Intelligent Tracking Control of Fixed-Base and Free-Flying Flexible Space Robots

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Abstract

Initially, tracking control of a fixed-base planar two-link rigid dynamics robot manipulator is simulated using inverse dynamics, linear quadratic regulator, fuzzy logic and linear quadratic Gaussian control strategies with a Jacobian transpose proportional-derivative control law.

The inverse dynamics strategy model is extended to tracking control of a fixed-base planar two-link robot manipulator with flexible dynamics derived using dominant cantilever and pinned-pinned assumed modes of vibration. Adaptation of the control law and transverse link vibration suppression is achieved by a fuzzy logic system within the control strategy.

A heuristic design ratio is determined to select optimal fuzzy logic system controllers with a low number of membership functions, high tracking precision and fast execution time. Using the inverse flexible dynamics control strategy simulated tracking results are obtained for a fuzzy logic system with three, five, seven and nine triangular and Gaussian membership functions providing a combination of type and number of membership functions for optimal tracking control and execution time. An optimal fuzzy logic system design ratio of five is achieved with three triangular membership functions and an output scaling gain of fifteen.

Repetitive learning performance is compared for inverse rigid dynamics control vs. fuzzy logic control and inverse flexible dynamics control vs. fuzzy logic system adaptive control strategies. Control strategies using fuzzy logic induce responsiveness to repetitive learning; whereas the conventional inverse dynamics control strategies induce no response.

Nonminimum phase behaviour of the flexible robot with dominant cantilever assumed mode dynamics is investigated to provide a method of achieving accurate end-effector tracking control in the presence of time delays and sensors noncollocated at alternate positions on the outboard link of a flexible robot manipulator. The effect of time-delayed control input is minimal compared to the overriding effect of fuzzy logic system adaptive control.

Hybrid control strategies are synthesized and simulated tracking results obtained for combinations of inverse flexible dynamics control with an extended Kalman filter or a fuzzy logic adaptive extended Kalman filter for noise filtering and a fuzzy logic system for vibration suppression. The effect of Kalman filtering is minimal while the fuzzy logic system has an overriding effect on diminishing transient vibration overshoot amplitudes at trajectory direction switches.

Finally, the end effector of a free-flying flexible space robot autonomously tracks a trajectory between two points in a space orbit using inverse flexible dynamics and fuzzy logic system adaptive control strategies for comparison. The robot autonomously tracks its end effector between initial and final positions in two-dimensional space while maintaining stability and station keeping the rigid spacecraft in its location in space. Simulated tracking results show inverse flexible dynamics control produces a significant drift and fails to reach its commanded final position. Fuzzy logic system adaptive control produces a more accurate direct trajectory with less drift and terminates closer to the commanded final position.

But, sporadic high magnitude driving torque “spikes” occur, similar to the “bursting phenomenon” often encountered with adaptive control systems, but they are damped by

the fuzzy logic system. The rigid spacecraft reactive translation and attitude change is minimal in each case but less so for fuzzy logic system adaptive control.

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List of Mathematical Symbols, Notation, Physical Constants and Units

$\mathbf{A}$	— continuous state matrix
$\mathbf{A}_i$	— homogeneous transformation and direction cosine matrices, $i = 0, 1, 2$
$\mathbf{A}_k$	— discrete state matrix
A	— mode shapes constant, 0.1
$\mathbf{B}$	— continuous control matrix
$\mathbf{B}_k$	— discrete control matrix
b_i	— distance from link centroid to distal end, $i = 0, 1, 2$
$\mathbf{C}$	— continuous state measurement connection matrix
$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$	— flexible dynamic coupling vector
$\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})$	— free-flying flexible space robot dynamic coupling matrix
$\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$	— rigid dynamic coupling vector
$\mathbf{C}_r(\mathbf{x})$	— rigid dynamic coupling vector as a function of state variables $\mathbf{x}$
$\mathbf{C}_r(\mathbf{x}_d)$	— rigid dynamic coupling vector about operating point vector $\mathbf{x}_d$
$\mathbf{C}_k$	— discrete measurement connection matrix
C_i	— element of rigid and flexible dynamic coupling vectors, $i = 1, 2, 3, 4$
C_{ij}	— element of flexible dynamic coupling matrix $\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})$
c	— wave velocity, 288.33, [m/s]
c_i	— link centroid, $i = 1, 2$

c_{ijk}	— Christoffel symbol
$\mathbf{D}$	— continuous direct transmission matrix
$\mathbf{D}_k$	— discrete direct transmission matrix
D_i	— body dimension of interest for integration, $i = 0, 1, 2$
d_i	— link offset distance, $i = 1, 2$
d_{t_i}	— simulation step time delay, [ss]
$\mathbf{E}_i$	— flexible deformation matrix, $i = 1, 2$
EI_{L_i}	— flexural rigidity, 1676, [Nm ²]
$\mathbf{e}$	— operational-space error vector
$\dot{\mathbf{e}}$	— operational-space error velocity vector
e_i	— element of the operational-space error vector
$\dot{e}_i$	— element of the operational-space error velocity vector
e_x	— x-coordinate position error, $x_c - x$
e_y	— y-coordinate position error, $y_c - y$
$\dot{e}_x$	— x-coordinate velocity error, $\dot{x}_c - \dot{x}$
$\dot{e}_y$	— y-coordinate velocity error, $\dot{y}_c - \dot{y}$
$\mathbf{F}$	— generalized force vector
F_i	— generalized force, $i = 0, 1, 2$
F_x	— force acting along x axis of spacecraft
F_y	— force acting along y axis of spacecraft

$\mathbf{G}_k$	— discrete dynamic process noise coefficient matrix
$\mathbf{g}$	— gravitational acceleration vector, 9.81, [m/s ²]
$\mathbf{H}_k$	— discrete measurement noise coefficient matrix
h	— Christoffel symbol element in dynamic coupling matrix
$\mathbf{I}_a$	— actuator inertia matrix
$\mathbf{I}_h$	— hub inertia matrix
I_{a_i}	— actuator inertia, $i = 1, 2$, [kg m ²]
I_{g_i}	— gear inertia, $i = 1, 2$, [kg m ²]
I_{h_i}	— hub inertia, $i = 1, 2, 5$ [kg m ²]
I_{L_i}	— link inertia, $i = 1, 2$, [kg m ²]
I_{r_i}	— rotor inertia, $i = 1, 2$, [kg m ²]
$\mathbf{J}(\boldsymbol{\theta})$	— rigid robot manipulator Jacobian
$\mathbf{J}_{c_i}(\boldsymbol{\theta})$	— Jacobian at the centroid of a rigid link, $i = 1, 2$
$\mathbf{J}^*(\boldsymbol{\theta})$	— generalized Jacobian matrix for a free-flying rigid space robot
$\mathbf{J}_M^*(\boldsymbol{\theta})$	— fixed-attitude-restricted motion Jacobian for a free-flying rigid space robot manipulator
$\mathbf{J}^*(\mathbf{q})$	— generalized Jacobian matrix for a free-flying flexible space robot
$\mathbf{J}_M^*(\mathbf{q})$	— fixed-attitude-restricted motion Jacobian for a free-flying flexible space robot manipulator
$\mathbf{J}_S^*(\boldsymbol{\theta})$	— spacecraft partition of the generalized Jacobian matrix for a free-flying rigid or flexible space robot

$\mathbf{J}_r(\mathbf{x}_d)$	— Jacobian of rigid dynamic coupling matrix about operating point $\mathbf{x}_d$
$\mathbf{J}_v(\boldsymbol{\theta})$	— translation partition of a rigid robot manipulator Jacobian
$\mathbf{J}_\omega(\boldsymbol{\theta})$	— rotation partition of a rigid robot manipulator Jacobian
$\mathbf{J}_{RM}(\boldsymbol{\theta})$	— Jacobian for a rigid robot manipulator with $\mathbf{J}_v(\boldsymbol{\theta})$ and $\mathbf{J}_\omega(\boldsymbol{\theta})$ partitions
$J_{ij}^T(\theta_i)$	— i-jth element of a rigid robot manipulator Jacobian
J	— optimal control performance index
$\mathbf{K}$	— stiffness matrix
$\mathbf{K}_I$	— LQR input gain matrix
$\mathbf{K}_O$	— LQR output gain matrix
$\mathbf{K}_k$	— discrete Kalman gain matrix
$\mathbf{K}_p$	— proportional gain matrix
$\mathbf{K}_d$	— derivative gain matrix
K_{d_i}	— derivative gain for joint i, $i = 1, 2$
K_i	— element of stiffness matrix $\mathbf{K}$, $i = 1, 2$, [kg/m ²]
K_{p_i}	— proportional gain for joint i, $i = 1, 2$
$\mathbf{k}_s$	— actuator stiffness matrix, [kg/m]
k_{a_i}	— actuator stiffness constant, $i = 1, 2$, [kg/m]
k_{c_j}	— mode shape parameter, $j = 1, \dots, n$
k_{r_i}	— rotor stiffness constant, $i = 1, 2$, [kg/m]
k_{s_i}	— joint stiffness constant, $i = 1, 2$, [kg/m]

L	— length of link, 4.5, [m]
L_i	— length of link: $L_0 = 3$ m, $L_1 = L_2 = 4.5$, [m]
$\mathcal{L}$	— Lagrangian function
$\mathbf{M}(\mathbf{q})$	— flexible dynamics inertia matrix
$\mathbf{M}_{\text{FF}}(\mathbf{q})$	— free-flying rigid spacecraft and flexible robot manipulator dynamics inertia matrix
$\mathbf{M}_r(\boldsymbol{\theta})$	— rigid dynamics inertia matrix
$\mathbf{M}_r(\mathbf{x})$	— rigid dynamics inertia matrix as a function of state variables $\mathbf{x}$
$\mathbf{M}_r(\mathbf{x}_d)$	— rigid dynamics inertia matrix about operating point vector $\mathbf{x}_d$
$M_{r,kp}$	— k-pth element of inertia matrix $\mathbf{M}_r(\boldsymbol{\theta})$
M_{ij}	— element of flexible dynamic coupling matrix $\mathbf{M}_{\text{FF}}(\mathbf{q}, \dot{\mathbf{q}})$
m_{h_i}	— hub mass, $i = 1, 2, 5$ [kg]
m_0	— spacecraft mass, 600, [kg]
m_i	— link mass, $i = 1, 2, 1.5075$, [kg]
m_p	— payload mass, [kg]
m_t	— total mass of FFFSRM, 603.015 [kg]
$m(x_i)$	— mass as a function of distance x_i , $i = 1, 2$, [kg/m]
$\mathbf{N}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$	— rigid dynamic equations of motion skew-symmetric property matrix
$\mathbf{N}$	— cross-weighting matrix
$\mathbf{N}_k$	— discrete cross-covariance matrix
O_i	— origin in frame of reference, $i = 0, 1, 2$

$O(n)$	— order- n
P_{k+1}^-	— projected error covariance matrix
P_k^-	— <i>a priori</i> error covariance matrix
P_k	— <i>a posteriori</i> error covariance matrix
${}^i p_i(x_i)$	— position vector to point x_i on a deformed link referenced to frame $X_i Y_i$
p_i	— absolute position vector to a point x_i referenced to frame $\hat{X}_0 \hat{Y}_0$
$p(x_i, t)$	— distributed dynamic load along distance x_i at time t , [kg/m]
Q	— LQR state weighting matrix
Q_k	— process noise covariance matrix
q	— generalized coordinate vector
q_i	— i th generalized coordinate, $i = 1, \dots, n$
R	— LQR control input weighting matrix
R_E	— FFSRM end-effector position vector $[x' \ y']^T$ relative to ECEF inertial frame of reference
R_k	— measurement noise covariance matrix
R_i	— position vector to a point on each body from its axis of rotation, $i = 0, 1, 2$
R_S	— spacecraft CM position vector $[x_S \ y_S]^T$ relative to ECEF inertial frame of reference
r_e	— absolute end-effector position vector referenced to frame $\hat{X}_0 \hat{Y}_0$
r_{r,c_i}	— link centroid position vector, $i = 1, 2$

${}^i\mathbf{r}_{i+1}$	— position to origin of frame $X_{i+1}Y_{i+1}$ referenced to frame X_iY_i
$\mathbf{r}_i$	— absolute position to origin of frame $X_{i+1}Y_{i+1}$ referenced to frame $\hat{X}_0\hat{Y}_0$
$\mathbf{S}$	— link deformation coefficient matrix
$\mathbf{T}_i^j$	— frame j to frame i transformation matrix
T	— total kinetic energy, [Joules]
T_{h_i}	— kinetic energy of hub for flexible dynamics, $i = 1, 2$, [Joules]
T_{L_i}	— kinetic energy of link for flexible dynamics, $i = 1, 2$, [Joules]
T_p	— kinetic energy of payload for flexible dynamics, [Joules]
T_r	— total kinetic energy for rigid dynamics, [Joules]
T_v	— translational component of kinetic energy, [Joules]
T_ω	— rotational component of kinetic energy, [Joules]
t_d	— delay time, [msec]
$t_{\max}$	— maximum overshoot time, [msec]
t_r	— rise time, [msec]
t_s	— settling time, [msec]
$\mathbf{U}_i$	— displacement vector for a point on a body, $i = 0, 1, 2$
U	— total potential energy, [Joules]
U_r	— total potential energy for rigid dynamics, [Joules]
$\mathbf{u}_i$	— link deformation function vector, $i = 1, 2$
$u(x_i, t)$	— link deformation function at distance x_i and time t

u_i	— link deformation function at $x_i = L_i$, $i = 1, 2$
u'_i	— link deformation function derivative, $i = 1, 2$
u'_e	— deformation function derivative at the end effector
V_i	— velocity vector for a point on a body, $i = 0, 1, 2$
$V(\dot{\theta}, e)$	— Lyapunov function candidate for a rigid dynamics robot manipulator
$V(q, e)$	— Lyapunov function candidate for a flexible dynamics robot manipulator
v_k	— measurement noise stream vector
w_k	— process noise stream vector
W	— power spectral density matrix associated with the control function
W_i	— global transformation matrix
W	— total work done on a robot joint by an external torque
X_i	— X coordinate in a rigid frame of reference, $i = 0, 1, 2$
$\hat{X}_i$	— X coordinate in a flexible deformation frame of reference, $i = 0, 1, 2$
x	— state variables vector
$\dot{x}$	— state variables derivative vector
x_d	— desired operating point vector for Lyapunov linearization
$\hat{x}_{k+1}^-$	— projected state estimate vector
$\hat{x}_k^-$	— <i>a priori</i> state estimate vector
$\hat{x}_k$	— <i>a posteriori</i> state estimate vector
x	— x-coordinate position of the end effector

x_c	— commanded x-coordinate position of the end effector
x_i	— x-coordinate distance of a point along a link, $i = 1, 2$
x_s	— spacecraft x-coordinate position
Y_i	— Y coordinate in a rigid frame of reference, $i = 0, 1, 2$
$\hat{Y}_i$	— Y coordinate in a flexible deformation frame of reference, $i = 0, 1, 2$
y	— y-coordinate position of the end effector
y_c	— commanded y-coordinate position of the end effector
y_s	— spacecraft y-coordinate position
Z_i	— Z coordinate in a rigid frame of reference, $i = 0, 1, 2$
$\hat{Z}_i$	— Z coordinate in a flexible deformation frame of reference, $i = 0, 1, 2$
$\mathbf{Z}$	— partitioned index matrix for Ω
$\mathbf{z}$	— state measurement output vector
$\mathbf{z}_k$	— discrete state measurement vector
$\hat{\mathbf{z}}_k$	— discrete predictive state measurement vector

Greek Symbols

α_0	— spacecraft attitude angle measured from fixed horizontal datum, [rad]
α_i	— link rotation angle measured from fixed horizontal datum, $i = 1, 2$, [rad]
α_{t_i}	— link angle of twist, $i = 1, 2$, [rad]
δ	— elastic deformation vector

δ_{ij}	— elastic deformation for beam i and mode j , $i = 1, \dots, n$, $j = 1, \dots, n$, [m]
δ_i	— elastic deformation for link i and dominant mode, $i = 1, 2$
ζ	— closed-loop damping ratio, 0.707
$\dot{\eta}_e$	— absolute angular velocity of frame $O_i X_i Y_i Z_i$
θ	— joint angles vector, [rad]
$\dot{\theta}$	— joint rates vector, [rad/s]
θ_a	— actuator angles vector, [rad]
θ_h	— hub angles vector, [rad]
θ_i	— robot body rotation angle, $i = 0, 1, 2$, [rad]
$\dot{\theta}_i$	— robot body angular velocity, $i = 0, 1, 2$, [rad]
θ_{a_i}	— actuator rotation angle, [rad]
θ_{h_i}	— hub rotation angle, $i = 1, 2$, [rad]
θ_{r_i}	— rotor rotation angle, $i = 1, 2$, [rad]
θ_t	— angular rotation of link distal end, [rad]
$\lambda_{c_j} L_i$	— cantilever mode characteristic root $[j - 0.5]\pi$, $j = 1, \dots, n$
$\lambda_{pp_j} L_i$	— pinned-pinned mode characteristic root, $j = 1, \dots, n$
ρ	— linear mass density of a flexible link, 0.335, [kg/m]
ρ_i	— linear mass density of a body, $i = 0, 1, 2$, [kg/m]
τ	— control law torque vector $[\tau_1 \ \tau_2]^T$

τ_i	— joint actuating torque, $i = 1, 2$, [Nm]
τ_k	— LQR and LQG dynamic regulator control law vector
τ_r	— reference control law vector
Φ_k	— state transition matrix
ϕ_{ij}	— mode shape for beam i and mode j , $i = 1, \dots, n$, $j = 1, \dots, n$
ϕ_i	— mode shape for link i and dominant mode, $i = 1, 2$
$\phi_i(x_i)$	— mode shape at distance x_i along link i for dominant mode, $i = 1, 2$
Ω	— partitioned matrix for calculating Φ_k and Q_k matrices
$lrp\Omega$	— lower-right partition of matrix Ω
$urp\Omega$	— upper-right partition of matrix Ω
$\bar{\omega}_i$	— skew-symmetric matrix of angular velocities $i = 0, 1, 2$
ω_{c_j}	— cantilever elastic mode frequency, $j = 1, \dots, n$, [Hz]
ω_{pp_j}	— pinned-pinned elastic mode frequency, $j = 1, \dots, n$, [Hz]
ω_{ij}	— modal frequency for elastic beam i and mode j , $i = 1, \dots, n$, $j = 1, \dots, n$, [Hz]

Fuzzy Logic Notation

A_i	— measured antecedent input signal to the FIS, $i = 1, \dots, n$
A_{ki}	— input variable for antecedent i of fuzzy rule k , $i = 1, \dots, n$, $k = 1, \dots, n$
a	— minimum outer limit of triangular membership function
B_k	— output variable for consequent of fuzzy rule, $k = 1, \dots, n$

b	— centre of triangular membership function
c	— maximum outer limit of triangular membership function
c	— centre of Gaussian membership function
$e(w)$	— fuzzy approximation error
F	— fuzzy set
$g(w)$	— functional mapping of w data points
K_N	— universe of discourse normalizing gain for FLS input variables δ_1 and δ_2
K_{N_i}	— universe of discourse normalizing gain for FLC input variable, $i = 1, \dots, n$
K_1	— FLC output scaling gain
K_2	— FLC output scaling gain
K_s	— FLS output scaling gain
L	— large
M	— medium
N	— negative
NH	— negative high
NL	— negative low
NVH	— negative very high
NVL	— negative very low
P	— positive
PH	— positive high
PL	— positive low
PVH	— positive very high

PVL	— positive very low
R	— overall fuzzy relation for all rules in a fuzzy inference system
R_k	— fuzzy rule implication, $k = 1, \dots, n$
R_C	— Mamdani fuzzy implication function
$\mathcal{R}^n$	— set of n-dimensional real numbers
$\mathcal{R}$	— set of real numbers
S	— small
U	— universe of discourse
$\{u\}$	— set of elements on antecedent universe of discourse U
u	— element of continuous antecedent universe of discourse
u_i	— element of discrete universe of discourse
v	— element of continuous consequent universe of discourse
W	— fuzzy subset
$\tilde{W}$	— crisp set
w	— crisp data points
Z	— zero
Z	— fuzzy subset
$\tilde{Z}$	— crisp set
$\cap$	— fuzzy intersection
$\cup$	— fuzzy union
$\wedge$	— AND (MIN) operator
$\vee$	— OR (MAX) operator

$\int$	— continuous fuzzy set union
$\sum$ or $+$	— discrete fuzzy set union
$/$	— fuzzy connection
α	— α -cut set
β_k	— fire strength of fuzzy rule, $k = 1, \dots, n$
γ	— FLAEKF output variable
λ	— FLS output variable
μ	— degree of membership
μ_A	— degree of membership for fuzzy rule antecedent
μ_B	— degree of membership for fuzzy rule consequent
μ_F	— degree of membership for fuzzy set F
σ	— Gaussian membership function width (shape) parameter
ψ	— fuzzy parameter

List of Acronyms

ARE	— algebraic Riccati equation
ACS	— attitude control system
CM	— centre-of-mass
CMG	— control moment gyro
COA	— centre-of-area
D - H	— Denavit-Hartenberg
dof	— degree-of-freedom
DPS	— distributed parameter system
ECEF	— earth-centred earth-fixed
EFL	— evolutionary fuzzy learning
EKF	— extended Kalman filter
FFFSRM	— free-flying flexible space robot manipulator
FAR	— fixed-attitude-restricted
FIS	— fuzzy inference system
FLAEKF	— fuzzy logic adaptive extended Kalman filter
FLC	— fuzzy logic control
FLS	— fuzzy logic system
FLSAC	— fuzzy logic system adaptive control
GJM	— generalized Jacobian matrix
GMP	— generalized modus ponens
GMT	— generalized modus tollens
GPS	— global positioning system

IFDC	— inverse flexible dynamics control
ILC	— iterative learning control
INS	— inertial navigation system
IRDC	— inverse rigid dynamics control
JTPD	— Jacobian transpose proportional-derivative
KF	— Kalman filter
LPTV	— linear periodically time-varying
LQ	— linear quadratic
LQR	— linear quadratic regulator
LQG	— linear quadratic Gaussian
LSS	— large space structures
LTI	— linear time invariant
MF	— membership function(s)
min	— minute
MIMO	— multiple-input multiple-output
MISO	— multiple-input single-output
NMF	— number of membership functions
NMP	— nonminimum phase
NOC	— natural orthogonal complement
OBSS	— orbiter boom sensor system
PD	— proportional-derivative
PID	— proportional-integral-derivative
PVCRI	— point-valued compositional rule of inference

RLC	—	repetitive learning control
RLIRDC	—	repetitive learning inverse rigid dynamics control
RLFLC	—	repetitive learning fuzzy logic control
RLIFDC	—	repetitive learning inverse flexible dynamics control
RLFLSAC	—	repetitive learning fuzzy logic system adaptive control
sec	—	second
SISO	—	single-input single-output
ss	—	simulation step
SSRMS	—	space shuttle remote manipulator system
WAC	—	wave-absorbing controller
2-D	—	two-dimensional
3-D	—	three-dimensional

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Chapter 1

Introduction

1.1 Thesis

Achieving autonomous intelligent positioning and tracking control of flexible robot manipulators is a desirable capability for future space operations yet presents a formidable challenge in synthesis and application. Flexural vibrations excited by robot joint actuators and spacecraft motions during space operations can generate large deformations at the end effector. These deformations present major difficulties in sensing the correct end-effector position and in applying the correct control action to achieve accurate and stable manoeuvrability.

This thesis presents research on the theory and tracking control of fixed-base rigid and flexible robot manipulators, and a free-flying flexible space robot manipulator.

The goals of the thesis are:

1. Synthesize, investigate and compare several control strategies for a robot manipulator modeled with linear and nonlinear rigid-link dynamics as a foundation in identifying a suitable strategy for further development towards the intelligent control of flexible space robot manipulators.
2. Synthesize and demonstrate an intelligent control strategy for a robot manipulator modeled with both reduced-order and full-order distributed parameter flexible-link dynamics models with cantilever and pinned-pinned elastic beam boundary conditions for comparison.
3. Establish a heuristic design ratio for modeling an optimal fuzzy inference system for high precision tracking at minimum computation time.

4. Investigate the effects of time delays in modeling nonminimum phase response and corrective tracking control and stability of a flexible robot manipulator.
5. Investigate the effects of noncollocated sensors positioned at different locations along the outboard flexible link on the tracking precision and stability of a flexible robot manipulator.
6. Investigate the effects of adding rigid component inertias into the flexible dynamics equations on the tracking precision and stability of a flexible robot manipulator.
7. Investigate and demonstrate the ability of an intelligent control strategy to repeatedly track the end effector of a rigid robot manipulator with precision and stability.
8. Investigate and demonstrate the ability of an intelligent control strategy to repeatedly track the end effector of a distributed parameter flexible robot manipulator with precision and stability.
9. Investigate and compare hybrid control strategies with a linear quadratic Gaussian dynamic regulator with, either an extended Kalman filter or, a fuzzy logic adaptive Kalman filter integrated within an intelligent control strategy on the tracking precision and stability of a flexible robot manipulator.
10. Investigate and demonstrate the ability of an intelligent control strategy to track the end effector of a free-flying flexible space robot manipulator using a fuzzy logic adaptive Jacobian transpose proportional-derivative control laws using a fixed-attitude-restricted motion Jacobian derived from flexible direct kinematics equations.

1.1.1 General Framework

Fuzzy logic is a form of autonomous intelligent approximate reasoning that can be applied to the position and tracking control of rigid and flexible robots performing terrestrial and space manipulation tasks, thereby providing an element of autonomy in their operation.

This thesis presents the theoretical syntheses and simulation of control strategies, in which fuzzy logic plays a significant role, either as a sole substitute for a dynamics model or an adjunct to other control strategies. Various other control strategies are synthesized and simulated for tracking results to provide a basis of comparison with fuzzy control and to direct further research.

1.1.2 Demonstration

Intelligent control of robot manipulators is demonstrated on three different applications.

Firstly, a fuzzy logic control (FLC) strategy in which two fuzzy inference systems (FIS) substitute for rigid robot dynamics equations. Secondly, a fuzzy logic adaptive extended Kalman filter (FLAEKF) is integrated within a linear quadratic Gaussian (LQG) control strategy for a rigid robot manipulator to provide a corrective measure against tracking divergence. Thirdly, a fuzzy logic system (FLS) is implemented in the feedback loop of an inverse flexible dynamics control (IFDC) strategy to adapt a Jacobian transpose proportional-derivative (JTPD) control law and suppress elastic vibrations of a flexible robot manipulator. This strategy demonstrates its effectiveness in providing intelligent and autonomous precision control. The strategy is further applied to a free-flying space robot manipulator and motivated by the prospect of an eventual substitute for the

currently teleoperated control of space robots using the resolved motion rate control method.

To provide a basis of comparison and a reliable foundation upon which suitable intelligent control techniques could be developed, modern control strategies are synthesized for the tracking control of both rigid and flexible fixed-base robot manipulators and a free-flying flexible space robot manipulator.

Modeling and simulation of all control strategies are performed with Matlab/Simulink™ software using the Control System and Fuzzy Logic Toolboxes [98].

1.1.2.1 Square Trajectory

Throughout Chapters 3, 4, 5 and 6 a commanded 12 m x 12 m square trajectory is consistently used in the simulation of all control strategies. The shoulder joint coincides with the fixed base of the robot located at the centre of the square trajectory. Compared to many other possible trajectories e.g. straight line, cloverleaf and circle, the square presents a greater challenge for control. In particular, the corners of the square present abrupt orthogonal direction switches for a robot tracking at constant velocity along each straight side. The end effector tracks the square trajectory clockwise starting at the lower left corner followed by the first, second and third direction switches. It is required to reach each corner position then instantaneously redirect itself along an orthogonal direction with minimum overshoot and settling time to a steady state straight trajectory while maintaining stability. To assist the end effector in following the desired trajectory sixteen x and y coordinate pairs, one at each corner position with others equispaced along each of the four straight sides, are commanded by the trajectory planner.

Square trajectories have been chosen by many researchers to demonstrate the

effectiveness and stability of their various robot control systems [2], [31], [85], [128], [157].

1.1.2.2 Point-to-Point Trajectory

For simulation of a free-flying flexible space robot a point-to-point trajectory is chosen to demonstrate the effectiveness and comparison of the IFDC and FLSAC strategies. The controller can autonomously direct the robot end effector to follow any trajectory between the initial and final positions.

1.1.3 Motivation

Accurate position control of a flexible robot in spacecraft operations is greatly affected by vibration and deformation of its links. Astronauts perform the control of robot manipulators on the space shuttle and international space station by operating servo controls with levers or computer keyboards. The procedure often results in frequent overcorrection and recorection until the target point is reached. Any task manoeuvre may take up to fifteen hours to complete. Therefore, accuracy in positioning the end effector is achieved primarily relying on the dexterity and ability of the astronaut. It would be desirable to eventually perform these operations expediently by relying solely upon autonomous intelligent control systems.

Also, manipulation of the robot generates reaction forces and moments that cause spacecraft translational and rotational motions resulting in inaccurate tracking and positioning of the end effector or even tumbling of the spacecraft. This often necessitates compensation by thruster jets or activation of control moment gyro (CMG) wheels requiring consumption of limited propellant fuel, electrical energy or both. The use of an intelligent control strategy may lead to a reduction or elimination of reactive spacecraft

Plate 1.1: Space shuttle remote manipulator system (SSRMS). (Courtesy Wikipedia, The Free Encyclopedia).

Plate 1.2: Orbiter boom sensor system (OBSS) and laser camera system (LCS) in use. (Courtesy of Wikipedia, The Free Encyclopedia).

motions, with potential savings in fuel and electrical energy consumption while achieving the desired manipulation accuracy in less time.

This thesis demonstrates an endeavour towards ultimately achieving autonomous intelligent control of a two-link flexible robot mounted on a rigid spacecraft tracking its end effector between two points in two-dimensional (2-D) space.

The free-flying flexible space robot manipulator is a nonholonomic distributed parameter system (DPS) for which link flexibility is modeled using the dominant assumed mode for Euler-Bernoulli cantilevered beam boundary conditions. Elastic vibrations coupled with large rotations and nonlinear interactions of the links presents a formidable challenge for control. Several challenges present difficulty in achieving intelligent autonomous control of large flexible space structures for which solutions and control syntheses, developed so far, have usually been based on linearized state-space dynamics models.

Flexible dynamics equations are most often derived as either reduced-order or lumped-parameter models. The control strategy is usually a composite of two subsystems in which the dynamics are decoupled into slow rigid dynamics and fast flexible dynamics subsystems. The results, though beneficial, remain far from the reality of a solution that embodies a coupled rigid and flexible dynamics model. The theoretical modeling and simulation results presented are an attempt to draw closer to that reality and provide the advantage of intelligent autonomous control.

Plate 1.1 shows the space shuttle remote manipulator system (SSRMS) and Plate 1.2 shows the SSRMS with an extension and orbiter boom sensor system (OBSS) mounted at its end for which an autonomous intelligent control strategy would provide a major

advantage in shuttle operations.

1.1.4 Thesis Outline

The thesis is organized into the following chapters:

Chapter 1: Introduction

This chapter provides an introduction, an outline, the motivation for conducting the research, a demonstration synopsis, highlights of thesis results and contributions.

Chapter 2: Background and Literature Review

This chapter presents a synopsis of the main referenced publications on various subtopics covered within the thesis.

Chapter 3: Fixed-Base Planar Two-Link Rigid Robot Manipulator

This chapter presents the theoretical formulation of a planar two-link rigid dynamics robot manipulator model. Inverse rigid dynamics (IRDC), linear quadratic regulator (LQR), fuzzy logic (FLC) and stochastic (LQG) control strategies with PD gains calculated on dominant assumed mode vibration frequencies for both cantilever and pinned-pinned flexible beam boundary conditions are derived and synthesized.

The PD gains form part of the JTPD control law used for all strategies within this chapter. The stochastic control strategies include a linear quadratic Gaussian (LQG) dynamic regulator with Kalman (KF), extended Kalman (EKF) and fuzzy logic adaptive Kalman filter (FLAEKF) subroutines. Simulation results are presented for each strategy used to control a rigid dynamics robot manipulator tracking a 12.6 m x 12.6 m square trajectory. Step input performance transients of planar displacements of the end effector are presented and compared for each control strategy.

Chapter 4: Fixed-Base Planar Two-Link Flexible Robot Manipulator

This chapter includes six major sections covering different aspects of the dynamics and control of a fixed-base planar two-link flexible robot manipulator.

Section 4.1 provides a brief overview of the underlying theory and derivation of flexible robot dynamics. The assumed modes concept is discussed as an acceptable approximation approach to modeling the flexible robot links as a distributed parameter system (DPS). Cantilever and pinned-pinned assumed modes are considered in modeling robot-link boundary conditions. Cantilever assumed modes are used to model each link as being fixed at the proximal and free at the distal end. Pinned-pinned assumed modes model each link as being free to rotate at both proximal and distal ends.

Section 4.2 presents a synthesis of inverse flexible dynamics (IFDC) and fuzzy logic system adaptive control (FLSAC) strategies incorporating a reduced-order dynamic coupling vector as given by Equation (3.7) for the rigid dynamics robot manipulator presented in Chapter 3. Simulated tracking results and step response transient performance characteristics are presented for both control strategies. Simulated tracking results and step response transient performance characteristics are presented for IFDC and FLSAC strategies with a full-order dynamics model for comparison with results for the reduced-order dynamics model.

Section 4.3 presents tracking results used in the determination of a heuristic design ratio for modeling an optimal fuzzy logic system used to suppress flexible robot vibrations. The ratio is considered in terms of a combination of; number and type of membership function, positive range of the universe of discourse, output scaling gain and minimum execution time. The heuristic design ratio is determined as the ratio of output

scaling gain to the number of membership functions for which, the output scaling gain values are multiples of the positive range of the universe of discourse.

Section 4.4 presents a theoretical analysis and tracking results for a fixed-base planar two-link flexible robot manipulator with collocated and noncollocated sensors. Time delays implemented in the feedback loop model the effect of nonminimum phase (NMP) response of the flexible links and time-delayed joint control action in the feed-forward loop is designed to correct the effects of the NMP response.

Section 4.5 presents a consideration of total system stiffness. The effect of modeling flexible joint drive transmission components coupled with multilink flexible dynamics is discussed and an approach to a theoretical model proposed.

Simulated tracking results are presented for IFDC and FLSAC strategies incorporating rigid joint inertias in the flexible link dynamics equations for both reduced- and full-order dynamic coupling matrices.

The effect of adding a rigid joint inertia at the elbow joint is considered tantamount to a payload at the distal end of a single link.

Section 4.6 presents a theoretical Lyapunov candidate function stability analysis of the planar two-link flexible robot manipulator model controlled by a JTPD control law.

Chapter 5: Repetitive Learning Control

This chapter presents two major sections covering repetitive learning control of fixed-base planar two-link rigid and flexible robot manipulators.

Section 5.1 presents simulation results for repetitive learning inverse rigid dynamics (RLIRDC) and repetitive learning fuzzy logic control (RLFLC) strategies for a fixed-base planar two-link rigid robot manipulator repetitively tracking a 12.6 m x 12.6 m square

trajectory.

Section 5.2 presents simulation results for repetitive learning inverse flexible dynamics (RLIFDC) and repetitive learning fuzzy logic system adaptive control (RLFLSAC) strategies for a fixed-base planar two-link flexible robot manipulator repetitively tracking a 12.6 m x 12.6 m square trajectory.

Chapter 6: Hybrid Control

This chapter presents the synthesis and application of hybrid control strategies applied to the control of a fixed-base planar two-link flexible robot manipulator.

Section 6.1 presents tracking results with an IFDC strategy augmented by a LQG dynamic regulator with an EKF or a FLAEKF.

Section 6.2 presents tracking results for the FLSAC strategy augmented by a LQG dynamic regulator with an EKF or a FLAEKF.

Section 6.3 presents tracking results for an IFDC strategy combined with NMP time delays and time-delayed corrective control for comparison with FLSAC alone.

Chapter 7: Intelligent Tracking Control of a Free-Flying Flexible Space Robot Manipulator

This chapter presents a theoretical description and analysis of the dynamics and control of a free-flying flexible space robot manipulator (FFFSRM).

Results are presented for tracking a trajectory between two points in a space orbit using an IFDC strategy with a fixed-attitude-restrictive (FAR) motion Jacobian transpose proportional-derivative (JTPD) control law and a FLSAC strategy with a FLS adaptive FAR motion JTPD control law.

Chapter 8: Summary, Conclusions and Future Research

This chapter presents summaries on the theoretical models for rigid and flexible robot

manipulator kinematics, dynamics, control laws and strategies. Summaries of motivation, problems are presented together with summaries of results and their validation.

Conclusions are presented on the control strategies, robot dynamics models and simulated tracking results.

A number of topics is identified and recommended for future research.

1.1.5 Highlights of Thesis Results

1. Stable tracking of a fixed-base two-link rigid robot manipulator with inverse dynamics, LQR, LQG with KF, EKF and FLAEKF dynamic regulators and fuzzy logic control strategies for both cantilever and pinned-pinned dominant assumed mode PD gains. Superior tracking control performance is achieved with FLC but the best closed-loop step input performance characteristics are achieved with LQR control with pinned-pinned PD gains.
2. Precision and stable repetitive tracking control of a fixed-base planar two-link rigid robot manipulator with a FLC strategy.
3. Stable tracking of a fixed-base planar two-link flexible robot manipulator with IFDC for both cantilever and pinned-pinned dominant assumed mode reduced-order dynamics models and PD gains. The cantilever model presents the greater challenge for further control improvement.
4. Precision and stable intelligent tracking of a fixed-base planar two-link flexible robot manipulator with a FLSAC strategy for both cantilever and pinned-pinned dominant assumed mode reduced-order dynamics models and PD gains. The cantilever model results demonstrate the greater improvement in performance.
5. Similar tracking results are obtained for a flexible robot manipulator

dynamics model with a full-order dynamic coupling vector. These results demonstrate the reduced-order dynamic coupling vector model may be a suitable substitute for the more mathematically complex full-order dynamics model.

6. Determination of a heuristic design ratio between FLS output scaling gain values and the number of membership functions spanning a universe of discourse range which is a factor of the output scaling gain and producing tracking results at minimum execution time.

7. Stable intelligent tracking control of a fixed-base planar two-link flexible robot manipulator with a FLSAC strategy and sensors located at different positions on the outboard link together with NMP response time delays and time-delayed corrective control action.

8. Stable intelligent tracking control of a fixed-base planar two-link flexible robot manipulator with joint inertia terms added to the dynamics model in a FLSAC strategy.

9. Stable repetitive intelligent tracking control of a fixed-base planar two-link flexible robot manipulator with a FLSAC strategy.

10. Stable intelligent tracking control of a fixed-base planar two-link flexible robot manipulator with EKF and FLAEKF noise filtering and FLSAC vibration suppression together with NMP response time delays and time-delayed corrective control action.

11. Derivation of a fixed-attitude-restricted motion Jacobian from flexible direct kinematics equations, and its application in accurate, stable and intelligent tracking control of a free-flying flexible space robot with a FLSAC strategy.

1.1.6 Thesis Contributions

1. Stable tracking of a 12.6 m x 12.6 m square trajectory by a planar two-link rigid dynamics robot manipulator with inverse dynamics, linear quadratic regulator and fuzzy logic control strategies using JTPD control laws with cantilever and pinned-pinned dominant assumed modes PD gains for comparison. The fuzzy logic control strategy is synthesized by substituting two fuzzy inference systems for the robot dynamics equations of motion where each FIS represents a robot link. Modeling these rigid dynamics strategies provides a platform from which to progress to modeling and simulation of a flexible robot dynamics model control strategy.

Tracking results demonstrate the FLC strategy produces greater control precision than the IRDC or LQR strategies and provides justification for fuzzy logic being a suitable form of intelligent control for flexible robots.

The IRDC strategy provides an environment for extensions to modeling planar nonlinear flexible robot manipulator dynamics.

2. A cross-covariance term is included in a FLAEKF subroutine to model cross-correlated process and measurement noise and provides control action to correct tracking divergence encountered with an EKF. The original subroutine, derived by others, excludes the cross-covariance term [124], [125]. The modified subroutine is used within a LQG control strategy for a fixed-base rigid dynamics robot manipulator subjected to different means and covariance values of process and measurement noise.

3. A fixed-base two-link flexible robot manipulator is modeled with both reduced- and full-order dynamic coupling vectors. Tracking results for the reduced-order model demonstrate close similarity to those for a full-order model but without the

mathematical complexity.

4. Greatly improved tracking results are obtained for a fixed-base two-link flexible robot manipulator with both reduced- and full-order dynamic coupling vectors using a FLSAC strategy.

5. Determination of a heuristic design ratio to enable expedient design of an optimal fuzzy inference system for elastic link vibration suppression in the tracking control of a two-link flexible robot manipulator.

6. Time delays are implemented in a FLSAC strategy to model nonminimum phase response for a fixed-base two-link flexible robot manipulator with rigid-link direct kinematics equations.

7. Stable tracking is achieved with sensors located at three alternate positions on the outboard link of a two-link flexible robot manipulator thereby circumventing hyperstability constraints.

8. Coupling of flexible robot joint drive and link dynamics is discussed.

Precision tracking results are obtained for a robot dynamics model with rigid joint drive inertias using a FLSAC strategy with both reduced- and full-order dynamics models and higher output scaling gains values. Using higher output scaling gain values for additional component inertias suggests future development of online automatic gain scheduling for different payloads and robot parameters.

9. Repetitive learning control is applied to rigid and flexible robot manipulators using IRDC, FLC, IFDC and FLSAC strategies. Tracking results demonstrate the IRDC and IFDC strategies are ineffective in reducing transient overshoot amplitudes whereas, FLC and FLSAC strategies produce much reduced transient

overshoot amplitudes.

10. An IFDC strategy is augmented by an LQG dynamic regulator with an EKF and a FLAEKF to form hybrid strategies for the tracking control of a two-link flexible robot manipulator. A comparison of results shows minimal effectiveness of either filter.

The FLSAC strategy is augmented by an LQG dynamic regulator with an EKF or a FLAEKF to form hybrid strategies for the tracking control of a two-link flexible robot manipulator. A comparison of results shows an overriding effect of FLSAC on tracking precision.

The FLSAC strategy produces an overriding effect on tracking the end effector of a two-link flexible robot manipulator compared to hybrid IFDC strategies augmented by an LQG dynamic regulator with an EKF and a FLAEKF together with NMP and corrective time-delayed control action.

11. Derivation of a FAR motion Jacobian from flexible direct kinematics equations, and formulation of a FAR motion JTPD control law for use in IFDC and FLSAC strategies applied to the tracking control of a free-flying flexible space robot manipulator.

12. Much improved tracking control with a FLSAC strategy than with an IFDC strategy for manoeuvring the end effector of a free-flying flexible space robot manipulator between two positions in a space orbit.

Chapter 2

Background and Literature Review

Single and multibody robot dynamics and control have received extensive research attention during the past two decades. Many robot control systems originally focused on classical transfer function, time domain and modern state-space techniques. However, with the recent proliferation of developments on intelligent control including; fuzzy logic, expert systems, neural networks and genetic algorithms, the prospect of completely autonomous intelligent control has become a major goal to be achieved in the development of spacecraft, space robotics and aerospace systems in general.

Flexural vibrations of robot manipulator links degrade performance in achieving commanded end-effector positions and the highly nonlinear dynamics present complexity in controller design. The dynamics are even more complicated for free-flying flexible space robots.

The references cited represent a limited range of available published literature. Those referenced are the main sources of information from which the current research was driven. The bibliography provides further background reading used to broaden the scope of understanding and deepen insight into the difficulties encountered and techniques used to control robot manipulators. They also provide the groundwork for cultivating ideas for further research towards achieving the goal of autonomous intelligent control. The following synopses highlight the main references.

Asada and Slotine [9], Brady et al [19], Craig [31], McKerrow [99], Paul [115], Schilling [127], Sciavicco and Siciliano [128], Spong and Vidyasagar [137], and Yoshikawa [157], are excellent references on the theory of dynamics and control of rigid

robots.

García de Jalón and Bayo [46] and Shabana [129] are excellent references on the theoretical development of the kinematics and dynamics of rigid and flexible multibody systems.

Brown and Hwang [20], Gelb [142], Grewal and Andrews [72], Lewis [92], Stengel [139], and Zarchan and Musoff [160], are excellent references on the theory, derivation and analysis of state-space systems, optimal control, state estimation and Kalman filtering.

Slotine and Li [134], and Vidyasagar [147], are excellent references on the theory of nonlinear systems and control.

Fraser and Daniel [45], and Junkins and Kim [81], provide sound theoretical derivations and application of the assumed modes method in formulating flexible robot manipulator dynamics models.

Kelkar and Joshi [84], provide sound theoretical derivations of dynamics and control models for nonlinear multibody flexible space structures.

Moallem, Patel and Khorasani [104], provide sound theoretical derivations of dynamics and control techniques for flexible robot manipulators.

Jassemi-Zargani and Neculescu [80], implement fused sensor data into an extended Kalman filter applied to acceleration outputs of an impedance-controlled robot arm.

Carusone, Buchan and D'Eleuterio [21], experimentally track a 75 cm x 75 cm square trajectory with a two-link flexible space robot using a series of steady-state linear regulators that produce good results.

Banerjee and Singhose [10], track a square trajectory 12.6 m x 12.6 m using an input

shaping method coupled with an inverse kinematics control scheme. Both linear and nonlinear control laws are used to simulate control of a two-link flexible robot manipulator and reduce residual vibrations. Full-order flexible dynamics equations are derived using a recursive $O(n)$ algorithm for a two-link flexible robot discretized into three rigid segments connected by torsion springs suitable for modeling large bending deformations.

They also include trajectory periodicity simulations in their studies with an input shaping technique for damping residual vibrations.

Their research stimulated the investigations presented in this thesis.

Beres and Sasiadek [11], formulate a Lagrange finite element dynamic model of an n -link flexible robot manipulator, in which large rigid-body rotational motion of the links and their small elastic deformations are coupled. The dynamic model for a two-link flexible robot manipulator with two finite elements per link may be extended to an arbitrary n -link manipulator with n -elements per link but at a high computational burden.

Sasiadek and Srinivasan [126], apply a model reference adaptive control (MRAC) strategy to investigate position and vibration control of a distributed mass, single-link flexible manipulator. The link is modeled using a modal expansion method based on a pinned-free elastic beam configuration to determine the first three significant vibration modes. The MRAC strategy, based on a state-space formulation, was used in an attempt to achieve smaller position errors and an alternative to proportional-integral-derivative (PID) control that fails to give satisfactory results. The elastic potential function is coupled to the rigid-link dynamics using Euler-Lagrange dynamics to formulate a generalized inertia matrix of the manipulator.

A simple feedback control law comprises a derivative gain, velocity feedback and gravity compensation terms. PID transient responses to step and impulse inputs are compared to MRAC responses. Simulation results demonstrate accuracy of the modal expansion method within an MRAC strategy by further reducing position errors and decreasing the settling time of transient responses to step inputs.

