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DYNAMIC RESPONSE OF CERTAIN TYPES OF HIGHWAY BRIDGES TO MOVING VEHICLES

by

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A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

November 1998

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The Doctor of Philosophy program in
Civil and Environmental Engineering
is a joint program with the University of Ottawa
administered by The Ottawa-Carleton Institute for Civil Engineering

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HIGHWAY BRIDGES TO MOVING VEHICLES**

submitted by
KAI DENG, B.S., M.S.

in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Dr. J.L. Humar, Thesis Supervisor and Chair
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Carleton University
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November 1998

Abstract

Vibrations of a bridge resulting from the passage of live loads across the span is an important consideration in design, because a vehicle moving across a bridge at a normal speed produces greater stresses than a vehicle that remains in a static position on the structure.

Several research studies have been published on the dynamic response of a single span beam model of the bridge traversed by a vehicle modeled as sprung and unsprung masses. Response of isotropic and orthotropic rectangular plate models under moving vehicle loads has also been studied by many researchers. The beam analytical models are able to represent the behaviour of some types of bridges but are not adequate for the study of the dynamic response of skew slab bridges and skew slab-on-girder bridges. A major component of the present study is the dynamic response analysis of these special types of bridges, on which only a limited research work exists.

In the present work, a skew slab bridge is represented by a finite element plate model and a skew slab-on-girder bridge is discretized by both plate elements and beam elements. The vehicle is represented by a single axle sprung mass. Analytical studies are carried out on the interaction between the bridge and the vehicle model. Governing parameters are identified and the effect of these parameters on the response is studied with a view to developing design recommendations.

As in the skew bridge, a limited number of published studies exist on the response of multispan continuous and cantilever bridges. Further work is carried out on such bridges in the present study. The bridge is modeled by a 2D beam or a beam frame structure, and the vehicle by a single axle sprung mass. Governing parameters are identified and systematic parametric studies are carried out.

Acknowledgements

I would like to express my gratitude to Dr. Jagmohan L. Humar, Professor and Chair, Department of Civil and Environmental Engineering, Carleton University, for his guidance and encouragement in the preparation and completion of this thesis.

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List of Symbols

A	the cross-section area of the beam
A_g	cross-section area of the girder
A_s	area of the slab, whose width is equal to the width of effective flange modified by the modulus ratio n
C	global damping matrix
$\hat{C}$	orthogonal damping matrix
C^*	damping matrix superimposed to the bridge equations of motion due to the vehicle inertia effect
D	flexural rigidity of the plate
$D, \dot{D}, \ddot{D}$	global nodal displacement, velocity, and acceleration vectors
D_x	longitudinal flexural rigidity per unit width of bridge deck
D_y	transverse flexural rigidity per unit length of bridge deck
D_{xy}	longitudinal torsional rigidity per unit width of bridge deck
D_{yx}	transverse torsional rigidity per unit length of bridge deck
E_g	modulus of elasticity of girder material
E_s	modulus of elasticity of slab material
EI	flexural rigidity of the beam
F	global nodal loading vector due to the interaction between the bridge and the vehicle
F_{MQ}	sum of the bending moments or shears in all girders at one section
F_{MQi}	maximum bending moment or shear in one girder at the section
G_g	shear modulus of girder material

G_s	shear modulus of slab material
$H(x)$	uneven road surface function of the approach
$\mathbf{I}$	identity matrix
I_b	moment of inertia of the beam element in the plate-beam model
I_b^*	moment of inertia of the composite longitudinal beam about its neutral axis
I_d	moment of inertia of diaphragm
I_g	moment of inertia of the girder about its own neutral axis
I_{mp}	impact factor
I_s	moment of inertia of the slab, whose width is equal to the width of effective flange modified by the modulus ratio n , about its own neutral axis
J_d	torsional inertia of the diaphragm
J_g	torsional inertia of the girder
$\mathbf{K}$	global stiffness matrix
K_b	element stiffness matrix
L	total length of the bridge
L_L	length of the longest span in the multispan bridge
$\mathbf{M}$	global mass matrix
$\mathbf{M}^*$	mass matrix augmented to the bridge equations of motion due to the vehicle inertia effect
M_1, M_2	principal moments of the bridge deck
M_n	bending moment per unit length of the section of the plate parallel to the n direction

M_p	$\max(M_1, M_2)$
M_x	longitudinal bending moment of the bridge
M_y	transverse bending moment of the bridge deck
M_{xy}	twisting moment of the bridge deck
N	a shape function row vector that relates γ to y
P_f	width of the effective flange
P_x	spacing between diaphragms
P_y	girder spacing
Q	modal matrix of size $N \times M$
Q_v	$\frac{1}{\sqrt{m_v}}$
R_d, R_s	maximum values of dynamic and static response, respectively
$T(t)$	harmonic function
T_1	fundamental period of the bridge
W	width of the bridge
W_v	weight of the vehicle
a	acceleration of the vehicle
$\frac{a}{b}$	skew aspect ratio
$\frac{a}{c}$	aspect ratio
c^*	damping matrix superimposed to the bridge equations of motion due to the vehicle inertia effect
c_h, c_v	time dependent matrices
c_v	damping constant in the vehicle model
e	distance between the neutral axis of the slab and the neutral axis of the girder

f	a time-dependent function representing the distribution of the load over the surface of the bridge
f_1	first natural frequency of the bridge
f_v	natural frequency of the vehicle in Hz
g	gravity constant
$h(x)$	function of pavement irregularities of the bridge
h_o, L_o	constants to describe the road roughness
$\mathbf{k}^*$	stiffness matrix augmented to the bridge equations of motion due to the vehicle inertia effect
$\mathbf{k}_h, \mathbf{k}_v$	time dependent matrices
k_v	stiffness of the vehicle spring
$\mathbf{m}^*$	mass matrix augmented to the bridge equations of motion due to the vehicle inertia effect
m_b	total mass of the bridge
m_t	unsprung mass of the vehicle
m_v	sprung mass of the vehicle
n	modulus ratio which compares the modulus of elasticity for the slab material with the modulus of elasticity for the girder material
$\mathbf{p}_1, \mathbf{p}_3$	forces vectors
t	thickness of the plate
$\bar{t}$	time variable
u_v	vertical displacement of the sprung mass relative to its position of equilibrium under self weight
$\tilde{u}_v$	transformed deflection of the vehicle spring

u_v^t	total vertical displacement of the sprung mass relative to the unformed position of the spring
v	speed of the vehicle
w	deflection of the bridge, positive upwards
$\bar{x}, \bar{y}$	oblique coordinates
x, y, z	rectangular coordinates
y	modal displacement coordinates
$\tilde{y}$	represents the M normal transformed coordinates
Λ	diagonal matrix of squares of frequencies of the bridge
α	speed parameter
δ	static deflection of the vehicle spring
ϕ	frequency parameter
γ	vertical displacement of the contact point between the vehicle and the bridge deck
η_o	wheel-load distribution factor
φ	mode shape function
$\varphi_i(x)$	shape function of the beam element
κ	mass ratio
λ	span ratio
ν	poisson's ratio
θ	skew angle
θ_x	rotation about x axis
θ_y	rotation about y axis
ρ	mass of the plate per unit area

σ_x	stress in x direction
σ_y	stress in y direction
τ_{xy}	shear stress
ω_1	first natural frequency of the bridge
ω_i	i th frequency of the bridge
ξ	percentage of critical damping
ξ_i	modal damping ratio of the bridge
ξ_v	vehicle damping ratio
ψ	parameter representing the location of the hinge
ζ, η	dimensionless oblique coordinates

Chapter 1

Introduction

1.1 Nature of the Problem

Vibrations of a bridge resulting from the passage of live loads across the span is an important consideration in design, because a vehicle moving across a bridge at a normal speed produces greater stresses than a vehicle that remains in a static position on the structure. The increase in stresses is often referred to the amplification of the dynamic effect. However, dynamic effect may also refer to the amplification of deflections, shears, or reactions. The term commonly used, among bridge designers and in bridge design specifications to signify dynamic effect, is impact.

The behaviour of highway bridges under the passage of heavy vehicles has been the subject of numerous investigations since the early 1950s. Dynamic response of a bridge to moving vehicles depends on the following factors:

1. dynamic characteristics of the vehicle, such as suspension stiffness and damping;
2. the speed, weight, and type of the vehicle;
3. bridge characteristics such as damping and natural frequencies of vibration;

4. surface roughness of the approach roadway and bridge deck;
5. traffic intensity;
6. braking and acceleration of the vehicle.

Among the aforementioned factors, the first four have the greater influence on the dynamic response of a bridge.

A number of research studies have been carried out in the past on single span right bridges. Governing parameters have been identified and design recommendations have been made. However, additional studies are needed on the dynamic characteristics of skew bridges and multispan bridges. These two types of bridge are studied in this work.

1.2 Review of Previous Analytical Studies

Solutions to the problem of bridge vibration produced by moving vehicles have been obtained by various researchers, using bridge and vehicle models of varying degrees of sophistication and considering various effects. Vehicles have been simulated by a constant force, an alternating force, point sprung or unsprung masses, and systems with distributed masses mounted on two or more axles. Bridges have been modeled as beams, as slab-girder bridges or as boxed girder bridges. A number of analyses and tests have been conducted. Previous studies relevant to the present research are reviewed in the following paragraphs.

1.2.1 Skew Bridges

A large number of bridge decks built today have some form of skew. Because of the increasing restriction on available space to serve the traffic flow, as well as due to the increasing speed of the vehicles, the alignment of a transport system can seldom be adjusted

for the purpose of reducing the skew or complexity of the bridge. In North America about 40% of the total bridge deck area is on skew alignment and about 10% of the total area is on heavy skews, ranging between 40° to 60° . In addition to introducing problems in the design details of a deck, skew has considerable effect on the deck's behaviour and critical design stresses. The special characteristics of skew of a slab deck can be summarized as follows:

1. variation in direction of maximum bending moment across width, from near parallel to span at edge, to near orthogonal to abutment in central regions;
2. hogging moments near obtuse corners;
3. considerable torsion of deck;
4. high reactions and shear forces near obtuse corners;
5. low reactions and possibly uplift at an acute corner.

The magnitude of these effects depends on the angle of skew, the ratio of width to span, and particularly on the type of construction of the deck and the supports.

The difficulty in the analysis and design of skew bridges has been overcome by extensive research and the use of digital computers. Many static analysis studies and experimental investigations on skew bridges have been carried out during the past two decades. In addition, the behaviour of isotropic and orthotropic plates of rectangular shape under moving vehicles has been studied by many researchers. A brief review of the literature on this subject can be found in Hutton and Cheung (1979), Gupta (1980), and Humar and Kashif (1995). However, there are only a few reports on the dynamic analysis of skew bridges.

Srinivasan and Munaswamy (1978) analyzed the dynamic response of a skew bridge deck by treating it as an orthotropic plate and using the finite strip method. The vehicle was represented as a moving force distributed over a small area. Response history curves and amplification spectra were drawn for three skew angles (i.e. 0° , 45° , and 60°). From their study, they found that when the force moves along an eccentric line, i.e. away from the central line of the bridge deck, the behaviour of the bridge deck becomes critical.

More recently, Wang et al. (1993) analyzed the vibration and impact characteristics of skewed steel bridges with skew angles ranging from 30° to 60° , under the passage of design vehicle loading. They included in their study a nonlinear vehicle model consisting of five rigid masses which represent the tractor, semi-trailer, steer-wheel/axle set, tractor wheel/axle set, and trailer wheel/axle set, respectively. The vehicle model was a 3D model with 12 degrees of freedom. The skewed multigirder bridge was treated as a grillage beam system. Dynamics response of the bridge was analyzed using the finite element method. The damping in the first and second modes of the bridge was set at one percent of critical. The power spectral density functions for highway surface roughness were used to produce four different classes of road surfaces (very good, good, average, and poor) for the purpose of analysis.

Wang et al. found that the first two frequencies of the bridge, which respectively corresponded with bending and torsion modes, were close to each other. In order to know the 3D impact characteristics, two loading cases, symmetric and asymmetric loadings produced by a single truck moving at a speed of 45 mph (72.41 km/hr), were considered. The lateral wheel-load distribution factors and impact factors for the bridges with skew angles ranging from 0° to 60° were computed. The wheel-load distribution factor was

defined as

$$\eta_o = \frac{F_{MQi}}{F_{MQt}}$$

in which $F_{MQt} = F_{MQ}/n$, F_{MQ} is the sum of the bending moments or shears in all girders at one section, n = number of wheel-loads in transverse direction, and F_{MQi} = maximum bending moment or shear in one girder at the section. The impact factor was defined as

$$I_{mp}(\%) = \left[\frac{R_d}{R_s} - 1 \right] * 100$$

in which R_d and R_s are the absolute maximum response values for dynamic and static studies, respectively. Wang et al. found that the impact factor for a girder in a steel multigirder bridge was closely related to the lateral loading position of the vehicle. The larger the static lateral distribution factor was, the smaller the impact factor would be. It was also found that the impact factors of girders along the transverse direction became more uniform with decreasing angle of skew.

In their report, based on 45° skew angle and asymmetric loading by two trucks, Wang et al. have provided the variation of the impact factors for exterior and internal girders, as related to the bending moment at midspan, with vehicle speed and road roughness. They have pointed out that the impact factors did vary with speed and road roughness. However under very good and good road surfaces, the variation of impact factors with speed was comparatively small. Very high impact would occur if the road surface was poor. The maximum impact factor for each girder increased with increasing angle of skew. The impact factor related to twisting moment near the ends of the supports having an obtuse skew angle could reach a high value, due to the effect of higher vibration modes.

1.2.2 Multispan Continuous and Cantilever Bridges

Continuous Bridges

Dynamic behaviour of continuous beams has been studied by Ayre et al. (1950) and Hoppmann (1950). They analyzed the natural modes and frequencies of vibration of continuous beams. Early field tests on actual continuous bridges were reported by Vandegrift (1944), Edgerton and Becroft (1958). Filho (1966) and Timoshenko and Woinowsky-Krieger (1974) analyzed the steady forced vibration of a continuous beam by the lumped mass method. Louw (1958) investigated the response of two-span highway bridges to the single-axle vehicle loading. Veletsos and Huang (1970) described the method of analysis which was developed in the University of Illinois studies based on the single-beam idealization of the structure. They treated the beam as multi-degree-of-freedom, linearly elastic system having distributed flexibility and concentrated point masses. The vehicle was represented as a three-axle sprung load unit, with due account taken of the effect of interleaf friction in its suspension system. The bridge damping was assumed to be of the absolute viscous type.

Hayashikawa and Watanabe (1981) treated the continuous beams as Bernoulli-Euler beams and determined their eigenvalues and eigenfunctions by using a general solution of a differential equation for the lateral vibration of the beams. Hayashikawa and Watanabe used a dimensionless parameter α to represent the speed of the vehicle. Parameter α ranged in value from 0.0 to 0.30 in their analysis for a three-span bridge. It was found that the dynamic amplification factor at the center of the second span was, for the range of α considered ($0.0 < \alpha < 0.3$), very complicated compared with that of the first span. Hayashikawa and Watanabe reasoned that this was because the vibration of the first span

affected the dynamic response of the second span. They also found that the dynamic amplification factors gradually decreased with increasing the logarithmic decrement of damping. Hayashikawa and Watanabe concluded that the dynamic amplification factors of simply-supported beams were, for similar values of the speed parameter, greater than those of two-span and three-span continuous beams.

Kennedy and Grace (1990) examined the dynamic response of two-equal-span continuous composite bridges with prestressed concrete deck slab over the negative moment region. In their dynamic analysis, they modeled the deck as an orthotropic plate. Kennedy and Grace compared two models of such bridges, one having a prestressed deck slab and the other with a reinforced deck slab. Bridge models (1) and (2) were tested under dynamic loads using:

1. The sweep or sine-wave test where the models were subjected to varying excitation frequencies over a range of 8–40 Hz, which included the first four models of vibration, namely, the first flexural mode, the first flexural-torsional mode, the second flexural-torsional mode. A Fast Fourier Transform program was used to obtain the vibration response charts from which the natural frequencies were determined.
2. The normal mode tests to determine the mode shapes corresponding to the frequencies found in the test.
3. The log-decrement or decay tests to determine the decaying free vibration response and thus the damping characteristics of the bridge models by first exciting the model to resonance and then turning off the hydraulic power to allow the model to undergo a decaying free vibration. The damping ratios were obtained by excluding the effect

of the hydraulic system.

Some codes of practice (Ontario Highway Bridge Design Code 1983) estimate the dynamic load allowance (DLA) for equal-span continuous bridges based on the first natural frequency of a single span. Therefore, Kennedy and Grace considered that when concrete deck slab was prestressed in the vicinity of the interior pier supports of a continuous composite bridge, its dynamic response could be estimated from a single-span composite bridge, since cracks were eliminated. The analytical results showed that the first and second mode shapes of the continuous composite bridge coincided with the first and second mode shapes of the single-span composite bridge with simply supported ends. Further, the third and fourth mode shapes of the former bridge coincided with the third and fourth mode shapes of the latter bridge, with one simple end and one fixed end. It was found that in all cases the ratios of the frequencies of the continuous and the single-span composite bridges were very close to unity. Kennedy and Grace concluded that this close correspondence between the dynamic response of the two types of bridges was attributed to the uniform rigidity along the entire span of the continuous composite bridge with a prestressed concrete deck, and such uniform rigidity could be readily achieved by prestressing a portion of the concrete deck in the vicinity of the interior pier supports.

Huang et al. (1992) analyzed the impact factors in six continuous multigirder bridges, with overall span lengths ranging from 130 ft to 260 ft (39.62 m to 70.25 m). These bridges were three-span steel bridges and were modeled as grillage beam systems. Analytical response was obtained for two trucks moving side by side and for varying transverse positions, vehicle speed, and road-surface roughness. It was found that the first and second modes dominated the response of middle span, while the impact factors in the side

span were principally affected by the third and fourth modes. On the other hand, the major effect on the response of the sections over the interior supports was due to the fifth and sixth modes, which was quite different from that of a simple beam, while somewhat like that of a fixed beam.

Huang et al. reported that impact factors for each girder of a continuous multigirder bridge were closely related to its wheel-load distribution factors. The larger the load carried by a girder, the smaller the impact factors would be. It was observed that the impact factors of sections over interior support were larger than the other sections, and the impact in the side span was generally larger than the impact in the middle span. Huang et al. thought that this was the results of influence of higher natural frequencies. The existence of damping decreased the response of the bridges, but the influence of damping ratio on the impact in individual components was different. The response of sections over interior supports was affected significantly by damping ratio, while the influence of damping ratio on sections within the spans was comparatively small.

Cantilever Bridges

Cantilever bridges are known to be very susceptible to vibrations caused by moving loads. The following is a brief review of the studies related to traffic induced vibration of cantilever bridges.

Oehler (1957) conducted field tests on several bridges and observed that bridges of the cantilever type showed relatively large amplitudes of vibration. Huang and Veletsos (1960) have reported an analytical study on the response of cantilever bridge under moving loads. Their study was limited to a single set of the ratios of the bridge spans. Wen and Toridis (1962) idealized the cantilever bridge as a lumped mass system with the entire mass of

the bridge concentrated at five points. They presented frequency values for the first three modes for different values of the aspect ratios. But, the number of masses considered was quite inadequate to expect accurate results for the higher modes.

Jagadish and Pahwa (1968) studied the natural frequencies of symmetrical double cantilever bridges and derived determinantal frequency equations for the symmetric and the antisymmetric modes of vibration. They compared the magnitudes of the various frequency parameters of some typical cantilever bridges to those of a simply-supported beam. It was seen that the frequencies of a double cantilever were quite close to each other and this dense distribution of frequencies was particularly evident when compared with the frequency spectrum for a simply-supported beam. It might be anticipated that the modes other than the fundamental would therefore contribute significantly to the response of the bridge under dynamic loads.

Veletsos and Huang (1970) carried out analytical studies on cantilever bridges idealized as single, linearly elastic beams with distributed flexibility and concentrated point masses. The vehicle was represented as a three-axle sprung load, with due account taken of the effect of interleaf friction in its suspension system. Veletsos and Huang found that cantilever bridges were the most susceptible to vibration, simple span bridges came next in order, and continuous bridges were the least susceptible. Even for a moving constant force, the cantilever structure was found to be more susceptible to vibration than the simple-span structure, but the increase in the response level was not as great in this case as for a sprung load. Their analysis also revealed that, in the absence of energy dissipation in the vehicle suspension system, the dynamic effects produced by an initially oscillating vehicle may be significantly greater than those induced by the same load smoothly moving across the span. Further, the interleaf friction in the vehicle suspension drastically changed the

detailed features of the response curves and significantly reduced the magnitudes of the peak effects. Therefore, Veletsos and Huang concluded that the effect of energy dissipation in the vehicle suspension should be considered in the analytical studies of the dynamics of highway bridges.

1.3 Design Codes Regarding Dynamic Response of Bridges

The term “dynamic load allowance” was used in 1983 edition of Ontario Highway Bridge Design Code to reflect the various sources of dynamic loading, including discrete and random irregularity of the loading surface, bridge static and vibratory deflections, and the dynamic effects of interaction between a moving vehicle and the bridge. The term “impact factor” was discarded as its literal interpretation is too narrow to be descriptive of the phenomenon of dynamic loading. The provisions were based on evidence obtained from field measurement and analysis, tempered with experience, and attempted to reflect the physical process of vehicle-bridge interaction. The relevant clauses are listed here

1. For a single axle or single wheel load, with no other highway live load considered in the design lane, the dynamic load allowance shall be 0.40.
2. For loading by any two or more axle loads of spans greater than 22 m in length, and spans continuous therewith, the dynamic load allowance shall be obtained from Fig. 2-4.3.2.4 of these provisions (*reproduced here as Fig. 1.1*). In the calculation of the first flexural frequency, the static values of material properties shall be used.
3. The dynamic load allowance shall be 0.30 for loading by other than a single wheel or single axle of all transverse members and spans, all simple spans not greater than 22 m in length, and all superstructures composed of continuous spans with no span

length greater than 22 m.

Central to those provisions was the use of the DLA/frequency relationship shown in Fig. 1.1 whose values were developed from tests (Csagoly et al. 1972, Wright and Green 1964, Green et al. 1984, Billing 1982, and Shepherd and Aves 1973).

In the current edition (1991) of OHBDC, the load model (OHBD truck) has been revised to reflect the latest vehicle survey data. This results in an increase in the weight of the design truck of about 6% from 700 KN to 740 KN. The dynamic load allowance has been refined and simplified by making it a function of the number of axles on the bridge rather than the natural frequency of the bridge, as shown in Table 1.1.

Table 1.1: Dynamic load allowance (OHBDC, 1991, clause 2-4.3.2.3)

Number of axles	DLA
1	0.40
2	0.30
3 or more	0.25

The Code recognizes that the dynamic load is caused by a combination of: (a) bumps in the riding surface or expansion joints which result in direct impact to the bridge deck; (b) dynamic variation in axle loads due to undulation and roughness in the riding surface; and (c) dynamic response of the main longitudinal bridge components to the moving vehicle loads. The effects of these factors are generally not independent, but the relative contribution from each may vary significantly depending upon the component loaded. The dynamic load allowance for a truck, or part thereof, is therefore specified according to the number of axles involved in generating the load effect.

In particular, the Code considered that for longer spans, for which the critical loading is due to three or more axles, a frequency match with a vehicle suspension is possible and the largest dynamic load effect is likely to involve dynamic interaction between the bridge structure and the vehicle. Tests (Csagoly et al. 1972, and Green et al. 1983) showed that there is increased dynamic response of the bridge superstructures having natural frequencies in the range of 2 to 5 Hz, a range typical of the bounce frequencies of vehicles. The increase is because of interaction between the vehicle and bridge, and has many of the characteristics of resonance in a simple oscillator.

1.4 Objectives and Scope of the Present Work

The isotropic and orthotropic plates and the slab-on-girder bridges of the rectangular shape under passage of the moving vehicles have been studied by many researchers. But only a few research studies have been carried out on the vibration of a skew bridge under the passage of a moving vehicle. The effect of the angle of skew must be investigated in detail. There are some published studies on dynamic response of multispan continuous and cantilever bridges, but they are not exhaustive. Field tests are always important in a study of the interaction of the bridge and the vehicle and the bridge and the pedestrian. Load models and bridge models need to be calibrated with field test results. Analytical studies have to be verified by field tests in order to draw reliable conclusions.

The objectives and scope of the current study are outlined below.

1. Study of dynamic response of skew bridges under the passage of a moving vehicle.

The parameters governing the dynamic response are identified, and design recommendations are developed.

2. Analysis of dynamic response of multispan continuous and cantilever bridges. An analytical case study on the dynamic response of Confederation bridge is also presented.

A general computer program has been developed for the analysis of skew slab bridges, skew slab-on-girder bridges, and framed beams for their dynamic response to a vehicle modeled as a mass supported by a spring and a dash-pot. The static response and free vibration response of bridges can also be obtained. The element library includes 3D beam element and thin plate and shell element. The program is designed so that it can easily include other element types. Gauss elimination method is used to obtain the solution of equilibrium equations. The determinant search or the subspace iteration techniques, depending on the size of the matrices, is used to solve the eigenvalue problem. The program is written in FORTRAN. The use of the standard overlay technique and external storage device (hard disk) makes the calculation of large structures possible. Portions of the program have been based on the source codes of the program SAP IV (Bathe et al. 1974) and those of Kashif (1992).

1.5 Layout of the Thesis

A general introduction and review of relevant literature are contained in the present Chapter. The provisions of two editions (1983, 1991) of Ontario Highway Bridge Design Code regarding the dynamic response of bridges are reviewed. The objectives and scopes of the current research are presented.

Chapter 2 deals with the vibration and impact characteristics of simply supported skew slab bridges, with skew angles ranging from 0° to 60° , under the passage of a moving vehicle. The bridge is simplified as an isotropic plate simply supported on the opposite

sides and free on the other edges. Plate bending elements with a quadrilateral shape are used for discretizing the deck. A vehicle is simulated by a single mass supported by a spring and a dashpot, moving with a constant velocity on the deck. Damping in both the bridge and the vehicle is neglected. A limited number of free vibration modes of the bridge are used for transforming the equation of the bridge deck in order to reduce the size of the problem in the finite element analysis. The resulting differential equations are solved by the average acceleration method. A number of bridges with practical geometries are analyzed and amplification factors are calculated.

