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MARKET STRUCTURE, RISK AND THE
OPTIMAL PATENT TERM

by

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A thesis submitted to
the Faculty of Graduate Studies and Research
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ABSTRACT

The optimal patent term depends on the proportion of the social benefits of invention which are privately appropriable. The degree of appropriability depends, in turn, on the nature of the market and broader institutional environment in which inventors must operate.

This thesis investigates the impact on the optimal patent term of alternative assumptions regarding the nature of the market in which inventors operate. In particular, it examines the consequences of the existence of post-patent competition within the context of both monopolistic and competitive pre-patent situations.

The thesis also examines the effect of the introduction of taxes on inventors' incomes on the optimal patent term.

The effect of additional pre-patent competition is generally to reduce the optimal patent term while the effect of additional post-patent competition is to increase it. The taxation of inventors' incomes also increases the optimal patent term.

The process of invention is, if nothing else, a risky one. Existing optimal patent term models have generally assumed a world of perfect certainty.

This thesis analyzes the impact of uncertainty and inventors' risk aversion on the optimal patent term. Three types of uncertainties are considered: uncertainty about the demand function of the industry which purchases a cost-reducing invention; uncertainty about the magnitude of the cost reduction; and uncertainty about the date of invention.

Under the assumptions of demand and cost reduction uncertainties the optimal patent terms are derived and the sensitivity of the latter to the changes in inventors' risk aversion are also analyzed for

unique as well as competitive inventors. It is demonstrated that, whether the pre-patent situation is characterized by monopoly (unique inventor) or competition, provided inventors are risk averse, an increase in either cost or demand uncertainty increases the optimal patent term. Simulation experiments also show the comparative static result that the optimal patent term is longer the more risk averse are inventors.

The optimal patent term in the case of uncertainty about the date of invention is also derived under the assumption that there is free entry into inventive activity. It is demonstrated that the optimal patent term in this case depends on the degree of ease in advancing the expected discovery date, and under some plausible assumptions, it is similar to the optimal patent term in the case of the perfectly certain, unique inventor.

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In memory of my late father who was a positive source
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CHAPTER 1

INTRODUCTION

In an influential article Arrow (1962) pointed out three reasons for market failure in the allocation of resources to invention. They are: the uncertainty attached to the outcome of inventive activity; the inappropriability of the returns to invention - meaning that in the absence of exclusive rights to their inventions (for example a patent right), inventors cannot appropriate the value of their inventions; and the indivisibility inherent in inventions - meaning that once a new invention (a new product or a new process) has been brought about, it can be spread to all consumers at zero marginal cost. The influence of each of these factors is such that under a competitive system there will be a tendency to underinvest resources in research and development (R&D) as compared to the social optimum.

What was assumed in Arrow's analysis was that individual inventors are unique. However, if this assumption is relaxed, i.e., if there is free entry into the market to invent, there may well be a level of investment in R&D under a competitive system that is beyond what is socially desirable.¹

Overinvestment may occur because of the competition for a patent. With free entry into the invention market, the firm inventing first will receive the patent and has the right to the private benefit from the invention. It has then been argued by Barzel (1968), Stigler (1968), Loury (1979), Dasgupta and Stiglitz (1980a, b) that competition for a patent could be such that all privately appropriable surplus inherent in inventive activity will be dissipated in the rush to appropriate it. In this case resources devoted to the race for the patent would just equal the value of the patent.

If there is free entry in R&D activities and if the first inventor is likely to reap the main benefits from the invention, the potential surplus may be dissipated because the race to invent leads invention to take place too early in order to preempt rivals (Barzel, 1968). In addition, firms do not take into account the parallel nature of their R&D activities, thus duplicate research efforts by identical rival firms will also assist in dissipating the potential surplus [Loury (1979), Dasgupta and Stiglitz (1980a, b)]. If all surplus is appropriable and all surplus is dissipated, new inventions will not be welfare-improving. New inventions become welfare-improving only to the extent that part of the value of inventions is not appropriable by inventors.

Following Arrow, much of the early work on patents began with the assumption of unique inventors. In this case the purpose of the patent grant is to increase the degree to which the benefits of inventors are privately appropriable (a solution to the market failure associated with the inappropriability of inventions) and thus to provide incentives to invent.² The undesirable feature of the patent is that it restricts the subsequent exploitation of the invention. The longer the patent term the greater is the value of the new inventions forthcoming. The longer the patent term the greater also is the value of the deadweight losses associated with the restriction of the use of inventions. Thus, the optimal patent term should be one at which the value of additional inventions induced is just equal to the additional surplus foregone by further restricting the use of existing inventions.

The optimal patent term in the case of a unique inventor was obtained by Nordhaus (1969). The optimal patent term in this case is, under plausible assumptions, quite close to the existing patent term.³ If

inventors are not unique and rivalry is such that the entire appropriable surplus is dissipated, it can be shown that the optimal patent term is quite short. Specifically, Berkowitz and Kotowitz (1979) have shown (using Nordhaus' methodology) that with homogenous inventors the optimal patent term is as short as six months. More recently they [Berkowitz and Kotowitz (1982)] have argued in an open economy context that no small country should have a patent system at all.

The above theoretical works on the optimal patent term employ many restrictive assumptions. It is not appropriate to draw any policy conclusion regarding a patent system from this type of analysis until other theoretical possibilities have been explored. There are a number of ways in which the assumptions of the model employed by these authors can be relaxed. The combinations of behavioural and environmental assumptions which might be employed are summarized in Table 1.1.

According to Table 1.1, as far as the invention market structure is concerned they include unique as well as rival (homogenous and non-homogenous) inventors. With free entry into inventing, the notion of 100 percent dissipation of appropriable surplus and hence a six-month optimal patent term is based on the assumption that rival inventors are homogenous. The inventors may well be nonhomogenous in the sense that they differ in quality.

The implications of the assumption of potential nonhomogenous rival inventors on the dissipation of appropriable surplus was analysed by Yu (1977). Following Cheung (1976) he argued that, if the potential inventors differ in quality, a superior inventor (e.g., in the case of a process invention, a superior invention is one in which the differential unit cost of production achieved from the invention is higher than that of the next

Table 1.1

Invention Market Structure and Environmental Conditions of Inventive Activity

Environmental Conditions		Perfect Certainty	Uncertainty	
Invention Market Structure	Unique Inventor		Demand	Cost
Invention Market Structure		Demand		Discovery Date
<u>Unique Inventor</u>				
a. No post-patent rivalry, constant elasticity invention possibility function		Nordhaus (1969) Tandon (1978, 1982)	Ch. 4	Ch. 4
b. No post-patent rivalry, variable elasticity invention possibility function		Dore et al. (1983)	-	-
c. Post-patent rivalry, constant elasticity invention possibility function		Ch. 3	-	-
<u>Rivalry</u>				
a. Free entry into inventing, homogeneous inventors, no taxes, no rivalry after the patent granted		Berkowitz & Kotowitz (1979)	Ch. 5	Ch. 5
b. Free entry into inventing, non-homogeneous inventors, no taxes, no post-patent rivalry		Yu (1981)	-	-
c. Free entry into inventing, homogeneous inventors, post-patent rivalry, taxes		Ch. 3	Ch. 5	Ch. 5
				Dasgupta and Stiglitz (1980), Kamien and Schwartz (1972, 1974, 1976), Ch. 5

best invention) will appropriate the differential rent i.e., the difference between his and the next best inventor's per unit post-invention cost of production. This differential rent will not be dissipated in the rush to patent. It is then implied that when the inventors are non-homogenous, the entire privately appropriable surplus resulting from inventive activity will not be dissipated. Some appropriable surplus in the form of differential rent will be preserved. The superior inventor can also appropriate more surplus, i.e., the size of the differential rent can be made larger if the superior inventor can form a cartel with the next best inventor. In such a situation additional surplus is preserved. Since in the presence of rivalry among nonhomogenous inventors, surplus is preserved in the form of differential rent, the surplus maximizing patent term will be longer than in the case of homogenous inventors. Yu does not attempt to determine the surplus maximizing patent term.

The existing models of the optimal patent term also assume that the successful inventor can extract the entire cost savings resulting from his invention in the form of royalties. If the entire royalty can be collected then it is implicit in the models that competition ceases once the patent is granted. From empirical work such as that of Beck (1976) one can gather that many major patented inventions had a number of noninfringing potential substitutes. The implications of such findings is that, although the best invention gets the patent, there are a number of inferior non-infringing potential substitute inventions which are available in the market.

We have mentioned earlier that the surplus which the winning inventor can extract is dissipated in the rivalry for the patent. The availability of potential substitute inventions will then limit the patentee to extract

the entire private benefits from his invention (most empirical studies of cost reducing process inventions confirm that royalties are generally less than one-third of the cost reduction obtained). Therefore some of the benefits of the invention are immediately passed on to its users. The portion of total surplus which is not appropriable by the inventor is preserved and thus dissipation is reduced. In sum, the existence of post-patent competition has the effect of reducing the surplus appropriable by the patentee. The amount of surplus which has been assumed to be privately appropriable in the optimal patent term models is then overstated. An alternative assumption would, therefore, allow rivalry after the grant of a patent (post-patent rivalry) instead of or in addition to rivalry for the patent itself (prepatent rivalry) and investigate the impact of post-patent competition on the optimal patent term.

It is also assumed in these models that inventions take place under the conditions of perfect certainty and thus the models ignore a more fundamental problem of resource allocation to invention that the outcome of inventive activity is uncertain. In the presence of uncertainty, employment of resources on inventive activity will depend on individual inventor's attitude towards risk as well as the degree to which inventors can appropriate the benefits from inventions. The analysis of the impact of uncertainty and inventors' attitudes towards risk is then essential to fill these theoretical gaps. To a world in which we have pre and/or post-patent rivalry with homogenous and nonhomogenous inventors, we can add possibilities that inventors' income is taxed (income taxes reduce the dissipation of surplus) and that there is uncertainty with regard either to the value of the invention or to the timing of the invention in the optimal patent term models.⁴

It is the purpose of this thesis to investigate the impact of alternative sets of assumptions regarding uncertainty, and rivalry on the optimal patent term. The thesis is not exhaustive with respect to theoretical possibilities considered. Nor does it purport to derive specific policy conclusions. What it does do is show the sensitivity of the results generated by a so-called "policy model" to changes in the structural and behavioural assumptions which underlie it.

We now proceed to give a brief summary of some of the work done to date in connection with the optimal patent term and thereafter we will indicate the direction of our own work.

1. Perfect certainty, unique inventor, no post-patent rivalry with constant and variable elasticity invention possibility functions.

As mentioned above, Nordhaus (1969) derives the optimal patent term for the case of a unique inventor under the conditions of perfect certainty and without allowing post-patent rivalry. For a given patent term, the inventor in this case can appropriate a portion of total surplus resulting from a cost-saving invention. Nordhaus assumes that the size of an invention measured in terms of unit cost reduction in production of an existing good is an increasing concave function of research input, and maximizes a social welfare function subject to the profit maximizing inventor's choice of the level of inventive input to obtain the optimal patent term. When a concave relationship between the inventive input and the unit cost reduction for production (i.e., the "invention possibility" function, as Nordhaus calls it) is specified, the numerical estimates of the optimal patent term depends on the size of invention, the elasticity of demand and the curvature of the invention possibility function which is constant for a given value of the elasticity of cost reduction with respect

to research (output elasticity of research). If the assumption of this concave relationship is relaxed, and an invention possibility function which exhibits both increasing and decreasing returns to scale is specified, the curvature of the invention possibility function is uniquely determined by the size of the invention. With perfect certainty and no post-patent competition, the optimal patent term will then depend on the elasticity of demand and the size of the invention [Dore et al. (1983)].

2. Perfect certainty, free entry into inventing, homogeneous inventors, no taxes, and no rivalry after the patent granted.

In the case of free entry into inventive activity, the competition for a patent among homogeneous inventors could be such that all the privately appropriable surplus resulting from an invention is dissipated. Under the assumption that 100 percent surplus is dissipated and inventors' incomes are not taxed, Berkowitz and Kotowitz (1979) obtain the optimal patent term under the conditions of perfect certainty, homogeneous inventors, and the winner of the patent does not face post-patent rivalry.

In their derivation of the optimal patent term, Berkowitz and Kotowitz also assume a concave relationship between the size of cost reduction and the inventive input, and specify an invention possibility function which is identical to that of Nordhaus for numerical estimation of the optimal patent term. In order to derive the welfare maximizing patent term, they maximize a social welfare function subject to the condition that the cost of research on any given inventive activity is just equal to the present value of the royalty income expected by the winner of the patent.

3. Rivalry for the patent, homogenous inventors, no taxes, no rivalry after the patent granted, uncertainty about the discovery date.

The modern theories of market structure and innovation popularized by Kamien and Schwartz (1972, 1974, 1976), Loury (1979), and Dasgupta and

Stiglitz (1980a, b) recognize rivalry in the invention market as a race to be the first to invent and to reap the entire benefits from the invention. Unlike Nordhaus, and Berkowitz and Kotowitz, they assume that the size of an invention (e.g., the amount of cost reduction in the case of a process invention) and the monetary reward (the royalty and other incomes to be received in the event that an inventor wins the race) from an invention do not depend on the amount of resources invested in inventive activity. Like Nordhaus, and Berkowitz and Kotowitz, they assume that the size of and the reward from an invention are perfectly certain. It is also assumed that the employment of resources in inventive activity affects the date of discovery of an invention and that the inventors face uncertainty either about the rivals' discovery dates (Kamien and Schwartz) or about their own discovery dates (Loury, Dasgupta and Stiglitz).

In the presence of pre-patent rivalry, when inventors face uncertainty about rivals' discovery dates and when inventors are risk neutral, the issues which have been raised are mainly about an expected profit maximizing innovative firm's decision of whether or not to develop an innovation; if the firm decides to innovate, at what speed it should do so (Kamien and Schwartz). Although no consideration has been given to obtain the welfare maximizing patent term, it is demonstrated by Kamien and Schwartz (1974) that the firm's decision on development and speed of an innovation will depend on some exogenous probability of the rival's (a composite rival) discovery date and the patent term. The smaller the probability that the rival will make the invention in a given period of time and the longer the length of the patent, the higher is the possibility for an innovating firm to undertake an innovative decision; and higher is also the speed of development if it does.

Recently Dasgupta and Stiglitz (1980a, b) and Loury (1979) investigated the relationship between market structure and innovation and introduced uncertainty about the potential inventors' own discovery dates. With free entry into inventive activity, it has been argued that competition for patents may be such as to result in an over-allocation of resources to R&D. Assuming that the size of invention is perfectly certain and that it does not depend on the employment of inventive resources, Dasgupta and Stiglitz (1980b) have shown that in the presence of uncertainty about the date of invention, the tendency of over-allocation of resources can be offset by setting the appropriate patent term. It is to be noted here that they do not give any numerical estimates of the optimal patent term.

As far as our own work is concerned, Chapter 2 develops the perfectly certain unique inventor (Nordhaus) model for the optimal patent term and extends it to the free entry, homogenous inventors, winner takes all (Berkowitz-Kotowitz) case.

Post-patent competition and taxation of inventors' incomes are introduced in Chapter 3. Under the conditions of perfect certainty, it is demonstrated that the post-patent competition and taxes on inventors' royalty income have the effect of increasing the optimal patent term when the inventors are unique as well as when there is free entry into the pre-patent phase of inventive activity.

In Chapter 4, we introduce uncertainty in the value of the inventions when the inventors are unique and examine its impact on the optimal patent term. Uncertainty is introduced in two ways: (a) uncertainty regarding the demand function of the industry which purchases the cost reducing inventions; and (b) uncertainty regarding the amount of cost

reduction associated with a given amount of inventive activity. In both cases the analysis is conducted under the assumptions that society (the patent authority) is risk neutral while inventors are either risk neutral or risk averse. Furthermore, uncertainty affects both the demand function and the magnitude of the cost reduction function multiplicatively.

It turns out that even when all parties are risk neutral, the introduction of uncertainty regarding the magnitude of cost reduction will influence the optimal patent term. When inventors are assumed to be risk averse, the optimal patent term is longer than the case of perfect certainty, whether uncertainty affects the demand function or the cost reduction function. Finally, given that inventors are risk averse, the optimal patent term is longer the higher the index of risk aversion.

Free entry into inventive activity in the presence of post-patent competition and taxes is reconsidered in Chapter 5 under the environment of uncertainty. The types of uncertainties considered are the same as those of Chapter 4. It turns out that under free entry into inventing, the introduction of uncertainty influences the optimal patent term considerably more than the optimal patent terms in Chapters 2 and 3. Specifically it is demonstrated that when inventors and society are risk neutral, the optimal patent term is longer than under perfect certainty case when uncertainty arises in the cost reduction function. This stands in direct contrast to the result derived for the optimal patent term when the inventors are unique. When inventors' risk aversion is added to the optimal patent term model, both cost reduction and demand uncertainty result in an optimal patent term that is longer than in the perfect certainty case.

We noted earlier that Dasgupta and Stiglitz (1980b) did not obtain

any numerical estimates of the optimal patent term. In Chapter 5, we remedy this feature of the Dasgupta-Stiglitz model by modifying it to incorporate a linear demand function for output of the using industry and a specific expected discovery date function.⁵ The optimal patent term in the presence of uncertainty about the inventors' discovery dates can be shown to depend upon the degree of easiness in advancing the expected discovery date. With these adjustments numerical estimates of the optimal patent term can also be obtained and it is shown that under some plausible assumptions, the optimal patent term in this case becomes similar to that of the unique inventor case.

Finally, Chapter 6 presents a summary of the results and concluding remarks. Some suggestions and direction for further research are also given.

FOOTNOTES TO CHAPTER 1

1. Recently Tandon (1983) has derived conditions under which there will be overinvestment in R&D given that there is free entry into inventive activity. See also Hirshleifer (1971).
2. Some alternative incentive mechanisms such as prizes, research contracts and their comparative advantages, if any, over patents from social welfare maximizing viewpoint has been analysed by Wright (1983). Hirschleifer (1971) has argued that the inventor not only appropriates part of the social benefits from his invention but also gains "fore knowledge", i.e., exclusive knowledge on facts which have an influence on the economy's price structure. This fore knowledge allows the inventor to earn speculative income and hence he has the incentive to invent.
3. For example, both Canada and U.S.A. maintain a 17-year patent term.
4. The value of the invention is to be interpreted as the value of the monetary reward anticipated by the inventor.
5. Dasgupta and Stiglitz consider a constant elasticity demand function and do not specify any expected discovery date function.

CHAPTER 2

THE ECONOMICS OF THE OPTIMAL PATENT LIFE

In his discussion of the allocation of resources to invention, Arrow (1962) asserted that under a competitive system there would be a tendency to underinvest private resources in research and development. More recently, it has been argued that there are factors which may lead a competitive system to invest excessive amounts of private resources in research and development.¹ One of the reasons for underinvesting is the public good character - or inappropriability of inventions. That is to say, that returns from inventions cannot be appropriated by their inventors without any legal protection (like a patent) for their inventions. Without any barrier to imitation, there will be a little incentive for firms to invest resources in inventive activities even though social returns from inventions may be much larger than private returns.² In order to increase the degree to which the benefits of inventions are privately appropriable and thus to encourage inventive activities, governments grant patents (exclusive rights) to inventors for significant new processes or products. The patent system is socially desirable because it promotes invention. It has the undesirable feature, however, of encouraging inventive activity by restricting the subsequent exploitation of it. On one hand, the longer the patent term the greater the value of benefits which is appropriable by inventors and the greater is the value of new inventions which will be forthcoming. On the other hand, due to the monopolistic restriction on the use of inventions, the longer patent term will imply a larger value of the associated deadweight losses. Thus the optimal patent term is one which jointly maximizes inventive activity and the use of inventions.

The explicit analysis of the welfare consequences of the trade-off between the level of inventive activity and the extent to which new inventions are used within the context of a formal optimizing model is a relatively recent phenomenon. The first major contribution was that of Nordhaus (1969) who calculated the optimal patent term in the case of a non-drastic process invention with a unique inventor.³ His results have been subsequently extended by Berkowitz and Kotowitz (1979), Tandon (1982) and Dore et al. (1983). The following analysis illustrates Nordhaus' theory of the optimal patent term and its related extensions.

2.1 Nordhaus' Theory of the Optimal Patent Term When the Inventors are Unique

In his optimal patent term model Nordhaus assumes that a non-drastic process invention is introduced into an industry which is competitive prior to the invention. The invention is assumed to take place under conditions of perfect certainty. Let us then assume that the demand curve faced by the industry may be approximated by a linear demand function:

$$X(P) = \bar{S} - \eta P \quad (2.1)$$

If production in the industry is characterized by constant returns to scale and if the industry is competitive prior to invention, the initial price is equal to the marginal and average cost of production. A firm in the industry may introduce a patented process invention which will reduce the unit cost of production by a fraction $B(R)$. $B(R)$ is assumed to be a concave function of R , where R is the number of units of inventive input employed. Nordhaus calls $B(R)$ the "invention possibility function". If C_0 is the cost per unit of output prior to invention and C_1 is the cost per unit of output after the invention, then it follows that $B(R) = \frac{C_0 - C_1}{C_0}$.

It is assumed that $B'(R) > 0$ and $B''(R) < 0$.

As illustrated in Figure 2.1, the maximum royalty which the inventor can charge for licencing all producers in the industry is equal to the total cost savings at the preinvention level of output X_0 given by the area of the rectangle C_0ABC_1 . So the inventor's royalty income is $(C_0 - C_1)X_0$ per period during the life of the patent.⁴ If C_0 and X_0 are normalized to 1, the royalty income of the inventor will be $B(R)$ per period. During the term of the patent grant, consumers do not benefit from the process invention because the price level remains at the preinvention level P_0 . When the patent period is over, however, the cost of production falls to C_1 , the industry output increases to X_1 per period and competition drives the price of the product down to P_1 . This process redistributes the monopoly profits of the inventor to consumers and allows the consumers to appropriate the full benefits produced by the cost savings. Thus after expiry of the patent there will be an increase in consumers' surplus composed of the area of the rectangle C_0ABC_1 , which had accrued to the inventor during the patent period, plus the area of the triangle ABD , which represents the gain in consumers' surplus from increased output at lower price.

An increase in the patent term increases the present value of the royalties to the inventor and hence increases his incentive to invent. By spending more resources in inventive activity larger cost savings are produced. The area of the rectangle C_0ABC_1 is increased as is the area of the triangle ABD . The longer is the patent term, however, the longer will be the time period before the additional triangle can be collected. Thus, the optimal patent term requires a balancing of the loss of current consumers' surplus (which arises from extending the patent term) against

Figure 2.1

Price and Output Consequences of a Cost Reducing Invention When Royalties
Equal the Cost Reduction

the incentive effects for invention and thus the larger sizes of future surpluses.

The welfare function which society wants to maximize is assumed to be:

$$W = \int_0^{\infty} B(R)X_0 e^{-\rho t} dt + \int_T^{\infty} (X_1 - X_0)B(R)e^{-\rho t} dt - sR \quad (2.2)$$

where

X_0 = preinvention level of output and is set equal to 1;

$B(R)$ = per unit cost saving = $\frac{C_0 - C_1}{C_0}$, sometimes $B(R)$ is expressed

for notational simplicity throughout the thesis as B ;

ρ = private and social discount rate;

T = term of the patent;

s = cost per unit of R .

The first term of (2.2) corresponds to the area of the rectangle C_0ABC_1 and is the present value of the private benefit to the inventor during the patent period and the benefit received by consumers when the patent period is over. The second term corresponds to the triangle ABD which is the present value of the additional triangle gain to consumers when the patent period expires. Assuming that all costs are incurred in the first period, the third term is the present value of the cost of resources invested in R&D.

If P_1 and C_1 are post-invention price and cost respectively and $P_1 = C_1$, then from the demand equation in (2.1), $X_1 - X_0 = \eta(P_1 - P_0) = \eta B(R)$, where η is the arc elasticity of demand at P_0 .⁵ When the above expression for $X_1 - X_0$ is substituted in (2.2), the integration of (2.2) yields:

$$W = \frac{B}{\rho} + \frac{\eta}{2\rho} B^2(1-\psi) - sR \quad (2.3)$$

where $\psi = 1 - e^{-\rho T}$.

The incentive for invention can be characterized as the profit maximizing inventor's choice of the level of research spending, R . Since the inventor is unique, he has the exclusive right to the royalties $B(R)$ on his own cost reducing process invention for T periods. In this case, the inventor will choose R in order to maximize his net profit function which is the present value of the royalties minus the resource cost,

$$\Pi = \int_0^T B(R) e^{-\rho t} dt - sR = \frac{\psi}{\rho} B - sR \quad (2.4)$$

The first order necessary condition is obtained by maximizing (2.4) with respect to R and the condition is

$$\frac{1}{\rho} B' \psi = s \quad (2.5)$$

The condition (2.5) states that the inventor will spend resources up to that level where present value of the income derived from an additional unit of research is equal to its unit cost.

In order to determine the optimal or welfare maximizing patent term one maximizes (2.3) subject to (2.5). Differentiating (2.3) with respect to ψ , noting from equation (2.5) that $\partial R / \partial \psi = -B'(R) / B''(R) \psi > 0$, we obtain

$$\frac{\partial W}{\partial \psi} = -\frac{B'^2}{\rho B'' \psi} - \frac{n}{2\rho} \left[\frac{2BB'^2}{B'' \psi} (1 - \psi) + B^2 \right] + s \frac{B'}{B'' \psi} = 0 \quad (2.6)$$

Substituting $\rho s = B' \psi$ [equation (2.5)] in equation (2.6) and solving for ψ , one obtains

$$\psi_M^* = \frac{1 + \eta B}{1 + \eta B(1 + \frac{k}{2})} \quad (2.7)$$

where $k = -B''B/B'^2 > 0$ is the degree of concavity of $B(R)$.

Nordhaus calls (2.5) the inventor's equilibrium and (2.7) the policy maker's equilibrium. The optimal patent term is the intersection of these two curves and the optimal patent life T is given by

$$T = -\frac{1}{\rho} \ln(1 - \psi^*) \quad (2.8)$$

where T ranges from 0 to ∞ as ψ^* ranges from 0 to 1 and ψ^* is the optimal value of ψ satisfying equations (2.5) and (2.7).

As mentioned above the optimal value of T is determined by the intersection of the inventor's equilibrium (2.5) and the policy maker's equilibrium (2.7). Treating ρ and η as parameters, we have two equations in determining B and T simultaneously. Figure 2.2 shows the equilibrium for two hypothetical curves.

To make this clearer, assume

$$B(R) = \beta R^\alpha \quad (2.8a)$$

so the equation (2.5) becomes

$$R = \left[\frac{\psi \alpha \beta}{\rho \beta} \right]^{1/(1-\alpha)} \quad (2.8b)$$

From equations (2.8a) and (2.8b), the size of the invention is:

$$B = \beta \left[\frac{\psi \alpha \beta}{\rho \beta} \right]^{\alpha/(1-\alpha)} \quad (2.8c)$$

When (2.8c) is substituted in (2.7), the optimal patent term ψ^* may be obtained from the solution of the equation:

Figure 2.2: Optimal Life of a Patent

Figure 2.3: The Optimal Patent Life for Different Industries

*Inventor equilibrium in industry

$$\psi + \psi^{1/(1-\alpha)} [\eta \beta \{(\beta \alpha) / \rho s\}^{\alpha/(1-\alpha)} (1 + k/2)] - \psi^{\alpha/(1-\alpha)} [\eta \beta \{(\beta \alpha) / (\rho s)\}^{\alpha/(1-\alpha)}] = 1 \quad (2.8d)$$

It is extremely difficult to solve for ψ from equation (2.8d). However, it is possible to compute values of T that would satisfy (2.7) for different values of B and η and determine whether the existing patent term is greater or less than the optimal patent term. The reasoning for this may be illustrated in Figure 2.3. In Figure 2.3, PP' represents the solution of the policy maker's equilibrium (2.7) and $\bar{\psi}$ represents the existing life of a patent (which is 17 years in Canada and in the United States). The curves OI_1 , OI_2 and OI_3 represent unknown inventor equilibria for different industries - I_1 , I_2 and I_3 . At the existing patent life $\bar{\psi}$, the observed size of the inventions in the three industries will be B_1 , B_2 and B_3 . By examining the observed equilibrium points - $(\bar{\psi}, B_1)$, $(\bar{\psi}, B_2)$ and $(\bar{\psi}, B_3)$ - we can determine whether the existing life is longer or shorter than the optimal. For I_1 , I_2 and I_3 the inventor equilibria are respectively to the right of, on and to the left of PP' . Thus, for I_1 , the existing life is longer than the optimal; for I_3 , the existing life is shorter than the optimal; whereas for I_2 , existing life is optimal.

