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PARAMETER IDENTIFICATION OF A NON-LINEAR ELECTRORHEOLOGICAL FLUID DAMPER MODEL

by

Glen Mui, B. Eng.

*A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering*

Department of Mechanical and Aerospace Engineering

Ottawa-Carleton Institute

for Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

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**"PARAMETER IDENTIFICATION OF A NON-LINEAR
ELECTRORHEOLOGICAL FLUID DAMPER MODEL"**

submitted by Glen Mui, B. Eng.

in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Co-Supervisor

Thesis Co-Supervisor

Chair, Department of Mechanical
and Aerospace Engineering

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Abstract

A controllable vehicle suspension system has performance advantages in ride comfort and road holding over the conventional passive suspension. The semi-active suspension, which can only modulate energy dissipation, can achieve comparable performance with controllable suspension systems that do not have this restriction. The hardware used in semi-active systems are usually controllable vibration dampers. A simple lumped parameter model of an Electrorheological (ER) fluid based damper was identified from dynamic experiments. The results of the analysis indicate that the ER effect on the model parameters is enhanced with increased electric field but is attenuated with increases in either mechanical excitation frequency or amplitude. Although the response of the ER effect is fast, the force from the damper continues to rise slowly with the application of a constant voltage. This time dependency may be an inherent property of ER flow mode devices rather than a property of the ER effect.

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Table of Contents

	Page
Acceptance Sheet	ii
Abstract	iii
Acknowledgements	iv
Table of Contents	v
List of Tables	ix
List of Figures	x
Nomenclature	xiii
 <u>Chapter 1: Introduction</u>	
1.0 Introduction	1
 <u>Chapter 2: Overview of Vehicle Suspension Design:</u>	
2.0 Introduction	4
2.1 The Quarter Car Model	5
2.2 The Passive Suspension	9
2.3 The Active Suspension	13
2.4 Slow-Active Suspensions	17
2.5 Semi-Active Suspension	19
2.6 Semi-Active Control	20
2.7 Comparison of Control Laws	25

2.8 The Estimation of States	29
2.9 Summary	30
<u>Chapter 3: Controllable Dampers and the Electrorheological Effect</u>	
3.0 Introduction	31
3.1 Semi-Active Suspension Hardware Alternatives	32
3.2 The ER Effect	35
3.2.1 Fibrillation	36
3.2.2 Interfacial Effects	37
3.2.3 The Role of Water:	38
3.3 ER Fluid Damper Arrangements	39
3.4 Rheological Properties	42
3.4.1 Properties in Shear Mode	43
3.4.2 Properties in Flow Mode	45
3.5 Fluid Analysis	48
3.6 Theoretical Predictions	51
3.7 Damper Design	55
3.8 Summary	58
<u>Chapter 4: Experiments and Model Identification</u>	
4.0 Introduction	59
4.1 Experiments	60
4.1.1 Mechanical	60
4.1.2 Electrical	63
4.1.3 Thermal	66

4.2 Procedure	67
4.2.1 Transient Tests	68
4.2.2 Steady State Oscillatory Tests	68
4.3 Modelling	69
4.3.1 Previous Damper Models	69
4.3.2 Proposed Damper Model	71
4.4 System Identification	73
4.4.1 Classical Identification Methods	73
4.4.2 Non-linear Least-Squares Algorithms	78
4.4.3 Comparison of Least-Squares Algorithms	83
4.5 Implementation	84
4.6 Summary	89
<u>Chapter 5: Results</u>	
5.0 Introduction	90
5.1 Qualitative Observations	91
5.2 Estimating Parameters	100
5.3 Quantitative Results	102
5.3.1 Assessment of Force Prediction Errors	102
5.3.2 Assessment of Parameter Values	106
5.4 Summary	118
<u>Chapter 6: Discussion</u>	
6.0 Introduction	120
6.1 Discussion	121
6.2 Considerations for Controller Design	124

6.3 Model Improvements	127
6.4 Summary	131
 <u>Chapter 7: Conclusions and Future Work</u>	
7.0 Conclusions	132
7.1 Future work	133
 References	 135
Appendix A: Data Plots	144
Appendix B: C++ Program	184
Appendix C: Results in Tabular Form	201

List of Tables

Table	Description	Page
2.1	Results of Simulations Comparing Control Laws Based on Ride Comfort . . .	26
3.1	Rheometry Results By Catania (1994)	52
4.1	Identification Sensitivity to Mechanical Excitation Amplitude	86
4.2	Identification Sensitivity to Mechanical Excitation Frequency	86
4.3	Identification Sensitivity to Sampling Period	87
4.4	Identification Sensitivity to Sample Size	88
C1	Mean Rectified Error in Predicted Damping Force Using Averaged Parameter Values Obtained Over the Entire Test Interval	202
C2	Mean Rectified Error in Predicted Damping Force Using Time Varying Parameter Values	203
C3	Inertial Parameter Values Identified From Transient Test Results	204
C4	Viscous Friction Parameter Values Identified From Transient Test Results . . .	205
C5	Coulomb Friction Parameter Values Identified From Transient Test Results	206
C6	Bias Parameter Values Identified From Transient Test Results	207
C7	Root Mean Squared Damping Force From Transient Test Results and Force Amplification From Transient Results to Steady State	208
C8	Parameter Values For Steady State Test Results Using the Fly-Back Converter	209

List of Figures

Figure	Description	Page
2.1	Schematic of the Quarter Car Model of a Vehicle Suspension System	5
2.2	Road Acceleration to Sprung Mass Acceleration Transfer Function Frequency Response for Various Values of Suspension Damping	10
2.3	Road Acceleration to Tyre Deflection Transfer Function Frequency Response for Various Values of Suspension Damping	10
2.4	Road Acceleration to Suspension Deflection Transfer Function Frequency Response for Various Values of Suspension Damping	12
2.5	Schematic of the Quarter Car Model of an Active Suspension System	14
2.6	Schematic of the Quarter Car Model of a Slow-Active Suspension System	18
2.7	Schematic of the Quarter Car Model of a Sky-Hook Suspension System	21
2.8	Proposed Control Hardware Implementation Using Check Valves	27
3.1	Venhovens (1993) Cone Mechanism for Anti-Roll/Pitch Control	33
3.2	Fibrillation Model	36
3.3	Double Layer Model	38
3.4	Traditional ER Damper Arrangements	40
3.5	Ball-Screw Damper Design Proposed by Coulter (1993)	41
3.6	Shear Stress versus Strain Graph	44
3.7	Mechanical Analogy of the ER Effect Proposed by Gamota (1991)	44
3.8	Shear Stress versus Shear Rate Graph of the Bingham Plastic Model	46
3.9	Bingham Flow in an Annular Gap	48
3.10	Shear Stress versus Shear Rate Graph of Rheometry Tests by Catania (1994) .	51

3.11	Comparison of Predicted and Experimental Results of Force for a Flow Mode Damper Using a Single Gap Arrangement	52
3.12	Comparison of Predicted and Experimental Results of Force for a Flow Mode Damper Using a Double Gap Arrangement	53
3.13	Comparison of Predicted and Experimental Results of Force for a Flow Mode Damper Using a Triple Gap Arrangement	53
3.14	Schematic of the TTRL ER Flow Mode Damper	55
3.15	Drawing of the Cylinder Construction	56
4.1	Schematic of the MTS Layout and Instrumentation	61
4.2	Load Cell Calibration Data Points and Resulting Fit	63
4.3	Electrical Network Used to Measure Voltage and Current	64
4.4	Operational Amplifier Calibration Data Points and Resulting Fit	65
4.5	Proposed ER Flow Mode Damper Model	72
4.6	Block Diagram of Correlation Identification Method	75
4.7	Block Diagram of Weiner Theory of Identification	77
5.1	Transient Test Results Using the Universal Voltronics Voltage Supply. Commanded Displacement is a 1Hz Sinusoid of Amplitude 10mm	92
5.2	Steady State Test Results Using the Universal Voltronics Voltage Supply. Commanded Displacement is a 1Hz Sinusoid of Amplitude 10mm	93
5.3	Step Down Transient Test Results Using the Universal Voltronics Voltage Supply Commanded Displacement is a 5Hz Sinusoid of Amplitude 10mm	94
5.4	Transient Test Results Using the Fly-Back Converter Voltage Supply. Commanded Displacement is a 1Hz Sinusoid of Amplitude 10mm	96
5.5	Steady State Test Results Using the Fly-Back Converter Voltage Supply Commanded Displacement is a 1Hz Sinusoid of Amplitude 10mm	97

5.6	Transient Test Results Using the Universal Voltronics Voltage Supply. Commanded Displacement is a 1Hz Saw-Tooth of Amplitude 10mm	99
5.7	Electrical Analogy of Damper Inertia	101
5.8	Mean Rectified Error in Predicted Damping Force Using Averaged Parameter Values Obtained Over the Entire Test Interval	104
5.9	Mean Rectified Error in Predicted Damping Force Using Time Varying Parameter Values	105
5.10	Measured Force, Predicted Force and Relative Velocity for an ER Damper at an Oscillation Freq. of 1Hz of Amplitude 10mm and Electric Field Strength 1kV/mm	107
5.11	Measured Force, Predicted Force and Relative Velocity for an ER Damper at an Oscillation Freq. of 9Hz of Amplitude 3mm and Electric Field Strength 1.5kV/mm	107
5.12	Inertial Parameter Values Identified From Transient Test Results	108
5.13	Viscous Friction Parameter Values Identified From Transient Test Results ...	109
5.14	Coulomb Friction Parameter Values Identified From Transient Test Results ..	110
5.15	Bias Parameter Values Identified From Transient Test Results	111
5.16	Root Mean Squared Damping Force From Transient Test Results	113
5.17	Force Amplification From Transient Results to Steady State	114
5.18	Parameter Values For Steady State Test Results Using the Fly-Back Converter	115
5.19	Measured and Predicted Force Using Tabulated Model Parameters. Oscillation Frequency of 5Hz of Amplitude 10mm	117
6.1	Possible Explanation of the ER behaviour in a Flow Mode Damper	122
6.2	Single Degree of Freedom System with ER Damper	125
6.3	Revised ER Damper Model Including Fluid Compressibility Effect	128

Nomenclature

Symbol	Definition
ζ	a vector of parameters
α	the ratio of the dynamic tyre deflection to the static tyre deflection
A, B, C, D	matrices used in the state space representation of a system
A_o	the piston area
A_v	the valve frontal area
β	weighting vector
b_i	the measured system outputs
C_o, C_i	damping coefficient
C_{hard}	hard damping coefficient
C_s	the suspension damping from the shock absorber.
C_{soft}	the lowest damping coefficient possible
C_t	the damping due to tyre hysteresis.
c', c	constants of integration
d	piston displacement
$\Delta\zeta$	change in parameters
p_i	fluid momentum
ΔP	the pressure differential across the gap
ΔP_{ER}	the pressure drop due to the ER effect,
ΔP_T	steady pressure drop across the ER valve
Δt	the sample period
ϕ	dimensionless parameter representing the fluid velocity
F	the force from the actuator
F_c	force contributions from Coulomb friction
F_v	force contributions from viscosity
F_d	another variable name for this control law of eq. 2.22
F_k	force contributions from non-linear stiffness
F_r	another variable name for this control law of eq. 2.21
g	the acceleration due to gravity
H	a vector of values of the impulse response
$H(s)$	the transfer function
$h(t)$	the impulse response
I	identity matrix
I_1, I_2	the inertia
J	the Jacobian matrix
κ	the ratio between the radial distance of the inside edge of the gap to R
K	the controller gain matrix
K_s	the suspension stiffness.
K_t	the tyre stiffness.

λ_o and λ_i	the ratios of the outer and inner edge of the plug region to R.
L and h	valve dimensions
m	the mass of the ER fluid in the valve
μ	the fluid viscosity
M_s	the sprung mass.
M_{us}	the unsprung mass.
n	the number of observations
γ	represents an exponent
θ	a matrix governed by the Ricatti equation
P(t)	the particular solution
Q	flow rate
q_1 , q_2 , and q_3	weighting factors
ρ	the fluid density
ψ	dimensionless parameter representing the radial distance
R	radial distance from the centre of the annulus to the outer edge of the gap.
r(t)	the homogeneous solution
R_1 , R_2 , R_3	friction parameters
s	the Laplace operator
sgn	the sign operator
t	time
τ	the shear stress,
T	dimensionless parameter representing the shear stress,
τ_y	the yield stress
T_y	dimensionless parameter representing the yield stress
u	the system input
u_0	the load cell bias
u_n	input expanded in a Laguerre function series
u_o	input amplitude
V	the fluid velocity
v(t)	disturbance
V_n	Hermite polynomials
w , w_1 , w_2	frequency, sprung mass and unsprung mass natural frequencies in rads/s
f , f_1 , f_2	frequency, sprung mass and unsprung mass natural frequencies in Hz
x	the state vector
Ξ	the parameter influence matrix
X_1	the suspension deflection, Z_1 - Z_2 .
X_2	the dynamic tyre deflection, Z_2 - Z_0 .
X_3	the time derivative of the suspension deflection
X_4	the time derivative of the tyre deflection
y	the system output
z(t)	measurement of the system
Z_0	the road surface position.
Z_1 , Z_2	the sprung mass and unsprung mass inertial positions.

Chapter 1: Introduction

1.0 Introduction:

Passive suspensions are the most widely used method of vibration isolation for automotive applications. Years of experience in selecting spring and shock absorber components have led to nearly optimized performance for this type of suspension system (Burton 1992). There are many theoretical and experimental studies, however, that demonstrate that further performance benefits can be realized using controllable elements in automobile suspensions. Elements which can dissipate as well as inject energy into the suspension system, making it an active suspension system, offer the most flexibility for control but in general are expensive and inefficient. A more restrictive but economical alternative would be to have controllable elements that only dissipate energy resulting in semi-active suspensions. The Transport Technology Research Laboratory (TTRL) at Carleton University has developed such a damper that is both simple in design as well as cost effective (Gorodkin et.al. 1979). This damper modulates energy dissipation by utilizing the electric field induced rheological properties of electrorheological (ER) fluids. The performance of this damper using skyhook and on-off control strategies has shown improvements over a passively damped system. Further improvements may be realized by using more sophisticated control strategies. Often these strategies require a model of the behaviour of the system under typical operating conditions.

Modelling of an ER fluid is difficult because the mechanism behind the ER effect is not fully understood. Most studies examine the behaviour of ER fluids in a gap formed

between two parallel plates separated by a fixed distance. Fluid flow is achieved by either fixing one plate and moving the other parallel to it, referred to as shear mode, or fixing both plates and inducing a pressure gradient within the gap which is referred to as flow mode. Although abundant information is available on the behaviour of ER fluids under unsteady shear mode conditions the same cannot be said for unsteady flow mode conditions. Previous attempts at modelling ER fluid behaviour in shear have encountered varying degrees of viscoelastic and plastic behaviour that depends upon the electric field applied and the rate of deformation. In flow mode devices hysteresis has been identified at low flow rates (Bullough 1989) as well as transient delays in the application of force (Nakano and Yonekawa 1994).

The objective of this work was the identification of a non-linear lumped parameter model for a flow mode ER damper. Chapter 2 will first establish the need for these types of suspension systems and will continue to describe the performance limitations of passive, active and semi-active automotive suspension systems through the study of a simple quarter car representation. Known semi-active control laws will be reviewed and compared using simulations. In Chapter 3 semi-active hardware alternatives including ER devices will be presented and our present understanding of the mechanism and behaviour of ER fluids will be reviewed. Chapter 4 describes the experimental apparatus, quantities measured and the procedure used to conduct the experiments. The model used to describe the ER damper will be given as well as a review of the methods available for identifying the parameters of this model. Results of the experiments and identification will be presented in Chapter 5 and a discussion of these results and their implications for the

control of ER damping devices will be given in Chapter 6.

Chapter 2: Overview of Vehicle Suspension Design

2.0 Introduction:

This chapter will describe the vehicle suspension system from the point of view of suspension design and controller design. Some control concepts and the quarter car model will be introduced in Section 2.1. Most of the discussion throughout this chapter will be based on this simple representation of the vehicle suspension system. Section 2.2 discusses the compromise in performance inherent in conventional passive suspension systems. Sections 2.3 through 2.5 introduce three types of controllable suspensions and the advantages and disadvantages of each system. Non-linear control laws are reviewed in Section 2.6 and simulation results are provided in Section 2.7. Section 2.8 illustrates how a model of the vehicle suspension can aid in controller design by estimating system quantities.

2.1 The Quarter Car Model:

A two degree of freedom model of a passenger vehicle suspension system is a popular choice by many suspension researchers. This model, often referred to as a quarter car model, is greatly simplified as it does not include the coupled response from the suspensions at the other three wheel stations. Despite this simplification, the model is a good reflection of the real performance of the full vehicle (Crolla et. al. 1989). Figure 2.1 shows a schematic of this model described as a system of masses, springs and dashpots.

Figure 2.1: Schematic of the quarter car model of a vehicle suspension system. The description of the model parameters are described, as follows, in the text.

The quarter car model parameters are:

- M_s is called the sprung mass and represents the mass of the vehicle chassis and payload.
- M_{us} is called the unsprung mass and represents the mass of the wheel and suspension linkages.
- K_s represents the suspension stiffness.
- K_t represents the tyre stiffness.
- C_s represents the suspension damping from the shock absorber.
- C_t represents the damping due to tyre hysteresis.
- Z_1 represents the sprung mass inertial position.
- Z_2 represents the unsprung mass inertial position.
- Z_0 represents the road surface position.

Often the tyre damping, C_t , has been omitted in order to simplify the analysis. The inertial positions of the sprung and unsprung masses were set to zero at their static equilibrium positions. Since the initial compression of the springs from their free lengths balances the constant forces of gravity on the masses, the force of gravity term in the equations of motion is not apparent.

Assuming the above elements are linear, a simple application of Newton's laws of motion to this system results in a set of first order equations which have been expressed in state space form in Equations 2.1 and 2.2.

$$\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_e}{M} & \frac{K_t}{M_{us}} & -\frac{C_e}{M} & 0 \\ \frac{K_s}{M_{us}} & -\frac{K_t}{M_{us}} & \frac{C_s}{M_{us}} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \ddot{z}_0 \quad (2.1)$$

$$M = \frac{M_{us} M_s}{M_{us} + M_s} \quad (2.2)$$

$$\frac{d}{dt} \bar{x} = A \bar{x} + B u$$

$$y = C \bar{x} + D u$$

C and D are matrices which operate on the state vector x and input u to obtain the system output y . X_1 represents the suspension deflection, Z_1 - Z_2 , X_2 represents the dynamic tyre deflection, Z_2 - Z_0 , X_3 and X_4 represent the time derivative of the suspension deflection and tyre deflection respectively. Selection of states is not unique. Depending upon how the system states are chosen the equations of motion may have a different appearance to the one presented. The transfer function, $H(s)$, which represents the relationship between input and output, can be obtained by Equation 2.3 where 's' represents the Laplace operator.

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B + D \quad (2.3)$$

$$= H(s)$$

The denominator of the transfer function is called the characteristic equation and the roots of this equation are the eigenvalues, sometimes called poles. The eigenvalues of the system represent the natural modes of the system and will remain unchanged regardless of how

the states are chosen. See Ogata (1990) for a more complete discussion of state space system representation and methods.

Often of interest is the response of the system to harmonic input. Assume the input, $u(t)$, is a cosine function with frequency ' w ' and an amplitude u_o given in the form of Equation 2.4.

$$u(t) = u_o \cos(w \cdot t) \rightarrow U(s) = \frac{u_o s}{s^2 + w^2} \quad (2.4)$$

The system output will take the form of Equation 2.5 at steady state.

$$y_{ss}(t) = A_1 e^{-j\omega t} + A_2 e^{j\omega t} \quad (2.5)$$

The amplitudes A_1 and A_2 may be computed as shown in Equation 2.6.

$$A_1 = \left. \frac{u_o s}{s - j\omega} H(s) \right|_{s = -j\omega}, \quad A_2 = \left. \frac{u_o s}{s + j\omega} H(s) \right|_{s = j\omega} \quad (2.6)$$

Alternatively, Equation 2.5 may be expressed as shown in Equation 2.7.

$$y_{ss}(t) = u_o |H(j\omega)| \cos(\omega t + \angle H(j\omega)) \quad (2.7)$$

Clearly from Equation 2.7 the magnitude of $H(j\omega)$ serves to amplify the input and the phase of $H(j\omega)$ describes the phase difference between input and output. Therefore Bode plots, which are plots of the magnitude and the phase of the transfer function against frequency, are often used to evaluate the performance of the system. The selection of transfer functions which best demonstrate the performance of the system is a critical step in this evaluation.

The purpose of a suspension system is to control the ride comfort and road holding performance of a vehicle while operating within the available suspension working space provided. Road holding describes the ability of a suspension to maximize vehicle control. This can be achieved by minimizing variations in tyre load. Ride comfort is a term describing the ability to isolate the passenger's movement from road irregularities. This term is subjective and depends upon an individual's level of tolerance to vibration. Statistical results published by the Society of Automotive Engineers (SAE) Ride and Vibration Manual (1965) and statistics compiled by Stikeleather et.al. (1973) have demonstrated that these levels are dependent upon frequency, amplitude, direction and duration of vibration. In light of the number of variables which influence an individual's perception of ride comfort, a certain amount of adjustability in a suspension system is warranted. The effects of adjusting the value of the suspension damping in a conventional automotive suspension system are the subject of Section 2.2. Note that the vertical motion of the suspension is examined only.

2.2 The Passive Suspension:

All the elements in the model discussed above are passive. A great deal of judgement must be exercised in selecting suspension elements based on the anticipated operating conditions of the vehicle. Consider the plots of road induced acceleration to sprung mass acceleration and road induced acceleration to tyre deflection transfer functions for various values of suspension damping shown in Figures 2.2 and 2.3 and one can appreciate how difficult this task is.

Figure 2.2: Road induced acceleration to sprung mass acceleration transfer function frequency response for various suspension damping. This transfer function describes the ride comfort characteristics of the suspension model.

Figure 2.3: Road induced acceleration to tyre deflection transfer function frequency response for various suspension damping. This transfer function describes the road holding characteristics of the suspension model. Note that the trends in road holding performance are identical to ride comfort except for frequencies above the second resonance.

The parameters are:

- $M_s = 44.2 \text{ kg}$
- $M_{us} = 6.2 \text{ kg}$
- $K_s = 24.5 \text{ kN/m}$
- $K_{us} = 2.846 \text{ kN/m}$
- $C_s = 14, 108, \text{ and } 216 \text{ N*s/m}$

These parameters were for a one seventh scale quarter car model based on the properties of a quarter car apparatus available in the TTRL.

