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Measurement of the Average B Hadron Lifetime in Z^0 Decays

by

Denis J. Dumas, B.Sc.

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

Department of Physics

Carleton University

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in Z^0 Decays**

submitted by
Denis J. Dumas, B.Sc.,

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Physics

Thesis Supervisor

External Examiner

Carleton University

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Abstract

A sample of 2610 electron candidates and 2762 muon candidates identified in hadronic Z^0 decays was used to measure the average b hadron lifetime. These data were recorded during 1990 and 1991 with the OPAL detector, one of the four large experiments operating at the Large Electron-Positron (LEP) collider at CERN, Geneva. Maximum likelihood fits to the lepton impact parameter distributions yielded an average b hadron lifetime of

$$\tau_b = 1\,523 \pm 34 \pm 37 \text{ fs},$$

where the first error is statistical and the second, systematic. This result is a weighted average over the semileptonic branching ratios and production rates of the b hadrons produced in Z^0 decays.

My Ph.D. Contributions to OPAL

The OPAL vertex detector was completed in 1988, and was first tested during that summer. This device, called a drift chamber, is described in detail in Chapter 3. I was involved in taking the wire tension measurements and mounting the high voltage distribution boards and preamplifiers onto the chamber. I also participated in the subsequent high voltage and cosmic ray tests, which included hooking up the detector to the data acquisition and high voltage systems, and collecting data generated by high energy cosmic ray muons.

I entered the Ph.D. program at Carleton University in the fall of 1988, and began working on the online monitoring software for the vertex chamber in the summer of 1989. My first task consisted in developing software to calculate the expected currents drawn by the vertex chamber power supplies, given the power supply voltages and the resistances connecting the various points of the chamber. The predicted currents were compared to the measured currents, and any serious anomaly would prompt the online system to turn off the chamber. I also developed a significant amount of the online software used to monitor the overall vertex chamber performance. I was primarily responsible for developing a menu driven histogramming package that gave the user easy access to hundreds of histograms displaying information such as the anode wire hit distributions, TDC distributions, and anode wire currents. This system also had the capability of lowering the vertex chamber voltages if, for example, the wire rates exceeded some preset limit. This software is still being used in the daily operation of the vertex chamber.

On August 13, 1989, OPAL recorded the first Z^0 decay at LEP, and shortly afterwards, on August 17, the vertex chamber was slowly powered on and recorded its first event. I participated in the commissioning of the vertex chamber, and in its daily operation during LEP's "pilot run" that summer.

In the summer of 1990 I moved to CERN and began developing the inclusive b hadron lifetime analysis for OPAL, in addition to helping with the daily operations of

the vertex chamber and, to a lesser extent, the OPAL detector. The results of the b hadron lifetime analysis were first published at the end of 1991 in *Physics Letters B* for the 1990 data sample, and recently in *Zeitschrift für Physik C* for the combined 1990–1991 data sample. I was the principal author for both papers. The more recent paper describing the 1990–1991 analysis was used as the skeleton for this dissertation.

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I especially wish to thank my wife, Pina, for editing portions of this dissertation and for understanding that when a graduate student says "I'll be finished in a month," it actually means "I'll be finished in a year."

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Chapter 1

Introduction

Our understanding of the fundamental building blocks of matter, and of their interactions with each other, has evolved at an ever increasing rate since the turn of the century. A multitude of theoretical ideas were proposed along the way to try to explain the phenomena observed in the subatomic world, but only a small subset of these ideas passed the “reality check” imposed by experimental measurements. The surviving theoretical ideas were merged and expanded into what is now a very successful model of the subatomic world which we call the *Standard Model*. The essential ingredients of this model are presented in the first section of this chapter.

The Standard Model has reached a point in its evolution where *all* of the observables it predicts agree with experimental measurements. To test the model for inconsistencies now requires high precision measurements, where “high precision” for a particle physicist means a precision ranging from a fraction of a percent to a few percent. One of the most precise measurements to date of the lifetime of a class of subatomic particles called *b hadrons* is presented in this dissertation, and is outlined in the second section of this chapter. The implications of this measurement for the Standard Model are discussed at the end of this dissertation.

1.1 The Standard Model

Our current theoretical understanding of nature and the supporting experimental evidence indicate that the fundamental building blocks of the Universe are the fermions called leptons and quarks, and the bosons that mediate all interactions between these fermions. The leptons and quarks and some of their properties are listed in Tables 1.1 and 1.2 respectively; the bosons are listed in Table 1.3. The fermions are grouped into three generations according to the way in which they couple to the bosons, as described below. For each fermion listed in these tables there exists an antifermion with exactly the same mass but of opposite charge. The theoretical framework describing the interactions between these fundamental particles is known as the Standard Model (SM) [5, 6, 7]. At present, this model incorporates only the electromagnetic, weak, and strong interactions. The gravitational force is at least forty orders of magnitude weaker than the electromagnetic, weak, and strong forces at the subatomic scale, and is therefore not incorporated into the Standard Model.

The electromagnetic interactions have been very successfully described by the theory of Quantum Electrodynamics, or QED [5]. The mediator of this interaction is the photon, which couples to all charged particles and has infinite range. In the Standard Model the theory of electromagnetic interactions has been merged with the theory of weak interactions to form what is called the theory of Quantum Flavour Dynamics, or QFD [6]. This is also referred to as the theory of electroweak interactions. In this model the weak force is mediated by the exchange of the $W^\pm$ and the Z^0 bosons. The range of the weak interactions is about 10^{-18} m. The Z^0 couples to fermion-antifermion pairs, whereas the coupling of the W is more complicated. In the lepton sector, the W couples to the leptons within a given generation. In the quark sector, the weak eigenstates are linear combinations of the mass eigenstates, and the transformation between the two is written in terms of the Cabbibo-Kobayashi-Maskawa (CKM) matrix [1]:

generation	name	symbol	charge $q/ e $	mass (MeV/c^2)
1	electron neutrino	ν_e	0	$< 7.3 \times 10^{-6}$
	electron	e	-1	$0.51099906 \pm 0.00000015$
2	muon neutrino	ν_μ	0	< 0.27
	muon	μ	-1	105.658389 ± 0.000034
3	tau neutrino	ν_τ	0	< 35
	tau	τ	-1	1777.0 ± 0.4

Table 1.1: General properties of the leptons [1, 2]. The mass limits for the electron and muon neutrinos are 90% confidence limits, whereas the mass limit on the tau neutrino is the 95% confidence limit.

generation	name	symbol	charge $q/ e $	mass (GeV/c^2)
1	up	u	$+2/3$	0.005 ± 0.003
	down	d	$-1/3$	0.010 ± 0.005
2	charm	c	$+2/3$	1.5 ± 0.2
	strange	s	$-1/3$	0.2 ± 0.1
3	top	t	$+2/3$	> 113
	bottom	b	$-1/3$	5.0 ± 0.3

Table 1.2: General properties of the quarks [1]. The masses are only estimates (see Reference [1] for details), except for the top quark mass which is the 95% confidence lower limit [3].

name	symbol	charge $q/ e $	mass (GeV/c^2)	interaction mediated
photon	γ	0	$< 3 \times 10^{-36}$	electromagnetic
W	$W^\pm$	± 1	80.24 ± 0.09	weak
Z	Z^0	0	91.187 ± 0.007	weak
gluon	g	0	0	strong

Table 1.3: General properties of the gauge bosons [1, 4].

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

where the primed quark labels are the weak eigenstates, the unprimed quark labels are the mass eigenstates, and the V_{ij} are the matrix elements giving the strength of the coupling between the two quarks i and j . By convention, only the charge $\frac{1}{3}$ quarks are mixed. The $W^+(W^-)$ therefore couples to a charge $\frac{2}{3}(-\frac{2}{3})$ quark(antiquark) and to a charge $\frac{1}{3}(-\frac{1}{3})$ antiquark(quark), with the strength determined by the corresponding CKM matrix element.

Four independent parameters are required to parameterize the CKM matrix elements [1]. A common parameterization is the Wolfenstein form [8], with real parameters A , λ , and ρ , and a phase δ :

$$\begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3\rho e^{i\delta} \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix}.$$

In this parameterization, $\lambda = \sin \theta_C$, where θ_C is the Cabibbo angle, and each term is expanded to $\mathcal{O}(\lambda^3)$ only.

The CKM matrix elements are determined from the weak decays of the relevant quarks, from deep inelastic neutrino scattering, and from the constraints imposed by the unitarity property of the CKM matrix. Assuming that only three generations of leptons and quarks exist in nature, the 90% confidence limits on the magnitude of the CKM matrix elements are [1]:

$$\begin{pmatrix} 0.9747 - 0.9759 & 0.218 - 0.224 & 0.002 - 0.007 \\ 0.218 - 0.224 & 0.9735 - 0.9751 & 0.032 - 0.054 \\ 0.003 - 0.018 & 0.030 - 0.054 & 0.9985 - 0.9995 \end{pmatrix}.$$

The leptons exist in a free state in nature, however, for reasons not yet fully understood, the quarks cannot. They combine in triplets to form baryons, such as

the proton and neutron, or in quark-antiquark pairs to form mesons, such as the pion. Collectively, baryons and mesons are called hadrons. The gluon is the carrier of the strong force and is responsible for binding the quarks within hadrons, and neutrons and protons within atomic nuclei. The theory describing this interaction is called Quantum Chromodynamics, or QCD [7]. In this model, the gluon and quarks carry a colour charge: red, green, or blue. The gluon couples to this colour charge, and therefore couples to the quarks and to other gluons. All hadrons are colourless, with baryons forming red-green-blue combinations, and mesons forming colour-anticolour combinations. The leptons have no colour and therefore do not couple to the gluon. The strong interaction is limited to nuclear scales, or about 10^{-15} m.

The main features of the theory of QCD deserve special mention here. Firstly, the self-coupling of gluons described above leads to the concept of *colour confinement*. This concept forms the basis of the quark fragmentation process discussed in the next chapter, and explains why quarks and gluons are only found in bound hadronic states. Secondly, the theory of QCD exhibits a behaviour called *asymptotic freedom*, which means that the QCD coupling constant, α_s , decreases logarithmically at shorter distances, or, equivalently, at higher energies. This is why QCD processes can be calculated at high energies using perturbation theory, which involves expansions in powers of α_s/π , but cannot be calculated at low energies, when α_s becomes too large and perturbation theory breaks down. The dependence of α_s on the energy scale μ is given by a first order differential equation [9], and a constant of integration Λ must be introduced when solving for α_s . The constant Λ must be determined from data. Therefore, the QCD coupling constant is a function of both the energy scale μ and the parameter Λ (see Reference [1] for the functional form of $\alpha_s(\mu^2, \Lambda)$).

There are 26 free parameters in the Standard Model: 6 lepton masses, 6 quark masses, 4 lepton mass mixing angles, 4 quark mass mixing angles, 3 electroweak parameters, 1 Higgs mass, 1 strong interaction CP violating phase, and 1 QCD coupling constant [1]. If the neutrino masses are set to zero, the 4 lepton mass mixing angles

description	symbol(s)	number
lepton masses	m_e, m_μ, m_τ	3
quark masses	$m_u, m_d, m_s, m_c, m_b, m_t$	6
electroweak parameters	$\alpha, G_F, \sin^2 \theta_W$	3
quark mass mixing	A, λ, ρ, δ	4
Higgs mass	m_H	1
QCD CP violating phase	θ_s	1
QCD coupling constant	α_s	1

Table 1.4: The Standard Model parameters assuming zero neutrino masses [1].

also vanish, leaving 19 free parameters. These 19 parameters are listed in Table 1.4. The electroweak parameters are the fine structure constant, $\alpha = 1/137$, the Fermi constant, $G_F = (8.96197 \pm 0.00015) \times 10^{-8} \text{ GeV} \cdot \text{fm}^3$, and the square of the sine of the Weinberg angle, $\sin^2 \theta_W = 0.2325 \pm 0.0008$ [1].¹ The quark mass mixing parameters listed in Table 1.4 correspond to the Wolfenstein parameterization discussed above.

A tremendous amount of work has been done during the past few decades to determine the Standard Model parameters as precisely as possible in order to test the model for any inconsistencies. So far, the Standard Model been extremely successful in describing the processes observed in nature. Most notably, the agreement between the experimental measurement and the QED prediction for the magnetic moment of the electron to 8 significant figures [5] is a phenomenal achievement, and precision tests [4] of the electroweak sector of the Standard Model have been consistent with the theory to within a fraction of a percent. However, there are two missing pieces of information. Firstly, the top quark has not yet been observed. With the current LEP data and the constraints on the top mass imposed by the Standard Model, we expect the top mass to be $m_t = 164_{-17}^{+16+18} \text{ GeV}/c^2$ [4]. The CDF Collaboration at Fermilab has recently placed a lower bound of 113 GeV/c² on the top mass [3], so there is hope that we will in fact discover the top quark in the near future. The critical missing

¹The Z^0 and W masses are included in the definition of these electroweak parameters.

piece of information is the Higgs particle. It is a key ingredient in giving mass to the Z^0 and $W^\pm$ bosons and has so far eluded discovery. To complicate matters, the Standard Model cannot predict the Higgs mass at all, and can easily accommodate a Higgs mass up to $1 \text{ TeV}/c^2$. The LEP experiments have pushed the lower bound on the Higgs mass to $48 \text{ GeV}/c^2$ [1]. CERN is planning to build the Large Hadron Collider within the next decade to bridge the gap between this lower mass bound and $1 \text{ TeV}/c^2$, and hopefully discover the Higgs.²

1.2 The B Hadron Lifetime

The b quark was discovered in 1977 at Fermilab (USA) with the first observation of the Υ resonances [10], which are $b\bar{b}$ quark resonances. The hadrons containing the b quark, collectively called b hadrons,⁴ have since been studied extensively at the e^+e^- storage rings CESR and PEP, located respectively at Cornell University and the Stanford Linear Accelerator Center in the United States, and DORIS and PETRA, located at the DESY laboratory in Germany. More recently, the experiments at the LEP e^+e^- collider at CERN (Switzerland) have made significant contributions to the overall understanding of the b hadron system, and the Tevatron proton-antiproton collider at Fermilab has been producing b physics results since 1992. Of particular interest to all of the above experimental groups has been the measurement of the b hadron lifetime, τ_b . The world average before LEP began operation in 1989 was $\tau_b = 1.18 \pm 0.11 \text{ ps}$ [11]. The individual measurements from PEP [12] and PETRA [13] that contribute to this average are shown in Figure 1.1.

One of the LEP measurements of the average lifetime of b hadrons produced in Z^0 decays is presented in this dissertation. Chapter 2 begins with a discussion of the

²The Superconducting Super Collider in Texas was also designed to discover and study the Higgs, but it was unfortunately cancelled by the US government on October 21, 1993.

³Any particle name with a line over it refers to its antiparticle.

⁴The notation “q hadrons” means hadrons containing the quark q, and is used throughout this dissertation.

Figure 1.1: Summary of inclusive b hadron lifetime measurements before LEP. The short error bars correspond to the statistical errors, and the long error bars correspond to the combination of the statistical and systematic errors.

production and decays of the Z^0 boson in high energy e^+e^- collisions, and is followed by a summary of our current understanding of the production of b hadrons in the fragmentation process following the Z^0 decay. The b hadron lifetime was determined from the kinematic properties of leptons produced in the semileptonic decay of b hadrons. Therefore, a brief description of these decays is also given. The final section of Chapter 2 discusses the theoretical motivation for the b hadron lifetime measurement and its relation to the parameters of the Standard Model.

The data used in this analysis were collected by the OPAL experiment at CERN. Chapter 3 includes a brief description of the LEP accelerator complex, followed by a detailed description of the OPAL detector, its data acquisition system, and the related online and offline analysis software.

The details of the b lifetime measurement are presented in Chapter 4 and the systematic errors associated with this measurement are discussed in Chapter 5.

All recent b hadron lifetime measurements are summarized in Chapter 6. These include the measurements from LEP and the Tevatron, and the indirect lifetime measurements from DORIS and CESR.

Chapter 2

B Physics at the Z^0 Resonance

The analysis presented in this dissertation was based on a sample of hadronic decays of the Z^0 produced in e^+e^- collisions. The Z^0 production and decay processes are discussed in Section 2.1. Section 2.2 reviews the quark fragmentation process, which describes the transition from two high energy quarks, produced in the decay $Z^0 \rightarrow q\bar{q}$, to the jets of hadrons observed in our detectors. Unfortunately, the fragmentation process is poorly understood and we must rely on the LUND model to simulate it. This model is incorporated into a Monte Carlo program called JETSET, and is described in Sections 2.2 and 2.3.

The production of b hadrons and their subsequent weak decays is then discussed in Section 2.4. Leptons produced in the semileptonic decays of b hadrons are used to extract the b hadron lifetime; these decays are described in detail in Section 2.5. The semileptonic decay models discussed in this section required modifications to the branching ratios used by the JETSET Monte Carlo. These modifications are listed in Section 2.6. Finally, in Section 2.7 the theoretical motivation for measuring the lifetime of b hadrons is presented, and the relation between the lifetime measurement and the Standard Model parameters is discussed.

2.1 Z^0 Production and Decays

The annihilation of an electron and its antiparticle, the positron, at a center-of-mass energy around the Z^0 mass, can proceed via the channels depicted in Figure 2.1. Figure 2.1(a) shows the s-channel process in which the electron and positron annihilate and couple to a virtual photon or a Z^0 , which subsequently decays into a fermion-antifermion pair ($f^+ f^-$). Figure 2.1(b) shows the t-channel process which is only possible for the $e^+ e^-$ final state. The t-channel process is not useful for this analysis, and will be discussed very briefly when the luminosity measurement is described in Chapter 3.

Figure 2.1: Feynman diagrams for $e^+ e^-$ annihilation around the Z^0 resonance. Figure (a) shows the annihilation into a virtual photon or a Z^0 , which then decays into a fermion-antifermion pair. Figure (b) shows the t-channel process for the $e^+ e^-$ final state.

The differential cross-section for $e^+ e^-$ annihilation into $f^+ f^-$ can be written as [14]

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 c^2 \alpha^2}{4E_{\text{cm}}^2} \left[C_1 (1 + \cos^2 \theta) + C_2 \cos \theta \right], \quad (2.1)$$

where

$$C_1 = e_e^2 e_f^2 + 2e_e e_f v_e v_f \mathcal{R}e(\chi) + (v_e^2 + a_e^2)(v_f^2 + a_f^2) |\chi|^2, \quad (2.2)$$

$$C_2 = 4e_e e_f a_e a_f \mathcal{R}e(\chi) + 8v_e a_e v_f a_f |\chi|^2, \text{ and} \quad (2.3)$$

$$\chi = \frac{cG_F M_Z^2}{\hbar^3 8\pi\alpha\sqrt{2}} \frac{E_{cm}^2}{(E_{cm}^2 - c^4 M_Z^2 + ic^2 M_Z \Gamma_Z)}. \quad (2.4)$$

Here, $\hbar$ is the reduced Planck constant, c is the speed of light, α is the electromagnetic fine structure constant, E_{cm} is the center-of-mass energy, θ is the angle between the electron and the fermion momentum vectors, e_e is the electron charge, e_f is the fermion charge, v_e and v_f are the electron and fermion vector coupling constants, a_e and a_f are the electron and fermion axial coupling constants, G_F is the Fermi constant, $\text{Re}(\chi)$ is the real component of χ , and M_Z and Γ_Z are the Z^0 mass and width respectively.

coupling constant	charge $+\frac{2}{3}$ quarks u, c	charge $-\frac{1}{3}$ quarks d, s, b
a	+1	-1
v	+0.38	-0.69

Table 2.1: Quark electroweak coupling constants.

The axial and vector coupling constants are related to the third components of the weak isospin of the fermions, t_{3L} and t_{3R} , as follows:

$$a_f = 2(t_{3L}^f - t_{3R}^f) \quad (2.5)$$

$$v_f = 2(t_{3L}^f + t_{3R}^f - 2e_f \sin^2 \theta_W). \quad (2.6)$$

In the Standard Model, $t_{3R}^f = 0$ for all fermions f , and t_{3L}^f is $+\frac{1}{2}$ for charge $+\frac{2}{3}$ quarks and neutrinos, and is $-\frac{1}{2}$ for charge $-\frac{1}{3}$ quarks and the charged leptons. The fermion charge e_f is in units of the electron charge $|e|$. Using $\sin^2 \theta_W = 0.2325$ yields the quark electroweak couplings listed in Table 2.1.

The cross-section for e^+e^- annihilation into hadrons as a function of center-of-mass energy E_{cm} is shown in Figure 2.2. The curve in this figure was obtained by integrating the Standard Model prediction given in Equation 2.1 over the solid angle Ω . The hadronic cross-section at the Z^0 peak is 41.56 ± 0.14 nb [4]. The measured branching ratios of the Z^0 are listed with the corresponding Standard Model predictions in

Figure 2.2: Cross-section for e^+e^- annihilation into hadrons. The curve is the Standard Model prediction. The data below 70 GeV were accumulated at the indicated e^+e^- storage rings, and the data around 91 GeV were accumulated at LEP.

decay mode	$BR_{\text{exp}} (\%)$	$BR_{\text{SM}} (\%)$
$Z^0 \rightarrow q\bar{q}$	69.9 ± 0.2	70.0
$Z^0 \rightarrow \nu\bar{\nu}$	20.0 ± 0.2	20.1
$Z^0 \rightarrow \ell^+ \ell^-$	3.37 ± 0.01	3.37
$Z^0 \rightarrow c\bar{c}$	12.0 ± 1.0	11.9
$Z^0 \rightarrow b\bar{b}$	15.4 ± 0.2	15.4

Table 2.2: Measured branching ratios (BR_{exp}) of the Z^0 , and the Standard Model predictions (BR_{SM}) [1, 4].

Table 2.2. Here, the $q\bar{q}$ represents all quark-antiquark combinations (except for top), the $\nu\bar{\nu}$ represents all neutrino-antineutrino combinations, the $\ell^+\ell^-$ represents any charged lepton-antilepton pair, the $b\bar{b}$ represents the quark-antiquark combination of the b quark, and the $c\bar{c}$ represents the quark-antiquark combination of the c quark. The average charged particle multiplicity in hadronic Z^0 decays is $\langle n_{ch} \rangle = 21.40 \pm 0.43$ [15].

2.2 Quark Fragmentation

In 1975, the Mark II Collaboration, working at SPEAR at center-of-mass energies of 6.2 and 7.4 GeV, found the first evidence for jet structure in the reaction $e^+e^- \rightarrow \text{hadrons}$ [16]. Later, when PETRA started producing e^+e^- annihilations in 1978 at center-of-mass energies around 17 GeV, the jet structure became spectacular with two back-to-back jets of hadrons clearly visible in most events. The natural explanation for this process was the production of a quark-antiquark pair, $e^+e^- \rightarrow q\bar{q}$, which then fragmented to produce jets of hadrons. Soon afterwards, events with three coplanar jets were observed, presumably due to one of the quarks radiating a hard gluon in the process $e^+e^- \rightarrow q\bar{q}g$. Events with higher jet multiplicities have since been observed, and are assumed to be manifestations of the processes $e^+e^- \rightarrow q\bar{q}gg$, $e^+e^- \rightarrow q\bar{q}q\bar{q}$, $e^+e^- \rightarrow q\bar{q}ggg$, and so on. The initial quarks and gluons produced in

e^+e^- annihilations are collectively called partons.

Unfortunately, the fragmentation process is not yet completely understood from first principles, so we must rely on models to help us understand the underlying physics. The LUND model [17, 18], the most successful model in e^+e^- physics, is used extensively throughout this analysis. Reference [14] gives an excellent overview of alternative models and where they fail to reproduce the data.

The LUND model is based on the concepts of parton showers [19] and string fragmentation [20]. The parton shower model is used to describe the first stage of the fragmentation process, where the initial quark and antiquark produced in the e^+e^- annihilation radiate gluons, which subsequently couple to other gluons or $q\bar{q}$ pairs, and so on, initiating a parton shower. These partons are produced off-shell, meaning they are virtual and have an effective mass Q^2 , where Q is the four-momentum of the parton. The branching of the partons continues until some cutoff scale, Q_0 , is reached, where $Q^2 \approx Q_0^2$. At this point, all gluons are converted to $q\bar{q}$ pairs and the remaining quarks and antiquarks are linked together by colour flux tubes, or strings, created by the confining QCD potential. The chromodynamic field lines linking the $q\bar{q}$ pairs are pulled together into a tube because of the coupling between gluons.

At this stage, the evolution of the event is described by a string model. As the quarks and antiquarks continue to move apart, the potential energy stored in the string linking each pair increases with their separation. If the quarks in a $q\bar{q}$ pair have sufficiently high relative momentum, it will become energetically favourable to break the string and create a new quark-antiquark pair, $q'\bar{q}'$. This will form two new strings, one linking the new quark q' with the original $\bar{q}$, and the other linking the new antiquark $\bar{q}'$ with the original q . This process is called string fragmentation. Baryon production is included by the production of a diquark-antidiquark pair at some of the string breaks. This process continues until only bound hadronic states remain, which subsequently decay into the hadrons, leptons and photons observed in our detectors. The first bound hadronic states containing the original quark and antiquark produced

Figure 2.3: The fragmentation process in e^+e^- annihilations. The horizontal axis represents time increasing from left to right. The different regions labelled (i)–(iv) are described in the text.

in the e^+e^- annihilation are called primary hadrons.

The fragmentation process is illustrated in Figure 2.3. Region (i) shows the e^+e^- annihilation into a $q\bar{q}$ pair, and Region (ii) shows the development of the parton shower. These processes can be calculated from electroweak theory and perturbative QCD respectively. Region (iii) shows a “black box”: the domain where fragmentation models (i.e., the string model) are necessary because of our lack of understanding of the transition from the parton shower to the bound hadronic states, which are depicted in Region (iv).

If the q' and $\bar{q}'$ created in the fragmentation process have no mass or transverse momentum, then the pair can be classically created at one point and pulled apart by the QCD field. However, if the quarks have mass and/or transverse momentum, they must be classically produced a certain distance apart so that the field energy between them can be converted into the sum of the two transverse masses, $m_\perp$. Here $m_\perp^2 = m^2 + p_\perp^2/c^2$, where m is the quark mass, and $p_\perp^2 = p_x^2 + p_y^2$ is the component of the quark's momentum transverse to the primary quark's direction. The LUND model accomplishes this process by using the concept of quantum mechanical tunnelling. The $q'\bar{q}'$ pair is first created at a point in order to preserve local flavour conservation, then the quarks are tunnelled out to the classically allowed region. The tunnelling probability is proportional to [18]

$$\exp\left(-\frac{\pi c^3}{\hbar} \frac{m_\perp^2}{\kappa}\right) = \exp\left(-\frac{\pi c^3}{\hbar} \frac{m^2}{\kappa}\right) \exp\left(-\frac{\pi c^3}{\hbar} \frac{p_\perp^2}{\kappa}\right), \quad (2.7)$$

where κ is the string constant with a value of typically 1 GeV/fm. Therefore the production of heavy quarks in the fragmentation process is highly suppressed, with the relative production rates predicted to be approximately

$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} : b\bar{b} = 1 : 1 : 0.3 : 10^{-11} : 10^{-35}. \quad (2.8)$$

The $s\bar{s}$ relative production rate has been measured [14] to be 0.33 ± 0.02 , which is in excellent agreement with this prediction. In the LUND model, the probability of producing either a $c\bar{c}$ or $b\bar{b}$ pair in the fragmentation process has been set to 0. However, a $c\bar{c}$ or $b\bar{b}$ pair can be produced in the gluon branching $g \rightarrow c\bar{c}$ or $g \rightarrow b\bar{b}$. For every 10^6 hadronic Z^0 decays, we expect approximately 25 000 $c\bar{c}$ pairs and 2 500 $b\bar{b}$ pairs to be produced from such branchings [21].

A variable commonly used to parameterize fragmentation is

$$z = \frac{(E + cp_\parallel)_{\text{hadron}}}{(E + cp_\parallel)_{\text{quark}}}, \quad (2.9)$$

where E is the energy and $p_\parallel$ is the momentum component along the initial quark direction. This definition for z is convenient since it is Lorentz invariant for boosts

along the initial quark direction. Experimentally, we measure the quantity

$$x_E = \frac{2E_{\text{hadron}}}{E_{\text{cm}}}, \quad (2.10)$$

where E_{hadron} is the hadron energy, and $x_E \leq z$ due to initial state QED corrections and final state QCD corrections.

Because of the inherent statistical nature of the fragmentation process the primary hadrons will be distributed in z according to some fragmentation function, $f(z)$. In the LUND model, the tunnelling process described above, combined with the requirement that the fragmentation result look the same whether the algorithm begins by forming hadrons at the q end or the $\bar{q}$ end of the string, leads to a unique form for $f(z)$ called the *LUND symmetric fragmentation function* [22]:

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(-\frac{bm_{\perp}^2 c^4}{z}\right), \quad (2.11)$$

where a and b are parameters determined from fits to the data. This function gives a good description of the light quark (uds) data, but has been shown [14] to be too “hard” for the heavier quarks (cb), meaning that the resulting $\langle x_E \rangle$ is too high compared to the data. For this reason, different fragmentation functions are used for heavy quarks, the most popular one being the *Peterson fragmentation function* [23]:

$$f(z) \propto \frac{1}{z [1 - 1/z - \epsilon_q / (1 - z)]^2}. \quad (2.12)$$

Although its theoretical foundation is still being debated, the Peterson fragmentation function is most commonly used because it requires a single free parameter, ϵ_q . More data will be required to determine which of the heavy quark fragmentation functions gives a better description of the data [14].

The most recent measurements of ϵ_q for b and c quarks produced in Z^0 decays yield $\epsilon_b = 0.0055^{+0.0040}_{-0.0030}$ [24] and $\epsilon_c = 0.050 \pm 0.020$ [25]. The central values for ϵ_b and ϵ_c are the averages of the measurements presented in References [24] and [25], and the errors are consistent with the range of these measurements. The corresponding values

of $\langle x_E \rangle$ are $\langle x_E \rangle_b = 0.70 \pm 0.02$ and $\langle x_E \rangle_c = 0.51 \pm 0.02$. The Peterson fragmentation functions for these two values of ϵ_q are shown in Figure 2.4. For comparison, the LUND fragmentation functions for b and c quarks are also shown, using the a and b parameters given below.

Figure 2.4: Peterson fragmentation function for 2 values of ϵ_q . The LUND fragmentation function for each quark is also shown. The distributions are normalized to have an area of 1.

2.3 The JETSET Monte Carlo

The LUND model described above has been incorporated into an excellent *event generator* called JETSET [18]. This program simulates complete e^+e^- annihilations into hadrons, including the decays of unstable particles such as b and c hadrons. The decays of longer-lived particles, such as the Λ^0 and the K_s^0 , are handled in the detector simulation program described in the next chapter since these particles can also interact with the material in our detectors. The JETSET program has literally thousands of

parameters, ranging from parameters that control the fragmentation process to most of the known particle branching ratios and masses (i.e., the content of the Particle Data Book [1]).

parameter	default	value used
$\sin^2 \theta_W$	0.23	0.2337
M_Z	91.2 GeV/ c^2	91.162 GeV/ c^2
Γ_Z	2.4 GeV	2.489 GeV
M_W	80.0 GeV/ c^2	79.8 GeV/ c^2
a	0.5	0.18
b	0.9 GeV $^{-2}$	0.34 GeV $^{-2}$
σ_p	0.35 GeV/ c	0.37 GeV/ c
Λ	0.40 GeV	0.29 GeV

Table 2.3: The JETSET 7.3 parameters modified for the OPAL event generation.

Most of the default parameters of JETSET version 7.3 were used in the OPAL event generation, which is described in the following chapter. However, several branching ratios important to this analysis were out of date and were modified to reflect the most recent data. These changes are discussed later in this chapter. Several electroweak parameters were also modified to the values listed in Table 2.3. In addition, the parameters controlling the fragmentation process were tuned to OPAL data [26] and used in the event generation. They are also listed in Table 2.3 and include the a and b parameters of the LUND symmetric fragmentation function, σ_p , which is the σ of the Gaussian used to give the quarks their transverse momentum $p_\perp$ during the fragmentation process, and the Λ parameter described in the previous chapter, which is used to determine the QCD coupling constant α_s .