In order to calculate the patent term numerically which would satisfy (2.7), we make parametric assumptions regarding the demand function of the industry and the invention possibility curve $B(R)$. Nordhaus assumes $B(R) = \beta R^\alpha$, where α is the constant elasticity of cost reduction with respect to research. From equation (2.7) we can then calculate the optimal patent term for given values of α , B , ρ and η . Under the assumption that $B(R) = \beta R^\alpha$ and because $k = -B''B/B'^2$, the value of k in equation (2.7) can be calculated as $k = (1-\alpha)/\alpha$. We allow B to take on values .005, .01, .05,

.1 and η takes on values .5, 1.0, 1.5, 2.0. ρ and α are set at .20 and .1 respectively.⁶ The results of our optimal patent term calculations are reported in Table 2.1. This calculation is the same as that reported by Nordhaus (1968, p. 81). From Table 2.1 it reveals that the optimal patent term ranges from 22.5 years to 4.3 years depending on the amount of cost reduction and the elasticity of demand. As was shown in Figure 2.2, given the above plausible values of B when the equilibrium T in table 2.1 (lower left number of each cell) is less than 17 years (which is the existing life of a patent both in Canada and the United States), the optimal patent life is shorter than 17 years. On the contrary, when the equilibrium T is more than 17 years, the optimal life is longer than 17 years.

It is interesting to note some comparative static results on the optimal patent term. Table 2.1 illustrates these results:

- (i) The higher the elasticity of demand η , the lower is T, everything else being constant.
- (ii) The larger the cost reduction B, the lower is T, everything else being constant.

There are basically two reasons why one might expect a shorter optimal patent life for large cost reductions. As Scherer (1980)⁷ points out these are: (a) large cost reductions quickly pay for themselves, and (b) with monopoly pricing a large cost reduction produces a large deadweight loss; therefore optimal social policy should call for an early termination of these deadweight losses. The same arguments apply to the elasticity of demand. The larger the value of η , the larger is the value of the associated deadweight losses. The optimal social policy should be then a shorter patent life in order to reduce the size of this deadweight loss.

The Nordhaus optimal patent term model has also been used by Tandon (1982), and Dore et al. (1983). In their analysis of the optimal patent term

Table 2.1

Optimal Patent Terms with Competition and Monopoly Inventing Satisfying
Policy Maker's Equilibrium, $\alpha = .10$, $\rho = .20$.

		η			
		.5	1.0	1.5	2.0
B	.005	.5 22.5	.5 19.1	.5 17.2	.5 15.8
	.01	.5 19.1	.5 15.8	.5 13.9	.5 12.6
	.05	.5 11.6	.5 8.7	.5 7.2	.5 6.2
	.10	.5 8.7	.5 6.2	.5 5.0	.5 4.3

Note: Lower left of cell is unique inventor, upper right cell
is competitive inventors.

Dore et al. (1983) consider a generalized invention possibility function $B(R)$ which exhibits increasing as well as decreasing returns to scale and incorrectly argues that the optimal patent term depends on the variable elasticity of cost reduction with respect to research (output elasticity of research). If a generalized invention possibility function is specified, it can easily be shown that the optimal patent term depends on the variable degree of sharpness (the curvature) of the invention possibility function rather than the output elasticity of research.

Following Dore et al. let us then specify a generalized invention possibility function of the form

$$B(R) = e^{-g/R}, \quad g > 0, \quad R > 0 \quad (2.8e)$$

where g represents the degree of easiness of an invention. The smaller the value of g the easier is the invention. The elasticity of B' with respect to B or roughly speaking, the curvature of the invention possibility function in (2.8e) is given by

$$k = -B''B/B'^2 = \frac{2 - (g/R)}{(g/R)} \quad (2.8f)$$

Since $B(R) = e^{-g/R}$, this implies that $g/R = -\ln B$, and from equation (2.8f), $k = -(\ln B + 2)/\ln B$. Thus k is a direct function of B . The larger the value of B the larger is the value of k . Given the invention possibility function in (2.8e) the optimal patent term is obtained by substituting $k = -(\ln B + 2)/\ln B$ in equation (2.7), and the result is

$$\psi_M = \frac{1 + \eta B}{1 + \eta B \left(\frac{1}{2} - \frac{1}{\ln B} \right)} \quad (2.8g)$$

From the above analysis, it is obvious that the derivation of the optimal patent term is the same as that of Nordhaus. The only difference

is that for Nordhaus, the curvature of the invention possibility function, k , is independent of B [for Nordhaus, $B(R) = \beta R^\alpha$ which implies that $k = -B''B/B'^2 = (1 - \alpha)/\alpha$ is a constant] but for Dore et al. it is not. This implies, in the case of Dore et al. that the optimal patent term is sensitive to changes in k .

The dependence of the optimal patent term on the degree of sharpness of the invention possibility function was not explicitly spelled out in the analysis of Dore et al. Given the invention possibility function in (2.8e) they calculate the output elasticity of research, $v = B'R/B = g/R = -\ln B$. This leads them to conclude that B is uniquely determined by v and hence the optimal patent term in equation (2.8g) depends on the output elasticity of research, v . For Nordhaus, when $B(R) = \beta R^\alpha$, $v = \alpha$, and thus output elasticity of research is constant at α . This leads them to conclude that B is independent of output elasticity of research in the case of Nordhaus. The consequence of the assumption of independence of B and v (as observed by Dore et al.) is that the optimal patent term in the case of Nordhaus is always longer than the case of Dore et al. For example, when $v = .5$, the corresponding $B = e^{-.5} \approx .605$. So the optimal patent term for $v = .5$ (hence $B = .605$), $\eta = 1.0$ and $\rho = .2$ is 5.11 for the Dore et al. case [from equation (2.8g)]. On the other hand, for $B = .605$, $\eta = 1.0$, $\rho = .2$, $\alpha = v = .5$, the optimal patent term is 9.20 for the Nordhaus case [from equation (2.7)].

From the above numerical examples, the conclusion of the longer optimal patent term in the Nordhaus case is not surprising. Dore et al. overlooked one of the comparative static results of the optimal patent term due to Scherer (1972) that the optimal patent life is shorter the sharper the curvature of the invention possibility function. It can then easily be shown that the longer patent term which was obtained in the above numerical

example in the case of Nordhaus is the direct consequence of the curvature of the invention possibility function. From the above numerical example, for $v = .5$ (hence $B = .605$), $\eta = 1.0$, $\rho = .2$, the optimal patent term in the case of Dore et al. was 5.11. Given the value of v (hence B), it is implicit in the example that the curvature of the invention possibility function for Dore et al. is $k_D = -(\ln B + 2)/\ln B = -(\ln .605 + 2)/\ln .605 \approx 2.98$. On the other hand, for $B = .605$, $\eta = 1.0$, $\rho = .2$, $\alpha = v = .5$, the optimal patent term in the case of Nordhaus was 9.20. Given the value of $\alpha = v = .5$, the curvature (implicit) of the invention possibility curve for Nordhaus is $k_N = (1 - \alpha)/\alpha = (1 - .5)/.5 = 1.0$. Since for same values of B , η and ρ the invention possibility function for Dore et al. is sharper than that of Nordhaus, i.e., $k_D > k_N$, the optimal patent term in the case of Dore et al. must be shorter than in the case of Nordhaus. As long as $k_D < k_N$, with same values of B , η and ρ the optimal patent term in the Dore et al. case will be longer than in the Nordhaus case. For example, for $B = .2$, $\eta = 1.0$, $\rho = .2$, the optimal patent terms for Dore et al. [equation (2.8g)] and Nordhaus [equation (2.7) with $\alpha = .1$] are respectively 19.6 and 4.2. The reason is that k_D is now smaller than k_N , where $k_D = -(\ln B + 2)/\ln B = -(\ln .2 + 2)/\ln .2 \approx .243$ and $k_N = (1 - \alpha)/\alpha = (1 - .1)/.1 = 9$.

Tandon (1982) used the optimal patent term model in connection with compulsory licensing of inventions. Compulsory licensing is a policy to reduce the restriction of use of inventions associated with the patent grant. The increased use of inventions under compulsory licensing also reduces the patentee's income, hence the incentive for investing in research and development. The social problem is to obtain an optimal royalty rate which would balance the negative effect on the investment of resources in R&D with the positive effect of reducing monopolistic restrictions on the use of inventions. Using Nordhaus' model, Tandon (1982) obtains the

optimal patent term and the optimal royalty rate simultaneously in the case of both process and product innovations. When inventors are unique, he concludes that for process as well as product innovations subject to compulsory licensing, the optimal patent has an infinite life.

2.2 The Optimal Patent Term When There is Free Entry into Inventive Activity

As we indicated in Chapter 1, with free entry into inventing, competition for a patent will be such that all privately appropriable surplus resulting from an invention will be dissipated in the rivalry for it. In this case, the value of the resources spent by rival inventors is just equal to the expected value of royalty income of the patentee. An increase in the patent term will then increase both the value of the surplus which is appropriable by the patentee and the value of the resources devoted in inventive activity. This increased surplus due to the increase in the patent term will be dissipated in the rivalry for the patent. As a result the surplus maximizing patent term will be shorter. The surplus maximizing patent term in this case was obtained by Berkowitz and Kotowitz (1979).

With free entry into inventive activity all the privately appropriable surplus which is given by the area of the rectangle C_0ABC_1 (Figure 2.1) is dissipated. This implies that during the length of the patent term the present value of the royalty income given by the area of the rectangle C_0ABC_1 is equal to the cost of the inventive activity. The amount of surplus which is realized is given by areas of the rectangle C_0ABC_1 and the triangle ABD summed over the period after the expiry of the patent. Then it follows that the area C_0ABC_1 is surplus only to the extent that it is not privately appropriable. An increase in the patent term increases C_0ABC_1 and ABD. An increase in the patent term also implies that society

has to wait longer to realize these surpluses given by the areas of C_0ABC_1 and ABD. Therefore, under free entry into inventing, the additional surplus generated by a given extension of the patent term is less than when inventors are unique. As a result, the surplus maximising patent term will be shorter when the inventors are competitive than when they are unique.

When there is free entry into inventing, the cost of research on any given invention is just equal to the present value of the royalty income. Thus investment in R by competitors will proceed until the aggregate rent arising from inventing falls to zero, i.e., from (2.4)

$$\int_0^T B(R)e^{-\rho t} dt - sR = \frac{1}{\rho} B\psi - sR = 0 \quad (2.9)$$

The nature of the circumstances which might underlie (2.9) requires some discussion.

The model assumes that the application of research effort to what is at any point of time a fixed stock of knowledge is subject to diminishing returns. Maximization of the implicit rent to the stock of knowledge requires that research effort be such that the value of the marginal product of research effort be equal to its marginal cost. With free entry research is undertaken until the value of the average product of research equals average cost and there is no rent to the stock of knowledge.

Insofar as individual researchers are concerned we envisage a situation in which there are n researchers each expending R_i units of research effort. Each has a probability of $1/n$ of winning the patent. Whoever wins takes all - that is, the winner gets the benefit of the losers' research efforts.

Under these circumstances the zero profit free entry equilibrium for the individual inventor requires that

$$1/n[\psi B(nR_i)/\rho R_i] = s.$$

Since $R = nR_1$ in aggregate the value of the average product of research equals average cost, or

$$\psi B(R)/\rho R = s.$$

It is important to note as Beck (1983) has, there is neither duplication nor premature invention in this model. From a private point of view, however, there is excessive research. That is, an individual with the exclusive right to the stock of relevant knowledge would engage in less research. This would be socially optimal providing the social and private benefits of research coincide. If they do not what is excessive from a private point of view may be an improvement from a social point of view. That is, given a big enough externality, society may prefer this "dissipation". While for some patent terms it is possible to improve welfare by allowing the type of free entry assumed here, given the unique inventor optimal patent term it is never welfare improving to replace the unique inventor with free entry.

The optimal patent term may be obtained by maximizing the welfare function (2.3) subject to (2.9).

Differentiating (2.9) with respect to ψ gives the competitive response to the level of resource spending to a change in ψ :

$$\frac{\partial R}{\partial \psi} = \frac{BR}{\psi(B - B'R)} \quad (2.10)$$

When (2.3) is differentiated with respect to ψ subject to (2.10), we obtain

$$\frac{\partial W}{\partial \psi} = \frac{RB}{\psi(B - B'R)} [B' + \eta BB'(1-\psi) - \rho s] - \frac{1}{2}\eta B^2 = 0 \quad (2.11)$$

By substituting $\rho s = B\psi/R$ from (2.9) in equation (2.11) and reorganizing

we obtain the patent term satisfying the policy maker's equilibrium in the case of competitive inventing as

$$T_C = -\frac{1}{\rho} \ln(1 - \psi_C^*) \quad (2.12)$$

where

$$\psi_C^* = \frac{1 + \eta B}{1 + \eta B \left[\frac{1}{2} + \frac{B}{RB'} \right] + \frac{B - RB'}{RB'}} \quad (2.13)$$

Comparison of expressions (2.7) and (2.13) reveals that given the concavity of $B(R)$ which implies $B_0 > RB'$, ψ_M^* will exceed ψ_C^* and thus the optimal patent term with unique inventors will exceed the optimal patent term with competitive inventors if

$$\frac{B}{R} - B' \geq \frac{B''B}{B'} \quad (2.14).$$

Since $B'' < 0$, whether the condition (2.14) holds depends on the invention possibility function $B(R)$. If it is of the form $B(R) = \beta R^\alpha$, as was assumed by Nordhaus, then (2.14) holds as an equality and $\psi_M^* > \psi_C^*$.

As before, for given values of α , B , ρ and η , we can calculate the patent term in the case of competitive inventing which would satisfy (2.13). The results of our patent term calculations are reported in Table 2.1. This calculation is the same as that reported by Berkowitz and Kotowitz (1979). It is clear from Table 2.1 that the patent term with unique inventors is generally close to the existing patent term while the patent term with competitive inventors is much shorter, approximately six months.

As mentioned in Chapter 1 that the assumptions of the optimal patent term models can be relaxed in many directions. We must then look at the Nordhaus, Berkowitz-Kotowitz models of the optimal patent term in an environment in which all the surplus is not dissipated as well as the portion of surplus which is appropriable is overstated. The models considered in this chapter ignore the effects of post-patent competition (which reduces the amount of surplus appropriable by the patentee), and taxation of inventors' income (which reduces the portion of appropriable surplus dissipated) which are surplus preserving features. These models also ignore the effects of uncertainty about the value of inventions on the optimal patent term. Introduction of these features in the Nordhaus, Berkowitz-Kotowitz models have the effect of increasing the optimal patent term.

FOOTNOTES TO CHAPTER 2

1. See, for example, Barzel (1968), Hirshleifer (1971), Kamien and Schwartz (1972), Loury (1979), Dasgupta and Stiglitz (1980), and Tandon (1983).
2. Social rates of return to research is greater than the corresponding private rate has been furnished by Mansfield et al. (1977).
3. See also Nordhaus (1972), Scherer (1972) and Stiglitz (1969).
4. For a non-drastic process invention (small invention) royalty rate equals to the total cost reduction. When the invention is drastic (large invention), the royalty rate will be less than the cost reduction [for this see Nordhaus (1969)]. If the inventor faces post patent competition from inferior substitute inventions, the royalty rate is also less than the cost reduction (the concept of post-patent competition will be elaborated in Chapter 3).
5. P_0 and C_0 respectively are the preinvention levels of price and cost and $P_0 = C_0$. When C_0 is normalized to 1, $P_0 - P_1 = C_0 - C_1 = B(R)$; slope of the demand function η is also the elasticity of demand when $P_0 = X_0 = 1$.
6. In his optimal patent term calculations Nordhaus assumes the elasticity α to be 0.1, although empirical literature confirms that α may vary from 0.05 to 0.12 [see Minasian (1962), Mansfield et al. (1965, 1968, 1977)].
7. See Scherer (1980), p. 448.

CHAPTER 3

SURPLUS PRESERVING FEATURES AND THE OPTIMAL PATENT TERM

We have seen in Chapter 2 that the low optimal patent term in the case of free entry into inventing is derived under the assumption that all appropriable surplus is dissipated because of rivalry for the patent. A six-month patent term based on these assumptions is in contrast with the existing patent term. There are a number of ways in which one can explain the observed length of the existing patent term. Firstly, it is to recognise that the privately appropriable surplus resulting from an invention is not entirely dissipated in the rivalry for the patent. Secondly, one could argue that the portion of surplus which is appropriable and thus available for dissipation is overstated.

In a recent paper, McPetridge and Rafiquzzaman (1983) pointed out a number of ways in which some appropriable surplus was preserved. There are several characteristics of the inventor's environment which act as surplus preserving mechanism. These surplus preserving features include: the taxation of inventors' royalty income, and the existence of post patent competition. The implications of these environmental limitations on dissipation of privately appropriable surplus due to tax system and post-patent competition and their impact on the optimal patent term are examined in this chapter.

3.1 Features of Inventor's Environment which Reduce Dissipation

3.1.1 Taxes

Competition among inventors will be such as to dissipate all privately appropriable surplus, the portion of the successful inventor's royalty income which is expected to be taxed away will therefore not be dissipated.

Our rivalry model has assumed that inventors, in aggregate, will just

cover their costs ex ante. If all costs are deductible for tax purposes and tax liabilities are a proportion of net income then inventors in aggregate will not expect to pay any taxes and their costs will again equal their total expected royalty income.

This outcome can be illustrated in the case of the type of rivalry described by Barzel (1968). In this case an inventor pre-empts his rivals by inventing at a point in time such that earlier invention would involve economic losses. If the respective opportunity costs of the resources used by this inventor are deductible, he will pay no taxes and the existence of the tax system will not affect the time at which pre-emptive invention occurs.

An alternative rivalry model assumes that there are n rival inventors who collectively make research expenditures equal to the present value of anticipated royalty income. One inventor gets the patent and can expect to pay taxes equal to some fraction of the differences between his royalty income and his own research expenditures. If the $n-1$ unsuccessful inventors are able to write off their expenditures against other income or to sell their deductions the total value of the deductions taken will be just equal to expected royalty income and there will be no net payment of tax ex ante.

The implication of these two examples is that if surplus is to be preserved in the form of income taxes which inventors expect to pay then either some of the costs of invention must be non-deductible for tax purposes or at least some part of the losses on unsuccessful inventive activities must be non-deductible from other income.

The existence of limitations on deductability such as these characterize the Canadian and most other income tax systems. The opportunity cost of equity is not deductible from taxable income. As a result income must exceed costs, including the opportunity cost of equity by the amount

$(tr_e E) / (1-t)$ ex ante where t is the income tax rate, r_e is the opportunity cost of equity and E is the value of the equity outstanding.

It is also the case in Canada and in a number of other jurisdictions that research costs can be deducted only from "related income" for tax purposes (Strain, 1981). An individual inventor cannot deduct research expenses from employment income. A corporation with no taxable income cannot transfer its deduction to another corporation even an affiliated one and must therefore carry it forward with the consequent loss in its present value.

These considerations lead to the conclusion that rivalry among inventors will be carried to the point at which the cost of research is equal to anticipated royalties less the inventors' expected tax liability. As a rough measure of the proportion of their gross income inventors might expect to pay in taxes we have taken income taxes as a fraction of personal income from the National Accounts. This proportion which we call τ averaged approximately 12 per cent between 1972 and 1978.

3.1.2 Post-Patent Competition: Theory

Much of the rivalry literature makes the implicit assumption that competition ceases once the patent is granted. In particular, the optimal patent term calculations of Berkowitz and Kotowitz (1979) assume that the successful inventor is free to extract the entire cost saving resulting from his invention in the form of royalties.

We will argue in this section that the ability of the patentee to extract royalties from the users of his invention is limited by potential competition from inferior but non-infringing substitutes. As a consequence, some of the benefits of the invention are immediately passed on to its users.

The proportion of surplus which is appropriable and thus available for dissipation is reduced.

Consider the case of patented invention i which reduces the unit cost of production in the using industry by B per cent. There are a large number of inferior but non-infringing substitutes for i , the invention of which becomes possible once the characteristics of i become known. Substitute j is inferior to invention i if there exists a royalty rate on i , $(1 - \theta)B_i$, $0 < \theta < 1$, which makes the development of j unprofitable while leaving the inventor of i with economic profits.

Any royalty in excess of $(1 - \theta)B_i$ will therefore bring one or more of the potential substitutes into existence. The inventor of i cannot, therefore, expect to extract royalties in excess of $(1 - \theta)$ per cent of the cost reduction which results from this invention. Cost savings in the amount of θB_i are passed on immediately to users and in present value terms the total value of resources expended in the rivalry for the patent on i cannot exceed $(1 - \theta)B_i$.

Consider a second case in which the possibility of developing a non-infringing substitute for invention i becomes apparent in the course of its use. It is in the mutual interest of the inventor and the users of invention i to avoid the expenditure of resources on a duplicate invention. If the duplicate is inferior there will be some positive royalty rate $(1 - \theta)B_i$ which leaves both the licensees and the patentee better off than if resources had been invested in the development of a substitute.

In this case it will again be known to all those competing for the patent on invention i that its appearance will make possible the invention of non-infringing substitutes and that, as a consequence, the royalty income of the successful inventor will amount to only $(1 - \theta)$ per cent of the cost

reduction resulting from the innovation. The balance of the cost saving, θB_1 , is passed on to users and cannot be dissipated in pre-patent rivalry.

The existence of post-patent competition has the effect of decreasing the surplus appropriable by inventors associated with any given level of inventive activity and, as we will demonstrate in Sections 3.3 and 3.4, increasing the optimal patent term.

3.2 Some Empirical Evidence on the Proportion of Surplus Appropriable by the Patentee

The empirical evidence we have been able to gather suggests that, insofar as cost-reducing process innovations are concerned, the patentee is likely to be able to appropriate something less than one-third of the cost reduction resulting from his invention.

Williamson (1963) surveyed the licensing practices of eleven U.S. companies and research organizations including Du Pont, General Electric and Raytheon. The evidence with respect to cost-reducing inventions was relatively meagre. Williamson found two licenses under which royalties were set at 20 per cent of the estimated net cost saving (1963, p. 36).

Williamson also examined the royalty rates awarded in thirty successful infringement suits. Of the thirty, four involved royalty rates based on the estimated savings arising from the use of the invention, the respective proportions of these savings which were awarded to the patentee in the form of royalties were 18 per cent, 100 per cent, approximately 50 per cent and over 57 per cent (1963, p. 236).

Williamson also considers the general division of rents between licensees and the licensor. He concludes:

The licensor's share of total (appropriable) surplus would generally seem to be between 20% and 50%. Two licenses with royalties based on net profits had rates of 34% and 50% respectively...This would tend to underestimate the percentage of the total surplus accruing to the licensor since "net profits" includes normal profits which are no part of the total surplus attributable to the patent rights. The same is true of the other two estimates that we came across of the normal division of profits between the two parties. One of these is the rule of thumb mentioned by several interviewees, that a licensor could expect to obtain about 25% of the expected profit margin on a patented item...The other is the estimate in a guide for inventors that 20% and 33-40% are the approximate percentages of profits an inventor can anticipate depending on whether or not he accepts the obligation of legally defending the patent (1963, pp. 243-244).

It is important to note that if the using industry is competitive as in our model, licensees will earn no profit in excess of their normal return and their observed share of rents to the innovation will be zero. This is not because the patentee has appropriated them but because any rents left in the hands of licensees will be competed away.

Enos (1962) has studied royalty rates on catalytic cracking patents. Enos reports that the royalty rate on the Burton process was 25 per cent of the cost saving it entailed while the royalty rate on the Fluid Process was 16 per cent of the cost saving (p. 215).

Some care should be taken in interpreting Enos' findings. His cost savings are in fact the net profits resulting from the use of the process given prevailing gasoline prices. Net profits are measured after the deduction of royalty payments and depreciation charges but before the deduction of a normal return to capital.

In order to obtain a measure of royalty payments as a function of the rents resulting from the use of the process it is necessary to derive rent estimates from the profit per barrel estimates reported by Enos (pp. 306-307). Reported royalties can then be expressed as a fraction of these rents. The results of this exercise are reported in Table 3.1.

Table 3.1

Catalytic Cracking Royalties as a Fraction of Economic Rents

	Burton Clark	Process Tube & Tank	Holmes Manley	Houdry 1939	Fluid 1942 40,000 BPSD
Profit/bbl. ¹	.98	.91	.19	.28	.31
+ Royalties/ bbl. ²	<u>.17</u>	<u>.10</u>	<u>.05</u>	<u>.04</u>	<u>.05</u>
Gross Profit/ bbl.	1.15	1.01	.24	.32	.36
- Opportunity cost of capital/bbl. ³	<u>.06</u>	<u>.04</u>	<u>.05</u>	<u>.06</u>	<u>.07</u>
Economic Rent/ bbl.	1.09	.97	.19	.26	.29
Royalty+ Economic Rent (%)	16	10	26	15	17

¹Enos, pp. 306-307.

²Enos, pp. 304-305.

³(.10)(cost of cracking plant)/bbl. yr. (average). This is equivalent to Enos' depreciation charges per bbl. which are 10% straight-line per year. See pp. 304-305.

Our interpretation of Enos' data is that the royalty rates on the various catalytic cracking processes ranged from 10 to 26 per cent of the economic rents attributable to their use. These calculations are quite rough. Enos himself concedes that the lump sum royalties on the Houdry process have not been properly annualized (p. 245). They do suggest, however, that a considerable fraction of the economic rents attributable to each of these cracking processes were left in the hands of licensees.

Layton (1972, pp. 80-93) studied the licensing of the float glass process by its inventor, Pilkingtons. The float glass process reduced the overall manufacturing cost of plate glass by 25 per cent. Pilkingtons typically received a lump sum payment plus running royalties in the amount of 6 per cent of sales for the first eight years and 4 per cent of sales for the following eight years (1972, p. 88).

Using the reported 6 per cent royalty rate, 25 per cent cost saving and the 5 per cent profit rates on sales of Pilkingtons' major licensees (p. 93) we can calculate the royalty rate as a percentage of the cost saving. That is,

$$\Delta C = .25C$$

$$S = C/(1-\Pi/S)$$

$$R/\Delta C = (R/S)(S/\Delta C) = .06/((.25)(1-.05)) = .253$$

where $\Delta C/C$ = the percentage change in manufacturing cost resulting from the introduction of the float glass process;

S = licensee annual sales;

Π = licensee annual profits;

$S = \Pi + C$;

R = annual royalties paid by licensee to Pilkingtons.

Thus, running royalties for float glass were approximately 25 per cent of the cost reduction resulting from the process. Lump sum royalties could have added another six percentage points to this. Specifically, the total, undiscounted lump sum royalty receipts of Pilkingtons from 1963 to 1969 amounted to £6,950,000. The implied annual sales of glass produced under license in 1969 is £98,050,000. (This is the value in £ of running royalties received by Pilkingtons in 1969 divided by .06.) The present value of annual sales at this level for seven years discounted at 10 per cent is £477,307,400. The implied percentage royalty on sales is 1.5 per cent ($6,950/477,307,400$). The implied royalty as a percentage of the cost saving is 6.5 per cent ($.015/((.26)(1-.05))$).

A generous estimate of Pilkingtons' royalties on the float glass process is that they amounted to 32 per cent of the cost reduction made possible by the process.

We must concede that observed royalty rates may be relatively low for reasons other than the existence of potential competition. If patents are cross-licensed the patentee may be receiving benefits from the licensee well in excess of observed royalty payments.

Alternatively, we may observe relatively low royalty rates when patent licenses are used as the basis for a cartel (Priest, 1977, pp. 326-330). There were allegations refuted by Priest (1977, pp. 364-376) that the catalytic cracking patents referred to above were the basis for a cartel arrangement among gasoline refiners. If cartel profits are attributable to a specific patent and these profits are shared with the patentee then the surplus available for dissipation in pre-patent rivalry is increased and the net surplus resulting from any given level of inventive activity is reduced.

