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Techniques for Aircraft Force and Moment Model Development Using Geometry and Stability and Control Derivative Estimates

by

Tari Eileen Kaye, B.Sc. (Physics)

A thesis submitted to the Faculty
of Graduate Studies and Research
in partial fulfilment of the
requirements for the degree of

Master of Engineering

Department of
Mechanical and Aerospace Engineering
Ottawa-Carleton Institute for Mechanical and Aerospace Engineering
Carleton University
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Development Using Geometry and Stability and
Control Derivative Estimates**

submitted by

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in partial fulfilment of the requirements
for the degree of Master of Engineering

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Abstract

Ground-based flight simulators are an important tool in pilot training, and are based on a force and moment model of the particular aircraft to be simulated. One type of model contains stability and control derivatives, which must be determined in order to accurately model the aircraft's response to given inputs.

Between 1991 and 1993, flight tests were performed on a Dash 8 Series 300 aircraft by personnel of the Flight Research Laboratory of the National Research Council of Canada, in conjunction with CAE Electronics, Ltd.. The data collected were used to create a simulator model. The stability and control derivatives in the simulator model were determined using a maximum likelihood estimation procedure.

A geometry-based force and moment model of the Dash 8 Series 300 aircraft is presented which uses values of the previously-determined stability and control derivatives to solve for several unknown model parameters. The resulting total aircraft forces and moments are compared to flight test values. Parameters occurring within the model equations are compared to empirical estimates. The geometry-based force and moment equations are substituted into the Dash 8 Series 300 flight simulator, several test cases are run, and the results are compared to flight test data. The model results are comparable to both flight test values and empirical estimates, and the flight simulator responses are within tolerance limits set by the United States Federal Aviation Administration for a level D simulator.

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Nomenclature

α	Lift-curve slope (= C_{L_α})
a	Acceleration
AR	Wing aspect ratio
b	Wing span
$\bar{c}$	Wing mean geometric chord
c_r	Wing root chord
c_t	Wing tip chord
$c(y)$	Wing chord length as a function of distance along wing
CG	Aircraft centre of gravity
C_D	Total drag coefficient
C_{D_0}	Profile drag coefficient
C_l	Total rolling moment coefficient
C_{l_p}	Rolling-moment-due-to-non-dimensional-roll-rate coefficient (SCD)
C_{l_r}	Rolling-moment-due-to-non-dimensional-yaw-rate coefficient (SCD)
C_{l_β}	Rolling-moment-due-to-sideslip-angle coefficient (SCD)
$C_{l_{\delta a}}$	Rolling-moment-due-to-aileron-deflection coefficient (SCD)
$C_{l_{\delta R}}$	Rolling-moment-due-to-rudder-deflection coefficient (SCD)
$C_{l_{\delta i}}$	Rolling-moment-due-to-inner-spoiler-deflection coefficient (SCD)
$C_{l_{\delta o}}$	Rolling-moment-due-to-outer-spoiler-deflection coefficient (SCD)

C_L	Total lift coefficient
C_{L_q}	Lift-due-to-non-dimensional-pitch-rate coefficient (SCD)
C_{L_α}	Lift-due-to-angle-of-attack coefficient (SCD)
$C_{L_{\delta_e}}$	Lift-due-to-elevator-deflection coefficient (SCD)
C_m	Total pitching moment coefficient
C_{m_q}	Pitching-moment-due-to-non-dimensional-pitch-rate coefficient (SCD)
C_{m_α}	Pitching-moment-due-to-angle-of-attack coefficient (SCD)
$C_{m_{\delta_e}}$	Pitching-moment-due-to-elevator-deflection coefficient (SCD)
C_n	Total yawing moment coefficient
C_{n_p}	Yawing-moment-due-to-non-dimensional-roll-rate coefficient (SCD)
C_{n_r}	Yawing-moment-due-to-non-dimensional-yaw-rate coefficient (SCD)
C_{n_β}	Yawing-moment-due-to-sideslip-angle coefficient (SCD)
$C_{n_{\delta_a}}$	Yawing-moment-due-to-aileron-deflection coefficient (SCD)
$C_{n_{\delta_R}}$	Yawing-moment-due-to-rudder-deflection coefficient (SCD)
$C_{n_{\delta_{s1}}}$	Yawing-moment-due-to-inner-spoiler-deflection coefficient (SCD)
$C_{n_{\delta_{s0}}}$	Yawing-moment-due-to-outer-spoiler-deflection coefficient (SCD)
$C_{thrusts}$	Total thrust coefficient
C_T	Engine thrust coefficient
C_Y	Total side force coefficient
C_{Y_p}	Side-force-due-to-non-dimensional-roll-rate coefficient (SCD)
C_{Y_r}	Side-force-due-to-non-dimensional-yaw-rate coefficient (SCD)
C_{Y_β}	Side-force-due-to-sideslip-angle coefficient (SCD)

$C_{Y_{\delta a}}$	Side-force-due-to-aileron-deflection coefficient (SCD)
$C_{Y_{\delta R}}$	Side-force-due-to-rudder-deflection coefficient (SCD)
$C_{Y_{\delta si}}$	Side-force-due-to-inner-spoiler-deflection coefficient (SCD)
$C_{Y_{\delta so}}$	Side-force-due-to-outer-spoiler-deflection coefficient (SCD)
D	Total drag
e	Oswald's efficiency factor
E	Inner spoiler roll correction factor
F	Outer spoiler roll correction factor
G	Aileron roll correction factor
h	Horizontal position of aircraft CG, as a percentage of mean geometric chord
h_n	Aircraft neutral point, as a percentage of mean geometric chord
H	Inner spoiler yaw correction factor
i	Incidence angle of appropriate aircraft component
I	Outer spoiler yaw correction factor
I	Moment of inertia about appropriate axis
J	Aileron yaw correction factor
k_1	Location of inner edge of aileron, as a percentage of wing semi-span
k_2	Location of outer edge of aileron, as a percentage of wing semi-span
k_3	Location of inner edge of outer spoiler, as a percentage of wing semi-span
k_4	Location of outer edge of outer spoiler, as a percentage of wing semi-span
k_5	Location of inner edge of inner spoiler, as a percentage of wing semi-span
k_6	Location of outer edge of inner spoiler, as a percentage of wing semi-span

K	Wing roll correction factor
l	Total rolling moment
l	Horizontal distance from CG to appropriate aircraft component's aerodynamic centre
l'_i	Horizontal distance from wing-body neutral point to horizontal tail aerodynamic centre
l_i^*	Horizontal distance from horizontal tail quarter-chord point to wing quarter-chord point
L	Total lift
m	Mass
M	Total pitching moment
N	Total yawing moment
p	Aircraft roll rate
q	Aircraft pitch rate
$\bar{q}$	Dynamic pressure
r	Aircraft yaw rate
S	Reference area (wing)
T	Total thrust
u	Velocity in x-direction
v	Velocity in y-direction
V	Volume coefficient of appropriate aircraft component
w	Velocity in z-direction
x	Axis pointing out aircraft nose
x	Wing-body side force neutral point

X	Total force in x-direction
y	Axis pointing out right wing
y	Wing dihedral neutral point
y_{engine}	Horizontal distance from CG to right or left engine location on wing
Y	Total side force
z	Axis pointing down
z	Vertical distance from CG to appropriate aircraft component's aerodynamic centre
z_T	Vertical distance from CG to engine thrust line
Z	Total force in z-direction
α	Angle of attack
α_m	Angle of attack measured during flight tests, from relative velocity to fuselage reference
α_{OL}	Zero-lift angle of attack, measured from the wing zero-lift line to the fuselage reference
β	Sideslip angle
γ	Wing dihedral angle
δa	Aileron deflection angle
δe	Elevator deflection angle
δR	Rudder deflection measurement
δsi	Inner spoiler deflection angle
δso	Outer spoiler deflection angle
ε	Downwash angle
ζ	Side force due to yaw rate correction factor

η	Efficiency factor (ratio of dynamic pressure at appropriate aircraft location to that at the wing)
θ	Aircraft pitch angle
θ_T	Angle between engine thrust centreline and fuselage reference
κ	Yawing moment due to yaw rate correction factor
λ	Rolling moment due to yaw rate correction factor
λ	Wing taper ratio (tip chord over root chord)
ξ	Yawing moment due to yaw rate correction factor
σ	Sidewash angle
Σ	Side force due to roll rate correction factor
τ	Effectiveness factor of appropriate aircraft component
ϕ	Bank angle
χ	Yawing moment due to roll rate correction factor
Ω	Drag factor of appropriate aircraft component

Subscripts

0	Conditions at zero control deflections, thrust, rates
a	Aileron
dyn	Aircraft dynamics (pitch, roll yaw)
e	Elevator
eff	Angles measured from zero-lift lines to references lines
F	Fin
$fuse$	Fuselage

<i>H</i>	Horizontal tail
<i>L</i>	Left
<i>R</i>	Right
<i>R</i>	Rudder
<i>s</i>	Stability axes
<i>si</i>	Inner spoilers
<i>so</i>	Outer spoilers
<i>t</i>	Horizontal tail
<i>T</i>	thrust
<i>trim</i>	At trim conditions
<i>vs</i>	Vertical strake
<i>w</i>	Wing
<i>wb</i>	Wing-body
<i>x</i>	Regarding x-axis
<i>y</i>	Regarding y-axis
<i>z</i>	Regarding z-axis

Special Notations

∂	Partial derivative
$\cdot$	Derivative
$\wedge$	Non-dimensional form

Acronyms, Designations, and Abbreviations

AIC	Akaike's Information Criteria
CAE	CAE Electronics Ltd., Montreal
Datcom	United States Air Force Stability and Control Datcom
ESDU	Engineering Sciences Data Unit International Ltd.
FAA	United States Federal Aviation Administration
FRL	Flight Research Laboratory of the Institute for Aerospace Research of the National Research Council of Canada
MLE	Maximum likelihood estimation
POM	Proof-of-match
SCD	Stability and control derivative
USAF	United States Air Force

Chapter 1

Introduction

1.1 Background

Ground-based flight simulators play an important role in research related to new aircraft designs and in pilot training. In recent years, they have become as important a design tool as wind tunnels and system test rigs, and have given designers the opportunity to explore the implications of different designs, without having to incur the cost and delay of building and testing a range of prototypes^[1]. In addition, simulators can be used to train new pilots, as well as to teach experienced pilots the intricacies of flying new aircraft types, without exposure to the dangers of actually flying an unfamiliar aircraft. Moreover, simulators are cost- and time-effective, since they are not affected by adverse weather conditions, fuel expenses, or availability of aircraft or airspace^[1]. They also allow specific training situations to be conducted which would be too expensive or dangerous to be performed in a real aircraft. The development of flight simulators for many different types of aircraft is therefore in high demand.

In order to simulate how any aircraft will fly, a valid set of equations of motion,

such as the Euler equations^[2], must be used in conjunction with a force and moment model of the aircraft. One type of aircraft force and moment model, which utilizes small perturbation assumptions and Taylor expansions, when combined with the Euler equations, results in a linearized, six degree-of-freedom set of equations that describes the aircraft's motion. The set of equations is decoupled, meaning that the longitudinal (x-z plane) forces and moments are separate from the lateral (x-y and y-z plane) forces and moments, and can be treated independently. The equations contain stability and control derivatives (SCDs), which are unique to each type of aircraft, and determine how that particular type of aircraft will respond to a given input. Thus, when this type of model is used, determination of the correct values of the SCDs is essential to the successful development of a ground-based flight simulator.

1.2 Methods for Determining SCDs

1.2.1 Flight Tests and Parameter Estimation

Flight tests and parameter estimation techniques can be used together to determine an aircraft's SCDs. First, flight tests of the aircraft in question are performed, which consist of two sets of specific manoeuvres. The first set is designed to excite all of the aircraft's characteristic modes of motion and is used in the parameter estimation process. The second set is used for final model validation purposes and consists of manoeuvres which provide information on specific areas of the flight envelope, such as take-offs, landings, and engine-out situations. The second set is not used in the initial parameter estimation procedure, but can be used to make small adjustments to the final SCD values.

Possible input manoeuvres for the collection of data for parameter estimation analyses range from simple pulses to frequency sweeps, where the control input is oscillated with increasing frequency. One of the most common flight test input manoeuvres is a 3-2-1-1 control input (see Fig. 1.1), which consists of a series of alternating step control inputs in a ratio of 3:2:1:1 seconds. This input is of relatively short duration, is easy to perform, and is easily repeatable, yet results in time histories which adequately describe the aircraft dynamics, since the input contains power over a wide frequency range^[3]. Overall, the flight tests result in the collection of large amounts of data related to atmospheric conditions, aircraft rotation rates, attitude and acceleration information, control surface positions, control forces, engine parameters, and other pertinent flight conditions.

To apply parameter estimation techniques to the flight test data, a representative model of the aircraft dynamics is needed. For a conventional aircraft design in non-extreme flight regimes, the standard linear decoupled set of equations which contain the SCDs is adequate^[4].

Once both the flight test data and a suitable dynamic model have been obtained, a time domain parameter estimation technique can be used to estimate the required SCD values. Batch techniques such as the equation error technique are based on the principle of least squares, and estimate the parameter values that minimize the sum of the squares of the differences between the measured and computed aircraft responses through a one-step mathematical process^[5]. These are the simplest methods to use, although they tend to give biased results when performed in the presence of measurement or sensor noise^[5]. Iterative methods, such as output error methods, minimize the errors between the

computed and actual aircraft response by using the same input and varying the parameter estimates until the outputs are comparable^[6]. An example of an output error method is analog matching. Here an operator chooses the best values of the parameters to match an oscilloscope trace of a computed response to that of the actual aircraft response^{[6], [7]}. Output error methods are highly dependent on the skill of the operator, and do not account for the presence of noise^{[6], [8]}.

Other parameter estimation processes are best classified as advanced statistical methods. Examples of these include extended Kalman filtering and maximum likelihood estimation (MLE). MLE is the most common method used to predict aircraft SCDs^[9]. An overall view of the MLE process is described in Iliff and Maine^[10]. The process involves maximizing the probability that the mathematically determined aircraft response matches that of the actual aircraft response. This probability is defined for each possible estimate of each SCD, and the estimate that results in the highest probability of match for any input is chosen as the best overall estimate. In practical terms, this means that a cost function is defined as a function of the difference between the measured and computed responses, and this cost function is minimized. A computational algorithm is used to find the estimates of the stability and control derivatives that minimize the cost function. These estimates are then used to update the mathematical model of the aircraft, which, in turn, updates the response and therefore the cost function. This process continues iteratively until a convergence criterion is satisfied. The resulting values are likely the most accurate estimates of all methods presently available^[9]. However, these values are valid only for the precise trim conditions at which they were evaluated. Thus, the results

of the MLE process are sets of SCDs for each trim condition studied. In order to create a model which is valid for all flight conditions, extensive curve-fitting of the SCDs against all possible flight variables is required.

It should be noted that the MLE process takes state and measurement noise (e.g. turbulence and sensor errors) into account. Since the aerodynamic model does not describe the aircraft motion exactly, there are also some modelling errors. These are usually ignored by treating them as state or measurement noise, or both^[10]. Measures of the relative accuracy of each estimate are also available, but they are largely dependent on the amount of flight test data available and the amount of noise added to each manoeuvre. Any anomalies found in the match can indicate problems such as ignored terms in the equations of motion, sensor problems, insufficient data, or nonlinearities. It is also important to realize that the MLE results are mathematically determined, and thus can be inconsistent with a physical model of the aircraft dynamics. That is, while a physical model shows the relationships which exist between the SCDs (e.g. dependence upon a common parameter), the MLE results for each SCD are calculated mathematically, and do not necessarily reflect those physical relationships.

1.2.2 Data Sheets

A second method to estimate the SCD values is the use of compiled data sheets. These usually make use of empirical relationships and plots created through theory and wind tunnel tests conducted on a variety of aircraft types to estimate values for some desired parameter. Data sheets are available in several forms, one of which can be used to estimate each SCD directly, and another of which can be used to find parameters which

are contained within equations that represent each SCD. Overall, this source of SCD data is limited, and often extrapolation is necessary to extend the predicted SCD values to appropriate conditions for an aircraft that does not fall within the limits of the original wind tunnel tests. Thus, SCD values obtained from data sheets are not always very accurate. However, they are often useful for preliminary design purposes. One set of data sheets commonly used is published by Engineering Sciences Data Unit (ESDU) International Ltd.^{[11], [12], [13], [14], [15]}

1.2.3 Geometric Equations

Force and moment calculations can be used to create a set of equations that adequately represents each SCD, yet includes only the known geometry of the aircraft and several unknown model parameters (e.g. neutral point, sidewash). This method is not usually implemented since the unknown model parameters are difficult or impossible to measure directly. It is also the least accurate method of estimating the SCD values, since the equations themselves are only simplified estimates of the actual forces and moments acting on each part of the aircraft at any given time. Etkin^[2] gives complete geometric derivations for the longitudinal set of SCDs (lift, drag, and pitching moment SCDs), along with partial derivations for some of the SCDs in the lateral set (side force, rolling moment, and yawing moment SCDs).

1.2.4 Combinations of Methods

While flight testing is the most widely-used method of determining aircraft SCDs, combinations of the other two methods (data sheets and geometric equations) are relatively common. United States Air Force Stability and Control Datcom^[16] and a group

of Fortran programs written by Smetana^[17] are only two of the many available packages that can be used to develop full force and moment models of aircraft. These methods rely either on wind tunnel data to determine the unknown parameters that appear in geometry-based equations of each SCD, or on purely empirical equations describing families of aircraft to determine the SCD values directly. Tests show that results from this type of method can be adequate for flight simulator use^[18].

1.3 Scope of Present Work

In 1991, the Flight Research Laboratory (FRL) of the Institute for Aerospace Research of the National Research Council of Canada, in cooperation with CAE Electronics Ltd., Montreal, conducted a series of flight tests on a Boeing deHavilland DHC-8 (Dash 8) Series 300 aircraft. The data obtained were used together with a MLE process and extensive curve-fitting to estimate values for the SCDs important to that particular aircraft. These, in turn, were used in the creation of a ground-based flight simulator model which was certified by the United States Federal Aviation Administration (FAA) to level D training standards in 1993.

Due to the nature of the MLE method and the subsequent curve fitting process, it is possible that discrepancies in the final force and moment model could exist. That is, the MLE method can produce values for the SCDs which are inconsistent with a physical model of the aircraft dynamics. The more important an individual SCD is to the aircraft dynamics, the more accurate is the MLE estimate. Thus, relatively unimportant SCDs could be estimated poorly, producing physically inconsistent results. A model

based on geometry would not contain these inconsistencies, and could be compared to the SCD model to highlight where problems may exist. In addition, the process of curve-fitting the MLE-derived SCD values to allow determination of the SCDs for every flight condition is time-consuming and laborious, since each SCD must be fit against all possible flight parameters. However, the unknowns in the geometric equations are dependent on fewer parameters. Thus, if a future parameter estimation procedure is used to determine these unknowns instead of the SCD values, the curve-fitting process would be simplified. Furthermore, by relating the SCDs through common geometrical parameters, the process of correcting the present simulator model to account for situations not explicitly modelled, such as flap and gear changes and ground effect, would be improved. For example, if it is known that a certain parameter varies during the process of raising the landing gear, each SCD containing that parameter could be quickly adjusted. Since the MLE process produces independent values for the SCDs, it does not explicitly indicate which SCDs are affected by a change in any particular parameter.

The present work develops a process which provides both a consistent and reliable methodology for developing a full flight envelope force and moment aircraft model from trim point SCD values, as well as the basis for future parameter estimation research to develop geometry-based trim point models. A set of geometry-based equations is presented which have been created to replace the existing SCD-based aircraft force and moment model. In addition, each SCD is itself described by a geometrical equation. These equations contain several unknown model parameters (such as neutral points and sidewash parameters) in addition to the aircraft's geometric measurements. The values

of the SCDs previously obtained from the MLE and curve fit processes are used to determine the values of the unknown model parameters in each geometry-based equation. Verification of the resulting values (and thus, the form of the equations) is obtained through comparison with empirical calculations of the same unknown model parameters. Further verification is obtained through comparison of the MLE-determined SCD values with the geometrically-calculated SCD values, through comparison of force and moment terms measured at a given moment in a particular flight test manoeuvre with those predicted by the geometric model, and through comparison of the geometry-based simulator model response with the original flight test data. In the final chapter, conclusions are drawn and recommendations are made for future work.

Chapter 2

Outline of Method

2.1 Setup

Throughout this project, the frame of reference used is one which is fixed to the aircraft, with the origin at the mass centre, and which moves with the aircraft. More specifically, a body axis system is used (see Fig. 2.1), in which the x-axis points out the nose of the aircraft and is parallel to the fuselage reference line (which is taken as the floor of the aircraft), the y-axis points out the right wing, and the z-axis points down. Fig. 2.1 also shows the notation for forces, velocities, rates, and moments, and indicates the directions of positive values of each. In this figure, (X, Y, Z) are the components of the resultant aerodynamic force, (u, v, w) are the components of the velocity of the aircraft centre of gravity (CG), l is the rolling moment, M is the pitching moment, N is the yawing moment, p is the roll rate, q is the pitch rate, and r is the yaw rate of the aircraft.