2.4 B Hadron Production and Decays

Most of the information regarding b hadrons produced in Z^0 decays comes from the LEP experiments. These experiments are relatively young and are just beginning to measure the properties of individual b hadrons, so we must rely on JETSET and theory to predict quantities such as the b hadron production rates and the b hadron masses, for example. The dominant ground state hadrons that are expected to be produced in Z^0 decays are listed in Table 2.4. The quark content of each hadron, its mass, and its production rate, which includes all contributions from the excited b hadron states, are listed in this table. The Ξ_b has recently been observed for the first time [27]; its production rate measurement is therefore very preliminary. The JETSET inputs for these parameters are in good agreement with the corresponding measurements.

b hadron symbol	quark content	mass (GeV/c^2)		production rate	
		JETSET	measured	JETSET	measured
B_u^+	bu	5.2776	5.2786 ± 0.0020	0.397	—
B_d^0	$\bar{b}d$	5.2794	5.2787 ± 0.0021	0.396	—
B_s^0	$\bar{b}s$	5.4797	5.3731 ± 0.0042	0.119	0.152 ± 0.042
Λ_b^0	bud	5.620	5.626 ± 0.026	0.078	0.122 ± 0.031
Ξ_b^+	$\bar{b}ds$	5.840	—	0.005	0.012 ± 0.006
Ξ_b^0	bus	5.840	—	0.005	0.012 ± 0.006

Table 2.4: The b hadron masses and production rates in JETSET, compared to the most recent measurements [1, 27]. The superscripts on the particle symbols always refer to the charge of the hadron. The subscripts on the meson symbols specify the flavour of the light quark in the b meson, and the subscripts on the baryon symbols specify that the heaviest quark in the baryon is the b quark.

The ground state b hadrons decay via the flavour-changing weak interaction. Furthermore, since there is no evidence for neutral flavour-changing currents, the decays must proceed via the charged-current weak interaction which involves the exchange

Figure 2.5: Feynman diagrams for two B_u^- decays. Both diagrams illustrate the spectator process, where the $\bar{u}$ quark does not participate in the decay process.

of a W boson. The decays

$$B_u^- \rightarrow D_u^0 \mu^- \bar{\nu}_\mu, \text{ and}$$

$$B_u^- \rightarrow D_u^0 \pi^-,$$

are illustrated in the Feynman diagrams shown in Figure 2.5.¹ In both cases the b quark is shown decaying into a c quark with the emission of a virtual W , which then couples to (a) a muon and its neutrino, or (b) a $\bar{u}d$ quark pair. Similar diagrams can be drawn for the other b hadrons. Decays which produce only hadrons are called hadronic decays, those which produce hadrons and leptons are called semileptonic decays, and those which produce only leptons are called leptonic decays. The average number of charged particles produced in b hadron decays is $\langle n_b \rangle = 5.0 \pm 0.5$ [28]. The corresponding JETSET prediction is $\langle n_b \rangle \approx 5.3$. The b hadrons are produced with a mean energy of 32 GeV, corresponding to $\langle x_E \rangle_b = 0.70$. The mean decay length at this energy is 2.7 mm for a mass of 5.3 GeV/ c^2 and a lifetime of 1.5 ps.

¹Throughout this dissertation, all references to particles and their decay modes include the corresponding charge conjugates. The symbol "D" is used to represent c mesons.

2.5 Semileptonic Decay Models

JETSET implements semileptonic b decays using a purely spectator quark description. In the decay $b \rightarrow c\ell\nu_\ell$,² the c quark is combined with the spectator antiquark or diquark to form one single hadron h . The overall process is then $H \rightarrow h\ell\bar{\nu}_\ell$, where H is a b antimeson or a b baryon. The decay product momenta are distributed according to a $V - A$ matrix element using the H and h hadron four-vectors instead of the quark four-vectors, and neglecting the decay product masses. The decay $b \rightarrow u\ell\nu_\ell$ is highly suppressed due to the small value of the CKM matrix element V_{ub} , and is not yet incorporated in JETSET. This approach to semileptonic b hadron decays is implemented in JETSET because of the lack of information on the exclusive decays of b hadrons.

In contrast, the semileptonic branching ratios and decay modes of the D_d^+ , D_u^0 , D_s^+ and Λ_c^+ hadrons are explicitly given in JETSET. The decay product momenta are distributed using the $V - A$ matrix element in the same manner as in semileptonic b hadron decays. However, when several hadrons are present in the final state, the sum of their four-vectors is used in the $V - A$ matrix element calculation to represent the single hadron h . In addition to this approximation, the branching ratios for the higher multiplicity decays (i.e., ≥ 4 particles) are not all known, and have been inferred from the inclusive semileptonic branching ratio and the exclusive branching ratios for the low multiplicity decays. For other charm hadrons such as the Ξ_c and Ω_c , little is known about their decays so the same approach used for semileptonic b hadron decays is implemented in JETSET.

The modelling of the momentum distributions of leptons produced in semileptonic decays of b and c hadrons by JETSET was found to be inadequate. In this analysis, the lepton momentum spectra produced by JETSET were replaced by the spectra predicted by the ACCMM model [29], and our sensitivity to the details of the semileptonic

²The notation " $b \rightarrow q\ell\nu_\ell$ " is used to denote the inclusive semileptonic decay of b hadrons, where the b quark decays into the quark q .

decays was estimated using the ISGW model [30]. These models are distinguished from other available models in their calculation of the entire lepton momentum spectrum. The CLEO Collaboration [31] has used these models to fit the lepton momentum spectra produced in the semileptonic decays of B_u^+ and B_d^0 mesons, and provide a self-consistent set of model parameters which were applied to OPAL data as described below. CLEO records data at the $\Upsilon(4S)$ resonance, where only B_u^+ and B_d^0 mesons are produced. The implications of the presence of B_s^0 and Λ_b^0 hadrons in OPAL data will be discussed later.

The ACCMM model [29] is a free-quark model which includes corrections for gluon emission. There are three free parameters which have to be determined from the data: the mass of the spectator quark in the parent meson, the mass of the quark produced in the weak decay, and a Fermi momentum, p_F , which describes the momentum distribution of the quarks in the decaying meson. CLEO uses three sets of masses and Fermi momentum parameters to describe their data, one each for $b \rightarrow c\ell\nu_\ell$, $b \rightarrow u\ell\nu_\ell$, and $c \rightarrow \ell$ decays.³ They find that the lepton momentum spectrum is insensitive to the mass of the spectator quark and fix this parameter at $150 \text{ MeV}/c^2$ for all three decays. For $b \rightarrow c\ell\nu_\ell$ decays, CLEO finds $p_F = 298 \text{ MeV}/c$ and $m_c = 1673 \text{ GeV}/c^2$ from fitting their data. In $b \rightarrow u\ell\nu_\ell$ decays, the mass of the u quark is fixed at $150 \text{ MeV}/c^2$, and the Fermi momentum is fixed to be the same as the Fermi momentum in $b \rightarrow c\ell\nu_\ell$ decays. CLEO finds that the results of their fits are very insensitive to the choice of parameters in $b \rightarrow u\ell\nu_\ell$ decays.

The ISGW model [30] is based on calculations of the form factors in a nonrelativistic quark model for the exclusive three-body final states $\bar{B} \rightarrow (D, D^*, D^{**})\ell^-\bar{\nu}_\ell$. Although this model attempts an honest calculation of the momentum spectra for these exclusive final states, it also includes an *ad hoc* correction which is required to improve agreement with data, and which the authors claim is motivated by relativistic effects. The relative branching ratios calculated by the ISGW model for the D, D^* ,

³The notation " $q \rightarrow \ell$ " refers to the fully inclusive semileptonic decay of quark q .

and D^{**} final states are 27%, 62%, and 11% respectively. CLEO also uses a modified version of the ISGW model to fit their data, referred to as ISGW⁺. In this case, CLEO allowed the fraction of $\bar{B} \rightarrow D^{**}\ell^-\bar{\nu}_\ell$ decays in the $b \rightarrow c\ell\nu_\ell$ sample to float freely in their fit, which resulted in a 32% D^{**} fraction instead of only 11%. This procedure was motivated by the poor fit of the ISGW model to their data.

For $c \rightarrow \ell$ decays, CLEO fits [31] the ACCMM model to the measured lepton momentum spectrum from semileptonic D decays obtained by DELCO [32], and obtains the parameters $m_s = 50 \text{ MeV}/c^2$ and $p_F = 282 \text{ MeV}/c$. DELCO data are used because CLEO finds that the parameters m_s and p_F are not well constrained by their own data. The ISGW prediction for the lepton momentum spectrum in semileptonic D meson decays is also used to fit OPAL data, as described below.

The modelling of cascade decays, $b \rightarrow c \rightarrow \ell$, is more involved because no reliable models exist for the inclusive $b \rightarrow c$ decay process. CLEO derived the $b \rightarrow c \rightarrow \ell$ momentum spectrum by combining their measured c meson momentum spectrum in b meson decays [33] with the ACCMM or ISGW prediction for the lepton momentum spectrum in the rest-frame of the parent c meson.

Electroweak radiative corrections to these models are significant. They are calculated in the rest-frame of the b mesons for $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ decays, and in the rest-frame of the c mesons for $c \rightarrow \ell$ decays. They vary substantially over the momentum range of the leptons and depend on the lepton mass. Near the end-point of the lepton spectra the magnitude of the corrections are almost 10% for electrons, and about 3% for muons.

The semileptonic decay model predictions for the lepton momentum spectra are shown in Figure 2.6. The $b \rightarrow \ell$ distributions include contributions solely from B_u^+ and B_d^0 mesons, the $b \rightarrow c \rightarrow \ell$ distributions include contributions solely from B_u^+ and B_d^0 mesons which decay into D_d^+ and D_u^0 mesons, and the $c \rightarrow \ell$ distributions include contributions solely from the decays of D_d^+ and D_u^0 mesons. The $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ momentum distributions were evaluated in the b meson rest-frame, and the $c \rightarrow \ell$

Figure 2.6: Lepton momentum spectra predicted by several semileptonic decay models.

momentum distributions were evaluated in the c meson rest-frame. The $b \rightarrow \ell$ spectra also include contributions from $b \rightarrow u\ell\nu_\ell$ decays, normalized by the branching ratios measured by CLEO in the fits described above. The JETSET prediction for the $b \rightarrow \ell$ spectrum is harder than the spectrum predicted by the other models, while the JETSET predictions for the $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$ spectra are softer. The significant differences between the $c \rightarrow \ell$ spectra are mostly the result of outdated charm branching ratios in the JETSET decay tables, which allow too many four and five-body semileptonic decays relative to three-body decays. Corrections to these branching ratios are discussed in the following section.

2.6 Branching Ratio Measurements

The CLEO measurements of the branching ratios $BR(b \rightarrow e)$ and $BR(b \rightarrow c \rightarrow e)$ were adopted, assuming that the branching ratios for $b \rightarrow e$ and $b \rightarrow \mu$, and for $b \rightarrow c \rightarrow e$ and $b \rightarrow c \rightarrow \mu$, are equal. CLEO also measured $BR(b \rightarrow \mu)$ from their muon momentum spectrum, but experimental difficulties in identifying low momentum muons did not allow a measurement of $BR(b \rightarrow c \rightarrow \mu)$. In addition, a measurement of $BR(b \rightarrow \ell)$ and $BR(b \rightarrow c \rightarrow \ell)$ was made from the combined electron and muon data, but the large experimental systematics on the CLEO muon data resulted in greater overall systematic uncertainties compared to the measurements using electrons alone.

Experimental difficulties identifying low momentum electrons are not as limiting as the problems with low momentum muons, nevertheless the CLEO analysis is most sensitive to leptons with rest-frame momenta in the upper half of the momentum spectrum. This introduces a strong correlation between the measured branching ratios and the decay model used to fit the data. In the analysis presented here, the lepton selection procedure introduced an acceptance similarly biased towards high lepton momenta in the decaying b hadron rest-frame. Therefore, the CLEO branching ratio measurements could be used, except that they had to be corrected for the presence

of B_s^0 and Λ_b^0 hadrons in LEP data. This procedure will be described in detail in Chapter 5.

In addition, the branching ratios for $b \rightarrow \tau \rightarrow \ell$, $b \rightarrow J/\psi \rightarrow \ell$, and $c \rightarrow \ell$ have changed considerably since the JETSET decay tables were last updated. The latest measurements of these branching ratios are also discussed in Chapter 5. The branching ratios used in this analysis are listed in Table 2.5.

decay mode	branching ratio (%)			
	JETSET	ISGW	ACCMM	ISGW**
$b \rightarrow \ell$	11.2	10.1	10.50 ± 0.50	11.1
$b \rightarrow c \rightarrow \ell$	10.2	10.3	9.0 ± 1.2	8.6
$b \rightarrow J/\psi \rightarrow \ell$	0.088	0.14 ± 0.04		
$b \rightarrow \tau \rightarrow \ell$	0.96	0.5 ± 0.2		
$c \rightarrow \ell$	10.3	9.6 ± 0.9		

Table 2.5: Branching ratios, in percent, for the different b and c hadron decays for different semileptonic decay models. The JETSET defaults are also shown.

2.7 B Hadron Lifetimes

The spectator quark approximation is the starting point for understanding the weak decays of hadrons containing a heavy quark Q . In this model, the light quark(s) in the decaying hadron does(do) not participate in the decay process. As a consequence, all hadronic states containing the quark Q are predicted to have identical lifetimes, determined by the intrinsic properties of Q . This hypothesis was first thought to apply to c hadrons, but experimentally it was found that the individual c hadron lifetimes differed substantially [1]:

$$\tau(D_d^+) : \tau(D_s^+) : \tau(D_u^0) : \tau(\Lambda_c^+) = 2.53 : 1.07 : 1 : 0.45. \quad (2.13)$$

This deviation from the spectator model prediction can be qualitatively understood by non-spectator mechanisms such as *W exchange* and *quark interference*, which will

be discussed below, but unfortunately a more quantitative understanding of this behaviour has so far not been achieved. However, theorists generally agree that the magnitude of these non-spectator effects should be proportional to some power of m_Q^{-1} , where m_Q is the mass of the heavy quark Q , so they should be less important in b hadron decays. Recent experimental measurements supporting this hypothesis are presented in Chapter 6. Only a brief discussion of the current theoretical estimates of the magnitudes of non-spectator effects in b hadron decays will be given here.

In the spectator approximation, the semileptonic decay width of b hadrons is written as [34]

$$\Gamma(b \rightarrow \ell) = \frac{G_F^2 c^4 m_b^5}{192 \pi^3 \hbar^6} (f_c |V_{cb}|^2 + f_u |V_{ub}|^2), \quad (2.14)$$

where $b \rightarrow \ell$ denotes the inclusive semileptonic decays of b hadrons, G_F is the Fermi constant, m_b is the b quark mass, f_c and f_u are coefficients involving phase space factors and QCD corrections, and V_{cb} and V_{ub} are CKM matrix elements. The parameters f_c and f_u can be approximated by [35]

$$f_q = \left(1 - 8a_q^2 + 8a_q^8 - 24a_q^4 \ln a_q\right) \times \left(1 - \frac{2\alpha_s(m_b^2)}{3\pi} \left[\left(\pi^2 - \frac{31}{4}\right)(1 - a_q)^2 + \frac{3}{2}\right]\right), \quad (2.15)$$

where $a_q = m_q/m_b$ for $q = u$ or $q = c$, and $\alpha_s(m_b^2)$ is the QCD coupling constant evaluated at the b quark mass. The first term in Equation 2.15 is a three-body phase space factor with the two lepton masses neglected, and the second term is a first order QCD correction. The semileptonic width given in Equation 2.14 is related to the b lifetime, τ_b , and the semileptonic branching ratio, $BR(b \rightarrow \ell)$, as follows:

$$\Gamma(b \rightarrow \ell) = \frac{\hbar BR(b \rightarrow \ell)}{\tau_b}. \quad (2.16)$$

Solving for the b hadron lifetime gives

$$\tau_b = \frac{192 \pi^3 \hbar^6}{G_F^2 c^4 m_b^5 (f_c |V_{cb}|^2 + f_u |V_{ub}|^2)} BR(b \rightarrow \ell). \quad (2.17)$$

Figure 2.7: Feynman diagrams for a B_u^- decay producing two identical quarks. (a) shows the spectator decay, and (b) shows the internal W conversion process leading to the same final state as (a).

Therefore, the measurement of the lifetime of b hadrons can be directly related to the CKM matrix elements V_{cb} and V_{ub} , thus providing constraints on the Standard Model parameters described in Chapter 1.

There are several non-spectator processes which can break the degeneracy of lifetimes predicted by the spectator quark model. By considering the flavour structure of the b hadrons and the pattern of the CKM matrix, theorists have identified those processes [36] which might have sizeable effects on the b hadron lifetime τ_b . There are three main classes of non-spectator processes which are frequently discussed in the literature: *quark interference*, *W exchange*, and *weak annihilation*.

Quark interference is the result of the Pauli exclusion principle acting on final states containing two identical quarks, and occurs predominantly in the channels

$$B_u^-(b\bar{u}) \rightarrow c\bar{u}d\bar{u}, \text{ and} \quad (2.18)$$

$$\Lambda_b^0(bud) \rightarrow c\bar{u}dud, \quad (2.19)$$

where the quarks produced in the decays are shown. The quarks produced in the B_u^- decay can combine to form a D_u^0 and a π^- in two ways, as shown in Figure 2.7. Figure 2.7(a) shows the spectator decay process where the W produces a π^- , and (b) shows what is called the *internal W conversion* process where the quarks coupling to the W are shared between the c and $\bar{u}$ quarks to produce the same final state

Figure 2.8: Feynman diagrams for two B_d^0 decays producing the same final state. (a) shows the spectator decay, and (b) shows the W exchange process.

as in (a). Similar diagrams can be drawn for Λ_b^0 decays. The amplitudes for these two decay modes have opposite signs and therefore add destructively, resulting in a slight lengthening the lifetime predicted by the spectator quark model. Altarelli and Petrarca [37] expect that this will alter the lifetime by less than 1%, although Bigi and Uraltsev [38] predict effects as large as a few percent. All other decay channels of the B_u^- and Λ_b^0 producing identical quarks or antiquarks in the final state are suppressed either by the Cabibbo angle, the small coupling between the b and u quarks, or phase space effects. Note that quark interference cannot occur in the decays of the B_d^0 and B_s^0 mesons. The decays of the Ξ_b and Ω_b baryons are also subject to Pauli interference, but these baryons are so rare that they are not considered in the present discussion.

W exchange can only occur in neutral b hadrons. The dominant channels are:

$$\bar{B}_d^0(b\bar{d}) \rightarrow c\bar{u}, \quad (2.20)$$

$$\bar{B}_s^0(b\bar{s}) \rightarrow c\bar{c}, \text{ and} \quad (2.21)$$

$$\Lambda_b^0(bud) \rightarrow cdd, \quad (2.22)$$

where the final state quarks subsequently fragment to produce hadrons. This process is shown in Figure 2.8 for the decay $\bar{B}_d^0 \rightarrow D_d^+ \pi^-$. Figure 2.8(a) shows the spectator decay where the W produces the π^- , and (b) shows the W exchange mechanism where a $d\bar{d}$ pair is generated in the fragmentation process to produce the same final state as in (a). Similar diagrams can be drawn for the W exchange decays of the B_s^0 and

Λ_b^0 hadrons. All other possible W exchange modes are suppressed by the Cabbibo angle or the small coupling between the b and u quarks. The W exchange process has the effect of decreasing the lifetime of the B_d^0 , B_s^0 , and Λ_b^0 hadrons relative to the spectator model prediction because of the additional decay channels made available. However, the effect is expected to be highly suppressed in b meson decays because of conservation of helicity at the light quark vertex, although Altarelli and Petrarca speculate that the effect could be as large as 10% if the W exchange is accompanied by gluon emission. On the other hand, Bigi and Uraltsev argue that gluon emission cannot significantly alter the fully inclusive rate given in Equation 2.14, and that this effect can only alter partial decay rates. In the Λ_b^0 system there is no helicity suppression mechanism for W exchange, and Cheng [39] predicts a 10% decrease in the Λ_b^0 lifetime.

Figure 2.9: Feynman diagram showing the weak annihilation process in a B_u^- . In this example the B_u^- decays leptonically into a muon-neutrino pair.

Finally, *weak annihilation* can only take place in the decays of the B_u^- and B_c^- mesons. This process is shown in Figure 2.9 for the leptonic decay of the B_u^- . However, this effect has no impact on the B_u^- lifetime because the coupling between the b and u quarks is so small. As with the Ξ_b and Ω_b baryons, the B_c^- meson is so rare that it is also ignored in this discussion.

When all of these non-spectator effects are considered together we expect the b hadron lifetimes to differ by 10–20% with the following hierarchy:

$$\tau(B_u^+) > \tau(B_s^0) \geq \tau(B_d^0) > \tau(\Lambda_b^0). \quad (2.23)$$

As indicated above, there is still much debate between theorists about the actual

magnitude of these effects on the b hadron states, and the current experimental measurements of the individual b hadron lifetimes, which are presented in Chapter 6, are not yet precise enough to conclusively determine the exact lifetime differences. Therefore, in Chapter 6 we will use the spectator model calculation of the b hadron semileptonic decay width, given in Equation 2.14, with the measured average b hadron lifetime presented in this dissertation to obtain the CKM matrix element $|V_{cb}|$.

Chapter 3

The OPAL Experiment

The data used in this analysis were recorded by the OPAL detector at LEP. Section 3.1 gives a brief description of the CERN laboratory and the LEP collider. The OPAL subdetectors, the data acquisition system, and the offline analysis software are then discussed in detail in Sections 3.2, 3.3, and 3.4 respectively. A description of the calibration of OPAL and its performance concludes this chapter.

3.1 CERN and LEP

CERN, the European Laboratory for Particle Physics, straddles the border between France and Switzerland, near Geneva. The laboratory was established during the 1950s and is currently operated and funded by its 18 member states: Austria, Belgium, the Czech and Slovak Federal Republic, Denmark, Finland, France, Germany, Greece, Hungary, Italy, the Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. In 1992, CERN employed 3 078 full-time staff [40] and provided research facilities to about 6 000 users, approximately 1 500 of these from non-member states such as Canada, the United States, Japan, and China. CERN's total budget for 1992 was a staggering 950 million Swiss francs.

The CERN accelerator complex is shown in Figure 3.1. The jagged line repre-

Figure 3.1: The CERN accelerator complex.

sents the France-Switzerland border running through the synchrotrons of the complex, which are represented by the circles. The largest synchrotron is the Large Electron-Positron (LEP) storage ring. The electrons and positrons circulating in LEP are accelerated in several stages by different machines, because each machine can only operate over a limited range of beam energy. The first stage of acceleration occurs in the electron linac (LIL), where electrons are accelerated to 200 MeV and directed into a target to produce positrons. The positrons are then accelerated up to 600 MeV in the second stage of the LIL, and are injected into the Electron-Positron Accumulator (EPA). The EPA stores and compresses the positron beam by synchrotron damping. About 1 000 positron bursts are needed to accumulate $\sim 2 \times 10^{11}$ positrons in 8 bunches. The beam is then injected into the Proton Synchrotron (PS), which accelerates it to 3.5 GeV. In turn, the Proton Synchrotron injects the beam into the Super Proton Synchrotron (SPS), which increases the energy to 20 GeV.¹ Finally, the

¹ As the name implies, the PS and SPS were originally used to accelerate protons for fixed target

positron beam is injected clockwise into LEP in four bunches. The r.m.s. bunch length in LEP is about 1.2 cm. The electrons go through the same acceleration and injection sequence as the positrons, and are injected counterclockwise into LEP. Electrons and positrons are injected into LEP alternately until the currents reach about 1.5 mA per beam (the design parameters called for 3 mA per beam). At this point the beams are accelerated to about 45 GeV.

The LEP ring is divided into eight straight sections, where the experimental halls and radio frequency (RF) cavities are located, separated by curved sections that are instrumented with dipole magnets to steer the beams around the ring. Quadrupole magnets are located all around the ring to focus the beams. The total path length traced by the beams is 26.658 km. With four bunches per beam there are eight collision points located in the middle of each straight section, but only four of these are equipped with detectors: ALEPH, DELPHI, L3, and OPAL. Their location on the LEP ring is indicated in Figure 3.1. In the unused straight sections the beams are separated by electrostatic separators to increase the beam lifetime, which is typically 10 hours on a good day. The RF cavities are located in the two straight sections near the L3 and OPAL experimental halls.

The reaction rate observed by each experiment at LEP is given by

$$R = \mathcal{L}\sigma, \quad (3.1)$$

where $\mathcal{L}$ is the luminosity (in units of $\text{cm}^{-2}\text{s}^{-1}$), and σ is the interaction cross-section (in units of cm^2). The luminosity depends on the beam characteristics and is given by

$$\mathcal{L} = fn \frac{N_1 N_2}{A}, \quad (3.2)$$

where f is the revolution frequency, n is the number of bunches per beam, N_1 and N_2 are the number of particles in the bunches of each beam, and A is the cross-sectional

experiments, and protons and antiprotons for colliding beam experiments. The fixed target work still continues when these machines are not being used as injectors for LEP.

Figure 3.2: The integrated luminosity recorded by OPAL during 1990 and 1991.

area of the beams, assuming the two beams overlap completely. The design luminosity of LEP was $1.6 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$, although typically only about 25% to 50% of this value was achieved during the 1990 and 1991 runs.² In practice, the luminosity is determined by measuring the rate of small-angle elastic scatters of electrons and positrons, called *Bhabha scattering*. The cross-section for this process can be calculated very precisely since it is a pure electroweak process. The luminosity is then determined using Equation 3.1, the measured rate, and the calculated cross-section (corrected for the detector acceptance).

The size of a data sample is often quoted in terms of the integrated luminosity. This is simply the integral of $\mathcal{L}$ over time and is given in units of barns (b), where $1 \text{ b} = 10^{-24} \text{ cm}^2$. The integrated luminosity recorded by OPAL for the 1990 and 1991 runs is shown in Figure 3.2 as a function of the number of days. The total for the two runs is 20.6 pb^{-1} . LEP delivered $\sim 10\%$ more luminosity to OPAL which was lost because of data acquisition problems, hardware problems, and readout deadtime.

²The design luminosity was reached this year when LEP was operating with 8 bunches per beam.

During 1990 about 53% of the luminosity was delivered on the Z^0 peak. The rest was divided between six other center-of-mass energies: three below the peak at 1 GeV intervals and three above the peak at 1 GeV intervals. The same procedure was applied in 1991, and in this case 66% of the luminosity was delivered on the Z^0 peak. The corresponding data points for the hadronic cross-section are shown in Figure 2.2.

3.2 The OPAL Detector

The Omni Purpose Apparatus for LEP (OPAL) is a general purpose detector [41] used to analyze the decays of the Z^0 produced in e^+e^- collisions. The completion of the OPAL Technical Proposal on May 16, 1983, marked the beginning of the prototyping and building phase of OPAL, which ended on August 13, 1989, when OPAL recorded the first Z^0 decay at LEP. Since the collaboration was first formed over ten years ago, it has grown to include over 300 members from 31 institutions from Canada, France, Germany, Israel, Italy, Japan, Switzerland, the United Kingdom, and the United States. The OPAL author list appearing with the publication of the work presented in this dissertation is shown in Appendix A.

OPAL was designed to be a versatile detector with nearly complete solid angle coverage. It was built using the conventional technologies available at that time, whereas the other LEP detectors incorporated novel technologies such as ring-imaging Čerenkov counters, for example, into their design. The main elements of the OPAL detector include a central tracking system in a solenoidal coil, a time-of-flight detector, an electromagnetic calorimeter system, a hadronic calorimeter system, a muon detection system, and forward detectors for the luminosity measurement. These components are further subdivided into the subdetector elements listed in Table 3.1. The two-letter symbols in this table are used throughout this dissertation to refer to the corresponding component of OPAL. To a large extent, the subdetectors were designed and tested independently at institutes away from CERN, and were assembled to form the complete detector at CERN. The readout electronics and high-voltage systems

for all subdetectors are housed in several huts located around the detector in the experimental hall. OPAL had a total of $\sim 179\,000$ readout channels in 1991.

detector system	subdetector		channels	Ref.
central tracking system	SI	microvertex detector	15 725	[41, 42]
	CV	vertex chamber	648	[41, 43]
	CJ	jet chamber	7 680	[41, 44]
	CZ	z chambers	2 304	[41, 45]
time-of-flight	TB	time-of-flight detector	320	[41]
electromagnetic calorimeter	EB	barrel calorimeter	9 440	[41]
	PB	barrel presampler	21 504	[41, 46]
	EE	endcap calorimeter	2 264	[41]
	PE	endcap presampler	6 080	[41, 47]
hadronic calorimeter	HT	towers	1 696	[41]
	HS	strips	56 146	[41]
	HP	pole tip towers	336	[41, 48]
		pole tip strips	10 240	
muon detector	MB	barrel	1 320	[41]
	ME	endcap	42 496	[41]
forward detectors	FD	tube chambers	768	[41]
		drift chambers	192	
		calorimeters/scintillators	216	
trigger system	TR	global trigger system		[41, 49]
	TT	track trigger system		[41, 50]

Table 3.1: Summary of OPAL subdetectors and the number of channels per subdetector. Also included are the trigger subsystems, TR and TT. The two letter abbreviations indicate the division of the subdetectors in the data acquisition system, described in the text.

A cross-section of OPAL is shown in Figure 3.3. In this figure the beams are perpendicular to the page and pass through the center of the concentric circles, which denote the boundaries of the OPAL subdetectors. A cross-section of OPAL parallel to the beam axis is shown in Figure 3.4. The electron and positron bunches are synchronized to collide in the center of the OPAL detector, referred to as the *interaction region*. The LEP beam at the OPAL interaction region, and also the OPAL solenoid

Figure 3.3: Cross-section of the OPAL detector.

Figure 3.4: Side view of the OPAL detector.

axis, are inclined at an angle of 13.9 mrad relative to the horizontal plane. The coordinate system is defined so that the z axis is in the direction of the electron beam, the x axis is horizontal and points approximately towards the center of the LEP ring, and the y axis is nearly vertical and points upward. The polar and azimuthal angles, θ and ϕ , are defined with respect to the $+z$ and $+x$ axes respectively. We refer to the x - y plane as the r - ϕ plane, and refer to the plane parallel to the beam axis as the r - z plane. Each component of OPAL is described below.

3.2.1 The Central Detector

A cross-section of the central detector parallel to the beam axis is shown in Figure 3.5. The vertex chamber, the jet chamber, and the z chambers are the three active components of the central detector. These devices are called drift chambers and use a gaseous mixture of 88.2% argon, 9.8% methane, and 2.0% isobutane at a pressure of 4 bar as the detection medium. This gas mixture is referred to as *OPAL gas*. The pressure vessel containing the central detector is formed by the beam pipe, an aluminum pressure vessel, and two aluminum pressure bells.

Drift chambers are sensitive to charged particles that pass through their active volume. The gas molecules are ionized along the path of a charged particle, and electric fields within the chambers drift the free electrons to anode wires where they initiate an ionization avalanche and produce a detectable signal, called a *hit*. The electrons drift through a gas with a well determined average velocity. Therefore, from the time the particle passed through the detector and the time the signal was detected on the anode wire, we can deduce how far the particle passed from the anode wire. Drift chambers usually have many anode wires within their active volume, thus providing enough measurements to reconstruct a charged particle's path, or *track*, through the detector.

The vertex chamber measures the spatial coordinates of charged particle tracks closest to the interaction region, and therefore determines the spatial resolution of

Figure 3.5: Cross-section of the OPAL central detector.

tracks as they are projected back to the interaction region. A good spatial resolution is required near the interaction region in order to resolve the decay vertices of short-lived particles such as the b hadrons. The jet chamber measures the charged particle momenta from the track curvature induced by the magnetic field, and is used to identify particles by measuring their average ionization energy loss per unit length, dE/dx . Finally, the z chambers measure the z coordinates of the tracks as they leave the jet chamber in order to improve both the polar angle and the invariant mass resolutions. The vertex chamber and the jet chamber provide the information necessary to construct the observable used to measure the average b hadron lifetime. The vertex chamber determines the precision of this measurement, and is therefore described in more detail than the other detectors in the following sections. The properties of the central detector are summarized in Table 3.2.

tracking device	vertex detector	jet chamber	z chambers
wire organization	36 axial sectors 12 wires/sector, 36 stereo sectors (4°) 6 wires/sector	24 axial sectors 159 wires/sector	24 chambers 8 cells/chamber 6 wires/cell
wire spacing (mm) wire stagger (μm)	5.3 (axial) and 5.0 (stereo) ± 41	10.0 ± 100	4.0 ± 250
max. drift distance (cm) drift velocity ($\mu\text{m}/\text{ns}$) max. drift time (μs) Lorentz angle (degrees)	1.4 39.5 0.35 5.5	25 53 4.7 20	25 53 4.7 0
gas	argon(88.2%), methane(9.8%), isobutane(2.0%) at 4 bar		
gas gain	2×10^4	10^4	$2 - 5 \times 10^4$
angular coverage	$23^\circ \leq \theta \leq 157^\circ$ 92% of 4π for axial+stereo $18^\circ \leq \theta \leq 162^\circ$ 95% of 4π for axial	$43^\circ \leq \theta \leq 137^\circ$ 73% of 4π for 159 wires $11^\circ \leq \theta \leq 169^\circ$ 98% of 4π for ≥ 8 wires	$44^\circ \leq \theta \leq 136^\circ$ 94% in ϕ 67% of 4π
coordinate determination	ϕ z ΔT and stereo wires	drift time charge division	charge division drift time
spatial resolution	ϕ z 4 cm (ΔT), 1.7 mm (stereo)	130 μm 10 cm	15 nm 100-350 μm
second hit resolution	2.0 mm	2.5 mm	2.5 mm

Table 3.2: Properties of the tracking chambers.