Skew slab-on-girder bridges are analyzed by using the finite element technique and the result are presented in Chapter 3. A plate-beam model is developed, and the slab is discretized by plate elements and the girders by beam elements. Seven governing parameters, namely, speed parameter α , mass ratio κ , frequency ratio ϕ , skew angle θ , aspect ratio, girder spacing, and number of girders, are identified, and their effects on the bridge response are studied. Response of internal girders and external girders are studied, and different vehicle moving paths are also considered. A comparison is made among the responses of the isotropic plate model, the orthotropic plate model, and the plate-beam model. Finally, a series of design envelope curves are derived.

In Chapter 4, the equations of motion of bridge-vehicle system are presented. Multi-span continuous and cantilever bridges are idealized as 2D linearly elastic multiple span beams and the mass of the beams is lumped at a number of finite element nodes. The vehicle is represented by a single sprung mass with one d.o.f. moving along the beam. The parameters which govern the response of continuous and cantilever bridges are found, and their practical ranges are determined. The effect of these parameters on the bridge is studied. Responses of the bridges with single, two, and three spans are compared, and

comparison is also made among different cantilever bridges. In addition, a case study is presented on an analytical evaluation of the vibration response of the Confederation Bridge, Prince Edward Island, under the passage of a moving vehicle.

A summary and conclusions of the current research, and suggestions for future work are presented in Chapter 5.

Figure 1.1: DLA/Frequency relationship, (OHBDC, 1983, clause 2-4.3.2.4)

Chapter 2

Skew Slab Bridges

2.1 Introduction

The part of study presented in this chapter deals with the dynamic response of a skew slab bridge. The purpose of this study is to analyze the vibration and dynamic characteristics of simply supported skew bridges, with skew angles ranging from 0° to 60° , under moving vehicles. The bridge is idealized as an isotropic plate simply supported on the opposite sides and free on the other edges. A simple vehicle model, which consists of a single mass supported by a spring and a dashpot, the latter representing a viscous damping mechanism (Fig. 2.1), is used. It is recognized that a single-axle model is a highly idealized representation of the vehicle. A more complicated modeling, including 3D multiple axle model, can be developed and has been used by some researchers (Huang et al. 1992; Wang et al. 1992a, 1992b). The advantage of a single axle model is that it allows the determination of important parameters that govern the response, while at the same time capturing the essential dynamic characteristics of the vehicle. Kashif (1992) has shown that in comparison to a multiple axle model, a single axle model gives conservative results.

2.2 Governing Differential Equations

2.2.1 Forced Vibration

The differential equation of motion for the middle surface of an isotropic parallelogram plate under the action of a vehicle represented by a moving sprung load of mass m_v and spring constant k_v is expressed in Cartesian coordinates as follows (Humar and Kashif, 1995):

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\rho}{D} \frac{\partial^2 w}{\partial t^2} = -\frac{m_v}{D} (g + \ddot{u}_v^t) f(x - vt, y) \quad (2.1)$$

in which w = deflection of the bridge, positive upwards; $D = \frac{t^3 E}{12}$ flexural rigidity of the plate; t = the thickness of the plate; x and y = rectangular coordinates; ρ = mass of the plate per unit area; f = a time-dependent function representing the distribution of the load over the surface of the bridge; v = speed of vehicle; u_v^t = absolute deflection of the sprung mass, positive upwards.

When considering a skew plate, it may often be more convenient to use coordinates parallel to the edges of the plate, namely the oblique coordinates $\bar{x}$ and $\bar{y}$ shown in Fig. 2.2. This has proved to be an effective method in the static analysis of plates. It is believed that the use of oblique coordinates in the dynamic bridge problem would be equally effective. By the transformation

$$\bar{x} = x - y \tan \theta$$

$$\bar{y} = y \sec \theta$$

in which θ is the skew angle defined as the angle between the longitudinal centerline of a bridge and a line normal to the centerline of bearings, Eq. 2.1 becomes, in oblique coordinates

$$\frac{\partial^4 w}{\partial \bar{x}^4} + \frac{\partial^4 w}{\partial \bar{y}^4} - 4 \sin \theta \left(\frac{\partial^4 w}{\partial \bar{x}^3 \partial \bar{y}} + \frac{\partial^4 w}{\partial \bar{x} \partial \bar{y}^3} \right) + 2(1 + 2 \sin^2 \theta) \frac{\partial^4 w}{\partial \bar{x}^2 \partial \bar{y}^2} + \cos^4 \theta \frac{\rho}{D} \frac{\partial^2 w}{\partial t^2} = \cos^4 \theta \cdot F \quad (2.2)$$

where $F = -\frac{m_v}{D}(g + \ddot{u}_v^t)f(\bar{x}, \bar{y}, t, v)$.

Introducing

$$\bar{x} = a\zeta$$

and

$$\bar{y} = b\eta$$

in which ζ and η are the dimensionless oblique coordinates, and a and b are the corresponding half oblique dimensions of the plate (Fig. 2.2), yields

$$\begin{aligned} & \left(\frac{b}{a}\right)^2 \frac{\partial^4 w}{\partial \zeta^4} + \left(\frac{a}{b}\right)^2 \frac{\partial^4 w}{\partial \eta^4} - 4 \sin \theta \left[\left(\frac{b}{a}\right) \frac{\partial^4 w}{\partial \zeta^3 \partial \eta} + \left(\frac{a}{b}\right) \frac{\partial^4 w}{\partial \zeta \partial \eta^3} \right] + \\ & 2(1 + 2 \sin^2 \theta) \frac{\partial^4 w}{\partial \zeta^2 \partial \eta^2} + \cos^4 \theta \frac{a^2 b^2 \rho}{D} \frac{\partial^2 w}{\partial t^2} = \cos^4 \theta a^2 b^2 F \end{aligned} \quad (2.3)$$

in which $\frac{a}{b}$ = the skew aspect ratio.

Noting that $2c = 2b \cdot \cos \theta$ (Fig. 2.2) where $2c$ is the width of the plate and substituting $b = c \cdot \sec \theta$ into Eq. 2.3 gives

$$\begin{aligned} & \left(\frac{c}{a}\right)^2 \frac{\partial^4 w}{\partial \zeta^4} + \cos^4 \theta \left(\frac{a}{c}\right)^2 \frac{\partial^4 w}{\partial \eta^4} - 4 \sin \theta \cos \theta \left[\left(\frac{c}{a}\right) \frac{\partial^4 w}{\partial \zeta^3 \partial \eta} + \cos^2 \theta \left(\frac{a}{c}\right) \frac{\partial^4 w}{\partial \zeta \partial \eta^3} \right] + \\ & 2(\cos^2 \theta + 2 \sin^2 \theta \cos^2 \theta) \frac{\partial^4 w}{\partial \zeta^2 \partial \eta^2} + \cos^4 \theta \frac{a^2 c^2 \rho}{D} \frac{\partial^2 w}{\partial t^2} = \cos^4 \theta a^2 c^2 F \end{aligned} \quad (2.4)$$

in which $\frac{a}{c}$ = the aspect ratio defined as the ratio of the span to the width of the bridge.

2.2.2 Free Vibration

The differential equation governing the free vibration of isotropic skew plate can be obtained by setting the right-hand side in Eq. 2.4 to be equal to zero yielding

$$\left(\frac{c}{a}\right)^2 \frac{\partial^4 w}{\partial \zeta^4} + \cos^4 \theta \left(\frac{a}{c}\right)^2 \frac{\partial^4 w}{\partial \eta^4} - 4 \sin \theta \cos \theta \left[\left(\frac{c}{a}\right) \frac{\partial^4 w}{\partial \zeta^3 \partial \eta} + \cos^2 \theta \left(\frac{a}{c}\right) \frac{\partial^4 w}{\partial \zeta \partial \eta^3} \right] +$$

$$2(\cos^2 \theta + 2 \sin^2 \theta \cos^2 \theta) \frac{\partial^4 w}{\partial \zeta^2 \partial \eta^2} + \cos^4 \theta \frac{a^2 c^2 \rho}{D} \frac{\partial^2 w}{\partial t^2} = 0 \quad (2.5)$$

The displacement function $w(\zeta, \eta, t)$ may be expressed as the product of two functions, one involving only the space coordinates ζ and η , called a mode shape function $\varphi(\zeta, \eta)$, and the other involving the variable time $T(t)$.

$$w(\zeta, \eta, t) = \varphi(\zeta, \eta) \cdot T(t) \quad (2.6)$$

It may be assumed that $T(t)$ varies sinusoidally with time. Substituting Eq. 2.6 in Eq. 2.5 and denoting the frequency of sinusoidal oscillations by ω , Eq. 2.5 can be expressed as

$$\begin{aligned} \left(\frac{c}{a}\right)^2 \frac{\partial^4 \varphi}{\partial \zeta^4} + \cos^4 \theta \left(\frac{a}{c}\right)^2 \frac{\partial^4 \varphi}{\partial \eta^4} - 4 \sin \theta \cos \theta \left[\left(\frac{c}{a}\right) \frac{\partial^4 \varphi}{\partial \zeta^3 \partial \eta} + \cos^2 \theta \left(\frac{a}{c}\right) \frac{\partial^4 \varphi}{\partial \zeta \partial \eta^3} \right] + \\ 2(\cos^2 \theta + 2 \sin^2 \theta \cos^2 \theta) \frac{\partial^4 \varphi}{\partial \zeta^2 \partial \eta^2} - \cos^4 \theta \frac{a^2 c^2 \rho \omega^2}{D} \cdot \varphi = 0 \end{aligned} \quad (2.7)$$

The coefficient in front of each term in Eq. 2.7 is a function of $\frac{a}{c}$ and θ . As a result, the mode shapes of isotropic skew plate are controlled by the aspect ratio $\frac{a}{c}$ and the skew angle θ . Two isotropic plates with the same aspect ratio and skew angle have the same mode shapes as well as similar frequencies to within a scale factor. The advantage of using the aspect ratio $\frac{a}{c}$ instead of the skew aspect ratio $\frac{a}{b}$ is that the total mass of the bridge will remain constant when the aspect ratio is constant but the skew angle is varying.

For rectangular plates, the skew angle $\theta = 0^\circ$, and Eq. 2.7 becomes

$$\left(\frac{c}{a}\right)^2 \frac{\partial^4 \varphi}{\partial \zeta^4} + \left(\frac{a}{c}\right)^2 \frac{\partial^4 \varphi}{\partial \eta^4} - 2 \frac{\partial^4 \varphi}{\partial \zeta^2 \partial \eta^2} - \frac{a^2 c^2 \rho \omega^2}{D} \cdot \varphi = 0 \quad (2.8)$$

Equation 2.8 shows that the mode shapes of the isotropic rectangular plates are controlled only by the aspect ratio $\frac{a}{c}$.

2.3 Finite Element Analysis

2.3.1 Formulation of the Equations of Motion

The standard finite element method is used in the analysis of the bridge model. The plate is discretized by the plate bending elements of a parallelogram shape (Fig. 2.3a). The plate bending element is based on the triangular element derived by Hsieh, Clough and Tocher (Clough and Tocher, 1966). This element ensures full compatibility of slopes and therefore provides monotonic convergence to the exact solution. The element shown in Fig. 2.3b has four nodes located at mid-thickness of the deck. Each node has three degrees of freedom, which are transverse displacement along z axis and two rotations about x and y axes, namely, w , θ_x and θ_y . The element stiffness matrix K_b is therefore of size 12×12 . When compared with other quadrilaterals having only twelve displacement coordinates at exterior nodes, the plate bending element considered provides the most accurate results (Clough and Felippa, 1968). The vehicle is represented by a single degree of freedom oscillator with a spring and a damping device. The latter accounts for the dissipation of energy in the oscillator. It is assumed that the vehicle remains in contact with the road surface and the surface of the bridge in the position of the static equilibrium is horizontal. The surface is also assumed to be even.

The equations of motion of the bridge are

$$[M]\{\ddot{\mathbf{D}}\} + [C]\{\dot{\mathbf{D}}\} + [K]\{\mathbf{D}\} = \{\mathbf{F}\} \quad (2.9)$$

in which $[M]$ = global mass matrix; $[K]$ = global stiffness matrix; $[C]$ = global damping matrix; $\{\mathbf{D}\}$, $\{\dot{\mathbf{D}}\}$, $\{\ddot{\mathbf{D}}\}$ = global nodal displacement, velocity, and acceleration vectors; and $\{\mathbf{F}\}$ = global nodal loading vector due to the interaction between the bridge and vehicle.

When the vehicle is represented by the simple mass shown in Fig. 2.1, the interaction force vector is given by

$$\mathbf{F} = \mathbf{N}^T [-m_t \ddot{\gamma} + c_v (\dot{u}_v^t - \dot{\gamma}) + k_v (u_v^t - \gamma) - m_t g] \quad (2.10)$$

in which k_v = the stiffness of the vehicle spring, m_t = the unsprung mass of the vehicle, c_v = the damping constant in the vehicle model, $\gamma = \mathbf{ND}$ the vertical displacement of the contact point between the vehicle and the bridge deck, $\mathbf{N}$ = a shape function row vector that relates γ to $\mathbf{D}$, and u_v^t = the total displacement of the sprung mass relative to the undeformed position of the spring.

The equation of motion of the sprung mass is given by

$$m_v \ddot{u}_v^t + c_v (\dot{u}_v^t - \dot{\gamma}) + k_v (u_v^t - \gamma) = -m_v g \quad (2.11)$$

To simplify the problem, several assumptions are made:

1. the vehicle moves with a constant speed v ;
2. mass m_t is taken to be negligible;
3. damping in both the vehicle and the bridge is ignored;
4. road roughness is not considered.

With these assumptions and a normal coordinate transformation represented by $\mathbf{D} = \mathbf{Qy}$, Eqs. 2.11 and 2.9 become

$$\begin{bmatrix} m_v & 0 \\ 0 & \mathbf{M}^* \end{bmatrix} \begin{bmatrix} \ddot{u}_v^t \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} k_v & -k_v \mathbf{N} \mathbf{Q} \\ -k_v \mathbf{Q}^T \mathbf{N}^T & \mathbf{K}^* + k_v \mathbf{Q}^T \mathbf{N}^T \mathbf{N} \mathbf{Q} \end{bmatrix} \begin{bmatrix} u_v^t \\ \mathbf{y} \end{bmatrix} = -m_v g \begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} \quad (2.12)$$

in which $\mathbf{M}^* = \mathbf{Q}^T \mathbf{M} \mathbf{Q}$; and $\mathbf{K}^* = \mathbf{Q}^T \mathbf{K} \mathbf{Q}$; $\mathbf{Q}$ is the modal matrix of size $N \times M$; and $\mathbf{y}$ represents modal displacement coordinates corresponding to the M normal modes included in the transformation.

Equations 2.12 are solved by using the Newmark's average acceleration method, with a time step equal to one-tenth of the fifteenth modal period of the plate which is the highest mode included in the analysis. A computer program has been developed for the solution of Eqs. 2.12. The eigenvalue problem is solved by using the subspace iteration method, and the subroutine programs are adapted from SAP IV.

Referring to Fig. 2.2, the boundary conditions of a skew plate that is simply supported along two opposite edges and free along the other two edges are

$$\begin{aligned} w &= 0 \\ \frac{\partial w}{\partial m} &= 0 \\ M_n &= 0 \end{aligned} \quad \text{at } \bar{x} = \pm a \quad (2.13)$$

where M_n is the bending moment per unit length of the section of the plate about m axis or parallel to the n direction (Fig. 2.2). The first condition ($w = 0$) is easily satisfied for any skew angle by setting $w = 0$ at finite element nodes along the two simply supported edges. For a rectangular plate the second condition is satisfied by setting θ_x , that is the rotation about an axis perpendicular to the simply supported edge, to be zero at nodes along the two simply supported edges, and the third condition is satisfied by providing a free d.o.f. θ_y to each finite element node along the simply supported edges. To model the second and third boundary conditions for a plate with arbitrary skew angle, several combinations of boundary conditions, namely, (a) $w = 0$; (b) $w, \theta_x = 0$; (c) $w, \theta_y = 0$; and (d) $w, \theta_x, \theta_y = 0$, are examined. For a bridge with a length of 20 m, width 10 m, thickness 0.525 m, and skew angle 45° , static responses at midpoint of the bridge and frequencies obtained by applying boundary conditions (b) and (c) respectively are close to results from

boundary conditions (d) which actually represent clamped boundary condition. Therefore, (b), (c), and (d) are incapable of modeling the simply supported boundary conditions for skew plate. In the present study, $w = 0$ is adopted to model the simply supported edges. In the finite element model, this should provide a fair representation of the condition at the boundary. Besides, approximation in the representation of the boundary condition is not likely to have much influence on the response behaviour measured away from the boundary.

Analytical results obtained from a solution of Eqs. 2.12 are generally expressed in terms of a dynamic amplification factor. In the present study, the dynamic amplification factor is defined as

$$DAF = \left(\frac{R_d}{R_s} \right)$$

where R_d and R_s are the maximum response values at any specified point obtained from a dynamic and a static analysis respectively.

2.3.2 Identification of Controlling Parameters

As stated in Section 2.2.2, mode shapes of isotropic skew plate are controlled by the aspect ratio and the skew angle θ . Two isotropic plates with the same aspect ratio and skew angle will have the same mode shapes as well as similar frequencies to within a scale factor. Therefore, the frequencies of two skew plates, A and B, with identical aspect ratio and θ are related by

$$\frac{\omega_{iA}}{\omega_{iB}} = \frac{\omega_{jA}}{\omega_{jB}}; \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, N \quad (2.14)$$

where ω_{iA} = the i th frequency of plate A and ω_{iB} = the i th frequency of plate B. Further,

if the transformed mass matrix of plate A is expressed as

$$\mathbf{M}_A^* = m_A \bar{\mathbf{M}} \quad (2.15)$$

where m_A = mass of plate A and $\bar{\mathbf{M}}$ is a normalized mass matrix, then

$$\mathbf{M}_B^* = m_B \bar{\mathbf{M}} \quad (2.16)$$

In a similar manner, if

$$\mathbf{K}_A^* = m_A \omega_{iA}^2 \bar{\mathbf{K}} \quad (2.17)$$

then

$$\mathbf{K}_B^* = m_B \omega_{iB}^2 \bar{\mathbf{K}} \quad (2.18)$$

We next define the speed parameter α , frequency parameter ϕ , and mass ratio κ as follows

$$\begin{aligned} \alpha &= \frac{v}{2f_1 L} = \frac{\pi v}{\omega_1 L} \\ \phi &= \frac{\omega_v}{\omega_1} \\ \kappa &= \frac{m_v}{m_b} \end{aligned} \quad (2.19)$$

where m_b = total mass of the plate; ω_1 = first natural frequency of the plate; L = length of the plate; ω_v = vehicle heave frequency. In addition, we define the coordinate transformation

$$\tilde{u}_v = \frac{u_v^t}{\delta} = \frac{\omega_v^2}{g} u_v^t$$

and

$$\bar{\mathbf{y}} = \frac{\omega_1^2}{\kappa g} \mathbf{y}$$

where δ = the static deflection of vehicle spring. Applying the coordinate transformation and substituting Eqs. 2.14 to 2.19 in Eqs. 2.12 yields

$$\begin{bmatrix} 1 & 0 \\ 0 & \kappa\phi^4\bar{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \frac{d^2\tilde{u}_v}{dt^2} \\ \frac{d^2\tilde{y}}{dt^2} \end{bmatrix} + \begin{bmatrix} 1 & -\kappa\phi^2\mathbf{N}\mathbf{Q} \\ -\kappa\phi^2\mathbf{Q}^T\mathbf{N}^T & \kappa^2\phi^4\mathbf{Q}^T\mathbf{N}^T\mathbf{N}\mathbf{Q} + \kappa\phi^2\bar{\mathbf{K}} \end{bmatrix} \begin{bmatrix} \tilde{u}_v \\ \tilde{y} \end{bmatrix} = - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.20)$$

where $\tilde{t} = \omega_v t$.

Referring to Fig. 2.4, it is seen that the vector $\mathbf{N}$ is a function of the relative location of the vehicle on the plate or a function of vt/L and y/W (or $\bar{y}/\bar{W}$). Since $vt/L = \alpha\omega_v t/\pi\phi = \alpha\tilde{t}/\pi\phi$, $\mathbf{N}$ becomes a function of α , ϕ , $\tilde{t}$ and y/W . Also, the mode shapes $\mathbf{Q}$ depend on the skew angle θ and aspect ratio. Equations 2.20 thus indicate that if two plates subjected to a sprung mass, moving along a direction parallel to span at the same relative position defined by y/W , have the same values of the aspect ratio, θ , α , ϕ , and κ , their deflections are identical to within a scale factor. The same scale factor also applies to the static deflections. Hence, if the dynamic deflections of such plates are normalized with respect to any representative static deflection, the normalized values are exactly equal. For right bridges, the governing parameters reduce to the aspect ratio, α , ϕ , and κ .

2.3.3 Calculation of Bending Moments

For an isotropic plate model, the bending and twisting moments per unit length parallel to the x and y axes may be expressed, in the conventional Cartesian coordinates (Fig. 2.5), as follows (Timoshenko and Woinowsky-Krieger, 1959):

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad (2.21)$$

$$M_y = -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad (2.22)$$

$$M_{xy} = -M_{yx} = D(1 - \nu) \frac{\partial^2 w}{\partial x \partial y} \quad (2.23)$$

where ν is the poisson's ratio of the plate material.

We now define at a given point in the plate orthogonal coordinate axes m and n as shown in the Fig. 2.5. The bending moment per unit length in the direction of n at the given point is

$$M_n = \int_{-t/2}^{t/2} z \sigma_n dz \quad (2.24)$$

Using for the stress component σ_n

$$\sigma_n = \sigma_x \cos^2 \vartheta + \sigma_y \sin^2 \vartheta + 2\tau_{xy} \sin \vartheta \cos \vartheta \quad (2.25)$$

where ϑ is the angle between the axes x and n , we can represent Eq. 2.24 in the following form:

$$M_n = M_x \cos^2 \vartheta + M_y \sin^2 \vartheta - 2M_{xy} \sin \vartheta \cos \vartheta \quad (2.26)$$

Equation 2.26 enables us to calculate the bending moment in any direction if we know M_x , M_y , and M_{xy} . To get the principal moment in the plate and the corresponding direction, Eq. 2.26 is differentiated with respect to ϑ and the derivative set equal to zero, i.e.

$$\frac{dM_n}{d\vartheta} = -(M_x - M_y) \sin 2\vartheta - 2M_{xy} \cos 2\vartheta = 0$$

Hence we get the direction of the principal moment

$$\vartheta = \frac{1}{2} \tan^{-1} \left(\frac{-2M_{xy}}{M_x - M_y} \right) \quad (2.27)$$

Substituting Eq. 2.27 in Eq. 2.26 gives the principal moment.

The equations giving the principal moment may also be obtained by using the similarity between stress and moment. Thus, by replacing σ_x , σ_y , and τ_{xy} with M_x , M_y , and M_{xy} in the following known equation

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad (2.28)$$

we obtain the principal moments as follow

$$M_{1,2} = \frac{M_x + M_y}{2} \pm \sqrt{\left(\frac{M_x - M_y}{2}\right)^2 + M_{xy}^2} \quad (2.29)$$

The principal moment that we are interested in is the one which has larger absolute value

$$M_p = \max(|M_1|, |M_2|) \quad (2.30)$$

In terms of oblique dimensionless coordinates, the longitudinal moment in Eq. 2.21 becomes

$$M_x = -\frac{D}{a^2} \left[(1 + \nu \tan^2 \theta) \frac{\partial^2 w}{\partial \zeta^2} - 2\nu \tan \theta \left(\frac{a}{c}\right) \frac{\partial^2 w}{\partial \zeta \partial \eta} + \nu \left(\frac{a}{c}\right)^2 \frac{\partial^2 w}{\partial \eta^2} \right] \quad (2.31)$$

Since w is function of the aspect ratio (a/c) and the skew angle θ , M_x is also a function of these two parameters. Similarly, M_y , M_{xy} , $M_{1,2}$, and ϑ can be expressed in the terms of oblique dimensionless coordinates. They are also the function of parameters θ and (a/c) .

2.4 Parametric Study

2.4.1 Information relating to the Bridges Studied

Four sets of concrete skew slab bridges are designed in accordance with the Ontario Highway Bridge Design Code. All bridges have a uniform cross section and are simply supported on the opposite sides and free on the other sides. The properties of bridge models are listed in Table. 2.1.

The skew angle θ of each bridge model is varied from 0° to 60° at an increment of 5° . Thus, each set of bridge models includes thirteen bridges with different skew angles. Modulus of elasticity $E = 2.85 \times 10^7 \text{ kN/m}^2$, Poisson's ratio $\nu = 0.15$, and unit mass of the plate is 2.4465 tonne/m^3 .

2.4.2 Modal Analysis

Tables 2.2 to 2.5 list the first ten frequencies of the bridge for several values of skew angle θ and several values of aspect ratio. Figure 2.6 shows the variation of the first two frequencies of bridge models with the skew angle. It is found that for all aspect ratios the first and second frequencies of the bridge become close to each other as θ increases. The first frequency of the bridge increases with increasing θ for all aspect ratios, namely, 1.0, 1.5, 2.0, and 2.5. The second frequency also increases with θ for the small aspect ratio of 1.0 but is almost independent of θ for the larger aspect ratios, namely, 1.5, 2.0, and 2.5. Generally, it is expected that the contribution to the dynamic amplification from second vibration mode increases with growing value of θ .