3.3 The Impact of Post-Patent Competition on the Optimal Patent Term When the Inventors are Unique

When the inventors are unique, the impact of post patent competition on the amount of decomposition of surplus is illustrated in Figure 3.1. The price of the output of the using industry falls immediately to $P_2 = C_2 = C_0 - \theta(C_0 - C_1)$, and output increases by the amount X_2 . When price goes down to P_2 , the additional surplus given by the area of the triangle ACD is realized by consumers and that given by the area of the rectangle CDFB is realized by the inventor during the life of the patent.

There is also a transfer of surplus given by the rectangle C_0ACC_2 from the inventor to the consumers during the life of the patent. After the expiry of the patent there will be an increase in consumers' surplus composed of the area of the rectangle C_2DFC_1 , which has accrued to the inventor during the life of the patent, plus the area of the triangle DEF which is the gain in consumers' surplus from increased output to X_1 at lower price P_1 .

Thus the welfare function which society wants to maximize in this case is given by

$$W = \int_0^{\infty} [B(R)X_0 + (X_2 - X_0)(P_2 - P_1) + \frac{1}{2}(X_2 - X_0)(P_0 - P_2)]e^{-\rho t} dt \\ + \int_T^{\infty} \frac{1}{2}(X_1 - X_2)(C_2 - C_1)e^{-\rho t} dt - sR \quad (3.1)$$

The first term of (3.1) corresponds areas in perpetuity given by the areas C_0ABC_1 , ACD, and CDFB. The second term corresponds to the triangle DEF which is the present value of the gain to consumers when the patent period expires. The third term is once again the present value of the cost of resources invested in R&D assuming that all resources are spent in the first period.

Figure 3.1

Price and Output Effects of a Cost Reducing Process Invention when
Royalties are Less than the Cost Reduction.

Assume again that the demand function is linear of the form $X(P) = \xi - \eta P$, and that X_0 and C_0 are normalized to 1. Then, it implies that $X_2 - X_0 = \eta(P_0 - P_2) = \eta\theta B(R)$; $(P_2 - P_1) = (1 - \theta)B(R) = C_2 - C_1$; $X_1 - X_2 = \eta(P_2 - P_1) = \eta(1 - \theta)B(R)$. Substituting these values in the above welfare function and simplifying, we obtain

$$W = \int_0^{\infty} B[1 + \eta\theta(1 - \theta)B + \frac{1}{2}\eta\theta^2 B]e^{-\rho t} dt + \int_T^{\infty} \frac{1}{2}\eta(1 - \theta)^2 B^2 e^{-\rho t} dt - sR \quad (3.2)$$

The integration of (3.2) yields:

$$W = \frac{B}{\rho} + \frac{\eta B^2}{2\rho} [1 - \psi + \theta(2 - \theta)\psi] - sR \quad (3.3)$$

As in Chapter 2.1, the profit maximizing inventor will choose that level of R which maximizes the present value of royalties minus the resource cost,

$$\begin{aligned} \Pi &= \int_0^T [(1 - \theta)B(R) + (X_2 - X_0)(P_2 - P_1)]e^{-\rho t} dt - sR \\ &= \frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi - sR \end{aligned} \quad (3.4)$$

The first order necessary condition is obtained by maximizing (3.4) with respect to R and the condition is

$$\frac{1}{\rho}(1 - \theta)(1 + 2\eta\theta B)B'\psi = s \quad (3.5)$$

The response of the unique inventor to a change in the patent term is given by the derivative of (3.5) with respect to ψ , which is

$$\frac{\partial R}{\partial \psi} = - \left[\frac{B'(1 + 2\eta\theta B)}{\{B'' + 2\eta\theta B(BB'' + B'^2)\}} \right] \psi \quad (3.6)$$

The welfare maximizing patent term is then determined by differentiating the welfare function (3.3) with respect to ψ subject to (3.6), setting the result equal to zero and solving for ψ . The result is [for the derivation of equation (3.7) see Appendix 3A]:

$$\psi_M^*|_{\theta} = \frac{1 + \eta B}{\Delta_1} \quad (3.7)$$

where

$$\Delta_1 = 1 + \eta B \left[1 + \frac{k}{2}(1-\theta)^2 \right] - \theta - \frac{\theta \eta B [\theta + \eta B(1 - 2\theta + 3\theta^2)]}{(1 + 2\theta \eta B)} \quad (3.8)$$

Comparison of expression (2.7) and (3.7) reveals that they are same if $\theta = 0$. For $0 < \theta < 1$, Δ_1 in (3.8) is less than the denominator of (2.7), hence $\psi_M^*|_{\theta > 0}$ exceeds $\psi_M^*|_{\theta = 0}$ and thus the introduction of post patent competition has increased the optimal patent term in the case of unique inventors.

The extent to which the optimal patent term is increased depends on the functional form of $B(R)$ and on the values of the parameters θ , η , B and ρ . Rather than calculating the numerical values of optimal patent term, we can ask the question: What is the maximum fraction $(1-\theta)$ of cost reduction in terms of royalty income the patentee should receive if a patent system allows an infinite patent term? This may be seen from (3.7) when $\psi_M^*|_{\theta} = 1$. With $\psi_M^*|_{\theta} = 1$ and after some algebraic simplification, the equation (3.7) reduces to

$$\begin{aligned} & \theta^3 \eta^2 B^2 (k-3) + \theta^2 [\eta B (\frac{k}{2} - 3) - 2\eta^2 B^2 (k-1)] \\ & + \theta [\eta^2 B^2 (k-1) - (1 + \eta B k)] + \frac{1}{2} \eta B k = 0 \end{aligned} \quad (3.9)$$

One can readily obtain the maximum value of $(1-\theta)$, i.e., the optimal degree

of appropriability by solving θ from equation (3.9). Solution of θ from equation (3.9) depends on the size of the invention B and the elasticity of demand η .

Following Nordhaus we specify $B(R) = \beta R^\alpha$ and allow B to take on values .005, .01, .05 and .1. η takes on values .5, 1.0, 1.5 and 2.0. Given an infinite patent life, the numerical estimates of the optimal degree of appropriability ($1 - \theta$) are presented in Table 3.2.

While our empirical investigations in Section 3.2 show that patentee typically gets 25-30% of the cost reduction, Table 3.2 shows that given an unlimited patent term, the optimal appropriability is not less than 65%. Since the observed degree of appropriability is much lower than the optimal degree of appropriability, in the case of unique inventors there is considerable underincentive to invent.

Table 3.2

The Optimal Appropriability of a Cost Reducing Process Invention With
an Infinite Patent Term

B	η .05	1.0	1.5	2.0
.005	.999	.998	.968	.959
.01	.978	.959	.940	.923
.05	.908	.842	.792	.753
.10	.842	.753	.693	.650

3.4 The Impact of Income Taxes and Post-Patent Competition on the Optimal Patent Term When There is Free Entry into Inventing

Our earlier discussions in Sections 3.1.1 and 3.1.2 lead to conclude that the portion of the successful inventor's royalty income which is to be taxed away will not be dissipated and the successful inventor cannot extract the entire royalties from the users of his invention due to the presence of post-patent competition. A consequence of post patent competition is then to reduce the amount of appropriable surplus and thus to reduce the dissipation of total surplus.

The implication of this type of post patent competition together with the taxation of inventors' income also has the effect of increasing the optimal patent term even if there is free entry into inventing.

With free entry into inventing the cost of research on any given invention is just equal to the present value of the after-tax royalty income. The zero after tax profit equilibrium condition of competitive inventors is then

$$\begin{aligned} \int_0^T [(1-\theta)B(R) + (X_2 - X_1)(P_2 - P_1)] e^{-\rho t} dt - \left(\frac{1}{1-\tau}\right) sR \\ = \frac{1}{\rho} (1-\theta)B(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right) sR = 0 \end{aligned} \quad (3.10)$$

where τ is the income tax rate.

The optimal patent term with competitive inventors is then obtained by maximizing the welfare function (3.3) subject to (3.10). The response of competitive inventors to a change in the patent term is obtained by taking the derivative of (3.10) with respect to ψ . The result is

$$\frac{\partial R}{\partial \psi} = \frac{RB(1 + \theta\eta B)}{[B(1 + \theta\eta B) - RB'(1 + 2\theta\eta B)]\psi} \quad (3.11)$$

When the welfare function (3.3) is differentiated with respect to ψ subject to (3.11) and solved for ψ , the welfare maximizing patent term is obtained from the expression [for the derivation of equation (3.12) see Appendix 3B]

$$\psi_{\theta, \tau}^* = \frac{1 + \eta B}{\Delta_2} \quad (3.12)$$

where

$$\Delta_2 = 1 + \eta B \left[\frac{1}{2} + \frac{B}{2RB'} \right] + \frac{B - RB'}{RB'} - \frac{B}{2RB'} (2 + \eta \theta B) \theta - \frac{\eta \theta B (2 - \theta)}{2(1 + \eta \theta B)} - \frac{\theta \eta^2 B^2}{2(1 + \eta \theta B)} - \frac{B \tau}{RB'} (1 - \theta) (1 + \eta \theta B) \quad (3.13)$$

When expressions (2.13) and (3.12) are compared, it reveals that they are same if θ and τ are zero. For $0 < \theta < 1$, and $0 < \tau < 1$, the denominator of (2.13) is greater than Δ_2 hence $\psi_{\theta, \tau}^* |_{\theta, \tau > 0}$ exceeds $\psi_{\theta, \tau}^* |_{\theta, \tau = 0}$. Thus the introduction of taxes and post patent competition has increased the optimal patent term.

To the extent that the optimal patent term is increased depends on the parameters η , ρ , θ and τ and the functional form of $B(R)$. We again specify $B(R) = \beta R^\alpha$. As before B takes on values .005, .01, .05 and .1, while η takes on values .5, 1.0, 1.5 and 2.0. Parameters θ , τ and ρ are set at .75, .12 and .20 respectively.

The optimal patent term calculations from (3.12) for various values of B and η and for given values of the parameters are reported in Table 3.3. The centre of each cell represents the optimal patent term in the absence of post patent competition and tax system, i.e., when $\theta = \tau = 0$. In the upper right hand corner of each cell is the optimal patent term under the assumption that $\theta = .75$, $\tau = 0$, while in the lower left hand corner of each

Table 3.3

The Optimal Patent Life with Competitive Inventing

($\alpha = .10$, $\theta = 0.75$, $\tau = 0.12$, $\rho = 20\%$)

		η			
		.5	1.0	1.5	2.0
B	.005	3.0 .5 2.6	3.0 .5 2.6	3.0 .5 2.6	3.0 .5 2.6
	.01	3.0 .5 2.6	3.0 .5 2.6	3.0 .5 2.6	3.0 .5 2.6
	.05	3.0 .5 2.6	3.0 .5 2.6	3.1 .6 2.6	3.1 .6 2.6
	.10	3.1 .5 2.6	3.0 .5 2.6	3.1 .6 2.6	3.1 .6 2.6

cell is the optimal patent term when $\theta = .75$ and $\tau = .12$. The values in Table 3.3 indicate that with competitive inventing, introduction of post patent competition and income tax on inventors' royalty income results in an optimal term about five times longer than when they do not exist. But the optimal patent term remains considerably shorter than the optimal patent term under the assumptions that inventors are unique (see Table 2.1).

APPENDIX 3A: The derivation of text equation (3.7).

From equation (3.3) the welfare function is written as

$$W = \frac{B}{\rho} + \frac{\eta B^2}{2\rho} [1 - \psi + \theta(2-\theta)\psi] - sR \quad (3.3)$$

$\partial W / \partial \psi = 0$ yields:

$$B' + \frac{\eta B^2}{2} [-1 + \theta(2-\theta)] \frac{\partial \psi}{\partial R} + \eta B B' [1 - \psi + \theta(2-\theta)\psi] - \rho s = 0 \quad (3A.1)$$

After substituting expressions for $\partial \psi / \partial R$ from text equation (3.6) and the value of ρs from text equation (3.5) in the above expression and then manipulating, we obtain

$$\begin{aligned} & 2B' [B' + 2\theta\eta B B'] - \eta B^2 [-1 + \theta(2-\theta)] [B''\psi + 2\theta\eta B (B B'' + B'^2)\psi] \\ & + 2\eta B B' [1 - \psi + \theta(2-\theta)\psi] [B' + 2\theta\eta B B'] \\ & - 2 [(1-\theta)B'\psi + 2\theta(1-\theta)\eta B B'\psi] [B' + 2\theta\eta B B'] = 0 \end{aligned}$$

When both sides of (3A.2) is multiplied by $1/B'^2$ and $-BB''/B'^2$ is replaced by k , (3A.2) may be rearranged to write

$$\begin{aligned} & \psi [-\frac{1}{2} \eta B k - \theta \eta^2 B^2 k + \theta \eta B k + 2\theta^2 \eta^2 B^2 k - \frac{1}{2} \eta \theta^2 B k - \theta^3 \eta^2 B^2 k + \theta \eta^2 B^2 \\ & - 2\theta \eta^2 B^2 - 2\theta^2 \eta^2 B^2 + 4\theta^2 \eta^2 B^2 - 4\theta^2 \eta^2 B^2 + \theta^3 \eta^2 B^2 - 2\theta^3 \eta^2 B^2 \\ & + 4\theta^3 \eta^2 B^2 - \eta B + 2\theta \eta B - 2\theta \eta B - \theta^2 \eta B + 2\theta^2 \eta B + 2\theta^2 \eta B - 1 + \theta] \\ & + 1 + \eta B + 2\theta \eta B (1 + \eta B) = 0 \quad (3A.3) \end{aligned}$$

$$\text{or, } \psi[-\frac{1}{2} \eta B k (1+2\theta \eta B) + \theta \eta B k (1+2\theta \eta B) - \frac{1}{2} \eta \theta^2 B k (1+2\theta \eta B) - \theta \eta^2 B^2 - 2\theta^2 \eta^2 B^2$$

$$+ 3\theta^3 \eta^2 B^2 - \eta B - 2\theta \eta B + 3\theta^2 \eta B - 1 + \theta] + (1+\eta B)(1+2\theta \eta B) = 0$$

$$\text{or, } \psi[(1+2\theta \eta B)(-\frac{1}{2} \eta B k + \theta \eta B k - \frac{1}{2} \eta \theta^2 B k) - 1(1+2\theta \eta B) - \eta B(1+2\theta \eta B)$$

$$+ \theta \eta^2 B^2 - 2\theta^2 \eta^2 B^2 + 3\theta^3 \eta^2 B^2 + 3\theta^2 \eta B + \theta] + (1+\eta B)(1+2\theta \eta B) = 0$$

$$\text{or, } \psi[-(1+2\theta \eta B)(1 + \eta B + \frac{1}{2} \eta B k - \theta \eta B k + \frac{1}{2} \eta \theta^2 B k) + \theta \eta^2 B^2 - 2\theta^2 \eta^2 B^2$$

$$+ 3\theta^3 \eta^2 B^2 + 3\theta^2 \eta B + \theta] + (1+\eta B)(1+2\theta \eta B) = 0$$

$$\text{or, } \psi[-(1+2\theta \eta B)\{1 + \eta B + \frac{\eta B k}{2}(1-\theta)^2\} + \theta \eta^2 B^2(1 - 2\theta + 3\theta^2) + \theta^2 \eta B$$

$$+ \theta(1 + 2\theta \eta B)] + (1+\eta B)(1+2\theta \eta B) = 0$$

$$\text{or, } \psi_{M|\theta}^* = \frac{1 + \eta B}{\Delta_1}$$

which is text equation (3.7), where

$$\Delta_1 = 1 + \eta B[1 + \frac{k}{2}(1 - \theta)^2] - \theta - \frac{\theta \eta B[\theta + \eta B(1-2\theta+3\theta^2)]}{1 + 2\theta \eta B}$$

APPENDIX 3B: The derivation of text equation (3.12)

From equation (3A.1) $\partial W/\partial \psi = 0$ yields:

$$B' + \frac{\eta B^2}{2}[-1 + \theta(2-\theta)]\frac{\partial \psi}{\partial R} + \eta B B'[1-\psi + \theta(2-\theta)\psi] - \rho s = 0 \quad (3A.1)$$

The competitive inventors' zero after tax equilibrium condition in text equation (3.10) and the response function to a change in the patent term in text equation (3.11) may be written as

$$\rho s = \frac{L\psi}{R\delta} \quad (3B.1)$$

$$\frac{\partial R}{\partial \psi} = \frac{RL}{(L-RM)\psi} \quad (3B.2)$$

where

$$\delta = \frac{1}{1-\tau} \quad (3B.3)$$

$$L = (1-\theta)B(1+\theta\eta B) \quad (3B.4)$$

$$M = (1-\theta)B'(1+2\theta\eta B) \quad (3B.5)$$

Substituting the values of $\partial \psi/\partial R$ and ρs in equation (3A.1), we obtain

$$B' + \frac{\eta B^2}{2}[-1 + \theta(2-\theta)] \frac{(L-RM)\psi}{RL} + \eta B B'[1 - \psi + \theta(2-\theta)\psi] - \frac{L\psi}{R\delta} = 0 \quad (3B.6)$$

Rearranging and solving for ψ from (3B.6), we obtain

$$\psi^*_{C|\theta, \tau} = (1 + \eta B)/\Delta_2 \quad (3B.7)$$

$$\text{where } \Delta_2 = \frac{L}{RB'\delta} - \frac{\eta B^2}{2}[\theta(2-\theta)-1]\frac{L-RM}{RLB'} + \eta B - \theta(2-\theta)\eta B \quad (3B.8)$$

Δ_2 may be simplified as follows: note from (3B.4) and (3B.5) that

$$\frac{L - RM}{RB'L} = \frac{B(1 + \theta\eta B) - RB'(1 + 2\theta\eta B)}{RB'B(1 + \theta\eta B)} \quad (3B.9)$$

$$\frac{L}{RB'} = \frac{(1-\theta)B(1+\theta\eta B)}{RB'} \quad (3B.10)$$

Using the above values, Δ_2 in (3B.8) may be simplified as:

$$\begin{aligned} \Delta_2 = & \frac{(1-\theta)(1+\theta\eta B)}{RB'\delta} - \frac{\eta B^2}{2} [\theta(2-\theta)-1] \left[\frac{B(1+\theta\eta B) - RB'(1+2\theta\eta B)}{RB'B(1+\theta\eta B)} \right] \\ & + \eta B - \theta(2-\theta)\eta B \end{aligned} \quad (3B.11)$$

After another long simplification (3B.11) may be written as

$$\begin{aligned} \Delta_2 = & \frac{(1-\theta)(1+\theta\eta B)}{RB'\delta} - \frac{\eta B^2\theta(2-\theta)}{2RB'} - \frac{\eta B\theta(2-\theta)}{2(1+\theta\eta B)} - \frac{\theta\eta^2 B^2}{2(1+\theta\eta B)} + \eta B \left[\frac{1}{2} + \frac{B}{2RB'} \right] \\ = & 1 + \eta B \left[\frac{1}{2} + \frac{B}{2RB'} \right] + \frac{B-RB'}{RB'} - \frac{B}{2RB'}(2+\theta\eta B)\theta \\ & - \frac{\eta\theta B(2-\theta)}{2(1+\theta\eta B)} - \frac{\theta\eta^2 B^2}{2(1+\theta\eta B)} - \frac{B\tau}{RB'}(1-\theta)(1+\theta\eta B) \end{aligned}$$

where $\delta = 1/(1-\tau)$ and hence the text equation (3.12).

CHAPTER 4

THE OPTIMAL PATENT TERM UNDER UNCERTAINTY WHEN THE INVENTORS ARE UNIQUE

Although in Chapter 3 we have discussed the impact of post-patent competition on the optimal patent term in the case of unique inventors, there are other features of the inventor's environment which affect the surplus maximizing optimal patent term. One line of investigation is to allow uncertainty about the value of inventions in Nordhaus' model and to evaluate the impact of uncertainty on the optimal patent term. As Scherer states:

For a special case Nordhaus has shown that terms larger than 15 years are optimal only for inventions yielding very modest cost savings or facing unusually elastic demand, but his analysis ignores such elements as uncertainty, risk aversion, and alternative incentives to innovation (1980, p. 454).

This suggests that the assumptions made in the optimal patent term models are somewhat restrictive. One question in particular deserves closer attention. The optimal patent term models of Nordhaus and others assume that conditions of perfect certainty prevail. The recognition of the impact of uncertainty on the optimal patent term is an essential step toward making this type of analysis more realistic.

The aim of this Chapter is to introduce uncertainty into the theory of the optimal patent term and to analyse its impacts on the optimal patent life. In what follows the inventor is unique and faces uncertainty about the value of his invention. Uncertainty is introduced in two ways: (a) Uncertainty regarding the demand function of the industry which purchases the cost reducing inventions; and (b) uncertainty regarding the amount of

cost reduction associated with a given amount of inventive activity. In both the cases the analysis is conducted under the assumptions that society is an expected welfare maximizer (risk neutral) while inventors are either risk neutral or risk averse. Uncertainty affects both the demand function and the magnitude of the cost reduction function multiplicatively.

It turns out that even when all parties are risk neutral, the introduction of uncertainty regarding the magnitude of the cost reduction resulting from a given invention does influence the optimal patent term. When inventors are assumed to be risk averse, the optimal patent term is longer than in the case of perfect certainty, whether uncertainty affects the demand function or the magnitude of the cost reduction function. Moreover, the patent term is longer the higher the index of the degree of risk aversion.

4.1 The Optimal Patent Term Under Demand Uncertainty

As noted above, Nordhaus and others have analyzed the optimal patent term under the conditions of perfect certainty about the demand function of the industry which uses the patented process invention. We now assume that the demand function faced by the using industry is uncertain, i.e., the quantity demanded is assumed to be a random variable with a probability distribution which depends upon price as a parameter. This means that the relevant demand is currently unknown, i.e., random or subject to an error term and that it does not shift over time. In such a case we may call the demand function in (2.1) the expected demand function. In the traditional theory, the decision maker has 100 percent confidence that the actual demand, say ϕ , will be equal to the expected demand $X(P)$, for this reason $X(P)$ is

called the riskless demand function. We introduce uncertainty in a multiplicative manner, i.e., $\phi = X(P)u$, where u is a random variable with a mean that is equal to 1 and a standard deviation that is equal to σ .¹ This implies that the mean of the random demand ϕ is equal to $X(P)$ and its standard deviation is equal to $\sigma X(P)$.

As illustrated in Figure 2.1, when the preinvention cost level is C_0 and the postinvention cost level is C_1 and there is no uncertainty about the industry demand function, the maximum royalty the monopolist inventor can charge for his invention is given by the area of the rectangle $C_0ABC_1 = B(R)X_0$ per period during the patent term. Once the demand function is assumed to be random, the inventor's royalty income per period is also random. One such example is shown as a dashed line in Figure 4.1 for a particular realization $u_0 > E(u)$, where E is the expectational operator. In this case, the actual per period royalty is the area of rectangle C_0EFC_1 . After expiry of the patent period, the consumers' surplus is increased by the areas of the rectangle C_0ABC_1 and triangle ABD (we note that the magnitude of this surplus is also random; EFG for the realization u_0 in Figure 4.1). As before, the optimal patent term requires that at the margin the incentive effects of additional patent protection and the marginally higher surpluses arising in the present and future must be balanced against the triangle loss in consumers' surplus which arises from extending the time before the additional triangle gain is realized.

If we assume that society is risk neutral (expected welfare maximizer),² the expected welfare function which society seeks to maximize is:

Figure 4.1

Price and Output Consequences of a Cost Reducing Invention Under
Uncertain Demand When Royalties Equal the Cost Reduction

$$E(W) = \bar{W} = \int_u [\int_0^\infty B(R) u X_0 e^{-\rho t} dt + \int_1^\infty B(R) u (X_1 - X_0) e^{-\rho t} dt - sR] f(u) du \quad (4.1)$$

where the integration is taken over the entire range of u and $f(u)$ is the probability density function of u . Normalizing X_0 to 1, we may substitute the expression for $X_1 - X_0 = \eta B(R)$ into equation (4.1) and integrate to yield:

$$\bar{W} = \frac{B}{\rho} + \frac{\eta}{2\rho} B^2 (1 - \psi) - sR \quad (4.2)$$

A comparison of (2.3) and (4.2) reveals that given uncertainty about the demand function of the using industry, the expected welfare function which society seeks to maximize is the same as in the perfect certainty case. For this reason we call (4.2) the certainty equivalent welfare function.

As mentioned in Chapter 2.1, in the perfect certainty case, the incentive for invention can be derived from the profit maximizing inventor's choice of the level of resource spending R . Under the condition of demand uncertainty, the inventor's royalties are random and so is his profit function. The present discounted value of his random profit is given by

$$V = \int_0^\infty B(R) u e^{-\rho t} dt - sR = \frac{\psi}{\rho} B(R) u - sR \quad (4.3)$$

where

$$E(V) = \frac{\psi}{\rho} B - sR \quad (4.4)$$

and

$$\text{Var}(V) = \frac{\psi^2}{\rho^2} B^2 \sigma^2 \quad (4.5)$$

The inventor's equilibrium choice of R will depend on his attitude towards risk.

4.1.1 Risk Neutral Inventor

If the inventor is risk neutral, the inventor will choose that level of R which maximizes the expected value of the discounted profit. In such a case he maximizes $E(V)$ in (4.4) with respect to R. Since expressions (2.4) and (4.4) are identical, the risk neutral inventor's equilibrium choice of R is determined by expression (2.5). In order to determine the expected welfare maximizing patent term, one differentiates the expression (4.2) with respect to ψ subject to (2.5) and the optimal ψ_{MRN}^* is the same as in expression (2.7).

Thus when the inventors are risk neutral, with monopoly inventing, the optimal patent term under demand uncertainty is identical to that of the perfect certainty case

4.1.2 Risk Averse Inventor

When the inventor is risk averse he will choose that level of R which maximizes the expected utility of his net discounted profit stream. That is, he will maximize

$$E[U(V)] = E[U(\frac{\psi}{\rho} B_u - sR)] \quad (4.6)$$

where U is the utility function operator. We assume a specific functional form for U(V), namely,

$$U(V) = a - be^{-\lambda V} \quad (4.7)$$

where $a, b, \lambda > 0$. This negative exponential utility function exhibits constant absolute risk aversion (the Arrow-Pratt measure of absolute risk aversion is λ) and satisfies a number of essential properties for a risk averse individual's behaviour.^{4,5} In particular,

$$U'(V) = b\lambda e^{-\lambda V} > 0, \quad (4.8a)$$

i.e., the marginal utility of profit is positive.

$$U''(V) = -b\lambda^2 e^{-\lambda V} < 0, \quad (4.8b)$$

i.e., marginal utility of profit diminishes with an increase in profit.

$$\frac{d}{dV} \left[-\frac{U''(V)}{U'(V)} \right] = 0, \quad (4.8c)$$

i.e., the marginal absolute risk aversion is non-decreasing with an increase in profit.

$$\frac{d}{dV} \left[-\frac{VU''(V)}{U'(V)} \right] = \lambda > 0, \quad (4.8d)$$

i.e., marginal relative risk-aversion increases with an increase in profit.

