Development and Testing of a GyroWheel Based Control System for the SCISAT-1 Scientific Satellite

by

**Paul Trevor Harrison,
B.A.Sc. Eng. Physics, UBC, 1998**

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfilment of the requirements for the degree of

Master of Applied Science

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Department of Mechanical and Aerospace Engineering

Carleton University

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Thesis Supervisor

Chair, Dept. of Mechanical and Aerospace Engineering

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April 2003

Abstract

GyroWheel is an innovative spacecraft attitude control device that provides actuating torques about three axes while measuring spacecraft angular rates about two axes. A demonstration unit will be flown on SCISAT-1, a Canadian scientific satellite, in 2003.

During normal operation of SCISAT-1, actuation is provided by a classical fixed-axis momentum wheel and magnetic torque rods. An alternative control system was developed, however, whereby GyroWheel provides actuation about all three axes, with the torque rods being required only for momentum dumping. The system is operated without angular momentum bias to suppress nutation.

Computer simulations of the system indicate that fine pointing control of SCISAT-1 using GyroWheel can be easily achieved, with a $3\text{-}\sigma$ pointing error of 0.02° and a maximum pointing error of 0.06° . This compares favourably with results for the momentum wheel and torque rod system, in which the pointing error can reach 0.2° during a typical orbit.

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Contents

Abstract	i
Acknowledgements	ii
Table of Contents	iii
List of Figures	vi
List of Tables	x
Nomenclature	xii
1 Introduction	1
1.1 Attitude Determination and Control of a Small Spacecraft	3
1.2 Attitude and Rate Sensors	4
1.2.1 Photometric Sensors	5
1.2.2 Magnetometers	5

1.2.3	Gyroscopic Sensors	6
1.3	Actuators	9
1.3.1	Reaction Thrusters	9
1.3.2	Magnetic Torquers	9
1.3.3	Reactions Wheels and Momentum Wheels	10
1.3.4	Control Moment Gyros	11
2	Introduction to GyroWheel	13
2.1	GyroWheel Operation	13
2.1.1	Torque Actuation	16
2.1.2	Rate Sensing	16
2.1.3	Operating Specifications	17
2.2	ADCS Interface	17
2.2.1	Momentum Command Mode	18
2.2.2	Torque Command Mode	18
3	Overview of SCISAT-1	21
3.1	Science Payload and Pointing Requirements	21
3.2	Satellite Configuration	23
3.3	Disturbance Torques	26

3.4	SCISAT-1 ADCS	26
3.4.1	Attitude and Rate Sensors	26
3.4.2	Sensor Outputs	28
3.4.3	Sensor Fusion	29
3.4.4	Actuators	30
3.4.5	Control Torque	32
3.4.6	ADCS Schematic	33
4	Development of the GyroWheel Control Law	35
4.1	Spacecraft Equations of Motion	36
4.2	Equations of Motion for SCISAT-1	39
4.2.1	Reference Frames	40
4.2.2	Attitude Errors	41
4.2.3	Rotation Matrices and Euler Angles	42
4.2.4	Spacecraft Angular Rate	45
4.2.5	Spacecraft Inertia with GyroWheel	47
4.2.6	Simplified Equations of Motion	49
4.3	GyroWheel Feedback Control	53
4.3.1	Gain Selection	57

5	Simulations and Results	59
5.1	GyroWheel Dynamic Model	60
5.1.1	Actuation Modeling	62
5.1.2	Rate Sensing Modeling	62
5.1.3	Model Validation	63
5.2	GyroWheel Acceptance Runs	63
5.3	Evaluation of GyroWheel Control System Performance	66
5.3.1	Roll Error	67
5.3.2	Pointing Error	68
5.3.3	GyroWheel Behaviour	71
5.3.4	Control Performance Statistics	72
5.4	Evaluation of GyroWheel Control System Stability	75
5.4.1	Roll Response	75
5.4.2	Pitch Step Response	78
5.4.3	Yaw Step Response	82
5.5	Evaluation of the GyroWheel Rate Feedback Method	84
5.6	Long Term GyroWheel Control Performance	85
5.7	Comparison with the Classical Wheel Based Control System	94
6	Conclusions and Recommendations	98

List of Figures

2.1	GyroWheel Rotor and Housing	14
2.2	Cutaway Diagram of GyroWheel	14
2.3	GyroWheel Gimbal System	15
3.1	SCISAT-1 Orbit	22
3.2	SCISAT-1	24
3.3	SCISAT-1 Attitude Determination and Control System (ADCS)	34
4.1	Quasi-Inertial, Desired, and Perturbed Pointing Frames	41
4.2	Block Diagram of GyroWheel Based ADCS for SCISAT-1	56
5.1	Comparison of Roll Axis Control Performance for Different Gain Sets (One orbit)	68
5.2	Comparison of Pitch Axis Control Performance for Different Gain Sets (One orbit)	69

5.3	Comparison of Yaw Axis Control Performance for Different Gain Sets (One orbit)	69
5.4	Comparison of Overall Pointing Control Performance for Different Gain Sets (One orbit)	70
5.5	Comparison of GyroWheel Tilt Behaviour for Different Gain Sets (One orbit)	72
5.6	Comparison of GyroWheel Angular Momentum Behaviour for Different Gain Sets (One orbit)	73
5.7	Roll Error as a Function of Time for a 10° Initial Roll Offset	76
5.8	Response of Spacecraft Pointing Error to a 10° Initial Roll Offset	77
5.9	Response of GyroWheel Tilt Angle to a 10° Initial Roll Offset	77
5.10	Response of GyroWheel Angular Momentum to a 10° Initial Roll Offset	78
5.11	Pitch Error as a Function of Time for a 2° Initial Pitch Offset	79
5.12	Response of Spacecraft Yaw Error to a 2° Initial Pitch Offset	80
5.13	Response of Spacecraft Roll Error to a 2° Initial Pitch Offset	80
5.14	Response of Spacecraft Pointing Error to a 2° Initial Pitch Offset	81
5.15	Response of GyroWheel Tilt Angle to a 2° Initial Pitch Offset	82
5.16	Response of GyroWheel Angular Momentum to a 2° Initial Pitch Offset	83
5.17	Response of Spacecraft Pointing Error to a 2° Initial Yaw Offset	83
5.18	Response of GyroWheel Tilt Angle to a 2° Initial Yaw Offset	84

5.19 Response of Spacecraft Pointing Error to a 2° Initial Pitch Offset Using GyroWheel Rate Feedback	85
5.20 Response of GyroWheel Tilt Angle to a 2° Initial Pitch Offset Using GyroWheel Rate Feedback	86
5.21 Long Term Behaviour of Spacecraft Roll Error (No Momentum Dumping)	88
5.22 Long Term Behaviour of Spacecraft Pitch and Yaw Errors (No Momentum Dumping)	89
5.23 Long Term Behaviour of Spacecraft Pointing Error (No Momentum Dumping)	89
5.24 Long Term Behaviour of GyroWheel Tilt Angle (No Momentum Dumping)	90
5.25 Long Term Behaviour of GyroWheel Angular Momentum (No Momentum Dumping)	90
5.26 Long Term Behaviour of Spacecraft Roll Error (With Momentum Dumping)	91
5.27 Long Term Behaviour of Spacecraft Pitch and Yaw Error (With Momentum Dumping)	91
5.28 Long Term Behaviour of Spacecraft Pointing Error (With Momentum Dumping)	92
5.29 Long Term Behaviour of GyroWheel Tilt Angle (With Momentum Dumping)	92
5.30 Long Term Behaviour of GyroWheel Angular Momentum (With Momentum Dumping)	93

5.31 Long Term Behaviour of Spacecraft Roll Error Using Classical Wheel Control (With Momentum Dumping)	95
5.32 Long Term Behaviour of Spacecraft Pitch and Yaw Errors Using Classical Wheel Control (With Momentum Dumping)	96
5.33 Long Term Behaviour of Spacecraft Pointing Error Using Classical Wheel Control (With Momentum Dumping)	96

List of Tables

2.1	GyroWheel Specifications (SCISAT-1 model)	17
3.1	SCISAT-1 Orbital Parameters	23
3.2	SCISAT-1 Mass and Inertia Properties	25
3.3	Expected Disturbance Torques for SCISAT-1	26
3.4	SciSat-1 Attitude and Rate Sensors	27
5.1	GyroWheel Model Inputs	60
5.2	GyroWheel Model Outputs	61
5.3	Attitude and Rate Filter Coefficients	64
5.4	Attitude and Rate Gains for Roll Axis Control	65
5.5	Attitude and Rate Gains for Pointing Control (100-s Settling Time) .	65
5.6	Attitude and Rate Gains for Pointing Control (200-s Settling Time) .	65
5.7	Attitude and Rate Gains for Pointing Control (300-s Settling Time) .	66
5.8	Commanded Roll Profile	67

5.9	Statistics for Roll Error (One Orbit)	73
5.10	Statistics for Pointing Error (One Orbit)	74
5.11	Statistics for GyroWheel Tilt Angle (One Orbit)	74
5.12	Statistics for the GyroWheel Angular Momentum (One Orbit)	75
5.13	Long-Term GyroWheel Control Performance (10 Orbits)	87
5.14	Statistics for Long-Term GyroWheel Control Performance (10 Orbits)	93
5.15	Control Gains for Classical Wheel Based Control	95
5.16	Statistics for Long-Term Classical Wheel Control Performance (10 Or- bits)	97

Nomenclature

$\underline{B}$	Earth magnetic field vector
D_r	Displacement tensor of GyroWheel centre of mass from spacecraft centre of mass
$F(z)$	z-transform of IIR filter
$F_p(s)$	Laplace transform matrix for the attitude IIR filter
$F_r(s)$	Laplace transform matrix for the rate IIR filter
$\underline{H}$	Total angular momentum of spacecraft with spinning components
$\underline{H}_{int}$	Angular momentum of internal spinning components of the spacecraft
$\underline{H}_{sc}$	Angular momentum of the spacecraft body
I_{gw}	Inertia tensor of GyroWheel rotor about the spacecraft centre of mass
$\tilde{I}_{sc}$	Measured spacecraft inertia
$\hat{I}_r$	Principal axis inertia tensor of the GyroWheel rotor
I_s	Spin axis inertia of the GyroWheel rotor
I_t	Transverse axis inertia of the GyroWheel rotor
I_w	Moment of inertia of a reaction wheel rotor about the spin axis

I_x	Principal moment of inertia about X axis
I_y	Principal moment of inertia about Y axis
I_z	Principal moment of inertia about Z axis
I_{gx}	Moment of inertia of the tuned rotor gyro gimbal
I_{sc}	Spacecraft moment of inertia tensor
I'_{sc}	Principal axis offset component of the spacecraft inertia tensor
I_{xx}	Moment of inertia about the spacecraft body X axis
I_{xy}	Product of inertia based on the spacecraft X and Y body axes
I_{yy}	Moment of inertia about the spacecraft body Y axis
I_{yz}	Product of inertia based on the spacecraft Y and Z body axes
I_{zx}	Product of inertia based on the spacecraft Z and X body axes
I_{zz}	Moment of inertia about the spacecraft body Z axis
ΔI_{sc}	Change in spacecraft moment of inertia tensor due to tilting of GyroWheel rotor
δI_{sc}	Error in measurement of the spacecraft moment of inertia tensor
K_h	Angular momentum gain matrix
K_p	Proportional (attitude) gain matrix
K_r	Derivative (rate) gain matrix
K_D	Momentum dumping gain matrix
K_{ff}	Feed-forward gain matrix

R_x	Rotation matrix about local X axis
R_y	Rotation matrix about local Y axis
R_z	Rotation matrix about local Z axis
R_{xyz}	Rotation matrix using the XYZ rotation sequence
T_z	Transformation matrix about the local Z axis
T_{xyz}	Transformation matrix using the XYZ rotation sequence
T_{yz}	Transformation matrix using the YZ rotation sequence
ω_d	Desired spacecraft spin rate
a_0	IIR filter numerator coefficient (0^{th} order)
a_1	IIR filter numerator coefficient (1^{st} order)
b_0	IIR filter denominator coefficient (0^{th} order)
b_1	IIR filter denominator coefficient (1^{st} order)
γ_y	GyroWheel rotor tilt angle about spacecraft body Y axis
γ_z	GyroWheel rotor tilt angle about spacecraft body Z axis
h	Angular momentum of the GyroWheel rotor
$\underline{h}^{(d)}$	Desired total spacecraft angular momentum
$\underline{h}_{gw}^{(d)}$	Desired GyroWheel angular momentum
$\underline{h}_{gw}^{(m)}$	Measured GyroWheel angular momentum
$\underline{h}_{gw}$	Angular momentum vector of GyroWheel rotor
$\underline{h}_r$	Angular momentum vector of tuned rotor gyro

$\underline{h}_{sc}^{(m)}$	Measured total spacecraft angular momentum
h_r	Angular momentum of rotor in tuned rotor gyro
h_w	Angular momentum of a reaction wheel rotor
k	GyroWheel discrete-time command index
k_f	Stiffness of flexure pivot
k_{hx}	Angular momentum gain for X axis component
k_{hy}	Angular momentum gain for Y axis component
k_{hz}	Angular momentum gain for Z axis component
k_{px}	Roll attitude gain term
k_{py}	Pitch attitude gain term
k_{pz}	Yaw attitude gain term
k_{rx}	Roll rate gain term
k_{ry}	Pitch rate gain term
k_{rz}	Yaw rate gain term
$\underline{\mu}$	Dipole moment of the spacecraft
m_r	Mass of GyroWheel rotor
ϕ	Rotation angle about local X axis
ψ	Rotation angle about local Z axis
s	Laplace transform variable
t	Time

Δt	Time interval over which the command is to be applied
τ_w	Torque applied to the spacecraft by a single reaction wheel
τ'_x	X component of external applied torque to the spacecraft (including off-axis inertia effects)
τ'_y	Y component of external applied torque to the spacecraft (including off-axis inertia effects)
τ'_z	Z component of external applied torque to the spacecraft (including off-axis inertia effects)
$\underline{\tau}'$	External applied torque to the spacecraft (including off-axis inertia effects)
τ_c	Spacecraft control torque
τ_w	Torque applied to the spacecraft by multiple reaction wheels
τ_D	Momentum dumping torque
$\tau_{\delta I}$	Disturbance torque due to error in the measurement of the spacecraft inertia tensor
$\tau_{\delta \omega}$	Disturbance torque due to error in the measurement of the rate offset
$\tau_{\delta \theta}$	Disturbance torque due to error in the measurement of the attitude offset
τ_{com}	Torque command sent to GyroWheel
τ_{dist}	Total disturbance torque acting on the satellite
$\tau_{dist}(s)$	Laplace transform of the total disturbance torque applied to the spacecraft
τ_{ext}	Torque applied externally to the spacecraft
τ_{ff}	Feed-forward torque

τ_{gw}	Torque applied to the spacecraft by GyroWheel
τ_{oa}	Disturbance torque due to spacecraft products of inertia
τ_r	Torque applied by rotor in tuned rotor gyro
τ_{tr}	Torque applied to spacecraft by magnetic torque rods
t^*	Time at which a command is sent to GyroWheel
θ	Rotation angle about local Y axis
$\Delta\theta_x$	Component of attitude error about the local X axis
$\Delta\theta_y$	Component of attitude error about the local Y axis
$\Delta\theta_z$	Component of attitude error about the local Z axis
$\delta\theta_m$	Error in sensor measurement of the attitude offset
$\underline{\theta}(s)$	Laplace transform of the spacecraft attitude vector
$\underline{\theta}_d$	Desired spacecraft attitude vector
$\underline{\theta}_d(s)$	Laplace transform of the desired spacecraft attitude vector
$\underline{\theta}_{sc}$	Spacecraft attitude vector
$\underline{\Delta\theta}^{(m)}$	Attitude error as measured by the spacecraft sensors
$\underline{\Delta\theta}$	Spacecraft attitude error vector
$\underline{\Delta\theta}_p$	Attitude error measurement made by secondary sensor
$\underline{\Delta\theta}_s$	Attitude error measurement made by primary sensor
$\underline{v}_p$	Vector of observability for primary attitude or rate sensor
ω_t	Tuned speed of a tuned rotor gyro

$\underline{\delta\omega}_m$	Error in sensor measurement of the rate offset
$\underline{\omega}$	Angular rate of body-fixed frame with respect to inertial space
$\underline{\omega}(0)$	Spacecraft angular rates at $t = 0$ (initial conditions)
$\underline{\omega}_d$	Desired spacecraft angular rate vector
$\underline{\omega}_h$	Homogenous solution to the spacecraft equation of motion
$\underline{\omega}_{nh}$	Non-homogenous solution to the spacecraft equation of motion
$\underline{\omega}_{sc}$	Spacecraft angular rate vector
ω_o	Constant satellite spin rate
ω_x	Component of the spacecraft angular rate in the body X axis
ω_x	Spacecraft angular rate in x axis
ω_y	Component of the spacecraft angular rate in the body Y axis
ω_y	Spacecraft angular rate in y axis
ω_z	Component of the spacecraft angular rate in the body Z axis
$\underline{\Delta\omega}^{(m)}$	Spacecraft angular rate vector
$\underline{\Delta\omega}_p$	Rate error measurement made by secondary sensor
$\underline{\Delta\omega}_s$	Rate error measurement made by primary sensor
x	Displacement from the centre of mass along the spacecraft body X axis
y	Displacement from the centre of mass along the spacecraft body Y axis
z	Displacement from the centre of mass along the spacecraft body Z axis

Chapter 1

Introduction

SCISAT-1 is the first in a series of scientific satellite missions being developed as part of the Canadian Space Agency's Small Scientific Satellite Program. Over its two-year mission lifetime, SCISAT-1 will conduct studies on the distribution of ozone and aerosols in the Earth's atmosphere. The prime contractor for the design and assembly of the satellite is Bristol Aerospace Ltd. of Winnipeg, Manitoba. As of the time of this writing, the satellite is slated for launch in the summer of 2003. SCISAT-1 will be a slow-spinning momentum-biased satellite, with a nominal actuating system consisting of a classical momentum wheel and three orthogonal magnetic torque rods.