The first two mode shapes of the bridges, which are simply supported on the opposite sides and free on the other edges, are shown in Figs. 2.7 and 2.8 in forms of contours for several values of skew angle and two aspect ratios, respectively, 1.0 and 2.0. The first mode of vibration of the bridge is pure bending mode when $\theta = 0$. As the value of θ increases, the first mode of vibration of the bridge includes a larger torsional component. For larger value of θ , modal amplitudes at the middle of two free edges in the first mode are much larger than the amplitude at the midpoint of the bridge, while for $\theta = 0^\circ$ the amplitudes at the middle of free edges are the same as the amplitude at the midpoint of the bridge. The second mode of vibration is a torsional mode for all skew angles from 0°

to 60° . From the results presented in Figs. 2.7, and 2.8, it can be expected that the first mode of vibration will make a greater contribution to the response at the middle of free edges of the skew bridge than to the response at the midpoint of the skew bridge.

The third to tenth vibration modes are drawn in Figs. 2.9 and 2.10 for bridge model with an aspect ratio = 2.0 and two values of θ , namely, 0° and 45° . Flexure modes and torsion modes can be clearly distinguished in Fig. 2.9 for the plate with $\theta = 0^\circ$ (i.e. rectangular plate). The vibration modes presented in Fig. 2.10 are coupled flexural-torsional modes, and none of them will make significant contribution to the response at the middle of the span. It is found that this is the case for all other skew bridge models. Based on the results presented in Figs. 2.7, 2.8, 2.9, and 2.10 one can expect that in a bridge with large skew angle the responses of the points near the center of the free edges will depend mainly on the first and second vibration modes.

2.4.3 Response of Bridges with Identical Values of Characterizing Parameters

As shown in the previous sections, isotropic bridge models with the same aspect ratios and skew angles and the same values of the parameters α , κ , and ϕ will have identical normalized response. It is of interest to verify this. Two 45° skew plates with identical values of aspect ratio, α , κ , and ϕ are analyzed for their responses to a vehicle moving along the centerline, as shown in Fig. 2.12. The material properties of the plates are given in Section 2.4.1. For plate X: $L_x = W_x = 10.0 \text{ m}$, $t_x = 0.325 \text{ m}$; and for plate Y: $L_y = W_y = 20.0 \text{ m}$, $t_y = 0.525 \text{ m}$. In order to keep the parameters α , κ , and ϕ identical for the two plates, the vehicle speed and vehicle properties m_v and k_v have to be changed appropriately. Figure 2.12 shows the amplification factors for deflection at the midpoints

of two plates. Analysis is carried out for $\kappa = 0.50$ and 0.90 , two different values of α , namely 0.15 and 0.30 and several values of ϕ . It is evident from the results presented in Fig. 2.12 that two plates with the same aspect ratios and skew angles and the same values of α , κ , and ϕ have identical dynamic amplification factors.

2.4.4 Effect of Skew Angle and Aspect Ratio on Response

Two right bridges ($\theta = 0^\circ$) of different aspect ratios but having identical values of α , κ , and ϕ will have approximately equal responses even though the frequency ratios are matched only for the fundamental frequencies of the bridges (Kashif, 1992). Since the response of right bridges is dominated by the lowest mode which is flexural in nature, this is to be expected. It is of interest to examine whether a similar behaviour may be expressed in skew bridges.

Bridges with four different aspect ratios, namely, 1.0 , 1.5 , 2.0 , and 2.5 , and different skew angles ranging from 0° to 60° are examined. Fig. 2.13a through 2.13d display the amplification factors for deflection at the midpoint of bridges as a function of θ for four speed parameters, namely, 0.1 , 0.15 , 0.2 , and 0.3 , and for $\kappa = 0.5$ and $\phi = 0.5$. The vehicle is moving along the centerline of the bridge. The dynamic amplification factor is defined as maximum dynamic response divided by maximum static response produced by the vehicle crawling across the bridge. Contrary to the generally held belief, dynamic amplification factors do not always increase with increasing value of θ . In the range $0^\circ \leq \theta \leq 30^\circ$, responses of bridges are almost independent of θ . Although the variation of amplification factors is a bit complicated for $\theta \geq 45^\circ$, the values of amplification factor are usually smaller than those for small skew angle θ . Generally, bridges with larger aspect ratios, namely, 1.5 , 2.0 , and 2.5 , have almost identical responses for $0^\circ \leq \theta \leq 30^\circ$. The response

of the bridge with aspect ratio = 1.0 is also close to those of bridges with larger aspect ratios, the maximum difference being of the order of 5%.

Figure 2.14 shows the maximum normalized deflections at three locations along skew central cross section due to off-center vehicles (i.e. vehicle moving at the centerline of a lane that is offset from the centerline of the bridge) for bridges with two aspect ratios, namely, 1.0 and 2.0, and for $\alpha = 0.15$, $\kappa = 0.5$ and $\phi = 0.5$. The maximum normalized deflections are plotted as a function of θ . The locations of two off-center vehicles and details of the skew central cross section are shown in Fig. 2.11. The dimensions shown in Fig. 2.11b are projected dimensions measured along an axis perpendicular to the longitudinal axis of the bridge, and do not therefore change with the skew angle. The maximum dynamic deflections are normalized with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle moving along the centerline of a lane. At location 2 (i.e. midpoint of the bridge), responses (or dynamic amplification factors) of two bridges with different aspect ratios are very close for all values of θ , and they are independent of the value of θ . This is to be expected based on modal analysis. Although the first vibration mode of the bridge has a larger torsional component for rising values of θ , while the second vibration mode is always a torsional mode for all values of θ , the response at the midpoint of the bridge is not affected much by torsional modes. On the other hand, points near the middle of free edges are expected to be affected by torsional modes. However, as seen in Fig. 2.14a, even at location 1 which is close to the free edge very small difference exists between the responses of two bridges with different aspect ratios and $\theta \leq 45^\circ$ when the vehicle is also off-center on the bridge, namely at #1. The difference gets larger when θ is greater than 45° . Similar phenomena can be seen for location 3 of the two bridges subjected to off-center vehicle #2. Deflections at

the locations farther away from the off-center vehicles (i.e. location 3 in Fig. 2.14a and location 1 in Fig. 2.14b) are generally smaller than that of midpoint, and are therefore not critical for the purpose of design.

The maximum normalized deflections for $\alpha = 0.3$ and for the same locations and values of aspect ratios, κ , and ϕ as those in Fig. 2.14 are displayed in Fig. 2.15. The results show the same features as those presented in Fig. 2.14.

Results presented in Figs. 2.13, 2.14 and 2.15 show that skew bridges with $\theta \leq 30^\circ$ would be expected to have a response that is similar to the rectangular bridges and bridges of different aspect ratios having identical values of θ , α , κ , and ϕ will have approximately equal responses at midpoint.

2.4.5 Effect of Controlling Parameters α , κ , and ϕ on Response

Bridge with Small Skew Angle

In the previous section, we found that bridges with identical α , κ , ϕ , and θ but different aspect ratios have almost similar responses when $0^\circ \leq \theta \leq 30^\circ$. The result was obtained for one value of ϕ , namely, 0.5. To verify whether this happens for all values of ϕ , two bridges with $\theta = 30^\circ$, width = 10 m, and different spans of 10 m and 20 m, respectively, are analyzed for a vehicle moving at the centerline of the bridge. Amplification factors for midpoint deflection of bridges are drawn in Fig. 2.16 as a function of ϕ for a value of $\kappa = 0.5$ and two values of α , namely, 0.15 and 0.3. The maximum difference between responses of two bridges is found to be of the order of 6%.

To further check the effect of aspect ratio on the response of skew bridges subjected to off-center vehicles, the two off-center vehicles shown in Fig. 2.11 are applied on the two bridges described in the proceeding paragraph. The maximum normalized deflections

at three locations along the skew central cross section are plotted as a function of θ in Figs. 2.17 and 2.18 for a value of $\kappa = 0.5$ and two values of α , namely, 0.15 and 0.3. Normalization is with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle moving along the centerline of a lane. In all cases, the normalized responses at location 2 which is the midpoint of the bridge, are similar for bridges with different aspect ratios, the maximum difference being of the order of 4%. At the location near the path of the off-center vehicle, bridges with different aspect ratios again have similar normalized responses. At the location farther away from the path of the off-center vehicle, the differences between normalized responses of two bridges are large, but these responses are smaller than that at the midpoint, and are not likely to govern the design.

Bridge with Large Skew Angle

The results presented in previous sections show that the behavior of bridges with large skew angles may be quite different from those of bridges with small skew angles. Further analytical studies are carried out to understand the behavior of such bridges.

A bridge with length = 10 m, width = 10 m, and $\theta = 45^\circ$ is analyzed for its response under the passage of a central vehicle. Figures 2.19a through 2.19d show the relationship between the dynamic amplification factor of midpoint deflection and the frequency ratio for different values of α and κ . Another bridge with length = 20 m, width = 10 m, and $\theta = 45^\circ$ is also analyzed for its response under the passage of a central vehicle. The results are presented in Figs. 2.20a through 2.20d. To cover a wide range of parameter values, κ has been varied from 0.1 to 1.5 in Figs. 2.19 and 2.20. However, in most practical cases, κ will be significantly smaller than 1.5.

Comparison of Figs. 2.19 and 2.20 shows that for both bridges variations of response with α , κ , and ϕ are very similar in nature. Although the difference between the responses of two bridges can be as large as 15% for certain values of α , κ , and ϕ , the change in aspect ratio does not produce much difference in the values of peaks in the corresponding curves of two bridges. Thus, for a given α and κ , the maximum amplification factors obtained by varying the value of ϕ for two bridge are quite close. This is an important feature, and will be considered in developing design recommendations.

Figure 2.21 displays the maximum normalized deflections at three locations along skew central cross section of these two skew bridges due to off-center vehicles for $\alpha = 0.15$ and $\kappa = 0.5$. As usual, normalization is with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle moving along the centerline of a lane. Responses at midpoints (location 2) of two bridges with different aspect ratios are found to be close. Fig. 2.22 shows responses of the same bridges for $\alpha = 0.3$ and $\kappa = 0.5$. Again, responses at midpoints (location 2) of two bridges with different aspect ratios are found to be close.

2.4.6 Comparison between Deflection and Moment Amplification Factors

Dynamic amplification factors for deflection, longitudinal moment M_x , and principal moment M_p at the midpoint of the bridge are shown as a function of θ in Figs. 2.23 and 2.24 for vehicle moving on three different paths and for two different values of α , namely, 0.15 and 0.3. The values of other parameters used in this analysis are: aspect ratio = 2.0, $\kappa = 0.5$, $\phi = 0.5$. Except for $\theta \geq 50^\circ$, the amplification factor for deflection is generally greater than the amplification factors for moments. The maximum difference between

amplification factor for deflection and amplification factor for M_p is of the order of 6%. The amplification factors for M_x and M_p are exactly the same when $\theta = 0^\circ$ and pretty close for $\theta \leq 30^\circ$. The difference becomes more noticeable for $\theta > 30^\circ$. The amplification factor for M_p is considered to include contributions of transverse moment M_y and twist moment M_{xy} . In the bridge with large skew angle, M_y and M_{xy} are much larger than those in right bridges.

A bridge with $\theta = 45^\circ$ and aspect ratio = 2 is analyzed next for its response for a number of different values of ϕ . Figures 2.25 and 2.26 show comparison of deflection and moment amplification factors for midpoint of the bridge as a function of ϕ for a vehicle moving on three different paths and for two different values of α , namely, 0.15 and 0.3. The values of $\kappa = 0.5, \phi = 0.5$ are used in the analysis. Again, the maximum difference between amplification factors for deflection and M_p is of the order of 6%.

2.4.7 Transverse Distribution of Dynamic Effects

Transverse distributions of maximum normalized deflections and maximum normalized M_x at two cross sections, the central cross section which is perpendicular to the centerline of the bridge and the skew central cross section which is parallel to the simply supported edges, are studied for several values of θ . Figure 2.27 shows the deflections of bridges due to a central vehicle for aspect ratio = 2, $\alpha = 0.15, \kappa = 0.5$, and $\phi = 0.5$. The deflections are normalized with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle moving along the centerline of the bridge. For bridges with the skew angle up to 45° , the distributions of deflections on both of two cross-sections are almost symmetrical. For the skew angle of $\theta = 60^\circ$ the deflections at the middle of free edges are significantly larger than that at the midpoint of the bridge.

Also the distribution of deflections along the skew central cross section is asymmetrical although the vehicle is traveling along the centerline of the bridge. The second vibration mode (torsional vibration mode) is considered to be responsible for this. It is clear from Fig. 2.6 that the second frequency is almost the same as the first frequency for all bridge models when $\theta = 60^\circ$. Therefore, the dominant vibration modes are first two modes for 60° skew bridges.

Figures 2.28 and 2.29 show the transverse distributions of maximum normalized deflections of bridges due to off-center vehicles. The deflections are normalized with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle crawling along the centerline of a lane. Comparison of Figs. 2.28 and 2.29 shows that for the same value of θ the transverse distributions of deflections produced by two different off-center vehicle are quite comparable. Again, for a skew angle of 60° the dynamic amplification for the free edge on the skew central plan is significantly larger than that at the center.

Figure 2.30 displays the transverse distributions of the maximum normalized M_x at the central cross section and the skew central cross section of bridges due to a central vehicle for several values of θ . The normalization is with respect to the maximum static M_x of the middle element due to the same vehicle model. Asymmetrical distributions of M_x are observed on both of cross sections for $\theta = 60^\circ$ although the vehicle is traveling along the centerline of the bridge. The distributions of M_x are nearly symmetrical on both of cross sections for other values of θ .

Figures 2.31 and 2.32 show the transverse distribution of maximum normalized moment M_x in bridges due to off-center vehicles. Comparison of Figs. 2.31 and 2.32 shows that the two different off-center vehicles produce transverse distributions of moments that are

quite comparable.

In summary it can be concluded that: the loading position significantly affects the transverse distribution of the dynamic responses; bridges with $\theta \leq 30^\circ$ have transverse distributions similar to that of right bridges.

2.4.8 Design Recommendation

Five parameters, namely, aspect ratio, skew angle θ , speed parameter α , mass ratio κ , and frequency ratio ϕ , control the response of isotropic slab bridge under the passage of a vehicle. In developing the design recommendation, all these parameters have to be considered. In addition, the different paths along which a vehicle may travel also influence the response of the bridge. However, for the response at the midpoint of the bridge, such influence is quite small. This can be proved by rearranging some results presented earlier. Figure 2.33 shows that the maximum difference of amplification factors at the midpoint of the bridge due to a vehicle moving along different paths is within 10%.

Responses of skew bridges with $\theta \leq 30^\circ$ are very close to those of right bridges. In the range $0^\circ \leq \theta \leq 30^\circ$, bridges with different aspect ratios have approximately the same response. Even for $\theta > 30^\circ$, the aspect ratio does not have a significant effect on the maximum dynamic amplification factor for a given α and κ and for a range of possible values of ϕ .

The speed parameter α may vary over a range of values depending on the speed of the vehicle. However, because the response, in general, increases with α , it is only the value of α corresponding to the maximum expected vehicle speed that is of interest. Like the speed parameter, the frequency ratio, ϕ , can vary over a range of values depending on the bounce frequency of the vehicle. The dynamic amplification does not vary systematically

with ϕ . This is clear from Figs. 2.19 and 2.20. Dynamic amplification should therefore be determined for the full range of possible values of ϕ .

Figure 2.34 shows the variation of amplification factor for midpoint deflection of the bridge with α for a number of different values of ϕ from 0 to 2.4 at a interval of 0.05 due to a central vehicle. The curves have been drawn for single values of θ and κ . Dynamic amplification is defined as the ratio of the maximum dynamic deflection at the midpoint of the bridge to the maximum static deflection at the midpoint of the bridge due to a central vehicle. The envelope to these curves gives the largest value of amplification factor for any given value of α . Curves such as the one in Fig. 2.34 can be used to select the design value of amplification factor for a given κ and θ , and similar curves can be generated for different θ and κ . In order to determine from such curves the dynamic amplification factor to be used in design, one needs to know the skew angle, α , and κ . The skew angle is of course known. For calculating α we need the expected maximum vehicle speed, the bridge span, and the fundamental frequency of the bridge. The expected maximum vehicle speed and the bridge span would be given. For calculating κ we need the vehicle and bridge masses. The mass of the design vehicle would be given. A preliminary design must be carried out to calculate the mass and fundamental frequency of the bridge.

It should be noted that the curves similar to Fig. 2.34 are related to deflection amplification. Their use for determining moment amplification leads to conservative results.

A few of the design envelope curves (obtained from bridges with aspect ratio = 2 due to a central vehicle) are shown in Figs. 2.35a through 2.35d. Response of a bridge with $\theta = 15^\circ$ is approximately the same as that of a right bridge. Response of a bridge with $\theta = 30^\circ$ is also close to that of a right bridge. Figure 2.36 presents a comparison of the envelope curves for five different skew angles, namely, 0° , 15° , 30° , 45° , and 60° . The value

of κ is 0.5. The difference in the amplification could be as much as 8%.

2.5 Summary

The finite element technique is used for the forced vibration analysis of skew slab bridges. Plate bending elements with a quadrilateral shape are used for discretizing the deck. A vehicle is simulated by a single mass supported by a spring and a dashpot, moving with a constant velocity on the deck. Damping in both the bridge and the vehicle is neglected. A limited number of free vibration modes of the bridge are used for transforming the equation of the bridge deck in order to reduce the size of the problem in the finite element analysis. The resulting differential equations are solved by the average acceleration method. A number of bridges with practical geometries are analyzed and amplification factors are calculated.

The following general conclusions can be drawn from the results presented in this chapter:

1. The skew angle θ , the aspect ratio, the mass ratio κ , the speed parameter α , and frequency ratio ϕ govern the response of skew bridges modeled by isotropic plates. Two bridges with the same values of θ , aspect ratio, α , κ , and ϕ will have identical responses. Two skew bridges with different aspect ratios will have approximately similar responses provided that they have the same values of θ , α , κ , and ϕ .
2. The first and second frequencies of the bridge become close to each other when the skew angle θ increases. The contribution to dynamic amplification from the second vibration mode is expected to increase with growing value of θ . The response of the point near the free edges of the skew bridge depends mainly on the first and second

vibration modes which are torsional in nature for larger skew angles.

3. In skew bridges, the amplification factor for deflection is greater than those for principal moment and longitudinal moment. The amplification factors for the principal and longitudinal moments are close to each other when $\theta \leq 30^\circ$.
4. For bridges with a large value of θ the transverse distribution of amplification factors is asymmetrical. The second vibration mode (torsional vibration mode) is considered to be responsible for this.
5. A series of design envelope curves are derived. The curves relate dynamic amplification factors to parameters θ , α , and κ . In using these curves a preliminary design must be carried out first to obtain the mass and fundamental of the bridge. α is obtained from the maximum expected travel speed of the vehicle, bridge length, and fundamental frequency of the bridge. κ is obtained from the mass of the vehicle and the mass of the bridge.
6. It is of interest to note that for $\theta \leq 30^\circ$, the design amplification factors are very close to those for the right bridge. In fact, the amplification in skew bridges is seen to be smaller than those in the right bridge. For larger skew angle, the design amplification factors may differ somewhat from that for the corresponding right bridge, but the difference is not large.

Table 2.1: Properties of skew bridges

Plate Type	A	B	C	D
W(m)	10.0	10.0	10.0	10.0
L(m)	10.0	15.0	20.0	25.0
t(m)	0.325	0.400	0.525	0.675
Mass(tonne)	79.511	146.790	256.883	412.847
Aspect ratio	1.0	1.5	2.0	2.5

Table 2.2: The lowest ten frequencies(Hz) of the plates $10.0 \times 10.0 \times 0.325(m)$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	5.061	5.378	6.504	9.144	14.491
2	8.813	8.822	9.072	10.324	14.520
3	19.499	18.335	16.569	15.966	22.172
4	20.308	21.660	25.391	24.477	26.911
5	24.812	26.564	30.428	33.502	33.372
6	37.558	33.614	30.655	39.808	40.453
7	38.998	41.539	43.538	45.060	49.014
8	45.787	48.012	44.148	45.576	60.810
9	50.486	53.131	58.675	56.789	74.856
10	58.271	53.783	59.375	58.472	76.940

Table 2.3: The lowest ten frequencies(Hz) of the plates $15.0 \times 10.0 \times 0.400(m)$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	2.763	2.926	3.498	4.839	7.815
2	6.562	6.384	6.122	6.277	7.966
3	11.088	11.949	13.267	11.979	13.588
4	16.179	14.996	14.629	18.200	18.414
5	18.921	19.963	21.735	23.371	23.550
6	24.994	26.508	23.125	24.330	30.927
7	30.284	26.777	31.518	30.459	40.377
8	30.538	33.933	36.340	36.738	41.352
9	42.968	41.340	37.084	41.696	43.329
10	44.496	43.384	44.386	47.705	49.635

Table 2.4: The lowest ten frequencies(Hz) of the plates $20.0 \times 10.0 \times 0.525(m)$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	2.037	2.152	2.549	3.466	5.657
2	6.216	5.856	5.348	5.136	5.969
3	8.175	8.983	11.101	11.104	11.447
4	14.237	13.450	12.174	14.922	15.895
5	18.429	19.807	20.879	19.140	21.769
6	22.222	22.659	21.760	23.403	29.428
7	25.318	23.578	25.499	26.796	31.363
8	31.498	32.208	32.588	35.562	32.160
9	32.805	35.572	33.568	37.157	40.693
10	40.082	36.705	41.153	38.406	44.420

Table 2.5: The lowest ten frequencies(Hz) of the plates $25.0 \times 10.0 \times 0.675(m)$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	1.675	1.767	2.078	2.786	4.567
2	6.235	5.591	4.969	4.600	5.028
3	6.722	7.691	9.492	10.537	10.595
4	13.716	12.654	11.446	12.800	14.903
5	15.164	16.815	19.635	17.706	22.376
6	23.419	21.870	19.987	24.060	24.192
7	26.363	26.450	26.660	25.451	29.378
8	27.017	29.116	30.751	32.914	31.439
9	34.456	33.700	32.283	35.441	41.395
10	35.946	35.118	37.777	35.998	43.638

Figure 2.1: Elevation of the bridge deck and vehicle model

Figure 2.2: Cartesian, oblique, and dimensionless oblique coordinates

(a) Plan of bridge deck showing the finite element discretization

(b) Plate bending element

Figure 2.3: Modeling of the bridge

Figure 2.4: Location of the vehicle on the bridge

Figure 2.5: Bending and twisting moments

Figure 2.6: Relationship between natural frequencies and skew angle

Figure 2.7: First two mode shapes of a plate of size 10×10 m with different skew angles

Figure 2.8: First two mode shapes of a plate of size 20×10 m with different skew angles

$f_3 = 8.175 \text{ Hz}$, LF 2

$f_4 = 14.237 \text{ Hz}$, TO 2

$f_5 = 18.429 \text{ Hz}$, LF 3

$f_6 = 22.222 \text{ Hz}$, LF-TO 1

$f_7 = 25.318 \text{ Hz}$, TO 3

$f_8 = 31.498 \text{ Hz}$, LF-TO 2

$f_9 = 32.805 \text{ Hz}$, LF 4

$f_{10} = 40.082 \text{ Hz}$, TO 4

TO -- Torsion LF -- Longitudinal Flexure

Figure 2.9: 3rd to 10th mode shapes of a rectangular plate of size $20 \times 10 \text{ m}$

Figure 2.10: 3rd to 10th mode shapes of a plate of size 20×10 m and skew angle $\theta = 45^\circ$

(a) Different paths of the vehicle

(b) Skew central cross section of the bridge (Projection on Y axis)

Figure 2.11: Position of the vehicle on the bridge

Figure 2.12: Amplification factors for midpoint deflection of two corresponding plate models

Figure 2.13: (a)(b) Relationship between skew angle and amplification factor for deflection at midpoint of the bridge due to a central vehicle ($\kappa = 0.5$, $\phi = 0.5$)

Figure 2.13: (c)(d) Relationship between skew angle and amplification factor for deflection at midpoint of the bridge due to a central vehicle ($\kappa = 0.5$, $\phi = 0.5$)

Figure 2.14: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.15: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.30$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.16: Comparison of amplification factors for midpoint deflection under a central vehicle for bridges with different aspect ratios but identical values of α , κ , ϕ , and θ ($\theta = 30^\circ$, $\kappa = 0.5$)

Figure 2.17: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.15$, $\kappa = 0.5$, $\theta = 30^\circ$)

Figure 2.18: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.30$, $\kappa = 0.5$, $\theta = 30^\circ$)

Figure 2.19: (a)(b) Amplification factor for midpoint deflection of bridges with aspect ratio = 1 and $\theta = 45^\circ$ due to a central vehicle

Figure 2.19: (c)(d) Amplification factor for midpoint deflection of bridges with aspect ratio = 1 and $\theta = 45^\circ$ due to a central vehicle

Figure 2.20: (a)(b) Amplification factor for midpoint deflection of bridges with aspect ratio = 2 and $\theta = 45^\circ$ due to a central vehicle

Figure 2.20: (c)(d) Amplification factor for midpoint deflection of bridges with aspect ratio = 2 and $\theta = 45^\circ$ due to a central vehicle

Figure 2.21: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.15$, $\kappa = 0.5$, $\theta = 45^\circ$)

Figure 2.22: Comparison of normalized deflections along skew central cross section due to off-center vehicles for bridges with different aspect ratios ($\alpha = 0.30$, $\kappa = 0.5$, $\theta = 45^\circ$)

Figure 2.23: Comparison of deflection and moment amplification factors for midpoint of the bridge for a vehicle moving on different paths. (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.24: Comparison of deflection and moment amplification factors for midpoint of the bridge for a vehicle moving on different paths. (Aspect ratio = 2, $\alpha = 0.30$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.25: Comparison of deflection and moment amplification factors for midpoint of the bridge for a vehicle moving on different paths. (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\theta = 45^\circ$)

Figure 2.26: Comparison of deflection and moment amplification factors for midpoint of the bridge for a vehicle moving on different paths. (Aspect ratio = 2, $\alpha = 0.30$, $\kappa = 0.5$, $\theta = 45^\circ$)

Figure 2.27: Response of bridges due to a moving central vehicle: distribution of normalized deflections (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.28: Response of bridges due to a moving off-center vehicle: distribution of normalized deflections (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.29: Response of bridges due to a moving off-center vehicle: distribution of normalized deflections (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.30: Response of bridges due to a moving central vehicle: distribution of normalized M_x (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.31: Response of bridges due to a moving off-center vehicle: distribution of normalized M_x (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.32: Response of bridges due to a moving off-center vehicle: distribution of normalized M_x (Aspect ratio = 2, $\alpha = 0.15$, $\kappa = 0.5$, $\phi = 0.5$)

Figure 2.33: Amplification factor for midpoint deflection of a bridge due to a vehicle moving along three different paths

Figure 2.34: Generation of envelope curves relating amplification factor, α , and κ

Figure 2.35: (a)(b) Design envelope curves relating amplification factor, α , and κ ($0 < \phi < 2.4$)

Figure 2.35: (c)(d) Design envelope curves relating amplification factor, α , and κ ($0 < \phi < 2.4$)

Figure 2.36: Envelope curves for different values of θ ($\kappa = 0.5$)

Chapter 3

Skew Slab-On-Girder Bridges

3.1 Introduction

Slab-on-girder bridges are the most prevalent type of highway bridge structure throughout the world. Their spans usually range from 10 m to 40 m. The ratio of the live load to the dead load is relatively large. Consequently, the vibration of girder bridges caused by moving vehicles is very important and has been studied by many researchers. However, most of these studies have dealt with right bridges. Slab-on-girder bridges with a skew have not been completely studied. Moreover, in most studies slab-on-girder bridges are modeled either as orthotropic plates or as assemblies of grillage beams. Not all slab-on-girder bridges can reasonably be treated as isotropic plates or orthotropic plates. The orthotropic plate theory in its present form is not capable of taking account of intermediate diaphragms, and the evaluation of slab and girder stresses from the results of plate analysis is still a problem. Although the grillage method, in which the slab and girder structure is replaced by a framework of intersecting beams whose stiffness are adjusted to approximate those of the slab and girders, has proved to be effective in analyzing the slab-on-girder bridge, the idealization of slab in its natural form as a plate and the girders as beam elements

would be more accurate. This is possible through a finite element analysis of the bridge structure.