When the utility of profit is specified in (4.7), for an optimal choice of R , the inventor will maximize

$$E[U(V)] = \int_{-\infty}^{\infty} [a - be^{-\lambda V}] f(V) dV = a - bM_V(\lambda) \quad (4.9)$$

where $f(V)$ is the probability density function of V and $M_V(\lambda)$ is the moment generating function of the V distribution.^{6,7}

Recall from expression (4.3) that the inventor's discounted value of the random profit V is a linear function of u . If we assume that u is normally distributed with mean equal to 1 and standard deviation equal to σ , then V is also normally distributed with mean $\mu = \frac{\psi}{\rho} B - sR$ and standard deviation $S = \frac{\psi}{\rho} B\sigma$. Under the above assumptions, one can write equation (4.9) as⁸

$$E[U(V)] = a - b \text{EXP}[-\lambda\mu + \frac{1}{2}\lambda^2 S^2] \quad (4.10)$$

The maximization of the above expression $E[U(V)]$ is therefore equivalent to maximizing (for $\lambda > 0$) the expression⁹

$$L = \mu - \frac{1}{2}\lambda S^2 = \frac{\psi}{\rho} B - sR - \frac{1}{2}\lambda \frac{\psi^2}{\rho^2} B^2 \sigma^2 \quad (4.11)$$

For an optimal level of R , the risk averse inventor then maximizes (4.11) with respect to R . Differentiating (4.11) with respect to R , one obtains the first order necessary condition as

$$\frac{\partial L}{\partial R} = \frac{\psi}{\rho} B' - \lambda \frac{\psi^2}{\rho^2} \sigma^2 B B' - s = 0,$$

i.e.,

$$\frac{\psi}{\rho} B' [1 - \lambda \frac{\psi}{\rho} \sigma^2 B] = s \quad (4.12)$$

From the perspective of an individual inventor the optimal patent length is a parameter in his decision calculus. Given this it should be noted that the risk averse inventor's optimality condition in equation (4.12) differs from the risk neutral inventor's optimality condition given in equation (2.5). The condition (4.12) indicates that for any given patent length the risk averse inventor will employ fewer resources in R&D

than his risk neutral counterpart. Equations (2.5) and (4.12) are compared diagrammatically in Figure 4.2. The solid curve and the dashed curve in Figure 4.2 represent the marginal benefit curves for the risk neutral and the risk averse inventor respectively. Since $B(R) = 0$, when $R = 0$, the two marginal benefit curves coincide at $R = 0$. Also, from (2.5) and (4.12) it can easily be seen that for any given R , the marginal benefit to the risk averse inventor is less than the marginal benefit to the risk neutral inventor, and this is essentially captured by the marginal adjustment to marginal benefits for risk aversion term $-\lambda \frac{\psi^2}{\rho^2} \sigma^2 BB'$ in equation (4.12). This implies that for the same patent term the marginal benefit curve for the risk averse inventor is below that of the risk neutral inventor. Thus, for a given patent term (and for a given ψ) and ρ , the profit maximizing risk neutral inventor employs R_{MRN}^* amount of resources at which the marginal benefit of employment of resources is equal to the marginal cost. When the inventor is risk averse, he employs R_{MRA}^* which is less than R_{MRN}^* and at which marginal benefit of employment of resources is equal to the marginal cost.

The response of a risk averse inventor to a change in the patent term is given by the derivative of (4.12) with respect to ψ which is

$$\frac{\partial R}{\partial \psi} = \frac{-\rho B' + 2\lambda \sigma^2 BB' \psi}{\rho B'' \psi - \lambda \sigma^2 (BB'' + B'^2) \psi^2} > 0 \quad (4.13)$$

The expected welfare maximizing patent term in this case is obtained by maximizing (4.2) with respect to ψ subject to (4.12). From (4.2), $\partial \bar{W} / \partial \psi = 0$ yields

$$B' + \eta BB'(1-\psi) - \frac{\eta}{2} B^2 \frac{\partial \psi}{\partial R} - \rho s = 0 \quad (4.14)$$

Figure 4.2

A Comparison of the Optimal Condition for the Risk Averse and the
Risk-Neutral Inventors when the Patent Term is Fixed

When the expression for $\partial R / \partial \psi$ is substituted in (4.14), the optimal patent term is obtained by the solution of the following equation [for the derivation of equation (4.15), see Appendix 4A]:

$$A_1 \psi^3 - A_2 \psi^2 + A_3 \psi - A_4 = 0 \quad (4.15)$$

where

$$A_1 = 4\lambda^2 \sigma^4 B^2 \quad (4.16a)$$

$$A_2 = \rho \lambda \sigma^2 B [6 + \eta B (3 + k)] \quad (4.16b)$$

$$A_3 = \rho [4\lambda \sigma^2 B (1 + \eta B) + \eta \rho B (2 + k) + 2\rho] \quad (4.16c)$$

$$A_4 = 2\rho^2 (1 + \eta B) \quad (4.16d)$$

$$k = -B''B/B'^2 \quad (4.16e)$$

$$B = B(R) \quad (4.16f)$$

In particular, when $\lambda = 0$, the inventor is risk neutral and the solution of equation (4.15) is the expression (2.7).

Nordhaus would call expression (4.15) the policy maker's equilibrium and the expression (4.12) the inventor's equilibrium under the condition of demand uncertainty and inventor's risk aversion. In principle, the optimal patent term is obtained by the intersection of the policy maker's and the inventor's equilibrium expressions.

It is to be noted that there is no neat analytical solution for the optimal ψ from equations (4.15) and 4.12). Values of ψ that would satisfy (4.15) will depend on parametric assumptions about the riskless demand function $X(P)$ and the "invention possibility" function $B(R)$. It will also depend on σ and the risk aversion index λ . For given values of the relevant parameters, the coefficients of equation (4.15) may be calculated and numerical solutions for optimal ψ ($0 \leq \psi \leq 1$) and hence the optimal patent term T^* satisfying the policy maker's equilibrium may be obtained.

To be consistent with Nordhaus, we specify $B(R)$ as βR^α and allow B to take on values .005, .01, .05, .1, and α to take on value .1.¹⁰ η takes on values .5, 1.0, 1.5, 2.0 while ρ is set at 2.0.

For a reasonable value of σ let us recall the assumption that u is normally distributed with mean equal to 1 and standard deviation equal to σ . Given this assumption, the random demand ϕ is also normally distributed with mean equal to $X(P)$ and standard deviation equal to $\sigma X(P)$. When $\sigma \leq 1/4$, the probability that ϕ is negative is negligible and for this reason we will not consider the cases where $\sigma > 1/4$ for our optimal patent term calculations. Nevertheless, $\sigma = 1/4$ would reflect a considerable degree of uncertainty. Under this set up, it follows that ϕ is normally distributed with mean equal to $X(P)$ and standard deviation equal to $\frac{1}{4}X(P)$.

Regarding the risk aversion index λ , one must quantify λ . Quantification of the parameter λ is not possible here. Rather we allow λ to take on different values to analyse some comparative static results regarding the optimal patent term.¹¹ A higher value of λ indicates an increase in risk aversion and hence indicates that the inventor is more risk averse.

The resulting optimal patent term calculations are reported in Tables 4C1 - 4C4 and corresponding simulation charts are represented in Figures 4C1 - 4C4 for different values of B , η and λ (see Appendix 4C). The numerical results show that patent terms satisfying the policy maker's equilibrium (4.15) under uncertainty is in general significantly longer than that under perfect certainty. The patent terms in Tables 4C1 - 4C4 satisfying equation (4.15) is in general greater than the existing patent life of 17 years which in turn imply that the optimal patent term under uncertainty is longer than the existing patent term if the assumed B values are considered as plausible.

Before leaving this model, however, it is interesting to note some comparative static results about the optimal patent term. Tables 4C1 - 4C4 and Figures 4C1 - 4C4 (see Appendix 4C) illustrate these results:

- (i) The higher the elasticity of demand η , the lower is T^* , everything else being constant.
- (ii) The larger the cost reduction B , the lower is T^* , everything else being constant.
- (iii) The larger the risk aversion index λ , the longer is T^* , everything else being constant.

The (i) and (ii) are analogous to Nordhaus' findings that the optimal patent term in this model would be shorter for higher η and for larger B . The reasons for this have been explained in Chapter 2.1 (p. 23). The illustration of (iii) is not immediately obvious. From Figure 4.2, note that $R_{MRA}^* < R_{MRN}^*$ for any $\lambda > 0$. This implies that $B(R_{MRA}^*) < B(R_{MRN}^*)$. That is, given the patent length that is optimal for the risk neutral case, $B(R_{MRA}^*) < B(R_{MRN}^*)$ and $B'(R_{MRA}^*) > B'(R_{MRN}^*)$. From this it follows that for each additional unit of R that is induced by an expansion in the patent term, the risk aversion case will produce a larger amount of cost savings and thus a larger welfare gain than would arise in the risk neutral case. This unambiguous welfare improving effect is tempered by the consideration that the costs of achieving additional units of R (in terms of extending patent length) may be higher under the risk aversion case. The simulation experiments which have been considered (see Tables 4C1 - 4C4 and Figures 4C1 - 4C4 in Appendix 4C), however, demonstrate that over a wide range of values the first effect dominates the second and that increases in risk aversion further increase the optimal length of the patent term.

4.2 The Optimal Patent Term When the Magnitude of Cost Reduction is Uncertain

It is likely that the inventor also faces uncertainty about the magnitude of cost reduction that will result from a given level of inventive activity. As in the case of demand, we assume that the actual magnitude of the amount of cost reduction is unknown to the inventor in the sense that the amount of cost reduction is random or subject to an error term. When the magnitude of the cost reduction is uncertain, the random cost reduction function may be written as $B(R, \tilde{u}) = B(R)(1 + \tilde{u})$, where $\tilde{u}$ is a random variable with mean equal to zero and standard deviation equal to $\tilde{\sigma}$. For a comparison with the perfect certainty setting, we call $B(R)$, the riskless cost reduction function. The techniques of this analysis are similar to that of Section 4.1, but the results are significantly different.

In the presence of uncertainty about the cost reduction, the expected welfare function which society wants to maximize is given by

$$E(W) = \tilde{W} = \int_{\tilde{u}} \left[\int_0^{\infty} B(R, \tilde{u}) X_0 e^{-\rho t} dt + \int_T^{\infty} \frac{1}{2} B(R, \tilde{u}) (X_1 - X_0) e^{-\rho t} dt - sR \right] f(\tilde{u}) d\tilde{u} \quad (4.17)$$

where the integration is taken over the entire range of $\tilde{u}$ and $f(\tilde{u})$ is the probability density function of $\tilde{u}$. Normalizing X_0 to 1 and substituting $(X_1 - X_0) = \eta B(R, \tilde{u}) = \eta B(R)(1 + \tilde{u})$ in equation (4.17) and evaluating the integral, we obtain

$$E(W) = \tilde{W} = \frac{B}{\rho} + \frac{\eta}{2\rho} (1 + \tilde{\sigma}^2) B^2 (1 - \psi) - sR \quad (4.18)$$

Unlike the demand uncertainty a comparison of (2.3) and (4.18) indicates that the expected welfare function under cost reduction uncertainty is different from the welfare function arising in the case of perfect certainty.

4.2.1 Risk Neutral Inventor

As in the demand uncertainty case, the risk neutral inventor will employ that level of R which maximizes the expected discounted value of his net profit, i.e., he will maximize

$$E(\tilde{V}) = E\left[\int_0^T B(R)(1 + \tilde{u})e^{-\rho t} dt - sR\right] = \frac{\psi}{\rho} B - sR \quad (4.19)$$

where

$$\tilde{V} = \frac{\psi}{\rho} B(R)(1 + \tilde{u}) - sR \quad (4.19a)$$

and

$$\text{Var}(\tilde{V}) = \frac{\psi^2}{\rho^2} B^2 \tilde{\sigma}^2 \quad (4.19b)$$

Since the inventor's profit functions are identical both under perfect certainty and cost reduction uncertainty [equations (2.4) and (4.19)], the inventor's optimal choice of R which maximizes (4.19) is given by expression (2.5). For the expected welfare maximizing patent term one then differentiates (4.18) with respect to ψ subject to the condition (2.5). From equation (4.18) and noting from equation (2.5) that $\partial R / \partial \psi = -B' / B''\psi > 0$, we obtain

$$\frac{\partial \tilde{W}}{\partial \psi} = -\frac{B'^2}{\rho B''\psi} - \frac{\eta}{2\rho} (1 + \tilde{\sigma}^2) \left[\frac{2BB'^2}{B''\psi} (1 - \psi) + B^2 \right] + \frac{sB'}{B''\psi} = 0 \quad (4.20)$$

When the value of $\rho s = B'\psi$ [equation (2.5)] is substituted in equation (4.20) and solved for ψ , the optimal ψ^* , satisfying the policy maker's equilibrium is given by

$$\tilde{\psi}_{MRN}^* = \frac{1 + \eta B(1 + \tilde{\sigma}^2)}{1 + \eta B(1 + \tilde{\sigma}^2)(1 + \frac{k}{2})} \quad (4.21)$$

where k is given in equation (4.16e).

Recall from Chapter 2 [equation (2.7)] that with perfect certainty, the optimal patent term when the inventor is unique is given by

$$\psi_M^* = \frac{1 + \eta B}{1 + \eta B(1 + \frac{k}{2})} \quad (2.7)$$

From (4.21) and (2.7) one can write

$$\begin{aligned} \psi_M^* - \tilde{\psi}_{MRN}^* &= \frac{1 + \eta B}{1 + \eta B(1 + \frac{k}{2})} - \frac{1 + \eta B(1 + \tilde{\sigma}^2)}{1 + \eta B(1 + \tilde{\sigma}^2)(1 + \frac{k}{2})} \\ &= \{(1 + \eta B)[1 + \eta B(1 + \tilde{\sigma}^2)(1 + \frac{k}{2})] \\ &\quad - [1 + \eta B(1 + \tilde{\sigma}^2)][1 + \eta B(1 + \frac{k}{2})]\} / \Delta_3 \end{aligned} \quad (4.22)$$

where

$$\Delta_3 = [1 + \eta B(1 + \frac{k}{2})][1 + \eta B(1 + \tilde{\sigma}^2)(1 + \frac{k}{2})] > 0 \quad (4.23)$$

With some algebraic manipulation, the expression (4.22) may be reduced to

$$\psi_M^* - \tilde{\psi}_{MRN}^* = \eta B \tilde{\sigma}^2 k / 2 \Delta_3 \quad (4.24)$$

Since $\Delta_3 > 0$, $k > 0$, it implies from (4.24) that $\psi_M^* > \tilde{\psi}_{MRN}^*$.

The following proposition may then be stated:

Proposition 4.1

When the inventor is risk neutral, and society maximizes expected welfare, uncertainty in the magnitude of the cost reduction associated with a given amount of inventive activity will yield an optimal patent life which is shorter than in the perfect certainty case.

It is quite easy to see why this holds. Essentially, for any given cost reduction function, the social problem is to minimize the deadweight loss associated with monopoly. When the inventor is risk neutral, his optimal condition for investing resources in R&D is equivalent to that of the certainty case; so his benefits from invention remain the same. But the expected value of the deadweight loss becomes larger than the deadweight loss in the perfect certainty case. Therefore, the longer the patent term, the longer the consumers have to wait to realize this extra expected benefit whose magnitude is greater than its certainty equivalent counterpart. So, by granting a shorter patent term, society can gain this extra benefit earlier and as a result the optimal patent term is shorter. Further discussion of Proposition 4.1 also appears in Chapter 5, pp. 111-116.

4.2.2 Risk Averse Inventor

When the inventor is risk averse, he will choose that level of R which maximizes the expected utility of his net discounted profit stream $\tilde{V}$, where $\tilde{V}$ is given in equation (4.19a). Assuming again a negative exponential form of utility function in (4.7) and also assuming that $\tilde{u}$ is normally distributed with mean equal to zero and standard deviation equal to $\tilde{\sigma}$ one can go through the same steps as in Section 4.1.2. It can easily be verified that equations (4.11), (4.12), and (4.13) hold when σ is replaced by $\tilde{\sigma}$.

As before the expected welfare maximizing patent term is obtained by maximizing (4.18) with respect to ψ subject to (4.12) (replacing σ by $\tilde{\sigma}$). From (4.18) $\partial \tilde{W} / \partial \psi = 0$ yields:

$$B' + \eta(1 + \bar{\sigma}^2)BB'(1 - \psi) - \frac{\eta}{2}(1 + \bar{\sigma}^2)B^2 \frac{\partial \psi}{\partial R} - \rho s = 0 \quad (4.25)$$

Replacing σ by $\bar{\sigma}$ in expressions for $\partial R/\partial \psi$ [equation (4.13)] and for ρs [equation (4.12)] and substituting these expressions in (4.25), the optimal patent term is obtained by the solution of the following equation [for the derivation of equation (4.26), see Appendix 4B]:

$$B_1 \psi^3 - B_2 \psi^2 + B_3 \psi - B_4 = 0 \quad (4.26)$$

where

$$B_1 = 4\lambda^2 \bar{\sigma}^4 B^2 \quad (4.27a)$$

$$B_2 = \rho \lambda \bar{\sigma}^2 B [\eta B(1 + \bar{\sigma}^2)(3+k) + 6] \quad (4.27b)$$

$$B_3 = \rho [4\lambda \bar{\sigma}^2 B \{1 + \eta B(1 + \bar{\sigma}^2)\} + \eta \rho B(1 + \bar{\sigma}^2)(2+k) + 2\rho] \quad (4.27c)$$

$$B_4 = 2\rho^2 [1 + \eta B(1 + \bar{\sigma}^2)] \quad (4.27d)$$

$$k = -B''B/B'^2 > 0 \quad (4.27e)$$

$$B = B(R) \quad (4.27f)$$

As in the case of demand uncertainty there is no neat analytical solution for ψ^* and hence for the optimal patent term from equations (4.26) and (4.12). For given values of the relevant parameters, numerical solutions for the patent term satisfying (4.26) can be obtained.

For numerical calculations of our optimal patent term we specify $B(R) = \beta R^\alpha$ and allow the parameters α , η , and ρ to take on same values as in Section 4.1.2. As before B takes on values .005, .01, .05, .1.

In order to use a reasonable value of $\bar{\sigma}$ for our numerical estimates of the optimal patent term, we have used $\bar{\sigma} = 1/4$. The reasons for this is similar to those in Section 4.1.2. Recall the assumption^o that $\tilde{u}$ is normally distributed with mean equal to zero and standard deviation equal to $\bar{\sigma}$,

Since $B(R, \tilde{u}) = B(R)(1 + \tilde{u})$, the random cost reduction is also normally distributed with mean equal to $B(R)$ and standard deviation equal to $B(R)\tilde{\sigma}$. When $\tilde{\sigma} \leq 1/4$, the probability that $B(R, \tilde{u})$ is negative is negligible and for this reason we do not consider $\tilde{\sigma} > 1/4$. However, $\tilde{\sigma} = 1/4$ would once again reflect a significant level of uncertainty.

The resulting optimal patent term calculations satisfying (4.26) are reported in Tables 4D1 - 4D4 and the corresponding simulation charts are represented in Figures 4D1 - 4D4 (see Appendix 4D). As in the case of demand uncertainty the numerical results in Tables 4D1 - 4D4 show that when the inventor is risk averse ($\lambda > 0$), the patent term under cost reduction uncertainty is also longer than that of the perfect certainty case. These patent terms are, in general, greater than 17 years which imply that the optimal patent term under cost reduction uncertainty is longer than the existing patent term if the B values are considered as plausible. When the inventor is risk neutral ($\lambda = 0$), the numerical estimates reported in Tables 4D1 - 4D4 also reveal that the optimal patent term is shorter than in the case of perfect certainty (comparing numerical estimates in Tables 4D1 - 4D4 for $\lambda = 0$ with those in Table 2.1). This confirms the Proposition 4.1.

The optimal patent term calculations in Tables 4D1 - 4D4 and the corresponding simulation charts in Figures 4D1 - 4D4 produces the following comparative static results which are consistent to those in Section 4.1.2 (see Appendix 4D):

- (i) The higher the elasticity of demand η , the lower is T^* , everything else being constant.
- (ii) The larger the cost reduction B , the lower is T^* , everything else being constant.
- (iii) The larger the risk aversion index λ , the longer is T^* , everything else being constant.

The explanations for these comparative static results are the same as those discussed in Section 4.2.1.

FOOTNOTES TO CHAPTER 4

1. Uncertainty may also be introduced in an additive manner with a mean that is equal to 0.
2. By risk neutral society I assume a risk neutral patent authority who seeks to maximize an individualistic social welfare function (sum of expected consumers and producers surplus) by granting patents to inventors. A similar approach has recently been taken by Wright (1983) in another context.
3. See footnote 4 of Chapter 2.
4. The Arrow-Pratt measure of absolute risk aversion = $-U''(V)/U'(V) = \lambda$.
5. See Arrow (1974) and Tsiang (1974). The negative exponential utility function does not strictly satisfy Arrow's postulates - in that it implies not a decreasing absolute risk-aversion; but merely a non-decreasing one.
6. If the distribution of V is not known, then by means of a Taylor's series expansion of $U(V)$ around $E(V) = \bar{V}$, one can obtain

$$E[U(V)] = U(\bar{V}) + \frac{1}{2}M_2(V)U''(\bar{V}) + \frac{1}{3}M_3(V)U'''(\bar{V}) + \frac{1}{4}M_4(V)U^{(4)}(\bar{V}) + \dots,$$
 where $M_i(V)$ is the i th central moment of V . Obviously, $E[U(V)]$ assumes a mean-variance form when third and higher moments are zero.
7. The moment generating function, i.e., the exponential transform $M_V(\lambda)$ for a probability distribution function $f(V)$ is defined by

$$M_V(\lambda) = \int_{-\infty}^{\infty} e^{-\lambda V} f(V) dV.$$
 so, $\int_{-\infty}^{\infty} (a - be^{-\lambda V}) f(V) dV = a - b \int_{-\infty}^{\infty} e^{-\lambda V} f(V) dV = a - bM_V(\lambda)$. For this see Keeney and Raiffa (1976), p. 201 and Mood et al. (1974).
8. If V is normally distributed with mean μ and standard deviation S , then $M_V(\lambda) = \text{EXP}[-\lambda\mu + \frac{1}{2}\lambda^2 S^2]$. For this see Keeney and Raiffa (1976) p. 202 and Mood et al. (1974).
9. See Hay (1976), p. 49 and Mood et al. (1974).
10. If $B(R) = \beta R^\alpha$, then $k = -B''B/B'^2 = \frac{1-\alpha}{\alpha} = 9$ when $\alpha = .1$.
11. We use those values of λ for which "standard deviation of V times λ " is less than one. For this see Tsiang (1972).

APPENDIX 4A: The derivation of text equation (4.15) for the optimal patent term under demand uncertainty

From equation (4.2) $\partial W / \partial \psi = 0$ yields text equation (4.14):

$$B' + \eta BB'(1 - \psi) - \frac{\eta}{2} B^2 \frac{\partial \psi}{\partial R} - \rho s = 0 \quad (4.14)$$

When expression for $\partial \psi / \partial R$ from text equation (4.13) is substituted in (4.14), we obtain

$$B' + \eta BB'(1 - \psi) - \frac{\eta}{2} B^2 \left[\frac{\rho B''\psi - \lambda \sigma^2 \{BB'' + B'^2\}\psi^2}{-\rho B' + 2\lambda \sigma^2 BB'\psi} \right] - \rho s = 0$$

or,

$$\begin{aligned} & B'[-\rho B' + 2\lambda \sigma^2 BB'\psi] + \eta BB'(1 - \psi)[- \rho B' + 2\lambda \sigma^2 BB'\psi] \\ & - \frac{\eta}{2} B^2 [\rho B''\psi - \lambda \sigma^2 \{BB'' + B'^2\}\psi^2] - \rho s[-\rho B' + 2\lambda \sigma^2 BB'\psi] = 0 \quad (4A.1) \end{aligned}$$

Noting from text equation (4.12) that $\rho s = \psi B'[1 - \lambda \frac{\psi}{\rho} \sigma^2 B]$, and substituting ρs in expression (4A.1), we obtain

$$\begin{aligned} & B'[-\rho B' + 2\lambda \sigma^2 BB'\psi] + \eta BB'(1 - \psi)[- \rho B' + 2\lambda \sigma^2 BB'\psi] \\ & - \frac{\eta}{2} B^2 [\rho B''\psi - \lambda \sigma^2 \{BB'' + B'^2\}\psi^2] \\ & - [\psi B' - \frac{\lambda \sigma^2}{\rho} BB'\psi^2] [-\rho B' + 2\lambda \sigma^2 BB'\psi] = 0 \end{aligned}$$

or,

$$\begin{aligned} & -\rho B'^2 + 2\lambda \sigma^2 BB'^2\psi - \eta \rho BB'^2 + 2\eta \lambda \sigma^2 B^2 B'^2\psi + \eta \rho BB'^2\psi - 2\eta \lambda \sigma^2 B^2 B'^2\psi^2 \\ & - \frac{\eta}{2} \rho B^2 B''\psi + \frac{\eta}{2} \lambda \sigma^2 \{B^3 B'' + B^2 B'^2\}\psi^2 - \rho B'^2\psi - 2\lambda \sigma^2 BB'^2\psi^2 \\ & - \lambda \sigma^2 BB'^2\psi^2 + \frac{2}{\rho} \lambda^2 \sigma^4 B^2 B'^2\psi^3 = 0 \quad (4A.2) \end{aligned}$$

When both sides of (4A.2) are multiplied by $1/B'^2$ and then reorganized, (4A.2) reduces to

$$\begin{aligned}
 & -2\rho^2 + 4\rho\lambda\sigma^2 B\psi - 2\eta\rho^2 B + 4\eta\rho\lambda\sigma^2 B^2\psi + 2\eta\rho^2 B\psi - 4\eta\rho\lambda\sigma^2 B^2\psi^2 \\
 & - \eta\rho^2 B^2 B''\psi/B'^2 + \eta\rho\lambda\sigma^2 (B^3 B''/B'^2 + B^2)\psi^2 + 2\rho^2\psi \\
 & - 4\rho\lambda\sigma^2 B\psi^2 - 2\lambda\rho\sigma^2 B\psi^2 + 4\lambda^2\sigma^4 B^2\psi^3 = 0
 \end{aligned} \tag{4A.3}$$

Note from text equation (4.16e) that $k = -B''B/B'^2$. Replacing $-B''B/B'^2$ by k and then simplifying expression (4A.3), we obtain

$$A_1\psi^3 - A_2\psi^2 + A_3\psi - A_4 = 0$$

which is the text equation (4.15), where

$$A_1 = 4\lambda^2\sigma^4 B^2$$

$$A_2 = \rho\lambda\sigma^2 B[6 + \eta B(3 + k)]$$

$$A_3 = \rho[4\lambda\sigma^2 B(1 + \eta B) + \eta\rho B(2 + k) + 2\rho]$$

$$A_4 = 2\rho^2(1 + \eta B)$$

APPENDIX 4B: The derivation of text equation (4.26) for the optimal patent term under cost reduction uncertainty.