Wong (1993) uses the road surface position to sprung mass position and road surface position to tyre deflection transfer functions to evaluate ride comfort and road holding respectively. However, the same trade offs in suspension performance are still evident. There exist two break or resonant frequencies in the response. The first resonant frequency, f_1 , often called the sprung mass natural frequency, occurs around 1 Hz in typical passenger vehicles. The second resonant frequency, f_2 , occurring around 10 Hz is often called the unsprung mass natural frequency.

Between f_1 and f_2 both ride comfort and road holding are degraded when suspension damping is increased but are improved for frequencies in the vicinity of f_1 and below. The compromise is between the frequency ranges, above or below f_1 , that are controlled. In the frequency region above f_2 , increasing C_s degrades ride comfort but improves road holding performance. In other words, above f_2 there is a conflict between ride quality and road holding.

One could optimally select C_s based on a single vehicle speed and single road

surface profile, but in a practical situation this damping selection will never be optimal. In general, if a vehicle were required to perform many high speed manoeuvres or if the road quality was poor, a high suspension damping would be needed. Conversely if a vehicle only travelled in straight paths and roads were of good quality, low suspension damping would be needed. A passive suspension is a compromise between road holding and ride comfort since in practice the vehicle requirements change depending upon the present operating conditions of the vehicle and the perception of ride comfort by the driver.

Another important consideration is the working space or 'rattle space' which refers to the amount of space available for tyre motion without encountering interference from the vehicle body. A set of road conditions may increase the amplitude of suspension travel or working space may be reduced with an increase in vehicle payload. The plot of road acceleration to suspension deflection in Figure 2.4 describes the suspension travel for various values of suspension damping.

Figure 2.4: Road acceleration to suspension deflection transfer function frequency response for various values of suspension damping. Increasing the suspension damping reduces the amount of working space required for operation.

In general, to prevent the suspension from being fully compressed, damping must be increased. Similar trade-offs can be observed when the suspension stiffness and the unsprung mass values are varied. The above discussion makes obvious the fact that a passive suspension will never be optimal because the vehicle operating environment changes continually. Suspension systems that replace some of the passive elements with controllable ones are the subject of Sections 2.3 to 2.5 and may offer an ability to adapt to these environmental changes.

2.3 The Active Suspension:

Active suspension systems contain elements such as hydraulic actuators which can be controlled such that energy can be added or removed from the system. Ideally such an active element would be capable of generating forces with arbitrary characteristics thus making possible the use of techniques such as state space control and optimal control. Furthermore, active elements are capable of adjusting for changes in payload thus maintaining sufficient suspension working space. A model of an active suspension system is shown in Figure 2.5

Figure 2.5: Schematic of the quarter car model of an active suspension system. The controllable element is a force generator located between the sprung and unsprung mass and due to this location an inherent limitation in suspension performance is encountered.

Suspension forces are always applied to M_s and M_{us} equally in magnitude but opposite in direction, therefore, a coupling of ride comfort and road holding performance should be expected. According to Karnopp (1986), the numerator terms of the transfer functions of this system are simply a reflection of this relationship. The equations of motion are given in state space form in Equation 2.8 where F represents the force from the actuator.

$$\frac{d}{dt} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_s}{M} & \frac{K_t}{M_{us}} & -\frac{C_s}{M} & 0 \\ \frac{K_s}{M_{us}} & -\frac{K_t}{M_{us}} & \frac{C_s}{M_{us}} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{M_{us}} \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \ddot{z} \quad (2.8)$$

If measurements of the states were available one could construct a controller based on the linear combination of those states. Let the actuator force be a linear combination of states shown in Equation 2.9.

$$F = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix} * \bar{X} \\ = K * \bar{X} \quad (2.9)$$

K is called the controller gain matrix. The resulting transfer functions to the input disturbance Z_0 are shown in Equations 2.10 to 2.12.

$$\frac{\ddot{Z}_1}{\ddot{Z}_0} = \frac{M_{us}(s k_4 M_{us} - K_t M + k_2 M_{us}) * (s C_s + K_s)}{\text{characteristic equation}} \quad (2.10)$$

$$\frac{X_2}{\ddot{Z}_0} = \frac{-M_{us}^2 (s^2 M + s(C_s + k_3) + K_s + k_1)}{\text{characteristic equation}} \quad (2.11)$$

$$\begin{aligned} \text{characteristic equation} = \\ M_s (s^4 M_{us}^2 M + s^3 M_{us} (M_{us} C_s + M_{us} k_3 - M k_4) \\ + s^2 M_{us} (K_t M - M k_2 + K_s M_{us} + k_1 M_{us}) \\ + s K_t (-C_s M - k_1 M + M_{us} C_s + M_{us} k_3) \\ + K_t (K_s M_{us} + k_1 M_{us} + K_s M + k_1 M)) \end{aligned} \quad (2.12)$$

In state space control the gain matrix elements are solved for by matching the coefficients in Equation 2.12 with a desired characteristic equation. This method is called pole placement. Unfortunately, once this equation is chosen the numerators of the transfer functions are fully determined. In a paper by Hall and Gill (1987), the authors used this pole placement technique with little success in improving the ride comfort of their system. Similar problems were encountered by Karnopp (1986) in his attempt to achieve an ideal filtering behaviour for both ride comfort and road holding. The difficulty is rooted in the unavoidable influence of the gains on the numerator terms.

The controller gain matrix cannot be optimally determined by selecting the characteristic equation or pole locations alone. One must also account for the numerator

terms which have a considerable contribution to the overall response of the suspension system. Optimal control may solve this selection problem by reformulating the selection process as a minimization problem. For instance, consider the quadratic function in Equation 2.13.

$$f = q_1 X_1^2 + q_2 X_2^2 + q_3 (\ddot{Z}_1)^2 \quad (2.13)$$

The terms represent the magnitudes of the suspension deflection, the tyre deflection and lastly the sprung mass acceleration. q_1 , q_2 , and q_3 represent multipliers which adjust the relative weighting of each term when the function is minimized. Selection of these weighting factors can be a difficult task, however, Thompson (1976) has devised a statistical approach to aid in this selection. The function can be rearranged as in Equation 2.14 into a matrix form where Q is a matrix operating on the product of states, R operates on the square of the input and N is a matrix operating on the product between the states and the input.

$$f = \frac{1}{2} (\bar{X}^T Q \bar{X} + \bar{U}^T R \bar{U} + 2 \bar{X}^T N \bar{U})$$

$$Q = 2 \begin{bmatrix} q_1 + \left(\frac{K_s}{M_s}\right)^2 & 0 & \frac{2C_s K_s}{M_s^2} & 0 \\ 0 & q_2 & 0 & 0 \\ \frac{2C_s K_s}{M_s^2} & 0 & \left(\frac{C_s}{M_s}\right)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = 2 \left[\frac{q_3}{M_s^2} \right], \quad N = \begin{bmatrix} -2 \frac{K_s}{M_s^2} \\ 0 \\ -2 \frac{C_s}{M_s^2} \\ 0 \end{bmatrix} \quad (2.14)$$

The system parameters appear in these matrices because the sprung mass acceleration is assembled from a linear combination of states and system parameters. The controller gain

matrix can be calculated as Equation 2.15.

$$K = BR^{-1}B^T\theta(t)\bar{x} \quad (2.15)$$

θ is a matrix governed by the Ricatti Equation (Eq. 2.16) which can be solved using 'Matlab TM' or any other control system software.

$$\dot{\theta} = -\theta A + \theta BR^{-1}B^T\theta - Q - A^T\theta + NR^{-1}B^T\theta \quad (2.16)$$

The systematic approach of designing controllers based on optimal control theory is appealing, however, actuators with the force and speed requirements demanded by these controllers may be expensive to acquire (Sharp and Hassan 1987). Two controllable vehicle suspension systems, slow-active and semi-active, offer alternatives to the expensive active suspension system.

2.4 Slow-Active Suspensions:

Typically active suspensions employ hydraulic actuators because they have the necessary bandwidth for controlling the effective frequency range of the road input, up to 10Hz (Margolis 1983), and the suspension system's natural modes. These actuators are also very expensive. One proposed system which would eliminate the need for such expensive actuators is called a slow-active suspension (Sharp and Hassan 1987). This system is illustrated in Figure 2.6.

Figure 2.6: Schematic of the quarter car model of a slow active suspension system. The active element is in series with passive suspension elements which allows the system to revert to a conventional suspension system.

The slow-active suspension contains an actuator placed in series with a parallel combination of a passive spring and damper. The suspension behaves like an active suspension for road excitation components within the bandwidth of the actuator. For road excitation components beyond this bandwidth the suspension exhibits passive behaviour. Sharp and Hassan (1987b) examined this type of suspension through simulation and have concluded that a 3Hz bandwidth actuator is optimum considering the degree of improvement in road holding and ride comfort achieved. This 3Hz bandwidth is not surprising considering that the frequencies which contribute most to the road power spectral density are the lower frequencies. Their performance graphs indicate that the damping in the system plays an important role in the performance of the system. If the system damping has such a dramatic influence then intuitively a controllable damper, which adapts to changing road conditions, may be a reasonable addition. Although

simulation results have been encouraging, concerns regarding the space requirements of the slow-active system remain an obstacle. Semi-active systems may avoid this obstacle by replacing the passive shock absorber with a controllable damper.

2.5 Semi-Active Suspension:

Semi-active suspension systems are similar to active ones in that both use controllable elements. The difference is that the controllable elements in semi-active systems are incapable of injecting energy into the suspension systems. These elements are usually controllable dampers, however, a variable stiffness element proposed by Venhovens et.al. (1993) is one exception. For a semi-active damper, in the instances where the controller demands the addition of energy the damper simply reverts to the lowest damping level possible. This restriction would certainly mean that an active suspension will out perform the semi-active system, however, the margin of improvement may not justify the prohibitive costs (Sharp and Hassan 1987) demanded by active suspensions. Sharp and Hassan (1986), Crolla et.al. (1989) and Margolis (1983), however, have demonstrated semi-active and active systems yield quite comparable performance given ample working space and optimally determined controllers. This result may be due to their method of comparison. The frequency response was evaluated at discrete frequencies between 0.25Hz and 15Hz using an input derived from the road spectral density. The frequency response was then used to construct the output spectral density which was numerically integrated over frequency. For non-linear systems, such as the semi-active suspension system, simulations were used to obtain a representative frequency

response by averaging the system response to each input over several periods of oscillation. This averaging tends to mask the harshness of ride performance caused by brief transients possibly resulting from the discontinuous nature of the semi-active control law. Surprisingly, Sharp and Hassan (1986) also found that an optimally configured passive suspension could also perform on par with the active and semi-active systems. The drawback of the passive suspension, when optimized for ride comfort, was that the suspension stiffness was softer than the active or semi-active systems (Crolla et.al. 1989). This may be of concern for maintaining adequate working space if the vehicle payload changes dramatically. In fact all suspension systems suffered degraded performance in both ride comfort and road holding when suspension working space was restricted (Sharp and Hassan 1986, Hedrick and Butsuen 1990). The active suspension appears to suffer the least of the three with working space restrictions.

The above discussion supports the notion that semi-active suspension systems can offer an economical and viable option for a controllable suspension with only a modest trade-off in performance. Since semi-active suspensions are non-linear in nature many non-linear control schemes have been proposed. Some of these control schemes will be introduced in Section 2.6.

2.6 Semi-Active Control:

Full state feedback control is but one approach to the design of a semi-active suspension control law. Most of the control law alternatives reviewed here were designed to optimize ride comfort for a single degree of freedom system but were adapted for two

degree of freedom systems. The ideal skyhook damping system is illustrated in Figure 2.7.

Figure 2.7: Schematic of the quarter car model of a sky-hook suspension system. The damping force, proportional to the sprung mass velocity, is applied to the sprung mass at all times through an inertial frame of reference.

A sky-hook damping system is an idealized quarter car system in which a damping force, proportional to the sprung mass velocity, is applied to the sprung mass at all times. This requirement can not be practically achieved because the damper would have to be attached to an inertial frame of reference. Margolis (1983) has indicated that inertial damping on the masses, including the unsprung mass, does result in the best vibration isolation with respect to resonance control and is often used as a benchmark system. Karnopp and Crosby (1974) first proposed the following control scheme (Eq. 2.17) as an approximation of an ideal skyhook damping system.

$$\begin{aligned} F &= C_s \dot{Z}_1 & \text{for } \dot{Z}_1 X_3 > 0 \\ &= C_{soft} X_3 & \text{for } \dot{Z}_1 X_3 < 0 \end{aligned} \quad (2.17)$$

C_0 represents a damping coefficient applied to the sprung mass velocity, $\dot{Z}_1$. C_{soft} represents the lowest damping coefficient possible and is invoked when the control law commands a force which cannot be achieved due to the relative motion of the damper. This condition occurs when the sprung mass velocity and the relative velocity across the damper, X_3 , are in opposite directions indicated by the product of these two quantities. This control law was found to produce a response close to an active suspension but was difficult to implement because of the need for a fast switching damper and a measurement of the sprung mass velocity. A variation of this scheme is to switch from either a hard, C_{hard} , or soft damping condition rather than continuously adjusting the damper force and is depicted in Equation 2.18.

$$\begin{aligned} F &= C_{hard} X_3 \text{ for } \dot{Z}_1 X_3 > 0 \\ &= C_{soft} X_3 \text{ for } \dot{Z}_1 X_3 < 0 \end{aligned} \quad (2.18)$$

This modification removes the need for an accurate measurement of sprung mass velocity (Krasnicki 1981, Margolis and Goshtasbpour 1984) and an expensive damper. The switching condition remains unchanged from that shown in Equation 2.17.

Rakheja and Sankar (described in Alanoly and Sankar 1985) proposed the control law shown in Equation 2.19.

$$\begin{aligned} F &= C_0 |X_3| \dot{X}_3 \text{ for } X_1 \dot{X}_3 < 0 \\ &= C_1 |X_3| \dot{X}_3 \text{ for } X_1 \dot{X}_3 > 0 \end{aligned} \quad (2.19)$$

The control force is modulated at all times by the square of the suspension velocity. Notice that the choice of damping coefficient, C_0 or C_1 , is based on the suspension deflection, X_1 , and the relative velocity between the sprung and unsprung mass. The net

effect is that the damping force always opposes the force of the suspension spring and this condition is monitored by the product of X_1 and X_3 . This involvement of the other states of the system is a departure from the sky-hook control philosophy. Implementation is greatly simplified but is penalized by less than optimal performance.

Wu et.al. (1994) proposed an on-off control shown in Equation 2.20 and is based on the same condition statement as Equation 2.19.

$$\begin{aligned} F &= C_{hard} X_3 \text{ for } X_1 X_3 < 0 \\ &= C_{soft} X_3 \text{ for } X_1 X_3 > 0 \end{aligned} \quad (2.20)$$

Alanoly and Sankar (1987) and Jolly and Miller (1989) have departed from the skyhook model and recognized that in order to minimize the acceleration of the sprung mass the damper should apply forces which oppose the suspension spring force. Jolly and Miller (1989) have designed an entirely mechanical semi-active damper with the damping characteristics of Equation 2.21 which would eliminate the need for any electronics

$$\begin{aligned} F &= F_r = -C_0 X_1 \text{ for } X_1 X_3 < 0 \\ &= C_{soft} X_3 \text{ for } X_1 X_3 > 0 \end{aligned} \quad (2.21)$$

This control law is used in the description of the composite control strategy proposed by Wu et.al. (1994) and, therefore, another variable name, F_r , is defined. The same switching condition in Equation 2.19 is used, however, the control forces are explicitly specified as opposing the suspension spring force. In this sense C_0 in this control law has the dimensions of a stiffness parameter. Lin and Willumeit (1991) have also designed a fully mechanically controlled damper which has a large damping coefficient at the first resonant frequency and a small damping coefficient at higher frequencies. Theoretically this

frequency characteristic is desirable for ride comfort control and this potential was demonstrated through quarter car simulations.

Schemes which optimize road holding rather than ride comfort are also included. So far only Wu et.al. (1994) has proposed the road holding control law shown in Equation 2.22.

$$\begin{aligned} F = F_d &= -K_s X_3 - M_{us} \ddot{Z}_2 \quad \text{for } F_d/X_3 > 0 \\ &= C_{sof} X_3 \quad \text{for } F_d/X_3 < 0 \end{aligned} \quad (2.22)$$

This control law is intended to minimize the net force on the unsprung mass by modulating the damping force to oppose the other forces on the unsprung mass derived from the system states and system parameters. F_d is another variable name for this control law and is used in the description of the composite control strategy proposed by Wu et.al. (1994). Note that the switching condition examines the current damping coefficient and if negative specifies the lowest damping value available.

Wu et.al. proposed a composite control strategy, Equation 2.23, which pairs his road holding control strategy with the ride comfort control law of Alanoly and Sankar (Eq. 2.21).

$$\begin{aligned} F &= \alpha F_d + (1-\alpha) F_r \\ \alpha &= \frac{|X_2|}{\left[\frac{(M_{us} + M_s) g}{K_t} \right]} \end{aligned} \quad (2.23)$$

where α represents the ratio of the dynamic tyre deflection to the static tyre deflection and 'g' represents the acceleration due to gravity. Note that if the dynamic tyre deflection, X_2 ,

approaches the static tyre deflection this implies that road holding is becoming degraded. The effect of α is, therefore, to bias the road holding strategy, F_r , with the ride comfort strategy, F_d .

This brief summary demonstrates that there are many variants to semi-active control, each slightly different from the other. Section 2.7 will compare the ride comfort control strategies through computer simulations.

2.7 Comparison of Control Laws:

In order to compare the ride comfort control schemes simulations of the quarter car model were implemented using 'Simulink' which is a simulation software program. A representative road acceleration time history was derived from the Van Deusen (1968) road spectral density function for runway conditions at a highway speed of 110 km/hr. This time history was calculated by sampling several sinusoidal functions of various frequencies, each covering a one third octave band (Wong 1993), such that most of the frequency region was represented up to 15Hz. The amplitude of road acceleration for each frequency was calculated based on the spectral density function which gives a root mean square (RMS) acceleration value when vehicle speed is factored in. This method of obtaining the road input is a variation of the method used by Lin and Willumeit (1991). All schemes were compared for the same ride comfort level with the RMS acceleration of the sprung mass at 0.0493 m/s^2 . Although it was mentioned earlier that averaging masks the harshness of ride quality this method will be used as a means of comparing the overall performance of each system. The parameters used in the simulation were the same as

those used in the transfer function analysis given in Section 2.2. The power and force values shown in Table 2.1 appear small due to the one seventh scale of the parameters.

Table 2. Results of simulations comparing control laws based on ride comfort.

Control law	avg. power dissipation (Watt)	RMS force (N)	RMS suspension deflection (mm)	RMS tyre deflection (mm)	max. force (N)
Alanoly & Sankar	2.59e-5	0.0755	0.125	0.013	0.440
Karnopp	3.29e-5	0.0592	0.108	0.013	0.297
Alanoly & Sankar on-off	2.45e-5	0.0480	0.114	0.013	0.267
Karnopp on-off	3.03e-5	0.0457	0.105	0.013	0.209
Full State Feedback Semi-Active	2.42e-5	0.0894	0.082	0.013	0.530
Rakheja & Sankar	3.12e-5	0.0480	0.106	0.013	0.325
Full State Feedback Active	-0.25e-5	0.0978	0.075	0.013	0.475

The full-state feedback active control was designed using optimal control theory described in Section 2.3. The semi-active full state feedback control was designed to be the same as the active case except that for instances where energy input was demanded by the control law the controller requested the lowest damping level possible.

Surprisingly the on-off strategies perform just as well in most of the tabulated categories when compared to their continuous counterparts. Crolla et.al. (1989) also observed this parity and further found that decreasing the difference between on and off

damping improved ride comfort at the expense of road holding performance. Increasing the time delay in applying control forces was also found to have a detrimental effect on ride comfort but could be compensated for by lowering the difference in damping between the on and off states. Redfield (1991) suggests the bandwidth of the damper be in the range of 50 to 100Hz in order to meet the switching requirements for controlling the highest road input frequency of consequence. For this reason a fast switching damper is necessary to implement semi-active control. Karnopp (1990) has suggested an alternative to fast switching dampers by having the transition from compression to tension handled mechanically with check valves. Check valves are mechanical valves which allow fluid flow in a single direction and are, therefore, analogous to diodes in an electrical circuit. Karnopp suggests the configuration shown in Figure 2.8 may result in a reduction in the required bandwidth to 10Hz thus allowing the use of cheaper controllable valves.

Figure 2.8: Proposed control hardware implementation using check valves. The check valves restrict flow to a single direction. Therefore, the path of the damper fluid is dependent upon the flow direction.

Unfortunately, this saving would be offset by the need for two controllable valves and two check valves. The implementation of the control laws described above would also be further complicated by the coordination of two controllable valves instead of one.

The table of simulation results also indicates that for the same ride comfort both full state controllers require approximately 20% less working space than the other alternatives. As the sprung mass acceleration was allowed to rise, working space requirements fell into line for all controllers. Not much difference in road holding performance was observed between the control laws, less than 0.0004 mm difference among the tyre deflection values. The average dissipated power was slightly less using full state feedback while nearly doubling the RMS damper force. As working space usage was also less for these full state feedback controllers, a lower average relative velocity and hence lower power consumption is achieved.

The semi-active and active full state controllers are quite comparable in performance. The active control is slightly better in terms of working space usage. Because of this lower working space requirement, one can conclude that using all the states of the system is beneficial in the design control laws. Measuring all these states can be quite expensive and in many cases impossible to do. One way of obtaining estimates of these states will be discussed in Section 2.8.

2.8 The Estimation of States:

The above control law alternatives, other than Karnopp's modified skyhook and optimal control, were suggested because they avoid fundamental problems in implementation such as the availability of measurable quantities. It has been demonstrated, however, that optimal control which uses all of the states of the system to arrive at a control law is a better method of control. The problem is how to measure all four states of the system.

A convenient method of determining the states of the system is to use the quarter car mathematical model as an estimator (Ogata 1990). For instance, all the states of the system could be estimated from measurements of suspension deflection using a potentiometer. This ability has considerable advantages in terms of practicality and cost. Accelerometer measurements suffer from drift and infinite d.c. gain when integrated (Karnopp 1986, Miller 1988). Estimating tyre deflection could possibly be done by measuring tyre pressure or using sonar but these methods are greatly affected by environmental disturbances. Relative displacement and relative velocity measurements are fairly simple quantities to measure according to Alanoly and Sankar (1987). However, the cost advantages of estimating states with a single sensor rather than four are clear.