Lee, Vukovich and Sasiadek [91], apply a fuzzy logic approach to the endpoint position control of an experimental flexible single-link manipulator. The fuzzy rules have position errors and rates as inputs, with motor hub speed as the output variable to drive the laboratory manipulator model that exhibits significant transient endpoint vibrations.

Moudgal, Passino and Yurkovich [106], use fuzzy controllers to vary voltage input to each motor of a two-link flexible robot in response to link tip accelerations and 90° slew angle position outputs with significant transient vibrations. Initially the links are controlled independently and uncoupled from each other. The fuzzy controllers have eleven triangular MF for each input and output variable. But, the output variable has adaptive MF where they are skewed in the vicinity of zero volts output to create a higher density for increased controller gain and the achievement of better control at the set point and to minimize overshoots.

A similar fuzzy controller design is used to simulate the dynamic interaction of coupled robot links. Then, a fuzzy supervisory controller is introduced to provide fine control of the robot that improves upon previous results.

de Silva [37], uses a fuzzy control technique to simulate control of a rigid two-link robot manipulator tracking a square trajectory and where the fuzzy inference system is used to tune the servo gains and achieve better control at direction switches.

Berstecher, Palm and Unbehauen [13], devise an adaptive fuzzy sliding mode controller by changing the location of triangular MF for nonlinear adaptation and causing changes in the moments of the defuzzification process, thereby producing a nonlinear and approximately correct output.

Layne and Passino [89], apply a fuzzy reference learning control method to a plant by using two fuzzy controllers with eleven triangular MF for joint position error and change-of-error inputs and torque outputs for a two-link flexible robot. The learning mechanism is a fuzzy inverse model also with triangular MF that adapts a direct fuzzy controller knowledge base according to an ideal reference model of the controlled plant.

Kim [86], uses an independent joint adaptive fuzzy controller comprising decentralized fuzzy logic and adaptive control to simulate asymptotic trajectory tracking with gross robot parameter variation. Good results are achieved without the requirement of an accurate robot dynamic model and joint angular acceleration measurements.

Chiu [27], uses a novel adaptive fuzzy control method to remove the disadvantages of traditional fuzzy approximation based control applied to highly uncertain rigid-link robot manipulators.

Vukovich and Yousefi-Koma [148], apply a wave-absorbing controller (WAC) utilizing a noncollocated piezofilm sensor bonded to a long flexible beam to suppress traveling waves in the beam and provide input to a piezoceramic actuator. This noncollocated sensor performs better than a collocated WAC. Also, performance improves as the length of the noncollocated piezosensor increases.

Stieber, Vukovich and Petriu [140], propose an extended hyperstability control concept for a flexible link robot. Conventional hyperstable control sensors, required to

satisfy the restrictive collocation requirement, are used in conjunction with performance sensors to enable vision-based stable control of large flexible space robot manipulators. Step response simulation results demonstrate the effectiveness of the vision-based scheme in overcoming conventional hyperstability restrictions.

Karkoub, Balas and Tamma [82], perform theoretical and experimental studies on a single-link flexible robot manipulator using the μ -synthesis control method. Sensor configurations include; collocated sensor feedback control with a hub angle sensor; noncollocated feedback control using both hub angle and endpoint acceleration sensors, and noncollocated feedback control using both hub and endpoint sensors for which the hub angle specification is relaxed to reduce hub response overshoot.

Experimental results verify the two theoretical noncollocated control designs are a dramatic improvement over the collocated hub angle sensor design and demonstrate robustness using endpoint acceleration feedback.

Liang and Balas [93], perform direct MRAC simulations for a single-link flexible robot manipulator based on a reduced-order model, Popov's hyperstability theory to represent large space structures (LSS) and four vibration modes. Angular displacement sensors are located at the hub, near mid-link and at the endpoint. Results show excellent model following, flexibility in choice of adaptive laws and avoidance of difficulties in using Lyapunov's stability function.

Hara, Yamamoto, Omata and Nakano [73], propose a new repetitive control scheme and derive sufficient conditions for its stability by applying a small gain theorem and the stability theorem for time-lag systems. Both state-space and coprime factorization synthesis algorithms are presented. The state-space approach utilizes a Kalman filter with

perfect regulation while the factorization approach achieves a solution to the Hankel norm approximation.

Sison and Chong [133], present a study on no-reset iterative learning control for linear time invariant (LTI), single-input single-output (SISO) plants that may be extended to linearly and periodically time-varying multiple-input multiple-output (MIMO) plants by introducing a finite-learning gain.

Horowitz [75], presents a discussion on the use of function identification and adaptive control algorithms in the formulation of learning controllers for robot manipulators. Similarities and differences between betterment learning schemes, repetitive controllers and adaptive learning schemes based on integral transforms are analyzed and compared. Stability and convergence properties of adaptive learning algorithms based on integral transforms are highlighted with experimental results to illustrate these properties.

Weiss [150], discusses the concept of a repetitive control system, the main stability theorem, a new concept of improved estimates on the error signal and a control system capable of tracking a superposition of several periodic signals with different periods.

Bonarini, [14], conducts fuzzy logic control (FLC) studies on learn behaviours including; techniques to modify the relationship between inputs and outputs through control actions based on a reference model and reinforcement learning algorithms, and evolutionary fuzzy learning (EFL) techniques for populations of fuzzy rules based on genetic algorithms.

Arimoto et al [6], update control laws with a proportional or derivative error term and learning gain. Typically, these adaptive algorithms successively improve performance to achieve asymptotic zero error tracking based on betterment learning laws.

Chen and Meirovitch [26], develop a control scheme for performing a docking manoeuvre of a flexible space robot with a moving target whose motion is unknown *a priori*. The Lagrange equations of motion of the space robot are separated into rigid and flexible dynamics. The rigid dynamics are controlled by a Lyapunov-like methodology. The flexible dynamics are controlled by a modified discrete-time linear quadratic regulator (LQR) feedback method with disturbance compensation performed by collocated piezoelectric actuator/sensor pairs along the flexible links.

Meirovitch and Chen [100], develop a perturbation control method to manoeuvre a flexible space robot ferrying a payload to dock with an orbiting target. The control synthesis includes robot trajectory optimization, rigid dynamics control using inverse dynamics and elastic vibrations controlled by a LQR.

Meirovitch and Lim [101], develop a perturbation control method for a manoeuvring operation of a flexible space robot, in which the problem is separated into a zero-order model problem for open-loop control, and a first-order problem for time-varying closed-loop LQR feedback control.

Saha [120], develops a methodology based on the natural orthogonal complement (NOC) and inverse dynamics for simulating the motion and control of a space robot with one or more rigid-link manipulators.

Yoshida and Umetani [155], [156], investigate the rigid kinematics and dynamics of free-flying multibody systems. They introduce a conservation of momentum law into the development of a new generalized Jacobian matrix (GJM), suitable for use with resolved motion rate and resolved acceleration control laws in free-flying space robot manipulator operations.

Masutani, Miyazaki and Arimoto [97], propose a sensory feedback control method for positioning the endpoint of a nonholonomic rigid dynamics space manipulator at a floating target, in accordance with the momentum conservation law and by utilizing the GJM.

Xu, Shum, Lee and Kanade [151], propose an adaptive control scheme to manipulate a space robot system modeled on rigid Lagrangian dynamics, the linear momentum conservation law and an attitude controlled base, in performing various tasks where unknown or unmodeled dynamics must be considered.

Nenchev, Umetani and Yoshida [108], present a method of analyzing a redundant free-flying spacecraft/manipulator system operating in a zero-gravity environment and applied to three configurations.

Walker and Wee [149], develop an adaptive control method to control a rigid dynamics space-based robot manipulator.

Sasiadek and Duleba [123], obtain simulation results to clarify and validate the purpose and use of thruster jets applied to free-flying space robot manipulators, and their influence on robot dynamics for space operations.

Papadopoulos and Dubowsky [39], [111], [112] and [113], present theoretical derivations and simulation results on free-floating and free-flying coordinated spacecraft/manipulator motion control. The kinematics and dynamics are derived as functions of rigid multibody system barycentric centre vectors. The Jacobian transpose control law is proposed as the primary method for coordinated control of both spacecraft and manipulator. The occurrence of dynamic singularities encountered with nonholomic systems is also investigated.

Chapter 3

Fixed-Base Planar Two-Link Rigid Robot Manipulator

The content of this chapter is based on published papers [50], [51], [52], [53], [55] and [61].

Six different control strategies are investigated by tracking the end effector of a two-link robot manipulator with long slender rigid links each 4.5 m in length. The results provide a basis of comparison for further development of an intelligent control strategy for a flexible robot manipulator in which the dynamics equations are closely representative of a distributed parameter system (DPS). The six strategies include; inverse rigid dynamics control (IRDC), linear quadratic regulator (LQR) control, fuzzy logic control (FLC), linear quadratic Gaussian (LQG) control with a Kalman filter, an extended Kalman filter (EKF) and a fuzzy logic extended Kalman filter (FLAEKF).

The fixed-base planar two-link rigid robot shown in Figure 3.1 has revolute shoulder and elbow joints. The links rotate clockwise in a plane and are oriented *elbow down*. The effects of gravity, friction, end-effector and joint inertias and payloads are neglected. Robot parameters and physical constants are the same as those of the two-link flexible robot manipulator presented in Reference [10].

3.1 Rigid Dynamics

A conventional closed form of the nonlinear dynamics for the planar two-link rigid robot manipulator is derived in terms of kinetic and potential energies stored in the system using the Euler-Lagrange formulation [9], [19], [31], [46], [99], [115], [127], [128], [129], [137], [157]. Given a generalized coordinates vector $\mathbf{q} = [q_1 \ q_2]^T$, total

Figure 3.1: Fixed-base planar two-link rigid robot manipulator.

kinetic and potential energies T_r and U_r stored in the system may be defined by the Lagrangian:

$$\mathcal{L}(q_i, \dot{q}_i) = T_r - U_r, \quad i = 1, 2 \quad (3.1)$$

Dynamic equations of motion for the system are derived by:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i, \quad i = 1, 2 \quad (3.2)$$

Kinetic and potential energies are derived as:

$$T_r = \frac{1}{2} \sum_{k=1}^0 \sum_{p=1}^r M_{r, kp} \dot{q}_i \dot{q}_j, \quad i = 1, 2, j = 1, 2, n = 2, r = 2 \quad (3.3)$$

and

$$U_r = \sum_{i=1}^n m_i \mathbf{g}^T \mathbf{r}_{0, C_i}, \quad n = 2 \quad (3.4)$$

Where; $U_r = 0$ for a space robot in a zero-gravity environment and $M_{r, kp}$ is the k -pth element of the robot inertia matrix $\mathbf{M}_r(\boldsymbol{\theta})$. Substituting Equation (3.3) into Equation (3.2) and by applying the differential operators, the Euler-Lagrange matrix form of the dynamic equations is given by [9]:

$$\boldsymbol{\tau} = \mathbf{M}_r(\boldsymbol{\theta}) \ddot{\boldsymbol{\theta}} + \mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \quad (3.5)$$

Where; slew angles $\theta_1 = q_1$ and $\theta_2 = q_2$ and form the vector $\boldsymbol{\theta}$. $\mathbf{M}_r(\boldsymbol{\theta})$ is the robot inertia matrix and $\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is a dynamic coupling vector comprising centrifugal and Coriolis force components given by [9], [19], [31], [99], [115], [128]:

$$\mathbf{M}_r(\boldsymbol{\theta}) = \begin{bmatrix} \frac{m_1 L^2}{3} + m_2 L^2 \left(\frac{4}{3} + \cos \theta_2 \right) & m_2 L^2 \left(\frac{1}{3} + \frac{1}{2} \cos \theta_2 \right) \\ m_2 L^2 \left(\frac{1}{3} + \frac{1}{2} \cos \theta_2 \right) & \frac{m_2 L^2}{3} \end{bmatrix} \quad (3.6)$$

$$\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = -\left(15.2634 \sin \theta_2\right) \begin{bmatrix} \dot{\theta}_2 (2\dot{\theta}_1 + \dot{\theta}_2) \\ -\dot{\theta}_1^2 \end{bmatrix} \quad (3.7)$$

The parameters of each link are; $L = L_1 = L_2 = 4.5$ m and mass $m_1 = m_2 = 1.5075$ kg.

A detailed derivation of the foregoing dynamics equations and a numerical form of matrix $\mathbf{M}_r(\boldsymbol{\theta})$ and vector $\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ for use in simulation is presented in Appendix A.

3.2 Fixed-Base Planar Two-Link Rigid Robot Manipulator Control Strategies

The strategies used to investigate control of the planar two-link rigid robot manipulator include; inverse rigid dynamics control (IRDC), linear quadratic regulator (LQR), fuzzy logic control (FLC), linear quadratic Gaussian (LQG) with Kalman filter (KF), extended Kalman filter (EKF) and fuzzy logic adaptive extended Kalman filter (FLAEKF) models.

3.2.1 Jacobian Transpose Control Law

An intuitive Jacobian transpose proportional-derivative (JTPD) control law is applied to all control strategies and provides a joint actuating torque vector given by [31], [128], [157]:

$$\boldsymbol{\tau} = \mathbf{J}^T(\boldsymbol{\theta}) \left[\mathbf{K}_p \begin{bmatrix} e_x \\ e_y \end{bmatrix} + \mathbf{K}_d \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} \right] \quad (3.8)$$

The control law vector $\boldsymbol{\tau} = [\tau_1 \ \tau_2]^T$ given by Equation (3.8) actuates robot joint 1 and joint 2. Proportional and derivative gains $\mathbf{K}_p$ and $\mathbf{K}_d$ are calculated on the basis of the dominant open-loop assumed mode frequency calculated for cantilever boundary conditions given by [30], [45], [119], [143], [144]:

$$\mathbf{K}_p = \begin{bmatrix} \omega_{c1}^2 & 0 \\ 0 & \omega_{c1}^2 \end{bmatrix} = \begin{bmatrix} 150.79 & 0 \\ 0 & 150.79 \end{bmatrix} \quad (3.9)$$

and

$$\mathbf{K}_d = \begin{bmatrix} 2\xi\omega_{c_1} & 0 \\ 0 & 2\xi\omega_{c_1} \end{bmatrix} = \begin{bmatrix} 17.364 & 0 \\ 0 & 17.364 \end{bmatrix} \quad (3.10)$$

For which; the dominant open-loop frequency $\omega_{c_1} = 12.28$ Hz.

Pinned-pinned boundary conditions are given by:

$$\mathbf{K}_p = \begin{bmatrix} \omega_{pp_1}^2 & 0 \\ 0 & \omega_{pp_1}^2 \end{bmatrix} = \begin{bmatrix} 1118.4 & 0 \\ 0 & 1118.4 \end{bmatrix} \quad (3.11)$$

and

$$\mathbf{K}_d = \begin{bmatrix} 2\xi\omega_{pp_1} & 0 \\ 0 & 2\xi\omega_{pp_1} \end{bmatrix} = \begin{bmatrix} 48.746 & 0 \\ 0 & 48.746 \end{bmatrix} \quad (3.12)$$

For which; the dominant open-loop frequency $\omega_{pp_1} = 34.474$ Hz.

The assumed modes method is presented in detail in Chapter 4 and Appendix C.

The Jacobian $\mathbf{J}(\boldsymbol{\theta})$ in Equation (3.8) is derived using the forward kinematics of the planar rigid robot manipulator and given by [9], [137]:

$$\mathbf{J}(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \quad (3.13)$$

For which, the end effector Cartesian coordinate positions are given by:

$$x = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) \quad (3.14)$$

$$y = L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) \quad (3.15)$$

3.2.2 Inverse Rigid Dynamics Control (IRDC)

The inverse rigid dynamics control (IRDC) strategy is synthesized as shown in Figure 3.2. Commanded end effector positions x_c and y_c input the control strategy from a

Figure 3.2: Inverse rigid dynamics control strategy.

Matlab/Simulink™ workspace matrix to form position errors used in computation of the control law given by Equation (3.8). After algebraic transposition of the dynamics equations and inversion of the inertia matrix, the joint angular accelerations are double integrated to give a joint rotation (slew angles) vector $[\theta_1 \ \theta_2]^T$. The slew angles transform into Cartesian coordinates in the direct kinematics Equations (3.14) and (3.15) and enter a summing block to form position errors $x_c - x$ and $y_c - y$. Similarly, a joint rates vector $[\dot{\theta}_1 \ \dot{\theta}_2]^T$ transforms into Cartesian velocities using a joint rate transformation algorithm given by [137]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \mathbf{J}(\boldsymbol{\theta}) \dot{\boldsymbol{\theta}} \quad (3.16)$$

These velocities enter a summing block to form velocity errors used in computation of the control law given by Equation (3.8).

Resulting trajectories are shown in Figures 3.3 (a) and (b) and 3.4 (a) and (b). Tracking results shown in Figure 3.3 (a) and (b) depict large transient overshoots at the direction switches for cantilever proportional-derivative (PD) gains but much smaller overshoots for pinned-pinned PD gains. Steady-state tracking along sides of the square is achieved in both cases. Joint torques shown in Figure 3.4 (a) and (b), experience sudden surges (spikes) at direction switches and are larger for pinned-pinned PD gains which demonstrates the cantilever PD gains model is much stiffer and the pinned-pinned PD gains model responds to the direction switch disturbance with larger vibration amplitude.

3.2.3 Optimal Control

Optimal control utilizes the linear quadratic regulator (LQR) strategy shown in Figure 3.5, [20], [21], [28], [29], [40], [44], [72], [76], [80], [92], [96], [124], [125],

Figure 3.3: Inverse rigid dynamics control trajectories (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

Figure 3.4: Inverse rigid dynamics control joint torques (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

[139], [142], [160].

This strategy has a similar feedback structure to the IRDC strategy, but the matrix dynamics equations given by Equations (3.5), (3.6) and (3.7) are linearized and transformed into state-space form given by Equations (3.17) and (3.18). The control law given in Equation (3.8) is operated on by the LQ input gain matrix and actuates each joint modeled within the state-space dynamics equations given by:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (3.17)$$

$$\mathbf{z} = \mathbf{Cx} + \mathbf{Du} \quad (3.18)$$

A detailed derivation of Equations (3.17) and (3.18) is presented in Appendix B.

The state vector $\mathbf{x}$ comprises two joint angles and two angular velocities. The model is configured in an operational space strategy comprising commanded end-effector positions, PD gains and direct kinematics equations to transform slew angles into Cartesian coordinate positions of the end effector. Torque vector $\boldsymbol{\tau}$ inputs to the LQR state-space dynamics equations for robot joint actuation. The state feedback system minimizes a performance index given by [40], [92], [139]:

$$J = \int_0^{\infty} (\mathbf{x}^T \mathbf{Q} \mathbf{x} + 2\mathbf{x}^T \mathbf{N} \mathbf{u} + \mathbf{u}^T \mathbf{R} \mathbf{u}) dt \quad (3.19)$$

The LQ-optimal gain matrix $\mathbf{K}$ is given by:

$$\mathbf{K} = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \quad (3.20)$$

Where; $\mathbf{P}$ is a definite positive solution of the Riccati equation:

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = \mathbf{0} \quad (3.21)$$

$\mathbf{Q}$ and $\mathbf{R}$ are arbitrary weighting matrices and $\mathbf{N}$ is a cross-weighting matrix of the performance index and given by:

Figure 3.5: Linear quadratic regulator control strategy.

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.22)$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.23)$$

$$\mathbf{N} = [\mathbf{0}] \quad (3.24)$$

Tracking results shown in Figure 3.6 (a) and (b) depict larger transient overshoots at the direction switches for cantilever PD gains. But, much smaller overshoots result using pinned-pinned PD gains. Steady-state tracking is achieved along the sides of each square. The results are an improvement over the IRDC strategy. But, the linearized state-space dynamics model avoids the complexity and difficulty in controlling a nonlinear multibody dynamics model.

3.2.4 Intelligent Control

In the context of this thesis, fuzzy logic is considered a constituent of intelligent control and published as an approach to approximate reasoning by Lofti Zadeh [159].

3.2.4.1 Fuzzy Logic Theory

The following theory and definitions pertain specifically to the design of strategies for the control of the planar two-link robot manipulators presented in this thesis. However, an extensive general body of literature on fuzzy logic theory is available. Some excellent titles on fuzzy logic and its application in control theory are listed in the references and bibliography [37], [38], [42], [90], [110], [114], [116], [141], [152], [154], [159], [161].

For a set of elements U denoted by $\{u\}$, U is a classical or 'crisp' set and referred to as

(a)

(b)

Figure 3.6: Linear quadratic regulator control trajectories (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

the universe of discourse in which u is an element of U . Classical sets are subsets of fuzzy sets. A fuzzy set F in a universe of discourse U is defined by a membership function (MF) μ_F in the interval $[0, 1]$, i.e. $\mu_F: U \rightarrow [0, 1]$, [77], [118], [152]:

For which;

$$F = \{(u, \mu_F(u)) | u \in U\} \quad (3.25)$$

For continuous U ;

$$F = \int_U \mu_F(u)/u \quad u \in U \quad (3.26)$$

For discrete U ;

$$F = \sum \mu_F(u_i)/u_i \quad u_i \in U \quad (3.27)$$

or

$$F = \mu_F(u_1)/u_1 + \mu_F(u_2)/u_2 + \mu_F(u_3)/u_3 + \dots + \mu_F(u_N)/u_N \quad (3.28)$$

Symbols ' $\int_U$, $\sum$ and $+$ ' denote fuzzy set union and $'/'$ denotes a connection between an element u and its degree of membership $\mu_F(u)$. These symbols should not be interpreted as mathematical integration, summation, addition and division.

A support set of F is the crisp set of all elements $u \in U$ such that $\mu_F(u) > 0$, and an element $u \in U$ at which $\mu_F(u) > 0.5$ is a crossover point of intersecting MF. When the support set is a single point in U it is called a fuzzy singleton. The crisp set of all $u \in U$ such that $\mu_F(u) \geq \alpha$ is the α -cut set of F . A fuzzy set F is normally given as a set of ordered pairs of elements and grade of membership.

Membership of an element of a fuzzy set may be defined in terms of its membership grade and specified heuristically from expert knowledge of a system. A MF can be any convex shape graphically represented as shown in Figures 3.7 and 3.8 and commonly used in fuzzy control applications.

A typical triangular MF is shown in Figure 3.7 together with its mathematical representation [152], [154].

For which;

$$\begin{aligned}
\mu_F(u: a, b, c) &= 0 && \text{if } u < a \\
\mu_F(u: a, b, c) &= (u-a)/(b-a) && \text{if } a \leq u \leq b \\
\mu_F(u: a, b, c) &= (c-u)/(c-b) && \text{if } b \leq u \leq c \\
\mu_F(u: a, b, c) &= 0 && \text{if } u > c
\end{aligned} \tag{3.29}$$

A typical Gaussian MF is shown in Figure 3.8 together with its mathematical representation with centre c and width (shape) parameter σ .

$$\mu_F(u: c, \sigma) = e^{-\frac{(u-c)^2}{2\sigma^2}} \tag{3.30}$$

3.2.4.2 Fuzzy Inference

In control applications, fuzzy sets may be used to construct a set of rules which form a fuzzy inference system (FIS). These rules take the form IF-THEN, which is a fuzzy relation depicted in Figure 3.9 for two typical rules with input variables A_{ki} and output variable B_i and given by [38], [78], [159], [114], [152], [154]:

IF u_1 is A_{11} AND u_2 is A_{12} THEN v is B_1

IF u_1 is A_{21} AND u_2 is A_{22} THEN v is B_2

For $A_{ki} \in U$, $u_i \in U$, $B_i \in V$ and $v \in U$.

The IF statement forms the antecedent of the k th fuzzy rule. The THEN statement forms the consequent of the k th fuzzy rule.

The initial step in fuzzy inference maps crisp inputs to fuzzy sets within the input variable universe of discourse discretized into several MF. Each MF is assigned a linguistic descriptor, e.g. positive high (PH) and negative low (NL). In Figure 3.9, the crisp values u_1 and u_2 have their fuzzy sets $A_1 \in U_1$, $u_1 \in U_1$, $A_2 \in U_2$, and $u_2 \in U_2$ for the k th rule. The connective AND is used to for point-valued intersection of antecedents

Figure 3.7: Triangular membership function.

Figure 3.8: Gaussian membership function.

denoted by $\cap$ that generates a point-valued fuzzy set of the k th rule given by [152].

$$A_k = \bigcap_{i=1}^n A_{ki}, \quad k = 1, \dots, N \quad (3.31)$$

Where; A_{ki} is the i th antecedent of the k th rule.

A fuzzy implication relation R_k in the form of an IF-THEN rule is either generalized modus ponens (GMP) or generalized modus tollens (GMT). The GMP inference process is forward data-driven and often preferred in control applications. It is used for all fuzzy logic applications in this thesis. For a multiple-input single-output (MISO) system, the antecedent $A_k = A_{k1} \times \dots \times A_{ki} \times \dots \times A_{kn}$ of the k th rule forms a fuzzy set in the product space $U = U_1 \times \dots \times U_i \times \dots \times U_n$, and the consequent is a control variable B_k given by the relation [152]:

$$R_k = A_k \rightarrow B_k \quad (3.32)$$

The Mamdani (MIN) fuzzy implication function is given by:

$$R_C = A \times B = \int_{U \times V} \mu_A(u) \wedge \mu_B(v) / (u, v) \quad (3.33)$$

R_C substitutes for R_k in Equation (3.32) and the resulting truncated MF is a 'fired' rule of strength β_k given by:

$$\beta_k = \mu_{A_{k1}}(u_1) \wedge \mu_{A_{k2}}(u_2) \quad (3.34)$$

Performing compositional operations to give an overall fuzzy relation R for N rules aggregates all rules in a FIS:

$$R = \bigcup_{k=1}^N R_k \quad (3.35)$$

The point-valued compositional rule of inference (PVCRI) has four main types of compositional operators of which the computationally simple and efficient 'sup-min' is used more frequently and given by [152]:

Figure 3.9: Fuzzy inference system.

$$\mu_B = \bigcup_{k=1}^N \left[\bigcup_u \left(\bigwedge_{i=1}^n \mu_{A_i} \right) \wedge \left(\bigwedge_{i=1}^n \mu_{A_{ki}} \wedge \mu_{B_k} \right) \right] \quad (3.36)$$

Used in conjunction with R_C given by Equation (3.33) then Equation (3.36) is given as:

$$\mu_B = \bigcup_{k=1}^N \left[\bigwedge_{i=1}^n \left(\bigcup_u (\mu_{A_i} \wedge \mu_{A_{ki}}) \right) \wedge \mu_{B_k} \right] \quad (3.37)$$

Where; $u = (A_1, \dots, A_i, \dots, A_n)$ is a set of n measured signals input to the FIS [152].

Fuzzy consequents given by Equation (3.37) are then mapped to crisp outputs or control action space. Mapping uses the center-of-area (COA) method to calculate the weighted average of an aggregate MF resulting in a value of μ_B corresponding to a continuous crisp value given by:

$$COA = \frac{\int \mu_B(u) u du}{\int \mu_B(u) du} \quad (3.38)$$

3.2.4.3 Choice of Membership Function

The choice of membership function for each FIS variable often results from an expected input data distribution or clusters. An MF may be any convex shape but triangular, trapezoid and Gaussian MF are often used. However, there are no established criteria for determining the best shape. The choice remains dependent upon experimentation considering all factors of the modeled system [103], [154].

In the following sub-subsections, Gaussian MF are selected and applied in the development of a fuzzy logic control strategy for tracking the end effector of a two-link rigid robot manipulator. Further discussion on the choice of MF type for flexible robot control applications is presented in Chapter 4, Subsection 4.2.4.

3.2.4.4 Number of Fuzzy Rules

The number of rules used in a fuzzy controller is strictly dependent upon the number

of MF assigned to each input variable and a decision faced with contradiction. It is desirable to have as many rules as possible to capture all input data, ensure completeness and have appropriate control resolution for accuracy [103], [154]. Conversely, as the number of rules increases generalization capability decreases while computational time increases.

There is no general procedure for determining an *optimal* number of fuzzy rules but several factors include; controller and system performance, computation time, type of MF, ease of construction and adaptability. In general, fuzzy controllers use fewer rules to achieve completeness due to fuzzy set overlap and soft matching. Several mathematical system identification and optimization techniques exist for fuzzy model identification based on data sets and are interwoven in the whole fuzzy model design process. But, in lieu of mathematical techniques, a fuzzy system can be modeled heuristically with *a priori* knowledge of the system application.

Optimality of a fuzzy logic system and choice of type and number of MF is given further discussion in Chapter 4, Subsection 4.2.4 leading to the determination of a heuristic design ratio. The design of the fuzzy inference systems substituting for the planar two-link rigid robot manipulator dynamics equations includes both intuition and the results of experimentation with different ranges of universe of discourse for each fuzzy variable.

3.2.4.5 Fuzzy Logic Control (FLC)

The fuzzy logic control strategy is shown in Figure 3.10 and the fuzzy logic robot manipulator with two fuzzy inference systems substituting for the robot dynamics equations is shown in Figure 3.11. Each FIS is dedicated to a link. The control law vector

$\tau = [\tau_1 \ \tau_2]^T$ given by Equation (3.8) actuates the robot joints through normalizing gains $K_{N_1} = 200$ and $K_{N_2} = 500$.

Link 1 has a single torque input $T(1)$ and link 2 has acceleration $A(1)$ and torque $T(2)$ inputs as shown in Figure 3.10. Acceleration $A(1)$ output from link 1 is input to link 2 through normalizing gain $K_{N_3} = 2$, to create a coupling effect between the links to capture the combined motion and interaction of both links. The motion of link 1 at the elbow joint significantly affects the motion of link 2 and the end effector position. However, the motion of link 2 does not significantly affect link 1.

Each FIS is a Mamdani type with nine Gaussian membership functions as shown in Figures 3.12 (a) and (b) and 3.13 (a) and (b) for cantilever PD gains. The membership functions have verbal descriptors Z (zero), negative very high (NVH), negative high (NH), negative low (NL), negative very low (NVL), positive very low (PVL), positive low (PL), positive high (PH) and positive very high (PVH). Link 1 has nine fuzzy rules and link 2 has eighty-one fuzzy rules. For cantilever PD gains torque $T(1)$ has a universe of discourse -500 to 500 Nm, torque $T(2)$ and acceleration $A(1-2)$ have universes of discourse -200 to 200 Nm and -2 to 2 rad/sec² respectively. Each link has acceleration outputs $A(1)$ and $A(2)$ universes of discourse of -5 to 5 rad/sec². For pinned-pinned PD gains the universes of discourse are; -5000 to 5000 Nm for torque $T(1)$, -1000 to 1000 Nm for torque $T(2)$, -2 to 2 rad/sec² for coupling acceleration $A(1-2)$ input to link 2 and -5 to 5 rad/sec² for acceleration outputs $A(1)$ and $A(2)$.

Acceleration outputs feed through output scaling gains K_1 and K_2 which are adjusted to modify the membership function base widths to obtain a square trajectory.

Simulations were performed with various combinations of K_1 and K_2 starting at unity

Figure 3.10: Fuzzy logic control strategy.

Figure 3.11: Fuzzy logic robot manipulator.

values for each gain, then incrementally increased until a square trajectory developed. The FLC strategy for cantilever and pinned-pinned PD gains produced initial and final square trajectories with the following ranges and stability limits of K_1 and K_2 given by:

$$\text{For cantilever PD gains:} \quad 8000 \leq K_1 \leq 200000 \quad (3.39)$$

$$2000 \leq K_2 \leq 80000 \quad (3.40)$$

$$\text{For pinned-pinned PD gains:} \quad 50000 \leq K_1 \leq 1000000 \quad (3.41)$$

$$5000 \leq K_2 \leq 100000 \quad (3.42)$$

Table 3.1 is a matrix of fuzzy rules of the form:

$$\text{IF } T(1) \text{ is NL THEN } A(1) \text{ is N} \quad (3.43)$$

$$\text{IF } T(1) \text{ is PH THEN } A(1) \text{ is PH} \quad (3.44)$$

Table 3.2 is a matrix of fuzzy rules of the form:

$$\text{IF } T(2) \text{ is PL and } A(1-2) \text{ is PL THEN } A(2) \text{ is PH} \quad (3.45)$$

$$\text{IF } T(2) \text{ is NVL and } A(1-2) \text{ is PVL THEN } A(2) \text{ is Z} \quad (3.46)$$

Figure 3.14 (a) and (b) and Figure 3.15 (a) and (b) show initial and final tracking results using minimum and maximum values of K_1 and K_2 . Initially, large overshoot amplitudes occur at the direction switches with straight zero-error steady-state tracking along the sides for cantilever PD gains. The final trajectory shows minimal overshoot amplitude at each direction switch.

Similarly, the initial trajectory for pinned-pinned PD gains shows more transient oscillations at the direction switches and a slower decay to a zero-error steady state along the sides of the square. The final trajectory shows greatly diminished overshoot amplitudes at the direction switches.

Table 3.1: Fuzzy rule matrix for link 1

T(1)	NVH	NH	NL	NVL	Z	PVL	PL	PH	PVH
A(1)	NVH	NH	NL	NVL	Z	PVL	PL	PH	PVH

Table 3.2: Fuzzy rule matrix for link 2

		T(2)								
		NVH	NH	NL	NVL	Z	PVL	PL	PH	PVH
A (1-2)	A(2)									
	NVH	NVH	NVH	NH	NH	NH	NL	NL	NVL	Z
	NH	NVH	NH	NH	NH	NL	NL	NVL	Z	PVL
	NL	NH	NH	NH	NL	NL	NVL	Z	PVL	PL
	NVL	NH	NH	NL	NL	NVL	Z	PVL	PL	PL
	Z	NH	NL	NVL	Z	Z	Z	PVL	PL	PH
	PVL	NL	NL	NVL	Z	PVL	PL	PL	PH	PH
	PL	NL	NVL	Z	PVL	PL	PL	PH	PH	PH
	PH	NVL	Z	PVL	PL	PL	PH	PH	PH	PVH
	PVH	Z	PVL	PL	PL	PH	PH	PH	PVH	PVH

3.2.5 Simulation Results Comparison for IRDC, LQR and FLC Strategies

Figures 3.16 (a) and (b) show combined transients occurring at the first direction switch for IRDC, LQR and FLC strategies. The IRDC strategy produces the largest overshoot amplitude with oscillations decreasing to a steady state for both cantilever and pinned-pinned PD gains. The LQR strategy produces lower overshoot amplitude that more rapidly decays to a steady state. The FLC strategy with cantilever PD gains produces even smaller overshoot amplitude, but with pinned-pinned PD gains, it fails to reach the corner point and gradually approaches the commanded steady state position along the side of the square trajectory.

(a)

(b)

Figure 3.12: Fuzzy logic control Gaussian membership functions (a) Link 1 torque input, (b) Link 2 torque input.

(a)

(b)

Figure 3.13: Fuzzy logic control Gaussian membership functions (a) Link 2 acceleration input, (b) Links 1 and 2 acceleration outputs.

Figure 3.14: Fuzzy logic control initial trajectories (a) Cantilever PD gains ($K_1 = 8000$, $K_2 = 2000$), (b) Pinned-pinned PD gains ($K_1 = 50000$, $K_2 = 5000$).

Figure 3.15: Fuzzy logic control final trajectories (a) Cantilever PD gains ($K_1 = 200000$, $K_2 = 80000$), (b) Pinned-pinned PD gains ($K_1 = 1000000$, $K_2 = 100000$).

Figure 3.16: IRDC, LQR and FLC transients compared at the first direction switch (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

Figure 3.17: End effector X-velocities comparison (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

(a) (b)
Figure 3.18: End effector Y-velocities comparison (a) Cantilever PD gains, (b) Pinned-pinned PD gains.

(a) (b)
Figure 3.19: Closed-loop step input transients for IRDC and LQR with cantilever PD gains (a) IRDC, (b) LQR.

Figure 3.20: Closed-loop step input transients for FLC with cantilever PD gains.

(a)

(b)

Figure 3.21: Closed-loop step input transients for IRDC and LQR with pinned-pinned PD gains (a) IRDC, (b) LQR.

Figure 3.22: Closed-loop step input transients for FLC with pinned-pinned PD gains.

End-effector linear velocity histories are presented in Figures 3.17 (a) and (b) and 3.18 (a) and (b) and all three strategies with cantilever and pinned-pinned PD gains are compared. Velocity spikes occurring at direction switches are largest for IRDC, lower for LQR and the lowest for FLC strategies.

Table 3.3: Closed-loop step input performance data for IRDC, LQR and FLC with cantilever and pinned-pinned PD gains

Control Strategy	PD Gains	Position	% Max Overshoot	t_d (msec)	t_r (msec)	t_s (msec)	t_{max} (msec)
IRDC	Cantilever	X	0	55	100	900	190
	Cantilever	Y	5	50	180	600	500
	Pinned-Pinned	X	2.85	20	35	300	50
	Pinned-Pinned	Y	10	10	80	400	130
LQR	Cantilever	X	1.4	50	100	750	200
	Cantilever	Y	6.25	70	225	750	490
	Pinned-Pinned	X	0	15	12	110	115
	Pinned-Pinned	Y	0	15	125	115	105
FLC	Cantilever	X	0	60	100	695	700
	Cantilever	Y	0	125	150	560	565
	Pinned-Pinned	X	2.85	100	55	300	150
	Pinned-Pinned	Y	0	10	90	245	250

The closed-loop performance transients shown in Figures 3.19 (a) and (b), 3.20, 3.21 (a) and (b) and 3.22 are obtained using step inputs ranging from $x = 9$ m to $x = 7$ m and $y = 0$ m to $y = 4$ m. These step input values are used to obtain all closed-loop performance transients throughout the thesis.

Performance characteristics are computed using classical time-domain analysis of control systems and include; percentage maximum overshoot, delay time t_d , rise time t_r , settling time t_s and the time at which the maximum overshoot is reached t_{max} [88].

From the results given in Table 3.3 it is evident that the LQR control strategy with pinned-pinned PD gains demonstrates the best performance. Table 3.4 shows the LQR

control strategy has the fastest stop clock time for tracking the square trajectory.

Table 3.4: Tracking simulation times for IRDC, LQR and FLC control strategies

Control Strategy	IRDC	LQR	FLC	
			Initial	Final
Cantilever PD Gains	65 sec	32 sec	2 min 36 sec	2 min 30 sec
Pinned-Pinned PD Gains	67 sec	39 sec	2 min 35 sec	2 min 26 sec

3.2.6 Stochastic Control (LQG with KF, EKF and FLAEKF)

The linear quadratic Gaussian (LQG) control strategy comprises a low pass filter within a linear state feedback system. The block diagram shown in Figure 3.23 depicts the main components and structure of the LQG control strategy. It includes a LQG dynamic regulator comprising a state estimator that filters output measurement noise (disturbance rejection) and calculates an estimated control law for addition to the reference control law [20], [72], [92], [139].

Three forms of filter are investigated; a linearized Kalman filter (KF), an extended Kalman filter (EKF) and a fuzzy logic adaptive extended Kalman filter (FLAEKF). Utilizing the *separation principle*, the linear state feedback and state estimation systems may be designed independently then connected together in the control loop by a LQG dynamic regulator control law calculated using updated state estimates [92], [139].

The state feedback system comprises linearized state-space dynamics equations for the robot manipulator dynamics and LQ input and output gains as derived in Appendix B. The LQG dynamic regulator comprises the state estimation subroutine equations with LQ output gains from the optimal LQR strategy presented in Subsection 3.2.3.

The model is configured in operational space comprising commanded end-effector positions, cantilever and pinned-pinned assumed mode PD gains, a Jacobian transpose

Figure 3.23: Linear quadratic Gaussian control strategy.

control law and direct kinematics equations given by Equations (3.8) to (3.15).

Slew angles θ_1 and θ_2 are fed back to form position errors for calculation of the torque applied to each joint. An ergodic stream of Gaussian distribution random numbers feeds into the robot dynamics and LQG dynamic regulator to simulate process and measurement noise respectively. Also, the slew angles θ_1 and θ_2 together with measurement noise $\mathbf{v}$ are processed through the LQG dynamic regulator in which state estimates are updated. They are then multiplied by negative LQ state feedback gains to give a supplemental control law vector $\boldsymbol{\tau}_k$ that is positively fed back to sum with the reference control law $\boldsymbol{\tau}_r$, as shown in Figure 3.23.

3.2.6.1 Linear Quadratic Gaussian with Kalman Filter (LQG with KF)

The KF subroutine is shown in Figure 3.24. State and measurement vectors of the LQG control strategy are given by [20], [72], [139]:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Gw} \quad (3.47)$$

$$\mathbf{z} = \mathbf{Cx} + \mathbf{Du} + \mathbf{v} \quad (3.48)$$

From these equations the state estimate is computed in the KF subroutine by integrating the following first-order linear differential equation given by:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{K}_k (\mathbf{z} - \mathbf{C}\hat{\mathbf{x}} - \mathbf{Du}) \quad (3.49)$$

For which the resulting LQG dynamic regulator control law is given by:

$$\boldsymbol{\tau}_k = -\mathbf{K}\hat{\mathbf{x}} \quad (3.50)$$

The LQG dynamic regulator minimizes a quadratic performance index of the form:

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q}_k \mathbf{x} + 2\mathbf{x}^T \mathbf{N}_k \mathbf{u} + \mathbf{u}^T \mathbf{R}_k \mathbf{u}) dt \quad (3.51)$$

in which; $\mathbf{Q}_k$ and $\mathbf{R}_k$ are specified weighting matrices that represent the relationship

Figure 3.24: LQG dynamic regulator with Kalman filter.

between regulation performance and control effort. Matrix $\mathbf{Q}_k$ is calculated as the process noise covariance and $\mathbf{R}_k$ is the measurement noise covariance for which the numerical values are given by:

$$\mathbf{Q}_k = \begin{bmatrix} -0.0001 & -0.0004 & 0.0003 & 0.0011 \\ 0.0004 & -0.0003 & -0.0004 & 0.0091 \\ 0.0006 & 0.0004 & -0.0014 & -0.0004 \\ -0.0004 & 0.0083 & -0.0004 & -0.0243 \end{bmatrix} \quad (3.52)$$

Gaussian random number variances are used in the simulation model to represent process and measurement noise covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$. Variances representing $\mathbf{Q}_k$ in the random number generator are elements selected from its diagonal in Equation (3.52) corresponding to the control law vector elements τ_1 and τ_2 . The variance representing $\mathbf{R}_k$ is arbitrarily chosen as the diagonal matrix of elements given by:

$$\mathbf{R}_k = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix} \quad (3.53)$$

Where; $\mathbf{Q}_k = E[\mathbf{w}_k \mathbf{w}_k^T]$ and $\mathbf{R}_k = E[\mathbf{v}_k \mathbf{v}_k^T]$.

$\mathbf{Q}_k$ is derived explicitly in Appendix B and $\mathbf{R}_k$ chosen arbitrarily for simulation purposes.

Process and measurement noise cross-covariance $\mathbf{N}_k$ is included in the state estimation equations to model the cross-correlated influence of control inputs and observed measurement outputs. $\mathbf{N}_k$ is calculated as the product of standard deviations of $\mathbf{Q}_k$ and $\mathbf{R}_k$ diagonal elements, i.e. $\mathbf{N}_k = E[\mathbf{w}_k \mathbf{v}_k^T] \sim \sigma_w \sigma_v$. $\mathbf{N}_k = 1$ for $\mathbf{w}_k = \mathbf{v}_k = N(0, 1)$.

Using $\mathbf{A}$, $\mathbf{C}$, $\mathbf{G}$, $\mathbf{Q}_k$ and $\mathbf{R}_k$, we obtain the Kalman gain matrix given by [2], [5], [6]:

$$\mathbf{K}_k = [\mathbf{P}_k \mathbf{C}^T \mathbf{R}_k^{-1}] \quad (3.54)$$

An explicit derivation is presented in Appendix B.

(a)

(b)

Figure 3.25: Linear quadratic Gaussian with Kalman filter and cantilever PD gains control trajectories for (a) $w_k = N(0,1)$, $v_k = N(0,1)$, (b) $w_k = N(0,Q_k)$, $v_k = N(0,R_k)$.

(a)

(b)

Figure 3.26: Linear quadratic Gaussian with Kalman filter and cantilever PD gains control trajectories for (a) $w_k = N(0,2Q_k)$, $v_k = N(0,R_k)$, (b) $w_k = N(0,Q_k)$, $v_k = N(0,2R_k)$.

(a)

(b)

Figure 3.27: Linear quadratic Gaussian with Kalman filter and cantilever PD gains control trajectories for (a) $w_k = N(0, Q_k)$, $v_k = N(25, R_k)$, (b) $w_k = N(25, Q_k)$, $v_k = N(25, 2R_k)$.

Tracking results are shown in Figures 3.25 (a) and (b), 3.26 (a) and (b) and 3.27 (a) and (b) for three sets of different process and measurement noise combinations with means of 0, 5 and 25 and covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$ multiplied by 2. In this way, the different means and covariance demonstrate the effect of different biases and variances in the noise stream distributions.

The results shown in Figures 3.25 (a) and (b) and 3.26 (a) and (b) depict no discernable difference for variation in process and measurement noise covariance and are very similar to those obtained using the LQR control strategy. However, when the mean of distribution values are increased from 0 to 25 and measurement noise covariance is doubled there are noticeable changes in overshoot magnitude as depicted in Figures 3.27 (a) and (b). The second direction switch overshoot in Figure 3.27 (a) is slightly larger for $\mathbf{v}_k = \mathcal{N}(25, \mathbf{R}_k)$, whereas the effect of both $\mathbf{w}_k = \mathcal{N}(25, \mathbf{Q}_k)$ and $\mathbf{v}_k = \mathcal{N}(25, 2\mathbf{R}_k)$ is to cause the first and third direction switch overshoot amplitudes to remain the same while the second direction switch overshoot amplitude is decreased as shown in Figure 3.27 (b). Otherwise, the KF is able to maintain stable tracking regardless of any variation in process and measurement noise.

3.2.6.2 Linear Quadratic Gaussian with Extended Kalman Filter (LQG with EKF)

While the LQG with KF is a fairly accurate and stable strategy for tracking the robot manipulator, the errors generated are always in reference to a predetermined nominal trajectory. But, for operations within a space environment, a nominal trajectory may be unreliable or redundant when the planned path deviates instantaneously or an alternative path must be tracked. For these unplanned trajectories, a more suitable strategy is the LQG with extended Kalman filter (EKF) [20], [139], [160].

An EKF is considered more suitable than a KF because linearization of the nonlinear dynamics equations occurs about a trajectory created by filter state estimate updates rather than a predetermined nominal trajectory. Interdependency between state estimates, measurement sequence and filter gain sequence is considered to be *on the average* better than a KF. However, an inherent anomaly of an EKF is its potentially high risk of divergence in unusual situations where the updated estimated trajectory has greater errors than an updated nominal trajectory. The EKF recursive state estimation subroutine is shown in Figure 3.28.

