxide-semiconductor interface. Assuming validity of the Einstein relation $D = \frac{kT}{q} \mu$, the current density for electrons can be written as

$$J_n = -q n \left[\frac{dV}{dy} - \frac{kT}{q} \frac{dn}{dy} \right] \quad (8.63)$$

The total channel current passing through any vertical cross section of the channel is then simply the integral of the current density over that cross section or

$$I_c = -\mu W \left[Q_m \frac{dV}{dy} - \frac{kT}{q} \frac{dQ_m}{dy} \right] \quad (8.64)$$

where Q is the absolute value of the mobile charge density (cm^{-2}) given by

$$Q_m = q \int_{x=0}^{x_d} n \, dx \quad (8.65)$$

In this section a numerical method is derived to solve for the mobile charge distribution assuming the surface potential distribution has

previously been determined. The system of mesh points of the previous section is employed to subdivide the channel into small intervals. The mobile charge distribution is obtained as the solution to a system of difference equations which serve to enforce current continuity between adjacent intervals. Such a system of equations could be symbolically written as

$$I_{ck-\frac{1}{2}} = I_{ck+\frac{1}{2}} \quad \text{for } K = 2, \dots, L-1 \quad (8.66)$$

where $I_{ck+\frac{1}{2}}$ for example, is the channel current over the interval between mesh points k and $k+1$.

The most obvious difference equation form of current continuity equation (8.64) is

$$I_{ck+\frac{1}{2}} = -\frac{W\mu_{k+\frac{1}{2}}}{H_{k+\frac{1}{2}}} \left[\left(\frac{Q_k + Q_{k+1}}{2} \right) (V_{k+1} - V_k) - \frac{kT}{q} (Q_{k+1} - Q_k) \right] \quad (8.67)$$

where $\mu_{k+\frac{1}{2}}$ refers to the mobility between mesh points k and $k+1$.

It can be shown however, that if equation (8.67) is used to form the system of difference equation (8.66), numerical instability results when $(V_{k+1} - V_k) > 2 \cdot \frac{kT}{q}$ [67].

To overcome this difficulty, an alternate form of current equation (8.64) can be adopted

$$I_c = W\mu \frac{kT}{q} e^{\frac{q}{kT} V} \frac{d}{dy} \left(e^{-\frac{q}{kT} V} Q_m \right) \quad (8.68)$$

Equation (8.68) reduces to equation (8.64) if the differentiation is carried out. Other authors [33, 34] have used a 2-D difference form of equation (8.68) for 2-D MOST simulation. Their difference equation for current can be written in 1-D form as

$$I_{c\ k+1/2} = \frac{kT}{q} \frac{W}{H_{k+1/2}} \left\{ Q_{k+1} \exp \left[-\frac{q}{kT} \left(\frac{V_{k+1} - V_k}{2} \right) \right] - Q_k \exp \left[\frac{q}{kT} \left(\frac{V_{k+1} - V_k}{2} \right) \right] \right\} \quad (8.69)$$

Although equation (8.69) leads to an attractively simple and stable system of difference equations, it drastically overestimates current whenever $(V_{k+1} - V_k) > 2 \cdot \frac{kT}{q}$ as pointed out by other authors [35, 71]. For the case where $Q_{k+1} = Q_k$, for example, it overestimates current by a factor of

$$\frac{2 \sinh \left[.5 \frac{q}{kT} (V_{k+1} - V_k) \right]}{\frac{q}{kT} (V_{k+1} - V_k)} \quad (8.70)$$

An alternative 2-D difference form of equation (8.68) has been successfully used for 2-D bipolar transistor simulator [71] but only for the case of low applied voltages. For the typical operating voltages of an MOST, this method would result in exponential overflow during computer computation.

Returning to equation (8.68), one notes that it can be written in the form

$$\frac{I_c}{W_{\mu}} \frac{q}{kT} e^{-\frac{qV}{kT}} dy = d \left[e^{-\frac{qV}{kT}} Q_m \right] \quad (8.71)$$

If, over the small interval $H_{k+1/2}$, the electric field is assumed equal to the constant value $(V_k - V_{k+1}) / H_{k+1/2}$, then integration of equation (8.71) yields

$$\begin{aligned}
& - \frac{I_{C \ k+1/2}}{W \mu_{k+1/2}} \frac{H_{k+1/2}}{(V_{k+1} - V_k)} \left[e^{-\frac{q}{kT} V_{k+1}} - e^{-\frac{q}{kT} V_k} \right] \\
& = \left[Q_{k+1} e^{-\frac{q}{kT} V_{k+1}} - Q_k e^{-\frac{q}{kT} V_k} \right] \quad (8.72)
\end{aligned}$$

which reduces to the following difference equation for current [67].

$$I_{C \ k+1/2} = \frac{-W \mu_{k+1/2} (V_{k+1} - V_k)}{H_{k+1/2}} \left[\frac{Q_{k+1}}{1 - e^{-\frac{q}{kT} (V_{k+1} - V_k)}} + \frac{Q_k}{1 - e^{-\frac{q}{kT} (V_{k+1} - V_k)}} \right] \quad (8.73)$$

When used to form the system of difference equations (8.66), equation (8.73) has been found to provide numerically stable and accurate current characteristics under all bias conditions. For the case where $V_{k+1} - V_k = 0$, equation (8.73) is replaced with

$$I_{C \ k+1/2} = W \frac{kT}{q} \frac{\mu_{k+1/2}}{H_{k+1/2}} (Q_{k+1} - Q_k) \quad (8.74)$$

which can be obtained from equation (8.73) by application of L'Hospital's Rule. The resulting system of difference equations can be written as

$$\begin{aligned}
0 = & -[EX_{k+1/2} \cdot \mu_{k+1/2}] Q_{k-1} + [EXI_{k-1/2} \cdot \mu_{k-1/2} + EX_{k+1/2} \cdot \mu_{k+1/2}] Q_k \\
& - [EXI_{k+1/2} \cdot \mu_{k+1/2}] Q_{k+1} \quad (8.75)
\end{aligned}$$

where for example

$$EX_{k+1/2} = \frac{V_{k+1} - V_k}{H_{k+1/2} \left\{ 1 - \exp \left[-\frac{q}{kT} (V_{k+1} - V_k) \right] \right\}} \quad (8.76)$$

$$EXI_{k+1/2} = \frac{V_{k+1} - V_k}{H_{k+1/2} \left\{ 1 - \exp \left[\frac{q}{kT} (V_{k+1} - V_k) \right] \right\}} \quad (8.77)$$

The electron mobility is assumed to have both surface field and lateral field dependence via

$$\mu_{k+\frac{1}{2}} = \frac{\mu_0}{\left\{ 1 + \left[\frac{\mu_0}{v_s} \frac{(V_{k+1} - V_k)}{H_{k+\frac{1}{2}}} \right]^2 \right\}^{\frac{1}{2}}} \cdot \frac{1}{\left\{ 1 + \theta \frac{C_{ox}}{\epsilon_{si}} \left[V_g - (V_{k+1} + V_k)/2 \right] \right\}} \quad (8.78)$$

This system of difference equations can be put into the matrix form

$$[B] [Q] = [D] \quad (8.79)$$

Since matrix B is tridiagonal, a solution for the mobile charge can be achieved by forward-backward substitution methods. From equation (8.75), it is evident that matrix B is diagonally dominant and therefore nonsingular. Thus a solution is guaranteed under all bias conditions.

At each end of the mesh, representative values for the boundary charge quantities Q_1 and Q_L must be established in order to solve matrix equation (8.79). From conventional MOST analysis, the equation of charge neutrality is

$$Q_m = \epsilon_{si} (E_s - E_d) \quad (8.80)$$

where E_s represents the gate induced surface field just inside the silicon and E_d is the field at the surface arising from the depletion charge. From Pao and Sah's equation of charge neutrality (equation B.6 of appendix B), one can also obtain

$$(\epsilon_{si} E_s)^2 = 2q \epsilon_{si} n_i e^{\frac{q}{kT}(V_s - \phi_f)} + (\epsilon_{si} E_d)^2 \quad (8.81)$$

Then at the source where $k = 1$ or at the drain where $k = L$

$$Q_k = [2q n_i \epsilon_{si} e^{\frac{q}{kT}(V_k - \phi_f)} + (\epsilon_{si} E_{dk})^2]^{\frac{1}{2}} - \epsilon_{si} E_{dk} \quad (8.82)$$

Assuming Boltzman statics at the source or drain, the quantity $(V_k - \phi_f)$ at both boundaries is

$$(V_k - \phi_f) = \frac{kT}{q} \ln \left\{ \frac{N_j}{n_i} \right\} \quad (8.83)$$

The depletion region electric field quantities E_{dl} and $E_{d\zeta}$ are calculated from the analysis of the previous section. In practice, the exponential in equation (8.82) dominates, so that for all practical purposes, the exact values of E_{dl} and $E_{d\zeta}$ are unimportant.

