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The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,

**"Bremsstrahlung of 10 to 30 MeV Electrons
Incident on Thick Targets"**

submitted by

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in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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September 21, 1990

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Bremsstrahlung of 10 to 30 MeV Electrons Incident on Thick Targets

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October 4, 1990

Preface

In 1987 I started work on a project that was suggested by my co-supervisors, Dave Rogers and Carl Ross. The intent of this project was to measure a set of bremsstrahlung spectra from thick targets in the energy range of interest to cancer therapy. The spectra would be used for benchmarking the best available calculation approach: Monte Carlo simulation of the radiation transport. During the course of the work abstracts were presented at several meetings. Most of the methodology and results have been summarized in a set of four papers for which I was the principal investigator. The articles have been incorporated directly into the thesis as follows:

1. Thesis chapter 2, "*Forward-directed bremsstrahlung of 10 to 30 MeV electrons incident on thick targets of Al and Pb*", to be published in **Medical Physics** 17(5) (1990).
2. Thesis chapter 3, "*Angular distributions of bremsstrahlung from 15 MeV electrons incident on thick targets of Be, Al and Pb*", submitted to **Medical Physics**. The appendix to this paper appears in appendix D of the thesis.
3. Thesis appendix A, "*Pile-Up corrections in pulsed-beam spectroscopy*", to be published in **Nuclear Instruments and Methods B** (1990).
4. Thesis appendix B, "*Precision response estimation, energy calibration, and unfolding of spectra measured with a large NaI detector*", to be published in **Nuclear Instruments and Methods A** (1990 or 1991). The appendix to this paper is contained in appendix C of the thesis.

The rest of the thesis includes an introductory chapter, a chapter on the measurement of the beam current, and an additional appendix with tables of the measured bremsstrahlung yields. Bibliographies are given at the end of each chapter and appendix.

Abstract

A comprehensive set of absolute measurements (photons per electron) of the bremsstrahlung yield from thick targets produced by electrons in the energy range 10 to 30 MeV are reported. The major purpose of these measurements was to provide data for benchmarking calculation approaches. Measurements were made over a wide range of electron energy (10 MeV to 30 MeV), angle (0° to 90°), and atomic number ($Z=4$ to $Z=82$). The targets were cylinders of Be, Al, and Pb with nominal thicknesses of 110% of the CSDA range of the electrons. A thin transmission detector was placed upstream of the target to measure the beam current. The bremsstrahlung spectrometer was a 20 cm diameter by 25 cm long NaI detector. Spectra were calculated to a high degree of precision using the EGS4 Monte Carlo system for simulating the radiation transport. The most recent values of the radiative and collision stopping powers from the ICRU were used. A recent modification to the code was utilized to obtain a more realistic angular distribution of bremsstrahlung about the direction of the radiating electron. The shapes of the measured and calculated spectra were generally in excellent agreement. The ratio of the total number of photons in each measured spectrum to those in the corresponding calculated spectrum varied from 0.92 ± 0.05 to 1.12 ± 0.06 , depending largely on the atomic number of the target. This verifies the use of EGS4 with the specified modifications for the calculation of absolute bremsstrahlung spectra to $\pm 10\%$. The angular distribution of bremsstrahlung from calculations based on some of the available analytical theories for thick targets were also compared to the measurements. The calculated angular distributions dropped off more quickly with angle than the measured distributions. The predictions would be improved by including

target-scattered photons in the theories.

Acknowledgements

This is the last and one of the most pleasant parts of this thesis to write. Here I have the opportunity to formalize my gratitude to the many people who have contributed to this work.

To the staff and my fellow students at Carleton University: Thank you for the academic enrichment, encouragement, and opportunities for personal development that you provided. A special thanks to the members of my steering committee: Dave Rogers, Peter Raaphorst, Bob Clarke, and Brian Logan.

To the first-rate scientists, technical experts, and support staff of the section of Ionizing Radiation Standards at the National Research Council of Canada: I have been greatly rewarded by the support and mentorship that every one of you provided me, both towards the technical side of this research endeavour and towards my overall development as a research scientist. Your own commitments to excellence in research had a strong influence on the quality of my own work, and for this I am truly grateful. I would like to single out four of you for special thanks. Thank you Leo Heistek for your reverant devotion to unconditional fundamental principles that in the end proved you dead right! Thank you Len Vander Zwan for your guidance in the details of scintillation spectrometry. Thank you Carl Ross for teaching me new experimental techniques and the rewards of attention to experimental detail. You were there, ready and willing, for all of the bremsstrahlung spectrometry, the preparations for conference presentations, the paper writing, and the thesis compilation. I will always be grateful for your large contribution to this thesis and my overall education. Thank you Dave Rogers for the hours you spent revising and debugging EGS, revising manuscripts, deal-

ing with administrative details, criticising presentations, and for your thoughtful contributions at all of the critical turning points from beginning to end. You have taught me a great deal over these past four years. Your mentorship will carry me through the rest of my career, wherever it takes me.

To Mom and Dad: I would not be here without you, either on this earth or completing this degree! You will soon have a doctor in the family. You should be proud of yourselves, and I am grateful to you.

To Cynthia: Thank you for returning the favour and typing *my* thesis. Thank you for supporting me for the long haul. But most of all, thank you for all of the things that you have done for me and the joy that you bring to my life.

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Chapter 1

Introduction

Radiation therapy with bremsstrahlung beams from megavoltage linear accelerators is one of the principal methods of cancer treatment. The treatment beam from these units is produced by monoenergetic beams of 5 to 50 MeV electrons incident on thick, high atomic-number targets. The target thicknesses are generally sufficient to fully stop all of the electrons in the incident beam. The electrons lose most of their energy through collisional and radiative processes. The photons produced in the radiative processes together comprise the bremsstrahlung beam.

The photon spectrum originating in the target strongly influences the distribution of dose in the patient and the response of dosimeters placed in the beam. For example, a sparse collection of measured photon spectra serve as a basis for the estimate of spectrum-averaged quantities required by the AAPM protocol [1], and detailed knowledge of the spectrum is important for accelerator design [2] and for development of accurate treatment planning algorithms for cancer therapy [3]. However, there are only a few measurements available of the bremsstrahlung produced by electrons in the energy range of interest to

radiotherapy.

The literature contains many theories of bremsstrahlung production. Many of these theories have been reviewed by Koch and Motz [4] and by Tsai [5]. Such theories generally fit into one of two groups: 1) theories restricted to incident electron energies of less than 2 MeV and 2) theories restricted to incident electron energies of greater than 50 MeV. Seltzer and Berger [6] have combined several theories in order to derive the bremsstrahlung cross-section, differential in emitted photon energy, for electrons in the range 1 keV to 10 GeV incident on neutral atoms with $Z=1$ to 100. For low energies Seltzer and Berger made use of the theory of Tseng and Pratt [7] who developed a numerical approach for energies below 2 MeV. At higher energies such calculations become prohibitively expensive in computer time and, therefore, cannot be carried out systematically. For energies greater than 50 MeV Seltzer and Berger utilized the Bethe-Heitler cross-section obtained in the Born approximation [8] modified to take into account the screening of the nuclear charge by the atomic electrons and the departure from the Born approximation. This was used to calculate the electron-nucleus bremsstrahlung. Contributions of the electron-electron bremsstrahlung were included using the cross-section of Haug [9] for bremsstrahlung in the field of a free electron at rest with corrections that take into account the effects of atomic binding and screening [10]. Seltzer and Berger resort to interpolation in the intermediate energy range of 2 to 50 MeV. Interpolated points were compared with available calculations by Tseng and Pratt [11] for Al and U targets at 5 and 10 MeV using their high-cost numerical approach. The interpolated values were within the estimated 5% uncertainty of the calculation. The estimated uncertainty of the interpolation procedure was 3 to 10%. In general all bremsstrahlung theories

have difficulty in this energy region.

Photon spectra from thick targets have been calculated by a variety of methods ranging from analytical calculations based on coarse simplifying assumptions, to Monte Carlo calculations which account for all of the physically significant phenomena, in particular, energy loss and scatter of electrons and attenuation and scatter of photons [12]. Even the spectra calculated by the Monte Carlo approach have potential inaccuracies due to systematic errors in basic physical quantities such as the bremsstrahlung cross-sections (which have just been discussed), errors and simplifications in specific algorithms, and approximations in the model geometry.

The purpose of this project was to measure the spectral distribution of bremsstrahlung from thick targets over a comprehensive range of energy and atomic number. Absolute spectra (photons per electron incident on the target) were measured in the forward direction (0°) for 10 to 30 MeV electrons incident on thick targets of Al and Pb. The measurements for 15 MeV electrons were extended to include eight angles from 0° to 90° and an additional target, Be. These measurements comprise the most comprehensive survey to date of absolute thick-target bremsstrahlung from relativistic electrons. The spectra cover a wider range of photon energy, contain more energy bins, and/or are more accurate than previous measurements. In addition, the measurements were absolute. The results were used as benchmark data for verification of state-of-the-art Monte Carlo calculations and analytical thick-target theories of bremsstrahlung. Spectra were determined from the Monte Carlo simulations to a high degree of precision. The comparisons of the measured and calculated spectral shapes and absolute yields were a stringent test of the Monte Carlo

approach.

1.1 Measurement of the bremsstrahlung spectra produced by megavoltage electrons

Spectral distributions of bremsstrahlung from megavoltage accelerators have been measured by a number of approaches. These techniques are discussed in this section. Approximate estimates of the resolution of the spectrometer and accuracy of the measured spectra are given to facilitate comparison of the methods. The spectral resolution is defined as the greater of the full-width at half-maximum of the detector response or the energy interval between points in the spectrum, and is generally quoted at 10 MeV. The citations in this section comprise a comprehensive survey of measurements of megavoltage bremsstrahlung spectra.

1.1.1 Pair production

Individual photons in the bremsstrahlung beam incident on a target interact with atomic nuclei, or occasionally with atomic electrons, to produce electron-positron pairs. The photon energy goes into the rest mass and kinetic energy of the two particles that are created. The kinetic energy of the photon is easily determined from the combined kinetic energies of the electron and positron. This technique has been used with a cloud chamber as the charged-particle spectrometer [13,14,15] and with a magnetic spectrometer [16]. The detector efficiency is calculated from the pair-production cross-section of the target material. Photons with energies less than 1.02 MeV do not produce pairs and are therefore not

detected. In practice, the detector is not useful for estimating the portion of the spectrum below about 4 MeV. The magnetic spectrometer used by Warner and Shrader [16] limited the energy resolution to 20% at 10 MeV; however, measurements were only made at 4 MeV intervals. Uncertainty in the pair production cross sections limits the accuracy of the spectral measurements to somewhat better than $\pm 10\%$.

1.1.2 Compton spectrometry

A Compton spectrometer consists of a thin-foil target used to scatter the bremsstrahlung photons and a spectrometer used to measure the energy of the photons or electrons which are scattered into a given range of angles. The bremsstrahlung spectrum is deduced from the energy and scattering angle of either scattered particle. Lasich et al [17] employed the method of Compton scattering using a cloud chamber in a magnetic field. The scattering angle and energy of electrons scattered from a 0.5 mm carbon plate were inferred from the initial direction and radius of curvature of the photographed cloud chamber tracks. The procedure was tedious and has poor resolution and poor accuracy.

Motz et al [18] and Robson and Gregg [19] used a magnetic spectrometer to measure the energy spectrum of electrons that were Compton scattered in the forward direction. The portion of the spectrum attributed to pair-produced electrons was measured by reversing the magnetic field and assuming the positron spectrum measured in this manner was identical to the pair-electron spectrum. This spectrum was subtracted from the total to yield the zero degree Compton-electron spectrum. Klein-Nishina cross sections were used to calculate the efficiency of the complete detection system. Corrections for electron scattering

in the foil and the finite resolution of the magnetic spectrometer were included in the efficiency calculation. The bremsstrahlung spectrum was calculated from a single analytical equation, essentially the Compton electron spectrum divided by the efficiency and converted from electron energy to incident photon energy. The accuracy of the spectrum was estimated to be $\pm 10\%$. The resolution of the Compton spectrum was limited by the magnetic spectrometer to 20% at 10 MeV. This translates into lower resolution after the energy conversion is applied to determine the bremsstrahlung spectrum.

Aitken and Henry [20] measured the spectral distribution of Compton-scattered photons to deduce the spectrum from a ^{60}Co source. Bentley et al [21] used this approach to measure x-ray spectra from 2 to 6 MV accelerators. A NaI(Tl) spectrometer was used to measure the energy spectrum of photons Compton-scattered into a fixed angular range. The detector response to monoenergetic photons was unfolded from the measured pulse-height distribution to yield the single-angle Compton-scattered photon spectrum. The Klein-Nishina cross-section was then used to estimate the bremsstrahlung spectrum from the scattered spectrum. Several minor measurement corrections were also applied. The spectral accuracy was about $\pm 10\%$. The resolution was about 20% at 5 MeV. This approach has been used by several other investigators for megavoltage bremsstrahlung spectroscopy [22,23,24] with similar accuracy and resolution.

Amols et al [25] used a pair of plastic scintillators to measure the energy of Compton-scattered electrons by a coincidence approach. In this approach, bremsstrahlung photons are incident directly on the first scintillator. Due to the low effective atomic number of the scintillator, photons that interact are predom-

inantly Compton scattered. Some of these scattered photons are intercepted by the second scintillator resulting in a coincidence signal. The pulse height of the signal from the first scintillator is only recorded when a coincidence occurs. This pulse height is related to the Compton scattered electron kinetic energy by the response function of the detector. The resolution achieved by this approach is at best 50% at ^{60}Co energies (1.25 MeV). This poor resolution is a result of poor definition of the scattering angle.

Compton spectrometry approaches are useful for spectrometry of high intensity bremsstrahlung beams in the energy range 0.3 to 12 MeV [26]. The upper energy is limited by the low cross-section for Compton scattering and the weak dependence of scattering angle on energy. These effects result in reduced accuracy and poor energy resolution at higher energies.

1.1.3 Photodisintegration of deuterium

Photons with sufficient energy to overcome the 2.22 MeV deuterium binding energy are absorbed by deuterium atoms by the $D(\gamma, n)p$ reaction. The two nucleons are split apart with a combined kinetic energy equal to the incident photon energy less the nuclear binding energy. In most circumstances the kinetic energy is approximately equally divided between the proton and the neutron. In deuterium photodisintegration spectrometry the energy spectrum of photoneutrons or photoprotons is measured. This is related to the bremsstrahlung spectrum through the photodisintegration cross-section and the simple energy relationship between the photonucleon and the incident photon. Sherman et al [27] provides a historical review of megavoltage photon spectrometry using this approach.

Photoproton energies have been deduced from track-length measurements in nuclear emulsions [13,28,29,30,31,32]. These researchers used nuclear emulsion films loaded with heavy water as a combined deuterium target and photoproton spectrometer. The proton kinetic energy is related to its track-length through the stopping power of the emulsion. The inaccuracy of identification and measurement of short track-lengths formed by low-energy photons limits this approach to photon energies greater than 5 MeV.

Photoneutron energies have been measured using a time-of-flight spectrometer [33]. The deuterium target consisted of heavy water D_2O contained in a 1 to 10 cm³ lucite vessel with 0.08 mm thick walls. Photoneutrons produced at 90° to the bremsstrahlung beam travelled 25 meters along an evacuated flight tube before reaching the NE213 liquid scintillator used for neutron detection. Measured temporal distributions of neutron events were corrected for detector efficiency and background and converted to energy to yield the neutron spectrum. An analytical expression for the photoneutron cross-section was used to determine the bremsstrahlung spectrum. The accuracy of the data was limited to $\pm 5\%$ to $\pm 10\%$. The uncertainty was somewhat larger at the highest energies where counting statistics became important. The energy resolution was approximately 8% at 10 MeV. The low energy cutoff of the spectrum measurement was limited to the $D(\gamma, n)p$ reaction threshold of 2.22 MeV. With neutron time-of-flight techniques this cutoff is around 4 to 5 MeV in practice. Sherman et al [27] and Lambert et al [34] used a refined time-of-flight technique to measure 25 MV bremsstrahlung spectra, achieving an energy resolution from 0.6% at 4 MeV to 4.5% at 34 MeV. The spectrum accuracy was still limited to $\pm 5\%$ to $\pm 10\%$ primarily due to uncertainty in the reaction cross-section.

1.1.4 Transmission spectrometry and depth-dose measurements

The transmission of bremsstrahlung beams through an absorber depends on the spectrum shape. Several researchers have used reiterative fitting approaches to determine bremsstrahlung spectra from transmission measurements (see [35] to [39]). A combination of lead and aluminum absorbers can be used to improve the accuracy of the spectral estimation [35]. Computer simulations demonstrated that if the approximate spectral shape was known, bremsstrahlung spectra could be reconstructed with an accuracy of the order of $\pm 10\%$ and a resolution of roughly 5%. A related approach involves the use of interactive fitting techniques to determine the bremsstrahlung spectrum that results in a measured depth-dose curve in water [40,41].

1.1.5 Photoactivation

Photons of sufficient energy to overcome the nuclear binding energy or to excite a nucleus into a higher energy state can cause stable nuclei to become radioactive. Typical reactions include $^{12}\text{C}(\gamma, n)^{11}\text{C}$, $^{89}\text{Y}(\gamma, 2n)^{87}\text{Y}$, and $^{115}\text{In}(\gamma, \gamma')^{115\text{m}}\text{In}$ which have threshold energies of 18.7 MeV, 20.8 MeV, and 1.0 MeV respectively. When a series of foils are irradiated by the same bremsstrahlung source the resulting set of foil activities depends strongly on the shape of the bremsstrahlung spectrum. The latter can then be reconstructed from a measure of the foil activities if the photoactivation cross-section of each foil is known.

The experiment requires a finite irradiation time, a waiting time between foil activation and activity measurements, and a finite counting time. Correc-

tions are required for these effects as well as for the detector efficiency and the number of γ -rays per nuclear decay. The result is an estimate of the saturation activity, which depends only on the fluence rate and the photoactivation cross-section. One approach to reconstructing the bremsstrahlung spectrum from the saturation foil activities involves the expansion of the spectrum into a set of orthonormal functions. The coefficients of the expansion are determined from the foil activities through an unfolding process which involves the photoactivation cross-sections, the orthonormal functions, and a weighting function. The weighting function is best chosen to be close to the bremsstrahlung spectrum in order to avoid unstable solutions.

Several investigators have used this approach [42,43,44,45,46,47]. Accuracies of $\pm 5\%$ to $\pm 50\%$ are achieved. Energy resolution is not quoted but is at best 1 MeV, the minimum energy interval used. The energy resolution depends strongly on energy and on the choice of foils. The spectral reconstruction is limited to energies greater than 5 MeV since all practical foils in use have threshold energies that equal or exceed this. Prior knowledge of the spectral shape is required in order to obtain a stable solution to the unfolding problem.

1.1.6 Scintillation spectrometry of primary photons

In this approach, individual bremsstrahlung photons are incident directly on a scintillating material. The photons interact with the atomic electrons of the material by the photoelectric effect, the Compton effect, or by producing electron-positron pairs. Part or all of the energy of the photon is transferred to the kinetic energy of the scattered electron, or in the case of pair production, to the rest mass and kinetic-energies of the electron and positron. These charged

particles generally deposit all of their kinetic energy in the scintillator. Energy is lost from the detector if the photon escapes, if bremsstrahlung photons emitted by the electrons or positrons escape, or if one or both of the positron-annihilation photons escapes. Small scintillators have a complex response, which may include prominent annihilation photon escape-peaks, Compton edges, characteristic x-ray escape-peaks, and backscatter peaks due to photons interacting in the material surrounding the crystal. Very large scintillators can trap most of the incident photon energy.

Some of the energy absorbed by the crystal is converted to low energy photons. The photons are generally collected by the photocathode of a photomultiplier tube which is optically coupled to the scintillator. Photons incident on the photocathode produce electrons which are accelerated to a dynode, producing more electrons on impact. In this manner the number of electrons is considerably multiplied through a chain of about 10 dynodes and the resulting electron pulse is collected by the anode at the end of the chain. Under ideal operating conditions the pulse height from the anode is linearly related to the energy absorbed by the scintillator.

The spectrum of bremsstrahlung is related to the absorbed energy spectrum through the detector response and the detector efficiency. There are many approaches available for unfolding the response of the detector from the measured spectrum. The methods vary in ease of implementation, processing speed, degree of sophistication, and accuracy.

Foote and Koch [48] were among the first to explore the possibility of megavoltage bremsstrahlung spectrometry with scintillators. Spectra measured by this approach have since been reported by many investigators (see [49] to [58]).

The scintillator of choice for megavoltage bremsstrahlung measurement is NaI. This is due to its high detection efficiency, the high efficiency of conversion of energy deposited in the detector to light, its good optical properties, and the existence of methods to grow large crystals. The detector resolution is about 5% at 10 MV. If large crystals of NaI are used then the detector response approaches a Gaussian of full-width at half-maximum equal to the detector resolution. In this case the unfolding procedure is quite accurate and bremsstrahlung spectra can be estimated with an accuracy better than $\pm 5\%$.

In this work a 20 cm diameter by 25 cm long NaI detector was used to measure the bremsstrahlung yield from thick targets and detailed comparisons were made to Monte Carlo calculated spectra. Forward-directed measurements are presented in chapter 2 for 10 to 30 MeV electrons incident on Al and Pb targets. Angular measurements are presented in chapter 3 for 15 MeV electrons incidents on Be, Al, and Pb. The method used for pile-up correction is discussed in appendix A. The methods used to determine the response and the energy calibration of the NaI detector and to unfold the detector response from the measured bremsstrahlung spectra are presented in appendices B and C. The total number of photons in the spectra were measured to better than a few percent. The detector was of high quality and excellent spectral resolution was achieved.

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Chapter 2

Forward-directed bremsstrahlung of 10 to 30 MeV electrons incident on thick targets of Al and Pb

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Abstract

Bremsstrahlung spectra from thick targets of Al and Pb have been measured absolutely (photons per incident electron) along the beam axis for electrons of 10, 15, 20, 25, and 30 MeV incident energy. The spectra have a 220 keV low-energy cutoff. The targets were cylinders with nominal thicknesses of 110% of the electron CSDA range. A thin transmission detector, calibrated against a toroidal-current monitor, was placed upstream of the target to measure the beam current. The spectrometer was a 20 cm diameter by 25 cm long cylindrical NaI detector. Measured spectra were corrected for pile-up, background, detector response, detector efficiency, attenuation in materials between the target and detector, and the collimator effect. Spectra were calculated using the EGS4 Monte Carlo system for simulating the radiation transport. The simulation model included the small amount of material upstream of the target. This material contributed about 40% of the spectrum, but its presence or absence had little effect on the calculated bremsstrahlung yield. The shapes of the measured and calculated spectra were in excellent agreement. The ratio of the total number of photons in each measured spectrum to those in the corresponding calculated spectrum varied from 0.97 ± 0.06 to 1.12 ± 0.06 , depending largely on the atomic number of the target. Absolute spectral measurements in the literature agreed with our calculations of spectral shape but showed a range of $\pm 30\%$ in the number of photons per incident electron relative to the calculated values, which is contrary to our result.

Keywords: absolute thick-target bremsstrahlung, bremsstrahlung, NaI spectrometry, radiotherapy, Monte Carlo.

2.1 Introduction

The photon spectrum originating in the target of a radiotherapy linear accelerator strongly influences the dose distribution in the patient and the response of dosimeters placed in the beam. Knowledge of the photon spectrum is a prerequisite for accurate treatment planning and dosimetry. For example, a sparse collection of measured photon spectra serve as a basis for the estimate of spectrum-averaged quantities required by the AAPM dosimetry protocol [1], and photon spectra are necessary for dose distribution calculations by the 'convolution' method [2].

Photon spectra have been calculated by a variety of methods ranging from analytical calculations based on coarse simplifying assumptions, to Monte Carlo calculations which account for all of the physically significant phenomena, in particular, that of electron transport. Even the spectra calculated by the Monte Carlo approach have potential inaccuracies due to systematic errors in basic physical quantities such as the bremsstrahlung cross-sections [3], errors and simplifications in specific algorithms, and approximations in the model geometry.

Various procedures have been used for measuring bremsstrahlung spectra from electrons with energies exceeding 5 MeV. These include Compton spectrometry [4], spectrometry based on the photodisintegration of the deuteron [5], transmission spectrometry [6], photoactivation spectrometry [7], and scintillation spectrometry with the detector placed directly in the beam [8,9]. Scintillation spectrometry provides energy resolution and accuracy competitive with the best of the other techniques and provides the widest energy range. However, in order to keep the event pile-up manageable, the method is restricted to low cur-

rents, several orders of magnitude lower than produced by linear accelerators in conventional use for radiotherapy.

High-accuracy spectral and absolute yield measurements made with a large NaI spectrometer are reported here. The results are intended to serve as benchmark data for verification of state-of-the-art Monte Carlo calculations and other measurement techniques.

2.2 Terminology

The photon spectrum produced by megavoltage electrons incident on a thick target is composed of a discrete spectrum superimposed on a continuous spectrum. The discrete spectrum consists of positron annihilation photons and characteristic fluorescent x-rays. By-products of the disintegration of nuclei activated by the electromagnetic cascade contribute little to the photon spectrum and are not considered here. The continuous spectrum consists of bremsstrahlung photons from primary electrons and secondary electrons and positrons, and scattered photons. The forward-directed photon spectrum consists primarily of bremsstrahlung photons.

Consider a target *in a vacuum*, irradiated by a narrow beam of monoenergetic electrons. The bremsstrahlung yield, dS/dE , is defined here as the number of photons of energy E per unit energy from the target which would reach a given point P in the vacuum per unit solid angle per electron incident on the target. The solid angle is defined from the point of intersection of the beam axis with

the upstream surface of the target. The bremsstrahlung yield is given by

$$\frac{dS}{dE} = \frac{1}{N_e} \frac{d^2 N_\gamma}{dE d\Omega}, \quad (2.1)$$

where $(d^2 N_\gamma / dE d\Omega) dE$ is the number of photons with energy between E and $E + dE$ which exit the target and reach point P per unit solid angle, and N_e is the number of electrons incident on the target. In principle, dS/dE depends on the distance of point P from the target, due to the finite source size. The bremsstrahlung yield is defined for a target in a vacuum, thus for in-air measurements consideration must be given to both attenuation and scatter in the air.

The photon fluence as defined by the ICRU [10] is related to the bremsstrahlung yield by

$$\frac{d\Phi}{dE} = \frac{d^2 N_\gamma}{dE da} = N_e \frac{dS}{dE} \frac{d\Omega}{da}, \quad (2.2)$$

where da is the cross-sectional area of the sphere at point P , through which the N_γ photons pass.

The integrated bremsstrahlung yield, S_{E_o} , is defined here as the total number of photons with energies greater than E_o which reach point P per unit solid angle per electron incident on the target. Then we have,

$$S_{E_o} = \int_{E_o}^{E_{max}} \frac{dS}{dE} dE = \frac{1}{N_e} \int_{E_o}^{E_{max}} \frac{d^2 N_\gamma}{dE d\Omega} dE = \frac{1}{N_e} \frac{d(N_\gamma)_{E_o}}{d\Omega}, \quad (2.3)$$

where $(N_\gamma)_{E_o}$ is the number of photons of energy greater than E_o which reach point P , and E_{max} is the maximum photon energy in the spectrum and is equal to the incident electron beam energy.

2.3 Materials and method

2.3.1 Electron Beam and Targets

The electron beam was provided by the NRCC Vickers research linear accelerator operated at 240 Hz. The width of each beam pulse was nominally 2 μ s. The spectrometry system employed was limited to mean photon-detection rates of 0.5 photons per beam pulse. Higher rates led to unacceptable levels of pulse pile-up. For our experimental arrangement, a 30 MeV electron beam with an average current of 50 fA, when incident on a thick target of Al or Pb, resulted in an average detection rate of 1 photon per beam pulse. Thus for 30 MV bremsstrahlung measurements we were restricted to currents below 25 fA, or 700 electrons per beam pulse. Higher currents were tolerated at lower incident electron energies or for off-axis measurements due to the lower bremsstrahlung cross-section. Techniques have been developed on the NRCC linac for accelerating electrons at currents as low as 1 electron per beam pulse [11], so the necessary currents were readily achievable.

Measurements were made for electron beams with mean energies of 10, 15, 20, 25, and 30 MeV as established by the calibrated 90° bending magnet. The energy spread of the electron beam was set at about 1.5% of the beam energy. The beam was focussed onto the 0.013 cm thick Ti exit window (figure 2.1) to a focal spot diameter of about 0.5 cm. Prior to hitting the exit window, the angular spread of the electrons in the beam was less than 0.5°. The beam then passed through a 0.015 cm thick Si transmission-current monitor and a 0.0051 cm thick stainless-steel entrance window on the evacuated target-chamber before impinging on the target. The targets were 7.26 cm diameter Al cylinders

Figure 2.1:

Cross-sections through the target chamber, spectrometer, and collimator. The electron beam passed through a 0.013 cm thick Ti exit window, a 0.015 cm thick Si current monitor, and entered the target chamber through a 0.0051 cm thick stainless steel window. The distance s to the target was 0.9 cm for the Al targets and 1.6 cm for the Pb targets. The bremsstrahlung produced in the target passed through a 0.254 cm thick Al window and 320 cm of air before reaching the NaI detector. The collimator insert had a 0.07 cm wide lip, 6.6 cm upstream of the NaI detector, which defined the solid angle. Distances are in centimeters.

Table 2.1:

Target thicknesses used for the measurements and Monte Carlo calculations of the bremsstrahlung yield. The 'CSDA ranges' column is the thickness of the target divided by the electron CSDA range. The measured densities of the targets are also shown.

Energy (MeV)	Al targets 2.705±0.008 g/cm ³			Pb targets 11.34±0.05 g/cm ³		
	Monte Carlo model (g/cm ²)	experiment (g/cm ²)		Monte Carlo model (g/cm ²)	experiment (g/cm ²)	
			CSDA ranges			CSDA ranges
10	6.48	6.48±.03	1.11	6.80	6.85±.06	1.12
15	9.73	9.73±.04	1.17	9.07	9.13±.07	1.15
20	11.62	11.63±.04	1.10	11.34	11.43±.08	1.22
25	15.13	15.14±.05	1.20	11.34	11.43±.08	1.08
30	16.21	16.21±.06	1.12	13.61	13.68±.08	1.18

and 3.17 cm diameter Pb cylinders with nominal thicknesses of 110% of the CSDA electron range. The target diameters were sufficient to prevent primary electrons with energies of 30 MeV or less from escaping from the sides. The target thicknesses are shown in table 2.1. The target materials Al and Pb were chosen to span a wide range of atomic number.

2.3.2 Beam current measurement

A transmission-current monitor (TCM) was used for routine monitoring of the beam current. The TCM, a 0.80 cm diameter Si semiconductor (Ortec TB-018-050-150) was placed in air, in tight geometry between the exit window of the

beam tube and the entrance window of the target chamber. This insured that most of the electrons in the beam were centered on the target. For bremsstrahlung and background measurements, the TCM was used to measure average beam currents from 20 to 400 fA at 240 Hz, or equivalently 500 to 10,000 electrons per linac pulse. The gain of the TCM was 1.3×10^4 , corresponding to a charge collection of between 1 and 20 pC per pulse. A Canberra 808 preamplifier was used to convert the charge pulse to a voltage pulse. This was amplified by an Ortec 410 shaping amplifier, digitized, and stored using a multi-channel analyzer (MCA). The TCM was calibrated for charge measurement by injecting a known charge into the preamplifier using a voltage pulse from a precision pulser fed into a calibrated capacitor. A set of calibrated 1 to 100 pF capacitors and calibrated 0.001 to 0.5 attenuators were available to enable the use of the most accurate range of the pulser. A measured correction factor of 0.988 ± 0.016 was applied to account for differences in the shape of the TCM charge signal and the pulser signal. This correction factor, which was independent of the amplitude of the signal, was specific for the width of the beam pulse, the decay-time constant of the shaping amplifier, and the decay time of the pulser signal used for these measurements. In addition to this correction, the measured charge was increased by a factor of 1.035 ± 0.004 to account for the finite capacitance of the Si detector (70 ± 7 pF) which was in parallel with the 2 nF decoupling capacitor of the preamplifier.

The number of electrons incident on the exit window N_e was monitored at 300 nA average current by a toroid. The high current was necessary to achieve 1% precision in the measurement. The toroid was placed upstream of the Ti beam exit window. Absolute calibration was achieved with a pulse

from a calibrated current-pulser fed through the toroid on a single wire in the arrangement used by Pruitt [12]. The TCM was removed during these high-current runs since these monitors could suffer radiation damage in less than a second when subjected to currents of 300 nA [13].

The block-current monitor (BCM) was used to transfer the current calibration made by the toroidal-current monitor at high currents to the TCM, which could only be used at low currents. The BCM consisted of the bremsstrahlung target, or block, and the charge measuring circuit. The target chamber was evacuated to 10 Pa to eliminate the signal due to air ionization. The BCM was used at nominal currents of 2 pA and 300 nA. An Ortec 142 preamplifier was used at the low current and a Canberra 2005 preamplifier, which was less sensitive than the Ortec 142, was used at the high current. Amplification was achieved with an Ortec 450 shaping amplifier. The charge calibration procedure was the same as that for the TCM, with a correction factor to account for differences in the BCM and pulser signal shapes of 1.002 ± 0.005 for the low current measurements and 0.996 ± 0.011 for the high current measurements. Both preamplifiers were equipped with $0.1 \mu\text{F}$ decoupling capacitors so that no correction for the block capacitance ($\sim 50 \text{ pF}$) was necessary.

The amount of charge collected by the BCM was sensitive to the potential difference between the block and the target chamber due to the presence of secondary electron emission from the downstream surface of the target chamber entrance window and the upstream surface of the block. This resulted in a 10% increase in current when the bias on the block was varied from -500 V to +500 V. Virtually all of the secondary emission electrons were collected with a potential of 200 V of either sign, consistent with the findings of others [14]. The average

of the BCM currents at ± 200 V bias was from 0 to 10% less than the beam current, depending on the beam energy and target material. This difference was attributed to the escape of primary and secondary electrons from the block and absorption of secondary electrons produced in materials around the block. Thus the BCM could not be used directly to obtain the current-calibration factor for the TCM.

The following procedure was used to calibrate the charge collected by the TCM in terms of N_e . The BCM was calibrated at a nominal current of 300 nA against the toroidal-current monitor, with the TCM removed. The TCM was calibrated at a nominal current of 2 pA against the charge collected by the BCM. Since the current from the block is proportional to the current in the beam tube for a given electron energy and target, the current-calibration factor for the TCM is given by

$$\frac{N_e}{Q_{TCM}} = \left(\frac{N_e}{Q_{BCM}} \right) \left(\frac{Q_{BCM}}{Q_{TCM}} \right), \quad (2.4)$$

where N_e/Q_{BCM} is the number of electrons in the beam tube per unit charge collected from the block (measured at high current) and Q_{BCM}/Q_{TCM} is the block current per unit TCM current (measured at low current). Monte Carlo studies that we performed using EGS4 [15] showed that delta-ray production in the TCM resulted in less than a 2% increase in the charge collected by the BCM per unit beam current for a 30 MeV beam. This justifies the use of eq 2.4 even though the TCM was removed during the runs at high currents (300 nA).

The TCM was preferable to the BCM for routine current measurement when measuring the bremsstrahlung yield in the forward direction due to the much higher signal to noise ratio, even though it added extra material in the path of

the electron beam.

2.3.3 Bremsstrahlung spectrometer

The spectrometry system consisted of a 20.3 cm diameter by 25.4 cm long right-cylindrical NaI scintillating crystal (Bicron) for photon counting and energy measurement. The detector was clad with 1.17 g/cm² equivalent thickness Al on the front face and enclosed by an outer cylindrical shield of Pb (figure 2.1). A hole with steps of three different radii in the centre of the Pb shield upstream of the NaI detector allowed the insertion of collimators of various diameters. A solid Pb plug was inserted for background measurements. The collimator inserted for the bremsstrahlung measurements had a diameter of 2.40 ± 0.02 cm (figure 2.1). The solid angle was defined by the lip on the downstream surface of the collimator. This lip was 315 cm from the target, resulting in a solid angle of 4.56×10^{-5} sr, which corresponds to a cone of half-angle 0.22° . The NaI detector and Pb shield, weighing close to 1000 kg, were mounted on a cart equipped with Aero-Go air-flotation pads to permit easy alignment to within a centimeter. Fine vertical and horizontal alignment was achieved with adjustment screws. The centre of the collimator aperture was positioned to within ± 0.4 cm ($\pm 0.07^\circ$) of the beam axis and the collimator axis was aligned to within $\pm 0.5^\circ$ of the beam axis.