Positive control surface deflections were defined to give negative corresponding moments. Thus, positive rudder deflection (δR) is to the left, positive elevator deflection

(δe) is down, and positive aileron deflection (δa) is right aileron down (with corresponding upward deflection of the left aileron). Inner and outer spoiler deflections (δsi and δso) are defined as left spoiler deflections positive and right spoiler deflections negative, since they only deploy upward.

In the following sections, all derivations assume a rigid body aircraft in gliding flight, with the addition of direct thrust effects (i.e. forces and moments produced directly by the engines). In addition, the x-z plane is assumed to be a plane of symmetry, and the mass of the aircraft is assumed to remain constant throughout any specific manoeuvre. Note that the effects of flaps and the indirect effects of thrust (i.e. effects of thrust on other model parameters, such as downwash) are not included here. They will be modelled later by curve fitting each model parameter with respect to thrust coefficient for each flap setting.

2.2 Longitudinal Geometry-Based Derivations

The longitudinal derivations are taken, for the most part, from Etkin^[2]. However, the present work makes use of a different elevator treatment, and deals with the problem of the varying zero-lift lines of the various aircraft components. Note that the drag equation is not included in this project.

2.2.1 Lift

The aircraft lift coefficient is defined as

$$C_L = \frac{L}{qS} \quad (2.1)$$

where: L is the total aircraft lift
 $\bar{q}$ is the dynamic pressure at the wing
 S is the wing reference area

Dividing the total aircraft lift into its various contributions gives

$$C_L = \frac{L_0 + L_{wb} + L_t + L_{dyn} + L_T}{\bar{q}S} \quad (2.2)$$

where the subscript 0 refers to the condition where the angle of attack, all control surface deflections, aircraft rates, and thrust are zero, wb refers to the wing-body combination, t to the horizontal tail, dyn to the relevant aircraft dynamics (in this case, pitch), and T to thrust. Effects of the vertical tail and vertical strake are neglected in the longitudinal equations. If these components are non-dimensionalized, the result is

$$C_L = C_{L_0} + C_{L_{wb}} + \eta_t \frac{S_t}{S} C_{L_t} + \eta_i \frac{S_i}{S} C_{L_{dyn}} + C_{L_T} \quad (2.3)$$

where: η_t is the horizontal tail efficiency factor (ratio of the dynamic pressure at the horizontal tail ($\bar{q}_t$) to the dynamic pressure at the wing ($\bar{q}$))
 S_t is the horizontal tail reference area

Even though the lift due to thrust is a direct thrust effect, it is neglected in order to reduce the number of unknowns in the lift equation and is thus deleted from equation 2.3.

For a linearized model, the lift due to angle of attack can be written as

$$C_L = a\alpha \quad (2.4)$$

where: a is the aircraft lift-curve slope
 α is the aircraft angle of attack, measured from the relative velocity vector to the aircraft zero-lift line

Thus the wing-body contribution to the lift can be written as

$$C_{L_{wb}} = a_{wb} \alpha_{wb} \quad (2.5)$$

where α_{wb} is measured from the relative velocity vector at the wing to the wing-body zero-lift line. With each angle of attack measured to its corresponding zero-lift line, C_{L_0} is zero, and it is deleted from equation 2.3. Also, the horizontal tail contribution can be written as

$$C_{L_t} = a_t \alpha_t \quad (2.6)$$

where α_t is measured from the relative velocity vector at the tail to the horizontal tail zero-lift line. Using Fig. 2.2, it can be shown that

$$C_{L_t} = a_t (\alpha_{wb} + \alpha_{eff_{wb}} - i_w + i_t - \epsilon - \alpha_{eff_t}) \quad (2.7)$$

where: $\alpha_{eff_{wb}}$ is the angle of attack of the wing-body combination, measured from the wing-body zero-lift line to the wing reference line
 i_w is the incidence angle of the wing, measured from the fuselage reference line to the wing reference line
 i_t is the incidence angle of the horizontal tail, measured from the fuselage reference line to the horizontal tail reference line
 ϵ is the downwash angle at the horizontal tail, measured from the relative velocity vector at the wing to the relative velocity vector at the tail
 α_{eff_t} is the angle of attack of the horizontal tail, measured from the horizontal tail zero-lift line to the horizontal tail reference line

Note that

$$\alpha_{eff} = \alpha_{0L_t} + i_t \quad (2.8)$$

where α_{0L_t} is the angle of attack of the horizontal tail, measured from the horizontal tail zero-lift line to the fuselage reference line (see Fig. 2.2).

Using a linear approximation, ϵ may be written as

$$\epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} \alpha_{wb} \quad (2.9)$$

and α_{0L_t} may be approximated as

$$\alpha_{0L_t} = \alpha_0 + \frac{\partial \alpha_{0L_t}}{\partial \delta e} \delta e \quad (2.10)$$

which assumes that the effect of the deflection of the elevator is purely a change in the horizontal tail zero-lift line. Then equation 2.7 can be written as

$$C_{L_t} = a_t \left(\alpha_{wb} + \alpha_{eff} - i_w + i_t - \epsilon_0 - \frac{\partial \epsilon}{\partial \alpha} \alpha_{wb} - \alpha_0 - \frac{\partial \alpha_{0L_t}}{\partial \delta e} \delta e - i_t \right) \quad (2.11)$$

The negative partial derivative of α_{0L_t} with respect to δe is given the symbol τ_e . Thus, rearranging equation 2.11 gives

$$C_{L_t} = a_t \alpha_{wb} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) + a_t (\alpha_{eff} - i_w - \epsilon_0 - \alpha_0) + a_t \tau_e \delta e \quad (2.12)$$

The pitch contribution to the lift can be thought of as originating from an increase in angle of attack of the horizontal tail due to the pitch rate, q . The fuselage and wing

contributions are assumed to be small. Thus,

$$C_{L_{\dot{\alpha}}} = a_t \Delta \alpha_t \quad (2.13)$$

or

$$C_{L_{\dot{\alpha}}} = a_t \frac{q l_t}{u_0} \quad (2.14)$$

where: l_t is the horizontal distance from the aircraft CG to the horizontal tail aerodynamic centre (see Fig. 2.3)
 u_0 is the x-component of the aircraft's initial undisturbed velocity

Substituting equations 2.5, 2.12, and 2.14 into equation 2.3 gives the total aircraft lift coefficient as

$$C_L = a_{wb} \alpha_{wb} \left[1 + \frac{a_t}{a_{wb}} \eta_t \frac{S_t}{S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] + a_t \eta_t \frac{S_t}{S} (\alpha_{effwb} - i_w - \epsilon_0 - \alpha_0) + a_t \eta_t \frac{S_t}{S} \tau_e \delta e + a_t \eta_t \frac{S_t}{S} \frac{q l_t}{u_0} \quad (2.15)$$

The first two terms of equation 2.15 represent the total aircraft lift due to angle of attack. Thus, for a linear model, they can be written as the product of the total aircraft lift-curve slope, a , and the aircraft angle of attack measured from the relative velocity vector at the wing to the aircraft zero-lift line, α . The equations are given as

$$a = a_{wb} \left[1 + \frac{a_t}{a_{wb}} \eta_t \frac{S_t}{S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] \quad (2.16)$$

and

$$\alpha = \alpha_{wb} - \frac{a_i}{a} \eta_i \frac{S_i}{S} (\epsilon_0 + \alpha_0 + i_w - \alpha_{eff_{wi}}) \quad (2.17)$$

Thus, the lift coefficient is

$$C_L = a\alpha + a_i \eta_i \frac{S_i}{S} \tau_e \delta e + a_i \eta_i \frac{S_i}{S} \frac{q l_i}{u_0} \quad (2.18)$$

Two things should be noted. First, α can be split into two angles: one which is measured from the relative velocity vector at the wing to the fuselage reference line (α_m), and one which is measured from the wing zero-lift line to the fuselage reference line (α_{0L}). α_m is usually the parameter measured during flight tests, while α_{0L} is an unknown which varies with both thrust and flap settings. Thus,

$$\alpha = \alpha_m - \alpha_{0L} \quad (2.19)$$

Second, a non-dimensional pitch rate ($\hat{q}$) is often more convenient to use than q . Thus, the pitch rate is multiplied by $\bar{c}/2u_0$ to get $\hat{q}$. This results in a term containing the horizontal tail volume coefficient, defined as

$$V_H = \frac{S_i l_i}{S \bar{c}} \quad (2.20)$$

If equation 2.16 is substituted for α , then the total aircraft lift coefficient is given by

$$C_L = a_{wb} \left[1 + \frac{a_i \eta_i}{a_{wb}} \frac{S_i}{S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] (\alpha_m - \alpha_{0L}) + a_i \eta_i \frac{S_i}{S} \tau_e \delta e + 2a_i \eta_i V_H \hat{q} \quad (2.21)$$

This is the final form of the lift equation. Next, equations for estimating the lift SCDs as functions of the unknown model parameters in equation 2.21 must be defined.

C_{L_α} is defined as the change in aircraft lift coefficient with a change in aircraft angle of attack while all other parameters are held constant. That is,

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} \quad (2.22)$$

Taking the partial derivative of equation 2.21 with respect to alpha gives

$$C_{L_\alpha} = a_{wb} \left[1 + \frac{a_t}{a_{wb}} \eta_t \frac{S_t}{S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] \quad (2.23)$$

which means C_{L_α} is identical to the total aircraft lift-curve slope, a , given by equation 2.16. This stability derivative will be positive, since an increase in angle of attack will result in an increase in lift (for angles below stall).

$C_{L_{\delta_e}}$ is defined as the change in aircraft lift coefficient with a change in elevator deflection angle, while all other parameters are held constant. That is,

$$C_{L_{\delta_e}} = \frac{\partial C_L}{\partial \delta_e} \quad (2.24)$$

From equation 2.21, this means that

$$C_{L_{\delta_e}} = a_t \eta_t \frac{S_t}{S} \tau_e \quad (2.25)$$

This control derivative will be positive, due to the definition of positive elevator

deflection (down).

Finally, C_{L_q} is defined as the change in aircraft lift coefficient with a change in the non-dimensional pitch rate, while all other parameters are held constant. That is,

$$C_{L_q} = \frac{\partial C_L}{\partial \dot{q}} \quad (2.26)$$

Equation 2.21 can be differentiated to yield

$$C_{L_q} = 2a_i \eta_i V_H \quad (2.27)$$

This stability derivative is usually positive.

2.2.2 Pitching Moment

The aircraft pitching moment coefficient is defined as

$$C_m = \frac{M}{\bar{q} S \bar{c}} \quad (2.28)$$

where M is the total aircraft pitching moment. This is divided into the contributions from each of the aircraft components to get

$$C_m = \frac{M_{wb} + M_i + M_{dyn} + M_T}{\bar{q} S \bar{c}} \quad (2.29)$$

or, in non-dimensional form,

$$C_m = C_{m_{wb}} + \frac{S_i}{S} \eta_i C_{m_i} + \frac{S_i}{S} \eta_i C_{m_{dyn}} + C_{m_T} \quad (2.30)$$

Note that the pitching moment at zero lift, zero control deflections, zero rates, and zero thrust (C_{m0}) is not explicitly shown in equation 2.30. Instead, it will be introduced in the derivations of the various contributions to the total pitching moment.

Fig. 2.4 shows a pitching couple independent of angle of attack and the forces acting on a wing in the x-z plane. From this, the pitching moment about the wing's CG can be described as

$$M_w = M_{0_w} + (L_w \cos \alpha_w + D_w \sin \alpha_w)(h - h_{n_w})\bar{c} + (L_w \sin \alpha_w - D_w \cos \alpha_w)z\bar{c} \quad (2.31)$$

where:

- M_{0_w} is the wing pitching moment at zero lift, zero control deflections, zero rates, and zero thrust
- L_w is the wing lift
- D_w is the wing drag
- α_w is the angle of attack of the wing, measured from the relative velocity vector at the wing to the wing zero-lift line
- $\bar{c}$ is the mean aerodynamic chord of the wing
- h is the CG position, measured in mean aerodynamic chord lengths, from the wing leading edge
- h_{n_w} is the neutral point of the wing, measured in mean aerodynamic chord lengths, from the wing leading edge
- z is the vertical distance from the CG to the neutral point of the wing

Equation 2.31 is made non-dimensional by dividing through by $\bar{q}S\bar{c}$, small angle assumptions are made ($\cos \alpha_w = 1$, $\sin \alpha_w = \alpha_w$), and negligible terms are dropped. In addition, the subscript w is changed to wb to account for the effect of the fuselage. (The form of the equation stays the same when adding the fuselage effect; only the values of the coefficients change.) Thus, the wing-body contribution to the pitching moment is given by

$$C_{m_{wb}} = C_{m_{0wb}} + \alpha_{wb} a_{wb} (h - h_{nwb}) \quad (2.32)$$

The horizontal tail contribution is modelled as

$$C_{m_t} = \frac{M_t}{q S_t c} \quad (2.33)$$

with

$$M_t = -l_t L_t \quad (2.34)$$

where l_t is the horizontal distance between the aircraft's CG and the horizontal tail aerodynamic centre (see Fig. 2.3). Thus

$$C_{m_t} = -\frac{l_t}{c} C_{L_t} \quad (2.35)$$

Substituting equation 2.12 into equation 2.35 gives

$$C_{m_t} = -\frac{l_t}{c} \left[a_t \alpha_{wb} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) + a_t (\alpha_{efwb} - i_w - \epsilon_0 - \alpha_0) + \tau_e \delta e \right] \quad (2.36)$$

The pitch rate contribution to the pitching moment is assumed to be due to the increase in lift coefficient of the horizontal tail. That is, the wing and fuselage contributions are neglected. Thus,

$$C_{m_{\dot{\alpha}}} = -\frac{l_t}{c} C_{L_{\dot{\alpha}}} \quad (2.37)$$

Substituting equation 2.14 into equation 2.37 and non-dimensionalizing q gives

$$C_{m_{\dot{\alpha}}} = -2a \left(\frac{l_t}{c} \right)^2 \dot{q} \quad (2.38)$$

The direct effect of thrust on the pitching moment arises from the fact that the thrust line is offset from the CG by a vertical distance z_T . The propeller normal force will also contribute to the pitching moment, but it is neglected here. Thus,

$$C_{m_T} = \frac{T z_T}{\bar{q} S c} \quad (2.39)$$

or

$$C_{m_T} = C_T \frac{z_T}{c} \quad (2.40)$$

where z_T is positive measured downward from the CG.

Thus, combining equations 2.30, 2.32, 2.36, 2.38, and 2.40 and simplifying gives

$$C_m = C_{m_{\alpha_{wb}}} + a_{wb} \alpha_{wb} \left[(h - h_{nwb}) - \frac{a_i \eta_i}{a_{wb}} V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] + a_i \eta_i V_H (\epsilon_0 + \alpha_0 + i_w - \alpha_{effwb}) \\ - a_i \eta_i V_H \left[\tau_e \delta e + 2 \frac{l_t}{c} \dot{q} \right] + C_T \frac{z_T}{c} \quad (2.41)$$

To get the pitching moment at zero lift, with no elevator deflection, pitch rate or thrust, set α to zero in equation 2.17 and δe , $\dot{q}$, and C_T to zero in equation 2.41, and substitute the resulting α_{wb} from equation 2.17 into equation 2.41. This gives

$$C_{m_0} = C_{m_{\alpha_{wb}}} + \left[\frac{a_i \eta_i}{a} \frac{S_i}{S} (\epsilon_0 + \alpha_0 + i_w - \alpha_{effwb}) \right] a_{wb} (h - h_{nwb}) - a_i \eta_i V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \\ + a_i \eta_i V_H (\epsilon_0 + \alpha_0 + i_w - \alpha_{effwb}) \quad (2.42)$$

If equation 2.41 is compared to equation 2.42, it can be shown that

$$C_m = C_{m_0} + a_{wb} \alpha \left[(h - h_{nwb}) - \frac{a_t \eta_t}{a_{wb}} V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] - a_t \eta_t V_H \left[\tau_e \delta e + 2 \frac{l_t}{c} \hat{q} \right] + C_T \frac{z_T}{c} \quad (2.43)$$

Finally, replacing α by equation 2.19 gives

$$C_m = C_{m_0} + a_{wb} \left[(h - h_{nwb}) - \frac{a_t \eta_t}{a_{wb}} V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] (\alpha_m - \alpha_{0L}) - a_t \eta_t V_H \left[\tau_e \delta e + 2 \frac{l_t}{c} \hat{q} \right] + C_T \frac{z_T}{c} \quad (2.44)$$

Equations relating the model parameters of this equation to estimates of the pitching moment SCDs are determined by taking the appropriate derivatives of equation 2.44

C_{m_α} is defined as the change in aircraft pitching moment coefficient with a change in aircraft angle of attack while all other parameters are held constant. That is,

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \quad (2.45)$$

Taking the partial derivative of equation 2.44 with respect to alpha gives

$$C_{m_\alpha} = a_{wb} \left[(h - h_{nwb}) - \frac{a_t \eta_t}{a_{wb}} V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] \quad (2.46)$$

With positive moments as defined in Fig. 2.1, this stability derivative will be negative for statically stable aircraft.

Equation 2.46 is a valid way to determine C_{m_α} , but it can be simplified by introducing the stick-fixed neutral point, h_n . This is the point where the pitching moment is independent of the angle of attack. In other words, h is equal to h_n when C_{m_α} is equal to zero. From equation 2.46, it can be seen that

$$h_n = h_{n_{wb}} + V_H \frac{a_t}{a_{wb}} \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \quad (2.47)$$

Fig. 2.3 shows that

$$l_t = l'_t - (h - h_{n_{wb}}) \bar{c} \quad (2.48)$$

where l'_t is the horizontal distance from the wing-body neutral point to the horizontal tail aerodynamic centre, and thus

$$V_H = V'_H - \frac{S_t}{S} (h - h_{n_{wb}}) \quad (2.49)$$

where V'_H is the horizontal tail volume coefficient corresponding to l'_t . Then

$$V_{H_a} = V'_H - \frac{S_t}{S} (h_n - h_{n_{wb}}) \quad (2.50)$$

and

$$h_n - h_{n_{wb}} = V'_H \frac{a_t}{a} \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \quad (2.51)$$

Equations 2.16, 2.46, 2.49, and 2.51 can be combined to simplify the equation for C_{m_α} to

$$C_{m_\alpha} = \alpha(h - h_n) \quad (2.52)$$

Equation 2.52 can be interpreted as Cl_α multiplied by a moment arm, the length of which is equal to the distance from the aircraft's CG to the aircraft's overall neutral point. This distance is defined as the negative of the aircraft stick-fixed static margin.

$C_{m_{\delta_e}}$ is defined as the change in aircraft pitching moment coefficient with a change in elevator deflection angle, while all other parameters are held constant. That is,

$$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e} \quad (2.53)$$

From equation 2.44, this means that

$$C_{m_{\delta_e}} = -a_i \eta_i V_H \tau_e \quad (2.54)$$

This control derivative will be negative for elevators aft of the aircraft's CG, and with downward deflection of the elevator defined as positive. It is often call "elevator effectiveness" or "elevator power".

Finally, C_{m_q} is defined as the change in aircraft pitching moment coefficient with a change in the non-dimensional pitch rate, while all other parameters are held constant. That is,

$$C_{m_q} = \frac{\partial C_m}{\partial \hat{q}} \quad (2.55)$$

Equation 2.44 can be differentiated to get

$$C_{m_q} = -2a_t \eta_t V_H \frac{l_t}{c} \quad (2.56)$$

This stability derivative must be negative for a dynamically stable aircraft, and is referred to as "pitch damping".

Equations 2.23, 2.25, 2.27, 2.46, 2.54, and 2.56 will be used, along with the MLE-derived lift and pitching moment SCDs, to determine values for the model parameters contained in each equation (i.e. a_{wb} , a_t , η_t , h_{nwb} , $\partial e / \partial \alpha$, and τ_t). Note that, in the preceding lift and pitching moment derivations, the terms a_t and η_t always appear together. Thus, they can be considered coupled, and it is not necessary to ascertain separate values for each of them.

2.3 Lateral Geometry-Based Derivations

The lateral derivations stem from several sources. The vertical tail contributions are taken from Etkin^[2], with the vertical strake contributions being derived in the same manner. The wing-body contributions have various origins, which will be referenced as the individual contributions are introduced.