2.9 Summary:

The topic of controllable vehicle suspensions has illuminated several points which support the work of this thesis.

- Controllable vehicle suspensions have an advantage over passive suspensions because the operating conditions of the vehicle continually change and the performance requirements of the suspension vary from one individual to another.
- Semi-active suspensions provide performance benefits comparable to active suspensions without the monetary costs associated with active systems or the implementation difficulties of slow-active suspensions.
- The use of a mathematical model can provide information regarding the performance and limitations inherent in a system.
- A model can aid in designing control laws.
- A model can help save money through reduced data acquisition hardware requirements.

The discussion so far has treated the actuators of these controllable systems as ideal elements. In practice these actuators have dynamic properties which may be an important consideration in the design of control laws. Alternative controllable elements for semi-active systems and the properties of ER dampers will be the subject of Chapter 3.

Chapter 3: Controllable Dampers and the Electrorheological Effect

3.0 Introduction:

This chapter will introduce the electrorheological phenomenon and its use in semi-active vehicle suspension systems. Section 3.1 will first review some alternative semi-active hardware that have been studied by other authors. The mechanisms behind the ER effect are reviewed in Section 3.2. Section 3.3 introduces the traditional arrangements for shear mode and flow mode dampers and Section 3.4 will describe the rheological observations of ER fluids subjected to both these deformation modes. A fluid analysis of an annular gap flow mode damper is provided in Section 3.5 and the results of a comparison between the predicted and measured damper performance are shown in Section 3.6. Section 3.7 describes the flow mode damper built by TTRL and the revisions that were made in response to the results of Section 3.6.

3.1 Semi-Active Suspension Hardware Alternatives:

For active suspensions, hydraulic actuators appear to be the dominant hardware solution because of their large force and bandwidth capability. Ryba (1993) has suggested that a rotational motor with either a rack and pinion, frictional drive or a swing arm linkage could be used as the active element. For semi-active suspensions, however, quite a few hardware schemes have been proposed. Hagele et.al. (1990) has specified the following requirements that a controllable damper should meet for use in typical passenger vehicles:

- maximum achievable force of 3 KN
- minimum achievable force of 0.2 KN
- response time of 5 ms or less
- maximum power consumption of 100 Watts
- operation between -40 and 120 degrees Celsius

One of the more popular ideas is the use of a shock absorber with a variable orifice. The design of an electromechanical version of such a shock absorber was examined by Hagele et.al. (1990). Their results indicate that a variable orifice shock absorber using an electromagnetic valve can have sufficient damping force and actuator speed for most semi-active control schemes. Examination of their damper schematic reveals a mechanism of great complexity which, therefore, may not be as rugged as the author claims. Not all such shock absorbers are as successful. Ivers and Miller (1989) tried

to confirm their simulated semi-active suspension performance with experimental results from a quarter car apparatus. They used a shock absorber with a variable orifice actuated by a stepper motor. Experimental results yielded less than ideal performance and part of the blame was attributed to mechanical valve delay. There are, however, commercially available servo-valves with the bandwidth capability for semi-active suspension control.

Another unique semi-active suspension scheme involves varying the stiffness properties of the suspension. One way of achieving this was proposed by Venhovens (1993) and is called the cone mechanism. Suspension stiffness changes were achieved by varying the length of the moment arm at which a control force acts. A schematic of this mechanism is shown in Figure 3.1.

Figure 3.1: Venhovens (1993) cone mechanism for anti-roll/pitch control. Link 'b' pivots about the Y-axis and link 'a' pivots about the local X_1 -axis. By rotating link 'a' the length of the moment arm can vary and, therefore, the force exerted on the wheel may be controlled.

Link 'b' pivots about the Y-axis and link 'a' pivots about the local X_1 -axis. By rotating link 'a' the length of the moment arm can vary and, therefore, the force exerted on the wheel may be controlled. The control force on the end of link 'a' is provided by a spring.

The rotation of link 'a' is accomplished by a motor which moves this spring in the shape of a cone. The motor axis, X_2 , and the X_1 -axis were intended to be co-linear which implies that the motor would only have to resist frictional forces. As the suspension deflects, however, the motor axis will be out of alignment with the X_1 -axis. This implies that the motor must occasionally do work on the spring in order to change the moment arm length. As a result of this energy input such a suspension may be better classified as an active suspension. This system was initially considered for use in anti-roll and pitch control in which suspension deflection is minimal and, therefore, was anticipated to consume little power. If the payload were to increase, however, the centre line of the cone mechanism and motor axis will be permanently displaced and the motor must expend additional energy. Considering the suspension deflection due to road excitation and changes in payload the system may not be as efficient as projected. Venhovens hopes to expand the use of this suspension system for vibration control.

Another damping device involves the use of electrorheological fluids.

Electrorheological fluids are fluids that change their flow properties when subjected to an electric field. Without the presence of an electric field the fluid behaves as a Newtonian fluid with a low viscosity (e.g. water). With an applied electric field the fluid congeals into a solid. ER fluids have a great deal of potential in the application of rapidly adjustable dampers. The design of the damper mechanism can be made extremely simple. Electrical and mechanical response is usually quite fast. Initial concerns were centred around an observed temperature sensitivity (Ushijima et.al. 1988, Bullough and Peel 1988) of the ER effect especially below freezing as most of the early ER fluids required the addition of

water. With the advancement of dry particle ER fluids this temperature dependency may soon be resolved. The main limitation of the use of these fluids appeared to be their inadequate electrical and mechanical properties (Coulter 1993). The feasibility of using ER fluids in vehicle dampers was investigated by Hagele et.al. (1990), and Bullough and Peel (1988). Their investigations have identified that a greater ER effect and lower electrical conductivity must be achieved before a practical vehicle application can be realized. The latest research by Weyenberg, Pialet and Petek (1995) has demonstrated that ER dampers designed with existing fluids may come close to meeting all of the controllable damper requirements except for the temperature range requirement. A Ford Thunderbird fitted with four prototype ER dampers was tested in a single wheel shake apparatus and showed noticeable improvements in ride comfort and road holding. A great deal of research continues to be done to resolve the remaining difficulties. Section 3.2 will describe the mechanism behind the ER effect.

3.2 The ER Effect:

In Chapter two the advantage of having a model of the system to be controlled was evident. Other than the slow active system, the controllable elements were assumed to be ideal. In reality these elements have their own dynamic properties and must be included in the system model. To develop a model of the dynamic properties of the ER damper it was helpful to understand the underlying mechanism behind the ER effect. Presently this mechanism is not fully understood. The consensus, however, is that the ER phenomenon involves some sort of induced polarization which is a redistribution of electric charge

within or around the ER particles (Block et.al. 1988).

3.2.1 Fibrillation:

This is a phenomenon closely associated with ER fluids under the influence of an electric field. Fibrillation is the formation of chain like structures which align with the electric field. This is thought to occur because of a difference in dielectric strength between the particulate phase and the liquid phase. This difference leads to unbalanced forces and particle migration depicted in Figure 3.2.

Figure 3.2: Fibrillation model. The difference in dielectric strength between the particulate phase and the liquid phase leads to unbalanced forces resulting in particle migration. ER particles when subjected to an electric field redistribute their charge such that the dipole forces cause chain structures to form.

Due to induced polarization the particles assemble end to end in a chain. The force necessary to separate these chains results in the observed yield phenomenon when a load is applied. Many authors believe fibrillation to be the primary mechanism behind the field induced rheological properties of these fluids (Zukoski 1993). Ginder et.al. (1994) was

able to verify the dielectric mismatch theory by comparison of their finite element model with rheometric data. Also, results of experiments conducted by Spreacher et.al. (1989) on a single chain of ER particles were explained well by electrostatic dipole theory. Others such as Klass and Martinek (1967) and Liang (1994) believe that fibrillation is simply a characteristic of the ER fluid and that chaining could not occur fast enough to explain the rapid changes in fluid properties. Ginder's optical probe study (1993), however, does show that structure formation could occur on the same time scale as the observed rheological properties. Also, the initial arrangement of the aggregating particles was found to influence the rate of structure formation.

3.2.2 Interfacial Effects:

Ginder concluded that conductivity mismatch, which describes charge carrier transport around the particle surface, was just as important in particle polarization as dielectric mismatch. The importance of interfacial effects on the enhancement of polarization was also mentioned by Jordan and Shaw (1989). Conductivity was shown by Felici et.al. (1994) to be necessary for the ER effect and may explain the coupled response in the electrical and mechanical domains (Whittle 1994, Sproston et.al. 1983, Conrad et.al. 1991). Another possible interfacial effect may be due to the distortion of a diffuse layer of ions extending from the particle surface into the fluid. This effect described by Jordan and Shaw (1989) was first proposed by Klass and Martinek (1967) and was named the double layer effect. This ionic layer, depicted in Figure 3.3, can be interpreted as an energy potential and any distortion of this layer due to flow or interaction from adjacent particles

results in energy dissipation.

Figure 3.3: Double layer effect. A layer of ions surrounding the ER particles is said to extend outward from the particle surface in the presence of the electric field. The distortion of one layer due to interference from others and from the flow of the fluid medium causes the dissipation of energy.

Unfortunately, because both double layer and fibrillation models predict a squared field strength dependence of rheological properties, delineation of the correct model cannot be easily made. The double layer theory, however, lacks an adequate explanation of the yield phenomenon commonly observed in ER fluids.

3.2.3 The Role of Water:

Before the introduction of non-aqueous ER fluids, water was thought to be an essential ingredient for the ER effect. Block et.al. (1988) has recently found alcohols to be equally effective. Stangroom (1995) suggested that water may act as a glue holding the ER particles together. Water molecules were said to assemble around mobile ions located

in the pores of the ER particles. As the electric field was increased the ions migrated to one end of the particle drawing the water molecules with them. This migration caused the end of the particle with the higher ion density to also have a higher concentration of water and hence water may form bridges between adjacent particles. The surface tension of the water must be overcome to separate the particles, leading to yielding. Jordan and Shaw (1989) also notes that water increases the permittivity of the particles and, therefore, aids in ionic polarization which is thought to be the mechanism behind the double layer effect. Wong and Shaw (1989) have performed experiments which suggest that water may be the primary agent in the ER effect. Their experiments showed that two very different ER fluids could produce identical ER effects with the addition of water. Excessive addition of water, however, would compromise the dielectric strength of an ER fluid (Jordan and Shaw 1989). Several devices have been proposed to utilize the ER effect for vehicle damping mechanisms, however, two basic arrangements have emerged. These arrangements will be discussed in Section 3.3.

3.3 ER Fluid Damper Arrangements:

There are two traditional damper arrangements utilizing either the flow mode or shear mode properties of the fluid. Squeeze deformation, involving the small amplitude compression of ER fluids between two electrodes, has been investigated by Sproston et.al. (1995). This deformation mode is more applicable to vibration mounts and is not within the scope of this thesis. In a *flow mode* damper the electrodes are fixed and fluid flow is initiated by inducing a pressure gradient. In a *shear mode* damper fluid flow is initiated by

moving one electrode, which is attached to the piston, relative to the other electrode. In order to prevent a pressure gradient from forming in the shear mode damper fluid is allowed to recirculate past the piston without passing through the gap formed by the electrodes. These damper arrangements, shown in Figure 3.4, have finite stroke length and tend to have poor material efficiency with respect to the ratio of the shear area to the total volume of ER fluid.

Figure 3.4: Traditional ER damper arrangements. The primary difference between the flow mode and shear mode dampers is that for the shear mode damper, fluid is allowed to circulate above and below the piston without having to pass through the electrode gap.

The shear area in a flow mode damper is the electrode area in contact with the energized ER fluid while in shear mode the shear area is the area of the moving electrode in contact with the energized ER fluid. Coulter and Duclos (1989) have shown that dampers based on flow mode are 40% less efficient than in shear mode. Designing shear mode dampers,

however, is difficult. To increase the shear area in a shear mode damper either the stroke length must be reduced or the physical size of the damper must be increased. A revised damper design by Coulter (1993), shown in Figure 3.5, was proposed to address these problems.

Figure 3.5: Ball-screw damper design proposed by Coulter (1993). The ball-screw assembly is used to transform translational motion of the screw into rotational motion of the shear plates. The disadvantage of this damper design is the necessity of using bearings.

This design utilizes a ball-screw assembly to transform translational motion of the screw into rotational motion of the shear plates. This design has several advantages. It makes very efficient use of the ER material available. The stroke length is only limited to the length of the screw. The ratio of the rotational speed of the plates to the translational speed of the piston rod can be modified by changing the lead of the screw. The electrode configuration is variable. The energy conversion of the ball-screw, however, is not one

hundred percent efficient due to the necessity of using thrust bearings (Coulter 1993). The use of these bearings in this design results in increased damping without the presence of the electric field and may limit the performance of the suspension. Also, backlash caused by any clearance between the bearings and their races may prevent the smooth transfer of control forces resulting in increased ride harshness. Tests of a prototype of this damper showed differences in compressive and tensile behaviour or a directional dependence. Whether this difference was by design or unintentional is unknown. Another possible shear mode damper design would be a rotary damper which may be located at pivot points or joints connecting the suspension linkages. Such a damper would efficiently use the ER fluid and could vary the shear area easily by using multiple plates. A disadvantage of this damper would be the difficulty of retro-fitting this damping into the limited space available in existing vehicle suspension systems. Although shear mode appears to be a preferred configuration, a flow mode damper offers the benefit of design simplicity. The rheological properties observed in both shear mode and flow mode will be reviewed in Section 3.4.

3.4 Rheological Properties:

Liang and Xu (1994) have demonstrated that the yield phenomenon can be approximately described by a rate insensitive stress that is a function of the electric field strength. Conrad (1993) describes this function as a power law with a coefficient varying between 1 and 2.5. Tao (1993) has indicated that there may also exist a threshold electric field strength which must be overcome before solidification can commence. Chenguan and Zhao (1994) observed a critical voltage above which the ER effect no longer was

enhanced and in some cases was diminished. Klingenberg et.al. (1991), however, has shown evidence that appears to support a strain rate dependence which may be explained by the structure of the ER material under deformation. Conrad (1993) has identified two deformation regimes: quasi-static and dynamic. Quasi-static is characterized by the fibrillated structure that remains intact under shear rates up to 10 rad/s. Korobko (1989) notes that the degree of ER fluid structure is unique for each electric field strength and shear rate combination. Above 10 rad/s is the dynamic regime which may be characterized by the formation an ER solid phase adhered to the electrode surfaces. Separating the two opposing solid phases is a layer of the liquid phase (Klingenberg 1991) which grows in thickness as the strain rate increases. The presence of this liquid layer would reduce the ability to transfer stress and is consistent with the observed degradation of the ER effect at high deformation rates (Marshall et.al. 1989, Korobko 1989).

3.4.1 Properties in Shear Mode:

Previous attempts at modelling ER fluids have identified varying degrees of viscoelastic and plastic behaviour under oscillating strain. Increasing the electric field strength and the strain amplitude has been shown to result in increased plastic rather than viscoelastic behaviour (Gamota 1991). The transition between these two rheological regimes is called the yield point, however, this point is ambiguous. These regions are depicted in the shear stress versus strain graph shown in Figure 3.6.

Figure 3.6: Shear stress versus strain graph.

Gamota (1991) has suggested that the viscoelastic regime may be modelled as a Zener element, which is analogous to a spring in series with a spring and damper in parallel, while the plastic regime may be modelled with Coulomb and viscous friction elements. A mechanical analogy of this system is shown in Figure 3.7.

Figure 3.7: Mechanical analogy of the ER effect proposed by Gamota (1991). The viscoelastic regime may be modelled as a Zener element, which is analogous to a spring in series with a spring and dashpot in parallel, while the plastic regime may be modelled with Coulomb and viscous friction elements.

The relevant rheological region for a specific situation could be identified from the shape

of shear stress versus shear strain graphs as well as the Fourier spectrum of the shear stress response to sinusoidal shearing (Gamota 1993). For vibratory systems such an identification impacts on how a system may be modelled and ultimately on control strategies. For instance, in situations where the strain rate rarely reverses, modelling of the plastic behaviour is most relevant. The transient response of ER fluids subjected to a step in electric field and steady shearing was studied by Takimoto, Minagawa and Koyama (1995) who found that the response time was faster when the shear rate was increased. This faster response time was attributed to a faster growth rate of ER particle clusters.

3.4.2 Properties in Flow Mode:

The Bingham plastic approximation for shear mode behaviour has been used with success by Conrad (1993), Klingenberg (1991) and Chen and Luckham (1993). Peel and Bullough (1994) has established that this model, depicted in Figure 3.8, is also a good approximation for flow mode behaviour and is able to capture the scaling dependence of these devices.

Figure 3.8: Shear stress versus shear rate graph of the Bingham plastic model. The Bingham plastic model is characterized by a rate independent term, the yield stress, which is shown as a vertical displacement from the Newtonian shear stress versus shear rate curve.

Lou et.al. (1992) has mentioned that shear and flow behaviour mimic one another which may imply that the rheological properties in flow mode may be inferred from shear mode test results. Recent attempts by Catania (1994) to relate the Bingham parameters in shear and in flow have met with limited success. Peel, Stanway and Bullough (1995) have presented a paper on a method of relating steady shear and flow data. One ramification of this performance prediction is the possibility of creating optimized designs of devices from a wider selection of available ER fluids. The Bingham plastic model assumes that the flow is fully developed, however, in oscillating movements this is clearly not the case.

Hysteresis has been observed by Bullough (1992) at near zero flow conditions and this may have ramifications on control strategies especially if the flow continually reverses direction. Lou et.al. (1992) has shown that the ER induced pressure drop saturates beyond

an oscillatory electric field strength of 50Hz. This saturation would certainly impose restrictions on the upper limit of frequencies that may be controlled in the system. The development of pressure upon the application of the electric field has been shown to be significant with rise times quoted at 0.1s by Peel et.al. (1994) and as high as 0.38s by Nakano (1994). This delay was attributed to fluid compressibility which may be a significant consideration for the design of controllers of ER flow mode devices considering that a pressure gradient in the fluid must be created in order to induce flow. For instance, as energy is dissipated the fluid temperature would certainly rise and coupled with a possible negative pressure gradient the creation of air bubbles in the fluid would be promoted. The inertial effects must certainly have a different impact on the dynamics of the fluid in shear mode than in flow mode. Studies on flow mode properties are relatively scarce when compared to the number of investigations of shear (Nakano 1994). This observation is disturbing when one considers that many devices proposed for ER fluids operate in flow mode (Brooks 1992). The development of a model of the flow mode damper, therefore, gains some importance as this information furthers our understanding of the ER phenomenon. The ER damper that was examined for this thesis was based on flow mode and a fluid analysis of this damper will be given in Section 3.5.

3.5 Fluid Analysis:

The damper to be examined channels ER fluid through annular gaps. Although Catania (1994) uses a flat plate analogy for a damper of almost identical geometry an annular plate treatment will be used here. For a more detailed description of this fluid analysis please refer to Fredrickson and Bird (1958). Consider the cross-section of the annular gap shown in Figure 3.9.

Figure 3.9: Bingham flow in an annular gap. The parameters R , κ , λ_+ and λ_- , L and ΔP are described in the text.

R represents the radial distance from the centre of the annulus to the outer edge of the gap. κ is the ratio between the radial distance of the inside edge of the gap to R . Notice that a solid plug forms in the middle of the gap due to an insufficient level of shear stress

required to develop a velocity gradient. λ_o and λ_i are the ratios of the outer and inner edge of the plug region to R . L is the length of the gap and ΔP is the pressure difference across the gap. If we consider the flow to be incompressible and fully developed then the Bingham fluid constitutive equation (Eq. 3.1) must hold.

$$\begin{aligned}\tau &= -\mu \frac{dV}{dr} - \tau_y, \text{ for } \kappa R \leq r \leq R\lambda_o \\ &= -\mu \frac{dV}{dr} + \tau_y, \text{ for } R\lambda_i \leq r \leq R\end{aligned}\quad (3.1)$$

τ represents the shear stress, τ_y represents the yield stress, μ represents the fluid viscosity and V represents the fluid velocity. From the conservation of momentum and mass, Equation 3.2 is obtained and is expressed in cylindrical coordinates.

$$\frac{\partial}{\partial r} r\tau = r \frac{\Delta P}{L} \rightarrow \tau = \frac{\Delta P}{2L} r + \frac{c'}{r} \quad (3.2)$$

For more detail on this step see White (1986). c' represents a constant of integration which may be derived from boundary conditions. The non-dimensional parameters shown in Equation 3.3 will be used.

$$T = \frac{\tau}{\frac{\Delta P R}{2L}}, \quad T_y = \frac{\tau_y}{\frac{\Delta P R}{2L}}, \quad \phi = \frac{\mu V}{\frac{\Delta P R^2}{2L}}, \quad \psi = \frac{r}{R}, \quad \lambda = \lambda_o, \lambda_i \quad (3.3)$$

The dimensionless parameters represent the shear stress, T , yield stress, T_y , fluid velocity, ϕ , and radial distance, ψ . Substituting these parameters into the continuity Equation 3.2 and evaluating the resulting expression at the inside and outside edge of the plug region gives Equation 3.4.

$$-T_y = \lambda_- + \frac{c}{\lambda_-}, \quad T_y = \lambda_+ + \frac{c}{\lambda_+} \quad (3.4)$$

c serves as a constant of integration similar to c' except that it is defined differently due to the non-dimensional notation. Equation 3.4 may be resolved to give an expression for T_y (Eq. 3.5).

$$T_y = \lambda_+ + \lambda_- \quad (3.5)$$

The non-dimensional velocity distribution can now be found by substituting Equations 3.3 and into the Bingham Equation (Eq. 3.1).

$$\phi_- = -T_y(\psi - \kappa) - \frac{1}{2}(\psi^2 - \kappa^2) + \lambda_-^2 \ln \frac{\psi}{\kappa} \quad \text{for } \kappa \leq \psi \leq \lambda_- \quad (3.6)$$

$$\phi_+ = -T_y(1 - \psi) + \frac{1}{2}(1 - \psi^2) + \lambda_+^2 \ln \psi \quad \text{for } \lambda_+ \leq \psi \leq 1 \quad (3.7)$$

The velocity distributions (Eq. 3.6 and 3.7) at the edge of the plug region must be equal because the plug moves with uniform velocity. Setting $\phi_-(\lambda_+)$ equal to $\phi_+(\lambda_+)$ leads to Equation 3.8.

$$2\lambda_+(\lambda_+ - T_y) \ln \left(\frac{\lambda_+ - T_y}{\kappa \lambda_+} \right) + (T_y + \kappa)^2 + 2T_y(1 - \lambda_+) = 1 \quad (3.8)$$

The velocity distributions can now be integrated to obtain flow rate (Eq. 3.9).

$$Q = \frac{\pi}{2} \frac{\Delta P R^4}{\mu L} \left[\frac{1 - \kappa^4}{4} - \frac{\lambda_+}{2} (\lambda_+ - T_y) (1 - \kappa^2) - \frac{1}{3} (1 + \kappa^3) T_y + \frac{1}{12} (2\lambda_+ - T_y)^3 T_y \right] \quad (3.9)$$

The pressure drop across the gap, ΔP , may be solved for a given gap geometry, yield stress, viscosity and flow rate by numerically solving Equations 3.5, 3.8 and 3.9. This

numerical solution was used in Section 3.6 in order to make theoretical predictions of the forces that are expected for the ER damper.