One might question the validity of equation (8.80) and (8.81) at the drain boundary in saturation. In the saturation mode, however, carriers in a region approaching the drain travel at or very near saturated drift velocity. This provides an overriding boundary value of $I_c / (v_s \cdot W)$ for the mobile charge. Thus under saturated drift velocity conditions, the value of the mobile charge at the drain is irrelevant to the solution for the mobile charge distribution elsewhere.

The SCMOST analysis contained in the previous sections was implemented in a FORTRAN program for use on a Xerox Sigma-9 computer. The program listing is given in appendix E. This program was used to determine the results presented in the next section.

8.5 Demonstration of the Analysis

The surface potential distribution predicted by this analysis for $V_{ds} = 2.0$ volts is shown in figures 8.3 and 8.4 for two MOS devices with channel lengths $L = 2.0 \mu m$ and $L = 1.5 \mu m$ respectively. For these results plus those shown in figures 8.5 through 8.9, the

Table 8.1 Data for Devices M1 and M2

n-channel, n-doped polysilicon gate MOSTs

Parameter	Value	Units	Device
t_{ox}	50.0	nm	M1 and M2
μ_0	800.0	$\text{cm}^2 \text{V}^{-1} \text{sec}^{-1}$	
θ	10^{-6}	$\text{cm} \text{V}^{-1}$	
v_s	$6.5 \cdot 10^6$	cm sec^{-1}	
x_j	0.3	μm	
N_{ss}	$2.0 \cdot 10^{10}$	cm^{-2}	
W	10.0	μm	M1
N	$4.0 \cdot 10^{15}$	cm^{-3}	
L	2.0	μm	
N	$2.0 \cdot 10^{15}$	cm^{-3}	M2
L	1.5	μm

Figure 8.3 Surface potential distribution within a $L = 2.0 \mu\text{m}$ MOST for a range of gate voltages.

Figure 8.4 Surface potential distribution within a $L = 1.5 \mu m$ MOST for a range of gate voltages.

device data listed in table 8.1 have been assumed. The general shape of the potential distribution plotted in figure 8.3 for the longer channel length device appears somewhat similar to the predictions of classical theory. For instance, the potential distribution for $V_{gs} = 0$ volts (in subthreshold) seems reasonably flat for $0.5 \mu\text{m} < y < 1.2 \mu\text{m}$ although the gently changing slope within this range bespeaks of non-negligible electric field strength. For $y > 1.2 \mu\text{m}$ on the same curve, the rapid rise in surface potential toward the drain is also characteristic of conventional MOST theory drain region analysis. Similar comments can be made with respect to the other curves plotted in figure 8.3. For comparison, the surface potential distribution for a shorter $L = 1.5 \mu\text{m}$ device are shown in figure 8.4. In these results, it is no longer possible to visually partition between a conventional channel region and a drain section. This behaviour is often described in terms of a general merging of source and drain regions or the strong penetration of lateral electric flux into all parts of the surface depletion region. When such effects are clearly evident in the surface potential distribution, very strong depletion charge sharing mechanisms are present.

A distinct difference between this analysis and conventional theory is the potential assumed at the source V_1 . In this analysis, the potential at the source $V_1 = V_{bi}$ is derived from Boltzmann statistics. The value and position of the potential minimum (or virtual cathode) is established by the numerical method as the solution simultaneously satisfying the requirements of current continuity and charge neutrality. Conventional MOST theory, on the other hand, assumes a negligible distance between the source and the

position of the potential minimum and a potential minimum value of $2\phi_f$ regardless of the bias conditions. Figures 8.3 and 8.4 imply that such an assumption could give rise to significant error particularly in short channel devices.

Drain induced barrier lowering is demonstrated in figure 8.5. As the drain bias is increased from $V_{ds} = 0$ to 5 volts, the position of the potential minimum is shown to move from the channel center towards the source. The value of the potential minimum V_{min} is also shown to increase with increasing drain bias. If one uses $V_{min} = 2\phi_f$ as the criterion for threshold, then this device is at the threshold condition when $V_{ds} = 0.5$ volts. Although the change in V_{min} with V_{ds} may not appear large, it does have a significant effect on drain current particularly in subthreshold. Drain current in subthreshold is known to rise exponentially with the value of the potential minimum.

A premise of all analytic subthreshold MOST models is that the drift current is negligible everywhere along the channel. In a SCMOST, this can not be true because the penetration of lateral electric flux gives rise to high electric fields along the channel length. The results in figure 8.6 show that only at one point in the channel (at the potential minimum) is the drift current negligible. Everywhere else, mobile charge transport is governed by a complex interplay of drift and diffusion current.

The distribution of drift and diffusion current shown in figure 8.7 for a device in saturation, indicates that over the majority of the channel length, drift current dominates. The interplay of drift and diffusion current near the source is analogous to low level current transport across a p-n junction in which the quasi,

Figure 8.5 SCMOST surface potential distribution for a range of drain voltages, demonstrating drain induced barrier lowering. Arrows mark the point of potential minimum.

Figure 8.6 Drift and diffusion current distribution within a SCMOST operating in subthreshold, $I_d = 5.7 \cdot 10^{-8}$ amps. The arrow marks the point of potential minimum where the electric field is zero.

Figure 8.7 Drift and diffusion current distribution within a SCMOST operating in the saturation mode, $I_d = 0.38$ mA.

Fermi levels remain virtually constant in the presence of extremely high but opposing drift and diffusion current. Once the charge is injected past the potential minimum, the diffusion current drops to a small fraction (typically less than 5%) of the total channel current. The rise in opposing diffusion current near the drain occurs due to the rapid increase in mobile charge density approaching the drain. There is no evidence of an assisting diffusion current near the drain as reported by Pao and Sah [44] from the results of their long channel constant mobility simulation. Carriers in this region of a SCMOST are saturated drift velocity limited, hence $Q_m \approx I_{ds}/(W \cdot v_s)$ and therefore there is not sufficient degree of freedom for a noticeable assisting diffusion current to develop.

The electric field strength and resultant carrier velocity are shown in figures 8.8 and 8.9 respectively for a device in saturation. The electric field strength is shown to be extremely high even for that rather moderate bias conditions $V_{gs} = 2.0$ volts, $V_{ds} = 2.0$ volts. Under these bias conditions, the electric field is within the range $\frac{dV}{dx} < E_c$ for only 33% of the entire channel length ($E_c = v_s/\mu_0$). Under the bias conditions, $V_{gs} = 4.0$ volts, $V_{ds} = 4.0$ volts, this region narrows to 7% of the channel length. Thus current flow is governed by the electrostatics within a rather small portion of the channel length. For all bias voltages shown, carrier velocity is within 10% of saturated drift velocity value v_s at the drain. For curves a and b, carrier velocity is within 50% of v_s over 85% of the channel length.

Figure 8.8 Electric field along the channel of a $L = 1.5$ μm MOST for (a) $V_{gs} = V_{ds} = 4.0$ volts, (b) $V_{gs} = V_{ds} = 2.0$ volts, and (c) $V_{gs} = V_{ds} = 1.0$ volt. $E_c = V_{gs}/\mu_0$.

Figure 8.9 Carrier velocity along the channel of a $L = 1.5$ μ m MOST for (a) $V_{gs} = V_{ds} = 4.0$ volts, (b) $V_{gs} = V_{ds} = 2.0$ volts, and (c) $V_{gs} = V_{ds} = 1.0$ volt.

8.6 Comparison of Theory with Experiment

This analysis was applied to an ion-implanted MOST with a channel length $L = 3.8 \mu\text{m}$. For computation, the device data listed in table 8.2 were assumed. Parameter values t_{ox} , L , W and x_j were known accurately from process characterization. Computed and measured[†] current characteristics are shown in figures 8.11, 8.12 and 8.13. During fabrication, the MOST was subject to a shallow implant through the oxide for threshold adjustment and therefore had a non-uniform substrate doping profile. From process evaluation, the substrate doping was known to be approximately $2.8 \cdot 10^{15} \text{ cm}^{-3}$ at $x = 0.4 \mu\text{m}$ and $2.0 \cdot 10^{15} \text{ cm}^{-3}$ at $x = 1.8 \mu\text{m}$.[†] Assuming a uniform substrate doping, an attempt was initially made to match the theoretical and experimental subthreshold characteristics of a longer channel length device, $L = 22.8 \mu\text{m}$, located on the same test insert. The results showed that it was always possible to obtain agreement at two of the three substrate bias levels $V_{\text{sb}} = 0, 2.0$ and 5.0 volts, but not all three simultaneously. This implied that it was necessary to take the non-uniform impurity distribution into account to obtain agreement. For simplicity of calculation, the following piecewise-linear substrate doping profile was assumed as approximation to the shallow ion-implant.

$$N(x) = 1.3 \times 10^{15} \left[1 - \frac{x}{10^{-4}} \right] + 1.9 \times 10^{15} \text{ cm}^{-3} \text{ for } 0 < x < 10^{-4} \text{ cm} \quad (8.84)$$

$$N(x) = 1.9 \times 10^{15} \text{ for } x > 10^{-4} \text{ cm} \quad (8.85)$$

The resulting agreement in subthreshold between theory and experiment for the $L = 22.8 \mu\text{m}$ device is shown in figure 8.10. This doping profile was then assumed for the $L = 3.8 \mu\text{m}$ device and used in

[†] Device data supplied by Bell Northern Research Ltd., Ottawa, Canada, courtesy of Dr. D. A. Vincent.