The linearized state-space equations of the rigid robot dynamics model are given by [20], [139], [160]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w} \quad (3.55)$$

$$\mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{v} \quad (3.56)$$

For these equations $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{G}$ are dynamic coefficient, control input coupling, process noise coupling, measurement sensitivity and output coupling matrices respectively. State, state derivative, control, measurement, process and measurement noise vectors are given by $\mathbf{x}$, $\dot{\mathbf{x}}$, $\mathbf{u}$, $\mathbf{z}$, $\mathbf{w}$, $\mathbf{v}$ respectively. Equations (3.55) and (3.56) are transformed to the discrete form given by [20]:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{\Gamma} \mathbf{u}_k + \mathbf{\Lambda} \mathbf{w}_k \quad (3.57)$$

$$\mathbf{z}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{v}_k \quad (3.58)$$

For time increments $k \geq 0$ and $\mathbf{D} = [\mathbf{0}]$.

Matrices $\mathbf{\Phi}_k$, $\mathbf{\Gamma}$, $\mathbf{\Lambda}$ and $\mathbf{C}_k$ are time-varying and $\mathbf{u}_k$ is a deterministic input vector; $\mathbf{w}_k \sim N(\mu, \mathbf{Q}_k) = N(0, 1)$ and $\mathbf{v}_k \sim N(\mu, \mathbf{R}_k) = N(0, 1)$ are zero-mean uncorrelated Gaussian process and measurement noise matrices [20], [139], [160]. EKF state estimate, error

Figure 3.28: LQG dynamic regulator with extended Kalman filter.

covariance and Kalman gain equations are derived from linearized discrete state-space equations given by Equations (3.57) and (3.58). The Kalman gain for the EKF with correlated process and measurement noise is given by [20]:

$$\mathbf{K}_k = (\mathbf{P}_k^- \mathbf{C}_k^T + \mathbf{N}_k) (\mathbf{C}_k \mathbf{P}_k^- \mathbf{C}_k^T + \mathbf{R}_k + \mathbf{C}_k \mathbf{N}_k + \mathbf{N}_k^T \mathbf{C}_k^T)^{-1} \quad (3.59)$$

The *a posteriori* state estimates are given in terms of *total* measurements on a corrected trajectory rather than *incremental* measurements on a nominal trajectory typical of a Kalman filter and given by:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k^-) \quad (3.60)$$

Where; $\hat{\mathbf{z}}_k^-$ and $\mathbf{z}_k - \hat{\mathbf{z}}_k^-$ are total predictive measurement and measurement residual vectors.

The *a posteriori* error covariance matrix is given by:

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{C}_k) \mathbf{P}_k^- - \mathbf{K}_k \mathbf{N}_k^T \quad (3.61)$$

Projected state estimates and error covariance matrix are given by:

$$\hat{\mathbf{x}}_{k+1}^- = \Phi_k \hat{\mathbf{x}}_k \quad (3.62)$$

$$\mathbf{P}_{k+1}^- = \Phi_k \mathbf{P}_k \Phi_k^T + \mathbf{Q}_k \quad (3.63)$$

The *a priori* state estimates and error covariance at time $k = 0$ are $\hat{\mathbf{x}}_0^-$ and $\mathbf{P}_0^-$. $\mathbf{K}_k$, $\mathbf{P}_{k+1}^-$, $\mathbf{P}_k^-$ and $\mathbf{P}_k$, are Kalman gain, projected error covariance, *a priori* error covariance, *a posteriori* error covariance matrices. $\hat{\mathbf{x}}_{k+1}^-$, $\hat{\mathbf{x}}_k^-$ and $\hat{\mathbf{x}}_k$ are projected, *a priori* and *a posteriori* state estimates. Measured $\mathbf{q}$ values output from the state-space dynamics and measurement noise stream $\mathbf{v}_k$ are together input to the state estimator. Process and measurement noise streams with $\mathbf{Q}_k$ and $\mathbf{R}_k$ noise covariance are represented by the

Gaussian distribution variance in the simulation model [20], [139], [160].

3.2.6.3 Linear Quadratic Gaussian with Fuzzy Logic Adaptive Extended Kalman Filter (LQG with FLAEKF)

The fuzzy logic adaptive extended Kalman filter (FLAEKF) state estimation subroutine is shown in Figure 3.29. It is an extension of the EKF subroutine presented in Subsection 3.2.6.2.

The weighted recursive equations for the FLAEKF are derived and presented in previous work where the EKF recursive equations are adapted with parameter γ operating on noise covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$, cross-covariance $\mathbf{N}_k$ and error covariance $\mathbf{P}_k$ [124], [125].

The equations originally derived by Sasiadek and Wang [124], [125], are further derived in this section to include a cross-covariance matrix $\mathbf{N}_k$ for correlation between process and measurement noise [53]. The following equations for process noise covariance, measurement noise covariance and cross-covariance matrices are given by:

$$\mathbf{Q}_\gamma = \mathbf{Q}_k \gamma^{-2(k+1)} \quad (3.64)$$

$$\mathbf{R}_\gamma = \mathbf{R}_k \gamma^{-2(k+1)} \quad (3.65)$$

$$\mathbf{N}_\gamma = \mathbf{N}_k \gamma^{-2(k+1)} \quad (3.66)$$

$$\mathbf{P}_k^{\gamma-} = \mathbf{P}_k^- \gamma^{2k}, \quad \gamma \geq 1 \quad (3.67)$$

Using the Kalman gain Equation (3.59) for an EKF and Equations (3.65), (3.66) and (3.67), the Kalman gain for the FLAEKF is derived and given by [53], [124], [125]:

$$\mathbf{K}_k = (\mathbf{P}_k^{\gamma-} + \mathbf{N}_k) \left(\mathbf{C}_k \mathbf{P}_k^{\gamma-} \mathbf{C}_k^T + \frac{\mathbf{R}_k}{\gamma^2} + \mathbf{C}_k \mathbf{N}_k + \mathbf{N}_k^T \mathbf{C}_k^T \right)^{-1} \quad (3.68)$$

The *a priori* state estimates are given by:

$$\hat{\mathbf{x}}_k^- = \Phi_k \hat{\mathbf{x}}_k \quad (3.69)$$

Figure 3.29: LQG dynamic regulator with fuzzy logic adaptive extended Kalman filter.

And the error covariance matrix is given by:

$$\mathbf{P}_{k+1}^{\gamma-} = \gamma^2 \Phi_k \mathbf{P}_k^{\gamma-} \Phi_k^T + \mathbf{Q}_k \quad (3.70)$$

The *a posteriori* state estimates are given by:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k^-) \quad (3.71)$$

The *a priori* state estimates and error covariance at time $k = 0$ are $\hat{\mathbf{x}}_0^-$ and $\mathbf{P}_0^{\gamma-}$.

The *a posteriori* error covariance matrix is given by:

$$\mathbf{P}_k^{\gamma} = (\mathbf{I} - \mathbf{K}_k \mathbf{C}_k) \mathbf{P}_k^{\gamma-} - \mathbf{K}_k \frac{\mathbf{N}_k^T}{\gamma^2} \quad (3.72)$$

For the initial condition at $k = 0$; $\mathbf{P}_0^{\gamma-} = \mathbf{P}_0$.

Assuming Gaussian statistics and from a *conditional density viewpoint* the mean-square estimation error is given by [20]:

$$\hat{\mathbf{x}}_k = E(\mathbf{x}_k | \mathbf{z}_k^*) \quad (3.73)$$

Where $\mathbf{z}_k^* = z_1, z_2, \dots, z_n$ is a stream of random measurements.

The mean state estimates are given by:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{M}_k (\mathbf{z}_k - \hat{\mathbf{z}}_k^-) \quad (3.74)$$

Where;

$$\mathbf{M}_k = \mathbf{P}_k^- \mathbf{C}_k^T (\mathbf{C}_k \mathbf{P}_k^- \mathbf{C}_k^T + \mathbf{R}_k)^{-1} \quad (3.75)$$

And the error covariance of the residuals is given by:

$$\mathbf{P}_z = \mathbf{C}_k \mathbf{R}_k^{-1} \mathbf{C}_k^T + \mathbf{R} \quad (3.76)$$

As k increases, $\mathbf{Q}_k$, $\mathbf{R}_k$ and $\mathbf{N}_k$ decrease so the most recent measurement is weighted higher. The FLAEKF adapts the value of γ according to the magnitude of $\mathbf{P}_z$ and non-zero means $\hat{\mathbf{x}}$ such that optimality and a zero-mean white noise condition is

(a)

(b)

(c)

Figure 3.30: FLAEKF FIS membership functions (a) $\hat{x}_k$, (b) P_z , (c) γ .

maintained. The FLAEKF behaves as an EKF when $\gamma = 1$. Process and measurement noise streams with $\mathbf{Q}_k$ and $\mathbf{R}_k$ noise covariance are represented by the Gaussian distribution variance in the simulation model [20], [139], [160].

Fuzzy variables $\hat{\mathbf{x}}_k$, $\mathbf{P}_z$, and γ forming the FIS each have four triangular MF and all universes of discourse fall into the positive right half-plane to satisfy the condition $\gamma \geq 1$. Input variables are mean state estimates and error covariance of the residuals given by Equations (3.74) and (3.76). Verbal descriptors are used to identify membership functions for zero (ZERO), small (S), medium (M) and large (L) shown in Figure 3.30 that generate fuzzy rules of the form:

$$\text{IF } \mathbf{P}_z \text{ is S AND } \hat{\mathbf{x}}_k \text{ is S THEN } \gamma \text{ is M} \quad (3.77)$$

The fuzzy rule matrix is given in Table 3.5.

Table 3.5: FLAEKF FIS fuzzy rule matrix

		$\hat{\mathbf{x}}_k$			
		Z	S	M	L
$\mathbf{P}_z$	γ				
	Z	Z	S	M	L
	S	S	M	L	L
	M	M	L	L	L
	L	L	L	L	L

For either an EKF or FLAEKF subroutine, the LQG dynamic regulator control law is given by the following equation:

$$\boldsymbol{\tau}_k = -\mathbf{K}\hat{\mathbf{x}}_k \quad (3.78)$$

Where $\mathbf{K}$ is a gain matrix determined by the algebraic Riccati equation (ARE) in the derivation of a LQR based on state-space rigid robot dynamics equations.

The LQG dynamic regulator control law is fed back to form the control law given by:

$$\tau = \tau_r + \tau_k \quad (3.79)$$

or

$$\tau = \mathbf{J}^T(\theta) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] - \mathbf{K} \hat{\mathbf{x}}_k \quad (3.80)$$

3.2.6.4 Simulation Results for LQG with EKF and FLAEKF

Simulation results are obtained to ascertain the extent of divergence that may be experienced with an LQG with EKF control strategy and how effectively a FLAEKF could reduce or eliminate this divergence.

Tracking results are shown in Figures 3.31 (a) and (b), 3.32 (a) and (b), 3.33 (a) and (b), 3.34 (a) and (b), 3.35 (a) and (b), 3.36 (a) and (b) and 3.37 (a) and (b) for combinations of process and measurement noise $\mathbf{w}_k = \mathbf{N}(0,1)$, $\mathbf{v}_k = \mathbf{N}(0,1)$; $\mathbf{w}_k = \mathbf{N}(5, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$; $\mathbf{w}_k = \mathbf{N}(5, 2\mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$; $\mathbf{w}_k = \mathbf{N}(5, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, 2\mathbf{R}_k)$; $\mathbf{w}_k = \mathbf{N}(5, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(25, \mathbf{R}_k)$; $\mathbf{w}_k = \mathbf{N}(25, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$ and $\mathbf{w}_k = \mathbf{N}(25, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(25, 2\mathbf{R}_k)$ respectively.

Figure 3.31 (a) and (b) shows the trajectories for $\mathbf{w}_k = \mathbf{N}(0,1)$, $\mathbf{v}_k = \mathbf{N}(0,1)$, i.e. zero-mean white noise. A slight reduction in transient overshoot amplitude is achieved at the third direction switch for the FLAEKF, whereas they are essentially the same at the other direction switches and tracking is stable for the trajectories overall.

Divergence is not evident in Figure 3.31 (a). The results shown in Figure 3.32 (a) and (b) for $\mathbf{w}_k = \mathbf{N}(5, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$; Figure 3.33 (a) and (b) for $\mathbf{w}_k = \mathbf{N}(5, 2\mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$; Figure 3.34 (a) and (b) for $\mathbf{w}_k = \mathbf{N}(5, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, 2\mathbf{R}_k)$ and Figure 3.36 (a) and (b) for $\mathbf{w}_k = \mathbf{N}(25, \mathbf{Q}_k)$, $\mathbf{v}_k = \mathbf{N}(5, \mathbf{R}_k)$ are similar.

However, noticeable deviations also occur in Figure 3.35 (a) for LQG with EKF at the first and third direction switches where the transient overshoot amplitudes are considerably enlarged. Also, a distinct divergence occurs along the third side of the square and at the third direction switch shown in Figure 3.37 (a).

These deviations from the commanded trajectory are effectively eliminated by the LQG with FLAEKF control strategy to a result closely similar to that obtained for zero-mean white noise in Figure 3.31 (b).

The closed-loop step input transients shown in Figures 3.38 (a) and (b) and 3.39 are representative of results for LQG with EKF and FLAEKF control strategies with process and measurement noise given by $\mathbf{w}_k = \mathbf{N}(0, \mathbf{Q}_k)$ and $\mathbf{v}_k = \mathbf{N}(0, 2\mathbf{R}_k)$.

This study verifies the capability of the LQG with FLAEKF control strategy to eliminate divergence errors caused by process and measurement noise disturbances with different distribution means and covariance encountered within the EKF when applied to the tracking control of a two-link rigid robot manipulator.

Results show higher values of the mean or multiple values of $\mathbf{R}_k$ have the greatest influence on generating divergence in an EKF. The FLAEKF proves to be an effective technique for precision tracking regardless of any divergence encountered. It performs consistently according to zero-mean white noise disturbance theory on which the extended Kalman filter is based. Closed-loop step input performance data for all stochastic control strategies is summarized in Table 3.6 and corresponding tracking simulation times are summarized in Table 3.7.

(a)

(b)

Figure 3.31: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(0,1)$, $v_k = N(0,1)$ (a) LQG with EKF, (b) LQG with FLAEKF.

(a)

(b)

Figure 3.32: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(5, Q_k)$, $v_k = N(5, R_k)$ (a) LQG with EKF, (b) LQG with FLAEKF.

(a)

(b)

Figure 3.33: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(5, 2Q_k)$, $v_k = N(5, R_k)$ (a) LQG with EKF, (b) LQG with FLAEKF.

(a)

(b)

Figure 3.34: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(5, Q_k)$, $v_k = N(5, 2R_k)$ (a) LQG with EKF, (b) LQG with FLAEKF.

Figure 3.35: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(5, Q_k)$, $v_k = N(25, R_k)$ (a) LQG with EKF, (b) LQG with FLAEKF.

Figure 3.36: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(25, Q_k)$, $v_k = N(5, R_k)$ (a) LQG with EKF, (c) LQG with FLAEKF.

Figure 3.37: Linear quadratic Gaussian with EKF, FLAEKF and cantilever PD gains control trajectories for $w_k = N(25, Q_k)$, $v_k = N(25, 2R_k)$ (a) LQG with EKF, (b) LQG with FLAEKF.

Figure 3.38: Closed-loop step input transients for LQG with KF and EKF control with cantilever PD gains, process and measurement noise $w_k = N(0, Q_k)$, $v_k = N(0, 2R_k)$ (a) LQG with KF, (b) LQG with EKF.

Figure 3.39: Closed-loop step input transients for LQG with FLAEKF control with cantilever PD gains, process and measurement noise $w_k = N(0, Q_k)$, $v_k = N(0, 2R_k)$.

Table 3.6: Closed-loop step input performance data for LQG with KF, EKF, FLAEKF and cantilever PD gains.

Control Strategy	Position	% Max Overshoot	t_d (msec)	t_r (msec)	t_s (msec)	t_{max} (msec)
LQG with KF	X	0	80	100	650	150
	Y	6.25	80	180	650	460
LQG with EKF	X	2.14	60	100	700	200
	Y	6.25	50	200	700	500
LQG with FLAEKF	X	2.85	80	90	700	200
	Y	6.25	60	220	700	500

Table 3.7: Tracking simulation times for stochastic control strategies.

Control Strategy	LQG with KF	LQG with EKF	LQG with FLAEKF
$w_k = N(0,1), v_k = N(0,1)$	36 sec	41 sec	41 sec
$w_k = N(0, Q_k), v_k = N(0, R_k)$	36 sec		
$w_k = N(0, 2Q_k), v_k = N(0, R_k)$	40 sec		
$w_k = N(0, Q_k), v_k = N(0, 2R_k)$	46 sec		
$w_k = N(5, Q_k), v_k = N(5, R_k)$		43 sec	47 sec
$w_k = N(5, 2Q_k), v_k = N(5, R_k)$		48 sec	46 sec
$w_k = N(5, Q_k), v_k = N(5, 2R_k)$		43 sec	46 sec
$w_k = N(0, Q_k), v_k = N(25, R_k)$	40 sec		
$w_k = N(5, Q_k), v_k = N(25, R_k)$		44 sec	49 sec
$w_k = N(25, Q_k), v_k = N(5, R_k)$		44 sec	48 sec
$w_k = N(25, Q_k), v_k = N(25, 2R_k)$	46 sec	44 sec	49 sec

3.3 Stability Analysis for Rigid Robot Dynamics Models

3.3.1 IRDC Model

Consider the flexible robot manipulator model of Equation (3.5) again:

$$\tau = \mathbf{M}_r(\theta)\ddot{\theta} + \mathbf{C}_r(\theta, \dot{\theta}) \quad (3.81)$$

The proposed controller given by Equation (3.8) is again:

$$\tau = \mathbf{J}^T(\theta) \left[\mathbf{K}_p \begin{bmatrix} e_x \\ e_y \end{bmatrix} + \mathbf{K}_d \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} \right] \quad (3.82)$$

Theorem 3.1: *The closed-loop system for a two-link rigid robot manipulator given by Equation (3.81), the Jacobian transpose control law given by Equation (3.82) and the control strategy shown in Figure 3.2 is asymptotically stable.*

For the rigid dynamics, robot equations given by Equation (3.81) it is desired to determine if the control structure is such that the operational space error $\mathbf{e} = \begin{bmatrix} x_c \\ y_c \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix}$

tends asymptotically to zero for the control law given by Equation (3.82).

Consider the positive definite quadratic Lyapunov function candidate given by [128], [134]:

$$V(\dot{\theta}, \mathbf{e}) = \frac{1}{2} \dot{\theta}^T \mathbf{M}_r(\theta) \dot{\theta} + \frac{1}{2} \mathbf{e}^T \mathbf{K}_p \mathbf{e} > 0 \quad \forall \dot{\theta}, \mathbf{e} \neq 0 \quad (3.83)$$

Where $\mathbf{M}_r(\theta)$ and $\mathbf{K}_p$ are positive definite matrices.

Differentiating Equation (3.83) we get:

$$\dot{V}(\dot{\theta}, \mathbf{e}) = \dot{\theta}^T \mathbf{M}_r(\theta) \ddot{\theta} + \frac{1}{2} \dot{\theta}^T \dot{\mathbf{M}}_r(\theta) \dot{\theta} + \dot{\mathbf{e}}^T \mathbf{K}_p \mathbf{e} \quad (3.84)$$

Since $\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \end{bmatrix} = 0$ then $\dot{\mathbf{e}} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} = -\mathbf{J}(\theta) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = -\mathbf{J}(\theta) \dot{\theta}$ according to Equation (3.16) and

substituting into Equation (3.84) we get:

$$\dot{V}(\dot{\theta}, e) = \dot{\theta}^T M_r(\theta) \ddot{\theta} + \frac{1}{2} \dot{\theta}^T \dot{M}_r(\theta) \dot{\theta} - \dot{\theta}^T J^T(\theta) K_p e \quad (3.85)$$

Introducing Equation (3.81) for the equations of motion and its skew-symmetric matrix property given by:

$$\dot{\theta}^T N_r(\theta, \dot{\theta}) \dot{\theta} = 0 \quad (3.86)$$

$$N_r(\theta, \dot{\theta}) = \dot{M}_r(\theta) - 2C_r(\theta, \dot{\theta}) \quad (3.87)$$

and

$$\dot{\theta}^T M_r(\theta) \ddot{\theta} + \frac{1}{2} \dot{\theta}^T \dot{M}_r(\theta) \dot{\theta} = \frac{1}{2} \dot{\theta}^T \{ \dot{M}_r(\theta) - 2C_r(\theta, \dot{\theta}) \} \dot{\theta} + \dot{\theta}^T \tau$$

We get:

$$\dot{V}(\dot{\theta}, e) = \dot{\theta}^T \{ \tau - J^T(\theta) K_p e \} \quad (3.88)$$

Substituting the control law of Equation (3.82) into Equation (3.88), we get:

$$\dot{V}(\dot{\theta}, e) = \dot{\theta}^T J^T(\theta) \{ [K_p e + K_d \dot{e}] - K_p e \} \quad (3.89)$$

Where; K_d is positive definite.

Then;

$$\dot{V}(\dot{\theta}, e) = \dot{\theta}^T J^T(\theta) K_d \dot{e}, \quad \text{i.e.} \quad \dot{V}(\dot{\theta}, e) \geq 0 \quad (3.90)$$

Since the property of negative definiteness does not hold and Lyapunov's method provides only a sufficient condition, nothing can be inferred on stability from this analysis. However, as a consequence of La Salle's theorem global asymptotic stability is ensured if the only trajectory for which $\dot{V}(\dot{\theta}, e) \equiv 0$ is the equilibrium trajectory on which $e \equiv 0$. Furthermore, since $\dot{V}(\dot{\theta}, e)$ can remain zero only along trajectories where $\ddot{\theta} = \dot{\theta} = 0$ then we get:

$$\dot{\theta}^T J^T(\theta) K_d \dot{e} = 0 \quad (3.91)$$

Since; $\dot{\theta}^T = \dot{\theta} = 0$, $|K_d| \neq 0$ and $J^T(\theta)$ has full rank, then $\dot{e} \equiv 0$ and by La Salle's theorem $e \equiv 0$. Then the robot will not get stuck when $e \neq 0$. Hence, the application of the control law given by Equation (3.82) to the rigid dynamics robot equations given by Equation (3.81) achieves global asymptotic stability. The closed-loop control strategy is stable.

This analytical stability proof is corroborated by all tracking simulation trajectories and closed-loop step input performance results obtained using the IRDC strategy.

3.3.2 State-Space Models

Lyapunov's second (direct) method for stability analysis may be applied to state-space models applied in the LQR and LQG strategies presented in Subsections 3.2.3 and 3.2.6 where the nonlinear dynamics equations are converted to a linear approximation as given in Equations (3.17) and (3.18) [134].

Consider the positive definite quadratic Lyapunov function candidate given by:

$$V(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x} \quad (3.92)$$

and

$$\dot{V}(\mathbf{x}(t)) = \mathbf{x}^T(t) (\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A}) \mathbf{x}(t) \quad (3.93)$$

We require:

$$\dot{V}(\mathbf{x}(t)) = -\mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) \quad (3.94)$$

Where; $\mathbf{P}$ is a symmetric positive definite matrix and $\mathbf{Q}$ is a positive semidefinite matrix.

Using the Lyapunov equation given by [20], [134], [139]:

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} = -\mathbf{Q} \quad (3.95)$$

Matrix $\mathbf{A}$ given by Equation (B.28) in Appendix B is reduced to the following 2 x 2 form:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 4.7179 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -13.3021 & 0 \end{bmatrix} \quad (3.96)$$

$\mathbf{Q}$ is arbitrarily chosen as:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.97)$$

Using Matlab™ the function $\text{lyap}(\mathbf{A}, \mathbf{Q})$ fails to provide a unique solution matrix $\mathbf{P}$ to Equation (3.95). Therefore, no conclusion can be drawn from the Lyapunov function of Equation (3.92).

Hence, no analytical determination can be achieved on the asymptotic or globally asymptotic stability of the linearized dynamic equations of the rigid robot manipulator.

Alternatively; letting matrix $\mathbf{P}$ be diagonal negative definite we have;

$$\mathbf{P} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (3.98)$$

and

$$\mathbf{Q} = -(\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A}) \quad (3.99)$$

Solving for $\mathbf{Q}$ we get:

$$\mathbf{Q} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 4.7179 & 0 \\ 0 & 4.7179 & 0 & -12.3021 \\ 0 & 0 & -12.3021 & 0 \end{bmatrix} \quad (3.100)$$

Which is not negative semidefinite because $|\mathbf{Q}| = 151.3417 > 0$.

Despite an inconclusive analytical stability proof, all tracking simulation trajectories and closed-loop step input performance transient results demonstrate local and global asymptotic stability for linearized dynamics state-space models.

3.3.3 Fuzzy Logic and Fuzzy Logic Adaptive Extended Kalman Control Models

A general framework for the stability analysis of fuzzy logic control systems is yet undeveloped. So far, several methods have been proposed towards constructing a general theory of stability for fuzzy logic control systems. They include; time-domain, hyperstability, bifurcation, frequency-domain (describing function), Lyapunov, interval matrix, linguistic, graphical, parameter plane and rule firing trajectory observation methods [38], [102], [110], [114], [118], [152], [154].

Of these, the Lyapunov direct method has achieved the greatest development for general wide acceptance. However, this method is specifically designed for application to Takagi-Sugeno-Kang type fuzzy inference systems in which the consequent fuzzy membership functions are represented by singletons or first-order polynomials. Its analysis is based upon linear first-order control systems and not well-suited to Mamdani type fuzzy inference systems.

Similarly, the majority of these methods contain many restrictions to universal use and are most often suited to control strategies in which the plant dynamics are linearized into first order state-space form. For example, the case in which the two fuzzy inference systems substitute for the rigid robot dynamical equations in Sub-subsection 3.2.4.5 is an anomalous control strategy for which it is difficult to identify and apply an existing fuzzy stability analysis method.

However, assessing stability primarily relies on the step input closed-loop

performance results pertaining to those control strategies in which fuzzy inference systems were implemented together with visual inspection of the simulated trajectory results.

Chapter 4

Fixed-Base Planar Two-Link Flexible Robot Manipulator

The content of this chapter is based on published papers [56], [58], [59], [60], [61], [62], [63], [66], [67], [68] and [70].

The control of a fixed-base planar two-link robot manipulator with flexible links is investigated in Subsection 4.2.1 using the inverse flexible dynamics control (IFDC) strategy shown in Figure 4.2. Tracking results are compared for both cantilever and pinned-pinned dominant assumed modes flexible dynamics models together with their respective PD gains [7], [32]. Actuation of the robot is achieved using the Jacobian transpose proportional-derivative (JTPD) control law given by Equation (3.8) in Chapter 3.

The inverse flexible dynamics control (IFDC) strategy is then further developed to include intelligent adaptation of the JTPD control law by a fuzzy logic system for vibration suppression of the flexible links [22]. The strategy is further investigated to: assess performance and stability in tracking with a full-order dynamic coupling vector in Subsection 4.2.3 and Sub-subsection 4.2.4.3, determine a heuristic design ratio in Section 4.3, obtain tracking results and assess stability for NMP response time delays and sensors located at different positions along link 2 in Section 4.4, and to obtain tracking results and assess stability for the inclusion of joint hub inertias in Section 4.5.

The two-degree-of-freedom (2-dof) two-link flexible robot manipulator shown in Figure 4.1 has the same parameters and dimensions as the planar two-link rigid robot manipulator described in Chapter 3. But, its links are modeled as a distributed parameter

system (DPS) using an assumed mode deformation function. The dynamics equations are derived using the dominant assumed mode of vibration for an Euler-Bernoulli cantilever beam [8], [15], [16], [23], [24], [25], [34], [35], [36], [45], [81], [105], [119], [126], [129], [143], [144]. Rotation and vibration motions are planar.

In contrast, other researchers have modeled elastic links using lumped parameter, finite element, linearized and discretized beam methods [10], [11], [12], [74], [121], [162].

Task forces, end-effector inertia, payload, gravity and friction effects are neglected. Shoulder and elbow joint inertias are initially excluded in the dynamics equations but subsequently added to simulate their effect.

Tracking control for a robot manipulator with flexible links only is presented in Sections 4.2, 4.3 and 4.4. Tracking control for a robot manipulator with flexible links and joint inertia is presented in Section 4.5.

4.1 Flexible Dynamics

The dynamics equations of the two-link flexible manipulator are derived using the theoretical Euler-Lagrange formulation in which the Lagrangian $\mathcal{L}$ is defined as the difference between kinetic T and potential U energies of the robot system given by [8], [11], [15], [16], [23], [34], [35], [36], [45], [47], [104], [105], [143]:

$$\mathcal{L} = T - U \quad (4.1)$$

The total kinetic energy T is given as the sum of kinetic energies attributable to the rigid hub T_{h_i} , links T_{L_i} and payload T_p given by:

$$T = \sum_{i=1}^n T_{h_i} + \sum_{i=1}^n T_{L_i} + \sum_{i=1}^n T_p, \quad n = 2 \quad (4.2)$$

Figure 4.1: Fixed-base planar two-link flexible robot manipulator.

The kinetic energy at hub i of mass m_{h_i} and mass moment of inertia I_{h_i} is given by:

$$T_{h_i} = \frac{1}{2} m_{h_i} \dot{\mathbf{r}}_i^T \dot{\mathbf{r}}_i + \frac{1}{2} I_{h_i} \dot{\eta}_i^2, \quad i = 1, 2 \quad (4.3)$$

The kinetic energy of link i of linear density ρ_i a distance x_i along link i and is given by:

$$T_{L_i} = \frac{1}{2} \int_0^{L_i} \rho_i(x_i) \dot{\mathbf{p}}_i^T(x_i) \dot{\mathbf{p}}_i(x_i) dx_i, \quad i = 1, 2 \quad (4.4)$$

The kinetic energy associated with the payload of mass m_p and mass moment of inertia I_p located at the end effector denoted by subscript e is given by:

$$T_p = \frac{1}{2} m_p \dot{\mathbf{r}}_e^T \dot{\mathbf{r}}_e + \frac{1}{2} I_p (\dot{\eta}_e + \dot{u}_e')^2 \quad (4.5)$$

Where; $\dot{\eta}_e = \sum_{j=1}^i \dot{\theta}_j + \sum_{k=1}^{i-1} \dot{u}_k'$, $i = 2$, is the absolute angular velocity of frame $X_i Y_i$.

Since payloads are neglected, Equation (4.5) is omitted from the total kinetic energy equation. Also, neglecting gravity the total potential energy is the elastic potential energy embodying the effects of distributed parameter flexibility of the links given by:

$$U = \sum_{i=1}^n U_i = \sum_{i=1}^n \frac{1}{2} \int_0^{L_i} EI_{L_i}(x_i) \left[\frac{d^2 u_i(x_i)}{dx_i^2} \right]^2 dx_i \quad (4.6)$$

Where U_i is the elastic energy stored in link i and EI_{L_i} is its flexural rigidity.

The foregoing equations represent a DPS without discretization of the links and are unsuitable for modeling and simulation purposes. Therefore, a finite-dimensional model of link deformation is obtained by introducing the dominant assumed mode deformation function for link i given by [36], [45], [143], [144]:

$$u(x_i, t) = \sum_{i=1}^n \phi_i(x_i) \delta_i(t), \quad n = 2 \quad (4.7)$$

Because of this assumed mode discretization, the Lagrangian $\mathcal{L}$ becomes a function of generalized coordinates vector $\mathbf{q}$. Applying Equation (4.7) to Equations (4.4) and (4.6) the dynamic equations of motion are obtained by satisfying the Euler-Lagrange equations in differential form given by:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \boldsymbol{\tau} \quad (4.8)$$

The resulting equations of motion in matrix form are given by [36]:

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}\mathbf{q} \quad (4.9)$$

Matrix $\mathbf{M}(\mathbf{q})$ is an inertia matrix comprising rigid and flexible-link elements, $\mathbf{q} = [\theta_1 \ \theta_2 \ \delta_1 \ \delta_2]^T$ is a generalized coordinate vector of joint angles θ_i and link deformations δ_i and $\mathbf{K}$ is a stiffness matrix. Small elastic deflections are an underlying assumption in applying the Euler–Bernoulli beam theory for calculating assumed modes of vibration. Second-order terms of interacting elastic modes can be neglected and because of the orthogonality properties of assumed modes, elastic Coriolis and centrifugal components may be omitted in $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ that simplifies to $\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ for rigid dynamics. Substituting for $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, Equation (4.9) may be expressed as:

$$\begin{bmatrix} \boldsymbol{\tau} \\ \mathbf{0} \end{bmatrix} = \mathbf{M}(\boldsymbol{\theta}, \boldsymbol{\delta}) \begin{bmatrix} \ddot{\boldsymbol{\theta}} \\ \ddot{\boldsymbol{\delta}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\delta} \end{bmatrix} \quad (4.10)$$

Zero-dynamics are the internal system dynamics, in which the inputs and outputs of the system are identically zero and include elastic deformations of the links δ_i . Explicit derivations of Equations (4.9) and (4.10) are presented in [36], [104], [105] and in Appendix C.

Assumed mode frequencies are generally higher than natural frequencies, but previous studies show them as equivalent for a cantilever beam where joint hub inertia resembles

the fixed end and is without mode approximation error [45]. A drawback in their use is the inherent neglect of stiffening as rotational speed increases and leads to the derivation of unreliable equations of motion that imply softening of the links. A corrective measure is to include a geometric stiffness matrix $\mathbf{K}$ as given in Equation (4.9). Alternatively, quadratic mode terms may be included in the derivation of Equations (4.4) and (4.6), thereby eliminating the need for $\mathbf{K}$ [45], [119].

In addition to the nonlinearity of rigid multibody dynamics, flexural deformation of the links, associated vibrations and NMP response compounds the difficulty in achieving accurate tracking control. An effective control strategy should include a dynamics model that embodies the nonlinear flexible dynamics of the robot and a control law that dampens residual vibrations while eliminating NMP response.

4.1.1 Cantilever Assumed Modes

From general transverse beam vibration theory, cantilever mode shapes for the links are given by [143], [144]:

$$\varphi_{c_j}(x_i) = A [\cosh \lambda_{c_j} x_i - \cos \lambda_{c_j} x_i - k_{c_j} (\sinh \lambda_{c_j} x_i - \sin \lambda_{c_j} x_i)] \quad (4.11)$$

Where; $A = 0.1$ is an arbitrary constant chosen for simulation, x_i is a distance along beam i , L_i is the length of beam and;

$$k_{c_j} = \frac{\cos \lambda_{c_j} L_i + \cosh \lambda_{c_j} L_i}{\sin \lambda_{c_j} L_i + \sinh \lambda_{c_j} L_i}, \text{ at } x_i = L_i \quad (4.12)$$

$\lambda_{c_j} L_i = (j - 0.5)\pi$, $j = 1, \dots, n$ are numerically approximated roots of the characteristic

$$\text{equation given by: } \cos(\lambda_{c_j} L_i) + \cosh(\lambda_{c_j} L_i) + 1 = 0 \quad (4.13)$$

Modal frequencies are given by:

$$\omega_{c_j} = (\lambda_{c_j} L_i)^2 \sqrt{\frac{EI_{L_i}}{\rho L_i^4}} \quad (4.14)$$

PD gains for the dominant assumed mode frequency, i.e. $j = 1$, are given by [137], [157]:

$$\mathbf{K}_p = \begin{bmatrix} \omega_{c_1}^2 & 0 \\ 0 & \omega_{c_1}^2 \end{bmatrix} = \begin{bmatrix} 150.79 & 0 \\ 0 & 150.79 \end{bmatrix} \quad (4.15)$$

and

$$\mathbf{K}_d = \begin{bmatrix} 2\xi\omega_{c_1} & 0 \\ 0 & 2\xi\omega_{c_1} \end{bmatrix} = \begin{bmatrix} 17.364 & 0 \\ 0 & 17.364 \end{bmatrix} \quad (4.16)$$

For which; the dominant open-loop frequency $\omega_{c_1} = 12.28$ Hz.

4.1.2 Pinned-Pinned Assumed Modes

Similarly, from transverse beam vibration theory pinned-pinned mode shapes are given by:

$$\varphi_{pp_j} = \sqrt{\frac{2}{\rho L_i}} \sin \lambda_{pp_j} x_i \quad (4.17)$$

For which; $\lambda_{pp_j} L_i = j\pi$ are roots of the characteristic equation $\sin \lambda_{pp_j} L_i = 0$, $j = 1, \dots, n$

at $x_i = L_i$.

Modal frequencies are given by:

$$\omega_{pp_j} = (\lambda_{pp_j} L_i)^2 \sqrt{\frac{EI_{L_i}}{\rho L_i^4}} \quad (4.18)$$

For pinned-pinned assumed modes, the distal end of a link coincides with that of a rigid link with angle θ_i subtended at the joint by the distal end. PD gains for the dominant assumed mode frequency, i.e. $j = 1$, are given by [137], [157]:

$$\mathbf{K}_p = \begin{bmatrix} \omega_{pp_1}^2 & 0 \\ 0 & \omega_{pp_1}^2 \end{bmatrix} = \begin{bmatrix} 1118.4 & 0 \\ 0 & 1118.4 \end{bmatrix} \quad (4.19)$$

and

$$\mathbf{K}_d = \begin{bmatrix} 2\xi\omega_{pp_1} & 0 \\ 0 & 2\xi\omega_{pp_1} \end{bmatrix} = \begin{bmatrix} 48.746 & 0 \\ 0 & 48.746 \end{bmatrix} \quad (4.20)$$

For which; the dominant open-loop frequency $\omega_{pp_1} = 34.474$ Hz.

4.1.3 Choice of Assumed Modes

In general, researchers prefer treating the links as cantilever beams because there is yet, no firm stipulation as to which set of assumed modes should be used. Ideally, the optimum set of assumed modes is that closest to the natural modes of a system. Natural modes depend on several factors within the robot system ensemble including; magnitude of hub inertia and payload mass. For large joint gearing inertia and relatively small payload mass, the link may be considered clamped at the proximal joint end and free at the distal end. Conversely, for smaller joint gearing inertia and larger payload mass both ends of the link may be considered pinned. The ultimate choice requires an assessment based on the actual robot structure and anticipated range of payloads together with its natural modes [45].

In this section, the behaviour of cantilever and pinned-pinned modes is demonstrated for a robot structure with link mass only. Hub inertias and payload mass are excluded from the robot dynamics given by Equation (4.9).

Figure 4.2: Inverse flexible dynamics control strategy.

4.2 Fixed-Base Planar Two-Link Flexible Robot Manipulator Control Strategies

4.2.1 Inverse Flexible Dynamics Control (IFDC)

The control strategy shown in Figure 4.2 is essentially the IRDC strategy shown in Figure 3.2, Chapter 3, except the equations of motion include assumed mode expansions to represent flexibility and embody additional elastic generalized coordinates for link deformations δ_1 and δ_2 . The strategy is referred to as inverse flexible dynamics control (IFDC). Its operation is similar to the IRDC strategy except link deformations δ_1 and δ_2 are fed back to the dynamics equations of motion. The direct kinematics are the same as those given by Equations 3.14 and 3.15, Chapter 3, Subsection 3.2.1, for the rigid dynamics model.

The IFDC strategy includes a robot dynamics model with multibody nonlinearities and flexible mode expansions. This complexity presents a major challenge in achieving precision control without linearization. The goal is to establish a foundation for developing a control strategy that embodies a realistic representation of DPS flexible manipulator dynamics while achieving accurate and stable tracking control. The intuitive JTPD control law provides initial results upon which further development can be made and greater control precision achieved.

4.2.2 Rigid vs. Flexible Robot Dynamics

Flexible dynamics models for both cantilever and pinned-pinned assumed modes are simulated to obtain tracking results for comparison.

Usually, the first three, modes of vibration are significant for control action and determined by assuming robot geometric boundary conditions that are consistent for

configuration changes in its operational workspace [45]. Only the dominant mode is used to demonstrate the difference in complexity between rigid and flexible dynamics models and to ascertain the capability of achieving precision tracking control. The investigation is also intended to ascertain what measures could be introduced to suppress flexible dynamics vibrations, so that a robust strategy may be developed to provide consistent precision steady state tracking when subjected to a variety of operating demands and disturbances.

Initial investigations presented in Chapter 3 produced results for a rigid dynamics model actuated by a JTPD control law with PD gains computed using the dominant mode frequency. In this chapter the same control law is applied to a flexible dynamics model formulated using the dominant assumed mode for cantilever and pinned-pinned flexible Euler-Bernoulli beam boundary conditions [36], [105].

4.2.2.1 Simulation Results for IFDC

Results obtained for both cantilever and pinned-pinned assumed modes flexible dynamics models are shown in Figures 4.3 to 4.7. Figure 4.3 (a) shows large overshoots occurring at the directions switches for the cantilever assumed mode model and is similar to that shown in Figure 3.3 for the rigid dynamics model. For the pinned-pinned assumed mode model, Figure 4.3 (b) shows much smaller overshoots with continued oscillation along each side of the square before converging to a steady state. However, the overshoots are smaller than shown in Figure 3.3 (b) for the rigid dynamics model.

For both cantilever and pinned-pinned assumed mode rigid dynamics models, a steady state is achieved rapidly after each direction switch and maintained until reaching the next direction switch as shown in Figure 4.3 (a) and (b). Even though the overshoot

amplitudes are larger for the cantilever assumed mode model, the pinned-pinned assumed mode model does not embody the same fixed-end restrictions at the proximal link ends and continuous oscillations occur as shown in Figures 4.3 (b) and 4.4 (b).

Tracking torque histories are shown in Figure 3.4 (a) and (b) for the rigid dynamics models and in Figure 4.5 (a) and (b) for the flexible dynamics models. The torque values for both flexible dynamics models are larger than for the rigid dynamics models because of the greater control effort required to suppress elastic vibrations.

Also, for rigid dynamics with pinned-pinned PD gains and for pinned-pinned assumed mode flexible dynamics, greater torque values are required to drive the robot manipulator than are required for rigid dynamics with cantilever PD gains or for cantilever assumed mode flexible dynamics.

For all models, the torque values peak at each direction switch, except for the pinned-pinned flexible dynamics model for which, the peak torques are constant to control oscillations throughout the tracking cycle after the first direction switch.

Link distal end deformation histories for cantilever assumed mode dynamics are shown in Figures 4.6 (a) and (b) for link 1 and link 2. Deformations for link 1 oscillate progressively with increasing magnitude throughout the entire tracking cycle while those for link 2 are initially large and continue to oscillate erratically, but at a considerably smaller magnitude than those for link 1.

In contrast, link distal end deformations for pinned-pinned assumed mode dynamics, shown in Figure 4.7, oscillate with higher frequency, increase in magnitude for both links after the first direction switch and are maintained at the same magnitude until completion of the tracking cycle.

(a) (b)
Figure 4.3: Inverse flexible dynamics control trajectories (a) Cantilever assumed mode dynamics, (b) Pinned-pinned assumed mode dynamics.

(a) (b)
Figure 4.4: Inverse flexible dynamics control transient at the first direction switch (a) Cantilever assumed mode dynamics, (b) Pinned-pinned assumed mode dynamics.

Closed-loop step input transients are presented in Figure 4.8 (a) and (b) for both cantilever and pinned-pinned assumed mode models and the performance characteristics are summarized in Table 4.2 [88].

In general, performance of the pinned-pinned assumed mode model is better than the cantilever assumed mode model. Only one overshoot is measurable for the cantilever model along the x coordinate. The pinned-pinned model exhibits a longer settling time in the x coordinate while delay times, rise times and maximum overshoot times all show better performance over the cantilever model.

These results may be interpreted as the cantilever model being more difficult to control and requiring an effective technique to dampen the vigorous residual vibrations of the links at direction changes and for other disturbances that excite link vibrations [17], [33], [131], [132].

4.2.3 Full-Order Flexible Cantilever Robot Dynamics

The full-order flexible dynamics model includes both rigid and flexible terms in the dynamic coupling vector for the dominant cantilever assumed mode of vibration and is derived in Appendix C. Tracking simulation results are shown in Figures 4.9 (a) and (b), 4.10, 4.11 and 4.12.

The general effect of introducing the full-order flexible dynamic coupling vector on tracking results is to produce lower magnitude oscillations at direction switches and cause them to decay more rapidly, as shown in Figure 4.9 (a) and (b). Input torque to each joint shown in Figure 4.11, is higher throughout the tracking cycle with higher peak values occurring at direction switches to control the added flexibility terms included in the dynamic coupling vector.

(a) (b)
Figure 4.5: Inverse flexible dynamics control torques (a) Cantilever assumed mode dynamics, (b) Pinned-pinned assumed mode dynamics.

(a) (b)
Figure 4.6: Cantilever assumed mode dynamics link deformations (a) Link 1 and link 2 deformation variables δ_1 and δ_2 , (b) Link 2 deformation variable δ_2 (zoomed).

Figure 4.7: Pinned-pinned assumed mode dynamics link 1 and link 2 deformation variables δ_1 and δ_2 .

(a)

(b)

Figure 4.8: Closed-loop step input transients for IFDC (a) Cantilever assumed mode dynamics, (b) Pinned-pinned assumed mode dynamics.

Figure 4.9: IFDC full-order flexible cantilever dynamics tracking results (a) Square trajectory, (b) Transient at the first direction switch.

Figure 4.10: Cantilever assumed mode full-order flexible dynamics link deformations (a) Link 1 and link 2 deformation variables δ_1 and δ_2 , (b) Link 2 deformation variable δ_2 (zoomed).

Figure 4.11: Full-order flexible cantilever dynamics control torques.

Figure 4.12: Closed-loop step input transients for the IFDC full-order flexible dynamics.

Link deformation values are higher in magnitude for both links as shown in Figure 4.10 (a) and (b) than is experienced with the reduced-order dynamics deformation values. Also, deformations for link 1 oscillate predominantly on the positive side of its neutral axis while deformations for link 2 oscillate with a slightly erratic behaviour.

Closed-loop step input transients are presented in Figure 4.12 with the associated performance characteristics summarized in Table 4.2. In general, the performance of the reduced-order cantilever dynamics model is better than for the full-order cantilever dynamics model. Maximum percentage overshoot, delay time, rise time and maximum overshoot time are much higher for the y coordinate. Therefore, the full-order dynamics model presents a greater challenge for control.

4.2.4 Fuzzy Logic System Adaptive Control (FLSAC)

Tracking of the planar two-link flexible robot manipulator is controlled by a fuzzy logic adaptive control (FLSAC) strategy. The strategy comprises the IFDC strategy shown in Figure 4.2, but includes a fuzzy logic system (FLS) with link deformation inputs δ_1 and δ_2 and a single output variable λ . The variable λ is adjusted by an output scaling gain K_s to adapt the JTPD control law given by Equation (3.8) in Chapter 3. This adaptive strategy is atypical in comparison with other adaptive control strategies applied to the control of flexible dynamic systems [13], [27], [83], [109], [130], [131], [132], [135].

4.2.4.1 Fuzzy Logic System (FLS)

Adaptive fuzzy control accommodates variations in system or process dynamics and may be achieved by several methods including; modifying existing rules, adding rules, modifying and/or adjusting MF, adjusting the universes of discourse and varying scaling

Figure 4.13: Fuzzy logic system adaptive control strategy.

Figure 4.14. Fuzzy logic system.

gains or control resolutions.

While it is desirable to achieve adaptive control automatically online, some methods can only be used offline. The adaptive approach in this case, includes offline scaling of the output MF together with online adaptation of the control law by an FLS output variable λ . Tuning the fuzzy output scaling gain K_s evenly redistributes MF centres across the universe of discourse [79], [109], [114].