From equation (4.18) $\partial\tilde{W}/\partial\psi = 0$ yields text equation (4.25):

$$B' + \eta(1 + \tilde{\sigma}^2)BB'(1 - \psi) - \frac{\eta}{2} B^2(1 + \tilde{\sigma}^2)\frac{\partial\psi}{\partial R} - \rho s = 0 \tag{4.25}$$

When expression for $\partial\psi/\partial R$ from text equation (4.13) (replacing σ by $\tilde{\sigma}$) is substituted in (4.25), we obtain

$$B' + \eta(1 + \tilde{\sigma}^2)BB'(1 - \psi) - \frac{\eta}{2}(1 + \tilde{\sigma}^2)\left[\frac{\rho B''\psi - \lambda\tilde{\sigma}^2\{BB'' + B'^2\}\psi^2}{-\rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi}\right] - \rho s = 0$$

or

$$\begin{aligned}
 & B'[-\rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] + \eta(1 + \tilde{\sigma}^2)BB'(1-\psi)[- \rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] \\
 & - \frac{\eta}{2} B^2(1 + \tilde{\sigma}^2)[\rho B''\psi - \lambda\tilde{\sigma}^2\{BB'' + B'^2\}\psi^2] \\
 & - \rho s[-\rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] = 0 \quad (4B.1)
 \end{aligned}$$

Noting from text equation (4.12) that $\rho s = \psi B'[1 - \lambda \frac{\psi}{\rho} \tilde{\sigma}^2 B]$, and substituting ρs in expression (4B.1), we obtain

$$\begin{aligned}
 & B'[-\rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] + \eta(1 + \tilde{\sigma}^2)BB'(1-\psi)[- \rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] \\
 & - \frac{\eta}{2} B^2(1 + \tilde{\sigma}^2)[\rho B''\psi - \lambda\tilde{\sigma}^2\{BB'' + B'^2\}\psi^2] \\
 & - [\psi B' - \lambda \frac{\psi^2}{\rho} \tilde{\sigma}^2 BB'][-\rho B' + 2\lambda\tilde{\sigma}^2 BB'\psi] = 0
 \end{aligned}$$

or

$$\begin{aligned}
 & -\rho B'^2 + 2\lambda\tilde{\sigma}^2 BB'^2\psi + \eta(1 + \tilde{\sigma}^2)(1-\psi)[- \rho BB'^2 + 2\lambda\tilde{\sigma}^2 B^2 B'^2\psi] \\
 & - \frac{\eta}{2}(1 + \tilde{\sigma}^2)[\rho B''B^2\psi - \lambda\tilde{\sigma}^2\{B^3 B'' + B^2 B'^2\}\psi^2] \\
 & - [-\rho B'^2\psi + 2\lambda\tilde{\sigma}^2 BB'^2\psi^2 + \lambda\tilde{\sigma}^2 BB'^2\psi^2 - 2\frac{\lambda^2}{\rho}\tilde{\sigma}^4 B^2 B'^2\psi^3] = 0 \quad (4B.2)
 \end{aligned}$$

When both sides of (4B.2) are multiplied by $1/B'^2$ and then reorganized, (4B.2) reduces to

$$\begin{aligned}
 & -2\rho^2 + 4\rho\lambda\tilde{\sigma}^2 B\psi - 2\rho^2\eta(1 + \tilde{\sigma}^2)B + 4\rho\eta(1 + \tilde{\sigma}^2)\lambda\tilde{\sigma}^2 B^2\psi \\
 & + 2\eta\rho^2(1 + \tilde{\sigma}^2)B\psi - 4\rho\lambda\eta(1 + \tilde{\sigma}^2)\tilde{\sigma}^2 B^2\psi^2 - \eta(1 + \tilde{\sigma}^2)\rho^2 B^2 B''\psi/B'^2 \\
 & + \eta\rho(1 + \tilde{\sigma}^2)\lambda\tilde{\sigma}^2(B^3 B''/B'^2 + B^2)\psi^2 + 2\rho^2\psi - 4\rho\lambda\tilde{\sigma}^2 B\psi^2 \\
 & = 2\rho\lambda\tilde{\sigma}^2 B\psi^2 + 4\lambda^2\tilde{\sigma}^4 B^2\psi^3 = 0 \quad (4B.3)
 \end{aligned}$$

Note from text equation (4.27e) that $k = -B''B/B'^2$. Replacing $-B''B/B'^2$ by k and then simplifying expression (4B.3), we obtain

$$B_1 \psi^3 - B_2 \psi^2 + B_3 \psi - B_4 = 0$$

which is the text equation (4.26), where

$$B_1 = 4\lambda^2 \tilde{\sigma}^4 B^2$$

$$B_2 = \rho \lambda \tilde{\sigma}^2 B [\eta B (1 + \tilde{\sigma}^2) (3 + k) + 6]$$

$$B_3 = \rho [4\lambda \tilde{\sigma}^2 B (1 + \eta B (1 + \tilde{\sigma}^2)) + \eta \rho B (1 + \tilde{\sigma}^2) (2 + k) + 2\rho]$$

$$B_4 = 2\rho^2 [1 + \eta B (1 + \tilde{\sigma}^2)]$$

Appendix 4C

Table 4C1: Value of Optimal Patent term Under Demand Uncertainty
Satisfying the Policy Maker's Equilibrium (4.15)

$B = 0.005$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	22.505	19.107	17.146	15.773
1.0	23.228	19.443	17.360	15.927
2.0	24.078	19.805	17.584	16.087
3.0	25.110	20.198	17.820	16.252
4.0	26.420	20.626	18.070	16.425
5.0	28.215	21.098	18.334	16.604
6.0	31.063	21.623	18.614	16.791
7.0	38.467	22.212	18.912	16.986
7.2	44.191	22.339	18.975	17.027
8.0		22.884	19.232	17.191
9.0		23.667	19.575	17.405
10.0		24.600	19.945	17.630
11.0		25.757	20.348	17.867
12.0		27.275	20.788	18.117
13.0		29.481	21.274	18.382
14.0		33.564	21.815	18.663
14.6		40.729	22.173	18.840
15.0			22.426	18.962
16.0			23.127	19.282
17.0			23.946	19.626
18.0			24.934	19.998
19.0			26.173	20.402
20.0			27.836	20.843
21.0			30.365	21.331
22.0			35.791	21.874
22.4			43.531	22.110
23.0				22.487
24.0				23.190
25.0				24.014
26.0				25.006
27.0				26.253
28.0				27.930
29.0				30.491
30.0				36.073
30.4				44.786

Appendix 4C

Table 4C2: Value of Optimal Patent Term Under Demand Uncertainty
Satisfying the Policy Maker's Equilibrium (4.15)

B = 0.01

Value of λ	Value of η 0.5	1.0	1.5	2.0
0.0	19.107	15.773	13.875	12.562
1.0	19.805	16.087	14.067	12.696
2.0	20.626	16.425	14.270	12.835
3.0	21.623	16.791	14.483	12.979
4.0	22.884	17.191	14.707	13.129
5.0	24.600	17.630	14.944	13.285
6.0	27.275	18.117	15.196	13.447
7.0	33.564	18.663	15.463	13.615
8.0		19.282	15.748	13.792
9.0		19.998	16.054	13.976
10.0		20.843	16.383	14.169
11.0		21.874	16.738	14.371
12.0		23.190	17.124	14.584
13.0		25.006	17.547	14.808
14.0		27.930	18.014	15.045
15.0		36.073	18.534	15.295
16.0			19.121	15.561
17.0			19.793	15.844
18.0			20.578	16.147
19.0			21.521	16.473
20.0			22.696	16.824
21.0			24.253	17.205
22.0			26.558	17.621
23.0			31.032	18.079
24.0				18.588
25.0				19.160

Appendix 4C

Table 4C3: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (4.15)

B = 0.05

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	11.568	8.673	7.158	6.184
1.0	12.097	8.854	7.242	6.227
2.0	12.712	9.044	7.326	6.269
3.0	13.441	9.243	7.411	6.310
4.0	14.327	9.452	7.495	6.348
5.0	15.448	9.670	7.579	6.385
6.0	16.952	9.897	7.660	6.419
7.0	19.204	10.134	7.740	6.450
8.0	23.612	10.378	7.816	6.477
8.65	37.631	10.540	7.863	6.493
9.0		10.628	7.887	6.501
10.0		10.882	7.953	6.520
11.0		11.134	8.011	6.534
12.0		11.381	8.061	6.542
12.8		11.569	8.093	6.544
13.0		11.614	8.100	
14.0		11.824	8.128	
15.0		12.001	8.142	
15.5		12.072	8.143	
16.0		12.131		
17.0		12.202		
17.7		12.211		

Appendix 4C

Table 4C4: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (4.15)

B = 0.1

Value of λ	Value of η		
	0.5	1.0	1.5
0.0	8.673	6.184	4.973
1.0	9.044	6.269	4.993
2.0	9.452	6.348	5.008
3.0	9.897	6.419	5.015
3.4	10.086	6.444	5.015
4.0	10.378	6.477	
5.0	10.882	6.520	
6.0	11.381	6.542	
6.4	11.569	6.544	
7.0	11.824		
8.0	12.131		
8.8	12.212		

Appendix 4C

Figure 4C1: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

($B = 0.005$)

Note: Charts I, II, III and IV correspond to $\eta = 0.5, 1.0, 1.5$ and 2.0 .

Appendix 4C

Figure 4C2: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

(B = 0.01)

Note: Charts I, II, III and IV correspond to $\eta = 0.5, 1.0, 1.5$ and 2.0 .

Appendix 4C

Figure 4C3: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index, λ varies

(B = 0.05)

Notes: Charts I, II, III and IV correspond to $\eta = 0.5, 1.0, 1.5$ and 2.0 .

Appendix 4C

Figure 4C4: Simulation Charts for the optimal patent terms, under demand uncertainty when the risk aversion index λ varies

($B = 0.1$)

Note: Charts I, II and III correspond to $\eta = 0.5, 1.0$ and 1.5 .

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Appendix 4D

Table 4D1: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (4.26)

B = 0.005

Value of λ	Value of n			
	0.5	1.0	1.5	2.0
0.0	22.206	18.812	16.855	15.486
1.0	22.882	19.126	17.055	15.630
2.0	23.668	19.463	17.264	15.778
3.0	24.607	19.827	17.483	15.932
4.0	25.772	20.221	17.713	16.091
5.0	27.305	20.652	17.956	16.257
6.0	29.544	21.126	18.213	16.429
7.0	33.744	21.653	18.486	16.608
7.7	46.783	22.060	18.687	16.738
8.0		22.246	18.775	16.795
9.0		22.923	19.085	16.990
10.0		23.711	19.416	17.194
11.0		24.654	19.773	17.408
12.0		25.824	20.160	17.633
13.0		27.365	20.581	17.870
14.0		29.620	21.045	18.119
15.0		33.881	21.558	18.383
15.5		39.508	21.837	18.521
16.0			22.134	18.664
17.0			22.789	18.963
18.0			23.547	19.282
19.0			24.447	19.625
20.0			25.553	19.996
21.0			26.984	20.398
22.0			29.011	20.839
23.0			32.505	21.324
23.9			44.851	21.808
24.0				21.865
25.0				22.475
26.0				23.175
27.0				23.993
28.0				24.977
29.0				26.213
30.0				27.868
31.0				30.377
32.0				35.706
32.5				51.263

Appendix 4D

Table 4D2: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (4.26)

B = 0.01

Value of λ	Value of η 0.5	1.0	1.5	2.0
0.0	18.812	15.486	13.595	12.289
1.0	19.463	15.778	13.774	12.413
2.0	20.221	16.091	13.961	12.542
3.0	21.126	16.429	14.157	12.674
4.0	22.246	16.795	14.364	12.812
5.0	23.711	17.194	14.581	12.954
6.0	25.824	17.633	14.810	13.102
7.0	29.620	18.119	15.052	13.256
7.8	48.493	18.578	15.270	13.391
8.0		18.664	15.309	13.415
9.0		19.282	15.583	13.581
10.0		19.996	15.875	13.755
11.0		20.839	16.188	13.936
12.0		21.865	16.526	14.125
13.0		23.175	16.891	14.324
14.0		24.977	17.290	14.532
15.0		27.868	17.727	14.752
16.0		35.706	18.211	14.983
16.2		42.929	18.314	15.031
17.0			18.753	15.228
18.0			19.366	15.487
19.0			20.073	15.763
20.0			20.905	16.057
21.0			21.915	16.372
22.0			23.197	16.712
23.0			24.944	17.079
24.0			27.690	17.478
25.0			34.410	17.916
25.2			38.634	18.009
26.0				18.400
27.0				18.941
28.0				19.553
29.0				20.255
30.0				21.080
31.0				22.077
32.0				23.335
33.0				25.036
34.0				27.659
35.0				33.598
35.4				46.036

Appendix 4D

Table 4D3: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (4.26)

B = 0.05

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	11.303	8.437	6.945	5.990
1.0	11.788	8.600	7.019	6.027
2.0	12.345	8.769	7.093	6.063
3.0	12.995	8.946	7.166	6.096
4.0	13.770	9.130	7.239	6.128
5.0	14.720	9.320	7.310	6.158
6.0	15.936	9.516	7.379	6.185
7.0	17.604	9.718	7.445	6.210
8.0	20.211	9.924	7.508	6.231
9.0	26.225	10.132	7.566	6.248
9.4	40.544	10.215	7.588	6.254
10.0		10.340	7.618	6.261
11.0		10.543	7.664	6.269
12.0		10.738	7.702	6.273
12.2		10.775	7.708	6.273
13.0		10.918	7.730	
14.0		11.076	7.748	
15.0		11.204	7.754	
16.0		11.294		
17.0		11.336		
17.2		11.339		

Appendix 4D

Table 4D4: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (4.26)

B = 0.1

Value of λ	Value of η		
	0.5	1.0	1.5
0.0	8.437	5.990	4.809
1.0	8.769	6.063	4.824
2.0	9.130	6.128	4.832
2.8	9.437	6.175	4.834
3.0	9.516	6.185	
4.0	9.924	6.231	
5.0	10.340	6.261	
6.0	10.738	6.273	
7.0	11.076		
8.0	11.294		
8.5	11.336		

Appendix 4D

Figure 4D1: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.005$)

Note: Charts I, II, III and IV correspond to $\eta = 0.5, 1.0, 1.5$ and 2.0 .

Appendix 4D

Figure 4D2: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

(B = 0.01)

Note: Charts I, II, III and IV correspond to $\eta = 0.5, 1.0, 1.5$ and 2.0 .

Appendix 4D

Figure 4D3: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.05$)

Note: Charts I, II, III and IV correspond to $\eta = 0.05, 1.0, 1.5$ and 2.0 .

Appendix 4D

Figure 4D4: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies
($B = 0.1$)

Note: Charts I, II and III correspond to $\eta = 0.5, 1.0$ and 1.5 .

CHAPTER 5

FREE ENTRY, UNCERTAINTY AND THE OPTIMAL PATENT TERM

It has been pointed out in Chapters 2 and 3 that free entry into inventive activity dissipates all the privately appropriable surplus resulting from an invention. Under the assumption that all privately appropriable surplus is dissipated, the optimal degree of appropriability is quite low. That is to say that the optimal patent term is as short as six months (Table 2.1).

We argued in Chapter 3 that the optimal patent term will be longer if either all appropriate surplus is not dissipated or only a small fraction of surplus is likely to be appropriable by inventors. When income taxes and existence of post-patent competition were introduced, in the case of free entry into inventive activity, the optimal patent term increased by a factor of five (see Table 3.3); although it remained considerably lower than in the case of unique inventors.

The purpose of this chapter is two-fold. The first one is to introduce uncertainty as to the value of inventions in the case of competitive inventing under the environment of post-patent competition and tax system. Even when there is free entry into inventing, the presence of uncertainty combined with post-patent competition and tax has the joint impact of increasing the optimal patent term quite significantly.

As in Chapter 4, uncertainty about the value of inventions is introduced in two ways: (a) Uncertainty about the demand function of the industry which purchases the cost reducing process invention; and (b) uncertainty about the magnitude of cost reduction associated with a given level of inventive activity. As before, uncertainty affects both the demand function and the cost reduction function multiplicatively. Specifically, it is shown that

even when all parties are risk neutral, the optimal patent term under cost reduction uncertainty is longer than the perfect certainty case. This stands in direct contrast to the result derived for the optimal patent term when individual inventors are unique. Furthermore, when inventors are assumed to be risk averse, the optimal patent term is longer than in the perfect certainty case; whether uncertainty affects the cost side or the demand side. Given that inventors are risk averse, the optimal patent term is longer the higher the index of risk aversion.

Another way to introduce uncertainty about the value of inventions is in the sense that the date at which the inventions will be made is not known with certainty, i.e., the date of discovery is uncertain. The final purpose of this Chapter is to investigate the impact of uncertainty about the date of the invention on the optimal patent term under the condition of free entry into inventive activity.

5.1 The Impact of Demand Uncertainty, Income Taxes and Post-Patent Competition on the Optimal Patent Term when There is Free Entry into Inventing

In this section we assume that the demand function faced by the industry which purchases the cost reducing process invention is uncertain and of the form $\phi = X(P)u$, where $X(P)$ is the riskless demand function given in equation (2.1) and u is a random variable with mean equal to 1 and standard deviation equal to σ .

The impact of post-patent competition on the amount and dissipation of surplus resulting from a given rate of cost reduction was illustrated in Figure 3.1. The price of output of the using industry falls to $P_2 = C_2 = C_0 - \theta(C_0 - C_1)$ and output expands to X_2 . As a consequence of decline in price, additional surplus in the amount given by the area of triangle ACD is realized by consumers per period and that given by rectangle CDFB is realized by the inventor per period during the term of the patent. There is, in addition, a transfer of surplus in the amount given by the rectangle C_0ACC_2 from the inventor to the consumers during the length of the patent. If there is uncertainty about the industry demand function, no matter which amount of total surpluses accrues to whom, all these surplus are ex-ante random during and after the expiry of the patent term. One such example is shown as the dashed line in Figure 5.1 for a particular realization $u_0 > E(u)$. During the patent term, in this case, the additional surplus received by the consumers and the inventor are the areas of triangle GLH and rectangle ILHJ respectively. The transfer of surplus from the inventor to the consumers is the rectangle C_0GLC_2 . When the patent period expires, the additional surplus given by the area of triangle HJK is also realized.

If the society maximizes expected welfare, the expected welfare function which society seeks to maximize is

Figure 5.1

Price and Output Effects of a Cost Reducing Process Invention Under
Demand Uncertainty when Royalties are Less than the Cost
Reduction

$$E(W) = \bar{W} = \int_u \left\{ \int_0^\infty [BuX_0 + (X_2 - X_0)u(P_2 - P_1) + \frac{1}{2}(X_2 - X_0)u(P_0 - P_2)]e^{-\rho t} dt + \int_{T_2}^\infty (X_1 - X_2)u(C_2 - C_1)e^{-\rho t} dt - sR \right\} f(u) du \quad (5.1)$$

where the integration is taken over the entire range of u and $f(u)$ is the probability density function of u . Normalizing X_0 and C_0 to 1, we may substitute the values of $(X_2 - X_0)$, $(P_0 - P_2)$, $(P_2 - P_1)$, $(X_1 - X_2)$ and $(C_2 - C_1)$ in equation (5.1) [see the derivation of equation (3.2), p. 43] and integrate (5.1) to yield:

$$E(W) = \bar{W} = \frac{B}{\rho} + \frac{\eta B^2}{2\rho} [1 - \psi + \theta(2 - \theta)\psi] - sR \quad (5.2)$$

The present discounted value of random after-tax net profit function for inventors is given by

$$\begin{aligned} V &= \int_0^T [(1 - \theta)Bu + (X_2 - X_0)u(P_2 - P_1)]e^{-\rho t} dt - \left(\frac{1}{1-\tau}\right)sR \\ &= \frac{1}{\rho}(1 - \theta)Bu(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right)sR \end{aligned} \quad (5.3)$$

where

$$\mu = E(V) = \frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right)sR \quad (5.4)$$

and

$$S^2 = \text{Var}(V) = \left[\frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi\right]^2 \sigma^2 \quad (5.5)$$

With free entry into inventing, the cost of any given invention is just equal to the present value of the expected after tax royalty income. Thus the zero-profit after-tax expected profit equilibrium is

$$\frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right)sR = 0 \quad (5.6)$$

Note that the expected welfare function in (5.2) and the zero expected after-tax profit equilibrium condition in (5.6) under demand uncertainty are identical to those of the perfect certainty case with post patent competition and taxes [see equations (3.3) and (3.10)]. Under the assumption that $\theta = \tau = 0$, the welfare function in (5.2) and the inventors' equilibrium condition in (5.6) are also identical to those of the perfect certainty case without post-patent competition and taxes [see equations (2.3) and (2.9)]. Thus, when inventors are risk neutral, with free entry into inventive activity, the optimal patent term under demand uncertainty is identical to that of the perfect certainty case.

Let us now turn to the case where inventors are assumed to be risk averse. Each inventor is assumed to maximize expected utility of his net discounted after-tax profit. That is, each inventor will maximize

$$E[U(V)] = EU\left[\frac{1}{\rho}(1 - \theta)Bu(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right)sR\right] \quad (5.7)$$

where U is the utility operator. We assume a negative exponential utility function of the form

$$U(V) = a(1 - e^{-\lambda V}), \quad (5.8)$$

where $a, \lambda > 0$. To the extent that this utility function can explain a risk averse individual's behaviour has been discussed in Chapter 4.1.2.

When the utility of the profit function is specified in (5.8), for an optimal choice of R , each inventor will maximize

$$E[U(V)] = \int_{-\infty}^{\infty} a(1 - e^{-\lambda V})f(V)dV = a(1 - M_V(\lambda)) \quad (5.9)$$

where $f(V)$ is the probability density function of V and $M_V(\lambda)$ is the moment generating function of the V distribution.

Recall from expression (5.3) that each inventor's discounted value of after-tax random profit is a linear function of u . If we assume that u is normally distributed with mean equal to 1 and standard deviation equal to σ , the expression (5.9) may be written as

$$E[U(V)] = a - a \text{EXP}[-\lambda\mu + \frac{1}{2} \lambda^2 S^2] \quad (5.10)$$

where μ and S^2 are given in equations (5.4) and (5.5) respectively.

With free entry into inventing, the zero expected utility condition is given by

$$E[U(V)] = a - a \text{EXP}[-\lambda\mu + \frac{1}{2} S^2 \lambda^2] = 0, \text{ i.e., } \mu - \frac{1}{2} \lambda S^2 = 0 \quad (5.11)$$

When the values of μ and S^2 from (5.4) and (5.5) are substituted in the above expression, the zero expected utility condition becomes

$$\frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi - \left(\frac{1}{1-\tau}\right)sR - \frac{1}{2}\left[\frac{1}{\rho}(1 - \theta)B(1 + \theta\eta B)\psi\right]^2 = 0 \quad (5.12)$$

The response of competitive inventors to a change in the patent term is given by the derivative of (5.12) with respect to ψ , which is

$$\frac{\partial R}{\partial \psi} = \frac{-\rho B N + \lambda \sigma^2 B^2 N^2 \psi}{\rho B' Q \psi - \rho^2 s \left(\frac{1}{1-\tau}\right) - \lambda \sigma^2 B B' N Q \psi^2} \quad (5.13)$$

where

$$N = (1 - \theta)(1 + \theta\eta B) \quad (5.14)$$

$$Q = (1 - \theta)(1 + 2\theta\eta B) \quad (5.15)$$

Assuming that society maximizes expected welfare the optimal patent term is obtained by maximizing (5.2) with respect to ψ subject to (5.12).

From (5.2) $\partial \bar{W} / \partial \psi = 0$ yields:

$$B' + \frac{\eta B^2}{2} [\theta(2-\theta)-1] \frac{\partial \psi}{\partial R} + \eta B B' [1 - \psi + \theta(2-\theta)\psi] - \rho s = 0 \quad (5.16)$$

When the expression $\partial \psi / \partial R$ from (5.13) and ρs from (5.12) are substituted in equation (5.16) and simplified, the optimal patent term is given by the solution of equations (5.17) and (5.12) [for the derivation of equation (5.17), see Appendix 5A]:

$$C_1 \psi^3 - C_2 \psi^2 + C_3 \psi - C_4 = 0 \quad (5.17)$$

where

$$C_1 = \frac{1}{5} \lambda^2 \sigma^4 B^2 k_1 N^3 \quad (5.18a)$$

$$C_2 = \rho \lambda \sigma^2 B [\eta B \{1 - \theta(2-\theta)\} \{2N(1 + \frac{k_1}{4}) - Q\} + \frac{1}{\delta} 3k_1 N^2] \quad (5.18b)$$

$$C_3 = 2\rho [\lambda \sigma^2 B N (1 + \eta B) + \frac{1}{2} \eta \rho B (2 + k_1 - Q/N) \{1 - \theta(2-\theta)\} + \frac{1}{\delta} \rho k_1 N] \quad (5.18c)$$

$$C_4 = 2\rho^2 (1 + \eta B) \quad (5.18d)$$

$$\delta = 1/(1 - \tau) \quad (5.18e)$$

$$k_1 = B/RB' \quad (5.18f)$$

It is to be noted that in the presence of post-patent competition and taxes, the optimal patent term under perfect certainty (which is the optimal patent term for risk neutral inventors in the case of demand uncertainty) may readily be obtained by substituting $\lambda = 0$ in equation (5.17). The optimal patent term in the absence of post-patent competition and taxes may also be obtained by substituting $\lambda = \theta = \tau = 0$ in equation (5.17). Absent post-patent competition and taxes, this is also the optimal patent term for risk neutral inventors under demand uncertainty.

As in Section 4.1, there is no neat analytical solution for the optimal

ψ and hence for the optimal patent term T from equations (5.17) and (5.12). Thus we try to find conclusions by making numerical calculations that would satisfy (5.17). The numerical solution of equation (5.17) depends on the parametric assumptions about the riskless demand function and the "invention possibility" function $B(R)$. It also depends on the risk aversion index λ and the parameters θ , τ , ρ and σ . For given values of the relevant parameters, the coefficients of equation (5.17) may be calculated and the numerical solutions for optimal $\psi(0 \leq \psi \leq 1)$ and hence the optimal patent term T may be obtained.

Following Section 4.1, we specify $B(R)$ as BR^{η} and allow B to take on values .005, .01, .05 and .1. η takes on values .5, 1.0, 1.5, 2.0 while θ , τ , ρ , and σ are set at .75, .12, .20 and .25. The resulting optimal patent term calculations are reported in Tables 5C1 - 5C4 and the corresponding simulation charts are represented in Figures 5C1 - 5C4 respectively for different values of B , η and the risk aversion index λ (see Appendix 5C).

What is explicit from our simulation results is that in the presence of pre and post-patent rivalry and taxation of inventors' royalty income, the impact of demand uncertainty and inventors' risk aversion is to increase the optimal patent term relative to the perfect certainty case with or without post-patent competition and taxes. For example, for $B = .1$, $\eta = 1.0$, Table 5C4 (p.146) reveals that with pre and post-patent competition and taxes the optimal patent term is 6.8 when $\lambda = 142$ while under perfect certainty, the optimal patent terms are 3.0 (see Table 3.3, p. 49) and .5 (see Table 2.1, p. 24) with post-patent competition and taxes and without them respectively. The results also show the comparative static property that as inventors become increasingly risk averse, i.e., the higher the value of the risk aversion index λ , the longer is the optimal patent term (see Tables 5C1 - 5C4 and Figures 5C1 - 5C4 in Appendix 5C).

The extent to which the optimal patent term is increased in the presence of demand uncertainty, pre and post-patent competition and taxation of inventors' royalty income in comparison with the perfect certainty case with and without post-patent competition and taxation may be seen in Table 5.1. Table 5.1 also indicates the amount to which the optimal patent term under uncertainty and perfect certainty in the case of free entry into inventive activity differs from that of a unique inventor case under similar conditions. In the centre of each cell is the optimal patent term in the case of competitive inventing under the assumption of perfect certainty with no post-patent competition and taxation of inventors' royalty income ($\theta = \tau = 0$). This is also the optimal patent term under free entry and demand uncertainty when the inventors are risk neutral. The upper lefthand corner of each cell refers to the free entry, perfect certainty optimal patent term in the presence of post-patent competition and taxes ($\theta = .75, \tau = .12$). This is also the optimal patent term under demand uncertainty in the presence of pre and post patent competition and taxes when the inventors are risk neutral ($\theta = .75, \tau = .12, \lambda = 0$). The lower lefthand corner of each cell refers to the optimal patent term with pre and post-patent competition, taxes, and demand uncertainty under the assumption that inventors are risk averse (for $\theta = .75, \tau = .12$; this optimal patent term corresponds to the largest value of the risk aversion index λ in Tables 5C1 - 5C4 in Appendix 5C). In the upper righthand corner of each cell is the perfect certainty optimal patent term when the inventors are unique (which is also the optimal patent term under demand uncertainty in the case of a risk neutral unique inventor) while the lower righthand corner of each cell refers to the optimal patent term under demand uncertainty in the case of a risk averse unique inventor (this optimal patent term corresponds to the largest value of the risk aversion index λ in

Table 5.1

Table for Comparison of the Optimal Patent Term with and without Demand

Uncertainty ($\alpha = .1$, $\theta = .75$, $\tau = .12$, $\rho = .20$)

			η					
			.5	1.0	1.5	2.0		
.005	3.0	22.5	3.0	19.1	3.0	17.2	3.0	15.8
	.5	5.9	.5	44.2	.5	40.7	.5	43.5
.01	3.0	19.1	3.0	15.8	3.0	13.9	3.0	12.6
	.5	5.9	.5	33.6	.5	36.1	.5	31.0
.05	3.0	11.6	3.0	8.7	3.0	7.2	3.0	6.2
	.5	8.3	.5	37.6	.5	12.2	.5	7.1
.10	3.0	8.7	3.0	6.2	3.0	5.0	3.0	4.3
	.5	7.5	.5	12.2	.5	6.4	.5	5.0

B

L

Tables 4C1 - 4C4 in Appendix 4C).

The optimal patent term under the assumptions that $\theta = .75$, $\tau = .12$ and $\lambda = 0$ (i.e., when inventors are risk neutral) is generally about six times longer than when both θ and τ are zero. The optimal patent term under the assumptions that $\theta = .75$, $\tau = .12$ and inventors' risk aversion is generally about 12 to 17 times longer than when both θ and τ are zero. In the presence of demand uncertainty, the free entry optimal patent term under the assumptions that $\theta = .75$, $\tau = .12$ and inventors' risk aversion remains lower than the optimal patent term in the case of a risk neutral unique inventor. It remains considerably lower than the case of a risk averse unique inventor.

5.2 The Impact of Cost Reduction Uncertainty, Income Taxes and Post-Patent Competition on the Optimal Patent Term when There is Free Entry into Inventing

In the presence of rivalry, it is likely that inventors face uncertainty about the magnitude of cost reduction resulting from a given invention. When the magnitude of cost reduction is uncertain, the cost reduction function may be written as $B(R, \tilde{u}) = B(R)(1 + \tilde{u})$, where $\tilde{u}$ is a random variable with mean equal to zero and standard deviation equal to $\tilde{\sigma}$. Given this type of uncertainty, the value of inventions become random.