Also included on SCISAT-1 is GyroWheel, an innovative attitude determination and control device developed by Bristol Aerospace that is being flown for the first time. It is included principally as a demonstration unit, with performance tests to be conducted during gaps in the science part of the mission, but will also serve as a backup actuator should the nominal attitude control system fail.

This thesis describes the development of a control law that allows GyroWheel to function as the primary actuator for SCISAT-1. When research for this thesis began, the attitude determination and control system on SCISAT-1 had already been largely

defined, with the control law for the classical wheel and torque rod system firmly in place. A control law for GyroWheel had been developed as well, which used the angular momentum of the GyroWheel rotor to apply a momentum bias to the system for rigidity. The focus of this thesis was the development of a parallel GyroWheel control law that operates the spacecraft as a non-biased system, which improves the slew capability. Part of this work has been already presented in [7]. This thesis, however, describes the derivation in greater detail, and includes more extensive results from computer simulations that demonstrate the viability of the control law under expected on-orbit conditions.

The remainder of Chapter 1 is an introduction to the fundamentals of designing an attitude determination and control system for a spacecraft, which will serve as a basis for the descriptions and derivations given in subsequent chapters. Chapter 2 is a detailed description of the GyroWheel unit itself and how it functions. Chapter 3 is an overview of the SCISAT-1 mission, the sensors and actuators that are available onboard, and the attitude determination and control system that ties them all together. Chapter 4 describes the derivation of the GyroWheel-specific control system using the spacecraft equations of motion. Chapter 5 describes the validation of the GyroWheel-based control law through the use of computer modeling and simulation. Finally, Chapter 6 presents the conclusions reached about the GyroWheel based control law performance based on the test results and offers recommendations for future work in this area.

1.1 Attitude Determination and Control of a Small Spacecraft

The purpose of an attitude determination and control system (ADCS) is to maintain the spacecraft in a desired orientation and/or spin state in the presence of disturbances. Sources of disturbance include gravity gradient torques, solar pressure torques, aerodynamic drag torques, and magnetic torques. Small spacecraft, particularly those with masses under 1000 kg, present a special challenge as their low moments of inertia make them dynamically more susceptible to the effects of these disturbances.

In terms of attitude control, spacecraft can generally be separated into two categories: spin stabilized and three-axis stabilized. Spin stabilized spacecraft take advantage of gyrodynamic stiffness, whereby a fast-spinning body will tend to maintain a fixed orientation in inertial space in the absence of large applied torques. The whole spacecraft body is therefore made to spin at a high angular rate, typically in the range of 20 to 90 RPM [9].

Three-axis stabilized spacecraft, on the other hand, rely on actuators to counteract disturbance torques. Included in this category are slow-spinning spacecraft, such as Earth-pointing and Sun-pointing satellites, that perform rotations in order to track their targets rather than for gyrodynamic stability.

A three-axis stabilized spacecraft may be momentum-biased or non-biased. Momentum-biased systems take advantage of gyrodynamic stiffness by having internal components spin at a high angular rate while the body of the spacecraft itself is held steady. In non-biased systems, the total nominal angular momentum of the spacecraft is zero, so no gyrodynamic effect is employed. Momentum-biased spacecraft are typically used for Earth-pointing or Sun-pointing missions, whereas non-biased systems

are typically used for missions with changing targets, such as astronomy missions, as the spacecraft must execute frequent attitude slews.

Regardless of the stabilization method used, a spacecraft generally requires a complement of both sensors and actuators in order to properly control its attitude. A central ADCS processor interprets the sensor data to determine the spacecraft attitude and angular rates and then calculates the torque required to correct for any offsets, using an appropriate control law. The ADCS processor then issues commands to the actuators to apply this torque to the spacecraft body.

The following sections describe the most common types of sensors and actuators used in small spacecraft.

1.2 Attitude and Rate Sensors

There are a number of different sensors available, both for determining spacecraft attitude and for determining angular body rates. If the sample frequency is sufficiently high, a sensor that is designed to measure attitude can function as a rate sensor as well by applying numerical differentiation. Additionally, a sensor that is specifically designed to measure rate can often be used to estimate attitude change using numerical integration, although some knowledge of attitude is required as well, to provide an initial attitude measurement and to periodically correct for integration error.

The following sensors represent the most common types used for small spacecraft:

- Photometric sensors
- Magnetometers
- Gyroscopic sensors.

1.2.1 Photometric Sensors

The most common types of photometric attitude sensors are sun sensors, Earth horizon sensors, and star sensors.

Sun sensors use arrays of photosensitive cells to detect the position of the Sun relative to the spacecraft body. They can be very accurate, but have the drawback that they cannot be used when a spacecraft is in eclipse. The output from a sun sensor is usually a vector representing the apparent direction of the Sun centroid as measured in the spacecraft body frame. This only gives two axes of attitude information, however, so a sun sensor must be used in combination with other sensors whenever three axes of observability is required.

Earth horizon sensors determine the apparent location of the Earth's limb by detecting the abrupt change in emitted infrared light between the Earth and black sky. An array of these sensors, combined with an accurate model of the Earth's shape, can be used to determine attitude about two axes. An obvious use for these sensors is in Earth-pointing spacecraft such as communication and resource satellites.

Star sensors, also referred to as star trackers, use patterns of fixed stars to compute spacecraft attitude and body rate. An image is taken of a star field and compared with a reference map in order to determine the attitude offset. Star sensors have the advantages of potentially high accuracy and full three-axis attitude determination capability, but can be expensive compared with other sensors due to the sophisticated optics that is often required.

1.2.2 Magnetometers

Magnetometers are commonly used in low-Earth orbiting satellites due to their low cost and high reliability. The output is typically a vector that describes the observed

direction of the Earth's local magnetic field in body-referenced co-ordinates. The attitude offset is calculated by comparing this measured field direction with the expected field direction if the spacecraft was at the desired attitude. The expected field direction is calculated using knowledge of the Earth's magnetic field and of the position of the spacecraft in its orbit. The accuracy of magnetometers is limited by the fact that knowledge of the Earth's magnetic field is far from perfect. Also, the field drifts noticeably over a time scale of months or years, so the field ephemeris must be updated regularly during a typical mission lifetime. Magnetometers are incapable of measuring attitude changes about the local field direction, so like sun sensors and Earth horizon sensors they only offer two axes of observability.

1.2.3 Gyroscopic Sensors

Another common sensor, and one with a very long flight heritage, is the gyroscope. The main component of the gyroscope is a fast-spinning rotor that is strapped down to the spacecraft body via a gimbal. If no external torques are applied to the rotor, it will maintain the same orientation in inertial space irrespective of the motion of the spacecraft. If the spacecraft body is perturbed about an axis other than the spin axis of the rotor, the gimbal angles of the rotor relative to the spacecraft will change as the rotor remains inertially fixed.

Theoretically, if a two-gimbal system was used, the spacecraft attitude could be read directly off the gimbals by measuring the shaft rotation angle. In practice, however, any gimbal joint that employs moving parts will experience a certain amount of friction, and this friction will impart a torque to the rotor that, over time, will alter its pointing direction. This is alleviated in simple gyros by either adding a torsional spring, which allows rate measurements, or a viscous damper, which turns the gyro into an attitude sensor. A detailed description of how these methods work can be

found in [10].

The tuned rotor gyro is a special type of two-gimbal gyroscope that uses flexure pivots for its gimbal joints rather than the traditional rotating “pins-and-jewels” joints. Because there are no moving parts involved, the joints are essentially frictionless. Instead, the pivots act like a torsional spring, exerting an opposing torque to the rotor when it tries to tilt away from the null position. If the rotor is spinning and tilted away from the null position, however, the centrifugal motion of the gimbal ring applies a torque in the opposite direction to the spring torque. At a certain spin rate the two torques will cancel each other out, and the rotor will operate as a free gyro¹. This spin rate is referred as the “tuned speed” of the rotor.

A full derivation of the tuned speed condition is given by Cain in [1]. Assuming that the gimbal angles are small, and that the flexures are all of equal stiffness, the tuned speed is given by the following equation:

$$\omega_t = \sqrt{\frac{4k_f}{I_{gx} + I_{gy} - I_{gz}}} \quad (1.1)$$

where k_f is the stiffness of each flexure pivot², I_{gx} is the principal axis spin inertia of the gimbal ring, and I_{gy} and I_{gz} are the transverse principal axis inertias.

In order to measure the spacecraft rates, external torques are applied to the rotor to maintain it in a fixed gimbal position, usually the null position. Since all other torques have been minimized by operating the rotor near its tuned speed, the amount of torque that has to be applied to the rotor to keep the gimbal angles constant will be proportional to the component of the spacecraft body rate that is perpendicular to the spin axis. This can be seen by examining the rotor behaviour in a co-ordinate system that is fixed to the spacecraft body, with the z-axis aligned with the rotor

¹There will be some residual torques due to the “fluttering” motion of the gimbal ring, as described in [1], but these can be minimized by appropriate design of the gimbal inertias.

²Each gimbal has two flexure pivots, for a total of four.

spin axis. The dynamic behaviour of the rotor in response to the applied torque is expressed as follows:

$$\underline{\tau}_r = \dot{\underline{h}}_r + \underline{\omega}_{sc} \times \underline{h}_r \quad (1.2)$$

where $\underline{h}_r$ is the angular momentum of the rotor, $\dot{\underline{h}}_r$ its time derivative, $\underline{\tau}_r$ the applied torque, and $\underline{\omega}_{sc}$ the spacecraft body rate. The cross product term results from the fact that the torque and angular momentum in (1.2) are defined with respect to a reference frame that is aligned with the spacecraft body, which is rotating with respect to inertial space.

The angular momentum of the rotor in this co-ordinate system is:

$$\underline{h}_r = \begin{bmatrix} 0 \\ 0 \\ h_r \end{bmatrix} \quad (1.3)$$

where h_r is the magnitude of the angular momentum. Inserting (1.3) into (1.2), and noting that $\dot{h}_r = 0$, gives:

$$\underline{\tau} = \begin{bmatrix} \omega_y h_r \\ -\omega_x h_r \\ 0 \end{bmatrix} \quad (1.4)$$

where ω_x and ω_y are the components of the spacecraft rate in the plane of the rotor. It is clear from this expression that spacecraft motion about the rotor spin axis cannot be sensed by this method, as there is no gyrodynamic effect in this direction. The tuned rotor gyro therefore functions as a two-axis rate sensor, with the output being the measured spacecraft rates, ω_x and ω_y , about the two axes perpendicular to the rotor spin axis.

1.3 Actuators

Spacecraft actuators can be divided into two main categories: passive and active. The most common form of passive actuator is the gravity gradient boom, which is often used for Earth-pointing satellites in low orbits. The majority of actuators in use today, however, fall under the active category. These include, but are not necessarily limited to, the following:

- Reaction thrusters
- Magnetic torquers
- Reaction wheels
- Momentum wheels
- Control moment gyros

1.3.1 Reaction Thrusters

Reaction thrusters can be used to apply torques in any required direction to the spacecraft body. The major drawback to this method is that the spacecraft must carry a supply of propellant sufficient to cover the needs of the ADCS, which adds weight and complexity and limits mission lifetime. The exhaust from the thrusters can also be potentially harmful to any sensitive instrumentation on board.

1.3.2 Magnetic Torquers

Magnetic torquers, also known as magnetorquers or torque rods, make use of the Earth's magnetic field to apply corrective torques. Each torque rod is an electromag-

net that generates a magnetic dipole moment when a current is passed through its coils. The torque applied to the spacecraft by the torque rods, $\underline{\tau}_{tr}$, will be equal to the cross product of the dipole moment, $\underline{\mu}$, with the local magnetic field vector, $\underline{B}$:

$$\underline{\tau}_{tr} = \underline{\mu} \times \underline{B} \quad (1.5)$$

The applied torque tends to align the rod with the local magnetic field. A spacecraft will typically have three torque rods, each aligned with a particular orthogonal body axis. It is clear from (1.5) that no torque can be applied in the direction of the local magnetic field. Magnetic torque rods therefore do not offer full three-axis controllability.

1.3.3 Reactions Wheels and Momentum Wheels

A reaction wheel consists of a rotor that is made to spin about a fixed axis. Control torques are applied to the spacecraft in the direction of the spin axis by varying the rotor spin rate. If a single wheel is used, the torque-momentum relationship is as follows:

$$\tau_w = -\dot{h}_w = -I_w \dot{\omega}_w \quad (1.6)$$

where τ_w is the applied wheel torque, $\dot{h}_w$ is the time rate of change of the wheel angular momentum, I_w is the rotor inertia about the spin axis, and $\dot{\omega}_w$ is the time rate of change of the rotor spin rate. Note the minus sign, which indicates that this is the torque applied to the spacecraft by the wheel, not the other way around.

A reaction wheel is limited in its operating range by the load capacity of the drive motor. The maximum achievable spin rate of the wheel is referred to as its saturation limit. In order to prevent the rotor from hitting this limit, a method of eliminating

momentum build-up is required. This is usually accomplished by applying small amounts of torque to the spacecraft using reaction thrusters or magnetic torque rods.

A momentum wheel operates on many of the same principles as a reaction wheel, except that the rotor is given a nominal momentum bias by maintaining its spin rate near some pre-defined value. A reaction wheel, by contrast, can potentially spin at any rate below its saturation limit, including zero. External momentum dumping torques are still required when using a momentum wheel, to prevent the momentum from drifting too far from its nominal value.

Each momentum wheel or reaction wheel has torquing authority only over its own spin axis. Full three-axis controllability therefore requires multiple wheels, or a single wheel with complementary actuators such as torque rods.

In the case of multiple wheels, a vector equation replaces the scalar one in (1.6):

$$\underline{\tau}_w = -\dot{\underline{h}}_w - \underline{\omega}_{sc} \times \underline{h}_w \quad (1.7)$$

where $\underline{\tau}_w$ is the total applied torque from all wheels, $\underline{h}_w$ is the total angular momentum, and $\underline{\omega}_{sc}$ is the body rate of the spacecraft with respect to inertial space. Note the signs used, which again indicate this is the torque applied **to** the spacecraft.

In this case where a single wheel is used, the torque applied in the direction of the wheel spin axis will be the same as that given by (1.6).

1.3.4 Control Moment Gyros

A control moment gyro is a momentum-biased device that can apply torques to two axes of the spacecraft, rather than just one. The rotor in a control moment gyro spins at a nominal rate like a conventional momentum wheel, but is gimbaled to allow it to tilt with respect to the drive shaft. Torques can therefore be generated in two axes by

adjusting the gimbal angles, which moves the angular momentum vector. The total applied torque is then given by (1.7). As with any momentum-based device, a control moment gyro requires external momentum dumping to stay within operating ranges, particularly since there will typically be upper limits to the gimbal angles in addition to the usual restrictions on the spin rate.

Chapter 2

Introduction to GyroWheel

GyroWheel is an innovative device that combines the actuating capabilities of the control moment gyro with the rate sensing capabilities of the tuned rotor gyro. It can generate control torques in three axes while also sensing the body rate of the spacecraft in two axes. This gives it a significant advantage over conventional fixed-axis momentum wheels, which provide control torques about only one axis and depend on external sensors for rate information.

Figure 2.1 is a photograph of the GyroWheel unit, with the upper lid removed to show the rotor. The diameter of the outer case is 23.5 cm. Figure 2.2 is a cutaway diagram that identifies the major components: rotor, gimbal, torque coil, permanent magnet, pattern scribe, and tilt sensor.