The NaI crystal was coupled to a 10-stage, 13 cm diameter, RCA S83006E photo-multiplier tube (PMT). The anode provided the fast-timing signal and the 7th dynode provided the energy signal. The 9th dynode signal exhibited an unexplained 25% non-linearity at 30 MeV when the PMT was operated at 1000 V. The 7th dynode was chosen for the energy signal because it gave much

improved linearity with adequate gain.

The PMT fast-timing signal was clipped by a 30 ns delay line and fed into an Ortec 454 filter/amplifier. The amplified and filtered signal was fed into a lower-level discriminator (LLD) which output a logic pulse for events above a nominal energy of 150 keV. The actual lower energy cutoff varied by ± 50 keV over the course of a 2000 s run. This prevented a reasonable analysis of the events with energy depositions below 0.2 MeV. The logic pulse was fed into a logic circuit which output an 'event' logic pulse each time the discriminator threshold was exceeded during a beam pulse. An 'inhibit' logic pulse was output when two or more 'event' logic pulses occurred during a beam pulse, and an 'accept' logic pulse was output when an 'event' occurred with no 'inhibit'. The number of 'event', 'inhibit', and 'accept' logic pulses were recorded individually on scalers. For the nominal mean rate of 0.5 detected events per pulse, the measured accept rate was 0.33, and the measured inhibit rate was 0.08.

The event-counting device described above was a paralyzable counter. The resolution time of the paralyzable counter, determined from the widths of fast-timing signals of various amplitudes produced by a 15 MV bremsstrahlung beam, was

$$\tau(E) = 0.045 \ln E + 0.22 \mu\text{s} \quad (\pm 10\%), \quad (2.5)$$

where E is in MeV. This corresponds to 230 ns for a 1.25 MeV event as from a ^{60}Co source. Since the resolution time was an order of magnitude shorter than the beam pulse time, reasonable event rates with accurate pile-up correction could be achieved.

The PMT energy signal was fed into a Canberra 2005 preamplifier in series

with an Ortec 572 shaping amplifier. The output signal was digitized into 4096 channels by a Canberra 8076 ADC and stored in a Canberra Series 90 MCA when received in coincidence with an 'accept' logic pulse from the fast-timing circuitry. The energy-deposition spectra of the bremsstrahlung were collected for 2000 s, the background for 400 s. The system gain was nominally 10 keV per channel, providing a dynamic range of 0.20 to 40 MeV.

The event-time distribution (ETD) is the time distribution of events within the beam pulse relative to the leading edge of the beam trigger. This was measured with a time-to-amplitude converter (TAC) which was started by the leading edge of the beam-trigger logic pulse and stopped by the arrival of an 'event' logic pulse. The TAC signal was digitized into 512 bins and stored in an MCA. The time distribution of events was approximately rectangular and had a nominal width of 2 μ s.

2.3.4 Measurement of integrated bremsstrahlung yield

The integrated bremsstrahlung yield was estimated from the measurements by (see eq 2.3)

$$S_{E_o} = \frac{(N_\gamma)_{E_o}}{\Omega N_e}. \quad (2.6)$$

The lower energy cutoff of S_{E_o} was set at $E_o = 220$ keV, just above the upper range of the LLD, to insure that no photon which deposited more than E_o of its energy was rejected by the LLD. For the purpose of the measurement of S_{E_o} (and dS/dE) we have taken $(N_\gamma)_{E_o}$ to be the number of photons of energy greater than E_o which would have passed *cleanly* through the collimator in the absence of attenuating material downstream of the target (in vacuum). Thus the 'point of interest' in the earlier definition of $d^2N_\gamma/dEd\Omega$ is now a surface defined by the

lip on the collimator, 6.6 cm upstream of the NaI detector (figure 2.1). The solid angle Ω is that subtended by the collimator lip at the point of intersection of the beam axis and the upstream surface of the target. The value of S_{E_0} obtained in this way is actually an average value over Ω . We expect that for the thick targets studied here, the bremsstrahlung yield varies little over Ω , and it is reasonable to assume that we have determined S_{E_0} for the differential solid angle along the beam axis. Higher angular-resolution could not be easily achieved since any angular variation in S_{E_0} across the collimator aperture (between 0° and 0.2°) was smeared out by the angular spread of the incident electron beam. In addition, the calculated values of S_{E_0} were also an average over a similar solid angle since higher angular-resolution would have required considerably longer computing time. Thus, it was appropriate to measure and calculate the averaged values of S_{E_0} and dS/dE for comparison purposes.

The method used to determine $(N_\gamma)_{E_0}$ from μ_p , the mean event-rate measured by the paralyzable counter, will now be presented. The mean number of photons per beam pulse which deposited an energy greater than E_0 in the NaI crystal was estimated by

$$(\eta_\gamma)_{E_0} = \mu_p C_{PUP} C_{LLD} . \quad (2.7)$$

The factor C_{PUP} is a pile-up correction factor, and C_{LLD} is a correction factor to eliminate events with energies below E_0 . Contamination of the bremsstrahlung beam by electrons, positrons, and collimator-scattered photons will be considered later.

The degree of pile-up depends on the event rate, the event resolution time, and the ETD [17]. The number of events per pulse was assumed to be Poisson

distributed with a constant mean event-rate, μ . The mean event-rate includes all energy-deposition events, that is, the rate we would measure if the resolution time of our paralyzable counter were zero. The pile-up correction factor is given by

$$C_{PUP} = \frac{\mu}{\mu_p}. \quad (2.8)$$

An iterative algorithm was developed which calculates μ from an equation relating μ_p to the ETD, the resolving time of the paralyzable counter, and μ . The ETD recorded during the spectral measurement for a given electron energy for *one* of the targets was used to estimate pile-up probabilities for the bremsstrahlung spectra from *both* of the targets (A. and Pb) *and* for the background spectra measured for that energy. The resolution time was taken as a constant equal to the mean event resolution time,

$$\bar{\tau} = \frac{1}{S_{E_o}} \int_{E_o}^{E_{max}} \tau(E) \frac{dS}{dE} dE, \quad (2.9)$$

where $\tau(E)$ from eq 2.5 was used. For the nominal mean rate of 0.5 events per pulse used for the spectral measurements, C_{PUP} was typically 1.06 ± 0.02 . Details of the calculation with experimental verification and analysis of the uncertainties can be found elsewhere [17].

The lower-level discriminator correction factor C_{LLD} removes those events in the energy region affected by the LLD, that is, below E_o . Let $\mathcal{L}(E)$ be the proportion of non-pile-up events from an energy-deposition of E in the NaI detector which trigger the discriminator. The function $\mathcal{L}(E)$ is roughly sigmoidal and goes to zero below ~ 100 keV and to unity above ~ 200 keV. Let $\tilde{M}(E)$ be

the spectrum of energy deposited in the NaI. Then,

$$C_{LLD} = \frac{\int_{E_0}^{\infty} \tilde{M}(E) dE}{\int_0^{\infty} \mathcal{L}(E) \tilde{M}(E) dE}. \quad (2.10)$$

The measured energy-deposition spectrum $M(E)$ can be used to approximate $\mathcal{L}(E)\tilde{M}(E)$ since $M(E)$ is naturally suppressed at lower energies by the LLD. Then we have,

$$C_{LLD} \simeq \frac{\int_{E_0}^{\infty} M(E) dE}{\int_0^{\infty} M(E) dE}. \quad (2.11)$$

This equation is an approximation since $M(E)$ was skewed to higher energies compared to $\tilde{M}(E)$. This was due both to event summing (pile-up) and the preferential rejection of low-energy events by the pile-up circuitry. It was important to keep E_0 low for the approximation in eq 2.11 to be valid. The cutoff energy E_0 was chosen to be 220 keV, just above the LLD noise level. For the worst case of 10 MeV electrons incident on an Al target, C_{LLD} was estimated to be 0.93 ± 0.02 . The uncertainty in C_{LLD} includes the approximation of eq 2.11 and the energy calibration uncertainty. For Al targets bombarded with higher-energy electrons and for all Pb targets there were proportionately fewer low-energy photons, C_{LLD} varied from 0.95 to 0.99, and the uncertainty in C_{LLD} was reduced.

The complete expression for $(N_\gamma)_{E_0}$ (used in eq 2.6) is

$$(N_\gamma)_{E_0} = \left[(\eta_\gamma^{TOT})_{E_0} - \frac{N_e^{TOT}}{N_e^{BKG}} (\eta_\gamma^{BKG})_{E_0} \right] C_{LOSS} C_{COLL}, \quad (2.12)$$

where the superscripts *TOT* and *BKG* refer to the measurements with the collimator open and plugged respectively. Roughly 2% of the events were from the background. The beam current for the background measurement was increased to approximately fifty times that for the bremsstrahlung yield measurement to provide good statistics in a short time. The number of 'event', 'accept', and

‘inhibit’ logic pulses were recorded during the background runs to correct the spectra for pile-up.

The quantity C_{COLL} in eq 2.12 is a correction for the collimator effect. The factor varied from 0.95 for the 30 MV Pb spectrum to 0.98 for the 10 MV Al spectrum. This correction is discussed in more detail later.

The quantity C_{LOSS} in eq 2.12 is a correction for the photons lost from the beam or undetected by the spectrometer:

$$\frac{1}{C_{LOSS}} = \frac{1}{S_{E_o}} \int_{E_o}^{E_{max}} \epsilon_{E_o}(E) \exp \{-\mu(E)x\} \frac{dS}{dE} dE. \quad (2.13)$$

The detector efficiency $\epsilon_{E_o}(E)$ is the probability that a photon of energy E , incident on the NaI detector, will deposit an energy of E_o or greater in the detector. The detector efficiency was calculated along with the response matrix using the EGS4 Monte Carlo system [15,18], as discussed later. The quantity $\exp \{-\mu(E)x\}$ in eq 2.13 is the transmission of the 0.69 g/cm² Al target chamber wall and the 0.39 g/cm² of air between the target chamber and the spectrometer. The transmission was calculated in a straightforward manner using tabulated attenuation coefficients. The factor C_{LOSS} varied from 1.08 for 30 MeV electrons on a Pb target to 1.12 for 10 MeV electrons on an Al target. For the latter case, 4.5% of the correction was due to attenuation in the target chamber exit window, 2.5% to air attenuation, and 5% to detector efficiency. The uncertainty in this correction was negligible.

2.3.5 Measurement of bremsstrahlung yield

The method used to determine the bremsstrahlung yield from the measured pulse-height distribution will now be presented. The photon spectrometer was

calibrated with γ -rays from a variety of sources (0.2 to 18 MeV), and with electrons from the linac (10 to 30 MeV). Details of the calibration are described elsewhere[16]. The system was linear to the greater of $\pm 1\%$ or ± 10 keV in the range 0.2 to 30 MeV. A linear calibration based on the 4.439 MeV line from an Am-Be source and the offset determined by precision pulser measurements was used to convert pulse height to energy. The calibrated spectrum was rebinned into bins of 25 keV widths. This was followed in sequence by pile-up correction, subtraction of the pile-up-corrected background spectrum, unfolding of the detector response, and application of the energy-dependent corrections for detector efficiency, attenuation in the target chamber wall and air, and the collimator effect. The magnitude of these corrections can be ascertained from the 10 MV bremsstrahlung example shown in figure 2.2. The pile-up correction to the measured spectrum was calculated adequately by assuming a constant resolution time, considering only double pile-up (two events detected as one), and using the measured pulse-height spectrum as the estimate of the spectrum with no pile-up when calculating the shape of the double pile-up spectrum. The effects of triple and higher order pile-up, pulse-height dependent event-resolution time, and iterations in the estimate of the pile-up correction were insignificant compared to the spectrum uncertainty due to counting statistics. Details of the pile-up correction approach used here will be reported elsewhere [17].

Let $M(E)$ be the number of accepts per unit energy between E and $E + dE$ including pile-up, that is, the distribution of pulse-heights after energy calibration of the measured spectrum. The pile-up corrected spectrum was calculated from (eq 26 of [17]).

$$\tilde{M}(E) = \mu_p L C_{PUP} \frac{M(E) - N_{C01}I(E)}{\int_{E_0}^{E_{ht}} [M(E) - N_{C01}I(E)]dE}, \quad (2.14)$$

Figure 2.2:

Data processing of the 10 MV bremsstrahlung spectrum that was measured for the Al target. The raw spectrum ($\bullet$) is shown normalized to 1 count above the low-energy cutoff of 0.22 MeV. All other spectra are plotted relative to this spectrum. The background (+) and pile-up (dotted line) spectra were subtracted from the raw spectrum. The resulting spectrum was renormalized to account for events lost to pile-up and was rebinned in preparation for unfolding (dashed histogram). The spectrum was then unfolded and corrections were applied for the detector efficiency and for the attenuation of the bremsstrahlung beam in the material between the target and the detector (solid histogram). Finally, the spectrum of photons from the collimator (lower histogram) was subtracted.

where L is the number of beam pulses, N_{C01} the number of accepts with one unresolved event, and

$$I(E) = \frac{\int_{E_o}^{E_{hi}} M(E)M(E - E')dE'}{\left[\int_{E_o}^{E_{hi}} M(E')dE'\right]^2}. \quad (2.15)$$

The upper bound of the integral E_{hi} was chosen roughly 10% higher than the electron beam energy to include most of the double pile-up spectrum. This approximation resulted in shorter computing times without significant loss of accuracy.

The background corrected pulse-height distribution was

$$\mathcal{M}(E) = \tilde{M}^{TOT}(E) - \frac{N_e^{TOT}}{N_e^{BKG}} \tilde{M}^{BKG}(E). \quad (2.16)$$

The superscripts *TOT* and *BKG* refer to the measurements with the collimator open and plugged respectively.

The bremsstrahlung yield is related to $\mathcal{M}(E)$ by

$$\frac{dS}{dE} = \frac{1}{N_e^{TOT}} \frac{\exp\{\mu(E)x\}}{\Omega\epsilon_{E_o}(E)} \mathcal{F}^{-1}\{\mathcal{M}(E)\}, \quad (2.17)$$

where $\mathcal{F}^{-1}$ is the unfolding function, that is, $\mathcal{F}^{-1}\{\mathcal{M}(E)\}$ is the photon spectrum which, when folded with the spectrometer response, gives the energy-deposition spectrum $\mathcal{M}(E)$. The response functions used to unfold $\mathcal{M}(E)$ were all normalized to unity above E_o so that the unfolding procedure did not change the normalization of the spectrum. The transmission factor and the detector efficiency, which appear in eq 2.17 have already been discussed. The corrections for the detector efficiency and the attenuation were applied to each bin of the spectrum *after* unfolding the detector response.

2.3.6 Detector response and unfolding

Response functions and detector efficiencies were calculated for our NaI detector using Monte Carlo techniques for photons with energies ranging from 0.2 to 30 MeV. The Monte Carlo simulations were done using the EGS4 system [15] with a user code developed for this purpose [18]. The simulations took into account the material on the front face of the spectrometer (important for detector efficiency) and the measured 10% increase in sensitivity in the back half of the crystal (important for unfolding). The latter was due to a deterioration in the epoxy layer which held the two halves of the crystal together. In the past, most large detectors such as ours were made up of two or more smaller crystals glued together. After the measurements reported here were completed the detector was re-glued at the factory (Bicron) and now exhibits a 3% increase in sensitivity in the back half.

The calculated response functions were folded with Gaussian functions with energy-dependent full-widths at half-maxima of Γ_G to account for line-broadening due to variations in light production, collection, and amplification. The degree of broadening was determined empirically from a fit of calculated monoenergetic response functions to measured pulse-height distributions of γ -rays and monoenergetic electrons from 0.2-30 MeV:

$$\Gamma_G = 0.077 E^{0.60} \text{ MeV}, \quad (2.18)$$

where E is in MeV. The measurement of Γ_G and the accuracy of the broadened response functions are discussed in detail elsewhere [16].

The element R_{ij} of the response matrix used for unfolding was defined as the response of the i th absorbed energy bin to the j th incident photon spectrum

given an energy deposition in the NaI crystal greater than the threshold, E_0 . Each photon spectrum was a rectangular distribution, that is, the fluence was constant over the width of the bin. The set of absorbed energy bins had the same bin-top energies as the incident photon spectra. The bin widths were set equal to the greater of 50 keV and Γ_G from eq 2.18.

The detector response was unfolded using FERDO [19]. This code is widely used in unfolding neutron spectra and has the advantage of providing an estimate of the uncertainty of the unfolding process. Our own studies have shown that FERDO can be used to reliably unfold bremsstrahlung spectra, and that the confidence intervals on the unfolded spectrum which are calculated by FERDO reasonably account for both the uncertainty in the measured spectrum and the uncertainty of the unfolding process [16]. The FERDO window functions, which limit the resolution of the unfolded spectrum [19], were Gaussian functions with full-widths at half-maxima (FWHM) equal to Γ_R , the FWHM of the response functions after Gaussian-broadening.

2.3.7 Collimator effect

Most previous authors have assumed that the collimator walls are perfectly absorbing (e.g., see [8,9]). An imperfect collimator would then result in an overestimate of the bremsstrahlung yield. Amols et al [24] performed Monte Carlo simulations of 1, 3, and 10 MeV point sources of photons placed 100 cm from 0.1, 0.3, and 1.0 cm diameter collimators of 20 cm and 30 cm thick Pb. The sources illuminated a 5 cm diameter circle centred on the collimator axis on the shield front face. On the basis of the results of Amols et al we have estimated that the bremsstrahlung beams were contaminated by between 2% and 10% collimator-

degraded photons. The large range in the estimate arises from the difference between our experiment and the Amols et al simulation, namely, the photon beams used in the simulations were monoenergetic, and the largest collimator had a diameter of 1 cm.

We performed our own Monte Carlo simulations with EGS4 using point sources of photons with spectral distributions which corresponded to the bremsstrahlung spectra that were measured in this study. The simulations were used to determine a correction for the contamination of each of the measured spectra by photons with paths which passed through the collimator walls. This included transmission and energy-degraded photons and annihilation photons from pair-produced positrons. The bremsstrahlung beams were modelled as point sources in these simulations. This was reasonable since, for the outer downstream rim of the large-radius Al targets, less than 5% of the collimator aperture at the lip was shadowed by the collimator, and only a relatively small number of photons originated from this region of the target. The effect of the diffuse source would have been more substantial for a smaller diameter, thicker collimator and/or a shorter source-to-detector distance.

The simulations showed that the shapes of the bremsstrahlung spectra were only modestly affected by the collimator, as shown for the 10 MV Al spectrum in figure 2.2. However, photons associated with the collimator made significant contributions to the integrated bremsstrahlung yields (see table 2.3 below). The calculated corrections for the collimator effect ranged from 2.4% for the 10 MV Al spectrum to 5.4% for the 30 MV Pb spectrum. Corrections for the collimator were calculated and applied to all of the measured spectra and to the integrated bremsstrahlung yields.

2.3.8 Monte Carlo calculations of bremsstrahlung yield

Forward-directed bremsstrahlung spectra were calculated with the EGS4 Monte Carlo system [15] using the PRESTA [20] algorithm. Two modifications of EGS4 made by Bielajew et al [21] were utilized in order to obtain more accurate spectra:

1. In EGS4 the angle between the bremsstrahlung photon and the primary electron of rest mass $m_e c^2$ and initial energy E_e is fixed at $m_e c^2 / E_e$. An option was provided to sample the photon angle from the distribution given by eq 2BS of Koch and Motz [22]. The largest effects were observed for thin targets near the angle $m_e c^2 / E_e$. For example, use of the Koch and Motz distribution resulted in 40% fewer photons at 3° for 30 MV bremsstrahlung from a 0.14 g/cm^2 Al target. The Koch and Motz distribution was used for thick-target calculations to insure that bremsstrahlung from the thin layers of Ti, Si, and Fe upstream of the target (pre-target material) was adequately estimated. For the example of 30 MV bremsstrahlung from a 16.21 g/cm^2 Al target shown in figure 2.3, the fixed-angle approach used in EGS4 resulted in $(14 \pm 3)\%$ more photons, mostly at low energies. The effect of the pre-target material will be discussed shortly.
2. To improve the precision of the calculations, N_{HIST} photon histories (table 2.2) were initiated for each bremsstrahlung event. Each of these photons was considered to represent $1/N_{HIST}$ photons in order to obtain proper normalization. Each photon energy was randomly selected from the bremsstrahlung energy distribution. The energy of the electron was reduced by the energy of only one of the photons. This resulted in the conservation of

Figure 2.3:

Bremsstrahlung yield calculated using the EGS4 Monte Carlo system for 30 MeV electrons incident on the pre-target material followed by a 16.21 g/cm^2 target of Al. Spectra were calculated between 0 and 0.5° . The two histograms plotted for each spectrum are the bounds of the 68% confidence intervals. The solid-line spectrum was calculated using eq 2BS of Koch and Motz for the angular distribution of bremsstrahlung photons (see text). The short-dashed-line spectrum was calculated using a fixed bremsstrahlung photon angle of $m_e c^2/E_e$. When this approximation was used, $(14 \pm 3)\%$ more photons were produced, mostly at low energies. The bremsstrahlung yield of the pre-target material, attenuated by 16.2 g/cm^2 of Al, is also shown (broken lines). The pre-target material contributed 40% of the bremsstrahlung yield from the thick targets.

Table 2.2:

Monte Carlo calculations of the bremsstrahlung yield. Number of photon histories started for each bremsstrahlung event N_{HIST} , number of electron histories N_e , μ VAX 3600 CPU time, and integrated bremsstrahlung yield with lower energy cutoff $E_o = PCUT = 0.1$ MeV.

Target	Energy (MeV)	N_{HIST}	N_e	CPU Time (hours)	$S_{E_o=0.1}$ (sr^{-1})	
					0-0.5°	0.5-1°
Al	10	30	300,000	7.7	1.58±.05	1.57±.03
	15	30	200,000	9.6	3.54±.07	3.53±.04
	20	30	150,000	11.7	6.54±.11	6.64±.06
	25	30	90,000	11.0	9.76±.14	10.04±.15
	30	30	75,000	11.8	14.83±.24	14.88±.16
Pb	10	30	125,000	11.6	1.41±.05	1.44±.04
	15	20	90,000	11.5	3.28±.12	3.24±.07
	20	20	45,000	11.0	5.64±.29	5.72±.17
	25	10	90,000	11.7	9.36±.22	9.57±.13
	30	5	125,000	9.7	12.33±.25	13.21±.22

energy in the limit of a large number of histories. Energy-loss straggling was also correctly handled by this approach.

Spectra were calculated for 0.1 cm diameter, 10, 15, 20, 25, and 30 MeV electron beams incident on cylindrical Al and Pb targets with radii of 7.26 cm and 3.17 cm respectively. Target thicknesses (table 2.1) were nominally the same as those used in the measurements. The photon fluence per unit energy was calculated in the 'scoring region'. This was a planar surface, perpendicular to the beam axis, 100 cm from the Ti exit window, and covering the angular range 0° to 0.5° . The photon fluence was converted to the bremsstrahlung yield using eq 2.2.

The pre-target material shown in figure 2.1 was included in the simulation model. Our own Monte Carlo studies have shown that this pre-target material accounts for approximately 40% of the bremsstrahlung yield at 0° (figure 2.3). In practice this made little difference in the calculated bremsstrahlung yields. This is due to the weak dependence of the bremsstrahlung yield on atomic number at small photon emission angles (see [23], and our own results, presented later). Since bremsstrahlung from a thin target is strongly forward directed, bremsstrahlung from the pre-target material is only important at small angles. For example, at 10° the pre-target material contributes less than 0.5% of the integrated bremsstrahlung yield from a 30 MV beam.

The following parameters, defined in the EGS4 documentation [15], were used for the spectral calculations: $AE = ECUT = 1.511$ MeV, $AP = PCUT = 0.10$ MeV. The parameter BLCMIN used in the PRESTA algorithm was set to 1.5 to insure that the electron step-size was much less than one mean-

free-path for a bremsstrahlung event at the first surface encountered by the electron beam, that is, the Ti window. This was a precaution since it was found that using the default value of BLCMIN in PRESTA resulted in a slight but statistically significant decrease in the integrated bremsstrahlung yield. Lower values of these parameters increase the computation time with no significant effect on the bremsstrahlung yield above 0.1 MeV, the lower limit of the measured spectra. Each spectrum calculation used approximately 10 hr of μ VAX 3600 CPU time. The corresponding number of histories are shown in table 2.2 along with the number of photon histories initiated for each bremsstrahlung event.

2.4 Results and discussion

2.4.1 Measurement uncertainties

The mean number of electrons incident on the target per pulse was obtained with an accuracy of $\pm 4\%$. The uncertainty estimate includes the $\pm 2\%$ precision in the measurement of the calibration factor, N_e/Q_{TCM} , the $\pm 0.5\%$ precision of the measurement of the charge, Q_{TCM} , and the dependence of the calibration factor on target material ($\sim 3\%$) and beam energy ($\sim 3\%$). A single calibration factor N_e/Q_{TCM} of $465 \pm 15/\text{pC}$ was used, independent of the beam current, electron energy, and target material.

The integrated bremsstrahlung yield S_{E_0} was obtained with an accuracy of $\pm 5\%$. The uncertainty was mainly due to the $\pm 4\%$ uncertainty in the measurement of the beam current. Other significant sources of uncertainty were: pile-up correction ($\pm 2\%$), lower level discriminator correction (negligible for Pb targets, ranging from $\pm 1\%$ at 30 MeV to $\pm 2\%$ at 10 MeV for Al targets), solid

angle measurement ($\pm 1\%$), and the collimator effect ($\pm 1\%$). Two other sources of systematic uncertainty were the target thicknesses and the beam alignment. Uncertainty in the target thickness results primarily in uncertainty in the attenuation of the bremsstrahlung beam by the target. The target thicknesses were measured with sufficient accuracy to result in negligible uncertainty in the bremsstrahlung yield. With regard to beam alignment, our own Monte Carlo studies (table 2.2) have shown that the integrated bremsstrahlung yield is insensitive to the angle uncertainty of $\pm 0.07^\circ$, assuming that dS/dE varies roughly linearly with angle from 0° to 1° . Misalignment of the collimator axis by 0.5° , the estimated accuracy of this quantity, results in a reduction of the bremsstrahlung yield of less than 1% (due to a reduced solid angle).

The main sources of uncertainty in the relative bremsstrahlung yield (spectral shape) measurements, $(1/S_{E_0})dS/dE$, were counting statistics, detector response unfolding, and the collimator effect. The uncertainty in the pile-up correction was negligible compared to the uncertainty due to counting statistics [17]. The standard deviations $\sigma_{M(E)}$ of the pile-up-corrected, background-subtracted spectra were calculated as follows. Consider $\sigma_{\tilde{M}(E)}$, the standard deviation in $\tilde{M}(E)$, the pile-up-corrected spectrum (eq 2.14). We estimated this quantity from

$$\sigma_{\tilde{M}(E)}^2 \simeq \begin{cases} \tilde{M}(E) e^\mu & \tilde{M}(E) > 1 \\ 1 & \tilde{M}(E) \leq 1 \end{cases} \quad (2.19)$$

The factor e^μ was required since $\tilde{M}(E)$ was normalized to the total events μL but was determined from the measured spectrum of no-pile-up events which contained $\mu e^{-\mu} L$ events. The latter spectrum was Poisson distributed, and the standard deviation of the counts in a bin was estimated as the square root of the counts in that bin. Then, using eq 2.16,

$$\sigma_{\mathcal{M}(E)}^2 = \tilde{M}^{TOT}(E)e^{\mu^{TOT}} + \tilde{M}^{BKG}(E)e^{\mu^{BKG}} \left(\frac{N_e^{TOT}}{N_e^{BKG}} \right)^2. \quad (2.20)$$

The standard deviations $\sigma_{\mathcal{M}(E)}$ were used by the unfolding routine FERDO [19] in the estimation of the 68% confidence intervals on the unfolded spectra.

The problem of contamination of the bremsstrahlung beam by charged-particles will now be considered. Sources of charged-particle contamination included the target, the air, and the collimator walls. We used EGS4 to perform our own Monte Carlo studies of the contamination of the beam with electrons and positrons with kinetic energies greater than 2 MeV. This is the minimum kinetic energy required by these particles to penetrate the cladding on the NaI detector and deposit enough energy in the scintillator to be detected. Contamination by charged particles from the target (in vacuum) was less than 1% of the integrated bremsstrahlung yield of 30 MeV electrons incident on Al or Pb targets. The effect would be less at lower accelerator energies or if air was included in the model. Contamination by charged particles produced in the air by a pure 30 MV bremsstrahlung beam was negligible. An upper limit of 1% was placed on the charged-particle contamination produced by 10 to 30 MV bremsstrahlung beams incident on the collimator. The latter result is consistent with the results of Amols et al [24]. The combined effects of the target, air, and collimator are less than 2%. Thus no corrections were necessary for charged-particle contamination.

The model used for the Monte Carlo simulations was different from the experimental set-up in some respects. This included a different source-to-detector distance (SDD), a different solid angle, a different separation between the Ti exit window and the target (1 cm in the simulation), and different target thick-

nesses. Our studies have shown that the calculated bremsstrahlung yield in the forward direction does not change significantly for an SDD from 100 to 300 cm (results not shown), or from a four-fold increase in the solid angle (see table 2.2). The different geometry of the pre-target material affected the yield by less than 1% based on the change in solid angle, assuming that the pre-target material contributed 40% of the yield. Differences in the measured and modelled target thicknesses, which primarily affect the attenuation of the bremsstrahlung beam, were too small to affect the yield. The model geometry was considered to represent the experimental geometry adequately.

2.4.2 Bremsstrahlung yield

The bremsstrahlung yields measured for 10, 15, 20, 25, and 30 MeV electrons incident on thick Al and Pb targets are shown in figure 2.4. These spectra have several advantages over previously published spectra:

1. Accurate current measurement simultaneous with photon counting giving high accuracy in the measurement of the bremsstrahlung yield.
2. A wide photon energy range, from 220 keV to the maximum energy in the spectrum.
3. High resolution.
4. Accurate spectral shapes with uncertainty estimates for each bin. The high accuracy is due to the small size of the corrections relative to the measured spectrum (see figure 2.2), and the accuracy to which corrections for the pile-up, detector response, and collimator effect were derived. Accurate unfolding of the detector response was facilitated by the nearly ideal

Figure 2.4: Measured bremsstrahlung spectra along the beam axis (from 0 to 0.2°) for 10, 15, 20, 25, and 30 MeV electrons incident on a) Al targets and b) Pb targets. The target thicknesses were nominally 110% of the CSDA electron range. The 68% confidence intervals are plotted for representative points. The estimated 4% uncertainty in the beam current measurement was *not* included in the uncertainty estimates. The measured spectra have been corrected for pile-up, background, detector response, attenuation of the beam in post-target material (Al and air), and the collimator effect.

response of the large NaI spectrometer, which absorbed most of the energy of the incident photons.

Comparison plots of the calculated and measured bremsstrahlung yield for four representative spectra are shown in figure 2.5. The measured and calculated spectral shapes agree well, including the turnover point at 0.5 MeV in the 10 MeV Pb spectrum and 1 MeV in the 30 MeV Pb spectrum. This turnover point is due to the high attenuation in the Pb target by the photoelectric effect at the lower photon energies.

The measured and calculated values of the integrated bremsstrahlung yields are shown in table 2.3 with a lower-energy cutoff of 0.22 MeV. The approach used to calculate the corrections for the collimator effect (table 2.3) was presented in an earlier section. The uncertainties in the ratios of the measured yields to the calculated yields varies from $\pm 5\%$ to $\pm 8\%$. The measured integrated yields are on average 8% higher than the calculated values for the Al targets and 2% higher for the Pb targets. This may be indicative of a trend with atomic number. For a given target, the ratios of the measured yields to the calculated yields are generally independent of electron energy to within the range of relative uncertainties of these ratios of $\pm 4\%$ to $\pm 6\%$. These results verify the use of EGS4 for absolute calculation of the bremsstrahlung yield in the forward direction to roughly 10%.

Plots of the ratios of the measured bremsstrahlung yield to the calculated bremsstrahlung yield are shown in figure 2.6. The measured spectral shapes agree with the calculations for both targets for all five energies for which measurements were made. This is strong evidence that EGS4 is suitable for cal-

Figure 2.5:

Comparison of measured (solid line) and calculated (dashed line) bremsstrahlung spectra for four cases: a) 10 and 30 MeV electrons on Al targets, and b) 10 and 30 MeV electrons on Pb targets. The comparisons are absolute. The histograms plotted are the bounds of the 68% confidence intervals. The uncertainty in the beam current measurement was *not* included in the uncertainty of the measured spectra.

Table 2.3: Measured and calculated integrated bremsstrahlung yields, in the forward direction, of 10 to 30 MeV electrons incident on Al and Pb targets. The correction applied for the collimator effect is also shown. The target thicknesses were nominally 110% of the CSDA electron range. Only photons with energies greater than $E_0 = 0.22$ MeV were included. The absolute uncertainty in the measurements was $\pm 5\%$, and the relative uncertainty was $\pm 3\%$. The calculational uncertainty was statistical only.

Target	Energy (MeV)	Collimator Effect (%)	$S_{E_0=0.22}$		
			measured (sr^{-1})	calculated (sr^{-1})	ratio
Al	10	2.4 ± 0.4	1.40	$1.25 \pm .03$	$1.12 \pm .06$
	15	2.7 ± 0.6	3.22	$2.94 \pm .05$	$1.10 \pm .05$
	20	3.5 ± 0.7	5.88	$5.43 \pm .08$	$1.08 \pm .06$
	25	4.2 ± 0.7	8.55	$8.11 \pm .12$	$1.05 \pm .05$
	30	4.3 ± 0.7	12.88	$11.89 \pm .17$	$1.08 \pm .06$
					1.08
Pb	10	3.3 ± 0.6	1.29	$1.22 \pm .05$	$1.06 \pm .08$
	15	3.8 ± 0.6	2.92	$2.87 \pm .10$	$1.02 \pm .06$
	20	4.2 ± 0.6	5.01	$5.18 \pm .17$	$0.97 \pm .06$
	25	4.6 ± 0.9	8.28	$8.25 \pm .22$	$1.00 \pm .06$
	30	5.4 ± 1.1	11.23	$10.66 \pm .25$	$1.05 \pm .06$
					1.02

Figure 2.6: Ratio of measured to calculated bremsstrahlung yield of 10 to 30 MeV electrons incident on thick targets of Al and Pb. The EGS4 Monte Carlo system was used for the calculations. The simulation model included the pre-target material shown in figure 1. The measured bremsstrahlung yield was linearly interpolated to the energy of the center of each calculation bin before taking the ratio. Error bars show the 68% confidence intervals. The uncertainty in the beam current measurement was *not* included. The dotted lines are the integrated bremsstrahlung yield ratios from table 2.3.

culating the shapes of forward-directed bremsstrahlung spectra from electrons incident on any thick target.

2.4.3 Bremsstrahlung spectral measurements of others

Few absolute thick target bremsstrahlung spectral measurements are available in the 5-50 MeV electron energy range. O'Dell et al [25] measured the spectra from electrons with energies from 5 to 21 MeV on a 0.2 radiation length (1.4 g/cm^2) Au/W target. The spectra were measured by using time-of-flight measurements to determine the energy of neutrons liberated in the photodisintegration of deuterium. This approach is limited to energies greater than about 3 MeV. The beam current ($\sim 1 \text{ A}$ during the pulse) was measured using the target as a Faraday cup. The authors compared their 10 MV spectrum to unpublished Monte Carlo calculations. Although the spectral shapes agreed, the calculated absolute spectra were approximately 35% higher than the measured spectra at 10 MeV. Sandifer and Taherzadeh [9] extended the results of O'Dell et al down to photon energies of 10 keV using scintillation spectrometry with a 13 cm diameter by 15 cm long cylindrical NaI detector. We have found that the 20.9 MV results of Sandifer and Taherzadeh are inconsistent with spectral shapes that we calculated using EGS4. However, as will be shown shortly, the measured spectral-shapes of the two other groups that we review here do agree with the calculated spectral shapes, as do our own measurements, which used the same spectrometry technique as Sandifer and Taherzadeh. In addition, Sandifer and Taherzadeh reported good agreement to the O'Dell et al spectra, but the agreement was actually quite poor for electron energies above 10 MeV. For these reasons, the Sandifer and Taherzadeh results are not included in these comparisons.

