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Image Filtering on Slotted Ring Multiple DSP Systems

by

D. Bruce Oakley, B.Eng.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering,
Carleton University,
Ottawa, Ontario, Canada, K1S 5B6

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research acceptance of the thesis

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submitted by D. Bruce Oakley, B.Eng., in partial fulfilment of the requirements for the degree of Master of Engineering.

Thesis Supervisor

Thesis Supervisor

Chair,
Department of Systems and
Computer Engineering

September 3, 1992

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University

Abstract

This thesis describes the partitioning of the linear image filtering problem for multiple DSP systems interconnected by slotted ring networks. An analysis of various partitioning methods is presented. The *block parallel transform method*, based on overlap-and-save processing in the frequency domain, is identified as an effective algorithm for image filtering with multiple processors. In this algorithm, image blocks are distributed to separate processors where filtering operations occur.

The slotted ring architecture is shown to effectively support the block parallel transform method. Other architectures are identified which can provide comparable performance.

An implementation of the block parallel transform method on a representative slotted ring system is described. Processing times correspond well with the analytical model and speed increases are roughly proportional to the number of processors used. A system with four processors filtered a 128x128 image 6 to 7 times faster than PC-based software.

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I gratefully recognize the help of my two supervisors, Dr. David Coll and Dr. Rafik Goubran. I must also thank George Yaremchuk for designing a wonderful multi-DSP system and for providing many patient hours of assistance. Finally, I am indebted to my fiancé, Lynn Fraser, for all of her support.

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A more recent example of a microprocessor built for use in arrays is the Texas Instruments TMS320C40 floating point DSP [54].

Arrays of microprocessors generally can't match the performance of cellular arrays, but they are useful for a broader class of problems. Such systems have, in fact, been used for intermediate and high level image processing [42, 55, 56, 57]. Microprocessor arrays have been suggested for low level processing, but actual implementations are few [43].

Systolic Arrays and Pipelines

Most low level image processing tasks consist of a static set of operations which are repeated over the entire data set. These operations generally don't vary with data values or location within the image. This facilitates the partitioning of tasks *algorithmically* instead of spatially. In other words, each processor can handle a particular operation rather than a piece of the image.

One popular method of algorithmic partitioning is the use of *systolic arrays*. A systolic array consists of a set of connected processing cells through which data is "pumped" or "pulsed" [58]. (The term "systolic" is based on the similarity to a heart pumping blood through a body). Systolic arrays are generally highly regular structures with nearest neighbour communications. A *linear* sequence of processing cells through which data passes *serially* is called a *pipeline*. A pipeline is a multiple instruction stream, single data stream (MISD) system.

There have been several successful implementations of systolic array and pipeline systems for low level image processing [59, 60, 61, 62]. Unfortunately, algorithmic partitioning doesn't allow the high degree of parallelism afforded by spatial partitioning. (The number of algorithm steps is usually far fewer than the number of pixels in an image). However, systolic arrays are not limited by I/O. Since data is pulsed through the processing cells at a steady rate, input and output occur

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Glossary

DMA Direct Memory Access.

DSP Digital Signal Processor.

FFT Fast Fourier Transform.

FLOP Floating Point Operation.

MA Memory Access.

PE Processing Element.

XFER Inter-PE Transfer.

Chapter 1

Introduction

Digital image processing can be accomplished quickly and effectively by multiple computers operating in parallel [1, 2, 3, 4, 5]. Recently, a broad range of digital signal processing tasks have been implemented using multiple digital signal processors (DSPs) with slotted ring interconnection architectures [6, 7, 8, 9, 10]. The use of slotted ring multiple DSP systems for image processing is studied in this thesis. The particular problem addressed is that of linear image filtering.

In Chapter 2, linear image filtering is reviewed and long processing times, due to large amounts of data, are identified as a significant problem.

A survey of parallel image processing techniques is presented in Chapter 3, with emphasis on *low level* processing such as linear filtering. The slotted ring architecture is discussed in Chapter 4, as well as recent usage for signal processing applications.

The slotted ring discussion completes the background chapters. The *specific* goals of the thesis are defined in Chapter 5, along with underlying assumptions.

In Chapter 6, the linear image filtering operation is analyzed in detail. Particular attention is paid to computational requirements. Frequency domain filtering of image blocks is identified as an efficient means of implementation in many cases.

In Chapter 7, methods for performing image filtering in parallel are presented. No single method is optimal under all conditions. The best *overall* performance is achieved by distributing image blocks to separate processors for frequency domain filtering.

The block distribution method has been implemented on a representative slotted ring system. The implementation is described in Chapter 8, with emphasis on performance.

Chapter 9 includes discussion of what has been achieved. Recommendations for future study are also presented.

Chapter 2

Linear Image Filtering

An *image* is perceived as a two-dimensional field of *colour*. Physically, colour is an interpretation of light energy and wavelength by the human visual system [11]. Some images are formed based on light energy only, averaged in some way over a desired band of wavelengths. Such images are called *monochromatic* and can be represented by a field of varying light intensity, or brightness. In this thesis, all images will be assumed to be monochromatic.

An image can be recorded digitally by sampling and quantizing intensity levels at regular points throughout the field [12]. There are many sampling structures which have proved beneficial for various applications. The simplest of these has sampling points located on a horizontally/vertically oriented rectangular grid. The *rectangular sampling* method will be assumed throughout this thesis. Since intensity is essentially a measure of energy, it is a non-negative quantity. A digital image is, thus, a finite-sized, two-dimensional array of strictly real, non-negative integers.

Once an image is in digital form, it can be manipulated by digital computers. The use of computers for imaging applications is now widespread [13, 14, 15]. Manufacturing, medical diagnosis and meteorology are just a few of the fields taking advantage of this [16, 17, 18].

Along with specific applications, much work has been done in an attempt to integrate still images with text, graphics, audio and video. As multimedia and ISDN grow, computer manipulation of images promises to become even more widespread [19, 20].

Unfortunately, many computer applications are hindered by a significant common problem. Generally, the acquisition process introduces some form of degradation into the acquired image. Many images contain flawed representations of their intended subjects. Common forms of degradation fall into three classes [21, 22]:

1. **Blurring:** Blurring is often caused by cameras that are moving relative to the object or that are not in perfect focus. Other causes include optical aperture effects or atmospheric effects for satellite and star field images [23]. Blurring is generally modelled as the linear convolution of the desired image with a blur operator or point spread function (PSF).
2. **Pointwise Nonlinearities:** Very often, non-linear distortion is introduced by sensors. For example, photographic film responds roughly logarithmically to incident light.
3. **Noise:** Noise is common in electrical systems. There are many potential sources, such as thermal noise from resistors. Noise can be introduced to images by, for example, video cameras or electrical storage devices. Also, quantization noise is introduced during digitization. Typically, noise is modelled as an additive random process, often with Gaussian distribution.

Reducing or eliminating these degradations to get a more accurate representation of the subject is called *image restoration*. Image restoration techniques are often based on linear filtering of the degraded image [24, 25, 26, 27]. Linear image filtering will be described in detail in Chapter 6.

In some cases, it is less useful to create an accurate representation of an image than to emphasize certain important aspects. This process is called *image enhancement*. While operations such as noise reduction are common to both restoration and enhancement, enhancement methods such as edge detection also make use of linear filtering [25].

Figure 2.1: Image Filtering in Fluorescence Microscopy

Figure 2.1 illustrates an image restoration application from the field of fluorescence microscopy. The original image, on the left, is distorted by scatter blur [22] and noise. The image on the right shows the result of Weiner filtering [25], based on an assumed raised-cosine weighted exponentially decaying point spread function. The filtered image is much clearer, particularly the bright spots representing fluorescently labelled areas within the cell nucleus.

Unfortunately, filtering an image can take a long time, mainly because of the large amounts of data involved. The images in Figure 2.1 have fairly low spatial resolution. They each consist of 128x128, for a total of 16,384 pixels. Still, the filtering shown took roughly 40 seconds on a 33 MHz 80486-based PC compatible computer, using software written in C [28].

Iterative deblurring of astronomical images is another example of a linear filtering application. Highly optimized software, written for this purpose, can perform a filtering task comparable to the above in under one second ¹ on a similar computer [29].

These computation times are compounded by two factors:

- Images are often much larger than those in Figure 2.1. An image with resolution comparable to a television picture would have over 300,000 pixels.

¹An estimate based on 3 second filtering time for a 256x256 image.

Satellite images can have on the order of 8 *million* pixels [18]. Processing time for most algorithms grows roughly proportionally to the number of pixels. Processing time for linear image filtering is described in Chapter 6.

- Many image restoration applications require iterative methods, usually because the degradation is not exactly known [21]. Therefore, an image may have to be filtered many times, either automatically or under human control.

The PC described above is typical of what is used in today's laboratories and offices. It would provide a rather cumbersome platform for iteratively restoring, say, a 512x512 image. There are computers which have the processing power to filter images very quickly, but they are expensive and often unavailable to researchers. It would be useful to provide a less expensive way to speed up image filtering.

Custom, dedicated image filtering hardware would provide faster processing. However, highly customized systems tend to be expensive and are not very flexible.

There are two important factors which may prove helpful in solving the above problem:

1. Recent years have seen great advances in the performance of single chip digital signal processors (DSPs) [30, 31]. These provide a relatively low cost means of solving certain types of problems with great speed. Image filtering is one of many suitable applications [32, 33].
2. Image processing is often very localized in nature. That is, processing of one area of an image is often independent of the processing of other areas. This suggests the use of spatial parallelism — processing different areas of an image *simultaneously* with different computers [1, 3].

These two ideas imply that several DSPs operating together could provide very efficient image filtering. Also, such a system promises to be useful for other applications, such as image coding and neural networks.

Chapter 3

Survey of Parallel Image Processing Techniques

In this Chapter, parallel methods for image processing are discussed, with particular emphasis on low level processing. Several existing systems are mentioned.

3.1 Image Processing Basics

The term *image processing* encompasses a very broad range of problems and techniques. These have been grouped into three classes [34, 35]:

1. **Low Level Processing:** Low level image processing includes numeric transformations of the image data itself, such as linear filtering.
2. **Intermediate Level Processing:** Intermediate level image processing is the conversion of spatial data into symbolic data. Objects are located within the image array and certain parameters, such as area and location, are measured.
3. **High Level Processing:** High level image processing approaches the problem of *computer vision*. Using features and characteristics extracted by an intermediate level process, the computer tries to interpret and understand the content of the image.

Each processing level deals with data of a different type [35]. Low level processing, for example, uses image data in a very regular, structured way. Intermediate level processing also uses spatial image data, but with far less regularity. The objects or features are generally not restricted in size or shape. High level processing handles symbolic data only. It does not access image data directly. These differences are important considerations when approaching these problems.

Many vision systems and applications will require all three levels of processing. This thesis will deal specifically with linear filtering, which is a low level process.

3.2 Parallel Processing Basics

One indication of the power of a parallel processing system is the number of processing elements (PEs) and the capability of each. Another factor is the *connectivity* or *topology* of the system. Topology illustrates how the PEs are connected together and how they can communicate. Topology is very important for purposes of classification [36, 37].

A *fully interconnected system* represents the most direct connection scheme, in which each PE is connected to every other PE. However, the number of connections grows with the square of the number of PEs. A fully interconnected system becomes extremely complex with even a moderate number of PEs.

In a practical system, PEs are connected to *some* of the other PEs in some regular fashion. How they are connected can greatly influence how well the system performs. For efficient operation, the connectivity should conform to the expected communication requirements [38, 39]. The communication requirements are determined by the application and how the problem is partitioned. So, the optimal connectivity is generally problem specific. The best topology for a given problem may be inappropriate for another problem.

It is difficult to conceive of a general purpose architecture which provides good efficiency for all problems. To be useful for all or most classes of problems, a system should allow any PE to communicate with any other PE. This is possible with a limited number of connections but, then, these connections must be shared. Sharing

can lead to contention, waiting and other problems which reduce efficiency.

As a prelude to discussing particular architectures, some important terms are presented [40]:

- **Single Instruction Stream, Multiple Data Stream (SIMD):** This describes a system in which all processors execute the same instruction synchronously on different sets of data. In other words, there is a single instruction stream but multiple data streams.
- **Multiple Instruction Stream, Multiple Data Stream (MIMD):** In MIMD systems, the processors run independently and can execute different instructions. That is, there are multiple instruction streams.
- **Loosely Coupled System:** In a loosely coupled system, processors communicate by passing messages back and forth. This message passing incurs some overhead.
- **Tightly Coupled System:** Tightly coupled systems have a shared, common memory. There is no communication overhead, but contention problems can occur.

3.3 Dedicated Architectures for Image Processing

A substantial amount of research has been done regarding parallel architectures for image processing [2, 4, 5, 41]. The vast majority of work has dealt with topologies optimized for specific problem classes. In particular, the three general classes of image processing each have specific requirements [35]. Systems designed for overall vision applications typically have three architectural levels, one for each processing level [35, 42]. Attempts have been made to combine two or more of the processing levels into a single topology [43].

The following sections include descriptions of particular methods and systems used for parallel image processing. All three processing levels are addressed, but

low level processing is emphasized.

3.3.1 Architectures for Low Level Processing

Low level processing involves numerical transformation of the image data itself. Data access is deterministic and often very localized. That is, the value of an output pixel is often a function of pixel values in a small, known surrounding area. This is ideal for partitioning problems spatially. In other words, each processor deals with a specific area of the image. This is the method most often used by low level architectures.

Cellular Arrays

A *bounded cellular automaton* is defined as an array of finite-state machines, called *cells*, each of which has the *same* transition function [44]. The transition of each cell depends on the state of the cell and its neighbours. This model bears a resemblance to low level image processing functions and has been used for image processing applications [44, 45].

In the simplest form, each cell represents an image pixel and its state represents the intensity value. In typical low level processes, the new value of a pixel is a function of its old value and those of its neighbours. Each cell can also represent a group of pixels.

Figure 3.1 shows part of a representative cellular array, with each PE corresponding to a cell. Note that each processor is connected only to its four closest neighbours. Any more distant communications must be passed on by other PEs. Cellular arrays are SIMD systems, since each processor executes identical instructions.

There have been a number of successful implementations of cellular array architectures, including the Massively Parallel Processor (MPP) [46, 47] and the Cellular Logic Image Processor (CLIP) series [47, 48, 49]. These architectures were implemented as large, dense arrays of very simple PEs. They were intended for use with one pixel per cell. Communication overhead was a problem, particularly when processing of any pixel required access beyond the 3x3 window of nearest neighbours.

Figure 3.1: Cellular Array (4-Connected)

This problem, and others, have been solved in more recent systems, such as SLIM [50]. Also, recent advances in VLSI technology have allowed construction of much larger arrays.

Cellular arrays allow massive parallelism, which can lead to extremely high performance. Performance is generally limited by I/O. Image data usually must be loaded into the array serially. These systems can be very expensive and are not often useful outside of low level spatial applications.

Array Processors

Cellular array systems exploit spatial parallelism on a very fine-grained scale. Each pixel, or small group of pixels, is represented by a simple PE. Spatial parallelism can also be used in a more coarse-grained fashion. This is done by using more sophisticated PEs, such as microprocessors, each dealing with a relatively large *block* of image data. The grid pattern of Figure 3.1 is still applicable, but far fewer processors are required for a given image size.