The control strategy shown in Figure 4.13 includes a multiple-input single-output (MISO) FLS. The FLS shown in Figure 4.14 is a Mamdani type with nine Gaussian MF assigned to each variable. Link distal end deformations δ_1 and δ_2 are output from the robot dynamics equations, normalized by gain $K_n = 5$ then feed into the FLS. Variable λ is output from the FLS with its MF base widths modified by output scaling gain K_s to adapt the JTPD control law given by:

$$\tau = K_s \lambda \left\{ \mathbf{J}^T(\boldsymbol{\theta}) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] \right\} \quad (4.21)$$

The FLS uses antecedent composition (MIN), implication (MIN), aggregation (MAX) and defuzzification (COA). Crisp values of δ_1 and δ_2 fuzzify to produce a degree of membership μ_A within the range $0 \leq \mu_A \leq 1$. The MIN operator determines the minimum value of μ_A causing the implication operator MIN to truncate the output MF. The MAX operator aggregates all truncated output MF resulting in a composite MF. Defuzzification of the aggregate MF is performed by the center-of-area (COA) method and computes a μ_B -value that corresponds to a crisp value for λ given by Equation (3.38) in Chapter 3, Sub-subsection 3.2.4.2.

Scaling gain K_s may be varied to adjust the output variable λ MF base widths for

(a)

(b)

Figure 4.15: Fuzzy logic system Gaussian membership functions (a) Input variables δ_1 and δ_2 , (b) Output variable λ .

improved tracking accuracy. Figure 4.15 (a) and (b) shows the FLS Gaussian MF for elastic link deformation input variables δ_1 and δ_2 , and output variable λ . Universes of discourse range from -5 m to 5 m for δ_1 and δ_2 and from 0 to 1 for λ .

Verbal descriptors for positive maximum (PMAX), positive very very high (PVVH), positive very high (PVH), positive high (PH), positive medium (PM), positive low (PL), positive very low (PVL), positive very very low (PVVL), zero (Z), negative very high (NVH), negative high (NH), negative medium (NM), negative low (NL) and negative very low (NVL) are assigned to MF across the universe of discourse and used to generate fuzzy rules of the form for which a symmetric fuzzy rule matrix for all possible rules is shown in Table 4.1.

$$\text{IF } \delta_1 \text{ is NL AND } \delta_2 \text{ is PL THEN } \lambda \text{ is PM} \quad (4.22)$$

$$\text{IF } \delta_1 \text{ is PVL AND } \delta_2 \text{ is PVH THEN } \lambda \text{ is PH} \quad (4.23)$$

The value of λ , for $\lambda > 0$, is determined by the fuzzy rules according to the resultant values of link deformations δ_1 and δ_2 and ranges from Z for zero deformation to PMAX for the maximum resultant deformation.

The FLS is developed intuitively by visualizing the dynamic behaviour of the links. As the magnitude of elastic deformation for each link varies positively or negatively, it complements or counters deformation of the other link, thereby providing a coupling effect to capture flexible dynamic interaction between the links and determine the net deformation to be controlled.

For example, as link 1 rotates and vibrates its distal end flexes positively and negatively about its neutral axis. The flexing motion excites vibrations in link 2 attached to it. Link 2 also rotates and vibrates so that its distal end flexes positively and negatively

about its neutral axis and is coupled to the vibrations excited by link 1. Deformation at the distal ends are measured and fed into the FLS.

The fuzzy rule matrix is designed such that when δ_1 and δ_2 fall into either PVH or NVH the value of λ is always PMAX, i.e. $\lambda = 1$, thereby adapting the control law to the maximum for each joint actuator. Fuzzy rules with either δ_1 or δ_2 having Z values produce a value of $\lambda < 1$, i.e. PVVL, PVL, PL, PM, PH, PVH or PVVH, to adapt the control law. When δ_1 and δ_2 are both Z the magnitude of control law adaptation is minimal or zero, i.e. λ is Z.

Table 4.1: Fuzzy logic system rule matrix

		δ_2								
λ		NVH	NH	NL	NVL	Z	PVL	PL	PH	PVH
δ_1	NVH	PMAX	PVVH	PVH	PH	PM	PH	PVH	PVVH	PMAX
	NH	PVVH	PVH	PH	PM	PL	PM	PH	PVH	PVVH
	NL	PVH	PH	PM	PL	PVL	PL	PM	PH	PVH
	NVL	PH	PM	PL	PVL	PVVL	PVL	PL	PM	PH
	Z	PM	PL	PVL	PVVL	Z	PVVL	PVL	PL	PM
	PVL	PH	PM	PL	PVL	PVVL	PVL	PL	PM	PH
	PL	PVH	PH	PM	PL	PVL	PL	PM	PH	PVH
	PH	PVVH	PVH	PH	PM	PL	PM	PH	PVH	PVVH
	PVH	PMAX	PVVH	PVH	PH	PM	PH	PVH	PVVH	PMAX

Offline determination of a suitable K_s value necessitate iterative simulation with unit increments of K_s . Initial results for tracking control with $K_s = 1$ were poor, but gradual improvement was achieved for successive iterations until the trajectories shown in Figure 4.16 (a) and (b) were achieved with $K_s = 50$.

4.2.4.2 Simulation Results for FLSAC

Tracking results for cantilever and pinned-pinned dynamics models are shown in Figure 4.16 (a) and (b) and first direction switch transients in Figure 4.17 (a) and (b). Much improvement is achieved for both cantilever and pinned-pinned assumed mode models compared to those obtained with the IFDC strategy. Overshoots are greatly diminished at all direction switches and zero-error steady-state tracking is achieved along each side of the trajectory for both cantilever and pinned-pinned assumed modes dynamics models.

Link deformations and corresponding values of λ are shown in Figures 4.18 (a) and (b) and 4.19 for the cantilever dynamics model, and Figure 4.20 (a) and (b) for the pinned-pinned dynamics model. Deformations for link 1, shown in Figure 4.18 (a) are similar to those for the IFDC strategy shown in Figure 4.6 (a). But, deformations for link 2 shown in Figure 4.18 (b), are cyclically smooth and less erratic than those shown in Figure 4.6 (b), thereby indicating a greater effect in controlling link 2 vibrations by the FLSAC strategy. Figure 4.19 depicts the cyclic variation of the FLS output variable λ that increases in magnitude with each oscillation to a maximum of 0.103 at the end of the tracking cycle.

Deformations for the pinned-pinned model shown in Figure 4.20 (a) are initially low at ± 0.1 m and ± 0.05 m for links 1 and 2 respectively along the first straight side of the trajectory. They increase sharply to ± 0.25 m and ± 0.18 m for links 1 and 2 respectively after the first direction switch then again increase sharply to ± 0.35 m and ± 0.3 m after the second direction switch. Then, a final increase occurs after the third direction switch to ± 0.4 m and ± 0.35 m for links 1 and 2 respectively along the fourth

(a)

(b)

Figure 4.16: FLSAC trajectories (a) Cantilever dynamics, (b) Pinned-pinned dynamics.

(a)

(b)

Figure 4.17: FLSAC transients at the first direction switch (a) Cantilever dynamics, (b) Pinned-pinned dynamics.

Figure 4.18: FLSAC cantilever dynamics input deformation variables (a) Link 1 and link 2 deformation variables δ_1 and δ_2 , (b) Link 2 deformation variable δ_2 (zoomed).

Figure 4.19: FLSAC output variable λ .

straight side of the trajectory.

Corresponding average peak cyclic values of λ shown in Figure 4.20 (b), are relatively low at 0.0825 along the first side of the trajectory. They increase to an average peak of 0.0865 after the first direction switch and continue at this magnitude along the second straight side of the trajectory. Further increases attain a peak average of 0.089 after the second direction switch and along the third straight side of the trajectory. Finally, even further increases occur after the third direction switch to peak at an average of 0.09 along the final straight side of the trajectory.

This pattern of incremental values of λ at each direction switch is interpreted as the need for greater control effort to dampen increasing residual vibrations of the links. The same is also true for the cantilever dynamics model but the frequency is lower and exhibits progressively higher amplitude oscillations throughout the tracking cycle. Control torques shown in Figure 4.21 (a) and (b); demonstrate an average magnitude for both links but those for the cantilever dynamics model are much lower than those for the pinned-pinned dynamics model. The cantilever dynamics model experiences torque spikes of 500 Nm at the direction switches, whereas the average torque required to drive both links of the pinned-pinned dynamics model is much higher at 1000 Nm then gradually increases throughout the tracking cycle to a maximum of 1500 Nm.

Closed-loop step input transients are presented in Figure 4.22 (a) and (b) with the associated performance characteristics summarized in Table 4.2 [88]. The performance of the pinned-pinned dynamics model is generally better than for the cantilever dynamics model. Maximum percentage overshoots, delay times, rise times and maximum overshoot times are all higher for both cantilever x and y coordinates.

4.2.4.3 FLSAC with Full-Order Flexible Cantilever Dynamics

Tracking results for the full-order cantilever dynamics model are shown in Figures 4.23 (a) and (b), 4.24 (a) and (b), 4.25, 4.26 and 4.27. Because of the prohibitively high computational time burden experienced when using nine Gaussian MF at $K_s = 50$, an optimal FLS with three triangular MF at $K_s = 15$, developed and presented in Section 4.3, is used to simulate tracking control for the full-order flexible cantilever dynamics model.

Despite the added mathematical complexity of the full-order flexible cantilever dynamics model, the trajectory shown in Figure 4.23 depicts a slight improvement over the reduced-order model shown in Figure 4.16 (a). Maximum overshoot amplitude at the direction switches are lower and a steady state is achieved more rapidly. This anomaly is attributed to the influence of the stiffness matrix $\mathbf{K}$ in the flexible dynamics equations.

Link deformations are shown in Figure 4.24 (a) and (b). The deformation oscillations are shifted on the positive side of the neutral axis of the link. Deformations for link 2 shown in Figure 4.24 (b) depict a significantly increasing trend in magnitude with each oscillation, whereas in comparison, the magnitude of oscillations depict a slightly decreasing trend with each oscillation for the reduced-order dynamics model as shown in Figure 4.18 (b).

Figure 4.25 depicts the cyclic variation of the FLS output variable λ that dramatically changes as the end effector tracks the trajectory. Initially, the value of λ increases erratically along the first side of the square trajectory then suddenly drops to 0.08 after the first direction switch. Dramatic increases occur up to a peak of 0.135 at subsequent direction switches then dramatically decrease to 0.08 along the straight sides of the square trajectory.

(a)

(b)

Figure 4.20: FLSAC pinned-pinned dynamics link deformation input and output variables (a) Input variables δ_1 and δ_2 , (b) Output variable λ .

(a)

(b)

Figure 4.21: FLSAC torques (a) Cantilever dynamics, (b) Pinned-pinned dynamics.

The maximum value of λ is 0.135 for the full-order dynamics model compared to 0.103 at the end of the tracking cycle for the reduced-order dynamics model. Hence, greater demands are placed on the FLS to adapt the control law and dampen residual vibrations at each direction switch for the full-order dynamics model.

Control torque histories are presented in Figure 4.26 and depict larger torque spikes occurring at the direction switches than those for the reduced-order dynamics model.

Closed-loop step input transients are presented in Figure 4.27 with the associated performance characteristics summarized in Table 4.2. The performance of the full-order dynamics model is an improvement over the reduced-order dynamics model. The maximum percentage overshoot is also zero but delay times, rise times and maximum overshoot times are all considerably lower for both x and y coordinates.

Greatly improved tracking results are achieved using a FLSAC strategy that overcomes the complexities of flexible dynamics and inadequacy of the IFDC strategy. The FLS produces an output variable λ to adapt the JTPD control law in response to elastic deformations of the links, so that tracking errors are reduced to a zero-error steady state. The FLSAC is effective for both cantilever and pinned-pinned models but more significantly for the cantilever model as it imposes a greater challenge for the control strategy. However, the results are obtained at a high computational time burden for both dynamics models.

The results apply only to a 2-dof system in this thesis. However, the strategy is easy to design and construct, it is effective in performance and may be extended to a broader range of applications and higher degree-of-freedom systems. The FLSAC strategy is effective for both reduced- and full-order dynamics models and greatly improves control

(a) (b)
Figure 4.22: Closed-loop step input transients for FLSAC (a) Cantilever assumed mode dynamics, (b) Pinned-pinned assumed mode dynamics.

(a) (b)
Figure 4.23: FLSAC full-order flexible cantilever dynamics tracking results (a) Control trajectory, (b) Transient at the first direction switch.

(a)

(b)

Figure 4.24: FLSAC full-order flexible cantilever dynamics link deformation input variables (a) Link 1 and link 2 deformation input variables δ_1 and δ_2 , (b) Link 2 input variable δ_2 (zoomed).

Figure 4.25: FLSAC full-order flexible cantilever dynamics output variable λ .

Figure 4.26: FLSAC full-order flexible cantilever dynamics control torques.

Figure 4.27: Closed-loop step input transients for FLSAC with full-order flexible cantilever dynamics model.

performance over the IFDC strategy described in Subsection 4.2.2.

Table 4.2: Closed-loop step input performance data for IFDC and FLSAC with cantilever and pinned-pinned assumed mode dynamics

Control Strategy	Assumed Mode	Position	% Max Overshoot	t_d (msec)	t_r (msec)	t_s (msec)	t_{max} (msec)
IFDC	Cantilever	X	3.75	100	200	975	500
	Cantilever	Y	0	100	200	615	250
	Pinned-Pinned	X	0	50	90	1200	430
	Pinned-Pinned	Y	0	70	100	400	160
	Full Dynamics	X	8.57	100	150	900	200
	Full Dynamics	Y	6.25	300	380	1400	625
FLSAC	Cantilever	X	0	100	200	630	630
	Cantilever	Y	0	80	250	600	600
	Pinned-Pinned	X	0	50	100	300	300
	Pinned-Pinned	Y	0	50	125	300	300
	Full Dynamics	X	0	100	125	450	450
	Full Dynamics	Y	0	100	200	575	575

Table 4.3: Tracking simulation times

Control Strategy	Reduced-Order Dynamics		Full-Order Dynamics
	Cantilever	Pinned-Pinned	Cantilever
IFDC	1 min 17 sec	1 min 46 sec	1 min 12 sec
FLSAC	3 min 45 sec	5 min 05 sec	12 min 32 sec

4.3 Heuristic Design Ratio for an Optimal Fuzzy Logic System

Minimizing flexural vibration while operating a flexible robot manipulator within strict time constraints requires a controller with high tracking precision and fast execution time.

Simulated tracking results are compared for different FLS designs to identify a simple heuristic ratio for the expedient design of an adaptive FLS controller for flexible robots.

The ratio can be used to select the minimum number of fuzzy rules and appropriate type of MF for precision control and an execution time suitable for real-time application.

The FLSAC presented in Section 4.2.3 is arbitrarily designed with nine Gaussian MF (81 rules) and $K_s = 50$. It achieves precision tracking at an execution time of 3 min 45 sec. An extension to this initial design is presented to include; offline selection of different type and number of MF, different values of K_s as multiples of the number of MF and upper limit of the FLS output universe of discourse and their influence on control accuracy and execution time.

The following 'ad hoc' experimental process is used to determine an optimal FLS design for tracking a flexible robot.

Sets of three, five, seven and nine triangular and Gaussian MF are simulated to ascertain their effect on tracking precision and execution time while K_s is adjusted for each MF to determine its value for precision control. A comparison of results leads to identifying a combination of type and number of MF for optimal tracking control and execution time. An optimal FLS is considered achieved when the ratio of K_s to the number of MF (NMF) is equal to the output variable universe of discourse.

The optimal design for tracking the 12.6 m x 12.6 m square trajectory is determined as three triangular MF at $K_s = 15$ leading to a design ratio of five and an execution time of 1 min 40 sec.

4.3.1 Choice of Membership Function

The type of MF is usually chosen to fit an expected input data distribution or clusters and greatly influences robot tracking accuracy and execution time. Any convex shape may be chosen but a triangular, trapezoidal or Gaussian MF is more common. However,

there are no established formal criteria for determining the best shape and reliance is placed upon knowledge, experience and experimentation in consideration of all factors of the system [103]. In this section, tracking results are restricted to triangular and Gaussian MF and compared for optimal performance [106], [118], [152], [154].

4.3.2 Optimal Number of Fuzzy Rules

Choosing an optimal number of fuzzy rules faces contradiction. It is desirable to have as many rules as possible to ensure completeness and appropriate control resolution for accuracy. Yet, not so many that the system is over parameterized, has reduced generalization capability, degraded approximation accuracy and increased execution time because the type and number of MF influences the size of a fuzzy approximation error $e(w)$. Adding MF may contribute to a larger fuzzy approximation error during fuzzification as the data is further discretized. For a given function $g: \tilde{W} \rightarrow \tilde{Z}$ where $\tilde{W} \subset \mathcal{R}^n$ and $\tilde{Z} \subset \mathcal{R}$, we can construct an FLS represented by a fuzzy approximation function $f: W \rightarrow Z$, where $W \subset \tilde{W}$ and $Z \subset \tilde{Z}$ are the domain and range of the robot system using a parameter vector ψ such that; $g(w) = f(w | \psi) + e(w)$. The parameter ψ may include MF centres, base widths, etc. A fuzzy approximation error $e(w)$ is subject to the existence of a bound given by $\sup_{w \in W} \{ |g(w) - f(w | \psi)| \}$. Evaluation of how closely $f(w | \psi)$ approximates $g(w)$, $\forall w, w \in W$, and a given ψ value would require computing the error between $g(w)$ and $f(w | \psi)$ at selected points $w \in W$ obtained from input-output data, i.e. *test set*.

However, the existence of $e(w)$ requires complete knowledge of $g(w)$. Since, $g(w)$ comprises an elastic displacement vector δ_i , where δ_i is an element of the generalized

coordinates vector $\mathbf{q}$ in Equation (4.9) and unknown prior to simulation, no *test set* can be generated. Hence, $e(w)$ is uncertain and the accuracy of approximation is unable to be evaluated analytically [8]. Determination of $e(w)$ is neglected and judicious selection of the most suitable type and number of MF is required to achieve rapid asymptotic convergence of the transient vibrations to a zero-error steady state along the trajectory. An *optimal* number of fuzzy rules depends upon the number of input variable MF, controller and system performance, execution time, type of MF, ease of construction and adaptability [106], [118], [152], [154].

Figure 4.28 (a) and (b) shows nine typical triangular MF for elastic link deformation input variables δ_1 and δ_2 and output variable λ . Figure 4.15 (a) and (b) show typical Gaussian MF for the same variables. Their structure is typical for the FLS designs with three, five and seven MF. Universes of discourse range from -5 m to 5 m for δ_1 and δ_2 and from 0 to 1 for λ . Scaling gain K_s adjusts output λ MF base widths to provide better tracking accuracy as K_s increases.

The FLS shown in Figure 4.14 has variables alternately constructed with three (9 rules), five (25 rules), seven (49 rules) and nine (81 rules) triangular and Gaussian MF. The corresponding symmetric fuzzy rule matrices are given in Tables 4.4 to 4.7.

Table 4.4: FLS rule matrix for three membership functions (9 rules)

		δ_2		
		N	Z	P
δ_1	λ			
	N	PMAX	PM	PMAX
	Z	PM	Z	PM
	P	PMAX	PM	PMAX

Table 4.5: FLS rule matrix for five membership functions (25 rules)

		δ_2				
		NH	NL	Z	PL	PH
δ_1	λ					
	NH	PMAX	PH	PM	PH	PMAX
	NL	PH	PM	PL	PM	PH
	Z	PM	PL	Z	PL	PM
	PL	PH	PM	PL	PM	PH
	PH	PMAX	PH	PM	PH	PMAX

Table 4.6: FLS rule matrix for seven membership functions (49 rules)

		δ_2						
		NVH	NH	NL	Z	PL	PH	PVH
δ_1	λ							
	NVH	PMAX	PVH	PH	PM	PH	PVH	PMAX
	NH	PVH	PH	PM	PL	PM	PH	PVH
	NL	PH	PM	PL	PVL	PL	PM	PH
	Z	PM	PL	PVL	Z	PVL	PL	PM
	PL	PH	PM	PL	PVL	PL	PM	PH
	PH	PVH	PH	PM	PL	PM	PH	PVH
	PVH	PMAX	PVH	PH	PM	PH	PVH	PMAX

Table 4.7: FLS rule matrix for nine membership functions (81 rules)

		δ_2								
λ		NVH	NH	NL	NVL	Z	PVL	PL	PH	PVH
δ_1	NVH	PMAX	PVVH	PVH	PH	PM	PH	PVH	PVVH	PMAX
	NH	PVVH	PVH	PH	PM	PL	PM	PH	PVH	PVVH
	NL	PVH	PH	PM	PL	PVL	PL	PM	PH	PVH
	NVL	PH	PM	PL	PVL	PVVL	PVL	PL	PM	PH
	Z	PM	PL	PVL	PVVL	Z	PVVL	PVL	PL	PM
	PVL	PH	PM	PL	PVL	PVVL	PVL	PL	PM	PH
	PL	PVH	PH	PM	PL	PVL	PL	PM	PH	PVH
	PH	PVVH	PVH	PH	PM	PL	PM	PH	PVH	PVVH
	PVH	PMAX	PVVH	PVH	PH	PM	PH	PVH	PVVH	PMAX

Table 4.8: Tracking simulation times for increasing K_s values

K_s	1	15	15	25	15
Membership Functions	3	3	5	5	7
Triangular	1 min	1 min	2 min	2 min	2 min
	40 sec	40 sec	03 sec	03 sec	48 sec
Gaussian	1 min	1 min	2 min	2 min	2 min
	40 sec	43 sec	06 sec	05 sec	49 sec

Table 4.9: Tracking simulation times for increasing K_s values.

K_s	25	35	15	35	45
Membership Functions	7	7	9	9	9
Triangular	2 min	2 min	3 min	3 min	3 min
	47 sec	48 sec	30 sec	32 sec	32 sec
Gaussian	2 min	2 min	3 min	3 min	3 min
	48 sec	50 sec	45 sec	45 sec	45 sec

Tracking results are shown in Figures 4.29 (a) and (b) to 4.38 (a) and (b). These results were obtained by iterative simulation using the specified number of triangular and Gaussian MF and increments of K_s , during which, improvement was observed until best tracking precision was achieved in each case.

Initial “benchmark” results given in Figure 4.29 (a) and (b) show poor tracking control for three triangular and Gaussian MF with $K_s = 1$. For $K_s = 15$ much improved tracking control is achieved as shown in Figure 4.30 (a) and (b). For five MF and $K_s = 15$ tracking control deteriorates as shown in Figure 4.31 (a) and (b) but improves with an increase of output scaling gain to $K_s = 25$ as shown in Figure 4.32 (a) and (b). This result is similar to that obtained with three MF and $K_s = 15$. For seven and nine MF at $K_s = 15$ tracking control again deteriorates as shown in Figures 4.33 (a) and (b) and 4.36 (a) and (b). For seven MF at $K_s = 25$ and nine MF at $K_s = 35$ a slight improvement in tracking control is achieved as shown in Figures 4.34 (a) and (b) and 4.37 (a) and (b). However, for seven MF at $K_s = 35$ and nine MF at $K_s = 45$ tracking control is much improved and similar to that for three MF at $K_s = 15$ as shown in Figures 4.35 (a) and (b) and 4.38 (a) and (b).

Execution time increases as the number of triangular or Gaussian MF increases as shown in Tables 4.8 and 4.9, thereby demonstrating a progressively slower fuzzy convergence. In general, Gaussian MF provide better tracking control over the same number of triangular MF but at higher execution times. Despite the slight disparity in execution times, both triangular and Gaussian MF, exhibit fast fuzzy tracking convergence to a steady-state trajectory when K_s is increased to an appropriate value. Therefore, adding MF at constant K_s values causes deterioration in tracking control and extends execution time.

Figure 4.28: Fuzzy logic system membership functions (a) Input variables δ_1 and δ_2 , (b) Output variable λ .

(a)

(b)

Figure 4.29: Tracking results for $K_s = 1$ (a) Three triangular MF, (b) Three Gaussian MF.

(a)

(b)

Figure 4.30: Tracking results $K_s = 15$ (a) Three triangular MF, (b) Three Gaussian MF.

Figure 4.31: Tracking results $K_s = 15$ (a) Five triangular MF, (b) Five Gaussian MF.

Figure 4.32: Tracking results $K_s = 25$ (a) Five triangular MF, (b) Five Gaussian MF.

(a)

(b)

Figure 4.33: Tracking results $K_s = 15$ (a) Seven triangular MF, (b) Seven Gaussian MF.

(a)

(b)

Figure 4.34: Tracking results $K_s = 25$ (a) Seven triangular MF, (b) Seven Gaussian MF.

(a)

(b)

Figure 4.35: Tracking results $K_s = 35$ (a) Seven triangular MF, (b) Seven Gaussian MF.

(a)

(b)

Figure 4.36: Tracking results $K_s = 15$ (a) Nine triangular MF, (b) Nine Gaussian MF.

(a)

(b)

Figure 4.37: Tracking results $K_s = 35$ (a) Nine triangular MF, (b) Nine Gaussian MF.

(a)

(b)

Figure 4.38: Tracking results $K_s = 45$ (a) Nine triangular MF, (b) Nine Gaussian MF.

These results are contrary to common experience where a low number of MF is usually considered suitable for coarse control and a greater number for fine control. Typically, by increasing the number of MF at a fixed output scaling gain the universe of discourse is further discretized and fuzzy control surface granularity increased, thereby capturing more random or clustered crisp data to provide fine control. Therefore, it would be expected that increasing the number of MF at constant K_s value further discretizes the universe of discourse and increases fuzzy control surface granularity to provide finer control.

However, a fuzzy approximation error may increase with added MF during fuzzification and defuzzification resulting in deteriorated control. The FLS in this case has data input from flexural link vibrations and is oscillatory. This would render additional MF redundant as further discretization fails to capture the continuous elastic vibrations causing deterioration in control. In contrast, increasing K_s widens the MF bases, encapsulates the continuous oscillatory behaviour of vibration in terms of the FLS output variable λ and improves control performance.

Therefore, tuning fuzzy controllers with an output scaling gain is considered a prime method of improving performance, whereas adding MF at constant K_s produces poor results because they do not fully capture the oscillatory response of continuous elastic link vibrations. However, adding MF might provide finer control where variation of the output scaling gain is limited.

Furthermore, it is demonstrated that precision tracking control is obtained consistently with K_s values as multiples of the NMF and the positive range of the universe of discourse, thereby establishing a heuristic design ratio given by:

$$\frac{K_s}{\text{NMF}} = 5 \quad (4.24)$$

The FLS with three MF at a value of $K_s = 15$ provides the optimal choice for expedient design, precision control and fast execution time.

Results show the FLSAC to be a robust, flexible and efficient method suitable for precision control of flexible robots and easy to design. Its design may be more expedient with the heuristic ratio to achieve precision control at fast execution time.

4.4 Nonminimum Phase Response and Sensor Location

4.4.1 Nonminimum Phase Response

Nonminimum phase (NMP) response is an inherent problem for flexible robot control [3], [4], [45], [104], [119], [134]. A characteristic of this phenomenon exists when a joint actuation response occurs in a flexible body opposite in sense to that expected. More precisely, when torque actuates a flexible robot joint it induces flexing and momentary acceleration of the end effector in a direction opposite in sense to that commanded by the actuating torque. Analytical control theory describes this behaviour by transfer function poles or zeros occurring in the right half s-plane and termed a phase shift, or transport lag, between the actuator and end effector. This phase shift creates a time delay between the actual end-effector positions and control action needed to correct feedback position errors and corresponds to the time for mechanical waves to propagate through the link from joint to end effector [3], [158].

Associated with NMP response is the distance between position sensor and actuator. Collocated sensors, typically rotation encoders, are mounted at joints to measure angular displacements that cause joint actuation. Typically, collocated control is suitable for fixed-base, rigid-link robots operating at speeds where flexibility and NMP response is

insignificant.

In contrast, flexible robots exhibit significant deformation at the end effector necessitating accurate control by adjusting for phase shift between actuator and the distal end of each link. Noncollocated sensors are usually position sensors located at the end effector or some intermediate point on the robot link to measure Cartesian coordinate positions.

NMP response is modeled by implementing time delays in the feedback loops of the control strategies shown in Figures 4.2 and 4.13, [3]. The FLS used is that described in Sub-subsection 4.2.4.1 comprising nine Gaussian MF for each of the variables δ_1 , δ_2 and λ shown in Figure 4.14. The universe of discourse for λ ranges between the limits $0 \leq \lambda \leq 20$ and the output scaling gain K_s is omitted.

Transport delay blocks are implemented in the Matlab/Simulink™ simulation models to represent NMP time delays [98]. As a correction for NMP response, the same time delay acts upon the control law in an attempt to achieve minimum phase response. Time delays are calculated using the transverse beam vibration wave velocity equation given by [3].

$$\text{Wave velocity} \quad c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{1745833}{21}} = 288.33 \text{ m/s} \quad (4.25)$$

Delay time from joint 1 to the end effector for two link lengths of 9 m is given by:

$$t_{dt1} = \frac{9}{288.33} = 0.0312 \text{ sec} \quad (4.26)$$

Delay time from joint 2 to the end effector for one link length of 4.5 m is given by:

$$t_{dt2} = \frac{4.5}{288.33} = 0.0156 \text{ sec} \quad (4.27)$$

Average trajectory simulation time = 402 sec for 16000 steps at 0.001 step size,
i.e. 0.0252 sec per simulation step.

Simulation time delay for joint 1:

$$dt_1 = \frac{0.0312}{0.0252} = 1.238 \text{ ss} \quad (4.28)$$

Simulation time delay for joint 2:

$$dt_2 = \frac{0.0156}{0.0252} = 0.619 \text{ ss} \quad (4.29)$$

Joint rates and angles are transformed into end-effector Cartesian velocities and positions after output from the robot manipulator dynamics. For simulation purposes, the time delay is implemented in the position feedback loop at the maximum delay time dt_1 (worst-case scenario).

Tracking results shown in Figures 4.39 (a) and (b) to 4.41 (a) and (b) depict the difference in performance between cantilever and pinned-pinned assumed modes for reduced-order flexible dynamics models using the IFDC control strategy. Time delays are included in the feedback loops to model NMP response and corrective time delays are included in the feed-forward loop. Results are included for tracking without time delays to model collocated sensors only at joints 1 and 2, to form a basis of comparison.

Tracking results are shown in Figure 4.39 (a) and (b) for collocated, NMP and NMP corrected cases. A forward shift in the NMP transient at the second direction switch is shown in Figure 4.39 (a). This is considered attributable to the effect of time-delayed feedback position data coinciding with an advanced commanded input position on the desired trajectory, thereby causing summation of asynchronous position data. The result is a larger error and delayed transient vibration motion of the end effector subsequently

Figure 4.39: IFDC tracking for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilever assumed modes, (b) Pinned-pinned assumed modes.

Figure 4.40: IFDC tracking at the first direction switch transient for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilever assumed modes, (b) Pinned-pinned assumed modes.

(a) (b)
Figure 4.41: IFDC tracking at the first direction switch transient (zoomed) for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilever assumed modes, (b) Pinned-pinned assumed modes.

(a) (b)
Figure 4.42: FLSAC tracking for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilever assumed modes, (b) Pinned-pinned assumed modes.

generating a delayed increase in control torque.

The pinned-pinned dynamics model trajectory, shown in Figure 4.39 (b), exhibits low amplitude transient vibrations with more oscillations that take longer to decay than the larger amplitude cantilever dynamics vibrations. This result corroborates the results presented in Figures 4.3 (a) and (b) and 4.4 (a) and (b), Sub-subsection 4.2.2.1.

Figure 4.41 (a) and (b) clarifies the results of implementing time delays to model the effects of NMP and corrected NMP control action at the first direction switch. The effect of NMP time delays on the cantilever dynamics model shown in Figure 4.41 (a), is to increase the overshoot amplitude while the NMP correction increases the amplitude even further.

In contrast, NMP time delays on the pinned-pinned dynamics model shown in Figure 4.41 (b) generate a similar increase in transient overshoot amplitude. But, the NMP corrective time delay reduces the amplitude by approximately one-third of the increase caused by the NMP time delay. The difference in performance is considered the result of the JTPD control law being inadequate to suppress the large errors generated by the more vigorous cantilever dynamics vibrations.

Tracking results shown in Figures 4.42 (a) and (b) to 4.44 (a) and (b) depict the difference in performance between cantilever and pinned-pinned assumed modes reduced-order flexible dynamics models using the FLSAC control strategy. Results are included for tracking without time delays to model collocated sensors at joints 1 and 2 and provide a comparison with NMP and NMP corrected results.

A significant reduction in overshoot amplitude is achieved for both cantilever and pinned-pinned modes. Small increases in the first direction switch overshoot amplitudes

Figure 4.43: FLSAC tracking at first direction switch for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilver assumed modes, (b) Pinned-pinned assumed modes.

Figure 4.44: FLSAC tracking at first direction switch transient (zoomed) for a sensor collocated at joint 2, a noncollocated (NMP) sensor at the end effector and NMP corrected (a) Cantilver assumed modes, (b) Pinned-pinned assumed modes.

are exhibited in Figure 4.44 (a) and (b). They are caused by the NMP time delay while smaller increases are caused by the NMP corrective time delay. However, the effects of implementing time delays to model NMP response and its correction are insignificant compared to the overriding effect of FLSAC.

4.4.2 Tracking Performance and Stability with Sensors at Alternate Locations

For a robot to be considered a *hyperstable* system the following conditions must be satisfied; sensors and actuators are equal in number, matching types of sensors and actuators and sensor/actuator collocation [93]. But, these conditions detract from the proper control of flexible robots because it is necessary to apply a noncollocated sensor at the end effector to capture real-time position data for NMP response [3]. Also, it is necessary to ensure system stability when the sensors are located at a distance from the actuator.

Therefore, to investigate performance and stability in the control of a flexible robot manipulator with noncollocated sensors, tracking results are obtained for sensors located at different positions along link 2.

The IFDC control strategy shown in Figure 4.2 and the FLSAC strategy shown in Figures 4.13 and 4.14 are used to simulate noncollocated control. The FLS shown in Figure 4.14, utilizes the optimal three triangular MF (9 rules) at $K_s = 15$ design in accordance with the fuzzy rule matrix given in Table 4.4. Universes of discourse for each variable are shown in Figure 4.45. Link distal end deformations δ_1 and δ_2 are normalized by gain $K_N = 5$ for input to the FLS.

Results are presented for tracking the end effector of a fixed-base planar two-link flexible robot manipulator using IFDC and FLSAC control strategies, initially with

collocated joint encoders, then with a position sensor located alternately at 0.5 m, 2.25 m and 4.0 m on the outboard link 2 from joint 2.

NMP response for sensors located at positions 0.5 m, 2.25 m, and 4.0 m on link 2 from joint 2 and at the end effector are modeled by implementing appropriate time delays in the feedback loop [3]. Time delays are calculated using the transverse beam vibration traveling wave velocity c given by Equation (4.25) [143].

Delay times t_{dti} and simulation time delays dt_i used to model NMP with a sensor noncollocated at the end effector are given by Equations (4.26), (4.27), (4.28) and (4.29) in Subsection 4.4.1. Delay times t_{dt_1} and simulation time delays dt_1 for the noncollocated sensor at 8.5 m, 6.75 m and 5.0 m from joint 1 are; 0.02948 sec (1.17 ss), 0.0234 sec (0.9289 ss), 0.01734 sec (0.688 ss) respectively and at 4.0 m, 2.25 m and 0.5 m from joint 2 are; 0.01387 sec (0.5505 ss), 0.0078 sec (0.3097 ss) and 0.001734 sec (0.0688 ss) respectively. For the purpose of simulation, worst case time delays dt_1 are implemented in the Matlab/Simulink™ control strategy models.

Different values of L_2 are substituted into the direct kinematics given by Equations (3.14) and (3.15), Chapter 3, Subsection 3.2.1 to represent noncollocated sensor locations. Direct kinematics equations are also included for a $L_2 = 4.5$ m to simultaneously track the end effector for comparison of collocated sensor results.

The results shown in Figure 4.46 (a) and (b) are similar for both control strategies with the sensor located at 4.0 m from joint 2. Tracking deviates from the commanded square trajectory for the sensor located at 2.25 m from joint 2, as shown in Figure 4.48 (a) and (b). With the sensor located 0.5 m from joint 2, the end effector tracks a smooth circular trajectory for the FLSAC strategy, whereas erratic vibratory tracking behaviour

(a)

(b)

Figure 4.45: Optimal FLS triangular membership functions (a) Input variables δ_1 and δ_2 , (b) Output variable λ .

occurs for the IFDC strategy. In each case, the trajectory tracked with the sensor at different locations passes within close proximity to the corner points of the square trajectory as they are the extremities of the fully extended robot manipulator.

The difference between NMP and NMP corrected results is minimal for both controls strategies as depicted in Figures 4.47 (a) and (b), 4.49 (a) and (b) and 4.51 (a) and (b). All trajectories are stable.

Angular displacement and velocity histories are shown in Figure 4.52 (a) and (b) for IFDC and FLSAC strategies with a sensor located at 0.5 m from joint 2. Angular velocities are of a high magnitude with minimal fluctuation throughout the tracking cycle for the IFDC strategy. However, angular velocity histories for the FLSAC strategy exhibit a cyclical fluctuation from a high magnitude of 8 rad/sec at direction switches to a lower 3 rad/sec between each direction switch.

The FLSAC strategy is effective in link vibration suppression, produces enhanced tracking control while maintaining stability with noncollocated sensors as demonstrated by the results presented in Subsections 4.4.1 and 4.4.2. Control action time delay has minimal effect on NMP correction. Noncollocated position sensors do not undermine stability and circumvent hyperstability constraints. Tracking results corroborate investigations by other researchers [93], [140], [148].

4.5 Total System Stiffness

Previous sections within Chapter 4 consider tracking control of the planar two-link flexible robot manipulator exclusively for link flexibility. While link flexibility is a major contributing factor to the difficulties encountered in controlling robots with long slender links, the stiffness of the system as a whole i.e. *total stiffness* (conversely *total flexibility*),

(a) (b)
Figure 4.46: IFDC and FLSAC tracking for a sensor collocated at joint 2, a sensor noncollocated 4.0 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

(a) (b)
Figure 4.47: IFDC and FLSAC tracking at first direction switch transient for a sensor collocated at joint 2, a sensor noncollocated 4.0 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

Figure 4.48: IFDC and FLSAC tracking for a sensor collocated at joint 2, a sensor noncollocated 2.25 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

Figure 4.49: IFDC and FLSAC tracking at third direction switch transient (zoomed) for a sensor collocated at joint 2, a sensor noncollocated 2.25 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

Figure 4.50: IFDC and FLSAC tracking with a sensor collocated at joint 2, a sensor noncollocated 0.5 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

Figure 4.51: IFDC and FLSAC tracking at first direction switch transient with a sensor collocated at joint 2, a sensor noncollocated 0.5 m from joint 2 (NMP) and NMP corrected (a) IFDC, (b) FLSAC.

Figure 4.52: IFDC and FLSAC link 2 angular displacement and angular velocity for a sensor noncollocated 0.5 m from joint 2 (NMP) (a) IFDC, (b) FLSAC.

Figure 4.53: Robot link flexible drive system.

presents an even greater challenge for control.

Depending on the robot manipulator design and drive system configuration, stiffness contributions may be inherent in joint actuators, gear trains and rotor inertias. Coupling actuators to robot links is determined by the specific drive system which could include; harmonic drives or DC motors with drive transmission reduction gearing where flexibilities are present in motor stator windings, gears, shafts and lubricants. Viscous, dynamic and Coulomb friction effects also contribute to the total stiffness of the system [136], [137], [138].

The simplified coupled actuator-link system shown in Figure 4.53 depicts a typical drive system for a robot manipulator that may be used as the basis for deriving a joint stiffness mathematical model.

4.5.1 Link Flexibility with Rigid Joint Inertia

The planar two-link flexible robot manipulator dynamics are modeled to include joint hub inertias with magnitudes that represent all joint actuator and drive transmission components mounted at the shoulder and elbow joints. Viscous, dynamic and Coulomb friction effects and flexibilities inherent in motor stator windings, gears, shafts and lubricants are excluded.

The stiffness of an actuator with inertia I_{a_i} may be modeled by a torsion spring constant k_{a_i} . The rotor transmits power directly from the actuator through the rotor with stiffness k_{r_i} and inertia I_{r_i} to the joint hub. The joint hub is a rigid structure with inertia I_{h_i} to which is connected the flexible link. For simplicity, hub inertia I_{h_i} is used as the total of all individual drive component inertias mounted at the shoulder and elbow joints,

i.e. actuators, gears and rotors, treated as rigid elements given by:

$$I_{h_i} = I_{a_i} + I_{g_i} + I_{r_i} \quad i = 1, 2 \quad (4.30)$$

Where; I_{g_i} is the gear inertia for gear i .

When the actuator rotates a desired angle θ_{a_i} , the rotor rotates angle θ_{r_i} to drive the rotation of the joint hub θ_{h_i} which is equivalent to link rotation angle θ_i . Since all components in the joint drive transmission components are treated as rigid elements then; $\theta_{a_i} = \theta_{r_i} = \theta_{h_i} = \theta_i$ and $k_{a_i} = k_{r_i} \rightarrow \infty$. Because the drive train of each joint is mounted at the joint the total hub inertia matrix $\mathbf{I}_h$ is included in Equation (4.9) where the joint hub inertia matrix terms are added to the flexible-link inertia terms and presented explicitly in in Appendix C.

Tracking results for the reduced-order flexible cantilever dynamics model are shown in Figure 4.54 (a) and (b). Tracking results for the full-order flexible cantilever dynamics model are shown in Figure 4.55 (a) and (b). To obtain the same precision tracking control as that obtained for the dynamics model without joint inertia terms a value of $K_s = 360$ is applied with the optimal FLS of three triangular MF. The increase in the K_s value suggests higher values may be required for payloads attached at the end effector.

Furthermore, it suggests the potential for an online automatic K_s scheduling subsystem to accommodate variations in payload and robot parameters.

Tracking times are presented in Table 4.10 and show minimal difference between reduced- and full-order flexible cantilever dynamics models but greater tracking times for the FLSAC strategy.

Figure 4.54: Trajectories for reduced-order cantilever dynamics with joint inertia (a) IFDC, (b) FLSAC ($K_s = 360$).

Figure 4.55: Trajectories for full-order cantilever dynamics with joint inertia (a) IFDC, (b) FLSAC ($K_s = 360$).

Table 4.10: Tracking simulation times for cantilever dynamics with joint inertia.

Control Strategy	Reduced-Order Dynamics	Full-Order Dynamics
IFDC	55 sec	55 sec
FLSAC ($K_s = 360$)	2 min 05 sec	2 min 12 sec

4.5.2 Combined Joint and Link Flexibilities

In this case, the actuator and rotor are considered to have distinct finite values of k_{a_i} and k_{r_i} represented by a common stiffness constant $k_{s_i} = k_{a_i} = k_{r_i}$ and where

$\theta_{a_i} = \theta_{r_i} \neq \theta_{h_i}$. Partitioning the actuator dynamics from the manipulator dynamics the

matrix form of the robot manipulator dynamics equations is given by [136], [137], [138]:

$$\begin{aligned} \mathbf{0} &= \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\dot{\mathbf{q}}, \mathbf{q}) + \mathbf{K}\mathbf{q} + \mathbf{k}_s(\boldsymbol{\theta}_a - \boldsymbol{\theta}_h) \\ \boldsymbol{\tau} &= \mathbf{I}_a\ddot{\boldsymbol{\theta}}_a - \mathbf{k}_s(\boldsymbol{\theta}_a - \boldsymbol{\theta}_h) \end{aligned} \quad (4.31)$$

Where; $\mathbf{I}_a$ is the actuator inertia matrix and $\mathbf{k}_s(\boldsymbol{\theta}_a - \boldsymbol{\theta}_h)$ is a restoring torque vector.

This is derived by adding actuator and rotor flexural rotation angles which gives:

$$(\theta_{a_i} - \theta_{h_i}) = (\theta_{a_i} - \theta_{r_i}) + (\theta_{r_i} - \theta_{h_i}) \quad (4.32)$$

Since; $k_{s_i} = k_{a_i} = k_{r_i}$ we get $k_{s_i}(\theta_{a_i} - \theta_{h_i}) = k_{a_i}(\theta_{a_i} - \theta_{r_i}) + k_{r_i}(\theta_{r_i} - \theta_{h_i})$

$$\text{and} \quad \mathbf{k}_s(\boldsymbol{\theta}_a - \boldsymbol{\theta}_h) = \begin{bmatrix} k_{s_1}(\theta_{a_1} - \theta_{h_1}) & 0 \\ 0 & k_{s_2}(\theta_{a_2} - \theta_{h_2}) \end{bmatrix} \quad (4.33)$$

Extensive research literature has been published on robot manipulators with flexible joints although most synthesized control strategies are based upon state-space dynamics equations models with rigid links [136], [137], [138].

One researcher derived an inverse dynamics strategy based on fourth-order dynamics

equations for a combined two-drive system located at the base of a single rigid link with a telescopic extension [43]. Consideration of total stiffness requires a more complex derivation of the dynamics equations beyond the scope of this thesis and is suggested as a topic for future research.

4.6 Stability Analysis for Flexible Robot Dynamics Models

Consider the flexible robot manipulator model of Equation (4.9) again:

$$\tau = M(q)\ddot{q} + C(q, \dot{q}) + Kq \quad (4.34)$$

The proposed controller given by Equation (3.8) is again:

$$\tau = J^T(\theta) \left[K_p \begin{bmatrix} e_x \\ e_y \end{bmatrix} + K_d \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} \right] \quad (4.35)$$

Theorem 4.1: The closed-loop system for a two-link flexible robot manipulator given by Equation (4.34), the Jacobian transpose control law given by Equation (4.35) and the control strategy shown in Figure 4.2 is asymptotically stable.

Consider the positive definite quadratic Lyapunov function candidate given by [128], [134]:

$$V(q, e) = T + U - \frac{1}{2} J_{ij}^T(\theta_i) \sum_{i=1}^2 K_{p_i} e_i^2 \quad (4.36)$$

Where; T and U are the total kinetic and potential energies, $J_{ij}^T(\theta)$ are elements of the Jacobian matrix pertaining to control error e_i , K_{p_i} is positive, e_i is an element of the error

vector $e = \begin{bmatrix} e_x \\ e_y \end{bmatrix}$ and $\dot{e}_i$ is an element of the error velocity vector $\dot{e} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix}$. Now, the total

work done by an external torque on each joint of the two-link flexible robot manipulator is given by [47]:

$$W = \sum_{i=1}^2 \int_0^t \tau_i(t) \dot{\theta}_i(t) dt \quad (4.37)$$

From the energy-work relationship, the increment of the system energy is equal to the work done by the external inputs we have:

$$\{T(t) + U(t)\} - \{T(0) + U(0)\} = \sum_{i=1}^2 \int_0^t \tau_i(t) \dot{\theta}_i(t) dt \quad (4.38)$$

Where; $T(t)$ and $U(t)$ are total kinetic and potential energies of the system at time t and $T(0)$ and $U(0)$ are constants representing kinetic and potential energies of the system at time 0.