2.1 GyroWheel Operation

The GyroWheel rotor is made of aluminum, with an annular-shaped gap on the underside (not visible in Figure 2.1). Permanent magnets are mounted to both interior

Figure 2.1: GyroWheel Rotor and Housing

Figure 2.2: Cutaway Diagram of GyroWheel

surfaces of the rotor, as indicated in Figure 2.2. A sawtooth pattern is etched onto the outside edge of the rotor to allow an optical sensor to measure the tilt angle and spin rate. Spin-up is provided by a brushless DC motor and connecting drive shaft, which are closed-loop controlled. In a classical momentum or reaction wheel, the rotor would be mounted directly to the drive shaft. For GyroWheel, a specially designed flexure gimbal system is used instead.

Figure 2.3 is a close-up view of the GyroWheel gimbal system, with the rotor removed. Each joint is a pair of flexure pivots arranged in a cross pattern. The cylindrical gimbal ring is connected to the drive shaft by two pairs of pivots, located 180° apart. The support ring for the rotor, visible at the top of the assembly in Figure 2.3, is connected to the gimbal ring through another twinned set of cross flexure pivots, oriented 90° with respect to those connecting the gimbal ring to the shaft (on the left and right sides as viewed in Figure 2.3). The gimbaling allows the rotor to be tilted up to 7° with respect to the drive shaft in each of two axes.

Figure 2.3: GyroWheel Gimbal System

The following subsections describe the basic methods by which GyroWheel functions

as both an actuator and a two-axis rate sensor. A more detailed examination of these methods can be found in [2], [3], [4], and [5].

2.1.1 Torque Actuation

Control torques about the spin axis of the GyroWheel rotor can be generated in a straightforward manner by varying the speed of the DC drive motor. Torques about the other two axes are generated by tilting the rotor while spinning. The tilt actuation is performed by two pairs of electromagnetic torque coils that are mounted to the case and positioned inside the rotor gap, between the two sets of permanent magnets. When a current is supplied to either pair of coils, a torque is generated that causes the rotor to tilt with respect to the drive shaft. An optical sensor mounted to the case scans horizontally across the pattern scribe as the rotor spins. The relative times between successive crossings of the sawtooth pattern are used to calculate the instantaneous gimbal angles, which provides feedback for closed-loop tilt control. Controlling both the spin rate and the tilt angle of the rotor means that the angular momentum vector can be completely specified, within the mechanical constraints of the system.

2.1.2 Rate Sensing

Two-axis rate sensing is accomplished by measuring the amount of current being supplied to the tilt coils. A calibrated scale factor is then applied to convert the measured current into a torque value. Rate sensing accuracy decreases whenever the rotor spin rate deviates from tuned speed, or whenever the momentum vector changes rapidly with time. If GyroWheel is operating within reasonable bounds, however, accurate rate sensing can be obtained at the same time GyroWheel is being used as an actuator.

2.1.3 Operating Specifications

Table 2.1 lists some of the operating specifications for GyroWheel.

Table 2.1: GyroWheel Specifications (SCISAT-1 model)

Rotor spin inertia	0.0197 kg-m ²
Rotor transverse inertia	0.0105 kg-m ²
Tuned rotor speed	1460 RPM
Tuned rotor momentum	3.01 N-m-s
Maximum rotor speed	6000 RPM
Maximum tilt angle	7°
Maximum torque capability (all axes)	0.1 N-m

2.2 ADCS Interface

GyroWheel has the capability of operating under several different command modes. The modes that are most commonly used are momentum command mode and torque command mode. The internal closed-loop rotor control is identical in both cases, but the form of the ADCS command to GyroWheel and how it is interpreted differs, as described in the following subsections.

The outputs from GyroWheel to the ADCS include the measured angular momentum of the rotor, the measured spacecraft rate, and, if requested, an integrated attitude estimate. GyroWheel also outputs status flags to indicate if an internal error has occurred or an operating limit has been exceeded.

2.2.1 Momentum Command Mode

In momentum command mode, the ADCS supplies a desired angular momentum vector and execution time to GyroWheel. The internal GyroWheel controller adjusts the tilt angle and spin speed of the rotor so that the momentum vector follows a straight-line path from its initial state to the commanded final state, at a constant slew rate. If torquing limits prevent the requested momentum change from occurring in the specified time, the rotor is steered toward the desired momentum at the maximum possible slew rate, and a status flag is sent to the ADCS indicating that the requested momentum change was only partially completed. If a tilt angle limit is reached, the rotor will hold its position and speed at the maximum tilt, and a similar status flag is sent to the ADCS.

2.2.2 Torque Command Mode

In the case of torque command mode, the inputs to GyroWheel from the ADCS are a desired torque, the measured spacecraft body rate, and an execution time. The torque command mode is particularly useful when ADCS processing resources are limited, as it allows the ADCS to issue the control torque directly to GyroWheel without having to convert it to a momentum command first. The conversion is instead performed internally by GyroWheel. What follows is a description of the implementation of torque command mode that was developed for use on SCISAT-1.

The relationship between the torque applied to the spacecraft by GyroWheel, $\underline{\tau}_{gw}$, the total angular momentum of the GyroWheel rotor and gimbal assembly, $\underline{h}_{gw}$, and the spacecraft body rate, $\underline{\omega}_{sc}$, is defined in the same manner as (1.7):

$$\underline{\tau}_{gw} = -\dot{\underline{h}}_{gw} - \underline{\omega}_{sc} \times \underline{h}_{gw} \quad (2.1)$$

Based on previous GyroWheel conventions, however, the torque command to be issued by the ADCS was defined to be the amount of torque to applied to the rotor by the spacecraft, rather than other way around. The commanded torque, τ_{com} , is therefore related to the rotor angular momentum and spacecraft body rate as follows:

$$\tau_{com} = \dot{h}_{gw} + \underline{\omega}_{sc} \times \underline{h}_{gw} \quad (2.2)$$

If a torque command is sent to GyroWheel by the ADCS at a time t^* , along with a specified command time interval of Δt , then the desired angular momentum at time $t^* + \Delta t$ is calculated by integrating (2.2):

$$\underline{h}_{gw}(t^* + \Delta t) = \underline{h}_{gw}(t) + \int_{t^*}^{t^* + \Delta t} [\tau_{com} - \underline{\omega}_{sc}(t) \times \underline{h}_{gw}(t)] dt \quad (2.3)$$

If the integration period, Δt , is equal to the ADCS command interval, then (2.3) can be written as follows, for commands issued on the k^{th} ADCS tick:

$$\underline{h}_{gw}(k + 1) = \underline{h}_{gw}(k) + \int_{t(k)}^{t(k+1)} [\tau_{com} - \underline{\omega}_{sc}(t) \times \underline{h}_{gw}(t)] dt \quad (2.4)$$

For SCISAT-1, a simple Euler integration is employed, using spacecraft rate measurements supplied as additional inputs by the ADCS. The relationship between the target angular momentum and the commanded torque is therefore as follows:

$$\underline{h}_{gw}(k + 1) = \underline{h}_{gw}(k) + [\tau_{com} - \underline{\omega}_{sc}(k) \times \underline{h}_{gw}(k)] \Delta t \quad (2.5)$$

where h_x , h_y , and h_z are the components of $\underline{h}_{gw}$ in each spacecraft axis, and ω_x , ω_y , and ω_z are the components of $\underline{\omega}_{sc}$. This implementation assumes constant values for $\underline{\omega}_{sc}$ and $\underline{h}_{gw}$ during integration. In reality, the GyroWheel momentum and spacecraft body rate can change significantly during the period Δt , so the actual applied torque will not be constant throughout the command interval. This problem could be alleviated by employing a more accurate integration method, such as a Modified Euler

or Runge-Kutta method. An even more promising possibility, however, would be to use the internal GyroWheel rate sensing to provide rate measurements about two of the axes and only use the rates supplied by the ADCS for the third axis. The advantage to this method is that internal rate measurements are available very frequently as compared with typical ADCS-supplied measurements, so the integration step size can be made smaller.

As will be discussed later, the simple Euler integration described above was found through simulation to be sufficiently stable for the pointing requirements of SCISAT-1. For future missions with more stringent attitude requirements, however, a more robust algorithm should probably be considered.

Chapter 3

Overview of SCISAT-1

This chapter is a description of the SCISAT-1 satellite, with emphasis on the attitude determination and control system used. The material presented in this chapter reflects the configuration of the satellite as already defined at the time that research for this thesis began. The constraints that were imposed on the development of a GyroWheelbased control law for SCISAT-1 due to the nature of the mission and due to the preset ADCS software algorithms are therefore indicated here.

3.1 Science Payload and Pointing Requirements

The science mission selected for the SCISAT-1 satellite is the Atmospheric Chemistry Experiment (ACE). The primary objectives of the ACE mission are to measure the distribution of ozone in the Earth's atmosphere and to determine some of the chemical processes involved in that distribution. Of particular concern to Canada, due to its geographic location, is the depletion of ozone that has been observed above the high Arctic in recent years, particularly during early spring. Data gathered by the ACE mission will be combined with results from similar studies, conducted both from orbit

and from the ground, to generate a more complete picture of ozone layer dynamics.

The scientific payload on board SCISAT-1 consists of two instruments: a Fourier Transform Spectrometer (ACE-FTS) and a second spectrometer known as MAESTRO (Measurements of Aerosol Extinction in the Stratosphere and Troposphere Retrieved by Occultation). A full description of each of these instruments can be found in [6].

The presence of ozone and various aerosols is indicated by absorption lines and bands in the spectrum of sunlight when it travels through the atmosphere. The orbit of SCISAT-1 has therefore been designed so that the satellite passes behind the Earth at frequent intervals, with science measurements made each time it goes through the “shadow” of the atmosphere, as shown in Figure 3.1.

Figure 3.1: SCISAT-1 Orbit

Table 3.1 lists the orbital parameters for SCISAT-1. The satellite will orbit the Earth

15 times each day, with two occultations per orbit. In order to take measurements, the instruments must be pointed continuously toward the sun during each occultation. Analysis of the instrument capabilities early in the mission planning phase determined that a $3\text{-}\sigma$ pointing accuracy of 1° from the Sun centroid was necessary in order to meet the science requirements [6].

Table 3.1: SCISAT-1 Orbital Parameters

Altitude	650 km
Inclination	73.9°
Period	5857 sec

3.2 Satellite Configuration

Figure 3.2 shows the basic configuration of SCISAT-1, with the X, Y, and Z body reference axes specified. For attitude control purposes, the X body axis of the spacecraft is also called the roll axis, while the Y and Z body axes are defined to be the pitch and yaw axes respectively. The instrument aperture and solar panels are both located on the +X face, which is the face that nominally points toward the Sun. To give a sense of scale, the +X face is just over 1.1 m in diameter.

Mounted onto the +Z face of the satellite is a cryo-radiator, which is used to remove excess heat from the ACE-FTS and help maintain the instrument at a desired temperature. In order to maximize heat dissipation, it is important to keep the radiator pointed toward black sky, away from both the Sun and the Earth. Mounting the radiator on the +Z face keeps it away from the Sun during normal operations. To keep it from pointing at the Earth, the satellite executes a slow rolling motion as it orbits.

Figure 3.2: SCISAT-1

As stated previously, the science instrument attitude requirement is Sun-pointing to within 1° . Restated with the axis definitions, this means that the combined pitch and yaw error must be less than 1° . The requirement for roll angle control depends on which science instrument is being used. The ACE-FTS requires that the radiator be continuously pointed away from the Earth. This means that the roll angle will change with respect to inertial space as the satellite orbits, with the average roll rate being equal to the orbit rate of about $0.06^\circ/\text{s}$, or 0.001 rad/s . In preliminary mission analysis of SCISAT-1, it was determined that the desired roll profile should be followed to within 5° ($1\text{-}\sigma$) and 15° ($3\text{-}\sigma$) for acceptable control of the radiator pointing direction [6].

For MAESTRO, the secondary science instrument, the requirement is that the instrument slit be kept tangent to the Earth's horizon during occultation to within $\pm 5^\circ$ ($1\text{-}\sigma$) [6]. The horizon at the sunrise occultation will be rotated approximately

180° with respect to the horizon at sunset occultation, so the satellite must be rotated through 180° during eclipse. In order to accommodate the requirements of both science instruments, the rolling motion for the cryoradiator is usually terminated immediately before each occultation and then restarted immediately following it.

Table 3.2 lists the mass and inertia properties for SCISAT-1. These properties are obtained from the most recent dynamic tests, performed at the David Florida Laboratory in Ottawa. The centre of mass is measured from a standard reference point using the co-ordinate axes defined earlier.

Table 3.2: SCISAT-1 Mass and Inertia Properties

Mass		152.57 kg
Moments of Inertia	I_{xx}	18.63 kg-m ²
	I_{yy}	12.58 kg-m ²
	I_{zz}	13.23 kg-m ²
	I_{xy}	0.26 kg-m ²
	I_{yz}	0.97 kg-m ²
	I_{zx}	0.16 kg-m ²
Centre of mass	X	-0.351 m
	Y	0.001 m
	Z	-0.023 m

3.3 Disturbance Torques

Table 3.3 lists the expected magnitudes of each of the four major sources of disturbance, based on simulation results. It can be seen that torques due to the residual magnetic moment of the satellite will be the primary source of disturbance.

Table 3.3: Expected Disturbance Torques for SCISAT-1

Disturbance Source	Magnitude (N-m)
Magnetic	4.5×10^{-5}
Gravity Gradient	1×10^{-5}
Solar Pressure	5×10^{-6}
Atmospheric	5×10^{-6}

3.4 SCISAT-1 ADCS

This section describes the different sensors and actuators that will be included on SCISAT-1 and the basic control law that will be employed.

3.4.1 Attitude and Rate Sensors

A total of five attitude and rate sensors are available on SCISAT-1: a Coarse Sun Sensor (CSS), a Fine Sun Sensor (FSS), a magnetometer, a star tracker, and GyroWheel. Each sensor is listed in Table 3.4 along with its expected accuracy.

The CSS consists of six photocells distributed across the outer surface of the satellite. The purpose of the CSS is to determine the approximate position of the Sun relative

Table 3.4: SciSat-1 Attitude and Rate Sensors

Sensor	3- σ Accuracy
Coarse Sun Sensor (CSS)	10°
Fine Sun Sensor (FSS)	0.02°
Magnetometer	3.3°
Star Tracker	0.56° (yaw) 0.17° (roll/pitch)
GyroWheel	drift < 1°/hr

to the satellite body axes during acquisition, or whenever it is tumbling. The CSS can find the Sun regardless of attitude, as long as the satellite is not in eclipse. The accuracy of the CSS is very low, however, and so this sensor is not very useful for fine pointing control.

The FSS, by comparison, has a much higher accuracy, but its field of view is more limited. It is intended for use during normal fine-pointing operations. The FSS is mounted on the Sun-facing +X face of the satellite and measures the attitude error in pitch and yaw during the sunlit part of the orbit. The FSS is insensitive to roll error, however, so it is generally used in combination with other sensors.

The magnetometer can measure attitude regardless of where the spacecraft is in its orbit, but is much less accurate than the FSS and can only measure attitudes and rates perpendicular to the local field direction.

The star tracker provides three-axis attitude and rate information and is the nominal source of such information when performing occultation science. The key limitation of the star tracker, however, is its high power consumption, which limits the amount of time it can be operated continuously.

Since GyroWheel is a demonstration unit on SCISAT-1, it is not included in the sensor list for the nominal ADCS. As will be discussed, using GyroWheel as a rate sensor during nominal operations also has implications for the momentum situation of the satellite. It can, however, be used for sensing whenever GyroWheel is acting as primary actuator. It should be noted that calibration of GyroWheel was still in progress as of the time of this writing, so formal values for rate sensing accuracy were not available. The design specification calls for a drift rate of less than 1° per hour, and calibration tests to date have indicated compliance with this requirement [8].

3.4.2 Sensor Outputs

The output from each sensor is a measured attitude error, $\underline{\Delta\theta}$, and a measured rate error, $\underline{\Delta\omega}$. The measured attitude error is equal to the actual attitude of the spacecraft minus the desired attitude plus any measurement error:

$$\underline{\Delta\theta}^{(m)} = \underline{\theta}_{sc} - \underline{\theta}_d + \underline{\delta\theta}_m \quad (3.1)$$

Similarly, the measured rate error is equal to the actual rate minus the desired rate plus any error in measurement:

$$\underline{\Delta\omega}^{(m)} = \underline{\omega}_{sc} - \underline{\omega}_d + \underline{\delta\omega}_m \quad (3.2)$$

The desired spacecraft rate, $\underline{\omega}_d$, is a slow roll about the X axis to keep the cryoradiator in an optimal orientation. It is typically implemented as a step profile, so that the roll rate remains constant over a given time interval. The average spacecraft rate over a single orbit is equal to the orbit rate, or about 0.001 rad/s. The desired spacecraft attitude is the integral of this rotation, which will be a series of ramp functions.

To determine attitude error, a series of polynomial approximations are used for each sensor to define what its output would be if the desired roll profile was being perfectly

followed. For example, the magnetometer uses a reference polynomial that dictates what the magnetic field should look like to the spacecraft at any given point in time. The attitude error is then calculated by comparing this desired magnetometer measurement from the real measurement. The rate error is determined by differentiating of the attitude measurements.