Table 8.2 Data for Devices 7BS and 7BL

n-channel, n-doped polysilicon gate MOSTs[†]

Parameter	Value from [†] Processing	Value Used	Units	Device ^{††}
t_{ox}	$107.0 \pm 2\%$	107.0	nm	7BS and 7BL
μ_0	~ 950	950	$\text{cm}^2 \text{V}^{-1} \text{sec}^{-1}$	
θ	—	10^{-6}	cm V^{-1}	
v_s	—	$8.0 \cdot 10^6$	cm sec^{-1}	
x_j	1.6	1.6	μm	
N_{ss}	—	$2.8 \cdot 10^{10}$	cm^{-2}	
N_i	ion-implanted $N_{av} = 2.4 \cdot 10^{15}$	explained in text	cm^{-3}	
W	24.0	24.0	μm	7BS
L	3.78 ± 0.1	3.88	μm	
W	100.0	100.0	μm	7BL
L	22.8	22.8	μm	

[†] Device data supplied by Bell Northern Research Ltd., Ottawa, Ontario, Canada, courtesy of Dr. D. A. Vincent.

^{††} Device numbers 7BS and 7BL correspond to Bell Northern Research device numbers 16S and 16B respectively.

Figure 8.10 Comparison of theory with experimentally measured subthreshold characteristics of a $L = 22.8 \mu\text{m}$ MOST.

the calculation of results plotted in figure 8.11, 8.12 and 8.13.

Comparison of calculated and experimentally measured I_d versus V_{ds} characteristics is shown in figure 8.11. The agreement in saturation is remarkable considering that this device model does not contain any specialized drain region analysis. On the average, each simulation required eighteen CPU seconds on a Xerox Sigma-9 computer time sharing system using the FLAG (Fortran Load And Go) compiler. In subthreshold, a solution was achieved in typically one CPU second. A speed improvement by approximately a factor of four would be expected on an IBM 370 model 168. Another thirty percent increase in speed can be achieved using a more efficient Fortran compiler. For comparison, a typical 2-D finite element or finite difference MOST simulation requires approximately two orders of magnitude more CPU time.

The experimental subthreshold current characteristics of the $L = 3.8 \mu m$ device in figure 8.12 show less substrate bias sensitivity than the long channel results of figure 8.10. This is typical of SCMOST behaviour. A discrepancy between theory and experiment is evident in the results of figure 8.12 for $V_{sb} = 2.0$ volts and $V_{sb} = 5.0$ volts. It is believed that the discrepancy is due to inaccuracies in the values computed for coefficients A_1 and B_1 as discussed in appendix C, and not inaccuracies in the basic model. Using $I_d = 10^{-6}$ amperes as a reference, this theory underestimates the ΔV_{gs} shift by 0.04 volts at $V_{sb} = 2.0$ volts and 0.11 volts at $V_{sb} = 5.0$ volts. For comparison, Yau's short channel threshold voltage formula [26] overestimates the ΔV_{gs} shift by 0.09 volts at $V_{sb} = 2.0$ volts and 0.20 volts at $V_{sb} = 5.0$ volts (assuming an

Figure 8.11 Comparison of theory with experimentally measured I_D versus V_{ds} characteristics of a $L = 3.8 \mu\text{m}$ MOST.

Figure 8.12 Comparison of theory with experimentally measured subthreshold characteristics of a $L = 3.8 \mu\text{m}$ MOST.

Figure 8.13 The effect of drain induced barrier lowering demonstrated in the measured subthreshold current of a $L = 3.8 \mu\text{m}$ MOST and compared with theory.

average substrate doping of $N = 2.4 \times 10^{15} \text{ cm}^{-3}$).

The effect of drain induced barrier lowering on subthreshold drain current is demonstrated in figure 8.13. In this figure, the theoretical and experimental results track exceptionally well. The minor discrepancies are believed to be attributable to surface state trapping mechanisms which are not accounted for in the model.

8.7 Conclusion

A SCMOST model has been presented which is quantitatively representative of the device internal operation rather than just its terminal characteristics. A novel SCMOST model concept has evolved which accounts for depletion charge sharing, drain induced barrier lowering, the threshold voltage dependance on device geometry and bias conditions as well as drain region analysis in a single electrostatic relation that applies universally along the channel from source to drain. Thus the 2-D essence of depletion charge sharing is captured in a 1-D electrostatic relationship. The analysis is also versatile enough to account for the non-uniform substrate doping of a typical ion-implanted device.

Although the solution for charge neutrality and current must be obtained numerically, the required CPU time is approximately two orders of magnitude less than an equivalent 2-D finite difference or finite element simulation would require. The versatility of this SCMOST modeling approach makes it an ideal vehicle for testing other SCMOST modeling concepts. The low computer time required together with the accuracy of the results should also make this model a

viable tool for the study of the consequences of channel length reduction and process changes in SCMOSTs. Comparison with experiment shows good agreement with the terminal characteristics of an ion-implanted, 3.8 μm channel length MOST, and good results are expected for even shorter channel length devices.

CHAPTER 9

CONCLUDING REMARKS

9.1 Summary

An overview of the problems pertinent to MOST miniaturization and a discussion of state-of-the-art MOST modeling methodology has been presented. Although the scaling rules are a useful guide to MOSFET miniaturization, there is reason to speculate that these rules will not be adhered to. For instance, the nonscaling properties of noise, the subthreshold characteristics, and the feasible limitations on process tolerances suggest that bias voltages will be maintained at levels higher than suggested by the scaling rules. Unfortunately this would serve to accentuate problems associated with high electric fields such as breakdown mechanisms, avalanche multiplication and hot electron effects. Moreover the nonscaling properties of the extrinsic debye length and the built-in potentials further enhances other undesirable mechanisms such as depletion charge sharing, drain induced barrier lowering, punch-through and the threshold dependence on drain bias, channel length and junction depth.

At this time, gradual channel theory is still the backbone of MOST modeling efforts chiefly because the assumptions relevant to the gradual channel approximation offer tremendous simplification to the MOST modeling problem. For linear circuits, longer channel length MOSTs will find continuing application due to their stability and

predictability of performance. For these devices, the gradual channel approximation is justifiable. In view of these facts, a versatile MOST model based on gradual channel theory was developed. The model features several refinements not commonly found in other MOST models such as subthreshold diffusion current analysis and a surface field dependant mobility. All derived expressions for the conductances and capacitances are smooth functions of the terminal voltages throughout all modes of device operation. Comparison with experimentally measured MOST output conductance, transconductance, gate capacitance and subthreshold current shows exceptionally good agreement.

Short channel characteristics occur in the DMOST. To demonstrate the mechanisms of this device, a simple numerical model has been presented. Diffusion current has been found to oppose the net channel current over the negatively graded portion of the channel, allowing significantly larger lateral electric fields to exist over this portion of the channel. This in part explains why anomalously low values of substrate doping must be assumed in simpler DMOST models in order to obtain agreement with measured device characteristics. The behaviour of the potential minimum (virtual cathode) and the charge injection mechanisms of the DMOST and SCMOST are found to be qualitatively similar.

A SCMOST model has been presented which is representative of the device internal operation rather than just its terminal characteristics. A novel SCMOST model concept is evolved which captures several SCMOST mechanisms within one electrostatic relation. This analysis also accounts for the non-uniform doping present in a typical ion-implanted MOST and the surface mobility dependance on

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both the normal and lateral electric fields. The low computer time required together with the accuracy of a device internal characterization, should make this model a useful tool for the study of the consequences of channel length reduction and process changes in SCMOSTs.

Comparison with experimentally measured current characteristics of a $L = 3.8 \mu\text{m}$ MOST demonstrates good agreement and good results are also expected for shorter channel length devices.