Sherman et al [5] used deuterium photodisintegration to measure the spectrum from 25 MeV electrons on 0.13 to 1 radiation length (0.084 to 6.74 g/cm²) Ta targets and a 0.07 radiation length (1.71 g/cm²) Al target. The beam current (~ 1 A during the pulse) was measured with a 0.036 g/cm² Al secondary emission monitor. The relative results were converted to absolute results using the measured bremsstrahlung yield of 0.18 /MeV/sr at 10 MeV for 25 MeV electrons incident on 0.84 g/cm² Ta, reported later by the same group [26]. The authors reported 20% fewer photons in their measurements than estimated by unpublished Monte Carlo calculations for the thickest Ta target used [27]. This result was considered within their experimental uncertainty.

We performed EGS4 Monte Carlo simulations of the experiments reported by O'Dell et al and Sherman et al to facilitate comparison of the bremsstrahlung yields measured by these two groups and our own group. Direct comparisons were of limited usefulness due to the different thicknesses and compositions of the targets used by the different groups. The 10 and 20.9 MeV electron bremsstrahlung spectra were calculated with a model which conformed to the O'Dell et al arrangement: a target composed of a 1.35 g/cm² W converter (no gold) and a 7.7 g/cm² Al filter. The 25 MeV electron bremsstrahlung spectra were calculated with a model which conformed to the Sherman et al arrangement: 0.036 g/cm² Al pre-target material with targets of 0.084 g/cm² Ta, 0.84 g/cm² Ta, 6.74 g/cm² Ta, and 1.71 g/cm² Al with no filter.

Results are shown in table 2.4. In all cases the spectral shapes agree; however, there are discrepancies between the measured and calculated bremsstrahlung yields. Uncertainties in the measurements are not quoted since neither group reported the accuracy of their current measurement. The mean ratio of

Table 2.4: Absolute measurements of thick-target bremsstrahlung yield of 10 to 30 MeV electrons from the literature are shown compared to our own calculations performed with the EGS4 Monte Carlo system. Representative points from each spectrum are shown. The mean ratio column is the uncertainty-weighted mean-ratio of each spectrum. Only calculational uncertainties are included. The mean ratio is plotted in figure 2.7.

Authors	Electron Energy (MeV)	Target	Photon Energy (MeV)	dS/dE		Ratio	Mean Ratio
				Measured (MeV ⁻¹ sr ⁻¹)	Calculated (MeV ⁻¹ sr ⁻¹)		
O'Dell et al [25]	10	Au/W	5	0.057	0.086±0.012	0.66	0.66
			8	0.022	0.034±0.006	0.65	
	20.9	Au/W	5	0.34	0.37±0.02	0.92	1.00
			10	0.145	0.126±0.014	1.15	
			18	0.046	0.044±0.007	1.05	
Sherman et al [5,26,27]	25	0.084 g/cm ² Ta	5	0.220	0.33±0.02	0.67	0.66
			10	0.087	0.125±0.007	0.69	
			20	0.032	0.057±0.008	0.56	
	25	0.84 g/cm ² Ta	5	0.484	0.56±0.05	0.86	0.80
			10	0.180	0.21±0.03	0.87	
			20	0.060	0.091±0.011	0.66	
	25	6.74 g/cm ² Ta	5	0.459	0.66±0.08	0.70	0.77
			10	0.149	0.21±0.03	0.71	
			20	0.043	0.034±0.010	1.26	
	25	1.71 g/cm ² Al	5	0.420	0.33±0.03	1.28	1.33
			10	0.158	0.114±0.012	1.39	
			20	0.051	0.038±0.004	1.34	

the measured yield to calculated yield is plotted in figure 2.7 along with our results. The 10 MV spectrum measured by O'Dell et al for the Au/W target ($Z=79$ and 74 , respectively) show 35% fewer photons than our calculated spectrum, in agreement with figure 9 of O'Dell et al. However, we measured 6% more photons than we calculated for the 10 MV Pb ($Z=82$) spectrum. The 20.9 MV measurement of O'Dell et al agrees with our calculation. This is in agreement with our 20 MV result for Pb. The measured spectrum for 25 MeV electrons incident on the Al target of Sherman et al shows 30% more photons than we calculated. We measured only 5% more photons for this target and energy. The measured spectra for 25 MeV electrons on the Ta targets ($Z=73$) show roughly 25% fewer photons compared to our calculated spectra for all three target thicknesses, in agreement with the calculation reported by Sherman et al for the thickest Ta target used [27]. In contrast, our measurement for 25 MeV electrons incident on the Pb show the same number of photons as the calculation. The discrepancies between our measurements and those of O'Dell et al and Sherman et al, when compared to a consistently-calculated set of spectra, may be due to uncertainty in the electron current measured by these other groups. Neither group gave details of their approach to electron current measurement, but Sherman et al did make a rough estimate of $\pm 20\%$ for the uncertainty in their bremsstrahlung yield measurements [27]. A larger uncertainty than this would be required to explain the differences between their results and ours.

2.5 Summary and conclusions

A procedure for measuring thick-target bremsstrahlung spectra from a linear accelerator operated at low current has been described. Forward-directed brems-

Figure 2.7: Ratios of measured to calculated integrated bremsstrahlung yield. Calculations are from this work using the EGS4 Monte Carlo system. Measurements from this work for targets of Al (Δ) and Pb (∇) are shown with the estimated 68% confidence intervals, including all sources of experimental uncertainty. The uncertainty-weighted mean ratio of the O'Dell et al ($\circ$) and Sherman et al ($\times$) spectra from table 2.4 are also shown. The uncertainty is not shown since the accuracy of the beam current measurement by these groups was not known.

strahlung spectra of 10, 15, 20, 25, and 30 MeV electrons on Al and Pb targets with nominal thicknesses of 110% the CSDA electron range have been obtained. These spectra are more accurate, of higher resolution, and/or cover a wider energy range than spectra which have been previously reported. In addition, spectra are absolute in the sense of number of photons per incident electron.

The measured spectra were compared to spectra calculated using the EGS4 Monte Carlo system. A modified version of the EGS4 code was utilized to obtain more accurate bremsstrahlung yield estimates in shorter computing times. The pre-target material was included in the model used for the Monte Carlo simulation. The comparison verifies the use of EGS4 for the calculation of the bremsstrahlung yield (photons per incident electron per unit energy) in the forward direction to a high degree of accuracy for the targets and range of electron energies used in this experiment.

Other investigators have reported bremsstrahlung yields in the forward direction for thick targets bombarded by electrons with energies in the range of 10 to 30 MeV. With the exception of Sandifer and Taherzadeh [9] these measurements agree with the EGS4 calculated spectral shapes, in agreement with our experience. However, the measured to calculated spectral ratios vary from approximately 0.7 to 1.3, which disagrees with our results. This may be explained by a large uncertainty in the beam current measured by these investigators.

Further work is underway to extend the atomic number range of the measurements and to measure the angular dependance of the bremsstrahlung spectra.

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Chapter 3

Angular distribution of bremsstrahlung from 15 MeV electrons incident on thick targets of Be, Al and Pb

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Abstract

Bremsstrahlung spectra from thick cylindrical targets of Be, Al and Pb have been measured at angles of 0° , 1° , 2° , 4° , 10° , 30° , 60° , and 90° relative to the beam axis for electrons of 15 MeV incident energy. The spectra are absolute (photons per incident electron) and have a 145 keV lower-energy cutoff. The target thicknesses were nominally 110% of the electron CSDA range. A thin transmission detector, calibrated against a toroidal current-monitor, was placed upstream of the target to measure the beam current. The spectrometer was a 20 cm diameter by 25 cm long cylindrical NaI detector. Measured spectra were corrected for pile-up, background, detector response, detector efficiency, attenuation in materials between the target and detector and collimator effects. Spectra were also calculated using the EGS4 Monte Carlo system for simulating the radiation transport. There was excellent agreement between the measured and calculated spectral shapes. The measured yield of photons per incident electron was 9% and 7% greater than the calculated yield for Be and Al respectively, and 2% less for Pb, all with an uncertainty of $\pm 5\%$. There was no significant angular variation in the ratio of the measured and calculated yields. The angular distributions of bremsstrahlung calculated using available analytical theories dropped off more quickly with angle than the measured distributions. The predictions of the theories would be improved by including target-scattered photons.

Keywords: absolute thick-target bremsstrahlung, bremsstrahlung, NaI spectrometry, radiotherapy, Monte Carlo.

3.1 Introduction

The photon spectrum originating in the target of a radiotherapy linear accelerator strongly influences the dose distribution in the patient and the response of dosimeters placed in the beam. Knowledge of the photon spectrum is a prerequisite for accelerator design (see, eg. [1]), accurate treatment planning (see, eg. [2]), and dosimetry (see, eg. [3]).

Measurements of the angular dependance of the bremsstrahlung yield of 15 MeV electrons incident on thick targets of Be, Al, and Pb are reported here. The measurements augment a previously reported set of measurements of the bremsstrahlung yield in the forward direction [4] which covered a wide range of atomic number (Al and Pb targets were used) and a wide range of incident electron energy (10 to 30 MeV). The measurements are intended to serve as benchmark data for verification of Monte Carlo calculations and other measurement techniques.

The bremsstrahlung yield, as defined in our previous paper, is the number of photons of energy E per unit energy from the target which would reach a given point P in a vacuum per unit solid angle per electron incident on the target. The solid angle is defined from the point of intersection of the beam axis with the upstream surface of the target. The bremsstrahlung yield is given by

$$\frac{dS}{dE} = \frac{1}{N_e} \frac{d^2 N_\gamma}{dE d\Omega}, \quad (3.1)$$

where $(d^2 N_\gamma / dE d\Omega) dE$ is the number of photons with energy between E and $E + dE$ which exit the target and reach point P per unit solid angle, and N_e is the number of electrons incident on the target. In principle, dS/dE depends on the distance of point P from the target, due to the finite source size. The brems-

strahlung yield is defined for a target in a vacuum, thus for in-air measurements consideration must be given to both attenuation and scatter in the air.

The integrated bremsstrahlung yield is

$$S_{E_o} = \int_{E_o}^{E_{max}} \frac{dS}{dE} dE = \frac{1}{N_e} \frac{d(N_\gamma)_{E_o}}{d\Omega}, \quad (3.2)$$

where $(N_\gamma)_{E_o}$ is the number of photons of energy greater than E_o which reach point P , and E_{max} is the maximum photon energy in the spectrum and is equal to the energy of the incident electron beam.

There are few published measurements of the angular dependance of bremsstrahlung generated by 10 to 30 MeV electrons incident on thick targets. Hirayama and Nakamura [5] and Nakamura and Hirayama [6] measured the bremsstrahlung yield from 15 MeV electrons incident on thick targets of Fe (10 g/cm²), W (13 g/cm²), and Pb (60 g/cm²). They also measured the bremsstrahlung yield of 15 and 25 MeV electrons incident on thick Pb targets with thicknesses of 60 and 30 g/cm², respectively. The measurements were carried out at 9° and 26° using activation detectors. The authors reported a 50% uncertainty in the measurements. They also found poor agreement between their measurements and their own calculations which they attributed to experimental difficulties.

Lambert et al [7] measured the bremsstrahlung yield of 25 MeV electrons incident on a 7 g/cm² Ta target at 0°, 1.7°, 3.4°, and 5.1°. The spectral shapes were accurate but were restricted to energies above 4 MeV. The absolute measurement (photons per electron) was limited to an uncertainty of ±30% by the beam current measurement.

Several others have reported measurements of the spectral shape [8,9]. These studies were limited to a single atomic number, a single energy, and a few angles.

There have been several reports of the relative intensity of the bremsstrahlung beam at various angles [10,11,12]. These require the conversion of depth-dose data to relative bremsstrahlung yield. The uncertainty due to the conversion can be relatively large.

Berger and Seltzer [13] discuss the need for using a Monte Carlo approach, as opposed to an analytical approach, in the calculation of the bremsstrahlung yield from highly relativistic electrons incident on thick targets. They have already shown agreement within $\pm 5\%$ between the results of Monte Carlo simulations and measurements of both relative intensity [14] and spectral shape [8]. Several approximate analytical theories have also recently been put forward (see, eg. [15,16]).

Our own measurements and Monte Carlo calculations are absolute and of higher accuracy and/or higher energy resolution than previous results. This allows for a more stringent evaluation of calculation approaches.

3.2 Materials and method

3.2.1 Electron beam and targets

The electron beam had an energy of 15 MeV with an energy spread of 1.5%, a pulse repetition rate of 240 Hz, and a nominal pulse width of 2 μ s. The accelerator was operated at currents ranging from 0.1 pA for bremsstrahlung yield measurements along the beam axis (0°) to 0.2 nA for background measurements at 90° . These currents provided roughly one incident photon on our collimated NaI detector for every three machine pulses. The experimental arrangement

used for the measurements of bremsstrahlung yield is shown in figure 3.1. The beam was focussed to a diameter of 0.35 ± 0.05 cm onto the 0.013 cm thick Ti exit window of the beam tube. The beam axis at this point was known to within ± 0.05 cm. The beam then passed through the transmission current monitor (TCM) and the target chamber (when present) and impinged on the target.

The targets were cylinders of Be (11.67 g/cm² thick, 6.72 g/cm² radius), Al (9.74 g/cm² thick, 9.81 g/cm² radius), and Pb (9.13 g/cm² thick, 17.95 g/cm² radius). The targets were chosen to span a wide range of atomic numbers. The target chamber had a 0.0051 cm thick entrance window of stainless steel upstream of the target, a 0.25 cm thick exit window of Al downstream of the target, and a 0.089 cm thick side-wall of stainless steel. The target chamber provided electrical isolation and was evacuated so that the charge deposited in the target could be collected and used to monitor the beam current.

3.2.2 Beam current measurement

The TCM, a 100 ± 1 μ m thick Si surface-barrier detector with a surface area of 3 cm² (Ortec AD-030-300-010), was used to monitor the beam current on a pulse-by-pulse basis. The current calibration of the TCM, in terms of number of electrons incident on the detector per unit charge collected, was $N_e/Q_{TCM} = 720 \pm 20$ /pC. We followed the same current measurement and calibration procedures that we used in our previous study with the following exceptions. In order to remove the dependance of the amplified signal from the TCM on the width of the beam pulse the feedback resistors of the pre-amplifiers were changed to provide a 1 ms decay time and the pre-amplifier signal was amplified by a delay-line amplifier with a 4.5 μ s delay. In order to eliminate any effect of the decay of

Figure 3.1: Experimental arrangement used for the measurements of the bremsstrahlung yield. The electron beam emerges from the beam tube through a thin Ti window, passes through a Si surface-barrier detector used for monitoring the beam current, and impinges on the target. Measurements at $\theta = 0^\circ, 1^\circ, 2^\circ, 4^\circ$, and 10° , were performed with the target mounted in an evacuated target chamber (not shown) and at $30^\circ, 60^\circ$, and 90° with the target mounted in air as shown in the diagram. The nominal distance from the target to the NaI detector was 300 cm.

the pulser signal on the charge calibration, a rectangular voltage pulse (infinite decay time) was used instead of the 1 ms decay time used previously. In our previous set of measurements, we were required to apply a correction factor of up to 1% to account for differences in the shape of the beam current signal and the pulser signal. These correction factors are no longer required. One other significant change to the current monitoring was an additional modification of the preamplifiers with 0.1 μF decoupling capacitors. This was done to prevent charge sharing between the TCM and the decoupling capacitor. In our previous measurements a 3.5% correction was required to account for this effect.

The current was monitored from the block (target) during background measurements at angles of 10° and greater. The TCM was removed during these runs since the high currents could affect its sensitivity. The ratio of the charge collected from the block to the charge collected from the TCM, Q_{BCM}/Q_{TCM} , was measured to be 1.16×10^{-4} for Be, 1.17×10^{-4} for Al, and 1.06×10^{-4} for Pb. The dependance of this ratio on the target material is likely due to variations in the degree of backscatter and Compton current with target atomic number and the dimensions of the target. The current calibration of the block was related to the current calibration of the TCM by

$$\frac{N_e}{Q_{BCM}} = \frac{N_e}{Q_{TCM}} \frac{Q_{TCM}}{Q_{BCM}}. \quad (3.3)$$

The removal of the TCM was shown experimentally and with Monte Carlo simulations to have less than a 2% effect on N_e/Q_{BCM} .

of the bremsstrahlung beam in the air and in the wall of the target chamber (when present), the detector efficiency, and the collimator effect. The procedures used for estimating and applying these corrections are discussed in more detail elsewhere [4,18,19].

The spectral distribution of photons from the target was determined from the distribution of pulse-heights from the NaI spectrometer. The procedures used are outlined in more detail elsewhere [4,18,19]. Briefly, the signal from the 7th dynode of the PMT was recorded on a multi-channel analyzer at a nominal conversion gain of 16 keV/channel each time one and only one event was detected by the paralyzable counter during a beam pulse. The pulse-height scale was calibrated in terms of energy using the 4.439 MeV γ -ray peak of an Am-Be source and the offset determined from precision pulser measurements. The calibrated spectrum was adjusted to bins of 25 keV widths, the pulse pile-up and background were subtracted, and the spectra were adjusted to bins of widths of Γ_G (eq 3.4). This was followed by unfolding of the detector response with FERDO [20], correction for attenuation of the bremsstrahlung beam in the air and target chamber walls (when present), and correction for the detector efficiency. The window functions used in the unfolding procedure, which limit the spectral resolution, had a FWHM of Γ_R , the FWHM of the lines in the measured γ -ray spectra that were used for the energy calibration of the detector. These were up to 50% larger than Γ_G , the FWHM of the broadening function. After the detector response was unfolded from the spectrum a correction for the collimator effect was applied to each bin and the spectrum was normalized to the measured value of the integrated bremsstrahlung yield.

The measurements from 0° to 10° were performed with the targets in the

3.2.3 Bremsstrahlung spectrometer

The NaI detector used in our previous set of measurements was modified by the manufacturer to improve the longitudinal uniformity of the detector sensitivity. This required cutting the detector in half and regluing the two halves. The modified detector, which was used in this study, was 20.3 cm in diameter and 25.1 cm long. A new photomultiplier-tube (PMT) was also installed. The minimum detectable energy for our spectrometry system under nominal operating conditions was 100 keV when this PMT was used. A thin Al window was also installed on the front face of the detector. The thickness of the detector cladding in the window region was 0.13 g/cm² of material of low atomic number. Thus the cladding was essentially transparent to photons of energy over the minimum detectable energy. An electron requires a minimum of 350 keV to penetrate the cladding and be detected. The effect of these changes on the measurement of the integrated bremsstrahlung yield is estimated at less than 1%.

The detector was calibrated using 0.2 to 18 MeV γ -rays. Briefly, the procedure used was to calculate the detector response to a given γ -ray using the EGS4 Monte Carlo system [17] and to use an automated fitting procedure to rebin and Gaussian-broaden the calculated response functions to give the best fit to the measured spectra [18]. The detector was found to be linear to the greater of 1% or 20 keV from 150 keV to 15 MeV, the energy range of interest in this study. The full-width-half-maxima (FWHM) of the broadening functions were found to be

$$\Gamma_G = 0.117E^{0.63}, \quad (3.4)$$

where E and Γ_G are in MeV. The detector had a 3% change in sensitivity

along its length, resulting in a lower resolution for γ -ray spectra than before the detector was modified. The detector was shielded by the same Pb housing used in our previous set of measurements. The bremsstrahlung beam was collimated to a diameter of 2.40 ± 0.02 cm by a 13.4 cm thick Pb collimator inserted into the slot in the 20 cm thick Pb shield. The aperture widened to a diameter of 2.54 cm in the region downstream of the collimator insert. The detector was placed at a nominal distance of 300 cm from the target. The actual distance varied between measurements.

3.2.4 Measurement of the bremsstrahlung yield

The bremsstrahlung yield was measured at 0° , 1° , 2° , 4° , 10° , 30° , 60° , and 90° relative to the beam axis. The acceptance angle of the collimator was 0.4° . The accuracy of the angle measurement was $\pm 0.07^\circ$ at 0° , and $\pm 0.2^\circ$ for off-axis measurements. The collimator axis was aligned to within $\pm 0.3^\circ$ of the line from the centre of the upstream surface of the target to the centre of the collimator aperture.

The integrated bremsstrahlung yield was determined from simultaneous measurements of the beam current and the event rate in the NaI detector [4,18,19]. The latter was measured by a paralyzable counter operating on a clipped signal from the anode of the PMT. The event counter was capable of detecting two 1.25 MeV events separated by 230 ns. For the nominal mean rate of 0.3 detected events per pulse we had a pulse pile-up rate of 3%. This low event-rate allowed us to estimate the pile-up corrections with negligible uncertainty ($<1\%$). Corrections were applied for pulse pile-up, drift and noise in the lower-level discriminator used for the event counting, background events, attenuation

of the bremsstrahlung beam in the air and in the wall of the target chamber (when present), the detector efficiency, and the collimator effect. The procedures used for estimating and applying these corrections are discussed in more detail elsewhere [4,18,19].

The spectral distribution of photons from the target was determined from the distribution of pulse-heights from the NaI spectrometer. The procedures used are outlined in more detail elsewhere [4,18,19]. Briefly, the signal from the 7th dynode of the PMT was recorded on a multi-channel analyzer at a nominal conversion gain of 10 keV/channel each time one and only one event was detected by the paralyzable counter during a beam pulse. The pulse-height scale was calibrated in terms of energy using the 4.439 MeV γ -ray peak of an Am-Be source and the offset determined from precision pulser measurements. The calibrated spectrum was adjusted to bins of 25 keV widths, the pulse pile-up and background were subtracted, and the spectra were adjusted to bins of widths of Γ_G (eq 3.4). This was followed by unfolding of the detector response with FERDO [20], correction for attenuation of the bremsstrahlung beam in the air and target chamber walls (when present), and correction for the detector efficiency. The window functions used in the unfolding procedure, which limit the spectral resolution, had a FWHM of Γ_R , the FWHM of the lines in the measured γ -ray spectra that were used for the energy calibration of the detector. These were up to 50% larger than Γ_G , the FWHM of the broadening function. After the detector response was unfolded from the spectrum a correction for the collimator effect was applied to each bin and the spectrum was normalized to the measured value of the integrated bremsstrahlung yield.

The measurements from 0° to 10° were performed with the targets in the

target chamber in order to maintain the same experimental geometry used for the calibration of the TCM against the block. The effects of the target chamber on the bremsstrahlung yield in this angular range were negligible except for attenuation of the bremsstrahlung beam in the Al exit window, a 4% correction which was straightforward to calculate. The bremsstrahlung yield measurements at angles greater than 10° were performed with the target chamber removed. This eliminated the effects of scatter and attenuation in the target chamber which proved to be more severe and difficult to account for at the larger angles. Scatter from the air and target chamber was shown both experimentally and with Monte Carlo simulations to be negligible from 0° to 10° . Similarly, the removal of the target chamber was shown to have negligible effect on the charge collected from the TCM per incident electron.

The measurements at 60° for Al and Pb and 90° for all three targets were complicated by scatter from objects near the target such as the flange at the end of the beam tube. To remove the scatter, these measurements were performed with a 5 cm thick, 5 cm diameter Pb pre-collimator placed midway between the target and the detector. The axis of the pre-collimator was offset 1.5 cm from the axis of the standard collimator so that the NaI detector had a clear line-of-sight to the thickest target (Be). The total room scatter (excluding the target but including the flange), determined by comparing the measurements of the integrated bremsstrahlung yield with and without pre-collimation, was 5%, 10%, and 20% of the yield at 90° and 0%, 4%, and 10% of the yield at 60° for the Be, Al and Pb targets, respectively.

At 90° the scatter from the flange was blocked with a 10 cm thick Pb brick so that all of the room scatter was effectively removed. The flange scatter was

measured to be 2%, 4% and 10% of the yield at 90° from the Be, Al, and Pb targets, respectively. The trend with atomic number is attributed to the increase in the number of backscattered photons relative to the 90° yield with increasing atomic number. The flange scatter could not be blocked at angles other than 90° . However, the scatter component of the photon fluence was expected to change much more slowly than the bremsstrahlung yield with angle. Assuming that the flange scatter was independent of angle, the flange scatter at 60° was estimated to be approximately 1%, 2%, and 4% of the yield from the Be, Al and Pb targets, respectively. The 60° measurements of the integrated bremsstrahlung yield were reduced by these amounts to correct for flange scatter. The flange scatter was negligible at smaller angles.

3.2.5 Monte Carlo Calculations

The EGS4 Monte Carlo system [17] with the PRESTA electron transport algorithm [21] was used with a modification [22] in which the angle of the bremsstrahlung photon relative to the electron direction was sampled from a realistic distribution, rather than using a fixed angle. This modification had a considerable effect on the angular distribution of the integrated bremsstrahlung yield (figure 3.2), but had little effect on the spectral shapes.

The following parameter values were used for the simulations: $AE = ECUT = 1.511$ MeV and $AP = PCUT = 0.10$ MeV, with the PRESTA parameter $BLCMIN = 1.5$. The Monte Carlo simulations made use of recently published tables of collision stopping powers [23,24] and mean bremsstrahlung cross-sections [25,26]. Use of the new collision stopping powers had negligible effect on the bremsstrahlung yield ($<1\%$) [24]. However, use of the new bremsstrahlung

Figure 3.2: The effect on the integrated bremsstrahlung yield of using a fixed angle of mc^2/E_e for the direction of the bremsstrahlung relative to the direction of the radiating electron as done in the default version of EGS4. This angle is 2° for an electron of energy $E_e = 15$ MeV. The integrated bremsstrahlung yield was calculated in Monte Carlo simulations for targets of Be ($\circ$), Al (Δ), and Pb (∇) in the experimental geometry used in this study. The figure shows the ratios of the fixed-angle results to the results obtained when the angular distribution of Schiff was used. The uncertainties are standard deviations. Some points are offset for clarity.

cross-sections had a significant affect. For example, in the case of 15 MeV electrons incident on the Be target used in this study, use of the new cross-sections resulted in changes in the integrated bremsstrahlung yield of 2% at 0° and 12% at 60° . The effect is expected to be less for higher atomic number targets and for higher incident electron energies.

The bremsstrahlung yield was calculated for 0.1 cm diameter, 15 MeV electron beams incident on Be, Al and Pb targets of the same dimensions as those used in the experiment. Spectra were accumulated along two orthogonal surfaces, one a plane and one a cylinder, both 300 cm from the target, in annular regions between 0° , 0.3° , 0.7° , 1.3° , 1.75° , 2.25° , 3.75° , 4.25° , 9.75° , 10.25° , 29.5° , 30.0° , 30.5° , 59.5° , 60° , 60.5° , 89.8° , 90° and 90.2° . The energy bins corresponded to those of the measured bremsstrahlung yield. Long runs were required (table 3.1) in order to obtain the bremsstrahlung yield with the angular resolution and energy resolution of the measurements with uncertainties comparable to those of the measurements. The two extreme angles had the largest calculational uncertainties. This was due to the small solid angle at 0° and the low intensity at 90° .

The pre-target material was included in the Monte Carlo simulations, although it had little effect on the bremsstrahlung yield. For example, the change in the integrated bremsstrahlung yield when the stainless-steel window is placed upstream of the target is shown in figure 3.3. The effect is small since the bremsstrahlung from the small amount of material upstream of the target is largely forward-directed, and the forward-directed bremsstrahlung is approximately independent of atomic number. The 2% increase in yield for the Be target is due to the larger bremsstrahlung cross-section in the stainless-steel and the increased

Table 3.1: Monte Carlo calculation of bremsstrahlung yield: Number of electron histories N_e , number of photon histories started for each bremsstrahlung event N_{HIST} , and CPU time on a SUN SPARC station 1. Geometry A includes the entrance window of the target chamber (0° to 10°). Geometry B is without the target chamber (30° to 90°).

Target	Geometry	N_e	N_{HIST}	CPU Time (hours)
Be	A	2,410,000	30	43
	B	3,350,100	30	60
Al	A	1,260,000	30	52
	B	1,580,000	30	65
Pb	A	340,000	20	64
	B	340,000	20	62

Figure 3.3: The effect of the stainless-steel window of the target chamber. The integrated bremsstrahlung yield ($E_0 = 100$ keV) was calculated in Monte Carlo simulations for targets of Be ($\circ$), Al (Δ), and Pb (∇) in the experimental geometry used in this study. The figure shows the ratios of the results obtained with the stainless-steel window placed upstream of the targets to those obtained without the stainless-steel window. The uncertainties are standard deviations. Some points are offset for clarity.

angular spread of the electrons caused by the presence of the stainless-steel. The stainless-steel window was removed for the measurements at angles larger than 10° .

3.3 Results and discussion

3.3.1 Measurement uncertainties

The major sources of uncertainty in the integrated bremsstrahlung yield were:

1. The factor N_e/Q_{TCM} which relates the charge collected by the TCM to the beam current ($\pm 3\%$).
2. The low-energy spectral cutoff which was 145 ± 5 keV. The uncertainty in the yield due to the low-energy cutoff was negligible for Pb targets, and ranged as high as $\pm 6\%$ for Be targets at 90° , the softest spectrum of the set.
3. The collimator effect, which varied from $0.2 \pm 0.2\%$ to $3.9 \pm 0.8\%$ (table 3.2). This effect is discussed in more detail in reference [4]. Charged-particles from the target, air, and collimator were negligible ($< 1\%$).
4. The flange scatter. This was only significant for the Pb spectrum measured at 60° . In this case the flange scatter was $4 \pm 4\%$. The flange scatter was removed at 90° with a pre-collimator.
5. The beam alignment and beam radius. These were only significant for the Pb spectrum measured at 90° . Monte Carlo simulations have shown that a 0.1 cm shift in the alignment of the beam and target axes results in

Table 3.2: Collimator effect relative to the integrated bremsstrahlung yield. Results are from Monte Carlo simulations. See reference [4] for details.

θ	Be (11.67 g/cm ²)	Al (9.74 g/cm ²)	Pb (9.13 g/cm ²)
	(%)	(%)	(%)
0°	3.9±.8	2.7±.6	3.8±.6
4°	3.6±.6	2.9±.6	3.1±.4
10°	2.5±.6	3.0±.4	2.9±.5
30°	1.5±.4	2.0±.4	2.8±.4
60°	0.7±.3	1.2±.3	2.3±.4
90°	0.2±.2	0.6±.3	2.9±.5

a 7% change in the integrated bremsstrahlung yield at 90° due to target attenuation. Changing the beam radius from 0.05 cm to 0.25 cm in the simulation resulted in a 5% decrease in the yield.

The major sources of uncertainty in the spectral shapes were counting statistics, unfolding of the response of the NaI detector, and the collimator effect [4,18,19].

3.3.2 Bremsstrahlung yield

The measured bremsstrahlung yields from the three targets are tabulated in appendix D. The uncertainties quoted in these tables include only the uncertainties due to counting statistics, unfolding, and the collimator correction. Measurements at representative angles are shown in figures 3.4, 3.5, and 3.6. Logarithmic plots were used to emphasize the low-energy part of the spectrum

Figure 3.4: The bremsstrahlung generated at angles of 1° , 4° , 10° , 30° , 60° , and 90° by 15 MeV electrons incident on a thick Be target. The measured bremsstrahlung yields (solid lines) are compared to the calculated bremsstrahlung yields (dashed lines). The calculations were done using the EGS4 Monte Carlo system. Results are absolute (photons per electron). The results at 0° and 2° are comparable to the 1° results and are left off of the plots for clarity. The spectral shapes determined from the Schiff spectra with target attenuation taken into account (dotted lines) are shown at 0° and 60° , normalized to the measured values of the integrated bremsstrahlung yield at those angles.

Figure 3.5: The bremsstrahlung generated at angles of 1° , 4° , 10° , 30° , 60° , and 90° by 15 MeV electrons incident on a thick Al target. The measured bremsstrahlung yields (solid lines) are compared to the calculated bremsstrahlung yields (dashed lines). The calculations were done using the EGS4 Monte Carlo system. Results are absolute (photons per electron). The results at 0° and 2° are comparable to the 1° results and are left off of the plots for clarity. The spectral shapes determined from the Schiff spectra with target attenuation taken into account (dotted lines) are shown at 0° and 60° , normalized to the measured values of the integrated bremsstrahlung yield at those angles.

Figure 3.6: The bremsstrahlung generated at angles of 1° , 4° , 10° , 30° , 60° , and 90° by 15 MeV electrons incident on a thick Pb target. The measured bremsstrahlung yields (solid lines) are compared to the calculated bremsstrahlung yields (dashed lines). The calculations were done using the EGS4 Monte Carlo system. Results are absolute (photons per electron). The results at 0° and 2° are comparable to the 1° results and are left off of the plots for clarity. The bremsstrahlung yield from Pb measured without pre-collimation (broken line) is shown where it differs markedly from the pre-collimated result. The spectral shapes determined from the Schiff spectra with target attenuation taken into account (dotted lines) are shown at 0° and 60° , normalized to the measured values of the integrated bremsstrahlung yield at those angles.

where the bremsstrahlung yield is greatest and the bin-widths are smallest and to accommodate the four decade range in the bremsstrahlung yield. In general, the yield increases with decreasing photon energy until the 'turnover point' of the spectrum is reached. This is the point at which the bremsstrahlung beam begins to be strongly attenuated in the target. The turnover point is seen at roughly 600 keV in the Pb spectra. By increasing the gain of our photo-multiplier tube (results not shown) the signal-to-noise ratio was improved sufficiently for us to observe the turnover in Al at about 90 keV, in agreement with separate Monte Carlo runs. There was no evidence of a turnover in the Be spectrum down to the lower-level-discriminator cutoff of 60 keV for the high-gain measurements.

Pre-collimation at 60° and 90° was used to effectively eliminate the room scatter, except for the flange scatter at 60°. The effect of pre-collimation on the bremsstrahlung yield at 60° and 90° from the Pb target is shown in figure 3.6. With no pre-collimation, the scattered photons significantly increase the yield below the turnover point.

A 511 keV peak is evident in the Pb spectra at all angles. The annihilation photons are more or less uniformly distributed and the bremsstrahlung yield is highly forward-directed. The peak becomes more prominent at the largest angles. For Al, a small 511 keV peak can be seen at 60° and 90°. This peak is absent from the Be spectra. The 511 keV photons are due to the annihilation of pair produced positrons in the target and the collimator. The trend with atomic number is due to the larger cross-section for pair production in higher atomic number targets. The measured yield of 511 keV photons per incident electron from the Pb target was a constant $0.014 \pm 0.002/\text{sr}$ from 0° to 10°. The yield dropped to $0.011 \pm 0.001/\text{sr}$ at 30°, $0.0068 \pm 0.0004/\text{sr}$ at 60° and $0.0019 \pm 0.0002/\text{sr}$

at 90°. This was partially due to the increased attenuation of the annihilation photon beam at large angles due to increased path-lengths through the Pb target. For the Al target the measured yield was $(4.5 \pm 1.6) \times 10^{-4}/\text{sr}$ at 60° and $(3.2 \pm 0.6) \times 10^{-4}/\text{sr}$ at 90°. At smaller angles the yield of annihilation photons from the Al target was too small relative to the bremsstrahlung yield for meaningful measurements.

The measured and calculated spectra are compared in figure 3.4, 3.5, and 3.6. Excellent agreement is obtained throughout the energy range of the spectra for all three targets at all eight angles except for the Pb spectra below 300 keV. The Monte Carlo simulation takes care of all of the important processes: energy loss and diffusion of electrons, bremsstrahlung production, photon attenuation and scatter, and positron annihilation. The yield of annihilation photons in the Al and Pb were predicted to $\pm 20\%$ in Pb and $\pm 50\%$ in Al. Annihilation photons from the collimator were not included in the calculations. The excessive number of low-energy counts in the measured spectra for Pb amount to only 1.8% and 1.2% of the integrated yield at 60° and 90° respectively. These counts are attributed to collimator effects and/or charged-particle contamination. At the smaller angles, where pre-collimation was not used, scatter from objects near the target such as the target chamber may also contribute to the excess of low-energy counts in the Pb spectra.