3.2 Finite Element Analysis

Figure 3.1a shows a bridge with prestressed concrete I-girders. The structure is idealized using the following assumptions:

1. all materials are elastic and homogeneous.
2. the slab has a constant thickness.
3. the supporting girders are equally spaced.
4. the slab and girders exhibit full composite action.
5. diaphragms are provided at supports and also inside the span, and diaphragms and the slab are fully composite.

A plate-beam model is developed. The slab is discretized by plate bending elements with a parallelogram shape as shown in Fig. 3.1b. Each element has four nodes, and each node has three degrees of freedom, which are vertical displacement along z axis and two rotations about x and y axes, namely, w , θ_x , and θ_y . Girders are modeled by beam elements with three degrees of freedom at each node. Since the slab and girders are composite, beam elements have the same d.o.f. as those of plate elements at the locations of girders. Diaphragms are also modeled by beam elements which have the same d.o.f. as those of adjacent plate elements. Therefore, the d.o.f. of only the plate elements appear in the assembled structure.

In order to develop a procedure for taking into account the composite action of the slab and the girders, it is useful to review the grillage method. In the conventional grillage analysis, the slab-on-girder bridge is represented by a plane grillage as shown in Fig. 3.1c. A certain width of the slab on top of girders and diaphragms acts as the flange as shown in Fig. 3.1d. The slab with an effective width and the girder act in a composite manner so that the combined section has a single neutral axis. The moment of inertia of the composite longitudinal beam with respect to this neutral axis can be expressed as

$$I_b^* = I_s + I_g + \frac{A_s A_g}{A_s + A_g} e^2 \quad (3.1)$$

where,

$I_s = \frac{nt^3 P_f}{12}$, the moment of inertia of the slab, whose width is equal to the width of effective flange modified by the modulus ratio n , about its own neutral axis.

I_g = the moment of inertia of the girder about its own neutral axis.

$A_s = nP_f t$, the area of the slab, whose width is equal to the width of effective flange modified by the modulus ratio n .

A_g = the area of the girder.

e = the distance between the neutral axis of the slab and the neutral axis of the girder.

n = the modulus ratio which compares the modulus of elasticity for the slab material with the modulus of elasticity for the girder material.

t = the thickness of the slab.

P_f = the width of effective flange.

The contributions of the slab and the girder are considered in I_s and I_g , respectively, and the composite action of slab and girder is considered in the last term of Eq. 3.1.

In the plate-beam model the effect of the slab has already been considered by the plate bending elements. The moment of inertia of the beam element can therefore be expressed as

$$I_b = I_g + \frac{A_s A_g}{A_s + A_g} e^2 \quad (3.2)$$

The torsional stiffness of the slab and girder structure has two parts. One part is contributed by the slab, and the other is from the girder. These two are simply added in the global stiffness matrix. Since the effect of the slab is already accounted for by plate bending elements in the plate-beam model, the torsional inertia of the girder alone need to be included in the beam element.

The intermediate diaphragms and diaphragms at span ends are also modeled by beam elements. The width of slab acting as flange is conservatively assumed to be 0.3 of the distance between longitudinal members as suggested by Hambly (1991). The cross section properties of the beam element are calculated following the same algorithm as described in the previous paragraphs.

A vehicle model, which consists of a single mass supported by a spring, is used. The vehicle load is applied on the plate elements. The equations of motion for the vehicle-bridge system are the same as Eqs. 2.12 rewritten here as follows.

$$\begin{bmatrix} m_v & 0 \\ 0 & \mathbf{M}^* \end{bmatrix} \begin{bmatrix} \ddot{u}_v^t \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} k_v & \mathbf{k}_h \\ \mathbf{k}_v & \mathbf{K}^* + \mathbf{k} \end{bmatrix} \begin{bmatrix} u_v^t \\ \mathbf{y} \end{bmatrix} = -m_v g \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (3.3)$$

A set of free vibration modes of the bridge is used in transforming Eqs. 3.3 to reduce the size of the problem. Equations 3.3 are solved by using the Newmark's average acceleration method, with a time step equal to one-tenth of the highest modal period of the bridge

considered in the analysis. The number of modes that need to be included in the analysis so as to provide adequate accuracy in calculating the girder moments is determined as follows. One girder with a certain width of slab acting as the flange is selected from the bridge being studied and is treated as a simply-supported beam discretized by beam elements. The simply-supported beam is analyzed for its response to a vehicle moving on it from one end to the other. The equations of motion of the vehicle-bridge system will be described in the next chapter. A convergence study is carried out to determine the minimum number of modes that should be included in the analysis to ensure enough accuracy in calculating moments in this beam. The corresponding modal period is then set to be the standard to determine the number of modes for the complete bridge. The total number of modes for the complete bridge is several times that for a single beam, and depends on the number of girders.

3.3 Characterizing Parameters

In Chapter 2 it was shown that the parameters governing the response of isotropic plate model of a bridge were α , κ , ϕ , θ , and aspect ratio. Both analytical studies and numerical examples were used to prove that these parameters indeed governed the dynamic response of bridge models. The governing parameters for a slab-on-girder bridge are different because the flexural and torsional rigidities of the slab-on-girder bridge deck are direction dependent.

Since the equations of motion of a system comprising a vehicle and a slab-on-girder bridge are the same as those of an isotropic plate model of a bridge, analytical studies can be carried out by following the procedures described in Section 2.3.2. For two slab-on-girder bridges having the same mode shapes and the same values of natural frequencies

to within a scaling factor, Equations similar to Eqs. 2.14 through 2.20 apply. Hence, the governing parameters for a slab-on-girder bridge include not only α , κ , and ϕ , but also those which control the frequencies and mode shapes of the bridge. In other words, two bridge models will have the same response if the parameters α , κ , and ϕ are the same and the dimensions of bridge cross sections are such that the bridges models have the same mode shapes and the same values of natural frequencies to within a scaling factor.

Among the parameters which control the frequencies and mode shapes of the bridge, the obvious two are aspect ratio and skew angle θ , which proved to be the only parameters governing the frequencies and mode shapes of isotropic plate models. Additional parameters for the slab-on-girder bridge are girder spacing, number of girders, etc.

3.4 Modeling of Bridges

Four sets of prestressed precast concrete slab-on-girder bridges as shown in Fig. 3.2 are designed in accordance with AASHTO. The bridges also satisfy Ontario Highway Bridge Design Code. The properties of bridges are listed in Table 3.1. The length ranges from 10 m to 40 m. All bridges have the same width, 10 m, and the same slab thickness, 230 mm. An additional 10 mm thick wearing coat of concrete is provided on the top surface of concrete deck. The wearing coat is assumed not to contribute to the structure stiffness but adds dead load. Parapets are assumed to have a cross-section area of 0.2186 m^2 . Each set of bridges includes thirteen bridge models with skew angle θ varying from 0° to 60° at an increment of 5° . Every bridge has five identical AASHTO girders, and the girder spacing is 2.0 m. Diaphragms are provided at span ends in all bridges. An intermediate diaphragm is provided at mid-span when the span is 20 m. Diaphragms are provided at third points when the span is 30 m and at quarter points when the span is 40 m.

The composite cross section of the bridge is shown in Fig. 3.2b. The girder concrete strength is $f'_c = 5000 \text{ psi} = 344.737 \text{ MPa}$. The slab concrete strength is $f'_c = 3300 \text{ psi} = 227.527 \text{ MPa}$. The effective flange width is controlled by the center-to-center girder spacing and is equal to it. In this case, the effective flange width is 2.0 m. This effective width is modified by the modular ratio n , which is the ratio of the modulus of elasticity for the slab concrete to the modulus of elasticity for the girder concrete. Using $E_c = 57,000\sqrt{f'_c}$, we obtain $E_s = 57,000\sqrt{3300} = 3,274,000 \text{ psi} = 22.573 \text{ GPa}$ for slab and $E_g = 57,000\sqrt{5000} = 4,031,000 \text{ psi} = 27.792 \text{ GPa}$ for girder. Therefore,

$$n = \frac{22.573}{27.792} = 0.812$$

The cross section properties of the composite girders and diaphragms, modeled by beam elements which exclude the effect of the slab, are calculated and listed in Table 3.2 for all four sets of bridges. The positions of the moving vehicle are shown in Fig. 3.3.

3.5 Modal Analysis

Tables 3.3 to 3.6 lists the first ten natural frequencies of a bridge model for different values of the skew angle θ and several values of aspect ratio, namely, 1.0, 2.0, 3.0, and 4.0. Figure 3.4 shows the variation of first two frequencies of bridge models with the skew angle. It is found that the first and second frequencies of the bridge are very close to each other for all values of θ and aspect ratio, and are practically independent of θ , particularly for the larger aspect ratios, namely, 2.0, 3.0, and 4.0. Figure 3.5 shows the first ten vibration modes for a bridge with $\theta = 0^\circ$ and aspect ratio = 2, and Fig. 3.6 shows the first ten vibration modes for a bridge with $\theta = 45^\circ$ and aspect ratio = 2. The first two modes in these two figures correspond to bending and torsional modes, respectively,

and it is found that this is the case for all slab-on-girder bridges with aspect ratio ≥ 1 .

3.6 Parametric Study

Finite element analyses are performed to determine the effect of the following parameters on the response of the bridge due to a moving vehicle.

1. The speed parameter, α
2. The mass ratio, κ
3. The frequency ratio, ϕ
4. The aspect ratio, $\frac{L}{W}$, see Fig. 3.3a
5. The skew angle, θ
6. Girder spacing
7. Number of girders

The path of the vehicle on the bridge affects the response of the bridge as we have seen in the previous chapter for slab bridges. During a bridge lifetime, the number, position and direction of travel of actual traffic lanes may change, depending upon the highway use. In a slab-on-girder bridge, the critical load effects on external girders are mostly due to a vehicle moving on a side lane and increase rapidly as the wheels approach the curb or barrier. In present study two vehicle positions, the central vehicle position and the off-center vehicle position as shown in Fig. 3.3, are considered. They emphasize on response of the central girder and the external girder, respectively. In determining the distance between the edge of the bridge and the gravity center of the off-center vehicle,

the Vehicle Edge Distance of 0.61 m or 2' as suggested by Huang et. al (1993) is adopted. The vehicle is assumed to have a wheel spacing of 1.83 m which match those of both the HS20-44 truck and the OHBD truck.

3.6.1 Dynamic Amplification of Deflection

3.6.1.1 Effects of Aspect Ratio and Skew Angle θ

Bridge Subjected to a Central Vehicle

Calculations have been carried out to determine the amplification factors of bridges due to a moving central vehicle which is defined as the vehicle moving along the centerline of the bridge. Amplification factors for the midpoint deflection are plotted in Fig. 3.7 as a function of θ for four values of aspect ratio, namely, 1, 2, 3, and 4, for two values of α , namely, 0.1 and 0.15, and for values of $\kappa = 0.2$ and $\phi = 0.5$. The important feature shown is that the larger the aspect ratio of the bridge, the less sensitive the midpoint response of the bridge to a change in the skew angle is.

To see the transverse distribution of the dynamic response, normalized maximum dynamic deflections along the central cross section of right bridges ($\theta = 0^\circ$) are shown in Fig. 3.8 for several values of α , κ , ϕ , and the aspect ratio. Similar results for distribution along the skew central cross section of bridges with $\theta = 45^\circ$ are shown in Fig. 3.9. Normalization is with respect to the maximum static midpoint deflection of the model of the bridge under consideration for a vehicle moving along the centerline of the bridge. It can be seen that the 45° skew bridges have symmetrical distributions of deflection along the skew cross section under a moving central vehicle. This is consistent with what we found in Section 2.4.7 for skew slab bridges with $\theta \leq 45^\circ$. In addition, it can be seen that bridges with larger value of aspect ratio, namely, 2.0, 3.0, and 4.0, have more uniform

transverse distribution of responses. Although bridges with different aspect ratios have similar responses at midpoints, responses near the free edges are quite different. It should be noted that for bridges of different aspect ratios having the same value ϕ , only the first frequency is matched. Also, from the model analysis, the first vibration mode is longitudinal flexural and the second is torsional. Therefore, the response of the midpoint of the bridge must mainly depend on the first vibration mode, and responses near free edges, especially in bridges with smaller value of aspect ratio, must be affected significantly by modes other than the first. In other words, modes other than the first one make larger contribution in a bridge with small value of aspect ratio than in a bridge with large value of aspect ratio.

Bridge Subjected to an Off-Center Vehicle

Following a procedure similar to that used for a central vehicle, the amplification factors for bridges due to an off-center vehicle have been calculated. Since the direction of the off-center vehicle affects the response of a skew bridge, the maximum response of the skew bridge is obtained by running the vehicle on the same path twice, once in each of the two opposite directions. Amplification factors for deflection at the middle of the external girder are plotted in Fig. 3.10 as a function of θ for four values of aspect ratio, namely, 1, 2, 3, and 4, for two values of α , namely, 0.1 and 0.15, and for values of $\kappa = 0.2$ and $\phi = 0.5$.

Normalized maximum dynamic deflections along the central cross section of right bridges ($\theta = 0^\circ$) are shown in Fig. 3.11 for several values of α , κ , ϕ , and the aspect ratio. Similar results for distribution along the skew central cross section of bridges with $\theta = 45^\circ$ are shown in Fig. 3.12. Normalization is with respect to the maximum static

deflection at the middle of the external girder of the model of the bridge under consideration for a vehicle moving along the same off-center path. The transverse distributions of the bridges with aspect ratio = 1.0 are different from bridges with other values of aspect ratio, and this is the case for both right bridge (Fig. 3.11) and skew bridge (Fig. 3.12). As stated in the case of bridge subjected to an off-center vehicle, the response near the middle of free edges, especially in bridges with smaller value of aspect ratio, must be affected significantly by modes other than the first.

3.6.1.2 Effects of α , κ , and ϕ

A rectangular bridge with dimensions 20×10 m is analyzed under the passage of a central vehicle. Figure 3.13 shows the relationship between the dynamic amplification factor for midpoint deflection and the frequency ratio ϕ for different values of α and κ . Another bridge with dimensions 20×10 m and $\theta = 45^\circ$ is also analyzed under the passage of a central vehicle, and the results are presented in Fig. 3.14. As for the skew slab bridge, a wide range of κ values, ranging from 0.1 to 1.0, has been covered. In most practical cases, κ is likely to be significantly smaller than 1.0.

Comparison of Figs. 3.13 and 3.14 shows that amplification factors for midpoint deflection of the rectangular bridge and the skew bridge have very similar variations with α , κ , and ϕ , and the influence of the skew angle on response of midpoint of the bridge is quite weak. Figure 3.15 provides a better comparison, and confirms the similarity of the amplification factors. This can be explained by the fact shown in Fig. 3.4 that first frequency, which corresponds to the flexural mode of the bridge, is practically independent of θ .

3.6.2 Dynamic Amplification of Moment

3.6.2.1 Bridge Subjected to a Central Vehicle

Amplification factors for midspan girder moment and normalized maximum dynamic girder moments are shown in Fig. 3.16 for bridges with different skew angles for values of $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$, and aspect ratio = 2. Figure 3.17 shows amplification factors for midspan girder moment and normalized maximum dynamic girder moments in the same manner as in Fig. 3.16 but for bridges with aspect ratio = 4. The amplification factor for midspan girder moment is defined as the ratio of the maximum dynamic moment at the midspan of the girder being considered to the maximum static moment at the same location due to the moving vehicle. Dynamic girder moment normalizations are with respect to the maximum static moment of the central girder of the model of the bridge under consideration for a central vehicle, and maximum dynamic girder moments in different girders do not have to occur at the same time. Several features are observed. (a) the transverse distribution of amplification factors of bridge with larger skew angle is more uneven than that of bridge with smaller skew angle; (b) the external girders of bridges, which are farther away from the path of the vehicle, tend to have larger value of amplification factor. Since the actual moments in external girders are quite small compared to those in internal girders as observed from Figs. 3.16b and 3.17b, the large values of amplification factor are not important.

3.6.2.2 Bridge Subjected to an Off-Center Vehicle

Figures 3.18 and 3.19 show amplification factors for midspan girder moment and normalized maximum dynamic girder moments for bridges with aspect ratio = 2 and 4, respectively, due to an off-center vehicle. Other parameters are the same as those in

Figs. 3.16 and 3.17. The amplification factor for midspan girder moment is defined as the ratio of the maximum dynamic moment at the midspan of the girder being considered to the maximum static moment at the same location due to the off-center moving vehicle. Dynamic girder moment normalizations are with respect to the maximum static moment in the external girder of the model of the bridge under consideration to an off-center vehicle. It is seen that the girder with maximum dynamic moment has the smallest value of amplification factor. In general, this would control the design. Girders located farther away from the one under the vehicle have large amplifications, but at the same time these girders have low moments. The large amplification are not likely to control the design.

3.6.3 Comparison of Deflection Amplification and Moment Amplification

Analyses are carried out to obtain comparative values of amplification factors for deflection and moment at the middle of the central girder. Figure 3.20 shows the amplification factors for deflection and moment as a function of θ for two values of aspect ratio, namely, 2 and 4. Two values of α , namely, 0.1 and 0.15, and the value $\kappa = 0.2$ are used in the analysis. The vehicle is moving along the centerline of the bridge. The amplification factor for moment is found to be smaller than the corresponding amplification factor for deflection, and they have very similar variations. To study the relationship between deflection and moment amplification factors as the frequency ratio ϕ is varied, skew bridges with $\theta = 45^\circ$ having two different aspect ratios, namely, 2 and 4, are analyzed for a vehicle moving on the centerline of the bridge. Figure 3.21 shows amplification factors for deflection and moment at the middle of the central girder as a function of ϕ . Two values of α , namely, 0.1 and 0.15, and the value $\kappa = 0.2$ are used in the analysis. The amplification factor

for moment is again found to be smaller than the corresponding amplification factor for deflection.

Similar analyses are carried out for the case of an off-center vehicle. Figures 3.22 and 3.23 show the amplification factors for deflection and moment at the middle of the external girder as a function of θ and ϕ , respectively. Other parameters are similar to those in Figs. 3.20 and 3.21. The moment amplification factor is generally smaller than the deflection amplification factor except for bridges with large skew angle, where the moment amplification is sometimes larger.

3.6.4 Comparison among Plate–Beam Model, Orthotropic Plate Model, and Isotropic Plate Model

The orthotropic and isotropic plate theory is the basis of many of analysis techniques that are currently used in practice. In methods that use the plate theory, the analysis of a bridge deck is carried out by replacing it by an equivalent orthotropic or isotropic plate, having the same stiffness characteristics as the actual bridge deck and having a uniform thickness. To appreciate the differences among plate–beam model, orthotropic plate model, and isotropic plate model, a set of bridges used in previous sections are analyzed by using these three different models. The bridges have dimensions 20×10 m and skew angle θ ranging from 0° to 60° at an interval of 5° . The longitudinal flexural rigidity per unit width D_x , the transverse flexural rigidity per unit length D_y , the longitudinal torsional rigidity per unit width D_{xy} , and the transverse torsional rigidity per unit length

D_{yx} of the plate-beam model are

$$D_x = \frac{E_g}{P_y} I_b^* \quad (3.4)$$

$$D_y = \frac{E_g}{P_x} I_d + \frac{E_s t^3}{12} \quad (3.5)$$

$$D_{xy} = \frac{G_g}{P_y} J_g + \frac{G_s t^3}{6} \quad (3.6)$$

$$D_{yx} = \frac{G_g}{P_x} J_d + \frac{G_s t^3}{6} \quad (3.7)$$

where

I_b^* is given by Eq. 3.1;

I_d is the moment of inertia of diaphragm;

J_g, J_d are respectively the torsional inertias of the girder and diaphragm;

E_g, E_s are respectively the moduli of elasticity of girder material and slab material;

G_g, G_s are respectively the shear moduli of girder material and slab material;

t is thickness of the slab;

P_x is the spacing between diaphragms;

P_y is the girder spacing.

The effect of the intermediate diaphragms is considered in D_y and D_{yx} as suggested by Jaeger et. al. (1989). In a right bridge, D_x and D_y correspond to the x and y axes which are perpendicular to each other. Therefore, D_{xy} and D_{yx} have the forms as shown in Eqs. 3.6 and 3.7. In a skew bridge, D_x corresponds to the longitudinal direction of the bridge, and D_y corresponds to the direction of the intermediate diaphragm, which is parallel to the skew supports or bearings. D_{xy} and D_{yx} in their present form are not capable of taking account of skew angle. But for simplicity, we still use Eqs. 3.6 and 3.7 to calculate D_{xy} and D_{yx} for the skew slab-on-girder bridge.

The thicknesses of both the orthotropic plate models and the isotropic plate models

are set to be 0.525 m. The orthotropic plate models have the same values of D_x, D_y, D_{xy} , and D_{yx} as those of the plate-beam models, and the isotropic plate models have the same D_x as those of the plate-beam models. Modal analysis is carried out to verify the modeling of the bridges. The first natural frequencies of these thirteen bridge models are listed in Table 3.7 for three different models. The first ten natural frequencies of the rectangular bridge are listed in Table 3.8 for three different models, and those of the skew bridge with $\theta = 45^\circ$ are listed in Table 3.9. Results from plate-beam models and results from orthotropic plate models are quite close for both the rectangular bridge and the 45° skew bridge.

3.6.4.1 History of Responses

The time histories of dynamic midpoint deflection are plotted in Fig. 3.25a for the rectangular bridge analyzed by using the three different models. The corresponding time histories for the skew bridge with $\theta = 45^\circ$ are plotted in Fig. 3.25b. A central vehicle is applied here, and the parameters $\alpha = 0.15, \kappa = 0.2$, and $\phi = 0.5$ are used. T in the figures is the time for the vehicle to traverse the bridge. The results from plate-beam model and orthotropic plate model are quite close for both rectangular and skew bridges.

Figure 3.26 shows the histories of dynamic deflection at the middle of the external girder that is close to the off-center vehicle. Again, the results from plate-beam model and orthotropic plate model have very similar variations for both bridges.

3.6.4.2 Bridge Subjected to a Central Vehicle

A set of bridges are analyzed by using three different models under the passage of a central vehicle. The amplification factor for midpoint deflection is shown as a function of θ in Fig. 3.27 for two values of α , namely, 0.1 and 0.15, and the values $\kappa = 0.2$ and

$\phi = 0.5$. The variations of amplification factor with ϕ are shown in Figs. 3.28 and 3.29 for a rectangular bridge and a skew bridge with $\theta = 45^\circ$, respectively. One can observe from Figs. 3.27, 3.28, and 3.29 that responses obtained from the three different models are very close. The maximum difference between plate-beam model and orthotropic plate model is of the order of 5% while the maximum difference between plate-beam model and isotropic plate model is of the order of 7%. This is expected since the first vibration modes of these bridge models are longitudinal flexural mode, and torsional modes do not affect the midpoint response.

Figure 3.30 shows the normalized maximum dynamic deflections along the central cross-section of the rectangular bridge models, and Fig. 3.31 shows similar results for the skew bridge models with $\theta = 45^\circ$. Normalization is with respect to the maximum static midpoint deflection of the model of the bridge under consideration due to a central vehicle. It is found that the transverse distribution of deflections of the plate-beam model and orthotropic plate model match very well, and the maximum difference is of order of 5%. Except at the midpoint, deflections along the central cross-section of the isotropic plate model are different from those of other two bridge models. Vibration modes other than first one must make important contribution to the response of a bridge, and unmatched higher-order modes between isotropic plate model and plate-beam model are responsible for the difference in results obtained from these two models.