3.6 Theoretical Predictions:

The fluid to be used was a 2 to 5 mixture, by mass, of zeolite particles in silicon oil. The particles have a mean diameter of 10 microns. The silicon oil had a kinematic viscosity of 10 centistokes and a density of 1145 Kg/m^3 . Previous work conducted by Catania (1994) found this formulation yielded the best trade off of performance at zero field and high field strength damping in the TTRL quarter car apparatus. Rheometry test results of shear stress versus shear rate produced by Catania are given in Figure 3.10 and the yield stress and viscosity for various field strengths are presented in Table 3.1 using a linear least squares approximation.

Figure 3.10: Shear stress versus shear rate graph of rheometry tests conducted by Catania (1994). The ER fluid was a 2 to 5 mixture, by mass, of zeolite particles in silicon oil.

Table 3.1: Rheometry Results by Catania (1994).

Field Strength, kV/mm	Viscosity, Pa-s	Yield Stress, Pa
0	0.095	0
1	0.103	0
2	0.102	80.7
3	0.114	149.4

This fluid tends to be shear thickening, that is the viscosity increases with the shear rate, for the most part except at high field strengths where the shear stress versus shear rate curve takes on a sigmoidal shape with viscosity initially decreasing with increased shear rate up to 1500s^{-1} . Since the fluid analysis assumes a rate independent viscosity discrepancies in steady state predictions are expected. Graphical results of the annular prediction and Catania's flat plate prediction are presented in Figures 3.11, 3.12 and 3.13.

One Electrode Gap

Figure 3.11: Comparison of predicted and experimental results of force for a flow mode damper using a single gap arrangement. Individual points represent the experimental data points for compression (squares) and tension (circles).

Figure 3.12: Comparison of predicted and experimental results of force for a flow mode damper using a double gap arrangement. Individual points represent the experimental data points for compression (squares), and tension (circles).

Figure 3.13: Comparison of predicted and experimental results of force for a flow mode damper using a triple gap arrangement. Individual points represent the experimental data points for compression (squares), and tension (circles).

These results are for a flow mode damper designed by Catania using an applied field strength of 2kV/mm. The flat plate prediction over estimates the annular prediction for two gaps or more which implies that the fluid analysis of this particular damper geometry is quite sensitive to the assumptions made. From the graphs, it is very difficult to make a determination of which approximation results in the better prediction.

Discrepancies between predictions and measured results appear to worsen as the number of gaps is increased. A possible explanation for this increased discrepancy may be due to a narrowing of the range of shear rates attained. The fluid properties that were selected were based on a broad range of shear rates and may no longer be representative for the current configuration of the damper. For instance, because of the shear thinning behaviour at low shear rates the fluid may be better modelled with a higher yield stress and lower viscosity. For the three gap arrangement the measured damping force data points appear to follow a sigmoidal shape similar to that observed in Figure 3.10 at a field strength of 2kV/mm. Unfortunately, due to the highly erratic behaviour of the fluid at this field strength and for this number of gaps it may be unlikely that a single value of yield stress and viscosity will adequately represent the fluid.

At the higher piston velocities most figures show that the measured damping forces in tension and compression differ from each other. For instance for the single gap results the tensile force tends to be less than the compressive force. Interestingly, with the two cylinder arrangement, the opposite relationship is true. This lack of symmetry may be the result of a flaw in the damper design. The damper used fluid gaps that extended almost the entire length of the damper cavity. The damper volume changes continually with the

piston position because the piston is single ended and, therefore, the damper cannot be filled to capacity at all times. The changing volume prevents the gaps from being filled at all times and, therefore, reduces the damper force. This condition would only occur in tension because the damper volume increases when the piston is moved upward. Further complicating the matter is the effect of splashing which, due to the proximity of the top of the gap to the fluid surface, may introduce air bubbles. Design modifications to help correct this asymmetrical behaviour will be described next.

3.7 Damper Design:

The ER damper that was examined operates on the flow mode principle. A schematic of the damper design is provided in Figure 3.14.

Figure 3.14: Schematic of the TTRL ER flow mode damper. As the piston is moved downward the ER fluid flows through the gaps in the direction of the arrows.

The damper consists of a stainless steel cylindrical bucket with a cavity 200mm deep and a diameter of 96mm. Inside are two concentric aluminum cylinders which, including the bucket, formed two annular 3mm gaps. Fluid is forced through these gaps due to the motion of a piston travelling inside the smaller cylinder. In order to allow fine adjustment of the damper performance, each cylinder was designed to have variable length. This was accomplished by manufacturing one tall and two shorter length cylinders which could be stacked together for full length or shortened with the removal of one or more of the short cylinders. In order to maintain the piston stroke length, any removed short cylinders were replaced by identical PVC versions which had equally spaced holes drilled along the curved surface of the cylinders to allow ER fluid to recirculate. Alignment of the stacked cylinders was maintained by cutting shoulders on the cylinders as shown in Figure 3.15.

Figure 3.15: Drawing of cylinder construction. Sections were stacked together for full length or shortened with the removal of one or more of the short cylinders. PVC versions of the cylinder sections had holes drilled along the periphery to allow ER fluid to recirculate.

The cylinders were held in place inside the bucket by two PVC end-caps placed above and below the cylinders. In order to accommodate an ER fluid reservoir, one half inch of space was left between the top of the bucket and the top end-cap. This reservoir was formed by

a PVC spacer which prevents the cylinders from moving vertically. The ER fluid in the reservoir serves two functions. The extra fluid would be able to accommodate the changes in internal volume of the damper as the piston rod travels thereby keeping the aluminum cylinders submerged in the ER fluid at all times. The second function was to prevent air bubbles from being drawn into the gaps since the formation of bubbles would occur further away from the gap entrance. An aluminum lid covers the open end of the bucket and is fastened in place by six screws. A central hole in the top of the lid accommodates the piston rod. Over this hole is a plastic bushing and an aluminum adapter which allows the damper to be used in the TTRL quarter car apparatus.

The bucket and the smaller internal cylinder act as the grounded electrodes while the remaining cylinder is attached to the high voltage supply. Grounding of the small cylinder was accomplished with a brass tab which is allowed to make contact with the bucket surface and is fastened to the cylinder with a brass set screw. The larger cylinder also uses a brass tab fastened in the same manner but has a high voltage wire soldered to it. This wire was then routed through a hole drilled in the lid of the damper. This hole also serves to vent the air in the damper cavity.

3.8 Summary:

The use of the ER effect in a controllable damper for vehicle applications has shown a great deal of potential, however, this chapter has shown a number of potential obstacles for designers of hardware and control systems which may prove difficult to surmount:

- The mechanism behind the ER effect is not fully understood as yet and the three dominant theories explaining this effect are all plausible.
- Observations of the rheological properties of ER fluids show non-linear effects, including hysteresis, which make modelling these devices difficult.
- Studies of ER flow mode properties are quite scarce which is disconcerting considering that many proposed ER devices operate in flow mode.
- The use of classical fluid analysis techniques to relate shear mode rheological properties with flow mode properties has achieved limited success.

The scarcity of available information on ER flow mode properties increases the value of this work. The difficulties highlighted above show that the modelling of an ER flow mode damper is a non-trivial task and that a different approach to modelling may be required. This approach is described in Chapter 4.

Chapter 4: Experiments and Model Identification

4.0 Introduction:

This chapter describes the approach used for estimating a model of the TTRL flow mode damper. The apparatus and data acquisition system used for the experiments are described in Section 4.1. The experimental procedure, including a description of the tests conducted, is given in Section 4.2. Section 4.3 reviews several models of ER devices proposed by other researchers and introduces the model this study will use. Section 4.4 will review several algorithms used for estimating parameters of a model from experimental data and will describe in more detail the quasi-linearization algorithm used in this study. The implementation and testing of this algorithm is described in Section 4.5.

4.1 Experiments:

Identification experiments require the ability to monitor all significant energetic interactions between the environment and the system in question. The three probable interactions involved with the ER damper are mechanical, electrical and thermal. All the measured signals were sampled with a Computer Boards Inc. Model CIO-AD08 analog to digital converter at 1kHz and a resolution of 12 bits. These energetic interactions are described in the following sections.

4.1.1 Mechanical:

The identification experiments were carried out on a Testline Mechanical Test System (MTS) located in the Mechanical and Aerospace Structures Laboratory at Carleton University. This apparatus, originally intended for fracture and fatigue studies, was found from previous research to be suitable for the relatively low frequency and high amplitude excitation required for the identification experiments. A schematic of the MTS layout and instrumentation is provided in Figure 4.1.

Figure 4.1: Schematic of MTS layout and instrumentation.

The MTS system consists of the following elements:

- Pump, Motor and Heat Exchanger model 510.23B.
- Service manifold model 290.
- Hydraulic actuator, servo valve and Linear Variable Differential Transducer (LVDT) model 244.12. The LVDT is a sensor for measuring the displacement.
- MTS Microconsole model 458.10 responsible for transfer of control, digital readout, cycle counter, run/stop, emergency stop, interlock control and closed loop control.
- DC controller model 458.12 responsible for load cell signal conditioning, scaling controls, servo loop response, stability adjustments and interlock detector adjustments.

- AC controller model 458.14 which performs the same functions as the DC controller except signal conditions the LVDT rather than the load cell.
- Function Generator model 458.90 which produces wave forms for the servo control.

The 25kN MTS model 661.19 load cell was substituted with a 1.3kN Lebow model 3397 Tension and Compression load cell. This was done in order to improve the signal to noise ratio of the load measurements. Comparison of the MTS reference manual numbers 833.53 and 831.42 and the Lebow Load Cell and Torque Sensor Handbook number 510A show that the load cells had compatible wiring and bridge resistances. This compatibility allowed the use of the MTS DC control unit for signal conditioning. The load cell was calibrated by suspending masses up to 16kg from the load cell and measuring the resulting voltage from the signal conditioner using a Beckman Industrial DM25 digital multimeter. Over 25 independent measurements were recorded and a least-squares fit was calculated to determine the amplification. The calibration data points and the resulting fit are shown in Figure 4.2.

Load-Cell Calibration

Figure 4.2: Load-Cell calibration data points and resulting fit. The slope (x-coefficient) and y-intercept are 134.25N/V and -0.0394N respectively using a linear least-squares fit.

Power for all electronic devices was kept on before and during the entire test session to allow the equipment time to achieve a thermal steady state condition, thereby, minimizing drift in the measurements.

4.1.2 Electrical:

The voltage across the damper was measured using a voltage divider consisting of a 400M Ω and a 100k Ω resistors in series. The 100k Ω resistor was used to sense the voltage across the damper. To measure the current, a 500 Ω resistor was placed in series with the damper. The current was anticipated to be on the order of milliamperes, therefore, the presence of this resistor should reduce the voltage reading by a negligible

amount. To prevent damage to the Analog to Digital (A-D) card due to voltage breakdown, a Metal Oxide Varistor (MOV) and a gas discharge tube were placed in parallel with the 500Ω resistor (not shown in Figure 4.3). This circuit is illustrated in Figure 4.3.

Figure 4.3: Electrical network used to measure voltage and current. To prevent damage to the A-D card due to voltage breakdown a Metal Oxide Varistor (MOV) and a gas discharge tube were placed in parallel with the 500Ω resistor (not shown in this figure).

The MOVs and gas discharge tubes are non-ohmic devices whose resistance is a function of the applied voltage. At low voltage their resistance is high and at high voltages their resistance is low. Upon a voltage spike these non-ohmic devices draw power from the voltage source and cause an immediate drop in voltage. In order to avoid using a resistor larger than 500Ω , an operational-amplifier (op-amp) circuit was built to increase the current measurement by a factor of 10. The amplifier was calibrated by applying voltages across the input and measuring the resulting output voltage using the multimeter. Twenty independent measurements were made and a least-squares fit of this data was calculated to determine the gain to be used for adjusting the current measurements as well as the op-

amp input bias. The data and resulting fit are shown in Figure 4.4

Op-Amp Calibration

Figure 4.4: Operational-amplifier calibration data points and resulting fit. The slope (x-coefficient) and y intercept were found to be 0.0937 and 0.0015V respectively using a least-squares fit.

The amplifier also adds some security as its position in the electrical circuit makes it more likely to be damaged in a voltage breakdown rather than the A-D. The electrical control was performed using two different types of voltage sources. The first source was the flyback converter developed by TTRL which offers the advantage of being computer controlled but generates fairly high frequency voltage and current data. The second was a Universal Voltronics voltage supply serial number 6-3-325. This was a more conventional voltage supply which generates smooth voltage and current data but must be operated manually. As a result, tests using this supply were restricted to step up and step down voltage commands.

4.1.3 Thermal:

The temperature of the ER fluid and the damper external surface were measured using a Yellow Springs Instrument (YSI) Inc. TH402 submersible and TH709 surface thermistor respectively. Thermistors were selected because of their high electrical resistance sensitivity in the temperature range the damper was expected to operate in. The signal conditioning unit for these sensors was built by ETC electronics and has controls for changing the bias, temperature range and filtering. The full range from 0 to 100 degrees Celsius produced a change in output voltage of 5 volts. A calibration chart of temperature versus resistance was provided by YSI for both probes. To test the calibration of the thermistors and signal conditioners the probes were first submerged in a Styrofoam cup of boiling water and the bias reset such that the output voltage was 5 volts. The probes were then submerged in ice water and the output voltage was recorded. The results indicate that the zero temperature voltage reading was off by 2.4% and 1% for the TH402 and TH709 probes respectively. This is fairly good agreement considering the resistance changes almost two orders of magnitude over this temperature range. The submersible probe was routed through the drainage hole located in the bottom of the damper casing. This placed the probe near the entrance of the fluid gaps giving a representative temperature of the fluid. The surface probe was attached to the outside of the damper wall using an elastic.

A description of the test procedure is given in Section 4.2.

4.2 Procedure:

The damper was supported in a rectangular steel frame which was bolted to the hydraulic actuator housing. The ends of this frame were composed of steel plates one of which provided a flat platform to which the damper was attached. This arrangement also ensures that the damper motion was parallel to the actuator motion. The damper was positioned directly beneath the actuator and fixed from below with an electrically insulated bolt. A plexiglass plate inserted between the bottom of the damper and the platform prevented grounding between the damper and the frame. The piston rod was initially fixed to the hydraulic actuator by a flexible coupling. The use of this coupling was found to produce an unusual step in the damping force data at zero crossings which was attributed to backlash. Subsequently, this coupling was replaced with a rigid connection. Once the piston rod was attached, the load cell voltage was reset to zero on the DC controller. This voltage was rechecked after several cycles of displacement input to ensure that residual loading, possibly due to misalignment of the piston rod and actuator, was not present. The MTS system was setup such that the actuator followed a commanded displacement waveform. The mechanical excitation frequency and shape were set in the function generator and the desired amplitude was obtained by adjusting the strain control on the AC controller. The mechanical excitation frequencies examined were 0.5, 1.0, 2.0, 5.0, and 9.0Hz. Tests for each frequency were attempted at 3, 5, and 10mm mechanical excitation amplitudes. Furthermore, sinusoid and sawtooth waveform shapes were used. Unfortunately, some amplitude and frequency combinations could not be achieved due to mechanical power limits. The field strength levels to be examined were 0, 0.5, 1.0 and

1.5kV/mm. The description of the tests is given below.

4.2.1 Transient Tests:

For each step voltage test, the command signal to the voltage supply was stepped from zero to the desired voltage level within a second after initiation. Due to computer memory limitations, each test was originally limited to three seconds of data acquisition at the fastest sampling rate. The voltage was allowed to continue well after acquisition to allow the data to be stored on disk and to start another test, this time stepping the voltage down to zero. These tests were initially done with the flyback converter, however, due to the high frequency content of the voltage and current signals the Universal Voltronics voltage supply was used in the second round of identification experiments.

Preliminary results of the transient tests showed that the damping force did not reach a steady state condition within the test duration. Subsequently, a series of steady state tests were scheduled as the second round of damper tests. Furthermore, there was little variation in the temperature readings and, therefore, measurements of these quantities were omitted from further tests. This decision also allowed the test duration to be extended to four seconds.

4.2.2 Steady State Oscillatory Tests:

The voltage was applied a full minute before the test was allowed to commence. A steady state condition was ensured by constant observation of the peak to peak damping force displayed by the MTS machine. Data was then recorded for four seconds at the

fastest sampling rate. The voltage level was reset and the procedure was repeated for all voltage levels, mechanical excitation amplitudes and frequencies up to 5Hz.

The analysis of the test data is described in Sections 4.3 and 4.4.

4.3 Modelling:

The fluid analysis in Section 3.5 produced a complicated set of non-linear equations which had to be solved numerically and was found to predict damping force rather poorly. Another approach is to design a lumped parameter model of the system and to estimate the parameters of the system from experimental data. The design of the model is often the most difficult stage.

4.3.1 Previous Damper Models:

Mechanical analogies of the ER effect, such as the one by Gamota (1991) described in chapter 3, give evidence of the kind of elements one might select and their placement in a lumped parameter model of an ER damper. Another mechanical analogy offered by Powell (1995) consists of force contributions from non-linear stiffness, F_k , viscous, F_v , and Coulomb friction elements, F_c . A mathematical description of the forces contributed from each element is given in Equation 4.1.

$$\begin{aligned}
 F_c &= F_{cs} \left(1 + \left(\frac{\bar{F}_{cd}}{F_{cs}} \right) e^{-d|X_3|} \right) \tanh(\epsilon X_3) \quad \text{quasi-static} \\
 &= F_{cd} (1 - e^{-d|X_3|}) \tanh(\epsilon X_3) \quad \text{dynamic} \quad (4.1) \\
 F_k &= i \tanh(d X_1) \\
 F_v &= c X_2
 \end{aligned}$$

where X_2 represents the relative velocity, X_1 represents the relative displacement, a , b , c , d , i , F_{cs} , F_{cd} and ε represents parameters to be identified. There are two expressions for Coulomb friction, each representing the quasi-static, F_{cs} , and dynamic, F_{cd} , deformation regimes. The hyperbolic tangent term was included in the Coulomb friction expressions as a smoothing function between the quasi-static and dynamic terms. This model was found to describe the ER behaviour in oscillatory shear quite well, however, this model was quite labourious to identify. Stanway et.al. (1986) showed that simple viscous and Coulomb friction elements could do a reasonable job of modelling an ER shear mode damper under oscillatory displacement. Their results showed that for high field strength tests above 1.5kV/mm the model estimate of displacement was deviating from the observed behaviour as a flattening of the waveform, resembling a clipped sinusoid, as the velocity passed through zero. This deviation may be evidence of viscoelastic behaviour. Further increases in the field strength above 2kV/mm showed an unusual direction dependence which remained unexplained. In general, both parameters were found to increase with the field strength. Stanway et.al. (1989) proposed another model of the same shear mode damper based on a power law (Eq. 4.2), however, the same model deviations from the observed behaviour were evident.

$$F = c|X_2|^\gamma \text{sgn}(X_2) \quad (4.2)$$

where γ represents the power and sgn represents the sign operator. Nakano (1995) has proposed the mathematical model, described in Equation 4.3, of a flow mode damper where L and h are valve dimensions, and A_v is the valve frontal area.

$$\begin{aligned}
 F &= A_0 \left(\frac{mA_0}{A_v} \dot{X}_v + \Delta P_T \right), \quad m = \rho L A_0 \\
 \Delta P_T &= c \frac{A_0}{A_v} \dot{X}_v + \Delta P_{ER} \operatorname{sgn}(\dot{X}_v) \\
 \Delta P_{ER} &= \left(\frac{3L}{h} \right) \left(aE^+ - (dE^+ - eE^-) (1 - e^{-\frac{F}{b(E)} \frac{A_0}{A_v}}) \right) \\
 b(E) &= \frac{f}{E^-}
 \end{aligned} \tag{4.3}$$

where ΔP_T represents the steady pressure drop across the ER valve, ΔP_{ER} is the pressure drop due to the ER effect, m is the mass of the ER fluid in the valve, ρ is the fluid density and A_0 is the piston area. Most of the parameter values selected for this model are based on steady state results which may not be valid when the model is used in a dynamic situation. No provisions were made in this model to account for the fluid compressibility which was found to be quite significant by this same author in an earlier paper (Nakano and Yonekawa 1994). Unfortunately, this model has not yet been validated under dynamic conditions.

4.3.2 Proposed Damper Model:

The flow mode damper was modelled as a mass with both viscous and Coulomb frictional forces acting upon it. The mass was an equivalent inertia that accounted for both the mass of the damper and the inertial effects of the fluid moving in the cylinder gaps. The dynamics of the system are written in state space form (Eq. 4.4) where the states are the momentum of the system, p_1 , and the position of the damper, d . The parameter u is the damper force in Newtons, A_0 is the piston area in m^2 and sgn represents the sign

function.

$$\frac{d}{dt} \begin{bmatrix} d \\ p_1 \\ I_1 \\ R_1 \\ R_2 \\ u_0 \end{bmatrix} = \begin{bmatrix} p_1 \\ \frac{u-u_0}{A_0} - \frac{R_1 p_1}{I_1} - R_1 \operatorname{sgn}(p_1) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.4)$$

A mechanical analogy is shown in Figure 4.5.

Figure 4.5: Proposed ER flow mode damper model. A description of the four model parameters is given in the text.