3.3.3 Mean energy of the bremsstrahlung beam

The mean energy

$$\bar{E} = \frac{1}{S_{E_o}} \int_{E_o}^{E_{max}} E \frac{dS}{dE} dE \quad (3.5)$$

is a useful beam quality indicator (see, eg. [1]). Strictly speaking, the whole spectrum should be used in the evaluation of the mean energy ($E_0 = 0$). This would require knowledge of the spectral shape down to energies well below the turnover point. In practice this is difficult to achieve, especially for targets of low atomic number. It is therefore useful to define $\bar{E}$ with a low-energy cutoff.

The mean energies of the measured and calculated spectra ($E_0 = 145$ keV) are compared in figure 3.7. For Be and Al, the measured spectra are slightly harder (increased number of high-energy photons) than the calculated spectra, giving mean energies roughly 2% higher than the calculated values. This is within two standard deviations of our estimated experimental uncertainty of $\pm 1\%$.

The Be spectra are harder than the Al spectra at angles below 10° . This is seen in both the measurements and calculations, and is attributed to the lower transmission of the Be target (figure 3.8), which in turn is due to the greater target thickness. The Pb spectra are much harder than the Be and Al spectra. This is due to the large degree of self-attenuation in the Pb target below 1 MeV. If the determination of the mean energy is restricted to energies above 1 MeV, where the different target materials have comparable attenuation coefficients, the Pb result agrees much more closely with the Al and Be results, as shown for the measured spectra in figure 3.7. In this case, the Pb spectra at angles less than 10° are actually softer than the Be and Al spectra. The calculated spectra (not shown) behave in the same manner.

At the larger angles (30° to 90°), the Pb spectrum is harder than the Al spectrum which in turn is harder than the Be spectrum. This trend is the result of the larger bremsstrahlung cross-section for higher atomic number targets and the details of the electron and photon transport in the target. The measured

Figure 3.7: The mean energies of the measured 15 MV bremsstrahlung spectra for energies above $E_0 = 145 \text{ keV}$ (solid lines) and $E_0 = 1 \text{ MeV}$ (broken lines) for Be ($\circ$), Al ($\triangle$), and Pb (∇). The mean energies from the spectra calculated by Monte Carlo at all eight angles (dashed lines) and from the thin-target spectra with target attenuation at 1° , 10° , and 60° ($*$ -Be, x -Al, $+$ -Pb) are shown for energies above $E_0 = 145 \text{ keV}$ only. Some points are offset for clarity.

Figure 3.8: Transmission of photons through 11.67 g/cm² Be (solid line), 9.74 g/cm² Al (dashed line), and 9.13 g/cm² Pb (broken line) targets. The factor $\exp\{-\mu t\}$ is plotted, where μ is the attenuation coefficient and t is the target thickness.

point at 90° for the Pb target has a larger uncertainty than the other points due to the greater sensitivity of the degree of attenuation in the Pb target (which is more dense than Be and Al) to beam alignment and focal spot size. Thus it is not surprising to see a mild discrepancy between the measurements and the calculations here.

A simple analytical approach was used to calculate bremsstrahlung spectra for thick targets. The bremsstrahlung cross-section formula of Schiff [27] was used to calculate the thin-target spectral distributions for 15 MeV electrons incident on Be, Al, and Pb. The thin-target spectra were then corrected for attenuation by the total thickness of the targets. This assumes that the major contribution to the bremsstrahlung yield is from electrons near the surface of the target. The rationale for this is that the electrons that have traversed a substantial portion of the target will produce less bremsstrahlung due to their lower energies, and only a fraction of the bremsstrahlung that is produced will be directed along the beam axis due to the large angular spread of the electrons from multiple scattering. This procedure has been used by others to estimate the shape of bremsstrahlung spectra from thick targets at small angles (see, eg. [3]). For the smaller angles (0° to 10°), the attenuated Schiff spectra are similar in shape to the measured spectra (figure 3.4, 3.5, and 3.6). The small differences in the spectral shapes result in an overestimate of the mean energies by about 10% (figure 3.7). This is not surprising given the assumption that the bremsstrahlung is all produced by electrons with energies equal to the beam energy.

At angles large compared to mc^2/E_e , the ratio of the rest mass of the electron to its energy (2° at 15 MeV), the Schiff formula for the thin-target spectra is

not valid. In addition, the bremsstrahlung spectra at large angles are largely composed of bremsstrahlung from multiply-scattered electrons which have lost significant amounts of energy in the target and from photons scattered in the target. Thus the simple approach of attenuating the thin-target spectrum is unsuitable for the calculation of spectral shapes at large angles (see figures 3.4 to 3.7).

Andreo and Brahme [28] and Findlay [16] have devised more sophisticated analytical approaches. The general scheme was to divide the target into a number of thin slabs. The thin-target bremsstrahlung yield for monoenergetic electrons with energies based on the depth of each slab was weighted by the transmission through the remaining target and an integration was performed over the target thickness. In Findlay's paper the spectral shapes, mean energies, and angular distributions of 5 to 20 MV bremsstrahlung from thick, high-Z targets were compared to ETRAN Monte Carlo calculations made by others [13,29]. Differences between the results of the analytical and Monte Carlo approaches demonstrate that there is still room for improvement in the analytical theories.

3.3.4 Integrated bremsstrahlung yield

The integrated bremsstrahlung yields from the measurements and Monte Carlo calculations are shown in table 3.3. The measured and calculated yields are in very good agreement (figure 3.9). The agreement is especially encouraging considering the dramatic decrease in the yield over the range of angles in the study. For example, the 90° result for Be is 0.4% of the 0° result. Most of the measured to calculated ratios for all 3 targets fall within $\pm 4\%$ of the mean ratio over the entire angular range. This is somewhat outside of the $\pm 2\%$ repeatability

Table 3.3: Measured and calculated integrated bremsstrahlung yields of 15 MeV electron incident on thick Be, Al and Pb targets. Only photons with energies greater than $E_{\gamma}=0.145$ MeV are included. The Gaussians fit to the data for each target by root-mean-square minimization are also shown. The uncertainty in the last digit is shown in brackets.

Angle	Be (/sr)		Al (/sr)		Pb (/sr)	
	measured	calculated	measured	calculated	measured	calculated
0°	2.73(14)	2.60(2)	3.42(17)	3.27(3)	2.92(15)	3.07(7)
1°	2.57(13)	2.378(5)	3.21(16)	3.032(8)	2.80(14)	2.82(2)
2°	2.14(11)	2.001(4)	2.78(14)	2.629(6)	2.48(12)	2.50(1)
4°	1.542(77)	1.388(2)	2.138(107)	1.969(4)	2.00(10)	2.00(1)
10°	0.630(32)	0.569(1)	1.060(53)	1.003(2)	1.199(60)	1.214(4)
30°	0.0949(48)	0.0865(1)	0.265(13)	0.247(1)	0.447(22)	0.450(1)
60°	0.0238(14)	0.0215(1)	0.0666(40)	0.0652(1)	0.1293(65)	0.1400(5)
90°	0.01061(74)	0.00989(5)	0.0287(17)	0.0251(1)	0.0519(36)	0.0533(9)
Fit	$2.4(3) \exp \{-\theta^2 / (8.6(7)^\circ)^2\}$		$3.2(2) \exp \{-\theta^2 / (8.8(11)^\circ)^2\}$		$2.8(2) \exp \{-\theta^2 / (10(2)^\circ)^2\}$

Figure 3.9: Ratios of measured and calculated integrated bremsstrahlung yields as a function of angle for Be ($\circ$), Al ($\triangle$), and Pb (∇). The calculations were done using the EGS4 Monte Carlo system. The relative uncertainty was $\pm 3\%$ from 0° to 60° , and up to $\pm 6\%$ at 90° . Points are offset for clarity.

of this measurement when the detector is repositioned and the target and electron beam are realigned.

The angular dependance of the integrated bremsstrahlung yield is shown in figure 3.10. The reduction in yield with angle is more pronounced with decreased atomic number. This is due to the increased angular spread of electrons in the higher atomic number targets. The yield at small angles is to first order independant of atomic number. In contrast, the yield at 10° varies by a factor of two between Be and Pb and the yield at 90° varies by a factor of five. The total yield, integrated over solid angle, is strongly influenced by the yield at the larger angles, due to the large solid angles subtended by these regions. Consequently, the total yield depends strongly on atomic number. This is a reflection of the dependance of the bremsstrahlung cross-section on atomic number.

Lambert et al [7] parameterized the angular distribution of bremsstrahlung from thick targets with a simple Gaussian. The Gaussian functions which give the minimum root-mean-square deviation to our data are shown in table 3.3. The result for Pb is compared to the measured angular distribution in figure 3.11. For the thick targets used in this study the Gaussian parameterizations are poor from 0° to 10° , and are completely inadequate at large angles.

Nordell and Brahme [15] developed an approximate analytical approach for determining the angular distribution of the integrated bremsstrahlung yield. In their approach, the target is divided into a number of thin slabs. The electron energy is taken as the average energy in each slab based on a simple model. The angular spread of photons from each slab is assumed to be Gaussian with contributions from the intrinsic angular distribution of the bremsstrahlung process and from the angular distribution of electrons. The yield from the target is

Figure 3.10: Measured angular distributions of the integrated bremsstrahlung yield of 15 MeV electrons incident on 11.67 g/cm^2 Be ($\circ$), 9.74 g/cm^2 Al (Δ), and 9.13 g/cm^2 Pb (∇) targets.

Figure 3.11: The measured angular distribution of the integrated bremsstrahlung yield of 15 MeV electrons incident on 9.13 g/cm² Pb (solid line) is compared to a Gaussian (dashed line) and to the analytical result of Nordell and Brahme for 17 MeV electrons incident on 8.6 g/cm² Au (broken line).

Table 3.4: Yield of scattered photons from the target relative to the integrated bremsstrahlung yield. Results are from Monte Carlo simulations.

θ	Be (11.67g/cm ²)	Al (9.74g/cm ²)	Pb (9.13g/cm ²)
	(%)	(%)	(%)
1°	2±1	3±2	4±4
4°	1±1	6±2	7±3
10°	9±2	11±2	10±1
30°	33±5	33±2	22±1
60°	72±9	54±6	38±1
90°	85±11	73±11	33±3

determined by integrating the attenuated yields over the target thickness. The result they obtained for a thick Au target is decidedly non-Gaussian, as shown in figure 3.11, and is a much better fit to our data than a Gaussian. However, the analytical theory underestimates the width of the angular distribution. The same is true for Findlay's result (which was discussed earlier) when compared to ETRAN.

We undertook a Monte Carlo study to determine the yield of photons that were scattered in the target relative to the integrated bremsstrahlung yield. The results are shown in table 3.4. The target scatter makes only a small contribution to the yield (<10%) at small angles (<10°) and is most significant at large angles and for targets of low atomic number. It is clearly necessary to include the target scatter in the calculation of the yield of photons from thick targets at large angles. The photon scatter inside the target was ignored by Nordell and Brahme and by Findlay in the development of their analytical expressions.

We have already reported measurements of the bremsstrahlung yield of 15 MeV electrons at 0° from thick Al and Pb targets [4]. Those measurements had a low-energy cutoff of $E_o=220$ keV. When this value of E_o is used for our present set of measurements, we obtain values of the integrated bremsstrahlung yield that are $2\pm3\%$ and $0\pm3\%$ lower than the previously-reported values for Al and Pb, respectively. Thus, the two points in the independent measurement sets which overlap agree to within the estimated uncertainty of the measurement.

If the results of both of our studies are combined, we see a trend in the discrepancies between the measurements and calculations. The measured yield for the low atomic-number targets used in both studies have about 10% more photons per electron in the measurements than in the calculations. This discrepancy vanishes for the Pb target. The excess photons are roughly uniformly distributed across the spectrum. In addition, no significant trend is seen with beam energy (10 to 30 MeV) or angle (0° to 90°). It is difficult to associate the excess counts with a collimator effect or scatter since these effects depend in a different way on atomic number, angle and energy than the observed trend. We have used EGS4 to explore the possibility that the assumption of a constant current calibration factor for the TCM, regardless of the composition or the dimensions of the target, was partially responsible for the observed trend. This possibility was considered since both the 100 μm and 144 μm thick Si detectors showed a 2.5% to 3.5% increase in N_e/Q_{TCM} when the Pb target was replaced with Be or Al. However, the Monte Carlo simulations showed that the presence of the target had less than a 1% effect on N_e/Q_{TCM} . Furthermore, the measured increase in N_e/Q_{TCM} could be explained by noise-related difficulties in the measurement of the block current. We therefore dismiss the possibility

that the current measurement was responsible for the excess counts. We are led to speculate that the source of the discrepancy is in the physics or in the application of the physics in the Monte Carlo code. One candidate is the bremsstrahlung cross-section data, which has an estimated accuracy of 3 to 10% in the energy range 2 to 50 MeV, with the larger uncertainty applicable at the lower energies [23,25] .

3.3.5 Summary

The bremsstrahlung yield of 15 MeV electrons incident on thick Be, Al, and Pb targets has been measured at eight different angles ranging from 0° to 90°. In a previous study we reported measurements of the forward-directed bremsstrahlung yield of 10, 15, 20, 25 and 30 MeV electrons incident on thick targets of Al and Pb. These two studies are the most comprehensive survey to date of absolute thick-target bremsstrahlung from relativistic electrons. The measurements cover a wider range of photon energy, contain more energy bins, and/or are more accurate in the absolute sense (photons per electron) than previous measurements.

The measurements were compared to the results of EGS4 Monte Carlo simulations. The agreement was excellent for all 34 spectra and show a marked improvement over previous benchmarking efforts. There were some small differences in the Pb spectra below 300 keV and in the mean energies of the spectra. In addition, the measurements of the integrated bremsstrahlung yields were about 10% higher than the calculated yields for the Be and Al targets at all angles and beam energies in contrast to the yields from Pb which were in agreement within the uncertainties at all angles and beam energies. This may be indicative of

a problem in the bremsstrahlung cross-section data or some other aspect of the Monte Carlo simulations. Nonetheless, the EGS4 system has been validated for the absolute calculation of the photon yield (photons per electron) from thick targets to 10%.

The bremsstrahlung yield that is obtained from some of the available analytical theories was in reasonable agreement with measurements and/or Monte Carlo calculations at small angles. The theories could be improved at large angles if photon scatter in the target and, to a lesser extent, photons from positron annihilation in the target were considered.

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Chapter 4

Calibration of the beam current monitors

4.1 Introduction

One of the significant aspects of the bremsstrahlung yield measurements reported here are that they are absolute (photons per incident electron). Since the accelerator current can drift unpredictably, absolute measurements require simultaneous measurements of the beam current and the photon fluence. The ideal geometry for the photon fluence measurements is a target placed in a vacuum. Any pre-target material will contribute bremsstrahlung to the photon beam. Any post-target material will attenuate and scatter the photon beam.

One approach that we used to measure of the beam current was to collect the charge deposited in the target (block). An absolute measurement could be made in this way if all of the electrons incident on the target were to stop in the target, and no electrons were liberated from the target by the photon field. Unfortunately, this is not the case. For a beam of 15 MeV electrons incident on our targets, with a thickness of roughly 110% of the CSDA electron range,

the charge collected from the Be, Al and Pb targets was 3%, 0%, and 7% less, respectively, than the mean charge of the incident electron beam per pulse. We believe that the main source of this electron loss was electrons from Compton events that were scattered out of the target.

The approach of using the block as the beam current monitor is further complicated by the requirement for electromagnetic isolation of the block in order to eliminate the overwhelming interference from the accelerator. In this arrangement, secondary electron emission from the surfaces of the exit window and target is a problem, and a bias of 50 V or more is required in order to collect these electrons (figure 4.1). It is undesirable to operate without bias since the block current is extremely sensitive to any potential difference between the block and the target chamber at or around 0 V. With bias applied it is necessary to evacuate the target chamber to eliminate the overwhelming signal from ionization produced in the air. A quiet pump was used to maintain the vacuum and the target chamber was mechanically isolated from the beam tube. This was done since, with bias applied, the block amplifies mechanical noise which can interfere significantly with the current measurement.

With the target chamber in place, the electron beam no longer has unimpeded passage to the target, but passes through an entrance window of some finite thickness. This pre-target material can contribute significant bremsstrahlung in the forward direction. In addition, the photon beam generated in the target is attenuated and scattered by the walls of the target chamber. This has a significant effect on the photon fluence at large angles. Another important consideration is that the target, as a detector, provides no amplification (one electron in is roughly one electron collected), the charge collected can be rather

Figure 4.1: The effect of bias on the block current for a 30 MeV electron beam incident on an Al target. This is an early measurement, and the experimental geometry was different than that used for the measurements of the current calibration factor. The graph is intended to show the sensitivity of the block current to the bias. The ratio of block current to toroid current was normalized to give an average value of unity.

small (less than 1000 electrons per beam pulse for the 30 MeV bremsstrahlung yield measurements in the forward direction), and the current signal is quite noisy at the lowest currents used in this study.

Another approach is to monitor the current with a thin solid state detector placed upstream of the target. We mounted a surface-barrier detector of Si between the exit window of the beam tube and the target, (or the window of the target chamber, when present). This added a minimum amount of material upstream of the target, and provided a relatively noise-free signal due to the high gain (roughly 10^4 ion pairs generated for each electron incident on the detector). The electrons in the beam lost only a small fraction of their energy in the detector (about 50 keV). Since the electrons were transmitted through the detector, we dubbed the detector the "Transmission Current-Monitor", or TCM. The target, when used for monitoring the current, was called the "Block Current-Monitor" or BCM. The TCM was our beam monitor of choice. This was due to its improved signal-to-noise over the BCM and its ability to monitor the beam current when the target was suspended in air with no target chamber to interfere with the photon beam.

4.2 Materials and method

The charge deposited in the detector (either BCM or TCM) was collected by a preamplifier, amplified, and the distribution of pulse-heights was accumulated on a multi-channel analyzer. Two calibrations were necessary for each current monitor in order to determine the mean number of electrons incident on the target from the mean channel number of the pulse-height distribution. First,

the detector must be calibrated in terms of charge collection. Second, a calibration factor must be measured to convert the collected charge to the number of electrons in a given beam pulse.

4.2.1 Charge calibration

The charge calibration was carried out using a precision pulser. A calibrated "test capacitor" of capacitance C was placed in series with the pulser so that a voltage pulse resulted in the injection of a known charge $Q=VC$ into the preamplifier. If care is taken, this approach can be used to measure charge deposition absolutely to better than $\pm 1\%$. The major refinements to the measurement of the charge deposition which were required to achieve this accuracy were:

1. Removal of the dependence of the charge measurement on the shape of the charge pulses from the detector and the pulser. This was done by increasing the decay time of the preamplifiers to 1 ms and using a delay-line amplifier with a $4.5 \mu\text{s}$ decay in order to eliminate the dependence of the charge collection on the width of the beam pulse (nominally $2 \mu\text{s}$). In addition, a rectangular pulse from the pulser was used to match the charge pulse from the detector.
2. Reduction of charge-sharing between the preamplifier and the detector capacitance. The detector capacitance was in parallel with the decoupling capacitor of the preamplifier. The capacitance of the block was roughly 50 pF, that of the $144 \mu\text{m}$ thick Si detector was roughly 65 pF and that of the $100 \mu\text{m}$ thick Si detector was roughly 390 pF. If the decoupling capacitor were 2 nF, for example, then the charge collected by the preamplifier

would be less than the deposited charge by 2.5% , 3.3%, and 20% for the three detectors, respectively. The preamplifiers were equipped with 0.1 μF decoupling capacitors, which reduces charge-sharing to less than 0.5% in the worst case (the 100 μm thick Si detector).

3. Reduction of the effect of the decoupling capacitor of the preamplifier on the voltage across the test capacitor during charge calibration. Test capacitors of 1 pF, 10 pF, 100 pF, and 450 pF were routinely used. With a 2 nF decoupling capacitor in series with the test capacitor of capacitance C , the charge injected into the preamplifier by a voltage pulse V was 0.05%, 0.5%, 5% and 23% less, respectively, than VC . With a 0.1 μF decoupling capacitor the charge on the voltage across the test capacitor was reduced by less than 0.5% of V in all of the cases.

We performed an experiment to validate the procedure used for the absolute measurement of the charge deposited in the TCM or BCM. The 144 μm Si detector was illuminated with the 5.477 MeV α -particle of an ^{241}Am source. The measurement was performed in an evacuated chamber to eliminate α -particle energy-losses in air. The Si detector had a 40 $\mu\text{g}/\text{cm}^2$ Au layer on the surface (manufacturers specification) which resulted in a small energy loss of 10 keV. The only other energy loss of consequence was 14 keV transferred to atomic motion, which is then unavailable for the creation of electron-hole pairs [2]). Thus the α -particle deposits 5.45 MeV in the TCM, which is in a comparable range to our bremsstrahlung yield measurements (1000 electrons deposit about 50 MeV). This corresponds to a charge deposition of $Q = eE/W = 0.241 \pm 0.001$ pC ($W = 3.62 \pm 0.02$ eV/ion pair [1]). This number is in excellent agreement with the measured charge of 0.242 ± 0.002 pC per α -particle. This confirms to better

than 1% that the measured charge is equal to the deposited charge. The good agreement between the calculated and measured charge deposition suggests that the charge measurement approach could be used to determine W/e for Si with an accuracy competitive with the 0.5% uncertainty in the currently accepted value.

The charge calibration procedure was also checked at high currents (300 nA). This was done by measuring the average current from the target relative to the average current through a toroid placed in the evacuated beam tube. The target (or block) current was measured with a current integrator and, in separate runs, with a preamplifier on a pulse-by-pulse basis. The latter approach made use of the charge-calibration procedure that we wanted to verify. The two results agreed to within the experimental uncertainty of $\pm 2\%$.

4.2.2 Current calibration

The block and transmission current monitors were calibrated in terms of the beam current using three different methods. Each of these methods will be discussed in turn.

Toroid to BCM to TCM

This method has already been discussed in the chapter on the measurement of bremsstrahlung yield in the forward direction. The ratio I_{beam}/I_{toroid} of the beam current to the toroid current was measured by simulating the beam current with a known current pulse. The toroid and block current monitors were then used simultaneously for beam current measurements at 300 nA. A charge Q_{toroid} was collected from the toroid during the course of the measurement. The calibration

factor for the BCM was calculated from

$$\frac{N_e}{Q_{BCM}} = \frac{1}{e} \frac{I_{beam} Q_{toroid}}{I_{toroid} Q_{BCM}}, \quad (4.1)$$

where N_e is the mean number of electrons per beam pulse, Q_{BCM} is the mean charge per pulse collected from the block, and e is the charge on the electron. The block and transmission current monitors were used simultaneously for beam current measurements at average beam currents ranging from 2 to 10 pA. The calibration factor for the TCM was calculated from

$$\frac{N_e}{Q_{TCM}} = \frac{1}{e} \frac{I_{beam} Q_{toroid} Q_{BCM}}{I_{toroid} Q_{BCM} Q_{TCM}}. \quad (4.2)$$

Single electron spectra

In this approach the linear accelerator was operated at very low currents of the order of one electron every beam pulse. The distribution of pulse-heights from the TCM were recorded on a multi-channel analyzer. The spectra were calibrated in terms of pulse-height per unit charge deposited in the detector using the pulser calibration technique discussed previously. The spectra were very noisy, and some ingenuity was required to overcome this problem. The noise spectrum was Gaussian with a FWHM of 40 keV for the 100 μm detector and 30 keV for the 144 μm detector. The thin detector proved to be unsuitable for these measurements due to the poor signal-to-noise ratio. The thick detector was adequate for measurement of the current calibration factor using single electrons.

Two approaches were used with comparable success. In the first approach, the current calibration factor was equated to the reciprocal of the mean charge.

This required the suppression of the background noise (noise above the lower-level-discrimination, unaccompanied by an electron event), and the removal of pulse pile-up. To accomplish this a large NaI detector was placed 10 cm downstream of the TCM. The analyzer for the current monitor was gated on the "accept" signal from the pile-up circuitry of the NaI detector. This guaranteed that at least one electron passed through the TCM during the beam pulse, and the background noise was completely suppressed. In addition, multi-electron pulses (pile-up) were only recorded when the time interval between the electrons was competitive with the nominal 350 ns reset time of the pile-up circuitry for 20 MeV events. The charge distribution was corrected for the remaining 3% pile-up using the methods described in Appendix A. Without this correction the current calibration factor would have been underestimated by 1%. The main source of uncertainty in this measurement is the possibility of a machine-related offset buried in the noise, beyond the level of detection. The minimum offset that we could detect was 2 keV. The mean energy deposited per electron was about 50 keV. An undetected 2 keV offset would thus result in a 4% error in the measurement of the mean charge.

The second approach to current calibration with the single electron spectrum involved the extraction of the offset and current calibration from a fit of the measured spectrum to the spectrum obtained from a Monte Carlo simulation. The spectrum was measured without pile-up rejection in order to obtain prominent pile-up peaks. Two spectra were accumulated at rates that differed by a factor of two to look for count-rate effects.

The use of Monte Carlo to calculate the energy loss in Si has been previously established [3]. The calculated spectrum was convolved with itself to obtain the

double pile-up spectrum. This spectrum was convolved with the calculated spectrum again to obtain the triple pile-up spectrum.

A root-mean-square-deviation minimization was done between the measured and calculated spectra to establish the relationship between the two charge scales. The fitting procedure described in appendix C was used. The peak weights were chosen by trial and error and had little influence on the offset and gain determined for the fit. The FWHM of the broadening (noise) function was fixed during the fit at the value obtained from a fit of the ungated spectrum and the calculated spectrum. This value was used since the ungated spectrum was free of background and pile-up and gave a much more accurate measurement of the noise. The use of the calculated spectrum modestly improved the precision of the measurements of the offset and of the centroids of the single-electron and pile-up peaks compared to the more conventional approach of fitting the peaks simultaneously with Gaussians. The main advantage of the Monte Carlo simulation was to obtain an estimate of the mean charge deposition in the detector relative to the features in the charge distribution.

The calibration factor was determined from the results of the fit as follows. The calculated distribution was fitted to the measured distribution to find the relationship between ΔE , the calculated energy deposition, and N , the channel number of the measured spectrum:

$$\Delta E = \frac{\Delta Q}{\Delta N} \beta_2 (N - \beta_3) \quad (4.3)$$

where β_2 and β_3 were parameters that were varied in the fit. The charge-calibration of the measured spectrum $\Delta Q/\Delta N$ was known to 1%, but the offset was uncertain. On the other hand, the offset of the calculated spectrum was

known exactly, and the charge calibration was uncertain, as will be discussed shortly. With these considerations, the offset from the fit, if reasonable, was taken as the offset of the measured distribution, and the measured charge was determined from

$$Q_{meas} = \frac{\Delta Q}{\Delta N}(N - \beta_3) = \frac{\Delta E}{\beta_2} \quad (4.4)$$

The main uncertainty in this measurement was a 3% uncertainty due to the background fit. The charge calibration contributed a 1% uncertainty. The fit was limited in accuracy by the width of the bins in the Monte Carlo spectrum, which was 1 keV. These uncertainties combine to give a 4% uncertainty in the calibration factor.

Calculation approach

The current calibration factor of the TCM was estimated from

$$\frac{N_e}{Q_{TCM}} = \frac{W/\epsilon}{\overline{\Delta E}}, \quad \frac{W}{\epsilon} = 3.68 \pm 0.02 \text{ eV/ion pair [1]}. \quad (4.5)$$

This conversion may be overly simplistic due to the contribution of low-energy electrons to the charge deposition and the possibility of systematic differences between the experiment and the simulation. An upper limit on $\overline{\Delta E}$ is the mean energy lost by the electron through ionizations and excitations, as calculated from the unrestricted stopping power. Since a portion of this energy is transferred to δ -rays which escape from the detector, Monte Carlo simulations were used to provide a more reliable estimate of $\overline{\Delta E}$. For 15 MeV electrons, 21% of the energy transferred through collision processes was found to leak from both the 100 μm and 144 μm thick Si detectors. In addition, δ -rays from the material surrounding the detector contributed 5% of the energy deposited in the detector. All of the

Table 4.1: Current calibration results. The ratio of the charge collected from the toroid to the charge collected from the target is Q_{toroid}/Q_{BCM} . The ratio of the number of electrons in the electron beam to the charge collected from the TCM is N_e/Q_{TCM} .

Energy (MeV)	Q_{toroid}/Q_{BCM}			N_e/Q_{TCM} (/pC)					
				144 μ m Si			100 μ m Si		
	Be	Al	Pb	Be	Al	Pb	Be	Al	Pb
10	—	1.014	1.109	—	454	457	—	—	—
15	1.006	0.992	1.069	470	472	460	729	730	705
20	—	—	1.047	—	—	463	—	—	—
30	—	—	1.029	—	—	456	—	—	—

pre-target material, including the window of the target chamber, was included in the simulations. The targets had a negligible effect and were not included in the simulations. The main uncertainties in this approach is the conversion of the energy deposited in the detector to the charge liberated in the detector and the detector thicknesses. The thicknesses of the detectors were known to 1% for the thin detector and 10% for the thick detector (manufacturers specifications).

4.3 Results and discussion

The measurements of Q_{toroid}/Q_{BCM} and N_e/Q_{TCM} by the toroid approach are summarized in table 4.1. There was up to a 4% range in the Q_{toroid}/Q_{BCM} measurements for a given target and beam energy. Repeated measurements of Q_{BCM}/Q_{TCM} have shown similar variations. The source of this variation was likely noise-related difficulties in the measurement of the current from the block (Q_{BCM}). The trend in Q_{toroid}/Q_{BCM} with energy and atomic number

is likely due to variations in the leakage of secondary electrons from the block (the Compton current). There is a 4% variation in the measured value of the calibration factor of the TCM with energy and atomic number. This variation is attributed to experimental variation in the measurement of the block current relative to the toroid and the TCM.

Monte Carlo simulations have been performed in which all of the pre-target material and each target have been included in the model. The results, which have an uncertainty of $\pm 1\%$, show that the mean energy deposition in the TCM is unaffected by the target but varies with beam energy by 2% from 10 to 30 MeV. The simulations also showed that the pre-target material increased the mean energy-deposition in the TCM by 5% for 15 MeV electrons for both the 100 μm and 144 μm Si detectors. Thus care must be taken when using a different geometry for the measurement of the bremsstrahlung yield than that used for the measurement of the current calibration factor. The stainless-steel entrance-window of the target chamber was both measured and calculated to contribute less than 1% of the energy deposited in the TCM. This justifies the use of the same current calibration factor whether or not the target chamber is in place. (The target chamber was removed for the 30°, 60° and 90° measurements of the bremsstrahlung yield).

The results of the various measurement and calculation approaches used to obtain the current calibration factor are summarized in table 4.2. We consider the toroid approach as the standard approach. The current distributions measured in the toroid approach are much less noisy than the single-electron distributions and consequently give the most accurate estimate of the calibration factor.

Table 4.2: Current calibration factors for the 144 μm thick TCM and the 100 μm thick TCM obtained by several methods: the toroid-to-BCM-to-TCM approach (toroid), the mean charge of the single-electron spectrum obtained with pile-up rejection ($\bar{Q}$), the fits of two different single electron spectra (different currents) obtained without pile-up rejection to the Monte Carlo spectrum (fit), and the Monte Carlo result (MC). The Monte Carlo simulations included the pre-target material. The ratio of the calibration factor obtained using the various methods to the toroid result is shown in the last column. The mean energy deposition was related to the current-calibration factor using the approximate equation $\overline{\Delta E} = (W/e)(N_e/Q_{TCM})$.

Approach	$\overline{\Delta E}$ (keV)	offset (fC)	N_e/Q_{TCM} (pC ⁻¹)	Ratio
<u>144μmSi</u>				
toroid	49.4	—	465 $\pm$ 3%	1.00
$\bar{Q}$	46.7	0	493 $\pm$ 4%	1.06
fit	51.3	0.10	448 $\pm$ 4%	0.96
fit	48.9	0.14	470 $\pm$ 4%	1.01
MC	53.3	—	431 $\pm$ 2%	0.93
<u>100μmSi</u>				
toroid	31.9	—	720 $\pm$ 3%	1.00
MC	36.7	—	626 $\pm$ 2%	0.87

The determination of the current calibration factor from the mean charge of the single-electron distribution depends strongly on the offset. The 4% uncertainty quoted for this approach is dependant on our ability to observe a 2 keV (0.1 fC) offset in a Gaussian noise distribution with a FWHM of 30 keV (1.3 fC). Our estimate is honest, but may be overly optimistic. Nevertheless, the result is in reasonable agreement with the toroid measurement. A spectral fit for the gated, single-electron distribution obtained with pile-up rejection is shown in figure 4.2. The charge scale on the calculated spectrum was adjusted to give the same mean charge-deposition per electron as the toroid result. The gain was then fixed in the fit. A 0.12 fC offset gave the best fit. This was near the threshold of detection. The current-calibration factor from this measurement is therefore consistent with the toroid result.

The single electron-distributions obtained without pile-up rejection are shown in figure 4.3 along with the calculated spectra. The calibration factor was insensitive to the mean energy of the calculated single-electron spectrum, the peak weights chosen for the calculated multi-electron spectra, and the FWHM of the broadening function. However, uncertainty in the background led to a 4% uncertainty in the calibration factor. This uncertainty was established with much more confidence than the offset-related uncertainty of the previous approach which made use of pile-up rejection. This was because the offset was more reliably estimated from spectra with two or more peaks.

The calculated current-calibration factor is 7% less than the result of the toroid measurement for the 144 μm Si detector. This is good agreement considering the 10% uncertainty in the thickness of the detector. The calculated current calibration factor is 13% less for the 100 μm Si detector. The thickness

Figure 4.2: The fit of the calculated spectrum (dashed line) to the measure current-distribution (solid line) that was gated on single events in the NaI detector placed downstream of the TCM. The calculated spectrum, determined using the EGS4 Monte Carlo system, was converted to charge-deposition by applying the W factor of 3.68 eV per ion pair. No background function was included in the fit since the background noise and pile-up in the measured spectrum was suppressed by gating the spectrum and a pile-up correction was applied. The charge-deposition of the calculated spectrum was increased by 8% to be consistent with the result of the toroid approach (table 4.2). The region of the fit is indicated by the arrows. With the gain fixed in the fit, the offset was determined to be 0.11 fC. This is near our detection threshold for the offset of roughly 0.1 fC.

Figure 4.3: Fits of calculated spectra (dashed lines), to current-distributions (solid lines) measured at beam currents that differed by a factor of two: a) lower current, b) higher current. The FWHM of the Gaussian noise function was fixed at 30 keV, the value obtained for the gated spectrum (figure 4.2). The offsets were 1.0 ± 0.4 fC and 0.9 ± 0.3 fC and the gain-multipliers were $1.06 \pm .02$ and $1.08 \pm .02$ for the lower and higher current spectra, respectively. The dotted line is the second-order background fit. The uncertainties quoted are the external errors of the fit. The region of each fit is indicated by the arrows.

of this detector was known to 1%. The discrepancy between the measurement and calculation is partly due to the difficulty in converting the energy deposited by the low-energy electrons (δ -rays) to the charge liberated as electron-hole pairs. In addition, the Monte Carlo calculation is further complicated by the material which surrounds the Si in our experimental arrangement. The δ -rays from this material increase the mean energy-deposition in the Si by approximately 5%. With these considerations, the 10% level of agreement with the calculations is considered to be reasonable. The measurements of the calibration factors are preferred to the calculated values due to the uncertainty in the calculated charge deposition, and the fact that the calculation cannot account for any unknown systematic uncertainties in the measurement such as the thickness of the detector.

4.4 Summary

The charge deposited in the TCM or the BCM was shown to be measured on a pulse-by-pulse basis to better than 1%.