Most general purpose microprocessors do not easily support the multiple communication links required for this architecture. Significant external hardware would be needed for each PE. Transputers, however, are an exception.

The transputer is a general purpose microprocessor with built-in communication links [51]. The internal hardware and instruction set are specially designed to allow easy communications with several other transputers. These devices would be well suited to an array system for image processing [52, 53].

A more recent example of a microprocessor built for use in arrays is the Texas Instruments TMS320C40 floating point DSP [54].

Arrays of microprocessors generally can't match the performance of cellular arrays, but they are useful for a broader class of problems. Such systems have, in fact, been used for intermediate and high level image processing [42, 55, 56, 57]. Microprocessor arrays have been suggested for low level processing, but actual implementations are few [43].

Systolic Arrays and Pipelines

Most low level image processing tasks consist of a static set of operations which are repeated over the entire data set. These operations generally don't vary with data values or location within the image. This facilitates the partitioning of tasks *algorithmically* instead of spatially. In other words, each processor can handle a particular operation rather than a piece of the image.

One popular method of algorithmic partitioning is the use of *systolic arrays*. A systolic array consists of a set of connected processing cells through which data is "pumped" or "pulsed" [58]. (The term "systolic" is based on the similarity to a heart pumping blood through a body). Systolic arrays are generally highly regular structures with nearest neighbour communications. A *linear* sequence of processing cells through which data passes *serially* is called a *pipeline*. A pipeline is a multiple instruction stream, single data stream (MISD) system.

There have been several successful implementations of systolic array and pipeline systems for low level image processing [59, 60, 61, 62]. Unfortunately, algorithmic partitioning doesn't allow the high degree of parallelism afforded by spatial partitioning. (The number of algorithm steps is usually far fewer than the number of pixels in an image). However, systolic arrays are not limited by I/O. Since data is pulsed through the processing cells at a steady rate, input and output occur

naturally as part of the sequence

Despite attempts to enhance flexibility [63, 64, 65], most systolic array systems are rather limited in their use. They can't be used easily to solve problems requiring different operations or data flows.

3.3.2 Architectures for Intermediate Level Processing

Intermediate level processing converts low level spatial data into high level symbolic data. As in low level processing, access to the full array of image data is required. This suggests that, again, mapping an array of processors onto the image would be appropriate. There are, however, significant differences from the low level case:

- Required data access is, in general, neither deterministic nor highly localized.
- Algorithms are often much more complicated than those used for low level processing.

The irregular data access requirements imply that a fine-grained network of PEs, such as a cellular array, would be inappropriate. More sophisticated processors would be required to handle intermediate level algorithms. Processors would run independently, implying the use of MIMD systems.

Array processors, including transputer arrays, have been used for intermediate level image processing [42, 55, 56, 57]. There have been attempts at integrating low and intermediate level processing on a single array [43, 53].

Other classes of architecture for intermediate level processing include the following:

- **Pyramid Systems:** An example of a pyramid system is shown in Figure 3.2. These systems provide an efficient combination of both local and global communications. Uhr [66] and Dyer [67] provide good overviews of pyramid systems.
- **Shared Memory (Tightly Coupled) Systems:** In a shared memory system, each processor has access to the entire image [68, 69]. However, memory

Figure 3.2: 2-D Pyramid System (3 Level, 4 Child)

contention is a potentially serious problem. The processor-memory interface is key to controlling memory contention [70, 71].

These system classes, especially the last one, are more flexible than those discussed earlier. This is useful because intermediate level processes are often not very strictly defined. Some researchers have tried to combine loosely and tightly coupled systems, in a partitionable or reconfigurable way, to improve efficiency [72].

3.3.3 Architectures for High Level Processing

High level image processing deals with symbolic data. Typically, data consists of sets of measured feature parameters. The high level image representation bears little resemblance to the original spatial array of intensity values.

High level algorithms are often quite complex and cannot be spatially partitioned easily. The parallel architectures discussed earlier would not be well suited to these

problems.

High level methods will not be discussed further, since this thesis deals with low level processing. High level architectural issues are covered by Voorhees, et al. [73] and by Chien and Lin [74], among others.

3.4 General Purpose Architectures Used for Image Processing

A general purpose system can, by definition, be used to solve many types of problems. This can be very useful in many cases, especially if financial resources are limited. However, this flexibility comes at the cost of performance. A general purpose system will rarely solve a problem as quickly as a system dedicated to that application.

The degree to which a multiprocessor system can be considered general purpose depends mainly on two factors:

1. **Programmability:** The degree to which processors can be flexibly and independently programmed.
2. **Connectivity:** The degree to which processors can communicate efficiently with *any* other processor.

It is impractical to directly interconnect all processors, except when their number is very small. General interconnection can be achieved in two ways:

1. **Shared Connections:** All processors are attached to a relatively simple interconnection network. The available bandwidth is shared among them.
2. **Reconfigurable Connections:** Again, all processors are attached to an interconnection network. However, this network can be *partitioned* in such a way that specific connections are dedicated to specific processors. In this way, interconnections can be matched to particular applications.

It should be noted that the aforementioned interconnection networks can be used to directly connect processors to each other *or* to connect processors to a shared memory.

The following sections describe examples of image processing systems based on general purpose architectures.

3.4.1 Loosely Coupled Systems

In loosely coupled systems, processors are connected to each other by a communication network. These systems are characterized by the nature of the interconnections:

- **Fixed Interconnections:** The most common scheme employs a single time-shared bus, usable by only one processor at a time [70, 75]. The ZMOB system [76, 77] used a ring-shaped shift register connection scheme. This allowed 256 processors to communicate without major bus contention problems.
- **Reconfigurable Interconnections:** The interconnection networks of these systems employ switches under some external control. This allows the topology to be adapted to suit the desired application. Some examples include the FLIP [78], PX-1 [79] and PASM [80] systems.

3.4.2 Tightly Coupled Systems

Tightly coupled systems employ shared memory for interprocessor communications. The interconnection between processors and memory is an important factor. Some connection methods are illustrated in the following paragraphs:

- Processors and memory can be connected very simply with a time-shared bus. The TAMIPS multiprocessor uses this method [81].
- More complex interconnection schemes, such as the pipelined network used by PSM [82], can provide better communication performance. Sunwoo and Aggarwal [72] have proposed a *flexibly coupled* system. This included reconfigurable connections between processors and memory.

- Orthogonal memory systems have been used effectively for parallel image processing [83, 84]. An orthogonal memory system consists of N processors and N^2 memory modules. The memory modules are arranged in a square matrix. Processor k has access to memory modules in row k and column k . So, any pair of processors has one module in common. The modules in the matrix diagonals serve as local memories.

Chapter 4

The Slotted Ring Architecture

The slotted ring is a loosely coupled MIMD architecture. This architecture can provide a good balance of flexibility and performance.

Ring architectures have often been used to implement local and wide area networks for computer communications [85, 86]. Also, a ring topology was the basis for the ZMOB multiprocessor system [77]. Ring architectures feature a number of advantages:

- Each element in a ring requires exactly two ports, one connected to each neighbour. Other topologies often require several ports, sometimes an increasing number as the network expands.
- Rings can be expanded very easily, and can include essentially any number of processors. Many other topologies require certain, fixed numbers of PEs.
- All connections in the ring are simple point-to-point links. This is an advantage mostly for networking applications, where processors are not in close physical proximity. However, single links *can* be made faster than, say, shared buses.

Unfortunately, some method is required to control or arbitrate access to the ring. This usually requires some additional hardware at each processor.

One type of ring, called the *token ring*, is quite popular for networking applications. When the ring is idle, a certain reserved data sequence, called the *token*,

is circulated. When a processor wants to transmit data, it must first remove the token from the ring. Then, the processor is free to transmit data. Upon completion, the token is restored to circulation. During busy periods, the token is passed from processor to processor. Each processor takes over the ring in turn.

Another type of ring, called the *slotted ring*, uses a different type of access. In this case, the ring is divided conceptually into a series of "bins" or "slots", each of which can contain data. Each slot has a flag, indicating whether it is full or empty. The slots circulate around the ring continuously.

To transmit data, a processor must wait for an empty slot to arrive. Then, it marks the slot as "full" and writes data into it. The full slot circulates until reaching the destination processor, which reads the data and marks the slot as "empty".

A slot can be designed to hold a single data item or a block of data. The simple full/empty scheme described above can be augmented by more elaborate access control methods.

For example, the ZMOB system [77] had one slot per processor on the ring. Each slot was specifically allocated to a particular processor, which could only write to *that* slot. All other processors could read the slot. This scheme limited the peak transmission throughput, but prevented a single processor from dominating the ring. This was well suited to applications involving simultaneous transmission by many or all processors.

In recent years, several slotted ring systems have been used for high performance applications:

1. The Ring Array Processor (RAP) was designed for neural network applications [6, 7]. It consisted of 16 floating point DSPs connected by a slotted ring. Initially, ring access was fixed to suit a specific class of problems. The Multi-

layer Perceptron (MLP) network requires two types of data transfer [6, 7]:

- **Forward Propagation:** This requires simultaneous broadcasts from all processors to all other processors.
- **Backward Propagation:** This requires that each processor add to and forward a partial sum. That is, a sum of products is calculated in pipeline fashion.

The initial RAP system implemented only these type of transfers. However, the ring interface hardware was built from a programmable gate array. This allowed future inclusion of more general transfer methods.

2. Bolch, et al. designed a multiprocessor system, MUPSI, for real time signal processing [8, 9]. It consisted of bit-slice processors connected by a slotted ring network.

The destination addressing was quite flexible, allowing *any set* of processors to be specified as targets for a transmission. MUPSI also featured a separate, fully buffered communications control unit. This allowed concurrent processing and communications. To the processor, the communications interface looked like a shared memory and the slotted ring structure was invisible.

MUPSI could be used to implement a wide variety of signal processing algorithms in real time, up to about 100 kHz. Problem partitioning was generally systolic, or algorithmic.

3. Yaremchuk has built a partitionable slotted ring system using floating point DSPs [10]. It is to be used for simulating signal processing algorithms for ISDN applications.

Yaremchuk's system is similar to MUPSI in that it has a buffered communications interface, independent of the processors. There is, with each processor, a dual-ported memory which is mapped into the memory space of all other processors. The processors themselves play no part in the data transfers over the ring.

The control of slot reads and writes is software programmable during ring initialization. This allows flexible slot access control. Further flexibility is provided by programmable logic hardware.

Yaremchuk's slotted ring system has been used to implement linear image filtering. Chapter 8 includes a description of the implementation and its performance, as well as a more detailed description of the system itself.

These systems, while similar in architecture, have provided very high performance over a range of problem types.

The ZMOB system was used effectively for image processing applications. It, however, used a large number of processors (256) based on technology which is no longer up to date.

It would be interesting to study how well a more recent slotted ring system could handle image processing problems. In Chapter 7, the use of a slotted ring system for linear image filtering will be considered.

Chapter 5

Problem Definition

The following points summarize what has been established so far:

1. Image processing, in particular linear image filtering, is a useful and necessary operation in a broad range of application areas.
2. Image filtering very often requires substantial computation time, particularly on serial computers. Researchers would benefit from improved processing speed.
3. Parallel processing methods provide a natural means of speeding up image filtering. Many existing systems exploit this, but the majority are very expensive and/or very specific in their range of uses. A more general purpose parallel system would be preferable in many cases.
4. DSPs are microcomputers specially designed for certain types of processing, including linear filtering. DSPs provide high performance at relatively low cost.
5. Slotted ring multiprocessor systems provide a very good balance of flexibility and performance. Existing systems have proven useful, especially in the field of signal processing.

A combination of DSP performance with parallel processing methods promises to be very effective for linear image filtering. The slotted ring architecture can

provide a flexible, expandable system without overly sacrificing performance. So, the goals of this thesis are:

1. To analyze quantitatively the problem of linear image filtering on a multi-DSP system with slotted ring architecture.
2. To implement a solution on a representative slotted ring multi-DSP system.
3. To discuss any benefits and problems regarding the above. To *address the suitability* of the slotted ring architecture for solving this problem.

The following assumptions are made:

1. Image data will initially be stored in memory on *one* of the processors on the ring. Typically, this one processor is connected to a host computer which provides the data. Thus, only a single external link is required. Also, this implies that any data partitioning is done *within* the slotted ring system.
2. All interprocessor communications are memory mapped and buffered. That is, a processor sends data to another processor by writing into *its own* memory space. The target processor retrieves it by reading from *its* memory space. Buffering prevents the processors from having to wait for the ring.
3. All processors have the use of at least one direct memory access (DMA) channel. DMA channels are common to most current DSPs. Use of DMA allows data transfers to occur concurrently with processing.
4. All images and spatial filter responses are strictly real, as is the case in virtually all real applications. This assumption simplifies processing and can decrease the required processing time.

Chapter 6

Computational Analysis of Image Filtering

6.1 Definition of Image Filtering

One common way to describe a digital image filter is to specify its unit sample response. (That is, its response to a single point of unit amplitude.) The result of filtering an image is given by the linear convolution operation [25, pg.13-20]:

$$\begin{aligned} y(m, n) &= x(m, n) * h(m, n) \\ y(m, n) &= \sum_{i=-L}^L \sum_{j=-L}^L x(m-i, n-j) h(i, j) \\ &\quad \forall m, n \in 0 \dots N-1 \end{aligned} \tag{6.1}$$

Where:

- x is the $N \times N$ source image
- y is the $N \times N$ result image
- h is the $M \times M$ filter response ($M = 2L + 1$)

From Fourier transform theory, the discrete Fourier transform (DFT) can be used to determine the circular convolution of two images [25, sect. 3.2].

$$\begin{aligned} y_c(m, n) &= x(m, n) \oplus h(m, n) \\ Y_c(k, l) &= X(k, l)H(k, l) \end{aligned} \quad (6.2)$$

Where:

- X, Y_c, H are the DFTs of x, y_c, h .

Also, under certain conditions, the circular convolution of two images is equal to the linear convolution.

$$x_p(m, n) \oplus h(m, n) = x_p(m, n) * h(m, n) \quad (6.3)$$

Where:

- x_p consists of the image x , zero-padded to a size of at least $(N + M - 1) \times (N + M - 1)$.

With sufficient zero-padding, circular convolution yields a result identical to linear convolution. So, the transform method can be used to perform linear filtering.

A comparison of the algorithm costs will provide some insight into why the transform method might be preferable to direct convolution. Computational cost is measured in terms of the number of floating point operations (FLOPs) and the number of memory accesses (MAs). These values give an approximate indication of the processing time required to complete a task.

The time required to complete a series of FLOPs may depend on the precedence, or ordering, of the operations. For example, many DSPs are designed to efficiently calculate a sum of products. These DSPs may take longer to calculate a product of sums, even though the number of FLOPs is the same for each task. In this analysis, precedence is *not* considered. It is assumed that simply totalling FLOPs will provide a sufficient indication of processing time.

6.1.1 Cost of Direct Convolution

Assume that the image is of size $N \times N$ and the filter response is of size $M \times M$.