The identification technique requires that the parameters of the model be added to the state equations as extra states whose values are held constant (see eq. 4.4). As the difficulties in modelling the system are primarily concerned with the ER fluid and its properties, the equivalent inertia was written in the fluid domain. That is, the inertia, I_1 , represents the rate of change of fluid flow rate per unit of applied pressure and, therefore, its units are $\text{N}\cdot\text{sec}^2/\text{m}^5$ or kg/m^4 . While unconventional, these units are a natural choice in

this problem. To convert the inertia to an equivalent mass in kilograms, multiply by the square of the piston area.

The resistance to flow was modelled as a Coulomb friction component which had a constant value of R_2 and always opposed the motion. This Coulomb friction was combined with a viscous force that was proportional to the relative velocity of the damper, where the resistance parameter is R_1 . This model is a simplified version of the Bingham plastic model in that the fluid analysis in chapter 3 demonstrated that the viscous and Coulomb friction parameters are interdependent in flow mode. This simplification has been successful in other ER modelling applications and the results of the present work indicate that a significant level of predictive performance can be achieved with this simplified version. A final parameter, the load cell bias, u_0 , was added to the model to account for a small amount of drift observed in the signal from the load cell from one data set to the next.

4.4 System Identification:

Given the input and output data from the system the estimation of the model can begin. The solution to this type of problem is the domain of System Identification and many tried and proven methods are available.

4.4.1 Classical Identification Methods:

One method called deconvolution estimates the impulse response of the system without actually arriving at an analytical expression. For instance, consider that the system

output is simply the integral of the impulse response multiplied by the system input (Eq. 4.5). Then each observation can be formed as in Equation 4.6.

$$y(t) = \int_0^t h(t-\tau) * u(\tau) d\tau \quad (4.5)$$

$$y(n\Delta t) = \Delta t \sum_{i=0}^{n-1} h\left(\frac{2n-1}{2}\Delta t - i\Delta t\right) u(i\Delta t) \quad (4.6)$$

Δt represents the sample period and $h(t)$ is the impulse response, $y(t)$ is the system output and n is the number of observations. With this discrete form of the system, a matrix equation of all observations can be formed as a linear combination of values of the input and the impulse response (Eq. 4.7).

$$Y = U*H = \begin{bmatrix} y(0) \\ y(\Delta t) \\ \vdots \\ y(n\Delta t) \end{bmatrix} = \begin{bmatrix} u(0) & 0 & \dots & 0 \\ u(\Delta t) & u(0) & \dots & 0 \\ \vdots & \vdots & \dots & 0 \\ u((n-1)\Delta t) & u((n-2)\Delta t) & \dots & u(0) \end{bmatrix} \begin{bmatrix} h(0) \\ h(\Delta t) \\ \vdots \\ h(n\Delta t) \end{bmatrix} \quad (4.7)$$

Notice that any form of input may be used, however, the matrix U , which is a matrix of input values, must be non-singular in order to solve for H , which is a vector of values of the impulse response. Also, as the number of observations increases the number of computations can become excessive as well as ill-conditioned.

Other identification methods called correlation techniques are a statistical approach and are outlined in Sage and Melsa (1971). The flow of information in these techniques is

shown in Figure 4.6.

Figure 4.6: Block diagram of Correlation identification method. Correlation of $u(t)$ and $x(t)$ may lead to a discrete time transfer function. The system input, $u(t)$, and the disturbance, $v(t)$, must be ergodic and Gaussian with zero mean.

Correlation of $u(t)$ and $x(t)$ may lead to a discrete time transfer function. No a-priori knowledge of the system is necessary and a relatively low input signal may be used to leave the system essentially undisturbed. This low input allows the system identification experiments to be conducted while the system is operated as usual, as if no experiments were being conducted at all. Unfortunately, the use of this technique is restricted to linear systems that are, at most, slowly time varying. The tests required are particularly long in duration and the system input, $u(t)$, and the disturbance, $v(t)$, must be ergodic and Gaussian with zero mean. If $v(t)$ is a stationary stochastic process then $v(t)$ is ergodic if the sample average over the ensemble is equal to the sample average over time. This may be a source of problems since $v(t)$ is not only comprised of the sensor noise but also the output signal of the normal operation of the system and, therefore, may not satisfy the above criteria.

Another method of identification is the sinusoidal response measurement. By

measuring the system response to steady sinusoidal input for various frequencies a bode diagram of gain and phase may be reconstructed. Asymptotic approximations may be applied to the bode diagram of gain to arrive at an analytical, but linear, expression of the transfer function. Brookfield and Dlodlo (1995) have applied this method to arrive at a linear model of an ER torque actuator with some success. There were, however, some observable deviations of their model from the measured transient response and in the bode diagram of phase which was attributed by the authors to the nature of the asymptotic approximation.

Learning models which adjust the model parameters according to errors between the system response and model response are another method of identification. The advantages of using this method are the use of several varieties of search procedures which may not necessarily be based on the model itself. Since these procedures can be independent of the model, this method enjoys a freedom to design for specific error criteria and inequality constraints. The system model must be designed before the identification begins and only a finite number of parameters may be adjusted simultaneously. The adjustment of parameters is a major difficulty due to the presence of multiple local minima and saddle points. Some search routines such as Genetic algorithms examine the entire solution space by picking, at random, multiple starting points. Genetic algorithms work well for many systems but are time consuming. The design of the model may also impose difficulties to convergence if the relative sensitivity of one parameter over another is too great. A lack of orthogonality, which means that the effect of one parameter in the model is poorly differentiated from another parameter, may also result in poor

convergence or multiple solutions.

The classical Wiener theory of non-linear systems offers a method of identification for a specific model design and is relatively systematic in implementation. The parameters of the model are coefficients of an operator which is expanded in Hilbert space and the input is expanded in a Laguerre function series. A flow diagram of this method is shown in the Figure 4.7.

Figure 4.7: Block diagram of Wiener theory of identification. The system input is subjected to a transformation into what are called Hermite polynomials, V_n . The system output, $y(t)$, is cross-correlated with these polynomials to arrive at the model coefficients.

If the input is a Gaussian white noise process, then u_n are uncorrelated Gaussian random processes with equal variances. Essentially the system input is subjected to a transformation into what are called Hermite polynomials, V_n , which serve as model inputs and have the property of being orthonormal. The system output, $y(t)$, is cross-correlated with these polynomials to arrive at the model coefficients. Unfortunately, the number of u_n , the number of Hermite polynomials and length of time required are theoretically infinite

and, therefore, a practical implementation requires truncation. Error analysis of this truncation is difficult to do. The Weiner theory does not account for time varying or unstable systems. Translating the resulting model coefficients into parameters of a more conventional model representation is also difficult.

The ideal identification algorithms for this study would be able to identify the system described in Section 4.3.2 and would be systematic in operation such that model design changes are accommodated easily.

4.4.2 Non-linear Least-Squares Algorithms:

Three of the more popular non-linear least-squares identification techniques are Gauss-Newton (Kalaba and Spingarn 1982), Non-linear Filtering (Detchmندی and Sridhar 1966), and Quasi-linearization (Kalaba and Spingarn 1982). Similar to learning model identification, a model of the system, expressed in state space form, must be designed beforehand. These non-linear least-squares algorithms are formulated in the continuous time domain and produce direct and simultaneous estimates of parameters. A continuous time formulation is preferred at this stage of model design because correspondence between the model and physical quantities is clearer than when using difference equations. Analytical expressions used to adjust parameters are based on the least-squares error between the model's response and the measured response and are approximated by linearizing the state equations. Consider a dynamic system described by a set of first order equations (Eq. 4.8).

$$\frac{d}{dt} \bar{x} = f(\bar{x}, \bar{z}, u) \quad (4.8)$$

where x is a vector of states, ζ is a vector of parameters and u is the model input

In the Gauss-Newton method the parameters are adjusted to obtain an improved estimate of the state vector. These adjustments in parameters are linearly approximated by applying a Taylor series expansion about the state vector x and ignoring the higher order terms. This gives Equation 4.9.

$$\begin{aligned}\bar{x}(\bar{\zeta} + \Delta\bar{\zeta}, t) &= \bar{x}(\bar{\zeta}, t) + \frac{\partial \bar{x}(t)}{\partial \bar{\zeta}} \Delta\bar{\zeta} \\ &= \bar{x}(\bar{\zeta}, t) + \Xi \Delta\bar{\zeta} \\ \Xi_{ij} &= \frac{\partial \bar{x}_i}{\partial \zeta_j}\end{aligned}\tag{4.9}$$

Ξ is called the parameter influence matrix. For details see Kalaba and Spingarn (1982).

The parameters are updated by simply adding the change in parameters, $\Delta\zeta$, to the previous estimate of the parameters. If we had a measurement of the system, $z(t)$, that was a linear combination of the states the error can be defined as Equation 4.10.

$$e_i = z(t_i) - C * \bar{x}(\bar{\zeta}, t_i)\tag{4.10}$$

Then $\Delta\zeta$ can be calculated by the solution shown in Equation 4.11.

$$\Delta\bar{\zeta} = (\Xi^T C^T C \Xi)^{-1} \Xi^T C^T \bar{e}\tag{4.11}$$

Notice that the maximum dimension of the vector e and the matrix Ξ is equal to the number of samples taken. For this reason the Gauss-Newton method is computationally expensive and is prone to numerical difficulties if the sample size is excessive.

Non-linear filtering is a considerably more complicated method of identification

and will not be presented here. The reader is referred to Detchmندی and Sridhar (1966) for more details on the theory behind this algorithm. In short, this method requires the user to solve a matrix Riccati Equation similar to that described in Chapter 2 to arrive at the initial estimate of the state of the system. In light of the nature of the solution, the parameters to be estimated must be included as part of the system state. This new state definition is called the augmented state.

The identification method used in this study is quasi-linearization as described in Kalaba and Spingarn (1982) and Sage and Melsa (1971). This is a least-squares technique applied to continuous time non-linear systems described in state space form. To illustrate the steps required the proposed damper model (Eq. 4.4) will be analysed. Consider a dynamic system described by a set of first order equations shown in Equation 4.12.

$$\begin{aligned}\frac{d}{dt}\bar{x} &= f(\bar{x}, \bar{\zeta}, u) \\ \bar{x} &= [d \quad p_1]^T \\ \bar{\zeta} &= [I_1 \quad R_1 \quad R_2 \quad u_c]^T\end{aligned}\quad (4.12)$$

The augmented state equation becomes Equation 4.13.

$$\begin{aligned}\frac{d}{dt}\bar{x} &= f(\bar{x}, u) \\ \bar{x} &= [d \quad p_1 \quad I_1 \quad R_1 \quad R_2 \quad u_c]^T\end{aligned}\quad (4.13)$$

An approximation of the $k+1$ derivative of the states can be obtained by expanding the state equations in a Taylor series (Eq. 4.14).

$$\frac{d}{dt}\bar{x}^{(k+1)} = f(\bar{x}^{(k)}, u) + J(\bar{x}^{(k)}, u) * (\bar{x}^{(k+1)} - \bar{x}^{(k)}) \quad (4.14)$$

J is called the Jacobian matrix where each element J_{ij} corresponds to the partial derivative of the i^{th} state equation with respect to the j^{th} state. For our system the Jacobian is given in Equation

4.15.

$$J(\bar{X}^{(k)}, u) = \begin{bmatrix} 0 & \frac{1}{I_1 * A_1} & -\frac{P_1}{A_1 * I_1} & 0 & 0 & 0 \\ 0 & -\frac{R_1}{I_1} & \frac{R_1 * P_1}{I_1^2} & -\frac{P_1}{I_1} & -\text{sgn}(p_1) & -\frac{1}{A_1} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.15)$$

The solution to the differential Equation 4.14 is Equation 4.16

$$\bar{X}^{(k+1)} = \bar{P}(t) + r(t) * \bar{X}(0) \quad (4.16)$$

where $P(t)$ represents the particular solution and the remaining term represents the homogeneous solution. By inspection the initial condition for r is an identity matrix and a zero vector for P .

Substitution of this solution into the linearized state equations give the differential Equations 4.17 governing P and r .

$$\begin{aligned} \frac{d}{dt} \bar{P} &= f(\bar{X}^{(k)}, u) + J(\bar{X}^{(k)}, u) * (\bar{P} - \bar{X}^{(k)}) \\ \frac{d}{dt} r &= J(\bar{X}^{(k)}, u) * r \end{aligned} \quad (4.17)$$

2

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To determine the updated parameters, the cost function, Equation 4.18, must be minimized.

$$S = \sum_{i=1}^n [\beta^T * \bar{x}(t_i) - b_i]^2 \quad (4.18)$$

where n is the number of data to be analysed, b_i represent the measured system outputs and β is a weighting vector which when multiplied by the state vector would give the model output. For instance, if we could measure the first state element of our system then the cost function becomes Equation 4.19.

$$S = \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i)^2 \quad (4.19)$$

Partial derivatives of Equation 4.19 with respect to the initial state elements are shown in Equation 4.20.

$$\begin{aligned} \frac{\partial S}{\partial d(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{1,1}(t_i) \\ \frac{\partial S}{\partial P_1(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{1,1}(t_i) \\ \frac{\partial S}{\partial I_{1,2}(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{1,2}(t_i) \\ \frac{\partial S}{\partial R_{1,2}(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{1,2}(t_i) \\ \frac{\partial S}{\partial R_{2,2}(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{2,2}(t_i) \\ \frac{\partial S}{\partial u_0(0)} &= \sum_{i=1}^n (P_1(t_i) + d(0) * r_{1,1}(t_i) + \dots - b_i) * r_{6,1}(t_i) \end{aligned} \quad (4.20)$$

In order to find the minimum of the cost function each of the partial derivatives in Equation 4.20 are set equal to zero. The resulting sums are linear in the initial augmented

state elements and can be solved using matrix algebra. Note that the above procedure also estimates the initial original state. If we already know some of these initial states then it is possible to reduce the number of equations to solve.

4.4.3 Comparison of Least-Squares Algorithms:

Stanway and Mottershead (1986) compared the performance of these three least-squares algorithms by identifying a model consisting of a viscous and Coulomb friction which is quite relevant to the model proposed for our ER damper. Their results indicate that without noise in the measurement the Gauss-Newton method was able to identify parameters with no more than 25 observations compared with 500 for the quasi-linearization method and 1600 using non-linear filtering. Increasing the level of noise in the measurement, however, resulted in large bias errors for Gauss-Newton, up to 40% in some cases, and required considerably more observations to converge. This is undesirable from a computational standpoint. The performances of the other two least-squares algorithms are considerably less affected by the level of noise. Quasi-linearization was found to produce slightly less accurate results compared to the other two during no noise conditions and tended to oscillate. Non-linear filtering, however, requires a greater number of computations for one iteration when compared to quasi-linearization. Consider that during the design stage the order of the model can grow and this makes quasi-linearization the most economically attractive alternative. For the above reasons quasi-linearization was selected to identify parameters from the experimental results. The implementation of this algorithm is described in Section 4.5.

4.5 Implementation:

The above algorithm was programmed in C++. This language was chosen for its powerful object oriented style which makes the reuse of code and the manipulation of data simple and less prone to error. For instance a class object called matrix was created which can perform the steps required for matrix and vector manipulations such as addition and multiplication and can even check for inconsistencies in vector and matrix sizes. Using ideas such as polymorphism and operator overloading allows mathematical operations normally associated with scalars to be applied to any class of object. The result being that mathematical formulas are entered as they are written without ever having the user keep track of how each element is manipulated. Being able to program in this manner promotes the reuse of code because algorithms can be entered in a most general manner. In fact f , J , and the cost function are the only user specified functions required and they are entered in a file that serves as a template. The computer program 'Maple TM' was used to calculate the Jacobian symbolically. 'Maple TM' also has provisions for generating C code which was of great help in transferring f and J into the template file.

Solution of the resulting linear equations was performed using singular value decomposition (SVD), described in the book 'Numerical Recipes in C' (Press et.al. 1992). A square matrix A of size M may be expressed as a product of a column-orthogonal matrix U , a diagonal matrix W and the transpose of an orthogonal matrix V , Equation 4.21.

$$A = U * W * V^T \quad (4.21)$$

The elements of matrix W are called the singular values. We wish to find a vector 'a' given the matrix 'A' and a vector 'b' which minimizes the following Equation 4.22.

$$\chi^2 = (A * a - b)^2 \quad (4.22)$$

The solution to this problem is shown in Equation 4.23, where $U_{(i)}$ and $V_{(i)}$ represent the i th column of matrix U and V respectively.

$$a = \sum_{i=1}^M \left(\frac{U_{(i)} * b}{w_i} \right) V_{(i)} \quad (4.23)$$

In many instances a model to be identified may contain more parameters than necessary to describe the system or perhaps the parameters chosen do not describe the system at all.

For both scenarios the system of equations to be solved in the quasi-linearization algorithm may become singular. SVD has an advantage in this situation because instead of generating numerical problems, as would result from LU decomposition or simple matrix inversion, SVD simply drives the parameters that result in a singular condition to zero and continues to solve for the others in a least-squares sense. Because of this ability SVD is the method of choice in solving most linear least-squares problems. Furthermore, SVD theory contains provisions for testing for ill-conditioned sets of equations and for estimating the covariance matrix of the estimate. The SVD condition number is defined as the ratio of the smallest singular value to the largest singular value. If this ratio is less than the smallest floating point number that can be represented in the computer then an ill-conditioned matrix exists.

Identical results were obtained when a numerical example in Chapter 7 of Kalaba and Spingarn (1982) was repeated as a test case. The proposed damper model of Section

4.3.2 was then implemented. The example data sets were generated with known parameters to demonstrate the quasi-linearization algorithm and to determine the sensitivity of the algorithm to mechanical excitation amplitude and frequency as well as sampling rate and sample size. The results are given in tabular form in Table 4.1 and 4.2.

Table 4.1: Identification sensitivity to mechanical excitation amplitude.

Amplitude, mm	3	5	10	15
% error I_1	0.74	0.72	0.68	0.67
% error R_1	0.21	0.32	0.38	0.39
% error R_2	1.2	0.87	0.87	0.87
% error u_0	13	11	11	11
RMS error in position, mm	0.0012	0.0010	0.0010	0.0010
Matrix Condition number	5.4e-18	7.3e-18	8.2e-17	2.8e-16

Table 4.2: Identification sensitivity to mechanical excitation frequency.

Frequency, Hz	1	2	5	9	15
% error I_1	1.62	0.97	0.72	0.66	0.54
% error R_1	0.01	0.06	0.33	1.45	3.61
% error R_2	0.12	0.14	0.87	0.49	6.50
% error u_0	1.40	1.50	11.3	24.3	35.7
RMS error in position, mm	0.0038	0.0005	0.0010	0.0006	0.0003
Matrix Condition number	5.4e-19	9.3e-18	7.3e-18	7.9e-18	3.9e-17

Notice that most of the parameter errors are less than 1% for I_1 , R_1 and R_2 which is quite acceptable. Increasing the excitation frequency appears to reduce the error in the estimate of inertia at the expense of R_1 and R_2 . Assuming that the velocity of excitation is sinusoidal, the ratio of the amplitude of acceleration to the amplitude of velocity is the frequency. Increasing the excitation frequency results in considerably higher acceleration levels and, therefore, more dominant inertial effects. The increased contribution of inertial effects tends to reduce the effects of the other parameters making them more difficult to identify. This would explain why simply increasing the amplitude of excitation has a less dramatic effect on the errors when compared to changing the frequency. For this same reason the bias parameter has a relatively higher error compared to the others but this effect is not expected to contribute very much to the overall response. Due to this small contribution these elevated errors in u_0 are not felt to be important.

Table 4.3: Identification sensitivity to sampling period.

Sampling Period, ms	0.5	1.0	2.0	5.0	10.0
% error I_1	0.35	0.72	1.42	3.43	5.76
% error R_1	0.15	0.33	0.86	1.36	4.70
% error R_2	0.52	0.87	0.53	10.6	11.8
% error u_0	7.1	11.3	2.2	71.6	10.0
RMS error in position, mm	0.0006	0.0010	0.0005	0.0060	0.0025
Matrix Condition number	6.9e-18	7.3e-18	7.5e-18	8.9e-18	1.2e-18

Table 4.4: Identification sensitivity to sample size.

Sample Size, periods	1 (200 pts)	2(400 pts)
% error I1	0.72	0.83
% error R1	0.33	0.27
% error R2	0.87	1.88
% error u0	11.3	1.2
RMS error in position, mm	0.0010	0.0018
Matrix Condition number	7.3e-18	2.4e-19

Tables 4.3 and 4.4 give an impression of how the sampling period and sample size affect parameter estimates. Selection of a sampling period of at most 1ms appears to keep the errors of the major parameters below 1%. A 2ms sampling period shows elevated levels of I_1 error with the others less than 1% while higher sampling periods produce estimate errors greater than 1%. When the sample size was increased to 600 data points or 3 periods of input excitation, not shown in Table 4.4, the identification algorithm did not converge within 5 iterations but instead was unstable. An explanation of this result may be that the system of equations is becoming ill-conditioned, possibly because some elements in the matrix are growing faster than others. Evidence of this ill-condition is demonstrated by the resulting lower matrix condition number in Table 4.4.

4.6 Summary:

This chapter introduced a simple non-linear model of the flow mode damper comprised of an inertial parameter, viscous friction and Coulomb friction parameters. The identification algorithm used to identify the parameters of this model was quasi-linearization which is a non-linear technique derived in the continuous time domain. Testing of this algorithm indicated that it is capable of identifying parameters for the proposed model, however, caution must be exercised in the selection of mechanical excitation frequency and amplitude, sampling period and sample size.

Chapter 5: Results

5.0 Introduction:

The results of the experiments and analysis are presented in this chapter. A qualitative analysis of the raw data is given in Section 5.1. Graphs of representative data show the damper force, displacement, voltage and current measurements as a function of time on four separate plots. Section 5.2 discusses the estimation of parameter values through fluid analysis and geometric considerations. This parameter estimate is used to initiate the identification procedure. Section 5.3 presents the results of the analysis. The identified parameter values and mean rectified model errors are presented in graphical form as a function of mechanical excitation frequency and amplitude, field strength and the type of voltage supply used.

5.1 Qualitative Observations:

Figures 5.1 through 5.3 show the measured quantities of a typical transient response and steady state response obtained from the MTS test sessions using the Universal Voltronics voltage supply. Measurements were taken at a sampling rate of 1kHz. At approximately the half second mark the voltage across the damper was stepped from zero to 3kV manually. This step has an immediate effect on the damping force and current. The damper force graph in particular has hardly any transition period once the voltage level has reached its final value. High frequency spikes in the damping force are observable in the compression stroke only, however, despite this effect the tension and compression results appear symmetrical and steady. Figure 5.1 may appear to reach steady state after the change in voltage, however, steady state results obtained a minute later (see Figure 5.2) show that the damping force almost doubles in amplitude and appears asymmetrical. The high frequency spikes observed in compression are more numerous than before and may be the cause of the inability to identify parameters for the remainder of the steady state results. For some steady state results a double peak is observed in the damping force measurement such as the one occurring just before the 3 second mark in Figure 5.2. Steps in the damping force in Figure 5.2 when the force changes direction are present only in this data set because of a backlash in the connection between the load cell and damper piston rod and, therefore, are not an intrinsic property of the damper. Unfortunately, no further data sets using this operating condition without this faulty connection were collected. The current signal contains ripples which appear to be in phase with the damping force.