3.4.3 Sensor Fusion

As most of the available sensors on SCISAT-1 can only observe attitude and rate errors about two axes, it is common practice to combine the measurements from two sensors to obtain an overall attitude and rate estimate. The more accurate of the two sensors is designated as the primary sensor, with the other designated as the secondary. When computing total attitude error, the attitude measured by the secondary sensor is projected onto the axis which the primary sensor cannot observe, and is then added vectorially to the component of the primary sensor attitude measurement lying in its own plane of observability. Mathematically, this is expressed as follows:

$$\underline{\Delta\theta}^{(m)} = \frac{\underline{v}_p \cdot \underline{\Delta\theta}_s}{\underline{v}_p \cdot \underline{v}_p} \underline{v}_p + \left[I - \frac{\underline{v}_p \underline{v}_p^T}{\underline{v}_p \cdot \underline{v}_p} \right] \underline{\Delta\theta}_p \quad (3.3)$$

where $\underline{\Delta\theta}_p$ is the attitude error as measured by the primary sensor, $\underline{\Delta\theta}_s$ is the attitude error as measured by the secondary sensor, $\underline{v}_p$ is the vector of non-observability for the primary sensor, and I is the identity matrix. For example, if the FSS is the primary sensor and the magnetometer is the secondary sensor, only the components of the FSS error perpendicular to the nominal sun direction are used. For the remaining axis, a component of the error measured by the magnetometer is used instead.

A similar projection is used for combining rate information from two different sensors:

$$\underline{\Delta\omega}^{(m)} = \frac{\underline{v}_p \cdot \underline{\Delta\omega}_s}{\underline{v}_p \cdot \underline{v}_p} \underline{v}_p + \left[I - \frac{\underline{v}_p \underline{v}_p^T}{\underline{v}_p \cdot \underline{v}_p} \right] \underline{\Delta\omega}_p \quad (3.4)$$

A problem clearly arises when the non-observable vectors of the two sensors become aligned. This can happen, for example, if the satellite is at a point in its orbit where the local magnetic field vector is aligned with the Sun vector, so that the magnetometer can no longer complement the measurements coming from the FSS. In such cases, the ADCS only has access to two axes of reliable attitude and rate information, and control performance is temporarily compromised. The amount of attitude drift that can occur in this situation depends on a number of factors, including the amount of time that elapses before the sensor axes diverge again.

Once the sensor data has been fused, the resulting values are passed through special discrete-time filters for control design purposes. Each axis has two filters: one for attitude and one for rate. Both are infinite impulse response (IIR) filters, with the z -transform being of the following form:

$$F(z) = \frac{a_1 z^{-1} + a_0}{b_1 z^{-1} + b_0} \quad (3.5)$$

where a_1 , a_0 , b_1 , and b_0 are constant coefficients. As will be described later, use of these filters allows flexibility in selecting appropriate feedback control gains.

3.4.4 Actuators

The nominal actuating system on SCISAT-1 consists of a classical IthacoTM momentum wheel and a set of three orthogonal magnetorquer rods. The momentum wheel is mounted so that its spin axis is parallel to the spacecraft roll axis. The nominal angular momentum of the wheel is 6.5 Nms, which was selected in order to keep the attitude drift during eclipse below 0.4°.

When the satellite is in full sunlight, the FSS provides accurate attitude and rate sensing about the pitch and yaw axes. The momentum wheel has full control authority about the roll axis at this point, while control about the remaining axes is provided

by the torque rods. During the short occultation periods, when the FSS is no longer available, the star tracker is used as the primary sensor. The star tracker cannot be used through the eclipse period, however, due to its high power consumption. GyroWheel can not be used for rate sensing without spinning the rotor, which would complicate momentum bias control. The magnetometer is therefore the only sensor available for attitude and rate determination during eclipse. The magnetometer is much less accurate in pitch and yaw than either the FSS or the star tracker, and provides only two axes of observability. Consequently, active control of the satellite during eclipse using the classical wheel is less effective than in sunlight. In fact, early dynamic analysis of SCISAT-1 found that improved pointing stability could be achieved by suspending active pointing control and minimizing drift solely using the momentum bias of the wheel. Simulations to date indicate that the total attitude drift during eclipse is typically about 0.2° .

Control of the satellite using GyroWheel presents somewhat different challenges. Like the classical momentum wheel, GyroWheel is mounted on the spacecraft so that its nominal spin axis (the zero-tilt axis) is aligned with the roll axis. Because of its tilt capability, however, GyroWheel is capable of providing control torques about all three axes, not just the roll axis. Therefore, there is no need to use the torque rods for pitch and yaw control, although they will still be required for momentum dumping.

The nominal momentum bias of GyroWheel is 3.01 Nms, or less than half that of the classical wheel, which means that the attitude drift during eclipse could reach almost 1° if a similar passive momentum-bias control technique was employed. The approach taken instead was to employ active pointing control and use GyroWheel itself as a sensor to provide pitch and yaw measurements during eclipse, with the magnetometer providing the roll measurement. For demonstration purposes, this has the added advantage of showing that GyroWheel can function as an actuator and a sensor simultaneously. As with any rate sensor, GyroWheel cannot provide

accurate attitude estimates indefinitely, due to build-up of integration error. When SCISAT-1 re-enters sunlight, however, it can switch back to the FSS for pointing error measurement. The measured attitude errors can therefore be re-initialized once per orbit.

3.4.5 Control Torque

The actuating (or control) torque, $\underline{\tau}_c$, is computed using a proportional-derivative (PD) control law, as follows:

$$\underline{\tau}_c = -\tilde{I}_{sc} (K_p \underline{\Delta\theta}^{(m)} + K_r \underline{\Delta\omega}^{(m)}) - \underline{\Delta\theta}^{(m)} \times K_h \underline{h}^{(m)} - K_{ff} \underline{\tau}_{ff} \quad (3.6)$$

where $\tilde{I}_{sc}$ is the spacecraft inertia as defined in the ADCS, K_p is a 3×3 proportional gain matrix, K_r a 3×3 rate gain matrix, K_h a 3×3 angular momentum gain matrix, $\underline{h}_{sc}^{(m)}$ the total measured angular momentum of the spacecraft, K_{ff} a 3×3 feed-forward gain matrix, and $\underline{\tau}_{ff}$ a feed-forward torque.

The control torque is distributed among the available actuators. When GyroWheel is the primary actuator, the momentum wheel will be disabled and the magnetorquer rods will be used only for momentum dumping. The commanded GyroWheel torque, $\underline{\tau}_{gw}$, will therefore be equal to the total control torque minus the dumping torque, $\underline{\tau}_D$:

$$\underline{\tau}_{gw} = \underline{\tau}_c - \underline{\tau}_D \quad (3.7)$$

The momentum dumping torque is proportional and of the opposite sense to the difference between the measured angular momentum and the desired angular momentum of the satellite as a whole:

$$\begin{aligned} \underline{\tau}_D &= -K_D (\underline{h}^{(m)} - \underline{h}^{(d)}) \\ &= -K_D (\tilde{I}_{sc} \underline{\Delta\omega}^{(m)} + \underline{h}_{gw}^{(m)} - \underline{h}_{gw}^{(d)}) \end{aligned} \quad (3.8)$$

where $\underline{h}_{gw}^{(m)}$ is the measured GyroWheel momentum, $\underline{h}_{gw}^{(d)}$ is the desired GyroWheel momentum, and K_D is 3×3 diagonal gain matrix. In general, the torque rods will not be able to supply all of the requested dumping torque, as they are only capable of generating torques perpendicular to the local magnetic field. Any error between the desired and applied dumping torque will therefore manifest itself as a disturbance to the system.

The total amount of torque that is to be applied to the spacecraft by GyroWheel is therefore:

$$\underline{\tau}_{gw} = -\tilde{I}_{sc}K_p\Delta\theta^{(m)} - \tilde{I}_{sc}K_r\Delta\omega^{(m)} - \Delta\theta^{(m)} \times K_h\underline{h}^{(m)} - K_{ff}\underline{\tau}_{ff} - \underline{\tau}_D \quad (3.9)$$

3.4.6 ADCS Schematic

The overall attitude determination and control system is shown in schematic form in Figure 3.3 for the case where GyroWheel is the primary actuator. The inputs and outputs for each element in the system are indicated in the diagram.

Figure 3.3: SCISAT-1 Attitude Determination and Control System (ADCS)

Chapter 4

Development of the GyroWheel Control Law

The approach used in developing a GyroWheel based control law for SCISAT-1 has been to separate the motion of the GyroWheel rotor from the motion of the spacecraft itself. This approach means that the spacecraft will have negligible momentum bias, which increases slew capability and reduces the effect of nutation on the spacecraft motion. Agility is not as an important a consideration for SCISAT-1, which will remain close to inertially-pointed throughout its mission, but damping nutation will definitely have a positive effect on fine-pointing control. Also, demonstrating that zero-biased control can be performed using GyroWheel as an actuator will make it a more attractive choice for future missions that may require large-angle slews.

In practice, the effects of momentum dumping and sensor errors mean that the dynamics of the GyroWheel rotor and the spacecraft can never be completely decoupled. If the magnitude of these effects is sufficiently small, however, they can be treated simply as external sources of disturbance for the purposes of control design.

The derivation of the control law begins with the general equations of motion for a spacecraft. Simplifications can then be made to these equations for the particular case of SCISAT-1 using GyroWheel as its primary actuator. The objective is to decouple the equations for roll, pitch, and yaw motion from one another, so that each axis of the spacecraft can be controlled independently. The control system is thereby reduced from a multi-variable system to three separate single-input single-output (SISO) systems, which simplifies feedback control design.

4.1 Spacecraft Equations of Motion

External torques acting on a body in space will cause the angular momentum of the body to change. If the torque and angular momentum vectors are both specified with respect to an inertial reference frame, then the time rate of change of angular momentum is equal to the applied torque:

$$\dot{\underline{H}} = \underline{\tau}_{ext} \quad (4.1)$$

For spacecraft, however, it is normal practice to develop the equations of motion in a reference frame that moves with the spacecraft body, as this is the reference frame in which sensors and actuators are typically fixed. If the angular momentum and torque vectors are to be specified in this rotating frame, then (4.1) must be modified as follows:

$$\dot{\underline{H}} + \underline{\omega} \times \underline{H} = \underline{\tau}_{ext} \quad (4.2)$$

where $\underline{\omega}$ is the angular rate of the body-fixed frame with respect to inertial space. Equation (4.2) is known as Euler's equation of motion.

The total angular momentum, $\underline{H}$, will be a vector sum of the angular momentum due to the motion of the spacecraft body and the angular momentum of any internal

spinning components such as reaction wheels, momentum wheels, or gyroscopes:

$$\underline{H} = \underline{H}_{sc} + \underline{H}_{int} \quad (4.3)$$

The angular momentum of the spacecraft is the product of its inertia tensor and its angular rates, so (4.3) becomes:

$$\underline{H} = I_{sc}\underline{\omega}_{sc} + \underline{H}_{int} \quad (4.4)$$

The inertia tensor, I_{sc} , is defined as follows:

$$I_{sc} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{zx} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{yz} & I_{zz} \end{bmatrix} \quad (4.5)$$

where the entries are integrals over the entire spacecraft mass of the products of the displacements, x , y , and z , from the spacecraft centre of mass:

$$\begin{aligned} I_{xx} &= \int (y^2 + z^2) dm & I_{xy} &= \int xy dm \\ I_{yy} &= \int (z^2 + x^2) dm & I_{yz} &= \int yz dm \\ I_{zz} &= \int (x^2 + y^2) dm & I_{zx} &= \int zx dm \end{aligned}$$

Differentiating (4.4) gives:

$$\begin{aligned} \dot{\underline{H}} &= \frac{d}{dt}(I_{sc}\underline{\omega}_{sc} + \underline{H}_{int}) \\ &= I_{sc}\dot{\underline{\omega}}_{sc} + \dot{I}_{sc}\underline{\omega}_{sc} + \dot{\underline{H}}_{int} \end{aligned} \quad (4.6)$$

Substituting (4.4) and (4.6) into (4.2) gives:

$$I_{sc}\dot{\underline{\omega}}_{sc} + \dot{I}_{sc}\underline{\omega}_{sc} + \dot{\underline{H}}_{int} + (\underline{\omega}_{sc} \times I_{sc}\underline{\omega}_{sc}) + (\underline{\omega}_{sc} \times \underline{H}_{int}) = \underline{\tau}_{ext} \quad (4.7)$$

This is the general vector equation of motion for a spacecraft with internal angular momentum. The solution to this equation will be a superposition of the general solution to the homogenous (i.e. torque-free) form of the equation and the particular solution to the non-homogenous form:

$$\underline{\omega}_{sc}(t) = \underline{\omega}_h(t, \underline{\omega}(0)) + \underline{\omega}_{nh}(t) \quad (4.8)$$

where $\underline{\omega}(0)$ represents the initial conditions of the system. There is no general analytical solution to either $\underline{\omega}_h(t, \underline{\omega}(0))$ or $\underline{\omega}_{nh}(t)$. However, simplifications can be made in most cases to allow insight into the dynamic behaviour of the spacecraft.

Consider the common case of a spacecraft spinning at a large, constant rate, ω_o , about one axis, using a momentum wheel for control about that axis. The wheel has a nominal angular momentum, h_w , that can be considered constant. To simplify matters further, it is also assumed that the body co-ordinate frame is aligned with the principal inertia axes. Using (4.7) and setting the applied torque to zero, the equations of motion for each axis are then:

$$\begin{aligned} I_x \dot{\omega}_x + (I_y - I_z) \omega_y \omega_z &= 0 \\ I_y \dot{\omega}_y + [(I_z - I_x) \omega_o + h_w] \omega_z &= 0 \\ I_z \dot{\omega}_z + [(I_x - I_y) \omega_o + h_w] \omega_y &= 0 \end{aligned} \quad (4.9)$$

where I_x , I_y , and I_z are the principal moments of inertia, ω_x , ω_y , and ω_z are the components of the spacecraft angular rate, $\underline{\omega}_{sc}$, and the x-axis has been chosen for the spacecraft spin axis. If ω_y and ω_z are much smaller than ω_o , which is typically the case, then their product will also be small and the $(I_y - I_z) \omega_y \omega_z$ term in the x-axis equation can be neglected. The x-axis equation is then:

$$I_x \dot{\omega}_x = 0 \quad (4.10)$$

which is in agreement with the assumption that ω_x is approximately constant, and equal to ω_o . The solutions to ω_y and ω_z are sinusoidal functions that are 90° out

of phase with respect to one another, as shown in [10]. Physically, the angular rate vector, $\underline{\omega}$, will appear to trace a cone in space about the angular momentum vector, when viewed in the spacecraft body frame. This coning motion is referred to as nutation, and the associated nutation frequency can be calculated using the following equation:

$$\omega_n = \sqrt{\frac{[(I_x - I_y)\omega_o + h_w][(I_x - I_z)\omega_o + h_w]}{I_y I_z}} \quad (4.11)$$

Typically, h_w will be much larger in magnitude than the momentum of the spacecraft body, so the momentum wheel will tend to dominate nutation dynamics. Nutation is generally to be avoided, if at all possible, as it can have a negative impact on pointing accuracy and stability.

In terms of the non-homogenous case, the effect of an external torque on the spacecraft will be to change the total angular momentum of the system. If the applied torque causes the direction of the angular momentum vector to change, this is referred to as precession. The long-term effect of environmental disturbance torques depends on whether those torques are cyclic or secular in nature. Cyclic torques tends to cancel out over long periods of time, whereas secular torques, or torques that act in a preferred direction, will cause a gradual build-up of pointing error if left uncorrected.

Designing a comprehensive attitude control law can often be quite difficult, as control schemes that attempt to minimize nutation tend to generate precession, and vice versa. A certain amount of tradeoff is therefore usually required.

4.2 Equations of Motion for SCISAT-1

The general equations of motion were given in vector form in (4.7). A number of simplifications can be made to these equations for the specific case of SCISAT-1

when using GyroWheel as the primary actuator that allow the desired decoupling of roll, pitch, and yaw to take place. These simplifications are outlined in this section.

4.2.1 Reference Frames

The first step in applying the spacecraft equations of motion to SCISAT-1 is to define appropriate reference frames. SCISAT-1 is a Sun-pointing mission, so a convenient reference frame would consist of an X-axis pointed continuously toward the centre of the Sun's disk, with the Y and Z axes both at some fixed angle with respect to the ecliptic plane. This reference frame will rotate with respect to inertial space as the Earth revolves around the sun, at a rate of 0.985° per day. The time scales of interest for satellite attitude control, however, are on the order of hours or minutes, so for analytical purposes this frame can be assumed to be an inertial, or "quasi-inertial", reference frame, and is given the designation 'i'.