9.2 Recommendations for Future Work

Modeling of the MOST has been a subject of study for two decades. In recent years, the importance of the MOST to integrated circuit technology has served to intensify this study. While improvements in fabrication capability have allowed considerable shrinking of MOST dimensions, the device mechanisms have become much more complex and interrelated. In the past, the main stream of device modeling efforts has been directed toward implementation in CAD network analysis programs. The constraints imposed by the requirements of CAD models however, substantially limit the sophistication of models that can be employed. It is the view of this author, that MOST modeling for this purpose may be reaching a point of diminishing returns. In fact, the decade to follow may witness a reversion to less sophisticated CAD device models. Such models will contain a number of adjustable parameters which can be obtained from automated device characterization or parameter extraction facilities or even more sophisticated 2-D and 3-D MOST simulations.

The DMOST model presented in chapter 7 could be used to supply

correction factors (fitting parameters) for use in less sophisticated CAD DMOST models. For this application, improved efficiency would result if the DMOST analysis of chapter 7 was implemented as a 'boundary value problem' rather than the 'initial value problem' approach as described.

Device models are also used to test device sensitivity to process changes. The SCMOST presented in chapter 8 could be used for this purpose. Further investigation into a more accurate means of determining values for the lateral electric field coefficients might also prove worthwhile. This analysis could be modified to account for surface state trapping. It could be employed as an efficient vehicle to study the effects of hot electron emission and the attendant carrier trapping mechanisms which degrade SCMOST performance.

Several modifications could be incorporated into the gradual channel model presented in chapter 6 such as drain region analysis and saturated drift velocity effects. It is expected that this model will have extensive application particularly for the CAD of linear circuits where longer channel device will be employed for the foreseeable future and accurate device characterization is often critical to circuit performance.

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APPENDIX A

A MODEL FOR THE RESISTIVE GATE MOST [43]

The resistive gate of the MOST illustrated in figure A.1 is biased with operational amplifiers so that the gate voltage along the channel is

$$V_{gs}(y) = \frac{V_{ds}y}{L} + V_{go} \quad (A.1)$$

where V_{go} is the input potential measured with respect to the source.

To achieve the desired linear output characteristics, this device is intentionally fabricated on a very lightly doped substrate. For this reason, the depletion charge can be ignored in the threshold voltage formula and the charge neutrality equation.

$$V_t = V_{fb} + 2\phi_f \quad (A.2)$$

$$Q_m = \int C_{ox} [V_{gs}(y) - V_{fb} - V_s] \quad (A.3)$$

Since this is a long channel MOST, to good approximation the mobility is constant and the diffusion current can be ignored.

Using equations (A.1), (A.2), (A.3) and (5.1) the current equation takes the form

$$\frac{I_d}{W} dy = \mu C_{ox} \left[\frac{V_{ds}}{L} y + V_{go} - V_t + 2\phi_f - V_s \right] dV_s \quad (A.4)$$

This equation is not in separated form, nor is it exact. Using equation (5.25) an integrating factor independent of y can be determined.

Figure A.1 Resistive gate MOST with linear I_d versus V_{ds} characteristics [43].

$$\lambda = \exp \left\{ - \frac{\mu W C_{ox} V_{ds}}{L I_d} V_s \right\} \quad (A.5)$$

If both sides of equation (A.4) are multiplied by λ the current equation becomes exact and therefore solvable. Moreover, the required integrations can be carried out in closed form. The resulting general solution is

$$\left\{ \frac{y}{L} + \frac{(V_{go} - V_t + 2\phi_f - V_s)}{V_{ds}} - \frac{L I_d}{\mu W C_{ox} V_{ds}^2} \right\} \cdot \frac{I_d L}{W} \exp \left\{ - \frac{\mu W C_{ox} V_{ds}}{L I_d} V_s \right\} = K^* \quad (A.6)$$

where K^* is an arbitrary constant yet to be determined. Using the boundary conditions $V_s = 2\phi_f$ at $y = 0$, and $V_s = 2\phi_f + V_{ds}$ at $y = L$, one can arrive at the following equation

$$\left[\frac{V_{go} - V_t}{V_{ds}} - \frac{L I_d}{\mu W C_{ox} V_{ds}^2} \right] \exp \left\{ - \frac{\mu W C_{ox} V_{ds}}{L I_d} \right\} = \left[\frac{V_{go} - V_t}{V_{ds}} - \frac{L I_d}{\mu W C_{ox} V_{ds}^2} \right] \quad (A.7)$$

which can only be true if

$$I_d = \mu C_{ox} \frac{W}{L} (V_{go} - V_t) V_{ds} \quad (A.8)$$

To determine the gate capacitance (as 'seen' by the operational amplifiers), it is necessary to substitute $L = y$ and $V_{ds} = V_s - 2\phi_f$ back into equation (A.8) with the result

$$V_s = \frac{I_d y}{\mu C_{ox} W (V_{go} - V_t)} + 2\phi_f \quad (A.9)$$

$$\text{and so } \frac{dV_s}{dy} = \frac{I_d}{\mu C_{ox} W (V_{go} - V_t)} \quad (A.10)$$

Using the current continuity equation (5.1) and (A.10) the charge density on the gate can be expressed as

$$Q_g = Q_m = C_{ox}(V_{go} - V_t) \quad (A.11)$$

The total capacitance of the gate is then

$$C_g = W \int_{y=0}^L \frac{dQ_g}{dV_{go}} dy = W L C_{ox} \quad (A.12)$$

One can also obtain the transconductance and output conductance directly from equation (A.8) as

$$g_m = \frac{dI_d}{dV_{go}} = \mu C_{ox} \frac{W}{L} V_{ds} \quad (A.13)$$

$$g_d = \frac{dI_d}{dV_{ds}} = \mu C_{ox} \frac{W}{L} (V_{go} - V_t) \quad (A.14)$$

Therefore according to first order theory, this MOST exhibits linear transconductance and output conductance as the inventors [43] purport.

This derivation was performed by this author in the spring of 1978. However, in December of 1979 a similar solution was reported by others [72].

APPENDIX B

BOUNDARY CONDITIONS FOR THE MOST CHANNEL

The equation of charge neutrality for an n-channel MOST as expressed by Pao and Sah [44] is

$$U_g' - U_s = \frac{C_D}{C_{ox}} \left\{ e^{U_s - \xi' - U_f} + e^{U_f - U_s} + (U_s - 1)e^{U_f} - (U_s + e^{-\xi'})e^{-U_f} \right\} \quad (B.1)$$

where the effective gate potential U_g' , the surface potential U_s , the hole fermi level U_f and the splitting of fermi levels ξ' are all normalized in units of $(\frac{kT}{q})$. The surface potential U_s is referenced to the substrate and $C_D = \epsilon_{si}/L_1$ where L_1 represents the intrinsic debye length. For all conditions of active MOST operation $U_s - U_f > 0$ and $U_s - U_f - \xi' > 0$. Except for the weakest region of subthreshold operation $U_s - U_f \gg 0$. Similarly except for saturation and the weakest mode of subthreshold operation, $U_s - U_f - \xi' \gg 0$. Therefore

$$\frac{e^{U_s - \xi' - U_f}}{e} + e^{U_f - U_s} \approx \frac{e^{U_s - \xi' - U_f}}{e} \quad (B.2)$$

Furthermore, since $U_f \gg 0$ and since $\xi' > 0$ except at the source where $\xi' = 0$ when $V_{sb} = 0$ then

$$(U_s - 1)e^{U_f} - (U_s + e^{-\xi'})e^{-U_f} \approx (U_s - 1)e^{U_f} \quad (B.3)$$

Thus equation (B.1) can be simplified to

$$U_g' - U_s = \frac{C_D}{C_{ox}} \left\{ e^{U_s - \xi' - U_f} + (U_s + 1)e^{U_f} \right\} \quad (B.4)$$

and conversion to unnormalized potentials yields

$$(V_g - V_s)^2 = \frac{2q\epsilon_{si} N}{C_{ox}^2} \left\{ \frac{kT}{q} e^{\frac{q}{kT} (V_s - \phi_n - \phi_f)} + (V_s - V_b - \frac{kT}{q}) \right\} \quad (B.5)$$

Furthermore since,

$$\phi_f = \frac{kT}{q} \ln \left(\frac{N}{n_i} \right) \quad (B.6)$$

$$\epsilon_{si} E_s = C_{ox} (V_g - V_s) \quad (B.7)$$

$$\epsilon_{si} E_d = \left\{ 2q\epsilon_{si} N (V_s - V_b - \frac{kT}{q}) \right\}^{\frac{1}{2}} \quad (B.8)$$

where E_s and E_d are the gate induced surface field and the surface field arising from the depletion charge respectively, then equation (B.5) can also be written as

$$(\epsilon_{si} E_s)^2 = 2q\epsilon_{si} n_i \left(\frac{kT}{q} \right) e^{\frac{q}{kT} (V_s - \phi_n)} + (\epsilon_{si} E_d)^2 \quad (B.9)$$