Taking derivatives of both sides of Equation (4.38) with respect to t we get:

$$\dot{T}(t) + \dot{U}(t) = \sum_{i=1}^2 \tau_i(t) \dot{\theta}_i(t) \quad (4.39)$$

Differentiating Equation (4.36) with respect to t and substituting Equation (4.39) for $\dot{T}(t) + \dot{U}(t)$ we get:

$$\dot{V}(q, e) = \sum_{i=1}^2 \tau_i(t) \dot{\theta}_i(t) - J_{ij}^T(\theta_i) K_{p_i} e_i \quad (4.40)$$

Substituting Equation (4.35) into Equation (4.40) yields:

$$\dot{V}(q, e) = J_{ij}^T(\theta_i) \sum_{i=1}^2 \left[K_{p_i} e_i + K_{d_i} \dot{e}_i \right] \dot{\theta}_i(t) - J_{ij}^T(\theta_i) \sum_{i=1}^2 K_{p_i} e_i \quad (4.41)$$

Where; $\sum_{i=1}^2 K_{d_i}$ is positive.

$$\text{Then} \quad \dot{V}(q, e) = \sum_{i=1}^2 \dot{\theta}_i J_{ij}^T(\theta_i) K_{d_i} \dot{e}_i, \quad \text{i.e.} \quad \dot{V} \geq 0 \quad (4.42)$$

Since the property of negative definiteness does not hold and Lyapunov's method

provides only a sufficient condition nothing can be inferred on stability from this analysis. However, because of La Salle's theorem global asymptotic stability is ensured if

the only trajectory for which $\dot{V}(q,e) \equiv 0$ is the equilibrium trajectory on which $\sum_{i=1}^2 e_i \equiv 0$.

Furthermore, since $\dot{V}(q,e)$ can remain zero only along trajectories where $\ddot{\theta} = \dot{\theta} = 0$ and $u(x_i, t) = 0$ we get:

$$\sum_{i=1}^2 \dot{\theta}_i J_{ij}^T(\theta_i) K_{d_i} \dot{e}_i = 0 \quad (4.43)$$

Since; $\sum_{i=1}^2 \dot{\theta}_i = 0$, $\left| \sum_{i=1}^2 K_{d_i} \right| \neq 0$ and $J^T(\theta)$ has full rank, then $\sum_{i=1}^2 \dot{e}_i \equiv 0$ and by La Salle's

theorem $\sum_{i=1}^2 e_i \equiv 0$. Then the robot will not get stuck when $\sum_{i=1}^2 e_i \neq 0$. Hence, the

application of the control law given by Equation (4.35) to the flexible dynamics robot equations given by Equation (4.34) achieves global asymptotic stability. The closed-loop control strategy is stable [47].

This analytical stability proof is corroborated by all simulated tracking trajectories and closed-loop step input performance results obtained using the IFDC and FLSAC strategies.

Chapter 5

Repetitive Learning Control

The content of this chapter is based on published papers [54] and [57].

Control strategies and simulation results presented in Chapter 3 and Chapter 4 are restricted to single tracking of the square trajectory i.e. rest-to-rest.

In Section 5.1 tracking the end effector of a rigid dynamics robot manipulator along a 12.6 m x 12.6 m square trajectory is performed repetitively using control strategies referred to as repetitive learning inverse rigid dynamics control (RLIRDC) and repetitive learning fuzzy logic control (RLFLC).

The RLIRDC strategy uses the IRDC strategy with the JTPD control law but the tracking cycle is repeated four times (five tracking cycles) without resetting to its initial state after each repetition. The RLFLC strategy uses the FLC strategy with the JTPD control law and the tracking cycle is repeated four times (five tracking cycles) without resetting to its initial state after each repetition.

In Section 5.2 tracking is repeated four times (five tracking cycles) using strategies referred to as; repetitive learning inverse flexible dynamics control (RLIFDC) and repetitive learning fuzzy logic system adaptive control (RLFLSAC) based on the IFDC and FLSAC strategies respectively.

The aim of repetitive learning control (RLC) is to train a robot to execute periodic motions or cycles, such that, its performance improves upon each repetition and tracks a desired trajectory for several or more repetitions without requiring a reset to its initial state.

Repetitive learning control is required when periodic signals input to a system and is

a natural control problem encountered in many engineering applications including space robotics. A vast amount of literature exists on robot learning control of which iterative learning control (ILC) pioneered by Arimoto, et al. [6], plays a significant part.

Iterative learning control (ILC) in which, the robot performs successive trajectory tracking iterations requires resetting back to its initial state prior to the next iteration. Resetting demands a high degree of dexterity, is very time-consuming and may be undesirable in operations where the reset process may be difficult, inaccurate or unreliable in updating the control law. Many proposed ILC systems typically use a proportional or derivative error term and learning gain to obtain control law updating algorithms.

RLC is an alternative to ILC where initial states of the robot are not reset at the start of repetitions. The robot is trained to gradually converge to the commanded trajectory based on the control performance of previous tracking cycles.

Essentially, RLC is a simple technique requiring less *a priori* knowledge of the controlled system and capable of modifying control input error signals automatically based on prior tracking cycles. The aim is to track a trajectory as close as possible to that commanded by increasing the number of repetitions. Principal published researches are Goldsmith [48], Hara, Yamamoto, Omata and Nakano [73], Horowitz [75], Sison and Chong [133], Weiss [150], and Yamamoto [153]. Typically, they include adaptive algorithms that successively improve performance to achieve asymptotic zero-error tracking based on the betterment learning laws proposed by Arimoto, et al [6].

FLC studies were conducted by Bonarini [14], Layne and Passino [89] and Moudgal, Passino and Yurkovich [114] on *learn behaviours*. They include techniques to modify the

relationship between inputs and outputs through control actions based on a reference model with reinforcement learning algorithms, and evolutionary fuzzy learning (EFL) techniques for populations of fuzzy rules based on genetic algorithms.

5.1 Rigid Dynamics Robot Manipulator

5.1.1 Repetitive Learning Inverse Rigid Dynamics Control (IRDC)

The RLIRDC strategy is the same as that described in Chapter 3, Subsection 3.2.2 and shown in Figure 3.2 with the control law given by Equation (3.8). For comparison, the results of four repetitions (five tracking cycles) are presented for RLIRDC and RLFLC strategies.

Using RLIRDC there is no improvement in tracking, as shown in Figure 5.1 (a), (b), (c) and (d) where the tracking of each cycle is not discernable. Very large overshoot amplitudes occur at direction switches in the initial cycle followed by a slight reduction at the lower left direction switch at the start of the first repetition. Tracking performance then reverts to follow the initial trajectory for all successive direction switches and repetitions. Tracking is stable for all cycles.

Tracking torque for each joint is shown in Figure 5.2 (a), (b) and (c) which depict torque spikes at each direction switch. Figure 5.2 (a) shows only the torque history with four spikes during the initial tracking cycle. Figure 5.2 (b) and (c) subsequently show the spikes at direction switches for the 2nd and 4th repetitions.

The magnitude of each spike is consistent at approximately ± 250 Nm for joint 1 and ± 40 Nm for joint 2 throughout all repetitions indicating no increase in demand on the controller and no improvement in tracking precision.

(a)

(b)

(c)

(d)

Figure 5.1: RLIRDC trajectories (a) Initial tracking with four repetitions, (b) Initial tracking at first direction switch (zoomed), (c) Initial tracking with 1st and 2nd repetitions at first direction switch, (d) Initial tracking with 1st, 2nd, 3rd and 4th repetitions at first direction switch (zoomed).

(a)

(b)

(c)

Figure 5.2: RLIRDC joint torques (a) Joint torques for initial tracking, (b) Joint torques for 2nd repetition, (c) Joint torques for 4th repetition.

5.1.2 Repetitive Learning Fuzzy Logic Control (RLFLC)

The RLFLC strategy is the same as that described in Chapter 3, Subsection 3.2.4 and shown in Figures 3.10 and 3.11 with the control law given by Equation (3.8).

Four repeated tracking cycles (five tracking cycles) are performed without initial state resets or implementation of a complex fuzzy reference model design with a learning mechanism, i.e. fuzzy inverse model and fuzzy rule modifier [89], [114].

RLFLC tracking results are shown in Figure 5.3 (a), (b), (c) and (d) in which very small transient vibration overshoot amplitudes occur at each direction switch for the initial tracking cycle. Subsequently, the tracking converges closer to the commanded trajectory on each repetition with a distinct improvement. Tracking is stable for all cycles.

Tracking torque for each joint is shown in Figure 5.4 (a), (b) and (c) which depict spikes at each direction switch. Figure 5.4 (a) shows only the torque history with four spikes during the initial tracking cycle. Figure 5.4 (b) and (c) also show the torque spikes at direction switches for the 2nd and 4th repetitions.

However, the torque spikes occurring at each direction switch show vigorous oscillations in an effort to reduce the transient vibrations. The average peak torque is much lower than that experienced with the RLIDC strategy at approximately ± 100 Nm for joint 1 and ± 20 for joint 2 throughout all repetitions. Hence, the vigorous oscillatory activity of the controller achieves a greater improvement in tracking precision at a much lower control torque at the direction switches.

The RLFLC strategy has demonstrated an ability to self-adapt in periodic tracking and asymptotically converges to a zero-error trajectory without an estimated learning

(a)

(b)

(c)

(d)

Figure 5.3: RLFLC trajectories (a) Initial tracking with four repetitions, (b) Initial tracking at first direction switch (zoomed), (c) Initial tracking with 1st and 2nd repetitions at first direction switch (zoomed), (d) Initial tracking with 1st, 2nd, 3rd and 4th repetitions at first direction switch (zoomed).

(a)

(b)

(c)

Figure 5.4: RLFLC joint torques (a) Joint torques for initial tracking, (b) Joint torques for 2nd repetition, (c) Joint torques for 4th repetition.

gain or rule base modification.

The results presented in Chapter 3, Subsection 3.2.5 demonstrate greater tracking precision can be achieved with a FLC strategy by substituting robot dynamics equations with an FIS for each robot link than with conventional control methods. The same performance is achieved when the FLC strategy is extended to repeated tracking without initial state reset or implementation of a complex fuzzy reference model design with a learning mechanism [89], [114].

There is no improvement in tracking when RLIRDC is used. In contrast, the RLFLC strategy demonstrates the requirement of only the initial tracking cycle to train the robot after which the trajectory converges to that commanded.

5.2 Flexible Dynamics Robot Manipulator

In this section the control strategies presented in Chapter 4, Subsections 4.2.1 and 4.2.4 are extended to repetitive learning control of the fixed-base planar two-link flexible robot manipulator described in Section 4.1 while tracking a 12.6 m x 12.6 m square trajectory.

An analysis and comparison between RLIFDC and RLFLSAC tracking results is presented.

5.2.1 Repetitive Learning Inverse Flexible Dynamics Control (RLIFDC)

The RLIFDC strategy is the same as that described in Chapter 4, Subsection 4.2.4 and shown in Figures 4.2 with the JTPD control law given by Equation (3.8).

Four repetitions (five tracking cycles) are performed without initial state resets.

There is no improvement in tracking, as shown in Figure 5.5 (a) and (b), where the tracking of each cycle is not discernable. A very large transient vibration overshoot

amplitude occurs at each direction switch in the initial cycle followed by a slight reduction at the lower left direction switch at the start of the first repetition. Tracking performance then reverts to follow the initial trajectory for all successive direction switches and repetitions. However, there is evidence of a small reduction in overshoot amplitude at the direction switches for each repetition shown in Figure 5.5 (c) and (d), but they are insignificant given the large amplitude of each overshoot. Tracking is stable for all cycles.

Tracking torque for each joint is shown in Figure 5.6 (a), (b) and (c) which depict spikes at each direction switch similar to that experienced with the RLIRDC strategy described in Subsection 5.1.1 and shown in Figure 5.2 (a), (b) and (c). Figure 5.6 (a) shows only the torque history with four spikes during the initial tracking cycle. Figure 5.6 (b) and (c) also show torque spikes at direction switches for the 2nd and 4th repetitions. The magnitude of each spike is consistent at approximately ± 250 Nm for joint 1 and ± 40 Nm for joint 2 throughout all repetitions, indicating no increase in demand on the controller and no improvement in tracking precision.

These results exemplify the difficulty faced in minimizing transient overshoot amplitudes at direction switches and the inability of the system to benefit from repetitive tracking with a RLIFDC strategy applied to a flexible robot manipulator. The intuitive JTPD control law alone fails to reduce overshoot amplitudes while the attempt to implement repetitive learning control is to no avail. Tracking is stable for all cycles.

5.2.2 Repetitive Learning Fuzzy Logic System Adaptive Control (RLFLSAC)

The RLFLSAC strategy is the same as that described in Chapter 4, Subsection 4.2.4 and shown in Figures 4.13 and 4.14 with the control law given by Equation (4.21).

(a)

(b)

(c)

(d)

Figure 5.5: RLIFDC trajectories (a) Initial tracking with four repetitions, (b) Initial tracking at first direction switch, (c) Initial tracking with 1st and 2nd repetitions at first direction switch, (d) Initial tracking with 1st, 2nd, 3rd and 4th repetitions at first direction switch.

(a)

(b)

(c)

Figure 5.6: RLIFDC joint torque (a) Joint torque for initial tracking, (b) Joint torque for 2nd repetition, (c) Joint torque for 4th repetition.

Four repetitions (five tracking cycles) are performed without initial state resets or implementation of a complex fuzzy reference model design with a learning mechanism, i.e. fuzzy inverse model and fuzzy rule modifier [89], [114].

RLFLSAC tracking results are shown in Figure 5.7 (a), (b), (c) and (d) in which a much reduced transient vibration overshoot amplitude occurs at each direction switch for the initial and all subsequent tracking cycle repetitions. Also, there is a distinct improvement where tracking converges closer to the commanded trajectory upon each repetition, as shown in Figure 5.7 (c) and (d). Tracking is stable for all cycles.

Tracking torque for each joint is shown in Figure 5.8 (a), (b) and (c) which depict spikes at each direction switch. Figure 5.8 (a) shows only the torque history with four spikes during the initial tracking cycle. Figure 5.8 (b) and (c) show the subsequent torque spikes at direction switches for the 2nd and 4th repetitions.

However, the torque spikes occurring at each direction switch show only one, yet smooth, oscillation necessary to reduce the overshoot amplitude. The average peak torque amplitude is much higher than that experienced with the RLIFDC strategy at approximately ± 400 Nm for joint 1 and ± 100 Nm for joint 2 throughout all repetitions.

Hence, the application of the higher torque produced by the controller effectively achieves a greater improvement in tracking precision at the direction switches.

It has been demonstrated in Chapter 4 that the FLSAC strategy is extremely effective in reducing residual vibrations and elastic deformations of a flexible dynamics robot manipulator. Even greater precision and stable tracking can be achieved by applying repetitive cycle inputs to the FLSAC strategy.

While repetitive tracking with the RLIFDC strategy achieves no improvement in

(a)

(b)

(c)

(d)

Figure 5.7: RLFLSAC trajectories (a) Initial tracking with four repetitions, (b) Initial tracking at first direction switch, (c) Initial tracking with 1st and 2nd repetitions at first direction switch, (d) Initial tracking with 1st, 2nd, 3rd and 4th repetitions at first direction switch.

(a)

(b)

(c)

Figure 5.8: RLFLSAC joint torque (a) Joint torque for initial trajectory, (b) Joint torque for 2nd repetition, (c) Joint torque for 4th repetition.

control precision, adaptation of the control law with a FLS achieves greater improvement.

Chapter 6

Hybrid Control

The content of this chapter is based on published papers [64], [65] and [69].

A hybrid control strategy in the context of this chapter refers to an IFDC or FLSAC strategy augmented by a LQG dynamic regulator with either an EKF or FLAEKF as described in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3. These strategies are used to track the end effector of a two-link flexible robot manipulator described in Chapter 4, Section 4.1. A JTPD control law, given by Equation (3.8) in Chapter 3, is used with the IFDC strategy while a FLS adaptive JTPD control law given by Equation (4.21) in Chapter 4 is used with the FLSAC strategy.

In Section 6.1, tracking results are presented for the IFDC strategy augmented by a LQG dynamic regulator with an EKF or FLAEKF to determine the effectiveness of filtering Gaussian white process and measurement noise on the tracking performance of a flexible dynamics robot manipulator.

In Section 6.2, tracking results are presented for the FLSAC strategy augmented by a LQG dynamic regulator with an EKF or FLAEKF to determine the effectiveness of residual vibration suppression combined with filtering Gaussian white process and measurement noise on the tracking performance of a flexible dynamics robot manipulator.

In Section 6.3, tracking results are presented for an IFDC strategy augmented by a LQG dynamic regulator with an EKF or FLAEKF to determine the effectiveness of filtering Gaussian white process and measurement noise together with the effect of a collocated sensor, a noncollocated sensor at the end effector (NMP) and a NMP

corrective time delay in the feed-forward loop. This hybrid strategy is compared to the tracking performance of FLSAC alone.

6.1 Hybrid IFDC with Noise Filtering Strategy

A LQG dynamic regulator with either an EKF or FLAEKF for noise filtering augments the IFDC strategy shown in Figure 6.1. Because of difficulties in deriving and modeling NMP (zero dynamics) state estimators, the EKF and FLAEKF subroutine equations are those derived in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3 for the rigid dynamics robot manipulator. They are implemented in a LQG dynamic regulator with LQ state feedback gains [72], [92], [96], [139]. For both EKF and FLAEKF the LQG dynamic regulator control law is given by:

$$\tau_k = -\mathbf{K}\hat{\mathbf{x}}_k \quad (6.1)$$

Where; $\mathbf{K}$ is the LQ gain matrix for rigid robot dynamics.

The LQG dynamic regulator control law positively sums with the feed-forward reference control law to form a resultant control law given by:

$$\tau = \tau_r + \tau_k \quad (6.2)$$

or in expanded form:

$$\tau = \mathbf{J}^T(\theta) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] - \mathbf{K}\hat{\mathbf{x}}_k \quad (6.3)$$

Tracking results are shown in Figure 6.3 (a) and (b) for Gaussian white process and measurement noise streams given by $\mathbf{w}_k = \mathbf{N}(0,1)$ and $\mathbf{v}_k = \mathbf{N}(0,1)$. The hybrid strategy with a FLAEKF produces a trajectory with significantly reduced overshoot amplitudes at each direction switch and constitutes a major improvement over the results achieved with the EKF.

Figure 6.1: Hybrid IFDC with noise filtering strategy.

6.2 Hybrid FLSAC with Noise Filtering Strategy

A LQG dynamic regulator with an EKF or FLAEKF for noise filtering augments the FLSAC strategy shown in Figure 6.2. As stated in Section 6.1, the EKF and FLAEKF subroutine equations are those derived in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3 for the rigid dynamics robot manipulator. They are implemented in a LQG dynamic regulator with LQ state feedback gains [72], [92], [96], [139].

The optimal FLS is chosen from the results of Table 4.4, Chapter 4, Section 4.3 for implementation in the control strategy. Three triangular MF are used for each of the input and output variables as shown in Figure 4.45. Input values of δ_1 and δ_2 are normalized by gain $K_N = 5$. The adaptive reference control law is given by:

$$\tau_r = K_s \lambda \left\{ \mathbf{J}^T(\boldsymbol{\theta}) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] \right\} \quad (6.4)$$

For which $K_s = 15$.

Adding the control law given by Equation (6.1) for the LQG dynamic regulator with either an EKF or a FLAEKF to Equation (6.4) provides the resultant control law given by:

$$\tau = K_s \lambda \left\{ \mathbf{J}^T(\boldsymbol{\theta}) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] \right\} - \mathbf{K} \hat{\mathbf{x}}_k \quad (6.5)$$

Tracking results shown in Figures 6.4 (a) and (b) depict greatly reduced transient vibration overshoot amplitudes at the direction switches. The effects of EKF and FLAEKF state estimators are not discernable as the FLSAC results override them.

Figure 6.2: Hybrid FLSAC strategy with noise filtering strategy.

Figure 6.3: Hybrid IFDC and a LQG dynamic regulator with EKF or FLAEKF trajectories (a) Hybrid IFDC and LQG dynamic regulator with EKF, (b) Hybrid IFDC and a LQG dynamic regulator with FLAEKF.

Figure 6.4: Hybrid FLSAC and LQG dynamic regulator with EKF or FLAEKF noise filtering trajectories (a) Hybrid FLSAC and LQG dynamic regulator with EKF, (b) Hybrid FLSAC and LQG dynamic regulator with FLAEKF.

6.3 Hybrid Noise Filtering and Nonminimum Phase Strategy vs. FLSAC

An IFDC strategy is augmented by a LQG dynamic regulator with an EKF or FLAEKF to determine the effectiveness of filtering Gaussian white process and measurement noise given by $\mathbf{w}_k = N(0,1)$ and $\mathbf{v}_k = N(0,1)$, together with a collocated sensor, a noncollocated sensor at the end effector (NMP) and a NMP corrective time delay in the feed-forward loop. The tracking accuracy of this hybrid strategy is compared to FLSAC alone, as shown in Figure 6.5 (a).

Figure 6.5 (b) shows zoomed views of transient vibration overshoots at the first direction switch. NMP time delays cause a minimal forward shift in the trajectory because of the asynchronicity between feedback and commanded position data. The effect of a NMP corrective time delay is negligible. In comparison, FLSAC alone drastically reduces transient vibration overshoot amplitudes at direction switches.

Similarly, for the IFDC strategy augmented by a LQG dynamic regulator with a FLAEKF shown in Figure 6.6 (a), there is an overall improvement in tracking precision with significant reductions in transient vibration overshoot amplitude at each direction switch. However, the effect of both NMP and NMP corrective time delays are insignificant as shown in Figure 6.6 (b). Again, FLSAC alone drastically reduces transient vibration overshoot amplitudes at direction switches.

Tracking simulation clock times are 1 min 18 sec for an EKF with NMP correction, 1 min 21 sec for a FLAEKF with NMP correction and 1 min 40 sec for FLSAC alone.

The FLSAC strategy alone achieves control accuracy superior to a hybrid IFDC strategy augmented by a LQG with either an EKF or a FLAEKF together with a NMP time delay and NMP corrective time-delayed control action.

Figure 6.5: Hybrid IFDC and LQG with EKF, NMP and NMP corrective control, and FLSAC trajectories (a) Square trajectories, (b) Vibration transient overshoots at first direction switch (zoomed).

Figure 6.6: Hybrid IFDC and LQG with FLAEKF, NMP and NMP corrective control, and FLSAC trajectories (a) Square trajectories, (b) Vibration transient overshoots at first direction switch (zoomed).

In general, FLSAC overrides noise filtering in achieving tracking accuracy. Compared to classical control or more advanced adaptive control strategies, FLSAC gives better trajectory tracking while execution time remains acceptable.

Chapter 7

Intelligent Tracking Control of a Free-Flying Flexible Space Robot Manipulator

The content of this chapter is based on a published paper [71].

Book [18] and Sasiadek [122] raise significant concerns and issues on the greater utilization of autonomous intelligent space robots in future space operations. They draw particular attention to the need for control systems that could autonomously, intelligently and accurately perform manipulation tasks without a man-in-the-loop and within an acceptable execution time.

The ability to do this requires controllers that quickly manoeuvre long slender manipulator structures while suppressing their inherent flexible vibrations and deformations. Achieving development of such a controller initially requires modeling the robot manipulator dynamics as a true nonlinear DPS without approximation or linearization for implementation within an effective control strategy.

The research presented in this chapter is intended to provide potential solutions in addressing some of the difficulties raised by researchers on space robotics.

The end effector of a free-flying flexible space robot manipulator (FFFSRM) is tracked along a planar trajectory using IFDC and FLSAC strategies for comparison. The robot is required to track its end effector between two points in 2-D space while maintaining the orbiting rigid spacecraft at a fixed position in space. Simulation results show the IFDC strategy produces a trajectory that fails to reach its commanded final position by a wide margin in both coordinates.

The FLSAC strategy produces a more accurate trajectory closer to the commanded

final position in both coordinates. Sporadic torque spikes occur that are similar to the “bursting phenomenon” encountered with adaptive control systems [5]. Spacecraft translation and rotation is minimal in each case but slightly greater for the IFDC strategy.

7.1 Free-Flying Flexible Space Robot Manipulator (FFFSRM)

The FFFSRM shown in Figure 7.1 is a 2-D nonholonomic multibody system with two distributed parameter flexible links denoted as body 1 and body 2 attached to a rigid spacecraft [26], [84], [100], [101]. The flexible links are capable of planar rotation and elastic vibration. Hence, without holonomic constraints the system has 7 dof including two translational, three rotational and two elastic deformations.

The rigid spacecraft, denoted as body 0, is free to translate but its attitude is considered restrained by control moment gyro (CMG) wheels, i.e. attitude control system (ACS) on. When operating in the “ACS on” mode, the spacecraft is subjected to a holonomic constraint thereby reducing the dof to six.

Flexural vibration deformation functions of the links are denoted by u_1 and u_2 while δ_1 and δ_2 denote actual link deformations. Task forces, payloads and friction effects are neglected and the system operates in a zero-gravity space environment.

End-effector mass is excluded from the dynamics equations but distributed link masses and joint inertias are included. The spacecraft is a 6 m x 6 m square body with a rigid connecting link $L_0 = 3$ m and mass $m_0 = 600$ kg.

The flexible links have lengths $L_1 = L_2 = 4.5$ m and mass $m_1 = m_2 = 1.5075$ kg. Joint hub masses $m_{h1} = m_{h2} = 5$ kg include the total mass of all drive components and inertias.

The flexible robot equations of motion are derived using the dominant assumed mode of vibration for an Euler-Bernoulli cantilever beam [26].

Figure 7.1: Free-flying flexible space robot manipulator.

The FFFSRM is located in a low earth orbit (LEO) relative to an earth-centered earth-fixed (ECEF) inertial frame of reference, enabling it to perform tasks within the reachable workspace of the end effector. Control torques τ_1 and τ_2 actuate the shoulder and elbow joints. The end effector is required to track between two points in space from $x = 500$ m and $y = 512$ m to $x = 503$ m and $y = 508$ m relative to the spacecraft CM located in orbit at $\mathbf{R}_S = [x_S \ y_S]^T$ where $x_S = 500$ m and $y_S = 500$ m from the ECEF inertial frame of reference at $X = 400$ km and $Y = 400$ km. It is desirable to minimize spacecraft translation and avoid thruster jet firing with implications of operating efficiency and fuel economies while station keeping the spacecraft during tracking operations.

7.1.1 FFFSRM Dynamics

The FFFSRM equations of motion are derived in previously published research literature [26], [100], [101] and presented explicitly in Appendix D. They are derived using the Euler-Lagrange formulation in terms of kinetic T and potential U energies stored in the system. Given a generalized coordinates vector $\mathbf{q} = [x_S \ y_S \ \theta_0 \ \theta_1 \ \theta_2 \ \delta_1 \ \delta_2]^T$ the total kinetic and potential energies stored in the system are given by [26]:

$$T = \sum_{i=0}^2 T_i = \frac{1}{2} \sum_{i=0}^2 \int \rho_i \mathbf{V}_i^T \mathbf{V}_i dD_i = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}_{FF} \dot{\mathbf{q}}, \quad i = 0, 1, 2 \quad (7.1)$$

$$U = \sum_{i=0}^2 \frac{1}{2} \delta_i^2 K_i = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{K} \dot{\mathbf{q}}, \quad i = 1, 2 \quad (7.2)$$

In which;

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_2 \end{bmatrix} \quad (7.3)$$

is a stiffness matrix and;

$$K_i = \int_0^{L_i} EI_{L_i} (\varphi_i'')^2 dD_i, \quad i = 1, 2 \quad (7.4)$$

The control force vector is given by $\mathbf{F} = [F_x \ F_y \ \tau_0 \ \tau_1 \ \tau_2]^T$, in which F_x and F_y are thrust forces acting on the spacecraft CM along the x- and y-coordinate axes and τ_0 , τ_1 and τ_2 are joint actuator torques. Since the ACS is on, any reaction moments generated by the robot manipulator motion and disturbing the spacecraft attitude are compensated by the CMG wheels to maintain the spacecraft attitude at $\theta_0 = 0$.

In the kinematics and dynamics equations derived in Reference [26], angles θ_0 , θ_1 and θ_2 are measured from a datum on each joint axis parallel to a fixed horizontal coordinate axis. To formulate the direct kinematics equations and Jacobian matrix in accordance with the Denavit-Hartenberg convention, angle θ_0 is measured from the fixed horizontal x-coordinate axis while θ_1 is measured from the spacecraft y-coordinate axis and θ_2 measured from the longitudinal axis of link 1. Therefore, assigning α_0 , α_1 and α_2 as the angles given in Reference [26], the spacecraft and robot manipulator rotation angles used in this analysis are given by $\theta_0 = \alpha_0$, $\theta_1 = \alpha_1 - \alpha_0$ and $\theta_2 = \alpha_2 - \alpha_1$.

Using Equations (7.1) and (7.2) and the dominant cantilever assumed mode presented in Chapter 4, Subsection 4.1.2, the Lagrangian function is given by [15], [16], [26], [34],

[35], [36], [84], [100], [101]:

$$\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = T - U \quad (7.5)$$

For the FFFSRM subjected to force vector $\mathbf{F}$ acting on the generalized coordinates vector $\mathbf{q}$ the dynamic equations of motion are derived by differentiation of the Lagrangian function in the form of the following equation.

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \mathbf{F} \quad (7.6)$$

The resulting Euler-Lagrange matrix equations of motion including the rigid spacecraft dynamics are given by:

$$\boldsymbol{\tau} = \mathbf{M}_{FF}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} \quad (7.7)$$

$\mathbf{M}_{FF}(\mathbf{q})$ is a matrix of rigid and flexible inertia terms for the spacecraft and robot manipulator links, $\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})$ is a matrix of Coriolis and centrifugal forces, $\mathbf{K}$ is a stiffness matrix and $\mathbf{q} = [x_s \ y_s \ \theta_0 \ \theta_1 \ \theta_2 \ \delta_1 \ \delta_2]^T$ is a generalized coordinate vector of joint angles and elastic deformations.

7.2 Control Strategies

7.2.1 Fixed-Attitude-Restricted (FAR) Motion Jacobian for a FFFSRM

The direct kinematics equations of the FFFSRM are complex, computationally intensive and derived from the Denavit-Hartenberg (D-H) convention [128], [137]. They include terms for the spacecraft CM position, spacecraft rigid connecting link and flexible link distal end deformation functions u_i added to L_i to account for flexibility and the effects of NMP. The end-effector position vector relative to the ECEF inertial frame of reference is given by [26], [121]:

$$\mathbf{R}_E = \mathbf{f}(\mathbf{q}) \quad (7.8)$$

The end-effector Cartesian positions and link deformations are considered measured by noncollocated vibration and global positioning system aided inertial navigation system (GPS aided INS) position sensors.

A partitioned generalized Jacobian matrix (GJM) for a rigid space robot manipulator derived in previously published work is given by [145], [155], [156]:

$$\mathbf{J}^*(\boldsymbol{\theta}) = \left[\mathbf{J}_S^*(\theta_0) : \mathbf{J}_M^*(\boldsymbol{\theta}) \right] \\ = \begin{bmatrix} -L_0 \sin \theta_0 & -L_1 \sin(\theta_0 + \theta_1) & -L_2 \sin(\theta_0 + \theta_1 + \theta_2) \\ L_0 \sin \theta_0 & L_1 \cos(\theta_0 + \theta_1) & L_2 \cos(\theta_0 + \theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} K_a : 0 & 0 \\ K_b : K_b & 0 \\ K_c : K_c & K_c \end{bmatrix} \quad (7.9)$$

Using the manipulator partition of Equation (7.9) and by partial differentiation of the equations given by Equation (7.8) the Jacobian $\mathbf{J}_M^*(\boldsymbol{\theta})$ for a rigid space robot manipulator is derived as:

$$\mathbf{J}_M^*(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial x}{\partial \theta_0} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ K_b & 0 \\ K_c & K_c \end{bmatrix} \quad (7.10)$$

Where; $K_a = \frac{m_t m_0 b_0}{L_0}$, $K_b = m_0 + \frac{m_t m_1 b_1}{L_1}$, $K_c = m_0 + m_1 + \frac{m_t m_2 b_2}{L_2}$ are inertia terms

pertaining to each body within the multibody system and $b_0 = 1.5$ m, $b_1 = b_2 = 2.25$ m are distances along each link from its CM to the distal end. The matrix $\mathbf{J}_M^*(\boldsymbol{\theta})$ is termed a “fixed-attitude restricted” (FAR) motion Jacobian for a free-flying rigid robot [41], [145].

Extending this derivation to a flexible space robot manipulator and using Equation (D.33), Section D.4, Appendix D, we get the Cartesian x and y positions of the end effector relative to the ECEF inertial frame of reference given by:

$$\begin{aligned}
x &= x_S + (c_0 c_1 - s_0 s_1)(4.5 + 0.2\delta_2)c_2 - (c_0 s_1 + s_0 c_1)(4.5 + 0.2\delta_2)s_2 + \dots \\
&\quad \dots + c_0 c_1(4.5 + 0.2\delta_1) - s_0 s_1(4.5 + 0.2\delta_1) + 3c_0 \\
y &= y_S + (s_0 c_1 + c_0 s_1)(4.5 + 0.2\delta_2)c_2 + (c_0 c_1 - s_0 s_1)(4.5 + 0.2\delta_2)s_2 + \dots \\
&\quad \dots + s_0 c_1(4.5 + 0.2\delta_1) + c_0 s_1(4.5 + 0.2\delta_1) + 3s_0
\end{aligned} \tag{7.11}$$

Where; $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$, x_S and y_S are spacecraft CM positions relative to the ECEF inertial frame of reference. Values for u_i are computed online as the robot manipulator performs its task using Equation (4.7) in Chapter 4, Section 4.1 and given by [26]:

$$u_i = \varphi_i(x)\delta_i(t) = 0.2\delta_i(t), \quad i = 1, 2 \tag{7.12}$$

Since the spacecraft is initially located in an orbital position in space to perform its task, the spacecraft partition $\mathbf{J}_S^*(\theta_0)$ of the GJM for a flexible robot manipulator given by:

$$\mathbf{J}^*(\mathbf{q}) = [\mathbf{J}_S^*(\theta_0) : \mathbf{J}_M^*(\mathbf{q})] \tag{7.13}$$

is not used in the control law [145].

Applying the Jacobian differential form to the direct kinematics given by Equation (7.11), we get:

$$\mathbf{J}_M^*(\mathbf{q}) = \begin{bmatrix} \frac{\partial x}{\partial \theta_0} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \alpha_1} & \frac{\partial x}{\partial \alpha_2} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \alpha_1} & \frac{\partial y}{\partial \alpha_2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ K_b & 0 \\ K_c & K_c \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7.14}$$

An explicit derivation of Equation (7.14) is presented in Appendix D.

Also, spacecraft translational and attitude coordinates are included in the kinematics equations and its attitude angle θ_0 is included in $\mathbf{J}_M^*(\mathbf{q})$. The spacecraft is considered stationary in orbit. It is free to translate, but its rotation is restricted by the ACS to

prevent tumbling if acted upon by internal reaction forces and moments generated by motion of the manipulator [94], [95].

The matrix $\mathbf{J}_M^*(\mathbf{q})$ is termed a “fixed-attitude-restricted” (FAR) motion Jacobian for a free-flying flexible robot.

7.2.2 Fixed-Attitude-Restricted (FAR) Motion Jacobian Transpose Proportional-Derivative (JTPD) Control Law

The control law used to control the FFFSRM is similar to the JTPD control law used to control the fixed-base two-link flexible robot manipulator presented in Chapter 4. But, the Jacobian for the fixed-base robot manipulator is now substituted by $\mathbf{J}_M^*(\mathbf{q})$ for which the FAR motion JTPD control law is given by:

$$\boldsymbol{\tau} = \mathbf{J}_M^{*T}(\mathbf{q}) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] \quad (7.15)$$

The PD gains $\mathbf{K}_p$ and $\mathbf{K}_d$ are computed for the dominant cantilever assumed mode frequency given by Equations (4.15) and (4.16) in Chapter 4, Subsection 4.1.1:

7.2.3 IFDC Strategy

The IFDC strategy shown in Figure 7.2 is similar to that presented in Figure 4.2, Chapter 4, Subsection 4.2.1, but substituted with the FFFSRM equations of motion and the control law given by Equation (7.15) [2], [26], [71]. Joint torques feed into the flexible dynamics equations to actuate motion of the flexible manipulator. Joint accelerations output from the dynamics equations and pass through double integration to produce link slew angles θ_1 and θ_2 together with flexural deformations δ_1 and δ_2 and spacecraft attitude angle θ_0 . These output variables are fed back into the flexible dynamics equations in an inner loop and transform into Cartesian end-effector positions

Figure 7.2: IFDC strategy.

Figure 7.3: FLSAC strategy.

through direct kinematics equations given by Equation (7.11). End-effector positions x and y , and joint rates $\dot{\theta}_1$ and $\dot{\theta}_2$ transformed to $\dot{x}$ and $\dot{y}$ by Equation (D.38), Appendix D, are fed back to form position errors e_x and e_y , and velocity errors $\dot{e}_x$ and $\dot{e}_y$.

7.2.4 FLSAC Strategy

The FLSAC strategy shown in Figure 7.3 is similar to that presented in Figures 4.13 and 4.14, Chapter 4, Subsection 4.2.4, but substituted with the FFFSRM equations of motion, the optimal FLS with three triangular MF for the variables δ_1 , δ_2 and λ and an output scaling gain of $K_s = 15$ [26], [71]. The control law given by Equation (7.15) is modified to provide the FLS adaptive FAR motion JTPD control law given by:

$$\tau = K_s \lambda \left\{ \mathbf{J}_M^*{}^T(\mathbf{q}) \left[\mathbf{K}_p \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \mathbf{K}_d \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} \right] \right\} \quad (7.16)$$

7.3 Simulation Results for a FFFSRM

All simulation results are shown in Figures 7.4 to 7.11. Results shown in Figure 7.4 (a) and (b) demonstrate the difference in tracking control between IFDC and FLSAC strategies. Simulation of the models is computationally intense because of the complex robot dynamics. Simulation times are extremely slow; i.e. 73 min for the IFDC strategy and 115 min for the FLSAC strategy.

The IFDC strategy exhibits a change of course from a 1 m straight drift path in the x direction then reverts towards its final position at $x = 502.1$ m and $y = 506.7$ m, thereby failing to reach the commanded final position $x = 503$ m and $y = 508$ m. Drift motion of the manipulator causes translation of the spacecraft in the positive x and negative

(a) (b)
Figure 7.4: Spacecraft position and robot manipulator end-effector tracking trajectories (a) IFDC strategy, (b) FLSAC strategy.

(a) (b)
Figure 7.5: Spacecraft translation (a) IFDC strategy, (b) FLSAC strategy.

(a)

(b)

Figure 7.6: IFDC strategy link distal end deformations (a) Link 1 - δ_1 , (b) Link 2 - δ_2 .

(a)

(b)

Figure 7.7: FLSAC strategy link distal end deformations (a) Link 1- δ_1 , (b) Link 2- δ_2 .

Figure 7.8: FLS output variable λ

Figure 7.9: IFDC strategy joint torques (a) Link 1- τ_1 , (b) Link 2- τ_2 .

(a)

(b)

Figure 7.10: FLSAC strategy joint torques, (a) Link 1- τ_1 , (b) Link 2- τ_2 .

Figure 7.11: Spacecraft attitude - θ_0 (a) IFDC strategy, (b) FLSAC strategy.

y directions to $x_s = 500.074$ m and $y_s = 499.882$ m, as shown in Figure 7.5 (a), and rotation $\theta_0 = 6.5$ rad, shown in Figure 7.11 (a).

The FLSAC strategy exhibits a more accurate trajectory by reverting from a 0.5 m straight drift path in the x direction to its final position at $x = 502.52$ m and $y = 507.7$ m, closer to the commanded final position. The spacecraft translates in the positive x and negative y directions to $x_s = 500.072$ m and $y_s = 499.883$ m, as shown in Figure 7.5 (b), and rotation $\theta_0 = 6$ rad, shown in Figure 7.11 (a). The spacecraft rotation appears staggered and is considered to be caused by the effectiveness of the FLS in reducing the reaction forces and moments on the spacecraft, thereby retarding spontaneous rotation.

Link deformation amplitudes for the FLSAC strategy are larger than for the IFDC strategy as shown in Figures 7.6 (a) and (b) and 7.7 (a) and (b). However, a large amplitude deformation occurs in link 1 at approximately 600 simulation steps (ss) for both control strategies with amplitude for the FLSAC strategy being greater. But, the large value of FLS output variable λ adapts the control law to suppress it and produce more even deformation cycles with much lower peaks, as shown in Figures 7.7 (a) and (b) and 7.8. In contrast, deformation amplitudes for the IFDC strategy, although lower, are unabated as shown in Figures 7.6 (a) and (b) and considered to cause the uncontrolled drift of the end effector.

Control torque is shown in Figure 7.9 for the IFDC strategy and Figure 7.10 (a) and (b) for the FLSAC strategy. The magnitude of torque applied to each link for the IFDC strategy is initially high then decreases to between ± 200 Nm until the occurrence of large amplitude vibrations at approximately 600 ss. Torque applied to each of the joints using the FLSAC strategy ranges between ± 200 Nm throughout the tracking cycle.

But, sporadic and unpredictable high magnitude spikes, similar to the “bursting phenomenon” often encountered with adaptive control systems, occur throughout the simulation with the largest magnitudes occurring within the first 200 ss [5], [83], [26]. However, the FLSAC strategy adequately suppresses any torque-excited vibrations that might cause instabilities in system operation.

Despite the spacecraft being modeled in the “ACS on” mode with $\theta_0 = 0$ in the Matlab/Simulink™ model, a slight rotation occurs for both control strategies shown in Figure 7.11 (a) and (b), with a marginally greater rotation for the IFDC strategy. Together with spacecraft translation, the rotation is partly responsible for the failure of the end effector to reach its commanded final position. However, the magnitude of reaction moments and forces generated by flexible link vibrations are not sufficient to tumble the spacecraft [94].

Hence, the law of conservation of linear and angular momentum (nonholonomic constraint) for the whole space robot system is preserved because there is insufficient torque applied to prevent spacecraft rotation. Therefore, to track the end effector to the commanded final position, a method of modeling an appropriate magnitude of restraining torque should be applied in the control strategy simulation model.

Chapter 8

Summary of Results, Conclusions and Future Research

8.1 Summary of Motivation and Problems

This research was motivated by the need to investigate and derive a control strategy for autonomous intelligent control of long slender flexible robot manipulators primarily for deployment in space operations, e.g. space shuttle and the international space station. Slewing motion during tracking and other manipulation tasks causes flexing of the links and vibration mode excitation resulting in end-effector position inaccuracies. Currently, this difficulty in operation requires excessive time and effort to correct. The main issues identified and addressed are:

- (i) *Synthesizing inverse dynamics, optimal, stochastic and intelligent control strategies for providing simulated tracking control results for a fixed-base planar rigid robot manipulator.* The primary objective was to obtain results for tracking a square trajectory to be used as a basis of comparison of control performance for both cantilever and pinned-pinned dominant assumed modes PD gains. The IRDC and FLC strategies were further simulated to obtain results and assess control performance for repetitive tracking cycles. This investigation provided the groundwork in identifying an approach to synthesizing an intelligent control strategy suitable for application to more dynamically complex fixed-base flexible robot manipulators and ultimately free-flying flexible space robot manipulators.
- (ii) *Synthesizing an intelligent control strategy suitable for providing precision control of a fixed-base planar two-link flexible dynamics robot manipulator.* This extends the

developments on the rigid dynamics robot control strategies to modeling the highly nonlinear dynamics of a robot manipulator with distributed parameter flexible links. It is intended to more closely model a flexible dynamics robot manipulator without resort to linearization, lumped-parameter models, order-reduction or approximation of the dynamics equations that are generally easier to model and control. Further investigations on the flexible dynamics model include:

- Obtaining tracking results for both cantilever and pinned-pinned dominant assumed mode dynamics and PD gains.
- Obtaining tracking results for reduced- and full-order dynamics models to ascertain their closeness to suffice with a less complex mathematical model.
- Deriving and experimenting with fuzzy inference systems with different types and number of membership functions over a given positive range of the universe of discourse, and the determination of a heuristic design ratio of output scaling gain to number of membership function at minimum execution time.
- Modeling and determining the effects of NMP response by deriving time delay calculations based on elastic beam travelling wave theory.
- Simulating the effects on system stability for noncollocated sensors positioned alternately at three different locations along the outboard link while tracking the end effector.
- Consideration of total system stiffness of a flexible robot manipulator by describing a mathematical approach to deriving coupled flexible joint and link dynamics for simulation within an inverse flexible dynamics control strategy.
- Obtaining tracking results with joint inertia terms included in the flexible

dynamics equations for both IFDC and FLSAC strategies.

- Performing repetitive tracking cycles to assess control performance and stability for IRDC, FLC, IFDC and FLSAC strategies.
 - Synthesizing hybrid inverse dynamics control strategies, which include combinations of an EKF or a FLAEKF with and without FLSAC, collocated sensors, nonminimum phase time delays and their correction to assess and compare tracking results.
- (iii) *Synthesizing and simulating IFDC and FLSAC strategies for tracking a free-flying flexible space robot manipulator.* Obtaining tracking results for IFDC and FLSAC strategies to compare their performance and effects on a flexible space robot position, attitude and stability.

8.2 Fixed-Base Planar Two-Link Rigid Robot Manipulator

8.2.1 Theoretical Models

The rigid dynamics and kinematics given by Equations (3.5), (3.6), (3.7), (3.14) and (3.15) are derived in Appendix A. Euler-Lagrange theory for multibody dynamics is applied to a planar robot manipulator model with rigid links of 4.5 m in length without joint hub, end-effector or payload inertias or the effects of gravity and friction. The robot manipulator parameters are taken from Banerjee and Singhose [10]. The rigid dynamics model is used for simulation in each of the following control strategies:

8.2.1.1 Inverse Rigid Dynamics Control (IRDC): The IRDC control strategy shown in Figure 3.2 is synthesized using a Jacobian transpose control law with PD gains computed from cantilever and pinned-pinned dominant assumed mode frequencies and given by Equations (3.8), (3.9), (3.10), (3.11) and (3.12). Cantilever and pinned-pinned

dynamics boundary conditions are chosen as close representations of a physical robot configuration. Cantilever boundary conditions treat the joints as fixed at the proximal and free at the distal ends of each link, whereas pinned-pinned boundary conditions treat the joints as free to rotate at each end of a link. Cantilever boundary conditions present the greater challenge for control.

The Jacobian given by Equation (3.13) is derived in Appendix A, Subsection A.1.3 from the direct kinematics given by Equations (3.14) and (3.15). The transposed robot dynamics equations give joint acceleration as the dependent variable requiring inversion of the inertia matrix given by Equation (3.6). Joint accelerations are twice integrated to give joint rates and rotational displacements then transform to Cartesian positions and velocities of the end effector given by Equations (3.14), (3.15) and (3.16). Position and velocity data sum with commanded position and velocity data provided at intervals along the prescribed 12.6 m x 12.6 m square trajectory to form tracking errors used in computation of the control law.