We again assume that society is expected welfare maximizer. Let us recall from equation (3.2) that the social welfare function in the presence of post-patent competition and taxes is written as

$$W = \int_0^{\infty} B(R) [1 + \eta\theta(1 - \theta)B(R) + \frac{1}{2}\eta\theta^2 B(R)^2] e^{-\rho t} dt + \int_T^{\infty} \frac{1}{2}\eta(1 - \theta)^2 [B(R)]^2 e^{-\rho t} dt - sR \quad (3.2)$$

From equation (3.2), the expected welfare function under cost reduction

uncertainty is then written as

$$E(W) = \bar{W} = \int_{\tilde{u}} \left[\int_0^{\infty} B(R, \tilde{u}) \{1 + \eta\theta(1 - \theta)B(R, \tilde{u}) + \frac{1}{2} \eta\theta^2 B(R, \tilde{u})\} e^{-\rho t} dt + \int_T^{\infty} \frac{1}{2} \eta(1 - \theta)^2 (B(R, \tilde{u}))^2 e^{-\rho t} dt - sR \right] f(\tilde{u}) d\tilde{u} \quad (5.19)$$

where the integration is taken over the entire range of $\tilde{u}$. Noting that $B(R, \tilde{u}) = B(R)(1 + \tilde{u})$; mean and standard deviation of $\tilde{u}$ respectively are zero and $\tilde{\sigma}$; the integration of (5.19) yields:

$$E(W) = \bar{W} = \frac{B}{\rho} + \frac{\eta B^2 (1 + \tilde{\sigma}^2)}{2\rho} [1 - \psi + \theta(2 - \theta)\psi] - sR \quad (5.20)$$

The present discounted value of random after tax profit for inventors is given by

$$\begin{aligned} \tilde{V} &= \left[\int_0^T (1 - \theta)B(R, \tilde{u}) + \theta(1 - \theta)\eta(B(R, \tilde{u}))^2 \right] e^{-\rho t} dt - \left(\frac{1}{1 - \tau} \right) sR \\ &= \frac{(1 - \theta)B(R, \tilde{u})\psi}{\rho} + \frac{\theta(1 - \theta)\eta(B(R, \tilde{u}))^2 \psi}{\rho} - \left(\frac{1}{1 - \tau} \right) sR \end{aligned} \quad (5.21)$$

where

$$E(\tilde{V}) = \bar{V} = \frac{(1 - \theta)B[1 + \theta\eta B(1 + \tilde{\sigma}^2)]\psi}{\rho} \quad (5.22)$$

$$\text{Var}(\tilde{V}) = \tilde{S}^2 = \frac{(1 - \theta)^2 \tilde{\sigma}^2 B^2 \psi^2}{\rho^2} [(1 + 2\theta\eta B)^2 + 2\theta^2 \eta^2 \tilde{\sigma}^2 B^2] \quad (5.23)$$

When the inventors are risk averse, each inventor chooses to invest that level of R in R&D which maximizes the expected utility of his net discounted after tax profit. That is, each inventor maximizes

$$E[U(\tilde{V})] = EU\left[\frac{1}{\rho}(1 - \theta)B(R, \tilde{u})(1 + \theta\eta B(R, \tilde{u}))\psi - \left(\frac{1}{1 - \tau}\right)sR\right] \quad (5.24)$$

Assuming that each inventor's preference is described by a utility function of the form (5.8) and following the same methodology and steps of Section 5.1, with free entry into inventing, the zero expected utility condition in the case of uncertainty in the magnitude of cost reduction is given by

$$\tilde{u} - \frac{1}{2} \lambda \tilde{S}^2 = 0 \quad (5.25)$$

where $\tilde{u}$ and $\tilde{S}^2$ are given in expressions (5.22) and (5.23) respectively.

When the values of $\tilde{u}$ and $\tilde{S}^2$ are substituted in equation (5.25), the zero expected utility condition becomes

$$\begin{aligned} \frac{(1-\theta)B}{\rho} [1 + \theta\eta B(1 + \tilde{\sigma}^2)] \psi - \left(\frac{1}{1-\tau}\right) sR - \frac{1}{2\rho^2} \lambda (1 - \theta)^2 \tilde{\sigma}^2 B^2 \\ [(1 + 2\theta\eta B)^2 + 2\theta^2 \eta^2 \tilde{\sigma}^2 B^2] \psi^2 = 0 \end{aligned} \quad (5.26)$$

Now, the response of competitive inventors to a change in the patent term is given by the derivative of (5.26) with respect to ψ , and the result is:

$$\frac{\partial R}{\partial \psi} = [-\rho B G + \lambda (1 - \theta)^2 \tilde{\sigma}^2 Z B^2 \psi] R / \Delta_4 \quad (5.27)$$

where

$$\begin{aligned} \Delta_4 = \rho R B' M \psi - \rho B G \psi + \frac{1}{2} \lambda (1 - \theta)^2 \tilde{\sigma}^2 B^2 Z \psi^2 \\ - \lambda (1 - \theta)^2 \tilde{\sigma}^2 R [Z B B' \psi^2 + 2B^2 \{(1 + 2\theta\eta B) + \theta\eta B' + \theta^2 \eta^2 \tilde{\sigma}^2 B B'\} \psi^2] \end{aligned} \quad (5.28)$$

$$G = (1 - \theta) \{1 + \theta\eta B(1 + \tilde{\sigma}^2)\} \quad (5.29)$$

$$Z = (1 + 2\theta\eta B)^2 + 2\theta^2 \eta^2 \tilde{\sigma}^2 B^2 \quad (5.30)$$

$$M = (1 - \theta) \{1 + 2\theta\eta B(1 + \tilde{\sigma}^2)\} \quad (5.31)$$

For the optimal patent term, an expected welfare maximizing society will maximize (5.20) subject to (5.26). From (5.20), $\partial \bar{W} / \partial \psi = 0$ yields:

$$B' + \eta(1+\tilde{\sigma}^2)\{1-\psi+\theta(2-\theta)\psi\}BB' + \frac{1}{2}\eta(1+\tilde{\sigma}^2)B^2\{\theta(2-\theta)-1\}\partial\psi/\partial R - \rho s = 0 \quad (5.32)$$

When the expressions for $\partial\psi/\partial R$ from equation (5.27) and ρs from equation (5.26) are substituted in (5.32), after simplification, the optimal patent term when the inventors are risk averse may be obtained from the solution of equations (5.33) and (5.26) [for the derivation of equation (5.33), see Appendix 5B]:

$$D_1\psi^3 - D_2\psi^2 + D_3\psi - D_4 = 0 \quad (5.33)$$

where

$$D_1 = \frac{1}{8}\lambda^2(1-\theta)^4\tilde{\sigma}^4B^2k_1Z^2 \quad (5.34a)$$

$$D_2 = \rho(1-\theta)^2\lambda\tilde{\sigma}^2B[\eta B(1+\tilde{\sigma}^2)Z\{1-\theta(2-\theta)\}\{1+\frac{1}{2}k_1-\frac{2B}{Z}\theta\eta(1+2\theta\eta B+\theta\eta B\tilde{\sigma}^2)\} + \frac{3}{8}k_1GZ] \quad (5.34b)$$

$$D_3 = 2\rho[\lambda(1-\theta)^2\tilde{\sigma}^2BZ\{1+\eta B(1+\tilde{\sigma}^2)\}+\eta B(1+\tilde{\sigma}^2)\rho\{1-\theta(2-\theta)\}\{G(1+\frac{1}{2}k_1) - \frac{1}{2}M\} + \frac{1}{8}\rho k_1G^2] \quad (5.34c)$$

$$D_4 = 2\rho^2G[1 + \eta B(1 + \tilde{\sigma}^2)] \quad (5.34d)$$

$$\delta = \left(\frac{1}{1-\tau}\right) \quad (5.34e)$$

$$k_1 = \frac{B}{RB'} \quad (5.34f)$$

Let us now assume that inventors are risk neutral. For simplicity, we also assume that inventors face no post-patent competition ($\theta = 0$) and pay no taxes on royalty income ($\tau = 0$), but face uncertainty about the magnitude of cost reduction. In this case, the optimal patent term may be obtained

from equation (5.33) when $\lambda = \theta = \tau = 0$. The optimal patent term when the inventors are risk neutral is given by [by substituting $\lambda = \theta = \tau = 0$ in equation (5.33)]:

$$\psi_{RNC}^* | \tau = \theta = \lambda = 0 = \frac{1 + \eta B(1 + \tilde{\sigma}^2)}{k_1 + \frac{\eta B}{2}(1 + \tilde{\sigma}^2)(1 + k_1)} \quad (5.35)$$

Recall from equation (2.13) that with perfect certainty, the optimal patent term in the case of competitive inventing may be rewritten as

$$\psi_C^* = \frac{1 + \eta B}{k_1 + \frac{1}{2}\eta B(1 + k_1)} \quad (2.13)$$

From (5.34) and (2.13),

$$\psi_C^* - \psi_{RNC}^* | \tau = \theta = \lambda = 0 = \frac{1}{2}\eta B^2 \tilde{\sigma}^2 (1 - k_1) / \Delta_5 \quad (5.36)$$

where

$$\Delta_5 = [k_1 + \frac{1}{2}\eta B(1 + k_1)] [k_1 + \frac{1}{2}\eta B(1 + \tilde{\sigma}^2)(1 + k_1)] \quad (5.37)$$

Since $\Delta_5 > 0$, $k_1 = B/RB' > 1$, it implies from equation (5.36) that

$$\psi_C^* - \psi_{RNC}^* | \tau = \theta = \lambda = 0 < 0, \text{ i.e., } \psi_C^* < \psi_{RNC}^* | \tau = \theta = \lambda = 0.$$

The following proposition may thus be stated:

Proposition 5.1

When society maximizes expected welfare and inventors are risk neutral, with free entry into inventive activity, the introduction of uncertainty regarding the magnitude of cost reduction yields an optimal patent term which is longer than the optimal patent term with perfect certainty.

The above proposition stands in direct contrast with Proposition 4.1. We may now proceed to give an intuitive explanation of the reasons why one may expect a shorter patent term in the case of unique inventors while a longer patent term in the case of competitive inventors under the conditions of cost reduction uncertainty and risk neutrality.

As illustrated in Figure 5.2, suppose that C_0 is the preinvention level of unit cost of production. Under cost reduction uncertainty, the post-invention level of cost of production is random. Suppose that random post invention level of production cost assumes values C_1 and C_2 with equal probabilities such that the expected value of the production cost is $\bar{C}_1$.

In the case of perfect certainty and unique inventors, for a given patent term (say T years), the social benefit and the social cost are respectively given by the area of the rectangle $C_0AG\bar{C}_1$ and the area of the triangle AGF . Under uncertainty, the expected social benefit is identical to its certainty equivalent counterpart (expected social benefit is given by the expected value of the rectangles C_0ABC_1 and C_0ADC_2 which is equal to $C_0AG\bar{C}_1$), but expected social cost is the expected value of the areas of the triangles ABC and ADE which is greater than the perfect certainty social loss by the amount given by the area of the triangle FKE .¹

Suppose now that the patent term is increased by one additional year (i.e., from T to $T+1$). This induces a higher level of cost reduction by inventors.² Suppose also that due to the increase in the patent term, the random production cost assumes values at C'_1 and C'_2 with equal probabilities, so that the average production cost is $\bar{C}'_1$.

Under the assumption of perfect certainty, when the patent term is extended to one additional year, i.e., the patent term is extended from T to $T+1$ years, the social gain may be obtained by summing the areas:

Figure 5.2

$\bar{C}_1 G \bar{C}_1$ (for all time); Agf (from the beginning of T+2 to infinitely many periods, ∞). The social cost due to the extension of the patent term for one additional year is given by: Area Agf (from 0 to T+2 periods) plus Area AGF (for T+1 to T+2 periods, because due to the extension of the patent term, society has to wait to realize this triangle as a gain for one more year) plus the resource cost necessary to achieve the production cost down to $\bar{C}_1$. The net present value of the social gain will depend on the relative magnitude of the above gross gains and losses, and also on the discount rate.

Under the assumption of uncertainty and when the patent term is increased from T to T+1 years, the expected social gain is given by: expected value of rectangles $\bar{C}_1 G n C'_1$ and $\bar{C}_1 G m C''_2$ (which equals to the area of the rectangle $\bar{C}_1 G \bar{C}_1$, for all periods) plus expected value of the areas of triangles And and ARl (which equals to the sum of the areas of the triangles Agf and ftl from the beginning of T+2 to ∞ periods). The expected social cost in this case is: expected value of area of triangles And and ARl (which is equal to Area Agf plus Area ftl from 0 to T+2 periods) plus expected value of the areas of triangles ABC and ADE (which equals to area AGF plus Area FKE from T+1 to T+2 periods, because if patent term were not extended these triangles could have been realized as gains a period earlier) plus the resource cost necessary to achieve the incremental random cost reductions.

When the social benefits and costs due to extending the patent term, for one additional year under uncertainty are compared to those under perfect certainty, one can immediately calculate that under uncertainty, the social benefits are increased by the area of the triangle flt from T+2 to ∞ periods while the social costs are increased by the sum of the areas of triangles flt from 0 to T+2 periods and FKE from T+1 to T+2. If we have a longer patent term to begin with (which is the case in monopoly inventing) and because

the areas of the triangles flt and FKE are the same (see Figure 5.2 and footnote 1), one can expect a larger incremental social cost (area of the triangle flt from 0 to $T+2$ periods plus that of the same triangle from $T+1$ to $T+2$ period) and a smaller incremental social benefit (area of the triangle flt from $T+2$ to ∞ periods) in terms of present values from extending the patent term for an additional year; and for this reason, society should call for a shorter patent term.

Let us now turn to the case of competitive inventing. In this case, under the assumption of perfect certainty, the total cost of resources is, by definition, equal to the present value of the royalties the successful inventor expects to earn. Thus the cost of an invention is the present value of the area of rectangle $C_0AG\bar{C}_1$ over the term of the patent. Since the privately appropriable surplus given by the area of rectangle $C_0AG\bar{C}_1$ is dissipated, the areas of rectangle $C_0AG\bar{C}_1$ and triangle AGF become social surplus after the expiry of the patent. Under the assumption of uncertainty, the expected social surplus which can be realized after the expiry of the patent term is given by the sum of the areas of rectangle $C_0AG\bar{C}_1$ and triangles AGF and FKE .

As before, assume that the patent term is extended to one additional year, i.e., from T to $T+1$. This will induce to produce proportionately more cost reduction than that is produced in the case of a unique inventor.³ Since the magnitude of cost reduction is random, let us then assume that the random cost of production in the case of free entry into inventive activity takes values at C_1'' and C_2'' with equal probabilities. So the expected production cost due to the extension of the patent term is $\bar{C}_1$.

When the patent term is extended for an additional year, the amount of social surplus which can be realized after the expiry of the patent is given

by the areas of rectangle $C_0AZ\bar{C}_1$ ($= C_0AG\bar{C}_1 + \bar{C}_1GZ\bar{C}_1$) and triangle AZP from from $T+2$ to ∞ periods. The social cost is then the sum of the areas of rectangle $C_0AZ\bar{C}_1$ (from 0 to the beginning of $T+2$ period); triangle AZP (from 0 to the beginning of $T+2$; because, if the patent term was not extended, society could gain this rectangle one period earlier); rectangle $C_0AG\bar{C}_1$ (from $T+1$ to $T+2$); and triangle AGF (from $T+1$ to $T+2$; because, if the patent term was not extended, society could also gain this triangle one period earlier). As in the case of unique inventors, the net present value of the gain in social surplus due to the extension of the patent term will depend on the relative magnitudes of the gross losses and gains and also on the discount rate.

Under the assumptions of uncertainty and free entry into inventing activity, when the patent period is extended, the gross social surplus is given by: expected value of rectangles C_0AmC_1'' and C_0AvC_2'' (which is equal to the area of the rectangle $C_0AZ\bar{C}_1$ from $T+2$ to ∞ periods) plus the expected value of the areas of triangles Amq and Avr (which is equal to the sum of the areas of triangles AZP and Pwr from $T+2$ to ∞ periods). The social cost in this case may be calculated as: expected value of the areas of rectangles C_0AmC_1'' and C_0AvC_2'' (which is equal to area of rectangle $C_0AZ\bar{C}_1$ from 0 to $T+2$ periods) plus expected value of the areas of triangles Amq and Avr (which is equal to the sum of the areas of triangles AZP and Pwr from 0 to $T+2$ periods) plus the expected value of rectangles C_0ABC_1 and C_0ADC_2 (which is equal to the area of rectangle $C_0AG\bar{C}_1$ from $T+1$ to $T+2$ period) plus expected value of triangles ABC and ADE (which is equal to the sum of the areas of triangles AGF and FKE from $T+1$ to $T+2$ period).

A comparison of social surplus and social cost under the assumptions of both perfect certainty and uncertainty reveals that, under uncertainty, extension of the patent term by a year increases the social surplus by the amount

given by the area of rectangle Pwr from $T+2$ to ∞ periods while the social cost is also increased by the areas of the triangles Pwr from 0 to $T+2$ periods and FKE from $T+1$ to $T+2$ period. If we have a smaller patent term to begin with (which is the case in competitive inventing) and because the area of triangle Pwr is larger than that of FKE (see Figure 5.2), under uncertainty, one can expect a larger incremental social surplus and a smaller incremental social cost in terms of present values; and for this reason, society should call for a longer patent term in this case.

In summary, we note that the introduction of uncertainty with risk neutrality does not alter the inventors' equilibrium conditions from the perfect certainty case under both competitive and monopoly inventing. So the constraints faced by society in deriving the optimal patent term are unchanged. When the patent term is extended for one year, society faces changes in the expected values of the welfare gains and losses in the same way for both competitive and monopoly inventing. Then the differences in the optimal patent term arise because starting points are not the same. If they are the same (as in Figure 5.2), competitive inventing with its large cost reduction will have greater tendency to extend the optimal patent term. If they are not the same, monopoly inventing with proportionately smaller cost reduction tends to contract the optimal patent term (e.g., longer initial T to begin with).

We now go back to equation (5.33). Although there is no neat analytical solution for the optimal ψ from equation (5.33) we can find our conclusions by making numerical calculations. For various values of the relevant parameters the numerical solutions for the optimal ψ and hence the optimal patent term may be obtained from equation (5.33).

For numerical calculations of the optimal patent term we specify

$B(R)$ as βR^α and use the same values for the parameters α , θ , τ , and ρ as in Section 5.1. Following similar arguments as in section 4.2 for the choice of a reasonable value of $\bar{\sigma}$, we use $\bar{\sigma} = 1/4$. We allow B to take on values .005, .01, .05, .1 and η to take on values .5, 1.0, 1.5, 2.0. The resulting optimal patent term calculations are reported in Tables 5D1 - 5D4 and the corresponding simulation charts are represented in Figures 5D1 - 5D4 respectively for different values of B , η and the risk aversion index λ (see Appendix 5D).

In the presence of free entry, post-patent rivalry, taxes, cost reduction uncertainty and inventors' risk aversion, our numerical calculations also reveal that the optimal patent term is considerably longer than the optimal patent term under perfect certainty with or without post-patent rivalry and taxes. Given the assumptions of free entry, post-patent rivalry and taxes the optimal patent term under cost reduction uncertainty, for example, for $B = .1$, $\eta = 1.0$ is 6.7 (Table 5D4, p. 154) when $\lambda = 125$. For the same values of B and η the free entry optimal patent terms are 3.0 (see Table 3.3, p. 49) and .5 (see Table 2.1, p. 24) with post-patent rivalry and taxes, and without them respectively. If inventors become increasingly risk averse, as shown by our numerical calculations, the optimal patent term becomes longer (see Tables 5D1 - 5D4 and Figures 5D1 - 5D4 in Appendix 5D).

Under the assumptions of free entry, post-patent competition, taxes and cost reduction uncertainty, the extent to which the optimal patent term is increased in comparison with the perfect certainty case (with and without post-patent competition and taxes) is summarized in Table 5.2. The amount to which the optimal patent term under the condition of free entry differs from that of a unique inventor is also given in Table 5.2. In the centre of each cell is the free-entry, perfect certainty optimal patent term under the assumption that $\theta = \tau = 0$ (since the magnitude of the optimal patent term in

Table 5.2

Table for Comparison of the Optimal Patent Term with and without

Cost Reduction Uncertainty

$r(\alpha = .1, \theta = .75, \tau = .12, \rho = .20)$

		η							
		.5		1.0		1.5		2.0	
.005		3.0	22.5	3.0	19.1	3.0	17.2	3.0	15.8
		.5		.5		.5		.5	
.01		10.2	46.9	9.7	39.5	9.4	44.9	9.2	51.3
		.5		.5		.5		.5	
.05		3.0	11.6	3.0	8.7	3.0	7.2	3.0	6.2
		.5		.5		.5		.5	
.10		8.2	40.5	7.5	11.3	7.0	7.8	6.7	6.3
		.5		.5		.5		.5	
		3.0	8.7	3.0	6.2	3.0	5.0	3.0	4.3
		.5		.5		.5		.5	
		7.5	11.3	6.7	6.0	6.3	4.8	6.0	-

the case of perfect certainty does not differ significantly from the optimal patent term under cost reduction uncertainty when the inventors are risk neutral, this is also the free entry optimal patent term when the inventors are risk neutral). In the upper lefthand corner of each cell is the perfect certainty optimal patent term under the assumptions of free entry, post patent rivalry and taxes ($\theta = .75$, $\tau = .12$, $\lambda = 0$). This is also the optimal patent term under cost reduction uncertainty when inventors are risk neutral ($\theta = .75$, $\tau = .12$, $\lambda = 0$). In the lower lefthand corner of each cell is the optimal patent term under the assumptions of free entry, post-patent rivalry, taxes and inventors' risk aversion (for $\theta = .75$, $\tau = .12$; this optimal patent term corresponds to the largest value of the risk aversion index in Tables 5D1 - 5D4 in Appendix 5D). The upper righthand corner of each cell refers to the optimal patent term in the case of a unique inventor under perfect certainty (which approximately equals the optimal patent term under cost reduction uncertainty when the inventor is risk neutral); while the lower righthand corner of each cell refers to the optimal patent term under cost reduction uncertainty in the case of a risk averse unique inventor (this optimal patent term corresponds to the largest value of the risk aversion index λ in Tables 4D1 - 4D4 in Appendix 4D).

The optimal patent term under the assumption that $\theta = .75$, $\tau = .12$, $\lambda = 0$ is generally six times longer than when both θ and τ are zero. When inventors are risk averse, under the assumption that $\theta = .75$, $\tau = .12$, the optimal patent term is generally 12 to 20 times longer than when both of these parameters are zero. In the presence of cost reduction uncertainty, the free entry optimal patent term under the assumptions that $\theta = .75$, $\tau = .12$ and inventors' risk aversion remains lower than the optimal patent term for a risk neutral unique inventor. It remains considerably lower than the case of a risk averse unique inventor.

5.3 The Impact of Uncertainty about the Date of an Invention on the Optimal Patent Term when There is Free Entry into Inventing

The recent theories of market structure and innovation developed by Dasgupta and Stiglitz (1980a, b), Loury (1979), and Lee and Wilde (1980) assume that the competition among potential rival inventors is a race to be the first. The winner of the race gets all the benefits from the invention over the duration of the patent. The amount of resources an inventor devotes to inventive activity affects the date of an invention. The date of an invention may be certain as well as uncertain. In the case of a process invention, inventive resources are employed in order to achieve a predetermined level of unit production cost reduction (which does not depend on the amount of resources invested in R&D). The greater is the amount of resource employment, the earlier is the date of discovery. This enables one to analyze both the level of resource employment and the date of invention of a firm.

In order to discuss the relationship of inventive activity to market structure, the above mentioned authors consider models of firm which incorporate rival inventors' strategic behaviours towards employment of resources to inventive activity (each inventor plays an oligopoly game with others in an industry). Each firm knows exactly its own as well as its rivals' levels of investment of resources to a specific inventive activity in any given length of time. Each firm knows that its decision on spending in R&D is invariant to the rivals' spending decisions. Given these assumptions, a particular firm can then take a strategy of employing a slightly more resources in R&D than its rivals to win the race.

Dasgupta and Stiglitz (1980b) consider the innovating firms' strategic behaviour regarding the allocation of resources to inventive activity and explore the possibility of a Cournot-Nash market equilibrium. Under the assumptions

of free entry into inventive activity and with perfect certainty about the date at which the invention will be made, they conclude that the number of firms will be engaged in R&D activity is nil. Given these assumptions, if any firm has an advantage over its rivals, then one can expect only one firm undertaking inventive activity in equilibrium. In the presence of uncertainty about the discovery date, as observed by them, if the uncertainties are mutually correlated,⁴ one can expect either no firm or at most one firm to engage in R&D activity in equilibrium. There will be several firms in equilibrium only when the date of discovery is uncertain and the inventive activities of rival firms are independent. This means, for example, that in the case of a process invention, there are several paths of achieving a predetermined level of cost reduction; so that no firm can be assured of being first to invent, and the date at which the invention will be made is not known with certainty.

When the date of discovery is uncertain, and the uncertainties faced by all firms are independent, two important questions may be addressed: (1) What are the socially optimal number of parallel research projects and amount of resources to be devoted to each? (2) If a market is characterized by n independent firms in the race to invent, and the winner gets the patent, will the market allocate socially optimal level of resources to inventive activity? Under the assumption of free entry into inventive activity when the date of discovery is uncertain, it can be shown that there is an optimal patent term for each industry and for each invention which will guarantee that the market undertakes the socially optimal level of inventive activity.

In order to determine an optimal patent term for an invention which will guarantee that the market/system allocates the correct level of private

resources to inventive activity we shall consider the model developed by Dasgupta and Stiglitz (1980b) and modify it to a certain extent. Unlike Dasgupta and Stiglitz (1980b) we consider a linear demand function for the output of the industry which purchases the cost reducing invention and specify an expected discovery date function. (Dasgupta and Stiglitz consider a constant elasticity industry demand function and do not specify any expected discovery date function.) The specification of an expected discovery date function will enable us to derive the numerical estimates of the optimal patent term for a particular invention. The optimal patent term will depend on the size of the invention, elasticity of demand, and the degree of easiness of advancing the expected date of discovery. The more distant the expected date of discovery, as our numerical estimates will show, the longer is the optimal patent term.

We assume throughout the following analysis that the existing technology of producing output is freely available to the users and that the unit cost of production is C_0 . We also assume that only one invention is to be made; this invention will reduce the unit cost of production from C_0 to the predetermined level C_1 . The time $\hat{T}$ it will take to make the invention is related to inventive input R by $\hat{T} = \hat{T}(R)$. It is also assumed that the demand function faced by the industry which uses the cost reducing invention is linear. The form of the demand function is given in equation (2.1).

As illustrated in Figure 5.3, if the economy is socially managed, the per period gain in gross social surplus from the invention is given by the area of the trapezoid $C_0ADBC_1 = (C_0 - C_1)X_0 + \frac{1}{2}(X_1 - X_0)(C_0 - C_1)$. If X_0 and C_0 are normalized to 1, then the gain in social surplus is given by $B(1 + \frac{1}{2}B)$ when $B = (C_0 - C_1)/C_0$.⁵ Note that the gain in surplus here is independent of the amount of resources invested in R&D.

Figure 5.3

If we assume that the present technology C_0 is freely available to all users but the inventor of the new technology C_1 is awarded a patent of length T , then during the period of the patent, the per period profit to the inventor is given by the area of the rectangle C_0ABC_1 (Figure 5.3). The magnitude of this profit is then $BX_0 = B$ (for $X_0 = 1$) per period during the length of the patent.

Let us consider an industry with n identical firms each competing to be the first to invent the new technology C_1 . The firm who invents first gets the reward of the amount B over the length of the patent; all other firms lose their investments in R&D. Under free entry into inventing, as we have noted earlier, several firms can undertake R&D activity if there is uncertainty about the discovery date and the uncertainties are independent.