The Magnet Coil and Beam Pipe

The magnet coil is made with a hollow, water-cooled, aluminum conductor fused together with a glass-epoxy. It surrounds the pressure vessel, which provides mechanical support, and has a mean diameter of 4.36 m. The coil operates with a current of about 7 000 A, providing an axial magnetic field of 0.435 T. The field was measured to be uniform in the central detector volume to within $\pm 0.5\%$. The solenoid thickness, including the pressure vessel, corresponds to about 1.7 radiation lengths.³

The OPAL beam pipe consists of a 0.1 mm thick aluminum tube onto which are epoxied a series of overlapping layers of carbon fiber. The aluminum skin is required for continuity of the conducting surface seen by the fields of the LEP beams, and provides electromagnetic shielding for the detector. It is also needed to provide a vacuum tight surface with minimum outgassing. The inner radius of the beam pipe is 78 mm and its thickness is 1.4 mm, including the thickness of the aluminum tube. This material represents a total of 0.66% of a radiation length.

The beam pipe described above was present in OPAL up to the end of the 1990 run. For the 1991 run, a high-precision silicon microvertex detector surrounding a 5.3 cm radius, 1.1 mm thick beryllium-composite beam pipe was inserted in OPAL. The old beam pipe was also replaced by an 8.0 cm radius carbon fiber pipe of 2.0 mm thickness. The total thickness of material replacing the old beam pipe was about 2.4% of a radiation length, an increase by a factor of 3.6.

The analysis presented in this dissertation does not include information from the silicon detector because the tracking precision of the wire vertex chamber is not an important limitation, and because the silicon detector was only operational for about 2/3 of the 1991 data taking. It does, however, include the effects of the additional material in the inner detector in the 1991 run.

³A radiation length is the length scale used to characterize the development of high energy electromagnetic interactions in matter. One radiation length is the mean distance over which a high energy electron loses all but 1/e of its energy by bremsstrahlung.

The Vertex Chamber

The vertex chamber [41, 43] is a 1 m long cylindrical drift chamber with inner and outer radii of 88 and 235 mm respectively, that surrounds the beam pipe. The volume of the chamber between radii of 90 and 170 mm is divided into 36 axial sectors with 12 anode wires each, stretched parallel to the beam, and the remaining volume is divided into 36 small angle (4°) stereo sectors with 6 anode wires each. The wires are strung between two 32 mm thick G10 fiberglass end-plates that are held 1 m apart by a single 1.5 mm thick carbon fiber tube located at a radius of 235 mm. The preamplifiers and high voltage distribution components are mounted onto the G10 end-plates, and are enclosed within a gas-tight container defined by two aluminum cylinders and an aluminum bulkhead. The cylinders are mounted onto the G10 end-plates at the inner and outer radii of the vertex chamber, and are about 345 mm long. High voltage and signal cable feedthroughs are inserted into the bulkhead. The vertex chamber is mechanically supported from the inner edge of the jet chamber and is mechanically independent of the beam pipe.

Figure 3.6 shows the wire layout for 3 axial cells and 3 stereo cells at one end-plate. In this figure, the beam is perpendicular to the page and is located at the intersection point of the wire planes. The cell geometry is determined by the topology of Z^0 decays and by the desired spatial resolution of the drift chamber. The $20\ \mu\text{m}$ diameter gold plated W-Rh anode wires are grouped in planes, and are separated by $200\ \mu\text{m}$ diameter gold plated Cu-Be potential wires that help shape the electric fields on the anode wire surfaces. The 12 anode wires in the axial cells are spaced 5.3 mm apart from radii of 103 to 162 mm. The 6 anode wires in the stereo cells are spaced 5.0 mm apart from radii of 188 to 213 mm. The cells are *bidirectional*, meaning that the electrons drift to the anode wires from both sides of the anode wire planes. For this reason, the anode wires are offset into the cell by a $\pm 41\ \mu\text{m}$ alternating stagger relative to the potential wire plane in order to determine on which side of the cell the track passed (called the *left-right drift ambiguity*). This staggering has been exaggerated in Figure 3.6. There

is an additional anode wire bow of $\sim 50 \mu\text{m}$ due to the electrostatic forces when the chamber voltages are applied. The cathode planes are composed of $125 \mu\text{m}$ diameter Cu-Be wires spaced 1 mm apart.

Figure 3.6: Wire layout of the OPAL vertex chamber.

The electrostatic field conditions within a cell are defined by the anode surface fields required to operate the chamber with a high detection efficiency, and by the drift fields. The electrostatic fields are established by applying the appropriate voltages to the cathode and potential wires; the anode wires are fixed at ground potential. The voltages are chosen to provide a uniform electric field over most of the drift cell with the field lines perpendicular to the anode wire planes. The field lines within

2–3 mm of the anode wire planes converge on the anode wires (some also converge on the potential wires), producing a very non-linear field near the anode wires. This field configuration is achieved by having the cathode wire potentials increase linearly with radius using a voltage divider network formed with resistors connecting adjacent cathode wires. The potential wires within an anode wire plane are all kept at the same voltage to ensure a uniform gain for all anode wires. The wires of the field shaping rings are biased to improve the field linearity near the edges of the cells. The foils are also kept at fixed potentials and serve to terminate the electrostatic fields.

The vertex chamber is operated with anode surface fields of 360 kV/cm, giving a single hit efficiency of $\sim 97\%$. This value is relatively constant for higher surface fields but decreases quickly below ~ 350 kV/cm, dropping to about 85% at 340 kV/cm. The gain at the operating surface field is $\sim 2 \times 10^4$, meaning that every electron arriving at the anode wire will, on average, produce an avalanche of about 2×10^4 ion pairs near the anode wire surface. Higher surface fields are not desirable because they require higher voltages, thus increasing the chance of a mechanical failure in the detector. In addition, higher voltages also give a higher gain, which accelerates the ageing process of drift chambers, whereby the anodes and/or cathodes become coated with deposits produced by the dissociation and polymerization of the gas molecules during the avalanche process. The deposits accumulate over time, changing the electrostatic properties of the chamber, and can eventually cause serious damage [51]. High counting rates have the same detrimental effect on the chamber as an increase in gain. For this reason, LEP constantly strives to maintain the background radiation level in OPAL as low as possible.

The vertex chamber is operated with a drift field of 2.5 kV/cm, resulting in a drift velocity of $39.5 \mu\text{m/ns}$ in OPAL gas. At this operating point the drift velocity decreases with increasing field with a slope of about $-6.4 (\mu\text{m/ns})/(\text{kV/cm})$. This is why a uniform drift field is essential—it results in a constant electron drift velocity across the cell, thus simplifying the calibration of the detector. The choice of drift

field, and hence drift velocity, directly affects the drift distance resolution in two ways. Firstly, the drift distance resolution improves with a slower drift velocity because the diffusion of the electrons as they drift to the anode wires decreases with velocity. Secondly, a significant contribution to the drift distance resolution comes from the timing resolution of the electronics, described below, and a slower drift velocity reduces this contribution. However, in this operating region an increase in drift field of almost 20% results in only a 1.6% improvement in the resolution. Therefore, as high a drift field as possible was chosen with the chamber stability and safety as the limiting factor. The drift path followed by the electrons is not parallel to the electric field lines, but is inclined at an angle of $\sim 5.5^\circ$ (called the *Lorentz angle*) due to the presence of the magnetic field.

Both ends of each anode wire are connected at the chamber end-plates to preamplifiers which transmit the signals down about 40 m of cable to the data acquisition electronics, where the signals are further amplified and fed into a constant fraction discriminator. The drift time measurement is obtained by combining the outputs from the two wire-ends in a mean-timer, which results in the measurement being independent of the position of the hit along the wire. This measurement is digitized and has an intrinsic bin width of 0.67 ns, or 26 μm when converted to a drift distance. The signals from the two wire ends are also passed to a unit that measures and digitizes the difference in their time of arrival with a precision of about 0.1 ns, which allows the z coordinate of the hit along the wire to be determined to ~ 4 cm.

In Z^0 decays many tracks can pass through a vertex chamber cell, so the electronics have the capability of recording up to 16 hits per wire per event. The electronics were carefully designed to shorten the fall time of the pulses to reduce the deadtime to 40–50 ns. This corresponds to a two-hit separation of about 2 mm, meaning that hits, or even entire tracks, are lost if particles pass within 2 mm of each other in a vertex chamber cell.

The drift distance resolution in the vertex chamber is $\sim 50 \mu\text{m}$ over most of the drift

space, and increases to over $100\ \mu\text{m}$ in the region near the anode plane corresponding to about 10% of the drift space. In addition, when multiple hits are recorded on an anode wire, all hits arriving after the first hit, collectively called *second hits*, will have significantly degraded resolutions of typically $90\ \mu\text{m}$ or worse. This is a result of the statistical fluctuations of the pulse height and shape which cause second hits arriving on the tails of previous hits to be discriminated at different points of their rising edge, thereby introducing time slewing effects. In addition, the hit positions for these tracks are found to be systematically biased by these timing errors. This point will be discussed again in Chapter 4 when the track selection requirements for this analysis are described.

The Jet Chamber

Surrounding the vertex chamber is a large volume jet chamber [41, 44]. The chamber has an inner radius of 0.25 m, a 1.85 m outer radius, is ~ 4 m long, and is divided into 24 azimuthal sectors. The geometry of each sector is very similar to the axial cell geometry of the vertex chamber, except that each anode wire plane consists of 159 wires spaced 1.0 cm apart. The anode wires are located between radii of 255 and 1835 mm, and are staggered by $\pm 100\ \mu\text{m}$ alternately to the left and right side of the plane defined by the potential wires. The wires are stretched between two conical end-plates that are held apart by a set of 24 hollow aluminum panels located at the outer radius of the chamber. There is no inner support tube, although, as indicated earlier, the vertex chamber is mounted onto the inner radius of the jet chamber end-plates. The jet chamber anode wires are also parallel to the beam.

Both ends of each anode wire are connected to preamplifiers mounted onto the end-plates of the jet chamber. The signals are transmitted to the readout electronics which digitizes them using 100 MHz flash analogue-to-digital converters (FADCs). The high speed sampling greatly improves the drift time measurement and the double hit resolution, and allows a good measurement of the total charge generated in a

hit. Each of the wires provides three-dimensional coordinates calculated from the wire position, from the drift time measurement in the x - y plane, and from a charge division measurement in the z direction. The total charge on each wire is also used to determine the mean ionization energy loss, dE/dx .

The chamber is operated with a gas gain of $\sim 10^4$ and a drift field of 890 V/cm. This operating point represents an optimization of the competing requirements for good particle identification through the dE/dx measurement, which requires a low gain to minimize saturation effects,⁴ and good z resolution through charge division, which requires a high gain to maximize the charge collected at the anodes, and hence the signal to noise ratio.

The average drift distance resolution of the jet chamber is 130 μm , and the average resolution in z is ~ 10 cm. The momentum resolution for charged particles in the region $|\cos \theta| < 0.7$ is

$$(\sigma_p/p)_{xy}^2 = 0.02^2 + (0.0015 \cdot p_{xy})^2, \quad (3.3)$$

where p_{xy} is the momentum in the x - y plane in GeV/ c . The constant in this expression is the contribution from multiple Coulomb scattering.

The identification of particles via the energy loss measurement, dE/dx , is a strong point of OPAL and is described in detail in Reference [52]. The energy loss of charged particles in matter is described by the Bethe-Bloch equation [53], which is a universal function of $\beta\gamma$ for all particle masses.⁵ The energy loss as a function of momentum shows a characteristic $1/\beta^2$ decrease at low $\beta\gamma$, reaches a minimum around $\beta\gamma = 4$, and continues with a logarithmic rise until it saturates. The region where the energy loss increases is called the *relativistic rise region*, and the region where it saturates is called the *Fermi plateau*. The mass of a particle can therefore be determined

⁴Saturation effects occur when the ions produced by the leading electrons in an ionization avalanche partly screen the incoming electrons from the full electric field, thus reducing the gas gain and the total charge produced in the avalanche. The effect increases with gain and has to be corrected for, adding to the uncertainty in the dE/dx measurement.

⁵Here $\beta = v/c$, where v is the particle speed and c is the speed of light, and $\gamma = 1/\sqrt{1 - \beta^2}$.

Figure 3.7: The average energy loss, dE/dx , in the jet chamber gas for electrons (e), pions (π), kaons (K), and protons (p) in hadronic Z^0 decays, and muons (μ) in Z^0 decays into $\mu^+\mu^-$. At least 130 samples were required for each dE/dx measurement. The theoretical predictions are also shown.

by measuring its momentum and average energy loss. Detailed models have been developed which allow the calculation of particle energy losses for many gasses [54]. The curves in Figure 3.7 show the predictions for five particle species in OPAL gas.

The energy loss distribution has a large inherent width due to the primary ionization statistics. Therefore, it is necessary to sample the energy loss frequently along a particle's track to determine the mean energy loss with reasonable accuracy. This is particularly important at higher momentum where the difference in energy loss between the particle species is small, as can be seen in Figure 3.7. For example, the difference in mean dE/dx between 10 GeV/c pions and electrons is only about 10%. Reduction of the fluctuations of the energy loss can be achieved by increasing the gas pressure, but this lowers the Fermi plateau and reduces the energy loss differences be-

tween particle species. Extensive studies have shown [52] that the particle separation power, expressed in terms of the resolution divided by the difference in energy loss between particles, is optimized at a pressure of 3 to 4 bar for commonly used drift chamber gases. This is the main reason why OPAL operates with a gas pressure of 4 bar.

The dE/dx distributions for all charged tracks produced in hadronic Z^0 decays and $Z^0 \rightarrow \mu^+ \mu^-$ decays are also shown in Figure 3.7. The dE/dx resolution for minimum ionizing pions in jets was found to be $\sigma(dE/dx)/(dE/dx) = 3.8\%$ for tracks with at least 130 good dE/dx samples. The power of the dE/dx measurement in electron identification will be discussed in Chapter 4.

The Z Chambers

The z chambers [41, 45] consist of 24 drift chambers, each 4 m long, 50 cm wide and 59 mm thick. The chambers are mounted onto the jet chamber support structure in the hollow aluminum panels described above, forming a 4 m long barrel with a diameter of 3.85 m. Each z chamber is divided in z into 8 bidirectional cells, resulting in a maximum drift distance of 25 cm in the z direction. The chambers use the same gas as the jet chamber, with the gas flowing in at one end and out the other.

Each cell of each z chamber has six 50 cm long anode wires spaced 4 mm apart at increasing radii, separated by potential wires. The anode wires are staggered at $\pm 250 \mu\text{m}$ from the potential wire plane to resolve the left-right drift ambiguity. The anode and potential wires are mounted onto a separate G10 window-frame structure called the anode module. Parallel to the potential wire plane, and ± 5 mm from it, are two planes of grid wires mounted onto the edges of the anode module. These grid wires provide a clear separation between the anode and drift regions, maintain a parallel and uniform drift field right up to the grid wire plane, and aid in establishing the required electric field at the anode wires. The signals from both ends of each anode wire are amplified and then digitized by a FADC system.

The drift field is established by a series of aluminum field-shaping electrodes spaced at 5 mm intervals. These window-frame shaped electrodes extend 5 mm into the chamber and are 0.5 mm thick. Thin rectangular cathode plates, perforated with small holes to permit free gas flow between cells, define the cell boundaries in the z direction. The cathodes are at the maximum drift potential, and a voltage divider network ensures that the potential applied to each field shaping electrode decreases linearly from the cathode to the grid wire planes. This electrode configuration forms a narrow drift slot 2.9 cm thick, 50 cm wide, and 25 cm long on both sides of the anode wire planes. The chambers operate with a drift field of 800 V/cm.

The intrinsic drift distance resolution is about $100\ \mu\text{m}$ at the shortest drift distance, and increases to about $200\ \mu\text{m}$ at the longest drift distance. The absolute resolution is determined by the precision of the survey of the anode modules and z chamber positions, and is expected to be about $300\ \mu\text{m}$. The total charge measured at each end of the anode wires is used to calculate the position of hits along the wires by the charge division technique. The resulting r - ϕ resolution is about 1.5 cm.

3.2.2 The Time-of-Flight Counters

The time-of-flight (TOF) system [41] is used for charged particle identification in the momentum range 0.6–2.5 GeV/ c by measuring the time of flight of charged particles from the interaction region, and is a very powerful tool to reject cosmic rays.

The TOF system consists of 160 scintillation counters forming a barrel of mean radius 2.36 m surrounding and coaxial with the magnet coil. Each counter is 6.84 m long and has a trapezoidal cross-section of 4.5 cm thickness and 8.9 to 9.1 cm width. The light is collected at both ends of each counter via 30 cm long plexiglass light guides glued directly to phototubes. The phototube signals are integrated to determine the total charge, and are fed into discriminators to determine the time of arrival of each signal.

The measured time of flight for muons from the decay $Z^0 \rightarrow \mu^+\mu^-$ compared

to their expected flight time has a resolution of 460 ps. The comparison of the z measurement from the TOF system with the z measurement from the barrel lead glass calorimeter, described below, indicates a z resolution of ~ 7.5 cm.

3.2.3 The Electromagnetic Calorimeter

The electromagnetic calorimeter [41] detects electrons and photons with energies ranging from tens of MeV to 100 GeV, measures their energies and positions, and provides electron-hadron discrimination in conjunction with the central tracking system. It is a total absorption calorimeter covering 98% of the solid angle, and consists of three large overlapping assemblies of lead glass blocks: the barrel and two endcaps. The energy measurement is derived from the Čerenkov photons⁶ generated by the electrons and positrons produced in electromagnetic showers initiated by the particles entering the lead glass.

The barrel lead glass calorimeter consists of a cylindrical array of 9440 lead glass blocks, with an inner radius of 2.455 m, surrounding the TOF system. It has full azimuthal coverage up to $|\cos\theta| < 0.82$. Each block has dimensions $\sim 10 \times \sim 10 \times 37.0$ cm³, with a depth corresponding to about 24.6 radiation lengths. The longitudinal axis of each block, when projected to the interaction region, is tangent to an imaginary cylinder of 3 cm radius and 30 cm length surrounding the interaction region. This pointing geometry minimizes the probability that a particle originating from the interaction region traverses more than one block, and prevents neutral particles from escaping through the gaps between the blocks. Each lead glass block is instrumented with a magnetic field tolerant phototube. The intrinsic energy resolution is $\sigma_E/E \simeq 0.2\% + 6.3\%/\sqrt{E}$, although this is substantially degraded by the presence of the material in front of the lead glass (by up to a factor of 2).

⁶Čerenkov radiation is emitted as the result of the coherent response of a medium to the passage of a relativistic charged particle. The speed of the particle must be larger than the phase velocity of the electromagnetic fields of frequency ω in that medium in order to have emission of Čerenkov radiation at that frequency.

The endcap electromagnetic calorimeter consists of two dome-shaped arrays, each of 1132 lead glass blocks, located between the pressure bells and the pole tip hadron calorimeter, which is described below. It has full azimuthal coverage in the region $0.81 < |\cos \theta| < 0.98$. Due to tight geometrical constraints, the lead glass blocks are coaxial with the beam axis and were manufactured in three lengths: 38, 42, and 52 cm. They are arranged to give a total depth of at least 20.5 radiation lengths (typically 22 radiation lengths) for all particles emerging from the interaction region. Each block is instrumented with a novel single stage multiplier known as a *vacuum photo-triode*, which was developed in conjunction with industry to be able to operate in the full axial magnetic field of the magnet. The energy resolution was $\sigma_E/E \sim 5\%/\sqrt{E}$ at low energy.

Since there are about two radiation lengths of material in front of the lead glass, due mostly to the coil and pressure vessel, most electromagnetic showers are initiated before the lead glass itself. Presampling devices are therefore installed in both the barrel and endcap regions immediately in front of the lead glass to measure the positions of the showers and to sample their energies. Presampling improves the electron-hadron discrimination, and also improves the electromagnetic energy resolution on a shower by shower basis. This is possible because the observed presampler pulse height is approximately proportional to the number of charged shower particles entering the device, which is itself roughly proportional to the energy deposited in the material in front of the presampler.

In the barrel region, the presampler [41, 46] consists of a cylinder of 6.623 m long tube chambers at a radius of 2.388 m, located between the TOF system and the barrel lead glass calorimeter. There are two layers of tubes arranged axially, each tube having a square cross-section of 9.6 mm². The two layers are offset by half a tube to avoid inefficiencies associated with the tube walls. These tubes operate with a gas mixture of 68% carbon dioxide and 32% n-pentane in the limited streamer mode. The positions of ionization showers in the tubes are determined from the charge induced on 1 cm

wide cathode strips placed on each side of each layer of tubes at an angle of 45° , such that the cathode strips associated with a layer of tubes are orthogonal to each other. The cathode strip information provides a positional resolution perpendicular to the electromagnetic showers ranging from 6 to 4 mm as the energy increases from 6 to 50 GeV. The charge collected on each wire is measured at both ends to allow a position measurement along the wire by charge division. A z resolution of about 10 cm is achieved with this method, and aids significantly in the removal of ambiguities during data analysis.

The endcap presampler [41, 47] is an umbrella type arrangement of thin multiwire chambers located between the pressure bells and the endcap lead glass calorimeters. It has full azimuthal coverage in the region $0.83 < |\cos \theta| < 0.95$. These chambers operate with a gas mixture of 55% carbon dioxide and 45% n-pentane in a high gain mode. The positional information is also extracted from cathode strips and pads. Resolutions comparable to the barrel presampler are achieved.

3.2.4 The Hadron Calorimeter

The hadron calorimeter [41] measures the energy of hadronic showers emerging from the electromagnetic calorimeter, assists in the identification of muons, and serves as the return yoke for the magnetic field of the coil. The calorimeter is constructed in three parts: the barrel, the endcaps, and the pole tips. The 10 m long barrel surrounds the electromagnetic calorimeter, is 3.4 to 4.4 m in radius, and covers the angular region $|\cos \theta| < 0.81$. It consists of eight 10 cm thick slabs of iron, with two extra slabs of iron covering the central 4 m section of the barrel to increase the amount of material seen by particles emerging in this region. The barrel is closed at each end by doughnut-shaped endcaps which consist of seven 10 cm thick slabs of iron covering the angular region $0.81 < |\cos \theta| < 0.91$. The pole tips [41, 48] are mounted between the endcaps of the hadron and electromagnetic calorimeters, and consist of nine 8 cm thick slabs of iron covering the region $0.91 < |\cos \theta| < 0.99$.

The slabs of iron in the barrel and endcaps are separated by 2.5 and 3.5 cm gaps respectively. The gaps are filled with planes of limited streamer tubes with the anode wires in adjacent tubes spaced 1.0 cm apart. The tubes are parallel to the beam in the barrel, are horizontal in the endcaps, and operate with a gas mixture of 75% isobutane and 25% argon. The signals are induced on large area pads on one side of the streamer tubes, typically 50 cm $\times$ 50 cm, and on the other side they are induced on 4 mm wide aluminum strips running parallel to the anode wires. The signals from the wires are only used for monitoring purposes. Layers of pads are grouped together to form towers, dividing the solid angle into 976 equal elements radiating from the interaction region.

The slabs of iron in the pole tips are separated by 1.0 cm gaps that are filled with thin multiwire chambers operating with a 55% carbon dioxide, 45% n-pentane gas mixture in a high gain mode. The chambers have cathode pads on one side, with areas of typically 500 cm², and 2.5 cm wide strips on the other side which fan out radially such that they are almost perpendicular to the anode wires. The corresponding pads from the ten layers of chambers are also grouped together to form towers pointing to the interaction region.

In this analysis, the hadron calorimeter is only useful as an absorber to stop all particles except muons from escaping the detector. The total amount of material seen by particles emanating from the interaction region exceeds 8 interaction lengths⁷ in the region $|\cos\theta| < 0.97$, including all the iron of the hadron calorimeter. The amount of material in the region $0.7 < |\cos\theta| < 0.9$ decreases to about 4 interaction lengths near the bottom portion of the OPAL detector, where all the support structures for the central detector and calorimeters are located. The 8 interaction lengths of material reduces the probability that a pion will not interact strongly in the material (sneakthrough) to less than 0.1% for all pion momenta. The probability that secondary particles produced during a hadronic shower leak out of the hadron

⁷An interaction length is the length scale appropriate to describe the development of a hadronic shower initiated by a high energy hadron entering the material.

calorimeter (punchthrough) is $\sim 0.8\%$ at 5 GeV/c, and increases approximately linearly up to $\sim 2\%$ at 40 GeV/c. A muon requires a momentum of at least 3 GeV/c to pass through all the material of OPAL.

3.2.5 The Muon Chambers

The muon detector [41] is designed to identify muons in the presence of a background of hadrons. It consists of a barrel section and two endcap sections, which combined cover 93% of the solid angle. The barrel consists of four layers of 10 m long, 1.2 m wide drift chambers forming a barrel of about 5 m radius. It covers the angular region $|\cos \theta| < 0.68$ with all four layers, and the region $0.68 < |\cos \theta| < 0.72$ with at least one layer. Each chamber is divided into two bidirectional drift cells with one anode wire per cell that runs the full length of the chamber in z , and which is parallel to the beam axis. These chambers operate with a 90% argon, 10% ethane gas mixture. The r - ϕ coordinate is measured from the drift time, and diamond-shaped cathode pads [55] running the whole length of the chamber next to each wire measure the z coordinate of each hit. The endcaps consist of 12 m $\times$ 12 m panels perpendicular to the beam axis. They are instrumented with four layers of limited streamer tubes and cover the angular region $\sim 0.67 < |\cos \theta| < \sim 0.98$. These tubes operate with a gas mixture of 75% isobutane and 25% argon. Two layers of streamer tubes run vertically, and the other two layers run horizontally. The position of the ionization avalanches on each wire are determined from 8 mm wide aluminum cathode strips running parallel to the streamer tubes on one side, and perpendicular to the tubes on the other side.

The identification of muons relies on extrapolating tracks seen in the central tracking system out to the muon chambers, taking into account the energy loss and multiple Coulomb scattering that occurs in the material preceding the muon chambers. A track segment reconstructed in the muon chambers that matches the extrapolated central detector track is then tagged as a muon. A positional match is required in both the

r - ϕ and r - z planes. The positional accuracy required for the muon segment measurement is determined by the multiple scattering of the highest energy muons of interest, and is about 2 mm. The positional resolutions achieved by the barrel muon chambers are ~ 1.5 mm in r - ϕ by the drift time measurement, and ~ 2 mm in z by the cathode pad measurement. The positional resolutions achieved by the endcap muon detector are ~ 1 mm in x and y using cathode strips perpendicular to the streamer tubes, and ~ 3 mm in x and y using cathode strips parallel to the streamer tubes.

3.2.6 The Forward Detectors

The forward detectors [41] are used to determine the luminosity of the LEP beams at the OPAL interaction region by measuring the rate of Bhabha scattering, and are also used to tag electrons from two-photon interactions. The two forward detectors surround the beam pipe and are located immediately after the pressure bells of the central detector, filling the holes at the center of each electromagnetic endcap calorimeter. They have a clean acceptance for particles emanating from the interaction region between 47 and 120 mrad from the beam axis. Each detector consists of tube chambers and drift chambers positioned in front of an electromagnetic calorimeter equipped with a presampler. The tube and drift chambers are used to precisely measure the angles of the scattered electrons and positrons, and the calorimeter measures the energy of the electromagnetic shower. The calorimeters consist of 35 layers of lead separated by layers of plastic scintillator, and are divided into 16 azimuthal segments. The information provided by the forward detectors is not used in this analysis. For a complete description of their performance see Reference [56].

3.3 Data Acquisition

The primary functions of the OPAL data acquisition (DAQ) system [57] are to select, collect, process, and store the raw data from the OPAL subdetectors. The

lowest layer of the online system consists of the subdetector and trigger computers which operate in parallel. These are all implemented in VME [58] and are called the *local system crates* (LSCs). The trigger system [49] selects events based on fast signals from some of the subdetectors. If an event is selected it broadcasts a readout request over a dedicated trigger bus to all LSCs. The *event builder* then merges the event data it collects from the subdetectors. The event builder forwards the events to the *filter* [59] which refines the trigger selection, compresses the data, and performs extensive event monitoring. Events selected by the filter are saved in a large disk buffer on the filter system. This buffer has a capacity corresponding to several hours of data taking and decouples the data acquisition from the event reconstruction and data recording. The *event reconstruction system* fetches events from the filter buffer over the Apollo Token Ring [60], records them on optical disk for long term storage, and reconstructs the events as soon as up-to-date calibration data are available. The reconstructed events are then sent over Ethernet [61] to other systems used for physics analysis. All the activities in the LSCs, the event builder, and the filter are controlled by a central DAQ control system. Other components of the online system, such as the *slow controls* system which supervises the operation of the subdetectors and monitors safety related quantities, will not be described here.

3.3.1 The Trigger

During the 1990 and 1991 LEP runs, the e^+e^- bunches collided at the OPAL interaction region at a rate of 45 kHz, or every 22.2 μ s. A flexible and programmable trigger system was designed to use fast information from a subset of the OPAL subdetectors to select crossings with a possible e^+e^- interaction. Its function was to reduce the high crossing rate to an event rate of less than 10 Hz which could be handled by the rest of the DAQ system.

The trigger system was designed to be as efficient as possible in selecting Z^0 decays and the Bhabha events used for the luminosity measurement. In addition, the trigger

system had to be sensitive to other processes such as $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ and $e^+e^- \rightarrow \gamma\gamma$, where only photons are detected, and had to be as open as possible to new and unexpected physics processes.

The main background sources are cosmic rays, interactions of the beam particles with residual gas molecules inside the beam pipe (beam-gas) or with the beam pipe material itself (beam-wall), synchrotron radiation, and detector noise. Cosmic rays pass through the detector at random locations, rarely coming near the interaction region. Similarly, beam-gas and beam-wall events are characterized by an event vertex displaced from the interaction region, as well as having low visible energies and track multiplicities. Good separation of the low multiplicity Z^0 decays from these backgrounds is achieved at the trigger level by using information on the vertex position along the beam line.

The OPAL subdetectors that provide trigger signals are the vertex chamber, the jet chamber, the time-of-flight detector, the barrel and endcap electromagnetic calorimeters, the barrel, endcap and pole tip hadron calorimeters, the barrel and endcap muon detectors, and the forward detectors. The trigger information from the hadron calorimeter is not used in the overall trigger decision. The trigger signals are divided into two categories: stand-alone trigger signals from individual subdetectors, and trigger signals formed by spatial coincidences between subdetectors in bins of θ and ϕ , called the θ - ϕ matrix. The full azimuthal range is divided into 24 ϕ bins, and the polar range is divided into 6 θ bins, 4 in the barrel region and 1 each for the endcaps, giving a total of 144 θ - ϕ bins. The θ - ϕ matrix is also divided into 5 layers defined by the track, time-of-flight, electromagnetic, hadron, and muon triggers. The stand-alone triggers are based on information such as the total energy or track multiplicities, and have relatively high thresholds to reduce the rate of spurious triggers. The θ - ϕ matrix triggers allow these thresholds to be lowered.

The track trigger [50] uses information from the vertex and jet chambers to look for charged particle tracks originating from the interaction region in both the r - z and

r - ϕ planes. This information is obtained from the 12 axial wires of each sector of the vertex chamber, and from three groups of 12 adjacent wires at different radii of each jet chamber sector. Signals formed by the vertex and jet chamber trigger electronics are combined by the track trigger processor, which detects the presence of tracks in any bins of the θ - ϕ matrix and provides various track multiplicity signals. In addition, simple counts of the number of wires hit are provided, and trigger signals depending on the specific ionization of particles are available from the jet chamber electronics.