Equation 6.1 indicates that each of the N^2 result values is formed from the sum of M^2 products. Each term in this product requires a multiplication, an addition and two memory reads. Neglecting less significant terms (assuming $M \gg 1$) provides the approximate result:

$$\begin{aligned} \text{Number of FLOPs} &= 2N^2M^2 \\ \text{Number of MAs} &= 2N^2M^2 \end{aligned} \quad (6.4)$$

6.1.2 Cost of Transform Method

Assume that the image is of size $N \times N$ and the filter response is of size $M \times M$. The padded size, N_p , is the smallest integral power of two which satisfies $N_p \geq N + M - 1$.

The cost of computing the 2-D DFT is discussed in Appendix A.1. Two methods are presented for efficiently computing the 2-D DFT, both based on Fast Fourier Transform (FFT) principles. The vector radix FFT method offers a small cost advantage, but the row-column decomposition method is easier to implement. More importantly, the latter method can make better use of the cost saving measures described in Section 6.1.3. The cost of an $N_p \times N_p$ DFT using the row-column method is:

$$\begin{aligned} \text{Number of FLOPs} &= 10N_p^2 \log_2 N_p \\ \text{Number of MAs} &= 10N_p^2 \log_2 N_p \end{aligned} \quad (6.5)$$

The transform filtering method requires three such transforms: transform the original image, transform the filter response and inverse transform the result image. (When several images are to be filtered with the same response, the filter need only be transformed once. If a *fixed* filter is being used, the computational cost is reduced by about one third.)

The filtering itself is performed by multiplying the transformed image by the transformed filter, element by element. For each of the N_p^2 result values, two memory reads, a multiplication and a memory write are performed. However, these are *complex* values, not real values. A complex multiply actually consists of four multiplications and two additions, and memory accesses are doubled. As a result, the filtering step requires:

$$\begin{aligned} \text{Number of FLOPs} &= 6N_p^2 \\ \text{Number of MAs} &= 6N_p^2 \end{aligned} \quad (6.6)$$

For the entire process, consisting of three image transforms and the image multiply:

$$\begin{aligned} \text{Number of FLOPs} &= 30N_p^2 \log_2 N_p + 6N_p^2 \\ \text{Number of MAs} &= 30N_p^2 \log_2 N_p + 6N_p^2 \end{aligned} \quad (5.7)$$

The number of FLOPs and the number of MAs happen to be identical in this case. These two cost measures will remain identical, or very nearly so, throughout the remainder of the analysis. Therefore, only FLOPs will be explicitly considered. The reader can infer memory access behaviour from floating point performance.

Figure 6.1 shows a comparison of the two methods. The "staircase effect" evident with the transform method is due to the required zero-padding. Since the padded size, N_p , of the image must be a power of two, the zero-padding is often significantly larger than required for linear convolution. Therefore, small increases in image size, N , often have no effect on N_p and the filtering cost. The same is true for increases in the filter response size, M . This is evident from the frequent convergence of the different lines.

There are certain redundancies involved in the Fourier transforms of real data. Fortunately, these can be used to reduce the complexity of calculating the transforms. Using these as a basis should provide a more useful comparison of our two methods. (Note that there is no similar redundancy for direct convolution).

Figure 6.1: Complexity Comparison of Two Basic Methods

6.1.3 Exploiting Redundancy in the Transform Method

The basic redundancy can be seen in the one-dimensional Fourier transform of a set of real data of length N . The real part of the transform is *even* about point $\frac{N}{2}$, while the imaginary part is *odd* [87]. (Assuming that the points are indexed from 0 to $N-1$). In other words, the transform is *conjugate symmetric* about its midpoint. This implies that only half of the elements (actually $\frac{N}{2} + 1$) are distinct.

By packing the N real data values into an $\frac{N}{2}$ length complex array, the transform can be calculated using an $\frac{N}{2}$ length FFT [87]. Actually, the $\frac{N}{2}$ length FFT result is used to generate the correct, symmetric transform of the original data. This process is described in Appendix A.2.

This redundancy also manifests itself in the two-dimensional Fourier transform of a set of real data ($N \times N$). After each real row is transformed, the useful information in each is stored in the first $\frac{N}{2} + 1$ elements. This implies that only the first $\frac{N}{2} + 1$ columns need to be transformed to get the result. (As it turns out, the transform of a real image is conjugate symmetric in the diagonal sense, as shown in Appendix A.3). The full image transform is determined by filling in the remaining columns with values from the transformed ones.

This two-dimensional redundancy is especially helpful for the transform filtering method, since convolving a real image with a real filter produces a real result image. This implies that only half of the image transform, the first $\frac{N}{2} + 1$ columns, needs to be maintained throughout the filtering operation.

These revisions to the transform filter method are summarized here in point form:

1. Transform each of the N_p rows using the real data method described in Appendix A.2. The total cost to transform all of the rows this way is:

$$\begin{aligned} \text{Number of FLOPs} &= N_p \left(\frac{5}{2} N_p \log_2 N_p + 2N_p \right) \quad (6.8) \\ &= N_p^2 \left(\frac{5}{2} \log_2 N_p + 2 \right) \end{aligned}$$

2. Transform each of the $N_p/2 + 1$ columns of the partially transformed image, using a conventional N_p -length FFT. The cost to transform all of the columns is:

$$\text{Number of FLOPs} = \left(\frac{N_p}{2} + 1 \right) \frac{10}{2} N_p \log_2 N_p \quad (6.9)$$

The total cost to transform the image is:

$$\begin{aligned} \text{Number of FLOPs} &= N_p^2 \left(\frac{5}{2} \log_2 N_p + 2 \right) + N_p^2 \left(\frac{5}{2} \log_2 N_p \right) \quad (6.10) \\ &\quad + \frac{10}{2} N_p \log_2 N_p \\ &= N_p^2 (5 \log_2 N_p + 2) + 5 N_p \log_2 N_p \\ &\simeq N_p^2 (5 \log_2 N_p + 2) \end{aligned}$$

3. Repeat steps (1) and (2), transforming the filter response zero-padded to a size of N_p .
4. Generate the transform of the result image by multiplying the image transform by the filter transform. These are all N_p rows by $N_p/2 + 1$ columns.

The cost to multiply the image by the filter is:

$$\begin{aligned} \text{Number of FLOPs} &= N_p \cdot \left(\frac{N_p}{2} + 1\right) \cdot 6 \\ &= 3N_p^2 + 6N_p \end{aligned} \quad (6.11)$$

5. Inverse transform the result image, using the reverse of steps (1) and (2) with inverse FFTs.

The total cost of the revised transform filter method is comprised, again, of three transform stages and one transform multiply:

$$\begin{aligned} \text{Number of FLOPs} &= N_p^2(3(5\log_2 N_p + 2) + 3) \\ &\quad + 5N_p \log_2 N_p + 6N_p \\ &\simeq N_p^2(15\log_2 N_p + 9) \end{aligned} \quad (6.12)$$

A revised comparison of the two methods is shown in Figure 6.2. The transform method still increases in steps, but has improved by about a factor of two compared to the earlier version.

One very noticeable difference between the two methods is the heavy dependence of the direct convolution cost on the filter response size, M . Equation 6.4 indicates that this increases with the square of M . Because of this quick increase, the transform method is more efficient for larger response sizes.

Appendix B includes more graphs, showing similar comparisons for different response sizes. The trade off point depends on the image size, but occurs at roughly $M = 13$ on average. Filter sizes of 7 or smaller *always* favour direct convolution.

Even for fairly small filter response sizes, the transform method compares quite well in certain cases. In particular, when the image size plus the necessary padding is a power of two, the transform method is at its most efficient. These points appear on the graph as the bottom corners, just before a step increase.

Figure 6.2: Comparison with Revised Transform Method

6.2 Block Methods

The image filtering process is often quite localized in nature. That is, the processing of one area of the image is often independent of the processing of other areas. This is because each result pixel depends only on the pixels in a region surrounding the corresponding pixel in the source image. To be precise, this *region of dependence* is the size and shape of the filter's impulse response, with its origin at the location of the result pixel.

Consider a square block of result pixels, called the *result block*. To calculate these results a second block, called the *source block*, is required. The source block

Figure 6.3: Source and Result Block Relationship

surrounds the result block and is comprised of the union of all regions of dependence of the result pixels. This is illustrated, for an impulse response with a square region of dependence, in Figure 6.3.

There are two basic methods for processing an image by separately processing its constituent blocks [25, sect. 3.2.3]:

1. Overlap-and-Add Method (Figure 6.4)

- The image is partitioned into a set of disjoint blocks of size $N_{rb} \times N_{rb}$.
- Each block is linearly convolved with the filter impulse response, creating a block of size $N_{sb} \times N_{sb}$.
- The processed blocks are overlapped and added together. The overlap is aligned such that the centre $N_{rb} \times N_{rb}$ regions of the processed blocks disjointly form the result image, as in the original partition.

The inter-block dependence is accounted for by adding together the overlapped areas. This is, essentially, an example of superposition, which is valid since linear convolution is used.

Figure 6.4: Overlap-and-Add Method

2. Overlap-and-Save Method (Figure 6.5)

- The image is divided into a set of overlapping blocks of size $N_{sb} \times N_{sb}$. The overlap alignment is identical to that of the processed blocks in the overlap-and-add method. (Where blocks overlap outside the image, the data is zero.)
- The blocks are convolved linearly or circularly. Either method can be used, since only the central $N_{rb} \times N_{rb}$ region will be kept.
- The central $N_{rb} \times N_{rb}$ regions are combined disjointly to form the result image.

This method relates directly to the earlier description of the source block and the result block. Each extracted, overlapping block contains all the information needed to generate the corresponding result block.

Figure 6.5: Overlap-and-Save Method

Both of these methods have the same memory requirements and the same convolution costs. The overlap-and-save method has the advantage that it doesn't require the extra cost of adding result blocks together.

6.2.1 Block Processing with Direct Convolution

Equation 6.4 indicates the number of operations required to convolve an $N \times N$ image. This cost is proportional to N^2 , for both FLOPs and MAs.

Using either block method, processing consists of one $N_{rb} \times N_{rb}$ linear convolution per block. If there are K blocks, the total cost is:

$$\text{Number of FLOPs} = K(2N_{rb}^2 M^2) = 2KN_{rb}^2 M^2$$

$$\text{Number of MAs} = K(2N_{rb}^2 M^2) = 2KN_{rb}^2 M^2$$

However, the K result blocks make up the $N \times N$ result image. Equating the total number of pixels yields:

$$KN_{rb}^2 = N^2 \quad (6.13)$$

In other words, the cost of block processing with direct convolution is identical to that of directly convolving the entire image.

6.2.2 Block Processing with the Transform Method

Transform filtering can be used with either block method. Circular convolution, of size $N_{sb} \times N_{sb}$, is sufficient for the overlap-and-save method, and can be done using $N_{sb} \times N_{sb}$ Fourier transforms. The linear convolution required by the overlap-and-add method can be performed using Fourier transforms by zero-padding the block to $N_{sb} \times N_{sb}$. (N_{sb} must be a power of 2 if the FFT algorithm is to be used.)

The results of Section 6.1.3 can be used to determine the cost of processing each block. The number of blocks required to complete the image must be calculated. The overlap-and-save method will be used.

Nominally, the number of blocks can be defined as the number of pixels in the image divided by the number of pixels per block. However, care must be taken when this ratio is not an integer. Figure 6.6 illustrates such a case, wherein the image size is not an integer multiple of the result block size. Typically, N_{sb} is a convenient power of 2 and M is application specific. So, the user doesn't have free control over N_{rb} ($N_{rb} = N_{sb} + 1 - M$). It is likely that $\frac{N}{N_{rb}}$ is *not* an integer.

Fortunately, the odd-sized edge blocks can be processed easily by using extra zero-padding to fill them out. This is shown by dotted lines in Figure 6.6.

The number of blocks needed to make up the image is:

$$K_b = \left(\lceil \frac{N}{N_{rb}} \rceil \right)^2 \quad (6.14)$$

Where $\lceil$ is the *ceiling* operator, which returns the smallest integer greater than or equal to the argument.

For each block, processing consists of a two-dimensional FFT, a transform multiply and a two-dimensional inverse FFT, each of size $N_{sb} \times N_{sb}$. Also, a single $N_{sb} \times N_{sb}$

Figure 6.6: Uneven Block Partitioning

FFT is required for the filter response. Using Equations 6.10 and 6.12, the total cost of the block method is:

$$\begin{aligned} \text{Number of FLOPs} = & (2K_b + 1)(N_{sb}^2(5\log_2 N_{sb} + 2)) \\ & + K_b(3N_{sb}^2 + 6N_{sb}) \end{aligned} \quad (6.15)$$

Figure 6.7 compares the block transform method with the whole image transform method. The comparison is based on a response size of 19 and includes a number of source block sizes. Comparisons for other response sizes are included in Appendix B. The graphs provide some useful insights regarding the relative performance levels of these methods:

- The whole image transform method rarely provides the best performance. This only occurs when the response size, M , is quite large and very little extra zero-padding is required. (The second condition, equivalent to $N + M$ slightly less than a power of 2, corresponds to the bottom right corner of a "step" on the whole image curves).

Figure 6.7: Block Transform Method for $M=19$

- As responses get larger, larger block sizes tend to outperform smaller ones. The different block sizes are roughly equivalent for the smallest response, but costs diverge as the response size increases.

Of all the methods considered, the block transform method has the best overall performance. Optimal block size varies with filter and image size, but larger blocks outperform smaller blocks in most cases.

It is important to note, however, that no single method is optimal for all image and filter sizes. This suggests that some form of adaptability may be useful. For example, rather than choosing a fixed block size, this can be set dynamically for each image/filter combination. The graphs indicate that the gain will not often be

great, but very little effort is required to do this.

Similarly, the whole image transform method could be chosen when conditions merit. However, these conditions are quite rare, and later chapters will illustrate some important benefits to using block-oriented methods only.

Section 6.1 showed that direct convolution is the most effective method for very small filter response sizes. If small responses are expected to be common, this may be a worthwhile option to be included in an adaptive system. Direct convolution *can* be used for block-oriented processing

Figure 6.8 shows the performance of the block transform method with dynamically chosen block size. This method provides the best performance, except for the specific cases previously described.

It is important to realize the cost of transforming the filter has been included. If a specific filter is to be used repeatedly, it need only be transformed once and saved. This scenario would benefit the whole image transform method.

The block transform method is best when new filters need to be created, such as in an iterative deblurring application.

Figure 6.8: Block Transform Method: Dynamically Chosen Block Size

Chapter 7

Parallel Image Filtering

In Chapter 6, the computational cost of linear image filtering was analyzed. In this chapter, the effects of performing steps in parallel are considered. The multiprocessor system is made up of DSPs communicating via a slotted ring network.