The current calibration factor of the 100 μm and 144 μm thick Si detectors were measured with the toroid-to-BCM-to-TCM technique to be:

$$\frac{N_e}{Q_{TCM}} = 720 \pm 20 / \text{pC for the } 100 \mu\text{m Si TCM}, \quad (4.6)$$

and

$$\frac{N_e}{Q_{TCM}} = 465 \pm 15 / \text{pC for the } 144 \mu\text{m Si TCM}. \quad (4.7)$$

The results are valid for Be, Al, and Pb targets from 10 to 30 MeV, the full range of our measurements. These calibration factors were verified to better

than 10% with low-current measurements (less than one electron per pulse) and Monte Carlo simulations.

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Appendix A

Pile-Up corrections in pulsed-beam spectroscopy

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Abstract

A previous theory for pile-up estimation, applicable when a paralyzable counter is used for pile-up rejection, is extended to pulsed beams. The variation of current within the beam pulse is dealt with explicitly. Approximate equations are developed to aid in estimating the effect of an energy-dependent resolution-time on the pile-up. Specific cases of spectroscopy with a NaI scintillation crystal are examined: 30 MV electron bremsstrahlung spectroscopy and 10 MeV electron spectroscopy. Given an 8% event loss to pile-up, the total events were estimated with this approach to better than $\pm 2\%$, and the pile-up corrections to the spectral shapes were estimated to within the uncertainty due to counting statistics.

A.1 Introduction

A project is underway to measure absolutely (photon yield per electron incident on the target) the bremsstrahlung spectra from a pulsed beam of 10-30 MeV electrons incident on thick targets of Al and Pb [1]. The photon yield is determined by counting the events which occur in a large NaI detector placed downstream of the target. When two or more events arrive in a time interval shorter than the resolution time of the event counter, they are recorded as a single event. This constitutes pulse pile-up, and results in a measured event rate which is lower than the actual event rate. Pile-up also distorts the shape of the pulse-height distribution. These effects are significant and must be accounted for in order to obtain accurate, absolute spectra.

Good introductions to pile-up can be found in Knoll [2] and Debertin and Helmer [3]. A variety of methods are available for electronically detecting pile-up, and either rejecting pulses in which pile-up was detected, or correcting for the pile-up automatically (eg, [4,5]). Other systems utilize a pulser of constant amplitude which does not interfere with the spectrum. Pile-up corrections are determined from the measured pile-up in the pulser peak (eg, [6]). Some of these methods, such as those utilizing pulsers, are not applicable to pulsed beams.

The salient features of our bremsstrahlung spectroscopy system with regard to pile-up effects are as follows:

1. The beam is pulsed.
2. Within the pulse, there is structure in the beam current, and this structure is repeated each pulse.

3. The spectrometer signal is output to two channels: a fast-timing channel, and an energy channel. The fast-timing channel is used for event counting. The energy channel is used to determine the amount of energy deposited in the detector. Pile-up circuitry is employed in the fast-timing channel to prevent the analysis of those pulses in the energy channel which are composed of more than one event. However, the detection of pile-up is limited by the resolving time of the circuitry in the fast-timing channel, and some pile-up will remain undetected. The resolution time is sufficiently short (nominally 200 ns) compared to the shaping time of the energy-signal amplifier ($1\ \mu\text{s}$) that, in the case of undetected double pile-up, the amplitude of the pile-up pulse in the energy channel is equal to the sum of the amplitudes of the two individual pulses to better than 2%.
4. Pulses in the fast-timing channel are passed through a discriminator. A count is registered when the signal exceeds a preset voltage level. If the time intervals between a sequence of events are less than the resolution time of the detector, only the first event is detected. This constitutes a paralyzable counter.
5. The pulse height of the energy signal is recorded in a multi-channel analyzer (MCA) each time one and only one event is detected by the paralyzable counter during a beam pulse. The MCA constitutes a non-paralyzable counter since it has ample time to reset between pulses (4 ms).

In our application the event rate is low, nominally 0.5 events per pulse, and the pulse-to-pulse variation in current is roughly $\pm 10\%$. The resolution time of the paralyzable counter depends on the amplitude of the signal and is approximately

200 ns, 10% of the width of the beam pulse.

Our approach to handling pile-up is to use the simple system described above to reject the majority of pulses in the energy channel which are composed of 2 or more events. We then estimate the actual event-rate from the event-rate measured by the paralyzable counter in the fast channel, and estimate a correction to remove the distortion of the pulse-height distribution due to pile-up. These corrections are based on a theory of pile-up developed by Tenney [7]. Tenney's theory is suitable for systems employing a paralyzable counter for pile-up rejection for which the superposition of piled-up events is additive, a reasonable assumption for our system. Tenney's work is suited to our application since it gives expressions for the actual event rate in terms of the measured event rate, and the pile-up corrected spectrum in terms of the measured spectrum. However, Tenney considered only radiation sources with a constant fluence rate.

In this work, Tenney's theory is extended to pulsed beams. The current within each pulse is allowed to vary, provided that the structure is repeated from pulse-to-pulse. The theory is presented first, followed by experimental evaluation of some of the theoretical results. Several different approaches are considered to account for the effect of structure in the beam current and the energy-dependence of the resolution time. A suitable approach to pile-up correction in the spectroscopy of bremsstrahlung is then devised.

A.2 Notation

Symbols that appear frequently in the text are repeated here for easy reference. This paper generally adopts the nomenclature used by Tenney.

$P(n)$ — probability of n events occurring during a single beam pulse.

$p_{m|n}$ — conditional probability that, given n events in a beam pulse, m events are detected by the paralyzable counter.

L — number of beam pulses.

μ — event rate (mean number of events per pulse).

N_o — total number of events.

μ_p — event rate as measured by the paralyzable counter.

N_p — number of events resolved by the paralyzable counter.

μ_a — 'accept' rate (one detected event per pulse).

N_{co} — number of acceptable counts.

N_{con} — number of acceptable counts with n *undetected* events.

μ_i — 'inhibit' rate (two or more detected events per pulse).

$\eta(t)dt$ — conditional probability that, given an event in a beam pulse, the event occurs between time t and $t + dt$.

τ — resolution time of the fast-timing signal.

$\tau(E)$ — energy-dependent resolution time.

$\bar{\tau}$ — spectrum-averaged resolution time.

T — width of beam pulse.

r — relative resolution time, τ/T .

E — pulse-height on an energy scale.

$M(E)dE$ — measured energy spectrum.

$f(E)dE$ — single-event spectrum, normalized to unity.

$I_0(E)$ — zeroeth order convolution of $f(E)$.

$I_n(E)$ — n th order convolution of $f(E)$.

$C(E)dE$ — pile-up correction spectrum.

$I'_0(E)$ — zeroeth order convolution of $f(E)$.

$I_n^*(E)$ — n th order convolution of $\tau(E)f(E)$.

A.3 Theory

Consider a spectrometry system with features as described in the introduction. Let $P(n)$ be the probability of n events occurring during a single beam pulse. Let μ be the mean number of events per pulse. For a constant μ , $P(n)$ is a Poisson distribution,

$$P(n) = \frac{\mu^n e^{-\mu}}{n!}. \quad (\text{A.1})$$

Let $p_{m|n}$ be the conditional probability that, given n events in a beam pulse, m events are detected by the paralyzable counter. Let L be the number of beam pulses. The total number of events is

$$N_o = L \sum_{n=1}^{\infty} n P(n) = L \sum_{n=1}^{\infty} n \frac{\mu^n e^{-\mu}}{n!} = L\mu. \quad (\text{A.2})$$

The number of events resolved by the paralyzable counter,

$$N_p = L \sum_{n=1}^{\infty} P(n) \sum_{m=1}^n m p_{m|n} = L \sum_{n=1}^{\infty} \frac{\mu^n e^{-\mu}}{n!} \sum_{m=1}^n m p_{m|n} = L\mu_p, \quad (\text{A.3})$$

where μ_p , defined in eq A.3, is the mean event rate detected by the paralyzable counter. The number of counts (pulses with one or more events) for which no pile-up events are detected, or the number of acceptable counts,

$$N_{co} = L \sum_{n=1}^{\infty} P(n) p_{1|n} = L \sum_{n=1}^{\infty} \frac{\mu^n e^{-\mu}}{n!} p_{1|n} = L\mu_a, \quad (\text{A.4})$$

where μ_a , defined in eq A.4, is the 'accept' rate.

When the paralyzable counter detects more than one event in a pulse, the gate on the MCA is closed. This inhibits the analysis of the energy signal. The

rate of occurrence of two or more detected events per pulse, or the 'inhibit' rate,

$$\mu_i = \sum_2^{\infty} (1 - p_{1|n}) \frac{\mu^n e^{-\mu}}{n!}. \quad (\text{A.5})$$

The quantities μ_p , μ_a and μ_i can be measured with the paralyzable counter using suitable logic. Then, if the set of conditional probabilities $\{p_{m|n}; m = 1, n; n = 1, \infty\}$ are known, the actual event rate μ can be determined, using an iterative algorithm, from eq A.3, A.4, or A.5. Since the measured event rate is higher than the accept and inhibit rates, it will provide the most accurate estimate of μ , assuming that counting statistics limit the accuracy. The accept and inhibit rates are useful for evaluating the theory.

The number of acceptable counts with n *undetected* events ($n + 1$ events in the beam pulse) is

$$N_{c0n} = LP(n+1) p_{1|n+1} = L \frac{\mu^{n+1} e^{-\mu}}{(n+1)!} p_{1|n+1}, \quad (\text{A.6})$$

and we have,

$$\sum_{n=0}^{\infty} N_{c0n} = N_{c0}. \quad (\text{A.7})$$

N_{c0n} is required for the determination of the pile-up distortion of the spectral shape.

A.3.1 Conditional pile-up probabilities

We now turn our attention to the determination of $\{p_{m|n}\}$. Let $\eta(t)dt$ be the event-time distribution, that is, the conditional probability that, given an event in a beam pulse initiated at $t = 0$ and of duration T , the event occurs between time t and $t + dt$. Let τ be the resolution time of the fast-timing signal. We

have

$$\int_0^T \eta(t) dt = 1 ; \eta(t) = 0 \text{ for } t < 0 \text{ and } t > T. \quad (\text{A.8})$$

The $\{p_{m|n}\}$ can be calculated by considering all permutations and combinations of the n events occurring at times $\{t_i, i = 1, n\}$ within the beam pulse which result in m events being detected by the paralyzable counter. An event is detected if no other events occur within the time interval τ before the event. Each event configuration must be weighted by the product of the $\{\eta(t_i)dt, i = 1, n\}$.

Given n events in a pulse, the probability of detecting between 1 and n events is unity:

$$\sum_{m=1}^n p_{m|n} = 1. \quad (\text{A.9})$$

The probability of resolving one event from a beam pulse with only one event is

$$p_{1|1} = 1. \quad (\text{A.10})$$

Constant resolution time

For now assume that the resolution time, τ , is a constant. Consider a beam pulse with two events, A and B. The probability of event A occurring between t and $t + dt$ is $\eta(t)dt$. Event B, to be unresolved from event A, must occur within the interval $(t - \tau, t + \tau)$. Thus we have,

$$p_{1|2} = \int_0^T \eta(t) dt \int_{\max(0, t-\tau)}^{\min(t+\tau, T)} \eta(t') dt', \quad (\text{A.11})$$

$$p_{2|2} = 1 - p_{1|2}, \quad (\text{A.12})$$

where $\max(t_1, t_2)$ and $\min(t_1, t_2)$ are respectively the maximum and minimum values of t_1 and t_2 .

For a 3-event pulse,

$$p_{1|3} = 2 \int_0^T \eta(t) dt \left[\int_{\max(0, t-\tau)}^t \eta(t') dt' \left\{ \int_{\max(0, t'-\tau)}^{t'} \eta(t'') dt'' + \int_t^{\min(t+\tau, T)} \eta(t'') dt'' \right\} + \int_t^{\min(t+\tau, T)} \eta(t') dt' \int_{t'}^{\min(t'+\tau, T)} \eta(t'') dt'' \right] \quad (\text{A.13})$$

$$p_{2|3} = 1 - p_{1|3} - p_{3|3} \quad (\text{A.14})$$

$$p_{3|3} = \int_0^T \eta(t) dt \left[\int_0^{\max(0, t-\tau)} \eta(t') dt' \left\{ \int_0^{\max(0, t'-\tau)} \eta(t'') dt'' + \int_{t'+\tau}^{\max(t-\tau, t'+\tau)} \eta(t'') dt'' \right\} + \int_{\min(t+\tau, T)}^T \eta(t') dt' \left\{ \int_{t+\tau}^{\max(t'-\tau, t+\tau)} \eta(t'') dt'' + \int_{t'+\tau}^{\max(t'+\tau, T)} \eta(t'') dt'' \right\} + 2 \left\{ \int_0^{\max(0, t-\tau)} \eta(t') dt' \right\} \left\{ \int_{\min(t+\tau, T)}^T \eta(t'') dt'' \right\} \right] \quad (\text{A.15})$$

The equation for $p_{1|3}$ was determined from the diagrams shown in figure A.1. The diagrams depict various configurations of 3 events, A, B, and C, which occur at times t , t' , and t'' , respectively. All three events must pile-up in order to contribute to $p_{1|3}$. Thus the time between any event and its nearest neighbour must not exceed the resolution time of the paralyzable counter. The origin of the three triple-integrals in eq A.13 are:

1. For the first triple-integral, event C precedes B which in turn precedes A (figure A.1). For event B to pile-up with A, B must occur in the region $(t - \tau, t)$. Then, for event C to pile-up with B, C must occur in the region $(t' - \tau, t')$.
2. For the second triple-integral, B precedes A which precedes C. Event B must occur in $(t - \tau, t)$ and event C must occur in $(t, t + \tau)$.
3. For the third triple-integral, A precedes B which precedes C. Event B must occur in $(t, t + \tau)$ and event C must occur in $(t', t' + \tau)$.

Figure A.1: Diagrams depicting various configurations of 3 events within a beam pulse which are detected as a single event by a paralyzable counter of resolution time τ . The 3 events, A, B, and C, occur at times t , t' , and t'' , respectively. The diagrams show representative configurations for 3 of the 6 possible different orders of events: a) $t'' < t' < t$, b) $t' < t < t''$, and c) $t < t' < t''$. For the particular event order of a given diagram, the area under the event-time distribution $\eta(t)$ which is shaded by parallel lines with positive slopes is the probability that B piles up with A, given that A occurs at time t . The area shaded by parallel lines with negative slopes is the probability that C piles up with B, given that B occurs at time t' . The probability $p_{1|3}$, that only one event is detected in a 3 event pulse, was deduced from these three diagrams (see text).

The factor of 2 in eq A.13 results since the events are indistinguishable. That is, the event orders CBA and BCA result in the same triple-integral, as do the event orders BAC and CAB, and the event orders ABC and ACB.

The complexity of the equations increases rapidly with each additional event in the pulse. The effect of pulses with four or more events will be considered later.

Rectangular event-time distribution (ETD)

Consider the case of constant beam current in a beam pulse of width T. The ETD is then rectangular:

$$\eta(t) = \begin{cases} 1/T & t \in (0, T) \\ 0 & \text{else} \end{cases} . \quad (\text{A.16})$$

Defining the relative resolution time $r = \tau/T$ and substituting $\eta(t)$ from eq A.16 into eqs A.10 to A.15 gives,

$$p_{1|1} = 1, \quad (\text{A.17})$$

$$p_{1|2} = 1 - (1 - r)^2 ; \quad p_{2|2} = (1 - r)^2 ; \quad r \leq 1, \quad (\text{A.18})$$

$$\begin{aligned} p_{1|3} &= 6r^2(1 - r) ; \quad p_{2|3} = 6r - 18r^2 + 14r^3 ; \\ p_{3|3} &= (1 - 2r)^3 ; \quad r \leq \frac{1}{4}. \end{aligned} \quad (\text{A.19})$$

The relative resolving time was restricted for ease in evaluating the integrals, which are complicated by their limits. There are no restrictions on τ or $\eta(t)$ in the general eqs A.10-A.15. For the spectrometry of 30 MV bremsstrahlung from a Pb target with our system, r varies from approximately 0.05 for the lowest energy events (100 keV) to 0.25 for the highest energy events (30 MeV).

A.3.2 Estimation of the event, accept, and inhibit rates

In this study the conditional probabilities $\{p_{m|n}\}$ are only calculated for pulses with fewer than four events. The upper and lower bounds on the mean event-rate μ , when the $p_{m|n}$ for pulses with four or more events are not known, can be determined from (see eq A.3)

$$\left[\sum_{n=1}^3 \frac{\mu^n e^{-\mu}}{n!} \sum_{m=1}^n m p_{m|n} \right] + \sum_{n=4}^{\infty} \frac{\mu^n e^{-\mu}}{n!} \leq \mu_p \leq \left[\sum_{n=1}^3 \frac{\mu^n e^{-\mu}}{n!} \sum_{m=1}^n m p_{m|n} \right] + \sum_{n=4}^{\infty} n \frac{\mu^n e^{-\mu}}{n!}. \quad (\text{A.20})$$

The quantity μ_p is the event rate as measured by the paralyzable counter. The only difference in the left-hand side and right-hand side of this equation is the factor n in the last sum on the right-hand side. The lower bound on μ , determined from the right-hand side, is the extreme of all events being resolved, that is, $p_{n|n} = 1$ for $n \geq 4$. This would occur if the resolution time approached zero. The upper bound, determined from the left-hand side, is the extreme of pile-up of all n events in each n -event pulse, that is, $p_{1|n} = 1$ for $n \geq 4$. This would occur if the resolution time exceeded the width of the beam pulse. For low event rates, the occurrence of 4 or more events in a pulse is rare, and the lower and upper bounds converge.

Similarly, the bounds on the accept rate μ_a , when the $\{p_{m|n}, n > 3\}$ are not known, are given by (see eq A.4)

$$\left[\sum_{n=1}^3 p_{1|n} \frac{\mu^n e^{-\mu}}{n!} \right] \leq \mu_a \leq \left[\sum_{n=1}^3 p_{1|n} \frac{\mu^n e^{-\mu}}{n!} \right] + \sum_{n=4}^{\infty} \frac{\mu^n e^{-\mu}}{n!}. \quad (\text{A.21})$$

The bounds on the inhibit rate μ , when the $\{p_{m|n}, n > 3\}$ are not known, are given by (see eq A.5),

$$(1 - p_{1|2}) \frac{\mu^2 e^{-\mu}}{2!} + (1 - p_{1|3}) \frac{\mu^3 e^{-\mu}}{3!} \leq \mu_i \leq (1 - p_{1|2}) \frac{\mu^2 e^{-\mu}}{2!} + (1 - p_{1|3}) \frac{\mu^3 e^{-\mu}}{3!} + \sum_4^{\infty} \frac{\mu^n e^{-\mu}}{n!}. \quad (\text{A.22})$$

A.3.3 Spectral shape

The distortion of the spectral shape by pile-up is now explored. The amplitude of the signal in the energy channel is recorded each time one and only one event is detected in the fast-timing channel during a beam pulse. The arrival time of an event is the time at which the leading edge of the signal in the fast-timing channel rises above the voltage level of the discriminator. The pulse-width of an event is the time interval in which the signal voltage is greater than the voltage level of the discriminator. Consider two successive events which occur during a beam pulse. Both events will be detected if the time between the arrival of the events is greater than the pulse-width of the *first* event. Pile-up detection is independent of the pulse-width of the second event due to the fast rise-time of the fast-timing signal.

Constant resolution time

When the resolution time is independent of energy, events of different energy are indistinguishable by the paralyzable counter. In this case, the results in the section of Tenney's paper in which spectral shape is discussed apply without modification to the pulsed-beam case. The measured energy spectrum is that given by eqs 40, 42, and 43 of Tenney for beams of constant fluence rate, rewritten

here for convenience (see section 4 of [7] for a derivation):

$$M(E)dE = \sum_{n=0}^{\infty} N_{c0n} I_n(E)dE, \quad (\text{A.23})$$

$$I_0(E) = f(E), \quad (\text{A.24})$$

$$I_n(E) = \int_0^E f(E - E') I_{n-1}(E') dE', \quad (\text{A.25})$$

where $M(E)dE$ is the number of events in the measured spectrum between E and $E + dE$, and $f(E)dE$ is the single-event spectrum (or pile-up corrected spectrum), normalized to unity. This result differs from Tenney's only in the calculation of the coefficients $\{N_{c0n}\}$ (eq A.6). For spectra composed of events with the same resolution time, the spectral dependence on the ETD is found solely in the $\{N_{c0n}\}$. The pile-up *correction* spectrum is defined here as

$$C(E)dE = \sum_{n=1}^{\infty} N_{c0n} I_n(E)dE. \quad (\text{A.26})$$

The single-event spectrum, which is the object of this analysis, can then be obtained from

$$f(E)dE = \frac{M(E)dE - C(E)dE}{N_{c00}}. \quad (\text{A.27})$$

A.3.4 Energy-dependent resolution time

The bremsstrahlung spectrum of 30 MeV electrons incident on a thick Pb target consists of a continuous distribution of photons from the maximum photon energy of 30 MeV down to roughly 200 keV. Below 200 keV the spectrum is highly attenuated by the target and drops rapidly in intensity. The resolution time of our paralyzable counter, which depends on the pulse-width of the leading event, varies from 100 ns to 500 ns over this energy range. This large variation in resolution time might have a large effect on the relationship between the

actual event rate and the measured event rate, and on the distortion of the spectrum shape due to pile-up. The theory presented in this section will be used to estimate the effect of the energy-dependent resolution time on these quantities, and to determine if it is sufficient to use a constant resolution time in the calculations.

Conditional pile-up probabilities

Let $b(\tau)$ be the proportion of the spectrum composed of events with resolution times greater than τ . To account for an energy-dependent resolution time, we rewrite the equations for the conditional pile-up probabilities for 2-event pulses with constant resolution time (eqs A.11 and A.12):

$$p_{1|2} = 1 - p_{2|2}, \quad (\text{A.28})$$

$$p_{2|2} = \int_0^T \eta(t) dt \int_0^T [1 - b(|t - t'|)] \eta(t') dt'. \quad (\text{A.29})$$

In our experiment, pulses with 3 or more events are relatively rare. Thus little error is introduced by ignoring the effect of the energy-dependent resolution time on the 3-event conditional probabilities $p_{1|3}$, $p_{2|3}$, and $p_{3|3}$.

In the case of constant resolution time, τ_o , $b(\tau)$ is a step function,

$$b(\tau) = \begin{cases} 1 & \tau \leq \tau_o \\ 0 & \tau > \tau_o \end{cases}, \quad (\text{A.30})$$

and eqs A.28 and A.29 reduce to eqs A.11 and A.12.

Spectral shape

Consider Tenney's quantity $\delta p_j dE$, the conditional probability that, given a leading event followed by n unresolved events, the sequence of energies of the events

is $E_0, E_1, \dots, E_n$, (the j th configuration) with the constraint of eq 29 of Tenney,

$$E < \sum_{k=0}^n E_k \leq E + dE. \quad (\text{A.31})$$

To estimate the effect of the energy-dependent resolution time on the spectral shape, we proceed with some simplifying assumptions. Consider the case of constant flux, or a rectangular ETD with $\tau(E) \ll T$. Then, analogous to eq 28 of Tenney, we use the approximate relation:

$$\begin{aligned} \delta p_j dE &= k f(E_0) dE_0 \tau(E_0) f(E_1) dE_1 \tau(E_1) \cdots \\ &\cdots f(E_{n-1}) dE_{n-1} \tau(E_{n-1}) f(E_n) dE_n ; \tau(E_i) \ll T, \end{aligned} \quad (\text{A.32})$$

In brief, the justification for eq A.32 is that a leading event with pulse-width τ_A will pile-up with the trailing event τ_A/τ_B times more frequently than a leading event with pulse-width τ_B . We have ignored terms of order $(\tau/T)^2$, which account for the boundaries of the ETD. An additional assumption is that the events which precede the $(i - 1$ th) event have no effect on the detection of the i th event.

Integrating eq A.32 over all possible energy configurations within the constraint that the energies sum to E (eq A.31) gives the quantity $p_n(E)dE$ (eq 30 of Tenney):

$$p_n(E)dE = \sum_j \delta p_j dE. \quad (\text{A.33})$$

The function $p_n(E)dE$ is the unity-normalized spectrum of pulse heights from the beam pulses with $(n + 1)$ events detected as a single event. In eq A.33 it is assumed that the amplitude of the energy signal is the sum of the amplitudes of the $(n + 1)$ events which comprise the signal, a reasonable assumption for our system. The sum is over all of the possible energy sequences within the

constraint of eq A.31. Since $p_n(E)dE$ is normalized to unity, integration of $p_n(E)dE$ over all energies gives the proportionality constant,

$$k = \left[\int_0^\infty \tau(E)f(E)dE \right]^{-n}. \quad (\text{A.34})$$

When $\tau(E)$ is set constant, eq A.32 reduces to the special case of constant resolution time:

$$\delta p_j dE = f(E_o)dE_o f(E_1)dE_1 \cdots f(E_n)dE_n, \quad (\text{A.35})$$

which is eq 28 of Tenney.

Starting from eqs A.32 and A.34, and following the procedure of Tenney (see section 4 of [7]), it is found that the previous equations for the pile-up corrected spectrum (eqs A.26 and A.27) still apply if the convolutions $\{I_n(E)\}$ (eqs A.24 and A.25) are replaced by $\{I_n^*(E)\}$:

$$I_0^*(E) = f(E), \quad (\text{A.36})$$

$$I_n^*(E) = \frac{\int_0^E \tau(E-E')f(E-E')I_{n-1}^*(E')dE'}{\int_0^\infty \tau(E)f(E)dE}. \quad (\text{A.37})$$

$I_n^*(E)$ reduces to $I_n(E)$ when $\tau(E)$ is set constant. For spectra composed of events with energy-dependent pulse-widths, the spectral shape dependence on the ETD is found in the shapes of the pile-up spectra, $\{I_n^*(E)\}$, as well as in the $\{N_{con}\}$.

A.4 Materials and method

The electron beam was provided by a Vickers linear accelerator capable of providing beam energies of 2 MeV to 50 MeV. The beam used for these experiments was pulsed at 240 Hz, with a nominal pulse-width of 2 μ s. Spectra of 10 MeV

electrons and 30 MV bremsstrahlung were measured with a 20 cm diameter by 25 cm long cylindrical NaI detector. In both measurements the detector was 320 cm from the source and the beam was collimated to a diameter of 2.4 cm by a 20 cm thick Pb collimator. The energy signal and the fast-timing signal were derived from the 7th dynode and the anode of the photomultiplier tube, respectively. The fast-timing signal was shortened to roughly 200 ns by passive clipping, and passed through a discriminator. The discriminator output a logic pulse when the input signal exceeded a preset voltage level. The discriminator was reset when the signal dropped below the preset voltage level. The resolution time of the counter was equal to the width of the signal as measured at the voltage level of the discriminator. The resolution time, determined from the signal width of the ^{60}Co lines with different photomultiplier-tube gains, measured from oscilloscope traces, was

$$\tau(E) = 0.079 \ln E(\text{MeV}) + 0.23 \mu\text{s} \quad (\pm 10\%), \quad (\text{A.38})$$

with E in MeV. Scalers were employed to record the event rate μ_p , the accept rate μ_a , and the inhibit rate μ_i .

A pile-up analysis was carried out for a 30 MV bremsstrahlung beam produced by a narrow beam of monoenergetic electrons incident on a 14 g/cm² thick Pb target. Spectra were recorded for 2000 s at nominal event rates of 0.1 and 0.5 events per pulse. The ETD recorded for the latter spectrum is shown in figure A.2. The ETD was measured with a time-to-amplitude converter which recorded the time interval between the beam trigger and the arrival of the first event in each pulse.

The pile-up correction factor is μ/μ_p . This could not be measured directly

Figure A.2:

The measured event-time distribution for the 30 MV bremsstrahlung example.

without using an extremely low count-rate and accumulating data for a long time in order to achieve reasonable statistics. However, the ratio of the pile-up correction factors at high and low event rates was easily measured by taking advantage of the fact that the bremsstrahlung fluence is proportional to the beam current, i . Then we have

$$\left. \frac{\mu}{\mu_p} \right|_{\mu_p'}^{\mu_p''} \equiv \frac{\mu''/\mu_p''}{\mu'/\mu_p'} = \frac{i''/\mu_p''}{i'/\mu_p'}, \quad (\text{A.39})$$

where the superscripts refer to different event-rates. The beam current was monitored with a thin Si detector upstream of the target.

Pile-up corrected spectra were estimated for the 30 MV bremsstrahlung spectrum and for the 10 MeV electron spectrum. Eqs A.23 to A.27 were used with a constant resolution time equal to the mean resolution time calculated from

$$\bar{\tau} = \int_0^\infty \tau(E) f(E) dE, \quad (\text{A.40})$$

using the measured value of $\tau(E)$ (eq A.38). An iterative algorithm was developed to solve these equations. In the 1st iteration, the no-pile-up spectrum $f(E)$ was estimated as the measured spectrum $M(E)$, normalized to unity, in order to obtain an estimate of the pile-up correction spectrum $C(E)$. In subsequent iterations the estimated pile-up corrected spectrum $M(E) - C(E)$, normalized to unity, was used for $f(E)$. To determine the effect of the energy-dependence of the resolution time on the correction spectrum, the same set of equations were used (eqs A.23 to A.27) with the convolutions I_n^r (eq A.37) in place of I_n (eq A.25).

A.5 Results and discussion

A.5.1 30 MV bremsstrahlung example

Calculated and experimental results of the bremsstrahlung spectroscopy example are summarized in table A.1. The first 6 lines in the table were calculated using the measured ETD and a constant resolution time. The mean resolution time was $0.31 \mu\text{s}$. The conditional probabilities for 2-event and 3-event pulses were then calculated (eqs A.11 to A.15). An iterative algorithm was used to obtain μ from the $\{p_{m|n}, n = 1, 3\}$ and μ_p using eq A.20. The accept and inhibit rates were then calculated from eqs A.21 and A.22.

For a mean detected-event rate of 0.51 (table A.1), about 75% of the 2-event pulses were resolved ($p_{2|2} \sim 0.75$) and about 40% of the 3-event pulses were resolved. About 25% of the 2-event pulses and 10% of the 3-event pulses were detected as single events and contributed to the measured spectrum.

The number of events detected by the paralyzable counter which arrive by themselves (no pile-up) can be estimated from,

$$N_{p0} = L [P(1) + 2P(2)p_{2|2} + 3P(3)p_{3|3} + P(3)p_{2|3} + \dots]. \quad (\text{A.41})$$

The number of detected events with 1 undetected event (double pile-up),

$$N_{p1} = L [P(2)p_{1|2} + P(3)p_{2|3} + \dots]. \quad (\text{A.42})$$

The number of detected events with 2 undetected events (triple pile-up),

$$N_{p2} = L [P(3)p_{1|3} + \dots]. \quad (\text{A.43})$$

From these equations, ignoring pulses with 4 or more events, the relative proportion of detected events to no pile-up, double pile-up, and triple pile-up events

Table A.1: Pile-up in a pulsed bremsstrahlung beam, showing the accept- rate (μ_a), inhibit-rate (μ_i), and the ratio of the pile-up correction factors for detected event rates of 0.091 and 0.507 events per pulse ($\mu/\mu_p|_{0.091}^{0.507}$). Calculations assume constant resolution times for a) the measured event-time distribution (ETD) of figure A.1 and b) a rectangular ETD of relative resolution-time, chosen to give a reasonable value of $p_{2|2}$. The calculation uncertainty due to lack of knowledge of the $\{p_{m|n}, n \geq 4\}$ is shown when significant. The bottom two rows of the table are the measured values.

Calculation			$p_{2 2}$	$p_{1 3}$	$p_{3 3}$
$\tau(E)$ (μs)	ETD	μ_p			
$\tau = 0.28$	measured	0.091 0.507	0.79	0.073	0.47
$\tau = 0.31$	measured	0.091 0.507	0.76	0.089	0.42
$\tau = 0.34$	measured	0.091 0.507	0.74	0.11	0.38
$\tau/T = 0.127$	rectangular	0.091 0.507	0.76	0.085	0.42

Calculation			μ_a	μ_i	μ/μ_p	$\mu/\mu_p _{0.091}^{0.507}$
$\tau(E)$ (μs)	ETD	μ_p				
$\tau = 0.28$	measured	0.091 0.507	0.0846 0.334 $\pm$.003	0.0032 0.083 $\pm$.001	1.010 1.060 $\pm$.008	1.050 $\pm$.008
$\tau = 0.31$	measured	0.091 0.507	0.0848 0.338 $\pm$.003	0.0031 0.081 $\pm$.001	1.011 1.067 $\pm$.010	1.055 $\pm$.010
$\tau = 0.34$	measured	0.091 0.507	0.085 0.341 $\pm$.003	0.0030 0.080 $\pm$.001	1.012 1.074 $\pm$.008	1.061 $\pm$.008
$\tau/T = 0.127$	rectangular	0.091 0.507	0.0848 0.338 $\pm$.003	0.0031 0.081 $\pm$.001	1.011 1.068 $\pm$.008	1.056 $\pm$.008
measured values		0.091	0.0843 $\pm$.0004	0.0033 $\pm$.0001	—	1.077 $\pm$.008
		0.507	0.334 $\pm$.001	0.082 $\pm$.001	—	

for a measured event rate of 0.51 was calculated to be 1 : 0.94 : 0.05 : 0.002.

The pile-up correction factor is the ratio of the actual event rate to the measured event rate, μ/μ_p . The calculated ratio of the pile-up correction factors at high and low event rates $\mu/\mu_p|_{0.09}^{0.51}$ is in reasonable agreement with the measured ratio (table A.1). No background subtraction was done for these measurements. The background rate was roughly 0.001 per beam pulse. For these calculations, the pulse-to-pulse variation in the beam current was negligible since the nominal $\pm 10\%$ fluctuation in the beam current led to only a $\pm 0.7\%$ range in the pile-up correction factor for a μ_p of 0.51.

If the dependence of the resolution time on energy is considered, this leads to a 2-event pile-up probability $p_{2|2}$ of 0.76 for a measured event rate of 0.51, using eq A.29. This is the same result as when the resolution time is assumed to be constant and equal to the mean resolution time $\bar{\tau}$ of $0.31\mu\text{s}$. The effect of the energy-dependent resolution time on pile-up probabilities of pulses with more than 2 events will have little effect on the pile-up correction factor since these pulses only contribute about 1% of the detected events. Thus, for this example, the effect of the energy-dependence of the resolution time on $\{p_{m|n}\}$ is adequately estimated by assuming a constant resolution time equal to the mean resolution time.

The $\pm 10\%$ uncertainty in the resolution time leads to an uncertainty in μ of $\pm 0.5\%$ for a μ_p of 0.51, as shown by the results for a τ of $0.28\mu\text{s}$ and τ of $0.31\mu\text{s}$.

The values of μ_a and μ_i , when calculated using a constant resolution time equal to the mean resolution time of $0.31\mu\text{s}$, compare favourably to the measured

values (table A.1). These results indicate that μ has been reasonably calculated from the measured event rate, and that the theory described here is applicable to our experimental situation. As a matter of general experimental practice, it is useful to monitor μ_a and μ_i to insure that the pile-up rejection circuitry is working properly, and as a check on the value of μ which has been obtained from μ_p .

Lines 7 and 8 in table A.1 were calculated assuming a rectangular ETD (eqs A.17 to A.22). The relative resolution time τ/T in table A.1 was chosen to match the $p_{2|2}$ calculated using the measured ETD with a τ of $0.31\mu\text{s}$. These results show that if τ/T is chosen carefully, the assumption of constant beam current is reasonable. If the ETD were not measured, one could assume a rectangular ETD and use the measured values of μ_p and μ_a to determine an 'effective' τ/T (using eqs A.20 and A.21). In this example, this would lead to error of 0.5% in the estimate of the pile-up correction factor.

The raw bremsstrahlung spectrum used in this example is shown in figure A.3. The pile-up corrected spectrum, also shown in figure A.3, oscillates randomly about zero counts above 30 MeV. This is consistent with the expectation that all events over 30 MeV, the maximum energy of the bremsstrahlung beam, are pile-up. Figure A.3 also shows that, for this example, with 2×10^5 counts in the spectrum, the effect of assuming constant resolution time is less than the uncertainty due to counting statistics. In addition, the effect of triple and higher order pile-up can be safely ignored, and only one iteration of the algorithm is necessary to estimate the pile-up correction spectrum.