3.6.4.3 Bridge Subjected to an Off-Center Vehicle

In previous section, we found that the results of plate-beam model and orthotropic model match very well in the case of a central vehicle. To verify this for an off-center vehicle, the same set of bridges is analyzed by using three different models due to an off-center

vehicle. The amplification factor for deflection at the middle of the external girder that is close to the path of the off-center vehicle is shown as a function of θ in Fig. 3.32 for two values of α , namely, 0.1 and 0.15, and the values $\kappa = 0.2$ and $\phi = 0.5$. The variations of amplification factor with ϕ are shown in Figs. 3.33 for a rectangular bridge and Fig. 3.34 for a skew bridge with $\theta = 45^\circ$, respectively. The maximum difference between plate-beam model and orthotropic plate model is of the order of 6% while the maximum difference between plate-beam model and isotropic plate model is of the order of 8%.

Figure 3.35 shows the normalized maximum dynamic deflections along the central cross-section of the rectangular bridge, and Fig. 3.36 shows similar results for the skew bridge models with $\theta = 45^\circ$. Normalization is with respect to the maximum static deflection at the middle of the external girder of the model of the bridge under consideration for an off-center vehicle. In all cases, plate-beam model and orthotropic model give results that show very similar variation trend and the differences between the two sets of results are fairly small. However, the results obtained with an isotropic model are significantly different.

3.6.5 Effect of Girder Spacing

Three 20-meter-long rectangular bridge models, having five AASHTO type III girders and the same slab thicknesses (230 mm) but different girder spacings, namely, 2.0, 2.4, 2.8 m, are analyzed for their responses to a vehicle moving along the centerline of the bridges. Diaphragms are provided at ends as well as at midspan. The aspect ratios of the three bridge models are 2.0, 1.67, and 1.43, respectively. Amplification factors for midspan girder moment and normalized maximum girder moment of five girders in each bridge model are plotted in Fig. 3.37a for $\alpha = 0.10$, $\kappa = 0.2$ and $\phi = 0.5$. Figure 3.37b shows

similar results but for $\alpha = 0.15$, $\kappa = 0.2$ and $\phi = 0.5$. The definition of amplification factor is the same as those in previous sections. Normalization is with respect to the maximum static moment of the central girder of the model of the bridge under consideration for a central vehicle. Some features are listed as follows: (a) The corresponding internal girders in the three bridge models have very similar amplification factors, the maximum difference being of the order of 3%. (b) The internal girders, which are near the path of the moving vehicle, take most of vehicle load. (c) The corresponding external girders in the three bridge models tend to have different responses, the maximum difference being of the order of 12%. (d) The moment amplification factor for external girders increases with increasing girder spacing or decreasing aspect ratio. This is because the static load (not shown) in the external girders significantly decreases with increasing girder spacing while the vehicle is on the centerline of the bridge. The large values of amplification factor in external girders are not important since the critical design moment in a girder is not likely to be produced by a vehicle that is moving along a path that is far from the girder. This is consistent with the conclusion drawn in Section 3.6.2.

The effect of girder spacing on skew bridges is also studied. Three bridge models of length = 20 m, $\theta = 45^\circ$, and slab thickness = 230 mm having five AASHTO type III girders are considered. They have different girder spacings, namely, 2.0, 2.4, and 2.8 m. The vehicle is moving along the centerline of the bridge. Analyses are carried out following the procedure described earlier, and results are presented in Figs. 3.38a and 3.38b. It is seen that in the skew bridges the girder spacing does not significantly affect the amplification factor of internal girders, which take most of dynamic load. However, it does affect the response of external girders which are farther away from the path of moving vehicle.

3.6.6 Effect of Number of Girders

Three 20-meter-long rectangular bridge models, one with three AASHTO type III girders, one with five, and the other with seven, are analyzed for their responses. Slab thicknesses are 230 mm, and girder spacings are 2.0 m. Diaphragms are provided at span ends as well as at midspan. Aspect ratios of three bridge models are 3.33, 2.0, and 1.43, respectively. The vehicle is moving along the centerline of the bridge. Amplification factors for midspan girder moment and normalized maximum girder moments in each bridge model are plotted against the girder number in Fig. 3.39a for $\alpha = 0.10$, $\kappa = 0.2$ and $\phi = 0.5$. Similar results are shown in Fig. 3.39b but for $\alpha = 0.15$, $\kappa = 0.2$ and $\phi = 0.5$. Normalization is with respect the maximum static moment of the central girder of the model of the bridge under consideration for a central vehicle.

Three skew bridge models with skew angle $\theta = 45^\circ$ but similar in aspect ratios to these rectangular bridges in previous paragraph are analyzed for their responses to a vehicle moving along the centerline of the bridge. The results are presented in Figs. 3.40a and 3.40b. The parameters are the same as those in Figs. 3.39a and 3.39b.

The conclusions from Figs. 3.39 to 3.40 are: (a) the change in number of girders does not significantly change the amplification factors of existing girders; (b) the ratio of maximum dynamic loads of different girders does not significantly change with the change of number of girders; (c) the larger the aspect ratio, the more even the distribution of amplification factors of girders is. This is consistent with the results presented in Section 3.6.1.

3.7 Design Recommendation

Seven parameters, namely, speed parameter α , mass ratio κ , frequency ratio ϕ , skew angle θ , aspect ratio, girder spacing, and number of girders, control the response of slab-on-girder bridge under passage of a vehicle. In developing the design recommendation, all these parameters have to be considered. In addition, the different paths of along which a vehicle may travel also influence the response of the bridge.

Based on results presented in Sections 3.6.5 and 3.6.6, one can conclude that girder spacing and number of girders are closely related to the aspect ratio. For the bridges studied the results from plate-beam model and orthotropic model match very well for both the central vehicle and the off-center vehicle. We could use orthotropic model without two parameters, namely, girder spacing and number of girders, to develop the design recommendation. This could reduce the number of variables and simplify the problem. It is recognized that when the number of girders is very small, for example 2, orthotropic plate model may not be adequate and the approximate result based on orthotropic plate model need to be refined for the purpose of final design.

Figure 3.41a shows the variation of amplification factor for midpoint deflection of a skew bridge with α for a number of different values of ϕ from 0 to 2.4 at a interval of 0.05 due to a central vehicle. The bridge has aspect ratio = 2 and $\theta = 45^\circ$ and is modeled by using the orthotropic model. The curves have been drawn for single values of θ and κ . The variation of amplification factor for deflection at middle of the external girder of the bridge due to an off-center vehicle with α is shown in Fig. 3.41b. The envelope to these curves in each figure gives the largest value of amplification factor for any given value of α for a certain vehicle position. Combining two envelopes from Figs. 3.41a and 3.41b, we

can obtain a curve which is an upper bound for any vehicle position. This curve can be used to select the design value of amplification factor for given κ and θ .

A few of the design envelope curves are shown in Figs. 3.42a through 3.42d for bridges with aspect ratio=2. Response of a skew bridge with $\theta = 15^\circ$ is approximately the same as that of a right bridge, and response of a skew bridge with $\theta = 30^\circ$ is also close to that of a right bridge. In fact, the design envelope curves of the skew bridges with $\theta = 45^\circ$ and 60° are not far away from that of the right bridge either. Considering that the first two natural frequencies of skew bridges are practically independent of skew angle as shown in Fig. 3.4 and that the influence of the skew angle on response of the bridge is quite weak as shown in Figs. 3.13, 3.14, and 3.15, the similar results presented in Figs. 3.42a to 3.42d are not surprising. In addition, the right bridge gives larger response than the skew bridges. For simplicity in application, the design envelope curves of the right bridge can be used for skew bridges, and this will result in a slightly conservative design.

Since the length of a slab-on-girder bridge can be up to 40 m in practice and the width of the bridge depends on the number of traffic lanes, the aspect ratio of the bridge can vary in a wide range, for example, 1.0 to 4.0. Figures 3.7 and 3.10 show no systematic variation among different values of aspect ratio. Similar procedure will apply if design recommendations for bridges with other values of aspect ratio are to be developed.

3.8 Conclusion

A series of parametric studies is carried out on skew slab-on-girder bridges to determine their dynamic response to a moving vehicle. The effect of several parameters on the response of the slab-on-girder bridge is studied. Some conclusions that can be drawn as a result of this study are as follows

1. The larger the aspect ratio of the bridge, the less sensitive to the change of skew angle the midpoint response of the bridge is. The bridge with a larger value of the aspect ratio has a more uniform transverse distribution of response. This is because the longitudinal flexural mode dominates the response.
2. The transverse distribution of amplification factors in a bridge with a larger skew angle is more uneven than that in a bridge with a smaller skew angle. This is true for all vehicle positions.
3. The moment amplification factor is generally smaller than the corresponding deflection amplification factor, but both have similar variations.
4. For the skew bridges studied, plate-beam model and the orthotropic plate model give similar responses in terms of absolute value of deflection, amplification factor, and transverse distribution of normalized deflections. However, the response obtained from an isotropic plate model is quite different in terms of the absolute value of deflection and transverse distribution of normalized deflections. It is recognized that the ability of an orthotropic plate model to provide a reasonable estimate of the response of a slab-on-girder bridge will be affected by the number of girders. If the number of girders is small, orthotropic model idealization may not be that accurate.
5. A change in girder spacing does not significantly change the amplification factor of those girders that take most of dynamic load.
6. An increase in number of girders does not significantly change the amplification factors of the girders.

7. A series of design envelops curves are derived on the basis of bridges with aspect ratio=2. The curves relate dynamic amplification factors to parameters θ , α , and κ . To use these curves a preliminary design must be carried out first to obtain the mass and fundamental frequency of the bridge. Parameters α and κ can then be calculated.

Table 3.1: Properties of slab-on-girder bridges

Bridge Set	A	B	C	D
Length(m)	10.0	20.0	30.0	40.0
Width(m)	10.0	10.0	10.0	10.0
Thickness of Slab(mm)	230	230	230	230
AASHTO Girder Type	I	III	V	VI
Int. Diaphragms	0	1	2	3
Mass(tonne)	93.772	232.242	460.417	618.428
Aspect Ratio	1.0	2.0	3.0	4.0

Table 3.2: Cross-section properties of composite girders and diaphragms

Bridge Set		A	B	C	D
Girder	$I_b (\times 10^{-2} m^4)$	4.0390	15.3603	41.0909	55.8479
	$J_b (\times 10^{-2} m^4)$	0.1139	0.4724	1.0296	1.0944
Intermediate Diaphragm	$I_b (\times 10^{-2} m^4)$		2.1237	7.0448	5.9665
	$J_b (\times 10^{-2} m^4)$		0.2297	0.4391	0.6475
Diaphragm at Ends	$I_b (\times 10^{-2} m^4)$	0.3975	0.5397	0.9063	1.6879
	$J_b (\times 10^{-2} m^4)$	0.0778	0.1039	0.1540	0.2306

Table 3.3: The lowest ten natural frequencies(Hz) of slab-on-girder bridges of size $10.0 \times 10.0m$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	12.328	12.350	12.422	12.615	13.066
2	12.331	12.387	12.582	12.950	13.333
3	15.769	15.928	16.469	17.623	20.346
4	23.902	24.022	24.441	25.340	27.050
5	38.300	38.304	38.276	38.018	29.553
6	43.019	43.026	42.951	41.447	29.572
7	43.020	43.033	42.993	41.616	36.134
8	53.341	53.632	54.328	47.843	44.197
9	55.203	55.677	56.646	47.943	44.203
10	57.507	57.334	56.737	56.327	55.302

Table 3.4: The lowest ten natural frequencies(Hz) of slab-on-girder bridges of size $20.0 \times 10.0m$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	5.381	5.383	5.391	5.418	5.542
2	5.498	5.522	5.607	5.796	6.154
3	11.650	11.421	10.835	10.200	10.029
4	20.705	20.734	20.831	20.748	18.545
5	20.774	20.791	20.836	21.037	21.356
6	23.952	24.033	23.912	21.841	21.425
7	26.400	25.916	24.852	25.481	27.767
8	29.139	29.285	29.754	30.587	30.284
9	40.380	40.483	40.263	38.856	32.336
10	40.874	40.758	40.725	40.762	32.377

Table 3.5: The lowest ten natural frequencies(Hz) of slab-on-girder bridges of size $30.0 \times 10.0m$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	3.396	3.396	3.397	3.404	3.441
2	3.580	3.593	3.638	3.741	3.975
3	13.359	13.357	12.266	10.209	8.327
4	13.523	13.519	13.453	13.492	13.528
5	14.337	13.813	13.507	13.575	13.884
6	19.719	19.385	18.503	17.462	16.614
7	29.264	29.284	29.348	25.803	20.289
8	29.278	29.298	29.356	29.052	24.875
9	32.727	32.791	30.870	29.492	29.722
10	34.668	33.898	32.462	29.711	29.766

Table 3.6: The lowest ten natural frequencies(Hz) of slab-on-girder bridges of size $40.0 \times 10.0m$

Mode No	Skew Angle (degree)				
	0°	15°	30°	45°	60°
1	2.173	2.173	2.171	2.170	2.183
2	2.375	2.385	2.422	2.507	2.700
3	8.670	8.681	8.672	8.661	8.508
4	8.684	8.681	8.717	8.798	8.686
5	19.076	18.614	16.208	12.705	9.572
6	19.440	19.080	18.488	15.744	13.309
7	19.459	19.430	19.167	19.227	19.249
8	21.269	20.556	19.393	19.321	19.483
9	27.103	26.665	25.444	23.846	22.443
10	33.088	33.112	33.187	32.932	23.588

Table 3.7: The first natural frequencies (Hz) of bridges of size $20 \times 10m$ calculated by using three different models

Skew angle	plate-beam	orthotropic plate	isotropic plate
0°	5.38	5.35	5.35
5°	5.38	5.35	5.39
10°	5.38	5.35	5.49
15°	5.38	5.36	5.67
20°	5.38	5.37	5.93
25°	5.39	5.38	6.27
30°	5.39	5.40	6.72
35°	5.40	5.42	7.30
40°	5.41	5.46	8.03
45°	5.42	5.51	8.98
50°	5.44	5.58	10.20
55°	5.48	5.69	11.76
60°	5.54	5.88	13.38

Table 3.8: The lowest ten natural frequencies (Hz) of a bridge of size $20 \times 10m$ and $\theta = 0^\circ$ calculated by using three different models

Mode	plate-beam	orthotropic plate	isotropic plate
1	5.38	5.34	5.35
2	5.50	6.01	16.88
3	11.65	10.75	21.42
4	20.71	21.28	38.15
5	20.77	21.68	48.23
6	23.95	22.66	56.88
7	26.40	24.47	67.14
8	29.14	33.04	81.14
9	40.38	41.16	85.85
10	40.87	45.51	105.61

Table 3.9: The lowest ten natural frequencies (Hz) of a bridge of size $20 \times 10m$ and $\theta = 45^\circ$ calculated by using three different models

Mode	plate-beam	orthotropic plate	isotropic plate
1	5.42	5.51	8.98
2	5.80	6.59	12.61
3	10.20	11.10	29.27
4	20.75	21.14	37.03
5	21.04	21.96	50.05
6	21.84	22.44	58.40
7	25.48	26.78	68.59
8	30.59	32.21	90.16
9	38.86	41.75	91.22
10	40.76	41.90	98.81

Figure 3.1: Modeling of a slab-on-girder bridge

(a) Plan

(b) Cross section of slab-on-girder bridge

Figure 3.2: Slab-on-girder bridge with five identical AASHTO girders

(a) Different paths of the vehicle

(b) Cross section of slab-on-girder bridge

Figure 3.3: Positions of the moving vehicle

Figure 3.4: First two frequencies of slab-on-girder bridges

Figure 3.5: First ten vibration modes of a slab-on-girder bridge with aspect ratio=2 and $\theta = 0^\circ$

Figure 3.6: First ten vibration modes of a slab-on-girder bridge with aspect ratio=2 and $\theta = 45^\circ$

Figure 3.7: Effects of aspect ratio and θ — amplification factor for midpoint deflection of bridges due to a central vehicle ($\kappa = 0.2, \phi = 0.5$)

Figure 3.8: Normalized maximum dynamic deflections along central cross-section of rectangular bridges due to a central vehicle ($\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.9: Normalized maximum dynamic deflections along skew central cross-section of bridges due to a central vehicle ($\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.10: Effects of aspect ratio and θ — amplification factors for deflection at the middle of the external girder of bridges due to an off-center vehicle ($\kappa = 0.2, \phi = 0.5$)

Figure 3.11: Normalized maximum dynamic deflections along central cross-section of rectangular bridges due to an off-center vehicle ($\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.12: Normalized maximum dynamic deflections along skew central cross-section of bridges due to an off-center vehicle ($\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.13: Effects of α , κ , and ϕ — amplification factor for midpoint deflection of a rectangular bridge with aspect ratio=2 due to a central vehicle

Figure 3.14: Effects of α , κ , and ϕ — amplification factor for midpoint deflection of a skew bridge with aspect ratio=2 and $\theta = 45^\circ$ due to a central vehicle

Figure 3.15: Comparison of amplification factors for midpoint deflection of bridges with different values of skew angle θ but identical values of α , κ , and ϕ due to a central vehicle

Figure 3.16: Amplification factors for moments, and normalized maximum moments at the midspan of each of the girders of a bridge due to a central vehicle (aspect ratio=2, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.17: Amplification factors for moments, and normalized maximum moments at the midspan of each of the girders of a bridge due to a central vehicle (aspect ratio=4, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.18: Amplification factors for moments, and normalized maximum moments at the midspan of each of the girders of a bridge due to an off-center vehicle (aspect ratio=2, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.19: Amplification factors for moments, and normalized maximum moments at the midspan of each of the girders of a bridge due to an off-center vehicle (aspect ratio=4, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.20: Comparison of amplification factors for deflection and moment at the middle of the central girder of bridges due to a central vehicle ($\kappa = 0.2, \phi = 0.5$)

Figure 3.21: Comparison of amplification factors for deflection and moment at the middle of the central girder of skew bridges due to a central vehicle ($\theta = 45^\circ, \kappa = 0.2, \phi = 0.5$)

Figure 3.22: Comparison of amplification factors for deflection and moment at the middle of the external girder of bridges due to an off-center vehicle ($\kappa = 0.2$, $\phi = 0.5$)

Figure 3.23: Comparison of amplification factors for deflection and moment at the middle of the external girder of skew bridges due to an off-center vehicle ($\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.24: Cross sections of a composite slab-on-girder bridge

Figure 3.25: Histories of midpoint deflection of bridges due to a central vehicle (aspect ratio=2, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.26: Histories of midpoint deflection of bridges due to an off-center vehicle (aspect ratio=2, $\alpha = 0.15$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.27: Comparison of amplification factors for midpoint deflection of plate-beam model, orthotropic plate model, and isotropic plate model. (central vehicle, aspect ratio=2, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.28: Comparison of amplification factors for midpoint deflection of plate-beam model, orthotropic plate model, and isotropic plate model. (central vehicle, aspect ratio=2, $\theta = 0^\circ$, $\kappa = 0.2$)

Figure 3.29: Comparison of amplification factors for midpoint deflection of plate-beam model, orthotropic plate model, and isotropic plate model. (central vehicle, aspect ratio=2, $\theta = 45^\circ$, $\kappa = 0.2$)

Figure 3.30: Distribution of normalized maximum dynamic deflection along the central cross-section of plate-beam model, orthotropic plate model, and isotropic plate model. (central vehicle, aspect ratio=2, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.31: Distribution of normalized maximum dynamic deflection along the central cross-section of plate-beam model, orthotropic plate model, and isotropic plate model. (central vehicle, aspect ratio=2, $\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.32: Comparison of amplification factors for deflection at the middle of the external girder of plate-beam model, orthotropic plate model, and isotropic plate model. (off-center vehicle, aspect ratio=2, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.33: Comparison of amplification factors for deflection at the middle of the external girder of plate-beam model, orthotropic plate model, and isotropic plate model. (off-center vehicle, aspect ratio=2, $\theta = 0^\circ$, $\kappa = 0.2$)

Figure 3.34: Comparison of amplification factors for deflection at the middle of the external girder of plate-beam model, orthotropic plate model, and isotropic plate model. (off-center vehicle, aspect ratio=2, $\theta = 45^\circ$, $\kappa = 0.2$)

Figure 3.35: Distribution of normalized maximum dynamic deflection along the skew cross-section of plate-beam model, orthotropic plate model, and isotropic plate model. (off-center vehicle, aspect ratio=2, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.36: Distribution of normalized maximum dynamic deflection along the skew cross-section of plate-beam model, orthotropic plate model, and isotropic plate model. (off-center vehicle, aspect ratio=2, $\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$)

Figure 3.37: Amplification factor for midspan girder moment and normalized maximum girder moment of rectangular bridges with different girder spacings but identical values of α, κ, ϕ , and number of girders due to a central vehicle. Number of girders = 5, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (a) $\alpha = 0.10$

Figure 3.37: Amplification factor for midspan girder moment and normalized maximum girder moment of rectangular bridges with different girder spacings but identical values of α, κ, ϕ , and number of girders due to a central vehicle. Number of girders = 5, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (b) $\alpha = 0.15$

Figure 3.38: Amplification factor for midspan girder moment and normalized maximum girder moment of skew bridges with different girder spacings but identical values of α, κ, ϕ , and number of girders due to a central vehicle. Number of girders = 5, $\theta = 45^\circ, \kappa = 0.2, \phi = 0.5$, (a) $\alpha = 0.10$

Figure 3.38: Amplification factor for midspan girder moment and normalized maximum girder moment of skew bridges with different girder spacings but identical values of α, κ, ϕ , and number of girders due to a central vehicle. Number of girders = 5, $\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (b) $\alpha = 0.15$

Figure 3.39: Amplification factor for midspan girder moment and normalized maximum girder moment of rectangular bridges with different number of girders but identical values of α , κ , ϕ , and girder spacing due to a central vehicle. Girder spacing = 2.0 m, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (a) $\alpha = 0.10$

Figure 3.39: Amplification factor for midspan girder moment and normalized maximum girder moment of rectangular bridges with different number of girders but identical values of α , κ , ϕ , and girder spacing due to a central vehicle. Girder spacing = 2.0 m, $\theta = 0^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (b) $\alpha = 0.15$

Figure 3.40: Amplification factor for midspan girder moment and normalized maximum girder moment of skew bridges with different number of girders but identical values of α, κ, ϕ , and girder spacing due to a central vehicle. Girder spacing = 2.0 m, $\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (a) $\alpha = 0.10$

Figure 3.40: Amplification factor for midspan girder moment and normalized maximum girder moment of skew bridges with different number of girders but identical values of α, κ, ϕ , and girder spacing due to a central vehicle. Girder spacing = 2.0 m, $\theta = 45^\circ$, $\kappa = 0.2$, $\phi = 0.5$, (b) $\alpha = 0.15$

Figure 3.41: Generation of envelope curves relating amplification factor, κ , and ϕ

Figure 3.42: (a)(b) Design envelope curves relating amplification factor, α , and κ ($0 < \phi < 2.4$)

Figure 3.42: (c)(d) Design envelope curves relating amplification factor, α , and κ ($0 < \phi < 2.4$)

Chapter 4

Multispan Continuous and Cantilever Bridges

4.1 Introduction

In Chapter 2, we proved that the controlling parameters in case of simply supported plate models are the speed parameter α , the mass ratio κ , the frequency ratio ϕ , the skew angle θ , and the aspect ratio. In multispan continuous and cantilever bridges, there are additional factors that influence the dynamic response under the passage of vehicles. The primary objective of present study is to carry out an analytical investigation of the dynamic response of such bridges. In addition, a case study is presented on an analytical evaluation of the vibration response of the Confederation bridge, Prince Edward Island, under the passage of a moving vehicle. In the present study, the bridge is idealized as a linearly elastic multiple span beam and the mass of the beam is lumped at a number of finite element nodes. The vehicle is represented by a single sprung mass with one d.o.f. moving along the deck. With these simple representations of the vehicle and the bridge it is possible to obtain results that would reveal the fundamental characteristics of the response.

4.2 Beam with a Moving Sprung Mass Model

In order to clearly identify the important parameters which govern the dynamic response of a multispan bridge, a simplified vehicle model shown in Fig. 4.1, which includes the important characteristics of the vehicle is adopted. The vehicle model has an unsprung mass m_t in contact with the deck and a sprung mass m_v supported by a spring of constant k_v and a damper with coefficient c_v . The multispan bridge is discretized by beam elements.