7.1 Goals and Criteria

The following factors are foremost in comparing parallel processing methods:

- **Decreased Processing Time:** Parallel methods are employed to decrease the required processing time. Ideally, processing will be K_{PE} times faster, where K_{PE} is the number of processors. For analysis, processing time will be measured in terms of parallel FLOPs and MAs.
- **Number of Inter-PE Transfers:** All parallel methods will require data to be transmitted between PEs. The number of inter-PE transfers (XFERs) should be minimized, since they represent an added cost (ie. they take time).
- **Transparency of Inter-PE Transfers:** All XFERs are memory-mapped and buffered. This implies that direct memory access (DMA) can be used effectively for XFERs of blocks stored contiguously in memory. In many cases, it will be possible to perform useful processing on one block of data while another block of data is being XFERed by DMA. Since the transfers are

concurrent with processing, their cost is "hidden". Any XFERs which can be hidden in this way are called *transparent*. As many XFERs as possible should be hidden, since they don't add to processing time. (A small, fixed cost is required to set up DMA, however).

- **Extendibility:** Some methods may perform poorly, or be impossible, when K_{PE} differs from a certain set of values. It would be preferable if any number of PEs could be used effectively. This allows *gradual* increase in system size, as performance requirements grow.

7.2 Parallel Methods

7.2.1 Block Parallel Method

Conceptually, this is the simplest method to be considered. In Section 6.2, it was shown that an image can be divided into blocks which can be processed independently. These blocks can also be processed *simultaneously* on separate PEs.

The block parallel method can be summarized as follows:

1. The master PE (MPE) distributes an image block to each slave PE (SPE).
2. Each PE, including the MPE, filters an image block.
3. The MPE collects a filtered block from each SPE and writes the blocks into a result image.
4. Steps 1 to 3 are repeated until all image blocks have been filtered.

Note:

- Each iteration of steps 1 to 3 is called a *stage*. If the number of blocks is not an integer multiple of K_{PE} , one or more PEs will be idle during the last stage.
- All PEs must have a copy of the filter response.
- Filtering is done in the frequency domain, using Fourier transforms. (Direct spatial convolution is more efficient for very small filter sizes only).

7.2.2 Row-Column Method

A 2-D DFT can be calculated by transforming all of the rows and then all of the columns. This suggests one way to partition the 2-D DFT problem over a set of K_{PE} processors (assume an $N \times N$ transform) [88]:

1. The N rows are distributed over the K_{PE} processors.
2. The rows are transformed. Each processor performs $\frac{N}{K_{PE}}$ 1-D FFTs.
3. The image is transposed. Each processor now holds $\frac{N}{K_{PE}}$ columns.
4. The columns are transformed. Again, each processor performs $\frac{N}{K_{PE}}$ 1-D FFTs.
5. If necessary, the image transform is transposed again to return to row major ordering.

Figure 7.1 illustrates the transpose step for four processors. Each processor exchanges an $\frac{N}{K_{PE}} \times \frac{N}{K_{PE}}$ block with each other processor. This can be seen as transposing a $K_{PE} \times K_{PE}$ matrix of blocks. After the interprocessor transfers, each block is itself transposed in the conventional sense to make the columns contiguous in memory. There has been research into transpose methods for parallel systems [89, 90]. However, this work is not applicable to the architecture being considered.

The result of this operation is that the rows (or columns) of the 2-D transform are distributed over the PEs. If the same method is used to transform the filter response, corresponding elements of the two transforms will always be on the same PE. This facilitates performing the filter multiplication step in parallel. The inverse transform can also be calculated using the row-column method.

The row-column filtering method can be summarized as follows:

1. Transform the filter response (full padded image size) in parallel, using the row-column method. The final transpose is unnecessary.
2. Transform the entire (padded) image in parallel, using the row-column method. Again, don't perform the final transpose.

Figure 7.1: Block Transpose for 4 Processors

3. Each PE now holds *corresponding* columns of the image and filter transforms. All image transform elements are multiplied by the corresponding filter transform elements.
4. Inverse transform the filtered image by reversing the row-column method (ie. inverse transform columns, transpose, inverse transform rows).

There is an important factor related to the above description of the row-column parallel 2-D FFT. It was assumed that the transform size, N , is an integer multiple of the number of processors, K_{PE} . Practically, this implies that K_{PE} is a power of 2.

The row-column method can still work if K_{PE} is *not* a power of 2. However, data distribution becomes unbalanced. The number of rows per PE becomes $\lceil \frac{N}{K_{PE}} \rceil$, but one processor will have *fewer* rows than the rest. This PE will have fewer transforms to perform and, thus, will be idle at times. In other words, the row-column method is less efficient if K_{PE} is not a power of 2.

A whole image transform was described above, but the row-column method can be used with block processing as well. The above filtering steps are repeated for each image block in sequence. (The filter response need only be transformed once). This process will be referred to as the *block row-column method*.

7.2.3 Vector Radix Method

The vector radix method for calculating the 2-D DFT is described in Appendix A.1. The basis for this method is the repeated partitioning of an $N \times N$ DFT into $\frac{N}{2} \times \frac{N}{2}$, $\frac{N}{4} \times \frac{N}{4}$ and smaller DFTs, down to 2×2 . The transform can be done in parallel by distributing these 2×2 DFTs over multiple processors [88].

Suppose that K_{PE} processors are available, where K_{PE} is the square of an integer. Furthermore, assume N is an integer multiple of K_{PE} . The $N \times N$ image data can be distributed, as a set of square blocks, over the K_{PE} processors. This is shown in figure 7.2 for the case $K_{PE} = 16$.

Recall from Appendix A.1 that the $N \times N$ FFT consists of $\log_2 N$ stages, each of which is composed of $\frac{N^2}{4}$ 2×2 butterflies. In this case, the butterflies of each stage are performed in parallel on the K_{PE} processors. The butterflies of each stage are independent, but there is significant spatial interdependence between stages.

Figure 7.3 shows the spatial orientation of the in-place butterflies required to transform an 8×8 image. (The orientation shown corresponds to bit-reversed ordering in both dimensions). Consider the following cases:

- $K_{PE} = 16$: Each processor does only one butterfly per stage. However, the

Figure 7.2: Partitioning Grid for 16 Processors

data must then be *realigned* between each stage. This realignment is needed to collect the data required for each butterfly together on the same processor.

- $K_{PE} = 4$: Each processor does four butterflies per stage. Note that realignment is required after the second stage but *not* after the first. The processors already hold the correct data to perform the second stage.
- $K_{PE} = 1$: The single processor does all sixteen butterflies of each stage. There is no realignment.

As K_{PE} increases, the stages get faster but more realignment is required. Of the $\log_2 N$ stages, $\log_2(\frac{N}{\sqrt{K_{PE}}})$ require no prior realignment. (This includes the first stage, which will be set up correctly by the original distribution). So, the number of realignment steps required is $\frac{1}{2} \log_2 K_{PE}$.

Figure 7.3: Spatial Orientation of 8x8 Butterfly Stages

As with the row-column method, the resulting transform is distributed over all PEs. This allows parallel filter multiplication, as long as the filter response was also transformed using the vector radix method. Also, the inverse transform can be performed by reversing the forward transform sequence. So, all filtering steps can be done in parallel.

The vector radix method requires a very uniform distribution of data over the PEs. In particular, the data on each PE should meet two criteria:

1. The blocks of data on all PEs must be identical in size and shape.
2. The lengths of both edges of the data block must be a power of 2.

These conditions allow an even distribution of 2×2 butterflies. If these conditions are not met, distribution and realignment become difficult and irregular.

If the transform size is a power of 2, the above conditions imply that the number of PEs must be a power of 2. (The earlier stipulation that K_{PE} be the square of an integer is unnecessary. The explanation was simpler with a *square* grid of blocks.) So, the vector radix method *does not* provide good extendibility.

The use of DMA requires that data to be transferred is stored contiguously in memory. This is *not* true of data to be used in distributed butterfly calculations. So, DMA cannot be used for inter-PE XFERs with the vector radix method and, thus, the XFERs are *not* transparent.

The vector radix method will not be considered further, due to its poor extendibility and lack of XFER transparency.

7.3 Computational Cost

In Chapter 6, computational cost was measured in terms of floating point arithmetic (FLOPs) and memory access (MAs). Actually, the two measures behaved very similarly. So, for simplicity, only FLOPs were explicitly analyzed.

In this chapter, similar measurements will be used. However, the parallel nature of the processing must be considered. During the time it takes one processor to perform a FLOP, all other processors can perform a FLOP as well.

A parallel task is complete when *all* processors have finished. So, the computational cost of a task is determined by *the greatest cost incurred by a single processor*.

7.3.1 Block Parallel Method

A stage has been defined as the simultaneous filtering of a block on each PE. The cost of each stage will be equal to the cost of processing one block on a single PE. The number of stages required to process an image is:

$$K_S = \lceil \frac{K_B}{K_{PE}} \rceil \quad (7.1)$$

Where:

- K_B = The number of blocks in the image
- K_{PE} = The number of PEs

The cost of the block parallel method is determined by substituting the number of stages for the number of blocks in Equation 6.15:

$$\begin{aligned} \text{Number of MAs} &= \\ \text{Number of FLOPs} &= (2K_s + 1)(N_{sb}^2(5\log_2 N_{sb} + 2)) \\ &\quad + K_s(3N_{sb}^2 + 6N_{sb}) \end{aligned} \quad (7.2)$$

Where N_{sb} is the source block size.

7.3.2 Row-Column Method

The number of rows distributed to each PE for the row-column method is:

$$N_r = \lceil \frac{N_p}{K_{PE}} \rceil \quad (7.3)$$

Where N_p is the padded image size.

Consider the effect this has on processing, in comparison to serial processing:

- The number of real row FFTs required on a processor is reduced from N_p to N_r . (This is true for *all* transform steps, forward and inverse).

- The number of complex column FFTs required on a processor is reduced from roughly $\frac{N_p}{2}$ to $\frac{N_r}{2}$. (Again, this is true for all transform steps.)
- The number of image transform columns to be multiplied by filter transform columns on a processor is reduced from roughly $\frac{N_p}{2}$ to $\frac{N_r}{2}$.

In other words, all costs have been reduced by the ratio $\frac{N_r}{N_p}$. The floating point arithmetic costs have, thus, been reduced to the following (refer to Equation 6.12):

$$\begin{aligned} \text{Number of FLOPs} &= \frac{N_r}{N_p} [N_p^2 (15 \log_2 N_p + 9)] \\ &= N_p \left(\lceil \frac{N_p}{K_{PE}} \rceil \right) (15 \log_2 N_p + 9) \end{aligned} \quad (7.4)$$

The same argument holds true for memory access. However, there is an additional cost to transpose a data block on each PE (the shaded $N_r \times N_r$ block in Figure 7.1). The other blocks can be transposed during the inter-PE transfers.

$$\begin{aligned} \text{Number of MAs} &= \frac{N_r}{N_p} [N_p^2 (15 \log_2 N_p + 9)] + N_r^2 \\ &= N_p \left(\lceil \frac{N_p}{K_{PE}} \rceil \right) (15 \log_2 N_p + 9) + \left(\lceil \frac{N_p}{K_{PE}} \rceil \right)^2 \end{aligned} \quad (7.5)$$

7.3.3 Block Row-Column Method

The row-column method is applied to each image block in sequence. In this case, the cost decrease ratio is $\frac{N_r}{N_{sb}}$, where:

- $N_r = \lceil \frac{N_{sb}}{K_{PE}} \rceil$
- N_{sb} = Source block size

Applying this decrease to the block method cost described in Equation 6.15:

$$\begin{aligned} \text{Number of FLOPs} = \frac{N_r}{N_{sb}} [(2K_b + 1)(N_{sb}^2(5\log_2 N_{sb} + 2)) \\ + K_b(3N_{sb}^2 + 6N_{sb})] \end{aligned} \quad (7.6)$$

Where K_b is the number of blocks in the image.

The number of memory accesses is identical, with the addition of a transpose for each block processed:

$$\begin{aligned} \text{Number of MAs} = \frac{N_r}{N_{sb}} [(2K_b + 1)(N_{sb}^2(5\log_2 N_{sb} + 2)) \\ + K_b(3N_{sb}^2 + 6N_{sb})] + K_b N_r^2 \end{aligned} \quad (7.7)$$

7.3.4 Cost Comparison

Figure 7.4 shows a comparison of the computational costs of the three methods being considered. This comparison illustrates two points:

1. The two block methods behave very similarly, but the block row-column method has slightly lower costs for virtually all image sizes.
2. The row-column method behaves similarly to the serial entire image method. That is, it has much higher costs except for certain specific image sizes.

It should be noted that source block size is dynamically selected based on minimized FLOPs.

Figure 7.4 shows a single, representative case. Several more cases are shown in Appendix C.

Figure 7.4: Parallel Computational Cost Comparison

7.4 Inter-PE Communication Cost

There are three type of inter-PE transfer (XFER) needed for the methods being considered:

1. **Distribution:** Initially, the entire image is stored on the master PE. Each parallel method requires that image data be distributed among all PEs. [**MPE writes to all SPEs**]
2. **Collection:** The result image should be stored on the master PE upon completion. So, filtered image data must be collected from all PEs. [**All SPEs write to MPE**]
3. **Transpose:** Row-column methods require a transpose step to collect columns together on PEs. The transpose involves an exchange of data between *each* pair of PEs. [**Each PE writes to all other PEs**]

Like FLOPs and MAs, XFERs originating on different PEs can occur in parallel. However, XFERs don't all necessarily take the same amount of time. Two XFERs started simultaneously on different PEs may be completed at different times because:

- One XFER may have to travel further around the ring than the other.
- One XFER may have to wait longer for an available slot.

Fortunately, because of buffering, the time to *initiate* an XFER is fixed. (In fact, only a memory write is needed to start an XFER.) For analysis purposes, XFERs will be said to occur in parallel if they can be initiated simultaneously.

7.4.1 Block Parallel Method

During distribution, all XFERs start from the master PE. Thus, none of these can be done in parallel. The number of source blocks distributed is the total number of blocks minus those processed on the master PE:

$$\text{Distribution XFERs} = (K_b - K_s)N_{sb}^2 \quad (7.8)$$

Where:

- K_b = The number of blocks in the image
- $K_s = \lceil \frac{K_b}{K_{PE}} \rceil$ = The number of stages
- N_{sb} = Source block size

The filter response must be distributed as well, but this can be done efficiently by sending filter parameters only. Each PE can then build its own filter response.

During collection, XFERs start on each slave PE and *can* be done in parallel. That is, N_{sb}^2 XFERs occur in parallel during collection of each stage:

$$\text{Collection XFERs} = K_s N_{rb}^2 \quad (7.9)$$

Where N_{rb} is the result block size.

There is no transpose required, so:

$$\text{Total XFERs} = (K_b - K_s)N_{sb}^2 + K_s N_{rb}^2 \quad (7.10)$$

7.4.2 Row-Column Method

Again, distribution XFERs must occur serially. Rows are sent to all PEs except the master PE:

$$\begin{aligned} \text{Distribution XFERs} &= N_p N_r (K_{PE} - 1) \\ &= N_p \left(\lceil \frac{N_p}{K_{PE}} \rceil \right) (K_{PE} - 1) \end{aligned} \quad (7.11)$$

Where N_p is the padded image size.