The time-of-flight trigger signals are generated from the mean timers, and require the discriminators at both ends of a counter to have fired within 50 ns of each other, and within 50 ns of the arrival time of a relativistic particle from the interaction region. These 160 signals are combined to produce 24 overlapping ϕ bins of 36° each, covering 4 of the 6 bins in θ . Three stand-alone trigger signals are also produced from the logical OR of the 160 counters, from counting the total multiplicity of the counters, and from counting the multiplicity of overlapping ϕ bins.

The electromagnetic calorimeter generates trigger signals based on the total energy recorded in different segmentations of the calorimeter. The energy is summed over the entire barrel, over each endcap, and over each of the 144 θ - ϕ cells. Each energy sum is discriminated at two different thresholds.

The muon detector provides a total of 120 θ - ϕ signals. Since the muon barrel detector has no z measurement for the trigger, the 24 ϕ signals which it provides are used in the θ - ϕ matrix to simultaneously set the 4 barrel θ bins. The endcap muon detector provides 96 θ - ϕ signals contributing 24 ϕ signals in 4 θ bins. The muon barrel detector provides one stand-alone signal which is simply the logical OR of its 24 ϕ signals. Each muon endcap also provides a stand-alone signal from the logical OR of its 48 θ - ϕ signals. A fourth stand-alone signal demands a hit in the θ - ϕ matrix of each endcap with a loose collinearity cut.

The forward detector trigger signals rely on the energy measurements of the calorimeters. The main luminosity trigger requires that both calorimeters register

at least 15 GeV each, averaged over the whole calorimeter. A coincidence of one calorimeter passing this threshold with other subdetector triggers provides a trigger for tagged two-photon interactions. A coincidence of back-to-back energy measurements in each calorimeter, with a total energy of at least 12 GeV, provides another trigger signal. A stand-alone trigger signal is generated when at least one of the calorimeters registers over 35 GeV.

The central trigger logic is programmable and allows a variety of logic conditions to be imposed on the subdetector signals. All subdetector stand-alone and θ - ϕ input signals are available at the central trigger logic electronics between 1 and 14 μ s after the bunch crossing, which is signalled to the LSCs by a beam pickup mounted directly onto the beam pipe near the OPAL detector. Internal signals are derived from the θ - ϕ signals, which are then combined with the stand-alone signals, and 7 μ s before the next bunch crossing a decision is taken whether to accept the associated event or reject it. If an event is accepted, a trigger signal is sent to the subdetectors to start the readout of the data. If an event is rejected, a reset signal is sent to the subdetectors to prepare the electronics for the next beam crossing. No new triggers are generated until all subdetectors have finished their readout, giving rise to a mean deadtime of about 15 ms.

For a typical machine luminosity of $0.4\text{--}0.8 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}$, the average trigger rate is 2.5–4.0 Hz. Approximately 6% of this rate (on the Z^0 peak) is from hadronic Z^0 decays, $\sim 1\%$ is from Z^0 decays into charged lepton pairs, and $\sim 10\%$ is from forward detector Bhabha events. Cosmic rays and triggers originating from noise contribute about 30% of the rate. A totally unbiased random trigger at a rate of 0.04 Hz provides samples of “empty” events which are used to monitor noise hits in some subdetectors, and for online pedestal adjustment. The rest of the rate is caused by beam-wall and beam-gas interactions, off-momentum particles, beam-halo muons produced at the various collimators, and synchrotron radiation.

The trigger efficiencies for the different physics channels are listed in Table 3.3. A

physics channel	$ \cos \theta $	efficiency (%)
$e^+e^- \rightarrow \text{hadrons}$	<1.0	100
$e^+e^- \rightarrow \mu^+\mu^-$	<0.95	99.9 ± 0.1
$e^+e^- \rightarrow \tau^+\tau^-$	<0.90	99.9 ± 0.1
$e^+e^- \rightarrow e^+e^-$	<0.85	100
$e^+e^- \rightarrow \gamma\gamma$	<0.90	>99.9
small angle Bhabha	0.989–0.999	>99.9

Table 3.3: Trigger efficiencies for different physics channels.

high trigger efficiency is achieved for these channels by having redundant trigger information generated by individual subdetectors, and by requiring coincidences between several subsets of subdetectors that are sensitive to single particles. The trigger efficiencies listed in Table 3.3 are for events within the corresponding region of $|\cos \theta|$, and were calculated from the data using the inherent redundancy of the trigger system.

3.3.2 Readout and Data Processing

When the subdetector LSCs receive the signal to read out their data, they first process it in order to reduce the data volume before transferring their subevent to the event builder. A considerable reduction in data volume is achieved by pedestal subtraction and zero suppression. The jet chamber, the z chamber, and the muon chamber DAQ subsystems perform this task at the hardware level, while the rest of the subdetectors perform the pedestal subtraction and zero suppression in software, usually by the LSC processors.

In addition to this data reduction, all drift chamber type subdetectors with FADC readout process the raw data to resolve multiple hits and to derive their space coordinates and charge. These include the jet chamber, the z chambers, and the muon detector. The raw data are subsequently discarded except when needed for test and monitoring purposes. The jet chamber LSC also executes a fast three-dimensional track fitting algorithm, which yields three-vectors and track charge information used

by the filter for second-level trigger decisions.

Calibration constants are also calculated during the readout and processing of the data by the LSCs. These are stored in a central data base, and are accessed by the event reconstruction system, described below, to reconstruct the events.

3.3.3 The Event Builder and Filter

The event builder collects the subevents from the 17 subdetector LSCs (listed in Table 3.1), merges them, and forwards the full events to the filter system. At this stage the average event size is 211 kbyte for hadronic events, and 60 kbyte for other types of events. The event builder is also based on the VME system, and is connected to the filter and LSCs via a dedicated VME Inter-Crate bus (VICbus) [57]. The data flow through the event builder is controlled by a single process that communicates with a process in each LSC and in the filter via an interrupt driven system. The event builder can handle a sustained rate of 25 Hz for 100 kbyte events.

Upon receiving the events from the event builder, the filter performs five separate functions:

- The integrity of the data structure of each event is verified.
- The events are partially reconstructed and classified into physics categories. This involves running a fast cluster finding algorithm on the electromagnetic calorimeter data, running a fast track reconstruction algorithm on the jet chamber raw data (if this wasn't already done in the jet chamber LSC), converting the forward detector signals into energies and clusters, reconstructing muon chamber track segments, and analyzing the trigger pattern. Using this information, backgrounds that were accepted by the hardware trigger are flagged and rejected, eliminating about 35% of the events processed by the filter. This processing step also adds filter analysis results to the event data structure and updates the event header. The final event header contains 64 words of basic

event information.

- The quality of the data is checked. The results of these checks are accumulated in histograms which are monitored by the shift crew for problems. A copy of the event data is also passed on to the online event display if a request for that particular event type is pending, enabling the shift crew to visually scan the events for anomalies.
- The data words of events accepted by the filter are compressed. Lists of the maximum possible ranges of all words defined in the data structure are used to store the compressed information in as few bits as possible, across word boundaries. Since every data word is scrutinized during compression, this process has the valuable secondary function of checking the data integrity at the finest level. After this procedure the average hadronic Z^0 decay is 46 kbyte in size and the other events are about 12 kbyte in size.
- The event data and associated information are written to disk. The compressed data are recorded in 20 Mbyte files, typically containing 2000 events each. The event headers of rejected events are also written to disk to verify the filter selection procedure offline. Two Gbyte of disk space are available for buffering the event files.

The filter system is based on an HP Apollo DN10000 workstation which provides all the CPU and I/O resources required to perform the filter tasks. The data compression procedure requires the bulk of the CPU resources, consuming roughly 200 ms of CPU time for low activity events, and 800 ms for hadronic Z^0 decays. The other processing requires another 100 ms of CPU time per event.

3.3.4 Online Event Reconstruction

The event reconstruction system reads the data files from the filter buffer disk and fully reconstructs the data as soon as up-to-date calibration parameters are available

from the subdetectors. The reconstructed quantities, such as the momenta of tracks and the associated energies found in the calorimeters, are written to a data structure called the *data summary tape*, or DST. At this stage the data is distributed to three remote computer systems for physics analysis: the CERN IBM mainframe, the OPAL offline VAX cluster, and a cluster of UNIX-based workstations. The reconstructed events are available for analysis within a few hours of being recorded. Before the reconstruction stage the data files are also written to 630 Mbyte optical disks for long term storage and for future iterations of the event reconstruction, as described in the following section.

The reconstruction system is based on a network of HP Apollo workstations. The event reconstruction is performed on three DN10000 workstations, each hosting four CPU boards which shared global memory and I/O resources, with a nominal performance of 17 SPECmark [62] per CPU board. Several less powerful workstations are used for data distribution and administrative tasks. Even with this powerful system, full event reconstruction requires 26 CPU seconds for the average hadronic event and 2 CPU seconds for the other types of events.

3.4 Offline Analysis Software

The event reconstruction performed during the data acquisition process is only temporary, allowing physicists to begin data analysis as soon as possible. After sufficiently large data samples are recorded, extensive offline analysis of the data is performed to fine-tune the calibration parameters in order to optimize the quality of the data. The data are then re-processed with these improved calibration parameters to yield the final data used in the published physics analyses.

OPAL uses many software tools in the event reconstruction and analysis stages. The event reconstruction program, called ROPE, is a collection of FORTRAN routines that produce the DST from the raw data and calibration constants. The events can be visually scanned using the OPAL event display, GROPE, which can also run

ROPE interactively for debugging purposes. The DST is analyzed using ROPE and HBOOK [63], a CERN software package that produces histograms and *ntuples* (a subset of the DST). The histogram and *ntuple* files require much less disk space than the full DST, and consequently can be processed quickly to speed up the analysis process. The final stages of analysis are done using PAW [64], a powerful CERN product that allows the user to manipulate histograms and *ntuples* interactively. More extensive fitting procedures can be performed in FORTRAN, using either the DST or *ntuple* information, with yet another CERN package called MINUIT [65].

As in most scientific work, our results must be compared with the current theoretical models in order to make progress in our understanding of the physical process we are studying. The models are incorporated into Monte Carlo programs called event generators, which were discussed in Chapter 2. The four-vectors produced by these generators, in our case the JETSET Monte Carlo, are processed by the OPAL Monte Carlo program, called GOPAL, which simulates the detector response to the particles produced in the Z^0 decays. The output of GOPAL is then processed by ROPE to produce a DST in the same format as the DST obtained by the reconstruction of real data, except that extra information, such as the generated momenta and energies of the particles, is included in the DST structure.

3.4.1 Event Reconstruction

ROPE is a collection of FORTRAN modules designed to reconstruct events from the raw data produced by the OPAL detector or by GOPAL. Each of the subdetectors listed in Table 3.1 has its own processor which controls the reconstruction of the data from that particular subdetector. In addition, there are many other processors within ROPE which attempt to merge the information from separate subdetectors, such as the central tracking processor described below, or attempt to associate tracks with energy clusters in the calorimeters or with muon chamber track segments. The end product of ROPE is the DST, which can then be read by ROPE for data analysis.

The charged particle tracks are the most important quantities used in the measurement of the average b hadron lifetime, so the track reconstruction will be described in some detail. The track reconstruction in OPAL proceeds in three stages:

1. Each of the three central detector chambers internally performs a pattern recognition process to form track segments within its sensitive volume.
2. The track segments are then passed to the central tracking processor which attempts to match the segments.
3. A global track fit is then performed using the space points in the subdetectors which have been associated through the matching process.

The initial pattern recognition is performed by means of a *triplet initiator*, which is a triplet of adjacent hits. Starting from one extremity of the chamber, the triplet initiator is used to define a *road* in the r - ϕ plane for the vertex and jet chambers, and in the r - z plane for the z chambers. The road is projected into the rest of the chamber, and the first hit encountered within this road is added to the segment to form a new track segment initiator. This process is iterated until no more hits can be added to the existing track segments. After the pattern recognition process track segments are discarded in the vertex chamber if they have fewer than six hits, and are discarded in the jet chamber if they have fewer than 10 hits. As discussed before, all three central detector chambers have bidirectional drift cells, so each track can appear to originate from either side of the anode wire planes. The pattern recognition process is performed on both drift ambiguities, but the jet chamber immediately rejects the wrong track segment on the basis of the quality of the track fit to the jet chamber hits. The wrong track segment will have a large χ^2 because of the staggering of the anode wires. In the case of the vertex and z chambers, both ambiguities are saved to be resolved in the matching process.

The central tracking processor collects all resulting track segments and attempts to merge the vertex chamber and z chamber segments with the jet chamber segments.

First, a match between a jet chamber segment and a vertex chamber axial segment is attempted, then with a z chamber segment, and finally with a vertex chamber stereo segment. The matching is accomplished by extrapolating the jet chamber tracks to the center of the chamber in question, and using the positional and angular information of the two track segments at that point to decide on the best match. The internal quality of the track segments in the vertex and z chambers is also used at this stage to help resolve the left-right ambiguities. The jet chamber track parameters, described below, are updated for each successful match, and the next stage of the merging uses these modified parameters.

Once the matching is complete, all track segment information is discarded and a global track fit is performed using the hits which were associated with each merged track. The resulting central detector tracks are determined by five parameters: κ , ϕ_0 , d_0 , $\tan \lambda$, and z_0 . They are defined as follows:

- κ is the track curvature, where

$$|\kappa| = \frac{1}{2\rho}, \quad (3.4)$$

and ρ is the radius of curvature of the track. κ is signed such that $\kappa > 0$ means that ϕ increases in moving along the track from the point of closest approach of the track to the origin in the r - ϕ plane (p.c.a.) For an axial magnetic field with B_z along the positive z axis, a positive κ corresponds to a particle with a negative physical charge q . This is formally written as

$$q = - \left(\text{sign of } \left(\frac{B_z}{\kappa} \right) \right). \quad (3.5)$$

- ϕ_0 is the azimuthal angle made by the track tangent at the p.c.a.
- d_0 is the signed impact parameter, where $|d_0|$ is the distance from the origin to the p.c.a. Defining $\vec{d}$ to be the vector from the origin to the p.c.a., $\hat{\phi}$ to be the unit track vector at the p.c.a., and $\hat{z}$ to be the unit vector along the z axis, then

$$d_0 = \hat{\phi} \times \vec{d} \cdot \hat{z}. \quad (3.6)$$

- $\tan \lambda = \cot \theta$, where θ is the track polar angle.
- z_0 is the z coordinate of the track at the p.c.a.

The parameter κ is related to the component of the track momentum perpendicular to the beam axis, p_T , by the relation

$$p_T = \frac{B_z}{2} \left| \frac{Q}{\kappa} \right|, \quad (3.7)$$

where Q is the track charge. The total momentum p and its components p_x , p_y , and p_z are therefore written as

$$p = p_T \sqrt{1 + \tan^2 \lambda}, \quad (3.8)$$

$$p_x = p_T \cos \phi_0, \quad (3.9)$$

$$p_y = p_T \sin \phi_0, \quad (3.10)$$

$$p_z = p_T \tan \lambda. \quad (3.11)$$

3.4.2 The OPAL Detector Simulation

The OPAL Monte Carlo [66] is based on the CERN GEANT simulation package [67], which provides a framework for defining the detector geometry and controls the tracking of particles through this geometry. After obtaining the four-vectors for an event, each particle in the event is tracked through the detector in small steps. At each step GEANT calculates the effects of multiple Coulomb scattering, energy loss via ionization and bremsstrahlung, and the interaction with the magnetic field, and the particle four-vector is modified accordingly. The probability of other discrete processes occurring is also computed at each step, and they are implemented if necessary. These processes include hadronic interactions with the detector materials, photon pair production, and particle decays.

When a particle traverses a sensitive volume of the detector, GEANT stores information which is used to calculate the detector response to the particle after all

particles in the event have been tracked through the detector. This procedure generates simulated raw data, which are called *digits* in GEANT terminology, and are stored for further processing. Detector resolution effects and inefficiencies are included at this stage. The output of GOPAL is a data structure identical to that produced by the OPAL online data acquisition system, and can be processed by ROPE to produce a DST. Extra information, such as the generated four-vectors and particle types, is added to the DST. The process of generating a fully reconstructed simulated hadronic Z^0 decay, excluding the generation of the four-vectors, takes about 450 CPU seconds on a machine running at 4-5 mips (millions of instructions per second).

There is also a SMEAR mode of GOPAL which provides a much faster simulation of the OPAL detector. In this simplified simulation, the number of hits assigned to each track is estimated, taking into account the single-hit efficiency and two-hit separation performance of each component of the central detector. The covariance matrix of the track parameters is then calculated, taking into account the estimated number of hits, their resolutions, and multiple Coulomb scattering. This covariance matrix is then used to smear the track parameters about their generated values. The simulation of the calorimeters and muon chambers is accomplished by a crude tracking of showers through these detectors. Each particle entering the outer region is tracked through a highly simplified representation of the detector geometry, with parameterizations for the energy deposited in each sensitive volume. The simulation operates at the cluster level in the calorimeters, producing acceptable results for single isolated particles but making it extremely difficult to combine the clusters when they overlap in a multiparticle environment such as a jet. The performance of the central detector is well described by the simplified simulation, but is not nearly as satisfactory for the calorimeters. The SMEAR mode of GOPAL runs about 100 times faster than the full simulation.

Comparisons of the Monte Carlo simulations with OPAL data will be shown when necessary in the following chapters.

3.5 Calibration and Performance of the OPAL Detector

There are two general classes of calibration constants required to convert the signals observed by the OPAL subdetectors into useful physics quantities. The first category includes all constants that determine the relative positions and orientations of the subdetectors with respect to each other, and the second category includes all constants used internally by each subdetector to convert its signals into quantities such as drift distances and energies. The positional calibration constants are obtained from measuring the positions of survey points on the subdetector structures, and/or from using data to optimize the matching of information between subdetectors. All subdetectors are aligned with respect to the jet chamber. The internal calibration constants are obtained from test beam data, from laser calibration systems, or from the data. The performance of the calorimeters and muon detectors was briefly discussed in Section 3.2, and a discussion of their calibration will not be given here—for complete details see the references listed in Table 3.1.

All three central detector drift chambers need calibration constants to parameterize the bow of the anode wires, to determine the drift time from the raw time measurement, and to convert the drift time to a drift distance. In addition, the vertex and z chambers require constants to precisely locate their anode wires relative to the jet chamber, and the jet chamber requires constants for its charge division measurement of the z coordinate and the dE/dx measurement. Only the calibration of the vertex and jet chambers will be discussed here since they are the most important detectors for the average b hadron lifetime measurement.

The calibration of the central detector begins with a calibration of the jet chamber, which is done in two ways [44]. One method uses a Nd:YAG laser [68] situated outside the detector, which injects two parallel ultraviolet laser beams with a spacing of 10 mm in the r - ϕ plane anywhere in the jet chamber cells. Since the laser beams produce a

straight line of ionization, this procedure calibrates the relative drift times for wires in a given anode wire plane, and is also used to measure the effective wire stagger (mechanical and electrostatic). The drift velocity and the Lorentz angle in the jet chamber are monitored by measuring the difference in arrival times of the signals produced by the two laser beams. The second method uses tracks from hadronic Z^0 decays that cross anode or cathode wire planes in the jet chamber. This procedure is useful because many miscalibration effects result in track discontinuities at these boundaries. In particular, tracks crossing anode planes are used to measure the global timing offset between the signals recorded by the jet chamber electronics and the beam crossing signal. This timing offset is required to calibrate the drift time measurement. Tracks crossing cathode planes are used to precisely measure the drift velocity and Lorentz angles. Finally, since the detectors surrounding the jet chamber have much better z resolutions (see Table 3.2), they are used to calibrate the z measurement of the jet chamber by constraining tracks in the r - z plane and adjusting the relevant calibration parameters.

Once the jet chamber is calibrated, the vertex chamber can be calibrated using the jet chamber tracks. The position of the vertex chamber within the jet chamber is determined by studying the angular difference in the r - ϕ plane, $\Delta\phi$, between the jet chamber and vertex chamber tracks in the center of the vertex chamber cells. If the vertex chamber is rotated in ϕ , the distribution of $\Delta\phi$ versus ϕ will be systematically shifted from 0. Similarly, the distribution of $\Delta\phi \sin \phi$ versus z will exhibit a slope if the vertex chamber is rotated in the x - z plane, and the distribution of $\Delta\phi \cos \phi$ versus z will exhibit a slope if the chamber is rotated in the y - z plane. The magnitude of the wire bows is determined by minimizing the average distance of the reconstructed space points in the vertex chamber from the jet chamber tracks as the wire bow calibration constants are varied. The drift time measurement and time-to-distance conversion are calibrated by studying the average d_0 distribution as a function of the local ϕ within each vertex chamber cell (measured with respect to the anode planes). An error in

Figure 3.8: Vertex chamber calibration distributions. All figures show the mean d_0 versus the local vertex chamber ϕ (in radians), measured with respect to the anode planes. All 36 sectors are projected into the same plot. Figure (a) shows the mean d_0 with optimized calibration parameters, (b) shows the discontinuity at the anode plane when the global timing offset, T_0 , is too high by 2 ns, and (c) shows the slope in the mean d_0 distribution when the drift velocity (c parameter) is too high by $0.5 \mu\text{m}/\text{ns}$, or by about 1%. The curves are linear fits which are allowed to have a discontinuity at the origin.

the drift time calibration will manifest itself as a discontinuity at the anode plane boundaries, whereas an error in the drift velocity measurement will introduce a slope in the distribution. These effects are shown in Figure 3.8.

Figure 3.9: d_0 sum distributions for leptons measured by (a) the jet chamber alone and (b) the vertex chamber and jet chamber combined.

The final quality of this calibration process is monitored by looking at the distribution of the sum of the d_0 s for tracks originating from the decays $Z^0 \rightarrow e^+e^-$ and $Z^0 \rightarrow \mu^+\mu^-$. Since both tracks originate from the same point and are parallel to each other (ignoring radiative effects), the spread of the d_0 sum distribution is a direct measure of the d_0 resolution. The d_0 sum distribution for these events using information from the jet chamber alone is shown in Figure 3.9(a), and the corresponding distribution for tracks with vertex chamber information is shown in Figure 3.9(b). The d_0 resolution is $113 \mu\text{m}$ for jet chamber tracks, and is $42 \mu\text{m}$ when the vertex chamber information is included. This resolution represents the best OPAL can do, without using the silicon microvertex detector, because the multiple scattering is negligible for these high momentum tracks and they are completely isolated.

Chapter 4

The B Lifetime Measurement

The average b hadron lifetime was extracted from the impact parameter distributions of electron and muon candidates identified in the hadronic decays of the Z^0 . Leptons were used for this measurement because they provide a very clean tag of b hadron decays, and the theoretical uncertainties associated with semileptonic b hadron decays are smaller than those associated with hadronic b hadron decays. The average impact parameter of leptons produced in b decays is directly proportional to the b hadron lifetime, and is of the order of a millimeter at the energies involved in Z^0 decays. Since the impact parameter resolution of the OPAL detector was better than $100\text{ }\mu\text{m}$, the impact parameter therefore provided excellent sensitivity to the b hadron lifetime.

The selection of hadronic events is described in Section 4.1. The procedures for identifying electrons and muons in these hadronic events are given in Section 4.2, as well as the cuts used to enrich these samples with leptons originating predominantly from the semileptonic decays of b hadrons. Many details of the JETSET simulation were updated using the most recent experimental measurements. These modifications are described in Section 4.3. The variable which is sensitive to the b lifetime is the *signed* impact parameter, and is defined in Section 4.4. The lifetime fit procedure is then discussed in detail in Section 4.5, and the results are presented in Section 4.6.

4.1 Event Selection

The identification of hadronic Z^0 decays is described in detail in Reference [56]. It is based on the energy deposited in the electromagnetic calorimeter and on the charged track multiplicity. Clusters in the barrel section of the calorimeter were required to have an energy of at least 100 MeV, and the clusters in the endcaps were required to have an energy of at least 200 MeV deposited in at least 2 adjacent lead glass blocks. Tracks were required to have at least 20 measured space points and a distance of closest approach to the interaction point of less than 2 cm in the direction perpendicular to the beam axis and less than 40 cm along the beam axis. Tracks were also required to have a minimum momentum component transverse to the beam direction of 50 MeV/c. Hadronic events were defined by the following four requirements: (i) at least 7 clusters, (ii) at least 5 tracks, (iii) a total energy deposited in the lead glass of at least 10% of the center-of-mass energy, $\sqrt{s}$,

$$R_{\text{vis}} \equiv \sum E_{\text{clus}} / \sqrt{s} > 0.1, \quad (4.1)$$

where E_{clus} is the energy of each cluster, and (iv) an energy imbalance along the beam direction which satisfied

$$R_{\text{bal}} \equiv \left| \sum (E_{\text{clus}} \cdot \cos \theta) \right| / \sum E_{\text{clus}} < 0.65, \quad (4.2)$$

where θ is the polar angle of the cluster. There were 146 936 (353 224) events which satisfied these criteria in 1990 (1991).

The cut on the number of clusters and the number of tracks eliminated Z^0 decays into charged lepton pairs. This multiplicity cut, coupled with the impact parameter requirement, served to eliminate cosmic ray events. The R_{vis} cut discarded two-photon and beam-gas events. The R_{bal} cut rejected beam-wall, beam-gas and beam-halo events, as well as cosmic rays passing through the endcaps. The efficiency of the event selection procedure was determined to be $98.4 \pm 0.4\%$, and the background contamination was dominated by $\tau^+ \tau^-$ events ($0.11 \pm 0.03\%$) and two-photon events ($0.07 \pm 0.03\%$).

The cuts listed above were used to select hadronic Z^0 decays for the lineshape analysis discussed in Reference [56]. In addition, the subdetectors used in this analysis were required to be in a good mode of operation when the events were recorded. These included the central tracking chambers, the barrel presampler, the electromagnetic calorimeters and the muon chambers. Approximately 9.5% (8.7%) of the 1990 (1991) sample was rejected by this requirement.

For the final step in the event selection, the tracks were required to pass within 5 cm of the interaction region in the x - y plane, to have a momentum component in the x - y plane above 0.15 GeV/ c , and a total measured momentum below 65 GeV/ c . Events were accepted if they contained at least 7 tracks passing these requirements, eliminating about 70% of the $\tau^+\tau^-$ contamination. Most of the remaining $\tau^+\tau^-$ events were accepted because a pion from a τ decay interacted hadronically in the silicon or beam pipe and generated several charged tracks. Only 0.53% (0.46%) of the 1990 (1991) sample was rejected by this cut, leaving a final sample of 132 323 (320 995) events.

Several Monte Carlo datasets were used in this analysis. Samples of 195 998 (408 477) hadronic events were generated using JETSET and were processed with the full GOPAL simulation of the OPAL detector in the 1990 (1991) configuration. The JETSET parameters discussed in Chapter 2 were used to generate these events, and the fragmentation of b and c quarks was done using the Peterson fragmentation function with parameters $\epsilon_b = 0.0035$ and $\epsilon_c = 0.060$. The event selection procedure described above was then applied to this Monte Carlo data, and 192 607 (401 911) events passed all cuts. Unless stated otherwise, any reference to Monte Carlo data refers to this sample of fully reconstructed Monte Carlo events. Other special purpose Monte Carlo samples used in this analysis will be described when they are first referenced.

The different detector configurations in the 1990 and 1991 runs produced negligible differences in this analysis, so the two datasets were analyzed as a single data sample.

4.2 Lepton Selection

The lepton selection was performed in three steps. In the first step kinematic cuts were applied to enrich the lepton samples with tracks produced predominantly in the decays of b hadrons. In the second step, variables sensitive to the unique properties of electrons and muons were defined and used to identify these leptons within hadronic Z^0 decays. And finally, several other track quality cuts were applied to ensure that the lepton candidate tracks had similar, well understood, impact parameter resolutions. The kinematic cuts are described in the following section, the electron and muon identification criteria are discussed in Sections 4.2.2 and 4.2.3 respectively, and the final track quality cuts are given in Section 4.2.4.

4.2.1 High p_t Lepton Selection

The fraction of selected leptons produced in b hadron decays was maximized in order to be as sensitive as possible to the b lifetime. This was accomplished by exploiting the kinematic properties of semileptonic decays, which were discussed in Chapter 2. The ACCMM lepton momentum distributions in the rest-frame of the parent hadron for $b \rightarrow \ell$ and $c \rightarrow \ell$ decays, and in the rest-frame of the b hadrons for $b \rightarrow c \rightarrow \ell$ decays, are shown in Figure 4.1. The leptons from $b \rightarrow \ell$ decays have a much higher mean momentum than leptons from $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$ decays because of the large b quark mass. This information is partially preserved when the system is boosted into the lab frame because the momentum component perpendicular to the hadron direction is invariant under a Lorentz boost. Therefore, a sample enriched with leptons from b decays was selected simply by requiring the lepton candidates to have a large momentum component transverse to the b hadron direction. Since we do not know the true b hadron direction it was approximated by the axis of the jet containing the lepton. The jet finding procedure is described below.

In each event, charged tracks and electromagnetic clusters which were not associ-

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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Figure 4.1: B tagging using semileptonic decay kinematics. The three curves represent the ACCMM lepton momentum spectra in the rest-frame of the parent hadron, copied from Figure 2.6. The cut which exploits this information to select the leptons from $b \rightarrow \ell$ decays is discussed in the text.

ated to a charged track were grouped into jets using the jet finding algorithm described in Reference [69]. The algorithm started by looking for pairs of tracks and/or clusters which formed the lowest invariant mass M_{ij} , where the labels i and j identify the tracks and clusters. This pair was then merged into a jet if

$$M_{ij}^2 < x_{\min}, \quad (4.3)$$

for some choice of the mass cutoff $\sqrt{x_{\min}}$. This process was repeated with the remaining tracks, clusters, and jets until the invariant mass of all possible combinations exceeded $\sqrt{x_{\min}}$, or until all tracks and clusters were merged into jets. A mass cutoff of $\sqrt{x_{\min}} = 7 \text{ GeV}/c^2$ was used because Monte Carlo studies indicated that the jet axis obtained in this manner gave the best angular resolution of the initial b hadron direction, which was 51 mrad in the x - y plane.

In addition to the requirements listed in Section 4.1, tracks considered as lep-

ton candidates satisfied the following conditions: (i) the momentum, p , was at least 4.0 GeV/ c , (ii) the transverse momentum with respect to the nearest jet, p_t , calculated with the lepton candidate included in the jet axis definition, was at least 0.8 GeV/ c for electron candidates and 1.0 GeV/ c for muon candidates, (iii) the absolute value of the cosine of the polar angle was less than 0.7, and (iv) at least 3 hits in the z chambers were associated with the track. Tracks satisfying these criteria were subjected to the lepton identification procedures described below.

4.2.2 Electron Identification

The electron selection was based on the work presented in Reference [70]. Electron candidates were required to be consistent with the expected signature for electrons in their measured dE/dx , in the pulse height registered in the electromagnetic pre-sampler, in the fraction of energy deposited in the electromagnetic calorimeter, and in the lateral spread of the associated electromagnetic shower.

The dE/dx of tracks in hadronic decays was shown in Figure 3.7. The dE/dx for electrons is clearly separated from the other particles over most of the momentum range. A normalized average energy loss, dE/dx_{norm} , was formed by subtracting the theoretical prediction from the measured value, and dividing this by the estimated uncertainty in the dE/dx measurement:

$$dE/dx_{\text{norm}} = \frac{dE/dx_{\text{measured}} - dE/dx_{\text{theory}}}{\sigma(dE/dx_{\text{measured}})}. \quad (4.4)$$

The distribution of dE/dx_{norm} for the electron sample is shown in Figure 4.2. The hadronic background is shown in the hatched area, and the fit to this distribution will be described below. Electron candidates were required to have at least 40 well measured dE/dx samples, and had to pass the one-sided cut of $dE/dx_{\text{norm}} > -2.0$.

Electromagnetic showers initiated by 10 GeV/ c electrons will be contained, on average, within a depth of about 32 cm of lead glass. On the other hand, 10 GeV/ c hadrons will only deposit about half of their energy in the lead glass if they begin to

Figure 4.2: The dE/dx_{norm} and E_{cone}/p distributions of the electron sample. The hatched areas indicate the background contributions to the fits, which are described in the text. The tracks included in these plots pass all electron selection cuts except the cut on the variable shown.

shower in the coil, and much less if they begin to shower in the calorimeters. Therefore electron candidates satisfied the condition that the measured electromagnetic energy divided by the reconstructed track momentum, E_{cone}/p , was between 0.7 and 1.4. Here E_{cone} is the energy of the electromagnetic cluster associated with the electron candidate, plus the energy recorded in all other lead glass blocks whose centers were within a 30 mrad^1 cone around the electron candidate track extrapolated to the electromagnetic calorimeter. The distribution of E_{cone}/p for the electron candidates is also shown in Figure 4.2 with the hadronic background distribution measured from the data. The fit will be discussed in more detail later.