The equations of motion of a finite element representation of the bridge deck are

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{C}\dot{\mathbf{D}} + \mathbf{K}\mathbf{D} = \mathbf{F} \quad (4.1)$$

where $\mathbf{D}$ is the vector of the nodal d.o.f. of the bridge, $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix of the bridge structure, and $\mathbf{F}$ is the effective force vector. Vector $\mathbf{F}$ is obtained from the information presented in Fig. 4.1 as

$$\mathbf{F} = \mathbf{N}^T [-m_t(\ddot{\gamma} + \ddot{h}(x)) + c_v(\dot{u}_v - \dot{\gamma} - \dot{h}(x)) + k_v(u_v - \gamma - h(x)) - (m_v + m_t)g] \quad (4.2)$$

where $\mathbf{N}$ is the row vector of shape functions which relate the displacement of the point of contact to the nodal d.o.f. of the bridge deck, $h(x)$ is the function of pavement irregularities, γ is the vertical displacement of the point of contact between the bridge deck and the vehicle, and u_v is the vertical displacement of the vehicle relative to its position of equilibrium under self weight. Displacement γ and its first and second derivatives are given by (Kashif, 1992)

$$\gamma = \mathbf{N}\mathbf{D}$$

$$\dot{\gamma} = \frac{\partial \mathbf{N}}{\partial x} \mathbf{D}\dot{x} + \mathbf{N}\dot{\mathbf{D}} \quad (4.3)$$

$$\ddot{\gamma} = \frac{\partial^2 \mathbf{N}}{\partial x^2} \mathbf{D}\dot{x}^2 + 2\frac{\partial \mathbf{N}}{\partial x} \dot{\mathbf{D}}\dot{x} + \frac{\partial \mathbf{N}}{\partial x} \mathbf{D}\ddot{x} + \mathbf{N}\ddot{\mathbf{D}}$$

The equation of vertical motion of the sprung mass is

$$m_v \ddot{u}_v + c_v(\dot{u}_v - \dot{\gamma} - \dot{h}(x)) + k_v(u_v - \gamma - h(x)) = 0 \quad (4.4)$$

On substituting for γ , $\dot{\gamma}$, and $\ddot{\gamma}$ into Eqs. 4.2 and 4.4, the equations of motion of the bridge-vehicle system are given by

$$\begin{aligned} & \begin{bmatrix} m_v & 0 \\ 0 & M + m_t \mathbf{N}^T \mathbf{N} \end{bmatrix} \begin{bmatrix} \ddot{u}_v \\ \ddot{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} c_v & -c_v \mathbf{N} \\ -c_v \mathbf{N}^T & \mathbf{C} + 2m_t v \mathbf{N}^T \frac{\partial \mathbf{N}}{\partial x} + c_v \mathbf{N}^T \mathbf{N} \end{bmatrix} \begin{bmatrix} \dot{u}_v \\ \dot{\mathbf{D}} \end{bmatrix} + \\ & \begin{bmatrix} k_v & -k_v \mathbf{N} - c_v v \frac{\partial \mathbf{N}}{\partial x} \\ -k_v \mathbf{N}^T & \mathbf{K} + m_t v^2 \mathbf{N}^T \frac{\partial^2 \mathbf{N}}{\partial x^2} + (c_v v + m_t a) \mathbf{N}^T \frac{\partial \mathbf{N}}{\partial x} + k_v \mathbf{N}^T \mathbf{N} \end{bmatrix} \begin{bmatrix} u_v \\ \mathbf{D} \end{bmatrix} = \\ & \begin{bmatrix} 0 \\ -(m_v g + m_t g) \mathbf{N}^T \end{bmatrix} + \begin{bmatrix} k_v h(x) + c_v \dot{h}(x) \\ (-k_v h(x) - c_v \dot{h}(x) - m_t \ddot{h}(x)) \mathbf{N}^T \end{bmatrix} \quad (4.5) \end{aligned}$$

in which $v = \dot{x}$ is the speed of the vehicle and $a = \ddot{x}$ is the acceleration of the vehicle. Equations 4.5 contain two force vectors on the right hand side. The first force vector, which consists of $m_v g$ and $m_t g$, represents the force caused by moving vehicle itself; the second force vector, which consists of uneven road surface function $h(x)$ and its derivatives, represents the extra force arising from road roughness. By applying the normal coordinate transformation represented by

$$\mathbf{D} = \mathbf{Q} \mathbf{y} \quad (4.6)$$

$$u_v = Q_v \hat{u}_v \quad (4.7)$$

where, $\mathbf{Q}_{(N \times M)}$ is the mass-orthonormal mode shapes of the bridge, Q_v is $\frac{1}{\sqrt{m_v}}$, and M

is the number of mode shapes considered in the analysis, Eqs. 4.5 become

$$\begin{bmatrix} 1 & 0 \\ 0 & \mathbf{I} + \mathbf{m}^* \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_v \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \frac{c_v}{m_v} & \mathbf{c}_h \\ \mathbf{c}_v & \widehat{\mathbf{C}} + \mathbf{c}^* \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_v \\ \dot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} \omega_v^2 & \mathbf{k}_h \\ \mathbf{k}_v & \Lambda + \mathbf{k}^* \end{bmatrix} \begin{bmatrix} \mathbf{u}_v \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{p}_1 \end{bmatrix} + \begin{bmatrix} p_2 \\ \mathbf{p}_3 \end{bmatrix} \quad (4.8)$$

where

$$\begin{aligned} \mathbf{m}_{(M \times M)}^* &= m_t \mathbf{Q}^T \mathbf{N}^T \mathbf{N} \mathbf{Q} \\ \mathbf{c}_{(M \times M)}^* &= 2m_t v \mathbf{Q}^T \mathbf{N}^T \mathbf{N}_{,x} \mathbf{Q} + c_v \mathbf{Q}^T \mathbf{N}^T \mathbf{N} \mathbf{Q} \\ \mathbf{k}_{(M \times M)}^* &= m_t v^2 \mathbf{Q}^T \mathbf{N}^T \mathbf{N}_{,xx} \mathbf{Q} + (c_v v + m_t a) \mathbf{Q}^T \mathbf{N}^T \mathbf{N}_{,x} \mathbf{Q} + k_v \mathbf{Q}^T \mathbf{N}^T \mathbf{N} \mathbf{Q} \\ \mathbf{c}_{h(1 \times M)} &= -c_v \mathbf{Q}_v \mathbf{N} \mathbf{Q} \\ \mathbf{c}_{v(M \times 1)} &= -c_v \mathbf{Q}^T \mathbf{N}^T \mathbf{Q}_v \\ \mathbf{k}_{h(1 \times M)} &= -k_v \mathbf{Q}_v \mathbf{N} \mathbf{Q} - c_v v \mathbf{Q}_v \mathbf{N}_{,x} \mathbf{Q} \\ \mathbf{k}_{v(M \times 1)} &= -k_v \mathbf{Q}^T \mathbf{N}^T \mathbf{Q}_v \\ \frac{c_v}{m_v} &= 2\omega_v \xi_v \\ \mathbf{p}_{1(M \times 1)} &= -(m_v + m_t) g \mathbf{Q}^T \mathbf{N}^T \\ p_2 &= \mathbf{Q}_v (k_v h(x) + c_v \dot{h}(x)) \\ \mathbf{p}_{3(M \times 1)} &= (-k_v h(x) - c_v \dot{h}(x) - m_t \ddot{h}(x)) \mathbf{Q}^T \mathbf{N}^T \end{aligned} \quad (4.9)$$

$\mathbf{I}_{(M \times M)}$ is the identity matrix; $\Lambda_{(M \times M)}$ is the diagonal matrix of squares of frequencies of the bridge; $\widehat{\mathbf{C}}_{(M \times M)}$ is the orthogonal damping matrix, in which elements on the diagonal correspond to the modal damping ratios and are of the form $2\xi_i \omega_i$; and ω_i is the i th frequency of free vibration of the bridge. Equation 4.8 can be solved by using the average-acceleration method.

Usually, the vehicle running on the approach to a bridge will have vertical movement because of uneven pavement. The movement of the vehicle when it just enters the bridge can be determined as follows. When the vehicle is running on the approach, the equation of motion of the vehicle is given by

$$m_v \ddot{u} + c_v (\dot{u} - \dot{H}(x)) + k_v (u - H(x)) = 0$$

or

$$m_v \ddot{u} + c_v \dot{u} + k_v u = k_v H(x) + c_v \dot{H}(x) \quad (4.10)$$

where $H(x)$ is the uneven road surface function of the approach. Equations 4.10 can be solved by a process of numerical integration. The values of u , $\dot{u}$, and $\ddot{u}$ obtained from Eq. 4.10 at the time of entry to the bridge can be used as the initial values in integrating Eqs. 4.8.

The pavement roughness is expressed by $h(x)$ in Eqs. 4.5 and 4.8 and by $H(x)$ in Eq. 4.10. The pavement roughness is random in nature. However, in the following derivation the pavement roughness is taken to vary in a sinusoidal manner (see Fig. 4.1b)

$$h(x) = h_o \sin \frac{2\pi x}{L_o} \quad (4.11)$$

where h_o is the amplitude and L_o the wave length of the sine function. The second force vector in Eqs. 4.8 can now be expressed in the form

$$\left[\begin{array}{c} Q_v(k_v h_o \sin \frac{2\pi x}{L_o} + c_v \frac{2\pi v h_o}{L_o} \cos \frac{2\pi x}{L_o}) \\ (h_o(-k_v + m_t \frac{4\pi^2 v^2}{L_o^2}) \sin \frac{2\pi x}{L_o} - \frac{2\pi h_o}{L_o} (c_v v + m_t a) \cos \frac{2\pi x}{L_o}) \mathbf{Q}^T \mathbf{N}^T \end{array} \right] \quad (4.12)$$

In the finite element analysis, the continuous bridge structure is divided into a number of 2-D beam elements of the type shown in Fig. 4.2a. If the bridge is continuous without any hinges in between the ends, each element will have six nodal degrees of freedom. Within each element, the vertical displacement of any point is expressed by the weighted superposition of four shape functions. Also, the consistent loads produced by a load acting between nodes are obtained by using the same shape functions. The four shape functions

used here are as follows;

$$\begin{aligned}
 \varphi_1(x) &= 1 - 3\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)^3 \\
 \varphi_2(x) &= x\left(1 - \frac{x}{l}\right)^2 \\
 \varphi_3(x) &= 3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3 \\
 \varphi_4(x) &= \frac{x^2}{l}\left(\frac{x}{l} - 1\right)
 \end{aligned} \tag{4.13}$$

In order to model the expansion joints, which may exist at a support and do exist at the junction of a cantilever beam and a suspended beam, the shape functions of the beam element which is connected to the joint have to be modified. Two beam elements connected by a hinge are shown in Fig. 4.2b. The four shape functions of the left element (m th element) are as follows;

$$\begin{aligned}
 \varphi_1(x) &= 1 - \frac{3}{2}\left(\frac{x}{l}\right)^2 + \frac{1}{2}\left(\frac{x}{l}\right)^3 \\
 \varphi_2(x) &= x\left(1 - \frac{3}{2}\frac{x}{l} + \frac{1}{2}\left(\frac{x}{l}\right)^2\right) \\
 \varphi_3(x) &= \frac{3}{2}\left(\frac{x}{l}\right)^2 - \frac{1}{2}\left(\frac{x}{l}\right)^3 \\
 \varphi_4(x) &= 0
 \end{aligned} \tag{4.14}$$

The four shape functions of the right element ($m+1$ th element) are as follows;

$$\begin{aligned}
 \varphi_1(x) &= 1 - \frac{3}{2}\left(\frac{x}{l}\right) + \frac{1}{2}\left(\frac{x}{l}\right)^3 \\
 \varphi_2(x) &= 0 \\
 \varphi_3(x) &= \frac{3}{2}\left(\frac{x}{l}\right) - \frac{1}{2}\left(\frac{x}{l}\right)^3 \\
 \varphi_4(x) &= -\frac{x}{2}\left(1 - \left(\frac{x}{l}\right)^2\right)
 \end{aligned} \tag{4.15}$$

Equations 4.8 are solved by using the Newmark's average acceleration method, with a time step equal to one-tenth of the highest modal period included in the analysis. A computer program has been developed for the solution of Eqs. 4.8. The eigenvalue problem is solved by using the subspace iteration method, and the subroutine programs are adapted from SAP IV.

4.3 Bridge Traversed by Several Moving Vehicles

In a multispan continuous bridges, it is very likely that there are several vehicles traveling on the bridge at the same time. The equations of motion of a finite element representation

of the bridge deck are in this case

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{C}\dot{\mathbf{D}} + \mathbf{K}\mathbf{D} = \mathbf{F}_1 + \dots + \mathbf{F}_i + \dots + \mathbf{F}_n \quad (4.16)$$

where n is the number of the vehicles on the bridge and $\mathbf{F}_i$ is the effective force vector produced by the i th vehicle. Vector $\mathbf{F}_i$ can be expressed in the form

$$\mathbf{F}_i = \mathbf{N}_i^T [-m_{ti}(\ddot{\gamma}_i + \ddot{h}(x_i)) + c_{vi}(\dot{u}_{vi} - \dot{\gamma}_i - \dot{h}(x_i)) + k_{vi}(u_{vi} - \gamma_i - h(x_i)) - (m_{vi} + m_{ti})g] \quad (4.17)$$

The equation of motion of the i th vehicle is

$$m_{vi}\ddot{u}_{vi} + c_{vi}(\dot{u}_{vi} - \dot{\gamma}_i - \dot{h}(x_i)) + k_{vi}(u_{vi} - \gamma_i - h(x_i)) = 0 \quad (4.18)$$

The displacement of the point of contact of i th vehicle and bridge deck and its first and second derivatives are

$$\begin{aligned} \gamma_i &= \mathbf{N}_i \mathbf{D} \\ \dot{\gamma}_i &= \frac{\partial \mathbf{N}_i}{\partial x} \mathbf{D} \dot{x} + \mathbf{N}_i \dot{\mathbf{D}} \\ \ddot{\gamma}_i &= \frac{\partial^2 \mathbf{N}_i}{\partial x^2} \mathbf{D} \dot{x}^2 + 2 \frac{\partial \mathbf{N}_i}{\partial x} \dot{\mathbf{D}} \dot{x} + \frac{\partial \mathbf{N}_i}{\partial x} \mathbf{D} \ddot{x} + \mathbf{N}_i \ddot{\mathbf{D}} \end{aligned}$$

where $\mathbf{N}_i$ is the row vector of the shape functions which relate the displacement of the point of contact of the i th vehicle to the nodal d.o.f. of the bridge deck. On substituting for γ , $\dot{\gamma}$, and $\ddot{\gamma}$ into Eqs. 4.17 and 4.18, substituting the resulting equations into Eq. 4.16, and introducing the coordinate transformations $\mathbf{D} = \mathbf{Q} \mathbf{y}$ and $u_{vi} = Q_{vi} \hat{u}_{vi}$ ($i = 1, \dots, n$), the equations of motion of the bridge-vehicles system are given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathbf{I} + \sum_{i=1}^n \mathbf{m}_i^* \end{bmatrix} \begin{bmatrix} \ddot{\hat{u}}_{v1} \\ \ddot{\hat{u}}_{vi} \\ \ddot{\hat{u}}_{vn} \\ \ddot{\mathbf{y}} \end{bmatrix} + \begin{bmatrix} 2\omega_{v1}\xi_{v1} & 0 & 0 & \mathbf{c}_{h1} \\ 0 & 2\omega_{vi}\xi_{vi} & 0 & \mathbf{c}_{hi} \\ 0 & 0 & 2\omega_{vn}\xi_{vn} & \mathbf{c}_{hn} \\ \mathbf{c}_{v1} & \mathbf{c}_{vi} & \mathbf{c}_{vn} & \hat{\mathbf{C}} + \sum_{i=1}^n \mathbf{c}_i^* \end{bmatrix} \begin{bmatrix} \dot{\hat{u}}_{v1} \\ \dot{\hat{u}}_{vi} \\ \dot{\hat{u}}_{vn} \\ \dot{\mathbf{y}} \end{bmatrix} +$$

$$\begin{bmatrix} \omega_{v1}^2 & 0 & 0 & \mathbf{k}_{h1} \\ 0 & \omega_{vi}^2 & 0 & \mathbf{k}_{hi} \\ 0 & 0 & \omega_{vn}^2 & \mathbf{k}_{hn} \\ \mathbf{k}_{v1} & \mathbf{k}_{vi} & \mathbf{k}_{vn} & \Lambda + \sum_{i=1}^n \mathbf{k}_i^* \end{bmatrix} \begin{bmatrix} \hat{u}_{v1} \\ \hat{u}_{vi} \\ \hat{u}_{vn} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sum_{i=1}^n \mathbf{p}_{1i} \end{bmatrix} + \begin{bmatrix} (p_2)_1 \\ (p_2)_i \\ (p_2)_n \\ \sum_{i=1}^n \mathbf{p}_{3i} \end{bmatrix} \quad (4.19)$$

where $\mathbf{m}_i^*$, $\mathbf{c}_i^*$, $\mathbf{c}_{vi}$, $\mathbf{c}_{hi}$, $\mathbf{k}_i^*$, $\mathbf{k}_{vi}$, $\mathbf{k}_{hi}$, $\mathbf{p}_{1i}$, $(p_2)_i$, and $\mathbf{p}_{3i}$ are given by expressions similar to those in Eqs. 4.9, and I, $\hat{\mathbf{C}}$, and Λ are similar to those in Eqs. 4.8.

4.4 Identification of Controlling Parameters

The differential equation of free vibration of a beam is (Timoshenko, 1972)

$$\frac{\partial^4 w}{\partial x^4} + \frac{A\rho}{EI} \frac{\partial^2 w}{\partial t^2} = 0 \quad (4.20)$$

in which w = the deflection of the beam, positive upwards; EI = the flexural rigidity of the beam; ρ = the beam mass per unit volume; A = the beam cross-section area.

Considering the normal modes of vibration which are obtainable by the process of separation of variables from Eq. 4.20, one may write

$$w(x, t) = \varphi(x) \cdot T(t) \quad (4.21)$$

in which $\varphi(x)$ is the function of the deflection curve for a mode of vibration. Substituting Eq. 4.21 in Eq. 4.20, it becomes

$$\frac{T''}{T} + \frac{EI}{A\rho} \frac{\varphi'''}{\varphi} = 0 \quad (4.22)$$

Denoting the frequency of sinusoidal oscillations by ω , Eq. 4.22 can be expressed as

$$\frac{T''}{T} = -\omega^2 \quad (4.23)$$

$$\frac{EI}{A\rho} \frac{\varphi'''}{\varphi} = \omega^2 \quad (4.24)$$

From Eq. 4.24

$$\frac{\partial^4 \varphi}{\partial x^4} - \frac{A\rho\omega^2}{EI}\varphi = 0 \quad (4.25)$$

Introducing the dimensionless coordinate

$$x = L\zeta$$

where L = a representative length in the multispan beam, say the total length. Equation 4.25 becomes, in dimensionless coordinate,

$$\frac{\partial^4 \varphi}{\partial \zeta^4} - K^4 \varphi = 0 \quad (4.26)$$

where

$$K^4 = \frac{A\rho L^4 \omega^2}{EI} \quad (4.27)$$

Since Equation 4.26 is in dimensionless unit and coordinate, beam A and beam B , both with the same number of spans, the same boundary conditions, the same span ratios, and the same relative locations of hinges, will have identical mode shapes as well as similar frequencies to within a scale factor. The scale factor is given by

$$\frac{\omega_{iA}}{\omega_{iB}} = \frac{\omega_{jA}}{\omega_{jB}} = \left(\frac{A_B \rho_B L_B^4 E_A I_A}{A_A \rho_A L_A^4 E_B I_B} \right)^{\frac{1}{2}} \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, N \quad (4.28)$$

The definitions of span ratio λ for a two-span continuous beam and a symmetrical three-span beam are shown in Fig. 4.3a. For a symmetrical three-span cantilever beam, we need two parameters to indicate the span ratio and the relative hinge locations. The parameters λ and ψ are defined in Fig. 4.3b. From the discussion above, it is evident that the mode shapes of the continuous beam are controlled by λ and the number of spans, while those of the cantilever beam are controlled by λ and an additional parameter ψ .

In Section 2.3.2, we defined three parameters related to the bridge-vehicle system for single span bridges. They are the speed parameter α , the mass ratio κ , and the frequency ratio ϕ . The speed parameter was defined as

$$\alpha = \frac{\pi v}{\omega_1 L} = \frac{T_1 v}{2L} \quad (4.29)$$

in which L is the length of the single span bridge, v is the constant speed of the vehicle, ω_1 is the fundamental frequency of the bridge, and T_1 is the fundamental period of the bridge. Since the vibration of a single span bridge depends mainly on the first vibration mode which is $\sin(\frac{\pi x}{L})$ or $\sin(\frac{\pi vt}{L})$ if the acceleration of the vehicle is zero, the speed parameter can be considered to be the ratio of $\frac{\pi v}{L}$ and ω_1 , or half of the ratio of the bridge fundamental period T_1 and the vehicle traversing time $\frac{L}{v}$.

In a multispan bridge structure the critical points are usually related to the span with the maximum length. Therefore, the definitions of parameters would be associated with that span. The parameters are thus defined for the case of a vehicle moving at the constant speed v as follows

$$\begin{aligned} \alpha &= \frac{\pi v}{\omega_1 L_L} \\ \phi &= \frac{\omega_v}{\omega_1} \\ \kappa &= \frac{m_v}{m_L} \end{aligned} \quad (4.30)$$

in which L_L is the length of the longest span in the multispan bridge, and m_L is the total beam mass of the longest span in the multispan bridge.

In order to identify the parameters governing the dynamic response of the multispan bridge, damping in both the bridge and the vehicle is neglected, and road surface roughness is not considered. Also, it is assumed that the unsprung mass m_t is negligible, and that the

vehicle is moving with a constant velocity v . The equations of the vehicle-bridge system, Eqs. 4.5, are reduced to

$$\begin{bmatrix} m_v & 0 \\ 0 & \mathbf{M} \end{bmatrix} \begin{bmatrix} \ddot{u}_v \\ \ddot{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} k_v & -k_v \mathbf{N} \\ -k_v \mathbf{N}^T & \mathbf{K} + k_v \mathbf{N}^T \mathbf{N} \end{bmatrix} \begin{bmatrix} u_v \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} 0 \\ -m_v g \mathbf{N}^T \end{bmatrix} \quad (4.31)$$

By applying the coordinate transformation

$$u_v = \delta \bar{u}_v \quad (4.32)$$

$$\mathbf{D} = \frac{\kappa g}{\omega_1^2} \mathbf{Q} \bar{\mathbf{y}} \quad (4.33)$$

where $\delta = \frac{m_v g}{k_v} = \frac{g}{\omega_v^2}$ is the static deflection of the vehicle under its own weight, ω_1 is the first frequency of the bridge, and $\mathbf{Q}$ is the modal matrix, Eqs. 4.31 become

$$\begin{bmatrix} 1 & 0 \\ 0 & \kappa \phi^4 \bar{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \frac{d^2 \bar{u}_v}{d\bar{t}^2} \\ \frac{d^2 \bar{\mathbf{y}}}{d\bar{t}^2} \end{bmatrix} + \begin{bmatrix} 1 & -\kappa \phi^2 \mathbf{N} \mathbf{Q} \\ -\kappa \phi^2 \mathbf{Q}^T \mathbf{N}^T & \kappa \phi^2 \bar{\mathbf{K}} + \kappa^2 \phi^4 \mathbf{Q}^T \mathbf{N}^T \mathbf{N} \mathbf{Q} \end{bmatrix} \begin{bmatrix} \bar{u}_v \\ \bar{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} 0 \\ -\kappa \phi^2 \mathbf{Q}^T \mathbf{N}^T \end{bmatrix} \quad (4.34)$$

where $\bar{t} = \omega_v t$; $\bar{\mathbf{M}} = \frac{1}{m_L} \mathbf{Q}^T \mathbf{M} \mathbf{Q}$ is the normalized mass matrix; and $\bar{\mathbf{K}} = \frac{1}{m_L \omega_1^2} \mathbf{Q}^T \mathbf{K} \mathbf{Q}$.

Referring to Fig. 4.1c, it is seen that the vector $\mathbf{N}$ is a function of the relative location of the vehicle on the beam or a function of vt/L . Since $vt/L = \alpha \omega_v t / \pi \phi = \alpha \bar{t} / \pi \phi$, $\mathbf{N}$ becomes a function of α , ϕ , and $\bar{t}$. Also, the mode shapes $\mathbf{Q}$ depend on λ and ψ . Equations 4.34 indicate that if two beams subjected to a sprung mass, moving along the same directions, have the same values of α , ϕ , κ , λ , ψ , and the number of spans, their deflections are identical to within a scale factor.

4.5 Practical Ranges of Controlling Parameters

Characteristic data for several existing continuous and cantilever bridges have been collected from a number of different sources. These data have been classified and listed

according to the number of spans and the span ratio in Tables. 4.1a and b. All of the bridges are girder or box type bridges. Description of these bridges is provided in as much detail as is available. It is seen from the data that three span bridge is the most popular type of continuous bridge, and the span ratio generally ranges from 0.5 to 1.0. Although the information relating to hinges in cantilever bridges is not available from the table, most of the bridges listed are symmetrical about the central line of the main span where the suspended span most likely exists. It can reasonably be assumed that the usual range of ψ is $0 < \psi < 0.5$.

A study was carried out by Kashif (1992) on the practical ranges of parameters α , κ , and ϕ of single span bridges. He found that the practical ranges for the values of parameters of α , κ , and ϕ were as follows: $\alpha \leq 0.30$, $\kappa \leq 1.50$, and $\phi \leq 1.20$. Although the conclusions reached by Kashif included data from single span truss bridges, which obviously have quite different total mass compared to the girder or box bridges having the same length and width, they provide a good starting point for the present study of multispan bridges. This is particularly so because the definitions of the parameters used in the present study are based only on one span of the multispan bridge.

Since the data related to the mass and the fundamental frequencies of the bridges listed in Table 4.1 are incomplete, further literature search was carried out to obtain adequate data. Some of the single-span and three-span bridge models developed by other researchers (Huang et. al. 1992, 1993, Wang et. al. 1995) are listed in the Table 4.2. The first three columns of Table 4.2 show the bridge span, dead weight, and fundamental frequency. Note that the dead weight corresponds to the longest span of the bridge. The remaining columns of the table display the values of the three controlling parameters α , κ ,

and ϕ which are calculated on the basis of the following information from Kashif (1992);

$$\begin{aligned} v : \quad \text{vehicle speed} &= 100 \text{ km/hr} \\ f_v : \quad \text{vehicle frequency} &= 1.5 - 4.0 \text{ Hz} \quad \text{average value } 2.75 \text{ Hz} \\ W_v : \quad \text{vehicle weight} &= 140 \text{ kips} \end{aligned}$$

Based on the data provided in Tables 4.1 and 4.2 and the conclusions drawn by Kashif (1992), the limits for the parameters can be established as follows;

$$\begin{aligned} 0.5 &\leq \lambda \leq 1.0 \\ 0 &< \psi < 0.5 \\ \alpha &\leq 0.3 \\ \kappa &\leq 0.5 \\ \phi &\leq 1.5 \end{aligned}$$

It is recognized that the value represent extremes, and the parameters for most bridges will lie in a narrow range.

4.6 Continuous Bridges

Two and three-span bridges are the most common types of continuous bridges. Therefore in this work a study of dynamic response of these bridges has been carried out. Models of two-span and symmetrical three-span continuous bridges with various span ratios are designed in accordance with AASHTO. Tables 4.3 and 4.4 list the information related to these models. Note that the dead weight corresponds to the longest span of the bridge. To simplify the analysis, all of the bridge models are assumed to have the same cross-sectional area or the same weight per unit length. The main span or central span of these bridge models is kept at 40 m, and the length of side span varies to make the span ratio different.