In the case of the DMOST, the internal substrate potential is not a particularly convenient potential reference since it varies with position along the channel. In order to establish the source as a potential reference, it is first necessary to interrelate the potentials V_g , ϕ_n and V_b . Thus

$$V_g - V_b = V_{gs} - V_{fb} + V_{sb} \quad (B.10)$$

where in the case of a n-doped polysilicon gate and p-type substrate

$$V_{fb} = \phi_{MS} - \frac{Q_{eff}}{C_{ox}} = -\frac{kT}{q} \ln \left\{ \frac{N_c N_g}{n_i^2} \right\} - \frac{Q_{eff}}{C_{ox}} \quad (B.11)$$

At the source,

$$\phi_n - V_b = \phi_f + V_{sb} \quad (B.12)$$

$$\text{hence, } \phi_n - \phi_f = V_b + V_{sb} \quad (B.13)$$

Similarly, at the drain one can obtain

$$\phi_n - \phi_f = V_b + V_{ds} + V_{sb} \quad (\text{B.14})$$

Then to establish the source as the reference for potentials V_s , V_g and ϕ_n , let

$$V_b = -V_{sb} - \frac{kT}{q} \ln \left\{ \frac{N_c N_{sr}}{n_i^2} \right\} \quad (\text{B.15})$$

From equations (B.5), (B.6), and (B.10), it follows that

$$C_{ox}^2 [V_{gs} - V_{fb} - V_{sb} - (V_s - V_b)]^2 = (2q\epsilon_{si} N_c) \left\{ \frac{kT}{q} \left(\frac{n_i}{N_c} \right)^2 \exp \left\{ \frac{q}{kT} [V_s - (\phi_n - \phi_f)] \right\} \right. \\ \left. + (V_s - V_b - \frac{kT}{q}) \right\} \quad (\text{B.16})$$

where V_b is obtained from equation (B.15) and the quantity $(\phi_n - \phi_f)$ is given by equation (B.13) at the source and (B.14) at the drain.

APPENDIX C

DETERMINATION OF COEFFICIENTS A_1 and B_1

In this analysis, the source and drain will be considered abrupt cylindrical junctions. The source region is illustrated in figure C.1. The potential difference between the source junction and the bulk is

$$V_1 - V_b = \frac{kT}{q} \ln \left\{ \frac{N_j N_b}{n_i^2} \right\} + V_{sb} \quad (C.1)$$

while at the drain

$$V_\zeta - V_b = \frac{kT}{q} \ln \left\{ \frac{N_j N_b}{n_i^2} \right\} + V_{sb} + V_{ds} \quad (C.2)$$

To good approximation, the radius of the cylindrical depletion layer boundary at the source is

$$r_{d1} = \left[\frac{2\epsilon_{si} (V_1 - V_b)}{qN_b} \right]^{1/2} + r_j \quad (C.3)$$

Using the Pythagorean Theorem, the depletion depth at $y = 0$ is

$$x_{d1} = (r_{d1}^2 - r_j^2)^{1/2} \quad (C.4)$$

Similarly, at the drain,

$$r_{d\zeta} = \left[\frac{2\epsilon_{si} (V_\zeta - V_b)}{qN_b} \right]^{1/2} + r_j \quad (C.5)$$

$$x_{d\zeta} = (r_{d\zeta}^2 - r_j^2)^{1/2} \quad (C.6)$$

Figure C.1 Diagram of source region for calculation of boundary coefficients.

At the source and drain, it will be further assumed that

$$\frac{\partial^2 V_s}{\partial y^2} = \frac{qN_{av}}{\epsilon_{si}} \quad (C.7)$$

where N_{av} is the average doping within surface depletion layer. From chapter 8 equation (8.37),

$$A_1 + B_1 = \frac{1}{\pi} \left\{ \int_0^{x_{d1}} \left[\frac{qN(x)}{\epsilon_{si}} \cdot x \right] dx - \frac{x_{d1}^2}{6} \cdot \frac{q N_{av}}{\epsilon_{si}} - (V_1 - V_b) \right\} \quad (C.8)$$

$$A_\zeta + B_\zeta = \frac{1}{\pi} \left\{ \int_0^{x_{d\zeta}} \left[\frac{qN(x)}{\epsilon_{si}} \cdot x \right] dx - \frac{x_{d\zeta}^2}{6} \cdot \frac{q N_{av}}{\epsilon_{si}} - (V_\zeta - V_b) \right\} \quad (C.9)$$

From equations (8.30) and (8.32) of the same chapter,

$$A_\zeta = A_1 \prod_{n=1}^{\zeta-1} \left[1 - \frac{H_{n+1/2}}{2} (p_n + \bar{p}_{n+1}) \right] \quad (C.10)$$

$$B_1 = B_\zeta \prod_{n=\zeta-1}^1 \left[1 - \frac{H_{n+1/2}}{2} (p_n + p_{n+1}) \right] \quad (C.11)$$

Thus coefficients A_1 and B_ζ can be determined from the four equations (C.8), (C.9), (C.10), and (C.11) since they contain only four unknowns A_1 , A_ζ , B_1 and B_ζ .

To be mathematically consistent, coefficients A_1 and B_ζ should be determined from fourier analysis at the source and drain rather than the method proposed here. Several such attempts have been made by this author. Some of these required more numerical computation. Unfortunately ~~the~~ results demonstrated worse agreement with experimentally measured device characteristics.

APPENDIX D

TWO DIMENSIONAL ELECTROSTATICS WITHIN THE OXIDE

Assuming a charge free gate oxide of thickness t_{ox} , an effective gate voltage V_g and a second order perturbation of surface potential around the point $y = y_k$, then within the oxide, $0 > x > -t_{ox}$,

$$\left. \frac{\partial^2 V}{\partial x^2} \right|_{y=y_k} = \left(1 + \frac{x}{t_{ox}} \right)^2 \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \quad (D.1)$$

This relation can be derived in exactly the same manner as equation (8.22) or simply abstracted from the result. By integration of equation

(D.1)

$$\left. \frac{\partial V}{\partial x} \right|_{y=y_k} = \frac{t_{ox}}{2} \left(1 + \frac{x}{t_{ox}} \right)^2 \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} + D_1 \quad (D.2)$$

$$V(x, y_k) = \frac{t_{ox}^2}{6} \left(1 + \frac{x}{t_{ox}} \right)^3 \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} + D_1 x + D_0 \quad (D.3)$$

Using equation (D.3) and the boundary constraint

$$V_g(-t_{ox}, y_k) - V(0, y_k) = V_g - V_s \quad (D.4)$$

then

$$D_1 = - \frac{(V_g - V_s)}{t_{ox}} - \frac{t_{ox}}{6} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \quad (D.5)$$

From equations (D.4) and (D.2), the gate induced surface field just inside the silicon is

$$-E_s = \frac{C_{ox}}{\epsilon_{si}} (V_g - V_s) - \frac{t_{ox} \epsilon_{ox}}{3 \epsilon_{si}} \left[\frac{\partial^2 V_s}{\partial y^2} \right]_{y=y_k} \quad (D.6)$$