When the ACE-FTS instrument is in use, SCISAT-1 must execute a rolling manoeuver to keep the cryo-radiator pointed away from the Earth. The desired pointing frame is therefore not the quasi-inertial frame, but one that is Sun-pointing but rotating with some desired angular rate, ω_d . This desired frame shares the same X axis with the quasi-inertial frame. At time t , the Y and Z axes will be rotated with respect to their quasi-inertial equivalents by an angle $\omega_d t$. The desired frame is given the designation 'd'.

In reality, the spacecraft body will not coincide exactly with the desired frame, due to the effects of disturbances. This "perturbed" pointing frame will be rotated with respect to the desired frame by a small angle $\Delta\theta_x$ about the X axis, a small angle $\Delta\theta_y$ about the Y axis, and a small angle $\Delta\theta_z$ about the Z axis. The perturbed pointing frame is given the designation 'p'.

The three different reference frames are shown graphically in Figure 4.1.

Figure 4.1: Quasi-Inertial, Desired, and Perturbed Pointing Frames

4.2.2 Attitude Errors

The spacecraft attitude error is defined as the difference between the actual attitude and the desired attitude. While this may seem like a straightforward definition, it is often complicated by the fact that rotations are in general not commutative.

In the case of SCISAT-1, the desired attitude is measured with respect to the quasi-inertial frame:

$$\underline{\theta}_d = \begin{bmatrix} \omega_d t \\ 0 \\ 0 \end{bmatrix} \quad (4.12)$$

Since the only motion of the desired pointing frame with respect to the quasi-inertial pointing frame is about the X axis, the time derivative of the desired attitude can be taken as follows:

$$\dot{\underline{\theta}}_d = \underline{\omega}_d = \begin{bmatrix} \omega_d \\ 0 \\ 0 \end{bmatrix} \quad (4.13)$$

where ω_d will be constant over a given time interval. This implies that the double-derivative of $\underline{\theta}_d$ will be zero:

$$\ddot{\underline{\theta}}_d = 0 \quad (4.14)$$

Deriving expressions for the perturbed attitude errors, $\Delta\theta_x$, $\Delta\theta_y$, and $\Delta\theta_z$, is a bit more complicated, due to the non-commutative nature of rotations. If the angles involved are sufficiently small, however, it can be shown that they are approximately commutative, and so the attitude errors can be obtained by simple vector subtraction.

4.2.3 Rotation Matrices and Euler Angles

A common way to represent angular displacements in space is through the use of rotation matrices. For example, the following matrix, $R_x(\phi)$, rotates a vector by an angle ϕ about the x-axis of the local co-ordinate system:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \quad (4.15)$$

The matrix is unitary, which means it can change the direction of a vector but will always preserve its length. Similar rotation matrices can be defined for the y- and

z-axes, using the rotation angles θ and ψ respectively:

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (4.16)$$

$$R_z(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.17)$$

An attitude offset can be specified using three sequential rotations about local coordinate axes. The angles of rotation used to specify the offset are known as Euler angles. Twelve different rotation sequences are possible, a common example being the XYZ rotation sequence:

$$R_{xyz}(\phi, \theta, \psi) = R_z(\psi)R_y(\theta)R_x(\phi) \quad (4.18)$$

Multiplying a vector by $R_{xyz}(\phi, \theta, \psi)$ causes it to be rotated about the x-axis by an angle ϕ , then rotated about the new y-axis by an angle θ , then rotated about the final z-axis by an angle ψ . Other rotation sequences can be represented in a similar manner.

To convert a vector from its representation in a initial reference frame to its representation in a rotated reference frame, a transformation matrix can be used. This transformation matrix is simply the product of the rotation matrices, but with the angles reversed in sign:

$$T_{xyz}(\phi, \theta, \psi) = R_z(-\psi)R_y(-\theta)R_x(-\phi) \quad (4.19)$$

Three Euler angle rates, $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$, can now be defined. For an XYZ rotation sequence, these represent the components of the angular rate error in the directions of the initial x-axis, the intermediate y-axis, and the final z-axis, respectively. The

total spacecraft rate is the sum of the desired rate and the three Euler angle rates, each transformed to the final body reference frame:

$$\underline{\omega}_{sc} = T_{xyz}(\phi, \theta, \psi)\underline{\omega}_d + T_z(\psi) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + T_{yz}(\theta, \psi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + T_{xyz}(\phi, \theta, \psi) \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} \quad (4.20)$$

When the Euler angle rate terms are expanded, (4.20) becomes:

$$\underline{\omega}_{sc} = \begin{bmatrix} \dot{\phi} \cos \theta \cos \psi + \dot{\theta} \sin \psi \\ -\dot{\phi} \cos \theta \sin \psi + \dot{\theta} \cos \psi \\ \dot{\phi} \sin \theta + \dot{\psi} \end{bmatrix} + R_z(-\psi)R_y(-\theta)R_x(-\phi)\underline{\omega}_d \quad (4.21)$$

In the case of SCISAT-1, the Euler angles represent the offsets in each axis from the desired pointing frame. The terms can therefore be replaced as follows:

$$\begin{aligned} \phi &= \Delta\theta_x \\ \theta &= \Delta\theta_y \\ \psi &= \Delta\theta_z \end{aligned} \quad (4.22)$$

where $\Delta\theta_x$, $\Delta\theta_y$, and $\Delta\theta_z$ represent the components of the attitude error. If the offsets are sufficiently small, the total offset can be treated as a vector, $\underline{\Delta\theta}$:

$$\underline{\Delta\theta}_{sc} = \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix} \quad (4.23)$$

The Euler angle rates are then:

$$\begin{aligned} \dot{\phi} &= \dot{\Delta\theta}_x \\ \dot{\theta} &= \dot{\Delta\theta}_y \\ \dot{\psi} &= \dot{\Delta\theta}_z \end{aligned} \quad (4.24)$$

4.2.4 Spacecraft Angular Rate

If (4.24) and (4.13) are substituted into (4.21), the expression for the total rate becomes:

$$\underline{\omega}_{sc} = \begin{bmatrix} (\omega_d + \dot{\Delta}\theta_x) \cos \Delta\theta_y \cos \Delta\theta_z + \dot{\Delta}\theta_y \sin \Delta\theta_z \\ -(\omega_d + \dot{\Delta}\theta_x) \cos \Delta\theta_y \sin \Delta\theta_z + \dot{\Delta}\theta_y \cos \Delta\theta_z \\ (\omega_d + \dot{\Delta}\theta_x) \sin \Delta\theta_y + \dot{\Delta}\theta_z \end{bmatrix} \quad (4.25)$$

An upper bound can be placed on the roll, pitch, and yaw errors in fine-pointing, based on the attitude control requirements. This is formalized as follows.

Assumption 1 $\Delta\theta_x < 5^\circ$, $\Delta\theta_y < 1^\circ$, and $\Delta\theta_z < 1^\circ$

A small-angle approximation can therefore be used. Simplifying the sine and cosine terms in (4.25) accordingly gives:

$$\underline{\omega}_{sc} = \begin{bmatrix} \omega_d + \dot{\Delta}\theta_x + \dot{\Delta}\theta_y \Delta\theta_z \\ \dot{\Delta}\theta_y - (\omega_d + \dot{\Delta}\theta_x) \Delta\theta_z \\ \dot{\Delta}\theta_z + (\omega_d + \dot{\Delta}\theta_x) \Delta\theta_y \end{bmatrix} \quad (4.26)$$

An assumption can be made about the rate errors as well. From a control design standpoint, the rate errors should be kept small. As a conservative estimate, it will be assumed that they will be at least an order of magnitude smaller than the orbit rate.

Assumption 2 $|\dot{\Delta}\theta_x|, |\dot{\Delta}\theta_y|, |\dot{\Delta}\theta_z| < 10^{-4} \text{ rad/s}$

Together with Assumption 1, this implies that the products of the rate errors and attitude errors will be at most on the order of 10^{-6} rad/s , which is three orders of magnitude smaller than the orbit rate. These terms can therefore be neglected from the expression for total spacecraft rate.

Assumption 3 $\dot{\Delta\theta}_y\Delta\theta_x \approx 0$, $\dot{\Delta\theta}_x\Delta\theta_y \approx 0$ and $\dot{\Delta\theta}_x\Delta\theta_z \approx 0$

The simplified expression of (4.26) is then:

$$\underline{\omega}_{sc} = \begin{bmatrix} \omega_d + \dot{\Delta\theta}_x \\ \dot{\Delta\theta}_y - \omega_d\Delta\theta_z \\ \dot{\Delta\theta}_z + \omega_d\Delta\theta_y \end{bmatrix} \quad (4.27)$$

The ω_d terms in the pitch and yaw components arise because the sensors are trying to measure attitude in a body-fixed frame that is spinning relative to inertial space at the roll rate, ω_d .

Taking the time derivative of (4.27) gives an expression for $\dot{\underline{\omega}}_{sc}$:

$$\dot{\underline{\omega}}_{sc} = \begin{bmatrix} \ddot{\Delta\theta}_x \\ \ddot{\Delta\theta}_y - \omega_d\dot{\Delta\theta}_z \\ \ddot{\Delta\theta}_z + \omega_d\dot{\Delta\theta}_y \end{bmatrix} \quad (4.28)$$

The desired spacecraft rate, $\underline{\omega}_d$, can be expressed in the perturbed body frame by applying an appropriate transformation:

$$\begin{aligned} \underline{\omega}_d &= T_{xyz}(\Delta\theta_x, \Delta\theta_y, \Delta\theta_z) \begin{bmatrix} \omega_d \\ 0 \\ 0 \end{bmatrix} \\ &\approx \begin{bmatrix} 1 & \Delta\theta_z & -\Delta\theta_y \\ -\Delta\theta_z & 1 & \Delta\theta_x \\ \Delta\theta_y & -\Delta\theta_x & 1 \end{bmatrix} \begin{bmatrix} \omega_d \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \omega_d \\ -\omega_d\Delta\theta_z \\ \omega_d\Delta\theta_y \end{bmatrix} \end{aligned} \quad (4.29)$$

The rate expression in (4.27) can therefore also be written as:

$$\underline{\omega}_{sc} = \underline{\dot{\Delta}\theta} + \underline{\omega}_d \quad (4.30)$$

Similarly, the expression for angular acceleration in (4.28) can be written as:

$$\underline{\dot{\omega}}_{sc} = \underline{\ddot{\Delta}\theta} + \underline{\dot{\omega}}_d \quad (4.31)$$

These expressions will be used later to evaluate the equations of motion.

4.2.5 Spacecraft Inertia with GyroWheel

The spacecraft moment of inertia tensor, I_{sc} , in (4.7) includes the moment of inertia of the GyroWheel rotor about the spacecraft centre of mass. If the rotor changes its tilt angle relative to the spacecraft, the value of I_{sc} will change as well. For the purposes of analysis, however, this effect is assumed to be negligible.

Assumption 4 I_{sc} is constant

To justify this assumption, consider the principal axis inertia tensor of the rotor, $\hat{I}_r$:

$$\hat{I}_r = \begin{bmatrix} I_s & 0 & 0 \\ 0 & I_t & 0 \\ 0 & 0 & I_t \end{bmatrix} \quad (4.32)$$

where I_s is the spin moment of inertia and I_t is the transverse moment of inertia. These moments of inertia can be rotated into the spacecraft body frame using the gimbal angles, γ_y , and γ_z . The moment of inertia of the GyroWheel rotor about the spacecraft centre of mass is then:

$$I_{gw} = R_y(\gamma_y)^T R_z(\gamma_z)^T \hat{I}_{gw} R_z(\gamma_z) R_y(\gamma_y) + m_r D_r \quad (4.33)$$

where m_r is the total mass of the rotor and D_r is a tensor representing the displacement of the GyroWheel rotor centre of mass from the spacecraft centre of mass. Since m_r and D_r will both be constant and all other components on the spacecraft have fixed moments of inertia, the change in I_{sc} due to rotor tilt is equal to the change in the moment of inertia of the rotor about its own centre of mass:

$$\Delta I_{sc} = R_y(\gamma_y)^T R_z(\gamma_z)^T \hat{I}_{gw} R_z(\gamma_z) R_y(\gamma_y) - \hat{I}_{gw} \quad (4.34)$$

Due to the operating constraints of GyroWheel, the tilt angles can be assumed to be less than 7° .

Assumption 5 $|\gamma_y| < 7^\circ$ and $|\gamma_z| < 7^\circ$

A small angle approximation can therefore be used for the rotation matrices $R_y(\gamma_y)$ and $R_z(\gamma_z)$. If the products of the tilt angles are neglected, the total rotation matrix, $R_z(\gamma_z)R_y(\gamma_y)$, is:

$$R_z(\gamma_z)R_y(\gamma_y) \approx \begin{bmatrix} 1 & \gamma_z & -\gamma_y \\ -\gamma_z & 1 & 0 \\ \gamma_y & 0 & 1 \end{bmatrix} \quad (4.35)$$

Inserting (4.32) and (4.35) into (4.34) gives:

$$\Delta I_{sc} = \begin{bmatrix} (\gamma_y^2 + \gamma_z^2)I_s & 0 & 0 \\ 0 & \gamma_z^2 I_t & -\gamma_y \gamma_z I_t \\ 0 & -\gamma_y \gamma_z I_t & \gamma_y^2 I_t \end{bmatrix} \quad (4.36)$$

If both tilt angles are at their limits, the maximum change in the spacecraft moment of inertia is:

$$\Delta I_{sc} = \begin{bmatrix} 5.88 \times 10^{-4} & 0 & 0 \\ 0 & 1.57 \times 10^{-4} & -1.57 \times 10^{-4} \\ 0 & -1.57 \times 10^{-4} & 1.57 \times 10^{-4} \end{bmatrix} \text{ kg-m}^2 \quad (4.37)$$

Comparing the magnitudes of the elements in (4.37) to the measured spacecraft moments of inertia listed in Table 3.2, it is clear that the effect of the GyroWheel rotor tilt on the overall spacecraft inertia is negligible. Assumption 4 is therefore valid.

It can also be assumed that the derivative of the spacecraft inertia tensor is zero.

Assumption 6 $\dot{I}_{sc} = 0$

This would seem to follow directly from Assumption 4. However, it also contains an implicit assumption that the rate of change of the GyroWheel tilt angles is small enough to be neglected. This can be seen if one takes the time derivative of the expression in (4.36). High tilt rates generally indicate large torque commands, however, which are not typically seen in fine-pointing control. Assumption 6 is therefore reasonable.

4.2.6 Simplified Equations of Motion

Using Assumption 6, the $\dot{I}_{sc}$ term can be eliminated from the vector equation of motion in (4.7):

$$I_{sc}\dot{\underline{\omega}}_{sc} + \dot{\underline{H}}_{int} + (\underline{\omega}_{sc} \times I_{sc}\underline{\omega}_{sc}) + (\underline{\omega}_{sc} \times \underline{H}_{int}) = \underline{\tau}_{ext} \quad (4.38)$$

If GyroWheel is serving as the primary actuator, it is assumed that the classical momentum wheel will not be spinning. GyroWheel will therefore be the only source of internal momentum and $\underline{H}_{int}$ in (4.38) can be replaced by $\underline{h}_{gw}$:

$$I_{sc}\dot{\underline{\omega}}_{sc} + \dot{\underline{h}}_{gw} + (\underline{\omega}_{sc} \times I_{sc}\underline{\omega}_{sc}) + (\underline{\omega}_{sc} \times \underline{h}_{gw}) = \underline{\tau}_{ext} \quad (4.39)$$

A good approximation to $\underline{h}_{gw}$ can be derived by applying the tilt rotation matrix

defined in (4.35) to the angular momentum vector:

$$\begin{aligned} \underline{h}_{gw} &= R_z(\gamma_z)R_y(\gamma_y) \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix} \\ &\approx \begin{bmatrix} h \\ \gamma_z h \\ -\gamma_y h \end{bmatrix} \end{aligned} \quad (4.40)$$

where h is the angular momentum of the GyroWheel rotor.