2.3.1 Side Force

The total aircraft side force coefficient is defined as

$$C_Y = \frac{Y}{qS} \quad (2.57)$$

where Y is the total aircraft side force. This can be separated into contributions from the

various aircraft components as

$$C_Y = \frac{Y_0 + Y_{wb} + Y_F + Y_{vs} + Y_{dyn} + Y_T}{\bar{q}S} \quad (2.58)$$

where the subscript 0 refers to the condition where the sideslip angle, as well as all control deflections, aircraft rates, and thrust are zero, wb refers to the wing-body combination, F to the vertical tail or fin, vs to the vertical strake, dyn to the relevant aircraft dynamics (in this case, roll and yaw), and T to thrust. Note that the horizontal tail is not considered in the lateral equation derivations. In non-dimensional form, equation 2.58 becomes

$$C_Y = C_{Y_0} + C_{Y_{wb}} + \frac{S_F}{S} \eta_F C_{Y_F} + \frac{S_{vs}}{S} \eta_{vs} C_{Y_{vs}} + C_{Y_{dyn}} + C_{Y_T} \quad (2.59)$$

Since the x-z plane was taken to be a plane of symmetry, then C_{Y_0} is equal to zero. Also, the side force due to thrust is neglected. This leaves wing-body, fin, vertical strake, and dynamic contributions to be determined.

Side force due to sideslip can be thought of in a similar manner to lift due to angle of attack. Thus, for the wing-body contribution, a linear model gives

$$C_{Y_{wb}} = -a_{fuse} \beta \quad (2.60)$$

where:

- a_{fuse} is a "side force-curve slope" for the wing-body combination
- β is the sideslip angle, measured from the aircraft centreline to the relative velocity vector at the wing (see Fig. 2.5)

The negative sign in the previous equation is a result of defining the y-axis positive out the right wing (see Fig. 2.1).

The wing-body contribution to the side force itself contains smaller contributions from the ailerons and spoilers. As one or more of these control surfaces is deflected, it creates an increase in side surface area, which, in turn, creates an increase in the total side force. However, for an aircraft with an unswept wing (such as the Dash 8) this increase is very small. Thus, all side force contributions due to ailerons and spoilers are neglected.

When an aircraft is sideslipping, the angle of attack of the vertical tail or fin, measured from the resultant velocity vector at the fin to the fin zero-lift line (see Fig. 2.6), is given by

$$\alpha_F = -\beta + \sigma - \alpha_{0L_F} \quad (2.61)$$

where: σ is the sidewash angle, similar to ε for a horizontal tail, measured from the relative velocity vector at the fin to the relative velocity vector at the wing
 α_{0L_F} is the angle of attack of the fin, measured from the zero-lift line of the fin to the aircraft centreline

Assuming that the effect of the rudder deflection is purely a change in the fin zero-lift line, α_{0L_F} can be approximated as

$$\alpha_{0L_F} = \alpha_0 + \frac{\partial \alpha_{0L_F}}{\partial \delta R} \delta R \quad (2.62)$$

where δR is the rudder deflection. However, since the fin is a symmetric airfoil, α_0 is zero. Also, the negative partial derivative of α_{0L_F} with respect to δR is given the symbol

τ_R . Thus, equation 2.61 can be written as

$$\alpha_F = -\beta + \sigma + \tau_R \delta R \quad (2.63)$$

This angle of attack gives a side force on the vertical tail, the coefficient of which is equal to

$$C_{Y_F} = a_F (-\beta + \sigma + \tau_R \delta R) \quad (2.64)$$

The vertical strake contribution to the side force can be derived in a manner similar to that of the fin (with no rudder), giving

$$C_{Y_v} = a_v (-\beta_v + \sigma_v) \quad (2.65)$$

However, in order to minimize the number of unknowns in the final equations, the vertical strake parameters η_v (from equation 2.59), β_v , and σ_v will be assumed to have the same values as their counterparts for the fin. The remaining geometric parameters for the vertical strake (those already encountered and those yet to be seen) will keep their values (e.g. S_v will maintain its value in equation 2.59). Thus,

$$C_{Y_v} = a_v (-\beta + \sigma) \quad (2.66)$$

The side force due to dynamics has two contributions, one arising from the aircraft roll rate, the other from the aircraft yaw rate. The fin is the major aircraft component involved in both cases, although the vertical strake, wing, and fuselage may also have some effect.

With reference to Fig. 2.7, it can be shown that a roll rate, p , causes a change in the angle of attack at the fin of the amount

$$\Delta \alpha_F = -\frac{pz_F}{u_0} + p \frac{\partial \sigma}{\partial p} \quad (2.67)$$

where z_F is the height of the fin aerodynamic centre above the aircraft's CG. (This height is used in order to minimize the number of unknowns in the model, even though it may not be the most appropriate mean height to use, since the angle of attack of the fin is not constant during a roll.) If the non-dimensional roll rate is defined as

$$\hat{p} = \frac{pb}{2u_0} \quad (2.68)$$

then

$$\Delta \alpha_F = -\hat{p} \left(2 \frac{z_F}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \quad (2.69)$$

Thus, the change in side force on the fin (due to roll rate) is

$$(C_{Y_{\dot{\sigma}}})_{p,F} = -a_F \frac{S_F}{S} \eta_F \left(2 \frac{z_F}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \hat{p} \quad (2.70)$$

The vertical strake contribution to the side force due to roll rate is derived in a similar fashion, resulting in

$$(C_{Y_{\dot{\sigma}}})_{p,vs} = -a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{z_{vs}}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \hat{p} \quad (2.71)$$

where the partial derivative of σ with respect to $\hat{p}$ for the vertical strake is assumed to be the same as that for the fin.

The wing-body contribution to the side force due to roll rate can be separated into a wing contribution and a fuselage contribution. The fuselage contribution is small, and is neglected. The wing contribution is a function of the wing lift coefficient, sweep, and aspect ratio. For a wing with high sweep, the lift vector has a component in the y -direction. For a wing with zero sweep, the wing contribution to the side force due to roll rate is caused by wing-tip suction^[19]. Goodman and Fisher^[19] shows that this contribution is a function of both the lift coefficient and the roll rate. Thus, it can be modelled as

$$(C_{Y_{\dot{p}}})_{p,wb} = \Sigma C_L \dot{p} \quad (2.72)$$

where Σ is some constant. However, as will be explained in chapter 5, this small contribution is not included in the final side force model.

A yaw rate, r , changes the angle of attack of the fin (see Fig. 2.8) by an amount

$$\Delta \alpha_F = \frac{r l_F}{u_0} + r \frac{\partial \sigma}{\partial r} \quad (2.73)$$

where l_F is the horizontal distance of the fin's aerodynamic centre behind the aircraft's CG. (Again, this distance is used to minimize the number of unknowns in the equations.) If the non-dimensional yaw rate is given by

$$\hat{r} = \frac{r b}{2 u_0} \quad (2.74)$$

then

$$\Delta \alpha_F = \hat{r} \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \hat{r}} \right) \quad (2.75)$$

Thus, the change in side force on the fin (due to yaw rate) is

$$(C_{Y_{\dot{\theta}}})_{r,F} = a_F \frac{S_F}{S} \eta_F \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \hat{r}} \right) \hat{r} \quad (2.76)$$

The vertical strake contribution to the side force due to yaw rate is derived similarly to get

$$(C_{Y_{\dot{\theta}}})_{r,vs} = a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{l_{vs}}{b} + \frac{\partial \sigma}{\partial \hat{r}} \right) \hat{r} \quad (2.77)$$

where the partial derivative of σ with respect to $\hat{r}$ for the vertical strake is assumed to be the same as that for the fin.

The wing-body contribution to the side force due to yaw rate can be divided into a wing contribution and a fuselage contribution. The fuselage contribution is small, and is thus neglected. The wing contribution is difficult to estimate. Goodman and Brewer^[20] gives results for several wing configurations, and Smetana^[17] curve fits them to get functions of lift coefficient and yaw rate. Thus, the wing contribution can be modelled as

$$(C_{Y_{\dot{\theta}}})_{r,wb} = \zeta C_L \hat{r} \quad (2.78)$$

where ζ is some constant. However, as will be explained in chapter 5, this small contribution is not included in the final side force model.

Thus, the total aircraft side force is obtained by substituting the contributions given by equations 2.60, 2.64, 2.66, 2.70, 2.71, 2.76, and 2.77 into equation 2.59. The result is

$$\begin{aligned}
 C_Y = & -a_{fuse}\beta - \left(a_F \frac{S_F}{S} + a_{vs} \frac{S_{vs}}{S} \right) \eta_F (\beta - \sigma) + a_F \frac{S_F}{S} \eta_F \tau_R \delta R \\
 & - \left[a_F \frac{S_F}{S} \eta_F \left(2 \frac{z_F}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) + a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{z_{vs}}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \right] \hat{p} \\
 & + \left[a_F \frac{S_F}{S} \eta_F \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \hat{p}} \right) + a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{l_{vs}}{b} + \frac{\partial \sigma}{\partial \hat{p}} \right) \right] \hat{p}
 \end{aligned} \tag{2.79}$$

Equations relating the unknown model parameters of equation 2.79 to estimates of the SCDs are obtained by taking the appropriate derivatives.

$C_{Y\beta}$ is defined as the change in the aircraft side force coefficient with a change in sideslip angle, while all other parameters are held constant. That is,

$$C_{Y\beta} = \frac{\partial C_Y}{\partial \beta} \tag{2.80}$$

Taking the derivative of equation 2.79 with respect to β gives

$$C_{Y\beta} = -a_{fuse} - \left(a_F \frac{S_F}{S} + a_{vs} \frac{S_{vs}}{S} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \tag{2.81}$$

This stability derivative is negative, due to the definition of positive sideslip angle.

C_{Yp} is defined as the change in aircraft side force coefficient with a change in the non-dimensional roll rate, while all other parameters are held constant. Thus,

$$C_{Y_p} = \frac{\partial C_Y}{\partial \dot{p}} \quad (2.82)$$

From equation 2.79, this means that

$$C_{Y_p} = -a_F \frac{S_F}{S} \eta_F \left(2 \frac{z_F}{b} - \frac{\partial \sigma}{\partial \dot{p}} \right) - a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{z_{vs}}{b} - \frac{\partial \sigma}{\partial \dot{p}} \right) \quad (2.83)$$

This stability derivative can be positive or negative, and is relatively insignificant.

C_{Y_r} is defined as the change in aircraft side force coefficient with a change in yaw rate, while all other parameters are held constant. That is,

$$C_{Y_r} = \frac{\partial C_Y}{\partial \dot{r}} \quad (2.84)$$

Taking the appropriate derivative of equation 2.79 gives

$$C_{Y_r} = a_F \frac{S_F}{S} \eta_F \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \dot{r}} \right) + a_{vs} \frac{S_{vs}}{S} \eta_F \left(2 \frac{l_{vs}}{b} + \frac{\partial \sigma}{\partial \dot{r}} \right) \quad (2.85)$$

This stability derivative is positive, due to the definition of positive yaw rate. However, it is often relatively insignificant.

$C_{Y_{\delta R}}$ is defined as the change in aircraft side force coefficient with a change in rudder deflection, with all other parameters constant. Thus,

$$C_{Y_{\delta R}} = \frac{\partial C_Y}{\partial \delta R} \quad (2.86)$$

From equation 2.79, this means that

$$C_{Y_{\delta R}} = a_F \frac{S_F}{S} \eta_F \tau_R \quad (2.87)$$

This control derivative is positive, due to the definition of positive rudder deflection (left).

It is also relatively insignificant.

$C_{Y_{\delta a}}$, $C_{Y_{\delta ao}}$, and $C_{Y_{\delta si}}$ are defined as the change in aircraft side force coefficient with a change in aileron, outer spoiler, or inner spoiler deflection, respectively. Since changes in side force due to aileron or spoiler deflections were neglected, these three control derivatives are zero.

In an attempt to reduce the number of unknowns in the SCD estimates, the following equations from Campbell and McKinney^[21] were utilized:

$$(C_{Y_p})_F = 2 \left(\frac{z_F}{b} \right) (C_{Y_{\dot{\delta}}})_F \quad (2.88)$$

and

$$(C_{Y_r})_F = -2 \left(\frac{l_F}{b} \right) (C_{Y_{\dot{\delta}}})_F \quad (2.89)$$

The fin contributions of equations 2.81 and 2.83, along with equation 2.88 can be solved to get

$$\frac{\partial \sigma}{\partial \dot{p}} = 2 \frac{z_F}{b} \frac{\partial \sigma}{\partial \dot{\beta}} \quad (2.90)$$

Similarly, the fin contributions of equations 2.81 and 2.85, along with equation 2.89 can

be solved to get

$$\frac{\partial \sigma}{\partial \beta} = -2 \frac{l_F}{b} \frac{\partial \sigma}{\partial \beta} \quad (2.91)$$

Equations 2.90 and 2.91 can be used to simplify equations 2.79, 2.83 and 2.85 to

$$\begin{aligned} C_Y = & -a_{fuse} \beta - \left(a_F \frac{S_F}{S} + a_{vs} \frac{S_{vs}}{S} \right) \eta_F (\beta - \sigma) + a_F \frac{S_F}{S} \eta_F \tau_R \delta R \\ & - 2 \left[a_F \frac{S_F}{S} \frac{z_F}{b} + a_{vs} \frac{S_{vs}}{S} \frac{z_{vs}}{b} \right] \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta \\ & + 2(a_F V_F + a_{vs} V_{vs}) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta \end{aligned} \quad (2.92)$$

$$C_{Y_p} = -2 \left(a_F \frac{S_F}{S} \frac{z_F}{b} + a_{vs} \frac{S_{vs}}{S} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \quad (2.93)$$

and

$$C_{Y_r} = 2(a_F V_F + a_{vs} V_{vs}) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \quad (2.94)$$

Note that a fin volume coefficient, V_F , and a vertical strake volume coefficient, V_{vs} , both defined similarly to the horizontal tail volume coefficient of equation 2.20, were substituted into the above equations where appropriate.

2.3.2 Rolling Moment

The total aircraft rolling moment coefficient is defined as

$$C_l = \frac{l}{q S b} \quad (2.95)$$

where l is the total aircraft rolling moment. Dividing the rolling moment into its various contributions gives

$$C_l = \frac{l_0 + l_{wb} + l_F + l_{ws} + l_{dyn} + l_T}{\bar{q} S b} \quad (2.96)$$

In non-dimensional form, equation 2.96 is equal to

$$C_l = C_{l_0} + C_{l_{wb}} + \frac{S_F}{S} \eta_F C_{l_F} + \frac{S_{ws}}{S} \eta_F C_{l_{ws}} + C_{l_{dyn}} + C_{l_T} \quad (2.97)$$

Once again, with the x-z plane a plane of symmetry, C_{l_0} is zero. Also, with the engines aligned with the x-axis, there will be no rolling moment due to thrust (i.e. $C_{l_T} = 0$).

The wing-body combination contributes to the aircraft rolling moment in several ways. Minor effects arise from the amount of wing sweep, the wing position on the fuselage, and the wing tip shape. The major contribution, however, is a result of the built-in geometric dihedral (γ) of the wing. Fig. 2.9 shows how, during a positive sideslip, the angle of attack of the right wing increases by an amount

$$\Delta \alpha_R = \beta \gamma \quad (2.98)$$

while the angle of attack of the left wing decreases by the same amount. This causes an increase in lift on the right wing and a decrease in lift on the left wing, each of the amount

$$L = a \beta \gamma \quad (2.99)$$

This results in a negative rolling moment. If y_R and y_L are some distances along the right

and left halves of the wing where the lift vectors can be modelled to give the correct rolling moment, and if they are measured from the centreline of the aircraft (with y_R positive and y_L negative), then the rolling moment of the wing-body combination due to dihedral is given by

$$(l_{wb})_r = -(L_R y_R + L_L y_L) \quad (2.100)$$

Thus, the wing-body rolling moment coefficient due to dihedral is

$$(C_{l_{wb}})_r = -\frac{(L_R y_R + L_L y_L)}{\bar{q} S b} \quad (2.101)$$

If equation 2.99 is substituted into equation 2.101 for L_R and L_L , then

$$(C_{l_{wb}})_r = \frac{a\beta\gamma}{b}(y_L - y_R) \quad (2.102)$$

The remaining wing-body contributions come from the ailerons and spoilers. Fig. 2.10 shows how a sample elliptic lift distribution over a wing changes when the ailerons are deflected. In this case, it is showing a negative aileron deflection, producing a positive rolling moment. Spoiler deflections cause the same type of lift distribution as an up-going aileron (with no corresponding down-going contribution on the opposite wing). Since the exact lift distribution is unknown, the strip integration method of Perkins and Hage^[22] is used to find the rolling moment due to ailerons and spoilers. Fig. 2.11 shows the basic geometry of the integration. The aileron case is treated first.

The section lift coefficient over the ailerons is modelled as

$$c_L = a_0 \tau_a \delta a \quad (2.103)$$

where: a_0 is the section lift-curve slope
 τ_a is the section aileron effectiveness factor
 δa is the aileron deflection

Using Fig. 2.11, it can be shown that the local rolling moment coefficient due to aileron deflection is equal to

$$d(C_{l_{wb}})_{\delta a} = \frac{-c(y)c_L y dy}{Sb} \quad (2.104)$$

where $c(y)$ is the wing chord as a function of y -distance out the wing. Substituting equation 2.103 into equation 2.104, integrating, and multiplying by two (to account for both ailerons) gives

$$(C_{l_{wb}})_{\delta a} = \frac{-2a_w \tau_a \delta a}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.105)$$

However, since this method assumes abrupt discontinuities in the spanwise lift distribution at each end of both ailerons, correction factors for each aileron, G_R and G_L , are used to give

$$(C_{l_{wb}})_{\delta a} = \frac{-2a \tau_a \delta a (G_R + G_L)}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.106)$$

Note that the correction factors are not modelled as functions of y since, without knowing what that functional relationship is, the integral cannot be calculated. Also note that the

total aircraft lift-curve slope is used in equation 2.106. This is because the wing lift-curve slope over the span containing the ailerons is not known, and the wing-body lift-curve slope is not known as well as that for the total aircraft. The difference is incorporated into the correction factors.

The outer and inner spoiler contributions to the wing-body rolling moment are derived in a manner similar to that for the ailerons, with a few minor differences. First, the integral will not be multiplied by two, since the spoilers on both wings never deploy at the same time. Second, only one correction factor is needed, since it should be the same for both wings (spoilers only deploy upward, whereas ailerons can deploy upward or downward). Finally, the correct integration limits and section flap effectiveness factors are used. Thus, the outer and inner spoiler contributions are modelled respectively as

$$(C_{l_{wb}})_{\delta so} = \frac{-a\tau_{so}\delta so F}{Sb} \int_{k_3 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.107)$$

and

$$(C_{l_{wb}})_{\delta si} = \frac{-a\tau_{si}\delta si E}{Sb} \int_{k_3 \frac{b}{2}}^{k_6 \frac{b}{2}} c(y) y dy \quad (2.108)$$

The rolling moment due to the fin is caused by a side force acting on the fin at the aerodynamic centre, when the aircraft is sideslipping. That is,

$$C_{l_f} = C_{y_f} \frac{z_F}{b} \quad (2.109)$$

where z_F is divided by b to non-dimensionalize equation 2.109. Equation 2.64 can be substituted into equation 2.109 to give

$$C_{l_F} = a_F \frac{z_F}{b} (-\beta + \sigma + \tau_R \delta R) \quad (2.110)$$

Similarly, the vertical strake contribution is modelled as

$$C_{l_{vs}} = a_{vs} \frac{z_{vs}}{b} (-\beta + \sigma) \quad (2.111)$$

The dynamic contribution again has two parts: roll rate (p) and yaw rate (r). Fig. 2.12 shows how a roll rate causes an asymmetric lift distribution over the wing that sets up a rolling moment in a direction opposite to that of the roll. With reference to Fig. 2.7, it is obvious that the roll rate causes a change in the angle of attack of the wing equal to

$$\Delta \alpha = \frac{py}{u_0} \quad (2.112)$$

Then the change in section lift coefficient at a distance y from the centreline is

$$c_L = \frac{a_0 py}{u_0} \quad (2.113)$$

and the section rolling moment coefficient is given, similar to equation 2.104, as

$$d(C_{l_{dyn}})_{p,wb} = \frac{-c_L c(y) y dy}{Sb} \quad (2.114)$$

Substituting equation 2.113 into 2.114, integrating over the entire wing, and substituting

the non-dimensional roll rate for p gives

$$(C_{l_{\dot{\phi}}})_{p,wb} = -\frac{4a_w \hat{p}^{b/2}}{Sb^2} \int_0^{b/2} c(y)y^2 dy \quad (2.115)$$

Again, a correction factor, K , is needed to account for the assumed discontinuities in the spanwise lift distribution at the ends of the wing, as well as for the difference between a_w and a . Thus, equation 2.115 becomes

$$(C_{l_{\dot{\phi}}})_{p,wb} = -\frac{4a\hat{p}K^{b/2}}{Sb^2} \int_0^{b/2} c(y)y^2 dy \quad (2.116)$$

Pearson and Jones^[23] demonstrates that the contribution to the rolling moment due to roll rate of a surface geometrically similar to the wing but with one quarter the span is 1/256 that of the wing contribution. Since the span of the fin is only about one sixth that of the wing, the fin (and vertical strake) contribution is neglected.