8.2.1.2 Optimal Control (LQR): The LQR control strategy shown in Figure 3.5 has rigid robot dynamics state-space equations given by Equations (3.17) and (3.18) that are Lyapunov-linearized as presented in Appendix B, Subsection B.1.1. Only end-effector positions are fed back to form position errors while velocity errors are computed using derivatives of the position errors. Arbitrary weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ given by Equations (3.22) and (3.23) are chosen to form the algebraic Riccati equation (ARE) given by Equation (3.21) for which a solution matrix $\mathbf{P}$ is determined for calculation of the LQ gains matrix $\mathbf{K}$ given by Equation (3.20). The state feedback system is designed to minimize a performance index J given by Equation (3.19).

8.2.1.3 Fuzzy Logic Control (FLC): The FLC strategy shown in Figures 3.10 and 3.11 is synthesized with the dynamics equations being substituted by fuzzy inference systems (FIS) each representing one link of the robot manipulator. The FIS algorithms given by Equations (3.43), (3.44), (3.45) and (3.46) are designed in accordance with the theory outlined in Chapter 3, Sub-subsections 3.2.4.1 and 3.2.4.2 and based upon the torque and acceleration variables $T(1)$, $T(2)$, $A(1)$, $A(2)$ and $A(1-2)$ with Gaussian MF, shown typically for cantilever PD gains in Figure 3.12 (a) and (b) and 3.13 (a) and (b). Tables 3.1 and 3.2 are fuzzy rule matrices formulated for different combinations of the fuzzy variables in Equations (3.43), (3.44), (3.45) and (3.46). Link 1 is substituted by a FIS with nine fuzzy rules (i.e. nine MF for the input variable) and link 2 is substituted by a FIS with eighty-one fuzzy rules (i.e. nine MF for each of two input variables). Equations (3.39), (3.40), (3.41) and (3.42) provide range limits for output scaling gains K_1 and K_2 to obtain initial and final trajectory results shown in Figures 3.14 (a) and (b) and 3.15 (a) and (b). These ranges are determined experimentally by successive simulations with unit increments of K_1 and K_2 values. Normalizing gains $K_{N_1} = 200$, $K_{N_2} = 500$ and $K_{N_3} = 2$ are used to normalize the input values to each FIS appropriate to their respective universes of discourse.

8.2.1.4 Stochastic Control (LQG with KF, EKF and FLAEKF): The LQG control strategy shown in Figure 3.23 is fundamentally similar to that for the LQR strategy shown in Figure 3.5, except it embodies an LQG dynamic regulator with a KF, EKF or FLAEKF in its feedback loop for Gaussian noise disturbance rejection shown in Figures 3.24, 3.28 and 3.29 respectively. The underlying theory applicable to each filter subroutine is outlined in Chapter 3, Sub-subsections 3.2.6.1, 3.2.6.2 and 3.2.6.3 with

explicit equations presented in Appendix B.

The FLAEKF fuzzy rule matrix given in Table 3.5 is based upon Equations (3.74) and (3.76) and fuzzy variables for $\hat{\mathbf{x}}_k$, P_z and γ , each comprising four triangular MF (16 fuzzy rules), shown in Figure 3.30 (a), (b) and (c). Their universes of discourse are selected intuitively from an understanding of the noise characteristics of the robot system then finalized after some initial experimentation with ranges of $\hat{\mathbf{x}}_k$ and P_z values. The universe of discourse for γ is restricted to $0 \leq \gamma \leq 1$ as a necessary condition. For $\gamma = 1$ the FLAEKF subroutine equations reduce to those for an EKF.

Cross-correlated noise is modeled by deriving and including a cross-covariance term $\mathbf{N}_k$ into the EKF and FLAEKF subroutine equations.

8.2.1.5 Stability Analysis for Rigid Robot Dynamics Models: A theoretical stability analysis is presented in Chapter 3, Subsection 3.3.1, for the IRDC strategy using the JTPD control law given by Equation (3.82) and a Lyapunov function candidate given by Equation (3.83). Even though the property of negative definiteness does not hold in the Lyapunov method, global asymptotic stability is achieved by applying La Salle's theorem.

In Chapter 3, Subsection 3.3.2, the Matlab™ `lyap(A, Q)` function is applied to the state-space, optimal and stochastic control models, but fails to provide a unique solution to the Lyapunov function candidate given by Equation (3.92). The stability analysis is inconclusive.

In lieu of a rigorous stability analysis for FLC and FLAEKF strategies, a brief discussion is presented in Chapter 3, Subsection 3.3.3, on the lack of a generalized framework applicable to fuzzy logic control systems. But, the tracking results

demonstrate stability for all trajectories for which fuzzy logic is implemented.

Tracking results and closed-loop step input performance characteristics demonstrate local and global asymptotic stability for all strategies.

8.2.2 Simulation Results

8.2.2.1 Inverse Rigid Dynamics Control (IRDC): Tracking results shown in Figure 3.3 (a) and (b) for the IRDC control strategy are stable and validate the theoretical kinematics and dynamics models, control law and strategy. The trajectories for cantilever and pinned-pinned PD gains models show the prescribed 12.6 m x 12.6 m square trajectory with larger overshoots occurring at direction switches for the cantilever PD gains. The trajectory for pinned-pinned PD gains is better-behaved as the vibrations are less vigorous. Figures 3.4 (a) and (b) show joint torques in which the cantilever PD gains model demonstrates lower torque spikes at the direction switches than the pinned-pinned PD gains model. The pinned-pinned PD gains model responds more quickly to torque actuation at direction switches as shown in Table 3.3.

8.2.2.2 Optimal Control (LQR): Tracking results shown in Figure 3.6 (a) and (b) are stable and validate the theoretical kinematics and dynamics models, control law and strategy for the LQR control strategy. The trajectories for cantilever and pinned-pinned PD gains models show the prescribed 12.6 m x 12.6 m square trajectory with larger overshoot amplitudes occurring at direction switches for the cantilever PD gains. The trajectory for pinned-pinned PD gains is better-behaved as shown in Table 3.3 because the vibrations are less vigorous.

8.2.2.3 Fuzzy Logic Control (FLC): Initial tracking results shown in Figure 3.14 (a) and (b) and final tracking results shown in Figure 3.15 (a) and (b) are stable and validate

the theoretical models, control law and strategy for the FLC strategy. The initial trajectory for the cantilever PD gains model shows the prescribed 12.6 m x 12.6 m square trajectory with considerably large overshoot amplitudes at each direction switch, but rapidly dampens to a steady state. The initial trajectory for the pinned-pinned PD gains model has lower amplitude overshoots but oscillations continue before settling to a steady state. The final tracking results for both PD gains models exhibit greatly reduced overshoot amplitudes at direction switches.

The FLC strategy is robust for output scaling gains having the following large values and wide ranges; $8000 \leq K_1 \leq 200000$ and $2000 \leq K_2 \leq 80000$ for cantilever PD gains and, $50000 \leq K_1 \leq 1000000$ and $5000 \leq K_2 \leq 100000$ for pinned-pinned PD gains.

8.2.2.4 Combined Simulation Results (IRDC, LQR and FLC): Combined tracking results are presented for the first direction switch transients for IRDC, LQR and FLC strategies in Figure 3.16 (a) and (b). The IRDC strategy exhibits the largest overshoot for both cantilever and pinned-pinned PD gains models, smaller for the LQR strategy and the smallest for the FLC strategy. However, the end effector for the FLC strategy with pinned-pinned PD gains does not reach the corner position. End-effector linear velocities are shown in Figures 3.17 (a) and (b) and 3.18 (a) and (b). Velocity spikes occurring at each direction switch are highest for the IRDC strategy, lower for the LQR strategy and lowest for the FLC strategy. The FLC strategy has a greater ability to dampen the effects of switching direction with minimal overshoots. Simulation times for each strategy to track the complete trajectory are given in Table 3.4 and show the LQR strategy as the fastest at 32 sec and 39 sec for cantilever and pinned-pinned PD gains models respectively. The FLC strategy is seen as the slowest at 2 min 30 sec and 2 min 26 sec for

cantilever and pinned-pinned PD gains models respectively.

Closed-loop step input transients for inputs from $x = 9$ m to $x = 7$ m and $y = 0$ m to $y = 4$ m are shown in Figures 3.19 (a) and (b), 3.20, 3.21 (a) and (b) and 3.22 for cantilever and pinned-pinned PD gains models. Table 3.3 shows their respective computed performance characteristics. The LQR strategy exhibits the worst performance of all strategies for the cantilever PD gains model. In contrast, it exhibits best performance for the pinned-pinned PD gains model.

8.2.2.5 Stochastic Control (LQG with KF, EKF and FLAEKF): Tracking results shown in Figures 3.25 (a) and (b), 3.26 (a) and (b) and, 3.27 (a) and (b) for different combinations of process and measurement noise are stable and validate the theoretical kinematics and dynamics models, KF subroutine equations, control law and strategy for the LQG with a KF control strategy. The results show virtually no difference for various combinations of noise except for slightly larger overshoot amplitudes occurring at the second and third direction switches in Figure 3.27 (a) and (b).

Similarly, tracking results shown in Figures 3.31 (a) and (b), 3.32 (a) and (b), 3.33 (a) and (b), 3.34 (a) and (b), 3.35 (a) and (b), 3.36 (a) and (b) and 3.37 (a) and (b) for different combinations of process and measurement noise are stable and validate the theoretical kinematics and dynamics models, EKF and FLAEKF subroutine equations, control law and strategy for the LQG with EKF and FLAEKF control strategies. The results show some slight reduction in overshoot amplitudes in Figures 3.31 (b), 3.32 (b) and 3.33 (b). A tendency for divergence occurs with a slightly larger overshoot at the third direction switch in Figure 3.35 (a), but is diminished by the LQG with FLAEKF strategy that is more effective than LQG with EKF when the mean of noise stream

distributions are equal or covariance are unequal. Pronounced evidence of divergence occurs in Figure 3.37 (a) for the LQG with EKF, but it is fully rectified by the LQG with FLAEKF as shown in Figure 3.37 (b).

Closed-loop step input transients for inputs from $x = 9$ m to $x = 7$ m and $y = 0$ m to $y = 4$ m are shown in Figures 3.38 (a) and (b) and 3.39. Table 3.6 shows their respective computed performance characteristics. The LQG with KF strategy exhibits the best performance while the LQG with FLAEKF strategy exhibits the worst performance of all strategies. Tracking simulation times are given in Table 3.7 and range between 36 sec to 49 sec for all combinations of process and measurement noise.

8.3 Fixed-Base Planar Two-Link Flexible Robot Manipulator

8.3.1 Theoretical Models

8.3.1.1 Inverse Flexible Dynamics Control (IFDC): Flexible robot equations of motion are derived from Euler-Lagrange theory. The dominant assumed modes for cantilever and pinned-pinned Euler-Bernoulli beam boundary conditions represent a distributed parameter system subjected to transverse vibrations, as presented in Chapter 4, Subsection 4.2.1 and Appendix C. A reduced-order dynamic coupling vector is used initially for simulation. It is equivalent to the rigid dynamic coupling vector $C_r(\dot{\theta}, \theta)$, easier to model and faster to simulate when used with intelligent control techniques.

The flexible dynamics equations are implemented within an IFDC strategy. The control law, PD gains, direct kinematics and operational sequence are the same as those described and used for the IRDC strategy presented in Chapter 3. A 12.6 m x 12.6 m square trajectory is used to demonstrate control strategy performance.

Also, a full-order cantilever flexible dynamics coupling vector containing flexible

terms is modeled to compare tracking results with those obtained for the reduced-order model.

8.3.1.2 Fuzzy Logic System Adaptive Control (FLSAC): Reduced-order flexible dynamics equations are initially implemented within a FLSAC strategy shown in Figure 4.13. The control law, PD gains, direct kinematics and operational sequence are the same as those described for the IRDC strategy presented in Chapter 3. A 12.6 m x 12.6 m square trajectory is used to demonstrate control strategy performance.

The FIS comprises two fuzzy input variables assigned to link deformations δ_1 and δ_2 and one output variable λ to adapt the JTPD control law given by Equation (3.8) in Chapter 3. Link deformations are normalized by a gain $K_N = 5$ for input to the FLS. An adjustable output scaling gain K_s is included to modify the MF base widths for greater tracking control precision. A value of $K_s = 50$, initially used for simulation, is determined incrementally in units from $K_s = 1$ to 50 to achieve the most accurate tracking precision. Each FIS variable comprises nine Gaussian MF (81 fuzzy rules) given in Table 4.1, Chapter 4, chosen arbitrarily with a universe of discourse $-5.0 < \delta_i < 5.0$ for link deformations and $0 \leq \lambda \leq 1$ for the output variable.

Full-order cantilever flexible dynamics are modeled to demonstrate the accuracy and stability of the FLSAC strategy and compare it to using the reduce-order dynamics model.

8.3.1.3 Heuristic Design Ratio for an Optimal FLS: The FLSAC strategy with reduced-order flexible dynamics equations of motion is used to determine an optimal FLS based on a combination of tracking precision and execution time. Fuzzy variables with three (9 fuzzy rules), five (25 fuzzy rules), seven (49 fuzzy rules) and nine (81 fuzzy

rules) triangular and Gaussian MF are modeled with different K_s values of 1, 15, 25, 35 and 45, given in Tables 4.4 to 4.7 to establish a heuristic design ratio defined as the quotient of K_s and the number of MF, i.e. $\frac{K_s}{\text{NMF}} = 5$, for the fastest execution time.

8.3.1.4 Nonminimum Phase (NMP) Response and Sensor Location: NMP response of the flexible robot manipulator described in Chapter 4 is modeled by implementing time delays in the feedback loops of the IFDC and FLSAC control strategies for cantilever and pinned-pinned assumed mode dynamics models. For the FLSAC strategy, the FIS initially comprises nine Gaussian MF and the output scaling gain K_s is omitted. Also, the universe of discourse range limits for the output variable are $0 \leq \lambda \leq 20$.

Time delays are calculated to represent the phase shift caused by a transverse beam vibration wave travelling from the actuator to the end effector where a sensor is located. A time delay is input to the control action to compensate for the NMP response. Time delays are calculated using the wave velocity equation given by Equation (4.25) in Chapter 4, Subsection 4.4.1. For maximum effect, the largest time delay for the total robot manipulator length is selected for implementation in the Matlab/Simulink™ model to represent the time for wave travel from joint 1 actuator to the end effector.

Time delays are also implemented in the FLSAC strategy with the optimal FIS of three triangular MF at $K_s = 15$, to control the effect of sensors noncollocated alternately at positions 0.5 m, 2.25 m and 4.0 m on the outboard link 2 from joint 2 on tracking accuracy and stability. The noncollocated sensor positions are modeled by substituting values for L_2 in the direct kinematics equations together with the appropriate time delays given in Chapter 4, Subsection 4.4.2.

8.3.1.5 Total System Stiffness: Joint inertias are included in the flexible cantilever

dynamics equations with a magnitude representing all drive train components, i.e. actuator, gears, rotors and hub, as a single rigid component.

A theoretical discussion is presented on the mathematical modeling of flexible-joint dynamics coupled with flexible-link dynamics. The derivation of a fourth-order differential equation to couple flexible-joint drive train dynamics with flexible-link dynamics for a flexible robot manipulator system is suggested as a topic for further research.

8.3.1.6 Stability Analysis for Flexible Robot Dynamics Models: A theoretical stability analysis is presented in Chapter 4, Section 4.6, for the IFDC and FLSAC strategies using the JTPD control law given by Equation (4.35) and a Lyapunov function candidate given by Equation (4.36). The property of negative definiteness does not hold in the Lyapunov method, but global asymptotic stability is achieved by the application of La Salle's theorem. This result is corroborated by the stable trajectories achieved for all tracking results and closed-loop step input performance characteristics.

8.3.2 Simulation Results

8.3.2.1 Inverse Flexible Dynamics Control (IFDC): Tracking results shown in Figure 4.3 (a) and (b) for the IFDC control strategy are stable and validate the theoretical kinematics and dynamics models, control law and strategy. The trajectories for both cantilever and pinned-pinned PD gains models show the prescribed 12.6 m x 12.6 m square trajectory with larger overshoots occurring at direction switches for the cantilever PD gains. The trajectory for pinned-pinned PD gains is more accurate with smaller vibration overshoot amplitudes at direction switches. Figures 4.4 (a) and (b) show a distinct difference in overshoot amplitudes between cantilever and pinned-pinned

assumed mode dynamics at the first direction switch. However, the transient vibrations for the pinned-pinned dynamics model are slow to decay and continue along the side of the square trajectory for approximately half its length.

Also, tracking results shown in Figure 4.9 (a) and (b) in Chapter 4, Subsection 4.2.3 are stable and validate the full-order flexible cantilever assumed mode dynamics model. The transient vibration overshoot peak amplitude is slightly less than that obtained with the reduced-order dynamics model. It is most probable, that the addition of elastic terms in the dynamic coupling vector causes the elastic deformations of link 1 to become skewed to the positive side of the neutral axis as shown in Figure 4.10 (a). Figure 4.12 also shows a slight fluctuation in the closed-loop step input transient for the y coordinate.

8.3.2.2 Fuzzy Logic System Adaptive Control (FLSAC): Tracking results shown in Figure 4.16 (a) and (b) for the FLSAC strategy are stable and validate the theoretical kinematics and dynamics models, control law and strategy. The trajectories are very close to those commanded. Transient overshoot amplitudes are greatly reduced at direction switches for both cantilever and assumed mode dynamics models compared to those obtained with the IFDC strategy. But, this precision is gained at the expense of a much higher computational time of 3 min 45 sec for the cantilever dynamics model and 5 min 5 sec for the pinned-pinned assumed mode dynamics model to complete the square trajectory as given in Table 4.3.

Similarly, the trajectory for the full-order dynamics model is even more accurate exhibiting lower transient vibration overshoot amplitudes at direction switches. The computational time burden is even greater at 12 min 32 sec to complete the square trajectory as given in Table 4.3.

Table 4.2 shows the best overall closed-loop step input performance is achieved for the FLSAC strategy with the pinned-pinned assumed mode dynamics model. The worst is the IFDC strategy with full-order dynamics. Also, the FLSAC strategy is best overall for all dynamics models.

8.3.2.3 Heuristic Design Ratio for an Optimal FLS: Tracking results shown in Figures 4.29 (a) and (b) to 4.38 (a) and (b), Chapter 4, Section 4.3, for the FLSAC control strategy using triangular and Gaussian MF with different K_s values are stable and validate the theoretical kinematics and dynamics models, control law and strategy. Gaussian MF generally demonstrate greater control precision but at a higher computational time burden.

A visual inspection of the tracking results and summary of execution times for each combination of universe of discourse range, NMF and K_s value enabled the determination of a heuristic design ratio of 5.

8.3.2.4 Nonminimum Phase Response and Sensor Location: Tracking results for the IFDC strategy with cantilever and pinned-pinned assumed mode dynamics models and sensors collocated at joint 2 and noncollocated at the end effector are shown in Figures 4.39 (a) and (b), 4.40 (a) and (b) and 4.41 (a) and (b), Chapter 4, Subsection 4.4.1. The trajectories are stable and exhibit minimal differences in performance when NMP time delays are implemented.

Tracking results shown in Figures 4.42 (a) and (b), 4.43 (a) and (b) and 4.44 (a) and (b) depict the difference in performance between cantilever and pinned-pinned assumed modes reduced-order dynamics models using the FLSAC control strategy. A significant reduction in vibration amplitude is achieved for both cantilever and pinned-pinned

modes. Small increases in the first direction switch transient vibration overshoot amplitudes are exhibited in Figure 4.44 (a) and (b) caused by the NMP time delay and even smaller increases caused by the NMP corrective time delay. However, the effects of implementing time delays to model NMP response and its correction are insignificant compared to the overriding effect of the FLS adaptive control action. For both strategies, results are included for tracking without time delays to model collocated sensors only at joints 1 and 2 for comparison.

Tracking results for a sensor noncollocated alternately at positions 0.5 m, 2.25 m and 4.0 m along the outboard link from joint 2 shown in Figures 4.46 (a) and (b), 4.47 (a) and (b), 4.48 (a) and (b), 4.49 (a) and (b), 4.50 (a) and (b) and 4.51 (a) and (b), Chapter 4, Subsection 4.4.2, are stable and exhibit minimal difference between NMP and NMP correction. It is evident from the results shown in Figure 4.50 (a) and (b), that the FLSAC strategy maintains smooth stable trajectories as the sensor is located closer to joint 2. In contrast, the IFDC strategy produces results that become erratic as the sensor is located closer to joint 2, caused by the elastic vibrations at the distal end of link 1.

8.3.2.5 Total System Stiffness: Tracking results for reduced- and full-order cantilever assumed mode dynamics models are shown in Figures 4.54 (a) and (b) and 4.55 (a) and (b) for IFDC and FLSAC strategies with a combined link flexibility and rigid joint inertia dynamics model. The IFDC strategy results are closely similar for both reduced- and full-order dynamics models and completely lack any precision in tracking the commanded square trajectory. However, extremely high precision tracking is achieved with the FLSAC strategy by using a FIS with three triangular MF at $K_s = 360$. The results for both strategies are stable and validate the theoretical kinematics, dynamics and

control strategy models.

8.4 Repetitive Learning Control

8.4.1 Theoretical Models

8.4.1.1 Repetitive Learning Inverse Rigid Dynamics Control (RLIRDC): The RLIRDC strategy is identical to the IRDC strategy described in Chapter 3, Subsection 3.2.2 and shown in Figure 3.2, but has four commanded repetitive trajectory input cycles.

8.4.1.2 Repetitive Learning Fuzzy Logic Control (RLFLC): The RLFLC strategy is identical to the FLC strategy described in Chapter 3, Sub-subsection 3.2.4.5 and shown in Figure 3.10 and 3.11, but has four commanded repetitive trajectory input cycles.

8.4.1.3 Repetitive Learning Inverse Flexible Dynamics Control (RLIFDC): The RLIFDC strategy is identical to the IFDC strategy described in Chapter 4, Subsection 4.2.1 and shown in Figure 4.2, but has four commanded repetitive trajectory input cycles.

8.4.1.4 Repetitive Learning Fuzzy Logic System Adaptive Control (RLFLSAC): The RLFLSAC strategy is identical to the FLSAC strategy described in Chapter 4, Subsection 4.2.4 and shown in Figure 4.13 and 4.14, but has four commanded repetitive trajectory input cycles.

8.4.2 Simulation Results

8.4.2.1 Repetitive Learning Inverse Rigid Dynamics Control (RLIRDC): Tracking results for the RLIRDC strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

Except for a slight reduction in the transient vibration overshoot amplitude at the lower left direction switch shown in Figure 5.1 (a), repetitive cycling of the trajectory produces negligible improvement as shown in Figure 5.1 (b), (c) and (d).

8.4.2.2 Repetitive Learning Fuzzy Logic Control (RLFLC): Tracking results for the RLFLC strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

Much reduced transient vibration overshoot amplitudes are achieved at each direction switch for the initial cycle, and then further reductions are gained on each trajectory cycle repetition, as shown in Figure 5.3 (a), (b), (c) and (d).

8.4.2.3 Repetitive Learning Inverse Flexible Dynamics Control (RLIFDC): Tracking results for the RLIFDC strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

The results are similar to those for RLIRDC. The overall reduction in overshoot amplitude at each direction switch is negligible, except for a slight reduction at the lower left direction switch shown in Figure 5.5 (a). However, minimal reductions in amplitude can be observed for each trajectory repetition as shown for the first direction switch in Figure 5.5 (b), (c) and (d).

8.4.2.4 Repetitive Learning Fuzzy Logic System Adaptive Control (RLFLSAC): Tracking results for the RLFLSAC strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

Greatly reduced transient vibration overshoot amplitudes are achieved at direction switches for the initial cycle, as shown in Figure 5.7 (a), (b), (c) and (d). Further reductions are gained upon each repetition. However, the overall reduction is not as large as that for RLFLC because of the inherently greater complexity and higher-order nonlinearity of the flexibility terms in the dynamics equations.

8.5 Hybrid Control

8.5.1 Theoretical Models

A hybrid control strategy in the context of Chapter 6 refers to an IFDC or FLSAC strategy augmented by a LQG dynamic regulator with either an EKF or FLAEKF as described in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3. These strategies are used to track the end effector of a two-link flexible robot manipulator described in Chapter 4, Section 4.1. A JTPD control law, given by Equation (3.8) in Chapter 3, is used with the IFDC strategy while a FLS adaptive JTPD control law, given by Equation (4.21) in Chapter 4, is used with the FLSAC strategy.

8.5.1.1 Hybrid IFDC with Noise Filtering Strategy: The hybrid IFDC strategy shown in Figure 6.1 and described in Chapter 6, Section 6.1, is augmented by a LQG dynamic regulator with an EKF or a FLAEKF described in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3.

8.5.1.2 Hybrid FLSAC with Noise Filtering Strategy: The hybrid FLSAC strategy shown in Figure 6.2 and described in Chapter 6, Section 6.2, is augmented by a LQG dynamic regulator with an EKF or FLAEKF described in Chapter 3, Sub-subsections 3.2.6.2 and 3.2.6.3.

8.5.1.3 Hybrid Noise Filtering and Nonminimum Phase Strategy vs. FLSAC: The hybrid IFDC strategy shown in Figure 6.1 and described in Chapter 6, Section 6.1, is augmented by a LQG dynamic regulator with an EKF or FLAEKF to determine the effectiveness of filtering Gaussian white process and measurement noise together with; a collocated sensor, a noncollocated sensor at the end effector (NMP) and NMP corrective time delays in the feed-forward loop. This hybrid IFDC strategy is compared to FLSAC

alone.

8.5.2 Simulation Results

8.5.2.1 Hybrid IFDC with Noise Filtering Strategy: Significantly reduced overshoot amplitudes are shown in Figure 6.3 (b) at each direction switch of a square trajectory for the IFDC strategy augmented by a LQG dynamic regulator with a FLAEKF to filter Gaussian white process and measurement noise streams given by $\mathbf{w}_k = N(0,1)$ and $\mathbf{v}_k = N(0,1)$. Less significant results are achieved with the strategy augmented by a LQG dynamic regulator with EKF.

Tracking results for the hybrid IFDC with noise filtering strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

8.5.2.2 Hybrid FLSAC with Noise Filtering Strategy: Tracking results shown in Figure 6.4 (a) and (b) appear nearly identical for both EKF and FLAEKF state estimators used to filter Gaussian white process and measurement noise streams given by $\mathbf{w}_k = N(0,1)$ and $\mathbf{v}_k = N(0,1)$. But, their effects are overridden by FLSAC that greatly reduces transient vibration overshoots at direction switches.

The results for the hybrid FLSAC with noise filtering strategy are stable and validate the theoretical kinematics, dynamics and control strategy models.

8.5.2.3 Hybrid Noise Filtering and Nonminimum Phase Strategy vs. FLSAC:

Tracking results are shown in Figures 6.5 (a) and (b) and 6.6 (a) and (b) for comparison between the effects of noise filtering using hybrid IFDC augmented by a LQG with an EKF or a FLAEKF together with NMP time delays and NMP time delay correction vs. FLSAC alone. Results demonstrate a minimal effect in transient vibration overshoot amplitude reduction caused by either LQG with an EKF or FLAEKF, or with a NMP

corrective time delay. A significant reduction in transient vibration overshoot amplitude is achieved with FLSAC alone.

The results shown in Figures 6.5 (a) and (b) and 6.6 (a) and (b) are stable and validate the theoretical kinematics, dynamics and control strategy models.

8.6 Intelligent Tracking Control of a Free-Flying Flexible Space Robot Manipulator

8.6.1 Theoretical Models

8.6.1.1 Free-Flying Flexible Space Robot Manipulator (FFFSRM): The dynamic equations of motion of a 2-D free-flying flexible space robot manipulator shown in Figure 7.1 are based on the Euler-Lagrange mathematical formulation in conjunction with the conservation of momentum law. The set of equations are those derived by Chen and Meirovitch [26], who decouple the rigid dynamics from the flexible dynamics to enable synthesizing the control strategy into two subsystems; inverse dynamics for the rigid dynamics and LQR for the flexible dynamics with rigid-body dynamics perturbations.

In chapter 7, fully coupled rigid and flexible dynamics equations are used to model a multibody DPS robot manipulator mounted on a rigid spacecraft. The FFFSRM equations of motion are presented in Chapter 7, Subsection 7.1.1 and derived in Appendix D. Flexible dynamics terms are calculated using the assumed modes method to model elastic link deformations. The fixed-attitude-restricted (FAR) motion Jacobian, derived using direct kinematics equations containing elastic deformation terms, is presented in Chapter 7, Subsection 7.2.1 and Appendix D.

The flexible manipulator is free to rotate and elastically deform about the two rotational joints, while the spacecraft is free to translate in two directions. The spacecraft attitude is considered held constant in the “ACS on” mode by control moment gyro

(CMG) wheels by inserting $\theta_0 = 0$ for spacecraft attitude in the Matlab/Simulink™ model.

8.6.1.2 IFDC Strategy: The control strategy shown in Figure 7.2 and described in Chapter 7, Subsection 7.2.3 is similar to the IFDC strategy shown in Chapter 4, Figure 4.2, but comprises the FFFSRM equations of motion and the FAR motion JTPD control law given by Equation (7.15) in Chapter 7, Subsection 7.2.2.

8.6.1.3 FLSAC Strategy: The control strategy shown in Figure 7.3 and described in Chapter 7, Subsection 7.2.4 is similar to the FLSAC strategy shown in Chapter 4, Figures 4.13 and 4.14, but comprises the FFFSRM equations of motion, the FLS adaptive FAR motion JTPD control law given by Equation (7.16) in Chapter 7, Subsection 7.2.4 and the optimal FLS with three triangular MF and $K_s = 15$.

8.6.2 Simulation Results

8.6.2.1 IFDC Strategy: Tracking results are stable and validate the theoretical kinematics, dynamics, control law and strategy.

The trajectory for the point-to-point end-effector tracking task shown in Figure 7.4 (a) depicts a tendency for the spacecraft and manipulator to drift linearly in the positive x direction for about 1 m before the joint actuators manoeuvre the end effector in the negative y direction towards the commanded final position.

The end effector fails to attain the commanded final position of $x = 503$ m and $y = 508$ m by $x = 0.9$ m and $y = 1.3$ m. Commensurate with this motion is the linear translation of the spacecraft CM of $x_s = 0.074$ m and $y_s = -0.128$ m shown in Figure 7.5 (a), and rotation $\theta_0 = 6.5$ rad, shown in Figure 7.11 (a). Link deformations and joint torques are shown in Figures 7.6 (a) and (b), and 7.9.

8.6.2.2 FLSAC Strategy: Tracking results are stable and validate the theoretical kinematics, dynamics, control law and strategy.

The trajectory for the point-to-point end-effector tracking task shown in Figure 7.4 (b) depicts a tendency for the spacecraft and manipulator to drift linearly in the positive x direction for 0.5 m and slightly in the negative y direction before the joint actuators manoeuvre the end effector in the negative y direction towards the commanded final position. Commensurate with this motion is the linear translation of the spacecraft CM of $x_s = 0.072$ m and $y_s = -0.117$ m shown in Figure 7.5 (b), and rotation $\theta_0 = 6$ rad, shown in Figure 7.11 (b). Link deformations, FLS output variable λ and joint torques are shown in Figures 7.7 (a) and (b), 7.8 and 7.10 (a) and (b). The end effector falls short of its commanded final position of $x = 503$ m and $y = 508$ m by $x = 0.48$ m and $y = 0.3$ m.

High magnitude input joint torque spikes occur during the tracking task similar to the “bursting phenomenon” encountered with adaptive control systems. The FLSAC strategy control law dampens the torque spikes and stability is maintained.

Despite the spacecraft being modeled in “ACS on” mode with $\theta_0 = 0$ in the Matlab/Simulink™ model, a slight spacecraft rotation occurs for both control strategies. The rotation for the IFDC strategy is marginally greater by 0.5 rad. The law of conservation of linear and angular momentum (nonholonomic constraint) for the space robot system is preserved because there is insufficient restraining torque to prevent spacecraft rotation. The spacecraft rotation is partly responsible for the failure of the end effector to reach the commanded final position.

Therefore, to achieve precise tracking of the end effector to the commanded final position, an appropriate magnitude of restraining torque should be modeled in the control

strategy simulation. However, the magnitude of reaction moments and forces generated by flexible link vibrations are not sufficient to tumble the spacecraft and stability is maintained [94].

Greater accuracy is achieved with the FLSAC strategy than with the IFDC strategy.

8.7 Conclusions

Six strategies are investigated for tracking the end effector of a fixed-base planar two-link rigid dynamics robot manipulator with long slender links along a square trajectory. The strategies include; inverse dynamics, LQR, FLC and LQG with KF, EKF and FLAEKF dynamic regulators. The FLC strategy is one form of intelligent control applicable to robot control and achieves the highest precision tracking results for both cantilever and pinned-pinned dominant assumed mode PD gains. The LQG dynamic regulator with a FLAEKF eliminates the divergent tracking experienced with an EKF and improves results generally.

Results obtained for the rigid dynamics robot manipulator provides a direction in applying intelligent control to DPS flexible multibody robot dynamics models that are more complex. The IRDC strategy is identified as having the greatest potential for substituting a DPS flexible multibody dynamics model and integrating an intelligent control subsystem for suppressing elastic link vibrations.

Tracking with a FLSAC strategy achieves much greater precision than IFDC for both cantilever and pinned-pinned assumed mode dynamics and demonstrates advancement in the intelligent control of DPS flexible multibody robots. The FLSAC strategy is further investigated with different FIS structures to establish a heuristic ratio for optimal design. Time delays are introduced into feedback loops of the IFDC and FLSAC strategies to

model NMP response and in the feed-forward loop to provide corrective control.

Stable tracking results are achieved with a sensor located at different positions along the outboard link of the robot manipulator for both IFDC and FLSAC strategies. However, the FLSAC strategy maintains greater control over the more severe effects of vibration disturbances occurring when the sensor is located closer to the elbow joint.

Repetitive tracking cycle results demonstrate the FLC strategy trains a rigid dynamics robot and the FLSAC strategy trains a flexible dynamics robot to converge towards the commanded trajectory after four repetitions. IRDC and IFDC strategies are ineffective in repetitive learning control.

The synthesis of hybrid IFDC/LQG with EKF or FLAEKF strategies and NMP time delays and time-delay corrective control have minimal effect on control precision compared to the overriding precision achieved with the FLSAC strategy.

Adding rigid joint hub inertias to both reduced- and full-order flexible dynamics models causes an irregular trajectory result using the IFDC strategy. But, the FLSAC strategy provides more precise and stable control.

When applied to the control of a free-flying flexible space robot manipulator, the FLSAC strategy directs the end effector more closely to its commanded final position with greater precision than the IFDC strategy but at a higher computational time burden.

Intelligent control in the form of three distinct fuzzy logic systems demonstrates a high degree of precision tracking within different strategies. The major advantages of intelligent control are; precision, robustness, versatility in controlling the effects of a variety of disturbances and robot system parameters, flexibility in combination with other control strategies, ease of development and stability in operation. However, the

computational time is high when using fuzzy logic.

8.8 Future Research

Future research considerations include; using dynamics models with higher modes for pinned-pinned, cantilever and pinned-free boundary conditions; an online automatic K_s scheduling subsystem to accommodate variations in payload and robot parameters; actuation of thruster jets for spacecraft manoeuvring and robot manipulator reaction control; a three-dimensional DPS dynamics model; multimodal vibration suppression; consideration of multimode effects on noncollocated sensors; different configurations of cooperating robot manipulators; obstacle avoidance; sensor fusion using GPS aided INS data within a hybrid FLSAC/LQG with FLAEKF control strategy; and the derivation of dynamics equations with coupled joint-link flexibilities together with the synthesis of a suitable intelligent control strategy.

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Appendix A

Fixed-Base Planar Two-Link Rigid Robot Manipulator Kinematics and Dynamics Equations

Consider the two-link robot manipulator shown in Figure A.1. Each link is of equal length $L_1 = L_2$, mass $m_1 = m_2$ and mass moment of inertia $I_{L1} = I_{L2}$ where the centroid of each link are respectively at points c_1 and c_2 located midway along each link. Each joint is driven by a torque τ_1 and τ_2 . Actuator, drive mechanism and hub inertias are excluded.

A.1 Kinematics

A.1.1 Position Kinematics

The planar two-link rigid manipulator shown in Figure A.1 has the fixed-base frame of reference (shoulder joint) with its origin at O_0 defined by $O_0X_0Y_0Z_0$ where the Z_0 axis is normal to the X_0Y_0 plane. By the Denavit-Hartenberg convention the frame of reference $O_1X_1Y_1Z_1$ (elbow joint) and frame of reference $O_2X_2Y_2Z_2$ (end effector) are established, where the Z_1 and Z_2 axes lie normal to the XY plane. Frames of reference (frames) $O_0X_0Y_0Z_0$ and $O_1X_1Y_1Z_1$ each have a single rotational degree-of-freedom whereas frame $O_2X_2Y_2Z_2$ has two translational degrees-of-freedom generated by rotation at frames $O_0X_0Y_0Z_0$ and $O_1X_1Y_1Z_1$. Selection of these frames of reference is achieved by assigning a homogeneous matrix A_i that transforms the coordinates of frame i to frame $i-1$ and A_i is a function of only joint variable θ_i , expressed as [9], [19], [115], [128], 137]:

$$A_i = A_i(\theta_i) \quad (A.1)$$

The matrix given by Equation (A.1) is obtained from the general theory of homogeneous transformations in which matrix A_i represents a product of four transformations given by:

Figure A.1: Fixed-base planar two-link rigid robot manipulator parameters.

$$\mathbf{A}_i = \text{Rot}_{Z,\theta_i} \cdot \text{Trans}_{Z,d_i} \cdot \text{Trans}_{X,L_i} \cdot \text{Rot}_{X,\alpha_{t_i}} \quad (\text{A.2})$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_{t_i} & -\sin\alpha_{t_i} & 0 \\ 0 & \sin\alpha_{t_i} & \cos\alpha_{t_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{t_i} & \sin\theta_i \sin\alpha_{t_i} & L_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{t_i} & -\cos\theta_i \sin\alpha_{t_i} & L_i \sin\theta_i \\ 0 & \sin\alpha_{t_i} & \cos\alpha_{t_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where; rotation angle θ_i , link length L_i , offset d_i and twist angle α_{t_i} are link parameters.

In general; from the theory on homogeneous transformations a homogeneous matrix that transforms the coordinates of a point from frame j to frame i is called a transformation matrix and consists of the product of homogeneous matrices pertaining to each frame comprising a serial link system denoted by [9], [19], [115], [137]:

$$\mathbf{T}_i^j = \mathbf{A}_{i+1} \mathbf{A}_{i+2} \dots \mathbf{A}_{j-1} \mathbf{A}_j \quad \text{if } i < j \quad (\text{A.3})$$

$$\mathbf{T}_i^j = \mathbf{I} \quad \text{if } i = j \quad (\text{A.4})$$

$$\mathbf{T}_i^j = (\mathbf{T}_i^j)^{-1} \quad \text{if } j > i \quad (\text{A.5})$$

However, the planar robot manipulator shown in Figure A.1 has only rotational degrees-of-freedom and no twist. Therefore, the only parameters required in the transformation computations are θ_i and L_i . Omitting the offset and twist parameters from the transformation equations we get:

$$\mathbf{A}_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.6})$$

Using transformation notation;

$$\mathbf{T}_0^1 = \mathbf{A}_1 \quad (\text{A.7})$$

$$\mathbf{A}_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.8})$$

Hence, the position of the end effector may be represented in terms of the fixed base frame by a homogeneous transformation generated by the product of homogeneous matrices and given by:

$$\mathbf{T}_0^2 = \mathbf{A}_1\mathbf{A}_2 = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1 + \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{A.9})$$

The first two entries of the fourth column in Equation (A.9) are the Cartesian x and y positions of the origin of frame $O_2X_2Y_2Z_2$ relative to base frame coordinates, i.e.:

$$x = L_1\cos\theta_1 + L_2\cos(\theta_1 + \theta_2) \quad (\text{A.10})$$

$$y = L_1\sin\theta_1 + L_2\sin(\theta_1 + \theta_2) \quad (\text{A.11})$$

Direct kinematics equations (A.10) and (A.11) are used in the simulations presented in Chapters 3, 4, 5 and 6.

A.1.2 Velocity Kinematics

Differentiating Equations (A.10) and (A.11), we obtain the Cartesian velocities of the end effector given by:

$$\dot{x} = -L_1 \sin \theta_1 \dot{\theta}_1 - L_2 \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \quad (\text{A.12})$$

$$\dot{y} = L_1 \cos \theta_1 \dot{\theta}_1 + L_2 \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \quad (\text{A.13})$$

A.1.3 Rigid Robot Manipulator Jacobian

In vector form Equations (A.12) and (A.13) may be stated as [9], [137]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J}_{\text{RM}}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{A.14})$$

Where; $\mathbf{J}_{\text{RM}}(\boldsymbol{\theta})$ is the robot manipulator Jacobian defined by the following matrix of partial differentials of joint variable elements:

$$\mathbf{J}_{\text{RM}}(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{J}_v(\boldsymbol{\theta}) \\ \mathbf{J}_\omega(\boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \sin \theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad (\text{A.15})$$

Where; $\mathbf{J}_v(\boldsymbol{\theta})$ is the end-effector linear velocity Jacobian and $\mathbf{J}_\omega(\boldsymbol{\theta})$ is the end-effector rotational velocity Jacobian. The first two rows of Equation (A.15) are partial differentials of Equations (A.10) and (A.11). The third row is the linear velocity of the end-effector in the direction normal to the plane of rotation, i.e. Z_2 . The last three rows are angular velocities of frame $O_2X_2Y_2Z_2$ in which the zeros and 1 are unit vectors in the direction along the axes of rotation, which in this case is about the Z_2 axis at velocity

$\dot{\theta}_1 + \dot{\theta}_2$. Also, the angular velocities of the end effector may be expressed in terms of $\mathbf{J}_\omega(\boldsymbol{\theta})$ given by [137]:

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \end{bmatrix} = \mathbf{J}_\omega(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{A.16})$$

However, as the orientation of the end effector is of no concern in this case, $\mathbf{J}_\omega(\boldsymbol{\theta})$ is neglected and the robot manipulator Jacobian reduces to the first two rows of Equation (A.15) and denoted by $\mathbf{J}(\boldsymbol{\theta})$. A more detailed analytical derivation of the robot manipulator Jacobian is given in references [9], [128], [137], [157].

A.2 Dynamics

The dynamics equations are derived based upon the theoretical Euler-Lagrange formulation in which the Lagrangian $\mathcal{L}$ is defined as the difference between kinetic T_r and potential U_r energies of the rigid link robot manipulator given by [9], [128], [137]:

$$\mathcal{L} = T_r - U_r \quad (\text{A.17})$$

Considering the two-link planar robot manipulator parameters shown in Figure A.1, the Euler-Lagrange equations of motion may be derived for an independent set of generalized coordinates θ_i using the following equation:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i} = \tau_i, \quad i = 1, 2 \quad (\text{A.18})$$

The kinetic energy is a quadratic function of $\dot{\boldsymbol{\theta}}$ given by:

$$T_r = \frac{1}{2} \sum_{k=1}^n \sum_{p=1}^r M_{r,kp} \dot{\theta}_i \dot{\theta}_j \equiv \frac{1}{2} \dot{\boldsymbol{\theta}}^T \mathbf{M}_r(\boldsymbol{\theta}) \dot{\boldsymbol{\theta}}, \quad i = 1, 2, j = 1, 2, n = 2, r = 2 \quad (\text{A.19})$$

Where $M_{r,kp}$ is the k -pth element of the 2×2 positive definite inertia matrix $\mathbf{M}_r(\boldsymbol{\theta})$. Also, the potential energy function $U_r = U_r(\boldsymbol{\theta})$ is independent of $\dot{\boldsymbol{\theta}}$ and summed for the two

links given by:

$$U_r = \sum_{i=1}^n m_i \mathbf{g}^T \mathbf{r}_{0,c_i}, \quad n = 2 \quad (\text{A.20})$$

Where; m_i is the mass of each link, $\mathbf{g}$ is the gravitational constant and $\mathbf{r}_{0,ci}$ is the position vector of the centroid of each link from its proximal frame origin.

The Lagrangian is given as:

$$\mathcal{L} = \frac{1}{2} \sum_{k=1}^n \sum_{p=1}^r M_{r,kp} \dot{\theta}_i \dot{\theta}_j - U_r(\theta) \quad (\text{A.21})$$

Since the gravitational potential is assumed zero for space operations then the potential energy is zero. Hence; $U_r = 0$ and the Lagrangian is reduced to the following equation:

$$\mathcal{L} = \frac{1}{2} \sum_{k=1}^n \sum_{p=1}^r M_{r,kp} \dot{\theta}_i \dot{\theta}_j \quad (\text{A.22})$$

At this point we invoke use of the robot manipulator Jacobian in deriving the equations of motion. The linear velocities of link centroids c_1 and c_2 are given by [128]:

$$\begin{bmatrix} \dot{x}_{c_1} \\ \dot{y}_{c_1} \end{bmatrix} = \mathbf{J}_{c_1}(\theta) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{A.23})$$

$$\mathbf{J}_{c_1}(\theta) = \begin{bmatrix} -\frac{L_1}{2} \sin \theta_1 & 0 \\ \frac{L_1}{2} \sin \theta_1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{A.24})$$

And; $\frac{L_1}{2}$ is the distance from the origin of frame $O_0X_0Y_0Z_0$ to point c_1 midway along link 1.

Similarly;

$$\begin{bmatrix} \dot{x}_{c_2} \\ \dot{y}_{c_2} \end{bmatrix} = \mathbf{J}_{c_2}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{A.25})$$

$$\mathbf{J}_{c_2}(\boldsymbol{\theta}) = \begin{bmatrix} -L_1 \sin \theta_1 - \frac{L_2}{2} \sin(\theta_1 + \theta_2) & -\frac{L_2}{2} \sin(\theta_1 + \theta_2) \\ L_1 \sin \theta_1 + \frac{L_2}{2} \cos(\theta_1 + \theta_2) & \frac{L_2}{2} \cos(\theta_1 + \theta_2) \\ 0 & 0 \end{bmatrix} \quad (\text{A.26})$$

Where; $\frac{L_2}{2}$ is the distance from the origin of frame $O_1X_1Y_1Z_1$ to point c_2 midway along link 2 [128].