Figure 5.1: Transient test results using the Universal Voltronics voltage supply.

Figure 5.2: Steady State test results using the Universal Voltronics voltage supply.

Figure 5.3: Step Down Transient test results using the Universal Voltronics voltage supply.

Notice also that the shape of these ripples mimic the profile of the force measurement as well. Whittle (1994) has obtained an expression which fits the pressure response to the current. This coupling may be expected since electrical energy is stored in the moving fluid. Block (1984) has indicated that ER fluids may be capable of generating electrical energy from the fluid motion, however, no research has been able to demonstrate this property. On some tests, where the mechanical excitation frequency is higher, current levels reach 0.2mA after the initial transient in voltage. Under these conditions the ripples observed in the current are also visible in the voltage. This is explained by considering that the increased power consumption affects the ability of the power supply to maintain a steady voltage. When the requested voltage is stepped down to zero, shown in Figure 5.3, the actual voltage follows a slow discharge path and the amplitude of the damping force as well as the current follows. Firoozian (1995) and Whittle (1994) observe a threshold voltage which must be exceeded before the ER effect is noticed. This threshold may be the result of Brownian motion which may disrupt the initial formation of ER structure. Firoozian also notes that the steady flow current draw follows a squared voltage dependency.

Figures 5.4 and 5.5 depict the same tests described above except the fly-back converter is substituted for the Universal Voltronics as the voltage supply. Apart from the expected high frequency content in the voltage and current measurements the damper force shows the same characteristics as before. Notice that the high frequency spikes observed in compression are smaller in magnitude. Turning our attention to the steady state results of Figure 5.5 the damping force is noticeably different from Figure 5.2.

Figure 5.4: Transient test results using the Fly-Back Converter voltage supply.

Figure 5.5: Steady State test results using the Fly-Back Converter voltage supply.

Damping force is more sinusoidal in shape and the high frequency spikes appear to be the same size and same number as in the transient case. Furthermore, the double peak seen in Figure 5.2 is absent. Observe, however, the last second of this test where the amplitude of the damper force is lower compared to the other cycles observed. This indicates that despite waiting a full minute the system has not reached steady state and may possibly be cycling. Figure 5.6 shows a transient test using the Universal Voltronics voltage supply but commanding a sawtooth mechanical excitation at 1Hz and an amplitude of 10mm. Notice that the damping force resembles a square wave which is expected considering that for much of each cycle the piston velocity is constant. This nearly constant damping force magnitude also results in the absence of any ripples in the observed current. The lack of character, or variation, in the force and displacement prevents all the significant modes of the system from being excited. This situation is analogous to trying to find the slope of a line when the only information given is the location of a single data point. For this reason, identification of parameters from the saw-tooth mechanical excitation and force records was found to be difficult because the viscous and Coulomb friction parameters could not be differentiated. High frequency spikes in the damping force at the zero crossings reflect the fact that the damper has inertial properties which require additional force to impart sudden changes in the piston velocity. These changes in velocity occupy a very short period of time compared to the entire test which will further complicate the task of identifying the inertial parameter. For the above reasons, not much faith is placed on the saw-tooth mechanical excitation results.

Figure 5.6: Transient test results using the Universal Voltronics voltage supply.

Upon closer inspection of the damping force plot the force is not constant in magnitude and may indicate a slow change in parameter values from oscillation to oscillation.

To examine this change the identification procedure was performed on segments of the data corresponding to the period of oscillation. The identification assumes the parameters are constant over that time period but by shifting the data window used forward by the sample time an indication of any changes in the parameters could be evaluated in an approximate way. The results of these studies indicated that there may be some pattern to the variation in parameter values and that a large degree of variability may be present in the parameters at high levels of electric field strength.

5.2 Estimating Parameters:

Quasi-linearization requires a guess at the initial augmented state vector in order to begin the identification. As little is known about the behaviour of the energized ER fluid in flow mode a good starting point would be to estimate the model parameters at zero field. The inertial parameters may be estimated by first assuming that the fluid is incompressible and that the geometry of the damper passages and the fluid density are known. From the schematic of the damper, Figure 3.13, the damper can be decomposed into a series of uniform fluid channels each representing an inertia calculated in Equation 5.1.

$$I = \frac{\rho L}{A_e} \quad (5.1)$$

ρ represents the fluid density, L is the channel length and A_e is the entrance area of the channel.

Inertia is analogous to an inductor in an electric circuit with fluid momentum and pressure represented by current and voltage respectively. Calculation of an equivalent inertia, therefore, follows the same additive laws for parallel and series inductors. The network of inductive elements is shown in Figure 5.7.

Figure 5.7: Electrical analogy of damper inertia. From the schematic of the damper, Figure 3.13, the damper can be decomposed into a series of uniform fluid channels each representing an inertia.

The equivalent inertia was calculated to be $216 \text{ kN}\cdot\text{s}^2/\text{m}^5$ which turns out to be very close to the inertia identified from the test results.

Viscous friction can be estimated from the fluid equations described in Chapter 3.

Assuming that the viscosity of the fluid is $0.1 \text{ Pa}\cdot\text{s}$ the viscous friction parameter was calculated to be $14 \text{ MPa}\cdot\text{s}/\text{m}^3$. This estimate turns out to be higher than the value identified by a factor of 3 which may reflect the dilatant behaviour observed in the fluid. A very low Coulomb friction is anticipated at zero field and therefore this parameter is initially set to zero. Unfortunately, this Coulomb friction parameter displays more variation with mechanical excitation than the other parameters but in general remains less than 500 Pa or approximately 2 N .

5.3 Quantitative Results:

From the standpoint of the identification method, having the performance index based on the error between the predicted and measured piston displacement rather than the damper force was a natural selection because displacement is an easy quantity to accommodate in the state definition. The damper model, in relation to controlling a system, may primarily be used to predict force rather than position in which case this ability should be assessed. The difficulty of estimating the first and second derivatives of the displacement measurements was resolved by differentiating the Fourier series representation of the displacement record. Coefficients of the Fourier series were calculated using the fast Fourier transform routine in 'Matlab'.

5.3.1 Assessment of Force Prediction Errors:

Figure 5.8 shows the mean rectified errors in the predicted damping force using a single parameter estimate averaged from the entire data set. Figure 5.9 shows errors based on the parameter values identified from single period segments of data. The data indicate that there is hardly any significant improvement in the quality of the model when parameters are identified on single period segments of the measured data which suggests that the parameter variation can be neglected at least within the 4 second test duration. Note that a change in the least significant bit of the A-D converter for the load cell measurement only corresponds to 0.33N and the differences between the two data sets are generally less than 1N. With only a few exceptions, the prediction error tends to grow with oscillation frequency and amplitude as well as electric field strength. These variables

all tend to increase the amplitude of the damping force and, therefore, one should expect a corresponding increase in prediction error. In terms of deciding which type of voltage supply characteristic is better for use in predicting damper force these results are inconclusive. The Universal Voltronics voltage supply, however, appears to have a slight edge in predictability for all frequencies except 1Hz at an amplitude of 10mm.

Figure 5.8: Mean rectified error in predicted damping force, (N), from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The average parameter values obtained over entire test interval were used to simulate the damper. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Figure 5.9: Mean rectified error in predicted damping force, (N), from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The time varying parameter values obtained over entire test interval were used to simulate the damper. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Examples of the measured force data and the predicted force data are shown in Figures 5.10 and 5.11. Except at zero crossings in the force a good agreement between the predicted and the measured forces is indicated. The general profile of the damping force is captured, however, where the model predicts a step in force when the damper passes through zero velocity the force measurement shows a slow transition. Considering that the static yield stress of ER fluids tend to be greater than the dynamic value, this transition seems inconsistent. The discrepancy at zero crossings represents a lag in the response of the real system, possibly due to fluid compressibility.

5.3.2 Assessment of Parameter Values:

Parameter values are listed in Figures 5.12 Inertial Parameter, 5.13 Viscous Friction, 5.14 Coulomb Friction, and 5.15 Bias Parameter. The independent variables are the frequency and amplitude of the damper motion, the strength of the electric field applied to the fluid of the damper and the voltage supply used. The missing entries at the damper oscillation frequency of 9Hz and 10mm amplitude are not available due to an inability to achieve this operating condition. These results indicate that the model parameters are a function of all these variables. In the first 1Hz plot of figure 5.12 the inertia parameter varies by almost a factor of four as the electric field strength is increased from 0 to 1.5kV/mm but also changes by a factor of four as the oscillation frequency is increased from 1Hz to 5Hz. At 5Hz and above, the inertial parameter appears to be insensitive to the strength of the electric field, however, the ER effect recovers slightly with lowering the mechanical excitation amplitude.

Figure 5.12. Inertial parameter, I_i ($\text{kN}\cdot\text{s}^2/\text{m}^5$), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Figure 5.13: Viscous friction parameter, R_1 ($\text{MPa}\cdot\text{s}/\text{m}^3$), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Figure 5 14: Coulomb friction parameter, R_2 (Pa), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Figure 5.15: Bias parameter, u_o (N), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Similar effects can be observed in both the resistance parameters although at the highest oscillation frequency for each amplitude there is an unexplained variation in the Coulomb friction parameter. Comparison of parameters based on the type of voltage supply used yields quite interesting results. For the inertia parameters tabulated only 6 out of 32 values were greater when using the fly-back converter voltage supply. For the friction parameters 18 of the viscous component and 25 of the Coulomb component were greater using the fly-back converter out of 32 tests. This implies that the characteristics of the input voltage have an effect on the model parameters.

The level of the load cell bias is small and does not have a significant effect on the results. The trend of the bias is to increase with field strength. This implies that there is some asymmetry in the damping force which increases the force in tension over compression. Note that, due to the damper construction, the high voltage electrode is in closer proximity to the bottom of the damper bucket than to the lid. The electric field will be quite complex at the entrances of the electrode gaps such that the ER fluid could possibly solidify before entering the 3mm electrode gaps. If this solid formation is large in volume it must be deformed to enter the gaps. The compression force may be lower for this damper because the ER solid forming at the bottom entrance of the gaps is in a more confined space and requires less deformation to enter the gaps compared to the other entrance.

Figure 5.16 shows the RMS damping force from the transient tests and the force amplification obtained at steady state in Figure 5.17. Figure 5.17 indicates that the steady state RMS damping force amplification averages 1.91 compared to the transient result.

Figure 5.16: Root mean squared damping force from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The data is incomplete at the 10mm amplitude since this operating condition could not be achieved at 9Hz.

Figure 5.17: the force amplification from transient results to steady state. The steady state data does not include the 9Hz operating condition.

Of the transient values available, the fly-back converter produces a higher damping force in 20 of the 33 combinations of mechanical excitation and non-zero average field strength. This observation may be the result of greater peak levels of electric field strength. Note that the ER effect has a squared dependence on electric field strength and although the average field strength is 1KV/mm the ER effect may be greater.

Figure 5.18 shows the major model parameter values at steady state using the fly-back converter. Identification results using the Universal Voltronics voltage supply were generally unstable and will not be presented here. Results for the 9Hz sinusoidal excitation are unavailable. The inertia and the viscous components of friction show increased values over the transient values while the Coulomb parameter is reduced. For instance at 2Hz and at an amplitude of 10mm the inertia has grown from 212 to 356 $\text{kN}\cdot\text{s}^2/\text{m}^3$ and the viscous parameter has grown from 9.86 to 15.07 $\text{MPa}\cdot\text{s}/\text{m}^3$.

Figure 5.18: Parameter values for steady state test results using the Fly-back converter. Sinusoidal mechanical excitation of amplitude 5mm and 10mm. The steady state data does not include the 9Hz operating condition.

The values in Figures 5.12 to 5.15 are used to linearly interpolate the parameters to predict the damper force for the step down voltage tests. Figure 5.19 is typical for most of these tests and shows that the predicted force under estimates the measured response. The first half second of this test where the voltage was kept constant indicates that the parameters have changed their values from that observed in the step up voltage tests. Remember that between the step up and step down voltage tests a certain amount of time was allowed to pass in order to allow data in the system memory to be stored on the hard drive. As half the tests were performed manually using the Universal Voltronics voltage supply this waiting period would have been longer than with the fly-back converter. During this time the parameters may have changed. In future, the step down voltage tests should be conducted separately from the step up tests so that this waiting period is eliminated.

Figure 5.19: Measured (solid) and Predicted (dashed) Force using Tabulated Parameter Values

5.4 Summary:

The results of the qualitative and quantitative analysis have illuminated interesting points regarding the behaviour of the ER flow mode damper:

- The step up transient test results appear to reach a steady state condition immediately after the voltage has reached its final value. In reality the force from the damper reaches its full potential at least a minute after the initial voltage step.
- The steady state results indicate an amplification of the RMS damping force of 1.91 over the transient damping force but the steady state force response using the Universal Voltronics voltage supply suffers from non-linear effects.
- The ER effect on the identified parameters is strengthened with increases in electric field but is attenuated by increases in mechanical excitation frequency and amplitude.
- The model parameters are also affected by the type of voltage supply is used. The Fly-back Converter, for instance, tends to generate less inertial effects while enhancing the Coulomb friction effects when compared to the Universal Voltronics voltage supply.
- Plots of the predicted and measured force response for the step up transient results show that the model has good predictive capability with differences occurring at zero-crossings in the force.
- When the identified parameters from the step up transient tests were used to predict the step down response, the model performed poorly. This result may be the result of a slow variation in the parameter values over time.

Possible causes of the non-linear behaviour demonstrated by the ER flow mode damper will be discussed in Chapter 6 as well as the ramification on the design of controllers.

Chapter 6: Discussion

6.0 Introduction:

The results in Chapter 5 indicate that the ER effect in a flow mode damper is non-linear in nature. Possible explanations for this behaviour are discussed in Section 6.1.

Section 6.2 examines the impact of these non-linearities on the design of controllers. In addition, the results of the analysis indicate that further improvements in the model design are required. These model improvements are discussed in Section 6.3.

6.1 Discussion:

The results of the identification from the simple ER damper model indicate that the ER effect is highly dependent on the mechanical excitation frequency and amplitude. This dependence may be understandable considering that all explanations of the ER effect point toward the solid structure formed under an electric field. Naturally the behaviour of the ER structure is highly dependent upon how the material is stressed, therefore, the dependence of the model parameters to mechanical excitation was expected. Increasing the oscillation frequency or amplitude raises the level of fluid agitation which would hinder the ER structure formation resulting in an observed decrease in ER effect. At the other extreme, a lower oscillation frequency or amplitude appears to increase the ER effect, most notably affecting the damper inertia. Recall that an ER solid layer was shown to be adhering to the inside surfaces of the electrodes under steady shear rate by Klingenberg et.al. (1991). In between these solid layers was ER fluid which grows in thickness as the increased shear rate erodes away the exposed surface of the ER solid. For shear mode devices, this variation in gap width would make difficult the prediction of performance because the actual conditions experienced by the fluid are difficult to measure. Similar events in a flow mode damper are conceivable. See Figure 6.1.

Solid layer adhered to electrode surface

Figure 6.1: Possible explanation of the ER behaviour in a flow mode damper. ER solid layer was shown by Klingenberg et.al. (1991) to adhere to the inside surfaces of the electrodes under steady shear rate. This solid layer may also form in a flow mode damper and cause of an apparent increase in the inertial and frictional effect.

A constriction in the gap width would affect the viscous friction parameter since an increase in mean fluid velocity in the gap would be required to support the same flow rate. In Chapter 5, the estimate of the fluid inertia was specified as a function of damper geometry and fluid density. If the effective electrode gap width is reduced with increased field strength and low shear rates then this would enhance the effective inertia since an equivalent change in pressure would act on a smaller area, thus imparting a lower change in fluid flow rate. Inertial effects appear significant for low frequencies of oscillation up to 2Hz at an amplitude of 10mm and 5Hz at an amplitude of 3mm.

Recall that the differences in ER effect between the two voltage supplies for the transient results indicates that Coulomb friction is more dominant when using the fly-back converter and that inertia is more dominant using the Universal Voltronics voltage supply.

The fly-back converter voltage characteristic displays a greater amount of variation. Given that the ER effect has shown in the past to have a squared dependence on the electric field the rate of chain formation may be greater using this voltage supply simply because the peak voltages are higher. On the other hand, because of the valleys in the applied voltage, the ER solid formation on the electrodes and in the plug region may not be as strong when compared to the solid formed using the Universal Voltronics voltage supply. This weakness would reduce the thickness of the solid formed on the electrodes and lead to the observed lower inertial properties. Curiously, steady state results using the fly-back converter display a lack of Coulomb characteristic which implies that the formation of fibrils is less active. One possible explanation may be that most of the ER particles have been removed from the fluid to form the ER structure on the electrodes and in the plug leaving nothing to form fibrils. If this were true then the steady state force response using the Universal Voltronics voltage supply should appear similar compared to the fly-back converter response but is rather noisy. This noise may be an indication that the solid structure formed under a constant electric field is more prone to sudden fracture. In this case, the high frequency spikes in the force response are simply the result of stick-slip phenomenon. A constantly varying field may provide some stress relaxation in the solid because there are some periods of fluidity when the electric field is low.

The force prediction errors shown in Tables 5.1 and 5.2 indicate that not much predictive performance is gained by estimating parameters more frequently than the test duration of four seconds once the electric field has been applied. With any extended length of time, however, the constant application of electric field can cause the model parameters

to change dramatically as was shown by the near doubling of the damping force amplitude at steady state demonstrated in Table 5.7. The electric field may be filtering ER particles in the electrode gap and thus fortifying the ER structure already formed there. It is important to realize that in this type of damper the fluid is continually passing through the electric field and will thus pass from areas of little or no electric field to areas of maximum electric field strength. The electrode gap is a small volume compared to the entire capacity of the damper and, therefore, the gap receives a nearly limitless supply of raw material from which it can draw. One would suspect that this accumulation of particles is a slow process considering it is driven by the rate that the ER fluid is exposed to the electric field. This process may explain why the damper force does not reach full potential until a minute has elapsed. There is some evidence that a cycling condition is present. Unfortunately, the damping force profile at this condition is noisy and the identification routine was unable to resolve the model parameters for many of the steady state test results. This kind of unpredictable behaviour is undesirable for the design of a controller and should be avoided.

6.2 Considerations for Controller Design:

Although all model parameters are smaller with increased levels of mechanical excitation the viscous and Coulomb friction contributions are still significant. In light of this result, the decision to include the enhancement in inertia into the controller design may be predicated by the level of mechanical excitation that is expected. The variation in the ER effect as a function of oscillation frequency and amplitude implies that, in a control

system, the gains used to determine the response of the damper to mechanical excitation must be a function of the frequency of excitation. Higher frequency inputs will require different gain levels due to the decreased effect of the electric field on the ER fluid. The dependence on frequency is evidence of a dependence of the current damping force on previous movement and activity of the damper. In this case, it may be possible to predict the future damping force by using a regression type of model. When the input is not repeated but is a general function of time, all that can be said is that there is a memory effect or hysteresis in the fluid.

Accommodating non-linear behaviour can be accomplished as long as the non-linearities can be described in a mathematical model (Stanway, Sproston and Wu 1989). One approach to a non-linear controller was proposed by J.J. Craig (described in Stanway et.al.1989) and is applied to a single degree of freedom system in Figure 6.2.

Figure 6.2: Single degree of freedom system with ER damper. M_s represents the mass of the system to be controlled and K_s represents the suspension stiffness.

There exists two feedback loops. One loop is used to estimate the external and non-linear forces on the mass, M_s , derived from the state of the system. This estimate is added to the output of a second feedback controller such that the system dynamics caused by the non-linearities are cancelled. The net effect is that the controller only regulates the mass position which can be done using simpler linear control techniques.

Cancellation of the non-linearities is also used by Nakano (1995). In this example, however, the electric field applied to the ER damper is modulated in order to eliminate the Coulomb friction characteristic in the force-velocity curve. This causes the ER damper to behave as a passive shock absorber. Through computer simulation of a quarter car model the transmissibility results showed improvements at both the first and second resonance frequency over that obtained by simply applying constant voltage. The friction coefficient was then modulated according to the sky-hook control law proposed by Karnopp and Crosby (1974) leading to a reduction in transmissibility observed around the first resonance.

To account for the slow variation in parameters that occurs progressively from cycle to cycle identification would have to be performed periodically. This requirement is more suitable to adaptive control techniques. If the mechanism for this slow variation is correctly described in Section 6.1 then there is a possibility that this effect can be reduced by the controller. A continually reversing electric field, for instance, has been reported by Chen et.al. (1993) to prevent material buildup on the electrode surfaces, possibly because the ER structure is allowed to disassemble and rebuild continuously. Although there is a great deal of high frequency content in the voltage profile of the fly-back converter this

characteristic is not the same because the electric field is not removed periodically.

Application of this type of electric field has been reported by Stevens et.al. (1985) and Boissy et.al. (1994) to have a lower ER effect than when applying a constant field.

Furthermore, there is a limit to the maximum frequency of the electric field which can be applied before the ER effect is no longer present. Some benefits can be realized with lower power consumption given that AC power consumption is less compared to DC for the same RMS voltage. Another possible solution would be to increase the ratio of the electrode gap volume to the damper capacity. This would limit the amount of ER particles entering the electrode gap since the supply of material is reduced.

6.3 Model Improvements:

The model had some difficulty in predicting the performance of the damper near zero crossings in the damper force. The model may be improved in this respect by adding hysteresis effects and the effects of fluid compliance. Building upon our existing model the modification may look similar to Figure 6.3.

Figure 6.3: Revised ER damper model including fluid compressibility effect. The plastic element of the model resembles the model described in Section 4.5. The viscoelastic behaviour is modelled as a simple Voigt element. A description of the revised model parameters is provided in the text.