These results suggest the following method for pile-up correction in pulsed bremsstrahlung spectroscopy:

Figure A.3:

The raw 30 MV bremsstrahlung spectrum (•) is shown after the spectrum was rebinned to 50 keV per bin. The number of counts in the raw spectrum which were due to pile-up was estimated in three ways: 1. Energy-dependent resolution time, triple pile-up included, 3 iterations of the algorithm (—). 2. Energy-dependent resolution time, double pile-up only, single iteration of the algorithm (— — —). 3. Constant resolution time, double pile-up only, single iteration of the algorithm (- - - -). The pile-up corrected spectrum (+) was obtained by subtracting the pile-up counts from the raw spectrum, assuming constant resolution time (the third approach). This spectrum was averaged over 5 bins for clarity.

1. Measure μ_p , μ_a , μ_i , and $\eta(t)$ along with the spectrum.
2. Use $\bar{\tau}$ (eq A.40) with the measured functions $\tau(E)$ and $\eta(t)$ to obtain $\{p_{m|n}, n = 1, 3\}$ from eqs A.11 to A.15.
3. Use these results with μ_p to calculate μ from eq A.20. Confirm that the calculated value of μ is reasonable by comparing the measured values of μ_a and μ_i with values calculated from eq A.21 and A.22 respectively.
4. Calculate the pile-up corrected spectrum from eq A.27 where the convolutions $I_0(E)$ and $I_1(E)$ (eqs A.24 and A.25) are calculated using the measured spectrum $M(E)$, normalized to unity, in place of $f(E)$. Triple and higher order pile-up is ignored in the estimate of the correction spectrum ($N_{con} = 0$ for $n > 1$ in eq A.23).

A.5.2 10 MeV electron example

The measured electron spectrum is shown in figure A.4. This spectrum shows clear double and triple pile-up peaks, making it ideal for assessing the accuracy of the estimate of the pile-up correction. The peak in the spectrum occurred about 2 MeV below the beam energy due to the ~ 1 g/cm² of material (primarily detector cladding) which the beam passed through before reaching the active region of the detector. The tail in the spectrum was due to a combination of energy-loss straggling of the electrons in the air and detector cladding, collimator scatter, and the escape of bremsstrahlung photons from the detector. None of these effects alter the pile-up analysis.

The pile-up correction spectrum was estimated assuming a rectangular ETD and using the energy-dependent resolution time of eq A.38. A pulse-width T of

Figure A.4: The 10 MeV electron spectrum, energy calibrated and adjusted to 50 keV per bin ($\bullet$) along with the spectrum of pile-up counts calculated in two ways: 1. Energy-dependent resolution time (—). 2. Constant resolution time (---). In both cases triple pile-up was included and the algorithm was iterated 3 times. The pile-up corrected spectrum (+) was obtained by subtracting the pile-up counts from the raw spectrum, assuming constant resolution time (the second approach). This spectrum was averaged over 5 bins for clarity.

2.45 μs was used. This value of T was chosen to give a value of μ_a comparable to the measured value (eq A.21). Both the double and triple pile-up peaks are adequately estimated by this approach. The pile-up correction spectrum was also calculated assuming a constant resolution time equal to the mean resolution time for this spectrum. Since the electron beam was essentially monoenergetic, it was not surprising that this assumption had little effect on the result, as shown in figure A.4.

A.5.3 Comparison to Tenney

It is interesting to compare the predictions of this work to Tenney's theory when the latter is applied without modification to a pulsed-beam spectroscopy example. Table A.2 summarizes the results of such a comparison for the bremsstrahlung example discussed previously. The quantities of interest are the measurable quantities μ_a and μ_i , and the quantities which affect the spectrum shape, $\{N_{con}\}$. The latter are the coefficients of the pile-up spectra (see eq A.23). These quantities were determined from Tenney's theory by setting the characteristic time of the slow counter (the MCA) to the pulse-period T . The equations used to calculate these quantities are rewritten here in the notation used in this work:

$$\mu = e^{\mu\tau/T} \mu_p \quad (\text{eq 10 of [7]}) \quad (\text{A.44})$$

$$\mu_a = e^{-2\mu} \mu_p \quad (\text{eq 19 of [7]}) \quad (\text{A.45})$$

$$\mu_i = e^{-\mu}(1 - e^{-\mu})\mu_p \quad (\text{eq 19 of [7]}) \quad (\text{A.46})$$

$$N_{con} = e^{-\mu\tau/T}(1 - e^{-\mu\tau/T})^n \mu_a L \quad (\text{eq 22 of [7]}) \quad (\text{A.47})$$

The mean event-rates calculated from the two theories compare favourably. This is because, for $\tau \ll T$, the dependence of μ_p on μ for a pulsed beam with a

Table A.2:

Pile-up in a pulsed bremsstrahlung beam. Comparison of Tenney's theory, based on constant beam fluence, to the pulsed-beam theory presented in this work, and to measurements. The results based on Tenney's theory were calculated using a slow counter characteristic time equal to the beam pulse-width of the rectangular ETD in table A.1 ($2.44 \mu\text{s}$). The results of the pulsed-beam theory were calculated using the measured ETD of figure A.1. The resolution time of the paralyzable counter was set to $0.31 \mu\text{s}$ for the calculation of both sets of results.

Approach	μ_p	μ	μ_a	μ_i	N_{c00}/N_{c0}	N_{c01}/N_{c0}	N_{c02}/N_{c0}
This Work	0.091	0.0920	0.0849	0.0030	0.988	0.0109	0.00012
Tenney	0.091	0.0921	0.0757	0.0073	0.988	0.0115	0.00013
Measurement	0.091		0.0843	0.0033			
This Work	0.507	0.542	0.338	0.081	0.932	0.0605	0.00405
Tenney	0.507	0.543	0.171	0.123	0.933	0.0622	0.00415
Measurement	0.507		0.334	0.082			

rectangular event-time-distribution of width T is the same as that for a beam of constant fluence-rate, measured with an MCA with a reset time of T . In both cases

$$\mu_p = \mu(1 - \mu\tau/T + O(\tau/T)^2). \quad (\text{A.48})$$

However, Tenney's theory underestimates the accept-rate and overestimates the inhibit-rate as seen by comparing the measured results of table A.1 to the calculated results of table A.2.

With regard to spectrum shape, Tenney's results agree with those of this work. That is, the relative weights of the single, double, and triple pile-up spectra agree reasonably, as shown in table A.2.

A.6 Conclusions

A theory of pile-up in pulsed beams has been presented. The theory accounts for variation of the beam current within the beam pulse. The theory has been used successfully for pile-up estimates in bremsstrahlung and electron spectroscopy. It was found that for our experimental system, the effect of the energy-dependent resolution time could be dealt with adequately by assuming a constant resolution time equal to the mean resolution time of the spectrum being measured. A method for pile-up correction in bremsstrahlung spectroscopy was presented. For our spectrometry system, for nominal event rates of 0.5 events per beam pulse, the method can be used to estimate the total number of events to $\pm 2\%$ and to estimate the distortion of the spectrum due to pile-up with negligible uncertainty.

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Appendix B

Precision response estimation, energy calibration, and unfolding of spectra measured with a large NaI detector

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Abstract

Methods are evaluated for determining the energy-calibration and the response of a 20 cm by 25 cm NaI detector, and for unfolding the detector response from measured spectra of 10 to 30 MV bremsstrahlung. A set of measured γ -ray and electron spectra with energies ranging from 1 to 30 MeV were fit to response functions calculated using Monte Carlo techniques (EGS4) to determine the energy-calibration of the spectrometer and the degree of broadening required for the calculated response functions. For electrons, the broadened response was in excellent agreement with the measurements. For γ -rays, there was evidence of collimator-degraded photons in the measured spectra. A matrix of broadened response-functions was generated for use in unfolding bremsstrahlung spectra in the energy range of 0.2 to 35 MeV. The unfolding routine FERDO was shown to provide reliable estimates of the spectrum of photons in the bremsstrahlung beam when this response matrix was used for unfolding. Collimator effects were ignored without significantly affecting the shape of the unfolded spectrum.

B.1 Introduction

A project is underway to measure a comprehensive set of bremsstrahlung spectra generated by electrons with energies ranging from 10 to 30 MeV which were incident on thick targets [1,2]. The measurements will provide benchmark data for verification of calculational approaches of thick-target bremsstrahlung and for application in radiotherapy treatment planning and radiation dosimetry. A large NaI crystal coupled to a photomultiplier tube (PMT) is employed for the spectrometry. The signal from the PMT is amplified, digitized, and stored in a multi-channel analyzer. The accumulated pulse-height distribution is corrected for pulse pile-up and the background is then subtracted to give $b(E)dE$, the measured spectrum, where E is the pulse-height. The measured spectrum is related to the bremsstrahlung spectrum $\phi(\mathcal{E})$ which is incident on the detector by (see, e.g., [3,4])

$$b(E)dE = dE \int_{E_0}^{\infty} \epsilon(\mathcal{E})R(E, \mathcal{E})\phi(\mathcal{E})d\mathcal{E}. \quad (\text{B.1})$$

where $\epsilon(\mathcal{E})$ is the probability that a photon of energy $\mathcal{E}$, incident on the NaI detector, will result in a pulse-height greater than E_0 , the minimum detectable pulse-height, and $R(E, \mathcal{E})dE$ is the probability that the pulse-height of the detected event is between E and $E + dE$, that is, the response of the detector to a photon of energy $\mathcal{E}$, normalized to unity above E_0 . Unfolding is the process by which eq B.1 is solved in order to obtain either the incident spectrum $\phi(\mathcal{E})$ or the efficiency-weighted incident spectrum $\epsilon(\mathcal{E})\phi(\mathcal{E})$.

In this paper we address the problem of determining the efficiency-weighted incident spectrum from the measured spectrum. Accurate determination of the incident spectrum requires an accurate energy calibration, a set of high quality

response functions which cover the whole energy range of the measurement, and a reliable unfolding routine. We evaluate methods for estimation of the response of our NaI detector to electrons and γ -rays, for energy-calibration of our detector with electrons and γ -rays, and for unfolding of the detector response from bremsstrahlung spectra measured with our detector.

B.1.1 Terminology

This section defines the terminology used for the various types of response functions discussed in this paper. Response functions which were obtained from measurements are referred to as 'measured spectra'. Response functions which were obtained directly from Monte Carlo simulations are referred to as 'calculated response functions'. A calculated response function that was folded with a broadening function is referred to as the 'broadened response function'.

B.1.2 Estimation of the detector response

The use of the Monte Carlo technique to estimate the response of NaI, BGO, and Ge detectors to γ -rays and electrons is well established (e.g., [5,6,7,8,9,10,11]). The response is calculated by simulating the interactions of one particle at a time, keeping track of the total energy deposited in the detector by the particle. Let $r(E', \mathcal{E})dE'$ be the probability of an energy deposition in the detector between E' and $E' + dE'$ from an incident particle of energy $\mathcal{E}$. Let $G(E, E')dE$ be the probability of a pulse of amplitude between E and $E + dE$ given an energy deposition of E' . The response of the spectrometer to a particle of energy $\mathcal{E}$ is

given by [5,6,8],

$$\hat{R}(E, \mathcal{E}) = \int_0^\infty G(E, E') r(E', \mathcal{E}) dE'. \quad (\text{B.2})$$

The broadened response function is normalized to unity above E_o to correspond to $R(E, \mathcal{E})$:

$$R(E, \mathcal{E}) = \hat{R}(E, \mathcal{E}) / \int_{E_o}^\infty \hat{R}(E, \mathcal{E}) \quad (\text{B.3})$$

For a NaI detector, the broadening-function $G(E, E')$ accounts for the processes of light production and collection, photo-electric conversion, and charge amplification. If the detector is of good quality these processes are stochastic, and a Gaussian form is appropriate for the broadening function:

$$G(E, E') = \frac{2\sqrt{\ln 2}}{\sqrt{\pi} \Gamma_G(E')} \exp \left\{ -4 \ln 2 \frac{(E - E')^2}{\Gamma_G(E')^2} \right\}, \quad (\text{B.4})$$

where Γ_G , the full-width at half-maximum (FWHM) of the broadening function, depends on the energy that was deposited. For example, at γ -ray energies up to a few MeV, where the photopeak is well resolved from the rest of the response function, $G(E, E')$ can be adequately estimated from the shape of the photopeak, which is Gaussian for a detector of good quality (see, eg, [12,13,14,15]). At these energies, Γ_G can be obtained accurately by a simple fit of each measured γ -ray spectrum to a Gaussian with background (see, e.g. [3]). For higher energies, the shape and FWHM of the full-energy peak is increasingly influenced by energy losses from the detector, and Γ_G is best determined by comparing the measured pulse-height distribution to the calculated response functions (see, e.g. [6,9,16,17,18,19]). The FWHM of the broadening function is expected to be independent of radiation type if the light production and collection is uniform throughout the crystal. For such detectors it is possible to use a combination of γ -rays and electrons of different energies to determine Γ_G .

For a non-uniform detector, the pulse-height depends on the region of energy-deposition in the detector. In such cases, the detector may suffer a loss of resolution and the broadening function may take on a non-Gaussian shape. In addition, Γ_G may depend on radiation type, since different particles deposit their energy in different regions of the detector. It is possible to account for the non-uniformity by including the measured variation in detector-sensitivity in the Monte Carlo simulation when calculating $r(E', \mathcal{E})$ (see, e.g. [18]) so that $G(E, E')$ is independent of radiation type. Such an approach does not adhere strictly to the definition of $r(E', \mathcal{E})$, but serves as a practical solution to the problem.

B.1.3 Energy calibration

For NaI detectors, measurements at γ -ray energies below 2 MeV have revealed non-linearities in the pulse-height per unit energy of the full-energy peak of 10% from 200 keV to 2 MeV, and of 20% from 20 keV to 2 MeV. This has been largely attributed to variations in the light yield of NaI with absorbed energy [13]. Calibrations from MeV energies up to GeV energies have deviated from linearity by up to $\pm 5\%$ due to non-linearities in both the crystal and PMT [17,20,21]. Larger non-linearities are possible if the crystal has defects or the PMT is operated in a non-linear region. Thus each detector must be individually calibrated using sources which span the energy range of interest.

Radioisotopes have been widely used for energy calibration for energies up to a few MeV, and γ -rays produced by protons accelerated by a Van de Graaff have been used to extend the calibration range to around 20 MeV. For γ -rays of energies up to a few MeV it is common to determine the mean channel number of the full-energy peak from a fit of a Gaussian (or a modified Gaussian) to the

measured pulse-height distribution (e.g., [22]). At higher energies, the single-escape peak interferes with the full-energy peak, and energy leakage may result in a shift in the full-energy peak to an energy below that of the incident γ -ray. An accurate calibration can still be obtained by fitting response functions calculated using Monte Carlo methods to the measured pulse-height distributions.

Mono-energetic beams of electrons, positrons, and other particles have also been used for energy calibrations. Charged-particle beams may lose a significant amount of energy in the material between the beam exit window and the active region of the detector. In addition, some energy may leak from the detector. For these situations, the position of the peak in the pulse-height distribution will correspond to an energy-deposition which is less than the beam energy. Several researchers have accounted for this by using analytical or Monte Carlo techniques to estimate the energy of the peak in the pulse-height distribution [21,23]. However, it is difficult to locate the peak accurately in the measured pulse-height distribution due to energy-loss straggling in the detector cladding and response-function broadening. Improved accuracy in the energy calibration can be achieved by fitting calculated response functions to measured response functions, as in the γ -ray case.

B.1.4 Unfolding

The method commonly used to unfold bremsstrahlung spectra measured with a NaI detector is matrix inversion (see, eg, [23,24,25]). The unfolding routine that we use, FERDO [26,27], utilizes a least-squares function-fitting approach. This routine, which has been widely used for unfolding the detector response from neutron spectra, has the advantage of providing uncertainty estimates.

B.2 Materials and method

B.2.1 Measurements

The spectrometer consisted of a 20 cm diameter by 25 cm long NaI crystal (Bicron) coupled to a 12.7 cm diameter 10-dynode PMT (SRC B125B01). The front face of the NaI crystal was clad with 0.02 g/cm² Cr, 0.86 g/cm² Al, 0.01 g/cm² sponge, 0.006 g/cm² polyethylene, and 0.2 to 0.4 g/cm² MgO [28]. The energy signal was tapped from the 7th dynode of the PMT since the dynodes further from the cathode exhibited large non-linearities. For example, with 1000 V applied to the PMT, the signal from the 9th dynode exhibited a 20% non-linearity between 4 and 20 MeV, whereas the signal from the 7th dynode was linear to $\pm 1\%$ in this region. The anode signal of the PMT was input to a fast-timing circuit which was used for pile-up rejection during the measurements of electron and bremsstrahlung spectra.

Spectra were measured for the sources listed in table B.1. These consisted of a ⁶⁰Co source, γ -rays from nuclear reactions induced by energetic protons from a Van de Graaff accelerator, and electrons from a linear accelerator. The energy of the electron beam was calibrated to $\pm 1\%$ [29]. The energy spread of the electrons in the beam was restricted to $\pm 0.2\%$ by a slit associated with the energy-analyzing magnet.

The spectral measurements were carried out using three different arrangements (figure B.1). The source-to-detector distances (SDD's) are shown in table B.1. The ⁶⁰Co spectrum was measured with the detector fully shielded in order to reduce the background and obtain accurate results in the region of the

a)

b)

c)

Figure B.1: Experimental arrangements used for spectral measurements: a) ^{60}Co and bremsstrahlung, b) 6-18 MeV γ -rays, and c) electrons. Distances are in centimeters.

Table B.1: Sources used for spectral measurements. The source-to-detector distance (SDD) used for each source is shown. The geometry column refers to the experimental arrangement shown in figure B.1.

Source	Energies (MeV)	SDD (cm)	Geometry
^{60}Co γ -rays	1.173, 1.332	40	a
$^{19}\text{F}(\text{p},\alpha\gamma)^{16}\text{O}$ $E_p = 0.49$ MeV	6.130, 7.117	52	b
$^7\text{Li}(\text{p},\gamma)^8\text{Be}$ $E_p = 0.49$ MeV	17.64	52	b
$^{19}\text{F}(\text{p},\alpha\gamma)^{16}\text{O}$ $E_p = 0.69$ MeV	11.85	52	b
$^{11}\text{B}(\text{p},\gamma)^{12}\text{C}$ $E_p = 2.53$ MeV	13.83, 18.27	52	b
electrons	10.0, 15.0, 20.0, 25.0, 30.0	1.8	c

Compton continuum (figure B.1a). The beam was collimated to a diameter of 2.4 cm. The other γ -ray spectra were measured with the beam collimated to a diameter of 5.1 cm by a pair of 5 cm thick Pb slabs (figure B.1b). The electron measurements (figure B.1c) were made without collimation at 1.8 cm from the 0.013 cm Ti exit window of the accelerator. The exit window defocussed the beam. In the short-SDD arrangement, the beam impinged on the central region of the detector without need for a collimator, eliminating collimator effects. The short SDD also minimized the energy that the electron beam lost to the air. The detector was shielded along the sides with Pb sheets to reduce the x-ray background from the modulator tank of the linear accelerator.

The natural background was measured and subtracted from the measured γ -ray spectra. The accelerator background from the Van de Graaff made only a

3

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3

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small contribution to the measured spectra and was neglected. The accelerator background from the linear accelerator was measured at zero beam current and subtracted from the electron spectra. The electron spectra were corrected for pulse pile-up prior to the background subtraction. Pile-up correction was not necessary for the γ -ray spectra due to the low count-rates. Details of the pile-up correction are presented elsewhere [30].

B.2.2 Response function estimation

An updated version of the user-code JACKET [8] was used with the EGS4 system [31] to calculate response functions for electrons and γ -rays. The JACKET code was developed with EGS3 for calculating response functions for cylindrically-symmetric detectors. For this study, the EGS4 parameters AP and PCUT were set equal to 0.01 MeV and the parameters AE and ECUT were set equal to 0.521 and 1.000 MeV respectively. Photons with energies less than PCUT and electrons with energies less than ECUT deposit all of their energy locally.

The NaI crystal was fabricated in 1981 from two 12.7 cm long crystals which were glued together with epoxy, a standard procedure at the time. The glue deteriorated, resulting in a discontinuity in the sensitivity of the detector, as seen in the longitudinal-uniformity scan with ^{60}Co , shown in figure B.2. An increase of 1.09 ± 0.01 in the sensitivity of the back half of the detector is evident. This non-uniformity produced an extra peak in the γ -ray pulse-height distributions which was in a higher channel than the full-energy peak, and caused the peak in the electron pulse-height distributions to spill over into higher channels. To account for this defect, we used the approach discussed in the introduction, that is, the measured non-uniformity was included in the Monte Carlo simulation.

Figure B.2: Longitudinal-uniformity scan of the NaI detector with a ^{60}Co source. The pulse-heights of the 1.173 (x) and 1.332 (+) MeV γ -ray photopeaks are shown relative to their respective means. The source was collimated to a diameter of 0.7 cm.

The NaI crystal was bisected, then for each incident particle, the energy deposited in the back half of the crystal was multiplied by 1.09 and added to the energy deposited in the front half. This gave the total pulse-height in terms of the energy of the most prominent peak in the detector response.

The model used to simulate the γ -ray experiments (figures B.1a and B.1b) consisted of a bisected NaI crystal with 1.18 g/cm^2 of Al on the front face. The source was situated at the appropriate SDD and was restricted to a radius equal to that defined by the aperture of the collimator used in the experiment. This assumes that the collimator was a perfect absorber. The ^{60}Co source was simulated as two γ -rays of 1.173 MeV and 1.332 MeV of equal intensity.

The electron experiments (figure B.1c) were modelled by a 0.1 cm diameter electron beam incident on 0.059 g/cm^2 of Ti followed by 0.0018 g/cm^2 of air, 1.18 g/cm^2 of Al, and a bisected NaI detector. For electron beams, the material upstream of the detector had a significant effect on the beam energy. Electrons lost approximately 2 MeV before reaching the active region of the detector, a 20% energy loss for a 10 MeV beam. To simplify the model, an equivalent thickness of Al, in mass per unit area, was used in place of the individual layers of cladding. Unfortunately, the thickness and density of the MgO reflecting powder was not accurately known. An equivalent thickness of Al of 1.18 g/cm^2 was chosen since it was within the factory specifications and gave a pulse-height-per-unit-energy at 8 MeV (10 MeV beam) comparable to the γ -ray result, which was insensitive to the thickness of the detector cladding.

B.2.3 Energy-calibration and broadening parameters

Several investigators have used calculated response functions to extract information from measured pulse-height distributions through iterative fitting techniques. Zobel and Hall [32] determined the energy calibration of an organic scintillator at a few MeV by fitting to the Compton edge of the pulse-height distribution, in the absence of the full-energy peak. Duvall et al [33] used an approach developed by Trombka and Schmadebeck [34] to find the relative peak intensities of a source with 3 γ -rays with energies between 6 and 7 MeV. Wender et al [10] fit response functions calculated with the CYLTRAN Monte Carlo system to the measured response of a BGO detector to γ -rays with energies from 6 to 40 MeV in order to determine Γ_G for their detector. The fitting approach that we used is described in appendix C. This approach differs from previous approaches in that Γ_G , the energy calibration, the background, and the line intensity can all be determined simultaneously.

B.2.4 Unfolding

A full set of response functions was calculated for our NaI detector for use in unfolding bremsstrahlung spectra in the energy range 0.2-35 MeV. The Monte Carlo simulation used a model which corresponded closely to the experimental arrangement used for the bremsstrahlung measurements, the details of which are reported elsewhere [1]. A polyenergetic source was used in the simulation instead of a monoenergetic source since this conformed more closely to the continuous nature of the bremsstrahlung beam.

The response matrix element r_{mj} was defined as the probability of an event

occurring in the m -th absorbed-energy bin given that a photon from the j -th photon-spectrum was incident on the detector and deposited an energy of E_o or greater in the NaI crystal. The j -th photon spectrum was a rectangular distribution which spanned the energy range $\mathcal{E}_{j-1}$ to $\mathcal{E}_j$. In terms of the differential response $r(E', \mathcal{E})$ (eq B.2):

$$r_{mj} = \int_{E'=E'_{m-1}}^{E'_m} \int_{\mathcal{E}=\mathcal{E}_{j-1}}^{\mathcal{E}_j} r(E', \mathcal{E}) d\mathcal{E} dE', \quad (\text{B.5})$$

where E'_m is the energy of the top of the m -th energy-deposition bin used in the Monte Carlo simulation.

The response matrix $\hat{R}_{ij}$ was composed of the response functions as calculated by Monte Carlo and broadened by $G(E, E')$:

$$\hat{R}_{ij} = \sum_m r_{mj} \int_{E_{i-1}}^{E_i} G(E, \frac{E'_{m-1} + E'_m}{2}) dE. \quad (\text{B.6})$$

This is a discrete form of eq B.2 where j refers to the incident energy, m refers to the energy deposited in the detector, and i refers to the pulse-height. The quantity E_i is the pulse-height of the top of the i -th bin. The width of the i -th bin was set equal to $\Gamma_G(E_{i-1})$, which was obtained from the fits of the measured γ -ray and electron spectra with the calculated response functions. These values of Γ_G were also used to obtain $G(E, E')$ from eq B.4. The incident photon spectra and the absorbed-energy distribution had the same bin-top energies as the pulse-height distribution, that is, $E_i = E'_m = \mathcal{E}_j$ for $i = m = j$. This was the highest resolution that could be achieved for the unfolded spectrum [35].

The measured spectra were unfolded using FERDO. The response matrix A_{ij} used for unfolding was composed of the broadened response functions, nor-

malized to unity above E_o , and divided by the widths of the pulse-height bins:

$$A_{ij} = \frac{R_{ij}}{E_i - E_{i-1}}, \quad R_{ij} = \hat{R}_{ij} / \sum_i \hat{R}_{ij}. \quad (\text{B.7})$$

The FERDO window functions, which limit the resolution of the unfolded spectrum [26], were Gaussian functions with FWHM of Γ_W . The window functions provided by the FERDO support software are calculated from

$$W_{kj} = \frac{2\sqrt{2}}{\sqrt{\pi}\Gamma_W(E_k)} \exp \left\{ -4 \ln 2 \frac{(E - \mathcal{E})^2}{\Gamma_W(E_k)^2} \right\}, \quad \mathcal{E} = \frac{\mathcal{E}_{j-1} + \mathcal{E}_j}{2}, \quad (\text{B.8})$$

where W_{kj} is the response of the k -th bin in the unfolded spectrum to the j -th photon spectrum. These window functions should not be used for high-resolution spectrometry where it is more appropriate to integrate the Gaussian. We utilized the following window function:

$$W_{kj} = \frac{2\sqrt{2}}{E_k - E_{k-1}} \int_{E_{k-1}}^{E_k} \frac{dE}{\sqrt{\pi}\Gamma_W(\mathcal{E})} \exp \left\{ -4 \ln 2 \frac{(E - \mathcal{E})^2}{\Gamma_W(\mathcal{E})^2} \right\}, \quad \mathcal{E} = \frac{\mathcal{E}_{j-1} + \mathcal{E}_j}{2}. \quad (\text{B.9})$$

This window function represents an ideal response function, that is, a Gaussian with a FWHM dependent only on the energy of the particle incident on the detector. The optimum approach for setting Γ_W is a subject of this study.

B.3 Results and discussion

B.3.1 Response functions

The function-fitting routine described in appendix C was used to determine the coefficients for the broadening and energy-calibration which gave the best-fit of the measured spectra to the calculated response functions. The measured spectra, except for the ^{60}Co spectrum, were all rebinned to bins with nominal

widths of 25 keV, assuming a linear energy-calibration based on the 4.439 MeV line of an Am-Be source and the offset from precision pulser measurements. The ^{60}Co spectrum was rebinned to bin-widths of 4 keV, based on the position of the two photopeaks in the spectrum and the pulser-determined offset. The offset parameter was fixed at zero during the function minimization since the fit was insensitive to this parameter and unrealistic values would result if it was allowed to vary (see appendix C). Since the background was subtracted from the measured response functions, the background coefficients were also fixed at zero during the function minimization. Results are shown in figure B.3 through B.9 and table B.2.

For the γ -ray spectra, there is good agreement between the measured spectra and the calculated response functions in the region of the full-energy peak (figures B.3 to B.7). The extra peak above the full-energy peak in the γ -ray spectra was reasonably accounted for by including the 9% increase in the sensitivity of the back-half of the detector in the model used in the Monte Carlo simulation. This peak is unresolved from the 1.332 MeV photopeak in the ^{60}Co spectrum where Γ_G/E is 6.2%. The spectrum shown in figure B.7 consists of a sharp 17.64 MeV line from the transition to the ground state in ^8Be and a broad line due to the transition to the broad state at 3 MeV. Two of the spectra had peaks which overlapped (figures B.4 and B.6). For these cases the broadening, energy-calibration, and scaling parameters were obtained as follows. The most prominent peak was fit first and the fitted response function was subtracted from the measured spectrum. The second peak was then fit and the two fitted response functions were added together for comparison to the measured spectrum.

Figure B.3: Comparison of measured spectrum (•) with calculated (broadened) response function (solid line): ^{60}Co . The measured spectrum was corrected for background. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.4: Comparison of measured spectrum ($\bullet$) with calculated (broadened) response function (solid line): 6.13 and 7.117 MeV γ -rays. The measured spectrum was corrected for background. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.5: Comparison of measured spectrum (•) with calculated (broadened) response function (solid line): 11.85 MeV γ -ray. The measured spectrum was corrected for background. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.6: Comparison of measured spectrum ($\bullet$) with calculated (broadened) response function (solid line): 13.83 and 18.27 MeV γ -rays. The measured spectrum was corrected for background. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.7: Comparison of measured spectrum (•) with calculated (broadened) response function (solid line): 17.64 MeV γ -ray. The measured spectrum was corrected for background. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.8: Comparison of measured spectrum (•) with calculated (broadened) response function (solid line): 10.0 MeV electrons. The measured spectrum was corrected for background and pulse pile-up. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Figure B.9: Comparison of measured spectrum (●) with calculated (broadened) response function (solid line): 30.0 MeV electrons. The measured spectrum was corrected for background and pulse pile-up. The calculated response function was Gaussian-broadened, rebinned, and scaled to obtain the best fit to the measured spectrum in the region indicated by arrows.

Table B.2: Results of the fit of the calculated response to the measured γ -ray and electron spectra. The electron spectra were corrected for pile-up. All of the spectra were background-subtracted and rebinned to nominal bin-widths of 25 keV (4 keV for ^{60}Co) prior to the fit. The '2-event pk' refers to the pile-up peak in the 10 and 15 MeV electron spectra, which occurs at 16 and 26 MeV, respectively. The energy E is the energy of the incident beam for γ -rays, and 2 MeV below the energy of the incident beam for electrons. The pulse-height per unit energy (P/E) is shown for each source relative to the 4.439 MeV line of an Am-Be source placed at the same SDD and in the same experimental geometry (table I). The Am-Be source was collimated to a diameter of 5 cm for measurements in geometry 'c' (electrons). The uncertainty in the last digit of P/E corresponds to ± 1 bin. The FWHM of the broadening functions (Γ_G) and the FWHM of the full-energy peaks in the measured spectra (Γ_R) are also shown.

E Energy (MeV)	Fitting Region (MeV)	Counts in Fit Region/ 10^4	P/E	Γ_G/E (%)	Γ_R/E (%)
<u>γ-rays</u>					
^{60}Co	1.06-1.46	4.21	—	6.4	6.4
6.130	4.01-6.51	5.96	0.994(4)	3.7	3.9
7.117	6.26-8.76	1.78	0.997(4)	4.3	4.0
11.85	10.0-12.5	1.59	0.998(2)	2.8	4.8
13.83	12.0-14.5	1.09	0.998(2)	2.7	4.8
17.64	16.0-18.5	3.53	0.997(1)	2.6	5.2
18.27	16.5-19.0	0.67	0.998(1)	2.3	5.2
<u>electrons</u>					
8	6.49-8.99	1.87	1.000(3)	3.4	9.3
13	11.8-14.2	1.74	1.007(2)	2.6	6.8
18	16.8-19.2	1.79	1.010(1)	2.3	5.9
23	22.0-24.9	1.56	1.009(1)	2.0	6.8
28	26.5-29.0	1.53	1.002(1)	1.7	6.7
<u>2-event pk</u>					
16	14.8-17.2	0.15	1.001(2)	2.4	9.1
26	25.0-27.5	0.14	1.004(1)	1.8	8.3

At energies below the full-energy peak in the measured γ -ray spectra there are more events than predicted by the calculation. For ^{60}Co , there are 20% more counts below 1.1 MeV, relative to the total counts in the spectrum. It was difficult to determine the number of excess counts in the other γ -ray spectra since the background rate was not accurately known. For example, the 6.13 MeV γ -ray spectrum shows anywhere from 0 to 20% more counts than the broadened response in the region from 3.5 to 5.9 MeV, depending on the amount of background subtracted. The discrepancy between measurement and calculation appears to persist up to the highest energies explored. Previous studies have shown good agreement between the measured and calculated response of NaI detectors to 6 to 60 MeV γ -rays measured without collimation [6,8]. This indicates that any excess counts are likely due to collimator-degraded photons. Our own preliminary Monte Carlo studies and the study of Amols et al [36] suggest that the actual number of collimator-degraded photons that would reach the detector is midway between the extremes estimated from the measurements.

For the electron spectra (figures B.8 and B.9), the agreement between the measured spectra and the broadened response functions is excellent down to 1 MeV with reduced chi-square values which range from 0.95 to 1.13 over the range of the fit. Only the 10 and 30 MeV spectra are shown in the figures. The 15, 20, and 25 MeV spectra compare equally well with the broadened response functions. This is an improvement on an earlier result reported by Berger and Seltzer for a 15 MeV electron beam incident on a 13 cm by 13 cm NaI detector [5] which shows an excess of counts in the measured spectrum below 10 MeV. The 9% discontinuity in the sensitivity of our detector caused a few extra counts to appear above the peak in the measured spectra. These were fully accounted for

by including the detector non-uniformity in the Monte Carlo simulations. The extra counts in the 10 and 30 MeV electron spectra below 1 MeV were present in all of the spectra. These were likely residual counts from the accelerator background.

B.3.2 FWHM of the broadening function

The FWHM of the broadening function, determined from the fit of the measured response functions to the calculated response functions, is shown in table B.2. The 2-event (pile-up) peaks in the 10 and 15 MeV electron spectra were used to obtain additional points. In this case, the calculated response functions were convolved with themselves before they were fit to the 2-event peaks in the measured spectra. The γ -ray and electron results agree quite well considering that a γ -ray and electron of the same energy deposit their energy in different regions of the detector. The sharp discontinuity in the sensitivity of our detector had little effect on Γ_G . We have recently had the detector repaired. The sensitivity of the repaired detector changes gradually by 3% over the length of the crystal. In this case it was found that Γ_G for electrons could be equated to Γ_G for γ -rays only if the non-uniformity was accurately simulated in the Monte Carlo calculation. The high value of Γ_G obtained from the fit for the 7.117 MeV γ -ray is attributed to difficulty in fitting due to interference from the nearby 6.13 MeV peak and inadequate background subtraction. A value of Γ_G/E of 3.6% for the 7.117 MeV line also gives an acceptable fit.

The power law

$$\Gamma_G = 0.077 E^{0.60}, \quad (\text{B.10})$$

with E in MeV, fits the data to $\pm 10\%$ throughout the energy range 0.3 MeV to

30 MeV. Data below 1 MeV was obtained by fitting a Gaussian to the photopeaks due to the γ -rays of ^{67}Ga , ^{137}Cs , and ^{22}Na . The radioisotope measurements were done in the experimental arrangement of figure B.1b at SDD's of 30, 22, and 10 cm respectively. The exponent of 0.60 in eq B.10 compares favourably to measurements in the literature for much smaller crystals [5].