During collection, the rows can be returned in parallel from all SPEs:

$$\begin{aligned} \text{Collection XFERs} &= N_p N_r \\ &= N_p \lceil \frac{N_p}{K_{PE}} \rceil \end{aligned} \quad (7.12)$$

The transpose step consists of a set of exchanges between each pair of PEs. After row transforms the row data is complex, but only half is needed because of the real data transform symmetry:

$$\begin{aligned} \text{Size of transpose block} &= N_r \cdot \frac{N_r}{2} \cdot 2 \\ &= N_r^2 \end{aligned} \quad (7.13)$$

Each PE must write a transpose block to all other PEs. The PEs can all write simultaneously (in parallel):

$$\begin{aligned} \text{Transpose XFERs} &= N_r^2 (K_{PE} - 1) \\ &= (\lceil \frac{N_p}{K_{PE}} \rceil)^2 (K_{PE} - 1) \end{aligned} \quad (7.14)$$

The total number of XFERs consists of distribution, collection and *three* transpose steps, for the three transforms:

$$\begin{aligned} \text{Total XFERs} &= N_p N_r (K_{PE} - 1) + N_p N_r + 3 N_r^2 (K_{PE} - 1) \\ &= N_p N_r K_{PE} + 3 N_r^2 (K_{PE} - 1) \end{aligned} \quad (7.15)$$

7.4.3 Block Row-Column Method

This method behaves like the row-column method, with two exceptions:

1. Image blocks, of size $N_{sb} \times N_{sb}$, are being transformed instead of the full padded image.

2. The operation is repeated for each block (K_b times). The filter transpose, however, happens only once.

So:

$$Total\ XFERs = K_b N_{sb} N_r K_{PE} + (2K_b + 1) N_r^2 (K_{PE} - 1) \quad (7.16)$$

Where $N_r = \lceil \frac{N_{sb}}{K_{PE}} \rceil$.

7.4.4 Cost Comparison

Figure 7.5 shows a comparison of the communication costs of the three methods being considered. The performance appears quite similar to the computational costs, except for the following:

1. The block parallel method has lower communication costs than the block row-column method, for most image sizes.
2. The costs of the block methods don't increase smoothly with image size. This is because source block size is chosen based on minimized FLOPs. Minimizing FLOPs *does not* necessarily minimize XFERs.

Figure 7.5 shows a single, representative case. Several more cases are shown in Appendix C.

7.5 Further Inter-PE Communication Considerations

The term *transparent* has been used to describe XFERs which can be initiated by DMA occurring simultaneously with processing. Transparency can be used to reduce the apparent communication costs and, thus, the processing time.

Processing time is also affected by *ring usage*. Ring usage describes how effectively the available slots are used. The data to be XFERed should be packed as

Figure 7.5: Parallel Communication Cost Comparison

densely as possible into the slots. Any unused slot is wasted, leading to a decrease in the overall communication transfer rate.

Transparency and ring usage are difficult to analyze quantitatively. These factors, and their effects on the methods being considered, are discussed in the following sections.

7.5.1 Transparency

There are two primary requirements for a set of XFERs to be transparent:

1. The source data must be stored contiguously in memory. A single contiguous memory block can be copied with one DMA call. Ideally, DMA can also be used to unload the buffer on the target PE. This requires that the destination also be a contiguous block of memory.
2. The use of DMA is only beneficial if other work is being done at the same time. XFERs are truly transparent only if *useful* processing is occurring simultaneously. (Useful processing actually *needs* to be done. It can't be added just to keep the processor busy.) For full transparency, the processing must last until the set of XFERs is *completely* done. This includes not only loading data into the buffer, but the ring transfers and unloading the destination PE buffer as well.

The term *transparency* may lead one to believe that XFERs started by DMA have no cost at all. This is not true, as the following illustrates:

- There is a small, fixed cost to start any DMA process. Typically, this consists of writing to a number of control registers.
- DMA and processing usually share the same memory bus. When the actual processing accesses memory, it contends with DMA for use of the bus. Processing normally has priority, but it still can be slowed down slightly. This is especially true if processing involves a lot of memory access.

Transparency with the Block Parallel Method

The block parallel method involves distribution and collection of square blocks. Blocks within an image are not stored contiguously, but the *rows* of the blocks are. Furthermore, the filtering process begins and ends with a set of row transforms. The following method, called the *row pipeline*, uses block rows to achieve a high degree of transparency:

Distribution (with Row Transforms):

1. The master PE distributes the first row of each block to the corresponding slave PE.

2. The master PE initiates the distribution of the next (2nd) set of rows using DMA. During DMA, the previous (1st) row on each PE is transformed. After each row arrives at a slave PE, DMA is started to unload the row from the buffer.
3. Step 2 is repeated, until all rows are distributed. During each row distribution, the previous row on each PE is transformed.
4. The final row is transformed on each PE.

Column Processing: On each PE, each column is transformed, multiplied by the corresponding filter response column and inverse transformed.

Collection (with Inverse Row Transforms):

1. The first row is inverse transformed on each PE.
2. Each slave PE initiates XFER of the previous (1st) row using DMA. During DMA, the next (2nd) row is inverse transformed. After each row arrives at the master PE, DMA is started to unload it from the buffer.
3. Step 2 is repeated until all rows are inverse transformed. Before transforming, each slave starts the DMA XFER of the previous row.
4. The final inverse transformed row on each slave PE is XFERed to the master.

So, almost all of the XFERs for the block parallel method can be made transparent. The effectiveness of the row pipeline depends heavily on *how many* XFERs can be completed during a row transform of given length. The effective DMA transfer rate, *during* a row FFT, must be sufficient to complete all DMA calls during the processing interval. The effective ring transfer rate must be greater than or equal to the DMA transfer rate. These transfer rates and, thus, the number of completable XFERs are determined by the system hardware.

Transparency with Row-Column Methods

During distribution and collection, the row-column methods can use the row pipeline described in the previous section. However, the row-column methods also require transposes.

The transpose operation involves a swap of blocks between each pair of PEs. Also, these blocks must be transposed themselves. There are two ways to accomplish this:

1. Transpose each block *during* the transfers. This can be done by loading the source PE buffer in row-major order and unloading the destination PE buffer in column-major order (or vice-versa). Unfortunately, DMA can only be used for row-major ordered access.
2. Explicitly transpose each block before or after the transfers. This allows DMA at each end, but *adds* the memory access cost of transposing each block.

NOTE: The computational cost analysis in Section 7.3 assumed that the *first* method was used.

The transpose steps are followed by column transforms. Unfortunately, the PEs don't hold any complete columns until after the transpose step is completed. This implies that entire columns can't be transformed simultaneously with the transpose XFERs. There are some other possibilities for useful simultaneous processing:

- The FFT algorithm is based on dividing a transform into pieces and combining the results. In a similar way, column "pieces" can be transformed as they arrive and combined when the transpose is complete. (This is only feasible if K_{PE} is a power of 2.)
- Transpose method 2, described earlier, required that blocks be explicitly before or after transfer. One block can be transposed while another is being transferred. However, a transpose consists entirely of memory accesses. Heavy memory contention between DMA and the transpose would be expected.

Neither of the above tasks can ideally be performed simultaneously with transpose XFERs. Also, it has been shown that DMA can't be used for all XFERs without adding to memory access costs. Transparency of the transpose XFERs is difficult at best.

7.5.2 Ring Usage

All XFERed data must travel over the ring network, from the source PE's outgoing buffer to the destination PE's incoming buffer. The time required for this transmission is an important part of the cost of XFERs. This time consists of two intervals:

1. An outgoing XFER must wait for an empty, available slot to arrive at the source PE.
2. The XFERed word must travel around the ring to the destination PE. This travel time is directly related to the separation of the two PEs.

A second XFER from the same PE can be started *before* the first has reached its destination. In other words, XFERs can be pipelined on the ring as long as enough empty slots are available. The XFERs are performed in the same order as they are written to the buffer (ie. the buffer is a *queue*).

To achieve efficient ring usage, the slot waiting time for XFERs should be minimized on all PEs. Also, unused slots should be as few as possible. Usage is primarily controlled by restricting ring access.

The following list describes the general types of ring access used in the systems mentioned in Chapter 4. This list is *not* intended to represent *all possible* ring access methods.

- **Open Access:** It is not actually necessary to restrict access to the ring. In this case, any PE can use any slot for any XFER, provided that the slot is empty.

- **Source Limited Access:** Ring access can be restricted by reserving slots for use by specific source PEs. A particular slot can only be written to by a single, specific PE.
- **Destination Limited Access:** Slots are reserved for XFERs to specific destinations. The destination PE can be specified in two ways:
 1. Absolute: The destination is an actual, physical PE. (eg. *This slot can XFER to PE number i*)
 2. Relative: The destination is specified *relative* to the source PE. (eg. *This slot can move data i locations along the ring*)

In terms of minimizing waiting times, *restricting* access may seem counter-productive. However, waiting times for *all* PEs must be considered. The restrictions can actually provide balance, ensuring that no PE is "locked out" by the other PEs.

The image filtering methods being considered all require that each PE both receive and transmit data. This fact has a bearing on access restrictions:

- If source limited access is used, there must be available slots for *all* PEs to write into.
- If destination limited access is used, there must be destination slots for *all* PEs.

The upcoming sections consider the effects of ring access on the three required XFER types described in Section 7.4

Distribution

During distribution, the master PE sends data to all slave PEs. The row pipeline, discussed earlier, allows a high degree of XFER transparency. Data is XFERred a row at a time to each PE.

Open Access:

Since there is only one source PE, open access is very effective for distribution. All slots will be filled and the full ring transfer rate will be realized.

Source Limited Access:

Each PE requires a reserved slot. So, all slave PE slots will be *unused*. The effective ring transfer rate is reduced by a factor of K_{PE} .

Destination Limited Access:

Again, each PE requires a reserved slot. Since all slave PEs are destinations, all slots except one will be used. However, the row pipeline dictates that data be XFERed a row at a time. During the XFER of a row, there is only *one* destination PE. So, the effective transfer rate is again reduced to $\frac{1}{K_{PE}}$ of the peak rate.

It is possible to use almost the full rate. However, this would require abandoning the row pipeline for an *interleaved* XFER method. (XFERs of rows to each slave are interleaved to match the arrival order of destination slots.) Transparency would no longer be possible.

Collection

Collection, like distribution, can be performed with good transparency using the row pipeline. During collection, however, all slave PEs write simultaneously to the master PE.

Open Access:

Open access ensures that all slots are used. However, it is important to consider order. Which slave PE finishes XFERing its row first?

Since all XFERs end at the master PE, the next PE along the ring will receive a steady stream of empty slots. Therefore, this particular slave will take over the ring. When it has finished, the next slave will take over the ring. In other words, each PE in turn takes over the ring and XFERs its entire row.

Source Limited Access:

All slave PEs are XFER sources, so only the master PE slot is unused. The effective transfer rate is $(1 - \frac{1}{K_{PE}})$ times the peak rate.

Destination Limited Access:

Destination PEs can be specified either absolutely or relatively. Both cases are considered:

- Absolute: All XFERs have the same absolute destination (the master PE).

So, all XFERs will require the same slot. The effective ring transfer rate is $\frac{1}{K_{PE}}$ times the peak rate.

- **Relative:** The relative destination is different for each slave PE. So, all slots are used and the peak ring transfer rate is realized.

Transpose

During transpose, all pairs of PEs exchange blocks of data. Each PE sends and receives roughly the same number of XFERs.

Open Access:

Again, open access will ensure that all slots are full. With all PEs writing and receiving XFERs, it seems difficult to predict how the slots will be used.

When an XFER arrives as the destination PE, the slot is read and becomes empty. If the PE is also sending XFERs, it can immediately write into the now empty slot before it moves on. With the ring so busy, incoming XFERs will likely be the only source of empty slots. So, each PE will be sending XFERs at the same rate at which incoming ones arrive. All PEs should finish at roughly the same time.

Source Limited Access:

All PEs send XFERs simultaneously. So, all of the slots will be full and the peak ring transfer rate is achieved.

Destination Limited Access:

All PEs receive XFERs simultaneously, providing that no two PEs send to the same destination at the same time. If this condition is met, the peak transfer rate is achieved once again.

Comparison of Ring Access Methods

The previous sections have produced two important facts:

1. Open access is the only type of ring access which works efficiently for all three XFER types.
2. All types of ring access work efficiently for transpose XFERs.

Regarding XFER types, the tranpose is the only difference between block parallel and row-column image filtering methods. So, ring usage considerations provide no basis for selecting a filtering method.

The fact that open access proved best may lead one to question why access restriction should even be considered. Some applications, such as real-time audio signal processing, can involve a *continuous* flow of data between PEs. In this case, it is vital to ensure that no PEs are locked out from using the ring. Access restrictions can prevent such lock outs very effectively.

The image filtering methods being considered *don't* involve continuous data flow. Data tends to be transfered in blocks, with ensuing processing requiring the entire block. It is acceptable for a PE to have to "wait its turn" to use the ring. So, restricting access provides no real benefits while slowing down the effective transfer rate.

It is noteworthy that distribution and collection are characterized by PEs "taking over" the ring for the duration of each row XFER. In other words, the XFERs occur as if on a token ring. The slotted access has little effect.

7.6 Comparison of Parallel Image Filtering Methods

Table 7.6 summarizes the results of the analysis done in this chapter. Two important points appear:

1. The row-column method suffers the same problem as the whole image transform method upon which it is based. That is, its costs are much higher than block methods except for certain, specific image sizes.
2. The performance of the two block methods are very similar. The block row-column method has slightly lower computational costs and the block parallel method has slightly lower communication costs.

None of the methods stand out as being superior for all cases or for all systems. The following statements can be made:

Criteria	Block Parallel Method	Row-Column Method	Block Row-Column Method
Computational Cost (FLOPs and MAs)	Gradual increase with image size. Slightly higher costs than block row-column method.	Increases in sudden steps. Costs much higher except for certain image sizes.	Best overall. Slightly outperforms block parallel method for all image sizes.
Communication Cost (XFERs)	Increase with image size is gradual, but unsmooth due to dynamic source block size selection. Lowest cost for almost all image sizes.	Increases in sudden steps. Costs are higher for most image sizes.	Increase curve is similar to block parallel method, but with generally higher costs.
Transparency of XFERs	Distribution and collection only. Almost all XFERs can be transparent, depending on system hardware.	Distribution and collection XFERs can be transparent, as in block parallel method. Transpose XFERs cannot be made transparent as effectively.	
Extendibility	Any number of PEs can be used. Efficiency suffers if K_{PE} is not an integer multiple of the number of image blocks.	Any number of PEs can be used, but efficiency is decreased slightly if K_{PE} is not a power of 2. Also, transpose XFER transparency is <i>very</i> difficult if K_{PE} is not a power of 2.	

Table 7.1: Comparison of Parallel Image Filtering Methods

- Block methods are better than the entire image method for most image sizes.
- System hardware is important in choosing a method. In particular, the relative strengths of *computation* and *communication* should be considered. If a system has powerful processing and a relatively slow ring transfer rate, inter-PE transfers represent a bottleneck. In this case, the block parallel method would be appropriate. If communication rates predominate, the block row column method would be better.
- Effective XFER transparency can be more easily realized with the block parallel method.
- Row-column methods benefit when K_{PE} is a power of 2. This may be important if general extendibility is not required.