The presampler barrel was calibrated so that $45 \text{ GeV}/c$ muons produced a signal with an amplitude of 2.0 units. Since hadrons tend to start showering in the calorime-

¹Thirty mrad extrapolated to the inner radius of the electromagnetic calorimeter at $\theta = 90^\circ$ corresponds to a cone of radius of 7.2 cm.

Figure 4.3: The presampler pulse height distribution for electron candidates. The Monte Carlo predictions for the electron and hadron contributions are also shown. The tracks pass all cuts except the presampler cut.

ters their presampler signals will tend be consistent with that of muons, whereas electrons most often initiate electromagnetic showers in the pressure vessel and coil and therefore generate much larger signals in the presampler. The distribution of the presampler pulse height for the electron candidates is shown in Figure 4.3 in two momentum bins. The amplitude of the pulse increases with the electron momentum, so a momentum dependent variable was defined as

$$PB_{\text{mult}} = \min(2.5 + p/2.0, 10.0), \quad (4.5)$$

where p is in units of GeV/c . Electron candidates were required to have a presampler pulse height of at least PB_{mult} .

Electromagnetic showers are narrower than hadronic showers. The typical maximum diameter for 10 GeV/c particles in lead glass is 7 cm for an electromagnetic shower, and 16 cm for a hadronic shower. Then, on average, electrons will deposit

Figure 4.4: The distributions of ΔE_{cone} and $E_{\text{cone}}/E_{\text{cone2}}$ for the electron candidates. The Monte Carlo predictions for the electron and hadron contributions are also shown. The tracks pass all cuts except the ΔE_{cone} and $E_{\text{cone}}/E_{\text{cone2}}$ cuts.

their energy within a few lead glass blocks whereas hadrons will distribute their energy over more blocks. Therefore, electron candidates were required to satisfy the condition $E_{\text{cone}}/E_{\text{cone2}} > 0.85$ or $\Delta E_{\text{cone}} < 2.0$ GeV, where E_{cone} was defined earlier, E_{cone2} is the energy contained in the blocks defining E_{cone} and in all adjacent blocks, and ΔE_{cone} is the difference between E_{cone2} and E_{cone} . This requirement will also be referred to as the *lateral spread cut*. The distributions of $E_{\text{cone}}/E_{\text{cone2}}$ and ΔE_{cone} are shown in Figure 4.4 for the electron candidates and the Monte Carlo electrons and hadrons. Note that this cut isolates electron candidates from other activity in the electromagnetic calorimeter, and therefore has a significant effect on their impact parameter distribution. This point will be revisited several times.

Finally, electrons produced in photon conversions were tagged and removed from the sample. They were identified using information from the central detector only. The identification required a pair of oppositely charged tracks whose momentum sum was

Figure 4.5: The reconstructed radius of photon conversions in the 1990 and 1991 data samples.

consistent with originating from the event vertex. The reconstructed radius of origin of the candidate pairs was required to be separated from the beam spot by at least 2 cm, and both tracks were required to have $dE/dx_{\text{norm}} > -2.5$. The most important identification criterion was that the invariant mass of the pair, assuming it was an e^+e^- pair, was required to be less than $50 \text{ MeV}/c^2$. The distributions of the reconstructed radius of conversion for both the data and Monte Carlo are shown in Figure 4.5, and show good agreement. The extra material in 1991 due to the silicon detector and new beam pipe is evident by the increase in the number of conversions at a radius of ~ 8 cm. According to Monte Carlo studies, the efficiency of the photon conversion identification procedure after the previous electron identification requirements was found to be $(84 \pm 4)\%$. The fraction of electrons misclassified as conversions was estimated to be $(1.7 \pm 0.5)\%$.

electron identification cuts	
1.	at least 40 good dE/dx samples
2.	$dE/dx_{\text{norm}} > -2.0$
3.	presampler pulse height $> PB_{\text{mult}}$
4.	$0.7 < E_{\text{cone}}/p < 1.4$
5.	$E_{\text{cone}}/E_{\text{cone2}} > 0.85$ or $\Delta E_{\text{cone}} < 2.0 \text{ GeV}$
6.	not a photon conversion

Table 4.1: Summary of the electron identification cuts.

The electron identification cuts are summarized in Table 4.1. The general features of the electron identification were reasonably modelled by the Monte Carlo, but the simulation was found to be inadequate for the calculation of the efficiencies. For this reason the electron identification efficiencies were calculated using various data samples; no Monte Carlo samples were used. The efficiency of the z chamber and dE/dx hits requirement was determined from a sample of muon candidates identified in hadronic Z^0 decays, as described below, and was corrected for the background contained in the muon sample. The efficiency of the dE/dx_{norm} cut was found to

p_t bin (GeV/c)	p bin (GeV/c)		
	4-6	6-10	>10
0.8-1.2	0.550 ± 0.021	0.508 ± 0.018	0.488 ± 0.021
1.2-1.6	0.644 ± 0.032	0.619 ± 0.024	0.539 ± 0.022
>1.6	0.664 ± 0.047	0.653 ± 0.036	0.588 ± 0.030

Table 4.2: The electron identification efficiencies as a function of p and p_t . They were determined directly from the data. The errors include both statistical and systematic contributions.

depend significantly on the track environment, and was determined from studying the dependence of the corresponding efficiency for a pure sample of low momentum pions. The uncertainty in the resulting dE/dx_{norm} efficiency takes into account possible differences between the electron and pion dE/dx_{norm} distributions. The efficiency of the E_{cone}/p requirement was determined from a sample of events containing only one good quality track identified as an electron, which were dominated by radiative Bhabha events and two-photon processes. This efficiency was then corrected for the effect of a high multiplicity track environment in hadronic Z^0 decays, and the fact that approximately 2% of tracks in hadronic events were matched to the wrong electromagnetic calorimeter cluster. Finally, the efficiencies of the lateral spread and presampler pulse height cuts were determined from the ratio of the number of electrons in the sample with and without the corresponding cut, after subtracting the hadronic background and photon conversions. The calculation of the hadronic background will be described later. Monte Carlo studies showed that the electron identification efficiencies differed significantly for each individual source of electrons. The numbers determined from the data are average efficiencies over the various electron sources. The efficiencies as a function of p and p_t are shown in Table 4.2.

4.2.3 Muon Identification

The muon selection was based on the work presented in Reference [70]. It exploits the fact that muons produced in Z^0 decays lose their energy only through ionization and easily penetrate the materials of the calorimeters to reach the muon chambers. The muon candidates were therefore selected from the subset of central detector tracks associated with track segments reconstructed independently in the muon chambers. The association was performed by extrapolating the central detector tracks to the muon chambers and calculating the positional matching variable

$$\chi_{\text{pos}}^2 = \left(\frac{\Delta\phi}{\sigma_{\Delta\phi}} \right)^2 + \left(\frac{\Delta\theta}{\sigma_{\Delta\theta}} \right)^2. \quad (4.6)$$

The angular deviations $\Delta\phi$ and $\Delta\theta$ were calculated at the point of closest approach between the extrapolated track and the muon segments, and the error estimates $\sigma_{\Delta\phi}$ and $\sigma_{\Delta\theta}$ were calculated from the errors in the muon segment position added in quadrature with the tracking and extrapolation errors. The muon segment with the smallest χ_{pos} was chosen as the match to the charged track. The distribution of χ_{pos} for muon candidates and the corresponding Monte Carlo prediction is shown in Figure 4.6. Muon candidates were required to satisfy the cut $\chi_{\text{pos}} < 3.0$.

In addition to the χ_{pos} cut, the dE/dx measured in the jet chamber was required to be consistent with the expected value for muons in order to reduce the background from $K \rightarrow \mu\nu$ decays. Candidates were rejected only if they had at least 20 good jet chamber charge samples used in the dE/dx calculation, and if the measured dE/dx was lower than a value corresponding to two standard deviations below that expected for a muon. Other backgrounds came from events where many muon segments were clustered near the candidate track, making it easy to misassociate a track with a muon segment. These events were either noise in the muon chambers or hadronic punchthrough, and the most pathological cases were rejected by requiring that there be less than 21 muon segments in a 300 mrad cone around the track.

The muon identification cuts are summarized in Table 4.3. Unlike the simulation

Figure 4.6: The positional matching variable χ_{pos} for muon candidates. The Monte Carlo predictions for the muon and hadron contributions are also shown. The tracks pass all muon selection cuts except the χ_{pos} cut.

of the electron identification variables, the muon identification variables simulated by the Monte Carlo were in good agreement with the data, allowing the muon identification efficiencies to be determined directly from the Monte Carlo. The resulting muon identification efficiencies as a function of p and p_t are listed in Table 4.4. The inefficiency arising from the geometrical acceptance of the muon chambers, multiple scattering, and calibration effects was cross-checked using data samples of $Z^0 \rightarrow \mu^+ \mu^-$

muon identification cuts	
1.	$\chi_{\text{pos}} < 3.0$
2.	dE/dx consistent with muon
3.	muon segment multiplicity < 21

Table 4.3: Summary of the muon identification cuts.

p_t bin (GeV/c)	p bin (GeV/c)		
	4-6	6-10	>10
1.0-1.2	0.712 ± 0.023	0.690 ± 0.020	0.661 ± 0.019
1.2-1.6	0.671 ± 0.022	0.708 ± 0.016	0.671 ± 0.015
>1.6	0.740 ± 0.041	0.694 ± 0.023	0.671 ± 0.016

Table 4.4: The muon identification efficiencies as a function of p and p_t . They were determined directly from the Monte Carlo simulation. The errors are statistical only.

and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ events. The inefficiency of the dE/dx requirement was cross-checked using data samples of pure low momentum pions, and pions from $K^0 \rightarrow \pi^+\pi^-$ decays. The muon segment multiplicity cut was estimated to be more than 99% efficient for prompt muons.²

4.2.4 Final Lepton Samples

The lepton candidate tracks were also required to (i) have at least 80 hits in the jet chamber associated with the track, (ii) have at least 6 hits in an axial sector of the vertex chamber associated with the track, and (iii) have less than half of the axial vertex chamber hits registered on the ionization tail of earlier hits (second hits). These cuts ensured that the track parameters of all lepton candidates had similar measurement uncertainties. The third criterion was applied because of the systematic problems with vertex chamber second hits, which were discussed in Chapter 3. Approximately 23% of the lepton candidates were rejected by the first-hit requirement after all other cuts were applied.

There were 2610 electron candidates and 2762 muon candidates in the data sample that passed all cuts. An electron candidate is shown in Figure 4.7, and a muon candidate is shown in Figure 4.8. These figures show event displays produced by GROPE, and are both projections in the x - y plane. The curved lines in the cen-

²Leptons produced in the decays $b \rightarrow \ell$, $b \rightarrow c \rightarrow \ell$, and $c \rightarrow \ell$ are called *prompt leptons*.

Figure 4.7: Event display of an electron candidate found in the 1991 data sample. The tagged electron is indicated by the symbol “e.” The electron energy is completely absorbed in the electromagnetic calorimeter.

Figure 4.8: Event display of a muon candidate found in the 1991 data sample. The tagged muon is indicated by the symbol “mu.” The muon deposits very little energy in the calorimeters and is associated with a muon segment (arrow) in the muon chambers.

tral region of these event displays represent the reconstructed central detector tracks. The first circle surrounding the central detector represents the inner boundary of the electromagnetic calorimeter, and the polygons drawn around this circle represent electromagnetic clusters and have an area proportional to the amount of energy associated with the cluster. The next circle represents the inner boundary of the hadron calorimeter with a corresponding set of energy clusters. Finally, the outer two near-semicircles show the inner and outer boundaries of the muon chambers. An arrow drawn in this region represents a muon track segment found in these detectors. The main features of the unique signatures left by these leptons are clearly displayed.

4.3 JETSET Modifications

The JETSET lepton momentum spectra shown in Figure 2.6 were changed to the spectra predicted by the ACCMM model to obtain the lifetime result of this analysis. This procedure was accomplished by weighting the Monte Carlo leptons from $b \rightarrow \ell$, $b \rightarrow c \rightarrow \ell$, and $c \rightarrow \ell$ decays by momentum-dependent weights calculated from the ratio of the ACCMM predictions to the JETSET predictions. This weighting procedure was necessary because OPAL did not have sufficient CPU resources at the time to generate a Monte Carlo sample for each set of parameters. The same momentum-dependent weights used for B_u^+ and B_d^0 mesons were applied to leptons from the semileptonic decays of B_s^0 and Λ_b^0 hadrons.

The JETSET branching ratios for the decays $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ were also changed to the values corresponding to the ACCMM model, and the JETSET branching ratios for the decays $b \rightarrow J/\psi \rightarrow \ell$, $b \rightarrow \tau \rightarrow \ell$, and $c \rightarrow \ell$ were modified according to the most recent experimental measurements. These branching ratios are listed in Table 2.5. The corrections were implemented by weighting the Monte Carlo leptons by the ratio of the desired branching ratios to the JETSET branching ratios.

Finally, the parameters for the Peterson fragmentation function for b and c quarks were changed from the values used for the Monte Carlo generation, which were $\epsilon_b =$

0.0035 and $\epsilon_c = 0.060$, to the average of the LEP measurements discussed in Chapter 2, which were $\epsilon_b = 0.0055$ and $\epsilon_c = 0.050$. This change was implemented by weighting the Monte Carlo events by z -dependent weights, where z is the fragmentation variable discussed in Chapter 2. The weights were calculated from the ratio of the Peterson fragmentation function obtained using the new value of ϵ_q to the corresponding function obtained using the value of ϵ_q used to generate the Monte Carlo data.

Unless stated otherwise, the corrections described above were always applied to *all* leptons generated with JETSET, regardless of whether the full detector simulation or the SMEAR detector simulation was used to process the Monte Carlo data. The fragmentation corrections were applied to all Monte Carlo tracks produced in primary b and c hadron decays, in addition to the leptons.

4.4 Definition of the Impact Parameter

The average b hadron lifetime was determined from maximum likelihood fits to the impact parameter distributions of the lepton samples. The impact parameter, δ , is the smallest distance in the plane transverse to the beam axis between the track in question and some origin. Ideally we would like the origin to be the production point of the b hadron, but in practice we have to approximate this point with some estimator. The estimator used in this analysis was the average beam position, or *beam spot*.

4.4.1 Determination of the Beam Spot

The procedure for measuring the beam spot in OPAL is described in Reference [71]. The algorithm used charged tracks from leptonic and hadronic decays of the Z^0 which passed the criteria $p_T > 0.25$ GeV/ c , $|d_0| < 1$ cm, and $\sigma_{d_0} < 0.1$ cm. Here p_T is the component of the track's momentum in the plane transverse to the beam axis, d_0 ,

is the impact parameter measured relative to the coordinate system origin (defined in Chapter 3), and σ_{d_0} is the estimated uncertainty in d_0 . The events were scanned in chronological order until 100 or more good tracks were found. Only complete events were included. A χ^2 fit was then performed to find a vertex in the x - y plane. If any tracks contributed more than 10 to the χ^2 of the fit, the worst track was discarded and the fit was repeated. This procedure was iterated until each remaining track contributed less than 10 to the χ^2 , and resulted in an average of 90 tracks per vertex. The beam position was calculated as the weighted mean of 30 vertex positions, corresponding to about 200 Z^0 decays. If consecutive beam positions were statistically consistent they were merged into a single beam position. This procedure resulted in 1 or 2 beam position measurements per run. The beam spot coordinates for the 1990 and 1991 runs are shown in Figure 4.9. The typical statistical accuracy is $6.7 \mu\text{m}$ in x , $4.5 \mu\text{m}$ in y , and 0.54 mm in z .

Once the beam coordinates were known, the distribution of impact parameters for tracks with small σ_{d_0} were used to determine the intrinsic size of the beam crossing region. The vertical beam size was expected to be much smaller than the detector resolution. Therefore, the width of the d_0 distribution from horizontal tracks was taken as a measure of the tracking resolution, and was subtracted in quadrature from the width measured from vertical tracks to calculate an average value of the beam spot size in the x -direction of $S_x = (157 \pm 5) \mu\text{m}$. This value of S_x was obtained from the data collected during 1990 and the first part of 1991. The beam spread in x for the rest of 1991 was determined to be $S_x = (147 \pm 4) \mu\text{m}$ using the silicon information. The width in the z direction implied a beam size of $S_z = (1.12 \pm 0.02) \text{ cm}$. These determinations were in excellent agreement with the range of values expected from the LEP beam optics. The beam spot dimensions used in this analysis were $S_x = (157 \pm 20) \mu\text{m}$ and $S_y = (27 \pm 10) \mu\text{m}$. The lifetime fit results are very insensitive to the value of S_y .

Figure 4.9: The beam spot coordinates for the 1990 and 1991 OPAL runs. The statistical errors are shown. The gap between runs 2000 and 2200 corresponds to the winter shutdown between the 1990 and 1991 data taking periods, when many test runs were performed.

4.4.2 Lepton Impact Parameter Distributions

The typical geometry of the production and semileptonic decay of a b hadron is shown in Figure 4.10. The beam ellipse is drawn approximately to scale in this diagram, which is a projection into the x - y plane. A Z^0 is produced somewhere in this ellipse and decays into a pair of $b\bar{b}$ quarks, but only the hemisphere containing the $\bar{b}$ quark is shown. The $b\bar{b}$ system fragments, producing the π^+ and π^- shown originating from the interaction region, and the B_u^+ which travels along the dotted line until it decays into a muon, the corresponding neutrino, and a $\bar{D}_u^0$. The $\bar{D}_u^0$ then travels along the second dotted line and subsequently decays into a kaon and a pion. The arrows in this figure represent the momentum vectors of the indicated particles, and are added to give the jet axis. The muon track is then projected back to the beam spot (dash-dotted line) and the shortest distance of approach of the track to the beam spot position in the x - y plane gives the magnitude of the impact parameter, δ .

The lepton candidate impact parameters were defined in this manner, and were given a positive sign if the angle in the x - y plane between the axis of the jet containing the lepton candidate and the vector from the beam spot to the point of closest approach of the track was less than 90° ; otherwise the impact parameters were given a negative sign. This convention is illustrated in Figure 4.11 for the two sign possibilities. This procedure ensured that, on average, tracks originating from the decays of long-lived hadrons were assigned a positive impact parameter, while tracks originating from the fragmentation process and light quark events were assigned negative impact parameters $\sim 50\%$ of the time. The impact parameter distributions for the muon and electron candidate samples are shown in Figure 4.12. These distributions display a clear positive asymmetry and have weighted means around $240 \mu\text{m}$, indicating a significant lifetime content.

The proportionality of the mean impact parameter to the b hadron lifetime is displayed in Figure 4.13. Each point in this distribution was obtained from 100 000 $b\bar{b}$

Figure 4.10: The definition of the impact parameter used to extract the b lifetime. The small ellipse represents the beam spot drawn to scale. The arrows represent the momentum vectors of the indicated particles, and the dotted lines represent the path followed by the corresponding particles, except for the dash-dotted extension of the muon momentum vector which is just the extrapolation of that vector to the beam spot. The definition of the jet axis is described in the text.

Figure 4.11: The convention for signing the impact parameters. One track is shown in each case with the corresponding jet axis.

Figure 4.12: The impact parameter distributions of the lepton samples. The weighted means were estimated using the impact parameter errors described in the text. The statistical errors on the weighted means are about $3 \mu\text{m}$.

events generated using JETSET. Only the four-vector information was used—no event reconstruction was performed. Each b hadron was forced to decay semileptonically into an electron, and each electron was required to satisfy the conditions $|\cos \theta| < 0.7$, $p > 4.0 \text{ GeV}/c$, and $p_t > 0.8 \text{ GeV}/c$. In this case the p_t was calculated with respect to the initial b hadron direction. The average of the impact parameters for all electrons was then calculated for the eleven different values of the b hadron lifetime shown in Figure 4.13. The impact parameters were calculated with respect to the Z^0 production point. The statistical error bars are approximately the size of the points. The linearity is very good, as shown by the straight line fit to the data points.

The remaining sections of this chapter discuss how the lepton impact parameter distributions shown in Figure 4.12 were parameterized to extract the average b hadron lifetime.

Figure 4.13: The mean impact parameter versus the b hadron lifetime. Each point was obtained by generating 100 000 $b\bar{b}$ events, forcing each b hadron to decay semileptonically into an electron. The calculation of the mean impact parameter is described in the text. The curve is a straight line fit to the data.

4.5 The Lifetime Fit

The observed impact parameter distributions were divided into the following five components:

1. Leptons arising from the semileptonic decay of b hadrons. These are referred to as *primary b decays*.
2. Leptons arising from the semileptonic decay of c hadrons or from the decay of τ leptons which themselves arise from b hadron decays. These are referred to as *cascade decays*.
3. Leptons arising from the semileptonic decay of c hadrons that do not come from b hadron decay. These are referred to as *primary c decays*.
4. Hadrons that are misidentified as leptons.
5. Decays-in-flight of charged pions and kaons to muons, photon conversions, and Dalitz decays of neutral mesons.

Separate probability density functions, which are simply normalized impact parameter distributions, were determined for each of these components. The probability density functions for leptons produced in $b \rightarrow \ell$, $b \rightarrow c/\tau \rightarrow \ell$, and $c \rightarrow \ell$ decays were described in terms of a convolution of the distributions expected from the underlying physics with a resolution function determined using the data. The underlying physics distributions, or *physics functions*, are described in Section 4.5.2, the resolution function is described in Section 4.5.3, and the convolution process is described in Section 4.5.4. The probability density functions for the remaining components of the lepton samples were obtained directly from the data or from the Monte Carlo, and are described in section 4.5.5.

The b lifetime was determined by fitting the observed impact parameter distributions with a maximum likelihood fit. The likelihood function $\mathcal{L}$ was defined as

$$\mathcal{L} = \prod_{j=1}^N \left[f_1 P_1(\delta^j, \sigma^j, \sigma_t^j, \tau_b) + f_2 P_2(\delta^j, \sigma^j, \sigma_t^j, \tau_b) + f_3 P_3(\delta^j, \sigma^j) + f_4 P_4(\delta^j, \sigma^j) + f_5 P_5(\delta^j, \sigma^j) \right], \quad (4.7)$$

where P_i is the probability density function for component i , f_i is the fraction of the lepton sample due to component i , described below, δ^j is the measured impact parameter of candidate j , σ^j and σ_t^j are uncertainties associated with δ^j , as described in Section 4.5.3, and N is the total number of lepton candidates. The only free parameter in the fit was the b hadron lifetime, τ_b .

4.5.1 Lepton Sample Compositions

The compositions of the lepton samples were obtained mostly from the Monte Carlo and are given in Table 4.5. The Monte Carlo lepton candidates were selected using all lepton selection cuts, and were weighted by the correction factors discussed in Section 4.3. An additional correction, described below, was applied to the Monte Carlo electrons to account for the poor modelling of the electron identification efficiencies. The hadronic background fraction for the electron sample was

electron sample components		% of total
1.	$b \rightarrow e$	78.5
2.	$b \rightarrow (c \text{ or } \tau) \rightarrow e$	10.9
3.	$c \rightarrow e$	6.8
4.	hadrons misidentified as electrons	3.5
5.	Dalitz decays and photon conversions	0.3
muon sample components		% of total
1.	$b \rightarrow \mu$	75.5
2.	$b \rightarrow (c \text{ or } \tau) \rightarrow \mu$	10.0
3.	$c \rightarrow \mu$	5.7
4.	hadrons misidentified as muons	4.8
5.	decay-in-flight of charged pions and kaons	4.0

Table 4.5: The compositions of the lepton samples.

determined directly from the data, and the muon sample fractions for components 4 and 5 were determined using both the data and Monte Carlo, as described below.

Monte Carlo studies showed that only 0.1 lepton candidates per lepton species were expected from the $Z^0 \rightarrow \tau^+ \tau^-$ background events discussed earlier, so this contribution to the sample fractions was ignored. The Monte Carlo also predicted that $\sim 0.3\%$ of the lepton candidates originated from b hadrons produced in the gluon branching $g \rightarrow b\bar{b}$, and that $\sim 0.2\%$ originated from c hadrons produced in the gluon branching $g \rightarrow c\bar{c}$. These components are included in the $b \rightarrow \ell$ and $c \rightarrow \ell$ sample fractions listed in Table 4.5.

The electron identification efficiencies obtained from the Monte Carlo were known to be wrong, and had to be corrected in order not to bias the electron sample fractions. The corrections were determined using the Monte Carlo efficiencies as a function of p and p_t , listed in Table 4.6, and the corresponding efficiencies obtained from the data, which are listed in Table 4.2. The Monte Carlo efficiencies were determined directly from the Monte Carlo by counting the number of electrons from $b \rightarrow e$, $b \rightarrow c/\tau \rightarrow e$, and $c \rightarrow e$ decays passing the identification cuts. The Monte Carlo electrons were then

p_t bin (GeV/c)	p bin (GeV/c)		
	4-6	6-10	>10
0.8-1.2	0.618 ± 0.016	0.560 ± 0.014	0.572 ± 0.015
1.2-1.6	0.698 ± 0.024	0.664 ± 0.017	0.659 ± 0.016
>1.6	0.684 ± 0.047	0.678 ± 0.025	0.668 ± 0.018

Table 4.6: The Monte Carlo electron identification efficiencies as a function of p and p_t . They were determined directly from the Monte Carlo. The errors include only statistical contributions.

weighted by the ratio of the efficiencies determined from the data to the efficiencies from the Monte Carlo. Unless stated otherwise, this correction was always applied whenever Monte Carlo electrons were used to obtain information about the electron candidates in the data.

The electron hadronic background fraction was obtained from fits to the dE/dx_{norm} and E_{cone}/p distributions from the data, which are shown in Figure 4.2, and was fixed in the calculation of the electron sample fractions. The background distributions in these fits were obtained from the data using anti-electron cuts. The dE/dx_{norm} background was obtained from tracks passing all the usual electron selection cuts, except the ratio of E_{cone}/p was required to be between 0.1 and 0.6. Similarly, the E_{cone}/p background was obtained from tracks passing all the usual electron selection cuts, except the dE/dx_{norm} was required to be less than -3.0 . The complete dE/dx_{norm} and E_{cone}/p distributions shown in Figure 4.2 were then fitted using these background distributions plus a Gaussian representing the electron signal. In both fits, the position and width of the Gaussian was fixed so that only the normalization of the background was allowed to vary. The fractions of the hadronic background distributions extending underneath the electron signals were then used to calculate the hadronic background fractions. Both fits provided consistent results, and their weighted mean of 31% was used as the hadronic background fraction for the electron sample.

Component 5 of the electron sample came principally from the Dalitz decays of π^0

Figure 4.14: The p and p_t distributions for the lepton samples. The Monte Carlo distributions have been normalized to the number of leptons in the data.

mesons, and included a few photon conversions. Photon conversions were efficiently eliminated from the electron sample by the photon conversion finder described earlier, the kinematic cuts, and the requirement of vertex chamber axial hits which limited the conversions to the beam pipe and silicon detector.

The Monte Carlo was found to give a reliable prediction of the probability of identifying a hadron as a muon, which was cross-checked using data samples of $K_s^0 \rightarrow \pi^+ \pi^-$ and $\tau \rightarrow 3\pi$ decays [70]. This information was used to determine the fraction of the muon sample composed of misidentified hadrons (component 4), which were predominantly punchthrough hadrons, and decays-in-flight of charged pions and kaons (component 5). The resulting fractions of 4.8% for component 4 and 4.0% for component 5 were fixed in the calculation of the muon sample fractions.

Figure 4.14 shows the p and p_t distributions for the lepton samples after all the corrections were applied, and using the sample fractions listed in Table 4.5. The distributions for the five components from the Monte Carlo are indicated in this figure. There is good agreement between the predicted and observed shapes of these distributions.

4.5.2 Physics Functions

The underlying physics distributions for the $b \rightarrow \ell$, $b \rightarrow c/\tau \rightarrow \ell$, and $c \rightarrow \ell$ decays were determined from 4.0 million $Z^0 \rightarrow b\bar{b}$, 3.0 million $Z^0 \rightarrow b\bar{b}$, and 1.0 million $Z^0 \rightarrow c\bar{c}$ JETSET events respectively. The b hadrons produced in the $b \rightarrow \ell$ sample were forced to decay semileptonically into an electron or a muon with equal probability. The $b \rightarrow c/\tau \rightarrow \ell$ events all included at least one of the decay chains $b \rightarrow c \rightarrow \ell$ or $b \rightarrow \tau \rightarrow \ell$ selected at the four-vector level from generic $Z^0 \rightarrow b\bar{b}$ decays, with equal numbers of electrons and muons. Similarly, the $c \rightarrow \ell$ events were selected at the four-vector level from generic $Z^0 \rightarrow c\bar{c}$ events and were also composed of equal numbers of electrons and muons. These events were then processed using the SMEAR simulation of the OPAL detector. The impact parameters were taken directly from the

generated information before the detector simulation, but only particles whose reconstructed tracks passed the track quality and kinematic cuts were used. The impact parameters were calculated relative to the event-by-event production point instead of the beam spot, and their signs were determined using the jet axes reconstructed from the charged tracks and unassociated electromagnetic clusters obtained after the detector simulation. This procedure was necessary because the signing of the impact parameters is determined by the other tracks in the jet, whereas the resolution function applies only to the lepton candidate track, and therefore cannot account for signing mistakes due to imprecise estimates of the b hadron direction.

The SMEAR simulation of the OPAL detector averaged the effects of many physics processes and applied them to all tracks. This procedure was acceptable in most cases, but was inadequate for the description of bremsstrahlung which affects only a small fraction of all charged tracks: the electrons. To properly incorporate the effects of electron bremsstrahlung into the SMEAR simulation, a Monte Carlo study using the full detector simulation was performed. In this study, $Z^0 \rightarrow b\bar{b}$ events were generated using JETSET and all b hadrons were required to decay semileptonically into either electrons or muons. The events were all produced at the coordinate system origin. Only the leptons with $p > 3.5 \text{ GeV}/c$ and $|\cos \theta| < 0.8$ were passed through the full detector simulation; all other particles in the events were discarded. The leptons used in the study passed all track quality and kinematic cuts, with the momentum cut applied at the four-vector level. No lepton identification cuts were applied. The electron sample included 15 221 events processed with the 1990 geometry and 24 473 events processed with the 1991 geometry. A muon sample was generated from 7 739 events processed with the 1991 geometry. The ratio of the momentum after bremsstrahlung to the initial momentum, p_f/p_{gen} , both taken before the detector simulation, is shown in Figure 4.15(a) for the 1991 electron sample. The curve is a fit using the sum of two exponential functions and a constant, and is given by

$$p(x) = 16.4 + \exp(0.22 + 5.66 x) + \exp(-47.0 + 55.1 x). \quad (4.8)$$

Figure 4.15: (a) The ratio of the momentum after bremsstrahlung over the initial generated momentum for electrons processed with the 1991 detector geometry and the full Monte Carlo. (b) The ratio of the measured momentum over the initial generated momentum of muons after the bremsstrahlung correction (points) compared to the same ratio for the Monte Carlo electrons (histogram).

To apply the bremsstrahlung correction to the physics function electrons, the fractional momentum loss of each electron was randomly generated according to the function $p(x)$, and the measured p and p_t were reduced accordingly. The correction was applied to 45% of the electrons in all physics function samples.

This procedure was checked by applying the correction to the Monte Carlo muons generated for this study. This check was possible because the only significantly different property for electrons and muons above 4 GeV/ c is their bremsstrahlung cross-section, which is completely negligible for muons. The ratio of the corrected measured momentum to the generated momentum, $p_{measured}/p_{gen}$, for these muons is shown in Figure 4.15(b) overlaid on the corresponding distribution for the electrons generated for this study. They agree very well. The electron impact parameter distribution

$ \cos \theta $ range	efficiency
0.00–0.48	1.00
0.48–0.62	0.80
0.62–0.70	0.60

Table 4.7: The muon chamber acceptance as a function of $|\cos \theta|$. The acceptances are averages over ϕ .

acquired a small tail because of the bremsstrahlung process, but this broadening was negligible compared to the smearing introduced by using the beam spot position as the vertex position, and was therefore ignored.