The fin and vertical strake contributions to the rolling moment due to yaw rate are, however, important. The fin contribution effectively results from a side force on the fin acting at the aerodynamic centre. Thus,

$$(C_{l_{\dot{\phi}}})_{r,F} = \frac{z_F}{b} (C_{Y_{\dot{\phi}}})_{r,F} \quad (2.117)$$

Substituting equation 2.76 into equation 2.117 gives

$$(C_{l_{\dot{\phi}}})_{r,F} = a_F \frac{S_F z_F}{S b} \eta_F \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \hat{p}} \right) \hat{p} \quad (2.118)$$

Similarly, the vertical strake contribution is

$$(C_{l_{\dot{\alpha}}})_{r,vs} = a_{vs} \frac{S_{vs}}{S} \frac{z_{vs}}{b} \eta_F \left(2 \frac{l_{vs}}{b} + \frac{\partial \sigma}{\partial \beta} \right) \dot{\beta} \quad (2.119)$$

Using equation 2.91, the above two equations can be simplified to

$$(C_{l_{\dot{\alpha}}})_{r,F} = 2a_F V_F \frac{z_F}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \dot{\beta} \quad (2.120)$$

and

$$(C_{l_{\dot{\alpha}}})_{r,vs} = 2a_{vs} V_{vs} \frac{z_{vs}}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \dot{\beta} \quad (2.121)$$

The wing-body combination also contributes to the rolling moment due to yaw rate. As the aircraft yaws positively (nose right), the left wing speeds up, while the right wing slows down. This causes an increase in lift on the left wing and a decrease in lift on the right wing, which results in a positive rolling moment. If the change in lift at each point along the wing could be integrated from wing-tip to wing-tip, the resulting rolling moment could be modelled. Since the wing contribution to the rolling moment due to yaw rate is a function of the lift coefficient and the yaw rate^[22], it is modelled as

$$(C_{l_{\dot{\alpha}}})_{r,wb} = \lambda C_L \dot{\beta} \quad (2.122)$$

where λ is a correction factor which accounts for the fact that the exact lift distribution is unknown.

Adding together equations 2.102, 2.106, 2.107, 2.108, 2.110, 2.111, 2.116, 2.120,

2.121, and 2.122 gives the total aircraft rolling moment coefficient as

$$\begin{aligned}
 C_l = & \frac{a\gamma\beta}{b}(y_L - y_R) - \frac{2a\tau_a\delta a(G_R + G_L)}{Sb} \int_{k_{1\frac{b}{2}}}^{k_{2\frac{b}{2}}} c(y)y dy \\
 & - \frac{a\tau_{so}\delta so F}{Sb} \int_{k_{3\frac{b}{2}}}^{k_{4\frac{b}{2}}} c(y)y dy - \frac{a\tau_{si}\delta si E}{Sb} \int_{k_{5\frac{b}{2}}}^{k_{6\frac{b}{2}}} c(y)y dy \\
 & + \left(a_F \frac{S_F z_F}{S b} + a_{vs} \frac{S_{vs} z_{vs}}{S b} \right) \eta_F (\sigma - \beta) + a_F \frac{S_F z_F}{S b} \eta_F \tau_R \delta R \\
 & - \frac{4aK\dot{p}}{Sb^2} \int_0^{b/2} c(y)y^2 dy \\
 & + \left[2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) + \lambda C_L \right] \hat{p}
 \end{aligned} \tag{2.123}$$

Equations relating the unknown model parameters of equation 2.123 to estimates of the SCDs are obtained by taking the appropriate derivatives.

C_{l_β} is defined as the change in aircraft rolling moment coefficient with a change in sideslip angle, while all other parameters are held constant. That is,

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta} \tag{2.124}$$

Taking the derivative of equation 2.123 with respect to β gives

$$C_{l_\beta} = \frac{a\gamma}{b}(y_L - y_R) + \left(a_F \frac{S_F z_F}{S b} + a_{vs} \frac{S_{vs} z_{vs}}{S b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \tag{2.125}$$

This stability derivative is usually negative, and is sometimes referred to as "effective

dihedral".

$C_{l_{\delta a}}$ is defined as the change in aircraft rolling moment with a change in aileron deflection while all other parameters are held constant, or

$$C_{l_{\delta a}} = \frac{\partial C_l}{\partial \delta a} \quad (2.126)$$

Taking the appropriate derivative of equation 2.123 gives

$$C_{l_{\delta a}} = -\frac{2a\tau_a(G_R + G_L)}{Sb} \int_{k_{1/2}}^{k_{3/2}} c(y)ydy \quad (2.127)$$

This control derivative is negative, due to the definition of positive aileron deflection, and is referred to as "aileron effectiveness" or "aileron power".

$C_{l_{\delta so}}$ is defined as the change in rolling moment coefficient with a change in outer spoiler deflection while all other parameters are held constant. Thus,

$$C_{l_{\delta so}} = \frac{\partial C_l}{\partial \delta so} \quad (2.128)$$

From equation 2.123, this means that

$$C_{l_{\delta so}} = -\frac{a\tau_{so}F}{Sb} \int_{k_{3/2}}^{k_{5/2}} c(y)ydy \quad (2.129)$$

This control derivative is usually negative.

$C_{l_{\delta i}}$ is defined as the change in aircraft rolling moment coefficient with a change in inner spoiler deflection while all other parameters are held constant. That is,

$$C_{l_{\delta i}} = \frac{\partial C_l}{\partial \delta_{si}} \quad (2.130)$$

Taking the derivative of equation 2.123 with respect to inner spoiler angle gives

$$C_{l_{\delta i}} = -\frac{a\tau_{si}E}{Sb} \int_{k_2 \frac{b}{2}}^{k_1 \frac{b}{2}} c(y)y dy \quad (2.131)$$

This control derivative is usually negative.

$C_{l_{\delta R}}$ is defined as the change in aircraft rolling moment coefficient with a change in rudder deflection while all other parameters are held constant. Thus,

$$C_{l_{\delta R}} = \frac{\partial C_l}{\partial \delta_R} \quad (2.132)$$

From equation 2.123, this means that

$$C_{l_{\delta R}} = a_F \frac{S_F z_F}{S b} \eta_F \tau_R \quad (2.133)$$

This control derivative is usually positive.

C_{l_p} is defined as the change in aircraft rolling moment coefficient with a change in the non-dimensional roll rate while all other parameters are held constant, or

$$C_{l_p} = \frac{\partial C_l}{\partial \dot{p}} \quad (2.134)$$

Taking the appropriate derivative of equation 2.123 gives

$$C_{l_p} = -\frac{4aK}{Sb^2} \int_0^{b/2} c(y)y^2 dy \quad (2.135)$$

This stability derivative is negative, and is often called "roll damping".

C_{l_r} is defined as the change in the aircraft rolling moment coefficient with a change in the non-dimensional yaw rate, with all other parameters constant. That is,

$$C_{l_r} = \frac{\partial C_l}{\partial \dot{r}} \quad (2.136)$$

Taking the derivative of equation 2.123 with respect to the non-dimensional yaw rate gives

$$C_{l_r} = 2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) + \lambda C_L \quad (2.137)$$

This stability derivative is usually positive.

2.3.3 Yawing Moment

The total aircraft yawing moment coefficient is defined as

$$C_n = \frac{N}{qSb} \quad (2.138)$$

where N is the total aircraft yawing moment. Dividing this into its various components gives

$$C_n = \frac{N_0 + N_{wb} + N_F + N_{vs} + N_{dyn} + N_T}{\bar{q}Sb} \quad (2.139)$$

or, in non-dimensional form,

$$C_n = C_{n_0} + C_{n_{wb}} + \frac{S_F}{S} \eta_F C_{n_F} + \frac{S_{vs}}{S} \eta_F C_{n_{vs}} + C_{n_{dyn}} + C_{n_T} \quad (2.140)$$

Again, since the x-z plane is a plane of symmetry, C_{n_0} is zero.

The wing-body contribution to the yawing moment is modelled as a side force (due to sideslip) acting at some point x along the fuselage, similar to a neutral point in the longitudinal derivations. Thus,

$$(C_{n_{wb}})_\beta = \frac{Y_{wb}x}{\bar{q}Sb} \quad (2.141)$$

In non-dimensional form, this is

$$(C_{n_{wb}})_\beta = C_{Y_{wb}} \frac{x}{b} \quad (2.142)$$

Substituting equation 2.60 into equation 2.142 gives

$$(C_{n_{wb}})_\beta = -a_{fuse} \beta \frac{x}{b} \quad (2.143)$$

Note that, in the above equation, x must be measured negative aft of the aircraft CG.

The wing-body combination also has smaller contributions to the yawing moment from the ailerons and the spoilers. As the ailerons or spoilers deflect, they change the lift distribution across the wing, and thus, they also change the drag distribution across the wing. Since the spoilers only deflect on one wing at a time, the wing with the deflected spoiler experiences an increase in drag, and thus, there will be a yawing moment in the direction of that wing. The ailerons deflect on both wings at the same time, one up and one down. The up-going aileron usually causes less drag than the down-going aileron (since it also creates less lift), and thus, there is usually a yawing moment in the direction of the down-going aileron (this is called "adverse yaw"). However, it is possible, depending on the aircraft configuration, that the opposite will occur ("proverse yaw").

Using a strip integration method similar to that used in the calculation of the aileron contribution to the rolling moment, it can be shown that

$$d(C_{n_{wb}})_{\delta a} = \frac{c(y)c_D y dy}{Sb} \quad (2.144)$$

where c_D is the section drag coefficient over the ailerons. It is modelled as

$$c_D = \Omega_a \delta a \quad (2.145)$$

where Ω_a is the section drag factor over the aileron. Integrating over one aileron and adding a correction factor to account for the assumed discontinuities gives

$$(C_{n_{wb}})_{\delta a} = \frac{\Omega_a \delta a}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.146)$$

Since the drag caused by each aileron is acting in the same direction, the result of integrating over both ailerons will not multiply the above equation by two. Instead, the result will be

$$(C_{n_{\omega}})_{\delta a} = \frac{\Omega_a \delta a (J_R - J_L)}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.147)$$

where each aileron has its own correction factor, J_R and J_L . Again, in order to perform the integration in equation 2.147, the correction factors are not functions of y .

The spoiler contributions are derived similarly to that of the ailerons. The results are

$$(C_{n_{\omega}})_{\delta so} = \frac{\Omega_{so} \delta so l}{Sb} \int_{k_3 \frac{b}{2}}^{k_4 \frac{b}{2}} c(y) y dy \quad (2.148)$$

and

$$(C_{n_{\omega}})_{\delta si} = \frac{\Omega_{si} \delta si H}{Sb} \int_{k_5 \frac{b}{2}}^{k_6 \frac{b}{2}} c(y) y dy \quad (2.149)$$

The yawing moment caused by the fin during a sideslip can be modelled as a side force acting at the aerodynamic centre of the fin (i.e. a distance l_F aft of the CG). Thus,

$$C_{n_r} = -C_{Y_F} \frac{l_F}{b} \quad (2.150)$$

where the negative sign is due to the definition of positive yawing moment (nose right).

Substituting equation 2.64 into the above equation gives

$$C_{n_F} = -a_F \frac{l_F}{b} (-\beta + \sigma + \tau_R \delta R) \quad (2.151)$$

Similarly, the vertical strake contribution is given as

$$C_{n_{vs}} = -a_{vs} \frac{l_{vs}}{b} (-\beta + \sigma) \quad (2.152)$$

The dynamic contribution to the aircraft yawing moment has both a roll rate and a yaw rate portion. A roll rate causes a change in the aircraft yawing moment, since, as the fin is rolled, a side force is developed which can be modelled as acting at the aerodynamic centre. Thus,

$$(C_{n_{\dot{\phi}}})_{p,F} = -\frac{l_F}{b} (C_{Y_{\dot{\phi}}})_{p,F} \quad (2.153)$$

Substituting equation 2.70 into equation 2.153 gives

$$(C_{n_{\dot{\phi}}})_{p,F} = a_F V_F \eta_F \left(2 \frac{z_F}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \hat{p} \quad (2.154)$$

The vertical strake contribution is defined similarly as

$$(C_{n_{\dot{\phi}}})_{p,vs} = a_{vs} V_{vs} \eta_F \left(2 \frac{z_{vs}}{b} - \frac{\partial \sigma}{\partial \hat{p}} \right) \hat{p} \quad (2.155)$$

Using equation 2.90, the above two equations can be simplified to

$$(C_{n_{\dot{\alpha}}})_{p,F} = 2a_F V_F \frac{z_F}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \hat{p} \quad (2.156)$$

and

$$(C_{n_{\dot{\alpha}}})_{p,vs} = 2a_{vs} V_{vs} \frac{z_{vs}}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta}\right) \hat{p} \quad (2.157)$$

The wing-body contribution to the yawing moment due to roll rate comes mostly from the wing. As the aircraft rolls, the angle of attack of the down-going wing increases, while that of the up-going wing decreases. Fig. 2.13 shows that the lift vectors on each wing tilt accordingly, resulting in a forward component on the down-going wing, and an aft component on the up-going wing. This causes a corresponding yawing moment. Using the strip integration method of Perkins and Hage^[22] results in

$$(C_{n_{\dot{\alpha}}})_{p,wb} = -\frac{2}{b} \int_0^{b/2} c_L \sin\left(\frac{py}{u_0}\right) dy \quad (2.158)$$

Assuming small angles, substituting in the non-dimensional roll rate, integrating, and adding a correction factor χ (for the assumed discontinuities at each end of the wing) gives

$$(C_{n_{\dot{\alpha}}})_{p,wb} = -\chi C_L \hat{p} \quad (2.159)$$

where χ has absorbed the constant.

The fin contribution to the yawing moment due to yaw rate can be modelled as a side force acting at the fin aerodynamic centre. That is,

$$(C_{n_{\dot{\gamma}}})_{r,F} = -\frac{l_F}{b}(C_{Y_{\dot{\gamma}}})_{r,F} \quad (2.160)$$

Substituting equation 2.76 into the above equation gives

$$(C_{n_{\dot{\gamma}}})_{r,F} = -a_F V_F \eta_F \left(2 \frac{l_F}{b} + \frac{\partial \sigma}{\partial \dot{\gamma}} \right) \dot{\gamma} \quad (2.161)$$

Similarly, the vertical strake contribution is

$$(C_{n_{\dot{\gamma}}})_{r,vs} = -a_{vs} V_{vs} \eta_F \left(2 \frac{l_{vs}}{b} + \frac{\partial \sigma}{\partial \dot{\gamma}} \right) \dot{\gamma} \quad (2.162)$$

Substituting equation 2.91 into the above two equations gives

$$(C_{n_{\dot{\gamma}}})_{r,F} = -2a_F V_F \frac{l_F}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \dot{\gamma} \quad (2.163)$$

and

$$(C_{n_{\dot{\gamma}}})_{r,vs} = -2a_{vs} V_{vs} \frac{l_{vs}}{b} \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \dot{\gamma} \quad (2.164)$$

The wing-body contribution to the yawing moment due to yaw rate also is mostly due to the wing. As the aircraft undergoes a positive yaw, the right wing slows down, while the left wing speeds up. This causes an increase in lift on the left wing, and a decrease on the right wing. This means that the drag on the left wing also increases, while that on the right wing decreases. Since the yawing moment of the wing due to yaw rate is a function of the drag coefficient and yaw rate^[21], and the drag coefficient is given as

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A R e} \quad (2.165)$$

where: C_{D_0} is the profile drag coefficient
 AR is the aspect ratio of the wing
 e is Oswald's efficiency factor

then

$$(C_{n_{\delta y}})_{r,wb} = (\xi C_L^2 + \kappa) \hat{r} \quad (2.166)$$

where ξ has absorbed the constant in the second term of equation 2.165, and κ has absorbed the first term in equation 2.165.

There will also be a yawing moment due to thrust, given by

$$C_{n_T} = \frac{y_{engine}}{b} (C_{T_L} - C_{T_R}) \quad (2.167)$$

where: y_{engine} is the distance, parallel to the y-axis, of the engines from the aircraft centreline
 C_{T_L} is the thrust coefficient of the left engine
 C_{T_R} is the thrust coefficient of the right engine

If the left and right thrust vectors are not aligned with the aircraft centreline (i.e. θ_T not zero), then there would also be a $\cos(\theta_T)$ term multiplied into equation 2.167. However, for the Dash 8, θ_T is zero.

Adding together equations 2.143, 2.147, 2.148, 2.149, 2.151, 2.152, 2.156, 2.157, 2.159, 2.163, 2.164, 2.166, and 2.167 gives the total aircraft yawing moment as

$$\begin{aligned}
C_n = & -a_{fuse} \beta \frac{x}{b} + \frac{\Omega_a \delta a (J_R - J_L)}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \\
& + \frac{\Omega_{so} \delta so l}{Sb} \int_{k_3 \frac{b}{2}}^{k_4 \frac{b}{2}} c(y) y dy + \frac{\Omega_{si} \delta si H}{Sb} \int_{k_5 \frac{b}{2}}^{k_6 \frac{b}{2}} c(y) y dy \\
& - (a_F V_F + a_{vs} V_{vs}) \eta_F (\sigma - \beta) - a_F V_F \eta_F \tau_R \delta R \\
& + \left[2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) - \chi C_L \right] \beta \\
& - \left[2 \left(a_F V_F \frac{l_F}{b} + a_{vs} V_{vs} \frac{l_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) - \xi C_L^2 - \kappa \right] \beta
\end{aligned} \tag{2.168}$$

Equations relating the unknown model parameters of equation 2.168 to estimates of the appropriate SCDs are obtained by taking the required derivatives.

C_{n_β} is defined as the change in aircraft yawing moment with a change in sideslip angle, while all other parameters are held constant. That is,

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta} \tag{2.169}$$

Taking the derivative of equation 2.168 with respect to β gives

$$C_{n_\beta} = -a_{fuse} \frac{x}{b} + (a_F V_F + a_{vs} V_{vs}) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \tag{2.170}$$

This stability derivative is usually positive, and is referred to as "weathercock stability".

$C_{n_{\delta a}}$ is defined as the change in aircraft yawing moment with a change in aileron deflection, while all other parameters are held constant. Thus,

$$C_{n_{\delta a}} = \frac{\partial C_n}{\partial \delta a} \quad (2.171)$$

Taking the appropriate derivative of equation 2.168 gives

$$C_{n_{\delta a}} = \frac{\Omega_a (J_R - J_D)}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \quad (2.172)$$

This control derivative can be positive or negative, but is usually negative.

$C_{n_{\delta a o}}$ is defined as the change in aircraft yawing moment with a change in outer spoiler deflection while all other parameters are held constant, or

$$C_{n_{\delta a o}} = \frac{\partial C_n}{\partial \delta s o} \quad (2.173)$$

From equation 2.168, this means that

$$C_{n_{\delta a o}} = \frac{\Omega_{s o} I}{Sb} \int_{k_3 \frac{b}{2}}^{k_4 \frac{b}{2}} c(y) y dy \quad (2.174)$$

This control derivative is usually negative.

$C_{n_{\delta a i}}$ is defined as the change in aircraft yawing moment with a change in inner spoiler deflection, while all other parameters are held constant. That is,

$$C_{n_{\delta a i}} = \frac{\partial C_n}{\partial \delta s i} \quad (2.175)$$

Taking the derivative of equation 2.168 with respect to inner spoiler angle gives

$$C_{n_{\delta s}} = \frac{\Omega_{ss} H}{Sb} \int_{k_{s\frac{b}{2}}}^{k_{o\frac{b}{2}}} c(y) y dy \quad (2.176)$$

This control derivative is usually negative.

$C_{n_{\delta R}}$ is defined as the change in aircraft yawing moment with a change in rudder deflection, while all other parameters are held constant, or

$$C_{n_{\delta R}} = \frac{\partial C_n}{\partial \delta_R} \quad (2.177)$$

Taking the appropriate derivative of equation 2.168 gives

$$C_{n_{\delta R}} = -a_F V_F \eta_F \tau_R \quad (2.178)$$

This control derivative is negative, and is often called "rudder effectiveness" or "rudder power".

C_{n_p} is defined as the change in aircraft yawing moment with a change in non-dimensional roll rate, while all other parameters are held constant. Thus,

$$C_{n_p} = \frac{\partial C_n}{\partial \hat{p}} \quad (2.179)$$

From equation 2.168, this means that

$$C_{n_p} = 2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) - \chi C_L \quad (2.180)$$

This stability derivative is usually negative.

C_{n_r} is defined as the change in aircraft yawing moment with a change in non-dimensional yaw rate, while all other parameters are held constant. That is,

$$C_{n_r} = \frac{\partial C_n}{\partial \hat{r}} \quad (2.181)$$

Taking the derivative of equation 2.168 with respect to the non-dimensional yaw rate gives

$$C_{n_r} = -2 \left(a_F V_F \frac{l_F}{b} + a_{vs} V_{vs} \frac{l_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) + \xi C_L^2 + \kappa \quad (2.182)$$

This stability derivative is negative, and is known as "yaw damping".

The lateral geometry-based equations will be compared to the MLE-derived values of the corresponding SCDs in order to determine values of the model parameters a_{fus} , a_F , a_{vs} , η_F , $\partial \sigma / \partial \beta$, τ_R , $(y_L - y_R)$, $\tau_a(G_R + G_L)$, $\tau_{so}F$, $\tau_{st}E$, K , λ , x , $\Omega_a(J_R - J_L)$, $\Omega_{so}J$, $\Omega_{st}H$, χ , ξ , and κ .