The plastic portion of the model resembles the model described in Section 4.5. The addition is a viscoelastic portion located in front of the plastic model. This revised model is similar to the one proposed by Gamota (1991) in Section 3.4.1, however, instead of using a Zener element to model the viscoelastic behaviour a simple Voigt element will be used. The Voigt element is analogous to a spring, C_1 , in parallel with a viscous friction element, R_3 . The Zener element may not be relevant in this model of the flow mode damper because C_1 , which was intended to model the rheological stiffness of the ER fluid, takes on the additional role of modelling compressibility resulting from the presence of air in the fluid. A further departure from the model of Gamota is the inclusion of an additional inertial element, I_2 , which would represent the inertia of the piston rod, the fluid above and below the piston, and a portion of the fluid inertia in the gap. Since the inertia in the gap

was found to be electric field strength dependent as well as deformation dependent, it will be difficult to relate a proportion of the equivalent damper inertia to either inertial parameters, I_1 or I_2 . The same argument may be made for the two viscous friction parameters, R_1 and R_3 . For this reason it is expected that all parameters will be electric field dependent and deformation dependent. The Coulomb friction parameter in our original model only had to respond to the piston velocity because compressibility was not modelled. The new model requires that the Coulomb friction parameter respond to the momentum and force applied to I_1 instead. This relationship introduces the complication of modelling the stick-slip phenomenon (Woestman 1993) and hysteresis (Bullough 1989). The six parameters plus the load cell bias and the position and momentum of each inertia would have to be identified. This is a difficult task because the number of unknowns raises the number of system states. The damper would have to be tested with an input with considerably more variation than what has been used so far such that all unknowns can be identified.

Kalaba and Spingarn (1982) have discussed a method of optimizing the input applied to a system to identify a given model of any order. The method involves identifying parameters of the desired model while the identification test is being conducted. The parameters are continually updated during the identification test. These values are also used to calculate the system input required to optimize the condition number of the solution matrix from Equation 4.20. This method of input optimization assumes that the model chosen reflects the behaviour of the system adequately such that identification is possible. It is, therefore, wise to test the validity of the model using any

input/output data with sufficient variation to identify the required number of parameters.

Also, having a good initial estimate of the parameter values is essential in order to prevent wide fluctuations in the commanded input at the beginning of the identification test which may otherwise cause damage to the system.

The current state of the model is not as useful for control as it could be due to the highly dependent nature of the model parameters to mechanical excitation frequency and amplitude, electric field strength and time. The results of the parameter identification and the results of other researchers suggest that the parameters may be velocity dependent. For instance, Nakano (1995) has suggested that the resistance to flow may have an exponential relationship and Stanway et.al. (1989) has investigated a power law to describe the resistance to flow. Research into the establishing of functional relationships between model parameters, such as fluid inertia, Coulomb friction and viscous properties, and operating variables such as mechanical excitation and electric field strength should be conducted. Condensing the results of the analysis to a few parameter values would be valuable in refining the model design. These refinements may reduce the number of identification tests that would have to be conducted in future.

6.4 Summary:

The observed behaviour of the ER flow mode damper may be explained a change in gap geometry resulting from an ER solid layer adhered to the electrode surfaces:

- The decrease in the electrode gap width would result in increased inertial effects as well as enhance frictional effects due to higher fluid velocities.
- Increases in the ER effect when the voltage is kept constant over a long period may be evidence of the electric field accumulating ER particles in the electrode gap.
- The dependence of the damper behaviour on the voltage supply may be explained by a higher fluidity of the ER solid when subjected to a high frequency electric field.
- Improvements in the model may involve the inclusion of the viscoelastic or the fluid compressibility effect. Further refinements in the model would involve investigating other forms of friction models and functional relationships between model parameters and operating variables such as mechanical excitation and electric field strength.
- The dependence of the model parameters on mechanical excitation frequency and amplitude indicate that a regression model of the ER damper may be more appropriate for this application.

Chapter 7: Conclusions and Future Work

7.0 Conclusions:

Controllable vehicle suspensions have an advantage over passive suspensions because the system parameters can be continually adjusted to suit varying operating conditions and individual driver preferences. The semi-active suspension was found to provide performance benefits comparable to active suspensions without the monetary costs associated with active systems or the implementation difficulties of slow-active suspensions. The design of any control system is enhanced by the use of a mathematical model of the system. A model can also help save money through reduced data acquisition hardware requirements. In practice, actuators have dynamic properties which may be an important consideration in the design of control laws. These dynamic properties are significant in ER dampers used in vehicle applications.

A number of potential obstacles are faced by designers of hardware and control systems of ER devices. The mechanism behind the ER effect is not fully understood. The observations of the ER rheological properties show non-linear effects, including hysteresis, which make modelling these devices difficult. The design of controllers for flow mode devices is hampered by the lack of a sufficient number of studies of ER flow mode properties. The use of classical fluid analysis techniques to relate shear mode rheological properties with flow mode properties has had limited success. This was demonstrated in this thesis when the damping force predictions, based on an annular and a flat plate

approximation, were unable to adequately predict the observed response. Furthermore, an asymmetry in the compression and tension magnitudes in the TTRL ER damper prompted a redesign of the damper components. These difficulties demonstrate that the modelling of an ER flow mode damper is a non-trivial task.

The results of the model identification indicate that a simple model containing inertia and both viscous forces and Coulomb friction can provide a good level of predictive capability within the first four seconds after the electric field has been applied. The parameter values used in the model were found to have a significant dependence on the frequency and amplitude of mechanical excitation. Due to this dependence on mechanical excitation, the current state of the model is not very useful for control. There is some evidence that there is a slow variation in the parameters over time. The degree of accumulation of ER particles on the surface of the electrodes may account for both these effects. This is an important feature to consider when designing controllers for dampers based on these fluids. For instance one may try to minimize this accumulation by using a sinusoidal voltage as the system input. To account for the mechanical excitation frequency and amplitude dependence a regression type of model could be used, and adaptive control techniques could be used to account for the slow changes in model parameters.

7.1 Future work:

Further work on this project should involve using a more appropriate voltage supply unit that would preferably be computer controlled. With such a supply one could examine the effects of voltage waveforms on the model parameter values which were

clearly seen to be affected differently when either the Universal Voltronics voltage supply unit was used or the fly-back converter was used. The use of a sinusoidal voltage waveform to minimize parameter variation over time should be assessed.

Further research into the establishing of functional relationships between model parameters, such as fluid inertia, Coulomb friction and viscous properties, and operating variables such as mechanical excitation and electric field strength is suggested. Condensing the results of the analysis in Chapter 5 to a few relevant parameter values would also be valuable in refining the model design. The revised model in Figure 6.3 of Chapter 6 and possible refinements due to velocity dependencies should be examined with appropriate input/output data. Such data should have sufficient waveform variation in order to allow identification of the model parameters. The test data that was collected may not be adequate for this purpose and, therefore, further experiments are required. These experiments may give an opportunity to use the input optimization technique proposed by Kalaba and Spingarn (1982).

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Appendix A: Data Plots

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Universal Voltronics Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Universal Voltronics Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Universal Voltronics Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Universal Voltronics Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Universal Voltronics Voltage Supply

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Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Fly-Back Converter Voltage Supply

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity, Fly-Back Converter Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity, Fly-Back Converter Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity, Fly-Back Converter Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Fly-Buck Converter Voltage Supply

Measured (solid) and Predicted (dashed) Force, and Estimated Velocity. Fly-Back Converter Voltage Supply

Appendix B: C++ Program

```

//
// This is the template file used to input the
// state equations and Jacobian of the system
//

#include <pc\c\matrix.cpp>

matrix Feval(matrix &, matrix &);
matrix Jeval(matrix &, matrix &);
matrix costfunc1(matrix &,matrix &,matrix &);
matrix costfunc2(matrix &,matrix &,matrix &);
long float costfunc3(matrix &,matrix &);

const unsigned int sample_size=917/2, sample_freq=50;
const long float A=0.004075;
const unsigned int states=6,unknowns=6,inputs=5;

#include <pc\c\mainq3.cpp>

matrix Feval(matrix &x,matrix &data) {
    matrix F(x.row());
    long float P1,I1,R11,R12,U,t1,t3,Uo;
    long float sgn=1.0;

    U=data.ele(1);
    P1=x.ele(1);
    I1=x.ele(2);
    R11=x.ele(3);
    R12=x.ele(4);
    Uo=x.ele(5);

    if (P1<0.0) sgn=-1.0;

    t1 = 1/I1;
    t3 = 1/A;
    F.assign(0, P1*t1*t3*1000.0);
    F.assign(1, 0.1342522E3*(0.7448667508E-2*U-Uo)*t3-R11*P1*t1-R12*sgn);

    return F;
};

matrix Jeval(matrix &x, matrix &data) {

    matrix J(x.row(),x.row());
    long float P1,I1,R11,R12,U,t1,t2,t4,t5,Uo;
    long float sgn=1.0;

    U=data.ele(1);
    P1=x.ele(1);
    I1=x.ele(2);
    R11=x.ele(3);
    R12=x.ele(4);

```

```

    Uo=x.ele(5),

    if (P1<0) sgn=-1.0,

    t1 = 1/11,
    t2 = 1/A,
    t4 = 11*t1,
    t5 = 1/t4,
    J.assign(0,1, t1*t2*1000.0),
    J.assign(0,2, -P1*t5*t2*1000.0);
    J.assign(1,1, -R11*t1);
    J.assign(1,2, R11*P1*t5);
    J.assign(1,3, -P1*t1);
    J.assign(1,4, -sgn);
    J.assign(1,5, -0.1342522E3*t2);

    return J;
},

matrix costfunc1(matrix &p,matrix &r,matrix &data){
    matrix temp(unknowns,unknowns),temp1(unknowns),temp2(unknowns);
    temp1 = r.row(0);
    temp2 = temp1.transpose();
    temp = temp1*temp2;
    return temp;
};

matrix costfunc2(matrix &p,matrix &r,matrix &data){
    matrix temp(unknowns);
    temp =r.row(0)*(data.ele(2)-p.ele(0));
    return temp;
};

long float costfunc3(matrix &x,matrix &data){
    long float temp;
    temp = pow(x.ele(0)-data.ele(2),2);
    return temp;
};

```

```

//
// This is the Main program in file mainq3.cpp
//

#include <fstream.h>

void integrate(matrix &x,matrix &p,matrix &r,matrix &data,matrix ddata),
void solve(matrix &s1,matrix &s2,matrix &b,matrix &v);
void quasi_linear(matrix &x,matrix &b,matrix &v,long float &Erms,matrix &subset,matrix &x0);

void main() {

```

```

matrix x(states),xo(states),subset(sample_size,inputs);
matrix b(unknowns),v(unknowns,unknowns);

char file_name[12];
int i,j,n;
long float temp,Erms,t;

// This section reads in the initial state vector.
//
cout << "\nEnter the initial state: \n";
for (i=0;i<states;i++) {
    cin >> temp;
    if ('cin'==0) x.assign(i,temp);
};

// This section reads in the input data file.
//
cout << "\nEnter the input data file name: ";
cin >> file_name;

ifstream f1(file_name);
if (!f1) cerr << "Cannot open " << file_name << " for input \n";

// This section reads in the output data file.
//
cout << "\nEnter the output data file name: ";
cin >> file_name;

ofstream f2(file_name);
if (!f2) cerr << "Cannot open " << file_name << " for output \n";

// Read in the first subset of data.
//
for (j=0;(j<sample_size)&&(f1.eof()==0);j++) {
    for (n=0;n<(inputs);n++) {
        f1>>temp;
        if (f1.eof()==0) subset.assign(j,n,temp);
    };
};

//
// Begin the identification.
//
while((f1.eof()==0)&&(i<=1000)) {
    i++;
    t=subset.ele(sample_size/2,0);

// Cycle through a number of times to converge to the best estimate
//
for (j=0;j<5;j++) {
    x.assign(0,subset.ele(0,2));
    quasi_linear(x,b,v,Erms,subset,xo);
};
};

```

```

        for (n=0;n<unknowns;n++) x.assign(states-unknowns+n,b.ele(n));
    };

    // Read in the next subset of data.
    //
    for (j=0;j<(sample_size-sample_freq);j++) {
        for (n=0;n<inputs;n++) {
            subset.assign(j,n,subset.ele(j+sample_freq,n));
        };
    };

    for (j=sample_size-sample_freq;(j<sample_size)&&(f1.cof()==0);j++) {
        for (n=0;n<(inputs);n++) {
            f1>>temp;
            if (f1.cof()==0) subset.assign(j,n,temp);
        };
    };

    // Print the results to the screen.
    //
    cout<<"\nThe new estimate is: ";
    x.print();
    xo=xo;

    // Write results to the output file.
    //
    f2 << t << " ";
    for (j=0;j<unknowns;j++) f2 << b.ele(j) << " ";
    f2 << Erms << "\n";

};

};

void quasi_linear(matrix &x,matrix &b,matrix &v,long float &Erms,matrix &subset,matrix &xo) {

    matrix p(states),r(states,unknowns);
    matrix data(inputs),ddata(inputs);
    matrix s1(unknowns,unknowns);
    matrix s2(unknowns);
    int i,j;

    for (i=0;i<(states-unknowns);p.assign(i,x.ele(i)),i++);
    for (i=0;i<unknowns;r.assign(states-unknowns+i,i,1.0),i++);
    Erms = 0.0;

    for (j=0;j<sample_size-1;j++) {
        data = subset.row(j);
        ddata = subset.row(j+1)-data;

        integrate(x,p,r,data,ddata);

        if (j==sample_freq) xo=x;
    }
}

```

```

        s1+=costfunc1(p,r,data);
        s2+=costfunc2(p,r,data);
        Erms+=costfunc3(x,data);
    };

    Erms = sqrt(Erms/j);
    cout <<"The root mean squared error is " <<Erms <<"\n".
    solve(s1,s2,b,v);

};

void integrate(matrix &x,matrix &p,matrix &r,matrix &data,matrix ddata) {

    matrix dx(x.row()),dp(p.row());
    matrix dr(r.row(),r.col());
    matrix sum_dx(x.row()),sum_dp(p.row());
    matrix sum_dr(r.row(),r.col());
    matrix F(x.row()),J(x.row(),x.row());

    long float g[4]={1.0,2.0,2.0,1.0};
    long float dt = ddata.ele(0)/1000.0;
    int i;
    for (i=0;i<4;i++) {
        F = Feval(x+dx/g[i],data+ddata/g[i])*dt;
        J = Jeval(x+dx/g[i],data+ddata/g[i])*dt;
        dp = (F + J*(p+dp/g[i] -x-dx/g[i]));
        dr = J*(r+dr/g[i]);
        dx = F;
        sum_dx += dx*g[i];
        sum_dp += dp*g[i];
        sum_dr += dr*g[i];
    };
    x += sum_dx/6.0;
    p += sum_dp/6.0;
    r += sum_dr/6.0;
    data += ddata;

};

void solve(matrix &s1,matrix &s2,matrix &b,matrix &cov){
    unsigned int i,j;
    long float d;
    unsigned int *indx;
    matrix temp=s1;
    matrix w(unknowns),v(unknowns,unknowns);
    svdcmp(temp,w,v);
    svbksb(temp,w,v,s2,b);
    svdvar(v,w,cov);
};

```

```

#include <math.h>
#include <iostream.h>
const long float TINY=1.0e-20;

class matrix{
    unsigned int rows;
    unsigned int cols;
    long float *ptr;
    void put(unsigned int,unsigned int,long float);
    long float get(unsigned int,unsigned int);

    public
    matrix(unsigned int=1,unsigned int=1);
    matrix(const matrix &);
    ~matrix();
    void reset();
    void assign(unsigned int,unsigned int,long float);
    void assign(unsigned int,long float);
    long float ele(unsigned int,unsigned int);
    long float ele(unsigned int);
    matrix col(unsigned int);
    matrix row(unsigned int);
    unsigned int col();
    unsigned int row();
    matrix transpose();
    void print();
    matrix &operator=(const matrix&);
    matrix operator* (const long float&);
    matrix operator* (matrix&);
    matrix operator+ (const matrix&);
    void operator+=(const matrix&);
    matrix operator/ (const long float&);
    matrix operator- (const matrix&);
    void operator-=(const matrix&);
    matrix ludcmp(unsigned int *,long float *d);
    matrix lubksb(unsigned int *,const matrix&);
};

inline void matrix::put(unsigned int indx,unsigned int indy,long float value) {
    ptr[indx*cols+indy]=value;
}

inline long float matrix::get(unsigned int indx,unsigned int indy) {
    return ptr[indx*cols+indy];
}

matrix::matrix(unsigned int n,unsigned int m) {
    unsigned int i;
    rows = n;
    cols = m;
    ptr = new long float[rows*cols+1];
}

```

```

        if (ptr==0) {cout<<"can't allocate".;};
        for (i=0;i<rows*cols;ptr[i]=0.0,i++);
};

matrix::matrix(const matrix &init) {
    unsigned int i;
    rows = init.rows;
    cols = init.cols;
    ptr = new long float[rows*cols];
    if (ptr==0) {cout<<"can't allocate".;};
    for (i=0;i<rows*cols;ptr[i]=init.ptr[i],i++);
};

matrix::~matrix() {
    delete[] ptr;
};

void matrix::reset() {
    unsigned int i;
    for (i=0;i<rows*cols;ptr[i]=0.0,i++);
};

void matrix::assign(unsigned int indx,unsigned int indy,long float value) {
    if (indy > cols-1) {indy=cols-1;};
    if (indx > rows-1) {indx=rows-1;};
    put(indx,indy,value);
};

void matrix::assign(unsigned int indx,long float value) {
    if (rows==1) {
        if (indx >= cols) indx=cols-1;
        put(0,indx,value);}
    else {
        if (indx >= rows) indx=rows-1;
        put(indx,0,value);}
};

long float matrix::ele(unsigned int indx,unsigned int indy) {
    if (indy > cols-1) {indy=cols-1;};
    if (indx > rows-1) {indx=rows-1;};
    return get(indx,indy);
};

long float matrix::ele(unsigned int indx) {
    if (rows==1) {
        if (indx >= cols) indx=cols-1;
        return get(0,indx);}
    else {
        if (indx >= rows) indx=rows-1;
        return get(indx,0);}
};

```

```

matrix matrix::col(unsigned int indx) {
    unsigned int i;
    matrix temp(rows);
    if (indx >= cols) indx=cols-1;
    for (i=0; i<rows; temp.put(i,0,get(i,indx)), i++);
    return temp;
};

```

```

matrix matrix::row(unsigned int indx) {
    unsigned int i;
    matrix temp(cols);
    if (indx >= rows) indx=rows-1;
    for (i=0; i<cols; temp.put(i,0,get(indx,i)), i++);
    return temp;
};

```

```

unsigned int matrix::col() {
    return cols;
};

```

```

unsigned int matrix::row() {
    return rows;
};

```

```

matrix matrix::transpose() {
    unsigned int i,j;
    matrix temp(cols,rows);
    for (i=0; i<cols; i++) {
        for (j=0; j<rows; temp.put(i,j,get(j,i)), j++);
    };
    return temp;
};

```

```

void matrix::print() {
    unsigned int i;
    for (i=0; i<rows*cols; i++) {
        if ((i%cols)==0) cout <<"\\n";
        cout << ptr[i] <<" ";
    };
};

```

```

matrix& matrix::operator=(const matrix &m) {
    unsigned int i;
    if (&m != this) {
        delete [] ptr;
        rows = m.rows;
        cols = m.cols;
        ptr = new long float[rows*cols];
        for (i=0; i<rows*cols; i++) {
            ptr[i]=m.ptr[i];
        };
    };
};

```

```

        return *this;
};

matrix matrix::operator*(const long float &c) {
    unsigned int i;
    matrix temp(rows,cols);
    for (i=0;i<rows;i++) {
        temp.ptr[i]=ptr[i]*c;
    };
    return temp;
};

matrix matrix::operator*(matrix &m) {
    unsigned int i,j,k;
    long float sum;
    matrix product(rows,m.cols);
    for (j=0;j<rows;j++) {
        for (k=0;k<m.cols;k++) {
            sum = 0.0;
            for (i=0;i<cols;i++) {
                sum+=get(j,i)*m.get(i,k);
            };
            product.put(j,k,sum);
        };
    };
    return product;
};

matrix matrix::operator+(const matrix &m) {
    unsigned int i,j;
    matrix sum(rows,cols);
    for (i=0;i<rows*cols;i++) {
        sum.ptr[i]=ptr[i]+m.ptr[i];
    };
    return sum;
};

void matrix::operator+=(const matrix &m) {
    unsigned int i;
    for (i=0;i<rows*cols;i++) {
        ptr[i]+=m.ptr[i];
    };
};

matrix matrix::operator/(const long float &c) {
    unsigned int i;
    matrix temp(rows,cols);
    for (i=0;i<rows*cols;i++) {
        temp.ptr[i]=ptr[i]/c;
    };
    return temp;
};

```

```

matrix matrix::operator-(const matrix &m) {
    unsigned int i,j;
    matrix sum(rows,cols),
    for (i=0;i<rows*cols;i++) {
        sum.ptr[i]=ptr[i]-m.ptr[i];
    };
    return sum;
},

void matrix::operator+=(const matrix &m) {
    unsigned int i;
    for (i=0;i<rows*cols;i++) {
        ptr[i]+=m.ptr[i];
    },
},

matrix matrix::ludcmp(unsigned int *indx,long float *d)
{
    unsigned int i,imax,j,k;
    long float big,dum,sum,temp;
    matrix vv(rows),copy(rows,cols);
    *d=1.0;
    for (i=0;i<rows*cols;copy.ptr[i]=ptr[i],i++),

    for (i=0;i<rows;i++) {
        big=0.0;
        for (j=0;j<cols;j++)
            if ((temp=fabs(get(i,j))) > big) big=temp;
        //if (big == 0.0) nrerror("Singular matrix in routine ludcmp");
        vv.put(i,0,1.0/big);
    }
    for (j=0;j<cols;j++) { // This is the loop over columns of Crout's method.
        for (i=0;i<j;i++) { // This is equation (2.3.12) except for i=j.
            sum=copy.get(i,j);
            for (k=0;k<i;k++) sum -= copy.get(i,k)*copy.get(k,j);
            copy.put(i,j,sum);
        }
        big=0.0; // Initialize the search for the largest pivot element.
        for (i=j;i<rows;i++) { // This is i=j of equation (2.3.12) and i=j+1...N
            sum=copy.get(i,j); // of equation (2.3.13).
            for (k=0;k<j;k++) sum -= copy.get(i,k)*copy.get(k,j);
            copy.put(i,j,sum);
            if ( (dum=vv.get(i,0)*fabs(sum)) >= big) {
                // Is the figure of merit for the pivot better than the best so far?
                big =dum;
                imax=i;
            }
        }
    }
    if (j != imax) { // Do we need to interchange rows?
        for (k=0;k<cols;k++) { // Yes, do so..
            dum=copy.get(imax,k);
            copy.put(imax,k,copy.get(j,k));