The SMEAR Monte Carlo gave a good description of the central detector, but its simulation of the response of the outer detectors was poor. For this reason the lepton identification cuts were not applied directly to the physics function tracks. Instead, tracks were discarded according to the lepton identification efficiencies as a function of p and p_t listed in Tables 4.2 and 4.4. In addition, the acceptance of the muon chambers was incorporated by discarding muon tracks using the efficiencies as a function of $|\cos \theta|$ listed in Table 4.7. When using these tables for the physics function track selection, the efficiencies were normalized to the largest single efficiency in the table in order to maximize the number of tracks accepted.

Although the SMEAR description of the outer detectors was poor, the two variables important for the determination of the physics functions were well modelled because they were insensitive to the inadequacies of the simulation. These variables were the angular separation between the jet axis and the corresponding b hadron momentum vector, $\Delta\phi$, and the lepton p_t . A good modelling of the $\Delta\phi$ distribution was important because its width determined the number of impact parameter sign errors. The distributions of these variables are shown in Figures 4.16(a) and (b) for the SMEAR and full Monte Carlo simulations. The data entering these distributions were obtained from the appropriate samples of $b \rightarrow e$ decays. The distributions are

Figure 4.16: Some SMEAR variables compared to the full Monte Carlo. Figure (a) shows the angular separation in the r - ϕ plane, in degrees, between the jet axis and the parent b hadron momentum vector. Figure (b) shows the lepton p_t distribution. All distributions were obtained from the appropriate $b \rightarrow e$ samples.

in good agreement.

The physics function distributions are shown in Figure 4.17. The distributions were expressed in terms of the impact parameters divided by the product of the speed of light and the average generated lifetime of the sample in question. For the $b \rightarrow \ell$ physics functions this was the generated b hadron lifetime, for the $b \rightarrow c/\tau \rightarrow \ell$ physics functions this was the sum of the generated b hadron lifetime and the average generated lifetime of c hadrons and the τ lepton, and for the $c \rightarrow \ell$ physics functions this was the average generated lifetime of primary c hadrons. The b hadrons were generated with a lifetime of 1500 fs. The average generated lifetime of c hadrons and τ leptons in the $b \rightarrow c/\tau \rightarrow \ell$ sample was 670 fs, and the average generated lifetime of c hadrons in the $c \rightarrow \ell$ sample was 717 fs.

The physics function distributions were parameterized using a sum of exponential

Figure 4.17: The physics functions. The curves show the results of the fits using the sums of exponential functions described in the text.

parameter	$b \rightarrow \ell$		$b \rightarrow c/\tau \rightarrow \ell$		$c \rightarrow \ell$	
	e	μ	e	μ	e	μ
a_1	0.339 ± 0.048	0.277 ± 0.046	n.a.	n.a.	n.a.	n.a.
b_1	0.919 ± 0.037	0.891 ± 0.041	n.a.	n.a.	n.a.	n.a.
a_2	2.39 ± 0.39	2.20 ± 0.46	n.a.	0.27 ± 0.18	n.a.	3.6 ± 1.2
b_2	3.18 ± 0.28	3.13 ± 0.32	n.a.	1.43 ± 0.45	n.a.	1.80 ± 0.23
a_3	9.61 ± 0.83	7.94 ± 0.88	1.97 ± 0.49	6.2 ± 1.8	10.7 ± 7.6	35.0 ± 9.6
b_3	10.81 ± 0.81	10.3 ± 1.1	3.33 ± 0.28	6.6 ± 1.1	2.59 ± 0.42	11.4 ± 1.9
a_4	29.2 ± 2.6	23.1 ± 2.0	12.5 ± 2.1	12.0 ± 2.6	80 ± 52	88 ± 40
b_4	71.7 ± 5.7	52.2 ± 4.2	15.4 ± 1.5	30.3 ± 8.2	15.9 ± 2.8	125 ± 66
c_1	33.0 ± 2.8	31.8 ± 2.9	18.4 ± 4.3	21.7 ± 5.2	139 ± 95	80 ± 21
d_1	40.7 ± 3.5	38.4 ± 3.5	2.96 ± 0.19	3.10 ± 0.21	17.4 ± 4.0	34.0 ± 7.4
c_2	26.6 ± 2.4	27.3 ± 2.7	10.0	10.0	66 ± 38	46 ± 10
d_2	5.48 ± 0.32	5.17 ± 0.33	1.401 ± 0.056	1.377 ± 0.064	3.13 ± 0.67	4.48 ± 0.52
c_3	22.2 ± 1.3	20.9 ± 1.3	n.a.	n.a.	10.0	10.0
d_3	1.669 ± 0.053	1.653 ± 0.062	n.a.	n.a.	1.02 ± 0.22	1.30 ± 0.10
c_4	5.0	5.0	n.a.	n.a.	n.a.	n.a.
d_4	0.736 ± 0.012	0.745 ± 0.013	n.a.	n.a.	n.a.	n.a.

Table 4.8: The physics function parameters. The numbers quoted without errors were fixed during the fit. The entries marked “n.a.” mean that the corresponding exponential function was not required to fit the physics function data.

functions in order to perform an analytic convolution with the resolution function. Up to eight exponential functions were used in each case, four for the positive impact parameter region and four for the negative region. The exact number of exponential functions was determined by the quality of the fit. These analytic physics functions had the form

$$PF_j(\delta, \tau_j) = \begin{cases} \sum_{i=1}^4 a_i \exp\left(+b_i \frac{\delta}{c\tau_j}\right) & \delta < 0 \\ \sum_{i=1}^4 c_i \exp\left(-d_i \frac{\delta}{c\tau_j}\right) & \delta \geq 0 \end{cases}, \quad (4.9)$$

where the label j refers to the physics function sample, a_i and c_i are the amplitudes for exponential function i , b_i and d_i are the exponents for exponential function i , δ is the impact parameter, c is the speed of light, and τ_j is the average generated lifetime appropriate for sample j , as described above. The fits were performed over the regions shown in Figure 4.17 using a χ^2 method. A finer binning than shown was used for the central region. In each case the analytic distribution was normalized to unity so one of the parameters was fixed during the fit. The resulting physics function parameters are listed in Table 4.8.

4.5.3 Resolution Function

The impact parameter resolution is a function of the variables used to select the lepton candidates. Therefore, the detector resolution function was measured with the set of tracks that passed all track quality and kinematic cuts, but failed the lepton identification. In addition, tracks were selectively discarded to bring the distributions of p , p_t , the number of jet chamber hits, and the number of vertex chamber hits into agreement with the distributions observed from the lepton samples for these quantities. Since the resolution function was applied to leptons from b and c hadron decays, the p and p_t distributions for components 4 and 5 of the lepton samples (obtained from the Monte Carlo) were subtracted from the corresponding distributions for the lepton samples before selectively discarding tracks. The distributions of the

parameter		data		Monte Carlo	
		electrons	muons	electrons	muons
p	a	0.120 ± 0.009	0.202 ± 0.009	0.105 ± 0.009	0.149 ± 0.009
	b	-0.05 ± 0.05	-0.26 ± 0.05	0.18 ± 0.05	0.13 ± 0.05
p_t	a	1.08 ± 0.04	1.23 ± 0.04	1.07 ± 0.04	1.26 ± 0.04
	b	-0.58 ± 0.04	-0.84 ± 0.04	-0.69 ± 0.04	-0.94 ± 0.04
CJ hits	a	0.02 ± 0.05	0.02 ± 0.05	0.03 ± 0.05	0.03 ± 0.05
	b	-2.81 ± 0.05	-2.25 ± 0.05	-2.14 ± 0.05	-2.44 ± 0.05
CV hits	a	0.062 ± 0.004	0.041 ± 0.004	0.091 ± 0.004	0.083 ± 0.004
	b	0.05 ± 0.08	0.26 ± 0.08	0.22 ± 0.08	0.17 ± 0.08

Table 4.9: Resolution function trimming parameters.

number of jet chamber and vertex chamber hits showed no significant differences between the prompt lepton sources and components 4 and 5. The selective discarding of tracks, or *trimming* procedure, was implemented by generating random numbers in the range (0,1) and discarding the tracks if the random numbers exceeded the result calculated using the function

$$p(x) = ax + b, \quad (4.10)$$

where x is p , p_t , the number of jet chamber hits, or the number of vertex chamber hits. The parameters a and b for each variable are listed in Table 4.9. These parameters were obtained from a simultaneous χ^2 fit to the four distributions described above. The tracks remaining after the fit are called *resolution function tracks*. The distributions of p , p_t , and number of vertex and jet chamber hits for the resolution function tracks before and after this trimming procedure are compared to the corresponding distributions for the electron candidates in Figures 4.18 and 4.19. Similar distributions were obtained for the muon selection, and for the Monte Carlo resolution function samples.

The distributions of the impact parameters divided by their calculated uncertainties for resolution function tracks from the data are shown in Figures 4.20(a) and (b).

Figure 4.18: The p and p_t distributions for the resolution function tracks before and after the trimming process compared to the electron candidate distributions.

Figure 4.19: The distributions of the number of jet and vertex chamber hits for the resolution function tracks before and after the trimming process compared to the electron candidate distributions.

The uncertainties were calculated according to the formula

$$\sigma^2 = a^2 \cdot V_{dd} + S_x^2 \cdot \sin^2 \phi + S_y^2 \cdot \cos^2 \phi, \quad (4.11)$$

where V_{dd} is the variance of the impact parameter from the track fit, which includes contributions due to multiple Coulomb scattering, S_x and S_y are the widths of the beam spot in x and in y respectively, ϕ is the azimuthal angle of the track momentum vector measured at the point of closest approach of the track to the beam spot, and a is a parameter which would be unity if the estimate of the impact parameter variance was consistent with the true reconstructed errors. In practice the true deviations of the measured impact parameters are not perfectly described by V_{dd} . Although σ describes the measurement accuracy on most tracks, there is a second, broader component to the distribution that comes from pattern recognition mistakes, large angle multiple scattering, imperfect detector calibrations, and the presence of long-lived particles in the data sample. Consequently, the resolution function was parameterized as the sum of two Gaussians,

$$R(\delta, \sigma, \sigma_t) = \frac{(1-f)}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) + \frac{f}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{\delta^2}{2\sigma_t^2}\right), \quad (4.12)$$

where f is the fractional area of the broad component of the distribution, and

$$\sigma_t^2 = w^2 \cdot a^2 \cdot V_{dd} + S_x^2 \cdot \sin^2 \phi + S_y^2 \cdot \cos^2 \phi \quad (4.13)$$

is the square of the width of the broad component. The parameters a , f , and w were determined from a maximum likelihood fit to the resolution function tracks. The width of the narrower Gaussian (in δ/σ) was constrained to unity in the fit. Only the entries with a negative impact parameter were used in the fit, since tracks from the decays of long-lived particles introduce an asymmetry in the distribution, as seen in Figures 4.20(a) and (b). Monte Carlo studies indicated that the resolution function track sample had an excess of entries in the region $\delta/\sigma < -15$ compared to the Monte Carlo leptons, so the fit was limited to the region $\delta/\sigma > -15$. The fitted

Figure 4.20: The fits to the resolution function track samples. The distributions show the impact parameter δ divided by the calculated uncertainty σ , which is described in the text. The standard selections in the data and Monte Carlo were fitted in the negative region only—the functions were then extrapolated into the position region (dashed curves) to show the excess of entries there. The Monte Carlo lepton data had the lifetime information removed, so the fit was performed over the entire range.

lep	par	Monte Carlo leptons	Monte Carlo	data	corrected data
e	<i>a</i>	0.953 ± 0.023	0.957 ± 0.016	1.053 ± 0.025	1.049 ± 0.040
	<i>f</i>	0.0185 ± 0.0043	0.0426 ± 0.0029	0.0679 ± 0.0052	0.0438 ± 0.0073
	<i>w</i>	5.64 ± 0.55	8.25 ± 0.31	7.20 ± 0.30	4.92 ± 0.55
μ	<i>a</i>	0.892 ± 0.021	0.935 ± 0.016	1.084 ± 0.025	1.034 ± 0.038
	<i>f</i>	0.0146 ± 0.0036	0.0408 ± 0.0028	0.0651 ± 0.0047	0.0389 ± 0.0065
	<i>w</i>	6.30 ± 0.68	8.77 ± 0.32	7.54 ± 0.29	5.42 ± 0.65

Table 4.10: The parameters describing the resolution functions. The errors are statistical only.

values of the parameters describing the resolution function appear in Table 4.10 under the column labelled “data.”

The resolution function obtained from the data had to be corrected for tracks produced in the decays of long-lived hadrons that were assigned negative impact parameters, and for any bias introduced by using hadrons to determine the resolution properties of leptons. The correction was determined from the Monte Carlo as follows. Shown in Figures 4.20(c) and (d) are the distributions of resolution function tracks from the Monte Carlo obtained using the same procedure used to select the resolution function tracks from the data. Figures 4.20(e) and (f) show the distributions for the Monte Carlo leptons after subtracting the contribution to the impact parameters from the lifetime of the parent hadron. This was done by calculating the true impact parameters of the tracks relative to the Z^0 production point, using only the generated information, and subtracting these from the measured impact parameters of the tracks after the detector simulation. The difference between the measured and true impact parameters contained only contributions from the detector resolution and from the width of the beam spot in the x - y plane. The Monte Carlo resolution functions were fitted following the same procedure used on the data, and the resulting resolution function parameters are listed in Table 4.10 under the column labelled “Monte Carlo.”

The Monte Carlo lepton distributions were fitted using the same resolution function parameterization, but over the region $|\delta/\sigma| < 15$ instead of only the negative half. The resulting resolution function parameters in this case are listed in Table 4.10 under the column labelled “Monte Carlo leptons.”

The a and w parameters from the data were then scaled by the ratios of these quantities obtained from the Monte Carlo lepton and the Monte Carlo resolution function tracks. The f parameters from the data were corrected by subtracting the f parameters obtained from the Monte Carlo resolution function fits, and adding the f parameters obtained from the Monte Carlo lepton fits. This procedure was necessary because the Monte Carlo was known to underestimate the number of tracks introduced in the tails of the resolution function by the degraded resolution near the vertex chamber anode planes. The corrected resolution function parameters for the data are given in Table 4.10 under the column labelled “corrected data,” and defined the resolution functions used in the likelihood fits for the lifetime.

4.5.4 The Convolution

As described earlier, the probability density functions for the $b \rightarrow \ell$, $b \rightarrow c/\tau \rightarrow \ell$, and $c \rightarrow \ell$ components were constructed by convoluting the appropriate physics function with the resolution function. The convolution was performed on a track-by-track basis, and had the form

$$P_i(\delta^j, \sigma^j, \sigma_t^j, \tau_i) = \int_{-\infty}^{+\infty} PF_i(x, \tau_i) R(x - \delta^j, \sigma^j, \sigma_t^j) dx, \quad (4.14)$$

where P_i is the probability density function for component i , δ^j is the impact parameter of track j , σ^j and σ_t^j are the impact parameter uncertainties described in the previous section, τ_i is the appropriate average lifetime for component i , PF_i is the physics function for component i as described in Section 4.5.2, and R is the resolution function described in the previous section. This convolution was performed analytically, and the details of the calculation are given in Appendix B.

4.5.5 Background Distributions

The probability density functions for component 4, the hadronic backgrounds, were determined directly from the data. The probability density function for the electron hadronic background was obtained from the set of tracks that passed all electron selection criteria, except the ratio E_{cone}/p was required to be in the range 0.1 to 0.6. The probability density function for the muon hadronic background was obtained from the set of tracks that passed all track quality and kinematic cuts but failed both the electron and muon identification criteria. Monte Carlo studies indicated that the resulting samples were dominated by hadron tracks. The probability density function for each hadron sample was formed from the corresponding impact parameters divided by their uncertainties σ , which were described in Section 4.5.3. These distributions are shown in Figure 4.21.

Figure 4.21: The fits to the hadronic background track samples. The distributions show the impact parameter δ divided by the calculated uncertainty σ , which is described in the text. The electron and muon selections are also described in the text.

parameter	electrons	muons
A	1000	1000
σ_g	1.008 ± 0.024	0.9945 ± 0.0070
a_1	n.a.	9.2 ± 1.6
b_1	n.a.	0.153 ± 0.010
a_2	25.9 ± 9.4	299 ± 39
b_2	0.277 ± 0.047	0.833 ± 0.043
c_1	123 ± 17	384 ± 26
d_1	0.238 ± 0.016	0.421 ± 0.014
c_2	n.a.	13.4 ± 2.2
d_2	n.a.	0.1035 ± 0.0055

Table 4.11: Hadronic background distribution parameters. The symbols for the parameters are defined in the text.

The probability density functions were parameterized as the sum of a Gaussian and exponential tails of the form

$$P_4(\delta, \sigma) = A \exp\left(-\frac{\delta^2}{2\sigma^2\sigma_g^2}\right) + \begin{cases} \sum_{i=1}^2 a_i \exp\left(+b_i \frac{\delta}{\sigma}\right) & \delta < 0.0 \\ \sum_{i=1}^2 c_i \exp\left(-d_i \frac{\delta}{\sigma}\right) & \delta \geq 0.0 \end{cases}, \quad (4.15)$$

where δ is the impact parameter and σ is the corresponding uncertainty, A and σ_g are the amplitude and width of the Gaussian, respectively, a_i and c_i are the amplitudes of exponential i , and b_i and d_i are the exponents of exponential i . The fits were performed using a maximum likelihood method, which required normalizing the functions P_4 to unity. The amplitudes and means of the Gaussians were fixed. The results of the fits are shown in Figure 4.21, and the corresponding parameters are listed in Table 4.11.

The probability density function for component 5 of the electron sample was chosen to be identical to the electron hadronic background probability density function. The dominant source of electrons in component 5 is the Dalitz decay of π^0 mesons produced during the fragmentation process. Consequently, these electrons have an impact parameter distribution very similar to the hadronic background which is composed

Figure 4.22: The fits to the $K/\pi \rightarrow \mu$ track sample. The distribution shows the impact parameter δ divided by the calculated uncertainty σ , which is described in the text.

of $\sim 50\%$ fragmentation tracks. The broadening of the impact parameter distribution due to extrapolating tracks from converted photons back to the origin was less than the width resulting from the combination of the tracking resolution and the width of the beam spot because of the high momentum cut and the axial vertex chamber hit requirement. These considerations, coupled with the small fraction (0.3%) of the data ascribed to this component, justified the use of the hadronic background distribution.

parameter	muons
A	100
a	72 ± 30
b	0.454 ± 0.060
c	76 ± 24
d	0.260 ± 0.025

Table 4.12: Parameters of the $K/\pi \rightarrow \mu$ background distribution. They are defined in the text.

The probability density function for component 5 of the muon sample was determined from a large sample of pion and kaon decays in hadronic events simulated by the Monte Carlo, in addition to the decays found in the standard Monte Carlo production. This probability density function was also expressed in terms of the impact parameter divided by the associated uncertainty σ , and was parameterized as the sum of exponential functions and a Gaussian with mean 0 and width 1,

$$P_5(\delta, \sigma) = A \exp\left(-\frac{\delta^2}{2\sigma^2}\right) + \begin{cases} a \exp\left(+b\frac{\delta}{\sigma}\right) & \delta < 0.0 \\ c \exp\left(-d\frac{\delta}{\sigma}\right) & \delta \geq 0.0 \end{cases}, \quad (4.16)$$

where δ is the impact parameter, σ is the impact parameter uncertainty, A is the amplitude of the Gaussian, and a , b , c , and d are the parameters of the two exponential functions. The probability density function data and the result of the maximum likelihood fit are shown in Figure 4.22. The corresponding parameters are listed in Table 4.12.

4.6 Results and Monte Carlo Checks

The results of the one parameter fits to the impact parameter distributions are given in Table 4.13. The curves in Figure 4.23 display the results of the fits superimposed on the data. The χ^2 were calculated from the binned data shown in Figure 4.23.

	electrons	muons
N_{cand}	2610	2762
τ_b (fs)	1528 ± 48	1513 ± 49
$\chi^2/\text{d.f.}$	33.6/37	31.2/37

Table 4.13: The lifetime fit results on data. The errors are statistical only.

The method described above was applied to Monte Carlo samples generated with b hadron lifetimes of 1300 and 2000 fs to check for possible biases in the analysis

Figure 4.23: The results of the lifetime fits. The data points show the distribution of impact parameters for the electron and muon candidates. The curve is the result of the maximum likelihood fit for τ_b described in the text. The shaded area shows the contribution from all sources other than leptons produced in the semileptonic decay of b hadrons and in cascade decays.

technique. This test was necessary to establish the validity of the method, and was crucial during the development of the analysis to identify and diagnose problems.

To apply the method to the Monte Carlo generated with a b lifetime of 1300 fs, the sample fractions and the probability density functions for components 4 and 5 were determined from the Monte Carlo using the procedures described above. The probability density functions for the $b \rightarrow \ell$, $b \rightarrow c/\tau \rightarrow \ell$, and $c \rightarrow \ell$ components were obtained from the SMEAR data samples discussed above, and the resolution function determined from the Monte Carlo leptons was used. The electron identification efficiencies determined from the Monte Carlo were used where necessary instead of the efficiencies determined from the data. The resulting fits are shown in Figure 4.24, and the lifetimes are listed in Table 4.14. The weighted mean of these fits gives $\tau_b = 1284 \pm 23$ fs, which is in good agreement with the generated lifetime. To

components	electrons		muons	
	N_{cand}	τ_b (fs)	N_{cand}	τ_b (fs)
1	3 618	$1\,279 \pm 31$	3 318	$1\,280 \pm 32$
2	452	$1\,379 \pm 121$	374	$1\,233 \pm 123$
1+2	4 070	$1\,286 \pm 30$	3 692	$1\,275 \pm 31$
1+2+3	4 254	$1\,298 \pm 31$	3 833	$1\,275 \pm 31$
1+2+3+4	4 531	$1\,280 \pm 31$	4 085	$1\,282 \pm 32$
all	4 541	$1\,280 \pm 31$	4 150	$1\,288 \pm 33$

Table 4.14: The results of fits to the components of the Monte Carlo lepton samples generated with a b hadron lifetime of 1300 fs. The lepton candidate sample components used in each fit are indicated. The fits to the complete electron and muon samples have χ^2 s of 41.4 and 42.9 for 37 degrees of freedom, respectively, and yield a weighted mean of $1\,284 \pm 23$ fs.

	electrons	muons
N_{cand}	1 183	1 036
τ_b (fs)	$1\,944 \pm 81$	$2\,195 \pm 97$
$\chi^2/\text{d.f.}$	29.4/36	28.7/37

Table 4.15: The results of fits to the Monte Carlo lepton samples generated with a b hadron lifetime of 2000 fs. The weighted mean of the two results is $2\,047 \pm 62$ fs.

verify that each component of the Monte Carlo data was properly parameterized a lifetime fit was performed with the $b \rightarrow e$ and $b \rightarrow \mu$ components alone, and with the $b \rightarrow c/\tau \rightarrow e$ and $b \rightarrow c/\tau \rightarrow \mu$ components alone. The lifetime fit was also performed on the sum of the $b \rightarrow \ell$ and $b \rightarrow c/\tau \rightarrow \ell$ components; the $c \rightarrow \ell$ component, the hadronic component, and component 5 were then included successively and the fit was repeated at each step. These results are included in Table 4.14, and are all consistent with the generated lifetime.

A small sample of $b\bar{b}$ events was also generated with a b hadron lifetime of 2 000 fs. Since there was practically no background in this sample the resolution functions and

Figure 4.24: The results of the lifetime fits for the Monte Carlo samples generated with a b hadron lifetime of 1300 fs and 2000 fs. The data points show the distribution of impact parameters for the electron and muon candidates from the Monte Carlo. The curve is the result of the maximum likelihood fit for τ_b described in the text. The shaded areas show the contribution from all sources other than those indicated.

probability density functions determined for the 1 300 fs Monte Carlo were used in this fit. The results are listed in Table 4.15, and the fits are shown in Figure 4.24. The weighted mean of these fits gives $\tau_b = 2047 \pm 62$ fs, which is in good agreement with the generated lifetime.

These Monte Carlo fits were an important test of the lifetime fit procedure. Recall that the physics function data were generated with a b hadron lifetime of 1 500 fs. The fits described above were performed on Monte Carlo data with generated lifetimes of 1 300 fs and 2 000 fs, demonstrating that the physics function scaling procedure was valid over a wide range of lifetimes.

Chapter 5

Systematic Effects

As seen in the previous chapter, many variables were required to parameterize the impact parameter distribution for the lifetime fit. Unfortunately, this also implies that there were many possible ways to bias the result of the fit. These had to be understood in order to determine and optimize the absolute precision of the measurement. The correlations of the impact parameter with various variables are discussed in Section 5.1, and are related to most of the systematic errors discussed in this chapter. The lepton samples were divided into two subsamples corresponding to two disjoint domains of several variables, such as the lepton p or p_t , to test the lifetime fit procedure and check for possible detector-based systematic effects. These consistency checks are described in Section 5.2. A detailed description of the determination of the dominant systematic errors for this analysis is then given in the remaining sections of this chapter.

5.1 Correlations of the Impact Parameter

The measurement presented in this analysis exploits the correlation of the impact parameter with the b hadron lifetime. Unfortunately, the impact parameter is also correlated to many other parameters ranging from the kinematic variables to the

geometrical acceptance of the selection cuts. The most important of these are shown in Figures 5.1 and 5.2, which were obtained from the $b \rightarrow \ell$ SMEAR lepton samples produced for the physics functions described in the previous chapter. The leptons entering these plots satisfied all selection cuts discussed in Chapter 4.

Figure 5.1(a) shows the dependence of the mean impact parameter, $\langle \delta \rangle$, on the mean b hadron momentum, $\langle p_B \rangle$, and the mean lepton momentum, $\langle p \rangle$. The explicit dependence of the impact parameter δ on the kinematic variables and the b hadron lifetime is easily shown to be

$$\delta \propto \left(\frac{p_B}{m_B} \right) \left(\frac{p_t}{p} \right) \tau_0, \quad (5.1)$$

where p_B and m_B are the b hadron momentum and mass respectively, p is the lepton momentum, p_t is the lepton transverse momentum with respect to the b hadron direction, and τ_0 is the proper decay time of the b hadron. This dependence on p_B and p is clearly displayed in Figure 5.1(a). The mean b hadron momentum is a function of the b quark fragmentation; the correlation displayed in this figure therefore implies that the lifetime will be sensitive to the value of the Peterson fragmentation parameter, ϵ_b . In the next section, the strong correlation between the impact parameter and the mean lepton momentum is shown to be properly incorporated into the analysis by repeating the fit in two separate momentum bins.

Figure 5.1(b) shows the mean impact parameter as a function of the mean lepton momentum in the parent b hadron rest-frame, and as a function of the mean lepton transverse momentum in the lab frame, $\langle p_t \rangle$. The lepton momentum in the b hadron rest-frame is a function of the semileptonic b decay model, therefore the lifetime will also be sensitive to the choice of model. Since the momentum perpendicular to the direction of a relativistic boost is invariant, the lepton p_t in the lab frame is highly correlated with the lepton momentum in the b hadron rest-frame, making the lifetime sensitive to the p_t of the lepton as well. In the following section, this correlation is shown to be properly incorporated into the analysis by repeating the lifetime fit in two separate p_t bins.

Figure 5.1: Systematic variations of the impact parameter as a function of (a) the mean b hadron momentum and the mean lepton momentum, (b) the mean lepton momentum in the parent b hadron rest-frame and the mean lepton transverse momentum in the lab frame, (c) the number of vertex chamber hits, and (d) the mean number of jet chamber hits. These plots were obtained from the $b \rightarrow \ell$ SMEAR data used to determine the physics functions.

Figure 5.2: Systematic variations of the impact parameter as a function of (a) the mean r - ϕ angle between the jet axis and lepton, (b) the mean angle from the nearest vertex chamber anode plane, (c) the absolute value of the cosine of the jet axis polar angle, θ_{jet} , and (d) the amount of smearing applied to the jet axis with and without signing the impact parameters.

Figure 5.1(c) shows that the impact parameter is independent of the number of vertex chamber hits associated with the track, whereas Figure 5.1(d) shows that it is strongly correlated with the number of jet chamber hits. This latter effect is related to the isolation of tracks in the r - ϕ plane from the jet core. Tracks in the jet core will have fewer jet chamber hits because of track overlap and confusion in the pattern recognition process in dense track environments. The average impact parameter for these tracks is smaller because the average angular r - ϕ separation from the jet axis is smaller. This correlation is clearly displayed in Figure 5.2(a), which shows the mean impact parameter as a function of the mean r - ϕ angle between the jet axis and the lepton.

Another track isolation effect is displayed in Figure 5.2(b). This figure shows the mean impact parameter as a function of the angle from the nearest vertex chamber anode plane, called the *local CV ϕ* . This correlation is a direct result of the 1st-hit requirement on vertex chamber tracks: the further the track is from the anode plane, the more isolated it has to be from other tracks for its ionization to reach the anode plane first. Therefore lepton candidates at large angles from the vertex chamber anode planes will also tend to be at larger angles from the jet axis in the r - ϕ plane (away from the jet core), and will consequently have larger impact parameters.

Figure 5.2(c) shows the mean impact parameter as a function of the absolute value of the cosine of the jet axis polar angle, θ_{jet} . The distribution is flat up to about $|\cos \theta_{\text{jet}}| = 0.7$, after which it drops sharply. This occurs because the jet axes extend into the region $|\cos \theta| > 0.7$ but the corresponding leptons are limited to $|\cos \theta| < 0.7$, thereby restricting the leptons in this region to the core of the jet, which reduces the mean impact parameter.

Finally, Figure 5.2(d) shows the mean impact parameter as a function of the amount the jet axis is smeared in the r - ϕ plane. The jet axis in this case was taken to be the parent b hadron momentum vector. The jet axis was smeared by a Gaussian with the angular width indicated on the horizontal axis. The top curve is the result of

smearing the jet axis without changing the sign of the impact parameter, and shows the magnitude of the effect due to the changing p_t . The negative slope is a result of the increasing mean impact parameter at the cut value of $p_t = 0.8$ GeV/c (see Figure 5.1(b)). When the impact parameter is given a sign using the smeared jet axis the mean impact parameter decreases further and follows the bottom curve shown in Figure 5.2(d). These distributions show that uncertainties in the jet axis smearing and in the number of impact parameter signing errors can also lead to biases in the measured lifetime.

5.2 Consistency Checks

Several checks were performed to further test the lifetime fit procedure by dividing each lepton sample into two subsamples corresponding to two disjoint domains of various variables. The results from these tests are listed in Table 5.1 and described below. Included in Table 5.1 are the number of lepton candidates and the resulting lifetimes for each subsample, the weighted mean of the electron and muon lifetimes for each subsample, and the test statistic defined by

$$\text{test statistic} = \frac{|\tau_1 - \tau_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}}, \quad (5.2)$$

where τ_1 and τ_2 are the weighted means of the lifetimes from the two subsamples for a given variable, and σ_1 and σ_2 are the corresponding statistical errors. Unless stated otherwise, the sample fractions, the resolution function (including the trimming parameters), the physics functions, and the background distributions for components 4 and 5 were re-evaluated for each subsample selection criteria. Only the fit parameters which were significantly different for the two corresponding subsamples in each test are shown here.

The lepton samples were split into positive and negative charges, $\pm|Q|$. This procedure was sensitive to detector calibration problems related to the tilted electron drift paths relative to the jet chamber anode planes caused by the Lorentz angle.

sample	electrons		muons		weighted mean (fs)	test statistic
	N_{cand}	τ_b (fs)	N_{cand}	τ_b (fs)		
$+ Q $	1253	1517 ± 69	1347	1561 ± 71	1538 ± 49	0.44
$- Q $	1357	1542 ± 67	1415	1475 ± 66	1508 ± 47	
$ \cos \phi > 0.7$	1286	1558 ± 67	1200	1572 ± 66	1565 ± 47	1.41
$ \cos \phi < 0.7$	1324	1500 ± 69	1562	1434 ± 72	1468 ± 50	
$p > 9 \text{ GeV}/c$	1265	1573 ± 73	1389	1544 ± 69	1558 ± 50	0.10
$p < 9 \text{ GeV}/c$	1345	1562 ± 63	1373	1538 ± 66	1551 ± 46	
$p_t > 1.3 \text{ GeV}/c$	1146	1542 ± 66	1582	1441 ± 58	1485 ± 44	1.15
$p_t < 1.3 \text{ GeV}/c$	1464	1507 ± 69	1180	1658 ± 87	1565 ± 54	
near CV anode	1328	1561 ± 71	1390	1520 ± 73	1541 ± 51	0.23
far CV anode	1282	1531 ± 66	1372	1519 ± 65	1525 ± 46	
1990 data	797	1580 ± 89	807	1618 ± 96	1598 ± 65	1.39
1991 data	1813	1508 ± 57	1955	1476 ± 56	1492 ± 40	
$ \cos \theta_{\text{jet}} < 0.4$	1278	1588 ± 70	1511	1580 ± 68	1584 ± 49	1.52
$ \cos \theta_{\text{jet}} > 0.4$	1332	1496 ± 66	1251	1462 ± 70	1480 ± 48	
data on Z^0 peak	2024	1520 ± 54	2104	1557 ± 57	1538 ± 39	1.07
data off Z^0 peak	586	1556 ± 102	658	1371 ± 92	1454 ± 68	

Table 5.1: Results of the consistency checks on the data described in the text. A brief description of each sample is given, followed by the number of lepton candidates in each sample and the resulting lifetime. A weighted mean of the electron and muon results is calculated and used to calculate the test statistic described in the text.