2.4 Determination of Model Parameters

2.4.1 Evaluation of Model Parameters From MLE-Derived SCD Data

Using the previous geometry-based equations which model the SCDs and the set of MLE-derived SCD data, it is possible to solve for unknown parameters such as h_{nb} or a_F . Often, more than one equation can be used to find a certain parameter. In these

cases, if the values from each equation are consistent, then the average of the values is used. If the values found from more than one equation vary widely, then either the equations have neglected an important effect that should be included, or the SCD data obtained from the MLE process are inconsistent with the physical model. If the equations are incomplete, all relevant effects are added until the resulting values of the unknown parameters are comparable. If the MLE-derived data for one SCD are inconsistent, the values obtained from the remaining (more important) SCD data are used.

2.4.2 Determination of Model Parameters From Trim Data

Once all the model parameters in the geometry-based SCD estimates are determined, it remains to assign values to the remaining model parameters in the force and moment model (i.e. α_{0L} and C_{m0}). To do this, the set of trim data is used. For example, the trim values of C_L for each flight test are used in equation 2.21, along with the corresponding trim values of α_m , δe , and q , and the determined values of the other model parameters, to determine values for α_{0L} . (Note that the α_{0L} values will be slightly offset, due to the fact that the lift due to thrust in equation 2.3 was neglected.) C_{m0} is found in a similar manner, using C_m data and corresponding trim data in equation 2.44.

2.4.3 Determination of Thrust and Flap Effects

Once the values of all previously-unknown model parameters are determined, a curve fit procedure is utilized in an attempt to include the indirect effects of thrust and the effects of flaps in the model, which had previously been neglected. Values obtained for each model parameter for each flight test are plotted against corresponding thrust coefficient values, and curve fits are performed. This is done independently for each flap

setting.

It is expected that all of the model parameters will be functions of thrust, and most will also depend on the flap setting, the exceptions being a_{wb} , a_n , a_{fuse} , and a_F . Lift-curve slope is not affected by a change in flap setting, since trailing-edge flaps only act to increase the effective camber of the wing, thereby increasing the amount of lift generated at a particular angle of attack. That is, flaps cause a shift in the lift-curve, but do not affect its slope. Thus, a plot of a_{wb} vs flap setting should show no correlation, unless account is not made of the change in effective wing area as the flaps are extended or retracted. a_n , a_{fuse} , and a_F should also show no flap dependence, since they are functions of aircraft components that are not altered by a change in flap setting.

2.5 Verification Procedures

2.5.1 Geometry-Based SCD Predictions vs MLE-Derived SCD Data

The first verification step is to plot the values of the SCDs predicted by the derived equations (after curve-fitting has been performed on the model parameters) against the original SCD values in the data set. This gives an estimate of how well the curve fits represent the actual flap and thrust effects. This does not, however, indicate how well the model parameters have been predicted.

2.5.2 Geometry-Based Force and Moment Predictions vs Flight Test Data

Using the values obtained for all model parameters, equations 2.21, 2.44, 2.92, 2.123, and 2.168 are evaluated for a variety of trim and non-trim conditions. The

resulting force and moment estimates are compared to the actual forces and moments measured at each specified condition during the flight tests. The actual force and moment values for the trim cases are given explicitly in the original data set. Values for the non-trim cases, however, must be determined from the following equations:

$$C_L = \frac{(ma_x - T)\sin\alpha_m - ma_z \cos\alpha_m}{\bar{q}S} \quad (2.183)$$

where: m is the mass of the aircraft
 a_x is the acceleration of the CG in the x-direction
 a_z is the acceleration of the CG in the z-direction

$$C_m = \frac{\dot{q}I_y}{\bar{q}Sc} \quad (2.184)$$

where: $\dot{q}$ is the pitch acceleration
 I_y is the moment of inertia about the y-axis

$$C_Y = \frac{ma_y}{\bar{q}S} \quad (2.185)$$

where: a_y is the acceleration of the CG in the y-direction

$$C_l = \frac{\dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz}}{\bar{q}Sb} \quad (2.186)$$

where: $\dot{p}$ is the roll acceleration
 $\dot{r}$ is the yaw acceleration
 I_x is the moment of inertia about the x-axis
 I_z is the moment of inertia about the z-axis
 I_{xz} is the product of inertia $\int xz \, dm$

and

$$C_n = \frac{-\dot{p}I_{xz} + \dot{r}I_z + pq(I_y - I_x) + qrI_{xz}}{\bar{q}Sb} \quad (2.187)$$

Values of the parameters in the preceding equations are taken from the original flight test data. Any parameters which are not listed explicitly are derived from other flight data. This procedure helps to indicate the accuracy of the geometry-based equations.

2.5.3 Model Parameter Values vs Theoretical Predictions

Theoretical sources, including ESDU data sheets and Datcom estimation procedures, are used to estimate values for as many of the model parameters in the geometry-based SCD equations as possible. These estimates serve to indicate how well the model parameters have been predicted by the derived equations. This also demonstrates the accuracy of the equations themselves.

2.5.4 Geometry-Based Simulator Model vs Flight Test Data

Final verification is provided by the flight simulator program previously created by FRL and CAE for the Dash 8 Series 300 aircraft. This program was developed using the original SCD model of the aircraft. To determine how well the geometry-based force and moment equations model the aircraft response, they are substituted into the simulator in place of the SCD model, input manoeuvres are added, and the predicted responses are compared to actual flight test responses. The geometry-based equations to be substituted into the flight simulator are summarized on the following pages.

Longitudinal

$$C_L = a_{wb} \left[1 + \frac{a_i \eta_i S_i}{a_{wb} S} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] (\alpha_m - \alpha_{0L}) + a_i \eta_i \frac{S_i}{S} \tau_e \delta e + 2a_i \eta_i V_H \hat{q} \quad (2.21)$$

$$C_m = C_{m_0} + a_{wb} \left[(h - h_{nwb}) - \frac{a_i \eta_i}{a_{wb}} V_H \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right] (\alpha_m - \alpha_{0L}) \\ - a_i \eta_i V_H \left[\tau_e \delta e + 2 \frac{l_i}{c} \hat{q} \right] + C_T \frac{z_T}{c} \quad (2.44)$$

Lateral

$$C_Y = -a_{fusv} \beta - \left(a_F \frac{S_F}{S} + a_{vs} \frac{S_{vs}}{S} \right) \eta_F (\beta - \sigma) + a_F \frac{S_F}{S} \eta_F \tau_R \delta R \\ - 2 \left[a_F \frac{S_F z_F}{S b} + a_{vs} \frac{S_{vs} z_{vs}}{S b} \right] \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta \\ + 2 [a_F V_F + a_{vs} V_{vs}] \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) \beta \quad (2.92)$$

$$C_l = \frac{a \gamma \beta}{b} (y_L - y_R) - \frac{2a \tau_a \delta a (G_R + G_L)}{Sb} \int_{k_1 \frac{b}{2}}^{k_2 \frac{b}{2}} c(y) y dy \\ - \frac{a \tau_{so} \delta so F}{Sb} \int_{k_2 \frac{b}{2}}^{k_1 \frac{b}{2}} c(y) y dy - \frac{a \tau_{sl} \delta sl E}{Sb} \int_{k_2 \frac{b}{2}}^{k_1 \frac{b}{2}} c(y) y dy \\ + \left(a_F \frac{S_F z_F}{S b} + a_{vs} \frac{S_{vs} z_{vs}}{S b} \right) \eta_F (\sigma - \beta) + a_F \frac{S_F z_F}{S b} \eta_F \tau_R \delta R \\ - \frac{4aK\hat{p}}{Sb^2} \int_0^{b/2} c(y) y^2 dy \\ + \left[2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) + \lambda C_L \right] \beta \quad (2.123)$$

$$\begin{aligned}
C_n = & -a_{fuse}\beta\frac{x}{b} + \frac{\Omega_a\delta a(J_R-J_L)}{Sb} \int_{k_{1/2}^{\frac{b}{2}}}^{k_{2/2}^{\frac{b}{2}}} c(y)ydy \\
& + \frac{\Omega_{so}\delta sol}{Sb} \int_{k_{3/2}^{\frac{b}{2}}}^{k_{4/2}^{\frac{b}{2}}} c(y)ydy + \frac{\Omega_{sl}\delta slH}{Sb} \int_{k_{5/2}^{\frac{b}{2}}}^{k_{6/2}^{\frac{b}{2}}} c(y)ydy \\
& - (a_F V_F + a_{vs} V_{vs}) \eta_F (\sigma - \beta) - a_F V_F \eta_F \tau_R \delta R \\
& + \left[2 \left(a_F V_F \frac{z_F}{b} + a_{vs} V_{vs} \frac{z_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) - \chi C_L \right] \dot{\beta} \\
& - \left[2 \left(a_F V_F \frac{l_F}{b} + a_{vs} V_{vs} \frac{l_{vs}}{b} \right) \eta_F \left(1 - \frac{\partial \sigma}{\partial \beta} \right) - \xi C_L^2 - \kappa \right] \dot{\tau} \\
& + \frac{y_{engine}}{b} (C_{thrusters_L} - C_{thrusters_R})
\end{aligned} \tag{2.168}$$

It should be noted that the simulator requires the forces and moments to be defined in a different set of axes than that which is used in this report. Specifically, the simulator defines them according to the aircraft stability axes, which are similar to the body axes, but with the x-axis parallel to the relative wind instead of the fuselage reference line (during steady symmetric flight), and the z-axis shifted accordingly. To transform the body axes to the stability axes, the body axes are rotated about the y-axis by an angle α_m (see Fig. 2.14). The equations for the pitching moment, side force, and lift are not affected by this rotation. However, the rolling moment and yawing moment equations must be redefined as follows (the subscript *s* refers to the stability axis system):

$$C_{l_s} = C_l \cos \alpha_m + C_n \sin \alpha_m \tag{2.188}$$

and

$$C_{n_s} = C_n \cos \alpha_m - C_l \sin \alpha_m \tag{2.189}$$

Chapter 3

SCD Data

The SCD data used in this project were produced by the Flight Research Laboratory (FRL) of the Institute for Aerospace Research of the National Research Council of Canada and CAE Electronics Ltd. of Montreal during a previous simulator creation project which used both flight test and parameter estimation techniques to produce the SCD values..

3.1 Overview

In July, 1991, FRL and CAE entered into an agreement to develop and validate a simulator model for the Boeing deHavilland DHC-8 (Dash 8) Series 300 aircraft^[24]. (This was a follow-up agreement to one previously undertaken in August, 1990, which was a similar project for the Dash 8 Series 100 aircraft.) This included flight testing a Dash 8 Series 300 aircraft and using a modified MLE process to determine the aircraft's SCD values, as well as creating a flight simulator mathematical model which used these SCD values to simulate the aircraft's response well enough to be certified to level D training standards by the FAA.

3.2 Aircraft Description

The Boeing deHavilland Dash 8 Series 300 aircraft used in the joint FRL/CAE project, bearing manufacturer's serial number 274 and Canadian Registration C-FKCU, is shown in Fig. 3.1. Fig. 3.2 shows the principal dimensions of the Dash 8 Series 300 aircraft. Values for all required geometric parameters are taken from several Dash 8 references,^{[25], [26], [27], [28]} and are summarized in Tab. 3.1. Note that the value quoted for the wing reference chord is the mean geometric chord, as opposed to the mean aerodynamic chord. This value was used throughout the original FRL/CAE project, and it was thus deemed appropriate to continue using it.

The elevator on the Dash 8 aircraft can be divided into a left and right elevator, with the left elevator being operated by the fore and aft movement of the pilot's control column, and the right elevator being operated in the same manner by the copilot's control column^[25]. For this project, only the case where the elevator is operated as a single control surface is being considered. The spoilers on each wing are divided into inner and outer spoilers which move together at low speeds. However, at airspeeds above 140 kts, the outer spoilers are disengaged because of increased roll sensitivity^[25]. Also, the rudder on the Dash 8 has two sections: a fore rudder and a trailing rudder. The fore rudder is hinged to the vertical stabilizer, while the trailing rudder is hinged to the fore rudder. The trailing rudder is geared to the vertical stabilizer such that the trailing rudder deflects twice as much as the fore rudder^[25]. Thus, a rudder deflection measured in degrees is difficult to define. Instead, the rudder deflection is measured in centimetres, from the trailing edge of the trailing rudder to a line intercepting the undeflected rudder position

(see Fig. 3.3). The MLE-derived SCD values were determined using this convention, so the only difference from measuring the rudder angle in degrees is that τ_R has units of degrees per centimetre, instead of being dimensionless. Fig. 3.4 gives a detailed view of the control surface placements. Note that the effects of tab deflections are ignored.

3.3 Flight Test Program

The Dash 8 Series 300 flight test program was conducted between August 2 and August 21, 1991. It consisted of 39.3 hours of flying over 20 flights. Included in these flights were 696 3-2-1-1 control input responses for MLE analysis and 161 manoeuvres whose time histories would later be used during the final validation of the model^[29]. These 3-2-1-1 manoeuvres were preceded and followed by a period of trimmed flight, and were modified so that the initial step was only two-thirds the magnitude of the subsequent steps. This allowed the aircraft to return closer to its initial trim conditions than it would when using the standard 3-2-1-1 manoeuvre^[3].

Data from cockpit instruments, sensors present on the aircraft for flight data recorder use, and sensors installed by FRL were recorded for all flights. Due to time restraints, an air data boom which would have been used to measure airflow angles could not be installed. Thus, the angle of attack, α , and the angle of sideslip, β , were not measured directly^[29]. Instead, they were reconstructed from the time histories of each aircraft manoeuvre. This flight path reconstruction technique was also used to determine the pitch angle, Θ , and the bank angle, ϕ , since the sensors installed to measure these attitudes were found to be prone to systematic errors^[24].

3.4 Parameter Estimation Procedure

Hui et al^[3] gives a detailed explanation of the parameter estimation procedure used in the FRL/CAE Dash 8 Series 100 project. The procedure for the Series 300 project was similar. It consisted of three separate steps: trim model derivation, stepwise regression analysis, and maximum likelihood estimation.

3.4.1 Trim Model Derivation

Data provided by flight path reconstruction of the trimmed flight periods preceding the 3-2-1-1 manoeuvres were used to obtain a preliminary aircraft trim model. This model used average values (over the trim period) of parameters such as angle of attack, and lift and drag coefficients, to develop estimates of the static SCDs. These initial estimates were used as a guide to judging the validity of subsequent MLE analysis^[24].

3.4.2 Stepwise Regression Analysis

Stepwise regression creates a linear model to best describe the correlations between input and output data sets. That is, it selects which parameters are important enough to be included in the model, and calculates the linear coefficients which give the best fit between measured and calculated quantities. This technique can produce results which are sufficiently accurate to be included in the final mathematical simulator model. However, it is prone to error due to measurement errors, so a MLE process was also used^[3].

3.4.3 Maximum Likelihood Estimation

A MLE procedure was used to provide the final estimates of the SCD values.

Since this procedure takes noise into account, it should produce the most accurate results. In the longitudinal model, only elevator inputs are important, so the analysis was performed using only the elevator time histories. However, in the lateral model, several different inputs can excite the same mode. Treating each input separately gave differing results for the SCDs, depending upon which input was being examined. To obtain a model which would more broadly represent the aircraft dynamics, a method was created whereby all four lateral inputs (i.e. ailerons, rudder, inner spoiler, and outer spoiler) time histories were concatenated into one^[3]. The SCD values thus obtained proved to be adequate, and were used in the remainder of the lateral analysis.

Overall, the use of trim data, stepwise regression, and MLE analysis provided similar results from a variety of sources, and was useful in tracking down analysis problems. The MLE results, in the form of SCD values for a particular aircraft configuration and trim, were tabulated into spreadsheet form, and, along with the trim data, became the database of information to be used in this project.

Chapter 4

Longitudinal Results

Due to the proprietary nature of the data used in this project, only the flaps 0° cases are presented. The other flap setting results were consistent with the flaps 0° cases, having different numerical values only.

4.1 Determination of Model Parameters

4.1.1 Horizontal Tail Parameters

Equations 2.27 and 2.56 were used concurrently to determine the product $a_i\eta_i$. Since all of the terms in these equations were known except for a_i (the lift-curve slope of the horizontal tail) and η_i (the horizontal tail efficiency factor), the equations reduced to the same form. Thus, there was only one equation with two unknowns. However, as mentioned previously, it was only necessary to obtain values for the combination $a_i\eta_i$, since the two parameters always occurred together.

The $a_i\eta_i$ values obtained from the C_{L_q} and C_{m_q} equations (equations 2.27 and 2.56, respectively) were not in good agreement. While the C_{m_q} equations gave results for the flaps 0° case that were relatively constant from about 0.075 to 0.1 per degree (with an

average value of 0.090 per degree), the C_{L_q} equation gave values that varied from about -0.03 to 0.15 per degree (with an average value of 0.038 per degree). Fig. 4.1 shows the results plotted against C_{thrust} . Two possible reasons for the disagreement were explored: first, that the model equation for one or both stability derivatives was in error, and second, that the MLE-derived SCD data were not accurate.

Equations 2.27 and 2.56 were derived from basic geometry and aerodynamics, and differ only in the inclusion of a negative sign and the term $l/\bar{c}$ in the C_{m_q} equation. The negative sign was a result of the definition of positive moments, and the other term was simply a moment arm. Thus, either both or neither of the equations should produce reasonable values for α, η . Since the difference between the C_{L_q} and C_{m_q} data was relatively large, this meant that the MLE-derived SCD data were suspect. Furthermore, since the C_{m_q} data produced reasonable results, it was concluded that the C_{L_q} data must be inconsistent.

A partial explanation for the misleading MLE results may be that the lift caused by the pitching velocity is a small portion of the total lift of the aircraft (as can be shown by inserting relevant values into equation 2.21 and solving for each of the contributions to the aircraft lift), and C_{L_q} is thus a relatively unimportant parameter. Therefore, it is conceivable that the parameter estimation routine would not return very accurate values for C_{L_q} . (Ultimately, it was discovered that the original C_{L_q} data was indeed inconsistent and that CAE had needed to alter the data to give a better representation of the actual lift due to pitching velocity.)

An expanded view of α, η , vs C_{thrust} from equation 2.56 is shown in Fig. 4.2. (Note

that the curve fits presented in Fig. 4.2 through Fig. 4.7 will be discussed in section 4.2.) These are the values that were used in the final geometric model, for both the lift and the pitching moment equations.

4.1.2 Elevator Effectiveness

τ_e (the negative partial derivative of α_{OL_e} with respect to δe) can be thought of as a measure of the elevator effectiveness. Equations 2.25 and 2.54 were used concurrently (substituting in the previously-determined a, η_e values) to determine τ_e . The values obtained from each equation agreed to within about 4 percent, and were averaged together to obtain the final values to be used in the geometric model. τ_e ranged from about 0.31 to 0.42 for the flaps 0° case, with an average value of 0.36. The final values are plotted against C_{thrust} in Fig. 4.3.

4.1.3 Aircraft Neutral Point

The neutral point of the aircraft, h_n , was found directly from equation 2.52, and its variation with C_{thrust} is shown in Fig. 4.4. Note that the neutral point is plotted as aircraft station number, rather than as a percentage of the wing mean geometric chord. This is because this form is easier to conceptualize, as well as program into the equations. (To convert from percent $\bar{c}$ to aircraft station number, simply multiply by $\bar{c}$ to get the distance behind the wing leading edge, and add 380 (the aircraft station number at the wing leading edge of the mean geometric chord).) Fig. 4.4 shows that the neutral point varies approximately from aircraft station number 441 to station number 422, with an average value of 430.61. Note that the aircraft neutral point is not used explicitly in the final geometric model, but is used to help with validation of the equations.

4.1.4 Downwash

Upon examination, it was noted that equations 2.23 and 2.46 contained three unknowns: a_{wb} , $h_{n_{wb}}$, and $\partial\epsilon/\partial\alpha$. With only two equations, these three unknowns could not be determined, and attempts to find a third equation were unsuccessful. As a last resort, an empirical formula^[30] for the downwash parameter, $\partial\epsilon/\partial\alpha$, was used to reduce the number of unknowns to two. Dommasch et al^[30] gives the following equation for $\partial\epsilon/\partial\alpha$ for power-off, low-speed flight, with the tail more than $\pm 0.5\bar{c}$ vertically from the centreline of the wake:

$$\frac{\partial\epsilon}{\partial\alpha} = 23C_{L_a} \frac{(1/\lambda)^{0.3}}{AR^{0.725}} \left(\frac{3\bar{c}}{l_t^*} \right)^{0.25} \quad (4.1)$$

where: λ is the wing taper ratio (tip chord over root chord)
 AR is the wing aspect ratio
 l_t^* is the horizontal distance from the horizontal tail quarter-chord point to the wing quarter-chord point

Values for the above parameters are included in Tab. 3.1.

Flaps 0° results for $\partial\epsilon/\partial\alpha$ ranged from about 0.27 to 0.35, with an average value of 0.32. A plot of $\partial\epsilon/\partial\alpha$ vs C_{thrust} is shown in Fig. 4.5.