In Chapter 6, filtering by direct convolution was described. This method outperforms the transform method for small filter response sizes. Direct convolution can be used with block parallel method but *not* with the row-column methods.

Under the most general conditions, the block parallel method would be the preferred choice. However, the performance of the block row-column method would be similar.

7.7 Some Finer Analysis Points

There are some easy ways to achieve small decreases in processing time. For simplicity, these have been excluded from the earlier quantitative analysis.

1. Zero padding is a part of all methods considered. It is wasteful to actually transfer zero data between PEs. Since the size and location of zero data is known, these transfers can be avoided.
2. Another result of zero padding is that full rows of zeroes exist (image rows or block rows). These rows don't need to be transformed, since the result is known to also consist of zeroes.

3. When inverse transforming a filtered image block, some of the row FFTs can be avoided. Only rows which are part of the smaller result block need by transformed.
4. During distribution and collection for block methods, the master PE is involved in every XFER. Thus, the master PE will take longer to finish than the slave PEs. This is important, because a stage is only as fast as the *slowest* PE. Also, as more PEs are added, the master time gets *slower* while the slave times remain the same. Eventually, filtering will be faster if the master PE *doesn't* do any processing but is dedicated to distribution and collection only.

Suppose:

- T_{MK} = stage processing time on the master PE with K PEs in the system.
- T_S = stage processing time on a slave PE (*excluding* wait for master PE).

If the master does processing, the time *per block* is $\frac{T_{MK}}{K}$. If master does *no* processing, the time per block is $\frac{T_S}{K-1}$.

Filtering will be faster with no master processing if:

$$\frac{T_S}{K-1} < \frac{T_{MK}}{K}$$

Or:

$$T_S < \left(\frac{K-1}{K}\right) T_{MK}$$

The above conditions are true *if* the dedicated master PE can perform distribution and collection fast enough to *not* hold up the slave PEs. The actual times will depend on the system.

Chapter 8

Implementation on a Representative System

This chapter describes an implementation of the block parallel image filtering method on Yaremchuk's system [10]. The ring is configured for source limited access, since unlimited access has not yet been implemented. The implementation will give an indication of actual processing times attainable. Also, performance will be compared to the analytical cost model. A diagram of the system is shown in Figure 8.1.

8.1 System Hardware

Figure 8.2 shows a simplified block diagram of a single PE. Some features, not used by the image filtering application, are not included in this description.

8.1.1 Processor and Memory

All PEs are based on the Texas Instruments TMS320C30 32-bit floating point DSP [91]. The TMS320C30 features highly pipelined execution and an instruction set optimized for floating point digital signal processing problems. On-chip peripherals include a DMA controller as well as two serial ports and two timers.

Figure 8.1: Diagram of Multiple DSP System (4 PEs)

Currently, external memory consists of 128 kwords of static RAM. The RAM is fast enough to prevent wait states. The TMS320C30 also has 2 kwords of zero wait state internal RAM.

8.1.2 Slotted Ring and Processor Interface

The ring itself is a loop of unidirectional point to point links. Ring data is shifted from PE to PE at a rate of one Gbit/sec. (32-bit parallel shifts occurring at 32 MHz).

Figure 8.2: PE Block Diagram

Incoming data is written, through a FIFO, to a dual-port RAM (DPRAM) which is mapped into the TMS320C30 address space. (Currently, DPRAM capacity is 1 kword.

Data is sent *to* the ring by writing it into another FIFO. This FIFO's input is also mapped into the TMS320C30 address space. Certain memory address bits are used to determine both the destination PE and the offset into that PE's DPRAM. The effect is that *the DPRAMs of all other PEs are mapped into the TMS320C30 address space*. In other words, all XFERs appear to the TMS320C30 to be memory writes. All ring activity is *transparent* to the processor.

8.1.3 Synchronization and Signalling

The slotted ring allows PEs to send data to each other. However, it is also important for PEs to maintain *synchronization* during the solution of a parallel problem. The following methods facilitate inter-PE program control:

1. **Synchronization Barriers:** The TMS320C30 has built in hardware to allow arbitration of memory shared by multiple processors. Specifically, there is a *request* signal, XF0, and an *acknowledge* signal, XF1.

In Yaremchuk's system, all open collector XF0 signals are wire-ANDed and connected back to all XF1 inputs. Any processor which signals a request will be halted until *all* processors signal a request. Then all processors can continue. (The TMS320C30 interlocked operation instructions can be used to signal and clear requests.) In this way, all processors can be forced to synchronize after completion of assigned tasks. This operation is called a *synchronization barrier*.

2. **DPRAM Signalling:** Each TMS320C30 can be interrupted by its local DPRAM. The interrupt is generated when the ring interface writes into a certain DPRAM location. This mechanism can be used by one PE to *signal* another PE. For example, one PE may alert another that a data block transmission has been completed.

8.2 Image Filtering Software

The block parallel image filtering method, including the row pipeline, has been implemented on Yaremchuk's system. A flowchart describing the software is shown in Figure 8.3. The software was written in the C programming language, except for FFT related routines. The FFT routines were written in assembly language.

The following sections include descriptions of key parts of the software.

8.2.1 Configuration

Only the master PE is connected directly to the host computer. So, the master PE must distribute certain configuration information to all of the slave PEs:

1. **PROCESS Configuration:** Process information includes image size, block size and filter response size. This must be transmitted only once for any image being filtered.

Figure 8.3: Flowchart of Image Filtering Software

2. **FILTER Configuration:** The spatial filter response is built, and then transformed, on each PE. To build the response, each PE must have the filter parameters. A new filter configuration is required each time an image is filtered.

3. **BLOCK Configuration:** Zero-padding is *not* distributed from the master PE to the slaves. Instead, any required zeroes are generated on the PE processing each block. In order to do this, the PE must know the original location of the block within the image. Only blocks on the edge of the image require padding. The edge itself — left or right, top or bottom — indicates the location of the padding. Block configuration consists of a single value indicating location within the image.

Some PEs may be idle during the final stage. The master PE can indicate this state to a slave by sending a NULL block configuration value. In this instance, the slave PE does no processing and simply maintains synchronization, as shown in Figure 8.3.

8.2.2 DMA Server

The image filtering software uses the row pipeline method to make XFERs transparent. DMA is used to send data to other PEs through the output FIFO and to unload incoming data from the DPRAM.

The DMA calls should occur "in the background" and be hidden from processing. To accomplish this, DMA calls are managed by interrupt service routines (ISRs) which read XFER parameters from a global data structure. These ISRs constitute the *DMA server* and the global data structure is called the *DMA server table*. The parameters in the DMA server table include the source address, destination address and size of each row to be XFERed.

The master PE is involved in several row XFERs during each row transform. (The master DMA server also performs a local copy. It moves an image block into a local processing area and copies the filtered block to the result image.) Therefore, the master DMA server table is actually a list, or queue. Slave DMA server tables contain only a single XFER description. The DMA server tables are initialized for distribution at the start of each stage and for collection in the middle of each stage, after column operations. The tables are updated after each row.

The following sections describe the operation of the DMA server.

Figure 8.4: DMA Server Distribution with 4 PEs

Distribution with the DMA Server

The operation of the DMA server during distribution of a single row is shown in Figure 8.4. The following points summarize the operation:

1. To start the DMA server, the master PE initiates the first DMA call in the table. This will usually be to load an image row into the master PE's processing area.
2. When the DMA call is completed, an interrupt occurs. The DMA ISR then does the following:
 - If the previous DMA call was *not* a local copy, send a SIGNAL to the destination PE. (A SIGNAL consists of writing a key value to the PE's DPRAM interrupt location.)
 - If the end of the DMA server table hasn't been reached, start the next DMA call.

This continues until all DMA calls are completed.

3. When a slave PE receives a SIGNAL, a DPRAM interrupt occurs. The DPRAM ISR then starts a DMA call to unload the row from the DPRAM.
4. When the row has been unloaded from the slave DPRAM, a DMA interrupt occurs. The DMA ISR sends an ACK to the master PE. (An ACK consists of writing a key value to location in the master PE DPRAM. Each slave has its own ACK location, *none* of which is the master PE DPRAM interrupt address.)
5. The master PE polls the slave ACK locations until all of the slaves have responded. Distribution of this row is now complete.

During the row distribution, the previous row is being transformed "in the foreground" on all PEs. On the master PE, the transform occurs between the start of the first DMA call and the update of the server table. On the slave PEs, the transform begins immediately and ends before the DMA server update. In Figure 8.4, solid boxes indicate foreground operations and dashed boxes indicate background (ISR) operations.

Figure 8.5: DMA Server Collection with 4 PEs

Collection with the DMA Server

DMA server operation during collection of a single row is shown in Figure 8.5. The following points summarize the operation:

1. The DMA server is started with a DMA call on every PE. All slave PEs are XFERring a result row to the master PE. (Each slave PE has a reserved area in the master PE DPRAM.) On the master PE, the DMA call copies a filtered row into the result image from the local processing area.
2. Upon completion of the slave DMA calls, the DMA ISR on each PE sends an ACK to the master PE. This is identical to the ACK used during distribution.
3. Upon completion of the local DMA on the master PE, the DMA ISR is called:
 - If the previous DMA call was *not* a local copy, send a RETURN to the source PE. (A RETURN is similar to a SIGNAL, but uses a different key value.)
 - If the end of the DMA server table hasn't been reached, poll the ACK location for the currently due PE until the ACK arrives. Then, start a DMA call to unload the row from the DPRAM into the result image.

The DMA ISR is called repeatedly until all slave rows arrive and are unloaded.

4. When a RETURN arrives at a slave PE, a DPRAM interrupt occurs. The DPRAM ISR sets a global flag to notify foreground processing. When the last slave PE receives the RETURN, collection of this row is complete.

During row collection, the next row is being inverse transformed in the foreground on all PEs. On each PE, the transform occurs between the start of the first DMA call and the DMA server table update. In Figure 8.5, solid and dashed boxes indicate foreground and background operations, respectively.

8.2.3 Processing

The actual processing of each block is essentially as described in Section 6.1.3. The basis is the radix-2 DIF FFT routine described in the TMS320C30 User's Guide [91]. Appendix D includes performance data for the processing steps.

There are a few important differences from the processing described in Section 6.1.3:

1. The transform of a real row of length N can be completely defined by the first $\frac{N}{2} + 1$ elements, due to symmetry. However, the first and last of these elements are always real. So, the transform can be stored in a complex array of length $\frac{N}{2}$ by storing the real part of element $\frac{N}{2}$ in the imaginary part of element 0. (Elements are indexed starting at 0.)

After all rows are transformed, the first complex column actually contains *two* sets of *real* data. These can be transformed together with a *single* FFT of length N [87]. Also, using the method described above, these two transforms can be stored in the same N -length complex column.

The image filtering software uses the above method of transforming and storing results. The decrease in processing time is small, but the transform can be conveniently stored *in place* of the original block.

2. Blocks taken from the top or bottom edge of the image will contain a number of rows which consist entirely of zeroes. These are not distributed, but are generated directly on slave PEs.

If a row consists entirely of zeroes, its transform is also made up of zeroes. So, there is no need to transform these pad rows. Their transforms are written directly into place after the distribution row pipeline is completed.

Also, not all of the block rows need to be inverse transformed after filtering. Rows which are not part of the *result* block are not inverse transformed.

8.3 Performance

Figure 8.6 illustrates the performance of the filtering system with 1, 2, 3 and 4 processors. The computational cost (FLOPs only) is also shown for comparison. Graphs illustrating performance with varying filter response sizes are included in Appendix D. Appendix D also includes a version of Figure 8.6 plotted on a linear, instead of logarithmic, scale.

The following observations pertain to Figure 8.6:

Figure 8.6: System Performance and Analytical Cost

- With three or four PEs, processing time for a 128x128 image is roughly 150 milliseconds. This is 6 to 7 times faster than the processing time mentioned in Chapter 2.
- Ideally, filtering on K processors should be K times faster than filtering on a single processor. This is approximately true for the analytical cost graph. The actual system performance curves have roughly the same shape as the cost curves, but provide a smaller gain over the single processor case. The system performance curves include overhead costs, such as maintaining the DMA server.
- For some image sizes, processing time (or computational cost) is equal for different numbers of processors. This occurs when both cases require the same number of stages, which can happen if the number of image blocks is small. For example, if there are 9 image blocks then 3 stages will be required for both 3 and 4 processors. Ultimately, this inefficiency is due to some processors being idle during the final stage.
- The shapes of the performance curves are similar to those of the FLOP analysis curves. The main difference is that the "steps" in the performance curves are slanted while those in the analysis curves are flat. This gradual increase is due to the software's treatment of zero padding. If image size is increased slightly, the number of blocks may remain the same but the amount of zero padding decreases. As zero padding decreases, the processing time for image edge blocks increases.

It is important to note that the performance graphs *do not* represent dynamic source block size selection based on minimum FLOPs. These graphs were generated by processing each image size with all available source block sizes (32, 64, 128 and 256) *and then* selecting the fastest for each case.

Figure 8.7 shows an example comparison of minimum FLOPs selection with the actual optimal speed. For several image sizes, minimum FLOPs selection does not provide the optimal performance. The increases in processing time for these cases

Figure 8.7: Optimal Processing Time vs. Minimum FLOPs Selection

range from about 10 % to 30 %. These differences imply that the FLOPs analysis model does not give an accurate indication of processing times. (The computational cost measures were only intended to be estimates.)

For dynamic source block size selection to be effective, the optimal block size must be selected *before* processing. To achieve this, a better selection criterion is needed. This problem has not been studied further.

Communication overhead is an important factor when considering multiprocessor performance. Table 8.1 shows the communication overhead incurred by the image filtering system under consideration.

N_{sb}	Absolute Overhead			Relative Overhead		
	$K_{PE} = 4$	$K_{PE} = 3$	$K_{PE} = 2$	$K_{PE} = 4$	$K_{PE} = 3$	$K_{PE} = 2$
32	81.0 %	63.8 %	47.0 %	20.5 %	21.3 %	23.5 %
64	44.3 %	34.7 %	26.0 %	11.1 %	11.6 %	13.0 %
128	23.4 %	18.3 %	14.3 %	7.6 %	6.1 %	7.2 %
256	12.8 %	10.2 %	8.3 %	3.3 %	3.4 %	4.2 %

Table 8.1: Communication Overhead of Image Filtering System

Communication overhead was calculated as follows:

$$\begin{aligned}
 \text{Absolute Overhead} &= \frac{T_S}{T_F} - 1 \\
 \text{Relative Overhead} &= \frac{1}{K_{PE}} \cdot \text{Absolute Overhead}
 \end{aligned} \tag{8.1}$$

Where:

- T_S = Time to perform a stage in which all blocks are full (ie. no zero padding).
- T_F = Time to filter a block on a single PE, with no DMA or inter-PE communication (again, no zero padding).

As K_{PE} increases, both communication overhead and the number of blocks processed per stage increase. Normalized overhead accounts for this fact. The original stage timing measurements are tabulated in Appendix D.