With expressions for the spacecraft rate and the GyroWheel momentum given by (4.27) and (4.40) respectively, it is now possible to substitute for $\underline{\omega}_{sc}$ and $\underline{h}_{gw}$ in the equations of motion and look at the roll, pitch, and yaw equations separately. A complication arises, however, from the fact that the SCISAT-1 body frame axes do not correspond exactly with the principal axes of the spacecraft moment of inertia. In fact, according to the most recent dynamic tests, the principal axis of maximum inertia will be offset from the nominal roll axis by nearly 10° . To accomodate the off-axis inertias while minimizing the number of terms that appear in the equations, the approach taken is to separate the spacecraft inertia tensor into a principal axis component, $\hat{I}_{sc}$, and an “offset” component, I'_{sc} :

$$I_{sc} = \hat{I}_{sc} + I'_{sc} \quad (4.41)$$

The vector equation of motion then becomes:

$$(\hat{I}_{sc} + I'_{sc})\dot{\underline{\omega}}_{sc} + \dot{\underline{h}}_{gw} + (\underline{\omega}_{sc} \times (\hat{I}_{sc} + I'_{sc})\underline{\omega}_{sc}) + (\underline{\omega}_{sc} \times \underline{h}_{gw}) = \underline{\tau}_{ext} \quad (4.42)$$

The terms involving I'_{sc} are temporarily combined with $\underline{\tau}_{ext}$ to form a new torque, $\underline{\tau}'$:

$$\hat{I}_{sc}\dot{\underline{\omega}}_{sc} + \dot{\underline{h}}_{gw} + (\underline{\omega}_{sc} \times \hat{I}_{sc}\underline{\omega}_{sc}) + (\underline{\omega}_{sc} \times \underline{h}_{gw}) = \underline{\tau}' \quad (4.43)$$

where $\underline{\tau}'$ is defined as:

$$\underline{\tau}' = \underline{\tau}_{ext} - I'_{sc}\dot{\underline{\omega}}_{sc} - (\underline{\omega}_{sc} \times I'_{sc}\underline{\omega}_{sc}) \quad (4.44)$$

Inserting (4.27) and (4.40) into (4.43), and using Assumption 3, produces the following three equations for roll, pitch, and yaw:

$$\begin{aligned} I_x\ddot{\Delta\theta}_x + (I_z - I_y)(\dot{\Delta\theta}_y - \omega_d\Delta\theta_z)(\dot{\Delta\theta}_z + \omega_d\Delta\theta_y) \\ + \dot{h} + (\dot{\Delta\theta}_y - \omega_d\Delta\theta_z)\gamma_y h - (\dot{\Delta\theta}_z + \omega_d\Delta\theta_y)\gamma_z h = \tau'_x \end{aligned} \quad (4.45)$$

$$\begin{aligned} I_y\ddot{\Delta\theta}_y + (I_x - I_y - I_z)\omega_d\dot{\Delta\theta}_z + (I_x - I_z)\omega_d^2\Delta\theta_y \\ + \gamma_z\dot{h} + \dot{\gamma}_z h + (\dot{\Delta\theta}_z + \omega_d\Delta\theta_y)h + \omega_d\gamma_y h = \tau'_y \end{aligned} \quad (4.46)$$

$$\begin{aligned} I_z\ddot{\Delta\theta}_z - (I_x - I_y - I_z)\omega_d\dot{\Delta\theta}_y + (I_x - I_y)\omega_d^2\Delta\theta_z \\ - \gamma_y\dot{h} - \dot{\gamma}_y h + \omega_d\gamma_z h - (\dot{\Delta\theta}_y - \omega_d\Delta\theta_z)h = \tau'_z \end{aligned} \quad (4.47)$$

where I_x , I_y , and I_z are the principal axis moments of inertia of the satellite and τ'_x , τ'_y , and τ'_z are the components of $\underline{\tau}'$.

Based on the profiles that have already been designed for SCISAT-1, the maximum roll rate of the spacecraft will be about 0.002 rad/s. An upper bound can therefore be placed on the desired roll rate, ω_d .

Assumption 7 $|\omega_d| < 0.002 \text{ rad/s}$

Using Assumptions 1, 2, and 7, and the principal inertias of the satellite derived from the measured inertias given in Table 3.2, it can be shown mathematically that the second term in (4.45) will be at most on the order of 10^{-8} N-m. It can therefore be assumed that this term will be negligible compared to the disturbance torques, whose expected values are on the order of 10^{-5} N-m, as was shown in Table 3.3.

Assumption 8 $|(I_z - I_y)(\dot{\Delta\theta}_y - \omega_d\Delta\theta_z)(\dot{\Delta\theta}_z + \omega_d\Delta\theta_y)| \ll |\tau_{ext}|$

Neglecting this term, then, the expression for roll motion becomes:

$$I_x \ddot{\Delta\theta}_x + \dot{h} + (\dot{\Delta\theta}_y - \omega_d \Delta\theta_z) \gamma_y h - (\dot{\Delta\theta}_z + \omega_d \Delta\theta_y) \gamma_z h = \tau'_x \quad (4.48)$$

Simplifications can be made to the pitch and yaw equations by comparing the magnitudes of the GyroWheel and spacecraft terms.

Assumption 9 $|h| \gg |(I_x - I_y - I_z)\omega_d|$

The magnitude of $(I_x - I_y - I_z)$ is 6.8 kg-m^{-2} , so the magnitude of $(I_x - I_y - I_z)\omega_d$ is at most about 0.014 N-m-s . This is much smaller than the tuned angular momentum of GyroWheel, h , which is 3.01 N-m-s , so Assumption 9 is valid.

Assumption 10 $|h| \gg |(I_x - I_z)\omega_d|$ and $|h| \gg |(I_y - I_x)\omega_d|$

The magnitudes of $(I_x - I_z)$ and $(I_y - I_x)$ are 5.7 kg-m^2 and 6.3 kg-m^2 respectively. Both $(I_x - I_z)\omega_d$ and $(I_y - I_x)\omega_d$ therefore have magnitudes less than or equal to 0.012 N-m-s , which is much smaller than h . Assumption 10 is therefore valid.

Using Assumptions 9 and 10, the pitch and yaw equations can now be simplified:

$$I_y \ddot{\Delta\theta}_y + \gamma_z \dot{h} + \dot{\gamma}_z h + (\dot{\Delta\theta}_z + \omega_d \Delta\theta_y) h + \omega_d \gamma_y h = \tau'_y \quad (4.49)$$

$$I_z \ddot{\Delta\theta}_z - \gamma_y \dot{h} - \dot{\gamma}_y h + \omega_d \gamma_z h - (\dot{\Delta\theta}_y - \omega_d \Delta\theta_z) h = \tau'_z \quad (4.50)$$

Taking (4.48), (4.49), and (4.50) and converting back to the vector form of the equations gives:

$$I_{sc} \ddot{\underline{\Delta\theta}} + \dot{\underline{h}}_{gw} + (\underline{\omega}_{sc} \times \underline{h}_{gw}) = \underline{\tau}_{ext} - \underline{\tau}_{oa} \quad (4.51)$$

where $\underline{\tau}_{oa}$ is a disturbance torque representing the effects of off-axis inertias.

Rearranging (4.51) by placing the GyroWheel terms on the right side of the equation gives:

$$I_{sc}\ddot{\underline{\theta}} = -\dot{\underline{h}}_{gw} - (\underline{\omega}_{sc} \times \underline{h}_{gw}) + \underline{\tau}_{ext} - \underline{\tau}_{oa} \quad (4.52)$$

The first two terms on the right hand side, when combined, are equal to the applied torque when GyroWheel is in torque command mode, as given by (2.1):

$$I_{sc}\ddot{\underline{\theta}} = \underline{\tau}_{gw} + \underline{\tau}_{ext} - \underline{\tau}_{oa} \quad (4.53)$$

The angular momentum of GyroWheel no longer appears explicitly in the equation of motion, which indicates that the dynamics of GyroWheel have become decoupled from the dynamics of the spacecraft body. The GyroWheel torque can therefore be viewed as an external torque applied to the spacecraft that corrects for the disturbances. The lack of cross product terms in (4.53) indicates that nutational motion has been effectively suppressed.

4.3 GyroWheel Feedback Control

The GyroWheel control torque, $\underline{\tau}_{gw}$, is calculated from the measured attitude and rates according to the ADCS control law given by (3.9). If (3.9) is inserted into (4.53), the resulting feedback equation is:

$$\begin{aligned} I_{sc}\ddot{\underline{\theta}} = & -\tilde{I}_{sc}(K_p\Delta\theta^{(m)} + K_r\Delta\omega^{(m)}) - \Delta\theta^{(m)} \times K_h\dot{\underline{h}}^{(m)} - K_{ff}\underline{\tau}_{ff} \\ & + \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D \end{aligned} \quad (4.54)$$

When using GyroWheel in torque command mode, there is no need to control the GyroWheel angular momentum explicitly, as it no longer directly affects the motion of the spacecraft. Therefore, the K_h term is set to zero. If no feedforward torque is

required either, the reduced feedback equation is:

$$I_{sc}\ddot{\underline{\Delta\theta}} = -\tilde{I}_{sc}(K_p\underline{\Delta\theta}^{(m)} + K_r\underline{\Delta\omega}^{(m)}) + \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D \quad (4.55)$$

The momentum dumping gain, K_D , from (3.8), will be selected so that the dumping torque, $\underline{\tau}_D$, does not impact the short-term control behaviour of GyroWheel. For analytical purposes, it can therefore be treated as a disturbance torque.

It is important to distinguish the “real” spacecraft moment of inertia tensor, I_{sc} , in (4.55) from the measured tensor, $\tilde{I}_{sc}$, that is used in the ADCS software. The latter is a result of dynamic testing and will contain some amount of error. $\tilde{I}_{sc}$ can be written as the sum of the real inertia tensor and an error matrix, δI_{sc} :

$$\tilde{I} = I_{sc} + \delta I_{sc} \quad (4.56)$$

Substituting (4.56) into (4.55) gives:

$$\begin{aligned} I_{sc}\ddot{\underline{\Delta\theta}} = & -I_{sc}(K_p\underline{\Delta\theta}^{(m)} + K_r\underline{\Delta\omega}^{(m)}) - \delta I_{sc}(K_p\underline{\Delta\theta}^{(m)} + K_r\underline{\Delta\omega}^{(m)}) \\ & + \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D \end{aligned} \quad (4.57)$$

These effects can be treated as a disturbance as well:

$$I_{sc}\ddot{\underline{\Delta\theta}} = -I_{sc}(K_p\underline{\Delta\theta}^{(m)} + K_r\underline{\Delta\omega}^{(m)}) + \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D - \underline{\tau}_{\delta I} \quad (4.58)$$

where:

$$\underline{\tau}_{\delta I} = \delta I_{sc}(K_p\underline{\Delta\theta}^{(m)} + K_r\underline{\Delta\omega}^{(m)}) \quad (4.59)$$

The following substitutions can be made for the attitude and rate error vectors:

$$\underline{\Delta\theta}^{(m)} = \underline{\theta}_{sc} - \underline{\theta}_d + \delta\underline{\theta}_m \quad (4.60)$$

$$\underline{\Delta\omega}^{(m)} = \dot{\underline{\theta}}_{sc} - \dot{\underline{\theta}}_d + \delta\underline{\omega}_m \quad (4.61)$$

$$\begin{aligned}\ddot{\underline{\theta}} &= \ddot{\underline{\theta}}_{sc} - \ddot{\underline{\theta}}_d \\ &= \ddot{\underline{\theta}}_{sc}\end{aligned}\tag{4.62}$$

Here $\ddot{\underline{\theta}}_d$ is zero because the desired roll rate is always constant for a given interval. Substituting these expressions into (4.58):

$$\begin{aligned}I_{sc}\ddot{\underline{\theta}}_{sc} &= -I_{sc}K_p(\underline{\theta}_{sc} - \underline{\theta}_d + \delta\underline{\theta}_m) - I_{sc}K_r(\dot{\underline{\theta}}_{sc} - \dot{\underline{\theta}}_d + \delta\underline{\omega}_m) \\ &\quad + \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D - \underline{\tau}_{\delta I}\end{aligned}\tag{4.63}$$

The terms involving measurement error can then be treated as disturbance torques:

$$\begin{aligned}I_{sc}\ddot{\underline{\theta}}_{sc} &= -I_{sc}K_p(\underline{\theta}_{sc} - \underline{\theta}_d) - I_{sc}K_r(\dot{\underline{\theta}}_{sc} - \dot{\underline{\theta}}_d) + \underline{\tau}_{ext} - \underline{\tau}_{oa} \\ &\quad - \underline{\tau}_D - \underline{\tau}_{\delta I} - \underline{\tau}_{\delta\theta} - \underline{\tau}_{\delta\omega}\end{aligned}\tag{4.64}$$

where:

$$\underline{\tau}_{\delta\theta} = I_{sc}K_p\delta\underline{\theta}_m\tag{4.65}$$

$$\underline{\tau}_{\delta\omega} = I_{sc}K_r\delta\underline{\omega}_m\tag{4.66}$$

The disturbance torques are now combined into a single term, $\underline{\tau}_{dist}$:

$$I_{sc}\ddot{\underline{\theta}}_{sc} = -I_{sc}K_p(\underline{\theta}_{sc} - \underline{\theta}_d) - I_{sc}K_r(\dot{\underline{\theta}}_{sc} - \dot{\underline{\theta}}_d) + \underline{\tau}_{dist}\tag{4.67}$$

where:

$$\underline{\tau}_{dist} = \underline{\tau}_{ext} - \underline{\tau}_{oa} - \underline{\tau}_D - \underline{\tau}_{\delta I} - \underline{\tau}_{\delta\theta} - \underline{\tau}_{\delta\omega}\tag{4.68}$$

Moving the terms involving $\underline{\theta}_{sc}$ and $\dot{\underline{\theta}}_{sc}$ to the left hand side gives:

$$I_{sc}\ddot{\underline{\theta}}_{sc} + I_{sc}K_r\dot{\underline{\theta}}_{sc} + I_{sc}K_p\underline{\theta}_{sc} = -I_{sc}K_p\underline{\theta}_d - I_{sc}K_r\dot{\underline{\theta}}_d + \underline{\tau}_{dist}\tag{4.69}$$

Multiplying both sides of (4.69) by I_{sc}^{-1} gives:

$$\ddot{\underline{\theta}}_{sc} + K_r\dot{\underline{\theta}}_{sc} + K_p\underline{\theta}_{sc} = I_{sc}^{-1}\underline{\tau}_{dist} - K_p\underline{\theta}_d - K_r\dot{\underline{\theta}}_d\tag{4.70}$$

Since K_p and K_r are implemented as diagonal matrices, (4.70) represents a system of three decoupled second-order equations. This means that each satellite axis can be controlled as a single-input single-output (SISO) system.

Taking the Laplace transform of (4.70) gives:

$$(s^2 + sK_r F_r(s) + K_p F_p(s))\underline{\theta}(s) = I_{sc}^{-1} \underline{\tau}_{dist}(s) - (sK_r F_r(s) + K_p F_p(s))\underline{\theta}_d(s) \quad (4.71)$$

where $F_r(s)$ and $F_p(s)$ are diagonal matrices that represent the continuous-time Laplace transforms of the IIR filters used for rate and attitude, respectively. The block diagram for this system is shown in Figure 4.2.

Figure 4.2: Block Diagram of GyroWheel Based ADCS for SCISAT-1

4.3.1 Gain Selection

Based on (4.71), the closed-loop transfer function based on the desired attitude, $\theta_d(s)$, is as follows:

$$\frac{\theta(s)}{\theta_d(s)} = \frac{sK_r F_r(s) + K_p F_p(s)}{s^2 + sK_r F_r(s) + K_p F_p(s)} \quad (4.72)$$

whereas the transfer function based on the disturbance torque is:

$$\frac{\theta(s)}{\tau_{dist}(s)} = \frac{I_{sc}^{-1}}{s^2 + sK_r F_r(s) + K_p F_p(s)} \quad (4.73)$$

The goal is to set the gain terms and filter coefficients to obtain a desired settling time given an offset from $\theta_d(s)$, while maintaining pointing stability in the presence of disturbance torques. These two requirements can not be satisfied concurrently using the gain terms alone, which is the reason the IIR filters were included in the design. The filters allow for flexibility in the placement of the dominant system poles.

For the roll axis, the choice of settling time is dictated by the need to slow down roll motion immediately before each occultation and to rotate the spacecraft 180° in between occultations. The settling time must therefore be significantly less than a quarter of an orbit, which is approximately 1500 seconds. For the classical momentum wheel control system, settling times between 400 and 1000 seconds were considered. There appears to be no reason to treat the roll axis any differently when using GyroWheel as primary actuator, so the gains and attitude filter coefficients calculated for classical wheel control are re-used.

In the case of pitch and yaw control, the original requirement for a settling time was dictated by the controllability of the magnetorquer rods. Since it normally takes a quarter of an orbit for both the pitch and the yaw axes to fully observe the magnetic field, the settling time was set to approximately 1500 seconds. GyroWheel, by contrast, has full control authority over the pitch and yaw axes, so a shorter settling time

can be selected to provide tighter pointing control.

While pointing performance generally improves with shorter settling times, it usually comes at the price of reduced stability margin. A high control bandwidth will respond extremely well to small disturbances, but will tend to become unstable in response to large disturbances. There will ultimately be a practical lower limit as well, due to the accuracy of the sensors. Additionally, the GyroWheel nutation frequency should be avoided, even though the control law has been designed to suppress nutation, as measurement errors in the rates supplied to GyroWheel may still cause some excitation at that frequency. Using (4.11), the nutation frequency for GyroWheel is:

$$\begin{aligned}\omega_n &\approx \sqrt{\frac{h_{gw}^2}{I_y I_z}} \\ &= \sqrt{\frac{3.01^2}{13.1 \cdot 12.5}} \\ &= 0.235 \text{ rad/s}\end{aligned}\tag{4.74}$$

This corresponds to a nutation period of about 27 seconds.