Because of these tilted drift paths, positively charged particles, which spiral in the $-\phi$ direction, deposited slightly more charge on each anode wire than negatively charged particles with equal momentum. This effect was particularly important in the calculation of the dE/dx . All default parameters and probability density functions were used for these fits.

The lepton samples were split into horizontal and vertical tracks according to $|\cos \phi|$, where ϕ is the azimuthal angle of the track momentum vector at the point of closest approach to the beam spot in the x - y plane. This procedure tested whether or not the beam spot dimensions used in the analysis were reasonable, and verified that the beam spot contribution to the impact parameter error had been correctly calcu-

sample components	subsample fractions (% of total)			
	p range (GeV/c)		p_t range (GeV/c)	
	> 9	< 9	> 1.3	< 1.3
$b \rightarrow e$	82.7	74.8	88.7	69.9
$b \rightarrow c/\tau \rightarrow e$	5.9	15.2	5.8	15.2
$c \rightarrow e$	5.7	7.8	2.8	10.2
misidentified hadrons	5.6	1.7	2.6	4.3
Dalitz decays, $\gamma \rightarrow e^+e^-$	0.1	0.5	0.1	0.4
$b \rightarrow \mu$	79.4	71.6	83.3	65.2
$b \rightarrow c/\tau \rightarrow \mu$	8.3	11.6	6.6	14.5
$c \rightarrow \mu$	5.8	5.5	3.5	8.6
misidentified hadrons	4.3	5.4	3.8	6.0
$K \rightarrow \mu\nu, \pi \rightarrow \mu\nu$	2.2	5.9	2.8	5.7

Table 5.2: Sample fractions for p and p_t consistency checks.

lated. Since this information was built into the resolution function and probability density functions, none of the fit parameters was re-evaluated for this test.

The lepton samples were divided into high and low ranges of p , and into high and low ranges of p_t . These were very powerful tests because the relative fractions of the sample components changed considerably for each subsample, as shown in Table 5.2, and because the mean impact parameter varies substantially with p and p_t , as seen in the previous section.

The lepton candidates were divided into tracks passing near the vertex chamber anode planes, and those passing far from the anode planes, or near the cathode planes. The selection was based on the local ϕ of the tracks at a radius of 13.25 cm, which is in the center of the vertex chamber axial system. The local ϕ , or ϕ_{CV} , covers the width of an axial cell (10°) with $\phi_{CV} = 0^\circ$ and $\phi_{CV} = 10^\circ$ at the cathode planes and $\phi_{CV} = 5^\circ$ at the anode planes. The *near CV anode* subset was composed of leptons satisfying the condition $3.2^\circ < \phi_{CV} < 6.8^\circ$, and the tracks that failed this requirement fell into the *far CV anode* category. Figure 5.3(a) shows the distribution of ϕ_{CV} for all lepton candidates. The concentration around the anode planes is a result of requiring

Figure 5.3: Local vertex chamber ϕ for (a) all lepton candidates, and (b) the resolution function tracks with $\delta/\sigma < -3.0$. The histograms show the corresponding distributions from the Monte Carlo.

1st-hit vertex chamber tracks in a jet environment, and clearly displays the isolation property of this cut. The data and Monte Carlo distributions are in good agreement, which shows that the two-hit resolution of the vertex chamber was well modelled by the Monte Carlo.

The division of the lepton samples into the near and far CV anode samples was a very good test of the resolution function correction procedure because the Monte Carlo was known to only partially reproduce the degraded resolution near the vertex chamber anode planes. This effect is clearly seen in Figure 5.3(b), which shows the distribution of ϕ_{CV} for the resolution function tracks with $\delta/\sigma < -3.0$. The Monte Carlo distribution in Figure 5.3(b) is lower than the data in the anode region, indicating that there are fewer poorly reconstructed Monte Carlo tracks near the vertex chamber anode planes. The consistency of the lifetime results for the two subsamples shows that this discrepancy is correctly compensated by the resolution functions, which are

resolution function parameters	near CV anode		far CV anode	
	e	μ	e	μ
a	1.242	1.194	1.035	0.879
f	0.0368	0.0607	0.0286	0.0305
w	5.26	5.34	2.73	3.85

Table 5.3: The corrected resolution function parameters applicable to the lepton subsamples divided according to the vertex chamber local ϕ defined in the text.

listed in Table 5.3.

The lepton samples were also split into the 1990 and 1991 samples. None of the fit parameters and probability density functions was re-evaluated for this test.

The lepton samples were divided according to the polar angle of the corresponding jet axis, θ_{jet} . The region $|\cos \theta_{\text{jet}}| < 0.4$ corresponds to the central section of the barrel, while the region $|\cos \theta_{\text{jet}}| > 0.4$ corresponds to the ends of the barrel. In the latter region the particles had longer path lengths in the jet chamber and consequently deposited more charge on each anode wire than particles which were nearly perpendicular to the beam axis. The particles in this region also passed through more material in the coil which changed the shower characteristics seen by the calorimeters. These two effects could potentially affect the identification of electrons. The jet chamber also suffered from long range cross-talk which severely degraded the z resolution as a function of θ . The consistent results between the two subsamples shows that these effects were well understood, and that the dependence of the impact parameter on θ_{jet} , shown in Figure 5.1(c), was properly incorporated into the probability density functions.

Finally, the lepton samples were divided into the data accumulated on the peak of the Z^0 cross-section, at a center-of-mass energy of 91.16 GeV, and the data accumulated off the peak. Approximately 77% of the hadronic events were produced on peak. The off-peak data produced b hadrons with a slightly different distribution of momentum than the on-peak data, but the combination of approximately equal

Figure 5.4: Lifetime versus impact parameter trim for (a) the electron sample and (b) the muon sample.

samples above and below the peak should cancel the effect. The consistent lifetime results for the two subsamples shows that there are no significant differences between the two subsamples.

The results of the consistency checks are summarized in the “test statistic” column of Table 5.1. All deviations between subsamples are acceptable, with the largest observed deviation being 1.52σ .

In addition to the consistency checks described above, the lifetime fits were repeated with truncated impact parameter distributions to test the sensitivity of the method to lepton candidates with large impact parameters. For these fits, the upper and lower impact parameter bounds were varied in steps until about 1.5% of the leptons were trimmed from the tails of the distributions. The bounds were chosen such that equal fractions of lepton candidates with positive and negative impact parameters were rejected with each step. The likelihood function was normalized within the impact parameter bounds for these fits. The results are displayed in Figures 5.4(a)

and 5.4(b) as a function of the percentage of discarded lepton candidates. The solid horizontal lines show the lifetime results using the full lepton samples, and the dashed lines show the corresponding $\pm 1 \sigma$ errors. The first points with non-zero trim correspond to discarding the two lepton candidates with the largest impact parameters, which were $-3.25 \text{ mm}/+4.76 \text{ mm}$ for the electrons, and $-3.52 \text{ mm}/+3.81 \text{ mm}$ for the muons. The lifetimes were very stable over the range shown.

The points with the highest fraction of discarded tracks correspond to impact parameter ranges of $(-0.56, +2.0) \text{ mm}$ for the electron sample, and $(-0.615, +2.053) \text{ mm}$ for the muon sample. The statistical errors for these points were 57 and 56 fs, and represent about a 17% increase from the statistical errors obtained using the full lepton samples. However, only a 0.8% increase in the statistical errors was expected, demonstrating that the lepton candidates with large impact parameters carry more weight in the lifetime fit than candidates with smaller impact parameters.

5.3 Common Systematic Errors

There are two general classes of systematic errors in this analysis. The first class includes all sources of systematic errors common to both the electron and muon candidates, such as uncertainties in the branching ratios, and are discussed below. The second class includes all sources of systematic errors specific to each lepton type, such as the uncertainty in the electron bremsstrahlung correction, and will be discussed in Section 5.4. The common systematic errors are summarized in Table 5.4. The total systematic error common to the electron and muon samples was 32 fs, determined by adding the individual contributions in quadrature.

5.3.1 Quark Fragmentation

The fragmentation process was discussed in Chapter 2. The values of $\langle x_E \rangle$ used by OPAL were averages of the published set of LEP measurements [24, 25] as of the

systematic source	error on τ_b (fs)		
	e	μ	ℓ
b quark fragmentation	17	16	17
c quark fragmentation	8	8	8
semileptonic b decay models	11	9	10
semileptonic c decay models	8	9	9
b hadron lifetime in the Monte Carlo	2	4	3
B^+/B^0 lifetimes and branching ratios in the Monte Carlo	8	8	8
$BR(b \rightarrow \ell)$	3	3	3
$BR(b \rightarrow c \rightarrow \ell)$	4	2	3
$BR(c \rightarrow \ell)$	8	7	8
$BR(b \rightarrow J/\psi \rightarrow \ell)$	1	1	1
$BR(b \rightarrow \tau \rightarrow \ell)$	2	0	1
vector/pseudoscalar c meson production in b decays	2	2	2
vector/pseudoscalar c meson production in $Z^0 \rightarrow c\bar{c}$ events	1	1	1
D_s production in $Z^0 \rightarrow c\bar{c}$ events	0	2	1
Λ_c production in $Z^0 \rightarrow c\bar{c}$ events	1	1	1
jet invariant mass cutoff	0	4	2
beam spot x position	1	1	1
beam spot y position	1	2	2
beam spot size in x	13	7	10
beam spot size in y	1	3	2
resolution function without trimming	12	8	10
resolution function 10 σ and 20 σ fits	8	7	8
resolution function statistics	8	7	8
total common systematic error	32		

Table 5.4: Summary of systematic errors common to the lepton samples.

end of 1992. In general, the b quark fragmentation measurements were based on fits to the p and p_t distributions of lepton candidates identified in hadronic Z^0 decays, and the c quark fragmentation measurements were based on fits to the $\langle x_E \rangle$ distribution of fully reconstructed D^{*+} mesons. The Peterson fragmentation function was used for b and c quark fragmentation within the JETSET framework, and the resulting average parameters were $\epsilon_b = 0.0055^{+0.0040}_{-0.0030}$ and $\epsilon_c = 0.050 \pm 0.020$.

Figure 5.5: Weights versus z for various Peterson fragmentation parameters. The weights in (a) are applied to all (immediate and secondary) decay products of b hadrons for the desired value of ϵ_b , and the weights in (b) are applied to all decay products of c hadrons produced in $Z^0 \rightarrow c\bar{c}$ decays.

All Monte Carlo tracks produced in the decays of b hadrons, including particles produced in the decays of the b hadron products, were weighted from the central value of ϵ_b to the two values of $\epsilon_b = 0.0095$ and $\epsilon_b = 0.0025$, and the mean shift in lifetime was taken as the error on τ_b associated with the uncertainty in the b quark fragmentation. The weights applied to the Monte Carlo are shown in Figure 5.5(a). Recall that the Monte Carlo data were generated using $\epsilon_b = 0.0035$, so the weights applied

to the Monte Carlo in order to obtain the central value were not unity. Similarly, all primary and secondary decay products of c hadrons in the Monte Carlo were weighted from the central value of ϵ_c to the two values of $\epsilon_c = 0.070$ and $\epsilon_c = 0.030$, and the mean shift in lifetime was taken as the error on τ_b associated with the uncertainty in the c quark fragmentation. The weights as a function of z are shown in Figure 5.5(b). Recall that the Monte Carlo was generated using $\epsilon_c = 0.060$. The fragmentation parameters ϵ_b and ϵ_c were varied independently. The sample fractions, the resolution functions, and the physics functions were re-evaluated for these fits. The resulting systematic errors were 17 and 8 fs for the b and c quark fragmentation respectively.

5.3.2 Semileptonic Decay Models

The semileptonic decay models were discussed in Chapter 2. The ACCMM model was used to obtain the lifetime result of this analysis. The ISGW model and its variant, ISGW⁺⁺, were used to evaluate the dependence of τ_b on the modelling of semileptonic decays. As discussed in Chapter 4, the different decay models were simulated by weighting the leptons from the rest-frame momentum spectrum predicted by JETSET to the desired spectrum. In $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ decays, the events were weighted according to the lepton momentum spectra in the rest frame of the b hadron. B_s^0 and Λ_b^0 hadrons were weighted by the same momentum-dependent factors as B_u^+ and B_d^0 mesons. The B_s^0 and Λ_b^0 masses used in this calculation are given in Table 2.4. Following the procedure adopted by CLEO, when the ACCMM or ISGW models were used to fit OPAL data, the $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ predictions from the same model were used simultaneously. When applying the ISGW⁺⁺ prediction for the $b \rightarrow \ell$ spectrum to OPAL data, the ISGW prediction for the $b \rightarrow c \rightarrow \ell$ decay was used. The $c \rightarrow \ell$ momentum spectrum was varied independently of the $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ spectra.

The weight distributions used for this procedure are shown in Figure 5.6. They were derived from the predictions for the lepton momentum distributions for the $b \rightarrow \ell$, $b \rightarrow c \rightarrow \ell$, and $c \rightarrow \ell$ decays shown in Figure 2.6 by dividing the distribution

Figure 5.6: The weights versus the lepton momentum in (a) the b hadron rest-frame for $b \rightarrow \ell$ decays, (b) the b hadron rest-frame for $b \rightarrow c \rightarrow \ell$ decays, and (c) the c hadron rest-frame for $c \rightarrow \ell$ decays.

for each model by the distribution obtained by JETSET. The fluctuations in the weight distributions were caused by statistical fluctuations in the JETSET distributions, and were obvious only near the endpoints of the distributions where there is very little data.

The b hadron lifetime was evaluated using both the ISGW and ISGW** predictions, and the maximum deviation from the result obtained using the ACCMM model was taken as a systematic error. The resulting average systematic error was 10 fs. The ISGW variant, ISGW**, had the softest momentum spectrum in the rest-frame of the parent hadron and resulted in the highest measured lifetime. The ISGW model was then used to simulate the primary c decays. The average deviation of the resulting b hadron lifetimes from the result obtained using the ACCMM model was taken as a systematic error, and was 9 fs. The ACCMM model had the harder rest frame momentum spectrum and gave the higher measured lifetime. In each case, the sample fractions, the physics functions, and the resolution functions were re-evaluated.

5.3.3 Monte Carlo B Hadron Lifetimes

The Monte Carlo used to correct the resolution function was generated with a b hadron lifetime of 1300 fs, and therefore introduced a bias in the measured lifetime. The lifetime distribution for a generated lifetime of $\tau_0 = 1300$ fs is shown in Figure 5.7(a). The Monte Carlo lifetime was increased to 1500 fs by weighting the Monte Carlo events by the ratio of the lifetime distribution generated with $\tau_0 = 1500$ fs to the distribution shown in Figure 5.7(a). The weight distribution as a function of the proper lifetime of the b hadron is shown in Figure 5.7(b). The resolution functions were re-evaluated and new lifetime measurements were obtained. The lifetimes increased by 2 and 4 fs for the electron and muon samples respectively. The measured b hadron lifetimes quoted in Chapter 4 were therefore corrected by $(+3 \pm 3)$ fs.

The effect of different exclusive b hadron lifetimes was investigated in two ways.

Figure 5.7: (a) The normalized lifetime distribution for a proper lifetime of $\tau_0 = 1.3$ ps, and (b) the weights obtained by taking the ratio of the distributions generated using proper lifetimes of $\tau_0 = 1.5$ ps and $\tau_0 = 1.3$ ps.

In the first test the lifetime and semileptonic branching ratio of the B_u^+ meson were increased by 20%, keeping the properties of all other b hadrons fixed. The branching ratio varies by the same amount as the lifetime because the semileptonic partial width is constant, as discussed in Chapter 2. The change in lifetime was simulated by weighting the events with a weight distribution similar to the one shown in Figure 5.7. The sample fractions, the physics functions, and the resolution functions were re-evaluated and used to obtain new measured lifetimes. The average shift from the nominal lifetime was 7 fs.

In the second test the lifetime and semileptonic branching ratio of the B_u^+ meson were varied by $\pm 10\%$, and the corresponding quantities for the B_d^0 meson were simultaneously varied by $\mp 10\%$, keeping the properties of all other b hadrons fixed. This procedure kept the average b hadron lifetime of the Monte Carlo samples fixed. The lifetime changes were also simulated by weighting the events with weight distri-

butions similar to the one shown in Figure 5.7. The sample fractions, the physics functions, and the resolution functions were re-evaluated in each case and used to obtain new measured lifetimes. The average shift from the nominal lifetime was 8 fs, and was taken as the systematic uncertainty in the measured lifetime associated with uncertainties in the exclusive b hadron lifetimes.

5.3.4 Branching Ratio Uncertainties

The branching ratios for the decays $b \rightarrow \ell$, $b \rightarrow c \rightarrow \ell$, $b \rightarrow J/\psi \rightarrow \ell$, $b \rightarrow \tau \rightarrow \ell$, and $c \rightarrow \ell$, listed in Table 2.5 under the “ACCMM” column, were used to obtain the lifetime results quoted in Chapter 4. The $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ branching ratios were obtained by CLEO from $\Upsilon(4S)$ decays, and had to be corrected for the presence of B_s^0 and Λ_b^0 hadrons in LEP data, since these hadrons are not produced in $\Upsilon(4S)$ decays. These corrections are described below. The $b \rightarrow J/\psi \rightarrow \ell$, $b \rightarrow \tau \rightarrow \ell$, and $c \rightarrow \ell$ branching ratios correspond to the most recent measurements of these parameters, and are also described in what follows.

In addition to these branching ratio corrections, several of the c hadron production rates were varied by their uncertainties. These include the relative production rates of vector to pseudoscalar c mesons, and the production rates of the D_s^+ and Λ_c^+ hadrons in $Z^0 \rightarrow c\bar{c}$ decays. The latter production rates were varied substantially since they have not yet been measured.

As usual, the changes in branching ratios and production rates were simulated by weighting the Monte Carlo events by the ratio of the new branching ratios or production rates to the default JETSET values.

The $b \rightarrow \ell$ Branching Ratio

The branching ratio $BR(B_d^0, B_u^+ \rightarrow \ell)$ was measured by CLEO using their electron sample and the ACCMM model, and was $(10.5 \pm 0.2 \pm 0.3)\%$ [31]. This central value was taken as the branching ratio $BR(b \rightarrow \ell)$ for this analysis. Reference [70] describes

how the recently measured B_s^0 and Λ_b^0 production rates [27], in conjunction with 10% shifts in the B_s^0 and Λ_b^0 semileptonic branching ratios, leads to a 3.4% relative error on $BR(b \rightarrow \ell)$ from the uncertain mixture of b hadrons. This uncertainty was added in quadrature to the uncertainties in the CLEO measurement to give a final branching ratio of $BR(b \rightarrow \ell) = (10.5 \pm 0.5)\%$.

The sample fractions and the resolution functions were re-evaluated for the minimum and maximum branching ratios, and the lifetime fit was repeated in each case. The average systematic shift from the nominal lifetime was 3 fs.

The $b \rightarrow c \rightarrow \ell$ Branching Ratio

The CLEO measurement of the branching ratio $BR(b \rightarrow c \rightarrow e)$ using the ACCMM decay model is $(9.7 \pm 0.8 \pm 0.6)\%$ [31]. Production of Λ_b^0 and B_s^0 states at LEP altered this branching ratio, since the Λ_c^+ and D_s^+ states expected to be produced dominantly from Λ_b^0 and B_s^0 decays, respectively, have smaller semileptonic branching fractions than the mixture of c hadrons produced from B_d^0 and B_u^+ decays. Using abundances measured by CLEO [33] of c hadrons from B_d^0 and B_u^+ decays, and assuming that B_s^0 and Λ_b^0 hadrons decay to D_s^+ and Λ_c^0 hadrons respectively, leads to shifts of typically 7% from the $BR(b \rightarrow c \rightarrow e)$ value measured by CLEO. Therefore a correction of 0.93 ± 0.07 was applied to the CLEO $BR(b \rightarrow c \rightarrow e)$ measurement, where the uncertainty was conservatively taken to be the full size of the effect. For the ACCMM model this resulted in a $b \rightarrow c \rightarrow \ell$ branching ratio of $(9.0 \pm 1.2)\%$.

The sample fractions, the resolution functions, and the $b \rightarrow c \rightarrow \ell$ physics functions were re-evaluated for the minimum and maximum branching ratios, and the lifetime fit was repeated in each case. The average systematic shift from the nominal lifetime was 3 fs.

The $b \rightarrow J/\psi \rightarrow \ell$ Branching Ratio

Combining the measured branching ratios for $B_d^0, B_u^+ \rightarrow J/\psi$ and $J/\psi \rightarrow \ell^+ \ell^-$ decays [1] gives

$$\begin{aligned} BR(B_d^0, B_u^+ \rightarrow J/\psi \rightarrow \ell^+ \ell^-) &= (0.07 \pm 0.01)\%, \\ BR(B_d^0, B_u^+ \rightarrow J/\psi \rightarrow \ell) &= (0.14 \pm 0.02)\%. \end{aligned}$$

The unknown effect of B_s^0 and Λ_b^0 decays was included by adding an additional 25% uncertainty to this branching ratio, giving an overall branching ratio of $BR(b \rightarrow J/\psi \rightarrow \ell) = (0.14 \pm 0.04)\%$.

The sample fractions, the resolution functions, and the $b \rightarrow c \rightarrow \ell$ physics functions were re-evaluated for the minimum and maximum branching ratios, and the lifetime fit was repeated in each case. The average systematic shift from the nominal lifetime was 1 fs.

The $b \rightarrow \tau \rightarrow \ell$ Branching Ratio

The branching ratio of $b \rightarrow \tau$ relative to $b \rightarrow e$ is predicted by the quark model [72] from phase space arguments. Allowing b and c quark masses to vary in the ranges 4.8–5.2 GeV/ c^2 and 1.3–1.7 GeV/ c^2 respectively, gives a prediction for the relative τ and e branching ratios of $(25 \pm 10)\%$. More sophisticated form-factor calculations [73] do not change this conclusion. Using the well-measured $BR(\tau \rightarrow e) = (17.9 \pm 0.3)\%$ [1] gives $BR(b \rightarrow \tau \rightarrow \ell) = (0.5 \pm 0.2)\%$, where differences between measurements of $BR(b \rightarrow \ell)$ for different decay models have been neglected.

The sample fractions, the resolution functions, and the $b \rightarrow c \rightarrow \ell$ physics functions were re-evaluated for the minimum and maximum branching ratios, and the lifetime fit was repeated in each case. The average systematic shift from the nominal lifetime was 1 fs.

The $c \rightarrow \ell$ Branching Ratio

For semileptonic decays of c hadrons, the overall branching ratio was calculated considering the measured branching ratios and lifetimes of the separate hadron types. The semileptonic branching ratios and lifetimes of D_u^0 and D_d^+ mesons are well measured [1], and the lifetimes of D_s^+ and Λ_c^+ states are also known with better than 10% precision. If the semileptonic partial widths of all these c hadrons are assumed to be the same, the semileptonic branching ratios are just proportional to the particle lifetimes. Combining the different measurements with the JETSET prediction for the relative abundances $D_d^+ : D_u^0 : D_s^+ : \Lambda_c^+$ of 3.1:6.8:1.5:1, gives a predicted average semileptonic branching ratio for a c hadron from Z^0 decay of $(9.6 \pm 0.9)\%$, including the errors on the measured branching ratios and lifetimes. The variation of the $c \rightarrow \ell$ branching ratio was considered independently of the $b \rightarrow c \rightarrow \ell$ branching ratio, because different mixtures of c hadrons were involved.

The sample fractions and the resolution functions were re-evaluated for the minimum and maximum branching ratios, and the lifetime fit was repeated in each case. The average systematic shift from the nominal lifetime was 8 fs.

Vector Versus Pseudoscalar c Meson Production

The D_d^+ and D_u^0 mesons (pseudoscalars) have very different lifetimes and branching ratios, and are the dominant sources of leptons in the decays $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$. Consequently, uncertainties in their relative production rates will result in systematic uncertainties in the measured b hadron lifetime. The D_d^+ and D_u^0 mesons contributing to the decays $b \rightarrow \ell$ and $b \rightarrow c \rightarrow \ell$ are either produced directly in b hadron decays and primary c quark fragmentation, or they are produced in the decays of the vector mesons D_d^{*+} and D_u^{*0} . If the vector meson states were never produced one would expect approximately equal numbers of D_d^+ and D_u^0 mesons in both $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$ decays. However, the D_u^{*0} meson only decays into the D_u^0 meson, whereas the D_d^{*+} decays into both the D_d^+ and D_u^0 mesons with approximately equal frequency (see

decay mode	branching ratio (%)
$D_d^{*+} \rightarrow D_u^0 \pi^+$	55 ± 4
$D_d^{*+} \rightarrow D_d^+ \pi^0$	27.2 ± 2.5
$D_d^{*+} \rightarrow D_d^+ \gamma$	18 ± 4
$D_u^{*0} \rightarrow D_u^0 \pi^0$	55 ± 6
$D_u^{*0} \rightarrow D_u^0 \gamma$	45 ± 6

Table 5.5: Branching ratios of the vector mesons, $D^*[1]$.

Table 5.5). Therefore it is the uncertainty in the relative production rates of vector to pseudoscalar c mesons that dominates the uncertainty in the final mixtures of D_d^+ and D_u^0 mesons. The contribution to this uncertainty from the D^{**} mesons was ignored because their decay modes are unknown and they are expected to be produced in $\sim 10\%$ of the events, compared to the expected $\sim 60\%$ production rates of D^* mesons in both the $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$ samples.

Measurements of the vector c meson production rates in $Z^0 \rightarrow c\bar{c}$ decays are not yet available, so we must (once again) rely on measurements obtained at lower center-of-mass energies. The TASSO and HRS Collaborations, operating at center-of-mass energies around 30 GeV, have measured the product branching ratio [74]

$$BR(c \rightarrow D_d^{*+}) \cdot BR(D_d^{*+} \rightarrow D_u^0 \pi^+) \cdot BR(D_u^0 \rightarrow K^- \pi^+) = (7.3 \pm 0.6) \times 10^{-3}.$$

The branching ratio for the decay $D_d^{*+} \rightarrow D_u^0 \pi^+$ is listed in Table 5.5, and the D_u^0 branching ratio is $BR(D_u^0 \rightarrow K^- \pi^+) = 0.0365 \pm 0.0021[1]$. Using these values one obtains

$$BR(c \rightarrow D_d^{*+}) = 0.364 \pm 0.046$$

as the D_d^{*+} production rate in the c quark fragmentation process. The default JETSET rate is 0.372. Assuming that the D_d^{*+} and D_u^0 mesons are produced with equal frequency, we then take the uncertainty in the D_d^{*+} production rate of 13% as the overall uncertainty in the vector c meson production rate. Keeping the sum of the vector and

pseudoscalar meson production rates constant, the uncertainty in the vector meson rate translates into a range of 1.9 to 5.2 in the ratio of vector to pseudoscalar meson production rates. The default JETSET ratio is 3.0. The ratio of D_d^+ to D_u^0 mesons in the $c \rightarrow \ell$ sample varies by $\pm 10.5\%$ over this range.

The sample fractions, the physics functions, and the resolution functions were re-evaluated by weighting the Monte Carlo events to correspond to the two extreme ratios, and the lifetime was evaluated in each case. The average systematic shift in the lifetimes from the nominal value was 1 fs.

For cascade decays we start with the measured inclusive branching ratios of [1]

$$BR(B \rightarrow D_d^+ X) = (22.7 \pm 3.3)\%,$$

$$BR(B \rightarrow D_u^0 X) = (46 \pm 5)\%,$$

where the X means all other possible decay products. Therefore the ratio of D_d^+ to D_u^0 mesons produced in b meson decays is 0.49 ± 0.09 , corresponding to a $\pm 18\%$ uncertainty. The default JETSET production rate is 0.452. Note that these branching ratios include all contributions from the decay chains $B \rightarrow D^* \rightarrow D$. Keeping the sum of the vector and pseudoscalar production rates constant, and fixing the ratios of charged to neutral mesons produced directly in b hadron decays, one finds that the uncertainty in the ratio of D_d^+ to D_u^0 mesons quoted above translates into a range of 1.1 to 4.4 in the ratio of vector to pseudoscalar c meson production rates in b hadron decays. The corresponding default ratio in JETSET is 2.9.

The sample fractions, the physics functions, and the resolution functions were re-evaluated by weighting the Monte Carlo events to correspond to the two extreme ratios, and the lifetime was evaluated in each case. The average systematic shift in the lifetimes from the nominal value was 2 fs.

D_s and Λ_c Production

The production rates of the D_s^+ and Λ_c^+ hadrons in $Z^0 \rightarrow c\bar{c}$ decays have not yet been measured, and were therefore varied substantially to account for possible biases

in the Monte Carlo. The D_s^+ production rate was varied from 10 to 20%, and the Λ_c^+ rate was varied from 5 to 15%. The default rates in JETSET were 12.0% and 7.8% for the D_s^+ and Λ_c^+ respectively. As usual, the different production rates were simulated by weighting the Monte Carlo D_s^+ and Λ_c^+ events by the appropriate factors. The other c hadrons were also weighted appropriately to keep the total number of c hadrons fixed. The sample fractions and $c \rightarrow \ell$ physics functions were re-evaluated, and the resulting average shift from the nominal lifetime was 1 fs in both cases.

5.3.5 Jet Energy Scale

The inadequate simulation of the electromagnetic calorimeter energy clusters by the SMEAR Monte Carlo was discussed in Chapters 3 and 4. It was shown that despite this problem, the simulation of the kinematic variables of interest by the SMEAR Monte Carlo was acceptable. However, the jet finding algorithm builds up jets from tracks and unassociated lead glass clusters until the invariant mass of the jet reaches the cutoff value of $\sqrt{x_{\min}}$, so a systematic error in the modelling of the lead glass energy will bias the number of jets found. The number of jets found per tagged hadronic Z^0 decay for the default jet finding algorithm is shown in Figure 5.8(a) for the data, the full Monte Carlo, and the SMEAR Monte Carlo. The average number of jets per event is 3.343 for the data, 3.348 for the full Monte Carlo, and 3.309 for the SMEAR Monte Carlo, so the poor SMEAR simulation reduces the average number of jets per event by about 1.0%.

To estimate the impact of this bias on the measured b hadron lifetime, the data were processed with the jet finding parameter $x_{\min}$ set to $51.8 \text{ (GeV}/c^2)^2$ instead of $49.0 \text{ (GeV}/c^2)^2$, without changing the full Monte Carlo or the SMEAR Monte Carlo (the jet finding algorithm was changed for the data instead of the SMEAR Monte Carlo for practical reasons). This increase in $x_{\min}$ reduced the average number of jets in the data by 1.0%. The number of jets found per event using the new value of $x_{\min}$ is shown in Figure 5.8(b) with the corresponding distributions for the full Monte Carlo

Figure 5.8: The number of jets in the data, the full Monte Carlo, and the SMEAR Monte Carlo. The data shown in (a) were processed using the default jet finding algorithm with the invariant mass cutoff of $x_{\min} = 49.0 \text{ (GeV}/c^2)^2$. In (b) the Monte Carlo distributions are identical to those in (a), but the data were processed using $x_{\min} = 51.8 \text{ (GeV}/c^2)^2$.

and SMEAR Monte Carlo using the default value of $x_{\min}$. The sample fractions, the resolution functions, and all probability density functions were re-evaluated and the lifetime fits were repeated. The average shift from the nominal lifetimes gave a jet energy scale systematic of 2 fs.