Table 8.1 shows that communication overhead decreases as source block size increases. This decrease occurs because communication costs remain roughly constant for different source block sizes. (See Tables D.3, D.4 and D.5 in Appendix D.) As source block size increases, block processing time increases and communication overhead time takes up a smaller percentage of the total.

The fact that communication overhead doesn't vary with source block size is important. This implies that all XFERs are completed when the master PE finishes each row transform during distribution and collection. (If the master PE had to pause and wait for XFER completion, the waiting time would increase as row length

increased.) In other words, *all XFERs are completely transparent* for the cases that have been profiled.

Table D.1 lists the number of DMA transfers which can occur behind a row FFT of given length. In each case, the number of transfers approximates or exceeds four times the FFT size. In other words, four rows can be transferred during the time required to transform a single row.

Table D.1 also lists the *rates* of DMA transfers behind row FFTs. They range from just over 1 transfer/ μsec to just over 2 transfers/ μsec . In a slotted ring system with destination limited access, the image filtering system would require $K_{PE} - 1$ destination slots. Therefore, the effective ring transfer rate from one PE to another would be:

$$\text{Effective Ring Rate} = \frac{1}{K_{PE} - 1} \cdot \text{Peak Ring Rate}$$

With a peak ring rate of 32 Mwords/sec, the effective ring rate *exceeds* the DMA rates for up to 16 PEs. So, transfer rate is limited by DMA, not by the ring itself. This implies that complete XFER transparency can exist with up to four PEs in the system, based on the number of transfers which can occur behind an FFT of given length (Table D.1).

Complete XFER transparency, as described earlier, has been observed for two, three and four PEs. Unfortunately, only four PEs were available for use. No data is available for performance with five or more PEs. If K_{PE} increased to five and beyond in Yaremchuk's system, the master PE would have to wait for completion of non-transparent DMA transfers. Communication overhead would increase.

If the master PE *only* did distribution and collection, with no processing, it would be able to perform more than 5 DMA transfers/ μsec [91]. This would speed up the stages by increasing the number of transparent XFERs behind each row transform. However, each stage would only process $K_{PE} - 1$ blocks. For large values of K_{PE} , it would likely be faster to dedicate the master PE to distribution and collection. This can't be investigated further without having more PEs to work with.

Chapter 9

Concluding Remarks

9.1 Summary of Contributions

Multiple DSP systems with slotted ring architectures have been shown to effectively support linear image filtering, in particular the block parallel transform algorithm. This algorithm has been implemented on a representative slotted ring system. Other architectures which can provide comparable performance have been identified.

Guidelines have been established for performing interprocessor transfers concurrently with computations. Also, block size selection has been identified as an important part of the block parallel transform method.

9.2 Conclusions

1. Linear image filtering can be performed effectively on a slotted ring system. A quantitative cost analysis has shown that the block parallel transform method is the most effective in most situations.
2. Interprocessor communications can be performed concurrently with computation using DMA. Processing times are limited by computations, as long as *all* interprocessor transfers can be done concurrently. As the number of processors grows, the number of transfers grows and eventually interprocessor communications become the limiting factor.

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3. The slotted ring provides the best effective transfer rate if slot access is not restricted at all. In this case, processors tend to take over the ring in sequence to complete block transfers. Only one processor at a time uses the ring. In other words, a token ring or common bus system may provide performance comparable to that of a slotted ring.
4. The computational cost analysis was done with varying image size, spatial filter response size and number of processors. The block parallel transform method was identified as the most efficient in most situations. In a few cases, however, other methods are more efficient:

- **Direct Convolution:** When a filter with a small spatial response is used, direct convolution is more efficient than frequency domain filtering for certain image sizes, as shown in Section 6.1.3. In these cases, direct convolution should be used with block parallel partitioning.
- **Entire Image Transform:** For some cases of image size and filter response size, the *padded* image size is equal to or slightly less than a power of 2. In many of these cases, transforming the entire image is more efficient than transforming a block at a time. As shown in Section 6.1.2, this is especially true when a fixed filter response is used and doesn't need to be transformed.

Transforming the entire image in parallel requires using row-column partitioning. This partitioning method incurs transpose costs, but these can be hidden *if* the number of processors is a power of 2.

9.3 Directions for Future Research

1. Many image processing applications are iterative, requiring many repetitions of the image filtering operation. It may be possible to reduce the processing time of these iterations. For example, suppose that the same source image is processed each time, with varying filter responses. If slave PEs have enough memory capacity, the source blocks need only be distributed *once*. In fact,

blocks only need to be transformed once. The general problem of iterative filtering on slotted rings should be analyzed.

2. The block parallel method can be expected to perform well on token ring and common bus systems. It would be useful to verify these expectations with actual implementations. Also, it would be interesting to study the use of the block parallel method on other multiple DSP ring architectures, such as the popular Banshee system [92]. The Banshee system allows direct communication between adjacent ring elements only.
3. For small filter sizes, the block parallel method should use direct convolution instead of Fourier transforms. However, direct convolution doesn't directly fit the row pipeline scheme. Some way is needed to perform interprocessor communications concurrently with direct convolution computations.
4. It has been shown that dynamic source block size selection based on minimum theoretical computational cost *does not* necessarily provide optimal processing time on a real system. A more effective selection criterion should be developed. This will likely prove to be system dependent to some degree.

Also, if non-square images are to be filtered, non-square blocks should be considered. In other words, optimal size and shape (or height and width) are required. Choosing these parameters effectively prior to filtering will minimize processing time.

5. In some slotted ring systems, including Yaremchuk's described in Chapter 8, access to the ring is restricted. The actual access to the slots is *statically* programmable. That is, access can be configured *once* before processing begins. If processing consists of different phases (such as distribution and collection), *dynamic* access configuration may be useful. Dynamic access configuration would allow ring restrictions to change "on the fly", in response to varying communication requirements. This may prove beneficial to slotted ring applications other than image filtering.

6. In some systems, the master PE may be dedicated to block distribution and collection only. Since no processing is being done, a DSP may not be the best choice for the master processor. A graphics processor, such as the Texas Instruments TMS34020 [93], would be better. The TMS34020 can perform image line or image block transfers with a single assembly language instruction. Also, it has a built in host computer interface. The master PE could easily have display capability if it were based on a graphics processor. This capability would reduce required communications with the host computer.
7. Texas Instruments has introduced a new floating point DSP, the TMS320C40 [54], designed specifically for multiprocessor systems. The TMS320C40 has six distinct communication ports with six DMA channels. Yaremchuk's system, particularly used for image filtering, may benefit from using the TMS320C40 instead of the TMS320C30. For example, the multiple DMA channels may decrease the overhead required for the DMA server.

Appendix A

The Two-Dimensional DFT: Computational Considerations

A.1 Computing the Two-Dimensional DFT

Determining an $N \times N$ 2-D DFT can be very computationally expensive, as the following definition shows [25, pg. 163]:

$$\mathbf{X}(k_1, k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) e^{-j \frac{2\pi}{N} (k_1 n_1 + k_2 n_2)} \quad (\text{A.1})$$

$\forall k_1, k_2 \in 0 \dots N - 1$

Each of the N^2 result values requires computing the sum of N^2 complex products. In other words, as N grows, the cost grows as N^4 . This can lead to extremely large numbers even for moderate values of N .

The well known Fast Fourier Transform (FFT) algorithm is a popular method of computing 1-D DFTs [94]. This principle can be extended to two dimensions with similar effectiveness.

Two possible methods of employing FFT principles in two dimensions are described here. In each case, it is assumed that all complex exponentials are pre-computed and can be simply read from memory. Also, the cost of bit-reversal is excluded. Bit-reversal can be performed when transform results are accessed.

1. Row-Column Decomposition [25, sect. 3.4.1]

Fortunately, the 2-D DFT can be computed by transforming all of the rows and then transforming all of the resulting columns. This allows direct use of the 1-D FFT algorithm

The cost of this method is $2N$ multiplied by the cost of a 1-D FFT of size N . The 1-D FFT consists of a sequence of two element, in-place computations called *butterflies*. Consider the radix-2, decimation-in-frequency (DIF) butterfly:

$$\begin{aligned} A &\leftarrow A + B \\ B &\leftarrow (A - B)W_N^k \end{aligned} \quad (\text{A.2})$$

Where $W_N^k = e^{-j\frac{2\pi}{N}k}$.

For a single butterfly calculation, the number of floating point operations (FLOPs) is:

$$\begin{aligned} 2 \text{ complex additions} &: 4\text{FLOPs} \\ 1 \text{ complex multiply} &: 6\text{FLOPs} \\ \text{Total} &: 10\text{FLOPs} \end{aligned} \quad (\text{A.3})$$

The number of memory accesses (MAs) is:

$$\begin{aligned} 2 \text{ complex data reads} &: 4\text{MAs} \\ 2 \text{ complex data writes} &: 4\text{MAs} \\ \text{look up complex exponential value (SIN, COS)} &: 2\text{MAs} \\ \text{Total} &: 10\text{MAs} \end{aligned} \quad (\text{A.4})$$

The decimation-in-time (DIT) butterfly incurs the same costs.

Each 1-D FFT requires $\frac{N}{2} \log_2 N$ butterflies. So, the total cost for an $N \times N$ 2-D DFT using radix-2 1-D FFTs is:

$$\begin{aligned}
 \text{Number of FLOPs} &= 2N * \frac{N}{2} \log_2 N * 10 & (A.5) \\
 &= 10N^2 \log_2 N \\
 \text{Number of MAs} &= 2N * \frac{N}{2} \log_2 N * 10 \\
 &= 10N^2 \log_2 N
 \end{aligned}$$

2. Vector Radix FFT [25, sect. 3.4.3]

The 1-D FFT is based on the fact that a DFT of size N can be generated from two DFTs of size $N/2$. This separation is repeated down to the butterfly computation, which is essentially a DFT of size 2.

It is also true that an $N \times N$ 2-D DFT can be generated from 4 DFTs of size $\frac{N}{2} \times \frac{N}{2}$. Following a path similar to the 1-D case leads to a 2x2 DFT, or 2x2 butterfly:

$$\begin{aligned}
 A &\leftarrow A + B + C + D & (A.6) \\
 B &\leftarrow (A - B + C - D)W_N^m \\
 C &\leftarrow (A + B - C - D)W_N^l \\
 D &\leftarrow (A - B - C + D)W_N^m W_N^l
 \end{aligned}$$

NOTE:

- By forming partial sums $A+B$, $C+D$, $A-B$ and $C-D$, only 8 complex additions are required.
- $W_N^m W_N^l = W_N^{m+l}$.

For a single 2x2 butterfly calculation, the number of FLOPs is:

$$\begin{aligned}
 8 \text{ complex additions} &: 16 \text{ FLOPs} \\
 3 \text{ complex multiplies} &: 18 \text{ FLOPs} \\
 \text{Total} &: 34 \text{ FLOPs} \quad (\text{A.7})
 \end{aligned}$$

The number of MAs is:

$$\begin{aligned}
 4 \text{ complex data reads} &: 8 \text{ MAs} \\
 4 \text{ complex data writes} &: 8 \text{ MAs} \\
 \text{look up 2 complex exponential values (SIN, COS)} &: 4 \text{ MAs} \\
 \text{Total} &: 20 \text{ MAs} \quad (\text{A.8})
 \end{aligned}$$

A total of $\frac{N^2}{4} \log_2 N$ of these butterflies are needed. So, the total cost for an $N \times N$ 2-D DFT using a 2x2 radix is:

$$\begin{aligned}
 \text{Number of FLOPs} &= \frac{N^2}{4} \log_2 N * 34 & (\text{A.9}) \\
 &= 8.5 N^2 \log_2 N \\
 \text{Number of MAs} &= \frac{N^2}{4} \log_2 N * 20 \\
 &= 5 N^2 \log_2 N
 \end{aligned}$$

Both of these methods reduce the DFT problem from an order of N^4 to one of $N^2 \log_2 N$. This can be a very significant saving for large values of N . The vector radix approach has slightly lower costs, but row-column decomposition is easier to implement. This is because it can make use of the large number of existing 1-D FFT implementations.

A.2 Efficient One-Dimensional Transform of Real Data

A.2.1 Algorithm

Consider the DFT of a 1-D set of real data, $\mathbf{x}(n)$, of length N :

$$\mathbf{X}(k) = \sum_{n=0}^{N-1} \mathbf{x}(n) e^{-j \frac{2\pi}{N} nk} \quad (A.10)$$

$\forall k \in 0 \dots N-1$

Now, split the sum into even and odd values of n :

$$\mathbf{X}(k) = \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}(2n) e^{-j \frac{2\pi}{N} 2nk} + \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}(2n+1) e^{-j \frac{2\pi}{N} (2n+1)k} \quad (A.11)$$

Define two new sequences, $\mathbf{x}_e(n)$ and $\mathbf{x}_o(n)$, each of length $N/2$, made up of the even and odd indexed elements of $\mathbf{x}(n)$, respectively:

$$\begin{aligned} \mathbf{X}(k) &= \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_e(n) e^{-j \frac{2\pi}{N/2} nk} + \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_o(n) e^{-j \frac{2\pi}{N/2} nk} e^{-j \frac{2\pi}{N} k} \\ &= \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_e(n) e^{-j \frac{2\pi}{N/2} nk} + e^{-j \frac{2\pi}{N} k} \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_o(n) e^{-j \frac{2\pi}{N/2} nk} \end{aligned} \quad (A.12)$$

The two sums are now in the form of DFTs of length $N/2$:

$$\mathbf{X}(k) = \mathbf{X}_E(k) + W_N^k \mathbf{X}_O(k) \quad (A.13)$$

Where:

- $\mathbf{X}_E(k)$, $\mathbf{X}_O(k)$ are the DFTs of $\mathbf{x}_e(n)$, $\mathbf{x}_o(n)$
- $W_N^k = e^{-j \frac{2\pi}{N} k}$

Since DFTs are periodic, $\mathbf{X}_E(k)$ and $\mathbf{X}_O(k)$ are extended to two cycles to make up the full length, N , of $\mathbf{X}(k)$. It should be noted that the breakdown shown above is the basis for the FFT algorithm.