APPENDIX B

LISTING OF PROGRAM FOR SCMOST SIMULATION

```

1.000 C      ...SHORT CHANNEL MOST SIMULATION
2.000 C
3.000      REAL*4 XP(101,11),YP(101,11)
4.000      REAL*8 AO(101),B(101),D(101),EX(101),EXI(101),
5.000      SH(101),HA(101),HB(101),HC(101),Q(101),SAAB(101),
6.000      SSUB(101),SUP(101),UC(101),V(101),VGE(101),
7.000      SVBX(101),VJV(101),XD(101),Y(101)
8.000      REAL*8 AVMVJ,BAB,CE,COX,
9.000      SEA,EFA,EFB,EFC,EN,EOX,ERR,ESI,F,FG,FF,
10.000     $HF,ID,IDL,IDT,IPN,IRT,KTDQ,L,MDVI,MDVO,NA,
11.000     SNI,NSS,PI,QE,QF,RKMO,RNPMO,UA,UB,UO,UODVS,
12.000     SVB,VBI,VEL,VDS,VGS,VS,VTC,W,WF,XJ,
13.000     $AOO,AR,BOO,BO,COE,DEVA,DFVB,DFVC,
14.000     SDV,DVF,DX,ESII,EXV,EXB,FD,FS,
15.000     SG,GIB,GN,HPI,IN,INX,LID,NAV,NSU,NX,
16.000     $QOKT,RN,RNPDT,ROI,ROTI,
17.000     $SAABF,SV,THA,TOX,XDE,XSCD,XSCS,ZA,ZN,
18.000     $XCR,SA,SB,XZ,INXS,INXD
19.000     INTEGER NPFC(11)
20.000 C
21.000 C      ...TECHNOLOGICAL PARAMETERS
22.000     ESI=11.7*8.86D-14
23.000     EOX=3.9*8.86D-14
24.000     FG=-.55D0
25.000     KTDQ=.0259D0
26.000     L=3.D-4
27.000     NI=1.45D10
28.000     NSS=.28D11
29.000     QE=1.602D-19
30.000     THA=1.0D-6
31.000     TOX=.05D-4
32.000     UO=850.D0
33.000     VS=6.5D6
34.000     W=100.D-4
35.000     XJ=.3D-4
36.000 C
37.000 C      ...SUBSTRATE DOPING DATA      N(X)=NA+SA+SB*X FOR X LT XCR
38.000 C      N(X)=NA      FOR X GT XCR
39.000     NA=2.5D15
40.000     NAV=2.9D15
41.000     XCR=1.0D-4
42.000     SA=1.0D15
43.000     SB=-SA/XCR
44.000 C

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```

45.000 NP=41
46.000 NPMO=NP-1
47.000 NPMT=NP-2
48.000 NPDT=.34*NP
49.000 NPMTT=NP-NPDT
50.000 RNPDT=DFLOAT(NPMTT-NPDT+1)
51.000 RNPMO=DFLOAT(NPMO)
52.000 PI=3.141592654D0
53.000 JC=1
54.000 C
55.000 C ...CALCULATED PARAMETERS
56.000 COX=EOX/TOX
57.000 COE=COX/ESI
58.000 ESII=1.D0/ESI
59.000 HPI=.5D0*PI
60.000 QOE=QE/ESI
61.000 QOKT=1.D0/KTDQ
62.000 ZN=1.D0
63.000 ROI=1.D0/(ZN+1.D0)
64.000 ROTI=1.D0/((ZN+1.D0)*(ZN+2.D0))
65.000 UODVS=UO/VS
66.000 ZA=.5D0*THA*COX/ESI
67.000 C
68.000 FF=KTDQ*DLOG(NA/NI)
69.000 WF=FG-FF
70.000 VBI=.55D0+FF
71.000 C
72.000 C ...DETERMINE MESH POINTS Y(K) & MESH INCREMENTS H(K)
73.000 DO20 K=1,NP
74.000 RKMO=DFLOAT(K-1)
75.000 Y(K)=.5D0*L*(1.D0-DCOS(PI*RKMO/RNPMO))
76.000 IF(K.GE.2) H(K-1)=Y(K)-Y(K-1)
77.000 20 CONTINUE
78.000 C
79.000 C ...DIFFERENCE TERMS USED IN SOL'N OF POISSON'S EQN
80.000 DG30 K=2,NPMO
81.000 HF=H(K)+H(K-1)
82.000 HA(K)=2.D0/(H(K-1)*HF)
83.000 HB(K)=2.D0/(H(K-1)*H(K))
84.000 30 HC(K)=2.D0/(H(K)*HF)
85.000 C
86.000 C

```

```

87.000 C      ...READ IN BIAS VOLTAGES
88.000 C      SET IAN=1 FOR ANOTHER SIMULATION, OTHERWISE SET IAN=0
89.000 40     WRITE(108,330)
90.000        READ,VGS,VDS,VB,IAN
91.000 C
92.000 C      ...SET FLAGS & INITIALIZE COUNTERS
93.000        ICF=0;IFST=1;NIT=0;NVIT=0
94.000        MDVI=0.DO;MDVO=0.DO;IDL=0.DO
95.000 C
96.000 C      ...CALCULATE EFFECTIVE GATE & SUBSTRATE POTENTIALS
97.000        DO50 K=1,NP
98.000        VGE(K)=VGS-WF+QE*NSS/COX
99.000 50     VBX(K)=-VB+KTDQ
100.000 C
101.000 C      ...LONG CHANNEL THRESHOLD VOLTAGE CALCULATION
102.000 C      ASSUMING N(X)=NA, FOR PRINTOUT ONLY
103.000        VTC=-QE*NSS/COX+WF+2.DO*FF
104.000        S=DSQRT(2.DO*ESI*QE*NA*(2.DO*FF+VB))/COX
105.000 C
106.000        PRINT,VGS,VDS,VB,VTC
107.000 C
108.000 C      ...INITIAL POTENTIAL DIST'N - MAPPED PARABOLA
109.000        Q(1)=.28D-5
110.000        SRG=DSQRT(2.DO*ESI*QE*NA)/COX
111.000        QYMIN=COX*((VGE(1)-2.*FF)-SRG*DSQRT(2.*FF-VBX(1)))
112.000        IF(QYMIN.LT..00001*Q(1)) QYMIN=.00001*Q(1)
113.000        VMNF=VBI-KTDQ*DLOG(Q(1)/QYMIN)
114.000        IF(VMNF.LT.2.*FF) VMNF=2.*FF
115.000        VCSAT=VGE(1)+SRG/2.
116.000        S=DSQRT(SRG*SRG/4.+SRG*(VGE(1)-VBX(1)))
117.000        IF(VCSAT.LE.VMNF) VMNF=VCSAT
118.000        TS=VBI-VMNF
119.000        TD=VBI+VDS-VMNF
120.000        TR=2.*ESI/(QE*NA)
121.000        XLS=SQRT(TR*TS)
122.000        XLD=SQRT(TR*TD)
123.000        CLTR=XLS+XLD
124.000        YMNF=CLTR/(1.+SQRT(TD/TS))
125.000        YCSAT=YMNF*(1.+SQRT((VCSAT-VMNF)/TS))
126.000        IF(YCSAT.GT..9*CLTR) YCSAT=.9*CLTR
127.000        DYCM=L-CLTR+YCSAT-YMNF
128.000        DO60 K=1,NP
129.000        YF=Y(K)*CLTR/L
130.000        IF(L.LE.CLTR) GO TO 62
131.000        YF=Y(K)
132.000        IF(Y(K).GT.YMNF) YF=YMNF+(Y(K)-YMNF)*(YCSAT-YMNF)/DYCM
133.000        IF(Y(K).GT.(L-CLTR+YCSAT)) YF=Y(K)-L+CLTR
134.000 62     V(K)=VMNF+(YF-YMNF)*(YF-YMNF)*TS/(YMNF*YMNF)
135.000 60     CONTINUE
136.000        AR=1.DO
137.000        IF(V(NP).GT.(1.DO+VCSAT)) AR=.5DO
138.000 C