4.1.5 Wing-Body Lift-Curve Slope

Once values had been calculated for $\partial\epsilon/\partial\alpha$, equation 2.23 was used to find the wing-body lift-curve slope, a_{wb} . Values ranged from about 0.08 per degree to 0.11 per degree for the flaps 0° case, with an average value of 0.10 per degree. A plot of a_{wb} vs C_{thrust} is shown in Fig. 4.6. Note that a_{wb} and $\partial\epsilon/\partial\alpha$ show identical variations with C_{thrust} .

This is because they are both dependent on Cl_α , and it is actually the variation of Cl_α with $C_{thrusts}$ that is seen in Fig. 4.5 and Fig. 4.6.

4.1.6 Wing-Body Neutral Point

The wing-body neutral point, $h_{n_{wb}}$, was found from equation 2.46, using the previously-determined values of both a_{wb} and $\partial\epsilon/\partial\alpha$. Once again, the results are presented in terms of aircraft station numbers, rather than percent of mean geometric chord. Flaps 0° results ranged from station numbers 363 to 333, with an average value of 355.25, and are shown plotted vs $C_{thrusts}$ in Fig. 4.7. Note that the wing-body neutral point lies forward of the leading edge of the wing at the mean geometric chord position, which is located approximately at aircraft station number 380.

4.1.7 Model Parameters Found From Trim Data

The trim values of C_L , C_m , α_m , $\delta\epsilon$, and q were used along with all of the previously-derived model values and the necessary geometric parameters in equations 2.21 and 2.44 to determine values for α_{0L} and C_{m_0} . The resulting values were averaged over all of the flight tests at each flap setting. The flaps 0° average value for α_{0L} is -3.25°, and the flaps 0° average value for C_{m_0} is 0.24. It should be noted that these two parameters are dependent on thrust. However, the average values gave reasonable results, so they were used in the final model equations.

4.2 Thrust and Flap Effects

In order to successfully complete the longitudinal geometric model, it was necessary to curve fit the values of each of the above parameters (excluding α_{0L} and C_{m_0})

with respect to C_{thrust} . This was done for each flap setting, although only the flaps 0° cases are presented. It was hoped that this would account for the effects of thrust and flaps, which were not explicitly included in the model equations.

A method known as Akaike's Information Criteria (AIC)^[31] was used to determine the optimal order of each curve fit. However, since each flap setting case should be consistent with the others, it was often necessary to judge which order curve fit best modelled all flap settings for each of the model parameters. That is, if three flap settings of the same model parameter were best modelled by a first order fit and two by a second order fit, all flap settings for that parameter were judged to be best described by a first order equation. Discrepancies such as this were common, due to the amount of scatter in the data. In the end, only h_n was modelled by a second-order fit. The rest were first-order. Flaps 0° fit equations are listed and shown graphically in Fig. 4.2 through Fig. 4.7.

4.3 Verification

4.3.1 Geometry-Based SCD Predictions vs MLE-Derived SCD Data

Replacing the parameters in the longitudinal geometry-based model SCD equations (equations 2.23, 2.25, 2.27, 2.46, 2.54, and 2.56) with their thrust- and flap-dependent curve fits allowed a prediction to be made of the SCDs. These predicted values were then compared to the MLE-derived SCD values. Obviously, the CL_q model values were not going to match the MLE CL_q values, so a new set of CL_q data was created by taking the a, η_i values obtained from the C_{m_q} data set, and calculating CL_q from equation 2.27. These

were then used as given values, so that the model results could be compared with some reasonable C_{L_q} data. (Recall that if the C_{L_q} results from the MLE calculations had been accurate, then equations 2.27 and 2.56 should have given similar results for α, η_r .) It should be noted that the model contains two equations for C_{m_α} : equation 2.46 and equation 2.52. Both equations were used to calculate model C_{m_α} values (MLE-derived values of C_{L_α} were inserted into equation 2.52 as α), and it was found that they gave comparable results, although the results from equation 2.52 appeared slightly better. Nevertheless, equation 2.46 was used in the final simulator model. Plots of all geometry-based model results vs MLE-derived results are shown in Fig. 4.8 through Fig. 4.13. Note that the line in these plots is not a curve fit. Instead, it is drawn to show where the model results are equal to the MLE results.

4.3.2 Geometry-Based Force and Moment Predictions vs Flight Test Data

Values for C_L and C_m for each trim case were calculated by inserting the correct curve fit, average, trim, or geometric value of each relevant parameter into equations 2.21 and 2.44 and solving. The resulting predictions were compared to the actual flight test measured values in Fig. 4.14 and Fig. 4.15. The lift coefficient values agreed, for the most part, to within about 10 percent. The pitching moment values often disagreed by orders of magnitude, although, since the measured values were extremely close to zero, this may not seriously affect the performance of the simulator model. On average, the geometry-based C_m predictions differed from the measured values by about 0.02.

Equations 2.183 and 2.184 were used to calculate non-trim flight test values of C_L

and C_m . All parameters in these two equations had been tabulated for several 3-2-1-1 elevator input manoeuvres, with the exceptions of $\bar{q}$ and $\dot{q}$. The dynamic pressure was calculated from tabulated values of pressure altitude (which, however, had to be estimated from a graph) and Mach number, and the pitch acceleration was estimated from a plot of pitch rate vs time. Geometry-based predictions of the non-trim lift and pitching moment coefficient values were determined by inserting the corresponding tabulated values of α_m , δe , q , and $C_{thrusts}$ into equations 2.21 and 2.44 (using curve fits for geometric model parameters). One flaps 0° elevator 3-2-1-1 case (flight H19207DE) was studied at three separate points in time: 15.062, 18.062, and 21.062 seconds. Tab. 4.1 contains the resulting non-trim values of C_L and C_m for both the geometry-based and the flight test equations. The agreement for both parameters seemed to improve as the values got larger (positively). Even though the values do not seem to agree well, it must be remembered that they are only estimates of the actual C_L and C_m values calculated by the simulator model from equations 2.21 and 2.44. Section 4.3.4 will show that the time histories of this particular 3-2-1-1 case match well. Thus, the actual force and moment values must be good enough, and the estimates are assumed to be reasonable.

4.3.3 Geometry-Based Model Parameter Values vs Theoretical Predictions

USAF Datcom, ESDU data sheets, and several other publications were searched for methods of estimating any of the longitudinal model parameters presented in this report. Values for the product $a_i \eta_i$ were not found, so a_i and η_i were estimated separately. All derivations assumed flaps undeflected (i.e. flaps 0°).

Both ESDU Wings 05.01.01^[11] and Datcom Section 4.4.1^[16] predicted a value of

1.0 for η_t . This seems appropriate, since the horizontal tail is located a relatively large distance above the wing, and the propellers are centred below the wing. The wing wake and propeller slipstreams would tend to descend as they moved back from the wing trailing edge^[32], and thus, the horizontal tail would likely be located outside their region of influence. If this is the case, then values obtained for a, η_t could be applied to a_t alone. ESDU 70011^[12] predicts a value for a_t in the range of 0.069 per degree, while Datcom Section 4.1.3.2^[16] says a_t should be about 0.075 per degree. The average flaps 0° value for a, η_t in the geometry-derived model was 0.090 per degree. If η_t is taken as 1.0, then the model-derived values for a_t are only somewhat comparable to the predicted ones. If η_t is greater than 1.0 (i.e. the horizontal tail is affected by the wing wake or propeller slipstreams), then the correlation is better. However, there is not enough data at present to be able to estimate the position of the wake or propeller slipstreams any better than by using the ESDU or Datcom methods. Thus, $\eta_t = 1.0$ is the best estimate possible.

Since $\partial \varepsilon / \partial \alpha$, a_{wb} , and $h_{n_{wb}}$ model values were all dependent on the empirical equation used to find $\partial \varepsilon / \partial \alpha$, it was important that their values match those predicted in the literature, so that the bias introduced by that empirical equation could be said to be minimal. ESDU 80020^[13] found that $\partial \varepsilon / \partial \alpha$ should be around 0.27, while Datcom Section 4.4.1^[16] predicted a value of 0.26. The model flaps 0° average value was 0.32. However, downwash^[26] and geometric^{[26], [27]} data for the Series 100 aircraft were available, and when these were compared to the ESDU 80020 estimate for the Series 100 aircraft, it was found that the ESDU estimate was approximately 0.03 too low. Thus, the ESDU estimate for the Series 300 aircraft was likely also about 0.03 too low, and the estimate was altered

to 0.30. This was much closer to the geometry-based model estimate.

There was no ESDU method available for estimating a_{wb} , but Datcom Section 4.3.1.2^[16] predicted a value of 0.11 per degree. The model flaps 0° average estimate was very close to this, at 0.10 per degree.

Estimates of h_{nwb} proved to be a problem. ESDU 76015^[14] predicted h_{nwb} to be at aircraft station number 391, and Datcom Section 4.3.2.2^[16] estimated it to be at station number 396. However, the geometry-derived model flaps 0° average value turned out to be aircraft station number 355. In other words, the model said it should be 25 inches forward of the wing leading edge (at the mean geometric chord position), while ESDU and Datcom estimates placed it 11 and 16 inches aft of the wing leading edge, respectively. Altering the empirical downwash equation to give values of h_{nwb} comparable to those in the literature did not appreciably alter values found for a_{wb} , but did increase $\partial \epsilon / \partial \alpha$ values to above 0.5. Since downwash data for the geometrically-similar Series 100 aircraft were known, and they were comparable to those given for the Series 300 aircraft by the original downwash equation, it was decided that the original downwash equation would be used. Additionally, the Dash 8 aircraft has a large horizontal tail volume coefficient (approximately 1.5), which was not necessarily taken into account in the ESDU and Datcom prediction methods for h_{nwb} . Thus, these methods may not be very accurate for this aircraft.

Estimates of the aircraft neutral point, h_n , were not found in either the ESDU data sheets or Datcom. Instead, a flight test method outlined in Etkin^[2] was used. The method is based on the relationship

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$$\frac{\partial \delta e_{trim}}{\partial C_{L_{trim}}} = - \frac{C_{m_\alpha}}{C_{L_\alpha} C_{m_{\delta e}} - C_{m_\alpha} C_{L_{\delta e}}} \quad (4.2)$$

which means that, when $h=h_n$ (and thus, C_{m_α} is zero), then $\partial \delta e_{trim} / \partial C_{L_{trim}}$ is equal to zero. The method consists of making in-flight measurements of δe_{trim} and $C_{L_{trim}}$ for a range of CG positions, and plotting the slope $\partial \delta e_{trim} / \partial C_{L_{trim}}$ vs each CG position. Then, extrapolation of the curve to zero allows the determination of h_n . However, this method is only strictly valid when the thrust is constant. Therefore, it had to be modified to account for thrust variations in the available data. To do this, plots of δe vs C_L were drawn and the thrust was noted for each point. (See Fig. 4.16.) Note that the longitudinal set of trim data did not contain a sufficient number of points to allow a reasonably accurate prediction of h_n . Thus, the set of lateral trim data was used as well. A curve fit was performed on each plot, and the slope of the curve was determined for each plot at a constant specific thrust value. This value was plotted vs CG position for each plot, and extrapolation yielded the value of h_n . Results of this procedure for the flaps 0° case are presented in Fig. 4.17 for thrust coefficients of approximately 0.045, 0.06, and 0.07 (although the amount of data for a thrust coefficient of 0.07 was somewhat limited). Neutral point positions were predicted at aircraft station numbers 433.0, 428.9, and 424.7 for each of the thrust coefficients respectively, while equation 2.52 results placed the corresponding aircraft neutral points approximately at station numbers 430.3, 429.6, and 432.6, and the neutral point curve fit of Fig. 4.4 gave the results as 430.8, 429.9, and 429.3. Thus, the agreement is quite good for the two lower thrust coefficients, and reasonable agreement exists between the predictions of equation 4.2 and the aircraft

neutral point curve fit for the highest thrust coefficient, even though the amount of data was limited. The neutral point prediction of equation 2.52 at the highest thrust coefficient did not agree well, but this was likely due to the amount of scatter in the original data (see Fig. 4.4).

No applicable method was found to estimate τ_n , α_{0L} , or C_{m0} . However, a rough check on the value of α_{0L} was obtained by plotting flaps 0° trim values of C_L vs α_m and extrapolating to $C_L = 0$. This is only a rough estimate because the tabulated C_L values contain all lift contributions, not just those due to angle of attack. The pitch rate and elevator angle contributions were eliminated, but it was not possible to subtract out the thrust effects. The results are presented in Fig. 4.18, which shows that the estimated value of α_{0L} was -3.23°, as compared to the geometry-based model average value, which was -3.25°.

4.3.4 Geometry-Based Simulator Model vs Flight Test Data

The final verification step was to replace the lift and pitching moment equations in the actual Dash 8 Series 300 flight simulator with equations 2.21 and 2.44. Each of the model parameters in the equations was replaced by its appropriate curve fit or average value. The known geometric parameters were inserted directly.

The flight simulator model containing the geometry-derived lift and pitching moment equations was tested using ten sets of flight test results: a longitudinal trim (cruise) case at flaps 0°, a longitudinal trim (approach) case at flaps 15°, a longitudinal trim (landing) case at flaps 35°, a phugoid dynamics case at flaps 0°, a short period dynamics case at flaps 0°, and five elevator 3-2-1-1 proof-of-match (POM) cases, one at

each flap setting (flaps 0° , 5° , 10° , 15° , and 35°). These POM cases are meant to check the validity of the flight simulator by providing actual flight test results for comparison that were not used in the original derivation of the simulator equations. Note that the FRL/CAE drag equation was not changed, and, for the longitudinal model validation only, the original FRL/CAE lateral equations were used. The flaps 0° results are presented below and in Fig. 4.19 through Fig. 4.21. Note that the x- and y-axis values are not shown in these figures, due to the proprietary nature of the data.

The longitudinal trim (cruise) case consisted of initializing the aircraft at a certain height, velocity, weight, CG position, etc., and letting the simulation run until the aircraft had been trimmed (i.e. no unbalanced moments). The model predicted an elevator deflection of 1.494° was necessary to trim the aircraft at a pitch angle of 0.661° and an angle of attack of 0.377° . The flight test results included an elevator angle of 1.420° , a pitch angle of 1.169° , and an angle of attack of 0.887° . The model results were within the $\pm 1^\circ$ tolerance limits consistent with FAA simulator certification requirements^[33].

The results of the phugoid and short period cases, and the flaps 0° elevator 3-2-1-1 POM case are presented in Fig. 4.19 through Fig. 4.21. The solid lines represent the actual flight test data recorded, while the dashed lines are the model predictions. The phugoid and short period oscillations were initiated in a trimmed aircraft by an elevator input. The initial offset in some of the parameters occurred because the model did not trim the aircraft to the exact values measured during the flight tests (since the model is not exact). The thrust was driven to match the actual thrust recorded during the flight test. The model period of oscillation and damping ratio for the phugoid case (see Fig.

4.19) were within the FAA set tolerance limits. The short period model results were also representative of the actual flight test results (see Fig. 4.20).

Results of the elevator 3-2-1-1 POM case can be seen in Fig. 4.21. The initial offsets were again due to the differences between model trim and actual flight test trim positions. The discontinuities in slope in the flight test results were due to the sparse sampling used in the original data collection. Note that, in addition to the thrust being driven, the model elevator was driven to match the flight test elevator, with the initial offset maintained throughout the manoeuvre. Thus, the discontinuities in slope also appeared in the model results for this parameter. Also, the flight test pressure altitude data were not tabulated in a form that could be compared to the geometry-based model results. Instead, pressure altitude was estimated from a plot which showed the change in altitude with time. Overall, the 3-2-1-1 POM model results were in good agreement with those of the flight test.

Chapter 5

Lateral Results

As in the longitudinal case, only the flaps 0° results are presented here.

5.1 Determination of Model Parameters

Before any lateral model parameters could be found, empirical estimates of some of them had to be determined, since there were too many unknowns for the number of equations to be solved. The aileron and spoiler equations each contained a product of two unknown model parameters. In each case, this product was treated as a single unknown, which allowed every aileron and spoiler equation to be solved without the need for an empirical estimate of any parameter. However, this could not be accomplished for any of the remaining SCD model equations. Several methods were attempted whereby one model parameter would be estimated, and the others then determined from the result. The β , p , and r derivative equations were studied with and without their wing-body contributions. Resulting values of the model parameters were compared to those determined from Datcom and ESDU sources, and the combination of empirical estimates that gave the best results was used in the final simulator model. This combination turned

out to be the determination of empirical estimates of a_F , a_{vs} , and $(C_{n_{wb}})_\beta$, keeping all wing-body contributions except for those of C_{Y_p} and C_{Y_r} .

5.1.1 Fin Lift-Curve Slope

Empirical values of the fin lift-curve slope, a_F , were calculated from both Datcom and ESDU sources. Datcom Section 4.1.1.2^[16] predicted a result of 0.0293 per degree, while ESDU 82010^[15] and 70011^[12] (combined) produced a result of 0.029 per degree. This close agreement was taken to be an indication of the reliability of the result, and the value of 0.029 per degree was used in the final model for a_F .

5.1.2 Vertical Strake Lift-Curve Slope

The same methods used to find a_F were used to determine values for a_{vs} . The Datcom result was 0.0061 per degree, and the ESDU result was 0.0065 per degree. The average of these, 0.0063 per degree, was used in the final model. Since the vertical strake contribution to any of the SCDs is small compared to that of the fin, it was decided that the above calculations would be accurate enough.

5.1.3 Sidewash

Even with the above values of a_F and a_{vs} , there were too many unknowns in each of the β , p , and r equations (excluding equation 2.135, which contained only one unknown, K , and was solved independently) to solve for the sidewash parameter, $\partial\sigma/\partial\beta$, or any other model parameter. Equations 2.81, 2.83, 2.85, 2.125, 2.137, 2.170, 2.180, and 2.182 were studied by deleting all the wing-body contributions (which were usually small) and obtaining values for the remaining model parameters, which turned out to be the product $\eta_F(1-\partial\sigma/\partial\beta)$. However, no two equations gave the same result. Then, each

equation in turn had its wing-body contribution deleted, was solved for the above product, and the result was substituted into each of the other equations. These equations were then solved for their respective wing-body contributions, which were compared to empirical estimates, where available. However, not one of the equations gave results that were in agreement with all of these empirical estimates. This meant that the wing-body contributions were important, or that the MLE-derived SCD data were inconsistent, or both.

If an empirical estimate existed for one of the wing-body contributions in the above eight equations listed, it was substituted into the appropriate equation (along with all other known geometric parameters and a_F and a_{v_s}), and the equation was solved for $\eta_F(1-\partial\sigma/\partial\beta)$. This was then used to calculate the wing-body contributions in each of the remaining seven equations. When $(C_{n_{wb}})_\beta$ was determined to be -0.0018 per degree (Perkins and Hage^[22]), and was used in the previously-described manner, the results for each of the remaining wing-body contributions except for those of equations 2.83 (C_{Y_p}) and 2.85 (C_{Y_r}) were found to agree at least reasonably well with their respective empirical estimates. Error estimates of the MLE-determined SCD values for both C_{Y_p} and C_{Y_r} were relatively large, and thus, these SCD values were deemed to be inconsistent. Since the roll rate and yaw rate contributions to the total aircraft side force are relatively minor, and the wing-body roll rate and yaw rate contributions are even less important, it was decided to use the values of a_F , a_{v_s} , and $\eta_F(1-\partial\sigma/\partial\beta)$ (determined as described above) in equations 2.83 and 2.85 without any wing-body contributions. This would result in new values for C_{Y_p} and C_{Y_r} that should be consistent with the remaining MLE-derived SCD values.

The final value of the sidewash parameter multiplied by the fin efficiency factor, $\eta_f(1-\partial\sigma/\partial\beta)$, determined from equation 2.170 ($C_{n\beta}$) ranged from about 1.05 to 1.45, with an average value of 1.29. The results are plotted against $C_{thrusts}$ in Fig. 5.1. (Note that the curve fits shown in Fig. 5.1 through Fig. 5.12 will be discussed in section 5.2.)

5.1.4 Fuselage "Side Force-Curve Slope"

With the values of the previously-mentioned model parameters, equation 2.81 was solved for the fuselage "side-force curve slope", or a_{fuse} . Values ranged from about 0.006 to 0.013 per degree for the flaps 0° case, with an average value of 0.0094 per degree. A plot of a_{fuse} vs $C_{thrusts}$ is shown in Fig. 5.2.

5.1.5 Wing-Body Side Force Neutral Point

The wing-body contribution to the yawing moment due to sideslip is modelled as a side force acting at some point, x , on the fuselage. This point is here called the wing-body side force neutral point. Values were determined from the wing-body contribution to equation 2.170 ($(C_{n_{wb}})_\beta$, previously found to be -0.0018 per degree), using the results for a_{fuse} described above. x ranged from about 160 inches to 340 inches forward of the aircraft's CG, with an average value of 211 inches, or 17.6 ft forward of the CG. The results are plotted vs $C_{thrusts}$ in Fig. 5.3. Note that this parameter does not explicitly appear in the final simulator model, since the wing-body contribution to equation 2.170 (-0.0018 per degree) is used instead.