The FWHM of the full-energy peaks in the measured spectra, Γ_R , is compared to the FWHM of the broadening function, Γ_G , in table B.2. A significant difference is apparent at γ -ray energies above 10 MeV and for all of the electron spectra. For the γ -rays, the single-escape peak is unresolved from the full-energy peak and contributes significantly to Γ_R . For the electrons, energy-loss straggling is largely responsible for the difference between the FWHM of the measured response functions and Γ_G . Due to these effects, a simple fit of a Gaussian to the spectra measured in this study was insufficient for determining Γ_G from the γ -ray spectra for energies above 10 MeV, and from any of the electron spectra.

B.3.3 Energy calibration

The energy calibration is shown in table B.2 in terms of the pulse-height per unit energy,

$$\frac{P}{E} = \frac{N - N_o}{E} \bigg/ \frac{N_{Am-Be} - N_o}{E_{Am-Be}}. \quad (\text{B.11})$$

The channel number N corresponds to a given energy-deposition of E . The channel offset N_o was determined from precision pulser measurements. The channel number N_{Am-Be} corresponds to an energy E_{Am-Be} of 4.439 MeV, a prominent line from an Am-Be source for which P/E was defined as unity.

A low-energy calibration performed with the radioisotopes ^{67}Ga , ^{137}Cs , ^{22}Na ,

and ^{60}Co showed the same 10% variation of P/E from 0.2 to 2 MeV reported by others (see, e.g., [13]). The calibration above 4 MeV was determined from the fits of the measured spectra with the calculated response functions as shown in appendix C. The results are listed in table B.2. The value of P/E is not shown for the ^{60}Co spectrum since a higher amplifier gain was used in order to bring the signal into the linear region of the electronics. The pulse-height varies linearly with energy to within $\pm 1\%$ in the range 4 to 30 MeV, which is comparable to other results in the literature [17,20,21]. The electronic linearity was better than $\pm 0.1\%$ in this region as determined from precision-pulser measurements. The accuracy of the fit was limited to ± 1 bin (see appendix C). This is the uncertainty quoted in table B.2. The uncertainties calculated by the function fitting routine were negligible in comparison to this. The quadratic trend in the electron results is indicative of a systematic error such as an error in the MgO thickness or in the calculated response functions.

B.3.4 Unfolding

An example of using FERDO to unfold a typical bremsstrahlung spectrum measured with our NaI detector is shown in figure B.10. The bremsstrahlung beam was generated by 30 MeV electrons incident on a 16 g/cm^2 Al target. The spectrum measured for this bremsstrahlung beam was corrected for pulse pile-up and the background was subtracted. The resulting spectrum, with 121 bins containing 150,000 counts, was used to represent $\epsilon(\mathcal{E})\phi(\mathcal{E})$ (eq B.1). This 'test' spectrum, was folded with the matrix of broadened response functions that was calculated for our NaI detector. The counts in each bin of the folded spectrum were randomized according to a Gaussian distribution with a standard deviation

Figure B.10: Example of unfolding using the FERDO unfolding routine. A 30 MV bremsstrahlung spectrum measured with our NaI detector was folded with the response matrix for our detector, randomized into 25 different spectra, and unfolded with the same response matrix. The means of the counts in each bin of the 25 randomized spectra (upper lines) are shown before (solid line) and after (dashed line) the spectra were unfolded. The spectral uncertainties before unfolding (solid line) are due entirely to counting statistics. The uncertainties of the unfolded spectrum that were estimated by FERDO are shown for two cases in which different widths were used for the window functions: Γ_R , the FWHM of the broadened response-functions (dashed line), and Γ_G , the FWHM of the Gaussian broadening-functions (dotted line). The uncertainty in the unfolded spectrum due to a 20% uncertainty in the peak-to-total ratio of the response functions is also shown (+).

equal to the square root of the counts in the bin. A total of 25 randomized spectra were generated. These randomized spectra represented a set of measured spectra, $b(E)$, that differed only by counting statistics. Each randomized spectrum was unfolded with the same response matrix used for folding the test spectrum.

The means of the randomized spectra before and after unfolding is shown in figure B.10 along with the standard deviations of the counts in each bin. The number of counts in the spectrum was unchanged after unfolding, as was expected, since the response functions were all normalized to unity. The object of unfolding is to move those events which involved some leakage of energy from the detector into the bins corresponding to total energy-absorption. Thus, unfolding reduces the number of events at the low-energy end of the spectrum and increases the number of events at the high-energy end of the spectrum.

One of the unfolded spectra was chosen arbitrarily for analysis of the spectral uncertainties that were provided by FERDO. The counts in each bin of this spectrum agreed with those of the test spectrum before folding within the uncertainty estimated by FERDO for 65% of the bins, which is in reasonable agreement with the expected result of 68% [26]. Furthermore, the uncertainty estimates of FERDO compared well to the standard deviations of the counts in each bin of the set of 25 unfolded spectra. Thus, if the response matrix is known exactly, the unfolded spectrum provided by FERDO agrees with the efficiency-weighted incident spectrum to within the spectral uncertainties estimated by FERDO.

When the widths of the window functions (Γ_W) match the widths of the response functions (Γ_R), the spectral uncertainties are barely changed by the

unfolding process (figure B.10). If, on the other hand, the widths of the window functions match the much smaller widths of the Gaussian broadening-function (Γ_G , see table B.2), the unfolding process contributes significantly to the overall spectral uncertainty. For example, for the 30 MV bremsstrahlung spectrum shown in figure B.10 with $\Gamma_W = \Gamma_G$ the unfolding process increased the spectral uncertainties by a factor of 1.1 at the lower energies and by a factor of 5 near the tip of the spectrum.

The spectral resolution is limited by the widths of the window functions. Window functions with widths larger than Γ_R improve the spectral uncertainties, but only to the extent that they would be improved by rebinning the spectrum to match the reduced resolution. Thus there is little to be gained by increasing Γ_W beyond Γ_R . The optimum FWHM for the window functions is Γ_R since this gives the maximum resolution for the unfolded spectrum with a minimal contribution of the unfolding process to the spectral uncertainties.

An upper bound on the uncertainty in the unfolded spectrum due to possible systematic errors in the response matrix was determined as follows. A response matrix was constructed from a set of modified response functions with 20% of the total counts redistributed evenly below the photopeak. The modified response represented the extreme of the possible response since fewer excess low-energy events were expected in the experimental arrangement used for the bremsstrahlung measurements than the 20% reported here for the Van de Graaff γ -rays. This was because the bremsstrahlung measurements were performed with a thicker collimator (20 cm) of smaller radius (1.2 cm) than the collimator used for the Van de Graaff measurements and the collimator was placed much further from the source (300 cm). The test spectrum was folded with the

modified response matrix. The resulting spectrum, which was not randomized, was unfolded with the unmodified response matrix. The absolute values of the differences between the unfolded spectrum and the test spectrum before folding is shown in figure B.10. These differences are an upper bound on the uncertainty due to possible errors in the response matrix used for unfolding. At energies above 1 MeV, the differences were competitive with the standard deviations of the unfolded spectrum. The response function estimates were therefore adequate for unfolding the test spectrum.

B.4 Summary

Response functions were calculated for a large NaI detector using the Monte Carlo routine EGS4 for γ -rays and electrons with energies ranging from 1 to 30 MeV. A fitting procedure was adopted to fit the calculated response to measured γ -ray and electron spectra. The fitting procedure worked effectively in extracting the pulse-height-to-energy conversion for the measured spectra and the FWHM of the Gaussian broadening-functions which were used to broaden the calculated response. The measured γ -ray spectra were in reasonable agreement with the broadened response functions in the region of the fit. There was evidence that collimator-degraded photons contributed to the spectra. The electron spectra, which were measured without collimation, fitted the broadened response functions very well down to 1 MeV.

The unfolding routine FERDO was tested for its ability to unfold a 30 MV bremsstrahlung spectrum. The unfolded spectrum agreed with the efficiency-weighted spectrum of incident photons to within the uncertainty estimates pro-

vided by FERDO, which were accurate estimates of the standard deviations of the counts in each bin of the unfolded spectrum. The optimum FWHM of the window function used by FERDO was Γ_R , the width of the broadened response functions.

The response matrix used for unfolding was constructed from a set of response functions calculated using the EGS4 Monte Carlo system and subsequently Gaussian-broadened. Collimator effects were ignored in the Monte Carlo simulations. This approach provided a response matrix which was perfectly adequate for unfolding the bremsstrahlung spectra measured with our NaI detector.

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Appendix C

FITMC: A routine for fitting calculated and measured spectral distributions

This appendix has been accepted for publication in Nuclear Instruments and Methods A as the appendix of the paper entitled "*Precision response function estimation,...*" which comprises appendix B of this thesis.

C.1 FITMC

The routine FITMC was used for fitting calculated response functions to measured pulse-height distributions. Details of the fitting approach are presented here.

Let y_i be the number of counts in the i -th bin of a measured pulse-height distribution. Let z_i be an estimate of y_i . The routine FITMC utilizes the function-minimization approach of Marquadt [1] as implemented in our labora-

tory [2] to minimize the quantity

$$\mathcal{R} = \sum_i (z_i - y_i)^2. \quad (\text{C.1})$$

A set of seven free parameters, $\{\beta_k; k = 1, 2, \dots, 7\}$ are provided in FITMC. The parameter β_1 defines Γ_G , β_2 and β_3 the slope and offset of the energy calibration, β_4 , β_5 , and β_6 the coefficients of a second-order background function, and β_7 the line intensity. The fitting function,

$$z_i = \beta_7 f_i(\beta_1, \beta_2, \beta_3) + \beta_4 + \beta_5 i + \beta_6 i^2, \quad (\text{C.2})$$

where $f_i(\beta_1, \beta_2, \beta_3)$ is the Gaussian-broadened, Monte Carlo calculated response function, rebinned into pulse-height bins. The pulse height is assumed to be proportional to the energy deposited in the detector in the region of the fit, that is,

$$E_i = \beta_2(i - \beta_3), \quad (\text{C.3})$$

where E_i is the energy of the centre of the i -th bin. The relationship between the pulse-height-per-unit-energy and the fitting parameters, obtained from eq B.11 with $N = i$ and substituting for i from eq C.3, is

$$P/E = \left[\frac{1}{\beta_2} + \frac{\beta_3 - N_o}{E} \right] \frac{E_{Am-Be}}{N_{Am-Be} - N_o}. \quad (\text{C.4})$$

In practice the spectra were rebinned to zero offset before fitting ($N_o = 0$) and the offset parameter β_3 was fixed at zero during the fit to avoid unrealistic values. In this case, the pulse-height-per-unit-energy is related to the fitting parameters by

$$P/E = (\beta_2)_{Am-Be} / (\beta_2)_E. \quad (\text{C.5})$$

The function $f_i(\beta_1, \beta_2, \beta_3)$ was obtained from a discrete form of eq B.2:

$$f_i(\beta_1, \beta_2, \beta_3) = \sum_{m=m_1}^{m_2} G_{im} \tilde{r}_m, \quad (\text{C.6})$$

where G_{im} is the broadening-function, centered on bin m , and integrated over the i -th pulse-height bin, and $\tilde{r}_m$ is the response function calculated by Monte Carlo for a given source, rebinned to correspond to the m -th pulse-height bin of the measured spectrum. The bounds of the summation, m_1 and m_2 , were chosen to limit the broadening of the calculated response function to bins within ± 1.3 times the FWHM of the broadening function. This reduces computing time while insuring that the calculated response is adequately broadened.

The broadening-function (eq B.4) was converted to the discrete form G_{im} by

$$G_{im} = \int_{\beta_2(i-\frac{1}{2}-\beta_3)}^{\beta_2(i+\frac{1}{2}-\beta_3)} G(E, E'_m) dE. \quad (C.7)$$

In terms of the error function,

$$G_{im} = \frac{1}{2} |\text{erf}(\omega_1) + \lambda_1 \text{erf}(\omega_2)|, \quad \lambda_1 = \begin{cases} +1 & ; m = i \\ -1 & ; m \neq i \end{cases}, \quad (C.8)$$

where

$$\text{erf}(\omega) = \frac{1}{\sqrt{\pi}} \int_{-\omega}^{+\omega} e^{-\omega^2} d\omega, \quad (C.9)$$

$$\omega_1 = \frac{2\sqrt{\ln 2} \beta_2}{\beta_1 E_m^n} |i - m + \frac{1}{2}|, \quad (C.10)$$

$$\omega_2 = \frac{2\sqrt{\ln 2} \beta_2}{\beta_1 E_m^n} |i - m - \frac{1}{2}|. \quad (C.11)$$

$$(C.12)$$

By definition, the FWHM of the broadening-function is related to the fitting-parameter β_1 by

$$\Gamma_G = \beta_1 E^n, \quad (C.13)$$

where n is a constant. This power-law relationship is common in the literature. We used $n = 0.60$, which is consistent with our own measurements (see eq B.10)

and those of others (e.g., [3]). This approach to response-function broadening is suitable if either the response function or the broadening function is reasonably constant over each pulse-height bin. The latter requirement is satisfied if the FWHM of the broadening function is greater than a few bin widths.

The response calculated by Monte Carlo, r_k , is rebinned to $\tilde{r}_m$ (eq C.6) by assuming that the calculated response is constant over each calculation bin. The bin widths used for the Monte Carlo calculation need not be constant and are best chosen to give high resolution where the response function varies rapidly. The precision of the energy calibration is roughly limited to the width of a pulse-height bin since the broadening is done after the rebinning.

The routine used for function minimization required the partial derivatives of z_i with respect to the free parameters, $\{\beta_k; k = 1, 2, \dots, 7\}$. The non-trivial partial derivatives are listed here:

$$\frac{\partial z_i}{\partial \beta_1} = -\frac{\beta_7}{\sqrt{\pi}\beta_1} \sum_{m=m_1}^{m_2} \lambda_2 \tilde{r}_m (\lambda_1 \omega_1 e^{-\omega_1^2} + \omega_2 e^{-\omega_2^2}),$$

$$\lambda_2 = \begin{cases} +1 & ; \omega_2 > \omega_1 \\ -1 & ; \omega_2 \leq \omega_1 \end{cases}, \quad (C.14)$$

$$\frac{\partial z_i}{\partial \beta_2} = \frac{\beta_7}{2} \sum_{m=m_1}^{m_2} \left\{ |\operatorname{erf}(\omega_2) + \lambda_1 \operatorname{erf}(\omega_1)| \left[\frac{r_{k_2}(m - \beta_3 + \frac{1}{2})}{E_{k_2}^{MC} - E_{k_2-1}^{MC}} - \frac{r_{k_1}(m - \beta_3 - \frac{1}{2})}{E_{k_1}^{MC} - E_{k_1-1}^{MC}} \right] \right. \quad (C.15)$$

$$\left. + \frac{4\sqrt{\ln 2} \tilde{r}_m}{\sqrt{\pi}\beta_1 E_m^n} \left[e^{-\omega_2^2} |i - m - \frac{1}{2}| + \lambda_1 e^{-\omega_1^2} |i - m + \frac{1}{2}| \right] \right\}, \quad (C.16)$$

$$\frac{\partial z_i}{\partial \beta_3} = \frac{\beta_2 \beta_7}{2} \sum_{m=m_1}^{m_2} |\operatorname{erf}(\omega_2) + \lambda_1 \operatorname{erf}(\omega_1)| \left[\frac{r_{k_1}}{E_{k_1}^{MC} - E_{k_1-1}^{MC}} - \frac{r_{k_2}}{E_{k_2}^{MC} - E_{k_2-1}^{MC}} \right], \quad (C.17)$$

where ω_1 , ω_2 , and λ_1 have been previously defined, E_k^{MC} is the energy of the top of the k -th bin used in the Monte Carlo simulation, and k_1 and k_2 are the calculation bins which encompass the i -th bin of the measured spectrum. The

partial derivatives $\partial z/\partial\beta_2$ and $\partial z/\partial\beta_3$ were calculated assuming that Γ_G varied slowly with E and ignoring the abrupt change of k_1 and k_2 by ± 1 when the edge of a pulse-height bin crosses the edge of a calculated response-function bin.

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Appendix D

Tables of bremsstrahlung yield measurements

The measured bremsstrahlung yields are tabulated here in units of $(\text{MeV}\cdot\text{sr})^{-1}$. The values are absolute in the sense of number of photons which emerge from the target into a given solid angle per electron incident on the target. The relative uncertainties of the measurements are reported. This includes counting statistics in the raw spectrum, random uncertainties in the unfolding of the detector response, and statistical uncertainties in the calculated collimator effect. Systematic uncertainties in the spectral measurements are not included, nor are uncertainties in the measurements of the beam current. Together these uncertainties contribute less than one standard deviation to the tabulated uncertainties. The forward-directed bremsstrahlung yields from 10 to 30 MeV electrons incident on Al targets (table D.1) and Pb targets (table D.2) are discussed in chapter 2. The angular bremsstrahlung yields from 15 MeV electrons incident on a Be target (table D.3), an Al target (table D.4), and a Pb target (table D.5) are discussed in chapter 3. These last three tables are an appendix to the paper “*Angular distribution of bremsstrahlung...*” which is to be published

in *Medical Physics* 17(5), 1990, and constitutes chapter 3 of this thesis.

Table D.1: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Al targets. The target thicknesses were nominally 110% of the CSDA electron range. The units are $(\text{MeV}\cdot\text{sr})^{-1}$. Only the relative uncertainties are shown. The energy column is the energy at the top of each bin in the spectral distribution.

Energy	10	15	20	25	30
0.20					
0.25	1510(25)E-3	269(5)E-2	396(9)E-2	449(13)E-2	590(22)E-2
0.30	1249(25)E-3	214(5)E-2	330(9)E-2	376(14)E-2	502(22)E-2
0.35	1058(16)E-3	181(4)E-2	285(7)E-2	328(11)E-2	452(18)E-2
0.40	999(17)E-3	176(5)E-2	267(12)E-2	306(22)E-2	40(4)E-1
0.45	929(17)E-3	166(5)E-2	255(12)E-2	297(22)E-2	39(4)E-1
0.50	855(18)E-3	152(5)E-2	245(13)E-2	279(23)E-2	39(4)E-1
0.55	753(15)E-3	141(5)E-2	226(12)E-2	258(22)E-2	36(4)E-1
0.60	677(17)E-3	131(5)E-2	206(12)E-2	239(22)E-2	33(4)E-1
0.66	663(17)E-3	127(4)E-2	207(8)E-2	252(13)E-2	373(14)E-2
0.72	635(14)E-3	113(3)E-2	186(7)E-2	234(12)E-2	335(12)E-2
0.78	560(13)E-3	100(3)E-2	164(7)E-2	208(12)E-2	302(12)E-2
0.85	544(13)E-3	1036(30)E-3	167(7)E-2	212(11)E-2	309(11)E-2
0.92	496(12)E-3	99(3)E-2	150(7)E-2	199(9)E-2	303(15)E-2
0.99	436(11)E-3	881(28)E-3	135(7)E-2	181(9)E-2	264(15)E-2
1.07	422(11)E-3	862(28)E-3	140(7)E-2	183(8)E-2	251(14)E-2
1.15	388(10)E-3	761(25)E-3	124(5)E-2	164(10)E-2	229(13)E-2
1.24	404(10)E-3	793(25)E-3	128(5)E-2	163(10)E-2	239(12)E-2
1.32	333(9)E-3	665(24)E-3	107(5)E-2	141(11)E-2	201(13)E-2
1.41	315(9)E-3	628(21)E-3	104(4)E-2	142(8)E-2	196(13)E-2
1.51	308(8)E-3	647(20)E-3	104(4)E-2	139(6)E-2	194(13)E-2
1.61	285(8)E-3	594(19)E-3	99(4)E-2	136(6)E-2	196(13)E-2
1.71	259(7)E-3	555(16)E-3	91(4)E-2	114(6)E-2	170(11)E-2
1.82	256(7)E-3	548(16)E-3	87(4)E-2	117(5)E-2	162(11)E-2
1.93	233(8)E-3	508(18)E-3	84(4)E-2	122(6)E-2	168(9)E-2
2.04	203(8)E-3	430(17)E-3	79(3)E-2	113(6)E-2	155(8)E-2
2.16	194(7)E-3	426(17)E-3	79(3)E-2	103(5)E-2	148(8)E-2
2.28	183(6)E-3	406(17)E-3	692(28)E-3	96(5)E-2	134(7)E-2
2.41	167(6)E-3	403(15)E-3	70(3)E-2	98(4)E-2	139(7)E-2
2.54	161(6)E-3	400(13)E-3	67(4)E-2	95(4)E-2	132(8)E-2
2.67	152(6)E-3	355(15)E-3	60(3)E-2	89(4)E-2	128(7)E-2
2.81	151(6)E-3	330(16)E-3	587(28)E-3	87(4)E-2	122(5)E-2
2.95	132(5)E-3	307(19)E-3	535(25)E-3	74(5)E-2	106(7)E-2
3.10	126(5)E-3	296(19)E-3	533(23)E-3	69(5)E-2	101(8)E-2
3.25	119(6)E-3	280(11)E-3	482(23)E-3	73(3)E-2	104(7)E-2
3.41	107(5)E-3	274(11)E-3	449(23)E-3	736(30)E-3	105(6)E-2
3.57	95(4)E-3	276(10)E-3	483(23)E-3	688(30)E-3	95(6)E-2
3.73	98(4)E-3	234(9)E-3	464(21)E-3	66(4)E-2	90(5)E-2

Table D.1: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Al targets. Continued from previous page.

Energy	10	15	20	25	30
3.90	94(4)E-3	217(10)E-3	418(20)E-3	58(4)E-2	90(4)E-2
4.08	91(5)E-3	211(12)E-3	408(20)E-3	57(4)E-2	96(4)E-2
4.26	82(4)E-3	209(11)E-3	412(19)E-3	570(30)E-3	82(6)E-2
4.44	73(4)E-3	192(9)E-3	391(19)E-3	576(28)E-3	66(6)E-2
4.63	66(4)E-3	194(9)E-3	330(18)E-3	526(27)E-3	71(6)E-2
4.82	64(4)E-3	177(9)E-3	333(18)E-3	491(25)E-3	79(5)E-2
5.02	62(3)E-3	174(8)E-3	319(18)E-3	486(25)E-3	78(5)E-2
5.22	60(3)E-3	152(9)E-3	296(17)E-3	483(27)E-3	65(4)E-2
5.43	52(3)E-3	146(9)E-3	299(17)E-3	458(28)E-3	59(4)E-2
5.64	461(30)E-4	143(8)E-3	265(16)E-3	418(27)E-3	60(4)E-2
5.86	473(27)E-4	142(8)E-3	281(16)E-3	393(24)E-3	66(4)E-2
6.08	467(28)E-4	133(8)E-3	247(15)E-3	396(24)E-3	63(3)E-2
6.31	433(25)E-4	125(7)E-3	242(14)E-3	364(20)E-3	600(30)E-3
6.54	381(23)E-4	110(7)E-3	233(13)E-3	364(20)E-3	527(28)E-3
6.78	349(23)E-4	115(7)E-3	216(13)E-3	366(19)E-3	533(28)E-3
7.02	308(21)E-4	109(7)E-3	199(12)E-3	355(19)E-3	497(26)E-3
7.27	307(21)E-4	107(7)E-3	184(11)E-3	300(19)E-3	486(28)E-3
7.53	298(20)E-4	96(6)E-3	180(11)E-3	293(19)E-3	481(28)E-3
7.78	245(18)E-4	78(6)E-3	184(11)E-3	296(18)E-3	421(26)E-3
8.05	217(17)E-4	75(6)E-3	188(11)E-3	297(18)E-3	396(26)E-3
8.32	190(16)E-4	86(5)E-3	167(11)E-3	264(21)E-3	399(25)E-3
8.59	160(14)E-4	80(5)E-3	165(10)E-3	245(20)E-3	405(25)E-3
8.87	135(13)E-4	76(4)E-3	151(10)E-3	227(20)E-3	389(23)E-3
9.16	117(11)E-4	68(4)E-3	122(9)E-3	208(17)E-3	382(23)E-3
9.45	80(9)E-4	64(4)E-3	124(8)E-3	235(15)E-3	357(22)E-3
9.74	33(6)E-4	56(4)E-3	150(8)E-3	254(15)E-3	330(21)E-3
10.04	13(5)E-4	56(4)E-3	141(8)E-3	229(14)E-3	333(21)E-3
10.35	3(3)E-4	60(3)E-3	123(9)E-3	196(13)E-3	310(20)E-3
10.66	23(26)E-5	51(3)E-3	109(8)E-3	173(12)E-3	304(20)E-3
10.98	0(15)E-5	421(29)E-4	98(8)E-3	193(12)E-3	291(19)E-3
11.31	27(16)E-5	500(30)E-4	113(7)E-3	191(10)E-3	295(19)E-3
11.64	37(14)E-5	389(26)E-4	111(7)E-3	164(9)E-3	243(18)E-3
11.97	30(13)E-5	285(23)E-4	98(7)E-3	149(9)E-3	227(17)E-3
12.31	9(7)E-5	276(26)E-4	91(6)E-3	150(8)E-3	252(16)E-3
12.66	15(12)E-5	314(26)E-4	89(6)E-3	159(8)E-3	255(16)E-3
13.01	11(7)E-5	308(24)E-4	77(5)E-3	159(8)E-3	229(15)E-3
13.37	0(0)E-3	272(17)E-4	73(5)E-3	139(7)E-3	222(15)E-3
13.74	0(0)E-3	203(15)E-4	68(5)E-3	126(7)E-3	211(14)E-3
14.11	0(0)E-3	113(12)E-4	75(5)E-3	116(7)E-3	202(14)E-3
14.49	0(0)E-3	69(10)E-4	68(4)E-3	116(6)E-3	189(12)E-3

Table D.1: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Al targets. Continued from previous page.

Energy	10	15	20	25	30
14.87	0(0)E-3	38(7)E-4	64(4)E-3	118(6)E-3	171(11)E-3
15.26	0(0)E-3	21(6)E-4	64(4)E-3	109(6)E-3	170(12)E-3
15.65	0(0)E-3	17(5)E-4	59(3)E-3	104(6)E-3	172(11)E-3
16.05	0(0)E-3	6(3)E-4	53(3)E-3	102(5)E-3	176(11)E-3
16.46	0(0)E-3	66(28)E-5	46(4)E-3	90(5)E-3	167(9)E-3
16.87	0(0)E-3	91(25)E-5	42(3)E-3	87(5)E-3	158(8)E-3
17.29	0(0)E-3	35(21)E-5	39(3)E-3	88(5)E-3	152(8)E-3
17.72	0(0)E-3	-1(14)E-5	35(3)E-3	82(5)E-3	142(8)E-3
18.15	0(0)E-3	0(0)E-3	307(27)E-4	82(4)E-3	139(8)E-3
18.59	0(0)E-3	0(0)E-3	256(18)E-4	78(4)E-3	131(7)E-3
19.03	0(0)E-3	0(0)E-3	201(16)E-4	70(4)E-3	130(7)E-3
19.49	0(0)E-3	0(0)E-3	153(14)E-4	69(4)E-3	129(7)E-3
19.94	0(0)E-3	0(0)E-3	102(11)E-4	65(4)E-3	121(6)E-3
20.41	0(0)E-3	0(0)E-3	38(8)E-4	60(4)E-3	121(7)E-3
20.88	0(0)E-3	0(0)E-3	8(6)E-4	56(3)E-3	105(6)E-3
21.35	0(0)E-3	0(0)E-3	8(5)E-4	50(3)E-3	89(6)E-3
21.84	0(0)E-3	0(0)E-3	10(4)E-4	415(28)E-4	94(6)E-3
22.33	0(0)E-3	0(0)E-3	13(4)E-4	400(27)E-4	100(6)E-3
22.82	0(0)E-3	0(0)E-3	5(3)E-4	405(25)E-4	89(5)E-3
23.33	0(0)E-3	0(0)E-3	-2(4)E-4	358(23)E-4	81(5)E-3
23.84	0(0)E-3	0(0)E-3	-2(3)E-4	282(20)E-4	88(5)E-3
24.35	0(0)E-3	0(0)E-3	0(0)E-3	173(16)E-4	90(5)E-3
24.87	0(0)E-3	0(0)E-3	0(0)E-3	78(12)E-4	83(5)E-3
25.40	0(0)E-3	0(0)E-3	0(0)E-3	44(10)E-4	74(4)E-3
25.94	0(0)E-3	0(0)E-3	0(0)E-3	29(8)E-4	66(4)E-3
26.48	0(0)E-3	0(0)E-3	0(0)E-3	12(6)E-4	62(4)E-3
27.03	0(0)E-3	0(0)E-3	0(0)E-3	12(4)E-4	55(3)E-3
27.59	0(0)E-3	0(0)E-3	0(0)E-3	21(5)E-4	447(29)E-4
28.15	0(0)E-3	0(0)E-3	0(0)E-3	17(4)E-4	315(23)E-4
28.72	0(0)E-3	0(0)E-3	0(0)E-3	3(5)E-4	206(18)E-4
29.30	0(0)E-3	0(0)E-3	0(0)E-3	-8(8)E-4	128(15)E-4
29.88	0(0)E-3	0(0)E-3	0(0)E-3	-10(8)E-4	64(11)E-4
30.48	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	13(7)E-4
31.07	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	-16(5)E-4
31.68	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	-23(5)E-4

Table D.2: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Pb targets. The target thicknesses were nominally 110% of the CSDA electron range. The units are $(\text{MeV}\cdot\text{sr})^{-1}$. Only the relative uncertainties are shown. The energy column is the energy at the top of each bin in the spectral distribution.

Energy	10	15	20	25	30
0.20					
0.25	190(14)E-3	20(3)E-2	244(26)E-3	34(8)E-2	38(15)E-2
0.30	300(16)E-3	25(4)E-2	28(3)E-2	42(8)E-2	36(15)E-2
0.35	422(12)E-3	46(3)E-2	503(20)E-3	76(7)E-2	60(14)E-2
0.40	496(26)E-3	670(30)E-3	69(7)E-2	9(3)E-1	89(20)E-2
0.45	565(26)E-3	88(3)E-2	102(7)E-2	12(3)E-1	142(20)E-2
0.50	647(27)E-3	105(4)E-2	135(8)E-2	16(3)E-1	202(21)E-2
0.55	657(26)E-3	114(3)E-2	152(8)E-2	20(3)E-1	242(20)E-2
0.60	629(27)E-3	113(3)E-2	150(8)E-2	21(3)E-1	251(21)E-2
0.66	666(18)E-3	118(4)E-2	161(6)E-2	249(11)E-2	284(26)E-2
0.72	612(15)E-3	110(4)E-2	156(6)E-2	248(10)E-2	267(26)E-2
0.78	546(15)E-3	103(4)E-2	150(6)E-2	236(10)E-2	265(26)E-2
0.85	559(14)E-3	108(4)E-2	162(5)E-2	259(10)E-2	296(24)E-2
0.92	554(13)E-3	106(3)E-2	153(6)E-2	226(15)E-2	271(20)E-2
0.99	489(12)E-3	949(30)E-3	143(5)E-2	206(14)E-2	266(19)E-2
1.07	475(12)E-3	912(29)E-3	149(5)E-2	209(14)E-2	283(19)E-2
1.15	430(11)E-3	856(24)E-3	134(5)E-2	205(10)E-2	254(19)E-2
1.24	445(12)E-3	914(23)E-3	140(5)E-2	230(8)E-2	273(19)E-2
1.32	383(11)E-3	802(21)E-3	121(5)E-2	200(8)E-2	233(20)E-2
1.41	356(9)E-3	722(22)E-3	120(4)E-2	191(6)E-2	237(13)E-2
1.51	343(9)E-3	703(22)E-3	118(4)E-2	185(6)E-2	240(9)E-2
1.61	326(9)E-3	693(22)E-3	111(4)E-2	168(6)E-2	228(9)E-2
1.71	295(9)E-3	602(24)E-3	104(4)E-2	155(9)E-2	206(9)E-2
1.82	297(9)E-3	583(23)E-3	101(4)E-2	159(9)E-2	210(9)E-2
1.93	269(9)E-3	561(24)E-3	92(5)E-2	154(6)E-2	202(8)E-2
2.04	242(8)E-3	491(24)E-3	82(5)E-2	143(5)E-2	185(8)E-2
2.16	230(8)E-3	486(21)E-3	80(4)E-2	132(5)E-2	175(8)E-2
2.28	197(7)E-3	458(18)E-3	74(4)E-2	114(4)E-2	151(8)E-2
2.41	189(6)E-3	445(16)E-3	71(3)E-2	114(4)E-2	153(8)E-2
2.54	187(6)E-3	425(14)E-3	695(28)E-3	114(4)E-2	146(8)E-2
2.67	172(6)E-3	381(13)E-3	646(28)E-3	102(4)E-2	139(7)E-2
2.81	170(6)E-3	386(13)E-3	628(27)E-3	98(4)E-2	139(6)E-2
2.95	153(5)E-3	347(14)E-3	560(27)E-3	91(4)E-2	128(5)E-2
3.10	144(5)E-3	315(14)E-3	535(26)E-3	87(4)E-2	132(4)E-2
3.25	127(5)E-3	316(11)E-3	544(22)E-3	85(3)E-2	120(5)E-2
3.41	118(6)E-3	286(11)E-3	506(21)E-3	85(3)E-2	105(5)E-2
3.57	110(6)E-3	278(11)E-3	477(21)E-3	73(3)E-2	106(5)E-2
3.73	106(5)E-3	261(11)E-3	436(20)E-3	66(4)E-2	103(4)E-2

Table D.2: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Pb targets. Continued from previous page.

Energy	10	15	20	25	30
3.90	100(4)E-3	237(10)E-3	418(19)E-3	64(4)E-2	98(3)E-2
4.08	98(4)E-3	221(10)E-3	413(19)E-3	65(4)E-2	93(3)E-2
4.26	90(4)E-3	224(9)E-3	356(19)E-3	67(4)E-2	84(4)E-2
4.44	76(3)E-3	207(11)E-3	317(19)E-3	61(4)E-2	79(4)E-2
4.63	80(3)E-3	185(12)E-3	322(19)E-3	53(4)E-2	81(4)E-2
4.82	76(4)E-3	185(9)E-3	310(20)E-3	49(3)E-2	75(4)E-2
5.02	69(4)E-3	183(8)E-3	316(20)E-3	54(3)E-2	63(4)E-2
5.22	57(3)E-3	164(8)E-3	278(17)E-3	502(28)E-3	63(3)E-2
5.43	58(3)E-3	143(8)E-3	266(14)E-3	451(25)E-3	645(28)E-3
5.64	588(30)E-4	137(7)E-3	277(14)E-3	442(24)E-3	608(29)E-3
5.86	498(26)E-4	143(6)E-3	267(13)E-3	432(19)E-3	52(3)E-2
6.08	435(24)E-4	145(6)E-3	238(12)E-3	393(19)E-3	51(3)E-2
6.31	40(3)E-3	121(7)E-3	221(12)E-3	358(21)E-3	542(28)E-3
6.54	394(28)E-4	100(7)E-3	203(12)E-3	326(21)E-3	529(27)E-3
6.78	345(26)E-4	91(7)E-3	206(12)E-3	338(21)E-3	477(26)E-3
7.02	362(25)E-4	103(6)E-3	186(11)E-3	312(20)E-3	446(26)E-3
7.27	304(21)E-4	102(5)E-3	174(10)E-3	294(19)E-3	403(30)E-3
7.53	280(20)E-4	85(5)E-3	175(10)E-3	317(19)E-3	387(29)E-3
7.78	265(19)E-4	85(5)E-3	174(9)E-3	258(18)E-3	367(28)E-3
8.05	227(18)E-4	88(5)E-3	163(9)E-3	254(17)E-3	363(26)E-3
8.32	188(15)E-4	80(5)E-3	149(9)E-3	249(14)E-3	371(19)E-3
8.59	185(14)E-4	70(4)E-3	142(9)E-3	242(13)E-3	329(18)E-3
8.87	159(13)E-4	62(4)E-3	132(9)E-3	235(12)E-3	317(17)E-3
9.16	110(11)E-4	65(5)E-3	117(8)E-3	226(13)E-3	300(19)E-3
9.45	84(10)E-4	61(5)E-3	118(8)E-3	195(13)E-3	285(20)E-3
9.74	44(9)E-4	56(5)E-3	121(7)E-3	191(13)E-3	266(20)E-3
10.04	19(7)E-4	51(4)E-3	115(7)E-3	179(12)E-3	252(18)E-3
10.35	11(4)E-4	51(3)E-3	112(6)E-3	184(10)E-3	237(14)E-3
10.66	29(24)E-5	47(3)E-3	97(6)E-3	172(10)E-3	248(14)E-3
10.98	40(22)E-5	43(3)E-3	90(5)E-3	161(9)E-3	220(12)E-3
11.31	-24(14)E-5	364(29)E-4	81(6)E-3	149(11)E-3	228(15)E-3
11.64	29(14)E-5	337(28)E-4	78(6)E-3	143(10)E-3	202(15)E-3
11.97	13(9)E-5	368(27)E-4	80(6)E-3	137(10)E-3	178(14)E-3
12.31	-17(8)E-5	349(22)E-4	80(5)E-3	146(10)E-3	176(12)E-3
12.66	-49(16)E-5	286(19)E-4	72(4)E-3	144(10)E-3	189(12)E-3
13.01	-29(10)E-5	242(17)E-4	63(4)E-3	119(9)E-3	183(11)E-3
13.37	0(0)E-3	220(16)E-4	56(4)E-3	93(9)E-3	158(11)E-3
13.74	0(0)E-3	212(14)E-4	57(3)E-3	87(9)E-3	142(10)E-3
14.11	0(0)E-3	176(13)E-4	54(4)E-3	104(8)E-3	142(11)E-3
14.49	0(0)E-3	98(10)E-4	53(5)E-3	106(6)E-3	149(11)E-3

Table D.2: Measured bremsstrahlung yields of 10 to 30 MeV electrons incident on Pb targets. Continued from previous page.