We now assume that for a given level of investment in R&D, the date $\hat{T}$ at which the invention will be made is not known with certainty. In particular, we assume that for a given level of resource employment R at time 0, the probability that the invention is made at or before $\hat{T}$ is described according to

$$F(\hat{T}) = P(t \leq \hat{T}) = 1 - e^{-h(R)\hat{T}} \quad (5.38)$$

The probability $F(\hat{T})$ is viewed as a function of the discovery date $\hat{T}$ in which R is a parameter of the distribution. So the expected discovery date is given by

$$E(\hat{T}) = \int_0^{\infty} \hat{T} F'(\hat{T}) d\hat{T} = [h(R)]^{-1} \quad (5.39)$$

The instantaneous conditional probability density that the invention will be made at $\hat{T}$ given that it has not been made by time $\hat{T}$ is

$$F'(\hat{T})/(1 - F'(\hat{T})) = h(R) \quad (5.40)$$

This means that the participant (firm) in the race experiences a constant instantaneous probability $h(R)$ that its research will produce a new technology (the predetermined level of unit production cost C_1). We assume that $h(R)$ is characterized by an initial range of increasing returns, followed by decreasing returns as depicted in Figure 5.4. It is also assumed that $h'(R) (= dh/dR) > 0$, i.e., more expenditure on R&D will yield greater conditional probability density of discovery by any given time $\hat{T}$. This in turn implies that a greater expenditure will bring about an earlier expected discovery date.

We now proceed to evaluate the probability of making the invention and thus winning the patent on the part of a particular firm (say the n th firm) in the presence of rivalry when the date of discovery is uncertain. Under the assumption that the research efforts of the participating n firms are statistically independent, the probability that none of the other $n-1$ firms make the invention on or before $\hat{T}$ is

$$\prod_{i=1}^{n-1} [1 - F(\hat{T})] = e^{-[\sum_{i=1}^{n-1} h(R_i)]\hat{T}} \quad (5.41)$$

where $F(\hat{T})$ is the probability that i th firm will discover on or before $\hat{T}$ depends on its lump-sum resource employment R_i . The probability that one of its $n-1$ rivals makes the invention on or before time $\hat{T}$, therefore is

$$1 - \prod_{i=1}^{n-1} [1 - F(\hat{T})] = 1 - e^{-[\sum_{i=1}^{n-1} h(R_i)]\hat{T}} \quad (5.42)$$

The above expression refers to the behaviour of a firm's (say the n th firm) $n-1$ rivals. The expression that refers to the behaviour of all n firms

Figure 5.4

can easily be obtained from expression (5.42) by replacing $n-1$ by n . The probability that the invention is made by time $\hat{T}$ is from (5.42) (replacing $n-1$ by n)

$$1 - e^{-[\sum_{i=1}^n h(R_i)]\hat{T}} \quad (5.43)$$

The corresponding probability density function associated with the race being won at $\hat{T}$, i.e., the invention is made at $\hat{T}$ is the derivative of (5.43) with respect to $\hat{T}$ and the result is

$$\sum_{i=1}^n h(R_i) e^{-[\sum_{i=1}^n h(R_i)]\hat{T}} \quad (5.44)$$

The expected discovery date is then $[\sum_{i=1}^n h(R_i)]^{-1}$. Note that if all participants are identical, then each invests exactly the same amount, $R_i = R$ for all i , and expression (5.44) simplifies to

$$nh(R)e^{-nh(R)\hat{T}} \quad (5.45)$$

When we have derived the probability of making the invention at time $\hat{T}$, our next problem is to obtain the socially optimal number of parallel research projects and the amount of resources to be devoted to each. We assume that in a socially managed economy, there are n independent and identical research units; each unit invests an amount R in R&D to produce the predetermined level of unit cost of production C_1 . As we have indicated earlier, there is a social gain from the invention given by the area C_0ADBC_1 (Figure 5.3) whose magnitude is $B(1 + \frac{1}{2}nB)$ if at least one research unit succeeds; otherwise society receives no benefits; the social cost is

nR . The social problem is then to obtain the number of parallel research units and the amount of resources to be devoted to each in order to maximize the net social benefits from the invention. To this we now turn.

It has already been noted that $\hat{T}$ is the date of discovery. The present discounted value of the gross social benefits from the invention is given by

$$\int_{\hat{T}}^{\infty} B(1 + \frac{1}{2}\eta B)e^{-\rho t} dt = \frac{1}{\rho} B(1 + \frac{1}{2}\eta B)e^{-\rho \hat{T}} \quad (5.46)$$

When the date of discovery is uncertain, in a socially managed economy, the planning problem is then to choose n and R so as to maximize the expected present value of the net social benefits $W(n, R)$, where

$$W(n, R) = \int_0^{\infty} \left[\frac{1}{\rho} B(1 + \frac{1}{2}\eta B)e^{-\rho \hat{T}} \cdot nh(R)e^{-nh(R)\hat{T}} \right] d\hat{T} - nR \quad (5.47)$$

The above expression is the expected discounted value of the invention to the society, multiplied by the probability density of discovering the invention at time $\hat{T}$, less the cost of n research units. The integration of (5.47) yields

$$W(n, R) = \frac{1}{\rho} \frac{B(1 + \frac{1}{2}\eta B)nh(R)}{nh(R) + \rho} - nR \quad (5.48)$$

To obtain the socially optimal number of research units and the socially optimal level of resource investment by each unit, one sets the derivatives of (5.48) with respect to R and n , respectively, equal to zero.

The first order necessary condition is obtained when $\partial W / \partial n = 0$ and $\partial W / \partial R = 0$. The conditions, respectively, are

$$\frac{B(1 + \frac{1}{2}\eta B)h(R)}{[nh(R) + \rho]^2} = R \quad (5.49)$$

and

$$\frac{B(1 + \frac{1}{2}\eta B)h'(R)}{[nh(R) + \rho]^2} = 1 \quad (5.50)$$

From equations (5.49) and (5.50) we can obtain the socially optimal number of research units (n^*) and the amount of resources (R^*) to be devoted to each research unit. Let us write the following expression from equations (5.49) and (5.50):

$$h'(R) = \frac{h(R)}{R} \quad (5.51)$$

Let R^* be the solution of (5.51). Hence the optimal level of investment in each research unit is independent of the number of research units (Figure 5.4). By substituting R^* in equation (5.49), the socially optimal number of research units n^* can be obtained. From equation (5.49) the socially optimal number of research units is then

$$n^* = \frac{1}{h(R^*)} \left[\frac{h(R^*)B(1 + \frac{1}{2}\eta B)}{R^*} \right]^{\frac{1}{2}} - \frac{\rho}{h(R^*)} \quad (5.52)$$

Our next task is to consider the case of a market economy in which n competitive firms undertake R&D expenditures. The aim of this is to obtain the optimal patent term for an invention which will guarantee the market system to allocate socially optimal level of resources to inventive activity. We assume that firms are identical and work independently. Suppose that the n th firm knows that $n-1$ other firms are undertaking R&D expenditures and also knows the level of investment R_i ($i = 1, 2, \dots, n$) of

rival firms.

Then the probability that the n th firm will be the winner (i.e., becomes the first to make the invention at $\hat{T}$) is given by the joint probability distribution

$$\prod_{i=1}^{n-1} (1 - F(\hat{T}(R_i))) F'(\hat{T}(R_n)) \quad (5.53)$$

where the first term in the above expression is the probability that none of the $n-1$ rival firms makes the invention on or before $\hat{T}$ and the second term is the n th firm's probability density of successful discovery at $\hat{T}$. When equations (5.39) and (5.42) are substituted in (5.53), expression (5.53) reduces to

$$h(R_n) e^{-[\sum_{i=1}^n h(R_i)] \hat{T}} \quad (5.54)$$

If the n th firm is the winner of the race, it will enjoy a flow of benefits of magnitude B per period during the length of the patent T . If the firm's date of discovery is $\hat{T}$, the present discounted value of this benefit is given by

$$\Pi[\hat{T}(R_n)] = \int_{\hat{T}}^{\hat{T}+T} B e^{-\rho t} dt = \frac{B}{\rho} \psi e^{-\rho \hat{T}} \quad (5.55)$$

where $\psi = 1 - e^{-\rho T}$.

Suppose firms are risk neutral. Since the date of discovery is uncertain, the n th firm will choose a level of R_n such that its expected discounted profit is maximized. The n th firm's expected value of net

discounted profit is given by

$$\begin{aligned}
 E[\Pi(\hat{T}(R_n))] &= \int_0^{\infty} \frac{B}{\rho} \psi e^{-\rho \hat{T}} h(R_n) e^{-[\sum_{i=1}^n h(R_i)] \hat{T}} - R_n \\
 &= \frac{B}{\rho} \frac{h(R_n) \psi}{(\sum_{i=1}^n h(R_i) + \rho)} - R_n \quad (5.56)
 \end{aligned}$$

We assume that the n th firm ignores the effects of its resource employment R_n on aggregate $\sum_{i=1}^n h(R_i)$, i.e., it takes the expected date of invention as given [note that the expected date of invention is $(\sum_{i=1}^n h(R_i))^{-1}$]. It varies R_n with a view to change the probability of making the invention first and thus to win the patent. The n th firm's optimal choice of R_n is obtained by maximizing (5.56) with respect to R_n . The first order necessary condition is [which is obtained by differentiating (5.56) with respect to R_n]:

$$\frac{B}{\rho} \frac{h'(R_n) \psi}{(\sum_{i=1}^n h(R_i) + \rho)} = 1 \quad (5.57)$$

Because all firms in the race are identical, each satisfies (5.57) at the same level of investment, $R_i = R$ (say). In equilibrium, therefore, (5.57) implies

$$\frac{B}{\rho} \frac{h'(R) \psi}{(nh(R) + \rho)} = 1 \quad (5.58)$$

With free entry into inventing, the number of firms is endogenous. The number of firms n_C in the market solution is defined by the condition

that the expected discounted value of profit for each firm is zero.

Hence from (5.56) and using symmetry, we get

$$\frac{B}{\rho} \frac{h(R)\psi}{(nh(R) + \rho)} = R \quad (5.59)$$

Equations (5.58) and (5.59) are the free entry Cournot-Nash equilibrium.

The optimal number of firms n_C and the level of spending R_e^* in R&D by each firm can be determined simultaneously from them. From (5.58) and (5.59) we may write

$$h'(R) = \frac{h(R)}{R} \quad (5.60)$$

Let R_e^* be solution of (5.60). Hence at the free entry equilibrium, each firm operates efficiently and invests at a socially optimal level $R^* = R_e^*$ [see Figure 5.4 and compare equations (5.51) and (5.60)].

Since in the case of competitive inventing, each firm operates at the socially optimal level, with a given patent term, whether the industry wide allocation of resources will be larger or smaller or equal (i.e., overinvestment or underinvestment or in balance) will depend on the socially optimal number of firms and the number of firms in the competitive equilibrium.

From (5.59), the number of firms in the free entry equilibrium is given by

$$n_C = \frac{B\psi}{\rho R^*} - \frac{\rho}{h(R^*)} \quad (5.61)$$

The optimal patent term for an invention (in our case the predetermined

level of cost reduction C_1 is the invention) is then to obtain ψ^* such that $n^* = n_C^*$. From equations (5.52) and (5.61) the optimal ψ and hence the optimal patent term T can be obtained from the following expression

$$\psi^* = \rho \left[\frac{1 + \frac{1}{2} \eta B}{h'(R^*)B} \right]^{\frac{1}{2}} < 1 \quad (5.62)$$

The optimal patent term from equation (5.62) cannot be calculated without specifying the expected discovery date function $h(R)$. We specify $h(R)$ as

$$h(R) = e^{-\epsilon/R}, \quad \epsilon > 0, R > 0 \quad (5.63)$$

where R is the R&D expenditures and ϵ represents the degree of easiness of advancing the expected date of invention. Since $1/h(R)$ is the expected date of invention, for a given value of R , the smaller the value of ϵ the greater is the conditional probability of making the invention and thus earlier is the expected date of discovery. Similarly, for a given value of R , larger value of ϵ implies smaller conditional probability of making the invention which in turn implies a longer expected date of discovery (Figure 5.5).

We have noted earlier that each firm in the competitive equilibrium operates efficiently and each firm undertakes socially optimal level of R&D expenditure. This is implied by the relation $h'(R^*) = h(R^*)/R^*$ in equilibrium. Therefore given the specification of $h(R)$ in (5.63), $h'(R^*) = 1/\epsilon R^*$.⁶ Thus from (5.62), the optimal patent term, given our specification of $h(R)$ can be obtained as

Figure 5.5

$$\psi^* = \rho \left[\frac{e\epsilon(1 + \frac{1}{2}\eta B)}{B} \right]^{\frac{1}{2}} < 1 \quad (5.64)$$

The optimal patent term in equation (5.64) depends on the discount rate ρ , magnitude of the invention B , elasticity of demand η , and the degree of easiness of forwarding the expected date of invention ϵ . The larger the value of ϵ , the longer is the optimal patent term.

As in Sections 5.1 and 5.2, for numerical estimates of the optimal patent term we allow B to take on values .005, .01, .05, .1 and η to take on values .5, 1.0, 1.5 and 2.0. ϵ takes on values .1, .2, .3, .4, .5, .6, .7, .8, .9 and 1.0 while ρ is set at .2.

Our optimal patent term calculations are reported in Table 5.3. The optimal patent term in this case is quite close to the optimal patent term in the case of a unique inventor under the condition of perfect certainty. For example, when $B = .1$, $\eta = 1.0$, $\rho = .2$, the optimal patent term in the case of a unique inventor is 6.2 (Table 2.1, p. 24) which is approximately the same as the optimal patent term in the case of rival inventors facing uncertainty about the date of invention when $\epsilon \approx .45$ (see Table 5.3). Also, when $B = .05$, $\eta = 1.0$, $\rho = .2$ the optimal patent term is 8.7 in the case of a unique inventor (Table 2.1, p. 24) which is, in the presence of uncertainty about the discovery date, about the same for rival inventors when $\epsilon \approx .3$ (see Table 5.3). The estimates of the optimal patent term in Table 5.3 also reveal the comparative static result that the larger the value of ϵ , i.e., the more distant is the expected date of discovery, the longer is the optimal patent term, everything else being constant.

Table 5.3: Value of optimal patent term when inventors face uncertainty about the date of the invention

($\eta = 1.0$, $\rho = .2$)

Value of ϵ	Value of B	
	0.05	0.1
.1	3.19	2.06
.2	5.51	2.61
.3	8.51	4.40
.4	14.43	5.63
.5	N/A	7.04
.6	N/A	8.79
.7	N/A	11.22
.8	N/A	15.58
.9	N/A	N/A
1.0	N/A	N/A

FOOTNOTES TO CHAPTER 5

1. Expected value of the area of triangles ABC and ADE with equal probabilities may be calculated as follows:

$$\begin{aligned} \frac{1}{2}(\text{area ABC}) + \frac{1}{2}(\text{area ADE}) &= \frac{1}{2}ABC + \frac{1}{2}ADE = \frac{1}{2}(AGF - BCFG) + \frac{1}{2}(AGF + GFED) \\ &= \frac{1}{2}AGF - \frac{1}{2}BCFG + \frac{1}{2}AGF + \frac{1}{2}GFED = AGF - \frac{1}{2}(BCHG + CFH) + \frac{1}{2}(GHID + HFKI + FKE) \\ &= AGF - \frac{1}{2}BCHG - \frac{1}{2}CFH + \frac{1}{2}GHID + \frac{1}{2}HFKI + \frac{1}{2}FKE \quad (\text{since } BCHG = GHID) \\ &= AGF - \frac{1}{2}CFH + \frac{1}{2}(2CFG) + \frac{1}{2}FEK = AGF + \frac{1}{2}CFH + \frac{1}{2}FEK = AGF + FEK \quad (\text{since } FEK = CFH). \end{aligned}$$

2. The first order condition is $\frac{B'}{\rho}\psi = s$. This implies $\frac{\partial R}{\partial \psi} > 0$.
3. The investment in R by competitive inventors will proceed until the zero profit condition is satisfied, i.e., $\frac{B}{\rho}\psi = sR$. Since by assumption, B is a concave function, for given ρ , s and ψ , $B/R > B'$. Thus, investment in R is greater under competitive inventing than under monopoly inventing. Since more resources are invested, more cost reduction will be forthcoming in the case of competitive inventing.
4. Uncertainties are mutually correlated in the sense that, for a given level of inventive input, the date at which the invention will be made is not known with certainty and that only one path of research strategy is available to all firms. Under these circumstances, given the R&D effort of rival firms, by spending sufficiently large, a particular firm can guarantee that it is the first to invent and thus win the patent although it cannot say the date of its invention with perfect certainty.
5. If the using industry is competitive before and after the invention, then the pre-invention per unit output price is $P_0 = C_0$; and the post-invention per unit output price is $P_1 = C_1$. For a linear demand function of the form $X(P) = \xi - \eta P$, and because $P_0 = C_0 = 1$, the area of the trapezoid is then $B(1 + \frac{1}{2}\eta B)$.
6. For $h(R) = e^{-\epsilon/R}$, $h'(R) = e^{-\epsilon/R}(\epsilon/R^2)$. The relation $h'(R) = h(R)/R$ implies $e^{-\epsilon/R}(\epsilon/R^2) = (e^{-\epsilon/R})/R$, which in turn implies $\epsilon = R$. Therefore, $h'(R) = e^{-\epsilon/R}(\epsilon/R^2) = e^{-\epsilon/\epsilon}(\epsilon/\epsilon^2) = 1/\epsilon$.

APPENDIX 5A: The derivation of text equation (5.17) for the optimal patent term under demand uncertainty

From equation (5.2), $\partial \bar{w} / \partial \psi = 0$ yields text equation (5.16):

$$B' + \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] \frac{\partial \psi}{\partial R} + \eta B B' [1 - \psi + \theta(2-\theta)\psi] - \rho s = 0 \quad (5.16)$$

Also from text equation (5.13)

$$\frac{\partial R}{\partial \psi} = \frac{-\rho B N + \lambda \sigma^2 B^2 N^2 \psi}{\rho B' Q \psi - \rho^2 s \delta - \lambda \sigma^2 B B' N Q \psi^2} \quad (5.13)$$

where $\delta = 1/(1-\tau)$; and N and Q are given in text equations (5.14) and (5.15). Note from text equation (4.12) that

$$\rho s = [\rho B N \psi - \frac{1}{2} \lambda \sigma^2 B^2 N^2 \psi^2] [\rho R \delta]^{-1} \quad (5A.1)$$

Substituting expression (5A.1) in (5.13) we obtain

$$\frac{\partial R}{\partial \psi} = [-\rho B N + \lambda \sigma^2 B^2 N^2 \psi] R [\rho R B' Q \psi - \rho B N \psi + \frac{1}{2} \lambda \sigma^2 B^2 N^2 \psi^2 - \lambda \sigma^2 R B B' N Q \psi^2]^{-1} \quad (5A.2)$$

When expression for $\partial \psi / \partial R$ from equation (5A.2) is substituted in (5.16), we can write equation (5.16) as

$$B' + \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] [\rho R B' Q \psi - \rho B N \psi + \frac{1}{2} \lambda \sigma^2 B^2 N^2 \psi^2 - \lambda \sigma^2 R B B' N Q \psi^2] [(-\rho B N + \lambda \sigma^2 B^2 N^2 \psi) R]^{-1} + \eta B B' [1 - \psi + \theta(2-\theta)\psi] - \rho s = 0 \quad (5A.3)$$

After substituting the value of ρs [equation (5A.1)] in equation (5A.3) and then expanding we obtain

$$\begin{aligned}
 & -\rho RB'B'N + \lambda \sigma^2 RB'B^2 N^2 \psi + \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] \rho RB'Q\psi - \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] \rho BN\psi \\
 & + \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] \lambda \sigma^2 B^2 N^2 \psi^2 - \frac{1}{2} \eta B^2 [\theta(2-\theta)-1] \lambda \sigma^2 RB'B'N Q \psi^2 \\
 & - \eta \rho RB^2 B'N + \eta \rho RB^2 B'N \psi - \rho \eta \theta(2-\theta) RB^2 B'N \psi + \eta \lambda \sigma^2 RB^3 B'N^2 \psi \\
 & - \eta \lambda \sigma^2 RB^3 B'N^2 \psi^2 + \eta \lambda \sigma^2 \theta(2-\theta) RB^3 B'N^2 \psi^2 + \frac{1}{6} [\rho B^2 N^2 \psi - \frac{3}{2} \lambda \sigma^2 B^3 N^3 \psi^2 \\
 & + \frac{1}{2\rho} \lambda^2 \sigma^4 B^4 N^4 \psi^3] = 0 \tag{5A.4}
 \end{aligned}$$

When both sides of (5A.4) are multiplied by $2\rho/(NRB'B)$, (5A.4) reduces to

$$\begin{aligned}
 & -2\rho^2 + 2\rho \lambda \sigma^2 BN\psi + \rho^2 \eta B [\theta(2-\theta)-1] (Q/N)\psi - \rho^2 \eta B [\theta(2-\theta)-1] (B/RB')\psi \\
 & + \frac{1}{2} \rho \eta B^2 [\theta(2-\theta)-1] \lambda \sigma^2 (B/RB') N \psi^2 - \rho \eta B^2 [\theta(2-\theta)-1] \lambda \sigma^2 Q \psi^2 - 2\eta \rho^2 B \\
 & + 2\eta \rho^2 B \psi - 2\rho^2 \eta \theta(2-\theta) B \psi + 2\rho \eta \lambda \sigma^2 B^2 N \psi - 2\rho \eta \lambda \sigma^2 B^2 N \psi^2 \\
 & + 2\rho \eta \lambda \sigma^2 \theta(2-\theta) B^2 N \psi^2 + \frac{1}{6} [2\rho^2 (B/RB') N \psi - 3\rho \lambda \sigma^2 (B^2/RB') N^2 \psi^2 \\
 & + \lambda^2 \sigma^4 (B^3/RB') N^3 \psi^3] = 0 \tag{5A.5}
 \end{aligned}$$

Note from text equation (5.18f) that $k_1 = B/RB'$. Replacing B/RB' by k_1 and then simplifying expression (5A.5) we obtain

$$C_1 \psi^3 - C_2 \psi^2 + C_3 \psi - C_4 = 0$$

which is the text equation (5.17), where

$$C_1 = \frac{1}{6} \lambda^2 \sigma^4 B^2 k_1 N^3$$

$$C_2 = \rho \lambda \sigma^2 B [\eta B \{1 - \theta(2-\theta)\} \{2N(1 + \frac{1}{2} k_1) - Q\} + \frac{3}{6} k_1 N^2]$$

$$C_3 = 2\rho [\lambda \sigma^2 BN(1+\eta B) + \frac{1}{2} \eta \rho B(2 + k_1 - Q/N) \{1-\theta(2-\theta)\} + \frac{1}{6} \rho k_1 N]$$

$$C_4 = 2\rho^2(1 + \eta B)$$

APPENDIX 5B: The derivation of text equation (5.33) for the optimal patent term under cost reduction uncertainty

From equation (5.20), $\partial W/\partial \psi = 0$ yields text equation (5.32):

$$B' + \eta(1+\tilde{\sigma}^2)\{1 - \psi + \theta(2-\theta)\psi\}BB' + \frac{1}{2} \eta(1+\tilde{\sigma}^2)B^2\{\theta(2-\theta)-1\}\partial\psi/\partial R - \rho s = 0 \quad (5.32)$$

When expression for $\partial\psi/\partial R$ from text equation (5.27) is substituted in (5.32) we obtain

$$B' + \eta(1+\tilde{\sigma}^2)\{1 - \psi + \theta(2-\theta)\psi\}BB' + \left[\frac{\Delta_4}{\{-\rho BG + \lambda(1-\theta)^2\tilde{\sigma}^2 Z B^2 \psi\}R} \right] - \rho s = 0$$

where the values of Δ_4 , G and Z are given in text equations (5.28), (5.29) and (5.30).

or

$$\begin{aligned} & B'[-\rho BG + \lambda(1-\theta)^2\tilde{\sigma}^2 Z B^2 \psi]R + [\eta(1+\tilde{\sigma}^2)\{1 - \psi + \theta(2-\theta)\psi\}BB'] \\ & [-\rho BG + \lambda(1-\theta)^2\tilde{\sigma}^2 Z B^2 \psi]R + \frac{1}{2} \eta(1+\tilde{\sigma}^2)B^2\{\theta(2-\theta)-1\}\Delta_4 - \rho s[-\rho BG \\ & + \lambda(1-\theta)^2\tilde{\sigma}^2 Z B^2 \psi]R = 0 \end{aligned} \quad (5B.1)$$

Note from text equation (5.26) that $\rho s = [\rho BG\psi - \frac{1}{2}\lambda(1-\theta)^2\tilde{\sigma}^2 B^2 Z \psi^2](\rho R\delta)^{-1}$,

where $\delta = 1/(1-\tau)$. When this value of ρs and the value of Δ_4 from text equation (5.28) are substituted in expression (5B.1), expression (5B.1) can be expanded to obtain

$$\begin{aligned}
 & -\rho B B' R G + \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^2 B' R \psi - \eta(1+\tilde{\sigma}^2) \rho B^2 B' R G + \eta(1+\tilde{\sigma}^2) \rho B^2 R B' G \psi \\
 & - \eta(1+\tilde{\sigma}^2) \rho B^2 R B' G \theta(2-\theta) \psi + \eta(1+\tilde{\sigma}^2) \lambda(1-\theta)^2 \tilde{\sigma}^2 Z R B^3 B' \psi - \eta(1+\tilde{\sigma}^2) \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^3 R B' \psi \\
 & + \eta(1+\tilde{\sigma}^2) (1-\theta)^2 \lambda \tilde{\sigma}^2 Z B^3 R B' \theta(2-\theta) \psi^2 + \frac{1}{2} \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \rho R B' M \psi \\
 & - \frac{1}{2} \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \rho B G \psi + \frac{1}{2} \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 B^2 Z \psi^2 \\
 & - \frac{1}{2} \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 R Z B B' \psi^2 - \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} B^2 \\
 & (1 + 2\theta \eta B) \theta \eta B' \lambda(1-\theta)^2 \tilde{\sigma}^2 R \psi^2 - \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 R B^2 \theta^2 \eta^2 \tilde{\sigma}^2 B B' \psi^2 \\
 & + \frac{1}{8} [\rho B^2 G^2 \psi - \frac{1}{2} \lambda(1-\theta)^2 \tilde{\sigma}^2 B^3 G Z \psi^2 - \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^3 G \psi^2 + \frac{1}{2\rho} \lambda^2 (1-\theta)^4 \tilde{\sigma}^4 B^4 Z^2 \psi^3] \\
 & = 0
 \end{aligned}
 \tag{5B.2}$$

where the value of M is given in text equation (5.31).