The translational component of T_r is given by:

$$T_v = \frac{1}{2} m_1 \begin{bmatrix} \dot{x}_{c_1} \\ \dot{y}_{c_1} \end{bmatrix}^T \begin{bmatrix} \dot{x}_{c_1} \\ \dot{y}_{c_1} \end{bmatrix} + \frac{1}{2} m_2 \begin{bmatrix} \dot{x}_{c_2} \\ \dot{y}_{c_2} \end{bmatrix}^T \begin{bmatrix} \dot{x}_{c_2} \\ \dot{y}_{c_2} \end{bmatrix} \quad (\text{A.27})$$

$$= \frac{1}{2} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}^T \left\{ m_1 \mathbf{J}_{c_1}^T \mathbf{J}_{c_1} + m_2 \mathbf{J}_{c_2}^T \mathbf{J}_{c_2} \right\} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

The rotational component of T_r is given by:

$$T_\omega = \frac{1}{2} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}^T \left\{ I_{L_1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + I_{L_2} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{A.28})$$

By adding the terms in $\{.\}$ in Equations (A.27) and (A.28) we get the inertia matrix $\mathbf{M}_r(\boldsymbol{\theta})$ given by:

$$\mathbf{M}_r(\boldsymbol{\theta}) = \left\{ m_1 \mathbf{J}_{c_1}^T \mathbf{J}_{c_1} + m_2 \mathbf{J}_{c_2}^T \mathbf{J}_{c_2} \right\} + \left\{ \begin{bmatrix} I_{L_1} + I_{L_2} & I_{L_2} \\ & I_{L_2} \end{bmatrix} \right\} \quad (\text{A.29})$$

Expanding the terms in Equation (A.29) and using trigonometric identities we get the following inertia matrix elements:

$$M_{11} = m_1 \left(\frac{L_1}{2} \right)^2 + m_2 \left\{ L_1 + \left(\frac{L_2}{2} \right)^2 + 2L_1 \left(\frac{L_2}{2} \right) \cos \theta_2 \right\} + I_1 + I_2 \quad (\text{A.30})$$

$$M_{12} = m_2 \left\{ \left(\frac{L_2}{2} \right)^2 + L_1 \left(\frac{L_2}{2} \right) \cos \theta_2 \right\} + I_2 = M_{21} \quad (\text{A.31})$$

$$M_{22} = m_2 \left(\frac{L_2}{2} \right)^2 + I_2 \quad (\text{A.32})$$

The Christoffel symbols are obtained from the following definition [128]:

$$c_{ijk} = \frac{1}{2} \left\{ \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{kij}}{\partial q_k} \right\} \quad (\text{A.33})$$

$$c_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0 \quad (\text{A.34})$$

$$c_{121} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = c_{121} = -m_2 L_1 \left(\frac{L_2}{2} \right) \sin \theta_2 \equiv h \quad (\text{A.35})$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = h \quad (\text{A.36})$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = -h \quad (\text{A.37})$$

$$c_{122} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = c_{212} = 0 \quad (\text{A.38})$$

$$c_{222} = \frac{1}{2} \frac{\partial d_{22}}{\partial q_2} = 0 \quad (\text{A.39})$$

Collecting and arranging inertia terms and Christoffel symbols we get the following dynamical equations in the form:

$$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + c_{121}\dot{\theta}_1\dot{\theta}_2 + c_{211}\dot{\theta}_2\dot{\theta}_1 + c_{221}\dot{\theta}_2^2 \quad (\text{A.40a})$$

$$\tau_2 = M_{21}\ddot{\theta}_1 + M_{22}\ddot{\theta}_2 + c_{111}\dot{\theta}_1^2 \quad (\text{A.40b})$$

Arranging inertia terms and Christoffel symbols into matrices the dynamical equations may be written in the form:

$$\boldsymbol{\tau} = \mathbf{M}_r(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \quad (\text{A.41})$$

Where; $\mathbf{M}_r(\boldsymbol{\theta})$ is the inertia matrix given by:

$$\mathbf{M}_r(\boldsymbol{\theta}) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \quad (\text{A.42a})$$

And; $\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is the dynamic coupling vector of Coriolis and centrifugal torques given by:

$$\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \eta \begin{bmatrix} \dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) \\ -\dot{\theta}_1^2 \end{bmatrix} \quad (\text{A.42b})$$

For use in simulation the parameters of each link are; $L = L_1 = L_2 = 4.5$ m and mass

$m_1 = m_2 = 1.5075$ kg. Equations (A.42a) and (A.42b) are given as:

$$\mathbf{M}_r(\boldsymbol{\theta}) = \begin{bmatrix} \frac{m_1 L^2}{3} + m_2 L^2 \left(\frac{4}{3} + \cos\theta_2 \right) & m_2 L^2 \left(\frac{1}{3} + \frac{1}{2} \cos\theta_2 \right) \\ m_2 L^2 \left(\frac{1}{3} + \frac{1}{2} \cos\theta_2 \right) & \frac{m_2 L^2}{3} \end{bmatrix} \quad (\text{A.43})$$

$$\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = -\left(\frac{1}{2} m_2 L^2 \sin\theta_2 \right) \begin{bmatrix} \dot{\theta}_2(2\dot{\theta}_1 + \dot{\theta}_2) \\ -\dot{\theta}_1^2 \end{bmatrix} \quad (\text{A.44})$$

Numerical values of $\mathbf{M}_r(\boldsymbol{\theta})$ and $\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ used in simulation are given by the inertia matrix:

$$\mathbf{M}_r(\boldsymbol{\theta}) = \begin{bmatrix} 50.8781 + 30.5268\cos\theta_2 & 10.1756 + 15.2634\cos\theta_2 \\ 10.1756 + 15.2634\cos\theta_2 & 10.1756 \end{bmatrix} \quad (\text{A.45})$$

and dynamic coupling vector:

$$\mathbf{C}_r(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = -(15.2634 \sin \theta_2) \begin{bmatrix} \dot{\theta}_2 (2\dot{\theta}_1 + \dot{\theta}_2) \\ -\dot{\theta}_1^2 \end{bmatrix} \quad (\text{A.46})$$

Appendix B

Rigid Dynamics State-Space, Linear Quadratic Regulator and Stochastic Control Equations

B.1 Linear Quadratic Regulator

B.1.1 State-Space Derivation

The planar rigid robot manipulator dynamic equations of motion are given in Appendix A as:

$$\tau = M_r(\theta)\ddot{\theta} + C_r(\theta, \dot{\theta}) \quad (B.1)$$

These equations are converted into state-space form suitable for simulation typically given by [40], [92], [134], [139]:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (B.2)$$

or

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{Cx} + \mathbf{Du} \end{bmatrix} \quad (B.3)$$

Where;

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} \quad (B.4)$$

And;

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} \quad (B.5)$$

are 4x1 vectors of state variables in which $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \dot{\theta}_2$ $\dot{x}_1 = \dot{\theta}_1 = x_2$,

$\dot{x}_2 = \ddot{\theta}_1$ and $\dot{x}_3 = \dot{\theta}_2 = x_4$, and $\dot{x}_4 = \ddot{\theta}_2$.

And;

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (\text{B.6})$$

is the vector of output measurements.

And;

$$\mathbf{u} = \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (\text{B.7})$$

is the input torque vector.

Using state-variables Equation (B.1) may be restated as:

$$\boldsymbol{\tau} = \mathbf{M}_r(\mathbf{x})\dot{\mathbf{x}} + \mathbf{C}_r(\mathbf{x}) \quad (\text{B.8})$$

Lyapunov-linearization of Equation (B.2) where $f(\mathbf{x}, \mathbf{u})$ is continuously differentiable and letting $f(\mathbf{0}, \mathbf{0}) = \mathbf{0}$ may be rewritten as [134], [147]:

$$\dot{\mathbf{x}} = \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{0}, \mathbf{u}=\mathbf{0}} \mathbf{x} + \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right|_{\mathbf{x}=\mathbf{0}, \mathbf{u}=\mathbf{0}} \mathbf{u} + f_h(\mathbf{x}, \mathbf{u}) \quad (\text{B.9})$$

Denoting the Jacobians $\mathbf{A} = \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{0}, \mathbf{u}=\mathbf{0}}$ and $\mathbf{B} = \left. \frac{\partial f(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} \right|_{\mathbf{x}=\mathbf{0}, \mathbf{u}=\mathbf{0}}$ and

neglecting higher-order terms $f_h(\mathbf{x}, \mathbf{u})$ Equation (B.9) may be written in the form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (\text{B.10})$$

This conforms to Equation (B.3).

Therefore, converting Equation (B.1) into state-variable form around a desired operating point $\mathbf{x}_d$ results in the following equation:

$$\boldsymbol{\tau}_d + \Delta\boldsymbol{\tau} = \mathbf{M}_r(\mathbf{x}_d + \Delta\mathbf{x})(\dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}) + \mathbf{C}_r(\mathbf{x}_d + \Delta\mathbf{x}) \quad (\text{B.11})$$

Where; $\boldsymbol{\tau} = \boldsymbol{\tau}_d + \Delta\boldsymbol{\tau}$ and $\mathbf{x} = \mathbf{x}_d + \Delta\mathbf{x}$ and Δ denotes control input and state-variable errors.

Also, neglecting higher-order terms of $\Delta\mathbf{x}$ and $\Delta\boldsymbol{\tau}$ and simplifying Equation (B.11) by

letting $\mathbf{M}_r(\mathbf{x}_d + \Delta\mathbf{x})\dot{\mathbf{x}} \approx \mathbf{M}_r(\mathbf{x}_d)\dot{\mathbf{x}}$ we obtain:

$$\boldsymbol{\tau}_d + \Delta\boldsymbol{\tau} = \mathbf{M}_r(\mathbf{x}_d)(\dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}) + \mathbf{C}_r(\mathbf{x}_d) + \mathbf{J}_c(\mathbf{x}_d)\Delta\mathbf{x} \quad (\text{B.12})$$

Putting Equation (A.46) in state-variable form, we get:

$$\mathbf{C}_r(\mathbf{x}_d) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -\{x_2 x_4 30.5268 \sin x_3 + x_4^2 15.2634 \sin x_3\} \\ x_2^2 15.2634 \sin x_3 \end{bmatrix} \quad (\text{B.13})$$

By neglecting state-variable products and higher-order terms $x_i x_j$ and $\dot{x}_i^2$ the Jacobian of $\mathbf{C}_r(\mathbf{x}_d)$ is given by:

$$\mathbf{J}_c(\mathbf{x}_d) = -\frac{\partial \mathbf{C}_r(\mathbf{x}_d)}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_d} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & C_{24} \\ 0 & 0 & 0 & 1 \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \quad (\text{B.14})$$

Where;

$$C_{21} = -\frac{\partial C_1}{\partial x_1} = 0$$

$$C_{22} = -\frac{\partial C_1}{\partial x_2} = 0$$

$$C_{23} = -\frac{\partial C_1}{\partial x_3} = -\{-30.5268 \cos x_3 - 15.2634 \cos x_3\}$$

$$C_{24} = \frac{\partial C_1}{\partial x_4} = 0$$

$$C_{41} = -\frac{\partial C_2}{\partial x_1} = 0$$

$$C_{42} = -\frac{\partial C_2}{\partial x_2} = 0$$

$$C_{43} = -\frac{\partial C_2}{\partial x_3} = -15.2634 \cos x_3$$

$$C_{44} = -\frac{\partial C_2}{\partial x_4} = 0$$

Letting $\cos(x_3) \approx 1$ at equilibrium point $x_3 = 0$ then the Jacobian of Equation (B.14):

$$\mathbf{J}_c(\mathbf{x}_d) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 45.7902 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -15.2634 & 0 \end{bmatrix} \quad (\text{B.15})$$

A specific solution of Equation (B.8) is given by:

$$\boldsymbol{\tau}_d = \mathbf{M}_r(\mathbf{x}_d) \dot{\mathbf{x}}_d + \mathbf{C}_r(\mathbf{x}_d) \quad (\text{B.16})$$

Equation (B.12) transforms into:

$$\Delta \boldsymbol{\tau} = \mathbf{M}_r(\mathbf{x}_d) \Delta \dot{\mathbf{x}} + \mathbf{J}_c(\mathbf{x}_d) \Delta \mathbf{x} \quad (\text{B.17})$$

Transposing Equation (B.17) in terms of angular acceleration as the dependent variable we get:

$$\Delta \dot{\mathbf{x}} = -\mathbf{M}_r^{-1}(\mathbf{x}_d) \mathbf{J}_c(\mathbf{x}_d) \Delta \mathbf{x} + \mathbf{M}_r^{-1}(\mathbf{x}_d) \Delta \boldsymbol{\tau} \quad (\text{B.18})$$

Equation (B.18) represents the nonlinear system given by Equation (B.1) for error terms and is valid for small deviations about an equilibrium point at $\mathbf{x} = \mathbf{0}$ for $\boldsymbol{\tau} = \mathbf{0}$. Neglecting the error terms Equation (B.18) may be restated using the variables in Equation (B.8):

$$\dot{\mathbf{x}} = -\mathbf{M}_r^{-1}(\mathbf{x}) \mathbf{J}_c(\mathbf{x}) \mathbf{x} + \mathbf{M}_r^{-1}(\mathbf{x}) \boldsymbol{\tau} \quad (\text{B.19})$$

Letting $\sin \theta_2 \approx \theta_2$, $\cos \theta_2 \approx 1$ for small θ_1 and θ_2 in $\mathbf{M}_r(\boldsymbol{\theta})$ we get:

$$\mathbf{M}_r(\mathbf{x}) = \begin{bmatrix} 81.405 & 25.44 \\ 25.44 & 10.175 \end{bmatrix} \quad (\text{B.20})$$

Dropping the variables in parenthesis, we get;

$$\mathbf{M}_r^{-1} = \frac{\text{adj}\mathbf{M}_r}{|\mathbf{M}_r|} \quad (\text{B.21})$$

$$\text{adj}(\mathbf{M}_r) = \begin{bmatrix} 10.175 & -25.44 \\ -25.44 & 81.405 \end{bmatrix} \quad (\text{B.22})$$

$$|\mathbf{M}_r| = 181.102 \quad (\text{B.23})$$

Hence;

$$\mathbf{M}_r^{-1} = \frac{\begin{bmatrix} 10.175 & -25.44 \\ -25.44 & 81.405 \end{bmatrix}}{181.102} = \begin{bmatrix} 0.0562 & -0.1405 \\ -0.1405 & 0.45 \end{bmatrix} \quad (\text{B.24})$$

In state-space form:

$$\mathbf{M}_r^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.0562 & 0 & -0.1405 \\ 0 & 0 & 1 & 0 \\ 0 & -0.1405 & 0 & 0.45 \end{bmatrix} \quad (\text{B.25})$$

For matrix **A** in Equation (B.3) we get:

$$\mathbf{A} = \mathbf{M}_r^{-1}(\mathbf{x}) \mathbf{J}_c(\mathbf{x}_d) \quad (\text{B.26})$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.0562 & 0 & -0.1405 \\ 0 & 0 & 1 & 0 \\ 0 & -0.1405 & 0 & 0.45 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 45.7902 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -15.2634 & 0 \end{bmatrix} \quad (\text{B.27})$$

Hence, in state-space form we get the matrices **A**, **B**, **C** and **D**:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 4.7179 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -13.3021 & 0 \end{bmatrix} \quad (\text{B.28})$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0.0562 & -0.1405 \\ 0 & 0 \\ -0.1405 & 0.45 \end{bmatrix} \quad (\text{B.29})$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (\text{B.30})$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{B.31})$$

Hence substituting in Equation (B.3) the state-space dynamics equations are given by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 4.7179 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -13.3021 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.0562 & -0.1405 \\ 0 & 0 \\ -0.1405 & 0.45 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (\text{B.32})$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (\text{B.33})$$

The weighting matrices are given by:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{B.34})$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{B.35})$$

Using the Matlab™ function $\mathbf{K} = \text{lqr}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R})$ for calculating the LQ gains we get [98]:

$$\mathbf{K} = \begin{bmatrix} 0.3272 & 3.2159 & 0.17 & 0.8121 \\ 0.9449 & 9.5692 & 0.5326 & 4.4425 \end{bmatrix} \quad (\text{B.36})$$

And the LQ gains used for simulation are selected from Equation (B.36) and given by:

$$\text{LQ gains in} = \begin{bmatrix} 3.2159 & 0 \\ 0 & 0.8121 \end{bmatrix} \quad (\text{B.37})$$

$$\text{LQ gains out} = \begin{bmatrix} 0.9449 & 0 \\ 0 & 0.5326 \end{bmatrix} \quad (\text{B.38})$$

B.2 Linear Quadratic Gaussian

B.2.1 Kalman Filter

The state-space equations including process and measurement noise are given by [20], [72], [139], [160]:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Gw} \quad (\text{B.39})$$

$$\mathbf{z} = \mathbf{Cx} + \mathbf{Du} + \mathbf{v} \quad (\text{B.40})$$

Where; $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are given by Equations (B.28), (B.29), (B.30) and (B.31).

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{B.41})$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (\text{B.42})$$

For the Kalman filter presented in Chapter 3 the state estimate is obtained by integrating the following first-order linear differential equation:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_k (\mathbf{z} - \mathbf{C}\hat{\mathbf{x}} - \mathbf{D}u) \quad (\text{B.43})$$

Using $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{G}$ and arbitrary process and measurement noise covariance matrices $\mathbf{Q}_k$

and $\mathbf{R}_k$, the continuous Kalman filter gains matrix is obtained by:

$$\mathbf{K}_k = [\mathbf{P}_k \mathbf{C}^T \mathbf{R}_k^{-1}] \quad (\text{B.44})$$

Where; $\mathbf{P}_k$ is the error covariance matrix and $\mathbf{R}_k$ is the measurement noise covariance.

The Kalman filter matrices are given by [20], [72], [139]:

$$\mathbf{A}_k = \mathbf{A} - \mathbf{K}_k \mathbf{C} \quad (\text{B.45})$$

$$\mathbf{B}_1 = \mathbf{B} - \mathbf{K}_k \mathbf{D} \quad (\text{B.46})$$

$$\mathbf{B}_k = [\mathbf{B}_1 \quad \mathbf{K}_k] \quad (\text{B.47})$$

$$\mathbf{C}_k = \mathbf{C} \quad (\text{B.48})$$

$$\mathbf{D}_k = \mathbf{D} \quad (\text{B.49})$$

$$\mathbf{A}_k = \begin{bmatrix} -1.4525 & 1 & 0.0666 & 0 \\ -0.8072 & 0 & 4.7018 & 0 \\ 0.0666 & 0 & -0.7288 & 1 \\ 0.1614 & 0 & -13.3199 & 0 \end{bmatrix} \quad (\text{B.50})$$

$$\mathbf{B}_k = \begin{bmatrix} 0 & 0 & 1.4525 & 0.0666 \\ 0.0562 & -0.1405 & 0.8072 & 0.0161 \\ 0 & 0 & -0.0666 & 0.7288 \\ -0.1405 & 0.45 & -0.1614 & 0.0178 \end{bmatrix} \quad (\text{B.51})$$

$$\mathbf{C}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (\text{B.52})$$

$$\mathbf{D}_k = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{B.53})$$

In deriving the state transition matrix Φ_k and process noise covariance $\mathbf{Q}_k$ matrices we use the following matrix equations [20], [49], [146]:

$$\mathbf{Z} = \begin{bmatrix} -\mathbf{A} & \mathbf{B}\mathbf{W}\mathbf{B}^T \\ \mathbf{ZERO} & \mathbf{A}^T \end{bmatrix} \quad (\text{B.54})$$

Where; **A** and **B** are given by Equations (B.38) and (B.39).

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad (\text{B.55})$$

$$\mathbf{ZERO} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{B.56})$$

$$\mathbf{\Omega} = e^{\mathbf{Z}} \quad (\text{B.57})$$

$$\mathbf{Z} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.7179 & 0 & 0 & -0.0047 & 0 & 0.0118 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13.3021 & 0 & 0 & 0.0174 & 0 & -0.0435 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.7179 & 0 & -13.3021 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (\text{B.58})$$

$$\mathbf{\Omega} = \begin{bmatrix} 1 & -1 & 0.665 & -0.4018 & 0.0001 & -0.0005 & -0.0001 & 0.0015 \\ 0 & 1 & 0.6265 & 0.665 & 0.0005 & 0.0028 & -0.0015 & -0.0073 \\ 0 & 0 & -0.8749 & 0.1328 & -0.0006 & 0.0007 & 0.0012 & -0.0029 \\ 0 & 0 & -1.7665 & -0.8749 & -0.0007 & -0.008 & 0.0029 & 0.0219 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.665 & -0.6265 & -0.8749 & 1.7665 \\ 0 & 0 & 0 & 0 & 0.4018 & 0.665 & -0.1328 & -0.8749 \end{bmatrix} \quad (\text{B.59})$$

The lower-right partition (lrp) of matrix $\mathbf{\Omega}$ is given by:

$$\text{lrp } \mathbf{\Omega} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0.6650 & -0.6265 & -0.8749 & 1.7665 \\ 0.4018 & 0.6650 & -0.1328 & -0.8749 \end{bmatrix} \quad (\text{B.60})$$

$$\Phi_k = \text{lrp } \Omega^T = \begin{bmatrix} 1 & 1 & 0.665 & 0.4018 \\ 0 & 1 & -0.6265 & 0.665 \\ 0 & 0 & -0.8749 & -0.1328 \\ 0 & 0 & 1.7665 & -0.8749 \end{bmatrix} \quad (\text{B.61})$$

The upper-right partition (urp) of matrix Ω is given by:

$$\text{urp } \Omega = \begin{bmatrix} 0.0001 & -0.0005 & -0.0001 & 0.0015 \\ 0.0005 & 0.0028 & -0.0015 & -0.0073 \\ -0.0006 & 0.0007 & 0.0012 & -0.0029 \\ -0.0007 & -0.008 & 0.0029 & 0.0219 \end{bmatrix} \quad (\text{B.62})$$

$$\mathbf{Q}_k = \Phi_k \text{urp } \Omega \quad (\text{B.63})$$

$$= \begin{bmatrix} 1 & 1 & 0.665 & 0.4018 \\ 0 & 1 & -0.6265 & 0.6650 \\ 0 & 0 & -0.8749 & -0.1328 \\ 0 & 0 & 1.7665 & -0.8749 \end{bmatrix} \begin{bmatrix} 0.0001 & -0.0005 & -0.0001 & 0.0015 \\ 0.0005 & 0.0028 & -0.0015 & -0.0073 \\ -0.0006 & 0.0007 & 0.0012 & -0.0029 \\ -0.0007 & -0.008 & 0.0029 & 0.0219 \end{bmatrix}$$

$$\mathbf{Q}_k = \begin{bmatrix} -0.0001 & -0.0004 & 0.0003 & 0.0011 \\ 0.0004 & -0.003 & -0.0004 & 0.0091 \\ 0.0006 & 0.0004 & -0.0014 & -0.0004 \\ -0.0004 & 0.0083 & -0.0004 & -0.0243 \end{bmatrix} \quad (\text{B.64})$$

Values selected for use in simulation are given by the diagonal elements of $\mathbf{Q}_k$:

$$\mathbf{Q}_k = \begin{bmatrix} -0.0001 & 0 & 0 & 0 \\ 0 & -0.003 & 0 & 0 \\ 0 & 0 & -0.0014 & 0 \\ 0 & 0 & 0 & -0.0243 \end{bmatrix} \quad (\text{B.65})$$

$$2\mathbf{Q}_k = \begin{bmatrix} -0.0002 & 0 & 0 & 0 \\ 0 & -0.006 & 0 & 0 \\ 0 & 0 & -0.0028 & 0 \\ 0 & 0 & 0 & -0.0486 \end{bmatrix} \quad (\text{B.66})$$

$$\mathbf{R}_k = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix} \quad (\text{B.67})$$

$$2\mathbf{R}_k = \begin{bmatrix} 18 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & 18 \end{bmatrix} \quad (\text{B.68})$$

B.2.2 Extended Kalman Filter

This program is executed as a Matlab/Simulink™ function block extended Kalman filter subroutine.

%%%

Extended Kalman Filter for 2-Link Rigid Robot Manipulator Control

%%%

function U = EKF(u);

Transition Matrix Phik:

Phik = [1 1 0.665 0.4018; 0 1 -0.6265 0.665; 0 0 -0.8749 -0.1328; 0 0 1.7665 -0.8749];

Process and Measurement Noise Covariances:

C = 1;

D = 1;

Qk = C*diag([-0.0001 -0.003 -0.0014 -0.0243]);

Rk = D*diag([9 9 9 9]);

sigmaW = sqrt(Qk);

sigmaV = sqrt(Rk);

Nk = sigmaW*sigmaV;

Initial Conditions:

$$q1 = u(1);$$

$$q2 = u(2);$$

$$\hat{x}Prev = u(3:6);$$

$$PPrev = u(7:22);$$

$$Pk = \text{reshape}(PPrev, 4, 4);$$

Projected State Estimate Matrix:

$$\hat{x}atkplus1 = Phik * \hat{x}Prev;$$

Compute Residual (Estimation Error):

$$zk = [q1; 0; q2; 0];$$

$$residual = zk - \hat{x}Prev;$$

Kalman Gain:

$$H = [1 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 0; 0 \ 0 \ 1 \ 0; 0 \ 0 \ 0 \ 0];$$

$$Kk = (Pk * H' + Nk) * \text{inv}(H * Pk * H' + Rk + H * Nk + Nk' * H');$$

Update State Estimate:

$$\hat{x}at = \hat{x}atkplus1 + Kk * residual;$$

$$\hat{x}at(:, :);$$

$$x1 = \hat{x}at(1, 1);$$

$$x2 = \hat{x}at(3, 1);$$

Error Covariance Update:

$$Pknew = Pk - Kk * (H * Pk * H' + Rk + H * Nk + Nk' * H') * Kk'$$

Projected Covariance Estimate:

$$Pkplus1 = (Phik * Pknew * Phik') + Qk;$$

Output:

$U = [x1; x2; \hat{x}(:); P_{k+1}(:)];$

B.2.3 Fuzzy Logic Adaptive Extended Kalman Filter

This program is executed as a Matlab/Simulink™ function block fuzzy logic adaptive extended Kalman filter subroutine.

%%%

Fuzzy Adaptive Extended Kalman Filter for 2-Link Rigid Robot Manipulator Control

%%%

function U = FEKFALPHA(u);

Transition Matrix Phik:

$Phik = [1 \ 1 \ 0.665 \ 0.4018; 0 \ 1 \ -0.6265 \ 0.665; 0 \ 0 \ -0.8749 \ -0.1328; 0 \ 0 \ 1.7665 \ -0.8749];$

Initial Conditions:

$q1 = u(1);$

$q2 = u(2);$

$\alpha = u(3);$

$k = u(4);$

$PPrev = u(5:20);$

$\hat{x}Prev = u(21:24);$

Process and Measurement Noise Covariances:

$C = 1; D = 1;$

{ $Q_k = \text{diag}([1 \ 1 \ 1 \ 1]);$ For use with Gaussian white process noise with unity covariance.}

{ $R_k = \text{diag}([1 \ 1 \ 1 \ 1]);$ For use with Gaussian white measurement noise with unity covariance.}

```

Qk = C*diag([-0.0001 -0.003 -0.0014 -0.0243]);

Rk = D*diag([9 9 9 9]);

sigmaW = sqrt(Qk);

sigmaV = sqrt(Rk);

Nk = sigmaW*sigmaV;

Pkminus = reshape(PPrev,4,4);

Pkalphaminus = Pkminus*(alpha^(2*k));

Ralpha = Rk*(alpha^-(2*(k+1)));

Qalpha = Qk*(alpha^-(2*(k+1)));

Nalpha = Nk*(alpha^-(2*(k+1)));

H = [1 0 0 0;0 0 0 0;0 0 1 0;0 0 0 0];

Projected State Estimate Matrix:

xhatkplus1 = Phik*xhatPrev;

Compute Residual (Estimation Error):

zk = [q1;0;q2;0];

residual = zk - xhatPrev;

Fuzzy Adapted Kalman Gain:

H = [1 0 0 0;0 0 0 0;0 0 1 0;0 0 0 0];

Kk = (Pkalphaminus*H' + Nk)*inv(H*Pkalphaminus*H' + (Rk/(alpha^2)) + H*Nk + .....
.....+ Nk'*H');
Mean of Residuals:

Mean = xhatPrev + Pkminus*H'*inv(H*Pkminus*H' + Rk)*(zk - H*xhatPrev);

Mean3 = Mean(3,1);

Covariance of Residuals:

```

$$P_z = H * R_k * H' + R_k;$$

$$P_{z3} = P_z(3,3);$$

Update State Estimate:

$$\hat{x} = \hat{x}_{\text{Prev}} + K_k * \text{residual};$$

$$x_1 = \hat{x}(1,1);$$

$$x_2 = \hat{x}(2,1);$$

$$x_3 = \hat{x}(3,1);$$

$$x_4 = \hat{x}(4,1);$$

Error Covariance Update:

$$P_{k+1} = (I - K_k * H) * P_k - K_k * N_k' / (\alpha^2);$$

Projected Covariance Estimate:

$$P_{k+1} = (\alpha^{2*(k+1)}) * P_k * P_{k+1} * P_k' + Q_k;$$

$$P_{k+1} = P_{k+1}(1,1);$$

$$P_{k+1} = P_{k+1}(2,1);$$

$$P_{k+1} = P_{k+1}(3,1);$$

$$P_{k+1} = P_{k+1}(4,1);$$

$$P_{k+1} = P_{k+1}(1,2);$$

$$P_{k+1} = P_{k+1}(2,2);$$

$$P_{k+1} = P_{k+1}(3,2);$$

$$P_{k+1} = P_{k+1}(4,2);$$

$$P_{k+1} = P_{k+1}(1,3);$$

$$P_{k+1} = P_{k+1}(2,3);$$

$$P_{k+1} = P_{k+1}(3,3);$$

$Pkplus1alpha12 = Pkplus1alpha(4,3);$

$Pkplus1alpha13 = Pkplus1alpha(1,4);$

$Pkplus1alpha14 = Pkplus1alpha(2,4);$

$Pkplus1alpha15 = Pkplus1alpha(3,4);$

$Pkplus1alpha16 = Pkplus1alpha(4,4);$

Output:

$k = k + 1;$

$U = [x1; x3; Mean3; Pz3; x1; x2; x3; x4; Pkplus1alpha1; Pkplus1alpha2; Pkplus1alpha3;$

$Pkplus1alpha4; Pkplus1alpha5; Pkplus1alpha6; Pkplus1alpha7; Pkplus1alpha8;$

$Pkplus1alpha9; Pkplus1alpha10; Pkplus1alpha11; Pkplus1alpha12; Pkplus1alpha13;$

$Pkplus1alpha14; Pkplus1alpha15; Pkplus1alpha16; k];$

Appendix C

Fixed-Base Planar Flexible Robot Kinematics and Dynamics Equations

C.1 Kinematics

Consider the planar two-link flexible robot manipulator with rotary joints subjected only to bending deformation without torsion about the longitudinal axes of the links depicted in Figure C.1. The parameters of the flexible manipulator are similar to the rigid robot manipulator described in Appendix A. Each link is of equal length $L_1 = L_2$, masses $m_1 = m_2$ and mass moments of inertia $I_{L_1} = I_{L_2}$.

The flexible robot manipulator has a fixed-base (inertial) frame of reference with its origin at O_0 defined by $\hat{X}_0\hat{Y}_0\hat{Z}_0 = X_1Y_1Z_1$ where the $\hat{Z}_0$ axis is normal to the $\hat{X}_0\hat{Y}_0$ plane. Planar rigid frames of reference X_1Y_1 and X_2Y_2 each have a single rotational degree-of-freedom whereas frame X_3Y_3 has two translational degrees-of-freedom generated by the rotation at frames X_1Y_1 and X_2Y_2 . Each link is connected at its proximal end at rigid body moving frame X_iY_i and its rigid motion is described by joint angles θ_i while flexible body moving frames of reference for each link i are denoted by $\hat{X}_i\hat{Y}_i$. Transverse deformation of the flexible links is denoted by $u_i(x_i)$ at abscissa x_i along each link, i.e. $0 \leq x_i \leq L_i$ and $i = 0, 1, 2$.

Let ${}^i\mathbf{p}_i(x_i) = \{x_i y_i(x_i)\}^T$ be a point on a deformed link i with reference to frame X_iY_i and $\mathbf{p}_i$ is its absolute position with reference to frame $\hat{X}_0\hat{Y}_0$ [15], [16], [35].

Figure C.1: Fixed-base planar two-link flexible robot manipulator parameters.

Similarly, ${}^i\mathbf{r}_{i+1} = {}^i\mathbf{p}_i(L_i)$ is the position of origin of frame $X_{i+1}Y_{i+1}$ with reference to frame X_iY_i and $\mathbf{r}_i$ is its absolute position with reference to frame $\hat{X}_0\hat{Y}_0$. The joint rigid rotation matrix and flexible deformation matrix of each link at the end of each link is given by:

$$\mathbf{A}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \quad (\text{C.1})$$

and

$$\mathbf{E}_i = \begin{bmatrix} 1 & -u'_i \\ u'_i & 1 \end{bmatrix} \quad (\text{C.2})$$

Where; $u'_i = \left. \frac{\partial u_i}{\partial x_i} \right|_{x_i=L_i}$ in which $u_i = \phi_i \delta_i$ from Equation (4.1) and $\arctan(u'_i) \approx u'_i$ is a

valid linear approximation under the assumption of small deformations. Therefore; it may be implied that all second-order terms of the products of deformations are neglected and the absolute position vectors expressed as:

$$\mathbf{p}_i = \mathbf{r}_i + \mathbf{W}_i {}^i\mathbf{p}_i \quad (\text{C.3})$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i + \mathbf{W}_i {}^i\mathbf{r}_{i+1} \quad (\text{C.4})$$

Where; $\mathbf{W}_i$ is the global transformation matrix from $\hat{X}_0\hat{Y}_0 = X_iY_i$ according to the recursive equation given by:

$$\mathbf{W}_i = \mathbf{W}_{i-1}\mathbf{E}_{i-1}\mathbf{A}_i = \hat{\mathbf{W}}_{i-1}\mathbf{A}_i, \quad \hat{\mathbf{W}}_0 = \mathbf{I} \quad (\text{C.5})$$

Where; $\mathbf{I}$ is an identity matrix of appropriate dimension.

On the basis of Equations (C.1) to (C.5), the kinematics at any point along the links is fully characterized. Applying Equation (C.5) to the two-link flexible robot manipulator

leads to the following displacement vectors for use in the direct kinematics formulation in the Matlab/Simulink™ simulation model [98].

$$\mathbf{W}_1 = \hat{\mathbf{W}}_0 \mathbf{A}_1 = \mathbf{I} \mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \quad (\text{C.6})$$

$$\begin{aligned} \mathbf{W}_2 = \mathbf{W}_1 \mathbf{E}_1 \mathbf{A}_2 = \hat{\mathbf{W}}_0 \mathbf{A}_2 &= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} 1 & -u'_1 \\ u'_1 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \\ &= \begin{bmatrix} \psi_1 \cos\theta_2 - \psi_2 \sin\theta_2 & -\psi_1 \sin\theta_2 - \psi_2 \cos\theta_2 \\ \psi_3 \cos\theta_2 - \psi_4 \sin\theta_2 & -\psi_3 \sin\theta_2 - \psi_4 \cos\theta_2 \end{bmatrix} \end{aligned} \quad (\text{C.7})$$

Where; $\psi_1 = (\cos\theta_1 - u'_1 \sin\theta_1)$, $\psi_2 = (u'_1 \cos\theta_1 + \sin\theta_1)$, $\psi_3 = (\sin\theta_1 + u'_1 \cos\theta_1)$

and $\psi_4 = (u'_1 \sin\theta_1 - \cos\theta_1)$.

C.2 Dynamics

The dynamics equations of the two-link flexible manipulator is derived using the theoretical Euler-Lagrange formulation in which the Lagrangian $\mathcal{L}$ is defined as the difference between kinetic T and potential U energies of the robot system given by [14], [15], [36]:

$$\mathcal{L} = T - U \quad (\text{C.8})$$

The total kinetic energy T is given as the sum of kinetic energies attributable to the rigid hub T_{h_i} , links T_{L_i} and payload T_p given by:

$$T = \sum_{i=1}^n T_{h_i} + \sum_{i=1}^n T_{L_i} + \sum_{i=1}^n T_p, \quad n = 2 \quad (\text{C.9})$$

The kinetic energy at hub i of mass m_{hi} and mass moment of inertia I_{hi} is given by:

$$T_{h_i} = \frac{1}{2} m_{h_i} \dot{\mathbf{r}}_i^T \dot{\mathbf{r}}_i + \frac{1}{2} I_{h_i} \dot{\eta}_i^2, \quad i = 1, 2 \quad (\text{C.10})$$

The kinetic energy of link i of linear density ρ_i a distance x_i along link i and is given by:

$$T_{L_i} = \frac{1}{2} \int_0^{L_i} \rho_i(x_i) \dot{\mathbf{p}}_i^T(x_i) \dot{\mathbf{p}}_i(x_i) dx_i, \quad i = 1, 2 \quad (\text{C.11})$$

The kinetic energy associated with the payload of mass m_p and mass moment of inertia I_p located at the end effector denoted by e is given by:

$$T_p = \frac{1}{2} m_p \dot{\mathbf{r}}_e^T \dot{\mathbf{r}}_e + \frac{1}{2} I_p (\dot{\eta}_e + \dot{u}_e')^2 \quad (\text{C.12})$$

Where; $\dot{\eta}_e = \sum_{j=1}^i \dot{\theta}_j + \sum_{k=1}^{i-1} \dot{u}_k'$, $i = 1, 2$, is the absolute angular velocity of frame $X_i Y_i$.

and
$$\dot{\mathbf{p}}_i = \dot{\mathbf{r}}_i + \dot{\mathbf{W}}_i^i \mathbf{p}_i + \mathbf{W}_i^i \dot{\mathbf{p}}_i \quad (\text{C.13})$$

Also;
$$\dot{\mathbf{A}}_i = \mathbf{S} \mathbf{A}_i \dot{\theta}_i \text{ and } \dot{\mathbf{E}}_i = \mathbf{S} \dot{u}_i' \text{ where } \mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (\text{C.14})$$

Equations (C.10) to (C.12) exploit the following identities:

$$\mathbf{A}_i^T \mathbf{A}_i = \mathbf{E}_i^T \mathbf{E}_i = \mathbf{S}^T \mathbf{S} = \mathbf{I}, \quad \dot{\mathbf{E}}_i = \mathbf{S} \dot{u}_i' \quad (\text{C.15})$$

and
$$\mathbf{A}_i^T \dot{\mathbf{A}}_i = \mathbf{S} \dot{\theta}_i, \quad \mathbf{E}_i^T \dot{\mathbf{E}}_i = (\mathbf{I} u_i' + \mathbf{S}) \dot{u}_i' \quad (\text{C.16})$$

Since payloads are neglected, Equation (C.12) is omitted from the total kinetic energy equation. Also, neglecting gravity the total potential energy is the elastic potential energy term embodying the effects of distributed parameter flexibility of the links given by:

$$U = \sum_{i=1}^n U_i = \sum_{i=1}^n \frac{1}{2} \int_0^{L_i} EI_{L_i}(x_i) \left[\frac{d^2 u_i(x_i)}{dx_i^2} \right]^2 dx_i \quad (\text{C.17})$$

Where U_i is the elastic energy stored in link i and EI_{L_i} is its flexural rigidity.

The foregoing equations represent a continuously distributed dynamical system without discretization of the link flexibility but is unsuitable for modeling and simulation

purposes. Therefore, a finite-dimensional model of link deformation is obtained by introducing the assumed modes method.

Modeling flexible robot dynamics with assumed modes captures the interaction between vibrations of distributed parameter flexible links and nonlinear rigid multibody dynamics. The flexible dynamic model of the robot manipulator includes a set of assumed vibration modes computed using Euler-Bernoulli beam theory boundary conditions rather than its natural modes. [45], [143], [144].

Assumed modes accommodate changes in robot manipulator configuration during operation or, for additional links, whereas natural modes must frequently be recomputed at each change in its configuration. Elastic deformations are modeled by a finite series of space-dependent, *admissible functions*, multiplied by a specific set of time-dependent amplitude functions, resulting in amplitudes that form the generalized configuration coordinates in the Euler-Lagrange dynamics formulation. Admissible functions satisfy, at least, system geometric boundary conditions. A chosen set of admissible functions form *basis functions* that apply to the robot throughout its operational workspace, provided geometric boundary conditions are consistent [45], [143], [144].

For the general case of an Euler-Bernoulli beam $i = 1, \dots, n$ subjected to a uniform distributed load along its length L_i , the equation of motion is given by [143]:

$$\frac{\partial^2}{\partial x_i^2} \left(E I_{L_i} \frac{\partial^2 u(x_i, t)}{\partial x_i^2} \right) dx_i + m(x_i) \frac{\partial^2 u(x_i, t)}{\partial t^2} = p(x_i, t) \quad \forall t, \quad 0 < x_i < L_i$$

(C.18)

The normal modes φ_{ij} , $j = 1, \dots, n$ must satisfy the following equation:

$$EI_{L_i} \frac{d^4 \varphi_{ij}(x_i)}{dx_i^4} - \omega_{ij}^2 m(x_i) \varphi_{ij}(x_i) = 0, \quad i = 1, \dots, n, j = 1, \dots, n \quad (C.19)$$

And its boundary conditions given by:

$$\varphi_{ij}(0) = \frac{d\varphi_{ij}(0)}{dx_i} = 0; \quad \frac{d^2 \varphi_{ij}(L_i)}{dx_i^2} = \frac{d^3 \varphi_{ij}(L_i)}{dx_i^3} = 0$$

For which, the solution is given by:

$$u(x_i, t) = \sum_{i=1}^n \sum_{j=1}^n \varphi_{ij}(x_i) \delta_{ij}(t), \quad i = 1, \dots, n, j = 1, \dots, n \quad (C.20)$$

Because of assumed mode discretization of the flexible links of the two-link flexible robot manipulator the Lagrangian $\mathcal{L}$ becomes a function of a generalized coordinates vector $\mathbf{q} = [\theta_1 \ \theta_2 \ \delta_1 \ \delta_2]^T$. Applying Equation (C.20) to Equations (C.11) and (C.17) and using the identities given by Equations (C.14), (C.15) and (C.16) the dynamical equations of motion are obtained by satisfying the Euler-Lagrange equations in differential form given by [36], [143]:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \boldsymbol{\tau} \quad (C.21)$$

The resulting dynamical equations of motion are given by:

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K} \mathbf{q} \quad (C.22)$$

Where;

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \quad (C.23)$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \quad (\text{C.24})$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & K_2 \end{bmatrix} \quad (\text{C.25})$$

Elements of the inertia matrix $\mathbf{M}(\mathbf{q})$, including joint hub inertia, given by Equation (C.23)

and for the dominant assumed mode of vibration are given by:

$$M_{11} = I_{h1} + I_{L1} + I_{h2} + m_{h2} L_1^2 + I_{L2} + m_2 L_1^2 + 2m_2 b_2 L_1 \cos \theta_2 + (2m_2 b_2 t_1 - 2L_1 t_2) \sin \theta_2,$$

$$M_{12} = M_{21} = I_{h2} + I_{L2} + m_2 b_2 L_1 \cos \theta_2 + (m_2 b_2 t_1 - 2L_1 t_2) \sin \theta_2,$$

$$M_{13} = M_{31} = \int_0^{L_1} \rho_1 \phi_1 x_1 dx_1 + (I_{h2} + I_{L2}) \dot{\phi}_1 + (m_{h2} + m_2) L_1 \phi_1 + \dots \\ \dots + m_2 b_2 (\phi_1 + L_1 \dot{\phi}_1) \cos \theta_2 - (\phi_1 + L_1 \dot{\phi}_1) t_2 \sin \theta_2,$$

$$M_{14} = M_{41} = \int_0^{L_2} \rho_2 \phi_2 x_2 dx_2 + \left(\int_0^{L_2} \rho_2 \phi_2 dx_2 \delta_2 \right) L_1 \cos \theta_2 + \left(\int_0^{L_2} \rho_2 \phi_2 dx_2 \delta_2 \right) t_2 \sin \theta_2,$$

$$M_{22} = I_{h2} + I_{L2},$$

$$M_{23} = M_{32} = (I_{h2} + I_{L2}) \dot{\phi}_1 + m_2 b_2 \phi_1 \cos \theta_2 - (t_2 + m_2 b_2 t_3) \phi_1 \sin \theta_2,$$

$$M_{24} = M_{42} = \int_0^{L_2} \rho_2 \phi_2 x_2 dx_2,$$

$$M_{33} = m_1 + 2m_2 b_2 \phi_1 \dot{\phi}_1 \cos \theta_2 - \phi_1 t_2 \sin \theta_2,$$

$$M_{34} = M_{43} = \left(\int_0^{L_2} \rho_2 \phi_2 x_2 dx_2 \right) \dot{\phi}_1 + \left(\int_0^{L_2} \rho_2 \phi_2 dx_2 \right) \phi_1 \cos \theta_2 - \left(\int_0^{L_2} \rho_2 \phi_2 dx_2 \right) \phi_1 t_3 \sin \theta_2,$$

$$M_{44} = m_2.$$

Where;

$$m_{h_1} = m_{h_2} = 5 \text{ kg}, I_{L_1} = \frac{m_1 L_1^2}{3}, I_{L_2} = \frac{m_2 L_2^2}{3}, I_{h_1} = 10I_{L_1}, I_{h_2} = 10I_{L_2}$$

$$t_1 = \varphi_1 - L_1 \dot{\varphi}_1 \delta_1, \quad t_2 = \int_0^{L_2} \rho_2 \varphi_2 dx_2 \delta_2, \quad t_3 = \dot{\varphi}_1 \delta_1$$

and b_i is the distance from the centre of mass of each link to its distal end.