Now, form a new complex sequence of length $N/2$ by placing the even indexed elements of $\mathbf{x}(n)$ in the real part, and the odd elements in the imaginary part:

$$\mathbf{h}(n) = \mathbf{x}_e(n) + j\mathbf{x}_o(n) \quad (\text{A.14})$$

Consider the DFT of this new sequence:

$$\begin{aligned} \mathbf{H}(k) &= \sum_{n=0}^{\frac{N}{2}-1} \mathbf{h}(n) e^{-j\frac{2\pi}{N/2}nk} \\ &= \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_e(n) e^{-j\frac{2\pi}{N/2}nk} + j \sum_{n=0}^{\frac{N}{2}-1} \mathbf{x}_o(n) e^{-j\frac{2\pi}{N/2}nk} \\ &= \mathbf{X}_E(k) + j\mathbf{X}_O(k) \end{aligned} \quad (\text{A.15})$$

Since $\mathbf{x}_e(n)$ and $\mathbf{x}_o(n)$ are strictly real:

$$\begin{aligned} \mathbf{X}_E\left(\frac{N}{2} - k\right) &= \mathbf{X}_E^*(k) \\ \mathbf{X}_O\left(\frac{N}{2} - k\right) &= \mathbf{X}_O^*(k) \end{aligned} \quad (\text{A.16})$$

Using this fact, the even and odd transforms can be extracted from the hybrid transform, $\mathbf{H}(k)$:

$$\begin{aligned} \mathbf{H}\left(\frac{N}{2} - k\right) &= \mathbf{X}_E\left(\frac{N}{2} - k\right) + j\mathbf{X}_O\left(\frac{N}{2} - k\right) \\ &= \mathbf{X}_E^*(k) + j\mathbf{X}_O^*(k) \\ &\quad \forall k \in 1 \dots \frac{N}{2} - 1 \end{aligned} \quad (\text{A.17})$$

And:

$$\begin{aligned} \mathbf{H}^*\left(\frac{N}{2} - k\right) &= \mathbf{X}_E(k) - j\mathbf{X}_O(k) \\ \forall k \in 1 \dots \frac{N}{2} - 1 \end{aligned} \quad (\text{A.18})$$

This results in:

$$\begin{aligned} \mathbf{H}(k) + \mathbf{H}^*\left(\frac{N}{2} - k\right) &= 2\mathbf{X}_E(k) \\ \mathbf{H}(k) - \mathbf{H}^*\left(\frac{N}{2} - k\right) &= 2j\mathbf{X}_O(k) \end{aligned} \quad (\text{A.19})$$

Using Equation A.19 with Equation A.13 yields:

$$\begin{aligned} \mathbf{X}(k) &= \frac{1}{2}(\mathbf{H}(k) + \mathbf{H}^*\left(\frac{N}{2} - k\right)) - j\frac{1}{2}W_N^k(\mathbf{H}(k) - \mathbf{H}^*\left(\frac{N}{2} - k\right)) \\ \forall k \in 1 \dots \frac{N}{2} - 1 \end{aligned} \quad (\text{A.20})$$

For the special case $k = 0$, the values $\mathbf{X}_E(k)$ and $\mathbf{X}_O(k)$ are real and can be easily separated:

$$\mathbf{X}(0) = \Re\{\mathbf{H}(0)\} + \Im\{\mathbf{H}(0)\} \quad (\text{A.21})$$

Substituting $\frac{N}{2} - k$ for k yields:

$$\begin{aligned} \mathbf{X}\left(\frac{N}{2} - k\right) &= \frac{1}{2}(\mathbf{H}\left(\frac{N}{2} - k\right) + \mathbf{H}^*(k)) + j\frac{1}{2}W_N^{-k}(\mathbf{H}\left(\frac{N}{2} - k\right) - \mathbf{H}^*(k)) \\ \forall k \in 1 \dots \frac{N}{2} - 1 \end{aligned} \quad (\text{A.22})$$

And:

$$\mathbf{X}\left(\frac{N}{2}\right) = \Re\{\mathbf{H}(0)\} - \Im\{\mathbf{H}(0)\} \quad (\text{A.23})$$

So, given a real set of data of length N , its DFT can be calculated using an FFT of length $N/2$. The FFT result is converted to the actual data transform using Equations A.20 to A.23. The conversion can be done in place, except that an extra element must be added to complete the actual transform. (The transform is of length N , but only the first $N/2 + 1$ elements are needed because of the real data symmetry).

A similar method can be used to perform an inverse DFT, if the result is to be strictly real.

Given a transform, $\mathbf{X}(n)$, of real data, form the hybrid transform:

$$\mathbf{H}(k) = \frac{1}{2}(\mathbf{X}(k) + \mathbf{X}^*(\frac{N}{2} - k)) + j\frac{1}{2}W_N^k(\mathbf{X}(k) - \mathbf{X}^*(\frac{N}{2} - k)) \quad (\text{A.24})$$

$$\forall k \in 1 \dots \frac{N}{2} - 1$$

Inverse transforming $\mathbf{H}(k)$ will give the hybrid signal $\mathbf{h}(n)$, in the form described in Equation A.14. The values of $\mathbf{x}(n)$ can be extracted easily from $\mathbf{h}(n)$. A comparison of Equations A.20 and A.24 indicates that these conversion processes are almost identical.

A.2.2 Computational Cost

Packing real $\mathbf{x}(n)$ into complex $\mathbf{h}(n)$

- Number of memory accesses (MAs)

$$\begin{aligned} \text{Read } N \text{ real values} &: N \text{ MAs} \\ \text{Write } N/2 \text{ complex values} &: N \text{ MAs} \\ \text{Total} &: 2N \text{ MAs} \end{aligned}$$

- Number of floating point operations (FLOPs)

$$\text{None are required.}$$

Converting hybrid $H(k)$ to transform $X(k)$

To do two elements in place:

- Number of MAs

$$\text{Read } H(k) \text{ and } H\left(\frac{N}{2} - k\right) : 4MAs$$

$$\text{Write } X(k) \text{ and } X\left(\frac{N}{2} - k\right) : 4MAs$$

$$\text{Look up } W_N^k (SIN, COS) : 2MAs$$

$$\text{Total} : 10MAs$$

- Number of FLOPs

$$H_1 = \frac{1}{2}(H(k) + H^*\left(\frac{N}{2} - k\right)) : 4FLOPs$$

$$H_2 = -j\frac{1}{2}(H(k) - H^*\left(\frac{N}{2} - k\right)) : 4FLOPs$$

$$T = H_2 W_N^k : 6FLOPs$$

$$X(k) = H_1 + T : 2FLOPs$$

$$X\left(\frac{N}{2} - k\right) = (H_1 - T)^* : 2FLOPs$$

$$\text{Total} : 18FLOPs \quad (A.25)$$

This loop is performed $N/4$ times for an approximate total cost of $4.5N$ FLOPs and $2.5N$ MAs.

So, the total cost of using this method, including the $\frac{N}{2}$ -length FFT, is:

$$\begin{aligned} \text{Number of FLOPs} &= \frac{10}{2} * \frac{N}{2} \log_2 \frac{N}{2} + \frac{9}{2}N & (A.26) \\ &= \frac{5}{2}N * (\log_2 N - 1) + \frac{9}{2}N \\ &= \frac{5}{2}N \log_2 N + 2N \\ \text{Number of MAs} &= \frac{10}{2} * \frac{N}{2} \log_2 \frac{N}{2} + \frac{5}{2}N + 2N \\ &= \frac{5}{2}N * (\log_2 N - 1) + \frac{9}{2}N \\ &= \frac{5}{2}N \log_2 N + 2N \end{aligned}$$

This is roughly half of the cost of doing a conventional N -length FFT. The inverse transform process is almost identical and has the same computational cost.

A.3 Symmetry in Two-Dimensional Transform of Real Data

Consider the DFT of an $N \times N$ set of real data, $\mathbf{x}(n_1, n_2)$:

$$\mathbf{X}(k_1, k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{(k_1 n_1 + k_2 n_2)} \\ \forall k_1, k_2 \in 0 \dots N-1$$

Where $W_N^m = e^{-j \frac{2\pi}{N} m}$.

Now, consider the DFT evaluated at a diagonally symmetric point:

$$\mathbf{X}(N - k_1, N - k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{((N-k_1)n_1 + (N-k_2)n_2)} \\ \forall k_1, k_2 \in 1 \dots N-1$$

This can be rewritten as:

$$\mathbf{X}(N - k_1, N - k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{(-k_1 n_1 - k_2 n_2)} W_N^{(N n_1 + N n_2)}$$

Since $W_N^{Nm} = 1$ for any integer m , the second complex exponential disappears:

$$\mathbf{X}(N - k_1, N - k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{(-k_1 n_1 - k_2 n_2)}$$

This differs from $\mathbf{X}(k_1, k_2)$ only in the sign of the complex exponential:

$$\mathbf{X}(N - k_1, N - k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{*(k_1 n_1 + k_2 n_2)}$$

Where the * signifies complex conjugation.

Since $\mathbf{x}(n_1, n_2)$ is strictly real:

$$\mathbf{X}(N - k_1, N - k_2) = \mathbf{X}^*(k_1, k_2)$$

So, the transform $\mathbf{X}(k_1, k_2)$ is diagonally conjugate symmetric about the point $\mathbf{X}(N/2, N/2)$ for all $k_1, k_2 \neq 0$.

Consider now the first row of the transform ($k_1 = 0$):

$$\begin{aligned} \mathbf{X}(0, k_2) &= \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{k_2 n_2} \\ &\quad \forall k_2 \in 1 \dots N-1 \end{aligned}$$

At a point located symmetrically along the first row:

$$\mathbf{X}(0, N - k_2) = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{(N-k_2)n_2}$$

Proceeding as before:

$$\begin{aligned} \mathbf{X}(0, N - k_2) &= \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{-k_2 n_2} W_N^{N n_2} \\ \mathbf{X}(0, N - k_2) &= \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} \mathbf{x}(n_1, n_2) W_N^{*k_2 n_2} \\ \mathbf{X}(0, N - k_2) &= \mathbf{X}^*(0, k_2) \end{aligned}$$

Similarly, for the first column:

$$\begin{aligned} \mathbf{X}(N - k_1, 0) &= \mathbf{X}^*(k_1, 0) \\ &\quad \forall k_1 \in 1 \dots N-1 \end{aligned}$$

X ₀₀	X ₀₁	X ₀₂	X ₀₃	X ₀₄	X* ₀₃	X* ₀₂	X* ₀₁
X ₁₀	X ₁₁	X ₁₂	X ₁₃	X ₁₄	X* ₇₃	X* ₇₂	X* ₇₁
X ₂₀	X ₂₁	X ₂₂	X ₂₃	X ₂₄	X* ₆₃	X* ₆₂	X* ₆₁
X ₃₀	X ₃₁	X ₃₂	X ₃₃	X ₃₄	X* ₅₃	X* ₅₂	X* ₅₁
X ₄₀	X ₄₁	X ₄₂	X ₄₃	X ₄₄	X* ₄₃	X* ₄₂	X* ₄₁
X* ₃₀	X ₅₁	X ₅₂	X ₅₃	X* ₃₄	X* ₃₃	X* ₃₂	X* ₃₁
X* ₂₀	X ₆₁	X ₆₂	X ₆₃	X* ₂₄	X* ₂₃	X* ₂₂	X* ₂₁
X* ₁₀	X ₇₁	X ₇₂	X ₇₃	X* ₁₄	X* ₁₃	X* ₁₂	X* ₁₁

The transform can be completely defined by:

i) Elements to the left of the heavy solid line (1st 5 columns)

or:

ii) Elements above the heavy dashed line (1st 5 rows)

Figure A.1: 2-D Transform Redundancy for $N=8$

So, both the first row and first column of the transform are themselves conjugate symmetric about element $N/2$.

The result of these symmetries is that the first $N/2 + 1$ columns (or the first $N/2 + 1$ rows) contain *all* of the information contained in the transform. This is illustrated in Figure A.1 for the case $N = 8$.

Appendix B

Additional Graphs for Serial Analysis

Figure B.1: Direct Convolution vs Basic Transform Method

Figure B.2: Direct Convolution vs Revised Transform Method

Figure B.3: Block Transform Method for $M=11$

Figure B.4: Block Transform Method for M=27

Appendix C

Additional Graphs for Parallel Analysis

Figure C.1: Computational Cost: M=11, 4 PEs

Figure C.2: Computational Cost: $M=27$, 4 PEs

Figure C.3: Computational Cost: $M=11$, 3 PEs

Figure C.4: Computational Cost: $M=19$, 3 PEs

Figure C.5: Computational Cost: M=27, 3 PEs

Figure C.6: Computational Cost: $M=11$, 2 PEs

Figure C.7: Computational Cost: M=19, 2 PEs

Figure C.8: Computational Cost: $M=27$, 2 PEs

Figure C.9: Communication Cost: M=11, 4 PEs

Figure C.10: Communication Cost: $M=27$, 4 PEs

Figure C.11: Communication Cost: M=11, 3 PEs

Figure C.12: Communication Cost: M=19, 3 PEs

Figure C.13: Communication Cost: $M=27$, 3 PEs

Figure C.14: Communication Cost: $M=11$, 2 PEs

Figure C.15: Communication Cost: $M=19$, 2 PEs

Figure C.16: Communication Cost: $M=27$, 2 PEs

Appendix D

Image Filtering System Performance Profiles

Size (N)	Column FFT (μ sec)	Row FFT w/o DMA (μ sec)	Row FFT w/ DMA (μ sec)	DMA Transfers Behind FFT	FFT Slowdown	DMA Rate (trans. per μ sec)
32	147	92.8	97.6	119	5.3 %	1.22
64	286	164	172	278	4.7 %	1.62
128	580	315	329	634	4.4 %	1.93
256	1207	632	659	1400	4.1 %	2.13

Table D.1: Performance of 1-D FFT and DMA

N_{ab}	2-D FFT Time (msec)	Block Filter Time (msec)
32	5.24	10.8
64	19.5	40.2
128	77.1	159
256	316	649

Table D.2: Performance of 2-D FFT and Block Filtering

N_{sb}	Row Pipeline Step	Distribution (μsec)	Collection (μsec)	Stage Time (msec)
32	Initialize Server	179	104	19.8
	Row Transform	187	196	
	Update Server	37.9	28.6	
64	Initialize Server	179	104	58.2
	Row Transform	261	278	
	Update Server	37.9	28.6	
128	Initialize Server	179	104	196
	Row Transform	416	456	
	Update Server	37.9	28.6	
256	Initialize Server	179	104	732
	Row Transform	741	823	
	Update Server	37.9	28.6	

Table D.3: Stage and Row Pipeline Timing with 4 PEs

N_{sb}	Row Pipeline Step	Distribution (μsec)	Collection (μsec)	Stage Time (msec)
32	Initialize Server	133	82.1	17.9
	Row Transform	164	174	
	Update Server	33.0	21.6	
64	Initialize Server	133	82.1	54.3
	Row Transform	237	253	
	Update Server	33.0	21.6	
128	Initialize Server	133	82.1	188
	Row Transform	391	430	
	Update Server	33.0	21.6	
256	Initialize Server	133	82.1	715
	Row Transform	714	795	
	Update Server	33.0	21.6	

Table D.4: Stage and Row Pipeline Timing with 3 PEs

N_{sb}	Row Pipeline Step	Distribution (μsec)	Collection (μsec)	Stage Time (msec)
32	Initialize Server	90.2	60.0	16.0
	Row Transform	143	153	
	Update Server	25.5	17.1	
64	Initialize Server	90.2	60.0	50.8
	Row Transform	217	234	
	Update Server	25.5	17.1	
128	Initialize Server	90.2	60.0	182
	Row Transform	373	412	
	Update Server	25.5	17.1	
256	Initialize Server	90.2	60.0	703
	Row Transform	698	777	
	Update Server	25.5	17.1	

Table D.5: Stage and Row Pipeline Timing with 2 PEs

Figure D.1: Image Filtering System Performance with 4 PEs

Figure D.2: Image Filtering System Performance with 3 PEs

Figure D.3: Image Filtering System Performance with 2 PEs

Figure D.4: Image Filtering System Performance with 1 PE

Figure D.5: Image Filtering System Performance: Linear Scale

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