4.6.1 Modal Analysis

Figure 4.4 shows the first ten vibration modes for two-span bridges with two different values of λ , namely, 0.5 and 1.0. The mode shapes for two different three-span bridges are

shown in Fig. 4.5. It is of interest to note a two-span bridge and a three span bridge have several identical frequencies and mode shapes if their λ values are identical. For $\lambda = 1.0$, 1st, 3rd, 5th, and 7th frequencies and modes of the two-span bridge are the same as the 1st, 4th, 7th, and 10th frequencies and modes respective of the three-span bridge. In fact, these are also the 1st, 2nd, 3rd, 4th frequencies of a single span bridge whose span is equal to one span of the multispans bridge and which has an identical cross-section. For $\lambda = 0.5$, 2nd, 5th, and 8th frequencies and modes of the two-span bridge are the same as the 2nd, 6th, and 10th frequencies and modes of the three-span bridge.

4.6.2 Bridges with Identical Values of Controlling Parameters

As shown in the previous sections of this chapter, continuous beam models with the same number of spans and the same values of the parameters α , κ , ϕ , and λ will have identical normalized responses. To verify this, two two-span continuous beams with a value of $\lambda = 0.5$ are analyzed for a vehicle moving from the short span to the long span. The beam dimensions and properties are as follows.

$$\rho = 2.4 \text{ tonne}/m^3, E = 2.78 \times 10^7 \text{ kN}/m^2$$

Beam X:

Spans:	$15m + 30m$
Span Ratio:	$0.5 : 1$
Cross-Section Area:	4.8384 m^2

Beam Y:

Spans:	$20m + 40m$
Span Ratio:	$0.5 : 1$
Cross-Section Area:	6.3947 m^2

Analyses are carried out for two values of $\kappa = 0.1$ and 0.5 , two values of $\alpha = 0.1$ and 0.2 , and several values of ϕ ranging between 0.1 and 1.2 . Figure 4.6 show the deflection amplification factors at the middle of the main span of the two continuous beams. It is

evident that responses of the two beams are identical.

4.6.3 Two-Span Bridges

The direction of movement of a vehicle affects the dynamic bridge response. This is clear from Figs. 4.7a and b which show dynamic responses of a beam with $\lambda = 0.5$ produced by a vehicle moving in two different directions for $\alpha = 0.2$, $\kappa = 0.1$ and several values of ϕ . In each figure the envelope curve is shown in dashed line and gives the maximum response of the beam for the specified values of the governing parameters. All further response results presented in this chapter for a given set of parameters represent the maximum of the values obtained corresponding to the movement of the vehicle from two different directions.

In the following paragraphs a study will be made of the effect of parameters λ , α , κ , and ϕ on the response of two-span bridges. To study the effect of a certain parameter, only that parameter will be varied while the others are kept fixed.

4.6.3.1 Deflection

Figures 4.8a through 4.8d present the relationship between ϕ and deflection amplification factor at the middle of main span for $\lambda = 0.5$ and different values of α and κ . The dynamic amplification factor is defined as the ratio of the maximum dynamic deflection at a certain location to the maximum static deflection at the same location produced by the given vehicle moving at a very slow speed. The latter deflection is essentially static in nature. One important feature is observed. There is a series of local peaks for different values of κ in each figure, and the location of this series of local peaks varies with α . For example, in Fig. 4.8a corresponding to $\alpha = 0.1$, peaks appear at $\phi = 0.36$; In Fig. 4.8b for $\alpha = 0.15$, they are at $\phi = 0.52$. It is apparent that the maximum bridge response

does not have to occur at $\phi=1.0$. The dependence of deflection amplification factor on the parameter λ for different combinations of $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and 0.5 , and $\phi = 0.5$ and 1.0 is shown in Fig. 4.9 where λ varies between 0.5 and 1.0 . No systematic variation is observed.

4.6.3.2 Positive moment

Figure 4.10 presents the relationship between ϕ and moment amplification factor at the middle of the main span for $\lambda = 0.5$, $\alpha = 0.1$ and 0.2 , and several values of κ . As for deflection, the moment amplification factor is defined as the ratio of the maximum dynamic moment at a given location to the maximum static moment at the same location, where the static moment results from the passing of the vehicle at a very slow speed. As in the case of deflection the maximum dynamic moment does not occur when $\phi = 1.0$. The variation of moment amplification factor with the parameter λ , for different combinations of α , κ , and ϕ , is shown in Fig. 4.11. Again, as for deflection no systematic variation is observed.

4.6.3.3 Negative moment

The relationships between ϕ and moment amplification factor at the intermediate support are illustrated in Fig. 4.12 for $\lambda = 0.5$, $\alpha = 0.1$ and 0.2 , and different values of κ . The variation of moment amplification factor with the parameter λ , for different combinations of α , κ , and ϕ , is shown in Fig. 4.13. Observations similar to these for deflection and moment amplifications apply as well to for negative moment amplification.

4.6.3.4 Comparison of Deflection and Moment Amplification Factors

Comparison of the results presented in Figs. 4.8, 4.10, and 4.12, and similarly in Figs. 4.9, 4.11, and 4.13 shows that the deflection and moment amplification factors may be varying in more or less similar manner with the variation in the values of governing parameters. Additional figures are presented here to assist in the comparison of deflection and moment amplification factors. Figure 4.14 shows the variation in the responses of the beam with ϕ for $\alpha = 0.1$ and 0.2 and $\kappa = 0.1$ and 0.5 , while Fig. 4.15 shows the variation in the responses of the beam with λ for $\phi = 1.0$, $\alpha = 0.1$ and 0.2 , and $\kappa = 0.1$ and 0.5 . The deflection amplification factor and the moment amplification factor at the middle of the main span vary in similar manner with variations in ϕ or λ , and the former is always greater than the latter. The maximum difference between the two is of the order of 18%. The variation of the negative moment amplification is somewhat of a different nature. It varies higher or lower than the deflection amplification factor, but is in all cases higher than the positive moment amplification factor.

4.6.4 Three-Span Bridges

Analytical studies similar to those for two-span bridges are carried out for symmetrical three-span bridges. The deflection and moment amplifications at the middle of central span and moment amplifications at the intermediate support are studied. Since the bridge structures are symmetrical, the vehicle is assumed to move only in one direction. However, dynamic responses at the two intermediate supports are compared and the larger value is picked up. The results obtained are described in the following paragraphs.

4.6.4.1 Deflection

Figures 4.16a through 4.16d present the relationship between ϕ and deflection amplification factor at the middle of central span for $\lambda = 0.5$ and different values of α and κ . As for two-span bridges, a series of local peaks is observed in each figure. The locations of the local peaks vary with α . Again, the maximum values do not necessarily occur at $\phi = 1.0$. The variation of deflection amplification factor with λ for different combinations of α , κ , and ϕ is shown in Fig. 4.17, whose λ varies between 0.5 and 1.0. No systematic variation is observed.

4.6.4.2 Positive moment

Figures 4.18 and 4.19 present the relationship between ϕ and moment amplification factor and the relationship between λ and moment amplification factor at the middle of central span, respectively. Observations similar to those for deflection amplification factors apply.

4.6.4.3 Negative moment

Figures 4.20 and 4.21 present the relationship between ϕ and moment amplification factor and the relationship between λ and moment amplification factor at the intermediate support, respectively. Again, observations similar to those for deflection amplification factors apply.

4.6.4.4 Comparison of Deflection and Moment Amplification Factors

In order to compare the deflection and moment amplifications, some of results presented previously are shown again in Figs. 4.22 and 4.23. Figure 4.22 shows the variation of amplification factors with ϕ for $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and 0.5 , and $\lambda = 0.5$. Figure 4.23 shows the variation of amplification factors with λ for $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and

0.5, and $\phi = 1.0$. As for the two-span bridges, deflection amplification factor and moment amplification factor at the middle of central span have very similar variation. The variation of amplification factor for negative moment is of a somewhat different nature. The moment amplification factor at the middle of central span has the smallest value in all the cases.

4.6.5 Comparison of Single, Two, and Three-Span Bridges

It is of interest to compare the dynamic response of a simply-supported bridge with those of continuous bridges having similar cross-section properties and span lengths. The simply-supported beam has a span of 40 m, a cross-section area of 6.3947 m^2 . A two-span beam with $\lambda = 1.0$ is chosen from Table 4.3, and a three-span beam with $\lambda = 1.0$ is chosen from Table 4.4. Since the length and cross-section area of each span of these two continuous beams are the same as those of the single-span beam, these three beam models will have the same fundamental frequency, that is, 1.89 Hz. The three beam models are analyzed for $\alpha = 0.1$ and 0.2, $\kappa = 0.1$ and 0.5, and several values of ϕ . Note that since the definitions of parameters α , κ , and ϕ of a multispan bridge are based on the longest span (in the present case all spans are equal), the weight, speed, and heave frequency of the moving vehicle will be identical for all three beam models. The deflection amplification factors at the middle of first span in each beam model are shown in Figs. 4.24a through 4.24d. The vehicle runs in only one direction as shown in figures. With only few exceptions, the single-span bridge has the maximum response.

4.6.6 The Effect of Damping in the Bridge

In Eqs. 4.8, the bridge damping is introduced through the orthogonal damping matrix $\hat{\mathbf{C}}$, in which elements on the diagonal correspond to the modal damping ratios and are of the form $2\xi_i\omega_i$ (ω_i is the i th frequency of free vibration of the bridge). A three-span

beam model with $\lambda = 0.5$ is analyzed for four values of ξ , namely, 1%, 2%, 5%, and 10%. In each case, all the mode shapes included in the analysis are assumed to have the same value of ξ . The deflection amplification factor at the middle of central span is shown in Fig. 4.25 as a function of ϕ for $\alpha = 0.1$ and 0.2, $\kappa = 0.1$ and 0.5, and different values of ξ . For comparison the responses corresponding to $\xi = 0$ are also drawn. Generally, The inclusion of bridge damping does not change the nature of the variation of response but reduces its amplitude. When $\xi = 10\%$, the maximum reduction is of the order of 4%. The moment amplification factor at the intermediate support is drawn in Fig. 4.26 for the same values of parameters as those of Fig. 4.26. Again, damping does not vary the nature of the variation of response but reduce its amplitude. The reduction in responses can be found to be as much as 10% when $\xi = 10\%$. Generally, the influence of bridge damping to the amplification factor for negative moment at the intermediate support is larger than that for the amplification factor for deflection.

4.6.7 The Effect of Damping in the Vehicle

The vehicle damping ratio is expressed as $\xi_v = \frac{c_v}{2k_v m_v}$. Figure 4.27 displays the effect of vehicle damping on the deflection amplification factor. Similarly, Fig. 4.28 shows the effect of vehicle damping on moment amplification factor. Several values of vehicle damping ratio ξ_v , namely, 1%, 2%, 3%, and 5%, are considered. Responses of bridge for $\xi_v = 0$ are also shown in both figures. Vehicle damping is seen to have very little effect on the bridge response. In general, vehicle damping results in a reduction in the peak.

4.6.8 Design Recommendations for Continuous Bridges

From the results presented in previous sections, it is seen that in continuous bridges, the largest dynamic amplification is either in the moment at the intermediate supports or in

the deflection at the middle of the main span. Design recommendations must therefore consider both of them. The procedure used to create a design amplification curve is discussed here through an example. A two-span bridge with $\lambda = 0.5$ is analyzed. Figure 4.29 shows the variations of dynamic amplification with α for a range of ϕ from 0 to 2.4. The curves have been drawn for a single value of κ . Figure 4.29a is for deflection amplification factor at the middle of main span, and Fig. 4.29b is for moment amplification factor at the intermediate support. Since the bridge is not symmetrical, the response of the bridge is different for different directions of vehicle movement. The vehicle is considered to run in both directions. The envelope in each figure gives the largest value of amplification factor for any given value of α . Combining two envelopes, we can get a design curve for a given value of κ .

Two sets of design recommendations are generated for two-span and three-span bridges separately. Figures 4.30a through 4.30f have been drawn for two-span bridges. Several different values of κ and λ are used. Figures 4.31a through 4.31f are for three-span bridges and correspond to the same values of κ and λ as those for the two-span bridges. Damping in both the bridge and the vehicle is not included, nor is pavement roughness.

As shown in Section 4.6.5, the response of a single span bridge is usually larger than that of a continuous bridge. It is therefore of interest to compare the design charts for single span and continuous bridges. Design envelopes for a two-equal-span and a three-equal-span continuous bridges are selected from Figs. 4.30d and 4.31d, and design envelopes for the single span right bridge are taken from Section 3.7 Fig. 3.42a. A comparison is presented in Fig. 4.32 among three bridges for different values of mass ratio κ , namely, 0.1, 0.3, and 0.5. In the range of $0 < \alpha < 0.18$ these three bridges have similar design amplification values. The single span bridge is found to have larger design amplification

value for most of the cases when $0.18 \leq \alpha < 0.30$. This shows that the design charts for single span bridges can also be used for the design of continuous bridges. This would simplify the use of such charts for the design of the bridge types studied here.

4.7 Cantilever Bridges

Cantilever bridges are usually built to allow free longitudinal displacement of the superstructure caused by temperature variation and concrete creep. The dynamic response of such bridges is likely to be different from both single-span and continuous bridges studied early. A set of bridge models whose details are provided in Table 4.5 are analyzed. All the models are symmetrical and have a 40 m central span and two 20 m side spans. Thus, the value of λ is 0.5 for all the models. The locations of hinges and hence ψ is assumed to vary from 0.05 to 0.45.

4.7.1 Modal Analysis

Figures 4.33a and b show the first ten vibration modes for bridge models with $\psi = 0.2$ and 0.4, respectively. It is of interest to note that in second modes the suspended spans rotate, as a rigid body, about their midpoints.

4.7.2 Deflection and Moment Amplification

4.7.2.1 Deflection

Figures 4.34a through 4.34d present the relationship between ϕ and deflection amplification factor at the tip of overhang for $\psi = 0.4$ and different values of α and κ . It is seen that the maximum response does not necessarily occur at $\phi = 1.0$. The variation of deflection amplification factor with parameter ψ for different combinations of α , κ , and ϕ is shown in Fig. 4.35, where ψ varies between 0.05 and 0.45. No systematic variation is observed.

4.7.2.2 Negative Moment

The relationships between ϕ and the amplification factor for moment at the intermediate support are illustrated in Figs. 4.36a and b for $\psi = 0.4$, $\alpha = 0.1$ and 0.2 , and different values of κ . As in the case of deflection, larger response is not necessarily at $\phi = 1.0$. The variation of moment amplification factor with parameter ψ for different combinations of α , κ , and ϕ is shown in Fig. 4.37. Again, there is no systematic variation.

4.7.2.3 Comparison of Deflection and Moment Amplification Factors

In order to compare the deflection amplification factor and the moment amplification factors, some of results presented previously are presented again in Figs. 4.38 and 4.39. Figure 4.38 shows the variation of amplification factors with ϕ for $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and 0.5 , and $\psi = 0.4$. The amplification factor for deflection at tip and the amplification factor for moment at the intermediate support have very similar variation, and the latter is always larger than the former. However, the maximum difference is not large, being of the order of 5%. Figure 4.39 shows the variation of amplification factors with ψ for $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and 0.5 , and $\phi = 1.0$. Again, the two sets of variations are very similar. There is a transition point ψ_0 between $\psi = 0.15$ and $\psi = 0.2$. The value of deflection amplification factor is greater than the moment amplification factor when $\psi < \psi_0$, and vice versa when $\psi > \psi_0$.

4.7.3 The Effect of Damping in the Bridge

A three-span cantilever beam with $\lambda = 0.5$ and $\psi = 0.4$ is analyzed for four values of bridge damping ξ , namely, 1%, 2%, 5%, and 10%. In each case, all the mode shapes included in the analysis are assumed to have the same value of ξ . The deflection amplification factor

at the tip of overhang is shown in Fig. 4.40 as a function of ϕ for $\alpha = 0.1$ and 0.2 , $\kappa = 0.1$ and 0.5 , and different values of ξ . For comparison the responses corresponding to $\xi = 0$ are also shown. Bridge damping reduces the response of the bridge without changing the nature of variation with ϕ . When $\xi = 10\%$, the maximum reduction can be as much as 12%. Amplification factors for moment at the intermediate supports are shown in Fig. 4.41 for the same values of parameters as those for deflection amplifications. The reduction in responses due to damping is similar to that for deflection amplifications.

4.7.4 The Effect of Damping in the Vehicle

Figures 4.42 and 4.43 display the effect of vehicle damping on deflection amplifications and moment amplifications respectively. Four values of vehicle damping ratio ξ_v are considered: 1%, 2%, 3%, and 5%. Responses of the bridge for $\xi_v = 0$ are also shown in both figures. As for continuous beams, vehicle damping reduces the peak response. However, the influence of vehicle damping is quite small.

4.7.5 Design Recommendations for Cantilever Bridges

Design amplification curves are generated for cantilever bridges with $\lambda = 0.5$ following a procedure similar to that described for continuous bridge. Envelope curves for dynamic amplifications have been drawn for different values of κ and ψ . Figures 4.44a through 4.44d show the results. Damping in both the bridge and the vehicle is not included, nor is the pavement roughness.

4.8 Comparison of Continuous and Cantilever Bridges

It is of interest to compare the responses of continuous and cantilever bridges of similar cross-section and spans. A three-span continuous beam and four three-span cantilever

beams are chosen. All the bridge models have the same main span of 40 m, side spans of 20 m, and cross-section area of 6.3947 m^2 . Thus, all the models have the same value of span ratio $\lambda = 0.5$. The values of ψ for the four cantilever beam are 0.1, 0.2, 0.3, and 0.4, respectively. Deflection amplification factors at the middle of main span of all five models are shown in Fig. 4.45. Moment amplification factors at the intermediate support are shown in Fig. 4.46. In general cantilever bridges show larger responses than continuous bridges.

4.9 Case Study—Confederation Bridge

4.9.1 Introduction

The Confederation Bridge, linking Prince Edward Island to mainland New Brunswick, is the longest bridge ever built over ice-covered water and one of Canada's greatest engineering accomplishments. The 13-kilometer bridge is designed to have a service life of 100 years. The bridge is likely to be subjected to significant dynamic loads due to heavy vehicles and long spans. It is to be noted that the design of such long-span bridge is not covered by design standards or codes.

4.9.2 Modeling of the Bridge

The main part of the Confederation Bridge consists of 45 spans in 22 repetitive units. Each unit is a one-bay rigid frame of 250 m long with two 97.5 m overhangs. Adjacent rigid frames are connected by a simply supported drop-in span of 55 m long.

A finite element method of analysis is used for the present study. Two rigid frames with three simply supported drop-in spans are modeled here by two-dimensional beam elements. One half of the structure is shown in Fig. 4.47. Each node has three degrees

of freedom, which are longitudinal and vertical displacements, namely, x and y , and a rotation θ . The bridge model is symmetrical with respect to the centerline passing through the mid point of the center span. In the analysis model, variations in the stiffness and mass properties of the box girder and bridge deck are all considered. The variation of the cross-sectional properties of the piers due to changes in the dimensions along their height is also considered. The masses of the pier shafts are taken into account by adding concentrated mass on the corresponding nodes. The expansion joints between drop-in span and overhang of the rigid frame are modeled as pins. The analysis model including the bridge superstructure and the piers has 126 nodes with 124 elements.

4.9.3 Modal Analysis

The vibration properties and characteristics of the bridge system are determined by a modal frequency analysis. The vibration frequencies and periods of the first 30 modes are listed in Table 4.6. The fundamental natural mode of the bridge has a vibration frequency of 0.508 Hz. The vibration frequencies of the 2nd to the 30th modes range from 0.6 Hz to 20 Hz. The first 10 vibration mode shapes are plotted in Figs. 4.48a and b.

4.9.4 Evaluation of Dynamic Response Due to the Vehicular Load

4.9.4.1 Deflection Amplification

Figures 4.49a to 4.49c show the variation of deflection amplification factor as a function of vehicle speed for different values of vehicle frequency. Responses of three different points are shown. These three points are the middle of rigid frame (point No.1, see Fig. 4.47), the tip of the overhang of rigid frame (point No.2) where the expansion joint is located, and the middle point of the simply supported drop-in span (point No.3). The vehicle speed is assumed to vary from 50 km/hr to 160 km/hr, and the vehicle frequencies are 1.0, 2.0,

3.0, and 4.0 Hz. The vehicle weight is taken as 60 tonne in all the cases. The results show that the vehicle frequency has little effect on responses of the bridge. Figure 4.50 shows the variation of deflection amplification factor with the vehicle speed for vehicle frequency = 3.0 Hz and for a series of vehicle weights ranging from 20 tonne to 100 tonne. It can be seen that the bridge responses corresponding to the different vehicle weights are almost the same. This is explained by the fact that the difference between vehicle mass and bridge mass is so large that neither vehicle weight nor vehicle frequency change the bridge response significantly. Based on Figs. 4.49 and 4.50, it can be concluded that only the vehicle speed has a significant effect on the bridge response expressed in terms of dynamic amplification.

Figure 4.51 displays the effect of damping in the bridge on its response. Damping is assumed as 2%, the vehicle weight as 60 tonne, and the vehicle frequency as 3.0 Hz. As expected, damping reduce the response, the maximum reduction being 10%.

4.9.4.2 Moment Amplification

Figures 4.52a to 4.52c show the variation of moment amplification factor as a function of vehicle speed for different values of vehicle frequency. Responses of three points are drawn. These three points are the middle of rigid frame (point No.1), the base of the overhang of the rigid frame (point No.4), and the intersection between beam and pier (point No.5). The vehicle weight is assumed to be 60 tonne, and the vehicle frequencies are 1.0, 2.0, 3.0, and 4.0 Hz. The results shows that the vehicle frequency has little effect on the response of the bridge.

Figure 4.53 shows the variation of moment amplification factor with the vehicle speed for vehicle frequency = 3.0 Hz and for a series of vehicle weight ranging from 20 tonne to

100 tonne. Again, the weight of vehicle has little effect on the amplification factor.

Figure 4.54 displays the effect of bridge damping on bridge responses. The damping ratio is 2%. The vehicle weight is 60 tonne, and the vehicle frequency is 3.0 Hz. Damping reduces the response and the reduction of moment amplification at point No.5 is as much as 15% for bridge damping ratio of 2%.

4.9.4.3 Comparison of Deflection and Moment Amplifications

Several results are presented again to display the relationship between deflection and moment amplification factors. The deflection and moment amplifications at point No.1 are shown in Fig. 4.55a, and the deflection amplification at point No.2 and the moment amplification at point No.4 are shown in Fig. 4.55b. The vehicle weight is taken as 60 tonne, and the vehicle frequency is 3.0 Hz. It can be seen that the amplification factor for positive moment is smaller than that for the deflection, and the amplification factor for negative moment is larger than that for the deflection, but the nature of their variations are similar.

4.9.4.4 Accelerations

Figures 4.56a and 4.56b show the variation of accelerations at the middle of the rigid frame (point No.1) and the tip of the overhang of the rigid frame (point No.2) as a function of vehicle speed for different values of vehicle frequency. The vehicle speed varies from 50 km/hr to 160 km/hr, and the vehicle frequencies are 1.0, 2.0, 3.0, and 4.0 Hz. The vehicle weight is 60 tonne in all the cases. The results show that the effect of vehicle frequency on the acceleration at each point is not strong and the accelerations increase slowly and steadily with the increase of the vehicle speed. The envelope in each figure gives the maximum accelerations of the bridge corresponding to a certain vehicle weight,

ie. 60 tonne. The acceleration at the tip of the overhead of the rigid frame (point No.2) is always larger than that at the middle of rigid frame (point No.1).

Figure 4.57 shows the envelope curves for a series of vehicle weight ranging from 20 tonne to 100 tonne at an interval of 20 tonne. The vehicle weight does affect the accelerations. The heavier the vehicle is, the larger the accelerations of the bridge are.

Figure 4.58 displays the effect of bridge damping on accelerations. The vehicle weight is 60 tonne, and the vehicle frequency is 3.0 Hz. The reduction of acceleration could be as much as 50% at point No.1 for bridge damping ratio of 2%.

4.10 Summary and Conclusions

Equations of motion are generated for the dynamic response of multispan continuous and cantilever bridges produced by the passage of a single vehicle as well as multiple vehicles. The governing parameters are identified and the effect of variation in these parameters on the response of the bridge is studied. The following conclusions can be drawn from the analytical results presented in this chapter.

1. Parameters α , κ , ϕ , λ , and the number of spans govern the response of a continuous bridge under the passage of a moving vehicle. For cantilever bridges, an additional parameter ψ is needed.
2. The practical ranges for the values of parameters of λ , ψ , α , κ , and ϕ are

$$0.5 \leq \lambda \leq 1.0$$

$$0 < \psi < 0.5$$

$$\alpha \leq 0.3$$

$$\kappa \leq 0.5$$

$$\phi \leq 1.5$$

3. The amplification of negative moment at the intermediate support is usually larger than that for the positive moment in the middle span.
4. The inclusion of bridge damping reduces the response of the bridge. Vehicle damping also reduces the peak response, but its effect is relatively small. In continuous bridges, the influence of bridge damping to the amplification factor for negative moment at the intermediate support is larger than that for the amplification factor for deflection.
5. A single span bridge has a larger dynamic amplification at midspan than a continuous bridge of similar cross-section and span.
6. As an application of the analytical procedure developed here, the dynamic response of the Confederation bridge is studied. Because of the large mass of the bridge and its low fundamental frequency, vehicle weight and vehicle frequency have little effect on the dynamic amplification of the bridge.