For testing the GyroWheel control law in simulation, the filter coefficients and gain values were selected using a software design tool provided by Bristol. The results of these tests are presented in the following chapter.

Chapter 5

Simulations and Results

The GyroWheel based attitude control system was evaluated using a dynamic model of SCISAT-1 that was developed by Bristol Aerospace Ltd. using the Matrix-X simulation software package. This model is designed to run in real time and consists of several different modules. These include a dynamics module, which computes the spacecraft motion based on applied torques, and an ADCS module, which runs ADCS flight software and interfaces with other modules for the sensors and actuators. The four main sources of environmental disturbance torques - aerodynamic drag, solar pressure, gravity gradients, and residual magnetic moments - are all simulated with this model. The behaviour of each actuator and sensor is also carefully modelled, including noise characteristics.

A dynamic model for GyroWheel was also made available by Bristol, designed to emulate the behaviour of the unit both as an actuator and as a rate sensor. When research into the GyroWheel based control law began, the model was only capable of running under momentum command mode. It was therefore necessary, as part of the research for this thesis, to modify the GyroWheel model to allow it to run under torque command mode and to integrate this new model with the SCISAT-1 model as

a whole.

5.1 GyroWheel Dynamic Model

The interface for the GyroWheel model consists of a series of inputs and outputs that connect to other modules. These inputs and outputs are listed in Tables 5.1 and 5.2 respectively.

Table 5.1: GyroWheel Model Inputs

Input	Units	Source
X Command	counts	ADCS module
Y Command	counts	ADCS module
Z Command	counts	ADCS module
Command Type	counts	ADCS module
X Measured Rate	counts	ADCS module
Y Measured Rate	counts	ADCS module
Z Measured Rate	counts	ADCS module
ADCS Sample Period	counts	ADCS module
X Body Rate	rad/s	Dynamics module
Y Body Rate	rad/s	Dynamics module
Z Body Rate	rad/s	Dynamics module
X Body Acceleration	rad/s ²	Dynamics module
Y Body Acceleration	rad/s ²	Dynamics module
Z Body Acceleration	rad/s ²	Dynamics module

Table 5.2: GyroWheel Model Outputs

Output	Units	Destination
X Momentum	counts	ADCS module
Y Momentum	counts	ADCS module
Z Momentum	counts	ADCS module
X Measured Rate	counts	ADCS module
Y Measured Rate	counts	ADCS module
Z Measured Rate	counts	ADCS module
X Measured Attitude Change	counts	ADCS module
Y Measured Attitude Change	counts	ADCS module
Z Measured Attitude Change	counts	ADCS module
Integration Time	counts	ADCS module
X Applied Torque	N-m	Dynamics module
Y Applied Torque	N-m	Dynamics module
Z Applied Torque	N-m	Dynamics module

5.1.1 Actuation Modeling

The commands received from the ADCS module will be either momentum commands or torque commands, with the values scaled to an integer number of counts. A flag indicating the command type is included as one of the inputs. A scaling is then applied to the command values to convert them back into engineering units.

If the input command is a momentum command, it is used directly as the target momentum value for the internal controller. If the input command is a torque command, however, the target momentum is calculated using the sample period and the measured spacecraft rate as provided by the ADCS using the equation given by (2.5). If the desired change in momentum can be accomplished in the requested amount of time without exceeding any operating limits the rotor momentum will slew along a straight-line path from the current value to the desired value. A detailed model of the rotor and gimbal dynamics could not be included in the GyroWheel module, as it would require very small integration time steps and prevent the model from running in real time. Instead, the dynamics are approximated with a 2-Hz Butterworth filter that is applied to the commanded momentum slew rate.

The apparent torque due to the motion of the spacecraft is calculated using the body rates and accelerations provided by the Dynamics module. The total applied torque sent as an output by the GyroWheel module to the Dynamics module will be the sum of this spacecraft torque and the momentum slew rate.

5.1.2 Rate Sensing Modeling

A low-fidelity model is also used for the GyroWheel rate sensing. The “measured” body rates are calculated by projecting the real spacecraft body rates onto the plane perpendicular to the rotor spin axis and then applying Gaussian noise to the output.

The noise model uses a standard deviation of 3.4×10^{-6} rad/s, which is based on recent hardware test results of GyroWheel by Schultz et al. [8].

5.1.3 Model Validation

As part of the thesis research, extensive tests were run on the GyroWheel model to ensure that both momentum command mode and torque command mode operated as expected. A high-fidelity model of the rotor and gimbal dynamics designed by Bristol was run in parallel with the low-fidelity model to compare performance, and produced positive results. A formal validation using actual hardware test results was not performed, however, due to the inavailability of this data at the time of research. This is a task that should eventually be pursued, in order to quantify the accuracy of the model. It should be stressed, though, that the behaviour of the model to date is fully consistent with that predicted by theory.

5.2 GyroWheel Acceptance Runs

A series of simulation runs was performed using the GyroWheel model fully integrated with the model of SCISAT-1. The objectives of these acceptance runs were as follows:

- To verify the operation of GyroWheel as an actuator
- To verify the operation of GyroWheel as a rate sensor
- To evaluate the fine-pointing performance of GyroWheel based control
- To evaluate the stability of GyroWheel based control

Control performance and stability were evaluated for three different pitch and yaw gain sets, each representing a desired pointing control bandwidth and associated

settling time. The same performance filter coefficients and roll attitude and rate gains were used in each case.

The filter coefficients used are listed in Table 5.3. The filter applied to the roll rate was identical to that applied to roll, pitch, and yaw attitude, and has an equivalent continuous-time cutoff frequency of 0.1 rad/s, or 0.016 Hz. The filter used for pitch and yaw rate has a cutoff frequency of 1.88 rad/s, or 0.3 Hz.

Table 5.3: Attitude and Rate Filter Coefficients

Coefficient	Roll Rate	Pitch Rate	Yaw Rate	Attitude
a_0	0.047619	0.579192	0.579192	0.047619
a_1	0.047619	0.579192	0.579192	0.047619
b_0	1.000000	1.000000	1.000000	1.000000
b_1	0.904762	-0.158384	-0.158384	0.904762

The roll attitude and rate gains used are listed in Table 5.4 and are designed to give a settling time of 200 seconds and a control bandwidth of 0.0045 Hz. The system is second-order, so the damping ratio was set to 0.707 for optimal transient response¹. These gain values have been used successfully in simulations of SCISAT-1 under classical wheel control. The roll axis dynamics of GyroWheel under torque command mode should be equivalent to that for the classical wheel, as there is no gyrodynamic coupling in that axis in either case. There is therefore no obvious need to adjust the roll gain values for GyroWheel based control.

The gains used for pitch and yaw control are given in Tables 5.5, 5.6, and 5.7. These represent settling times of 100, 200, and 300 seconds and control bandwidths of 0.0090, 0.0045, and 0.0030 Hz, respectively. The damping ratio in each case is 0.707. Control

¹The optimal transient response occurs when the integral over the response of the square of the attitude error, $\int_0^\infty (\theta_{sc} - \theta_d)^2 dt$, is minimized.

Table 5.4: Attitude and Rate Gains for Roll Axis Control

k_{rx}	0.141712
k_{px}	0.00165474

gains using higher bandwidths were found through simulation to be unstable when responding to significant disturbances.

Table 5.5: Attitude and Rate Gains for Pointing Control (100-s Settling Time)

k_{ry}	0.292
k_{py}	0.0100966
k_{rz}	0.292
k_{pz}	0.0100966

Table 5.6: Attitude and Rate Gains for Pointing Control (200-s Settling Time)

k_{ry}	0.146
k_{py}	0.00252415
k_{rz}	0.146
k_{pz}	0.00252415

Table 5.7: Attitude and Rate Gains for Pointing Control (300-s Settling Time)

k_{ry}	0.097333333
k_{py}	0.001121844
k_{rz}	0.097333333
k_{pz}	0.001121844

5.3 Evaluation of GyroWheel Control System Performance

The performance of the GyroWheel based control system over a typical orbit was evaluated for each of the three pointing gain sets. The orbit used has a zero “beta angle”, which means that ascending nodes of the orbit plane are located along the Earth-Sun line. At the start of the simulation the satellite is midway on the sunlit side of Earth, with all attitude errors and rates initialized to zero. The full sequence of roll manoeuvres for the first orbit is given in Table 5.8. The values in the time column indicate the point in the simulation where the corresponding roll rate becomes the desired rate.

The first 1798 seconds is an attitude hold phase, so the desired roll rate is set to zero. The roll profile assumes an initial roll angle of -16° with respect to the Sun-pointing frame. This angle allows the satellite to perform the hold phase and be in the correct attitude for science operations when the first occultation occurs. During this first phase, the FSS and magnetometer are used in combination to provide attitude and rate feedback. When the spacecraft enters eclipse, the FSS is replaced as primary sensor by GyroWheel. Large roll manoeuvres are performed during eclipse to reorient the spacecraft for the sunrise occultation. For the occultations themselves, however,

Table 5.8: Commanded Roll Profile

Time (sec)	Roll Rate (rad/s)	Remarks
0	0	Midpoint of sunlit side
1798	-0.000002	Sunset occultation
1898	-0.000971	Spacecraft enters full eclipse
2928	-0.002071	Midpoint of eclipse
3961	-0.000003	Sunrise occultation
4060	-0.000572	Spacecraft enters full sunlight
5857	-0.001175	Midpoint of sunlit side

the roll rate is reduced to allow science measurements to occur, as seen at 1798 and 3961 seconds. When the spacecraft emerges from sunrise occultation, it switches back to the FSS for pitch and yaw sensing and begins another roll manoeuvre. Momentum dumping was not used during these runs.

5.3.1 Roll Error

Figure 5.1 shows the roll error as a function of time for the three different pointing control gain sets. The three plots are virtually identical, which demonstrates that roll axis control is decoupled from pointing control, as expected. The maximum error is about 3.1° , which is well within the $1\text{-}\sigma$ limit imposed by the roll attitude requirements. The peaks at 700, 2600, 3600, and 5200 seconds correspond to points in the orbit where the magnetic field becomes closely aligned with the Sun vector and roll measurement accuracy is relatively poor. Ripple behaviour can be seen in the plot at points near 1900, 3000, and 4000 seconds, which correspond to step changes in the desired rate.

Figure 5.1: Comparison of Roll Axis Control Performance for Different Gain Sets (One orbit)

5.3.2 Pointing Error

Figures 5.2 and 5.3 show, respectively, the pitch error and yaw error as a function of time for each gain set. As would be expected, higher control bandwidth leads to lower average pointing error. Figures 5.2 and 5.3 also demonstrate the differences in pointing performance when the FSS is used for attitude and rate feedback as opposed to GyroWheel. GyroWheel is active as primary sensor during the stretch of time between 1870 seconds and 4000 seconds. The curves are smoother, due to the lower measurement noise of GyroWheel as compared with the FSS. The effects of rate integration error can be clearly seen, however, as the pointing error steadily increases until the point at which the spacecraft leaves eclipse and switches back to the FSS. Even in the lowest bandwidth case, though, the maximum drift is less than 0.2° , which is well within pointing requirements.

Figure 5.2: Comparison of Pitch Axis Control Performance for Different Gain Sets (One orbit)

Figure 5.3: Comparison of Yaw Axis Control Performance for Different Gain Sets (One orbit)

Figure 5.4 shows the spacecraft pointing trajectory during the full orbit for each gain set. The trajectories were generated by graphing the Y-offset of the spacecraft attitude with respect to the Sun-pointing frame against the Z-offset. The origin of the plot represents the Sun vector direction and, for small angles, the lines traced show the path of the spacecraft pointing direction in inertial space. This is not exactly equivalent to plotting pitch error versus yaw error, as those quantities are measured with respect to the spacecraft body frame, which is rotating with respect to inertial space through most of the orbit. The total magnitude of the pointing error will be the same in both cases, however. Figure 5.4 confirms the effect of control bandwidth on the tightness of pointing control. The large lobes to the lower right represent the effects of GyroWheel rate integration.

Figure 5.4: Comparison of Overall Pointing Control Performance for Different Gain Sets (One orbit)

5.3.3 GyroWheel Behaviour

Figure 5.5 is a plot of the GyroWheel spin axis trajectories during the full orbit. The trajectories were generated by graphing the Y tilt angle of the rotor with respect to spacecraft body frame against the Z tilt angle. For small angles, this type of plot shows the path of the rotor spin axis relative to the spacecraft body, with the origin representing the null position. Ideally, the rotor axis should remain inertially fixed. The Sun-pointing frame is rotating with respect to inertial space at a rate of $0.986^\circ/\text{s}$ as the Earth revolves about the Sun, so an apparent drift of 0.67° per orbit as seen in body frame is expected. In reality, disturbance torques and errors in torque application will cause additional drift. In Figure 5.5, the effects of these errors and disturbances can clearly be seen. The initial drift from the null position is due to the motion of the Sun-pointing frame while the spacecraft is maintaining a constant attitude. A larger drift occurs while the spacecraft is in eclipse and using GyroWheel as a sensor. Rate integration error causes the attitude being supplied to the ADCS to become less and less accurate over time, so that the control torques computed by the ADCS do not fully compensate for the real errors. The total drift of the spin axis during eclipse, however, is less than 0.5° in the worst case, and in normal operation this drift will be limited by momentum dumping. It is interesting to note that the drift of the rotor spin axis under GyroWheel sensing increases with the control bandwidth. This is due to the fact that the pitch and yaw error measurements, which are afflicted by rate integration error, are being given greater weight by the higher pointing control gains.

Figure 5.6 is a plot of the GyroWheel rotor angular momentum during the full orbit. There is negligible difference between the three control gain sets, which demonstrates that rotor spin control is unaffected by the pointing error, as would be expected due to decoupling of roll control and pointing control. The plot clearly shows the effects of step changes in the desired roll rate on the spin speed. There is an overall increase

Figure 5.5: Comparison of GyroWheel Tilt Behaviour for Different Gain Sets (One orbit)

in the spin speed over the orbit, due to build-up of disturbance torques. This should be corrected when momentum dumping is enabled.

5.3.4 Control Performance Statistics

Table 5.9 lists the mean, $1\text{-}\sigma$, $3\text{-}\sigma$, and maximum roll error for the three pointing control gain sets, based on the runs discussed in the previous section. As the gains used for the roll axis were the same in all cases, there is minimal difference in the results.

Table 5.10 lists the corresponding results for the pointing error. As indicated in Figure 5.4, higher bandwidth leads to tighter pointing control, with the maximum error in the 100-second case being less than 0.06° . The results from all three gain sets are well within the pointing requirements.

Figure 5.6: Comparison of GyroWheel Angular Momentum Behaviour for Different Gain Sets (One orbit)

Table 5.9: Statistics for Roll Error (One Orbit)

	100 s	200 s	300 s
Mean	0.9151°	0.9193°	0.8955°
1- σ	0.7784°	0.7773°	0.7568°
3- σ	2.3353°	2.3319°	2.2705°
Max	3.1021°	3.0493°	2.8985°

Table 5.10: Statistics for Pointing Error (One Orbit)

	100 s	200 s	300 s
Mean	0.0212°	0.0372°	0.0896°
1-σ	0.0131°	0.0206°	0.0416°
3-σ	0.0393°	0.0618°	0.1247°
Max	0.0587°	0.0884°	0.1984°

Table 5.11 lists the statistics for the GyroWheel tilt angle. It is interesting to note that while there is some dependence on bandwidth, the effect is not nearly as pronounced as with the spacecraft pointing error. This is because a higher control bandwidth means higher gains, but also lower attitude and rate errors, so the total control torque computed by the ADCS remains approximately the same.

Table 5.11: Statistics for GyroWheel Tilt Angle (One Orbit)

	100 s	200 s	300 s
Mean	0.2565°	0.2601°	0.2691°
1-σ	0.1172°	0.1244°	0.1406°
3-σ	0.3517°	0.3732°	0.4217°
Max	0.4527°	0.4665°	0.4915°

Table 5.12 lists the statistics for the angular momentum of the GyroWheel rotor. As was indicated in the discussion of Figure 5.6, when the rotor is near the null position the rotor spin rate is affected only by roll axis control. The pointing control gains therefore do not have a noticeable effect.

Table 5.12: Statistics for the GyroWheel Angular Momentum (One Orbit)

	100 s	200 s	300 s
Mean	3.0410 Nms	3.0407 Nms	3.0407 Nms
1-σ	0.0137 Nms	0.0138 Nms	0.0137 Nms
3-σ	0.0412 Nms	0.0413 Nms	0.0412 Nms
Max	3.0746 Nms	3.0744 Nms	3.0745 Nms

5.4 Evaluation of GyroWheel Control System Stability

The stability of GyroWheel based control was evaluated by plotting the response of the system to initial attitude offsets. The control performance evaluations confirmed that all three gain sets were robust to the effects of disturbance torques. It is necessary as well, however, to establish that each case offers a reasonable margin of stability to large displacements. Both roll control and pointing control were tested for this.