5.1.6 Wing Dihedral Neutral Points

The dihedral of the wing contributes to the rolling moment through a lift force on the right and left halves of the wing acting at points y_R and y_L , respectively. The

parameter $y_L - y_R$ can be found from equation 2.125 (C_{l_β}) using the previously-determined values of a_F , a_{vs} , $\eta_F(1 - \partial\sigma/\partial\beta)$, the remaining known geometric parameters, and α (or C_{L_α}) from the longitudinal information. The results ranged from approximately -300 to -600 inches, with an average of -428.8 inches, and are plotted vs C_{thrust} in Fig. 5.4. The negative sign is due to the convention of y positive out the right wing. If it is assumed that $y_R = y_L$, then the lift vectors are modelled as acting at 214.4 inches, or 17.9 ft, out each wing. Since the span of each wing is 45 ft, the lift vectors are modelled as acting at a distance about 40 percent of the way out each wing from the fuselage centreline.

5.1.7 Rudder Effectiveness

τ_R (the negative partial derivative of $\alpha_{\theta_{LF}}$ with respect to δR) is a measure of the rudder effectiveness. Equations 2.87, 2.133, and 2.178 were used concurrently to determine the product $\tau_R \eta_F$, since no value had been found for the fin efficiency factor on its own. The average value of the results from equation 2.87 ($C_{Y_{\delta R}}$) was 0.90, the average of the results from equation 2.133 ($C_{l_{\delta R}}$) was 0.37, and the average of the results from equation 2.178 ($C_{n_{\delta R}}$) was 0.62. The overall average was 0.63. Since the error estimates of the MLE-derived SCD data were much lower for $C_{n_{\delta R}}$ than for the other two derivatives, and the average of the values from the three equations was so close to the values from equation 2.178, it was decided that the average value would be adequate. The results range from about 0.5 to 0.76 and are shown plotted against C_{thrust} in Fig. 5.5.

5.1.8 Aileron Parameters

The aileron contribution to the rolling moment is given by equation 2.127 ($C_{l_{\delta a}}$). Since the wing chord as a function of y position was known, the integral in equation

2.127 could be evaluated. Again, a value of the lift-curve slope was obtained from the longitudinal equations and substituted into equation 2.127, along with the other known geometric parameters. The unknown product, $\tau_a(G_R + G_L)$, remained, with τ_a the aileron efficiency factor. The results ranged from about 0.22 to 0.35, with an average value of 0.28, and are shown plotted against $C_{thrusts}$ in Fig. 5.6.

The second aileron parameter, $\Omega_a(J_R - J_L)$ (with Ω_a the aileron drag factor), was determined from equation 2.172. The results ranged from approximately -0.26 to -0.60, and had an average value of -0.39. Fig. 5.7 shows the results plotted against $C_{thrusts}$.

5.1.9 Spoiler Parameters

The two outer spoiler parameters were derived from equations 2.129 ($C_{l_{\delta so}}$) and 2.174 ($C_{n_{\delta so}}$). $\tau_{so}F$ is the outer spoiler efficiency factor multiplied by a constant, and ranged from about 0.18 to 0.24, with an average of 0.20. $\Omega_{so}I$ is the outer spoiler drag factor multiplied by a constant, and ranged from about -0.20 to -0.45. It had an average value of -0.36. Results for the above two parameters are plotted against $C_{thrusts}$ in Fig. 5.8 and Fig. 5.9. Note that there are fewer points for the outer spoiler parameters than for any other parameters. This is because the inner spoilers can deflect without the outer spoilers deflecting, and so there are several flight tests without outer spoiler information.

The inner spoiler parameters, $\tau_i E$ and $\Omega_i H$, are found in equations 2.131 ($C_{l_{\delta si}}$) and 2.176 ($C_{n_{\delta si}}$). $\tau_i E$, the inner spoiler efficiency factor multiplied by a constant, ranged from approximately 0.20 to 0.27, with an average value of 0.23. It is plotted vs $C_{thrusts}$ in Fig. 5.10. $\Omega_i H$, the inner spoiler drag factor multiplied by a constant, ranged from about -0.28 to -0.42, with an average value of -0.38. It is plotted vs $C_{thrusts}$ in Fig. 5.11.

Note that the outer and inner spoiler parameters described above tend to be similar. That is, $\tau_o F$ is approximately the same as $\tau_i E$, and $\Omega_o I$ is very close to $\Omega_i H$. Since both the outer and inner spoilers are modelled as the same size and shape, and are located directly next to each other on the wing, the results are reasonable.

5.1.10 Wing-Body Contribution to Rolling Moment Due to Roll Rate

The wing-body contribution to C_{l_p} is the only contribution modelled, and is actually only a wing contribution, given by equation 2.135. It contains a correction factor, K , which was determined in the same manner as the aileron and spoiler parameters described previously. K ranged from about 0.67 to 0.96, with an average of 0.76. It is shown in Fig. 5.12, plotted against $C_{thrusts}$.

5.1.11 Wing-Body Contribution to Rolling Moment Due to Yaw Rate

The wing-body contribution to C_{l_r} is found in equation 2.137, and is modelled only as a wing contribution. λC_L was found by substituting in all of the other known parameters (either geometric or determined previously). The results are plotted against C_L in Fig. 5.13. That is, λ was determined by plotting the results of solving equation 2.137 against C_L and curve-fitting. For this parameter, no curve fits vs $C_{thrusts}$ were performed. The results for $(\lambda C_L + \text{constant})$ ranged from about 0.08 to 0.3 with the actual curve fit being $0.1844C_L + 0.0439$. (Note that, if the results of equation 2.137 had simply been divided by C_L , the resulting average value of λ would have been 0.25.)

5.1.12 Wing-Body Contribution to Yawing Moment Due to Roll Rate

The wing-body contribution to C_{n_p} is also modelled only as a wing contribution, which is found in equation 2.180. $-\lambda C_L$ was treated in the same manner as λC_L was

previously. It is plotted against C_L in Fig. 5.14, and ranged from about -0.13 to -0.25. The actual curve fit is $-0.1301C_L - 0.0969$.

5.1.13 Wing-Body Contribution to Yawing Moment Due to Yaw Rate

The wing-body contribution to C_{n_r} is given in equation 2.182 as $\xi C_L^2 + \kappa$. This is again actually only a wing contribution. The results from equation 2.182 are plotted this time against C_L^2 in Fig. 5.15. The results ranged from about -0.10 to -0.26, and the actual curve fit is given as $-0.0242C_L^2 - 0.1508$.

5.2 Thrust and Flap Effects

In order to account for the effects of thrust and flaps not explicitly included in the model equations, the lateral model parameters were curve fit with respect to C_{thrust} for each flap setting. (Although only the flaps 0° cases are presented.) As in the longitudinal section, Akaike's Information Criteria (AIC)^[31] was used to determine the best order of each fit, taking into account the condition that each of the five flap settings had to be modelled by the same order fit. As a result, all of the lateral model parameters that were curve fit with respect to C_{thrust} were modelled by first-order fits. Flaps 0° fit equations are listed and shown graphically in Fig. 5.1, Fig. 5.2, and Fig. 5.4 through Fig. 5.12.

5.3 Verification

5.3.1 Geometry-Based SCD Predictions vs MLE-Derived SCD Data

All lift-, thrust-, and flap-dependent curve fits were substituted back into the

equations for each of the lateral SCDs (equations 2.81, 2.87, 2.93, 2.94, 2.125, 2.127, 2.129, 2.131, 2.133, 2.135, 2.137, 2.170, 2.172, 2.174, 2.176, 2.178, 2.180, and 2.182), and the values of all geometric parameters, lift coefficient, thrust coefficient, and empirically-estimated model parameters were added. This allowed a prediction to be made of each of the 18 lateral SCDs. These predicted values were then compared to the MLE-derived SCD values. The C_{Y_p} and C_{Y_r} values were obviously not going to match. Nevertheless, they were compared to their MLE-derived values, in order to see how far apart they actually were. The comparisons are shown in Fig. 5.16 through Fig. 5.33. Again, the plots do not show curve fit lines, but rather lines which indicate the location where the model values are equal to the MLE-derived values.

5.3.2 Geometry-Based Force and Moment Predictions vs Flight Test Data

Values for C_Y , C_b , and C_n for each trim case were calculated by inserting the trim values of the aircraft rotation rates and control deflections, along with all of the curve fit, lift, thrust, empirical or geometric parameters into equations 2.92, 2.123, and 2.168 and solving. Since there were no trim values for C_Y , C_b , or C_n in the original data set, C_Y was calculated from equation 2.185, and C_l and C_n were assumed to be zero. The results are shown in Fig. 5.34 through 5.36. The C_Y values differed by up to about 0.04, but the C_l values were within ± 0.005 of zero, and the C_n values were within ± 0.006 of zero. The overall agreement was taken to be good.

For non-trim cases, equations 2.185, 2.186, and 2.187 were used to calculate values for C_Y , C_b , and C_n . As in the longitudinal cases, some of the variables in these equations had been tabulated during the flight tests, but some had not. The dynamic

pressure ($\bar{q}$) was calculated from tabulated values of the pressure altitude and Mach number, and the roll acceleration ($\dot{p}$), yaw acceleration ($\dot{r}$), and pitch rate (q) were estimated from plots of roll rate, yaw rate, and pitch angle vs time. The y-acceleration values were estimated from a smoothed version of the tabulated y-acceleration values. Finally, α_m was extracted from the simulator longitudinal model by running the model to the desired time and asking for the angle of attack. Note that this was the value determined by the model equations, and not the flight test measured value. Geometry-based predictions of C_Y , C_D , and C_n were calculated from equations 2.92, 2.123, and 2.168, using values of all necessary parameters that had been tabulated during the flight tests. One flaps 0° aileron 3-2-1-1 case (flight H05306DA) was studied at three separate points in time: 7.0, 9.0, and 11.0 seconds. Tab. 5.1 contains the resulting non-trim values for both the geometry-based model equations and equations 2.185, 2.186, and 2.187. The agreement is reasonable for the C_Y and C_D values, but the C_n values given by the model are seemingly inconsistent with those given by equations 2.185, 2.186, and 2.187. This may be a result of the C_n values all being near zero, where the model equations seem to be somewhat inaccurate. Also, C_n is affected more by the longitudinal parameter α_m than either C_Y or C_D . Since α_m was only a rough estimate, the values of C_n may have been poorly estimated. Section 5.3.4 will show that the time histories for this case match the flight test time histories, so the model equations must give reasonable results.

5.3.3 Geometry-Based Model Parameter Values vs Theoretical Predictions

As was done for the longitudinal model parameters, USAF Datcom, ESDU data sheets, and other publications were searched for methods of estimating any of the lateral

model parameters. All derivations assumed flaps undeflected.

a_F and a_{w_0} had already been derived from empirical relations. No methods other than those already discussed in sections 5.1.1 and 5.1.2 were used to find values for these two parameters.

A method of estimating $\eta_F(1 + \partial\sigma/\partial\beta)$ was found in Datcom Section 5.4.1^[16]. The plus sign in this equation is a result of the fact that Datcom defines σ in a direction opposite to that employed in chapter 2 of this report. Thus, the results should be comparable to values obtained previously for $\eta_F(1 - \partial\sigma/\partial\beta)$. Datcom predicts a value of 1.03, where section 5.1.3 of this report stated that the model flaps 0° average value was 1.29. The agreement is better if it is considered that the Datcom prediction assumes power-off flight. Datcom Section 4.4.1^[16] predicts a power-off value of η_F to be about 0.9 (this is an estimated value over a range of lift coefficients and heights above the aircraft CG). Using the value of 1.29 for the combined factor, this would mean that the power-off sidewash factor ($\partial\sigma/\partial\beta$) must be about -0.15. The power-on value should be greater than this. If the value is assumed to be -0.20 (a reasonable increase), and the model value for the combined parameter is still 1.29, then the power-on value of the fin efficiency factor (η_F) would be 1.08. A simple propeller streamtube calculation^[30] using the average flaps 0° thrust coefficient value results in a power-on value of 1.06 for η_F (in the slipstream). This is a much better agreement, and thus, the model value of 1.29 for $\eta_F(1 - \partial\sigma/\partial\beta)$ is reasonable.

Datcom Section 5.2.1 predicted a value of 0.0044 per degree for a_{fus} . The model flaps 0° average estimate was 0.0094 per degree. The two values are, at least, of the

same order of magnitude.

No methods were found for predicting the product $\tau_R \eta_F$, or just the rudder effectiveness parameter itself. Also, no methods were found for predicting any of the aileron or spoiler model parameters, or $(y_L - y_R)$ or x , but these last two can at least be compared to what conventional engineering judgement says about them. Since these parameters are modelled as points on the aircraft where a force is acting, they must at least lie somewhere on the aircraft. If $y_L = y_R$, then the lift vectors are modelled as acting at points 17.9 ft out each wing. This is about 40 percent of the way out each wing, which seems to be a reasonable distance at which a resultant lift vector could be modelled. The wing-body side force is modelled as acting at a point $x = 17.6$ ft forward of the aircraft's CG. The aircraft CG is always located somewhat near the horizontal position of the wings on the body. 17.6 ft in front of this is well within the fuselage length. Thus, the value of x is reasonable.

Perkins and Hage^[22] gives a value for the wing-body contribution to C_l (λC_L) of $0.25C_L$, for an elliptic lift distribution over the wing. Datcom Section 7.1.3.2^[16] predicts $0.25C_L + 0.0002$ for flaps 0° . These compare reasonably well to the curve fit given in section 5.1.11 as $0.1844C_L + 0.0439$, especially since the lift distribution over the wing of the Dash 8 is not elliptic. (Note, however, that the average value of λ given in section 5.1.11 is 0.25.) The constant term in the curve fit likely represents some small contributions to the rolling moment that were neglected in the model equations.

Datcom Section 7.3.2.3^[16] predicts a value for the wing-body contribution to C_{n_p} , or $-\lambda C_L$, of $-0.126C_L$. Perkins and Hage^[22] predicts $-0.125C_L$ for an elliptic lift

distribution over the wing. These values compare well to the model curve fit equation from section 5.1.12 of $-0.1301C_L - 0.0969$. The constant term is likely due to the omission of small contributions to the yawing moment in the model equations.

The model wing-body contribution to C_n was previously given in section 5.1.13 as $-0.0242C_L^2 - 0.1508$. Datcom Section 7.3.3.3^[16] predicts this contribution to be given by the equation $-0.03C_L^2 - 0.0094$, while Campbell and McKinney^[21] and Toll and Queijo^[34] predict $-0.02C_L^2 - 0.3C_{D_0}$. Using equation 2.165 and the trim data, C_{D_0} was estimated to be about 0.3. Thus, the prediction of Campbell and McKinney^[21] and Toll and Queijo^[34] becomes $-0.02C_L^2 - 0.09$. Both of the above predictions agree well with the model curve fit equation, at least for the lift coefficient relationship. The disagreement in the constant term is likely again due to neglected terms in the model equations.

5.3.4 Geometry-Based Simulator Model vs Flight Test Data

The final verification was provided by replacing the side force, rolling moment, and yawing moment equations in the actual Dash 8 Series 300 flight simulator with equations 2.92, 2.123, and 2.168. Each of the model parameters in the equations was replaced by its appropriate curve fit, derived, or known geometric value.

Seventeen sets of flight test results were used to test the model equations: a Dutch Roll case at flaps 0° , a Dutch Roll case at flaps 15° , one aileron 3-2-1-1 POM case at each flap setting of 0° , 5° , 10° , 15° , and 35° , one spoiler 3-2-1-1 POM case at each flap setting, and one rudder 3-2-1-1 POM case at each flap setting. The longitudinal model equations described in chapter 4 for the lift and pitching moment coefficients were left in place. Thus, these cases were a test of the full model. (Note that the longitudinal test

cases were re-run with the model lateral equations in place of the original simulator lateral equations, and the results were indistinguishable from those presented in chapter 4.) The flaps 0° results are presented in Fig. 5.37 through Fig. 5.40. The solid lines represent the flight test results, while the dashed lines are the model predictions. Again, the x- and y-axis values are not shown in these figures, due to the proprietary nature of the data.

The flaps 0° Dutch Roll case consisted of trimming the aircraft at a certain altitude, velocity, weight, etc., and then initiating an oscillation with a rudder input. Results can be seen in Fig. 5.37. Note that the thrust was driven to exactly match the actual thrust measured during the flight test. The model values of the period and damping ratio were within FAA set tolerance limits, as were the model predictions of the time to half amplitude and the phase of the bank angle (ϕ) with respect to the sideslip angle (β).

Results of the aileron, spoiler, and rudder 3-2-1-1 POM cases are shown in Fig. 5.38 through Fig. 5.40. Initial offsets occurred because the model equations did not trim the aircraft to precisely the same orientation as was recorded during the flight tests, since the model is not exact. The thrust and spoilers were driven to exactly match the flight test values. The remaining control surfaces were driven to match the movement that occurred during the flight tests, with the initial trim offset maintained throughout the manoeuvres. The model results matched those of the flight test data very well.

Chapter 6

Conclusions and Recommendations

Using only geometrical considerations and SCD values produced by a MLE process, an acceptable simulator model (to FAA level D standards) of the Dash 8 Series 300 aircraft has been produced. The method is applicable to any type of aircraft, providing the SCD values and the aircraft geometry are known.

Part of the original motivation for this project was to determine which, if any, of the MLE-derived SCD values for the Dash 8 Series 300 were inconsistent with a geometry-based force and moment model. It has been shown that the SCD values produced by a MLE process for C_{L_q} , C_{Y_p} , and C_{Y_r} did not correspond to values predicted by the geometry-based model. Furthermore, values for $C_{Y_{\delta a}}$, $C_{Y_{\delta a_0}}$, and $C_{Y_{\delta a_1}}$ were not needed, as they were not included in the final simulator model, and did not appreciably affect the results. Also, several other terms, most significantly the wing-body contributions to C_{Y_p} and C_{Y_r} , were neglected in the final model, but again, this did not appreciably affect the results.

At first, the curve fits of the geometry-based model parameters did not seem to model the parameters very well at all, since the plots in chapters 4 and 5 were so

scattered. Also, the agreement between the geometry-based model predictions of the SCDs to the MLE-derived SCD values often seemed poor. However, the agreement between the trim point and non-trim point total forces and moments was relatively good for values that were not close to zero, and the final simulator results were within tolerance limits set by the FAA. Thus, the disagreement in total forces and moments when close to zero does not affect the final results significantly.

None of the results discussed above can verify that the model equations are correct. They only indicate that the curve fits and modelling are good enough to produce acceptable final simulator results. The ESDU, Datcom, and other sources' predictions of the various model parameters do, however, demonstrate the accuracy of the geometry-based model. Since almost all of the model parameters agree reasonably well with their predicted values, the geometry-based equations are, indeed, correct.

If this geometry-based method is to be used in the future as the basis for a force and moment flight simulator model when values of the SCDs are not known, or even as a method for predicting aircraft SCD values, the complete lift distribution across the wing of the aircraft in question must be known. Without it, many of the SCD values (or terms in the force and moment equations) cannot be predicted, notably C_{l_p} and the rolling and yawing moments due to aileron and spoiler deflections. It is recommended that some method of determining the lift distribution, whether it be wind tunnel or in-flight measurements, or a numerical method, be used. A possible improvement to the geometry-based simulator model would be to find some way of modelling the aircraft lift distribution based on geometrical considerations.

It is further recommended that efforts be made to measure any model parameters (e.g. downwash, sidewash, or neutral points), either in a wind tunnel or during a flight test. This would decrease the number of model parameters that had to be empirically-determined in order to obtain a final force and moment model. Ideally, enough information would be known so that no parameters needed to be empirically-determined. The resulting model would then be truly geometry-based.

References

- [1] Rolfe, J. M., and Staples, K. J., *Flight Simulation*, Cambridge Aerospace Series, Cambridge University Press, 1986.
- [2] Etkin, B., *Dynamics of Flight; Stability and Control*, 2nd ed., John Wiley & Sons Inc., 1982.
- [3] Hui, K., Baillie, S. W., and deLecuw, J. H., *Application of Parameter Estimation to Identify the Aerodynamic Derivatives of the Dash 8 Series 100*, CASI Flight Test Symposium, Ottawa, Ontario, May 1992.
- [4] Klein, V., *On the Adequate Model for Aircraft Parameter Estimation*, CIT, Cranfield Report Aero No. 28, March 1975.
- [5] Klein, V., *Identification Evaluation Methods*, AGARD-LS-104, Paper No. 2, November 1979.
- [6] Wünnenberg, H., Friedrich, H., von Meier, U., and Munser, H.-J., *Determination of Stability Derivatives from Flight Test Results - Comparison of Five Analytical Techniques*, AGARD-CP-172, Paper No. 10, May 1975.
- [7] Chaquin, J.-P., *Five Identification Methods Applied to Flight Test Data*, AGARD-CP-172, Paper No. 11, May 1975.
- [8] Sorenson, H. W., *Parameter Estimation; Principles and Problems*, Marcel Dekker Inc., 1980.
- [9] Hamel, P. G., *Aircraft Parameter Identification Methods and their Applications - Survey and Future Aspects*, AGARD-LS-104, Paper No. 1, November 1979.
- [10] Iliff, K. W., and Maine, R. E., *More Than You May Want to Know About Maximum Likelihood Estimation*, NASA-TM-85905, January 1985.
- [11] *Kinetic Pressure in the Wake Behind a Wing*, ESDU Wings 05.01.01, Engineering Sciences Data Unit International Ltd., 1990.
- [12] *Lift-Curve Slope and Aerodynamic Centre Position of Wings in Inviscid Subsonic Flow*, ESDU 70011, Engineering Sciences Data Unit International Ltd., 1990.