Energy	10	15	20	25	30
14.87	0(0)E-3	41(7)E-4	55(5)E-3	99(5)E-3	142(11)E-3
15.26	0(0)E-3	15(6)E-4	48(4)E-3	99(5)E-3	142(11)E-3
15.65	0(0)E-3	7(4)E-4	40(4)E-3	91(5)E-3	128(10)E-3
16.05	0(0)E-3	7(3)E-4	39(4)E-3	78(5)E-3	108(10)E-3
16.46	0(0)E-3	87(27)E-5	391(24)E-4	73(7)E-3	116(9)E-3
16.87	0(0)E-3	51(18)E-5	331(21)E-4	72(7)E-3	114(8)E-3
17.29	0(0)E-3	15(14)E-5	302(20)E-4	68(7)E-3	100(8)E-3
17.72	0(0)E-3	4(10)E-5	285(19)E-4	65(7)E-3	98(8)E-3
18.15	0(0)E-3	0(0)E-3	240(17)E-4	59(6)E-3	103(7)E-3
18.59	0(0)E-3	0(0)E-3	192(15)E-4	61(4)E-3	102(6)E-3
19.03	0(0)E-3	0(0)E-3	157(13)E-4	64(4)E-3	89(5)E-3
19.49	0(0)E-3	0(0)E-3	121(12)E-4	55(3)E-3	74(5)E-3
19.94	0(0)E-3	0(0)E-3	70(9)E-4	51(3)E-3	73(5)E-3
20.41	0(0)E-3	0(0)E-3	28(7)E-4	51(3)E-3	81(5)E-3
20.88	0(0)E-3	0(0)E-3	9(5)E-4	439(29)E-4	80(5)E-3
21.35	0(0)E-3	0(0)E-3	10(4)E-4	411(27)E-4	73(4)E-3
21.84	0(0)E-3	0(0)E-3	10(3)E-4	424(27)E-4	70(4)E-3
22.33	0(0)E-3	0(0)E-3	30(21)E-5	359(24)E-4	72(4)E-3
22.82	0(0)E-3	0(0)E-3	-5(28)E-5	283(21)E-4	69(4)E-3
23.33	0(0)E-3	0(0)E-3	6(4)E-4	248(19)E-4	61(4)E-3
23.84	0(0)E-3	0(0)E-3	9(3)E-4	204(17)E-4	53(4)E-3
24.35	0(0)E-3	0(0)E-3	0(0)E-3	139(14)E-4	42(5)E-3
24.87	0(0)E-3	0(0)E-3	0(0)E-3	84(11)E-4	39(6)E-3
25.40	0(0)E-3	0(0)E-3	0(0)E-3	45(9)E-4	43(6)E-3
25.94	0(0)E-3	0(0)E-3	0(0)E-3	18(6)E-4	43(6)E-3
26.48	0(0)E-3	0(0)E-3	0(0)E-3	9(5)E-4	42(3)E-3
27.03	0(0)E-3	0(0)E-3	0(0)E-3	4(4)E-4	381(25)E-4
27.59	0(0)E-3	0(0)E-3	0(0)E-3	-4(3)E-4	339(23)E-4
28.15	0(0)E-3	0(0)E-3	0(0)E-3	-113(28)E-5	291(21)E-4
28.72	0(0)E-3	0(0)E-3	0(0)E-3	-8(5)E-4	221(17)E-4
29.30	0(0)E-3	0(0)E-3	0(0)E-3	0(7)E-4	150(14)E-4
29.88	0(0)E-3	0(0)E-3	0(0)E-3	6(7)E-4	100(12)E-4
30.48	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	66(10)E-4
31.07	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	38(7)E-4
31.68	0(0)E-3	0(0)E-3	0(0)E-3	0(0)E-3	14(5)E-4

Table D.3: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Be target (11.67 g/cm² thick, 6.72 g/cm² radius). The units are (MeV·sr)⁻¹. Only the relative uncertainties are shown. The energy column is the energy at the top of each bin in the spectral distribution.

Energy	0°	1°	2°	4°
0.14				
0.18	214(4)E-2	208(4)E-2	183(3)E-2	1456(28)E-3
0.22	206(4)E-2	209(4)E-2	175(3)E-2	1337(24)E-3
0.27	180(3)E-2	175(3)E-2	1479(27)E-3	1140(22)E-3
0.32	165(3)E-2	1578(29)E-3	1359(24)E-3	1036(20)E-3
0.38	148(9)E-2	142(8)E-2	120(7)E-2	915(25)E-3
0.44	129(12)E-2	118(11)E-2	102(9)E-2	806(30)E-3
0.51	115(11)E-2	104(11)E-2	91(9)E-2	732(28)E-3
0.58	101(11)E-2	95(10)E-2	82(9)E-2	617(27)E-3
0.67	103(5)E-2	94(5)E-2	79(4)E-2	562(16)E-3
0.76	894(25)E-3	869(23)E-3	721(19)E-3	534(12)E-3
0.86	827(24)E-3	795(22)E-3	656(18)E-3	501(12)E-3
0.96	773(24)E-3	708(23)E-3	575(19)E-3	447(15)E-3
1.08	649(23)E-3	635(22)E-3	547(18)E-3	401(14)E-3
1.20	624(18)E-3	579(17)E-3	498(14)E-3	366(12)E-3
1.33	575(16)E-3	536(15)E-3	452(13)E-3	322(11)E-3
1.47	509(22)E-3	473(20)E-3	404(17)E-3	305(9)E-3
1.62	470(21)E-3	438(20)E-3	355(16)E-3	286(9)E-3
1.78	461(12)E-3	414(11)E-3	344(9)E-3	236(9)E-3
1.95	405(12)E-3	368(11)E-3	322(9)E-3	219(8)E-3
2.12	365(12)E-3	346(12)E-3	289(10)E-3	215(7)E-3
2.31	335(11)E-3	327(11)E-3	255(9)E-3	188(6)E-3
2.51	320(9)E-3	312(9)E-3	238(7)E-3	167(6)E-3
2.72	293(9)E-3	274(8)E-3	232(7)E-3	148(6)E-3
2.94	269(10)E-3	245(9)E-3	208(7)E-3	142(6)E-3
3.17	234(9)E-3	220(9)E-3	187(7)E-3	131(5)E-3
3.41	229(8)E-3	222(7)E-3	176(6)E-3	125(6)E-3
3.66	208(10)E-3	182(9)E-3	153(8)E-3	120(3)E-3
3.93	193(8)E-3	177(7)E-3	157(6)E-3	109(4)E-3
4.20	171(7)E-3	166(7)E-3	137(6)E-3	96(3)E-3
4.49	163(7)E-3	154(6)E-3	125(5)E-3	90(3)E-3
4.80	155(6)E-3	137(6)E-3	109(5)E-3	73(4)E-3
5.11	133(5)E-3	127(5)E-3	107(4)E-3	711(30)E-4

Table D.3: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Be target. Continued from previous page.

Energy	0°	1°	2°	4°
5.44	128(5)E-3	125(5)E-3	102(4)E-3	642(23)E-4
5.78	116(4)E-3	115(4)E-3	97(3)E-3	629(24)E-4
6.13	103(6)E-3	90(5)E-3	79(4)E-3	561(17)E-4
6.49	103(4)E-3	97(4)E-3	74(3)E-3	513(18)E-4
6.88	89(3)E-3	90(3)E-3	710(26)E-4	468(20)E-4
7.27	87(3)E-3	833(30)E-4	639(24)E-4	413(19)E-4
7.68	86(3)E-3	732(27)E-4	577(22)E-4	395(19)E-4
8.10	697(28)E-4	656(26)E-4	541(21)E-4	359(17)E-4
8.54	666(29)E-4	604(27)E-4	456(22)E-4	314(13)E-4
8.99	606(28)E-4	552(25)E-4	437(21)E-4	287(12)E-4
9.45	539(23)E-4	507(21)E-4	435(18)E-4	273(10)E-4
9.94	501(22)E-4	483(20)E-4	376(16)E-4	249(9)E-4
10.43	460(24)E-4	422(22)E-4	338(18)E-4	218(8)E-4
10.94	402(23)E-4	356(21)E-4	296(18)E-4	195(7)E-4
11.47	374(19)E-4	357(17)E-4	249(14)E-4	174(7)E-4
12.02	333(17)E-4	288(15)E-4	215(12)E-4	140(6)E-4
12.58	270(14)E-4	239(13)E-4	180(11)E-4	104(5)E-4
13.15	221(13)E-4	201(12)E-4	148(10)E-4	80(4)E-4
13.75	172(12)E-4	151(11)E-4	122(9)E-4	53(4)E-4
14.35	109(11)E-4	98(10)E-4	70(8)E-4	372(25)E-5
14.98	45(9)E-4	43(9)E-4	23(7)E-4	182(17)E-5
15.63	142(23)E-5	149(22)E-5	86(16)E-5	68(10)E-5
16.29	48(13)E-5	49(12)E-5	54(11)E-5	17(5)E-5
16.96	-4(11)E-5	12(6)E-5	26(7)E-5	0(4)E-5

Table D.3: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Be target. Continued from previous page.

Energy	10°	30°	60°	90°
0.14				
0.18	758(17)E-3	1910(25)E-4	829(14)E-4	532(5)E-4
0.22	649(17)E-3	1686(20)E-4	699(11)E-4	440(4)E-4
0.27	568(15)E-3	1318(18)E-4	511(9)E-4	311(3)E-4
0.32	495(14)E-3	1117(15)E-4	412(8)E-4	2396(26)E-5
0.38	440(12)E-3	940(15)E-4	337(7)E-4	1829(22)E-5
0.44	380(10)E-3	785(15)E-4	271(6)E-4	1372(18)E-5
0.51	334(9)E-3	683(13)E-4	224(5)E-4	952(14)E-5
0.58	302(8)E-3	583(12)E-4	186(5)E-4	562(11)E-5
0.67	279(7)E-3	498(10)E-4	144(4)E-4	287(7)E-5
0.76	237(6)E-3	435(9)E-4	112(3)E-4	182(6)E-5
0.86	210(5)E-3	354(8)E-4	855(27)E-5	120(5)E-5
0.96	194(4)E-3	325(7)E-4	672(23)E-5	79(4)E-5
1.08	175(4)E-3	289(6)E-4	456(19)E-5	57(3)E-5
1.20	156(4)E-3	245(5)E-4	298(15)E-5	500(28)E-6
1.33	142(4)E-3	212(5)E-4	235(13)E-5	396(24)E-6
1.47	127(4)E-3	187(5)E-4	167(11)E-5	334(22)E-6
1.62	115(3)E-3	155(4)E-4	128(10)E-5	267(19)E-6
1.78	101(3)E-3	135(4)E-4	102(9)E-5	198(16)E-6
1.95	922(30)E-4	121(3)E-4	72(7)E-5	148(14)E-6
2.12	843(27)E-4	101(3)E-4	68(7)E-5	109(12)E-6
2.31	742(27)E-4	84(3)E-4	71(7)E-5	96(11)E-6
2.51	680(22)E-4	756(26)E-5	48(5)E-5	82(10)E-6
2.72	597(21)E-4	609(23)E-5	41(5)E-5	75(9)E-6
2.94	574(20)E-4	518(20)E-5	26(4)E-5	42(7)E-6
3.17	520(18)E-4	472(19)E-5	25(4)E-5	42(7)E-6
3.41	438(18)E-4	427(18)E-5	20(3)E-5	36(6)E-6
3.66	411(16)E-4	358(17)E-5	169(29)E-6	25(5)E-6
3.93	389(14)E-4	280(15)E-5	149(29)E-6	17(4)E-6
4.20	338(13)E-4	241(13)E-5	112(24)E-6	12(3)E-6
4.49	286(13)E-4	191(12)E-5	92(21)E-6	18(4)E-6
4.80	279(13)E-4	183(11)E-5	77(18)E-6	11(3)E-6
5.11	245(14)E-4	163(9)E-5	104(20)E-6	12(3)E-6

Table D.3: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Be target. Continued from previous page.

Energy	10°	30°	60°	90°
5.44	234(10)E-4	125(8)E-5	75(16)E-6	10(3)E-6
5.78	209(10)E-4	104(8)E-5	43(14)E-6	84(26)E-7
6.13	173(10)E-4	91(7)E-5	11(9)E-6	13(3)E-6
6.49	170(9)E-4	82(6)E-5	34(13)E-6	74(22)E-7
6.88	140(9)E-4	73(5)E-5	55(14)E-6	75(23)E-7
7.27	131(8)E-4	56(5)E-5	39(11)E-6	48(18)E-7
7.68	128(7)E-4	35(4)E-5	55(13)E-6	45(16)E-7
8.10	118(7)E-4	31(4)E-5	33(9)E-6	31(12)E-7
8.54	95(6)E-4	294(29)E-6	30(10)E-6	7(5)E-7
8.99	74(5)E-4	183(22)E-6	22(8)E-6	3(5)E-7
9.45	72(6)E-4	184(22)E-6	34(8)E-6	4(4)E-7
9.94	53(5)E-4	119(16)E-6	17(5)E-6	9(6)E-7
10.43	43(4)E-4	71(13)E-6	16(5)E-6	2(4)E-7
10.94	46(4)E-4	58(11)E-6	18(5)E-6	-8(29)E-8
11.47	31(3)E-4	30(8)E-6	66(25)E-7	4(4)E-7
12.02	30(3)E-4	15(6)E-6	20(17)E-7	4(3)E-7
12.58	165(24)E-5	27(7)E-6	6(16)E-7	6(27)E-8
13.15	166(20)E-5	16(5)E-6	-1(15)E-7	-2(26)E-8
13.75	36(12)E-5	40(23)E-7	0(15)E-7	-1(25)E-8
14.35	55(12)E-5	0(16)E-7	-2(15)E-7	-5(25)E-8
14.98	29(8)E-5	-3(15)E-7	0(14)E-7	0(24)E-8
15.63	31(25)E-6	14(18)E-7	13(14)E-7	26(28)E-8
16.29	-2(14)E-6	7(15)E-7	6(13)E-7	13(24)E-8
16.96	-3(13)E-6	0(15)E-7	0(14)E-7	0(23)E-8

Table D.4: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of an Al target (9.74 g/cm² thick, 9.81 g/cm² radius). The units are (MeV·sr)⁻¹. Only the relative uncertainties are shown. The energy column is the energy at the top of each bin in the spectral distribution.

Energy	0°	1°	2°	4°
0.14				
0.18	310(6)E-2	286(6)E-2	261(5)E-2	209(4)E-2
0.22	298(5)E-2	279(5)E-2	244(4)E-2	202(4)E-2
0.27	245(5)E-2	237(5)E-2	206(4)E-2	170(3)E-2
0.32	221(4)E-2	209(4)E-2	187(3)E-2	1499(30)E-3
0.38	193(5)E-2	183(4)E-2	163(4)E-2	127(3)E-2
0.44	170(5)E-2	167(5)E-2	146(4)E-2	114(4)E-2
0.51	149(5)E-2	147(4)E-2	125(4)E-2	100(3)E-2
0.58	138(4)E-2	131(4)E-2	114(4)E-2	89(3)E-2
0.67	125(3)E-2	118(3)E-2	1031(26)E-3	791(28)E-3
0.76	1119(28)E-3	1052(26)E-3	933(23)E-3	716(26)E-3
0.86	1032(27)E-3	941(25)E-3	839(22)E-3	643(24)E-3
0.96	942(27)E-3	894(25)E-3	770(22)E-3	615(15)E-3
1.08	842(25)E-3	820(24)E-3	687(20)E-3	547(14)E-3
1.20	763(22)E-3	726(21)E-3	621(18)E-3	505(15)E-3
1.33	693(21)E-3	637(19)E-3	581(17)E-3	462(15)E-3
1.47	628(18)E-3	611(17)E-3	540(14)E-3	412(12)E-3
1.62	582(16)E-3	551(15)E-3	490(13)E-3	360(11)E-3
1.78	555(13)E-3	505(13)E-3	443(11)E-3	338(7)E-3
1.95	500(14)E-3	454(13)E-3	411(11)E-3	325(8)E-3
2.12	455(14)E-3	414(13)E-3	362(12)E-3	284(9)E-3
2.31	408(14)E-3	394(13)E-3	322(11)E-3	265(8)E-3
2.51	381(10)E-3	373(10)E-3	318(8)E-3	226(8)E-3
2.72	356(12)E-3	322(11)E-3	293(9)E-3	223(7)E-3
2.94	321(14)E-3	281(13)E-3	240(11)E-3	201(6)E-3
3.17	281(14)E-3	259(13)E-3	230(11)E-3	183(6)E-3
3.41	286(8)E-3	270(7)E-3	234(6)E-3	165(5)E-3
3.66	275(7)E-3	249(6)E-3	211(5)E-3	160(5)E-3
3.93	243(7)E-3	227(7)E-3	192(6)E-3	135(6)E-3
4.20	202(8)E-3	187(8)E-3	166(7)E-3	125(5)E-3
4.49	194(6)E-3	186(6)E-3	158(5)E-3	120(4)E-3
4.80	183(6)E-3	181(5)E-3	143(5)E-3	109(4)E-3
5.11	173(5)E-3	155(5)E-3	133(4)E-3	104(3)E-3

Table D.4: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of an Al target. Continued from previous page.

Energy	0°	1°	2°	4°
5.44	149(6)E-3	134(6)E-3	123(5)E-3	87(3)E-3
5.78	139(5)E-3	134(5)E-3	118(4)E-3	77(3)E-3
6.13	130(5)E-3	124(5)E-3	104(4)E-3	74(3)E-3
6.49	126(4)E-3	117(4)E-3	95(3)E-3	685(27)E-4
6.88	105(5)E-3	99(4)E-3	85(4)E-3	619(23)E-4
7.27	99(4)E-3	88(4)E-3	77(3)E-3	570(23)E-4
7.68	88(4)E-3	82(4)E-3	72(3)E-3	523(22)E-4
8.10	78(4)E-3	78(3)E-3	671(29)E-4	509(21)E-4
8.54	761(25)E-4	735(24)E-4	614(20)E-4	444(17)E-4
8.99	704(24)E-4	646(22)E-4	557(18)E-4	391(16)E-4
9.45	647(26)E-4	575(24)E-4	479(20)E-4	366(13)E-4
9.94	605(24)E-4	545(23)E-4	453(19)E-4	315(11)E-4
10.43	568(20)E-4	512(18)E-4	410(15)E-4	289(11)E-4
10.94	495(17)E-4	433(15)E-4	375(13)E-4	250(9)E-4
11.47	401(15)E-4	397(14)E-4	333(12)E-4	215(13)E-4
12.02	374(14)E-4	367(13)E-4	296(11)E-4	193(12)E-4
12.58	338(18)E-4	305(17)E-4	235(14)E-4	164(7)E-4
13.15	272(15)E-4	232(14)E-4	198(12)E-4	115(6)E-4
13.75	194(9)E-4	176(8)E-4	140(7)E-4	77(4)E-4
14.35	117(7)E-4	113(6)E-4	83(5)E-4	45(3)E-4
14.98	56(4)E-4	46(4)E-4	336(29)E-5	166(18)E-5
15.63	141(19)E-5	111(17)E-5	80(14)E-5	46(10)E-5
16.29	9(6)E-5	11(8)E-5	25(7)E-5	14(5)E-5
16.96	16(9)E-5	-1(4)E-5	20(8)E-5	8(5)E-5

Table D.4: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of an Al target. Continued from previous page.

Energy	10°	30°	60°	90°
0.14				
0.18	1261(20)E-3	478(6)E-3	1933(22)E-4	1071(11)E-4
0.22	1177(17)E-3	420(5)E-3	1650(18)E-4	931(8)E-4
0.27	946(15)E-3	331(4)E-3	1226(15)E-4	686(7)E-4
0.32	841(13)E-3	279(4)E-3	1009(13)E-4	560(6)E-4
0.38	738(14)E-3	236(4)E-3	775(11)E-4	430(5)E-4
0.44	636(16)E-3	203(5)E-3	651(11)E-4	329(4)E-4
0.51	547(15)E-3	177(5)E-3	555(10)E-4	257(4)E-4
0.58	508(14)E-3	156(4)E-3	491(9)E-4	210(3)E-4
0.67	456(11)E-3	1346(29)E-4	366(7)E-4	1279(24)E-5
0.76	397(10)E-3	1135(22)E-4	298(6)E-4	909(20)E-5
0.86	353(9)E-3	996(21)E-4	240(5)E-4	681(17)E-5
0.96	317(7)E-3	839(18)E-4	199(4)E-4	552(14)E-5
1.08	287(7)E-3	742(17)E-4	154(4)E-4	423(12)E-5
1.20	248(7)E-3	654(15)E-4	120(3)E-4	358(11)E-5
1.33	218(7)E-3	578(14)E-4	980(30)E-5	277(10)E-5
1.47	199(6)E-3	495(14)E-4	854(26)E-5	237(9)E-5
1.62	179(5)E-3	420(13)E-4	721(24)E-5	190(8)E-5
1.78	160(5)E-3	382(11)E-4	611(21)E-5	155(7)E-5
1.95	147(5)E-3	329(10)E-4	486(18)E-5	124(6)E-5
2.12	135(5)E-3	299(9)E-4	423(17)E-5	105(6)E-5
2.31	130(3)E-3	266(8)E-4	381(16)E-5	90(5)E-5
2.51	109(5)E-3	241(8)E-4	314(15)E-5	80(5)E-5
2.72	96(4)E-3	206(7)E-4	268(14)E-5	61(4)E-5
2.94	923(29)E-4	185(6)E-4	232(12)E-5	52(3)E-5
3.17	881(27)E-4	160(6)E-4	187(11)E-5	39(3)E-5
3.41	793(26)E-4	141(5)E-4	173(11)E-5	316(26)E-6
3.66	741(21)E-4	122(5)E-4	154(10)E-5	302(26)E-6
3.93	654(24)E-4	104(4)E-4	114(8)E-5	247(23)E-6
4.20	590(21)E-4	100(4)E-4	99(7)E-5	185(18)E-6
4.49	555(17)E-4	80(5)E-4	97(7)E-5	161(17)E-6
4.80	489(17)E-4	73(5)E-4	82(6)E-5	102(13)E-6
5.11	427(18)E-4	67(3)E-4	60(5)E-5	85(12)E-6

Table D.4: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of an Al target. Continued from previous page.

Energy	10°	30°	60°	90°
5.44	396(16)E-4	58(3)E-4	48(4)E-5	77(11)E-6
5.78	376(13)E-4	507(24)E-5	51(4)E-5	58(9)E-6
6.13	308(11)E-4	473(23)E-5	35(3)E-5	34(9)E-6
6.49	272(13)E-4	412(22)E-5	246(28)E-6	46(8)E-6
6.88	250(13)E-4	321(21)E-5	201(24)E-6	22(5)E-6
7.27	248(12)E-4	249(18)E-5	162(22)E-6	9(3)E-6
7.68	208(11)E-4	226(16)E-5	111(17)E-6	9(3)E-6
8.10	178(10)E-4	204(15)E-5	99(16)E-6	10(3)E-6
8.54	166(8)E-4	171(12)E-5	67(13)E-6	88(29)E-7
8.99	162(7)E-4	139(10)E-5	73(13)E-6	43(18)E-7
9.45	140(6)E-4	108(8)E-5	60(11)E-6	19(13)E-7
9.94	117(5)E-4	77(7)E-5	34(8)E-6	0(8)E-7
10.43	102(6)E-4	58(6)E-5	22(6)E-6	21(14)E-7
10.94	83(6)E-4	47(5)E-5	15(5)E-6	22(12)E-7
11.47	66(4)E-4	33(4)E-5	10(4)E-6	3(7)E-7
12.02	546(29)E-5	26(4)E-5	9(4)E-6	5(7)E-7
12.58	449(26)E-5	97(19)E-6	41(22)E-7	9(7)E-7
13.15	292(20)E-5	35(13)E-6	21(17)E-7	2(6)E-7
13.75	191(16)E-5	51(15)E-6	6(14)E-7	0(5)E-7
14.35	107(11)E-5	23(8)E-6	0(14)E-7	0(5)E-7
14.98	42(7)E-5	0(4)E-6	0(13)E-7	6(6)E-7
15.63	19(5)E-5	-2(4)E-6	0(13)E-7	3(5)E-7
16.29	85(29)E-6	3(4)E-6	0(12)E-7	0(5)E-7
16.96	2(19)E-6	5(4)E-6	0(13)E-7	0(5)E-7

Table D.5: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Pb target (9.13 g/cm² thick, 17.95 g/cm² radius). The units are (MeV·sr)⁻¹. Only the relative uncertainties are shown. The energy column is the energy at the top of each bin in the spectral distribution.

Energy	0°	1°	2°	4°
0.14				
0.18	15(3)E-2	12(3)E-2	96(27)E-3	112(22)E-3
0.22	16(3)E-2	14(3)E-2	133(28)E-3	123(22)E-3
0.27	18(3)E-2	18(3)E-2	153(27)E-3	151(21)E-3
0.32	32(3)E-2	304(30)E-3	270(26)E-3	254(21)E-3
0.38	54(3)E-2	508(29)E-3	459(26)E-3	386(20)E-3
0.44	739(30)E-3	747(29)E-3	640(25)E-3	571(20)E-3
0.51	957(29)E-3	956(29)E-3	867(25)E-3	763(20)E-3
0.58	1147(29)E-3	1145(28)E-3	1044(25)E-3	891(20)E-3
0.67	109(3)E-2	109(3)E-2	941(28)E-3	805(19)E-3
0.76	108(3)E-2	107(3)E-2	953(28)E-3	790(18)E-3
0.86	105(3)E-2	104(3)E-2	897(27)E-3	771(17)E-3
0.96	990(27)E-3	998(26)E-3	856(23)E-3	727(17)E-3
1.08	941(26)E-3	914(25)E-3	811(22)E-3	643(16)E-3
1.20	885(21)E-3	845(20)E-3	772(17)E-3	621(16)E-3
1.33	822(19)E-3	806(18)E-3	718(16)E-3	584(16)E-3
1.47	715(19)E-3	726(18)E-3	629(16)E-3	535(12)E-3
1.62	640(18)E-3	641(18)E-3	582(16)E-3	473(10)E-3
1.78	606(20)E-3	565(19)E-3	521(17)E-3	416(9)E-3
1.95	547(20)E-3	509(19)E-3	472(17)E-3	379(9)E-3
2.12	516(19)E-3	469(18)E-3	413(16)E-3	349(8)E-3
2.31	476(14)E-3	446(14)E-3	379(12)E-3	321(7)E-3
2.51	423(11)E-3	396(11)E-3	359(10)E-3	288(7)E-3
2.72	397(10)E-3	374(10)E-3	338(8)E-3	267(7)E-3
2.94	348(10)E-3	345(10)E-3	304(8)E-3	248(6)E-3
3.17	306(10)E-3	312(10)E-3	271(8)E-3	213(6)E-3
3.41	304(8)E-3	288(8)E-3	248(7)E-3	189(7)E-3
3.66	276(8)E-3	248(8)E-3	225(7)E-3	181(5)E-3
3.93	249(8)E-3	232(7)E-3	209(6)E-3	161(5)E-3
4.20	235(7)E-3	212(6)E-3	183(6)E-3	138(5)E-3
4.49	199(8)E-3	193(7)E-3	169(6)E-3	123(4)E-3
4.80	183(7)E-3	176(7)E-3	156(6)E-3	121(4)E-3
5.11	169(5)E-3	163(5)E-3	144(4)E-3	113(3)E-3

Table D.5: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Pb target. Continued from previous page.

Energy	0°	1°	2°	4°
5.44	148(6)E-3	142(5)E-3	132(5)E-3	107(3)E-3
5.78	138(4)E-3	137(4)E-3	117(4)E-3	895(30)E-4
6.13	139(4)E-3	132(4)E-3	107(3)E-3	809(29)E-4
6.49	112(4)E-3	108(4)E-3	93(4)E-3	752(27)E-4
6.88	102(4)E-3	102(4)E-3	85(4)E-3	709(20)E-4
7.27	103(3)E-3	88(3)E-3	840(29)E-4	598(22)E-4
7.68	936(28)E-4	839(27)E-4	755(23)E-4	560(23)E-4
8.10	868(27)E-4	735(25)E-4	684(22)E-4	517(21)E-4
8.54	763(27)E-4	743(27)E-4	624(23)E-4	497(17)E-4
8.99	674(25)E-4	659(25)E-4	558(21)E-4	432(15)E-4
9.45	54(3)E-3	56(3)E-3	506(28)E-4	400(13)E-4
9.94	53(3)E-3	53(3)E-3	451(27)E-4	351(11)E-4
10.43	539(23)E-4	488(22)E-4	422(19)E-4	308(12)E-4
10.94	490(20)E-4	413(19)E-4	357(16)E-4	272(11)E-4
11.47	416(19)E-4	370(18)E-4	303(15)E-4	247(9)E-4
12.02	371(17)E-4	350(17)E-4	280(14)E-4	215(8)E-4
12.58	317(12)E-4	298(11)E-4	257(10)E-4	176(9)E-4
13.15	248(10)E-4	235(10)E-4	199(8)E-4	151(8)E-4
13.75	176(8)E-4	172(8)E-4	152(7)E-4	111(6)E-4
14.35	117(6)E-4	107(6)E-4	95(5)E-4	67(4)E-4
14.98	55(4)E-4	46(4)E-4	320(27)E-5	262(21)E-5
15.63	175(22)E-5	173(23)E-5	75(14)E-5	64(10)E-5
16.29	76(15)E-5	60(12)E-5	33(8)E-5	8(5)E-5
16.96	8(10)E-5	6(5)E-5	17(7)E-5	5(5)E-5

Table D.5: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Pb target. Continued from previous page.

Energy	10°	30°	60°	90°
0.14				
0.18	88(12)E-3	94(4)E-3	244(12)E-4	40(4)E-4
0.22	105(13)E-3	102(4)E-3	260(11)E-4	46(4)E-4
0.27	140(12)E-3	100(4)E-3	259(10)E-4	46(4)E-4
0.32	221(12)E-3	120(4)E-3	361(11)E-4	57(4)E-4
0.38	338(12)E-3	157(5)E-3	510(13)E-4	91(6)E-4
0.44	449(11)E-3	205(6)E-3	681(14)E-4	165(7)E-4
0.51	551(11)E-3	261(6)E-3	1000(15)E-4	308(8)E-4
0.58	654(11)E-3	332(6)E-3	1288(17)E-4	431(8)E-4
0.67	533(14)E-3	243(5)E-3	801(15)E-4	299(6)E-4
0.76	500(14)E-3	216(5)E-3	714(15)E-4	292(5)E-4
0.86	479(13)E-3	203(5)E-3	648(14)E-4	290(5)E-4
0.96	442(12)E-3	186(3)E-3	602(12)E-4	273(4)E-4
1.08	402(11)E-3	167(3)E-3	532(11)E-4	247(4)E-4
1.20	381(10)E-3	148(3)E-3	474(10)E-4	228(4)E-4
1.33	349(9)E-3	1321(30)E-4	410(10)E-4	195(4)E-4
1.47	325(7)E-3	1193(26)E-4	368(9)E-4	174(3)E-4
1.62	281(6)E-3	1073(24)E-4	311(8)E-4	1485(29)E-5
1.78	248(7)E-3	936(23)E-4	281(7)E-4	1303(28)E-5
1.95	227(6)E-3	844(20)E-4	242(7)E-4	1147(26)E-5
2.12	203(5)E-3	733(18)E-4	208(6)E-4	991(25)E-5
2.31	188(4)E-3	673(16)E-4	184(5)E-4	841(26)E-5
2.51	163(4)E-3	578(17)E-4	164(5)E-4	699(21)E-5
2.72	149(4)E-3	523(15)E-4	134(5)E-4	633(20)E-5
2.94	134(3)E-3	473(13)E-4	120(4)E-4	577(18)E-5
3.17	129(3)E-3	400(12)E-4	101(4)E-4	474(14)E-5
3.41	1105(29)E-4	363(12)E-4	94(3)E-4	396(13)E-5
3.66	95(4)E-3	294(13)E-4	85(3)E-4	357(12)E-5
3.93	911(28)E-4	280(10)E-4	678(29)E-5	301(11)E-5
4.20	840(23)E-4	245(9)E-4	611(23)E-5	257(10)E-5
4.49	734(23)E-4	237(8)E-4	532(24)E-5	216(9)E-5
4.80	706(21)E-4	197(8)E-4	478(22)E-5	194(9)E-5
5.11	602(20)E-4	177(6)E-4	429(18)E-5	156(6)E-5

Table D.5: Measured bremsstrahlung yields of 15 MeV electrons incident on the central axis of a Pb target. Continued from previous page.

Energy	10°	30°	60°	90°
5.44	539(22)E-4	162(7)E-4	368(18)E-5	133(5)E-5
5.78	510(18)E-4	151(5)E-4	294(14)E-5	123(5)E-5
6.13	448(19)E-4	137(6)E-4	242(13)E-5	97(4)E-5
6.49	387(16)E-4	119(4)E-4	232(17)E-5	75(4)E-5
6.88	383(16)E-4	103(4)E-4	204(14)E-5	68(4)E-5
7.27	347(12)E-4	96(4)E-4	150(12)E-5	58(3)E-5
7.68	306(10)E-4	85(3)E-4	118(11)E-5	428(26)E-6
8.10	268(9)E-4	70(3)E-4	112(10)E-5	310(21)E-6
8.54	242(10)E-4	61(3)E-4	105(7)E-5	266(19)E-6
8.99	230(9)E-4	569(30)E-5	82(6)E-5	226(17)E-6
9.45	201(7)E-4	483(27)E-5	61(5)E-5	195(15)E-6
9.94	178(6)E-4	415(25)E-5	44(4)E-5	138(12)E-6
10.43	152(6)E-4	330(18)E-5	38(3)E-5	74(17)E-6
10.94	133(5)E-4	274(15)E-5	308(30)E-6	50(17)E-6
11.47	112(5)E-4	232(19)E-5	177(21)E-6	48(8)E-6
12.02	98(4)E-4	206(18)E-5	166(21)E-6	33(5)E-6
12.58	81(6)E-4	142(17)E-5	83(13)E-6	24(4)E-6
13.15	65(5)E-4	85(14)E-5	38(10)E-6	124(29)E-7
13.75	460(25)E-5	50(5)E-5	18(5)E-6	52(18)E-7
14.35	220(16)E-5	26(4)E-5	7(4)E-6	32(15)E-7
14.98	74(9)E-5	60(18)E-6	23(26)E-7	3(10)E-7
15.63	21(5)E-5	-7(7)E-6	-15(22)E-7	2(7)E-7
16.29	73(26)E-6	-13(9)E-6	-20(22)E-7	0(7)E-7
16.96	50(25)E-6	-24(12)E-6	-13(22)E-7	-2(7)E-7

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