5.4.1 Roll Response

To test the stability of the roll control system, a series of simulations were performed using an initial roll angle offset of 10° . Initial pitch and yaw errors were set to zero, as were all body rates. The stability of the gains used has already been confirmed through simulation of the classical wheel control system. For completeness, however, this test was repeated for GyroWheel based control.

Figure 5.7 shows the roll angle response to the offset using the 100-second pointing control gain set. The roll axis clearly stabilizes, and settles after approximately 400 seconds. There is an apparent steady-state offset of about 0.3° , which can easily be

accounted for by the inaccuracy of the magnetometer. As was indicated by the results of the control performance evaluations, the choice of gains for pointing control should have no impact on roll axis behaviour. It was therefore considered unnecessary to test roll response with the 200- and 300-second pointing gains.

Figure 5.7: Roll Error as a Function of Time for a 10° Initial Roll Offset

Figure 5.8 shows the spacecraft pointing error during the run. The amplitude of the pointing error is on the same order of magnitude as that due to disturbance torques, as can be seen by comparison with Figure 5.4. This demonstrates that roll axis control and pointing control have been effectively decoupled.

Figure 5.9 shows the tilt angle of the GyroWheel rotor during the run. The drift of the rotor spin axis is due almost entirely to disturbance torques, as roll axis error is not compensated for by GyroWheel tilt.

Figure 5.10 shows the angular momentum of the rotor during the run. The variation in the rotor spin speed provides the torque to bring the spacecraft back to its desired

Figure 5.8: Response of Spacecraft Pointing Error to a 10° Initial Roll Offset

Figure 5.9: Response of GyroWheel Tilt Angle to a 10° Initial Roll Offset

attitude. The settling time is about the same as seen for the roll angle in Figure 5.7.

Figure 5.10: Response of GyroWheel Angular Momentum to a 10° Initial Roll Offset

5.4.2 Pitch Step Response

To test pitch axis stability, an initial offset of 2° was used. The results were compared for all three pointing control gain sets.

Figure 5.11 plots the pitch error as a function of time for the three different gain sets. In each case the pitch error stabilizes with appropriate damping and settling time. The amount of overshoot decreases with the control bandwidth, as is expected.

Figure 5.12 plots the yaw error as a function of time. This plot indicates that there is still some coupling between pitch and yaw control, despite attempts to eliminate it in the system design. The coupling arises because of errors in the measured body

Figure 5.11: Pitch Error as a Function of Time for a 2° Initial Pitch Offset

rates that are being supplied to GyroWheel by the ADCS. The relative errors in the FSS rate measurements are reasonably small, but can still translate into large absolute errors when the body rates are higher than normal, as is the case here when correcting for a large initial offset. Without accurate rate measurements, GyroWheel can not apply the appropriate torque for decoupling. This should not be a problem during eclipse, as the ADCS will be using GyroWheel measured rates for pitch and yaw control. It should also be noted that the system passes the test for stability in each of the three cases examined, despite the coupling. This effect of rate measurement error is, however, a further argument in favour of using GyroWheel-measured rates as feedback for torque command mode rather than exclusively those supplied by the ADCS.

Figure 5.13 plots the roll error as a function of time. There is clear evidence of coupling here as well, again likely due to the ADCS rate measurement errors.

Figure 5.14 shows the spacecraft pointing trajectories during the 500 second run.

Figure 5.12: Response of Spacecraft Yaw Error to a 2° Initial Pitch Offset

Figure 5.13: Response of Spacecraft Roll Error to a 2° Initial Pitch Offset

The pitch-yaw coupling can be clearly seen here, with the pointing direction of the spacecraft following a spiral path toward the origin.

Figure 5.14: Response of Spacecraft Pointing Error to a 2° Initial Pitch Offset

Figure 5.15 shows the path of the GyroWheel rotor spin axis during each run. The paths are approximately the mirror images of the corresponding spacecraft pointing trajectories, as can be seen by comparing the plot with Figure 5.14. This effect is due to the tendency of GyroWheel to maintain a fixed inertial orientation as the spacecraft moves around it. The effects of disturbance torques are difficult to observe on this plot, as they are dwarfed by the correcting torques applied to the spacecraft by GyroWheel. In normal operation, the GyroWheel rotor would not remain at a 2° offset indefinitely, as momentum dumping would tend to pull it back toward the null position.

Figure 5.16 shows the response of the GyroWheel momentum to the initial pitch error. There is a noticeable effect here, due to the fact that the rotor moves away from the

Figure 5.15: Response of GyroWheel Tilt Angle to a 2° Initial Pitch Offset

null position. A small portion of the spin rate is therefore involved in pointing control.

5.4.3 Yaw Step Response

Tests of yaw axis stability were conducted in the same manner as those conducted for pitch axis stability, using an initial offset of 2° . The results were analogous to those of the pitch axis tests, as was expected. Only two of the plots from these runs are presented here.

Figure 5.17 shows the path of the spacecraft pointing direction. The shape of the trajectories is similar to those given in Figure 5.14 and shows the same pitch-yaw coupling.

Figure 5.18 shows the path of the rotor spin axis. Again, the rotor appears to follow

Figure 5.16: Response of GyroWheel Angular Momentum to a 2° Initial Pitch Offset

Figure 5.17: Response of Spacecraft Pointing Error to a 2° Initial Yaw Offset

almost a mirror path to that of the spacecraft pointing direction, due to inertial effects.

Figure 5.18: Response of GyroWheel Tilt Angle to a 2° Initial Yaw Offset

5.5 Evaluation of the GyroWheel Rate Feedback Method

The results of the pitch and yaw axis stability tests indicate that pitch and yaw axes are not fully decoupled using the GyroWheel based control system as currently designed. To determine if the coupling could be reduced by using GyroWheel rate measurements in place of the pitch and yaw rate measurements from the ADCS, the GyroWheel model was temporarily modified so that the GyroWheel rate outputs were fed back directly into the GyroWheel spacecraft rate inputs. In this manner, only the roll rate was being supplied by the ADCS. Using the 100-second pointing gain set, the

model was run with a 2° initial pitch offset. The resulting spacecraft and GyroWheel tilt trajectories are shown in Figures 5.19 and 5.20, respectively. Comparison of these plots with those in Figures 5.14 and 5.15 demonstrate that pitch and yaw have become more effectively decoupled using this rate feedback scheme.

Figure 5.19: Response of Spacecraft Pointing Error to a 2° Initial Pitch Offset Using GyroWheel Rate Feedback

5.6 Long Term GyroWheel Control Performance

The tests described so far establish the performance of the GyroWheel based control system over a single orbit and confirm stability in the presence of large attitude offsets. It is necessary, however, to demonstrate that this performance is maintained over longer time periods. Simulations were performed for a period of ten orbits, using the 100-second pointing control gain set and zero initial attitude and rate. A key aspect of this testing was evaluating the effect of momentum dumping on GyroWheel

Figure 5.20: Response of GyroWheel Tilt Angle to a 2° Initial Pitch Offset Using GyroWheel Rate Feedback

based control.

Table 5.13 lists the portions of the run during which the spacecraft is in either sunlight or eclipse. In sunlight, the FSS is used for pitch and yaw measurements. During eclipse, GyroWheel measurements are used instead. A repeating roll profile is used, based on that given in Table 5.8, but with no attitude hold phase after the first orbit.

Figures 5.21 through 5.25 show the results from a ten-orbit simulation with momentum dumping turned off. Despite appearances, the increase in roll error seen toward the end in Figure 5.21 is due mostly to a transient effect of the magnetic field, combined with the specific orbit that was selected. Figure 5.22, however, shows a definite deterioration in the pointing control over time, particularly when GyroWheel is being used for attitude and rate sensing. The effect is also noticeable in Figure 5.23, where severe excursions from the desired pointing direction can be clearly seen. Figure 5.24 shows the steady growth of the GyroWheel tilt angle due to build-up of distur-

Table 5.13: Long-Term GyroWheel Control Performance (10 Orbits)

Orbit	Phase	Start (s)	End (s)
0	Sunlight	0	1870
1	Eclipse	1875	3995
1	Sunlight	3995	7730
2	Eclipse	7730	9855
2	Sunlight	9855	13590
3	Eclipse	13590	15715
3	Sunlight	15715	19450
4	Eclipse	19450	21575
4	Sunlight	21575	25310
5	Eclipse	25310	27435
5	Sunlight	27435	31170
6	Eclipse	31170	33295
6	Sunlight	33295	37030
7	Eclipse	37030	39155
7	Sunlight	39155	42890
8	Eclipse	42890	45015
8	Sunlight	45015	48750
9	Eclipse	48750	50875
9	Sunlight	50875	54610
10	Eclipse	54610	56735
10	Sunlight	56735	59995

bance torques. These disturbances also have a long-term effect on the spin speed of GyroWheel, as can be seen in Figure 5.25.

Figure 5.21: Long Term Behaviour of Spacecraft Roll Error (No Momentum Dumping)

Figures 5.26 through 5.30 show the results from the identical simulation, but with momentum dumping switched on. The transient effect is still visible in the roll error in Figure 5.26, but the pointing control is much more stable, as Figures 5.27 and 5.28 show. The effectiveness of momentum dumping on controlling the GyroWheel rotor tilt angle and spin is also evident, as seen in Figures 5.29 and 5.30.

Table 5.14 summarizes the performance statistics for GyroWheel based control using the results from the ten-orbit simulation with momentum dumping. Both the roll error and pointing error are well within the satellite specifications, as indicated by the 1- and 3- σ results.

Figure 5.22: Long Term Behaviour of Spacecraft Pitch and Yaw Errors (No Momentum Dumping)

Figure 5.23: Long Term Behaviour of Spacecraft Pointing Error (No Momentum Dumping)

Figure 5.24: Long Term Behaviour of GyroWheel Tilt Angle (No Momentum Dumping)

Figure 5.25: Long Term Behaviour of GyroWheel Angular Momentum (No Momentum Dumping)

Figure 5.26: Long Term Behaviour of Spacecraft Roll Error (With Momentum Dumping)

Figure 5.27: Long Term Behaviour of Spacecraft Pitch and Yaw Error (With Momentum Dumping)

Figure 5.28: Long Term Behaviour of Spacecraft Pointing Error (With Momentum Dumping)

Figure 5.29: Long Term Behaviour of GyroWheel Tilt Angle (With Momentum Dumping)

Figure 5.30: Long Term Behaviour of GyroWheel Angular Momentum (With Momentum Dumping)

Table 5.14: Statistics for Long-Term GyroWheel Control Performance (10 Orbits)

	Roll Error	Pointing Error	GyroWheel Tilt Angle	GyroWheel Angular Momentum
Mean	1.0787°	0.0126°	0.0832°	3.0149 Nms
1- σ	1.2903°	0.0080°	0.0460°	0.0101 Nms
3- σ	3.8710°	0.0240°	0.1379°	0.0302 Nms
Max	10.2308°	0.0538°	0.2568°	3.0624 Nms

5.7 Comparison with the Classical Wheel Based Control System

An important aspect to the evaluation of GyroWheel as a control actuator is a comparison of its performance to that of the nominal actuating system on SCISAT-1, consisting of the classical momentum wheel in combination with magnetorquer rods. Of particular interest was a comparison of the pointing error of the two systems when the satellite is emerging from eclipse. In nominal operations SCISAT-1 will not use active pitch and yaw control during eclipse, but will instead rely solely on the momentum bias of the wheel to minimize drift. Simulations performed to date indicate the maximum drift using this control method is typically about 0.2° , and can theoretically be as high as 0.4° . Using GyroWheel as an attitude and rate sensor during eclipse, however, allows for active pointing control when using GyroWheel as the primary actuator, and the results using the 100-sec pointing gain set show that the pointing errors at the end of the eclipse period are less than 0.06° .

Results from a 10-orbit simulation of SCISAT-1 using the classical wheel and magnetorquer for control was made available by Bristol Aerospace. Apart from the sensors, actuators and control gains used, this simulation was identical to that performed for GyroWheel based control. Table 5.15 lists the control gains that were used for the classical wheel run. The roll control gains are identical to those used for GyroWheel based control. These gains are only used during the sunlight portion of the orbit. In eclipse, the k_{ry} , k_{rz} , and k_{hz} terms are set to zero.

Figures 5.31 through 5.33 show the results of the classical wheel simulation. The roll axis control is comparable to that for GyroWheel, indicating that GyroWheel rotor is as effective as the classical wheel rotor in exerting control torques about that axis. The pointing control, however, is noticeable poorer, particularly during eclipse, as seen in Figures 5.32 and 5.33.

Table 5.15: Control Gains for Classical Wheel Based Control

k_{rx}	0.141712
k_{px}	0.00165474
k_{ry}	0.01
k_{py}	0
k_{rz}	0.01
k_{pz}	0
k_{hx}	0.0067
k_{hy}	0
k_{hz}	0

Figure 5.31: Long Term Behaviour of Spacecraft Roll Error Using Classical Wheel Control (With Momentum Dumping)

Figure 5.32: Long Term Behaviour of Spacecraft Pitch and Yaw Errors Using Classical Wheel Control (With Momentum Dumping)

Figure 5.33: Long Term Behaviour of Spacecraft Pointing Error Using Classical Wheel Control (With Momentum Dumping)

Table 5.16 lists the statistical results of the classical wheel control run. In comparing these results with Table 5.14, it can be seen that while the roll axis control performance is approximately equivalent, the pointing performance of GyroWheel is a considerable improvement over that offered by the classical wheel and magnetorquer system. This confirms the observations made earlier based on the plots of the simulation outputs.

Table 5.16: Statistics for Long-Term Classical Wheel Control Performance (10 Orbits)

	Roll Error	Pointing Error
Mean	1.1149°	0.0993°
1-σ	1.3546°	0.0574°
3-σ	4.0639°	0.1723°
Max	10.8856°	0.2241°

Chapter 6

Conclusions and Recommendations

The results of the acceptance runs for GyroWheel based control indicate that fine-pointing control of SCISAT-1 can be easily achieved. The predicted $3\text{-}\sigma$ pointing accuracy, when using a 0.0090 Hz control bandwidth, is less than 0.03° . This compares very favourably with the $3\text{-}\sigma$ pointing accuracy under classical wheel and torque rod control, which is just under 0.2° . More importantly, from a science point of view, is that the maximum pointing error during the sunrise occultation is less than 0.06° for GyroWheel based control using GyroWheel measured rates for pitch and yaw feedback, as compared with nearly 0.2° when using the classical wheel for gyrodynamic stability. GyroWheel pointing control using the 0.0090 Hz bandwidth was also found to be stable in response to initial attitude offsets of 2° , as were the 0.0060 Hz and 0.0045 Hz bandwidths. Roll axis control performance using GyroWheel was found to be comparable to that obtained using the classical wheel, with a $1\text{-}\sigma$ roll error of about 1.3° and a $3\text{-}\sigma$ error of less than 4° . The gain set used for roll, representing a control bandwidth of 0.0045 Hz, was found to be stable in response to an initial roll offset of as much as 10° .

As these predictions of performance and stability are based on computer modelling

and simulation, they should be considered tentative rather than final. The noise model used for the GyroWheel rate sensing was based on early hardware test results and should be updated as soon as more complete data is available. The simulations can then be easily re-run in order to determine if control performance and stability will be affected by any new changes or refinements in the model. The results presented in this paper are, however, a good indication that GyroWheel based control will both achieve and exceed the requirements imposed on it by the SCISAT-1 mission, and that it will offer considerable advantages in performance over the standard classical momentum wheel and torque rod system.

The performance of the GyroWheel based control system could be improved even further, however, through the introduction of some modifications. In its present form, the torque command mode of GyroWheel requires the ADCS to provide rate measurements about all three axes in order to properly apply the requested amount of torque to the spacecraft. This makes the system susceptible to error and noise in the measured rates coming from external sensors. A more logical alternative would be to make use of the internal GyroWheel rate measurements for the two axes perpendicular to the rotor spin axis, so that the spacecraft ADCS only needs to provide rate information for the remaining axis. This would reduce the effects of external sensor error, assuming that the GyroWheel rate measurements are more accurate, which appears to be the case with SCISAT-1. Another significant advantage to using internal GyroWheel rate measurements is that they are available far more frequently than ADCS commands typically are, so integration of the commanded torque to determine the required angular momentum can be performed with smaller time steps, reducing error even further. Rather than replace the present torque command mode, however, it is recommended instead that a new command mode be added that uses internally measured rates for command processing in place of ADCS-supplied rates. The user therefore retains the option of using external rate sensor measurements when operating GyroWheel, should it in fact be beneficial to do so. In the case of SCISAT-1,

it is not essential to implement such a mode, as the present torque command mode is more than adequate for meeting the attitude control requirements. The error in the rate measurements coming from the FSS prevents fully-decoupled control from occurring, but it does not appear to affect the overall stability of the system, and the pointing control performance is still well within specification. Increasing the performance capabilities and operational flexibility of GyroWheel will, however, make it a more attractive option as an actuator for future missions which may have different sensor capabilities or pointing control requirements.

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