- [13] *Average Downwash at the Tailplane at Low Angles of Attack and Subsonic Speeds*, ESDU 80020, Engineering Sciences Data Unit International Ltd., 1990.
- [14] *Aerodynamic Centre of Wing-Fuselage Combinations*, ESDU 76015, Engineering Sciences Data Unit International Ltd., 1990.
- [15] *Contribution of Fin to Sideforce, Yawing Moment and Rolling Moment Derivatives Due to Sideslip. $(Y_{\dot{\beta}})_F$, $(N_{\dot{\beta}})_F$, $(L_{\dot{\beta}})_F$ in the Presence of Body, Wing and Tailplane*, ESDU 82010, Engineering Sciences Data Unit International Ltd., 1990.
- [16] Hoak, D. E., and Ellison, D. E., *USAF Stability and Control Datcom*, October 1960. (Rev. January 1975.)
- [17] Smetana, F. O., *Computer Assisted Analysis of Aircraft Performance Stability and Control*, McGraw-Hill Inc., 1984.
- [18] Baarspul, M., and Mulder, J. A., *The Composition of Flight Simulation Models: Flight Testing versus Datcom Techniques*, AIAA-90-3120-CP, 1990.
- [19] Goodman, A., and Fisher, L. R., *Investigation at Low Speeds of the Effect of Aspect Ratio and Sweep on Rolling Stability Derivatives of Untapered Wings*, NACA TR-968, 1950.
- [20] Goodman, A., and Brewer, J. D., *Investigation at Low Speeds of the Effect of Aspect Ratio and Sweep on Static and Yawing Stability Derivatives of Untapered Wings*, NACA TN-1669, 1943.
- [21] Campbell, J. P., and McKinney, M. O., *Summary of Methods for Calculating Dynamic Lateral Stability and Response and for Estimating Lateral Stability Derivatives*, NACA TR-1098, 1952.
- [22] Perkins, C. D., and Hage, R. E., *Airplane Performance Stability and Control*, John Wiley & Sons, Inc., 1949.
- [23] Pearson, H. A., and Jones, R. T., *Theoretical Stability and Control Characteristics of Wings with Various Amounts of Taper and Twist*, NACA TR-635, 1938.
- [24] Srinivasan, R., Hui, K., and Baillie, S., *Flight Testing and Simulator Model Development for the deHavilland Dash 8 Aircraft*, CASI Journal, Vol. 39, No. 4, December 1993.
- [25] *Dash 8 Series 300 Operating Data*, PSM 1-83-1, Section 5, Boeing Canada, deHavilland Division, May 1994.

- [26] *DHC-8-100 Flight Characteristics Manual*, Boeing Canada, deHavilland Division, October 1990.
- [27] *Dash 8 Series 100 Maintenance Manual*, Sections 6-10-00 and 6-20-00, Boeing Canada, deHavilland Division, 1993.
- [28] *Jane's All the World's Aircraft*, 1993-94 ed, Jane's Information Group Limited, Sentinel House, Surrey, UK, 1993.
- [29] Baillie, S., Hui, K., and deLeeuw, Dr. J. H., *The Flight Test and Data Analysis Program for the Development of a Boeing/deHavilland Dash 8 Simulator Model*, AGARD-CP-519, Paper No. 30, October 1992.
- [30] Dommasch, D. O., Sherby, S. S., and Connolly, T. F., *Airplane Aerodynamics*, Pitman Publishing Corporation, 1967.
- [31] Akaike, H., *Use of an Information Theoretic Quantity for Statistical Model Identification*, Proc. 5th Hawaii International Conference on System Science, pp. 249-250, 1972.
- [32] Silverstein, A., Katzoff, S., and Bullivant, W. K., *Downwash and Wake Behind Plain and Flapped Airfoils*, NACA TR-651, 1939.
- [33] *FAA Advisory Circular, Airplane Simulator Qualification*, AC 120-40B, July, 1991.
- [34] Toll, T. A., and Queijo, M. J., *Approximate Relations and Charts for Low-Speed Stability Derivatives of Swept Wings*, NACA TN-1581, 1948.

Tab. 3.1: Geometric Parameters**Longitudinal**

Parameter	Description	Value	Units	Reference
S_t	horizontal tail reference area	150	ft ²	[26]
S	wing reference area	605	ft ²	[28]
AR	wing aspect ratio	13.39	none	[28]
$\bar{c}$	wing mean geometric chord	80.64	in	derived from [25], [28]
c_r	wing root chord	8.33	ft	[25]
c_t	wing tip chord	3.92	ft	[25]
λ	wing taper ratio (tip chord over root chord)	0.47	none	derived from [25]
h	horizontal position of aircraft CG (CG_x when given in inches)	variable	% of $\bar{c}$	flight test data
l_t	horizontal distance from CG_x to horizontal tail aerodynamic centre	$909.56 - CG_x$	in	[26]
l_t^*	horizontal distance from horizontal tail quarter-chord to wing quarter-chord	42.5	ft	derived from [25]

Tab. 3.1 (continued): Geometric Parameters**Lateral**

Parameter	Description	Value	Units	Reference
S_F	vertical tail reference area	152	ft ²	[26]
b	wing span	90	ft	[25]
γ	geometric wing dihedral	2.5	deg	[27]
CG_z	vertical position of aircraft CG	variable	in	flight test data
z_F	vertical distance from CG_z to vertical tail aerodynamic centre	$300.0 - CG_z$	in	derived from [27]
l_F	horizontal distance from CG_x to vertical tail aerodynamic centre	$845.12 - CG_x$	in	[26]
z_{vs}	vertical distance from CG_z to centroid of vertical strake	$192.68 - CG_z$	in	derived from [27]
l_{vs}	horizontal distance from CG_x to centroid of vertical strake	$676.94 - CG_x$	in	derived from [27]
S_{vs}	vertical strake reference area	34.83	ft ²	derived from [27]
$c(y)$	wing chord as a function of y position	8.33 ($0 \leq y \leq 12.52$) $10.105 - 0.137y$ ($12.42 \leq y \leq 45$)	ft	derived from [25]
k_1	inner edge of aileron	0.75	% of b/2	derived from [27]
k_2	outer edge of aileron	1.00	% of b/2	derived from [27]
k_3	inner edge of outer spoiler	0.62	% of b/2	derived from [27]
k_4	outer edge of outer spoiler	0.74	% of b/2	derived from [27]
k_5	inner edge of inner spoiler	0.49	% of b/2	derived from [27]
k_6	outer edge of inner spoiler	0.61	% of b/2	derived from [27]

Tab. 4.1: Non-trim C_L and C_m Values for Flight H19207DE (Flaps 0°)

Parameter	Units	t = 15.062 sec	t = 18.062 sec	t = 21.062 sec	Origin of Data
T	lbs	2908.52490	2931.52515	2952.27808	tabulated
δe	deg	-1.59503	-1.60937	1.93977	tabulated
α_m	deg	4.32386	0.97329	1.72411	tabulated
q	deg/sec	3.26638	4.05962	0.21552	tabulated
M	none	0.33500	0.33024	0.32566	tabulated
$C_{thrusts}$	none	0.03869	0.04028	0.04179	derived
pressure altitude	ft	7939.125	8036.125	8088.125	read from graph
density	lbs/ft ³	0.06025	0.06007	0.05997	derived
aircraft weight	lbs	37370.0	37370.0	37370.0	tabulated
speed of sound	ft/sec	1087.4626	1087.0739	1086.8752	derived
I_y	slugs*ft ²	276516.66	276516.66	276516.66	tabulated
$\dot{q}$	rad/sec ²	-0.0506	0.1257	-0.0745	read from graph
x-acceleration	ft/sec ²	3.19842	0.65699	1.07912	tabulated
$\ddot{q}$	lbs/ft ²	124.2626	120.3095	116.7581	derived
normal load factor	G's	1.70267	0.78926	1.01925	tabulated
Flight Test C_L	none	0.8448	0.4047	0.5382	equation 2.183
Model C_L	none	0.8373	0.4674	0.5698	equation 2.21
Flight Test C_m	none	-0.0277	0.0711	-0.0434	equation 2.184
Model C_m	none	-0.0490	0.0733	-0.0737	equation 2.44

Tab. 5.1: Non-trim C_y , C_l , and C_n Values for Flight H05306DA (Flaps 0°)

Parameter	Units	t = 7.0 sec	t = 9.0 sec	t = 11.0 sec	Origin of Data
T	lbs	3054.06787	3064.73145	3074.36621	tabulated
δe	deg	1.24589	1.34684	1.47385	tabulated
α_m	deg	0.4716	0.3724	0.4361	simulator model
q	deg/sec	-0.01031	-0.06875	-0.21142	read from graph
M	none	0.36612	0.36552	0.36516	tabulated
$C_{thrusts}$	none	0.037336	0.037598	0.037798	derived
pressure altitude	ft	10239.77246	10244.14746	10249.92090	tabulated
density	lbs/ft ³	0.05607	0.05606	0.05605	derived
aircraft weight	lbs	32811.91	32811.91	32811.91	tabulated
speed of sound	ft/sec	1075.91	1075.89	1075.87	derived
I_x	slugs*ft ²	267335.31	267335.31	267335.31	tabulated
I_y	slugs*ft ²	181505.31	181505.31	181505.31	tabulated
I_z	slugs*ft ²	421920.84	421920.84	421920.84	tabulated
I_{xz}	slugs*ft ²	26495.68	26495.68	26495.68	tabulated
p	deg/sec	-7.12481	10.10020	2.18217	tabulated
r	deg/sec	-0.00549	-0.83612	1.34160	tabulated
$\bar{q}$	lbs/ft ²	135.20548	134.73365	134.43940	derived
β	deg	0.20066	-0.82032	1.05693	tabulated
$\dot{r}$	rad/sec ²	0.00601	-0.03879	0.05171	read from graph
$\dot{p}$	rad/sec ²	-0.00317	0.10908	0.63467	read from graph
y-acceleration	ft/sec ²	-0.5	0.8	-1.5	tabulated

Tab. 5.1 (continued): Non-trim C_Y , C_l , and C_n Values for Flight H05306DA (Flaps 0°)

Parameter	Units	t = 7.0 sec	t = 9.0 sec	t = 11.0 sec	Origin of Data
$\delta\alpha$	deg	3.611940	-6.083090	-7.840641	tabulated
δR	cm	-1.39089	-1.26113	-1.26175	tabulated
δs_o	deg	0.15646	0.16685	0.18105	tabulated
δs_i	deg	-0.29995	-0.29997	-0.29997	tabulated
CG_x	in	399.72	399.72	399.72	tabulated
CG_z	in	157.92	157.92	157.92	tabulated
Flight Test C_Y	none	-0.006234	0.009974	-0.018701	equation 2.185
Model C_Y	none	-0.008186	0.006249	-0.026139	equation 2.92
Flight Test C_l	none	-0.000100	0.002840	0.015548	equation 2.186
Model C_l	none	-0.000260	0.002264	0.012606	equation 2.123
Flight Test C_n	none	0.000356	-0.002627	0.000681	equation 2.187
Model C_n	none	0.003506	0.000836	0.005498	equation 2.168

Fig. 1.1: The 3-2-1-1 Control Input
(taken from reference [24])

Fig. 2.1: Axes and Notation
(adapted from reference [2])

Fig. 2.2: Relevant Angles at the Wing and Horizontal Tail

Fig. 2.3: Reference Lengths
(adapted from reference [2])

Fig. 2.4: Moment About the CG in the Plane of Symmetry
(adapted from reference [2])

Fig. 2.5: Sideslip Angle
(adapted from reference [2])

Fig. 2.6: Relevant Angles at the Fin

Fig. 2.7: Angle of Attack Changes Due to Roll Rate
(taken from reference [2])

Fig. 2.8: Angle of Attack Changes Due to Yaw Rate
(adapted from reference [2])

Fig. 2.9: Dihedral Effect
(taken from reference [2])

Fig. 2.10: Lift Distribution with Deflected Ailerons
(taken from reference [22])

Fig. 2.11: Strip Integration Over Wing
(taken from reference [22])

Fig. 2.12: Lift Distribution Due to Roll Rate
(adapted from reference [22])

Fig. 2.13: Inclination of Lift Vector Due to Roll Rate
(adapted from reference [2])

Fig. 2.14: Transformation of Body Axes to Stability Axes

Fig. 3.1: The Dash 8 Series 300 Test Aircraft, C-FKCU
(taken from reference [24])

Fig. 3.2: Dash 8 Series 300 Principal Dimensions
(taken from reference [25])

Fig. 3.3: Definition of Rudder Measurement

Fig. 3.4: Flight Control Surfaces Location
(taken from reference [25])

Fig. 4.1: Flaps 0° Variation of Horizontal Tail Parameters with Thrust (Both C_{Lq} and C_{mq} Data)

Fig. 4.2: Flaps 0° Variation of Horizontal Tail Parameters with Thrust (C_{mq} Data Only)

Fig. 4.3: Flaps 0° Variation of Elevator Effectiveness with Thrust

Fig. 4.4: Flaps 0° Variation of Aircraft Neutral Point with Thrust

Fig. 4.5: Flaps 0° Variation of Downwash with Thrust

Fig. 4.6: Flaps 0° Variation of Wing-Body Lift-Curve Slope with Thrust

Fig. 4.8: Flaps 0° C_{L_α} Comparison

Fig. 4.9: Flaps 0° C_{m_α} Comparison

Fig. 4.10: Flaps 0° $C_{L_{\delta e}}$ Comparison

Fig. 4.11: Flaps 0° $C_{m_{\delta e}}$ Comparison

Fig. 4.12: Flaps 0° CL_q Comparison

Fig. 4.13: Flaps 0° C_{m_q} Comparison

Fig. 4.14: Flaps 0° Trim C_L Comparison

Fig. 4.15: Flaps 0° Trim C_m Comparison

Fig. 4.17: Flaps 0° Aircraft Neutral Point Locations for Various Thrust Coefficients

Fig. 4.18: Estimate of Zero-Lift Angle of Attack

ATG PROOF OF MATCH Josh 8 - 300 FRL/NRC 10-NOV-94-16:14:02
PHUGOID DYNAMICS E0331151

**Fig. 4.19: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Phugoid Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 10-NOV-94-16:14:02
 PHUGOID DYNAMICS E0331151

**Fig. 4.19 (continued): Geometry-Based Simulator Model vs Flight Test Results
 (Flaps 0° Phugoid Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 10-NOV-94-16:22:39
ATG POM F132125H

**Fig. 4.20: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Short Period Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 10-NOV-94-16:22:39
 ATG POM F132125H

**Fig. 4.20 (continued): Geometry-Based Simulator Model vs Flight Test Results
 (Flaps 0° Short Period Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 10-NOV-94-16:32:50
ELEVATOR 3211 POM H192070E

Fig. 4.21: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Elevator 3-2-1-1 POM Case)
— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 10-NOV-94-16:32:50
ELEVATOR 3211 POM H19207DE

**Fig. 4.21 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Elevator 3-2-1-1 POM Case)**
— Flight Test - - - Model

Fig. 4.21 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Elevator 3-2-1-1 POM Case)
—— Flight Test - - - Model

Fig. 5.1: Flaps 0° Variation of Sidewash with Thrust

Fig. 5.2: Flaps 0° Variation of Fuselage "Side Force-Curve Slope" with Thrust

Fig. 5.3: Flaps 0° Variation of Wing-Body Side Force Neutral Point with Thrust

Fig. 5.4: Flaps 0° Variation of Wing Dihedral Neutral Points with Thrust

Fig. 5.5: Flaps 0° Variation of Rudder Effectiveness with Thrust

Fig. 5.6: Flaps 0° Variation of Aileron Roll Parameters with Thrust

Fig. 5.7: Flaps 0° Variation of Aileron Yaw Parameters with Thrust

Fig. 5.8: Flaps 0° Variation of Outer Spoiler Roll Parameters with Thrust

Fig. 5.9: Flaps 0° Variation of Outer Spoiler Yaw Parameters with Thrust

Fig. 5.10: Flaps 0° Variation of Inner Spoiler Roll Parameters with Thrust

Fig. 5.11: Flaps 0° Variation of Inner Spoiler Yaw Parameters with Thrust

Fig. 5.12: Flaps 0° Variation of Wing Roll Correction Factor with Thrust

Fig. 5.13: Flaps 0° Variation of Wing Roll Due to Yaw Parameter with Lift

Fig. 5.14: Flaps 0° Variation of Wing Yaw Due to Roll Parameter with Lift

Fig. 5.15: Flaps 0° Variation of Wing Yaw Due to Roll Parameter with Lift

Fig. 5.16: Flaps 0° $C_{Y\beta}$ Comparison

Fig. 5.17: Flaps 0° C_{Yp} Comparison

Fig. 5.18: Flaps 0° C_Y Comparison

Fig. 5.19: Flaps 0° C_{Y6R} Comparison

Fig. 5.20: Flaps 0° C_D Comparison

Fig. 5.21: Flaps 0° C_L Comparison

Fig. 5.22: Flaps 0° C_l Comparison

Fig. 5.23: Flaps 0° $C_{l_{SR}}$ Comparison

Fig. 5.24: Flaps 0° $C_{l_{\alpha}}$ Comparison

Fig. 5.25: Flaps 0° $C_{l_{\alpha_0}}$ Comparison

Fig. 5.26: Flaps 0° $C_{l_{\alpha}}$ Comparison

Fig. 5.27: Flaps 0° $C_{n_{\beta}}$ Comparison

Fig. 5.28: Flaps 0° C_{np} Comparison

Fig. 5.29: Flaps 0° C_n Comparison

Fig. 5.30: Flaps 0° $C_{n_{\delta R}}$ Comparison

Fig. 5.31: Flaps 0° $C_{n_{\delta 0}}$ Comparison

Fig. 5.32: Flaps 0° $C_{n_{\delta so}}$ Comparison

Fig. 5.33: Flaps 0° $C_{n_{\delta s1}}$ Comparison

Fig. 5.34: Flaps 0° Trim C_Y Comparison

Fig. 5.35: Flaps 0° Trim C_l Comparison

Fig. 5.36: Flaps 0° Trim C_n Comparison

**Fig. 5.37: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Dutch Roll Case)**

— Flight Test - - - Model

**Fig. 5.37 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Dutch Roll Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 9-NOV-94-16:13:42
 DUTCH ROLL 2d7 (6H) - FL 0 GU E036146H

**Fig. 5.37 (continued): Geometry-Based Simulator Model vs Flight Test Results
 (Flaps 0° Dutch Roll Case)**

— Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 22-NOV-94-18:03:29
 LATERAL 3211 POM H053060A

Fig. 5.38: Geometry-Based Simulator Model vs Flight Test Results
 (Flaps 0° Aileron 3-2-1-1 POM Case)
 — Flight Test - - - Model

**Fig. 5.38 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Aileron 3-2-1-1 POM Case)**

— Flight Test - - - Model

**Fig. 5.38 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Aileron 3-2-1-1 POM Case)**
—— Flight Test - - - Model

ATG PROOF OF MATCH Josh 8 - 300 FRL/NRC 22-NOV-94-18:11:05
LATERAL 3211 POM H0530610

Fig. 5.39: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Spoiler 3-2-1-1 POM Case)
— Flight Test - - - - Model

ATG PROOF OF MATCH Dash 9 - 300 FRL/NRC 22-NOV-94-18:11:05
 LATERAL 3211 POM H0530610

**Fig. 5.39 (continued): Geometry-Based Simulator Model vs Flight Test Results
 (Flaps 0° Spoiler 3-2-1-1 POM Case)**

— Flight Test - - - Model

Fig. 5.39 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Spoiler 3-2-1-1 POM Case)
—— Flight Test - - - - Model

ATG PROOF OF MATCH Josh 8 - 300 FRL/NRC 22-NOV-94-18:20:09
 LATERAL 3211 POM H05306DR

Fig. 5.40: Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Rudder 3-2-1-1 POM Case)
 — Flight Test - - - Model

ATG PROOF OF MATCH Dash 8 - 300 FRL/NRC 22-NOV-94-18:20:09
 LATERAL 3211 POM H05306DR

Fig. 3.40 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Rudder 3-2-1-1 POM Case)
 — Flight Test - - - Model

Fig. 5.40 (continued): Geometry-Based Simulator Model vs Flight Test Results
(Flaps 0° Rudder 3-2-1-1 POM Case)
— Flight Test - - - Model

END

3 1 - 0 5 - 9 5

FIN