```

```

139.000 C      ...MOBILE CHARGE AT SOURCE & DRAIN
140.000      XSCS=XJ+DSQRT(2.D0*ESI*(V(1)-VBX(1))/(QE*NA))
141.000      XSCD=XJ+DSQRT(2.D0*ESI*(V(NP)-VBX(NP))/(QE*NA))
142.000      XD(1)=DSQRT(XSCS*XSCS-XJ*XJ)
143.000      XD(NP)=DSQRT(XSCD*XSCD-XJ*XJ)
144.000      EXB=DEXP((VBI-FF)/KTDQ)
145.000      GIB=2.D0*ESI*QE*NI
146.000      FS=2.D0*ESI*(V(1)-VBX(1))/XSCS
147.000      FD=2.D0*ESI*(V(NP)-VBX(NP))/XSCD
148.000      Q(1)=DSQRT(GIB*EXB+FS*FS)-FS
149.000      Q(NP)=DSQRT(GIB*EXB+FD*FD)-FD
150.000 C
151.000 C      ...DETERMINATION OF THE SUM A(K)+B(K) AT SOURCE AND DRAIN
152.000      INXS=.5D0*QOE*NA*XD(1)*XD(1)
153.000      INXS=INXS+QOE*XCR*XCR*(.5D0*SA+SB*XCR/6.D0)
154.000      NSU=QOE*NAV
155.000      AOO=INXS-XD(1)*XD(1)*ROTI*NSU-(V(1)-VBX(1))
156.000      INXD=.5D0*QOE*NA*XD(NP)*XD(NP)
157.000      INXD=INXD+QOE*XCR*XCR*(.5D0*SA+SB*XCR/6.D0)
158.000      BOO=INXD-XD(NP)*XD(NP)*ROTI*NSU-(V(NP)-VBX(NP))
159.000 C
160.000 C      ....INITIAL DEPLETION DEPTH XD(K)
161.000      DO63 K=2,NPMO
162.000      XD(K)=DSQRT(2.D0*ESI*(V(K)-VBX(K))/(QE*NA))
163.000 63      CONTINUE
164.000 C
165.000 C      ...INITIAL VALUES OF SAAB(K)=A(K)+B(K)
166.000      AO(1)=1
167.000      DO65 K=2,NP
168.000      AO(K)=AO(K-1)*(1.D0+HPI*H(K-1)/XD(K-1))
169.000      $*(1.D0+HPI*H(K-1)/XD(K))
170.000 65      CONTINUE
171.000      GN=AO(NP)
172.000      RN=1.D0/AO(NP)
173.000      AO(1)=(BOO-RN*AOO)/(GN-RN)
174.000      BO=AOO-AO(1)
175.000      SAAB(1)=AOO
176.000      DO67 K=2,NP
177.000      SAAB(K)=AO(1)*AO(K)+BO/AO(K)
178.000 67      CONTINUE
179.000 C
180.000 C

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181.000 C    ...READ OUTPUT MODE & MAX NUMBER OF ITERATIONS
182.000 C    IOPM=0 OUTPUTS ID & NUMERICAL INFO TO TERMINAL
183.000 C    IOPM=1 OUTPUTS INTERNAL MOST SIMULATION RESULTS
184.000 C    TO TERMINAL /
185.000 C    IOPM=10 TO 20 OUTPUTS DATA FOR PLOTTING
186.000 C    EXAMINE END OF PROGRAM FOR DETAILS
187.000 70    WRITE(108,340)
188.000      READ,IOPM,MNIT
189.000      IF((MNIT.LT.0).OR.(IOPM.LT.0)) STOP
190.000      IF(MNIT.EQ.0) GO TO 200
191.000 C
192.000 C
193.000 C    ...BEGIN ITERATIVE SOLUTION
194.000      DO190 K=1,MNIT
195.000      NIT=NIT+1
196.000      IF(IFST.EQ.1) GO TO 110
197.000      DO71 K=2,NPMO
198.000      VJV(K)=V(K)
199.000 71    CONTINUE
200.000 C
201.000 C    ...DETERMINE SAAB(K)=A(K)+B(K)
202.000      DO75 KD=1,2
203.000      AO(1)=1.D0
204.000      DO72 K=2,NP
205.000      AO(K)=AO(K-1)*(1.D0+HPI*H(K-1)/XD(K-1))
206.000      S*(1.D0+HPI*H(K-1)/XD(K))
207.000 72    CONTINUE
208.000      GN=AO(NP)
209.000      RN=1.D0/AO(NP)
210.000      AO(1)=(BOO-RN*AOO)/(GN-RN)
211.000      BO=AOO-AO(1)
212.000      DO73 K=2,NP
213.000      SAAB(K)=AO(1)*AO(K)+BO/AO(K)
214.000 73    CONTINUE
215.000 C
216.000 C    ...SOLVE FOR XD(K)
217.000      DO74 K=2,NPMO
218.000      NX=QOE*NA*XD(K)
219.000      INX=.5D0*QOE*NA*XD(K)*XD(K)
220.000      XZ=XD(K)
221.000      IE(XD(K).GT.XCR) XZ=XCR
222.000      NX=NX+QOE*(SA+SB*XZ)*XZ
223.000      INX=INX+QOE*(.5D0*SA+SB*XZ/3.D0)*XZ*XZ
224.000      CE=HA(K)*V(K-1)-HB(K)*V(K)+HC(K)*V(K+1)
225.000      IF(CE.GT.NSU) GE=NSU
226.000      G=VBX(K)-V(K)+INX-SAAB(K)-CE*XD(K)*XD(K)*ROTI
227.000      DGXDI=1.D0/(NX-2.D0*XD(K)*ROTI*CE)
228.000      XD(K)=XD(K)-DGXDI*G
229.000 74    CONTINUE
230.000 75    CONTINUE
231.000 C

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232.000 C    ...SOLVE FOR POTENTIAL DIST'N ITERATIVELY AS
233.000 C    SOL'N TO POISSON'S EQN
234.000      DO105 JV=1,20
235.000      NVIT=NVIT+1
236.000      DO90 K=2,NPMO
237.000      IN=QOE*NA*XD(K)
238.000      XZ=XD(K)
239.000      IF(XD(K).GT.XCR) XZ=XCR
240.000      IN=IN+QOE*(SA+.5D0*SB*XZ)*XZ
241.000      XDE=XD(K)*ROI-TOX*EOX/(3.D0*ESI)
242.000      CE=V(K-1)*HA(K)-V(K)*HB(K)+V(K+1)*HC(K)
243.000      SAABF=2.D0*SAAB(K)/XD(K)
244.000 C    ...SERIES EXPANSION FOR EXP((V(K)-VJV(K))/KTDQ)
245.000      QF=Q(K)*ESII
246.000      IF(JV.EQ.1) GO TO 85
247.000      SV=V(K)-VJV(K)
248.000      DVF=AR*SV*QOKT
249.000      IF(SV.LT.0.D0) DVF=-DVF
250.000      EXV=1.D0+DVF*(1.D0+.5D0*DVF*(1.D0+(DVF/3.D0)
251.000      S*(1.D0+.25D0*DVF*(1.D0+.2D0*DVF)))
252.000      IF(SV.LT.0.D0) EXV=1.D0/EXV
253.000      QF=QF*EXV
254.000 C    ...SET UP TRIDIAGONAL MATRIX ELEMENTS
255.000 85    F=-COE*(VGE(K)-V(K))+QF+IN-SAABF-XDE*CE
256.000      DFVA=-XDE*HA(K)
257.000      DFVB=COE+QF*QOKT+XDE*HB(K)
258.000      DFVC=-XDE*HC(K)
259.000      IF(K.GT.2) SUB(K-1)=DFVA
260.000      D(K-1)=DFVB
261.000      IF(K.LT.NPMO) SUP(K-1)=DFVC
262.000      B(K-1)=-F
263.000 90    CONTINUE
264.000 C    ...SOLVE TRIDIAGONAL MATRIX EQN
265.000      CALL TRID(SUB,D,SUP,B,NPMT)
266.000      MDVI=0.D0
267.000      DO100 K=2,NPMO
268.000      BAB=DABS(B(K-1))
269.000      IF(BAB.GT.MDVI) MDVI=BAB
270.000 100    V(K)=B(K-1)+V(K)
271.000 C    ...TEST FOR CONVERGENCE
272.000      IF(MDVI.LE.1.D-4) GO TO 107
273.000 105    CONTINUE
274.000 C    ...DETERMINE MAX V(K) SHIFT
275.000 107    MDVO=0.D0
276.000      DO108 K=2,NPMO
277.000      AVMVJ=DABS(V(K)-VJV(K))
278.000 108    IF(AVMVJ.GT.MDVO) MDVO=AVMVJ
279.000 C
280.000 110    CONTINUE
281.000      IFST=0
282.000 C

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283.000 C      ...SOLVE FOR MOBILE CHARGE DIST'N AS SOL'N TO
284.000 C      THE CURRENT CONTINUITY EQN
285.000 140    DO150 K=1,NPMO
286.000 C      ...CALCULATE EXPONENTIALS
287.000      DV=V(K+1)-V(K)
288.000      EA=DV*QOKT
289.000      EX(K)=1.D0+.5D0*EA
290.000      EXI(K)=1.D0-.5D0*EA
291.000      IF((EA.LT.1.D-4).AND.(EA.GT.-1.D-4)) GO TO 145
292.000      DX=DEXP(EA)
293.000      EX(K)=EA*DX/(DX-1.D0)
294.000      EXI(K)=EA/(DX-1.D0)
295.000 C      ...CALCULATE MOBILITY
296.000 145    UB=ZA*(VGE(K+1)-V(K+1)+VGE(K)-V(K))
297.000      IF(UB.LT.0.D0) UB=0.D0
298.000      UA=UODVS*DV/H(K)
299.000      UC(K)=UO/(DSQRT(1.D0+UA*UA)*(1.D0+UB)*H(K))
300.000 150    CONTINUE
301.000 C      ...SET UP TRIDIAGONAL MATRIX ELEMENTS
302.000      DO160 K=2,NPMO
303.000      EFA=UC(K-1)*EX(K-1)