Elements of the dynamic coupling vector $C(\mathbf{q}, \dot{\mathbf{q}})$ given by Equation (C.24) and for the dominant assumed mode of vibration are given by:

$$C_1 = \left\{ \begin{array}{l} \left[-2m_2 b_2 L_1 \dot{\theta}_2 + 2m_2 b_2 (\varphi_1 - L_1 \dot{\varphi}_1) \dot{\delta}_1 - 2v_2 L_1 \dot{\delta}_2 \right] \dot{\theta}_1 + \dots \\ \dots + \left[-2m_2 b_2 L_1 \dot{\theta}_2 - 2m_2 b_2 L_1 \dot{\varphi}_1 \dot{\delta}_1 - 2v_2 L_1 \dot{\delta}_2 \right] \dot{\theta}_2 + \dots \\ \dots + \left[-2v_2 L_1 \dot{\varphi}_1 \dot{\delta}_2 \right] \dot{\delta}_1 \end{array} \right\} \sin \theta_2 + \dots$$

$$\dots + \left\{ \begin{array}{l} \left[2m_2 b_2 \dot{\theta}_1 + 2m_2 b_2 \dot{\theta}_2 - 2v_2 \dot{\delta}_2 \right] t_1 + \dots \\ \dots + \left[-2L_1 \dot{\theta}_1 - L_1 \dot{\theta}_2 - (\varphi_1 + L_1 \dot{\varphi}_1) \dot{\delta}_1 \right] t_2 \end{array} \right\} \dot{\theta}_2 \cos \theta_2$$

$$C_2 = (m_2 b_2 L_1 \dot{\theta}_1 + 2m_2 b_2 \varphi_1 \dot{\delta}_1) \dot{\theta}_1 \sin \theta_2 + \dots$$

$$\dots + \left\{ \begin{array}{l} \left[(-2m_2 b_2 L_1 \dot{\theta}_1 - 2v_2 \dot{\delta}_2) t_1 + \dots \right. \\ \dots + \left[L_1 \dot{\theta}_1 + (\varphi_1 + L_1 \dot{\varphi}_1) \dot{\delta}_1 \right] t_2 \left. \right] \dot{\theta}_1 + \dots \\ \dots + (\varphi_1 \dot{\varphi}_1 \dot{\delta}_1 t_2 + v_2 \dot{\varphi}_1 t_3) \dot{\delta}_1 \end{array} \right\} \cos \theta_2$$

$$C_3 = \left\{ \begin{array}{l} \left[2m_2 b_2 (\varphi_1 - L_1 \dot{\varphi}_1) \dot{\theta}_1 - 2m_2 b_2 \varphi_1 \dot{\theta}_2 - 2v_2 \varphi_1 \dot{\delta}_2 \right] \dot{\theta}_1 + \dots \\ \dots + \left[-2m_2 b_2 \dot{\theta}_2 - 2m_2 b_2 \varphi_1 \dot{\varphi}_1 \dot{\delta}_1 - 2v_2 \varphi_1 \dot{\delta}_2 \right] \dot{\theta}_2 + \dots \\ \dots + \left[-2v_2 \varphi_1 \dot{\varphi}_1 \dot{\delta}_2 \right] \dot{\delta}_1 \end{array} \right\} \sin \theta_2 + \dots$$

$$\dots + \left\{ \begin{array}{l} \left[-(\varphi_1 + L_1 \dot{\varphi}_1) \dot{\theta}_1 - \varphi_1 \dot{\theta}_2 - 2\varphi_1 \dot{\varphi}_1 \dot{\delta}_1 \right] t_2 + \dots \\ \dots + \left[-m_2 b_2 \varphi_1 \dot{\theta}_2 - v_2 \varphi_1 \dot{\delta}_2 \right] t_3 \end{array} \right\} \dot{\theta}_2 \cos \theta_2$$

$$C_4 = (v_2 L_1 \dot{\theta}_1 + 2v_2 \varphi_1 \dot{\delta}_1) \dot{\theta}_1 \sin \theta_2 + (v_2 t_2 \dot{\theta}_1 - v_2 \varphi_1 \dot{\delta}_1 t_3) \dot{\theta}_2 \cos \theta_2.$$

Where; $v_1 = \int_0^{L_1} \rho_1 \phi_1 dx_1$, $v_2 = \int_0^{L_2} \rho_2 \phi_2 dx_2$

Elements of the stiffness matrix in Equation (C.25) are given by:

$$K_1 = m_1 \omega_c^2 \quad \text{and} \quad K_2 = m_2 \omega_c^2$$

C.3 Numerical Form of Dynamics Equations for Simulation

Inserting numerical values into the inertia matrix $\mathbf{M}(\mathbf{q})$ given by Equation (C.23) and $\phi_1 = \phi_2 = 0.2$ for the cantilever dominant assumed mode, excluding hub inertia terms for simulation in Matlab/Simulink™ we get:

$$M_{11} = 50.8781 + 30.5269 \cos \theta_2 - (0.5108 \delta_1 + 0.000066462 \delta_2) \sin \theta_2$$

$$M_{12} = 10.1756 + 15.2634 \cos \theta_2 - (0.2554 \delta_1 + 0.000033231 \delta_2) \sin \theta_2$$

$$M_{13} = 1.9793 + 1.6122 \cos \theta_2 - 0.00000351 \delta_2 \sin \theta_2$$

$$M_{14} = 0.000033233(1 + \cos \theta_2) - 0.00000055607 \delta_1 \sin \theta_2$$

$$M_{23} = 0.6225 + 0.6784 \cos \theta_2 - (0.000001477 \delta_2 + 0.0415 \delta_1) \sin \theta_2$$

$$M_{21} = M_{12}, M_{31} = M_{13}, M_{32} = M_{23}, M_{41} = M_{14}, M_{42} = M_{24}, M_{43} = M_{34},$$

$$M_{22} = 10.1756, M_{34} = 0.000033233, M_{44} = 1.5075$$

Inserting numerical values into the inertia matrix $\mathbf{M}(\mathbf{q})$ given by Equation (C.23) and $\phi_1 = \phi_2 = 0.2$ for the cantilever dominant assumed mode, including hub inertia terms for simulation in Matlab/Simulink™ we get:

$$M_{11} = 345.465 + 30.5269 \cos \theta_2 - (0.5108 \delta_1 + 0.000066462 \delta_2) \sin \theta_2;$$

$$M_{12} = 111.9319 + 15.2634 \cos \theta_2 - (0.2554 \delta_1 + 0.000066462 \delta_2) \sin \theta_2$$

$$M_{13} = 13.3833 + 1.6122 \cos \theta_2 - 0.00000351 \delta_2 \sin \theta_2$$

$$M_{14} = 0.000033233(1 + \cos \theta_2) - 0.00000055607 \delta_1 \sin \theta_2$$

$$M_{23} = 6.8479 + 0.6784 \cos \theta_2 - (0.000001477 \delta_2 + 0.0415 \delta_1) \sin \theta_2$$

$$M_{21} = M_{12}, M_{31} = M_{13}, M_{32} = M_{23}, M_{41} = M_{14}, M_{42} = M_{24}, M_{43} = M_{34},$$

$$M_{22} = 111.9319, M_{34} = 0.000033233, M_{44} = 1.5075$$

Inserting numerical values into the dynamic coupling vector $C(\mathbf{q}, \dot{\mathbf{q}})$ given by Equation (C.24) and $\varphi_1 = \varphi_2 = 0.2$ for the cantilever dominant assumed mode for simulating the full-order dynamics in Matlab/Simulink™ we get:

$$\begin{aligned} C_1 = & ((-30.5269 \dot{\theta}_2 - 0.5108 \dot{\delta}_1 - 0.000066462 \dot{\delta}_2) \dot{\theta}_1 + (-15.2634 \dot{\theta}_2 - 0.9338 \dot{\delta}_1 - \dots \\ & \dots - 0.000066462 \dot{\delta}_2) \dot{\theta}_2 - 0.0000040661 \dot{\delta}_1 \dot{\delta}_2) \sin \theta_2 + ((-0.5108 \dot{\theta}_1 - 0.2554 \dot{\theta}_2 + \dots \\ & \dots + 0.00000055607 \dot{\delta}_2) \dot{\delta}_1 + (-0.000066462 \dot{\theta}_1 - 0.000033231 \dot{\theta}_2 - \dots \\ & \dots - 0.0000035100 \dot{\delta}_1) \dot{\delta}_2) \dot{\theta}_2 \cos \theta_2 \end{aligned}$$

$$\begin{aligned} C_2 = & (15.2634 \dot{\theta}_1 + 1.3568 \dot{\delta}_1) \dot{\theta}_1 \sin \theta_2 + (((0.2554 \dot{\theta}_1 + 0.00000055607 \dot{\delta}_2) \dot{\delta}_1 + \dots \\ & \dots + (0.000033231 \dot{\theta}_1 + 0.0000035100 \dot{\delta}_1) \dot{\delta}_2) \dot{\theta}_1 + (0.000000090361 \dot{\delta}_1 \dot{\delta}_2 + \dots \\ & \dots + 0.000000027640 \dot{\delta}_1) \dot{\delta}_1) \cos \theta_2 \end{aligned}$$

$$\begin{aligned} C_3 = & ((-0.2554 \dot{\theta}_1 - 1.3568 \dot{\theta}_2 - 0.0000029540 \dot{\delta}_2) \dot{\theta}_1 + (-3.3918 \dot{\theta}_2 - 0.083 \dot{\delta}_1 - \\ & 0.0000029540 \dot{\delta}_2) \dot{\theta}_2 - 0.00000018072 \dot{\delta}_2 \dot{\delta}_1) \sin \theta_2 + \\ & ((-0.0000035100 \dot{\theta}_1 - 0.0000014770 \dot{\theta}_2 - 0.00000018072 \dot{\delta}_1) \dot{\delta}_2 + \\ & (-0.0415 \dot{\theta}_2 - 0.000000090361 \dot{\delta}_2) \dot{\delta}_1) \dot{\theta}_2 \cos \theta_2 \end{aligned}$$

$$\begin{aligned} C_4 = & (0.000033231 \dot{\theta}_1 + 0.000002954 \dot{\delta}_1) \dot{\theta}_1 \sin \theta_2 + (-0.00000055607 \dot{\delta}_1 \dot{\theta}_1 - \dots \\ & \dots - 0.000000090361 \dot{\delta}_1^2) \dot{\theta}_2 \cos \theta_2 \end{aligned}$$

The numerical elements of the stiffness matrix in Equation (C.25) are given by:

$K_1 = 227.322\delta_1$ and $K_2 = 227.322\delta_2$ based on a dominant mode frequency of 12.28 Hz and link mass of 1.5075 kg.

Similarly, a numerical form of the dynamics equations may be obtained for the dominant pinned-pinned assumed mode.

Appendix D

Free-Flying Flexible Space Robot Manipulator Kinematics and Dynamics Equations

D.1 Kinematics

Kinematics and dynamics equations are derived explicitly for a space robot comprising a rigid spacecraft, two flexible links and a rigid end effector [26].

The kinematics and dynamics equations used in Chapter 7 and presented in this appendix are adopted from Reference [26], but modified for the free-flying flexible space robot configuration shown in Figure D.1. The end effector mass is excluded. Also, the angles used in Reference [26] are measured from a fixed horizontal axis, whereas in this analysis the angles are measured from each preceding link to the next link in the serial chain.

Hence, we denote the angles given in Reference [26] by α_i , and the angles in this analysis by θ_i . Then by substitution we have $\theta_0 = \alpha_0$, $\theta_1 = \alpha_1 - \alpha_0$ and $\theta_2 = \alpha_2 - \alpha_1$. For the space robot shown in Figure D.1 the rigid spacecraft and link is denoted Body 0 and the two flexible serial links Body 1 and 2 respectively.

The spacecraft is capable of two planar translations, x_s and y_s and rotation θ_0 . The two flexible links are capable of rotations, θ_1 and θ_2 together with elastic vibrations u_1 and u_2 . The lengths of each link are $L_0 = 3$ m, $L_1 = 4.5$ m and $L_2 = 4.5$ m.

For Body 0 the displacement and velocity vectors are given by [26]:

$$\mathbf{U}_0 = \mathbf{R}_S + \mathbf{A}_0^T \mathbf{R}_0 \quad (\text{D.1})$$

$$\mathbf{V}_0 = \dot{\mathbf{R}}_S + \mathbf{A}_0^T \bar{\boldsymbol{\omega}}_0 \mathbf{R}_0 \quad (\text{D.2})$$

For Body 1 the displacement and velocity vectors are given by:

Figure D.1 Two-dimensional free-flying flexible space robot manipulator parameters.

$$\mathbf{U}_1 = \mathbf{R}_S + \mathbf{A}_0^T \mathbf{L}_0 + \mathbf{A}_1^T (\mathbf{R}_1 + \mathbf{u}_1) \quad (\text{D.3})$$

$$\mathbf{V}_1 = \dot{\mathbf{R}}_S + \mathbf{A}_0^T \bar{\boldsymbol{\omega}}_0 \mathbf{L}_0 + \mathbf{A}_1^T \bar{\boldsymbol{\omega}}_1 (\mathbf{R}_1 + \mathbf{u}_1) + \mathbf{A}_1^T \dot{\mathbf{u}}_1 \quad (\text{D.4})$$

For Body 2 the displacement and velocity vectors are given by:

$$\mathbf{U}_2 = \mathbf{R}_S + \mathbf{A}_0^T \mathbf{L}_0 + \mathbf{A}_1^T (\mathbf{L}_1 + \mathbf{u}_1) + \mathbf{A}_2^T (\mathbf{R}_2 + \mathbf{u}_2) \quad (\text{D.5})$$

$$\mathbf{V}_2 = \dot{\mathbf{R}}_S + \mathbf{A}_0^T \bar{\boldsymbol{\omega}}_0 \mathbf{L}_0 + \mathbf{A}_1^T \bar{\boldsymbol{\omega}}_1 (\mathbf{L}_1 + \mathbf{u}_1) + \mathbf{A}_1^T \dot{\mathbf{u}}_1 + \mathbf{A}_2^T \bar{\boldsymbol{\omega}}_2 (\mathbf{R}_2 + \mathbf{u}_2) + \mathbf{A}_2^T \dot{\mathbf{u}}_2 \quad (\text{D.6})$$

Where;

$$\mathbf{A}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}, \quad i = 0, 1, 2 \quad (\text{D.7})$$

are matrices of direction cosines.

$$\bar{\boldsymbol{\omega}}_i = \begin{bmatrix} 0 & -\dot{\theta}_i \\ \dot{\theta}_i & 0 \end{bmatrix}, \quad i = 0, 1, 2 \quad (\text{D.8})$$

are matrices of skew-symmetric angular velocities.

A position vector to any point on each of the bodies is given by:

$$\mathbf{R}_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \quad \mathbf{R}_1 = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}, \quad \mathbf{R}_2 = \begin{bmatrix} x_2 \\ 0 \end{bmatrix} \quad (\text{D.9})$$

Deformation vectors are given by:

$$\mathbf{u}_1 = \begin{bmatrix} 0 \\ u_1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ u_2 \end{bmatrix} \quad (\text{D.10})$$

$$u_i = \varphi_i(x_i) \delta_i \Big|_{x_i=L_i}, \quad i = 1, 2 \quad (\text{D.11})$$

and

$$u_1 = \varphi_1 \delta_1, \quad u_2 = \varphi_2 \delta_2 \quad (\text{D.12})$$

An approximate deformation function applied to an elastic link i subjected to transverse vibrations of the dominant assumed mode is given by [26], [143]:

$$u(x_i, t) = \sum_{i=1}^n \phi_i(x_i) \delta_i(t), \quad n = 2 \quad (D.13)$$

Where; $\phi_i(x_i)$ denotes the dominant assumed mode shape at a distance along link i with cantilever boundary conditions. From transverse beam vibration theory, cantilever assumed mode shapes are given by [143], [144].

$$\phi_{c_j}(x_i) = A [\cosh \lambda_{c_j} x_i - \cos \lambda_{c_j} x_i - k_{c_j} (\sinh \lambda_{c_j} x_i - \sin \lambda_{c_j} x_i)] \quad (D.14)$$

where; $A = 0.1$ is an arbitrary constant, $k_{c_j} = \cos \lambda_{c_j} L_i + \cosh \lambda_{c_j} L_i / \sin \lambda_{c_j} L_i + \sinh \lambda_{c_j} L_i$ and

$\lambda_{c_j} L_i = (j - 0.5)\pi$ are numerically approximate roots of the characteristic equation

$\cos(\lambda_{c_j} L_i) + \cosh(\lambda_{c_j} L_i) + 1 = 0$. Modal frequencies are given by:

$$\omega_{c_j} = (\lambda_{c_j} L_i)^2 \sqrt{\frac{EI_{L_i}}{\rho L_i^4}} \quad (D.15)$$

The deformation function $u(x_i, t)$ is used in the derivation of the flexible Euler-Lagrange equations of motion.

D.2 Dynamics

The dynamics equations used to model the flexible space robot are explicitly derived in published research literature. Using the kinematics equations presented in Section D.1, the flexible space robot equations of motion are derived using the Euler-Lagrange formulation in terms of kinetic T and potential U energies stored in the system. Given a generalized coordinates vector $\mathbf{q} = [x_s \ y_s \ \theta_0 \ \theta_1 \ \theta_2 \ \delta_1 \ \delta_2]^T$ of joint angles and elastic deformations, the total kinetic and potential energies stored in the system are given by:

$$T = \sum_{i=0}^2 T_i = \frac{1}{2} \sum_{i=0}^2 \int_0^{L_i} \rho_i \mathbf{V}_i^T \mathbf{V}_i dD_i = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}_{FF} \dot{\mathbf{q}}, \quad i = 0, 1, 2 \quad (D.16)$$

$$U = \sum_{i=0}^2 \frac{1}{2} \delta_i^2 K_i = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{K} \dot{\mathbf{q}}, \quad i = 1, 2 \quad (\text{D.17})$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_2 \end{bmatrix} \quad (\text{D.18})$$

is a stiffness matrix and;

$$K_i = \int_0^{L_i} EI_{L_i} (\phi_i'')^2 dx_i, \quad i = 1, 2 \quad (\text{D.19})$$

The control force vector is given by $\mathbf{F} = [F_x \ F_y \ \tau_0 \ \tau_1 \ \tau_2]^T$ in which F_x and F_y are thrust forces acting on the spacecraft CM along the x and y coordinate axes and τ_0 , τ_1 and τ_2 are joint actuator torques.

Using Equations (D.16) and (D.17) the Lagrangian function is given by:

$$\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = T - U \quad (\text{D.20})$$

For the robot subjected to the force vector $\mathbf{F}$ acting on the vector of generalized coordinates $\mathbf{q}$ the dynamic equations of motion are derived by differentiation of the Lagrangian function in the form of the following equation.

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \mathbf{F} \quad (\text{D.21})$$

The resulting Euler-Lagrange equations of motion are given by:

$$\boldsymbol{\tau} = \mathbf{M}_{FF}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} \quad (\text{D.22})$$

$\mathbf{M}_{FF}(\mathbf{q})$ is a matrix of rigid and flexible-link inertia elements for the spacecraft and robot manipulator links, $\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})$ is a matrix of Coriolis and centrifugal forces and $\mathbf{K}$ is a stiffness matrix.

$$\mathbf{M}_{FF}(\mathbf{q}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} & M_{37} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} & M_{47} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} & M_{57} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} & M_{67} \\ M_{71} & M_{72} & M_{73} & M_{74} & M_{75} & M_{76} & M_{77} \end{bmatrix} \quad (\text{D.23})$$

$$M_{11} = M_{22} = m_0 + m_1 + m_2, M_{12} = M_{21} = 0,$$

$$M_{13} = M_{31} = -\left\{ \left(\int_0^L \rho_0 x_0 dx_0 \right) \sin \theta_0 + \left(\int_0^L \rho_0 y_0 dy_0 \right) \cos \theta_0 + (m_1 + m_2) L_0 \cos \theta_0 \right\},$$

$$M_{23} = M_{32} = -\left\{ -\left(\int_0^L \rho_0 x_0 dx_0 \right) \cos \theta_0 + \left(\int_0^L \rho_0 y_0 dy_0 \right) \sin \theta_0 + (m_1 + m_2) L_0 \sin \theta_0 \right\},$$

$$M_{33} = \int_0^L \rho_0 x_0^2 dx_0 + \int_0^L \rho_0 y_0^2 dy_0 + (m_1 + m_2) L_0^2,$$

$$M_{14} = M_{41} = -\left(\int_1^L \rho_1 x_1 dx_1 + m_2 L_1 \right) \sin(\theta_0 + \theta_1) - \left(\int_1^L \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 \cos(\theta_0 + \theta_1),$$

$$M_{15} = M_{51} = -\left(\int_2^L \rho_2 x_2 dx_2 \right) \sin(\theta_0 + \theta_1 + \theta_2) - \left(\int_2^L \rho_2 \varphi_2 dx_2 \right) \delta_2 \cos(\theta_0 + \theta_1 + \theta_2),$$

$$M_{16} = M_{61} = -\left(\int_1^L \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \sin(\theta_0 + \theta_1),$$

$$M_{17} = M_{71} = -\left(\int_2^L \rho_2 \varphi_2 dx_2 \right) \sin(\theta_0 + \theta_1 + \theta_2),$$

$$M_{24} = M_{42} = \left(\int_1^L \rho_1 x_1 dx_1 + m_2 L_1 \right) \cos(\theta_0 + \theta_1) - \left(\int_1^L \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 \sin(\theta_0 + \theta_1),$$

$$M_{25} = M_{52} = \left(\int_2^L \rho_2 x_2 dx_2 \right) \cos(\theta_0 + \theta_1 + \theta_2) - \left(\int_2^L \rho_2 \varphi_2 dx_2 \right) \delta_2 \sin(\theta_0 + \theta_1 + \theta_2),$$

$$M_{34} = M_{43} = \left(\int \rho_1 x_1 dx_1 + m_2 L_1 \right) L_0 \sin \theta_1 + \left(\int \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 L_0 \cos \theta_1,$$

$$M_{35} = M_{53} = \left(\int \rho_2 x_2 dx_2 \right) L_0 \sin(\theta_1 + \theta_2) + \left(\int \rho_2 \varphi_2 dx_2 \right) \delta_2 L_0 \cos(\theta_1 + \theta_2),$$

$$M_{45} = M_{54} = \left(\int \rho_2 x_2 dx_2 \right) L_1 \cos \theta_2 + \left(\int \rho_2 x_2 dx_2 \right) \varphi_1 \delta_1 \sin \theta_2 - \dots \dots$$

$$\dots \dots - \left(\int \rho_2 \varphi_2 dx_2 \right) \delta_2 L_1 \sin \theta_2 + \left(\int \rho_2 \varphi_2 dx_2 \right) \delta_2 \varphi_1 \delta_1 \cos \theta_2,$$

$$M_{44} = \int \rho_1 x_1^2 dx_1 + m_2 L_1^2 + \delta_1 \left\{ \left(\int \rho_1 x_1 \varphi_1^2 dx_1 \right) + m_2 \varphi_1^2 \right\} \delta_1,$$

$$M_{55} = \int \rho_2 x_2^2 dx_2 + \delta_2 \left\{ \left(\int \rho_2 x_2 \varphi_2^2 dx_2 \right) \right\} \delta_2,$$

$$M_{26} = M_{62} = \left(\int \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \cos(\theta_0 + \theta_1),$$

$$M_{36} = M_{63} = \left(\int \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) L_0 \sin \theta_1, \quad M_{46} = M_{64} = \int \rho_1 x_1 \varphi_1 dx_1 + m_2 L_1 \varphi_1,$$

$$M_{56} = M_{65} = \left(\int \rho_2 x_2 dx_2 \right) \varphi_1 \cos \theta_2 - \varphi_1 \left(\int \rho_2 \varphi_2 dx_2 \right) \delta_2 \sin \theta_2,$$

$$M_{66} = \left(\int \rho_1 \varphi_1^2 dx_1 \right) + m_2 \varphi_1^2,$$

$$M_{27} = M_{72} = \left(\int \rho_2 \varphi_2 dx_2 \right) \cos(\theta_0 + \theta_1 + \theta_2), \quad M_{37} = M_{73} = \varphi_1 \cos \theta_2,$$

$$M_{47} = M_{74} = \left(\int \rho_2 \varphi_2 dx_2 \right) L_1 \cos \theta_2 + \left(\int \rho_2 \varphi_2 dx_2 \right) \varphi_1 \delta_1 \sin \theta_2,$$

$$M_{57} = M_{75} = \left(\int \rho_2 x_2 \varphi_2 dx_2 \right), \quad M_{67} = M_{76} = \left(\int \rho_2 \varphi_2 dx_2 \right) \varphi_1 \cos \theta_2,$$

$$M_{77} = \int \rho_2 \varphi_2^2 dx_2,$$

$$\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} & C_{27} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} & C_{37} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} & C_{47} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} & C_{57} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} & C_{67} \\ C_{71} & C_{72} & C_{73} & C_{74} & C_{75} & C_{76} & C_{77} \end{bmatrix} \quad (\text{D.24})$$

Where;

$$C_{11} = C_{12} = C_{21} = C_{22} = C_{31} = C_{32} = C_{41} = C_{42} = C_{51} = C_{52} = C_{61} = C_{62} = C_{71} = C_{72} = C_{11} = \\ C_{22} = C_{33} = C_{44} = C_{55} = C_{66} = C_{77} = 0,$$

$$C_{13} = \left\{ - \left(\int_0 \rho_0 x_0 dx_0 \right) \cos \theta_0 + \left(\int_0 \rho_0 y_0 dy_0 \right) \sin \theta_0 + (m_1 + m_2) L_0 \sin \theta_0 \right\} \dot{\theta}_0,$$

$$C_{14} = \left\{ - \left(\int_1 \rho_1 x_1 dx_1 + m_2 L_1 \right) \cos(\theta_0 + \theta_1) + \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 \sin(\theta_0 + \theta_1) \right\} (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{15} = \left\{ - \left(\int_2 \rho_2 x_2 dx_2 \right) \cos(\theta_0 + \theta_1 + \theta_2) + \dots \dots \dots \right. \\ \left. \dots \dots \dots + \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \sin(\theta_0 + \theta_1 + \theta_2) \right\} (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{16} = -2 \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \cos(\theta_0 + \theta_1) (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{17} = -2 \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \cos(\theta_0 + \theta_1 + \theta_2) (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{23} = - \left\{ \left(\int_0 \rho_0 x_0 dx_0 \right) \sin \theta_0 + \left(\int_0 \rho_0 y_0 dy_0 \right) \cos \theta_0 + (m_1 + m_2) L_0 \cos \theta_0 \right\} \dot{\theta}_0,$$

$$C_{24} = \left\{ - \left(\int_1 \rho_1 x_1 dx_1 + m_2 L_1 \right) \sin(\theta_0 + \theta_1) - \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 \cos(\theta_0 + \theta_1) \right\} (\dot{\theta}_0 + \dot{\theta}_1)$$

$$C_{25} = \left\{ - \left(\int_2 \rho_2 x_2 dx_2 \right) \sin(\theta_0 + \theta_1 + \theta_2) - \dots \dots \dots \right. \\ \left. \dots \dots \dots - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \cos(\theta_0 + \theta_1 + \theta_2) \right\} (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{26} = -2 \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \sin(\theta_0 + \theta_1) (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{27} = -2 \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \sin(\theta_0 + \theta_1 + \theta_2) (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{34} = \left\{ \left(\int_1 \rho_1 x_1 dx_1 + m_2 L_1 \right) L_0 \cos \theta_1 - \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 L_0 \sin \theta_1 \right\} (\dot{\theta}_0 + \dot{\theta}_1)$$

$$C_{35} = \left\{ \begin{aligned} & \left(\int_2 \rho_2 x_2 dx_2 \right) L_0 \cos(\theta_1 + \theta_2) - \dots \\ & \dots - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 L_0 \sin(\theta_1 + \theta_2) \end{aligned} \right\} (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{36} = 2 \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) L_0 \cos \theta_1 (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{37} = 2 \left(\int_2 \rho_2 \varphi_2 dx_2 \right) L_0 \cos(\theta_1 + \theta_2) (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{43} = \left\{ - \left(\int_1 \rho_1 x_1 dx_1 + m_2 L_1 \right) L_0 \cos \theta_1 + \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) \delta_1 L_0 \sin \theta_1 \right\} \dot{\theta}_0,$$

$$C_{45} = \left\{ \begin{aligned} & \left(- \int_2 \rho_2 x_2 dx_2 \right) L_1 \sin \theta_2 - \dots \\ & \dots - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 L_1 \cos \theta_2 + \dots \\ & \dots + \left(\int_2 \rho_2 x_2 dx_2 \right) \varphi_1 \delta_1 \cos \theta_2 - \dots \\ & \dots - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \varphi_1 \delta_1 \sin \theta_2 \end{aligned} \right\} (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{46} = 2 \delta_1 \left(\int_1 \rho_1 \varphi_1^2 dx_1 + m_2 \varphi_1^2 \right) (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{47} = 2 \left(-L_1 \sin \theta_2 \int_2 \rho_2 \varphi_2 dx_2 + \varphi_1 \delta_1 \cos \theta_2 \int_2 \rho_2 \varphi_2 dx_2 \right) (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{53} = \left\{ \left(- \int_2 \rho_2 x_2 dx_2 \right) L_0 \cos(\theta_1 + \theta_2) + \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 L_0 \sin(\theta_1 + \theta_2) \right\} \dot{\theta}_0,$$

$$C_{54} = \left\{ \begin{aligned} & \left(\int_2 \rho_2 x_2 dx_2 \right) L_1 \sin \theta_2 + \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 L_1 \cos \theta_2 - \dots \\ & \dots - \left(\int_2 \rho_2 x_2 dx_2 \right) \varphi_1 \delta_1 \cos \theta_2 + \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \varphi_1 \delta_1 \sin \theta_2 \end{aligned} \right\} (\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{56} = \left\{ 2 \left(\int_2 \rho_2 x_2 dx_2 \right) \sin \theta_2 \varphi_1 + \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \cos \theta_2 \varphi_1 \right\} (\dot{\theta}_0 + \dot{\theta}_1),$$

$$\begin{aligned}
C_{57} &= 2\delta_2 \left(\int_2 \rho_2 \varphi_2^2 dx_2 \right) (\dot{\theta}_0 + \dot{\theta}_1), \\
C_{63} &= - \left(\int_1 \rho_1 \varphi_1 dx_1 + m_2 \varphi_1 \right) L_0 \cos \theta_1 \dot{\theta}_0, \\
C_{64} &= - \left(\int_1 \rho_1 \varphi_1^2 dx_1 + m_2 \varphi_1^2 \right) \delta_1 (\dot{\theta}_0 + \dot{\theta}_1), \\
C_{65} &= \left\{ - \left(\int_2 \rho_2 x_2 dx_2 \right) \sin \theta_2 \varphi_1 - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \delta_2 \cos \theta_2 \varphi_1 \right\} (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2), \\
C_{67} &= -2\varphi_1 \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \sin \theta_2 (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2), \\
C_{73} &= - \left(\int_2 \rho_2 \varphi_2 dx_2 \right) L_0 \cos(\theta_1 + \theta_2) \dot{\theta}_0, \\
C_{74} &= \left(L_1 \sin \theta_2 \int_2 \rho_2 \varphi_2 dx_2 - \varphi_1 \delta_1 \cos \theta_2 \int_2 \rho_2 \varphi_2 dx_2 \right) (\dot{\theta}_0 + \dot{\theta}_1), \\
C_{75} &= - \left(\int_2 \rho_2 \varphi_2^2 dx_2 \right) \delta_2 (\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2), \\
C_{76} &= 2 \left(\int_2 \rho_2 \varphi_2 dx_2 \right) \varphi_1 \sin \theta_2 (\dot{\theta}_0 + \dot{\theta}_1).
\end{aligned}$$

Where;

$$\varphi_i = \varphi_i(x_i) \Big|_{x_i=L_i}, \quad i = 1, 2$$

D.3 Numerical Form of Dynamics Equations for Simulation

Inserting numerical values for the space robot parameters into the inertia matrix $\mathbf{M}_{FF}(\mathbf{q})$ given by Equation (D.23), and $\varphi_1 = \varphi_2 = 0.2$ for the cantilever dominant assumed mode for simulation in Matlab/Simulink™ we get:

$$M_{11} = 603.015, M_{12} = M_{21} = 0, M_{13} = M_{31} = -(450 \sin \theta_0 + 454.5225 \cos \theta_0 + 10),$$

$$M_{14} = M_{41} = -10.1756 \sin(\theta_0 + \theta_1) - 0.603 \delta_1 \cos(\theta_0 + \theta_1),$$

$$M_{15} = M_{51} = -3.3919 \sin(\theta_0 + \theta_1 + \theta_2) - 0.0000073846 \delta_2 \cos(\theta_0 + \theta_1 + \theta_2),$$

$$M_{16} = M_{61} = -0.603 \sin(\theta_0 + \theta_1), M_{17} = M_{71} = -0.0000073846 \cos(\theta_0 + \theta_1 + \theta_2),$$

$$M_{22} = 603.015, M_{23} = M_{32} = 450 \cos \theta_0 + 454.5225 \cos \theta_0 + 203.5126,$$

$$\begin{aligned}
M_{24} &= M_{42} = 10.1756\cos(\theta_0 + \theta_1) - 0.603\delta_1\sin(\theta_0 + \theta_1), \\
M_{25} &= M_{52} = 3.3919\cos(\theta_0 + \theta_1 + \theta_2) - 0.0000073846\delta_2\sin(\theta_0 + \theta_1 + \theta_2), \\
M_{26} &= M_{62} = 0.603\cos(\theta_0 + \theta_1), M_{27} = M_{72} = 0.0000073846\cos(\theta_0 + \theta_1 + \theta_2), \\
M_{33} &= 1837.1, M_{34} = M_{43} = 30.5268\sin\theta_1 + 1.8091\delta_1\cos\theta_1, \\
M_{35} &= M_{53} = 10.1757\sin(\theta_1 + \theta_2) + 0.000022154\delta_2\cos(\theta_1 + \theta_2), M_{36} = 1.8091\sin\theta_1, \\
M_{37} &= M_{73} = 0.2\cos\theta_2, M_{44} = 40.7025 + 0.0783\delta_1^2, \\
M_{45} &= M_{54} = 15.2636\cos\theta_2 + 0.6784\delta_1\sin\theta_2 - 0.000033231\delta_2\sin\theta_2 + \dots \\
&\quad \dots + 0.0000014769\delta_2\delta_1\cos\theta_2, \\
M_{46} &= M_{64} = 0.9204, M_{47} = M_{74} = 0.000033231\cos\theta_2 + 0.0000014769\delta_1\sin\theta_2, \\
M_{55} &= 10.1756 + 0.018\delta_2^2, M_{56} = 0.6784\cos\theta_2 - 0.0000014770\delta_2\sin\theta_2, \\
M_{57} &= M_{75} = 0.000033231, M_{63} = 1.8091\sin\theta_1, M_{65} = 0.6784\cos\theta_2, M_{66} = 0.0783, \\
M_{67} &= M_{76} = 0.0000014770\cos\theta_2, M_{77} = 0.018.
\end{aligned}$$

Where; $m_{h_1} = m_{h_2} = 5\text{ kg}$, $I_{L_1} = \frac{m_1 L_1^2}{3}$, $I_{L_2} = \frac{m_2 L_2^2}{3}$, $I_{h_1} = 10I_{L_1}$, $I_{h_2} = 10I_{L_2}$

Inserting numerical values for the space robot dynamic parameters into the dynamic coupling matrix $\mathbf{C}_{FF}(\mathbf{q}, \dot{\mathbf{q}})$ given by Equation (D.24), and $\varphi_1 = \varphi_2 = 0.2$ for the cantilever dominant assumed mode for simulation in Matlab/Simulink™ we get:

$$\begin{aligned}
C_{11} &= C_{12} = C_{21} = C_{22} = C_{31} = C_{32} = C_{41} = C_{42} = C_{51} = C_{52} = C_{61} = C_{62} = C_{71} = C_{72} = C_{11} = \\
C_{22} &= C_{33} = C_{44} = C_{55} = C_{66} = C_{77} = 0,
\end{aligned}$$

$$C_{13} = -(6.03\cos\theta_0 + 10.552\sin\theta_0 + 1804.5\sin\theta_0)\dot{\theta}_0,$$

$$C_{14} = (-10.1756\cos(\theta_0 + \theta_1) + 0.603\delta_1\sin(\theta_0 + \theta_1))(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{15} = (-3.3919\cos(\theta_0 + \theta_1 + \theta_2) + 0.0000073846\delta_2\sin(\theta_0 + \theta_1 + \theta_2))(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{16} = -1.206\cos(\theta_0 + \theta_1)(\dot{\theta}_0 + \dot{\theta}_1), C_{17} = -0.4\cos(\theta_0 + \theta_1 + \theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{23} = (-6.03\sin\theta_0 - 10.552\cos\theta_0 - 1804.5 \cos\theta_0)\dot{\theta}_0,$$

$$C_{24} = (-10.1757\sin(\theta_0 + \theta_1) - 0.603 \delta_1 \cos(\theta_0 + \theta_1))(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{25} = (-3.3919\sin(\theta_0 + \theta_1 + \theta_2) - 0.0000073846\delta_2 \cos(\theta_0 + \theta_1 + \theta_2))(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{26} = -1.206\sin(\theta_0 + \theta_1)(\dot{\theta}_0 + \dot{\theta}_1), C_{27} = -0.000014769\sin(\theta_0 + \theta_1 + \theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{34} = (30.527\cos\theta_1 - 1.8091\delta_1 \sin\theta_1)(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{35} = (10.1757\cos(\theta_1 + \theta_2) - 0.000022154\delta_2 \sin(\theta_1 + \theta_2))(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{36} = 3.6181\cos\theta_1(\dot{\theta}_0 + \dot{\theta}_1), C_{37} = 0.000044308\cos(\theta_1 + \theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{43} = (-30.5270\cos\theta_1 + 1.8091\delta_1 \sin\theta_1)\dot{\theta}_0,$$

$$C_{45} = (-15.2636\sin\theta_2 - 0.000033231\delta_2 \cos\theta_2 + \dots + 0.6784\delta_1 \cos\theta_2 - 0.0000014769\delta_2 \delta_1 \sin\theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{46} = 0.1566\delta_1(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{47} = (-0.000066462\sin\theta_2 + 0.0000029539\delta_1 \cos\theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{53} = (-10.1757\cos(\theta_1 + \theta_2) + 0.000022154\delta_2 \sin(\theta_1 + \theta_2))\dot{\theta}_0,$$

$$C_{54} = (15.2636\sin\theta_2 + 0.000033231\delta_2 \cos\theta_2 - 0.6784\delta_1 \cos\theta_2 + \dots + 0.0000014769\delta_2 \delta_1 \sin\theta_2)(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{56} = (1.3568\sin\theta_2 + 0.0000029539\delta_2 \cos\theta_2)(\dot{\theta}_0 + \dot{\theta}_1), C_{57} = 0.036\delta_2(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{63} = -1.8091\cos\theta_1 \dot{\theta}_0, C_{64} = -0.0783\delta_1(\dot{\theta}_0 + \dot{\theta}_1),$$

$$C_{65} = (-0.6784\sin\theta_2 - 0.0000014770\delta_2 \cos\theta_2)(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{67} = -0.0000029540\sin\theta_2(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2), C_{73} = -0.000022154\cos(\theta_1 + \theta_2)\dot{\theta}_0,$$

$$C_{74} = (0.000033231\sin\theta_2 - 0.0000014769\delta_1 \cos\theta_2)(\dot{\theta}_0 + \dot{\theta}_1), C_{75} = -0.018\delta_2(\dot{\theta}_0 + \dot{\theta}_1 + \dot{\theta}_2),$$

$$C_{76} = 0.0000029539\sin\theta_2(\dot{\theta}_0 + \dot{\theta}_1).$$

D.4 Direct Kinematics Equations for a FFSRM

Applying the Denavit-Hartenburg convention to the flexible robot manipulator, we get [137]:

$$\begin{aligned}
 \mathbf{A}_i &= \text{Rot}_{Z,\theta_i} \cdot \text{Trans}_{Z,d_i} \cdot \text{Trans}_{X,L_i+u_i} \cdot \text{Rot}_{X,\alpha_i} \\
 &= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_i + u_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (\text{D.25}) \\
 &= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & \sin\theta_i & (L_i + u_i)\cos\theta_i \\ \sin\theta_i & \cos\theta_i & -\cos\theta_i & (L_i + u_i)\sin\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

For which, link deformation vectors are given by Equation (D.10) [26]:

$$\mathbf{u}_1 = \begin{bmatrix} 0 \\ u_1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ u_2 \end{bmatrix} \quad (\text{D.26})$$

$$u_i = \varphi_i(x_i)\delta_i \Big|_{x_i=L_i} \quad (\text{D.27})$$

$$u_1 = \varphi_1\delta_1, \quad u_2 = \varphi_2\delta_2 \quad (\text{D.28})$$

$$\varphi_1 = \varphi_2 = 0.2 \quad (\text{D.29})$$

$$A_0 = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & L_0\cos\theta_0 \\ \sin\theta_0 & \cos\theta_0 & 0 & L_0\sin\theta_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(D.30)

$$= \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 3\cos\theta_0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 3\sin\theta_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & (L_1 + u_1)\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & (L_1 + u_1)\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(D.31)

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & (4.5 + 0.2\delta_1)\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & (4.5 + 0.2\delta_1)\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & (L_2 + u_2)\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & (L_2 + u_2)\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(D.32)

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & (4.5 + 0.2\delta_2)\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & (4.5 + 0.2\delta_2)\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence;

$$T_0^3 = A_0 A_1 A_2 \begin{bmatrix} (c_0 c_1 - s_0 s_1)c_2 + (-c_0 s_1 - s_0 c_1)s_2 & -(c_0 c_1 - s_0 s_1)s_2 + (-c_0 s_1 - s_0 c_1)c_2 & 0 & \dots\dots\dots \\ \dots\dots\dots (c_0 c_1 - s_0 s_1)c_2(L_2 + u_2) + (-c_0 s_1 - s_0 c_1)s_2(L_2 + u_2) + \dots\dots\dots & \dots\dots\dots + c_0 c_1(L_1 + u_1) - s_0 s_1(L_1 + u_1) + c_0 L_0 & & \\ (s_0 c_1 + c_0 s_1)c_2 + (c_0 c_1 - s_0 s_1)s_2 & -(s_0 c_1 + c_0 s_1)s_2 + (c_0 c_1 - s_0 s_1)c_2 & 0 & \dots\dots\dots \\ \dots\dots\dots (s_0 c_1 + c_0 s_1)c_2(L_2 + u_2) + (c_0 c_1 - s_0 s_1)s_2(L_2 + u_2) + \dots\dots\dots & \dots\dots\dots + s_0 c_1(L_1 + u_1) + c_0 s_1(L_1 + u_1) + s_0 L_0 & & \\ & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \end{bmatrix} \quad (D.33)$$

$$= \begin{bmatrix} (c_0 c_1 - s_0 s_1)c_2 + (-c_0 s_1 - s_0 c_1)s_2 & -(c_0 c_1 - s_0 s_1)s_2 + (-c_0 s_1 - s_0 c_1)c_2 & 0 & \dots\dots\dots \\ \dots\dots\dots (c_0 c_1 - s_0 s_1)c_2(4.5 + 0.2\delta_2) + (-c_0 s_1 - s_0 c_1)s_2(4.5 + 0.2\delta_2) + \dots\dots\dots & \dots\dots\dots + c_0 c_1(4.5 + 0.2\delta_1) - s_0 s_1(4.5 + 0.2\delta_1) + 3c_0 & & \\ (s_0 c_1 + c_0 s_1)c_2 + (c_0 c_1 - s_0 s_1)s_2 & -(s_0 c_1 + c_0 s_1)s_2 + (c_0 c_1 - s_0 s_1)c_2 & 0 & \dots\dots\dots \\ \dots\dots\dots (s_0 c_1 + c_0 s_1)c_2(4.5 + 0.2\delta_2) + (c_0 c_1 - s_0 s_1)s_2(4.5 + 0.2\delta_2) + \dots\dots\dots & \dots\dots\dots + s_0 c_1(4.5 + 0.2\delta_1) + c_0 s_1(4.5 + 0.2\delta_1) + 3s_0 & & \\ & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \end{bmatrix}$$

The first two entries of the fourth column in Equation (D.33) are the Cartesian x and y positions of the end effector relative to the ECEF inertial frame of reference given by:

$$\begin{aligned} x &= x_S + (c_0 c_1 - s_0 s_1)(4.5 + 0.2\delta_2)c_2 - (c_0 s_1 + s_0 c_1)(4.5 + 0.2\delta_2)s_2 + \dots\dots\dots \\ &\quad \dots\dots\dots + c_0 c_1(4.5 + 0.2\delta_1) - s_0 s_1(4.5 + 0.2\delta_1) + 3c_0 \\ y &= y_S + (s_0 c_1 + c_0 s_1)(4.5 + 0.2\delta_2)c_2 + (c_0 c_1 - s_0 s_1)(4.5 + 0.2\delta_2)s_2 + \dots\dots\dots \\ &\quad \dots\dots\dots + s_0 c_1(4.5 + 0.2\delta_1) + c_0 s_1(4.5 + 0.2\delta_1) + 3s_0 \end{aligned} \quad (D.34)$$

Where; $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$, x_S and y_S are spacecraft CM positions.

D.5 Fixed-Attitude-Restricted (FAR) Motion Jacobian for a FFFSRM

Applying the Jacobian differential form to the direct kinematics given by

Equation (D.34), we get:

$$\mathbf{J}_M^*(\mathbf{q}) = \begin{bmatrix} \frac{\partial x}{\partial \theta_0} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial u_1} & \frac{\partial x}{\partial u_2} \\ \frac{\partial y}{\partial \theta_0} & \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial u_1} & \frac{\partial y}{\partial u_2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ K_b & 0 \\ K_c & K_c \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{D.35})$$

$$= \begin{bmatrix} -1.995(c_0 s_1 + s_0 c_1) c_2 (L_2 + u_2) + 0.9963(s_0 s_1 - c_0 c_1) s_2 (L_2 + u_2) - \dots \\ 1.995(c_0 c_1 - s_0 s_1) c_2 (L_2 + u_2) - 0.9963(c_0 s_1 + s_0 c_1) s_2 (L_2 + u_2) + \dots \\ \dots - 0.9963 c_0 s_1 (L_1 + u_1) - 0.9963 s_0 c_1 (L_1 + u_1) - \dots \\ \dots + 0.9963 c_0 c_1 (L_1 + u_1) - 0.9963 s_0 s_1 (L_1 + u_1) - \dots \\ \dots - 0.9988(c_0 c_1 - s_0 s_1) s_2 (L_2 + u_2) & -0.9988(c_0 c_1 - s_0 s_1) s_2 (L_2 + u_2) - \dots \\ \dots - 0.9988(s_0 c_1 + c_0 s_1) s_2 (L_2 + u_2) & -0.9988(s_0 c_1 + c_0 s_1) s_2 (L_2 + u_2) - \dots \\ & \dots - 0.9988(c_0 c_1 + s_0 s_1) c_2 (L_2 + u_2) \\ & \dots + 0.9988(c_0 c_1 - s_0 s_1) c_1 (L_2 + u_2) \end{bmatrix} \quad (\text{D.36})$$

$$= \begin{bmatrix} -1.995(c_0 s_1 + s_0 c_1) c_2 (4.5 + 0.2\delta_2) + 0.9963(s_0 s_1 - c_0 c_1) s_2 (4.5 + 0.2\delta_2) - \dots \\ 1.995(c_0 c_1 - s_0 s_1) c_2 (4.5 + 0.2\delta_2) - 0.9963(c_0 s_1 + s_0 c_1) s_2 (4.5 + 0.2\delta_2) + \dots \\ \dots - 0.9963 c_0 s_1 (4.5 + 0.2\delta_1) - 0.9963 s_0 c_1 (4.5 + 0.2\delta_1) - \dots \\ \dots + 0.9963 c_0 c_1 (4.5 + 0.2\delta_1) - 0.9963 s_0 s_1 (4.5 + 0.2\delta_1) - \dots \\ \dots - 0.9988(c_0 c_1 - s_0 s_1) s_2 (4.5 + 0.2\delta_2) & -0.9988(c_0 c_1 - s_0 s_1) s_2 (4.5 + 0.2\delta_2) - \dots \\ \dots - 0.9988(s_0 c_1 + c_0 s_1) s_2 (4.5 + 0.2\delta_2) & -0.9988(s_0 c_1 + c_0 s_1) s_2 (4.5 + 0.2\delta_2) - \dots \\ & \dots - 0.9988(c_0 s_1 + s_0 c_1) c_2 (4.5 + 0.2\delta_2) \\ & \dots + 0.9988(c_0 c_1 - s_0 s_1) c_1 (4.5 + 0.2\delta_2) \end{bmatrix} \quad (\text{D.37})$$

Also, the joint rate transformation from joint to Cartesian variables is given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J}_M^*(\mathbf{q}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (\text{D.38})$$