5.3.6 Beam Spot Size and Position

The measurement of the average beam position and its horizontal and vertical spread was discussed in Chapter 4. The positions were known to about $15 \mu\text{m}$ in x and $10 \mu\text{m}$ in y , and the beam ellipse size was known to about $20 \mu\text{m}$ in x and $10 \mu\text{m}$ in y . These uncertainties were used to estimate the effect of a systematic bias in the beam position and size on the measured b lifetime. The beam size in

x was changed to 137 and 177 μm , and the lifetime was re-evaluated for each case. Similarly, new lifetimes were obtained with the beam size in y changed to 17 and 37 μm . The variations in x and in y were done independently. The average shifts in the measured lifetimes from the nominal value were 10 fs when changing S_x , and 2 fs when changing S_y . The mean beam spot x and y coordinates were also systematically shifted by $\pm 15 \mu\text{m}$ in x and $\pm 10 \mu\text{m}$ in y , which resulted in average lifetime shifts from the nominal value of 1 fs for the x shifts and 2 fs for the y shifts. The resolution functions, the probability density functions for the hadronic backgrounds, and the probability density function for the pion and kaon decays to muons were re-evaluated for each of the above fits.

5.3.7 Resolution Function Uncertainties

Three sources of systematic biases introduced by the resolution function were investigated. One source was the limited statistics used to determine the resolution function parameterizations. The sensitivity of the measured lifetime to the statistical uncertainties in the resolution function parameters was estimated by varying each parameter describing the corrected resolution function by ± 1 statistical standard deviation, and repeating the lifetime fit in each case. The parameter correlations were ignored. This produced a maximum variation in τ_b of 8 fs in the electron sample and 7 fs in the muon sample. The average value of 8 fs was taken as the statistical systematic uncertainty in the resolution function parameterization.

Another source of systematic bias in the measured lifetime was the resolution function correction procedure described in Chapter 4. Ideally, this procedure should correctly remove any biases introduced in the resolution function by the use of hadrons instead of leptons, and by the residual lifetime information on the negative side of the resolution function distributions. If this was true then the trimming procedure used to select the resolution function tracks would not be necessary. However, when the resolution function tracks were not selectively discarded before they were used in the

resolution function fits the lifetimes shifted by an average of 10 fs. This was taken as a systematic error to account for biases in the resolution function correction procedure.

The third source of systematic error is the fit procedure itself. As described in Chapter 4, the resolution function fits were limited to the region $|\delta/\sigma| < 15$ because the corresponding distributions for the Monte Carlo leptons did not extend beyond about $|\delta/\sigma| = 10$. This limit was changed from $|\delta/\sigma| < 10$ to $|\delta/\sigma| < 20$ and resulted in an average lifetime shift of 7 fs.

5.4 Lepton-Specific Systematic Errors

The systematic errors specific to one lepton species or the other are given in Table 5.6, and are described below. Adding each contribution in quadrature results in the total uncorrelated systematic errors of 32 and 24 fs for the electron and muon samples respectively.

5.4.1 Hadronic Background Uncertainties

The electron hadronic background content was measured from the data using the dE/dx_{norm} and E_{cone}/p fits described in Chapter 4. The uncertainty in the electron sample's hadronic background content was derived from uncertainties in the fit normalizations and errors induced by the uncertainty in the mean and width of the Gaussian distributions. The average of the errors from the dE/dx_{norm} and E_{cone}/p fits could be used since these were determined to be uncorrelated. An additional error was added to account for a possible momentum dependent systematic difference between the two fits. Other systematic errors were studied with different fit ranges and by using slightly different background selection requirements to determine the uncertainty in the shape of the hadronic background. They were found to be small compared to the other errors and could be neglected. Combining these effects with the statistical uncertainties in the hadronic background measurements gave a

source	error on τ_b (fs)	
	electrons	muons
background fractions	3	10
$b \rightarrow c/\tau \rightarrow e$ electron identification relative efficiencies	3	—
$c \rightarrow e$ electron identification relative efficiencies	20	—
electron bremsstrahlung	10	—
$b \rightarrow \ell$ physics function statistics	19	20
$b \rightarrow c/\tau \rightarrow \ell$ physics function statistics	9	8
$c \rightarrow \ell$ physics function statistics	3	2
hadronic background function statistics	6	2
$K/\pi \rightarrow \mu$ function statistics	—	4
total	32	24

Table 5.6: Summary of systematic errors specific to each lepton sample.

total systematic uncertainty in the electron sample's hadronic background content of $\pm 10\%$ [70]. The electron sample fractions were re-evaluated for each case and the lifetime fit was repeated. The average shift from the nominal value was 3 fs.

The fractions of the muon sample composed of misidentified hadrons and of decays-in-flight of charged pions and kaons were calculated using the fake probability per track determined from the data, as described in Chapter 4. The Monte Carlo predicted that approximately 75% of these tracks were pions, and approximately 20% were kaons. The accuracy of the Monte Carlo modelling of the fake probability per track was studied by comparing Monte Carlo predictions with data in various control samples. The same selections were applied to data and simulated events for each of the samples, and the accuracy of the modelling of track yields and matching distributions was tested. The background from charged pions was tested with identified $K^0 \rightarrow \pi^+\pi^-$ decays, complemented by $\tau \rightarrow 3\pi$ decays in $Z^0 \rightarrow \tau^+\tau^-$ events. The modelling of tracks matched incorrectly between the central tracking chambers and muon detectors was studied with samples passing the muon selection cuts except failing the $\chi_{\mu\mu} < 3$ requirement, or failing the criteria used to resolve matching ambiguities. The resulting

total uncertainty in the muon hadronic and decay backgrounds was $\pm 14\%$ [70]. While a wide range of control samples was available for these tests, not all components of the hadronic and decay backgrounds could be tested individually, so the quoted uncertainty contains an estimate of the remaining ambiguity, as well as contributions from the statistical precision of the test samples. The muon sample fractions were re-evaluated with the increase and decrease in the background normalization, and the lifetime fit was repeated in each case. The average shift from the nominal measured lifetime was 10 fs.

5.4.2 Electron Identification Relative Efficiencies

Electrons from primary b decays, cascade decays, and primary c decays have different isolation properties, and as a result the electron identification efficiency is expected to differ for each source. According to the Monte Carlo simulation, the efficiency for electrons from primary c decays differs from the efficiency for electrons from primary b decays by -40% , and the efficiency for electrons from cascade decays differs from that of electrons from primary b decays by -15% . It was unclear if this effect existed in the data, so the full size of the predicted differences were used in assessing the systematic error. The electron sample fractions were re-evaluated with the appropriate changes to the $b \rightarrow c/\tau \rightarrow e$ and $c \rightarrow e$ components, and the lifetime fit was repeated in each case. The resulting average shifts from the nominal measured lifetime were 3 fs for the $b \rightarrow c/\tau \rightarrow e$ variation, and 20 fs for the $c \rightarrow e$ variation.

5.4.3 Electron Bremsstrahlung Correction

The physics function correction for electron bremsstrahlung was discussed in Chapter 4. The shift in lifetime induced by the correction was 10 fs, and was taken as a systematic error.

5.4.4 Probability Density Function Statistics

Each parameter of the $b \rightarrow \ell$ physics functions, the $b \rightarrow c/\tau \rightarrow \ell$ physics functions, the $c \rightarrow \ell$ physics functions, the hadronic background distributions, and the function describing the distribution of charged pion and kaon decays to muons was varied by its statistical uncertainty. The correlations between parameters were not taken into account, and the maximum shift from the nominal lifetime was taken as a systematic error corresponding to the statistical uncertainties in the probability density functions. These systematic errors are summarized in Table 5.6. The largest uncertainties come from the $b \rightarrow \ell$ physics functions.

5.5 The Combined Lifetime

The results from the electron and muon fits were combined, weighting them by their statistical errors and by the systematic errors that were not correlated between the two samples. The result is

$$\tau_b = 1523 \pm 34 \pm 37 \text{ fs}, \quad (5.3)$$

where the first error is statistical and the second, systematic.

Chapter 6

Summary and Conclusions

The result of this analysis is summarized in Section 6.1. During the past year, a considerable amount of effort has been devoted to measuring the inclusive and exclusive b hadron lifetimes within the LEP experiments. These results, including some results from the CDF, CLEO, and ARGUS Collaborations, are summarized in Section 6.2. In Section 6.3, the world average for the inclusive b hadron lifetime measurement is used to derive the CKM matrix element $|V_{cb}|$, and the conclusions are given in Section 6.4.

6.1 The OPAL Inclusive B Hadron Lifetime

The combined result from the electron and muon lifetime fits is

$$\tau_b = 1\,523 \pm 34 \pm 37 \text{ fs},$$

where the first error is statistical and the second, systematic. This result corresponds to the average lifetime of b hadrons produced in $Z^0 \rightarrow b\bar{b}$ events:

$$\tau_b = \frac{\sum_i \epsilon_i f_i b_i \tau_i}{\sum_i \epsilon_i f_i b_i}, \quad (6.1)$$

where ϵ_i is the overall efficiency to tag the semileptonic decay of b hadron i , f_i is the production rate of b hadron i , b_i is the semileptonic branching ratio of b hadron i , and

τ_i is the lifetime of b hadron i . The analysis presented in this dissertation constitutes OPAL's current inclusive b hadron lifetime measurement [75], and supersedes OPAL's previously published result [76].

There are a few minor differences between the numbers given in this dissertation and the numbers appearing in Reference [75]. Most of them involved some fine-tuning of the systematic errors and produced a negligible difference in the overall systematic error (1 fs). The largest difference is in the muon sample fractions, and deserves special mention here. The published fractions for components 4 and 5, the hadronic background and the decays-in-flight of charged pions and kaons, were 7.0% and 1.8% respectively, while the numbers appearing in this dissertation are 4.8% and 4.0%. The cause of this discrepancy was partly from a "pseudo-bug" in the GEANT code, and partly from my oversight. The pseudo-bug in GEANT is that *all* neutrinos produced in the weak decays of very long-lived hadrons, such as charged pions and kaons, are labelled as *electron* neutrinos because the GEANT authors thought that nobody would care about the neutrinos! Unfortunately, this information was used in the Monte Carlo to find decays-in-flight of charged pions and kaons into *muons* that occurred at large radii within the jet chamber volume. The net effect was to shift this set of tracks from component 5 to component 4, thus increasing the hadronic background fraction. Fortunately, the tracks reconstructed in the central detector which were associated with these decays were mostly produced by the hadrons before they decayed, so the corresponding impact parameter distribution was well represented by the hadronic background probability density function. Therefore no bias was introduced by calling these tracks hadrons instead of decays-in-flight. The lifetime measurement changed by -3 fs when this error was corrected. This shift was compensated by applying the $+3$ fs correction due to the Monte Carlo lifetime, which was not done in the publication.

6.2 Summary of B Lifetime Measurements

The b hadron lifetime measurements summarized here were presented at the 5th International Symposium on Heavy Flavour Physics held in Montréal on July 6–10, 1993 [77]. Most of these measurements are relatively new and are updated frequently. The numbers shown in the following figures therefore represented a snap-shot of the situation in July, and are *not* all up-to-date.

The other three LEP experiments have also measured the inclusive b hadron lifetime [78, 79, 80] using analyses very similar to the one presented in this dissertation. In addition to the lepton impact parameter analysis, the DELPHI experiment has extended this method to fit the impact parameter distribution of all charged tracks with large p and p_t [79]. These measurements, including the OPAL result presented in this dissertation, are shown in Figure 6.1(a). A common systematic error of 0.03 ps was assumed when calculating the average of 1.46 ± 0.04 ps to account for the uncertainties in the heavy quark fragmentation and semileptonic decay models that were common to the LEP experiments.

There are a number of inclusive measurements that extract the lifetime from the decay length of partially reconstructed b hadron decays. The CDF [81] and LEP [79, 82] experiments have applied this technique using reconstructed $J/\psi \rightarrow e^+e^-$ and $J/\psi \rightarrow \mu^+\mu^-$ decays, and the DELPHI [79] experiment has used generic vertices in a similar analysis. The ALEPH [83] experiment has also used the *dipole method*, which extracts the lifetime from the distance between generic vertices in opposite hemispheres. These results are shown in Figure 6.1(b). Combining all the inclusive measurements gives a world average inclusive b hadron lifetime of

$$\tau_b = 1.49 \pm 0.03 \text{ ps.}$$

The lifetimes of the B_u^+ and B_d^0 mesons have been measured using several techniques. The CDF [84] and ALEPH [78] Collaborations have used the decay lengths of small samples of fully reconstructed B_u^+ and B_d^0 decays in several decay modes.

Figure 6.1: Inclusive b lifetime measurements (a) using the impact parameter techniques, and (b) using partially reconstructed decays. Entries labelled with the * symbol are preliminary.

Figure 6.2: B_u^+ and B_d^0 meson lifetime measurements. Entries labelled with the * symbol are preliminary.

To increase the statistics, the LEP experiments [85] have used D-lepton correlations to select samples enriched in B_u^+ and B_d^0 meson decays. In this case the lifetimes were obtained from the decay lengths formed using the reconstructed D meson and the lepton. Finally, the DELPHI experiment [86] has also performed a similar decay length analysis using generic vertices, where the charge of the B meson was determined from the total charge associated with the reconstructed vertices. The results of these analyses are shown in Figure 6.2, and yield the world average values of

$$\begin{aligned}\tau(B_u^+) &= 1.60_{-0.11}^{+0.12} \text{ ps}, \\ \tau(B_d^0) &= 1.42 \pm 0.12 \text{ ps}.\end{aligned}$$

As indicated in Chapter 2, measurements of the individual b hadron lifetimes will give us important insight into the dynamics of b hadron decays. To test the equality of the B_u^+ and B_d^0 lifetimes the CDF and L $^+L^-$ experiments also provide the ratio $\tau(B_u^+)/\tau(B_d^0)$. It is important that each experiment take the ratio of their own measurements because the B_u^+ and B_d^0 samples discussed above have a large uncertainty in the cross-contamination introduced by the selection procedure, which leads to anticorrelated errors between the two lifetime measurements for each experiment. The ARGUS [87] and CLEO [88] Collaborations have obtained indirect measurement of the ratio $\tau(B_u^+)/\tau(B_d^0)$ by measuring the semileptonic branching ratios of these mesons. Assuming that the semileptonic decay rates of the B_u^+ and B_d^0 are equal, it then follows that the ratio of the branching ratios is equal to the ratio of the lifetimes. All measurements of the ratio $\tau(B_u^+)/\tau(B_d^0)$ are shown in Figure 6.3, and are combined to give a world average of

$$\frac{\tau(B_u^+)}{\tau(B_d^0)} = 1.02_{-0.08}^{+0.09}.$$

The LEP experiments have also measured the B_s^0 lifetime [78, 79, 89] using D $_s$ -lepton correlations to identify the B_s^0 . As with the B_u^+ and B_d^0 , the lifetime was extracted from the B_s^0 decay length obtained from the fully reconstructed D $_s$ and the

Figure 6.3: B_u^+ and B_d^0 lifetime ratios. Entries labelled with the * symbol are preliminary.

charged lepton. These results are shown in Figure 6.4(a), and yield a world average of

$$\tau(B_s^0) = 1.39^{+0.24}_{-0.20} \text{ ps.}$$

Finally, the LEP experiments [90, 91] have employed similar techniques to the ones discussed above to obtain the average b baryon lifetime from Λ -lepton correlations. The ALEPH [78] and DELPHI [90] Collaborations have also measured the Λ_b^0 baryon lifetime using Λ_c^+ -lepton correlations, where the Λ_c^+ was fully reconstructed. The inclusive b baryon measurements are shown in Figure 6.4(b), and the Λ_b^0 measurements

Figure 6.4: B_s^0 and b baryon lifetime measurements. Entries labelled with the * symbol are preliminary.

are shown in Figure 6.4(c). Combining these measurements gives the world averages of

$$\tau(\text{b baryon}) = 0.97^{+0.19}_{-0.17} \text{ ps},$$

$$\tau(\Lambda_b^0) = 1.19^{+0.42}_{-0.29} \text{ ps}.$$

The b baryon lifetime is expected to be dominated by the Λ_b^0 , which is predicted to be the most common baryon produced at LEP.

All exclusive b hadron lifetime averages are shown in Figure 6.5, and are compared to the average inclusive b hadron lifetime. The exclusive lifetimes do exhibit the expected hierarchy discussed in Chapter 2, but they are not sufficiently precise to conclusively determine that they are different from each other. However, the theoretical expectation that the exclusive b hadron lifetimes should be more similar to each other than the exclusive c hadron lifetimes has proven to be correct.

Figure 6.5: Summary of b hadron lifetime measurements.

6.3 Calculation of $|V_{cb}|$

The observation that the individual b hadron lifetimes are equal within the current uncertainties allows us to extract the CKM matrix element $|V_{cb}|$ using the equations given in Section 2.7.

The average inclusive b hadron lifetime of $\tau_b = 1.49 \pm 0.03$ ps and the semileptonic branching ratio of b hadrons $BR(b \rightarrow \ell) = 0.105 \pm 0.005$ [1] are used in this calculation. Note that the error on this branching ratio covers the range of branching ratios quoted by CLEO for the various semileptonic decay models. We also need the masses of the b, c, and u quarks. These are taken to be $m_b = 5.0 \pm 0.3$ GeV/ c^2 , $m_c = 1.5 \pm 0.2$ GeV/ c^2 , and $m_u = 0.005 \pm 0.003$ GeV/ c^2 [1]. The strong coupling constant α_s evaluated at the b quark mass scale is also required. This is obtained by extrapolating the LEP result at the Z^0 mass, $\alpha_s(M_Z^2) = 0.1134 \pm 0.0035$ [1], to the b quark mass scale, which results in $\alpha_s(m_b^2) = 0.21 \pm 0.03$. The last ingredient required for this calculation is the ratio $|V_{ub}/V_{cb}|$. This is obtained from the CLEO and ARGUS measurements summarized in Reference [92]. The value used here is $|V_{ub}/V_{cb}| = 0.10 \pm 0.05$, which covers the range of values quoted in Reference [92] due to the model dependence and the discrepancy of the CLEO results with their previous published values.

The result of this calculation is

$$|V_{cb}| = 0.0372 \pm 0.0077,$$

and is displayed visually in Figure 6.6 which shows the intersection of the solution for Equation 2.17 and the value of $|V_{ub}/V_{cb}|$ quoted above (the two solid curves). The dotted curves show the one sigma uncertainties. The individual contributions to the uncertainty in $|V_{cb}|$ are listed in Table 6.1. The dominant source of uncertainty is our poor knowledge of the quark masses, and the contribution from the uncertainty in the inclusive b lifetime measurement is negligible in comparison. The value of $|V_{cb}|$ given above is consistent with the range of values allowed within the CKM matrix [1], which was given in Chapter 2.

Figure 6.6: $|V_{ub}|$ versus $|V_{cb}|$. The world average curve was obtained using the world average inclusive b hadron lifetime, and the other parameters described in the text. The ratio $|V_{ub}/V_{cb}|$ is also described in the text.

source	error
m_b	0.0070
m_c	0.0030
m_u	0.0000
$BR(b \rightarrow \ell)$	0.0009
τ_b	0.0004
$ V_{ub}/V_{cb} $	0.0003
$\alpha_s(m_b^2)$	0.0003

Table 6.1: The individual contributions to the uncertainty in $|V_{cb}|$.

6.4 Conclusions

The combined result from the electron and muon lifetime fits is

$$\tau_b = 1\,523 \pm 34 \pm 37 \text{ fs},$$

where the first error is statistical and the second, systematic. This result corresponds to the average lifetime of b hadrons produced in $Z^0 \rightarrow b\bar{b}$ events. The analysis presented in this dissertation constitutes OPAL's current inclusive b hadron lifetime measurement [75], and supersedes OPAL's previously published result [76].

The measurement of the lifetimes of b hadrons continues to be of great interest, with the emphasis currently shifting from the inclusive measurements which are now limited by systematic uncertainties, to the exclusive measurements which are still limited by statistics. The exclusive measurements will provide valuable insight into the dynamics of b hadron decays, and will help settle the arguments between theorists which were discussed in Chapter 2.

The world average inclusive b hadron lifetime can be used to obtain the CKM matrix element $|V_{cb}|$, which places constraints on the Standard Model parameters. However, this calculation is limited by theoretical uncertainties, so a better understanding of bound hadronic systems is required in order to use the improved precision of the b lifetime measurement to test the Standard Model.

In the meantime, the precise (2%) world average inclusive b hadron lifetime measurement will be useful to the LEP experiments which need this number for their current high-statistics b physics analyses.

Appendix A

The OPAL Collaboration

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Appendix B

The Analytical Convolution

The convolution of the underlying physics distributions with the detector resolution function was done analytically. The underlying physics distributions, or physics functions, were represented by a sum of exponential functions:

$$PF(\delta, \tau) = \begin{cases} \sum_{i=1}^{N_-} a_i \exp\left(+b_i \frac{\delta}{c\tau}\right) & \delta < 0 \\ \sum_{j=1}^{N_+} c_j \exp\left(-d_j \frac{\delta}{c\tau}\right) & \delta \geq 0 \end{cases}, \quad (\text{B.1})$$

where δ is the impact parameter, c is the speed of light, τ is the appropriate average lifetime, N_- is the number of exponential functions valid for negative δ , N_+ is the number of exponential functions valid for positive δ , and a_i , b_i , c_j and d_j are the parameters of the underlying physics distributions listed in Table 4.8. The resolution function was represented by the sum of two Gaussian distributions written as

$$R(\delta, \sigma, \sigma_t) = \frac{1-f}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) + \frac{f}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{\delta^2}{2\sigma_t^2}\right), \quad (\text{B.2})$$

where f is the fraction of data under the tail Gaussian, listed in Table 4.5, and σ and σ_t are the uncertainties associated with the impact parameter as described in Section 4.5.3.

The convolution of these two functions gives

$$C(\delta, \sigma, \sigma_t, \tau) = \int_{-\infty}^{+\infty} PF(x, \tau) R(x - \delta, \sigma, \sigma_t) dx \quad (B.3)$$

$$\begin{aligned} &= \sum_{i=1}^{N_-} \frac{a_i}{\sqrt{2\pi}} \left[\frac{1-f}{\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) \int_0^\infty \exp\left(-\frac{x^2}{2\sigma^2} - \left\{\frac{\delta}{\sigma^2} + \frac{b_i}{c\tau}\right\}x\right) dx + \right. \\ &\quad \left. \frac{f}{\sigma_t} \exp\left(-\frac{\delta^2}{2\sigma_t^2}\right) \int_0^\infty \exp\left(-\frac{x^2}{2\sigma_t^2} - \left\{\frac{\delta}{\sigma_t^2} + \frac{b_i}{c\tau}\right\}x\right) dx \right] + \\ &\quad \sum_{j=1}^{N_+} \frac{c_j}{\sqrt{2\pi}} \left[\frac{1-f}{\sigma} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) \int_0^\infty \exp\left(-\frac{x^2}{2\sigma^2} + \left\{\frac{\delta}{\sigma^2} - \frac{d_j}{c\tau}\right\}x\right) dx + \right. \\ &\quad \left. \frac{f}{\sigma_t} \exp\left(-\frac{\delta^2}{2\sigma_t^2}\right) \int_0^\infty \exp\left(-\frac{x^2}{2\sigma_t^2} + \left\{\frac{\delta}{\sigma_t^2} - \frac{d_j}{c\tau}\right\}x\right) dx \right]. \quad (B.4) \end{aligned}$$

To obtain Equation B.4 the integrals in the region $x < 0$ were transformed to integrals in the region $x > 0$ by reversing the order of integration and making the change of variable $x \rightarrow -x$.

The general solution to these integrals [93] is written as

$$\int_0^\infty \exp(-\beta x^2 - \gamma x) dx = \frac{1}{2} \sqrt{\frac{\pi}{\beta}} \exp\left(\frac{\gamma^2}{4\beta}\right) \operatorname{erfc}\left(\frac{\gamma}{2\sqrt{\beta}}\right) \quad (B.5)$$

for $\operatorname{Re}(\beta) > 0$, and where $\operatorname{erfc}(x)$ is the complementary error function which is related to the error function $\operatorname{erf}(x)$ by the relation $\operatorname{erfc}(x) = 1.0 - \operatorname{erf}(x)$. This result reduces Equation B.4 to the following:

$$\begin{aligned} C(\delta, \sigma, \sigma_t, \tau) &= \\ &\sum_{i=1}^{N_-} \frac{a_i}{2} \left[(1-f) \exp\left(\frac{1}{2} \left\{\frac{b_i\sigma}{c\tau}\right\}^2 + \frac{b_i\delta}{c\tau}\right) \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \left\{\frac{b_i\sigma}{c\tau} + \frac{\delta}{\sigma}\right\}\right) + \right. \\ &\quad \left. f \exp\left(\frac{1}{2} \left\{\frac{b_i\sigma_t}{c\tau}\right\}^2 + \frac{b_i\delta}{c\tau}\right) \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \left\{\frac{b_i\sigma_t}{c\tau} + \frac{\delta}{\sigma_t}\right\}\right) \right] + \\ &\sum_{j=1}^{N_+} \frac{c_j}{2} \left[(1-f) \exp\left(\frac{1}{2} \left\{\frac{d_j\sigma}{c\tau}\right\}^2 - \frac{d_j\delta}{c\tau}\right) \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \left\{\frac{d_j\sigma}{c\tau} - \frac{\delta}{\sigma}\right\}\right) + \right. \\ &\quad \left. f \exp\left(\frac{1}{2} \left\{\frac{d_j\sigma_t}{c\tau}\right\}^2 - \frac{d_j\delta}{c\tau}\right) \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \left\{\frac{d_j\sigma_t}{c\tau} - \frac{\delta}{\sigma_t}\right\}\right) \right]. \quad (B.6) \end{aligned}$$

Unfortunately, these equations suffer from the limited precision of floating point numbers available on computers. In some cases the parameter of an exponential function in

Resolution test chart showing patterns of vertical and horizontal lines with numerical values: 1.0, 1.1, 1.25, 1.4, 1.6, 1.8, 2.0, 2.2, 2.5, 2.8, 3.2, 3.6, 4.0, 4.5.

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Equation B.6 is large enough to cause a floating point overflow, and at the same time the parameter of the corresponding complementary error function is large enough to cause a floating point underflow, but the product of the two results is a well behaved number! This occurs mostly when evaluating the convolution of the tail Gaussian of the resolution function with the sharply peaked exponentials of the underlying physics distributions.

To overcome this problem we use the rational approximations for the error function $\text{erf}(x)$ and the complementary error function $\text{erfc}(x)$ given in Reference [94]. The approximation for $\text{erf}(x)$ is

$$\text{erf}_1(x) = x \left(\sum_{j=0}^3 p_{1j} x^{2j} \right) \left(\sum_{j=0}^3 q_{1j} x^{2j} \right)^{-1}, \quad (\text{B.7})$$

and is valid in the range $|x| \leq 0.47$. The complementary error function has two approximations. The first is

$$\text{erfc}_1(x) = \exp(-x^2) f_1(x), \quad (\text{B.8})$$

$$f_1(x) = \left(\sum_{j=0}^7 p_{2j} x^j \right) \left(\sum_{j=0}^7 q_{2j} x^j \right)^{-1}, \quad (\text{B.9})$$

and is valid in the range $0.47 \leq x \leq 4.0$. The second approximation for $\text{erfc}(x)$ is

$$\text{erfc}_2(x) = \exp(-x^2) f_2(x), \quad (\text{B.10})$$

$$f_2(x) = \frac{1}{x} \left[\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} \left(\sum_{j=0}^4 p_{3j} x^{-2j} \right) \left(\sum_{j=0}^4 q_{3j} x^{-2j} \right)^{-1} \right], \quad (\text{B.11})$$

and is valid in the range $x \geq 4.0$. The parameters for these functions are listed in Tables B.1, B.2, and B.3, and are suitable for double precision floating point numbers in FORTRAN.

The exponential term in the approximations for $\text{erfc}(x)$ has the desired effect of controlling the numerical problems, as we will see below. The relations $\text{erf}(-x) = -\text{erf}(x)$, and $\text{erfc}(-x) = 2 - \text{erfc}(x)$ are used to evaluate the functions for negative arguments. Therefore, the entire range of x can be covered using the following combinations of the approximations

described above:

$$\text{erfc}(x) = \begin{cases} 2.0 - \text{erfc}_2(|x|) & x \leq -4.0 \\ 2.0 - \text{erfc}_1(|x|) & -4.0 \leq x \leq -0.47 \\ 1.0 - \text{erf}_1(x) & |x| \leq 0.47 \\ \text{erfc}_1(x) & 0.47 \leq x \leq 4.0 \\ \text{erfc}_2(x) & x \geq 4.0 \end{cases} \quad (\text{B.12})$$

The convolution for the range $x \leq 0.47$ is obtained by substituting the appropriate approximation from Equation B.12 into Equation B.6, and will not be expanded here. However, important cancellations occur for the case $x \geq 0.47$. For $0.47 \leq x \leq 4.0$ the convolution reduces to the following:

$$\begin{aligned} C(\delta, \sigma, \sigma_t, \tau) = & \sum_{i=1}^{N_-} \frac{a_i}{2} \left[(1-f) \exp\left(-\frac{1}{2} \left\{ \frac{\delta}{\sigma} \right\}^2\right) f_1\left(\frac{1}{\sqrt{2}} \left\{ \frac{b_i \sigma}{c\tau} + \frac{\delta}{\sigma} \right\}\right) + \right. \\ & \left. f \exp\left(-\frac{1}{2} \left\{ \frac{\delta}{\sigma_t} \right\}^2\right) f_1\left(\frac{1}{\sqrt{2}} \left\{ \frac{b_i \sigma_t}{c\tau} + \frac{\delta}{\sigma_t} \right\}\right) \right] + \\ & \sum_{j=1}^{N_+} \frac{c_j}{2} \left[(1-f) \exp\left(-\frac{1}{2} \left\{ \frac{\delta}{\sigma} \right\}^2\right) f_1\left(\frac{1}{\sqrt{2}} \left\{ \frac{d_j \sigma}{c\tau} - \frac{\delta}{\sigma} \right\}\right) + \right. \\ & \left. f \exp\left(-\frac{1}{2} \left\{ \frac{\delta}{\sigma_t} \right\}^2\right) f_1\left(\frac{1}{\sqrt{2}} \left\{ \frac{d_j \sigma_t}{c\tau} - \frac{\delta}{\sigma_t} \right\}\right) \right], \end{aligned} \quad (\text{B.13})$$

which is now a well behaved function. For the case of $x \geq 4.0$ the function f_1 in Equation B.13 is replaced by the function f_2 . f_1 and f_2 are defined in Equations B.9 and B.11.

j	p_{1j}	q_{1j}
0	+2.42667 95523 05318 E+2	+2.15058 87586 98612 E+2
1	+2.19792 61618 29415 E+1	+9.11649 05404 51490 E+1
2	+6.99638 34886 19136 E+0	+1.50827 97630 40779 E+1
3	-3.56098 43701 81539 E-2	+1.00000 00000 00000 E+0

Table B.1: Parameters of the error function approximation $\text{erf}_1(x)$. Valid in the range $|x| \leq 0.47$.

j	p_{2j}	q_{2j}
0	+3.00459 26102 01616 E+2	+3.00459 26095 69833 E+2
1	+4.51918 95371 18729 E+2	+7.90950 92532 78980 E+2
2	+3.39320 81673 43437 E+2	+9.31354 09485 06096 E+2
3	+1.52989 28504 69404 E+2	+6.38980 26446 56312 E+2
4	+4.31622 27222 05674 E+1	+2.77585 44474 39876 E+2
5	+7.21175 82508 83094 E+0	+7.70001 52935 22947 E+1
6	+5.64195 51747 89740 E-1	+1.27827 27319 62942 E+1
7	-1.36864 85738 27167 E-7	+1.00000 00000 00000 E+0

Table B.2: Parameters of the complementary error function approximation $\text{erfc}_1(x)$. Valid in the range $0.47 \leq x \leq 4.0$.

j	p_{3j}	q_{3j}
0	-2.99610 70770 35422 E-3	+1.06209 23052 84679 E-2
1	-4.94730 91062 32507 E-2	+1.91308 92610 78298 E-1
2	-2.26956 59353 96869 E-1	+1.05167 51070 67932 E+0
3	-2.78661 30860 96478 E-1	+1.98733 20181 71353 E+0
4	-2.23192 45973 41847 E-2	+1.00000 00000 00000 E+0

Table B.3: Parameters of the complementary error function approximation $\text{erfc}_2(x)$. Valid for $x \geq 4.0$.

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