304.000      EFB=UC(K-1)*EXI(K-1)+UC(K)*EX(K)
305.000      EFC=UC(K)*EXI(K)
306.000      IF(K.GT.2) SUB(K-1)=-EFA
307.000      D(K-1)=EFB
308.000      IF(K.LT.NPMO) SUP(K-1)=-EFC
309.000      B(K-1)=0.D0
310.000      IF(K.EQ.2) B(1)=EFA*Q(1)
311.000      IF(K.EQ.NPMO) B(NPMT)=EFC*Q(NP)
312.000 160    CONTINUE
313.000 C      ...SOLVE TRIDIAGONAL MATRIX EQN
314.000      CALL TRID(SUB,D,SUP,B,NPMT)
315.000      DO170 K=2,NPMO
316.000 170    Q(K)=B(K-1)
317.000 C
318.000 C      ...AVERAGE CURRENT OVER CENTER 3RD OF CHANNEL
319.000      ID=0.D0
320.000      DO180 K=NPDT,NPMTT
321.000 180    ID=UC(K)*(Q(K)*EX(K)-Q(K+1)*EXI(K))+ID
322.000      ID=W*KTDQ*ID/RNPDT
323.000 C      ...TEST FOR CONVERGENCE
324.000      DELID=DABS(ID-IDL)
325.000      IF(ID.LE..0D0) ID=1.D-12
326.000      ERRF=DELID/ID
327.000      IF(ERRF.GE.1.D-5) ICF=0
328.000      IF(ERRF.LT.1.D-5) ICF=1+ICF
329.000      IDL=ID
330.000      IF((NP*(3.*NIT+NVIT)).GT.25000.) ICF=3
331.000      IF(ICF.GE.3) GO TO 200
332.000 190    CONTINUE
333.000 C
334.000 C

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335.000 C      ...OUTPUT DATA TO TERMINAL
336.000 200    LID=0.D0
337.000        IF(ID.GT.0.D0) LID=DLOG10(ID)
338.000        PRINT, ID, LID, DELID, NIT
339.000        PRINT, MDVO, MDVI, NVIT
340.000        IF(IOPM.NE.1) GO TO 230
341.000        DO220 K=1, NP
342.000        VEL=0.D0
343.000        IF(Q(K).NE.0.D0) VEL=ID/(W*Q(K))
344.000 215    WRITE(108, 310) Y(K), V(K), Q(K), VEL, XD(K)
345.000        IF(K.EQ.NP) GO TO 220
346.000        EN=(V(K+1)-V(K))/H(K)
347.000        IDT=W*KTDQ*UC(K)*(Q(K)*EX(K)-Q(K+1)*EXI(K))
348.000        IFN=W*KTDQ*UC(K)*(Q(K)-Q(K+1))
349.000        IRT=IDT-IFN
350.000        WRITE(108, 320) IDT, IRT, IFN, EN
351.000 220    CONTINUE
352.000 C
353.000 C      ...OUTPUT DATA FOR PLOTTING
354.000 230    IF((IOPM.LT.10).OR.(IOPM.GT.15)) GO TO 250
355.000        DO240 K=1, NP
356.000        XP(K, JC)=Y(K)
357.000        IF(IOPM.EQ.10) YP(K, JC)=V(K)
358.000        IF(IOPM.EQ.11) YP(K, JC)=Q(K)
359.000        IF((IOPM.EQ.12).AND.(Q(K).NE.0.D0)) YP(K, JC)=ID/(W*Q(K))
360.000 240    CONTINUE
361.000        NPFC(JC)=NP
362.000 250    IF((IOPM.LT.16).OR.(IOPM.GT.20)) GO TO 270
363.000        DO260 K=1, NPMO
364.000        XP(K, JC)=Y(K)+.5*H(K)
365.000        IDT=W*KTDQ*UC(K)*(Q(K)*EX(K)-Q(K+1)*EXI(K))
366.000        IFN=W*KTDQ*UC(K)*(Q(K)-Q(K+1))
367.000        IRT=IDT-IFN
368.000        IF(IOPM.EQ.16) YP(K, JC)=(V(K+1)-V(K))/H(K)
369.000        IF(IOPM.EQ.17) YR(K, JC)=IRT
370.000        IF(IOPM.EQ.18) YP(K, JC)=IFN
371.000 260    CONTINUE
372.000        NPFC(JC)=NPMO
373.000 C
374.000 270    IF((IOPM.GE.10).AND.(IOPM.LE.20)) JC=JC+1
375.000        IF((ICF.LT.3).AND.(IOPM.NE.30)) GO TO 70
376.000        IF(IAN.EQ.1) GO TO 40
377.000        JC=JC-1
378.000        IF(JC.GE.1) CALL PLOT(XP, YP, NPFC, JC)
379.000        STOP
380.000 310    FORMAT(' ', 5D14.6)
381.000 320    FORMAT(' ', 10X, 4D14.6)
382.000 330    FORMAT('0', 'VGS, VDS, VB,  ANOTHER ?')
383.000 340    FORMAT(' ', 'O/P MODE, MAX. ITERATIONS ?')
384.000        END
385.000 C
386.000 C

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387.000      SUBROUTINE TRID(SUB,DIAG,SUP,B,MD)
388.000 C      ...SOL'N OF "A*Z=B" WHERE "A" IS A TRIDIAGONAL MATRIX
389.000      REAL*8 SUB(101),DIAG(101),SUP(101),B(101),G,GI
390.000      IF(DIAG(1).EQ.0.D0) GO TO 490
391.000      GI=1.D0/DIAG(1)
392.000      B(1)=B(1)*GI
393.000      IF(MD.LE.1) RETURN
394.000      SUP(1)=SUP(1)*GI
395.000 C      ...FORWARD SUBSTITUTION
396.000      DO430 K=2,MD
397.000      G=DIAG(K)-SUB(K)*SUP(K-1)
398.000      IF(G.EQ.0.D0) GO TO 490
399.000      GI=1.D0/G
400.000      B(K)=(B(K)-SUB(K)*B(K-1))*GI
401.000      IF(K.GE.MD) GO TO 430
402.000      SUP(K)=SUP(K)*GI
403.000 430      CONTINUE
404.000 C      ...BACKWARD SUBSTITUTION
405.000      MDMO=MD-1
406.000      DO450 K=1,MDMO
407.000      KB=MD-K
408.000      B(KB)=B(KB)-SUP(KB)*B(KB+1)
409.000 450      CONTINUE
410.000      RETURN
411.000 490      WRITE(108,495)
412.000 495      FORMAT('0','ERROR --- DIVISION BY ZERO IN TRID')
413.000      STOP
414.000      END
415.000 C
416.000 C

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417.000 SUBROUTINE PLOT(X,Y,LO,LT)
418.000 C ...PLOT ROUTINE FOR USE WITH HP DIGITAL PLOTTER
419.000 C X(I,J),Y(I,J) - COORDINANTS OF POINT I ON LINE J
420.000 C LO(J) - NUMBER OF POINTS ON LINE J
421.000 C LT - NUMBER OF LINES TO BE PLOTTED
422.000 REAL*4 X(101,11),Y(101,11)
423.000 INTEGER IA(6),LO(11)
424.000 DATA IA/0,1999,3999,5999,7999,9999/
425.000 C ...DETERMINE MAX & MIN VALUES
426.000 XMAX=X(1,1);XMIN=X(1,1);YMAX=Y(1,1);YMIN=Y(1,1)
427.000 DO620 J=1,LT
428.000 LNP=LO(J)
429.000 DO630 I=1,LNP
430.000 IF(X(I,J).GT.XMAX) XMAX=X(I,J)
431.000 IF(X(I,J).LT.XMIN) XMIN=X(I,J)
432.000 IF(Y(I,J).GT.YMAX) YMAX=Y(I,J)
433.000 IF(Y(I,J).LT.YMIN) YMIN=Y(I,J)
434.000 630 CONTINUE
435.000 620 CONTINUE
436.000 PRINT,XMAX,XMIN,YMAX,YMIN
437.000 READ,XMAX,XMIN,YMAX,YMIN,NB
438.000 XD=XMAX-XMIN
439.000 YD=YMAX-YMIN
440.000 C ...PLOT BORDER
441.000 IF(NB.EQ.1) GO TO 680
442.000 WRITE(108,600)
443.000 DO640 J=1,4
444.000 DO650 I=1,6
445.000 IB=7-I
446.000 IF(J.EQ.1) WRITE(108,606) IA(1),IA(I)
447.000 IF(J.EQ.2) WRITE(108,606) IA(I),IA(5)
448.000 IF(J.EQ.3) WRITE(108,606) IA(6),IA(IB)
449.000 IF(J.EQ.4) WRITE(108,606) IA(IB),IA(1)
450.000 650 CONTINUE
451.000 640 CONTINUE
452.000 WRITE(108,604)
453.000 C ...SCALE & PLOT DATA
454.000 680 DO660 J=1,LT
455.000 WRITE(108,602)
456.000 LNP=LO(J)
457.000 DO670 I=1,LNP
458.000 IX=9999.5*(X(I,J)-XMIN)/XD
459.000 IF((IX.GT.9999).OR.(IX.LT.0)) GO TO 670
460.000 IY=9999.5*(Y(I,J)-YMIN)/YD
461.000 IF((IY.GT.9999).OR.(IY.LT.0)) GO TO 670
462.000 WRITE(108,606) IX,IY
463.000 670 CONTINUE
464.000 WRITE(108,604)
465.000 660 CONTINUE
466.000 RETURN
467.000 600 FORMAT('PLTL')
468.000 602 FORMAT('PLTP')
469.000 604 FORMAT(' PLTT')
470.000 606 FORMAT(2I5)
471.000 END

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END

0	1	H	0	9	H	8	2
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FIN