Table 4.1: (a) Span ratios of some existing bridges

Bridge Description	Span Length (m)	Span Ratios
Turkey Run State Park, Indiana, 1977	54.9-54.9	1-1
Gull Lake, Hwy 11, ON, 4 Steel Plate Girders	48.77-48.77	1-1
Townline Road over Hwy 402, ON, 4 Box Girders	30.48-30.48	1-1
Bostwick Road over Hwy 402, ON, Voided Slab	36.58-38.10	0.96-1
IBDAS Girder Bridge	25.03-26.83	0.93-1
Commercial Drive Bridge, Vancouver, multi-Cell Box	19-37	0.51-1
King George Flyover, BC, Two-Cell Box	36-43-36	0.84-1-0.84
Little Rouge Creek, Hwy 7, ON, 6 AASHTO II Girders	15.24-18.29-15.24	0.83-1-0.83
CNR Overhead, AASHTO II Girder, 1980	14.6-17.7-14.6	0.82-1-0.82
Medicine River Bridge	19.5-24.4-19.5	0.8-1-0.8
Modeste Creek Bridge	19.5-24.4-19.5	0.8-1-0.8
South Nation River, Hwy 417, ON, 2 Steel Box Girders	53.04-66.45-53.04	0.8-1-0.8
Conestogo River Bridge, Waterloo, ON	34.7-44.2-34.7	0.56-1-0.56
Deibuel up-hill Bridge, Switzerland	36.95-41-32.35	0.9-1-0.79
Willow Creek Bridge	24.4-33.5-24.4	0.73-1-0.73
Kentucky River, Kentucky, 1979	69.6-97.5-69.6	0.71-1-0.71
South Muskoka R., Hwy 11, ON, 3 Steel Box Girders	25.91-36.58-25.91	0.71-1-0.71
Illinois River, Illinois	118.87-167.64-118.87	0.71-1-0.71
Three-span bridge, Lethbridge, Alberta	12.2-18.3-12.2	0.67-1-0.67
Cavanville Creek, Hwy 28, ON, Slab	8.53-12.80-8.53	0.67-1-0.67
Ketchum Bridge, Box Girder	86.87-137.16-86.87	0.63-1-0.63
Pefferlaw Brook, Hwy 48, ON, 8 AASHTO II Girders	12.19-21.34-12.19	0.57-1-0.57
Madawaska River, Hwy 17, ON, 4 Steel Plate Girders	68.58-121.92-68.58	0.56-1-0.56
Sucker River Bridge, 1981	12-21.3-12	0.56-1-0.56
Little Sturgeon Bridge, 1982	17-32-17	0.53-1-0.53
Pike County, Kentucky	28.5-56.34-28.5	0.51-1-0.51
St. Louis Missouri Bridge	30.48-60.96-30.48	0.5-1-0.5
Lievre River, Quebec, 1967	39.6-79.2-39.6	0.5-1-0.5
JFK Memorial Causeway, Texas, 1973	30.5-61.0-30.5	0.5-1-0.5
Muscatuck River, U.S.50, Indiana, 1975	29-58-29	0.5-1-0.5
Sugar Creek, Indiana, 1976	27.6-55-27.6	0.5-1-0.5
Parke County, Indiana	27.43-54.86-27.43	0.5-1-0.5
Scottdale Bridge, Michigan	29.57-62.94-29.57	0.47-1-0.47

Table 4.1: (b) Span ratios of some existing bridges

Bridge Description	Span Length (m)	Span Ratios
St. Vincent Street Bridge, Hwy 400, ON	19.50-2@24.38-19.50	0.8-2@1-0.8
Lock Street Bridge, Phoenix, NY	42.67-2@53.34-42.67	0.8-2@1-0.8
Vail Pass, Colorado 1977(1)	46.6-2@64-47	0.73-2@1-0.73
Highwood River Bridge	17.7-2@25.0-17.7	0.71-2@1-0.71
Vail Pass, Colorado, 1977(2)	40.8-2@61-40.8	0.67-2@1-0.67
Vail Pass, Colorado, 1977(3)	40.8-2@61-44	0.67-2@1-0.72
South Muskoka R.(N), Hwy 11, ON	7.62-20.52-30.79-7.70	0.25-0.67-1-0.25
Povencher Bridge, Winnipeg, Manitoba	51-56.5-59-58-41.8	0.86-0.96-1-0.98-0.71
Mississippi River, Hwy 7, ON	21.34-3@27.43-21.34	0.78-3@1-0.78
Oldman River Bridge(1)	25.3-3@35.0-25.3	0.73-3@1-0.73
Bow River Bridge(2)	25.3-3@35.0-25.3	0.73-3@1-0.73
Oldman River Bridge(2)	27.4-3@37.8-27.4	0.72-3@1-0.72
Vail Pass, Colorado, 1977	46-47.2-2@64-47	0.72-0.74-2@1-0.73
Kishwaukee River, Illinois, 1979	51.8-3@76.2-51.8	0.68-3@1-0.68
AASHTO I girders, Trinidad and Tobago	13.7-22.4-13.7-22.4-13.9	0.61-1-0.61-1-0.61
Macdonald-Cartier Bridge, Ottawa, ON	88.4-141.3-159-141.3-88.4	0.56-0.89-1-0.89-0.56
Continuous RC-INSUIT, Trinidad	9.1-11.9-2@18-11.9-9.1	0.51-0.66-2@1-0.66-0.51
Wabash River, Indiana, 1978	28.5-4@57-28.5	0.5-4@1-0.5
Bear River, Nova Scotia, 1972	62.1-6@80.77-62.1	0.77-6@1-0.77
Islington Avenue Extension, ON, 1979	2@49-61-5@83	2@0.59-0.73-5@1
Linne Cove, North Carolina, 1978	30-49.7-4@54.9-49.7-30	0.55-0.91-4@1-0.91-0.55
Lake Oahe Crossing Missouri River, ND	54.56-10@80.77-54.56	0.68-10@1-0.68
Long Key, Florida, 1978	34.4-101@36-34.4	0.96-101@1-0.96

Table 4.2: Description of slab-on-girder bridge models studied by other researchers

Spans (m)	Weight (kN)	f_b (Hz)	ϕ	κ	α
24.38	2517.72	3.843	1.397	0.247	0.148
30.48	3701.19	2.995	1.089	0.168	0.152
36.58	5083.52	2.414	0.878	0.122	0.157
42.67	6175.63	2.030	0.738	0.101	0.160
24.38	1569.56	4.018	1.460	0.396	0.142
36.58	2321.06	2.731	0.993	0.268	0.139
17.07-21.34-17.07	1502.80	4.591	1.669	0.414	0.142
19.51-24.38-19.51	1765.16	3.843	1.397	0.353	0.148
21.95-27.43-21.95	2065.15	3.234	1.176	0.301	0.157
24.38-30.48-24.38	2407.95	2.745	0.998	0.259	0.166

Table 4.3: Description of two-span continuous bridge models

Spans (m)	λ	Cross-Section Area (m^2)	Weight (kN)
20-40	0.50	6.3947	6016
22-40	0.55	6.3947	6016
24-40	0.60	6.3947	6016
26-40	0.65	6.3947	6016
28-40	0.70	6.3947	6016
30-40	0.75	6.3947	6016
32-40	0.80	6.3947	6016
34-40	0.85	6.3947	6016
36-40	0.90	6.3947	6016
38-40	0.95	6.3947	6016
40-40	1.00	6.3947	6016

Table 4.4: Description of symmetrical three-span continuous bridge models

Spans (m)	λ	Cross-Section Area (m^2)	Weight (kN)
20-40-20	0.50	6.3947	6016
22-40-22	0.55	6.3947	6016
24-40-24	0.60	6.3947	6016
26-40-26	0.65	6.3947	6016
28-40-28	0.70	6.3947	6016
30-40-30	0.75	6.3947	6016
32-40-32	0.80	6.3947	6016
34-40-34	0.85	6.3947	6016
36-40-36	0.90	6.3947	6016
38-40-38	0.95	6.3947	6016
40-40-40	1.00	6.3947	6016

Table 4.5: Description of symmetrical three-span cantilever bridge models, $\lambda = 0.5$

Spans (m)	Suspended Span (m)	ψ	Cross-Section Area (m^2)	Weight (kN)
20-40-20	36	0.05	6.3947	6016
20-40-20	32	0.10	6.3947	6016
20-40-20	28	0.15	6.3947	6016
20-40-20	24	0.20	6.3947	6016
20-40-20	20	0.25	6.3947	6016
20-40-20	16	0.30	6.3947	6016
20-40-20	12	0.35	6.3947	6016
20-40-20	8	0.40	6.3947	6016
20-40-20	4	0.45	6.3947	6016

Table 4.6: Modal frequencies and periods of the Confederation bridge (2D model)

No.	FREQUENCY rad/second	FREQUENCY Hz	PERIOD second
1	3.193	0.508	0.19680E+01
2	3.766	0.599	0.16682E+01
3	5.679	0.904	0.11064E+01
4	6.805	1.083	0.92326E+00
5	10.433	1.660	0.60224E+00
6	16.441	2.617	0.38216E+00
7	19.390	3.086	0.32404E+00
8	19.861	3.161	0.31635E+00
9	21.039	3.349	0.29864E+00
10	26.669	4.245	0.23559E+00
11	28.445	4.527	0.22089E+00
12	31.381	4.994	0.20022E+00
13	41.178	6.554	0.15259E+00
14	47.185	7.510	0.13316E+00
15	49.563	7.888	0.12677E+00
16	51.009	8.118	0.12318E+00
17	57.077	9.084	0.11008E+00
18	58.761	9.352	0.10693E+00
19	63.495	10.106	0.98956E-01
20	63.839	10.160	0.98423E-01
21	73.915	11.764	0.85006E-01
22	80.711	12.846	0.77848E-01
23	81.890	13.033	0.76727E-01
24	88.190	14.036	0.71246E-01
25	91.632	14.584	0.68569E-01
26	95.105	15.136	0.66066E-01
27	102.523	16.317	0.61286E-01
28	109.225	17.384	0.57525E-01
29	122.808	19.546	0.51163E-01
30	125.151	19.918	0.50205E-01

(a) Vehicle moving on the uneven pavement

(b) Function describing an uneven pavement

(c) Location of the vehicle on the bridge

Figure 4.1: Vehicle moving on uneven pavement

(a) Small beam element

(b) Two beam elements connected by a hinge

Figure 4.2: Beam elements

 Figure 4.3: Definition of parameters λ and ψ for continuous and cantilever bridges

Figure 4.4: First ten vibration modes of two-span continuous bridges

Figure 4.5: First ten vibration modes of three-span continuous bridges

Figure 4.6: Amplification factor for deflection at the middle of the main span of two 2-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.7: The effect on response of the direction of movement of a vehicle on a 2-span bridge, $\alpha = 0.2$, $\kappa = 0.1$, (a) deflection at the middle of the main span, (b) moment at the intermediate support

Figure 4.8: Amplification factor for deflection at the middle of the main span of two-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.15$

Figure 4.8: Amplification factor for deflection at the middle of the main span of two-span continuous bridges with $\lambda = 0.5$, (c) $\alpha = 0.2$, (d) $\alpha = 0.3$

Figure 4.9: Amplification factor for deflection at the middle of the main span of two-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.10: Amplification factor for moment at the middle of the main span of two-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.11: Amplification factor for moment at the middle of the main span of two-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.12: Amplification factor for negative moment at the intermediate support of two-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.13: Amplification factor for negative moment at the intermediate support of two-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.14: Comparison of deflection and moment amplification factors for two-span continuous bridge, $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.5, \kappa = 0.1$, (d) $\alpha = 0.5, \kappa = 0.5$

Figure 4.15: Comparison of deflection and moment amplification factors for two-span continuous bridges, $\phi = 1.0$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.16: Amplification factor for deflection at the middle of the central span of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.15$

Figure 4.16: Amplification factor for deflection at the middle of the central span of three-span continuous bridges with $\lambda = 0.5$, (c) $\alpha = 0.2$, (d) $\alpha = 0.3$

Figure 4.17: Amplification factor for deflection at the middle of the central span of three-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.18: Amplification factor for moment at the middle of the central span of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.19: Amplification factor for moment at the middle of the central span of three-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.20: Amplification factor for negative moment at the intermediate support of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.21: Amplification factor for negative moment at the intermediate support of three-span continuous bridges, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.22: Comparison of deflection and moment amplification factors for three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.1$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.23: Comparison of deflection and moment amplification factors for three-span continuous bridges, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.24: Comparison of amplification factors for deflection at the middle of the first span of single, two, and three-span bridges, $\kappa = 0.1$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.24: Comparison of deflection amplification factors at the middle of first span of single, two, and three-span bridges, $\kappa = 0.5$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.25: Effect of bridge damping on amplification factor for deflection at the middle of the central span of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.2, \kappa = 0.1$, (c) $\alpha = 0.2, \kappa = 0.5$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.26: Effect of bridge damping on amplification factor for moment at the intermediate support of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.1$, (c) $\alpha = 0.5, \kappa = 0.2$, (d) $\alpha = 0.1, \kappa = 0.2$, $\kappa = 0.5$

Figure 4.27: Effect of vehicle damping on amplification factor for deflection at the middle of the central span of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.1$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.28: Effect of vehicle damping on amplification factor for moment at the intermediate support of three-span continuous bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.29: Generation of design envelope curves relating amplification factor, α , and κ . $\lambda = 0.5$, $\kappa = 0.1$, $\phi = 0.0-2.4$, (a) deflection at the middle of the main span, (b) negative moment at the intermediate support

Figure 4.30: Design envelope curves relating amplification factor, α , and κ for two-span bridges. $0 < \phi < 2.4$, (a) $\lambda = 0.5$, (b) $\lambda = 0.6$

Figure 4.30: Design envelope curves relating amplification factor, α , and κ for two-span bridges. $0 < \phi < 2.4$, (c) $\lambda = 0.7$, (d) $\lambda = 0.8$

Figure 4.30: Design envelope curves relating amplification factor, α , and κ for two-span bridges. $0 < \phi < 2.4$, (e) $\lambda = 0.9$, (f) $\lambda = 1.0$

Figure 4.31: Design envelope curves relating amplification factor, α , and κ for three-span bridges. $0 < \phi < 2.4$, (a) $\lambda = 0.5$, (b) $\lambda = 0.6$

Figure 4.31: Design envelope curves relating amplification factor, α , and κ for three-span bridges. $0 < \phi < 2.4$, (c) $\lambda = 0.7$, (d) $\lambda = 0.8$

Figure 4.31: Design envelope curves relating amplification factor, α , and κ for three-span bridges. $0 < \phi < 2.4$, (e) $\lambda = 0.9$, (f) $\lambda = 1.0$

Figure 4.32: Comparison of design charts for single span bridge, two-equal-span continuous bridge and three-equal-span continuous bridge

Figure 4.33: First ten vibration modes of three-span cantilever bridges

Figure 4.34: Amplification factor for deflection at the tip of the overhang of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1$, (b) $\alpha = 0.15$

Figure 4.34: Amplification factor for deflection at the tip of the overhang of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (c) $\alpha = 0.2$, (d) $\alpha = 0.3$

Figure 4.35: Amplification factor for deflection at the tip of the overhang of three-span cantilever bridges with $\lambda = 0.5$, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.36: Amplification factor for negative moment at the intermediate support of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1$, (b) $\alpha = 0.2$

Figure 4.37: Amplification factor for negative moment at the intermediate support of three-span cantilever bridges with $\lambda = 0.5$, (a) $\phi = 0.5$, (b) $\phi = 1.0$

Figure 4.38: Comparison of deflection and moment amplification factors for three-span cantilever bridge with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1, \kappa = 0.5$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.5$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.39: Comparison of deflection and moment amplification factors for three-span cantilever bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.40: Effect of bridge damping on amplification factor for deflection at the tip of the overhang of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.41: Effect of bridge damping on amplification factor for moment at the intermediate support of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.42: Effect of vehicle damping on amplification factor for deflection at the tip of the overhang of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.43: Effect of vehicle damping on amplification factor for moment at the intermediate support of three-span cantilever bridges with $\lambda = 0.5$ and $\psi = 0.4$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.44: Design envelope curves relating amplification factor, α , and κ for three-span cantilever bridges. $0 < \phi < 2.4$, (a) $\psi = 0.1$, (b) $\psi = 0.2$

Figure 4.44: Design envelope curves relating amplification factor, α , and κ for three-span cantilever bridges. $0 < \phi < 2.4$, (c) $\psi = 0.3$, (d) $\psi = 0.4$

Figure 4.45: Comparison of amplification factors for deflection at the middle of the central span of continuous and cantilever bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.1, \kappa = 0.5$, (c) $\alpha = 0.2, \kappa = 0.1$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.46: Comparison of amplification factors for moment at the intermediate support of continuous and cantilever bridges with $\lambda = 0.5$, (a) $\alpha = 0.1, \kappa = 0.1$, (b) $\alpha = 0.2, \kappa = 0.1$, (c) $\alpha = 0.1, \kappa = 0.5$, (d) $\alpha = 0.2, \kappa = 0.5$

Figure 4.47: Modeling of the Confederation bridge

Figure 4.48: (a) 1st to 5th mode shapes of the Confederation bridge

Figure 4.48: (b) 6th to 10th mode shapes of the Confederation bridge

Figure 4.49: Deflection amplification factors for Confederation bridge. Vehicle weight = 60 tonne, vehicle frequency = 1.0, 2.0, 3.0, 4.0 Hz, (a) point #1, (b) point #2, (c) point #3

Figure 4.50: Deflection amplification factors for Confederation bridge. Vehicle weight = 20, 40, 60, 80, 100 tonne, vehicle frequency = 3.0 Hz (a) point#1, (b) point #2, (c) point #3

Figure 4.51: Effect of bridge damping on deflection amplification factors for Confederation bridge. $\xi=2\%$, vehicle weight = 60 tonne, vehicle frequency = 3.0 Hz, (a) point#1, (b) point #2, (c) point #3

Figure 4.52: Moment amplification factors for Confederation bridge. Vehicle weight = 60 tonne, vehicle frequency = 1.0, 2.0, 3.0, 4.0 Hz, (a) point #1, (b) point #4, (c) point #5

Figure 4.53: Moment amplification factors for Confederation bridge. Vehicle weight = 20, 40, 60, 80, 100 tonne, vehicle frequency = 3.0 Hz (a) point#1, (b) point #4, (c) point #5

Figure 4.54: Effect of bridge damping on moment amplification factors for Confederation bridge. $\xi_b=2\%$, vehicle weight = 60 tonne, vehicle frequency = 3.0 Hz, (a) point #1, (b) point #4, (c) point #5

Figure 4.55: Comparison of deflection and moment amplification factors for Confederation bridge. Vehicle weight = 60 tonne, vehicle frequency = 3.0 Hz

Figure 4.56: Accelerations of Confederation bridge. Vehicle weight = 60 tonne, vehicle frequency = 1.0, 2.0, 3.0, 4.0 Hz, (a) point#1, (b) point #2

Figure 4.57: Envelope curves for accelerations of Confederation bridge. Vehicle weight = 20, 40, 60, 80, 100 tonne, (a) point#1, (b) point #2

Figure 4.58: Effect of bridge damping on accelerations of Confederation bridge. $\xi_b=2\%$, vehicle weight=60 tonne, vehicle frequency = 3.0 Hz, (a) point#1, (b) point #2

Chapter 5

Summary and Conclusions

5.1 Summary of the Research

The primary objective of this research is to study the dynamic response of certain types of bridges to a moving vehicle. Skew slab bridges, skew slab-on-girder bridges, and multispan continuous and cantilever bridges have been investigated for their response to moving vehicles. A general computer program has been developed for the dynamic analysis of various bridge structures.

A skew slab bridge is idealized as an isotropic plate, simply supported on the opposite sides and free on the other edges. Analytical study of free vibration of the bridge is carried out in oblique coordinates. The finite element method is then used to model the bridge, and the equations of motion of vehicle and bridge system are derived. Plate bending elements with a quadrilateral shape are used for discretizing the deck. A vehicle is simulated by a single mass supported by a spring and a dashpot, moving with a constant velocity on the deck. Damping in both the bridge and the vehicle is neglected. A limited number of free vibration modes of the bridge are used for transforming the equation of the bridge deck in order to reduce the size of the problem in the finite element analysis. The

resulting differential equations are solved by the average acceleration method. The effect of governing parameters on bridge response is studied. The distributions of deflection and moment amplifications along the central cross-section of the bridge are also analyzed. The development of a design procedure is described, and design charts are generated for slab bridges with different skew.

A plate-beam model is developed for the analysis of slab-on-girder bridges. The slab is discretized by plate elements and the girders by beam elements. Parameters which govern the response of a skew slab-on-girder bridge under the passage of a moving vehicle are identified. The deflection and moment amplifications for both the internal girders and the external girders are calculated, and the transverse distributions of deflection and moment at the central cross-section are analyzed. A comparison is made among the response of an isotropic plate model, an orthotropic plate model, and a plate-beam model. It is seen that the results of the orthotropic plate model and the finite element plate-beam model are close. Based on the orthotropic plate model, a set of design envelope curves are derived.

Equations of motion of bridge-vehicle system are derived for multispan continuous and cantilever bridges. The effect of road roughness and damping in both the bridge and the vehicle is included in the equation. The parameters which govern the response of such a bridge due to a moving vehicle are found, and their practical ranges are discussed. A parametric study is conducted to determine the natural frequencies and mode shapes of both continuous and cantilever bridges. The effect of different parameters on bridge response is investigated. As a case study, a sector of the Confederation Bridge, Prince Edward Island is modeled as a 2-D framed beam model, and both free and forced vibrations of the bridge are studied.

5.2 Conclusions of the Present Study

The main conclusions drawn from the studies undertaken in this research are listed in the sequence in which they appear in the thesis:

Skew Slab Bridges

1. The skew angle θ , the aspect ratio, the mass ratio κ , the speed parameter α , and frequency ratio ϕ govern the response of skew bridges modeled by isotropic plates. Two bridges with the same values of θ , aspect ratio, α , κ , and ϕ will have identical responses. Two skew bridges with different aspect ratios will have approximately similar responses provided that they have the same values of θ , α , κ , and ϕ .
2. The first and second frequencies of the bridge become close to each other when the skew angle θ increases. The contribution to dynamic amplification from the second vibration mode is expected to increase with growing value of θ . The response of the point near the free edges of the skew bridge depends mainly on the first and second vibration modes which are torsional in nature for larger skew angles.
3. In skew bridges, the amplification factor for deflection is greater than those for principal moment and longitudinal moment. The amplification factors for the principal and longitudinal moments are close to each other when $\theta \leq 30^\circ$.
4. For bridges with a large value of θ the transverse distribution of amplification factors is asymmetrical. The second vibration mode (torsional vibration mode) is considered to be responsible for this.
5. A series of design envelope curves are derived. The curves relate dynamic amplification factors to parameters θ , α , and κ . In using these curves a preliminary design

must be carried out first to obtain the mass and fundamental of the bridge. α is obtained from the maximum expected travel speed of the vehicle, bridge length, and fundamental frequency of the bridge. κ is obtained from the mass of the vehicle and the mass of the bridge.

6. It is of interest to note that for $\theta \leq 30^\circ$, the design amplification factors are very close to those for the right bridge. In fact, the amplification in skew bridges is seen to be smaller than those in the right bridge. For larger skew angle, the design amplification factors may differ somewhat from that for the corresponding right bridge, but the difference is not large.

Skew Slab-on-Girder Bridges

1. The larger the aspect ratio of the bridge, the less sensitive to the change of skew angle the midpoint response of the bridge is. The bridge with a larger value of the aspect ratio has a more uniform transverse distribution of response. This is because the longitudinal flexural mode dominates the response.
2. The transverse distribution of amplification factors in a bridge with a larger skew angle is more uneven than that in a bridge with a smaller skew angle. This is true for all vehicle positions.
3. The moment amplification factor is generally smaller than the corresponding deflection amplification factor, but both have similar variations.
4. For the skew bridges studied, plate-beam model and the orthotropic plate model give similar responses in terms of absolute value of deflection, amplification factor, and transverse distribution of normalized deflections. However, the response obtained

from an isotropic plate model is quite different in terms of the absolute value of deflection and transverse distribution of normalized deflections. It is recognized that the ability of an orthotropic plate model to provide a reasonable estimate of the response of a slab-on-girder bridge will be affected by the number of girders. If the number of girders is small, orthotropic model idealization may not be that accurate.

5. A change in girder spacing does not significantly change the amplification factor of those girders that take most of dynamic load.
6. An increase in number of girders does not significantly change the amplification factors of the girders.
7. A series of design envelopes curves are derived on the basis of bridges with aspect ratio=2. The curves relate dynamic amplification factors to parameters θ , α , and κ . To use these curves a preliminary design must be carried out first to obtain the mass and fundamental frequency of the bridge. Parameters α and κ can then be calculated.

Multispan Continuous and Cantilever Bridges

1. Parameters α , κ , ϕ , λ , and the number of spans govern the response of a continuous bridge under the passage of a moving vehicle. For cantilever bridges, an additional parameter ψ is needed.

2. The practical ranges for the values of parameters of λ , ψ , α , κ , and ϕ are

$$0.5 \leq \lambda \leq 1.0$$

$$0 < \psi < 0.5$$

$$\alpha \leq 0.3$$

$$\kappa \leq 0.5$$

$$\phi \leq 1.5$$

3. The amplification of negative moment at the intermediate support is usually larger than that for the positive moment in the middle span.
4. The inclusion of bridge damping reduces the response of the bridge. Vehicle damping also reduces the peak response, but its effect is relatively small. In continuous bridges, the influence of bridge damping to the amplification factor for negative moment at the intermediate support is larger than that for the amplification factor for deflection.
5. A single span bridge has a larger dynamic amplification at midspan than a continuous bridge of similar cross-section and span.
6. As an application of the analytical procedure developed here, the dynamic response of the Confederation bridge is studied. Because of the large mass of the bridge and its low fundamental frequency, vehicle weight and vehicle frequency have little effect on the dynamic amplification of the bridge.

5.3 Recommendations for Future Work

1. A study of the response of skew and continuous bridges under a vehicle model with multiple axles would be useful and will provided more refined guidelines for design.

2. A parametric study of the effect of the road roughness could be carried out. A road roughness profile could be created by a random generator and should be validated on the basis of actual field surveys.
3. Analytical investigation of bridge vibration caused by pedestrian or a moving crowd could be carried out to examine the magnitude of response and the conditions under which such vibrations could be perceptible. The results obtained from such analytical studies could be compared with field test results.
4. The present studies may be extended to cover curved bridges.

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