```

```

        copy.put(j,k,dum);
    }
    *d = -(*d); // ...and change the parity of d.
    vv.put(imax,0,vv.get(j,0)); // Also interchange the scale factor.
}
indx[j]=imax;
if (copy.get(j,j) == 0.0) copy.put(j,j,TINY);
// If the pivot element is zero the matrix is singular (at least to the precision of the
// algorithm). For some applications on singular matrices, it is desirable to substitute
// TINY for zero.
if (j != cols-1) { // Now finally divide the pivot element
    dum=1.0/(copy.get(j,j));
    for (i=j+1;i<rows;i++) copy.ptr[i*cols+j] *= dum;
}
} // Go back for the next column in the reduction.
return copy;
};

matrix matrix::lubksb(unsigned int *indx, const matrix &b)
{
    unsigned int i,ii=0,ip,j;
    long float sum;
    matrix copy(b.rows,b.cols);
    copy = b;

    for (i=1;i<=rows;i++) { // When ii is set to a positive value, it will become the
        ip=indx[i-1]; // index of the first nonvanishing element of b. We now
        sum=copy.get(ip,0); // do the forward substitution, equation (2.3.6). The
        copy.put(ip,0,copy.get(i-1,0)); // only new wrinkle is to unscramble the permutation
        if (ii) // as we go.
            for (j=ii;j<=i-1;j++) sum -= get(i-1,j-1)*copy.get(j-1,0);
        else if (sum) ii=i; // A nonzero element was encountered, so from now on we
        copy.put(i-1,0,sum); // will have to do the sums in the loop above
    }
    for (i=rows;i>0;i--) { // Now we do the backsubstitution, equation (2.3.7).
        sum=copy.get(i-1,0);
        for (j=i+1;j<=rows;j++) sum -= get(i-1,j-1)*copy.get(j-1,0);
        copy.put(i-1,0,sum/get(i-1,i-1)); // Store a component of the solution vector X
    }
    // All done!
    return copy;
};

static long float maxarg1,maxarg2;
#define FMAX(a,b) (maxarg1=(a),maxarg2=(b),(maxarg1) > (maxarg2) ? \
(maxarg1) : (maxarg2))

static int iminarg1,iminarg2;
#define IMIN(a,b) (iminarg1=(a),iminarg2=(b),(iminarg1) < (iminarg2) ? \
(iminarg1) : (iminarg2))

#define SIGN(a,b) ((b) >= 0.0 ? fabs(a) : -fabs(a))

```

```

void svdcmp(matrix &a, matrix &w, matrix &v)
/* Given a matrix a[1..m][1..n], this routine computes its singular value decomposition
   A=U*W*V'. The matrix U replaces a on output. The diagonal matrix of singular values
   W is output as a vector[1..n]. The matrix V(not the transpose Vt) is output as
   v[1..n][1..n].
*/
{
    long float pythag(long float a, long float b);
    int flag, i, its, j, jj, k, l, nm, m, n;
    long float anorm, c, f, g, h, s, scale, x, y, z, tmp;

    m=a.row();
    n=a.col();
    matrix rv1(n);
    g=scale=anorm=0.0;
    for (i=0; i<n; i++) {
        l=i+1;
        rv1.assign(i,0,scale*g);
        g=s=scale=0.0;
        if (i<m) {
            for (k=i; k<m; k++) scale += fabs(a.ele(k,i));
            if (scale) {
                for (k=i; k<m; k++) {
                    a.assign(k,i,a.ele(k,i)/scale);
                    s += a.ele(k,i)*a.ele(k,i);
                }
                f=a.ele(i,i);
                g=-SIGN(sqrt(s),f);
                h=f*g-s;
                a.assign(i,i,f-g);
                for (j=l; j<n; j++) {
                    for (s=0.0, k=i; k<m; k++) s += a.ele(k,i)*a.ele(k,j);
                    f=s/h;
                    for (k=i; k<m; k++) a.assign(k,j,a.ele(k,j)+f*a.ele(k,i));
                }
                for (k=i; k<m; k++) a.assign(k,i,a.ele(k,i)*scale);
            }
        }
        w.assign(i,0,scale*g);
        g=s=scale=0.0;
        if (i<m && i!=n-1) {
            for (k=l; k<n; k++) scale += fabs(a.ele(i,k));
            if (scale) {
                for (k=l; k<n; k++) {
                    a.assign(i,k,a.ele(i,k)/scale);
                    s += a.ele(i,k)*a.ele(i,k);
                }
                f=a.ele(i,i);
                g=-SIGN(sqrt(s),f);
                h=f*g-s;
                a.assign(i,i,f-g);
                for (k=l; k<n; k++) rv1.assign(k,0,a.ele(i,k)/h);
            }
        }
    }
}

```

```

        for (j=1;j<m;j++) {
            for (s=0.0,k=1;k<n;k++) s+= a.ele(j,k)*a.ele(i,k);
            for (k=1;k<n;k++) a.assign(j,k,a.ele(j,k)+s*rvl.ele(k,0));
        }
        for (k=1;k<n;k++) a.assign(i,k,a.ele(i,k)*scale);
    }
    anorm=FMAX(anorm,(fabs(w.ele(i,0))+fabs(rvl.ele(i,0))));
}
for (i=n-1;i>=0;i--) { //Accumulate right hand transformations.
    if (i<n-1) {
        if (g) {
            for (j=1;j<n;j++) //Double division to avoid possible underflow
                v.assign(j,i,(a.ele(i,j)/a.ele(i,1))/g);
            for (j=1;j<n;j++) {
                for (s=0.0,k=1;k<n;k++) s+= a.ele(i,k)*v.ele(k,j);
                for (k=1;k<n;k++) v.assign(k,j,v.ele(k,j)+s*v.ele(k,1));
            }
        }
        for (j=1;j<n;j++) {
            v.assign(i,j,0.0);
            v.assign(j,i,0.0);
        }
        v.assign(i,i,1.0);
        g=rvl.ele(i,0);
        l=i;
    }
}
for (i=IMIN(m,n)-1;i>=0;i--) { // Accumulate left hand transformations
    l=i+1;
    g=w.ele(i,0);
    for (j=1;j<n;j++) a.assign(i,j,0.0);
    if (g) {
        g=1.0/g;
        for (j=1;j<n;j++) {
            for (s=0.0,k=1;k<m;k++) s+= a.ele(k,i)*a.ele(k,j);
            f=(s/a.ele(i,i))*g;
            for (k=i;k<m;k++) a.assign(k,j,a.ele(k,j)+f*a.ele(k,i));
        }
        for (j=i;j<m;j++) a.assign(j,i,a.ele(j,i)*g);
    } else for (j=i;j<m;j++) a.assign(j,i,0.0);
    tmp=a.ele(i,i);
    a.assign(i,i,++tmp);
}
for (k=n-1;k>=0;k--) { // Diagonalization of the bidiagonal form:
    for (its=1;its<=30;its++) { // loop over the singular values,
        flag=1; // and over the allowed iterations
        for (l=k;l>=0;l--) { //Test for splitting.
            nm=l-1;
            if ((long float)(fabs(rvl.ele(l,0))+anorm) == anorm) {
                flag = 0;
                break;
            }
        }
    }
}

```



```

    }
    if ((long float)(fabs(w. ele(nm,0))+anorm) == anorm) break;
}
if (flag) {
    c=0.0;    // Cancellation of rv1(l), if l>1.
    s=1.0;
    for (i=1; i<=k; i++) {
        f=s*rv1. ele(i,0);
        rv1. assign(i,0,c*rv1. ele(i,0));
        if ((long float)(fabs(f)+anorm) == anorm) break;
        g=w. ele(i,0);
        h=pythag(f,g);
        w. assign(i,0,h);
        h=1.0/h;
        c=g*h;
        s=-f*h;
        for (j=0; j<m; j++) {
            y=a. ele(j,nm);
            z=a. ele(j,i);
            a. assign(j,nm,y*c+z*s);
            a. assign(j,i,z*c-y*s);
        }
    }
}
z=w. ele(k,0);
if (l==k) { //Convergence.
    if (z<0.0) { // Singular value is made nonnegative.
        w. assign(k,0,-z);
        for (j=0; j<n; j++) v. assign(j,k,-v. ele(j,k));
    }
    break;
}
if (its == 30) break; //error ("no convergence in 30 svdcmp iterations");
x=w. ele(1,0);
nm=k-1;
y=w. ele(nm,0);
g=rv1. ele(nm,0);
h=rv1. ele(k,0);
f=((y-z)*(y+z)+(g-h)*(g+h))/(2.0*h*y);
g=pythag(f,1.0);
f=((x-z)*(x+z)+h*((y/(f+SIGN(g,f)))-h))/x;
c=s=1.0;    // Next QR transformation:
for (j=1; j<=nm; j++) {
    i=j+1;
    g=rv1. ele(i,0);
    y=w. ele(i,0);
    h=s*g;
    g=c*g;
    z=pythag(f,h);
    rv1. assign(j,0,z);
    c=f/z;
    s=h/z;
}

```

```

f=x*c+g*s;
g=g*c-x*s;
h=y*s;
y*=c;
for (jj=0;jj<n;jj++) {
    x=v.ele(jj,j);
    z=v.ele(jj,i);
    v.assign(jj,j,x*c+z*s);
    v.assign(jj,i,z*c-x*s);
}
z=pythag(f,h);
w.assign(j,0,z); // Rotation can be arbitrary if z=0
if (z) {
    z=1.0/z;
    c=f*z;
    s=h*z;
}
f=c*g+s*y;
x=c*y-s*g;
for (jj=0;jj<m;jj++) {
    y=a.ele(jj,j);
    z=a.ele(jj,i);
    a.assign(jj,j,y*c+z*s);
    a.assign(jj,i,z*c-y*s);
}
}
rvl.assign(1,0,0.0);
rvl.assign(k,0,f);
w.assign(k,0,x);
}
}

long float pythag(long float a,long float b)
// Computes (a^2+b^2)^0.5 without destructive underflow or overflow.
{
    long float absa,absb,
    absa=fabs(a);
    absb=fabs(b);
    if (absa > absb) return absa*sqrt(1.0+pow(absb/absa,2));
    else return (absb==0.0 ? 0.0: absb*sqrt(1.0+pow(absa/absb,2)));
}

void svbksb(matrix &u,matrix &w,matrix &v,matrix &b,matrix &x)
/* Solves A*X=B for a vector X, where A is specified by the arrays u(1..m,1..n],
w(1..n],v(1..n,1..n] as returned by svdcmp, m and n are the dimensions of a and
will be equal for square matrices, b(1..m] is the input right hand side x(1..n]
is the output solution vector. No input quantities are destroyed, so the routine
may be called sequentially with different b's.
*/
{
    int jj,j,i,n,m;

```

```

long float s;

m=u.row();
n=u.col();
matrix tmp(n);
for (j=0;j<n;j++) { // Calculate U'*B.
    s=0.0;
    if (w. ele(j,0)) {
        for (i=0;i<m;i++) s+=u. ele(i,j)*b. ele(i,0);
        s/=w. ele(j,0);
    }
    tmp assign(j,0,s);
}
for (j=0;j<n;j++) {
    s=0.0;
    for (ij=0;jj<n;jj++) s+=v. ele(j,jj)*tmp. ele(jj,0);
    x. assign(j,0,s);
}
}

void svdvar(matrix &v,matrix &w,matrix &cvm)
/* To evaluate the covariance matrix cvm[1..ma][1..ma] of the fit for
ma parameters obtained by svdfit, call this routine with matrices
v[1..ma][1..ma],w[1..ma] as returned from svdfit.
*/
{
    int k,j,i,ma;
    float sum;
    ma = v.row();
    matrix wti(ma);

    for (i=0;i<ma;i++) {
        if (w. ele(i)) wti. assign(i,0.1.0/(w. ele(i)*w. ele(i)));
    }
    for (i=0;i<ma;i++) { //Sum the contributions to covariance matrix
        for (j=0;j<=i;j++) {
            for (sum=0.0,k=0;k<ma;k++) sum +=v. ele(i,k)*v. ele(j,k)*wti. ele(k,0);
            cvm. assign(j,i,sum);
            cvm. assign(i,j,sum);
        }
    }
}

```

Appendix C: Results in Tabular Form

Table C1: Mean rectified error in predicted damping force, (N), from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The average parameter values obtained over entire test interval were used to simulate the damper. (- represents an operating condition that could not be achieved)

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Mean rectified error, N					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	1.5	1.5	1.6	1.5	1.5	1.6
	0.5	1.5	1.5	1.7	1.0	1.1	2.9
	1.0	1.4	1.8	2.5	1.9	1.8	1.9
	1.5	5.0	2.6	3.9	4.5	3.7	4.1
2	0.0	2.3	1.8	1.6	2.3	1.8	1.6
	0.5	2.3	1.7	1.5	3.0	2.5	2.0
	1.0	2.6	2.1	2.8	4.6	3.0	2.6
	1.5	4.9	4.6	4.0	6.0	4.2	3.9
5	0.0	7.1	4.1	3.2	7.1	4.1	3.2
	0.5	7.3	4.0	3.1	6.8	2.9	2.2
	1.0	7.1	4.3	3.5	8.8	4.5	3.6
	1.5	9.0	5.7	5.6	10.2	6.8	5.9
9	0.0	-	7.5	6.0	-	7.5	6.0
	0.5	-	7.6	5.5	-	7.1	3.9
	1.0	-	8.5	6.8	-	10.7	6.0
	1.5	-	10.3	8.5	-	14.1	8.5

Table C2: Mean rectified error in predicted damping force, (N), from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The time varying parameter values obtained over entire test interval were used to simulate the damper. (- represents an operating condition that could not be achieved)

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Mean rectified error time-vary, N					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	1.5	1.5	1.6	1.5	1.5	1.6
	0.5	1.5	1.5	1.7	1.0	1.1	2.9
	1.0	1.3	1.2	1.8	1.7	1.5	1.6
	1.5	3.3	2.3	3.0	4.9	3.6	4.0
2	0.0	2.3	1.8	1.6	2.3	1.8	1.6
	0.5	2.0	1.4	1.0	2.5	2.3	1.7
	1.0	2.3	1.6	2.1	3.8	2.6	2.1
	1.5	3.8	2.7	2.8	5.2	4.1	3.9
5	0.0	7.0	4.1	3.2	7.0	4.1	3.2
	0.5	7.2	4.0	3.1	6.2	2.4	1.9
	1.0	6.2	4.2	3.2	8.4	4.8	3.3
	1.5	7.7	4.9	5.0	9.7	7.6	6.1
9	0.0	-	7.5	6.0	-	7.5	6.0
	0.5	-	7.3	5.4	-	6.8	4.0
	1.0	-	7.4	5.8	-	10.9	5.9
	1.5	-	7.3	6.8	-	15.6	8.5

Table C3: Inertial parameter, I_1 ($\text{kN}\cdot\text{s}^2/\text{m}^5$), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. (- represents an operating condition that could not be achieved)

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Inertial parameter, $\text{kN}\cdot\text{s}^2/\text{m}^5$					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	212	185	112	212	185	112
	0.5	210	213	276	222	308	453
	1.0	382	387	1080	350	499	803
	1.5	767	888	1086	510	612	591
2	0.0	214	213	208	214	213	208
	0.5	221	218	232	227	213	215
	1.0	275	297	526	251	278	346
	1.5	326	523	568	291	342	407
5	0.0	204	213	216	204	213	216
	0.5	210	225	214	212	215	223
	1.0	220	248	288	222	231	224
	1.5	227	291	336	234	220	224
9	0.0	-	214	216	-	214	216
	0.5	-	227	223	-	223	223
	1.0	-	225	234	-	203	227
	1.5	-	236	257	-	199	226

Table C4: Viscous friction parameter, R_1 (MPa*s/m³), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. (- represents an operating condition that could not be achieved).

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Viscous friction, MPa *s/m ³					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	4.52	4.88	5.97	4.52	4.88	5.97
	0.5	7.37	8.65	16.91	6.72	12.96	21.23
	1.0	12.09	14.04	29.78	9.44	14.20	32.02
	1.5	19.39	26.93	46.78	12.45	18.53	34.58
2	0.0	4.62	4.62	4.83	4.62	4.62	4.83
	0.5	6.53	6.61	9.26	9.86	13.49	17.07
	1.0	11.51	13.20	25.62	13.83	15.88	19.55
	1.5	15.76	21.25	25.17	17.28	14.42	21.48
5	0.0	4.79	4.82	4.89	4.79	4.82	4.89
	0.5	5.90	6.30	7.00	6.30	7.02	9.13
	1.0	8.65	10.85	15.37	8.95	11.92	14.59
	1.5	12.32	15.00	20.18	11.72	14.44	18.44
9	0.0	-	4.96	5.06	-	4.96	5.06
	0.5	-	6.22	7.08	-	6.64	7.94
	1.0	-	8.35	10.07	-	10.72	11.30
	1.5	-	10.56	14.29	-	8.05	17.98

Table C5: Coulomb friction parameter, R_2 (Pa), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. (- represents an operating condition that could not be achieved).

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Coulomb friction, Pa					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	337	339	285	337	339	285
	0.5	593	511	486	1588	1612	1540
	1.0	1829	1458	2655	3021	3030	2512
	1.5	3703	4023	4142	4208	4476	4083
2	0.0	291	264	245	291	264	245
	0.5	604	538	456	560	1106	1554
	1.0	1044	1614	2120	1199	2460	3144
	1.5	1908	3294	4277	1834	4107	4244
5	0.0	446	186	141	446	186	141
	0.5	933	615	418	1661	1210	1027
	1.0	764	1358	1535	2187	1362	1984
	1.5	273	1870	2607	1571	2060	2125
9	0.0	-	492	187	-	492	187
	0.5	-	1204	491	-	1590	1026
	1.0	-	764	1317	-	260	2015
	1.5	-	1776	2366	-	4691	1142

Table C6: Bias parameter, u_0 (N), values identified from transient test results using the Universal Voltronics voltage supply and Fly-back converter. Sinusoidal mechanical excitation. (- represents an operating condition that could not be achieved, * indicate that the value of force is less than the least significant bit, 0.33N, of the A-D converter.)

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Bias, N					
		Universal power supply			Fly-back power supply		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	-0.03*	-0.11*	-0.12*	-0.03*	-0.11*	-0.12*
	0.5	0.17*	0.01*	-0.03*	1.06	1.11	-0.00*
	1.0	0.85	0.63	0.70	1.10	1.27	1.11
	1.5	0.78	0.40	0.27*	1.70	1.34	1.34
2	0.0	-0.07*	0.00*	-0.14*	-0.07*	0.00*	-0.14*
	0.5	0.16*	0.00*	0.01*	-0.61	-0.42	-0.23*
	1.0	1.05	0.76	0.57	-0.61	-0.37	-0.36
	1.5	1.50	1.22	0.84	-0.51	0.10*	-0.32*
5	0.0	0.93	0.06*	-0.08*	0.93	0.06*	-0.08*
	0.5	1.42	0.28*	0.07*	0.35	-0.43	-0.11*
	1.0	2.47	1.00	0.81	1.82	-0.43	-0.22*
	1.5	3.25	1.69	1.16	2.34	1.12	0.10*
9	0.0	-	0.57	-0.02*	-	0.57	-0.02*
	0.5	-	1.05	0.22*	-	0.61	0.12*
	1.0	-	2.28	0.96	-	2.14	0.22*
	1.5	-	2.69	1.33	-	-1.00	1.34

Table C7: Root mean squared damping force from transient test results using the Universal Voltronics voltage supply and Fly-back converter. The number in the brackets indicates the force amplification from transient results to steady state. (- represents an operating condition which could not be achieved.

Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Damping Force, N					
		Universal Voltronics power supply			Fly-back supply power		
		Ampl.			Ampl.		
		10mm	5mm	3mm	10mm	5mm	3mm
1	0.0	4.8	3.1	2.4	4.8	3.1	2.4
	0.5	7.9(2.3)	5.4(2.9)	6.0	11.2(1.7)	11.5(1.1)	11.1
	1.0	15.9(2.5)	11.0(2.9)	16.9	18.6(1.8)	17.1(1.3)	17.2
	1.5	30.1(1.7)	25.9 (1.6)	26.5	27.7(1.8)	25.9(1.4)	24.8
2	0.0	9.3	5.0	3.3	9.3	5.0	3.3
	0.5	12.5(2.1)	7.4(2.7)	6.0	17.5(1.4)	14.5(1.3)	13.6
	1.0	22.5(2.4)	16.4(2.3)	19.5	26.6(1.7)	21.4(1.5)	20.5
	1.5	31.9(2.4)	28.8(1.9)	27.6	34.7(1.9)	27.4(1.8)	25.8
5	0.0	31.4	16.8	10.5	31.4	16.8	10.5
	0.5	36.6(1.4)	20.0(1.8)	12.6	38.4(1.4)	21.7(1.8)	15.8
	1.0	42.8(1.9)	29.8(2.0)	25.6	50.0(1.8)	30.8(2.0)	24.2
	1.5	55.0(2.2)	38.8(2.1)	34.3	56.5(1.9)	38.5(2.2)	30.7
9	0.0	-	44.4	27.5	-	44.4	27.5
	0.5	-	51.3	30.2	-	50.0	29.3
	1.0	-	54.6	38.1	-	59.2	37.3
	1.5	-	66.1	49.9	-	66.2	44.9

Table C8: Parameter values for steady state test results using the Fly-back converter.
Sinusoidal mechanical excitation of amplitude 5mm and 10mm.

Parameter values at steady state							
Amplitude		5 mm			10 mm		
Damper Oscillation Frequency (Hz)	Field Strength (kV/mm)	Inertia (kN*s ² /m ⁵)	Viscous (MPa*s/ m ³)	Coulomb (Pa)	Inertia (kN*s ² /m ⁵)	Viscous (MPa*s/ m ³)	Coulomb (Pa)
1	0.5	552	29.00	514	409	22.60	395
	1.0	773	55.99	470	659	45.22	479
	1.5	825	95.84	543	784	59.65	1661
2	0.5	394	21.40	546	356	15.07	563
	1.0	445	39.22	615	406	29.22	458
	1.5	411	64.28	480	429	41.23	814
5	0.5	323	15.14	1197	257	10.83	1558
	1.0	354	29.28	1477	258	22.27	673
	1.5	306	36.87	5927	200	30.52	-2465

END

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FIN