Multiplying both sides of (5B.2) by $2\rho/RB'$, we obtain

$$\begin{aligned}
 & - 2\rho^2 B G + 2\rho \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^2 \psi - 2\rho^2 \eta(1+\tilde{\sigma}^2) B^2 G + 2\rho^2 \eta(1+\tilde{\sigma}^2) B^2 G \psi \\
 & - 2\rho^2 (1+\tilde{\sigma}^2) B^2 G \theta(2-\theta) \psi + 2\rho \eta(1+\tilde{\sigma}^2) \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^3 \psi - 2\rho \eta(1+\tilde{\sigma}^2) \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B^3 \psi^2 \\
 & + 2\rho \eta(1+\tilde{\sigma}^2) (1-\theta)^2 \lambda \tilde{\sigma}^2 Z B^3 \theta(2-\theta) \psi^2 + \rho^2 \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} M \psi \\
 & - \rho^2 \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} (B/RB') G \psi + \frac{1}{2} \rho \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 (B^2/RB') Z \psi^2 \\
 & - \rho \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 Z B \psi^2 - 2\rho \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} B^2 (1 \\
 & + 2\theta \eta B) \theta \eta \lambda(1-\theta)^2 \tilde{\sigma}^2 \psi^2 - 2\rho \eta B^2 (1+\tilde{\sigma}^2) \{ \theta(2-\theta) - 1 \} \lambda(1-\theta)^2 \tilde{\sigma}^2 B^3 \theta^2 \eta^2 \tilde{\sigma}^2 \psi^2 \\
 & + \frac{1}{8} [2\rho^2 (B^2/RB') G^2 \psi - \rho \lambda(1-\theta)^2 \tilde{\sigma}^2 (B^3/RB') G Z \psi^2 - 2\rho \lambda(1-\theta)^2 \tilde{\sigma}^2 Z (B^3/RB') G \psi^2 \\
 & + \lambda^2 (1-\theta)^4 \tilde{\sigma}^4 (B^4/RB') Z^2 \psi^3] = 0
 \end{aligned}
 \tag{5B.3}$$

Note from text equation (5.34f) that $k_1 = B/RB'$. Replacing B/RB' by k_1 and then simplifying expression (5B.3) we obtain

$$D_1 \psi^3 - D_2 \psi^2 + D_3 \psi - D_4 = 0$$

which is the text equation (5.33), where

$$D_1 = \frac{1}{\delta} \lambda^2 (1-\theta)^4 \tilde{\sigma}^4 B^2 k_1 Z^2$$

$$D_2 = \rho (1-\theta)^2 \lambda \tilde{\sigma}^2 B [\eta B (1+\tilde{\sigma}^2) Z \{1 - \theta(2-\theta)\} \{1 + \frac{1}{2} k_1 \\ - \frac{2B}{Z} \theta \eta (1 + 2\theta \eta B + \theta \eta B \tilde{\sigma}^2)\} + \frac{3}{\delta} k_1 G Z]$$

$$D_3 = 2\rho [\lambda (1-\theta)^2 \tilde{\sigma}^2 B Z \{1 + \eta B (1+\tilde{\sigma}^2)\} + \eta B (1+\tilde{\sigma}^2) \rho \{1 - \theta(2-\theta)\} \{G(1+\frac{1}{2} k_1) - \frac{1}{2} M\} \\ + \frac{1}{\delta} \rho k_1 G^2]$$

$$D_4 = 2\rho^2 G [1 + \eta B (1+\tilde{\sigma}^2)]$$

Appendix 5C

Table 5C1: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (5.17)
 $B = 0.005$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.008	3.009	3.010	3.011
60.0	3.031	3.032	3.033	3.034
120.0	3.053	3.054	3.055	3.056
180.0	3.077	3.078	3.079	3.080
240.0	3.101	3.102	3.103	3.104
300.0	3.125	3.127	3.128	3.129
360.0	3.151	3.152	3.153	3.154
420.0	3.177	3.178	3.179	3.181
480.0	3.204	3.205	3.207	3.208
540.0	3.232	3.233	3.234	3.236
600.0	3.260	3.262	3.263	3.265
660.0	3.290	3.291	3.293	3.294
720.0	3.320	3.322	3.324	3.325
780.0	3.352	3.354	3.355	3.357
840.0	3.385	3.386	3.388	3.390
900.0	3.419	3.420	3.422	3.424
960.0	3.454	3.456	3.458	3.460
1020.0	3.490	3.492	3.494	3.496
1080.0	3.528	3.530	3.532	3.535
1140.0	3.568	3.570	3.572	3.574
1200.0	3.609	3.611	3.614	3.616
1260.0	3.652	3.654	3.657	3.659
1320.0	3.697	3.699	3.702	3.704
1380.0	3.744	3.747	3.749	3.752
1440.0	3.793	3.796	3.799	3.802
1500.0	3.845	3.848	3.851	3.854
1560.0	3.900	3.903	3.906	3.909
1620.0	3.957	3.960	3.964	3.967
1680.0	4.018	4.021	4.025	4.028
1740.0	4.083	4.086	4.090	4.094
1800.0	4.151	4.155	4.159	4.163
1860.0	4.225	4.229	4.233	4.237
1920.0	4.303	4.308	4.312	4.317
1980.0	4.388	4.393	4.397	4.402
2040.0	4.479	4.484	4.489	4.494
2100.0	4.578	4.584	4.589	4.595
2160.0	4.687	4.693	4.699	4.705
2220.0	4.807	4.813	4.820	4.826
2280.0	4.940	4.947	4.954	4.960
2340.0	5.089	5.097	5.104	5.111
2400.0	5.259	5.267	5.275	5.283
2460.0	5.455	5.464	5.473	5.481
2520.0	5.687	5.697	5.706	5.715
2560.0	5.868	5.878	5.888	5.897
2570.0		5.927	5.937	5.946
2575.0			5.962	5.972
2585.0				6.024

Appendix 5C

Table 5C2: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (5.17)
 $B = 0.01$

Value of λ	Value of η 0.5	1.0	1.5	2.0
0.0	3.009	3.011	3.013	3.015
30.0	3.032	3.034	3.036	3.038
60.0	3.054	3.056	3.059	3.061
90.0	3.078	3.080	3.082	3.084
120.0	3.102	3.104	3.106	3.109
150.0	3.127	3.129	3.131	3.134
180.0	3.152	3.154	3.157	3.159
210.0	3.178	3.181	3.183	3.186
240.0	3.205	3.208	3.211	3.213
270.0	3.233	3.236	3.239	3.241
300.0	3.262	3.265	3.268	3.270
330.0	3.291	3.294	3.297	3.301
360.0	3.322	3.325	3.328	3.332
390.0	3.354	3.357	3.360	3.364
420.0	3.386	3.390	3.393	3.397
450.0	3.420	3.424	3.428	3.431
480.0	3.456	3.460	3.463	3.467
510.0	3.492	3.496	3.500	3.504
540.0	3.530	3.535	3.539	3.543
570.0	3.570	3.574	3.579	3.583
600.0	3.611	3.616	3.620	3.625
630.0	3.654	3.659	3.664	3.669
660.0	3.699	3.704	3.710	3.715
690.0	3.747	3.752	3.757	3.763
720.0	3.796	3.802	3.807	3.813
750.0	3.848	3.854	3.860	3.866
780.0	3.903	3.909	3.915	3.921
810.0	3.960	3.967	3.974	3.980
840.0	4.021	4.028	4.035	4.042
870.0	4.086	4.094	4.101	4.109
900.0	4.155	4.163	4.171	4.179
930.0	4.229	4.237	4.246	4.254
960.0	4.308	4.317	4.326	4.334
990.0	4.393	4.402	4.412	4.421
1020.0	4.484	4.494	4.505	4.515
1050.0	4.584	4.595	4.606	4.617
1080.0	4.693	4.705	4.717	4.728
1110.0	4.813	4.826	4.839	4.851
1140.0	4.947	4.960	4.974	4.988
1170.0	5.097	5.111	5.126	5.141
1200.0	5.267	5.283	5.299	5.315
1230.0	5.464	5.481	5.499	5.516
1260.0	5.697	5.715	5.733	5.751
1285.0	5.927	5.946	5.965	5.982
1290.0		5.998	6.016	6.034
1305.0			6.182	6.198
1370.0				7.196

Appendix 5C

Table 5C3: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (5.17)
 $B = 0.05$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.017	3.026	3.035	3.044
10.0	3.055	3.065	3.074	3.084
20.0	3.094	3.105	3.115	3.125
30.0	3.136	3.147	3.158	3.169
40.0	3.179	3.192	3.204	3.215
50.0	3.225	3.238	3.251	3.264
60.0	3.273	3.288	3.301	3.315
70.0	3.324	3.339	3.354	3.369
80.0	3.378	3.394	3.411	3.427
90.0	3.435	3.453	3.470	3.488
100.0	3.496	3.515	3.534	3.553
110.0	3.560	3.581	3.602	3.622
120.0	3.630	3.652	3.675	3.697
130.0	3.704	3.729	3.753	3.777
140.0	3.784	3.811	3.838	3.864
150.0	3.872	3.901	3.930	3.959
160.0	3.967	3.999	4.031	4.063
170.0	4.071	4.107	4.142	4.177
180.0	4.187	4.226	4.265	4.304
190.0	4.316	4.359	4.403	4.446
200.0	4.461	4.510	4.558	4.607
210.0	4.628	4.682	4.736	4.790
220.0	4.821	4.883	4.943	5.002
230.0	5.051	5.120	5.186	5.249
240.0	5.331	5.407	5.477	5.541
250.0	5.685	5.764	5.831	5.882
260.0	6.157	6.220	6.254	6.257
270.0	6.827	6.789	6.707	6.597
280.0	7.745	7.346	7.040	6.781
285.0	8.137	7.511	7.107	6.794
290.0	8.312	7.568	7.106	

Appendix 5C

Table 5C4: Value of optimal patent term under demand uncertainty
Satisfying the Policy Maker's Equilibrium (5.17)
 $B = 0.1$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.026	3.044	3.060	3.075
5.0	3.065	3.084	3.101	3.118
10.0	3.105	3.125	3.145	3.163
15.0	3.147	3.169	3.190	3.210
20.0	3.192	3.215	3.238	3.259
25.0	3.238	3.264	3.288	3.312
30.0	3.288	3.315	3.341	3.367
35.0	3.339	3.369	3.398	3.425
40.0	3.394	3.427	3.458	3.488
45.0	3.453	3.488	3.521	3.554
50.0	3.515	3.553	3.589	3.625
55.0	3.581	3.622	3.662	3.702
60.0	3.652	3.697	3.741	3.784
65.0	3.729	3.777	3.825	3.873
70.0	3.811	3.864	3.917	3.969
75.0	3.901	3.959	4.017	4.074
80.0	3.999	4.063	4.127	4.190
85.0	4.107	4.177	4.248	4.317
90.0	4.226	4.304	4.382	4.458
95.0	4.359	4.446	4.532	4.616
100.0	4.510	4.607	4.701	4.793
105.0	4.682	4.790	4.893	4.991
110.0	4.883	5.002	5.113	5.211
115.0	5.120	5.249	5.362	5.449
120.0	5.407	5.541	5.640	5.691
125.0	5.764	5.882	5.929	5.903
130.0	6.220	6.257	6.184	6.037
134.0	6.669	6.538	6.317	6.065
138.0	7.145	6.732	6.359	
142.0	7.487	6.795		
146.0	7.563			

Appendix 5C

Figure 5C1: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

($B = 0.005$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5C

Figure 5C2: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

($B = 0.01$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5C

Figure 5C3: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

($B = 0.05$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5C

Figure 5C4: Simulation Charts for the optimal patent terms under demand uncertainty when the risk aversion index λ varies

($B = 0.1$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5D

Table 5D1: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (5.33)
B = 0.005

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.008	3.009	3.010	3.012
60.0	3.031	3.032	3.033	3.034
120.0	3.054	3.055	3.056	3.057
180.0	3.077	3.078	3.080	3.081
240.0	3.101	3.103	3.104	3.106
300.0	3.126	3.128	3.129	3.131
360.0	3.151	3.153	3.155	3.157
420.0	3.178	3.180	3.182	3.184
480.0	3.205	3.207	3.209	3.211
540.0	3.233	3.235	3.237	3.240
600.0	3.261	3.264	3.267	3.269
660.0	3.291	3.294	3.297	3.299
720.0	3.322	3.325	3.328	3.331
780.0	3.354	3.357	3.360	3.363
840.0	3.387	3.390	3.394	3.397
900.0	3.421	3.424	3.428	3.432
960.0	3.456	3.460	3.464	3.468
1020.0	3.493	3.497	3.501	3.506
1080.0	3.531	3.536	3.540	3.545
1140.0	3.571	3.576	3.581	3.586
1200.0	3.612	3.617	3.623	3.628
1260.0	3.655	3.661	3.667	3.673
1320.0	3.701	3.707	3.713	3.719
1380.0	3.748	3.755	3.761	3.768
1440.0	3.798	3.805	3.812	3.820
1500.0	3.850	3.858	3.866	3.874
1560.0	3.905	3.913	3.922	3.930
1620.0	3.963	3.972	3.981	3.991
1680.0	4.024	4.034	4.044	4.054
1740.0	4.090	4.100	4.111	4.122
1800.0	4.159	4.171	4.183	4.195
1860.0	4.233	4.246	4.259	4.272
1920.0	4.313	4.327	4.341	4.355
1980.0	4.398	4.414	4.429	4.445
2040.0	4.491	4.508	4.525	4.543
2100.0	4.592	4.611	4.630	4.649
2160.0	4.702	4.723	4.744	4.766
2220.0	4.824	4.847	4.871	4.895
2280.0	4.959	4.986	5.013	5.040
2340.0	5.111	5.142	5.173	5.204
2400.0	5.285	5.320	5.356	5.393
2460.0	5.487	5.528	5.570	5.613
2520.0	5.726	5.775	5.825	5.876
2580.0	6.018	6.078	6.140	6.203
2640.0	6.391	6.468	6.546	6.625
2700.0	6.904	7.005	7.107	7.208
2760.0	7.708	7.842	7.960	8.056
2820.0	9.344	9.217	9.086	8.958
2865.0	10.219	9.737	9.413	9.613
2870.0	10.220	9.742	9.416	

Appendix 5D

Table 5D2: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (5.33)
 $B = 0.01$

Value of λ	Value of n			
	0.5	1.0	1.5	2.0
0.0	3.009	3.012	3.014	3.016
30.0	3.032	3.034	3.036	3.039
60.0	3.055	3.057	3.060	3.062
90.0	3.078	3.081	3.084	3.087
120.0	3.103	3.106	3.109	3.112
150.0	3.128	3.131	3.134	3.138
180.0	3.153	3.157	3.161	3.164
210.0	3.180	3.184	3.188	3.192
240.0	3.207	3.211	3.216	3.220
270.0	3.235	3.240	3.245	3.249
300.0	3.264	3.269	3.274	3.279
330.0	3.294	3.299	3.305	3.311
360.0	3.325	3.331	3.337	3.343
390.0	3.357	3.363	3.370	3.376
420.0	3.390	3.397	3.404	3.411
450.0	3.424	3.432	3.439	3.447
480.0	3.460	3.468	3.476	3.484
510.0	3.497	3.506	3.515	3.523
540.0	3.536	3.545	3.554	3.564
570.0	3.576	3.586	3.596	3.606
600.0	3.617	3.628	3.639	3.650
630.0	3.661	3.673	3.685	3.697
660.0	3.707	3.719	3.732	3.745
690.0	3.755	3.768	3.782	3.796
720.0	3.805	3.820	3.834	3.849
750.0	3.858	3.874	3.890	3.906
780.0	3.913	3.930	3.948	3.965
810.0	3.972	3.991	4.009	4.028
840.0	4.034	4.054	4.075	4.096
870.0	4.100	4.122	4.144	4.167
900.0	4.171	4.195	4.219	4.243
930.0	4.246	4.272	4.299	4.325
960.0	4.327	4.355	4.384	4.414
990.0	4.414	4.445	4.477	4.510
1020.0	4.508	4.543	4.578	4.614
1050.0	4.611	4.649	4.688	4.729
1080.0	4.723	4.766	4.810	4.855
1110.0	4.847	4.895	4.945	4.996
1140.0	4.986	5.040	5.096	5.154
1170.0	5.142	5.204	5.269	5.335
1200.0	5.320	5.393	5.467	5.545
1230.0	5.528	5.613	5.701	5.792
1260.0	5.775	5.876	5.982	6.092
1290.0	6.078	6.203	6.332	6.465
1320.0	6.468	6.625	6.786	6.944
1350.0	7.005	7.208	7.397	7.553
1380.0	7.842	8.056	8.166	8.175
1410.0	9.217	8.958	8.710	8.476
1420.0	9.571	9.104	8.769	8.489
1425.0	9.670	9.142	8.779	
1430.0	9.723	9.160		
1435.0	9.742			

Appendix 5D

Table 5D3: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (5.33)
 $B = 0.05$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.018	3.027	3.037	3.046
10.0	3.057	3.069	3.080	3.091
20.0	3.098	3.112	3.126	3.140
30.0	3.141	3.158	3.174	3.190
40.0	3.186	3.206	3.225	3.244
50.0	3.234	3.257	3.279	3.301
60.0	3.285	3.310	3.336	3.362
70.0	3.338	3.368	3.397	3.427
80.0	3.395	3.428	3.462	3.497
90.0	3.455	3.493	3.532	3.572
100.0	3.519	3.563	3.607	3.653
110.0	3.588	3.637	3.688	3.741
120.0	3.661	3.718	3.776	3.837
130.0	3.741	3.806	3.873	3.942
140.0	3.828	3.901	3.978	4.059
150.0	3.922	4.006	4.095	4.189
160.0	4.026	4.123	4.226	4.335
170.0	4.140	4.253	4.373	4.502
180.0	4.268	4.399	4.541	4.694
190.0	4.413	4.567	4.735	4.918
200.0	4.578	4.761	4.963	5.183
210.0	4.770	4.991	5.237	5.501
220.0	4.997	5.269	5.570	5.876
230.0	5.274	5.614	5.978	6.281
240.0	5.625	6.055	6.451	6.609
250.0	6.091	6.616	6.874	6.714
260.0	6.751	7.207	7.036	
270.0	7.658	7.489		
280.0	8.242			

Appendix 5D

Table 5D4: Value of optimal patent term under cost reduction uncertainty
Satisfying the Policy Maker's Equilibrium (5.33)
 $B = 0.1$

Value of λ	Value of η			
	0.5	1.0	1.5	2.0
0.0	3.027	3.046	3.063	3.079
5.0	3.069	3.091	3.113	3.138
10.0	3.112	3.140	3.166	3.191
15.0	3.158	3.190	3.222	3.253
20.0	3.206	3.244	3.282	3.319
25.0	3.257	3.301	3.346	3.390
30.0	3.310	3.362	3.414	3.466
35.0	3.368	3.427	3.488	3.549
40.0	3.428	3.497	3.567	3.639
45.0	3.493	3.572	3.653	3.738
50.0	3.563	3.653	3.747	3.846
55.0	3.637	3.741	3.850	3.966
60.0	3.718	3.837	3.964	4.100
65.0	3.806	3.942	4.090	4.250
70.0	3.901	4.059	4.231	4.420
75.0	4.006	4.189	4.392	4.615
80.0	4.123	4.335	4.574	4.838
85.0	4.253	4.502	4.786	5.094
90.0	4.399	4.694	5.031	5.377
95.0	4.567	4.918	5.315	5.664
100.0	4.761	5.183	5.636	5.893
105.0	4.991	5.501	5.960	5.985
110.0	5.269	5.876	6.204	
115.0	5.614	6.281	6.276	
120.0	6.055	6.609		
125.0	6.616	6.714		
130.0	7.207			
135.0	7.489			

Appendix 5D

Figure 5D1: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.005$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5D

Figure 5D2: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.01$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5D

Figure 5D3: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.05$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

Appendix 5D

Figure 5D4: Simulation Charts for the optimal patent terms under cost reduction uncertainty when the risk aversion index λ varies

($B = 0.1$; $\eta = 0.5, 1.0, 1.5$ and 2.0)

CHAPTER 6

SUMMARY AND CONCLUSIONS

It is well known that society gets benefits from production and generation of knowledge. The patent system is one of the various government instruments for providing incentives to invent. Being a government sanctioned monopoly, it will, in the absence of perfect price discrimination, have the tendency to lead to a less-than-optimal use of available resources through restricting the use of inventions. Given that one cannot have both the incentives for invention and the optimal utilization of inventions with a patent system, one can then look for a second-best optimum by seeking that length of patent life that will maximize the incentive to invent and minimize the underutilization of inventions.

This thesis has dealt with the optimal patent life under alternative sets of assumptions. The existing literature on the subject makes a number of restrictive assumptions about the nature of inventive activity and the appropriability of the rents on inventions. In the past it has been assumed that the outcome of inventive activity is perfectly certain and either that inventors can appropriate all privately appropriable surplus (e.g., unique inventors) or all privately appropriable surplus is dissipated in the rivalry for the patent. A relatively longer patent term for unique inventors and a very short patent term for homogenous rival inventors are the direct consequence of these assumptions.

Chapter 2 of this thesis considers the optimal patent term models of Nordhaus and Berkowitz and Kotowitz and related extensions. These authors have found that with perfect certainty the optimal patent term is longer in the case of unique inventors than when there is free entry into inventing. Under the assumption that the entire privately appropriable surplus is

preserved, the optimal patent term is quite close to the existing patent term. But if the entire privately appropriable surplus is dissipated, as in the case of free entry into inventing, the optimal patent term is less than a year.

In Chapter 3 it is argued that the patentee's royalty rate will generally be less than the cost reduction made possible by his invention because of the availability of non-infringing substitute inventions. This situation is said to be one involving post-patent competition. The existence of post-patent competition has the effect of lengthening the optimal patent term both in the unique inventor and free entry cases. Thus, the optimal patent term depends on the competitive situation both before and after the patent is awarded.

Taxation of inventors' royalty income as another surplus preserving feature is also discussed in Chapter 3. Since the patentee pays taxes on his royalty income, the potential after-tax royalty income rather than the entire royalty income enters the decision calculus of inventors. Surplus is preserved in the form of tax revenues and thus the privately appropriable surplus available for dissipation is reduced. It is then demonstrated that under free entry into inventing the optimal patent term with post-patent competition and taxation of inventors' royalty income is longer than when there is no post-patent competition and taxes on inventors' royalty income.

Chapters 4 and 5 deal with a more fundamental problem of allocation of resources to invention: the uncertainty about the outcome of inventive activity. Like any other investment decisions, investment in R&D is risky. The incentives for invention depends on the inventors' attitude towards risk as well as on the degree of appropriability of inventions. Both of

these factors in turn affect the optimal length of a patent term. In order to analyse the impact of uncertainty on the optimal patent term, three types of uncertainty are considered: uncertainty about the demand function of the industry which purchases the cost-reducing invention; uncertainty about the magnitude of the cost reduction; and uncertainty about the date of invention.

The impact of uncertainty and the inventors' attitude towards risk on the optimal patent term in the case of unique inventors is analysed in Chapter 4. Numerical estimates of the optimal patent term are obtained and the respective effects of changes in the elasticity of demand, the magnitude of the cost reduction and the degree of inventors' risk aversion on the optimal patent term are also reported.

When inventors are risk neutral, it is demonstrated that

- (1) With demand uncertainty, the optimal patent term is identical to that of the perfect certainty case. This is because uncertainty does not affect the unique inventor's and the policy maker's equilibrium conditions.
- (2) With cost reduction uncertainty, the optimal patent term is shorter than the perfect certainty case. Since the inventor's equilibrium condition under cost reduction uncertainty is identical to that of the perfect certainty case, when the patent life is extended the incentives for further research and thus the amount of cost savings forthcoming are the same for both the cases. But the extension of the patent term increases the associated deadweight losses more under uncertainty than under perfect certainty. So the reduction in the length of the patent term will reduce the associated deadweight losses.

Under the assumption of inventor's risk aversion several results follow:

- (1) With both demand and cost reduction uncertainty, a risk averse inventor allocates fewer resources to inventive activity than his risk neutral counterpart. This implies that the incentive to invent is greater if the inventor is risk neutral than if he is risk averse. This is because for a given patent term, the marginal private benefit from research is less under risk aversion than under risk neutrality.
- (2) With both demand and cost reduction uncertainty the optimal patent term is longer when the inventor is risk averse than when he is risk neutral. The reason for this is that for each additional unit of research effort that is induced by an increase in the patent term, the risk aversion case will produce a larger amount of cost savings and thus a larger welfare gain than would arise in the risk neutral case.
- (3) With inventor's risk aversion, in the presence of demand uncertainty the optimal patent term, as the numerical estimates show, ranges from 5 to 45 years. In the presence of cost reduction uncertainty, it ranges from 5 to 51 years.
- (4) With both demand and cost reduction uncertainty and inventor's risk aversion, the numerical estimates of the optimal patent term also have the following comparative static properties: (i) the higher the elasticity of demand, the lower is the optimal patent term, everything else being constant; (ii) the larger the amount of cost reduction, the lower is the optimal patent term, everything else being constant; and (iii) the more averse to risk is the inventor, the longer is the optimal patent term, everything else being constant.

Chapter 5 analyses the impact of uncertainty on the optimal patent term when there is free entry into the market for invention. As in the case of the unique inventor, the optimal patent term is also sensitive to inventors' attitudes towards risk. The optimal patent term is again derived under the assumptions of either demand or cost reduction uncertainty, and of either inventor risk neutrality or risk aversion. Numerical estimates and the respective comparative static effects on the optimal patent term of changes in the elasticity of demand, the magnitude of the cost reduction and the degree of inventors' risk aversion are also obtained.

When the inventors are assumed to be risk neutral, it is demonstrated that

(1) With demand uncertainty, the optimal patent term is identical to that of the perfect certainty case. The reason for this, as in the unique inventor case, is that the inventors' and policy maker's equilibrium conditions are invariant to uncertainty.

(2) With cost reduction uncertainty the optimal patent term is longer than the perfect certainty case. This stands in direct contrast with the optimal patent term derived for the case of unique inventors.

This is because when the patent term is extended the net gain in welfare in the case of competitive inventing is larger under uncertainty than under perfect certainty while the net gain in welfare in the case of monopoly inventing is smaller under uncertainty than under perfect certainty.

When the inventors are assumed to be risk averse the following results are obtained:

(1) With demand uncertainty, post-patent competition and taxation on inventors' royalty income, the optimal patent term approximately ranges from 3 to 8 years. This is approximately 6 to 17-times longer than the optimal patent term in the absence of post-patent competition, taxation on inventors' royalty income and inventors' risk aversion.

(2) With cost reduction uncertainty, post-patent competition, and taxation on inventors' royalty income, the optimal patent term approximately ranges from 3 to 10 years. This is 6 to 20 times longer than the optimal patent term with no post-patent competition, no taxation on inventors' royalty income and no risk aversion.

(3) With both demand and cost reduction uncertainty, the numerical estimates of the optimal patent term reveal the comparative static result that the greater the degree of inventor risk aversion the longer is the optimal patent term.

Uncertainty about the value of invention arising from uncertainty about the date of invention is introduced in Chapter 5. The optimal patent term with free entry and an uncertain invention date is derived and it is demonstrated that it depends on the size of the invention, the elasticity of demand and the expected date of invention. By specifying an expected discovery date function, numerical estimates of the optimal patent term are obtained, and it is shown that under some plausible assumptions the optimal patent term derived under these assumptions is similar to the case of the unique inventor.

In this thesis the implications of a set of assumptions such as post-patent competition, taxation on inventors' income, uncertainty and risk aversion for the optimal patent term are examined. The effects of these

assumptions have been ignored in the existing literature. It is seen that these assumptions have direct influence on the optimal patent term. The impact of these assumptions is to increase the optimal length of a patent term both in the case of unique inventors and when there is free entry into inventing. This suggests that the results in the existing literature are sensitive to changes in the structural and behavioural assumptions. These models do not by themselves provide a basis for policy formulation. With further development they may help us to understand the considerations underlying the observed characteristics of existing intellectual property rights.

While the analysis of this thesis suggests that uncertainty, post-patent competition and income taxes play a key role in explaining the sensitivity of the results of the existing optimal patent term models, the conclusions must be approached with caution. Many simplifying assumptions have been made and it is not clear that these results will carry over into more complex models. For example, in this thesis, inventors face uncertainty either about the demand function of the industry which purchases the cost reducing invention or about the amount of cost reduction. Inventors are in fact likely to face uncertainty on the demand and cost reduction sides simultaneously. Similarly, no consideration has been given to the dynamic nature of the problem. In this thesis, demand does not shift over time. Inventors are uncertain only about its initial position. While actual inventors do face this type of uncertainty, they are also likely to face additional uncertainty over future demand. The thesis may be extended along these lines.

When inventors face uncertainty about the date of invention, it has been assumed that the size of invention, i.e., the level of cost reduction

is perfectly certain and does not depend on the amount of resources invested in R&D. By investing more in R&D, inventors can advance the date of invention but cannot produce any further cost reduction. It is also possible that by investing more, a greater cost reduction can be achieved. Since inventive activity is uncertain, it is more likely that inventors face uncertainty about both the date and magnitude of invention simultaneously. On one hand, more resource employment will bring about inventions earlier than what is socially desirable. On the other hand, more resource employment will produce more cost reductions. One can then readily find a trade-off between earlier invention and more cost reduction. It is then possible to obtain a patent term which will guarantee an optimal date of invention as well as an optimal size of invention. The thesis may also be extended along this important line of investigation.

With uncertainty about the date of invention, it is also assumed that every inventor conducts his inventive activity in isolation. In the present model there is no scope for individual inventors to learn from other inventors' research efforts or to transmit their own experiences to others. The impact of these spillover effects on the competition for the patent and on the allocation of resources to R&D may also be a promising line of investigation.

As was assumed by Yu (1977), if inventors are nonhomogenous in the sense that they differ in quality, some privately appropriable surplus will be preserved in the form of differential skill rents. Neither Yu nor this thesis considers the case of nonhomogenous inventors and analyses the impact of the existences of differential skill rents on the optimal patent term. It may prove possible to obtain the optimal patent term in the case of nonhomogenous inventors, under the various structural and behavioral assumptions employed in this thesis.

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